An experimental approach to dike propagation : the effects of stress, solidification and internal flow

Pansino, Stephen

2019

Pansino, S. (2019). An experimental approach to dike propagation : the effects of stress, solidification and internal flow. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/104237>

<https://doi.org/10.32657/10356/104237>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**AN EXPERIMENTAL APPROACH TO DIKE PROPAGATION: THE
EFFECTS OF STRESS, SOLIDIFICATION AND INTERNAL FLOW**

STEPHEN PANSINO

Earth Observatory of Singapore

2019

**AN EXPERIMENTAL APPROACH TO DIKE PROPAGATION: THE
EFFECTS OF STRESS, SOLIDIFICATION AND INTERNAL FLOW**

STEPHEN PANSINO

Earth Observatory of Singapore

A thesis submitted to the Nanyang Technological University in partial
fulfilment of the requirement for the degree of Doctor of Philosophy

2019

(Blank page)

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

31 May 2019

.....
Date

.....
Stephen Pansino

(Blank page)

Nanyang Technological University
Interdisciplinary Graduate School
Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

3rd June 2019

.....
Date

.....
Benoit Taisne

(Blank page)

**Nanyang Technological University
Interdisciplinary Graduate School
Authorship Attribution Statement**

This thesis contains material from one paper from the following peer-reviewed journals where I was the first and corresponding author.

Chapter 2 is published as Pansino, S. & Taisne, B. (2019). How magmatic storage regions attract and repel propagating dikes. *J. Geophysical Res.: Solid Earth* **124**, 274-290. DOI: 10.1029/2018JB016311

The contributions of the co-authors are as follows:

- Asst. Prof. Taisne provided the initial project direction and edited the manuscript drafts
- I prepared the manuscript drafts
- I designed the experimental apparatus and performed all experimental work in the Magma Transport Dynamics Laboratory in the Earth Observatory of Singapore and subsequently analyzed the data

This thesis also contains material from three papers in preparation, two of which I am the first and corresponding author.

Chapter 3 is in preparation to be published as Pansino, S., Emadzadeh, A. & Taisne, B. Dike channelization and solidification: Time scale controls on the geometry and placement of magma migration pathways. To be submitted to *J. Geophysical Res.: Solid Earth*

The contributions of the co-authors are as follows:

- Asst. Prof. Taisne and Dr. Emadzadeh edited the manuscript drafts
- I prepared the manuscript drafts
- Dr. Emadzadeh performed Particle Image/Tracking Velocimetry measurements on the experiments and analyzed the associated data
- I designed the experimental apparatus and ran all experiments in the Magma Transport Dynamics Laboratory in the Earth Observatory of Singapore and subsequently analyzed all other data
- Dr. Emadzadeh and I set up the Particle Image/Tracking Velocimetry equipment

Chapter 4 is in preparation to be published as two separate manuscripts:

Emadzadeh, A., Pansino, S. & Taisne, B. 4D PTV/PIV measurements of experimental dikes and the surrounding solid medium. To be submitted to *Tectonophysics*

(Blank page)

Pansino, S., Taisne, B. & Emadzadeh, A. Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns. To be submitted to *Earth and Planetary Sci. Lett.*

The first manuscript focuses on our experimental technique, since it is the first time being used in our field. The second focuses on the results from our experiments. The contributions of the co-authors are as follows:

- Asst. Prof. Taisne provided the project direction edited the manuscript drafts
- Dr. Emadzadeh prepared the first manuscript draft
- I prepared the second manuscript draft
- Dr. Emadzadeh performed Particle Image/Tracking Velocimetry measurements on the experiments and analyzed the associated data
- I designed the experimental apparatus and ran the experiments in the Magma Transport Dynamics Laboratory in the Earth Observatory of Singapore and subsequently performed analysis on all other data
- Dr. Emadzadeh and I set up the Particle Image/Tracking Velocimetry equipment

We the undersigned agree with the above stated “proportion of work undertaken” for each of the above published (or submitted) peer-reviewed manuscripts contributing to the thesis:

Signed:

Stephen Pansino
Student
Interdisciplinary Graduate School
Nanyang Technological University
Date: 3rd June 2019

Asst Prof Benoit Taisne
Supervisor
Asian School of the Environment
Nanyang Technological University
Date: 3rd June 2019

Assoc Prof Emma Hill
Associate Chair (Research)
Asian School of the Environment
Nanyang Technological University
Date: 6 June 2019

(Blank page)

Additional Information that will be reflected on Supplementary Transcript

NAME OF CANDIDATE	: STEPHEN PANSINO	DATE OF BIRTH:	6 APRIL 1987
MATRICULATION NO.	: G1402609E	DATE OF ADMISSION:	27 JULY 2014
DEGREE CONFERRED	: DOCTOR OF PHILOSOPHY		
THEMATIC RESEARCH-BASED PROGRAMME	: SUSTAINABLE EARTH	RESEARCH CENTRE:	EARTH OBSERVATORY OF SINGAPORE
Thesis Title	: AN EXPERIMENTAL APPROACH TO DIKE PROPAGATION: THE EFFECTS OF STRESS, SOLIDIFICATION AND INTERNAL FLOW		
Summary of Work	: Steve used small, laboratory-scale experiments to investigate magmatic propagation, in which magma migrates through the Earth's crust via cracks. He g experience in various image processing techniques, which were used to measur geometry and deformation in his experiments. His work has contributed to understanding of how dikes grow and how underground plumbing systems develop		
Main Supervisor	: Assistant Professor, Benoit Taisne, Earth Observatory of Singapore		
Co-supervisor	: Assistant Professor, Lorenzo Masia, School of Mechanical & Aerospace Engineering		
Mentor	: Associate Professor, Fidel Costa, Earth Observatory of Singapore		

We confirm that the above information indicated by the candidate is correct and noted that it will be reflected in the supplementary transcript.

Name & Signature of Co-Supervisor/Date

(Blank page)

Acknowledgements

I would like to take this opportunity to thank everyone who supported me throughout the duration of my studies, including all of my friends who have come and gone through the EOS.

First and foremost, I would especially like to thank my wife, Dayana Schonwalder, who has tirelessly supported me through these years. She has been a source of emotional support and has given me plenty of advice, both academic and emotional. She has been a key mentor for me in understanding the petrological concepts of volcanology, which are not my strong suit. Whenever I was unsure about something, anything, I went to her first.

I would also like to thank my supervisor, Dr. Benoit Taisne, for taking me under his wing. He has given me an awesome opportunity to do my PhD in this young, cutting-edge research group. He has supported my education, helping me to see new parts of the world, and has supported my curiosity, giving me flexibility to pursue avenues of research that I found interesting. I am also proud to say that I completed this PhD without any major arguments (possibly without minor arguments too, though my memory isn't so good) with him. :D

I thank the anonymous reviewers of my thesis, who gave me a lot of positive and constructive feedback to improve the document's overall quality. I appreciate that they took time from their busy schedules to review my work and they made me feel that my findings are indeed interesting and important.

Thank you too all the friends I've made along the way. I have had an awesome time meeting people from many different countries and cultures and have had some trips I'll never forget! Thanks to Paula Burgi, Fred Leclerc, Paul Morgan, Fabio Manta, Sara Contu, Priya Nanjundiah and Antoinette Jade, who I lived with and sometimes vacationed with. Also thank you to my laboratory mates, Fabio (again) and Adel Emadzadeh, and other group members, like Anna, Chiou Ting, David, Yizhou, Lauriane and Dorianne (to name a few), with whom I shared many conversations and laughs. I'd also like to thank my junior PhD students, who it has been a pleasure to meet and hang out with, including Marcus, Olivier, Charlotte, George and Sri.

I would like to thank my family, who has supported my move from the eastern USA to the other side of the world. I've missed my family a lot and I'm sure they've missed my smiling face too. Thanks Mom and Dad, Chris, Julie and Greg for being patient with me while I'm away.

I thank Nanyang Technological University and the Agency for Science, Technology and Research (A*STAR), as well as the Earth Observatory of Singapore, who provided funding for me to perform my PhD studies for this many years.

Finally, I would like to thank Coca-Cola. Coca-Cola: it's the freshest. Ba da ba ba baaaa I'm loving it.

(Blank page)

Table of contents

Originality form	III
Supervisor statement.....	V
Authorship statement	VII
IGS Supplementary Transcript	XI
Acknowledgements.....	XIII
Table of contents	XV
List of Figures	XIX
List of Tables.....	XXII
Thesis abstract	XXIII
Introduction to this thesis, for a general audience.....	1
Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements	15
1.1 Introduction	15
1.2 Methods.....	17
1.2.1 Experimental setup	18
1.2.2 Dike geometry measurements.....	21
1.2.3 Surface deformation measurements and data processing.....	24
1.3 Results & Discussion.....	32
1.3.1 Assessment of the dike intrusion.....	32
1.3.2 Assessment of solid deformation.....	33
1.3.3 Integrating to retrieve 1D and 2D probability distributions	35
1.3.4 Quantifying error.....	37
1.3.5 Strengths and limitations of the Okada model	39
1.3.6 Comparison to the 2007 dike intrusion in Northern Tanzania	46
1.4 Conclusion	48
1.5 Chapter Notation	49

Chapter 2: How magmatic storage regions attract and repel propagating dikes.....	52
2.1 Introduction	52
2.2 Methods.....	55
2.2.1 Experimental procedures.....	56
2.2.2 Scaling	59
2.3 Results	62
2.3.1 Qualitative observations	62
2.3.2 Quantifying dike response	64
2.3.3 Quantifying the effective radius of interaction.....	67
2.4 Discussion.....	70
2.4.1 Dimensions of the critical radius.....	70
2.4.2 Spherical nature of the reservoir	71
2.4.3 Relation to various volcanic behavior	72
2.4.4 Driving pressure	76
2.4.5 Limitations and future work.....	77
2.5 Conclusions	78
2.6 Chapter Notation	78
Chapter 3: Dike channelization and solidification: time scale controls on the geometry and placement of magma migration pathways	81
3.1 Introduction	81
3.1.1 Hot spot volcanoes settings	82
3.1.2 Monogenetic volcanic field settings	82
3.1.3 Experimental and numerical studies and theoretical framework	83
3.1.4 Our contribution	86
3.2 Methods.....	86
3.2.1 Experimental setup	86
3.2.2 Scaling	89
3.3 Results	91
3.3.1 Time-dependent behaviors & flux-dependent behaviors.....	91

3.3.2	Image processing	94
3.3.3	Dimensionless analysis.....	96
3.4	Discussion.....	99
3.4.1	Channelization due to solidification or due to preexisting flow paths	99
3.4.2	Plumbing region distribution	100
3.4.3	Stable conduit development at depth	106
3.4.4	Shield volcano rift zones	110
3.4.5	Experimental limitations	112
3.5	Conclusions	113
3.6	Chapter Notation	114
Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns.....		118
4.1	Introduction	118
4.2	Methods.....	120
4.2.1	Experimental setup	120
4.2.2	Flow visualization using Particle Image/Tracking Velocimetry	124
4.2.3	Scaling	126
4.3	Results.....	130
4.3.1	Controls on circulation	130
4.3.2	Controls on propagation velocity.....	135
4.3.3	Effects of pressure	137
4.4	Discussion.....	139
4.4.1	Dike shape and stability	139
4.4.2	Linking to crystal growth patterns	141
4.4.3	Linking to geophysical measurements.....	143
4.4.4	Pressure-geometry relationships	145
4.4.5	Pressure-propagation relationships.....	147
4.4.6	Future work: horizontally-propagating dikes.....	149
4.5	Conclusion.....	150

4.6 Chapter Notation	151
Summaries from this thesis, perspectives and future work	153
Appendix A: Gelatin preparation	155
Appendix B: Shear wave measurements to estimate the gelatin's Young's modulus.....	159
B.1 Obtaining shear wave velocity measurements.....	159
B.2 Comparison with deflection tests	164
B.3 Limitations to the technique.....	166
Appendix C: Other gelatin properties	167
C.1 Latent heat content	167
C.2. Thermal diffusivity	170
C.3 Temperature-dependent viscosity.....	171
Appendix D: Chapter-specific appendices	172
Appendix D.1. Chapter 1: End-of-injection frame of reference.....	172
D.2.1. Chapter 2: The balloon-less reservoir construction technique	175
D.2.2. Chapter 2: Measurements of $\partial x/\partial z$ and $\partial^2 x/\partial z^2$	178
References	180

List of Figures

Fig. 1.1	A flow chart describing our processing methodology.....	18
Fig. 1.2	A schematic of the experimental set-up.....	21
Fig. 1.3	Dike geometry measurements.....	23
Fig. 1.4	Dike orientation measurements.....	24
Fig. 1.5	Vertical deformation measurements and artefacts.....	27
Fig. 1.6	Horizontal deformation measurements and artefacts.....	28
Fig. 1.7	Okada displacement schematic.....	30
Fig. 1.8	Deformation measurements, model and residual values.....	31
Fig. 1.9	Signal-to-noise ratio analysis.....	34
Fig. 1.10	Integrating an 8D matrix to 1D uncertainties.....	36
Fig. 1.11	1D and 2D marginal uncertainty distribution.....	37
Fig. 1.12	Dike measurements, inversion estimate comparison.....	38
Fig. 1.13	Measurement-inversion error, statistical analysis.....	40
Fig. 1.14	Comparison of Okada vs. experimental surface deformation.....	40
Fig. 1.15	Plots showing the error estimate against depth, d^*	43
Fig. 1.16	Plots showing the error estimate against pressure, P^*	45
Fig. 2.1	Dike orientation definitions.....	56
Fig. 2.2	Young's modulus estimation via shear wave velocity.....	57
Fig. 2.3	Experimental setup.....	58
Fig. 2.4	Photographs of resulting dike deflection.....	63
Fig. 2.5	Methodology to quantify dike deflection.....	65
Fig. 2.6	Deflection due to dimensionless stress, σ^*	67
Fig. 2.7	Method to model lateral propagation critical radius.....	69
Fig. 2.8	Dimensionless critical radius, r_{crit}^* , for radial dikes.....	70
Fig. 2.9	Critical radius, r_{crit} , dimensional estimates.....	71
Fig. 2.10	Model of under pressure driven emplacement patterns.....	74
Fig. 2.11	Time scale of viscoelastic relaxation of critical radius.....	75

Fig. 2.12	Model of eruption triggering.....	76
Fig. 3.1	Experimental setup.....	86
Fig. 3.2	Photographs showing the effect of repose time.....	92
Fig. 3.3	Photographs showing the effect of flux.....	93
Fig. 3.4	Method to quantify dike thickness and surface area.....	95
Fig. 3.5	Aspect ratio due to repose time and flux.....	97
Fig. 3.6	Intruded volume ratio due to repose time and flux.....	98
Fig. 3.7	Schematic of thermal evolution of a fissure eruption.....	100
Fig. 3.8	Time scales for conduit or dike swarm construction.....	101
Fig. 3.9	Schematic for 2D numerical model of conduction.....	103
Fig. 3.10	Results from the 2D numerical model.....	104
Fig. 3.11	Illustration of crust temperature profile near a conduit.....	108
Fig. 3.12	Model of rift zone evolution.....	111
Fig. 4.1	Experimental setup.....	123
Fig. 4.2	2D PIV of liquid circulation in a dike.....	132
Fig. 4.3	Circulation parameter, v_c^* against flux and buoyancy.....	133
Fig. 4.4	Model of flow evolution due to buoyancy.....	135
Fig. 4.5	Propagation parameter, v_p^* against flux and buoyancy.....	136
Fig. 4.6	Experimental pressure measurement.....	137
Fig. 4.7	Comparison of pressure measurements.....	138
Fig. 4.8	2D propagation velocity model against measurements.....	141
Fig. 4.9	Model of crystal circulation path.....	142
Fig. 4.10	Constraining magma rheology via flux and buoyancy.....	144
Fig. 4.11	Thickness scales, fracture- and viscous-limited propagation.....	146
Fig. 4.12	Flux and propagation velocity models.....	148
Fig. A.1	Photograph of bacteria-derived gas bubbles in gelatin.....	156
Fig. A.2	Cooling-driven convection, viscoelastic behavior of gelatin.....	157
Fig. B.1	Photograph of rainbow fringes in a stressed gelatin.....	159

Fig. B.2	Shear wave measurement setup.....	160
Fig. B.3	Shear waves in layered gelatins.....	161
Fig. B.4	Wave measurement video processing technique.....	162
Fig. B.5	Threshold value of light differential.....	163
Fig. B.6	Artefact of a wave entering or leaving the field of view.....	164
Fig. B.7	Shear waves technique vs surface deflection measurements.....	165
Fig. B.8	Wall effects for shear wave measurements.....	167
Fig. C.1	Latent heat estimate for gelatin.....	169
Fig. C.2	Thermal diffusivity estimate for gelatin	170
Fig. C.3	Temperature-dependent viscosity of gelatin.....	172
Fig. D.1.1	Surface deformation for a post-injection reference frame.....	173
Fig. D.1.2	Statistical error analysis for post-injection reference frame.....	174
Fig. D.2.1	Methodology to construct a balloon-less reservoir.....	177
Fig. D.2.2	Methodology to quantify dike deflection	179

List of Tables

Table 1.1	Experimental conditions for chapter 1.....	19
Table 1.2	Application to the 2007 Lake Natron dike intrusion.....	47-48
Table 1.3	Notation for chapter 1.....	49-50
Table 2.1	Experimental conditions for chapter 2.....	61
Table 2.2	Scaling comparison for experiments and nature.....	62
Table 2.3	Notation for chapter 2.....	79-79
Table 3.1	Experimental conditions for chapter 3.....	87-88
Table 3.2	Scaling comparison for experiments and nature.....	90-91
Table 3.3	Dike dimension measurement results.....	96
Table 3.4	Notation for chapter 3.....	114-116
Table 4.1	Experimental conditions for chapter 4.....	121-122
Table 4.2	Scaling comparison for experiments and nature.....	129
Table 4.3	Notation for chapter 4.....	151-152

Thesis abstract

This thesis addresses dike propagation from various vantage points, in order to construct a holistic conceptual model of how they behave in nature. Each chapter explores a different aspect of propagation using analogue experiments, in which gelatin models the solid medium. Dikes are important to study, in that they are fundamental to magma migration. Most basaltic eruptions (e.g. the large 2018 eruption at Kilauea, Hawai'i and the 2014 eruption of Bardarbunga, Iceland) are fed by dikes. Recharge events, which supplies fresh magma from depth to shallow magma reservoirs and are associated to eruptions (e.g. the 1991 eruption of Pinatubo, Philippines), are commonly considered to be fed by propagating dikes.

To start, I investigated the surface deformation due to buoyantly propagating, oil-filled dikes, in order to quantify the uncertainties associated to the Okada model, which models the surface deformation as functions of the geometry and position. To do so, I performed Bayesian inversions on deformation data from laboratory experiments, to estimate the dike geometry. I then compared these estimates with visual measurements, to quantify which parameters (e.g. length, depth) well-resemble reality. I found that the Okada model produces good, first-order approximations for most parameters, but crucially, the dike depth is not well-constrained, especially for shallow dikes. The mismatch is primarily due to the shape simplification of the Okada model, which assumes the dike to have a rectangular shape, whereas real dikes are elliptical.

Next, I discuss how dike propagation is affected by the stress distribution in the surrounding medium, with a focus on the stresses around magmatic reservoirs. I made experiments modeling a reservoir, using a simple balloon embedded in gelatin, and injected buoyant, oil-filled dikes from below, allowing them to reorient due to the stress field. I found that a compressional stress field around an inflated, over-pressurized reservoir encourages dikes to propagate toward the reservoir, since the orientation of principal stresses cause dikes to propagate radial to, and therefore toward, the reservoir. By contrast, an extensional stress field around a deflated, under-pressurized reservoir encourages dikes to avoid the reservoir, since the principal stresses cause dikes to propagate tangentially around the reservoir. The magnitude of stress influences how close a dike can approach a reservoir before it begins to change orientation and, at the natural scale, this corresponds to a maximum of a few tens of kilometers radius.

In Chapter 3, I explore magma migration from a thermal perspective, in order to understand the pathway geometry that develops over a long-term scale. I performed a series of experiments, repeatedly injecting a volume of warm, liquid gelatin into cold, solid gelatin and observed what types of structures developed with time. Liquid gelatin is a convenient material to use as it is transparent, has a temperature-dependent viscosity and seems to have a shear-independent viscosity; in short we can visually measure its thickness and it behaves like a gas-poor magma. When the time interval between injections was short, the initial dike rapidly channelized into a sub-cylindrical conduit, elliptical in cross-section. This structure remained 'open', allowing persistent liquid flux to

pass from the source to the surface. In contrast, a long time interval between injections allowed dikes to completely solidify, prompting subsequent liquid injections to form new dikes. Over the course of an experiment, such behavior resulted in the intrusion of a dike swarm, in which each injection made a discrete dike. I found that the injection flux controlled the thermal stability of an injection, in which a critical flux rate is necessary for a dike to propagate to the surface before solidifying. Dikes with a lower flux exhibit unstable, piecewise propagation, in which the leading tip of the dike solidifies and forces propagation to resume at the base, near the point of injection.

Finally, I investigated the internal liquid flow of a propagating, and subsequently erupting, dike. In this case, I injected either vegetable oil (buoyant, intermediate-viscosity Newtonian liquid), water (weakly-buoyant, low-viscosity Newtonian liquid), or liquid gelatin (buoyant, temperature-dependent viscosity), in order to model the internal flow depending on the buoyancy and viscous forces, as well as cooling in the dike. We seeded the liquids with particles that were tracked using Particle Image Velocimetry and Particle Tracking Velocimetry, to serve as a proxy for the internal flow field evolution. When the liquid is initially injected, the dike's internal flow takes on a vertically circulating pattern, with a central upward jet and return flow along the lateral fringes. As the dike grows, the circulation pattern weakens and transitions to simple, ascending flow. This change of behavior corresponds to the buoyancy of the liquid, which builds in magnitude as the dike elongates and favors upward motion. The form of propagation relates to these flow patterns, in which the dike initially propagates radially, in the form of a penny-shaped crack. When buoyancy forces become dominant, the propagation becomes vertical, with little horizontal growth.

This thesis provides a broad view on the factors controlling dike propagation and evolution, from analysis on the driving forces of propagation to the effect of the stresses in the medium to the thermal viability and vulnerability to solidification. Since magmatic dikes propagate in response to such various processes, this approach offers valuable and fundamental insights into their nature.

Introduction to this thesis, for a general audience

Research goals and hypotheses

This thesis focuses on dikes, which are magma-filled cracks that propagate through the Earth's crust. Dikes are important structures via which magma migrates from depth towards the surface. But like any topic, gaining an understanding of how dikes behave in nature requires that we study them from different, complementary perspectives. For example, we may want to understand how the flow in a dike evolves due to cooling or how we can detect and characterize a propagating dike through geophysical monitoring. In each chapter, I will walk through some of these topics, discussing relevant aspects of dike propagation, in order to understand how we can detect them at the surface, how they can grow into complex geometries and how their internal flow may evolve as they grow and cool. Specifically, I want to address:

- 1) How surface deformation relates to a dike intrusion via the Okada model, in order to quantify the model's strengths and limitations
- 2) How dikes respond to the stresses around a pressurized magma reservoir, from the hypothesis that the sign of the stress (i.e. an inflated or deflated reservoir) controls whether dikes are attracted to or repelled by the reservoir
- 3) How magma migrates through the crust, depending on injection frequency, from the hypothesis that frequent injections provide sufficient heat to maintain a liquid pathway, so that an initial dike develops into a stable, persistently-active conduit. Infrequent injections allow enough time for dikes to cool off and solidify, so that each injection makes a new dike
- 4) The form with which magma flows inside of a dike, from the hypothesis the flow pattern depends on the magnitudes of the forces acting on the dike

General methodology

Throughout this thesis, I rely heavily on analogue modeling, which entails performing small, laboratory-scale experiments to understand what happens at the much-larger, natural, volcanic scale. In each experiment, I prepare a block of gelatin, which represents Earth's crust, and inject a fluid of some kind to represent the migrating magma. Since gelatin is brittle and elastic (like the Earth's crust on the spatial- and temporal-scale of a propagating dike), the injected fluid fractures the gelatin to create a small dike. This solid medium, by its very nature, produces planar dike geometries, similar to those found around volcanoes. In each chapter, I discuss in more detail the particulars of each specific

experimental setup, but they are all variations of this basic design. General experiment preparation procedures (i.e. gelatin cooking) are detailed in appendix A.

For laboratory experiments to be applicable to the natural scale, I use dimensionless analysis, which helps to maintain similar proportions between the two sizes. This often takes the form of force ratios, in which a driving force causes one behavior and a resisting force causes another; however dimensionless numbers can also be ratios of other parameters, such as velocity or size ratios (e.g. aspect ratio). For example, a commonly used dimensionless number is the Reynolds number, which is a ratio of a liquid's momentum (the driving force) to its internal viscous forces (the resisting force). When the momentum is the stronger force, the Reynolds number has a value greater than 1 and if it is much, much greater than 1, the liquid undergoes turbulent flow (the precise value depends on the geometry and flow pattern, so that turbulent flow occurs for a different magnitude of the Reynolds number in a pipe geometry and a river geometry). When the momentum is weaker than the viscous force, the value is less than 1 and the flow takes on a laminar form. There can be many combinations of force ratios, so it is important to determine which forces are most important for an experiment, and then analyze those. Throughout this thesis I tend to represent dimensionless numbers with a variable followed by an "*" sign (e.g. P^* , for dimensionless pressure), a convention that is sometimes used in this field of research.

At times, I supplement the experimental results with numerical modeling, which entails programming in Matlab a hypothetical region of the Earth's crust and applying some stress or thermal dynamic to that region. I do this in circumstances where performing a lab-scale experiment is not feasible, perhaps when the required time scales are too long for an experiment (i.e. an experiment would take months to years to complete) or when the numerical technique assists analysis of the analogue results (e.g. helping to frame dimensionless results in the natural scale).

Background

As introduced, dikes are magma-filled cracks, which have an internal pressure that allows them to propagate. They are essentially planar in geometry, in that two of their dimensions are much larger than the third (i.e. they are thin, but very long and wide). Dikes have been identified in many volcanic settings: at stratovolcanoes, which come to mind when one imagines a volcano (Porreca et al., 2006; Schonwalder et al., 2018); at rifting centers, where the crust pulls apart and encourages magma to ascend (Maccaferri et al., 2014; Valentine & Connor, 2015); and at hot spot volcanoes, like Hawaii (Furumoto, 1978; Owen et al., 2000). They can technically be filled with any magma, however they predominantly contain basalt, which is relatively fluid and easily flows through cracks (Rivalta et al., 2015). As such, dikes that contain more-viscous magma tend to be wider, to accommodate the flow (Wada, 1994). When they erupt, they can make fissure eruptions, in which a curtain of lava is ejected from an extended kilometers-long fissure

(Wylie et al., 1999; Jones et al., 2017), and can also construct monogenetic cones, which are mounds of lava that build up during a single eruption and then immediately go extinct (Shane & Coote, 2018). Their importance extends beyond eruptive activity, since they are a fundamental magma transport structure. Their flat shape is, in fact, mechanically efficient for large volumes of magma (or any other fluid) to migrate through the crust (Keating et al., 2008).

Dikes have traditionally been investigated through field studies, in which ones that are old and have solidified are measured for their position, geometry and orientation. They can be exposed where they intersect the Earth's surface, such as along the ground or cliffs (Porreca et al., 2006; or see the below photograph for an example), or by preferential erosion of the surrounding crust, which can happen at old, dead volcanic systems (Anderson, 1937; Poland et al., 2008). In the latter case, removal of the surrounding rock can expose the volcano's underground plumbing system and provides an excellent opportunity to study its form. For example, at Summer Coon volcano in the USA, erosion has exposed a radial system of dikes extending outward from a central intrusive complex (Poland et al., 2008). Newer techniques, which involve characterizing the magnetic properties and orientations of microscopic crystals, show the ability to reconstruct the flow direction (Leslie et al., 2004; Magee et al., 2012).

In the last several decades, monitoring agencies have developed methods of detecting active dike propagation. As dikes propagate through the crust, their volume pushes aside the surrounding rock, generating a displacement which can be measured via GPS instruments and satellite-based techniques (Okada, 1985; Okada et al., 2000; Wnuk & Wauthier, 2017). The magma's migration and the associated stresses exerted on the surrounding rock generates many small-magnitude earthquakes, whose source locations can be recovered (Ayele et al., 2007; Alparone et al., 2010; Rivalta et al., 2015). This gives researchers a sense of the position and the velocity of migrating magma, while the displacement indicates its size and geometry (Hayashi & Morita, 2003; Battaglia et al., 2005; Morita et al., 2006).

A photograph I took of a dike (the sub-vertical, linear feature) cutting through large pillow lavas (round features), part of the Troodos ophiolite sequence in Cyprus. A notebook is included for scale. This dike was exposed by the construction of a road cut.

Dikes are inherently complicated in that they form due to the interaction of two states of matter: a liquid (magma) intruding into a solid host medium (the crust). The solid is fractured and separated in a way that is in part determined by its strength, since a relatively strong material needs more pressure to fracture and to deform (Lister & Kerr, 1991; Rubin, 1995; Gonnerman & Taisne, 2015). The internal magma flows at a rate determined in part by its viscosity (Taisne et al., 2011). A third important factor is the dike's geometry, in that thicker dikes place more stress on the surrounding crust but facilitate magma flow (Maccaferri et al., 2019). There is a fundamental coupling between the solid and liquid, so that propagation can be limited by the viscous resistance in the magma, the resistance of the crust to fracturation, or both. In this sense, magma may be *able* to flow fast, but it does not if the solid is very resistant to fracture, so the dike propagates slowly on average. Conversely, the solid may be easily fractured, but if the magma is too viscous to quickly flow, the dike again propagates slowly on average (Rivalta et al., 2015).

As dikes ascend to the surface they undergo additional thermal processes. Magma is hotter than the surrounding crust and dikes are thin and have a large surface area, so they are very susceptible to cooling and solidification (Einarsson & Brandsdóttir, 1980; Delaney & Pollard, 1982; Fialko & Rubin, 1998; Wylie et al., 1999). They are considered to be able to propagate a limited distance, known as the "thermal entry length", after which solidification causes it to arrest. However, magma has a temperature-dependent viscosity, in that hotter magma is significantly more fluid than cooler magma. In the

geometry of a dike, this can lead to a process of “fingering”, in which hotter magma flows at higher velocity than the surrounding cooler magma (Whitehead & Helfrich, 1991; Wylie et al., 1999). A fast-flowing “finger” of hot magma therefore extends through the surrounding cooler magma, which complicates the flow field and results in the formation of a viable channel for continued flow.

In terms of compositional processes, magma is a substance that is strongly temperature- and pressure-dependent (Cashman & Sparks, 2013; Rogers, 2015). Perhaps the most-relevant process for a propagating dike is the exsolution of volcanic volatile gases, which preferentially forms bubbles at low pressures (Sparks, 1978; Burton et al., 2007; Cashman & Sparks, 2013; Gonnermann & Taisne, 2015). As a dike ascends, its gas fraction increases due to the exsolution and decompression of bubbles, which imparts a buoyancy force that can help to drive the dike upwards (Taisne & Jaupart, 2011). Their expansions increase the bulk volumetric flux, which causes the dike propagation velocity to increase and the thickness to decrease. In addition to exsolution, magma can slowly “evolve” as it cools. This occurs due to fractional crystallization, in which crystals form in the magma and rob the remaining melt of elements, causing it to become depleted (Rogers, 2015). The resulting evolved magma becomes increasingly viscous and resistant to flow. This process can generally be ignored in the context of dike propagation, since dikes usually contain fresh, fluid, basaltic magma (Rivalta et al., 2015).

Fractures, and by extension dikes, are sensitive to the stresses in the surrounding medium. The orientation of stresses is quantified by principal stresses, in which the orientation of the highest stress is referred to as σ_1 , the intermediate stress as σ_2 and the lowest stress as σ_3 . These stresses are orientated perpendicular to each other (e.g., consider a cube with a high pressure applied to the top, less pressure applied to the side and very little pressure applied to the front) and their orientation controls how a dike orients (Anderson, 1937; Delaney, 1986). A dike propagates in whatever form expends the least amount of energy, or as my supervisor Dr. Taisne puts it, “nature is lazy”. Pushing the crust open in the direction of σ_3 requires the least amount of work, so the thickness dimension indicates the orientation of σ_3 (Chadwick & Dieterich, 1995; Bagnardi et al., 2013; Gudmundsson et al., 2014). Propagation therefore occurs in a direction that is perpendicular to σ_3 .

A productive area of research is analytical and numerical modeling, which investigate dike shape and/or velocity, usually using 2D models (Weertman, 1971; Lister & Kerr, 1991; Dahm, 2000b; Roper & Lister, 2007; Pinel et al., 2017; Maccaferri et al., 2019). They consider growth to occur in the length dimension and the dike pushes apart the crust in the thickness dimension. The third, breadth dimension is neglected, assuming that there is little variation in flow and shape along it. This allows simplifications to be made, like the assumption of Hagen-Poiseuille flow, which estimates the flow velocity via the viscous forces and driving pressure drop (Dahm, 2000b).

These models can be subdivided depending on the assumption of dike geometry. The first model is the “Weertman” model (Weertman, 1971), which considers the dike to be a

discrete volume that adopts a static, flat, tear-drop shape (in cross section) as it ascends, like a flat bubble rising up through the crust (see the left schematic in the image below). The dike maintains a constant volume, a constant shape (when propagating through a homogenous medium), and is driven entirely by its own internal buoyancy (Dahm, 2000b). Since there is no additional driving pressure, propagation can only proceed when the dike is sufficiently long and thus has a sufficiently large buoyancy force. The propagation velocity depends on the magma viscosity, in which magma ascends through the center of the dike and a no-slip condition constrains flow along the walls; at the bottom fringe of the dike, the magma is considered to be squeezed upward and the tail pinches shut, or alternately leaves a very small proportion of its volume behind.

Two 2D models of dike propagation. To the left is the Weertman model, in which an isolated volume of magma propagates due to its own buoyancy. To the right is the lubrication model, in which the dike is connected to its source reservoir and can propagate due to influx and buoyancy. Each model has strengths and weaknesses.

The second model is based on “lubrication” theory, which considers the dike to be a long feature that extends upward (or laterally for a horizontally-propagating dike) from its source reservoir. In this case, the dike has an inflated, upper “head” region (near the top) and a thinner, basal “tail” region (Lister, 1990; Roper & Lister, 2007). As magma ascends, it accumulates in the head and generates stress on the surrounding crust and flow is limited by the viscosity in the tail (Roper & Lister, 2005). Since the dike remains connected with its source reservoir, the driving pressure can be comprised of a

Introduction to this thesis, for a general audience

combination of the source pressure and/or magma buoyancy (Roper & Lister, 2005; Roper & Lister, 2007; Taisne & Tait, 2011).

Analogue modeling, in contrast with analytical models, is capable of fully representing 3D behaviors and is the main technique employed in my thesis. An analogue model recreates a small-scale version of a natural process, by carefully considering the behaviors of the model materials and the processes acting upon them (Galland et al., 2015). The ratio of driving to resisting forces (e.g. the ratio of liquid momentum to viscous forces) remains the same for small and large scales, so that the resulting behavior (e.g. if a liquid has turbulent flow) is independent of size; everything is the same, only smaller and weaker (Schlichting, 1979; Di Giuseppe et al., 2009). Analogue models are described using dimensionless number, which are geometry or force ratios that remain the same between the small, laboratory scale and the large, natural scale (Schlichting, 1979; Pioli et al., 2012; Kavanagh et al., 2013; Pansino & Taisne, 2019). Some common examples are aspect ratio (a ratio of length to width) and the Reynolds number (a ratio of liquid inertia to viscous forces).

For all of the experiments presented in this thesis, I used gelatin as an analogue solid medium, which represents the Earth's crust. The behavior of the crust, and of solids in general, is time-dependent, in that it responds to sudden forces differently than prolonged forces (Di Giuseppe et al., 2009). Solids have an elastic component of deformation, in which they deform and then return to their initial shape, when a force is quickly applied and released. They also have a viscous component, in which they permanently deform when a force is applied for an extended period of time (Segall, 2010). On the time scale of a propagating dike, which is relatively fast, the upper part of the Earth's crust (i.e. the shallowest 10 or so km) behaves like a brittle (i.e. they can fracture), elastic material, whereas the lower part has a viscous component (Di Giuseppe et al., 2009). Gelatin is similarly brittle and elastic on the time scale of propagating dikes, meaning it qualitatively behaves like the upper crust (Di Giuseppe et al., 2009; Kavanagh et al., 2013). When liquid is quickly intruded into such a material, it fractures the solid and makes a dike that is similar to a magmatic dike, only smaller (and less dangerous). We characterize the medium strength using the Young's modulus (its response to compression), which is related to the shear modulus (its response to shear) via the Poisson's ratio (Segall, 2010). The Poisson's ratio for gelatin is 0.5, which indicates that it is theoretically incompressible, whereas for the crust it is ~ 0.25 and has a degree of compressibility (Kavanagh et al., 2013). The difference in Poisson's ratio between experiments and nature manifests in the aspect ratio of an experimental dike, which is relatively thick in comparison with magmatic dikes (Rivalta et al., 2005).

The liquid, magma analogue has a bit more flexibility than the solid medium. Depending on the liquid we choose, we can model a range of rheologies, which control how the liquid flows. For example, water and vegetable oil are Newtonian liquids, which are simple and flow at a rate depending on the driving pressure. Due to their simplicity, Newtonian liquids are commonly used in analogue experiments (Menand & Tait, 2002;

Pioli et al., 2012; Galland et al., 2015; Kavanagh et al., 2018). We could use liquids with other rheologies, like shear-thinning liquids, which flow more easily when more force is applied (Parfitt & Wilson, 2008; e.g. Ketchup, which resists coming out of the bottle until you hit the bottom, at which point too much comes out). When we choose a liquid, we have to consider what type of condition we are modelling. Magma migrating deep underground is usually assumed to have a Newtonian rheology, in which the concentration of volatiles and crystals is relatively small. Choosing to use a shear-thinning liquid might better model a shallow magma, which is rich in bubbles that can deform and reduce the effective viscosity (Llewellyn & Manga, 2005).

In addition to the solid and liquid analogue rheologies, we need to consider the forces acting on the dike. The relative size of a dike is quantified by a critical length scale known as the “buoyancy length”, which is the necessary length for a dike to propagate due only to its buoyancy forces (Takada, 1990; Taisne & Tait, 2009). This is related to the Weertman model of dike propagation introduced above, in that the buoyancy length is the length at which buoyancy forces balance with the fracture resistance pressure of the medium (Lister, 1990). Buoyancy is commonly quantified via $\Delta\rho gL$, for liquid-medium density difference, $\Delta\rho$, gravitational constant, g , and dike length, L . The fracture resistance pressure, $K_c/L^{1/2}$, is quantified by the fracture toughness (a material property of the solid medium, a measure of its strength), K_c , and the dike length. When the forces are in balance, $\Delta\rho gL = K_c/L^{1/2}$, so the buoyancy length is $L = (K_c/\Delta\rho g)^{2/3}$ (Lister, 1990; Taisne & Tait, 2009). A small value of the buoyancy length indicates that the dike becomes buoyant at a smaller size and is equivalent to “zooming out”, so that we observe a relatively long propagation; a large value converse indicates the dike needs to grow large to become buoyant and is equivalent to “zooming in”, so that we observe how a dike initiates.

If we choose to study a dike with a constant influx, we can non-dimensionalize the flux in different ways. A typical method is to consider the influx into the tail, so that magma flux, Q , and viscosity, μ , correspond to a thicker tail, whereas buoyancy draws magma upwards and corresponds to a thinner tail. The scale tail thickness, H_∞ , is quantified by $H_\infty = (Q\mu/\Delta\rho g)^{1/3}$ (Roper & Lister, 2007). The scale influx Q_∞ is then quantified by $Q_\infty = Q/H_\infty$. Note that in this methodology, Q is a 2D flux, meaning the flux per unit length (Roper & Lister, 2007; Rivalta et al., 2015). When the parameter is large, it indicates that the dike is relatively thick. Another way to non-dimensionalize the flux is thermally, via a balance between heating and cooling. The influx provides heat to the dike at a rate determined by the influx, Q (now regular 3D flux) and dike thickness, H , while the cooling via conduction of heat into the surrounding crust depends on the surface area and the thermal diffusivity, α . If the dike has an approximate surface area of its vertical length, L , by its horizontal breadth, B , the dimensionless flux, Φ , is quantified by $\Phi = QH/(\alpha LB)$ (Taisne & Tait, 2011). When the value is large, it indicates that the flow is high and heat flows upward through the dike; when small, the heat is lost into the crust. Via the material selection and dimensionless numbers, we can control how the liquid and solid interact, as well as the relative size, shape and dynamics of a dike, so that they match nature.

Introduction to this thesis, for a general audience

Thesis layout

Each chapter can be considered as a self-contained study. They all follow the classic format of scientific publications, which begins with an introduction providing the necessary background for that chapter, followed by a description of the methodology, results, discussion and finally a summarizing conclusion.

I try to maintain cohesion between the chapters, by using consistent terminology and symbols; however this thesis contains many symbols and some are unavoidably repeated, sometimes with different meanings (e.g. P means probability in chapter 1, but means pressure in the other chapters). In order to facilitate the reader's understanding, I explain the meaning of each symbols the first time it is used in a chapter and include a list of symbols at the end of each chapter, which can be used as references. This also helps to control the size of the list of symbols, so that they include only the relevant ones for that chapter.

Parts of my thesis represent work that was done as part of a team. For chapters 1 and 2, I performed the experiments and much of the analysis on my own, but extensively discussed the context with my supervisor Dr. Taisne, who gave me guidance on how to frame the results. For chapters 3 and 4, Dr. Adel Emadzadeh joined our research team and assisted running experiments. His expertise lay in fluid dynamics and flow imaging techniques, so he was able to help me gather and process velocity data of the dikes in these chapters. As in chapters 1 and 2, Dr. Taisne helped to frame the discussion here as well. In terms of content, the writing in this thesis is my own, with the exception of the flow imaging methodology in Chapter 4, which was written by Dr. Emadzadeh and edited by myself. Similarly, the majority of the figures are of my own design and include data analysis that I performed in Matlab and illustrations that I made in Adobe Illustrator. Again, Dr. Emadzadeh assisted to provide figures of flow velocity data in Chapter 4.

Chapter overviews

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

A figure of surface deformation measurements (left column), a well-fitting Okada model (center column) and the residual error between the two (right column).

I will start by discussing geophysical inversions using the Okada model, which relates surface deformation to the opening of a simple rectangular crack. The Okada model is a series of equations which incorporate the size, location and orientation of a crack to estimate the surface deformation at a position on the Earth's surface. By solving these equations, it is possible to construct a map of surface deformation. Performing a geophysical inversion reverses the process, feeding surface deformation data into the model to understand the geometry of the crack. The Okada model makes quick-to-compute geometry estimates, with the trade-off of being a simplification of the reality, so that it does not very well match the shape of a dike and produces an approximation of the actual physical problem. Throughout this chapter, I perform an experimental analysis of the Okada model to quantify the uncertainties associated to it, in order to detail how well it performs and under which conditions it is, or is not, applicable. To do so, I injected oil-filled dikes and recorded the surface deformation with an array of above-mounted cameras, whose images were used to create 3D models of the surface. I also recorded the dike itself with a side-mounted camera, to measure its actual geometry. This study is not intended for complex, computer intensive models, but to help researchers in observatory

Introduction to this thesis, for a general audience

setting to interpret their quick inversions and know to what degree they can trust their results.

Chapter 2: How magmatic storage regions attract and repel propagating dikes

An example of a dike propagating around, then away from, a deflated reservoir.

Here, I append the contents of a published manuscript that details how dikes react to the stress field around a pressurized magma storage region (Pansino & Taisne, 2019, Journal of Geophysical Research: Solid Earth). The formatted version of the publication is included at the end of the thesis. The concept is that a pressurized reservoir exerts a stress field on the surrounding crust, which affects how dikes propagate. Dikes prefer to open against the least-compressive stress and the orientation of this stress depends on the sign of the pressure in the reservoir, such that dikes propagate towards inflated reservoirs and around deflated reservoirs. I modeled the reservoir with a simple balloon, which was inflated or deflated and then injected an oil-filled dike from below. I analyze how effectively the propagation responds to the sign and magnitude of the stress, for different initial dike orientations. This is important, because it provides fundamental insight on how dikes propagate near magma storage regions, whether they are likely to intersect and empty into the said reservoir, or bypass the reservoir and erupt at a

Introduction to this thesis, for a general audience

volcano's flank. Such findings help us to anticipate whether future dikes are likely to migrate inward toward the reservoir, which may favor summit activity at the associated edifice, or be repelled away, favoring flank eruptions.

Chapter 3: Dike channelization and solidification: time scale controls on the geometry and placement of magma migration pathways

Solidification of gelatin (darkened regions) during continuous influx causes a dike to develop a central conduit (flow marked by the arrow).

In chapter 3, I discuss how solidification affects the preferred geometry of magma migration. When a dike propagates through a homogeneous solid medium and erupts, it creates an open pathway via which further magma can pass. This work is also published (Pansino et al., 2019) and included at the end of the thesis. If magma continues to pass through the dike, it quickly channelizes into a sub-cylindrical conduit (i.e. the flow becomes confined to a relatively narrow path), which conserves heat, as shown in the above photograph. In fact, there is evidence that this channelization may be facilitated by dike propagation, in that the influx generates a jet which provides heat along the dike's central axis. This jet is relatively wide in comparison with the resulting conduit and better discussed in the context of chapter 4. If magma ceases to flow for a significant period of time, the dike solidifies and the pathways seals shut. Further magma migration must then take the form of a new dike. To perform these experiments, I worked with Dr.

Emadzadeh, and repeatedly injected a fixed volume of warm, liquid gelatin into the cold, solid, gelatin medium, with a chosen flux and time interval between injections. When dikes intrude into a solid, they generate a stress field that can affect subsequent dikes. In order to mitigate the accumulation of stress, we constructed water-filled, vertical free surfaces along the experiment walls, which allow the gelatin to deform and thereby relieve stress. We explored how the flux of, and time interval between, magma migration events causes different forms of emplacement, from conduit flow to compound diking (in which a dike intrudes inside of a previous dike) to dike swarming (in which a group of dikes intrude into the crust, one by one). I then hypothesize that this time interval has an important influence on magma emplacement in the crust, in which high frequency corresponds to a centralized plumbing system, whereas a low frequency corresponds to spatially-distributed volcanism. In this sense, a low frequency of magma flux manifests at the surface as a monogenetic volcanic field and higher frequency manifests as a centralized volcanic edifice.

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns

Particle image velocimetry measurements of a water-filled dike. Arrows indicate the flow direction and color indicates upward (red) or downward (blue) flow. Note the strong jet to the right side. This dike is about 15 cm wide.

Finally, I conclude the thesis with a study on the internal flow of a propagating dike. It is recently becoming clear that dikes can have an internal circulation, in which magma ascends and descends as the dike grows towards the surface. Together with Dr. Emadzadeh, I investigated what causes this circulation, with the hypothesis that it is a function of the driving forces. We seeded a liquid (either oil or water) with small particles (~ 1 mm diameter) and injected it into gelatin. The seeding particles could be visually tracked with a camera, which we used to perform particle image velocimetry analysis and thereby measure the flow field. They have a density of 1030 kg/m^3 and have a sufficiently slow settling velocity (which depends on the liquid density and viscosity); in practice, the liquid flows at a rate at least 5 times faster than the particle settling, but could be on the order of 50 times more). The forces acting on a dike both drive circulation and the form of propagation (i.e. radial dike growth vs vertical dike growth). I discuss the results within the context of petrology, in which crystal textures have the potential to reveal information on how magma flows inside of a dike. I similarly discuss within the context of geophysical measurements, in which geodetic and seismic data can indicate the dike's geometry and form of propagation. Both of such data essentially indicate how forces are balanced in a propagating dike, which in turn can give us insight into other information, such as the magma's rheology or the degree to which the source reservoir pressure controls propagation.

Summaries, perspectives and future work

Following the chapters, I wrap up the thesis with a conclusion to summarize and contextualize the work. As is evident from the chapter subjects, the goal of my doctoral study is simply to gain a better understanding of how fluid-filled cracks operate. My interest lay in a volcanic context, as volcanoes and their plumbing systems are fascinating and complex phenomena. The significance is deeper however, in that fluid-filled cracks are studied in other contexts, such as the hydraulic fracturing of rocks to extract oil and natural gas. Since fluid-filled crack propagation entails many complementary and competing processes, like fracture mechanics, fluid dynamics and heat transfer, there are indeed many perspectives to follow. For this reason, I pursue a well-rounded approach and study these processes individually, in order to thoroughly conceptualize how they behave in nature.

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

Abstract

We present here the results from analogue experiments, which were designed to quantify the uncertainties associated to the Okada model. In our experiments, we injected a volume of oil into a solid gelatin medium and measured the resulting surface deformation. We then performed Bayesian inversions of the data, using the Okada model, to estimate the dike's size, location and orientation. These estimates are compared with laboratory measurements, which we obtained by visually measuring the dike in the transparent medium.

We find that the Okada model produces good, first-order approximations of a dike, with the caveat that certain parameters (the dike's orientation) are more-reliably estimated than others (the dike's opening width and depth). The uncertainty associated to the dike's length and depth are functions of depth, in that the model loses accuracy if the dike is too shallow. This is likely due to the shape mismatch between a rectangular Okada displacement patch and a penny-shaped or elliptically-shaped dike, which have somewhat different surface deformation patterns.

1.1 Introduction

In geophysics, mathematical models have been derived, and used to relate physical processes (e.g. fault slipping during an earthquake, magma storage regions changing volume) with their resulting surface deformation. Volcanologists commonly invoke the Mogi model, which describes surface deformation due to a point source that changes in volume, and the Okada displacement model, which describes surface deformation due to a rectangular crack either slipping or opening, respectively described by Mogi (1958) and Okada (1985). Due to their relative simplicity, they are computationally quick to solve and therefore commonly used for monitoring purposes (Ayele et al., 2007; Battaglia et al., 2013). For example, Cameron et al. (2014) at the USGS invoked the Mogi model to describe the deformation at Akutan volcano in 2014 and found that it produced a good fit for the observations. In this way, the measured surface deformation is used to describe what happens underground, by determining the position, geometry and displacement of the source structure that would best-match the observed deformation (Ayele et al., 2007).

To the best of our knowledge, in nature, the full shape of dikes are not well-resolved, since usually only a linear cross-section is exposed. However, Daniels et al. (2012)

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

measured variations in dike width, in both the vertical and horizontal (along-strike) directions, at the Isle of Rum (Scotland) and Helam Mine (South Africa) and reconstructed the 3D shape of such basaltic and kimberlitic dikes. The width distributions illustrate that dikes' profiles do not generally fit to an elastic model, in that they tend to be thinner in the middle and thicker towards the fringes than expected. This implies preservation of the active shape near the fringes, due to cooling and solidification, and post-emplacement narrowing of the middle. The profiles they present also indicate the dikes had some vertical elongation, in that they are not very broad (~2 km). A dike of such breadth could be penny-shaped if it originated from a source depth of a similar length scale, which is unlikely for kimberlites. It is possible such dikes have abstract shape, in which they are broader at depth and narrower near the surface. This has been experimentally shown to occur when a dike passes from a relatively strong and ridged medium to a weaker one (Rivalta et al., 2005), but to our knowledge, this has not yet been shown with field evidence.

It is known that the Okada model is simplifications of the processes it represents. Even considering simple, analogue models, there is an obvious discrepancy in the shape mismatch between a rectangular Okada displacement and experimental dikes, which can be penny-shaped, elliptical, tabular, or even segmented (Takada, 1990; Kavanagh et al., 2018; Urbani et al., 2018). All experimental dikes have a curvature, in that the leading edge is rounded, and therefore have no distinct corners. Furthermore, dikes are not of uniform opening width and orientation, since they have a thicker, upper head region and a thinner, deeper tail (Taisne et al., 2011) and may curl or twist to accommodate to the stresses in the crust as they ascend to the surface (Pansino & Taisne, 2019).

A further simplification is that the Okada and Mogi models were originally derived for a single source of strain operating in an infinite homogeneous half-space, with some distance between the source and the surface (Mogi, 1958; Okada, 1985; Pasal et al., 2014). In nature however, observed patterns of deformation can be due to multiple interacting processes. For example, if magma were to migrate from one storage reservoir to a shallower one, it would generate two overlapping signals, respectively due to the deflation and inflation of the lower and upper reservoirs. In such cases, the system is represented as a collection of sources, like two, nearby Mogi sources (Tryggvason, 1986). Modeling two sources very close together neglects the assumption that each component is operating in an infinite, homogeneous half-space, which can lead to varying degrees of error. Pascal et al. (2014) show that sources closer than a few reservoir radii away from each other are not well-modeled by simply superimposing multiple single-source models together. However, they show that this is not the case for Okada displacements superimposed along shared plane, in that it is reasonable to model a dike as a patchwork of Okada displacements. Via such a methodology, it is possible to constrain the width distribution in a dike and determine where the bulk of the magma intrudes (Nobile et al., 2012), at the cost of computational efficiency.

Even for a simple, single ascending dike, the crust does not necessarily behave like an infinite, homogeneous half-space, partly due to topographic effects and anisotropies in the rock's properties. Considering that dikes have been shown to laterally propagate away from topographic high toward topographic lows (Pinel & Jaupart, 2004; Urbani et al., 2018), the stress field due to the topography clearly affects the dikes, causing them to propagate laterally rather than vertically. To the best of our knowledge, it is not yet known if a localized stress field (e.g. due to topography) in the crust modifies the surface deformation due to a propagating dike.

In spite of these simplifications and challenges, the inversion procedure for a single source is considered to be well-conditioned (Nunnari et al., 2005). Given enough quality data, the surface deformation is considered to be sufficiently and uniquely sensitive to the parameters that go into the Okada model, which are the dike dimensions, position and orientation as well as the rheological properties of the crustal medium. In other words, a well-constrained inversion will correctly estimate the geometry of the dike.

Here, we take an experimental approach to investigate the uncertainty associated to the Okada model. While the process of inverting synthetic data is considered to be accurate, we do not actually know how well the model performs for real dikes. Since it is not possible to physically measure the geometry of an active, natural dike, we will employ analogue experiments. We make a series of experiments with gelatin as the host medium and oil as the liquid magma analogue and generate dikes whose surface deformation can be measured using photogrammetry techniques. We then invert the surface deformation data to estimate the dimensions and position of the dike. We statistically compare the inversion estimates with visual measurements, in order to quantify the uncertainty associated to the Okada model.

1.2 Methods

We perform analogue experiments of dike propagation, which yield images of the dike, as well as the surface. We process the surface images to produce vertical and horizontal deformation data, which we use to perform Bayesian style inversions and estimate the dike geometry. We then compare the inversion estimates to images to determine the accuracy. This process is illustrated in Fig. 1.1.

Fig. 1.1. A flow chart describing our processing methodology. Descriptions are given in the indicated sections of this document.

1.2.1 Experimental setup

We made a series of analogue experiments of dikes propagating through a gelatin medium (made with Xiamen Yasin brand gelatin, 250 bloom, 8-15 mesh), housed in a 50x50x50 cm rectangular tank. We use gelatin because it is elastic at the time scale of the experiment and scales to the Earth's crust properties (Di Giuseppe et al., 2009; Kavanagh et al., 2013). Additionally, it is transparent, so we can visually observe and make measurements of the propagating dike. Gelatin is an incompressible material and has a Poisson's ratio of 0.5. For each experiment, we prepared 75 L of varying gelatin concentration, from 1.8 wt% to 3.75 wt% (Table 1.1), which resulted in a range of gelatin

strengths. We injected a volume of oil, which then buoyantly ascended through the gelatin.

Table 1.1 Experimental conditions.

Exp	Conc %	Gel. Vol. L	Gel. Temp. °C	E kPa	Dike Vol. mL
1	3.75	75	15	3.1	50
2	2.5	75	15	1.3	40
3	1.8	75	15	0.5	40
4	3.1	75	15	1.7	40
5	3.75	100	15	2.8	50

To prepare each gelatin, we heated deionized water to 60°C and dissolved granules of 250 bloom gelatin. We then added a splash of bleach (~20-30 mL) to mitigate bacteria growth and therefore improve the gelatin clarity and covered the liquid gelatin with a layer of oil to prevent evaporation and the formation of a hard crust at the upper surface; we assume that this small volume does not substantially affect the gelatin rheology. We finally placed the gelatin into a cold room at 15°C and allowed it to cool for a few days to solidify and harden (gelatin hardens for some time, reaching its final strength after two days). Usually, gelatin is cooled to 5 to 10 °C to scale to the Earth’s crust (Di Giuseppe et al., 2009; Kavanagh et al., 2013), however our cold room has a minimum temperature of 15 °C. In comparison with Kavanagh et al. (2013) our gelatin behaves with a similar rheology (brittle and elastic), but with a weaker Young’s modulus (e.g. ~3 kPa for our 3.75 wt% gelatin, compared to 14 kPa for theirs); this may be due to a combination of the ambient temperature (ours is 5 to 10 °C warmer) and the gelatin quality (ours is 250 bloom, compared to their 260 bloom). Fortunately, our cold room is large enough to perform the experiments, so our gelatin maintains this ambient temperature throughout the dike propagation.

We measure the gelatin strength using shear waves, which become visible using polarizing filters (Ramesh, 2000; Taisne & Tait, 2009). Placing a filter in front of, and behind, the gelatin allow the deviatoric stress to become visible as a rainbow pattern. Gently tapping the gelatin surface therefore generates shear waves that move slow enough (~1 m/s) to be visible to the unaided eye and recorded by a 50 frame/second video (Pansino & Taisne, 2019). The wave velocity, v_s , corresponds to Young’s modulus, E , via the shear modulus, G , medium density, ρ , and Poisson’s ratio, ν :

$$G = \rho v_s^2, \quad (1.1)$$

$$E = 2G(1+\nu). \quad (1.2)$$

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

This is a new measurement technique developed in our laboratory. A traditional way of measuring the Young's modulus is by placing a known mass of a known diameter on the gelatin surface and measuring the deflection. Our technique is compared with this deflection test and its benefits and limitations are described further in Appendix B.

For the dike liquid, we used store-bought canola oil which was mixed with chili powder for pigment, which allowed us to estimate the width distribution using colorimetry techniques (detailed in section 1.2.2). Prior to injection, we made a precut in the base of the gelatin to ensure a vertical orientation of the dike, with its broad, flat side facing in our preferred orientation for observation (Taisne & Tait, 2009). We then used a peristaltic pump to inject a volume of oil over a period of 1 to 2 minutes, to allow the dike to initially grow in steady, controlled way; we have noted anecdotally that injecting too quickly threatens to force the dike to suddenly reorient, fracturing the overlying gelatin at a $\sim 45^\circ$ angle from the tip of the initial dike. Upon reaching the desired volume, we turned off the pump and allowed the dike to ascend only due to its own buoyancy. During the injection phase, the dikes had grown to 9 to 18 cm in length, depending on the Young's modulus, which left relatively little room for constant-volume propagation (the gelatin depth above the injector was 24 to 34 cm).

As the experiment proceeded, we used an array of DSLR cameras, mounted both above and on the side, to record the surface deformation as well as the dike geometry (Fig. 1.2). The single side camera was placed to view the dike's broad side, while the above-mounted array was positioned with two cameras on either side of the tank, focused on the far side of the gelatin surface, in order to view the surface obliquely and better-resolve vertical deformation. The gelatin is transparent, so to be able to measure surface deformation, we dusted the surface with a thin layer of multi-colored sand, which creates a heterogeneous pattern for later image processing. However, during gelatin preparation, we had applied a layer of vegetable oil which first needed to be removed. To do so, propped up one wheel of the tank apparatus using a small plank (causing the oil to flow towards one corner) and scooped away the oil using a plastic, rectangular box. After scooping away the oil, we dabbed the remaining oil residue with paper towels to leave the surface dry, to which we applied the sand layer. We also placed a series of calibration targets on the edges and corners of the surface, which also helps for later image processing. We set up two 40 x 30 cm² LED light panels (NanGuang brand, 72W) on opposing sides of the tank (under and behind the cameras), to illuminate the surface and ensure that we captured clear, quality images. We adjusted their angle and brightness ($\sim 30\%$ of the max power) to homogeneously illuminate the surface, which is important for surface deformation measurements and relies on the correlation between subsequent photographs (Jones, 2015). Dikes ascended through the gelatin somewhat slowly (*i.e.* tens of minutes to hours) and so we recorded a time lapse of each experiment, with 10 second intervals. The above-mounted cameras were synchronized using an Arduino programmable controller, which sends a signal for the cameras to shutter on a defined frequency. The side camera was controlled separately and tended to become out of sync

with the others. When comparing photos from these two sets, we chose photographs that were temporally closest together. The maximum five second offset is relatively small in comparison with the slow propagation speed of the dikes, which usually ascended on the order of ~ 0.1 mm/s and no faster than ~ 1 mm/s.

Fig. 1.2. A schematic of the experimental set-up. (a,b) Front and side perspectives of a volume of oil pumped into the gelatin, which forms a dike and ascends only due to buoyancy. The dike causes the gelatin surface to deform, which is recorded using a synchronized array of above-mounted cameras. The dike is also recorded using a side-mounted camera, to measure its position and dimensions. (c) An example photograph of the surface, from experiment 1. Since gelatin is transparent, we visualize the surface by applying a thin layer of multi-colored heterogeneous sand. Calibration targets assist with later image processing. (d) An example photograph of the dike from the perspective of the side camera, also from experiment 1.

1.2.2 Dike geometry measurements

The dimensions of the dike were visually measured using a camera positioned on the side of the tank apparatus (using a variable-zoom, Canon EFS 18-55 mm lens), and processed using a script that we wrote in Matlab. We selected a subset of frames and processed them to automatically identify the location of the dike's top, bottom, sides and opening width distribution, which allowed us to determine dimensions and depth. For each frame of the time lapse, we use the green color channel (from RGB images), which shows a high contrast between the dike and the gelatin medium, allowing us to clearly distinguish between the two. We then subtracted the frames from an initial reference frame taken before the dike was injected, which essentially removes much of the background and

further highlights the dike (Fig. 1.3b). In this way, pixels that show the dike have a range of values and pixels that show only the gelatin have near-zero values.

For the width distribution, we employed the Beer-Lambert law, which relates the intensity of transmitted light through the dike to its width, so that the darker regions are estimated to be thicker (Fig. 1.3b; Taisne *et al.*, 2011). The light transmittance diminishes logarithmically with the width, such that:

$$I = I_{ref} e^{-cW} . \quad (1.3)$$

For a photo, I , and an initial reference photo, I_{ref} , we can estimate the relative width of each pixel, W , scaled by a constant, c . For a pixel area, A , the sum of these values, gives an estimate of the dike volume, V_{est} ,

$$V_{est} = \sum cWA = A \sum [-\ln(I/I_{ref})] , \quad (1.4)$$

which scales linearly with the actual volume, V , measured during the experiment. The width of each pixel located at point (x,z) thus scales as:

$$W(x,z) = -\ln[I(x,z)/I_{ref}(x,z)] [V/V_{est}] . \quad (1.5)$$

This assumes that the light passes through the same amount of gelatin before and after the dike emplacement, which is technically not true. The intrusion of the dike displaces a small amount of the gelatin, which makes a bias in the width estimate. This results in a slight underestimation of dike width; a few more millimeters of gelatin on either side of the dike would make the dike appear a bit darker and thus give an estimate of a slightly wider dike. Since the width of the dike is very small relative to the entire block of gelatin ($\sim 1\%$ of the total gelatin width), this effect is assumed to be minimal. The actual error due to gelatin displacement depends on the light source brightness and absorption of the gelatin medium. About half of the luminosity of the light source in our photos is absorbed by our gelatin, such that for equation 1.3, $I/I_{ref} \approx 0.5$. The measured light intensity is affected by the percentage of the gelatin width occupied by the dike, which as noted is about 1%. The accuracy is therefore $(I/I_{ref})^{0.01} = (0.5)^{0.01} = 99.3\%$.

Fig. 1.3. (a) A photo from experiment 1 of a propagating, oil-filled dike, pigmented red with chili powder. Note there are faint radial patterns that developed along the dike surface are plumose structures, which indicate a high stress intensity at the dike tip (Schofield et al., 2012). (b) The light transmitted through the dike diminishes with dike width. The color represents the change from an initial reference photo taken before the dike was injected. (c) The width distribution is estimated using the known volume of the dike. We also automatically determine the dike dimensions and position (represented by the arrows).

The top and sides of the dike were automatically detected in a straightforward manner. The first and last column with non-zero width measurements indicate the sides of the dike, while the first non-zero row indicates the top. Determining the location of the dike's bottom was not as simple, since the dike tail never completely shuts, so there is no clear boundary between dike and gelatin. Instead, we set an arbitrary volume limit by estimating the position above which 98% of the dike's volume is contained (i.e. we throw out the deepest, thinnest 2% of the dike's tail; Fig. 1.3c).

Finally, to measure the dike's dip and strike, we used supplementary photographs taken after each experiment (Fig. 1.4). We took a photograph viewing the dike in cross-sectional view to measure its dip, such that a 90° angle represents a vertical dike. For the strike, we took a photo from directly above the fissure and measured the angle relative to the look direction of the main, side-mounted camera, such that dikes had a strike of 90° , similar to that of the dip. This simplifies our inversion process, in which a strike of 90° implies a West-East oriented dike on a map, which qualitatively matches the perspective from our side camera.

Fig. 1.4. The dike orientation is determined after the experiment has completed. (a) The dip is estimated from a simple photo from experiment 3 taken from the side, as in Fig. 1.2b. Note the clamps visible at the bottom of the image were acting as a temporary fix for our injector piece, which had flexible sheets that peeled apart from the ridged structure. (b) Similarly, the strike is estimated directly from above, for experiment 4. Note the purple textured appearance is due to a thin layer of multi-colored sand. During eruption, the oil easily flows out from the dike and therefore only makes a small fissure on the surface. The bright, orange feature at the center shows this fissure, illuminated by a light shining from below.

1.2.3 Surface deformation measurements and data processing

We used photogrammetry techniques similar to those presented by Galland et al. (2016) to measure the surface deformation in our experiments. We mounted four cameras above the experiment to measure the surface deformation. We positioned the cameras approximately 50 cm above the tank, located approximately above the tank walls. Due to the large size of the tank apparatus and the relatively low ceiling in our work space, it was not possible to view the entire surface in a single photograph (using a fixed-zoom, Canon EF 50 mm lens), so we focused two cameras to one end of the tank and two to the opposite end, with significant overlap in the center. As mentioned, in the experimental setup, we lightly dusted the top of the gelatin with a layer of mixed-colored sand, which is heterogeneous and provides enough visual information for image processing. We also set a series of unique, known (to the software) calibration targets (see Fig. 1.2c and Fig. 1.6a) which are used to improve the automatically align the cameras and to improve the accuracy of the output. We initially recorded a time lapse with 10 sec frequency, but due to the dike's very slow ascent velocity, we then chose a representative subset of these photos for further processing. The following techniques for measuring surface

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

deformation depend on the resolution of the photographs, and in these experiments allowed us to detect sub-millimeter deformation.

We first processed the vertical component of the surface deformation using Agisoft PhotoScan (Photoscan, 2017), a photogrammetry software which makes 3D models and digital elevation models (DEMs) from a set of photographs. This uses the structure from motion methodology, in which the perceived distortion of an object viewed from different perspectives can be used to determine its 3D geometry (Koenderick & van Doorn, 1991). Our work flow began by organizing the photographs from each camera into a separate folder (4 folders, one for each camera). We then uploaded the folders into the software, which can be read as a time lapse with four perspectives. The software can understand the relative location of each camera using the calibration targets, however we also measured and double-checked the position to ensure accuracy. We then followed the software's default workflow, which involves creating a 3D point cloud of matched tie points in the photos, making a mesh and finally a set of DEM. In this way, we were able to generate a DEM for each time step of an experiment. Afterwards, we also output orthorectified photographs of the entire surface, which were used to recover the horizontal components of deformation.

We read the DEMs using a script we wrote for Matlab to determine the deformation change between frames and noticed two artefacts in the outputs. First, the resulting deformation yielded an artefact that we believe arises due to the camera configuration, in which data in the center of the surface differs to the data to the sides of the tank (Fig. 1.5a). The deformation data at either end of the tank was constructed from two cameras viewing the surface from slightly different angles, in which parallax gives a sense of depth but is not well-constrained. By comparison, the 3D geometry of the surface at the center of the tank was much-better resolved, since it was constructed from four cameras viewing the surface from opposing sides. Indeed, Koenderink & van Doorn (1991) show that at least three views are necessary to resolve the 3D shape of an object and indicate that more views increase the reconstruction quality. We therefore crop out the data at the sides of the tank, to keep the better resolved central part; this is disappointing, since we had arranged the camera orientation to recover the entire surface deformation, but in fact we limited our quality coverage.

We also noticed that the surface deformation contained a signal due to the dike propagation convolved with another signal resembling an inflation of the surface. We suspect this is due to the volume of oil injected into the confined block of gelatin (Fig. 1.5a, b). When a volume of a material is inserted into an incompressible medium such as gelatin, the medium must deform outwards to accommodate that volume (i.e. there is a conservation of mass). For our experimental setup, in which the gelatin is confined by ridged tank walls, the deformation must be solely upward. The inflation signal seems to have a maximum at the center of the tank and decreases near the walls. We imagine that this in part due to the central location of the dike, such that the uplift is highest closest to

the dike, and in part due to the gelatin being stuck to tank walls, which inhibits uplift very close to the wall. These two factors can be observed in Fig. 1.5a, in that the surface seems to have a gradual deformation gradient near the center of the tank (due to the closeness of the dike) and a sudden decrease at the walls (due to the gelatin sticking to the walls). There is also the possibility that the walls are generating a sort of surface deformation, in that they push back on the gelatin strain field that surrounds the dike. In this case, we would expect that the deformation signal would be barrel-like in shape, in which it has a curvature in the dike-parallel direction (which has a relatively small component of stress) and fairly uniform deformation in the dike-perpendicular-direction (which has a larger component of stress), since the walls apply an equal and opposite force to the dike. As noted, the deformation pattern seems to be more like a rounded bump and therefore does not resemble this barrel shape.

The inflation signal interferes with our ability to use the Okada model and therefore needs to be removed. We initially considered to choose the end of the injection (a few minutes into the experiment, after which the dike ascended only due to buoyancy) as the reference time for surface deformation measurements; essentially time = 0 is set as the moment when we turned off the pump. If we had done so, it would have removed the inflation signal due to the dike volume intruding into the gelatin. However, during this time, the dike grew significantly (9 to 18 cm) and so using this reference time would have cut out a significant proportion of the propagation. Instead, we opted to account for this intrusion signal by fitting the surface inflation to a simple parabolic surface, with a broad central peak (Fig. 1.5c,d). We took cross sections of data through the center of the surface deformation (in both the dike-parallel and dike-perpendicular directions) and smoothed them to identify the surface deformation directly above the dike. We identified the equation of each cross section and combined them into a surface, Z , such that $Z = C_1X^2 + C_2Y^2 + C_3$, by solving for the constants C_1 to C_3 . The signal was removed from all subsequent frames, leaving only the typical dike lobe pattern of deformation (Fig. 1.5e). Integrating this signal over the entire surface area matches the dike volume to a fairly good degree, and yields somewhat higher volumes (best match models 40.9 mL for a 40 mL dike, worst match models 27 mL for 20 mL dike). We focus on this corrected surface deformation data for the main part of this text, however using the end of injection as a reference time is also possible. In general, this produces somewhat more-scattered, less-accurate results, however the differences are not great. A comparative summary is provided in Appendix D.1.

Fig. 1.5. Vertical deformation due to a dike injection in our experiments. This example is from experiment 1, at the final time step before eruption, in which the dike was vertical and had a depth of 7 mm. (a) A central rectangular feature is likely an artefact due to camera overlap. Also a broad inflation signal can be observed. (b) The overlap is first cropped, then we extract a cross section in the dike-perpendicular direction (shown as a dashed blue line) and dike-parallel direction (not shown) to fit the inflation signal. (c) A cross section of the deformation data corresponding to the dashed blue line in (b). The raw data (solid blue line) is smoothed (solid red line) to find the deformation at the center of the surface, which then allows us to find the equation of a parabola (dashed black line). We then repeat this for the perpendicular cross section to generate a simple parabolic surface. (d) The parabolic surface, which is removed from the data. (e) The resulting dike signal shows symmetrical inflation lobes on either side of the dike and a narrow subsidence pattern directly over the dike.

We then processed the horizontal deformation using digital image correlation (DIC). This technique allows us to track the movement between photographs, but solves only for orthogonal movement to the look direction. We used orthorectified photos from Agisoft to ensure we measured solely horizontal displacement (Fig. 1.6a), as opposed to using the raw photographs viewing the surface from an angle. To perform the DIC analysis, we used a freely-available DIC toolbox for Matlab, written by Elizabeth Jones at the University of Illinois (Jones, 2015), based on a code by Eberl et al. (2006). This code compares a small region of interest in a photograph with a larger look region in the subsequent photograph. Within the look region, it scans for highest-correlating match, which indicates the new location of that region and thus the displacement between photos. The correlation, χ , is quantified by,

$$\mu = \Sigma[(I - \bar{I})(I' - \bar{I}')] / \{ \Sigma[(I - \bar{I})^2] \Sigma[(I' - \bar{I}')^2] \}^{1/2}, \quad (1.6)$$

for a light intensity, I , at reference location (x_1, y_1) , a mean intensity of the surrounding region of interest $\bar{I}$, light intensity of the deformed subset at new location (x_2, y_2) , I' , and a mean light intensity of the new surrounding region, $\bar{I}'$. We defined a look region to exclude the calibration targets (Fig. 1.6b) and then performed the analysis over a coarse grid of 100 pixel increments to quantify large displacements (Fig. 1.6c). We then refined the grid to 30 pixel increments to determine the fine scale displacements.

The resulting dike-parallel, x direction (Fig. 1.6d) and dike-perpendicular, y direction (Fig. 1.6e) components of displacement show similar artefacts to the DEM outputs. The camera-overlap artefact is particularly evident in the x direction and we similarly cropped the data to use the best-resolved data from the center of the image. The volume inflation artefact is not immediately evident in these data, so such displacement is assumed to be primarily vertical and therefore neglected. Note that this technique outputs displacement in pixel values, so we converted to millimeters using the calibration targets, which are each 5 cm wide and have a 40 cm gap between opposing targets (e.g. Fig. 1.6a, the gap between the upper central and lower central targets is 40 cm).

Fig. 1.6. DIC processing, from experiment 1. (a) An original, orthorectified photo from Agisoft output. We process an area enclosed by the white dashed box. (b) In the Matlab toolbox, we choose the bounds of the image to exclude the calibration targets. (c) We define a coarse grid of 100 pixel increments (shown) to detect large displacements and then use a fine grid of 30 pixel increments to determine the final displacement field. (d)

DIC analysis yields surface displacement data in the dike-parallel, X direction. A similar artefact to the DEM is evident in the form of discontinuities, due to how displacement data is resolved for 4 cameras (center of the surface) compared to 2 cameras (left and right sides). The left and right sides are therefore cropped. The inflation artefact is not clear in this data and is therefore neglected. (e) Displacement in the dike-perpendicular, Y direction. A similar artefact is present in this data, though it is not very pronounced.

This methodology has been successfully implemented, via an open-source software named MicMac (Galland et al., 2016), to measure the surface deformation in similar experiments, with compacted silica flour as the solid analogue and low viscosity oil as the magma analogue. They were able to measure the surface deformation due to a sill intrusions, as well as strike-slip fault motion. The DIC technique relies on the same logic as particle image velocimetry (White et al., 2003; Abdelmalak et al., 2012), which has also been used to measure the crustal deformation due to dike intrusion in experiments made with silica flour (Abdelmalak et al., 2012) and gelatin (Kavanagh et al., 2018). To the best of our knowledge, this is the first study to measure the surface deformation of a gelatin medium and compare with in-situ measurements of the dike geometry. Inversion technique

To perform an inversion, we wrote a script that loads the surface deformation data, then repeated calls a function that computes the surface deformation data and calculates the probability of that model, using a Bayesian approach to optimize the probability. We first aligned the horizontal and vertical data onto a common reference frame and downsampled the data to reduce processing time. We did this by defining a surface grid, roughly 100x100 pixels in dimensions (the exact dimensions varied a bit as we preserved the aspect ratio of the overlapping images), and took the mean value of all measurements that fall into each pixel. We repeated this process for each component of surface deformation (vertical and horizontals), thereby obtaining three components of deformation for each pixel of the surface.

For each run of the inversion, we computed the displacement field due to an opening crack via the Okada displacement model. We did this by calling a function that calculates the Okada (1985) displacement equations in UniCycle, which is a toolbox written for Matlab that can be used to compute the Greens functions associated to different forms of fault slip (Barbot, 2013). The Okada model uses the dike's geometry (vertical length, horizontal breadth & opening width), position (depth & horizontal location of the dike's leading tip), orientation (dip & strike) and crust rheological parameters (shear modulus & Poisson's ratio) to estimate the deformation at any point on the surface (Fig. 1.7).

Fig. 1.7. A schematic for a rectangular Okada displacement. The orientation is characterized by the dip and strike angles, θ_{dip} and θ_{strike} . The strike is with reference to our camera look direction in the lab, while dip is with reference to horizontal. The dimensions are characterized by the length, L , breadth, B , and width, W , which are respectively the down-dip dimension, along-strike dimension and opening aperture of the crack. The position is characterized by the depth, d , dike-parallel horizontal location, X , and dike-perpendicular location, Y . The X and Y locations are the upper, left corner of the crack, with reference to the dike tip at the time of eruption. The X position is therefore related to the dike breadth, whereas the Y position is independent and trends towards 0.

We generated values for these parameters semi-randomly, computed the corresponding deformation and determine the residual error by subtracting the measured data from the forward model (Fig. 1.8). For each run of the inversion, the probability, Ω , that the model well-describes the deformation data is calculated by the sum of the square of the residuals, normalized by the number of measurements (i.e. the number of pixels), σ :

$$\Omega = \exp\{-1/2 \sum[(\text{model-data})^2/\sigma]\} \quad (1.7)$$

Fig. 1.8. An example of an inversion, near the end of experiment 3. The rows show individual components of surface deformation, which respectively are dike-parallel, dike-perpendicular and vertical. The columns are, from left to right, the data measurements from the overhead camera array, the best-fitting Okada model, and the residual between the two.

In order to choose values to test for each parameter, we use the Bayesian method of inversion, which is detailed by Anderson and Segall (2013). This method randomly generates sets of model parameters, using Monte Carlo approach, and test the resulting modeled deformation data with the observed one. When a new set of parameters is generated, it tests if the results are better than the previous iteration; if so it keeps them, if not it discards them. At the start of each inversion, the parameters are defined by a “random walk” and allowed to vary over a broad, predefined range. As the inversion progresses, the relative step size of the walk becomes narrower, focusing to a best-matching value. In such a way, the method investigates the parameter space in coarse detail to determine broad, best-fitting values and then focuses its investigation to the nearby area to refine the results. This produces best-fitting inversions in a computationally-efficient way. We perform each inversion using 40,000 runs and determining the best-fitting values for the various parameters. Since the dike grows throughout each experiment, we repeated the inversion process for each frame of the time lapse, producing a time series of independent inversions.

1.3 Results & Discussion

1.3.1 Assessment of the dike intrusion

Each experiment began with a phase of constant-flux injection, in which we set the pump to inject 40 to 50 mL of vegetable oil over a period of 1 to 2 minutes. For gelatins that had a relatively high Young's modulus, the dike grew radially and maintained a penny-shaped form during this phase. The propagation proceeded primarily due to the influx and buoyancy forces only became significant enough to fracture the gelatin as the dike reached its final volume. Afterward, we stopped the pump and let the dike propagate upwards only due to buoyancy. For gelatins that had a relatively low Young's modulus, the dikes began to take on a distinct vertical component of propagation before the pump turned off, indicating the buoyancy forces became strong enough to drive propagation at an earlier stage. As the pump continued to inject oil, such dikes continued to grow a bit in horizontal breadth. After we turned off the pump, they ascended only due to buoyancy. The dike form and propagation were very similar to other investigations using liquid as the magma analogue, propagating through a homogenous, unstressed medium (Takada, 1990; Menand & Tait, 2002; Taisne & Tait, 2009).

The critical length scale for the buoyant propagation of a dike, sometimes known as the buoyancy length, L_b , is that at which propagation can be driven entirely by buoyancy (Taisne & Tait, 2009). This is derived from a balance between the buoyancy pressure, P_b , and the fracture pressure of the medium, P_f , and is dependent on the liquid-medium density contrast, $\Delta\rho$, the medium's fracture toughness, K_c , and the dike length, L :

$$\begin{aligned} P_b &\sim P_f, \\ \Delta\rho g L &\sim K_c/L^{1/2}, \\ \text{and} \\ L_b &\sim (K_c/\Delta\rho g)^{2/3}, \end{aligned} \tag{1.8}$$

in which g is the gravitational constant (Weertman, 1980; Lister & Kerr, 1991; Rubin, 1995). Note the fracture toughness is related to the Young's modulus, such that $K_c = (2\gamma E)^{1/2}$, where γ is the surface energy and has a value of $\sim 1 \text{ J/m}^2$ for gelatin (Menand & Tait, 2002). As a dike approaches L_b , it becomes sufficiently buoyant to self-propagate and begins to decouple from its injection source. Dikes that began to decouple before we turned off the pump had lengths somewhat longer than L_b , so that the stress intensity at the dike tip was relatively high (Weertman, 1980; Dahm, 2000b).

1.3.2 Assessment of solid deformation

The surface deformation pattern in our experiments is qualitatively similar to satellite-based observations of magmatic dikes (e.g. Nunnari et al., 2005; Ayele et al., 2007; Ebinger et al., 2008) and to the deformation predicted by the Okada model (e.g. Okada, 1985; Segall, 2010; Pascal et al., 2014), in which the solid medium on either side of the dike displaced symmetrically upward and outward, while the medium directly along the dike plane subsided. In nature however, magmatic dikes tend to produce deformation fields that are asymmetrical, due to the dike dip orientation (Nunnari et al., 2005), the superposition of multiple signals (Biggs et al., 2009; Nobile et al., 2012) and pre-existing stress shadows, which show relatively small displacements during an intrusion (Ebinger et al., 2008).

In terms of experimental studies, we note that analogue experiments made with silica flour (used in other studies) produce qualitatively different dikes than gelatin (used here). Silica flour is a brittle, but weakly cohesive medium which fractures in a jagged shape, so that dikes resemble a series of interconnected segments (e.g. Abdelmalak et al., 2012; Galland, 2012). By comparison, the fracturation of gelatin, a brittle, elastic and cohesive medium generates dikes that have a smooth, continuous shape (Menand & Tait, 2002; Rivalta et al., 2005; Taisne & Tait, 2009). That said, our measured displacements are on the same order of magnitude (millimeter to centimeter) as those presented in other investigations, including both silica flour and gelatin experiments (Abdelmalak et al., 2012; Galland, 2012; Trippanera et al., 2015; Galland et al., 2016; Kavanagh et al., 2018).

Our ability to model a dike's orientation depends largely on the quality of the surface deformation signal, which can be weak and obscured by measurement noise. The signal-to-noise ratio is quantified by the ratio of the signal power to the noise power (Sim et al., 2013) and in our experiments, a dike's deformation signal becomes more distinct as it ascends to the surface (see Fig. 1.9 a and b, as well as d and e). The associated deformation distribution changes from a symmetrical, narrow, Gaussian distribution centered at zero at the beginning of an experiment, in which the deviation represents the noise, to an asymmetrical, wider distribution centered above zero as the dike displaces the surface (see Fig. 1.9 c and f). As a dike approaches the surface, the median surface deformation fluctuates (sometimes positively, sometime negatively) and the distribution widens (Fig. 1.9g). We therefore estimate the signal-to-noise ratio, SNR , via the standard deviation at a time step, SD , of the data against its initial value, SD_0 :

$$SNR = SD/SD_0 . \quad (1.9)$$

In this sense, the widening distribution captures the uplift and subsidence imposed on the surface by the dike (Fig. 1.9h). The SNR indicates that the dikes make a clear surface signal in the second half of each set of photographs, which corresponds to depths approaching 10 cm.

Fig. 1.9. Signal-to-noise analysis for experiments 1 (Young's modulus of 3.1 kPa) and 4 (Young's modulus of 1.7 kPa). (a, b) The vertical component of surface deformation, after inflation correction described in section 1.2.3, from the beginning and end of experiment 1. (c) Corresponding deformation distributions for each time step. Blue represents the

beginning of the experiment and red represents the end. (d-f) Similar graphics for experiment 4. (g) The standard deviation associated to the surface deformation of experiment, as a function of frame number. For experiment 1, the elapse time was 81 min, while for experiment 4 it was 8 min. (h) The associated signal-to-noise ratio, SNR.

1.3.3 Integrating to retrieve 1D and 2D probability distributions

We will start with an initial “sanity check” assessment of the inversion by looking at their probability density functions (PDFs) with respect to the observed, or known parameter. A PDF from a Bayesian inversion can be generated in different ways and we choose to derive them from the measured residuals. We store the values of each parameter, from all inversion runs, as well as their corresponding probabilities (equation 1.7). If we were to plot the probability against each of the eight parameters, it would generate an 8D (plus probability, 9D total) space, in which some regions are heavily populated with values while others are not; this is because the Bayesian inversion generates a random walk in the parameter’s space and then progressively focuses in on well-matching set of values. To be able to view the error values independent of population density, we divide this 8D space into 16 evenly-spaced segments per parameter and then find the mean error in each 8D segment. This is difficult to visualize, so imagine dividing a 2D grid into 16x16 blocks ($16^2 = 256$ individual blocks) and inside of each we calculate the mean error (Fig. 1.10a); in reality, we do this for an eight dimensional matrix ($16^8 = 4.3$ billion individual blocks, containing varying numbers of probability measurements).

With our 8D matrix computed, we then reduce the number of dimensions by integration along one of the dimensions, via the trapezoid rule:

$$\Omega_{s1,s2,s3,s4,s5,s6,s7} = (s8(n)-s8(n+1))\Sigma(\Omega_{s1,s2,s3,s4,s5,s6,s7,n} + \Omega_{s1,s2,s3,s4,s5,s6,s7,n+1})/2 . \quad (1.10)$$

Here, Ω is the probability matrix with some number of dimensions (one to eight), variables $s1$ through $s8$ represent the eight parameters of the inversion and n is an index along the parameter for which integration is being done. For illustration purposes, imagine a 3D cube of probabilities: integrating along one of the dimensions of the cube accumulates the information as sum of the probabilities and transforms it into a 2D plane (Fig. 1.10b). By choosing which dimensions we integrate along, we can reduce the 8D matrix into separate 1D and 2D PDFs, which are much easier to visualize.

Fig. 1.10. (a) We convert a point cloud of errors (circles) to a grid (background tiles). Color denotes magnitude in which the red end of the color spectrum represents well-matching, low errors. For each square of the grid, we find the mean value of all points that fall within that region. We show 2D here for simplicity, but in reality we do this for an 8D space. (b) An illustration showing how integration can convert an N-dimensional matrix to an N-1-dimensional matrix. In this example, integrating the 3D matrix along the direction of parameter 3 generates the shown 2D matrix. We integrate using the trapezoidal rule, illustrated oblique to the 3D matrix, by summing the areas labeled a, b, and c, under each integration curve. Integration is then repeated along parameters 1 and 2 to generate 1D PDFs for parameters 2 and 1, respectively. In this way, we can reduce the original 8D probability matrix into separate 1D and 2D PDFs.

We will visualize the inversion estimates for the dike's dimensions and the depth of its upper tip, as these parameters are consequential for estimating a dike's volume and proximity to the surface, thus its eruptive potential. Their 1D and joint 2D PDFs (Fig. 1.11) show us that inversion produces its best-fitting models of surface deformation over a small range of values, which has some sense. Perhaps more-importantly, the PDFs have clear peaks and distributions, which indicates that the inversion has a best fitting value and that very-different values indeed produce bad models. This also helps us to visualize if the parameters are interdependent, which would cause the 2D distributions to present a linear trend (e.g. both deep, wide dikes and narrow, shallow dikes could produce similarly good fits to the data). Also of note is the position of the best-fitting values, which at times lay at the border of the test range of values. This indicates a limitation of the Okada model, since in these situations, we controlled the inversion to only allow sensible estimates based on a priori knowledge (e.g. in our experiments the dike should not be larger than the size of the tank), so the best-fitting model parameters do represent reasonable values.

Fig. 1.11. An example from experiment 1, near the end of the experiment (frame 19 of 22, or 81 minutes into the experiment). We show the 1D and 2D PDFs for the dike's vertical length, horizontal breadth, opening width and depth of the upper tip. The 1D plots are as labeled and 2D plots have vertical and horizontal axes consisting of the parameters directly to the left and directly below (see example to lower left). Each of these parameters have a peak and a distribution, which give a sense of the values produce good models of surface deformation. The 1D plots have actual values represented by the vertical red lines and, in the width plot, the solid line and dashed lines respectively represent the maximum and mean dike width. Note the vertical axes' magnitudes reflect the integration described by equation 1.7, which incorporates parameter magnitudes and stems from the probability histogram shown in (b).

1.3.4 Quantifying error

The ability of the Okada model to accurately represent reality is the focus of this study and any limitations are better discussed from a statistical perspective. We compile all of the geometry measurements with inversion results for each of our model parameters. For some parameters, the inversions produce estimates that jump around in non-monotonic ways, indicating the inversion struggles to make a sensible estimate through time (Fig.

1.12a). For example, as a dike grows, the length estimate varies from millimeters to tens of centimeters and back to millimeters, as though the surface deformation does not provide enough information to recover the dike's length. Other parameters, like the horizontal location of the dike tip (in the dike-perpendicular, Y-direction) are very consistently and accurately measured (Fig. 1.12b). In practice, inversion estimates could be made more-consistent by using an ensemble Kalman filter, which accumulates information at various time steps to produce a sensible time series evolution of each parameter (Bato et al., 2017). In this sense, an inversion from one time step acts as a priori information for the subsequent step and new data updates it. However, this technique relies on access to a combination of high temporal resolution data (e.g. GPS) and high spatial resolution data (e.g. satellite-based deformation observations), which may not always be available. Since we focus on the efficacy of the Okada model to represent source parameter producing deformation field, we choose to analyze each time step independently to strengthen our statistical analysis of the results.

Fig. 1.12. The measurements (red dots) and inversion estimate (blue dots) of the inversion parameters against the frame number of the time lapse, for experiment 1 (see Table 1.1 for experimental conditions). Note that time interval between frames is about 4.5 min, so that the final red data point (frame 19) corresponds to an elapse time of 81 min, since the

beginning of the experiment. Some parameters (e.g. Y Location) show good agreement while others show a lot of scatter (e.g. Length) or systematic bias (e.g. Breadth).

To quantitatively assess the uncertainty of each parameter, we calculate the errors, ϵ , simply as the accuracy of the inversion, s_{inv} , against measurements, s_{meas} , for each parameter:

$$\epsilon = s_{inv}/s_{meas} , \quad (1.11)$$

where $\epsilon = 1$ constitutes a perfect match. Since the measurements for Y position approach 0, we quantify the error proportional to the dike breadth, such that

$$\epsilon_Y = (Y_{inv}-Y_{meas})/B_{meas} + 1 , \quad (1.12)$$

which also gives values approaching 1.

We note two types of error between the measurements and inversions. The first is due to the model, in which a parameter is always over- or under-estimated by the inversion (e.g. Fig. 1.12 d and e) and indicates a mismatch between the Okada model and the experimental dike. The second is signal-related error, in which a weak signal-to-noise ratio corresponds to a poor inversion and tends to occur at the beginning of an experiment (e.g. Fig. 1.12 f, g, and h).

1.3.5 Strengths and limitations of the Okada model

By accumulating all the error estimates for each time step of each experiment, a total of 102 values, we can statistically assess how accurately each parameter is estimated, via the median and standard deviation (Fig. 1.13). On the whole, approximating a dike to an Okada displacement yields fairly good results, as the error tends to be between 50% and 200% of the actual values. The inversion particularly well-estimates the dike's orientation (dip and strike) and dike-perpendicular horizontal location (Y-Location), since these parameters are distinctly exhibited in the surface deformation; the Y-location accuracy shows a lot of scatter in the error estimates due to the way they are calculated, but are accurate when directly comparing inversions with measurements (Fig. 1.12h). The length estimate shows a large degree of scatter in comparison to the other parameters (Fig. 1.12c). This is not entirely surprising, since it depends on the inversion resolving the depth of the dike's lower fringe; the deformation due to the dike tail is perhaps too weak to be adequately constrained. The estimate for the dike breadth is consistently estimated to be too short (Fig. 1.12d) and, similarly, the dike-parallel X-Location tends to be underestimated. When we process inversions, we set the dike tip at the time of eruption as the reference point and find one of the upper corners of the dike (assuming it takes the rectangular shape of an Okada displacement). The dike breadth and X position are thereby interrelated and the difference, $X-B/2$, fairly-well locates the dike tip, by a median error of 12 mm, with a median absolute deviation of 5 cm. While the Okada model can accurately represent the horizontal location of the dike tip, the mismatch in

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments:
Comparing inversion estimates to lab measurements

geometry between a rectangular Okada displacement and a penny-shaped dike ensures that we poorly estimate the dike breadth and tip depth. The shape mismatch results in a corresponding deformation mismatch (Fig. 1.14), in that the upper tip makes a pronounced deformation signal.

Fig. 1.13. Boxplots for the log of the errors, ϵ , (for all experiments) of each parameter from the inversion, with 1 representing a perfect match. The red lines and blue boxes indicate the median error and the 25 and 75 percentiles. The red + marks are the remaining errors. We assess the estimated dike width with the mean and maximum measurements separately to see which is more accurate. Well-matching parameters are marked by green boxes, while the others indicate a bias in the model.

Fig. 1.14. The deformation due to a synthetic, rectangular patch (a to c) compared with a real dike (e to g, from experiment 1). Shown are the strike-parallel (x-directional), strike-perpendicular (y-directional) and vertical (z-directional) components of the deformation. For reference, we show the laboratory dike (d) and its synthetic counterpart (solid black box), which was made with the same conditions (e.g. dimensions, placement, gelatin rheology). By contrast, the best-fitting inversion for this deformation data (dashed box on photograph, surface data not shown) yields a smaller, deeper dike. The deformation pattern due to a rectangular patch has a distinct horizontal breadth, whereas that due to a rounded dike does not. For deeper dikes, this contrast is not as apparent (see deformation pattern in Fig. 1.8).

The errors reveal important information about the dike dimensions and volume. The inversions consistently overestimate the mean width of the dike, but they better-estimate its maximum width, which is located in the upper, bulged head of the dike (see Fig. 1.3 for the width distribution). This has some intuitive sense, since the head region is both nearest to the surface and has a greater magnitude of ground displacement than the rest of the dike, essentially allowing it to overshadow the stress due to the thinner and deeper tail region. Since the inversion only recovers the width of the dike head, one would expect the total dike volume to be overestimated by the factor between the head width and entire dike's average width (which in our experiments tends to be less than three). However, the median volume estimates are surprisingly accurate, albeit with a large degree of scatter. The Okada model tends to underestimate the vertical length and horizontal breadth and overestimate the opening width, so that the combined volume is more-accurately estimated than any of the individual dimensions. Perhaps the surface deformation reflects the geometry of the head region only; this would explain why the length is underestimated (the whole dike is longer than just the head), the mean width is overestimated (the dike is on average thinner than the head region), while the volume is well-estimated (the head contains most of the dike's volume).

To analyze the errors in a more in-depth fashion, we will inspect whether they have any relationship with the position or the forces acting on the dike. We employ two dimensionless parameters, which will assist us to frame our results in the context of the natural scale. The first is the dimensionless depth, d^* , which scales the depth of the dike tip, d , against its horizontal breadth, B :

$$d^* = d/B . \quad (1.13)$$

See Fig. 1.7 for a schematic indicating these variables. The second is the dimensionless buoyancy forces, P^* , which scales the liquid buoyancy that drives the dike upward against the fracture pressure that the dike needs to overcome to propagate:

$$P^* = \Delta \rho g L / (K_c / L^{1/2}) = L / L_b . \quad (1.14)$$

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

As in equation 1.8, L is the dike's vertical length, $\Delta\rho$ is the density difference between the ascending liquid and the gelatin medium, g is acceleration of gravity, K_c is the fracture toughness and L_b is the buoyancy length. Note that the dike may have a dip and that L quantifies the vertical component; our dikes were nearly-vertical, so this also quantifies the total length.

A dike's surface deformation is likely maximized when d^* is small (the dike is near to the surface) and P^* is big (the driving forces are high), but significant surface deformation does not necessarily correspond to an accurate inversion. In terms of the d^* parameter, the Okada model assumes some distance between the crack top and the surface, in that the rectangular shape better approximates the surface deformation at a distance; however, the deformation diminishes with distance between the dike and surface, so there is potentially a depth at which the error is minimized. Similarly for the P^* parameter, Okada's (1985) equations are written for a static displacement and do not account for the forces (e.g. buoyancy) acting on the dike. The P^* parameter represent a simple, first-order approximation of the driving force and indicates whether the dike is growing predominantly vertically or radially (as discussed via observations in section 1.3.1). We would expect that the inversion would be more accurate for dikes with relatively weak buoyancy forces, which have low values of P^* . As the dike grows and its buoyancy force increases, the similarity to the Okada model decreases.

In our experiments, some parameters (e.g., the dike's upper tip depth, vertical length, opening width) are better or worse estimated as function of d^* or P^* , while others seem to be independent of those parameters (e.g., Fig. 1.15 and Fig. 1.16 dip, strike, horizontal breadth). Inversion estimates seem to be most-accurate for $d^* > 0.3$, which for a 1 km broad dike corresponds to a depth of 300 m or more. For shallower dikes, the inversion overestimates the depth and under-estimates the length (Fig. 1.15, subplots III and VI), implying that the Okada model cannot well resolve deformation due to near-surface cracks, which is known, since the model was developed for an infinite, elastic half-space (Okada, 1985). As mentioned above, the model also struggles to accurately predict the dike's breadth and the X location of the dike tip (especially for very shallow dikes), due to the geometry mismatch between an Okada displacement and a penny-shaped dike (Fig. 1.15, subplots IV and VIII).

In terms of P^* , the dike's volume and the tip's Y location are better estimated for high P^* (i.e. when the dike is long), but its depth, X location and width are better estimated at low values of P^* . The length parameters does not have clear relationships with the pressure and seems to be more accurate, but with a wider distribution at low P^* . The breadth is against consistently underestimated, regardless of the value of P^* . In total, the P^* parameter indicates that the buoyancy affects the stress and strain fields around the dike and causes the width and depth to be overestimated and the length to be underestimated.

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments:
Comparing inversion estimates to lab measurements

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments:
Comparing inversion estimates to lab measurements

Fig. 1.15. (Previous page) A set of plots showing the error estimate, ϵ , between the inversion and visual measurements, against dimensionless depth, d^ , for all parameters of all experiments. An ϵ of 10^0 indicates a perfect match. Note the vertical scale differs between the upper 2 rows and the bottom row. The black markings show the moving median (circle) and deviation (upward, downward pointing triangles) for nearby values of d^* (within half an order of magnitude). In the subplot V, the ‘.’ and ‘+’ markings respectively indicate the mean and maximum dike width measurements (see Fig. 1.3c for an example of width distribution) and the median and deviation markings correspond by color. Errors tend to be more scattered, but also more accurate at greater d^* .*

Fig. 1.16 (Next page) A similar set of plots showing the error against the dimensionless pressure, P^ . Again, note the vertical scale differs between rows. We expected better error estimates at low values of P^* , since such values are associated with weaker buoyancy forces, which may impose a different deformation signal than a static Okada displacement. Indeed, parameters like length, width and depth are better-estimated for low values of P^* .*

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments:
Comparing inversion estimates to lab measurements

Arguably, the most important parameters from an inversion, in terms of monitoring and decision making, are the location of the dike and its volume. Unfortunately, the tendency of the inversion to overestimate a dike's depth when the dike is shallow may be a source of uncertainty for monitoring agencies (Fig. 1.15, subplot VI). A deep dike can be accurately estimated to be deep, while a shallow dike is inaccurately estimated to also be deep; so, if dikes are always estimated to be deep regardless of their actual depth, how can we be sure of their location? Based on the error analysis, it is safe to say that the inversion of dike depth represents an upper bound estimate, meaning that the dike is not deeper than expected. The analysis also indicates that in reality, dikes may be an order of magnitude shallower than predicted via the Okada model. This result indicates that a dike may (or may not) be much shallower than expected and makes a strong case for combining results from such inversions with other geophysical data like seismicity measurements. Such findings seem to contrast with the arguments made by Nunnari et al. (2005), who show that the Okada model is well-conditioned and accurate, both for synthetic and natural magmatic dikes. However, they argue that there is "broad" agreement between ground deformation inversions and other measurements, which we assume means that they match within a factor of 3 or 4; our general results show the same degree of agreement.

On the other hand, estimates of the dike volume are fairly accurate (Fig. 1.15, subplot VII, median and deviation markings). Regardless of the dike's location, the volume tends to be estimated within a factor of 2 of the actual value (i.e. 50%-200% of the real volume). Looking closer however, the inversions most-accurately estimate the dike volume when it is relatively shallow, $d^* < 0.1$. For dikes that are deeper, $d^* > 0.1$, the errors are more scattered, but remain centered near the actual value, within a factor of 2. Framing this in the context of tip depth uncertainty, a researcher would need other measurement to constrain the dike depth and therefore constrain the uncertainty on the volume estimate.

1.3.6 Comparison to the 2007 dike intrusion in Northern Tanzania

We can consider the results through an example. In July 2007, a dike intruded near Lake Natron in Northern Tanzania, in an immature segment of the African Rift Valley (Biggs et al., 2009). The dike did not erupt, however the event was captured through satellite observations (Envisat, ALOS and Radarsat), which Biggs et al. (2009) used to perform a set of geophysical inversions and determine the sequence of events. They found that the event was preceded by normal faulting, after which a Mogi source deflated and a corresponding dike intruded.

We will consider one of the interferograms that Biggs et al. (2009) presented, which captured the dike intrusion but not the fault slip, and therefore is the most-similar to our experimental setup. They model the dike intrusion to have a depth of 4.0 km, a vertical length of 4.3 km, a horizontal breadth of 7.6 km and an opening width of 0.8 m. Assuming the dike had a buoyancy of 10 kg/m^3 and propagated through a crust with a Young's

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments: Comparing inversion estimates to lab measurements

modulus of 30 GPa (with a fracture toughness of $4 \times 10^5 \text{ Pa} \cdot \text{m}^{1/2}$), then the dike had a d^* value of 0.53 and a P^* value of 70. The high magnitude of P^* is larger than any values in our experiments, which is problematic for scaling purposes as there should be overlap between experimental and natural conditions. This is a limitation of our experimental setup, in that an oil-filled dike would need to be on the order of a few meters long to have a proportional buoyancy force. For the moment, the best we can do is compare with the errors associated to our highest values of $P^* = 2$.

The median errors, ϵ , as well as the upper and lower bounds for $d^*=0.53$ and $P^*=2$ are presented in Table 1.2. To estimate ranges of values for each parameter, S_{meas} , we use equation 1.11 and estimates from Biggs et al. (2009), S_{inv} . As previously discussed, our error measurements associated to d^* indicate that the dike may be a shallower, broader and thinner than expected, respectively by $\sim 1 \text{ km}$, $\sim 10 \text{ km}$ and $\sim 40 \text{ cm}$ in this example. Using P^* , the dike is again expected to be shallower, broader, thinner and also longer respectively by $\sim 3 \text{ km}$ $\sim 9 \text{ km}$, $\sim 40 \text{ cm}$ and $\sim 2 \text{ km}$. These estimates generally agree that the dike may have been shallower and thinner than estimates by Biggs et al. (2009). We can gain a bit more insight through the seismic moment associated to the intrusion event, which Biggs et al. (2009) report was $7.2 \times 10^{17} \text{ N} \cdot \text{m}$ during the time period. For a shear modulus of 30 GPa, the dike volume was on the order of $2.4 \times 10^7 \text{ m}^3$, which closely matches the volume estimate associated to P^* . Since our largest estimates P^* are smaller than those of nature, we cannot be sure if the volume match is due to accurately scaled errors or if it is merely coincidental (i.e. in the Fig 1.16 volume subplot, we do not know if the trend continues to increase for $P^* \gg 1$ or if it plateaus for all $P^* > 1$). If we consider the inversion performed by Biggs et al. (2009), together with the seismic data they present and our error analysis, it is likely the dike had dimensions similar to what they estimate, but was a bit narrower and broader, with an overall smaller volume and emplaced at a shallower depth.

Table 1.2 Errors associated to d^* and P^* for the 2007 Lake Natron dike intrusion. We provide the parameter estimates from Biggs et al. (2009) and apply the errors from our experiments to quantify the uncertainty. We omit the dike's map position and orientation and instead focus on geometry and depth. For the width parameter, we employ the maximum width errors, which better matched the surface deformation in our experiments.

Parameter	ϵ $d^*=0.53$	ϵ $P^*=10^{-1}$	S_{inv} (Biggs et al., 2009)	S_{meas} via d^*	S_{meas} via P^*
Length	0.93 +0.28/-0.33	0.70 +0.23/-0.18	4.3 km	4.6 km +2.5/-1.1 km	6.2 km +2.2 /-1.6 km
Breadth	0.42 +0.24/-0.08	0.46 +0.20/-0.08	7.6 km	18.2 km +4.4/-6.6 km	16.5 km +3.6/-5.0 km

Width	1.93 +0.58/-0.82	1.89 +0.63/-0.75	0.8 m	0.4 m +0.3/-0.1 m	0.4 m +0.3/-0.1 m
Depth	1.40 +0.80/- 0.40	3.23 +1.06/-0.85	4.0 km	2.9 km +1.1/-1.0 km	1.2 km +0.4/-0.3 km
Volume	1.45 +0.63/-0.80	1.15 +0.49/-0.35	$2.6 \times 10^7 \text{ m}^3$	$1.8 \times 10^7 \text{ m}^3$ +2.2/-0.54 $\times 10^7 \text{ m}^3$	$2.3 \times 10^7 \text{ m}^3$ +1.0/-0.7 $\times 10^7 \text{ m}^3$

1.4 Conclusion

We performed a series of analogue experiments to determine the uncertainty associated to the Okada model. To do so, we injected oil in gelatins of various Young's moduli and recorded the propagation with an array of cameras. A camera mounted at the side measured a time series of the dike's geometry and above-mounted cameras tracked the surface deformation. We inverted the ground deformation measurements using a Bayesian inversion procedure and compared the resulting geometry estimates with actual measurements.

We found that the Okada model statistically well-estimates the geometry of a dike. For most parameters of the inversion, the best 50% of the inversion estimates fall within a factor of two of the actual values, while the remaining 50% tend to fall within a factor of 10. For the dike opening width and tip depth, which tend to be overestimated, and the horizontal breadth, which tends to be underestimated, these worsen to factors of 5 and 30. There is some depth-dependence of the results, in that dike's vertical length, as well as the depth and horizontal location of its upper tip are better estimated when the dike is relatively deep (i.e. deeper than it is broad). Conversely, volume estimates are better (with a narrower distribution) when the dike is shallow. Since the error of the depth measurement is itself a function of depth, there is an inherent uncertainty in this parameter, in that dikes may be an order of magnitude shallower than predicted by the Okada model. We argue that depth measurements should be constrained jointly using multiple types of measurements, which has been successfully performed in other studies (Anderson & Segall, 2013; Riel et al., 2015).

The results of these experiments also indicate how ground deformation is linked to the geometry of the dike. The inversion estimates better-match measurements of the maximum opening width of the dike rather than the mean width. Similarly, the inversion tends to underestimate the dike's length. These details imply that the ground

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments:
Comparing inversion estimates to lab measurements

deformation is sensitive to the geometry of the dike's upper head region, which is thicker and shorter than the dimensions of the dike as a whole. Similarly, the volume of the dike is well-estimated by the Okada model, since the head contains most of the dike's volume in our experiments.

1.5 Chapter Notation

Table 1.3. A list of parameters for this study.

Symbol	Units	Meaning
<u><i>Inversion parameters</i></u>		
θ_{dip}	rad	Dike's dip angle
θ_{sstrike}	rad	Dike's strike angle (relative to camera look direction)
L	m	Dike's vertical length
B	m	Dike's horizontal breadth
W	m	Dike's opening width
d	m	Depth of the dike's upper tip
X	m	Surface location (dike-parallel direction)
Y	m	Surface location (dike-perpendicular direction)
<u><i>Probability & statistical parameters</i></u>		
Ω	-	Probability
s	-	Placeholder symbol for the inversion parameters listed above
σ	-	Number of elements in a matrix
n	-	Index, representing a position along a vector or matrix
N	-	Index, representing the number of dimensions of a matrix
Σ	-	Symbol of summation
η	-	Correlation variable, for digital image correlation
SD	-	Standard deviation
SNR	-	Signal-to-noise ratio
ϵ	-	Error between inversion estimates and lab measurements
<u><i>Dike width distribution parameters</i></u>		
I, I_{ref}	-	Photograph and reference photo
c	-	Constant of proportionality
A	m^2	Surface area that light passes through
<u><i>Other physical parameters</i></u>		
V	m^3	Dike volume
Z	m	Vertical location
$\Delta\rho$	kg/m^3	Density contrast between the dike liquid and medium
g	m/s^2	gravity
E	Pa	Young's modulus

Chapter 1: Application of the Okada model to propagating dikes in analogue experiments:
Comparing inversion estimates to lab measurements

K_c	$\text{Pa}\cdot\text{m}^{1/2}$	Fracture toughness
γ	J/m^2	Surface energy
P_b	Pa	Buoyancy pressure
P_b	Pa	Fracture pressure

Dimensionless parameters

d^*	-	Dimensionless depth, normalized by dike breadth
p^*	-	Dimensionless pressure, ratio of buoyancy to elastic force

Future work and a segue to chapter 2

The results presented in chapter 1 are, in my opinion, interesting, in that they indicate how the solid medium deforms due to a dike intrusion. As we saw, the deformation seems to be sensitive to the maximum dike width, which occurs in the upper head of the dike, but misrepresents the dike's volume. There are dynamics which we have not explored, which are worthy of future work, like the effect of the liquid's buoyancy and viscosity. In chapter 1, we had only used vegetable oil as the magma analogue, so it would be useful to explore how a more viscous liquid like silicone oil would affect the deformation, since the resulting dikes would likely be thicker and easier to detect. Similarly, we could investigate more- and less-buoyant fluids like air and water, which would allow us to achieve a wider range for the P^* parameter that we estimated in the results. For example, if we used air, we could inject less volume and mitigate the surface inflation signal that complicated our analysis. As we will see much later in chapter 4, the buoyancy of the liquid has a control on the dike's internal flow and on its propagation pattern. It is feasible that strong flow dynamics inside of a dike generates a characteristic surface deformation pattern, similar to but different from the Okada model, but such has yet to be identified. In fact, the ability to uncover new dynamics or relationships is a key benefit of analogue modeling, in that complex behaviors can be difficult to identify in nature or model through numerical methods.

We have seen that the Okada model approximately describes how stresses due to a dike opening are distributed in the crust. This mode of displacement generates compression to the sides of the dike and extension above it, along its plane of orientation. At the surface, this causes deformation patterns like those seen in the previous chapter, in which the region just above the dike displaces downward and the regions to either side displace upward and away.

For a propagating dike, stress can be externally or internally derived: a dike has an internal pressure and exerts stress on the surrounding crust, however the crust also has a stress field, which can affect how the dike propagates. This is because stresses concentrate along the dike's fringe, which fractures in the most mechanically-efficient way. Stress fields can be large and exert influence on a regional scale, for those generated by tectonic forces in the geologic setting. They can also be smaller and more localized to individual volcanoes. In the upcoming chapter, we will discuss how dikes react in the presence of a localized stress field, which occurs around pressurized, magma storage regions. This work has already been published in the *Journal of Geophysical Research: Solid Earth* (Pansino & Taisne, 2019).

Chapter 2: How magmatic storage regions attract and repel propagating dikes

Abstract

We investigate the effect of magmatic reservoir pressure on the propagation of dikes that approach from below, using analogue experiments. We injected oil into gelatin and observed how dike propagation responded to the stress field around a pressurized, spherical reservoir, filled with water. The reservoir was modeled using two different setups: one simply using an inflatable rubber balloon; the other by constructing a liquid-filled cavity.

We find that the dike's response is dependent on the sign of the reservoir pressure (i.e. inflated/ over-pressurized & deflated/ under-pressurized) as well as on the dike's initial orientation (i.e. if its strike is radially, circumferentially or obliquely oriented to the reservoir). Dikes that are initially strike-radial respond respectively by propagating toward or away from over-pressurized or under-pressurized reservoirs, taking advantage of the reservoir's hoop stresses. Otherwise-oriented dikes respond by changing orientation, twisting and curling into a form dictated by the principal stresses in the medium. For over-pressurized reservoirs, they are coaxed to propagate radially to, and therefore approach, the reservoir. For under-pressurized reservoirs, they generally re-orient to propagate tangentially, which causes them to avoid the reservoir.

The magnitude of reservoir pressure controls at which distance dikes can be affected and, at natural scales, we estimate that this occurs within a radius of a few tens of kilometers. This diminishes with time, due to viscous stress relaxation of the crust, which will occur on a time scale of hundreds of years.

2.1 Introduction

When magma ascends from depth toward a shallow magmatic storage region, it can migrate either via an existing pathway (i.e. a conduit) or it can fracture open a new pathway in the form of a dike. Deeply-sourced feeder dikes can supply large volumes of mafic magma, which may either arrive to a shallower reservoir and mingle with the existing magma or bypass the reservoir and erupt at the surface. For example, some monogenetic eruptions at Sanganguey volcano (Mexico) emit lavas that come directly from a deep reservoir, while others emit lavas that represent a mixture of deeply and shallowly residing magmas (Crabtree & Waters, 2017). The path that a feeder dike takes depends on the state of stresses in the crust and one significant source of stress can be a pressurized magmatic reservoir, which has been shown can attract feeder dikes (Karlstrom et al., 2009). Generally speaking, when fresh magma arrives to a magmatic storage region, it is thought to remobilize the residing magma, which can lead to an

eruption (Murphy et al., 2000; Ruth et al., 2016). Whether a dike propagates toward a shallow reservoir and mingles with the shallow magma, or bypasses the reservoir, has an effect on the probability, location and style of eruption, which has important implications for hazards and risk management. We therefore want to understand how dikes propagate in the presence of a pressurized or under-pressurized magmatic reservoir.

Magma reservoirs become pressurized and depressurized via a few mechanisms. Pressurization is predominantly due to the intrusions of new magma from depth, which increases the available magma volume and strains the surrounding crust. This is thought to occur either due to influx via a persistently-active feeder conduit (e.g. Anderson & Segall, 2013) or via a series of dikes, which generate stress on the surrounding crust and attract subsequent dikes (Kühn & Dahm, 2008). For a body of magma residing in a reservoir, the pressure evolves as the magma evolves. As magma cools, it forms crystals and the remaining magma undergoes second boiling in a process of crystallization which causes the pressure to increase (Cashman & Sparks, 2013; Edmonds & Woods, 2018). A reservoir can lose pressure due to volume extraction (either via a dike or sill intrusion into the surrounding crust or due to eruption), cooling or viscous relaxation of the surrounding crust (Segall, 2010; Degrueter & Huber, 2014; Townsend et al., 2019).

Dikes tend to propagate at a velocity governed by their driving forces (i.e. their source over-pressure and internal buoyancy) and impeding forces (fracture toughness of the crust and magma viscosity). For constant-flux dikes, which receive a constant influx of magma, the propagation velocity is dependent on the influx, magma buoyancy and host medium strength (Taisne & Tait, 2011). For constant-volume dikes, with no influx, the velocity depends on the viscous pressure drop in the lower, tail region of a dike (Taisne et al., 2011). As the dike grows in length, the tail region thins and the viscous pressure drop increases, which causes the propagation to slow down. A constant-flux dike can effectively transition to a constant-volume dike if the buoyancy forces become dominant and pull the dike head away from the source reservoir, during which the velocity evolves through several phases (Dahm et al., 2010; Taisne et al., 2011).

Dikes are known to be oriented according to the stresses in the crust and open against the least compressive principal stress, σ_3 , and propagate perpendicular to this direction (Delaney, 1986). If a dike enters a localized stress field, such as that due to a pressurized magma storage region, it can twist and curl into its preferred orientation. For example, Porreca et al. (2006) show that dikes in the Mt. Somma scarp near Vesuvius take a range of orientations, from radial to circumferential, from inward to outward dipping, and posit that this is due to varying reservoir pressures at the time of emplacement. This is because a pressurized reservoir favors radial emplacement, while a depressurized one favors tangential orientation (Bianco et al., 1998). Indeed, at the well-exposed Ardnamurchan dike swarm, Anderson (1937) interprets inward dipping circumferential dikes as sourced from an over-pressurized reservoir and outward dipping dikes sourced from an under-pressurized reservoir. More-recently at the same location, Magee et al. (2012) show via the anisotropy of magnetic susceptibility that magma flow was dominantly horizontal and

that the flow aligned similarly to nearby NW-SE trending regional dikes. They proposed that the circumferential and regional dikes are linked and that the local circumferential pattern was due to the reservoir's pressure.

The competition between local and regional stress fields can be evident from a dike's orientation. On the volcanoes of the Galapagos Islands for example, swarms of dikes tend to align either circumferentially near the edifices or radially a bit further away (Chadwick & Dieterich, 1995). This is because many of the volcanoes have calderas, whose formation causes unloading of the underlying crust. This generates an extensional force that causes dikes, originating from the reservoir, to emplace in such orientations (Bagnardi et al., 2013). This also would cause any subsequent feeder dikes, ascending from below, to deflect laterally away and eventually erupt at the caldera rim (Chadwick & Dieterich, 1995; Corbi et al., 2016). Away from the collapse structure, the stress field is configured differently and dikes take on a radial pattern. For another example, using geophysical monitoring techniques, Gudmundsson et al. (2014) show that during the 2014 Bárðarbunga eruption (Iceland), a dike intruded into the crust from a pressurized reservoir and began to propagate radially away. Some kilometers away, the dike rotated by $\sim 30^\circ$ to match the orientation of the regional stresses.

Dikes can also change orientation due to other factors. Crustal heterogeneities (e.g., layers) play a significant role deflecting dikes into sills, in that a strong overlying layer may cause dike propagation to come to arrest (Rivalta et al. 2005). If the two layers are not strongly coupled together, pressure accumulation at the interface may lead to failure and decoupling, allowing the magma to form a sill (Kavanagh et al. 2015). Furthermore, a propagating dike or sill can abruptly change orientation at its leading edge, due to the balance of internal forces and the cohesive strength of the surrounding crust (Galland et al., 2014; Schmeidel et al., 2017). Galland et al. (2017) show using analogue experiments with silica powder that, if a dike or sill is shallow (overlying compressional forces are low) and has a high viscosity and/or high influx rate (internal pressure is high), it can divert into a cone sheet.

The direction in which a dike propagates depends on the orientation of the principal stresses in the crust, as well on the distribution of stresses (i.e. if there is a vertical or horizontal gradient in magnitude). Dahm (2000a) used numerical modeling to study the propagation direction of quasi-static cracks (i.e. those with no driving internal pressure) and found that dikes tend to propagate from regions of high stress to low stress and that dikes prefer to propagate perpendicularly to σ_3 . When the stress gradient is perpendicular to σ_3 , such that these two conditions compete with each other, dikes propagate almost-perpendicular to σ_3 , and gradually descend along the stress gradient at a high angle. This results in energy minimization, in which fracturation at the dike tip occurs in the orientation that takes the least energy. Watanabe et al. (2002) showed similar behavior with analogue experiments, by injecting silicon oil into gelatin, and found that a surface load (e.g. the load due to a volcanic edifice) re-orientes the stress field to attract dikes. In this case, the dikes propagated towards high stresses entirely because of

the orientation of σ_3 . Maccaferri et al. (2011) show similar results numerically, by modeling a boundary-element liquid-filled crack, for various loadings and dike angles of ascent. In both cases, the strike of the dike is oriented circumferentially to the loading and the dike curls inward toward the edifice load. Since an edifice's load induces a compressive state on the underlying crust, it can also inhibit dike propagation (Pinel & Jaupart, 2003). Dikes that propagate into such a region may either come to arrest or begin horizontal propagation at their lateral edges. Urbani et al. (2018) used analogue modeling to extend this concept to topography more-generally and show that dikes propagate away from topographic highs to erupt in topographic lows.

It is clear that dikes respond to stresses in the crust in a complex fashion and that their ability to interact with existing magma bodies has important hazards implications, such as influencing the location of future eruptions. We therefore investigate how dikes respond (i.e. how they bend and change orientation or how they propagate horizontally) to a pressurized (i.e. inflated or deflated) magmatic reservoir using analogue experiments. This allows us to explore such responses simply by varying the initial conditions, which are the initial dike orientation and the magnitude of reservoir inflation or deflation. For a small subset of experiments, we use a different setup to secondarily investigate how dikes behave when they intersect with the reservoir. These simple models can be intuitive, descriptive, and allow us to simulate complex three-dimensional behavior that cannot readily be modeled via numerical methods.

2.2 Methods

Throughout this study we refer to dikes in different orientations, relative to the reservoir (Fig. 2.1). The following terminology will be adopted:

Radial or strike-radial: We use in the conventional sense to indicate dikes that are vertically oriented and whose strikes are radial to the reservoir.

Circumferential or strike-circumferential: We generally use this in the conventional sense as well, for dikes whose strikes are oriented tangential to the map-view projection of the reservoir. Such dikes can propagate radially to the reservoir, in the way that a cone sheet does, but are not 'radial dikes'. We use this for both inward and outward dipping dikes.

Oblique: Dikes whose strikes are initially oblique (45°) to the map-view projection of the reservoir.

Coaxial: Dikes whose vertical leading tips are aligned with the center of the map-view projection of the reservoir. They are positioned directly below the reservoir.

Non-radial: We use this to mean dikes in our experiments that were not radial dikes, meaning circumferential, oblique and coaxial dikes.

Figure 2.1. An illustration of different dike orientations (map view) we consider in this study. Dikes are the narrow dark features and the reservoir footprint is indicated by the dashed line.

2.2.1 Experimental procedures

We performed the experiments in 40x40x30 cm (length, width, height) and 50x50x50 cm tanks, filled with 3.75 wt% industrial gelatin (250 bloom). The retailer did not specify the animal source beyond beef or pork, though the two sources have been shown to be mechanically similar (Nur Hanani et al., 2012). We prepared each gelatin in a similar way, by mixing gelatin granules into a pot of purified water and slowly heating the pot to 60 °C, while constantly stirring with an overhead stirrer (Tosh et al., 2003; Di Giuseppe et al., 2009). We added a small splash of bleach to the gelatin to mitigate bacteria growth, which cause the gelatin to become cloudy in appearance. The resulting liquid gelatin is then placed in our cold room (15 °C) to solidify for 72 hours, with a layer of oil on the surface to prevent evaporation.

Gelatin is an ideal medium for such experiments as it can scale to the Earth's crust and is transparent, so we can observe what occurs inside (Di Giuseppe et al., 2009; Kavanagh et al., 2013; Galland et al., 2015; Rivalta et al., 2015). Due to the slight variations in the preparation (initial temperature, cooling time, cooking time/evaporation), it varies

somewhat in strength from experiment to experiment, which we measure. Gelatin is a photoelastic material, so we can measure the gelatin's strength at the beginning of each experiment via the velocities of shear waves, which become visible using a pair of polarizing filters (Fig. 2.2, Taisne & Tait, 2011). When light passes through these filters, it becomes linearly polarized to a particular incidence angle. As light passes through the gelatin, its retardation depends on the stress level and the light's wavelength (Ramesh, 2000). This enables us to see the deviatoric stress in the gelatin and is sensible enough to visualize propagating stress perturbation generated at the surface. The shear modulus of the gelatin, G , is directly proportional to the shear wave velocity, v_s , via

$$G = \rho v_s^2, \quad (2.1)$$

where ρ is the gelatin's density (Lee et al., 2017). For a weak material like gelatin, these shear waves can be quite slow (~ 1 m/s) and are easy to track with a simple camera or the unaided eye. Finally, the Young's modulus, E , can be found via

$$E = 2G(1+\nu), \quad (2.2)$$

where ν is the Poisson's ratio of the medium.

Fig. 2.2. (a) A drawing of the setup to visualize shear waves. Light passes through polarizing filters, placed in front of and behind the experimental tank. Stresses perturbation within the gelatin become visible and, by gently exciting the gelatin surface, we can see shear waves propagating. (b,c) Two sequential images of a shear wave (dark region) propagating downward through a solid gelatin. The velocity is used to estimate the gelatin's Young's modulus.

We injected dikes from the base of the tank using an injection piece attached to a pump, which we used to supply the liquid. The piece contains a retractable blade inside, which form an initial precut (Fig. 2.3a), forcing the dikes into our preferred, vertical orientation. We set the balloon and injector position so that the dike's initial strike is oriented either radial (experiments 1 & 2), circumferential (experiments 3 & 4), oblique at a 45° angle (experiments 5 & 6) or coaxial (experiment 7) to the reservoir (Fig. 2.3b). For the injected liquid, we used vegetable oil mixed with chili powder (to provide pigment), which is less dense than gelatin and rises buoyantly. We generally injected constant volume dikes, in that we pumped in the oil with a moderately-fast influx over a period of a few minutes,

turned off the pump and allowed the dikes to ascend solely due to their buoyancy. Dikes extended in length and the tail region remained filled with the initial liquid. For two experiments, we had constant-flux conditions, in that we never stopped injecting oil, so the dike propagated primarily due to the driving pressure (from the pump) and less due to buoyancy.

Fig. 2.3. Schematic of our experimental setup. A balloon embedded in gelatin is pressurized and a pump injects oil. We record the experiments with cameras from different perspectives. (a) Our dike injection piece, shown attached to the bottom of an empty tank. It contains a sliding blade that makes an initial cut in the gelatin, into which oil is pumped. (b) Plan view showing the balloon and injector positioning and offset, for different dike orientations. The injector location is somewhat constrained and can be rotated in 90° increments, so we move the balloon to control distance and orientation (c) An embedded balloon in a gelatin is pressurized or depressurized via a vertical tube (here we show experiment 5, see Table 2.1 for experimental conditions). (d) The corresponding deviatoric stress field shown using polarized filters. The repeating rainbow pattern indicates the stress gradient and allows us to minimize boundary effects. (e) An apparatus allows us to construct a liquid reservoir without a balloon. Hot water is circulated into a balloon to re-melt a layer of surrounding gelatin. The balloon is deflated and surrounded with dyed water. (f) A photo showing the final cavity in a pressurized state (experiment 1).

To model the reservoir, we used two different setups, both of which achieved the primary goal of generating a stress field around the reservoir. For the first setup we embedded, from above, a simple latex balloon in the gelatin (while still warm and liquid), with a diameter of ~10 cm and with its center positioned ~15 cm below the surface. We accessed the balloon using a vertical tube (Fig. 2.3c), which we filled with water to apply hydrostatic pressure to the balloon and thereby maintain its diameter. After the gelatin solidified, the balloon could be further inflated or deflated (via the tube) to apply a compressive or extensional deviatoric stress to the system (Fig. 2.3d). With this setup, the balloon acts as a physical barrier, so we could not observe interchange of liquids between the dike and reservoir, but we could model the stress field around the reservoir. We used polarized film to visualize the extent of the stress distribution and minimize their

interaction with the tank walls, thereby mitigating boundary-effects. Stress visualization helped us ensure that we injected the dikes into a region of low stress, which allowed us to observe their behavior as they entered the stress field around the balloon. For the second setup, we wanted to be able to assess the fluid exchange between the reservoir and dike and so we designed a specific apparatus to construct a balloon-less reservoir. This piece allowed us to embed a pressurized balloon in the gelatin, then deflate it and replace it with dyed water (Fig. 2.3e). The liquid-filled cavity could then be further pressurized to perform the experiment (Fig. 2.3f). The setup is a bit complicated and so we explain it further in this chapter's appendix, Appendix D.2.1.

We recorded the experiments using typical cameras, which took a time lapse of photographs from different perspectives and allowed us to track the dikes' position and geometry (see Pansino & Taisne, 2018 for data repository). We took photos in 18 MP resolution at a frequency of 6 photos per minute, which was high relative to the dike velocity (generally <1 mm of propagation between photos).

2.2.2 Scaling

The scaling and analysis of this study rely on the deviatoric stresses in the medium, in which the magnitude at any radial distance from the center of reservoir, r , is a function of the reservoir's internal pressure and radius, respectively ΔP and r_{res} . This has components that are radial and tangential to the reservoir, respectively denoted as σ_{rr} and σ_{tt} :

$$\sigma_{rr}(r) = \Delta P (r_{\text{res}}/r)^3 \quad (2.3)$$

and

$$\sigma_{tt}(r) = -(1/2)\Delta P (r_{\text{res}}/r)^3. \quad (2.4)$$

This assumes the medium behaves as an infinitely-thick-walled, spherical pressure vessel (Segall, 2010). A pressurized reservoir exerts compressive forces on the medium in the radial direction, but places it in a state of extension in the tangential direction. A negative pressure contracts the medium, causing the reverse effect. The minimum compressive stress, σ_3 , which a dike opens against, is the lesser of these two components, so a nearby dike's preferred orientation (radial or tangential) depends on the sign of the reservoir pressure. The hoop stress (i.e. the σ_{tt} component) can have a secondary effect on radially-oriented dikes, either helping or hindering propagation along the nearest edge of the dike.

For comparison with natural systems, we scale the stress distribution against the fracture pressure of the dike, P_f , which quantifies the amount of pressure necessary for a dike to fracture the medium, and therefore propagate:

$$\sigma^* = |\sigma_3|/P_f. \quad (2.5)$$

The dimensionless stress, σ^* , essentially maintains that the magnitude of stress is proportional to the strength of the material. This is a similar approach as taken by Daniels

and Menand (2015), who scale regional extensional stresses against dike buoyancy forces; we will also incorporate buoyancy into the scaling. Following Lister and Kerr (1991),

$$P_f = K_c / L_b^{1/2} , \quad (2.6)$$

Menand and Tait (2002),

$$K_c = (2\gamma E)^{1/2} , \quad (2.7)$$

and Taisne and Tait (2009),

$$L_b = (K_c / \Delta\rho g)^{2/3} . \quad (2.8)$$

L_b is the buoyancy length, the minimum length a dike must surpass to buoyantly rise through the crust without any external driving pressure. K_c and γ are respectively the fracture toughness and surface energy of the medium (both material properties), $\Delta\rho$ is the fluid-medium density difference and g is gravity. Equations 2.6-2.8 can be combined to relate P_f to the medium's strength properties and the dike's buoyancy:

$$P_f = (2\gamma E \Delta\rho g)^{1/3} . \quad (2.9)$$

This quantifies the internal pressure in the upper, head region of head of a dike, which is useful for comparing the ratio of external to internal stress. In our experiments, we injected vegetable oil ($\sim 910 \text{ kg/m}^3$) into gelatins of 3.75 wt% concentration ($\sim 1010 \text{ kg/m}^3$, measured via a sample's mass and volume offset when submerged in water), leading to a $\Delta\rho$ of 100 kg/m^3 for all experiments (Table 2.1). We assume γ to be 1 J/m^2 (Kavanagh et al., 2013) and E ranges between 3.3 kPa and 5.3 kPa (derived from direct measurement of G). For ΔP , we estimate the reservoir pressure using its change of radius due to inflation or deflation, Δr_{res} . The strain associated to the reservoir expansion or contraction, is simply related to the stress at its outer margin via the Young's modulus, assuming small radius variation:

$$\Delta P = E \Delta r_{\text{res}} / r_{\text{res}} . \quad (2.10)$$

We label $\sigma^*(r)$ at the reservoir wall margin as ΔP^* :

$$\Delta P^* = \sigma^*(r_{\text{res}}) = \Delta P / P_f . \quad (2.11)$$

Under these conditions, $|\Delta P^*|$ ranges from 10^{-1} -10. This overlaps with natural systems, which are estimated to have a similar range of 10^0 - 10^3 at the reservoir boundary, neglecting viscoelastic effects around the reservoir and using values listed in Table 2.2. The maximum magnitude of σ^* is ΔP^* at the reservoir boundary and approaches 0 at large radii.

Table 2.1. Experimental conditions †

Chapter 2: How magmatic storage regions attract and repel propagating dikes

Parameter (units)	Exp 1	Exp 2	Exp 3	Exp 4	Exp 5	Exp 6	Exp 7
E (Pa)	4642	2750	5272	3424	3311	3719	3827
Gelatin Volume (L)	112	100	48	112	100	100	100
Cold Room Temperature (°C)	15	15	15	15	15	15	15
Cooling Time (days)	5	3	3	4	3	3	4
Dike Initial Orientation	Radial	Radial	Circum.	Circum.	Oblique	Oblique	Coaxial
$\Delta\rho$ (kg/m ³)	100	100	100	100	100	100	100
Reservoir Diameter (cm)	9.9	8.3	10.1	7.6	10.7	7.9	8.6
Reservoir Depth (cm)	16.6	8.6	8.8	16.7	14.9	14.3	13.1
Dike - Reservoir Horizontal Offset (cm)	16.3	10.3	11.0	16.1	14.1	14.1	0.0
P_f (Pa)	209	175	218	189	187	194	196
Injection Type‡	CV	CV	CF§	CV	CV	CV	CF
Volume or Rate (mL, mL/min)	120	50	4.25	100	72	144	20
Over/Under- Pressured	Over	Under	Over	Under	Over	Under	Under
ΔP (Pa)	124	-378	1079	-96	325	-545	-356
ΔP^*	0.6	-2.2	5.0	-0.5	1.7	-2.8	-1.8

† Symbols are E : Young's modulus, $\Delta\rho$: gelatin-dike density contrast, P_f : gelatin fracture pressure, ΔP : reservoir pressure, ΔP^* : dimensionless reservoir pressure ‡ Constant volume (CV) or constant flux (CF). § At this moment, we did not have good control over the driving pressure of the dike. We believe the pressure was fairly constant, but there was likely some fluctuation.

Table 2.2. Comparison of values between experiments and natural systems

Symbol (unit)	Meaning	Experiments†	Nature
ΔP (Pa)	reservoir pressure	10^2 - 10^3	10^5 - 10^8 ^A
E (Pa)	Young's modulus	~2500-5500	$10^{10.5}$
$\Delta\rho$ (kg/m ³)	medium-liquid density difference	100	100 ^B
γ (J/m ²)	surface energy	1 ^C	1 ^C
ν	Poisson's ratio	0.5	0.25
ΔP^*	dimensionless reservoir pressure	10^{-1} - 10^1	10^0 - 10^3

† All values in experiments column were measured in our lab, unless otherwise noted

A Karlstrom et al. (2009). B Taisne & Tait (2009). C Kavanagh et al. (2013).

2.3 Results

2.3.1 Qualitative observations

We note two classifications of dike response, which depend on the dike's initial orientation. Strike-radial dikes propagated vertically at the outset of each experiment and, as they approached the reservoir, began to propagate horizontally as well (Fig. 2.4a,b). We believe that the dikes took advantage of the hoop stresses around the reservoir, which encouraged lateral movement. For the over-pressurized experiment (experiment 1; Fig. 2.4a), the hoop stresses were extensional, which helped the dike propagate toward the reservoir, while the reverse occurred for the under-pressurized experiment (experiment 2; Fig. 2.4b). In the under-pressure experiment, the dike propagated horizontally away from the reservoir upon encountering its stress field. After the dike passed around the stress field, it resumed vertical propagation.

Fig. 2.4. Annotated photos (all in cross-sectional view) depicting different dike responses, depending in the initial dike orientation (each row) and reservoir pressure state (each column). Photos a, b, c, etc. correspond to experiments 1, 2, 3, etc. unless otherwise noted; for parameters, see Table 2.1. For scale, the reservoir in all photos is 10 (+/- 2) cm in diameter. We label the horizontal and vertical positions as x and z. Black dashed lines

outline the dikes and black arrows show their movement. Photo (a) shows an example from reservoir setup 2 (see section 2.2.1) while the others show setup 1. (a,b) Hoop stresses around the reservoir (shown by white dashed lines) encourage or inhibit dike opening, causing the dike to propagate laterally. (c,d) Experiment 3, over- and then under-pressurized. Dikes change orientation to open against σ_3 and the preferred path is traced by the dashed white lines. (e,f) Two views are presented to have a sense of the 3D geometry of oblique dikes. The stars indicate contact between the dike and balloon. The dikes rotate horizontally and vertically into their preferred orientations. (g) The overpressure scenario is omitted, since the dike would go straight into the reservoir. The dike initially curls into a concentric shell, but the driving pressure forces it to escape this path.

Experiment 1 was set up in a way that allowed us to assess how liquid exchange occurred between the dike and reservoir. We observed that the dike intersected with the reservoir and that the two immiscible fluids stratified due to density contrast. The dike allowed a relief of pressure in the reservoir (observed using polarized film) and afterwards continued propagating vertically toward the surface. We do not have pressure data for the dike to be able to quantify our results, but we do discuss these observations later in section 2.4.

Other experiments, with non-radial dikes had a second class of response, in which dikes curled and twisted into new orientations (Fig. 2.4c-g). They reoriented according to σ_3 , so for over-pressurized experiments (experiments 3 & 5; Fig. 2.4c,e), the dikes curled inward to propagate toward the reservoir. For under-pressurized experiments (experiments 4, 6, & 7; Fig. 2.4d, f, g), σ_3 is radial to the reservoir, so the dike reoriented to propagate tangentially around it. After these dikes passed around the reservoir, they began to escape its stress field and their paths transitioned from curved to linear. As a result, they never formed perfect shells around the reservoir; instead they took more of a tulip-flower-petal like shape. Both oblique and coaxial dikes changed orientation to become circumferential dikes.

In addition to these two classifications of dike response, we noticed that dikes generally propagate either toward or away from the reservoir, depending on the reservoir's pressure state (i.e. over- or under-pressurized). Over-pressurized reservoirs tend to attract dike, either due to hoop stresses or σ_3 orientation, and in each case the dike intersected with the reservoir. Conversely, under-pressurized reservoirs prevented dikes from approaching the reservoir and in each case the dike was forced horizontally away, to then erupt at the surface.

2.3.2 Quantifying dike response

We characterize dike response via visual analysis. As noted, radial dikes responded by propagating laterally, while other, non-radial dikes responded by changing orientations. We analyze these two responses separately. For experiments with non-radial dikes

(experiments 3-7), only a single photo taken at the end was necessary to quantify the results, since the shape of the dike records its history of interaction with the stress field (Fig. 2.5a). We traced its shape in an illustration software to assist with detection of the dike, which sometimes was obscured by features that would hinder automated dike tracking (e.g., polarized light fringes; dike overlapping the reservoir). We processed the output in Matlab to measure the dike's position as a function of depth, relative to the balloon reservoir (Fig. 2.5b). These dikes respond to reservoir stress by changing orientation, so we analyze the magnitude of orientation-change as a proxy for dike response (Fig. 2.5c), using the parameter $\partial^2 x / \partial z^2$, where x and z are respectively the horizontal and vertical components of the dike's position. In short, we quantify the location of each pixel in a photo of the dike and see how it compares with its neighboring pixels.

Fig. 2.5. (a) In some experiments, the dike was not easy to see (experiment 7). (b) In such cases, we manually traced the dike in an illustrating software (black region), which helped us to quantify its path in Matlab (yellow + signs). For dikes that were distinct from the background, we did no manual tracing. (c) For non-radial dikes, the response is quantified by how abruptly it changes orientation, via the parameter $\partial^2 x / \partial z^2$. (d) Sequential photos (overlapped) of a propagating dike (experiment 2). Two important points on the dike: that nearest to the reservoir center (red star) and the geometric center of where fracturation is occurring between sequential frames (marked by the green star). Also included is the center of the roughly-penny-shaped head region of the dike (blue star). (e) In Matlab, subtracting one photo from the next in a sequence highlights the differences between the two and illustrates where the dike was growing. The 'two important points' are

automatically located (green/red dots). (f) We analyze strike-radial dikes by tracking their change of horizontal position, represented by $\partial x/\partial z$.

For dikes with initially strike-radial orientations, we note that two locations on the dike are important to describe how the dikes behaved: the point nearest to the reservoir and a point that indicates where the dike fractured the medium and grows the crack's surface as it propagates (Fig. 2.5d). We used each frame of a time lapse to determine the location of the nearest point on the dike to the reservoir, and pairs of sequential frames to determine where the dike was in the process of fracturation. We analyzed these frame-pairs in Matlab and, by taking the difference between the two, we could highlight this crescent-shaped new surface (Fig. 2.5e). We calculated the geometric center to represent this area and track how it moves horizontally due to the reservoir's stress field, which we quantify using the parameter $\partial x/\partial z$ (Fig. 2.5f). The methodology for quantification of $\partial x/\partial z$ and $\partial^2 x/\partial z^2$ are explained in greater detail in this chapter's appendix, Appendix D.2.2.

To be able to compare our experiments with nature, we first normalize the radius, r , the x and z components of the radius and the dike breadth, B , by the reservoir radius, r_{res} , into dimensionless forms, respectively r^* , x^* , z^* , B^* :

$$r^* = r/r_{\text{res}} , \quad (2.12)$$

$$x^* = x/r_{\text{res}} , \quad (2.13)$$

$$z^* = z/r_{\text{res}} , \quad (2.14)$$

$$B^* = B/r_{\text{res}} . \quad (2.15)$$

We similarly use the dimensionless form of the dike response parameters, so instead of $\partial^2 x/\partial z^2$, we use $\partial^2 x^*/\partial z^{*2}$; $\partial x/\partial z$ is inherently dimensionless.

As non-radial dikes ascend from below, they change orientation as a function of σ^* (introduced in section 2.2.2). In the case of over-pressurized experiments, this is a straightforward, power-law relationship. In the case of under-pressurized reservoirs, when these dikes bypass the reservoir, they leave its stress field and transition from curved to linear trajectory. When this occurs, the change of orientation response, $\partial^2 x^*/\partial z^{*2}$, drops. The relationship between σ^* and $\partial^2 x^*/\partial z^{*2}$ for a dike entering the stress field is different from the relationship for one leaving the stress field. We therefore analyze data for the first relationship only, to be able to compare with over-pressurized experiments and to be able to characterize where a dike 'feels' the stress due to the reservoir. All non-radial experiments are plotted together (Fig. 2.6a) and form a common power-law trend:

$$\partial^2 x^*/\partial z^{*2} = 61.4\sigma^{*2.4} . \quad (2.16)$$

Fig. 2.6. (a) Results from the non-radial dikes. The rate at which the dike changes orientation, $\partial^2 x^* / \partial z^{*2}$, is a function of dimensionless stress, σ^* . We show this as a semi log plot since some points are a bit less than zero on the vertical axis. We include a best-fitting power-law curve (black, dashed line). (b) The stress difference between two points on a dike, $\sigma^*_{near} - \sigma^*_{frac}$, form a relationship with the degree to which it deflects horizontally, $\partial x / \partial z$. We again plot a best-fitting curve.

For the strike-radial dikes, we performed a similar analysis. These dikes remained vertically oriented and preferred to rise vertically, due to their buoyancy. They only deviated from vertical propagation because of the stress field due to the reservoir (Fig. 2.4a,b), as hoop stresses close to the reservoir are relatively high. In the under-pressurized experiments, the dike eventually passed around the stress field and continued to propagate vertically. Similar to non-radial dikes, we analyzed the response as the dike entered the stress field to compare the two experiments and quantify where the dike begins to ‘feel’ the stress due to the reservoir. As noted, we found that the dike’s response, $\partial x / \partial z$, is a function of the stress at two points on the dike’s surface: its nearest point to the reservoir, σ^*_{near} (Fig. 2.5d, red star), and the geometric center of where fracturation is occurring, σ^*_{frac} (Fig. 2.5d, green star). When we view its behavior against the difference in stress between these two points, a power law relationship emerges (Fig. 2.6b):

$$\partial x / \partial z = 1.73(\sigma^*_{near} - \sigma^*_{frac})^{0.69}. \quad (2.17)$$

2.3.3 Quantifying the effective radius of interaction

At this moment, we can describe the degree to which a dike deflects as a function of the stresses in the crust. Dikes that feel very little stress deflect to a very small degree. We want to understand how close a dike needs to be to a reservoir before it is significantly

affected by the stress field and the radius at which this occurs we define as a critical radius, r_{crit}^* .

2.3.3.1 Non-radial dikes

For circumferential, oblique, and coaxial orientations, dikes respond to the medium stresses by changing orientation, which becomes pronounced as they approach σ^* of 0.1 (Fig. 2.6a). This magnitude of stress well-matches results by Maccafferi et al. (2019), who studied dike response to edifice loading and found that dikes change orientation when $\sigma^* \geq 0.09$. We therefore define this value as a critical stress, σ_{crit}^* , which informs us of the location of r_{crit}^* . For σ^* at the reservoir's wall, ΔP^* , we use equations 2.3, 2.5 and 2.12 to derive:

$$r_{crit}^* \sim (\Delta P^* / \sigma_{crit}^*)^{1/3}. \quad (2.18)$$

For $\sigma_{crit}^* = 0.1$, the critical radius is therefore a function of only ΔP^* . In section 2.4.1, we will convert these dimensionless findings into plausible values for nature.

2.3.3.2 Radial dikes

We perform a similar analysis to scale up the results from the radially-oriented dikes (Fig. 2.4a,b). In section 2.3.2, we noted that the degree to which a dike is horizontally-deflected depends on the stress difference between nearest point on the dike (Fig. 2.5d, red star) and a point at the geometric center of newly created surface (Fig. 2.5d, green star). For a dike whose center (Fig. 2.5d, blue star) is located at x_{cent} and z_{cent} , assuming a penny-shaped crack, the dimensionless stress of the nearest point, σ_{near}^* , is

$$\sigma_{near}^* = \Delta P^* / [r^*(x_{cent}, z_{cent}) - B^*/2]^3. \quad (2.19)$$

The geometric center of where fracturation is occurring, located at x_{frac} and z_{frac} , does not have a set location; we only know that it is somewhere on the dike. However, we can easily calculate the stress, σ_{frac}^* , for any point on the dike:

$$\sigma_{frac}^* = \Delta P^* / r^*(x_{frac}, z_{frac})^3. \quad (2.20)$$

With σ_{near}^* and σ_{frac}^* , we can estimate $\partial x / \partial z$ at any point, via equation 2.17. The deflection of the dike is also defined as the degree to which the center of the crack moves horizontally as it ascends:

$$\partial x / \partial z = (x_{frac} - x_{cent}) / (z_{frac} - z_{cent}). \quad (2.21)$$

When combining equations 2.17, 2.18, 2.20 and 2.21, no analytical solution can be derived for the location of x_{frac} and z_{frac} . Instead, we numerically model its location in Matlab by finding the point which best allows equations 2.17 and 2.21 to be equal, for a corresponding initial location of the dike center (Fig. 2.5d, blue star), as function of ΔP^* and B^* (some solutions are shown in Fig. 2.7). In this way, for any point at x_{cent} and z_{cent} , we locate the point at x_{frac} and z_{frac} , estimate σ_{near}^* and σ_{frac}^* , via equations 2.19 and 2.20,

and finally estimate $\partial x/\partial z$ using either equation 2.17 or 2.21. This methodology allows us to map out $\partial x/\partial z$ for any ΔP^* and B^* (Fig. 2.7). It is worth noting that in the negative ΔP case, x_{frac} is further away than x_{cent} , since dikes deflect away from the reservoir, while the reverse is true for positive ΔP ; even so, the stress difference tends to be dominated by σ_{near}^* , so the $\partial x/\partial z$ maps for both $-\Delta P$ and $+\Delta P$ look similar.

Fig. 2.7. Maps of $\partial x/\partial z$ (color bar) for different dimensionless dike sizes, B^* , and dimensionless reservoir pressures, ΔP^* . These are generated numerically using Matlab, by finding the location of x_{frac} and z_{frac} that allows equations 2.17 and 2.21 to be equal, for any location of the dike center, then displaying the corresponding value of $\partial x/\partial z$. We define that the dike begins to respond where $\partial x/\partial z > 10^{-1}$ (which we set as the lower limit of the color bar). The horizontal and vertical axes are dimensionless positions with the reservoir center at (0,0). The white region in the upper, left corner represents the locations where the dike touches the reservoir and is therefore larger for large-breadth dikes (right column). Deflection is larger at higher pressures (bottom row).

To find the r_{crit}^* for these radial dikes, we define a critical deflection at $\partial x/\partial z = 0.1$, in which the dike deflects horizontally one meter for every ten meters it ascends; we choose this value since it is at the lower limit of our experimental observations (Fig. 2.6b). The ΔP^* and B^* parameters affect r_{crit}^* , so we attempt to define an empirical relationship between the three, so that we can know if the dike is within the r_{crit}^* , without having to consider its nearest point or the geometric center of new surface creation. We consider

B^* on the range of 10^{-1} - 10^1 , to analyze dikes much smaller and much bigger than the reservoir. They follow a power law, such that $r_{crit}^* = a\sigma^{*b}B^{*c}$, so we find a best-fitting surface, for the variables a , b , and c , which allows us to make an estimate of critical radius for a range of pressures and dike size (Fig. 2.8). We will scale this information to natural systems in the next section.

$$r_{crit}^* = 2.87\Delta P^{*0.24}B^{*0.15}. \quad (2.22)$$

Fig. 2.8. A surface showing at what distance strike-radial dikes begin to respond, r_{crit}^* (color bar), against dike size, B^* , and reservoir pressure, ΔP^* . Over- and under-pressure conditions yield similar r_{crit}^* .

2.4 Discussion

2.4.1 Dimensions of the critical radius

We scale the dimensionless results presented above for plausible natural conditions, using equations 2.18 and 2.22 respectively for non-radial and radial dikes. For these equations, r_{crit}^* can be described as functions of ΔP^* and B^* . We performed an analysis for $10^0 < \Delta P^* < 10^3$, which correspond to ΔP of 10^5 - 10^8 Pa, assuming values of E and Δp listed in Table 2.2 and therefore a fracture pressure magnitude of 10^5 Pa. The parameters

r_{crit}^* and B^* are proportional to the reservoir's radius and can be easily converted back to dimensions via equations 2.12 and 2.15. In this example we consider the dike's horizontal breadth to be 1 km or 10 km. We therefore view the critical radius against reservoir radius, reservoir pressure, and, for radial dikes, dike breadth (Fig. 2.9). For example, a one kilometer radius reservoir will likely affect dikes within a 2-20 km radius, depending on the reservoir pressure, numbers on the same order of magnitude as previous numerical models (Karlstrom et al., 2009).

Fig. 2.9. We estimate the radius, r_{crit} , at which dikes begin to deflect, for varying reservoir radius, r_{res} , and pressure (color shading) at natural scale. (a) For non-radial dikes, estimated via equation 2.18. Line shading represent estimates for different reservoir pressures. (b) For radial dikes via equation 2.22. Shadings and pressures are as in (a). Dashed and solid lines represent horizontal dike breadths of 1 and 10 km, respectively.

2.4.2 Spherical nature of the reservoir

We model the magma storage region as a spherical reservoir because it is practical to construct in the lab using a balloon and, although the balloon reservoir in our experiments didn't always deform perfectly uniformly (probably due to heterogeneous strength of the balloon material; see Fig. 2.4b), it models a simple, spherical (or nearly-spherical) stress field. It should be noted however, that the shape of the reservoir controls the shape of the surrounding stress field and, in turn, how dikes will propagate in its vicinity.

Unfortunately, we arrive to the problem that no one really knows the geometry of an active magma storage region. Field evidence of extinct systems indicates that intrusive bodies are built incrementally by layering of sills (Menand et al., 2010; Menand, 2011; Barnett & Gudmundsson, 2014). Ground deformation data however, imply that reservoirs commonly undergo inflation when pressurized, which at the surface can be modeled as a

point expansion (Pascal et al., 2014). Whether the actual form of a reservoir is roughly elliptical or is a stack of hydraulically connected sills and dikes remains open to debate. Gudmundsson (2012) points out that elliptical reservoirs are thermally stable, while protrusions (dikes, sills) extending from such a body are prone to solidification. We feel that a hydraulically-connected network of dikes and sills, when pressurized, may exert a stress field on the crust that, at a distance, can be approximated to a point expansion. At any rate, the stresses due to inflation are evident on the earth's surface, so we can be sure that they exist elsewhere in the crust as well and that dikes will be encouraged to deflect wherever they are present.

It is likely that aspect ratio of a natural magmatic reservoir dictates the degree to which our findings are applicable. Highly-elongated (e.g. very flat, very tall and narrow) reservoirs will generate a stress field that is fundamentally different to what we model and dikes would therefore have a different response from what we observed. We expect low to moderate aspect ratio reservoirs to affect dikes in a similar fashion to our study. Moderately-oblate ellipses concentrate stress on their out horizontal margins (Gudmundsson, 2006; Gudmundsson, 2012), which should affect dike further away than a spherical geometry. Conversely, prolate ellipsoids concentrate stress on the vertical margins of the reservoir, meaning dikes would be affected only when nearer.

2.4.3 Relation to various volcanic behavior

2.4.3.1 Reservoir-avoiding dikes

Our findings that dikes can propagate around a reservoir (Fig. 2.4d,f) relate to some observations made in nature. Anderson (1937) documented outward dipping, circumferential dikes at Ardnamurchan dike swarm and hypothesized their orientation was due to an under-pressurized reservoir, though he modeled the dikes as originating from the reservoir (ring dikes), rather than below. Magee et al. (2012) proposed that those dikes indeed flowed around the reservoir (albeit laterally), supplied from regional dikes. At the Mt. Somma scarp (next to Vesuvius), Porreca et al. (2006) made similar observations of dikes that have a variety of orientations, ranging from circumferential to oblique to radial and determine that these orientations may have been a result of the reservoir pressure (over- or under-pressured) at the time of emplacement. In either example, we would expect a dike originating from below the reservoir (passing around it) to be mafic in composition. This seems to be the case at the Mt. Somma scarp, in which exposed dikes and lavas are more mafic than many of the other products (both Plinian and inter-Plinian) in the locality (Ayuso et al., 1998).

We can attempt to characterize the reservoir pressure in these systems *ex post facto*. As we observed in the experiments, dikes with relatively low driving pressure conformed to the external stresses by deflecting around the reservoir at an approximately-constant radius (Fig. 2.4d). When they reached to the same depth as the reservoir, they began sub-vertical ascent towards the surface. For the circumferential dikes at Ardnamurchan and at

the Mt. Somma scarp, we will assume their radii from the reservoir characterizes that at which they were affected by the reservoir stress. As observed in our results, dikes react to an external stress when $\sigma^* \sim 0.1$. We will therefore use equation 2.18 to solve for the value of ΔP^* , such that $\Delta P^* \sim 0.1r^{*3}$, and, by estimating the driving pressure in the dike via equation 2.9 (using values from Table 2.2) estimate the magnitude of ΔP .

At Mt. Somma, the dikes tend to be between ~ 1 and 2 km from the inferred position of the edifice (Porreca et al., 2006). Assuming the reservoir at the time of emplacement was a bit smaller, ~ 800 m in radius, then $1.25 < r^* < 1.88$ and so the reservoir pressure was $0.20 < \Delta P^* < 0.66$, implying the reservoir was weakly under-pressured. If we estimate the pressure the dike internal pressure is on the order of 40 kPa, then the associated reservoir under-pressure was 7 to 26 kPa, indeed quite low.

At Ardnamurchan, dikes are located between 1 and 8 km (though most are from 3 to 5 km) from the magmatic center (Magee et al., 2012), which is elliptical and respectively has a short and long axis radii of 3 and 5 km. If we consider the most-distant dikes (8 km away) and assume the reservoir has an effective radius of 4 km, then $r^* \sim 2$ and so $\Delta P^* \sim 0.80$. If we estimate the driving pressure in the dikes was similar to those at Mt. Somma (40 kPa), then $\Delta P \sim 31$ kPa, again quite low.

These estimates are reliant on a number of assumptions, particularly the driving pressure in the dike. Dikes that propagate from a shallow magma reservoir to the surface are commonly modeled to have driving pressures on the order of 10^6 to 10^8 Pa. If we assume that the dikes at Ardnamurchan and Mt. Somma were driven by pressures on that order of magnitude, then the reservoir under-pressure would had to have been similarly stronger.

2.4.3.2 Eruptive cycles

We can continue to consider the effects of under-pressure reservoirs in the context of observations made by Takada (1997), who documents a cyclic pattern at several volcanoes, including Fuji, Etna, and Mauna Loa. Takada finds that flank vents following a major eruption tend to be located far away from the central vent and that subsequent events become progressively nearer as the central edifice approaches another period of activity, which occurs on a time frame of 400-1000 years. An eruption deflates the reservoir to some degree and places the surrounding crust in a state of extension, which inhibits dike propagation close to the reservoir. In this case, an ascending dike will likely be deflected away from the volcanic center and could then erupt as a flank eruption (Fig. 2.10). Further eruptions sourced from the reservoir would be unlikely until either the stresses in the crust relaxed, thus allowing dikes to penetrate the magma storage region, or magma somehow managed to enter the reservoir and re-pressurize it (i.e. magma supplied to the reservoir by a stable, long-lived conduit). As the reservoir re-pressurizes, the extensional stresses weaken and the deflection of dikes becomes less pronounced. This pattern of behavior is difficult to explain by assuming the dikes are sourced from an

over-pressurized reservoir. After a major eruption, a volcano would not have the high internal pressure necessary to drive a dike far away.

Fig. 2.10. Proposed mechanism for the cyclic pattern of flank eruptions, driven by reservoir depressurization. Summit eruptions depressurize the reservoir and place the crust in a state of extension, which deflects dikes away. As the stress decreases, either due to repressurization or stress relaxation, the deflection becomes less pronounced. Eventually dikes are no longer deflected and the cycle can repeat.

If we assume the crust behaves like a Maxwell material, the stress surrounding the reservoir relaxes with time, t , such that

$$\sigma(t) = 2 G \epsilon_0 \exp(-G t/\eta) = [\Delta P/(1+\nu)] \exp(-G t/\eta), \quad (2.23)$$

where, ϵ_0 is the initial strain and G and η are the crust's shear modulus and viscosity (Segall, 2010). This assumes the initial strain is linearly related to the reservoir pressure, $\epsilon_0 \approx \Delta P/E$, since the material behaves elastically on this short initial time scale, and that $G = E/2(1+\nu)$. Using equations 2.18 and 2.22, we estimate the r_{crit}^* as a function of time and the initial pressure (Fig. 2.11), for values of $\eta = 1020 \text{ Pa}\cdot\text{s}$ (Giuseppe et al., 2009) and $G = 30 \text{ GPa}$. The initial magnitude of r_{crit}^* depends strongly on the initial pressure, but the long-term value of r_{crit}^* becomes small on the order of one hundred years for any initial pressure. This has some agreement with Takada's (1997) findings, in which dikes cycle through this relaxation/pressurization pattern on the order of hundreds to one thousand years.

Fig. 2.11. Time evolution of the critical radius of non-radial dike deflection due to viscoelastic relaxation. Shading represents different initial magnitudes of reservoir pressure. The initial stress relaxes with time, so the critical radius diminishes.

2.4.3.3 Eruption triggering

An important observation from our experiments is that dikes that breach a magmatic reservoir do not necessarily cease propagation (Fig. 2.12a, b). In experiment 1, the dike was attracted to the over-pressurized reservoir and, after it made contact, the dike continued to propagate. In our experiment, the two immiscible liquids stratified by density and the reservoir liquid (neutrally buoyant in gelatin) provided no buoyant force to continue propagation; therefore propagation was driven by buoyancy of the oil (the liquid filling the dike) and by the slight over-pressure of the reservoir. In a natural analogue, the magma in a reservoir would be equally- or more-buoyant than the magma in a mafic dike. It could therefore flow inside of the dike and take advantage of the existing fracture to propagate towards the surface (Fig. 2.12c).

Fig. 2.12. (a,b) An example from experiment 1 (cross-sectional view), showing that dike propagation may continue after a dike intersects a reservoir. (c) A model for the extraction of viscous reservoir magma into a basaltic dike. The basalt lubricates and propagates the fracture, while the over-pressure and buoyancy of the reservoir magma open the fracture wide.

This has implications for eruption triggering, in that a basaltic dike may help to open a pathway along which more-evolved magmas from a shallow reservoir can migrate. Takeuchi (2004) shows that some cataclysmic eruptions of very viscous magma were preceded by precursory eruptions of magma generally a few orders of magnitude less-viscous. It is conceivable that such eruptions are initiated by deeply-sourced basaltic dikes which, upon breaching the reservoir, encourage eruption via a combination of mechanisms: (1) remobilization of the residing magma (Murphy et al., 2000; Ruth et al., 2016); (2) providing a low-viscosity lubricating layer inside of the dike, which the viscous evolved magma slides along (Watts et al., 1999); (3) providing a large fracture, which requires less pressure to open widely than a small fracture (Rubin, 1995).

2.4.4 Driving pressure

We note that our results have been influenced to some degree by the dikes' driving pressures, which is especially obvious in experiment 7 (Fig. 2.4g). Dikes with a significant internal pressure seem to propagate according to their own internally-derived local stress field (i.e. they propagate along a straight path) and only alter course when affected by a significant external stress (e.g., reservoir stresses, regional stresses). As they escape the external stress field, they continue propagating straight. This behavior is also exhibited to a lesser degree for other experiments that had an under-pressurized reservoir. It seems that this balance between internal and external pressure sources may dictate the degree to which a dike deviates from its quasi-static path.

We account for internal pressure to some degree in the experiment scaling, in that σ^* is scaled by the dike's fracturation pressure (which itself is a function of buoyancy and the Young's modulus), however the internal pressure can be much higher than the fracturation pressure, which is likely the case in a constant-flux experiment (experiment 7). Constant-volume experiments should have an internal pressure close to the fracturation pressure. Dahm (2000b) estimates the pressure, P_i , in a constant-volume dike

via a similar methodology to ours, which sums the critical fracture pressure (equation 2.6) and buoyancy pressure. In their model, the entire dike contributes to fracturation, such that $P_i = K_c/L + \Delta\rho gL$, rather than just the head region, which we assume maintains a relatively constant length. The pressure estimated in this way is dependent on the dike length and is effectively double to triple our estimates, for dikes of length 5 to 40 cm (recall in our tank, dikes are not longer than the gelatin medium thickness, 39 cm). Estimating the internal pressure in this way is likely most-accurate for small dikes, which grow radially and do not have a defined head or tail region. As the dikes grow and the tail becomes relatively long, we expect this formulation would over-estimate the pressure. Similarly to our analysis, this would not fully-recover the pressure in constant-flux dikes.

2.4.5 Limitations and future work

As noted previously in this thesis, analogue experiments come with inherent simplifications, which are imposed by design to analyze the effects of one or two influencing factors on dike propagation. We have already described some of the key factors which may influence dike propagation in nature (e.g. layering of the crust, the competition between internal driving pressure and external stress).

One factor we have neglected, but may be pertinent to this study, is the reservoir depth. Throughout our experiments, we constructed reservoirs that were similar in diameter (from 7 to 11 cm; see Table 2.1) and had a bit of variation in depth (from 8 to 17 cm); these experiments had diameter to depth ratios of 0.9 to 2.2. This was largely due to space constraints, in which we wanted to construct a reservoir of similar size to our dikes (which tend to be 10-15 cm in horizontal breadth, for oil-filled dikes) and leave sufficient space for a dike to propagate toward, and potentially around, it. However, using our current equipment, it would be possible to explore the effect of depth, by injecting air-filled dikes. Such dikes tend to be smaller (~5 cm) than the oil-filled dikes we presented here. Maintaining a similar dike to reservoir size ratio, we could therefore construct smaller reservoirs that have a wider range of diameter to depth ratios (up to a magnitude of 6 or so). This would yield information on dikes that deflect away from an under-pressured reservoir, in that we would be able to observe how they behave when they leave the reservoir's stress field.

A second factor we have neglected is the liquid rheology. We used oil as the magma analogue for all of the experiments, since it has a moderate buoyancy and viscosity in comparison to other easily-accessible liquids. However, since we did not vary the liquid between experiments, we cannot observe how buoyancy and viscosity contribute to the dike's behavior. We expect that more-buoyant liquids have a stronger internal pressure and are therefore less-affected by the external stress field. Similarly, we expect that higher-viscosity liquids propagate more-slowly and adjust to the external stresses relatively abruptly. It is known that more-viscous liquids produce thicker dikes (Wada, 1994), which place large stresses on the surrounding medium, to a degree proportional to

the thickness to length aspect ratio. It is possible that the interaction between stresses due to the dike and those due to the reservoir may lead to unforeseen behavior.

2.5 Conclusions

We present an investigation on how dikes respond to the stress field around a pressurized magma storage region and found two classifications of response. Non-radial dikes re-orient to open against σ_3 and therefore propagate either radially or tangentially to an over- or under-pressurized reservoir, respectively. Radial dikes do not change orientation, but rather propagate laterally at their fringes due to the hoop stresses around the reservoir. Over-pressurized reservoirs generate extensional hoop stresses, which help a dike to propagate toward the reservoir, while the reverse occurs for an under-pressurized reservoir.

These two classes of interaction behave as functions of the stress distribution due to the reservoir and, in both cases, the higher the stress, the more pronounced the reaction. For non-radial dikes, it is a function of the stress at the dike's vertical leading tip. For radial dikes, the stress has a more complex role and we find that lateral propagation functions according to the stress difference between two points: the point nearest to the reservoir and a point at the center of where fracturation is occurring as it propagates.

Dikes that ascend near a reservoir are likely to cause volcanic crises, regardless of the state of pressure. By quantifying a volcano's reservoir pressure however, we can anticipate whether dike will propagate toward or away from it and therefore whether it is likely to cause an eruption near to, or far from, the central vent; information that is vital for hazards management purposes. The degree to which we accurately estimate competing stress sources, that is, the driving pressure of a dike, the local stresses due to the reservoir and broader regional stresses, will dictate our ability forecast where dikes are likely to erupt.

2.6 Chapter Notation

Table 2.3. Definitions of Symbols Used in this Study.

symbols	units	meaning
<u>crust rheology parameters</u>		
G	Pa	shear modulus
E	Pa	Young's modulus
ν		Poisson's ratio
K_c	Pa m ^{1/2}	fracture toughness
γ	J/m ²	surface energy

Chapter 2: How magmatic storage regions attract and repel propagating dikes

η	Pa s	crust viscosity
v_s	m/s	shear wave velocity
<u>geometry & propagation parameters</u>		
x	m	horizontal position (relative to reservoir)
z	m	vertical position (relative to reservoir)
r	m	radius to reservoir center
$x/z/r_{cent}$	m	position of dike center (penny shape assumed)
$x/z/r_{near}$	m	nearest point on dike to the reservoir
$x/z/r_{frac}$	m	geometric center of where fracturation is occurring
r_{res}	m	reservoir radius
r_{crit}	m	distance at which a dike deflects due to reservoir
$x/z/r^*$		normalized by the reservoir radius
$\partial x/\partial z$		magnitude of horizontal propagation, per unit of vertical propagation
$\partial^2 x/\partial z^2$	1/m	magnitude of orientation change, per unit of vertical propagation
L_b	m	dike length needed for buoyant propagation
B	m	dike horizontal breadth
B^*		breadth normalized by reservoir radius
t	s	time
<u>pressure & stress parameters</u>		
$\Delta\rho$	kg/m ³	dike-medium density contrast
g	m/s ²	gravity
ΔP	Pa	reservoir pressure
ΔP^*		dimensionless reservoir pressure
P_f	Pa	dike internal pressure needed for fracturation
P_i	Pa	dike internal pressure, including fracturation and buoyancy
$\sigma_{rr,tt}(r)$	Pa	stress due to reservoir (radial/tangential components)
$\sigma_3(r)$	Pa	minimum principal stress
$\sigma^*(r)$		dimensionless stress due to reservoir
$\sigma^*_{near,frac}$		dimensionless stress at r_{near}, r_{frac}
ϵ_0		initial strain on crust due to reservoir pressure

Segue to chapter 3

The experiments in chapter 2 describe how a dike can be encouraged to propagate towards a reservoir, which is favorable when the reservoir is pressurized. Conversely, if a reservoir is depressurized, magma is deflected away until the stresses in the crust relax or until it is re-pressurized. This leads to a question... how does a reservoir pressurize, or re-pressurize, in the first place?

One possibility is that a gradual influx of magma, via a long-lived supply conduit, pressurizes the system, and so we explore the conditions under which such a structure can form and persist. We can deduce that subterranean conduits (i.e. below a volcanic edifice) are secondary geometric features, in the sense that they develop from some other initial geometry, such as a dike. For example, fissure eruptions are commonly observed to evolve from an extended, linear geometry into one or several vents. The corresponding supply pathway likely evolves from the planar geometry of an initial dike to one or several branching cylindrical geometries. However it is not clear if the cylindrical forms extend all the way to the reservoir.

If a shallow conduit can develop to allow magma to exit a reservoir, can a deeper one develop to allow magma to enter? Under what conditions would this supply conduit solidify and close? In the next chapter, we attempt to constrain this supply geometry from the hypothesis that this is a function of cooling and solidification. We want to see if we can develop a lab-scale conduit from an initial dike and determine under what conditions that path is favorable.

Chapter 3: Dike channelization and solidification: time scale controls on the geometry and placement of magma migration pathways

Abstract

Our team investigates the conditions under which magma prefers to flow via a dike or a conduit-like structure. To address this, we perform a series of analogue experiments, repeatedly injecting warm, liquid gelatin, into a cold, solid gelatin medium and allowing the structure to evolve with time. We vary the liquid flux and the time interval between discrete injections of gelatin.

The time interval controls the geometry of the migration, in that long intervals (Fourier number, $Fo > 10^2$) allow the intrusions to solidify, favoring the propagation of new dikes. Short time intervals ($Fo < 10^0$) allow the magma to channelize into a 'conduit'. Between these time scales, a transitional behavior exists, in which new dikes nest inside of previous dikes. The flux controls the distance a dike can propagate before solidifying, in that high fluxes (dimensionless flux, $\Phi > 10^0$) favor propagation to the surface and eruption, whereas low fluxes ($\Phi < 10^{-1}$) favor intrusion. Again, within this range, a transitional behavior exist, in which dikes may erupt at the surface in an unstable, pulse-like fashion.

We supplement the experimental findings with a 2D numerical model of thermal conduction to characterize the temperature gradient in the crust as a function of intrusion recurrence frequency. We find that for very infrequent intrusions ($Fo > 10^4$ to 10^5) all thermal energy due to the intrusion is lost, while more-frequent intrusions allow heat to build up nearby.

3.1 Introduction

Fissure eruptions have been documented to evolve with time, from initially-elongated fissures to centralized vents (Wylie et al., 1999; Keating et al., 2008; Jones et al., 2017). The change of geometry from planar to cylindrical reflects the easiest form of magma transport; a planar form is most efficient for propagation through a brittle, elastic medium; a cylindrical form is the most thermally efficient (Keating et al., 2008). Dikes which supply such eruptions may also evolve with time at depth, as they have a high ratio of surface area to volume and therefore are very vulnerable to cooling and, eventually, solidification. Cooling is heterogeneous along the dike surface, in that regions that become cool also become more viscous, encouraging flow toward the hotter regions of the dike, which therefore remain warm and active. As they transition to cylindrical geometries, the surface area and heat loss drops allowing thermally-efficiently magma

transport to the surface (Fukushima et al., 2010). For example, an eruption on Heimaey, Iceland began as a 1.5 km long fissure, which focused to a single central vent over a period of a day (Wylie et al., 1999). Similarly, exposed shallow plumbing systems at monogenetic volcanoes in the southwestern USA display conduit structures, which likely formed due to excavation, then infill, of the surrounding crust (Keating et al., 2008).

There is an element of time necessary to achieve such a geometry, since channelization results from the prolonged effusion of magma. Dikes that do not develop a focused vent remain planar during solidification. In some cases, dikes even become narrower at their centers, as they evacuate magma and deflate while their lateral fringes solidify and preserve their initial, active state (Daniels et al., 2012). The necessary time scale for flow channelization is not immediately clear; Keating et al. (2008) suggest that the size of the eruption may be important, in that small eruptions can focus on a scale of hours, while larger eruptions do so more gradually.

3.1.1 Hot spot volcanoes settings

The channelization of fissure eruptions into focused vents has been documented in a number of settings, primarily characterized by basaltic intrusions. Fissure eruptions commonly occur in the rift zones of hot spot volcanoes like Kilauea and Piton de la Fournaise, in which dike intrusions tend to be linked to deformation of the larger edifices and therefore preferentially occur in certain regions (Michon et al., 2015; Jones et al., 2017). Propagating dikes (and after the fact, solidified swarms) have been imaged via geophysical data at both of these volcanoes and in some cases have been observed to evolve into centralized vents (Owens et al., 2000; Leslie et al., 2004; Fukushima et al., 2010; Jones et al., 2017). External forces (i.e. local and regional stresses) clearly guide dikes to intrude in such zones periodically, but magma migration may also take the form of long-lived channels, which must exist to allow structures like Pu'u O'o vent to persist (Patrick et al., 2019). These small rift zones can therefore support both persistent magma flow and more-voluminous, intermittent dike propagation events.

3.1.2 Monogenetic volcanic field settings

If hot spot volcanoes represent semi-continuously active settings, then their opposite would be infrequently-erupting volcanic fields. Such regions can have eruption recurrence frequencies of one eruption per 10^4 yr to 10^5 yr, but can periodically increase in activity to one eruption per 10^3 yr or less (Valentine & Connor, 2015). Since these events are so infrequent, cooling dominates the system and intrusions have time to thoroughly solidify. Subsequent intrusions must create new paths via dike propagation. It has been shown that dike swarms have natural spacing between dikes, as each one induces a compressive stress on the surrounding crust, which inhibits later dikes from propagating too close; the final spacing between dikes is energetically optimized (Bunger et al., 2013). As such, the infrequent timing of events causes dikes (and their eruptive products) to be spatially

separated. It should be noted that volcanic fields have relatively small long-term outputs of lava and are, at their most voluminous, similar to arc-related stratovolcanoes ($10^2 \text{ km}^3/\text{Myr}$) but can be orders of magnitude less (Valentine & Connor, 2015). Even though this type of setting is characterized by low magma output, with long repose times between events, a single event can display dike flow channelization, similar to the other examples we previously introduced (Keating et al., 2008). Volcanic fields, along with hot spot volcano rift zones, demonstrate that the geometry of magma transport depends on the magma flux, the duration of the injection and the time interval between injections. These parameters dictate if a dike can develop into a conduit-like pathway and, depending on the repose time, whether a subsequent pulse needs to propagate a new dike.

3.1.3 Experimental and numerical studies and theoretical framework

In this study, we assume that heat loss from a dike occurs primarily via conduction (into the country rock) and advection (magma erupting from the dike). In engineering literature, conduction is commonly simplified to a 1-D problem for finite geometries that are very tall and wide in comparison to their thickness (Çengel, 2008a); this conveniently describes a dike's geometry. Since a dike has a high ratio of surface area to volume, it is very susceptible to heat loss. However, when a dike erupts, magma flows upward through the crust and cool as it approaches the surface. In this case, heat loss from the dike is split between conduction into the crust and magma flow onto the surface.

In the context of volcanism, experimental studies on dike solidification can be subdivided into two categories: the ability of a dike or sill to propagate into a cold medium (Fialko & Rubin, 1998; Taisne & Tait, 2011; Chanceaux & Menand, 2014) and the stability of an erupting dike (Bruce & Huppert 1989; Wylie & Lister, 1995). Since dikes have a large surface area, they can potentially come to arrest due to solidification before propagating a significant distance; the distance a dike can propagate before solidifying is termed the "thermal entry" length (Fialko & Rubin, 1998). The thermal entry length in fact is a combination of the cooling time and propagation velocity, in that a dike undergoing 1-D heat conduction into the surrounding medium cools with a characteristic thermal diffusion time scale, t_d , in which

$$t_d = H^2/\alpha, \quad (3.1)$$

where H is the dike thickness and α is the crust's thermal diffusivity (Fialko & Rubin, 1998). Note that the value of H in this equation is for an idealized slab of uniform thickness, whereas dikes in nature have a thickness distribution (Daniels et al., 2012); we discuss how we quantify the value in our experiments later, in section 3.3.2. The thermal diffusion time scale is also characterized by the Fourier number, Fo , which is a measure of the degree to which heat has transferred from a warm region into a cooler region over time, t , (Çengel, 2008b):

$$Fo = t\alpha/H^2 . \quad (3.2)$$

The thermal diffusion time scale in equation 3.1 therefore follows when $Fo=1$. Note that the thermal diffusivity relates to other material properties, the thermal conductivity, k , density, ρ , and specific heat capacity, C_p :

$$\alpha = k/(\rho C_p) . \quad (3.3)$$

A material with a high α cools off quickly. For a dike driven by excess pressure, ΔP , the velocity depends on the balance between pressure and viscous forces, P_v , such that

$$\Delta P = P_v = \mu v L / H^2 , \quad (3.4)$$

where μ is the magma viscosity, v is velocity and L is propagation length (Fialko & Rubin, 1998; Taisne et al., 2011). Assuming that the thermal entry length, L_{cool} , relates to the velocity and cooling time, such that $L_{cool} = vt$, then the length can be estimated and depends primarily on the dike's thickness:

$$L_{cool} = H^2 (\Delta P / \alpha \mu)^{1/2} . \quad (3.5)$$

It stands to reason that a warmer dike can propagate farther and so equation 3.5 is modified (Fialko & Rubin, 1998):

$$L_{cool} = \lambda H^2 (\Delta P / \alpha \mu)^{1/2} , \quad (3.6)$$

where the λ modifier is a function of two dimensionless parameters, the dimensionless solidus temperature, Θ , and the Stefan number, S , and is solved for numerically:

$$\Theta = (T_s - T_\infty) / (T_0 - T_\infty) , \quad (3.7)$$

$$S = \gamma / C_p (T_0 - T_\infty) , \quad (3.8)$$

and

$$\lambda = \exp(-\lambda^2) / \pi^{1/2} S [\{\Theta / \text{erfc}(-\lambda)\} - \{(1-\Theta) / \text{erfc}(\lambda)\}] . \quad (3.9)$$

The temperatures T_s , T_∞ and T_0 are respectively those of the solidus, the ambient host rock and the magmatic source (Fialko & Rubin, 1998), γ is the specific latent heat and the erfc is the error function.

Taisne & Tait (2011) show through analogue experiments that, while solidification can stop propagation, further influx of magma into the center of the dike can cause it to inflate and resume propagation. They injected paraffin wax into gelatin and observed that the dike's chilled margins provided sufficient insulation to maintain flow in the dike's core. The propagation dynamic varied between no propagation, step-wise propagation and continuous propagation, as a function of Θ and the dimensionless flux, Φ . This second parameter is a ratio of heating via influx and cooling via conduction:

$$\Phi = QH / \alpha L B , \quad (3.10)$$

for volumetric flux, Q , and dike horizontal breadth, B . The propagation of a dike becomes less affected by thermal conditions (i.e. continuous propagation) for very hot magmas or very high fluxes, while low fluxes of cold magmas may not yield any propagation at all. For transitional conditions, dikes undergo step-wise propagation, in which an arrested dike needs to inflate and build pressure to resume propagation.

Chanceaux & Menand (2014) made similar experiments focusing on sill formation. They show that a dike, arriving at the interface between two layers of the crust, can deflect into a sill under certain temperature and flux conditions. These conditions coincide with step-wise propagation described by Taisne & Tait (2011), whereas dikes in the 'continuous propagation' regime tend to pass through the interface and continue propagating vertically. Considering these two studies together from a thermal perspective, it is clear that dikes that are very hot and have a very high flux propagate in a fast and stable way, whereas magma closer to the solidus, flowing at a lower flux, is prone to unstable, step-wise propagation or solidification.

For a dike that can propagate to the surface, its form of eruption is inherently unstable, in that magma flow has a tendency to focus from a sheet of lava into one or a few vents over time. Wylie et al. (1999) show through numerical modeling that a magma with temperature-dependent viscosity develops viscosity contrasts within a dike, which are the dominant mechanism causing flow channelization into a discrete vent. The viscosity contrasts cause the flow to channelize on a timescale of ~ 10 hr, consistent with the timescale observed in the 1973 fissure eruption on Heimaey, Iceland.

The size of an erupting vent evolves as a function of magma flux; if the flux is 'high', the vent walls will melt and the dike widens until the supply is exhausted, but if the supply is 'low', the magma will solidify and block the vent (Bruce & Huppert, 1989). In a 2-D framework, Bruce & Huppert (1989) numerically modeled this fluctuation and found it to be a function of the dike's initial width. Over some critical size on the order of meters, a basaltic dike driven by a constant overpressure can continue to migrate and erupt until its source has emptied.

When the surrounding crust melts due to magma heat and flux, it can be thermally eroded away, which results in a conduit that is wider than the initial dike. As alluded to above, such geometries have been observed in volcanic fields in the southwestern USA, in which excavation of the surrounding crust formed a conduit (Keating et al., 2008). In such locations, narrow, basal feeder dikes (which are now exposed due to land erosion) can be observed to transition to shallower, wider conduits. To the best of our knowledge, this process has not yet been quantified in analogue experiments. When it occurs, the conduit becomes more thermally stable, as the ratio of volume to surface increases (Holmes-Cerfon & Whitehead, 2011).

3.1.4 Our contribution

We present below a series of analogue experiments investigating how dikes solidify and evolve with subsequent influxes of magma. We repeatedly injected hot, liquid gelatin in to a cold, solid gelatin medium, varying the flow rate and the time between discrete injections. We then identify the conditions under which dikes can channelize into conduit-like structures, how they remain stable and what types of structures develop otherwise. By quantifying such behaviors, we can gain insights of how the rate and frequency of pulses of magma may shape the underlying plumbing system of a volcano.

3.2 Methods

3.2.1 Experimental setup

We performed a series of analogue experiments, in which we repeatedly injected warm, liquid gelatin into a solid gelatin medium (both made from the same granular gelatin, 250 bloom, 8-15 mesh, Xiamen Yasin brand), housed in a 50x50x50 cm³ cubic tank. We made five injections per experiment, each 500 mL in volume, providing sufficient volume to propagate up through the gelatin and erupt. Between experiments, we varied only the magnitude of flux and the time interval between injections. Below the tank, we attached a small 10 cm tall, 10 cm diameter cylindrical reservoir, which contained a copper heating coil and maintained a constant temperature of the liquid injection (Fig. 3.1a).

Fig. 3.1. (a) A schematic of our experimental setup. A peristaltic pump draws liquid gelatin from a supply beaker to our tank apparatus. A thermal bath keeps the system at a constant temperature of 32 °C. We constructed water-filled free surfaces on both sides of the tank to minimize the stress build up in the medium due to repeated intrusions. (b)

Gelatin-supply tubing is braided with thermal bath tubing and then insulated to keep the gelatin warm while in transit. (c) Prior to the first injection of an experiment, we make an initial cut using a retractable blade to ensure that the first dike propagates vertically. Subsequent injections are not controlled in this way.

We chose gelatin as an analogue for both the solid medium and the injected liquid because it is very versatile. Gelatin is commonly used as an analogue for the Earth's crust because its strength can be controlled by varying the concentration during preparation and because it is transparent, so we can observe what occurs inside the medium (or a dike intruded behind another dike). Additionally, it behaves like the Earth's upper crust, in that it is a brittle, elastic material on the timescale of a propagating dike and a viscoelastic material on a longer time scale (Giuseppe et al., 2009; Kavanagh et al., 2013). As an injected liquid, gelatin conveniently and quickly solidifies at the experimental temperature, so we can observe flow paths that develop as a dike cools heterogeneously.

We prepared each experiment in a similar fashion (see Table 3.1 for experimental conditions), in which the gelatin medium was either 3.5 wt% or 4.0 wt% of gelatin and the dike was either 4.0 wt% or 4.57 wt% of gelatin (maintaining a consistent concentration ratio). The high concentration of the dike, relative to the medium, ensured that a solidified dike was stronger than the surrounding medium, preventing subsequent injections from following the previous dike's path. Since liquid gelatin is not very buoyant in solid gelatin, we added 5.0 wt% to 6.0 wt% of salt or sugar to the host medium to increase its density and thus make the dikes positively buoyant. We used one material or the other out of convenience, however we noted that salt had a tendency to weaken the gelatin to a degree and so we measured relatively lower values of the Young's modulus (see Table 3.1). To measure the medium density, we cut out a sample after performing an experiment and measured its mass on a scale, followed by its volume by immersing it in water. We estimated the density contrast, $\Delta\rho$, assuming the liquid gelatin had a density similar to water, 1000 kg/m^3 . In reality, the liquid gelatin density varies a bit as a function of temperature and measurements in our lab indicate it ranges from 995 to 1005 kg/m^3 . We added food dye to the injected gelatin for pigment, to help distinguish the dikes from the medium and give a sense of where they are thicker (darker looking) or thinner (lighter looking). The concentration of the food dye was low and did not otherwise affect the liquid's properties. Before running an experiment, we measured the Young's modulus, E , via the methodology described in Appendix B.

Table 3.1. Experimental conditions. From left to right are the gelatin concentration of the liquid and solid medium, additive salt/sugar concentration, Young's modulus, flux rate, time interval and density difference.

Exp	$C_{l,gel}$	$C_{s,gel}$	$C_{s,sug}$	E	Q	t_{int}	$\Delta\rho$
#	[wt%]	[wt%]	[wt%]	[Pa]	[mL/min]	[s]	[kg/m ³]

1	4.0	3.5	Salt, 6	1550	12	2	80
2	4.57	4.0	Sugar, 6	4050	12	210	80
3	4.0	3.5	Salt, 5	1590	12	20700	70
4	4.0	3.5	Salt, 5	1920	1.2	2	70
5	4.0	3.5	Salt, 5	2480	170	2	70
6	4.0	3.5	Salt, 5	4570	3.8	210	70
7	4.0	3.5	Sugar, 5	3990	38	210	70
8	4.0	3.5	Sugar, 5	4110	12	2070	70
9	4.0	3.5	Sugar, 5	4580	38	2070	70
10	4.0	3.5	Sugar, 5	6650	3.8	20700	70
11	4.0	3.5	Sugar, 5	3850	38	6600	70

For the magma analogue, we injected liquid gelatin at a temperature of 31°C to 33°C, marginally hotter than the melting temperature of 28 °C, which allows it to quickly cool as it is injected into the solid medium. As far as we can tell, it has a Newtonian rheology with a temperature-dependent viscosity. We maintained the temperature of our gelatin supply using a thermal bath into which we submerged our supply beaker. The thermal bath also circulates heated water through flexible tubes, which we attach to the basal, cylindrical reservoir's copper heating coil (Fig. 3.1a). We also threaded these tubes with the gelatin supply tube, to maintain its temperature between the supply beaker and the experimental tank (Fig. 3.1b). When we were ready to start an experiment, we made an initial cut into the gelatin using a retractable blade (Fig. 3.1c), to force the dike to be vertically oriented, and perpendicular to the visualization system. After making this cut, we removed the blade and pumped liquid gelatin into the cut, which then grew into a dike. We controlled the injection's flux, duration and time interval, using a programmable peristaltic pump. The flux range achievable with our current set-up ranges from 0.8 ml/min to 170 ml/min.

These experiments had the potential to intrude a lot of material into the solid medium, which would build up large strains on the medium. We attempted to mitigate strain accumulation by constructing vertical free surfaces, filled with water, on opposing ends of the tank. As a dike is intruded and pushed the medium apart laterally, the free surfaces flexed outward to accommodate the strain. To prepare these surfaces, we embedded vertical plastic sheets, 1 cm in thickness, during preparation of the gelatin medium. After

the medium cooled and solidified, we removed the sheets and replaced them with water. However, gelatin became firmly attached to the sheets during solidification, so we cut them out in order to remove them without badly damaging the gelatin. We adhered a metal wire to the plastic sheet prior to solidification and pulled it upward to make a smooth, continuous cut along the interface between the gelatin and the sheet. The cut was never perfectly flat and had a slightly undulating form. Removal of the sheets tended to leave the water filled boundaries littered with fragments of gelatin, which impeded our view of the dikes. We therefore flushed the boundaries with fresh water until these fragments were sufficiently cleaned out.

We recorded each experiment using a set of cameras positioned to look at the dike's broad side (Fig. 3.1a, the x-z plane). Each time an injection erupted and each time an injection terminated, we took several photos. We processed each photo against an initial reference photo to determine the geometry of the dike as a whole. We also processed each photo against the previous injection to see the change between injections, in order to determine which regions of the dike were actively flowing and which had solidified.

3.2.2 Scaling

For the scaling of these experiments we consider two primary factors, the time interval between pulses of activity and the volumetric flux of magma during a pulse, and to a lesser degree, the temperature of the dike. For the time interval we scale our experiments simply using a dimensionless time term, the Fourier number, Fo , shown in equation 3.2 (Çengel, 2008b). This is a measure of the amount of heat lost from a heated slab via conduction versus what is stored, assuming one-dimensional heat transfer; *i.e.* the slab is much taller and broader than it is thick, so heat transfer is perpendicular to and homogenous along its surface. For $Fo \ll 1$, much of the initial heat remains in the slab and the system is thermally heterogeneous. For $Fo \gg 1$, heat has been conducted away and the system become thermally homogeneous (see Table 3.2 for a comparison between experiments and nature). Values for Fo on the order of 10^2 or 10^3 are fundamentally no different than $Fo = \infty$, since in both cases, the dike has completely cooled down and solidified.

For the magnitude of flux, we use a dimensionless flux term, Φ , estimated following Taisne and Tait (2011), which balances heat into the system via influx and advection of heat out of the system via conduction (equation 3.10). If Φ is large, the system is dominated by heating, while if Φ is small, the system is dominated by cooling. This is inherently related to the dimensionless time duration of the injection, Fo_{inj} , such that

$$\Phi = QH/\alpha LB = QH^2/\alpha V = H^2/\alpha t_{inj} = 1/Fo_{inj} . \quad (3.11)$$

Here, t_{inj} is the injection duration and V is the volume of the dike, assuming $V \approx LBH \approx Qt_{inj}$.

The relationship between flux and time poses a question for the experiment: do we hold flux constant and test the time and thus volume of an injection or do we do the reverse and test the flux of an injection, holding volume constant? A good argument can be made for either approach. We would argue that by varying the flux, we can observe the effect of driving pressure on the geometry of the plumbing system that develops in the medium. In such a way, high driving pressures may favor dikes with relatively large dimensions, which grow both vertically long and horizontally broad, while small driving pressures favor dikes that grow primarily vertically.

We thermally scale the experiments following Delaney and Pollard (1982) and Taisne and Tait (2011), via the dimensionless solidus temperature, Θ , shown in equation 3.7. The injection is above or below the solidus respectively when $\Theta < 1$ or $\Theta > 1$. Delaney and Pollard (1982) make the argument that magma is probably injected at temperatures near the liquidus, such that in nature $0.9 < \Theta < 0.95$. For a gelatin melting point of 28 °C (melting is rapid, so we assume this characterizes the solidus and liquidus temperature) and ambient temperature of 15°C, we therefore choose injection temperatures of ~32°C. In practice, the temperature in our setup fluctuated by a few degrees as liquid flow through the system. As described in the previous section, we configured experimental setup to minimize this fluctuation and keep the system thermally stable.

The parameters Fo and Φ are highly dependent on the geometry of the intrusion, in that for a broad, thin dike, Fo may initially be large and Φ small. If the flow in the dike channelizes into a conduit-like feature, the parameters could evolve so that Fo decreases and Φ increases. When we show the scaling for these parameters, we show ranges encompassing both dike flow and conduit flow.

Note that in nature, the thermal evolution of a dike is dependent on another dimensionless parameter, the Stefan number (equation 3.8), which is a ratio of the available latent heat to sensible heat in a system. The proportion of latent heat in a basaltic magma can be large and represent the majority of available enthalpy (Morse, 2011). In the context of our experiments, this term is neglected because the latent heat release from our gelatin is relatively small, such that $S \ll 1$ (see Appendix C).

Table 3.2. Values for both our analogue experiments and basaltic dikes in nature.

<i>Parameter</i> <i>[units]</i>	Meaning	Experiments	Nature (Basalt)
α [m^2/s]	Thermal diffusivity (crust)	7.4×10^{-8} ^k	10^{-6} ^a
t_{int} [s]	Time between injections	10^0 - 10^4	10^6 * - 10^{11} ^b
H [m]	Dike opening thickness	10^{-3} - 10^{-2}	10^{-1} - 10^1 ^{c,d}
L [m]	Dike vertical length	0.27-0.38	10^3 - 10^4
B [m]	Dike horizontal breadth	0.08-0.31	10^2 - 10^4 ^{c,d}
Q [m^3/s]	Magma flow rate	10^{-8} - 10^{-6}	10^{-1} - 10^3 ^{d,e}
V [m^3]	Dike volume	10^{-4} - 10^{-3}	10^6 - 10^8 ^{c,d,f,g}
T_0 [$^{\circ}C$]	Dike initial temperature	30-32	-

T_{∞} [°C]	Ambient temperature	15-17	<500
T_s [°C]	Solidus temperature	28	1200-1300 ^h
Fo	Fourier number	10^{-4} - 10^3	10^{-2} - 10^7
Φ	Flux parameter	10^{-2} - 10^1	10^{-3} - 10^2
Θ	Temperature parameter	0.76-1.00	0.9-0.95 ⁱ
S	Stefan number	0.04 ^k	~ 4 ^j

a Douglas et al. (2016); b Molloy et al. (2009); c Keating et al. (2008); d Wylie et al. (1999); e Bunger et al. (2013); f Wnuk & Wauthier (2017); g Wauthier et al. (2015); h Green & Falloon (2015); i Taisne & Tait (2011); j Morse (2011); k This study (appendix C)

* Assuming a lower bound on the order of weeks

3.3 Results

3.3.1 Time-dependent behaviors & flux-dependent behaviors

Each experiment began with an initial dike and the systems evolved as a function of the time interval between liquid injections. For experiments with a short time interval (seconds to minutes), the flow in the dike quickly channelized into a narrow conduit (Fig. 3.2a), which tended to be elliptical in cross section, a few centimeters across and approximately half a centimeter thick. We were not able to detect any clear sign of thermal erosion of the gelatin medium due to flow through the conduit, even for the highest flux; this is probably due to the low injection temperature we used. Experiments with a moderate time interval (30-60 minutes) had enough time for the dike to partially solidify. Such dikes were still soft at the start of the next injection, which resulted in a new dike propagating inside the original dike (Fig. 3.2b). We assume that, since the liquid migrated in the form of a dike, the previous injection was indeed solid; had it been liquid, we expect the flow would have instead formed a conduit. We will refer to this as ‘nested dike’ behavior. For experiments with a very long time interval (3+ hours), the dike had time to completely solidify. Subsequent injections created new dikes, leading to a dike swarm being generated (Fig. 3.2c,d). Sequentially emplaced dikes did not follow an obvious pattern, other than that they tended to avoid the previous dike(s). Their pattern is likely due to the stresses in the medium, in that each dike imposed a localized stress on the surrounding medium, so that their spacing is energetically optimized, as discussed in the section 3.1 (Bunger et al., 2013). For example, in Fig. 3.2d, the large dike closest to the bottom of the photo was emplaced first, followed by the right, then left, then smaller, central dikes.

Fig. 3.2. The effect of repose time, which yields different types of intrusions. Each experiment begins with a single dike. (a) Conduit formation (experiment 7). For experiments with a short repose time (on the order of seconds to minutes), the dike heterogeneously solidifies and maintains a central liquid conduit (highlighted by dashed line). (b) Nested dike formation (experiment 8). For experiments with a moderate repose time (~ 30 minutes), the initial dike partially solidifies. It is still soft at the time of the next injection, so the new dike forms inside of the original dike. (c,d) Two different views for dike swarm formation (experiment 3). For a long repose time (on the order of hours), the initial dike completely solidifies. The subsequent injection is forced to make a new dike, which solidifies, leading to a collection of dikes.

The flux seemed to have less control on the form of the system (*i.e.* conduit vs. dike swarm formation) and more control on the amount of material that remained solidified in the gelatin medium. Fluxes that were sufficiently high ($>10 \text{ mL/min}$) allowed the dike to propagate to the surface and begin erupting (Fig. 3.3a), which continued steadily until the injection finished. Very low flux experiments ($\sim 1 \text{ mL/min}$) could not propagate quickly enough to reach the surface before solidifying. In such cases, intrusion restarted at the

base of the experiment and propagated alongside (in the same plane) the previous failed dike lobe (Fig. 3.3c). This created a fingering appearance, with each lobe originating from the same source location. As the complex dike became wide and full of lobes, the liquid began to flow inside the older lobes, widening them, without significantly advancing the dike front (Fig. 3.3d). We were also able to detect a transitional behavior at semi-low flow rates (~ 3 mL/min), which formed a larger-scale version of the fingering behavior. Large dike lobes sometimes propagated all the way to the surface (other times no) and erupted in an unsteady fashion, flowing for tens of minutes before solidifying and forcing propagation to begin again at the base of the tank (Fig. 3.3b). In such cases, it seems as though the viscous forces in the increasingly cold dike eventually overwhelmed the flux and propagation of a new dike became the easiest path for the liquid.

Fig. 3.3. The effect of flux, which controls the ability of a dike to erupt. (a) Sufficiently-high flux (>10 mL/min) allows the dike to reach the surface before solidifying (experiment 7). (b) A transitional behavior at semi-low flux (3.8 mL/min), in which the dike can reach the surface, but is unstable (experiment 10). Eruptions can last for tens of minutes, but eventually solidify and force intrusion to begin again in a new dike. (c,d) Images showing two stages of dike propagation (both from experiment 4), for experiments with low flux

(1.2 mL/min). The dike cannot propagate to the surface before solidifying, so new lobes begin to propagate again from the base. The complex dike eventually grows wide and thick, but struggles to reach the surface.

Note that when dikes erupted, they expelled a large proportion of their volume onto the surface, similar to results presented by Kavanagh et al. (2018), who used PIV visualization to show the same behavior. However, liquid along the lateral fringes of the dike tended to solidify, so that the expelled liquid originated from the center of the dike. This can be seen in Fig. 3.2a, in which the left and right sides of the dike appear somewhat dark. Ignoring the conduit that developed later, the post-eruptive cross-sectional shape was thicker towards the lateral fringes and thinner toward the middle, which matches field observations of magmatic dike, made by Daniels et al. (2012), who were able to measure the thickness distribution of basaltic and kimberlitic dikes. In our experiments (Fig. 3.2a), it is possible to detect the darker central conduit, flanked by two lighter regions representing the deflated dike, which are in turn flanked by darker regions of solidified gelatin. The space between the thick, lateral regions is relatively broad in comparison with the conduit.

3.3.2 Image processing

Our dimensionless analysis is highly dependent on the geometry of the intrusion. To quantify this, we used image processing techniques to identify the thickness distribution and surface area of each intrusion, for each experiment. These techniques generally rely on comparing an image of interest with some reference image. Results from such measurements are presented in Table 3.3.

3.3.2.1 Dike thickness distribution

To estimate the dike's opening thickness, we use the Beer-Lambert law, which relates the light transmission through a diffusing medium to its thickness (Taisne et al., 2011):

$$H = -\ln(I/I_0)V/\sum[-\ln(I/I_0)A_{px}] . \quad (3.12)$$

The light transmitted through a dike, I , relative to a reference photo taken before the dike is intruded, I_0 , decreases logarithmically as the thickness increases. For a pixel of area, A_{px} , summing the $-\ln(I/I_0)A_{px}$ term yields a value that is proportional to the actual volume. Scaling this value with the known volume, V , yields the width distribution of the dike. Via this method, we can estimate the thickness of a single dike or of a dike that channelized into a conduit. This technique provides us with a thickness distribution, however we characterize the dike thickness via the median value. For a typical dike, the histogram of the thickness distribution shows a high frequency of values for the thicker part of a dike and a low frequency for the thinner part. For an example of injection 1 from experiment 7, we measured a median thickness of 4.6 mm, mean thickness of 4.8 mm and modal

thickness of 6.1 mm. We chose the median value as it is representative of the whole dike and is less-sensitive to outlier measurements than the mean.

For nested dike behavior, using the Beer-Lambert law can result in misleading thickness estimates, since it identifies all gelatin (both solidified and liquid) in the estimate. To determine the thickness distribution of the liquid region of each injection, we employ the Beer-Lambert law for two separate times, for one photograph taken during the injection and for one taken before the injection. Doing so yields a thickness distribution of all of the intruded gelatin, both liquid and solid, H_{total} , and one of only the solidified portion that remained in the medium, H_{solid} . The difference between the two reveals the thickness distribution of the liquid region, $H = H_{total} - H_{solid}$. Using these photo pairs, we could determine the geometry change between each subsequent intrusion, after the first, assuming no remobilization of the solidified gelatin (Fig. 3.4). If such dikes had remobilized the solidified gelatin, we expect the post-experiment photos would reflect gelatin loss, but in fact they appear to contain the same amount or more gelatin.

Fig. 3.4. To estimate the thickness of a dike intruded inside of a previous dike, we use the Beer-Lambert law. We find the volume of intruded gelatin before and during an injection via the expression $-\ln(I/I_0)$ and the difference between the two yields the thickness distribution of the liquid region. We set a threshold value to automatically distinguish the active dike from the remaining gelatin, after which the surface area can be easily estimated by the number of pixels and the area of a pixel.

For multiple layers of intrusions (*i.e.* dike swarm behavior), the Beer-Lambert law fails entirely, as the previous dikes pollute the image and complicate our estimation of the active dike's thickness. Employing the Beer-Lambert law for multiple dikes tends to yield nonsensical results, which take the form of sudden thickness differences where dikes overlap. Since we can still estimate the surface area, A_s , of such dikes to a high degree of confidence and we know the volume injected, we simply estimate the average thickness via $H = V/A_s$. This methodology works well for individual dikes. Using the example of injection 1 of experiment 7 again, this produces an estimate thickness of 4.6 mm (recall the median and mean thickness were respectively 4.6 and 4.8 mm).

3.3.2.2 Surface area of the active region of an intrusion

As each experiment progresses, the intrusion evolves to become more complex. We are primarily interested in the active region of the intrusion, whether it takes the form of a dike or conduit. For dike swarm experiments and nested dike experiments, the active region is the newest dike (or embedded dike), so we could simply compare a photo from one injection, I_i , to the previous, I_{i-1} , to highlight where the newest dike intruded. Viewing the ratio I_i/I_{i-1} in Matlab makes the new dike apparent and its surface area can be estimated, $A_s = A_{px} * N$, for a dike identified in N pixels. In practice, we did this simultaneously with the thickness distributions, by defining a threshold degree of change. The pixels with significant change indicated the active surface area of the dike, while those that were lower than the threshold indicated the inactive regions (Fig. 3.4).

For conduit-forming experiments, comparing a photo from one injection with the previous tends to reveal little change, since the conduit maintains a stable geometry. We manually masked the photos of conduit-forming experiments to highlight the conduit shape and, by processing the masked image in Matlab, identify its surface area and thickness.

Table 3.3. Dike dimension measurements. From left to right are the median thickness, length and surface area of the active region of the dike, the total intruded volume, as well as the form of the active intrusion. Each pair of values indicate the initial (left) and final (right) measurements. The vertical length does not change between intrusions in an experiment. The form references types of intrusions shown in Figures 2 and 3.

Exp	H (cm)	L (cm)	A_s (cm ²)	V (L)	Form
1	0.57 → 0.78	31.8	552 → 52	0.33 → 0.50	Conduit
2	0.60 → 0.88	32.5	685 → 57	0.40 → 0.47	Conduit
3	0.58 → 0.11	35.2	590 → 326	0.34 → 0.98	Dike swarm
4	0.75 → 2.05	39.4	692 → 777	0.50 → 1.96	No eruption
5	0.68 → 0.67	34.8	205 → 341	0.18 → 0.95	Unstable
6	0.48 → 0.65	34.1	877 → 107	0.40 → 0.44	Conduit
7	0.54 → 0.16	33.1	536 → 127	0.29 → 0.42	Nested
8	0.13 → 0.10	35.5	241 → 261	0.10 → 0.13	Nested
9	0.43 → 0.61	36.4	542 → 481	0.24 → 0.42	Dike swarm
10	0.20 → 0.07	21.7	475 → 248	0.09 → 0.17	Dike swarm

3.3.3 Dimensionless analysis

To perform our dimensionless analysis, we employ the time parameter, Fo , and the flux parameter, Φ , introduced in section 3.1. The dike thickness distributions determined via image analysis provide us with a range of values, so we take the median thickness as

described in the previous section. We use the horizontal aspect ratio, Ψ , (opening thickness to the median horizontal breadth) of the actively flowing region of the intrusion to characterize what kind of feature it is:

$$\Psi = H/B . \quad (3.13)$$

Cylindrical conduits therefore have low aspect ratios close to 1, while dikes have aspect ratios that approach 0. Note that the Ψ shares H/B terms with Φ and, in order to eliminate like terms, we plot a form of the dimensionless flux, such that $\Phi/\Psi = Q/(\alpha L)$. By plotting Ψ against Fo and $Q/(\alpha L)$ (Fig. 3.5), we can see that the intrusion shape depends on both parameters. To do so, we plot an empirical, best-fitting surface of the form of $\Psi = C_1 Fo^{C_2} [Q/(\alpha L)]^{C_3}$, and solve for the values of C_1 , C_2 and C_3 by minimizing the sum of the square of the residuals (in log scale) between measurements and the model surface of Ψ . The equation of the surface (shown in Fig. 3.5a) indicates that Ψ is dependent on both parameters, in that conduits need both a moderate flux and frequent intrusion to remain active. For intrusions that have sufficient flux to be able to erupt, conduits tend to develop for $Fo < 1$ and $Q/(\alpha L) > 10$, unstable conduits form (and then solidify) for $Q/(\alpha L) \sim 0.3$, nested dike behavior occurs for $Fo \sim 10$ and finally dike swarms occur for $Fo \gg 10$.

Fig. 3.5. The aspect ratio, Ψ , against dimensionless time, Fo , and a form of the dimensionless flux, $\Phi/\Psi = Q/(\alpha L)$. The parameters Φ and Ψ share an H/B term, which we eliminate to show the independent effect of flux. Colors correspond to experiments that were conduit-forming (red), unstable-flow (magenta), nested dike (green), dike swarms (blue) and finally dikes that did not erupt (blue x). We do not use non-eruptive dikes to analyze the aspect ratio, since they do not match eruptive data and lay visibly below the

trend in (b). (a) The parameters are plotted in 3D. A best-fitting surface is applied to the data to see the relative contribution of Fo and $Q/(\alpha L)$. (b-d) 2D plots showing how the parameters compare to each other.

We also explore the effect of these two parameters on the amount of material intruded into the crust. For each 500 mL injection, we measured the volume of material that erupted, V_{erupt} and simply analyze the intruded volume ratio, V^* , such that

$$V^* = (500 - V_{erupt})/500 . \quad (3.14)$$

Plotting V^* against Fo and Φ and applying another best-fitting surface as done for Figure 5, we see that the intruded volume strongly depends on the flux (Fig. 3.6). This makes sense, in that a low flux allows more time for the intruded liquid to cool and solidify inside the medium. The weak dependence on Fo stems from the plumbing evolution of each type of behavior. For example, the value of Fo did not change much throughout conduit-forming experiments, even though they progressively extruded more and more liquid as the conduit developed into a stable form. For nested dike experiments by contrast, each intrusion was thinner than the previous, causing the corresponding values of Fo to increase and V^* to decrease (they were thin, so less volume intruded). Similarly, for dike swarms, the first dikes to intrude could freely deform the surrounding medium, resulting in greater thicknesses, lower values of Fo and greater values of V^* . Subsequent dikes interacted with the local stresses due to the previous ones and therefore were relatively thin.

Fig. 3.6. How dimensionless time, Fo , and dimensionless flux, Φ , affect the ratio of intruded volume, V^ . Markings are as in Fig. 3.4. (a) The parameters are plotted in 3D and a best-fitting surface is shown. The intruded volume is almost entirely a function of Φ . (b-d) The parameters are shown against each other in 2D.*

Regarding the best-fitting surfaces shown in Fig. 3.5a and 3.6a, we plot these to determine the degree to which Fo and Φ , or alternately $Q/(\alpha L)$, have an effect on ψ and V^* ; however for various reasons we hesitate to use them as predictive tools and therefore do not further discuss them in this article. For example, the relationship between Fo , Φ and V^* is not linear. As we noted, the amount of intruded material decreased as each experiment proceeded, so in Fig. 3.6b, each category of experiment has a negative trend between Fo and V^* . However, between categories, there is a positive trend; conduit experiment intruded less material than nested dikes, which intruded less than dike swarms. To forecast the value of V^* , one would require a complete knowledge of the stress and temperature distribution in the medium, as well as the dynamics of the intrusion. Similarly for ψ , we cannot make any predictions as functions of Fo and $Q/(\alpha L)$, since the aspect ratio in gelatin experiments is different from natural magmatic dikes due to the difference in the Poisson's ratio, ν ; values are 10^{-2} to 10^{-1} for gelatin ($\nu=0.5$) and 10^{-4} to 10^{-3} for in nature ($\nu=0.25$ to 0.3 ; Rivalta et al., 2005; Kavanagh et al., 2013).

What we can do is quantitatively distinguish between types of behavior. For example, conduit-forming experiments had $Fo < 1$, so we can broadly say that liquid migrates via a conduit geometry when the time interval is short. Dike swarms by contrast had $Fo > 10^2$. Similarly, experiments that were able to make stable eruptions had $Q/(\alpha L) > \sim 10$, whereas those that arrested due to solidification had $Q/(\alpha L) < \sim 1$.

3.4 Discussion

3.4.1 Channelization due to solidification or due to preexisting flow paths

It is important to consider that dikes have complex internal flow, which may affect how heat is distributed throughout the dike. Kavanagh et al. (2018) identified several stages of flow within a dike including: (1) initial injection and radial growth of a penny-shaped crack, (2) growth and internal circulation of the magma and (3) eruption and ejection of magma from the dike. The assumption that magma flows uni-directionally away from the source may not be true during the propagation stage of a dike and instead may only apply for syn-eruptive, channelized flow. In this context, it is possible that, during dike propagation, a central upward jet provides heat to the center of a dike, whilst cooler magma circulates downward along the lateral edges, imposing a thermal distribution that favors channelization (see Chapter 4 results for figures of a dike's internal flow field).

One way to check for this would be by measuring a fissure eruption's temperature distribution (a time series). Kavanagh et al. (2018) show that, prior to eruption, dikes can have an internal magma circulation, consisting of an upward central jet and lateral downward flow. As we described in section 3.3.1, as a dike breaches the surface, it contracts and squeezes much of the liquid upward onto the surface. Assuming, the pre-eruptive circulation described by Kavanagh et al. (2018) imposes a thermal distribution that is warmer towards the center and cooler towards the sides, the distribution would be reflected in the initial stage of a fissure eruption (Fig. 3.7). The fissure would initially emit magma from the upward central jet, followed by the remaining somewhat-cooler magma. As the eruption continues and channelizes, the temperature would increase to reflect the magma's direct path from depth to the surface. To the best of our knowledge, an initial drop in temperature following the onset of a fissure eruption has not yet been investigated, however the progressive increase in lava temperature as a fissure localizes has been shown at the Holuhraun lava field associated to the 2014 eruption of Bardarbunga volcano (Aufaristama et al., 2018).

Fig. 3.7. A schematic view for the temperature evolution of a fissure eruption. The top panels represent the surface lava flow temperature and the bottom panels represent the dike temperatures. The temperature initially drops between time 1 and time 2, as the dike first evacuates magma from a warmer, central region, followed by the cooler lateral regions. As the dike channelizes, magma flows from depth to the surface relatively quickly, so the eruptive temperature increases.

3.4.2 Plumbing region distribution

Perhaps the most important discussion from the results shown in section 3.3.1 is that of how a plumbing system initiates and is distributed, which is a rapidly advancing topic of research (Cashman & Sparks, 2013; Edmonds et al., 2019). A key finding is the effect of

the frequency of magma flux on its form of migration. When there is a long repose time between magma pulses, migration takes the form of dike propagation. For very infrequent influx of magma, heat can entirely conduct away from any intrusion. In such a case, there is no preferred path (thermally-speaking) along which a dike will propagate. Sporadic intrusions of magma would be randomly distributed throughout the region, favoring the small, decentralized eruptions associated to volcanic fields (Fig. 3.8a).

However, a repose time may be long enough that migration occurs via dike propagation, but short enough that the region gradually builds up heat. Subsequent dike propagation is favored in the relatively-warmer (decreasing the value of Θ) and relatively-softer regions of the crust and it is likely that a dike can migrate laterally along a horizontal temperature gradient, further focusing magmatism into a central region. The long-term result of such magma influx frequency would be the development of a centralized volcanic plumbing system, along with an associated large, central edifice (Fig. 3.8b). Since dikes in this scenario would be focused to varying degrees (i.e. dikes do not horizontally migrate all the way to the center of the region), the edifice would likely build up into an oblong structure. If the repose time of magma influx is very short, we imagine the system would be more-centralized to a narrow region (Fig. 3.8c) and construct a fairly conical edifice. In terms of natural examples, those described in section 3.1 come to mind. Volcanic fields have repose times > 1 kyr (Valentine and Connor, 2015), while persistently active volcanoes like Kilauea have negligible repose times (Patrick et al., 2019).

Fig. 3.8. A model of how volcanic activity is horizontally distributed, depending on the frequency of magma influx defined by the dimensionless time, Fo . The boundary between (a) and (b) is numerically derived (see below) and the boundary between (b) and (c) is experimentally derived (see section 3.3). (a) A very low frequency allows all heat from an

intrusion to be lost. Dikes propagate through the crust, independently to each other, creating small monogenetic cones. (b) A moderate frequency maintains a hot central region, but no molten path. Dikes preferably propagate through the hotter region and build a less-centralized plumbing system and a large edifice. (c) Very high frequency maintains a molten pathway along which magma migrates. The resulting plumbing system would be very centralized and would build a large edifice.

With this logic in mind, we attempt to define a critical frequency of influx which separates these different types of systems. To do so, we use a simple 2D model of transient heat conduction through the crust, written in Matlab, in which heat transfer is described by Fourier's law, in which

$$\rho C_p (dT/dt) = k \nabla^2 T, \quad (3.15)$$

for density, ρ , specific heat capacity, C_p , and thermal conductivity, k (Turcotte & Schubert, 2002; Daniels et al., 2014). The rate of temperature increase dT/dt , therefore depends on the temperature distribution in the crust, $\nabla^2 T = (\partial^2 T / \partial x^2 + \partial^2 T / \partial z^2)$, for heat flow in the horizontal (x) and vertical (z) directions. The thermal diffusivity, α , is related to other crustal properties via equation 3.3 ($\alpha = k / \rho C_p$) and so,

$$(dT/dt) = \alpha \nabla^2 T. \quad (3.16)$$

This applies for a dike with little heat loss due to advection (i.e. eruption of magma onto the surface), with negligible hydrothermal fluid circulation, which accelerates heat transport and cooling (Cherkaoui et al., 2003) and little heat generation due to friction or latent heat release.

This model assumes that a vertical column of 1 m half-width, $H_{1/2}$, representing a dike, becomes hot (temperature of 1400 K) at time, $t=0$, and conducts heat away with time, over a spatial extent of 1 x 8 km (horizontal x vertical). For any element at location (j,i) , heat transfer, Δq , is calculated by the temperature difference with neighboring elements (Fig. 3.9), the material properties of the crust, k , the distance between elements, Δx , and the time step over which heat conducts, Δt , $\Delta q_{ij} = k A_{px} (\Delta t / \Delta x) ([T_{i,j-1} - T_{i,j}] + [T_{i,j+1} - T_{i,j}] + [T_{i-1,j} - T_{i,j}] + [T_{i+1,j} - T_{i,j}])$, in which $A_{px} = \Delta x^2$. The new temperature, T_{new} , at time $t + \Delta t$, of that element is then found using the previous temperature, T_{prev} , and the heat transfer, $T_{new, ij} = T_{prev, ij} + \Delta q_{ij} / \Delta x^3 \rho C_p$, for an element of volume Δx^3 (Turcotte & Schubert, 2002; Çengel, 2008). The cooling of the elements representing the dike and the heating of those representing the surrounding crust depend on the heat transfer, as well as the density and specific heat capacity of those elements. For simplicity, we assume the density and heat capacity of the dike and crust to be the same and to be temperature-independent, respectively 2600 kg/m³ and 1200 J/kg·K. Similarly we assume the thermal conductivity and thermal diffusivity are uniform and respectively have values of 3 W/m·K and 10⁻⁶ m²/s. These values are typical for basalt. The rate of heat transfer depends on the thermal conductivity and diffusivity, so assuming a higher value for either speeds up heat transfer

in the model. The density and specific heat capacity indicate how susceptible a material is to temperature change, and increasing these values causes the heating of an element to take longer.

Fig. 3.9. Illustrations of how heat transfers in our 2D conduction model. The color represents the relative temperature of each element in the model grid, in which red is hot and blue is cold. (a) We set the left boundary of the model grid to magmatic temperature and allow heat to conduct away. The crust has an initial geothermal gradient and the top surface is constrained to atmospheric, surface temperature. (b) The temperature difference between neighboring elements allows us to estimate the heat transfer, q , across a distance of Δx , through a surface area of Δx^2 , over time step of Δt .

We impose a boundary condition on the top of the model, requiring that the upper surface has a temperature of 300 K, and apply an initial geothermal gradient of 30°C/km throughout the crust. The left boundary of the model is defined by the dike intrusion and the right and bottom sides are free to change. We reheat the dike on a defined frequency (every 10^7 sec, 10^8 sec, etc.) and let the system evolve for 10^{12} s (~ 30 kyr).

We employ a few procedures in the model to increase the computational efficiency. We use a variable time step, Δt , which is dependent on the maximum heat flux at any time step in the model. When the left boundary is reheated, Δt is set to 10^4 s and then allowed to increase as heat conducts away and the flux diminishes, to a maximum value of 10^9 s. If a recurrence intrusion would occur during Δt , the time is cut down to the remaining time. For the spatial scale, we use a large element size of 5 m, which is larger than the half-width, $H_{1/2}$, of the dike. When heating occurs, we estimate the temperature of the left boundary of the model grid as an average between magmatic temperature (1400 K) and the nearby crust, which we approximate using the adjacent element, such that $T_{i,1} = [1400H_{1/2} + T_{i,2}(\Delta x - H_{1/2})]/\Delta x$. These simplifications do not model the highest heat flux,

which occurs when a dike first comes into contact with the surrounding, cooler crust, but do allow us to model long-term heating of the crust.

Note that we do not make any assumption of sills, plutons, or other magmatic bodies (of which there could be various shapes, sizes, numbers, or locations), that would naturally develop in such plumbing systems and instead consider only vertical intrusions. Such intrusions would undoubtedly change the heat distribution in the crust, but develop due to other, non-thermal processes such as fracturing and deformation of the crust. At any rate, such bodies develop with time and would not exist when a plumbing system first initiates.

The modeling indicates that a repetitive intrusions can build up heat in a narrow band of the crust. For a recurrence interval on the time scale less than 10^{10} seconds (~ 300 yr), the crust builds up a distinct temperature profile, in which the horizontal temperature gradient is clear (Fig. 3.10a-d). In the context of our conceptual model, this frequency would build a temperature gradient that would encourage intrusions to propagate near the center of the region, focusing into a centralized plumbing region. The temperature gradient that develops due to an intrusion recurrence interval of 10^{11} seconds (~ 3 kyr) appears quite weak by contrast (Fig. 3.10e), while that due to 10^{12} seconds (~ 30 kyr) has vanished entirely and returned the crust to background temperatures (Fig. 3.10f).

Fig. 3.10. Results for the 2D numerical model of heat conduction. An intrusion at the left side of the space is heated to 1400 K on a time interval, t_{int} , specified in each subfigure; the model simulates a total time of 10^{12} s, so the final subfigure records the temperature distribution due to a single intrusion. Contours represent isotherms every 50°C. We impose an initial geothermal gradient of 30°C/km and a surface temperature of 300 K. The vertical and horizontal axes are identical for all figures and represent the depth and horizontal distance from the intrusion.

Our model informs us of the intrusion recurrence frequency that is necessary for the focusing of a plumbing system, assuming that heating of the crust indeed promotes dike propagation into a narrow region (i.e. Fig. 3.8b). Such focusing can occur when the intrusion recurrence interval is less than 1 kyr, whereas a less frequent recurrence allows intrusions to be spatially dispersed. This agrees with Valentine & Connor (2015), who indicate that volcanic fields typically have eruption recurrence intervals 10-100 kyr, but may be as short as 1 kyr. Plumbing systems with shorter recurrence intervals are perhaps characterized by centralization of intrusive events into a focused plumbing system, which in turn constructs a large, centralized edifice. As such edifices are constructed, their loading on the crust further focuses intrusions towards the edifice (Roman & Jaupart, 2014). However, it should be reiterated that the model is simplistic, so it does not account for complicating factors, like a hydrothermal system, which would greatly accelerate heat transport, distributing the heat from a dike over a relatively wide area in a relatively short time (Chevallier & Verwoerd, 1990; Cherkaoui et al., 2003; Pashkevich & Taskin, 2009).

Note that temperature profiles estimated by our model are fairly straightforward and show agreement with other studies. For example, Delaney (1988) developed a numerical model for 1-D conduction from a dike with a temperature-dependent thermal properties and found that it rapidly cools from magmatic temperature to ambient temperature on a time scale of $\sim 10^7$ s (~ 4 months); in our model a single dike cools off on the same time scale. In terms of 2D modeling, our model generates temperature gradients similar to those modeled by Rudman & Epp (1983), on the same time scales and spatial scales, though with a different shape (they model a relatively wide dike, which narrows to a point). Our model results also resemble the heat accumulation modeled by Daniels et al. (2014), who investigated the heating due to repeated dike intrusions during continental rifting. They found that a recurrence interval of 3×10^{10} s can significantly heat the nearest ~ 500 m to the dike on a time frame of 20 kyr, which nearly matches the conditions and results for the model in Fig. 3.10d.

It is tempting to compare the numerical treatment to the analogue experiments, however there is little overlap in dimensionless space. The numerical model simulates heat loss from dikes with negligible advection and volume extrusion, implying low fluxes of $\Phi < 10^{-1}$. The time intervals range from $10 < Fo < 10^6$. In Fig. 3.3d, these conditions represent the bottom right corner of the plot, where there is little experimental data. This is because

such experiments are time consuming to perform, as they can take weeks to years to complete. This is, in fact, the purpose of the numerical model, to learn about the temperature distribution for conditions that are difficult to replicate experimentally.

Of interest is how the different scenarios in our conceptual model (Fig. 3.8) compare with real volcanoes. We have already noted that volcanic fields have repose times on the order of 10 to 100 kyr. Assuming that such eruptions are fed by dikes with thicknesses on the order of 1 to 10 m, these events would be associated with $10^3 < Fo < 10^6$, in general agreement with the arguments from the numerical model. This can be compared with stratovolcanoes, which have shorter repose times. For example, Mullineaux (1986) investigated the tephra stratigraphy of products from Mt. St. Helens and found that the repose times were typically 100 to 1000 yr, during the last several millennia. Periods of activity tended to last several decades and were characterized by the explosive and extrusive eruption of andesitic magma (Swanson et al., 1985). Due to the evolved nature of the magma, we may consider that it flows through a relatively-wide aperture of 10 to 100 m and so the andesitic magma can be characterized by $10^{-1} < Fo < 10^2$. Any basaltic intrusions migrating through the system would likely be narrower (Wada, 1994) and be characterized by $10^2 < Fo < 10^4$.

We can also consider stratovolcanoes with shorter repose times. Schonwalder et al. (2018) investigated the petrology of the monogenetic scoria cones at Llaima volcano (Chile), which is a highly-active, basalt-andesite volcano that erupts once per decade, on average. They found through crystal size distribution analysis that its reservoir has a generally-shorter residency time (1 to 12 yr) than eruption repose time (~10 yr), implying more-frequent magma recharge. It is difficult to gauge the geometry of the feeding system for the volcano, but if we again assume that intrusions are from 1 to 10 m wide and occur once per year to once per decade, the volcano would be characterized by $10^{-1} < Fo < 10^2$. In the framework of our model and our experimental results, the reservoir may be fed by a persistently-active conduit-like structure. If we consider even more-active volcanoes, like the persistently-active Kilauea system, it would have even smaller values of $Fo \sim 0$.

3.4.3 Stable conduit development at depth

Our experiments instruct provide insight into the conditions for which a stable, molten conduit may develop in the crust ($Fo < 1$ and $\Phi > 1$). Considering that the dimensionless numbers contain details of the plumbing geometry, we can rewrite equations 3.2 and 3.10 to solve for the necessary conditions to maintain a molten channel. We label these as the critical flux, Q_{crit} , and critical repose time, t_{crit} . Assuming steady state heat transfer in a conduit geometry with radius, R , the heat into the conduit due to magma flux balances with loss due to conduction:

$$Q_{crit} \rho C_p T_0 = 2\pi L k (T_0 - T_\infty) / \ln(r/R) , \quad (3.17)$$

for a magma and ambient temperatures T_0 and T_∞ and conduction occurring over a radial distance into the crust, r (Çengel, 2002). The Q_{crit} is dependent on temperature and the temperature gradient. For values of $2 < r/R < 1000$, there is a constricted range of values of $0.7 < \ln(r/R) < 7$. Since it varies over a small range, we assume it to be constant, so

$$Q_{crit} = 2\pi \alpha L (1 - T_\infty/T_0) . \quad (3.18)$$

For magma much hotter than the ambient temperature (so that $T_\infty/T_0 \approx 0$), which ascends through a 1-10 km long conduit, a relatively low mean flux of only 10^{-2} - 10^{-1} m³/s is required to maintain a liquid conduit.

Perhaps unsurprisingly, the time parameter has stricter conditions for a conduit to remain molten. The time for solidification implies a transient heat conduction problem, such that

$$V \rho C_p \partial T / \partial t = k A_s \partial T / \partial r \quad (3.19)$$

assuming the flow has stopped, in a conduit of surface area, A_s , and volume, V . Therefore,

$$\partial T / \partial t = (2\alpha/R) \partial T / \partial r , \quad (3.20)$$

in which $\partial T / \partial t$ and $\partial T / \partial r$ decrease as time goes by and the conduit cools. For solidification to occur, the magma only needs to cool to the solidus temperature, and we integrate the cooling rate over time, t_{crit} , so that $\partial T / \partial t = (T_0 - T_s) / t_{crit}$ for a solidus temperature, T_s . This approximates the temperature gradient to a time-averaged gradient. If we integrate again with respect to r , over a range of R to r , then

$$(T_0 - T_s) / t_{crit} = 2\alpha (T_0 - T_\infty) / R(r - R) . \quad (3.21)$$

If r is proportional to R , such that $R^* = r/R$, then

$$t_{crit} = (R^2 / 2\alpha) (R^* - 1) [(T_0 - T_s) / (T_0 - T_\infty)] . \quad (3.22)$$

In the above equation, characteristic time scale represented by $R^2 / 2\alpha$ has a strong dependence on the conduit radius, in that large conduits have a large ratio of volume to surface area. For a 1 m radius conduit, the time scale is several days, whereas for a 10 m radius conduit, it is around 19 months. However the other terms in the equation modify the time estimate to a significant degree. For an initial temperature defined by Θ (Table 3.2), the temperature terms in this equation together have a value of 0.28 to 0.38, suggesting quicker cooling. The temperature gradient term, $R^* - 1$, meanwhile suggests longer cooling times. It is difficult to judge which radial distance approximates the temperature gradient, but if conduction occurs over tens or hundreds of meters, this term could be large (Fig. 3.11).

Fig. 3.11. An illustration for the temperature gradient, $\partial T/\partial r$, in the crust sounding a conduit. The gradient controls conduction, so we approximate it to a linear function over the temperature range of T_0 to T_∞ , which indicates the magnitude of R^* . The steeper the slope, the faster heat is lost from the conduit.

In terms of sensible heat, reasonable estimates for conduit cooling would be weeks to decades, depending on the magma temperature, crust temperature and conduit geometry. Latent heat however can provide a large amount of thermal energy and represent the majority of available enthalpy (Morse, 2011). In this context, as magma crystallizes, it likely enters an approximately steady state of heat loss, in which conduction balances with the energy released during phase changes. If the available amount of latent heat is indeed several times larger than the sensible heat, then solidification would take a similar factor longer to conclude. In the context of our experiments, it is expected that the latent heat does not play a large role, since most of the available heat is sensible heat (see Appendix C).

The conditions for solidification in a conduit geometry have also been addressed in previous studies. Solidification occurs by the freezing of magma along the annulus of conduit, which proceeds when the heat loss due to conduction exceeds heat supply due to advection, for laminar flow conditions with a constant pressure drop; when heat loss balances with supply, the conduit maintains a steady-state radius (Zerkle & Sunderland, 1968; Mulligan & Jones, 1975). Holmes-Cerfon & Whitehead (2010) show that the critical flux necessary for steady state flow depends on the pressure drop, the Stefan number (equation 3.8), the dimensionless flux Φ (equation 3.10, modified for a conduit geometry, in which $H/B = 1$), and the dimensionless temperature difference, Θ_d , similar to Θ described above, in which $\Theta_d = (T_s - T_\infty)/(T_0 - T_s)$. The Θ_d corresponds to the degree of solidification, in which $\Theta_d \gg 1$ indicates that solidification should occur fairly rapidly. For magma driven by a constant flux (which is the driving condition for our experiments), they find that any flux can maintain magma flow, but may cause large driving pressures.

However, for magma flow driven by a pressure drop (either constant or somewhat variable), they find that stable flow in a conduit requires a minimum critical flux of $\Phi \sim 6000$, significantly higher than that observed in our experiments ($\Phi \sim 1$). This could be partially explained by the time-dependent solidification of gelatin in our experiments, which is discussed later in section 3.4.5 and allows gelatin to temporarily (~ 10 minutes) remain liquid as it cools below the solidus. However, we expect this yields a similar effect as latent heat release, which also delays the solidification process. Although there is not a significant latent heat release from our gelatin, the time dependent solidification effectively takes its place to a degree.

A natural setting that indicates the necessary critical flux for a conduit is a lava lake, which represents the uppermost surface of a molten conduit. Lava lakes are thought to persist due to magma convection, in which bidirectional flow supplies heat from depth throughout the conduit and maintains a molten pathway (Harris et al., 1999; Oppenheimer et al., 2004; Sweeney et al., 2008). Depending on their level of activity, the magma flux can vary in magnitude, but tends to be greater than $0.01 \text{ m}^3/\text{s}$. For example, Harris et al. (1999) estimated that Mt. Erebus lava lake in Antarctica had a magma flux of $\sim 0.01 \text{ m}^3/\text{s}$ during measurements taken in both 1986 and 1989; similarly Erta' Ale lava lake in Ethiopia had fluxes varying from $5 \text{ m}^3/\text{s}$ in 1973 to $0.01 \text{ m}^3/\text{s}$ in 1986. They reported that other lava lakes tended to have higher fluxes. Although such systems are characterized by bidirectional flow, rather than the unidirectional flow that we analyze in this study, we can assume this $0.01 \text{ m}^3/\text{s}$ base value provides an order-of-magnitude estimate of the necessary critical flux for stable flow. Assuming that such conduits are between 1 and 10 km long and that $\alpha = 10^{-6} \text{ m}^2/\text{s}$, then the minimum value of Φ is 1-10, similar to what we estimate for our experiments (solving equation 3.10, in which $H/B = 1$).

If we consider deeper magma migration, in the form of magma supply from deep sources into shallow crustal reservoirs, these estimates hint that the form of magma migration ultimately depends on how magma enters the crust from the mantle. A slow, steady trickle of fresh magma may be sufficient to maintain a molten conduit, thereby continuously feeding shallower bodies. Conversely, if magma accumulates at the bottom of the crust in infrequent, discrete pulses, a previously-existing conduit would not have the necessary heat to remain liquid, so the magma would have to migrate in the form of a new dike. In this sense, magma supply into shallow reservoirs may depend on how effectively and how steadily partial melt can form and percolate into the crust.

A good example of the effect of supply frequency and cooling on deep magma migration may be Piton de la Fournaise. Prior to the 1998 eruption, magma migration from depth produces a series of volcano tectonic earthquakes, whose hypocenters tracked the migration front (Battaglia et al., 2005). The lateral distribution of these earthquakes indicated that the migration followed a fairly narrow path below sea level and then a sheet-like path above sea level. This sudden change in geometry was considered to be due to the magma flowing through a pre-existing conduit (perhaps partially-molten) at

depth and then rapidly propagating a shallow dike. This deep conduit structure was reactivated during the 2015 seismic episode, which also led to an eruption (Lengliné et al., 2016). The small magnitudes of the deep seismic events are evidence that magma flowed through the existing, partially-molten structure and therefore implies that it was persistently active on the 1998-2015 time frame. We argue that there may be a thermal discontinuity located at sea level, in which the underlying crust is well-insulated and maintains an open, persistently-active conduit. Above this discontinuity, the crust may be generally cooler, so that intrusions are thoroughly solidified and require migration in the form of a dike.

3.4.4 Shield volcano rift zones

In addition to identifying the nature of supply pathways, we can apply our findings to rift zones (namely those that occur on the flanks of shield volcanoes; *e.g.*, Kilauea's east rift zone, Piton de la Fournaise rift zones) to gain some sense of the internal structure. Such rift zones are known to be frequently active pathways along which magma repeatedly travels.

In the case of Kilauea, magma frequently drains from the summit region along the rift zone and this background activity is occasionally punctuated by more-intense fissure eruptions (Furumoto, 1978; Owen et al., 2000; Patrick et al., 2019). The frequent, relatively-low flux of the background phase likely favors the form of a sub-horizontal conduit, which minimizes the heat loss due to conduction. The higher-intensity fissure eruptions occur due to some combination of increased magma flux (internal pressure) and subsidence of the volcano's flank (external extension), which cause the conduit to fail and propagate a dike.

Two differences between this setting and our experiments are the flow direction (*i.e.* horizontal vs vertical) and the stress field (predominantly extensions vs neutral). Dikes that propagate horizontally are likely following along the level of neutral buoyancy, in the overlying and underlying crust are respectively less dense and denser than the magma (Lister and Kerr, 1991). Dikes propagating along this level flow in a qualitatively-similar fashion to vertically-ascending dikes, in which they grow primarily in the direction of flow. For example, Taisne and Tait (2011) investigated the effect of solidification on a vertically propagating dike, whereas Daniels and Menand (2015) performed experiments for horizontally propagating dikes. The dikes presented by Daniels and Menand (2015) are very similar in appearance to those by Taisne and Tait (2011), albeit rotated 90°.

An extensional stress field seems to have a greater effect on the form of dike intrusions, in that it accommodates liquid intrusion. Daniels and Menand (2015) repeatedly injected Vegetaline into a gelatin medium under extensional stress and reported that dikes exert localized compression on the surrounding host medium, which causes subsequent dikes to rotate and propagate away. This effect is mitigated by the magnitude of extension and

the spacing between dikes, which both promote the parallel intrusion of dikes. In the context of Kilauea's East Rift Zone, where dikes seems to intrude parallel to each other, along a fairly narrow band, this implies that the extensions is quite significant (Furumoto, 1978; Leslie et al., 2004; Wright & Marsh, 2016).

Over the lifetime of the rift zone, it is plausible that the geometry would oscillate between cylindrical and planar, depending on the level of activity, slowly accumulating layers into a dike complex (Fig. 3.12). Indeed, geophysical measurements (seismic reflections, gravity, magnetic data) indicate that a dense intrusive body spans the length of Kilauea's east rift zone to the submarine Puna Ridge and contains a narrower, hot ($\sim 1000^\circ\text{C}$) region along its axis (Furumoto, 1978; Leslie et al., 2004). Subsidence of the volcano's flank generates extensional forces that cause dikes to orient perpendicularly to the direction of subsidence, ensuring the intrusive dike complex is sub-vertical, with dimensions much taller and longer than wide.

Fig. 3.12. A model for the development of a rift zone's internal structure. Frequent influx of magma favors intrusion along a planar region. (a) An initial dike propagates through the region. (b) As influx decreases, flow may take the form of a horizontal conduit. (c) A subsequent increase in flux concentrates stress along the conduit, causing the propagation of a nested (or alternately, a parallel) dike. (d,e) Rift zones tend to be found in extensional settings, ensuring propagation of sub-vertically oriented dikes. With time, these dikes may build thick, tabular intrusive complexes. Periods of low flux favor magma flow in confined channel (Patrick et al., 2019), while high flux favors dike propagation and growth of the intrusion.

3.4.5 Experimental limitations

Our experimental setup has the inherent limitation that gelatin solidifies as a function of temperature, time and shear strain. We consistently measured eruptive temperatures below the solidus of gelatin, indicating that either solidification requires time or is inhibited due to the liquid's motion. In either case, the resulting dike's form could be somewhat different from a magmatic dike (perhaps thinner, with less material intruded). Anecdotally-speaking, we have observed in experiments not related to this thesis that imposing continuous motion (e.g. constant stirring) on a solidifying gelatin causes the material to take on a paste-like rheology. The gelatin takes on an amorphous, opaque appearance, which is solid, but capable of flow. We did not observe any similarly-appearing gelatin in these experiments, nor did we observe any flow instabilities that could be associated with a shear-thinning liquid, however we recognize the possibility that the shear rate can affect the gelatin's characteristics. A material that solidifies independently of time and shear strain may have grown a thicker chilled margin and restricted the internal flow. Additionally, the time dependence of gelatin solidification implies that its viscosity may also have some time-dependence. This potentially complicates comparison with other studies of flow channelization in a fissure, in which it has been shown that channelization strongly depends on the viscosity ratio between warmer and cooler regions of a dike.

That said, we believe our results are fundamentally sound. Taisne & Tait (2011) made analogue experiments of solidifying dikes using paraffin wax, which is strongly temperature-dependent. The general form of their dikes were similar to ours, albeit with greater thickness ($H/B = \psi \sim 0.1$; in our experiments $0.01 < \psi < 0.1$ for dike shaped intrusions) and distinct chilled margins. By comparison, Chanceaux and Menand (2014) observed similar aspect ratios of $\psi \sim 0.1$ to Taisne & Tait (2011) while Daniels & Menand (2015) recorded even large values of $\psi \sim 0.3$, both using Vegetaline. In terms of dike length and breadth, the measurements from this study are similar to those of Taisne and Tait (2011) and Daniels and Menand (2015), in which all of our dikes had lengths (in the direction of flow) of 20-30 cm and breadths (in the perpendicular direction) of 5 to 20 cm; the results from Chanceaux and Menand (2014) are not comparable, since they studied sill intrusions. It is likely that the inner fluid core of such dikes undergo flow patterns similar to those observed in this study, those presented in the next chapter, and in Kavanagh et al. (2018). Furthermore, it is promising that we were able to replicate behaviors observed in nature, such as the heterogeneous solidification and the subsequent channelization of a dike.

Quantitatively speaking, the characteristic time for solidification, H^2/α , for our experimental dikes is on the time scale of ~ 10 min, which matches the time frame for channelization that we observed in our experiments, which seems to imply the time dependence did not play a large role in the solidification of a thin dike. For experiments which did not result in conduit formation, either the time interval between injections or

the flux duration (or both) tended to be much larger (> 30 min) than this 10 minute time frame. All things considered, it seems as though the gelatin's time dependence had no effect on the form of liquid migration. Rather, its effects were confined to the aspect ratio of the dikes, which may have been somewhat thinner than for a material with a time-independent phase transition.

While liquid gelatin behaves as a Newtonian liquid, in that it seems to have a shear-independent viscosity, these observations warrant a further detailed investigation into the gelatin rheology, in order to understand how it evolves during solidification, as function of temperature, time and stress. For example, anecdotal observations suggest that the shear rate affects gelatin solidification, but we do not have the equipment to properly test this. Such an investigation would help us to understand precisely how our results relate to magmatic dikes. In nature, magma can have either simple or complex rheologies, depending in part on the size and concentration of crystals, which inhibit flow, and bubbles, which can either inhibit or promote flow (Llewellyn & Manga, 2005; Molina et al., 2012).

3.5 Conclusions

The experiments we present in this research illustrate the effects of solidification magma migration. For very frequent intrusions ($Fo < 10^0$), magma maintains a hot, open pathway from depth to the surface. This path initiates as a dike and, upon eruption, quickly channelizes into a conduit. For very infrequent intrusions ($Fo > 10^2$), the dike can completely solidify, so a subsequent intrusion forms a new dike and the system eventually accumulates a dike swarm. We also show a transitional behavior ($Fo \sim 10^1$), in which a dike intrusion solidifies but remains soft, so subsequent intrusion propagate new dikes inside the original dike.

We find that the effect of flux primarily controls the stability of flow through the crust. Intrusions that propagate with low fluxes ($\Phi < 10^{-1}$) struggle to reach the surface and form thick dikes that are comprised of many small, individual lobes, that are compounded together. Dikes with sufficiently-high flux ($\Phi > 1$) propagate to the surface and can steadily flow until the supply stops. Between these two regimes, the flux has a transitional regime, in which a dike may (or may not) propagate to the surface and any conduit that forms is temporary, and eventually closes due to solidification.

We expand these results using a 2D numerical model of thermal conduction. For injections more-frequent than 10^{11} to 10^{12} s (about 3 to 30 kyr; $Fo \sim 10^4$ to 10^5), the crust accumulates heat near the intrusive region, which we argue softens the crust and encourages subsequent dikes propagate nearby. For less-frequent intrusions, heat is lost and the crust returns to its background temperature. Considering the experimental and

numerical evidence together, the frequency of migration dictates whether magma flows through a conduit or a series of spatially-focused or spatially-dispersed dikes.

3.6 Chapter Notation

Table 3.4. Symbols and meanings used in this chapter

Symbol	Units	Meaning
t	s	time
t_d	s	characteristic time scale for a dike to solidify due to conduction
t_{int}	s	time interval, repose time between injections
t_{inj}	s	time duration of an injection
t_{crit}	s	time scale for a conduit to solidify due to conduction
dt	s	time step for thermal conduction model
dx	m	pixel size for thermal conduction model
L	m	dike vertical length
L_{cool}	m	thermal entry length
B	m	dike horizontal breadth
H	m	dike opening width
H_{solid}, H_{total}	m	total width and solid layers width, for compound dikes
R	m	conduit radius
r	m	radial distance from conduit
R^*	-	distance characterizing temperature gradient slope
A_s	m^2	surface area
A_{px}	m^2	pixel area
V	m^3	volume

Chapter 3: Dike channelization and solidification: time scale controls on the geometry and placement of magma migration pathways

V_{erupt}	m^3	erupted volume
V^*	-	proportion of injected volume remaining intruded in the medium
v	m/s	velocity
Q	m^3/s	volume flux
Q_{crit}	m^3/s	flux scale necessary to maintain a molten conduit
T	$^{\circ}\text{C}$	temperature
T_s	$^{\circ}\text{C}$	magma solidus temperature
T_0	$^{\circ}\text{C}$	magma initial temperature
T_{∞}	$^{\circ}\text{C}$	host rock ambient temperature
$T_{\text{prev}}, T_{\text{new}}$	$^{\circ}\text{C}$	temperature in thermal conduction model, before/after time step
Δq	J	heat transfer to an element in thermal conduction model
i, j	-	element indices in thermal conduction model
I, I_0	-	pixel, reference brightness values for Beer-Lambert law
k	$\text{J}/(\text{m}\cdot\text{s}\cdot\text{K})$	thermal conductivity
α	m^2/s	thermal diffusivity
ρ	kg/m^3	density
μ	$\text{Pa}\cdot\text{s}$	viscosity
C_p	$\text{J}/(\text{kg}\cdot\text{K})$	specific heat capacity
ΔP	Pa	driving pressure
P_v	Pa	viscous forces
λ	-	modifier function for thermal entry length
γ	J/kg	specific latent heat
S	-	Stefan number
Fo	-	Fourier number (for time interval between injections)
Fo_{inj}	-	Fourier number (for injection duration)
Φ	-	dimensionless flux

Chapter 3: Dike channelization and solidification: time scale controls on the geometry and placement of magma migration pathways

Θ	-	dimensionless temperature
Ψ	-	dike aspect ratio

Segue to chapter 4

During the experiments of chapter 3, we acquired a new imaging system to measure magma flow fields, as well as strain in the crust, via particle image/tracking velocimetry. We had managed to perform measurements on some of those experiments and decided to make a dedicated series of experiments, focusing on a dike's internal flow. We did not discuss any such observations or results in chapter 3, because it has a long-term perspective, focusing on the types of structures that develop over time, for a given injection frequency.

This next and final chapter has a short-term perspective, focusing on how a dike grows and evolves, depending on the forces that drive it. For example, if the dominant force acting on a dike is the source reservoir's overpressure, we would expect the dike to grow in a penny-shaped geometry and have a vigorous internal flow. With our new capabilities in the laboratory, we can observe both the crack geometry evolution as well as the flow field evolution, and fully characterize the forces acting on it; such experiments lay at the heart of dike propagation theory.

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns

Abstract

Magma ascent towards the Earth's surface occurs through dike propagation. The magma's buoyancy affects the internal flow dynamics and consequently the crystals that erupt. A series of analog experiments were conducted in this study, examining flux-driven and buoyancy-driven magma ascent. The fluid dynamics were measured using 2D (PIV) and 3D (PTV) techniques. The results of this study show that the dike's buoyancy has a strong control on the form of its internal flow. Dikes that are small, relative to their buoyancy length scale, do not have significant buoyancy forces and the resulting flow takes on a circulating pattern, with a central upward jet and lateral return flow. Dikes that are long enough to surpass this scale become buoyancy driven and the resulting flow takes on a simpler, ascending form.

4.1 Introduction

Magma migration towards the Earth's surface commonly occurs through dike propagation. This is because fractures are mechanically efficient geometries for transporting large volumes of magma (Keating et al., 2008). Dikes have traditionally been assumed to have a relatively simple velocity profile, in which magma flows unidirectionally forward (either upward for ascending dikes or horizontally for laterally propagating dikes) with a laminar flow profile (Whitehead & Helfrich, 1991; Fialko & Rubin, 1998; Dahm, 2000b; Pinel et al., 2017).

Dikes have traditionally been described using 2D models, which investigate the shape and/or flow as functions of the magma flow dynamics and host rock response. They assume minimal variation in shape and flow along the direction perpendicular to the 2D plane. This approach allows simplifications, such as the assumption of Hagen-Poiseuille flow through a channel, which describes the liquid velocity as a function of the viscous pressure and the driving pressure drop (Dahm, 2000a). These models are themselves divided into three "schools of thought" that model the dike flow in different ways (Rivalta et al., 2015; Townsend et al., 2017). The first school describes dikes as teardrop-shaped structures in cross section. These are essentially detached from their source reservoir and migrate at a rate depending in part on their buoyancy. The magma remains in a discrete volume near the front of the dike and, as the magma migrates, the tail closes shut and so dikes that are sufficiently long have enough buoyancy to propagate without any additional driving force (Weertman, 1971; Dahm, 2000b; Rivalta et al., 2015). The second

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns

school models dikes as driven by a constant influx or a constant pressure from the source reservoir and the dike maintains a relatively thick, upper head region and a thinner basal tail region. The flow velocity and dike shape depend on the characteristics of both the magma injection and the surrounding crustal medium (Lister, 1990; Roper and Lister, 2007; Townsend et al., 2017). Finally, the third school focuses on the propagation of horizontal cracks, in which lateral propagation occurs because a rheological barrier (e.g. a strong overlying layer, a less dense layer) or a stress barrier prevents vertical propagation to the surface (Lister, 1990; Lister & Kerr, 1991; Pinel & Jaupart, 2004; Townsend et al., 2017). A common feature of all of these models is that the liquid is assumed to flow forward (i.e. either upward for a vertically-propagating dike or laterally for a horizontal dike). A typical feature in many studies is equilibrium, in which the fracture propagates with a steady velocity and shape, independent of how far it propagates through the crust (e.g. Weertman, 1971; Roper & Lister, 2007). Dahm (2010) investigated the transitional forms that occur as a series of several phases, when a dike is initially injected and then begins ascent as a Weertman fracture.

However, it is recently becoming clear that dikes may exhibit internal circulation, in which magma cycles between shallow and deep via vortices that develop along the dike plane (Albert et al., 2018; Kavanagh et al., 2018). The crystal growth patterns associated with monogenetic eruptions can indicate multiple forms of magma flow, sometimes occurring during a single eruption. For example, Albert et al. (2018) analyzed the olivine crystals from Parícutin and showed that they formed in a dynamic environment. Crystals from the initial, propagating phase of the eruption have a variety of zoning patterns, sometimes oscillatory zoning, which grew on the timescale of dike propagation (hours to days). These zoning patterns indicate that the crystals were exposed to a range of temperatures, pressures, and oxygen fugacities and they cycled between such conditions. By contrast, crystals representing the later stages of the eruption are simpler and entirely characterized by normally-zoned olivines, which ascend directly from their source to the surface. This evidence is interpreted by Albert et al. (2018) to mean that the dike had an initial phase of circulating flow, followed by a phase of simple, ascending flow.

The internal circulation in a dike has been shown experimentally as well. Kavanagh et al. (2018) made analogue experiments of propagating dikes in gelatin, focusing on the flow field in the dike. They show through Particle Image Velocimetry (PIV) that the internal flow evolves as the dike propagates and then erupts. As dikes grow, they may have a strong circulation, in which the liquid ascends and descends many times before erupting. As a dike erupts, a significant proportion of the internal volume is expelled, leaving it narrower than its actively-propagating geometry, due to the elastic response of the host material.

However based on crystal textures, it can be inferred that dikes do not always leave a record of circulation. Monogenetic eruptions, both in volcanic fields and as parasitic cones have frequently shown that they simply tap magma associated to their storage

region conditions (Coote et al., 2018; Schonwalder-Angel et al., 2018). In such cases the propagation phase is not well-marked in the crystal textures and are only evidenced in the number of microlites. Moreover, crystals that do have complex zoning do not always indicate circulation. Normally-zoned and reverse-zoned crystals are usually interpreted to form respectively due to fractionation of the source reservoir and to recharge events (Coote et al., 2018; Shane & Coote, 2018). Similarly, oscillatory-zoned crystals can be interpreted as forming due to several recharge events or possibly due to bulk of the magma progressively degassing and cooling (Crabtree & Waters, 2017; Gordeychik et al., 2018).

With these evidences in mind, we would like to investigate under which conditions a dike can exhibit internal circulation. We perform analogue experiments and use particle imaging velocimetry (PIV) to track the internal flow, varying the flux and liquid properties between experiments. We investigate this behavior with the hypothesis that the circulation occurs as a function of the forces acting on the dike. In such a way, a dike that is strongly driven by overpressure at the source and the influx of liquid into the dike may exhibit circulation, whereas a dike that is driven by its internal buoyancy forces may not. By linking the circulation to the forces acting on the dike, we can therefore relate crystal zoning patterns to the forces that drive propagation.

4.2 Methods

4.2.1 Experimental setup

We conducted a series of scaled laboratory experiments with gelatin as the crust analogue and different fluids (water, liquid gelatin and vegetable oil) as the magma analogue. Gelatin is commonly used to model dike propagation, as it is brittle and elastic on the timescale of a propagating dike, so that it scales to the Earth's crust (Kavanagh et al., 2013). It is transparent, so we can observe the geometry of, and fluid dynamics within, the propagating dike. It is also photoelastic, so we can visualize the stresses within the gelatin medium using polarized light.

We prepared each gelatin medium by dissolving 3 wt% to 3.5 wt% of granular gelatin in 112 L of deionized water and heating the solution to 60°C. For a subset of experiments, we added 5 wt% concentration of white sugar to the solution, in order to increase its density (small concentrations of sugar do not significantly affect the resulting rheology). We poured the gelatin into a 50x50x50 cm³ tank and covered it with a layer of oil to prevent evaporation (and the formation of a hard skin at the surface). Finally, we placed the tank into a cold room to solidify at 15°C for 3 days. This procedure is commonly used in the analogue modeling of dike propagation and makes the gelatins' rheology scale to nature (e.g. Menand & Tait, 2002; Kavanagh et al., 2013; Pansino & Taisne, 2019). As

described in chapter 3, we measured the density of the gelatin after an experiment, by cutting a sample from the medium and measuring the mass, then submerging it in water and measuring the volume displacement. For gelatins with sugar, the density was about 1070 kg/m^3 , while for gelatins without, the density was about 1010 kg/m^3 .

The strength of the gelatin is an important parameter controlling dike propagation. It is characterized by the Young's modulus, which is dependent on many factors, like gelatin concentration, temperature, cooling history, the addition of sugar or salt, or even water quality. We therefore measured the Young's modulus of each gelatin before performing the experiment (so that it is at ambient temperature), by measuring the velocity at which shear waves propagate through the medium (Pansino & Taisne, 2019). The shear waves, which are made visible using polarizing filters, are slow enough to view with the unaided eye and can be recorded by a typical camera, at 50 frames/second (fps). The square of the shear wave velocities, v_s , are proportional to the shear modulus, G , which allows us to estimate the Young's modulus, E , and fracture toughness, K_c (Lee et al., 2017; Menand and Tait, 2002):

$$G = \rho v_s^2, \quad (4.1)$$

$$E = 2G(1+\nu), \quad (4.2)$$

and

$$K_c = (2\gamma E)^{1/2}. \quad (4.3)$$

In the above equations, ν is Poisson's ratio and γ is the medium's surface energy, which we assume to be 1 J/m^2 (Kavanagh et al., 2013). Further details on wave velocity measurements can be found in Appendix B.

Table 4.1. Conditions for the analogue experiments. We label vegetable oil as "Veg oil". From left to right are the concentration of granular gelatin in the medium, concentration of the sugar additive for the medium (to increase its density and impose buoyancy on the dike), Young's modulus, fracture toughness, the analogue liquid, liquid-medium density difference, liquid viscosity, influx and the driving force mechanism. Note that S1 and S2 are also used in chapter 3 (respectively experiments 9 and 11 in Table 3.1) and represent the first injection from their respective experiments.

Exp	C_{gel}	C_{sugar}	E	K_c	Liquid	$\Delta\rho$	μ	Q	Driving force
	Wt %	Wt %	Pa	$\text{Pa}\cdot\text{m}^{1/2}$		kg/m^3	$\text{Pa}\cdot\text{s}$	mL/min	
C1	3.0	0.0	3620	85	Veg oil	100	10^{-1}	11	Pump

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns

C2	3.0	0.0	3520	84	Veg oil	100	10^{-1}	170	Pump
C3	3.0	0.0	1800	60	Veg oil	100	10^{-1}	2	Pump
C4	3.0	0.0	1420	53	Veg oil	100	10^{-1}	21	Gravity
C5	3.5	0.0	1840	61	Water	10	10^{-3}	15-350 †	Gravity
S1	3.5	5.0	4580	96	Gelatin	70*	10^{-2}	38	Pump
S2	3.5	5.0	3850	88	Gelatin	70*	10^{-2}	38	Pump
S3	3.1	0.0	3600 ‡	85	Gelatin	10*	10^{-2}	3	Pump

* For liquid gelatin, which has a density similar to water, the $\Delta\rho$ value was achieved by increasing the density of the gelatin medium, by adding sugar during preparation. † The range of flux is due to our gravity-driven setup (see Fig. 4.1a). Water was allowed to flow from an elevated beaker at a variable flux. By contrast, we had a gravity-driven experiment of oil, which had an extremely stable flux. ‡ This value is estimated from other similar gelatin mediums, experiments C1 and C2.

Our magma analogues were water, vegetable oil and liquid gelatin. The density of water is 1000 kg/m^3 , vegetable oil has 910 kg/m^3 and liquid gelatin is assumed to have a density of 1000 kg/m^3 for simplicity. As noted in chapter 3, the density of liquid gelatin actually varies with temperature and our measurements indicate it ranges from 995 to 1005 kg/m^3 at temperatures of 25 to 50°C . We use these values, together with the solid medium density indicated above, to estimate the density contrast shown in Table 4.1. The water and oil are Newtonian liquids and the liquid gelatin, as described in chapter 3, seems to have a temperature-dependent, shear-independent viscosity. We classify experiments with liquid gelatin as the ‘S’ series and those with water or vegetable oil as the ‘C’ series (Table 4.1). These two series have several differences. In the S series, liquid gelatin in the dike may solidify, whereas in the C series, phase change cannot occur. In addition, in order to increase the buoyancy of the liquid for some experiments in the S series, we added either sugar and/or salt to the solid medium during preparation, which increased its density. In the C series, the buoyancy varied only due to the physical characteristics of the magma analogue.

When we start an experiment, we first make an initial cut in the gelatin, which imposes an orientation on the ascending dike, so that it is vertical and that it is perpendicular to our cameras. We then inject the magma analogue into the cut. For most experiments (in both the S and C series), we injected the liquid at a constant flux using a peristaltic pump (Fig. 4.1; see Table 4.1, “pump” driven experiments). We could estimate the dike volume at any point in time simply using the flux rate. For experiments C4 and C5, we adjusted the setup to be gravity-driven, in which we placed a beaker of liquid high above the

gelatin (1.25 m difference between the liquid surface in the beaker and the gelatin surface) and allowed gravity to pull the liquid down and force it into the gelatin (Fig. 1a). We placed the beaker on a scale and recorded the mass loss with time, which allowed us to directly measure the dike volume (mass divided by density) as well as the flux (differential of volume with respect to time). We assume the driving pressure to be constant, since the liquid surface dropped by less than a centimeter during the experiment; the pressure decreased by less than 1%.

Note, that since the S series can solidify, the injected liquid gelatin was kept warm and fluid using a thermal bath. We placed a thermocouple in a basal supply reservoir to monitor its temperature (Fig. 4.1) and we attempted to maintain a target temperature of 32 °C. This temperature is slightly above the gelatin liquidus (~28 °C) and therefore resembles the likely injection temperature for basaltic dikes (Delaney and Pollard, 1982; Taisne and Tait, 2011).

Fig. 4.1. The experimental setup. Liquid (here shown in red for visibility) is seeded with PIV tracer particles and injected into a gelatin medium. The particles are illuminated by an LED light panel and 4 synchronized high-speed cameras records them. Each camera has a different angle of view, which can potentially allow us to track them in 3D (for oil, the angle of refraction prohibits 3D measurements). A pressure transducer at the base of the tank records the pressure time series. Liquid is forced into the gelatin at a constant flux using a peristaltic pump. (a) Alternatively, we use a gravity-driven setup to attempt to supply a constant pressure and variable flux. In this scenario, we record a mass time series to track the volume and volumetric flux.

To measure the overpressure associated to each injection, we attached a pressure transducer to the side of a small cylindrical reservoir, which was attached to the bottom of our tank and mimicked a source reservoir (Fig. 4.1). The pressure transducer was calibrated prior to the injection and recorded the pressure just below the location of injection. It should be noted that the pressure measurements were only conducted for experiments (C1-C5).

4.2.2 Flow visualization using Particle Image/Tracking Velocimetry

The fluid circulation within the dike was measured with Particle Image Velocimetry (PIV). This image-based velocimetry technique is nonintrusive, meaning that the measuring equipment does not affect the flow field, and based on processing successive images taken of the location of interest. The flow is seeded with passive tracer particles, which follow the motion of the fluid, and the displacement of these tracer particles over a known period of time can be determined to calculate their velocities. PIV data provides a statistical description of the flow field, which we found useful to quantify the flow velocity for later analysis. Our visualization system offers the potential to measure the flow field in 3D using particle tracking velocimetry (PTV), in which the system tracks the 3D position of each particle with time, but in practice this was only successful when we injected liquid gelatin, which has the same optical properties as solid gelatin. We will describe the 3D setup here, but our results rely only on 2D PIV data.

The PIV technique is an Eulerian method that statistically estimate the liquid velocity field inside a rectangular grid (Adrian, 1991; Schroeder & Willert, 2008; Raffel et al., 2013). Similar to DIC described in chapter 1, this uses pattern matching, in which a region of interest in an image is compared to nearby regions in the subsequent image and the direction of movement is determined by cross-correlation (Zhang & Porfiri, 2019). The main difference is DIC tends to be used for solids, assuming small strains from an initial reference image, whereas PIV is used for liquids, assuming continuous flow between images (Zhang & Porfiri, 2019). The concentration needs to be such that the pattern matching can recognize groups of particles. We varied the concentration between experiments from ~ 1 g of particles for each liter of liquid (which looked to have too few particles) to ~ 5 g/L (which seemed fine). For 3D PTV, the concentration should be lower; if there are many particles, they can obscure each other and inhibit measurements (Adrian, 1991).

The visualization setup for PIV consists of an illumination source, four high-speed cameras fitted with c-mount, $f=4$ mm lenses (with no additional filters), a synchronization board and a computer with a dedicated software package, which analyzes the images and estimates the flow velocities. The illumination system was a 300W, LED-Flashlight 300 (product of LaVision, www.lavision.de), composed of an array of 72 high-power LEDs fitted in an area of 300×100 mm². The illumination system provided short pulses of high

intensity light synchronized with the high-speed cameras by a Programmable Timing Unit (PTU), operated and controlled by the DaVis 10 software package (Davis, 2018).

The four high-speed cameras used were fitted and aligned in a single unit (Mini-shaker L, product of LaVision, www.lavision.de) as illustrated in Fig. 4.1. The cameras were synchronized with the LED light by the PTU and controlled by the DaVis 10 software. The recording frequency (and by extension, the LED pulse frequency) varied between experiments from 10 to 60 Hz, depending on the rate of liquid injection. Prior to each experiment, the cameras were calibrated for 3D visualization in liquid gelatin and all focused on the area of interest (height x width x depth of 40 x 35 x 10 cm³) in the center of the tank. To calibrate, we submerged a specialized plate (product of LaVision) into the gelatin at the expected location of the dike. The plate has a series of rectangular ridges that run horizontally along the surface, so that in cross-section it resembles a square wave. There are circular markings along the top and bottom of each ridge and the relative location of each marking is mapped into the Davis 10 software, which accounts for distortion due to the tank and the gelatin independently for each camera. Then the calibration plate was removed and the gelatin was left to solidify in the cold room for 72 hours without moving the tank or changing the position of the cameras or their settings. Note that we performed each experiment entirely in our cold room, so we did not notice any issues like condensation affecting the cameras.

Gelatin scatters light and therefore the intensity of light decreases from the side near to the LED light source to the other side. To overcome this issue, small mirrors were positioned on the opposing side of the tank to reflect the light back to the observation area. The wall of tank on the LED side was also partly covered with opaque, black paper to let the light shine through a relatively narrow gap (5 to 10 cm) into the gelatin and diminish the light pollution due to scattering outside of the region of interest. With this arrangement, the light source would only illuminate the region of interest with nearly-uniform light intensity.

We initially used spherical polyamide particles of 55 and 100 μm diameter (products of LaVision). However our gelatin scatters more light than we expected, which introduced noise to the measurements and made it difficult to detect the particles. For the experiments presented here, we switched to larger 800-1000 μm Pliolite particles (products of Omnova), which we were better-able to detect. These particles have a density of 1030 kg/m³ and a refractive index of 1.558. They have a sufficiently slow settling velocity (which we measured) in comparison to the liquid velocity, such that the liquid flows faster by a factor of 5 to 50. They were most prone to fall out of suspension in experiments with water. We mixed these particles into the liquid using a simple stirring rod and added a drop of dish soap to weaken the surface tension and help the particles submerge. We then covered the container with plastic wrap and let the liquid rest overnight, to allow the particles to settle to the bottom and completely immerse. When we performed an experiment, we affixed an overhead stirrer to the supply beaker to

maintain homogeneous suspension of the particles as they were pumped into the tank apparatus.

The images captured by the four high-speed cameras were analyzed by the DaVis 10 software to measure the displacement of the tracer particles in two or more consecutive images. DaVis software divides each image into small sub-regions called interrogation areas and measures the displacement of particles within them using 2D or 3D cross correlation. In this study, two sizes of interrogation areas 64 x 64 and 32 x 32 pixels were selected and the displacement of particles within each interrogation region was statistically determined by measuring a 2D or 3D dimensional correlation of the interrogation region in successive images. The image analysis used multi-pass cross correlation with decreasing interrogation window size. The evaluation starts in the first pass with the initial interrogation window size (64 x 64 pixels) and calculates a reference vector field. In the next pass the window size is half the size of the previous pass (32 x 32 pixels) and the vector calculated in the first pass is used as a best-choice window shift. The interrogation areas had 50% overlap and two iterations were carried out for each window, in which the velocity vector calculated from the first iteration was used as a reference vector for the second iteration. This procedure helps to correlate the right particles and significantly improves the signal-to-noise ratio of the measurements.

As noted, we were unable to use 3D PTV measurements for experiments with water and vegetable oil, since they have different optical properties from gelatin. When we attempted to combine the measurements from the 4 cameras to into a 3D motion measurement, the calibration failed; essentially our camera became cross-eyed and could not understand the particle positions. When we injected gelatin, for S series experiments, the calibration succeeded and 3D measurements were possible.

In order to compare all the experimental results, we chose to focus on the 2D PIV measurements. We choose the clearest-looking video from each experiment (usually from one of the four camera looking “away” from the light source), and choose to analyze the region of the video where the dike was captured. The regions of the image outside of the dike were screened out to prevent noisy measurements.

4.2.3 Scaling

We performed a dimensionless analysis of our results to compare them with natural systems. Dike propagation is driven by a combination of buoyancy (P_b) and source reservoir overpressure (ΔP). It is resisted by a combination of the fracture pressure (P_f) at the dike tip (i.e. resistance to fracturation), elastic pressure (P_e) in the medium (i.e. resistance of the medium to deformation) and viscous loss (P_v) (i.e. resistance to flow), such that

$$\Delta P + P_b \sim P_f + P_e + P_v . \quad (4.4)$$

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns

Each of these forces (except the overpressure) are typically quantified by individual pressure scales that are dependent on the dike's geometry (Lister, 1990; Lister and Kerr 1991; Taisne et al., 2011b):

$$P_b = \Delta\rho g L, \quad (4.5)$$

$$P_f = K_c / \min(L, B)^{1/2}, \quad (4.6)$$

$$P_e = EH / \min(L, B), \quad (4.7)$$

and

$$P_v = \mu v L / H^2 = \mu Q L / (B H^3). \quad (4.8)$$

In the above equations, $\Delta\rho$ is the liquid-medium density contrast, g is the gravitational acceleration, L , B , and H are the dike's vertical length, horizontal breadth and opening thickness, μ is viscosity, v is the liquid's characteristic velocity scale and Q is the volumetric flux ($Q=BHv$). The denominators of equations 4.6 and 4.7 use the smaller of the L and B dimensions, which we assume characterizes the magnitude of their respective stresses, following Taisne et al. (2011).

As a dike grows, the pressure balance can evolve, so that the dominating pressures vary with time. For example, the ΔP is the dominant driving force when the dike is small, whereas the P_b becomes significant as the dike grows in length. For the resisting forces, the P_f is initially large and becomes smaller as the dike grows. The P_e and P_v are dependent on both the length, L , or breadth, B , and the thickness, H . For dikes that become thinner as they grow, the P_e starts large and decreases as the dike grows. The P_v , by contrast, increases as the dike grows.

The dike is characterized as “small” or “large” via a parameter known as the buoyancy length, L_b . When the dike reaches the buoyancy length, the buoyancy pressure become strong enough to propagate the dike without any additional overpressure and balances with the fracturation pressure, such that $P_b \sim P_f$ (Menand & Tait, 2002). The buoyancy is dependent on the vertical length, L , while the fracturation is dependent on the smaller of the length and breadth, B , dimensions. For a penny-shaped crack, which is commonly defined as a crack with $L = B$, the fracturation is generally presented using the crack length, $P_f = K_c / L^{1/2}$. The balance, $P_b \sim P_f$ therefore defines the buoyancy length as,

$$L_b = (K_c / \Delta\rho g)^{2/3}. \quad (4.9)$$

When the dike length reaches the buoyancy length, the buoyancy pressure becomes significant and the dike begins to become vertically-elongated. The propagation becomes driven by the upper, head of the dike, which is roughly penny-shaped, as has been observed by Taisne et al. (2011). If we assume a penny-shaped head, then the buoyancy pressure can be defined as $P_b \approx \Delta\rho g B$. In the lower, thinner, tail region by contrast,

buoyancy balances with viscous forces and therefore does not contribute to propagation (Weertman, 1980). According to this framework, we therefore define the pressure ratio, P^* , to determine the buoyancy pressure's relative magnitude:

$$P^* = P_b/P_f = \Delta\rho g B^{3/2}/K_c = B^{3/2}/L_b^{3/2} = B/L_b . \quad (4.10)$$

When the buoyancy pressure is less than the fracturation pressure ($P^* < 1$), the dike cannot propagate without an additional overpressure from the source reservoir. In this case, propagation is radial and the dike maintains a penny shape (Menand and Tait, 2002; Taisne et al., 2011). When the dike reaches the buoyancy length, L_b , $P^* = 1$ and as it continues to grow the buoyancy pressure becomes larger than the fracturation pressure and P^* shows its relative magnitude. Note that, for a constant-volume dike, with no influx, the penny-shaped head does not significantly grow in length, so the P^* parameter remains nearly constant. However, if there is influx, the dike head may enlarge, giving rise to an increase in P^* .

We also define the dimensionless length, L^* , which is simply a length ratio, showing how long the dike is, relative to its buoyancy length:

$$L^* = L/L_b . \quad (4.11)$$

The liquid flow within the dike is governed by the Reynolds number, Re , which is a ratio between inertial forces, defined as defined as $\rho v^2/L$, and the viscous forces, defined as $\mu v/H^2$ (Schlichting, 1979):

$$Re = (\rho v^2/L)/(\mu v/H^2) . \quad (4.12)$$

The dynamic pressure, P_d , of a liquid is the kinetic energy per unit volume and is defined as $P_d = \rho v^2$. The Reynolds number in equation 4.12 can therefore be written as the ratio of dynamic pressure to the viscous pressure, shown in equation 4.8:

$$Re = P_d/P_v = \rho v^2/(\mu v L/H^2) = \rho QH/(\mu BL) , \quad (4.13)$$

which is how we estimate it in our experiments.

We will scale the time using the other dimensionless numbers. The Reynolds number quantifies the velocity to indicate whether it is relatively high or low and so we will treat it as a dimensionless velocity. The dimensionless time, t^* , is simply scaled via the dimensionless length and Reynolds number via $t^* = L^*/Re$. For our experiments, t^* has a range of 3×10^{-2} to 5×10^4 (Table 4.2), while for nature it ranges from 10^{-4} to 10^{14} , due to the large range for Re .

Our analogue experiments are scaled with natural conditions for basaltic dikes, via the dimensionless parameters, P^* , L^* and Re , which depend on the geometry of the dike. We measured the dike length and breadth from the video recordings. Since we were unable

to attain 3D data for all of our experiments, we estimated the mean thickness using the dike volume (which for a constant flux dike depends on the time) and surface area. Table 2 shows typical values for basaltic magma properties and dike geometries in nature, as well as their analogues in our experiments. There is good overlap for P^* and Re between experiments and natural systems. The L^* overlaps to a lesser degree with magmatic systems, mainly due to their size proportions. Magmatic dikes reach their buoyancy length when they grow to several hundreds of meters in length, but may continue to propagate for kilometers. In our experimental setting, the buoyancy length for oil dikes is half the size of our tank, therefore the maximum $L^* \approx 2$. For water dikes, the buoyancy length is greater than the size of our tank and therefore models fairly small dikes ($L^* \approx 0.6$).

Table 4.2. Comparison of experimental and natural basaltic dikes. We estimate the range for dimensionless numbers in natural systems using equations 4.11 to 4.13 and values from this table (e.g. $10^2 < \mu < 10^4$).

Parameter	Experiments	Nature
L (m)	0.04* - 0.35	$10^{1*} - 10^4$
B (m)	0.04* - 0.28	$10^{1*} - 10^4$
H (m)	0.004 - 0.016	0.1 - 10
ρ (kg/m ³)	910 - 1000	2600
$\Delta\rho$ (kg/m ³)	10 - 100	$10^1 - 10^2$
μ (Pa·s)	$10^{-3} - 10^{-1}$	$10^2 - 10^4$
K_c (Pa·m ^{1/2})	60 - 100	10^6
Q (m ³ /s)	0.03 - 2.8×10^{-6}	1 - 100
L_b (m)	0.14 - 0.91	50 - 1200
L^*	0.1 - 2	$10^{-2} - 10^2$
P^*	0.06 - 1.0	$10^{-2} - 10^0$
Re	$4 \times 10^{-6} - 3$	$10^{-10} - 10^2$
t^*	$3 \times 10^{-2} - 5 \times 10^4$	$10^{-4} - 10^{14}$

* Lower estimates given to estimate dimensionless numbers when the dike is small

4.3 Results

4.3.1 Controls on circulation

When we injected water, we made similar observations to a previous study (Kavanagh et al., 2018), in which the dike displayed a strong circulation throughout the entire propagation (Fig. 4.2b1 to b3). This took the form of a centralized upward jet, an outward spreading flow near the upper fringe of the dike and a downward return flow along the lateral fringes. Since the experiment was gravity-driven at a constant pressure, it naturally had a range of fluxes, from 15 to 350 mL/min. At the beginning of the experiment, the influx had a peak value of 350 mL/min, however shortly afterward (~1 minute into the experiment), the particles formed a clog in a kink in the supply tube and restricted flow into the dike. We then shook the tube to unclog it and the volume influx peaked again to 250 mL/min and then gradually decreased to 15 mL/min. We analyzed data from before the clog occurred, representing the beginning of the propagation, and data from after we freed the clog, representing the middle and end of propagation, as well as the eruption (Fig. 4.2b2 to b5). We did not notice any instabilities in the upward jet that Kavanagh et al. (2018) observed as the dike approached eruption, nor did we observe any pinching of the dike tail, leading to bifurcation of the basal part of the jet. This is surprising, as the conditions of their experiments were very similar to C5 and we cannot come up with an explanation for this small discrepancy. When the dike erupted, it began with an initial expulsion of the liquid from the dike, followed by a steady-state ascending flow pattern. Note that we did not perform multiple experiments with water to examine the flux, but the variable flux that resulted allows us to investigate its effect.

When we injected oil, the flow pattern showed only an initial circulation when the dike was small (Fig. 4.2a1), which was accompanied by radial, penny-shaped propagation. However, as the dikes continued to grow, they became vertically elongated and the circulation stopped entirely, exhibiting only upward flow (Fig. 4.2a2 and a3). Though the flux varied over two orders of magnitude, the propagation and overall flow pattern varied little between experiments (C1-C4, Table 4.1). The C Series experiments suggest that there is rheology control on circulation and that the flux has little to no effect. The dike erupted in a similar fashion to C5, in that they rapidly expelled much of the intruded volume and then took on a steady state ascending flow (Fig. 4.2a4 and a5).

The S series experiments represent a variation on the C series, since the liquid is prone to solidification. For experiments S1 and S2, the magma analogue was fairly buoyant and evolved in a similar way to the experiments made with oil. They began with an initial phase of internal circulation, coupled with radial dike growth and, as buoyancy became pronounced, they similarly began to propagate vertically. However, solidification at the lateral fringes of the dike encouraged flow near the warmer, central region (Fig. 4.2c1 to c3), and the circulation pattern evolved to take the form of an upward jet, positioned between two bands of return flow. This circulation persisted throughout the experiments.

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to crystal growth patterns

Experiment S3, which was not buoyant, evolved in a similar way to the experiment C5, which was made with water. It maintained circulation throughout the entire propagation and similarly maintained a penny-shaped form with radial growth. Solidification constrained the flow to a lesser degree than S1 and S2 (but in a similar form to that presented in Fig. 4.2c1 to c3) and was most evident just before eruption. The eruptions in the S series were similar to the C series and began with an initial expulsion of the liquid, followed by a steady-state, unidirectional flow along the dike's central axis (Fig. 4.2c4 and c5). Their upward flow fields were comparatively narrower however, as solidification restricted the available space for liquid movement, causing the flow to channelize.

Experiments S1 and S2 are also presented in chapter 3 (respectively experiments 9 and 11 in Table 3.1) and the PIV data are from the first injection of their respective experiments. These results show that the dikes were able to channelize to a degree during the first injection, even prior to eruption, and had a width of 5 to 10 cm. By comparison, experiments in chapter 3 that formed a stable conduit tended to narrow to 1 to 3 cm. These experiments S1 and S2 respectively correspond to a nested dike experiment and a dike swarm. The subsequent injection from the nested dike experiment strongly resembles the steady state eruptive flow pattern shown in Fig. 4.2c5, implying the flow pattern strongly influenced the shape of the next intrusion.

Fig. 4.2. 2D PIV results from selected experiments, to contrast the flow evolution between oil-, water- and gelatin-filled dikes (respectively experiments C1, C5 and S1). Color represents the vertical component of velocity (on a log scale), in which red and blue respectively correspond to upward and downward velocity. All images a common scale bar shown to the bottom left. Vectors indicate flow field profiles. Boxes mark the locations of measurements for the mean upward and downward velocity. All dikes displayed internal circulation when small, while only weakly-buoyant and solidification-prone dikes (C5 and the S series) maintain it when larger. Oil-filled (C1) dikes transitioned to a simple ascending flow profile as they grew.

As we see in Fig. 4.2, circulation is exhibited in the dike when there are upward and downward liquid movement. In order to quantify the circulation, liquid velocity

measurements are taken from certain parts of the dike, which are the central, upward jet, v_{cent} , and the flow at the dike's lateral peripheries, v_{periph} , which may be either upward or downward. We quantify these velocities by measuring the mean velocities within regions that best characterizes the flow in the central jet and along the lateral peripheries (represented by purple boxes in Fig. 4.2). This is a form of validating our data, since our setup sometimes produces noisy results, for regions of the dike have a low particle concentration or are obscured by plumose structures that develop along the dike surface (see Fig. 1.3 in chapter 1) or shadowing due to the dike itself (i.e. the side of the dike closest to the light source is well-lit, while the opposite side is shadowed). All of these factors are evident in Fig. 4.2a3 to a5, in that the right side of the dike appears to have noisy measurements. This dike had a somewhat low seeding concentration and shadowing limited particle detection on that side, so the flow field appears noisy, especially during the steady state phase of the eruption. A plumose structure also developed along the central axis of the dike, which is most evident at the onset of eruption as a discontinuity in the velocity field.

To non-dimensionalize the internal flow, we define the degree of circulation, v_c^* , as the difference between v_{cent} and v_{periph} , scaled by v_{cent} :

$$v_c^* = (v_{cent} - v_{periph}) / v_{cent} . \quad (4.14)$$

In this way, $0 \leq v_c^* \leq 2$, in which circulation is represented by values approaching 2, when v_{cent} and v_{periph} have the same magnitude and opposite directions. When $v_c^* < 1$, there is no circulation, since the flow at the periphery is upward. We compare v_c^* against the dimensionless parameters P^* and Re , introduced in section 2.3, to determine how it is affected by buoyancy and influx (Fig. 4.3).

Fig. 4.3. (a) The circulation parameter, $v_c^* = (v_{cent} - v_{periph}) / v_{cent}$, against the (a) dimensionless buoyancy, $P^* = P_b / P_f$ (B / L_b , equation 4.10) and (b) the Reynolds number. For convenience, we also show P^* against Re (c) and the three parameters together in 3D (d). Blue

markings represent the C series experiments and white markings represent the S series. The circle symbols represent the buoyant experiments (oil injected or liquid gelatin injected into a solid gelatin with a boosted density; C1-C4 and S1-S2) and triangles represent non-buoyant experiments (water injected or liquid gelatin into a plain gelatin; C5 and S3). For each experiment, three measurements are presented, representing when the dikes were small, medium or large (before eruption) and boxes (in c and d) indicate the range of values estimated for each experiment. The surface shown in (d) is fitted to the C series data by minimizing the sum of the squares of the residuals (equation shown). Vertical black lines between data points and the surface indicate the mismatch. The S series vertical lines are dashed to illustrate they are not used to fit this surface.

To determine the relative influence of P^* and Re on v_c^* , we plot them in 3D and fit a surface (Fig. 4.4d), which helps to visualize their 3D distribution. Since v_c^* varies between 0 and 2, we set up a function that approaches 2 at large values of Re and small values of P^* ; conversely it approaches 0 as Re approaches 0 and P^* becomes large. This equation takes the form of two convoluted exponential functions, shown below in separate brackets:

$$v_c^* = [(2-A_1)\exp(-A_2P^*)+A_1] [-\exp(-A_3Re^{C_4}+A_5)+2] . \quad (4.16)$$

We then fit this equation to our data by minimizing the sum of the squares of the residuals, solving for the constants A_1 , A_2 , A_3 , A_4 and A_5 (values shown in Fig. 4.4d). This procedure shows v_c^* is strongly dependent on P^* and very weakly dependent on Re , which is essentially removed from the equation by the small values of A_3 and A_4 . This agrees with observations in which the flux does not seem to affect the dike's internal flow pattern.

The S series data do not follow the same relationship as the C series (equation 4.15), which is apparent in Fig. 4.3d. We expect that this is due to the solidification in the dike liquid, which modulates its internal flow. As we discussed for Fig. 4.2, the flow pattern in the S series entailed a degree of circulation throughout propagation, which was absent in C1-C4 for dikes longer than the buoyancy length. In the S series, the central upward jet supplies heat and maintains a high velocity, while the peripheries cool off and solidify. Solidification at the peripheries constrains the available space for return flow and causes it to take a relatively high downward velocity. As a result, all of the S series experiments maintained a similar degrees of circulation to each other and did not follow equation 4.16. Considering all of the experiments, we determine that the P^* parameter has a strong control on circulation. When P^* is small, the buoyancy forces are negligible, the dike is likely driven by overpressure at its source and the internal flow has a strong circulation. However, solidification modifies the circulation and adds complexity to the flow dynamics. The length scale over which buoyancy forces are weak and circulation must occur is described by equation 4.9 (Fig. 4.4). For basaltic dikes that have a density difference with the medium between 10^0 and 10^2 kg/m³, propagating through a medium

with a fracture toughness of $1 \text{ MPa}\cdot\text{m}^{1/2}$ (Meredith & Atkinson, 1985), internal circulation occurs to a minimum length of a tens of meters to around two kilometers.

Fig. 4.4. (a) An illustration of a dike at different stages of growth and driven by buoyancy. Dikes shorter than the buoyancy length, L_b , circulate, whereas longer dikes do not. The dimensionless length, $L^*=L/L_b$, is shown for our experiments. In all experiments, the dikes grew to a length of ~ 40 cm; however their L_b and thus L^* are different. Oil is the most buoyant fluid, so it has the smallest L_b and largest L^* . (b) Estimates of L_b and L^* for basaltic magma, assuming a K_c of $1 \text{ MPa}\cdot\text{m}^{1/2}$ and final dike length of 3 km. Both parameters are therefore a function of the magma-crust density difference.

4.3.2 Controls on propagation velocity

A dike's propagation velocity is more-readily monitored (via seismicity) than is the internal fluid velocity. We therefore incorporate propagation velocities into our analysis. We define the dimensionless crack propagation velocity, v_p^* , as the proportion of the horizontal component of its propagation, $\partial B/\partial t$, in which it grows in breadth, against the vertical component, $\partial L/\partial t$, in which it grows in length:

$$v_p^* = (\partial B/\partial t)/(\partial L/\partial t) . \quad (4.17)$$

For dikes that grow radially, in a penny-shaped form, v_p^* approaches a value of 1, whereas dikes that grow primarily vertically, in a tabular form, have values approaching 0. This propagation velocity ratio conveniently relates to the same dimensionless parameters discussed in section 4.3.1 (Fig. 4.5). As the dike grows, the horizontal component of propagation velocity drops, which coincides both with a gain of buoyancy

force (P^* increases) and of viscous force (the dike's opening width and therefore Re decrease as the dike grows). When the dike surpasses the buoyancy length, the internal flow velocity responds by losing circulation, whereas the propagation responds by growing primarily vertically (Fig. 4.5a).

Fig. 4.5. (a) The dimensionless propagation velocity, v_p^* , against dimensionless buoyancy, P^* . Markings are as in Fig. 4.3. The v_p^* parameter is the ratio of its horizontal propagation velocity to its vertical. As P^* increases, the dike's horizontal growth diminishes and becomes dominated by vertical growth. (b) The v_p^* parameter against Reynolds number. As the dike grows, viscous forces dominate. The internal flow thereby becomes driven by buoyancy forces, causing vertical growth. (c) The dimensionless parameter space for the experiments. (d) The three parameters plotted in 3D. The v_p^* parameter decreases as both a function of P^* and Re , thus the oblique view angle. A best-fitting exponential surface is provided (equation shown) and the mismatch with data are shown by the black vertical lines.

The propagation velocity seems to have a more-defined relationship with Re than does the internal flow (compare Fig. 4.3b and Fig. 4.5b). We can visualize this influence on v_p^* by obliquely viewing the 3D data presented in Fig. 4.5d, which shows a well-defined trend; the v_p^* is dependent on the convolution of P^* and Re . By comparison, the relationships are more-scattered when comparing v_p^* to either P^* or Re individually, especially for small values of v_p^* (Fig. 4.5a & b).

We quantify the influence of each parameter by plotting a surface to the data, which takes the form of two convoluted exponential functions (similar to the function for v_c^*):

$$v_p^* = \exp[-A_1(P^*)^{A_2}][1 - \exp(-A_3Re^{A_4})] . \quad (4.18)$$

This equation allows v_p^* to approach a value of 1 at low P^* and high Re ; conversely it approaches 0 at high P^* and low Re . We then solve for the constant A_1, A_2, A_3 and A_4 , by minimizing the sum of the square of the residuals, between the data and the model surface (constant values shown in Fig. 4.5d). Indeed, the exponent associated to Re is larger for v_p^* than v_c^* , reinforcing the idea that it has a greater effect on the propagation velocities than the degree of circulation.

4.3.3 Effects of pressure

The source overpressure is an important parameter which initiates dike growth and can drive propagation. We measured its time series evolution for our C series experiments. Most of our experiments were driven by a peristaltic pump, which maintains a constant flux and allows pressure to fluctuate. Such pumps provide whatever pressure is necessary to maintain the flow rate and therefore the transducer records the resistance of the medium to fracture propagation (equations 4.6 and 4.7), as well as viscous pressure drop due to the flow (equation 4.8). This is exhibited in the pressure time series, which recorded a spike at the start of the experiments, when the dikes began to propagate (Fig. 4.6). After the initial spike, there is a period over which the pressure rapidly, then gradually, decreased, which corresponded to the dike growing to the buoyancy length (Fig. 4.6c, “regime change” marking). Once the dike reached this length, the pressure either remained constant (for lower fluxes) or increased (for higher fluxes).

Fig. 4.6. (a) Raw data from our pressure transducer for experiment C1. The pressure spikes when the dike propagation starts. (b) A zoomed-in view of the region designated by

narrow black box in (a). As the pump rotates, it induces a cyclic pressure pattern. (c) We smooth the cyclic pattern by taking the median value over a 40 s moving window. For faster flux experiments, we used a smaller smoothing window. As the dike propagates, the pressure initially drops until it reaches a steady value, which coincides with the dike becoming buoyantly controlled. Afterwards, the pressure likely indicates the force needed to supply liquid into the gelatin.

The resistance to fracture propagation is also evident when comparing experiments. Plotting the time series together reveals little variation in pressure between experiments, even though they had very different fluxes (Fig. 4.7a). In fact, the pressure seems to be primarily a function of the Young's modulus, which plays a role in the dike's geometry and thus the elastic and viscous pressures that resist flow into the dike. This is evident when we scale the pressure measurements by the Young's modulus, which shows an initial peak in pressure, followed by values close to one (Fig. 4.7b). Any variations in pressure with time reflects the evolution of elastic pressure and viscous pressure drop, which change as the dike geometry evolves. For example, the gradual increase of pressure in experiments C2 and C5 is likely due to an increase of viscous pressure drop, since the dikes become thinner as they grow in length.

Fig. 4.7. (a) Comparison of pressure measurements from the C series of experiments, with normalized time scales, in which 0 is the experiment start and 1 is the time of eruption. Color represents the relative flux (dark is high, > 100 mL/min, light is low, ~ 1 mL/min) and solid and dashed lines respectively indicate pump-driven and gravity-driven experiments. Experiments all begin with a peak pressure, followed by a drop and sometimes followed by another gradual increase. Some show a pressure drop at the end, corresponding to when we stopped the liquid flow. Note, during C5, a blockage in our supply tube cut off flow to the dike, affecting the pressure time series; we freed the flow midway through the experiment, causing a secondary spike. (b) A similar plot, with the pressure scaled by the

Young's modulus. After the initial peaks, the normalized pressure measurements have values close to one, indicating the strong control of the Young's modulus on the reservoir pressure.

Interestingly, our gravity-driven experiments (C4 & C5), which were an attempt to force a constant-pressure, variable flux condition, yielded similar results to the pump-driven experiments. Both had an initial pressure spike at the start of the experiment, followed by a decreases as the dike approached the buoyancy length. Again, this likely demonstrates that the pressure recorded the ability of liquid to enter the dike, so as the dike grew, its resistance to influx evolved. Surprisingly, C4 (oil injected) maintained a virtually constant flux, varying by less than 1 mL/min over the course of the experiment. This is likely due to the viscous pressure drop in our supply tubing, which has a greater magnitude than the resisting forces in the dike.

Unfortunately, the effect of the source reservoir's overpressure on liquid circulation and crack growth remains unclear. If we look at the circulation parameter, v_c^* , against overpressure, we notice that there is no consistent correlation between experiments. In each experiment, the circulation diminished with time, as the dikes grew and their buoyancy forces became significant. Similarly, the propagation velocity parameter v_p^* decreased as each dike grew. The pressure measurements however, show an initial peak and sharp drop, followed by either flat or increasing magnitudes. The pressure profiles essentially show varying overpressure, while v_c^* and v_p^* decrease monotonically.

The effect of overpressure on circulation and propagation is somewhat intuitive and we would expect that, when overpressure dominates, the circulation is strong and that propagation takes the form of radial, penny-shaped growth. Since our scaled pressure measurements (Fig. 4.7b) do not show a large variability (even including the initial pressure peak), we believe there is potential for future research examining the effect of very high pressures, much higher than the Young's modulus. This would require the construction of a truly constant-pressure experimental setup, one which could achieve a wider range of overpressures.

4.4 Discussion

4.4.1 Dike shape and stability

Here we discuss the dike shape. We had planned to use 3D measurements from our visualization system to describe the full 3D shape of the dike, but as we described in

section 4.2.2, the 3D calibration failed for oil and water-filled dikes. 3D measurements of the S series experiments indicate the dikes are thicker near the upper tip and thinner in the tail, but the depth resolution is not high enough to fully understand the shape. For dimensionless analysis introduced in 4.2.3, we quantified the thickness using the volume and surface area, such that the thickness represents a mean value. As such it is difficult for us to comment on the cross-sectional shape of these dikes. We know from observations of oil-filled dikes in chapter 1 that the dikes reaching the buoyancy length began to decouple from the injection location and take on a larger component of vertical growth than horizontal, even as the injection continued. They took on a tear drop shape, with a relatively thicker head region and a long, thinner tail. This shape is similar to what is described by 2D models of propagation, in which the Weertman model describes the dike as tear drop shaped with a closed tail (Weertman, 1971; Dahm, 2000b) and lubrication theory describes it as lens-like shaped head with a tail of uniform thickness (Lister, 1990; Roper & Lister, 2007). Such a shape has been observed in many other studies investigating constant volume dikes (Takada, 1900; Watanabe et al., 2002; Rivalta et al., 2005; Taisne & Tait, 2009) and has also been observed in constant flux experiments (Kavanagh et al., 2018). The tear drop shape is usually attributed to a significant buoyancy force, but as Kavanagh et al. (2018) show using analogue experiments with water-filled dikes, it can develop for weakly buoyant dikes as well. While we do not have the full shape of the dikes characterized for the experiments in this chapter, there is no reason to think that they differ from any of these other observations.

We note that the both the 3D geometry and the flow pattern in all dikes evolved with time, so that propagation transitions from radial and penny-shaped to vertically-dominated, while the flow transitions from circulation to vertical ascent; these transitions occur when a dike grows to the buoyancy length. However, we note that even relatively long dikes continued to increase in breadth and velocity (both the liquid and propagation velocity) and evolve in flow pattern so that the value of v_c^* monotonically decreased. This implies that, on the length scale associated to our experiments, $L^* \leq 2$, constant flux dikes do not reach the equilibrium conditions. Instead, they undergo a shape and flow evolution in a series of phases, similar to what Dahm (2000b) describes for constant volume dikes. The applicability of the lubrication theory 2D model, introduced in section 4.1, depend on the stage of growth. The circulation we observed during influx for dikes with $L^* < 1$ conflicts with the 2D simplification, in which flow is uniform along the dike's breadth axis. However, as the dike becomes very long, such that $L^* \gg 1$, we assume the flow become closer to uniform, such that $v_c^* \ll 1$; experiments with highly-buoyant liquid would need to be conducted to confirm this. Furthermore, we would expect that dikes can eventually reach an equilibrium, due to the balance of buoyancy and viscous pressure drop. Roper and Lister (2007) defined a steady propagation rate, c , which is derived from this balance, including the magma influx. The vertical rate of propagation, $\partial L / \partial t$, in our experiments is generally lower than this rate, however as the dikes grew long, they began to approach the theoretical value (Fig. 4.8a). Similarly, the dike propagation velocity

more-closely resembles the theoretical value when its flow profile was closest to uniform (lowest values of v_c^* , Fig. 4.8b)

Fig. 4.8. The steady state propagation velocity, c , described by Roper & Lister (2007), scaled by our measured propagation velocity, $\partial L / \partial t$, shown against (a) dimensionless length, L^* , and (b) the circulation parameter, v_c^* . Markings are as in Fig. 4.3. Longer dikes show a better agreement with the c parameter, in which $c / (\partial L / \partial t) = 1$ (horizontal dashed line). Similarly, the parameter has better agreement for lower values of v_c^* , in which $v_c^* = 0$ represents uniform flow.

4.4.2 Linking to crystal growth patterns

A key advancement we can make is to draw a relationship between the forces acting on a dike to the circulation that occurs inside. Such dynamics can potentially be recorded in crystal textures, in which concentric layers indicate the conditions of their surrounding magma and diffusion profiles indicate the associated time scales (Gordeychik et al., 2018). For a crystal to record changes of environment, it needs sufficient time to grow a layer and recent studies have shown that olivine crystals can record such information on the time scale of a propagating dike (Albert et al., 2018; Gordeychik et al., 2018). Similar patterns have been identified in plagioclase crystals from volcanic fields in Mexico (Crabtree & Waters, 2017) and New Zealand (Coote et al., 2018; Shane & Coote, 2018), but were attributed either to recharge and mixing events or to progressive degassing and cooling of the magma.

We know from our observations that the velocity of an experimental dike's central upward jet increases as the dike grows. Meanwhile, the velocity at the lateral fringes reverse from downward to upward flow as the dike surpasses the buoyancy length. In nature, if a dike has an internal magma circulation, entrained crystals may record pressure swings due to their ascent and descent (Fig. 4.9a). Similarly, they may record

temperature swings due to cooling in the dike and re-warming in the central, upward supply jet. If natural dikes behave similarly to our experiments, then for a dike driven by a relatively-small reservoir overpressure (i.e. buoyancy forces are dominant to overpressure), the crystals cease circulation after the dike reaches the buoyancy length. The pressure associated to the supply reservoir (i.e. the crystals' initial depth) and to the oscillating phase of propagation (i.e. the depth above which crystals no longer circulate) allow us to constrain the buoyancy length of the dike (Fig. 4.9b). The volume of magma, V , associated to this circulating phase is characterized by the buoyancy length, such that $V = \pi H L_b^2 / 4$, and for the range of values of H given in Table 4.2 is less than 10^7 m^3 .

Fig. 4.9. (a) An illustration of the path a crystal may follow, inside of a propagating dike. The blue dashed line indicates a crystal's hypothetical path, the black solid contours indicate how the dike's shape changes as it grows and the black arrows indicate magma flow pattern evolution. The blue x markings indicate example locations where a layer may grow and the blue circle indicates the location at the start of eruption. The red x mark shows where the crystal starts to make its final descent, which can be used as a proxy for the buoyancy length. (b) The depth time series associated to the path in (a). By constraining the depth range of the circulation pattern, we may be able to estimate some of the forces acting on the dike.

However, as we described in section 4.3.3, the overpressure in our experiments scaled to the Young's modulus of the gelatin and therefore did not change much with flux. Since we were unable to produce a wide range of overpressures, we cannot well-deduce its effect on circulation. In all likelihood, the pressure has a strong control on circulation, driving influx and radial dike propagation/growth, in the same way that the buoyancy

forces inhibit circulation and cause vertical dike propagation. If a hypothetical crystal indicates that circulation occurred over a length scale much longer than 1 km (i.e. longer than the maximum L_b in Fig. 4.4b), this would be strong evidence that the overpressure can overwhelm the buoyancy forces and promote circulation, even for long dikes. In that case, the crystals would constrain the ratio of overpressure to buoyancy forces.

4.4.3 Linking to geophysical measurements

As described in the results, the dimensionless parameters P^* and Re also relate with the propagation velocity parameter, v_p^* , such that propagation is predominantly vertical for large P^* and small Re . Conversely, it is predominantly radial for small P^* and large Re . Both P^* and Re are dependent on the dike geometry, which can be estimated via geophysical measurements. Surface deformation measurements can inform us of the dike's length, breadth, opening width and even volumetric flux (Ayele et al., 2007; Pascal et al., 2014), while volcano-tectonic earthquake hypocenters can inform the length, breadth, dike shape and propagation velocity (Hayashi & Morita, 2003; Morita et al., 2006; Taisne et al., 2011). For example, Hayashi & Morita (2003) mapped the hypocenters associated to the 1998 Izu Peninsula seismic swarm and inferred that the hypocenters were temporally clustered at the actively propagating fringes of the dike. Such measurements provided information not only of the dike's propagation velocity, but also of the dimensions and shape, which was interpreted to grow both upwards and downwards from a level of neutral buoyancy. We would argue that solidification may also play a role, by halting propagation at one fringe of a dike and encouraging it along another; for example Taisne and Tait (2011) showed through analogue experiments that, when solidification causes a dike to pause, propagation may initiate again along the lateral margins.

Such measurements, together with our results, can help us to estimate a range of the magma rheology for unerupted intrusions. By measuring the vertical and horizontal components of the propagation velocity through seismic data, we can gain a sense of the magnitude of P^* and Re via equation 4.18 (Fig. 4.10a). By also measuring the dike dimensions and volumetric flux through geodetic means, we can acquire enough information to constrain the magma rheology via equations 4.10 and 4.13 (Fig. 4.10b & c). For example, if we hypothetically measure the propagation velocity, so that $v_p^* = 0.6$, we can estimate that $P^* < 1$ and $Re > 10^{-3}$, via the equation 4.18 (Fig. 10a, white dashed line). If we additionally estimate the dike dimensions to be $500 \times 500 \times 1 \text{ m}^3$ and the flux to be $100 \text{ m}^3/\text{s}$, we can estimate the corresponding magma rheology, $\mu/\rho < 1 \text{ m}^2/\text{s}$ via equation 4.13 (Fig. 4.10b, dashed and solid blue line). However, some rheologies are unlikely (e.g. magma that has a viscosity similar to water) or statistically impossible based on a priori knowledge when available, so we can in turn further constrain the likely magnitude of Re (Fig. 4.10b, solid blue line). We could do a similar exercise to constrain the host rock fracture toughness, using P^* and equation 4.10 (Fig. 4.10c). For the example dike of the

given dimensions, the density difference and fracture toughness should be characterized by $\Delta\rho/K_c < 10^{-5} \text{ kg/Pa}\cdot\text{m}^{5/2}$, signifying the fracture toughness is 5 orders of magnitude larger than the density difference. Using estimates of a dike's geometry and propagation velocity from geophysical data (with the associated uncertainties from such data), we can thereby produce broad estimates of the magma rheology and shed light on the dynamics that drive the magmatic system.

Fig. 4.10. (a) A map of v_p^* against P^* and Re . For any measured value of v_p^* , there are many potential combinations of P^* and Re , which can help us to constrain the magma rheology. We plot the contour for a hypothetical measurement of $v_p^* = 0.6$ (white dashed line), described in the text. The white box shows a further constrained range of P^* and Re and corresponds to the solid blue lines in (b) and (c). (b) For a dike with a Re range from (a), we can estimate the rheology depending on the influx and dimensions, which can be estimated via geophysical measurements. The example in the main text is shown as the blue line. The solid segment shows typical basalt rheologies and the dashed segment shows unrealistic rheologies. (c) Similarly, for a dike with a P^* range shown in (a), we can estimate the density and host fracture toughness depending on the dike dimensions. The solid blue line indicates typical values while the dashed segment indicates values that are unrealistic.

It should be noted that this method works for dikes that have significant horizontal growth, with $v_p^* > 0.1-0.2$. As dikes become completely dominated by vertical growth, and thus become dominated by buoyancy, we lose the ability to determine magma rheology. Looking at the surface in Fig. 4.10a, we can see that, for $P^* > 1$, the surface is approximately uniform (the blue region). All values of $P^* > 1$ and all values of Re return values of $v_p^* \approx 0$, so we can determine the dike is longer than the buoyancy length, but we cannot constrain the buoyancy length's magnitude. Similarly, we cannot estimate a range of Re , even in broad terms like laminar or turbulent, so neither can we constrain the magma rheology.

4.4.4 Pressure-geometry relationships

Our experimental setup provides us with a range of measurements, including the dike geometry, fluid velocity field, propagation velocity, and source overpressure. As such we have access to all the information needed to completely characterize the pressure acting on the dikes. This allows us to define certain scaling parameters that describe the dike's geometry and propagation as functions of these governing pressures. Dike thickness and propagation velocity scales are useful for forward modeling dike propagation as well as relating dike geometry and growth to the forces which control them.

Typically, such analysis uses pressure balances to determine geometry or velocity scales, which are functions of the properties of the system, but independent of scale. For example, Taisne & Tait (2011) show how the mean dike tip propagation velocity can be derived from the balance between buoyancy and fracture pressure in the head of the dike, such that $\partial L / \partial t \sim QG\Delta\rho g / [(1-\nu)K_c^2]$ (see Table 4.3 for symbols), assuming the magma has a Newtonian rheology and propagates through a homogenous half space. We note however that the different pressures controlling propagation (equations 4.4-4.8) evolve with time, as they are all geometry-dependent. Similarly, the dikes in our experiments evolved in shape and propagation velocity as they grew. From this perspective, we derive dike thickness scales which have dependence on the other dimensions; with knowledge of a dike's length or breadth, it is possible to estimate the thickness.

The thickness can be described by various force ratios, depending on which one dominates the system. We will describe here the cases when the viscous pressure drop or elastic pressure are negligible, or when neither are negligible. When viscous pressure drop is negligible, the driving pressure (reservoir pressure plus buoyancy pressure) balances with the elastic pressure:

$$\Delta P + P_b \sim P_e . \quad (4.19)$$

Combining the previous equation with equation 4.7, we obtain the following thickness scale, H_e :

$$H_e \sim B(\Delta P + P_b) / E . \quad (4.20)$$

By contrast, when the elastic pressure is negligible, the driving pressure balances with viscous pressure drop:

$$\Delta P + P_b \sim P_v . \quad (4.21)$$

Combining the previous equation with equation 4.8, we derive the thickness scale, H_v :

$$H_v \sim [\mu QL / B(\Delta P + P_b)]^{1/3} , \quad (4.22)$$

which is similar to the c parameter from Roper & Lister (2007) and compared to in Fig. 4.8. Finally, when no forces are negligible, the thickness is proportional to the force balance between all forces (equation 4.4). In this scenario, the thickness minimizes the pressure due to these forces, such that an increase of thickness corresponds to an increase of P_e and a decrease of P_v :

$$\partial P / \partial H = 0 = \partial P_e / \partial H + \partial P_v / \partial H + \partial P_f / \partial H - \partial \Delta P / \partial H - \partial P_b / \partial H = E/B - 3\mu QL / (BH^4) . \quad (4.23)$$

Only the P_e and P_v contain the H term (equations 4.7 and 4.8, respectively), so the partial derivatives of the other forces equal zero. We therefore define the thickness scale, H_{ev} , as that which minimizes the total pressure in the system:

$$H_{ev} = (3\mu QL/E)^{1/4} , \quad (4.24)$$

which has similarities to the ' K ' parameter presented by Roper & Lister (2007) as the dimensionless fracture toughness. These three scales, H_e , H_v , and H_{ev} can be compared with experimental measurements to verify under which conditions they are true (Fig. 4.a10a).

Fig. 4.11. (a) Comparison of scale thickness estimates H_e , H_{ev} and H_v , against lab measurements, H . We show these estimates against the ratio of viscous pressure drop to fracture pressure, which governs if the dike propagation is fracturation-controlled or viscously-controlled. Marker colors are as indicated in Fig. 4.3, in which blue and white are respectively the C and S series. Note we do not have pressure data for the S series, so the H_e/H and H_v/H trends only include C series measurements. Marker shapes indicate different scale equations. We plot trend lines, in which $H_e/H \sim (P_v/P_f)^{1/3}$, $H_{ev}/H \sim (P_v/P_f)^{1/3}$ and $H_v/H \sim (P_v/P_f)^{1/5}$ for H_v . A perfect match is represented by $H_{e, ev, v}/H = 1$. In our experiments, P_v/P_f was lowest at the onset of propagation and highest just before eruption, since viscous pressure drop increases as the dike grows. (b) A similar plot, with

the axes zoomed out and the trend lines extrapolated. In our experiments, the viscous forces were relatively small, so $10^{-5} < P_v/P_f < 10^{-2}$, however for basaltic dikes, viscous pressure drop is more significant: $10^{-2} < P_v/P_f < 10^3$. This indicates that H_{ev} best-estimates the dike thickness for most basaltic dikes.

We analyze the agreement between the thickness scales and lab measurements, H_e/H , H_{ev}/H and H_v/H , against the ratio of viscous pressure drop to fracture pressure, P_v/P_f , which indicates if the propagation is fracturation-controlled or viscous-controlled. Assuming that a dike is fracturation- or viscous-controlled allows one to simplify the dynamics operating within the dike and, in nature, either case can be true depending on the liquid viscosity and flow rate. In our experiments however, we only achieved fracturation-controlled propagation, $P_v/P_f < 1$ (Fig. 4.10b). We can deduce from our measurements that the H_{ev} scale best-matches the true dike thickness for all but the most-fracturation-dominated experiments. Furthermore, these different thickness ratios follow clear trend lines against P_v/P_f . If we extrapolate these trend over the range of P_v/P_f for basaltic dike, we can predict that H_{ev} best-estimates the thickness of all but the most-viscous-dominated basaltic dikes.

4.4.5 Pressure-propagation relationships

We also derive a propagation velocity argument, to be able to describe growth of the L and B dimensions. We do this using geometric arguments, in that the growth of the dike must match the influx, or, put another way, the volume influx is expended by propagating the dike vertically and horizontally by some proportions, as well as inflating the dike in the H dimension. In the following paragraphs we will investigate how influx and geometry are related, assuming that one has full knowledge of the dimensions, L , B and H , and their growth with time, respectively $\partial L/\partial t$, $\partial B/\partial t$ and $\partial H/\partial t$. We will also discuss the case in which we neglect $\partial H/\partial t$ (if it is small in comparison to H), because it can be a difficult term to analytically define as well as retrieve via monitoring techniques. Our experimental measurements indicate that it tends to have a small magnitude ($\sim 10^{-4} < \partial H/\partial t < 10^{-4}$ m/s), especially in comparison with the magnitude of H , which is proportionally much larger (by a factor of 10^2 to 10^4).

To start, we use a geometric approximation, assuming that the dike takes on an elliptical surface area with a uniform average thickness. The growth is inherently linked to the influx and geometry, so that $Q = \partial V/\partial t$, for a dike of volume, V . Assuming one knows the dike's dimensions and growth rate, the influx can be estimated, Q_{est} , via,

$$Q_{est} = [\pi/(4\Delta t)] [(L + \Delta t \partial L/\partial t) (B + \Delta t \partial B/\partial t) (H + \Delta t \partial H/\partial t) - LBH] , \quad (4.25)$$

over time interval, Δt . For our measurements of the dimensions and growth, the estimated flux indeed very well approximates true values (Fig. 4.12a). As discussed, if we neglect the $\partial H/\partial t$ term, the influx tends to be somewhat overestimated, since the dike

thins out as it grows in length. Regardless, this formulation allows us to make a first-order relationship between the dike influx and growth. Furthermore, this shows that the geometry approximation in equation 4.25 adequately resembles reality.

Fig. 4.12. (a) Estimations of the influx, Q_{est} , against lab measurements, Q , via a simplification of the dike geometry, assuming it is elliptical, with a uniform thickness, using measurements of L , B , H , $\partial L/\partial t$, $\partial B/\partial t$ and $\partial H/\partial t$ (red dots), and comparing with the true influx, Q . The line for $Q_{est}=Q$ is shown. Since $\partial H/\partial t$ is relatively small, it could be approximated to zero (blue dots) and the resulting flux is slightly overestimated. (b) We also show estimates of the propagation velocity in the vertical direction, $\partial L/\partial t_{est}$, against lab measurements, $\partial L/\partial t$. We make the estimates using measurements of Q , L , B , H and $\partial H/\partial t$ (red dots), assuming $\partial B/\partial t$ is a function of $\partial L/\partial t$, P^ and Re . The line for $\partial L/\partial t_{est}=\partial L/\partial t$ is shown. Once again, assuming $\partial H/\partial t=0$ (blue dots) is possible, but results in somewhat underestimated values of $\partial L/\partial t$.*

We will reorganize equation 4.25 to solve for $\partial L/\partial t$. In section 4.3.2, we showed that the propagation dominates in the L direction when P^* is large and Re is small. Since $\partial B/\partial t$ is roughly proportional to $\partial L/\partial t$ as a function of P^* and Re :

$$\begin{aligned} [(4Q\Delta t/\pi + LBH)/(H + \Delta t\partial H/\partial t)] &= (L + \Delta t\partial L/\partial t)(B + \Delta t\partial B/\partial t) = \\ &= (L + \Delta t\partial L/\partial t)[B + F(P^*, Re)\Delta t\partial L/\partial t] . \end{aligned} \quad (4.26)$$

The function $F(P^*, Re)$ is the relationship shown in Fig. 4.5d, in that $\partial B/\partial t = F(P^*, Re)\partial L/\partial t$. The relationship can be further reworked to estimate the vertical propagation, $\partial L/\partial t_{est}$, as

$$\Delta t^2(\partial L/\partial t_{est})^2 + [F(P^*, Re)L\Delta t]\partial L/\partial t_{est} + \{LB - [(4Q\Delta t/\pi + LBH)/(H + \Delta t\partial H/\partial t)]\} = 0 , \quad (4.27)$$

which follows the form of $a_1x^2 + a_2x + a_3 = 0$ and can be solved via the quadratic formula, $x = [-a_2 \pm (a_2^2 - 4a_1a_3)^{1/2}]/(2a_1)$. This yields two possible results for $\partial L/\partial t_{est}$, one of which is negative (therefore nonsensical) and the other matches experimental data (Fig. 4.12b). If we again neglect $\partial H/\partial t$, then the ' a_3 ' term of the equation reduces to $-4Q\Delta t/\pi H$ and the resulting $\partial L/\partial t_{est}$, while having the right order of magnitude, tend to underestimate the

propagation velocity by up to a factor of two, especially for relatively high true values of $\partial L/\partial t$.

Finally, if we return to the relationship in Fig. 4.5d, we can estimate the value of $\partial B/\partial t$, which we will label as $\partial B/\partial t_{est}$, and compare them with true values, which again show a good match (Fig. 4.12c). For the case in which we neglect $\partial H/\partial t$, since $\partial B/\partial t_{est}$ is proportional to $\partial L/\partial t_{est}$, it similarly mismatches true values by up to a factor of 2.

These propagation velocity arguments are useful in the context of forward modeling. By beginning with a dike of some initial geometry, which receives an influx, it is possible to estimate how the dike will evolve in length and breadth, with time (assuming it propagates through a homogeneous medium). The formulation includes driving forces, which determines the proportion of vertical growth to horizontal. Furthermore, it is possible to neglect inflation or deflation of the dike in the H dimensions, with the caveat that the estimated growth rates may be underestimated, though not to a large degree. Such models have clear benefits for hazards management and risk assessment purposes, during an unfolding volcanic unrest.

This formulation is also useful for influx-estimates during a crisis. As discussed in section 4.4.2, seismicity data has been shown to indicate how magma migrates up through the crust. Information of the dimensions and propagation velocities can be used to estimate the magnitude of influx. With high-quality data, there is the potential to even estimate the influx time series, which may reveal if a dike is continuously fed by a large source volume or consists of a discrete injection of magma.

4.4.6 Future work: horizontally-propagating dikes

Our experiments are configured to analyze continuously injected, vertically propagating dikes. However, many dikes in nature are observed to propagate sub-horizontally: intrusions at spreading centers (Fialko & Rubin, 1998); dikes in a volcano's associated rift zones (Michon et al., 2015; Wright & Marsh, 2016); the dike associated to 2014 Bárðarbunga fissure eruption (Gudmundsson et al., 2014); and the dike associated to the 2012 Tolbachik fissure eruption (Lundgren et al., 2015), to name a few. Analogue experiments show that dikes are dominated by lateral propagation for a variety of reasons, including reaching a level of neutral buoyancy (LNB; Lister & Kerr, 1991), coming into contact with a highly rigid overlying layer (Rivalta et al., 2005; Taisne & Jaupart, 2009), and encountering highly compressive local stresses (Pinel & Jaupart, 2004; Menand et al., 2010). An ascending dike may experience some or all of these factors as they approach the surface and encounter rock layers of contrasting rheology to the basement rocks (Geishi et al., 2012) and/or local stresses due to an edifice's load.

In the case of a dike reaching the LNB, it loses its driving buoyancy force and can no longer significantly ascend. The region above the LNB is negatively buoyant and wants to

sink downward, whereas the region below is positively buoyant and wants to ascend. The competing buoyancy forces accumulate at the level of neutral buoyancy and propagate the dike horizontally. We imagine that the flow is maximized along the level of neutral buoyancy, but the potential flow patterns are difficult to predict.

By contrast, dikes that are stopped by a rigid layer or a stress boundary maintain their buoyancy forces. Such dikes are forced to propagate horizontally while buoyancy forces push the magma upward against the barrier. Since the driving forces are acting in different directions, we imagine the propagation would be slower than for a non-buoyant dike and that the flow has a maximum velocity at a depth that is a bit below the barrier. Again, the flow pattern in such a dike is difficult to predict.

For either scenario, further work is needed to describe the internal flow pattern and the form of propagation, as functions of the driving forces like P^* and Re , as well as the source overpressure. However, they both provide avenues to gain further insight on how such dynamics are linked to their driving forces. Additionally, such work would inform us of the dynamics which erupted products undergo as they migrate from their source to their eruptive vent.

4.5 Conclusion

We performed a series of analogue experiments to determine what driving forces control the internal flow in a propagating dike. We injected liquids of various densities and viscosities (water, oil and liquid gelatin) into a solid gelatin medium, at varying flux rates. The liquids were seeded with particles that we tracked with a 4-camera assembly, which we used to perform PIV and PTV analysis of the flow field. The flow pattern evolves as the dike grows, from a circulating pattern when the dike is small to a unidirectional ascending flow when the driving force is dominated by the buoyancy. These flow patterns coincide with the form of propagation, in that small dikes tend to grow radially in a penny-shaped form, while longer dikes grow predominantly vertically, in a tabular form. As dikes erupt, they deflate and expel much of their volumes onto the surface, after which flow takes on a steady, unidirectional flow from the source to the surface.

We find that the dimensionless parameters, P^* (the ratio of buoyancy forces to fracturation resistance), and the Reynolds number (the ratio of momentum to viscous forces) control how the dike evolves. The degree of circulation (i.e. whether the return flow magnitude is similar to the upward flow) depends almost exclusively on P^* . The influx plays almost no role, in that high fluxes drive faster flow, but the overall pattern remains unaffected. However, since we were unable to maintain a constant-pressure at

the reservoir, we are unable to determine the effect of the reservoir overpressure; we think that circulation would be strongest when the overpressure is much higher than the buoyancy forces. The propagation form, by comparison, is dependent on both P^* and Re . As dikes grow and gain buoyancy, the propagation begins to be primarily vertical, instead of radial (i.e. in a penny-shape). We were similarly unable to determine the effect of overpressure on the propagation, though it is likely the propagation is radial when the overpressure is dominant, we cannot say so with certainty.

In the context of natural systems, dikes that are longer than a kilometer start to feel the effect of buoyancy force, with propagation that then tends to become vertically dominated, which is reflected in the simple, ascending internal flow pattern. We posit that the crystals in erupted lavas may provide some insight on the forces acting on a dike, in that oscillatory zoned crystals have been shown to record fluctuating conditions that could be associated with such circulation on the time scale of hours to days. The associated range of pressures may indicate how long a dike was (vertically) when it became driven by buoyancy, which helps to constrain the driving forces. For dikes that exhibit circulation on length scale longer than the buoyancy length, the overpressure is likely the dominant driving force.

4.6 Chapter Notation

Table 4.3. Symbols and meanings used in this study.

Symbol	Units	Meaning
$\Delta\rho$	kg/m^3	Liquid - medium density contrast
$\rho_{\text{crust,melt}}$	kg/m^3	Density of the host rock or melt
μ	$\text{Pa}\cdot\text{s}$	Dynamic viscosity
ϵ	-	Gas fraction
g	m/s^2	Gravity
G	Pa	Shear modulus
E	Pa	Young's modulus
ν	-	Poisson's ratio
γ	J/m^2	Surface energy
K_c	$\text{Pa}\cdot\text{m}^{1/2}$	Fracture toughness

Chapter 4: Internal circulation in a propagating dike: Linking analogue experiments to

L	m	Length (vertical)
L_b	m	Buoyancy length scale
B	m	Breadth (horizontal)
H	m	Opening thickness
V	m^3	Volume
v	m/s	Velocity
c	m/s	Equilibrium velocity scale from Roper & Lister (2007)
v_s	m/s	Shear wave velocity
v_{cent}	m/s	Upward velocity along dike's central jet
v_{periph}	m/s	Velocity at the dike's lateral peripheries
$v_{p,L}$	m/s	Propagation velocity in the vertical, L direction
$v_{p,B}$	m/s	Propagation velocity in the horizontal, B direction
A_{1-5}	-	Constants to fit exponential surfaces to our data
Q	m^3/s	Volumetric flux
ΔP	Pa	Driving overpressure
P_b	Pa	Buoyancy forces
P_f	Pa	Fracture pressure
P_e	Pa	Elastic force
P_v	Pa	Viscous forces
P_m	Pa	Momentum forces
L^*	-	Dimensionless length; L/L_b
p^*	-	Dimensionless buoyancy force; P_b/P_f
Re	-	Reynolds number; P_m/P_v
v_c^*	-	The degree of circulation; $(v_{cent}-v_{periph})/v_{cent}$
v_p^*	-	Propagation velocity ratio; $v_{p,B}/v_{p,L}$

Summaries from this thesis, perspectives and future work

In my view, this research represents a holistic view on how dikes behave in nature. Even in their simplest form, dikes are complex structures whose propagation depend on the coupling between the fracturation of the solid medium and the internal flow of viscous magma. They are complicated to study for the same reason that they are interesting, because they operate due to various competing and complementary physical and thermal dynamic processes.

The rate and direction of propagation depends in part on the magnitude, orientation and distribution of stresses in the crust, especially near the dike's leading tip, which magnifies these stresses. For this reason, we see dike propagation respond to localized stresses, such as those surrounding a pressurized magma reservoir, like the results presented in chapter 2. Such behavior can occur for a variety of stress sources, like an edifice's load pushing down on the underlying crust or due to broader, regional stresses defined by the geologic setting. The orientation of external stresses, superimposed on the dike's own, internally-derived stresses controls fracturation at the dike's tip, which in turn controls the dike's propagation direction. For example, the combination of external and internal stresses in experiment 7 of chapter 2, caused the dike to propagate around the reservoir and subsequently propagate sub-radially away. By comparison, the dike in experiment 4 of this chapter had a much weaker internal pressure (since the dike had no input flux), enabling it to curve around the reservoir over a longer arc, before propagating vertically away. If either of these dikes were quasi-static, with no internal pressure, they would had completely encircled the reservoir.

If the orientation of stresses controls the direction of propagation, then the sign and magnitude of stress has a control on if, and how fast, a dike propagates. Highly compressive stresses under an edifice can create a barrier to vertical growth and instead encourage the dike to propagate horizontally (Pinel & Jaupart, 2004). In contrast, we see in experiment 1 of chapter 2 that a high pressure around a magmatic reservoir generates compression in the radial direction, but extension in the tangential direction, which eases and encourages dike propagation toward the reservoir. Fracture along the dike's perimeter is sensitive to these stress gradients and ends up driving the dike in whichever direction is energetically most-efficient, be it towards a pressurized reservoir or towards topographic lows. In all likelihood, such behavior also extends to crustal rigidity gradients, either from thermal gradients, encouraging dikes inward towards a plumbing system, or compositional gradients, inhibiting propagation through metamorphic regions.

However stresses are not everything and the rheology of the magma also plays a significant role on a dike's growth. Propagation can be limited by the magma's viscous forces, in that it cannot proceed until the magma flows forward. As dikes grow in length, they tend to thin out, which causes the viscous forces to increase dramatically and slow propagation. This is especially true for constant-volume dikes, which ascend only due to buoyancy and have no additional magma influx (Taisne & Tait, 2009). Even for a dike with

extremely fluid magma, the rheology is important in terms of its buoyancy forces. As we saw in chapter 4, the buoyancy governs how the magma flows inside of a dike, in that strong buoyancy forces generate a simple, ascending flow pattern, whereas the lack of buoyancy allows the magma to circulate in large advective cells. Similarly, the buoyancy drives vertical propagation of the dike, while the lack of buoyancy force allows the dike to grow radially.

To add another layer of complexity, we can also consider thermal effects. Dikes are thin, with large surface areas and are therefore vulnerable to rapid cooling. As magma cools off, it gains viscosity and eventually solidifies, which can arrest propagation before the dike ever reaches the surface. We saw in chapter 3 that this is characterized by the dimensionless flux, which basically states that a critical flow rate is necessary for the dike to reach the surface before it solidifies. Experiment 4 of this chapter had a low, continuous flux of liquid gelatin, which resulted in the formation of a large, thick dike intrusion. The experiment occasionally erupted a few drops of liquid, but these extrusions were short-lived and intrusion resumed each time.

For dikes that do erupt, small variations in the temperature distribution cause rapid flow channelization, as cooler regions gain viscosity and encourage flow through the warmer regions (Wylie et al., 1999). When the magma supply finally ceases, the geometry of the next magma migration event becomes dependent on the cooling time interval. In chapter 3, we showed that a short time interval between migration events favors flow through a long-lived, sub-cylindrical conduit that connects the source to the surface. For moderate time intervals, the pathway partially solidifies, but remains soft, providing a mechanically-efficient path for the next dike to propagate along. For long time intervals, the supply pathway completely solidifies and forces further migration to open a brand new path, via a dike. If such behavior were to occur steadily for many millennia, then the spatial distribution of volcanism would indicate migration frequency. Short-to-moderate time intervals would favor construction of centralized volcanic edifices, while long time intervals would favor the development of volcanic fields.

By providing insight on these different types of behavior, I hope that future readers will find inspiration in interpreting monitoring data, as well as structural and petrological field evidence. Although individual dikes are intruded on a short time scale, I feel that these studies make a case for considering the long term evolution of a system, in terms of whether repeat intrusions are likely to focus inward into a geographically confined area (either due to thermal or stress factors) or be spatially dispersed. I also feel that this work emphasizes the need for quality, diverse monitoring data, which help us to well-constrain a propagating dike's geometry and driving forces, as well as the need to characterize the plumbing system, whose pressure state affects the dike's preferred path. I hope this research can refine how hazards are considered, from the emplacement and eruption of active dikes to constraining eruption evolution.

Appendix A: Gelatin preparation

Gelatin is a common experimental material for use as an analogue for the Earth's crust. It is an ideal material, in that it is elastic (behaves like the earth on short time scales), can be prepared to have a range of Young's moduli, is transparent, and is photoelastic, a property that helps us to visualize stresses. In the context of volcanology, it is commonly used in experiments focusing on the growth of dikes or sills, as both gelatin and the earth's crust behave in a brittle, elastic manner (Giuseppe et al, 2009; Kavanagh et al., 2013).

I performed a variety of different types of experiments during these studies, but I used the same basic preparation procedure to cook all the gelatins. So without further ado...

Steve's exquisite gelatin recipe

Preparation time: 15 minutes

Cooking time: 4 hours

Total time: 4 hours, 15 minutes

Yields: 112 L of gelatin

This morning I went for a run with my dog and the cool, breezy autumn air reminded me of one of my favorite seasonal treats: lab-grade gelatin! When I got home, I set right to work on this simple, rustic recipe.

The preparation of gelatin does not vary much between experiments, though the resulting Young's modulus can vary. I generally prepared all of my gelatins for the same 50x50x50 cm tank and filled it to a depth of 30-45 cm, thereby requiring 75-112.5 L of gelatin. For the materials, I used 250 bloom "beef or pork" gelatin, as specified by the manufacturer, and dissolved it in deionized water. Deionized water is preferred over regular tap water, since it is purer and therefore contains less bacteria and chemicals. Failing to use deionized water results in cloudy-looking, opaque gelatins that are prone to growing life (Fig. A.1).

Fig. A.1. Using tap water for gelatin preparation can lead to less-than-useful results. Bacteria make the gelatin their home and produce lots of gaseous byproducts, which then form tiny dikes. These dikes smell bad when they leave the gelatin. This gelatin could not be used for any experiments, but on the other hand, it looks kind of neat.

I varied the gelatin concentration between 2.5-5.0 wt%. The concentration, C , is simply the ratio of mass of the raw granular gelatin material, m_g , to the total mass, m_{tot} , including the mass of the water, m_w :

$$C = m_g/m_{tot} = m_g/(m_g+m_w) . \quad (A.1)$$

Therefore, for a mass of water and desired concentration, the necessary mass of the granular gelatin is:

$$m_g = Cm_w/(1-C) , \quad (A.2)$$

which I measured out each time.

I placed the granular gelatin into a large pot (60L capacity) with 40-50L of deionized water. The pot was placed on a hot plate with an overhead stirrer attached. Sometimes the water was somewhat warm when I poured in the granular gelatin, other times the water was cold. Placing the granular gelatin in early, when the water is cold, allows plenty of time for small bubbles to rise up out of the gelatin solution, which is said to increase the clarity of the gelatin; though to my eye, I did not see much difference. I then turned up the heat to a high temperature to warm the gelatin solution over a period of 3-4 hours, to a target temperature of 60°C, at which gelatin molecules separate to their

Appendix A: Gelatin preparation

smallest building blocks. Meanwhile, I left the overhead stirrer constantly running to help dissolve the gelatin granules and mix the solution. I frequently had to check the gelatin and stir manually with a long stick to suspend the granules, which tended to sink and stick to the bottom of the pot as a viscous syrup and then burn. After several hours of heating and mixing, this pot contained a high-concentration solution of gelatin. At this time, I added a splash of bleach (I never measured how much, but it was ~20-100 mL) to inhibit bacterial growth.

On the side, I gathered the remaining deionized water and placed it into the experimental tank. When ready, I mixed it with the high-concentration gelatin to yield whatever the desired concentration was for the experiment. I played with the temperature of the remaining water between experiments to see if it affected the Young's modulus of the gelatin. Based on strength measurement over many experiments, using water that was room temperature cooled off the hot gelatin solution quickly and lead to somewhat weaker gelatins. Perhaps the molecules did not have time to build long, complex networks before solidifying. Conversely, mixing the hot gelatin with warm or hot water lead to somewhat stronger gelatins. However, using hot water ($> 50^{\circ}\text{C}$) seemed to have adverse effects on the gelatin medium. Anecdotally speaking, the dikes I injected for gelatins made with hot water were difficult to control, in terms of orientation. It seems to me that hot gelatin solutions developed an internal convection as they cooled, which caused the resulting medium to develop an anisotropic strength. As such, dikes that I intended to inject vertically (I made a precut accordingly to control the orientation) ended up propagating at a significant angle. Perhaps the best visualization I have of this behavior comes from an early experiment, which is not related to the results presented in this thesis (Fig. A.2). In this experiment, I had placed a layer of sugary syrup at the base of the gelatin solution to act as a free surface. The hot gelatin circulated and suspended some of this syrup a generated a plume-like structure.

Appendix A: Gelatin preparation

Fig. A.2. (a) Very hot gelatin convects while cooling in our cold room. Here a dense sugar syrup placed at the bottom of the experiment was entrained by the convection, forming this plume-like structure. (b) The rheology of the gelatin in the plume seemed elastic, but not brittle. Injections of dense syrup from above created diapir-like channels for liquid migration, rather than a dike.

After mixing the gelatin solution to its final concentration, I covered it with a layer of vegetable oil, to prevent evaporation and the formation of a hard crust on the surface. I then placed the experimental tank in our cold room to cool at 15°C for a few days. The necessary cooling time depends on the size of the tank, as a large volume needs more time to completely cool off and harden. For the volumes I used (75 - 112.5 L) the gelatin was solid but soft after 1 day. After 2 days, they were at or near their peak hardness and after 3 days I would perform the experiment.

I found it very difficult to well-control the Young's modulus of each gelatin. I believe there are many factors that affect the gelatin strength, from the temperature, time and overall cooling history of the gelatin, to its concentration and the purity of the water being used. Experiments that were prepared very similarly resulted in strengths that varied by as much as a factor of 2. Since dike behavior depends on the strength of the medium, I therefore had to measure the strength each time. See Appendix B of this thesis for such measurements.

Appendix B: Shear wave measurements to estimate the gelatin's Young's modulus

B.1 Obtaining shear wave velocity measurements

Knowledge of the strength of gelatin is important for scaling analogue data to natural systems. Traditionally, the strength of a gelatin is measured by placing a weight of known mass and diameter onto the gelatin's surface and measuring how much it deflects downward. With more sophisticated equipment, the viscous and elastic properties of a sample can be directly measured, although not in situ.

My supervisor, Dr. Taisne, and I developed an alternative technique to measure the gelatin's Young's modulus using shear waves. Gelatin is photoelastic, which allows visualization of the deviatoric stress via the use of polarized filters (Fig. B.1). If there is no stress within the gelatin, light passes through the system and the filters merely cause the gelatin to look a bit dimmer. In regions where there is stress, the light retards as a function of the stress level and wavelength, which results in the appearance of fringes with color (Ramesh, 2000). This enables one to visualize a propagating stress perturbation, a wave, by tapping the surface.

Fig. B.1. The photoelastic behavior of gelatin. Here we applied a surface load and, with the aid of polarized filters, the deviatoric stress becomes apparent as a repeating rainbow pattern. The spacing of these rainbow fringes indicates the stress gradient.

The shear wave velocity is proportional to the shear modulus of the gelatin, which can be converted to the Young's modulus assuming a given Poisson's ratio (Makov, 2009). In a solid, pressure waves tend to propagate on the order of km/s and shear waves are slower. In a very weak material like gelatin, they propagate at velocities on the order of m/s and, using polarized filters, are readily observable by the unaided eye or a basic camera (Fig. B.2).

Fig. B.2. Shear wave measuring setup. (a) We place a polarizing filter against the front and back walls of the tank apparatus. (b,c) A shear wave propagates downward through a block of gelatin. The velocity is used to determine the strength of the gelatin.

The shear wave velocity relates to the shear modulus of the material, G , such that

$$G = \rho v_s^2, \quad (\text{B.1})$$

where ρ is the gelatin's density and v_s is the shear wave velocity. The density of gelatin is similar to water; for example, a 5.0 wt% gelatin has a density of $\sim 1020 \text{ kg/m}^3$. The shear modulus relates to the Young's modulus, E , via

$$E = 2G(1+\nu), \quad (\text{B.2})$$

where ν is the Poisson's ratio of the gelatin, which is incompressible and therefore has a value of 0.5.

To perform a measurement, I placed a polarizing filter both in front of and behind the experimental tank apparatus and shined a light through the system (Fig. B.2a). To generate shear waves, I placed a small, plastic pedestal on the gelatin surface and tapped it repeatedly, producing sets of 40-50 waves, which radiate away in all directions from the pedestal (Fig. B.3). Gelatin is isotropic, so the wave travels at the same velocity in all directions. I recorded each set of waves with a basic DSLR camera, with 1280x800 px spatial resolution and 50 frame/sec time resolution.

Fig. B.3. Wave are generated in different directions by tapping a pedestal placed on the gelatin surface. (a,b) Respectively vertical and horizontal wave propagation in a homogenous block of gelatin. (c,d) Vertical propagation in two contrasting layers in the same experiment (an experiment not discussed in this thesis). (e) A broad, horizontally propagating wave in the same gelatin shown in (c) and (d). I generated this by striking a sheet of plastic embedded along the left wall of the tank. The wave separates into two, due to the strength contrast between the layers.

I processed the videos in Matlab by tracking the change of light intensity between successive frames, ΔI , in which regions of high change generally correspond to the position of the wave. For the green channel, I , of a video frame, i , ΔI is simply

$$\Delta I = I_i - I_{i-j}, \quad (\text{B.3})$$

across an interval of j frames. It is best to maximize the number of measurements from a wave, so the interval tends to be over one or two frames; generally I used an interval of two frames to lessen noisy results. I cropped the videos down to a narrow strip, along which the wave was clearly visible, both to aid in the processing speed and accuracy of the measurements. This usually took the form of a horizontal or vertical strip of the video, originating from the location of the wave source and ending at the far wall or bottom of the tank (Fig. B.4a). I used a simple smoothing filter to remove any high-frequency noise from the video and make the wave more-distinct from the background. I then tracked the wave position from the ΔI (Fig. B.4b) by picking the centers of high magnitude regions. The velocity is simply estimated via the travel distance and the camera frame rate (Fig. B.4c), in units of pix/s. I converted to cm/s using a visual scale that appears in the picture, which usually was a structure of known size embedded in the gelatin. Since a pair of frames generates a single measurement and each wave is captured over a handful of frame-pairs, each group of waves potentially generates hundreds of measurements, depending on how clearly the wave can be distinguished from the background noise.

Fig. B.4. A processing example. (a) Frames from a video of a propagating wave are cropped to a strip in the direction of the wave travel. (b) The differential of light intensity between successive frames allows the wave to be mapped. The blue and yellow regions respectively represent where the wave is in the current frame and where it was in the previous. The white and black + marking are automatically picked 'locations' of the wave, taken from the center of these regions (above some threshold value). (c) The corresponding velocity is plotted against the travel distance along the processing strip. In this example, the gelatin is homogenous, so there is no velocity gradient.

To control the quality of the results via this technique, I used several criteria to accept or reject velocity measurements. The consistency of these measurements is a function of the magnitude of ΔI , in which a high magnitude tends to produce less-scattered results (Fig. B.5a). By setting an acceptance threshold and throwing out those from weak signals, we can strongly improve the final velocity estimate (Fig. B.5b, c). Other criteria impose realistic results. For example, I throw out bad velocity measurements, which are much too fast or too slow to reflect reality. Gelatins have a fairly limited range of Young's moduli, so they also have a limited range of velocities. If a velocity measurement is an order or magnitude out of this range, the automated wave-location detection either confused two separate waves or a wave with background noise. Another criteria imposes that the wave must travel in a clear sequence, such that it is detected in 3 separate frames moving in one direction. If a wave-like signal is detected to move forward and then suddenly backward, it is likely background noise and not a real wave (sometimes striking a gelatin too hard causes the whole block to jiggle, generating lots of anomalies that interfere with the wave detection).

Fig. B.5. (a) Velocity measurements shown against the differential in the magnitude of light. Blue and red dots respectively represent negative and positive differentials. The measurements are more-consistent when the differential has a large magnitude. (b) With no threshold set, the velocity measurements are scattered. (c) Measurements between the magnitude of ± 30 (see vertical dashed lines in subfigure a) are thrown out. The results are more consistent.

I have noticed that this shear wave technique tends to produce some artifact measurements. This is mostly a function of the wave's position, in which a wave entering or leaving the camera frame produces unreliable velocity results (Fig. B.6). A wave that has not fully-entered (or fully-left) the view can still be detected, even though its center is not on-screen. As the wave becomes fully visible in subsequent frames, the technique mis-judges the distance the wave has travelled. I therefore reject measurements taken near the boundaries of the video field of view (omitting the first and last 10% of the field of view) and only use measurements taken from the center of the frame.

Appendix B: Shear wave measurements to estimate the gelatin's Young's modulus

Fig. B.6. a) Artifacts appear in the results near each end of the horizontal axis. b) These are due to mis-measurements that occur when a wave comes into, or leaves, the frame.

With lots of velocity measurements available from a video, I can statistically quantify the gelatin's Young's modulus. I first find the median velocity from the accepted measurements along with the median absolute deviation (i.e. the standard deviation associated to the median), which gives a range of plausible velocities. I then convert the median, upper and lower deviations to individual estimates of the shear modulus and Young's modulus, via equations B.1 and B.2. Since the moduli are proportional to the square of velocity, their lower deviations are smaller than their upper deviations (e.g. $E = 3000 + 310/-290$ Pa).

B.2 Comparison with deflection tests

Since this is a new technique, I compared the results with an established method for measuring the Young's modulus. I performed mass deflection tests, following Kavanagh et al. (2013), in which I placed a known mass, m , on a pedestal of a known diameter, D , and measured how the gelatin surface deflected downwards, w , due to the weight:

$$E = mg(1-v^2)/(Dw) , \quad (B.4)$$

where g is gravity. For this method to be statistically meaningful, it is necessary to repeat the test many times varying the mass and diameter of the pedestal. This technique produces its most-accurate results when the load diameter is small, relative to the size of the surface. Kavanagh et al. (2013) find that when the diameter is less than 10% of the gelatin surface width, boundary effects become negligible. I have noticed that this technique is difficult in practice, since the load and deflection need to be significant enough to be measured with little error; however, using a heavy load generates a lot of deflection and tends to leave the gelatin fractured, which may contaminate results. It

Appendix B: Shear wave measurements to estimate the gelatin's Young's modulus

requires a bit of time to find the ideal pressure that causes the gelatin to deflect without damaging it and doing so for many repeat measurements can be tedious. By comparison, the shear wave technique is much easier, since it only requires a video recording of the user tapping the gelatin for a minute or so, and processing the results using our own scripts.

For gelatins that have fully hardened, there is general agreement between the two methods, while for gelatins that have not, the deflection method consistently estimates higher Young's moduli than does the wave method (Fig. B.7). This is likely due to heterogeneous cooling, in that the gelatin top surface cools and hardens more quickly than does the center of the gelatin. The deflection test directly samples the hardness of the upper surface, while the shear wave test tends to sample the bulk velocity of the gelatin (as the wave propagates through the medium). I believe this points to how effective the shear wave technique is, since it well-measures the Young's modulus of the gelatin's interior and generally matches the deflection tests for fully-hardened gelatins (which we use for experiments anyhow).

Fig. B.7. Compiled results from both shear wave ($\cdot$ marking) and deflection ($\times$ marking) measurements, shown against the amount of cooling time from the time the warm gelatin is poured into the tank. Each marking represents the median E value for a shear wave test and we omit error bars for clarity (which tend to be $\sim +20\%/-10\%$ of the median). As the

gelatin sets, it gets harder. The two measurement techniques match more closely as the gelatin fully hardens.

B.3 Limitations to the technique

I list here a few limitations to this technique that one will have to consider in order to increase the level of confidence in the results.

1) Number of frames recording a wave.

The shear wave technique is most-effective for waves that can be recorded over many frames, yielding many measurements. When recording video, it is best to prioritize a high frame-rate over a high spatial resolution. Waves are generally wide and easily seen with the naked eye, however in hard gelatins, they move quickly. Using a low frame rate reduces the number of measurements that can be made. Similarly, when using the technique in small containers ($< \sim 20$ cm in its longest direction), it may be necessary to record with a higher frame rate.

2) Background noise

As noted above, striking the gelatin to produce a shear wave may also cause the bulk gelatin to jiggle, producing very noisy results. This is especially true for strong gelatins, which are difficult to accurately measure. Conversely, soft gelatins exhibit very clear, slow-moving waves, with little background noise, yielding very nice measurements. For example, all of the photos shown here are of soft gelatins, since the waves are so distinct.

3) Wall effects

I have noticed that the wave velocity may be affected by the tank walls. Using the shear wave technique in a very narrow tank can produce a velocity gradient, in which the core of the gelatin displays slower waves than those close to the wall. For example, I performed shear wave measurements in a cylindrical tube (1.5 m tall, 20 cm diameter) and detected a small gradient, though it is unclear if it is a true gradient due to wall effects or a visual artifact due to refraction of the light through a cylindrical geometry. The gradient is small and difficult to visually observe, however it shows up statistically. By subdividing such a video into vertical strips (i.e. a strip at the tank wall, one slightly inward, and so on to the tank center) and processing them individually, we can detect a gradient (Fig. B.8). More work needs to be done to determine if this is just a visual artifact and, if real, what distance from the wall produces reliable measurements that indicate the gelatin's true Young's modulus.

Fig. B.8. In a narrow cylindrical tank, a velocity gradient appears, in which the wave velocity is higher near the wall (left side) and lower in the tank center (right side). Measurements are noisier near the wall, but the median values (solid black line) and deviations (dashed black lines) show the gradient. It is unclear if this is a true gradient due to wall effects or an artifact of light refraction as it passes through our tank.

Appendix C: Other gelatin properties

C.1 Latent heat content

We designed simple tests to measure two of the thermal properties of our gelatin: the latent heat and the thermal diffusivity. Both test involved heating a gelatin and measuring the temperature time series using Omega brand thermocouples. In this section we will describe how we estimated the latent heat in our gelatin. In section C.2, we describe how we estimated thermal diffusivity. Equations in these sections follow general principals of heat transfer described by Turcotte and Schubert (2002) and Çengel (2008).

Appendix C: Other gelatin properties

We warmed a small 1x1x1 cm³ cube of gelatin on a hot plate and measured its temperature time series. We cut the gelatin cube from a larger 0.5 L sample of 3.5 wt% gelatin, which had been made the previous day and allowed to cool in a cold room refrigerator for 24 hr at 15 °C. During cooling, it was also covered with a layer of vegetable oil, to prevent evaporation and the formation of a hard crust. To warm the gelatin, we heated the hot plate surface to 35.5 °C and covered it with a paper towel, which absorbs the melting gelatin and keeps the solid gelatin in place; when melting, the gelatin is prone to sliding on the hot plate and can therefore lose contact with the thermocouple. We positioned the tip of the thermocouple near the base of the gelatin, to maintain steady contact as the gelatin block melted and shortened.

The temperature time series is shown as the blue measurements in Fig. C.1. As the gelatin warmed, it increased in temperature at a fairly constant pace, until it reached the melting point (~28.5 °C). The temperature increase slowed for some 45 s, during which the block melted. Afterwards, the liquid increased in temperature at the initial pace. We assume that the temperature increase slowed due to latent heat capture. We estimate the heat flux, q , by first measuring the temperature increase rate, dT/dt , which we accomplished using the “polyfit” function in Matlab, which estimates the slope of a line. We measured the slope across a 20 s moving window and calculated the heat flux as

$$q = \rho V C_p (dT/dt) , \quad (C.1)$$

for a gelatin of volume, V , density, ρ , and specific heat capacity, C_p . We use a V of 1 cm³, ρ of 1000 kg/m³ and C_p of 4050 J/(kg·K). The heat flux time series is shown as the red measurements in Fig. C.1.

Fig. C.1. A temperature time series (blue points) used to estimate the latent heat of our gelatin material. We heated a 1 cm^3 block of gelatin, which melted at $\sim 28.5^\circ\text{C}$, from time stamps 855 s to 900 s (vertical black dashed lines). The estimated heat flux into the gelatin is also plotted (red points). We use the initial heat flux and apparent drop in heat flux (characterized by the horizontal black, dashed lines) to quantify the latent heat.

We estimate the latent heat by assuming that the initial and final heat flux quantify the actual heat flux, q_{act} , into the gelatin, so that the apparent drop in heat flux during melting quantifies the latent heat capture. The gelatin latent heat, γ , is therefore

$$\gamma = (q_{act} - q_{app})t_m / (\rho V), \quad (\text{C.2})$$

for apparent heat flux, q_{app} , during a melting time period of t_m . The measurements of q_{act} , q_{app} and t_m are marked by dashed black lines in Fig. C.1 and are respectively 0.092 J/s, 0.026 J/s and 45 s. Using these measurements, we estimate that the latent heat of our gelatin is 2970 J/kg.

As described in section 3.2.2, the Stefan number, S , characterizes a ratio of latent heat to sensible heat in a system. For our experiments, which were performed with an injection temperature, T_0 , of $\sim 32^\circ\text{C}$ and ambient temperature, T_∞ , of $\sim 15^\circ\text{C}$, the Stefan number, $S = \gamma / [C_p(T_0 - T_\infty)]$, is therefore on the order of 4×10^{-2} , indicating that latent heat is a small component of the total heat in the system.

C.2. Thermal diffusivity

To estimate the thermal diffusivity of our gelatin, we performed a similar heating test with a larger 50x50x30 cm³ block of gelatin, in which water placed on the gelatin's upper surface provided heat to the system. We made the block using 3.5 wt% concentration of gelatin, covered it with a layer of vegetable oil (to prevent evaporation and the formation of a hard crust) and then allowed it to cool for 3 days in our cold room at 15 °C. To measure the temperature time series in the gelatin, we inserted a series of thermocouples at different depths (0, 3.9, 7.0, 10.0 and 11.9 cm). We positioned the tip of each thermocouple along the gelatin's central vertical axis and avoided placing them very deep in order to resemble a 1-D heat transfer condition, in which heat diffuses primarily downward through the gelatin. To start the test, we flood the gelatin's upper surface with relatively warm water (21-22 °C) and measured how the heat conducted downward into the relatively cold medium. Note the water temperature was less than the melting point of the gelatin, so the gelatin did not melt. We circulated the surface water through a thermal bath in order to maintain a constant heat supply. The temperature measurements for each thermocouple are shown in Fig. C.2 (black curves). The shallower thermocouples recorded more-rapid temperature increase than the deeper ones.

Fig. C.2. Temperature time series measurements (black curves) for thermocouples embedded at different depths (~4-12 cm) in gelatin medium. We placed relatively warm water on the gelatin surface and the thermocouples recorded the heating via conduction. The shallowest thermocouples recorded the most-rapid temperature increase. Note the measurements have a weak oscillation due to the ambient conditions (performed in a cold room refrigerator, which periodically turns on and off). We made a simple 1-D numerical model to estimate the temperature increase due to conduction for various magnitudes of thermal diffusivity (blue curves). We fit these curves by minimize the sum of the square of the residual between the model and measurements. The associated best-fitting value of the thermal diffusivity is shown.

In order to estimate the thermal diffusivity of the gelatin, we treat each thermocouple time series as a separate 1-D heat transfer problem, in which the heat flux can be modeled via Fourier's law:

$$q = k(dT/dx) , \quad (S5)$$

and

$$dT/dt = q/(pVC_p) = (\alpha/V)(dT/dx) , \quad (S6)$$

for a thermal conductivity, k , thermal diffusivity, α . A thermocouple's temperature increase over a time step, dt , can therefore be modeled by the temperature difference between it and the surface water. We solve this numerically for 1-D heat conduction, applying a heat source (with the water's temperature) to an otherwise homogenous medium (with the gelatin's initial temperature). Note that in Fig. C.2, each thermocouple records a slightly different initial temperature, so we model each thermocouple separately, using their initial temperatures to represent the gelatin's initial temperature. We then allow the heat to conduct along the profile at a rate determined by α and compared the model with the measurements. We performed this model for various magnitudes of α and found the best-fitting value by minimizing the sum of the square of the residuals between the measurements and the model (best-fitting model shown by the blue curves in Fig. C.2). In this way, we estimate that the thermal diffusivity of our gelatin medium is $7.4 \times 10^{-8} \text{ m}^2/\text{s}$.

C.3 Temperature-dependent viscosity

We performed a small test to determine how the viscosity of liquid gelatin depends on its temperature. We heated a small batch of 4.0 wt% gelatin to ~80 °C and poured 40 mL into a small container. We measured this sample using an A&D brand, model SV-10, tuning fork viscometer, which simultaneously measures viscosity and temperature. Since the sample volume was small, it quickly cooled down and we recorded a series of measurements (Fig. C.3). The test shows that, above 60 °C, gelatin has a temperature-independent viscosity. Between 30 °C and 60 °C, gelatin has an approximately linear,

Appendix D: Chapter-specific appendices

temperature-dependent viscosity. Below 30 °C, the viscosity rapidly increases as solidification proceeds.

Fig. C.3. Measurements of liquid gelatin viscosity against temperature. Gelatin has a temperature-dependent viscosity in the temperature range (<32 °C) used in the experiments presented in chapter 3.

Appendix D: Chapter-specific appendices

Appendix D.1. Chapter 1: End-of-injection frame of reference

We will discuss here how the results change when we use a different reference time frame to measure surface deformation. In this document, we discussed the results of inverting the surface deformation due to a dike, which was forcibly injected into the gelatin. The volume injection causes the gelatin surface to bulge upward, which yields a complementary surface deformation signal that needs to be accounted for and removed (see section 1.2.4). Instead of modeling and removing this signal, as in the main text, here we simply set the reference time to the end of injection, such that the dike volume is constant between measurements (Fig. D.1.1d,e).

Fig. D.1.1. An illustrative comparison of the reference time frame, t_0 . A pre-injection reference considers the gelatin before any liquid has been injected (a-c), while a post-injection reference considers the gelatin after the injection has completed and a constant volume of liquid ascends due to buoyancy (d-f). The example time of measurement, t_i , is the same for both rows. The relative change is illustrated simply as the difference between t_i and t_0 . A post-injection reference frame has a hidden deflation signal due to the dike tail contracting (f, dark blue region). When assessing the inversion uncertainty, we compare with the entire dike (e.g. L , W , V) as well as to the geometry of the upper migrating part (e.g. L_m , W_m , V_m) shown in yellow in (f). (g-i) For reference we show the vertical component of surface deformation associated to (d-f). We subtract the deformation in (g) from (h) to retrieve (i), which we invert to estimate the dike dimensions.

For the surface deformation measurements, this process is simple. The DIC horizontal measurements are made simply by changing the initial surface orthophoto to the one after we turned off the pump. For the vertical deformation, we subtract the DEM at this new reference time from the DEM at each time step. We then inverted this data in

exactly the same way as in the main part of this text, using a single Okada displacement tile. By not needing to correct the deformation data (nor make any assumption of the shape of the volume inflation signal), it would be expected that the inversions more-accurately estimate the dike geometry. In reality, this approach comes with its own limitations. The surface deformation depends on the change of geometry of the dike, so each measurement exhibits a convolution of the signal due to the dike propagating upward and the signal due to the bottom of the dike deflating and losing volume (Fig. D.1.1f). Modeling this joint opening & closing behavior as a single Okada displacement therefore can produce inaccurate dike geometry estimates.

To quantify the accuracy of the inversions, we compare with the absolute dimensions of the dike (L , B , W) as in the main text, but we also compare with the dimensions of the propagation growth, relative to the initial size (Fig. D.1.1f, label L_m). In this way, we check if the inversion better-models the newly-propagated part of the crack (its length, L_m , thickness, W_m , and volume, V_m) or the crack as a whole.

The post-injection reference time generally yields somewhat less-accurate errors (Fig. D.1.2) with a somewhat greater degree of scatter than the pre-injection reference time discussed in the main text (Fig. 1.13), although the differences are not large. Specifically, the dip angle, strike angle, horizontal breadth and X location have similar errors and distributions between the two methods, whereas the main text method yields better estimates of the length, depth and volume.

Fig. D.1.2. A plot similar to Fig. 1.13. Changing to a post-injection reference time lead to greater uncertainties on the inversion. A direct comparison can be made for the dip, strike, length (L), breadth (B), mean and maximum widths (W), depth (d), volume (V) horizontal location (X , Y locs), which all fare worse. We can also investigate the length (L_m), width (W_m) and volume (V_m) of the upper propagating portion of the dike, which are visualized in Fig. D.1.1f.

The post-injection reference better-estimates the opening thickness. This may be due to the way we remove the inflation signal, by fitting and removing a parabolic surface; this assumption of the inflation signal's shape may not perfectly reflect reality. It also may be due to the way we statistically analyze the data, in which the post-injection frame of reference cuts out the early phase of dike propagation, when the dike tip is deep and the associated errors are relatively large. By analyzing the errors only when the dike is relatively shallow, the results reflect only the better inversions.

As discussed in the main text, perhaps the most important parameters are the tip depth and the volume, which are related to the onset and intensity of an eruption. The post-injection reference time produces somewhat less-accurate inversions of the dike depth than the method in the main text, however it is less scattered, such that dikes are always deeper than the inversion estimates, by a factor of 2.4 (+/- 1); the inversion is less-accurate, but the uncertainty is better-constrained. By contrast, the volume is less-accurately estimated, with a larger degree of uncertainty. The absolute volume of the dike tends to be underestimated, whereas the volume of the upper, migrating region is overestimated. This is presumably due to the argument that the strain field around the dike reflects the geometry of the dike head. By using a post-injection reference time, the deformation due to the dike head is well-represented only when the dike has propagated significantly from its initial position (i.e. in Fig. D.1.1f, when the yellow regions has propagated significantly upward and no longer has a crescent shape). Similarly, the other measurements of the propagation, L_m and W_m , are likely better-approximated by the Okada geometry when the head of the dike has ascended significantly from its reference location.

We would suspect that, in nature, the Okada model better-estimates the geometry of the dike than our experiments indicate, in part due to the volume inflation signal. In nature, there is no gain in volume due to magma being forcibly intruded into the crust, as is the case for our experiments. We are forced to cope with this either by modeling and removing the signal, which may warp the surface deformation pattern so that it does not well-match the Okada pattern, or we may use a late reference time, which requires that the dike propagates significantly from its already-shallow, initial position. That said, the fact that these two separate methods produce similar results indicates that the Okada model is limited in its ability to accurately represent a penny-shaped or elliptical dike, especially in terms of its depth. As stated in the main text, these results reinforce the importance of analyzing multiple types of data (e.g. seismic data) to constrain the geometry of the dike.

D.2.1. Chapter 2: The balloon-less reservoir construction technique

We describe here in greater detail the balloon-less reservoir apparatus, referred to in the chapter 2 as reservoir setup 2. Using the following procedures, we were able to create a

water-filled cavity in a gelatin (Fig. D.2.1, a more-detailed version of Fig. 2.2e). We designed this apparatus to examine how a dike exchanges liquid with a pressurized reservoir and therefore understand how pressure is distributed between the two structures and how propagation proceeds afterwards. This apparatus is similar to reservoir setup 1 (Fig. 2.2c, d), in that we use a balloon as a mold, to create the spherical shape of the reservoir cavity, however it has additional features.

The balloon mold rests at the top of a conduit pipe, which supplies the liquid that eventually fills the cavity. The conduit is 4 cm in diameter: big enough to house a few internal structures, but less than half the diameter of the balloon. It is ~25 cm long, so that the balloon center is positioned ~15 cm below the gelatin's upper surface. The balloon is fixed to a rod that slides up and down through the conduit pipe and allow us to pull the balloon tightly down to the conduit top (Fig. D.2.1a). We chose to fill the cavity with dyed water, which is similar in density to the gelatin and does not apply a buoyant force on the overlying gelatin, allowing us to examine only the stresses due to inflation. The water is slightly less dense than the gelatin however (it wants to rise up out of the conduit pipe), so to keep the liquids from exchanging and to inhibit gelatin from clogging the pipe, we initially filled the pipe with a denser sugar solution. After the gelatin solidified, we could drain the sugar solution and replace it with our preferred dyed water. As gelatin solidifies, it becomes firmly attach to the balloon surface; deflating the balloon causes the gelatin to tear. We mitigate this damage by routing a separate closed channel for hot water through the pipe conduit and into the balloon interior (Fig. D.2.1b). We circulated hot water into the balloon to re-melt a thin layer of gelatin along the balloon's surface, to help it detach. With the balloon separate from the gelatin, we attached the outlet of the closed hot water loop to the inlet of the conduit, to deflate the balloon at the same rate that we filled the surrounding cavity (Fig. D.2.1c).

During this final stage, we deflated the balloon a bit before starting to fill the exposed cavity, to apply a small suction force on the liquid in the conduit, which mitigates accidental dikes from forming at the top of the conduit. Similarly, after the cavity was prepared, we injected only a small amount of water to apply a weak pressure on the gelatin (Fig. D.2.1d). We keep the inflation to a minimum, since a dike could easily propagate from the reservoir and defeat the purpose of the experiment. For example, during a failed attempt, pressurizing the reservoir initiated a dike where the balloon meets the conduit top; further influx of liquid into the reservoir merely caused the dike to propagate and pressure was lost.

Fig. D.2.1. Schematic diagrams of the balloon-less reservoir preparation (reservoir setup 2 in the main article). (a) A balloon rests on top of a conduit pipe. It is attached to a sliding rod, to control its position and help to secure it to the conduit pipe, and connected in a closed circuit to a hot water bath. This closed circuit lets us inflate or deflate the balloon separately from the conduit. The conduit is initially filled with dense sugar solution. (b) After the gelatin solidifies, we replace the sugar solution with dyed water. We also begin to circulate hot water into the balloon, to re-melt a layer of gelatin along the balloon surface. (c) We deflate the balloon at the same rate that we fill the remaining cavity with water. (d) We then inject more water to inflate the cavity and apply stress to the gelatin.

D.2.2. Chapter 2: Measurements of $\partial x/\partial z$ and $\partial^2 x/\partial z^2$

Once we have extracted the information of dike position, we further process the data by estimating the local stresses and differential positions. For an example of a non-radial dike, let us look at experiment 9 (Fig. D.2.2a). Due to the high resolution of the position data, using adjacent pixels to find the differential can lead to noisy results; some pixels are align horizontally, others vertically. We therefore estimate the differential, $\partial x/\partial z$, at each pixel, i , on the dike, over a window size, j . We use Matlab's polyfit function, which finds the best-fitting linear slope over the window, which is roughly equal to

$$(\partial x/\partial z)_i = (x_{i+j/2} - x_{i-j/2}) / (z_{i+j/2} - z_{i-j/2}) . \quad (D.2.1)$$

The second differential, $\partial^2 x/\partial z^2$, is done similarly and is roughly

$$(\partial^2 x/\partial z^2)_i = [(\partial x/\partial z)_{i+j/2} - (\partial x/\partial z)_{i-j/2}] / (z_{i+j/2} - z_{i-j/2}) . \quad (D.2.2)$$

When we display these differentials, we do so against a stress term σ^* (introduced in the next section) estimated locally at pixel i . Estimating $\partial x/\partial z$ and $\partial^2 x/\partial z^2$ in this way respectively shortens our data vectors by $j/2$ and j points at each end of the vector. This has some dependence on the window size. Window sizes that are too small yield noisy results, while those that are too large yield results that are too smoothed and too shortened (Fig. D.2.2c). Ideally, we choose the smallest size that shows a distinct trend, but in practice we simply use a window size that is function of the vector size, such that:

$$j = \text{length}(x)^{1/2} . \quad (D.2.3)$$

For the radial set of dikes, the number of data points from an experiment depends on the number of frames in our time lapse, rather than on the pixel resolution of a single photo from the end of the experiment. These dikes seem to propagate in a way that can be described by where they make new surface and so, to accurately show where new surface was being created, there needed to be a significant amount of growth between frames (Fig. D.2.2e). As such, our data vectors for these experiments were significantly shorter than for the other dikes (tens of points, compared with hundreds).

We were able to maximize how much we could use these few points by estimating partials using different window sizes and accumulating the information. Similar to the non-radial dikes, the variable window size is that which shows the clearest signal without omitting much data. In practice the window size range we used was:

$$\text{length}(x)/5 < j < \text{length}(x)/3 . \quad (D.2.4)$$

In comparison with the non-radial dikes, the large spacing between data points for radial dikes produced clear $\partial x/\partial z$ signals, even for small window sizes.

Depending on the initial dike orientation, we found either $\partial x/\partial z$ or $\partial^2 x/\partial z^2$ to be important. The resulting partial derivatives from our data preparation however require more steps to show any trends. First, the data does not necessarily have a monotonic relationship with the stress. As dikes enter the near-field region of the reservoir, the local

stresses become high and they rapidly adjust their orientation or position. The resulting $\partial x/\partial z$ and $\partial^2 x/\partial z^2$ decrease in magnitude precipitously, in a way that does not back-track along the relationship with the local stress. We choose to analyze the data in which the dike enters the stress field and crop out the rest (Fig. D.2.2d,h). Finally, we view the absolute value of the data to be able to easily compare our over- and under-pressure experiments.

Fig. D.2.2. How we estimate partial derivatives. (a) A processing photo for experiment 9, with the dike traced. (b) The dimensionless position, x^* and z^* (normalized against reservoir radius), of each pixel along the path is read in Matlab. Data outside of the region bounded by the two cyan squares is lost via our $\partial^2 x/\partial z^2$ estimation. (c) Estimation of $\partial^2 x/\partial z^2$ with different window sizes: red is adjacent pixels, blue is half the size of the data vector. (d) Once the dike enters its preferred orientation, $\partial^2 x/\partial z^2$ drops (blue curve). We crop it to keep data of the dike entering the stress field (purple curve). We use absolute values (red curve) to compare experiments ($+\sigma^*$ and $-\sigma^*$). (e-h) Similar data processing for a radially-oriented dike. Each data point represents a photo in the time lapse, so the resolution is low.

References

References

- Abdelmalak, M., M., Mourgues, R., Galland, O. & Bureau, D. (2012). Fracture mode analysis and related surface deformation during dike intrusion: Results from 2D experimental modelling. *Earth Planet. Sci. Lett.*, 359-360, 93-105. doi: 10.1016/j.epsl.2012.10.008
- Adrian, R., J. (1991). Particle-imaging techniques for experimental fluid mechanics. *Ann. Rev. Fluid Mech.*, 23, 1, 261-304.
- Albert, H., Larrea, P., Costa, F., Widom, E. & Siebe, C. The tip of the dike: crystals record the conduit opening from the source to the surface. *Cities on Volcanoes 10*, Naples, Italy, 2018. Invited oral contribution.
- Alparone, S., Cannata, A., Gambino, S., Gresta, S., Milluzzo, V. & Montalto, P. (2010). Time-space variation of volcano-seismic events at La Fossa (Vulcano, Aeolian Islands, Italy): new insights into seismic sources in a hydrothermal system. *Bull. Volcanol.*, 72, 803-816. doi: 10.1007/s00445-010-0367-6
- Anderson, E., M. (1937). IX. - The dynamics of the formation of cone-sheets, ring-dykes, and caldron-subsidences. *Proc. Roy. Soc. Edinburgh*, 56, 128-157. doi: 10.1017/S0370164600014954
- Anderson, K. & Segall, P. (2013). Bayesian inversion of data from effusive volcanic eruptions using physics-based models: Application to Mount St. Helens 2004-2008. *J. Geophys. Res. Sol. Earth*, 118, 2017-2037. doi: 10.1002/jgrb.50169
- Aufaristama, M., Hoskuldsson, A., Jonsdottir, I., Ulfarsson, M., O. & Thordarson, T. (2018). New insights for detecting and deriving thermal properties of lava flow using infrared satellite during 2014-2015 effusive eruption at Holuhraun, Iceland. *Rem. Sens.*, 10, 151, doi: 10.3390/rs10010151
- Ayele, A., Jacques, E., Kassim, M., Kidane, T., Omar, A., Tait, S., Nercessian, A., de Chaballier, J., B. & King, G. (2007). The volcano-seismic crisis in Afar, Ethiopia, starting September 2005. *Earth Planet. Sci. Lett.*, 255, 177-187. doi: 10.1016/j.epsl.2006.12.014
- Ayuso, R., A., De Vivo, B., Rolandi, G., Seal, R., R. & Paone, A. (1998). Geochemical and isotopic (Nd-Pb-Sr-O) variations bearing on the genesis of volcanic rocks from Vesuvius, Italy. *J. Volcanol. Geotherm. Res.*, 82, 53-78. doi: 10.1016/S0377-0273(97)00057-7
- Bagnardi, M., Amelung, F. & Poland, M. (2013). A new model for the growth of basaltic shields based on deformation of Fernandina volcano, Galápagos Islands. *Earth Planet. Sci. Lett.*, 377-378. 358-366. doi: 10.1016/j.epsl.2013.07.016

References

- Barbot, S. (2013). UniCycle (Unified Cycle of Earthquakes), v1.
www.bitbucket.org/sbarbot
- Barnett, Z. A. & Gudmundsson, A. (2014). Numerical modelling of dykes deflected into sills to form a magma chamber. *J. Volcanol. Geotherm. Res.*, 281, 1-11. doi: 10.1016/j.jvolgeores.2014.05.018
- Bato, M., G., Pinel, V. & Yan, Y. (2017). Assimilation of deformation data for eruption forecasting: Potentiality assessment based on synthetic cases. *Front. Earth Sci.*, 5, 48, doi: 10.3389/feart.2017.00048
- Battaglia, J., Ferrazzini, V., Staudacher, T., Aki, K. & Cheminée, J.-L. (2005). Pre-eruptive migration of earthquakes at Piton de la Fournaise volcano (Réunion Island). *Geophys. J. Int.*, 161, 549-558. doi: 10.1111/j.1365-246X.2005.02606.x
- Battaglia, M., Cervelli, P., F. & Murray, J., R. (2013). dMODELS: A MATLAB software package for modeling crustal deformation near active faults and volcanic centers. *J. Volcanol. Geotherm. Res.*, 254, 1-4 doi: 10.1016/j.jvolgeores.2012.12.018
- Bianco, F., Castellano, M., Milan, G., Ventura, G. and Vilardo, G. (1998). The Somma-Vesuvius stress field induced by regional tectonics: evidences from seismological and mesostructural data. *J. Volcano. Geotherm. Res.*, 82, 199-218. doi: 10.1016/S0377-0273(97)00065-6
- Biggs, J., Amelung, F., Gourmelen, N., Dixon, T., H. & Kim, S.-W. (2009). InSAR observations of 2007 Tanzania rifting episodes reveal mixed fault and dyke extension in an immature continental rift. *Geophys. J. Int.*, 179, 1, 549-558. doi: 10.1111/j.1365-246X.2009.04262.x
- Bruce, P., M. and Hupper, H., E. (1989). Thermal control of basaltic fissure eruptions. *Nature*, 342, 665-667. doi: 10.1016/j.jvolgeores.2015.06.012
- Bunger, A., P., Menand, T., Cruden, A., Zhang, X. and Halls, H. (2013). Analytical predictions for a natural spacing within dyke swarms. *Earth Planet. Sci. Lett.*, 375, 270-279. doi: 10.1016/j.epsl.2013.05.044
- Burton, M., R., Mader, H., M. & Polacci, M. (2007). The role of gas percolation in quiescent degassing of persistently active basaltic volcanoes. *Earth Planet. Sci. Lett.*, 264, 46-60. doi: 10.1016/j.epsl.2007.08.028
- Cameron, C., E., Dixon, J., P., Neal, C., A., Waythomas, C., F., Schaefer, J., R. & McGimsey, R., G. (2014). 2014 Volcanic activity in Alaska: Summary of events of the Alaska Volcano Observatory. Report by the US Department of the Interior.
- Cashman, K., V. & Sparks, R., S., J. (2013). How volcanoes work: A 25 year perspective. *Geol. Soc. Amer. Bull.*, 125, 5/6, 664-690. doi: 10.1130/B30720.1

References

- Çengel, Y., A. (2002). Heat transfer: A practical approach. 2nd ed. McGraw-Hill, New York. ISBN-10: 9780072458930
- Çengel, Y., A. (2008). Introduction to thermodynamics and heat transfer, second edition. 217-256. New York, NY: McGraw-Hill. ISBN-13: 978-0071287739
- Chadwick Jr., W., W. and Dieterich, J., H. (1995). Mechanical modeling of circumferential and radial dike intrusion on Galapagos volcanoes. *J. Volcanol. Geotherm. Res.*, 66, 37-52. doi: 10.1016/0377-0273(94)00060-T
- Chanceaux, L. & Menand, T. (2014). Solidification effects on sill formation: An experimental approach. *Earth Planet. Sci. Lett.*, 403, 79-88. doi: 10.1016/j.epsl.2014.06.018
- Cherkaoui, A., S., M., Wilcock, W., S., D., Dunn, R., A. & Toomey, D., R. (2003). A numerical model of hydrothermal cooling and crustal accretion at a fast spreading center mid-ocean ridge. *Geochem. Geophys. Geosys.*, 4, 9. doi: 10.1029/2001GC00215
- Chevallier, L. & Verwoerd, W., J. (1990). Influence of temperature on the distribution of stress and displacement in a volcano: a numerical approach. *Bull. Volc.*, 52, 413-425.
- Coote, A., Shane, P., Stirling, C. & Reid, M. (2018). The origin of plagioclase phenocrysts in basalts from continental monogenetic volcanoes of the Kaikohe-Bay of Islands field, New Zealand: implications for magmatic assembly and ascent. *Contrib. Mineral. Petr.*, 173, 14. doi: 10.1007/s00410-018-1440-y
- Corbi, F., Rivalta, E., Pinel, V., Maccaferri, F., & Acocella, V. (2016). Understanding the link between circumferential dikes and eruptive fissures around calderas based on numerical and analog models. *Geophys Res Lett*, 43, 6212-6219. doi: 10.1002/2016GL068721
- Crabtree, S., M. & Waters, L. E. (2017). The petrologic history of the Sanganguey volcanic field, Nayarit, Mexico: Comparisons in a suite of crystal-rich and crystal-poor lavas. *J. Volcanol. Geotherm. Res.*, 336, 51-67. doi: 10.1016/j.volgeores.2017.02.005
- Dahm, T. (2000a). Numerical simulations of the propagation path and the arrest of fluid-filled fractures in the Earth. *Geophys. J. Int.*, 141, 623-638. doi: 10.1046/j.1365-246x.2000.00102.x
- Dahm, T. (2000b). On the shape and velocity of fluid-filled fractures in the Earth. *Geophys. J. Int.*, 142, 181-192. doi: 10.1046/j.1365-246x.2000.00148.x
- Dahm, T., Hainzl, S. & Fischer, T. (2010). Bidirectional and unidirectional fracture growth during hydrofracturing: Role of driving stress gradients. *J. Geophys. Res.*, 115, B12322. doi: 10.1029/2009JB006817

References

- Daniels, K., A., Kavanagh, J., L., Menand, T. & Sparks, R., S., J. (2012). The shapes of dikes: Evidence for the influence of cooling and inelastic deformation. *G. S. A. Bull.*, 124, 7/8, 1102-1112. doi: 10.1130/B30537.1
- Daniels, K., A., Bastow, I., D., Keir, D., Sparks, R., S., J. & Menand, T. (2014). Thermal models of dyke intrusion during development of continent-ocean transition. *Earth Planet. Sci. Lett.*, 385, 145-153. doi: 10.1016/j.epsl.2013.09.018
- Daniels, K., A. & Menand, T. (2015). An experimental investigation of dyke injection under regional extensional stresses. *J. Geophys. Res. Solid Earth*, 120, 2014-2035, doi: 10.1002/2014JB011627
- Davis (2018). Göttingen, Germany: Lavision.
- Degruyter, W. & Huber, C. (2014). A model for eruption frequency of upper crustal silicic magma chambers. *Earth Planet. Sci. Lett.*, 403, 117-130. doi: 10.1016/j.epsl.2014.06.047
- Delaney, P., T. & Pollard, D., D. (1982). Solidification of basaltic magma during flow in a dike. *Amer. J. Sci.*, 282, 856-885
- Delaney, P. (1986). Field relations between dikes and joints: Emplacement processes and paleostress analysis. *J. Geophys. Res.*, 91, B5, 4920-4938. doi: 10.1029/JB091iB05p04920
- Delaney, P., T. (1988). Fortran 77 programs for conductive cooling of dikes with temperature-dependent thermal properties and heat of crystallization. *Comp & Geosci*, 14, 2, 181-212
- Di Giuseppe, E., Funiciello, F., Corbi, F., Ranalli, G. & Mojoli, G. (2009). Gelatins as rock analogs: A systematic study of their rheological and physical properties. *Tectonophys.*, 473, 391-403. doi: 10.1016/j.tecto.2009.03.012
- Douglas, M., M., Geyer, A., Álvarez-Valero, A., M. and Martí, J. (2016). Modeling magmatic accumulations in the upper crust: Metamorphic implications for the country rock. *J. Volcanol Geotherm. Res.*, 319, 78-92. doi: 10.1016/j.jvolgeores.2016.03.008
- Eberl, C., Thompson, R., Gianola, D. & Bundschuh, S. (2006). Digital image correlation and tracking with Matlab, v 1.2. <http://www.mathworks.com/matlabcentral/fileexchange/12413>.
- Ebinger, C., J., Keir, D., Ayele, A., Calais, E., Wright, T., J., Belachew, M., Hammond, J., O., S., Campbell, E. & Buck, W., R., (2008). Capturing magma intrusion and faulting processes during continental rupture: seismicity of the Dabbahu (Afar) rift. *Geophys. J. Int.*, 174, 1138-1152. doi: 10.1111/j.1365-246X.2008.03877.x

References

- Edmonds, M. & Woods, A. (2018). Exsolved volatiles in magma reservoirs. *J. Volcanol. Geotherm. Res.*, 368, 13-30. doi: 10.1016/j.jvolgeores.2018.10.1018
- Edmonds, M., Cashman, K., Holness, M. & Jackson, M. (2019). Architecture and dynamics of magma reservoirs. *Phil. Trans. Roy. Soc. A*, 377, 20180298. doi: 10.1098/rsta.2018.0298
- Eichelberger, J., C. & Izbekov, P., E. (2000). Eruption of andesite triggered by dyke injection: Contrasting cases at Karymsky volcano, Kamchatka and Mt Katmai, Alaska. *Phil. Trans. R. Soc. Lond. A*, 358, 1465-1485. doi: 10.1098/rsta.2000.0599
- Einarsson, P. & Brandsdóttir, B. (1980). Seismological evidence for lateral magma intrusion during the July 1978 deflation of Krafla volcano in NE-Iceland. *J. Geophys.*, 47, 160-165.
- Fialko, Y., A & Rubin, A., M. (1998). Thermodynamics of lateral dike propagation: Implications for crustal accretion at slow spreading mid-ocean ridges. *J. Geophys. Res.*, 103, B2, 2501-2514. doi: 10.1029/97JB03105
- Fierstein, J. and Hildreth, W. (1992). The plinian eruptions of 1912 at Novarupta, Katmai National Park, Alaska. *Bull. Volcanol.*, 54, 646-684. doi: 10.1007/BF00430778
- Fukushima, Y., Cayol, V., Durand, P. and Massonnet, D. (2010). Evolution of magma conduits during the 1998-2000 eruptions of Piton de la Fournaise, Réunion Island. *J. Geophys. Res.*, 115, B10. doi: 10.1029/2009JB007023
- Furumoto, A., S. (1978). Nature of the magma conduit under the East Rift Zone of Kilauea volcano, Hawaii. *Bull. Volcanol.*, 41, 4, 435-453. doi: 10.1007/BF02597376
- Galland, O. (2012). Experimental modelling of ground deformation associated with shallow magma intrusions. *Earth Planet. Sci. Lett.*, 317-318, 145-156. doi: 10.1016/j.epsl.2011.10.017
- Galland, O., Burchardt, S., Hallot, E., Mourgues, R. and Bulois, C. (2014). Dynamics of dikes versus cone sheets in volcanic systems. *J. Geophys. Res. Solid Earth*, 119, 6178-6192. doi: 10.1002/2014JB011059
- Galland, O., Holohan, E., van Wyk de Vries, B., & Burchardt, S. (2015). Laboratory modelling of volcano plumbing systems: A review. In *Adv. in Volcanology*, Berlin, Heidelberg. doi: 10.1007/11157_2015_9
- Galland, O., Bertelsen, H., S., Guldstrand, F., Girod, L., Johannessen, R., F., Bjugger, F., Burchardt, S. & Mair, K. (2016). Application of open-source photogrammetric software MicMac for monitoring surface deformation in laboratory models. *J. Geophys. Res.*, 121, 2852-2872. doi: 10.1002/2015JB012564

References

- Geishi, N., Kusumoto, S. & Gudmundsson, A. (2012). Effects of mechanical layering of host rocks on dike growth and arrest. *J. Volcanol. Geotherm. Res.*, 223-224, 74-82. doi: 10.1016/j.jvolgeores.2012.02.004
- Giuseppe Di E, Funicello F, Corbi F, Ranalli G, Mojol G (2009). Gelatins as rock analogs: A systematic study of their rheological and physical properties. *Tectonophys*, 473, 391-403.
- Gonnermann, H. & Taisne, B. (2015). Magma transport in dikes. In *The Encyclopedia of Volcanoes 2nd ed.*, Elsevier Inc. doi: 10.1016/B978-0-12-385938-9.00010-9
- Gordeychik, B., Churikova, T., Kronz, A., Sundermeyer, C., Simakin, A. & Wörner, G. (2018). Growth of, and diffusion in, olivine in ultra-fast ascending basalt magmas from Shiveluch volcano. *Sci. Rep.*, 8, 11775, doi: 10.1038/s41598-018-30133-1
- Green, D., H. and Falloon, T., J. (2015). Mantle-derived magma: intraplate, hot-spots and mid-ocean ridges. *Sci. Bull.*, 60(22), 1873-1900. doi: 10.1007/s11434-015-0920-y
- Gudmundsson, A. (2006). How local stresses control magma-chamber ruptures, dyke injections, and eruptions in composite volcanoes. *Earth-Sci. Rev.*, 79, 1-31. doi: 10.1016/j.earscirev.2006.06.006
- Gudmundsson, A. (2012). Magma chambers: Formation local stresses, excess pressures, and compartments. *J. Volcanol. Geotherm. Res.*, 237-238, 19-41. doi: 10.1016/j.jvolgeores.2012.05.015
- Gudmundsson, A., Lecoeur, N., Mohajeri, N., & Thordarson, T. (2014). Dike emplacement at Bardarbunga, Iceland, induces unusual stress changes, caldera formation, and earthquakes. *Bull. Volc.*, 76, 869, doi: 10.1007/s00445-014-0869-8
- Harris, A., J., L., Flynn, L., P., Rothery, D., A., Oppenheimer, C. & Sherman, S., B (1999). Mass flux measurements at active lava lakes: Implications for magma recycling. *J. Geophys. Res.*, 104, B4, 7117-7136.
- Hayashi, Y. & Morita, Y. (2003). An image of a magma intrusion process inferred from precise hypocentral migrations of the earthquake swarm east of the Izu Peninsula. *Geophys. J. Int.*, 153, 159-174. doi: 10.1046/j.1365-246X.2003.01892.x
- Hildreth, W. (1987). New perspectives on the eruption of 1912 in the Valley of Ten Thousand Smokes, Katmai National Park, Alaska. *Bull. Volcanol.*, 49, 680-693. doi: 10.1007/BF01080359.
- Holmes-Cerfon, M., C. & Whitehead, J., A. (2011). Instability and freezing in a solidifying melt conduit. *Physica D*, 240, 131-139. doi: 10.1016/j.physd.2010.10.009
- Hou, G., Kusky, T., M., Wang, C., & Wang, Y. (2010). Mechanics of the giant radiating Mackenzie dyke swarm: A paleostress field modeling. *J. Geophys. Res.*, 115, B02402, doi: 10.1029/2007JB005475

References

- Karlstrom, L., Dufek, J., & Manga, M. (2009). Organization of volcanic plumbing through magmatic lensing by magma chambers and volcanic loads. *J. Geophys. Res.*, 114, B10204. doi: 10.1029/2009JB006339
- Kavanagh, J., L., Menand, T. & Daniels, K., A. (2013). Gelatin as a crustal analogue: Determining elastic properties for modelling magmatic intrusions. *Tectonophys.*, 582, 101-111. doi: 10.1016/j.tecto.2012.09.032
- Kavanagh, J., L., Boutelier, D. and Cruden, A. R. (2015). The mechanics of sill inception, propagation and growth: Experimental evidence for rapid reduction in magmatic overpressure. *Earth Planet. Sci. Lett.*, 421, 117-128. doi: 10.1016/j.epsl.2015.03.038
- Kavanagh, J., L., Burns, A., J., Hazim, S., H., Wood, E., P., Martin, S., A., Hignett, S. & Dennis, D., J., C. (2018). Challenging dyke ascent models using novel laboratory experiments: Implications for reinterpreting evidence of magma ascent and volcanism. *J. Volcanol. Geotherm. Res.*, 354, 87-101. doi: 10.1016/j.jvolgeores.2018.01.002
- Keating, G., N., Valentine, G., A., Krier, D., J. and Perry, F., V. (2008). Shallow plumbing systems for small-volume basaltic volcanoes. *Bull. Volcanol.*, 70, 563-582. doi: 10.1007/s00445-007-0154-1
- Koenderink, J., J. & van Doorn, A., J. (1991). Affine structure from motion. *J. Opt. Soc. Am. A*, 8, 2, 377-385. doi: /10.1364/JOSAA.8.000377
- Kühn, D. & Dahm, T. (2008). Numerical modelling of dyke interaction and its influence on oceanic crust formation. *Tectonophys.*, 447, 53-65. doi: 10.1016/j.tecto.2006.09.018.
- Jones, E., M., C. (2015). Matlab-based DIC code, v4.
- Jones, T., J., Llewellyn, E., W., Houghton, B., F., Brown, R., J. and Vye-Brown, C. (2017). Proximal lava drainage controls on basaltic fissure eruption dynamics. *Bull. Volcanol.*, 79, 81. doi: 10.1007/s00445-017-1164-2
- Lee, B., J., Kee, S.-H., Oh, T., & Kim, Y.-Y. (2017). Evaluating the dynamic elastic modulus of concrete using shear-wave velocity measurements. *Adv. Mat. Sci. Eng.*, 2017, #1651753. doi: 10.1155/2017/1651753
- Lengliné, O., Dutuel, Z. & Ferrazzini, V. (2016). Uncovering the hidden signature of a magmatic recharge at Piton de la Fournaise volcano using small earthquakes. *Geophys. Res. Lett.*, 43, 4255-4262. doi: 10.1002/2016GL068383
- Leslie, S., C., Moore, G., F. and Morgan, J., K. (2004). Internal structure of Puna Ridge: evolution of the submarine East Rift Zone of Kilauea, Hawai'i. *J. Volcanol. Geotherm. Res.*, 129, 237-259. doi: 10.1016/S0377-0273(03)00276-2

References

- Lister, J., R. (1990). Buoyancy-driven fluid fracture: Similarity solutions for the horizontal and vertical propagation of fluid-filled cracks. *J. Fluid Mech.*, 217, 213-239.
- Lister, J., R. & Kerr, C. (1991). Fluid-mechanical models of crack propagation and their application to magma transport in dykes. *J. Geophys. Res.*, 96, B6, 10049-10077. doi: 10.1029/91jb00600
- Llewellyn, E., W. & Manga, M. (2005). Bubble suspension rheology and implications for conduit flow. *J. Volcanol. Geotherm. Res.*, 143, 205-217. doi: 10.1016/j.jvolgeores.2004.09.108
- Lundgren, P., Kiryukhin, A., Milillo, P. & Samsonov, S. (2015). Dike model for the 2012-2013 Tolbachik eruption constrained by satellite radar interferometry observations. *J. Volcanol. Geotherm. Res.*, 307, 79-88. doi: 10.1016/j.jvolgeores.2015.05.011
- Maccaferri, F., Bonafede, M., & Rivalta, E. (2011). A quantitative study of the mechanisms governing dike propagation, dike arrest and sill formation. *J. Volcanol. Geotherm. Res.*, 208, 39-50. doi: 10.1016/j.jvolgeores.2011.09.001
- Maccaferri, F., Rivalta, E., Keir, D., & Acocella, V. (2014). Off-rift volcanism in rift zones determined by crustal unloading. *Nature Geosci*, 7, 297-300. doi: 10.1038/NGEO2110
- Maccaferri, F., Smittarello, D., Pinel, V. & Cayol, V. (2019). On the propagation path of magma-filled dikes and hydrofractures: The competition between external stress, internal pressure and, crack length. *Geochem, Geophys, Geosyst*, 20. doi: 10.1029/2018GC007915.
- Magee, C., Stevenson, C., O'Driscoll, B., Schofield, N., & McDermott, K. (2012). An alternative emplacement model for the classic Ardnamurchan cone sheet swarm, NW Scotland, involving lateral magma supply via regional dykes. *J. Struc. Geol.*, 43, 73-91, doi:10.1016/j.jsg.2012.08.004
- Makov G (2009). *The rock physics handbook: Tools for seismic analysis of porous media*, 2nd ed. Leiden: Cambridge Univ. Press.
- Menand, T. & Tait, S. (2002). The propagation of a buoyant liquid-filled fissure from a source under constant pressure: An experimental approach. *J. Geophys. Res.*, 107, B112306, doi: 10.1029/2001JB000589
- Menand, T., Daniels, K., A. & Benghiat, P. (2010). Dyke propagation and sill formation in a compressive tectonic environment. *J. Geophys. Res.*, 115, B08201. doi: 10.1029/2009JB006791
- Menand, T. (2011). Physical controls and depth of emplacement of igneous bodies: A review. *Tectonophys.*, 500, 11-19. doi: 10.1016/j.tecto.2009.10.1016

References

- Meredith, P., G. & Atkinson, B., K. (1985). Fracture toughness and subcritical crack growth during high-temperature tensile deformation of Westerly granite and Black gabbro. *Phys. Earth Planet. Inter.*, 39, 33-51. doi: 10.1016/0031-9201(85)90113-X.
- Michon, L., Ferrazzini, V., Di Muro, A., Villeneuve, N. and Famin, V. (2015). Rift zones and magma plumbing system of Piton de la Fournaise volcano: How do they differ from Hawaii and Etna? *J. Volcanol. Geotherm. Res.*, 303, 112-129. doi: 10.1016/j.jvolgeores.2015.07.031
- Mogi, K. (1958). Relations between the eruptions of various volcanoes and the deformations of the ground surface around them. *Bull. Earthq. Res.*, 36, 99 – 134.
- Molina, I., Burgisser, A. & Oppenheimer, C. (2012). Numerical simulations of convection in crystal-bearing magmas: A case study of the magmatic system at Erebus, Antarctica. *J. Geophys. Res.*, 117, B07209. doi: 10.1029/2011JB008760
- Molloy, C., Shane, P. and Augustinus, P. (2009). Eruption recurrence rates in a basaltic volcanic field based on tephra layers in maar sediments: Implications for hazards in the Auckland volcanic field. *Geol. Soc. Amer. Bull.*, 121, 11/12, 1666-1677. doi: 10.1130/B26447.1
- Morita, Y., Nakao, S. & Hayashi, Y. (2006). A quantitative approach to the dike intrusion process inferred from a joint analysis of geodetic and seismological data for the 1998 earthquake swarm off the coast of Izu Peninsula, central Japan. *J. Geophys. Res.*, 111, B06208. doi: 10.1029/2005JB003860
- Morse, S., A. (2011). The fractional latent heat of crystallizing magmas. *Amer. Mineral.*, 96, 682-689. doi: 10.2138/am.2011.3613
- Mulligan, J., C. & Jones, D., D. (1975). Experiments on heat transfer and pressure drop in a horizontal tube with internal solidification. *Int. J. Heat Mass Trans.*, 19, 213-219.
- Mullineaux, D., R. (1986). Summary of pre-1980 tephra-fall deposits erupted from Mount St. Helens, Washington State, USA. *Bull. Volcanol.*, 48, 17-26.
- Murphy, M., D., Sparks, R., J., S., Barclay, J., Carrol, M., R., & Brewer, T., S. (2000). Remobilization of andesite magma by intrusion of mafic magma at the Soufriere Hills volcano, Montserrat, West Indies. *J. Petrol.*, 41, 1, 21-42. doi: 10.1093/petrology/41.1.21
- Nobile, A., Pagli, C., Keir, D., Wright, T., J., Ayele, A., Ruch, J. & Acocella, V. (2012). Dike-fault interaction during the 2004 Dallol intrusion at the northern edge of the Erta Ale Ridge (Afar, Ethiopia). *Geophys. Res. Lett.*, 39, L19305. doi: 10.1029/2012GL053152
- Nunnari, G., Puglisi, G. & Guglielmino, F. (2005). Inversion of SAR data in active volcanic areas by optimization techniques. *Nonlinear Proc. Geophys.*, 12, 863-870

References

- Nur Hanani, Z., A., Roos, Y., H. & Kerry, J., P. (2012). Use of beef, pork and fish gelatin sources in the manufacture of films and assessment of their composition and mechanical properties. *Food Hydrocolloids*, 29, 144-151.
- Okada, Y. (1985). Surface deformation due to shear and tensile faults in a half-space. *Bull. Seismol. Soc. Amer.*, 75 (4), 1135-1154.
- Okada, Y., Yamamoto, E. & Ohkubo, T. (2000). Coswarm and preswarm crustal deformation in the eastern Izu Peninsula, central Japan. *J. Geophys. Res.*, 105, 1, 681-692.
- Oppenheimer, C., McGonigle, A., J., S., Allard, P., Wooster, M., J. & Tsanev, V. (2004). Sulfur, heat, and magma budget of Erta 'Ale lava lake, Ethiopia. *Geol. Soc. Amer.*, 32, 6, 509-512. doi: 10.1130/G20281.1
- Owen, S., Segall, P., Lisowski, M., Miklius, A., Murray, M., Bevis, M. and Foster, J. (2000). January 30, 1997 eruptive event on Kilauea Volcano, Hawaii, as monitored by continuous GPS. *Geophys. Res. Lett.*, 27, 17, 2757-2760. doi: 10.1029/1999GL008454
- Pansino, S. & Taisne, B. (2019). How magmatic storage regions attract and repel propagating dikes. *J. Geophys. Res. Sol. Earth*, 124, 274-290. doi: 10.1029/2018JB016311
- Pansino, S., Emadzadeh, A. & Taisne, B. (2019). Dike channelization and solidification: Time scale controls on the geometry and placement of magma migration pathways. *J. Geophys. Res. Sol. Earth*, 124. doi: 10.1029/2019JB018191
- Parfitt, E., A. & Wilson, L. (2008). *Fundamentals of Physical Volcanology*. Blackwell Publishing Company, Malden MA, USA. ISBN 978-0-063205443-5
- Pascal, K., Neuber, J. & Rivalta, E. (2014). On precisely modelling surface deformation due to interacting magma chambers and dykes. *Geophys. J. Int.*, 196, 253-278. doi: 10.1093/gji/gg343
- Pashkevich, R., I. & Taskin, V., V. (2009). Heat transfer in a geothermal system of Mutnovsky volcano: The influence of the form, discharge of magma chamber degassing and rocks permeability. *Proceedings from Workshop on Geothermal Reservoir Engineering*.
- Patrick, M., Orr, T., Anderson, K. & Swanson, D. (2019). Eruptions in sync: Improved constraints on Kilauea Volcano's hydraulic connection. *Earth Planet. Sci. Lett.*, 507, 50-61. doi: 10.1016/j.epsl.2018.11.030
- Photoscan (2017). St. Petersburg, Russia: Agisoft.

References

- Pinel, V. & Jaupart, C. (2004). Magma storage and horizontal dyke injection beneath a volcanic edifice. *Earth Planet. Sci. Lett.*, 221, 245-262. doi: 10.1016/S0012-821X(04)00076-7
- Pinel, V., Carrara, A., Maccaferri, F., Rivalta, E. & Corbi, F. (2017). A two-step model for dynamical dike propagation in the two dimensions: Application to the July 2001 Etna eruption. *J. Geophys. Res.*, 122. doi: 10.1002/2016JB013630
- Pioli, L., Bonadonna, C., Azzopardi, B., J., Phillips, J., C. & Ripepe, M. (2012). Experimental constraints on the outgassing dynamics of basaltic magmas. *J. Geophys. Res.*, 117, B03204. doi: 10.1029/2011JB008392
- Polland, M., P., Moats, W., P. & Fink, J., H. (2008). A model for radial dike emplacement in composite cones based on observations from Summer Coon volcano, Colorado, USA. *Bull. Volcanol.*, 70, 861-875. doi: 10.1007/s00445-007-0175-9
- Porreca, M., Acocella, V., Massimi, E., Mattei, M., Funcicello, R., & De Benedetti, A., A. (2006). Geometric and kinematic features of the dike complex at Mt. Somma, Vesuvio (Italy). *Earth Planet. Sci. Lett.* 245, 389-407. doi: 10.1016/j.epsl.2006.02.027
- Raffel, M., Willert, C., E. & Kompenhans, J. (2013). Particle image velocimetry: a practical guide. Springer.
- Ramesh, K. (2000). Digital photoelasticity – Advanced techniques and applications. Springer-Verlag Berlin Heidelberg. doi: 10.1007/978-3-642-59723-7
- Riel, B, Milillo, P., Simons, M., Lundgren, P., Kanamori, H. & Samsonov, S. (2015). The collapse of Bárðarbunga caldera, Iceland. *Geophys. J. Int.*, 202, 446-453. doi: 10.1093/gji/ggv157
- Rivalta, E., Böttinger, M. & Dahm, T. (2005). Buoyancy-driven fracture ascent: Experiments in layered gelatin. *J. Volcanol. Geotherm. Res.*, 144, 273-285. doi: 10.1016/j.jvolgeores.2004.11.030
- Rivalta, E., Taisne, B., Bunger, A., P., & Katz, R., F. (2015). A review of mechanical model of dike propagation: Schools of thought, results and future directions. *Tectonophysics*, 638, 1-42. doi: 10.1016/j.tecto.2014.10.003
- Rogers, N. (2015). The composition and origin of magmas. In *The Encyclopedia of Volcanoes 2nd ed.*, Elsevier Inc. doi: 10.1016/B978-0-12-385938-9.00004-3
- Roman, A. & Jaupart, C (2014). The impact of a volcanic edifice on intrusive and eruptive activity. *Earth Planet Sci. Lett.*, 408, 1-8. doi: 10.1016/j.epsl.2014.09.016
- Roper, S., M. & Lister, J., R. (2005). Buoyancy-driven crack propagation from an over-pressured source. *J. Fluid Mech.*, 536, 79-98. doi: 10.1017/S0022112005004337

References

- Roper, S., M., & Lister, J., R. (2007). Buoyancy-driven crack propagation: the limit of large fracture toughness. *J. Fluid Mech.*, 580, 359-380. doi: 10.1017/S0022112007005472
- Rubin, A., M. (1995). Propagation of magma-filled cracks. *Annu. Rev. Earth Planet. Sci.*, 23, 287-336. doi: 10.1146/annurev.ea.23.050195.001443
- Rudmann, A., J. & Epp, D. (1983). Conduction models of the temperature distribution in the East Rift Zone of Kilauea volcano. *J. Volcanol. Geotherm. Res.*, 16, 189-203.
- Ruth, D., C., S., Cottrell, E., Cortés, J., A., Kelley, K., A., & Calder, E., S. (2016). From passive degassing to violent strombolian: The case of the 2008 eruption of Llaima volcano, Chile. *J. Petrol*, 57 (9), 1833-1864. doi: 10.1093/petrology/egw063
- Schlichting, H. (1979). *Boundary Layer Theory*. 7th Edition, McGraw-Hill, New York.
- Schmiedel, T., Galland, O. & Breitzkreuz, C. (2017). Dynamics of sill and laccolith emplacement in the brittle crust: Role of host rock strength and deformation mode.
- Schofield, N., J., Brown, D., J., Magee, C. & Stevenson, C., T. (2012). Sill morphology and comparison of brittle and non-brittle emplacement mechanisms. *J. Geol. Soc. London*, 169, 127-141. doi: 10.1144/0016-76492011-078
- Schonwalder-Angel, D., Cortés, J., A. & Calder, E., S. (2018). The interplay of magmatism and tectonics: An example based on the satellite scoria cones at Llaima volcano, Chile. *J. Volcanol. Geotherm. Res.*, 367, 31-45. doi: 10.1016/j.volgeores.2018.10.020
- Schroeder, A. & Willert, C., E. (2008). *Particle image velocimetry: New developments and recent applications*. Springer. ISBN 978-3-540-73528-1.
- Segall, P. (2010). *Earthquake and volcano deformation*. Princeton Univ. Press, p203-205. ISBN: 9781400833856
- Shane, P. & Coote, A. (2018). Thermobarometry of Whangarei volcanic field lavas, New Zealand: Constraints on plumbing systems of small monogenetic basalt volcanoes. *J. Volcanol. Geotherm. Res.*, 354, 130-139. doi: 10.1016/j.jvolgeores.2018.02.013
- Sim, K., S., Nia, M., E. & Tso, C., P. (2013). Review of single image signal-to-noise ratio estimation for SEM image. *A.I.P. Conf. Proc.*, 1528, 365. doi: 10.1063/1.4803626
- Sparks, R., S., J. (1978). The dynamics of bubble formation and growth in magmas: a review and analysis. *J. Volcanol. Geotherm. Res.*, 3, 1-2, 1-37.
- Swanson, D., A., Casadevall, T., J., Dzurisin, D., Holcomb, R., T., Newhall, C., G., Malone, S., D. & Weaver, C., S. (1985). Forecasts and predictions of eruptive activity at Mount St. Helens, USA: 1975-1984. *J. Geodyn.*, 3, 397-423.

References

- Sweeney, D., Kyle, P., R. & Oppenheimer, C. (2008). Sulfur dioxide emissions and degassing behavior of Erebus volcano, Antarctica. *J. Volcanol. Geotherm. Res.*, 177, 725-733. doi: 10.1016/j.jvolgeores.2008.01.024
- Taisne, B. & Jaupart, C. (2009). Diike propagation through layered rocks. *J. Geophys. Res.*, 114, B09203. doi: 10.1029/2008JB6228
- Taisne, B. & Tait, S. (2009). Eruption versus intrusion? Arrest of propagation of constant volume, buoyant, liquid-filled cracks in an elastic, brittle host. *J. Geophys. Res.*, 114, B06202. doi: 10.1029/2009JB006297
- Taisne, B., Brenguier, F., Shapiro, N., M. & Ferrazzini, V. (2011). Imaging the dynamics of magma propagation using radiated seismic intensity. *Geophys. Res. Lett.*, 38, L04304. doi: 10.1029/2010GL046068
- Taisne, B. & Jaupart, C. (2011). Magma expansion and fragmentation in a propagating dyke. *Earth Planet. Sci. Lett.*, 301, 146-152. doi: 10.1016/j.epsl.2010.10.038
- Taisne, B. & Tait, S. (2011). Effect of solidification on a propagating diike. *J. Geophys. Res.*, 116, B01206. doi: 10.1029/2009JB007058
- Taisne, B., Tait, S. & Jaupart, C. (2011). Conditions for the arrest of a vertical propagating dyke. *Bull. Volcanol.*, 73, 191-204. doi: 10.1007/s00445-010-0440-1
- Takada, A. (1990). Experimental study on propagation of liquid-filled crack in gelatin: shape and velocity in hydrostatic stress condition. *J. Geophys. Res. Sol. Earth*, 95, B6, 8471-8481. doi: 10.1029/JB095iB06p08471
- Takada, A. (1997). Cyclic flank-vent and central-vent eruption patterns. *Bull. Volcanol.*, 58, 539-556. doi: 10.1029/JB095iB06p08471
- Takeuchi, S. (2004). Precursory diike propagation control of viscous magma eruptions. *Geology*, 32, 11, 1001-1004. doi: 10.1130/G20792.1
- Tosh, S., M., Marangoni, A., G., Ross Hallett, F. & Britt, I., J. (2003). Aging dynamics in gelatin gel microstructure. *Food Hydrocolloids*, 17, 503-513.
- Townsend, M., R., Pollard, D., D. & Smith, R., P. (2017). Mechanical models for dikes: A third school of thought. *Tectonophys.*, 703-704, 98-118. doi: 10.1016/j.tecto.2017.03.008
- Townsend, M., Huber, C., Degruyter, W. & Bachmann, O. (2019). Magma chamber growth during intercaldera periods: Insights from thermo-mechanical modeling with applications to Laguna del Maule, Campi Flegrei, Santorini, and Aso. *Geochem., Geophys., Geosyst.*, 20, 1574-1591. doi: 10.1029/2018GC008103
- Trippanera, D., Ruch, J., Acocella, V. & Rivalta, E. (2015). Experiments of diike- diike-induced deformation: Insights on the long-term evolution of divergent plate boundaries. *J. Geophys. Res.*, 120, 6913-6942. doi: 10.1002/2014JB011850

References

- Tryggvason, E. (1986). Multiple magma reservoirs in a rift zone volcano: Ground deformation and magma transport during the September 1984 eruption of Krafla, Iceland. *J. Volcanol. Geotherm Res.*, 28, 1-44.
- Turcotte, D., L. and Schubert, G. (2002). *Geodynamics*, 2nd ed. Cambridge University Press.
- Urbani, S., Acocella, V. & Rivalta, E. (2018). What drives the lateral versus vertical propagation of dikes? Insights from analogue models. *J. Geophys. Res. Sol. Earth*, 123, 5, 3680-3697. doi: 10.1029/2017JB015376
- Valentine, G., A. and Connor, C. B. (2015). Basaltic volcanic fields. In *The Encyclopedia of Volcanoes*, 2nd ed., Academic Press. doi: 10.1016/B978-0-12-385938-9.00023-7
- Wada, Y (1994). On the relationship between dike width and magma viscosity. *J. Geophys. Res.*, 99, B9, 17743-17755.
- Watanabe, T., Masuyama, T., Nagaoka, K., & Tahara, T. (2002). Analog experiments on magma-filled cracks: Competition between external stresses and internal pressure. *Earth Planet. Space*, 54, 1247-1261. doi: 10.1186/bf03352453
- Watts, R., B., de Silva, S., L., Jimenez de Rios, G. & Croudace, I. (1999). Effusive eruption of viscous silicic magma triggered and driven by recharge: a case study of the Cerro Chascon-Runtu Jarita Dome Complex in Southwest Bolivia. *Bull. Volcanol.*, 60, 241-264. doi: 10.1007/s004450050274
- Wauthier, C., Cayol, V., Smets, B., d'Oreye, N. and Kervyn, F. (2015). Magma pathways and their interactions inferred from InSAR and stress modeling at Nyamulagira volcano, D.R. Congo. *Remote Sens.*, 2015, 7, 15179-15202. doi: 10.3390/rs71115179
- Weertman, J. (1971). Velocity at which liquid-filled cracks move in the Earth's crust or in glaciers. *J. Geophys. Res.*, 76, 35, 8544-8553.
- Weertman, J. (1980). The stopping of a rising, liquid-filled crack in the Earth's crust by a freely slipping horizontal joint. *J. Geophys. Res.*, 85, B2, 967-976.
- White, D., J., Take, W., A. & Bolton, M., D. (2003). Soil deformation measurement using particle image velocimetry (PIV) and photogrammetry. *Geotechniques*, 53, 619-631
- Whitehead, J., A. & Helfrich, K., R. (1991). Instability of flow with temperature-dependent viscosity: A model of magma dynamics. *J. Geophys. Res.*, 96, B3, 4145-4155.
- Wnuk, K. and Wauthier, C. (2017). Surface deformation induced by magmatic processes at Pacaya Volcano, Guatemala revealed by InSAR. *J. Volcanol. Geotherm. Res.*, 344, 197-211. doi: 10.1016/j.jvolgeores.2017.06.024

References

- Wright, T., L. & Marsh, B. (2016). Quantification of the intrusion process at Kīlauea volcano, Hawai'i. *J. Volcanol. Geotherm. Res.*, 328, 34-44. doi: 10.1016/j.jvolgeores.2016.09.019
- Wylie, J., J., Helfrich, K., R., Dade, B., Lister, J., R. and Salzig, J., F. (1999). Flow localization in fissure eruptions. *Bull. Volcanol.*, 60, 432-440. doi:10.1007/s004450050243
- Wylie, J., J & Lister, J., R. (1995). The effects of temperature-dependent viscosity on flow in a cooled channel with application to basaltic fissure eruptions. *J. Fluid Mech.*, 305, 239-261. doi: 10.1017/S0022112095004617
- Zerkle, R., D. & Sunderland, J., E. (1968). The effect of liquid solidification in a tube upon laminar-flow heat transfer and pressure drop. *J. Heat Trans.*, 90, 2, 183-189. doi: 10.1115/1.3597471
- Zhang, P & Porfiri, M. (2019). A combined digital image correlation/particle image velocimetry study of water-backed impact. *Composite Struct.*, 224, 111010

RESEARCH ARTICLE

10.1029/2018JB016311

Key Points:

- Dike propagation is generally attracted to overpressure magma reservoirs and repelled by underpressure reservoirs
- The mechanism that attracts or repels dikes differs, depending on the dike's initial orientation
- The likelihood of satellite/off-axes eruptions increases following large eruptions and evolves with time following viscous stress relaxation

Supporting Information:

- Supporting Information S1
- Movie S1
- Movie S2
- Movie S3
- Movie S4

Correspondence to:

S. Pansino,
steve.pansino@gmail.com

Citation:

Pansino, S., & Taisne, B. (2019). How magmatic storage regions attract and repel propagating dikes. *Journal of Geophysical Research: Solid Earth*, 124, 274–290. <https://doi.org/10.1029/2018JB016311>

Received 2 JUL 2018

Accepted 17 DEC 2018

Accepted article online 20 DEC 2018

Published online 19 JAN 2019

How Magmatic Storage Regions Attract and Repel Propagating Dikes

Stephen Pansino¹ and Benoit Taisne^{1,2}
¹Earth Observatory of Singapore, Nanyang Technological University, Singapore, Singapore, ²Asian School of the Environment, Nanyang Technological University, Singapore, Singapore

Abstract We investigate the effect of magmatic reservoir pressure on the propagation of dikes that approach from below, using analogue experiments. We injected oil into gelatin and observed how dike propagation responded to the stress field around a pressurized, spherical reservoir, filled with water. The reservoir was modeled using two different setups: one simply using an inflatable rubber balloon and the other by constructing a liquid-filled cavity. We find that the dike's response is dependent on the sign of the reservoir pressure (i.e., inflated/overpressurized and deflated/underpressurized) as well as on the dike's initial orientation (i.e., if its strike is radially, circumferentially, or obliquely oriented to the reservoir). Dikes that are initially strike radial respond, respectively, by propagating toward or away from overpressurized or underpressurized reservoirs, taking advantage of the reservoir's hoop stresses. Otherwise-oriented dikes respond by changing orientation, twisting and curling into a form dictated by the principal stresses in the medium. For overpressurized reservoirs, they are coaxed to propagate radially to, and therefore approach, the reservoir. For underpressurized reservoirs, they generally reorient to propagate tangentially, which causes them to avoid the reservoir. The magnitude of reservoir pressure controls at which distance dikes can be affected, and, at natural scales, we estimate that this occurs within a radius of a few tens of kilometers. This diminishes with time, due to viscous stress relaxation of the crust, which will occur on a timescale of hundreds of years.

Plain Language Summary Magma commonly moves up toward the surface by creating cracks in the crust. It flows inside of the cracks and propagates by applying pressure that drives the flow and damages surrounding rocks. Nature always finds the easiest path for the crack, so if it takes less pressure to push apart the ground vertically or horizontally, the crack will grow accordingly. As it makes its way to the surface, it may encounter local stress variations that change its propagating direction. This applies near magma storage regions, below volcanoes. If such a region is highly pressurized or deflated, then nearby cracks will “feel” the change in their surrounding conditions and react by aligning in a direction of favorable stress. This makes it look like they are growing toward or circling around the storage region, respectively. We studied this behavior using scaled model experiments in laboratory conditions. We use different types of materials to represent nature, such as gelatin as rock and oil as magma. We were able to show how these cracks change shape for different reservoir pressures. We found that after a large eruption, subsequent eruptions are more likely to occur farther from the summit of a volcano.

1. Introduction

When magma ascends from depth toward a shallow magmatic storage region, it can either migrate via an existing pathway (i.e., a conduit) or fracture open a new pathway in the form of a dike. Deeply sourced feeder dikes can supply large volumes of mafic magma, which may either arrive to a shallower reservoir and mingle with the existing magma or bypass the reservoir and erupt at the surface. For example, some monogenetic eruptions at Sanganguey volcano (Mexico) emit lavas that come directly from a deep reservoir, while others emit lavas that represent a mixture of deeply and shallowly residing magmas (Crabtree & Waters, 2017). The path that a feeder dike takes depends on the state of stresses in the crust, and one significant source of stress can be a pressurized magmatic reservoir, which has been shown can attract feeder dikes (Karlstrom et al., 2009). Generally speaking, when fresh magma arrives to a magmatic storage region, it is thought to remobilize the residing magma, which can lead to an eruption (Murphy et al., 2000; Ruth et al., 2016). Whether a dike propagates toward a shallow reservoir and mingles with the shallow magma, or bypasses the reservoir, has an effect on the probability, location, and style of eruption, which has important

©2018. The Authors.

This is an open access article under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

implications for hazards and risk management. We therefore want to understand how dikes propagate in the presence of a pressurized or underpressurized magmatic reservoir.

Dikes are known to be oriented according to the stresses in the crust and open against the least compressive principal stress, σ_3 , and propagate perpendicular to this direction (Delaney, 1986). If a dike enters a localized stress field, such as that due to a pressurized magma storage region, it can twist and curl into its preferred orientation. For example, Porreca et al. (2006) show that dikes in the Mt. Somma scarp near Vesuvius take a range of orientations, from radial to circumferential, from inward to outward dipping, and posit that this is due to varying reservoir pressures at the time of emplacement. This is because a pressurized reservoir favors radial emplacement, while a depressurized one favors tangential orientation (Bianco et al., 1998). Indeed, at the well-exposed Ardnamurchan dike swarm, Anderson (1937) interprets inward dipping circumferential dikes as sourced from an overpressurized reservoir and outward-dipping dikes sourced from an underpressurized reservoir. More recently at the same location, Magee et al. (2012) show via the anisotropy of magnetic susceptibility that magma flow was dominantly horizontal and that the flow aligned similarly to nearby NW-SE trending regional dikes. They proposed that the circumferential and regional dikes are linked and that the local circumferential pattern was due to the reservoir's pressure.

The competition between local and regional stress fields can be evident from a dike's orientation. On the volcanoes of the Galapagos Islands, for example, swarms of dikes tend to align either circumferentially near the edifices or radially a bit further away (Chadwick & Dieterich, 1995). This is because many of the volcanoes have calderas, whose formation causes unloading of the underlying crust. This generates an extensional force that causes dikes, originating from the reservoir, to emplace in such orientations (Bagnardi et al., 2013). This also would cause any subsequent feeder dikes, ascending from below, to deflect laterally away and eventually erupt at the caldera rim (Chadwick & Dieterich, 1995; Corbi et al., 2016). Away from the collapse structure, the stress field is configured differently, and dikes take on a radial pattern. For another example, using geophysical monitoring techniques, Gudmundsson et al. (2014) show that during the 2014 Bárðarbunga eruption (Iceland), a dike intruded into the crust from a pressurized reservoir and began to propagate radially away. Some kilometers away, the dike rotated by $\sim 30^\circ$ to match the orientation of the regional stresses.

Dikes can also change orientation due to other factors. Crustal heterogeneities (e.g., layers) play a significant role, deflecting dikes into sills, in that a strong overlying layer may cause dike propagation to come to arrest (Rivalta et al., 2005). If the two layers are not strongly coupled together, pressure accumulation at the interface may lead to failure and decoupling, allowing the magma to form a sill (Kavanagh et al., 2015). Furthermore, a propagating dike or sill can abruptly change orientation at its leading edge, due to the balance of internal forces and the cohesive strength of the surrounding crust (Galland et al., 2014; Schmiedel et al., 2017). Galland et al. (2014) show using analogue experiments with silica powder that if a dike or sill is shallow (overlying compressional forces are low) and has a high viscosity and/or high influx rate (internal pressure is high), it can divert into a cone sheet.

The direction in which a dike propagates depends on the orientation of the principal stresses in the crust, as well on the distribution of stresses (i.e., if there is a vertical or horizontal gradient in magnitude). Watanabe et al. (2002) shows with analogue experiments, injecting silicon oil into gelatin, that a surface load reorients the stress field to attract dikes. Maccaferri et al. (2011) show similar results numerically, by modeling a boundary element liquid-filled crack, for various loadings and dike angles of ascent. In both cases, the strike of the dike is oriented circumferentially to the loading, and the dike curls inward toward the edifice load. An edifice's load, however, can induce a compressive state on the underlying crust and inhibit dike propagation (Pinel & Jaupart, 2003). Dikes that propagate into such a region may either come to arrest or begin horizontal propagation at their lateral edges. Urbani et al. (2018) use analogue modeling to extend this concept to topography more generally and show that dikes propagate away from topographic highs to erupt in topographic lows.

It is clear that dikes respond to stresses in the crust in a complex fashion and that their ability to interact with existing magma bodies has important hazards implications, such as influencing the location of future eruptions. We therefore investigate how dikes respond (i.e., how they bend and change orientation or how they propagate horizontally) to a pressurized (i.e., inflated or deflated) magmatic reservoir using analogue experiments. This allows us to explore such responses simply by varying the initial conditions, which are the initial

Figure 1. (a) A drawing of the setup to visualize shear waves. Light passes through polarizing filters, placed in front of and behind the experimental tank. Stresses perturbation within the gelatin become visible, and, by gently exciting the gelatin surface, we can see shear waves propagating. (b and c) Two sequential images of a shear wave (dark region) propagating downward through a solid gelatin. The velocity is used to estimate the gelatin's Young's modulus.

dike orientation and the magnitude of reservoir inflation or deflation. For a small subset of experiments, we use a different setup to secondarily investigate how dikes behave when they intersect with the reservoir. These simple models can be intuitive and descriptive and allow us to simulate complex three-dimensional behavior that cannot readily be modeled via numerical methods.

2. Methods

Throughout this study we refer to dikes in different orientations, relative to the reservoir (Figure S4, in the supporting information). The following terminology will be adopted:

1. *Radial or strike radial*: We use in the conventional sense to indicate dikes that are vertically oriented and whose strikes are radial to the reservoir.
2. *Circumferential or strike circumferential*: We generally use this in the conventional sense as well, for dikes whose strikes are oriented tangential to the map-view projection of the reservoir. Such dikes can propagate radially to the reservoir, in the way that a cone sheet does, but are not “radial dikes.” We use this for both inward and outward dipping dikes.
3. *Oblique*: Dikes whose strikes are initially oblique (45°) to the map-view projection of the reservoir.
4. *Coaxial*: Dikes whose vertical leading tips are aligned with the center of the map-view projection of the reservoir. They are positioned directly below the reservoir.
5. *Nonradial*: We use this to mean dikes in our experiments that were not radial dikes, meaning circumferential, oblique, and coaxial dikes.

2.1. Experimental Procedures

We performed the experiments in $40 \times 40 \times 30$ cm (length, width, and height) and $50 \times 50 \times 50$ cm tanks, filled with 3.75 wt% industrial gelatin (250 bloom). The retailer did not specify the animal source beyond beef or pork (see the supporting information), though the two sources have been shown to be mechanically similar (Nur Hanani et al., 2012). We prepared each gelatin in a similar way, by mixing gelatin granules into a pot of purified water and slowly heating the pot to 60°C , while constantly stirring with an overhead stirrer (Di Giuseppe et al., 2009; Tosh et al., 2003). We added a small splash of bleach to the gelatin to mitigate bacteria growth, which causes the gelatin to become cloudy in appearance. The resulting liquid gelatin is then placed in our cold room (15°C) to solidify for 72 hr, with a layer of oil on the surface to prevent evaporation.

Gelatin is an ideal medium for such experiments, as it can scale to the Earth's crust and is transparent, so we can observe what occurs inside (Di Giuseppe et al., 2009; Galland et al., 2015; Kavanagh et al., 2013; Rivalta et al., 2015). Due to the slight variations in the preparation (initial temperature, cooling time, and cooking time/evaporation), it varies somewhat in strength from experiment to experiment, which we measure. Gelatin is a photoelastic material, so we can measure the gelatin's strength at the beginning of each experiment via the velocities of shear waves, which become visible using a pair of polarizing filters (Figure 1; Taisne & Tait, 2011). When light passes through these filters, it becomes linearly polarized to a particular incidence angle. As light passes through the gelatin, its retardation depends on the stress level and the

Figure 2. Schematic of our experimental setup. A balloon embedded in gelatin is pressurized and a pump injects oil. We record the experiments with cameras from different perspectives. (a) Our dike injection piece, shown attached to the bottom of an empty tank. It contains a sliding blade that makes an initial cut in the gelatin, into which oil is pumped. (b) Plan view showing the balloon and injector positioning and offset, for different dike orientations. The injector location is somewhat constrained and can be rotated in 90° increments, so we move the balloon to control distance and orientation. (c) An embedded balloon in a gelatin is pressurized or depressurized via a vertical tube (here we show experiment 5; see Table 1 for experimental conditions). (d) The corresponding deviatoric stress field shown using polarized filters. The repeating rainbow pattern indicates the stress gradient and allows us to minimize boundary effects. (e) An apparatus allows us to construct a liquid reservoir without a balloon. Hot water is circulated into a balloon to remelt a layer of surrounding gelatin. The balloon is deflated and surrounded with dyed water. (f) A photo showing the final cavity in a pressurized state (experiment 1).

light's wavelength (Ramesh, 2000). This enables us to see the deviatoric stress in the gelatin and is sensible enough to visualize propagating stress perturbation generated at the surface. The shear modulus of the gelatin, G , is directly proportional to the shear wave velocity, v_s , via

$$G = \rho v_s^2, \quad (1)$$

where ρ is the gelatin's density (Lee et al., 2017). For a weak material like gelatin, these shear waves can be quite slow (~ 1 m/s) and are easy to track with a simple camera or the unaided eye. Finally, the Young's modulus, E , can be found via

$$E = 2G(1 + \nu), \quad (2)$$

where ν is the Poisson's ratio of the medium.

We injected dikes from the base of the tank using an injection piece attached to a pump, which we used to supply the liquid. The piece contains a retractable blade inside, which forms an initial precut (Figure 2a), forcing the dikes into our preferred, vertical orientation. We set the balloon and injector position so that the dike's initial strike is oriented either radial (experiments 1 and 2), circumferential (experiments 3 and 4), oblique at a 45° angle (experiments 5 and 6), or coaxial (experiment 7) to the reservoir (Figure 2b). For the injected liquid, we used vegetable oil mixed with chili powder (to provide pigment), which is less dense than gelatin and rises buoyantly. We generally injected constant volume dikes, in that we pumped in the oil with a moderately fast influx over a period of a few minutes, turned off the pump, and allowed the dikes to ascend solely due to their buoyancy. Dikes extended in length, and the tail region remained filled with the initial liquid. For two experiments, we had constant-flux conditions, in that we never stopped injecting oil, so the dike propagated primarily due to the driving pressure (from the pump) and less due to buoyancy.

To model the reservoir, we used two different setups, both of which achieved the primary goal of generating a stress field around the reservoir. For the first setup we embedded, from above, a simple latex balloon in the gelatin (while still warm and liquid), with a diameter of ~ 10 cm and with its center positioned ~ 15 cm below the surface. We accessed the balloon using a vertical tube (Figure 2c), which we filled with water to apply hydrostatic pressure to the balloon and thereby maintain its diameter. After the gelatin solidified, the balloon could be further inflated or deflated (via the tube) to apply a compressive or extensional deviatoric stress to the system (Figure 2d). With this setup, the balloon acts as a physical barrier, so we could not observe interchange of liquids between the dike and reservoir, but we could model the stress field around the

reservoir. We used polarized film to visualize the extent of the stress distribution and minimize their interaction with the tank walls, thereby mitigating boundary effects. Stress visualization helped us ensure that we injected the dikes into a region of low stress, which allowed us to observe their behavior as they entered the stress field around the balloon. For the second setup, we wanted to be able to assess the fluid exchange between the reservoir and dike, and so we designed a specific apparatus to construct a balloon-less reservoir. This piece allowed us to embed a pressurized balloon in the gelatin, then deflate it and replace it with dyed water (Figure 2e). The liquid-filled cavity could then be further pressurized to perform the experiment (Figure 2f). The setup is a bit complicated, and so we explain it further in the supporting information.

We recorded the experiments using typical cameras, which took a time lapse of photographs from different perspectives and allowed us to track the dikes' position and geometry (see Pansino & Taisne, 2018, for data repository). We took photos in 18-MP resolution at a frequency of 6 photos per minute, which was high relative to the dike velocity (generally <1 mm of propagation between photos).

2.2. Scaling

The scaling and analysis of this study rely on the deviatoric stresses in the medium, in which the magnitude at any radial distance from the center of reservoir, r , is a function of the reservoir's internal pressure and radius, respectively, ΔP and r_{res} . This has components that are radial and tangential to the reservoir, respectively, denoted as σ_{rr} and σ_{tt} :

$$\sigma_{rr}(r) = \Delta P (r_{res}/r)^3 \quad (3)$$

and

$$\sigma_{tt}(r) = -(1/2) \Delta P (r_{res}/r)^3. \quad (4)$$

This assumes the medium behaves as an infinitely-thick-walled, spherical pressure vessel (Segall, 2010). A pressurized reservoir exerts compressive forces on the medium in the radial direction but places it in a state of extension in the tangential direction. A negative pressure contracts the medium, causing the reverse effect. The minimum compressive stress, σ_3 , which a dike opens against, is the lesser of these two components, so a nearby dike's preferred orientation (radial or tangential) depends on the sign of the reservoir pressure. The hoop stress (i.e., the σ_{tt} component) can have a secondary effect on radially oriented dikes, either helping or hindering propagation along the nearest edge of the dike.

For comparison with natural systems, we scale the stress distribution against the fracture pressure of the dike, P_f , which quantifies the amount of pressure necessary for a dike to fracture the medium, and therefore propagate

$$\sigma^* = |\sigma_3|/P_f. \quad (5)$$

The dimensionless stress, σ^* , essentially maintains that the magnitude of stress is proportional to the strength of the material. This is a similar approach as taken by Daniels and Menand (2015), who scale regional extensional stresses against dike buoyancy forces; we will also incorporate buoyancy into the scaling. Following Lister and Kerr (1991),

$$P_f = K_c/L_b^{1/2}, \quad (6)$$

Menand and Tait (2002),

$$K_c = (2\gamma E)^{1/2}, \quad (7)$$

and Taisne and Tait (2009),

$$L_b = (K_c/\Delta\rho g)^{2/3}. \quad (8)$$

L_b is the buoyancy length, the minimum length a dike must surpass to buoyantly rise through the crust without any external driving pressure. K_c and γ are, respectively, the fracture toughness and surface energy of the

Table 1
Experimental Conditions

Parameter (units)	Exp 1	Exp 2	Exp 3	Exp 4	Exp 5	Exp 6	Exp 7
E (Pa)	4,642	2,750	5,272	3,424	3,311	3,719	3,827
Gelatin volume (L)	112	100	48	112	100	100	100
Cold room temperature (°C)	15	15	15	15	15	15	15
Cooling time (days)	5	3	3	4	3	3	4
Dike initial orientation	Radial	Radial	Circum.	Circum.	Oblique	Oblique	Coaxial
$\Delta\rho$ (kg/m ³)	100	100	100	100	100	100	100
Reservoir diameter (cm)	9.9	8.3	10.1	7.6	10.7	7.9	8.6
Reservoir depth (cm)	16.6	8.6	8.8	16.7	14.9	14.3	13.1
Dike—reservoir horizontal offset (cm)	16.3	10.3	11.0	16.1	14.1	14.1	0.0
P_f (Pa)	209	175	218	189	187	194	196
Injection type ^a	CV	CV	CF ^b	CV	CV	CV	CF
Volume or rate (ml, ml/min)	120	50	4.25	100	72	144	20
Overpressured/underpressured	Over	Under	Over	Under	Over	Under	Under
ΔP (Pa)	124	−378	1,079	−96	325	−545	−356
ΔP^*	0.6	−2.2	5.0	−0.5	1.7	−2.8	−1.8

Note. E = Young's modulus; $\Delta\rho$ = gelatin-dike density contrast; P_f = gelatin fracture pressure; ΔP = reservoir pressure; ΔP^* = dimensionless reservoir pressure. ^aConstant volume (CV) or constant flux (CF). ^bAt this moment, we did not have good control over the driving pressure of the dike. We believe the pressure was fairly constant, but there was likely some fluctuation.

medium (both material properties). $\Delta\rho$ is the fluid-medium density difference, and g is gravity. Equations (6)–(8) can be combined to relate P_f to the medium's strength properties and the dike's buoyancy:

$$P_f = (2\gamma E \Delta\rho g)^{1/3}. \quad (9)$$

In our experiments, we injected vegetable oil (~ 910 kg/m³) into gelatins of 3.75 wt% concentration ($\sim 1,010$ kg/m³, measured via a sample's mass and volume offset when submerged in water), leading to a $\Delta\rho$ of 100 kg/m³ for all experiments (Table 1). We assume γ to be 1 J/m² (Kavanagh et al., 2013), and E ranges between 3.3 and 5.3 kPa (derived from direct measurement of G). For ΔP , we estimate the reservoir pressure using its change of radius due to inflation or deflation, Δr_{res} . The strain associated to the reservoir expansion or contraction is simply related to the stress at its outer margin via the Young's modulus, assuming small radius variation:

$$\Delta P = E \Delta r_{res} / r_{res}. \quad (10)$$

We label $\sigma^*(r)$ at the reservoir wall margin as ΔP^* :

$$\Delta P^* = \sigma^*(r_{res}) = \Delta P / P_f. \quad (11)$$

Under these conditions, $|\Delta P^*|$ ranges from 10^{-1} to 10. This overlaps with natural systems, which are estimated to have a similar range of 10^0 – 10^3 at the reservoir boundary, neglecting viscoelastic effects around the reservoir and using values listed in Table 2. The maximum magnitude of σ^* is ΔP^* at the reservoir boundary and approaches 0 at large radii.

Table 2
Comparison of Values Between Experiments and Natural Systems

Symbol (unit)	Meaning	Experiments ^a	Nature
ΔP (Pa)	Reservoir pressure	10^2 – 10^3	10^5 – 10^{8b}
E (Pa)	Young's modulus	$\sim 2,500$ – $5,500$	$10^{10.5}$
$\Delta\rho$ (kg/m ³)	Medium-liquid density difference	100	100^c
γ (J/m ²)	Surface energy	1^d	1^d
ν	Poisson's ratio	0.5	0.25
ΔP^*	Dimensionless reservoir pressure	10^{-1} – 10^1	10^0 – 10^3

^aAll values in experiments column were measured in our lab, unless otherwise noted. ^bKarlstrom et al. (2009). ^cTaisne and Tait (2009). ^dKavanagh et al. (2013).

3. Results

3.1. Qualitative Observations

We note two classifications of dike response, which depend on the dike's initial orientation. Strike-radial dikes propagated vertically at the outset of each experiment and, as they approached the reservoir, began to propagate horizontally as well (Figures 3a and 3b). We believe that the dikes took advantage of the hoop stresses around the reservoir, which encouraged lateral movement. For the overpressurized experiment (experiment

Figure 3. Annotated photos (all in cross-sectional view) depicting different dike responses, depending in the initial dike orientation (each row) and reservoir pressure state (each column). Photos a, b, c, etc. correspond to experiments 1, 2, 3, etc. unless otherwise noted; for parameters, see Table 1. For scale, the reservoir in all photos is 10 ($\pm$ 2) cm in diameter. We label the horizontal and vertical positions as x and z . Black dashed lines outline the dikes and black arrows show their movement. Photo (a) shows an example from reservoir setup 2 (see section 2.1) while the others show setup 1. (a and b) Hoop stresses around the reservoir (shown by white dashed lines) encourage or inhibit dike opening, causing the dike to propagate laterally. (c and d) Experiment 3, overpressurized and then underpressurized. Dikes change orientation to open against σ_3 , and the preferred path is traced by the dashed white lines. (e and f) Two views are presented to have a sense of the 3-D geometry of oblique dikes. The stars indicate contact between the dike and balloon. The dikes rotate horizontally and vertically into their preferred orientations. (g) The overpressure scenario is omitted, since the dike would go straight into the reservoir. The dike initially curls into a concentric shell, but the driving pressure forces it to escape this path.

1; Figure 3a), the hoop stresses were extensional, which helped the dike propagate toward the reservoir, while the reverse occurred for the underpressurized experiment (experiment 2; Figure 3b). In the underpressure experiment, the dike propagated horizontally away from the reservoir upon encountering its stress field. After the dike passed around the stress field, it resumed vertical propagation.

Experiment 1 was set up in a way that allowed us to assess how liquid exchange occurred between the dike and reservoir. We observed that the dike intersected with the reservoir and that the two immiscible fluids stratified due to density contrast. The dike allowed a relief of pressure in the reservoir (observed using polarized film) and afterward continued propagating vertically toward the surface. We do not have pressure data for the dike to be able to quantify our results, but we do discuss these observations later in section 4.

Other experiments, with nonradial dikes, had a second class of response, in which dikes curled and twisted into new orientations (Figures 3c–3g). They reoriented according to σ_3 , so for overpressurized experiments (experiments 3 and 5; Figures 3c and 3e), the dikes curled inward to propagate toward the reservoir. For underpressurized experiments (experiments 4, 6, and 7; Figures 3d, 3f, and 3g), σ_3 is radial to the reservoir, so the dike reoriented to propagate tangentially around it. After these dikes passed around the reservoir, they began to escape its stress field, and their paths transitioned from curved to linear. As a result, they never formed perfect shells around the reservoir; instead, they took more of a tulip-flower-petal like shape. Both oblique and coaxial dikes changed orientation to become circumferential dikes.

In addition to these two classifications of dike response, we noticed that dikes generally propagate either toward or away from the reservoir, depending on the reservoir's pressure state (i.e., overpressurized or underpressurized). Overpressurized reservoirs tend to attract dike, either due to hoop stresses or σ_3 orientation, and in each case the dike intersected with the reservoir. Conversely, underpressurized reservoirs prevented dikes from approaching the reservoir, and in each case the dike was forced horizontally away, to then erupt at the surface.

3.2. Quantifying Dike Response

We characterize dike response via visual analysis. As noted, radial dikes responded by propagating laterally, while other, nonradial dikes responded by changing orientations. We analyze these two responses separately. For experiments with nonradial dikes (experiments 3–7), only a single photo taken at the end was necessary to quantify the results, since the shape of the dike records its history of interaction with the stress field (Figure 4a). We traced its shape in an illustration software to assist with detection of the dike, which sometimes was obscured by features that would hinder automated dike tracking (e.g., polarized light fringes; dike overlapping the reservoir). We processed the output in Matlab to measure the dike's position as a function of depth, relative to the balloon reservoir (Figure 4b). These dikes respond to reservoir stress by changing orientation, so we analyze the magnitude of orientation change as a proxy for dike response (Figure 4c), using the parameter $\partial^2 x / \partial z^2$, where x and z are, respectively, the horizontal and vertical components of the dike's position. In short, we quantify the location of each pixel in a photo of the dike and see how it compares with its neighboring pixels.

For dikes with initially strike-radial orientations, we note that two locations on the dike are important to describe how the dikes behaved: the point nearest to the reservoir and a point that indicates where the dike fractured the medium and grows the crack's surface as it propagates (Figure 4d). We used each frame of a time lapse to determine the location of the nearest point on the dike to the reservoir and pairs of sequential frames to determine where the dike was in the process of fracturation. We analyzed these frame pairs in Matlab, and, by taking the difference between the two, we could highlight this crescent-shaped new surface (Figure 4e). We calculated the geometric center to represent this area and track how it moves horizontally due to the reservoir's stress field, which we quantify using the parameter $\partial x / \partial z$ (Figure 4f).

To be able to compare our experiments with nature, we first normalize the radius, r ; the x and z components of the radius; and the dike breadth, B , by the reservoir radius, r_{res} , into dimensionless forms, respectively, r^* , x^* , z^* , B^* :

$$r^* = r / r_{res}, \quad (12)$$

$$x^* = x / r_{res}, \quad (13)$$

Figure 4. (a) In some experiments, the dike was not easy to see (experiment 7). (b) In such cases, we manually traced the dike in an illustrating software (black region), which helped us to quantify its path in Matlab (yellow + signs). For dikes that were distinct from the background, we did no manual tracing. (c) For nonradial dikes, the response is quantified by how abruptly it changes orientation, via the parameter $\partial^2 x / \partial z^2$. (d) Sequential photos (overlapped) of a propagating dike (experiment 2). Two important points on the dike: that nearest to the reservoir center (red star) and the geometric center of where fracturation is occurring between sequential frames (marked by the green star). Also included is the center of the roughly penny-shaped head region of the dike (blue star). (e) In Matlab, subtracting one photo from the next in a sequence highlights the differences between the two and illustrates where the dike was growing. The “two important points” are automatically located (green/red dots). (f) We analyze strike-radial dikes by tracking their change of horizontal position, represented by $\partial x / \partial z$.

$$z^* = z / r_{res}, \quad (14)$$

$$B^* = B / r_{res}. \quad (15)$$

We similarly use the dimensionless form of the dike response parameters, so instead of $\partial^2 x / \partial z^2$, we use $\partial^2 x^* / \partial z^{*2}$; $\partial x / \partial z$ is inherently dimensionless.

As nonradial dikes ascend from below, they change orientation as a function of σ^* (introduced in section 2.2). In the case of overpressurized experiments, this is a straightforward, power law relationship. In the case of underpressurized reservoirs, when these dikes bypass the reservoir, they leave its stress field and transition from curved to linear trajectory. When this occurs, the change of orientation response, $\partial^2 x^* / \partial z^{*2}$, drops. The relationship between σ^* and $\partial^2 x^* / \partial z^{*2}$ for a dike entering the stress field is different from the relationship for one leaving the stress field. We therefore analyze data for the first relationship only, to be able to compare with overpressurized experiments and to be able to characterize where a dike “feels” the stress due to the reservoir. All nonradial experiments are plotted together (Figure 5a) and form a common power law trend:

$$\partial^2 x^* / \partial z^{*2} = 61.4 \sigma^{*2.4}. \quad (16)$$

For the strike-radial dikes, we performed a similar analysis. These dikes remained vertically oriented and preferred to rise vertically, due to their buoyancy. They only deviated from vertical propagation because of the stress field due to the reservoir (Figures 3a and 3b), as hoop stresses close to the reservoir are relatively high. In the underpressurized experiments, the dike eventually passed around the stress field and continued to propagate vertically. Similar to nonradial dikes, we analyzed the response as the dike entered the stress field to compare the two experiments and quantify where the dike begins to “feel” the stress due to the

Figure 5. (a) Results from the nonradial dikes. The rate at which the dike changes orientation, $\partial^2 x^*/\partial z^{*2}$, is a function of dimensionless stress, σ^* . We show this as a semi log plot since some points are a bit less than zero on the vertical axis. We include a best-fitting power law curve (black, dashed line). (b) The stress difference between two points on a dike, $\sigma_{near}^* - \sigma_{frac}^*$, form a relationship with the degree to which it deflects horizontally, $\partial x/\partial z$. We again plot a best-fitting curve.

reservoir. As noted, we found that the dike's response, $\partial x/\partial z$, is a function of the stress at two points on the dike's surface: its nearest point to the reservoir, σ_{near}^* (Figure 4d, red star), and the geometric center of where fracturation is occurring, σ_{frac}^* (Figure 4d, green star). When we view its behavior against the difference in stress between these two points, a power law relationship emerges (Figure 5b):

$$\partial x/\partial z = 1.73(\sigma_{near}^* - \sigma_{frac}^*)^{0.69}. \quad (17)$$

3.3. Quantifying the Effective Radius of Interaction

At this moment, we can describe the degree to which a dike deflects as a function of the stresses in the crust. Dikes that feel very little stress deflect to a very small degree. We want to understand how close a dike needs to be to a reservoir before it is significantly affected by the stress field and the radius at which this occurs we define as a critical radius, r_{crit}^* .

3.3.1. Nonradial Dikes

For circumferential, oblique, and coaxial orientations, dikes respond to the medium stresses by changing orientation, which becomes pronounced as they approach σ^* of 0.1 (Figure 5a). We define this value as a critical stress, σ_{crit}^* , and it tells the location of r_{crit}^* . For σ^* at the reservoir's wall, ΔP^* , we use equations (3), (5), and (12) to derive:

$$r_{crit}^* \sim (\Delta P^*/\sigma_{crit}^*)^{1/3}. \quad (18)$$

For $\sigma_{crit}^* = 0.1$, the critical radius is therefore a function of only ΔP^* . In section 4.1, we will convert these dimensionless findings into plausible values for nature.

3.3.2. Radial Dikes

We perform a similar analysis to scale up the results from the radially oriented dikes (Figures 3a and 3b). In section 3.2, we noted that the degree to which a dike is horizontally deflected depends on the stress difference between nearest point on the dike (Figure 4d, red star) and a point at the geometric center of newly created surface (Figure 4d, green star). For a dike whose center (Figure 4d, blue star) is located at x_{cent} and z_{cent} , assuming a penny-shaped crack, the dimensionless stress of the nearest point, σ_{near}^* , is

$$\sigma_{near}^* = \Delta P^*/[r^*(x_{cent}, z_{cent}) - B^*/2]^3. \quad (19)$$

Figure 6. Maps of $\partial x/\partial z$ (color bar) for different dimensionless dike sizes, B^* , and dimensionless reservoir pressures, ΔP^* . These are generated numerically using Matlab, by finding the location of x_{frac} and z_{frac} that allows equations (17) and (21) to be equal, for any location of the dike center, then displaying the corresponding value of $\partial x/\partial z$. We define that the dike begins to respond where $\partial x/\partial z > 10^{-1}$ (which we set as the lower limit of the color bar). The horizontal and vertical axes are dimensionless positions with the reservoir center at (0,0). The white region in the upper, left corner represents the locations where the dike touches the reservoir and is therefore larger for large-breadth dikes (right column). Deflection is larger at higher pressures (bottom row).

The geometric center of where fracturation is occurring, located at x_{frac} and z_{frac} , does not have a set location; we only know that it is somewhere on the dike. However, we can easily calculate the stress, σ_{frac}^* , for any point on the dike:

$$\sigma_{frac}^* = \Delta P^* / r^*(x_{frac}, z_{frac})^3. \quad (20)$$

With σ_{near}^* and σ_{frac}^* , we can estimate $\partial x/\partial z$ at any point, via equation (17). The deflection of the dike is also defined as the degree to which the center of the crack moves horizontally as it ascends:

$$\partial x/\partial z = (x_{frac} - x_{cent}) / (z_{frac} - z_{cent}). \quad (21)$$

When combining equations (17), (18), (20), and (21), no analytical solution can be derived for the location of x_{frac} and z_{frac} . Instead, we numerically model its location in Matlab by finding the point which best allows equations (17) and (21) to be equal, for a corresponding initial location of the dike center (Figure 4d, blue star), as function of ΔP^* and B^* (some solutions are shown in Figure 6). In this way, for any point at x_{cent} and z_{cent} , we locate the point at x_{frac} and z_{frac} , estimate σ_{near}^* and σ_{frac}^* , via equations (19) and (20), and finally estimate $\partial x/\partial z$ using either equations (17) or (21). This methodology allows us to map out $\partial x/\partial z$ for any ΔP^* and B^* (Figure 6). It is worth noting that in the negative ΔP case, x_{frac} is further away than x_{cent} , since dikes deflect away from the reservoir, while the reverse is true for positive ΔP ; even so, the stress difference tends to be dominated by σ_{near}^* , so the $\partial x/\partial z$ maps for both $-\Delta P$ and $+\Delta P$ look similar.

To find the r_{crit}^* for these radial dikes, we define a critical deflection at $\partial x/\partial z = 0.1$, in which the dike deflects horizontally 1 m for every 10 m it ascends; we choose this value since it is at the lower limit of our experimental observations (Figure 5b). The ΔP^* and B^* parameters affect r_{crit}^* , so we attempt to define an

Figure 7. A surface showing at what distance strike-radial dikes begin to respond, r_{crit}^* (color bar), against dike size, B^* , and reservoir pressure, ΔP^* . Overpressure and underpressure conditions yield similar r_{crit}^* .

empirical relationship between the three, so that we can know if the dike is within the r_{crit}^* , without having to consider its nearest point or the geometric center of new surface creation. We consider B^* on the range of 10^{-1} – 10^1 , to analyze dikes much smaller and much bigger than the reservoir. They follow a power law, such that $r_{crit}^* = a\sigma^{*b}B^{*c}$, so we find a best-fitting surface, for the variables a , b , and c , which allows us to make an estimate of critical radius for a range of pressures and dike size (Figure 7). We will scale this information to natural systems in the next section.

$$r_{crit}^* = 2.87\Delta P^{*0.24}B^{*0.15}. \quad (22)$$

4. Discussion

4.1. Dimensions of the Critical Radius

We scale the dimensionless results presented above for plausible natural conditions, using equations (18) and (23), respectively, for nonradial and radial dikes. For these equations, r_{crit}^* can be described as functions of

ΔP^* and B^* . We performed an analysis for $10^0 < \Delta P^* < 10^3$, which correspond to ΔP of 10^5 – 10^8 Pa, assuming values of E and $\Delta\rho$ listed in Table 2 and therefore a fracture pressure magnitude of 10^5 Pa. The parameters r_{crit}^* and B^* are proportional to the reservoir's radius and can be easily converted back to dimensions via equations (12) and (15). In this example we consider the dike's horizontal breadth to be 1 or 10 km. We therefore view the critical radius against reservoir radius; reservoir pressure; and, for radial dikes, dike breadth (Figure 8). For example, a 1-km radius reservoir will likely affect dikes within a 2- to 20-km radius, depending on the reservoir pressure, numbers on the same order of magnitude as previous numerical models (Karlstrom et al., 2009).

4.2. Spherical Nature of the Reservoir

We model the magma storage region as a spherical reservoir because it is practical to construct in the lab using a balloon and, although the balloon reservoir in our experiments did not always deform perfectly

Figure 8. We estimate the radius, r_{crit} , at which dikes begin to deflect, for varying reservoir radius, r_{res} , and pressure (color shading) at natural scale. (a) For non-radial dikes, estimated via equation (18). Line shading represents estimates for different reservoir pressures. (b) For radial dikes via equation (23). Shadings and pressures are as in (a). Dashed and solid lines represent horizontal dike breadths of 1 and 10 km, respectively.

uniformly (probably due to heterogeneous strength of the balloon material; see Figure 3b), it models a simple, spherical (or nearly spherical) stress field. It should be noted, however, that the shape of the reservoir controls the shape of the surrounding stress field and, in turn, how dikes will propagate in its vicinity.

Unfortunately, we arrive to the problem that no one really knows the geometry of an active magma storage region. Field evidence of extinct systems indicates that intrusive bodies are built incrementally by layering of sills (Barnett & Gudmundsson, 2014; Menand, 2011; Menand et al., 2010). Ground deformation data, however, imply that reservoirs commonly undergo inflation when pressurized, which at the surface can be modeled as a point expansion (Pascal et al., 2014). Whether the actual form of a reservoir is roughly elliptical or is a stack of hydraulically connected sills and dikes remains open to debate. Gudmundsson (2012) points out that elliptical reservoirs are thermally stable, while protrusions (dikes and sills) extending from such a body are prone to solidification. We feel that a hydraulically connected network of dikes and sills, when pressurized, may exert a stress field on the crust that at a distance, can be approximated to a point expansion. At any rate, the stresses due to inflation are evident on the earth's surface, so we can be sure that they exist elsewhere in the crust as well and that dikes will be encouraged to deflect wherever they are present.

It is likely that aspect ratio of a natural magmatic reservoir dictates the degree to which our findings are applicable. Highly elongated (e.g., very flat, very tall, and narrow) reservoirs will generate a stress field that is fundamentally different to what we model, and dikes would therefore have a different response from what we observed. We expect low to moderate aspect ratio reservoirs to affect dikes in a similar fashion to our study. Moderately oblate ellipses concentrate stress on their out horizontal margins (Gudmundsson, 2006, 2012), which should affect dike further away than a spherical geometry. Conversely, prolate ellipsoids concentrate stress on the vertical margins of the reservoir, meaning dikes would be affected only when nearer.

4.3. Relation to Various Volcanic Behavior

4.3.1. Reservoir-Avoiding Dikes

Our findings that dikes can propagate around a reservoir (Figures 3d and 3f) relate to some observations made in nature. Anderson (1937) documented outward-dipping, circumferential dikes at Ardnamurchan dike swarm and hypothesized their orientation was due to an underpressurized reservoir, though he modeled the dikes as originating from the reservoir (ring dikes), rather than below. Magee et al. (2012) proposed that those dikes indeed flowed around the reservoir (albeit laterally), supplied from regional dikes. At the Mt. Somma scarp (next to Vesuvius), Porreca et al. (2006) made similar observations of dikes that have a variety of orientations, ranging from circumferential to oblique to radial, and determine that these orientations may have been a result of the reservoir pressure (overpressured or underpressured) at the time of emplacement. In either example, we would expect a dike originating from below the reservoir (passing around it) to be mafic in composition. This seems to be the case at the Mt. Somma scarp, in which exposed dikes and lavas are more mafic than many of the other products (both plinian and interplinian) in the locality (Ayuso et al., 1998).

4.3.2. Eruptive Cycles

We can continue to consider the effects of underpressure reservoirs in the context of observations made by Takada (1997), who documents a cyclic pattern at several volcanoes, including Fuji, Etna, and Mauna Loa. Takada finds that flank vents following a major eruption tend to be located far away from the central vent and that subsequent events become progressively nearer as the central edifice approaches another period of activity, which occurs on a time frame of 400–1,000 years. An eruption deflates the reservoir to some degree and places the surrounding crust in a state of extension, which inhibits dike propagation close to the reservoir. In this case, an ascending dike will likely be deflected away from the volcanic center and could then erupt as a flank eruption (Figure 9). Further eruptions sourced from the reservoir would be unlikely until either the stresses in the crust relaxed, thus allowing dikes to penetrate the magma storage region, or magma somehow managed to enter the reservoir and repressurize it (i.e., magma supplied to the reservoir by a stable, long-lived conduit). As the reservoir repressurizes, the extensional stresses weaken, and the deflection of dikes becomes less pronounced. This pattern of behavior is difficult to explain by assuming the dikes are sourced from an overpressurized reservoir. After a major eruption, a volcano would not have the high internal pressure necessary to drive a dike far away.

Figure 9. Proposed mechanism for the cyclic pattern of flank eruptions, driven by reservoir depressurization. Summit eruptions depressurize the reservoir and place the crust in a state of extension, which deflects dikes away. As the stress decreases, either due to repressurization or stress relaxation, the deflection becomes less pronounced. Eventually, dikes are no longer deflected and the cycle can repeat.

If we assume the crust behaves like a Maxwell material, the stress surrounding the reservoir relaxes with time, t , such that

$$\sigma(t) = 2 G \varepsilon_0 \exp(-G t / \eta) = [\Delta P / (1 + \nu)] \exp(-G t / \eta), \quad (23)$$

where ε_0 is the initial strain and G and η are the crust's shear modulus and viscosity (Segall, 2010). This assumes the initial strain is linearly related to the reservoir pressure, $\varepsilon_0 \approx \Delta P / E$, since the material behaves elastically on this short initial timescale, and that $G = E / 2(1 + \nu)$. Using equations (18) and (23), we estimate the r_{crit}^* as a function of time and the initial pressure (Figure 10), for values of $\eta = 1,020$ Pa/s (Di Giuseppe et al., 2009) and $G = 30$ GPa. The initial magnitude of r_{crit}^* depends strongly on the initial pressure, but the long-term value of r_{crit}^* becomes small on the order of 100 years for any initial pressure. This has some agreement with Takada's (1997) findings, in which dikes cycle through this relaxation/pressurization pattern on the order of hundreds to 1,000 years.

4.3.3. Eruption Triggering

An important observation from our experiments is that dikes that breach a magmatic reservoir do not necessarily cease propagation (Figures 11a and 11b). In experiment 1, the dike was attracted to the overpressurized reservoir, and, after it made contact, the dike continued to propagate. In our experiment, the two immiscible liquids stratified by density and the reservoir liquid (neutrally buoyant in gelatin) provided no buoyant force to continue propagation; therefore, propagation was driven by buoyancy of the oil (the liquid filling the dike) and by the slight overpressure of the reservoir. In a natural analogue, the magma in a reservoir would be equally or more buoyant than the magma in a mafic dike. It could therefore flow inside of the dike and take advantage of the existing fracture to propagate toward the surface (Figure 11c).

This has implications for eruption triggering, in that a basaltic dike may help to open a pathway along which more-evolved magmas from a shallow reservoir can migrate. Takeuchi (2004) shows that some cataclysmic eruptions of very viscous magma were preceded by precursory eruptions of magma generally a few orders of magnitude less viscous. It is conceivable that such eruptions are initiated by deeply sourced basaltic dikes which, upon breaching the reservoir, encourage eruption via a combination of mechanisms: (1) remobilization of the residing magma (Murphy et al., 2000; Ruth et al., 2016); (2) providing a low-viscosity lubricating layer inside of the dike, which the viscous evolved magma slides along (Watts et al., 1999); (3) providing a large fracture, which requires less pressure to open widely than a small fracture (Rubin, 1995).

Figure 10. Time evolution of the critical radius of nonradial dike deflection due to viscoelastic relaxation. Shading represents different initial magnitudes of reservoir pressure. The initial stress relaxes with time, so the critical radius diminishes.

Figure 11. (a and b) An example from experiment 1 (cross-sectional view), showing that dike propagation may continue after a dike intersects a reservoir. (c) A model for the extraction of viscous reservoir magma into a basaltic dike. The basalt lubricates and propagates the fracture, while the overpressure and buoyancy of the reservoir magma open the fracture wide.

4.4. Driving Pressure

We note that our results have been influenced to some degree by the dikes' driving pressures, which is especially obvious in experiment 7 (Figure 3g). Dikes with a significant internal pressure seem to propagate according to their own internally derived local stress field (i.e., they propagate along a straight path) and only alter course when affected by a significant external stress (e.g., reservoir stresses and regional stresses). As they escape the external stress field, they continue propagating straight. This behavior is also exhibited to a lesser degree for other experiments that had an underpressurized reservoir. It seems that this balance between internal and external pressure sources may dictate the degree to which a dike deviates from its quasi-static path. We account for this to some degree in our scaling of experiments, in that σ^* is scaled by the dike's fracturation pressure (which itself is a function of buoyancy and the Young's modulus); however, the internal pressure can be much higher than the fracturation pressure, which is likely the case in a constant-flux experiment (experiment 7). Constant-volume experiments should have an internal pressure close to the fracturation pressure.

5. Conclusions

We present an investigation on how dikes respond to the stress field around a pressurized magma storage region and found two classifications of response. Nonradial dikes reorient to open against σ_3 and therefore propagate either radially or tangentially to an overpressurized or underpressurized reservoir, respectively. Radial dikes do not change orientation, but rather propagate laterally at their fringes due to the hoop stresses around the reservoir. Overpressurized reservoirs generate extensional hoop stresses, which help a dike to propagate toward the reservoir, while the reverse occurs for an underpressurized reservoir.

These two classes of interaction behave as functions of the stress distribution due to the reservoir and, in both cases, the higher the stress, the more pronounced the reaction. For nonradial dikes, it is a function of the stress at the dike's vertical leading tip. For radial dikes, the stress has a more complex role, and we find that lateral propagation functions according to the stress difference between two points: the point nearest to the reservoir and a point at the center of where fracturation is occurring as it propagates.

Dikes that ascend near a reservoir are likely to cause volcanic crises, regardless of the state of pressure. By quantifying a volcano's reservoir pressure, however, we can anticipate whether dike will propagate toward or away from it and therefore whether it is likely to cause an eruption near to, or far from, the central vent, information that is vital for hazards management purposes. The degree to which we accurately estimate competing stress sources, that is, the driving pressure of a dike, the local stresses due to the reservoir and broader regional stresses, will dictate our ability forecast where dikes are likely to erupt.

References

- Anderson, E. M. (1937). IX.—The dynamics of the formation of cone-sheets, ring-dykes, and caldron-subsidences. *Proceedings. Royal Society of Edinburgh*, 56, 128–157. <https://doi.org/10.1017/S0370164600014954>
- Ayuso, R. A., De Vivo, B., Rolandi, G., Seal, R. R., & Paone, A. (1998). Geochemical and isotopic (Nd-Pb-Sr-O) variations bearing on the genesis of volcanic rocks from Vesuvius, Italy. *Journal of Volcanology and Geothermal Research*, 82(1-4), 53–78. [https://doi.org/10.1016/S0377-0273\(97\)00057-7](https://doi.org/10.1016/S0377-0273(97)00057-7)

Acknowledgments

This work comprises Earth Observatory of Singapore contribution no. 222. This research is supported by the National Research Foundation Singapore (award NRF2015-NRF-ISF001-2437) and the Singapore Ministry of Education under the Research Centres of Excellence initiative. It has also been partly funded by the Dr. Stephen Riady Geoscience Scholars Fund. Our data are available at <https://doi.org/10.26074/5zwv-7j56>. This is under DR-NTU, a local instance of DataVerse, supported by Nanyang Technological University. For more information, please see the supporting information.

- Bagnardi, M., Amelung, F., & Poland, M. (2013). A new model for the growth of basaltic shields based on deformation of Fernandina volcano, Galápagos Islands. *Earth and Planetary Science Letters*, 377–378, 358–366. <https://doi.org/10.1016/j.epsl.2013.07.016>
- Barnett, Z. A., & Gudmundsson, A. (2014). Numerical modelling of dykes deflected into sills to form a magma chamber. *Journal of Volcanology and Geothermal Research*, 281, 1–11. <https://doi.org/10.1016/j.jvolgeores.2014.05.018>
- Bianco, F., Castellano, M., Milan, G., Ventura, G., & Vilardo, G. (1998). The Somma-Vesuvius stress field induced by regional tectonics: Evidences from seismological and mesostructural data. *Journal of Volcanology and Geothermal Research*, 82(1–4), 199–218. [https://doi.org/10.1016/S0377-0273\(97\)00065-6](https://doi.org/10.1016/S0377-0273(97)00065-6)
- Chadwick, W. W. Jr., & Dieterich, J. H. (1995). Mechanical modeling of circumferential and radial dike intrusion on Galapagos volcanoes. *Journal of Volcanology and Geothermal Research*, 66(1–4), 37–52. [https://doi.org/10.1016/0377-0273\(94\)00060-T](https://doi.org/10.1016/0377-0273(94)00060-T)
- Corbi, F., Rivalta, E., Pinel, V., Maccaferri, F., & Acocella, V. (2016). Understanding the link between circumferential dikes and eruptive fissures around calderas based on numerical and analog models. *Geophysical Research Letters*, 43, 6212–6219. <https://doi.org/10.1002/2016GL068721>
- Crabtree, S. M., & Waters, L. E. (2017). The petrologic history of the Sanganguey volcanic field, Nayarit, Mexico: Comparisons in a suite of crystal-rich and crystal-poor lavas. *Journal of Volcanology and Geothermal Research*, 336, 51–67. <https://doi.org/10.1016/j.volgeores.2017.02.005>
- Daniels, K. A., & Menand, T. (2015). An experimental investigation of dyke injection under regional extensional stresses. *Journal of Geophysical Research: Solid Earth*, 120, 2014–2035. <https://doi.org/10.1002/2014JB011627>
- Delaney, P. (1986). Field relations between dikes and joints: Emplacement processes and paleostress analysis. *Journal of Geophysical Research*, 91(B5), 4920–4938. <https://doi.org/10.1029/JB091iB05p04920>
- Di Giuseppe, E., Funicello, F., Corbi, F., Ranalli, G., & Mojoli, G. (2009). Gelatin as rock analogs: A systematic study of their rheological and physical properties. *Tectonophysics*, 473(3–4), 391–403. <https://doi.org/10.1016/j.tecto.2009.03.012>
- Galland, O., Burchard, S., Hallot, E., Mourgues, R., & Bulois, C. (2014). Dynamics of dikes versus cone sheets in volcanic systems. *Journal of Geophysical Research: Solid Earth*, 119, 6178–6192. <https://doi.org/10.1002/2014JB011059>
- Galland, O., Holohan, E., van Wyk de Vries, B., & Burchard, S. (2015). Laboratory modelling of volcano plumbing systems: A review. In *Advances in volcanology* (pp. 1–68). Berlin, Heidelberg: Springer. https://doi.org/10.1007/11157_2015_9
- Gudmundsson, A. (2006). How local stresses control magma-chamber ruptures, dyke injections, and eruptions in composite volcanoes. *Earth Science Reviews*, 79(1–2), 1–31. <https://doi.org/10.1016/j.earscirev.2006.06.006>
- Gudmundsson, A. (2012). Magma chambers: Formation local stresses, excess pressures, and compartments. *Journal of Volcanology and Geothermal Research*, 237–238, 19–41. <https://doi.org/10.1016/j.jvolgeores.2012.05.015>
- Gudmundsson, A., Lecoer, N., Mohajeri, N., & Thordarson, T. (2014). Dike emplacement at Bardarbunga, Iceland, induces unusual stress changes, caldera formation, and earthquakes. *Bulletin of Volcanology*, 76(10), 869. <https://doi.org/10.1007/s00445-014-0869-8>
- Karlstrom, L., Dufek, J., & Manga, M. (2009). Organization of volcanic plumbing through magmatic lensing by magma chambers and volcanic loads. *Journal of Geophysical Research*, 114, B10204. <https://doi.org/10.1029/2009JB006339>
- Kavanagh, J., L., Boutelier, D., & Cruden, A. R. (2015). The mechanics of sill inception, propagation and growth: Experimental evidence for rapid reduction in magmatic overpressure. *Earth and Planetary Science Letters*, 421, 117–128. <https://doi.org/10.1016/j.epsl.2015.03.038>
- Kavanagh, J., L., Menand, T., & Daniels, K., A. (2013). Gelatin as a crustal analogue: Determining elastic properties for modelling magmatic intrusions. *Tectonophysics*, 582, 101–111. <https://doi.org/10.1016/j.tecto.2012.09.032>
- Lee, B. J., Kee, S.-H., Oh, T., & Kim, Y.-Y. (2017). Evaluating the dynamic elastic modulus of concrete using shear-wave velocity measurements. *Advances in Materials Science and Engineering*, 2017, 1651753. <https://doi.org/10.1155/2017/1651753>
- Lister, J. R., & Kerr, C. (1991). Fluid-mechanical models of crack propagation and their application to magma transport in dykes. *Journal of Geophysical Research*, 96(B6), 10,049–10,077. <https://doi.org/10.1029/91JB00600>
- Maccaferri, F., Bonafede, M., & Rivalta, E. (2011). A quantitative study of the mechanisms governing dike propagation, dike arrest and sill formation. *Journal of Volcanology and Geothermal Research*, 208(1–2), 39–50. <https://doi.org/10.1016/j.jvolgeores.2011.09.001>
- Magee, C., Stevenson, C., O'Driscoll, B., Schofield, N., & McDermott, K. (2012). An alternative emplacement model for the classic Ardnamurchan cone sheet swarm, NW Scotland, involving lateral magma supply via regional dykes. *Journal of Structural Geology*, 43, 73–91. <https://doi.org/10.1016/j.jsg.2012.08.004>
- Menand, T. (2011). Physical controls and depth of emplacement of igneous bodies: A review. *Tectonophysics*, 500(1–4), 11–19. <https://doi.org/10.1016/j.tecto.2009.10.1016>
- Menand, T., Daniels, K. A., & Benghiat, P. (2010). Dyke propagation and sill formation in a compressive tectonic environment. *Journal of Geophysical Research*, 115, B08201. <https://doi.org/10.1029/2009JB006791>
- Menand, T., & Tait, S. (2002). The propagation of a buoyant liquid-filled fissure from a source under constant pressure: An experimental approach. *Journal of Geophysical Research*, 107(B11), 2306. <https://doi.org/10.1029/2001JB000589>
- Murphy, M. D., Sparks, R. S. J., Barclay, J., Carroll, M. R., & Brewer, T. S. (2000). Remobilization of andesite magma by intrusion of mafic magma at the Soufriere Hills volcano, Montserrat, West Indies. *Journal of Petrology*, 41(1), 21–42. <https://doi.org/10.1093/ptrology/41.1.21>
- Nur Hanani, Z. A., Roos, Y. H., & Kerry, J. P. (2012). Use of beef, pork and fish gelatin sources in the manufacture of films and assessment of their composition and mechanical properties. *Food Hydrocolloids*, 29(1), 144–151. <https://doi.org/10.1016/j.foodhyd.2012.01.015>
- Pansino, S., & Taisne, B. (2018). Datasets relating to: How magmatic storage regions attract and repel propagating dikes. <https://doi.org/10.26074/5zwv-7j56>
- Pascal, K., Neuberg, J., & Rivalta, E. (2014). On precisely modelling surface deformation due to interacting magma chambers and dykes. *Geophysical Journal International*, 196(1), 253–278. <https://doi.org/10.1093/gji/ggt343>
- Pinel, V., & Jaupart, C. (2003). Magma storage and horizontal dyke injection beneath a volcanic edifice. *Earth and Planetary Science Letters*, 221(1–4), 245–262. [https://doi.org/10.1016/S0012-821X\(04\)00076-7](https://doi.org/10.1016/S0012-821X(04)00076-7)
- Porreca, M., Acocella, V., Massimi, E., Mattei, M., Funicello, R., & De Benedetti, A. A. (2006). Geometric and kinematic features of the dike complex at Mt. Somma, Vesuvio (Italy). *Earth and Planetary Science Letters*, 245(1–2), 389–407. <https://doi.org/10.1016/j.epsl.2006.02.027>
- Ramesh, K. (2000). *Digital photoelasticity—Advanced techniques and applications*. Berlin Heidelberg: Springer. <https://doi.org/10.1007/978-3-642-59723-7>
- Rivalta, E., Böttlinger, M., & Dahm, T. (2005). Buoyancy-driven fracture ascent: Experiments in layered gelatin. *Journal of Volcanology and Geothermal Research*, 144(1–4), 273–285. <https://doi.org/10.1016/j.jvolgeores.2004.11.030>
- Rivalta, E., Taisne, B., Bunger, A., P., & Katz, R., F. (2015). A review of mechanical model of dike propagation: Schools of thought, results and future directions. *Tectonophysics*, 638, 1–42. <https://doi.org/10.1016/j.tecto.2014.10.003>

- Rubin, A. M. (1995). Propagation of magma-filled cracks. *Annual Review of Earth and Planetary Sciences*, 23(1), 287–336. <https://doi.org/10.1146/annurev.ea.23.050195.001443>
- Ruth, D. C. S., Cottrell, E., Cortés, J. A., Kelley, K. A., & Calder, E. S. (2016). From passive degassing to violent strombolian: The case of the 2008 eruption of Llaima volcano, Chile. *Journal of Petrology*, 57(9), 1833–1864. <https://doi.org/10.1093/petrology/egw063>
- Schmiedel, T., Galland, O. & Breitzkreuz, C. (2017). Dynamics of sill and laccolith emplacement in the brittle crust: Role of host rock strength and deformation mode.
- Segall, P. (2010). *Earthquake and volcano deformation* (pp. 203–205). Princeton, NJ: Princeton University Press.
- Taisne, B., & Tait, S. (2009). Eruption versus intrusion? Arrest of propagation of constant volume, buoyant, liquid-filled cracks in an elastic, brittle host. *Journal of Geophysical Research*, 114, B06202. <https://doi.org/10.1029/2009JB006297>
- Taisne, B., & Tait, S. (2011). Effect of solidification on a propagating dike. *Journal of Geophysical Research*, 116, B01206. <https://doi.org/10.1029/2009JB007058>
- Takada, A. (1997). Cyclic flank-vent and central-vent eruption patterns. *Bulletin of Volcanology*, 59(B6), 8471–8556. <https://doi.org/10.1029/JB095iB06p08471>
- Takeuchi, S. (2004). Precursory dike propagation control of viscous magma eruptions. *Geology*, 32(11), 1001–1004. <https://doi.org/10.1130/G20792.1>
- Tosh, S. M., Marangoni, A. G., Ross Hallett, F., & Britt, I. J. (2003). Aging dynamics in gelatin gel microstructure. *Food Hydrocolloids*, 17, 503–513.
- Urbani, S., Acocella, V., & Rivalta, E. (2018). What drives the lateral versus vertical propagation of dikes? Insights from analogue models. *Journal of Geophysical Research: Solid Earth*, 123, 3680–3697. <https://doi.org/10.1029/2017JB015376>
- Watanabe, T., Masuyama, T., Nagaoka, K., & Tahara, T. (2002). Analog experiments on magma-filled cracks: Competition between external stresses and internal pressure. *Earth, Planets and Space*, 54(12), e1247–e1261. <https://doi.org/10.1186/bf03352453>
- Watts, R. B., de Silva, S. L., Jimenez de Rios, G., & Croudace, I. (1999). Effusive eruption of viscous silicic magma triggered and driven by recharge: a case study of the Cerro Chascon-Runtu Jarita Dome Complex in Southwest Bolivia. *Bulletin of Volcanology*, 61(4), 241–264. <https://doi.org/10.1007/s004450050274>

RESEARCH ARTICLE

10.1029/2019JB018191

Key Points:

- A dike rapidly channelizes into a conduit geometry after its eruption initiates
- If the time interval between migration events is short, magma will follow a preexisting conduit; otherwise, a new dike will propagate
- A critical flux is necessary for a dike to propagate to the surface without solidifying

Supporting Information:

- Supporting Information S1

Correspondence to:

S. Pansino,
stepheng001@e.ntu.edu.sg

Citation:

Pansino, S., Emadzadeh, A., & Taisne, B. (2019). Dike channelization and solidification: Time scale controls on the geometry and placement of magma migration pathways. *Journal of Geophysical Research: Solid Earth*, 124. <https://doi.org/10.1029/2019JB018191>

Received 12 JUN 2019

Accepted 28 AUG 2019

Accepted article online 1 SEP 2019

Dike Channelization and Solidification: Time Scale Controls on the Geometry and Placement of Magma Migration Pathways

S. Pansino¹ , A. Emadzadeh² , and B. Taisne^{1,2}
¹Earth Observatory of Singapore, Nanyang Technological University, Singapore, ²Asian School of the Environment, Nanyang Technological University, Singapore

Abstract We investigate the conditions under which magma prefers to migrate through the crust via a dike or a conduit geometry. We performed a series of analogue experiments, repeatedly injecting warm, liquid gelatin, into a cold, solid gelatin medium and allowing the structure to evolve with time. We varied the liquid flux and the time interval between discrete injections of gelatin. The time interval controls the geometry of the migration, in that long intervals allow the intrusions to solidify, favoring the propagation of new dikes. Short time intervals allow the magma to channelize into a conduit. These times are characterized by the Fourier number (Fo), a ratio of time and thermal diffusion to dike thickness, so that long times scales have $Fo > 10^2$ and short time scales have $Fo < 10^0$. Between these time scales, a transitional behavior exists, in which new dikes nest inside of previous dikes. The flux controls the distance a dike can propagate before solidifying, in that high fluxes favor continual propagation, whereas low fluxes favor dike arrest due to solidification. For vertically propagating dikes, this indicates whether or not a dike can erupt. A transitional behavior exist, in which dikes may erupt at the surface in an unstable, on-and-off fashion. We supplemented the experimental findings with a 2-D numerical model of thermal conduction to characterize the temperature gradient in the crust as a function of intrusion recurrence frequency. For very infrequent intrusions ($Fo > 10^4$ to 10^5) all thermal energy is lost, while more frequent intrusions allow heat to build up nearby.

Plain Language Summary When magma ascends through the Earth's crust to the surface, it tends to do so via cracks, which propagate upward with the magma inside. After these magmatic “dikes” erupt, they quickly start to channelize into a centralized vent, transforming a long, fissure eruption into a focused lava fountain. Underground, such a dike begins to partially solidify, forming a cylindrical conduit where the magma flows fastest and solidifying elsewhere. After the eruption stops, a new eruption can follow this same conduit if it happens quickly enough, preventing the conduit from solidifying and the pathway from closing. Otherwise, if the conduit solidifies, new magma needs to make another dike to be able to ascend through the crust. This time interval between magma ascent events likely affects volcano formation over a long time. If magma erupts frequently, it tends to follow the same path repeatedly, allowing lava to eventually build up a large, centralized volcano. If it is very infrequent, magma needs to make new dikes with new pathways each time, building a group of smaller, separate volcanic cones.

1. Introduction

Fissure eruptions have been documented to evolve with time, from initially elongated fissures to centralized vents (Jones et al., 2017; Keating et al., 2008; Wylie et al., 1999). The change of geometry from planar to cylindrical reflects the easiest form of magma transport; a planar form is most efficient for propagation through a brittle, elastic medium; a cylindrical form is the most thermally efficient (Keating et al., 2008). Dikes that supply such eruptions may also evolve with time at depth, as they have a high ratio of surface area to volume and therefore are very vulnerable to cooling and, eventually, solidification. Cooling is heterogeneous along the dike surface, in that regions that become cool also become more viscous, encouraging flow toward the hotter regions of the dike, which therefore remain warm and active. As they transition to cylindrical geometries, the surface area and heat loss drop, allowing thermally efficient magma transport to the surface (Fukushima et al., 2010). For example, an eruption on Heimaey, Iceland, began as a 1.5-km-long fissure, which focused to a single, central vent over a period of a day (Wylie et al., 1999). Similarly, exposed

©2019. The Authors.

This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

shallow plumbing systems at monogenetic volcanoes in the southwestern United States display conduit structures, which likely formed in part due to excavation, then infill, of the surrounding crust (Keating et al., 2008).

There is an element of time necessary to achieve such a geometry, since channelization results from the prolonged effusion of magma. Dikes that do not develop a focused vent remain planar during solidification. In some cases, dikes even become narrower at their centers, as they evacuate magma and deflate, while their lateral fringes solidify and preserve their initial, active state (Daniels et al., 2012). The necessary time scale for flow channelization is not immediately clear; Keating et al. (2008) suggest that the size of the eruption may be important, in that small eruptions can focus on a scale of hours, while larger eruptions do so more gradually.

1.1. Hot Spot Volcano Settings

The channelization of fissure eruptions into focused vents has been documented in a number of settings, primarily characterized by basaltic intrusions. Fissure eruptions commonly occur in the rift zones of hot spot volcanoes like Kilauea and Piton de la Fournaise, in which dike intrusions tend to be linked to deformation of the larger edifices and therefore preferentially occur in certain regions (Jones et al., 2017; Michon et al., 2015). Propagating dikes (and after the fact, solidified swarms) have been imaged via geophysical data at both of these volcanoes and, in some cases, have been observed to evolve into centralized vents (Fukushima et al., 2010; Jones et al., 2017; Leslie et al., 2004; Owen et al., 2000). External forces (i.e., local and regional stresses) clearly guide dikes to intrude in such zones periodically, but magma migration may also take the form of long-lived channels, which must exist to allow structures like Pu'u O'o vent to persist (Patrick et al., 2019). These small rift zones can therefore support both persistent magma flow and more voluminous, intermittent dike propagation events.

1.2. Volcanic Field Settings

If hot spot volcanoes represent semicontinuously active settings, then their opposite would be infrequently erupting volcanic fields. Such regions can have eruption recurrence frequencies of one eruption per 10^4 to 10^5 years but can periodically increase in activity to one eruption per 10^3 years or less (Valentine & Connor, 2015). Since these events are so infrequent, cooling dominates the system and intrusions have time to thoroughly solidify. Subsequent intrusions must create new paths via dike propagation. It has been shown that dike swarms have natural spacing between dikes, as each one induces a compressive stress on the surrounding crust, which inhibits later dikes from propagating too close; the final spacing between dikes is energetically optimized (Bunger et al., 2013). As such, the infrequent timing of events causes dikes (and their eruptive products) to be spatially separated. It should be noted that volcanic fields have relatively small long-term outputs of lava and are, at their most voluminous, similar to arc-related stratovolcanoes (10^2 km³/Myr) but can be orders of magnitude less (Valentine & Connor, 2015). Even though this type of setting is characterized by low magma output, with long repose times between events, a single event can display dike flow channelization, similar to the other examples we previously introduced (Keating et al., 2008). Volcanic fields, along with hot spot volcano rift zones, demonstrate that the geometry of magma transport depends on the magma flux, the duration of the injection and time interval between injections. These parameters dictate if a dike can develop into a conduit-like pathway and, depending on the repose time, whether a subsequent pulse needs to propagate a new dike.

1.3. Experimental Studies

Experimental studies on dike solidification can be subdivided into two categories: the ability of a dike or sill to propagate into a cold medium (Chanceaux & Menand, 2014; Delaney & Pollard, 1982; Fialko & Rubin, 1998; Rubin, 1993; Taisne & Tait, 2011) and the stability of an erupting dike (Bruce & Huppert, 1989; Wylie & Lister, 1995). A propagating dike is subject to cooling and solidification as a function of the temperature difference with the host crust, the magma flow rate, the dike geometry, and the magma's latent heat (Delaney & Pollard, 1982; Fialko & Rubin, 1998). When the magma is very hot and the flow rate is high, the dike geometry evolves by thermal erosion, in which the host rock melts. When the magma is relatively cool and flow rate is low, solidification can proceed. Dikes gain surface area as they propagate and therefore become increasingly susceptible to heat loss via conduction. They therefore have an inherent limit on how

far they can propagate before solidifying, which is termed the “thermal entry length” (Delaney & Pollard, 1982; Einarsson & Brandsdóttir, 1978; Fialko & Rubin, 1998).

Taisne and Tait (2011) show through analogue experiments that, while solidification can stop propagation, further influx of magma into the center of the dike can cause it to inflate and resume propagation. They injected paraffin wax into gelatin and observed that the dike's chilled margins provided sufficient insulation to maintain flow in the dike's core. The propagation dynamic varied between no propagation, step-wise propagation, and continuous propagation, as a function of the temperature and the dimensionless flux, Φ . This flux parameter is a ratio of heating via influx and cooling via conduction:

$$\Phi = QH/\alpha LB, \quad (1)$$

for volumetric flux, Q , thermal diffusivity, α , and dike vertical length, horizontal breadth, and thickness, respectively, L , B , and H . The propagation of a dike becomes less affected by thermal conditions (i.e., continuous propagation) for very hot magmas or very high fluxes, while low fluxes of cold magmas may not yield any propagation at all. For transitional conditions, dikes undergo step-wise propagation, in which an arrested dike needs to inflate and build pressure to resume propagation.

Chanceaux and Menand (2014) made similar experiments focusing on sill formation. They show that a dike, arriving at the interface between two layers of the crust, can deflect into a sill under certain temperature and flux conditions. These conditions coincide with step-wise propagation described by Taisne and Tait (2011), whereas dikes in the “continuous propagation” regime tend to pass through the interface and continue propagating vertically. Considering these two studies together from a thermal perspective, it is clear that dikes that are very hot and have a very high flux propagate in a fast and stable way, whereas magma closer to the solidus, flowing at a lower flux, is prone to unstable, step-wise propagation or solidification.

For a dike that can propagate to the surface, its form of eruption is inherently unstable, in that magma flow has a tendency to focus from a sheet of lava into one or a few vents over time. Wylie et al. (1999) show through numerical modeling that a magma with temperature-dependent viscosity develops viscosity contrasts within a dike, which are the dominant mechanism causing flow channelization into a discrete vent. The viscosity contrasts cause the flow to channelize on a time scale of ~10 hr, consistent with the time scale observed in the 1973 fissure eruption on Heimaey, Iceland.

The size of an erupting vent evolves as a function of magma flux; if the flux is “high,” the vent walls will melt and the dike widens until the supply is exhausted, but if the supply is “low,” the magma will solidify and block the vent (Bruce & Huppert, 1989). In a 2-D framework, Bruce and Huppert (1989) numerically modeled this fluctuation and found it to be a function of the dike's initial width. Over some critical size on the order of meters, a basaltic dike driven by a constant overpressure can continue to migrate and erupt until its source has emptied.

1.4. Our Contribution

We present below a series of analogue experiments investigating how dikes solidify and evolve with subsequent influxes of magma. We repeatedly injected hot, liquid gelatin into a cold, solid gelatin medium, varying the flow rate and the time between discrete injections. We then identify the conditions under which dikes can channelize into conduit-like structures, how they remain stable and what types of structures develop otherwise. By quantifying such behaviors, we can gain insights of how the rate and frequency of pulses of magma may shape the underlying plumbing system of a volcano.

2. Methods

2.1. Experimental Setup

We performed a series of analogue experiments, in which we repeatedly injected warm, liquid gelatin into a solid gelatin medium (both made from the same granular gelatin, 250 bloom), housed in a $50 \times 50 \times 50\text{-cm}^3$ cubic tank. We made five injections per experiment, each 500 ml in volume, providing sufficient volume to propagate up through the gelatin and erupt. Between experiments, we varied only the magnitude of flux and the time interval between injections. Below the tank, we attached a small 10-cm-tall, 10-cm-diameter cylindrical reservoir, which contained a copper heating coil and maintained a constant temperature of the liquid injection (Figure 1a).

Figure 1. (a) A schematic of our experimental setup. A peristaltic pump draws liquid gelatin from a supply beaker to our tank apparatus. A thermal bath keeps the system at a constant temperature of 32 °C. We constructed water-filled free surfaces on both sides of the tank to minimize the stress buildup in the medium due to repeated intrusions. (b) Gelatin supply tubing is braided with thermal bath tubing and then insulated to keep the gelatin warm while in transit. (c) Prior to the first injection of an experiment, we make an initial cut using a retractable blade to ensure that the first dike propagates vertically. Subsequent injections are not controlled in this way.

We chose gelatin as an analogue for both the solid medium and the injected liquid because it is very versatile. Gelatin is commonly used as an analogue for the Earth's crust because its Young's modulus can be controlled by varying the concentration during preparation and because it is transparent, so we can observe what occurs inside the medium (or a dike intruded behind another dike). Additionally, it behaves like the Earth's upper crust, in that it is a brittle, elastic material on the time scale of a propagating dike and a viscoelastic material on a longer time scale (Di Giuseppe et al., 2009; Kavanagh et al., 2013). As an injected liquid, gelatin quickly solidifies at the experimental temperature, so we can observe flow paths that develop as a dike cools heterogeneously.

We prepared each experiment in a similar fashion (see Table 1 for experimental conditions), in which the gelatin medium was either 3.5 wt% or 4.0 wt% of gelatin and the dike was either 4.0 wt% or 4.57 wt% of gelatin (maintaining a consistent concentration ratio). The high concentration of the dike, relative to the medium, ensured that a solidified dike was stronger than the surrounding medium, preventing subsequent injections from following the previous dike's path. Since liquid gelatin is not very buoyant in solid gelatin,

Table 1
Experimental Conditions

Exp	$C_{l,gel}$ (wt%)	$C_{s,gel}$ (wt%)	$C_{s,sug}$ (wt%)	E (Pa)	K_c (Pa/m ^{1/2})	ν (—)	Q (ml/min)	t_{int} (s)	$\Delta\rho$ (kg/m ³)
1	4.0	3.5	6	1,550	56	0.5	12	2	80
2	4.57	4.0	6	4,050	90	0.5	12	210	80
3	4.0	3.5	5	1,590	56	0.5	12	20,700	70
4	4.0	3.5	5	1,920	62	0.5	1.2	2	70
5	4.0	3.5	5	4,570	96	0.5	3.8	210	70
6	4.0	3.5	5	3,990	89	0.5	38	210	70
7	4.0	3.5	5	4,110	91	0.5	12	2,070	70
8	4.0	3.5	5	4,580	96	0.5	38	2,070	70
9	4.0	3.5	5	6,650	115	0.5	3.8	20,700	70
10	4.0	3.5	5	3,850	88	0.5	38	6,600	70

Note. From left to right are the gelatin concentration of the liquid and solid medium, additive sugar concentration, Young's modulus, fracture toughness, Poisson's ratio, flux rate, time interval and density difference.

Figure 2. The effect of repose time, which yields different types of intrusions. Each experiment begins with a single dike. (a) Conduit formation (Experiment 6). For experiments with a short repose time (on the order of seconds to minutes), the dike heterogeneously solidifies and maintains a central liquid conduit (highlighted by dashed line). (b) Nested dike formation (Experiment 7). For experiments with a moderate repose time (~ 30 min), the initial dike partially solidifies. It is still soft at the time of the next injection, so the new dike forms inside of the original dike. (c, d) Two different views, from the side and from above, for dike swarm formation (Experiment 3). For a long repose time (on the order of hours), the initial dike completely solidifies. The subsequent injection is forced to make a new dike, which solidifies, leading to a collection of dikes.

we added 5.0 wt% to 6.0 wt% of sugar to the medium to increase its density and thus make the dikes positively buoyant. We added food dye to the injected gelatin for pigment, to help distinguish the dikes from the medium and give a sense of where they are thicker (darker looking) or thinner (lighter looking). The concentration of the food dye was low and did not otherwise affect the liquid's properties.

For the magma analogue, we injected liquid gelatin at a temperature of 31 to 33 °C, marginally hotter than the melting temperature of 28 °C, which allows it to quickly cool as it is injected into the solid medium. We maintained the temperature of our gelatin supply using a thermal bath into which we submerged our supply beaker. The thermal bath also circulates heated water through flexible tubes, which we attach to the basal, cylindrical reservoir's copper heating coil (Figure 1a). We also threaded these tubes with the gelatin supply tube, to maintain its temperature between the supply beaker and the experimental tank (Figure 2b). When we were ready to start an experiment, we made an initial cut into the gelatin using a retractable blade (Figure 3c), to force the dike to be vertically oriented, and perpendicular to the visualization system. After making this cut, we removed the blade and pumped liquid gelatin into the cut, which then grew into a dike. We controlled the injections flux, duration, and time interval, using a programmable peristaltic pump. The flux range achievable with our current setup ranges from 0.8 to 170 ml/min.

These experiments had the potential to intrude a lot of material into the solid medium, which would build up large strains of the medium. We attempted to mitigate strain accumulation by constructing vertical free

Figure 3. The effect of flux, which controls the ability of a dike to erupt. (a) Sufficiently high flux (>10 mL/min) allows the dike to reach the surface before solidifying (Experiment 6). (b) A transitional behavior at semilow flux (3.8 mL/min), in which the dike can reach the surface, but is unstable (Experiment 9). Eruptions can last for tens of minutes but eventually solidify and force intrusion to begin at the base of the medium. (c, d) Images showing two stages of dike propagation (both from Experiment 4), for experiments with low flux (1.2 mL/min). The dike cannot propagate to the surface before solidifying, so new lobes begin to propagate again from the base. The complex dike eventually grows wide and thick, but struggles to reach the surface.

surfaces, filled with water, on opposing ends of the tank. As a dike is intruded and pushed the medium apart laterally, the free surfaces flexed outward to accommodate the strain. To prepare these surfaces, we embedded vertical plastic sheets, 1 cm in thickness, during preparation of the gelatin medium. After the medium cooled and solidified, we removed the sheets and replaced them with water. However, gelatin became firmly attached to the sheets during solidification, so we cut them out in order to remove them without badly damaging the gelatin. We adhered a metal wire to the plastic sheet prior to solidification and pulled it upward to make a smooth, continuous cut along the interface between the gelatin and the sheet. The cut was never perfectly flat and had a slightly undulating form. Removal of the sheets tended to leave the water-filled boundaries littered with fragments of gelatin, which impeded our view of the dikes. We therefore flushed the boundaries with fresh water until these fragments were sufficiently cleaned out.

We recorded each experiment using a set of cameras positioned to look at the dike's broad side. Each time an injection erupted and each time an injection terminated, we took several photos. We processed each photo against an initial reference photo to determine the geometry of the dike as a whole. We also processed each photo against the previous injection to see the change between injections, in order to determine which regions of the dike were actively flowing and which had solidified.

2.2. Scaling

For the scaling of these experiments we consider two primary factors, the time interval between pulses of activity and the volumetric flux of magma during a pulse, and to a lesser degree, the temperature of the

Table 2
Values for Both Our Analogue Experiments and Basaltic Dikes in Nature

Parameter (units)	Meaning	Experiments	Nature (basalt)
α (m ² /s)	Thermal diffusivity (crust)	7.4×10^{-8} ^a	10^{-6} ^b
t_{int} (s)	Time between injections	10^0 – 10^4	10^6 – 10^{11} ^{*,c}
H (m)	Dike opening thickness	10^{-3} – 10^{-2}	10^{-1} – 10^1 ^{d,e}
L (m)	Dike vertical length	0.27–0.38	10^3 – 10^4
B (m)	Dike horizontal breadth	0.08–0.31	10^2 – 10^4 ^{d,e}
Q (m ³ /s)	Magma flow rate	10^{-8} – 10^{-6}	10^{-1} – 10^3 ^{e,f}
V (m ³)	Dike volume	10^{-4} – 10^{-3}	10^6 – 10^8 ^{d,e,g,h}
T_0 (°C)	Dike initial temperature	30–32	1,100–1,200 ^{i,j}
T_∞ (°C)	Ambient temperature	15–17	100–300 ⁱ
T_s (°C)	Solidus temperature	28	1,000 ^k
Fo	Fourier number $\alpha t/H^2$	10^{-4} – 10^3	10^{-2} – 10^7
Φ	Flux parameter $QH/(\alpha LB)$	10^{-2} – 10^1	10^{-3} – 10^2
$Q/(\alpha L)$	Alternate flux parameter	10^{-1} – 10^2	10^1 – 10^6
Θ	Temperature parameter $(T_s - T_\infty)/(T_0 - T_\infty)$	0.76–1.00	0.9–0.95 ^j
S	Stefan number $\gamma/[C_p(T_0 - T_\infty)]$	0.04 ^a	~ 4 ^l

^aThis study, see supporting information. ^bDouglas et al. (2016). ^cMolloy et al. (2009). ^dKeating et al. (2008). ^eWylie et al. (1999). ^fBunger et al. (2013). ^gWnuk and Wauthier (2017). ^hWauthier et al. (2015). ⁱChevallier and Verwoerd (1990). ^jDelaney and Pollard (1982). ^kNatland and Dick (2009). ^lMorse (2011).
*Assuming a lower bound on the order of weeks.

dike. For the time interval we scale our experiments simply using a dimensionless time term, the Fourier number, Fo :

$$Fo = \alpha t/H^2, \quad (2)$$

where t is the time interval between injections of magma (Çengel, 2008; Turcotte & Schubert, 2002). This is a measure of the amount of heat lost from a heated slab via conduction versus what is stored, assuming one-dimensional heat transfer; that is, the slab is much taller and broader than it is thick, so heat transfer is perpendicular to and homogenous along its surface. For $Fo \ll 1$, much of the initial heat remains in the slab and the system is thermally heterogeneous. For $Fo \gg 1$, heat has been conducted away and the system become thermally homogeneous (see Table 2 for a comparison between experiments and nature). Values for Fo on the order of 10^2 or 10^3 are fundamentally no different than $Fo = \infty$, since in both cases, the dike has completely cooled down and solidified. Sometimes the Fo is used to define a characteristic time scale for cooling in a system, t_d , at which heat loss is balanced with stored heat, such that $Fo = 1$ and therefore $t_d = H^2/\alpha$ (Fialko & Rubin, 1998). Note that the thermal diffusivity relates to other material properties, the thermal conductivity, k , density, ρ , and specific heat capacity, C_p , such that $\alpha = k/(\rho C_p)$ (Çengel, 2008).

For the magnitude of flux, we use a dimensionless flux term, Φ , estimated following Taisne and Tait (2011), which balances heat into the system via influx and advection of heat out of the system via conduction (equation (1)). If Φ is large, the system is dominated by heating, while if Φ is small, the system is dominated by cooling. This is inherently related to the dimensionless time duration of the propagation, Fo_{prop} , such that

$$\Phi = QH/\alpha LB = QH^2/\alpha V = H^2/\alpha t_{\text{prop}} = 1/Fo_{\text{prop}}. \quad (3)$$

Here, t_{prop} is the propagation duration and V is the volume of the dike, assuming $V \approx LBH \approx Qt_{\text{prop}}$.

The relationship between flux and time poses a question for the experiment: Do we hold flux constant and test the time and thus volume of an injection or do we do the reverse and test the flux of an injection, holding volume constant? A good argument can be made for either approach. We would argue that by varying the flux, we can observe the effect of driving pressure on the geometry of the plumbing system that develops in the medium. In such a way, high driving pressures may favor dikes with relatively large dimensions, which grow both vertically long and horizontally broad, while small driving pressures favor dikes that grow primarily vertically.

The parameters Fo and Φ are highly dependent on the geometry of the intrusion, in that for a broad, thin dike, Fo may initially be large and Φ small. If the dike channelizes into a conduit-like feature, the parameters could evolve so that Fo decreases and Φ increases. When we show the scaling for these parameters (Table 2), we show ranges encompassing both dike flow and conduit flow.

We thermally scale the experiments following Delaney and Pollard (1982) and Taisne and Tait (2011), via the dimensionless solidus temperature, Θ :

$$\Theta = (T_s - T_\infty) / (T_0 - T_\infty), \quad (4)$$

where T_s , T_∞ , and T_0 are, respectively, the temperature of the solidus, the ambient host rock, and the magmatic source. The injection is above or below the solidus respectively when $\Theta < 1$ or $\Theta > 1$. Delaney and Pollard (1982) make the case that magma is probably injected at temperatures near the liquidus, such that $0.9 < \Theta < 0.95$. For a gelatin melting point of 30 °C (melting is rapid, so we assume this characterizes the solidus and liquidus temperature) and ambient temperature of 15 °C, we therefore choose an injection temperature of ~32 °C. In practice, the temperature in our setup fluctuated by a few degrees as liquid flowed through the system. As described in the previous section, we configured experimental setup to minimize this fluctuation and keep the system thermally stable.

Note that in nature, the thermal evolution of a dike is dependent on another dimensionless parameter, the Stefan number, S :

$$S = \gamma / [C_p(T_0 - T_\infty)], \quad (5)$$

where γ is the latent heat (Fialko & Rubin, 1998). The S is a ratio of the available latent heat to sensible heat in a system. The proportion of latent heat in a basaltic magma can be large and represent the majority of available enthalpy (Morse, 2011). In the context of our experiments, this term is neglected because the latent heat release from our gelatin is relatively small, such that $S = 0.04$ (see the supporting information).

3. Results

3.1. Time-Dependent Behaviors and Flux-Dependent Behaviors

Each experiment began with an initial dike and the systems evolved as a function of the time interval between liquid injections. For experiments with a short time interval (seconds to minutes), the flow in the dike quickly channelized into a narrow conduit (Figure 2a), which tended to be elliptical in cross section, a few centimeters across and approximately half a centimeter thick. We were not able to detect any clear sign of melt back of the gelatin medium due to flow through the conduit, even for the highest flux; this is probably due to the low injection temperature we used. Experiments with a moderate time interval (30–60 min) had enough time for the dike to partially solidify. Such dikes were still soft at the start of the subsequent injection, which resulted in a new dike propagating inside the original dike (Figure 2b). We will refer to this as “nested dike” behavior. For experiments with a very long time interval (3+ hr), the dike had time to completely solidify. Subsequent injections created new dikes, leading to a dike swarm being generated (Figures 2c and 2d).

The flux seemed to have less control on the form of the plumbing system (i.e., conduit vs. dike swarm formation) and more control on the amount of material that remained solidified in the gelatin medium. Fluxes that were sufficiently high (>10 ml/min) allowed the dike to propagate to the surface and begin erupting (Figure 3a), which continued steadily until the injection finished. Very low flux experiments (~1 ml/min) could not propagate quickly enough to reach the surface before solidifying. In such cases, intrusion restarted at the base of the experiment and propagated alongside (in the same plane) the previous failed dike lobe (Figure 3c). This created a fingering appearance, with each lobe originating from the same source location. As the complex dike became wide and full of lobes, the liquid began to flow inside the older lobes, widening them, without significantly advancing the dike front (Figure 3d). We were also able to detect a transitional behavior at semilow flow rates (~3 ml/min), which formed a larger-scale version of the fingering behavior. Large dike lobes sometimes propagated all the way to the surface (other times no) and erupted in an unsteady fashion, flowing for tens of minutes before solidifying and forcing propagation to begin again at the base of the tank (Figure 3b). In such cases, it seems as though the viscous forces in the increasingly cold dike eventually overwhelmed the flux and propagation of a new dike became the easiest path for the liquid.

Table 3
Dike Dimension Measurements

Exp	H (cm)	L (cm)	A_s (cm ²)	V (L)	Form
1	0.57 → 0.78	31.8	552 → 52	0.33 → 0.50	Conduit
2	0.60 → 0.88	32.5	685 → 57	0.40 → 0.47	Conduit
3	0.58 → 0.11	35.2	590 → 326	0.34 → 0.98	Dike swarm
4	0.75 → 2.05	39.4	692 → 777	0.50 → 1.96	No eruption
5	0.68 → 0.67	34.8	205 → 341	0.18 → 0.95	Unstable
6	0.48 → 0.65	34.1	877 → 107	0.40 → 0.44	Conduit
7	0.54 → 0.16	33.1	536 → 127	0.29 → 0.42	Nested
8	0.13 → 0.10	35.5	241 → 261	0.10 → 0.13	Nested
9	0.43 → 0.61	36.4	542 → 481	0.24 → 0.42	Dike swarm
10	0.20 → 0.07	21.7	475 → 248	0.09 → 0.17	Dike swarm

Note. From left to right are the median thickness, length, and surface area of the active region of the dike, the total intruded volume, as well as the form of the active intrusion. Each pair of values indicate the initial (left number; from the first injection) and final (right number; last injection) measurements. For dikes that erupted, these measurements are taken after the onset of eruption. For the dike that did not, these numbers represent the end of the first and the final injections. The vertical length does not change between intrusions in an experiment. The form references types of intrusions shown in Figures 2 and 3.

3.2. Image Processing

The dimensionless numbers Fo and Φ , which we use to analyze our results, are highly dependent on the geometry of the intrusion. To quantify this, we used image processing techniques to identify the thickness distribution and surface area of each intrusion, for each experiment. These techniques generally rely on comparing an image of interest with some reference image. Estimates for the dike geometry via the following methods are shown in Table 3.

3.2.1. Dike Thickness Distribution

To estimate the dike's opening thickness, we use the Beer-Lambert law, which relates the light transmission through a diffusing medium to its thickness (Taisne et al., 2011):

$$H = -\ln(I/I_0)V/\sum[-\ln(I/I_0)A_{px}]. \quad (6)$$

The light transmitted through a dike, I , relative to a reference photo taken before the dike is intruded, I_0 , decreases logarithmically as the thickness increases. For a pixel of area, A_{px} , summing the $-\ln(I/I_0)A_{px}$ term yields a value that is proportional to the actual volume. Scaling this value with the known volume, V , yields the width distribution of the dike. Via this method, we can estimate the thickness of a single dike or of a dike that channelized into a conduit.

For nested dike behavior, using the Beer-Lambert law can result in misleading thickness estimates, since it identifies all gelatin (both solidified and liquid) in the estimate. To determine the thickness distribution of the liquid region of each injection, we employ the Beer-Lambert law for two separate times, for one photograph taken during the injection and for one taken before the injection. Doing so yields a thickness distribution of all of the intruded gelatin, both liquid and solid, H_{total} , and one of only the solidified portion that remained in the medium, H_{solid} . The difference between the two reveals the thickness distribution of the liquid region, $H = H_{total} - H_{solid}$. Using these photo pairs, we could determine the geometry change between each subsequent intrusion, after the first (Figure 4).

For multiple layers of intrusions (i.e., dike swarm behavior), the Beer-Lambert law fails entirely, as the previous dikes pollute the image and complicate our estimation of the active dike's thickness. Employing the Beer-Lambert law for multiple dikes tends to yield nonsensical results, which take the form of sudden thickness differences where dikes overlap. Since we can still estimate the surface area, A_s , of such dikes to a high

Figure 4. To estimate the thickness of a dike intruded inside of a previous dike, we use the Beer-Lambert law. We find the volume of intruded gelatin before and during an injection via the expression $-\ln(I/I_0)$ and the difference between the two yields the thickness distribution of the liquid region. We set a threshold value to automatically distinguish the active dike from the remaining gelatin, after which the surface area can be easily estimated by the number of pixels and the area of a pixel.

degree of confidence and we know the volume injected, we simply estimate the average thickness via $H = V/A_s$.

3.2.2. Surface Area of the Active Region of an Intrusion

As each experiment progresses, the intrusion evolves to become more complex. We are primarily interested in the active region of the intrusion, whether it takes the form of a dike or conduit. For dike swarm experiments and nested dike experiments, the active region is the newest dike (or embedded dike), so we could simply compare a photo from one injection, I_i , to the previous, I_{i-1} , to highlight where the newest dike intruded. Viewing the ratio I_i/I_{i-1} in Matlab causes the new dike to appear very apparent and its surface area can be estimated, $A_s = A_{px}N$, for a dike identified in N pixels. In practice, we did this simultaneously with the thickness distributions, by defining a threshold degree of change. The pixels with significant change indicated the active surface area of the dike, while those that were lower than the threshold indicated the inactive regions (Figure 4).

For conduit-forming experiments, comparing a photo from one injection with the previous tends to reveal little change, since the conduit maintains a stable geometry. We manually masked the photos of conduit-forming experiments to highlight the conduit shape and, by processing the masked image in Matlab, identify its surface area and thickness.

3.3. Framing the Results Through Dimensionless Numbers

To compare our results with natural systems, we employ the dimensionless time parameter, Fo , and the dimensionless flux parameter, Φ . The dike thickness distributions determined via image analysis provide us with a range of values, so we take the median thickness. We use the horizontal aspect ratio, Ψ , (median opening thickness to the median horizontal breadth) of the actively flowing region of the intrusion to quantitatively characterize what kind of feature it is:

$$\Psi = H/B. \quad (7)$$

Cylindrical conduits therefore have low aspect ratios close to 1, while dikes have aspect ratios that approach 0. Note that the Ψ shares H/B terms with Φ and, in order to eliminate like terms, we plot a form of the dimensionless flux, such that $\Phi/\Psi = Q/(\alpha L)$. By plotting Ψ against Fo and $Q/(\alpha L)$ (Figure 5), we can see to what degree the intrusion shape depends on either parameter. To do so, we plot an empirical, best fitting surface of the form of $\Psi = C_1 Fo^{C_2} [Q/(\alpha L)]^{C_3}$ and solve for the values of C_1 , C_2 , and C_3 by minimizing the sum of the square of the residuals (in log scale) between measurements and the model surface of Ψ . The equation of the surface (shown in Figure 5a) indicates that Ψ is dependent on both parameters, in that conduits need both a moderate flux and frequent intrusion to remain active. Conduits tend to develop for $Fo < 1$, nested, compound dikes form when $1 < Fo < 10$ and dike swarms occur for $Fo \gg 10$. The $Q/(\alpha L)$ parameter determines the thermal stability or propagation, in which solidification causes dikes arrest when $Q/(\alpha L) < 1$ and eruption can proceed unimpeded when $Q/(\alpha L) > \sim 10$. In terms of Φ , these two end members are respectively quantified by $\Phi < 10^{-1}$ and $\Phi > 3 \times 10^{-1}$ (though for simplicity we will later use $\Phi > 1$).

We also explore the effect of these two parameters on the amount of material intruded into the crust. For an injection volume, V_{inj} , of 500 ml, we measured the volume of material that erupted, V_{erupt} and simply analyze the intruded volume ratio, V^* , such that

$$V^* = (V_{inj} - V_{erupt})/V_{inj}. \quad (8)$$

Plotting V^* against Fo and Φ and applying another best fitting surface as done for Figure 5, we see that the intruded volume strongly depends on the flux (Figure 6). This makes sense, in that a low flux allows more time for the intruded liquid to cool and solidify inside the medium. The weak dependence on Fo stems from the plumbing evolution of each type of behavior. For example, the value of Fo did not change much throughout conduit-forming experiments, even though they progressively extruded more and more liquid as the conduit developed into a stable form. For nested dike experiments by contrast, each intrusion was thinner than the previous, causing the corresponding values of Fo to increase and V^* to decrease (they were thin, so less volume intruded). Similarly, for dike swarms, the first dikes to intrude could freely deform the surrounding

Figure 5. The aspect ratio, Ψ , against dimensionless time, Fo , and dimensionless flux divided by the aspect ratio, such that $\Phi/\Psi = Q/(\alpha L)$. The Φ contains the same H/B terms as the aspect ratio, so we use $Q/(\alpha L)$ to show the independent effect of flux. Colors correspond to experiments that were conduit forming (red), unstable flow (magenta), nested dikes (green), dike swarms (blue), and finally dikes that did not erupt (blue x). We do not use noneruptive dikes to analyze the aspect ratio, since they do not match eruptive data and lay visibly below the trend in (b). (a) The parameters are plotted in 3-D. A best fitting surface is applied to the data to see the relative contribution of Fo and $Q/(\alpha L)$. (b–d) Two-dimensional plots showing how the parameters compare to each other.

Figure 6. How dimensionless time, Fo , and dimensionless flux, Φ , affect the ratio of intruded volume, V^* . Markings are as in Figure 5. (a) The parameters are plotted in 3-D, and a best fitting surface is shown. The intruded volume is almost entirely a function of Φ . (b–d) Two-dimensional plots showing how the parameters compare to each other. Fo has little influence on V^* , so there is no apparent trend in (b). By comparison in (c), low values of Φ correspond to V^* values approaching 1.

medium, resulting in greater thicknesses, lower values of Fo and greater values of V^* . Subsequent dikes interacted with the local stresses due to the previous ones and therefore were relatively thin.

Regarding the best fitting surfaces shown in Figures 5a and 6a, we plot these to determine the degree to which Fo and Φ , or alternately $Q/(\alpha L)$, have an effect on Ψ and V^* ; however, for various reasons we hesitate to use them as predictive tools and therefore do not further discuss them in this article. For example, the relationship between Fo , Φ , and V^* is not linear. As we noted, the amount of intruded material decreased as each experiment progressed, so in Figure 6b, each category of experiment has a negative trend between Fo and V^* . However, between categories, there is a positive trend; conduit experiment intruded less material than nested dikes, which intruded less than dike swarms. To forecast the value of V^* , one would require a complete knowledge of the stress and temperature distribution in the medium, as well as the dynamics of the intrusion. Similarly for Ψ , we cannot make any predictions as functions of Fo and $Q/(\alpha L)$, since the aspect ratio in gelatin experiments is different from natural magmatic dikes; values are 10^{-2} to 10^{-1} for gelatin and 10^{-4} to 10^{-3} for in nature (Kavanagh et al., 2013).

What we can do is quantitatively distinguish between types of behavior. For example, conduit-forming experiments had $Fo < 1$, so we can broadly say that liquid migrates via a conduit geometry when the time interval is short. Dike swarms by contrast had $Fo > 100$. Similarly, experiments that were able to make stable eruptions had $Q/(\alpha L) > \sim 10$, whereas those that arrested due to solidification had $Q/(\alpha L) < \sim 1$.

4. Discussion

4.1. Channelization Due to Solidification or Due to Preexisting Flow Paths

It is important to consider that dikes have complex internal flow, which may affect how heat is distributed throughout the dike. Kavanagh et al. (2018) identified several stages of flow within a dike including: (1) initial injection and radial growth of a penny-shaped crack, (2) growth and internal circulation of the magma, and (3) eruption and ejection of magma from the dike. The assumption that magma flows unidirectionally away from the source may not be true during the propagation stage of a dike and instead may only apply for syneruptive, channelized flow. In this context, it is possible that, during dike propagation, a central upward jet provides heat to the center of a dike, while cooler magma circulates downward along the lateral edges, imposing a thermal distribution that favors channelization.

One way to check for this would be by measuring a time series of a fissure eruption's temperature distribution. Kavanagh et al. (2018) show that, prior to eruption, dikes can have an internal magma circulation, consisting of an upward central jet and lateral downward flow. As the dike breaches the surface, it contracts and squeezes much of the liquid upward onto the surface. Assuming that the preeruptive circulation imposes a thermal distribution that is warmer toward the center and cooler toward the sides, the distribution would be reflected in the initial stage of a fissure eruption (Figure 7). The fissure would initially emit magma from the upward central jet, followed by the remaining somewhat cooler magma. As the eruption continues and channelizes, the temperature would increase to reflect the magma's direct path from depth to the surface. To the best of our knowledge, an initial drop in temperature following the onset of a fissure eruption has not yet been identified, however, the progressive increase in temperature as a fissure channelizes has been shown at the Holuhraun lava field associated with the 2014 eruption of Bardarbunga volcano (Aufaristama et al., 2018).

4.2. Plumbing Region Distribution

Perhaps, the most important discussion from these results is that of how a plumbing system initiates and is distributed. A key finding from our results is the effect of injection frequency on the form of migration. When there is a long repose time between magma pulses, migration occurs via dike propagation, since heat can entirely conduct away from any intrusion. In such a case, there is no preferred path (thermally speaking) along which a dike will propagate. Sporadic intrusions of magma would be randomly distributed throughout the region, favoring the small, decentralized eruptions associated to volcanic fields (Figure 8a).

However, a repose time may be long enough that migration occurs via dike propagation but short enough that the region gradually builds up heat. Subsequent dike propagation is favored in the relatively warm (decreasing the value of Θ) and relatively soft regions of the crust and it is likely that a dike can migrate laterally along a horizontal temperature gradient, further focusing magmatism into a central region. The long-

Figure 7. A schematic view for the temperature evolution of a fissure eruption. The upper panels show the surface lava flow. The temperature initially drops between Time 1 and Time 2, as the dike first evacuates magma from a warmer, central region, followed by the cooler lateral regions. As the dike channelizes, magma flows from depth to the surface relatively quickly, so the eruptive temperature increases.

term result of such magma influx frequency would be the development of a centralized volcanic plumbing system, along with an associated large, central edifice (Figure 8b). Since dikes in this scenario would be focused to varying degrees (i.e., dikes do not horizontally migrate all the way to the center of the region), the edifice would likely build up into an oblong structure. If the repose time of magma influx is very short, we imagine the system would be more centralized to a narrow region (Figure 8c) and construct a fairly conical edifice.

Figure 8. A model of how volcanic activity is horizontally distributed, depending on the frequency of magma influx defined by the dimensionless time, Fo . The boundary between (a) and (b) is numerically derived (see below) and the boundary between (b) and (c) is experimentally derived (see section 3). (a) A very low frequency allows all heat from an intrusion to be lost. Dikes propagate through the crust, independently to each other, creating small monogenetic cones. (b) A moderate frequency maintains a hot central region but no molten path. Dikes preferably propagate through the hotter region and build a less centralized plumbing system and a large edifice. (c) Very high frequency maintains a molten pathway along which magma migrates. The resulting plumbing system would be very centralized and would build a large edifice.

Figure 9. Two-dimensional temperature contours for the crust near an intrusive region, from the numerical model. These model the heat distribution after a fixed time frame of 10^{12} s (~ 30 kyr), which is similar to the repose time of volcanic fields. The model allows an intrusion to reheat the left side of the region on a recurrence time interval, t_{int} , shown in each subfigure and the system evolves via conduction. The surface is kept at a constant temperature of 300 K, and we impose a geothermal gradient of 30 °C/km. For $t_{\text{int}} \leq 10^{10}$ s, the region stays warm near the intrusion, whereas for $t_{\text{int}} \geq 10^{12}$ s, all heat is lost and the temperature profile returns to the geothermal gradient.

With this logic in mind, we attempt to define a critical frequency of influx, which separates these different types of systems. To do so, we use a simple 2-D numerical model of transient heat conduction through the crust, in which heat transfer between elements in the model is described by Fourier's law:

$$\rho \Delta x^3 C_p (dT/dt) = k \Delta x^2 (dT/dx), \quad (9)$$

for density ρ , specific heat capacity C_p , thermal conductivity k , element volume Δx^3 , and surface area Δx^2 (Çengel, 2008; Turcotte & Schubert, 2002). This model is essentially a simplified version of the model presented by Daniels et al. (2014) and aims to quantify the time scale over which heat is completely lost from a dike. It assumes that a vertical column of 1-m half-width, representing a dike, becomes hot at time, $t = 0$, with a temperature of 1400 K and conducts heat away with time over a horizontal spatial scale of 1,000 m and depth of 8,000 m. The boundary is then reheated on a recurrence frequency of 10^7 s, 10^8 s, etc., and the temperature distribution is allowed to evolve for 10^{12} s (~ 30 kyr). The model imposes a surface temperature of 300 K and a geothermal gradient of 30 °C/km. For more details on the model setup, please see the supporting information.

Note that we do not make any assumption of sills, plutons, or other magmatic bodies (of which there could be various shapes, sizes, numbers, or locations) that would naturally develop in such plumbing systems and instead consider only vertical intrusions. Such intrusions would undoubtedly change the heat distribution in the crust but develop due to other, nonthermal processes such as fracturing and deformation of the crust. At any rate, such bodies develop with time and would not exist when a plumbing system first initiates.

The conduction model indicates that repetitive intrusions can build up heat in a narrow band of the crust. For a recurrence interval on the time scale less than 10^{10} s (~ 300 years), the crust builds up a distinct temperature profile (Figure 9a). In the context of our conceptual model (Figure 8), this frequency would build a temperature gradient that would encourage intrusions to propagate near the center of the region, focusing into a centralized plumbing region and corresponds to $Fo \sim 10^4$. The temperature gradient that develops due to an intrusion recurrence interval of 10^{11} s (~ 3 kyr; $Fo \sim 10^5$) appears quite weak (Figure 9b), while that due to 10^{12} s (~ 30 kyr; $Fo \sim 10^6$) has vanished entirely and returned the crust to the background thermal gradient (Figure 9c).

Our model informs us of the intrusion recurrence frequency that is necessary for the focusing of a plumbing system, assuming that heating of the crust indeed promotes dike propagation into a narrow region (i.e., Figure 8b). Such focusing can occur when the intrusion recurrence interval is less than 1 kyr, whereas a less frequent recurrence allows intrusions to be spatially dispersed. This agrees with Valentine and Connor (2015), who indicate that volcanic fields typically have eruption recurrence intervals 10–100 kyr but may be as short as 1 kyr. Plumbing systems with shorter recurrence intervals are perhaps characterized by

centralization of intrusive events into a focused plumbing system, which in turn constructs a large, centralized edifice. As such edifices are constructed, their loading on the crust further focuses intrusions toward the edifice (Roman & Jaupart, 2014). However, it should be reiterated that the model is simplistic, so it does not account for complicating factors, like a hydrothermal system, which would greatly accelerate heat transport at shallow depth, distributing the heat from a dike over a relatively wide area in a relatively short time (Cherkaoui et al., 2003; Chevallier & Verwoerd, 1990; Pashkevich & Taskin, 2009).

Note that temperature profiles estimated by our model are fairly straightforward and show agreement with other studies. For example, Delaney (1988) developed a numerical model for 1-D conduction from a dike with a temperature-dependent thermal properties and found that it rapidly cools from magmatic temperature to ambient temperature on a time scale of $\sim 10^7$ s (~ 4 months); in our model a single dike cools off on the same time scale. In terms of 2-D modeling, our model generates temperature gradients similar to those modeled by Rudman and Epp (1983), on the same time scales and spatial scales, though with a different shape (they model a relatively wide dike, which narrows to a point). Our model results also resemble the heat accumulation modeled by Daniels et al. (2014), who investigated the heating due to repeated dike intrusions during continental rifting. They found that a recurrence interval of 3×10^{10} s can significantly heat the nearest ~ 500 m to the dike on a time frame of 20 kyr, which nearly matches the conditions and results for the model in Figure 9a.

4.3. Stable Conduit Development at Depth

Our experiments provide insight into the conditions for which a stable, molten conduit may develop in the crust ($Fo < 1$ and $\Phi > 1$). Considering that the dimensionless numbers control the plumbing geometry, we can rewrite equations (1) and (2) to solve for the necessary conditions to maintain a molten channel. We label these as the critical flux, Q_{crit} , and critical repose time, t_{crit} . Assuming steady-state heat transfer in a conduit geometry with radius, R , the heat into the conduit due to magma flux balances with loss due to conduction:

$$Q_{crit} \rho C_p T_0 = 2\pi L k (T_0 - T_\infty) / \ln(r/R), \quad (10)$$

for a magma and ambient temperatures T_0 and T_∞ and conduction occurring over a radial distance into the crust, r (Çengel, 2008). The Q_{crit} is dependent on temperature and the temperature gradient. For values of $2 < r/R < 1,000$, there is a constricted range of values of $0.7 < \ln(r/R) < 7$. Since it varies over a small range, we assume it to be constant, so

$$Q_{crit} = 2\pi \alpha L (1 - T_\infty/T_0). \quad (11)$$

For magma much hotter than the ambient temperature (so that $T_\infty/T_0 \approx 0$), which ascends through a 1- to 10-km-long conduit, a relatively low mean flux of only 10^{-2} – 10^{-1} m³/s is required to maintain a liquid conduit.

Perhaps unsurprisingly, the time parameter has stricter conditions for a conduit to remain molten. The time for solidification implies a transient heat conduction problem, such that

$$V \rho C_p (\partial T / \partial t) = k A_s (\partial T / \partial r). \quad (12)$$

assuming the flow has stopped, in a conduit of surface area, A_s , and volume, V . Therefore,

$$\partial T / \partial t = (2\alpha / R) (\partial T / \partial r), \quad (13)$$

in which $\partial T / \partial t$ and $\partial T / \partial r$ decrease as time goes by and the conduit cools. For solidification to occur, the magma only needs to cool to the solidus temperature, and we integrate the cooling rate over time, t_{crit} , so that $\partial T / \partial t = (T_0 - T_s) / t_{crit}$ for a solidus temperature, T_s . This approximates the temperature gradient to a time-averaged gradient. If we integrate again with respect to r , over a range of R to r , then

$$(T_0 - T_s) / t_{crit} = 2\alpha (T_0 - T_\infty) / [R(r - R)]. \quad (14)$$

If r is proportional to R , such that $R^* = r/R$, then

Figure 10. An illustration for the temperature gradient, $\partial T/\partial r$, in the crust sounding a conduit. The gradient controls conduction, so we approximate it to a linear function over the temperature range of T_0 to T_∞ , which indicates the magnitude of R^* . The steeper the slope, the faster heat is lost from the conduit.

$$t_{\text{crit}} = (R^2/2\alpha)(R^*-1)[(T_0-T_s)/(T_0-T_\infty)]. \quad (15)$$

In the above equation, characteristic time scale represented by $R^2/2\alpha$ has a strong dependence on the conduit radius, in that large conduits have a large ratio of volume to surface area. For a 1-m-radius conduit, the time scale is several days, whereas for a 10-m-radius conduit, it is around 19 months. However, the other terms in the equation modify the time estimate to a significant degree. For an initial temperature defined by Θ (Table 2), the temperature terms in this equation together have a value of 0.28 to 0.38, suggesting quicker cooling. The temperature gradient term, R^*-1 , meanwhile suggests longer cooling times. It is difficult to judge which radial distance approximates the temperature gradient, but if conduction occurs over tens or hundreds of meters, this term could be large (Figure 10).

In terms of sensible heat, reasonable estimates for conduit cooling would be weeks to decades, depending on the magma temperature, crust temperature and conduit geometry. Latent heat however can provide a large amount of thermal energy and represent the majority of available enthalpy (Morse, 2011). In this context, as magma crystallizes, it likely enters an approximately steady state of heat loss, in which conduction balances with the energy released during phase changes. If the available amount of latent heat is indeed several times larger than the sensible heat, then solidification would take a similar factor longer to conclude. In the context of our experiments, it is expected that the latent heat does not play a large role, since most of the available heat is sensible heat (see the supporting information).

The conditions for solidification in a conduit geometry have also been addressed in previous studies. Solidification occurs by the freezing of magma along the annulus of conduit, which proceeds when the heat loss due to conduction exceeds heat supply due to advection, for laminar flow conditions with a constant pressure drop; when heat loss balances with supply, the conduit maintains a steady-state radius (Mulligan & Jones, 1976; Zerkle & Sunderland, 1968). Holmes-Cerfon and Whitehead (2011) show that the critical flux necessary for steady-state flow depends on the pressure drop, the Stefan number (equation (5)), the dimensionless flux Φ (equation (1), modified for a conduit geometry, in which $H/B = 1$) and the dimensionless temperature difference, Θ_d , similar to Θ described above, in which $\Theta_d = (T_s - T_\infty)/(T_0 - T_s)$. The Θ_d corresponds to the degree of solidification, in which $\Theta_d \gg 1$ indicates that solidification should occur fairly rapidly. For magma driven by a constant flux (which is the driving condition for our experiments, in which pressure is variable), they find that any flux can maintain magma flow, but may cause large driving pressures. However, for magma flow driven by a pressure drop (either constant or somewhat variable), they find that stable flow in a conduit requires a minimum critical flux of $\Phi \sim 6,000$, significantly higher than that observed in our experiments ($\Phi \sim 1$). This could be partially explained by the time-dependent solidification of gelatin in our experiments, which is discussed later in section 4.5 and allows gelatin to temporarily (~ 10 min) remain liquid as it cools below the solidus. However, we expect this yields a similar effect as

latent heat release, which also delays the solidification process. Although there is not a significant latent heat release from our gelatin, the time-dependent solidification effectively takes its place to a degree.

A natural setting that indicates the necessary critical flux for a conduit is a lava lake, which represents the uppermost surface of a molten conduit. Lava lakes are thought to persist due to magma convection, in which bidirectional flow supplies heat from depth throughout the conduit and maintains a molten pathway (Harris et al., 1999; Oppenheimer et al., 2004; Sweeney et al., 2008). Depending on their level of activity, the magma flux can vary in magnitude, but tends to be greater than $0.01 \text{ m}^3/\text{s}$. For example, Harris et al. (1999) estimated that Mt. Erebus lava lake in Antarctica had a magma flux of $\sim 0.01 \text{ m}^3/\text{s}$ during measurements taken in both 1986 and 1989; similarly Erta' Ale lava lake in Ethiopia had fluxes varying from $5 \text{ m}^3/\text{s}$ in 1973 to $0.01 \text{ m}^3/\text{s}$ in 1986. They reported that other lava lakes tended to have higher fluxes. Although such systems are characterized by bidirectional flow, rather than the unidirectional flow that we analyze in this study, we can assume this $0.01\text{-m}^3/\text{s}$ base value provides an order-of-magnitude estimate of the necessary critical flux for stable flow. Assuming that such conduits are between 1 and 10 km long and that $\alpha = 10^{-6} \text{ m}^2/\text{s}$, then the minimum value of Φ is 1–10, similar to what we estimate for our experiments (solving equation (1), in which $H/B = 1$).

If we consider deeper magma migration, in the form of magma supply from deep sources into shallow crustal reservoirs, these estimates hint that the form of magma migration ultimately depends on how magma enters the crust from the mantle. A slow, steady trickle of fresh magma may be sufficient to maintain a molten conduit, thereby continuously feeding shallower bodies. Conversely, if magma accumulates at the bottom of the crust in infrequent, discrete pulses, a previously existing conduit would not have the necessary heat to remain liquid, so the magma would have to migrate in the form of a new dike. In this sense, magma supply into shallow reservoirs may depend on how effectively and how steadily partial melt can form and percolate into the crust.

A good example of the effect of supply frequency and cooling on deep magma migration may be Piton de la Fournaise. Prior to the 1998 eruption, magma migration from depth produces a series of volcano tectonic earthquakes, whose hypocenters tracked the migration front (Battaglia et al., 2005). The lateral distribution of these earthquakes indicated that the migration followed a fairly narrow path below sea level and then a sheet-like path above sea level. This sudden change in geometry was considered to be due to the magma flowing through a preexisting conduit (perhaps partially molten) at depth and then rapidly propagating a shallow dike. The deep conduit structure was reactivated during the 2015 seismic episode, which also led to an eruption (Lengliné et al., 2016). The small magnitudes of the deep seismic events are evidence that magma flowed through the existing, partially molten structure and therefore implies that it was persistently active on the 1998–2015 time frame. We argue that there may be a thermal discontinuity located at sea level, in which the underlying crust is well insulated and maintains an open, persistently active conduit. Above this discontinuity, the crust may be generally cooler, so that intrusions are thoroughly solidified and require migration in the form of a dike.

4.4. Shield Volcano Rift Zones

In addition to identifying the nature of supply pathways, we can apply our findings to rift zones (namely those that occur on the flanks of shield volcanoes; for example, Kilauea's east rift zone, Piton de la Fournaise rift zones) to gain some sense of the internal structure. Such rift zones are known to be frequently active pathways along which magma repeatedly travels.

In the case of Kilauea, magma frequently drains from the summit region along the rift zone and this background activity is occasionally punctuated by more intense fissure eruptions. The frequent, relatively low flux of the background phase likely favors the form of a subhorizontal conduit, which minimizes the heat loss due to conduction. The higher-intensity fissure eruptions occur due to some combination of increased magma flux (internal pressure) and subsidence of the volcano's flank (external extension), which cause the conduit to fail and propagate a dike.

Over the lifetime of the rift zone, it is plausible that the geometry would oscillate between cylindrical and planar, depending on the level of activity, slowly accumulating layers into a dike complex (Figure 11). Indeed, geophysical measurements (seismic reflections, gravity, and magnetic data) indicate that a dense intrusive body spans the length of Kilauea's east rift zone to the submarine Puna Ridge and contains a narrower, hot ($\sim 1000^\circ\text{C}$) region along its axis (Furumoto, 1978; Leslie et al., 2004). Subsidence of the volcano's

Figure 11. A model for the development of a rift zone's internal structure. Frequent influx of magma favors intrusion along a planar region. (a) An initial dike propagates through the region. (b) As influx decreases, flow may take the form of a horizontal conduit. (c) A subsequent increase in flux concentrates stress along the conduit, causing the propagation of a nested (or alternately, a parallel) dike. (d, e) Rift zones tend to be found in extensional settings, ensuring propagation of subvertically oriented dikes. With time, these dikes may build thick, tabular intrusive complexes. Periods of low flux favor magma flow in confined channel (Patrick et al., 2019), while high flux favors dike propagation and growth of the intrusion.

flank generates extensional forces that cause dikes to orient perpendicularly to the direction of subsidence, ensuring the intrusive dike complex is subvertical, with dimensions much taller and longer than wide.

4.5. Experimental Limitations

Our experimental setup has the inherent limitation that gelatin solidifies as a function of temperature, time and shear strain. We consistently measured eruptive temperatures below the solidus of gelatin, indicating that solidification either requires time or is inhibited due to the liquid's motion. In either case, the resulting dike's form could be somewhat different from a magmatic dike (perhaps thinner, with less material intruded). A material that solidifies independently of time and shear strain may have grown a thicker chilled margin and restricted the internal flow. Additionally, the time dependence of gelatin solidification implies that its viscosity may also have some time dependence. This potentially complicates comparison with other studies of flow channelization in a fissure, in which it has been shown that channelization strongly depends on the viscosity ratio between warmer and cooler regions of a dike.

That said, we believe our results are fundamentally sound. Taisne and Tait (2011) made analogue experiments of solidifying dikes using paraffin wax, which is strongly temperature-dependent. The general form of their dikes were similar to ours, albeit with greater thickness and distinct chilled margins. It is likely that the inner fluid core of the dike undergoes flow patterns similar to those observed in this study and those described by Kavanagh et al. (2018). Furthermore, we were able to replicate behaviors observed in nature, such as the heterogeneous solidification and the subsequent channelization of a dike.

Quantitatively speaking, the characteristic time for solidification, H^2/α , for our experimental dikes is on the time scale of ~ 10 min, which matches the time frame for channelization that we observed in our experiments, which seems to imply the time dependence did not play a large role in the solidification of a thin dike. For experiments that did not result in conduit formation, either the time interval between injections or the flux duration (or both) tended to be much larger (> 30 min) than this 10-min time frame. All things considered, it seems as though the gelatin's time dependence had no effect on the form of liquid migration. Rather, its effects were confined to the aspect ratio of the dikes, which may have been somewhat thinner than for a material with a time-independent phase transition.

5. Conclusions

The experiments we present in this paper illustrate the effects of solidification on magma migration. For very frequent intrusions ($Fo < 10^0$), magma maintains a hot, open pathway from depth to the surface. This path initiates as a dike and, upon eruption, quickly channelizes into a conduit. For very infrequent intrusions ($Fo > 10^2$), the dike can completely solidify, so a subsequent intrusion forms a new dike and the system eventually accumulates a dike swarm. We also show a transitional behavior ($Fo \sim 10^1$), in which a dike intrusion solidifies but remains soft, so subsequent intrusion propagate new dikes inside the original dike.

We find that the effect of flux primarily controls the stability of flow through the crust. Intrusions that propagate with low fluxes ($\Phi < 10^{-1}$) struggle to reach the surface and form thick dikes that are composed of many small, individual lobes that are compounded together. Dikes with sufficiently high flux ($\Phi > 1$) propagate to the surface and can steadily flow until the supply stops. Between these two regimes, the flux has a transitional regime, in which a dike may (or may not) propagate to the surface and any conduit that forms is temporary, and eventually closes due to solidification.

We expand these results using a 2-D numerical model of thermal conduction. For injections more frequent than 10^{11} to 10^{12} s (about 3 to 30 kyr; $Fo \sim 10^4$ to 10^5), the crust accumulates heat near the intrusive region, which we argue softens the crust and encourages subsequent dikes propagate nearby. For less frequent intrusions, heat is lost and the crust returns to its background temperature. Considering the experimental and numerical evidence together, the frequency of migration dictates whether magma flows through a conduit or a series of spatially focused or spatially dispersed dikes.

Acknowledgments

This work comprises Earth Observatory of Singapore Contribution 256. This research is supported by the National Research Foundation Singapore (Award NRF2015-NRF-ISF001-2437) and the Singapore Ministry of Education under the Research Centres of Excellence initiative. Our data are available online (<https://doi.org/10.21979/N9/KI33LP>). This is under DR-NTU, a local instance of DataVerse, supported by Nanyang Technological University. For more information about the data, please see the supporting information document.

References

- Aufaristama, M., Hoskuldsson, A., Jonsdottir, I., Ulfarsson, M. O., & Thordarson, T. (2018). New insights for detecting and deriving thermal properties of lava flow using infrared satellite during 2014–2015 effusive eruption at Holuhraun, Iceland. *Remote Sensing*, 10(2), 151. <https://doi.org/10.3390/rs10010151>
- Battaglia, J., Ferrazzini, V., Staudacher, T., Aki, K., & Cheminée, J.-L. (2005). Pre-eruptive migration of earthquakes at the Piton de la Fournaise volcano (Réunion Island). *Geophysical Journal International*, 161(2), 549–558. <https://doi.org/10.1111/j.1365-246X.2005.02606.x>
- Bruce, P. M., & Huppert, H. E. (1989). Thermal control of basaltic fissure eruptions. *Nature*, 342(6250), 665–667. <https://doi.org/10.1038/342665a0>
- Bunger, A. P., Menand, T., Cruden, A., Zhang, X., & Halls, H. (2013). Analytical predictions for a natural spacing within dyke swarms. *Earth and Planetary Science Letters*, 375, 270–279. <https://doi.org/10.1016/j.epsl.2013.05.044>
- Çengel, Y. A. (2008). *Introduction to thermodynamics and heat transfer*, (2nd ed.). Dubuque, IA: McGraw-Hill, c2008.
- Chanceaux, L., & Menand, T. (2014). Solidification effects on sill formation: An experimental approach. *Earth and Planetary Science Letters*, 403, 79–88. <https://doi.org/10.1016/j.epsl.2014.06.018>
- Cherkaoui, A. S., Wilcock, W. S., Dunn, R. A., & Toomey, D. R. (2003). A numerical model of hydrothermal cooling and crustal accretion at a fast spreading mid-ocean ridge. *Geochemistry, Geophysics, Geosystems*, 4(9), 8616. <https://doi.org/10.1029/2001GC000215>
- Chevallier, L., & Verwoerd, W. (1990). Influence of temperature on the distribution of stress and displacement in a volcano: A numerical approach. *Bulletin of Volcanology*, 52(6), 413–425. <https://doi.org/10.1007/BF00268923>
- Daniels, K. A., Bastow, I. D., Keir, D., Sparks, R. S. J., & Menand, T. (2014). Thermal models of dyke intrusion during development of continent–ocean transition. *Earth and Planetary Science Letters*, 385, 145–153. <https://doi.org/10.1016/j.epsl.2013.09.018>
- Daniels, K. A., Kavanagh, J. L., Menand, T., & R. Stephen, J. S. (2012). The shapes of dikes: Evidence for the influence of cooling and inelastic deformation. *GSA Bulletin*, 124(7–8), 1102–1112. <https://doi.org/10.1130/b30537.1>
- Delaney, P. T. (1988). FORTRAN 77 programs for conductive cooling of dikes with temperature-dependent thermal properties and heat of crystallization. *Computers & Geosciences*, 14(2), 181–212. [https://doi.org/10.1016/0098-3004\(88\)90004-0](https://doi.org/10.1016/0098-3004(88)90004-0)
- Delaney, P. T., & Pollard, D. D. (1982). Solidification of basaltic magma during flow in a dike. *American Journal of Science*, 282(6), 856–885. <https://doi.org/10.2475/ajs.282.6.856>
- Di Giuseppe, E., Funicello, F., Corbi, F., Ranalli, G., & Mojoli, G. (2009). Gelatins as rock analogs: A systematic study of their rheological and physical properties. *Tectonophysics*, 473(3–4), 391–403. <https://doi.org/10.1016/j.tecto.2009.03.012>
- Douglas, M. M., Geyer, A., Álvarez-Valero, A. M., & Martí, J. (2016). Modeling magmatic accumulations in the upper crust: Metamorphic implications for the country rock. *Journal of Volcanology and Geothermal Research*, 319, 78–92. <https://doi.org/10.1016/j.jvolgeores.2016.03.008>
- Einarsson, P., & Brandsdóttir, B. (1978). Seismological evidence for lateral magma intrusion during the July 1978 deflation of the Krafla volcano in NE-Iceland. *University of Iceland, Reykjavik, IS*.
- Fialko, Y. A., & Rubin, A. M. (1998). Thermodynamics of lateral dike propagation: Implications for crustal accretion at slow spreading mid-ocean ridges. *Journal of Geophysical Research*, 103(B2), 2501–2514. <https://doi.org/10.1029/97JB03105>
- Fukushima, Y., Cayol, V., Durand, P., & Massonnet, D. (2010). Evolution of magma conduits during the 1998–2000 eruptions of Piton de la Fournaise volcano, Réunion Island. *Journal of Geophysical Research*, 115, B10204. <https://doi.org/10.1029/2009JB007023>
- Furumoto, A. S. (1978). Nature of the magma conduit under the east rift zone of Kilauea Volcano, Hawaii. *Bulletin Volcanologique*, 41(4), 435–453. <https://doi.org/10.1007/bf02597376>
- Harris, A. J., Flynn, L. P., Rothery, D. A., Oppenheimer, C., & Sherman, S. B. (1999). Mass flux measurements at active lava lakes: Implications for magma recycling. *Journal of Geophysical Research*, 104(B4), 7117–7136. <https://doi.org/10.1029/98JB02731>
- Holmes-Cerfon, M. C., & Whitehead, J. A. (2011). Instability and freezing in a solidifying melt conduit. *Physica D: Nonlinear Phenomena*, 240(2), 131–139. <https://doi.org/10.1016/j.physd.2010.10.009>

- Jones, T. J., Llewellyn, E. W., Houghton, B. F., Brown, R. J., & Vye-Brown, C. (2017). Proximal lava drainage controls on basaltic fissure eruption dynamics. *Bulletin of Volcanology*, 79(11), 81. <https://doi.org/10.1007/s00445-017-1164-2>
- Kavanagh, J. L., Burns, A. J., Hilmi Hazim, S., Wood, E. P., Martin, S. A., Hignett, S., & Dennis, D. J. C. (2018). Challenging dyke ascent models using novel laboratory experiments: Implications for reinterpreting evidence of magma ascent and volcanism. *Journal of Volcanology and Geothermal Research*, 354, 87–101. <https://doi.org/10.1016/j.jvolgeores.2018.01.002>
- Kavanagh, J. L., Menand, T., & Daniels, K. A. (2013). Gelatine as a crustal analogue: Determining elastic properties for modelling magmatic intrusions. *Tectonophysics*, 582, 101–111. <https://doi.org/10.1016/j.tecto.2012.09.032>
- Keating, G. N., Valentine, G. A., Krier, D. J., & Perry, F. V. (2008). Shallow plumbing systems for small-volume basaltic volcanoes. *Bulletin of Volcanology*, 70(5), 563–582. <https://doi.org/10.1007/s00445-007-0154-1>
- Lengliné, O., Duputel, Z., & Ferrazzini, V. (2016). Uncovering the hidden signature of a magmatic recharge at Piton de la Fournaise volcano using small earthquakes. *Geophysical Research Letters*, 43, 4255–4262. <https://doi.org/10.1002/2016GL068383>
- Leslie, S. C., Moore, G. F., & Morgan, J. K. (2004). Internal structure of Puna Ridge: evolution of the submarine East Rift Zone of Kilauea Volcano, Hawaii. *Journal of Volcanology and Geothermal Research*, 129(4), 237–259. [https://doi.org/10.1016/s0377-0273\(03\)00276-2](https://doi.org/10.1016/s0377-0273(03)00276-2)
- Michon, L., Ferrazzini, V., Di Muro, A., Villeneuve, N., & Famin, V. (2015). Rift zones and magma plumbing system of Piton de la Fournaise volcano: How do they differ from Hawaii and Etna? *Journal of Volcanology and Geothermal Research*, 303, 112–129. <https://doi.org/10.1016/j.jvolgeores.2015.07.031>
- Molloy, C., Shane, P., & Augustinus, P. (2009). Eruption recurrence rates in a basaltic volcanic field based on tephra layers in maar sediments: Implications for hazards in the Auckland volcanic field/Auckland volcanic field. *GSA Bulletin*, 121(11–12), 1666–1677. <https://doi.org/10.1130/b26447.1>
- Morse, S. A. (2011). The fractional latent heat of crystallizing magmas. *American Mineralogist*, 96(4), 682–689. <https://doi.org/10.2138/am.2011.3613>
- Mulligan, J., & Jones, D. (1976). Experiments on heat transfer and pressure drop in a horizontal tube with internal solidification. *International Journal of Heat and Mass Transfer*, 19(2), 213–219. [https://doi.org/10.1016/0017-9310\(76\)90115-0](https://doi.org/10.1016/0017-9310(76)90115-0)
- Natland, J. H., & Dick, H. J. (2009). Paired melt lenses at the East Pacific Rise and the pattern of melt flow through the gabbroic layer at a fast-spreading ridge. *Lithos*, 112(1–2), 73–86. <https://doi.org/10.1016/j.lithos.2009.06.017>
- Oppenheimer, C., McGonigle, A., Allard, P., Wooster, M., & Tsanev, V. (2004). Sulfur, heat, and magma budget of Erta Ale lava lake, Ethiopia. *Geology*, 32(6), 509–512. <https://doi.org/10.1130/G20281.1>
- Owen, S., Segall, P., Lisowski, M., Miklius, A., Murray, M., Bevis, M., & Foster, J. (2000). January 30, 1997 eruptive event on Kilauea Volcano, Hawaii, as monitored by continuous GPS. *Geophysical Research Letters*, 27(17), 2757–2760. <https://doi.org/10.1029/1999GL008454>
- Pashkevich, R. I., & Taskin, V. V. (2009). Heat transfer in a geothermal system of Mutnovsky Volcano: The influence of the form, discharge of magma chamber degassing and rocks permeability, paper presented at Proceedings, Thirty-Fourth Workshop on Geothermal Reservoir Engineering, Stanford University, Stanford, California.
- Patrick, M., Orr, T., Anderson, K., & Swanson, D. (2019). Eruptions in sync: Improved constraints on Kilauea Volcano's hydraulic connection. *Earth and Planetary Science Letters*, 507, 50–61. <https://doi.org/10.1016/j.epsl.2018.11.030>
- Roman, A., & Jaupart, C. (2014). The impact of a volcanic edifice on intrusive and eruptive activity. *Earth and Planetary Science Letters*, 408, 1–8. <https://doi.org/10.1016/j.epsl.2014.09.016>
- Rubin, A. M. (1993). On the thermal viability of dikes leaving magma chambers. *Geophysical Research Letters*, 20(4), 257–260. <https://doi.org/10.1029/92GL02783>
- Rudman, A. J., & Epp, D. (1983). Conduction models of the temperature distribution in the East Rift Zone of Kilauea volcano. *Journal of Volcanology and Geothermal Research*, 16(3–4), 189–203. [https://doi.org/10.1016/0377-0273\(83\)90029-X](https://doi.org/10.1016/0377-0273(83)90029-X)
- Sweeney, D., Kyle, P. R., & Oppenheimer, C. (2008). Sulfur dioxide emissions and degassing behavior of Erebus volcano, Antarctica. *Journal of Volcanology and Geothermal Research*, 177(3), 725–733. <https://doi.org/10.1016/j.jvolgeores.2008.01.024>
- Taisne, B., & Tait, S. (2011). Effect of solidification on a propagating dike. *Journal of Geophysical Research*, 116, B01206. <https://doi.org/10.1029/2009JB007058>
- Taisne, B., Tait, S., & Jaupart, C. (2011). Conditions for the arrest of a vertical propagating dyke. *Bulletin of Volcanology*, 73(2), 191–204. <https://doi.org/10.1007/s00445-010-0440-1>
- Turcotte, D., & Schubert, G. (2002). *Geodynamics*, (2nd ed., edited). Cambridge: Cambridge University Press.
- Valentine, G. A., & Connor, C. B. (2015). Chapter 23—Basaltic volcanic fields. In H. Sigurdsson (Ed.), *The encyclopedia of volcanoes*, (Second ed.pp. 423–439). Amsterdam: Academic Press. <https://doi.org/10.1016/B978-0-12-385938-9.00023-7>
- Wauthier, C., Cayol, V., Smets, B., d'Oreye, N., & Kervyn, F. (2015). Magma pathways and their interactions inferred from InSAR and stress modeling at Nyamulagira Volcano, D.R. Congo. *Remote Sensing*, 7(11), 15,179–15,202. <https://doi.org/10.3390/rs71115179>
- Wnuk, K., & Wauthier, C. (2017). Surface deformation induced by magmatic processes at Pacaya Volcano, Guatemala revealed by InSAR. *Journal of Volcanology and Geothermal Research*, 344, 197–211. <https://doi.org/10.1016/j.jvolgeores.2017.06.024>
- Wylie, J. J., Helfrich, K. R., Dade, B., Lister, J. R., & Salzig, J. F. (1999). Flow localization in fissure eruptions. *Bulletin of Volcanology*, 60(6), 432–440. <https://doi.org/10.1007/s004450050243>
- Wylie, J. J., & Lister, J. R. (1995). The effects of temperature-dependent viscosity on flow in a cooled channel with application to basaltic fissure eruptions. *Journal of Fluid Mechanics*, 305, 239–261. <https://doi.org/10.1017/S0022112095004617>
- Zerkle, R. D., & Sunderland, J. (1968). The effect of liquid solidification in a tube upon laminar-flow heat transfer and pressure drop. *Journal of Heat Transfer*, 90(2), 183–189. <https://doi.org/10.1115/1.3597471>