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**NANYANG
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SINGAPORE

**DESIGN, MODELLING AND
CHARACTERISATION OF A
MAGNETIC VARIABLE STIFFNESS
MECHANISM**

**LIM ZHEN YI
SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

2019

**DESIGN, MODELLING AND
CHARACTERISATION OF A
MAGNETIC VARIABLE STIFFNESS
MECHANISM**

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School of Mechanical and Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
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2019

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*Design, Modelling and
Characterisation of a
Magnetic Variable Stiffness Mechanism*

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

20 August 2019

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Date

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Zhen Yi Lim

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I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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This thesis does not contain any materials from papers published in peer-reviewed journals or from papers accepted at conferences in which I am listed as an author.

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Zhen Yi Lim

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Contents

Statement of Originality	1
Supervisor Declaration Statement	3
Authorship Attribution Statement	5
Acknowledgements	7
Table of Contents	9
List of Figures	13
List of Tables	59
Abbreviations	75
Symbols	81
Abstract	85
1 Introduction	87
1.1 Background	87
1.2 Motivation	91
1.3 Objective	93
1.4 Scope	94
1.5 Organization	94

2	Literature Review	97
2.1	Design of Variable Impedance Actuators	97
2.1.1	Fixed Compliance	98
2.1.2	Variable Stiffness/ Compliance	100
2.1.3	Fixed Damping	113
2.1.4	Variable Damping	114
2.1.5	Inertial Actuators	119
2.1.6	Combination	119
2.2	Parallel Elastic Actuators	121
2.3	Technology Gap	122
3	Proposed Mechanism Design	125
3.1	Overview	125
3.2	Mathematical Models – Halves Configuration	126
3.2.1	1-DOF Model	126
3.2.2	2-DOF Model	127
3.2.3	3-DOF Model	137
3.2.4	Simple Simulation Model	146
3.2.5	Ring of Magnets Simulation Model	159
3.3	Optimal Size	174
3.3.1	Simple Simulation Model	175
3.3.2	Ring of Magnets Simulation Model	177
3.4	Mathematical Models – Alternating Configuration	181
3.4.1	Simple Simulation Model	193
3.5	Mathematical Models – Discrete Alternating Configuration	207
3.5.1	Simple Simulation Model	218
3.6	Optimal Size – Discrete Alternating Configuration	231
3.6.1	Simple Simulation Model	236
3.6.2	Ring of Magnets Simulation Model	249
3.7	Conclusion	290

4	Experimental Results	293
4.1	Prototype Specifications	293
4.2	Prototype Simulation	297
4.3	Experimental Setup	299
4.4	Experimental Results	311
4.5	Sources of Errors	325
5	Conclusion	329
5.1	Contributions	330
5.2	Future Work	331
5.2.1	Improvements to Mechanical Design	331
5.2.2	Investigate Dynamic Performance of the Prototype . .	331
5.2.3	Design Suitable Control Algorithm for the Mechanism .	331
5.2.4	Investigate the Scaling Performance of the Mechanism	332
5.2.5	Test other Magnetisation Configurations	332
A	Optimal Size – Halves Configuration	333
A.1	Simple Simulation Model	334
B	Alternating Configuration – Simple Simulation Model	341
B.1	Outer:Diametrical—Inner:Diametrical	342
B.2	Outer:Radial—Inner:Diametrical	350
B.3	Outer:Radial—Inner:Radial	360
B.4	Outer:Halbach—Inner:Diametrical	370
C	Discrete Alternating Configuration – Simple Simulation Model	379
C.1	Outer:Radial—Inner:Diametrical	380
C.2	Outer:Radial—Inner:Radial	390
C.3	Outer:Halbach—Inner:Diametrical	396
D	Discrete Alternating Configuration – Ring of Magnets Sim- ulation Model	405
D.1	Outer:Diametrical—Inner:Diametrical	406

D.2 Outer:Radial—Inner:Diametrical 414

D.3 Outer:Radial—Inner:Radial 424

D.4 Outer:Halbach—Inner:Diametrical 430

E Prototype Torque Profile Comparisons 439

References 439

List of Figures

1.1	Illustration of the working principle of the proposed magnetic variable stiffness mechanism. The rotor magnet is coloured orange, and the ring magnets are divided into two groups, coloured red and blue. The oppositional rotation of the two groups of ring magnets allow the magnetic field strength experienced by the rotor to decrease and hence modulates the restoring torque on the rotor magnet when it is deflected from it's equilibrium position.	93
2.1	Concept of the Series Elastic Actuator (Series Elastic Actuator (SEA)) [1].	98
2.2	Prototype of the ComPact © Soft Actuator (ComPact ©), a rotary SEA [2].	99
2.3	Concept of the Safe Link Mechanism (SLM) [3].	100
2.4	Configurations with antagonistic springs and antagonistic motors: (a) uni-directional, (b) cross-coupled and (c) bi-directional [4].	102
2.5	Configurations with antagonistic springs and independent motors: (a) partial decoupling, (b) complete decoupling with positioning motor at the joint and (c) complete decoupling without motor at joint [4].	103
2.6	Controlling the transmission ratio by changing: (a) spring position, (b) force application point and (c) pivot point [4]. . . .	104

2.7	CAD model of the Variable Stiffness Joint using Leaf Springs (VSJ) using Leaf Springs [5].	106
2.8	Concept for variables stiffness using granular jamming [6]. . .	107
2.9	Computer-aided Design (CAD) model of the Magnetic Variable Stiffness Actuator (MVSA) [7].	108
2.10	Concept for the MAgnetic Variable stiffnEss spRIng-Clutch (MAgnetic Variable stiffnEss spRIng-Clutch (MAVERIC)) device [8].	108
2.11	CAD model of the Variable Stiffness Unit (VSU) [9].	109
2.12	Prototype of the Mechanical-Magnetic Variable Stiffness Actuator (Mechanical-Magnetic Variable Stiffness Actuator (M ² -VSA)) [10].	109
2.13	Various principles for variable damping: (a) controlled friction, (b) controlled rheology (magnetic), (c) fluid dynamics using variable orifice, (d) eddy current and (e) laminar fluid dynamics damping [4].	115
2.14	Configurations for combining variable stiffness and variable dampers: (a) in parallel, (b) in parallel with damper connected to an external frame, and (c) and (d) in series [4].	120
3.1	Illustration of the magnetic spring concept. The magnetic dipole is coloured orange and situated at the centre of the Cartesian coordinate system, deflected about the z-axis by an angle θ relative to the $\mathbf{B}$ field oriented along the x-axis. $\boldsymbol{\tau}$ is the restoring torque about the z-axis generated when $\theta \neq 0$ . .	126

- 3.2 Simplified model of a novel magnetic variable stiffness actuator. An outer ring of magnets represented by a circle of magnetic dipoles with positions $\mathbf{r}$ defined by the angle γ is composed of two parts: the red semi-circle forming the virtual north pole, and the blue semi-circle forming the virtual south pole. Dipoles on both halves are initially oriented in the x-direction to produce an effective diametrical magnetisation when the actuator is in an unmodulated configuration. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ 130
- 3.3 3D view of the simplified model of a novel magnetic variable stiffness actuator with effective diametrical magnetisation. τ is the restoring torque about the z-axis generated when the rotor is deflected. 131
- 3.4 Simplified 2-DOF model of the variable stiffness actuator with effective diametrical magnetisation. For modulation of the magnetic coupling strength, the magnetic dipoles within the outer ring are rotated by an angle ϕ and $-\phi$ about their local z-axes for the virtual north pole and the virtual south pole respectively. 131
- 3.5 An alternative setup where the dipoles within the virtual north pole are oriented to point away, and those within the virtual south pole are oriented to point towards the centre of the circle. This produces an effective radial magnetisation when the actuator is in an unmodulated configuration. 133
- 3.6 3D view of the simplified model of a novel magnetic variable stiffness actuator with effective radial magnetisation. τ is the restoring torque about the z-axis generated when the rotor is deflected. 133

- 3.7 Simplified 2-DOF model of the variable stiffness actuator with effective radial magnetisation. The effective radial magnetisation is achieved by rotating the magnetic dipoles within the outer ring originally oriented to point in the x-direction by an angle γ and $\pi + \gamma$ about their local z-axes for the virtual north pole and the virtual south pole respectively. For modulation of the magnetic coupling strength, the magnetic dipoles within the outer ring are further rotated by an angle ϕ and $-\phi$ about their local z-axes for the virtual north pole and the virtual south pole respectively. 134
- 3.8 Halbach cylinders with magnetisation patterns characterised by the parameter k [11]. 134
- 3.9 Another setup where the dipoles in both the virtual north pole and virtual south pole are oriented at an angle 2γ relative to the x-axis as specified to form a Halbach cylinder. 136
- 3.10 3D view of the simplified model of a novel magnetic variable stiffness actuator with Halbach cylinder magnetisation. τ is the restoring torque about the z-axis generated when the rotor is deflected. 136
- 3.11 Simplified 2-DOF model of the variable stiffness actuator with Halbach cylinder magnetisation where the dipoles in both the virtual north pole and virtual south pole are oriented at an angle of 2γ relative to the x-axis. For modulation of the magnetic coupling strength, the magnetic dipoles within the outer ring are further rotated by an angle ϕ and $-\phi$ about their local z-axes for the virtual north pole and the virtual south pole respectively. 137

3.12	Simplified 3-DOF model of the variable stiffness actuator with effective diametrical magnetisation. The equilibrium position of the actuator is adjusted by further rotating the magnetic dipoles within the outer ring by an angle ψ about their local z-axes.	140
3.13	Simplified 3-DOF model of the variable stiffness actuator with effective radial magnetisation. The equilibrium position of the actuator is adjusted by further rotating the magnetic dipoles within the outer ring by an angle ψ about their local z-axes. .	141
3.14	Setting of ψ for any particular desired equilibrium orientation θ_d	142
3.15	Simplified 3-DOF model of the variable stiffness actuator with Halbach cylinder magnetisation. The equilibrium position of the actuator is adjusted by further rotating the magnetic dipoles within the outer ring by an angle $-\psi$ about their local z-axes.	145
3.16	Top view of the geometry of the simple simulation setup. The direction of magnetisation of the materials for the “Halves” configuration are defined in Table 3.4.	148
3.17	Isometric view of the geometry of the simple simulation setup with a diametrically magnetised rotor magnet. Magnetisation of the materials for the Halves configuration are as defined in Table 3.4.	149
3.18	Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi = 0$ rad for the simple simulation model.	149
3.19	Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	150
3.20	Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi = 0$ rad for the simple simulation model.	151

3.21 Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	151
3.22 Torque profiles generated by the Havles Radial-Diametrical magnetisation setup for $\psi = 0$ rad for the simple simulation model.	152
3.23 Torque profiles generated by the Halves Radial-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	153
3.24 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Halves Radial-Diametrical magnetisation for the simple simulation model.	153
3.25 Torque profiles generated by the Havles Radial-Radial magnetisation setup for $\psi = 0$ rad for the simple simulation model.	154
3.26 Torque profiles generated by the Halves Radial-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	155
3.27 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Halves Radial-Radial magnetisation for the simple simulation model.	156
3.28 Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi = 0$ rad for the simple simulation model.	156
3.29 Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	157
3.30 Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi = 0$ rad for the simple simulation model.	158
3.31 Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	158

- 3.32 Ring of Magnets geometry for number of ring magnets $n = 4$.
The solid curves represents the rotor and ring magnets, while the dashed curves represents the space needed for each magnet to maintain the desired gap δ between all magnets, with radii $\frac{\delta}{2}$ larger than their solid counterparts. r_{rtr} is the radius of the rotor magnet, and r_{rng} is the radius of the ring magnets. . . . 160
- 3.33 Top view of the geometry of the ring of magnets simulation setup, with ring magnets labelled for the Diametrical or Halbach magnetisation configurations. The direction of magnetisation of the materials for the “Halves” configuration are defined in Table 3.8. Magnetisation directions are illustrated for $\psi = 25^\circ$, $\phi = 80^\circ$. The magnets belonging to the virtual “north pole” and “south pole” are drawn in red and blue respectively. 163
- 3.34 Top view of the geometry of the ring of magnets simulation setup, with ring magnets labelled for the Radial magnetisation configuration. The direction of magnetisation of the materials for the “Halves” configuration are defined in Table 3.8. Magnetisation directions are illustrated for $\psi = 25^\circ$, $\phi = 80^\circ$. The magnets belonging to the virtual “north pole” and “south pole” are drawn in red and blue respectively. 164
- 3.35 Isometric view of the geometry of the ring of magnets simulation setup with a radially magnetised rotor magnet. Magnetisation of the materials for the Halves configuration are as defined in Table 3.8. 165
- 3.36 Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model. 165
- 3.37 Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model. 166

3.38 Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	166
3.39 Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	167
3.40 Torque profiles generated by the Halves Radial-Diametrical magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	167
3.41 Torque profiles generated by the Halves Radial-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	168
3.42 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Halves Radial-Diametrical magnetisation for the ring of magnets simulation model.	169
3.43 Torque profiles generated by the Halves Radial-Radial magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	169
3.44 Torque profiles generated by the Halves Radial-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	170
3.45 Torque deviation when $\theta = -\psi$ for $\phi = 0^\circ$ for the Halves Radial-Radial magnetisation for the ring of magnets simulation model.	171
3.46 Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	171
3.47 Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	172

3.48	Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	173
3.49	Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	173
3.50	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration. . . .	176
3.51	Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.	177
3.52	Holding Torques generated for the Ring of Magnets Simulation Model for varying number of ring magnets ($2n$) for the Halves magnetisation configurations.	179
3.53	Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying number of ring magnets ($2n$) for the Halves magnetisation configurations.	180
3.54	Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the "Virtual North Pole" for the "Halves" configuration.	182
3.55	Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the "Virtual South Pole" for the "Halves" configuration.	182
3.56	Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the "Virtual North Pole" for the "Alternating" configuration.	183
3.57	Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the "Virtual South Pole" for the "Alternating" configuration.	183

- 3.58 Illustration of the working principle of the proposed magnetic variable stiffness mechanism in alternating configuration. The rotor magnet is coloured orange, and the ring magnets are divided into two groups, coloured red and blue. The oppositional rotation of the two groups of ring magnets allow the magnetic field strength experienced by the rotor to decrease and hence modulates the restoring torque on the rotor magnet when it is deflected from it's equilibrium position. 184
- 3.59 Illustration of the “Alternating” magnetisation configuration for the effective Diametrical and Halbach magnetisation configurations for $n = 6$. The circle of magnetic dipoles are grouped into the virtual north pole and the virtual south pole, coloured red and blue respectively. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ 185
- 3.60 Illustration of the “Alternating” magnetisation configuration for the effective Radial magnetisation configuration for $n = 6$. The circle of magnetic dipoles are grouped into two semi-circular halves, forming the virtual north pole and the virtual south pole, coloured in shades of red and blue, and labelled i_n and i_s respectively. Within each half the magnetic dipoles that will be rotated by $-\phi$ rad are assigned a lighter colour than those that will be rotated by ϕ rad, labelled i_- and i_+ respectively. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ 190

- 3.61 Illustration of the “Alternating” magnetisation configuration for the effective Radial magnetisation configuration for $n = 5$. The circle of magnetic dipoles are grouped into two semi-circular halves, forming the virtual north pole and the virtual south pole, coloured in shades of red and blue, and labelled i_n and i_s respectively. Within each half the magnetic dipoles that will be rotated by $-\phi$ rad are assigned a lighter colour than those that will be rotated by ϕ rad, labelled i_- and i_+ respectively. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ 191
- 3.62 Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model. 196
- 3.63 Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model. 197
- 3.64 Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model. 198
- 3.65 Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model. 198
- 3.66 Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model. 200
- 3.67 Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model. 200

3.68	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.	201
3.69	Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	202
3.70	Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	202
3.71	Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	204
3.72	Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	205
3.73	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.	205
3.74	Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	206
3.75	Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	206
3.76	Torque profiles generated by the Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	208
3.77	Torque profiles generated by the Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	208

3.78	Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	209
3.79	Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	209
3.80	Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	221
3.81	Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	221
3.82	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.	222
3.83	Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	223
3.84	Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	224
3.85	Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	225
3.86	Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	225
3.87	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.	226

3.88	Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	226
3.89	Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	227
3.90	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the simple simulation model.	227
3.91	Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	228
3.92	Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	228
3.93	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the simple simulation model.	229
3.94	Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	230
3.95	Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	230
3.96	Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	231
3.97	Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	232

3.98 Holding Torques generated for the Simple Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.	238
3.99 Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.	238
3.100 Holding Torques generated for the Ring of Magnets Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.	250
3.101 Holding Torque to Magnets' Volume Ratio for the Ring of Magnets Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.	251
3.102 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	252
3.103 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	253
3.104 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	253
3.105 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	254

- 3.106 The holding torques with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 256
- 3.107 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. . . . 257
- 3.108 Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model. 259
- 3.109 Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model. 259
- 3.110 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the ring of magnets simulation model. 260
- 3.111 Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model. 261
- 3.112 Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model. 261

- 3.113 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 264
- 3.114 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 266
- 3.115 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 267

- 3.116 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 268
- 3.117 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 270
- 3.118 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 271
- 3.119 Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model. 273
- 3.120 Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model. 273

3.121 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the ring of magnets simulation model.	274
3.122 Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	274
3.123 Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	275
3.124 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the ring of magnets simulation model.	275
3.125 Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	276
3.126 Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	276
3.127 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the ring of magnets simulation model.	277
3.128 The holding torques with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.	278

- 3.129 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 279
- 3.130 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model. 280
- 3.131 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model. 281
- 3.132 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model. 282
- 3.133 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model. 282
- 3.134 The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 284

- 3.135 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 285
- 3.136 The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 288
- 3.137 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation. 289

4.1	CAD drawing of the magnetic region of the prototype with the sidewalls of the magnet holders removed for clarity. The ring magnets are coloured red and blue for the “Virtual North Pole” and the “Virtual South Pole” respectively. The central rotor magnet is coloured orange, and the gears are coloured yellow.	295
4.2	Transverse cross-section of the CAD drawing of the prototype with the sidewalls of the magnet holders removed for clarity. The yellow circles represent the pitch circle of the spur gears. The central rotor magnet is coloured orange. The ring magnets belonging to the “Virtual North Pole” are coloured red and are each attached to a small ring spur gear that meshes with a larger central spur gear.	296
4.3	Transverse cross-section of the CAD drawing of the prototype with the sidewalls of the magnet holders removed for clarity. The yellow circles represent the pitch circle of the spur gears. The central rotor magnet is coloured orange. The ring magnets belonging to the “Virtual South Pole” are coloured blue and are each attached to a small ring spur gear that meshes with a larger central spur gear.	296
4.4	Predicted torque profiles for the prototype using the Discrete Alternating Halbach-Diametrical-6 setup with the ring of magnets simulation model for $\psi = 0$ rad.	298
4.5	Predicted torque profiles for the prototype using the Discrete Alternating Halbach-Diametrical-6 setup with the ring of magnets simulation model for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad.	298
4.6	Proof-of-concept prototype.	300

- 4.7 A power flow diagram for the mechanism. In this cross-sectional view, the gear assemblies are coloured yellow, the rotor output shaft is coloured green, the hollow shafts connecting the worm wheels to their respective main spur gears are coloured pink, the Rotor Magnet is coloured orange, and the ring magnets are colour-coded blue and red for the “Virtual North Pole” and “Virtual South Pole” respectively. Note that for convenience the power from both inputs are each shown to flow through a single blue-coloured ring magnet, while in reality the blue-coloured and red-coloured ring magnets are two separate groups each controlled by one of the inputs. 300
- 4.8 A single magnet used in the ring of magnets setup. 301
- 4.9 3D-printed tool used to mark the ring magnets for alignment with their holders. 301
- 4.10 First, the magnet was placed on a metal sheet with one end protruding slightly over the edge of the metal sheet. 302
- 4.11 Next, the tool was then fitted onto the protruding end of the magnet, making sure that the flat surface of the tool is flush with the tabletop. 302
- 4.12 Finally, the magnet was pushed into the tool until the proximal end was flush with the slotted flat surface at the front of the tool. A marker was then used to mark the magnet through the top half of the slot. 302
- 4.13 Ring magnets in their brass holders. 303
- 4.14 3D-printed tool used to mark the brass ring magnet holders for alignment with their holders. 303
- 4.15 An example of the marking made on a brass magnet holders to indicate the desired orientation of the ring magnet. 304

4.16	CUI AMT203 Modular Absolute Encoders. The central unit provides readings of the rotor orientation while remaining two is responsible for providing the orientation of the two groups of ring magnets.	305
4.17	3D-printed tool used to locate the default position for both groups of ring magnets, accomplished by resting the notch of the tool on the side of the mechanism and rotating the ring magnets via their coupled worm gear such that the marking on the magnet holder lines up with the bottom edge of the distal end of the tool.	305
4.18	The experimental setup used to collect the torque readings for various settings of θ , ψ and ϕ	307
4.19	The wiring box connecting the encoder and torque sensor to a multifunction I/O device on a windows computer.	308
4.20	The LabView software interface used to take readings for the characteristaion of the prototype.	308
4.21	A set of Polyvinyl Chloride (PVC) blocks machined to specific dimensions to produce a regular set of angular positions θ at intervals of 7.5° for the rotor magnet when its output lever rests on their top left/ right edge.	309
4.22	The output lever deflects the rotor magnet 45° from its equilibrium position, held in place by a machined PVC block. . . .	309
4.23	The output lever deflects the rotor magnet -45° from its equilibrium position, held in place by a machined PVC block. . . .	310
4.24	The rotor magnet is at its equilibrium position, with the output lever resting gently on the side of the mechanism, in preparation for readings to be taken for positive deflection of the rotor magnet.	310

4.25	The rotor magnet is at its equilibrium position, with the output lever resting gently on the side of the mechanism, in preparation for readings to be taken for negative deflection of the rotor magnet.	311
4.26	Levers coupled to the worm gear connected to each group of ring magnets, for manual manipulation of the ring magnets into precise positions for torque load readings to be taken. Motors can be used in the place of the levers to turn the variable stiffness mechanism into an actuator, but this will not be covered in this work.	312
4.27	Torque profiles generated by the prototype for $\psi = 0^\circ$	313
4.28	Torque profiles generated by the prototype for $\psi = 45^\circ$	313
4.29	Torque profiles generated by the prototype for $\psi = 90^\circ$	314
4.30	Torque profiles generated by the prototype for $\psi = 135^\circ$	314
4.31	Torque profiles generated by the prototype for $\psi = 180^\circ$	315
4.32	Torque profiles generated by the prototype for $\psi = -135^\circ$ . . .	315
4.33	Torque profiles generated by the prototype for $\psi = -90^\circ$	316
4.34	Torque profiles generated by the prototype for $\psi = -45^\circ$	316
4.35	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$	317
4.36	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	318
4.37	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$	318
4.38	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	319
4.39	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$	319
4.40	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$	321

4.41	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	321
4.42	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$	322
4.43	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	322
4.44	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$	323
4.45	Marking used to realign the spur gear and the magnet holder, to ensure that the magnets are their desired orientationn during the assembly of the prototype.	326
4.46	Approximate locations of the ring magnets encased within their holders. Note the unintended small axial offset between the two groups of magnets.	328
A.1	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Diametrical configuration. .	335
A.2	Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Diametrical configuration.	335
A.3	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Radial configuration. . . .	336
A.4	Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Radial configuration.	336

A.5	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Diametrical configuration.	337
A.6	Holding Torque to Magnets' Volume Ratio for the Simple Sim- ulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Diametrical con- figuration.	337
A.7	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Radial configuration.	338
A.8	Holding Torque to Magnets' Volume Ratio for the Simple Sim- ulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Radial configu- ration.	338
A.9	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.	339
A.10	Holding Torque to Magnets' Volume Ratio for the Simple Sim- ulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.	339
A.11	Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Radial configuration.	340
A.12	Holding Torque to Magnets' Volume Ratio for the Simple Sim- ulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Radial config- uration.	340

B.1	Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	343
B.2	Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	343
B.3	Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	344
B.4	Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	344
B.5	Torque profiles generated by the Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	345
B.6	Torque profiles generated by the Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	345
B.7	Torque profiles generated by the Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	346
B.8	Torque profiles generated by the Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	346
B.9	Torque profiles generated by the Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	347
B.10	Torque profiles generated by the Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	347

B.11 Torque profiles generated by the Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	348
B.12 Torque profiles generated by the Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	348
B.13 Torque profiles generated by the Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	349
B.14 Torque profiles generated by the Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	349
B.15 Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	351
B.16 Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	351
B.17 Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	352
B.18 Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	352
B.19 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.	353
B.20 Torque profiles generated by the Alternating Radial-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	353

B.21 Torque profiles generated by the Alternating Radial-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	354
B.22 Torque profiles generated by the Alternating Radial-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	354
B.23 Torque profiles generated by the Alternating Radial-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	355
B.24 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-5 magnetisation setup for the simple simulation model.	355
B.25 Torque profiles generated by the Alternating Radial-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	356
B.26 Torque profiles generated by the Alternating Radial-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	356
B.27 Torque profiles generated by the Alternating Radial-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	357
B.28 Torque profiles generated by the Alternating Radial-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	357
B.29 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-7 magnetisation setup for the simple simulation model.	358
B.30 Torque profiles generated by the Alternating Radial-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	358

B.31 Torque profiles generated by the Alternating Radial-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	359
B.32 Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	361
B.33 Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	361
B.34 Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	362
B.35 Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	362
B.36 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.	363
B.37 Torque profiles generated by the Alternating Radial-Radial-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	363
B.38 Torque profiles generated by the Alternating Radial-Radial-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	364
B.39 Torque profiles generated by the Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	364
B.40 Torque profiles generated by the Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	365

B.41 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-5 magnetisation setup for the simple simulation model.	365
B.42 Torque profiles generated by the Alternating Radial-Radial-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	366
B.43 Torque profiles generated by the Alternating Radial-Radial-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	366
B.44 Torque profiles generated by the Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	367
B.45 Torque profiles generated by the Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	367
B.46 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-7 magnetisation setup for the simple simulation model.	368
B.47 Torque profiles generated by the Alternating Radial-Radial-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	368
B.48 Torque profiles generated by the Alternating Radial-Radial-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	369
B.49 Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	371
B.50 Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	371

B.51 Torque profiles generated by the Alternating Halbach-Diametrical- 3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	372
B.52 Torque profiles generated by the Alternating Halbach-Diametrical- 3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	372
B.53 Torque profiles generated by the Alternating Halbach-Diametrical- 4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	373
B.54 Torque profiles generated by the Alternating Halbach-Diametrical- 4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	373
B.55 Torque profiles generated by the Alternating Halbach-Diametrical- 5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	374
B.56 Torque profiles generated by the Alternating Halbach-Diametrical- 5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	374
B.57 Torque profiles generated by the Alternating Halbach-Diametrical- 6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	375
B.58 Torque profiles generated by the Alternating Halbach-Diametrical- 6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	375
B.59 Torque profiles generated by the Alternating Halbach-Diametrical- 7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	376
B.60 Torque profiles generated by the Alternating Halbach-Diametrical- 7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	376

B.61	Torque profiles generated by the Alternating Halbach-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	377
B.62	Torque profiles generated by the Alternating Halbach-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	377
C.1	Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	381
C.2	Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	381
C.3	Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	382
C.4	Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	382
C.5	Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.	383
C.6	Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	383
C.7	Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	384
C.8	Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	384

C.9 Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	385
C.10 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-5 magnetisation setup for the simple simulation model.	385
C.11 Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	386
C.12 Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	386
C.13 Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	387
C.14 Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	387
C.15 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-7 magnetisation setup for the simple simulation model.	388
C.16 Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	388
C.17 Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	389
C.18 Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	391

C.19 Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	391
C.20 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.	392
C.21 Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	392
C.22 Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	393
C.23 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the simple simulation model.	393
C.24 Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	394
C.25 Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	394
C.26 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the simple simulation model.	395
C.27 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	397
C.28 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	397

C.29 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	398
C.30 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	398
C.31 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	399
C.32 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	399
C.33 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	400
C.34 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	400
C.35 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	401
C.36 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	401
C.37 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	402
C.38 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	402

C.39	Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.	403
C.40	Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.	403
D.1	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	407
D.2	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	407
D.3	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	408
D.4	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	408
D.5	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	409
D.6	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	409
D.7	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	410
D.8	Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	410

D.9 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	411
D.10 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	411
D.11 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	412
D.12 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	412
D.13 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	413
D.14 Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	413
D.15 Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	415
D.16 Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	415
D.17 Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	416
D.18 Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	416

D.19 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the ring of magnets simulation model.	417
D.20 Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	417
D.21 Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	418
D.22 Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	418
D.23 Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	419
D.24 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-5 magnetisation setup for the ring of magnets simulation model.	419
D.25 Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	420
D.26 Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	420
D.27 Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	421
D.28 Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	421

D.29 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-7 magnetisation setup for the ring of magnets simulation model.	422
D.30 Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	422
D.31 Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	423
D.32 Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	425
D.33 Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	425
D.34 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the ring of magnets simulation model.	426
D.35 Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	426
D.36 Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	427
D.37 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the ring of magnets simulation model.	427
D.38 Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	428

D.39 Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	428
D.40 Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the ring of magnets simulation model.	429
D.41 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	431
D.42 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	431
D.43 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	432
D.44 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	432
D.45 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	433
D.46 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	433
D.47 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	434
D.48 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	434

D.49 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	435
D.50 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	435
D.51 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	436
D.52 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	436
D.53 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.	437
D.54 Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.	437
E.1 Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$	440
E.2 Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	440
E.3 Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$	441
E.4 Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	441
E.5 Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$	442
E.6 Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$	442

E.7	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	443
E.8	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$	443
E.9	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	444
E.10	Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$	444
E.11	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$	445
E.12	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	445
E.13	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$	446
E.14	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	446
E.15	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$	447
E.16	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$	447
E.17	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	448
E.18	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$	448
E.19	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	449
E.20	Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$	449
E.21	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$	450

E.22	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	450
E.23	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$	451
E.24	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	451
E.25	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$	452
E.26	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$	452
E.27	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$	453
E.28	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$	453
E.29	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$	454
E.30	Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$	454

List of Tables

2.1	Comparison of variable stiffness actuators/ mechanisms. . . .	110
2.2	Comparison of variable stiffness actuators/ mechanisms, continued.	111
3.1	Comparison of the holding torques derived from the 2-Degree(s) of Freedom (DOF) mathematical models. Their magnitudes relative to the Diametrical magnetisation configuration are also shown.	136
3.2	Comparison of the holding torques derived from the 3-DOF mathematical models. Their magnitudes relative to the Diametrical magnetisation configuration are also shown.	146
3.3	Parameters of the simple simulation setup.	147
3.4	Definitions of the magnetic material in <i>ANSYS Maxwell</i> for the Halves magnetisation configurations for the simple simulation model. The Z component of is set to 0 in all cases.	147
3.5	Properties of magnetic material used for Finite Element Method (FEM) analysis.	148
3.6	Ring magnet size r_{rng} , radial position of ring magnets $r_{pos,rng}$ and overall setup size r_{setup} for $r_{rtr} = 6$ mm, $\delta = 0.25$ mm and $n \in [2, 3, 4, 5, 6, 7, 8]$	160
3.7	Parameters of the ring of magnets simulation setup.	161

3.8	Definitions of magnetic material in <i>ANSYS Maxwell</i> for the Halves magnetisation configurations for the ring of magnets simulation model. The ring of magnets is composed of $2n$ magnets divided evenly into 2 “poles”, with individual magnets from each “pole” numbered $i \in [0, n - 1]$, counting up from the ones located proximal to the negative y-axis, counter-clockwise for the virtual “north pole” and clockwise for the virtual “south pole” for the Diametrical and Halbach magnetisation configurations. For the Radial magnetisation configuration, the virtual “south pole” counts up from the one proximal to the y-axis in a counter-clockwise fashion instead. The Z component is set to 0 in all cases.	162
3.9	Optimal radius ratio $x = \frac{r_{out}}{r_{in}}$ and their corresponding torque generated per unit volume $R_{\tau,v}$ for each Halves magnetisation configuration for the Simple Simulation Model.	178
3.10	Sampling intervals for θ for setups with discrete magnets for varying n	178
3.11	Optimal number of ring magnets and their corresponding torque generated per unit volume $R_{\tau,v}$ for each Halves magnetisation configuration for the Ring of Magnets Simulation Model. . . .	180
3.12	Multiplier for the holding torque with respect to the maximum holding torque of the $n \in [3, 5, 7]$ setups, and the deviation of the setting of ϕ for the freewheeling state from $\frac{\pi}{2}$ rad for the even numbered Alternating Radial magnetisation configurations.	189
3.13	Comparison of the holding torques derived for the “Alternating” configurations. Their magnitudes relative to the Diametrical magnetisation configuration are also shown. The Diametrical magnetisation configuration for $n = 2$, Radial magnetisation configurations for $n \in [2, 4, 6, 8]$, and Halbach magnetisation configuration for $n = 2$ are omitted as their rotor equilibrium positions are not easily controlled.	193

3.14	Additional sampling points for ϕ for the setups in which a deviation of the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, for the simple simulation model. Also included are the expected values of ϕ to attain the freewheeling state as obtained from the mathematical models for the Alternating configurations.	194
3.15	Definitions of the magnetic material in <i>ANSYS Maxwell</i> for the Alternating magnetisation configurations for the simple simulation model. The z-component of is set to 0 in all cases. .	195
3.16	Holding torques with no modulation for the Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model.	196
3.17	Holding torques with no modulation for the Alternating Radial-Diametrical magnetisation configurations for the simple simulation model. The holding torques for $n \in [2, 4, 6, 8]$ as a ratio of that of $n = 3$ are also shown, along with the multiplier values predicted by the mathematical model.	199
3.18	The settings of ϕ at which the freewheeling effect occurs, and the expected setting as derived from the mathematical model, for the Alternating Radial-Diametrical magnetisation configurations for the simple simulation model.	201
3.19	Holding torques with no modulation for the Alternating Radial-Radial magnetisation configurations for the simple simulation model. For $n \in [2, 4, 6, 8]$, the holding torque reading is taken at $\phi = 0$ rad for which the “constant torque spring” torque profile is observable. The holding torques for $n \in [2, 4, 6, 8]$ as a ratio of that of $n = 3$ are also shown, along with the multiplier values predicted by the mathematical model. . . .	203
3.20	Holding torques with no modulation for the Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model.	207

3.21	Overall multiplier and the holding torque multiplier with respect to the maximum holding torque of the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configuration, and the constant of the $\sin \theta \cos \psi$ for the odd numbered Discrete Alternating Radial magnetisation configurations.	212
3.22	Multiplier for the holding torque with respect to the maximum holding torque of the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configuration, and the deviation of the setting of ϕ for the freewheeling state from $\frac{\pi}{2}$ rad, for the even numbered Discrete Alternating Radial magnetisation configurations.	214
3.23	Overall multiplier with respect to the $n \in [1, 3, 4, 5, 6, 7, 8]$ setups of the Alternating Halbach magnetisation configuration, for the $n \in [3, 4, 5, 6, 7, 8]$ Discrete Alternating Halbach magnetisation setups.	216
3.24	Comparison of the holding torques derived for the “Discrete Alternating” configurations. Their magnitudes relative to the Diametrical magnetisation configuration, from multiplying their “Multiplier” values (from Tables 3.21 and 3.23) with the appropriate “Multiple” from Table 3.13, are also shown. The Diametrical magnetisation configuration for $n = 2$, Radial magnetisation configurations for $n \in [2, 4, 6, 8]$, and Halbach magnetisation configuration for $n = 2$ are omitted as their rotor equilibrium positions are not easily controlled.	217
3.25	Additional sampling points for ϕ for the setups in which a deviation of the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, for the simple simulation model. Also included are the expected values of ϕ to attain the freewheeling state as obtained from the mathematical models for the Discrete Alternating configurations.	218

3.26	Definitions of the magnetic material in <i>ANSYS Maxwell</i> for the Discrete Alternating magnetisation configuration for both the simple simulation model and the ring of magnets simulation model. The Z component of is set to 0 in all cases.	219
3.27	Holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model. The holding torques for $n \in [2, 3, 4, 5, 6, 7, 8]$ as a ratio of that of the $n = 3$ setup of the Alternating Radial-Diametrical configuration are also shown, along with the multiplier values predicted by the mathematical model.	220
3.28	The settings of ϕ at which the freewheeling effect occurs, and the expected setting as derived from the mathematical model, for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model.	223
3.29	Holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model. The holding torques for $n \in [2, 3, 4, 5, 6, 7, 8]$ as a ratio of that of the $n = 3$ setup of the Alternating Halbach-Diametrical configuration are also shown, along with the multiplier values predicted by the mathematical model.	229
3.30	Ratio $x = \frac{r_{out}}{r_{in}}$ for $\delta = 0$ from Equation (3.119), along with the corresponding $\ln x$ and $\frac{\ln x}{x^2}$ values.	233
3.31	Multiplier values for the holding torques for Discrete Alternating Diametrical magnetisation, with the benchmark Halves Diametrical magnetisation configuration assigned a multiplier value of 1. Indicators for the predicted holding torques and holding torque to volume ratios, derived by combining $\ln x$ and $\frac{\ln x}{x^2}$ from Table 3.30 with the Multiplier values, are also shown.	234

- 3.32 Multiplier values for the holding torques for Discrete Alternating Radial magnetisation, with the benchmark Halves Diametrical magnetisation configuration assigned a multiplier value of 1. Indicators for the predicted holding torques and holding torque to volume ratios, derived by combining $\ln x$ and $\frac{\ln x}{x^2}$ from Table 3.30 with the Multiplier values, are also shown. . . . 235
- 3.33 Multiplier values for the holding torques for Discrete Alternating Halbach magnetisation, with the benchmark Halves Diametrical magnetisation configuration assigned a multiplier value of 1. Indicators for the predicted holding torques and holding torque to volume ratios, derived by combining $\ln x$ and $\frac{\ln x}{x^2}$ from Table 3.30 with the Multiplier values, are also shown. 236
- 3.34 Settings of ϕ at which the highest torque is generated for the Discrete Alternating configurations with varying outer ring radii for which the required setting deviates from $\phi = 0$, for the simple simulation model, for $\theta = 90^\circ$ and $\psi = 0^\circ$. Also included are the values of ϕ at which the maximum torque was expected to occur, derived from the mathematical models for the Discrete Alternating configurations. 237
- 3.35 Optimal number of ring segments and their corresponding holding torque generated per unit volume $R_{\tau,v}$ for each Discrete Alternating magnetisation configuration for the Simple Simulation Model. 239
- 3.36 Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model with varying outer ring radii. 239

- 3.37 The holding torques with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 240
- 3.38 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 241
- 3.39 Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model with varying outer ring radii. 241
- 3.40 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 242

- 3.41 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 243
- 3.42 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 243
- 3.43 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 244
- 3.44 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. 244

- 3.45 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 245
- 3.46 Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model with varying outer ring radii. 246
- 3.47 The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 246
- 3.48 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 247

- 3.49 The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 248
- 3.50 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 248
- 3.51 Additional sampling points for ϕ for the setups in which a deviation of the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, for the ring of magnets simulation model. Also included are the expected values of ϕ to attain the freewheeling state as obtained from the mathematical models for the Discrete Alternating configurations. 249
- 3.52 Settings of ϕ at which the highest torque is generated for the Discrete Alternating configurations with varying outer ring radii for which the required setting deviates from $\phi = 0$, for the ring of magnets simulation model, for $\theta = 90^\circ$ and $\psi = 0^\circ$. Also included are the values of ϕ at which the maximum torque was expected to occur, derived from the mathematical models for the Discrete Alternating configurations. 250

3.53	Optimal number of ring segments and their corresponding holding torque generated per unit volume $R_{\tau,v}$ for each Discrete Alternating magnetisation configuration for the Ring of Magnets Simulation Model.	251
3.54	Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model with varying numbers of ring magnets. .	254
3.55	The holding torques with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.	255
3.56	The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. . . .	258
3.57	The settings of ϕ at which the freewheeling effect occurs, and the expected setting as derived from the mathematical model, for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model. . . .	262
3.58	Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model with varying numbers of ring magnets.	262

- 3.59 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as the ratio of the former to the latter less one. 263
- 3.60 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as the ratio of the former to the latter less one. . . . 265
- 3.61 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 265
- 3.62 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. . . . 269

- 3.63 The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. . . . 269
- 3.64 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 272
- 3.65 Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model with varying outer ring radii. 277
- 3.66 The holding torques with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 278

- 3.67 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 279
- 3.68 Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model with varying outer ring radii. 283
- 3.69 The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. 283
- 3.70 The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. . . . 286

3.71	The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.	287
3.72	The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.	287
3.73	Comparison of the setups with the best holding torque to volume ratios for each ring magnetisation pattern, from the finite element simulations. The Diametrical magnetisation setup for $n = 2$, Radial-Diametrical magnetisation setups for $n \in [2, 4, 6, 8]$, the Radial-Radial magnetisation setups and Halbach magnetisation setup for $n = 2$ are omitted as their rotor equilibrium positions are not easily controlled.	290
4.1	Dimensions of magnets used in the prototype, and the gap between the magnets.	295
4.2	Properties of magnetic material used for FEM analysis of the prototype.	297
4.3	Coefficients of Determination comparing the simulation results with the fitted model.	299

4.4	Coefficients of Determination comparing the experimental results with the simulation results via the fitted model for $\phi_{Pro} = \phi_{Sim}$	320
4.5	Coefficients of Determination comparing the experimental results with the simulation results via the fitted model for $\phi_{Pro} = \phi_{Sim} + 15^\circ$	320
4.6	Abridged comparison of variable stiffness actuators/ mechanisms.	324
4.7	Abridged comparison of variable Stiffness actuators/ mechanisms, continued.	324

Abbreviations

AC Alternating Current. 135

AMASC Actuator with Mechanically Adjustable Series Compliance. 102, 109, 111

ANLES Actuator with Non-Linear Elastic System. 103

AVASTT Actuator with VArable Stiffness and Torque Threshold. 105, 109, 111

AwAS Actuator with Adjustable Stiffness. 105, 109, 111

AwAS-II Actuator with Adjustable Stiffness II. 105, 109, 111

CAD Computer-aided Design. 14, 33, 34, 107, 108, 294

CCEA Continuous-state Coupled Elastic Actuation. 103, 109, 111

cHRI Cognitive and Social Human-Robot Interaction. 88

ComPact© ComPact© Soft Actuator. 13, 99, 115

CompAct-VSA Compact Compliant Actuator. 105, 109, 111

CTE Compliant Transmission Element. 101, 109, 111

CVT Continuously Variable Transmissions. 106

DEA Differential Elastic Actuator. 89, 98

DECMMA Distributed Elastically Coupled Macro Mini Actuation. 99

DLR German Aerospace Center. 101, 102, 103

DM² Distributed Macro-Mini Actuation. 90, 99

DOF Degree(s) of Freedom. 59, 126, 135, 145, 146, 150, 155, 161, 163, 170

EAP Electroactive Polymers. 103

ECB Eddy Current Brakes. 116

ER Electrorheological. 117, 118, 119

FEM Finite Element Method. 59, 73, 145, 146, 159, 175, 236, 297

FlexCVA Flex Continuously Variable Actuator. 99, 109, 111

FSCA Force Sensing and Compliant Actuators. 89, 98

FSJ Floating Spring Joint. 104, 109, 111

HCM Hybrid Control Module. 105, 109, 111

HDAU Hybrid Dual Actuator Unit. 105

HMA Hot-Melt-Adhesive. 115

HRI Human-Robot Interaction. 88

HSVA Hybrid Variable Stiffness Actuator. 105

HypoSEA Hypocycloid-based Series Elastic Actuator. 100

IVT Infinitely Variable Transmissions. 106

M²-VSA Mechanical-Magnetic Variable Stiffness Actuator. 14, 91, 108, 110, 111, 122, 320, 323, 324, 329

MACCEPA Mechanically Adjustable Compliance and Controllable Equilibrium Position Actuator. 104, 109, 111

- MACCEPA-VD** Mechanically Adjustable Compliance and Controllable Equilibrium Position Actuator with Variable Damping. 116, 120
- MACCEPA 2.0** Mechanically Adjustable Compliance and Controllable Equilibrium Position Actuator 2.0. 104, 109, 111
- MARIONET** Moment Arm Adjustment for Remote Induction Of Net Effective Torque. 104, 109, 111
- MAVERIC** MAgnetic Variable stiffnEss spRIng-Clutch. 14, 91, 108, 110, 111, 122, 146, 324, 329
- MCAU** Modular Compliant Actuation Unit. 120
- MIA** Mechanical Impedance Adjuster. 106, 109, 111
- MR** Magnetorheological. 117, 118, 119
- multiANLES** Joint with Multiple Actuator with Non-Linear Elastic System. 101, 109, 111
- MVSA** Magnetic Variable Stiffness Actuator. 14, 107, 110, 111
- mVSA-UT** University of Twente Miniaturised Variable Stiffness Actuator. 105, 110, 111
- NNEA** Novel Nonlinear Elastic Actuator. 101, 103, 109, 111
- PAM** Pneumatic Artificial Muscle. 101, 103, 105
- PEA** Parallel Elastic Actuator. 121
- pHRI** Physical Human-Robot Interaction. 88
- PID** Proportional-Integral-Derivative Controller. 331
- PPAM** Pleated Pneumatic Artificial Muscle. 103
- PVC** Polyvinyl Chloride. 36, 306, 307

QA-Joint Quasi-Antagonistic Joint. 102, 103, 109, 111

SCA Series Clutch Actuator. 119

SDA Series Damper Actuator. 113, 120

SEA Series Elastic Actuator. 13, 89, 98, 99, 98, 99, 100, 120

SLM Safe Link Mechanism. 13, 100

SPEA Series-Parallel Elastic Actuator. 121

V2E2 Very Versatile Energy Efficient Actuator. 106

VIA Variable Impedance Actuators. 85, 90, 94, 97, 103, 120

VPDA Variable Physical Damping Actuator. 115

VSA-HD Variable Stiffness Actuator - Harmonic Drive. 103, 105, 110, 111

vsaUT University of Twente Variable Stiffness Actuator. 105, 110, 111

vsaUT-II University of Twente Variable Stiffness Actuator II. 105, 110, 111

VS-Joint Variable Stiffness Joint. 101, 104, 110, 111

VSA Variable Stiffness Actuators. 85, 90, 97, 120, 329

VSA-Cube Variable Stiffness Actuator Cube. 101, 110, 111, 117, 120

VSA-II Variable Stiffness Actuator II. 101, 103, 109, 111

VSA-I Variable Stiffness Actuator I. 101, 103, 109, 111

VSJ Variable Stiffness Joint using Leaf Springs. 13, 106, 110, 111

VSJPM Variable-stiffness Joint Based on A Permanent Magnetic Mechanism. 105, 110, 111

VSM Variable Stiffness Mechanisms. 85

VSU Variable Stiffness Unit. 14, 91, 108, 110, 111, 122, 323, 324

VTs Variable Torsion Stiffness. 106

WMSDs Work-related Musculoskeletal Disorders. 87

Symbols

$F_{D,V}$ force due to viscous damping. 117

$F_{D,ER}$ damping force due to the electrorheological effect. 117

$\mathbf{m}_{in}$ magnetic dipole vector representing the rotor magnet. 126

m_{in} scalar strength of the magnetic dipole representing the rotor magnet.
126

$\mathbf{B}$ B-field acting on the rotor dipole. 126

B scalar strength of the magnetic field acting on the rotor dipole. 126

θ rotor angular position of the proposed mechanism. 126

τ torque acting on the rotor dipole. 126

k_z stiffness of the rotor about the z-axis. 127

τ_z z-component of the torque acting on the rotor dipole. 127

θ_{max} setting of θ at which the holding/ maximum resistive torque of the
mechanism occurs. 127

ϕ parameter for changing the stiffness of the proposed mechanism. 127

$\mathbf{m}_{out}^l$ magnetic moment per unit length. 129

$\mathbf{r}(\gamma)$ vector connecting the rotor dipole to any generic magnetic dipole in the
outer ring. 129

γ angular position of the ring dipoles. 129

r radius of the ring of magnetic dipoles. 129

μ_0 permeability of free space. 129

k Halbach parameter. 133

ψ parameter for changing the equilibrium position of the proposed mechanism. 137

ζ angular deflection of the rotor relative to its equilibrium position. 138

θ_d desired equilibrium rest orientation of the rotor. 141

$\tau_{hold,max}$ maximum holding torque for the radial magnetisation configuration. 143

$\tau_{hold,min}$ minimum holding torque for the radial magnetisation configuration. 143

r_{rtr} radius of the rotor magnet. 159

n number of magnets in each of the two groups of ring magnets in the mechanism. 159

r_{rng} radius of the ring magnets. 159

$r_{pos,rng}$ radial position of the ring magnets. 159

r_{setup} overall radius of the magnetic setup within the mechanism. 159

δ desired gap between magnets in the setup. 159

B_w strength of the uniform magnetic flux in the cavity of a $k = 2$ Halbach cylinder. 174

r_{out} outer radius of a hollow cylinder. 174

r_{in} bore radius of a hollow cylinder. 174

B_r flux density of the magnetic material. 174

x ratio of the outer radius to the cavity radius of a hollow cylinder. 174

e Euler's number. 175

$R_{\tau,v}$ holding torque generated per unit volume. 175

Abstract

The development of Variable Impedance Actuators (VIA) has gained prominence in recent years, driven by the desire for robots to work more closely with humans to improve work productivity. This class of actuators work to vary the dynamical characteristics of robots during operation, allowing them to be more pliant when executing high-speed manoeuvres while remaining stiff and firm for tasks requiring precision, helping robots satisfy the necessary safety requirements for operating in close quarters with human workers without sacrificing too much of their performance.

The sub-class of VIAs that have received the most attention thus far has been the Variable Stiffness Actuators (VSA): actuators that can modulate their apparent stiffness by manipulating elastic elements within their design. Their popularity is likely due to the ability to transmit forces without excessive energetic losses (in contrast to variable dampers) and their relative ease of construction (compared to the need to interface with flywheels and other similar devices for inertial management). Among these are a small number of magnetic VSAs leverages the interactions between permanent magnets as a contactless spring, potentially eliminating an important source of fatigue failure in VSAs that comes with the use of physical springs.

Thus far, all of the magnetic Variable Stiffness Mechanisms (VSM) rely on varying the axial overlap of a rotor magnet with surrounding magnets to achieve the change in magnetic coupling strength and thus the torque and stiffness generated by the mechanism. This linear motion is achieved through the use of lead screws. As a result, the input that goes into changing the stiffness of the mechanism does not and cannot contribute to moving the

equilibrium position of the mechanism.

This work looks to introduce a new magnetic variables stiffness mechanism design. The proposed design retains the internal permanent magnet rotor common to all magnetic variable stiffness mechanisms thus far, with the variation of stiffness achieved through the manipulation of an outer ring of magnets. This allows the compliance of the resulting magnetic spring system to be varied without the need to linearly insert or retract magnets, while allowing the two inputs that go into varying the stiffness of the mechanism to both also work to move the equilibrium position of the mechanism. It also avoids the need to allocate space for the axial translation of magnets in the linear displacement design. The use of identical motors and transmissions also improves operational robustness since inventory for spare parts can be reduced and simplified. In more extreme circumstances the cannibalisation of units for spares is also more broadly useful.

Mathematical models were constructed to explore various configurations of the proposed design and used to find configurations that are the most space/ volume/ mass efficient per unit of holding torque transmittable. Simulation studies were used to corroborate the findings of the mathematical models and to identify the limits of their applicability.

A proof-of-concept prototype of the most promising configuration found was designed, built, and characterised, showing that the proposed mechanism functions as expected.

Chapter 1

Introduction

1.1 Background

Robots have long been prominently featured in science fiction, depicted as intelligent and able companions of humans both in work and play [12, 13, 14, 15]. Despite the progress made in the past few decades that saw the proliferation of industrial robots beginning in the 1990s and the increasingly accepted use of robots in surgery, this vision is still a long way off from being realised, with robots overwhelmingly being operated while segregated from humans, typically autonomously in a fenced-off area, or in a master-slave configuration with the human in command.

This can be attributed in large part to two factors: The failure thus far to develop viable general artificial intelligence, and general discomfort of humans with working with robots, arising from both safety concerns where high-powered robotic arms and actuators seen as a potential threat to human limbs and lives, as well as uneasiness due to uncertainty about what to expect of robots, their intelligence and capabilities, a sentiment accentuated by the presence of the uncanny valley [16]; the dissonance that arises when the behaviour of animal or human-like robots do not match our mental models of how they should behave [17].

Setting aside the issues with and the difficulty of developing a general artificial intelligence, the next step forward in robotics development is in

enabling closer and more intimate interactions between humans and robots in order to leverage their complementary strengths: Robots can be many times as strong as humans, and are capable of performing routine but tedious tasks for prolonged periods of time without degradation in ability, but fall far short of humans in sensing and perception, as well as ability to adapt to novel situations; Marrying the strengths of humans and robots can allow for higher productivity than manual labour or fully automated robotic work cells or machines alone can achieve. This is of particular interest to countries facing an ageing population where a shrinking and geriatrifying workforce threatens to undermine economic growth and prosperity. Another potential benefit is the reduction in the occurrence of work-related injuries, such as Work-related Musculoskeletal Disorders (WMSD) from performing repetitive tasks [18], and back and spinal injuries from accidents when handling heavy or bulky parts [19].

Intensive research in this direction has been conducted in recent years, constituting the field of Human-Robot Interaction (HRI) which is commonly divided into two branches: Physical Human-Robot Interaction (pHRI) and Cognitive and Social Human-Robot Interaction (cHRI) [20]. The former is concerned with the physical problems that arise when interaction between humans and robots occur; these include the design of robots for the minimisation of potential injuries, path planning for obstacle avoidance in situations where full knowledge of the environment is not available, to providing assistance when moving heavy loads while retaining tactile feedback for a human operator. The latter aims to understand HRI from a social and psychological point-of-view; how effective communication between humans and robots can be accomplished with an eye towards more intuitive and fluid control of and collaboration with robots, and enhancing safety by accounting for the two-way perception between humans and robots.

For simple human-robot collaboration where complex interactions and advanced communication skills are not required, the safety aspects emphasised in pHRI are of prime importance. A number of strategies have been devel-

oped to deal with such concerns, amongst them more intelligent and nuanced ways to monitor and restrict the workspaces of robots [21, 22, 23, 18, 24, 25] to avoid injuries to workers. Another approach takes the view that collisions should be thought of as unavoidable due to the limitations of sensors, unexpected movements, faults or otherwise. This paradigm places the focus on minimising the damage that can result from inadvertent collisions, beginning with enhancing the intrinsic safety of robots by reducing their mass and eliminating sharp or pinch-prone geometries in their design [18, 25, 26, 20, 27].

A common thread that has emerged is the introduction of compliance in robots, starting with the introduction of soft coverings on robot links. A well-explored approach is the use of active force controllers with traditional rigid actuators to modulate the output impedance with respect to sensed interactions [28, 29, 30, 31]. This approach is reliant on having numerous force/ torque sensors for the detection of impacts; portions of the robot that are not so equipped may prove to be blind to collisions [18]. At the same time, inherent actuator [32] and bandwidth [33, 34] limitations exist that can prevent the robot from exhibiting the desired impedance profile, especially during hard and fast impacts when compliance is valued at a premium.

Alternatively, passive compliance can be introduced into the actuators of the robot, typically in the form of an elastic element placed in series with a traditional stiff actuator. Such SEA [1] helps to decouple the actuator's rotor inertia [18], the reflected inertia of which usually accounts for much of the effective inertia as seen from the end effector, from the links of the robot in the event of an impact. Alternative implementations include the Force Sensing and Compliant Actuators (FSCA) [35] and the Differential Elastic Actuator (DEA) [36]. Aside from the issue of safety, the presence of passive compliance also helps resolve the issue of the robustness of robotic interaction with its environment where large reaction forces can result from minor errors in the modelling of both the environment and robot, and also of the control of the robot when attempted with traditionally minimally compliant actuators, particularly when kinematic constraints are present such as in the opening of

a door [37].

These advantages come at the expense of response speed and settling times, lowering overall performance. While much work has been done [38, 39, 40, 41, 42] to improve the performance of such actuators, intrinsic limitations of such setups remains. One way to recover the lost performance is through the Distributed Macro-Mini Actuation (DM²) design scheme [43]. Here, large, high-torque and low-frequency series elastic actuators installed at the base of the robot are paired with small, high-frequency actuators installed in parallel collocated at each joint. The former is responsible for generating slow time-varying torques such as that for working against gravity, while the latter enables rapid and precise positioning of the end-effector and disturbance rejection without significantly increasing the overall impedance of the system [18, 24]. These advantages however come at the cost of increased system mass and complexity.

Another approach looks to implement variable mechanical compliance or stiffness of the actuators, inspired by how the joints in the limbs of humans and animals are controlled in nature, staying soft when moving quickly and becoming stiff when precise manipulation is needed. This type of behaviour has been shown to emerge from the solution to the Safe Brachistochrone problem (an optimal control problem that takes into account limits in motor torques, transmission impedance and safety conditions) for simple one-dimensional systems [44], and is speculated to hold true for more complicated systems as well. This prompted the development of a number of Variable Stiffness Actuators (VSA), a sub-class of Variable Impedance Actuators (VIA) that also include mechanisms capable of imposing variable damping and changing the inertia experienced at a joint.

Aside from safety, these VIAs and VSAs are also useful for robotic tasks where robust interaction with the environment is required, such as the opening of doors or drawers, and in cyclic and explosive movements where the ability to tune the eigenfrequency of the system allows greater energy efficiency to be achieved alongside higher peak torques and velocities beyond

what the actuating motors are capable of [45].

1.2 Motivation

While a variety of mechanisms for variable stiffness actuation have been designed, none are clearly superior to the others, with the optimal choice heavily dependent on the requirements for a particular design, trading off properties such as the ability to achieve theoretically infinite stiffness, the capacity for free infinite rotation, or linearity and resolution in the control of the stiffness of the mechanism. Under such conditions, the development and availability of different variable stiffness actuator designs may prove useful in satisfying the requirements of different robotic designs.

Most of the existing variable stiffness mechanisms use physical springs for their compliance, with only a small number such as the VSU [9], the M²-VSA [10], and the MAVERIC [8] making use of magnetic springs for their operation.

These designs possess some advantages over their more typical counterparts, stemming chiefly from their non-contact nature which enables highly reliable, quiet and efficient operation, while also ameliorating the problem of fatigue that plagues mechanisms that relies on the deformation of physical elastic elements for compliance, eliminating an important potential source of failure, particularly when high speed operation of a repetitive nature is involved [45].

Magnetic variable stiffness mechanisms also admit a wide range of stiffness settings, up to and including having the rotor/ output be in a free wheeling state, allowing great flexibility in commanding desired joint characteristic.

These advantages are however also associated with some disadvantages, stemming chiefly from the increased mass and volume of the mechanism as the mass and volume of the permanent magnets required to achieve a certain maximum stiffness setting is larger than that of a physical spring element. Such mechanisms are thus less suitable for use in the distal joints of dynamic

agile robots where the added mass is energetically costly due to frequent changes in velocity and direction. This downside can be partially mitigated with cable driven designs that allow the heaviest portions of the mechanism to be placed at the base of the robotic device, with the distal links remaining light and agile.

All of the above mentioned magnetic variable stiffness mechanisms rely on varying the axial overlap of a rotor magnet with surrounding magnets to achieve the change in magnetic coupling strength and thus the torque and stiffness generated by the mechanism. This linear motion is achieved through the use of lead screws actuated by a small motor. A separate larger motor is responsible for rotating the mechanism to change its reset/ equilibrium position. With this type of design, the input that goes into changing the stiffness of the mechanism does not and cannot contribute to moving the equilibrium position of the mechanism.

In this work, a novel design for a magnetic variable stiffness mechanism is proposed. By manipulating a ring of smaller magnets surrounding a larger rotor magnet and thus the magnetic field acting upon it the compliance of the resulting magnetic spring system can be varied without the need to linearly insert or retract the magnets. This idea is illustrated in Figure 1.1.

This design allows the two inputs that go into varying the stiffness of the mechanism to both also work to move the equilibrium position of the mechanism. In this way, two smaller identical motors can replace the one large and one small motor pairing configuration (responsible for changing the mechanism's equilibrium/ rest position and the mechanism's stiffness setting, respectively) in the linear displacement design, which helps to minimise the overall mass and volume of the motors required. It also avoids the need to allocate space for the axial translation of magnets in the linear displacement design. Finally, the use of identical motors and transmissions improves operational robustness since inventory for spare parts can be reduced and simplified. In more extreme circumstances the cannibalisation of units for spares is also more broadly useful.

Figure 1.1: Illustration of the working principle of the proposed magnetic variable stiffness mechanism. The rotor magnet is coloured orange, and the ring magnets are divided into two groups, coloured red and blue. The oppositional rotation of the two groups of ring magnets allow the magnetic field strength experienced by the rotor to decrease and hence modulates the restoring torque on the rotor magnet when it is deflected from it's equilibrium position.

1.3 Objective

The objectives of this project are as follows:

1. Construct mathematical models to explore potential ring magnetisation configurations for the proposed magnetic variable stiffness mechanism, and use the models to find the configurations that are the most space/ volume/ mass efficient per unit of holding torque transmittable.
2. Develop a physical prototype of the proposed magnetic variable stiffness mechanism based on the most promising configuration identified using the mathematical models, characterise its behaviour under load, and verify that it possess the ability to realise a large range of stiffness settings, including very low stiffness settings that are not achievable

with variable stiffness mechanisms that utilise physical compliant components.

1.4 Scope

The scope of this project is as follows:

1. The rotor magnet is represented in the mathematical models as a single magnetic dipole. Hence the effect of using rotor magnets with more complex magnetisation patterns will not be captured. In this work, one such rotor magnetisation pattern, “Radial” magnetisation, will be examined with the aid of finite element software.
2. The optimality of the various configurations of the proposed mechanism (defined as the ratio of the holding torque of the mechanism to the mass of the mechanism or the volume that the mechanism occupies) will only consider the mass/ volume of the magnets.
3. While the proposed mechanism has the potential to be used as an actuator by connecting a motor to each of its inputs, this work will be limited to the investigation of its characteristics as a variable stiffness mechanism.
4. This initial investigation will not attempt to account for the dynamic effects of moving magnetic fields, and hence will be conducted under quasi-static conditions.

1.5 Organization

Chapter 1 provides a brief introduction and motivation for the development of the variable stiffness actuator.

Chapter 2 presents an overview of VIAs.

Chapter 3 introduces the mathematical models that describe the working principles of the variable stiffness actuator, analyses them to find efficient configurations, and presents results of simulations studies of the actuator.

Chapter 4 introduces the physical prototype developed, details the experimental setup and procedures, and presents their results, illustrating the behaviour of the actuator.

Chapter 5 concludes the report by summarising the contribution and suggesting future work to be done.

Chapter 2

Literature Review

2.1 Design of Variable Impedance Actuators

Traditionally actuators are designed to have high and ideally infinite impedance, typically achieved with high gearing ratios as is in servo motors. This allows them to achieve precise positioning and excellent trajectory tracking with high bandwidth, acting as a position source. However such actuators have limitations such as not being capable of absorbing impact shocks or storing energy. In addition, position control alone for situations where the environment is unknown and unmodelled is also not a well-posed problem [4].

Since the impedance of an actuator defines its behaviour to an external force, it is possible to obtain better actuator characteristics if their impedance can be changed dynamically to suit the task at hand. This can be done via three avenues: the changing of stiffness, damping and inertial characteristics. Such actuators form a class known as Variable Impedance Actuators (VIA) and are especially suited for applications requiring efficient cyclic motions, explosive motions, robustness to perturbations and modelling errors, force accuracy such as in haptics or with exoskeletons, and where safety is of paramount importance [46]. The addition of impedance as a controllable factor also makes interaction with an unknown environment a well-posed problem [4].

Although a large number of VIAs have been designed, no particular design

Figure 2.1: Concept of the Series Elastic Actuator (SEA) [1].

has emerged as being completely superior to the rest. The selection of optimal actuators depends strongly on the application in question [4, 47]. One relevant consideration is the power flow and energy efficiency of the actuator [48]. More generally, a VSA datasheet template akin to that for electric motors was developed to capture the essential characteristics of VSAs and present them in a transparent, easily referenceable and comparable format to ease adoption by end-users [49]. However, most of the actuators reported in the literature thus far are immature prototype designs whose potential have not been fully exploited and/ or characterised. Hence the information required is often not available or maybe misleading by selling short the potential of the design.

2.1.1 Fixed Compliance

One way to adapt traditional stiff actuators for use beyond that in industrial robotic arms is to couple elastic elements with the actuator. The Force Sensing and Compliant Actuator (FSCA) for instance connects the stator of the motor elastically to the chassis of the robot and uses a force/ torque sensor affixed to the chassis for sensing [35]. The Differential Elastic Actuator (DEA) on the other hand differentially couples the motor, load and spring for a relatively compact mechanism that retains good mechanical impedance characteristics [36].

More commonly, the elastic element is introduced in series with the motor, such as the air springs used Raibert's hopping machines [50]. This config-

Figure 2.2: Prototype of the ComPact©, a rotary SEA [2].

uration came to be known as the Series Elastic Actuator (SEA), which was studied in detailed by Williamson [1], shown in Figure 2.1. It was designed for force control applications using fixed springs whose stiffness cannot be changed during operation. It is, however, possible to perform active control such that the actuator displays a desired stiffness or force by controlling the equilibrium position of the spring [51], though actuator and bandwidth limitations exist as well which if exceeded exposes the true compliance characteristics of the underlying fixed spring.

The design of the SEA allows the motor's rotor inertia to be decoupled from the link, allowing for the absorption of shocks during collisions [52]. The elastic element also allows energy to be stored for efficient cyclic or explosive motions. These advantages come at the expense of response speed and settling times which limits the SEA's performance. While much work has been done [38, 39, 40, 41, 42] to improve the performance of such actuators, intrinsic limitations of such a setup remains.

Efforts to overcome the limitations of SEAs have seen them being used in conjunction with smaller motors to recover the bandwidth performance, such as in the Distributed Elastically Coupled Macro Mini Actuation (DECMMA) and the Distributed Macro-Mini Actuator (DM²) setups.

Other compliant mechanism designs include the ComPact© soft actu-

Figure 2.3: Concept of the SLM [3].

ator [2], shown in Figure 2.2, and the Flex Continuously Variable Actuator (FlexCVA) [53], both of which are rotary series elastic actuators, with the former being a very compact compliant actuator with a limited range of compliance while the latter uses two parallel compliant mechanisms for an unlimited range of compliance.

Nonlinear SEAs have also been developed. The Hypocycloid-based Series Elastic Actuator (HypoSEA) creates a nonlinear virtual spring by stretching a linear spring with a hypocycloid mechanism [54], creating a nonlinear stiffness profile for jumping and hopping applications [55]. The Safe Link Mechanism (SLM) uses geometrical properties with a spring that allows it to maintain very high stiffness until a critical load is reached at which point it “buckles” and becomes very compliant for impact absorption [3]. Its threshold impact “buckling” force can be set by varying the resting angle of the link connecting its two sliders and the properties of the spring. This concept is illustrated in Figure 2.3.

2.1.2 Variable Stiffness/ Compliance

Adaptable compliance is the logical follow-up to the SEA, allowing the actuator’s characteristics to adapt to a situation as required. For instance, it allows the actuator to mimic the control of limbs and joints in humans and animals in nature, staying soft when moving quickly and becoming stiff

when precise manipulation is needed. This type of stiffness trajectory profile that has been shown to emerge from the solution to the Safe Brachistochrone problem (an optimal control problem that takes into account limits in motor torques, transmission impedance and safety conditions) for simple single joint systems [44], and is speculated to hold true for more complicated systems as well.

An additional benefit of such actuators is in robotic manipulation, where softening of an elastic element installed in series with the motor upon impact can help to prevent contact chattering due to high frequency oscillations in cases where the motor's rotor inertia greatly exceeds that of the load's, that may otherwise be uncorrectable with software control alone [56], while preserving performance at all other times.

A number of adaptable compliance mechanisms and actuators have been designed, using two actuators to allow the decoupling of the equilibrium position and stiffness. They can be broadly classified by their working principles [4]: altering the spring preload, changing the transmission between load and spring and altering the physical properties of the spring.

Spring Preload

Mechanisms in this category effect changes in stiffness by changing the pre-tension or preload applied on the elastic element. This can be achieved using agonist-antagonist setups, inspired by how muscles in animals work in combination to achieve complex actuation, with the typical example being the biceps and triceps pair on the human arm. The biggest downside of this approach is the low energy efficiency due to motors working against each other, with the reduced energy storage capacity also a concern [4].

With the antagonistic springs and antagonistic motors setup, 2 nonlinear springs are used in conjunction with 2 motors that when driven in opposite directions changes the overall stiffness of the mechanism, and when driven in the same direction changes the position of equilibrium. Three different embodiments of this design are possible, namely uni-directional, cross-coupled

Figure 2.4: Configurations with antagonistic springs and antagonistic motors: (a) uni-directional, (b) cross-coupled and (c) bi-directional [4].

and bi-directional, illustrated in Figure 2.4. A uni-directional design is the simplest of the three but is limited by the torque output of the weaker motor and is not able to provide any net torque on a load at the maximum stiffness setting. Examples of this include the Pneumatic Artificial Muscle (PAM) [57], the robotic arm with antagonistic actuated joints in [58], the Novel Nonlinear Elastic Actuator (NNEA) [59], the Joint with Multiple Actuator with Non-Linear Elastic System (multiANLES) [60], and the Variable Stiffness Soft Dexterous Gripper in [61]. Cross-coupling introduces a third spring across the two prime movers, allowing the preload for nominal working conditions to be set using only a fraction of the two motors torque, and can be seen in the design of the Variable Stiffness Actuator I (VSA-I) [62] and the Compliant Transmission Element (CTE) [63, 64]. The bi-directional design adds a fourth spring to connect each actuator to the output link via compliant elements, thus allowing the combined torque of both motors to perform work on the load. However, the joint range is limited by the extension or compression of the springs. This design can be seen in the Variable Stiffness Actuator II (VSA-II) [65], the Variable Stiffness Actuator Cube (VSA-Cube) [66, 67] and the Variable Stiffness Joint (VS-Joint) [68] used in the German Aerospace Center (DLR) hand/ arm [69, 70].

Alternatively, antagonistic springs can be used with independent motors arranged such that partial or full decoupling of the control of the stiffness from that of the equilibrium position is achieved, as illustrated in Figure 2.5.

Figure 2.5: Configurations with antagonistic springs and independent motors: (a) partial decoupling, (b) complete decoupling with positioning motor at the joint and (c) complete decoupling without motor at joint [4].

This allows the motors to be dimensioned separately for each task, leading to lower cost and energy requirements. This design can be found in the DLR Quasi-Antagonistic Joint (QA-Joint) [71], a partially decoupled mechanism where a change in stiffness requires actuation by the other motor for the equilibrium output position to remain unchanged, and the Actuator with Mechanically Adjustable Series Compliance (AMASC) which achieves full decoupling with the use of a lever arm connecting the ends of the two springs [72, 73, 74, 75, 76].

The nonlinear elastic characteristics required for variable stiffness using antagonist setups [77] are produced using many different approaches, including the use of non-hookean rubbers and polymers, springs with varying diameters and pitches along their lengths and hence variable spring rate [4], Electroactive Polymers (EAP) [78, 79, 80], the use of structures to create geometrical nonlinearities (VSA-I, DLR hand/ arm, Actuator with Non-Linear Elastic System (ANLES)) [62, 69, 81, 60], cam mechanisms (Continuous-state Coupled Elastic Actuation (CCEA), QA-Joint, NNEA) [71, 82, 59], four-bar linkages (VSA-II, Variable Stiffness Actuator - Harmonic Drive (VSA-HD)) [65, 83, 84] and even the properties of air (PAM, Pleated Pneumatic Artificial Muscle (PPAM)) [85, 86, 57, 87, 88, 89].

Figure 2.6: Controlling the transmission ratio by changing: (a) spring position, (b) force application point and (c) pivot point [4].

The use of antagonistic springs reduces the energy efficiency and energy storage capacity of the actuator as energy needs to be injected into the springs or transferred between them instead of being fed directly to the joint being controlled [4, 90]. A measure was developed for a systematic comparison of the energy efficiency of various VIAs [48].

To avoid the problems associated with antagonistic spring setups, preloading can be done on a single linear spring instead, with the other motor controlling the equilibrium position of the mechanism while a nonlinear link between the output and the elastic element allows for the change in stiffness to be effected. This setup is implemented in the Mechanically Adjustable Compliance and Controllable Equilibrium Position Actuator (MACCEPA) with a lever arm [91, 92, 93], the Mechanically Adjustable Compliance and Controllable Equilibrium Position Actuator 2.0 (MACCEPA 2.0) with a cam that allows for custom stiffness profiles to be designed [94], the VS-Joint and the Floating Spring Joint (FSJ) with profiled cam sliders [68, 95] and the Moment Arm Adjustment for Remote Induction Of Net Effective Torque (MARIONET) [96].

Transmission between Load and Spring

An alternative to the preloading of springs is to change the transmission ratio between the output link and the elastic elements. Since no preloading of springs occurs, no energy bar that to overcome the effects of friction is required to change the stiffness of the mechanism when it is at equilibrium.

A common way to achieve this is through the use of a lever in an adjustable moment-arm. This approach allows for energetically efficient stiffness variation as the displacements involved in the adjustment is perpendicular to the force generated by the springs, but with the downside of limited deflection range. Three parameters come into play in this approach: the position of the pivot, the position of the spring and the position of the force application point, as shown in Figure 2.6.

Changing of the pivot position theoretically allows for an infinite range of stiffness values with certain implementations that allows the pivot and force application points to coincide, a property that allows shorter levers and softer springs to be used for more compact mechanisms. This design is used by the Actuator with Adjustable Stiffness II (AwAS-II) [97, 98], the Compact Compliant Actuator (CompAct-VSA) [99], the University of Twente Variable Stiffness Actuator II (vsaUT-II) [100, 101] and the Miniaturised University of Twente Miniaturised Variable Stiffness Actuator (mVSA-UT) [102].

Changing of the spring position was done in the Actuator with Adjustable Stiffness (AwAS) [103] and in the Hybrid Control Module (HCM) in the Hybrid Dual Actuator Unit (HDAU) or Hybrid Variable Stiffness Actuator (HSVA) [104, 105] but is not commonly used since the effects of friction is increased along with potentially undesirable dynamical behaviours. Finally, changing of the force application point was implemented by the University of Twente Variable Stiffness Actuator (vsaUT) [106, 107, 48, 108].

Nonlinear mechanical interlinks can be used in conjunction with the above mechanisms to provide a secondary source of variation of the transmission ratio and hence the stiffness of the mechanism. This concept has been used to improve the stiffness profile of the PAMs used in the biped Lucy by varying the angle and length of the lever arms of the pull rod mechanism connecting them [109]. The Actuator with VArIable Stiffness and Torque Threshold (AVASTT) uses a lever arm to vary the geometry that dictates the travel path of a roller attached to the end of a spring. A torque threshold can be specified by preloading the spring, preventing the roller from moving and

Figure 2.7: CAD model of the VSJ using Leaf Springs [5].

hence producing infinite stiffness until the applied force is sufficiently great to overcome the preload [110]. Other realisations includes the use of variable radius pulleys [111], cams [68, 94, 95] and four-bar linkages [112]. The VSA-HD in particular uses the nonlinearity inherent of four-bar linkages to serve as the sole source of variable stiffness [83, 84]. The Variable-stiffness Joint Based on A Permanent Magnetic Mechanism (VSJPMM) leverages the inherently nonlinear force/ stiffness characteristics of permanent magnets and combines it with a trapezoidal joint layout to achieve the same effect [113].

Continuously Variable Transmissions (CVT) have also been proposed as a means to achieve the change in transmission ratio. The conceptual design of the Very Versatile Energy Efficient Actuator (V2E2) for instance proposes the use of Infinitely Variable Transmissions (IVT) with zero and negative gearing ratios between the output link and the elastic elements for the adjustment of the transmission ratio without the need for a lever [114].

Physical Properties of the Spring

Another way to achieve variable stiffness is to manipulate the physical properties of the elastic elements. The most common approach involves varying the active length of the elastic elements, as seen in the Jack Spring [115], the Variable Stiffness Joint using Leaf Springs (VSJ) [116, 5] as illustrated in Figure 2.7, the Mechanical Impedance Adjuster (MIA) [117], the Variable

Figure 2.8: Concept for variable stiffness using granular jamming [6].

Torsion Stiffness (VTS) [118] and in robotic fingers with variable stiffness [119, 120].

For some materials application of heat can effect changes to their elastic modulus, though this process tends to be slow and impractical [4]. For mechanisms that utilise the bending of beams as the elastic component the second moment of inertia in play can be altered simply by rotating the beam by 90/degree about the axis orthogonal to the bending axis and the direction of bending [4]. This method provides for two distinct stiffness settings, with intermediate settings ruled out due to the lateral buckling problem. Selective locking of flat plates, wire bundles or granules (illustrated in Figure 2.8) also allows resistance to bending to be modified, typically achieved through the application of pressure by the creation of a vacuum or electrostatic attraction [121, 122, 6, 123, 124], though precise control of stiffness is difficult due to friction and hysteresis, with the actual stiffness achieved dependant on the state of bending at the moment the vacuum is applied [4]. The application of favourable stresses on leaf springs in flexural mechanism using piezoelectric actuators to induce stress stiffening or softening has also been studied, leading to a design capable of changing stiffness rapidly while being largely immune from assembly error and wear [125, 126, 127].

The use of magnetic springs in VIAs has also received some attention. The Magnetic Variable Stiffness Actuator (MVSA), shown in Figure 2.9, is a linear magnetic variable stiffness spring developed to enable rapid changes in stiffness of the leg of a planar hopper at any point in its gait cycle [7]. The design uses two opposing permanent magnets, with one attached to the

Figure 2.9: CAD model of the MVSA [7].

Figure 2.10: Concept for the MAgnetic Variable stiffnEss spRIng-Clutch (MAVERIC) device [8].

proximal end of the shank of the leg and the other allowed to move with the distal end making contact with the ground. An additional coil wound around the fixed magnet allows partial cancellation of its magnetic flux and hence allows the strength of the magnetic interactions between the two magnets to vary.

Rotary versions of variable stiffness magnetic springs have also been explored, typically using the actuation concept as illustrated by the concept for the MAgnetic Variable stiffnEss spRIng-Clutch (MAVERIC) device in Figure 2.10. The magnetic Variable Stiffness Unit (VSU), developed to enhance the safety of humans should proximity sensing fail to prevent unintended collisions with a manipulator from occurring, constructs its magnetic spring out of two concentric rings of 4 arc-shaped permanent magnets each arranged with alternating pole positions, with the stiffness of the spring varied by

Figure 2.11: CAD model of the VSU [9].

Figure 2.12: Prototype of the Mechanical-Magnetic Variable Stiffness Actuator (M^2 -VSA) [10].

changing the overlapping area between the two rings with a lead screw [9]. The Mechanical-Magnetic Variable Stiffness Actuator (M^2 -VSA), a prototype of which is shown in Figure 2.12, utilises the same concept, using permanent magnets mounted along the core and the shell of the joint as part of a three-stage compliance mechanism, with its stiffness regulated by the relative translational displacement of the core and shell [10]. A more comprehensive exploration of the use of magnetic couplers for variable stiffness actuation was performed in the design of the MAVERIC device [8].

Comparison of Variable Stiffness Actuators/ Mechanisms

Table 2.1: Comparison of variable stiffness actuators/ mechanisms.

Name	Overload	L x W x H [mm]	Mass [kg]	Notes
AMASC [72, 74, 75]	Hard Stop	N.R.	N.R.	-
AwAS [103]	Hard Stop	270 x 130 x N.R. ^a	1.8	-
AwAS-II [97, 98]	Hard Stop	180 x 140 x N.R.	1.1	-
AVASTT [110]	Hard Stop	N.R.	4.5	^c
CCEA [82]	Hard Stop	60 x 600 x 74	0.8	-
CompAct-VSA [99, 128]	Hard Stop	93 x 93 x 106	1.8	-
CTE [63, 64]	Hard Stop	N.R. x 80 x 50	N.R.	-
FlexCVA [125, 126, 127]	Hard Stop	N.R.	N.R.	-
FSJ [95]	Hard Stop	119 x 119 x 92	1.41	-
HCM [104, 105]	Hard Stop	100 x 100 x 258	2.36	-
Jack Spring [115]	Hard Stop	N.R.	0.84	-
MACCEPA [91, 92]	Hard Stop	N.R.	2.4	-
MACCEPA 2.0 [94]	Hard Stop	N.R.	N.R.	-
MARIONET [96]	Limited Slip	N.R.	N.R.	-
MIA [117]	Failure	0.07 x N.R. x N.R.	N.R.	-
multiANLES [60]	Hard Stop	458 x 216 x 199	N.R.	-
NNEA [59]	Limited Slip	N.R.	N.R.	-
QA-Joint [71]	Hard Stop	N.R.	1.2	-
VSA-I [62]	Failure	N.R.	N.R.	-
VSA-II [65]	Hard Stop	120 x 120 x 85	0.345	-
VSA-Cube [66, 67]	Failure	55 x 55 x 55	0.26	^d
VSA-HD [65, 83, 84]	Hard Stop ^e	N.R.	1.7	-
vsaUT [106, 107, 48, 108]	Hard Stop	N.R.	N.R.	-
vsaUT-II [100, 101]	Hard Stop	N.R.	2.4	-
mVSA-UT [102]	Hard Stop	30 x 32 x 49	0.1	-
VSJ [116, 5]	Hard Stop	146 x 146 x 144	4.95	-

Continued on next page

Table2.1 – continued from previous page

Name	Overload	L x W x H [mm]	Mass [kg]	Notes
VS-Joint [68, 69, 70]	Hard Stop	97 x 97 x 106	1.4	-
VSJPM [113]	Hard Stop	N.R.	N.R.	-
MVSA [7]	Hard Stop	N.R.	N.R.	^f
VSU [9]	Slip	150 x 150 x 230	5.2	-
M ² -VSA [10]	Slip	87 x 87 x 45	N.R.	^g
MAVERIC [8]	Slip	20.6 x 20.6 x 12.5	N.R.	^f

^a Not Reported.^b Not Applicable.^c Torque threshold through pretension of cables.^d Designed for modular assembly with multiple VSA-Cubes.^e Also used to avoid singularity in four-bar linkage.^f Data is based on physical testing with magnets. No actuated prototype was reported.^g Uses magnets to provide ability to slip under overload conditions.

Table 2.2: Comparison of variable stiffness actuators/ mechanisms, continued.

Name	Stiffness ^c [Nm/rad]	Speed ^d [s]	Deflection ^e [rad]	Rotation ^f [rad]
AMASC	2.5 – 18 (7.2x) ^g	N.R. ^a	N.R.	N.R.
AwAS	30 – 1300 (43x)	3.5	±0.2/ ± 0.03	± $\frac{2\pi}{3}$
AwAS-II	0 – ∞ (∞x)	2.5	±0.3/ ± 0.04	± $\frac{5\pi}{6}$
AVASTT	0 ^h – ∞ (∞x)	N.R.	±0.94/0	±∞ ⁱ
CCEA	62 – 171 (2.8x)	N.R.	±1.26/N.R.	±∞
CompAct-VSA	0 – ∞ (∞x)	0.1	±0.35/0	±∞
CTE	0.3 – 120 (400x)	N.R.	± $\frac{\pi}{6}$ /N.R.	±∞

Continued on next page

Table 2.2 – continued from previous page

Name	Stiffness ^c [Nm/rad]	Speed ^d [s]	Deflection ^e [rad]	Rotation ^f [rad]
FlexCVA	5 – 25 (5x) ^g	0.01	$\pm 30 / \pm 30^j$	N.A. ^b
FSJ	52.4 – 826 (15.8x)	0.33	$\pm 0.26 / \pm 0.05$	π
HCM	4 – 126 (31.5x)	0.14	$\pm 0.53 / \text{N.R.}$	$\pm \infty$
Jack Spring	13 – 28 (2.2x) ^g	N.R.	N.R.	N.A.
MACCEPA	0.52 – 1.15 (2.2x)	N.R.	N.R.	$\pm \pi$
MACCEPA 2.0	2.86 – 5.21 (1.8x)	N.R.	N.R.	$\pm \pi$
MARIONET	0 – 15 (∞ x)	0.065	N.R.	$\pm \infty$
MIA	0.57 – ∞ (∞ x)	N.R.	$\pm 0.7 / 0$	N.R.
multiANLES	10.9 – 21.2 (1.9x)	N.R.	N.R.	N.R.
NNEA	0.00057 – 0.04 (70x)	N.R.	N.R.	N.R.
QA-Joint	20 – 750 (37.5x)	0.12	$\pm 0.252 / \pm 0.005$	$\pm \infty$
VSA-I	200 – 2200 (11x)	N.R.	N.R.	$\pm \infty$
VSA-II	0.5 – 0.95 (19x)	N.R.	$\pm 0.3 / \pm 0.03$	$\pm \infty$
VSA-Cube	3 – 14 (4.7x)	0.32	$\pm 0.28 / \pm 0.15$	$\frac{2\pi}{3}$
VSA-HD	0.38 – 8360 (∞ x)	0.55	$\pm 1.05 / \text{N.R.}$	$\pm \infty$
vsaUT	N.R.	5	N.R.	N.R.
vsaUT-II	0.7 – ∞ (∞ x) ^k	0.9	$\pm 0.70 / 0^l$	± 0.5
mVSA-UT	0 – ∞ (∞ x)	0.54	$\pm \frac{\pi}{4} / 0$	$\pm \infty$
VSJ	83.54 – 3096 (36.7x)	0.155	N.R.	$\pm \infty$
VS-Joint	38 – 55 (1.5x)	0.3	$\pm 0.25 / \pm 0.25$	$\pm \infty$
VSJPM	1.3 – 239.8 (184x)	0.4	$\pm 0.35 / \pm 0.35$	$\pm 1.22 / \pm 1.22$
MVSA	0.175 – 0.345 (1.97x) ^g	0.08	0.06 ^m	N.A.
VSU	4.08 – 49.4 (12.1x)	N.R.	$\pm \infty / \pm \infty^n$	$\pm \infty$
M ² -VSA	0 – 2 (∞ x)	N.R.	$\pm \infty / \pm \infty$	$\pm \infty$
MAVERIC	0 – 0.11 (∞ x)	N.A. ^o	$\pm \infty / \pm \infty$	$\pm \infty$

Continued on next page

Table 2.2 – continued from previous page

Name	Stiffness ^c [Nm/rad]	Speed ^d [s]	Deflection ^e [rad]	Rotation ^f [rad]

^a Not Reported.

^b Not Applicable.

^c Maximum stiffness attained at the minimum and maximum stiffness settings.

^d Measured by the time required for a step change in stiffness.

^e Range of elastic deflection at minimum and maximum stiffness settings.

^f Range of equilibrium positions that can be commanded.

^g Linear stiffness measured in [kN/m].

^h Only for the initial deflection at the lowest stiffness setting.

ⁱ Any angular position can be commanded, with multiple rotations allowed.

^j Linear deflection measured in [μ m]

^k Measured maximum stiffness of 948 N/rad due to finite structural stiffness.

^l Measured maximum deflection of 0.035 rad at maximum stiffness due to finite structural stiffness.

^m Linear deflection measured in [m].

ⁿ Due to ability to slip under overload conditions.

^o No actuated prototype was reported.

2.1.3 Fixed Damping

The introduction of inherent compliance into actuators comes with several performance penalties, amongst them a large increase in phase lag, reduced stability margins and the presence of an anti-resonance that complicates the control of the motor [4], creating problems with the control of the actuator at frequencies well below its resonance regime [129]. Bandwidth limitations on the actuator's performance are also imposed [56, 33, 32] along with undamped oscillations that are energetically expensive to quell by active control [4].

Physical damping can be used to alleviate such problem. Passive damping devices have been created for the control of robotic joints, amongst them the

Series Damper Actuator (SDA) which provides viscous damping [130] and the pseudo damper in the TWENDY-ONE's arms which provide visco-elastic damping with the coupling of a rotary damper with an elastic torsion bar [131].

2.1.4 Variable Damping

Depending on the instantaneous configuration and dynamics of a robotic manipulator, and the load it is manipulating, the load experienced locally by the actuator at each joint can change dramatically. Optimal actuator performance while providing damping for the suppression of oscillations requires critical damping to be achieved [56], which in ever-changing conditions, in turn, requires that the stiffness and/ or the damping be adaptable [132]. Since the stiffness requirements at a joint may be constrained by other factors, the additional degree of freedom afforded by a variable damper can be very useful. Excessive energy loss can also be prevented if the damping effect can be turned off, allowing more energy to be stored in the elastic element when necessary [128]. This approach is named semi-active damping for the small amount of energy required to effect a large damping effect, in contrast to the energetically expensive active damping which uses active control to present a damping phenomenon at the output of a robotic manipulator.

A number of mechanisms with adaptable damping have been designed. They can be broadly classified by their working principles [4]: controlled friction, the use of eddy currents, fluid dynamic dampers and controlled rheology. Some of these are illustrated in Figure 2.13.

Controlled Friction

Friction dampers produce damping forces from the friction between two parts with relative motion. This frictional force typically scales linearly with the normal force the two parts exert on each other. Modulation of this normal force allows the damping force generated to be varied, which with the proper control algorithm can be used to emulate nonlinear damping such as viscous

Figure 2.13: Various principles for variable damping: (a) controlled friction, (b) controlled rheology (magnetic), (c) fluid dynamics using variable orifice, (d) eddy current and (e) laminar fluid dynamics damping [4].

damping. This type of damping can produce large forces at low velocities [4] but suffers from hysteresis and the presence of a static friction band [133, 134]. The large frictional forces involved also leads to high operational noise levels and also to wear and tear which in time will cause the performance of the damper to deteriorate as well as complicating its control.

Friction dampers are in common use today, ranging from freight train suspensions to seismic hazard protection for structures [135], and particularly in the automotive sector where it forms the basis of operation for both the drum and disk brakes. This design is also used in the Variable Physical Damping Actuator (VPDA) [136] and ComPact© [128, 137], with both using piezoelectric actuators for the application of the braking force [138]. Other implementations include magnetic particle/ powder brakes [139, 140] and a novel damper using Hot-Melt-Adhesive (HMA) when in its solid phase [141].

Eddy Current and Electrical Damping

Eddy currents are induced within conductors when subject to a changing magnetic field, with the currents producing magnetic fields that oppose the source magnetic field that created them. In the case of a conductor moving relative to a magnetic source generating a constant magnetic field this results in a force opposing said motion proportional to the relative velocity between

them in low relative velocity regimes, a phenomenon that can be exploited to create eddy current-based dampers. Both permanent magnets and electromagnets can be used to realise dampers based on this working principle, with electromagnets having the advantage of containing no moving parts at the cost of power consumption even when holding a fixed damping value. The contact-free nature of this damping phenomenon makes for reliable damper designs by helping to eliminate the frictional wear and tear common in other brake designs while also obviating the use of fluids that complicates the design of the dampers. However, the relatively low damping torques that can be generated typically requires the use of additional transmission devices to match the requirement of most applications.

The strength of the damping force depends on the geometries of the conductor and the magnet, the gap separating them, the specific conductivity of the conductor and the magnetic flux across the conductor. These parameters provide avenues through which potential variable dampers can be designed. One such design, the Eddy Current Brakes (ECB) presented in [142, 143], uses a pair of electromagnets positioned atop an aluminium blade and varies the strength of damping provided with the switching on and off of the electromagnet, and was used for haptic texture rendering [144].

Electrical damping works more directly by using a generator to convert motion into electrical energy which is then dissipated in an electrical circuit of variable resistance, thereby providing physical damping in the system. As is the case with eddy current damping, the generator is typically paired with a gearbox to achieve higher damping ratios. This design has been used in haptic devices [145, 146], and used to provide variable damping in robotics in the Mechanically Adjustable Compliance and Controllable Equilibrium Position Actuator with Variable Damping (MACCEPA-VD) [147].

Fluid Dynamics

Fluid dynamic dampers rely on the properties of fluid flow for the generation of damping forces. Aside from the density, viscosity and compressibility of

the fluids used, the damping forces produced largely depends on the dominant flow regime in play within the force generation portion of the damper, indicated by its Reynolds number [4]. Laminar flows in the case of low Reynolds numbers result in linear damping while turbulent flows at high Reynolds numbers produce quadratic damping.

In the case of linear damping, the forces generated additionally depends on the area of contact between the fluid and the mechanical surfaces of the damper and the height of the fluid chamber [4], parameters through which variable dampers can be designed. An example is the variable damping module based on changing the area of fluid contact with the use of an aperture mechanism was designed for use in conjunction with the VSA-Cube variable stiffness actuator [132].

Quadratic damping is typically achieved, especially in the automotive industry, with the use of a constricting orifice through which fluids are forced to flow at high speeds. Such damper designs are capable of providing high damping forces at high speeds, but generates low damping forces at low speeds, resulting in long-lasting residual oscillations that are disadvantageous for most applications [148]. Here, the adjustment of the size of the orifice provides an avenue for the design of variable dampers.

Controlled Rheology

Electrorheological (ER) and Magnetorheological (MR) fluids are Bingham fluids whose properties such as yield strength and initial viscosity depend on the strength and direction of the electric or magnetic field passing through them, respectively. This makes them suitable for the design and construction of variable damper devices. Such dampers can be seen as an extension of variable fluid dynamic dampers where the variable component is the properties of the fluid instead of the area of contact or orifice size. These fluids are suspensions whose mediums used are of low viscosity for fast response to changing electric or magnetic fields, and also to accentuate the ER or MR effect over that of normal viscous damping for good controllability.

ER fluids use polar particles that, in the presence of an electric field organises into long fibrils when stationary, which when aligned orthogonal to the shear direction in an electrorheological damper manifests as an increase in yield strength of the Bingham fluid. During motion, this alignment also causes the drag on fluid motion and hence the damping coefficient to increase substantially [149]. The damping force generated can be split into two components; that due to viscous damping, $F_{D,V}$, and that due to the ER effect, $F_{D,ER}$. For good controllability, the force ratio of $F_{D,V}$ to $F_{D,ER}$ should be small [149]. This force ratio can be reduced by increasing the electric field strength applied. Since reducing the size of the annulus through which the ER fluid is made to flow and across which the electric field is applied also produces an increase in $F_{D,V}$ alongside the increase in $F_{D,ER}$ and hence yields little net benefit, high operating voltages are required instead [149].

Magnetorheological fluids, on the other hand, use ferromagnetic particles, typically of soft iron, that form chains in the presence of a magnetic field much like that seen in ER fluids. When used in a damper increase in yield strength and viscosity is observed alongside decreasing marginal viscosity with increasing magnetic field strength until magnetic saturation is reached, producing an approximately constant force damper at high magnetic field strengths [149]. The magnetic field required is typically generated using a solenoid but at a voltage much lower than that required for ER dampers, making them more suitable for practical applications.

Much of the advantages with ER and MR dampers lie in the significantly reduced number of moving parts in their design which bodes well for their reliability. However, significant problems still exist, such as the electrophoretic effect in ER fluids where the drift of charged particles result in current leakage, and that of MR fluids to settle after periods of disuse, and to thicken when no magnetic field is applied over a long period of time [149]. MR dampers, in addition, have been found to possess high hysteresis in the torque to velocity relationship similar to that found in friction dampers [133] while also possessing hysteretic torque to field strength relationship due to

magnetic remnance in the particles [149]. ER fluids, on the other hand, have been found to suffer from stiction that produces force overshoots during directional changes [149].

A number of ER-based and MR-based clutches have been developed [150, 151, 152, 153], including the Series Clutch Actuator (SCA) that leverages an MR damper's torque transmission saturation characteristic for a safety device that reliably decouples the motor's inertia from its output [154], with some ER-based clutches have developed for force feedback and haptic rendering [155, 156]. MR-based and ER-based dampers were also designed for semi-active control of vibrations [157, 138] which can be especially useful for lightweight robots. Other applications of ER or MR dampers include their use in smart adaptive prosthetics [158, 159] and in passively compliant robotic arms [160]. MR-based dampers have additionally found commercial success in the automotive sector for use heavy vehicles and luxury vehicles [161, 149], as well as for the control of seismic vibrations in structures [162, 161].

2.1.5 Inertial Actuators

Another way to manipulate the impedance experienced by an end effector is to change the inertia of the system. This most commonly involves the controlling of the coupling with an external mass, such as that in a flywheel which also provides an alternative to springs for the storage and release of energy in the robotic system.

2.1.6 Combination

Given the different and distinct advantages associated with variable stiffness actuators and variable damping actuators, a combination of the two promises to provide the best of both worlds for actuation. For the simplest system using only a single elastic element and damper connecting a single output link to a prime actuator and/ or ground reference frame there are 3 main ways in which the different elements can be connected: in series, in parallel,

Figure 2.14: Configurations for combining variable stiffness and variable dampers: (a) in parallel, (b) in parallel with damper connected to an external frame, and (c) and (d) in series [4].

or in parallel with the damper connected to an external frame and the output link [132, 4], as illustrated in Figure 2.14.

Connecting a damper and elastic element in series best insulates the output link from an actuating motor's inertia but requires continuous rotation of the motor and hence large energy expenditure for the application of a constant torque as seen in the Series Damper Actuator (SDA) [130].

A parallel connection where both ends of the damper and elastic element are connected as seen in the Modular Compliant Actuation Unit (MCAU) [163] is the typical way to utilise both modules to enable fast and accurate motion of compliant joints without excessive oscillations. This setup however usually requires that some engineering effort is made to integrate the two components.

If the output link of an existing VSA is fitted with a damper the final configuration is achieved. This configuration can be seen with the VSA-Cube fitted with a variable damping module and the MACCEPA-VD [132, 147], but comes with the disadvantage of also damping motion that does not originate from the joint being controlled. This can be avoided if the damping can be turned off entirely, or if the frame of the damper is connected to that of the VSA which changes its configuration to one that is purely parallel as described above.

2.2 Parallel Elastic Actuators

While SEAs and VIAs are successful in changing the speed profile required of drive motors, the load borne by the motor has not changed since the compliant element is placed in series with the motor [164]. For robotic joints where high torques and low speeds are typical this means that motor selection remains a choice between that of using a large bulky motor that will largely operate well outside its high efficiency operating window of high speed and low torque, or coupling smaller motors with large reduction ratios where frictional losses greatly affects the efficiency of the joint [164].

This can potentially be avoided by placing an elastic element in parallel with the motor to bear part of the load placed on the joint. This results in what is known as the Parallel Elastic Actuator (PEA). For this configuration to function properly, the elastic element and the link between the elastic element and the motor need to be carefully selected and designed to result in an appropriate equilibrium position and torque profile [165]. They have been used for gravity compensation [166, 167], in exoskeletons to reduce motor torque and power requirements during load lifting [168], and for efficient locomotion with large payloads [169].

An always engaged parallel elastic elements can, however, limit the useful range of motion of a joint since it may at times resist desired motions. Efforts to circumvent this limitation includes the invention of clutchable PEAs [170, 171, 172] and the Series-Parallel Elastic Actuator (SPEA) with variable recruitment of parallel elastic elements reminiscent of the workings of biological muscles that allows the actuator's torque profile to be tuned for a greater variety of tasks [164].

With the development of parallel elastic actuators locking devices in robotics have also gained attention, beyond their typical role as dampers for energy management, as reconfiguration mechanisms like those required in the aforementioned variable PEAs. A comprehensive overview is provided by Plooi et al in [173].

2.3 Technology Gap

While a variety of mechanisms for variable stiffness actuation have been designed, none are clearly superior to the others, with the optimal choice heavily dependent on the requirements for a particular design, trading off properties such as the ability to achieve theoretically infinite stiffness, the capacity for free infinite rotation, or linearity and resolution in the control of the stiffness of the mechanism. Under such conditions, the development and availability of different variable stiffness actuator designs may prove useful in satisfying the requirements of different robotic designs.

Most of the existing variable stiffness mechanisms use physical springs for their compliance and hence are vulnerable to fatigue during high-speed operations. In such scenarios, the use of magnetic couplers is advantageous, chiefly due to their non-contact nature which enables highly reliable, quiet and efficient operation while avoiding reliance on the deformation of physical elastic elements for compliance, eliminating an important potential source of failure [45]. Such couplers also provide an inherent level of protection against overloading as slippage will occur when their torque limit is exceeded, helping to greatly limit the damage sustained by both the robot and its environment during collisions or otherwise.

The inherent compliance present in magnetic couplers allows them to be used as springs whose stiffness depends on the strength of the magnetic coupling. Mechanisms that vary the strength of the couplings can be used for the design of magnetic variable stiffness actuators. Such designs include the Variable Stiffness Unit (VSU) [9], the Mechanical-Magnetic Variable Stiffness Actuator (M²-VSA) [10] and the MAgnetic Variable stiffnEss spRING-Clutch (MAVERIC) [8], all of which relies on varying the overlap between concentric rings of magnets to achieve changes in stiffness.

In this work I propose a novel design for a magnetic variable stiffness mechanism. By manipulating a ring of smaller magnets surrounding a larger rotor magnet and thus the magnetic field acting upon it, the compliance

of the resulting magnetic spring system can be varied without the need to linearly insert or retract the magnets. This idea is illustrated in Figure 1.1. Here, the two inputs that go into varying the stiffness of the mechanism both also work to move the equilibrium position of the mechanism. In this way, two smaller motors can replace the one large and one small motor pairing configuration (responsible for changing the mechanism's equilibrium/ rest position and the mechanism's stiffness setting, respectively) in the linear displacement design, which helps to minimise the overall mass and volume of the motors required. It also avoids the need to allocate space for the axial translation of magnets in the linear displacement design. Finally, the use of identical motors and transmissions improves operational robustness since inventory for spare parts can be reduced and simplified. In more extreme circumstances the cannibalisation of units for spares is also more broadly useful.

Chapter 3

Proposed Mechanism Design

3.1 Overview

This chapter presents mathematical models that seek to capture the essential characteristics of the proposed magnetic variable stiffness mechanism. The magnets will be represented as simple dipoles for their analytical simplicity, and are expected to provide good predictive performance since only a small region along the boundary of the ring and rotor magnets violate the distance assumption for the use of dipoles, and the quantity of the magnetic material located in this region is particularly small due to the cylindrical shape of the ring magnets used.

These models will then be used to explore potential ring magnetisation configurations, as well as to find the configurations that are the most space/ volume/ mass efficient per unit of holding torque transmittable. The strengths and weaknesses of the models will also be examined through comparison with finite element simulations.

In this work, bold mathematical symbols denote vectors, while regular unbolded symbols denote scalar variables.

3.2 Mathematical Models – Halves Configuration

3.2.1 1-DOF Model

Figure 3.1: Illustration of the magnetic spring concept. The magnetic dipole is coloured orange and situated at the centre of the Cartesian coordinate system, deflected about the z-axis by an angle θ relative to the $\mathbf{B}$ field oriented along the x-axis. τ is the restoring torque about the z-axis generated when $\theta \neq 0$.

To illustrate the working principle of a magnetic spring, a simple 1-DOF model is presented. Here, the magnetic rotor is represented by single magnetic dipole $\mathbf{m}_{in}$ with magnitude m_{in} , and the external magnetic field by $\mathbf{B}$ with magnitude B in the x-direction. Assuming that the external magnetic field is oriented along the x-axis of a three-dimensional Cartesian coordinate system, if the magnetic rotor is rotated counter-clockwise relative to the external magnetic field by an angle θ as shown in Figure 3.1, the resulting magnetic field and dipole moment can be represented as:

$$\mathbf{m}_{in} = m_{in} \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix} \quad (3.1)$$

$$\mathbf{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \quad (3.2)$$

The magnetic torque $\boldsymbol{\tau}$ acting on the rotor dipole is then:

$$\boldsymbol{\tau} = \mathbf{m}_{in} \times \mathbf{B} = m_{in} \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix} \times \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} = -m_{in} B \begin{bmatrix} 0 \\ 0 \\ \sin \theta \end{bmatrix} \quad (3.3)$$

This is a restoring torque that opposes the displacement θ . Hence, a nonlinear spring is produced, with stiffness k_z :

$$k_z = \frac{d\tau_z}{d\theta} = -m_{in} B \cos \theta \quad (3.4)$$

where τ_z is the z-component of $\boldsymbol{\tau}$. Note that slippage occurs when θ exceeds π radians as the torque changes direction and the rotor is accelerated in the direction of deflection instead.

The holding torque τ_{hold} can be found by:

$$\frac{d\tau_z}{d\theta} = -m_{in} B \cos \theta = 0 \quad (3.5)$$

$$\theta_{max} = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z} \quad (3.6)$$

$$\tau_{hold} = |\tau_z(\theta_{max})| = |-m_{in} B \sin \theta_{max}| = m_{in} B \quad (3.7)$$

where θ_{max} is the setting of θ at which the holding/ maximum resistive torque of the mechanism occurs.

3.2.2 2-DOF Model

To vary the strength of the magnetic coupling and hence the stiffness of the magnetic spring, the strength of the magnetic field $\mathbf{B}$ needs to be changed. One way to accomplish this is through the use of electromagnets producing magnetic fields proportional to the magnitude of the current running through them. This method, however, requires current and power to be continuously supplied, making the mechanism less energy efficient. A method to manipulate permanent magnets to produce the same effect is required instead.

If the external magnetic field is instead composed of two components, $\mathbf{B}_1$ and $\mathbf{B}_2$, both of which are half the strength and are initially oriented in the same direction but can be simultaneously rotated in opposite directions by

an angle of modulation ϕ about the z-axis, the resulting magnetic field acting on the rotor dipole is:

$$\mathbf{B}_1 = \frac{1}{2}B \begin{bmatrix} \cos \phi \\ \sin \phi \\ 0 \end{bmatrix}, \mathbf{B}_2 = \frac{1}{2}B \begin{bmatrix} \cos \phi \\ -\sin \phi \\ 0 \end{bmatrix}, \quad (3.8)$$

$$\mathbf{B}_{net} = \mathbf{B}_1 + \mathbf{B}_2 = B \begin{bmatrix} \cos \phi \\ 0 \\ 0 \end{bmatrix} \quad (3.9)$$

The restoring torque acting on the rotor, stiffness of the magnetic spring and holding torque are then:

$$\boldsymbol{\tau} = -m_{in}B \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin \theta \end{bmatrix} \quad (3.10)$$

$$k_z = \frac{d\tau_z}{d\theta} = -m_{in}B \cos \theta \cos \phi \quad (3.11)$$

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| = m_{in}B \cos \phi \quad (3.12)$$

Note that only the component of the net magnetic field in-line with the original magnetic fields' orientation remains. This provides a way to modulate the strength of the magnetic field acting on the rotor without the need to manipulate the strength of each magnetic field directly, allowing permanent magnets manipulated by non-backdrivable mechanisms to efficiently produce the effect required.

One way to realise this actuation scheme is to use a pair of magnets, each represented by a magnetic dipole for simplicity, both positioned equidistant from the rotor and opposite of each other, oriented in the same direction as the rotor dipole and rotated in opposite direction about their local z-axes located at the centre of each magnet by a single motor through a gear system.

Diametrical Magnetisation

For stronger magnetic coupling for higher maximum possible stiffness and transmissible torque more magnets can be used, expanding each magnet into

an outer semi-circle ring of magnets around the rotor all oriented in the same direction, henceforth referred to as the “Halves” configuration. This produces a virtual diametrically magnetised hollow cylindrical magnet when in the unmodulated configuration, which when represented as a continuous circle of magnetic dipoles can be visualised as in Figures 3.2, 3.3 and 3.4. The magnetic flux density due to the ring of magnets at the position of the rotor dipole can be found using [174, 8]:

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \left(\frac{3(\mathbf{m}_{out}^l \cdot \mathbf{r}(\gamma))}{|\mathbf{r}|^5} \mathbf{r}(\gamma) - \frac{\mathbf{m}_{out}^l}{|\mathbf{r}|^3} \right) r d\gamma \quad (3.13)$$

where $\mathbf{m}_{out}^l$ is the magnetic moment per unit length:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos \phi \\ \sin \phi \\ 0 \end{bmatrix} & \gamma \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ m_{out}^l \begin{bmatrix} \cos \phi \\ -\sin \phi \\ 0 \end{bmatrix} & \gamma \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right] \end{cases} \quad (3.14)$$

$\mathbf{r}(\gamma)$ is the vector connecting the rotor dipole to any generic magnetic dipole in the outer ring:

$$\mathbf{r}(\gamma) = r \begin{bmatrix} \cos \gamma \\ \sin \gamma \\ 0 \end{bmatrix}, |\mathbf{r}|^5 = r^5, |\mathbf{r}|^3 = r^3 \quad (3.15)$$

where γ represents the angular position on the ring of magnets, r is the radius of the ring of magnetic dipoles, and μ_0 is the permeability of free space. Evaluating $\mathbf{B}$ using (3.14) and (3.15) produces:

$$\mathbf{B} = \frac{\mu_0 m_{out}^l}{4r^2} \begin{bmatrix} \cos \phi \\ 0 \\ 0 \end{bmatrix} \quad (3.16)$$

The torque experienced by the rotor, the stiffness of the magnetic spring and the holding torque using (3.1) are then:

$$\boldsymbol{\tau} = -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin \theta \end{bmatrix} \quad (3.17)$$

$$k_z = \frac{d\tau_z}{d\theta} = -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \theta \cos \phi \quad (3.18)$$

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| = \frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \quad (3.19)$$

Figure 3.2: Simplified model of a novel magnetic variable stiffness actuator. An outer ring of magnets represented by a circle of magnetic dipoles with positions $\mathbf{r}$ defined by the angle γ is composed of two parts: the red semi-circle forming the virtual north pole, and the blue semi-circle forming the virtual south pole. Dipoles on both halves are initially oriented in the x-direction to produce an effective diametrical magnetisation when the actuator is in an unmodulated configuration. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ .

Radial Magnetisation

Instead of using diametrical magnetisation, the magnets in each semi-circular ring can be oriented such that their magnetic moments point either directly

Figure 3.3: 3D view of the simplified model of a novel magnetic variable stiffness actuator with effective diametrical magnetisation. τ is the restoring torque about the z-axis generated when the rotor is deflected.

Figure 3.4: Simplified 2-DOF model of the variable stiffness actuator with effective diametrical magnetisation. For modulation of the magnetic coupling strength, the magnetic dipoles within the outer ring are rotated by an angle ϕ and $-\phi$ about their local z-axes for the virtual north pole and the virtual south pole respectively.

towards or away from the rotor, creating a radial magnetisation as visualised

in Figures 3.5, 3.6 and 3.7. $\mathbf{m}_{out}^l$ is then:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(\gamma + \phi) \\ \sin(\gamma + \phi) \\ 0 \end{bmatrix} & \gamma \in (-\frac{\pi}{2}, \frac{\pi}{2}] \\ m_{out}^l \begin{bmatrix} -\cos(\gamma - \phi) \\ -\sin(\gamma - \phi) \\ 0 \end{bmatrix} & \gamma \in (\frac{\pi}{2}, -\frac{\pi}{2}] \end{cases} \quad (3.20)$$

Evaluating $\mathbf{B}$ then produces:

$$\mathbf{B} = \frac{2\mu_0 m_{out}^l}{\pi r^2} \begin{bmatrix} \cos \phi \\ 0 \\ 0 \end{bmatrix} \quad (3.21)$$

The torque experienced by the rotor, the stiffness of the magnetic spring and holding torque are then:

$$\boldsymbol{\tau} = -\frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin \theta \end{bmatrix} \quad (3.22)$$

$$k_z = \frac{d\tau_z}{d\theta} = -\frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \theta \cos \phi \quad (3.23)$$

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| = \frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \quad (3.24)$$

This radial magnetisation configuration produces a magnetic coupling that is $\frac{8}{\pi}$ times stronger than that achieved with the diametrical magnetisation configuration.

Halbach Magnetisation

Another potential magnet configuration is the Halbach cylinder, a cousin to the more famous Halbach array [175]. Both utilise a rotating pattern of magnetisation to augment the magnetic field on one side of the magnets while cancelling out the magnetic field on the other side. In the case of

Figure 3.5: An alternative setup where the dipoles within the virtual north pole are oriented to point away, and those within the virtual south pole are oriented to point towards the centre of the circle. This produces an effective radial magnetisation when the actuator is in an unmodulated configuration.

Figure 3.6: 3D view of the simplified model of a novel magnetic variable stiffness actuator with effective radial magnetisation. τ is the restoring torque about the z -axis generated when the rotor is deflected.

the Halbach cylinder, the concentration of the magnetic fields produced by the ring of magnets into the bore where the rotor magnet sits will increase maximum possible stiffness and transmissible torque at no increase in the mass of the magnets used and the volume of space occupied.

The patterns of magnetisation in Halbach cylinders are characterised by the parameter k ; the direction of magnetisation of the magnetic material within the cylinder is its angular position γ multiplied by k , as illustrated in Figure 3.8. The magnetic field produced within the bore has $k - 1$ poles. The

Figure 3.7: Simplified 2-DOF model of the variable stiffness actuator with effective radial magnetisation. The effective radial magnetisation is achieved by rotating the magnetic dipoles within the outer ring originally oriented to point in the x-direction by an angle γ and $\pi + \gamma$ about their local z-axes for the virtual north pole and the virtual south pole respectively. For modulation of the magnetic coupling strength, the magnetic dipoles within the outer ring are further rotated by an angle ϕ and $-\phi$ about their local z-axes for the virtual north pole and the virtual south pole respectively.

particular case of $k = 2$ is of particular interest since it produces a largely uniform field that is ideal for coupling with a uniformly magnetised rotor magnet.

Splitting the Halbach cylinder into halves produces the configuration vi-

Figure 3.8: Halbach cylinders with magnetisation patterns characterised by the parameter k [11].

sualised in Figures 3.9, 3.10 and 3.11. $\mathbf{m}_{out}^l$ is then:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(2\gamma + \phi) \\ \sin(2\gamma + \phi) \\ 0 \end{bmatrix} & \gamma \in (-\frac{\pi}{2}, \frac{\pi}{2}] \\ m_{out}^l \begin{bmatrix} \cos(2\gamma - \phi) \\ \sin(2\gamma - \phi) \\ 0 \end{bmatrix} & \gamma \in (\frac{\pi}{2}, -\frac{\pi}{2}] \end{cases} \quad (3.25)$$

$\mathbf{B}$ is then:

$$\mathbf{B} = \frac{3\mu_0 m_{out}^l}{4r^2} \begin{bmatrix} \cos \phi \\ 0 \\ 0 \end{bmatrix} \quad (3.26)$$

The torque, stiffness and holding torque are:

$$\boldsymbol{\tau} = -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin \theta \end{bmatrix} \quad (3.27)$$

$$k_z = \frac{d\tau_z}{d\theta} = -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \theta \cos \phi \quad (3.28)$$

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| = \frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \quad (3.29)$$

Note that this cylindrical Halbach magnetisation configuration produces a magnetic coupling that is 3 times that of the diametrical magnetisation configuration, and $\frac{3\pi}{8}$ times stronger than that achieved with the radial magnetisation configuration.

Comparison

Table 3.1 shows a comparison of the mathematical models derived thus far. On the basis of torque generation, the Halbach magnetisation configuration appears to be the most promising thus far.

Figure 3.9: Another setup where the dipoles in both the virtual north pole and virtual south pole are oriented at an angle 2γ relative to the x-axis as specified to form a Halbach cylinder.

Figure 3.10: 3D view of the simplified model of a novel magnetic variable stiffness actuator with Halbach cylinder magnetisation. τ is the restoring torque about the z-axis generated when the rotor is deflected.

Table 3.1: Comparison of the holding torques derived from the 2-DOF mathematical models. Their magnitudes relative to the Diametrical magnetisation configuration are also shown.

Configuration	τ_{hold}	Multiple
Diametrical	$\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	1
Radial	$\frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi$	$\frac{8}{\pi} \approx 2.55$
Halbach	$\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	3

Figure 3.11: Simplified 2-DOF model of the variable stiffness actuator with Halbach cylinder magnetisation where the dipoles in both the virtual north pole and virtual south pole are oriented at an angle of 2γ relative to the x-axis. For modulation of the magnetic coupling strength, the magnetic dipoles within the outer ring are further rotated by an angle ϕ and $-\phi$ about their local z-axes for the virtual north pole and the virtual south pole respectively.

3.2.3 3-DOF Model

For the mechanism to function as an actuator, the net effective direction of the magnetic field $\mathbf{B}$ and hence the equilibrium rest position of the rotor needs to be adjustable. If electromagnets are used the magnetic field can be made to rotate as in an Alternating Current (AC) motor. If permanent magnets are to be used for their strength and persistent magnetic fields, a different solution is required.

For the system with the magnetic field composed of two components, $\mathbf{B}_1$ and $\mathbf{B}_2$, if both components are rotated by an additional angle ψ about the z-axis, we get:

$$\mathbf{B}_1 = \frac{1}{2}B \begin{bmatrix} \cos(\psi + \phi) \\ \sin(\psi + \phi) \\ 0 \end{bmatrix}, \mathbf{B}_2 = \frac{1}{2}B \begin{bmatrix} \cos(\psi - \phi) \\ \sin(\psi - \phi) \\ 0 \end{bmatrix}, \quad (3.30)$$

$$\mathbf{B}_{net} = \mathbf{B}_1 + \mathbf{B}_2 = B \begin{bmatrix} \cos \phi \cos \psi \\ \cos \phi \sin \psi \\ 0 \end{bmatrix} = B \cos \phi \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix} \quad (3.31)$$

$$\mathbf{m}_{in} = m_{in} \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix} \quad (3.32)$$

$$\begin{aligned} \boldsymbol{\tau} &= \mathbf{m}_{in} \times \mathbf{B}_{net} = -m_{in} B \cos \phi \begin{bmatrix} 0 \\ 0 \\ \cos \theta \sin \psi - \sin \theta \cos \psi \end{bmatrix} \\ &= -m_{in} B \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta - \psi) \end{bmatrix} \end{aligned} \quad (3.33)$$

$$k_z = \frac{d\tau_z}{d\theta} = -m_{in} B \cos(\theta - \psi) \cos \phi \quad (3.34)$$

Hence no restoring torque is produced when $\theta = \psi$, the equilibrium rest position of the rotor. Letting $\zeta = \theta - \psi$ be the deflection of the rotor away from its equilibrium position, the holding torque can be found by:

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d\zeta} = 0 \right) \right| = m_{in} B \cos \phi \quad (3.35)$$

For the setup using a ring of magnets, the most immediate and obvious solution is to rotate the ring of magnets about the global z-axis with a “positioning” motor. While this concept is simple it likely requires the motor and gearing system used for modulating the strength of the magnetic coupling to rotated as well, making the mechanism’s design more complex while also impacting its performance due to the higher inertia the “positioning” motor needs to deal with.

Diametrical Magnetisation

Alternatively, if the entire ring of magnets are rotated about their respective local z-axes by an angle ψ , for the configuration with effective diametrical

magnetisation shown in Figure 3.12 we get:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(\psi + \phi) \\ \sin(\psi + \phi) \\ 0 \end{bmatrix} & \gamma \in (-\frac{\pi}{2}, \frac{\pi}{2}] \\ m_{out}^l \begin{bmatrix} \cos(\psi - \phi) \\ \sin(\psi - \phi) \\ 0 \end{bmatrix} & \gamma \in (\frac{\pi}{2}, -\frac{\pi}{2}] \end{cases} \quad (3.36)$$

$$\mathbf{B} = \frac{\mu_0 m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix} \quad (3.37)$$

$$\begin{aligned} \boldsymbol{\tau} &= -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \cos \theta \sin \psi - \sin \theta \cos \psi \end{bmatrix} \\ &= -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta - \psi) \end{bmatrix} \end{aligned} \quad (3.38)$$

$$k_z = \frac{d\tau_z}{d\theta} = -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos(\theta - \psi) \cos \phi \quad (3.39)$$

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d(\zeta = \theta - \psi)} = 0 \right) \right| = \frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \quad (3.40)$$

Comparing (3.38) to (3.33), it can be seen that the same effect is produced. By using independent actuators to drive each half of the ring of magnets the need to rotate the entire stiffness modulating mechanism is eliminated.

Figure 3.12: Simplified 3-DOF model of the variable stiffness actuator with effective diametrical magnetisation. The equilibrium position of the actuator is adjusted by further rotating the magnetic dipoles within the outer ring by an angle ψ about their local z-axes.

Radial Magnetisation

When a similar scheme is applied to the configuration with effective radial magnetisation as shown in Figure 3.13 we get:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(\gamma + \psi + \phi) \\ \sin(\gamma + \psi + \phi) \\ 0 \end{bmatrix} & \gamma \in (-\frac{\pi}{2}, \frac{\pi}{2}] \\ m_{out}^l \begin{bmatrix} -\cos(\gamma + \psi - \phi) \\ -\sin(\gamma + \psi - \phi) \\ 0 \end{bmatrix} & \gamma \in (\frac{\pi}{2}, -\frac{\pi}{2}] \end{cases} \quad (3.41)$$

$$\mathbf{B} = \frac{\mu_0 m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 2 \cos \psi \\ -\sin \psi \\ 0 \end{bmatrix} \quad (3.42)$$

$$\boldsymbol{\tau} = -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \cos \theta \sin \psi + 2 \sin \theta \cos \psi \end{bmatrix} \quad (3.43)$$

$$k_z = \frac{d\tau_z}{d\theta} = -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} (2 \cos \theta \cos \psi - \sin \theta \sin \psi) \cos \phi \quad (3.44)$$

Figure 3.13: Simplified 3-DOF model of the variable stiffness actuator with effective radial magnetisation. The equilibrium position of the actuator is adjusted by further rotating the magnetic dipoles within the outer ring by an angle ψ about their local z-axes.

The expression derived for the torque generated by the mechanism suggests that a direct mapping from a desired equilibrium rest orientation of the rotor θ_d onto the ψ angle setting cannot be made. The appropriate mapping can be found by taking:

$$\cos \theta_d \sin \psi + 2 \sin \theta_d \cos \psi = 0 \quad (3.45)$$

$$\tan \psi = -2 \tan \theta_d \quad (3.46)$$

$$\psi = \arctan(-2 \tan \theta_d) = -\arctan(2 \tan \theta_d) \quad (3.47)$$

The resulting mapping can be seen in Figure 3.14. Counter-intuitively the desired equilibrium position θ_d requires setting ψ to an opposite value of almost equal magnitude. Comparing the graph to the straight line of slope negative one we can see that ψ leads (in magnitude) θ_d for $\theta_d \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ while lagging for $\theta_d \in [-\pi, -\frac{\pi}{2}] \cup [\frac{\pi}{2}, \pi]$. It is clear that the control of a variable stiffness mechanism with this configuration is more complicated than in the case where effective diametrical magnetisation is used.

Figure 3.14: Setting of ψ for any particular desired equilibrium orientation θ_d .

If the simple linear mapping of $\psi = -\theta_d$ is made, a residual torque will be present for $\psi \neq n\frac{\pi}{2}$, $n \in \mathbb{Z}$:

$$\tau_{res} = \frac{\mu_0 m_{in} m_{out}^l}{2\pi r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin 2\psi \end{bmatrix} \quad (3.48)$$

This residual torque is largest when $\psi = \frac{\pi}{4} + n\frac{\pi}{2}$, $n \in \mathbb{Z}$.

Another important parameter is the minimum holding torque that can be generated by the variable stiffness mechanism. For the setup with effective diametrical magnetisation, this holding torque value is the same across all values for θ_d . For the setup with effective radial magnetisation however, the expression derived for torque suggests that the holding torque varies depending on the setting for ψ as well. Finding the minimum holding torque (independent of the modulation of the magnetic coupling) produced by such a mechanism requires:

$$\min_{\psi}(\max_{\theta}(\tau_z)) \implies \min_{\psi}(\max_{\theta}(\cos \theta \sin \psi + 2 \sin \theta \cos \psi)) \quad (3.49)$$

Evaluating the inner condition gives the rotor position θ that generates the largest restoring torque in magnitude for any set value of ψ :

$$\frac{d}{d\theta}(\cos \theta \sin \psi + 2 \sin \theta \cos \psi) = 0 \quad (3.50)$$

$$2 \cos \theta \cos \psi - \sin \theta \sin \psi = 0 \quad (3.51)$$

$$\theta = \arctan(2 \cot \psi) \quad (3.52)$$

Substituting into (3.49) produces:

$$\min_{\psi}(\cos(\arctan(2 \cot \psi)) \sin \psi + 2 \sin(\arctan(2 \cot \psi)) \cos \psi) \quad (3.53)$$

which simplifies to:

$$\min_{\psi}(\sqrt{3 \cos^2 \psi + 1}) \quad (3.54)$$

Substituting (3.52) back into (3.43) gives the holding torque of the mechanism:

$$\tau_{hold} = |\tau_z(\theta = \arctan(2 \cot \psi))| = \frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \sqrt{3 \cos^2 \psi + 1} \cos \phi \quad (3.55)$$

The maximum holding torque $\tau_{hold,max}$ is achieved when $\cos^2 \psi = 1$:

$$\psi = n\pi, \quad n \in \mathbb{Z} \quad (3.56)$$

$$\tau_{hold,max} = \frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \quad (3.57)$$

and the minimum holding torque $\tau_{hold,min}$ occurs when $\cos^2 \psi = 0$:

$$\psi = n\pi + \frac{\pi}{2}, \quad n \in \mathbb{Z} \quad (3.58)$$

$$\tau_{hold,min} = \frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \quad (3.59)$$

For reference, the holding torque produced by a setup with effective diametrical magnetisation is reproduced from (3.40):

$$\tau_{hold} = \frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \quad (3.60)$$

Note that the minimum holding torque generated is half the magnitude of the maximum holding torque, while also being slightly larger than the holding

torque generated by the setup with effective diametrical magnetisation by a factor of $\frac{4}{\pi}$.

The residual torque from (3.48) is:

$$\tau_{res} = \frac{\mu_0 m_{in} m_{out}^l}{2\pi r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin 2\psi \end{bmatrix} \quad (3.61)$$

The residual torque when using the simple mapping is significant, having half the magnitude of the minimum holding torque at its maximum, hence pointing to the necessity of using the more complex mapping in (3.47).

Halbach Magnetisation

For the Halbach magnetisation as shown in Figure 3.15 we get:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(2\gamma + \psi + \phi) \\ \sin(2\gamma + \psi + \phi) \\ 0 \end{bmatrix} & \gamma \in (-\frac{\pi}{2}, \frac{\pi}{2}] \\ m_{out}^l \begin{bmatrix} \cos(2\gamma + \psi - \phi) \\ \sin(2\gamma + \psi - \phi) \\ 0 \end{bmatrix} & \gamma \in (\frac{\pi}{2}, -\frac{\pi}{2}] \end{cases} \quad (3.62)$$

$$\mathbf{B} = \frac{3\mu_0 m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} \cos \psi \\ -\sin \psi \\ 0 \end{bmatrix} \quad (3.63)$$

$$\begin{aligned} \boldsymbol{\tau} &= -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \cos \theta \sin \psi + \sin \theta \cos \psi \end{bmatrix} \\ &= -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta + \psi) \end{bmatrix} \end{aligned} \quad (3.64)$$

$$k_z = \frac{d\tau_z}{d\theta} = -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos(\theta + \psi) \cos \phi \quad (3.65)$$

$$\tau_{hold} = \left| \tau_z \left(\frac{d\tau_z}{d(\zeta = \theta + \psi)} = 0 \right) \right| = \frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \quad (3.66)$$

Equation (3.64) shows that the effect of rotating the outer magnets by an additional angle ψ in a Halbach magnetisation configuration has the counter-intuitive effect of shifting the equilibrium rest position by $-\psi$. Hence $-\psi$ should be used as the equilibrium rest position setting instead.

Note that the holding torque of the Halbach magnetisation configuration is $\frac{3\pi}{8}$ times larger than the maximum holding torque in the radial magnetisation case while retaining the simplicity of the torque profile of the diametrical magnetisation configuration.

Figure 3.15: Simplified 3-DOF model of the variable stiffness actuator with Halbach cylinder magnetisation. The equilibrium position of the actuator is adjusted by further rotating the magnetic dipoles within the outer ring by an angle $-\psi$ about their local z-axes.

Comparison

Table 3.2 shows a comparison of the 3-DOF mathematical models. With the addition of the ψ parameter for adjusting the equilibrium position of the mechanism, an additional weakness of the radial magnetisation configuration in the form of lower holding torques for $\psi \neq n\pi$ is revealed. Hence the Halbach magnetisation configuration remains the most promising thus far.

Table 3.2: Comparison of the holding torques derived from the 3-DOF mathematical models. Their magnitudes relative to the Diametrical magnetisation configuration are also shown.

Configuration	τ_{hold}	Multiple
Diametrical	$\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	1
Radial (max)	$\frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi$	$\frac{8}{\pi} \approx 2.55$
Radial (min)	$\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi$	$\frac{4}{\pi} \approx 1.27$
Halbach	$\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	3

3.2.4 Simple Simulation Model

While the mathematical models presented in Section 3.2 illustrate the basic principle behind the workings of the mechanism, they do not account for the complex geometry that physical mechanisms have. Simulation studies using the FEM are thus necessary to verify that the magnetic modulation predicted is still present in a more realistic setup geometry, and also to provide an estimate of the performance possible from such mechanisms. *ANSYS Maxwell 3D* was used to perform the magnetostatic FEM analysis.

Taking advantage of the ability in *ANSYS Maxwell* to specify the direction of magnetisation of materials in vectorial or radial form, a relatively simple simulation setup consisting of two nested concentric rings of magnetic material was created to test the working principles of the mechanism. The dimensions of the setup used in the MAVERIC paper [8] were adopted and slightly modified to equalise the heights of the outer and inner magnets, as detailed in Table 3.3, and illustrated in Figures 3.16 and 3.17. The magnetisations of the material used in this set of simulations were defined as specified in Table 3.4.

The NdFe35 material in the default materials library was adapted to represent the permanent magnets to be used in the mechanism, with properties as shown in Table 3.5. The variables for the FEM analysis are θ from -180° to 180° in steps of 22.5° , and ψ and ϕ each from 0° to 90° in steps of 22.5° .

Table 3.3: Parameters of the simple simulation setup.

Parameter [unit]	Outer Magnet	Inner Magnet
Inner Radius [mm]	6.25	1
Outer Radius [mm]	10.4	6
Height [mm]	10	10

Table 3.4: Definitions of the magnetic material in *ANSYS Maxwell* for the Halves magnetisation configurations for the simple simulation model. The Z component of is set to 0 in all cases.

Configuration				Magnetic Coercivity
Ring	Magnetisation	Pole	Component	
Outer	Diametrical	North	X	$\cos(\psi + \phi)$
			Y	$\sin(\psi + \phi)$
		South	X	$\cos(\psi - \phi)$
			Y	$\sin(\psi - \phi)$
	Radial	North	X	$\cos(\gamma + \psi + \phi)$
			Y	$\sin(\gamma + \psi + \phi)$
		South	X	$-\cos(\gamma + \psi - \phi)$
			Y	$-\sin(\gamma + \psi - \phi)$
	Halbach	North	X	$\cos(2\gamma + \psi + \phi)$
			Y	$\sin(2\gamma + \psi + \phi)$
		South	X	$\cos(2\gamma + \psi - \phi)$
			Y	$\sin(2\gamma + \psi - \phi)$
Inner	Diametrical	Both	X	$\cos \theta$
			Y	$\sin \theta$
	Radial	North	R	1
			Phi	0
		South	R	-1
			Phi	0

Table 3.5: Properties of magnetic material used for FEM analysis.

Name	NdFe35
Magnitude of Magnetic Coercivity [kA m^{-1}]	890
Relative Permeability	1.0997785406
Bulk Conductivity [$\text{k}\Omega \text{m}^{-1}$]	625

Figure 3.16: Top view of the geometry of the simple simulation setup. The direction of magnetisation of the materials for the “Halves” configuration are defined in Table 3.4.

Outer:Diametrical—Inner:Diametrical

For the case where both the outer and inner magnets are diametrically magnetised corresponding to the 3-DOF Model with Diametrical Magnetisation in Section 3.2.3, the torque profiles generated for $\psi = 0$ rad is shown in Figure 3.18. The effect of changing the parameter ψ responsible for setting the equilibrium rest position of the mechanism is visualised in Figure 3.19 where the torque profiles are plotted for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad.

Note how the holding torque produced by the mechanism changes with variation of ϕ , going from 101 mNm with no modulation ($\phi = 0^\circ$) to being

Figure 3.17: Isometric view of the geometry of the simple simulation setup with a diametrically magnetised rotor magnet. Magnetisation of the materials for the Halves configuration are as defined in Table 3.4.

Figure 3.18: Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.19: Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

non-existent for $\phi = 90^\circ$, indicating that the rotor can rotate freely. The general shape of the torque curves shown in both figures is also in line with that predicted by Equation (3.38).

Outer:Diametrical—Inner:Radial

It is also possible for a radially magnetised magnet to be used as the inner magnet. The torque profiles that are generated with such a configuration is shown in Figure 3.20 for $\psi = 0$ rad, with the effect of changing ψ shown in Figure 3.21.

Compared to the original configuration where both magnets used were diametrically magnetised the holding torque is about 28% lower at 73 mNm versus 101 mNm. Since the general characteristics of both configurations are similar, the Diametrical-Radial is overall an inferior design.

Figure 3.20: Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.21: Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.22: Torque profiles generated by the Havles Radial-Diametrical magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Outer:Radial—Inner:Diametrical

For the case where the outer magnet is radially magnetised corresponding to the 3-DOF Model with Radial Magnetisation in Section 3.2.3, the torque profiles generated for $\psi = 0$ rad is shown in Figure 3.22, with the effect of changing ψ shown in Figure 3.23.

Compared to the Diametrical-Dimetrical magnetisation configuration the maximum holding torque produced is 3.5 times as large at 349 mNm while the minimum holding torque produced is 2.2 times as large at 219 mNm. The benefits of an outer radially magnetised magnet ring in the simple simulation thus appears to be significantly larger than that the $\frac{4}{\pi}$ advantage predicted by the mathematical models in either Section 3.2.2 or Section 3.2.3. The ratio of the maximum holding torque to the minimum holding torque also deviates from the mathematical model with a value of is 1.59 versus the predicted value of 2.

Figure 3.24 shows that the residual torque that is still present when

Figure 3.23: Torque profiles generated by the Halves Radial-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.24: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Halves Radial-Diametrical magnetisation for the simple simulation model.

Figure 3.25: Torque profiles generated by the Havles Radial-Radial magnetisation setup for $\psi = 0$ rad for the simple simulation model.

$\theta = -\psi$ matches that predicted by equation (3.48) in shape. It has a maximum magnitude of 65 mNm at $\psi = 45^\circ$. While this is considerably smaller than the 50% predicted in Section 3.2.3, it is still significant with a magnitude of approximately 30% of that of the minimum holding torque.

Outer:Radial—Inner:Radial

The torque profiles generated if the inner rotor magnet is instead radially magnetised is shown in Figure 3.25 for $\psi = 0$ rad, with the effect of changing ψ shown in Figure 3.26.

For $\psi = 0$ rad this configuration becomes an adjustable constant torque torsion spring with a maximum torque capacity of 193 mNm as shown in Figure 3.25. This phenomenon is however not preserved when ψ is adjusted, as visualised in Figure 3.26, creating a spring with initially small restoring torque that accelerates up to a peak of 273 mNm at larger rotor deflections. Hence if an adjustable constant torque spring with adjustable equilibrium position is needed, the entire variable stiffness mechanism will need to be

Figure 3.26: Torque profiles generated by the Halves Radial-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

rotated instead of manipulating the parameter ψ as in the previous configurations.

The same counter-intuitive reverse relationship between the equilibrium position of the rotor and the setting of ψ , as expected from the use of a radially oriented outer ring of magnets, is also present. Figure 3.27 shows the residual torque present when $\theta = -\psi$. Its shape is somewhat skewed from that in Figure 3.24, with a sampled peak of 113 mNm.

Outer:Halbach—Inner:Diametrical

For the case where the outer magnet is a Halbach cylinder ($k = 2$) corresponding to the 3-DOF Model with Halbach Magnetisation in Section 3.2.3, the torque profiles generated for $\psi = 0$ rad is shown in Figure 3.28, with the effect of changing ψ shown in Figure 3.29.

Compared to the Halves Diametrical-Diametrical setup, the Halves Halbach-Diametrical setup produces a maximum holding torque that is almost 4.4 times larger at 443 mNm, easily exceeding the predicted advantage of 3

Figure 3.27: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Halves Radial-Radial magnetisation for the simple simulation model.

Figure 3.28: Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.29: Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

predicted in Section 3.2.2. Additionally, Figure 3.29 confirms the counter-intuitive relationship between ψ and the equilibrium rest position of the rotor magnet.

Outer:Halbach—Inner:Radial

For the pairing of a Halbach cylinder with a radially magnetised rotor magnet, the torque profiles that are generated with such a configuration is shown in Figure 3.30 for $\psi = 0$ rad, with the effect of changing ψ shown in Figure 3.31.

A similar relationship as that between the Halves Diametrical-Diametrical and Halves Diametrical-Radial setups exist between the Halves Halbach-Diametrical and Halves Halbach-Radial setups, with the latter's maximum holding torque of 303 mNm being about 32% smaller. Hence it appears that the Halbach-Diametrical configuration is the most promising examined thus far.

Figure 3.30: Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.31: Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

3.2.5 Ring of Magnets Simulation Model

A more realistic indication of the mechanism's potential performance can be obtained if the geometry of the mechanism is modelled in more detail, at the expense of significantly higher computational costs. Here, the ring of magnet setup was modelled.

The geometry of the setup is constrained by the desire to pack the magnets closely together for higher torques and stiffness within a smaller volume. If the separation between all magnets is kept the same, a setup similar to that illustrated in Figure 3.32 is produced. For configuration with rotor magnet of radius r_{rtr} and $2n$ ring magnets, we get:

$$\sin \frac{\pi}{2n} = \frac{(r_{rng} + \frac{\delta}{2})}{(r_{rtr} + \frac{\delta}{2}) + (r_{rng} + \frac{\delta}{2})} \quad (3.67)$$

The radius of the ring magnet r_{rng} , its radial position in the setup $r_{pos,rng}$ and the overall radius of the setup r_{setup} are given by (3.68), (3.69) and (3.70) respectively.

$$r_{rng} = (r_{rtr} + \frac{\delta}{2}) \frac{\sin \frac{\pi}{2n}}{1 - \sin \frac{\pi}{2n}} - \frac{\delta}{2} \quad (3.68)$$

$$r_{pos,rng} = (r_{rtr} + \frac{\delta}{2}) \left(1 + \frac{\sin \frac{\pi}{2n}}{1 - \sin \frac{\pi}{2n}} \right) \quad (3.69)$$

$$r_{setup} = (r_{rtr} + \frac{\delta}{2}) \frac{2 \sin \frac{\pi}{2n}}{1 - \sin \frac{\pi}{2n}} \quad (3.70)$$

Table 3.6 shows the size of the required ring magnets r_{rng} , radial position of the ring magnets $r_{pos,rng}$ and the overall size of the setup r_{setup} for a rotor magnet with radius $r_{rtr} = 6$ mm, a desired gap $\delta = 0.25$ mm and number of ring magnets $2n$ for $n \in [2, 3, 4, 5, 6, 7, 8]$.

To retain a similar overall size as in Section 3.2.4, the setup with 12 ring magnets $n = 6$ was chosen. The full geometrical parameters for the setup are given in Table 3.7 and illustrated in Figures 3.33, 3.34 and 3.35. The direction of magnetisation of the material used in the simulations defined in Table 3.8.

The same NdFe35 material with properties as shown in Table 3.5 was again used for this set of simulations. The variables for the FEM analysis

Figure 3.32: Ring of Magnets geometry for number of ring magnets $n = 4$. The solid curves represents the rotor and ring magnets, while the dashed curves represents the space needed for each magnet to maintain the desired gap δ between all magnets, with radii $\frac{\delta}{2}$ larger than their solid counterparts. r_{rtr} is the radius of the rotor magnet, and r_{rng} is the radius of the ring magnets.

Table 3.6: Ring magnet size r_{rng} , radial position of ring magnets $r_{pos,rng}$ and overall setup size r_{setup} for $r_{rtr} = 6$ mm, $\delta = 0.25$ mm and $n \in [2, 3, 4, 5, 6, 7, 8]$.

n	r_{rng} [mm]	$r_{pos,rng}$ [mm]	r_{setup} [mm]
2	14.66	20.91	35.70
3	6.125	12.25	18.38
4	3.797	9.922	13.72
5	2.739	8.864	11.60
6	2.139	8.264	10.40
7	1.753	7.878	9.631
8	1.485	7.610	9.094

similar to that used for the Simple Simulation Model, with ψ and ϕ each from 0° to 90° in steps of 22.5° . θ is sampled more finely to illuminate the effect of using discrete magnets in the outer ring, with 2 sampling points to “frame” each ring magnet, one sampling point to capture its peak and additional 2 in between to flesh out the shape of the torque profile. Since 12 outer ring magnets were used ($n = 6$), the sampling gap was set as $\frac{360^\circ}{2n(5-1)} = 7.5^\circ$. Hence we have θ from -180° to 180° in steps of 7.5° .

Table 3.7: Parameters of the ring of magnets simulation setup.

Parameter [unit]	Value
Inner Radius of Rotor Magnet [mm]	1
Outer Radius of Rotor Magnet [mm]	6
Height of Rotor Magnet [mm]	10
Number of Ring Magnets	$2n$
Radius of Ring Magnets [mm]	r_{rng}
Height of Ring Magnets [mm]	10
Radial Position of Ring Magnets [mm]	$r_{pos,rng}$

Outer:Diametrical—Inner:Diametrical

For the case where the outer ring of magnets are arranged to produce an effective diametrical magnetisation corresponding to the 3-DOF Model with Diametrical Magnetisation in Section 3.2.3, and paired with a diametrically magnetised inner rotor magnet, the torque profiles generated for $\psi = 0$ rad is shown in Figure 3.36. The effect of changing the parameter ψ responsible for setting the equilibrium rest position of the mechanism is visualised in Figure 3.37 where the torque profiles are plotted for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad.

Compared to the results from Section 3.2.4 the holding torque with no modulation was found to be about 25% smaller at 76 mNm. This reduction is expected due to the smaller amount of magnetic material in play, as well as the variation in relative size between the rotor and ring magnets.

Table 3.8: Definitions of magnetic material in *ANSYS Maxwell* for the Halves magnetisation configurations for the ring of magnets simulation model. The ring of magnets is composed of $2n$ magnets divided evenly into 2 “poles”, with individual magnets from each “pole” numbered $i \in [0, n - 1]$, counting up from the ones located proximal to the negative y-axis, counter-clockwise for the virtual “north pole” and clockwise for the virtual “south pole” for the Diametrical and Halbach magnetisation configurations. For the Radial magnetisation configuration, the virtual “south pole” counts up from the one proximal to the y-axis in a counter-clockwise fashion instead. The Z component is set to 0 in all cases.

Configuration				Magnetic Coercivity
Magnet	Magnetisation	Pole	Component	
Ring	Diametrical	North	X	$\cos(\psi + \phi)$
			Y	$\sin(\psi + \phi)$
		South	X	$\cos(\psi - \phi)$
			Y	$\sin(\psi - \phi)$
	Radial	North	X	$\cos(\frac{\pi}{n}(i_N + \frac{1}{2}) - \frac{\pi}{2} + \psi + \phi)$
			Y	$\sin(\frac{\pi}{n}(i_N + \frac{1}{2}) - \frac{\pi}{2} + \psi + \phi)$
		South	X	$\cos(-\frac{\pi}{n}(i_S + \frac{1}{2}) + \frac{\pi}{2} + \psi - \phi)$
			Y	$\sin(-\frac{\pi}{n}(i_S + \frac{1}{2}) + \frac{\pi}{2} + \psi - \phi)$
	Halbach	North	X	$\cos(2(\frac{\pi}{N}(i_n + \frac{1}{2}) - \frac{\pi}{2}) + \psi + \phi)$
			Y	$\sin(2(\frac{\pi}{n}(i_N + \frac{1}{2}) - \frac{\pi}{2}) + \psi + \phi)$
		South	X	$\cos(-2(\frac{\pi}{n}(i_S + \frac{1}{2}) + \frac{\pi}{2}) + \psi - \phi)$
			Y	$\sin(-2(\frac{\pi}{n}(i_S + \frac{1}{2}) + \frac{\pi}{2}) + \psi - \phi)$
Rotor	Diametrical	Both	X	$\cos \theta$
			Y	$\sin \theta$
	Radial	North	R	1
			Phi	0
		South	R	-1
			Phi	0

Figure 3.33: Top view of the geometry of the ring of magnets simulation setup, with ring magnets labelled for the Diametrical or Halbach magnetisation configurations. The direction of magnetisation of the materials for the “Halves” configuration are defined in Table 3.8. Magnetisation directions are illustrated for $\psi = 25^\circ$, $\phi = 80^\circ$. The magnets belonging to the virtual “north pole” and “south pole” are drawn in red and blue respectively.

Outer:Diametrical—Inner:Radial

When the inner rotor magnet is radially magnetised instead, the resulting torque profiles are as shown in Figures 3.38 and 3.39.

The “wavy” appearance of the torque profiles is due to the discrete magnetisations of each ring magnet, as opposed to the continuous variance of magnetisation direction defined for the outer ring material in 3.2.4, when used in conjunction with a radially magnetised rotor magnet. Compared to when the inner rotor magnet is diametrically magnetised the maximum holding torque is 24% smaller at 58 mNm.

Figure 3.34: Top view of the geometry of the ring of magnets simulation setup, with ring magnets labelled for the Radial magnetisation configuration. The direction of magnetisation of the materials for the “Halves” configuration are defined in Table 3.8. Magnetisation directions are illustrated for $\psi = 25^\circ$, $\phi = 80^\circ$. The magnets belonging to the virtual “north pole” and “south pole” are drawn in red and blue respectively.

Outer:Radial—Inner:Diametrical

For the case where the outer ring of magnets are arranged to produce an effective radial magnetisation configuration corresponding to the 3-DOF Model with Radial Magnetisation in Section 3.2.3, the torque profiles generated are shown in Figures 3.22 and 3.23.

Compared to the Diametrical-Diametrical magnetisation configuration the maximum holding torque produced is about 3.3 times as large at 247 mNm while the minimum holding torque produced is about 2 times as large at 150 mNm. The benefits of an outer radially magnetised magnet ring in the simple simulation thus appears to be significantly larger than that the $\frac{4}{\pi}$ advantage predicted by the mathematical models in either Section 3.2.2 or Section 3.2.3. The ratio of the maximum holding torque to the minimum

Figure 3.35: Isometric view of the geometry of the ring of magnets simulation setup with a radially magnetised rotor magnet. Magnetisation of the materials for the Halves configuration are as defined in Table 3.8.

Figure 3.36: Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.37: Torque profiles generated by the Halves Diametrical-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.38: Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.39: Torque profiles generated by the Halves Diametrical-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.40: Torque profiles generated by the Halves Radial-Diametrical magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.41: Torque profiles generated by the Halves Radial-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

holding torque also deviates from the mathematical model with a value of about 1.6 versus the value of 2 predicted.

Figure 3.42 shows the residual torque that is still present when $\theta = -\psi$, which matches that predicted by equation (3.48) in shape. It has a maximum magnitude of 48 mNm at $\psi = 45^\circ$. While this is considerably smaller than the 50% of the minimum holding torque predicted in Section 3.2.3, it is still significant with a magnitude of 32%.

Outer:Radial—Inner:Radial

When the inner rotor magnet is radially magnetised instead, the resulting torque profiles are as shown in Figures 3.43 and 3.44.

For $\psi = 0$ rad this configuration becomes similar to an adjustable constant torque torsion spring with a maximum torque capacity of 137 mNm as shown in Figure 3.43. The torque produced however is not as consistent due to the discrete magnetisation of the individual magnets, with the “wavy” appearance carrying over from the Halves Diametrical-Radial configuration. As

Figure 3.42: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Halves Radial-Diametrical magnetisation for the ring of magnets simulation model.

Figure 3.43: Torque profiles generated by the Halves Radial-Radial magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.44: Torque profiles generated by the Halves Radial-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

is the case with the simple simulation model, this “constant” torque spring phenomenon is not preserved when ψ is adjusted, as shown in Figure 3.44, creating a spring with initially small restoring torques that accelerates up to a peak of 223 mNm at larger rotor deflections.

The reverse relationship between the equilibrium position of the rotor and the setting of ψ from the use of a radially magnetised outer ring of magnets is present. Figure 3.45 shows the residual torque present when $\theta = -\psi$. Its shape is somewhat skewed from that in Figure 3.42, with a sampled peak of 85 mNm.

Outer:Halbach—Inner:Diametrical

For the case where the outer ring of magnets are arranged to produce an effective Halbach cylinder ($k = 2$) corresponding to the 3-DOF Model with Halbach Magnetisation in Section 3.2.3, the torque profiles generated for $\psi = 0$ rad is shown in Figure 3.46, with the effect of changing ψ shown in Figure 3.47.

Figure 3.45: Torque deviation when $\theta = -\psi$ for $\phi = 0^\circ$ for the Halves Radial-Radial magnetisation for the ring of magnets simulation model.

Figure 3.46: Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.47: Torque profiles generated by the Halves Halbach-Diametrical magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Compared to the Halves Diametrical-Diametrical setup, the Halves Halbach-Diametrical setup produces a maximum holding torque that is almost 4.1 times larger at 309 mNm, easily exceeding the predicted advantage of 3 predicted in Section 3.2.2. Additionally, Figure 3.29 confirms the counter-intuitive relationship between ψ and the equilibrium rest position of the rotor magnet.

Outer:Halbach—Inner:Radial

When the inner rotor magnet is radially magnetised instead, the torque profiles that are generated with such a configuration is shown in Figure 3.48 for $\psi = 0$ rad, with the effect of changing ψ shown in Figure 3.49.

A similar relationship as that between the Halves Diametrical-Diametrical and Halves Diametrical-Radial setups exist between the Halves Halbach-Diametrical and Halves Halbach-Radial setups, with the latter's maximum holding torque of 214 mNm being about 31% smaller. Hence it appears that the Halbach-Diametrical configuration is the most promising examined thus

Figure 3.48: Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.49: Torque profiles generated by the Halves Halbach-Radial magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

far.

3.3 Optimal Size

An optimal mechanism of the proposed design seeks to deliver the largest range of stiffness for the mass of the mechanism or the volume that the mechanism occupies. Setting aside the question of the optimal selection of gears and housing due to the complexity involved, only the magnets used in the mechanism design will be considered here.

Since the magnetic field generated by a magnet decays in strength as one moves away from it, with a scaling of r^3 for a point dipole and a scaling of r^2 for a linear magnet of infinite length, magnetic material in the ring of magnets that is further away from the rotor must necessarily contribute less to the generation of torque upon it, and is hence a less “efficient” use of the magnetic material. At the same time, the torque generated also declines with the reduction of magnetic material used in the ring of magnets. It stands to reason then that there exists an optimal rotor magnet to ring magnet size ratio that generates torque on the rotor most efficiently.

For a Halbach cylinder ($k = 2$) it is known [175] that the strength of the uniform magnetic flux B_w in its cavity is related to its outer radius r_{out} and inner radius r_{in} , and the flux density of the magnetic material B_r by:

$$B_w = B_r \ln \frac{r_{out}}{r_{in}} \quad (3.71)$$

Using the 2-dimensional geometrical setup of the simple simulation model, assuming that the rotor and outer magnet are made of the same material (and hence density) and neglecting the gap between the rotor and the outer ring as well as the bore of the rotor magnet, a relationship between the torque generated to volume ratio, and the ratio $x = \frac{r_{out}}{r_{in}}$ is given by:

$$|\tau| \propto B_w r_{out}^2 \quad (3.72)$$

$$\begin{aligned} \frac{|\tau|}{volume} &\propto B_w \frac{r_{in}^2}{r_{out}^2} \\ &\propto \frac{\ln x}{x^2} \end{aligned} \quad (3.73)$$

The first derivative of (3.73) finds a turning point at:

$$\frac{d}{dx} \frac{\ln x}{x^2} = \frac{1 - 2 \ln x}{x^3} = 0 \quad (3.74)$$

$$\ln x = \frac{1}{2} \quad , \quad x = \sqrt{e} \approx 1.6487 \quad , \quad x \in \mathbb{R}^+ \quad (3.75)$$

The second derivative produces a negative value at $x = \sqrt{e}$ where e is Euler's number, indicating it is a maximum:

$$\frac{d^2}{dx^2} \frac{\ln x}{x^2} = \frac{6 \ln x - 5}{x^4} = -\frac{2}{e^2} \quad , \quad x = \sqrt{e} \quad (3.76)$$

For a rotor magnet of radius 6 mm, this translates into a setup of overall radius approximately 9.89 mm. For a ring of magnets setup with no gaps between the magnets ($\delta = 0$), the radius of the ring magnets is:

$$r_{rng} = \frac{r_{out} - r_{in}}{2} \quad (3.77)$$

The ring magnet radius is thus approximately 1.95 mm. Substituting into equation (3.67) and solving for n produces a value of approximately 6.35. Hence the configurations $n = 6, 7$ are potentially optimal.

Although this result was derived for Halbach magnetisation, since the fundamental property of decaying magnetic field strength over distance and the rate at which this occurs is similar for most magnetisation configurations, it may, in general, be approximately valid. This will be examined in this section through the simulation results for the Diametrical and Radial magnetisation configurations.

3.3.1 Simple Simulation Model

To check this result with FEM simulation, the simple simulation model was run again for $\theta = 90^\circ$, $\psi = 0^\circ$ and $\phi = 0^\circ$, with the outer radius defined as a ratio of the rotor magnet's radius instead, ranging from 1.05 to 7. The absolute value of the rotor torque results divided by the sum of the volumes of the magnetic material used gives the torque generated per unit volume $R_{\tau,v}$, from which the most efficient ring size can be found. This was done for

Figure 3.50: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.

all magnetisation configurations (in addition to the Halbach configurations for which the mathematical result was derived) to check if the result remains valid for other ring magnetisation configurations.

For illustration, the results for the Halbach-Diametrical magnetisation configuration is shown in Figures 3.50 and 3.51 are included here. The plots for the other magnetisation configurations can be found in Appendix A. Note that the torque generated appears to grow logarithmically as the radius of the outer ring increases. Since the volume of magnetic material used grows at a polynomial rate of r^2 , this produces a torque generated to volume of magnetic material efficiency graph that shows a peak at a radius ratio of approximately 1.6 and has a shape similar to that of the $\frac{\ln x}{x^2}$ expression expected.

The optimal radius ratio $x = \frac{r_{out}}{r_{in}}$ found for each magnetisation configuration can be seen in Table 3.9, ranging from 1.57 for both of the Halbach magnetisation configurations to 1.62 for the Radial-Radial magnetisation configuration. These results deviate slightly from the expected value of approx-

Figure 3.51: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.

imately 1.65, possibly due to the hole and gap present in the simulation model as well as its 3-dimensional nature with end-effects that the mathematical result neglects taken into account. This difference is small enough to not matter for the design of a prototype since the implied optimal number of ring magnets to use remains the same at 12 or 14.

3.3.2 Ring of Magnets Simulation Model

To check if a more realistic geometry affects the recommendation for the number of ring magnets to use, the Ring of Magnets Simulation Model was run again for $\psi = 0^\circ$ and $\phi = 0^\circ$. For the configurations with a diametrically magnetised rotor magnet, the simulation was sampled at $\theta = 90^\circ$. For the configurations with a radially magnetised rotor magnet, θ was sampled for a full rotation, with sampling intervals determined as detailed in Section 3.2.5 with the formula $\frac{360^\circ}{2n(5-1)}$ to help the search for the maximum absolute value of

Table 3.9: Optimal radius ratio $x = \frac{r_{out}}{r_{in}}$ and their corresponding torque generated per unit volume $R_{\tau,v}$ for each Halves magnetisation configuration for the Simple Simulation Model.

Configuration	$x = \frac{r_{out}}{r_{in}}$	$R_{\tau,v}$ [mNm/mm ³]
Diametrical-Diametrical	1.60	0.0312
Diametrical-Radial	1.59	0.0226
Radial-Diametrical	1.61	0.1086
Radial-Radial	1.62	0.0594
Halbach-Diametrical	1.57	0.1381
Halbach-Radial	1.57	0.0947

torque generated. The resulting sampling intervals are shown in Table 3.10.

Table 3.10: Sampling intervals for θ for setups with discrete magnets for varying n .

n	Interval ($^\circ$)
2	$\frac{360^\circ}{2*2*(5-1)} = 22.5^\circ$
3	$\frac{360^\circ}{2*3*(5-1)} = 15^\circ$
4	$\frac{360^\circ}{2*4*(5-1)} = 11.25^\circ$
5	$\frac{360^\circ}{2*5*(5-1)} = 9^\circ$
6	$\frac{360^\circ}{2*6*(5-1)} = 7.5^\circ$
7	$\frac{360^\circ}{2*7*(5-1)} \approx 6.429^\circ$
8	$\frac{360^\circ}{2*8*(5-1)} = 5.625^\circ$

The results are illustrated in Figures 3.52 and 3.53, and also presented in Table 3.11. Aside from the configurations where the outer ring of magnets are arranged to produce an effective diametrical magnetisation pattern, the optimal number of ring magnets was found to be 12 or 14. This comports well with both the mathematical results and that from the Simple Simulation Model in the preceding section. The optimal torque to volume ratios found are also slightly smaller than that from the Simple Simulation Model, which

Figure 3.52: Holding Torques generated for the Ring of Magnets Simulation Model for varying number of ring magnets ($2n$) for the Halves magnetisation configurations.

can be attributed to the less efficient positioning of the magnetic material as the bulk of the material resides in a central band in the ring of magnets some distance away from the rotor magnet, and potentially also to the discrete magnetisation direction of each ring magnet as opposed to the continuous magnetisation variation used in the Simple Simulation Model.

Note that “depressions” can be observed in Figure 3.53 for the configurations with a radially magnetised rotor magnet for even n . This is again due to the “wavy” torque profile generated when discretely magnetised ring magnets are used in conjunction with a radially magnetised rotor magnet, which causes the “expected” peak torque at $\theta = 90^\circ$ to “disappear”. For a visualisation of this compare Figures 3.43 and 3.25.

Figure 3.53: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying number of ring magnets ($2n$) for the Halves magnetisation configurations.

Table 3.11: Optimal number of ring magnets and their corresponding torque generated per unit volume $R_{\tau,v}$ for each Halves magnetisation configuration for the Ring of Magnets Simulation Model.

Configuration	n	$R_{\tau,v}$ [mNm/mm ³]
Diametrical-Diametrical	4	≈ 0.0298
Diametrical-Radial	5	≈ 0.0283
Radial-Diametrical	6	≈ 0.0938
Radial-Radial	6	≈ 0.0579
Halbach-Diametrical	6	≈ 0.1174
Halbach-Radial	7	≈ 0.0842

3.4 Mathematical Models – Alternating Configuration

While the Halves magnetisation configurations show some promise for the design of a Variable Stiffness Mechanism, they pose some difficulties for the design of a physical prototype. From Figure 1.1, the most direct way to actuate this mechanism is through the use of a system of spur gears as illustrated in Figures 3.54 and 3.55, with a large centrally mounted main spur gear meshing with a cluster of smaller ring spur gears each attached to a ring magnet for each half of the mechanism. This allows the changes in the ring magnets' orientation to be synchronised. The desired close packing of the magnets for efficient torque generation as described in Section 3.2.5 however places a hard limit on the size of ring spur gears that can be used, which in turn places a lower bound on the size of the mechanism based on the smallest sizes of commercially available spur gears. The resulting large gear ratio between the main spur gear and the ring spur gears also necessitates the use of larger motors for the actuation of the mechanism. These problems are especially pronounced when the ring magnets to be rotated in synchrony are immediately adjacent to one another.

An alternative configuration that groups the ring of magnets alternately instead of in semicircles, illustrated in Figure 3.58 may help alleviate this problem. The spatial constraints on the gear system responsible for rotating the ring magnets are eased allowing the use of larger gears that are spaced further apart as illustrated in Figures 3.56 and 3.57. Figure 3.59 shows the same concept in the abstract ring of dipoles form. This “Alternating” configuration will be explored in this section.

Diametrical Magnetisation

For the configuration with effective diametrical magnetisation, the dipole segments (or ring magnets) are labelled from 0 to $n - 1$ for each virtual pole, starting at the negative y-axis and proceeding counter-clockwise for

Figure 3.54: Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the “Virtual North Pole” for the “Halves” configuration.

Figure 3.55: Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the “Virtual South Pole” for the “Halves” configuration.

Figure 3.56: Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the “Virtual North Pole” for the “Alternating” configuration.

Figure 3.57: Illustration of a system of spur gears used to synchronise the changes in orientation of the ring magnets of the “Virtual South Pole” for the “Alternating” configuration.

Figure 3.58: Illustration of the working principle of the proposed magnetic variable stiffness mechanism in alternating configuration. The rotor magnet is coloured orange, and the ring magnets are divided into two groups, coloured red and blue. The oppositional rotation of the two groups of ring magnets allow the magnetic field strength experienced by the rotor to decrease and hence modulates the restoring torque on the rotor magnet when it is deflected from it's equilibrium position.

the virtual north pole and clockwise for the virtual south pole, as shown in Figure 3.59. Modulation of the stiffness of the mechanism involves rotating the dipoles/ magnets in the virtual north pole by an angle ϕ rad and those in the virtual south pole by $-\phi$ rad. Evaluation of the mathematical model was limited to $n = 1, 2, 3, 4, 5, 6, 7, 8$ as higher values of n likely corresponds to ever declining torque generation efficiency in a physical prototype, as suggested

Figure 3.59: Illustration of the “Alternating” magnetisation configuration for the effective Diametrical and Halbach magnetisation configurations for $n = 6$. The circle of magnetic dipoles are grouped into the virtual north pole and the virtual south pole, coloured red and blue respectively. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ .

by the results obtained in Section 3.3.2. This gives:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(\psi + \phi) \\ \sin(\psi + \phi) \\ 0 \end{bmatrix} & \begin{aligned} \gamma &\in (2i_N \frac{\pi}{n} - \frac{\pi}{2}, (2i_N + 1) \frac{\pi}{n} - \frac{\pi}{2}], \\ i_N &\in [0, 1, \dots, n-1] \end{aligned} \\ m_{out}^l \begin{bmatrix} \cos(\psi - \phi) \\ \sin(\psi - \phi) \\ 0 \end{bmatrix} & \begin{aligned} \gamma &\in (-(2i_S + 1) \frac{\pi}{n} - \frac{\pi}{2}, -2i_S \frac{\pi}{n} - \frac{\pi}{2}], \\ i_S &\in [0, 1, \dots, n-1] \end{aligned} \end{cases} \quad (3.78)$$

$$\mathbf{B} = \begin{cases} \frac{\mu_0 m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix} & n \in [1, 3, 4, \dots, 8] \\ \frac{\mu_0 m_{out}^l}{4\pi r^2} \begin{bmatrix} \cos \psi (\pi \cos \phi - 6 \sin \phi) \\ \sin \psi (\pi \cos \phi + 6 \sin \phi) \\ 0 \end{bmatrix} & n = 2 \end{cases} \quad (3.79)$$

$$\boldsymbol{\tau} = \begin{cases} -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta - \psi) \end{bmatrix} & n \in [1, 3, 4, \dots, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \begin{bmatrix} 0 \\ 0 \\ \sin \theta \cos \psi (\pi \cos \phi - 6 \sin \phi) \\ -\cos \theta \sin \psi (\pi \cos \phi + 6 \sin \phi) \end{bmatrix} & n = 2 \end{cases} \quad (3.80)$$

$$k_z = \frac{d\tau_z}{d\theta} = \begin{cases} -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos(\theta - \psi) \cos \phi & n \in [1, 3, 4, \dots, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \begin{pmatrix} \cos \theta \cos \psi (\pi \cos \phi - 6 \sin \phi) \\ + \sin \theta \sin \psi (\pi \cos \phi + 6 \sin \phi) \end{pmatrix} & n = 2 \end{cases} \quad (3.81)$$

$$\tau_{hold} = \frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi, \quad n \in [1, 3, 4, \dots, 8] \quad (3.82)$$

The expressions obtained, aside for $n = 2$, match the results obtained in Section 3.2.3. Note that $n = 1$ is simply the Halves Diametrical magnetisation setup. For $n = 2$, the parameters ψ and ϕ do not play distinct roles in modulating the stiffness and shifting the equilibrium position of the mechanism. If this $n = 2$ setup is desired for some reason (for instance, to maximise the torque generation from the mechanism within a cubic space), its holding torque for $\psi = 0$ rad is:

$$\begin{aligned} \tau_{hold} &= \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| \\ &= \frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} (\pi \cos \phi - 6 \sin \phi) \\ &= \frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \sqrt{36 + \pi^2} \sin \left(\phi - \arctan \frac{\pi}{6} \right), \quad n = 2 \end{aligned} \quad (3.83)$$

The maximum holding torque thus occurs at $\phi = \arctan \frac{\pi}{6} - \frac{\pi}{2} \approx -1.09$ rad, and has a maximum magnitude $\frac{\sqrt{36+\pi^2}}{\pi} \approx 2.2$ times as large as that for the other setups, making it a potentially attractive option for situations where a large range of stiffness is desirable but where the equilibrium position of the rotor does not need to be adjusted. The setting of ϕ for which the setup is in a freewheeling state is $\phi = \arctan \frac{\pi}{6} \approx 0.48$ rad.

Radial Magnetisation

For the configuration with effective radial magnetisation, the basis of its property lies in the separation of the ring of dipoles/ magnets into two semi-circles, with one forming the virtual north pole with magnetisation oriented to point away from the rotor magnet, and the other forming the virtual south pole with magnetisation oriented to point towards the rotor magnet, as previously illustrated in Figures 3.5 and 3.6. For the Alternating configuration to work, each virtual pole needs to be split into two groups, the first to be rotated by an angle ϕ rad and the other by $-\phi$ rad. The dipoles/ magnets are labelled in ascending order in a clockwise fashion, starting from the ones closest to the negative y-axis for the virtual north pole and the ones closest to the positive y-axis for the virtual south pole. This produces a slightly different setup for odd or even n , as illustrated in Figures 3.60 and 3.61 for $n = 6$ and $n = 5$ respectively. This gives:

$$\mathbf{m}_{out}^l = \left\{ \begin{array}{l} m_{out}^l \begin{bmatrix} \cos(\gamma + \psi + \phi) \\ \sin(\gamma + \psi + \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} -\cos(\gamma + \psi + \phi) \\ -\sin(\gamma + \psi + \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} \cos(\gamma + \psi - \phi) \\ \sin(\gamma + \psi - \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} -\cos(\gamma + \psi - \phi) \\ -\sin(\gamma + \psi - \phi) \\ 0 \end{bmatrix} \end{array} \right. \begin{array}{l} \gamma \in (2i_{N,+}\frac{\pi}{n} - \frac{\pi}{2}, (2i_{N,+} + 1)\frac{\pi}{n} - \frac{\pi}{2}], \\ i_{N,+} \in [0, 1, \dots, \frac{n}{2} - 1], \quad n = 2k, \quad k \in \mathbb{N} \\ \text{or } i_{N,+} \in [0, 1, \dots, \frac{n-1}{2}], \quad n = 2k + 1, \quad k \in \mathbb{N} \\ \gamma \in ((2i_{S,+} + 1)\frac{\pi}{n} + \frac{\pi}{2}, (2i_{S,+} + 2)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{S,+} \in [0, 1, \dots, \frac{n}{2} - 1], \quad n = 2k, \quad k \in \mathbb{N} \\ \text{or } i_{S,+} \in [0, 1, \dots, \frac{n-3}{2}], \quad n = 2k + 1, \quad k \in \mathbb{N} \\ \gamma \in ((2i_{N,-} + 1)\frac{\pi}{n} + \frac{\pi}{2}, (2i_{N,-} + 2)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{N,-} \in [0, 1, \dots, \frac{n}{2} - 1], \quad n = 2k, \quad k \in \mathbb{N} \\ \text{or } i_{N,-} \in [0, 1, \dots, \frac{n-3}{2}], \quad n = 2k + 1, \quad k \in \mathbb{N} \\ \gamma \in (2i_{S,-}\frac{\pi}{n} + \frac{\pi}{2}, (2i_{S,-} + 1)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{S,-} \in [0, 1, \dots, \frac{n}{2} - 1], \quad n = 2k, \quad k \in \mathbb{N} \\ \text{or } i_{S,-} \in [0, 1, \dots, \frac{n-1}{2}], \quad n = 2k + 1, \quad k \in \mathbb{N} \end{array} \right. \quad (3.84)$$

$$\mathbf{B} = \begin{cases} \begin{bmatrix} \frac{\mu_0 m_{out}^l}{\pi r^2} \begin{bmatrix} \cos \psi (2 \cos \phi - \sin \phi \tan \frac{\pi}{2n}) \\ -\sin \psi (\cos \phi - 2 \sin \phi \tan \frac{\pi}{2n}) \\ 0 \end{bmatrix} \\ \frac{\mu_0 m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 2 \cos \psi \\ -\sin \psi \\ 0 \end{bmatrix} \end{bmatrix} & n \in [2, 4, 6, 8] \\ \begin{bmatrix} \frac{\mu_0 m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 2 \cos \psi \\ -\sin \psi \\ 0 \end{bmatrix} \end{bmatrix} & n \in [3, 5, 7] \end{cases} \quad (3.85)$$

$$\boldsymbol{\tau} = \begin{cases} \begin{bmatrix} -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \begin{bmatrix} 0 \\ 0 \\ \cos \phi (2 \sin \theta \cos \psi + \cos \theta \sin \psi) \\ -\sin \phi \tan \frac{\pi}{2n} (\sin \theta \cos \psi + 2 \cos \theta \sin \psi) \end{bmatrix} \\ -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ 2 \sin \theta \cos \psi + \cos \theta \sin \psi \end{bmatrix} \end{bmatrix} & n \in [2, 4, 6, 8] \\ \begin{bmatrix} -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ 2 \sin \theta \cos \psi + \cos \theta \sin \psi \end{bmatrix} \end{bmatrix} & n \in [3, 5, 7] \end{cases} \quad (3.86)$$

$$k_z = \frac{d\tau_z}{d\theta} = \begin{cases} -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \begin{bmatrix} \cos \phi (2 \cos \theta \cos \psi - \sin \theta \sin \psi) \\ -\sin \phi \tan \frac{\pi}{2n} (\cos \theta \cos \psi - 2 \sin \theta \sin \psi) \end{bmatrix} & n \in [2, 4, 6, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi (2 \cos \theta \cos \psi - \sin \theta \sin \psi) & n \in [3, 5, 7] \end{cases} \quad (3.87)$$

Note that the expressions found for $n \in [3, 5, 7]$ match those in Section 3.2.3, and the same analysis applies. For $n \in [2, 4, 6, 8]$ the expressions found are more complicated, with an additional $-\sin \phi \tan \frac{\pi}{2n} (\sin \theta \cos \psi + 2 \cos \theta \sin \psi)$ term added. If these setups are to be used without the parameter ψ , its holding torque is:

$$\begin{aligned} \tau_{hold} &= \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| \\ &= \frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \left(2 \cos \phi - \sin \phi \tan \frac{\pi}{2n} \right) \\ &= \frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \sqrt{4 + \tan^2 \frac{\pi}{2n}} \sin \left(\phi - \arctan \frac{2}{\tan \frac{\pi}{2n}} \right), \quad n \in [2, 4, 6, 8] \end{aligned} \quad (3.88)$$

The maximum holding torque thus varies depending on the setup, occurring at $\phi = \arctan \frac{2}{\tan \frac{\pi}{2n}} - \frac{\pi}{2} + m\pi$, $m \in \mathbb{Z}$, with maximum magnitudes that also

varies, and are $\frac{\sqrt{4+\tan^2 \frac{\pi}{2n}}}{2}$ times as large as that for the $n \in [3, 5, 7]$ setups. As n grows, the magnitude of the holding torque tends towards that of the $n \in [3, 5, 7]$ setups since:

$$\lim_{n \rightarrow \infty} \frac{\sqrt{4 + \tan^2 \frac{\pi}{2n}}}{2} = 1 + \quad (3.89)$$

Similarly, the setting of ϕ for which the setup is in a freewheeling state converges onto that of the $n \in [3, 5, 7]$ setups:

$$\lim_{n \rightarrow \infty} \arctan \frac{2}{\tan \frac{\pi}{2n}} = \frac{\pi}{2} - \quad (3.90)$$

The multiplier value and deviation of ϕ from $\frac{\pi}{2}$ rad for $m = -1$ for $n \in [2, 4, 6, 8]$ are shown in Table 3.12.

Table 3.12: Multiplier for the holding torque with respect to the maximum holding torque of the $n \in [3, 5, 7]$ setups, and the deviation of the setting of ϕ for the freewheeling state from $\frac{\pi}{2}$ rad for the even numbered Alternating Radial magnetisation configurations.

n	Multiplier ($\frac{\sqrt{4+\tan^2 \frac{\pi}{2n}}}{2}$)	Deviation of ϕ [rad]
2	≈ 1.12	≈ -0.464
4	≈ 1.02	≈ -0.204
6	≈ 1.01	≈ -0.133
8	≈ 1.00	≈ -0.099

Figure 3.60: Illustration of the “Alternating” magnetisation configuration for the effective Radial magnetisation configuration for $n = 6$. The circle of magnetic dipoles are grouped into two semi-circular halves, forming the virtual north pole and the virtual south pole, coloured in shades of red and blue, and labelled i_n and i_s respectively. Within each half the magnetic dipoles that will be rotated by $-\phi$ rad are assigned a lighter colour than those that will be rotated by ϕ rad, labelled i_- and i_+ respectively. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ .

Halbach Magnetisation

For the configuration with effective Halbach magnetisation ($k = 2$), the layout is the same as that in the effective Diametrical magnetisation examined earlier, shown in Figure 3.59. This gives:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos(2\gamma + \psi + \phi) \\ \sin(2\gamma + \psi + \phi) \\ 0 \end{bmatrix} & \begin{aligned} \gamma &\in (2i_N \frac{\pi}{n} - \frac{\pi}{2}, (2i_N + 1) \frac{\pi}{n} - \frac{\pi}{2}], \\ i_N &\in [0, 1, \dots, n-1] \end{aligned} \\ m_{out}^l \begin{bmatrix} \cos(2\gamma + \psi - \phi) \\ \sin(2\gamma + \psi - \phi) \\ 0 \end{bmatrix} & \begin{aligned} \gamma &\in (-(2i_S + 1) \frac{\pi}{n} - \frac{\pi}{2}, -2i_S \frac{\pi}{n} - \frac{\pi}{2}], \\ i_S &\in [0, 1, \dots, n-1] \end{aligned} \end{cases} \quad (3.91)$$

Figure 3.61: Illustration of the “Alternating” magnetisation configuration for the effective Radial magnetisation configuration for $n = 5$. The circle of magnetic dipoles are grouped into two semi-circular halves, forming the virtual north pole and the virtual south pole, coloured in shades of red and blue, and labelled i_n and i_s respectively. Within each half the magnetic dipoles that will be rotated by $-\phi$ rad are assigned a lighter colour than those that will be rotated by ϕ rad, labelled i_- and i_+ respectively. The magnetic rotor is represented by a single dipole, coloured orange, situated at the centre of the Cartesian coordinate system rotated about the z-axis by an angle θ .

$$\mathbf{B} = \begin{cases} \frac{3\mu_0 m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix} & n \in [1, 3, 4, \dots, 8] \\ \frac{\mu_0 m_{out}^l}{4\pi r^2} \begin{bmatrix} \cos \psi (3\pi \cos \phi + 2 \sin \phi) \\ \sin \psi (-3\pi \cos \phi + 2 \sin \phi) \\ 0 \end{bmatrix} & n = 2 \end{cases} \quad (3.92)$$

$$\tau = \begin{cases} -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta + \psi) \end{bmatrix} & n \in [1, 3, 4, \dots, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \begin{bmatrix} 0 \\ 0 \\ \sin \theta \sin \psi (3\pi \cos \phi + 2 \sin \phi) \\ -\cos \theta \cos \psi (-3\pi \cos \phi + 2 \sin \phi) \end{bmatrix} & n = 2 \end{cases} \quad (3.93)$$

$$k_z = \frac{d\tau_z}{d\theta} = \begin{cases} -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \sin(\theta + \psi) \cos \phi & n \in [1, 3, 4, \dots, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \begin{pmatrix} \cos \theta \sin \psi (3\pi \cos \phi + 2 \sin \phi) \\ + \sin \theta \cos \psi (-3\pi \cos \phi + 2 \sin \phi) \end{pmatrix} & n = 2 \end{cases} \quad (3.94)$$

$$\tau_{hold} = \frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi, \quad n \in [1, 3, 4, \dots, 8] \quad (3.95)$$

As with the Alternating Diametrical magnetisation configuration, the expressions obtained, aside for $n = 2$, match the results obtained in Section 3.2.3. Note that $n = 1$ is simply the Halves Halbach magnetisation setup. For $n = 2$, the holding torque for $\psi = 0$ is:

$$\begin{aligned} \tau_{hold} &= \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| \\ &= \frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} (-3\pi \cos \phi + 2 \sin \phi) \\ &= \frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \sqrt{4 + 9\pi^2} \sin \left(\phi - \arctan \frac{3\pi}{2} \right), \quad n = 2 \end{aligned} \quad (3.96)$$

The maximum holding torque thus occurs at $\phi = \arctan \frac{3\pi}{2} - \frac{\pi}{2} \approx -3.35$ rad, and has a maximum magnitude $\frac{\sqrt{4+9\pi^2}}{3\pi} \approx 1.02$ times as large as that of the other setups, and also approximately 1.39 times that of the $n = 2$ diametrical magnetisation setup and hence even more attractive for the static equilibrium position situation as described earlier. The setting of ϕ for which the setup is in a freewheeling state is $\phi = \arctan \frac{3\pi}{2} \approx -1.78$ rad.

Comparison

Table 3.13 shows a comparison of the mathematical models derived for the “Alternating” configurations. Since the expressions found are largely the

same as in Section 3.2.3, the Halbach magnetisation configuration remains the most promising thus far.

Table 3.13: Comparison of the holding torques derived for the “Alternating” configurations. Their magnitudes relative to the Diametrical magnetisation configuration are also shown. The Diametrical magnetisation configuration for $n = 2$, Radial magnetisation configurations for $n \in [2, 4, 6, 8]$, and Halbach magnetisation configuration for $n = 2$ are omitted as their rotor equilibrium positions are not easily controlled.

Configuration	n	τ_{hold}	Multiple
Diametrical	[3, 4, 5, 6, 7, 8]	$\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	1
Radial (max)	[3, 5, 7]	$\frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi$	$\frac{8}{\pi} \approx 2.55$
Radial (min)	[3, 5, 7]	$\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \cos \phi$	$\frac{4}{\pi} \approx 1.27$
Halbach	[3, 4, 5, 6, 7, 8]	$\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	3

3.4.1 Simple Simulation Model

The Simple Simulation Model as defined in Section 3.2.4 was run to check this set of mathematical results. The same NdFe35 material definition with properties as shown in Table 3.5 was also used for this set of simulations. The dimensions as detailed in Table 3.3 were also retained. The sampling intervals for θ set done according to the formula $\frac{360^\circ}{2n(5-1)}$ as set out in Section 3.3.2, and detailed in Table 3.10. Additional samples for θ were also taken at 22.5° intervals to capture the overall sinusoidal shape expected. ψ and ϕ were each sampled from 0° to 90° in steps of 22.5° . For the setups for which a deviation from the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, a series of readings for the setup using a diametrically magnetised rotor magnet were taken for $\theta = 90^\circ$, $\psi = 0^\circ$ and ϕ varying in steps of 0.1° from -90° to 90° to determine the offset required. An additional 5 sampling points for ϕ starting from the offset value in decrements of 22.5° were then added to the simulation runs to better illustrate the intended effect

of the stiffness modulation, as detailed in Table 3.14. The magnetisation of the material used in this set of simulations were defined as specified in Table 3.15.

The Diametrical-Radial and Halbach-Radial configurations were omitted as Sections 3.2.4 and 3.2.5 had shown them to be generally inferior to the other configurations. The Radial-Diametrical configuration was retained for comparison with the mathematical model.

Table 3.14: Additional sampling points for ϕ for the setups in which a deviation of the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, for the simple simulation model. Also included are the expected values of ϕ to attain the freewheeling state as obtained from the mathematical models for the Alternating configurations.

Outer	n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
Diametrical	2	$-70.3^\circ, -47.8^\circ, -25.3^\circ, -2.8^\circ, 19.7^\circ$	$\approx 27.6^\circ$
Radial	2	$-32.3^\circ, -9.8^\circ, 12.7^\circ, 35.2^\circ, 57.7^\circ$	$\approx 63.4^\circ$
Radial	4	$-14.8^\circ, -7.7^\circ, 30.2^\circ, 52.7^\circ, 75.2^\circ$	$\approx 78.3^\circ$
Radial	6	$-9.5^\circ, 13^\circ, 35.5^\circ, 58^\circ, 80.5^\circ$	$\approx 82.4^\circ$
Radial	8	$-7.1^\circ, 15.4^\circ, 37.9^\circ, 60.4^\circ, 82.9^\circ$	$\approx 84.3^\circ$
Halbach	2	$-171.8^\circ, -149.3^\circ, -126.8^\circ, -104.3^\circ, -81.8^\circ$	$\approx -102.0^\circ$

Outer:Diametrical—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.62 and 3.63, and are also representative of the torque profiles for $n \in [4, 5, 6, 7, 8]$. The other graphs can be found in Appendix B.

The holding torques with no modulation are detailed in Table 3.16, and have magnitudes of approximately 101 mNm for $n \in [3, 4, 5, 6, 7, 8]$, corresponding very closely to the result of 101 mNm found in Section 3.2.4, as also predicted by the mathematical model.

The torque profiles for $n = 2$ are shown in Figures 3.64 and 3.65.

Table 3.15: Definitions of the magnetic material in *ANSYS Maxwell* for the Alternating magnetisation configurations for the simple simulation model. The z-component of is set to 0 in all cases.

Configuration				Magnetic Coercivity
Ring	Magnetisation	Pole	Component	
Outer	Diametrical	North	X	$\cos(\psi + \phi)$
			Y	$\sin(\psi + \phi)$
		South	X	$\cos(\psi - \phi)$
			Y	$\sin(\psi - \phi)$
	Radial	North, +	X	$\cos(\gamma + \psi + \phi)$
			Y	$\sin(\gamma + \psi + \phi)$
		South, +	X	$-\cos(\gamma + \psi + \phi)$
			Y	$-\sin(\gamma + \psi + \phi)$
		North, -	X	$\cos(\gamma + \psi - \phi)$
			Y	$\sin(\gamma + \psi - \phi)$
		South, -	X	$-\cos(\gamma + \psi - \phi)$
			Y	$-\sin(\gamma + \psi - \phi)$
	Halbach	North	X	$\cos(2\gamma + \psi + \phi)$
			Y	$\sin(2\gamma + \psi + \phi)$
		South	X	$\cos(2\gamma + \psi - \phi)$
			Y	$\sin(2\gamma + \psi - \phi)$
Inner	Diametrical	Both	X	$\cos \theta$
			Y	$\sin \theta$
	Radial	North	R	1
			Phi	0
		South	R	-1
			Phi	0

Table 3.16: Holding torques with no modulation for the Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model.

n	τ_{hold} [mNm]
2	≈ 281
3	≈ 102
4	≈ 102
5	≈ 101
6	≈ 101
7	≈ 101
8	≈ 101

Figure 3.62: Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.63: Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Note that the freewheeling effect for $\psi = 0$ rad occurs at $\phi \approx 19.7^\circ$, similar to the value of 27.6° predicted by the mathematical model. The holding torque with no modulation was found to be 281 mNm, approximately 2.8 times as large as that for $n \in [3, 4, 5, 6, 7, 8]$, compared to the magnitude of 2.2 times predicted.

Outer:Radial—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.66 and 3.67, and are also representative of the torque profiles for $n \in [5, 7]$. The other graphs can be found in Appendix B.

The holding torques with no modulation are detailed in Table 3.17. For $n \in [3, 5, 7]$ the magnitudes of the maximum and minimum holding torques are approximately 350 mNm and 218 mNm respectively, corresponding very closely to the results of 349 mNm and 219 mNm found in Section 3.2.4, as also predicted by the mathematical model.

Figure 3.68 shows that the residual torque that is still present when

Figure 3.64: Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.65: Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

$\theta = -\psi$ for $n = 3$. This is also representative of that for $n \in [5, 7]$, the graphs of which can be found in Appendix B. Their maximum magnitudes of approximately 65 mNm, 66 mNm and 65 mNm are similar to that in Section 3.2.4.

Table 3.17: Holding torques with no modulation for the Alternating Radial-Diametrical magnetisation configurations for the simple simulation model. The holding torques for $n \in [2, 4, 6, 8]$ as a ratio of that of $n = 3$ are also shown, along with the multiplier values predicted by the mathematical model.

n	$\tau_{hold,max}$ [mNm]	$\tau_{hold,min}$ [mNm]	Multiplier	Expected Multiplier
2	≈ 412	N.A.	≈ 1.18	≈ 1.12
3	≈ 350	≈ 218	N.A.	N.A.
4	≈ 361	N.A.	≈ 1.03	≈ 1.02
5	≈ 350	≈ 218	N.A.	N.A.
6	≈ 354	N.A.	≈ 1.01	≈ 1.01
7	≈ 349	≈ 218	N.A.	N.A.
8	≈ 353	N.A.	≈ 1.01	≈ 1.00

The torque profiles for $n = 2$ are shown in Figures 3.69 and 3.70, and are also representative of the torque profiles for $n \in [4, 6, 8]$. The other graphs can be found in Appendix B.

From Table 3.17, the holding torque with no modulation was found to tend towards that of $n \in [3, 5, 7]$ as n grows, as predicted by the mathematical model in equation (3.89). The mathematical model does however systematically under predicts the magnitude of the multipliers, by up to 5% for $n = 2$.

The deviation of the setting of ϕ at which the freewheeling effect occurs from $\frac{\pi}{2}$ rad for each setup are shown in Table 3.18. The general trend of smaller deviations from $\phi = 90^\circ$ as predicted by the mathematical model holds, although the mathematical model systematically under predicts the magnitude of the deviation.

Figure 3.66: Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.67: Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.68: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.

Table 3.18: The settings of ϕ at which the freewheeling effect occurs, and the expected setting as derived from the mathematical model, for the Alternating Radial-Diametrical magnetisation configurations for the simple simulation model.

n	ϕ [°]	Expected ϕ [°]
2	$\approx 57.7^\circ$	$\approx 63.4^\circ$
4	$\approx 75.2^\circ$	$\approx 78.3^\circ$
6	$\approx 80.5^\circ$	$\approx 82.4^\circ$
8	$\approx 82.9^\circ$	$\approx 84.3^\circ$

Figure 3.69: Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.70: Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Outer:Radial—Inner:Radial

The torque profiles for $n = 3$ are shown in Figures 3.71 and 3.72, and are also representative of the torque profiles for $n \in [5, 7]$. The other graphs can be found in Appendix B.

The holding torques with no modulation are detailed in Table 3.19. For $n \in [2, 4, 6, 8]$, the holding torque reading is taken at $\phi = 0$ rad for which the “constant torque spring” torque profile is observable.

For $n \in [3, 5, 7]$, the torque profiles match those in Section 3.2.4, with the “constant torque spring” torque profile preserved with changing values of ϕ . The holding torques for these setups are approximately 193 mNm. As is in Section 3.2.4, the “constant torque spring” is not preserved when ψ is varied.

Figure 3.73 shows that the residual torque that is still present when $\theta = -\psi$ for $n = 3$. This is also representative of that for $n \in [5, 7]$, the graphs of which can be found in Appendix B. Their maximum magnitudes of approximately 112 mNm, 113 mNm and 113 mNm are similar to that in Section 3.2.4.

Table 3.19: Holding torques with no modulation for the Alternating Radial-Radial magnetisation configurations for the simple simulation model. For $n \in [2, 4, 6, 8]$, the holding torque reading is taken at $\phi = 0$ rad for which the “constant torque spring” torque profile is observable. The holding torques for $n \in [2, 4, 6, 8]$ as a ratio of that of $n = 3$ are also shown, along with the multiplier values predicted by the mathematical model.

n	τ_{hold} [mNm]
2	≈ 193
3	≈ 194
4	≈ 194
5	≈ 194
6	≈ 194
7	≈ 194
8	≈ 194

Figure 3.71: Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

The torque profiles for $n = 2$ are shown in Figures 3.74 and 3.75, and are also representative of the torque profiles for $n \in [4, 6, 8]$. The other graphs can be found in Appendix B.

It is immediately apparent that the “constant torque spring” torque profile only exists for $\psi = 0$ and $\phi = 0$. Hence these setups are wholly unsuitable for acting as an adjustable constant torque spring. The holding torque for the “constant torque spring” torque profile is approximately 193 mNm, similar to those for $n \in [3, 5, 7]$.

Outer:Halbach—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.76 and 3.77, and are also representative of the torque profiles for $n \in [4, 5, 6, 7, 8]$. The other graphs can be found in Appendix B.

The holding torques with no modulation are detailed in Table 3.20, and have magnitudes of approximately 443 mNm for $n \in [3, 4, 5, 6, 7, 8]$, corresponding very closely to the result of 443 mNm found in Section 3.2.4, as

Figure 3.72: Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.73: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.

Figure 3.74: Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.75: Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

also predicted by the mathematical model.

Table 3.20: Holding torques with no modulation for the Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model.

n	τ_{hold} [mNm]
2	≈ 448
3	≈ 443
4	≈ 443
5	≈ 445
6	≈ 443
7	≈ 443
8	≈ 443

The torque profiles for $n = 2$ are shown in Figures 3.78 and 3.79.

Note that the freewheeling effect for $\psi = 0$ rad occurs at $\phi \approx -81.8^\circ$, similar to the value of -102.0° predicted by the mathematical model. The holding torque with no modulation was found to be 448 mNm, approximately 1.01 times as large as that for $n \in [3, 4, 5, 6, 7, 8]$, compared to the magnitude of 1.02 times predicted.

3.5 Mathematical Models – Discrete Alternating Configuration

In Section 3.4 the magnetisation of the segments of the ring magnet was defined continuously. In reality, however, magnets can only be easily magnetised in a single direction. In this section, the effects of each segment having a single direction of magnetisation will be investigated. Note that the formulation for the effective diametrical magnetisation configuration remains exactly as presented in Section 3.4 and is hence omitted from this analysis. For the other two configurations, to preserve the symmetry about the y-axis of the pattern of magnetisation at $\psi = 0^\circ$ and $\phi = 0^\circ$, the angular position of the

Figure 3.76: Torque profiles generated by the Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.77: Torque profiles generated by the Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.78: Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.79: Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

centre of each segment is used to set its magnetisation direction, derived from the formulations set out in Section 3.4. The labelling of the dipole segments (or ring magnets) remains the same as shown in Figures 3.60, 3.61 and 3.59.

Radial Magnetisation

For the configurations with effective radial magnetisation, we get:

$$\mathbf{m}_{out}^l = \left\{ \begin{array}{l} m_{out}^l \begin{bmatrix} \cos(\frac{(4i_{N,+}+1-n)\pi}{2n} + \psi + \phi) \\ \sin(\frac{(4i_{N,+}+1-n)\pi}{2n} + \psi + \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} -\cos(\frac{(4i_{S,+}+1+n)\pi}{2n} + \psi + \phi) \\ -\sin(\frac{(4i_{S,+}+1+n)\pi}{2n} + \psi + \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} -\cos(\frac{(4i_{S,+}+3+n)\pi}{2n} + \psi + \phi) \\ -\sin(\frac{(4i_{S,+}+3+n)\pi}{2n} + \psi + \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} \cos(\frac{(4i_{N,+}+3-n)\pi}{2n} + \psi - \phi) \\ \sin(\frac{(4i_{N,+}+3-n)\pi}{2n} + \psi - \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} -\cos(\frac{(4i_{S,-}+3+n)\pi}{2n} + \psi - \phi) \\ -\sin(\frac{(4i_{S,-}+3+n)\pi}{2n} + \psi - \phi) \\ 0 \end{bmatrix} \\ m_{out}^l \begin{bmatrix} -\cos(\frac{(4i_{S,-}+1+n)\pi}{2n} + \psi - \phi) \\ -\sin(\frac{(4i_{S,-}+1+n)\pi}{2n} + \psi - \phi) \\ 0 \end{bmatrix} \end{array} \right. \begin{array}{l} \gamma \in (2i_{N,+}\frac{\pi}{n} - \frac{\pi}{2}, (2i_{N,+}+1)\frac{\pi}{n} - \frac{\pi}{2}], \\ i_{N,+} \in [0, 1, \dots, \frac{n}{2}-1], \quad n=2k, \quad k \in \mathbb{N} \\ \text{or} \\ i_{N,+} \in [0, 1, \dots, \frac{n-1}{2}], \quad n=2k+1, \quad k \in \mathbb{N} \\ \gamma \in ((2i_{S,+}+1)\frac{\pi}{n} + \frac{\pi}{2}, (2i_{S,+}+2)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{S,+} \in [0, 1, \dots, \frac{n}{2}-1], \quad n=2k, \quad k \in \mathbb{N} \\ \gamma \in ((2i_{S,+}+1)\frac{\pi}{n} + \frac{\pi}{2}, (2i_{S,+}+2)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{S,+} \in [0, 1, \dots, \frac{n-3}{2}], \quad n=2k+1, \quad k \in \mathbb{N} \\ \gamma \in ((2i_{N,-}+1)\frac{\pi}{n} + \frac{\pi}{2}, (2i_{N,-}+2)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{N,-} \in [0, 1, \dots, \frac{n}{2}-1], \quad n=2k, \quad k \in \mathbb{N} \\ \text{or} \\ i_{N,-} \in [0, 1, \dots, \frac{n-3}{2}], \quad n=2k+1, \quad k \in \mathbb{N} \\ \gamma \in (2i_{S,-}\frac{\pi}{n} + \frac{\pi}{2}, (2i_{S,-}+1)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{S,-} \in [0, 1, \dots, \frac{n}{2}-1], \quad n=2k, \quad k \in \mathbb{N} \\ \gamma \in (2i_{S,-}\frac{\pi}{n} + \frac{\pi}{2}, (2i_{S,-}+1)\frac{\pi}{n} + \frac{\pi}{2}], \\ i_{S,-} \in [0, 1, \dots, \frac{n-1}{2}], \quad n=2k+1, \quad k \in \mathbb{N} \end{array} \right. \quad (3.97)$$

$$\mathbf{B} = \begin{cases} \begin{bmatrix} \frac{\mu_0 m_{out}^l}{4\pi r^2} \begin{bmatrix} \frac{\cos \psi}{n} [\cos \phi \csc \frac{\pi}{2n} (\pi + 3n \sin \frac{\pi}{n}) + \sin \phi \sec \frac{\pi}{2n} (\pi - 3n \sin \frac{\pi}{n})] \\ 2 \sin \psi [-3 \cos(\frac{\pi}{2n} + \phi) + \frac{\pi}{n} \cos(\frac{\pi}{2n} - \phi) \csc \frac{\pi}{n}] \\ 0 \end{bmatrix} \\ \frac{\mu_0 m_{out}^l}{4n\pi r^2} \csc \frac{\pi}{2n} \cos \phi \begin{bmatrix} \cos \psi (\pi + 3n \sin \frac{\pi}{n}) \\ \sin \psi (\pi - 3n \sin \frac{\pi}{n}) \\ 0 \end{bmatrix} \end{bmatrix} & n \in [2, 4, 6, 8] \\ \begin{bmatrix} \frac{\mu_0 m_{out}^l}{4n\pi r^2} \csc \frac{\pi}{2n} \cos \phi \begin{bmatrix} \cos \psi (\pi + 3n \sin \frac{\pi}{n}) \\ \sin \psi (\pi - 3n \sin \frac{\pi}{n}) \\ 0 \end{bmatrix} \end{bmatrix} & n \in [3, 5, 7] \end{cases} \quad (3.98)$$

$$\boldsymbol{\tau} = \begin{cases} \begin{bmatrix} -\frac{\mu_0 m_{in} m_{out}^l}{4n\pi r^2} \begin{bmatrix} 0 \\ 0 \\ 6n \sin \theta \cos \psi \cos \phi \cos \frac{\pi}{2n} \\ -2 \cos \theta \sin \psi [-3n \cos(\frac{\pi}{2n} + \phi) + \pi \csc \frac{\pi}{n} \cos(\frac{\pi}{2n} - \phi)] \\ -\sin \theta \cos \psi [-\pi \cos \phi \csc \frac{\pi}{2n} + \sin \phi \sec \frac{\pi}{2n} (-\pi + 3n \sin \frac{\pi}{n})] \end{bmatrix} \end{bmatrix} & n \in [2, 4, 6, 8] \\ \begin{bmatrix} -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \frac{3n \sin \frac{\pi}{n} - \pi}{4n \sin \frac{\pi}{2n}} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin \theta \cos \psi \frac{3n \sin \frac{\pi}{n} + \pi}{3n \sin \frac{\pi}{n} - \pi} \\ + \cos \theta \sin \psi \end{bmatrix} \end{bmatrix} & n \in [3, 5, 7] \end{cases} \quad (3.99)$$

$$k_z = \frac{d\tau_z}{d\theta} = \begin{cases} -\frac{\mu_0 m_{in} m_{out}^l}{4n\pi r^2} \begin{bmatrix} 6n \cos \frac{\pi}{2n} \cos \theta \cos \psi \cos \phi \\ -2 \sin \theta \sin \psi [-3n \cos(\frac{\pi}{2n} + \phi) + \pi \csc \frac{\pi}{n} \cos(\frac{\pi}{2n} - \phi)] \\ -\cos \theta \cos \psi [-\pi \cos \phi \csc \frac{\pi}{2n} + \sin \phi \sec \frac{\pi}{2n} (-\pi + 3n \sin \frac{\pi}{n})] \end{bmatrix} & n \in [2, 4, 6, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \frac{3n \sin \frac{\pi}{n} - \pi}{4n \sin \frac{\pi}{2n}} \cos \phi \begin{bmatrix} \cos \theta \cos \psi \frac{3n \sin \frac{\pi}{n} + \pi}{3n \sin \frac{\pi}{n} - \pi} \\ -\sin \theta \sin \psi \end{bmatrix} & n \in [3, 5, 7] \end{cases} \quad (3.100)$$

The expressions found for $n \in [3, 5, 7]$ are similar to those in Sections 3.2.3 and 3.4, with an overall multiplier of $\frac{3n \sin \frac{\pi}{n} - \pi}{4n \sin \frac{\pi}{2n}}$ applied, and the constant of 2 for the $\sin \theta \cos \psi$ term changed to $\frac{3n \sin \frac{\pi}{n} + \pi}{3n \sin \frac{\pi}{n} - \pi}$. As the number of segments used in increased, the expressions tend towards that in Section 3.2.3 as:

$$\lim_{n \rightarrow \infty} \frac{3n \sin \frac{\pi}{n} - \pi}{4n \sin \frac{\pi}{2n}} = 1^- \quad (3.101)$$

$$\lim_{n \rightarrow \infty} \frac{3n \sin \frac{\pi}{n} + \pi}{3n \sin \frac{\pi}{n} - \pi} = 2^+ \quad (3.102)$$

For $\psi = 0$, the holding torque is:

$$\tau_{hold,max} = \frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \frac{3n \sin \frac{\pi}{n} + \pi}{8n \sin \frac{\pi}{2n}} \cos \phi \quad (3.103)$$

For $\psi = \frac{\pi}{2}$ rad, the holding torque is:

$$\tau_{hold,min} = \frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \frac{3n \sin \frac{\pi}{n} + \pi}{8n \sin \frac{\pi}{2n}} \cos \phi \quad (3.104)$$

The holding torque multiplier also has limit:

$$\lim_{n \rightarrow \infty} \frac{3n \sin \frac{\pi}{n} + \pi}{8n \sin \frac{\pi}{2n}} = 1^- \quad (3.105)$$

The particular values of the overall multiplier, constant and multiplier for the holding torque $\frac{3n \sin \frac{\pi}{n} + \pi}{4n \sin \frac{\pi}{2n}}$ for $n \in [3, 5, 7]$ are shown in Table 3.21. The increasing value of the holding torque multiplier as n grows suggests that with discrete magnetisation of the ring magnets the configuration is less efficient overall. This effect is ameliorated as a larger number of ring magnets is used as a closer approximation of the continuous distribution of magnetisation orientation as described in Sections 3.2.3 and 3.4 is achieved. Note that this recommendation does not factor in the optimal size analysis for torque/stiffness generated to volume/ mass ratio as described in Section 3.3. This will be covered in a Section 3.6.

Table 3.21: Overall multiplier and the holding torque multiplier with respect to the maximum holding torque of the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configuration, and the constant of the $\sin \theta \cos \psi$ for the odd numbered Discrete Alternating Radial magnetisation configurations.

n	Multiplier $(\frac{3n \sin \frac{\pi}{n} + \pi}{4n \sin \frac{\pi}{2n}})$	Constant $(\frac{3n \sin \frac{\pi}{n} + \pi}{3n \sin \frac{\pi}{n} - \pi})$	τ_{hold} Multiplier $(\frac{3n \sin \frac{\pi}{n} + \pi}{8n \sin \frac{\pi}{2n}})$
3	≈ 0.775	≈ 2.35	≈ 0.911
5	≈ 0.918	≈ 2.11	≈ 0.967
7	≈ 0.958	≈ 2.05	≈ 0.983

The appropriate mapping of a desired rotor magnet equilibrium orientation θ_d to the setting ψ is:

$$\psi = -\arctan\left(\frac{3n \sin \frac{\pi}{n} + \pi}{3n \sin \frac{\pi}{n} - \pi} \tan \theta_d\right) \quad (3.106)$$

For $n \in [2, 4, 6, 8]$, the expressions found are more complicated. If these

setups are to be used without the parameter ψ , its holding torque is:

$$\begin{aligned}
 \tau_{hold} &= \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| \\
 &= \frac{\mu_0 m_{in} m_{out}^l}{4n\pi r^2} \left(\left(-6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n} \right) \cos \phi + \left(\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi) \right) \sin \phi \right) \\
 &= \frac{\mu_0 m_{in} m_{out}^l}{4n\pi r^2} \sqrt{\left(6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n} \right)^2 + \left(\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi) \right)^2} \sin \left(\phi - \arctan \frac{6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n}}{\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi)} \right)
 \end{aligned} \tag{3.107}$$

The maximum holding torque thus varies depending on the setup, occurring at $\phi = \arctan \frac{6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n}}{\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi)} - \frac{\pi}{2} + m\pi$, $m \in \mathbb{Z}$, with maximum magnitudes that also varies, and are $\frac{\sqrt{\left(6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n} \right)^2 + \left(\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi) \right)^2}}{8n}$ times as large as that for the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configuration. The setting of ϕ for which the setup is in a freewheeling state is $\phi = \arctan \frac{6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n}}{\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi)}$. As n grows, the magnitude of the holding torque tends towards that of the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configurations since:

$$\lim_{n \rightarrow \infty} \frac{\sqrt{\left(6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n} \right)^2 + \left(\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi) \right)^2}}{8n} = 1^- \tag{3.108}$$

Similarly, the setting of ϕ for which the setup is in a freewheeling state converges onto that of the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configuration as:

$$\lim_{n \rightarrow \infty} \arctan \frac{6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n}}{\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi)} = \frac{\pi}{2}^- \tag{3.109}$$

The particular multiplier value and deviation of ϕ from $\frac{\pi}{2}$ rad for $m = -1$ for $n \in [2, 4, 6, 8]$ are shown in Table 3.22.

Table 3.22: Multiplier for the holding torque with respect to the maximum holding torque of the $n \in [3, 5, 7]$ setups of the Alternating Radial magnetisation configuration, and the deviation of the setting of ϕ for the freewheeling state from $\frac{\pi}{2}$ rad, for the even numbered Discrete Alternating Radial magnetisation configurations.

n	Multiplier ($\frac{\sqrt{\left(6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n}\right)^2 + \left(\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi)\right)^2}}{8n}$)	Deviation of ϕ [rad]
2	≈ 0.847	≈ -0.303
4	≈ 0.967	≈ -0.188
6	≈ 0.985	≈ -0.129
8	≈ 0.992	≈ -0.097

Halbach Magnetisation

For the configurations with effective Halbach magnetisation ($k = 2$), we get:

$$\mathbf{m}_{out}^l = \begin{cases} m_{out}^l \begin{bmatrix} \cos((4i_N + 1)\frac{\pi}{n} + \psi + \phi) \\ \sin((4i_N + 1)\frac{\pi}{n} + \psi + \phi) \\ 0 \end{bmatrix} & \begin{aligned} \gamma &\in (2i_N\frac{\pi}{n} - \frac{\pi}{2}, (2i_N + 1)\frac{\pi}{n} - \frac{\pi}{2}], \\ i_N &\in [0, 1, \dots, n-1] \end{aligned} \\ m_{out}^l \begin{bmatrix} \cos(-(4i_N + 1)\frac{\pi}{n} + \psi - \phi) \\ \sin(-(4i_N + 1)\frac{\pi}{n} + \psi - \phi) \\ 0 \end{bmatrix} & \begin{aligned} \gamma &\in (-(2i_S + 1)\frac{\pi}{n} - \frac{\pi}{2}, -2i_S\frac{\pi}{n} - \frac{\pi}{2}), \\ i_S &\in [0, 1, \dots, n-1] \end{aligned} \end{cases} \quad (3.110)$$

$$\mathbf{B} = \begin{cases} \frac{3n\mu_0 m_{out}^l}{4\pi r^2} \sin \frac{\pi}{n} \cos \phi \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix} & n \in [3, 4, \dots, 8] \\ \frac{\mu_0 m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix} & n = 1 \\ \frac{\mu_0 m_{out}^l}{4\pi r^2} \begin{bmatrix} \cos \psi (6\pi \cos \phi + \pi \sin \phi) \\ \sin \psi (-6\pi \cos \phi + \pi \sin \phi) \\ 0 \end{bmatrix} & n = 2 \end{cases} \quad (3.111)$$

$$\boldsymbol{\tau} = \begin{cases} -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \frac{n}{\pi} \sin \frac{\pi}{n} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta + \psi) \end{bmatrix} & n \in [3, 4, \dots, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi \begin{bmatrix} 0 \\ 0 \\ \sin(\theta - \psi) \end{bmatrix} & n = 1 \\ -\frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \begin{bmatrix} 0 \\ 0 \\ 6 \cos \phi \sin(\theta + \psi) + \pi \sin \phi \sin(\theta - \psi) \end{bmatrix} & n = 2 \end{cases} \quad (3.112)$$

$$k_z = \frac{d\tau_z}{d\theta} = \begin{cases} -\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \frac{n}{\pi} \sin \frac{\pi}{n} \cos(\theta + \psi) \cos \phi & n \in [3, 4, \dots, 8] \\ -\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos(\theta - \psi) \cos \phi & n = 1 \\ -\frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} (6 \cos \phi \cos(\theta + \psi) + \pi \sin \phi \cos(\theta - \psi)) & n = 2 \end{cases} \quad (3.113)$$

$$\tau_{hold} = \begin{cases} \frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \frac{n}{\pi} \sin \frac{\pi}{n} \cos \phi & n \in [3, 4, \dots, 8] \\ \frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi & n = 1 \end{cases} \quad (3.114)$$

Note that $n = 1$ is simply the Halves Diametrical magnetisation setup. The expressions found for $n \in [3, 4, 5, 6, 7, 8]$ are similar to those in Sections 3.2.3

and 3.4, with an overall multiplier of $\frac{n}{\pi} \sin \frac{\pi}{n}$ applied. Since:

$$\lim_{n \rightarrow \infty} \frac{n}{\pi} \sin \frac{\pi}{n} = 1^- \quad (3.115)$$

the expressions tend towards those in Section 3.4 as n grows. The particular multiplier value for $n \in [3, 4, 5, 6, 7, 8]$ are shown in Table 3.23.

Table 3.23: Overall multiplier with respect to the $n \in [1, 3, 4, 5, 6, 7, 8]$ setups of the Alternating Halbach magnetisation configuration, for the $n \in [3, 4, 5, 6, 7, 8]$ Discrete Alternating Halbach magnetisation setups.

n	Multiplier ($\frac{n}{\pi} \sin \frac{\pi}{n}$)
3	≈ 0.827
4	≈ 0.900
5	≈ 0.935
6	≈ 0.955
7	≈ 0.967
8	≈ 0.974

For $n = 2$, the expressions found are more complicated. If this setup is to be used without the parameter ψ , its holding torque is:

$$\begin{aligned} \tau_{hold} &= \left| \tau_z \left(\frac{d\tau_z}{d\theta} = 0 \right) \right| \\ &= \frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} (6 \cos \phi + \pi \sin \phi) \\ &= \frac{\mu_0 m_{in} m_{out}^l}{4\pi r^2} \sqrt{36 + \pi^2} \sin \left(\phi + \arctan \frac{6}{\pi} \right), \quad n = 2 \end{aligned} \quad (3.116)$$

The maximum holding torque thus occurs at $\phi = -\arctan \frac{6}{\pi} - \frac{\pi}{2} \approx -2.66$ rad, and has a maximum magnitude $\frac{\sqrt{36+\pi^2}}{3\pi} \approx 0.719$ times as large as that of the Halves Halbach magnetisation setup. This result suggests that the $n = 2$ setup holds no particular advantage over the other setups, in opposition to the result found in Section 3.4. The setting of ϕ for which the setup is in a freewheeling state is $\phi = -\arctan \frac{6}{\pi} \approx -1.09$ rad.

Comparison

Table 3.24 shows a comparison of the mathematical models derived for the “Discrete Alternating” configurations. Since the “Multiplier” values do not deviate much from 1, the Halbach magnetisation configuration remains the most promising thus far.

Table 3.24: Comparison of the holding torques derived for the “Discrete Alternating” configurations. Their magnitudes relative to the Diametrical magnetisation configuration, from multiplying their “Multiplier” values (from Tables 3.21 and 3.23) with the appropriate “Multiple” from Table 3.13, are also shown. The Diametrical magnetisation configuration for $n = 2$, Radial magnetisation configurations for $n \in [2, 4, 6, 8]$, and Halbach magnetisation configuration for $n = 2$ are omitted as their rotor equilibrium positions are not easily controlled.

Configuration	n	τ_{hold}	Multiple
Diametrical	[3, 4, 5, 6, 7, 8]	$\frac{\mu_0 m_{in} m_{out}^l}{4r^2} \cos \phi$	1
Radial (max)	3	$\frac{2\mu_0 m_{in} m_{out}^l}{\pi r^2} \frac{3n \sin \frac{\pi}{n} + \pi}{8n \sin \frac{\pi}{2n}} \cos \phi$	≈ 2.32
	5		≈ 2.46
	7		≈ 2.50
Radial (min)	3	$\frac{\mu_0 m_{in} m_{out}^l}{\pi r^2} \frac{3n \sin \frac{\pi}{n} + \pi}{8n \sin \frac{\pi}{2n}} \cos \phi$	≈ 1.16
	5		≈ 1.23
	7		≈ 1.25
Halbach	3	$\frac{3\mu_0 m_{in} m_{out}^l}{4r^2} \frac{n}{\pi} \sin \frac{\pi}{n} \cos \phi$	≈ 2.48
	4		≈ 2.70
	5		≈ 2.81
	6		≈ 2.87
	7		≈ 2.90
	8		≈ 2.92

3.5.1 Simple Simulation Model

The Simple Simulation Model specified in Section 3.2.4 was again used to check this set of mathematical results, in particular the holding torque multiplier and the deviation of the setting ϕ from $\frac{\pi}{2}$ rad, without taking into account the optimal size analysis for torque/ stiffness generated to volume/ mass ratio. The material definition, dimension of the setup and the sampling regimes for θ, ψ and ϕ were as described in Section 3.4.1. The additional 5 sampling points for ϕ are detailed in Table 3.25. The magnetisation of the material used in this set of simulations were defined as specified in Table 3.26.

In addition to the Diametrical-Radial and Halbach-Radial configurations, the Diametrical-Diametrical configuration was omitted as it is identical to that in Section 3.4. For the Radial-Radial configuration, only the setups for $n \in [3, 5, 7]$ are included as the simulation results from Section 3.4 have shown that the setups for $n \in [2, 4, 6, 8]$ are not suitable for a variable stiffness mechanism.

Table 3.25: Additional sampling points for ϕ for the setups in which a deviation of the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, for the simple simulation model. Also included are the expected values of ϕ to attain the freewheeling state as obtained from the mathematical models for the Discrete Alternating configurations.

Outer	n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
Radial	2	$-25.3^\circ, -2.8^\circ, 19.7^\circ, 42.2^\circ, 64.7^\circ$	$\approx 72.6^\circ$
Radial	4	$-13.9^\circ, 8.6^\circ, 31.1^\circ, 53.6^\circ, 76.1^\circ$	$\approx 79.2^\circ$
Radial	6	$-9.2^\circ, 13.3^\circ, 35.8^\circ, 58.3^\circ, 80.8^\circ$	$\approx 82.6^\circ$
Radial	8	$-7^\circ, 15.5^\circ, 38^\circ, 60.5^\circ, 83^\circ$	$\approx 84.4^\circ$
Halbach	2	$-160.3^\circ, -137.8^\circ, -115.3^\circ, -92.8^\circ, -70.3^\circ$	$\approx -62.4^\circ$

Table 3.26: Definitions of the magnetic material in *ANSYS Maxwell* for the Discrete Alternating magnetisation configuration for both the simple simulation model and the ring of magnets simulation model. The Z component of is set to 0 in all cases.

Configuration				Magnetic Coercivity
Ring	Magnetisation	Pole	Component	
Outer	Radial	North, +	X	$\cos(\frac{\pi}{n}(2i_{N,+} + \frac{1}{2}) - \frac{\pi}{2} + \psi + \phi)$
			Y	$\sin(\frac{\pi}{n}(2i_{N,+} + \frac{1}{2}) - \frac{\pi}{2} + \psi + \phi)$
		South, +	X	$-\cos(\frac{\pi}{n}(2i_{S,+} + \frac{1}{2}) + \frac{\pi}{2} + \psi + \phi)$
			Y	$-\sin(\frac{\pi}{n}(2i_{S,+} + \frac{1}{2}) + \frac{\pi}{2} + \psi + \phi)$
		South, +	X	$-\cos(\frac{\pi}{n}(2i_{S,+} + \frac{3}{2}) + \frac{\pi}{2} + \psi + \phi)$
			Y	$-\sin(\frac{\pi}{n}(2i_{S,+} + \frac{3}{2}) + \frac{\pi}{2} + \psi + \phi)$
		North, -	X	$\cos(\frac{\pi}{n}(2i_{N,-} + \frac{3}{2}) - \frac{\pi}{2} + \psi - \phi)$
			Y	$\sin(\frac{\pi}{n}(2i_{N,-} + \frac{3}{2}) - \frac{\pi}{2} + \psi - \phi)$
		South, -	X	$-\cos(\frac{\pi}{n}(2i_{S,-} + \frac{3}{2}) + \frac{\pi}{2} + \psi - \phi)$
			Y	$-\sin(\frac{\pi}{n}(2i_{S,-} + \frac{3}{2}) + \frac{\pi}{2} + \psi - \phi)$
		South, -	X	$-\cos(\frac{\pi}{n}(2i_{S,-} + \frac{1}{2}) + \frac{\pi}{2} + \psi - \phi)$
			Y	$-\sin(\frac{\pi}{n}(2i_{S,-} + \frac{1}{2}) + \frac{\pi}{2} + \psi - \phi)$
	Halbach	North	X	$\cos(2(\frac{\pi}{n}(2i_N + \frac{1}{2}) - \frac{\pi}{2}) + \psi + \phi)$
			Y	$\sin(2(\frac{\pi}{n}(2i_N + \frac{1}{2}) - \frac{\pi}{2}) + \psi + \phi)$
		South	X	$\cos(-2(\frac{\pi}{n}(2i_S + \frac{1}{2}) + \frac{\pi}{2}) + \psi - \phi)$
			Y	$\sin(-2(\frac{\pi}{n}(2i_S + \frac{1}{2}) + \frac{\pi}{2}) + \psi - \phi)$
Inner	Diametrical	Both	X	$\cos \theta$
			Y	$\sin \theta$
	Radial	North	R	1
			Phi	0
		South	R	-1
			Phi	0

Outer:Radial—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.80 and 3.81, and are also representative of the torque profiles for $n \in [5, 7]$. The other graphs can be found in Appendix C.

The holding torques with no modulation are detailed in Table 3.27. For $n \in [3, 5, 7]$ the magnitudes of the maximum and minimum holding torques was found to tend towards that of the $n \in [3, 5, 7]$ setups of the Alternating Radial-Diametrical configuration as n grows, as predicted by the mathematical model in equation (3.105).

Figure 3.82 shows that the residual torque that is still present when $\theta = -\psi$ for $n = 3$. This is also representative of that for $n \in [5, 7]$, the graphs of which can be found in Appendix C. Their maximum magnitudes of approximately 69 mNm, 67 mNm and 66 mNm are similar to that in found in Section 3.2.4.

Table 3.27: Holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model. The holding torques for $n \in [2, 3, 4, 5, 6, 7, 8]$ as a ratio of that of the $n = 3$ setup of the Alternating Radial-Diametrical configuration are also shown, along with the multiplier values predicted by the mathematical model.

n	$\tau_{hold,max}$ [mNm]	$\tau_{hold,min}$ [mNm]	Multiplier	Expected Multiplier
2	≈ 304	N.A.	≈ 0.869	≈ 0.847
3	≈ 316	≈ 179	≈ 0.903	≈ 0.911
4	≈ 341	N.A.	≈ 0.974	≈ 0.967
5	≈ 338	≈ 204	≈ 0.966	≈ 0.967
6	≈ 346	N.A.	≈ 0.989	≈ 0.985
7	≈ 343	≈ 211	≈ 0.980	≈ 0.983
8	≈ 348	N.A.	≈ 0.994	≈ 0.992

The torque profiles for $n = 2$ are shown in Figures 3.83 and 3.84, and are also representative of the torque profiles for $n \in [4, 6, 8]$. The other graphs

Figure 3.80: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.81: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.82: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.

can be found in Appendix C.

From Table 3.27, the holding torque with no modulation was found to tend towards that of the $n \in [3, 5, 7]$ setups of the Alternating Radial-Diametrical configuration as n grows, as predicted by the mathematical model in equation (3.108).

The deviation of the setting of ϕ at which the freewheeling effect occurs from $\frac{\pi}{2}$ rad for each setup are shown in Table 3.28. The general trend of smaller deviations from $\phi = 90^\circ$ as predicted by the mathematical model holds true.

Outer:Radial—Inner:Radial

The torque profiles for $n \in [3, 5, 7]$ are shown in Figures 3.85, 3.86, 3.88, 3.89, 3.91 and 3.92.

Note that the shape of the torque profiles tends towards that in Section 3.2.4 as n grows, with the “undulations” smoothed out as the number of

Table 3.28: The settings of ϕ at which the freewheeling effect occurs, and the expected setting as derived from the mathematical model, for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model.

n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
2	$\approx 64.7^\circ$	$\approx 72.6^\circ$
4	$\approx 76.1^\circ$	$\approx 79.2^\circ$
6	$\approx 80.8^\circ$	$\approx 82.6^\circ$
8	$\approx 83^\circ$	$\approx 84.4^\circ$

Figure 3.83: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.84: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

segments increases, with a maximum holding torque of approximately 195 mNm similar to the 193 mNm found previously.

Figures 3.87, 3.90 and 3.93 shows that the residual torque that is still present when $\theta = -\psi$. Their maximum magnitudes of approximately 80 mNm, 100 mNm and 110 mNm are similar to that in Section 3.2.4.

Outer:Halbach—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.94 and 3.95, and are also representative of the torque profiles for $n \in [4, 5, 6, 7, 8]$. The other graphs can be found in Appendix C.

The holding torques with no modulation are detailed in Table 3.29.

For $n \in [3, 4, 5, 6, 7, 8]$ the magnitudes of the maximum and minimum holding torques was found to tend towards that of the $n \in [3, 4, 5, 6, 7, 8]$ setups of the Alternating Halbach-Diametrical configuration as n grows, as predicted by the mathematical model in equation (3.115),

Figure 3.85: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.86: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.87: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.

Figure 3.88: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.89: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.90: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the simple simulation model.

Figure 3.91: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.92: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.93: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the simple simulation model.

Table 3.29: Holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model. The holding torques for $n \in [2, 3, 4, 5, 6, 7, 8]$ as a ratio of that of the $n = 3$ setup of the Alternating Halbach-Diametrical configuration are also shown, along with the multiplier values predicted by the mathematical model.

n	τ_{hold} [mNm]	Multiplier	Expected Multiplier
2	≈ 304	≈ 0.686	≈ 0.719
3	≈ 371	≈ 0.837	≈ 0.827
4	≈ 404	≈ 0.912	≈ 0.900
5	≈ 420	≈ 0.948	≈ 0.935
6	≈ 427	≈ 0.964	≈ 0.955
7	≈ 432	≈ 0.975	≈ 0.967
8	≈ 435	≈ 0.982	≈ 0.974

Figure 3.94: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure 3.95: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure 3.96: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

The torque profiles for $n = 2$ are shown in Figures 3.78 and 3.79.

Note that the freewheeling effect for $\psi = 0^\circ$ occurs at $\phi \approx -70.3^\circ$, similar to the value of -62.5° predicted by the mathematical model. The holding torque with no modulation was found to be 304 mNm, approximately 0.686 times as large as that of the $n = 3$ setup of the Alternating Halbach-Diametrical configuration, compared to the magnitude of 0.719 times predicted.

3.6 Optimal Size – Discrete Alternating Configuration

Since the number of dipole segments/ ring magnets used affects the torque and stiffness generated by the mechanism distinct from the size effects as discussed in Section 3.2.5, it is worth examining if this fact has any bearing on the theoretical optimal number of segments/ magnets.

Figure 3.97: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

The relationship between the number of ring magnets $2n$ and the ratio $x = \frac{r_{out}}{r_{in}}$ can be found from equations (3.67) and (3.77), reproduced here for convenience:

$$\sin \frac{\pi}{2n} = \frac{(r_{rng} + \frac{\delta}{2})}{(r_{rtr} + \frac{\delta}{2}) + (r_{rng} + \frac{\delta}{2})} r_{rng} = \frac{r_{out} - r_{in}}{2} \quad (3.117)$$

For $\delta = 0$ and $r_{in} = r_{rtr}$ (or alternatively, $r_{rtr} + \frac{\delta}{2} = r_{in}$ and $r_{rng} + \frac{\delta}{2} = \frac{r_{out} - r_{in}}{2}$):

$$\begin{aligned} \sin \frac{\pi}{2n} &= \frac{\frac{r_{out} - r_{in}}{2}}{r_{in} + \frac{r_{out} - r_{in}}{2}} \\ &= \frac{r_{out} - r_{in}}{r_{out} + r_{in}} \\ &= \frac{x - 1}{x + 1}, \quad x = \frac{r_{out}}{r_{in}} \end{aligned} \quad (3.118)$$

$$n = \frac{\pi}{2 \arcsin \frac{x-1}{x+1}} \quad (3.119)$$

$$x = \frac{1 + \sin \frac{\pi}{2n}}{1 - \sin \frac{\pi}{2n}} \quad (3.120)$$

Table 3.30 shows the values of x , $\ln x$, and $\frac{\ln x}{x^2}$ for $n \in [2, 3, 4, 5, 6, 7, 8]$. Assuming that the relationship between the size of the outer ring and its particular magnetisation configuration is a simple multiplicative one, indicators for holding torques and holding torque to volume ratios that are comparable across the various magnetisation configurations can be derived for each setup by combining the $\ln x$ and $\frac{\ln x}{x^2}$ values with appropriate multiplier values for each magnetisation configuration. The holding torque of the Halves Diametrical magnetisation configuration is used as the benchmark and assigned a multiplier value of 1. This allows the predictions of the mathematical models to be checked against the simulation results. If the predictions turn out to be accurate, it may be possible to use these models with arbitrarily defined magnetisation directions to find novel magnetisation patterns for optimal performance through optimisation algorithms such as genetic algorithm or particle swarm optimisation.

Table 3.30: Ratio $x = \frac{r_{out}}{r_{in}}$ for $\delta = 0$ from Equation (3.119), along with the corresponding $\ln x$ and $\frac{\ln x}{x^2}$ values.

n	x	$\ln x$	$\frac{\ln x}{x^2}$
2	≈ 5.83	≈ 1.76	≈ 0.0519
3	3	≈ 1.10	≈ 0.122
4	≈ 2.24	≈ 0.806	≈ 0.161
5	≈ 1.89	≈ 0.639	≈ 0.178
6	≈ 1.70	≈ 0.530	≈ 0.184
7	≈ 1.57	≈ 0.453	≈ 0.183
8	≈ 1.48	≈ 0.395	≈ 0.179

Diametrical Magnetisation

For Discrete Alternating Diametrical magnetisation, Section 3.4 shows $n \in [3, 4, 5, 6, 7, 8]$ to have multiplier values of 1. Within this range of n the relationship in (3.73) remains unchanged and the prediction of optimality at

$x = \sqrt{e}$, $n \approx 6.35$ carries over. For $n = 2$, the multiplier value was found to be $\frac{\sqrt{36+\pi^2}}{\pi} \approx 2.16$. These are summarised in the second column of Table 3.31.

Combining these values multiplicatively with the $\ln x$ and $\frac{\ln x}{x^2}$ values from Table 3.30 gives us the indicators for the predicted holding torques and holding torque to volume ratios in the third and fourth columns of Table 3.31.

Table 3.31: Multiplier values for the holding torques for Discrete Alternating Diametrical magnetisation, with the benchmark Halves Diametrical magnetisation configuration assigned a multiplier value of 1. Indicators for the predicted holding torques and holding torque to volume ratios, derived by combining $\ln x$ and $\frac{\ln x}{x^2}$ from Table 3.30 with the Multiplier values, are also shown.

n	Multiplier	Multiplier * $\ln x$	Multiplier * $\frac{\ln x}{x^2}$
2	≈ 2.16	≈ 3.80	≈ 0.112
3	1	≈ 1.10	≈ 0.122
4	1	≈ 0.806	≈ 0.161
5	1	≈ 0.639	≈ 0.178
6	1	≈ 0.530	≈ 0.184
7	1	≈ 0.453	≈ 0.183
8	1	≈ 0.395	≈ 0.179

Radial Magnetisation

For Discrete Alternating Radial magnetisation, from 3.5 the multiplier for $n \in [2, 4, 6, 8]$ is $\frac{\sqrt{\left(6n \cos \frac{\pi}{2n} + \pi \csc \frac{\pi}{2n}\right)^2 + \left(\sec \frac{\pi}{2n} (3n \sin \frac{\pi}{n} - \pi)\right)^2}}{8n}$. Application to equation (3.73) and substituting (3.119) gives:

$$\frac{|\tau|}{volume} \propto \frac{\ln x \arcsin \frac{x-1}{x+1} \sqrt{\frac{(x+1)^4}{x(x-1)^2} + \frac{36}{\arcsin^2 \frac{x-1}{x+1}} - \frac{12\sqrt{x}(x+1)(x^2-6x+1)}{x(x-1) \arcsin \frac{x-1}{x+1}}}}{8x^2} \quad (3.121)$$

The maximum of this expression was numerically found to be approximately 0.182, at $x \approx 1.63$. Solving for n using equation (3.119) produces $n \approx 6.51$, suggesting that the $n = 6$ configuration is optimal.

For $n \in [3, 5, 7]$, the multiplier is $\frac{3n \sin \frac{\pi}{n} + \pi}{4n \sin \frac{\pi}{2n}}$. Applying it to equation (3.73) and substituting (3.119) gives:

$$\frac{|\tau|}{\text{volume}} \propto \frac{(x+1) \ln x \left(2 \arcsin \frac{x-1}{x+1} + 3 \sin(2 \arcsin \frac{x-1}{x+1}) \right)}{8x^2(x-1)} \quad (3.122)$$

The maximum of this expression can be numerically found to be approximately 0.180, occurring at $x \approx 1.62$. Solving for n using equation (3.119) produces $n \approx 6.59$, suggesting that the $n = 7$ configuration is optimal.

The multiplier values are summarised in the second column of Table 3.32.

Combining these values multiplicatively with the $\ln x$ and $\frac{\ln x}{x^2}$ values from Table 3.30 gives us the indicators for the predicted holding torques and holding torque to volume ratios in the third and fourth columns of Table 3.32.

Table 3.32: Multiplier values for the holding torques for Discrete Alternating Radial magnetisation, with the benchmark Halves Diametrical magnetisation configuration assigned a multiplier value of 1. Indicators for the predicted holding torques and holding torque to volume ratios, derived by combining $\ln x$ and $\frac{\ln x}{x^2}$ from Table 3.30 with the Multiplier values, are also shown.

n	Multiplier	Multiplier * $\ln x$	Multiplier * $\frac{\ln x}{x^2}$
2	≈ 2.16	≈ 3.80	≈ 0.112
3	≈ 1.82	≈ 2.00	≈ 0.222
4	≈ 2.46	≈ 1.98	≈ 0.396
5	≈ 1.93	≈ 1.24	≈ 0.344
6	≈ 2.51	≈ 1.33	≈ 0.461
7	≈ 1.97	≈ 0.890	≈ 0.360
8	≈ 2.53	≈ 0.998	≈ 0.453

Halbach Magnetisation

For Discrete Alternating Halbach magnetisation, the multiplier value from Section 3.5 for $n \in [3, 4, 5, 6, 7, 8]$ is $\frac{3n}{\pi} \sin \frac{\pi}{n}$. Applying it to equation (3.73)

and substituting (3.119) gives:

$$\frac{|\tau|}{volume} \propto \frac{\ln x \sin(2 \arcsin \frac{x-1}{x+1})}{2x^2 \arcsin \frac{x-1}{x+1}} \quad (3.123)$$

The maximum of this expression was numerically found to be approximately 0.177 at $x \approx 1.59$. Solving for n using equation (3.119) produces $n \approx 6.82$, suggesting that one of the $n = 6, 7$ configurations is optimal.

For $n = 2$, the multiplier is $\frac{\sqrt{36+\pi^2}}{\pi} \approx 2.16$. This is summarised in Table 3.33.

Combining these values multiplicatively with the $\ln x$ and $\frac{\ln x}{x^2}$ values from Table 3.30 gives us the indicators for the predicted holding torques and holding torque to volume ratios in the third and fourth columns of Table 3.33.

Table 3.33: Multiplier values for the holding torques for Discrete Alteranting Halbach magnetisation, with the benchmark Halves Diametrical magnetisation configuration assigned a multiplier value of 1. Indicators for the predicted holding torques and holding torque to volume ratios, derived by combining $\ln x$ and $\frac{\ln x}{x^2}$ from Table 3.30 with the Multiplier values, are also shown.

n	Multiplier	Multiplier * $\ln x$	Multiplier * $\frac{\ln x}{x^2}$
2	≈ 2.16	≈ 3.80	≈ 0.112
3	≈ 2.48	≈ 2.73	≈ 0.303
4	≈ 2.70	≈ 2.18	≈ 0.434
5	≈ 2.81	≈ 1.79	≈ 0.500
6	≈ 2.86	≈ 1.52	≈ 0.526
7	≈ 2.90	≈ 1.31	≈ 0.531
8	≈ 2.92	≈ 1.16	≈ 0.524

3.6.1 Simple Simulation Model

These results were checked with FEM simulation using the simple simulation model. The outer radii of the ring magnets were calculated using equation (3.70), as detailed in Table 3.6.

All configurations were sampled at $\theta = 90^\circ$ and $\psi = 0^\circ$, and at the value of ϕ that generates the largest torque. For the configurations for which the required setting deviates from $\phi = 0$, the appropriate setting for ϕ is found by sampling ϕ in steps of 0.1° from -180° to 180° for $\theta = 90^\circ$ and $\psi = 0^\circ$, as detailed in Table 3.34. The magnetisation of the material used in this set of simulations were defined as specified in Table 3.26.

The Diametrical-Diametrical configurations were included alongside the Radial-Diametrical and Halbach-Diametrical configurations for comparison with the mathematical results described above.

Table 3.34: Settings of ϕ at which the highest torque is generated for the Discrete Alternating configurations with varying outer ring radii for which the required setting deviates from $\phi = 0$, for the simple simulation model, for $\theta = 90^\circ$ and $\psi = 0^\circ$. Also included are the values of ϕ at which the maximum torque was expected to occur, derived from the mathematical models for the Discrete Alternating configurations.

Outer	n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
Diametrical	2	-70.7°	$\approx -62.4^\circ$
Radial	2	-25.7°	$\approx -17.4^\circ$
Radial	4	-14.2°	$\approx -10.8^\circ$
Radial	6	-9.8°	$\approx -7.4^\circ$
Radial	8	-8.8°	$\approx -5.6^\circ$
Halbach	2	-161.3°	$\approx -152.4^\circ$

The results are illustrated in Figures 3.98 and 3.99, and the optimal numbers of ring magnets found are presented in Table 3.35.

Outer:Diametrical—Inner:Diametrical

For the Diametrical-Diametrical magnetisation configurations, their holding torques and holding torque to volume ratios are shown in Table 3.36.

Table 3.37 rescales the simulation results for holding torque relative to the $n = 6$ setup (chosen as the basis for comparison since a setup of the same size

Figure 3.98: Holding Torques generated for the Simple Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.

Figure 3.99: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.

Table 3.35: Optimal number of ring segments and their corresponding holding torque generated per unit volume $R_{\tau,v}$ for each Discrete Alternating magnetisation configuration for the Simple Simulation Model.

Configuration	Predicted n	n	$R_{\tau,v}$ [mNm/mm ³]
Diametrical-Diametrical	6	6	≈ 0.0310
Radial-Diametrical	6	8	≈ 0.108
Halbach-Diametrical	7	8	≈ 0.135

Table 3.36: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model with varying outer ring radii.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
2	≈ 711	≈ 0.0178
3	≈ 191	≈ 0.0183
4	≈ 150	≈ 0.0259
5	≈ 122	≈ 0.0297
6	≈ 101	≈ 0.0310
7	≈ 86.0	≈ 0.0309
8	≈ 74.9	≈ 0.0303

was used in Section 3.2.5) and provides a comparison with the corresponding values predicted by the mathematical model (rescaled from Table 3.31). The magnitudes of the predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. Table 3.38 does the same for the holding torque to volume ratios.

Table 3.37: The holding torques with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 7.02	≈ 7.17	≈ 2.15
3	≈ 1.89	≈ 2.07	≈ 9.83
4	≈ 1.48	≈ 1.52	≈ 2.99
5	≈ 1.20	≈ 1.21	≈ 0.353
6	1	1	0
7	≈ 0.849	≈ 0.854	≈ 0.621
8	≈ 0.739	≈ 0.746	≈ 0.908

For both holding torque and holding torque to volume ratio, the predictions from the mathematical model are largely in line with the simulation results, with the largest deviation being 12.7%. The optimal number of ring segments is 12, with torque to volume ratio $R_{\tau,v}$ also in line with the results in Section 3.3.

Outer:Radial—Inner:Diametrical

For the Radial-Diametrical magnetisation configurations, their holding torques and holding torque to volume ratios are detailed in Table 3.39.

Table 3.40 rescales the simulation results for holding torque relative to the $n = 6$ setup and provides a comparison with the corresponding values

Table 3.38: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.576	≈ 0.609	≈ 5.80
3	≈ 0.590	≈ 0.665	≈ 12.7
4	≈ 0.836	≈ 0.875	≈ 4.70
5	≈ 0.959	≈ 0.970	≈ 1.12
6	1	1	0
7	≈ 0.997	≈ 0.997	≈ -0.0333
8	≈ 0.979	≈ 0.976	≈ -0.306

Table 3.39: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model with varying outer ring radii.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
2	≈ 711	≈ 0.112
3	≈ 567	≈ 0.222
4	≈ 487	≈ 0.396
5	≈ 399	≈ 0.344
6	≈ 346	≈ 0.461
7	≈ 297	≈ 0.360
8	≈ 266	≈ 0.453

predicted by the mathematical model (rescaled from Table 3.32). The magnitudes of the predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. Table 3.41 does the same for the holding torque to volume ratios.

Table 3.40: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 2.05	≈ 2.86	≈ 39.2
3	≈ 1.64	≈ 1.51	≈ -8.08
4	≈ 1.41	≈ 1.49	≈ 6.12
5	≈ 1.15	≈ 0.930	≈ -19.2
6	1	1	0
7	≈ 0.857	≈ 0.670	≈ -21.9
8	≈ 0.767	≈ 0.751	≈ -2.08

The deviations found here are noticeably larger than those for the Diametrical-Diametrical configurations. There appears to be a divide between even and odd n , with the latter seeing larger deviations aside from $n = 2$. If the figures were rescaled relative to $n = 7$ instead we get Tables 3.42 and 3.43 and the opposite pattern is be observed.

If the values for even and odd n were compared within their own groups, the deviations found, aside from $n = 2, 3$, were all under 10%. Hence the mathematical model may be useful for rough comparisons within the group for larger values of n but may produce misleading results if a comparison between the two groups is attempted. It also suggests that the mathematical model may not work well for small n , and should be used with caution for those configurations.

To evaluate the usefulness of the mathematical model for comparing con-

Table 3.41: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.168	≈ 0.243	≈ 44.2
3	≈ 0.512	≈ 0.483	≈ -5.64
4	≈ 0.796	≈ 0.859	≈ 7.88
5	≈ 0.918	≈ 0.746	≈ -18.5
6	1	1	0
7	≈ 1.01	≈ 0.781	≈ -22.4
8	≈ 1.02	≈ 0.983	≈ -3.25

Table 3.42: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier $\ast \ln x$	Deviation (%)
2	≈ 2.40	≈ 4.27	≈ 78.2
3	≈ 1.91	≈ 2.25	≈ 17.6
4	≈ 1.64	≈ 2.23	≈ 35.8
5	≈ 1.34	≈ 1.39	≈ 3.44
6	≈ 1.17	≈ 1.49	≈ 28.0
7	1	1	0
8	≈ 0.895	≈ 1.12	≈ 25.3

Table 3.43: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.167	≈ 0.311	≈ 85.7
3	≈ 0.509	≈ 0.618	≈ 21.5
4	≈ 0.791	≈ 1.10	≈ 38.9
5	≈ 0.912	≈ 0.957	≈ 4.91
6	≈ 0.994	≈ 1.28	≈ 28.8
7	1	1	0
8	≈ 1.01	≈ 1.26	≈ 24.6

figurations utilising different magnetisation patterns, the multiplier values were rescaled relative to the Diametrical-Diametrical $n = 6$ setup, producing Tables 3.44 and 3.45.

Table 3.44: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison.

n	τ_{hold} Multiple	Multiplier $\ast \ln x$	Deviation (%)
2	≈ 7.02	≈ 7.17	≈ 2.15
3	≈ 5.61	≈ 3.78	≈ -32.6
4	≈ 4.81	≈ 3.75	≈ -22.2
5	≈ 3.94	≈ 2.33	≈ -40.7
6	≈ 3.42	≈ 2.51	≈ -26.6
7	≈ 2.93	≈ 1.68	≈ -42.7
8	≈ 2.62	≈ 1.88	≈ -28.2

Table 3.45: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.576	≈ 0.609	≈ 5.80
3	≈ 1.75	≈ 1.21	≈ -30.8
4	≈ 2.72	≈ 2.15	≈ -20.9
5	≈ 3.14	≈ 1.88	≈ -40.2
6	≈ 3.42	≈ 2.51	≈ -26.6
7	≈ 3.44	≈ 1.96	≈ -43.0
8	≈ 3.47	≈ 2.47	≈ -29.0

For both holding torque and holding torque to volume ratio, the deviations are large at around 30%, indicating that the mathematical model is not suitable for comparison between setups using different magnetisation patterns.

Overall, the optimal number of ring segments was found to be 16, with the configurations with 14 ring segments very near optimal as well. While the configurations for $n = 9$ may be a more efficient setup, the difference appears unlikely to be large as the change from $n = 7$ to $n = 8$ is very small.

Outer:Halbach—Inner:Diametrical

For the Halbach-Diametrical magnetisation configurations, their holding torques and holding torque to volume ratios are shown in Table 3.46.

Table 3.47 rescales the simulation results for holding torque relative to the $n = 6$ setup and provides a comparison with the corresponding values predicted by the mathematical model (rescaled from Table 3.33). The magnitudes of the predictions' deviations from the simulation results are expressed

Table 3.46: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model with varying outer ring radii.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
2	≈ 711	≈ 0.0178
3	≈ 659	≈ 0.0629
4	≈ 573	≈ 0.0990
5	≈ 495	≈ 0.121
6	≈ 427	≈ 0.131
7	≈ 376	≈ 0.135
8	≈ 334	≈ 0.135

as their ratio of the former to the latter less one. Table 3.48 does the same for the holding torque to volume ratios.

Table 3.47: The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 1.67	≈ 2.50	≈ 50.4
3	≈ 1.54	≈ 1.80	≈ 16.5
4	≈ 1.34	≈ 1.44	≈ 7.07
5	≈ 1.16	≈ 1.18	≈ 2.06
6	1	1	0
7	≈ 0.879	≈ 0.865	≈ -1.62
8	≈ 0.781	≈ 0.761	≈ -2.50

As with Radial-Diametrical magnetisation, larger deviations were found for $n = 2, 3$, suggesting again that the mathematical model does not work

Table 3.48: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.137	≈ 0.213	≈ 55.8
3	≈ 0.482	≈ 0.576	≈ 19.6
4	≈ 0.758	≈ 0.825	≈ 8.84
5	≈ 0.924	≈ 0.950	≈ 2.84
6	1	1	0
7	≈ 1.03	≈ 1.01	≈ -2.26
8	≈ 1.03	≈ 0.996	≈ -3.67

well for small n . For the other configurations, the predictions from the mathematical model are largely in line with the simulation results, with the largest deviation being 8.84%. The optimal number of ring segments is 12, with torque to volume ratio $R_{\tau,v}$ also in line with the results in Section 3.3.

When the multiplier values were rescaled relative to the Diametrical-Diametrical $n = 6$ setup, producing Tables 3.49 and 3.50.

Here, the deviations were again found to be large, indicating that the mathematical model should not be directly used for comparison between setups using different magnetisation patterns. The similar magnitude of the deviations found across both Radial and Halbach magnetisation patterns suggests that the addition of a simple multiplier of 0.3 may allow for rough comparisons across the board.

Overall, the optimal number of ring segments was found to be 16, with the configurations with 14 ring segments very near optimal as well.

Table 3.49: The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 7.02	≈ 7.17	≈ 2.17
3	≈ 6.50	≈ 5.15	≈ -20.9
4	≈ 5.65	≈ 4.11	≈ -27.3
5	≈ 4.88	≈ 3.39	≈ -30.7
6	≈ 4.22	≈ 2.86	≈ -32.1
7	≈ 3.71	≈ 2.48	≈ -33.2
8	≈ 3.29	≈ 2.18	≈ -33.8

Table 3.50: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the simple simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier * $\frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.576	≈ 0.609	≈ 5.81
3	≈ 2.03	≈ 1.65	≈ -18.8
4	≈ 3.20	≈ 2.36	≈ -26.1
5	≈ 3.89	≈ 2.72	≈ -30.1
6	≈ 4.22	≈ 2.86	≈ -32.1
7	≈ 4.35	≈ 2.89	≈ -33.6
8	≈ 4.36	≈ 2.85	≈ -34.6

3.6.2 Ring of Magnets Simulation Model

The Ring of Magnets Simulation Model was used to check the above results and those in Section 3.5 when a more realistic geometrical setup is used. The material definition and dimension of the setup were as described in Sections 3.2.5 and 3.3.2. The sampling regimes for θ , ψ and ϕ were as described in Section 3.4.1. The additional 5 sampling points for ϕ are detailed in Table 3.51. The magnetisation of the material used in this set of simulations were defined as specified in Table 3.26.

Table 3.51: Additional sampling points for ϕ for the setups in which a deviation of the setting of $\phi = 90^\circ$ to achieve the freewheeling effect is expected, for the ring of magnets simulation model. Also included are the expected values of ϕ to attain the freewheeling state as obtained from the mathematical models for the Discrete Alternating configurations.

Outer	n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
Diametrical	2	$-72^\circ, -49.5^\circ, -27^\circ, -4.5^\circ, 18^\circ$	$\approx 27.6^\circ$
Radial	2	$-27^\circ, -4.5^\circ, 18^\circ, 40.5^\circ, 63^\circ$	$\approx 72.6^\circ$
Radial	4	$-13.7^\circ, 8.8^\circ, 31.3^\circ, 53.8^\circ, 76.3^\circ$	$\approx 79.2^\circ$
Radial	6	$-9.6^\circ, 12.9^\circ, 35.4^\circ, 57.9^\circ, 80.4^\circ$	$\approx 82.6^\circ$
Radial	8	$-7.1^\circ, 15.4^\circ, 37.9^\circ, 60.4^\circ, 82.9^\circ$	$\approx 84.4^\circ$
Halbach	2	$-162^\circ, -139.5^\circ, -117^\circ, -94.5^\circ, -72^\circ$	$\approx -62.4^\circ$

Additionally, all configurations were sampled at $\theta = 90^\circ$ and $\psi = 0^\circ$, and at the value of ϕ that generates the largest torque. For the configurations for which the required setting deviates from $\phi = 0$, the appropriate setting for ϕ is found by sampling ϕ in steps of 0.1° from -180° to 180° for $\theta = 90^\circ$ and $\psi = 0^\circ$, as detailed in Table 3.52.

The results are illustrated in Figures 3.100 and 3.101, and the optimal numbers of ring magnets found are presented in Table 3.53.

Table 3.52: Settings of ϕ at which the highest torque is generated for the Discrete Alternating configurations with varying outer ring radii for which the required setting deviates from $\phi = 0$, for the ring of magnets simulation model, for $\theta = 90^\circ$ and $\psi = 0^\circ$. Also included are the values of ϕ at which the maximum torque was expected to occur, derived from the mathematical models for the Discrete Alternating configurations.

Outer	n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
Diametrical	2	-71.6°	$\approx -62.4^\circ$
Radial	2	-26.6°	$\approx -17.4^\circ$
Radial	4	-13.7°	$\approx -10.8^\circ$
Radial	6	-7.7°	$\approx -7.4^\circ$
Radial	8	-5.7°	$\approx -5.6^\circ$
Halbach	2	-159.9°	$\approx -152.4^\circ$

Figure 3.100: Holding Torques generated for the Ring of Magnets Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.

Figure 3.101: Holding Torque to Magnets' Volume Ratio for the Ring of Magnets Simulation Model for varying number of ring segments ($2n$) for the Discrete Alternating magnetisation configurations.

Table 3.53: Optimal number of ring segments and their corresponding holding torque generated per unit volume $R_{\tau,v}$ for each Discrete Alternating magnetisation configuration for the Ring of Magnets Simulation Model.

Configuration	n	$R_{\tau,v}$ [mNm/mm ³]
Diametrical-Diametrical	6	≈ 0.0287
Radial-Diametrical	6	≈ 0.0953
Radial-Radial	7	≈ 0.0565
Halbach-Diametrical	6	≈ 0.118

Figure 3.102: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Outer:Diametrical—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.102 and 3.103, and are also representative of the torque profiles for $n \in [4, 5, 6, 7, 8]$. The other graphs can be found in Appendix D.

The torque profiles are broadly similar to those in Section 3.2.5, with the holding torque for $n = 6$ at 75 mNm being very similar to the 76 mNm found for the Halves Diametrical-Diametrical magnetisation configuration for the ring of magnets simulation model.

The torque profiles for $n = 2$ are shown in Figures 3.104 and 3.105.

Note that the freewheeling effect for $\psi = 0$ rad occurs at $\phi \approx 18.0^\circ$, similar to the value of 27.6° predicted by the mathematical model in Section 3.4.

The holding torques and holding torque to volume ratios are shown in Table 3.54.

Figure 3.103: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.104: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.105: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Table 3.54: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model with varying numbers of ring magnets.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
2	≈ 607	≈ 0.0216
3	≈ 148	≈ 0.0188
4	≈ 114	≈ 0.0254
5	≈ 91.9	≈ 0.0283
6	≈ 74.7	≈ 0.0284
7	≈ 63.0	≈ 0.0278
8	≈ 55.7	≈ 0.0275

Table 3.55 rescales the simulation results for holding torque relative to the $n = 6$ setup and provides a comparison with the corresponding values predicted by the mathematical model (rescaled from Table 3.31). The magnitudes of the predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. Table 3.56 does the same for the holding torque to volume ratios. Figures 3.106 and 3.107 show these results graphically.

Table 3.55: The holding torques with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 8.13	≈ 7.17	≈ -11.7
3	≈ 1.98	≈ 2.07	≈ 4.60
4	≈ 1.52	≈ 1.52	≈ -0.0489
5	≈ 1.23	≈ 1.21	≈ -1.89
6	1	1	0
7	≈ 0.843	≈ 0.854	≈ 1.33
8	≈ 0.745	≈ 0.746	≈ 0.0931

For both holding torque and holding torque to volume ratio, the predictions from the mathematical model are largely in line with the simulation results, with the largest deviation being 19.8%. The optimal number of ring magnets is 12, as expected. The generally lower torque generated volumetric efficiency as compared to those from the simple simulation model is also expected due to the less ideal positioning of the bulk of the magnetic material in the ring of magnets.

Figure 3.106: The holding torques with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

Figure 3.107: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

Table 3.56: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Diametrical-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.760	≈ 0.609	≈ -19.8
3	≈ 0.661	≈ 0.665	≈ 0.570
4	≈ 0.892	≈ 0.875	≈ -1.87
5	≈ 0.995	≈ 0.970	≈ -2.60
6	1	1	0
7	≈ 0.978	≈ 0.997	≈ 1.89
8	≈ 0.966	≈ 0.976	≈ 1.08

Outer:Radial—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.108 and 3.109, and are also representative of the torque profiles for $n \in [4, 5, 6, 7, 8]$. The other graphs can be found in Appendix D.

Figure 3.110 shows that the residual torque that is still present when $\theta = -\psi$ for $n = 3$. This is also representative of that for $n \in [5, 7]$, the graphs of which can be found in Appendix D. Their maximum magnitudes are approximately 99 mNm, 60 mNm and 41 mNm, respectively.

Note that the residual torque for $n = 5$, with ring magnets of an overall size similar to that in Section 3.2.4 is close to the value previously found. The less efficient packing of the magnetic material of the ring around the rotor magnet contributes to the lower value of the residual torque.

The torque profiles are broadly similar in shape to those in Section 3.2.5, with the maximum holding torque for $n = 6$ at 251 mNm being very similar to the 247 mNm found for the Halves Radial-Diametrical magnetisation

Figure 3.108: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.109: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.110: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the ring of magnets simulation model.

configuration for the ring of magnets simulation model.

The torque profiles for $n = 2$ are shown in Figures 3.111 and 3.112, and are also representative of the torque profiles for $n \in [4, 6, 8]$. The other graphs can be found in Appendix D.

The deviation of the setting of ϕ at which the freewheeling effect occurs from $\frac{\pi}{2}$ rad for each setup are shown in Table 3.57. The general trend of smaller deviations from $\phi = 90^\circ$ as predicted by the mathematical model continues to hold true.

The holding torques and holding torque to volume ratios are shown in Table 3.58.

Table 3.59 rescales the simulation results for holding torque relative to the $n = 6$ setup and provides a comparison with the corresponding values predicted by the mathematical model (rescaled from Table 3.32). The magnitudes of the predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. Table 3.60 does the same

Figure 3.111: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.112: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Table 3.57: The settings of ϕ at which the freewheeling effect occurs, and the expected setting as derived from the mathematical model, for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model.

n	ϕ [$^\circ$]	Expected ϕ [$^\circ$]
2	$\approx 63.0^\circ$	$\approx 72.6^\circ$
4	$\approx 76.3^\circ$	$\approx 79.2^\circ$
6	$\approx 80.4^\circ$	$\approx 82.6^\circ$
8	$\approx 82.9^\circ$	$\approx 84.4^\circ$

Table 3.58: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model with varying numbers of ring magnets.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
2	≈ 607	≈ 0.0216
3	≈ 445	≈ 0.0365
4	≈ 369	≈ 0.0823
5	≈ 296	≈ 0.0910
6	≈ 251	≈ 0.0953
7	≈ 212	≈ 0.0935
8	≈ 185	≈ 0.0912

for the holding torque to volume ratios. Figures 3.113 and 3.114 show these results graphically.

Table 3.59: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as the ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 2.42	≈ 2.86	≈ 18.0
3	≈ 1.78	≈ 1.51	≈ -15.2
4	≈ 1.47	≈ 1.49	≈ 1.32
5	≈ 1.18	≈ 0.930	≈ -21.1
6	1	1	0
7	≈ 0.845	≈ 0.670	≈ -20.7
8	≈ 0.738	≈ 0.751	≈ 1.75

As with the simple simulation model, there appears to be a divide between even and odd n , with the latter seeing larger deviations aside from $n = 2$. If the figures were rescaled relative to $n = 7$ instead we get Tables 3.61 and 3.62 and the opposite pattern is be observed. Figures 3.115 and 3.116 show these results graphically.

Again, when the values for even and odd n were compared within their own groups, the deviations found, aside from $n = 2$, were all under 10%. Hence the mathematical model may be useful for rough comparisons within the group for larger values of n but may produce misleading results if a comparison between the two groups is attempted. It also suggests that the mathematical model may not work well for small n , and should be used with caution for those configurations.

To evaluate the usefulness of the mathematical model for comparing configurations utilising different magnetisation patterns, the multiplier values were rescaled relative to the Diametrical-Diametrical $n = 6$ setup, producing

Figure 3.113: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Table 3.60: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as the ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.227	≈ 0.243	≈ 7.14
3	≈ 0.592	≈ 0.483	≈ -18.5
4	≈ 0.863	≈ 0.859	≈ -0.523
5	≈ 0.955	≈ 0.748	≈ -21.7
6	1	1	0
7	≈ 0.980	≈ 0.781	≈ -20.3
8	≈ 0.956	≈ 0.983	≈ 2.76

Table 3.61: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier $\ast \ln x$	Deviation (%)
2	≈ 2.87	≈ 4.27	≈ 48.8
3	≈ 2.10	≈ 2.25	≈ 6.92
4	≈ 1.74	≈ 2.23	≈ 27.8
5	≈ 1.40	≈ 1.39	≈ -0.515
6	≈ 1.18	≈ 1.49	≈ 26.1
7	1	1	0
8	≈ 0.874	≈ 1.12	≈ 28.4

Figure 3.114: The holding torque to volumne ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Figure 3.115: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Figure 3.116: The holding torque to volumne ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Table 3.62: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.231	≈ 0.311	≈ 34.4
3	≈ 0.604	≈ 0.618	≈ 2.24
4	≈ 0.881	≈ 1.10	≈ 24.8
5	≈ 0.974	≈ 0.957	≈ -1.77
6	≈ 1.02	≈ 1.28	≈ 25.5
7	1	1	0
8	≈ 0.976	≈ 1.26	≈ 28.9

Tables 3.63 and 3.64. Figures 3.117 and 3.118 show these results graphically.

Table 3.63: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier $\ast \ln x$	Deviation (%)
2	≈ 8.13	≈ 7.17	≈ -11.7
3	≈ 5.96	≈ 3.78	≈ -36.6
4	≈ 4.94	≈ 3.75	≈ -24.2
5	≈ 3.95	≈ 2.33	≈ -41.0
6	≈ 3.35	≈ 2.51	≈ -25.2
7	≈ 2.83	≈ 1.68	≈ -40.7
8	≈ 2.47	≈ 1.88	≈ -23.8

Figure 3.117: The holding torques with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Again, for both holding torque and holding torque to volume ratio, the deviations are large at around 30%, indicating that the mathematical model is not suitable for comparison between setups using different magnetisation patterns.

Overall, the optimal number of ring segments was found to be 12, with the configurations with 14 ring segments very near optimal as well.

Outer:Radial—Inner:Radial

The torque profiles for $n \in [3, 5, 7]$ are shown in Figures 3.119, 3.120, 3.122, 3.123, 3.125 and 3.126.

As in Section 3.5, the shape of the torque profiles becomes smoother as n

Figure 3.118: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Table 3.64: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.760	≈ 0.609	≈ -19.8
3	≈ 1.99	≈ 1.21	≈ -39.0
4	≈ 2.89	≈ 2.15	≈ -25.6
5	≈ 3.20	≈ 1.88	≈ -41.4
6	≈ 3.35	≈ 2.51	≈ -25.2
7	≈ 3.29	≈ 1.96	≈ -40.3
8	≈ 3.21	≈ 2.47	≈ -23.1

grows. However, they retain the wavy pattern also seen in Section 3.2.5.

Figures 3.121, 3.124 and 3.127 shows that the residual torque that is still present when $\theta = -\psi$. Their maximum magnitudes are approximately 129 mNm, 106 mNm and 70 mNm show that trend of the Radial-Radial magnetisation configuration having larger torque residues than the Radial-Diametrical magnetisation configuration carries over from Sections 3.2.4 and 3.2.5.

For the Radial-Radial magnetisation configurations, their holding torques and holding torque to volume ratios are detailed in Table 3.65.

Table 3.66 rescales the simulation results for holding torque relative to the $n = 7$ setup and provides a comparison with the corresponding values predicted by the mathematical model (rescaled from Table 3.32). The magnitudes of the predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. Table 3.67 does the same for the holding torque to volume ratios. Figures 3.128 and 3.129 show these results graphically.

Figure 3.119: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.120: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.121: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the ring of magnets simulation model.

Figure 3.122: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.123: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.124: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the ring of magnets simulation model.

Figure 3.125: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.126: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure 3.127: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the ring of magnets simulation model.

Table 3.65: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model with varying outer ring radii.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
3	≈ 286	≈ 0.363
5	≈ 183	≈ 0.563
7	≈ 128	≈ 0.565

Table 3.66: The holding torques with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
3	≈ 2.10	≈ 2.25	≈ 0.716
5	≈ 1.40	≈ 1.39	≈ -2.74
7	1	1	0

Figure 3.128: The holding torques with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Table 3.67: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
3	≈ 0.604	≈ 0.618	≈ -3.69
5	≈ 0.974	≈ 0.957	≈ -3.97
7	1	1	0

Figure 3.129: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Radial-Radial magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 7$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Figure 3.130: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

A close alignment of the multiple obtained from the simulation results from the ring of magnets simulation model with those derived from the mathematical model continues to be observed. The optimal number of ring segments is 10 or 14. Part of the deviation may be due to the discrete magnetisation of the ring segments combining with a radially magnetised rotor to form wavy patterns in the torque profiles of the setups that elevates or depresses the maximum torque generated, as seen in Figures 3.119, 3.122 and 3.125.

Outer:Halbach—Inner:Diametrical

The torque profiles for $n = 3$ are shown in Figures 3.130 and 3.131, and are also representative of the torque profiles for $n \in [4, 5, 6, 7, 8]$. The other graphs can be found in Appendix D.

The torque profiles are broadly similar to those in Section 3.2.5, with the holding torque for $n = 6$ at 309 mNm being very similar to that found for

Figure 3.131: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

the Halves Halbach-Diametrical magnetisation configuration for the ring of magnets simulation model.

The torque profiles for $n = 2$ are shown in Figures 3.132 and 3.133. Note that the freewheeling effect for $\psi = 0$ rad occurs at $\phi \approx -72.0^\circ$. While this deviates somewhat from the value of -102° predicted by the mathematical model in Section 3.5, the general direction and magnitude are as expected.

The holding torques and holding torque to volume ratios are shown in Table 3.68. Table 3.69 rescales the simulation results for holding torque relative to the $n = 6$ setup and provides a comparison with the corresponding values predicted by the mathematical model (rescaled from Table 3.33). The magnitudes of the predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one. Table 3.70 does the same for the holding torque to volume ratios. Figures 3.134 and 3.135 show these results graphically.

Aside from $n = 2$, the predictions from the mathematical model are

Figure 3.132: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure 3.133: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Table 3.68: Holding torques and holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model with varying outer ring radii.

n	τ_{hold} [mNm]	$R_{\tau,v}$ [mNm/mm ³]
2	≈ 608	≈ 0.0216
3	≈ 520	≈ 0.0659
4	≈ 434	≈ 0.0968
5	≈ 364	≈ 0.112
6	≈ 309	≈ 0.117
7	≈ 265	≈ 0.117
8	≈ 231	≈ 0.114

Table 3.69: The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier * $\ln x$	Deviation (%)
2	≈ 1.97	≈ 2.50	≈ 27.2
3	≈ 1.68	≈ 1.80	≈ 6.68
4	≈ 1.41	≈ 1.44	≈ 1.97
5	≈ 1.18	≈ 1.18	≈ 0.279
6	1	1	0
7	≈ 0.857	≈ 0.865	≈ 0.897
8	≈ 0.748	≈ 0.761	≈ 1.82

Figure 3.134: The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Figure 3.135: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Table 3.70: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.184	≈ 0.213	≈ 15.5
3	≈ 0.561	≈ 0.576	≈ 2.57
4	≈ 0.824	≈ 0.825	≈ 0.120
5	≈ 0.954	≈ 0.950	≈ -0.449
6	1	1	0
7	≈ 0.995	≈ 1.01	≈ 1.45
8	≈ 0.969	≈ 0.996	≈ 2.82

largely in line with the simulation results, with the largest deviation being 6.68%.

When the multiplier values were rescaled relative to the Diametrical-Diametrical $n = 6$ setup, producing Tables 3.71 and 3.72. Figures 3.136 and 3.137 show these results graphically.

Here, the deviations were again found to be large, indicating that the mathematical model should not be directly used for comparison between setups using different magnetisation patterns. As with the simple simulation model, the similar magnitude of the deviations found across both Radial and Halbach magnetisation patterns suggests that the addition of a simple multiplier of 0.3 may allow for rough comparisons across the board.

Overall, the optimal number of ring segments was found to be 12.

Comparison

Table 3.73 shows a comparison of the holding torque to volume ratios of the various magnetisation configurations as predicted by the finite element

Table 3.71: The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	τ_{hold} Multiple	Multiplier $\ast \ln x$	Deviation (%)
2	≈ 8.13	≈ 7.17	≈ -11.8
3	≈ 6.95	≈ 5.15	≈ -26.0
4	≈ 5.81	≈ 4.11	≈ -29.3
5	≈ 4.87	≈ 3.39	≈ -30.4
6	≈ 4.13	≈ 2.86	≈ -30.6
7	≈ 3.54	≈ 2.48	≈ -30.0
8	≈ 3.09	≈ 2.18	≈ -29.4

Table 3.72: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one.

n	$R_{\tau,v}$ Multiple	Multiplier $\ast \frac{\ln x}{x^2}$	Deviation (%)
2	≈ 0.760	≈ 0.609	≈ -19.9
3	≈ 2.32	≈ 1.65	≈ -28.9
4	≈ 3.40	≈ 2.36	≈ -30.6
5	≈ 3.94	≈ 2.72	≈ -30.9
6	≈ 4.13	≈ 2.86	≈ -30.6
7	≈ 4.11	≈ 2.89	≈ -29.6
8	≈ 4.00	≈ 2.85	≈ -28.7

Figure 3.136: The holding torques with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

Figure 3.137: The holding torque to volume ratios $R_{\tau,v}$ with no modulation for the Discrete Alternating Halbach-Diametrical magnetisation configurations for the ring of magnets simulation model rescaled relative to the Diametrical-Diametrical $n = 6$ setup. The corresponding values predicted by the model are also provided for comparison. The predictions' deviations from the simulation results are expressed as their ratio of the former to the latter less one, and are plotted in absolute terms for ease of visualisation.

simulations. The Radial-Diametrical configurations for even n are omitted as their characteristics do not allow for independent control of the stiffness and equilibrium position. The Radial-Radial configurations are also omitted as their equilibrium position cannot be directly controlled with the parameter ψ .

Table 3.73: Comparison of the setups with the best holding torque to volume ratios for each ring magnetisation pattern, from the finite element simulations. The Diametrical magnetisation setup for $n = 2$, Radial-Diametrical magnetisation setups for $n \in [2, 4, 6, 8]$, the Radial-Radial magnetisation setups and Halbach magnetisation setup for $n = 2$ are omitted as their rotor equilibrium positions are not easily controlled.

Magnetisation Configuration	n	$R_{\tau,v}$ [mNm/mm ³]
Diametrical-Diametrical	6	≈ 0.0284
Radial-Diametrical	6	≈ 0.0953
Halbach-Diametrical	6	≈ 0.117

The Halbach-Diametrical configuration once again appears to be the most promising, and hence was chosen as the design for the prototype.

3.7 Conclusion

This chapter presented mathematical models that captures the essential characteristics of the proposed magnetic variable stiffness mechanism. They were used to explore potential ring magnetisation configurations, as well as to find the configurations that are the most space/ volume/ mass efficient per unit of holding torque transmittable. The Halbach magnetisation setup for $n = 6$ was found to be the most optimal.

The strengths and weaknesses of the models were also be examined through comparison with finite element simulations. The models were found to be reliable in predicting the relative performance of setups of the same magnetisation pattern, justifying the use of simple dipole models to represent

the magnets in the setup over more accurate but analytically more complex quadrupole and higher order models. Cross-comparison between setups of different magnetisation patterns however proved to be inaccurate with errors of up to 40%. Hence the models are not suitable for general optimisation with randomised ring magnetisation directions.

Chapter 4

Experimental Results

This chapter presents the design specifications of the prototype developed to test the proposed magnetic variable stiffness mechanism and its functional characteristics, showing that the mechanism indeed works as intended.

4.1 Prototype Specifications

Of the various configurations examined in Chapter 3, the Discrete Alternating Halbach-Diametrical Magnetisation configuration appears to be the most promising, delivering the largest range of controllable stiffness and holding torques per unit volume of magnetic material used. Further, the setup in which 12 ring magnets are used ($n = 6$) emerged as theoretically the most efficient design within that subgroup (see Table 3.68). This setup was thus chosen as the basis for the design of a prototype to test the viability of the proposed actuator.

For safety, the size of the rotor magnet was limited to a diameter of 50 mm. On advice from magnet manufacturers that the dimensions of magnets orthogonal to their magnetisation direction should not exceed that in their magnetised direction to reduce the self-demagnetisation effect, the height of the rotor magnet was set to 50 mm as well. This is particularly important for the rotor magnet since the formation of a Halbach cylinder with the outer ring magnets has the effect of concentrating their magnetic fields onto it,

making the possibility of demagnetisation a more pressing concern than for the ring magnets.

Two sets of spur gears were to be used to actuate the two groups of ring magnets, situated on either side of the magnets along their axes. Each set of gears consisted 6 small spur gears, each attached to a ring magnet, arranged around and meshing with a central spur gear connected mechanically to a crank or motor through which the rotational position of the ring magnets could be manipulated. Ideally, the ring spur gears should be as large as possible relative to the central spur gear to reduce the amount of torque required to actuate the ring magnets. The presence of the ring magnets belonging to the other set of magnets and their locating shafts though limits the size of the ring spur gears that can be used.

Since the radial position of the ring magnets relative to the rotor magnet $r_{pos,ring}$ must be equal to the sum of the pitch radii of the ring spur gears and the central spur gear, it was constrained by the limited selection of standard gear sizes available. Hence, an appropriate set of gears was first selected, whose sum of pitch radii determined the ring magnets' radial position, from which their radii and the gap between the magnets can be determined from (3.68) and (3.69). The heights of the ring magnets were kept the same as that of the rotor magnet. The dimensions of the magnets used in the prototype are detailed in Table 4.1.

These constraints and their resulting parameters produced a design illustrated in Figure 4.1, with the sidewalls of the magnet holders removed for clarity. Figures 4.2 and 4.3 show the transverse cross-sections said region where the meshing of the ring spur gears with the central main spur gear can be seen from their representative yellow pitch circles.

Neodymium magnets of high operating temperatures were selected for their high intrinsic coercivity to aid in resisting demagnetisation. In particular, magnets of grade N38EH were procured from *China Rare Earth Magnet Limited*, with magnetic coercivity of approximately 900 kA m^{-1} .

To avoid interfering with the magnetic operation of the actuator, all

Table 4.1: Dimensions of magnets used in the prototype, and the gap between the magnets.

Parameter [unit]	Value
Radius of Rotor Magnet [mm]	25
Height of Rotor Magnet [mm]	50
Number of Ring Magnets	12
Radius of Ring Magnets [mm]	6.9
Height of Ring Magnets [mm]	50
Radial Position of Ring Magnets [mm]	37.5
Gap Between Magnets [mm]	≈ 5.6

Figure 4.1: CAD drawing of the magnetic region of the prototype with the sidewalls of the magnet holders removed for clarity. The ring magnets are coloured red and blue for the “Virtual North Pole” and the “Virtual South Pole” respectively. The central rotor magnet is coloured orange, and the gears are coloured yellow.

Figure 4.2: Transverse cross-section of the CAD drawing of the prototype with the sidewalls of the magnet holders removed for clarity. The yellow circles represent the pitch circle of the spur gears. The central rotor magnet is coloured orange. The ring magnets belonging to the “Virtual North Pole” are coloured red and are each attached to a small ring spur gear that meshes with a larger central spur gear.

Figure 4.3: Transverse cross-section of the CAD drawing of the prototype with the sidewalls of the magnet holders removed for clarity. The yellow circles represent the pitch circle of the spur gears. The central rotor magnet is coloured orange. The ring magnets belonging to the “Virtual South Pole” are coloured blue and are each attached to a small ring spur gear that meshes with a larger central spur gear.

Table 4.2: Properties of magnetic material used for FEM analysis of the prototype.

Name	N38EH
Magnitude of Magnetic Coercivity [kA m^{-1}]	900
Relative Permeability	1.0997785406
Bulk Conductivity [$\text{k}\Omega \text{m}^{-1}$]	625

parts of the prototype were made of non-ferromagnetic materials such as Aluminium, Stainless Steel, Brass or Zirconium.

4.2 Prototype Simulation

For comparison with the physical prototype, the simulations for the Discrete Alternating Halbach-Diametrical magnetisation configuration for $n = 6$ in Section 3.6.2 were repeated with geometrical parameters as detailed in Table 4.1. The NdFe35 material in the default materials library was adapted to represent the permanent magnets to be used in the mechanism, with properties as shown in Table 4.2. The sampling regimes for θ , ψ and ϕ were as detailed in Table 3.4.1.

The predicted torque profiles for the prototype are shown in Figures 4.4 and 4.5. A holding torque of 15.7 Nm was predicted, which translates into a holding torque to magnets' volume ratio $\mathbf{R}_{\tau,v}$ of approximately 0.148 mNm/mm^3 .

The simulation results can be fitted to the mathematical model by replacing the A constant in the $A \cos \phi \sin \theta + \psi$ expression with the predicted holding torque. The model can then be compared to the simulation results by comparing predicted torque values at the sampled θ , ψ and ϕ values. The calculated Coefficients of Determination, shown in Table 4.3, are all close to 1, with an overall value of approximately 0.9999979, indicating that the fitted model corresponds very closely to the simulation result. The abnormal values for $\phi = 90^\circ$ are due to the flat torque profiles at that ϕ setting that corresponds very closely to the constant mean value used as the basis of

Figure 4.4: Predicted torque profiles for the prototype using the Discrete Alternating Halbach-Diametrical-6 setup with the ring of magnets simulation model for $\psi = 0$ rad.

Figure 4.5: Predicted torque profiles for the prototype using the Discrete Alternating Halbach-Diametrical-6 setup with the ring of magnets simulation model for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad.

Table 4.3: Coefficients of Determination comparing the simulation results with the fitted model.

ψ ($^{\circ}$)	ϕ ($^{\circ}$)				
	0	22.5	45	67.5	90
0	0.9999993	0.9999994	0.9999976	0.9999917	-0.0137916
45	0.9999994	0.9999988	0.9999983	0.9999893	-0.0183736
90	0.9999994	0.9999993	0.9999966	0.9999926	-0.0487378
Overall	0.9999979				

comparison.

4.3 Experimental Setup

The proof-of-concept prototype is shown and labelled in Figure 4.6. On the far left of the prototype is the output lever that is linked to the central cylindrical rotor magnet through a torque sensor and clamp couplers. The rotor magnet is surrounded by a ring of smaller cylindrical ring magnets divided alternately into two groups. Each of these ring magnets is attached to a small spur gear that meshes with a larger central spur gear corresponding to their assigned group on either side of the magnets. The two central spur gears are in turn linked via hollow shafts to worm wheels that are driven by the input worm gears, shown with stainless steel levers coupled onto their ends.

Figure 4.7 gives a cross-sectional view of the CAD model of the mechanism, with the approximate flow of power through the mechanism shown with arrows. It starts from the two inputs, through the worm and spur gear assemblies to the ring magnets, and through their combined magnetic fields to the central rotor magnet and finally out of the mechanism through the rotor shaft.

The ring magnets form the core of the mechanism, the orientation and manipulation of which generate the characteristics of the mechanism. Each

Figure 4.6: Proof-of-concept prototype.

Figure 4.7: A power flow diagram for the mechanism. In this cross-sectional view, the gear assemblies are coloured yellow, the rotor output shaft is coloured green, the hollow shafts connecting the worm wheels to their respective main spur gears are coloured pink, the Rotor Magnet is coloured orange, and the ring magnets are colour-coded blue and red for the “Virtual North Pole” and “Virtual South Pole” respectively. Note that for convenience the power from both inputs are each shown to flow through a single blue-coloured ring magnet, while in reality the blue-coloured and red-coloured ring magnets are two separate groups each controlled by one of the inputs.

Figure 4.8: A single magnet used in the ring of magnets setup.

Figure 4.9: 3D-printed tool used to mark the ring magnets for alignment with their holders.

ring magnet was labelled by the manufacturer with a red line showing its magnetisation direction, as seen in Figure 4.8. For assembly, a marking on the flat surface of the magnet showing its magnetisation direction was required for alignment with the magnet holder. A 3D-printed tool, pictured in Figure 4.9 was developed for this purpose. The process of doing is illustrated and described in Figures 4.10 through 4.12. Note that the original marking provided by the manufacturer is slightly slanted and hence does not show the direction of magnetisation accurately. The marking process also helps to compensate for this error.

To set the desired orientation for each magnet in the mechanism, six brass magnet holders were attached to their respective ring spur gears with set screws and meshed with a larger spur gear at the centre of the constellation. In this orientation, markings were made on both the magnet holders and the

Figure 4.10: First, the magnet was placed on a metal sheet with one end protruding slightly over the edge of the metal sheet.

Figure 4.11: Next, the tool was then fitted onto the protruding end of the magnet, making sure that the flat surface of the tool is flush with the tabletop.

Figure 4.12: Finally, the magnet was pushed into the tool until the proximal end was flush with the slotted flat surface at the front of the tool. A marker was then used to mark the magnet through the top half of the slot.

Figure 4.13: Ring magnets in their brass holders.

Figure 4.14: 3D-printed tool used to mark the brass ring magnet holders for alignment with their holders.

ring spur gears, and on the spur gears and main spur gears, to enable their subsequent realignment. A second 3D-printed tool, shown in Figure 4.14 was then used to mark the sides of the magnet holders to indicate the desired orientation of each ring magnet. An example of this marking is visible from Figure 4.15. The markings were extended to the rim of the magnet holders to facilitate alignment.

The ring magnets were then encased in the magnet holders, with each magnet's orientation determined by aligning the marking on their top surface with that of the magnet holder, and firmly attached with *3M DP100 Scotch-Weld Epoxy Adhesive*, as shown in Figure 4.13. An additional locating shaft was subsequently attached to the exposed end of the magnets. The two groups of ring magnets were then assembled separately and then brought together with the rotor magnet (also attached with adhesive to a load shaft on one end and an encoder shaft on the other, to constitute the variable stiffness mechanism.

Figure 4.15: An example of the marking made on a brass magnet holders to indicate the desired orientation of the ring magnet.

Three 12-bit *CUI AMT203 Modular Absolute Encoders*, pictured in Figure 4.16 were used to measure the rotational orientation of the rotor magnets and the two sets of ring magnets (with one magnet from each group standing in for the orientation of the entire group of magnets).

The default positions of the two groups of ring magnets were found by aligning the blue line markings on two of the brass holders with the end of a second 3D-printed tool, as shown in Figure 4.17. These readings were taken $\psi + \phi = 0$ and $\psi - \phi = 0$, equivalent to $\psi = 0$ and $\phi = 0$, and used to zero the position readings of the two encoders. The zero position for the rotor was taken to be its resting position at $\psi = 0$ and $\phi = 0$.

A *Kistler Type 4520A50 Basic Line Torque Sensor* with a maximum torque rating of 50 Nm was used to measure the torque acting on the rotor magnet via the rotor shaft.

The experimental setup is shown in Figure 4.18. The encoders and torque sensor were connected to a *National Instruments PCI-6289 Multifunction I/O Device* through a wiring box as shown in Figure 4.19. A LabView program, shown in Figure 4.20 was written to display the angular orientation of the rotor and each of the two groups of ring magnets. This allows any combination of the parameters ψ and ϕ to be set precisely for the characterisation of the mechanism's torque profile, by manipulating a couple of levers mechanically

Figure 4.16: CUI AMT203 Modular Absolute Encoders. The central unit provides readings of the rotor orientation while remaining two is responsible for providing the orientation of the two groups of ring magnets.

Figure 4.17: 3D-printed tool used to locate the default position for both groups of ring magnets, accomplished by resting the notch of the tool on the side of the mechanism and rotating the ring magnets via their coupled worm gear such that the marking on the magnet holder lines up with the bottom edge of the distal end of the tool.

linked to the central spur gears of both groups of the ring magnets, as shown in Figure 4.26. ϕ was sampled at 7.5° intervals from -7.5° to 105° , while ψ was sampled at intervals of 45° from -135° to 180° .

A set of PVC blocks, pictured in Figure 4.21, were machined to specific dimensions to produce a regular set of angular positions θ at intervals of 7.5° for the rotor magnet when its output lever rests on their top left/ right edge, as illustrated in Figures 4.22 and 4.23. The zero position is when the lever rests gently on either sides of the mechanism's walls, shown in Figures 4.24 and 4.25. The sampling regime for θ ranged from -75° to 75° in steps of 7.5° .

For a particular ψ setting, the experimental readings were taken in this manner:

1. Enter the desired/ target ψ setting in degrees into the software interface.
2. Enter the desired/ target ϕ setting in degrees into the software interface.
3. Remove any PVC block propping up the output lever.
4. Decouple the output lever from the torque sensor such that the rotor magnet is able to rotate freely.
5. Manipulate the levers connected to the “Virtual North Pole” and the “Virtual South Pole” such that the “Psi” and “Phi” values at the bottom of the interface match the desired/ target values.
6. Allow the output lever to rest gently on the right side of the mechanism (as seen from the top, in Figure 4.24).
7. Reconnect the output lever to the torque sensor. This sets a “zero” position from which the PVC blocks can be used to shift the rotor magnet's angular position in regular intervals in the counter-clockwise direction.
8. Click on the “READ” button to record the readings for that combination of θ , ψ and ϕ settings.

Figure 4.18: The experimental setup used to collect the torque readings for various settings of θ , ψ and ϕ .

9. Repeat Steps 2 and 5 for every desired ϕ value.
10. Using one of the PVC blocks, prop up the output lever so that the rotor is deflected by a fixed angle, as shown in Figure 4.22. Repeat Steps 8 and 9 to take the readings for the particular combination of θ and ψ values.
11. Repeat Steps 3 through 10, with the output lever resting gently on the left side of the mechanism instead (Figure 4.25), to get the readings for clockwise deflection of the rotor.
12. Repeat the entire process for each new ψ setting.

Due to concerns over the integrity of the glue fixing the rotor magnet to the output shaft, some of the readings were omitted when creaking sounds were produced while approaching the desired θ value. Hence the actual maximum holding torque of the actuator is not known. This is not of great concern as the holding torque of the actuator should not be exceeded during operation to avoid having the rotor snap around in a complete rotation.

Figure 4.19: The wiring box connecting the encoder and torque sensor to a multi-function I/O device on a windows computer.

Figure 4.20: The LabView software interface used to take readings for the characterisation of the prototype.

Figure 4.21: A set of PVC blocks machined to specific dimensions to produce a regular set of angular positions θ at intervals of 7.5° for the rotor magnet when its output lever rests on their top left/ right edge.

Figure 4.22: The output lever deflects the rotor magnet 45° from its equilibrium position, held in place by a machined PVC block.

Figure 4.23: The output lever deflects the rotor magnet -45° from its equilibrium position, held in place by a machined PVC block.

Figure 4.24: The rotor magnet is at its equilibrium position, with the output lever resting gently on the side of the mechanism, in preparation for readings to be taken for positive deflection of the rotor magnet.

Figure 4.25: The rotor magnet is at its equilibrium position, with the output lever resting gently on the side of the mechanism, in preparation for readings to be taken for negative deflection of the rotor magnet.

As there exists a small gap in the interface between the output lever and the output shaft, the intervals between readings of θ are not exactly 7.5° , but this should not have an adverse effect on the representativeness of the following results.

4.4 Experimental Results

Figures 4.32 through 4.31 show the torque profiles generated by the prototype for $\psi \in [-135^\circ, -90^\circ, -45^\circ, 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ]$.

Note that the torque profiles generated have a roughly sinusoidal shape, intercepting the x-axis in the vicinity of $\theta = -\psi$, as predicted by both the mathematical analyses and the simulation results. The maximum torque observed is approximately 15 Nm at $\theta = 75^\circ$ and $\phi = 15^\circ$. This is close to the value of 15.7 Nm predicted in Section 4.2. The maximum value would likely have been higher and closer to the predicted value had it been possible to take readings for $\theta = 90^\circ$ for $\phi \in [0^\circ, 7.5^\circ, 15^\circ]$.

Figure 4.26: Levers coupled to the worm gear connected to each group of ring magnets, for manual manipulation of the ring magnets into precise positions for torque load readings to be taken. Motors can be used in the place of the levers to turn the variable stiffness mechanism into an actuator, but this will not be covered in this work.

Figure 4.27: Torque profiles generated by the prototype for $\psi = 0^\circ$. Figure 4.28: Torque profiles generated by the prototype for $\psi = 45^\circ$.

Figure 4.29: Torque profiles generated by the prototype for $\psi = 90^\circ$. Figure 4.30: Torque profiles generated by the prototype for $\psi = 135^\circ$.

Figure 4.31: Torque profiles generated by the prototype for $\psi = 180^\circ$. Figure 4.32: Torque profiles generated by the prototype for $\psi = -135^\circ$.

Figure 4.33: Torque profiles generated by the prototype for $\psi = -90^\circ$.

Figure 4.34: Torque profiles generated by the prototype for $\psi = -45^\circ$.

Figure 4.35: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$.

Note that the mechanism's freewheeling setting (as characterised by a flat torque profile with respect to θ) was found to be at $\phi = 105^\circ$ as opposed to the $\phi = 90^\circ$ expected.

Visually comparing the prototype's torque profile with the simulation results for the same ϕ settings we get Figures 4.35 through 4.39. While the deviation is small for low values of ϕ , it grows to a maximum at $\phi = 90^\circ$.

Since the fitted model corresponds very closely to the simulation results, it can be used as a proxy to compare the experimental results with the simulation results even if their sampling regimes for θ differs. The result is shown in Table reftab:CoDProFit0. Note the drop-off in the coefficient values as ϕ approaches 90° .

If the comparison was made with the prototype's torque profile at values of ϕ that are 15° larger, we get Figures 4.40 through 4.44, and Table 4.5 instead. Here, the deviations are noticeably smaller, with the maximum deviation falling from 5 Nm to 2 Nm, with the overall Coefficient of Determination reduced to approximately 0.989 from 0.930. This is in line with the finding

Figure 4.36: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure 4.37: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure 4.38: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure 4.39: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$.

Table 4.4: Coefficients of Determination comparing the experimental results with the simulation results via the fitted model for $\phi_{Pro} = \phi_{Sim}$

ψ ($^{\circ}$)	ϕ_{Sim} ($^{\circ}$) ($\phi_{Pro} = \phi_{Sim}$)				
	0	22.5	45	67.5	90
0	≈ 0.996	≈ 0.996	≈ 0.968	≈ 0.829	≈ -0.000108
45	≈ 0.982	≈ 0.988	≈ 0.963	≈ 0.833	≈ -0.00885
90	≈ 0.992	≈ 0.996	≈ 0.971	≈ 0.854	≈ -0.0242
Overall	≈ 0.930				

Table 4.5: Coefficients of Determination comparing the experimental results with the simulation results via the fitted model for $\phi_{Pro} = \phi_{Sim} + 15^{\circ}$

ψ ($^{\circ}$)	ϕ_{Sim} ($^{\circ}$) ($\phi_{Pro} = \phi_{Sim} + 15^{\circ}$)				
	0	22.5	45	67.5	90
0	≈ 0.997	≈ 0.998	≈ 0.996	≈ 0.981	≈ -0.0971
45	≈ 0.985	≈ 0.987	≈ 0.990	≈ 0.981	≈ -0.228
90	≈ 0.996	≈ 0.996	≈ 0.998	≈ 0.992	≈ -0.645
Overall	≈ 0.989				

that maximum torque generation occurred at around $\phi = 15^{\circ}$. Hence the deviation can be compensated for through a straightforward adjustment in software. More comparisons of the torque profile of the prototype with the simulation results can be found in Appendix E.

Note too that there exist a small residual torque even at $\phi = 105^{\circ}$ that was not predicted by the mathematical models and simulation results. This is likely due to the errors introduced in the assembly of the prototype, as detailed in the next section.

Abridged versions of the comparison tables of variable stiffness mechanisms presented in Section 2.1.2 are shown here in Tables 4.6 and 4.7 and to compare the properties and characteristics of the various magnetic variable stiffness mechanisms with this work.

Compared to the M²-VSA, the prototype occupies a volume that is 24.7

Figure 4.40: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure 4.41: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure 4.42: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure 4.43: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure 4.44: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$.

times larger while being capable of providing a stiffness range that is 5.75 times as large. While this suggests a poor volumetric efficiency in terms of torque/ stiffness generation, a good portion of this deficit is likely due to the poor scaling of magnetic effects over distance, the choice of a large-sized gap between the magnets, and unoptimised mechanism design. A smaller and more tightly engineered prototype using the concept presented in this work can conceivably match the volumetric efficiency of the M²-VSA. The prototype presented here also has the potential to be an actuator since it is capable of adjusting its equilibrium position, whereas the entire M²-VSA mechanism needs to be rotated to accomplish that.

The prototype presented here is capable of providing a holding torque that is similar to the VSU, of approximately 15 Nm. For the VSU however, the holding torque is reached after only 20° of rotor deflection, compared to 75° here. This results in a much larger stiffness range for the VSU, but also limits the lower range of stiffness that can be commanded. Unlike the M²-VSA the VSU can adjust its equilibrium position. However, the input

that goes into changing the stiffness of the mechanism does not contribute to moving the equilibrium position of the mechanism, unlike the mechanism presented in this work.

Table 4.6: Abridged comparison of variable stiffness actuators/ mechanisms.

Name	Overload	L x W x H [mm]	Mass [kg]	Notes
VSU [9]	Slip	150 x 150 x 230	5.2	-
M ² -VSA [10]	Slip	87 x 87 x 45	N.R.	^a
MAVERIC [8]	Slip	20.6 x 20.6 x 12.5	N.R.	^b
This Work	Slip	332 x 189 x 134	N.R.	-

^a Uses magnets to provide ability to slip under overload conditions.

^b Data is based on physical testing with magnets. No actuated prototype was reported.

Table 4.7: Abridged comparison of variable Stiffness actuators/ mechanisms, continued.

Name	Stiffness ^c [Nm/rad]	Speed ^d [s]	Deflection ^e [rad]	Rotation ^f [rad]
VSU	4.08 – 49.4 (12.1x)	N.R. ^a	$\pm\infty / \pm\infty^c$	$\pm\infty$
M ² -VSA	0 – 2 (∞ x)	N.R.	$\pm\infty / \pm\infty$	$\pm\infty$
MAVERIC	0 – 0.11 (∞ x)	N.A. ^{bd}	$\pm\infty / \pm\infty$	$\pm\infty$
This Work	0 – 11.5 (∞ x)	N.R.	$\pm\infty / \pm\infty$	$\pm\infty$

^a Not Reported.

^b Not Applicable.

^c Due to ability to slip under overload conditions.

^d No actuated prototype was reported.

The experimental setup presented in the MAVERIC paper was on a much smaller scale than the prototype presented here and was also not built into

a functional device. Hence a comparison with its performance is omitted.

4.5 Sources of Errors

While the characteristics of the prototype were found to be generally in agreement with the mathematical analyses and the simulation results, some small discrepancies were still found, as observed in 4.4. A number of sources of errors have contributed to these deviations.

Labelling of Magnets' Magnetisation Direction

Since the magnetic field acting on the rotor magnet was manipulated by changing the orientations of the ring magnets, it was important that the magnetisation directions of the ring magnets were accurately marked. However, the labelling provided by the manufacturers were found to be inaccurate and inconsistent.

An example of this can be seen in Figure 4.10. Here, a magnet was placed on a ferromagnetic metal sheet. The presence of the metal sheet orientates the magnet such that its poles are aligned vertically, orthogonal to the surface of the desk. The red coloured marking provided by the manufacturer can be seen to deviate slightly from the ideal marking shown in green. This error was partially mitigated by making a second marking, in blue, on the flat surface of the magnet with the aid of a ruler.

Aligning of Ring Magnets in their Holders

Another related source of error is the alignment of the ring magnets with their holders. This involved translating the blue vertical line along the axis of the magnet holder into a radial marking on its rim, and then aligning this second marking with that on the ring magnet, as shown in Figures 4.13 and 4.15.

Both of these steps were performed manually with only visual guidance. While care was taken, errors were nevertheless likely introduced during this

Figure 4.45: Marking used to realign the spur gear and the magnet holder, to ensure that the magnets are their desired orientationn during the assembly of the prototype.

process.

Affixing of Magnets and their Holders to the Ring Spur Gears

Positional errors were also introduced when the magnet holders were affixed to their corresponding spur gears. The alignment of the two parts was also accomplished by visually lining up a marking on the spur gear with that on the magnet holder, as seen in Figure 4.45, and is hence another source of error in the ring magnets' final orientation.

Loosening of the Set Screws of the Spur Gears

The spur gears were affixed to the magnet holders using two set screws, each aligned with one of two flat faces cut into the shaft of the magnet holders. They were found to have loosened after an extended period of operation of the prototype. The strong magnetic forces acting on the ring magnets made realignment of the markings on the spur gears and on the magnet holders difficult and necessitated the acceptance of a small visual misalignment in some cases.

Backlash within the Gear System

While the gears used in the prototype were of good quality with little apparent backlash, some backlash was nevertheless still present. As the magnetic forces acting on individual magnets are different at each setting of θ , ψ and ϕ , this allows the orientation of ring magnets within each group of magnets to become slightly “out of sync” with one another. This fact could not be captured since the rotational position of a single magnet from each group was taken to be representative of all the magnets in its group.

Axial offset of the Ring Magnetes

Figure 4.46 shows the current axial positioning of the ring magnets, with the approximate positioning of the magnets within their holders illustrated with red lines. The two groups of ring magnets appear to have a small axial offset relative to each other. This is due to the existence of gaps between the spur gears and the mounting plates, allowing the magnetic forces to push the two groups of ring magnets apart. This axial displacement likely had an effect on the magnetic field acting on the rotor magnet.

Axial tilt of the Ring Magnets

From Figure 4.46 it is also clear that some of the ring magnets’ orientations are tilted slightly with respect to the others due to flaws in the assembly process. This may too have contributed to the errors observed.

Figure 4.46: Approximate locations of the ring magnets encased within their holders. Note the unintended small axial offset between the two groups of magnets.

Chapter 5

Conclusion

There currently exist a wide variety of mechanisms designed for variable stiffness actuation. However, none are currently clearly superior to the others, with the optimal choice heavily dependent on the requirements for a particular design, trading off properties such as the ability to achieve theoretically infinite stiffness, the capacity for free infinite rotation, or linearity and resolution in the control of the stiffness of the mechanism. Under such conditions, the availability of different variable stiffness actuator designs may prove to be useful for the design of advanced robots in the future.

Although VSAs that utilise magnetism in its working principle has received some attention as seen by the development of the M²-VSA and the MAVERIC, there is still a relative paucity of designs that capitalises on the unique strengths such as its non-contact nature and inherent protection from overloading that magnetism brings to the table.

In this work, a novel design for a novel magnetic variable stiffness mechanism was presented. The proposed design allows the two inputs that go into varying the stiffness of the mechanism to both also work to move the equilibrium position of the mechanism, an advantage over existing magnetic variable stiffness mechanisms in which the inputs to changing the equilibrium position and the stiffness of the mechanism cannot contribute to the action of the other. It also avoids the need to allocate space for the axial translation of magnets in the linear displacement design. The use of identical motors

and transmissions also improves operational robustness since inventory for spare parts can be reduced and simplified. In more extreme circumstances the cannibalisation of units for spares is also more broadly useful.

Mathematical models that capture the essential characteristics of a variety of configurations of said concept were developed. Further analyses revealed their various strengths and weaknesses and how they scaled with size, allowing the most promising designs to be identified.

Simulation studies were conducted and showed general agreement with the trends predicted by the mathematical model. The models were found to be reliable in predicting the relative performance of setups of the same magnetisation pattern, but cross-comparison between setups of different magnetisation patterns proved to be inaccurate with errors of up to 40%.

Overall, the Halbach ring magnetisation configuration as most promising. This configuration was adapted to design and build a proof-of-concept prototype. The prototype was characterised and was found to work as expected. Its torque profile was found to be generally in line with the predictions of both the mathematical models and the results of the simulation studies.

5.1 Contributions

The contributions of my work are as follows:

1. Constructed mathematical models to explore potential ring magnetisation configurations for the proposed magnetic variable stiffness mechanism.
2. Used the models to find the configurations that are the most space/ volume/ mass efficient per unit of holding torque transmittable.

5.2 Future Work

5.2.1 Improvements to Mechanical Design

As a rough estimate of the possible improvements in terms of the size of the prototype, if the torque sensor and the part of the mechanisms housing used to contain it were removed, the dimensions of the prototype would be reduced to approximately 200 x 189 x 134 mm from 332 x 189 x 134 mm, with the mass reduced by about 2 kg.

With more effort and resources devoted to redesigning and optimising the mechanical layout and transmission of the prototype, it should be possible to shrink its dimensions by a further 30%.

5.2.2 Investigate Dynamic Performance of the Prototype

The experimental data described in this report were static readings obtained at fixed settings of θ , ψ and ϕ . It is known, however, that significant dynamic effects such as the generation of eddy currents are present when magnetic fields change quickly, such as when the prototype is actuated at high speeds. These effects are likely complex and have significant effects on the performance of the device and hence needs to be investigated.

5.2.3 Design Suitable Control Algorithm for the Mechanism

Given the deviations of the prototype's behaviour from the ideal, a direct implementation of a fitted version of the mathematical model in feedforward control is likely to result in inaccurate or even unexpected performance. The addition of a Proportional-Integral-Derivative Controller (PID) controller may suffice to achieve reasonable performance, but higher performance may require detailed modelling of the errors within the system.

5.2.4 Investigate the Scaling Performance of the Mechanism

Although a theoretical investigation of the scaling performance of the concept was conducted in this work, the analyses did not take the practicalities of mechanical design into consideration. The building of a few additional prototypes of varying sizes will provide a much better indication of the true scaling performance of this type of magnetic variable stiffness mechanism.

5.2.5 Test other Magnetisation Configurations

In this work, only the Halbach-Magnetisation for $n = 6$ was examined experimentally. While the finding that the prototype largely behaves as expected leads us to expect that the other configurations will also behave as the mathematical models and simulations predicted, further experimental testing may reveal shortcomings of the mathematical models that need to be rectified.

Appendix A

Optimal Size – Halves Configuration

A.1   Simple Simulation Model

Figure A.1: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Diametrical configuration.

Figure A.2: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Diametrical configuration.

Figure A.3: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Radial configuration.

Figure A.4: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Diametrical-Radial configuration.

Figure A.5: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Diametrical configuration.

Figure A.6: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Diametrical configuration.

Figure A.7: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Radial configuration.

Figure A.8: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Radial-Radial configuration.

Figure A.9: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.

Figure A.10: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Diametrical configuration.

Figure A.11: Holding Torques generated for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Radial configuration.

Figure A.12: Holding Torque to Magnets' Volume Ratio for the Simple Simulation Model for varying Ring Outer Radius r_{out} to Rotor Outer Radius r_{in} Ratios for the Halves Halbach-Radial configuration.

Appendix B

Alternating Configuration – Simple Simulation Model

B.1 Outer:Diametrical—Inner:Diametrical

Figure B.1: Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.2: Torque profiles generated by the Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.3: Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.4: Torque profiles generated by the Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.5: Torque profiles generated by the Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.6: Torque profiles generated by the Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.7: Torque profiles generated by the Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.8: Torque profiles generated by the Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.9: Torque profiles generated by the Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.10: Torque profiles generated by the Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.11: Torque profiles generated by the Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.12: Torque profiles generated by the Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.13: Torque profiles generated by the Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.14: Torque profiles generated by the Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

B.2 Outer:Radial—Inner:Diametrical

Figure B.15: Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.16: Torque profiles generated by the Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.17: Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.18: Torque profiles generated by the Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.19: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.

Figure B.20: Torque profiles generated by the Alternating Radial-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.21: Torque profiles generated by the Alternating Radial-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.22: Torque profiles generated by the Alternating Radial-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.23: Torque profiles generated by the Alternating Radial-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.24: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-5 magnetisation setup for the simple simulation model.

Figure B.25: Torque profiles generated by the Alternating Radial-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.26: Torque profiles generated by the Alternating Radial-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.27: Torque profiles generated by the Alternating Radial-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.28: Torque profiles generated by the Alternating Radial-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.29: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Diametrical-7 magnetisation setup for the simple simulation model.

Figure B.30: Torque profiles generated by the Alternating Radial-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.31: Torque profiles generated by the Alternating Radial-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

B.3 Outer:Radial—Inner:Radial

Figure B.32: Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.33: Torque profiles generated by the Alternating Radial-Radial-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.34: Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.35: Torque profiles generated by the Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.36: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.

Figure B.37: Torque profiles generated by the Alternating Radial-Radial-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.38: Torque profiles generated by the Alternating Radial-Radial-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.39: Torque profiles generated by the Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.40: Torque profiles generated by the Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.41: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-5 magnetisation setup for the simple simulation model.

Figure B.42: Torque profiles generated by the Alternating Radial-Radial-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.43: Torque profiles generated by the Alternating Radial-Radial-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.44: Torque profiles generated by the Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.45: Torque profiles generated by the Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.46: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Alternating Radial-Radial-7 magnetisation setup for the simple simulation model.

Figure B.47: Torque profiles generated by the Alternating Radial-Radial-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.48: Torque profiles generated by the Alternating Radial-Radial-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

B.4 Outer:Halbach—Inner:Diametrical

Figure B.49: Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.50: Torque profiles generated by the Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.51: Torque profiles generated by the Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.52: Torque profiles generated by the Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.53: Torque profiles generated by the Alternating Halbach-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.54: Torque profiles generated by the Alternating Halbach-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.55: Torque profiles generated by the Alternating Halbach-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.56: Torque profiles generated by the Alternating Halbach-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.57: Torque profiles generated by the Alternating Halbach-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.58: Torque profiles generated by the Alternating Halbach-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.59: Torque profiles generated by the Alternating Halbach-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.60: Torque profiles generated by the Alternating Halbach-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure B.61: Torque profiles generated by the Alternating Halbach-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure B.62: Torque profiles generated by the Alternating Halbach-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Appendix C

Discrete Alternating Configuration – Simple Simulation Model

C.1 Outer:Radial—Inner:Diametrical

Figure C.1: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.2: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.3: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.4: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.5: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the simple simulation model.

Figure C.6: Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.7: Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.8: Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.9: Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.10: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-5 magnetisation setup for the simple simulation model.

Figure C.11: Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.12: Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.13: Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.14: Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.15: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-7 magnetisation setup for the simple simulation model.

Figure C.16: Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.17: Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

C.2 Outer:Radial—Inner:Radial

Figure C.18: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.19: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.20: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the simple simulation model.

Figure C.21: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.22: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.23: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the simple simulation model.

Figure C.24: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.25: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.26: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the simple simulation model.

C.3 Outer:Halbach—Inner:Diametrical

Figure C.27: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.28: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.29: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.30: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.31: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.32: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.33: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.34: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.35: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.36: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.37: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.38: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Figure C.39: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the simple simulation model.

Figure C.40: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the simple simulation model.

Appendix D

Discrete Alternating Configuration – Ring of Magnets Simulation Model

D.1 Outer:Diametrical—Inner:Diametrical

Figure D.1: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.2: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.3: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.4: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.5: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.6: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.7: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.8: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.9: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.10: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.11: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.12: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.13: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.14: Torque profiles generated by the Discrete Alternating Diametrical-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

D.2 Outer:Radial—Inner:Diametrical

Figure D.15: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.16: Torque profiles generated by the Discrete Alternating Radial-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.17: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.18: Torque profiles generated by the Discrete Alternating Radial-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.19: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-3 magnetisation setup for the ring of magnets simulation model.

Figure D.20: Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.21: Torque profiles generated by the Discrete Alternating Radial-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.22: Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.23: Torque profiles generated by the Discrete Alternating Radial-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.24: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-5 magnetisation setup for the ring of magnets simulation model.

Figure D.25: Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.26: Torque profiles generated by the Discrete Alternating Radial-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.27: Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.28: Torque profiles generated by the Discrete Alternating Radial-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.29: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Diametrical-7 magnetisation setup for the ring of magnets simulation model.

Figure D.30: Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.31: Torque profiles generated by the Discrete Alternating Radial-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

D.3 Outer:Radial—Inner:Radial

Figure D.32: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.33: Torque profiles generated by the Discrete Alternating Radial-Radial-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.34: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-3 magnetisation setup for the ring of magnets simulation model.

Figure D.35: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.36: Torque profiles generated by the Discrete Alternating Radial-Radial-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.37: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-5 magnetisation setup for the ring of magnets simulation model.

Figure D.38: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.39: Torque profiles generated by the Discrete Alternating Radial-Radial-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.40: Torque deviation when $\theta = -\psi$ for $\phi = 0$ rad for the Discrete Alternating Radial-Radial-7 magnetisation setup for the ring of magnets simulation model.

D.4 Outer:Halbach—Inner:Diametrical

Figure D.41: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.42: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-2 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.43: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.44: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-3 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.45: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.46: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-4 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.47: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.48: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-5 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.49: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.50: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-6 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.51: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.52: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-7 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Figure D.53: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi = 0$ rad for the ring of magnets simulation model.

Figure D.54: Torque profiles generated by the Discrete Alternating Halbach-Diametrical-8 magnetisation setup for $\psi \in [\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}]$ rad for the ring of magnets simulation model.

Appendix E

Prototype Torque Profile Comparisons

Figure E.1: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure E.2: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure E.3: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure E.4: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure E.5: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$.

Figure E.6: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure E.7: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure E.8: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure E.9: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure E.10: Comparison at $\psi = 0^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$.

Figure E.11: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure E.12: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure E.13: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure E.14: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure E.15: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$.

Figure E.16: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure E.17: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure E.18: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure E.19: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure E.20: Comparison at $\psi = 45^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$.

Figure E.21: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 0^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure E.22: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 22.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure E.23: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 45^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure E.24: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 67.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure E.25: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 90^\circ$ with the simulation results for $\phi = 90^\circ$.

Figure E.26: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 15^\circ$ with the simulation results for $\phi = 0^\circ$.

Figure E.27: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 37.5^\circ$ with the simulation results for $\phi = 22.5^\circ$.

Figure E.28: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 60^\circ$ with the simulation results for $\phi = 45^\circ$.

Figure E.29: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 82.5^\circ$ with the simulation results for $\phi = 67.5^\circ$.

Figure E.30: Comparison at $\psi = 90^\circ$ of the prototype's torque profile for $\phi = 105^\circ$ with the simulation results for $\phi = 90^\circ$.

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