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A Generalized CAPM Model with Asymmetric Power Distributed Errors with an Application to Portfolio Construction

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Abstract

We estimate the CAPM model on European stock market data, allowing for asymmetric and fat-tailed return distributions using independent and identically asymmetric power distributed (IIAPD) innovations. The results indicate that the generalized CAPM with IIAPD errors has desirable properties. It is substantially less likely to be rejected than the traditional CAPM with normally distributed errors and, moreover, a backtest shows that portfolios constructed using the IIAPD errors outperform the portfolio constructed with normally distributed errors in terms of commonly used performance measures.

JEL Classification: C12, C14, C22, C52

Keywords: CAPM, Non-Gaussian Distribution, Fat Tails, Power Distribution, Minimum Variance Portfolio

1 Introduction

Although the Gaussian (normal) distribution is widely used in capital asset pricing and portfolio selection (see e.g., Markowitz, 1952, 1959; Sharpe, 1964; Lintner, 1965; Mossin, 1966; Black, 1972; Campbell *et al.*, 1997), there is ample empirical evidence that returns are not usually normally distributed (see, e.g., Fama, 1965a; Mandelbrot, 1967; Blattberg and Gonedes, 1974; Affleck-Graves and McDonald, 1989). According to Campbell *et al.* (1997), specifically taking asymmetry and fat-tails of financial data into account is relevant for asset pricing.

To cope with fat tails and skewness of financial data, Komunjer (2007) constructs the Asymmetric Power Distribution (APD), extending the Generalized Power Distribution (GPD) by accommodating asymmetry. Likewise, the Asymmetric Exponential Power Distribution (AEPD) proposed by Zhu and Zinde-Walsh (2009) includes two tail parameters to describe the decay of the tail densities. Table 1 indicates how a number of well-known distributions are nested as special cases of the APD and the AEPD.

[Table 1 about here.]

Despite the vast empirical evidence on non-normality of returns (see Sharpe, 2007b), most existing literature on empirical asset pricing adheres to the assumption of normally distributed returns. In this paper, we consider the seminal Capital Asset Pricing Model (CAPM) under the more general assumption that the error terms are independent and identically asymmetric power distributed (IIAPD) with zero mean, variance σ_ε^2 , skewness parameter α , and tail parameter(s) to accommodate the asymmetry and fat tails of the returns distribution. Our work provides a generalization of CAPM because the previously assumed normal return distributions can be considered as special cases of IIAPD. Zeckhauser and Thompson (1970) consider the estimation of linear models with power distributions as highly desirable and expect the effects on the estimated coefficients to be substantial. Thus, considering IIAPD errors potentially is empirically highly relevant to researchers and practitioners.

To examine whether two tail parameters (p_1 and p_2 in the AEPD capture the decay of the left and right tail, respectively) can better describe the distributions of asset returns than one (λ in the APD determines the decay of both tails), we develop two versions of the generalized CAPM with IIAPD errors, namely, the CAPM-IIAPD (independent and identically asymmetric power distributed with one tail parameter) and the CAPM-IAEPD (independent and

identically asymmetric exponential power distributed with two tail parameters). Using these new models as well as the traditional CAPM, we address the following research questions:

- (i) To what extent does the specification of the error term affect whether the CAPM model is ‘alive’ (working) or ‘dead’ (rejected)?
- (ii) Does the generalized CAPM with non-normal errors outperform the traditional CAPM with normal errors in terms of fitness measures?
- (iii) Does the generalized CAPM with two tail shape parameters (CAPM-IAEPD) have better fitness and predictive power than the CAMP with only one tail shape parameter (CAPM-IAPD)?
- (iv) What does the assumption of IIAPD errors imply for the practitioners in the fields of security valuation and portfolio management? Do the portfolios constructed under this assumption outperform those employing the normality assumption?

Our results show that the specification with IIAPD errors has very nice properties. First, it helps to ‘save’ the CAPM in the sense that the CAPM is substantially less likely to be rejected with IIAPD errors than with traditional normally distributed errors, in particular when using weekly and monthly returns data. Second, the generalized CAPM with IIAPD errors outperforms the traditional CAPM in terms of fitness measures in particular with weekly and daily data. Meanwhile, incorporating two parameters for the tail shape does not seem to further improve the fitness or predictive power of the model relative to the model with one parameter for the tail shape.

As indicated in research question (iv), we also study what the assumption of IIAPD errors implies for practitioners in the fields of security valuation and portfolio management. Do the portfolios constructed using this assumption outperform those employing the normality assumption? Our results suggest the answer is ‘yes’ ; the portfolios constructed using the IIAPD errors outperform the portfolio employing the normality errors, according to the Sharpe Ratios, Jensen’s Alphas, and Treynor Ratios.

By assuming the possibility of lending and borrowing at a risk-free rate of interest, the Sharpe-Lintner version of the CAPM is given by

$$\mathbb{E}(R_i) = R_f + \beta_{iM}(\mathbb{E}(R_M) - R_f), \quad (1)$$

where $\mathbb{E}[R_i]$ is the expected return of asset i , R_f is the return on the risk-free asset, and $\mathbb{E}[R_M]$ is the expected return on the market portfolio. The compact form of Eq (1), in terms of excess returns (i.e. returns in excess of the risk-free rate), is expressed by

$$\mathbb{E}(Z_i) = \beta_{iM} \mathbb{E}(Z_M), \quad (2)$$

where Z_i represents the return on the i th asset in excess of the risk-free rate, $Z_i \equiv R_i - R_f$, Z_M represents the excess return on the market portfolio of assets, $Z_M \equiv R_M - R_f$, and $\beta_{iM} = \frac{\text{Cov}(Z_i, Z_M)}{\text{Var}(Z_M)}$. The data-generating process (DGP) of the CAPM is assumed to be given by

$$Z_{it} = \alpha_{iM} + \beta_{iM} Z_{Mt} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \text{NID}(0, \sigma_{\varepsilon_i}^2), \quad (3)$$

where i denotes the asset, t denotes the time period ($t = 1, \dots, T$), Z_{it} and Z_{Mt} are the excess returns in time period t for the asset i and the market portfolio, respectively, $Z_{it} \equiv R_{it} - R_{ft}$, $Z_{Mt} \equiv R_{Mt} - R_{ft}$, α_{iM} represents the asset excess return intercept, β_{iM} is the proportionality factor that reflects the sensitivity of asset i relative to the market risk, and ε_{it} is a normally, independent distributed (NID) random variable with mean 0 and variance $\sigma_{\varepsilon_i}^2$.

As in several previous papers (see e.g., Fama and French, 1996), here ‘alive’ implies that α_{iM} is statistically insignificant and β_{iM} is statistically significant; thus, if at least one of them does not hold, the CAPM is called ‘dead’. Therefore, Question (i) will be studied via the 95% confidence intervals of α_{iM} and β_{iM} , which simplifies both the likelihood ratio test for the coefficients and the Kolmogorov-Smirnov test for residuals. In addition, the Akaike Information Criterion (AIC), density plots of the residuals, and distribution tests are used to address Questions (ii) and (iii). Finally, an applicable portfolio strategy, the Minimum Variance Portfolio (MVP), will be presented in Section 7 with a backtest based on historical data. The MVP is applied because its construction does not require the estimation of expected returns, which are difficult to estimate, and the optimization of MVP depends on the estimates of β_{iM} . Moreover, it performs well in practice and is popular among investors.

The empirical analysis is conducted on the European market, where the number of stocks in the market portfolio is smaller than in the US market. The EURO STOXX 50 index is used as a proxy of the market portfolio to examine the European stock market. Maximum Likelihood is used to analyze its constituents. We estimate the CAPM using daily, weekly

as well as monthly returns. After that, a backtest on a monthly basis is used to investigate portfolio construction strategies.

This paper is organized as follows: Section 2 provides a literature review. The models are presented in Section 3. Section 4 shows the log-likelihood functions of models. Section 5 describes the data. Section 6 presents the results and the goodness of fit of the estimated models. Section 7 examines the implications of the different error distributions to portfolio constructions. Finally, Section 8 concludes.

2 Literature review

To our knowledge, there is very little previous work in which IIAPD errors are applied to model asset returns conditional on market portfolios. Li and Lin (2014) checked the validity of CAPM for the French stock market based on a model called CAPM-AEPD, however, they used the location parameter μ and the scale parameter σ of the AEPD as the proxies of the mean and standard deviation of the distribution. Although μ and σ indeed correspond to the mean and standard deviation in the special case of a normal distribution, they are not suitable proxies for the mean and standard deviation in the APD or AEPD.

Our paper is also related to a broad class of studies on the application of other non-Gaussian distributions in finance. The class of symmetric stable Paretian distributions has, for instance, been used to model fat-tailed returns distributions (see e.g., Mandelbrot, 1963a, 1963b; Fama, 1965b, 1971). In addition, portfolio theory under stable Paretian laws has been developed (see Rachev and Mitnik, 2000). However, Cootner (1964) argues that the evidence of Paretian distributed security returns is too casual. Moreover, the probability density functions (PDFs) of the stable Paretian distributions cannot be expressed analytically, and their higher order moments are undefined. Campbell *et al.* (1997) deem that the stable Paretian distributions are too fat-tailed and that the return distribution may be fat-tailed with both finite variance and finite higher moments. In contrast, the stable Paretian distributions have more probability mass in the tail areas and their variance and higher moments are infinite. Moreover, Thomas and Gup (2010) argue that the empirical patterns of stock returns are inconsistent with the stable Paretian theory of constant ‘alpha peakedness’ and ‘beta skewness’ for returns over different time intervals. As a result, the stable Paretian does not appear to be the ideal distribution to model the fat tails of financial returns.

A number of alternative distributions have been proposed in the literature. For the sake of flexibility in modeling fat tails, the seminal Exponential Power Distribution (EPD, also named the Generalized Laplace distribution, the Generalized Power Distribution, or the Generalized Error Distribution) family proposed by Subbotin (1923) has also been applied in finance (see, e.g., Harvey, 1981; Nelson, 1991). However, the EPD does not allow for asymmetry in financial returns data. The Skew Normal (SN) distribution and the Skew Exponential Power distribution (SEPD) proposed by Azzalini (1985, 1986) can fit data with skew distributions. Despite this attractive property, the skewness parameter may be estimated to be infinite. Moreover, consistency of the maximum likelihood estimators (MLEs) has not been established for the SN distribution and the SEPD. In contrast, Fernández *et al.* (1995), Theodossiou (2000) and Komunjer (2007) extend the EPD family in the sense of accommodating asymmetry, categorized by Zhu and Zinde-Walsh (2009) as the second method of constructing SEPD. For the second method of constructing SEPD family, Zhu and Zinde-Walsh (2009) prove consistency of MLEs when the tail shape parameter is larger than one. Besides, Komunjer (2007) also proves consistency of MLEs in the context of the APD. Since the APD can provide consistent parameter estimates and Komunjer's (2007) method to skew EPD is conceptually simpler and readily applicable to the nested densities, the APD as a sort of SEPD is used for modeling excess returns. Furthermore, Zhu and Galbraith (2010) apply a generalized asymmetric Student-t distribution to asset pricing and Zhang *et al.* (2011) apply non-Gaussian distributions to study market volatility.

3 The CAPM-IAPD and the CAPM-IAEPD

Under the mean-variance framework of modern portfolio theory, $\mathbb{E}(\varepsilon_{it}) = 0$ and $\mathbb{E}(\varepsilon_{it}^2) = \sigma_{\varepsilon_i}^2$. A random variable (r.v.) ε with mean zero and variance σ_{ε}^2 can be obtained by standardizing a r.v. X , as

$$\varepsilon = \sigma_{\varepsilon} \frac{X - \omega}{\delta}, \quad (4)$$

where σ_{ε} is the standard deviation of ε and X is a r.v. with mean ω and variance δ^2 .

Assume that the random variable X has an arbitrary standard asymmetric power distribution. Specifically, let X be a standard APD random variable in the context of the CAPM-IAPD and a standard AEPD random variable in the context of CAPM-IAEPD. Then, X can

be expressed by a function of the error term, that is

$$\begin{aligned} X &= \omega + \frac{\delta}{\sigma_\varepsilon} \varepsilon \\ &= \omega + \frac{\delta}{\sigma_\varepsilon} (Z - \alpha_M - \beta_M Z_M), \end{aligned} \quad (5)$$

where Z is the r.v. of excess returns and Z_M a non-stochastic predetermined term. As the density of the return variables is of interest, let $f_X(x)$ represent the density of a given asymmetric power distribution, then the PDF of the return variable Z is given by

$$\begin{aligned} f_Z(z) &= f_X(x) \left| \frac{dz}{dx} \right|^{-1} \\ &= f_X \left(\omega + \delta \cdot \frac{Z - \alpha_M - \beta_M Z_M}{\sigma_\varepsilon} \right) \cdot \frac{\delta}{\sigma_\varepsilon}. \end{aligned} \quad (6)$$

The data-generating process (DGP) of the CAPM-IAPD reads

$$\begin{aligned} Z_{it} &= \alpha_{iM} + \beta_{iM} Z_{Mt} + \varepsilon_{it}, & \varepsilon_{it} &\sim \text{IAPD}(\alpha, \lambda, 0, \sigma_{\varepsilon_i}^2) \\ \varepsilon_{it} &= \sigma_{\varepsilon_i} \frac{X - \omega}{\delta}, \\ \mathbb{E}(\varepsilon_{it}) &= 0, \\ \mathbb{E}(\varepsilon_{it}^2) &= \sigma_{\varepsilon_i}^2, \end{aligned} \quad (7)$$

where ε_{it} denotes the IAPD error term with mean 0, variance $\sigma_{\varepsilon_i}^2$, that can be expressed by the standard APD random variable X . According to Komunjer (2007), the mean and variance of X are given by

$$\begin{aligned} \omega &= \mathbb{E}(X) = \frac{\Gamma(2/\lambda)}{\Gamma(1/\lambda)} (1 - 2\alpha) \delta_{\alpha, \lambda}^{-1/\lambda}, \\ \delta^2 &= \text{Var}(X) = \frac{\Gamma(3/\lambda) \Gamma(1/\lambda) (1 - 3\alpha + 3\alpha^2) - \Gamma(2/\lambda)^2 (1 - 2\alpha)^2}{\Gamma(1/\lambda)^2} \delta_{\alpha, \lambda}^{-2/\lambda}, \end{aligned} \quad (8)$$

respectively, where α is the skewness parameter, $\alpha \in (0, 1)$ measures the degree of asymmetry, $\lambda > 0$ is the tail parameter which controls the tail decay, $\delta_{\alpha, \lambda} \equiv \frac{2\alpha^\lambda(1-\alpha)^\lambda}{\alpha^\lambda + (1-\alpha)^\lambda}$ with $\delta_{\alpha, \lambda} \in (0, 1)$, and $\Gamma(\cdot)$ is the Gamma function.

According to Komunjer (2007), the density of X is given by

$$f_X(x) = \begin{cases} \left(\frac{\delta_{\alpha,\lambda}^{1/\lambda}}{\Gamma(1+1/\lambda)} \right) \exp\left(-\frac{\delta_{\alpha,\lambda}}{\alpha^\lambda} |x|^\lambda\right), & \text{for } x \leq 0 \\ \left(\frac{\delta_{\alpha,\lambda}^{1/\lambda}}{\Gamma(1+1/\lambda)} \right) \exp\left(-\frac{\delta_{\alpha,\lambda}}{(1-\alpha)^\lambda} |x|^\lambda\right), & \text{for } x > 0. \end{cases} \quad (9)$$

The DGP of the CAPM-IAEPD is

$$\begin{aligned} Z_{it} &= \alpha_{iM} + \beta_{iM} Z_{Mt} + \varepsilon_{it}, & \varepsilon_{it} &\sim \text{IAEPD}(\alpha, p_1, p_2, 0, \sigma_{\varepsilon_i}^2) \\ \varepsilon_{it} &= \sigma_{\varepsilon_i} \frac{Y - \omega}{\delta}, \\ E(\varepsilon_{it}) &= 0, \\ E(\varepsilon_{it}^2) &= \sigma_{\varepsilon_i}^2, \end{aligned} \quad (10)$$

where ε_{it} denotes the IAEPD error term with mean 0 and variance variance $\sigma_{\varepsilon_i}^2$, can be expressed by the standard AEPD random variable Y . According to Zhu and Zinde-Walsh (2009), the mean and the variance of Y are given by

$$\begin{aligned} \omega &= E(Y) = \frac{1}{B} \left[(1-a)^2 \frac{p_2 \Gamma(2/p_2)}{\Gamma^2(1/p_2)} - \alpha^2 \frac{p_1 \Gamma(2/p_1)}{\Gamma^2(1/p_1)} \right] \\ \delta^2 &= \text{Var}(Y) \\ &= \frac{1}{B^2} \left\{ (1-a)^3 \frac{p_2^2 \Gamma(3/p_2)}{\Gamma^3(1/p_2)} + \alpha^3 \frac{p_1^2 \Gamma(3/p_1)}{\Gamma^3(1/p_1)} - \left[(1-a)^2 \frac{p_2 \Gamma(2/p_2)}{\Gamma^2(1/p_2)} - \alpha^2 \frac{p_1 \Gamma(2/p_1)}{\Gamma^2(1/p_1)} \right]^2 \right\}, \end{aligned} \quad (11)$$

respectively, where α is the skewness parameter, $\alpha \in (0, 1)$, $p_1 > 0$ is the left tail parameter and $p_2 > 0$ the right tail parameter. $B \equiv \alpha K_{EP}(p_1) + (1-\alpha) K_{EP}(p_2) = (\frac{\alpha}{\alpha^*}) K_{EP}(p_1) = (\frac{1-\alpha}{1-\alpha^*}) K_{EP}(p_2)$, where $\alpha^* = \frac{\alpha K_{EP}(p_1)}{\alpha K_{EP}(p_1) + (1-\alpha) K_{EP}(p_2)}$ to ensure continuity and if $p_1 = p_2$, $\alpha^* = \alpha$. $K_{EP}(p) \equiv 1 / [2p^{1/p} \Gamma(1+1/p)]$, where $\Gamma(\cdot)$ is the Gamma function. Based on Zhu and Zinde-Walsh (2009), the PDF of a standard AEPD random variable Y can be defined as

$$f_Y(y) = \begin{cases} \left(\frac{\alpha}{\alpha^*} \right) K_{EP}(p_1) \exp\left(-\frac{1}{p_1} \left| \frac{y}{2\alpha^*} \right|^{p_1}\right), & \text{for } y \leq 0 \\ \left(\frac{1-\alpha}{1-\alpha^*} \right) K_{EP}(p_2) \exp\left(-\frac{1}{p_2} \left| \frac{y}{2(1-\alpha^*)} \right|^{p_2}\right), & \text{for } y > 0. \end{cases} \quad (12)$$

According to Zhu and Zinde-Walsh (2009), the APD and the AEPD are skewed to the right if $\alpha < 1/2$ and to the left if $\alpha > 1/2$, and p_1 controls the left tail while p_2 controls the right

tail. Specifically, $p_1 < 2$ leads to a fat left tail, which becomes fatter for smaller p_1 (the same applies to p_2 regarding the right tail).

4 Maximum Likelihood Estimation

The method of Maximum Likelihood (ML) is used and the BFGS algorithm is employed. This is because the approximation of the Hessian¹ updated by the BFGS algorithm is particularly robust and the BFGS algorithm is preferred over successive iterations (see Brooks, 2008). The log-likelihood for the usual CAPM with normal innovations takes the form

$$L_T(\theta) = \sum_{t=1}^T \log \left(\frac{1}{\sigma_{\varepsilon_i} \sqrt{2\pi}} \exp \left(-\frac{(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})^2}{2\sigma_{\varepsilon_i}^2} \right) \right), \quad (13)$$

where ‘log’ denotes the natural logarithm, the parameter vector $\theta = (\alpha_{iM}, \beta_{iM}, \sigma_{\varepsilon_i})^T$ and $\alpha_{iM} \in \mathbb{R}$, $\beta_{iM} \in \mathbb{R}$, and $\sigma_{\varepsilon_i} > 0$. To maximize the loglikelihood with respect to the parameters in θ , $\hat{\theta}$ is the solution to the problem $\max_{\theta \in \Theta} L_T(\theta)$ where Θ is a parameter set, $\Theta \subset \mathbb{R} \times \mathbb{R} \times \mathbb{R}_+^*$.

In terms of the CAPM-IAPD, log-likelihood function, $L_T(\theta)$ reads as

$$\begin{aligned} L_T(\theta) = & T \log \left(\frac{\delta}{\sigma_{\varepsilon_i}} \left[\frac{\delta_{\alpha, \lambda}^{1/\lambda}}{\Gamma(1 + 1/\lambda)} \right] \right) \\ & + \sum_{t=1}^T \left[-\frac{\delta_{\alpha, \lambda}}{\alpha^\lambda} \left| \omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}} \right|^\lambda \mathbf{1} \left(\omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}} \leq 0 \right) \right. \\ & \left. - \frac{\delta_{\alpha, \lambda}}{(1 - \alpha)^\lambda} \left| \omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}} \right|^\lambda \mathbf{1} \left(\omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}} > 0 \right) \right], \end{aligned} \quad (14)$$

where the notation is consistent with that of Eqs (7)–(9). Since $\alpha_{iM} \in \mathbb{R}$, $\beta_{iM} \in \mathbb{R}$, $0 < \alpha < 1$, $\lambda > 0$ and $\sigma_{\varepsilon_i} > 0$, and $\theta = (\alpha_{iM}, \beta_{iM}, \alpha, \lambda, \sigma_{\varepsilon_i})^T$ is the parameter vector, $\hat{\theta}$ will be the solution to the problem $\max_{\theta \in \Theta} L_T(\theta)$ where Θ is a parameter set, $\Theta \subset \mathbb{R} \times \mathbb{R} \times (0, 1) \times \mathbb{R}_+^* \times \mathbb{R}_+^*$.

¹The confidence interval of the parameter is built by this empirical Hessian in section 6.

Similarly, the log-likelihood function $L_T(\theta)$ of the CAPM-IAEPD is given by

$$\begin{aligned}
L_T(\theta) = & T \log \left(\frac{\delta}{\sigma_{\varepsilon_i}} B \right) \\
& + \sum_{t=1}^T \left[-\frac{1}{p_1} \left| \frac{\omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}}}{2\alpha^*} \right|^{p_1} 1 \left(\omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}} \leq 0 \right) \right. \\
& \left. - \frac{1}{p_2} \left| \frac{\omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}}}{2(1 - \alpha^*)} \right|^{p_2} 1 \left(\omega + \frac{\delta(Z_{it} - \alpha_{iM} - \beta_{iM} Z_{Mt})}{\sigma_{\varepsilon_i}} > 0 \right) \right],
\end{aligned} \tag{15}$$

where the notation is consistent with that of Eqs (10)–(12), the parameter vector is given by $\theta = (\alpha_{iM}, \beta_{iM}, \alpha, p_1, p_2, \sigma_{\varepsilon_i})^T$ and $\alpha_{iM} \in \mathbb{R}$, $\beta_{iM} \in \mathbb{R}$, $0 < \alpha < 1$, $p_1 > 0$, $p_2 > 0$ and $\sigma_{\varepsilon_i} > 0$. Therefore, $\hat{\theta}$ will be the solution to the problem $\max_{\theta \in \Theta} L_T(\theta)$ where Θ is a parameter set, $\Theta \subset \mathbb{R} \times \mathbb{R} \times (0, 1) \times \mathbb{R}_+^* \times \mathbb{R}_+^* \times \mathbb{R}_+^*$.

5 Data description

In studies using US stock returns a broad-based stock index, such as the S&P 500 or the Center for Research in Security Prices (CRSP) market index, is usually taken as the proxy of the market portfolio. In this paper, the EURO STOXX 50 Index is chosen as the proxy of the market portfolio of the European Market. In fact, the EURO STOXX 50 Index is highly correlated with the S&P 500, and the correlation was pegged at 0.865². In addition, the EURO STOXX 50 Index is viewed as Europe's leading Blue-chip index for the Eurozone with currency consistency. Furthermore, the constituents of this index are supersector leaders in the Eurozone with representativeness and diversification³, and the interval effect influences big firms less.

As recommended by the European Money Markets Institute (EMMI), we use the Euro Interbank Offered Rate (Euribor) and the Euro OverNight Index Average (Eonia) as proxies of the risk-free rate to take into account the effects of tax, regulatory, and liquidity conditions for the European market. Besides, Eonia is used as the risk-free rate for the realized daily returns based on the argument by Hull (2011) that investors gradually perceive the overnight

²John W. Labuszewski, Inter-Market Stock Index Spreads. CME GROUP (August 2013): <https://www.cmegroup.com/trading/equity-index/files/intermarket-stock-index-spreads.pdf>.

³The index covers 50 stocks from Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, The Netherlands, Portugal and Spain.

index swap rate⁴ as the risk-free rate for high frequency financial data after the recent financial crisis. As to the rate of return, for the assets that pay dividend regularly, we use the total return index from Datastream (Thomson Reuters) to account for capital appreciation and income. As usual we define the (log) return R_t on an asset at time t as

$$R_t \equiv \log \text{TR}_t - \log \text{TR}_{t-1}, \quad (16)$$

where TR_t is the total return index value at time t and ‘log’ denotes the natural logarithm. The excess return of any asset Z_t ,

$$Z_t \equiv 100 \times (R_t - R_{f,t-1}), \quad (17)$$

where R_t is as defined above and $R_{f,t-1}$ denotes the risk-free rate expressed at time $t - 1$. In the empirical analysis we will use daily, weekly as well as monthly data. Therefore, a period may refer to a trading day, a week or a month depending on the frequency of data used.

Since the sub-prime crisis in 2008 may introduce a large structural break in the specification of the CAPM model, we use data starting from 2009 when the market more or less stabilizes. Therefore, we can focus on the specification of error terms. Table2 summarizes the descriptive statistics of the sample data. All observed values of the skewness are different from 0 and all values of the kurtosis are larger than 3, although the p -values of Jarque-Bera test shows that there are 7 weekly return series and 20 stocks’ monthly total returns that cannot be distinguished from being normally distributed at the 5% significance level. Therefore, Table 2 shows evidence that most of the return series do not follow a normal distribution, in particular at higher data frequencies.

[Table 2 about here.]

6 Empirical results and goodness of fit

This section reports the empirical results. Goodness of fit is measured by the Akaike Information Criterion (AIC), density plots of the residuals and the estimated parametric distribution, and corresponding distribution tests.

⁴Nevertheless, a well-discussed convention based on a comprehensive dimension for estimating the risk-free rate seems to be a hole in the theoretical literature.

6.1 Estimates of coefficients and confidence intervals

To address Question (i), Tables 3 to 5 show the coefficient estimates and whether they are statistically significant. Since minus the inverse of the Hessian is a way to estimate the covariance matrix of the ML estimator, evaluated at the vector of ML estimates (see Davidson and MacKinnon, 2004), the standard error (SE) for the j th estimate can be calculated as

$$SE_j = \sqrt{-(e_j^T H^{-1} e_j)}, \quad (18)$$

where e_j is a vector and consists of zeros and a single one on the j th position and H denotes the Hessian at $\hat{\theta}$. Due to the asymptotic normality and consistency of the MLE⁵, an empirical 95% confidence interval (CI) of parameter j is given by

$$CI_j = (e_j^T \hat{\theta} e_j - 1.96 \cdot SE_j, e_j^T \hat{\theta} e_j + 1.96 \cdot SE_j), \quad (19)$$

where $\hat{\theta}$ denotes the solution⁶ to the problem $\max_{\theta \in \Theta} L_T(\theta)$.

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

Following the commonly used definition, we say that the CAPM is ‘dead’ or rejected for a stock unless the estimation generates both a significant $\hat{\beta}_{iM}$ and a insignificant $\hat{\alpha}_{iM}$. The results show that of our 50 stocks in the market portfolio, the CAPM is ‘alive’ for 27 stocks in the traditional CAPM with normal error terms, 30 in CAPM-IAPD and 35 in CAPM-IAEPD with daily data. For weekly data, these numbers are only 10 for the traditional CAPM, 36 for CAPM-IAPD and 30 for CAPM-IAEPD, while for monthly data, these numbers are only 6 for traditional CAPM, 32 for CAPM-IAPD and 21 for CAPM-IAEPD. To conclude, the assumption of IAPD errors makes rejection of the CAPM substantially less likely, in particular for weekly and monthly data.

⁵See Zhu and Zinde-Walsh (2009) for the MLE properties of the AEPD and Komunjer (2007) for the MLE properties of the APD.

⁶The command ‘fmincon’ is used for the optimization procedure.

6.2 Goodness of fit

The AIC values, which we use to address Questions (ii) and (iii), are presented in Table 6. As defined by Akaike (1974)

$$AIC = 2k - 2L_T(\hat{\theta}), \quad (20)$$

where $L_T(\hat{\theta})$ is the value of the log-likelihood function, evaluated at $\hat{\theta} = \operatorname{argmax}_{\theta \in \Theta} L_T(\theta)$ and k is the number of components of $\hat{\theta}$. The smaller the AIC value, the better the model.

[Table 6 about here.]

For Question (ii), the evidence from the AIC values indicates that the generalized CAPM with non-normally distributed errors performs better than the CAPM with normally distributed errors, especially for the high frequency data⁷. In particular, all the AIC values from the CAPM are larger than the generalized CAPMs (the CAPM-IAPD and the CAPM-IAEPD) for daily returns, and 48 out of 50 values obtained from the CAPM are larger with regard to weekly returns. It seems that the AIC values are not always largest in the CAPM for monthly data. However, the average value of the AIC for the CAPM is the largest regardless of data frequency, so in that sense the CAPM is outperformed. As for Question (iii), first, the estimated AIC does not lend support to the statement that CAPM-IAEPD performs better than CAPM-IAPD. In other words, the inclusion of two tail parameters does not add much explanatory power to the model. The average AICs in these two model specifications are very close to each other, and the order is not fixed. In general, the average AIC is substantially larger in the traditional CAPM than in CAPM-IAPD and CAPM-IAEPD, while the difference between the latter two is less pronounced. These results provide unambiguous evidence that the generalized CAPM with IAPD errors provides better fitness measure than the traditional CAPM. Secondly, to visualize the actual and a given theoretical distribution, the probability density function (PDF) of the actual random variable can be estimated by the Kernel Density Estimation in light of Silverman (1986). The Gaussian Kernel is used for the density estimation and the bandwidth h , $h = 1.06 \times \operatorname{Std}(\hat{u}_t) \times T^{-1/5}$, where Std denotes the standard deviation and T denotes the number of time periods. The black solid line represents the empirical distribution and the red dotted line represents the given distribution. Com-

⁷However, due to the method of Maximum Likelihood the AIC values for monthly returns are restricted by the sample size.

pared with the normal, the APD and the AEPD display a very close fit, although the values of the tail parameters in the CAPM-IAPD and the CAPM-IAEPD are similar ($\hat{\lambda} = 0.8851$ and $\hat{p}_1 = 0.8973$, $\hat{p}_2 = 0.8824$, see Appendix A). Without loss of generality, some typical and representative density plots are presented in this paper⁸. Fig. 1 suggests that the generalized CAPM under the IIAPD errors assumption is more realistic than the CAPM under the normality errors assumption. Similarly, there is no remarkable evidence showing that the CAPM-IAEPD is better than the CAPM-IAPD.

[Figure 1 about here.]

Thirdly, a distribution test is performed to investigate Questions (ii) and (iii). The empirical 95% confidence interval (see Section 6.1) plays an important role in determining the statistical significance as well as the distribution test. The reason is that the AEPD and the APD can be reduced to the normal distribution, provided that $\alpha = 0.5$, $p_1 = p_2 = 2$ and $\alpha = 0.5$, $\lambda = 2$ respectively. As a result, if 0.5 lies outside the CI of $\hat{\alpha}$ (i.e. $\alpha \neq 0.5$ at the 95% significant level) or 2 lies outside the CI of $\hat{\lambda}$ (for the CAPM-IAPD) or of $\hat{p}_1$ or $\hat{p}_2$ (for the CAPM-IAEPD), the null hypothesis of a normal distribution can be rejected at the 5% significance level and thus the result is statistically significant. Besides, the CIs can test some non-normal distributions nested in these two distributions, e.g. the Laplace distribution. This is because these two can be reduced to the Laplace distribution if $\alpha = 0.5$, $p_1 = p_2 = 1$ and if $\alpha = 0.5$, $\lambda = 1$. Table 7 summarizes the distribution test results.

[Table 7 about here.]

Table 7 shows that the generalized CAPM can capture the asymmetry of return distributions. The skewness parameter α is statistically significant among 50 return series regardless of the data frequency. When modeling daily returns, there are 16 $\hat{\alpha}$ s statistically significant corresponding to the CAPM-IAPD, comparable to the case of the CAPM-IAEPD (18 $\hat{\alpha}$ s). However, the asymmetry of 23 weekly return series and 17 monthly return series can be captured by the CAPM-IAEPD, while these numbers are respectively 3 and 9 in the case of the CAPM-IAPD; thus the CAPM-IAEPD is more sensitive to fit asymmetry, especially when modeling weekly and monthly returns. To sum up, the generalized CAPM with IIAPD

⁸The others are available from the authors upon request.

errors outperforms the traditional CAPM in the sense of accommodating asymmetry. Table 7 also demonstrates that the generalized CAPM can describe the tail behavior and measure the degree of fat tailedness of return distributions. The vast majority of tail parameters are statistically significant when performing the normal distribution test for daily and weekly returns. As a result, a number of the daily returns and weekly returns are not normally distributed and are characterized by asymmetry and fat tails. Even for monthly returns, which are usually considered to be normally distributed, the numbers of the last row in Table 7 indicate that some of them are not normally distributed (14 out of 50 estimated by the CAPM-IAPD; 25 out of 50 by the CAPM-IAEPD). Moreover, the CAPM-IAEPD is more sensitive in terms of the number of rejections of the null hypothesis (for the normal distribution), because the number of 25 is larger than the number of 14. However, due to the small number of observations (only 60 monthly returns) versus the econometric methodology (the method of ML), a statement that two tail parameters are better is controversial. Nonetheless, based on the evidence provided by daily and weekly returns, the CAPM-IAPD and the CAPM-IAEPD are comparable according to the normal distribution test. From the above it can be concluded that the generalized CAPM with IAPD errors outperforms the CAPM since the null hypothesis that the error terms are normally distributed can be rejected several times.

Obviously, the tail parameter estimates are concentrated around 1 and the skewness parameter estimates are distributed around 0.5 (see Appendix A, which exports these estimates and their 95% confidence intervals); this suggests that the returns might be distributed according to the Laplace distribution. Therefore, the Laplace distribution test is conducted as stated above. The last two columns of Table 7 summarize the result. The assumption of IAPD errors is more relevant owing to this significant result. In terms of the daily returns, there is no distinct difference between the CAPM-IAPD and the CAPM-IAEPD because the corresponding numbers of significant results are comparable. For instance, the number of significant skewness parameters is 16 for the CAPM-IAPD, and then the number becomes 18 for the CAPM-IAEPD. Similarly, there are 26 daily return series significant according to the CAPM-IAPD and this number becomes 28 when estimated by the CAPM-AEPD. For weekly returns, more series estimated by the CAPM-IAEPD (H_0 is rejected 31 times) do not follow the Laplace distribution at the 5% significance level. This is mainly caused by the

presence of skewness parameters; there are 3 significant skewness parameters estimated by the CAPM-IAPD while this number amounts to 23 when estimated by the CAPM-IAEPD. Likewise, there are more significant parameter estimates estimated by the CAPM-IAEPD when modeling monthly returns. As argued by Zhu and Zinde-Walsh (2009), the APD is a sort of SEPD and the skewness parameter in the SEPD cannot be rich enough to capture all of the asymmetry of distributions of returns, especially the asymmetry in the tails, because the SEPD's left tail is always thinner than its right tail. Hence, for the in-sample weekly and monthly returns, the AEPD might better capture the characteristics of financial data, like asymmetry, heavy tails, and leptokurtic or fat-tails. However, a backtest of portfolio performance on a monthly basis can show us some evidence from a pragmatic point of view.

7 Application to portfolio strategy and a simulated backtest

The analysis above indicates that the IAPD errors assumption outperforms the NID assumption in the CAPM estimation. This section studies whether the specification of the error distribution also generates different levels of profitability in portfolio construction. The minimum variance portfolio (MVP), the lowest point on the efficient frontier, is applied to construct a portfolio strategy for the following reasons: (1) The MVP is very popular among investors, because it performs well and is easy to apply. (2) Although MVPs are typically heavily concentrated in the lowest volatility stocks (e.g., DeMiguel *et al.*, 2009), low volatility stocks come with higher returns (see e.g., Ang *et al.*, 2006; Bali and Cakici, 2008). Hence, many investors who wish to invest in low-risk stocks believe that the MVPs concentrated in low volatility stocks will perform well. (3) The only required inputs in the optimization of the MVPs are correlations and volatilities, i.e. only depending on risk (see e.g., Amenc and Martellini, 2002); therefore, the systematic risk, quantified by β_{iM} can be employed. (4) It is well known that estimating the expected returns is difficult, and the composition of the MVP is not affected by expected returns.

7.1 Portfolio strategy

From an active portfolio management point of view, allocating money into the constituents of a broad market index is a smart approach to reduce the likelihood of excessive trading and

the liquidity issue of offsetting positions. As mentioned above, the constituents of the EURO STOXX 50 index are Europe's leading Blue-chip stocks and supersector leaders. Assume that investors allocate all money into the European stock market and wish to minimize risk, namely, invest in the MVP. The portfolio construction, i.e. the asset allocation problem, involves the maximization of a quadratic function of the decision variables, subject to a set of linear constraints, some of which are inequalities (see, e.g., Markowitz, 1956; Sharpe, 1963; Sharpe, 1970; Sharpe, 1971; Elton *et al.*, 1978). However, according to Sharpe (1978), the approaches developed by Sharpe (1963, 1970, 1971) and by Elton *et al.* (1978) for a simplified model hardly accommodate a different model of covariance relationships. Therefore, the algorithm developed by Sharpe (1978) is employed to construct MVP, exploiting the simplicity of the constraints but allowing as much complexity as desired in the covariance matrix. According to Sharpe (2007b), in terms of the MVP, an investor who wishes to minimize risk will have a risk tolerance of zero, no matter how much expected return is sacrificed in the process. Thus, the asset allocation problem of the MVP in terms of relative risk aversion can be written as

$$u = x^T - 0.5 \text{rra} (x^T C x), \quad \text{subject to :} \quad \text{sum}(x) = 1, lb \leq x \leq ub, \quad (21)$$

where 'rra' denotes relative risk aversion, x is a $(n \times 1)$ vector of decision variables, the weights of holdings to represent a portfolio, e denotes a $(n \times 1)$ vector of expected returns (the mean of the historical returns), lb denotes a $(n \times 1)$ vector of lower bounds, and ub denotes a $(n \times 1)$ vector of upper bounds (all elements of x are non-negative if short sales are not allowed, i.e. $lb = 0$ and $ub = \iota$), C denotes the $(n \times n)$ covariance matrix defined as

$$C = \begin{cases} \sigma_{ij} = \sigma_i^2, & \text{if } i = j \\ \sigma_{ij} = \beta_{iM} \beta_{jM} \sigma_M^2, & \text{if } i \neq j, \end{cases} \quad (22)$$

where σ_i^2 denotes the variance of security i 's return, σ_M^2 denotes the variance of excess return on the market portfolio. β_{iM} and β_{jM} are yielded by the CAPMs. Based on this security selection process, this portfolio strategy can, to some extent, realize the optimization according to the optimally selected securities.

7.2 Backtest results

Tracing the historical data of the EURO STOXX 50 index and its constituents as long as possible (i.e. from 14 July 2001), there are 49 stocks that can be included into the portfolio (because the data of GDF SUEZ is only available from 29 July 2005). The investment horizon is approximately five years (59 months), from 14 June 2009 to 14 April 2014. From the perspective of investment policy statement, for convenience some assumptions are made. Ideally the investment constraints exclude the investor’s taxation level and liquidity needs. Moreover, there are no transaction costs and investor’s net cash inflow of the portfolio is 0 (i.e. there are no funding payments and no withdrawals). Due to restrictions on short sales for most investors, short sales are not allowed here. Rebalancing is done on a monthly basis to reduce the likelihood of excessive trading. The data used for estimating β_{iM} and β_{jM} are disposed as described in Section 4. Assume that the market value of the portfolio at the beginning is the amount of money that the investor wishes to allocate into the MVP and there is no cash position at the beginning. The initial value V_t of the portfolio, $V_t = TR_t \times x_{t-1}$ where TR_t is the $(1 \times n)$ vector of the total return index value at period t , x_{t-1} is the vector of weights determined at period $t - 1$, based on the historical data till period $t - 1$ ($t \geq 96$ if start from July 14th, 2001).

[Table 8 about here.]

Table 8 reports the result of the backtest for performance evaluation. The methodology of average monthly total return, the Sharpe Ratio and corresponding annualized calculations are in line with Morningstar (2005). The Standard Deviation (a measure of risk) is estimated by the sample standard deviation. As a robustness check we also tried using a HAC estimator of the variance, to take into account possible autocorrelation in returns; this gave very similar results. The Sharpe Ratio, the ratio of the difference between expected return and the riskless rate to standard deviation, is another risk-adjusted performance measure. Normally, the greater the Sharpe Ratio, the better the performance (a given number greater than 1 is categorized as good) is. The Information Ratio is a measure of the risk-adjusted return. For quants, the annualized Information Ratio is calculated by $(\text{annualized excess return})/(\sqrt{12} \times \text{monthly } \sigma_{te})$, where excess return is relative to the benchmark portfolio’s annualized return, and σ_{te} denotes tracking error, a measure of how closely a portfolio follows the index to which

it is benchmarked. The manager's skill is considered as good if the Information Ratio is larger than 0.5, and implies that an investment historically delivered 0.5% in excess returns for every unit of risk above the benchmark. As discussed by Jensen (1968), Jensen's Alpha is the intercept term, α_j , in $Z_{jt} = \alpha_j + \beta_j Z_{Mt} + u_{jt}$, where Z_{jt} denotes the excess return on an arbitrary actual portfolio j , which is another risk-adjusted measure to evaluate the abnormal return of a portfolio, and the higher the better. Finally, the Treynor Ratio T , $T = Z_j/\beta_j$, where Z_j represents the annualized portfolio return in excess of the risk-free rate, is a measure of the return per unit of systematic risk, the higher the better as well.

From the perspective of return/risk analysis, these three portfolio strategies developed in this paper outperform the benchmark portfolio. Regardless of the realized total return, the average monthly total return, or the annualized total return, Strategy B (based on CAPM-IAPD) is superior to Strategy A (based on traditional CAPM), Strategy C (CAPM-IAEPD) and the benchmark portfolio. In contrast, Strategy C is the superb strategy in the sense of standard deviation and annualized standard deviation, with the greatest annualized Sharpe Ratio (1.1218). In addition, Strategy B, with an annualized Sharpe Ratio of 1.1185 also outperforms the CAPM based strategy. A Sharpe Ratio that is greater than 1 implies good risk-adjusted performance. Therefore, utilizing the IIAPD errors assumption for portfolio construction can lead to 'good' risk-adjusted performance. Compared with these three strategies, the benchmark portfolio has a wider range of monthly returns (from -14.31% to 13.34%). Besides a strategy's standard deviation, drawdown is important because it represents loss. The Maximum Drawdown (MD) describes the worst circumstance of loss occurred during the course of trading, $MD = \max(D_i - D_j)/D_i$ where D_i is the portfolio value in period i , D_j is the portfolio value in period j , and $i > j$. The lower MD represents the less loss in percentage. Due to the assumption about the initial value of investment made above, the comparison of MD makes little sense. However, we can compare it during the whole investment horizon to obtain some related conclusion. Fig.2 illustrates the investment value in the case of different strategy and the corresponding drawdown in percentage (take negative value for comparison). These active strategies can hedge the downside market risk to a certain extent.

[Figure 2 about here.]

Regarding the EURO STOXX 50 as the market portfolio, the portfolio in the case of Strategy B is first-rank in terms of relative performance. It has the greatest excess return

(8.25% relative to EURO STOXX 50), statistically the largest significant Jensen's Alpha (1.02%), the greatest annualized Information Ratio (0.54), and the highest Treynor Ratio (0.38). A Jensen's Alpha of 1.02 indicates that manager's picks added returns versus the benchmark are 1.02%. An Information Ratio of 0.54 (greater than 0.5) identifies that the performance is 'good' and reflects that an investment historically delivered 0.54% in excess returns for every unit of risk above the benchmark. A Treynor Ratio of 0.38 means that the return earned in excess of a riskless investment per unit systematic risk is 0.38%. Since all R-squared values are lower than 0.5 and the percentage of portfolio's movements that can be explained by movements in EURO STOXX 50 is low (less than 0.7), each of the portfolios does not act much like the benchmark portfolio. Overall, these three portfolios are defensive, especially the portfolio using Strategy C (with the lowest beta, 0.46). In general, the frequently used performance measures (the Sharpe Ratio, Jensen's Alpha, and the Treynor Ratio) indicate that the portfolios constructed using the IIAPD errors outperform the one using normally distributed errors (Strategy A). The positive alpha of Strategy B (1.02%) or C (0.84%) is statistically significant and larger than that of Strategy A. Strategy B and C also have higher Treynor Ratios (both 0.38 and 0.34 are larger than 0.32) and Sharpe Ratios (both 1.1185 and 1.1218 are larger than 0.9986).

8 Conclusion

The modeling of non-normally distributed returns attracts more and more attention from academia and policy makers after the recent financial crisis (Thomas and Gup, 2010). This paper proposes modeling asymmetric and fat-tailed distributed returns using a family of non-Gaussian distributions. By assuming IIAPD errors, the CAPM is generalized to two new specifications: CAPM-IIAPD and CAPM-IAEPD.

The application of the generalized CAPM models to the European stock market from 2009 to 2013 suggests that the assumption of IIAPD errors has very nice properties. First, it makes the rejection of CAPM much less likely than under the assumption of normally distributed errors. Second, the generalized CAPMs outperform the traditional CAPM with normally distributed errors in terms of AIC values, density plots through the Kernel Density Estimation, and related distribution tests, in particular for higher frequency data. Third, the generalized CAPMs under the IIAPD errors assumption are more realistic in the sense

of accommodating the asymmetry and fat tails of return distributions. Their density plots display a very close fit between actual residuals and the given distributions.

Meanwhile, there is little evidence that CAPM-IAEPD outperforms the CAPM-IAPD model in terms of the Akaike Information Criterion, though AEPD is supposed to be able to capture a larger fraction of the asymmetry of the distributions of weekly and monthly returns, especially the asymmetry in the tails (Zhu and Zinde-Walsh, 2009). The advantage of CAPM-IAPED may be more pronounced with larger samples, e.g. stocks in the S&P500 index or in the FTSE Global Equity Index Series (GEIS), however, this investigation is beyond the scope of this paper.

As stated by Zeckhauser and Thompson (1970), the incorporation of IIAPD errors causes a substantial change on the estimated coefficients of the regression. We propose a portfolio strategy to apply this notion. The backtest result shows that the portfolios constructed by using the IIAPD errors can outperform the portfolio employing the normality errors in the sense of commonly used performance measures: the Sharpe Ratio, Jensen's Alpha, and the Treynor Ratio. This result confirms that the IIAPD error does not only improve the theoretical modeling of asset prices, but may also help to construct more profitable trading strategies for the industry.

Maximum likelihood estimation performs well when the sample size is large. Hence, from the sample size point of view, higher frequency returns are preferable within the same time frame. This leaves open the question what effects the frequency (daily, weekly or monthly) has on the portfolio returns. For finite samples, the bootstrap can be used to assess the statistical stability of these outcomes. Thus, some bootstrapping methods can be used for investigating the difference between using the IIAPD errors and the normality. For instance, the Bias-corrected and accelerated (BC_a) method developed by DiCiccio and Efron (1996) and the corresponding BC_a confidence intervals. This open question requires further research.

Besides, these models can be used for a broader data set and applied to other asset classes, such as bonds, other fixed income securities, and real estate. Last but not least, these distributions can be applied to investigate the pricing of financial derivatives, e.g., options, as well as the corresponding investment strategies.

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Appendix A

Parameter estimates and confidence intervals for the distribution test

The tables in this Appendix present the estimates of the parameters that describe the non-normality of the distributions (the skewness parameter α and the tail parameters λ or p_1 and p_2). CI_j denotes the empirical 95% confidence interval of the estimate j . In terms of the skewness parameter α , which can accommodate for the asymmetry, the sign * stands for statistically significant, i.e., 0.5 lies outside of $CI_{\hat{\alpha}}$. According to Zhu and Zinde-Walsh (2009), the parameter α determines that the distribution skews to the right if $\alpha < 0.5$ and to the left if $\alpha > 0.5$, and $p < 2$ leads to a fat tail, which gets fatter for smaller p . Therefore, if 2 lies outside $CI_{\hat{\lambda}}$, $CI_{\hat{p}_1}$, or $CI_{\hat{p}_2}$, the tail parameter can capture the fat tail of return distribution. Since different distribution test has different tail parameter(s)'s confidence intervals, we did not label the significance of tail parameters.

[Table 9 about here.]

[Table 10 about here.]

[Table 11 about here.]

[Table 12 about here.]

[Table 13 about here.]

[Table 14 about here.]

Figures

Figure 1: The PDF plots of actual residuals and the given distribution (ALLIANZ, daily returns, from 2/1/2009 to 3/1/2014)

Figure 2: Comparison of historical drawdowns

Tables

Table 1: Special cases of the APD and the AEPD.

Distributions	Parameters in APD	Parameters in AEPD
Normal	$\alpha = 1/2, \lambda = 2$	$\alpha = 1/2, p_1 = p_2 = 2$
GPD	$\alpha = 1/2, \lambda > 0$	$\alpha = 1/2, p_1 = p_2$
Laplace	$\alpha = 1/2, \lambda = 1$	$\alpha = 1/2, p_1 = p_2 = 1$
Asymmetric Laplace	$\alpha \neq 1, \lambda = 1$	$\alpha \neq 1/2, p_1 = p_2 = 1$
Two-piece normal	$\alpha \neq 1/2, \lambda = 2$	$\alpha \neq 1/2, p_1 = p_2 = 2$
SEPD		$\alpha \neq 1/2, p_1 = p_2$

The GPD allows flexibility in modeling the tail behavior. According to Komunjer (2007), the GPD can be the uniform distribution when $\lambda = \infty$, be short-tailed distributions when $2 \leq \lambda < \infty$, and be the fat-tailed ones when $0 < \lambda < 2$. According to Zhu and Zinde-Walsh (2009), the APD is a sort of SEPD (Skew Exponential Power Distribution) due to the quantification of asymmetry; however, the AEPD corresponds with the SEPD if $p_1 = p_2$.

Table 2: Discriptive statistics of log returns on EURO STOXX 50 and on its 50 constituents over five years

	Panel A: Daily return			Panel B: Weekly return			Panel C: Monthly return		
	from 2/1/2009 to 3/1/2014	from 2/1/2009 to 3/1/2014	from 2/1/2009 to 3/1/2014	from 2/1/2009 to 3/1/2014	from 2/1/2009 to 3/1/2014	from 2/1/2009 to 3/1/2014	from 1/2009 to 12/2013	from 1/2009 to 12/2013	from 1/2009 to 12/2013
	Sk.	Kurt.	Prob.	Sk.	Kurt.	Prob.	Sk.	Kurt.	Prob.
EURO STOXX 50	0.0616	5.9714	0.0000	-0.4616	4.9730	0.0000	-0.8186	5.6805	0.0000
AIR LIQUIDE	0.2951	5.3009	0.0000	-0.2991	3.7947	0.0045	0.0074	4.4052	0.0847
AIRBUS GROUP	-0.0856	5.0470	0.0000	-0.0220	4.5850	0.0000	-0.5776	4.7551	0.0040
ALLIANZ(XET)	0.0913	7.3102	0.0000	-0.1116	5.9164	0.0000	-0.1885	4.7057	0.0221
ANHEUSER BUSCH INBEV	0.1090	6.8945	0.0000	0.1058	4.8449	0.0000	0.8421	4.2764	0.0038
ASML HOLDING	0.1608	4.1224	0.0000	-0.3035	3.5362	0.0279	-0.5478	3.3693	0.1881
ASSICURAZIONI GENERALI	0.0875	5.5597	0.0000	-0.3816	3.8715	0.0007	-0.9150	5.7328	0.0000
AXA	0.1766	10.0268	0.0000	-0.5043	7.1913	0.0000	-1.2351	9.6548	0.0000
BANCO SANTANDER	0.5785	9.0338	0.0000	-0.1258	3.9170	0.0072	0.1307	4.5933	0.0384
BASF(XET)	0.2700	5.2173	0.0000	0.0626	4.4234	0.0000	-0.6024	5.1457	0.0005
BAYER(XET)	-0.0510	6.7235	0.0000	-0.1158	3.4598	0.2354	-0.8487	4.7446	0.0006
BBV ARGENTARIA	0.4689	7.6930	0.0000	-0.2712	4.0840	0.0003	-0.1679	4.8006	0.0151
BMW(XET)	0.1496	5.9912	0.0000	0.0493	3.9183	0.0095	0.1542	4.1121	0.1892
BNP PARIBAS	0.6275	9.3714	0.0000	0.2460	7.6167	0.0000	0.3785	4.5714	0.0223
CARREFOUR	0.2865	4.3608	0.0000	-0.1395	4.0145	0.0024	-0.3086	3.9733	0.1901
CRH DUB	-0.1828	6.6617	0.0000	-0.2919	4.3250	0.0000	-0.6461	3.5661	0.0831
DAIMLER(XET)	0.2404	5.9642	0.0000	-0.2787	4.5601	0.0000	-0.0773	4.7718	0.0192
DANONE	-0.1024	4.6780	0.0000	-0.2981	4.0697	0.0003	-0.5826	3.0018	0.1832
DEUTSCHE BANK(XET)	0.4858	8.3499	0.0000	0.3953	6.2084	0.0000	1.1714	7.0347	0.0000
DEUTSCHE POST(XET)	0.0799	5.4619	0.0000	-0.3387	4.7162	0.0000	-1.2428	8.9574	0.0000
DEUTSCHE TELEKOM(XET)	0.3247	7.6724	0.0000	0.0478	3.6487	0.0957	0.0956	3.0079	0.9553
E.ON(XET)	-0.2726	7.0589	0.0000	-0.3194	5.2088	0.0000	-0.1940	3.5715	0.5508
ENEL	-0.2163	4.8145	0.0000	-0.3304	3.4298	0.0337	-0.7189	4.4606	0.0052
ENI	-0.1502	6.0236	0.0000	-1.4288	11.7019	0.0000	-1.6701	9.2087	0.0000
ESSILOR INTL	0.0127	5.3871	0.0000	-0.1092	3.5386	0.1582	-0.2577	3.1489	0.6978
GDF SUEZ	-0.1463	5.9872	0.0000	-0.0895	3.7857	0.0289	-0.3802	2.9806	0.4851
IBERDROLA	0.0759	7.2581	0.0000	-0.2422	3.9365	0.0023	-0.2691	3.2916	0.6261
INDITEX	0.1236	6.5094	0.0000	-0.3414	5.5630	0.0000	-0.4718	4.5043	0.0194
ING GROEP	0.1799	9.4022	0.0000	-0.5989	7.0294	0.0000	-0.8092	9.7542	0.0000
INTESA SANPAOLO	-0.1969	6.7454	0.0000	-0.7072	5.5960	0.0000	-0.1894	4.4073	0.0703
L'OREAL	0.1432	4.6142	0.0000	-0.1944	5.2606	0.0000	-0.5837	4.1969	0.0304
LVMH	0.1062	6.3517	0.0000	0.1433	3.9796	0.0034	0.0044	2.9648	0.9984
MUENCHENER RUCK(XET)	-0.0414	7.9765	0.0000	-0.2123	7.2215	0.0000	-1.0818	6.7718	0.0000
ORANGE	0.1372	4.7477	0.0000	0.0389	3.1625	0.8377	0.3584	3.3833	0.4378
PHILIPS ELTN KONINKLIJKE	0.0944	5.0956	0.0000	-0.0029	3.4804	0.2837	-0.7400	6.7715	0.0000
REPSOL YPF	0.1961	6.0883	0.0000	-0.5155	4.4335	0.0000	-0.8524	4.3232	0.0030
RWE(XET)	-0.1855	6.3848	0.0000	-0.0639	4.3280	0.0001	-0.3374	3.6638	0.3262
SAINT GOBAIN	-0.1857	6.3245	0.0000	-0.2644	4.5544	0.0000	-0.7750	5.4861	0.0000
SANOFI	-0.1711	5.2049	0.0000	-0.9564	6.6454	0.0000	-0.5388	3.2087	0.2217
SAP(XET)	-0.4580	5.8075	0.0000	-0.3859	3.7148	0.0024	0.0310	3.2823	0.9008
SCHNEIDER ELECTRIC	0.1336	4.7638	0.0000	-0.0639	3.8900	0.0121	-0.7510	4.6776	0.0018
SIEMENS(XET)	-0.1266	6.8107	0.0000	-0.3580	4.9728	0.0000	-0.8352	6.5379	0.0000
SOCIETE GENERALE	0.1665	7.6073	0.0000	-0.1421	4.4022	0.0000	0.1733	4.2605	0.1181
TELEFONICA	0.1279	6.8824	0.0000	0.0327	3.4303	0.3555	0.0255	2.2951	0.5356
TOTAL	0.0608	5.7416	0.0000	-0.0723	3.2455	0.6421	-1.1868	4.2732	0.0001
UNIBAIL RODAMCO	0.0368	5.5696	0.0000	-0.5987	5.1569	0.0000	-0.3618	4.2982	0.0632
UNICREDIT	-0.1563	6.7061	0.0000	-1.1639	8.8242	0.0000	0.3982	6.4558	0.0000
UNILEVER CERTS	0.3115	6.7401	0.0000	0.0120	4.5573	0.0000	-0.7735	5.5590	0.0000
VINCI	0.1254	5.2638	0.0000	-0.3472	4.1257	0.0001	-0.7045	4.9382	0.0008
VIVENDI	-0.0709	4.8741	0.0000	-0.3428	4.2161	0.0000	-0.7709	3.4228	0.0410
VOLKSWAGEN PREF(XET)	-0.0750	6.8137	0.0000	0.1390	4.1186	0.0007	0.6517	5.3641	0.0001

Panel A has 1306 observations, Panel B has 262 observations, and Panel C has 60 observations. Sk. denotes skewness, Kurt. denotes kurtosis, and Prob. is the values of Jarque-Bera test with the probabilities.

Table 3: Estimation results for coefficients

	Daily returns					
	CAPM-IAPD		CAPM-IAEPD		CAPM	
	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$
AIR LIQUIDE	0.0317	0.7811	0.0295	0.7922	0.0283*	0.7893
AIRBUS GROUP	0.0949	0.7968	0.0943	0.7972	0.0948*	0.7828
ALLIANZ(XET)	0.0216*	1.0595	0.0221	1.0640	0.0190	1.1716
ANHEUSER-BUSCH INBEV	0.1002*	0.5878	0.1005	0.5880	0.1005*	0.5905
ASML HOLDING	0.1035*	0.7191	0.1043	0.7189	0.1040*	0.7221
ASSICURAZIONI GENERALI	-0.0400	1.0925	-0.0406*	1.0911	-0.0414	1.1140#
AXA	-0.0121*	1.4411	-0.0119*	1.4466	-0.0180	1.6383
BANCO SANTANDER	-0.0134	1.3244	-0.0158	1.3202	-0.0180	1.3914
BASF(XET)	0.0602*	1.0085	0.0590*	1.0080	0.0584*	1.0313
BAYER(XET)	0.0479	0.8263	0.0495	0.8314	0.0507*	0.7966
BBV.ARGENTARIA	-0.0197	1.3659	-0.0226	1.3523	-0.0244	1.4102
BMW(XET)	0.0777	0.9706	0.0788	0.9672	0.0772*	1.0219
BNP PARIBAS	0.0117	1.5621#	0.0106	1.5585#	0.0066	1.6742
CARREFOUR	-0.0052	0.9105#	-0.0056	0.9021	-0.0062	0.9134
CRH(DUB)	-0.0208	1.2056	-0.0135	1.2144	-0.0112*	1.1596
DAIMLER	0.0395*	1.1225	0.0380	1.1217	0.0371*	1.1610
DANONE	0.0058	0.5625	0.0076	0.5665	0.0082	0.5476
DEUTSCHE BANK(XET)	-0.0236*	1.3645	-0.0158*	1.3668	-0.0204	1.5251
DEUTSCHE POST(XET)	0.0485	0.8484	0.0487*	0.8480	0.0478*	0.8739
DEUTSCHE TELEKOM(XET)	0.0171*	0.6226	0.0172	0.6226	0.0176	0.6113
E.ON(XET)	-0.0645	0.8709	-0.0642	0.8659	-0.0646*	0.8736
ENEL	-0.0212	0.9292	-0.0213	0.9314#	-0.0215*	0.9392
ENI	-0.0026	0.8776	-0.0027	0.8779	-0.0032	0.8927
ESSILOR INTL.	0.0521	0.3964	0.0529	0.3959	0.0523	0.4205
GDF SUEZ	-0.0616	0.9319	-0.0616	0.9319	-0.0601*	0.8839
IBERDROLA	-0.0359	0.9235	-0.0351	0.9278	-0.0367	0.9825
INDITEX	0.0838*	0.6856	0.0847	0.6859	0.0847*	0.6862
ING GROEP	-0.0198	1.7323	-0.0131	1.7333#	-0.0187	1.9298
INTESA SANPAOLO	-0.0640	0.7811	-0.0613	1.5783	-0.0632*	1.6522
L'OREAL	0.0365	0.7968	0.0378	0.6786	0.0386	0.6531
LVMH	0.0524*	1.0595	0.0518*	0.9712	0.0526	0.9541
MUENCHENER RUCK.(XET)	0.0185	0.5878	0.0180	0.7699	0.0165	0.8181
ORANGE	-0.0545*	0.7191	-0.0548	0.7131	-0.0540*	0.6963
PHILIPS ELTN.KONINKLIJKE	0.0330*	1.0925	0.0333	0.9908#	0.0330*	0.9960
REPSOL YPF	0.0021	1.4411	-0.0001	1.0249	-0.0006	1.0458
RWE(XET)	-0.0708*	1.3244	-0.0699	0.7542	-0.0721*	0.8255
SAINT GOBAIN	-0.0123*	1.0085	-0.0128	1.3642	-0.0132	1.3699
SANOFI	0.0302*	1.5819	0.0320	0.6925#	0.0337*	0.6409
SAP(XET)	0.0513	0.6786	0.0542	0.5493#	0.0539*	0.5574
SCHNEIDER ELECTRIC	0.0380	0.9641	0.0369	1.2519	0.0371*	1.2478
SIEMENS(XET)	0.0325	0.7614	0.0318	0.9094	0.0301	0.9625
SOCIETE GENERALE	-0.0346	0.7119	-0.0343	1.6924	-0.0380	1.8105
TELEFONICA	-0.0322*	0.9913	-0.0319	0.8380#	-0.0318	0.8359
TOTAL	0.0039	1.0164	0.0026	0.8189	0.0024	0.8223
UNIBAIL-RODAMCO	0.0447	0.7529	0.0449	0.7767	0.0443	0.7949
UNICREDIT	-0.0927*	1.3649	-0.0919	1.5540	-0.0972	1.7209
UNILEVER CERTS.	0.0365*	0.6879	0.0367	0.4397	0.0366*	0.4437
VINCI	0.0176	0.5559	0.0153	1.0928	0.0154	1.1000
VIVENDI	-0.0154	1.2523	-0.0163	0.8343#	-0.0151	0.7906
VOLKSWAGEN PREF (XET)	0.1029	0.9039	0.1030*	0.9898	0.1047	0.9411
The number of significant results	18	48	7	42	22	49
The number of joint significance	48		42		49	
The number of 'alive'	30		35		27	

The label '*' stands for statistically significant in the $\hat{\alpha}_{iM}$ columns while '#' means insignificant at the 5% significance level (i.e., opposite to the meaning of *) in the $\hat{\beta}_{iM}$ columns. In the case of the value of $\hat{\alpha}_{iM}$ without * and the value of $\hat{\beta}_{iM}$ without #, it implies the CAPM is 'alive'. The hypothesis of $\hat{\alpha}_{iM} = \hat{\beta}_{iM} = 0$ indicates that $\hat{\alpha}_{iM}$ without * and $\hat{\beta}_{iM}$ with #, otherwise the null is rejected at the 5% significance level and the result is statistically significant. The results show that the estimates are affected by time intervals and different distributional assumptions.

Table 4: Estimation results for coefficients

	Weekly returns					
	CAPM-IAPD		CAPM-IAEPD		CAPM	
	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$
AIR LIQUIDE	0.1654	0.6609	0.1655	0.6746	0.1667*	0.6486
AIRBUS GROUP	0.4888*	0.7570	0.4948	0.7573	0.5000*	0.7221
ALLIANZ(XET)	0.0793	1.1837	0.0806	1.2149#	0.0736*	1.2628
ANHEUSER-BUSCH INBEV	0.5099	0.5646	0.5135	0.5644#	0.5207*	0.5244
ASML HOLDING	0.5177*	0.7065	0.5248	0.7170#	0.5259*	0.7028
ASSICURAZIONI GENERALI	-0.2175	1.1766	-0.2257	1.1841	-0.2235*	1.1724
AXA	-0.1012	1.6490	-0.1032	1.6985#	-0.1185*	1.7930
BANCO SANTANDER	-0.0996	1.3707	-0.1003	1.3718#	-0.1063*	1.4046
BASF(XET)	0.2835	1.0585	0.3076	1.0616	0.2966*	1.0740
BAYER(XET)	0.2600*	0.8162	0.2698	0.8152	0.2663*	0.8029
BBV.ARGENTARIA	-0.1448	1.5104	-0.1526	1.5259	-0.1610*	1.5548
BMW(XET)	0.3955	1.0310	0.3782	1.0142	0.3902*	0.9599
BNP PARIBAS	0.0074	1.6840	0.0093	1.7102	0.0106	1.7211
CARREFOUR	-0.0272	0.9301	-0.0307	0.9290	-0.0349	0.9471
CRH(DUB)	-0.0707	1.0903#	-0.0621	1.0937	-0.0589	1.0823
DAIMLER	0.2036	1.2281	0.1802	1.2294	0.1894*	1.2049
DANONE	0.0444	0.4874	0.0479	0.5081#	0.0567	0.4541
DEUTSCHE BANK(XET)	-0.0794	1.4493	-0.0788	1.4491#	-0.0941	1.5549
DEUTSCHE POST(XET)	0.2244	0.9959#	0.2191	0.9812	0.2238*	1.0539
DEUTSCHE TELEKOM(XET)	0.1068*	0.5234	0.1033	0.5254#	0.1017*	0.5319
E.ON(XET)	-0.2966	0.8220	-0.2718	0.8278	-0.3036	0.8760
ENEL	-0.1153	0.9553	-0.1109	0.9486	-0.1113*	0.9499
ENI	-0.0147	0.8850	-0.0106	0.8826	-0.0203	0.9376
ESSILOR INTL.	0.2827*	0.3699	0.2815	0.3589	0.2778*	0.3660
GDF SUEZ	-0.2956	0.9484	-0.2901	0.9199	-0.2811*	0.8917
IBERDROLA	-0.1673	0.9554	-0.1731	0.9904	-0.1731*	0.9903
INDITEX	0.4195*	0.6634	0.4198	0.6450#	0.4179*	0.6737
ING GROEP	-0.0858	1.8498	-0.0981	1.8461#	-0.1521*	2.1568
INTESA SANPAOLO	-0.3505	1.6817	-0.3376	1.6761	-0.3403*	1.6942
L'OREAL	0.2084	0.6141	0.1771	0.6132#	0.1945	0.6213#
LVMH	0.2746	0.9011	0.2759*	0.9008	0.2793*	0.8803
MUENCHENER RUCK.(XET)	0.0975	0.7533#	0.0982	0.7446	0.0875*	0.8045
ORANGE	-0.2769	0.7100	-0.2724	0.7064	-0.2640*	0.6503
PHILIPS ELTN.KONINKLIJKE	0.1502	1.0046	0.1583	1.0053	0.1559*	1.0096
REPSOL YPF	-0.0148	1.0762	-0.0074	1.0625	-0.0016	1.0428
RWE(XET)	-0.3392	0.7934#	-0.3447	0.7853	-0.3540*	0.8408
SAINT GOBAIN	-0.0655	1.4192	-0.0504	1.3327	-0.0659	1.4084#
SANOFI	0.1747	0.6024	0.0499	0.4997#	0.1896*	0.5526
SAP(XET)	0.2636	0.6386	0.2744*	0.6215	0.2768*	0.5964
SCHNEIDER ELECTRIC	0.1848	1.2035	0.1986	1.2006	0.2005*	1.1962
SIEMENS(XET)	0.1474	0.9790	0.1478	0.9770	0.1478*	0.9930
SOCIETE GENERALE	-0.1895	1.9618	-0.1972*	1.9726	-0.2217*	2.0000
TELEFONICA	-0.1619	0.8195#	-0.1591	0.8054	-0.1580	0.7964#
TOTAL	0.0119	0.7995	0.0142	0.7900#	0.0181	0.7923
UNIBAIL-RODAMCO	0.2261	0.8024	0.2243	0.8033#	0.2118*	0.8606
UNICREDIT	-0.4936	1.7979	-0.4908*	1.7344	-0.5328*	1.9625
UNILEVER CERTS.	0.1836	0.4320#	0.1900*	0.4387	0.1933*	0.3953
VINCI	0.0969	1.0593	0.0831	1.0671#	0.0915*	1.0265
VIVENDI	-0.0677*	0.7211	-0.0665	0.7436	-0.0602	0.6990
VOLKSWAGEN PREF (XET)	0.5128*	1.0146	0.5136	1.0070	0.5200*	0.9683
The number of significant results	8	44	5	35	37	47
The number of joint significance	44		35		47	
The number of 'alive'	36		30		10	

The label '*' stands for statistically significant in the $\hat{\alpha}_{iM}$ columns while '#' means insignificant at the 5% significance level (i.e., opposite to the meaning of *) in the $\hat{\beta}_{iM}$ columns. In the case of the value of $\hat{\alpha}_{iM}$ without * and the value of $\hat{\beta}_{iM}$ without #, it implies the CAPM is 'alive'. The hypothesis of $\hat{\alpha}_{iM} = \hat{\beta}_{iM} = 0$ indicates that $\hat{\alpha}_{iM}$ without * and $\hat{\beta}_{iM}$ with #, otherwise the null is rejected at the 5% significance level and the result is statistically significant. The results show that the estimates are affected by time intervals and different distributional assumptions.

Table 5: Estimation results for coefficients

	Monthly returns					
	CAPM-IAPD		CAPM-IAEPD		CAPM	
	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$	$\hat{\alpha}_{iM}$	$\hat{\beta}_{iM}$
AIR LIQUIDE	0.7519	0.5819	0.7476*	0.5796	0.7878*	0.5196
AIRBUS GROUP	1.8731*	0.8606	2.0208	0.9613	1.8926*	0.8308
ALLIANZ(XET)	0.3731	1.3033	0.3785	1.3031	0.3665*	1.3182
ANHEUSER-BUSCH INBEV	2.7440*	0.2999	2.6874	0.3373	2.6809*	0.3540#
ASML HOLDING	2.4662	0.7913#	2.4402*	0.8779	2.5211*	0.7266
ASSICURAZIONI GENERALI	-0.9247	1.3659	-0.9232	1.3656#	-0.9858*	1.3929
AXA	-0.4299	2.1274	-0.4428	1.9804#	-0.7623*	2.0817
BANCO SANTANDER	-0.4880	1.5473	-0.4746	1.5450#	-0.4417*	1.4966
BASF(XET)	1.4795*	1.0719	1.4610	1.0874	1.4819*	1.0618
BAYER(XET)	1.2922	0.8490	1.3903	0.8383	1.1531*	0.8198
BBV.ARGENTARIA	-0.6778*	1.8176	-0.6995*	1.7998	-0.6454*	1.7144
BMW(XET)	1.7084	0.7209	1.7136	0.7209	1.9165*	0.7647
BNP PARIBAS	-0.4827	1.5365	-0.3868	1.5729#	-0.4126*	1.5364
CARREFOUR	-0.2001	0.8700	-0.2195	0.8626	-0.2145	0.8850
CRH(DUB)	-0.2046	0.7739	-0.0700	0.8203#	-0.1435	0.7581
DAIMLER	0.8695	1.1418	0.6678	1.2123#	0.9060*	1.2060
DANONE	0.3758	0.4108	0.4596*	0.4197	0.2583*	0.4422
DEUTSCHE BANK(XET)	-0.2416	1.3540	-0.3533	1.4029#	-0.3284	1.4036
DEUTSCHE POST(XET)	0.9991	1.1971#	1.0031	1.1926	0.9834*	1.2200
DEUTSCHE TELEKOM(XET)	0.2948*	0.4460	0.3123	0.4606	0.3155*	0.4457
E.ON(XET)	-1.0712	0.8194	-1.1098*	0.8057	-1.0992*	0.8048
ENEL	-0.3363	1.0234	-0.4640	1.0296#	-0.4050*	1.0051
ENI	-0.1489	0.9178	-0.1671	0.9503	-0.1664	0.9284
ESSILOR INTL.	1.2908	0.4172	1.3573*	0.4110	1.2965*	0.4143
GDF SUEZ	-1.0922	0.9653	-0.9795	0.8540	-0.9948*	0.8353
IBERDROLA	-0.6276	1.0100#	-0.5808	1.0098	-0.6220*	1.0057
INDITEX	1.9584*	0.7491	1.9623	0.6973#	1.9798*	0.7179
ING GROEP	-0.5525	2.2609#	-0.6098	2.2713#	-0.9122*	2.7406
INTESA SANPAOLO	-1.4058*	1.8151	-1.3742*	1.8127	-1.4084*	1.8271
L'OREAL	0.7540	0.6889#	0.7998	0.6741#	0.8868*	0.6352
LVMH	1.5870	0.6641	2.1234*	0.5772#	1.4276*	0.6939
MUENCHENER RUCK.(XET)	0.3120	0.8173	0.3621	0.7997	0.3190*	0.8197
ORANGE	-0.9414*	0.3989	-0.9293	0.4061	-0.9501*	0.4375
PHILIPS ELTN.KONINKLIJKE	0.8374	0.8643	0.8241	0.8654	0.8549*	0.8567
REPSOL YPF	0.0338	1.0417	0.0867	0.9697	0.1067	0.9312
RWE(XET)	-1.4657	0.8490#	-1.3618	0.8485#	-1.4115*	0.8928
SAINT GOBAIN	-0.3733	1.4819	-0.3711	1.5407#	-0.3364*	1.4472
SANOFI	1.1237	0.4643#	0.9377	0.4643#	0.7975*	0.5679
SAP(XET)	1.0008*	0.6380	1.0154	0.6543	0.9523*	0.6862
SCHNEIDER ELECTRIC	1.0298	0.9201	0.9999*	0.9035	1.0393*	0.9068
SIEMENS(XET)	0.6866	0.9714	0.7619	0.9851#	0.7225*	0.9773
SOCIETE GENERALE	-0.9926	2.0684	-0.9731	2.0682	-1.0191*	2.1331
TELEFONICA	-0.2964	0.5071	-0.2941	0.5071	-0.5254*	0.5784
TOTAL	0.1556	0.5961	0.2920	0.5219#	0.1097	0.6660
UNIBAIL-RODAMCO	1.1779	0.7962	1.0232	0.8136#	1.0969*	0.7976
UNICREDIT	-2.8231	2.2772#	-2.3811*	2.3194	-2.4823*	2.3315
UNILEVER CERTS.	0.7462	0.4444	1.0377	0.4312	0.7832*	0.4139
VINCI	0.4435	0.8985	0.3402	0.9285#	0.4542*	0.8550
VIVENDI	-0.2170	0.5539	-0.1737	0.5928#	-0.1767	0.5599
VOLKSWAGEN PREF (XET)	2.5502*	1.0295	2.5497	1.0371	2.5447*	1.0448
The number of significant results	10	42	10	30	43	49
The number of joint significance	42		31		50	
The number of 'alive'	32		21		6	

The label '*' stands for statistically significant in the $\hat{\alpha}_{iM}$ columns while '#' means insignificant at the 5% significance level (i.e., opposite to the meaning of *) in the $\hat{\beta}_{iM}$ columns. In the case of the value of $\hat{\alpha}_{iM}$ without * and the value of $\hat{\beta}_{iM}$ without #, it implies the CAPM is 'alive'. The hypothesis of $\hat{\alpha}_{iM} = \hat{\beta}_{iM} = 0$ indicates that $\hat{\alpha}_{iM}$ without * and $\hat{\beta}_{iM}$ with #, otherwise the null is rejected at the 5% significance level and the result is statistically significant. The results show that the estimates are affected by time intervals and different distributional assumptions.

Table 6: AIC values of all models

	Panel A: Daily (2/1/2009:3/1/2014)			Panel B: Weekly (2/1/2009:3/1/2014)			Panel C: Monthly (1/2009:12/2013)		
	CAPM	CAPM-IAPD	CAPM-IAEPD	CAPM	CAPM-IAPD	CAPM-IAEPD	CAPM	CAPM-IAPD	CAPM-IAEPD
AIR LIQUIDE	3404.7792	3282.5789	3273.3300	1071.0754	1053.1739	1053.4466	318.5227	319.1901	321.0973
AIRBUS GROUP	5239.4814	5118.6279	5120.5969	1477.6521	1455.0802	1456.9297	416.9211	418.6490	415.7329
ALLIANZ(XET)	3936.3325	3603.1652	3605.1154	1162.2661	1143.9197	1144.3393	356.2161	357.2208	359.2180
ASML HOLDING	4911.7388	4847.6203	4849.6203	1358.3971	1349.0144	1350.8045	400.6253	403.6490	405.2052
ASSICURAZIONI GENERALI	4067.5945	3904.9562	3904.5743	1244.7638	1235.0569	1236.5694	363.8638	367.3110	369.3109
AXA	4708.5050	4372.5305	4343.5449	1343.0419	1313.3811	1310.7202	397.0854	400.5601	391.8591
BANCO SANTANDER	4345.0254	4191.2021	4188.2104	1344.5264	1326.9169	1328.3008	393.0753	385.6875	387.1642
BASF(XET)	4040.7210	3898.4428	3900.0790	1232.4929	1231.7500	1232.3148	363.2755	366.1828	367.6716
BBVA(XET)	4223.4670	4021.8981	4021.9999	1238.6052	1238.6461	1240.2267	377.8441	380.0820	380.3903
BBV ARGENTARIA	4359.4215	4227.0034	4218.1860	1336.3989	1333.0669	1333.7667	401.5845	402.0574	403.3554
BMW(XET)	4827.7379	4668.0833	4669.2428	1404.1748	1391.2474	1391.4627	391.9551	386.7233	388.7228
BNP PARIBAS	5039.2188	4660.0230	4661.1340	1454.2134	1370.1373	1368.1584	406.2638	406.9220	402.2350
CARREFOUR	4682.0249	4505.6741	4505.4638	1348.9085	1340.9425	1342.9018	388.0063	391.7355	393.6014
CRH(DUB)	5283.4545	5135.4567	5118.6000	1456.8142	1433.5111	1433.7177	384.3730	381.2503	375.9582
DAIMLER	4690.5223	4571.8386	4573.5910	1387.4205	1382.4527	1382.9792	397.8190	398.9164	399.4867
DANONE	3892.2674	3754.0163	3753.7876	1198.7669	1178.6293	1176.6313	353.1643	355.5174	355.7278
DEUTSCHE BANK(XET)	4982.3960	4667.3736	4663.9118	1457.3866	1394.9743	1396.3611	433.6488	431.2898	431.7422
DEUTSCHE POST(XET)	4171.5837	4001.8009	4003.7845	1275.3501	1262.9302	1250.2071	361.7067	365.4598	367.4503
DEUTSCHE TELEKOM(XET)	3972.1759	3672.1741	3674.1314	1220.0266	1204.7331	1206.6488	364.7031	366.8557	368.5905
E.ON(XET)	4328.7577	4022.7278	4026.7350	1349.6999	1302.8363	1297.0884	393.7058	394.7580	396.8198
ENEL	4058.1883	3812.2301	3813.5376	1243.0138	1218.3821	1219.9810	368.2645	371.4945	371.4763
ENI	3464.5987	3237.1494	3239.0743	1129.5922	1071.7415	1070.2499	340.7860	336.6173	336.5278
ESSILOR INTL.	3857.7706	3735.1194	3737.0009	1151.4506	1150.9133	1146.7520	369.1882	373.0727	374.4033
GDF SUEZ	4192.3990	3890.3929	3892.3926	1246.2112	1200.0635	1200.6748	373.7930	373.9302	372.8901
IBERDROLA	3957.5703	3677.2674	3675.9632	1249.3856	1220.7357	1222.0571	355.0820	358.8171	359.9763
INDITEX	4563.2441	4440.6878	4442.6278	1298.2055	1276.7538	1278.7223	375.7284	377.7895	376.2194
ING GROEP	5662.1819	5249.5844	5250.4103	1530.3149	1463.6679	1465.6324	439.8121	428.1230	429.8627
INTESA SANPAOLO	5288.4946	5196.8393	5195.5371	1440.7111	1429.3927	1428.0978	404.6019	406.7886	408.6941
L'OREAL	3753.1063	3589.6571	3591.0100	1151.2560	1141.1982	1133.1213	345.3132	345.3563	346.9598
LMVH	4161.4799	3987.3615	3987.8295	1238.1088	1218.4942	1220.4864	365.7603	367.6444	364.9497
MUENCHENER RUCK(XET)	3863.2036	3609.4476	3611.1722	1172.7860	1134.1315	1134.6489	341.7563	345.6664	347.3030
ORANGE	3843.5331	3618.2881	3624.5904	1210.6081	1180.5587	1177.7156	364.2943	364.1301	366.0758
PHILIPS ELTN.KONINKLIJKE	4468.6409	4277.3814	4278.6606	1271.6785	1260.9423	1262.7055	396.0723	398.9605	400.9315
REPSOL YPF	4275.1793	3912.1535	3913.6831	1285.5763	1247.2936	1242.3500	381.9375	369.5314	370.2363
RWE(XET)	4307.7329	4013.7808	4014.9623	1320.2225	1301.1124	1302.7612	400.0470	402.9541	403.0425
SAINT GOBAIN	4766.5813	4434.2789	4435.9417	1367.0306	1331.0066	1329.9246	369.6516	371.5671	371.5442
SANOFI	4033.3433	3834.3528	3831.3354	1203.7546	1172.1759	1151.7765	343.2414	329.3709	331.2858
SAP(XET)	3895.6489	3716.6206	3707.9152	1230.4176	1212.3774	1212.1600	359.0922	361.1816	362.7946
SCHNEIDER ELECTRIC	4367.4042	4277.1392	4279.0675	1293.9589	1286.4785	1285.6321	358.0319	359.3865	360.2324
SIEMENS(XET)	3801.0419	3539.6663	3541.3629	1178.7649	1178.7608	1179.9576	337.5384	340.7186	340.6336
SOCIETE GENERALE	5510.6206	5294.2833	5293.3718	1504.5681	1476.4378	1470.1409	428.2042	431.0748	433.0140
TELEFONICA	3482.5221	3257.6321	3258.5265	1166.2414	1134.2753	1136.0249	354.9537	341.7658	343.7637
TOTAL	3002.4824	2886.1874	2887.4341	1043.1823	1045.5476	1045.2078	335.9980	337.3031	335.8815
UNIBAIL-RODAMCO	4186.5678	4016.2840	4018.2744	1236.0314	1219.9705	1220.3930	368.0987	371.3323	370.9766
UNICREDIT	5840.9692	5591.6484	5592.2060	1646.4487	1566.5937	1565.4769	554.3365	462.8040	464.8667
UNILEVER CERTS.	3775.9479	3547.1560	3548.5824	1160.6792	1150.6445	1152.0599	339.6822	332.1331	331.9772
VINCI	3407.4328	3284.8754	3284.6742	1103.4640	1091.7427	1089.6642	329.1005	327.6920	324.4626
VIVENDI	4110.2505	3912.1863	3912.0853	1221.3243	1188.9673	1187.9891	360.0927	362.9131	362.3890
VOLKSWAGEN PREF.(XET)	5613.5494	5367.4037	5366.8780	1541.5877	1533.3005	1535.1940	453.0682	452.8570	454.8644
Average value of AIC	4345.3696	4134.3684	4134.3860	1289.9884	1266.7742	1265.7050	379.3894	377.3335	377.7010

Panel A displays the AIC values of daily data of 50 constituents estimated by these models, with 1306 observations. Panel B shows the AIC values using weekly data (262 observations). Panel C presents the AIC values for monthly data (60 observations). The last row summarizes the mean value of the AIC.

Table 7: Distribution test results of the error terms

	CAPM-IAPD	CAPM-IAEPD	CAPM-IAPD	CAPM-IAEPD
	Normal distribution	Normal distribution	Laplace distribution	Laplace distribution
	$H_0 : \alpha = 0.5, \lambda = 2$	$H_0 : \alpha = 0.5, p_1 = p_2 = 2$	$H_0 : \alpha = 0.5, \lambda = 1$	$H_0 : \alpha = 0.5, p_1 = p_2 = 1$
Daily:				
Significant α s	16	18	16	18
Significant λ s	49		16	
Significant p_1 s		41		15
Significant p_2 s		41		14
H_0 are rejected	49	43	26	28
Weekly:				
Significant α s	3	23	3	23
Significant λ s	42		9	
Significant p_1 s		34		13
Significant p_2 s		29		15
H_0 are rejected	42	40	10	31
Monthly:				
Significant α s	9	17	9	17
Significant λ s	10		6	
Significant p_1 s		14		8
Significant p_2 s		7		13
H_0 are rejected	14	25	15	24

The skewness parameter α accommodates for the asymmetry, and ‘significant α s’ stands for how many α s out of 50 are statistically significant, i.e., 0.5 lies out of $CI_{\hat{\alpha}}$. Zhu and Zinde-Walsh (2009) indicate that the parameter p determines to the right if $\alpha < 0.5$ and to the left if $\alpha > 0.5$, and $p < 2$ leads to a fat tail, fatter for smaller p ; therefore, if 2 lies outside $CI_{\hat{\lambda}}$, $CI_{\hat{p}_1}$, or $CI_{\hat{p}_2}$, the tail parameter can capture the fat tail of the return distributions.

Table 8: Backtest report of performance analysis on the investment based on three portfolio strategies

Investment horizon: 14/06/2009 to 14/04/2014	Benchmark: EURO STOXX 50 Index	Strategy A: CAPM approach	Strategy B: CAPM-IAPD approach	Strategy C: CAPM-IAEPD approach
Initial market value(14/06/2009)	695.34	11780.42	9778.48	9296.56
Market value in the end(14/04/2014)	1053.32	23612.47	21735.70	18517.74
Return/Risk Analysis				
Total return (realized)	51.48%	100.44%	122.28%	99.19%
Average monthly total return	0.88%	1.30%	1.48%	1.26%
Annualized total return	11.07%	16.73%	19.32%	16.27%
Standard deviation	5.68%	4.35%	4.46%	3.77%
Annualized standard deviation	19.68%	15.07%	15.45%	13.06%
Monthly Sharpe Ratio	14.69%	28.83%	32.29%	32.38%
Annualized Sharpe Ratio	0.5088	0.9986	1.1185	1.1218
Best month	13.34%	11.77%	12.55%	9.64%
Worst month	-14.31%	-7.48%	-7.37%	-8.10%
% of Up month	62.07%	60.34%	60.34%	63.79%
% of Down month	37.93%	39.66%	39.66%	36.21%
Avg. Monthly Gain	4.40%	4.11%	4.38%	3.49%
Avg. Monthly Loss	-4.89%	-2.99%	-2.93%	-2.67%
Gains Std. Dev.	3.15%	2.98%	2.88%	2.50%
Loss Std. Dev.	3.72%	1.73%	2.09%	1.82%
Maximum Drawdown	28.56%	12.41%	13.79%	12.89%
Relative Performance				
Annualized excess return (relative to the benchmark portfolio)		5.65%	8.25%	5.20%
Jensen's Alpha		0.83%* (1.9079)	1.02%** (2.2265)	0.84%** (2.2853)
Beta		0.51*** (6.7127)	0.49*** (6.2722)	0.46*** (7.0017)
Tracking Error (on monthly basis)		4.25%	4.44%	4.14%
Annualized Information Ratio		0.38	0.54	0.36
R-Squared		0.45	0.41	0.47
Treynor Ratio		0.32	0.38	0.34

A label ‘*’ indicates significance at the 10% level, ‘**’ at the 5% level and ‘***’ at the 1% level. T -values are given in parentheses. The benchmark portfolio is the EURO STOXX 50 Index and the risk-free rate is the 1 month Euribor rate. Assume the market value of the portfolio at the beginning is the amount of money that the investors wish to allocate in the MVP. The initial weights are decided in the one period ahead of 14/06/2009 and the initial market value is calculated by $V_t = TR_t \times x_{t-1}$.

Table 9: Estimation results for parameters

Daily:	CAPM-IAPD					
	Normality, $H_0: \alpha = 0.5$ and $\lambda = 2$					
	Laplace, $H_0: \alpha = 0.5$ and $\lambda = 1$					
	$\hat{\alpha}$	$CI_{\hat{\alpha}}$	$\hat{\lambda}$	$CI_{\hat{\lambda}}$		
AIR LIQUIDE	0.4976	0.4796	0.5156	1.2250	1.1176	1.3324
AIRBUS GROUP	0.4821	0.4546	0.5096	1.1456	0.5563	1.7349
ALLIANZ(XET)	0.4926*	0.4922	0.4931	0.8851	0.8256	0.9446
ANHEUSER-BUSCH INBEV	0.4743	0.4403	0.5083	0.9577	0.8712	1.0442
ASML HOLDING	0.4673*	0.4346	0.4999	1.3072	1.1851	1.4293
ASSICURAZIONI GENERALI	0.4836	0.4646	0.5025	1.0922	0.9415	1.2430
AXA	0.5036*	0.5023	0.5048	0.8793	0.7320	1.0266
BANCO SANTANDER	0.4941	0.4771	0.5111	1.0695	0.9880	1.1509
BASF(XET)	0.4816*	0.4643	0.4988	1.1080	0.9902	1.2258
BAYER(XET)	0.4861	0.3038	0.6683	1.0704	0.9725	1.1682
BBV.ARGENTARIA	0.4515	0.3966	0.5063	1.1788	1.0705	1.2872
BMW(XET)	0.4918*	0.4856	0.4980	1.0721	0.9754	1.1689
BNP PARIBAS	0.4705	0.1100	0.8311	0.8909	0.2975	1.4842
CARREFOUR	0.4682	-5.6372	6.5736	1.0795	0.9619	1.1971
CRH(DUB)	0.4550	-0.1660	1.0759	1.1517	0.8668	1.4365
DAIMLER	0.4907*	0.4817	0.4997	1.1339	1.0290	1.2387
DANONE	0.5150	0.3627	0.6673	1.1606	0.9307	1.3905
DEUTSCHE BANK(XET)	0.5031	0.4986	0.5076	0.8755	0.7156	1.0354
DEUTSCHE POST(XET)	0.4855	0.4580	0.5129	1.0716	0.9264	1.2168
DEUTSCHE TELEKOM(XET)	0.4936	0.4865	0.5007	0.9574	0.8922	1.0226
E.ON(XET)	0.5188	0.2133	0.8244	0.9830	0.8827	1.0832
ENEL	0.5066*	0.5065	0.5068	0.9970	0.9102	1.0838
ENI	0.4982	0.4924	0.5039	1.0176	0.9406	1.0946
ESSILOR INTL.	0.4834	0.4487	0.5181	1.1439	1.0932	1.1945
GDF SUEZ	0.5189*	0.5060	0.5318	0.9555	0.8629	1.0482
IBERDROLA	0.4918	0.4171	0.5665	0.9343	0.8455	1.0232
INDITEX	0.4790*	0.4605	0.4976	1.1706	1.1026	1.2385
ING GROEP	0.5037	0.4985	0.5090	0.8468	0.7208	0.9729
INTESA SANPAOLO	0.4745	0.4230	0.5261	1.1997	1.0884	1.3109
L'OREAL	0.4882	0.3873	0.5892	1.1114	0.8880	1.3348
LVMH	0.5162*	0.5078	0.5245	1.1076	1.0038	1.2114
MUENCHENER RUCK.(XET)	0.4958	0.4760	0.5157	0.9682	0.4089	1.5276
ORANGE	0.4762*	0.4675	0.4848	1.0016	0.9116	1.0916
PHILIPS ELTN.KONINKLIJKE	0.4760*	0.4657	0.4862	1.0801	0.5911	1.5692
REPSOL YPF	0.4994	0.4567	0.5420	0.8737	-6.5483	8.2956
RWE(XET)	0.5279*	0.5275	0.5283	0.8986	0.8005	0.9967
SAINT GOBAIN	0.5031*	0.5027	0.5036	0.9351	0.8584	1.0118
SANOFI	0.4987	0.4719	0.5255	1.0634	1.0326	1.0942
SAP(XET)	0.5007	0.4740	0.5275	1.0657	0.9763	1.1552
SCHNEIDER ELECTRIC	0.4897	0.4639	0.5154	1.2317	1.1543	1.3090
SIEMENS(XET)	0.4884	0.3712	0.6056	0.9361	0.7603	1.1118
SOCIETE GENERALE	0.5063*	0.5010	0.5115	0.9934	0.8937	1.0930
TELEFONICA	0.5071	0.4858	0.5285	0.9801	0.8925	1.0676
TOTAL	0.4931	0.4761	0.5100	1.1912	1.0384	1.3439
UNIBAIL-RODAMCO	0.4921	0.4708	0.5134	1.0879	1.0019	1.1739
UNICREDIT	0.5043	0.4891	0.5195	0.9782	0.8848	1.0716
UNILEVER CERTS.	0.4892*	0.4877	0.4907	1.0143	0.9326	1.0961
VINCI	0.4826	0.4383	0.5270	1.1823	1.0023	1.3622
VIVENDI	0.4917	0.4817	0.5018	1.0704	0.9863	1.1544
VOLKSWAGEN PREF. (XET)	0.5155	0.4593	0.5717	1.0048	0.9100	1.0996

Table 10: Estimation results for parameters

Weekly:	CAPM-IAPD					
	Normality, $H_0: \alpha = 0.5$ and $\lambda = 2$					
	Laplace, $H_0: \alpha = 0.5$ and $\lambda = 1$					
	$\hat{\alpha}$	$CI_{\hat{\alpha}}$	$\hat{\lambda}$	$CI_{\hat{\lambda}}$		
AIR LIQUIDE	0.5141	0.3859	0.6423	1.2552	0.7775	1.7330
AIRBUS GROUP	0.4810	0.4473	0.5146	1.1180	0.9026	1.3335
ALLIANZ(XET)	0.4514	0.3743	0.5286	1.1451	0.2388	2.0514
ANHEUSER-BUSCH INBEV	0.4342	0.2945	0.5740	1.1894	0.9704	1.4085
ASML HOLDING	0.5286	0.4516	0.6057	1.2596	0.9616	1.5575
ASSICURAZIONI GENERALI	0.4874	0.3880	0.5868	1.3155	1.0479	1.5831
AXA	0.4309*	0.3646	0.4972	1.0775	1.0071	1.1479
BANCO SANTANDER	0.4608	0.4078	0.5138	1.1527	0.8204	1.4850
BASF(XET)	0.4319	0.3594	0.5043	1.5314	1.1733	1.8894
BAYER(XET)	0.4345	0.3563	0.5127	1.6447	0.8978	2.3915
BBV.ARGENTARIA	0.4907	0.4197	0.5617	1.4280	1.0772	1.7787
BMW(XET)	0.4742	0.4181	0.5303	1.2146	0.8467	1.5826
BNP PARIBAS	0.4925*	0.4922	0.4928	0.9059	0.4462	1.3657
CARREFOUR	0.5418	0.4630	0.6206	1.3122	0.9292	1.6952
CRH(DUB)	0.5047	-12.5904	13.5998	1.1252	-14.0179	16.2683
DAIMLER	0.4947	0.4298	0.5597	1.3494	0.9603	1.7384
DANONE	0.5059	0.4402	0.5716	1.1773	0.7648	1.5897
DEUTSCHE BANK(XET)	0.4443*	0.4379	0.4508	0.9027	0.6357	1.1697
DEUTSCHE POST(XET)	0.4815	0.2794	0.6836	1.3266	-3.2532	5.9064
DEUTSCHE TELEKOM(XET)	0.4898	0.4576	0.5221	1.1827	0.9226	1.4428
E.ON(XET)	0.5089	0.2695	0.7483	1.0238	0.2451	1.8024
ENEL	0.5429	0.3492	0.7367	1.1025	0.7929	1.4121
ENI	0.5218	-0.3269	1.3705	1.0723	0.7811	1.3635
ESSILOR INTL.	0.5485	0.4870	0.6101	1.5478	1.1762	1.9193
GDF SUEZ	0.5254	0.4798	0.5710	1.0661	0.8468	1.2855
IBERDROLA	0.4889	0.4570	0.5208	1.0139	0.6035	1.4242
INDITEX	0.4819	0.4455	0.5183	1.1781	0.9567	1.3996
ING GROEP	0.4824	0.4301	0.5348	0.8098	0.6290	0.9906
INTESA SANPAOLO	0.5149	0.4483	0.5815	1.2886	0.0381	2.5391
L'OREAL	0.5176	-0.2517	1.2869	1.3235	-0.7290	3.3760
LVMH	0.4976	0.4516	0.5436	1.1772	0.9616	1.3927
MUENCHENER RUCK.(XET)	0.4500	0.3741	0.5259	0.9875	0.6927	1.2823
ORANGE	0.4502	0.3885	0.5119	1.0425	0.7434	1.3416
PHILIPS ELTN.KONINKLIJKE	0.4611	0.2943	0.6279	1.2679	0.9331	1.6026
REPSOL YPF	0.5009	0.0144	0.9874	1.0686	0.6555	1.4817
RWE(XET)	0.5249	-2.4508	3.5007	1.1360	0.8060	1.4659
SAINT GOBAIN	0.4755	0.4281	0.5230	1.0833	0.8608	1.3057
SANOFI	0.5627	0.1877	0.9377	1.2358	0.6760	1.7956
SAP(XET)	0.4994	0.4593	0.5396	1.1293	0.8625	1.3961
SCHNEIDER ELECTRIC	0.4349	0.3420	0.5279	1.3880	1.0900	1.6861
SIEMENS(XET)	0.5246	0.4405	0.6087	1.5761	0.8744	2.2779
SOCIETE GENERALE	0.5199	0.4652	0.5746	1.1214	0.9092	1.3337
TELEFONICA	0.4464	0.2047	0.6881	1.0681	0.2332	1.9029
TOTAL	0.4800	0.3860	0.5741	1.6990	1.4034	1.9947
UNIBAIL-RODAMCO	0.5445	0.3800	0.7090	1.1753	0.3769	1.9737
UNICREDIT	0.5430	0.3755	0.7105	0.9600	0.1593	1.7606
UNILEVER CERTS.	0.5268	0.3280	0.7256	1.3061	0.3278	2.2844
VINCI	0.4981	0.4453	0.5509	1.2947	0.9704	1.6189
VIVENDI	0.5050	0.4988	0.5112	1.0901	0.8615	1.3187
VOLKSWAGEN PREF. (XET)	0.4956	0.4123	0.5788	1.3233	1.0117	1.6349

Table 11: Estimation results for parameters

Monthly:	CAPM-IAPD					
	Normality, $H_0:\alpha = 0.5$ and $\lambda = 2$					
	Laplace, $H_0:\alpha = 0.5$ and $\lambda = 1$					
	$\hat{\alpha}$	$CI_{\hat{\alpha}}$	$\hat{\lambda}$	$CI_{\hat{\lambda}}$		
AIR LIQUIDE	0.4975	-0.0856	1.0807	1.0632	0.1881	1.9382
AIRBUS GROUP	0.6081	0.4376	0.7787	1.6847	0.9594	2.4100
ALLIANZ(XET)	0.5747	0.4490	0.7005	1.4044	0.7718	2.0370
ANHEUSER-BUSCH INBEV	0.2838*	0.2037	0.3640	1.2691	0.7182	1.8199
ASML HOLDING	0.5348	0.0816	0.9881	1.5200	-6.9279	9.9679
ASSICURAZIONI GENERALI	0.5277	0.0274	1.0279	2.9875	-5.0501	11.0251
AXA	0.6902	0.4333	0.9471	2.9581	-19.3723	25.2885
BANCO SANTANDER	0.4225*	0.4083	0.4367	0.9506	0.1735	1.7276
BASF(XET)	0.6042	0.4163	0.7920	2.0806	0.7159	3.4452
BAYER(XET)	0.6641	0.4935	0.8347	3.0122	0.8212	5.2033
BBV.ARGENTARIA	0.4708	0.3728	0.5687	1.2070	0.6508	1.7631
BMW(XET)	0.0000*	-0.1035	0.1036	5.2074	-26.7221	37.1369
BNP PARIBAS	0.5155	0.2928	0.7382	1.4202	0.8842	1.9562
CARREFOUR	0.5409	0.3693	0.7125	1.8011	-0.2750	3.8772
CRH(DUB)	0.6440	-0.1328	1.4208	1.4446	0.8806	2.0086
DAIMLER	0.2913	-0.2746	0.8572	2.6184	-1.1951	6.4319
DANONE	0.6348	0.4123	0.8572	2.8713	1.4093	4.3333
DEUTSCHE BANK(XET)	0.4612	-0.0748	0.9973	1.1717	0.6058	1.7376
DEUTSCHE POST(XET)	0.5551	0.1090	1.0011	2.0240	0.4342	3.6138
DEUTSCHE TELEKOM(XET)	0.5245	0.4735	0.5756	1.4210	0.8344	2.0075
E.ON(XET)	0.5175	0.1027	0.9323	1.3449	-0.6829	3.3726
ENEL	0.5809	0.4243	0.7374	1.4214	0.5889	2.2538
ENI	0.6089*	0.5878	0.6301	1.1289	0.6118	1.6460
ESSILOR INTL.	0.4667	0.2054	0.7279	1.9224	0.2417	3.6031
GDF SUEZ	0.5114	0.3652	0.6577	1.1009	-1.3087	3.5104
IBERDROLA	0.4696	0.1778	0.7613	1.7535	1.0080	2.4990
INDITEX	0.3882*	0.2827	0.4937	1.5516	0.9255	2.1777
ING GROEP	0.5108	-0.4674	1.4890	0.7224	-12.0446	13.4893
INTESA SANPAOLO	0.3793	0.2177	0.5409	1.7954	0.6915	2.8993
L'OREAL	0.4859	-0.4672	1.4389	4.3269	-16.6918	25.3455
LVMH	0.5962	-1.6503	2.8428	3.2896	-5.5359	12.1152
MUENCHENER RUCK.(XET)	0.4921	0.2963	0.6879	1.8135	1.3787	2.2482
ORANGE	0.4177*	0.3450	0.4903	1.2427	0.7320	1.7533
PHILIPS ELTN.KONINKLIJKE	0.4797	0.2726	0.6867	2.8092	1.5487	4.0698
REPSOL YPF	0.6402*	0.5454	0.7351	0.9950	-2.7201	4.7102
RWE(XET)	0.4346	0.2495	0.6197	1.4567	0.7389	2.1744
SAINT GOBAIN	0.5740	0.4130	0.7351	1.5495	0.6534	2.4455
SANOFI	0.0007	-1.9593	1.9607	98.0142	96.0542	99.9742
SAP(XET)	0.3559	0.0948	0.6170	2.2727	0.7298	3.8156
SCHNEIDER ELECTRIC	0.6624	0.4843	0.8406	2.2271	0.7197	3.7345
SIEMENS(XET)	0.4947	0.4097	0.5798	1.4794	-0.0201	2.9789
SOCIETE GENERALE	0.4080	0.2701	0.5459	1.6986	1.0843	2.3128
TELEFONICA	0.0000*	-0.1177	0.1178	3.6444	-23.2429	30.5318
TOTAL	0.7068	0.4971	0.9166	2.7335	0.7715	4.6955
UNIBAIL-RODAMCO	0.5728	0.3667	0.7789	1.4885	0.4029	2.5740
UNICREDIT	0.4599	-0.4302	1.3501	0.6371	-112.0506	113.3248
UNILEVER CERTS.	0.8009	-0.0232	1.6250	1.7409	-5.4355	8.9173
VINCI	0.6423	0.3967	0.8878	1.1461	0.0590	2.2333
VIVENDI	0.5257	-0.0676	1.1191	1.5978	-3.0827	6.2784
VOLKSWAGEN PREF. (XET)	0.4197*	0.3432	0.4961	1.1808	0.5044	1.8572

Table 12: Estimation results for parameters

Daily:	CAPM-IAEPD								
	Normality, $H_0: \alpha = 0.5$ and $p_1 = p_2 = 2$								
	Laplace, $H_0: \alpha = 0.5$ and $p_1 = p_2 = 1$								
	$\hat{\alpha}$	$CI_{\hat{\alpha}}$	$\hat{p}_1$	$CI_{\hat{p}_1}$	$\hat{p}_2$	$CI_{\hat{p}_2}$			
AIR LIQUIDE	0.6420	0.2896	0.9943	1.6197	-0.0069	3.2462	0.9773	-0.2213	2.1759
AIRBUS GROUP	0.4841	0.4464	0.5218	1.1533	1.0950	1.2115	1.1386	1.0862	1.1910
ALLIANZ(XET)	0.4949	0.4645	0.5253	0.8973	0.8224	0.9723	0.8824	0.7877	0.9772
ANHEUSER-BUSCH INBEV	0.4777	0.4548	0.5007	0.9723	0.8375	1.1070	0.9472	0.8503	1.0440
ASML HOLDING	0.4624	0.4110	0.5138	1.2924	1.1248	1.4599	1.3212	1.2488	1.3936
ASSICURAZIONI GENERALI	0.5306*	0.5291	0.5320	1.1696	1.0402	1.2990	0.9966	0.8928	1.1003
AXA	0.5038*	0.5037	0.5040	0.8839	0.8758	0.8921	0.8810	0.8799	0.8820
BANCO SANTANDER	0.5238	0.3707	0.6768	1.1580	0.5218	1.7942	0.9850	0.8734	1.0965
BASF(XET)	0.4863	0.4582	0.5144	1.1341	1.0424	1.2258	1.0897	0.9841	1.1953
BAYER(XET)	0.4748	0.2765	0.6731	1.0229	0.5396	1.5061	1.1185	0.9880	1.2490
BBV.ARGENTARIA	0.5423*	0.5300	0.5547	1.3541	1.1945	1.5138	0.9862	0.8872	1.0853
BMW(XET)	0.4728	0.3570	0.5886	1.0199	0.8196	1.2203	1.1044	0.9877	1.2211
BNP PARIBAS	0.4195*	0.4064	0.4326	0.8194	0.7226	0.9162	0.9645	0.8542	1.0747
CARREFOUR	0.5222*	0.5007	0.5436	1.1761	1.0514	1.3008	0.9799	0.9000	1.0598
CRH(DUB)	0.3389*	0.3384	0.3395	0.8976	0.8183	0.9769	1.4029	1.3576	1.4482
DAIMLER	0.4949	0.3968	0.5929	1.1564	0.9505	1.3624	1.1176	-0.8899	3.1250
DANONE	0.4837	0.4361	0.5313	1.0895	1.0039	1.1751	1.2401	1.1480	1.3322
DEUTSCHE BANK(XET)	0.5245*	0.5243	0.5248	0.9414	0.9407	0.9421	0.8192	0.8190	0.8194
DEUTSCHE POST(XET)	0.4843*	0.4774	0.4911	1.0682	0.9359	1.2005	1.0773	0.9476	1.2071
DEUTSCHE TELEKOM(XET)	0.4953	0.4251	0.5655	0.9646	0.8618	1.0674	0.9522	0.5742	1.3302
E.ON(XET)	0.5889	0.3529	0.8248	1.0617	0.8901	1.2333	0.8709	0.7410	1.0007
ENEL	0.4725	-7.3323	8.2773	0.9544	-1.1887	3.0975	1.0807	-0.0538	2.2152
ENI	0.5031	0.4662	0.5399	1.0239	0.3421	1.7057	1.0053	0.3188	1.6918
ESSILOR INTL.	0.4800	0.4552	0.5049	1.1281	1.0430	1.2132	1.1555	1.0183	1.2927
GDF SUEZ	0.5191*	0.5119	0.5263	0.9565	0.8946	1.0184	0.9553	0.8953	1.0152
IBERDROLA	0.5313*	0.5058	0.5568	0.9897	0.8703	1.1091	0.8587	0.7358	0.9817
INDITEX	0.4769	0.4398	0.5140	1.1603	1.1064	1.2142	1.1802	1.1578	1.2025
ING GROEP	0.5124	-2.5601	3.5850	0.8747	-11.2517	13.0011	0.8228	-4.4043	6.0499
INTESA SANPAOLO	0.4016	0.2734	0.5297	1.0440	0.9538	1.1341	1.3513	1.3233	1.3792
L'OREAL	0.4809	0.3670	0.5948	1.0830	0.9262	1.2399	1.1424	1.0292	1.2555
LVMH	0.4696*	0.4645	0.4746	1.0154	0.7631	1.2677	1.1621	1.0318	1.2924
MUENCHENER RUCK.(XET)	0.4891*	0.4816	0.4965	0.9517	0.8499	1.0535	0.9867	0.8893	1.0841
ORANGE	0.3796*	0.2998	0.4594	0.8715	0.6737	1.0693	1.1493	0.9576	1.3410
PHILIPS ELTN.KONINKLIJKE	0.4982	0.3492	0.6473	1.1079	-5.8457	8.0615	1.0415	-1.4610	3.5440
REPSOL YPF	0.4946	-0.3448	1.3340	0.8570	-11.3505	13.0646	0.8889	-5.8541	7.6320
RWE(XET)	0.5461	0.3422	0.7499	0.9256	0.7558	1.0953	0.8667	0.6193	1.1140
SAINT GOBAIN	0.4854	0.4588	0.5119	0.9128	0.7950	1.0307	0.9755	0.8313	1.1198
SANOFI	0.4698	0.4644	0.4753	0.9857	-0.2024	2.1739	1.1502	0.7363	1.5641
SAP(XET)	0.4425	-0.5772	1.4623	0.9425	-0.5701	2.4552	1.2392	-16.3975	18.8759
SCHNEIDER ELECTRIC	0.4922	0.4704	0.5140	1.2440	0.4930	1.9950	1.2204	0.9389	1.5020
SIEMENS(XET)	0.4706	0.4027	0.5385	0.9092	0.7317	1.0867	0.9791	0.6921	1.2660
SOCIETE GENERALE	0.5377*	0.5158	0.5596	1.0902	0.9662	1.2142	0.9302	0.8298	1.0305
TELEFONICA	0.5213	-1.0826	2.1252	1.0083	0.2125	1.8041	0.9408	-2.6411	4.5228
TOTAL	0.5052*	0.0545	0.9560	1.2301	-0.8313	3.2915	1.1512	0.3331	1.9692
UNIBAIL-RODAMCO	0.4913	0.4650	0.5175	1.0845	0.7523	1.4167	1.0914	0.8710	1.3117
UNICREDIT	0.5267*	0.5171	0.5364	1.0078	0.7668	1.2488	0.9203	0.5612	1.2794
UNILEVER CERTS.	0.4807*	0.4686	0.4928	0.9851	0.8904	1.0797	1.0365	0.7692	1.3038
VINCI	0.5059	0.4453	0.5665	1.2654	1.1505	1.3803	1.1253	1.0065	1.2442
VIVENDI	0.5166	0.3068	0.7265	1.1163	-2.2789	4.5116	1.0065	-0.3413	2.3543
VOLKSWAGEN PREF. (XET)	0.4613*	0.4608	0.4619	0.9106	0.7922	1.0291	1.0721	0.8712	1.2729

Table 13: Estimation results for parameters

Weekly:	CAPM-IAEPD								
	Normality, $H_0: \alpha = 0.5$ and $p_1 = p_2 = 2$								
	Laplace, $H_0: \alpha = 0.5$ and $p_1 = p_2 = 1$								
	$\hat{\alpha}$	$CI_{\hat{\alpha}}$	$\hat{p}_1$	$CI_{\hat{p}_1}$	$\hat{p}_2$	$CI_{\hat{p}_2}$			
AIR LIQUIDE	0.6571*	0.5246	0.7896	1.5595	-0.1139	3.2330	0.9935	0.6761	1.3109
AIRBUS GROUP	0.4710	0.3636	0.5784	1.0880	0.8602	1.3157	1.1609	0.8268	1.4951
ALLIANZ(XET)	0.3525*	0.3342	0.3708	0.9369	-5.7830	7.6567	1.3790	-8.4849	11.2428
ANHEUSER-BUSCH INBEV	0.3892	-11.8438	12.6222	1.0611	-2.1989	4.3211	1.2564	-0.8731	3.3859
ASML HOLDING	0.5028	-0.6205	1.6261	1.1929	0.6016	1.7842	1.3412	0.8578	1.8246
ASSICURAZIONI GENERALI	0.5329	0.4225	0.6432	1.4378	0.9956	1.8799	1.2068	0.9631	1.4506
AXA	0.2967	-1.7494	2.3429	0.8387	0.4779	1.1995	1.4086	0.3409	2.4763
BANCO SANTANDER	0.5527	-1.6617	2.7671	1.3336	-1.3157	3.9829	0.9863	0.8251	1.1474
BASF(XET)	0.3597	0.1954	0.5240	1.2582	0.7620	1.7545	1.7716	1.5064	2.0368
BAYER(XET)	0.3796	0.2171	0.5422	1.4444	0.7734	2.1154	1.8129	1.3255	2.3004
BBV.ARGENTARIA	0.6460*	0.5047	0.7873	1.8518	1.3264	2.3772	1.0624	0.8092	1.3156
BMW(XET)	0.5657*	0.5282	0.6031	1.5133	-0.8737	3.9003	1.0251	0.6113	1.4389
BNP PARIBAS	0.6175	-0.1587	1.3938	1.0827	1.0811	1.0843	0.7344	0.0321	1.4367
CARREFOUR	0.5523	0.4709	0.6336	1.3350	1.1862	1.4837	1.2720	1.0632	1.4808
CRH(DUB)	0.4216*	0.4136	0.4296	0.9479	0.7523	1.1435	1.3032	1.0838	1.5225
DAIMLER	0.5777	0.2700	0.8853	1.5880	1.3779	1.7981	1.1184	0.5146	1.7223
DANONE	0.3688	0.2138	0.5238	0.9013	0.2884	1.5142	1.4771	1.2492	1.7049
DEUTSCHE BANK(XET)	0.3736	0.1605	0.5867	0.7884	-0.2356	1.8124	0.9639	0.2891	1.6388
DEUTSCHE POST(XET)	0.1866*	0.1865	0.1866	0.6505	0.3610	0.9399	1.8490	1.4641	2.2340
DEUTSCHE TELEKOM(XET)	0.5001	-0.5196	1.5198	1.2144	0.6302	1.7986	1.1504	0.6654	1.6355
E.ON(XET)	0.3387*	0.3385	0.3388	0.7538	0.6382	0.8695	1.3377	0.7987	1.8767
ENEL	0.5161	-1.7933	2.8256	1.0432	-1.5106	3.5970	1.1764	-0.2047	2.5575
ENI	0.4076*	0.3908	0.4245	0.9307	0.5254	1.3360	1.4003	0.5339	2.2667
ESSILOR INTL.	0.8320	-9.9870	11.6510	2.4762	-55.1412	60.0936	0.7151	-8.4345	9.8647
GDF SUEZ	0.2486*	0.2486	0.2486	0.6575	0.3265	0.9884	1.8116	1.3503	2.2730
IBERDROLA	0.4193	0.2951	0.5436	0.8937	0.2310	1.5563	1.1342	0.8394	1.4290
INDITEX	0.3923	-4.2173	5.0019	1.0238	-4.2213	6.2689	1.3376	0.6513	2.0240
ING GROEP	0.4703	-1.3584	2.2991	0.7883	-2.9592	4.5359	0.8366	-6.2657	7.9390
INTESA SANPAOLO	0.3569*	0.2416	0.4722	0.9530	0.8931	1.0130	1.6651	1.2482	2.0820
L'OREAL	0.8063	-1.1537	2.7663	2.0677	0.1077	4.0277	0.6589	-1.3011	2.6189
LVMH	0.4957	0.4516	0.5397	1.1693	0.8285	1.5101	1.1857	0.8303	1.5411
MUENCHENER RUCK.(XET)	0.4091*	0.3772	0.4410	0.8831	0.4542	1.3121	1.0945	0.8137	1.3752
ORANGE	0.3325*	0.2867	0.3782	0.8177	0.6158	1.0196	1.2694	0.9433	1.5956
PHILIPS ELTN.KONINKLIJKE	0.4447*	0.4076	0.4818	1.2119	0.9074	1.5163	1.3262	0.9438	1.7086
REPSOL YPF	0.2751*	0.1862	0.3640	0.7374	0.5396	0.9352	1.7357	1.3517	2.1198
RWE(XET)	0.5441	0.4738	0.6143	1.1940	0.8479	1.5401	1.0736	0.7792	1.3679
SAINT GOBAIN	0.6446*	0.6090	0.6802	1.3318	1.0044	1.6593	0.7959	0.6322	0.9595
SANOFI	0.0613*	0.0613	0.0613	0.3640	-0.8397	1.5678	3.4932	2.7220	4.2644
SAP(XET)	0.4086*	0.4064	0.4109	0.9794	0.7378	1.2210	1.4065	0.9789	1.8342
SCHNEIDER ELECTRIC	0.3108*	0.1774	0.4442	0.9728	0.4647	1.4809	1.5830	1.1525	2.0135
SIEMENS(XET)	0.3662*	0.2906	0.4417	1.2161	0.8804	1.5519	2.1261	1.5114	2.7409
SOCIETE GENERALE	0.7940*	0.6127	0.9753	1.7703	1.5561	1.9844	0.6004	0.0748	1.1259
TELEFONICA	0.4956	0.4598	0.5313	1.1516	0.8571	1.4461	0.9975	0.7054	1.2897
TOTAL	0.2592	-1.3788	1.8973	1.0393	-1.6582	3.7368	2.4251	1.7398	3.1103
UNIBAIL-RODAMCO	0.6984	-1.2616	2.6584	1.5184	3.4784	2.8406	0.8806	-1.0794	2.8406
UNICREDIT	0.4517*	0.4512	0.4521	0.8296	0.6952	0.9640	1.2003	0.9915	1.4091
UNILEVER CERTS.	0.6853*	0.6718	0.6989	1.6350	1.2422	2.0277	0.9894	0.8074	1.1715
VINCI	0.6658	-5.5376	6.8691	1.8535	0.1370	3.5700	0.9736	-2.2276	4.1749
VIVENDI	0.6495*	0.6308	0.6683	1.2211	0.9650	1.4772	0.8324	0.6701	0.9947
VOLKSWAGEN PREF. (XET)	0.5234	0.3260	0.7208	1.3850	1.1113	1.6587	1.2624	-0.0882	2.6130

Table 14: Estimation results for parameters

Monthly:	CAPM-IAEPD							
	Normality, $H_0: \alpha = 0.5$ and $p_1 = p_2 = 2$							
	Laplace, $H_0: \alpha = 0.5$ and $p_1 = p_2 = 1$							
	$\hat{\alpha}$	$CI_{\hat{\alpha}}$	$\hat{p}_1$	$CI_{\hat{p}_1}$	$\hat{p}_2$	$CI_{\hat{p}_2}$		
AIR LIQUIDE	0.5153	0.4469	0.5836	1.1308	0.3861	1.8755	1.0110	0.4134
AIRBUS GROUP	0.2270*	0.2260	0.2279	0.7105	0.0950	1.3260	3.4043	-10.6646
ALLIANZ(XET)	0.5718	0.3547	0.7888	1.3928	0.6956	2.0901	1.4221	0.4679
ANHEUSER-BUSCH INBEV	0.3821	-2.7648	3.5290	1.6659	-8.1127	11.4444	1.1004	-3.6620
ASML HOLDING	0.3991	0.0824	0.7159	0.9756	-6.8874	8.8386	1.5370	-22.4167
ASSICURAZIONI GENERALI	0.5256	0.0168	1.0343	2.9700	-1.2502	7.1903	3.0022	-0.2985
AXA	0.0764*	0.0764	0.0764	0.4781	-2.9960	3.9523	5.3292	2.3831
BANCO SANTANDER	0.4570	-7.6770	8.5910	1.2668	-1.4185	3.9521	0.9417	-1.8229
BASF(XET)	0.3161*	0.1490	0.4833	1.2916	0.2154	2.3678	3.5888	1.3600
BAYER(XET)	0.2292*	0.1391	0.3194	1.1616	0.4303	1.8929	7.2625	-7.5537
BBV.ARGENTARIA	0.5555	-0.1739	1.2850	1.4557	0.8783	2.0331	1.0000	-0.2324
BMW(XET)	0.0001*	-0.1997	0.1998	88.4822	85.1641	91.8002	5.2173	-0.6374
BNP PARIBAS	0.1874	-1.7726	2.1474	0.6380	-1.3220	2.5980	0.5476	4.4676
CARREFOUR	0.6196	-0.4397	1.6788	2.0284	-0.8694	4.9263	1.4814	-1.6333
CRH(DUB)	0.2743*	0.2743	0.2743	0.6105	0.2584	0.9626	2.6096	1.1754
DAIMLER	0.7013	-0.6259	2.0284	4.9098	-5.8343	15.6540	1.0853	-1.3927
DANONE	0.2560*	0.2471	0.2648	1.1503	0.8127	1.4879	5.0316	1.1139
DEUTSCHE BANK(XET)	0.5724	-1.3876	2.5324	1.5863	-0.3737	3.5463	0.9510	-1.0090
DEUTSCHE POST(XET)	0.5853	0.1639	1.0068	2.0795	0.3515	3.8074	1.8831	-0.4377
DEUTSCHE TELEKOM(XET)	0.4406	0.0448	0.8363	1.2228	-0.2397	2.6854	1.7671	0.9966
E.ON(XET)	0.4067	0.0708	0.7425	1.2148	-0.2409	2.6706	1.5882	0.2180
ENEL	0.7149	-1.1335	2.5632	1.8266	-1.4425	5.0957	0.8826	-3.3960
ENI	0.7533	0.3456	1.1610	1.3034	-1.5164	4.1233	0.7276	0.1592
ESSILOR INTL.	0.2990*	0.1929	0.4052	1.2324	0.2720	2.1929	2.5595	0.2693
GDF SUEZ	0.2183*	0.2182	0.2183	0.6777	0.2920	1.0634	2.1285	1.5141
IBERDROLA	0.2901*	0.1025	0.4776	1.1946	-0.5148	2.9040	2.5123	-1.2337
INDITEX	0.0909*	0.0909	0.0909	0.6571	-10.3193	11.6334	3.2645	-1.9298
ING GROEP	0.4787	-0.0108	0.9683	0.6892	-2.1659	3.5443	0.7780	-1.5638
INTESA SANPAOLO	0.3102	0.0734	0.5470	1.4094	0.7888	2.0299	1.9314	1.3700
L'OREAL	0.8956	-1.0644	2.8556	7.3158	5.3558	9.2758	-1.0023	2.9177
LVMH	0.1991	-1.7609	2.1591	1.2710	-0.6890	3.2310	98.0399	101.9599
MUENCHENER RUCK.(XET)	0.3638	-0.3804	1.1081	1.2984	-3.5293	6.1261	2.2726	0.4858
ORANGE	0.4072	0.2946	0.5198	1.1867	0.3143	2.0592	1.2899	0.3478
PHILIPS ELTN.KONINKLIJKE	0.5648	0.2503	0.8792	3.3288	-0.0209	6.6786	2.3832	0.5025
REPSOL YPF	0.4411*	0.4287	0.4535	0.8238	0.6320	1.0157	1.7863	0.8553
RWE(XET)	0.2629	-1.6971	2.2229	0.8501	-1.1099	2.8101	1.9072	-0.0528
SAINT GOBAIN	0.2215	-1.7385	2.1815	0.7732	-1.1868	2.7332	3.1634	1.2034
SANOFI	0.0001	-1.9599	1.9601	86.6638	84.7038	88.6238	26.1538	24.1938
SAP(XET)	0.2038*	0.0704	0.3372	1.2399	0.8747	1.6051	2.6954	0.5451
SCHNEIDER ELECTRIC	0.8700*	0.8661	0.8740	2.7488	1.4968	4.0007	0.8530	0.7450
SIEMENS(XET)	0.3070	-1.6530	2.2670	0.8572	-1.1028	2.8172	2.0404	0.0804
SOCIETE GENERALE	0.3433*	0.2328	0.4539	1.4345	0.2140	2.6550	1.8619	1.0728
TELEFONICA	0.0000*	-0.0996	0.0998	87.7993	74.5538	101.0270	3.6394	-186.3123
TOTAL	1.0000	-0.9600	2.9600	4.0309	2.0709	5.9909	96.3701	94.4101
UNIBAIL-RODAMCO	0.7215	-0.0724	1.5154	2.0315	-0.0864	4.1494	0.9297	-1.9190
UNICREDIT	0.4296	-1.5304	2.3896	0.6838	-1.2762	2.6438	0.7342	-1.2258
UNILEVER CERTS.	0.4828	0.3455	0.6200	1.2432	0.4080	2.0785	99.9999	65.5483
VINCI	0.8335*	0.5593	1.1077	1.3877	0.4616	2.3138	0.5707	-7.1830
VIVENDI	0.2049	-0.7039	1.1138	0.7901	-0.3240	1.9043	2.7470	-26.4145
VOLKSWAGEN PREF. (XET)	0.4434	-0.0817	0.9686	1.2323	-2.1833	4.6479	1.1429	-1.0300