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# Design of frequency interleaving ADC with mismatch compensation

L. Qiu, Y.J. Zheng and L. Siek

In this paper, a frequency-interleaving based multi-channel ADC with mismatch compensation is presented, which is immune from the time skew problem existed in time-interleaved ADC (TI-ADC). The channel mismatches, such as bandwidth mismatch, gain mismatch, offset mismatch and filter bank mismatch, are addressed and modelled in the reconstruction optimization. A prototype of 4-channel 1GS/s 12-bit frequency-interleaved ADC (FI-ADC) is designed to demonstrate the mismatch compensation. Simulation results show that the mismatches in FI-ADC can be compensated effectively.

**Introduction:** By using the multi-channel ADCs, the sampling speed can be increased proportionally with the number of channels [1]. However, the strict requirement of matching between channels makes the performance worse than expected. The architecture of TI-ADC is sensitive to time skew. Although calibration can be used, the resultant performance is still not satisfactory [2]. Besides TI-ADC, there is another multi-channel ADC called frequency-interleaved ADC, whose architecture is depicted in Figs. 1, that splits input signal in the frequency domain. There were some applications using hybrid filter bank (HFB) to design FI-ADC [3-4]. However, those previous designs relied on high order analysis filters, increasing the design complexity [3]. In [4], a new method of design synthesis filters was proposed, which reduced high order requirement of the analysis filters. In this paper, the FI-ADC is introduced first, followed by the proposed calibration scheme and simulation results.

**Frequency-interleaved ADCs:** In FI-ADCs, the input signal  $X(j\Omega)$  is split into  $M$  equal frequency bands shown in Fig. 2. After zero-order sampling and holding, sub-ADCs and interpolation, the split signals are filtered by digital synthesis filter bank. The output signal is given by

$$Y(e^{j\omega}) = \frac{1}{M} \sum_{p=0}^{M-1} X(j\frac{\omega}{T} - j\frac{2\pi p}{MT}) T_p(e^{j\omega}), \quad (1)$$

where  $T$  is the sample period of the system,  $p$  indicates the frequency-shifted versions after sampling. And the  $T_p(e^{j\omega})$  is

$$T_p(e^{j\omega}) = \frac{1}{M} \sum_{m=0}^{M-1} H_m(j\frac{\omega}{T} - j\frac{2\pi p}{MT}) F_m(e^{j\omega}), \quad (2)$$

where  $H_m(j\omega/T)$  is  $m$ th channel analog analysis filter, and  $F_m(e^{j\omega})$  is  $m$ th digital synthesis filter.  $T_0(e^{j\omega})$  is the transfer function of original signal and  $T_p(e^{j\omega})$  ( $p=1, \dots, M-1$ ) are the transfer functions of aliasing signals. And if the  $T_p(e^{j\omega})$  satisfies the condition like

$$T_{ideal}(e^{j\omega}) = \begin{cases} c \cdot e^{-j\omega d}, & p=0 \\ 0, & p=1, \dots, M-1 \end{cases} \quad (3)$$

where  $c$  is a non-zero constant and  $d$  is the system delay, the system can then be regarded as perfect-reconstruction system.

**Design of frequency-interleaved ADC:** To build the analysis filter bank, low order lowpass and bandpass filters are adopted, with expression

$$\begin{cases} H_{LP}(s) = \frac{1}{RC_1s + 1} \\ H_{BP}(s) = \frac{RC_2s}{R^2C_1C_2s^2 + RC_2s + 1} \end{cases} \quad (4)$$

Keeping the ratio of  $C_1/C_2$  a constant, the shape the frequency response is unchanged. Therefore, only the resistors need to be trimmed to compensate the process variation. The process variation parameters of resistor and capacitor are set to  $\alpha_1$  and  $\alpha_2$ , respectively.  $H_{BP}(s)$  in (4) can be rewritten as

$$H_{Non-ideal}(s) = \frac{kRC_2s}{k^2R^2C_1C_2s^2 + kRC_2s + 1}, \quad (5)$$

where  $k$  is equal to  $\alpha_1\alpha_2$ . Incorporated with other mismatches, the non-ideal model of FI-ADC shown in Fig. 1 encompasses gain mismatch  $a_m$ , offset mismatch  $b_m$  and  $H'_m(s)$ , which includes bandwidth mismatch and filter bank mismatch described as

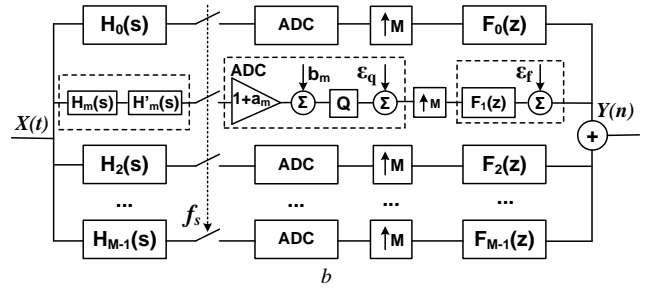


Fig. 1 Architecture of frequency-interleaved ADCs with mismatch model.

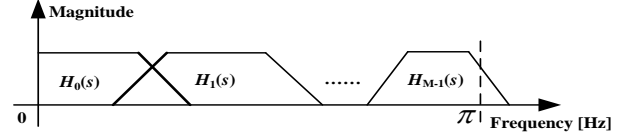


Fig. 2 The frequency distribution of  $M$ -channel analysis filter bank.

$$\begin{aligned} H'_m(s) &= \frac{k_m(R^2C_1C_2s^2 + RC_2s + 1)}{k_m^2R^2C_1C_2s^2 + k_mRC_2s + 1} \cdot \frac{1}{1 + s/c_m\omega_0} \\ &= H_{Afb}'_m(s) \cdot H_{Bw}'_m(s) \end{aligned} \quad (6)$$

In (6),  $H_{Afb}'_m(s)$  and  $H_{Bw}'_m(s)$  represent the analog analysis filter bank mismatch and bandwidth mismatch, respectively. The filter bank mismatch parameter of  $m$ th channel is  $k_m$ . The bandwidth of the sample circuit in each channel is modelled under a first order approximation with  $c_m$  indicating the bandwidth mismatch and  $\omega_0$  denoting the reference bandwidth of each channel. In addition, the quantization in sub-ADC and limited word length in FIR synthesis filters are added as sources of noise  $\epsilon_q$  and  $\epsilon_f$ . With all the mismatches mentioned above taken into account, the output signal in (1) can be rewritten as

$$Y(e^{j\omega}) = \frac{1}{M} \sum_{p=0}^{M-1} X(j\frac{\omega}{T} - j\frac{2\pi p}{MT}) T_p'(e^{j\omega}) + O(e^{j\omega}), \quad (7)$$

where

$$T_p'(e^{j\omega}) = \sum_{m=0}^{M-1} H_m \cdot H_{Afb}'_m(j\frac{\omega}{T_s} - j\frac{2\pi p}{MT_s}) F_m(e^{j\omega}) \cdot \frac{1}{1 + s/c_m\omega_0} \cdot (1 + a_m)$$

and

$$O(e^{j\omega}) = \frac{1}{M} \sum_{p=0}^{M-1} \sum_{m=0}^{M-1} b_m \cdot F_m(e^{j\omega}) \delta(j\frac{\omega}{T_s} - j\frac{2\pi p}{MT_s}).$$

It can be seen in (7) that the  $T_p'(e^{j\omega})$  includes the coefficient variation of analysis filters  $H_{Fb}'_m(j\omega)$ , bandwidth mismatch  $c_m$  and gain mismatch  $a_m$ , which cause both distortion ( $p=0$ ) and aliasing ( $p=1, 2, \dots, M-1$ ). The offset mismatch multiplied by  $F_m(e^{j\omega})$  generates spectral components at  $2\pi p f_s/M$ . Given the analysis filter bank, the coefficients of synthesis filter bank can be obtained through optimization programme. The error of perfect reconstruction is

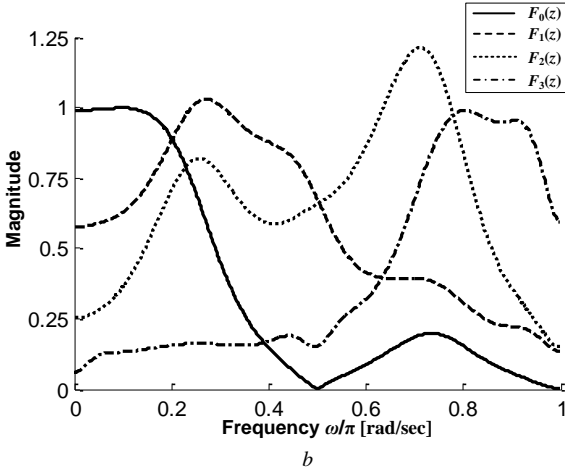
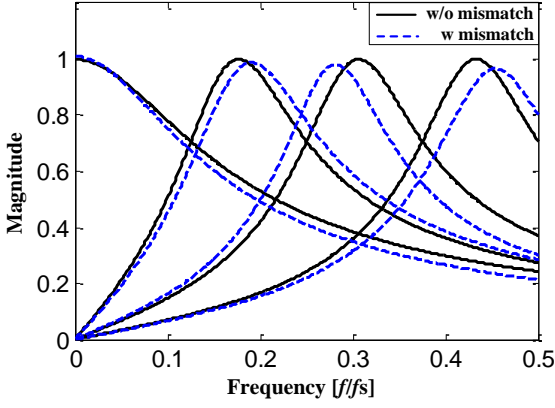
$$e_p(\omega) = |T_p'(e^{j\omega}) - D_p(e^{j\omega})|. \quad (8)$$

The min-max criterion is adopted to optimize the coefficients of synthesis filters bank. The optimization expression is

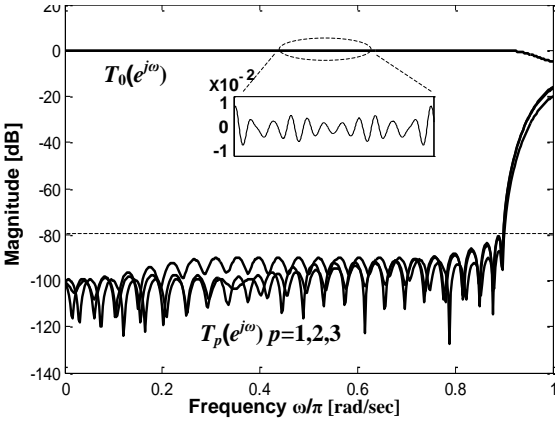
$$\min_f \{ \max_{\omega} W_p(\omega) | e_p(\omega) | \}, \quad \omega \in [0, \pi], \quad p=0, \dots, M-1 \quad (9)$$

where  $f$  represents the coefficients of digital synthesis filters.  $W_p(\omega)$  is the weight indicating importance of different frequency band. The optimization is solved by standard second-order cone programming (SOCP) solver [4].

**Simulation Results:** To demonstrate the proposed ADC architecture with mismatch compensation, a 4-channel 1GS/s 12-bit FI-ADC is designed. Firstly, the frequency response of analysis filter bank with mismatch is shown in Figs. 3a, which are 1<sup>st</sup> order low-pass and 2<sup>nd</sup> order band-pass Butterworth filters. The corresponding mismatch parameters are listed in Table 1. The reference bandwidth  $\omega_0$  of each channel is set to  $2\pi \cdot 10$ GHz. To achieve a significant attenuation with short-tap FIR filters, the idea of weighted filter design [4, 5] is applied, where the "don't care" band of optimization is  $[0.9\pi, \pi]$ . As mentioned in (3), the target  $c$  and  $d$  are set to 1 and 20, respectively. The



**Fig. 3** Frequency response of hybrid filter bank.  
a Analog analysis filter bank  $H_m(s)$  ( $m=0, 1, 2, 3$ )  
b Digital synthesis filter bank  $F_m(z)$  ( $m=0, 1, 2, 3$ )



**Fig. 4** Reconstruction results.

optimization results (Table 2) show that 41-tap FIR synthesis filters can satisfy the design requirement. The frequency response of optimized synthesis filter bank is illustrated in Figs. 3b, where  $F_m(z)$  represents the frequency response of synthesis filter in  $m$ th channel. The offset mismatch is calibrated by inserting an accumulation and average block in each channel. In Fig. 4, the maximum distortion of the reconstruction function  $T_0(e^{j\omega})$  is around 0.01 dB. And the attenuation  $T_m(e^{j\omega})$  ( $m=1, 2, 3$ ) is below -80 dB in the frequency range  $[0, 0.9\pi]$ .

**Conclusion:** In this paper, a multi-channel FI-ADC architecture is presented, which can avoid the time skew problem existed in TI-ADC. The simple implementation of active analog analysis filters make the frequency-interleaved architecture promising for integration into chip.

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**Table 1:** Mismatch parameters of the 4-channel FI-ADC.

$m$	Bandwidth (MHz) (RC)	Filter bank mismatch ( $k_m$ )	Bandwidth mismatch ( $c_m$ )	Gain error ( $a_m$ )	Offset error ( $b_m$ )
0	[0, 125]	0.90	1.06	0.010	0.006
1	[125, 250]	1.08	0.98	-0.005	-0.007
2	[250, 375]	0.92	0.92	-0.002	0.002
3	[375, 500]	1.05	1.03	0.003	0.008

**Table 2:** Coefficient values of synthesis filter bank  $F_m(z)$  ( $m=0, 1, 2, 3$ ).

n	$F_0(z)$	$F_1(z)$	$F_2(z)$	$F_3(z)$
0	0.000284	-0.00111	0.001529	-0.00105
1	-0.00062	0.002171	-0.00256	0.000817
2	0.000232	-0.00197	0.003666	-0.0017
3	-0.00104	0.002835	-0.00512	0.004344
4	0.001551	-0.00896	0.013266	-0.01556
5	-0.00132	0.007409	-0.01076	0.01374
6	0.001364	-0.00776	0.01151	-0.0159
7	-0.00182	0.01139	-0.01671	0.024618
8	0.004584	-0.02894	0.041644	-0.06398
9	-0.00306	0.021188	-0.03027	0.050278
10	0.002813	-0.02065	0.029359	-0.05185
11	-0.00416	0.030083	-0.04238	0.074845
12	0.013245	-0.08637	0.121297	-0.19534
13	-0.00515	0.045954	-0.06447	0.129817
14	0.002948	-0.03719	0.052417	-0.12136
15	-0.01021	0.076221	-0.10686	0.191251
16	0.118302	-0.52567	0.695288	-0.70434
17	0.830889	-2.24757	2.150652	-0.14132
18	0.972155	-0.67685	-1.15837	0.801142
19	1.131472	0.506672	-0.47326	-1.09394
20	1.029907	0.597894	0.316101	1.246276
21	0.230605	0.265168	-1.01813	-0.66734
22	0.039804	-0.04178	0.161458	0.22793
23	-0.13079	-0.1011	0.307556	0.053862
24	-0.22871	0.014495	-0.16186	-0.04759
25	-0.07004	-0.13625	0.313922	0.004337
26	-0.03417	-0.05122	-0.02054	0.069202
27	0.008401	0.024006	-0.09504	-0.11566
28	0.057719	0.055405	0.040543	0.158905
29	0.010473	0.024458	-0.06246	-0.07875
30	0.006153	-0.00214	-0.00575	0.025519
31	-0.00206	-0.00868	0.01989	-0.00083
32	-0.0094	0.00211	-0.00615	0.019693
33	-0.0022	-0.00504	0.013895	-0.002
34	-0.00155	-0.00284	-0.00011	0.00124
35	-0.00009	0.0015	-0.00509	-0.00299
36	0.002427	0.003178	0.001478	0.010767
37	0.000259	0.000444	-0.00157	-0.0029
38	0.000089	-0.00049	-0.00104	4.72E-05
39	0.000145	-0.00026	0.001008	0.000389
40	-0.0001	0.000046	0.000312	0.001737

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