Optimizing a recover-and-assemble remanufacturing system with production smoothing

Zahraei, Seyed Mehdi; Teo, Chee-Chong

2018

Zahraei, S. M., & Teo, C.-C. (2018). Optimizing a recover-and-assemble remanufacturing system with production smoothing. *International Journal of Production Economics*, 197, 330-341. doi:10.1016/j.ijpe.2018.01.016

<https://hdl.handle.net/10356/103426>

<https://doi.org/10.1016/j.ijpe.2018.01.016>

© 2018 Elsevier B.V. All rights reserved. This paper was published in *International Journal of Production Economics* and is made available with permission of Elsevier B.V.

Downloaded on 26 Apr 2023 18:20:21 SGT

OPTIMIZING A RECOVER-AND- ASSEMBLE REMANUFACTURING SYSTEM WITH PRODUCTION SMOOTHING

Seyed Mehdi Zahraei ^{a,*}

Chee-Chong Teo ^b

^a School of Business, Singapore University of Social Sciences, 463 Clementi Road, 599494, Singapore

^b School of Civil & Environmental Engineering, Nanyang Technological University, Nanyang Avenue, 639798, Singapore

* Corresponding author.

E-mail addresses: zahraei@suss.edu.sg (S.M. Zahraei), teocc@ntu.edu.sg (C.-C. Teo).

Abstract

We consider a recover-and-assemble remanufacturing system in which an upstream stage recovers components from used products before a downstream stage performs assembly using the recovered components. In such systems, the inventory variances of the recovered components depend significantly on the production variabilities of the recovery and assembly processes. The production variabilities, in turn, are functions of the uncertain arrivals of used products (at the recovery) and demand for remanufactured products (at the assembly). We study how best to employ production smoothing to mitigate the uncertainties. We first build a network model that incorporates the inter-stage material and demand flows. We then derive the safety stock formulae as functions of the smoothing behavior and uncertainties. We embed the variables characterized by the model into an optimization procedure that optimizes the smoothing and the safety stocks. Through numerical examples, we demonstrate the application of the model, the value of production smoothing in this context and related managerial insights.

Keywords: remanufacturing, recover-and-assemble, production smoothing, safety stocks.

1. Introduction

There has been heightened interest in product recovery in recent years. One of the most widely practiced forms of product recovery is remanufacturing, which is commonly referred to as the process of restoring used, end-of-life or defective products to like-new condition. Hagerty and Glader (2011) report that the U.S. market for remanufactured goods is estimated to be worth more than \$100 billion per year. A recent survey by Parker et al. (2015) shows that the remanufacturing activities within the European Union generate about €30 billion in turnover. The sector is widely expected to grow rapidly due to its economic benefits as well as increased regulatory pressure to enhance environmental performance. The operational challenges unique to remanufacturing have been well documented. The challenges are mainly driven by uncertainties in timing, quantity and quality of returned products, which subsequently leads to difficulty in balancing the returned items with the stochastic demands (see e.g., Fleischmann et al. 1997 and Guide 2000 for a discussion).

In this paper, we consider a *recover-and-assemble* remanufacturing system. In this configuration, the upstream stage recovers components from returned products (widely known as *cores*), which typically involves disassembling, cleaning, inspection and restoration. Subsequently, the downstream stage

assembles the product using the recovered components, usually together with other components that could be externally sourced or in-house built. In the recover-and-assemble system, the inventories of recovered components are replenished by the recovery process and are consumed by the assembly. Hence the inventory variability (and thus the safety stocks) depends considerably on the production variabilities at the recovery and assembly processes, which in turn are respective functions of the stochastic arrivals of returned products and demand for assembled products. The intent of this research is to study how best to manage the uncertainties in the recover-and-assemble systems by the use of production smoothing. Production smoothing is the leveling of production output across time, usually to ease capacity loading when faced with variable demand. In this paper, we first develop a model that optimizes the amount of smoothing at the recovery and assembly processes. Additionally, we characterize the optimal safety stocks that correspond to the smoothing tactic, for both the recovered components and the finished remanufactured products.

The recover-and-assemble configuration is extensively employed in a large variety of industries. One example is the remanufacturing of consumer electronics. The worn-out components are replaced, whereas durable components are recovered and reused in the assembly of the remanufactured products. Electronics remanufacturing has been in the limelight lately due to the rapid growth of e-waste (electronic waste). In the U.S. alone, as much as 4.5 million tons of old consumer electronic goods have been disposed (Minter, 2013). It is projected to increase due to shortening of life-cycles of consumer electronics. In order to reduce the e-waste and to recover material cost, many manufacturers have begun to collect used products from consumers and explore the value-added product recovery. Sony has created the Green Fill Program that provides its retailers collection kiosks for used electronics. LG Electronics establishes the LG Electronics Recycling Program that offers drop-off sites and free shipping for returning end-of-life products. Product take-back legislation has also been prevalent in recent years. For example, the European Commission had passed the Waste Electrical and Electronic Equipment (WEEE) Directive such that European Union members are required to establish e-waste collection schemes. In the U.S., more than 25 states have enacted product take-back legislations (Electronics Take-back Coalition, 2011), mandating state-wide e-waste collection and recycling.

The remanufacturing of automotive parts, which has been in existence for decades, is also mostly based on the recover-and-assemble operation. The useful parts (such as engine housings) are recovered, and the worn-out or defective parts are either restored or disposed. After the usual processes of cleaning and inspection, the recovered parts are then assembled with other parts to form

the completed product. The APRA (Automotive Parts Remanufacturing Association) approximates that there are more than 15,000 companies in the U.S. alone that remanufacture motor vehicle parts and the industry is estimated to hire more than 350,000 employees (U.S. International Trade Commission, 2012). Besides the automotive industry, the U.S. International Trade Commission (2012) reports several other examples from diverse industries that operate with the recover-and-assemble configuration. Examples are aerospace parts, medical devices, office furniture and restaurant equipment. Clearly, the recover-and-assemble operation is prevalent in practice and the aforementioned examples are certainly far from exhaustive. The importance of remanufacturing has also given rise to the concept of *Design for Remanufacturing (DfRem)* in which the products are designed to facilitate the remanufacturing process, particularly in the form of the modular product architecture where modules can be easily disassembled, restored and re-assembled (see Yang et al. 2014). The *DfRem* trend will lead to more widespread adoption of the recover-and-assemble systems.

Despite the prevalence of the recover-and-assemble configuration, there have been limited studies on such configuration as compared to other remanufacturing settings. In most literature, the remanufacturing activities are modeled aggregately as a single step, without considering the individual processes and their interplay. (See Akcah and Cetinkaya 2011 for a review of models categorized according to network configurations.) Readers interested in the remanufacturing literature can refer to an abundance of review papers such as Junior and Filho (2012) and Steeneck and Sarin (2013) (on production planning and control), Morgan and Gagnon (2013) (on scheduling of remanufacturing operations), Pourghadim et al. (2013) (on inventory management), Schenkel et al. (2015) (on value creation through the recovery of returned products), and Lin et al. (2015) (on plant selection in multi-plant remanufacturing environment). Like our paper, there are others that study the recovery of components. Ferrer and Whybark (2001) characterize the production planning for the fundamental problem of uncertain rate of returns of used components that is subject to stochastic demand for remanufactured products. Under similar problem setting, Depuy et al. (2007) include the considerations of the quality variation of the returned components and uncertainty of the processing times. DeCroix and Zipkin (2005) consider a single-product, multi-component assembly system where some of the components in the returned products can be recovered and returned to inventory; they establish conditions under which the component recovery is equivalent to a serial system and develop heuristics that can be employed when these conditions are not satisfied. The main contrast of our paper to the aforementioned works is that our model explicitly incorporates the downstream assembly process and inventories, as our focus is to capture the system-wide interactions and trade-offs in the

recover-and-assemble setting. To our knowledge, there are very few papers in the research stream on component recovery that characterize the individual operations of recovery and assembly. Bayindir et al. (2003) model a multi-stage system with both recovery and assembly stages where parts required for assembly can either be externally supplied or restored from returned items. However, their objective considerably differs from ours as their aim is to find the optimal “return ratio”, which is the probability that a sold item is returned for recovery after their useful life. In addition, their model is based on the $M/G/\infty$ queuing model and thus, the underlying modeling approach and assumptions are substantially different. Han et al. (2013) also include the recovery and assembling processes in their model. They consider a deterministic, finite rolling horizon problem for which they develop a linear optimization program. In contrast, our model focuses on tactical decisions of production smoothing in a stochastic environment and how safety stocks should be planned to correspond to the smoothing tactics. In addition, both Bayindir et al. and Han et al. do not characterize the recovered component inventory and its safety stock. More recently, similar to our focus on reducing variability in a remanufacturing network, Li et al. (2017) studies a three-echelon hybrid manufacturing and remanufacturing system. But their paper differs in that they consider variability from a supply chain viewpoint (retailer-manufacturer-supplier) while ours considers the production planning aspects in the recover-and-assemble context.

We believe the main reason behind the lack of works on recover-and-assemble is due to the tractability problems in modeling. The main difficulty lies in the characterization of the inventory random variable for the recovered components, which is essential to characterize the optimal safety stock levels. In modeling such systems, the inventory variable should incorporate the influence of production planning from both recovery and assembly processes that are subject to the uncertainties of arrival of returned products and demand respectively. Besides, in order to be realistic with real-life systems, the model should capture the multi-component setting in which usually a subset are recovered components while others are externally sourced and/or internally built. This is important as the number of components and how many of which are recovered components can considerably affect how smoothing should be performed (as will be shown). The linear-system modeling framework employed in our model enables the aforementioned features to be incorporated. The framework is based on the TPM (Tactical Planning Model) for a job shop by Graves et al. (1986). Other TPM-based linear-system models include Graves (1988), Graves and Fine (1989), Graves et al. (1998), Teo et al. (2011, 2012) and Zahraei and Teo (2017). The core of these models is the linear production function that depicts the first-order smoothing action. Balakrishnan et al. (2004) and Boute and Van Mieghem

(2015) use the same linear function for modeling order smoothing. In particular, Boute and Van Mieghem (2015) show that the function “closely tracks” the optimal (but intractable) dual base-stock policy. In this paper, the model’s linearity offers considerable tractability in modeling the recover-and-assemble network and its multiple components.

The contributions of this paper are as follows. To our knowledge, this is the first paper that studies the use of the commonly employed tactic of production smoothing to mitigate the uncertainties in the remanufacturing environment. As will be demonstrated, the optimal extent of smoothing carried out in the recover-and-assemble system depends significantly on inter-stage interactions and trade-offs between production capacities versus inventories. Therefore, rather than considering a single-stage or a sub-network in isolation, we model the processing stages and inventories as a network that capture the essential inter-stage interactions and cost trade-offs, which is a key methodological contribution of this paper. Another significant contribution is that we derive the safety stock formulae as a function of the smoothing parameters and uncertainties. In addition, we show how the variables characterized in the model can be embedded into an optimization procedure to obtain optimal smoothing and safety stock levels. Through numerical experiments, we illustrate the application of the model and demonstrate the value of smoothing. In addition, we seek to answer the following questions. What are the conditions under which we should perform smoothing of the recovery rather than to process all available returned items? What are the considerations in a multi-component setting when choosing the optimal extent of smoothing? In achieving global optimality, what are the local sub-optimality? In reducing system variability, how does production smoothing compare with the approach of smoothing the arrivals of returned items?

Another contribution of our model is the characterization of correlation between demand and arrivals of returned items. In the literature, it is usually assumed that demand and arrivals are independent for tractability. In many remanufacturing systems, there often exist such correlations. For example, some short life-cycle products (e.g., single-use camera and reusable containers, see e.g., Kelle and Silver 1989 and Toktay et al. 2000) may have considerable degree of correlation while repairable items may exhibit perfect positive correlation. Two of the exceptions are van der Laan and Salomon (1997) and Mitra (2012), which allow demand and arrivals of returned items to be correlated. Mitra (2012) studies the optimal inventory level for an inventory of item that are either remanufactured or procured from supplier; van der Laan and Saloman (1997) consider a pull-and-push system for an inventory with production, remanufacturing and disposal operations. Our work considerably differs from theirs in terms of intention, system configuration and decision variables.

The rest of the paper is organized as follows. In the next section, we describe the model development. In §3, we report the numerical experiments that were performed to demonstrate the model's application, the value of the production smoothing and the managerial insights. We provide concluding remarks and a discussion on the limitations of the work in §4.

2. Model

We begin by providing a list of notations in Table 1.

Table 1. List of notations

i :	index for stage
L_i :	risk period for inventory of component i
$\hat{L}$:	maximum value of L_i 's among components
φ_{0i} :	number of units of component i needed for every unit of product processed at ASSEMBLY
φ_{1i} :	number of units of components restored at RECOVERY for every unit of returned item
n_i :	control parameter (planned lead-time) at ASSEMBLY ($i = 0$) and RECOVERY ($i = 1$)
m_i :	nominal production capacity at ASSEMBLY ($i = 0$) and RECOVERY ($i = 1$) per period
c_{Pi} :	expediting cost per unit of capacity shortages at ASSEMBLY ($i = 0$) and RECOVERY ($i = 1$)
c_{ii} :	expediting cost per unit of inventory shortages at stage i
h_i :	unit holding cost at stage i per period
α_i :	safety factor at stage i
N_{0t} :	demand at processed inventory of ASSEMBLY at start of period t
N_{1t} :	number of returned items at RECOVERY at start of period t
μ_i :	expected N_{it} at ASSEMBLY ($i = 0$) and RECOVERY ($i = 1$)
σ_i :	standard deviation of N_{it} at ASSEMBLY ($i = 0$) and RECOVERY ($i = 1$)
P_{it} :	production requirements at ASSEMBLY ($i = 0$) and RECOVERY ($i = 1$) and order quantity for component i ($i = 2, \dots, q$) in period t
Q_{0t} :	inventory shortfall at ASSEMBLY at start of period t after demand observation
Q_{1t} :	inventory level of returned items at RECOVERY at start of period t
D_{it}^p :	demand (net demand) over risk period for orders placed at period t for single-source (dual-source) component i
B_i :	base stock level at stage i
EL_i :	expected inventory level at stage i

$q+1$:	number of stages in the network
$\mathbf{D}$:	diagonal matrix with elements $1/n_i$ ($i = 0, 1, \dots, q$)
Φ :	square matrix with φ_{ij} as its elements
μ :	column vector with μ_i ($i = 0$ for ASSEMBLY and $i = 1$ for RECOVERY) as only non-zero elements
Σ :	covariance matrix consisting of covariance of N_{it}
$\mathbf{P}$:	column vector with P_{it} as its elements ($i = 0, 1, \dots, q$)
$\mathbf{A}$:	column vector with A_{it} as its elements ($i = 0, 1, \dots, q$)
$\mathbf{N}_t$:	column vector that comprises N_{0t} and N_{1t} as the only non-zero elements
$\mathbf{I}$:	identity matrix

PROBLEM SETTING AND ASSUMPTIONS

We consider a remanufacturing system that consists of two processing stages, namely *RECOVERY* and *ASSEMBLY*. The returned products, which we referred to as *returns* hereafter, arrive at RECOVERY to wait for reprocessing. We assume no yield loss of the returns or equivalently, the returns that arrived have been inspected and those that do not comply with required quality standard are disposed. RECOVERY performs tasks that may include disassembling, cleaning and restoring the disassembled parts. Upon completion, the restored parts are then stocked as components, waiting to be assembled. ASSEMBLY processes the components into finished products, which are then stored as *processed inventory* to serve external customer demand. We define the processed inventory to be part of ASSEMBLY.

There are two types of components, namely *single-source components* and *dual-source components*. Single-source components are replenished solely by suppliers (either internal or external suppliers). Dual-source components are replenished by RECOVERY and the rest ordered from suppliers (either internal or external suppliers). In particular, we assume the quantity of dual-source components recovered is smaller than that required by ASSEMBLY in serving the demand in every period and the excess is ordered from suppliers. This assumption contributes to considerable tractability in characterizing the inventory variable. We expect it to be valid if the demand rate is much higher than that of the arrival rate of the returns, which is commonly observed in practice and frequently assumed in the literature (see e.g., Fleishmann et al. 1997 for a discussion). We also note that if the probability for returns quantity to exceed demand in each period is low, the modeling complexity resulted from relaxing this assumption would have yielded limited benefit.

Thus the system consists of three types of inventories - returns, component inventories (single- and dual-source) and processed inventories. Figure 1 shows an *example* of the network with one single-source and one dual-source component. (Note that the model allows for multiple single- and dual-source components.)

Figure 1. An example of the recover-and-assemble production network.

We consider the penalty cost for service failures of stock-outs and capacity shortages. In the literature, there are two main streams of models categorized according to how a system addresses service failures – guaranteed-service models and stochastic-service models. Refer to Graves and Willems (2003) and Klosterhalfen et al. (2013) for reviews and comparison of both streams of models. Our model is inclined towards the guaranteed-service models, as we assume services are *guaranteed* by taking expediting actions (incurring extra costs) in times of capacity shortage and stock-outs. Specifically, we assume a nominal capacity level at RECOVERY and ASSEMBLY. When production requirements (defined as production quantities scheduled to be completed) exceed the nominal capacity, expediting actions (e.g., overtime or use of temporary labor) are carried out at an additional cost that is guaranteed to satisfy the excess requirements. Furthermore, in times of stock-outs, expediting actions are performed that is always able to meet the excess demand. For example, the stock-out quantity can be expedited from an external supplier by faster transport mode at a premium or from another more responsive but costlier supplier. The guaranteed-service models typically do not exactly address how the excess demand can be met and assume that other responses are always available to achieve the guaranteed service. In the same spirit, we do not attempt to specify exactly how the capacitated expediting should respond to stock-outs in order to achieve the guaranteed service. The guaranteed-service assumption enables considerable tractability and allows the characterization of smoothing, network flows and the related trade-offs, which are the focus of this

research. (Note that since the processed inventory serves external demand, its stock-out penalty can be in the form of backordering instead of expediting).

For the component inventories, we assume they are replenished according to the standard base-stock policy. Each inventory faces a deterministic lead-time and a trade-off between inventory holding and stock-out penalty (i.e., the expediting cost), which justifies the base-stock policy. Note that the processed inventory is not managed by the base-stock policy but is replenished by the ASSEMBLY stage whose output can be smoothed (which we will discuss later).

The demand at the processed inventory as well as the returns arrivals at RECOVERY are exogenous and are each assumed to be i.i.d. (independent and identically distributed) over disjoint time periods. However the demand and returns arrivals are allowed to be correlated.

COST FUNCTION

We index the stages by $i = 0, 1, 2, \dots, q$: ASSEMBLY by $i = 0$, RECOVERY by $i = 1$ and the components by $i = 2, 3, \dots, q$ (i.e., a total of $q - 1$ types of components). In the following, we present other notations and discuss the cost incurred by each type of stages. Note that in some instances, the same notations are defined differently for the different stage types. As will be seen, this allows the use of the equations of similar forms for the different stage types, which facilitates explanations and comparisons (as will be seen).

ASSEMBLY. We let m_0 denotes the finite nominal capacity at ASSEMBLY (and m_1 for that at RECOVERY). If its production requirements P_{0t} in period t exceed its nominal capacity m_0 , expediting actions are carried out at a cost of c_{p0} for each unit of $P_{0t} - m_0$. We let B_0 denotes the initial stock level at the start of the time horizon, i.e., at $t = 0$. In addition, Q_{0t} denotes the shortfall of processed inventory in t , which we define as the difference between the actual net inventory level and B_0 after the demand arrival at the start of the period. Therefore $Q_{0t} > B_0$ implies occurrence of stock-out and accordingly, $Q_{0t} - B_0$ is the stock-out quantity for which expediting cost of c_{p0} is incurred for every unit of stock-out.

RECOVERY. Similar to ASSEMBLY, expediting actions are carried out for the excess requirements when its production requirements P_{1t} exceed m_1 . Likewise, the expedited production incurs a cost of c_{p1} for every unit of shortage. Furthermore, we let Q_{1t} denotes the number of returned items waiting to be processed at RECOVERY after the returns arrive at the start of period t .

Components. For single-source components, we consider the *demand over risk period* denoted by D_{it}^p for orders placed in period t for inventory of component i . As widely defined, the *risk period* represents the total duration of demand realization that the base stock should be designed to meet. We assume a review interval of one period and thus the risk period includes the one-period review in addition to the supplier's lead-time. For dual-source components, D_{it}^p represents the *net* demand over risk period, which we define as the demand over risk period for stage i *minus* the replenishments received from RECOVERY over the risk period. With the base stock denoted by B_i , $D_{it}^p > B_i$ implies stock-out and as such $D_{it}^p - B_i$ is the quantity that needs to be expedited. Each unit of inventory expedited incurs a cost of c_{li} .

All stages. We let EI_i denotes the expected inventory level at stage i that incurs a unit holding cost of h_i per period. (Note that EI_0 and EI_1 denote the expected processed inventory at ASSEMBLY and the expected inventory of returns at RECOVERY respectively.)

By letting $x^+ = \max(x, 0)$, we express the expected total cost per period as follows:

$$\sum_{i \in \{0,1\}} c_{pi} E[P_{it} - m_i]^+ + c_{i0} E[Q_{0t} - B_0]^+ + \sum_{i \in \{2,3,\dots,g\}} c_{li} E[D_{it}^p - B_i]^+ + \sum_{all\ i} h_i EI_i. \quad (1)$$

The expected total cost captures the trade-offs between inventory holding cost, capacity shortage cost and stock-out penalties. To compute the expected total cost, we need to characterize Q_{it} , D_{it}^p , P_{it} and EI_i . In particular, we characterize the first two moments of Q_{it} , D_{it}^p and P_{it} (which determine B_i and EI_i). Note that in the model, there is no distributional assumption needed for the demand and returns arrivals when characterizing the first two moments. Due to the model's linearity, if the external demands and the return arrivals are assumed to be normally distributed, Q_{it} , D_{it}^p and P_{it} are also normally distributed. We can then compute the expected capacity and inventory shortages by the normal linear loss function. Thus the model's aim is to characterize these variables to compute the expected total cost which is also the objective function of the optimization program.

MODEL DEVELOPMENT

In this section, we characterize the variables P_{it} , Q_{it} , D_{it}^p , B_i and EI_i . We first consider the functions for production requirements and replenishments quantities. The derived linear functions allow us to subsequently combine all the stages into a single linear model that characterizes the variables.

ASSEMBLY

At ASSEMBLY, demand arrives at its processed inventory at the start of each period. The stage reviews its inventory shortfall at the start of every period after the demand arrival and then sets its production requirements. The production is controlled by a linear function given by:

$$P_{0t} = (1/n_0)Q_{0t}, \quad (2)$$

where P_{0t} is the production requirements at the ASSEMBLY in period t , Q_{0t} is the shortfall of processed inventory at ASSEMBLY at the start of period t , and n_0 ($n_0 \geq 1$) is a control parameter. Equation (2) is the same as the linear control rule in Graves (1986), Graves et al. (1986), Graves (1988), Fine and Graves (1988), Teo et al. (2011, 2012), and Zahraei and Teo (2017). Balakrishnan et al. (2004) refer to (2) as the *Equal-Weight Moving Average* policy; they prove that among all *Moving Weighted Average* policies, (2) maximizes order smoothing. Boute and Van Mieghem (2015) find that the scaled total cost resulted from (2) “closely tracks” that of the optimal dual base-stock policy; in addition, the dual base-stock policy is not analytically tractable and has to be solved numerically even for a simple production stage that faces i.i.d. demand.

We show that n_0 is a control parameter for first-order smoothing. We first consider the standard balance equation for the ASSEMBLY, which can be expressed as:

$$Q_{0t} = Q_{0,t-1} - P_{0,t-1} + N_{0t}, \quad (3)$$

where N_{0t} denotes the demand at the processed inventory that arrives at the start of t . We show that $1/n_0$ is a smoothing parameter by substituting (2) into (3) to obtain the first-order smoothing equation; that is:

$$P_{0t} = (1 - 1/n_0)P_{0,t-1} + (1/n_0)N_{0t}. \quad (4)$$

Note that (4) entails that under smoothing, P_{0t} is correlated over time.

RECOVERY

Equations (2) to (4) also apply to RECOVERY but with different interpretations. That is, RECOVERY also sets its production requirements according to (2) but unlike ASSEMBLY (and as defined earlier), Q_{1t} denotes the inventory of returns at the start of t . Thus RECOVERY reviews Q_{1t} and then sets its output according to (2). Similarly, n_1 is also regarded as a control parameter and $(1/n_1)$ as a first-order smoothing parameter. Furthermore, N_{1t} denotes the quantity of returns that arrive at the start of period t . We re-express the balance equation (3) and the first-order smoothing equation (4) for the RECOVERY in the following:

$$Q_{1t} = Q_{1,t-1} - P_{1,t-1} + N_{1t}, \quad (3')$$

and

$$P_{1t} = (1 - 1/n_1)P_{1,t-1} + (1/n_1)N_{1t}. \quad (4')$$

Single- and Dual-Source Components

The net demand at component i that occurs at the start of period t is given by:

$$\varphi_{i0}P_{0t} - \varphi_{i1}P_{1t} \quad \text{for } i = 2, 3, \dots, q. \quad (5)$$

The first term models the demand requirements from ASSEMBLY where φ_{i0} indicates the number of units of component i needed for every unit of product processed at ASSEMBLY. The second term is applicable only to the dual-source components; it depicts the replenishments to be received (i.e., *negative* demand arrival) from RECOVERY in period t where φ_{i1} is the number of components that are recovered for every unit of returned item. Clearly, $\varphi_{i1} = 0$ for single-source components.

As with a typical standard base-stock policy, the components are ordered at the start of every period by a quantity equals to the demand received in the same period. By defining P_{it} as the order placed for component i at the start of period t , and Q_{it} be the inventory shortfall at the start of t , this implies that:

$$P_{it} = Q_{it} \quad \text{for } i = 2, 3, \dots. \quad (6)$$

Note that (6) is of the same form as (2) with n_i equals 1. Given that (5) is the net demand arrival, the balance equation for the component's inventory shortfall Q_{it} is given by:

$$Q_{it} = Q_{i,t-1} - P_{i,t-1} + \varphi_{i0}P_{0t} - \varphi_{i1}P_{1t}. \quad (7)$$

For the dual-source components, we recognize that Q_{it} is the *net* shortfall at the start of period t that takes into account the to-be-received supplies (i.e., $\varphi_{i1}P_{1t}$) from RECOVERY in the same period. By substituting (6) to replace Q_{it} and $Q_{i,t-1}$ in (7), it is easy to see that:

$$P_{it} = \varphi_{i0}P_{0t} - \varphi_{i1}P_{1t}. \quad (8)$$

That is, the order in every period equals the net demand received by the dual-source component inventory.

MATRIX-VECTOR MODEL

As mentioned earlier, under smoothing, production at ASSEMBLY and RECOVERY (and consequently the inventory levels) are correlated over time. To incorporate the correlations, we develop the network model in matrix-vector form that is based on the TPM approach.

We first express the demand arrivals at the processed inventory and the components as well as the returns arrivals at RECOVERY in a single matrix-vector notation. Specifically, we let the column vector $\mathbf{A}_t = \{A_{0t}, \dots, A_{qt}\}'$, where elements A_{it} denotes the respective arrival at stage i . We can then express the arrival vector by:

$$\mathbf{A}_t = \mathbf{\Phi} \mathbf{P}_t + \mathbf{N}_t, \quad (9)$$

where $\mathbf{P}_t = \{P_{0t}, \dots, P_{qt}\}'$ and $\mathbf{\Phi}$ is a square matrix with elements φ_{ij} . The first term on the right-hand side of (9) models the demand arrivals at the components (5) while the second term characterizes the demand arrivals at the processed inventory and returns arrivals at RECOVERY. $\mathbf{N}_t$ is a column vector that comprises N_{0t} and N_{1t} as the only non-zero elements. The variable N_{it} is i.i.d. over time by definition and has a mean μ_i and standard deviation σ_i .

Next, we combine the first-order smoothing for ASSEMBLY and RECOVERY in (4) and (4'), which we express as:

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{D}) \mathbf{P}_{t-1} + \mathbf{D} \mathbf{A}_t, \quad (10)$$

where $\mathbf{D}$ is diagonal matrix with elements $1/n_i$ for $i \in \{0, 1\}$ and 1 else. By substituting (9) into (10), we obtain:

$$\mathbf{P}_t = \mathbf{B} \mathbf{P}_{t-1} + \mathbf{H} \mathbf{N}_t, \quad (11)$$

where $\mathbf{H} = (\mathbf{I} - \mathbf{D} \mathbf{\Phi})^{-1} \mathbf{D}$ and $\mathbf{B} = (\mathbf{I} - \mathbf{D} \mathbf{\Phi})^{-1} - \mathbf{H}$. We note that the matrix $(\mathbf{I} - \mathbf{D} \mathbf{\Phi})$ is invertible provided that the spectral radius of workflow matrix $\mathbf{\Phi}$ is less than one, which is the necessary condition for the system to reach steady state (as will be further explained). By repeatedly iterating $\mathbf{P}_t$ and assuming an infinite history of the system, we obtain the following geometric series:

$$\mathbf{P}_t = \sum_{s=0}^{\infty} \mathbf{B}^s \mathbf{H} \mathbf{N}_{t-s}. \quad (12)$$

The vector $\mathbf{N}_t$ has a mean $\boldsymbol{\mu}$, which is a column vector comprising the expected external demand μ_0 at ASSEMBLY and the expected return arrivals μ_1 at RECOVERY as the only non-zero elements. The covariance matrix of $\mathbf{N}_t$ is given by $\boldsymbol{\Sigma}$ that consists of covariances of external demands at ASSEMBLY and the returns arrivals at RECOVERY.

We can obtain the expectation and variance of $\mathbf{P}_t$ from (12) by:

$$\mathbb{E}[\mathbf{P}_t] = (\mathbf{I} - \mathbf{\Phi})^{-1} \boldsymbol{\mu}, \quad (13)$$

and

$$\text{Var}(\mathbf{P}_t) = \sum_{s=0}^{\infty} \mathbf{B}^s \mathbf{H} \Sigma \mathbf{H}' \mathbf{B}'^s. \quad (14)$$

Equation (14) provides the variances of production and replenishment quantities at each stage as well as the covariances for each pair of stages. The series in (14) can be computed analytically or numerically using the approaches discussed in Graves (1986). Graves (1986) shows that the series equivalent to (14) will converge if and only if the spectral radius of Φ is less than one. This condition assures that every unit processed cannot eventually result in an average of more than one unit processed at the same stage; otherwise, production quantity grows perpetually and the system will not reach a steady state. In the context of this paper, this could only happen in the case in which the average arrival rate of returns is higher than the average demand rate, which is improbable in the remanufacturing setting.

Next we derive the first two moments of Q_{it} for $i \in \{0, 1\}$. We express the inventory shortfalls or the inventory of returns in (2) in matrix form as:

$$\mathbf{Q}_t = \mathbf{D}^{-1} \mathbf{P}_t,$$

from which we can observe that:

$$\mathbf{E}[\mathbf{Q}_t] = \mathbf{D}^{-1} \mathbf{E}[\mathbf{P}_t], \quad (15)$$

and

$$\text{Var}(\mathbf{Q}_t) = \mathbf{D}^{-1} \text{Var}(\mathbf{P}_t) \mathbf{D}^{-1}. \quad (16)$$

With the obtained expected value and variance of Q_{0t} given by (15) and (16) respectively, one can set B_0 for the processed inventory by:

$$B_0 = \mathbf{E}[Q_{0t}] + z_0 \sqrt{\text{Var}(Q_{0t})},$$

where z_0 is the safety factor and its value represents the desired protection offered by the safety stock.

In addition, we can find the expected processed inventory level by:

$$EI_0 = z_0 \sqrt{\text{Var}(Q_{0t})}.$$

Note that we do not account for cycle inventory in EI_0 since it is independent of the decision variables. For the inventory of returns at RECOVERY waiting to be processed, the expected inventory level can be found from (15) as:

$$EI_1 = \mathbf{E}[Q_{1t}].$$

The $\text{Var}(P_{it})$ and $\text{Var}(Q_{it})$ are measures of production and inventory variability respectively. At ASSEMBLY and RECOVERY, a greater extent of smoothing entails smaller $\text{Var}(P_{it})$ and hence fewer occurrences of capacity shortages. However, the production at ASSEMBLY would be less responsive to the stock level of processed inventory, which necessitates more safety stocks. More smoothing at

RECOVERY results in a higher expected stock level of returns. Figure 2(a) and 2(b) illustrate the trade-offs between the standard deviations $\sigma(P_{it})$ and EI_i as n_i is varied for ASSEMBLY and RECOVERY respectively.

Figure 2. Trade-off between $\sigma(P_{it})$ and EI_i for (a) ASSEMBLY and (b) RECOVERY.

DEMAND OVER RISK PERIOD AT COMPONENTS STAGES

Next we consider the demand over risk period at the components. Here we let L_i denotes the length of risk period for component i . We begin our explanation by first looking at an example on the dual-source components.

Example: Suppose $L_i = 2$ for a dual-source component i . As shown in Figure 3, the net demand over risk period covers the demands received from ASSEMBLY in t and $t + 1$ minus the replenishments to be received from RECOVERY in t and $t + 1$.

Figure 3. Net demand over risk period at dual-source component i .

Thus the (net) demand over risk period $D_{it}^{\mathcal{P}}$ can be expressed as follows:

$$D_{it}^{\mathcal{P}} = \varphi_{i0}P_{0t} - \varphi_{i1}P_{1t} + \varphi_{i0}P_{0,t+1} - \varphi_{i1}P_{1,t+1} = \mathcal{A}_{it} + \mathcal{A}_{i,t+1}$$

The above example illustrates that a component with risk period L_i has its risk period spans as far as $t + L_i - 1$. For dual-source components, the net demand over risk period at the start of period t equals to the total demands from t to $t + L_i - 1$ *minus* the supplies to be received over the same periods. For single-source components, only the demands over periods t to $t + L_i - 1$ are included. We can express the demand (or net demand) over risk period for each component i as:

$$D_{it}^{\mathcal{P}} = \sum_{k=1}^{L_i} \mathcal{A}_{i,t+k-1}. \quad (17)$$

The expected demand over risk period can be obtained by

$$\mathbb{E}[D_{it}^{\mathcal{P}}] = L_i \times \mathbb{E}[\mathcal{A}_{it}]. \quad (18)$$

We can find $\mathbb{E}[\mathcal{A}_{it}]$ in (18) by taking the expectation of (9) to obtain $\mathbb{E}[\mathbf{A}_t] = \mathbf{\Phi} \mathbb{E}[\mathbf{P}_t] + \boldsymbol{\mu}$.

Since production at ASSEMBLY and RECOVERY are correlated over time due to smoothing, demands and replenishments of the components are also correlated. Hence we need to incorporate the correlation of demand and replenishment over the risk period at these stages. We express (17) in matrix-vector form to capture the correlation. We find that:

$$\mathbf{D}_t^{\mathcal{P}} = \sum_{k=1}^{\hat{L}} \boldsymbol{\pi}_k \mathbf{A}_{t+k-1}, \quad (19)$$

where $\hat{L}$ is the maximum value of L_i among all components stages, $\boldsymbol{\pi}_k$ are diagonal matrices with elements π_{ik} . For each component i , π_{ik} 's equal to one for $k = 1, 2, \dots, L_i$, and zero else. By substituting (9) and (11) and by repeatedly iterating, we can find $\mathbf{D}_t^{\mathcal{P}}$ as follows:

$$\mathbf{D}_t^{\mathcal{P}} = \mathbf{U} \mathbf{P}_t + \sum_{k=0}^{\hat{L}-1} (\mathbf{V}_k + \boldsymbol{\pi}_{k+1}) \mathbf{N}_{t+k}, \quad (20)$$

where $\mathbf{U} = \sum_{k=1}^{\hat{L}} \boldsymbol{\pi}_k \mathbf{\Phi} \mathbf{B}^{k-1}$,

$$\text{and } \mathbf{V}_k = \begin{cases} \mathbf{0} & k = 0 \\ \left(\sum_{n=k+1}^{\hat{L}} \boldsymbol{\pi}_n \mathbf{\Phi} \mathbf{B}^{n-k-1} \right) \mathbf{H} & \text{else.} \end{cases}$$

The derivations for the matrices $\mathbf{U}$ and $\mathbf{V}_k$ can be found in *Appendix*. Now we can express the variance of $D_{it}^{\mathcal{P}}$ as:

$$\text{Var}(\mathbf{D}_t^{\mathcal{P}}) = \mathbf{U} \text{Var}(\mathbf{P}_t) \mathbf{U}' + \sum_{k=0}^{\hat{L}-1} (\mathbf{V}_k + \boldsymbol{\pi}_{k+1}) \boldsymbol{\Sigma} (\mathbf{V}_k + \boldsymbol{\pi}_{k+1})'. \quad (21)$$

With the mean and variance of D_{it}^p , given by (18) and (21) respectively, we can set the required base stock level for component i by:

$$B_i = E[D_{it}^p] + z_i \sqrt{\text{Var}(D_{it}^p)}.$$

In addition, the expected inventory level for component i is given by:

$$EI_i = z_i \sqrt{\text{Var}(D_{it}^p)}. \quad (22)$$

Note that EI_i in (22) does not account for cycle stock for the same reason as that for EI_0 . We show in Figure 4 how EI_i changes with n_0 and n_1 , i.e., how EI_i relies on the extent of smoothing at both ASSEMBLY AND RECOVERY.

Figure 4. EI_i versus n_0 and n_1

OPTIMIZATION

To minimize the expected total cost (1), we formulate a nonlinear program with n_i and the safety factors z_i as decision variables. The only constraints are $n_i \geq 1$ as defined in (2). By searching from multiple start points in test problems, we encountered multiple local minima, thereby indicating that the problem is nonconvex. Hence we solve the optimization by the approach of searching from multiple start points, which is facilitated by the Global Search solver in MATLAB. Note that the continuous decision variables and the differentiable objective function allow the efficient gradient-based algorithm to be employed to find the local minimum for each start point.

3. Numerical Examples and Insights

Using numerical examples, we illustrate the model’s application and capability, specifically in capturing the intricacies of inter-stage interactions, cost trade-offs and multi-component setting. In addition, we discuss the related insights. We employ a production network that comprises a single dual-source component in addition to ASSEMBLY and RECOVERY. (We exclude the single-source component in the meantime but will study the effect of its inclusion in one of the experiments.) Table 2 shows the input data for the stages. Furthermore, we assume $\varphi_{i0} = \varphi_{i1} = 1$.

Table 2. Input data for numerical experiments

Stage	Demand/Returns $\mu_i (\sigma_i)$	L_i	m_i	b_i	c_{pi}	c_{li}
ASSEMBLY	1,000 (250)	-	1,200	5	30	70
RECOVERY	350 (170)	-	470	1	30	-
Dual-source	-	1	-	3	-	50

3.1. BASE CASE 1: NO SMOOTHING AT BOTH ASSEMBLY AND RECOVERY

We first illustrate the system-wide trade-offs when performing smoothing. We employ a base case in which both ASSEMBLY and RECOVERY do not carry out any smoothing. The base-stock levels in the base case are optimized using the first two moments of (unsmoothed) P_{it} at both stages. Table 3 shows the comparison between the model’s results with those of the base case.

Table 3. Model versus Base Case 1 (model’s results in parentheses)

Stage	n_i	$\sigma(P_{it})$	$\sigma(Q_{it})$	$\sigma(D^{\pi}_{it})$	Production Expediting Cost	Inventory Expediting Cost	Inventory Holding Cost	Expected Total Stage Cost
ASSEMBLY	1.000(1.991)	250(145)	250(288)	-	902(166)	557(642)	1,830(2,109)	3,289(2,917)
RECOVERY	1.000(1.879)	170(102)	170(192)	-	721(182)	-	350(658)	1,071(840)
Dual-Source	-	302(177)	-	302(177)	-	391(229)	1,410(827)	1,801(1,056)
Expected Total Cost					1,623(348)	948(871)	3,590(3,594)	6,161(4,813)

The model’s results indicate a lower variability of the demand over risk period at the dual-source component caused by the smoothing at both ASSEMBLY and RECOVERY (reducing its expected cost from \$1,801 to \$1,056). Comparing with the results of the base case allows us to observe the cost trade-offs when setting n_i . Smoothing at ASSEMBLY results in less frequent capacity shortages, which leads to a decrease in its expected production expediting cost. However, a smoother production implies lower responsiveness to changes in inventory levels. Accordingly, it leads to an increase in the expected inventory holding cost and expected inventory expediting cost. At RECOVERY, in addition to the effect of lowering the variability at the dual-source component, the smoothing decision involves

similar trade-off between the holding cost of returns and the production expediting cost; a larger n_i at RECOVERY leads to less frequent capacity shortages but causes the expected inventory holding cost of the returns to increase.

This example illustrates the significant effect of smoothing on reducing the variability of the component inventory. The above analysis also illustrated the model's capability in capturing the intricate trade-offs for minimizing system-wide cost.

3.2. BASE CASE 2: SMOOTHING ONLY AT ASSEMBLY

We show the resulting sub-optimality when we achieve system-wide optimality. We compare the model's optimal solution with a base case in which smoothing is performed only at ASSEMBLY (but not at RECOVERY). In this base case, ASSEMBLY is locally optimized, i.e. n_0 is optimized by incorporating only its production expediting cost, and the holding and expediting of the processed inventory. The base-stock levels are set by the use of the first two moments of P_{it} of the optimized ASSEMBLY and the (unsmoothed) RECOVERY. The results are shown in Table 4.

Table 4. Model versus Base Case 2 (model's results in parentheses)

Stage	n_i	$\sigma(P_{it})$	$\sigma(Q_{it})$	$\sigma(D^P_{it})$	Production Expediting Cost	Inventory Expediting Cost	Inventory Holding Cost	Expected Total Stage Cost
ASSEMBLY	1.615(1.991)	167(145)	270(288)	-	285(166)	602(642)	1,979(2,109)	2,866(2,917)
RECOVERY	1.000(1.879)	170(102)	170(192)	-	721(182)	-	350(658)	1,071(840)
Dual-Source	-	239(177)	-	239(177)	-	309(229)	1,113(827)	1,422(1,056)
Expected Total Cost (\$ per period)					1,006(348)	911(871)	3,442(3,594)	5,359(4,813)

In this base case, the trade-off in locally optimizing n_0 at ASSEMBLY involves the inventory holding and expediting costs of the processed inventory *versus* the production expediting cost. However, in applying the model, the trade-off involves the additional benefit of smoothing demand variability at the dual-source component. Thus the model's solution suggests a larger n_i than the base case for the additional purpose of reducing upstream variability. The results indicate that the expected cost of ASSEMBLY is higher than that in the base case, as it undertakes local sub-optimality in performing more smoothing in order to attain global optimality. Given the presence of sub-optimality, if the stages are each independently managed (even if they are wholly-owned), the managerial goals and incentives of the stages should be properly aligned to bring about global optimality.

Smoothing in Multi-Component Setting

We consider Base Case 2 but with an additional single-source component (with unit holding cost of \$2). Note that the inventory of single-source component only faces a single source of variability, which is the demand from ASSEMBLY (while dual-source component is subject to the additional variability of replenishment from RECOVERY). The results (Table 5) indicate that the optimal n_0 increases to 2.333 (from 1.991 in the last experiment) because ASSEMBLY needs to smooth more for the additional benefit of reducing variability at the single-source component inventory (besides that at the dual-source component inventory). In doing so, it leads to greater local sub-optimality at ASSEMBLY as the difference of its expected cost between the model and the base case becomes larger. However, the percentage reduction in the expected total cost of the model over the base case gets larger at 11.1%. The greater extent of smoothing at ASSEMBLY improves the expected cost of the dual-source component more substantially, with 30.7% reduction over the base case. This example shows the significant influence of the number and type of components (both single- or dual-source) on the optimal smoothing.

Table 5. Additional single-source component (model's results in parentheses)

Stage	n_i	$\sigma(P_{it})$	$\sigma(Q_{it})$	$\sigma(D^{\pi}_{it})$	Production Expediting Cost	Inventory Expediting Cost	Inventory Holding Cost	Expected Total Stage Cost
ASSEMBLY	1.615(2.333)	167(131)	270(305)	-	285(107)	602(678)	1,979(2,229)	2,866(3,015)
RECOVERY	1.000(1.899)	170(102)	170(193)	-	721(178)	-	350(665)	1,071(843)
Dual-Source	-	239(165)	-	239(165)	-	309(215)	1,113(771)	1,422(986)
Single-Source	-	167(131)	-	167(131)	-	136(106)	585(457)	721(563)
Expected Total Cost (\$ per period)					1,006(285)	1,047(999)	4,027(4,122)	6,090(5,407)

3.3. SIGNIFICANCE OF SMOOTHING AT RECOVERY

As stated earlier, the usual use of production smoothing is to ease capacity loading by leveling output. We will demonstrate that even if production expediting is costless (or equivalently, production capacity is infinite) and with the prior assumption that recovered components are always fewer than ASSEMBLY's requirements, it is still optimal to perform smoothing at RECOVERY under a range of operating conditions.

We conduct an experiment in which we set the unit production expediting cost at RECOVERY to be zero. The results show that it is still optimal to smooth at RECOVERY, even though the optimal n_1 decreases from 1.879 to 1.282. The results highlight the significance of smoothing at RECOVERY to achieve lower variability at the component. We carry out further experiments by adopting different values of RECOVERY's unit holding cost of the returns b_1 as well as the unit holding cost, unit

expediting cost and replenishment lead-time of the dual-source component (b_2 , c_{12} and L_2 respectively). The results (Table 6) indicate that even with costless production expediting at RECOVERY, it can be optimal to perform substantial amount of smoothing with changes made independently to b_1 , b_2 , c_{12} and L_2 . For example, $b_1 = 0.50$ ($n_1 = 1.656$), $b_2 = 7.0$ ($n_1 = 1.654$), $c_{12} = 90$ ($n_1 = 1.338$) and $L_2 = 4$ ($n_1 = 2.255$).

Table 6. Optimal n_1 at different b_2 , b_1 , c_{12} and L_2 at zero unit production expediting cost

b_1	n_1	b_2	n_1	c_{12}	n_1	L_2	n_1
0.50	1.656	2.00	1.144	40	1.259	1	1.282
1.00	1.282	3.00	1.282	50	1.282	2	1.951
1.50	1.122	4.00	1.397	60	1.301	3	2.126
2.00	1.024	5.0	1.495	70	1.319	4	2.255
2.50	1.000	6.0	1.576	80	1.332		
3.00	1.000	7.0	1.654	90	1.338		

3.4. SMOOTHING OF RETURN ARRIVALS

Since the uncertainty of returns arrivals is a major challenge in managing a remanufacturing system, reducing this source of variability has been reported in literature to be significant in cost improvement (see e.g. van der Laan et al. 1999 and Habibi et al. 2017). We perform an experiment to determine how lowering the returns variability alone compares with production smoothing.

In this experiment, we assume that the variability of returns arrivals can be smoothed, which we refer to hereafter as *returns smoothing*. Inherently, we assume that the purchase of the returns or their deliveries can be leveled across time. We model the returns smoothing by including a dummy stage that levels the returns arrivals by the linear production function (2). By modeling the smoothing behavior using (2) (as opposed to simply lowering the variance of arrivals), the model more realistically captures the arrivals that should rightly be correlated over time as the returns arrivals are leveled across time. We note that unlike production smoothing, returns smoothing does not increase the average inventory of returns at RECOVERY. However, we expect other costs may be incurred for implementing such smoothing, e.g., to incentivize the upstream external stage to prolong the holding of the returns. We do not attempt to incorporate such costs in the experiment as this would require assumptions pertaining to the specificity of the return policy. Rather, we experiment only the effect of returns smoothing on the cost components that we have so far considered.

We compare the results of the following cases: *model without returns smoothing* (i.e., control parameter n at dummy = 1), *model with returns smoothing* and the two base cases (Base Case 1 and 2), both now with returns smoothing. Figure 5 shows the expected total costs as n at the dummy is increased from 1.0.

Figure 5. Expected total cost for the model and the base cases

The expected total cost of *model without return smoothing* is lower than that of Base Case 1 (with returns smoothing) over the range of n considered; that is, even with large amount of smoothing of the return arrivals for Base Case 1, it is still outperformed by the model. This illustrates the limitation of returns smoothing simply because it is only able to reduce the variability at the RECOVERY. We can also observe that the difference in the expected total cost between Base Case 2 and *model with returns smoothing* is smaller. More notably, at higher n of the dummy, the cost of Base Case 2 tracks closely that of *model with returns smoothing*; their difference is due to local optimization at ASSEMBLY in the base case (as discussed in §3.2). Furthermore, the expected total cost of Base Case 2 becomes lower than that of *model without return smoothing* when the return arrivals are sufficiently smoothed (i.e., when n_i is higher than about 1.4). We note that the difference between the total expected costs of *model with returns smoothing* and *model without returns smoothing* enables managers to quantify the cost of implementing returns smoothing in addition to production smoothing.

We also observe that the expected total costs of both base cases undergo steep declines during the initial increase in n . For instance, increasing n from 1.0 to 2.0 reduces the expected total cost of Base Case 1 by 12.4% and that of Base Case 2 by 15.5% but a corresponding increase of n for *model with returns smoothing* only reduces the cost by 6.4%. As n increases further, the rate of reduction in the expected total costs decreases. For example, an increase in n from 3.0 to 4.0 leads to only a cost reduction of only 0.97% for Base Case 1 and 1.37% for Base Case 2.

The example illustrates production smoothing at both ASSEMBLY and RECOVERY tends to outperform returns smoothing. Furthermore, although a small extent of returns smoothing can bring

about significant cost saving (as illustrated by Base Case 1), but similar extent of savings should not be expected if production smoothing is already in place.

3.5. CORRELATION BETWEEN DEMAND AND RETURN ARRIVALS

We show an application of the model for the scenario in which demand and returns arrivals are positively correlated. For example, such positive correlation can occur for short life-cycle products when customers purchase new products (generating demand at the processed inventory) and subsequently trade-in their used or end-of-life products (leading to returns arrivals). We compare the model's results based on coefficients of correlation 0, 0.25, 0.50 and 0.75 in Table 7. Each of ASSEMBLY and RECOVERY is separately subject to the randomness of demand and return arrivals respectively, and hence is not affected by the changes in correlation. However the dual-source component experiences both sources of variability (via ASSEMBLY and RECOVERY). The model's solution suggests less amount of smoothing at both ASSEMBLY and RECOVERY as correlation increases. This is because the resulting lower variability at the dual-source component necessitates less smoothing. Also, compared to ASSEMBLY and RECOVERY, the expected cost of the dual-source component decreases more significantly with the increase in correlation. The lower variability leads to a decrease in its base-stock level; for instance, the base-stock level reduces by 9.96% when the coefficient increases from 0.25 to 0.75.

Table 7. Results for positive correlation between demand and returns

Stage	n_i				Stage	Expected stage cost			
	Correlation coefficient					Correlation coefficient			
	0	0.25	0.50	0.75		0	0.25	0.50	0.75
ASSEMBLY	1.991	1.961	1.928	1.887	ASSEMBLY	2,917	2,910	2,903	2,895
RECOVERY	1.879	1.820	1.749	1.657	RECOVERY	1,056	833	827	822
Dual-Source	-	-	-	-	Dual-Source	840	936	790	594
Base stock at Dual-Source	926	894	856	805	Expected total cost	4,813	4,680	4,520	4,312

The above analysis shows the importance of identifying the correlation level between demand and return arrivals, as smoothing and base-stocks should be regulated according to the correlation.

4. Concluding Remarks

This paper presents a model that supports the variability mitigation for recover-and-assemble remanufacturing systems through production smoothing. We first describe how we model the

recover-and-assemble configuration as a linear matrix-vector system. We characterize the production and inventory random variables needed to compute the expected total cost, which forms the objective function of the optimization procedure that determines the optimal smoothing and safety stocks. We report on numerical examples to demonstrate the model's application and to draw insights on smoothing. In particular, we show how the tactics should be planned by considering inter-stage cost trade-offs, holding cost levels relative to stock-out penalties, recovered versus sourced components in the multi-component setting and the sub-optimality at individual processing stages. In doing so, we also demonstrate the underlying intricacies of planning and the model's capability in addressing these complexities.

There remain limitations and questions regarding the model. Like many works in the literature, we imposed the assumption of demand netting to achieve tractability. We have not addressed how to relax this assumption, which would likely to result in a much less tractable model. In addition, even though the model structure allows a characterization of multi-product environment, we have not studied this setting and the associated interactions. For this setting, it would be worthwhile to study how smoothing should be carried out in assembling the different products using both non-identical and common recovered components. In addition, for the single-product problem in this paper, the numerical approach of employing multiple start points of the optimization has been efficient for the test problems we encountered. However, in extending the model to characterize more complex operations (e.g., a multi-product setting), one may need to study the resulting model structure and the resulting computational complexity for the optimization. It would be worthwhile to consider the aforementioned areas in the future research.

5. References

- Akcah, E., S. Cetinkaya. 2011. Quantitative models for inventory and production planning in closed-loop supply chains. *International Journal of Production Research*. **49** 2373-2407.
- Balakrishnan, A., J. Geunes, M.S. Pangburn. 2004. Coordinating supply chains by controlling upstream variability propagation. *Manufacturing & Service Operations Management*. **6** 163–183.
- Bayındır, Z.P., N. Erkip, R. Güllü. 2003. A model to evaluate inventory costs in a remanufacturing environment. *International Journal of Production Economics*. 81/82: 597–607.
- Boute R., J.A. Van Mieghem. 2015. Global dual sourcing and order smoothing: The impact of capacity and leadtimes. *Management Science*. **61**(9) 2080–2099.
- DeCroix, G., P. Zipkin. 2005. Inventory management for an assembly system with product or component returns. *Management Science*. **51** 1250–1265.

- DePuy, G.W. J.S. Usher, R.L. Walker, G.D. Taylor. 2007. Production planning for remanufactured products. *Production Planning & Control*. **18**(7) 573-583.
- Daly, L., 2006. Recycling Technology Products: An Overview of E-Waste Policy Issues. Report for the Office of Technology Policy, Technology Administration, United States Department of Commerce. Office of Technology Policy, Washington, D.C.
- Electronics Takeback Coalition, 2011. Available at <http://www.electronicstakeback.com/promote-good-laws/state-legislation/> (accessed 1st November 2015).
- Ferrer, G., Whybark D.C. 2001. Material planning for a remanufacturing facility. *Production and Operation Management*. **10**(2) 112-124.
- Fine, C.H., S.C. Graves. 1989. A tactical planning model for manufacturing subcomponents in mainframe computers. *Journal of Manufacturing & Operations Management*. **2** 4-34.
- Fleischmann, M., J.M. Bloemhof-Ruwaard, R. Dekker, E. Van der Laan, J. A.E.E. Van Nunen, L.N. Van Wassenhove. 1997. Quantitative models for reverse logistics: a review. *European Journal of Operational Research*. **103**(1) 1-17.
- Graves, S.C. 1986. A tactical planning model for a job shop. *Operations Research*. **34** 522-533.
- Graves, S.C., H.C. Meal, S. Dasu, Y. Qiu. 1986. Two-stage production planning in a dynamic environment. S. Axsater, C. Schneeweiss, E. Silver, eds. *Multi-Stage Production Planning and Inventory Control*. Springer-Verlag, Berlin. 9-43.
- Graves, S. C. 1988. Safety stocks in manufacturing systems. *Journal of Manufacturing & Operations Management*. **1** 67- 101.
- Graves, S. C., D.B. Kletter, W.B. Hetzel. 1998. A dynamic model for requirements planning with application to supply chain optimization. *Operations Research*. **46** 35-49.
- Graves, S. C., S.P. Willems. 2000. Optimizing strategic safety stock placement in supply chains. *Manufacturing & Service Operations Management*. **1** 68-83.
- Graves, S.C., S.P. Willems. 2003. Supply chain design: safety stock placement and supply chain configuration. A.G. de Kok, S.C. Graves, eds. *Handbooks in Oper. Res. & Mgmt. Sci. Vol. 11., Supply Chain Management: Design, Coordination & Operation*. Elsevier, Amsterdam, The Netherlands. Chapter 3.
- Guide Jr., V.D.R., 2000. Production planning and control for remanufacturing: industry practice and research needs. *Journal of Operations Management*. **18** 467-483.
- Habibi, M.K.K., O. Battaia, V.-D. Cung, A. Dolgui. 2017. Collection-disassembly problem in reverse supply chain. *International Journal of Production Economics*. **181** 334-344.
- Hagerty, J.R., P. Glader. 2011. From trash heap to store shelf. Available at <http://online.wsj.com/article/SB10001424052748704115404576095881429852432.html> (accessed 15th Jan. 2016).
- Junior, M. L., M.G., Filho. 2012. Production planning and control for remanufacturing: literature review and analysis. *Production Planning & Control*. **23**(6) 419-435.

- Kelle, P., & Silver, E. A. 1989. Forecasting the returns of reusable containers. *Journal of Operations Management*. **89**(1) 17–35.
- Klosterhalfen, S.T., D. Dittmar and S. Minner. 2013. An integrated guaranteed- and stochastic-service approach to inventory optimization in supply chains. *European Journal of Operational Research*. **231**(1) 109–119.
- Li, Z., M.M. Naim and S.M. Disney. 2017. The impact of product returns and remanufacturing uncertainties on the dynamic performance of a multi-echelon closed-loop supply chain. *International Journal of Production Economics*. **183** 487–502.
- Lin, D., C.C. Teo, C. Lee. 2015. Heuristics for Integrated Job assignment and Scheduling for a Multi-Plant Remanufacturing. *International Journal of Production Research*. **53**(9), 2674–2689.
- Minter, A. 2013. U.S. isn't Flooding the Third World with E-Waste. <https://www.bloomberg.com/view/articles/2013-05-26/stop-the-baseless-panicking-over-u-s-e-waste> (accessed 15 May 2016)
- Mitra, S. 2012. Inventory management in a two-echelon closed-loop supply chain with correlated demands and returns. *Computers & Industrial Engineering*. **62** 870–879.
- Morgan, S. D., R. J., Gagnon. 2013. A systematic literature review of remanufacturing scheduling. *International Journal of Production Research*. **51** (16), 4853–4879.
- Parker, D., K. Riley, S. Robinson, H. Symington, J. Tewson, K. Jansson, S. Ramkumar, D. Peck. 2015. Remanufacturing Market Study. Retrieved from <https://www.remanufacturing.eu/wp-content/uploads/2016/01/study.pdf> (accessed: 8th June 2017).
- Pourghadim, V.A., M.Y. Noordin, K.Y. Wong, 2014. A Review of Inventory Management for Remanufacturing Environment. *Advanced Materials Research*. **845** 618–622.
- Schenkel, M., M.C.J. Caniels, H. Krikke, E. van der Laan. 2015. Understanding value creation in closed loop supply chains – Past findings and future directions. *Journal of Manufacturing Systems*. **37** 729–745.
- Steenneck, D.W., S.C. Sarin. 2013. Pricing and production planning for reverse supply chain: a review. *International Journal of Production Research*. **51**(23) 23–24.
- Teo, C.C., R. Bhatnagar, S.C. Graves. 2011. Setting planned lead times for a make-to-order production system with master schedule smoothing. *IIE Transactions* **43** 399–414.
- Teo, C.C., R. Bhatnagar, S.C. Graves. 2012. Application of master schedule smoothing and planned lead times control. *Production and Operations Management*. **21** 211–223.
- Toktay, L.B., Wein, L.M., Zenios, S.A. 2000. Inventory management of remanufacturable products. *Management Science*. **46**(11) 1412–1426.
- U.S. International Trade Commission. 2012. Remanufactured Goods: An Overview of the U.S. and Global Industries, Markets, and Trade. Investigation No. 332-525. USITC Publication 4356.
- van der Laan, E. and M. Salomon. 1997. Production planning and inventory control with remanufacturing and disposal. *European Journal of Operational Research*. 102:264–6787.
- van der Laan, E., M. Salomon, R. Dekker, L.N. Van Wassenhove. 1999. Inventory control in hybrid systems with remanufacturing. *Management Science*. 45(5), 733–747.

- Yang, S.S., S.K. Ong and A.Y.C. Nee. 2014. Product design for remanufacturing. A.Y.C. Nee ed. Handbook of Manufacturing Engineering and Technology. Vol. 6. Springer-Verlag, London. 3195-3217.
- Zahraei S.M. and C. C. Teo. 2017. Optimizing a Supply Network with Production Smoothing, Freight Expediting and Safety Stocks: An Analysis of Tactical Trade-Offs. *European Journal of Operational Research*. 262(1) 75–88.

Appendix

First we rewrite $\mathbf{D}_t^{\mathcal{P}}$ in (20) and then we present its derivation.

$$\mathbf{D}_t^{\mathcal{P}} = \mathbf{U}\mathbf{P}_t + \sum_{k=0}^{\hat{L}-1} (\mathbf{V}_k + \boldsymbol{\pi}_{k+1})\mathbf{N}_{t+k}, \quad (\text{A.1})$$

$$\text{where } \mathbf{U} = \sum_{k=1}^{\hat{L}} \boldsymbol{\pi}_k \boldsymbol{\Phi} \mathbf{B}^{k-1} \text{ and } \mathbf{V}_k = \begin{cases} \mathbf{0} & k=0 \\ \left(\sum_{n=k+1}^{\hat{L}} \boldsymbol{\pi}_n \boldsymbol{\Phi} \mathbf{B}^{n-k-1} \right) \mathbf{H} & \text{else} \end{cases}.$$

For the benefits of clarity, we first derive $\mathbf{D}_t^{\mathcal{P}}$ for $\hat{L}=1, 2$ and 3 before generalizing $\mathbf{D}_t^{\mathcal{P}}$ as a function of $\hat{L}$.

For $\hat{L}=1$, $\mathbf{D}_t^{\mathcal{P}}$ in (19) can be written as:

$$\mathbf{D}_t^{\mathcal{P}} = \boldsymbol{\pi}_1 \mathbf{A}_t. \quad (\text{A.2})$$

Now by substituting (9) one can rewrite (A.2) as

$$\mathbf{D}_t^{\mathcal{P}} = \boldsymbol{\pi}_1 \boldsymbol{\Phi} \mathbf{P}_t + \boldsymbol{\pi}_1 \mathbf{N}_t.$$

Therefore, in general, for $\hat{L}=1$, $\mathbf{D}_t^{\mathcal{P}}$ can be expressed as:

$$\mathbf{D}_t^{\mathcal{P}} = \mathbf{U}\mathbf{P}_t + (\mathbf{V}_0 + \boldsymbol{\pi}_1)\mathbf{N}_t,$$

where $\mathbf{U} = \boldsymbol{\pi}_1 \boldsymbol{\Phi} \mathbf{P}_t$ and $\mathbf{V}_0 = \mathbf{0}$.

Similar to the derivation for $\hat{L}=1$, $\mathbf{D}_t^{\mathcal{P}}$ in (19) for $\hat{L}=2$ can be written as:

$$\mathbf{D}_t^{\mathcal{P}} = \sum_{k=1}^2 \boldsymbol{\pi}_k \mathbf{A}_{t+k-1} = \boldsymbol{\pi}_1 \mathbf{A}_t + \boldsymbol{\pi}_2 \mathbf{A}_{t+1}. \quad (\text{A.3})$$

Now by substituting (9) one can rewrite (A.3) as

$$\mathbf{D}_t^{\mathcal{P}} = \boldsymbol{\pi}_1 (\boldsymbol{\Phi} \mathbf{P}_t + \mathbf{N}_t) + \boldsymbol{\pi}_2 (\boldsymbol{\Phi} \mathbf{P}_{t+1} + \mathbf{N}_{t+1}).$$

Next, by substituting (11) for $\mathbf{P}_{t+1}$, we can observe that:

$$\mathbf{D}_t^{\mathcal{P}} = (\boldsymbol{\pi}_1 \boldsymbol{\Phi} + \boldsymbol{\pi}_2 \boldsymbol{\Phi} \mathbf{B}) \mathbf{P}_t + \boldsymbol{\pi}_1 \mathbf{N}_t + (\boldsymbol{\pi}_2 \boldsymbol{\Phi} \mathbf{H} + \boldsymbol{\pi}_2) \mathbf{N}_{t+1}. \quad (\text{A.4})$$

In general, $\mathbf{D}_t^{\mathcal{P}}$ in (A.4) can be re-expressed as

$$\mathbf{D}_t^{\mathcal{P}} = \mathbf{U}\mathbf{P}_t + \sum_{k=0}^1 (\mathbf{V}_k + \pi_{k+1}) \mathbf{N}_{t+k},$$

where $\mathbf{U} = \sum_{k=1}^2 \pi_k \Phi \mathbf{B}^{k-1}$ and $\mathbf{V}_k = \begin{cases} \mathbf{0} & k=0 \\ \pi_2 \Phi \mathbf{H} & k=1 \end{cases}$.

Similarly, for $\hat{L} = 3$, one can rewrite $\mathbf{D}_t^{\mathcal{P}}$ in (19) as:

$$\mathbf{D}_t^{\mathcal{P}} = \sum_{k=1}^3 \pi_k \mathbf{A}_{t+k-1} = \pi_1 \mathbf{A}_t + \pi_2 \mathbf{A}_{t+1} + \pi_3 \mathbf{A}_{t+2}, \quad (\text{A.5})$$

Now by substituting (9) and (11), and repeating iterating, one can rewrite $\mathbf{D}_t^{\mathcal{P}}$ in (A.5) as:

$$\mathbf{D}_t^{\mathcal{P}} = (\pi_1 \Phi + \pi_2 \Phi \mathbf{B} + \pi_3 \Phi \mathbf{B}^2) \mathbf{P}_t + \pi_1 \mathbf{N}_t + ((\pi_2 \Phi + \pi_3 \Phi \mathbf{B}) \mathbf{H} + \pi_2) \mathbf{N}_{t+1} + (\pi_3 \Phi \mathbf{H} + \pi_3) \mathbf{N}_{t+2}. \quad (\text{A.6})$$

In general, $\mathbf{D}_t^{\mathcal{P}}$ in (A.6) can be re-expressed as

$$\mathbf{D}_t^{\mathcal{P}} = \mathbf{U}\mathbf{P}_t + \sum_{k=0}^2 (\mathbf{V}_k + \pi_{k+1}) \mathbf{N}_{t+k},$$

where $\mathbf{U} = \sum_{k=1}^3 \pi_k \Phi \mathbf{B}^{k-1}$ and $\mathbf{V}_k = \begin{cases} \mathbf{0} & k=0 \\ \left(\sum_{n=k+1}^3 \pi_n \Phi \mathbf{B}^{n-k-1} \right) \mathbf{H} & k=1 \text{ and } k=2 \end{cases}$.

For $\hat{L} \geq 4$, one can use similar substitution and repeating iteration as described above to find $\mathbf{D}_t^{\mathcal{P}}$ given by (A.1). Therefore, from the above results, we obtain the general form of $\mathbf{D}_t^{\mathcal{P}}$ in (19).