RSS indoor localization and tracking with INS assistance

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RSS Indoor Localization and Tracking with INS

Assistance

Wen Kai

**SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING**

2019

RSS Indoor Localization and Tracking with INS Assistance

Wen Kai

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

21-Aug-2019

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Date

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Tan Soon Yim

Authorship Attribution Statement

This thesis contains material from 1 paper accepted at conferences in which I am listed as an author.

Chapter 3 is published as K. Wen, C. K. Seow, and S. Y. Tan, “Inertial navigation system positioning error analysis and Cramér-Rao lower bound,” in Proc. IEEE/ION PLANS 2016 Position Location and Navigation Symposium (PLANS’16), Savannah, GA, Apr. 11-14, 2016, pp. 213-218.

The contributions of the co-authors are as follows:

- A/Prof. Tan Soon Yim provided the project conceptualization and supervision. He also co-developed the proposed methodology to analysis localization root mean square error and edited the manuscript draft.
- I co-developed the proposed methodology to analysis localization root mean square error with A/Prof. Tan and Dr. Seow Chee Kiat. I collected data from inertial measurement unit, performed the simulation based on the data, and analyzed the results. I also prepared the manuscript draft.
- Dr. Seow co-developed the proposed methodology to analysis localization root mean square error and revised manuscript.

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Abstract

In recent years, various indoor localization and tracking systems have received a significant amount of attention, due to the increasing demand for location-aware applications in indoor environments. Wireless localization technologies, such as received signal strength-based (RSS-based) localization systems, have been widely used due to their low implementation cost. However, such systems suffer from poor localization accuracy due to multipath and non-line-of-sight (NLOS) propagation. Unlike wireless localization systems, inertial navigation system (INS) does not suffer from multipath and NLOS propagation issues. However, INS accuracy degrades as time elapses, because of accumulated noise in accelerometers and gyroscopes. Furthermore, the inaccurate estimation of inertial measurement unit (IMU) orientation leads to the incorrect projection of gravitational acceleration in accelerometers, which causes large localization errors. Several fusion systems, such as INS with RSS, INS with mapping, etc., have been explored as a way of mitigating these issues. Existing INS-based techniques use an INS zero velocity update (ZUPT) with extended Kalman filter (EKF) to mitigate orientation and localization error. ZUPT requires users to physically attach the IMU to one foot to leverage the zero velocity measurement and continuously calibrate the IMU. To further improve localization accuracy, particle filtering (PF) is used to model the nonlinear system, and maps are used

to confine particle movement. The placement of the IMU to utilize ZUPT, the computation complexity of PF, and the uncertain availability of accurate maps all make the fusion system challenging for implementation.

This thesis presents a novel weighted RSS indoor localization and tracking scheme that uses INS projection. The proposed scheme does not require the IMU to be physically attached to the user's foot, and mapping information is not required. The scheme also reduces computation time versus PF. Simulation results of the proposed scheme show that it significantly outperforms both conventional INS and an RSS fusion scheme using EKF.

This thesis derives a closed form equation to analyze the noise of acceleration along the navigation frame, which leads to errors in stand-alone INS positioning estimations. The results are used to calculate appropriate weight for the proposed scheme in later chapters. For the case when there is perfect knowledge of IMU orientation with respect to the navigation frame coordinates, this thesis proposes a novel weighted nonlinear RSS localization and tracking scheme that integrates INS positioning estimation projections. Due to the cumulative effect of noise on the gyroscope, INS-based localization performance deteriorates rapidly even when the initial orientation is known. Thus, an algorithm is proposed to continuously mitigate the effect of noise on gyroscopes. Simulation results obtained from three different scenarios show that when fused with INS projection, the proposed RSS localization scheme outperforms conventional fusion schemes that use EKF by up to 56%.

To develop a more practical scheme that does not require perfect knowledge of IMU orientation with respect to the navigation frame coordinates, an algorithm is proposed to estimate the initial orientation during the calibration stage. To further mitigate noise from the IMU, another algorithm is proposed to continuously mitigate the effect of noise on accelerometers. Once all of these proposed noise mitigation algorithms and initial calibrations are incorporated, the simulation results obtained from three different scenarios show that the proposed RSS localization scheme fused with INS projection significantly outperforms conventional fusion schemes that use EKF by up to 65% (82%) for 2D (3D) environments. The proposed scheme was tested on different routes at a real environment, and the results show that its localization is up to 53% (62%) more accurate than the other two localization schemes in 2D (3D) environments.

Table of Contents

Statement of Originality	I
Supervisor Declaration Statement.....	II
Authorship Attribution Statement.....	III
Acknowledgements.....	IV
Abstract	V
Table of Contents	VIII
List of Figures	XI
List of Tables	XVIII
List of Abbreviations	XIX
List of Symbols.....	XXI
Chapter 1	1
Introduction.....	1
1.1 Background and Motivation	1
1.2 Objective and Contributions of the Thesis.....	5
1.3 Organization of the Thesis	6
Chapter 2.....	8

Literature Review.....	8
2.1 RSS-based Localization Scheme	8
2.2 Inertial Navigation Fundamentals.....	10
2.3 Pedestrian Inertial Navigation.....	14
2.3.1 Step-based PDR	14
2.3.2 Strapdown INS PDR	15
Chapter 3.....	18
Inertial Navigation System Localization Error Analysis	18
3.1 Theory and Formulation	19
3.1.1 INS Localization Equation.....	20
3.1.2 Derivation of INS Localization RMSE.....	24
3.2 Noise Characterization and Acquisition	27
3.2.1 Error sources in MEMS IMU sensors.....	28
3.2.2 Noise Characteristic Analysis Using Allan Standard Deviation.....	30
3.3 INS EKF Error Propagation and Cramér-Rao Lower Bound	35
3.4 Results and Comparison	40
3.5 Conclusion	44
Chapter 4.....	46
RSS Localization and Tracking Scheme with INS Assistance under Known Initial Orientation.....	46
4.1 Proposed RSS Localization Scheme	47
4.2 Gyroscope Noise Mitigation	54
4.3 Simulation Results and Comparison with EKF	58
4.3.1 Environment 1.....	59
4.3.2 Environment 2.....	70
4.3.3 Environment 3.....	80
4.4 Conclusion	89

Chapter 5	91
RSS Localization and Tracking Scheme with INS Assistance without Initial Knowledge of Orientation.....	91
5.1 Proposed Initial Orientation Angle Estimation Technique	92
5.2 Accelerometer Noise Mitigation.....	96
5.3 Simulation Results and Comparison with EKF	98
5.3.1 Environment 1	99
5.3.2 Environment 2.....	111
5.3.3 Environment 3.....	120
5.4 Experimental Results and Comparison	130
5.4.1 Experiment Environment and Setup	131
5.4.2 Comparison of Proposed RSS Measurement Improvement and Localization Results.....	135
5.5 Conclusion	144
Chapter 6.....	145
Conclusion and Future Work	145
6.1 Conclusion	145
6.2 Future Work	147
Author's Publications.....	149
Bibliography	150
Appendix A.....	168
Standard Kalman Filter and Extended Kalman Filter.....	168
A.1 Standard Kalman Filter	168
A.2 Extended Kalman Filter	170

List of Figures

Figure 2.1 Illustration of the navigation frame and strapdown INS body frame	12
Figure 3.1 Indoor floorplan, navigation frame (blue), and body frame (green)	21
Figure 3.2 A sample log-log plot of Allan standard deviation [106].....	31
Figure 3.3 Gyroscopes: Allan standard deviation	32
Figure 3.4 Accelerometers: Allan standard deviation.....	32
Figure 3.5 IMU movement: Path 1	41
Figure 3.6 Localization RMSE comparison: Path 1	42
Figure 3.7 IMU movement: Path 2	43
Figure 3.8 Localization RMSE comparison: Path 2	44
Figure 4.1 Illustration of INS projection.....	48
Figure 4.2 Histogram of measurements in four different environments [37]	51
Figure 4.3 E1 path (red) and AP locations (green)	59
Figure 4.4 Velocity profile of user following the path in E1	60
Figure 4.5 INS path (blue)	61
Figure 4.6 RSS localization (blue), $\sigma_{RSS} = 6dB$	62
Figure 4.7 INS+RSS+EKF (blue)	63
Figure 4.8 INS with gyroscope noise mitigation (blue).....	64
Figure 4.9 Proposed RSS scheme with gyroscope noise mitigation (blue)	65

Figure 4.10 Cumulative error: proposed (blue) vs EKF (red)	66
Figure 4.11 2 nd run, INS path (blue)	66
Figure 4.12 2 nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$	67
Figure 4.13 2 nd run, INS+RSS+EKF (blue)	67
Figure 4.14 2 nd run, INS with gyroscope noise mitigation (blue).....	68
Figure 4.15 2 nd run, proposed RSS scheme with gyroscope noise mitigation (blue)	68
Figure 4.16 2 nd run, cumulative error: proposed (blue) vs EKF (red)	69
Figure 4.17 Average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow)	70
Figure 4.18 E2 path (red) and AP locations (green)	71
Figure 4.19 Velocity profile of user following the path in E2.....	71
Figure 4.20 E2 INS path (blue).....	72
Figure 4.21 E2 RSS localization (blue), $\sigma_{RSS} = 6dB$	73
Figure 4.22 E2, INS+RSS+EKF (blue)	73
Figure 4.23 E2, INS with gyroscope noise mitigation (blue)	74
Figure 4.24 E2, Proposed RSS scheme with gyroscope noise mitigation (blue)	74
Figure 4.25 E2, cumulative error: proposed (blue) vs EKF (red)	75
Figure 4.26 E2, 2 nd run, INS path (blue).....	76
Figure 4.27 E2, 2 nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$	76
Figure 4.28 E2, 2 nd run, INS+RSS+EKF (blue)	77
Figure 4.29 E2, 2 nd run, INS with gyroscope noise mitigation (blue)	77
Figure 4.30 E2, 2 nd run, proposed RSS scheme with gyroscope noise mitigation (blue)	78

Figure 4.31 E2, 2 nd run, cumulative error: proposed (blue) vs EKF (red).....	78
Figure 4.32 E2, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow)	79
Figure 4.33 E3, original map [88].....	80
Figure 4.34 E3, path (red) and AP locations (green)	81
Figure 4.35 Velocity profile of user following the path in E3.....	81
Figure 4.36 E3, INS path (blue).....	82
Figure 4.37 E3, RSS localization (blue), $\sigma_{RSS} = 6dB$	83
Figure 4.38 E3, INS+RSS+EKF (blue)	83
Figure 4.39 E3, INS with gyroscope noise mitigation (blue)	84
Figure 4.40 E3, proposed RSS scheme with gyroscope noise mitigation (blue)	84
Figure 4.41 E3, cumulative error: proposed (blue) vs EKF (red).....	85
Figure 4.42 E3, 2 nd run, INS path (blue).....	85
Figure 4.43 E3, 2 nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$	86
Figure 4.44 E3, 2 nd run, INS+RSS+EKF (blue)	86
Figure 4.45 E3, 2 nd run, INS with gyroscope noise mitigation (blue)	87
Figure 4.46 E3, 2 nd run, proposed RSS scheme with gyroscope noise mitigation (blue).....	87
Figure 4.47 E3, 2 nd run, cumulative error: proposed (blue) vs EKF (red).....	88
Figure 4.48 E3, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow)	89
Figure 5.1 Illustration of ψ_x and G projection	92
Figure 5.2 E1 path (red) and AP locations (green)	100
Figure 5.3 Velocity profile of user following the path in E1	100
Figure 5.4 INS path without initial orientation estimation (blue).....	101

Figure 5.5 RSS localization (blue), $S_{RSS} = 6dB$	102
Figure 5.6 INS+RSS+EKF (blue)	102
Figure 5.7 INS with all improvements applied (blue)	103
Figure 5.8 Proposed RSS scheme with all techniques applied	104
Figure 5.9 Cumulative error: proposed (blue) vs EKF (red)	104
Figure 5.10 2 nd run, INS path without initial orientation estimation (blue).....	105
Figure 5.11 2 nd run, RSS localization (blue), $S_{RSS} = 6dB$	106
Figure 5.12 2 nd run, INS+RSS+EKF (blue)	106
Figure 5.13 2 nd run, INS scheme with all techniques applied without known initial orientation (blue) vs INS scheme with gyroscope noise mitigation and known initial orientation (black)	107
Figure 5.14 2 nd run, proposed RSS scheme with all techniques applied without known initial orientation (blue) vs proposed RSS scheme with gyroscope noise mitigation and known initial orientation (black)	107
Figure 5.15 2 nd run, cumulative error: proposed (blue) vs EKF (red)	108
Figure 5.16 Average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow) vs NLOS selection (green) vs INS (black)	110
Figure 5.17 Average 3D cumulative error: proposed (blue) vs EKF (red) vs NLOS selection (Green).....	110
Figure 5.18 E2 path (red) and AP locations (green)	111
Figure 5.19 Velocity profile of user following the path in E2.....	112
Figure 5.20 E2, INS path without initial orientation estimation (blue)	113
Figure 5.21 E2, RSS localization (blue), $\sigma_{RSS} = 6dB$	113
Figure 5.22 E2, INS+RSS+EKF (blue)	114
Figure 5.23 E2, INS with all improvements applied (blue)	114

Figure 5.24 E2, proposed RSS scheme with all techniques applied (blue)	115
Figure 5.25 E2, cumulative error: proposed (blue) vs EKF (red)	115
Figure 5.26 E2, 2 nd run, INS path without initial orientation estimation (blue)	116
Figure 5.27 E2, 2 nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$	117
Figure 5.28 E2, 2 nd run, INS+RSS+EKF (blue)	117
Figure 5.29 E2, 2 nd run, INS with all improvements applied without known initial orientation (blue) vs INS with gyroscope noise mitigation and known initial orientation (black)	118
Figure 5.30 E2, 2 nd run, proposed RSS scheme with all techniques applied without known initial orientation (blue) vs proposed RSS scheme with gyroscope noise mitigation and known initial orientation (black)	118
Figure 5.31 E2, 2 nd run, cumulative error: proposed (blue) vs EKF (red)	119
Figure 5.32 E2, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow) vs INS (black)	120
Figure 5.33 E3 original map [88]	121
Figure 5.34 E3 path (red) and AP locations (green)	121
Figure 5.35 Velocity profile of user following the path in E3	122
Figure 5.36 E3, INS path without initial orientation estimation (blue)	123
Figure 5.37 E3, RSS localization (blue), $\sigma_{RSS} = 6dB$	123
Figure 5.38 E3, INS+RSS+EKF (blue)	124
Figure 5.39 E3, INS with all improvements applied (blue)	124
Figure 5.40 E3, proposed RSS scheme with all techniques applied (blue)	125
Figure 5.41 E3, cumulative error: proposed (blue) vs EKF (red)	125

Figure 5.42 E3, 2 nd run, INS path without initial orientation estimation (blue)	126
Figure 5.43 E3, 2 nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$	127
Figure 5.44 E3, 2 nd run, INS+RSS+EKF (blue)	127
Figure 5.45 E3, 2 nd run, INS with all improvements applied without known initial orientation (blue) vs INS with gyroscope noise mitigation and known initial orientation (black)	128
Figure 5.46 E3, 2 nd run, proposed RSS scheme with all techniques applied without known initial orientation (blue) vs proposed RSS scheme with gyroscope noise mitigation and known initial orientation (black)	128
Figure 5.47 E3, 2 nd run, cumulative error: proposed (blue) vs EKF (red)	129
Figure 5.48 E3, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow) vs INS (black)	130
Figure 5.49 Experiment environment	131
Figure 5.50 First experiment route, including AP locations (green) and movement path (red)	132
Figure 5.51 Second experiment route, including AP locations (green) and movement path (red)	133
Figure 5.52 First experiment route localization paths: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)	139
Figure 5.53 First experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)	140

Figure 5.54 3D First experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)	140
Figure 5.55 Second experiment route localization paths: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)	141
Figure 5.56 Second experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)	142
Figure 5.57 3D Second experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)	143

List of Tables

Table 3-1 Gyroscope noise sources	33
Table 3-2 Accelerometer noise sources	33
Table 3-3 Gyroscope noise standard deviation.....	34
Table 3-4 Accelerometer noise standard deviation.....	34
Table 5-1 Proposed solution 1 for RSS'_i under different RSS measurement scenarios at t_{i-1} and t_i	136
Table 5-2 Proposed solution 2 for RSS'_i under different RSS measurement scenarios at t_{i-1} and t_i	137
Table 5-3 Modified weight scenarios in solution 2	138

List of Abbreviations

AOA	Angle of arrival
AP	Access point
CRLB	Cramér-Rao lower bound
EKF	Extended Kalman filter
FCC	Federal Communication Commission
GNSS	Global navigation satellite system
GPS	Global positioning system
IMU	Inertial measurement unit
INS	Inertial navigation system
KF	Kalman filter
LOS	Line of sight
LS	Least squares
MD	Mobile device
MEMS	Micro-electromechanical system
NLOS	Non-line of sight
PDR	Pedestrian dead reckoning
RFID	Radio-frequency identification

RMSE	Root mean square error
RSS	Received signal strength
TDOA	Time difference of arrival
TOA	Time of arrival
UWB	Ultra-wideband
WLAN	Wireless local area network

List of Symbols

$\left[\omega_{x,k}^b, \omega_{y,k}^b, \omega_{z,k}^b \right]^T$	Angular velocity along body frame x,y,z axes at time instance k
$\left[a_{x,k}^b, a_{y,k}^b, a_{z,k}^b \right]^T$	Specific force along body frame x,y,z axes at time instance k
$\left[f_{X,k}^N, f_{Y,k}^N, f_{Z,k}^N \right]^T$	Specific force along navigation frame X,Y,Z axes at time instance k
$\left[a_{X,k}^N, a_{Y,k}^N, a_{Z,k}^N \right]^T$	Acceleration along navigation frame X,Y,Z axes at time instance k
$\left[v_{X,k}^N, v_{Y,k}^N, v_{Z,k}^N \right]^T$	Velocity along navigation frame X,Y,Z axes at time instance k
$\left[p_{X,k}^N, p_{Y,k}^N, p_{Z,k}^N \right]^T$	Position along navigation frame X,Y,Z axes at time instance k
$\left[\sigma_{\omega x}, \sigma_{\omega y}, \sigma_{\omega y} \right]^T$	Standard deviation of white Gaussian noise on x,y,z axes of gyroscope

$\left[\sigma_{ax}, \sigma_{ay}, \sigma_{az}\right]^T$	Standard deviation of white Gaussian noise on x, y, z axes of accelerometer
$\left[AP_X, AP_Y, AP_Z\right]$	AP's coordinates
$\left[\psi_x, \psi_y, \psi_z\right]^T$	Angle rotated along body frame x, y, z axes
$\Delta\psi_{x,j}$	Angle rotated along body frame x axis during cluster j
$\mathbf{C}_{b,k}^N$	Coordinates frame transformation matrix at time instance k
n_p	Path loss exponent
t_i	Time at i^{th} second
RSS_i^-	Measured RSS at t_i
RSS_i^o	True RSS at t_i
RSS_i'	Improved RSS at t_i
ΔRSS_i	RSS difference between t_{i-1} and t_i
RSS_r	Mean RSS value at reference distance
d_r	Reference distance
d_i^o	True distance between user and APs at t_i
d_i^-	Maximum likelihood estimate of d_i^o based on RSS_i^-
d_i'	Maximum likelihood estimate of d_i^o based on RSS_i'
σ_{ins}	Standard deviation of INS localization error

σ_{RSS}	Standard deviation of simulated RSS
G	Gravitational acceleration
$\bar{W}_{x,j}^b$	Moving average angular velocity on x axis during cluster j
Δt	Sampling interval
(i, j)	i^{th} row and j^{th} coloum element of a matrix
$ \bullet $	Absolute value of the argument
$E(\bullet)$	Expectation of the argument

Chapter 1

Introduction

1.1 Background and Motivation

Localization and tracking have become extremely important due to their various applications in cellular systems, military guidance systems, and asset-tracking systems. In the United States, the Enhanced 911 (E-911) service mandated by the Federal Communication Commission (FCC) over cellular systems is required to estimate the position of an emergency caller with an error of less than 100 meters [1]. Thanks to the rapidly increasing number of available sensors, localization technologies have developed swiftly, and the demand for location-based services has grown significantly [2]-[7]. Global navigation satellite systems (GNSS), such as the Global Positioning System (GPS), GLONASS, BEIDOU, and Galileo, provide good localization accuracy in many outdoor scenarios [5]. However, accuracy degrades severely in GNSS-poor environments, such as indoor locations, urban canyons, and underground, due to channel fading, low signal-to-noise ratios, and multipath propagation [8]. Developing accurate localization techniques for those environments, especially indoor environments, would not

only have great commercial value; it would also benefit soldiers, security personnel, and first responders in hostile environments.

Tremendous progress has been made in improving indoor wireless localization performance over the last decade [9]-[50]. Infrastructure-based indoor localization systems utilize various types of existing wireless infrastructure, such as wireless local area networks (WLAN) [13]-[14], radio-frequency identification (RFID) [15], [17]-[18], and ultra-wideband (UWB) [19]-[21]. Generally, infrastructure-based localization schemes can be divided into geometrical and statistical approaches. Geometrical approaches make use of information such as time of arrival (TOA) [22]-[25], time difference of arrival (TDOA) [26]-[27], angle of arrival (AOA) [28]-[31], and received signal strength (RSS) [32]-[40]. TOA and TDOA techniques, for instance, convert time to distance, and the RSS method uses a propagation model to convert signal strength to distance to locate users. The AOA method estimates the angle of a signal to calculate its position. Among statistical approaches, RSS fingerprinting is one of the most widely used [41]-[45]. This requires a training stage to construct an RSS database at a predetermined location; a pattern recognition technique is then applied when localization is needed.

Conventional wireless localization systems use TOA, AOA, TDOA, and RSS information to determine subjects' positions. In general, the localization is accurate when line of sight (LOS) signal path or accurate propagation model is available. However, in indoor environments, these techniques suffer from poor localization accuracy, due to the multipath effect and non-line of sight (NLOS)

signal propagation. Further, the channel parameters used in RSS-based localization systems are environment-dependent, which also degrades localization performance. Much research has been carried out to mitigate the error caused by the multipath effect and NLOS signal propagation [46]-[50].

Unlike infrastructure-based indoor localization systems, infrastructure-less indoor localization systems, such as the inertial navigation system (INS), do not need sensors to be pre-installed. Inertial navigation is a navigation technology that makes use of measurements from an inertial measuring unit (IMU), which normally includes accelerometers, gyroscopes, magnetometers, and pressure sensors. These techniques are also known as “dead reckoning,” as the IMU’s velocity, location, and orientation relative to previous time instances are constantly tracked. IMU sensors are most common in aerospace applications, and most commonly involve bulky gimbals or laser gyroscopes. With the development of micro-machined electromechanical systems (MEMS), it is possible to make small, low-cost IMUs that can feasibly be carried by humans [60]-[62]. One category of pedestrian dead reckoning (PDR) is step-based PDR, which estimates step length and heading angle at each detected step to calculate the user’s location [63]-[75]. Another is strapdown INS PDR, which transforms and integrates coordinates based on tri-axial accelerations and angular velocity measurements to compute a user’s location and orientation [76]-[95]. Localization using a step-based PDR scheme involves step detection and gait-length estimations, which are error-prone due to users’ unique walking patterns and gait length. Strapdown INS mechanisms do not have such problems, as they register accelerations and rotations on all three axes, which in theory can be used

to calculate any location and orientation in three-dimensional space. Pressure sensors will not provide valuable information if the user is tracked across only one floor of a building, while magnetometers are affected by magnetic disturbance in many indoor environments. Therefore, in this thesis, a strapdown INS that leverages accelerometers and gyroscopes is used as part of the proposed localization system.

MEMS IMUs are subject to different types of noise, which have been characterized by a number of previous researchers [96]-[107]. The propagation of measurement error through the projection and integration of random noise causes location and orientation errors to accumulate rapidly [108]. To mitigate this potential for error, the use of zero velocity updates (ZUPT) with extended Kalman filters (EKF) has been proposed [76]-[95], [110]. ZUPT requires users to tie the IMU to one foot, so that zero-velocity measurements can be leveraged to continuously calibrate it. Various fusion systems, such as INS with RSS, INS with TOA, and INS with mapping, have been extensively explored in conjunction with ZUPT [82]-[91]. To further improve localization accuracy, particle filtering (PF) has been used to model nonlinear systems and to reduce errors caused by INS [92], [93], [110]-[118]. However, the computational complexity of PF and the frequent unavailability of accurate maps make implementation of the fusion system challenging. Among all available fusion systems, INS and RSS fusion systems show the best potential, especially given that RSS systems are readily available if not ubiquitous nowadays, and that a signal is usually still available even when LOS is not present.

1.2 Objective and Contributions of the Thesis

The objective of the research described here was to develop novel and efficient localization schemes that use RSS and INS instead of ZUPT, and that perform well when the initial orientation is unknown and when the user holds the IMU in a more natural position. As part of this research, simulations were performed in different environments with various access point (AP) setups and initial orientation angles.

The main contributions of this thesis are:

- 1) Deriving a closed form equation to directly analyze acceleration noise characteristics along the navigation frame. Stand-alone INS localization root mean square error (RMSE) is derived; this not only helps users evaluate the quality of the IMU, but also provides information for setting a weighting factor in a fused system involving INS.
- 2) Proposing a weighted nonlinear RSS localization scheme with INS projection, based on an assumption of perfect knowledge of the IMU's initial orientation with respect to the navigation frame. Furthermore, a method is proposed to continuously mitigate the effect of random noise on gyroscopes.
- 3) Developing a technique to accurately estimate the initial orientation with respect to the navigation frame during the calibration stage. To further mitigate noise from the IMU, another method is introduced to continuously mitigate the effect of random noise on accelerometers.

1.3 Organization of the Thesis

The remainder of this thesis is organized as follows:

Chapter 2 presents a detailed review of the literature on inertial navigation and the error characteristics of MEMS IMUs. Different filtering techniques, such as EKF and PF, are reviewed and ZUPT is discussed.

Chapter 3 presents the derivation of a closed form equation to analyze stand-alone INS localization RMSE. The derivation breaks down the transformation matrix and includes up to the second order of Taylor expansion. The accuracy of the derivation is compared with Cramér-Rao lower bound (CRLB).

Chapter 4 presents a novel weighted nonlinear RSS localization scheme with INS projection, in the case when the IMU's initial orientation is known. Strapdown INS PDR utilizing ZUPT or PDR based on approximate step length has been densely researched. Strapdown INS PDR utilizing ZUPT can provide reasonable localization results even without fusing with other localization systems. Nevertheless, the IMU must be mounted on the user's foot to achieve best performance. Handheld PDR, in contrast, is generally considered more practical and easier to use. However, it requires a user-specific training phase to estimate step length and uses heading information for localization. In general, handheld PDR localization is not as accurate as ZUPT-based schemes, mainly due to the inaccuracy of the step length estimation model. In addition, it applies only to

forward movement. This thesis proposes a handheld strapdown INS PDR fused with an RSS localization system to avoid the limitations of ZUPT-based schemes and handheld PDR. The scheme combines adjacent RSS values with the help of INS. The weighting factor used to combine RSS values is laid out and the proposed technique for continuously mitigating the effect of random noise on gyroscopes is explained. The performance of the proposed localization scheme using the noise-mitigation technique is analyzed through simulations in three different indoor environments. Comparisons are made with existing INS and RSS fusion systems that use EKF.

The findings suggest that the estimation of the initial orientation angles will project gravitational acceleration onto the body frame of the x and y axes. Initial orientation angle estimations are the largest source of error in INS localization schemes. Chapter 5 demonstrates a technique for accurately estimating initial orientation angles. Another continuous random noise-mitigation technique is introduced and applied to accelerometers. The performance of the proposed localization scheme, incorporating these noise-mitigation techniques and the initial orientation estimation, is compared across three different indoor environments with existing INS and RSS fusion systems that use EKF. The proposed scheme was also tested on different routes at real environment, and performance comparisons are presented.

Finally, Chapter 6 concludes the thesis and indicates directions for future work.

Chapter 2

Literature Review

This chapter reviews RSS-based localization schemes and discuss inertial navigation fundamentals. Section 2.3 presents an analysis of pedestrian inertial navigation methods and Bayesian filters in particular; extended Kalman filtering and particle filtering models with ZUPT are also discussed. The chapter concludes with a discussion of state-of-the-art INS fusion systems.

2.1 RSS-based Localization Scheme

Among wireless localization systems, RSS-based localization has several advantages over other systems. These include low implementation cost and the simple measurements acquisition. It does not require the presence of LOS signal, instead an accurate propagation model is needed for good localization result. RSS-based localization systems can be divided into RF propagation approaches and RSS fingerprinting approaches. RF propagation approaches estimate the range between the mobile device (MD) and APs according to a propagation path-

loss model, then use these to perform multi-lateration to obtain the location [39], [51]. Techniques involving least squares (LS) are commonly seen in this approach. In RSS fingerprinting approaches, a training phase is necessary before localization. During this training, a database is constructed to record RSS values from APs at each predetermined training location. The MD location is then computed by matching measured RSS values with the training database.

Of the two approaches, RSS fingerprinting generally has better localization results. However, its performance suffers when the environment is less stable than in the training phase. Pattern recognition algorithms are widely used to compare measured RSS values with those stored in the database. [53] treats this as a classification problem, seeking to determine which location gives the maximum likelihood; [39] averages the coordinates of the K-nearest matches of known locations to determine a position. In [54], the authors expand the fingerprint vector's dimensions using channel impulse response. While these and other efforts have been made to improve localization accuracy, [34] also try to reduce the high training costs involved in such techniques by determining the optimum value for the time-space sampling period. [57] offers an adaptive recalibration technique that can be performed in real time.

In contrast, RF propagation approaches do not require a training phase, but their localization accuracy is affected by the estimation accuracy of the path-loss model parameters used in complex indoor environments. To address this issue, various authors have suggested ways to improve the estimation accuracy of the path-loss model parameters. [52] derived analytical propagation path loss

formulae for microcellular communications in urban street grid environments and [58] proposed a linear regression method to determine the RSS-distance relationship, while also considering the RSS measurements' correlations. [35] proposes both a direct and indirect localization scheme, with the direct one using weighted LS and the indirect one maximizing the joint probability distribution to estimate location. [59] estimates APs' locations based on measured RSS values such that the RF propagation approaches can operate without knowledge of the true location of APs. The localization accuracy of RF propagation approaches is still poor, however, in complex indoor environments where multipath is dominant and path-loss model parameters cannot be accurately estimated. In summary, when the environment is stable, RF propagation approaches generally perform worse than RSS fingerprinting approaches in terms of localization accuracy.

This discussion highlights both the advantages and disadvantages of using RSS-based localization systems. The most difficult challenge is to improve localization accuracy while avoiding a time- and labor-intensive training phase whenever possible.

2.2 Inertial Navigation Fundamentals

Inertial navigation is a relative localization and tracking technique, in which location and orientation are continuously tracked relative to a starting point; this

approach requires no external signals beyond the IMU measurements. The IMU measures the three-dimensional specific force and angular velocities of the device. Specific force is equivalent to the acceleration supplied by all external forces other than gravity [97]. Inertial navigation systems (INS) process IMU measurements to continuously track the device's orientation and location within a given frame of reference, which will be called the navigation frame throughout this thesis. In general, there are two types of INS: stable platform INS and strapdown INS [60]. Stable platform INS places accelerometers and gyroscopes on a platform that is already aligned with the navigation frame. Gimbals are mounted to allow free rotation along all three axes. The gimbals detect the platform's rotation relative to the navigation frame and then automatically adjust to counteract any misalignments, thereby ensuring that the platform remains aligned with the navigation frame. Acceleration due to gravity will be subtracted directly in the navigation frame. Temporal integration is performed to obtain the velocity and location of the navigation frame. In strapdown INS, by contrast, the accelerometers and gyroscopes are fixed along its own frame of reference, also known as the body frame. To understand the orientation of the IMU relative to the navigation frame, the angular velocity measured by the gyroscope is used to constantly update the body frame's orientation. As a result, body frame specific force, as measured by the accelerometers, is transformed into a navigation frame. After acceleration due to gravity has been subtracted, the true acceleration of the IMU is acquired. Velocity and location along the navigation frame can be computed by doing simple integration with respect to time. Figure 2.1 provides an illustration of the navigation frame and strapdown INS body frame. The body

frame will rotate following the rotation of the IMU while the navigation frame remains fixed with respect to the environment.

Figure 2.1 Illustration of the navigation frame and strapdown INS body frame

Stable platform INS is more mechanically complex, and is typically used on aircraft or submarines. By contrast, strapdown INS is smaller and lighter due to its simpler mechanical construction, but more computationally complex. Despite this complexity, its size advantages make it more appealing than stable platform systems, especially as computing power grows according to Moore's law. The advances in MEMS construction have further reduced the size of strapdown INS, such that humans can now carry the device relatively easily [61], [62].

MEMS IMU's portability comes with a tradeoff: a large amount of noise. Both deterministic and stochastic noises have been identified [96]-[109]. Deterministic noise, such as initial bias and scale factors, can be estimated and compensated for before the system is used for navigation. A handful of techniques have been proposed to estimate deterministic noise. [107] uses turntables and flight simulators for more accurate estimation, while [102]-[104]

estimate deterministic noise without using external equipment. Some companies, such as Xsens, perform calibration procedures before shipping out their products. Stochastic or random noise, however, such as angle/velocity random walk and bias instability, perturbs the IMU sensors once the device is turned on.

Angle/velocity random walk can be modeled as white Gaussian noise in accelerometer and gyroscope measurements, and bias instability can be modeled as a random walk process in bias [96]. The characteristics of random noise are normally captured by Allan variance or Allan standard deviation techniques [105], [106]. The standard deviation of various random noises can be extracted from the Allan standard deviation plot, for use in deriving localization error growth or implementing fusion systems.

An infrastructure-less localization system such as INS is accurate in the initial stages of movement, but its localization accuracy deteriorates as time goes on. In contrast, infrastructure-based localization techniques such as RSS-based localization have the advantage of statistically time-independent localization accuracy. However, the accuracy of this technique is environmentally dependent and it requires RSS values from at least three APs and accurate propagation model. In this thesis, an RSS-based localization technique is fused with INS, to harness the advantages of infrastructure-based and infrastructure-less schemes, and thus achieve better localization results than is possible using either scheme individually.

2.3 Pedestrian Inertial Navigation

Pedestrian initial navigation is also known as pedestrian dead reckoning (PDR). PDR systems fall into two main categories: step-based PDR [63]-[75] and strapdown INS based PDR [76]-[95].

2.3.1 Step-based PDR

The major building blocks of step-based PDR are step detection, step or gait length estimation, and heading estimation. Various step-detection techniques have been proposed [67]-[73]. [71] and [73] use peaks in the signal to detect steps. Step-length estimation performance varies with the algorithms used. Individual characteristics can affect performance as well, as gait lengths, walking patterns, and walking speeds can differ tremendously between subjects. [68] suggests step length is influenced by the walking frequency, variance of the accelerometer signals during one step period, ground inclination, etc. [74] presents an on-the-fly training process to estimate step length. However, this system takes a significant amount of time to train and does not self-adjust if the user changes his or her step length or walking speed mid-course. Headings are normally obtained through magnetometer measurements. The major limitation is magnetic disturbance, which can seriously deteriorate heading information, as shown in [70]. Gyroscope values provide a relative heading with respect to previous steps. If no noise-mitigation techniques are applied or if the initial orientation of the

gyroscope is unknown, the heading information provided will not be helpful for step-based PDR. Apart from these issues, step-based PDR assumes that users only walk in the direction they are facing. Unusual movements, such as walking backwards or side-to-side, are not handled well by step-based PDR. Step-based PDR can operate with low-grade IMU sensors, but it must be fused with other more accurate localization systems to reduce errors.

2.3.2 Strapdown INS PDR

Strapdown INS PDR systems use a full tri-axial INS for localization and tracking. This can track all types of movement in three-dimensional space, thus placing fewer constraints on users' movements. Localization errors, however, accumulate rapidly due to the noise in IMU sensors discussed in Section 2.2. Various publications [97]-[99] have shown that noise generated by gyroscopes and incorrect alignment information are the major sources of localization error, as incorrect gravitational acceleration is projected into the body frame axes. Zero velocity updating (ZUPT) has been proposed as a way of mitigating the error [76].

ZUPT requires users to mount the IMU on their foot, and takes advantage of the fact that the foot's velocity is zero when it is grounded; it thus provides an external measurement that can be used to correct localization and orientation tilt errors [76]-[95]. This technique is most commonly implemented using Bayesian filters, such as EKF and PF, to trace correlations between velocity, location, and

orientation information. EKF is preferred when efficiency is the main concern. The general form of EKF is shown in Appendix A. [81] illustrates a detailed implementation of ZUPT-based tracking using a 15-element error-state vector EKF. ZUPT can reduce localization error significantly; however, it fails to provide heading information and, as a result, localization errors still accumulate with time [76], [81], [88]. [110] argues that using ZUPT for estimations may introduce new calibration errors, as the actual velocity may not be exactly zero during the grounded period. To further improve tracking accuracy, two-level filtering systems have been introduced, using EKF and PF as lower- and higher-level filtering systems, respectively [90], [113]-[115]. PF uses environmental obstacles such as walls to confine particles' movement. Different resampling algorithms [119], [120] and wall-crossing detection methods [82], [92] have been proposed to support PF. However, computational complexity is still a burden when the particles' sample size is too large. Furthermore, PF relies heavily on map constraints, which makes it challenging to implement in cases of inaccurate mapping or in environments with scarce physical constraints, such as open halls.

State-of-the-art INS localization and tracking systems are fusion systems, in which INS is integrated with various other system types, among which RSS-based systems [75], [82], [88], [93], [115]-[116] are the most commonly used, for the reasons detailed in Section 2.1. RSS-based systems use RSS either from RFID tags or from Wi-Fi APs. [88] propose a tight-coupling fusion system. After ZUPT is applied, this system uses the calculated range between RFIDs to the INS location and the estimated range at that time using a path-loss model. The range differences are then fed into EKF to mitigate errors in orientation and location.

The error growth is about 10 times slower than in ZUPT-only systems, and the error is eventually bounded by the RFID ranging system.

RSS-based and inertial navigation system—representing infrastructure-based and infrastructure-less localization schemes, respectively—are reviewed in this chapter. The next chapter presents a detailed characterization of various types of noise, such as angle/velocity random walk and bias instability, based on Allan standard deviation methodology. Based on this characterization, noise parameters are exploited to set the weighting factor for the fusion with RSS-based systems outlined in chapters 4 and 5.

Chapter 3

Inertial Navigation System

Localization Error Analysis

INS has received substantial attention in the indoor localization and tracking sector in recent years, due to the multiple benefits discussed in Chapter 2. Advances in micro-electromechanical systems (MEMS) technology have led to the development of rugged, low-cost, small, lightweight inertial devices, which allow them to be carried by pedestrians. However, various sources of error in IMUs cause orientation and localization errors to accumulate during the IMU-active period. Thus, understanding the theoretical background for localization performance not only helps users to evaluate the quality of a given IMU, but also sets localization error benchmarks for different paths and moving patterns, before experiments are conducted using a real device. In addition, the standard deviation study of localization error can provide valuable information for fusion systems, such as EKF and the scheme proposed in the following chapter.

In this chapter, the effect of white Gaussian noise on the INS is investigated. The standard deviations of Gaussian noise in tri-axial accelerometers and gyroscopes are acquired through the Allan standard deviation technique. A theoretical analysis of INS localization RMSE along three local navigation frame axes is presented. The second order of the Taylor expansion approximation for the transformation matrix is used. Finally, comparisons are made with Cramér-Rao lower bound (CRLB) for INS perturbed by the same white Gaussian noise.

3.1 Theory and Formulation

The strapdown navigation algorithm and its localization RMSE are analyzed in this section. Body frame specific force $\mathbf{a}_{xyz}^b(t)$ and angular velocity $\boldsymbol{\omega}_{xyz}^b(t)$ undergone by the device along the time dimension are used to compute navigation frame acceleration $\mathbf{a}_{XYZ}^N(t)$, velocity $\mathbf{v}_{XYZ}^N(t)$, and position $\mathbf{p}_{XYZ}^N(t)$. These values are measured at a fixed sampling rate, which is 100 Hz for the device used in this thesis. In this way, $\mathbf{a}_{XYZ,k}^N$, $\mathbf{v}_{XYZ,k}^N$, and $\mathbf{p}_{XYZ,k}^N$, representing discrete time values at time instance k , are derived. These discrete time values form a state vector that transforms the localization to a state machine, which will be implemented using EKF.

3.1.1 INS Localization Equation

The gyroscope in an MEMS IMU measures angular velocity, whereas the accelerometer measures the specific force applied to the IMU. The measurements are made along the IMU's body frame, because the inertial sensors are rigidly attached to its body. In this thesis, the local navigation frame is the reference frame, in which two axes are fixed along the direction of the environment and the third points locally upward. The INS calculation must be initialized; this initial state is assumed to be known, in terms of the orientation, velocity, and position of the device with respect to the navigation frame. Hence, the origins of the two coordinates are not the same; however, they can be connected through the initial known state and transformation matrix $\mathbf{C}_b^N$. This is illustrated in Figure 3.1. The local navigation frame is a valid approximation of the inertial frame, given that the tracking period is relatively short and MEMS IMU gyroscope errors are significantly larger than the error introduced by the earth's rotation.

Figure 3.1 Indoor floorplan, navigation frame (blue), and body frame (green)

The three-dimensional IMU measurements at time k can be written as $\boldsymbol{\omega}_{xyz,k}^b = [\omega_{x,k}^b, \omega_{y,k}^b, \omega_{z,k}^b]^T$ and $\mathbf{a}_{xyz,k}^b = [a_{x,k}^b, a_{y,k}^b, a_{z,k}^b]^T$ to represent the tri-axial angular velocity and specific force, respectively. The superscript b indicates the body frame. The navigation frame acceleration, velocity, and position at time k are given as $\mathbf{a}_{XYZ,k}^N = [a_{X,k}^N, a_{Y,k}^N, a_{Z,k}^N]^T$, $\mathbf{v}_{XYZ,k}^N = [v_{X,k}^N, v_{Y,k}^N, v_{Z,k}^N]^T$, and $\mathbf{p}_{XYZ,k}^N = [p_{X,k}^N, p_{Y,k}^N, p_{Z,k}^N]^T$, respectively. The superscript N indicates the navigation frame, and X , Y , and Z represent the three local frame axes. The accelerations on the frames with different coordinates are connected through transformation matrix $\mathbf{C}_b^N$, which transforms body frame measurements $\mathbf{a}_{xyz}^b$ onto the navigation frame. The rate with which $\mathbf{C}_b^N$ changes needs to be tracked when updating the orientation of the inertial sensors. It is given as [60], [97]:

$$\dot{\mathbf{C}}_{b,k}^n = \lim_{\Delta t \rightarrow 0} \frac{\mathbf{C}_{b,k+1}^n - \mathbf{C}_{b,k}^n}{\Delta t} \quad (3.1)$$

where Δt is the INS sampling interval of 0.01 seconds between time instance k and $k+1$.

In addition, $\mathbf{C}_{b,k+1}^n$ can be expressed as a product of two matrices, as follows:

$$\mathbf{C}_{b,k+1}^n = \mathbf{C}_{b,k}^n \mathbf{A}_k \quad (3.2)$$

where $\mathbf{A}_k$ is the rotation matrix that relates the body frame at t_k to t_{k+1} .

The complete expression of $\mathbf{A}_k$ can be given as a product of three matrices:

$$\begin{aligned} \mathbf{A}_k &= \begin{bmatrix} \cos \psi_{z,k} & -\sin \psi_{z,k} & 0 \\ \sin \psi_{z,k} & \cos \psi_{z,k} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \psi_{y,k} & 0 & \sin \psi_{y,k} \\ 0 & 1 & 0 \\ -\sin \psi_{y,k} & 0 & \cos \psi_{y,k} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_{x,k} & -\sin \psi_{x,k} \\ 0 & \sin \psi_{x,k} & \cos \psi_{x,k} \end{bmatrix} \\ &= \begin{bmatrix} \cos \psi_{y,k} \cos \psi_{z,k} & -\cos \psi_{x,k} \sin \psi_{z,k} & \sin \psi_{x,k} \sin \psi_{z,k} \\ \cos \psi_{y,k} \sin \psi_{z,k} & -\cos \psi_{x,k} \cos \psi_{z,k} & -\sin \psi_{x,k} \cos \psi_{z,k} \\ -\sin \psi_{y,k} & \sin \psi_{x,k} \cos \psi_{y,k} & \cos \psi_{x,k} \cos \psi_{y,k} \end{bmatrix} \end{aligned} \quad (3.3)$$

where $\psi_{x,k}$, $\psi_{y,k}$, and $\psi_{z,k}$ are the angles of rotation according to the body frame between time t_k and time t_{k+1} .

Because the measurements are taken every 0.01 s, small angle approximation can be used for pedestrian movement, so that $\cos \psi \approx 1$ and $\sin \psi \approx \psi$ [81], [97]. For small angle rotations, $\mathbf{A}_k$ can be written as:

$$\mathbf{A}_k = \begin{bmatrix} 1 & -\psi_{z,k} & \psi_{y,k} \\ \psi_{z,k} & 1 & -\psi_{x,k} \\ -\psi_{y,k} & \psi_{x,k} & 1 \end{bmatrix} \quad (3.4)$$

$$= \mathbf{I} + \boldsymbol{\Psi}_{xyz,k}$$

where $\mathbf{I}$ is a 3×3 identity matrix.

Now we have:

$$\dot{\mathbf{C}}_{b,k}^n = \mathbf{C}_{b,k}^n \lim_{\Delta t \rightarrow 0} \frac{\boldsymbol{\Psi}_{xyz,k}}{\Delta t} \quad (3.5)$$

$$\text{where } \lim_{\Delta t \rightarrow 0} \frac{\delta \boldsymbol{\Psi}}{\Delta t} = \boldsymbol{\Omega}_k = \begin{bmatrix} 0 & -\omega_{z,k}^b & \omega_{y,k}^b \\ \omega_{z,k}^b & 0 & -\omega_{x,k}^b \\ -\omega_{y,k}^b & \omega_{x,k}^b & 0 \end{bmatrix}.$$

The solution to the preceding equation is [60], [97]:

$$\mathbf{C}_{b,k+1}^N = \mathbf{C}_{b,k}^N \exp \left(\int_{\Delta t} \boldsymbol{\Omega}_k dt \right) \quad (3.6)$$

Equation (3.6) can be further expanded using the rectangular rule after introducing another skew-symmetric matrix $\mathbf{B}$:

$$\mathbf{B}_k = \int_{\Delta t} \boldsymbol{\Omega}_k dt = \begin{bmatrix} 0 & -\omega_{z,k}^b \Delta t & \omega_{y,k}^b \Delta t \\ \omega_{z,k}^b \Delta t & 0 & -\omega_{x,k}^b \Delta t \\ -\omega_{y,k}^b \Delta t & \omega_{x,k}^b \Delta t & 0 \end{bmatrix} \quad (3.7)$$

The rectangular rule is used due to the high sampling frequency and high noise level inside commercial IMUs. Thus it is a general practice to use the rectangular rule to gain acceptable accuracy and computational efficiency [60], [81], [87], and [97]. By substituting Equation (3.7) into (3.6) and performing the Taylor expansion, Equation (3.6) can be expressed as:

$$\mathbf{C}_{b,k+1}^N = \mathbf{C}_{b,k}^N \left(\mathbf{I} + \mathbf{B}_k + \frac{\mathbf{B}_k^2}{2!} + \frac{\mathbf{B}_k^3}{3!} + \dots \right) \quad (3.8)$$

Thus, the navigation frame acceleration at time k can be expressed as

$$\begin{aligned}\mathbf{f}_{XYZ,k}^N &= \mathbf{C}_{b,k}^N \mathbf{a}_{xyz,k}^b \\ \mathbf{a}_{XYZ,k}^N &= \mathbf{f}_{XYZ,k}^N - [0, 0, G]^T\end{aligned}\quad (3.9)$$

where G is the gravitational acceleration and $\mathbf{f}_{XYZ,k}^N$ is the specific force.

Once the acceleration is obtained, $\mathbf{v}_{XYZ}^N$ and $\mathbf{p}_{XYZ}^N$ are calculated using single and double integration with respect to time:

$$\mathbf{v}_{XYZ,k}^N = \mathbf{v}_{XYZ,0}^N + \int_0^{k\Delta t} \mathbf{a}_{XYZ,k}^N dt \quad (3.10)$$

$$\mathbf{p}_{XYZ,k}^N = \mathbf{p}_{XYZ,0}^N + \int_0^{k\Delta t} \mathbf{v}_{XYZ,k}^N dt \quad (3.11)$$

The analysis of variance of $\mathbf{a}_{XYZ}^N$ naturally serves as a stepping stone to the final location's RMSE.

3.1.2 Derivation of INS Localization RMSE

The transformation matrix can also be expressed solely with the known initial orientation $\mathbf{C}_{b,0}^N$ and angular velocity values, beginning at the start time, as:

$$\mathbf{C}_{b,k+1}^N = \mathbf{C}_{b,0}^N \prod_{i=1}^k \left(\mathbf{I} + \mathbf{B}_i + \frac{\mathbf{B}_i^2}{2!} + \frac{\mathbf{B}_i^3}{3!} + \dots \right) \quad (3.12)$$

where $\mathbf{C}_{b,0}^N$ is the initial orientation of the IMU.

Keeping up to the second order of the Taylor expansion remains valid when updating $\mathbf{C}_b^N$ in equation (3.12) if the user is holding the device in his or her

hands and pointing it in the direction in which he or she is moving. It is assumed that the device does not undergo large swings in random directions within a sampling period of 0.01 s, whereas small tilting angles that occur naturally when the user is moving forward are acceptable. The truncated version of $\mathbf{C}_b^N$ can be expressed as:

$$\mathbf{C}_{b,k+1}^N = \mathbf{C}_{b,0}^N \left(I + \sum_{i=1}^k \mathbf{B}_i + \sum_{i=1}^k \frac{\mathbf{B}_i^2}{2} + \sum_{j=2}^k \sum_{i=1}^{j-1} \mathbf{B}_i \mathbf{B}_j \right) \quad (3.13)$$

The compositions for $a_{X,k}^N$, $a_{Y,k}^N$, and $a_{Z,k}^N$ are similar, and $a_{X,k}^N$ is used for illustration. Substituting equation (3.13) into (3.9), a_X^N at time instance $k+1$ can be written in three parts, as:

$$\begin{aligned} \begin{bmatrix} a_{X,k+1}^N \\ a_{Y,k+1}^N \\ a_{Z,k+1}^N \end{bmatrix} &= \begin{bmatrix} \mathbf{C}_{b,k+1}^N(1,1) & \mathbf{C}_{b,k+1}^N(1,2) & \mathbf{C}_{b,k+1}^N(1,3) \\ \mathbf{C}_{b,k+1}^N(2,1) & \mathbf{C}_{b,k+1}^N(2,2) & \mathbf{C}_{b,k+1}^N(2,3) \\ \mathbf{C}_{b,k+1}^N(3,1) & \mathbf{C}_{b,k+1}^N(3,2) & \mathbf{C}_{b,k+1}^N(3,3) \end{bmatrix} \begin{bmatrix} a_{x,k+1}^b \\ a_{y,k+1}^b \\ a_{z,k+1}^b \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ G \end{bmatrix} \\ \Rightarrow a_{X,k+1}^N &= \mathbf{C}_{b,k+1}^N(1,1)a_{x,k+1}^b + \mathbf{C}_{b,k+1}^N(1,2)a_{y,k+1}^b + \mathbf{C}_{b,k+1}^N(1,3)a_{z,k+1}^b \end{aligned} \quad (3.14)$$

where i and j represent the row/column index of the matrix: $\mathbf{C}_{b,k+1}^N(i, j)$.

The three parts that combine to give $a_{X,k+1}^N$ are as follows:

$$\begin{aligned} &\mathbf{C}_{b,k+1}^N(1,1)a_{x,k+1}^b \\ &= \Delta t^2 \left(1 - \frac{\sum_{i=1}^k (\omega_{z,i}^b)^2 - \sum_{i=1}^k (\omega_{y,i}^b)^2}{2} - \sum_{j=2}^k \sum_{i=1}^{j-1} \omega_{z,i}^b \omega_{z,j}^b - \sum_{j=2}^k \sum_{i=1}^{j-1} \omega_{y,i}^b \omega_{y,j}^b \right) a_{x,k+1}^b \\ &= (\mathbf{C}_{b,k}^N(1,1) - \frac{\omega_{z,i}^{b\,2} - \omega_{y,i}^{b\,2}}{2} \Delta t^2 + \sum_{i=1}^{k-1} \omega_{z,i}^b \omega_{z,k}^b \Delta t^2 - \sum_{i=1}^{k-1} \omega_{y,i}^b \omega_{y,k}^b \Delta t^2) a_{x,k+1}^b \end{aligned}$$

$$\begin{aligned}
& \mathbf{C}_{b,k+1}^N(1,2)a_{y,k+1}^b \\
&= \left(-\sum_{i=1}^k \omega_{z,i}^b \Delta t - \frac{\sum_{j=2}^k \sum_{i=1}^{j-1} \omega_{y,i}^b \omega_{x,j}^b \Delta t^2}{2} - \sum_{j=2}^k \sum_{i=1}^{j-1} \omega_{y,i}^b \omega_{x,j}^b \Delta t^2 \right) a_{y,k+1}^b \\
&= (\mathbf{C}_{b,k}^N(1,2) - \omega_{z,k}^b \Delta t - \frac{\omega_{y,k}^b \omega_{x,k}^b}{2} \Delta t^2 - \sum_{i=1}^{k-1} \omega_{y,i}^b \omega_{x,k}^b \Delta t^2) a_{y,k+1}^b
\end{aligned}$$

$$\begin{aligned}
& \mathbf{C}_{b,k+1}^N(1,3)a_{z,k+1}^b \\
&= \left(-\sum_{i=1}^k \omega_{y,i}^b \Delta t - \frac{\sum_{j=2}^k \sum_{i=1}^{j-1} \omega_{z,i}^b \omega_{x,j}^b \Delta t^2}{2} - \sum_{j=2}^k \sum_{i=1}^{j-1} \omega_{z,i}^b \omega_{x,j}^b \Delta t^2 \right) a_{z,k+1}^b \\
&= (\mathbf{C}_{b,k}^N(1,3) - \omega_{y,k}^b \Delta t - \frac{\omega_{z,k}^b \omega_{x,k}^b}{2} \Delta t^2 - \sum_{i=1}^{k-1} \omega_{z,i}^b \omega_{x,k}^b \Delta t^2) a_{z,k+1}^b
\end{aligned}$$

Because of the high sampling rate, the rectangular rule was used again to calculate the navigation frame location. When initial speed is assumed to be zero, the location at time instance k is given as:

$$p_{X,k}^N = \sum_{i=1}^k (k-i+0.5) a_{X,i}^N \Delta t^2 \quad (3.15)$$

The three parts of a_X^N shown in equation (3.14) are correlated due to the accumulated angular velocity values. The variance of $p_{X,k}^N$ thus consists of two parts: the first is the variation in each time instance's acceleration, and the second is the covariance between each of the accelerations. This can be expressed as:

$$\begin{aligned}
Var(p_{X,k}^N) = & \sum_{i=1}^k (k-i+0.5)^2 Var(a_{X,i}^N) \Delta t^4 + \\
& 2 \sum_{j=3}^{k-1} \sum_{i=2}^{j-1} (k-i+0.5)(k-j+0.5) Cov(a_{X,i}^N, a_{X,j}^N) \Delta t^4
\end{aligned} \tag{3.16}$$

where $E(\bullet)$ stands for expectation and

$$Cov(a_{X,i}^N, a_{X,j}^N) = E(a_{X,i}^N \cdot a_{X,j}^N) - E(a_{X,i}^N)E(a_{X,j}^N)$$

The variance of the three parts in equation (3.14) and the covariance between them provide the variance of the navigation frame position in equation (3.16). No shortcuts were found when calculating the covariance between accelerations on the same axis at different time instances. Each individual component of $a_{X,i}^N$ and $a_{X,j}^N$ is correlated. Covariance between all parts at different time instances are calculated to complete the second part of equation (3.16). The final location RMSE at time k is expressed as:

$$RMSE_k = \sqrt{Var(p_{X,k}^N) + Var(p_{Y,k}^N) + Var(p_{Z,k}^N)} \tag{3.17}$$

3.2 Noise Characterization and Acquisition

In this section, various error sources inside the IMU are first described, followed by a detailed explanation of Allan standard deviations and how they can be acquired from the plot. Finally, the standard deviations of different types of noise are compared and discussed.

3.2.1 Error sources in MEMS IMU sensors

Various types of error sources in both gyroscopes and accelerometers have impacts on measurement accuracy. Deterministic noise does not change over time, which permits calibration and compensation before the device is used for navigation. Stochastic noise, however, must be modeled by appropriate stochastic processes. The major types of stochastic noise are discussed in what follows.

3.2.1.1 Thermo-mechanical Noise/Angle (Velocity)

Random Walk

MEMS gyroscopes and accelerometers are perturbed by thermo-mechanical noise. This noise fluctuates at a much higher rate than the sampling rate of inertial sensors, and the spectrum of such noise is approximately white [106]. Hence, the measurements are perturbed by a white noise sequence that is a zero-mean uncorrelated sequence of random variables with finite variance σ^2 . The result of integrating the white noise sequence in gyroscopes and accelerometer measurements is random walks in angle and velocity, respectively. Thus, this noise is also called angle random walk and velocity random walk. It is standard practice to assume that random noise follows a Gaussian distribution when it is modeled in an INS [60]. This noise is then modeled as an additive Gaussian white noise.

3.2.1.2 Flicker Noise/Bias Instability

The biases of MEMS sensors tend to wander over time due to flicker noise, even when the temperature of the device is kept constant. Flicker noise, also known as $1/f$ noise, is present in many systems' electronic components, and is usually observed at low frequencies [106]. It is often modeled as a random walk process [97]. Another sequence of white Gaussian noise is used to generate bias instability. The resulting error in the integrated accelerometer and gyroscope bias instability is a second-order random walk in velocity and angle.

3.2.1.3 Temperature Effect

Sensor self-heating or changes in the environment also produce small fluctuations in sensor bias other than bias instability. The relationship between bias and temperature is often highly nonlinear [97]. It is possible to compensate for the fluctuation if the IMU is equipped with a temperature sensor.

Generally speaking, angle/velocity random walk and bias fluctuation are the major error sources inside an IMU. The relative importance of each error source depends on the values of parameters extracted using the Allan standard deviation, which is explained next.

3.2.2 Noise Characteristic Analysis Using Allan Standard Deviation

There are three types of stochastic modeling techniques: the PSD approach in the frequency domain, correlation methods, and variance techniques. The PSD approach is difficult for non-system analysts to understand, and correlation methods are model-sensitive and work best with prior knowledge of a few terms of the model. Variance techniques differ in terms of signal processing techniques such as weighting function, window functions, etc. [106]. The Allan standard deviation used with variance techniques is a time-domain analysis technique that can be applied to any signal to determine the character of the underlying noise processes [105]. Assume that N is the total number of consecutive data points collected at a given sampling time Δt . All points are divided into clusters, each of which consists of n data points. Then the Allan variance is computed as [106]:

$$\sigma^2(T) = \frac{1}{2(N-2n)} \sum_{k=1}^{N-2n} \left[\frac{1}{T} \int_{t_k}^{t_k+T} M(t)dt - \frac{1}{T} \int_{t_{k+1}}^{t_{k+1}+T} M(t)dt \right]^2 \quad (3.18)$$

where $T=n\Delta t$, $t_{k+1}=t_k + \Delta t$, and $M(t)$ represents the measured data.

Different random processes can be identified by different gradients of the curve plotted on the Allan standard deviation plot, as shown in Figure 3.2.

Figure 3.2 A sample log-log plot of Allan standard deviation [106]

The Allan standard deviation plot shows that the angle/velocity random walk and bias instability noise can be identified, while other types of noise are more obscure. Angle/velocity random walk can be identified with a gradient of -0.5, whereas the bias instability noise has a gradient of 0. The device used in this thesis is the IMU Mtw from Xsens. The device was left stationary for 3 hours for data collection. The Allan standard deviation plots for the 3 hours of gyroscope and accelerometer data from the Xsens IMU are shown in Figures 3.3 and 3.4.

Figure 3.3 Gyroscopes: Allan standard deviation

Figure 3.4 Accelerometers: Allan standard deviation

The random walk parameter can be obtained directly by fitting a straight line through the slope and reading its value at $t = 1$. The angle unit used in Figure 3.4 is radians. The numerical value of the bias instability parameter is the value

taken when the flat region around the bottom begins [106]. The values of these parameters for gyroscopes and accelerometers are given in Tables 3-1 and 3-2.

Body frame	Angle Random Walk (ARW) Parameter	Bias Instability (BI) Parameter
x axis	$0.029^{\circ} / \sqrt{s}$	$6.7 \times 10^{-3} / s$
y axis	$0.029^{\circ} / \sqrt{s}$	$6.7 \times 10^{-3} / s$
z axis	$0.029^{\circ} / \sqrt{s}$	$4.7 \times 10^{-3} / s$

Table 3-1 Gyroscope noise sources

Body frame	Velocity Random Walk (VRW) Parameter	Bias Instability (BI) Parameter
x axis	$0.0015 m / s / \sqrt{s}$	$1.8 \times 10^{-3} m / s^2$
y axis	$0.0015 m / s / \sqrt{s}$	$1.8 \times 10^{-3} m / s^2$
z axis	$0.0015 m / s / \sqrt{s}$	$5.8 \times 10^{-3} m / s^2$

Table 3-2 Accelerometer noise sources

After extracting the parameters ARW, VRW, and BI from the Allan standard deviation plot, it is necessary to convert the parameters to obtain the σ of the white Gaussian noise that drives angle/velocity random walk and bias instability. Gyroscope and accelerometer random walk parameters are converted using the following equations [97]:

$$\sigma_{\omega} = \frac{ARW}{\sqrt{\Delta t}}, \quad \sigma_a = \frac{VRW}{\sqrt{\Delta t}} \quad (3.19)$$

where Δt is the INS sampling interval.

The bias instability parameters for the gyroscope and accelerometer are converted using the following equations [97]:

$$\sigma_{BI} = \frac{BI}{\sqrt{\frac{t}{\Delta t}}} \quad (3.20)$$

where t is the time when BI (the bias instability parameter) is taken from the Allan standard deviation plot.

The converted standard deviation numbers are given in Tables 3-3 and 3-4.

Body frame	ARW White Gaussian noise σ_{ω}	BI random walk $\sigma_{\omega, BI}$
x axis	$0.29^{\circ}/s$	$9.5 \times 10^{-5}^{\circ}/s$
y axis	$0.29^{\circ}/s$	$9.5 \times 10^{-5}^{\circ}/s$
z axis	$0.27^{\circ}/s$	$5.3 \times 10^{-5}^{\circ}/s$

Table 3-3 Gyroscope noise standard deviation

Body frame	VRW White Gaussian noise σ_a	BI random walk $\sigma_{a, BI}$
x axis	$0.015 m/s^2$	$5.7 \times 10^{-5} m/s^2$
y axis	$0.015 m/s^2$	$6.7 \times 10^{-5} m/s^2$
z axis	$0.036 m/s^2$	$1.9 \times 10^{-4} m/s^2$

Table 3-4 Accelerometer noise standard deviation

It is worth mentioning that the bias instability white Gaussian noise is respectively 3 orders and 4 orders smaller than the ARW and VRW white Gaussian noise. Even though bias instability has some impact on INS localization performance when the movement period is long, ARW and VRW white Gaussian noise are dominant when the period is short, if not effectively mitigated. Furthermore, in the fusion system proposed in the next chapter, trust in the INS solution decreases over time, which means the impact of noise inside the IMU becomes weaker as the fusion system leans more towards RSS localization

results. Thus, this thesis focuses on the effect of ARW and VRW white Gaussian noise on localization performance and various techniques are proposed in the following chapters to mitigate that effect.

3.3 INS EKF Error Propagation and Cramér-Rao Lower Bound

Bayesian filters provide a framework for estimating the state of a dynamic system based on noisy measurements from different sensors [124]. Thus they can be used to combine PDR with other measurements such as ZUPT or RSS values. The state of a PDR system normally contains the pedestrian's orientation, velocity and position. To make this state trackable, the dynamic systems investigated in this thesis, including PDR, are considered as first-order Markov processes, in which the current state depends only on the propagation of the previous state and measurements at the current state. Among all the variants of Bayesian filters, the Kalman filter is the most widely used to estimate the state of a linear system, largely due to its computational efficiency, whereas EKF is used to estimate the state of a non-linear system. The method for using EKF to estimate the errors of the PDR state, also known as the error-state, is illustrated in [76] and [81].

The true state of the IMU device at time instance k is defined according to its transformation matrix, velocity, and position with respect to the navigation frame:

$$[\mathbf{C}_k, \mathbf{v}_k, \mathbf{p}_k]$$

The INS solution provided by the equations described in Section 3.1.1 diverges from the true state due to the propagation of errors inside IMU measurements, and can be expressed as:

$$[\hat{\mathbf{C}}_k, \hat{\mathbf{v}}_k, \hat{\mathbf{p}}_k]$$

where the hat symbols are used to represent the INS solution.

Error-state vector $\delta\mathbf{x}$ is defined as the difference between the true state and INS solution, and it is shown as:

$$\delta\mathbf{x}_k = [\delta\boldsymbol{\psi}_k^T, \delta\mathbf{v}_k^T, \delta\mathbf{p}_k^T]^T \quad (3.21)$$

where

$$\delta\boldsymbol{\psi}_k = [\delta\psi_{X,k}, \delta\psi_{Y,k}, \delta\psi_{Z,k}]^T, \quad \delta\mathbf{v}_k = [\delta v_{X,k}, \delta v_{Y,k}, \delta v_{Z,k}]^T, \quad \text{and}$$

$\delta\mathbf{p}_k = [\delta p_{X,k}, \delta p_{Y,k}, \delta p_{Z,k}]^T$, representing the orientation error, velocity error, and position error with respect to the navigation frame.

The relationship between the true state and INS solution is then shown as [76], [81], [128]:

$$\mathbf{C}_k = \boldsymbol{\Phi}_k \hat{\mathbf{C}}_k \quad (3.22)$$

$$\mathbf{v}_k = \hat{\mathbf{v}}_k + \delta\mathbf{v}_k \quad (3.23)$$

$$\mathbf{p}_k = \hat{\mathbf{p}}_k + \delta\mathbf{p}_k \quad (3.24)$$

where

$$\boldsymbol{\Phi}_k = \begin{bmatrix} 1 & \delta\psi_{Z,k} & -\delta\psi_{Y,k} \\ -\delta\psi_{Z,k} & 1 & \delta\psi_{X,k} \\ \delta\psi_{Y,k} & -\delta\psi_{X,k} & 1 \end{bmatrix}$$

The estimated error state from EKF can be propagated as:

$$\delta \hat{\mathbf{x}}_k = \mathbf{F}_k \delta \hat{\mathbf{x}}_{k-1} \quad (3.25)$$

and its covariance matrix can be updated as:

$$\mathbf{P}_k = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{Q}_k \quad (3.26)$$

where $\mathbf{F}_k$ is the error state transition matrix and $\mathbf{Q}_k$ is the process noise covariance matrix.

According to [60] and [76], $\delta \mathbf{v}_k$ can be updated using:

$$\delta \mathbf{v}_k = \delta \mathbf{v}_{k-1} + S(\mathbf{f}_{XYZ,k}^N) \delta \boldsymbol{\psi}_{k-1} \Delta t \quad (3.27)$$

where

$$S(\mathbf{f}_{XYZ,k}^N) = \begin{bmatrix} 0 & -f_{Z,k}^N & f_{Y,k}^N \\ f_{Z,k}^N & 0 & -f_{X,k}^N \\ -f_{Y,k}^N & f_{X,k}^N & 0 \end{bmatrix}$$

This $\delta \mathbf{v}_k$ updating equation causes orientation errors $\delta \boldsymbol{\psi}_{k-1}$ to propagate into linear velocity errors in the navigation frame, and thus lets the EKF construct the appropriate correlations in the covariance matrix [76]. Hence, $\mathbf{F}_k$ is expressed as [76], [81]:

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \Delta t S(\mathbf{f}_{XYZ,k}^N) & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Delta t \mathbf{I} & \mathbf{I} \end{bmatrix} \quad (3.28)$$

where $\mathbf{I}$ is a 3×3 identity matrix and $\mathbf{0}$ is a 3×3 matrix containing all zeros.

The process noise covariance, $\mathbf{Q}_k$, is adaptively tuned to express the INS's error growth due to sensor noise [76]:

$$\mathbf{Q}_k = \begin{bmatrix} \mathbf{T}_k & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (3.29)$$

where T_k and V_k represent errors in the gyroscope and accelerometer, respectively, and

$$\mathbf{T}_k = \Delta t^2 (\mathbf{C}_{b,k}^N) \begin{bmatrix} (\sigma_{\omega_x}^b)^2 & 0 & 0 \\ 0 & (\sigma_{\omega_y}^b)^2 & 0 \\ 0 & 0 & (\sigma_{\omega_z}^b)^2 \end{bmatrix} (\mathbf{C}_{b,k}^N)^T$$

$$\mathbf{V}_k = \Delta t^2 (\mathbf{C}_{b,k}^N) \begin{bmatrix} (\sigma_{a_x}^b)^2 & 0 & 0 \\ 0 & (\sigma_{a_y}^b)^2 & 0 \\ 0 & 0 & (\sigma_{a_z}^b)^2 \end{bmatrix} (\mathbf{C}_{b,k}^N)^T.$$

σ_{ω}^b and σ_a^b represent the standard deviations of the white Gaussian noise perturbing the IMU sensors' measurement, which are obtained using the Allan standard deviation technique.

The CRLB provides a lower bound for the mean square error. This is used as a benchmark to assess the performance of the suboptimal filtering algorithms. The Bayesian version of CRLB is also called posterior CRLB. For each time instance k , the CRLB is given by:

$$E \left\{ [g(\mathbf{Z}_k) - \mathbf{X}_k] [g(\mathbf{Z}_k) - \mathbf{X}_k]^T \right\} \geq \mathbf{J}_k^{-1} \quad (3.30)$$

where $\mathbf{Z}_k$ represents all measurements up to time instance k , $g(\mathbf{Z}_k)$ is an unbiased estimator of $\mathbf{X}_k$, and $\mathbf{J}_k$ is the Fisher information matrix (FIM).

Based on equation (3.30), the inverse of FIM is the lower performance benchmark. In order to find the lower benchmark for a propagating state machine such as EKF, recursive FIM is proposed. The recursive updating of FIM tailored for the EKF model is given as [93], [105]:

$$\mathbf{J}_{k+1} = \mathbf{J}_k^Z + \left\{ (\mathbf{Q}_k)^{\frac{1}{2}} \left[(\mathbf{Q}_k)^{\frac{1}{2}} \right]^T + \mathbf{F}_k \mathbf{J}_k^{-1} \mathbf{F}_k^T \right\}^{-1} \quad (3.31)$$

where J_k^Z is the FIM for the external measurements other than IMU.

However, there are no external measurements considered if the stand-alone INS performance is investigated: J_k^Z will be zero throughout the calculation. If we set J_0^{-1} to the value of initial covariance matrix of EKF P_0 , equation (3.31) is identical to equation (3.26). More generally, it can be shown that posterior CRLB for the linear Gaussian filtering problem is equivalent to the covariance matrix of the Kalman filter [124], [125]. According to the definition of $\mathbf{P}_k$, the position variances along the navigation frame XYZ axes are stored at $\mathbf{P}_k(7,7)$, $\mathbf{P}_k(8,8)$, and $\mathbf{P}_k(9,9)$ respectively. Thus, the 3D position RMSE can also be represented using equation (3.26) as

$$RMSE_k^{2nd} = \sqrt{\mathbf{P}_k(7,7) + \mathbf{P}_k(8,8) + \mathbf{P}_k(9,9)} \quad (3.32)$$

where i and j represent the row/column index of the 9×9 matrix $\mathbf{P}_k$.

In chapters 4 and 5, $\mathbf{P}_k(7,7)$ and $\mathbf{P}_k(8,8)$ are used to calculate 2D INS localization RMSE in a similar manner to equation (3.17) and labeled as σ_{ins} in the weight function. $\mathbf{Q}_k$ is formed by the gyroscope noise variance, the accelerometer noise variance, and $\mathbf{C}_b^N$. First-order and small-angle

approximations are used to derive the Kalman filter. The performance of this Kalman filter not only depends on the IMU signal noise, but also on how we choose $\mathbf{C}_b^N$: different approximations of $\mathbf{C}_b^N$ yield different covariance matrices.

3.4 Results and Comparison

Two paths are used here to test the accuracy of the derived RMSE equations. The first is a closed loop. It is assumed that the initial orientation of the IMU is known. As shown in Figure 3.5, the IMU starts moving from the stationary position at point $[0, 0]$ and proceeds in a counterclockwise loop. The IMU accelerates at 1 m/s^2 eastward for one second, followed by moving 12 m in a straight line at a constant speed. For the rest of the loop, the speed remains constant. There are four 90° turning sections, with a radius of 7.5 m , during which the z -axis angular velocity is set at 0.1333 rad/s and the centripetal acceleration sensed by the IMU's y -axis is set at 0.1333 m/s^2 to keep the IMU on the designated path at a constant speed. The IMU eventually returns to the starting point.

Figure 3.5 IMU movement: Path 1

The simulation results for Path 1 are shown in Figure 3.6. In Figures 3.6 and 3.8, the black curve represents the derived positioning RMSE results as discussed in Section 3.1.2. The blue and green curves stand for positioning error CRLB, as elaborated in Section 3.3. The difference is that the blue curve represents the position error CRLB when $\mathbf{C}_b^N$ is updated using second-order approximation, whereas the green curve shows the results when the built-in Matlab function “expm” is used to update $\mathbf{C}_b^N$ without additional approximation. All curves remain very close up to the 50 second mark, when the IMU finishes the second turn. The estimated difference in error between the proposed method and CRLB (expm) is 1.5 m, 3% of the distance covered up to the 50 s mark. After the third turn, the difference is 10.4 m, 14.3% of the distance covered at that time. When one loop is complete, the positioning error of the proposed method reaches 186.2 m, whereas the CRLB (expm) is at 146.2 m. The CRLB (2nd order) begins to

deviate further from the derived second-order RMSE after 70 seconds. For the stand-alone INS, there is no correction from other sources.

Figure 3.6 Localization RMSE comparison: Path 1

Path 2, shown in Figure 3.7, involves more turns than Path 1. The initial alignment of the IMU remains the same as in Path 1. The IMU also begins to move from the stationary position at point $[0, 0]$ and accelerates at 1 m/s^2 along the X direction for 1 second. It maintains the same speed before decelerating back to a standstill at the end of the path. At the turning points, the centripetal acceleration and angular velocity are adjusted accordingly to satisfy the linear velocity. After traveling to point $[8.5, 0]$, the IMU travels a circular path with a 1.5 m radius in a clockwise direction. It decelerates -1 m/s^2 eastward and stops at $[10, 0]$.

Figure 3.7 IMU movement: Path 2

The simulation results for Path 2 are shown in Figure 3.8. The curves start to diverge at around 12 seconds, which is also when the IMU is just past the four relatively sharp 90° turning points. The split accelerates during the 360° turn. However, the derived RMSE curve remains close to the CRLB (expm). When Path 2 is completed, the positioning error of the proposed derivation reaches 6.8 *m*, whereas the CRLB (expm) has an error of 5.2 *m*. The difference is 6.5% of the total distance traveled. This suggests that the second-order approximation is reasonable for both paths.

Figure 3.8 Localization RMSE comparison: Path 2

3.5 Conclusion

The variance analysis of $\mathbf{p}_{XYZ}^N$ at different time instances is integral to setting weighting factors when INS is fused with other localization systems. In this chapter, a method has been proposed to calculate the growth in stand-alone INS positioning error. The proposed method breaks down the transformation matrix and provides a theoretical closed-form equation to analyze the noise in $\mathbf{a}_{XYZ}^N$ caused by the body frame accelerometer and gyroscope. The error in $\mathbf{p}_{XYZ}^N$ is then acquired by performing a double integration of $\mathbf{a}_{XYZ}^N$ over the time of traveling. The proposed method gives reasonable results for both paths in the test. It also serves as an alternative to posterior CRLB, as it depends on the values assigned to $\mathbf{C}_b^N$.

Overall, in this chapter the strapdown INS localization equations are shown first, followed by the localization error analysis under two scenarios. Although the errors during the initial 10 seconds of both scenarios are below 1 *m*, which shows that INS localization is valuable, the error exceeds 100 *m* at 98 seconds, indicating that localization results are not satisfactory. This indicates the need to improve localization accuracy through fusion with another system, such as RSS-based localization; such a system is elaborated in subsequent chapters. In this chapter, through the experimental characterization of the INS noise performance, the standard deviation of the bias random walk noise is found to be 3 orders and 4 orders smaller than the standard deviation of white Gaussian noise in the accelerometer and gyroscope respectively from the Allan standard deviation plot. As such, only white Gaussian noise will be extracted for use in setting weighting factors for the RSS and INS fusion in subsequent chapters.

Chapter 4

RSS Localization and Tracking

Scheme with INS Assistance under

Known Initial Orientation

Chapter 3 showed that localization errors grow continuously, which makes INS alone incapable of indoor localization. This chapter introduces a novel weighted nonlinear RSS localization and tracking scheme that uses INS positioning estimation projections, based on the assumption that the initial orientation of the IMU is known. The weighting used to combine RSS values with INS positioning projections is explained and a method to continuously mitigate the effect of noise on angular velocities is also shown. The chapter concludes by presenting a detailed comparison of the proposed scheme with conventional integrations of INS with RSS using EKF across three distinct environments.

4.1 Proposed RSS Localization Scheme

In this section, a novel recursive RSS localization scheme is introduced. Due to the extra information provided by INS and the appropriate weighting, the RSS value at each second is more accurate, resulting in improved RSS localization results.

The fusion scheme consists of two parts. INS projection is conducted first, followed by the combination of previous and current RSS values linked by INS projection. The projection is illustrated in Figure 4.1 at first. Assume IMU starts moving at the white dot near AP_1 and follow the true (red) path. Blue path is one example of INS calculated path. The true moving direction and INS calculated moving direction are shown using red and blue arrows accordingly. Red, blue and yellow circles represent true location, INS calculated location and improved location after applying proposed scheme at t_{j-1} and t_{i-1} respectively whereas their projected location at time t_j and t_i are depicted using star symbols. One thing worth noting is that during the initial stage of movement, the three sets of locations mentioned above are practically the same hence the symbols representing the locations at t_{j-1} and t_j are overlapped. The reason for this observation is that initially INS projection (blue arrow) is accurate, as a result the projected location (yellow star) of MD is accurate with respect to true location. This information will be used to improve RSS measurements at t_j . The three sets of locations gradually separate with each other over time which is shown at

t_{i-1} and t_i as INS projection is getting less accurate. It is due to the reason that INS projection performance deteriorates with time. However the projected location is still statistically closer to true location as compared to the one without applying projection. It will help to improve RSS measurements, which eventually results in the improved location (yellow circle) at t_i . The detailed formulation of obtaining improved location will be articulated as follow.

Figure 4.1 Illustration of INS projection

Suppose that the recursive scheme propagates to time t_{i-1} and the distances $d'_{i-1,1...m}$ between the improved locations, and that all m APs are calculated. To improve the RSS value at t_i , INS readings collected between t_{i-1} and t_i are first

used to provide a position projection based on where it was located at t_{i-1} . The distances between the projected location and APs, $d_{i,1...m}$, can then be calculated. An illustration of this idea can be seen in Figure 4.1. The RSS difference can be calculated based on $d'_{i-1,1...m}$ and $d_{i,1...m}$, with which the improved RSS at t_i can be obtained after collecting RSS measurements at t_i . Finally, the improved RSS is fed into the equations to calculate locations. The recursive RSS value improvement technique is presented in the rest of this section.

The recursive RSS value improvement technique is shown here. At time t_i there are two sets of RSS values that can be utilized. One of them is the measured RSS value at time t_i , which is simulated using the following equation:

$$RSS_{i,1...m}^- = RSS_r - 10n_p \log_{10} \left(\frac{d_{i,1...m}^o}{d_r} \right) + \nu \text{ dB} \quad (4.1)$$

where RSS_r is the mean RSS value at reference distance d_r , $d_{i,1...m}^o$ is the true distance at t_i , n_p is the path-loss exponent, and ν is a Gaussian random variable used to simulate multipath and shadowing effect [37] and [127]. We use $\sigma_\nu = 6\text{dB}$ in the simulation, because this value has been found to be realistic in indoor environments [88].

The other set of RSS values comes from combining improved RSS values $RSS'_{i-1,1...m}$ at t_{i-1} and RSS difference $\Delta RSS_{i,1...m}$ between t_{i-1} and t_i due to INS projection. The RSS difference between t_{i-1} and t_i is calculated as follows:

$$\begin{aligned}\Delta RSS_{i,1...m} &= \left[RSS_r - 10n_p \log_{10}(d_{i,1...m} / d_r) \right] - \left[RSS_r - 10n_p \log_{10}(d'_{i-1,1...m} / d_r) \right] \\ &= -10n_p \log_{10}(d_{i,1...m} / d'_{i-1,1...m}) \text{ dB}\end{aligned}\quad (4.2)$$

where $d_{i,1...m}$ is the distance between all APs to the location calculated by INS at time t_i and $d'_{i-1,1...m}$ is the distance between all APs to the estimated location using the improved RSS value at time t_{i-1} .

The improved RSS value at time t_i is obtained by combining the two sets of RSS values using weighting factors:

$$RSS'_{i,1...m} = w1_{i,1...m} (RSS'_{i-1,1...m} + \Delta RSS_{i,1...m}) + w2_{i,1...m} RSS_{i,1...m} \text{ dB} \quad (4.3)$$

where $w1$ and $w2$ are weighting factors.

It is worth mentioning that equation (4.3) starts at t_0 and that the starting point and APs' coordinates are known; thus, the true distance between them can be expressed as $d^o_{0,1...m}$. The true RSS value at starting point $RSS^o_{0,1...m}$ can be acquired using the distance to the RSS model, as follows:

$$RSS^o_{0,1...m} = RSS_r - 10n_p \log_{10} \left(\frac{d^o_{0,1...m}}{d_r} \right) \text{ dB} \quad (4.4)$$

The $RSS'_{0,1...m}$ will be replaced by $RSS^o_{0,1...m}$ to kick-start the proposed recursive scheme.

The Gaussian model used in equation (4.1) was verified in a real environment [37]. The histogram of a total of 18,000 measurements at four different indoor environments is shown in Figure 4.2, where -52 in the x axis label is the RSS value at the reference distance and P_R is the received RSS value.

Figure 4.2 Histogram of measurements in four different environments [37]

The Gaussian fit in the graph shows the practicality of setting ν as a Gaussian random variable to model the shadowing effect. Although it cannot model every indoor environment accurately, it would appear to be suitable for a normal office environment, because the measurements in [37] were made at the Higher Technical School of Telecommunications, University of Valladolid, which features a typical indoor layout of cubicles, meeting rooms, and hallways. This Gaussian variable assumption generally holds true for a heavy multipath environment.

INS localization is initially more accurate when noises are effectively mitigated. As shown in Figure 4.1, INS projection from t_{j-1} to t_j follows the true path very closely; therefore the initial value for w_1 should be much larger than w_2 .

However, w_2 increases as the system relies more on raw RSS values because of the deterioration in INS performance, as demonstrated by the INS projection from t_{i-1} to t_i . The INS direction projection at t_{i-1} already begins to deviate from the red true direction. Eventually, w_2 takes over completely as the contributing weighting factor, as the error will be bounded by the RSS localization system if no further calibration is applied to the INS. Hence, the proposed weighting factor at t_i is

$$w_{i,1...m} = \frac{|d_{i,1...m}^- - d_{i,1...m}|}{|d_{i,1...m}^- - d_{i,1...m}| + \sigma_{ins,i}}, \quad w_{i,1...m}^2 = 1 - w_{i,1...m} \quad (4.5)$$

where $\sigma_{ins,i}$ is the standard deviation of the INS positioning error, as discussed in the previous chapter.

At the beginning of the trajectory, $\sigma_{ins,i}$ is much smaller compared to $|d_{i,1...m}^- - d_{i,1...m}|$, because $d_{i,1...m}^-$ is based on the measured RSS value. The values of w_1 and w_2 are close to 1 and 0, respectively. As time goes by, $\sigma_{ins,i}$ begins to diverge more quickly than $|d_{i,1...m}^- - d_{i,1...m}|$, as illustrated in Section 3.3. As a result, the value of w_2 is increasing, which is desirable because the system will place more trust in RSS measurements than INS projection, as represented by w_1 . Eventually the value of w_2 will approach 1, indicating that RSS measurements are almost the only source for localization, due to the unbounded error growth of the INS unless a reset or calibration is performed on it.

The improved RSS values need to be converted to distances before localization can be performed. At time t_i , the probability density function of measurement RSS value $RSS_{i,1...m}^-$ in equation (4.1) is:

$$p = \frac{1}{(2\pi\sigma_v)^{\frac{N}{2}}} \exp \left[-\frac{1}{2\sigma_v^2} \sum_{n=1}^N (RSS_{i,1...m}^- - \left\{ RSS_r - 10n_p \log_{10} \left(\frac{d_{i,1...m}^o}{d_r} \right) \right\})^2 \right] \quad (4.6)$$

where N is the number of samples emitted from each AP.

Letting $RSS_{i,1...m}^t = RSS_r - 10n_p \log_{10} \left(\frac{d_{i,1...m}^o}{d_r} \right)$ and setting the derivative of the log-likelihood function of equation (4.6) to zero yields the maximum likelihood estimate of RSS, $RSS_{i,1...m}^t$, as follows:

$$\frac{\partial \ln p}{\partial RSS_{i,1...m}^t} = \frac{1}{\sigma_v^2} \sum_{n=1}^N (RSS_{i,1...m}^- - RSS_{i,1...m}^t) = 0 \quad (4.7)$$

$$RSS_{i,1...m}^t = \frac{1}{N} \sum_{n=1}^N RSS_{i,1...m}^- \quad (4.8)$$

Only one RSS is measured at a time for each AP:

$$RSS_{i,1...m}^t = RSS_{i,1...m}^- \quad (4.9)$$

Therefore $d_{i,1...m}^-$, the maximum likelihood estimate of true distances at t_i , can be calculated as follows:

$$d_{i,1...m}^- = d_r \cdot 10^{\frac{RSS_r - RSS_{i,1...m}^-}{10n_p}} \quad (4.10)$$

With the improved RSS value $RSS'_{i,1...m}$, $d'_{i,1...m}$ can be easily calculated in a manner similar to equation (4.10). Weighted nonlinear LS using $d'_{i,1...m}$ is applied to obtain the final location estimation $[p_{X,i}, p_{Y,i}]$:

$$f_{\min} = \sum_1^m \left\{ w_{i,1\dots m} \left[\sqrt{(p_{X,i} - AP_{X,1\dots m})^2 + (p_{Y,i} - AP_{Y,1\dots m})^2} - d'_{i,1\dots m} \right] \right\}^2 \quad (4.11)$$

where $w_{i,1\dots m} = \frac{1}{d'_{i,1\dots m}}$.

The Levenberg-Marquardt method [121] was chosen to perform the calculation, due to its ability to adaptively vary the parameters between the gradient descent and Gauss-Newton updates [122], [123].

4.2 Gyroscope Noise Mitigation

Though ZUPT has been widely adopted in INS research for indoor localization, the requirement that the IMU be mounted on the user's foot makes implementation challenging—and perhaps even unnecessary, given the current ubiquity of MEMS IMUs. In this thesis, this issue is worked around through the use of a general handheld device, which the user holds in a manner similar to a cellphone. The body frame x -axis of the device is thus closely aligned with the direction of the user holding the device, and the IMU remains static with respect to the user. These requirements will be discussed in detail in the next chapter.

Various sources have shown that noise in gyroscope measurements is the main source of error in the overall INS error [60], [97], [98], [99] and [108]. Even with a known initial orientation, INS positioning estimation still drifts away from the true path as a result of gyroscope noise, specifically on the body frame x and y

axes. After a period of time, the rotation angle along the body frame axes becomes

$$\psi_{xyz} = \int_t \omega_{xyz}^b \quad (4.12)$$

The noise inside ψ_x and ψ_y , also known as tilting error, causes gravitational acceleration G , 9.8 m/s^2 , to be incorrectly projected onto the body frame x and y axes. Equation (4.12) also indicates that noise in ω_x^b and ω_y^b at the early stages of the path will accumulate until its completion, and thus has a larger effect compared to noise in later stages. To mitigate noise on ω_x^b and ω_y^b as quickly as possible, a calibration stage, spanning 10 seconds before the user starts moving, is included. This aids in mitigating noise in gyroscopes, and it serves the purpose of estimating the initial orientation, as will be explained in next chapter. The user is required to hold the device and remain stationary throughout the calibration stage. This stationary calibration stage is also used to estimate initial bias inside gyroscope measurements when a real IMU device is used, as discussed in Section 5.4; thus, the $n_c = 1000$ set of three axis gyroscope values is

$$\begin{bmatrix} \omega_{x,0}^b, \omega_{y,0}^b, \omega_{z,0}^b \end{bmatrix}^T = \begin{bmatrix} 0, 0, 0 \end{bmatrix}^T \quad (4.13)$$

One cluster is defined as a collection of continuous samples of accelerometer and gyroscope measurements after the calibration stage. A cluster size of K samples of angular velocities on x and y axes $\begin{bmatrix} \omega_{x,1...K}^b, \omega_{y,1...K}^b \end{bmatrix}^T$ are investigated in this section. K is set to 100 because IMU sampling frequency is 100 Hz and RSS values are measured every second, so that INS denoising techniques can be applied before fusing with the RSS localization scheme. From the results in

Chapter 3, it is known that $[\sigma_{\omega_x}^b, \sigma_{\omega_y}^b]^T$ are $[0.29^\circ / s, 0.29^\circ / s]^T$. If all $|\omega_x^b|$ values within one cluster are less than $[3\sigma_{\omega_x}^b, 3\sigma_{\omega_y}^b]^T$, then the IMU is assumed not to be rotating along the x -axis, and the moving average value can be applied to the x -axis to mitigate noise by replacing all sample values inside that cluster. The moving average equation for angular velocity along the x -axis for cluster j is given as:

$$\bar{\omega}_{x,j}^b = \frac{[n_c + K(j-1)]\bar{\omega}_{x,j-1}^b + E(\omega_{x,K(j-1)+1 \dots Kj}^b)K}{n_c + Kj} \quad (4.14)$$

where $E(\bullet)$ stands for expectation and $\bar{\omega}_{x,0}^b = \omega_{x,0}^b$.

For illustration, the average for the first cluster is computed as:

$$\bar{\omega}_{x,1}^b = \frac{n_c \bar{\omega}_{x,0}^b + E(\omega_{x,1 \dots K}^b)K}{n_c + K} = \frac{E(\omega_{x,1 \dots K}^b)K}{n_c + K} \quad (4.15)$$

where $\bar{\omega}_{x,0}^b = \omega_{x,0}^b = 0$.

If the cluster size is 100, the angle of rotation along the x -axis during j second is expressed as:

$$\Delta\psi_{x,j} = \Delta t K \bar{\omega}_{x,j}^b \quad (4.16)$$

where $\Delta t = 0.01$ and $K = 100$.

The total accumulated angle along the x -axis up to cluster j is:

$$\psi_{x,j} = \sum_{m=1}^j \Delta\psi_{x,m} = \Delta t K \sum_{m=1}^j \bar{\omega}_{x,m}^b \quad (4.17)$$

Equations (4.14) and (4.16) show that the angle of rotation $\Delta\psi_{x,j}$ will tend to approach 0 because ω_x^b inside each cluster is a white Gaussian sequence and $\bar{\omega}_{x,0}^b$ equals 0. The impact of newly collected clusters decreases as $\frac{K}{n_c + Kj}$ decreases when j increases. There are several scenarios where noise mitigation

techniques can be applied differently when no rotation is detected. If the angle is accumulated along one direction either positive or negative, noise mitigation techniques need to be applied so that the angle does not accumulate. The updated $\psi_{x,j}$ can be expressed as:

$$\psi'_{x,j} = \begin{cases} \Delta\psi_{x,j}j - \psi'_{x,j-1}(j-1) & | \Delta\psi_{x,j} | \leq | \psi'_{x,j-1} | \\ \psi'_{x,j-1}j - \Delta\psi_{x,j}(j-1) & | \Delta\psi_{x,j} | > | \psi'_{x,j-1} | \end{cases} \quad (4.18)$$

where $\psi'_{x,0} = 0$

If the signs of $\Delta\psi_{x,j}$ and $\psi'_{x,j-1}$ are different, the incremental angle during time j , $\Delta\psi_{x,j}$, can be added to $\psi'_{x,j-1}$:

$$\psi'_{x,j} = \Delta\psi_{x,j} + \psi'_{x,j-1} \quad (4.19)$$

The updated mean angular velocity for cluster j is expressed as:

$$\bar{\omega}_{x,j}^b = \frac{\psi'_{x,j} - \psi'_{x,j-1}}{K\Delta t} \quad (4.20)$$

All values of $\omega_{x,j}^b$ are replaced by $\bar{\omega}_{x,j}^b$ during the second and substituted into the fusion system proposed in Section 4.1. Equations (4.18) to (4.20) help to mitigate the cumulative effect in (4.17). A similar technique can also be applied to the y axis. Currently, the noise on ω_z^b is not mitigated because it contains heading information that is necessary for the INS to determine location.

After updating the proposed RSS scheme detailed in Section 4.1 with the improvements from the two-step moving average technique developed in Section 4.2, the proposed scheme is expected to outperform other INS and RSS fusion schemes that use EKF. A detailed comparison of the schemes' performance is presented in the next section.

4.3 Simulation Results and Comparison with EKF

In total, three simulation environments are used. As shown in [37], [88], [93], and [128], the shadowing effect of the RSS signals is modeled by a Gaussian random variable with 6 dB standard deviation and the path-loss exponent is 2.3, as discussed in Section 4.1. The modeling of the shadowing effect and path-loss exponent remains the same for the three simulated environments and newly added experiment environment in Chapter 5. These environments differ in various ways. Environments 1 and 2 are simulations created to demonstrate the performance of the proposed localization scheme. The movement path in environment 1 is shorter and contains fewer turning sections compared to environment 2. Hence, environment 2 poses more challenges for stand-alone INS localization. After the proposed denoising technique is applied, the stand-alone INS localization paths stay relatively close to the true path in environment 1 compared to environment 2. Environment 3 and the path of movement are a reconstruction of the environment and path of movement in publication [88]. Environment 3 is a real office environment in which the deployment of APs differs vastly from the other two environments. Experiment environment is also an office environment which will be elaborated in detail at Chapter 5. Localization performance is thus tested in these four environments under different levels of difficulty, in terms of path length, path complexity, and AP deployment.

4.3.1 Environment 1

The first environment (E1), with the true path and all 15 AP locations, is shown in Figure 4.3; the velocity profile of the user is depicted in Figure 4.4. The path starts at $[0, 0]$ and the total traveling time is 45 seconds. The velocity changes model real-life situations.

Figure 4.3 E1 path (red) and AP locations (green)

Figure 4.4 Velocity profile of user following the path in E1

Figure 4.5 presents an individual run of the simulation using INS alone without any improvement technique. Both the E1 path and the INS path have 9 numbers indicating the time (seconds) at 9 points along the path. INS is accurate up to 15 seconds. However, the accumulated noise on the IMU has a greater effect after that point.

Figure 4.5 INS path (blue)

Figure 4.6 shows the RSS localization results without applying any improvement techniques. The RSS-based localization results are expected to fluctuate from time to time, as these are independent of each other. Unlike INS, RSS localization results are bounded within a certain range as long as enough RSS signals are received to calculate the location. The localization bound is dependent on the number and placement of APs and the environment path loss exponent. Furthermore, it is time-independent.

Figure 4.6 RSS localization (blue), $\sigma_{RSS} = 6dB$

The INS and RSS fusion system using EKF is shown in Figure 4.7. This INS+RSS+EKF system was originally proposed by [88] and is a tight coupling of the EKF system with ZUPT included. Because foot-mounted IMUs are not considered in this thesis, ZUPT is excluded from the comparison.

Figure 4.7 INS+RSS+EKF (blue)

The system is accurate through the first 15 seconds, as the INS is weighted more heavily during this period. Gradually, more weight shifts to the RSS and both systems contribute significantly to the localization results. The EKF system calculates Kalman gain based on the INS and RSS systems' standard deviations and distributes the weight accordingly. If the system runs long enough, it will eventually be bounded by the RSS system's output.

Figure 4.8 shows the performance of the gyroscope noise-mitigation scheme proposed in Section 4.2.

Figure 4.8 INS with gyroscope noise mitigation (blue)

A comparison with Figure 4.5 shows that the INS localization performance is greatly improved. The error at 31 seconds in Figure 4.7 is comparable to that at 16 seconds in Figure 4.4. Other types of noise, especially from the z -axis gyroscope, still make the INS deviate from the true path. The proposed RSS localization scheme with INS projections further improves the performance. This result is shown in Figure 4.9.

Figure 4.9 Proposed RSS scheme with gyroscope noise mitigation (blue)

A comparison with Figure 4.7 shows that the proposed RSS scheme generally outperforms the EKF scheme, especially in the first half of the path. The proposed scheme also runs more smoothly than EKF. The cumulative error along the path is given in Figure 4.9. The cumulative error at the end of the path is 37.61 m for the proposed scheme and 58.77 m for EKF. The improvement is 36%.

Figure 4.10 Cumulative error: proposed (blue) vs EKF (red)

Figures 4.11–4.16 demonstrate another run of the simulation under the same true path, but with a different set of noise.

Figure 4.11 2nd run, INS path (blue)

Figure 4.12 2nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 4.13 2nd run, INS+RSS+EKF (blue)

Figure 4.14 2nd run, INS with gyroscope noise mitigation (blue)

Figure 4.15 2nd run, proposed RSS scheme with gyroscope noise mitigation
(blue)

Figure 4.16 2nd run, cumulative error: proposed (blue) vs EKF (red)

Figures 4.11–4.16 reveal that the proposed scheme generally achieves better results than the EKF. The cumulative error at the end of the path is 21.1 *m* for the proposed scheme and 60.4 *m* for EKF. The improvement is 65%.

To conclude this performance comparison, a total of 50 independent runs were simulated. To the best of our knowledge, 50 runs is sufficient to generalize the averaged cumulative error, which is shown in Figure 4.17. The average cumulative error at 45 seconds is 141.10 *m* for the RSS localization scheme, 68.3 *m* for the INS+RSS+EKF scheme, and 34.8 *m* for the proposed RSS localization with INS projection scheme. The proposed scheme shows improvements of 75% and 49% over the RSS localization scheme and INS+RSS+EKF scheme, respectively.

To further test the proposed scheme, another two environments are introduced in the following section.

Figure 4.17 Average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow)

4.3.2 Environment 2

The second constructed environment (E2), with true path and all 25 AP locations, is shown in Figure 4.18; the user's velocity profile is depicted in Figure 4.19. The path starts at $[0, 0]$ and the total traveling time is 103 seconds.

Figure 4.18 E2 path (red) and AP locations (green)

Figure 4.19 Velocity profile of user following the path in E2

Similar to Section 4.3.1, this section presents a detailed comparison of the proposed scheme and the EKF scheme. Figures 4.20–4.25 provide comparisons of one run of the path; this is followed by comparisons of a second run, in Figures 4.26–4.31.

Figure 4.20 E2 INS path (blue)

For ease of visualization, a larger map range is used to display the path generated by stand-alone INS, due to the cumulative error effect. The simulated INS path runs out of figure range after 46 seconds.

Figure 4.21 E2 RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 4.22 E2, INS+RSS+EKF (blue)

Figure 4.23 E2, INS with gyroscope noise mitigation (blue)

Figure 4.24 E2, Proposed RSS scheme with gyroscope noise mitigation (blue)

Figure 4.25 E2, cumulative error: proposed (blue) vs EKF (red)

Figures 4.19–4.25 again demonstrate the advantages of the proposed scheme over EKF in terms of accuracy. The cumulative error improves from 255.0 m to 165.7 m , which is 35% at the 103 second end point.

Figure 4.26 E2, 2nd run, INS path (blue)

Figure 4.27 E2, 2nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 4.28 E2, 2nd run, INS+RSS+EKF (blue)

Figure 4.29 E2, 2nd run, INS with gyroscope noise mitigation (blue)

Figure 4.30 E2, 2nd run, proposed RSS scheme with gyroscope noise mitigation
(blue)

Figure 4.31 E2, 2nd run, cumulative error: proposed (blue) vs EKF (red)

Figures 4.26–4.31 show the advantage of the proposed scheme over EKF in terms of accuracy. The cumulative error improves from 279.4 *m* to 147.8 *m*, which is 47% at the 103 second end point.

The average cumulative error over 50 independent runs is presented in Figure 4.32. The average cumulative error at 103 seconds is 359.5 *m* for the RSS localization scheme, 226.8 *m* for the INS+RSS+EKF scheme, and 178.6 *m* for the proposed RSS localization with INS projections scheme. The proposed scheme improves 51% and 23% on the RSS localization and INS+RSS+EKF schemes, respectively.

Figure 4.32 E2, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow)

4.3.3 Environment 3

The third environment (E3) is a reconstruction of the one in [88]. The original map is shown in Figure 4.33, and the reconstructed true path and all 71 AP locations are depicted in Figure 4.34. The user's velocity profile is depicted in Figure 4.35. The path starts at [37, 34] and the total traveling time is 91 seconds. The illustration in Figure 4.33 does not differentiate doors and walls. The user needs to go through doors to complete the path.

Figure 4.33 E3, original map [88]

Figure 4.34 E3, path (red) and AP locations (green)

Figure 4.35 Velocity profile of user following the path in E3

The rest of this section compares the proposed scheme and RSS scheme. Figures 4.36–4.41 show the comparisons for one run, while Figures 4.42–4.47 present another run.

Figure 4.36 E3, INS path (blue)

Figure 4.37 E3, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 4.38 E3, INS+RSS+EKF (blue)

Figure 4.39 E3, INS with gyroscope noise mitigation (blue)

Figure 4.40 E3, proposed RSS scheme with gyroscope noise mitigation (blue)

Figure 4.41 E3, cumulative error: proposed (blue) vs EKF (red)

Using the proposed scheme, the cumulative error improves from 180.4 m to 72.5 m , a 60% improvement at the 91 second end point.

Figure 4.43 E3, 2nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 4.44 E3, 2nd run, INS+RSS+EKF (blue)

Figure 4.45 E3, 2nd run, INS with gyroscope noise mitigation (blue)

Figure 4.46 E3, 2nd run, proposed RSS scheme with gyroscope noise mitigation
(blue)

Figure 4.47 E3, 2nd run, cumulative error: proposed (blue) vs EKF (red)

With the proposed scheme, the cumulative error decreases from 157.8 *m* to 87.5 *m*, a 45% improvement, by the 91 second end point.

The average cumulative error over 50 independent runs is presented in Figure 4.48. The average cumulative error at 91 seconds is 265.5 *m* for the RSS localization scheme, 181.5 *m* for the INS+RSS+EKF scheme, and 79.9 *m* for the proposed RSS localization with INS projection scheme. The proposed scheme shows a 70% and 56% improvement over the RSS localization scheme and the INS+RSS+EKF scheme, respectively.

Figure 4.48 E3, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow)

4.4 Conclusion

In this chapter, a weighted nonlinear RSS localization scheme with INS projections is proposed. First, a novel technique to combine adjacent RSS values using INS projections is discussed in detail. Next, a method is elaborated to mitigate noise in angular velocity measurements by the gyroscope. To test the accuracy and robustness of the proposed scheme, extensive simulations were performed across three environments, each involving different paths and AP deployment. In terms of average cumulative error over 50 independent runs, the proposed scheme demonstrates improvements of 49%, 23%, and 56% over the

INS+RSS+EKF fusion system across the three environments. The next chapter proposes a technique to address the situation when the initial orientation is unknown in advance. It also introduces a technique for mitigating body frame acceleration noise.

Chapter 5

RSS Localization and Tracking

Scheme with INS Assistance

without Initial Knowledge of

Orientation

Chapter 4 showed that the proposed weighted nonlinear RSS scheme outperforms the INS+RSS+EKF fusion system when perfect knowledge of the initial orientation is available. This chapter introduces an angle estimation technique for cases when the initial orientation of the IMU is unknown. Next, a method to mitigate noise in accelerometer measurements is presented. This method is the final component of the proposed weighted nonlinear RSS scheme, incorporating all of the novel techniques introduced in chapters 4 and 5. A detailed comparison of the proposed scheme against conventional fusions of INS with RSS using EKF across three environments is presented. The proposed

scheme is also tested in a real environment, and comparisons are made between the various schemes. Conclusions are presented at the end of the chapter.

5.1 Proposed Initial Orientation Angle Estimation Technique

The tilting angle, which is the angle between the body frame z -axis and the navigation frame Z -axis, will cause gravitational acceleration G to project onto the body frame x and y axes. The tilting angle is formed by the rotation angles along the body frame x and y axes, which are ψ_x and ψ_y . Together with ψ_x and ψ_y , the heading angle ψ_z (the rotation on the body frame z axis) forms the initial orientation of a device. An illustration of how ψ_x affects the projection of G is given in Figure 5.1.

Figure 5.1 Illustration of ψ_x and G projection

Suppose that the body frame x , y , and z axes are all aligned with the navigation X , Y , and Z axes. Under a zero noise condition, the z -axis accelerometer will register a value of G due to the gravitation acceleration based on how the MEMS IMU works, as explained in Chapter 3. A rotation of ψ_x along the x -axis will drive the y and z axes to y' and z' axes. As a result, the z' -axis and y' -axis accelerometer will register values of $G \cos \psi_x$ and $-G \sin \psi_x$. Similarly, the effect of ψ_y can be shown in the same way. As discussed in Chapter 4, the tilting angle error will result in an incorrect projection of G on the x and y axes. One of the error sources, the effect of white Gaussian noise on gyroscope measurements, was already mitigated with techniques introduced in the previous chapter. Nevertheless, errors caused by inaccurate initial orientation estimations are much greater: the initial orientation error will serve as a constant error throughout the whole INS navigation process. As will be seen later in the chapter, failure to estimate the initial orientation will result in thousands of meters of error within the span of the first minute. Thus, an accurate estimation of initial orientation is crucial for achieving better localization results. A technique is presented next to address this issue.

Frequently, when the user is holding the device, the body frame x -axis is not perfectly aligned with his or her direction. Devices have a small tilting angle. To model the initial orientation realistically, it is assumed the initial rotation angles $\psi_{x,0}$, $\psi_{y,0}$, and $\psi_{z,0}$ all follow a zero-mean Gaussian distribution. As suggested by other researchers using EKF, 1.8° is a reasonable standard deviation for the initial orientation angles [92]. To make use of the IMU's characteristics when it is stationary, a 10-second standing calibration stage is introduced. During this

period, the accelerometers record values as the user holds the device without moving. There are two relationships that can be used to estimate the initial orientation angle, as follows:

$$\sqrt{(a_{x,0}^{bo})^2 + (a_{y,0}^{bo})^2 + (a_{z,0}^{bo})^2} = G \quad (5.1)$$

$$\frac{\psi_{x,0}^o}{\psi_{y,0}^o} \cong -\frac{a_{y,0}^{bo}}{a_{x,0}^{bo}} \quad (5.2)$$

where $a_{x,0}^{bo}$, $a_{y,0}^{bo}$, and $a_{z,0}^{bo}$ represent the true specific forces along each axis, as registered by accelerometers at the calibration stage.

Equation (5.2) is valid when $\psi_{x,0}$ and $\psi_{y,0}$ are small. This can be safely applied if the preceding assumptions hold true. During the calibration stage, $a_{x,0}^{bo}$ and $a_{y,0}^{bo}$ can be approximated using $a_{x,0}'^b$ and $a_{y,0}'^b$:

$$a_{x,0}'^b = E([a_{x,0}^b(1), \dots, a_{x,0}^b(N)]), \quad a_{y,0}'^b = E([a_{y,0}^b(1), \dots, a_{y,0}^b(N)]) \quad (5.3)$$

where k indicates the k^{th} sample of the N samples in the calibration stage and $N = 1000$.

As the results in Chapter 3 indicate, Xsens Mtw IMUs have more noise in their z -axis accelerometer measurements compared to the other two axes. However, it is known that the total specific force will be G if the IMU is stationary. By substituting Equation (5.3) into (5.1), a more accurate $a_{z,0}'^b$ can be obtained, instead of finding the expected value for $a_{z,0}^b$ over 1,000 samples to be similar to (5.3):

$$a_{z,0}'^b = \sqrt{G^2 - (a_{x,0}'^b)^2 - (a_{y,0}'^b)^2} \quad (5.4)$$

When the IMU is stationary, the true specific force at the navigation frame is $[0, 0, G]^T$. Using this information and the Euler transformation matrix described in [60], we can form a relationship:

$$\begin{bmatrix} 0 \\ 0 \\ G \end{bmatrix} = C_{b,0}^N \begin{bmatrix} a_{x,0}^{bo} \\ a_{y,0}^{bo} \\ a_{z,0}^{bo} \end{bmatrix} \cong C_{b,0}^N \begin{bmatrix} a_{x,0}^{rb} \\ a_{y,0}^{rb} \\ a_{z,0}^{rb} \end{bmatrix} \quad (5.5)$$

where

$$C_{b,0}^N = \begin{bmatrix} \cos \psi_{y,0} \cos \psi_{z,0} & -\cos \psi_{x,0} \sin \psi_{z,0} & \sin \psi_{x,0} \sin \psi_{z,0} \\ \cos \psi_{y,0} \sin \psi_{z,0} & -\cos \psi_{x,0} \cos \psi_{z,0} & -\sin \psi_{x,0} \cos \psi_{z,0} \\ -\sin \psi_{y,0} & \sin \psi_{x,0} \cos \psi_{y,0} & \cos \psi_{x,0} \cos \psi_{y,0} \end{bmatrix}$$

is the transformation matrix at the calibration stage.

Substituting equation (5.2) into (5.5) and solving the equation, $\mathcal{Y}_{x,0}^o$ and $\mathcal{Y}_{y,0}^o$ can be estimated using the third row of the matrix. The more rigorous solution to estimating the rotation angles $\psi_{x,0}^o$ and $\psi_{y,0}^o$ can be obtained by using optimization techniques such as the expectation-maximization (EM) algorithm [129]-[131]. Numeric estimation is adopted here because the main focus of the thesis is to propose an accurate and robust fusion system using strapdown INS and an RSS-based system. The estimation error standard deviation is 0.003° and 0.0025° for $\mathcal{Y}_{x,0}^o$ and $\mathcal{Y}_{y,0}^o$, respectively, over 100 randomly generated initial angles. This demonstrates that the proposed scheme can accurately estimate initial orientation angles.

5.2 Accelerometer Noise Mitigation

As discussed in the previous chapter, gyroscope noise is a greater source of error than accelerometer noise. However, improvements can still be made to accelerometers to further enhance localization accuracy.

First, assume that the user is walking on a flat surface and that there is no extra acceleration along the Z-axis other than gravitational acceleration. Further, assume that the user is holding the IMU steady, which also means that the tilting angle will not change much even with noise perturbing the device. It is then reasonable to set

$$a_{z,k}^{rb} = a_{z,0}^{rb} \quad (5.6)$$

Second, the clusters proposed in Chapter 4 are used to mitigate gyroscope noise. Each cluster has K samples. Within the cluster, no technique is applied if the absolute values of a_x^b or a_y^b fluctuate more than 10 times out of their standard deviations, because this suggests that the user is accelerating within the cluster. In other instances, when the user is walking in a straight line, a_x^b and a_y^b should only fluctuate within a relatively small range because of the Gaussian noise within the cluster. At the same time, when turning is detected, nothing should be done to the clusters. It is assumed that $|\omega_{z,k}^b| < 2^\circ/s$ implies that the IMU is not subject to any turning. Given that no turning is detected at cluster j , we can calculate

$$\Delta \bar{a}_j^b = E(a_{x,K \cdot (j-1)+1 \dots K \cdot j}^b) - E(a_{y,K \cdot (j-1)+1 \dots K \cdot j}^b) \quad (5.7)$$

Then for each sample in the cluster, the updated $a_x'^b$ and $a_y'^b$ is presented as

$$a_{x,k}'^b = a_{x,k}^b - \frac{a_{x,k}^b - a_{y,k}^b - \Delta \bar{a}_j^b}{2}, \quad a_{y,k}'^b = a_{y,k}^b + \frac{a_{x,k}^b - a_{y,k}^b - \Delta \bar{a}_j^b}{2} \quad (5.8)$$

Equations (5.7) and (5.8) are applied when the device is moving at a constant acceleration in a straight line, such that $a_{x,k}^b$ and $a_{y,k}^b$ in one cluster follow the

Gaussian distribution $a_{x,k}^b \sim N(\mu_{ax}, (\sigma_{ax})^2)$ and $a_{y,k}^b \sim N(\mu_{ay}, (\sigma_{ay})^2)$. The

$E(a_{x,K \cdot (j-1)+1 \dots K \cdot j}^b)$ and $E(a_{y,K \cdot (j-1)+1 \dots K \cdot j}^b)$ terms in equation (5.7) represent the

true value of accelerations along body frame xy axes. The term $\frac{a_{x,k}^b - a_{y,k}^b - \Delta \bar{a}_j^b}{2}$

within one cluster follow white Gaussian distribution,

$\frac{a_{x,k}^b - a_{y,k}^b - \Delta \bar{a}_j^b}{2} \sim N(0, \frac{(\sigma_{ax})^2 + (\sigma_{ay})^2}{4})$. The subtraction and addition of this

term to the original a_x^b and a_y^b is done in equation (5.8) to mitigate the white

Gaussian noise perturbing each sample within one cluster for a_x^b and a_y^b .

Finally, a new two-step method is proposed to improve $a_{z,k}^b$ again after the noise

on $a_{x,k}^b$, $a_{y,k}^b$, $\omega_{x,k}^b$ and $\omega_{y,k}^b$ has been mitigated. With all of the noise-

mitigation techniques introduced previously, we have

$$\begin{bmatrix} f_{X,k}^N \\ f_{Y,k}^N \\ f_{Z,k}^N \end{bmatrix} = C_{b,k}^N \begin{bmatrix} a_{x,k}'^b \\ a_{y,k}'^b \\ a_{z,k}'^b \end{bmatrix} \quad (5.9)$$

where $\begin{bmatrix} f_{X,k}^N \\ f_{Y,k}^N \\ f_{Z,k}^N \end{bmatrix}$ is the navigation frame acceleration without subtracting G .

Because the tilting angle varies within a small range along the path, we can assume that it remains the same as the initial tilting angle and its sine value is

$$\sin \theta = \frac{\sqrt{(a'_{x,0})^2 + (a'_{y,0})^2}}{G} \quad (5.10)$$

The navigation frame's specific force value is transferred back to the body frame z-axis:

$$a''_{x,k} = a'_{z,k} + \sqrt{(f_{X,k}^N)^2 + (f_{Y,k}^N)^2} \sin \theta \quad (5.11)$$

When the user is accelerating, deaccelerating, or turning, the errors affecting the tilting angle will project the wrong acceleration occurring on the navigation XY plane to the body frame z axis, which in term affects body frame measurements. Therefore, equations (5.6) and (5.11) are proposed to mitigate this concern. Equation (5.11) is substituted back into equation (5.9) to replace $a'_{z,k}$. The calculation is then completed to obtain the navigation frame velocity and location.

5.3 Simulation Results and Comparison with EKF

In this chapter, the three environments introduced in Chapter 4 are used again to test the scheme proposed throughout chapters 4 and 5. To test the robustness of the initial orientation angle estimation technique, different initial orientation angles will be used for each run. The IMU noise and Gaussian random variable used to generate the RSS are set to the random values used in Chapter 4 and show how accurate the initial orientation angle estimation is, followed by a comparison between the final proposed scheme and INS with RSS using EKF. The validity of the proposed scheme was tested in a real environment, and the results of the performance comparison are discussed at the end of the chapter.

5.3.1 Environment 1

The E1 used in Chapter 4 is used here as the first test environment. The velocity profile also remains the same. The path starts at $[0, 0]$ and the total traveling time is 45 seconds. At the end of this section, the average 3D cumulative errors of the various schemes are compared. Performance comparisons for a 3D environment will be presented again in the discussion of the real environment experiment in Section 5.4.

Figure 5.2 E1 path (red) and AP locations (green)

Figure 5.3 Velocity profile of user following the path in E1

The first run of the simulation with the true initial orientation angles $\gamma_{x,0}^o = -1.187^\circ$ and $\gamma_{y,0}^o = -0.845^\circ$ is shown in Figures 5.4–5.10. The estimated initial orientation angles are $\gamma_{x,0}' = -1.188^\circ$ and $\gamma_{y,0}' = -0.848^\circ$. Figure 5.4 depicts the INS path without estimating the initial orientation. The INS path goes out of figure range after just 14 seconds. Figure 5.5 shows the RSS localization results without applying improvements. The RSS signal from Chapter 4 is used to test the accuracy of the initial angle-estimation. Figure 5.6 illustrates the performance of the INS and RSS fusion system using EKF in Chapter 4. No initial orientation estimation is applied to this system.

Figure 5.4 INS path without initial orientation estimation (blue)

Figure 5.5 RSS localization (blue), $S_{RSS} = 6dB$

Figure 5.6 INS+RSS+EKF (blue)

Figure 5.7 shows the performance of the INS system that includes the initial orientation estimation and all noise-mitigation techniques introduced in chapters 4 and 5.

Figure 5.7 INS with all improvements applied (blue)

The performance in Figure 5.7 shows a significant improvement over that in Figure 5.4. This comparison also reveals that the initial orientation angle is the biggest source of error if it is not estimated. Figure 5.8 shows the proposed RSS scheme with all of the techniques introduced in chapters 4 and 5. The cumulative error is shown in Figure 5.9. The cumulative error at the end of the path is 33.3 *m* for the proposed scheme with all techniques applied and 97.2 for EKF. The improvement is 63%.

Figure 5.8 Proposed RSS scheme with all techniques applied

Figure 5.9 Cumulative error: proposed (blue) vs EKF (red)

Figures 5.10–5.15 present another run of simulations with different initial orientation angles and different noise sources from the first run. The true initial orientation angles are $\mathcal{Y}_{x,0}^o = 4.190^\circ$ and $\mathcal{Y}_{y,0}^o = 0.271^\circ$, and the estimated initial orientation angles are $\mathcal{Y}_{x,0}' = 4.190^\circ$ and $\mathcal{Y}_{y,0}' = 0.268^\circ$. For comparison, Figures 5.13–5.15 include the results when initial orientations are known, as presented in Chapter 4.

Figure 5.10 2nd run, INS path without initial orientation estimation (blue)

The simulated INS path runs out of figure range after 6 seconds.

Figure 5.11 2nd run, RSS localization (blue), $S_{RSS} = 6dB$

Figure 5.12 2nd run, INS+RSS+EKF (blue)

Figure 5.13 2nd run, INS scheme with all techniques applied without known initial orientation (blue) vs INS scheme with gyroscope noise mitigation and known initial orientation (black)

Figure 5.14 2nd run, proposed RSS scheme with all techniques applied without known initial orientation (blue) vs proposed RSS scheme with gyroscope noise mitigation and known initial orientation (black)

Figure 5.15 2nd run, cumulative error: proposed (blue) vs EKF (red)

The cumulative error at the end of the path is 18.8 *m* for the proposed scheme with all techniques applied and 92.7 *m* for EKF. The improvement is 80%.

In addition to the proposed fusion scheme and EKF, another RSS with INS localization scheme [126] is added for comparison in one simulated environment and one experiment environment. The authors of [126] apply user-specific step-length based PDR, which requires a training phase, to provide preliminary localization as the first part of the system. The four APs with highest RSS values are selected and three out of four are used to formulate standard LS to estimate the location. After computing locations from all four combinations of three APs, the distance between estimated locations is computed. The NLOS condition is considered to be detected if the maximum value of the distances is larger than a given threshold, which leads to discarding the RSS-based localization results. Otherwise the center point of LS localization results is used to replace the PDR

localization computed at the first stage. The threshold for the NLOS selection scheme is set as 2 *m*, as the localization error is the minimum for thresholds ranging from 1 *m* to 6 *m* in this environment.

Again, a total of 50 independent trials with different initial orientation angles were simulated. The average cumulative error at 45 seconds is 5891 *m*, 209.9 *m*, 141.1 *m*, 99.5 *m*, and 34.8 *m* for the INS localization scheme, NLOS selection scheme, RSS localization scheme, EKF scheme, and the final proposed RSS localization–INS projection scheme with noise mitigation. As shown in Figure 5.16, only a few degrees of initial orientation angle without estimation can cause the localization error to be more than 5000 *m* within 45 seconds of the true path. The proposed scheme shows a 99%, 83%, 75%, and 65% improvement on the INS localization scheme, NLOS selection scheme, RSS localization scheme, and EKF scheme, respectively. The average cumulative errors when initial orientation is known are added as dotted lines for the sake of comparison with results without known initial orientation. Initial orientation information makes a 30% improvement in EKF error, from 99.5 *m* to 68.3 *m*. Figure 5.16 also demonstrates the effectiveness of the proposed initial orientation estimation technique as dotted blue line overlaps with solid blue line.

To test the performance in a 3D environment, the height of APs is assumed to be 3 *m* above the floor and the MD is assumed to be held 1 *m* above the floor. The average 3D cumulative errors are shown in Figure 5.17. Errors at 45 seconds are 925.3 *m*, 375.1 *m*, and 70.1 *m* for the NLOS selection scheme, EKF scheme, and

proposed RSS localization–INS projection scheme with noise mitigation. The proposed scheme improves 93% and 82% on the other schemes, respectively.

Figure 5.16 Average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow) vs NLOS selection (green) vs INS (black)

Figure 5.17 Average 3D cumulative error: proposed (blue) vs EKF (red) vs NLOS selection (Green)

5.3.2 Environment 2

The E2 used in Chapter 4 is the second test environment here. The velocity profile also remains the same. The path starts at $[0, 0]$ and the total traveling time is 103 seconds.

Figure 5.18 E2 path (red) and AP locations (green)

Figure 5.19 Velocity profile of user following the path in E2

Figures 5.20–5.25 present one run of simulations. The true initial orientation angle $\psi_{x,0}^o = 1.698^\circ$ and $\psi_{y,0}^o = 1.770^\circ$, while the estimated initial orientation angles are $\psi_{x,0}' = 1.696^\circ$ and $\psi_{y,0}' = 1.773^\circ$. Figures 5.22–5.25 illustrate that the proposed scheme offers significant improvements in accuracy over EKF. The cumulative error improves from 294.2 m to 172.6 m, which is a 41% improvement overall.

Figure 5.20 E2, INS path without initial orientation estimation (blue)

Figure 5.21 E2, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 5.22 E2, INS+RSS+EKF (blue)

Figure 5.23 E2, INS with all improvements applied (blue)

Figure 5.24 E2, proposed RSS scheme with all techniques applied (blue)

Figure 5.25 E2, cumulative error: proposed (blue) vs EKF (red)

Figures 5.26–5.31 present another run of simulations in E2. The true initial orientation angles are $\psi_{x,0}^o = -0.100^\circ$ and $\psi_{y,0}^o = 2.252^\circ$, and the estimated initial orientation angles are $\psi_{x,0}^e = -0.103^\circ$ and $\psi_{y,0}^e = 2.256^\circ$. Figures 5.29–5.32 also include the results when initial orientations are known, as presented in Chapter 4.

Figure 5.26 E2, 2nd run, INS path without initial orientation estimation (blue)

Figure 5.29 E2, 2nd run, INS with all improvements applied without known initial orientation (blue) vs INS with gyroscope noise mitigation and known initial orientation (black)

Figure 5.30 E2, 2nd run, proposed RSS scheme with all techniques applied without known initial orientation (blue) vs proposed RSS scheme with gyroscope noise mitigation and known initial orientation (black)

Figure 5.31 E2, 2nd run, cumulative error: proposed (blue) vs EKF (red)

The cumulative error decreases from 316.5 *m* to 131 *m*, which is a 59% improvement. The average cumulative error information is shown in Figure 5.32. The average cumulative errors at 103 seconds are 69520 *m*, 359.5 *m*, 258.9 *m*, and 166.5 *m* for the INS localization scheme, RSS localization scheme, INS+RSS+EKF scheme, and final proposed RSS localization–INS projection scheme with noise mitigation, respectively. The proposed scheme offers cumulative improvements of 99%, 54%, and 36% over the other schemes, respectively. Initial orientation information makes a 12% improvement in EKF error from 258.9 *m* to 226.8 *m*.

Figure 5.32 E2, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow) vs INS (black)

5.3.3 Environment 3

The E3 used in Chapter 4 is used here as the last test environment. The velocity profile also remains the same. The path starts at [37, 34] and the total traveling time is 91 seconds.

Figure 5.33 E3 original map [88]

Figure 5.34 E3 path (red) and AP locations (green)

Figure 5.35 Velocity profile of user following the path in E3

As in Section 4.3.3, a comparison between the proposed scheme and the EKF scheme is presented in this section. Figures 5.36–5.41 present one run of simulations. The true initial orientation angles are $\psi_{x,0}^o = 3.059^\circ$ and $\psi_{y,0}^o = 0.787^\circ$, while the estimated initial orientation angles are $\psi_{x,0}' = 3.062^\circ$ and $\psi_{y,0}' = 0.788^\circ$.

Figure 5.36 E3, INS path without initial orientation estimation (blue)

Figure 5.37 E3, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 5.38 E3, INS+RSS+EKF (blue)

Figure 5.39 E3, INS with all improvements applied (blue)

Figure 5.40 E3, proposed RSS scheme with all techniques applied (blue)

Figure 5.41 E3, cumulative error: proposed (blue) vs EKF (red)

The cumulative error improves from 195.7 *m* to 78.8 *m* at 91 seconds, which is 60%.

Figures 5.42–5.46 present another run of simulations. The true initial orientation angles are $\psi_{x,0}^o = -0.446^\circ$ and $\psi_{y,0}^o = -2.278^\circ$, and the estimated initial orientation angles are $\psi_{x,0}^e = -0.443^\circ$ and $\psi_{y,0}^e = -2.276^\circ$. Figures 5.45–5.48 also include the results when initial orientations are known, as presented in Chapter 4.

Figure 5.42 E3, 2nd run, INS path without initial orientation estimation (blue)

Figure 5.43 E3, 2nd run, RSS localization (blue), $\sigma_{RSS} = 6dB$

Figure 5.44 E3, 2nd run, INS+RSS+EKF (blue)

Figure 5.45 E3, 2nd run, INS with all improvements applied without known initial orientation (blue) vs INS with gyroscope noise mitigation and known initial orientation (black)

Figure 5.46 E3, 2nd run, proposed RSS scheme with all techniques applied without known initial orientation (blue) vs proposed RSS scheme with gyroscope noise mitigation and known initial orientation (black)

Figure 5.47 E3, 2nd run, cumulative error: proposed (blue) vs EKF (red)

The cumulative error improves from 180.3 *m* to 88.3 *m* at 91 seconds, which is an improvement of 51%. The average cumulative errors at 91 seconds are 47790 *m*, 265.5 *m*, 205.1 *m*, and 80.8 *m* for the INS localization scheme, RSS localization scheme, INS+RSS+EKF scheme, and the final proposed RSS localization–INS projection scheme with noise mitigation. The proposed scheme shows improvements of 99%, 70%, and 61% over the other schemes, respectively. Initial orientation information leads to a 12% improvement in EKF error from 205.1 *m* to 181.5 *m*.

The dotted blue curve and solid curve stay close throughout the whole movement process in all three environments, indicating that the initial orientation estimation is effective.

Figure 5.48 E3, average cumulative error: proposed (blue) vs EKF (red) vs RSS (yellow) vs INS (black)

5.4 Experimental Results and Comparison

This section presents the results of testing the proposed RSS–INS fusion scheme in a real indoor environment. In Section 5.4.1, the environment layout and AP locations are presented, followed by descriptions of the path of movement, the selection of IMU device, AP and MD. The experiment’s results are given in Section 5.4.2, and the proposed fusion scheme, EKF and NLOS selection scheme results are compared and discussed. The proposed fusion scheme outperforms the others in both the 2D and 3D scenarios.

5.4.1 Experiment Environment and Setup

The indoor environment was a section of level B2 of the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. In this experiment, the user's movements remained on the same building level, and all APs were positioned on the ceiling of that level. Two sets of experiment performed at different locations of the environment are proposed. Figures 5.49 shows a photo of the experiment environment. The floor plan of the environment including two different routes are illustrated in Figures 5.50 and 5.51.

Figure 5.49 Experiment environment

Figure 5.50 First experiment route, including AP locations (green) and movement path (red)

Figure 5.51 Second experiment route, including AP locations (green) and movement path (red)

An experiment campaign was conducted in a typical office environment with dimension of $40\text{ m} \times 100\text{ m}$ using measured IMU values from Xsens device and RSS signals from 14 APs to show the validity of the proposed scheme. The height of the APs is either 3.3 m or 2.6 m , depending on the location. All APs are

Alcatel-Lucent Enterprise OmniAccess OAW-AP225 IEEE 802.11ac wireless access points. The walking path with its start and end points is shown in red. The MD used to measure RSS at every second was a Sony XZ Premium handphone running Android 8 OS, and an Xsens Mtw IMU was used to measure INS data. There are in total 84 RSS measurement points. The sampling frequencies are 1 Hz and 100 Hz for MD and IMU respectively. Path loss exponent n_p is 2.3 and the standard deviation of the Gaussian random variable ν is 6 dB [88]. The reference RSS_r , which is AP dependent, can be obtained using the following equation:

$$RSS_{r,1..m} = RSS_{1..m}^- + 10n_p \log_{10} \left(\frac{d_{1..m}^o}{d_r} \right) \quad (5.12)$$

where $RSS_{1..m}^-$ is the measured RSS values at known MD locations and $d_{1..m}^o$ is the Euclidean distances between MD and APs.

Letting reference d_r be equal to 1 m, a set of reference RSS values RSS_r can be acquired. The mean of RSS_r is found to be -53 dBm, which is used in the experiment environment. The standard deviations of IMU white Gaussian noise used in the simulations remain unchanged, to construct the weighting factor to improve RSS values in equation (4.3) for the experiment data. During the 10 second stationary calibration stage, the mean values of accelerometer measurements were used to estimate initial orientation, as elaborated in section 5.1. The mean values of gyroscope measurements, which are the bias when device is turned on, were also calculated and subtracted from the measured angular velocities after the calibration stage.

5.4.2 Comparison of Proposed RSS Measurement Improvement and Localization Results

In Section 5.2, the IMU is assumed to be held steady throughout the movement. Experimentally, it has been found the mean values of body frame x and y rotation angles are 0.43° and -0.42° for the first 40 seconds, reaching 1.42° and -1.30° respectively during total 84 seconds of movement of the first route. Hence, this assumption of only a small change in tilting angle is reasonable.

There is one noticeable difference between the simulated RSS values in previous sections and measured ones. Whereas RSS values were received by the user from all APs in the simulations, RSS values from some APs were not measured by the MD in the experiment, due to large transmission loss at some locations. The improved RSS shown in equation (4.3) needs to be modified to account for this phenomenon. Two solutions are proposed to address this issue.

The first solution is outlined in Table 5-1, which contains four different scenarios for RSS measurements. In scenario 1, RSS is measured at both t_{i-1} and t_i . The improved RSS is calculated following equation (4.3). In scenario 2, RSS is measured at t_{i-1} but disappears at t_i , and the improved RSS value at t_i relies on the previous improved RSS value at t_{i-1} and INS projection as described in

Section 4.1. In scenario 3, RSS is not measured at t_{i-1} but appears at t_i , and the improved RSS will solely rely on the measurement at t_i . In scenario 4, RSS values are not measured at either t_{i-1} or t_i , so those APs are excluded from the localization calculation.

	RSS_{i-1}^-	RSS_i^-	RSS'_i in (4.3)
1	✓ ¹	✓	$w1_i(RSS'_{i-1} + \Delta RSS_i) + w2_i RSS_i^-$
2	✓	✗ ²	$RSS'_{i-1} + \Delta RSS_i$
3	✗	✓	RSS_i^-
4	✗	✗	Dropped

¹ Measured. ² Not Measured.

Table 5-1 Proposed solution 1 for RSS'_i under different RSS measurement scenarios at t_{i-1} and t_i

The second solution is shown in Table 5-2. For scenario 1, RSS is measured at both t_{i-1} and t_i . The improved RSS is also calculated following equation (4.3). In scenario 2, RSS is measured at t_{i-1} but disappears at t_i , the improved RSS value at t_i relies on the previous improved RSS value at t_{i-1} and INS projection. In scenario 3, RSS is not measured at t_{i-1} but appears at t_i , the improved RSS will rely on INS projection and the RSS measurement at t_i . In scenario 4, RSS values are not measured at either t_{i-1} or t_i , so the improved RSS value at t_i also relies on the improved RSS value at t_{i-1} and INS projection.

	RSS_{i-1}^-	RSS_i^-	RSS'_i in (4.3)
1	✓	✓	$w1_i(RSS'_{i-1} + \Delta RSS_i) + w2_i RSS_i^-$
2	✓	✗	$RSS'_{i-1} + \Delta RSS_i$
3	✗	✓	$w1_i(RSS'_{i-1} + \Delta RSS_i) + w2_i RSS_i^-$
4	✗	✗	$RSS'_{i-1} + \Delta RSS_i$

Table 5-2 Proposed solution 2 for RSS'_i under different RSS measurement scenarios at t_{i-1} and t_i

Because INS projection accuracy deteriorates along the path, the original weighting factor $w_i = \frac{1}{d'_i}$ in equation (4.11) is modified to take this effect into account by introducing a new term $w_{loc,i}$ into the equation, as shown in Table 5-3. The new weighting factor is expressed as:

$$w'_i = w_{loc,i} w_i \quad (5.13)$$

In scenario 1, RSS are measured at both t_{i-1} and t_i , and the $w_{loc,i}$ equals 1. In scenario 2, RSS measurements start to disappear at t_i , and the value of $w_{loc,i}$ is set to $w1_{i-1}$. Let N be the number of consecutive times RSS are not measured; then the term $(w1_{i-N})^N$ is introduced if RSS measurements keep disappearing. In scenario 3, if RSS measurement changes from disappearing to appearing, $w_{loc,i}$ equals to the greater value of $(w1_{i-N})^N$ and $w2_i$. In scenario 4, RSS values are not measured at either t_{i-1} and t_i , so that $w_{loc,i}$ equals $(w1_{i-N})^N$. The value of $w_{loc,i}$ is decreasing, which also means that the weight of RSS relative to INS projection is decreasing.

	RSS_{i-1}^-	RSS_i^-	Weight $w_{loc,i}$
1	✓	✓	1
2	✓	✗	$w1_{i-1}$
3	✗	✓	$\max\{(w1_{i-N})^N, w2_i\}$
4	✗	✗	$(w1_{i-N})^N$

Table 5-3 Modified weight scenarios in solution 2

First localization path from the various schemes are shown in Figure 5.52. The true walking path, represented as a black dashed line, is used as a reference. Two blue paths are depicted to describe the proposed fusion scheme. The solid blue path represents the first proposed solution shown in Table 5-1 without a modified weight factor, while the dotted blue path is the second solution, with w_{loc} as part of the modified weighted LS. The dotted blue line stays closer to the true path, especially around the first and second turn. The RSS reception is poor at the first turn; thus, this result shows that the proposed INS projection and w_{loc} can provide valuable information for localization. At the end of the path, both blue lines almost overlap with each other, indicating that the weighting for RSS is much higher than INS projection at the end. The EKF result is shown in red, and the NLOS selection scheme introduced in section 5.3.1 is depicted in green. The threshold for the NLOS selection scheme is set at 3 m , as the localization error is the minimum for thresholds ranging from 1 m to 6 m .

Figure 5.52 First experiment route localization paths: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)

In the environments using 2D and 3D modelling which are referred to as 2D and 3D environments in this thesis, the cumulative errors of first experiment route are shown in Figures 5.53 and 5.54. The proposed scheme with modified weighted LS shows a 30% (49%) and 31% (61%) improvement over the NLOS selection scheme and EKF, respectively, in a 2D (3D) environment. Aside from the performance advantage of the proposed fusion scheme over other schemes in both 2D and 3D environments, the proposed scheme also fluctuates less when switching between 2D and 3D calculations. This result shows that the proposed scheme can handle 3D environments better than other schemes.

Figure 5.53 First experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)

Figure 5.54 3D First experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)

For the first experiment route, the mean value of the number of measured RSS at each location is 9.6 and the minimum number of measured RSS is 6. In order to further test the robustness of our proposed scheme under more RSS challenging scenarios, the second experiment route has been proposed. For the second route, the mean value of the number of measured RSS at each location is 4.7 and the minimum number of measured RSS is 3. Furthermore, the number of measured RSS is less than 5 for 27 seconds, accounting for half of total walking duration of the second route. There is no APs on the left side of the second route, which is another factor to make RSS localization more challenging. Second localization path from the various schemes are shown in Figure 5.55. The dotted blue line depicting results using our proposed scheme with w_{loc} as part of the modified weighted LS again stays closer to the true path for majority of the time.

Figure 5.55 Second experiment route localization paths: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)

The cumulative errors of second experiment route are shown in Figures 5.56 and 5.57. The proposed scheme with modified weighted LS shows a 75% (75%) and 42% (43%) improvement over the NLOS selection scheme and EKF, respectively, in a 2D (3D) environment. This result shows that the proposed scheme can handle environment that is more RSS challenging better than other schemes.

Figure 5.56 Second experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)

Figure 5.57 3D Second experiment route cumulative error: proposed with modified weighted LS (dotted blue) vs proposed without modified weighted LS (blue) vs NLOS selection (green) vs EKF (red)

5.5 Conclusion

In this chapter, an initial orientation angle estimation technique is proposed. The correct initial orientation angle estimation is shown to be crucial for improving the localization results of fusion systems. Furthermore, a technique to mitigate noise in the specific forces measured by accelerometers is explained. Simulations are performed across three environments with different initial orientation angles and sources of noise. The final form of the proposed scheme, which includes all of the techniques outlined in chapters 4 and 5, significantly outperform INS and the RSS fusion system using EKF. In terms of the average cumulative error over 50 independent runs, the final proposed scheme offers improvements of 65%, 36%, and 61% over the INS+RSS+EKF fusion system across the three environments examined here. Finally, the validity of the proposed scheme was tested in a real environment. The results indicate that the scheme has an average of 53% (62%) and 37% (52%) improvement over NLOS selection scheme and EKF respectively in two 2D (3D) environment.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This thesis has proposed a novel weighted nonlinear RSS localization and tracking scheme using INS projection. The proposed scheme offers a more realistic holding position for the IMU and improves localization accuracy compared with conventional INS and RSS fusion systems using EKF.

Chapter 3 presented a theoretical closed-form equation to analyze the navigation frame localization RMSE. This was achieved by breaking down the transformation matrix $\mathbf{C}_b^N$ and using up to second-order Taylor expansion approximations. An IMU navigation frame-localization equation was introduced and IMU sensors' noise characteristics were obtained using the Allan standard deviation technique, which was then fed into the derived equation for calculation. The result was compared with posterior CRLB using different $\mathbf{C}_b^N$

representations. The proposed method outperformed other commonly used schemes and can thus be used as an alternative to posterior CRLB.

Chapter 4 revealed a novel weighted RSS localization and tracking scheme with INS projections, in cases when the IMU's initial orientation is known. This method combines RSS values at adjacent sampling times with the help of INS localization estimations during that period. The weighting factor for combining RSS values was also elaborated. The combined RSS values were then converted to distance and weighted nonlinear LS and an algorithm applied to estimate location. A method was also proposed to mitigate white Gaussian noise on angular velocity measured by the gyroscope, which accounts for the majority of total error. Simulations were performed in three different environments. The results show that up to 56% improvement over conventional INS and RSS fusion systems using EKF can be achieved.

Chapter 5 first addressed the situation in which the initial orientation is unknown. A method was proposed for accurately estimating the initial orientation angles, which are responsible for gravitational accelerations on the body frame x and y axes. Errors in initial orientation estimations were shown to be the largest source of error; as such, it is crucial to have an accurate estimation to improve localization results. The technique used to mitigate noise on specific forces measured by accelerometers was also explained. Simulations were performed across three environments with different initial orientation angles. The final proposed scheme with all noise mitigation techniques significantly outperformed INS and RSS fusion systems with EKF. The results show that up to 65%

improvement on EKF can be obtained. The performance of the proposed scheme was also tested in a real environment. The proposed scheme with modified weighted LS has 53% (62%) and 37% (52%) improvement over the NLOS selection scheme and EKF, respectively, in 2D (3D) environments.

6.2 Future Work

Indoor localization using infrastructure-less and infrastructure-based systems is a topic that still requires more research.

Our proposed fusion system can be updated by taking more previous INS projections and improved RSS values into consideration. More sophisticated weighting factors need to be acquired accordingly.

Other systems could be added to form new fusion systems. 5G networks will soon be implemented for both indoor and outdoor environments. As a result, device-to-device communication will become commonplace. Cooperative localization will become essential. As such, the present work can be further extended to include the fusion of 5G cellular network location services into the current RSS fusion system to enhance the localization accuracy of peer-to-peer localization. Scenarios in which there are multiple users inside the same environment, when their relative positions are known, should also be investigated. Cooperative schemes involving multiple users are worth exploration.

Our proposed scheme relies more on RSS measurements as time elapses because its weighting factor is approaching 1. Hence IMU needs to be recalibrated to reset the fusion system such that INS projection becomes effective again. Light-fidelity (LiFi) that uses light emitting diodes (LED) for high speed wireless communication is one option to recalibrate IMU. LED lighting is readily available in indoor environments nowadays due to its low implementation cost and low energy consumption. LiFi use visible light spectrum such that it can avoid RF interference which makes it more attractive in areas such as hospitals and nuclear power plants where WiFi is forbidden. It is also helpful to address the issue when insufficient RSS values are received by MD for fusion.

Furthermore, as detailed map information becomes more prevalent, especially in 5G environments, localization can be further improved with usage of few landmarks with fewer APs, as the current localization is bounded by RSS performance. Map information can also help to re-calibrate IMUs.

Author's Publications

Journal papers:

[1] K. Wen, C. K. Seow, and S. Y. Tan, "A novel weighted RSS localization and tracking scheme using INS projection," *Sensors 2019*, submitted.

Conference papers:

[1] K. Wen, C. K. Seow, and S. Y. Tan, "Inertial navigation system positioning error analysis and Cramér-Rao lower bound," in *Proc. IEEE/ION PLANS 2016 Position Location and Navigation Symposium (PLANS'16)*, Savannah, GA, Apr. 11-14, 2016, pp. 213-218.

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Appendix A

Standard Kalman Filter and Extended Kalman Filter

This appendix describes the fundamentals of the standard Kalman filter (KF) and the extended Kalman filter (EKF). Both KF and EKF propagate Bayesian belief as a Gaussian distribution with their means and covariances.

A.1 Standard Kalman Filter

The true state x of the systems propagates according to the following linear equation:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{n}_k \quad (\text{A.1})$$

where $\mathbf{n}_k$ is process noise assumed to follow a zero mean Gaussian distribution with covariance $\mathbf{Q}_k$:

$$\mathbf{n}_k \sim N(0, \mathbf{Q}_k) \quad (\text{A.2})$$

Measurements can be formulated using a linear relationship with the true state:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \quad (\text{A.3})$$

where $\mathbf{v}_k$ is measurement noise assumed to follow a zero mean Gaussian distribution with covariance $\mathbf{R}_k$:

$$\mathbf{v}_k \sim N(0, \mathbf{R}_k) \quad (\text{A.4})$$

Given the preceding assumptions, linear optimal Bayesian propagation and correction equations can be implemented using matrix operation. The estimated state's mean and covariance can be propagated as:

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_k \hat{\mathbf{x}}_{k-1}^- \quad (\text{A.5})$$

$$\hat{\mathbf{P}}_k^- = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{Q}_k \quad (\text{A.6})$$

where $\mathbf{P}_{k-1}$ is the corrected covariance matrix at time instance $k-1$.

Then measurement $\mathbf{z}_k$ is used to correct the propagated state:

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (\text{A.7})$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-) \quad (\text{A.8})$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^- \quad (\text{A.9})$$

where $\mathbf{K}_k$ is called Kalman gain.

A.2 Extended Kalman Filter

The standard Kalman filter assumes that both the propagation and correction equation are linear. EKF is used to deal with nonlinear situations by calculating Jacobian matrices to linearize the equation. Now the propagation and measurement are written as

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}) + \mathbf{n}_k \quad (\text{A.10})$$

$$\mathbf{z}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k \quad (\text{A.11})$$

The Jacobian matrices of f_k and h_k are

$$\mathbf{F}_k = \left[\frac{\partial f_k}{\partial \mathbf{x}} \right]_{\hat{\mathbf{x}}_{k-1}} \quad (\text{A.12})$$

$$\mathbf{H}_k = \left[\frac{\partial h_k}{\partial \mathbf{x}} \right]_{\hat{\mathbf{x}}_k^-} \quad (\text{A.13})$$

Finally, the estimated state's mean and covariance can be propagated as

$$\hat{\mathbf{x}}_k^- = f_k(\hat{\mathbf{x}}_{k-1}^-) \quad (\text{A.14})$$

$$\hat{\mathbf{P}}_k^- = \mathbf{F}_k \mathbf{P}_{k-1} \mathbf{F}_k^T + \mathbf{Q}_k \quad (\text{A.15})$$

The measurement update equations are

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1} \quad (\text{A.16})$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - h_k(\hat{\mathbf{x}}_k^-)) \quad (\text{A.17})$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^- \quad (\text{A.18})$$