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Automatic Generation of Anisotropic Quadrilateral Meshes on Three Dimensional Surfaces using Metric Specifications

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Summary

A new algorithm for constructing full quadrilateral anisotropic meshes on 3D surfaces is proposed in this paper. The proposed method is based on the advancing front and the systemic merging techniques. Full quadrilateral meshes are constructed by systemically converting triangular elements in the background meshes into quadrilateral elements. By using the metric specifications to describe the element characteristics, the proposed algorithm is applicable to convert both isotropic and anisotropic triangular meshes into full quadrilateral meshes. Special techniques for generating anisotropic quadrilaterals such as new selection criteria of base segment for merging, new approaches for the modifications of the background mesh and construction of quadrilateral elements, are investigated and proposed in this study. Since the final quadrilateral mesh is constructed from a background triangular mesh and the merging procedure is carried out in the parametric space, the mesh generator is robust and no expensive geometrical computation that is commonly associated with direct quadrilateral mesh generation schemes is needed.

KEYWORDS: *Anisotropic mesh generation; Indirect method; Advancing front technique; Systemic merging technique; Quadrilateral mesh generation*

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1 Introduction

Since model and data preparation is often the most time consuming process in large-scale finite element analysis (FEA), a robust automatic mesh generator becomes an indispensable tool in such applications. Towards this end, the generation of well-graded high quality meshes for 3D surfaces plays a critical role in the analysis of thin-walled structures and full 3D solid mesh generation. Nowadays, the generation of triangular meshes on 3D surfaces by either the *Delaunay method* or the *advancing front technique* (AFT) are generally considered matured techniques [1-9]. However, in many applications, quadrilateral meshes are more efficient and accurate than triangular meshes. For this reason, there is a constant demand to develop robust quadrilateral mesh generator for 3D surface meshing. In the last decade, much research efforts had been directed to the development of quadrilateral mesh generation and different approaches have been proposed especially in the area of unstructured quadrilateral mesh generation. Virtually all existing quadrilateral meshing algorithms suggested can be grouped into two main categories, namely, the *direct* and the *indirect* approaches.

In the direct algorithms, quadrilateral elements are directly created over the problem domains. Talbott and Parkinson [10] proposed an algorithm to generate quadrilateral meshes in 2D domain by bi-sectioning and the use of pre-designed templates. Zhu et al. [11] proposed an AFT for forming and merging two triangular elements at a time. Blacker and Stephenson [12] developed the well-known *paving algorithm* and a set of delicate measures and operations to control the formation of quadrilaterals. This paving algorithm was then extended to 3D surfaces by Cass et al. [13]. In addition, White and Kinney [14] modified the paving algorithm and increased its stability by controlling the element formation procedure in an element-by-element instead of row-by-row manner.

In the indirect algorithms, quadrilaterals are formed by converting two or more triangular elements at a time in an existing triangular mesh. The original triangular mesh or the so-called *background mesh* can be created by any triangular mesh generator. Lo [15] first suggested an algorithm converting the triangular mesh into a mixed mesh with dominant amount of quadrilateral elements. Johnston et al. [16] then developed addition procedures such as local splitting and swapping to further increase the proportion of quadrilateral elements in the final mesh. Rank et al. [17] proposed a technique transforming triangular mesh into quadrilateral mesh by splitting two neighboring triangles. Lo, Lee and Lau [18,19] later extended their work to generate full quadrilateral meshes in 2D planes and 3D surfaces by introducing the

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systemic merging technique (SMT). By combining the SMT with concepts and techniques used in the paving algorithm, Owen et al [20] successfully developed the *Q-MORPH* algorithm to generate fully quadrilateral meshes with well-aligned rows of elements and fewer number of internal irregular nodes.

In order to construct high quality meshes, effort has been devoted to quantify and control various element characteristics, such as the grading, stretching effect and directions, of the meshes. Initially, the required characteristics are manually defined by users in terms of simple parameters [21,22]. As applications to more complicated problems are needed, more generalized approaches such as the *node spacing function* were introduced [23,24]. However, in some applications, it is required to control both the element size and the anisotropic characteristics of elements in the mesh [25-27] and thus a more general description is needed. Recently, Borouchaki et al [28,29] proposed the *metric specifications* for describing and quantifying the anisotropic characteristics of a finite element mesh. Based on this general approach, a few metric triangular mesh generators have been developed [30-34].

The main objective of this study is to present a new indirect quadrilateral mesh generation scheme over 3D curved surfaces using the metric specifications for element characteristics control. The proposed method is partially similar to the techniques proposed by Lee and Lo [18,19] and Owen et al. [20]. Additional measures and algorithms specially designed for the anisotropic case are investigated. The selection of base segment for quadrilateral formation, which turns out to be the most important step affecting the results of the conversion phase, has been carefully investigated and a new scheme to select the base segment in the general anisotropic case will be proposed in this paper.

2 Overview of the quadrilateral mesh conversion procedure

The presented method is an indirect method with the basic principle that a full quadrilateral mesh can be formed by carefully converting and combining the triangles in the background triangular mesh to quadrilateral elements. As shown in Fig. 1, a quadrilateral element can be constructed by modifying the local structure of the background triangular mesh and converting *two or more* triangular elements into a quadrilateral element. By repeating such operations, quadrilateral elements can be constructed to cover the whole problem domain provided that the number of segments enclosing the background triangular mesh is even (and hence with even number of elements) as shown in Fig. 2.

The whole mesh conversion procedure can be divided into the following key steps:

2.1 Background triangular mesh generation

The 3D surface is first discretized into a triangular mesh using the metric advancing front triangulator described in reference [32]. Triangular meshes defined in the 3D and the parametric spaces of the geometrical models will be generated [32]. In addition, the metric tensor at the nodal points of the background triangular mesh will also be defined.

2.2 Formation of initial front

Before the conversion process is started, an initial merging front will be set up. External boundary segments will be arranged in anti-clockwise direction while the interior openings will be in a clockwise manner.

2.3 Base segment selection for quadrilateral formation

The first step of quadrilateral formation is to determine where the new quadrilateral element should be constructed. The frontal segment on which a new quadrilateral element will be formed is called the *base segment*. It should be pointed out that the selection of the base segment will greatly affect the performance of the conversion procedure and the quality of the final quadrilateral mesh. Therefore, a careful investigation has been carried out in the selection of base segment and a more detailed description will be given in Section 4.

2.4 Quadrilateral formation

After the base segment is selected, a quadrilateral will be formed by merging a number of triangles bounded by four side edges. The quadrilateral formation process can be further divided into two steps:

(a) Side edges preparation

Before the quadrilateral is formed, the four side edges enclosing it must be first defined. The base segment will be used as the "bottom" side edge of the quadrilateral (Fig. 3a). According to configurations of the base segment and its neighboring edges and triangles, the side (left and right) edges can be defined by a front segment or an internal edge or they will be created by some local modification operations (Fig. 3b). Finally, the "top" edge of the quadrilateral is defined by using an existing edge or it can be obtained by an edge recovery procedure (Fig. 3c).

(b) Quadrilateral element formation

After the four side edges are defined, a new quadrilateral will be formed by connecting these edges and eliminating/merging all the triangles inside (Fig. 3d).

2.5 Updating of merging front

After a new quadrilateral element is formed, the merging front will be updated accordingly. Deleted front segments will be removed from the merging front database and newly created front segments will be added. In addition, the structures of the background mesh adjacent to the new quadrilateral elements will also be updated.

2.6 Local post-processing procedures

Since during the quadrilateral element formation step, the structures of the background mesh adjacent to the base segment may be modified and highly distorted elements (with respect to the metric specification) may be formed. After a new quadrilateral element is formed, some local post-processing procedures will be employed to enhance the quality of the local mesh surrounding the element. These local enhancement procedures, including local structural enhancement, local node smoothing, loop splitting and edge seaming, will restore the quality and ensure the *validity* of the local mesh around the merging front so that subsequent merging operations can be performed smoothly.

2.7 Global post-processing

After all the elements in the background mesh have been converted to quadrilaterals, some traditional clean-up operations and smoothing procedures are then employed to further enhance the quality of the final quadrilateral mesh.

3 Geometrical model used and metric specifications

In this section, brief summaries of the geometrical model used and the metric specification are given in order to facilitate the detailed description of the mesh conversion scheme. Details of the geometrical model used and the metric specification approach can be found in references [32] and [28], respectively.

3.1 Definition of geometry of the mesh domain

As the metric triangulator developed in reference [32] is adopted for the generation of anisotropic background triangular meshes, the problem domain under consideration will again be represented as a union of bi-variate surface patches and the non-uniform rational B-spline (NURBS) surfaces will be used to define the support surface mapping.

$$(x, y, z)^T = \mathbf{r}(u, v) \quad (1)$$

In the geometrical model described in reference [32], trimming curves, repeated lines, crack lines and surface branching will all be defined as NURBS curves in the parametric spaces of the support surfaces. In addition, whenever a singular point (degenerated edge) is present, a secondary mapping will be automatically employed to transform the domain of the parametric space from a unit square to an isosceles triangle to avoid the difficulties of generating boundary segments along the degenerated edge. This geometrical model was found to be highly flexible and robust and model most commonly encountered geometrical features in engineering applications.

3.2 Metric Specifications, length and angle measures

In this study, the metric specifications approach will be used to control the element characteristics of the mesh. In the 3D space, the metric tensor is expressed as a symmetric positive definite matrix, $\mathbf{M}_{3D}$, such that $\text{Det}(\mathbf{M}_{3D}) > 0$ and the eigenpairs of $\mathbf{M}_{3D}$, $(\mathbf{g}_i, \lambda_i)$, $i=1,2,3$, will define the principal stretching directions and node spacing in the mesh. By using $\mathbf{M}_{3D}$, the length scale transformation between, $d\boldsymbol{\xi}$, the elementary vector in the normalize space and $d\mathbf{x}$, the corresponding vector in the 3D space is given by

$$d\boldsymbol{\xi}^T d\boldsymbol{\xi} = d\mathbf{x}^T \mathbf{M}_{3D} d\mathbf{x} \quad (2)$$

In the parametric space, by using Eqn. 1, the 2×2 surface metric tensor, $\mathbf{M}$, that combines the effects of the user specifications of node density, $\mathbf{M}_{3D}$, and the surface mapping can be defined as

$$\mathbf{M} = \begin{pmatrix} \frac{\partial \mathbf{r}}{\partial u} & \frac{\partial \mathbf{r}}{\partial v} \end{pmatrix}^T \mathbf{M}_{3D} \begin{pmatrix} \frac{\partial \mathbf{r}}{\partial u} & \frac{\partial \mathbf{r}}{\partial v} \end{pmatrix} \quad (3)$$

Let $\mathbf{p}_1(u_1, v_1)$ and $\mathbf{p}_2(u_2, v_2)$ be two points in the parametric space, then the distance between $\mathbf{p}_1$ and $\mathbf{p}_2$ with respect to $\mathbf{M}$, $l(\mathbf{M}, \mathbf{p}_1, \mathbf{p}_2)$, will equal to

$$l(\mathbf{M}, \mathbf{p}_1, \mathbf{p}_2) = \sqrt{\begin{pmatrix} u_2 - u_1 \\ v_2 - v_1 \end{pmatrix}^T \mathbf{M} \begin{pmatrix} u_2 - u_1 \\ v_2 - v_1 \end{pmatrix}} \quad (4)$$

The angle between two infinitesimal vectors $d\mathbf{u}$ and $d\mathbf{v}$ with respect to $\mathbf{M}$, $\theta(\mathbf{M}, d\mathbf{u}, d\mathbf{v})$, is defined as

$$\theta(\mathbf{M}, \mathbf{du}, \mathbf{dv}) = \begin{cases} \cos^{-1} \left(\frac{(\mathbf{du})^T \mathbf{M} \mathbf{dv}}{l(\mathbf{M}, \mathbf{du}) \cdot l(\mathbf{M}, \mathbf{dv})} \right) & \text{if } \mathbf{du} \times \mathbf{dv} > 0 \\ 2\pi - \cos^{-1} \left(\frac{(\mathbf{du})^T \mathbf{M} \mathbf{dv}}{l(\mathbf{M}, \mathbf{du}) \cdot l(\mathbf{M}, \mathbf{dv})} \right) & \text{otherwise} \end{cases} \quad (5)$$

It should be emphasized that since the proposed mesh conversion scheme is designed to generate full quadrilateral anisotropic meshes on 3D surfaces, *all* the length and angle measurements carried out during the mesh conversion process will be referred to the normalize space by using Eqns. (4) and (5) respectively. In addition, by using the metric specifications to describe the mapping (parametrization) of the surface, both the triangulator and the quadrilateral conversion scheme can be applied to a wide range of surfaces include those with rapid changing surface derivatives and singular points.

4 Quadrilateral conversion algorithm and implementation

In this section, the detailed descriptions of the quadrilateral conversion algorithm will be presented. The basic terminology and the elementary mesh modification procedures will be first introduced, as they will be frequently referred to during the detailed description of the conversion algorithm.

4.1. Basic terminology

4.1.1 Nodes, elements, edges and segments

Nodes and elements are the fundamental entities in mesh generation. An element is simply a list of nodes arranged in the anti-clockwise orientation. In general, a node in the mesh has two sets of coordinates: one for the parametric space and one for the 3D space. The 3D coordinates of the node are always computed from its parametric coordinates and the definition of the NURBS surface on which it is lying. An *edge* is simply a side of an element that connects the two end nodes and can be located at anywhere in the mesh. To distinguish from a general edge, a *segment* generally refers to edges that are lying on the *boundary* of a certain region. For example, a boundary segment of the mesh is an edge lying on the boundary of the mesh (Fig. 4a) while a front segment is an edge lying on the unconverted region of the mesh (Fig. 4b).

4.1.2 Merging front, converted and unconverted domain

Figure 4 shows the configuration of an intermediate mesh during mesh conversion. The whole mesh is divided into two distinct regions. The *unconverted region* is simply the collection of all unconverted triangles in the background mesh while the *converted regions* are covered by quadrilateral elements generated (or triangular elements used) during the conversion process. Triangles in the unconverted regions are referred to as *live triangles* since new quadrilateral elements will be generated from them in subsequent conversion steps. The triangles in the converted region are already used and cannot be reused in the future and therefore, they are referred as *dead triangles*. The *merging front* is simply the union of all the boundary segments of the unconverted regions. The merging front will divide the converted region from the unconverted regions and must form at least one closed loop. The edges on the merging front are known as *front segments*. *Internal edges* in the unconverted region are called *live edges* as during the quadrilateral formation steps they can be swapped, divided or even deleted. The edges in the converted region and the merging front are known as *dead edges* as they cannot be modified in subsequent merging operations (Fig. 4b).

4.2 Elementary mesh modification operations

Elementary mesh modification operations are simple operations that will be frequently used in the quadrilateral formation step, the local post-processing procedures and the global processing procedures. In general, they will enhance the quality of a given mesh (either the background triangular mesh or the local mixed mesh during the quadrilateral formation step, or the final quadrilateral mesh) by modifying the structure (connectivity) of the mesh. Four kinds of primary mesh modifications are used in the current implementation.

4.2.1 Edge swapping

The edge swapping operation replaces the edge shared by two elements with a new edge linking the opposite nodes as shown in Fig. 5a. For a valid swapping operation, the two newly formed angles should be less than 180° (with respect to the metric space *and* the parametric space by using Eqn. 5).

4.2.2 Edge collapsing (Element deletion)

In the edge collapsing operation, an edge shared by only two elements will be removed and the two nodes at the ends of the edge will be merged into one node. As a result, the two elements adjacent to the edge will also be deleted as shown in Fig. 6.

4.2.3 Edge division

In the edge division process, an edge shared by only two elements will be divided at the midpoint of the edge. As a result, the two adjacent elements will be divided into four elements as shown in Fig. 7.

4.2.4 Node deletion

The node deletion process is carried out when a node is surrounded by exactly three (Fig. 8a) or four triangular elements (Fig. 8b). As shown in Fig. 8a, if the node is surrounded by three elements, they will be combined into one element. For the four-element case, a new edge spanning the shorter diagonal will be created as shown in Fig. 8b.

4.3 Implementation of the quadrilateral conversion algorithm

4.3.1 Generation of background triangular mesh

Theoretically, any triangulation technique can be employed to construct the background triangular mesh. However, as the proposed scheme is designed to generate unstructured anisotropic quadrilateral meshes, it is assumed that the input background triangular mesh should be created to have proper anisotropic characteristics and already satisfy the user specifications. In this study, the general surface anisotropic mesh generator developed by Lee [32] will be used for the generation of the background mesh. The geometry of the support surfaces is defined by using the NURBS curves and the surface metric tensor $\mathbf{M}$ is defined at all the nodal points of the background triangular mesh.

4.3.2 Formation of initial merging front

As mentioned in Section 4.1.2, the merging front is simply the boundary of the unconverted region. Hence, at the beginning of the conversion process, the merging front is taken as the boundary of the background mesh. In addition to the definition of the merging front itself, four additional sets of data will be set up to improve the efficiency of the merging process by facilitating local searching operations. These data sets are

- (a) The lengths, l , of all front segments (Fig. 9).
- (b) Elements (and hence nodes and edges) that are connected to the front segments.
- (c) The internal (frontal) angle, θ , associated with all the nodes on the merging front (Fig. 9).
- (d) A linked list database that stores the connectivity information of the front segments.

4.3.3 Base segment selection

In order to form a quadrilateral element, it is first necessary to determine where the new element will be formed. Towards this end, at the beginning of each element formation (merging) step, a *base segment* must be selected from the merging front. Note that the base segment will determine both the *location* of the new element and the *advancing direction* of the merging front. Hence, the selection criterion for the base segment will dominate the propagation direction of the merging front. In fact, it is found that the selection criterion used can greatly affect the quality of the final mesh. Traditionally, the selection is based on some heuristic approaches such as the lengths of the front segments [18,19] or the levels of the edges [20] and no unique and theoretically neat criterion is available. In fact, after testing many commonly used criteria, it is found that no single predefined criterion will always be able to yield optimal results under all different element size (from uniform isotropic mesh to strongly graded anisotropic mesh) requirements. However, it is found that the following general rules will apply in most cases:

(1) *Strongly graded background mesh*

For both isotropic and anisotropic cases, if the background triangular mesh is strongly graded, priority should be given to the *shortest front segment*, to create smallest quadrilateral first, as this will help to avoid the chance of a big element crossing with small elements.

(2) *Nearly uniform background mesh*

In the case that the background mesh is composed of nearly uniform elements, the optimal choice of base segment is the one with the *smallest frontal angles* at its two end nodes. Selecting the base segment with the minimum frontal angle will enable the algorithm to remove concave regions of the merging front. In addition, it will also reduce the number of front segments, hence increasing the speed of the merging process.

(3) *Boundary offsetting*

In the initial stage of the merging process, it is sometimes preferred to first generate a number of *layers* of quadrilateral elements [11,20] along the boundary of the problem domain (Fig. 10) before the merging process is propagated into the interior of the problem domain. Note that while the boundary offsetting procedure will tend to enhance the shape quality of the boundary elements in the final mesh, it will normally impair the element grading quality of the mesh produced, especially when strongly graded and anisotropic elements are located near the domain boundary.

Based on the above observations, it is suggested that for the general anisotropic case, the base front segment should be selected based on *both* the length of the segment and values of the two frontal angles at its end points. In the current implementation, the following selection algorithm is found to be versatile and yields good results in most situations:

- (i) Locate the current front segments with the minimum *H-value* and denote the minimum value as $H_{\min}$. For a given front segment AB, the H-value of the segment, H_{AB} is defined as

$$H_{AB} = \frac{1}{2} \left(\frac{1}{\sqrt{\max(\lambda_1^A, \lambda_2^A)}} + \frac{1}{\sqrt{\max(\lambda_1^B, \lambda_2^B)}} \right) \quad (6)$$

where λ_i^A and λ_i^B $i=1,2$ are the eigenvalues of the metric tensors defined at node A ($\mathbf{M}_A$) and B ($\mathbf{M}_B$), respectively (Fig. 11).

- (ii) Check whether there is a front segment whose length is less than $cH_{\min}$ and with *both* frontal angles θ_A and θ_B (Fig. 11) less than a tolerance angle, θ_{rect} . If there is such a front segment, take this segment as the base segment. Otherwise, go to the next step.
- (iii) Check whether there is a frontal segment whose length is less than $cH_{\min}$ and with the left frontal angle θ_A is less than θ_{rect} . If there is such a front segment, take this segment as the base segment. Otherwise, repeat the checking using the right frontal angle, θ_B . Finally, if no segment can satisfy the angle requirement, the frontal segment with the smallest H-value will be used.

In steps (ii) and (iii), c is a constant scale factor between 1.5 and 2.0 and θ_{rect} is the critical angle for forming rectangular elements. In the current implementation, c is set equal to 2.0 and $\theta_{\text{rect}}=135^\circ$ is used (Fig. 12). The priority of selecting base segment for different cases of the above scheme are summarized in Table 1.

Note that the H-value defined in Eqn. 6 is developed for the use in general anisotropic case. To have a deeper understanding on its physical meanings, one can consider the isotropic case when $\lambda_1^A = \lambda_2^A$ and $\lambda_1^B = \lambda_2^B$, Eqn. 6 will reduce to

$$H_{AB} = \frac{1}{2} \left(\frac{1}{\sqrt{\lambda_1^A}} + \frac{1}{\sqrt{\lambda_1^B}} \right) = \frac{1}{2} (h_A + h_B) \quad (7)$$

Since by the definition of the metric tensor [28], $\lambda_1^A = 1/(h_A)^2$ and $\lambda_1^B = 1/(h_B)^2$ (h_A and h_B are

the required element sizes at nodes A and B respectively), in the isotropic case, the H-value is equivalent to the averaged target element size of the segment. Finally, if the background mesh is a uniform mesh with $h_A=h_B$, the H-value equal to the length of the segment and step (i) is simply to locate the shortest segment on the merging front.

The advantage of using the H-value over the shortest physical length for the selection of the base segment is demonstrated in Fig. 13 in which the intermediate mesh formed during the merging process of an anisotropic triangular mesh is shown. As shown Figure 13, if the physically shortest segment is chosen as the base segment for merging, the merging front would advance a direction perpendicular to the shortest segment (direction 1-1). As a result, the merging front will propagate like a narrow peninsula into the unconverted region and the geometry could be excessively complicated. However, if the edge of the smallest H-value is used, the two highlighted edges in Fig. 13 will then have similar H-values. Hence, both the 1-1 and the 2-2 directions will be equally likely to be the advancing direction and the chance of forming an excessively narrow converted region will be reduced.

Finally, it should be pointed out that it is difficult to recommend a single universal criterion that works for all cases. An extensive study to investigate the influence of the base selection criteria on the overall conversion behaviors had been carried out. It was found that the usefulness of the segment level criterion [20] is highly depended on the gradient of element size rather than the anisotropy of elements. When a considerable element size gradation exists, it is not a good idea to use the level as the base segment selection criterion for the *whole* meshing procedure. However, for a background mesh with boundary layers of graded elements, it may be beneficial to apply the level criterion until a number of boundary layers are formed. Thus, in order to provide additional flexibility of the merging algorithm, an option of boundary offsetting is implemented in the base segment selection scheme. The user can specify explicitly to first generate a number of boundary layers before the merging front propagates into the interior of the problem domain. If the boundary offsetting option is used, steps (i) to (iii) will be applied first to all the boundary segments before any interior edge is used to guarantee that offset layers of boundary quadrilaterals will be formed.

4.3.4 Formation of new quadrilateral

After the base segment is selected, a new quadrilateral element will be created by using the base segment as the bottom side edge of the new quadrilateral element. The new quadrilateral is then created by the following steps:

- (i) Prepare the left and right side edges of the new quadrilateral.

- (ii) Prepare the top edge connecting the two top nodes of the side edges.
- (iii) Merge all the triangular elements enclosed by the four side edges into a quadrilateral.

Owen et al. [20] proposed a systematic approach to prepare the four sides for a new quadrilateral. They defined the side (left and right) edges first and then recovery the top edge connecting the two top nodes of the side edges. In this study, the procedure for preparing side edges is similar to that of the Q-Morph algorithm except that the metric calculations are consistently used in all the length and angle computations and all the operations are carried out in the parametric space.

Definition of left and right side edges

After the base segment (bottom edge) is selected, there are in general two ways to define the side edges of the quadrilateral (Fig. 3). That is, by either using an existing edge as the side edge or to create a new edge by some elementary mesh modification operations. The procedure for defining the side edges is given below.

- (i) Consider the left base node (node A in Fig. 3) on the base segment AB.
- (ii) If the frontal angle at A, θ_A (Fig. 3a) is less than θ_{rect} (Fig. 12), the first adjacent edge (measured clockwise from AB) at A will be used as the left side edge and goto step (vi).
- (iii) Collect all adjacent edges at node A and calculate their angles with the base segment AB. For each angle θ_i , compute its deviation from the right angle (with respect to the metric specifications), $|\theta_i - 90^\circ|$. The edge which corresponding to a minimum deviation and satisfies the condition

$$|\theta_i - 90^\circ| \leq 30^\circ \quad (8)$$

will be selected and goto step (vi). If no such edge exists, go to step (iv).

- (iv) Check all the adjacent elements of node A for swapping (Section 4.2.1). If the element can be swapped, calculate the angle between the swapped edges and the base front. Find the *best swapping element* corresponding to the *best swapped edge* such that its angle with the base segment is closest to the right angle (with respect to the metric specifications).
- (v) If an existing adjacent edge forms an angle closer to the right angle than the best swapped edge, take that existing edge as the left side edge. Otherwise, perform the

swapping operation and take the best swapped edge as the left side edge.

- (vi) If the selected side edge is longer than $1.5l_{AB}$, where l_{AB} is the metric length of the base segment (Fig. 3b), or if the selected edge splits the merging front into two sub-fronts with odd numbers of segments, divide the selected side edge using the edge division operation described in Section 4.2.3.
- (vii) Repeat the above procedure for the node B to establish the right side edge.

Definition of top edge

After the left and right side edges are defined, the edge linking the two top nodes of the two side edges is the last edge to be prepared. In the case that the edge linking the two top nodes of the side edges already exists, it can be used as the top edge of the quadrilateral element. However, in some cases such an edge may not exist (e.g. the dotted line in Fig. 14a) and a recovery procedure, which consists of a series of edge swapping and node removal operations, will be carried out to construct the top edge. A summary of the top edge recovery procedure is given below:

- (i) The top node on the left side edge (node C in Fig. 14) will be selected as the starting node for the recovery procedure. Construct the vector $\overrightarrow{CD}$ that links the starting node with the end node D.
- (ii) Find the adjacent element of node C that contains the direction of vector $\overrightarrow{CD}$ and hence locate the corresponding edge that intersects with $\overrightarrow{CD}$ (edge EF in Fig. 14a).
- (iii) Check whether it is first required to carry out the node removal operation so that a valid swapping operation can be performed (Fig. 14c)
- (iv) Perform an edge swapping operation for the element (Fig. 14b).
- (v) Perform local smoothing (to be described in the following section) for the local mesh around node C.
- (vi) If the newly formed edge links node C with node D, it will be used as the top edge.

Otherwise, find the next edge intersects with $\overrightarrow{CD}$ and goto step (iii).

Numerical experiments done in this research indicate that the existence of the recover edge between the top nodes of two sides is *not* guaranteed, and this is an important difference of the current work from the edge recovery in the constrained Delaunay triangulation [28]. An typical example of such a case is that edge EF in Fig. 14 is a frontal edge that has already been used in previous quadrilateral formation. In such a case, it is impossible to recover edge between nodes C and D for the given configuration (since it is impossible to swap the edge

EF) and the edge recovery procedure could be failed. In most cases, such difficulty could be solved by using the node removal and the local smoothing operations. In the current implementation, the top edge recovery procedure is first carried out using the prepared side edges. Whenever it is detected that the edge recovery procedure is failed, the right side edge will be replaced by its LHS adjacent edge which sharing the same base node. The top edge recovery procedure will then be repeated again until the top edge is recovered. This procedure will ensure that the robustness of the recovery procedure at the cost of element quality. However, it is a reasonable choice because, compared with normal recovery, changing the right side edge is exceptionally rare. It should be pointed out that the node removal operation (step iii) is effective to remove the configurations that will not permit swapping to carry out. In addition, it also improves the effectiveness of the local smoothing operation (step v) by reducing the number of highly distorted elements formed during the swapping operation, especially when the background mesh is highly anisotropic.

Construction of quadrilateral element

After all the four side edges are defined, a new quadrilateral element is created by eliminating all the triangular elements bounded by the side edges. Note that in this case, two or more triangles may be merged to become a quadrilateral. However, it can be shown easily that the total number of front segments will always remain even.

4.3.5 Updating the merging front

After a new quadrilateral is created, the merging front will be updated in a usual manner. In addition, the status of edges and elements adjacent to the newly formed element and other data sets used (Section 4.3.2) will also be updated. An example is given in Fig. 15.

4.3.6 Local mesh post-processing

Since during quadrilateral formation step it is frequently required to carry out some elementary mesh modification operations, some highly distorted elements may be formed and the quality of the local mesh (in the unconverted region) around the newly formed quadrilateral element may be deteriorated. If the distorted elements are left untreated, the robustness and the stability of the subsequent quadrilateral formation steps may be affected. In order to improve the quality of the local mesh and the robustness of the conversion scheme, a series of local mesh post-processing schemes is implemented and applied whenever a new

quadrilateral is generated before the next base segment is selected. The mesh post-processing step is composed of the following four basic operations.

Local structural enhancement

In the local structural enhancement, edges and elements around the new quadrilateral will be examined. Node deletion and edge swapping procedures will be carried out to eliminate elements with very big or very small internal angles. The actual procedure will be conducted in the following steps.

- (i) Examine all neighboring nodes around the new quadrilateral, all the nodes that are connected with three or four adjacent triangular elements will be removed by the node deletion procedure described in Section 4.2.4 (Fig. 16c)
- (ii) Examine all neighboring edges around the new quadrilateral, swap the edge if the maximum internal angle of the resulting elements after swapping is smaller than the maximum internal angle of the elements before swapping (Fig. 16b). If any edge is swapped, goto step (i). Otherwise, the local structural enhancement process is considered to be finished.

Local mesh smoothing

The main function of the local mesh smoothing procedure is to reposition the nodes surrounding the new quadrilateral element so that the shape qualities of both the newly formed quadrilateral element and its neighbouring elements will be improved. Two smoothing algorithms, namely, the Laplacian smoothing scheme [35] and the edge length smoothing scheme [31] were tested. However, it was found that these two schemes, which are respectively designed for use in isotropic case and meshes with triangular elements only, can sometimes lead to the formation of invalid elements. Hence, a new area-weighted smoothing algorithm is implemented in this study. This smoothing algorithm will move nodes to the weighed-centroid of its surrounding elements. As shown in Fig. 17, $\mathbf{P}'$, the new position of the node $\mathbf{P}$ will be computed as

$$\mathbf{P}' = \sum_{i=1}^{N_p} \frac{A_i \mathbf{C}_i}{A_T}, \quad A_T = \sum_{i=1}^{N_p} A_i \quad (9)$$

In Eqn. 9, N_p is the number of elements adjacent to node $\mathbf{P}$. $\mathbf{C}_i$ is the centroid of the i th element, E_i , surrounding node $\mathbf{P}$ while A_i is the area of E_i . If E_i is a triangular element, A_i is given by

$$A_i = \frac{1}{2} l_i l_{i+1} \sin \theta_i \quad (10a)$$

Otherwise, if E_i is a quadrilateral

$$A_i = \frac{1}{2} (l_i l' \sin \alpha_i + l' l_{i+1} \sin \beta_i) \quad (10b)$$

In Eqn. 10b, l' is the length of the diagonal of the quadrilateral and α_i and β_i are the internal angles of the two triangular elements formed by dividing the quadrilateral along its diagonal (Fig. 17). Since the initial position of $\mathbf{P}$ is inside the polygon formed by its surrounding elements, the above scheme is applicable to both isotropic and anisotropic cases.

In addition to the position of point $\mathbf{P}$, the sequence to carry out node repositioning will also affect the quality of the local mesh. It is found that the best result can usually be obtained by first reposition nodes that are connected to or on the triangular elements and then followed by nodes that are connected to the newly formed quadrilateral element. Finally, nodes on the newly generated quadrilateral element will be smoothed.

Frontal segment seaming

Frontal segment seaming is employed to eliminate the formation of seriously distorted quadrilaterals when the frontal angle between two adjacent front segments is very small. The seaming operation will be performed if two adjacent front segments are forming a small angle less than or equal to $\theta_{\text{seam}}=30^\circ$ (Fig. 12). The edge seaming operation is carried out in two steps as shown in Fig. 18. In the first step, if there is no edge linking the two end nodes (nodes B and C) of the adjacent segments, an edge recovering procedure similar to the one used for top edge recovery in the quadrilateral formation step will be carried out to recover the edge (Fig. 18b). In the second step, the seaming operation is completed by using an edge collapsing operation (Fig. 18c). Note that in some cases, the lengths of two edges to be seamed could be very different. In these cases, the transition seam techniques proposed by Blacker and Stephenson [12] and Owen et al. [20] could be employed to obtain more optimal results.

Merging front splitting

Merging front splitting is another front modification operation employed to avoid the formation of seriously distorted quadrilaterals. As shown in Fig. 19, in some cases after a new quadrilateral is formed, the merging front may enclose a very narrow region. As a result, excessively slender elements may be formed in subsequent merging processes. The merging

front splitting operation is thus implemented to avoid the formation of slender elements under such a configuration and is summarized as follow.

(i) Checking of elements adjacent to the newly formed quadrilateral:

After a new quadrilateral element is formed, all its neighbouring triangular elements will be checked. All triangle elements that have all their nodes on the merging front, will be connected into a set denoted as T .

(ii) Detection of slender element:

Loop over all the elements inside T and search for any slender element that satisfy the following conditions:

- (a) The three nodes of the elements are not the end points of two adjacent front segments.
- (b) The area of the element, A_e , (in the metric space) is less than a certain tolerance value, $\varepsilon_1=0.1$.
- (c) The width to height ratio of the element, r_e , is larger than a certain critical ratio $\varepsilon_2=5.0$.
- (d) If there is more than one element in T that satisfy condition (a)-(c), collect them into a list V . If there is more than one element in V , arrange the elements in ascending order of A_e/r_e such that the element with the minimum value of A_e/r_e is the first element in V . The first element in V is selected as the slender element for deletion.

For example, in Fig. 19a, T will be the set $\{FAC, ABC, ADE, EBA\}$. However, only the subset $\{FAC, ABC, EBA\}$ satisfies condition (a) while $V=\{AEB, ABC\}$.

(iii) Division of merging front

- (a) Locate and check the two side-edges of the slender element and determine the edge that makes the numbers of segments on both resulting sub-fronts even.
- (b) If such an edge can be found, divide the merging front into two sub-fronts by collapsing that edge.
- (c) Otherwise, if there is more than one element in V , the next element with minimum A_e/r_e value is selected as the slender element and goto step (iii-a).

For example, in Fig. 19, $V=\{AEB, ABC\}$ and if AEB is first selected as the slender element, since both the edges AE and AB *cannot* be collapsed, ABC is then selected as the slender element and eventually edge AC is collapsed as shown in Fig. 19b.

4.3.7 Global Post Processing

By repeating the merging operation, the background triangular mesh will be converted into a full quadrilateral mesh. After the full quadrilateral mesh is formed, a set of standard mesh enhancement and mesh smoothing procedures [11,31,32] can then be applied to further enhance the quality of the output mesh.

5 Mesh generation examples

In this section, mesh generation examples will be presented to demonstrate the robustness and the performance of the proposed mesh generator. As in the previous study [32], the anisotropic characteristics of the meshes are specified by the 3D metric tensor $\mathbf{M}_{3D}$ (Eqn. 2) of the form

$$\mathbf{M}_{3D} = [\mathbf{g}_1 \quad \mathbf{g}_2 \quad \mathbf{g}_3] \begin{bmatrix} 1/(h_1)^2 & 0 & 0 \\ 0 & 1/(h_2)^2 & 0 \\ 0 & 0 & 1/(h_3)^2 \end{bmatrix} [\mathbf{g}_1 \quad \mathbf{g}_2 \quad \mathbf{g}_3]^T \quad (11)$$

in which h_i and $\mathbf{g}_i$ $i=1,2,3$ are the i th principal size and stretching directions respectively.

5.1 Example 1: Square plate with hole

In the first example, a simple 2D anisotropic mesh is used to demonstrate the overall process of the anisotropic SMT. The domain under consideration is a rectangular plate with internal hole. The metric tensor, $\mathbf{M}_1$, used is defined as

$$\mathbf{M}_1(x, y): \begin{cases} h_1 = 0.005 + 0.15r(x, y), h_2 = 15h_1 e^{-5.5r(x, y)} \\ \mathbf{g}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \mathbf{g}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \end{cases} \quad (12)$$

where r is the shortest distance between the point (x, y) and the line $x=y$. One can expect, as shown in Fig. 20, the mesh generated is highly stretched along the line $x=y$. Figure 20 also shows the propagation of the merging front, note that in this case only the H-value was used to control the selection of the base segment and no explicit instruction was used to force the generation of a boundary layer of quadrilaterals. However, one can see in Fig. 20 that a layer of elements was formed automatically along the external boundary of the square before the merging front propagated into the interior of the problem domain.

5.2 Example 2: Cylindrical plate

In the second examples, the problem domain under consideration is a cylindrical plate. The geometry of the plate and the definition of metric tensor used is given in Fig. 21. Note that in this example, the final metric tensor used can be regarded as the intersection of two metric tensors that lead to highly stretched elements in two perpendicular directions. In this example, a scale factor c was used to control the overall mesh density. The initial triangular and final quadrilateral meshes generated corresponding to $c=1.0$, 0.8 and 0.64 are shown in Fig. 22. Figure 23 shows the distributions of edge lengths (with respect to the metric specifications) for the final quadrilateral meshes generated. From Fig. 23, it can be seen that the edges have good length distributions with most of them have lengths close to the ideal value of 1.0 . For the qualities of the meshes produced, the minimum and maximum internal angles (again, with respect to the metric specifications) for the case of $c=0.80$ is plotted in Fig. 24. It can be seen that the internal angles are all within the range of $[30^\circ, 160^\circ]$ with majority of them within the optimal range of $[60^\circ, 120^\circ]$.

5.3 Example 3: Problem with internal openings

In the third example, a 3D surface with internal opening is used as an example to demonstrate the effect of boundary layer offsetting. In this example, the metric tensor was specified in such a way that small elements will be created along the internal openings of the surface. The background triangular mesh used is shown in Fig. 25a. In this example it is again found that by using the H -value to control the merging operation, a nearly complete layer of boundary elements are formed during the initial stage of the merging process (Fig. 25b) and this eventually improves the quality of the final mesh. The final quadrilateral mesh generated is shown in Fig. 25c.

5.4 Examples 4 and 5: Problems with more than one support surface

In Examples 4 and 5, the problem domains under consideration consist of more than one support surface and anisotropic meshes were generated by performing the merging procedure in a surface-by-surface manner. In Example 4, (Fig. 26) the problem domain is formed by the intersection of two planes while in Example 5 (Fig. 27), by the intersection of a vertical plane and a horizontal cylinder. From Figs. 26 and 27, it can be seen that the grading of the background triangular meshes were preserved and the number of nodes in the final quadrilateral meshes are similar to the number of nodes in the background meshes. Note that since the triangulator [32] used in this study generates the background triangular mesh in a

hierarchical (point-curve-surface) manner and the boundary segments of the background mesh will never be modified once they are created by the triangulator, the outputs (for both the triangulator and the quadrilateral mesh conversion schemes) will be independent of the sequence of connections of the sub-surfaces and their order of generations.

5.5 Example 6: Beam with openings

In the final example, a more complicated problem domain is considered. A beam with internal openings in its web is used to demonstrate the use of the present algorithm to problem domain with more complicated geometry. In this example, the problem domain consists of totally three support surfaces (top and bottom flange and the web). The initial mesh generated is shown in Fig. 28a. The metric tensor used is defined in such a way that small and highly stretched elements are generated along the flange-web intersections and the internal openings of the web. The final mesh generated is shown in Fig. 28b. From these figures it can be seen that the mesh grading is largely preserved.

6 Conclusions

A new indirect technique has been developed to convert anisotropic triangular meshes to full quadrilateral meshes. The proposed scheme can be used for the generation of both isotropic and anisotropic quadrilateral meshes. The proposed scheme is developed based on a systemic merging technique similar to the Q-MORPH algorithm and the advancing front technique that are originally designed for the isotropic applications. A new base front segment selection scheme for anisotropic case is suggested and tested in this study. By using a series of carefully designed local structural modification operations and a new quadrilateral element formation scheme, the mesh conversion procedure developed is robust for the formation of strongly anisotropic quadrilateral meshes. As the mesh conversion scheme use an indirect method and a well-graded background triangular mesh is used as the skeleton for the formation of the quadrilateral meshes, the anisotropic characteristics of the original meshes will be well preserved in the final meshes. Numerical examples given indicated that in the final quadrilateral meshes generated, most of the edges generated have normalized edge lengths closed to the perfect value of unity and with internal angles within the optimal range. Regarding the speed of the conversion scheme, as more complicated calculation steps are required for the implementation of the general anisotropic case using the metric specifications, the speed of the present conversion scheme is slower than the corresponding conversion scheme used in isotropic case. Detailed timings shown that the operational

complexity of the conversion scheme is of order $O(NQ^{1.2})$ where NQ is the number of quadrilaterals generated. However, the overall speed of the present scheme is fast enough for most practical applications. By using a low-ended PC equipped with a Pentium III 500MHz CPU, the present conversion scheme can generate a highly graded anisotropic quadrilateral mesh with 10,000 elements within 20 seconds.

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Figure 1. Basic mesh conversion principle.

Figure 2. Generation for full quadrilateral mesh from background triangular mesh.

Figure 3. Formation of quadrilateral element: (a) base segment as bottom side edge; (b) preparation of left and right side edges; (c) recovery of top side edge; (d) merging for quadrilateral formation.

Figure 4. Definition of basic entities: (a) edges, segments, converted and unconverted regions; (b) merging front and classification of edges and elements.

Figure 5. Swapping of two triangles: (a) the swapping operation; (b) conditions for valid operation.

Figure 6. Edge collapsing (element deletion) operation.

Figure 7. Edge division operation.

Figure 8. Node deletion: (a) three-element case; (b) four-element case.

Figure 9. Information for the i th front segment and the j th front node.

Figure 10, Generation of boundary layer during merging: (a) initial merging front; (b) merging front after the generation of boundary layer; (c) merging front propagated to the interior region.

Figure 11. H-value and frontal angles for segment AB.

$\theta_{seam}=45^\circ$, Critical angle for seaming operation

$\theta_{rect}=135^\circ$, Critical angle for forming rectangular element

$\theta_{revr}=225^\circ$, Critical angle for checking reversal of front segment

Figure 12. Classification of frontal angle.

two segments with similar H-value

Figure 13. Merging of an anisotropic mesh.

Figure 14. Top edge recovery procedure.

Figure 15. Updating of merging front.

Figure 16. Local structural enhancement: (a) original local mesh; (b) swapping of edge; (c) node deletion.

Figure 17. Area-weighted smoothing for point P .

Figure 18. Frontal segment seaming: (a) adjacent segment with small frontal angle; (b) edge recovery operation (in this case simple swapping); (c) edge collapsing operation.

Figure 19. Merging front division: (a) detection of slender element; (b) division of merging front.

Figure 20. Example 1: Square plate with hole (NQ= Number of quadrilaterals generated).

Figure 21. Example 2: Cylindrical plate and metric tensor definition.

$c=1.00$

$c=0.80$

$c=0.64$

NQ=274

NQ=387

NQ=602

Figure 22. Example 2: Final quadrilateral meshes generated with different value of c .

Figure 23. Edge length distribution for meshes generated in Example 2.

Figure 24. Minimum and maximum angle distributions for mesh generated in Example 2 ($c=0.80$).

(a)

(b)

(c)

Figure 25. Example 3: Surface with internal openings: (a) background triangular mesh (NN=2630, NT=4570); (b) intermediate mesh after boundary offsetting; (c) Final quadrilateral mesh (NN=3350, NQ=3005). (Note: NN=Number of nodes, NT=Number of triangular elements, NQ=Number of quadrilateral elements)

(a)

(b)

Figure 26. Example 4: Intersection of two plates: (a) background triangular mesh (NN=593, NT=1096); (b) Final quadrilateral mesh (NN=525, NQ=460).

(a)

(b)

Figure 27. Example 5: Intersection of plate and cylinder: (a) background triangular mesh (NN=908, NT=1800); (b) Final quadrilateral mesh (NN=905, NQ=804).

(a)

(b)

Figure 28 Example 6: Beam with openings: (a) background triangular mesh (NN=3536, NT=6962);
(b) Final quadrilateral mesh (NN=4585, NQ=4082).

Priority	Configuration of segment
1 both $\theta_A, \theta_B < \theta_{rect}$
2 $\theta_A < \theta_{rect}$
3 $\theta_B < \theta_{rect}$
4 front segment with smallest H- value

Table 1. Priority of selection of base segment.