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Shear Lag Analysis by Adaptive Finite Element Method Part 1: Analysis of Simple Plated Structures

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Summary

Due to the presence of sharp stress gradients, traditional finite element analysis using uniform meshes for the solution of shear lag problems for thin-walled structures is inefficient and will result in inaccurate values of effective breadth ratio. By using the adaptive finite element analysis, it is possible to obtain results with predetermined accuracy with a minimum amount of computational cost. An adaptive finite element analysis procedure is used to deal with the problem of shear lag effects of plated structures with arbitrary dimension and geometry. This study consists of two parts. In Part 1 of the study, shear lag effects in simple plated structures, such as straight rectangular, single-cell box girders are studied. In Part 2 of the study, the adaptive refinement procedure will be extended to the shear lag analysis of complex plated structures such as core walls with openings, multi-cell box girders and box girders with curve flanges.

Keywords: *Shear Lag analysis, adaptive finite element analysis, thin-walled structures*

1. Introduction

Thin-walled components are extremely efficient structures in supporting and distributing loads and their associated effects. The advantage of thin-walled structures is that the material is efficiently used both in flexure and torsion, and thus leads to considerable saving in material, particularly where self-weight is important. In plated structures of low aspect ratios, shear flows will be developed between the web and the flange panels when the structure is subjected to shear forces. Owing to the shear deformation of the panels, the longitudinal displacements remote from the web-flange junctions will lag behind those at the junctions. As a result, nonuniform longitudinal stress distribution or shear lag will be developed within the flange (Fig. 1).

Over the years, many methods have been developed to deal with the shear lag problem. Existing methods for shear lag analysis include

- (1) The harmonic analysis method [1],
- (2) The folded-plate method [2],
- (3) The finite stringer analysis [3]
- (4) The Single Fourier Series approach [4]
- (5) Other semi-empirical methods [5].

However, the formulations of most analytical or semi-empirical methods involve many simplified assumptions. As a result, they can only tackle shear lag problems for plated structures with simple geometry and cannot be easily extended to structures with complex geometry. Hence, numerical method, in particular, the finite element method, remains the only feasible approach for the analysis of shear lag in complex thin-walled structures. In Part 1 of the present study, an adaptive finite element analysis procedure provided by the commercial finite element package LUSAS is adopted to solve the shear lag problem of the plated structures with simple geometry. Parametric studies will also be carried out to investigate the effect of some key geometrical

parameters on the shear lag effect of the structures. In Part 2 of the study, concentration will then be focused on the shear lag analysis of complex structures using the adaptive refinement procedure.

2. Modelling method and adaptive finite element analysis

Three-dimensional plated structure with simple geometry can often be simplified to 2D plane stress models under the premise that the accuracy of shear lag analysis is acceptable. In this paper, the parallel processing method [6] will be used as the main method for shear lag analysis. In this method, a thin-walled box structures as shown in Fig. 2 will be decomposed into flange and web substructures (Fig. 3). The out-of-plane stiffness of each substructure is assumed to be negligible when comparing with the in-plane stiffness. Each plated component is considered to be in a state of plane stress. It is assumed that the external loads are applied at the interfaces between the substructures, and they can be decomposed into components acting in-plane at the adjacent thin-walled substructures [7]. As a result, the top flange can be isolated and regarded as an independent structures and the box girder is simplified into a 2D top flange plate loaded by inplane edge shear forces. Regarding the distribution of stresses in a box-girder subjected to arbitrary bending-moment and shear-force distribution along the span, the usual linear elastic engineer's beam theory is applied in this study. That is, the shear flow, q , is given by the formula

$$q(y) = -\frac{s(y)}{I} Q \quad (1)$$

where $s(y)$ is the shear force at the section y . I and Q are the second moment of area and the first moment of area above the centroid at section y respectively.

When the finite element method is used for the analysis of shear lag problem, the finite element mesh used will obviously affect the accuracy of the solution obtained. When the mesh is very

coarse, the solution obtained may not be accurate enough. Crisfield [8] pointed out that the degree of refinement of the mesh near the loading point would affect the results. The use of highly refined uniform meshes is clearly impractical since they are too expensive in terms of computational cost. Towards this end, the use of an adaptive algorithm will make it possible to obtain solution with predetermined accuracy with a minimum amount of computational cost [9-11]. A typical adaptive mesh refinement procedure will consist of the following steps.

- (1) Start with an initial mesh and carry out the finite element analysis.
- (2) Based on the result of the analysis, estimate the current error of the solution.
- (3) If the error is acceptable, the adaptive analysis procedure is stopped.
- (4) A refinement mesh is generated based on the estimated error distribution.
- (5) Carry out the finite element analysis based on the new mesh and go to step (2).

In this study, the commercial software LUSAS, version 12 [12-14], which has the feature of adaptive analysis, is employed as a tool to carry out adaptive finite element analysis. This software allows a model to be remeshed from a previous analysis to get an accurate result. The remeshing procedure will use the background grid meshing approach [12-15]. The new element size distribution of the next refinement mesh will be determined from the discontinuous error of some pre-selected results (e.g. stresses and displacements) at the nodal points of the current mesh. The discontinuity error at a given node is defined as the difference between the nodal values common to that node from the average value at the same node. For simple plate structures in which the shear lag problem can be reduced to a linear elastic 2D plane stress problem, the six-node plane stress elements (T6 element) will be employed in the adaptive analysis. The discontinuous error of longitudinal stress obtained will be used to determine the new element size during the adaptive refinement.

In an adaptive analysis, the new element size, h_{new} , is computed from the current element size, h_{old} , according to the current discontinuity error, e_d (expressed in terms of percentage of the average nodal stress) and the acceptable discontinuity error, e_a , prescribed by the user [12-14]. For values of discontinuity error greater than e_a , h_{new} will be defined as

$$h_{\text{new}} = \frac{h_{\text{old}}}{1 + S_r e_d} \quad (2)$$

Otherwise, h_{new} will be given by

$$h_{\text{new}} = \frac{h_{\text{old}}}{1 + S_r (e_a - e_d)} \quad (3)$$

Hence, e_a will set the limit below which the element size will be increased (de-refinement). The two constants S_r and S_l are the element size reduction and increasing scale factors respectively. They are used to control and limit the refinement and de-refinement rates of the adaptive procedure and are determined by users according to practical situation [12].

3 Computation of effective breadth ratio by adaptive finite element analysis

The concept of *effective breadth*, b_e , was first proposed by Von Karman [16] in order to measure the effect of shear lag in thin-walled structures and has been widely adopted. It is defined as the breadth along the flange over which the longitudinal stress, σ_y , is assumed to be uniformly distributed and equal to the maximum stress σ_{max} (Fig. 1) and can be expressed as

$$b_e = \frac{\int_0^b \sigma_y dx}{\sigma_{\text{max}}} \quad (4)$$

Eqn. 4 simply implies that the analysis of shear lag is equivalent to the analysis of σ_y . Following this definition of effective breadth, the *effective breadth ratio*, ψ , is defined as the ratio of the effective breadth to the actual breadth. That is

$$\psi = \frac{b_e}{b} = \frac{\int_0^b \sigma_y dx}{b \sigma_{\max}} \quad (5)$$

When the degree of shear lag is small, the distribution of longitudinal stress across the flange is approximately uniform and ψ will close to unity. When the shear lag effect is severe, the longitudinal stress in the flange will lag behind that given by the uniform distribution and ψ will close to zero.

In actual computation, the distribution of the longitudinal stress can be obtained by using the stress-displacement relation of the finite element solution. In order to calculate the effective breadth ratio using Eqn. 5, the longitudinal stress distribution along the flange must be defined first. In the present study, the method of least squares is used to construct a polynomial to describe the variation of longitudinal stress across the section under consideration. In general, σ_y is assumed to take the form

$$\sigma_y = \sum_{i=1}^m a_i (x - \bar{x})^{i-1}, \quad \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad (6)$$

In Eqn. 6, x is the coordinate along the flange of the structure (Fig. 4). a_i , $i=1, \dots, m$, are coefficients to be found from the fitting procedure. It should be noted that m must not be greater than the number of distinct points on the line, N (Fig. 4). $\bar{x}$ is the centroid of the sampling points. As a high degree polynomial may not always lead to accurate result [17], the maximum degree of polynomial adopted in this study is limited to 6 (i.e. $m \leq 7 \leq N$).

After a_i are found, b_e and ψ can be obtained by using Eqn. 4 and Eqn. 5 respectively. In practice, it is often required to compute the effective breadth ratio along the span. This can be achieved by creating a command file inside the LUSAS [14] environment to output all the necessary data at different cross sections of the structure to a file. An external program is then executed to compute the effective breadth ratios at different locations.

When adaptive finite element method is used to solve the shear lag problem, two factors, namely the allowable discontinuous error e_a and the number of adaptive refinements carried out, will largely affect the final accuracy of the solution. In practice, the initial mesh should be neither too fine, making the preparation of data excessively time consuming, nor too coarse, so that too many iterations are needed to achieve the target accuracy. For shear lag problems, the acceptable accuracy can be determined through the comparison of different practical results. A case of a simply supported box girder (Fig. 5) with aspect ratio $R=L/b=1$ under a central point load is selected to illustrate how to deal with these factors. A coarse mesh with 50 elements is used as the initial mesh as shown in Fig. 6a and three refinements are carried out (Figs. 6b to 6d). Table 1 shows the parameters used for adaptive refinement and the results obtained. It can be seen from Fig. 6 that when the mesh is refined, the density of elements near the support becomes higher. This is due to the jump in the longitudinal stress near the support (see the examples given Section 4.1 for more details). However, it can be shown that the magnitude of the longitudinal stress near the support is much less than that in other parts of the problem domain. Therefore, a too much refined mesh is not optimal in terms of computational cost needed. Fig. 7 shows the effective breadth ratio obtained from different values of e_a . It can be seen that the effective breadth ratio obtained practically converged to the final value when $e_a \leq 5\%$. Hence, in this study, a value of $e_a = 5\%$ will be used in all the adaptive finite element analyses.

	e_a	No. of elements
Initial mesh	---	50
Mesh 1	10%	240
Mesh 2	5%	692
Mesh 3	3%	1049
$S_r=1.5, S_l=2.0$		

Table 1 Parameters for adaptive refinement and results obtained

4 Parametric studies for simple plated structures

4.1 Shear lag in a simply supported box girder

In this section, the effective breadth ratios for simply supported box girders (Fig. 5) with different aspect ratio and subject to different loading conditions were computed using the adaptive refinement procedure described in the previous section.

Figs. 8 and 9 shows the effective breadth ratio of a simply supported box girder under the actions of a central concentrated point load and a uniformly distributed loading for the cases of $R=5$ and $R=1$ respectively. It can be seen that the variation of effective breadth mainly takes place within an interval of $b/2$ on both sides of the point load and near the supports. It should be pointed out that the longitudinal stress is very small near the support (Fig. 10), comparing with the stress away from the support. Therefore, even though ψ is close to zero near the supports, it will not cause too much effect on practical design.

This example was also studied by N. Tahan et al [4] by means of the Single Fourier Series (SFS) method. Fig. 11 shows the SFS results for ψ along the span under the same loading cases. When comparing Fig. 11 with Figs. 8 and 9, it is evident that the present results are different from the SFS counterparts near the support of the beam, but in good agreement elsewhere. The results of SFS depict that the stress distribution near the support is uniform under point loading and shear

lag occurs only near the loading point. However, this is not compatible with the stress distribution obtained by the finite element method (Fig. 10). Furthermore, one can see that there is a strong gradient of stress across the cross-section near the support of the beam. As a result, during the adaptive analysis, more elements are generated there to capture the stress variation.

Fig. 12 shows the longitudinal stress variations at the flange-web junction and the middle of the top flange plate. It can be seen that when $R=5$ and under point load (Fig. 12a), severe shear lag occurs near the loading point and the support. For $R=1$ (Fig. 12c), the stress at the junction is much higher than that at the middle, indicating a small ψ at all sections along the span. For uniformly distributed loading and $R=5$, the stress at the junction and the middle are of comparable magnitudes and increase at the same rate except near the support. For $R=1$, the longitudinal stress in the middle is also much smaller than that at the junction. It should be noted that for $R=1$ under both loading conditions, a small magnitude of tensile stress occurs in flange-web junction, especially at the middle of the beam. This phenomenon is first discussed by N. Tahan [18] and it was suggested that it would take place when the aspect ratio is smaller than 0.738. But in fact, it can be seen that even when the aspect ratio equals to 1, tensile stress has developed along the whole span.

In order to cover a wider range of practical cases, the influence of load superposition on effective breadth ratio had also been studied. Fig. 13 shows the results from a simply support box girder with $R=5$, under the actions of two- and six-point symmetric equal loadings. It is evident that the effective breadth ratios drop near the loading positions. Fig. 14 shows the shear lag effects of two- and six-point *antisymmetric* equal loadings for a box-girder with $R=5$. In this case, the effective breadth ratio is reduced not only near the loading point position but also in the middle of the span. There is a reduction of effective breadth ratio in the middle of the span for the six-point

antisymmetric case. Finally, when comparing Fig. 14 with Fig. 13, it can be seen that the reductions of ψ near the loading positions for the antisymmetric case are larger than the symmetric case.

4.2 Negative shear lag in plated structures

In its classical meaning, shear lag is associated with the increase of the longitudinal stresses at the flange-web junction when comparing with those away from the junction (Fig. 15a). However, in some previous studies [19-20] it was found that the longitudinal stress at the middle of the flange sometimes may exceed those at the web-flange junction (Fig. 15b). This phenomenon is commonly known as *negative shear lag* and was first identified by Nakai and Murayama [21] in the case of cantilever box-girders under uniformly distributed loading and may cause some minor effect in a small part of a simply supported box-girders. Among the methods used to deal with the negative shear lag, the concept of a “substitute beam” introduced by Kristek and Studnicka [20] is thought to be useful. However, this method requires two analyses of the beam under different configurations. Herein, the adaptive finite element method will again be used to analyze the negative shear lag phenomenon.

Figs. 16a to 16c show a case of cantilever box girder with aspect ratio $R=2.5$ under the action of a uniformly distributed loading while Fig. 16d shows the final adaptive mesh used which consists of 5414 elements. The distributions of longitudinal stress across the flange are shown in Figs. 17 and 18. It can be seen that the stress distributions near the fixed end (Figs. 19i to 19k) have the usual character of positive shear lag. However, in the cross sections remote from the support (at a distance $b/2$ away from the support, b is the width of the flange), the maximum stress does not occur at the flange-web junctions. It is observed that the longitudinal stresses across the flange

gradually decrease as the distance from the support increases. Particularly, the stress at the junction continues to drop to zero and even changes sign from negative to positive near the free end, where the longitudinal stress changes from the compressive to tensile at the junction.

This cantilever box girder was also studied under the action of a point load at the free end. The results are shown in Fig. 19. It is found that the longitudinal stress across the flange are almost uniform except for a small part near the two ends of the girder which indicates that there is little shear lag effect along the span.

In order to find out the factors that affect the negative shear lag for a cantilever beam, a parametric study was carried out by the adaptive analysis procedure. The box girder shown in Fig. 16 under the action of uniformly distributed loading is considered again. In general, the effect of the shear lag is evaluated by the concept of effective breadth. However, for negative shear lag, the change of longitudinal stresses from the support to the free end is large and the effective breadth concept may not be the best way to reflect the stress variation. Thus, the ratio $\sigma_{\text{jun}}/\sigma_{\text{mid}}$, where σ_{jun} and σ_{mid} are the longitudinal stresses at the flange-web junction and the middle of the flange respectively, is used as a indicator to study the shear lag effect. The difference between $\sigma_{\text{jun}}/\sigma_{\text{mid}}$ and unity will reflect the effect of shear lag. The greater the difference, the larger the shear lag effect.

Ten cases of different breadth to length ratio, b/L , from 0.1 to 1.0 are studied. The number of elements in the final meshes obtained (with $e_a=5\%$) for the ten cases are in turn equal to 364, 685, 939, 1230, 1564, 2070, 2340, 2471, 2914, and 3082. The results of parametric studies are shown in Fig. 20, where $\sigma_{\text{jun}}/\sigma_{\text{mid}}$ is plotted against the aspect ratio of the beam for different cross sections located at distances of $0.15L$, $0.30L$, $0.60L$ and $0.90L$ from the free end. Note that negative shear lag occurs at where $\sigma_{\text{jun}}/\sigma_{\text{mid}} < 1$ while positive shear lag develops in the regions

at where $\sigma_{jun}/\sigma_{mid} > 1$. In some cases, such as $b/L=1.0$ and $y=0.15$ (Fig. 20a), $\sigma_{jun}/\sigma_{mid} < 0$ but $|\sigma_{jun}/\sigma_{mid}| > 1$. This is also considered as negative shear lag. From Fig. 20, the following conclusions can be drawn.

- (a) Negative shear lag effect is severe in the region close to the free end of the girder.
- (b) The effect of the negative shear lag is strongly influenced by the aspect ratio. When the value of b/L increases, just as the aspect ratio R decreases, the negative shear lag effect increases. When b/L is small, the negative shear lag effect may be neglected.
- (c) When $b/L > 0.5$, the longitudinal stress begins to change its sign at the cross section $y=0.15L$. Particularly, the longitudinal stress changes its sign at $y=0.30L$ if $b/L = 1.0$.
- (d) Positive shear lag takes place at the cross section $y=0.90L$ when $b/L > 0.4$ and almost no shear lag occurs if $b/L < 0.4$. In fact, positive shear lag will occur near the support for a uniformly distributed loading.

4.3 Shear lag problem for tapered box girder

In this section, the shear lag problem of a beam with varying depth (i.e. a tapered beam, Fig. 21), which also can be simplified into 2D plane stress problem, is studied. Previous studies [22-24] have shown that an inclined compression flange tends to reduce the net vertical shear force in the web. Guyon [24] proposed a simple and accurate method to determine the shear flow between the web and the flange of a tapered beam and will be used here. Once the shear flow between the web and the flange is known, the adaptive refinement procedure can again be applied to the simplified 2D model.

4.3.1 Edge shear flow and geometric parameter for a tapered beam

From Guyon's theory [24], the edge shear flows may be obtained by the following equations for different loading cases.

(a) Edge shear flow for an end point load

The expression for the shear flow to horizontal flange is given by (Fig. 22a)

$$q(y) = \frac{Qd_q}{d_s^2} = \frac{Qd_q}{(d_q + y\tan\theta)^2} \quad (7)$$

where Q is the magnitude of the point load and d_q is the depth at the loading position.

(b) Edge shear flow for uniformly distributed loading

For a uniformly distributed loading (Fig. 22b), the shear flow is given by:

$$q(y) = \frac{wy(2d_q + y\tan\theta)}{2(d_q + y\tan\theta)} = \frac{wy(d_q + d_s)}{2d_s^2} \quad (8)$$

where w is the density of the uniformly distributed loading.

Fig. 23 represents two linearly tapered cantilever boxes, loaded by a point load, Q , with the same value of y_t . The taper angles of these two cantilevers are equal to α and β respectively, such that $\tan\beta = c\tan\alpha$ and c is a constant. From Eqn. 7, the edge shear flows for these two beams will equal to

$$\text{Taper angle} = \alpha \quad q(y) = Q \frac{d_q}{d_s^2} \quad (9)$$

$$\text{Taper angle} = \beta \quad q(y) = Q \frac{(cd_q)}{(cd_s)^2} = Q \frac{d_q}{cd_s^2} \quad (10)$$

Thus, the change in the taper angle does not alter the shape of the edge shear loading curve, but only scales it by $1/c$. Since the distribution of effective breadth is only affected by the shape of the edge shear flow, any change in slope which does not alter L/y_t will not change the effective breadth distribution. Thus, the dimensionless parameter L/y_t will determine the tapering effect of the beam during the shear lag analysis. Figs. 24 shows the effect of L/y_t on the edge shear flow of the beam. In Fig. 24, different ratios of L/y_t are achieved by fixing L , d_q and changing d_l .

4.3.2 The effect of L/y_t on effective breadths

Note that when using the adaptive procedure for the solution of shear lag problems with multiple loading cases, there is no need to create different models for different loading configurations. In fact, it is only required to refine the mesh according to the largest discontinuous error obtained from all the loading cases. Different loading conditions are then applied to the model created and as a result leads to significant savings in computational cost.

Fig. 25 shows the variation of the effective breadth ratio along the beam with aspect ratio $L/b=5$ for different values of L/y_t under a point load. The final mesh of the adaptive analysis contains 915 elements. These curves indicate that in this case any change of L/y_t does not severely affect the effective breadth. In general, when L/y_t increases, the effective breadth increases near the fixed end while opposite situation occurs near the free end.

Fig. 26 shows the effective breadth ratio for a beam with $R=1$ under the action of a point load with different values of L/y_t . (The final mesh contains 890 elements.) For $R=1$, the changing of L/y_t has larger effect than the $R=5$ case. The variations of effective breadth ratios for different values of L/y_t at the free end are similar in both cases. Figs. 25 and 26 also show that the effective breadth ratio of a tapered beam is close to that of a beam with constant depth when L/y_t is small.

Figs. 27 and 28 present the results for ψ for various values of L/y_t under uniformly distributed loading for the cases of $R=5$ (312 elements in the final mesh), and $R=1$ (295 elements in the final mesh.) respectively. For the case of $R=5$, different values of L/y_t mainly affect the value of ψ in the range of $L/5$ from the free end (Fig. 27). The difference of gradients of shear flows in this range is very large (Fig. 24c), which may be the reason for this result. From Fig. 28, it can be seen that effective breadth first increases as L/y_t increases until $L/y_t = 7/3$, then the effective breadth begins to decrease as L/y_t increases in the region from the free end to $y=0.5L$. Outside this region the effect of changing L/y_t is small. When comparing Fig. 27 with Fig. 28, it is again found that the smaller the aspect ratio, the larger the shear lag effect.

5 Conclusions

In this paper, the adaptive finite element method using the parallel processing analysis has been used to deal with the shear lag effect of simple plated structures including simply supported box girders, cantilevered box girders and beams with varying depth. The effective breadth concept is employed to measure the shear lag effect. A procedure for the analysis of shear lag is developed by using the commercial finite element software LUSAS to obtain accurate results. By using the adaptive refinement procedure, parametric studies have been carried out to investigate the effect of different geometry, loading and boundary conditions on the effective breadth ratio of various simple plated structures. Furthermore, it is found that the adaptive refined procedure is convenient to use and can produce accurate results quickly. Comparing with the traditional analytical method, the present method can yield a solution with predetermined accuracy in a minimum amount of computational cost, especially for the case of multiple loading conditions.

Reference

1. V. Kristek, J. Studnicka and M. Skaloud, Shear lag in wide flanges of steel bridges, *ACTA Tech. CSAV* **26**, 464-488 (1981)
2. V. Kristek, Folded plate approach to analysis of shear wall systems and frame structures, *Proc., Instn. Civ. Engrs.*, Part 2, **67**, Thomas Telford Services Ltd., London, 1065-1075 (1979)
3. H. R. Evans, and A. R. Taherian, A design aid for shear lag calculations, *Proc., Instn. Civ. Engrg.*, Part 2, **69**, Thomas Telford Services Ltd., London, 403-424 (1980)
4. N. Tahan and M. N. Pavlovic, Shear lag revisited: the use of Single Fourier Series for determining the effective breadth in plated structures, *Compt. Struc.*, **63**, 759-767 (1997)
5. A. K. H. Kwan, Simple method for approximate analysis of framed tube structures, *J. Struct. Eng.*, ASCE, **120**, 1221-1239 (1994)
6. R. H. Dodds, and L. A. Lopez, Substructuring in linear and nonlinear analysis, *Int. J. Num. Meth. Eng.*, **15**, 583-97 (1980)
7. A. Fafitis and A. Y. Rong Analysis of thin-walled box girders, *Thin-walled structures*, **21**, 233-240 (1995)
8. M. A. Crisfield, Discussion of "Shear lag in steel box girder bridges", *Struct. Eng.*, **53**, 439-448, (1976)
9. O. C. Zienkiewicz and J. Z. Zhu, The three R's of engineering analysis and error estimation and adaptivity, *Comput. Methods Appl. Mech. Engrg.*, **82**, 95-113 (1990)
10. O. C. Zienkiewicz and J. Z. Zhu, The superconvergent patch recovery (SPR) and adaptive finite element refinement, *Comput. Methods Appl. Mech. Engrg.*, **101**, 207-224, (1992)
11. C. K. Lee and S. H. Lo, An automatic adaptive refinement finite element procedure for 2D elastostatic analysis, *Int. J. Numer. Methods Engrg.*, **35**, 1967-1989 (1992)
12. LUSAS, MYSTRO User Guide, FEA, Ltd, UK, 1997
13. LUSAS, LUSAS User Guide, FEA, Ltd, UK, 1997.
14. LUSAS, MYSTRO Command Reference I and II, FEA, Ltd, UK, 1997
15. P. L. George, Automatic mesh generation, application to finite element methods, John Wiley & Sons Publishing Company, 1991.
16. Karman, Th. V., Die Mittragende Poreite, Springer, Berlin, 1924

17. P. Lancaster and K. Salkauskas, Curve and surface fitting, Department of mathematics and statistics, University of Calgary, Canada (1986)
18. N. Tahan, Single Fourier series for two dimensional analysis of plates and box girders, Ph.D. thesis, University of London (1991)
19. S. T. Chang, and F. Z. Zheng, Negative shear lag in cantilever box girder with constant depth, *J. Struct. Eng.*, ASCE, **113**, 20-35 (1987)
20. V. Kristek, and J. Studnicka, Discussion of “Negative shear lag in cantilever box girder with constant depth” by Chang S. T. and Zheng F. Z. *J. Struct. Eng.*, ASCE, **114**, 2168-2173 (1988).
21. H. Nakai and Y. Murayama, Researches on negative shear lag of cantilever beams and application to bridge design, *Trans. JSCE* **8**, 107-109 (1976).
22. G. Davies, R. S. Lamb, and C. Snell, Stress distribution in beams of varying depth, *The Structural Engineering*, **51**, Nov, (1973)
23. R. Lamb, Stress distribution in Tapered concrete beams, Ph.D. Thesis, Dept. of Civ. Eng. Univ. of Nottm, (1971)
24. Y. Guyon, Beams of variable depth, Prestressed concrete, London, Contractors record and Municipal Engineering, 367-375 (1953)

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Fig. 1 Stress distribution affected by shear lag

Fig. 2 Thin-walled box girder
under point load

Fig. 5 A simply supported box girder

Fig. 6 Case study: (a) initial mesh, 50 elements (b) $e_a=10\%$, 240 elements
(c) $e_a=5\%$, 692 elements (d) $e_a=3\%$, 1049 elements

Fig. 7 Effective breadth ratios obtained from different value of e_a

Fig. 8 Effective breadth ratio for $R=5$

Fig. 9 Effective breadth ratio for $R=1$.

Fig. 10 Longitudinal stress distribution for $R=5$ at $y/b=0, 0.1, 0.2, 0.3, 0.5, 0.8$, and 1.0 (central point loading case)

Fig. 11 SFS results of effective breadth ratio along the span of $R=1$ and $R=5$ (Fig. 8 of reference [4])

Fig. 12 Longitudinal stress variation at the flange-web junction and middle of the top flange
 --- junction, — middle. (a) $R=5$, point load (b) $R=5$, uniformly distributed loading
 (c) $R=1$, point load (d) $R=1$, uniformly distributed loading

Fig. 13 Two different loading arrangements for point loads ($R=5$) (a) two-point symmetric equal loading (b) six-point symmetric equal loading (c) effective breadth ratio along the span

Fig. 14 Another two different loading arrangements of point loads ($R=5$) (a) two-point antisymmetric equal loading (b) six-point antisymmetric equal loading (c) effective breadth ratio along the span

Fig. 15 Distribution of longitudinal stress across the flange of box-girder
(a) positive shear lag (b) negative shear lag.

Fig. 16 Cantilever box girder: (a) cross section (b) loading
(c) flange-plate loaded by in-plane edge shear forces (d) final mesh

Fig. 17 Contour of longitudinal stress in flange (uniformly distributed loading)

Fig. 18 Longitudinal stress distributions across the flange (uniform distributed loading)

Fig. 19 Longitudinal stresses at the junction and middle of the flange (point load)

Fig. 20 Results of parametric study: shear lag factor of different aspect ratios.
(a) $y=0.15L$ (b) $y=0.30L$ (c) $y=0.60L$ (d) $y=0.90L$ (uniformly distributed loading)

Fig. 21 Tapered box girder: (a) dimensions conditions
(b) flange-plate loaded by in-plane edge shear forces

Fig. 22 Notations and sign conventions: (a) point load (b) uniformly distributed loading

Fig. 23 Cantilever with constant L/y_t

Fig. 24 Tapered beam and edge shear flow for parametric study (a) Tapered beam (b) point load (c) uniformly distributed loading (Fractions represent L/y_t)

Fig. 25 ψ (effective breadth ratio) along the beam for different L/y_t ($R=5$, point load)

Fig. 26 ψ along the beam for different L/y_t ($R=1$, point load)

Fig. 27 ψ along the beam for different L/y_t ($R=5$, uniformly distributed loading)

Fig. 28 ψ along the beam for different L/y_t ($R=1$, uniformly distributed loading)