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GENERIC JORDAN TYPE OF THE SYMMETRIC AND EXTERIOR POWERS

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ABSTRACT. We prove a result relating the stable generic Jordan types of the symmetric and exterior powers of the Heller translations of a module for a finite elementary abelian p -group. In the case of the trivial module, the stable generic Jordan types of the symmetric and exterior powers of its Heller translations are completely described.

1. INTRODUCTION

Fix an algebraically closed field k of positive characteristic p . Let $E = \langle g_1, \dots, g_n \rangle$ be an elementary abelian p -group of order p^n and M be a finite-dimensional kE -module. The generic Jordan type of M is the Jordan type of

$$u_\alpha = \sum_{i=1}^n \alpha_i (g_i - 1)$$

on M for a generic element $\alpha = (\alpha_1, \dots, \alpha_n) \in \mathbb{A}^n(k) \setminus \{\mathbf{0}\}$ (see [6]). The module M is of constant Jordan type if the Jordan type of u_α on M does not depend on $\alpha \in \mathbb{A}^n(k) \setminus \{\mathbf{0}\}$ (see [5]).

In [5, Proposition 4.7], Friedlander, Pevtsova and Suslin proved that the generic Jordan type of the tensor product of kE -modules is the tensor product of their generic Jordan types. In [3, Corollary 4.3], Carlson, Friedlander and Pevtsova proved that the tensor product of two kE -modules of constant Jordan type is again a module of constant Jordan type. Motivated by these results, we asked, if the similar results hold, when tensor product is replaced by symmetric power S^m or exterior power Λ^m . The answers are negative. Suppose that $m = p$. We provide the regular kE -module kE as a counter-example of our queries.

Fix a positive integer m . Our main result relates the stable generic Jordan types of S^m and Λ^m of various Heller translations of a module for an elementary abelian p -group. As an application, we obtain a complete description of the stable generic Jordan types of $S^m \Omega^i k$ and $\Lambda^m \Omega^i k$.

Throughout the article, we have omitted many known results which the readers can find in the standard texts [1, 4].

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2. PRELIMINARIES

Let E be an elementary abelian p -group and M be a kE -module. Suppose that the generic Jordan type of M is $[1]^{a_1}[2]^{a_2} \cdots [p]^{a_p}$. We call $[1]^{a_1}[2]^{a_2} \cdots [p-1]^{a_{p-1}}$ the stable generic Jordan type of M . In the case M is generically free; i.e. $a_1 = \cdots = a_{p-1} = 0$, we write $\emptyset$ for its stable generic Jordan type.

Lemma 1. *Let G be a finite group and N be the direct sum of r copies of the regular module kG . Let m be a positive integer such that $1 \leq m \leq r|G|$. Suppose that $lp^d = \gcd(m, |G|)$ and $p \nmid l$. Then*

(i)

$$V_G(\Lambda^m N) = \bigcup_{\substack{H \leq G, \\ |H| \mid lp^d}} \text{Res}_{G,H}^* V_H(k) = V_G(S^m N).$$

In particular, the cohomological varieties $V_G(\Lambda^m N), V_G(S^m N)$ have dimensions not more than d .

Suppose further that G is an elementary abelian p -group. Then

- (ii) *the cohomological varieties $V_G(\Lambda^m N), V_G(S^m N)$ have dimensions exactly d , and*
- (iii) *in the case of $m = s|G|$, the stable generic Jordan types of $\Lambda^m N, S^m N$ are $[1]^{\binom{r}{s}}, [1]^{r^s}$ respectively.*

Proof. (i) For each $1 \leq j \leq r$, let $N_j = kG$. The module $N = N_1 \oplus \cdots \oplus N_r$ has a canonical basis $\{g_{1,1}, \dots, g_{|G|,1}, \dots, g_{1,r}, \dots, g_{|G|,r}\}$ where $g_{i,j} \in G \cap N_j$. Then $\Lambda^m N$ has a monomial basis Ω given by

$$\{g_{i_1,j_1} \wedge \cdots \wedge g_{i_m,j_m} \mid j_1 \leq \cdots \leq j_m, i_1 \leq \cdots \leq i_m, g_{i_a,j_a} \neq g_{i_b,j_b} \text{ for } a \neq b\}.$$

For each $x = g_{i_1,j_1} \wedge \cdots \wedge g_{i_m,j_m} \in \Omega$, let $H_x = \{g \in G \mid gx = \pm x\}$. Then $\Lambda^m N \cong \bigoplus_{x \in \Omega/G} \text{Ind}_{H_x}^G k_x$ for some one-dimensional kH_x -module k_x . Since H_x acts fixed-point freely on the set $\{g_{i_1,j_1}, \dots, g_{i_m,j_m}\}$, we have $|H_x| \mid m$; namely, $|H_x| \mid lp^d$. Conversely, given a subgroup $H = \{h_1, \dots, h_t\}$ of G with $t \mid lp^d$, we let $m = tl' + s|G|$ where $1 \leq tl' \leq |G|$, for some non-negative integers s, l' . Since $l' \leq |G|/t$, we may choose $\{f_1, \dots, f_{l'}\} \subseteq H \setminus G$ and let

$$x = g_{1,1} \wedge \cdots \wedge g_{|G|,1} \wedge \cdots \wedge g_{1,s} \wedge \cdots \wedge g_{|G|,s} \wedge h_1 f_1 \wedge \cdots \wedge h_t f_1 \wedge \cdots \wedge h_1 f_{l'} \wedge \cdots \wedge h_t f_{l'}.$$

Clearly, $x \in \Omega$ and $H \subseteq H_x$. We now apply [4, Propositions 8.1.4, 8.2.4] to get the description of the cohomological variety $V_G(\Lambda^m N)$.

By using the permutation basis

$$\Omega' = \{g_{i_1,j_1} \cdots g_{i_m,j_m} \mid j_1 \leq \cdots \leq j_m \text{ and } i_1 \leq \cdots \leq i_m\}$$

for $S^m N$, and arguing similarly, we get the other desired result.

(ii) In this case, G has a subgroup E of order p^d and $\dim V_E(k) = d$.

(iii) Notice that $\text{Ind}_{H_x}^G k_x$ is not generically free if and only if $H_x = G$. In this case, its generic Jordan type is $[1]$ and there are precisely $\binom{r}{s}$ of such $x \in \Omega$ fixed by G up to a sign. Similarly, there are precisely r^s of such $x \in \Omega'$ fixed by G . $\square$

Example 1. Suppose that E is elementary abelian of order p^2 . The regular module kE is a module of constant Jordan type. On the other hand, by Lemma 1 (i), $\Lambda^p kE$ has stable generic Jordan type non- $\emptyset$ on the points of $\text{Res}_{E,E'}^* V_{E'}(k) \setminus \{0\}$ for some subgroup E' of order p of E , and $\emptyset$ otherwise. Thus $\Lambda^p kE$ is not a module of constant Jordan type despite the fact that kE is. Similarly, we obtain that $S^p kE$ is not a module of constant Jordan type.

Example 2. Again, let E be an elementary abelian p -group of order p^2 . As we have seen in Example 1, $\Lambda^p kE$ and $S^p kE$ are generically free; namely their stable generic Jordan types are $\emptyset$. On the other hand, generically kE has Jordan type $[p]^p$. Taking the p th exterior power (respectively, symmetric power) of the generic Jordan type $[p]^p$, by Lemma 1 (iii), we obtain the stable generic Jordan type

$$[1]^{(p)} = [1]^p \text{ (respectively, } [1]^{p^1} = [1]^p).$$

Thus, in general, generic Jordan type does not ‘commute’ with exterior power or symmetric power.

The remainder of this section is devoted to prove Lemma 3 which will be the key idea in the proof of Theorem 5.

Let C be a chain complex of kG -modules $\cdots \rightarrow C_i \xrightarrow{d_i} C_{i-1} \rightarrow \cdots$. For a positive integer m , let X be the total complex of the m -fold tensor product of C over k ; namely, the n th component X_n is the direct sum of $C_{a_1} \otimes \cdots \otimes C_{a_m}$ where $a_1 + \cdots + a_m = n$, and the differential $\partial_n : X_n \rightarrow X_{n-1}$ is given by

$$\partial_n(c_1 \otimes \cdots \otimes c_m) = \sum_{i=1}^m (-1)^{a_1 + \cdots + a_{i-1}} c_1 \otimes \cdots \otimes c_{i-1} \otimes d_{a_i} c_i \otimes c_{i+1} \otimes \cdots \otimes c_m$$

where $c_i \in C_{a_i}$. Clearly $\partial_{n-1} \circ \partial_n = 0$ and hence X is a chain complex of kG -modules. For each n , we have also a right $\mathfrak{S}_m$ -action on X_n given by

$$(c_1 \otimes \cdots \otimes c_m)(i, i+1) = (-1)^{a_i a_{i+1}} c_1 \otimes \cdots \otimes c_{i+1} \otimes c_i \otimes \cdots \otimes c_m$$

for any $1 \leq i \leq m-1$. Direct calculation shows that the $\mathfrak{S}_m$ -action commutes with ∂_n for any $n \in \mathbb{Z}$ and hence X is a chain complex of kG - $k\mathfrak{S}_m$ -bimodules.

Let $0 \rightarrow M_1 \xrightarrow{\iota} M_2 \xrightarrow{\pi} M_3 \rightarrow 0$ be a short exact sequence of kG -modules. Specialize the above discussion to the chain complex C where

$$C_i = \begin{cases} M_2 & i = 0, \\ M_3 & i = -1, \\ 0 & \text{otherwise;} \end{cases}$$

and $d_0 = \pi : M_2 \rightarrow M_3$, we get the following lemma.

Lemma 2. *Let $0 \rightarrow M_1 \xrightarrow{\iota} M_2 \xrightarrow{\pi} M_3 \rightarrow 0$ be a short exact sequence of kG -modules. For every positive integer m , we have an exact sequence X^m*

of kG - $k\mathfrak{S}_m$ -bimodules

$$0 \rightarrow X_1^m \xrightarrow{\partial_1} X_0^m \xrightarrow{\partial_0} X_{-1}^m \xrightarrow{\partial_{-1}} \dots \xrightarrow{\partial_{-m+1}} X_{-m}^m \rightarrow 0,$$

where $X_1^m = T^m(M_1)$ is the m -fold tensor product of the module M_1 , $\partial_1 = \iota^{\otimes m}$ and, for each $0 \leq j \leq m$, X_{-j}^m is the direct sum of the tensor products of exactly $m-j$ and j copies of M_2 and M_3 respectively.

Proof. Let $0 \leq j \leq m$. We want to prove that $\ker \partial_{-j} \subseteq \operatorname{im} \partial_{-j+1}$. It is done by induction on m . For $1 \leq j \leq m$, we have $X_{-j}^m = X_{-j}^{m-1} \otimes_k M_2 \oplus X_{-j+1}^{m-1} \otimes_k M_3$. Let $\{m_1, \dots, m_{r+s}\}$ be a basis of M_2 extending a basis $\{m_1, \dots, m_r\}$ of M_1 , and Ω, Ω' be bases of $X_{-j}^{m-1}, X_{-j+1}^{m-1}$ respectively. Suppose that

$$\begin{aligned} 0 &= \partial_{-j} \left(\sum_{x \in \Omega, 1 \leq i \leq r+s} a_{x,i}(x \otimes m_i) + \sum_{x' \in \Omega', r+1 \leq i \leq r+s} b_{x',i}(x' \otimes \pi(m_i)) \right) \\ &= \sum_{x \in \Omega, 1 \leq i \leq r+s} a_{x,i}(\partial_{-j} x \otimes m_i) + (-1)^j \sum_{x \in \Omega, r+1 \leq i \leq r+s} a_{x,i}(x \otimes \pi(m_i)) + \\ &\quad \sum_{x' \in \Omega', r+1 \leq i \leq r+s} b_{x',i}(\partial_{-j+1} x' \otimes \pi(m_i)). \end{aligned} \quad (1)$$

This shows that, for each $1 \leq i \leq r+s$, $\sum_{x \in \Omega} a_{x,i} \partial_{-j} x = 0$ and hence, by induction, there exists $y_i \in X_{-j+1}^{m-1}$ such that $\partial_{-j+1} y_i = \sum_{x \in \Omega} a_{x,i} x$. Substitute

$$\partial_{-j+1} \left(\sum_{1 \leq i \leq r+s} y_i \otimes m_i \right) = \sum_{x \in \Omega, 1 \leq i \leq r+s} a_{x,i}(x \otimes m_i) + (-1)^{j-1} \sum_{i \geq r+1} y_i \otimes \pi(m_i)$$

into Equation (1), we are reduced to the case of $a_{x,i} = 0$ for all $x \in \Omega$. Similar argument shows that $\partial_{-j+2} y'_i = \sum_{x' \in \Omega'} b_{x',i} x'$ for some $y'_i \in X_{-j+2}^{m-1}$ and hence we obtain that $\operatorname{im} \partial_{-j+1} = \ker \partial_{-j}$. The exactness at X_0^m can be proved similarly. $\square$

We continue our discussion on the exact sequence X^m in Lemma 2. For each $-1 \leq j \leq m$, consider the tensor products $X_{-j}^m \otimes_{k\mathfrak{S}_m} k$ and $X_{-j}^m \otimes_{k\mathfrak{S}_m} \operatorname{sgn}$ where sgn denotes the sign representation of $\mathfrak{S}_m$. Since the components M_2, M_3 are in degrees 0, -1 respectively, we have

$$\begin{aligned} X_{-j}^m \otimes_{k\mathfrak{S}_m} k &= S^{m-j} M_2 \otimes_k \Lambda^j M_3 \quad (j \neq -1), \\ X_{-j}^m \otimes_{k\mathfrak{S}_m} \operatorname{sgn} &= \Lambda^{m-j} M_2 \otimes_k S^j M_3 \quad (j \neq -1), \\ X_1^m \otimes_{k\mathfrak{S}_m} k &= S^m M_1, \\ X_1^m \otimes_{k\mathfrak{S}_m} \operatorname{sgn} &= \Lambda^m M_1. \end{aligned}$$

Also we obtain the maps $\bar{\partial}_{-j} = \partial_{-j} \otimes 1$ and $\bar{\partial}'_{-j} = \partial'_{-j} \otimes 1$ upon taking tensor products. Effectively, under the identification, for any $x_1, \dots, x_{m-j} \in M_2$

and $y_{m-j+1}, \dots, y_m \in M_3$, we have

$$\begin{aligned} & \bar{\partial}_{-j}(x_1 \cdots x_{m-j} \otimes y_{m-j+1} \wedge \cdots \wedge y_m) \\ &= \sum_{1 \leq i \leq m-j} x_1 \cdots \widehat{x_i} \cdots x_{m-j} \otimes \pi(x_i) \wedge y_{m-j+1} \wedge \cdots \wedge y_m, \\ & \bar{\partial}'_{-j}(x_1 \wedge \cdots \wedge x_{m-j} \otimes y_{m-j+1} \cdots y_m) \\ &= \sum_{1 \leq i \leq m-j} (-1)^{m-j-1} (x_1 \wedge \cdots \wedge \widehat{x_i} \wedge \cdots \wedge x_{m-j} \otimes \pi(x_i) y_{m-j+1} \cdots y_m), \end{aligned}$$

here $\widehat{x_i}$ means that x_i is missing from the terms.

Lemma 3. *Let $0 \rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow 0$ be a short exact sequence of kG -modules. For each positive integer m , we have long exact sequences of kG -modules*

$$\begin{aligned} 0 \rightarrow S^m M_1 \xrightarrow{\bar{\partial}_1} S^m M_2 \xrightarrow{\bar{\partial}_0} S^{m-1} M_2 \otimes_k \Lambda^1(M_3) \xrightarrow{\bar{\partial}_{-1}} \cdots \xrightarrow{\bar{\partial}_{-m}} \Lambda^m(M_3) \rightarrow 0, \\ 0 \rightarrow \Lambda^m M_1 \xrightarrow{\bar{\partial}'_1} \Lambda^m M_2 \xrightarrow{\bar{\partial}'_0} \Lambda^{m-1} M_2 \otimes_k S^1(M_3) \xrightarrow{\bar{\partial}'_{-1}} \cdots \xrightarrow{\bar{\partial}'_{-m}} S^m(M_3) \rightarrow 0. \end{aligned}$$

Proof. Let $0 \leq j \leq m$. By standard algebra, we know that $\ker \bar{\partial}_{-j} = (\ker \partial_{-j}) \otimes_{k\mathfrak{S}_m} k$ (see [2, §II.3.6, Proposition 6]). So that

$$\ker \bar{\partial}_{-j} = (\operatorname{im} \partial_{-j+1}) \otimes_{k\mathfrak{S}_m} k = \operatorname{im} \bar{\partial}_{-j+1}.$$

The map $\bar{\partial}_1$ is clearly injective. The exactness of the second sequence can be proved similarly. $\square$

3. MAIN RESULTS

We are now ready to prove our main result. Before doing this, we deal with a simple case which basically states that the generic Jordan type of the exterior power (respectively, symmetric power) of a module depends solely on the generic Jordan types of the various exterior powers (respectively, symmetric powers) of its non-projective part.

In the following lemma, we use the convention that the value of $\binom{r}{j}$ is 0 if $j > r$, and 1 if $r = j = 0$. Also, we use the convention that the value of 0^j is 0 if $j > 0$, and 1 if $j = 0$.

Lemma 4. *Let M be a kE -module where E is an elementary abelian p -group of order p^n and $1 \leq m \leq \dim_k M$ be a positive integer. Suppose that, for each integer j such that $0 \leq jp^n \leq m$, the stable generic Jordan type of $\Lambda^{m-jp^n} \Omega^0 M$ (respectively, $S^{m-jp^n} \Omega^0 M$) is $[1]^{a_{1,j}} \cdots [p-1]^{a_{p-1,j}}$. Then the stable generic Jordan type of $\Lambda^m M$ (respectively, $S^m M$) is $[1]^{m_1} \cdots [p-1]^{m_{p-1}}$ where, for each $1 \leq s \leq p-1$, we have*

$$m_s = \sum_{0 \leq jp^n \leq m} a_{s,j} \binom{r}{j} \quad (\text{respectively, } m_s = \sum_{0 \leq jp^n \leq m} a_{s,j} r^j)$$

and $rp^n = \dim_k M - \dim_k \Omega^0 M$.

In particular, in the case of $1 \leq m \leq p^n - 1$, the stable generic Jordan types of $\Lambda^m M$ and $\Lambda^m \Omega^0 M$ (respectively, $S^m M$ and $S^m \Omega^0 M$) are identical.

Proof. Let $M = \Omega^0 M \oplus P$ where P is the largest free kE -submodule of M so that $rp^n = \dim_k P$. It is well-known that

$$\Lambda^m M \cong \bigoplus_{i=0}^m \Lambda^{m-i} \Omega^0 M \otimes_k \Lambda^i P.$$

By Lemma 1 (ii), $\Lambda^i P$ is not generically free if and only if $\dim(V_E(\Lambda^i P)) = n$ if and only if $i = jp^n$ for some non-negative integer j . In this case, by Lemma 1 (iii), the stable generic Jordan type of $\Lambda^i P$ is $[1]^{(r)}_j$. So, upon taking tensor product, the stable generic Jordan type of $\Lambda^{m-jp^n} \Omega^0 M \otimes_k \Lambda^{jp^n} P$ is

$$[1]^{a_{1,j}}_j \dots [p-1]^{a_{p-1,j}}_j.$$

The case of symmetric power can be obtained similarly.

Suppose that $1 \leq m \leq p^n - 1$. For the case of exterior power, we have $m_s = a_{s,0} \binom{r}{0} = a_{s,0}$. For the case of symmetric power, we have $m_s = a_{s,0} r^0 = a_{s,0}$. $\square$

Let M, N be kE -modules. We say that the stable generic Jordan types of M, N are complementary if the stable generic Jordan types of M, N are

$$[1]^{a_1} [2]^{a_2} \dots [p-2]^{a_{p-2}} [p-1]^{a_{p-1}},$$

$$[1]^{a_{p-1}} [2]^{a_{p-2}} \dots [p-2]^{a_2} [p-1]^{a_1}$$

respectively.

Theorem 5. *Let E be an elementary abelian p -group of order p^n and M be a kE -module. Let m be a positive integer such that $1 \leq m \leq p^n - 1$.*

- (i) *Suppose that m is even. Let i, j be any integers. Then the stable generic Jordan types of $\Lambda^m \Omega^{2i} M$ and $S^m \Omega^{2j+1} M$ are identical. Similarly, the stable generic Jordan types of $S^m \Omega^{2i} M$ and $\Lambda^m \Omega^{2j+1} M$ are identical.*
- (ii) *Suppose that m is odd. Then the stable generic Jordan types of $\Lambda^m \Omega^{2i} M$ (respectively, $S^m \Omega^{2i+1} M$, $\Lambda^m \Omega^{2i+1} M$ and $S^m \Omega^{2i} M$) are identical for all integers i . Furthermore, for any integers i, j , the stable generic Jordan types of $\Lambda^m \Omega^{2i} M, S^m \Omega^{2j+1} M$ (respectively, $\Lambda^m \Omega^{2i+1} M, S^m \Omega^{2j} M$) are complementary.*

In both cases, the stable generic Jordan types of $\Lambda^m M$ and $\Lambda^m \Omega^0 M$ (respectively, $S^m M$ and $S^m \Omega^0 M$) are identical.

Proof. Let P_M be the projective cover of M so that we have a short exact sequence $0 \rightarrow \Omega M \rightarrow P_M \rightarrow M \rightarrow 0$. Consider the long exact sequence obtained in Lemma 3,

$$0 \rightarrow S^m \Omega M \xrightarrow{\phi_{m+1}} S^m P_M \xrightarrow{\phi_m} S^{m-1} P_M \otimes_k \Lambda^1 M \rightarrow \dots \xrightarrow{\phi_1} \Lambda^m M \rightarrow 0.$$

Since P_M is a free kE -module, we obtain that, by Lemma 1 (ii), $S^i P_M$ are generically free and thus the kE -modules $S^i P_M \otimes_k \Lambda^{m-i} M$ are generically free for all $1 \leq i \leq m$. By using the short exact sequence, for $2 \leq i \leq m+1$,

$$0 \rightarrow \ker \phi_i \rightarrow S^i P_M \otimes_k \Lambda^{m-i} M \rightarrow \ker \phi_{i-1} \rightarrow 0$$

and arguing inductively, we obtain that the stable generic Jordan types of $\Lambda^m M, S^m \Omega M$ are identical if m is even, and complementary if m is odd. Similarly, by applying another long exact sequence of Lemma 3, we obtain that the stable generic Jordan types of $S^m M, \Lambda^m \Omega M$ are identical if m is even, and complementary if m is odd. Repeat the argument with the short exact sequence $0 \rightarrow M \rightarrow I_M \rightarrow \Omega^{-1} M \rightarrow 0$ and using the fact that $\Omega \Omega^{-1} \simeq \Omega^0 \simeq \Omega^{-1} \Omega$, we have all the desired results. The final statement is precisely given by Lemma 4. $\square$

Specializing to the case of the trivial module k , we can describe the stable generic Jordan types of the exterior and symmetric powers of the various Heller translations of k .

Corollary 6. *Let k be the trivial kE -module where E is an elementary abelian p -group of order p^n . Let $1 \leq m \leq p^n - 1$ and i be an integer. Then the kE -module $\Lambda^m \Omega^i k$ has stable generic Jordan type*

$$\begin{cases} \emptyset & \text{if } i \text{ is even and } m \geq 2, \\ [1] & \text{if } i \text{ is odd and } m \text{ is even, or } i \text{ is even and } m = 1, \\ [p-1] & \text{if both } m, i \text{ are odd;} \end{cases}$$

meanwhile, the kE -module $S^m \Omega^i k$ has stable generic Jordan type

$$\begin{cases} \emptyset & \text{if } i \text{ is odd and } m \geq 2, \\ [1] & \text{if } i \text{ is even,} \\ [p-1] & \text{if } i \text{ is odd and } m = 1. \end{cases}$$

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