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Digital Holography to Light Field

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ABSTRACT

Holography uses wave (physical) optical principles of interference and diffraction to record and display images. Interference allows us to record the amplitude and phase of the optical wave emanating from an object on a film or recording medium and diffraction enables us to see this wave-field, i.e. the amplitude and phase of the object. Visually this corresponds to both perspective and depth information being reconstructed as in the original scene. Digital Holography has enabled quantification of phase which in some applications provides meaningful engineering parameters. There is growing interest in reconstructing this wavefield without interference. Thus the non-interferometric Transport of Intensity Equation (TIE) method is gaining increased research, which uses two or more defocused images to reconstruct the phase. Due to its non-interferometric nature, TIE relaxes the stringent beam-coherence requirements for interferometry, extending its applications to various optical fields with arbitrary spatial and temporal coherence.

The alternate school of thought emerges from the computer science community primarily deals with ray optics. In a normal imaging system all rays emerging from an object point into are focused to a conjugate image point. Information of ray direction is lost and thus the perspective and depth information. A light field image is one that has information of both amplitude and direction of rays fanning from any object point and thus provides perspective (or what could be termed as phase) of the object wave as well. It would thus be possible to extract phase as we know it from this albeit for a coherent illumination case.

Keywords: digital holography; transport-of-intensity equation; Light field; phase retrieval;

1. INTRODUCTION

The concept of phase is deeply instilled in our mind from a very early age and its measurement seems inevitably intertwined with the need to perform interferometry for a long time. Phase is an important component of an optical wavefield because all waves are characterized by an amplitude and a phase. Since the phase of a wavefield is not accessible directly, phase measurement and retrieval become a central problem in many areas of physics and optics.

In the basic theory of partial coherence, it is recognized that waves are always a superposition of waves with different frequencies and different phases. The result is that the concept of phase tends to lose meaning unless strenuous efforts are made to retain the coherence of the wave. Phase measurement has therefore been traditionally taken to implicitly require light that is highly coherent (spatially and temporally). However, research into astronomical adaptive optics and ophthalmology, which attempts to develop techniques that are able to determine phase perturbations induced by the passage of light waves through an aberrating medium, which usually has a very broad range of wavelengths and therefore does not itself have a well-defined phase. On another note, recently growing research in transport-of-intensity equation (TIE) further confirmed that quantitative phase can be retrieved only through object field intensities at two or more axially displaced planes, without stringent beam-coherence requirements. This suggests it is clearly possible to perform phase measurement even though the conventional idea of phase has broken down.

The alternate school of thought emerges from the computer scientist group who primarily deal with ray optics. In a normal imaging system all rays emerging from an object point into are focused to a conjugate image point. Information of ray direction is lost and so to the perspective and depth information. In ray optics, there is no concept of phase, instead, the quantities interested here is so-called the light field – which has information of both amplitude and direction of rays fanning from any object point and thus provides perspective (or what could be termed as phase) of the object wave as well. This now brings us to a central issue of the paper - what is the meaning of the term “phase” and how it relates to the light field? For a coherent complex scalar wave, its definition is clear: the “phase” is equal to the argument of the

complex function describing the field. Such a field, which is perfectly spatially and temporally coherent, has wavefront corresponding to surfaces of constant phase. For a non-coherent field, this definition breaks down and we need a better and more general description of what is meant by phase for a partially coherent field. We will consider this interesting issue after a short introduction about phase measurement and phase retrieval. After introduction the general definition of the phase, we will show its tight connection with the light field. We will show through some experiments that with this new definition, the phase can be easily recovered from the light field. This process also can be reversed, for a smooth object under certain simplified illumination.

2. PHASE MEASUREMENT AND RETRIEVAL

Complex scalar fields are completely specified by their amplitude and phase at each point of space-time. Because of the rapidity of field oscillations at optical and higher frequencies, only the mean-square modulus of such fields (averaged over many cycles) is directly measurable. Interferometry and holography is an extremely well known technique for phase measurement. The basis of the interferometry and holography is to overlay one coherent beam with another. The resulting coherent superposition results in amplitude modulation, so-called interference fringes which can be seen by a camera. Here we briefly introduce the basic principle of the digital holography [1]: In both transmission and reflective setups for holography, a coherent laser beam is split into two parts – the reference beam illuminates the CCD directly. The object beam either passes through or reflects off the sample and interferes with the reference beam at the CCD plane with a small angle to generate the off-axis hologram. The intensity distribution recorded by the camera can be written as [2, 3]

$$I_H(x, y) = |O|^2 + |R|^2 + RO^* + R^*O \quad (1)$$

$R(x, y)$ and $O(x, y)$ are the reference and object waves respectively, *denotes the complex conjugate. The whole information of the object wave is encoded into the +1(real) or -1(virtual) interference term, described by

$$R^*O = |O||R|\cos(\phi_O - \phi_R) \quad (2)$$

The hologram is sampled by the CCD array and then transferred into a computer as an array of numbers. For the off-axis holography, filtering the hologram's two-dimensional Fourier spectrum can eliminate the virtual image and the zero-order term. The diffracted field, including amplitude and phase distribution at the image plane is then numerically propagated from the hologram plane using Fresnel transform, convolution, or angular spectrum methods. It should be noted the phase measured via holography is actually the phase difference between the object beam and the reference beam, and the phase for the reference beam must be known a priori for the determination of object phase. Secondly, note that only the cosine term is measured and this only yields the phase modulo 2π . Phase unwrapping is needed to remove these phase jumps, which however is not a simple task particularly if the phase distribution is not continuous or the data contain significant levels of noise.

Fig. 1 Schematics of the digital holography and TIE phase retrieval. (a) Digital holography. (b) TIE.

The TIE uses only object field intensities at multiple axially displaced planes without any interference with a separate reference beam. The experimental setup for TIE typically involves a $4f$ imaging system. By translating the camera or the object, multiple intensity images at different image distance can be obtained. Due to its non-interferometric nature, the

illumination can be quasi-monochromatic and partially-coherent. TIE determines the object-plane phase from the first derivative of intensity in the near Fresnel region [4]

$$-k \frac{\partial I}{\partial z} = \nabla \cdot (I \nabla \phi) \quad (3)$$

Where k is the wave number. ∇ is the gradient operator over (x, y) . z denotes position along the optics axis perpendicular to the x - y plane. If $I > 0$ and ϕ is continuous in a region with smooth boundaries, the solution to TIE is unique. That is, the phase can be uniquely determined by solving TIE with I and $\partial I / \partial z$. Experimentally, the intensity is easy to obtain and the intensity derivative is estimated by finite differences between two close separated images. Then the phase can be obtained by solving the TIE by treating it as a modified Poisson equation or expanding it into a complete set of Zernike polynomials.

3. PHASE FOR PARTIALLY COHERENT FIELD

The conventional definition of phase breaks down if the waves is not coherent and so for the following discussion to be properly defined we need a general definition of what is meant by phase which is valid for non-coherent fields. Let $U(\mathbf{x})$ be a stationary and ergodic quasi-monochromatic paraxial scalar field with arbitrary coherence, where $\mathbf{x}$ is the two-dimensional spatial vector. This partially coherent field can be characterized by the Wigner distribution function (WDF) [5]

$$W(\mathbf{x}, \mathbf{u}) = \int \Gamma\left(\mathbf{x} + \frac{\mathbf{x}'}{2}, \mathbf{x} - \frac{\mathbf{x}'}{2}\right) \exp(-i2\pi \mathbf{u} \mathbf{x}') d\mathbf{x}', \quad (4)$$

where $\mathbf{x} = (x, y)$ and $\mathbf{u} = (u, v)$ are the two-dimensional spatial and spatial frequency vectors, respectively, Γ is the mutual intensity related to the ensemble average $\Gamma(\mathbf{x}_1, \mathbf{x}_2) = \langle U(\mathbf{x}_1) U^*(\mathbf{x}_2) \rangle$. Here we prefer to use the phase-space formulation in terms of the WDF because its convenience and closely resembles the ray concept in geometrical optics. With use of the Liouville transport equation [5], we can derive a generalized transport of intensity equation (GTIE) which is valid under partially coherent field [6]

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\lambda \nabla_{\mathbf{x}} \cdot \int \mathbf{u} W(\mathbf{x}, \mathbf{u}) d\mathbf{u}. \quad (5)$$

In completely coherent case, the field can be fully described by the 2D complex amplitude $U(\mathbf{x}) = \sqrt{I(\mathbf{x})} \exp[i\phi(\mathbf{x})]$, where ϕ is the (conventional) phase. From the time(space)-frequency analysis perspective, the completely coherent field can be regarded as a mono-component signal, and the first conditional frequency moment of WDF (instantaneous frequency) relates to the transverse gradient of the phase of the complex field [7]:

$$\frac{\int \mathbf{u} W(\mathbf{x}, \mathbf{u}) d\mathbf{u}}{\int W(\mathbf{x}, \mathbf{u}) d\mathbf{u}} = \frac{1}{2\pi} \nabla \phi(\mathbf{x}). \quad (6)$$

Substitution of Eq. (6) into Eq. (5) leads to coherent TIE

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\frac{1}{k} \nabla \cdot [I(\mathbf{x}) \nabla \phi(\mathbf{x})]. \quad (7)$$

The partially coherent field does not have a well-defined phase since the field experiences statistical fluctuations over time. However, the phase-space representation on LHS of Eq. (6) is still valid, leading to a new meaningful and more general definition of “phase”. Here we refer the new “phase” $\phi(\mathbf{x})$ defined by Eq. (6) as the generalized phase of partially coherent fields to distinguish it from its coherent counterpart. The generalized phase behaving precisely as the conventionally defined phase and directly reduces to conventional phase when the field is fully coherent. It is seen from Eq. (6) that the generalized phase is a scalar potential whose gradient yields the conditional frequency moment of the WDF. It is clear from a distribution point of view, that quantity is the average frequency at a particular location. Furthermore, the simultaneous space-frequency description of WDF closely resembles the ray concept in geometrical optics, which conveys information about the amplitude and direction of flow of energy (Poynting vector). However, the WDF is not a rigorous probability distribution in phase space since it can take negative values. But it is rather instructive to adopt this simple interpretation as link between physical optics and geometrical optics. Furthermore, this simple

interpretation provides an illuminating physical picture of beam propagation and energy transport with reasonable accuracy where the diffraction effects and the vectorial nature of the field can be safely ignored.

4. SHACK HARTMANN SENSOR AND LIGHT FIELD IMAGING

For completely coherent fields, the phase-space representation is highly redundant because the complex field is defined only over the 2D plane $\mathbf{x}$. The 2D intensity and the reconstructed phase distribution gives total knowledge about the complex field so that the behavior of the field can be perfectly predicted. If the phase varies slowly such that the approximations $\phi(\mathbf{x} + \mathbf{x}/2) - \phi(\mathbf{x} - \mathbf{x}/2) \approx \mathbf{x} \cdot \nabla \phi(\mathbf{x})$ is valid, the phase space redundancy becomes more apparent since the signal occupies only a single slice in phase space

$$W(\mathbf{x}, \mathbf{u}) = I(\mathbf{x}) \delta \left[\mathbf{u} - \frac{1}{2\pi} \nabla \phi(\mathbf{x}) \right]. \quad (8)$$

The form of WDF given above now is a true energy probability distribution in phase-space, telling us the geometric ray or energy flow at single position travels only along single direction described by the phase normal. This is an advantageous feature to allow phase measurement simply by measuring the directions of the rays, using the Shack-Hartmann sensor [8].

Fig. 2 Principle of the Shack-Hartmann sensor and Light field Imaging. (a) Shack-Hartmann sensor. (b) Light field Imaging.

The Shack-Hartmann sensor is an array of lenslets, each of which brings the incident field to a focus. When normally incident plane waves are shone onto the sensor, each lenslet brings the light to a focus at the center of its associated detector. If an aberrated wave is incident onto the sensor, then the location of each spot will be displaced by a vector proportional to the average phase gradient $\nabla \phi(\mathbf{x})$ over the lenslet. This displacement may be sensed by, for example, a quadrant detector. The resulting signals should be integrated to create an estimate of the phase distribution. Though its conceptual simplicity, the disadvantage of the Shack-Hartmann sensor is obvious: the spatial resolution over the wavefront is limited by the size and number of the lenslets.

For partially coherent field (include the condition of the totally incoherent fields), the 4D WDF is generally non-redundant. From the geometrical optics perspective, the geometric ray at single position does not travel only in one direction; instead, it fans out to make a 2D distribution, which account for the higher dimensionality of the partially coherent field. The light field camera, as a counterpart of the Shack-Hartmann sensor in computer graphics community, allows joint measurement of the spatial and directional distribution of light [9]. The “light field” is a term commonly used in the computer graphics literature to represent a collection of light rays in geometric optics. It is parameterized by a four-variable function $L(\mathbf{x}, \boldsymbol{\theta})$, taking into account both the geometrical position of the rays $\mathbf{x}$ and also their directions $\boldsymbol{\theta}$. It approaches the WDF at geometric optics limit [10]. Light field imaging enables us to apply ray-tracing techniques to compute synthetic photographs, change the focus and perspective view flexibly. However, it requires elaborate optical setups and significantly sacrifices spatial resolution (traded for angular information) as compared to conventional imaging technique.

5. FROM LIGHT FIELD TO PHASE

As we mentioned previously, the phase information can be quantitatively retrieved from a set of defocused images through solving the transport of intensity equation. Usually acquiring the image data for TIE is time-consuming, as the object stage or camera has to be moved between image captures. Though several configurations have been developed to eliminate the mechanic motion [11, 12], light field imaging enables a totally new way to collect the whole image stack, as the intensity images at an arbitrary plane of focus can be easily reconstructed from the raw light field. By using the reconstructed stack, the TIE can be solved to recover the phase, allowing for the capture of defocus information and phase imaging in single-shot. This method suggests one viable way to convert the light-field to the phase.

It would be more interesting to consider the problem from a different perspective. It is seen from Eq. (10) that the generalized phase is a scalar potential whose gradient yields the conditional frequency moment of the WDF. Under geometric optics limit, we can apply the approximation $L(\mathbf{x}, \boldsymbol{\theta}) \approx W(\mathbf{x}, \lambda \mathbf{u})$ to Eq. (10),

$$\frac{\int \boldsymbol{\theta} L(\mathbf{x}, \boldsymbol{\theta}) d\boldsymbol{\theta}}{\int L(\mathbf{x}, \boldsymbol{\theta}) d\boldsymbol{\theta}} = \frac{1}{2\pi} \nabla \phi(\mathbf{x}). \quad (9)$$

It is clear from the above definition that, that the quantity on LHS is the centroid of the light field- the average directions of light at one given position. This means the phase gradient can be easily recovered by a simple centroid detect scheme applied to the raw light field image, which is similar with the standard procedure for processing the Shack-Hartmann sensor images. Compared with the first method which solves the TIE explicitly, the second method, which employs the new definition of the “phase” here, is inherently much easier and straightforward.

Fig. 3 Image captured with a light field microscope with different illumination NA. (a) Condenser NA=0.05, (b) NA=0.15, (c) NA=0.2 (d) NA=0.25. The second row shows the enlarged images corresponding to the boxed regions.

Fig. 4 Phases reconstructed from the light fields (Unit: rad). (a) Condenser NA=0.05, (b) NA=0.15, (c) NA=0.2 (d) NA=0.25

We show one typical example to verify the correctness of Eq. (9). The experiment is carried out based on a light field microscope (Olympus BX-41) with a microlens array (pitch 150 μ m, ROC 10.518mm) inserted in the intermediate image plane. The sample measured is a plano-convex microlens array, which is imaged with a 20 $\times$ objective with NA=0.40. The illumination from the built-in halogen lamp was filtered by a green interference filter with a central wavelength of 550 nm and a pass-band of 45 nm. The coherence of the illumination depends on the condenser setting. We captured 4 groups of light field images with the condenser NA ranging from 0.05 – 0.25, as shown in Fig. 3. From the enlarged image, we can clearly see the intensity changes corresponding to each lenslet, from a focus spots array to a 2-D sub-aperture image. According to Eq. (9), we extract the centroid for each subimage and followed by an integration procedure, the phase images can be reconstructed as shown in Fig. 3. The results confirm that the phase can be reconstructed from the light field; even the field is not completely coherent. Note that for the case when the condenser aperture is open up to NA=0, the sub-aperture image is too large and some of them overlap, which poses a problem for the centroid locating, resulting in some artifacts in the final phase image. In Figure 4, we show another example to examine the validity of the approach for a more complex sample (stem cell). The result looks nice except the resolution is too low. Figure 3(c) shows the result after cubic interpolation, showing less mosaic effect.

Fig. 5 Result of the stem cell (Unit: rad). (a) Raw light field image NA=0.2, (b) recovered phase (rad), (c) result after cubic interpolation.

6. FROM PHASE TO LIGHT FIELD

It is quite understandable that the phase can be recovered from the light field since the 4D light field is inherently in a higher dimensionality which totally cover the 2-D phase information. By employing the method discussed above, the phase can be extracted from the light field without great efforts. But it would be quite interesting to consider whether the inverse of this process can be realized: with the knowledge of phase, is it possible to recover the whole light field? Apparently the answer should be NO for the general cases. However, under certain conditions, we can indeed convert the phase to the light field. Besides the trivial case when the field is purely coherent, let us consider that the sample is illuminated by a spatial stationary illumination that is generally true for the experimental arrangements in optical microscopy [5]. The fully spatially incoherent primary source (usually at the condenser aperture plane for an optical microscope) featured by the intensity distribution $|P_c(\mathbf{x})|^2$ and the positional cross-spectral density $\Gamma(\mathbf{x} + \mathbf{x}'/2, \mathbf{x} - \mathbf{x}'/2) = |P_c(\mathbf{x})|^2 \delta(\mathbf{x}')$ produces the illumination WDF $W_{in}(\mathbf{x}, \mathbf{u})$ at the far-zone

$$W_{in}(\mathbf{x}, \mathbf{u}) = |P_c(\mathbf{u})|^2. \quad (10)$$

Equation (16) is in fact an expression of the Van Cittert-Zernike theorem. If the phase varies slowly such that the approximations $\phi(\mathbf{x} + \mathbf{x}/2) - \phi(\mathbf{x} - \mathbf{x}/2) \approx \mathbf{x} \cdot \nabla \phi(\mathbf{x})$, the resultant field just leaving the object can be represented as

$$L(\mathbf{x}, \boldsymbol{\theta}) = cI(\mathbf{x}) \left| P_c \left[\boldsymbol{\theta} - \frac{1}{k} \nabla_{\mathbf{x}} \phi(\mathbf{x}) \right] \right|^2, \quad (11)$$

where c is a constant ensuring $I(\mathbf{x}) = \int L(\mathbf{x}, \boldsymbol{\theta}) d\boldsymbol{\theta}$. Note the last step we used the approximation $L(\mathbf{x}, \boldsymbol{\theta}) \approx W(\mathbf{x}, \lambda \mathbf{u})$. Equation (18) represents exactly the geometric optical behavior of the specimen: for each incident ray, it leaves the specimen from the same position but its direction is shifted as a function of the phase gradient of the object. The specimen, can be regarded as a spread-less system, does not change the angular distribution of the incident field, which is fully determined by the source intensity distribution. Thus, with the knowledge of the source intensity distribution and the phase of the object $\phi(\mathbf{x})$ (can be retrieved from TIE with only 2 images), the 4D light-field can be fully characterized.

7. CONCLUSIONS

In conclusion, we discuss the connection between the phase measurement/retrieval method and the light field imaging method, especially we clarify the meaning of phase for a non-coherent field and proved they can be interconverted under certain conditions. Hopefully, it could serve as a new bridge between ray-optics and physical optics, or physics and computer graphics, inspiring more works to be recognized, reformulated or reinterpreted in either field wherein their parallels are not yet presented or recognized.

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