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Fatigue behaviors of Square-to-Square Hollow Section T-joint with corner crack. II: Numerical Modeling

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Abstract: Based on the experimental results obtained in the first part of the study, a geometrical model and a mesh generation procedure are suggested for the numerical modeling of the uncracked and cracked square-to-square hollow section (SSHS) T-joints. In the proposed model, the surface crack is represented as an unsymmetrical 3D curved surface with the deepest point located at the corner of the brace-chord intersection. The mesh generation scheme developed is capable to generate well graded finite element (FE) meshes for cracked SSHS T-joints with multiple cracks. The validity of the proposed geometrical model and the reliability of the proposed mesh generation scheme are checked by comparing the predicted SIFs and the residual life against those computed based on the actual measurements. It is found that the suggested modeling method is reliable in the sense that conservative predictions for the SIFs and the residual life are obtained from the numerical simulations.

Key words: Fatigue performance; square-to-square hollow sections T-joints; Surface crack modeling, fracture mechanics method

1. INTRODUCTION

In many previous studies of circular hollow section (CHS) tubular joints [1-5], it was accepted that the fracture mechanic method (FMM) is a reliable approach for the fatigue assessment and residual life prediction of a cracked joint. In order to obtain a realistic prediction of the stress intensity factor (SIF) and the residual fatigue life for a cracked joint, a valid and consistent geometrical model for the welding profile and the surface crack shape are essential. Toward this end, most of the previous research efforts had been directed to study the fatigue performances of cracked CHS T, Y and K joints but virtually no study was carried out for the case of rectangular hollow sections (RHS) joints. In Part I of the current study [6], four full-size SSHS T-joints were tested under combined cyclic loading until fatigue failure. During the test, the propagation of cracks along the chord surface and their penetration in the chord thickness direction were monitored by the alternating current potential drop (ACPD) technique. The test results indicated that when a SSHS T-joint is subjected to combined cyclic loading, the first surface crack will appear at one of the four corners of the brace-chord intersection and will mainly propagate along a line parallel to the chord wall. Furthermore, near the end of the fatigue test, branches and secondary cracks may be formed. Further post-failure crack shape measurements showed that the crack surface is a smooth 3D curved surface and cannot be described using the models employed for CHS tubular joints. The main objective of this paper is to make use of the information and test results presented in Part I to develop a geometrical model and a FE mesh generation scheme for the numerical modeling of uncracked and cracked SSHS T-joints under static and cyclic loadings. It is hoped that the numerical model developed from the suggested modeling method will produce some reasonably *conservative predictions* for the responses (e.g. stress concentration factors (SCF), SIF and residual fatigue life) of the joint. In order to validate the reliability of the proposed geometrical model and the mesh generation procedure, the numerical results obtained will be compared with the actual measurements recorded during the static and fatigue tests.

2 THE PROPOSED GEOMETRICAL MODEL

In this study, the definition of the geometrical model will be established in the sequence of (i) weld toe definition, (ii) crack surface geometry and (iii) crack front equation.

2.1 Definition of weld toe

The basic configuration for a SSHS T-joint is shown in Fig. 1. The brace-chord intersection curve consists of four straight lines and four corners (CP1-CP4). The global coordinate system (x - y - z) of the joint is defined with the origin O located at the intersecting point between the brace centroid and the chord face (Fig. 1). Since the surface crack will be initiated at the weld toe on the chord surface near one of the corners of the brace, one may consider the typical corner CP3 for the definition of the weld toe (Fig. 2). In Fig. 2, the arc LH defines the brace corner at CP3 and sustains an angle $\omega \leq \pi/2$ at F such that $|FL| = |FH| = r_b$. Furthermore,

$$\angle HFQ = \pi/4 - \omega/2, \quad |FQ| = r_b \cos(\pi/4 - \omega/2), \quad |HQ| = r_b \sin(\pi/4 - \omega/2) \quad (1)$$

Hence, the co-ordinates of F, (x_F, y_F, z_F) , are given by

$$x_F = -y_F = -b_I/2 + r_b \cos(\pi/4 - \omega/2), \quad z_F = 0 \quad (2)$$

If the *exceeding* welding thickness, W , is constant along the whole brace-chord intersection, then the arc B'D' is the weld toe corresponding to the arc LH. However, in practice, the weld toe is usually fabricated as a smooth curve which sustains a right angle around the corner. Hence, in this study, a full right angle arc BD with centre at G is used instead such that the line GHB is perpendicular to HQ. From Fig. 2,

$$\begin{aligned} |GH| &= |LG| = |FQ| - |HQ| = r_b (\cos(\pi/4 - \omega/2) - \sin(\pi/4 - \omega/2)) = \sqrt{2} r_b \sin(\omega/2) \\ |GB| &= |GH| + |HB| = \sqrt{2} r_b \sin(\omega/2) + W = r_w \end{aligned} \quad (3a)$$

The co-ordinates of G, (x_G, y_G, z_G) , are then given by

$$x_G = -y_G = -b_I/2 + r_w - W = -b_I/2 + \sqrt{2} r_b \sin(\omega/2), \quad z_G = 0 \quad (3b)$$

Finally, from Eqn. 3, equations of the weld toe for the quarter M₃-CP3-M₄ can be summarized as

$$\begin{aligned} \text{From M}_3 \text{ to D:} \quad & x + b_I/2 + W = z = 0 \\ \text{For arc B to D:} \quad & (x - x_G)^2 + (y - y_G)^2 - (r_w)^2 = z = 0 \\ \text{From B to M}_4: \quad & y - b_I/2 - W = z = 0 \end{aligned} \quad (4)$$

Equations for other quarters of the joint can be obtained in a similar manner. In this study, in order to generate a conservative model for SCF and SIF estimations, values of W and r_b are set equal to the *minimum* values specified in the American Welding Society specification [7] and the EN standard [8] and are equal to $0.5t_l$ and $1.5t_l$ respectively. In addition, a value of $\omega=\pi/3$ is used which is based on actual measurements. It should be noted that Eqns. 1 to 4 can be extended to the cases of RHS T/K joints with modest modifications.

2.2 Definition of crack surface profile

Fig. 3a shows a typical surface crack with tips $\mathbf{p}_1$ and $\mathbf{p}_2$ at the corner CP4. The local coordinate s' is defined in such a way that at point M, $s'=0$, while at $\mathbf{p}_1$, $s'(\mathbf{p}_1)=s'_1$ and at $\mathbf{p}_2$, $s'(\mathbf{p}_2)=s'_2$ so that $s'_2 - s'_1 = l$, the length of the surface crack. From the results obtained in Part I, the crack profile, $\mathbf{O'd'}$ for the section $l'-l'$ perpendicular to the weld toe at $\mathbf{O'}=(x_{O'}, y_{O'}, 0)$ is a 3D curve (Fig. 3b). A local coordinate system $\mathbf{x'-y'-z'}$ is defined with origin at $\mathbf{O'}$ in such a way that the $\mathbf{x'}$ and $\mathbf{y'}$ axes are, respectively, perpendicular and tangential to the weld toe at $\mathbf{O'}$. $\mathbf{d'}$ is the deepest point such that a and a' are, respectively, the *curved* and *projected* lengths (to the $\mathbf{z'}$ axis) of $\mathbf{O'd'}$. The angle between the global $\mathbf{x}$ and local $\mathbf{x'}$ axes is equal to $\pi-\phi(s')$ such that the relationship between the $\mathbf{x'-y'-z'}$ and the $\mathbf{x-y-z}$ coordinate system is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\cos(\phi(s')) & \sin(\phi(s')) & 0 \\ -\sin(\phi(s')) & -\cos(\phi(s')) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} x_{O'} \\ y_{O'} \\ 0 \end{bmatrix} \quad (5)$$

For any given cross section where $y'=0$, in order to define the crack profile, it is required to express x' in terms of z' for $0 \leq z' \leq t_0$. From the measurements obtained in Part I, it was found that for a typical point $\mathbf{q'}$ on the crack surface, the angle $\theta_s(z')$ increases as z' increases and the gradient of the curve $\mathbf{O'd'}$ often converges to a constant value near the bottom of the chord. Hence, the equation which describes the geometry of the surface crack will be deduced from the following assumptions:

(1) The crack profile will be modeled as a curve for $0 \leq z' \leq z'_c$ and a straight line for $z'_c \leq z' \leq t_0$. The value z'_c is known as the *critical depth* for the crack and a value of $z'_c = 0.8t_0$ is adopted in this study.

(2) For $0 \leq z' \leq z'_c$, $\theta_s(z')$ will increase linearly with z' from $\theta_s(0)$ to $\theta_s(z'_c)$ (Fig. 3c) so that

$$\theta_s(z') = \theta_s(0) + [\theta_s(z'_c) - \theta_s(0)]z'/z'_c, \quad 0 \leq \theta_s(0) < \theta_s(z'_c) < \pi/2 \quad (6)$$

(3) For $z'_c \leq z' \leq t_0$, the crack profile will be a straight line with slope equal to the gradient of the curve when $z' = z'_c$. That is, in Fig. 3c, the angle ϕ_c is given by

$$\tan(\phi_c) = \left. \frac{dz'}{dx'} \right|_{z' = z'_c} \quad (7)$$

By using the above assumptions, equations for the crack profile with respect to the x' - y' - z' coordinate system can be written as

$$\text{For } 0 \leq z' \leq z'_c: \quad x' = z' \tan(\theta_s(0) + [(\theta_s(z'_c) - \theta_s(0))z'/z'_c]) \quad (8a)$$

$$\text{For } z'_c \leq z' \leq t_0: \quad \frac{z' - z'_c}{x' - z'_c \tan(\theta_s(z'_c))} = \tan(\phi_c) = \left. \frac{dz'}{dx'} \right|_{z' = z'_c} \quad (8b)$$

From Eqn. 8a,

$$\tan(\phi_c) = \left. \frac{dz'}{dx'} \right|_{z' = z'_c} = \left[\left. \frac{dx'}{dz'} \right|_{z' = z'_c} \right]^{-1} = \frac{\cos^2(\theta_s(z'_c))}{\sin(\theta_s(z'_c))\cos(\theta_s(z'_c)) + (\theta_s(z'_c) - \theta_s(0))} \quad (9)$$

and from Eqns. 6 and 9, since $\theta_s(0) < \theta_s(z'_c) < \pi/2$ and $\tan(\phi_c) > 0$, the surface crack will reach the bottom of the chord ($z' = t_0$) when

$$x' = \cotan(\phi_c)(t_0 - z'_c) + z'_c \tan(\theta_s(z'_c)) = t_0 \tan(\theta_s(z'_c)) + \frac{(t_0 - z'_c)(\theta_s(z'_c) - \theta_s(0))}{\cos^2(\theta_s(z'_c))} \quad (10)$$

From the measurements done in Part I, it was found that the angles $\theta_s(0)$ and $\theta_s(z'_c)$ vary along the crack length (s') direction. Actual measurements indicate that $\theta_s(0)$ usually starts from a small initial value $\theta_s^i(0) \approx \pi/12$ at M (Fig. 4a) and increases almost linearly until point B ($s' = L_w = b_l/2 - \sqrt{2}r_b \sin(\omega/2)$). After that $\theta_s(0)$ varies more rapidly and attends a maximum value $\theta_s^{max}(0) \approx \pi/6$ at point P ($s' = L_w + \pi r_w/8$) and then decreases until point D ($s' = L_w + \pi r_w/2$). In fact, from the test results obtained in Part I, the extent of the surface

crack is often limited within points M and D (i.e. for $0 \leq s' \leq L_w + \pi r_w/2$). The variation of $\theta_s(z'_c)$ with s' follows a similar trend. For $z'_c = 0.8t_0$, it starts with an initial value of $\theta_s^i(z'_c) \approx \pi/6$ at M and attends a maximum value of $\theta_s^{max}(z'_c) \approx \pi/3$ at point P. Based on the above observations, the variations of $\theta_s(0)$ and $\theta_s(z'_c)$ will be modeled using the following conditions (Fig. 4b):

(1) Graphs of $\theta_s(0)$ and $\theta_s(z'_c)$ against s' will be a straight line for $0 \leq s' \leq L_w$ with

$$\theta_s^i(0) = \pi/12 \text{ and } \theta_s^i(z'_c) = \pi/6.$$

(2) The variations will be modeled as a quadratic curve for $L_w \leq s' \leq L_w + \pi r_w/2$ with maximum values of $\theta_s^{max}(0) = \pi/6$ and $\theta_s^{max}(z'_c) = \pi/3$ at the point P ($s' = L_w + \pi r_w/8$).

(3) The straight line and quadratic curve are continuous and smooth at point B.

From the above conditions, the variation of $\theta_s(0)$ with s' , $\theta_s(0, s')$, can be expressed by the following piecewise smooth equations:

$$\theta_s(0, s') = \begin{cases} \theta_s^i(0) + \frac{16(\theta_s^{max}(0) - \theta_s^i(0))}{16L_w + \pi r_w} s' & \text{for } 0 \leq s' \leq L_w \\ \theta_s^{max}(0) - \frac{(\theta_s^{max}(0) - \theta_s^i(0))}{\pi r_w (16L_w + \pi r_w)} (8(s' - L_w) - \pi r_w)^2 & \text{for } L_w \leq s' \leq L_w + \pi r_w/2 \end{cases} \quad (11)$$

Similarly, the variation of $\theta_s(z'_c)$ with s' , $\theta_s(z'_c, s')$, can be obtained by replacing the terms $\theta_s^i(0)$ and $\theta_s^{max}(0)$ in Eqn. 11 by $\theta_s^i(z'_c)$ and $\theta_s^{max}(z'_c)$ respectively. Combining Eqns. 8, 9 and 11, the crack surface at any cross section with $y'=0$ can be expressed as

$$\text{For } 0 \leq z' \leq z'_c: \quad x' = z' \tan(\theta_s(0, s')) + [(\theta_s(z'_c, s') - \theta_s(0, s')) z' / z'_c] \quad (12a)$$

$$\text{For } z'_c \leq z' \leq t_0: \quad x' = \left[\tan(\theta_s(z'_c, s')) + \frac{(\theta_s(z'_c, s') - \theta_s(0, s'))}{\cos^2(\theta_s(z'_c, s'))} \right] (z' - z'_c) + z'_c \tan(\theta_s(z'_c, s')) \quad (12b)$$

Finally, expression of the crack surface is obtained by applying Eqn. 4 to determine $(x_{O'}, y_{O'})$ and then Eqn. 5 to transform the results to the global coordinate system.

2.3 Definition of crack front

Actual measurements of crack fronts done in Part I indicated that the shape of a crack front resembles an elliptical curve on the projected $s'-z'$ plane (Fig. 5). Hence, the shape of the

crack front will be modeled as a piecewise unsymmetrical bi-elliptical curve on the projected $s'-z'$ plane of the form

$$\text{For } 0 \leq s' \leq s'_l + l_d \text{ and } z' \geq 0: \quad \frac{(s' - (s'_l + l_d))^2}{(l_d)^2} + \frac{(z')^2}{(a')^2} = 1 \quad (13a)$$

$$\text{For } s'_l + l_d < s' \leq s'_2 \text{ and } z' \geq 0: \quad \frac{(s' - (s'_l + l_d))^2}{(l - l_d)^2} + \frac{(z')^2}{(a')^2} = 1 \quad (13b)$$

In Eqn. 13 and Fig. 5b, the deepest point **d'** is located at $s' = s'_l + l_d$ on the crack front with curved and projected depth equal to a' and $z' = a$ respectively. The bi-elliptical curve attains a maximum value of a' when $s' = s'_l + l_d$ and becomes a single elliptical curve when $l_d = l/2$. Furthermore, the points on the crack front can be generated when s'_l , l and l_d are known and their values can be measured from experiments for validation. It is found that for the four specimens tested, Eqn. 13 led to conservative models such that the modeled crack fronts are deeper than the measured results. An example for the measured and modeled final crack front shapes for Specimen I is given in Fig. 6.

3. MESH GENERATION

After the geometry of the welding profile and the surface crack are defined, FE meshes will be generated by a tailor-make mesh generator consists of two distinct steps, namely, (i) mesh generation for welded joints and (ii) mesh generation for cracked joints.

3.1 Mesh generation for SCF study

Due to the geometry of RHS tubular joints, the procedure to generate a 3D solid FE mesh for a welded joint is relatively simple when compared with CHS joint [3-5]. In order to obtain well shaped elements and gradation mesh towards the joint intersection, the whole joint will be divided into a number of distinct zones (Fig. 7). Templates of FE mesh with different element density are then applied to these zones. In order to obtain good numerical results, finer meshes are generated near the intersection (e.g. zones CF and F3) with three layers of elements in the chord thickness direction. In order to obtain a conservative estimation of SCFs, the zone CF is constructed with the parameters $W=0.5t_l$ and $h=0.5t_l$ (Fig. 1a). In the regions far away from the intersection (e.g. Zones K1, K2 and K3) only

one layer of larger elements will be employed. Transitions regions (e.g. zones F1, G1 and H1, H2) are inserted to provide smooth transition of element size. After all the distinct zones are generated, they are merged together to generate the full model as shown in Fig. 8.

3.2 *Mesh generation for SIF study*

Steps to generate a mesh with surface crack are more complicated than uncracked joint and are summarized below:

- (i) A mesh without surface crack is first generated.
- (ii) Elements in zone CF which intersect with the surface crack will be identified (Fig. 9), extracted and a sub-zone will be formed for further processing. Note that more than one such sub-zone could be formed if multiple cracks are required.
- (iii) A tube of “spider web” elements will be generated around the crack front. The tube will consist of three layers of elements and each layer will consist of eight elements surrounding the crack opening (Fig. 10b). In order to capture the stress singularity near the crack front, elastic singular quarter-point prism elements are used in the first layer of the tube [9] and they are enclosed by two layers of quadratic brick elements (Fig. 10b).
- (iv) The spider web tube will first be generated as a straight tube. In order to blended with the 3D curved crack front, a local coordinate system $\mathbf{u}'\text{-}\mathbf{v}'\text{-}\mathbf{w}'$ is defined along the crack front (Figs. 5b and 10a). Final mesh around the crack front will be obtained by mapping the straight tube according to variation of the $\mathbf{u}'\text{-}\mathbf{v}'\text{-}\mathbf{w}'$ system along the crack front (Fig. 10a).
- (v) After the “twisted” tube is generated, it will then be inserted back to the extracted sub-zone. Tetrahedral and pyramid elements are employed to link the tube to the adjacent elements in zone CF (Fig. 10c).
- (vi) Finally, the whole mesh will be formed with the processed zone CF inserted back to the welded zone (Fig. 11).

4 **MODEL VALIDATION**

In this section, the validities of the proposed geometrical model and the mesh generation procedure will be investigated by comparing the numerical results obtained by using the

suggested model with the static and fatigue testing results obtained from the full scale tests described in Part I. In all the numerical simulations presented, the material properties and the loading conditions were corresponding to the actual measured values. The boundary conditions are that the two ends of the chord (Ends G and H) were completely fixed while the loading end of the brace (End F) is free (Fig. 8). All the numerical analyses were carried by the commercial FE software ABAQUS [10]. Numerical models for both uncracked and cracked joints will be created and the performance on the numerical model will be assessed by considering the following results:

- (1) SCFs distribution at the T-joint intersections,
- (2) SIF at the crack front for surface cracks, and
- (3) fatigue life prediction.

It should be stressed that the prime target of the numerical modeling is *not* to reproduce exactly the interested responses (e.g. SCF, SIF and residual fatigue life) of the joint but rather to provide reasonably *conservative predictions* for fatigue assessments.

4.1 Validation of numerical results for uncracked joint

In order to validate the weld toe model and the uncracked joint mesh generation procedure, the experimental SCF distributions around the joint intersection [6] were compared with the corresponding values obtained by the FE modeling. It is found that for all the four specimens, under the basic loading conditions (axial, AX; in-plane bending, IPB; out-of-plane bending, OPB), the purposed modeling consistently leaded to reasonably conservative SCF predictions for both the chord and brace of the joints. Fig. 12 shows an example for the SCF distributions on the chord for Specimen I. This implies that by using the superposition method, the presented uncracked model can be used to predict the SCF values under combined static loadings.

4.2 Validation of numerical results for cracked joint

4.2.1 Experimental and numerical SIFs

The propagation of surface crack fronts on tubular joints is well known to be a mixed mode fracture problem. Hence, the mode I, II and III SIFs, K_I , K_{II} and K_{III} obtained from the analyses should be combined together to form the *equivalent SIF*, K_e defined as [4]

$$K_e = \left[K_I^2 + K_{II}^2 + K_{III}^2/(1-\nu) \right]^{1/2} \quad (14)$$

which is the main driving parameter for surface crack growth. In Eqn. 14, $\nu=0.3$ is the Poisson's ratio of the material. In order to compare with the K_e computed from the FE model, data obtained from the full scale test were converted to SIFs by using the Paris' law

$$\frac{da'}{dN} = C(\Delta K)^m \quad (15)$$

where a' and N are respectively, the curved crack depth and number of cyclic load applied. C and m are two material parameters and were taken as $m=2.92$ and $C = 8.02 \times 10^{-12} (\text{m/cycle})(\text{MPa} \times \text{m}^{1/2})^{-2.92}$ [11]. ΔK is the range of SIFs and is equivalent to K_e since the magnitude of the minimum applied cyclic loading in the tests is equal to zero.

4.2.2 Validation of the numerical method and finite element meshes

In general, when the FEM is employed for tubular joint modeling, two types of errors can be identified. The *geometrical modeling error* is caused by the different between the exact geometry of the joint and the geometrical model adopted. As the exact geometries of the joints vary from one joint to another, no matter how much details are included in the geometrical model, this type of error is difficult to control and can never be eliminated. The *discretization error* is caused by the finite degree of freedoms of the FE mesh, the uncertainty of the material properties and the inexactness of loading and boundary conditions used. In practice, for tubular joint modeling, values of the material properties can be determined with high accuracy from coupon test. For the incompleteness of the numerical model and the error in boundary and loading conditions, this can be controlled by adopting some good modeling practices such as careful construction of high quality meshes with realistic boundary and loading conditions according to the actual conditions.

In this section, the experimental SIF values (Eqn. 15) will be compared with the corresponding numerical values (Eqn. 14) from the FE analyses based on the *exact* welding and surface crack geometrical details obtained from the ACPD measurements. This means that most the geometrical modeling error will be eliminated and the discrepancy between the numerical and the measured results will be mainly due to discretization error. Figs. 13 to 16 compare the numerical and measured SIF values at the deepest points for Specimens I

to IV respectively. From these figures, it can be seen that the numerical results obtained is accurate with the maximum relative error less than 10%. Furthermore, consistently conservative approximations were obtained for almost the whole range of a'/t_0 except for Specimens I and III where slightly underestimated values are obtained when $a'/t_0 \approx 0.1$. Hence, it can be concluded that the FE meshes generated are adequate and the values of material properties employed are valid.

Note that during the crack initiation phase when $a' < 0.1\text{mm}$, the shape of the crack cannot be measured by the ACPD technique accurately. Since the numerical model was created from the geometrical model based on the ACPD measurements, it is believed that during the crack initiation phase *both the experimental and numerical results* will not be as accurate as the values obtained during the crack propagation phase. Hence, in Figs. 13 to 16, no numerical SIF value was compared with the experimental results for $a'/t_0 \leq 0.1$. Similarly, such condition also occurred at regions near the two cracks tips where the ACPD measurements are not as accurate as the measurements made at other locations. In fact, detailed analyses showed that the numerical results agreed well with the experimental results along the *whole* crack front except at regions near the crack tips.

It should also be noted that for Specimens II and IV, secondary crack and branches were observed during the test [6]. However, the FE analysis results shown in Figs. 14 and 16 were obtained by using *only* a single crack. Further analyses involving secondary crack and branches indicated that the SIF values at the deepest point of the major surface crack are insensitive to the presences of these features. The main reason is that they only appeared near the end of the fatigue test. Hence, in subsequent numerical modelings, these features will not be included.

A final remark for the numerical results is that during the computation of K_e , it was found that K_e was almost dominated by the first mode (K_I) while the second mode (K_{II}) remained at or near zero and such result agrees with the findings obtained from CHS T joints [4].

4.2.3 Validation of the geometrical model

Despite the exact geometrical model can reproduce the experimental results with high accuracy, it requires detailed information of the crack shape for construction and has limited practical values. In practice, during fatigue assessment, the only information of the

crack available for assessment is the extent (i.e. locations of point $\mathbf{p}_1$ and $\mathbf{p}_2$ in Fig. 3a) and the depth (a') of the surface crack which could be obtained by visual and X-ray inspections. In such a case, it is necessary to assume a realistic geometrical model to define the shape of the crack surface implicitly from the field measurements for the numerical simulation.

The proposed geometrical model was adopted to generate meshes for the prediction of SIFs. The SIFs values at the deepest points computed by using the proposed geometrical model were then compared with those obtained from the experiments and those generated by the exact geometrical model. The results obtained are shown in Figs. 17 to 20 for Specimens I to IV respectively. In Figs. 17 to 20, results obtained from a number of simpler geometrical models based on the assumption that the angle $\theta_s(z')$ (Fig. 3b) is constant along the z' direction are also shown. Note that this kind of models are commonly used in the modeling of CHS tubular joints [1,3-5]. From Figs. 17 to 20, it can be concluded that

- (1) Among all the numerical models tested, the exact geometrical model led to the most accurate results for all the specimens.
- (2) All the models with constant $\theta_s(z')$ did not lead to satisfactory prediction of SIFs. In general, models based on small value of $\theta_s(z')$ ($\theta_s(z') \leq \pi/6$) resulted in overestimation when a'/t_0 is small but severe underestimation when $a'/t_0 \geq 0.6$. Models based on large value of $\theta_s(z')$ ($\theta_s(z') \geq \pi/4$) led to severe overestimation when $a'/t_0 \geq 0.6$.
- (3) For all the specimens, the proposed model resulted in accurate and slightly conservative prediction of SIF for the whole range of a'/t_0 . The maximum relative error is about 15% when compared with the experimental results. Furthermore, the proposed model tends to over-predict the SIF values near end of the fatigue life of the joint (i.e. for $a'/t_0 \geq 1.0$).

4.2.4 Fatigue life prediction

In order to estimate the fatigue life of the cracked joint, one may rearrange Eqn. 15 so that dN becomes the object and integrating both sides of the equation and obtains

$$\int dN = \int \frac{da'}{C(\Delta K)^m} \quad (16a)$$

Suppose that an approximation K_e (Eqn. 14) for ΔK is available when a' is within a given range $[a'_{initial}, a'_{final}]$ such that $0 \leq a'_{initial} < a'_{final}$, then by Eqn 16a the residual life, $N_r(a')$,

which is the number of load cycles for the crack to penetrate from a curved depth of a' to a'_{final} can be estimated as

$$N_r(a') = \int_{a'}^{a'_{final}} \frac{da'}{C(K_e)^m} \quad \text{for } a'_{initial} \leq a' \leq a'_{final} \quad (16b)$$

As K_e is computed by numerical simulation, its value is only defined at some selected values of a' in the range $[a'_{initial}, a'_{final}]$. Hence, numerical integration is required to compute Eqn. 16b. The accuracy of the residual life predicted by Eqn. 16b depends on the accuracy of K_e used. In case that the actual crack penetration history is available (e.g. by ACPD measurement), the accuracy of the predicted residual life can be assessed by the factor of safety of the prediction, FOS_N , which is defined as the ratio between $N_{act}(a')$, the actual number of cycle recorded for the crack to penetrate from a' to a'_{final} , and $N_r(a')$. That is

$$FOS_N(a') = \frac{N_{act}(a')}{N_r(a')} \quad (17)$$

A value of $FOS_N < 1.0$ indicates that prediction is not conservative. In practice, a slightly conservative result of FOS_N is preferred.

By using Eqn. 16, the SIFs obtained by using different models were employed to predict the residual life of the four specimens tested in Part I. Figs. 21 to 24 compare the resulted obtained by using different geometrical models (measured, proposed, constant $\theta_s(z') = \pi/3$ and $\pi/4$) with the actual ACPD measurements. The values of $a'_{initial}$ and a'_{final} employed are the values corresponding to the range of a' for the geometrical model used in the SIF calculations and are listed in Table 1. Results for the FOS_N obtained using the predictions from Figs. 21 to 24 are shown in Figs. 25 to 28 respectively. From Figs. 21 to 28, it can be concluded that

- (1) In general, all the models are conservative when compared with the actual measurements. The only exceptions are the proposed model and the model based on exact geometry for Specimen I, which gave almost exact but slightly underestimated residual life for $8\text{mm} \leq a' \leq 11\text{mm}$.
- (2) As expected, the model based on exact geometry is the most accurate with values of FOS_N closest to unity. The proposed model is slightly more conservative than the exact model. In fact, for Specimens I, II and IV, the FOS_N for the proposed model are only

slightly higher than the results from the exact model. For Specimen III, slight divergence of the FOS_N occurred when a' is close to a'_{final} . Hence, it can be concluded that the proposed model can lead to practical prediction of residual life.

- (3) Models based on constant $\theta_s(z')=\pi/3$ and $\pi/4$ are too conservative, especially for Specimen III. Hence, despite that they are simpler than the proposed model, they are not suitable to be used in practice.

5. CONCLUSIONS

A geometrical model and mesh generation procedure for the numerical modeling of cracked SSHS T-joints was proposed. The proposed model and mesh generation procedure were developed based on the results obtained from the testing of four full scale SSHS joints. The proposed modeling procedure is flexible in the sense that they allow the modeling of single and multiple surface cracks and can be extended to the case of SSHS Y-joint and RHS T and Y-joints with modest modifications. Numerical simulations for the predictions of SIF and residual life indicated that geometrical models that are commonly used for the modeling of CHS tubular joints should not be used for the modeling of RHS tubular joints. Furthermore, it is shown that the proposed modeling procedure could lead to accurate and more importantly, reasonably conservative results when comparing with the experimental measurements. Thus, the proposed model could be used in the practical fatigue assessments of SSHS T-joints.

REFERENCE

1. Bowness, D., and Lee, M. M. K., (1995), "The development of an accurate model for the fatigue assessment of doubly curved cracks in tubular joints", *International Journal of Fracture*, Vol. 73, pp 129-147.
2. Zhao, X.L., Herion, S., Packer, J.A., Puthli, R., Sedlacek, G., Wardenier, J., Weynand, K., van Wingerde, A. and Yeomans, N., (2000). "Design Guide for Circular and Rectangular hollow Section Joints under Fatigue Loading". *CIDECT*, TUV Germany.
3. Lie, S.T., Lee, C. K. and Wong S.M., (2003), "Model and mesh generation of cracked tubular Y-joints", *Engineering Fracture Mechanics*, Vol. 70, p161-184.
4. Lie S.T., Chiew, S.P., Lee C.K. and Huang, Z.W., (2004), "Fatigue performance of cracked tubular T-joints under combined loads - Part II - Numerical", *Journal of Structural Engineering*, 130(4), p572-581.
5. Lee, C.K., Lie S.T., Chiew S.P and Shao Y.B., (2005), "Numerical models verification of cracked tubular T, Y and K-joints under combined loads", *Engineering Fracture Mechanics*, Vol. 72 p983-1009.
6. Chiew S. P, Lee, C.K., Lie S.T. and Ji H.L., "Fatigue behavior of Square-to-Square Hollow Section T-joint. I: Experimental studies", Submitted to the *Engineering Fracture Mechanics*.
7. American Welding Society (AWS) (2000), "Structural Welding Code-Steel", *ANSI/AWS D1.1-2000*. American Welding Society, Inc, Miami.
8. EN 10210-2, (1997), "Hot finished structural hollow sections of non-alloy and fine grain structural steels. Part 2, Tolerances, dimensions and sectional properties", *British Standards Institution. BSI*
9. Lie S.T., Lee C.K., Chiew, S.P. and Yang, Z.M., (2005), "Static strength of cracked square hollow section T joints under axial loads- Part II - Numerical", To appear in the *Journal of Structural Engineering*.
10. ABAQUS *Theory Manual and User Manual*, version 6.4, (2004). Hibbit, Karlsson & Sorensen Inc., USA.
11. Vosikovsky, O., Bell, R., Burns, D.J. and Mohaupt, U.H., Fracture mechanics assessment of fatigue life of welded plate T-joints, including thickness effect." *Proc 4th Int Conf on Behaviour of Offshore Structures, BOSS 85* Delft, The Netherlands, Elsevier, Amsterdam (1987), pp. 453-464.

Fig. 1. Configuration of the joint and global coordinate system (x - y - z)

Fig. 2. Modelling of weld toe

Fig. 3. Geometrical model for the crack surface

Fig. 4. Assumed variation of $\theta_s(z')$ with s'

(a)

(b)

Fig. 5. Modelling of crack font.

Fig. 6. Shape of final crack front for Specimen I

Fig. 7. Mesh generation without surface crack.

Fig. 8. Final mesh without crack

Fig. 9. Elements in zone CF are extracted to form sub-zone A

Fig. 10. Generation of mesh around the crack front in sub-zone A

Fig. 11. Mesh at weld toe after surface crack mesh generation

Fig. 12. SCF distributions on chord for Specimen I under different basic loads (Note: Strain gauge positions shown in Fig. 4 of reference [Chiew et al. 2006].)

Fig. 13. SIF at the deepest point using exact geometrical model, Specimen I

Fig. 14. SIF at the deepest point using exact geometrical model, Specimen II

Fig. 15. SIF at the deepest point using exact geometrical model, Specimen III

Fig. 16. SIF at the deepest point using exact geometrical model, Specimen IV

Fig. 17. Predictions of SIF at the deepest point by different models, Specimen I

Fig. 18. Predictions of SIF at the deepest point by different models, Specimen II

Fig. 19. Predictions of SIF at the deepest point by different models, Specimen III

Fig. 20. Predictions of SIF at the deepest point by different models, Specimen IV

Fig. 21. Prediction of residual fatigue life, Specimen I

Fig. 22. Prediction of residual fatigue life, Specimen II

Fig. 23. Prediction of residual fatigue life, Specimen III

Fig. 24. Prediction of residual fatigue life, Specimen IV

Fig. 25. $FOS_N(a')$ for the prediction of residual fatigue life, Specimen I

Fig. 26. $FOS_N(a')$ for the prediction of residual fatigue life, Specimen II

Fig. 27. $FOS_N(a')$ for the prediction of residual fatigue life, Specimen III

Fig. 28. $FOS_N(a')$ for the prediction of residual fatigue life, Specimen IV

Table 1. Initial and final curved crack depths used in residual fatigue life prediction

	Specimen I	Specimen II	Specimen III	Specimen IV
$a'_{initial}$ (mm)	2.10	1.75	1.42	1.27
a'_{final} (mm)	19.33	19.40	20.00	16.43