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Sampling moiré as a special windowed Fourier ridges algorithm in demodulation of carrier fringe patterns

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Abstract. In recent times, both the windowed Fourier ridges (WFR) and sampling moiré (SM) algorithms have been extensively used due to their high effectiveness in the demodulation of carrier fringe patterns. As they are developed independently, they are mostly recognized as completely different techniques, but we theoretically prove that SM is a special WFR with a specific window shape and a preset local frequency. This unifies the two different algorithms and enhances the understanding of their theoretical aspects, which helps to simplify the selection of these algorithms in real applications. © 2018 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.OE.57.10.100503]

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1 Introduction

High-accuracy phase extraction is essential for fringe-based optical measurement methods,¹ for which, the pixelwise phase shifting (PS),² the global Fourier transform (FT),³ and the local windowed Fourier transform (WFT)^{4–7} algorithms are the “continuous spectrum” of techniques for fringe pattern analysis.⁸ While PS extracts phase accurately and robustly from multiple phase-shifted fringe patterns, FT and WFT can do so from a single carrier fringe pattern. On the other hand, sampling moiré (SM)^{9–11} is a hybrid method to extract the phase from a single carrier fringe pattern using PS. Although it uses an inaccurate phase shift, the phase can be extracted accurately. We have compared SM and windowed Fourier ridge (WFR) using both simulated and experimental fringe patterns and observed almost identical performance. In this letter, for the first time, SM will be theoretically proved to be a special WFR algorithm with a specific window shape and a preset local frequency.

A carrier fringe pattern is usually two-dimensional (2-D), which can either be processed by a 2-D algorithm directly or

a one-dimensional (1-D) algorithm in a row-by-row manner. Since SM is primarily 1-D, the following 1-D fringe model is used:

$$f(x) = a(x) + b(x) \cos[\varphi(x) + \omega_g x], \quad (1)$$

where $x = 0, 1, \dots, M-1$ is the pixel index with M as the pixel number of the image width, $f(x)$, $a(x)$, $b(x)$, and $\varphi(x)$ are the captured fringe intensity, the background intensity, the fringe amplitude, and the phase to be determined, respectively, and ω_g is a global carrier frequency and corresponds to the fringe pitch P by $\omega_g = 2\pi/P$. This model has been commonly used,³ including in SM.^{9–11} Since $\varphi(x)$ varies spatially, it introduces an additional frequency $d\varphi(x)/dx$, which can be combined with ω_g to form the local frequency $\omega_x(x) = \omega_g + d\varphi(x)/dx$. This concept is used in WFR.^{4–7} Based on the above discussions, from now on, both SM and WFR are by default 1-D algorithms.

The content of this letter is structured as follows: Secs. 2 and 3 briefly introduce SM and WFR, respectively. In Sec. 4, SM is proved to be a special WFR. Section 5 provides further discussions, and finally, Sec. 6 concludes the letter.

2 Sampling Moiré Technique

To evaluate the fringe pattern shown in Eq. (1), SM preselects a sampling pitch T (an integer value) close to P . The carrier fringe pattern $f(x)$ is then downsampled every T -pixels into a total of T -step phase-shifted fringe patterns, and the intensity values of the pixels in between are interpolated from these downsampled pixels, resulting in the following moiré fringe patterns:^{9–11}

$$f_{m,t}(x) = \begin{cases} f(x), & \text{for } x = kT + t \\ \text{interpolated,} & \text{otherwise} \end{cases}, \quad (2)$$

where the subscript m indicates “moiré,” the subscript $t = 0, 1, \dots, T-1$ is the index of these phase-shifted fringe patterns, and k is an integer running from 0 to floor $[(M-t-1)/T]$. These moiré fringe patterns have the same size as the original carrier fringe pattern and can be written as^{9–11}

$$f_{m,t}(x) = a(x) + b(x) \cos[\varphi(x) + (\omega_g - \omega_s)x + \omega_s t], \quad (3)$$

where $\omega_s = 2\pi/T$ denotes the sampling frequency.

It is noted that (i) the global frequency ω_g is reduced by ω_s into $\omega_g - \omega_s$ due to the downsampling. Because T is close to P , the reduced global frequency is usually very small, the moiré fringe patterns with much wider fringe spacing are generated and can be interpolated with high accuracy; (ii) $\omega_s t$ is introduced due to different starting points of the downsampling and can be treated as a phase shift. Consequently, the phase of the moiré fringe patterns can be obtained using the T -step PS algorithm¹² written in a complex number form as

$$\varphi(x) + (\omega_g - \omega_s)x = \angle \sum_{t=0}^{T-1} [f_{m,t}(x) \exp(-j2\pi t/T)], \quad (4)$$

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where $j = \sqrt{-1}$ and $\angle$ takes the argument (or angle) of a complex number. To recover the phase with the original global frequency, the complex field in the right of Eq. (4) is multiplied by $\exp(j2\pi x/T)$, resulting in

$$\begin{aligned}\varphi_{\text{SM}}(x) &= \varphi(x) + \omega_g x \\ &= \angle \sum_{t=0}^{T-1} \{f_{m,t}(x) \exp[-j2\pi(t-x)/T]\},\end{aligned}\quad (5)$$

which will be used as the formal representation of SM for further analysis.

3 Windowed Fourier Ridges Algorithm

WFR has been explained in detail in Refs. 4–6 and is outlined for the completeness of this letter. Given a fringe pattern $f(x)$, its WFT is defined as

$$Sf(u; \xi_x) = \int_{-\infty}^{\infty} f(x)g(x-u) \exp[-j\xi_x(x-u)]dx, \quad (6)$$

where x and u are the spatial coordinates, ξ_x is the frequency coordinate, and $g(x)$ is a window function. Based on the WFT spectrum, WFR first estimates the local frequency from the ridge of the spectrum as

$$\hat{\omega}_x(u) = \arg \max_{\xi_x} |Sf(u; \xi_x)|, \quad (7)$$

and subsequently estimates the phase as

$$\hat{\varphi}(u) = \angle Sf[u; \hat{\omega}_x(u)], \quad (8)$$

$$f_{m,t}(u) = \begin{cases} \frac{1}{T} [(T+t-t_0) \cdot f(u+t-t_0) + (-t+t_0) \cdot f(u+T+t-t_0)] & \text{if } t < t_0 \\ \frac{1}{T} [(t-t_0) \cdot f(u-T+t-t_0) + (T-t+t_0) \cdot f(u+t-t_0)] & \text{if } t \geq t_0 \end{cases} \quad (11)$$

Substituting the above equation into Eq. (5) yields

$$\varphi_{\text{SM}}(u) = \angle \left(\begin{aligned} &\frac{1}{T} \sum_{t=0}^{t_0-1} \{(T+t-t_0) \cdot f(u+t-t_0) \exp[-j2\pi(t-u)/T]\} \\ &+ \frac{1}{T} \sum_{t=0}^{t_0-1} \{(-t+t_0) \cdot f(u+T+t-t_0) \exp[-j2\pi(t-u)/T]\} \\ &+ \frac{1}{T} \sum_{t=t_0}^{T-1} \{(t-t_0) \cdot f(u-T+t-t_0) \exp[-j2\pi(t-u)/T]\} \\ &+ \frac{1}{T} \sum_{t=t_0}^{T-1} \{(T-t+t_0) \cdot f(u+t-t_0) \exp[-j2\pi(t-u)/T]\} \end{aligned} \right). \quad (12)$$

Using a substitution $x = u + t - t_0$ for the first summation S_1 in Eq. (12), followed using the relationship $\exp[-j2\pi(-u+t_0)/T] = \exp(j2k\pi) = 1$ due to Eq. (10), gives

where $\wedge$ indicates that they are the estimations of the true values. Substituting the discrete version of Eq. (6) into Eq. (8) gives

$$\varphi_{\text{WFR}}(u) = \angle \sum_{x=u-W}^{u+W} \{f(x)g(x-u) \exp[-j\hat{\omega}_x \cdot (x-u)]\}, \quad (9)$$

where a limited window size of $[-W, W]$ has been assumed. In this representation, the window shape and size are general.

4 Sampling Moiré as a Special Windowed Fourier Ridges Algorithm

SM is based on downsampling, interpolation, and PS, whereas WFR is based on WFT. They look completely different. To explore their inherent relationship, we analyze SM in Eq. (5) by explicitly incorporating the intensity interpolation. For simplicity, first-order (i.e., linear) interpolation is examined first.

Let us consider a pixel at a general position,

$$u = kT + t_0, \quad (10)$$

where k is an integer and $0 \leq t_0 \leq T-1$. This pixel's intensity is retained in $f_{m,t_0}(x)$ during downsampling and thus does not require interpolation, i.e., $f_{m,t_0}(u) = f(u)$. In other phase-shifted moiré fringe patterns ($t \neq t_0$), all $f_{m,t}(u)$ are generated by interpolation. Two downsampled pixels, one on the left and one on the right, should be carefully located and then used for the linear interpolation:

$$S_1 = \sum_{x=u-t_0}^{u-1} \{(T+x-u) \cdot f(x) \exp[-j2\pi(x-u)/T]\}. \quad (13)$$

By doing the same manipulations for the other three summations, we have

$$\varphi_{\text{SM}}(u) = \angle \left(\begin{aligned} &\frac{1}{T} \sum_{x=u-t_0}^{u-1} \{(T+x-u) \cdot f(x) \exp[-j2\pi(x-u)/T]\} \\ &+ \frac{1}{T} \sum_{x=u+T-t_0}^{u+T-1} \{(T-x+u) \cdot f(x) \exp[-j2\pi(x-u)/T]\} \\ &+ \frac{1}{T} \sum_{x=u-T}^{u-t_0-1} \{(T+x-u) \cdot f(x) \exp[-j2\pi(x-u)/T]\} \\ &+ \frac{1}{T} \sum_{x=u}^{u+T-t_0-1} \{(T-x+u) \cdot f(x) \exp[-j2\pi(x-u)/T]\} \end{aligned} \right). \quad (14)$$

The first and the third terms can now be combined, so do the second and fourth terms, resulting in

$$\varphi_{\text{SM}}(u) = \angle \left(\begin{aligned} &\frac{1}{T} \sum_{x=u-T}^{u-1} \{(T+x-u) \cdot f(x) \exp[-j2\pi(x-u)/T]\} \\ &+ \frac{1}{T} \sum_{x=u}^{u+T-1} \{(T-x+u) \cdot f(x) \exp[-j2\pi(x-u)/T]\} \end{aligned} \right). \quad (15)$$

It can finally be arranged into

$$\varphi_{SM}(u) = \angle \sum_{x=u-T}^{u+T} \{f(x)g(x-u) \exp[-j\omega_s \cdot (x-u)]\}, \quad (16)$$

with $\omega_s = 2\pi/T$ being the angular frequency of downsampling and $g(x-u)$ being a triangular window as

$$g(x-u) = 1 - |x-u|/T, \quad -T \leq x-u \leq T. \quad (17)$$

Note that, when $x = u + T$, we have $g(x-u) = 0$, indicating that this pixel has no contribution to the summation, which allows us to change the upper summation limit from $u + T - 1$ to $u + T$, as shown in Eq. (16). Compared with WFR in Eq. (9), it is now clear that SM (first order) is in the same form of WFR with a triangular window of half size $W = T$ and a local frequency preset at $\hat{\omega}_x = \omega_s$.

SM (third order) using cubic interpolation is also often used, where a third-order polynomial is generated using four downsampled pixels, two on the right and two on the left. By the same derivation as SM (first order), it can again be led to the form of Eq. (16), but the window is slightly different: the window shape is smoother, and the window size is doubled. The windows for first- and third-order interpolations are shown in Fig. 1, with a Gaussian window shown for reference.

To verify this theoretically derived equivalence, we simulate a fringe pattern (256×256 pixels) with a phase of k times the peaks phase generated using the MATLAB peaks function, together with a carrier of fringe pitch $P = 8.1$ pixels as an example. Additive noise is added with different noise levels (NLs) (the ratio of the noise standard deviation to the fringe amplitude in percentage). The sampling pitch is set at $T = 8$ pixels. To exclude border effects around image boundaries for both methods, the central 200×200 pixels are involved for computing the root-mean-square (RMS) phase error. The SM is implemented according to Ref. 9. The WFR has already been used since the early work¹³ and has very minor changes since then. We merely modify the WFR code by changing the Gaussian window to

Fig. 1 Windows corresponding to intensity interpolation for SM (first order/third order), and the Gaussian window for WFR, where T expresses the sampling pitch.

Fig. 2 Simulation results of SM (first order/third order) and WFR (with first/third-order SM windows): (a) the phase complexity changes and (b) NL changes.

the windows corresponding to first- and third-order interpolations as shown in Fig. 1. We turn off the frequency scanning in WFR by setting both the lowest and highest frequencies to ω_s . The RMS phase errors obtained using SM and WFR for (i) $k = 0:0.5:4$, $NL = 0$ and (ii) $k = 1$, $NL = 0:0.2:1$ are shown in the left and right of Fig. 2, respectively. The form $a = b:c:d$ means that the parameter a changes from b to d with an increment of c . Although the principles and implementations of SM and WFR are completely different, their performances are observed to be identical in all these cases, verifying our theoretical analysis. The RMS phase error of SM (third order) is not significantly different from that of SM (first order), which can now be explained easily by their similar window shapes as shown in Fig. 1.

5 Discussion on Methodology Errors

The most obvious difference between SM and WFR is the local frequency determination. In WFR, the best local frequency is always scanned and searched using Eq. (7) and then used for phase extraction in Eq. (8). In SM, the sampling pitch is preset manually, which could be very different from the exact fringe pitch. No frequency scanning may sacrifice the phase accuracy dramatically, but it does not happen.

To quantitatively find it out, we resort to the theoretical WFR result where a Gaussian window with σ_x representing the window size⁵ is used. The windowed Fourier spectrum can be written as

$$Sf(u; \hat{\omega}_x) = f(u) \left(\frac{4\pi\sigma_x^2}{1 + \sigma_x^4 c_{xx}^2} \right)^{\frac{1}{4}} \exp \left[\frac{-\sigma_x^2 (\hat{\omega}_x - \omega_x)^2}{2(1 + \sigma_x^4 c_{xx}^2)} - j \frac{\sigma_x^4 c_{xx} (\hat{\omega}_x - \omega_x)^2}{2(1 + \sigma_x^4 c_{xx}^2)} + j \frac{1}{2} \arctan(\sigma_x^2 c_{xx}) \right], \quad (18)$$

where we have considered $f(u) = 0.5b(u) \exp[j\varphi(u)]$ as the fundamental component of the carrier fringe pattern.⁷ From $Sf(u; \hat{\omega}_x)$, the phase can be obtained as

$$\angle Sf(u; \hat{\omega}_x) = \varphi(u) - \frac{\sigma_x^4 c_{xx} (\hat{\omega}_x - \omega_x)^2}{2(1 + \sigma_x^4 c_{xx}^2)} + \frac{1}{2} \arctan(\sigma_x^2 c_{xx}). \quad (19)$$

The first term in the right of the above equation is exactly what we desire, but the second and third terms appear as methodology errors. For convenience, they are labeled as the frequency mismatch and the curvature errors, respectively. In WFR, the frequency mismatch error is suppressed by finding $\hat{\omega}_x$ that is closest to ω_x while the curvature error is suppressed by estimating c_{xx} and then deducting it by $\frac{1}{2} \arctan(\sigma_x^2 c_{xx})$.

The following further examines these methodology errors: (i) when $c_{xx} = 0$, i.e., the phase is locally linear, thus both the frequency mismatch error and the curvature error disappear. In such a case, SM can be used with perfect accuracy, even with a rough and inaccurate estimation of the sampling pitch. Nevertheless, a good estimation of the local frequency ω_x increases the amplitude of $Sf(u; \hat{\omega}_x)$ and consequently increases the robustness of phase extraction from noisy fringe patterns. (ii) When $c_{xx} \neq 0$, we consider the following typical values: a phase curvature of $c_{xx} = 0.01 \text{ rad}/(\text{pixel})^2$, a frequency mismatch error of $\hat{\omega}_x - \omega_x = 0.2 \text{ rad/pixel}$, and a window size of $\sigma_x = 5 \text{ pixels}$. The corresponding frequency mismatch and curvature errors are both around 0.12 rad (i.e., 2% of 2π). These errors are seen to be quite small, although their significance depends on specific applications. Overall, this analysis explains why SM still has high accuracy even without frequency scanning. Nevertheless, SM should be used with caution, and indeed, incorporating scanning into SM is being considered by the authors.

6 Conclusions

In this letter, SM is theoretically proved to be a special WFR algorithm when set with specific windows and without

frequency scanning, thus leading to two seemingly completely different techniques to be linked up and unified. This helps us to enhance the understanding of the algorithms, simplifies the technique selection for real applications, and enables further insights for developing even better fringe analysis techniques.

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