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2014

Ji, C., & Zhao, D. (2014). Lattice Boltzmann investigation of acoustic damping mechanism and performance of an in-duct circular orifice. *The Journal of the Acoustical Society of America*, 135(6), 3243-3251.

<https://hdl.handle.net/10356/104656>

<https://doi.org/10.1121/1.4876376>

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Lattice Boltzmann investigation of acoustic damping mechanism and performance of an in-duct circular orifice

Chenzhen Ji and Dan Zhao^{a)}

*Aerospace Engineering Division, College of Engineering, Nanyang Technological University,
50 Nanyang Avenue, Singapore 639798, Singapore*

(Received 16 August 2013; revised 20 March 2014; accepted 2 May 2014)

In this work, three-dimensional numerical simulations of acoustically excited flow through a millimeter-size circular orifice are conducted to assess its noise damping performance, with particular emphasis on applying the lattice Boltzmann method (LBM) as an alternative computational aeroacoustics tool. The model is intended to solve the discrete lattice Boltzmann equation (LBE) by using the pseudo-particle based technique. The LBE controls the particles associated with collision and propagation over a discrete lattice mesh. Flow variables such as pressure, density, momentum, and internal energy are determined by performing a local integration of the particle distribution at each time step. This is different from the conventional numerical investigation attempting to solve Navier-Stokes (NS) equations by using high order finite-difference or finite-volume methods. Compared with the conventional NS solvers, one of the main advantages of LBM may be a reduced computational cost. Unlike frequency domain simulations, the present investigation is conducted in time domain, and the orifice damping behavior is quantified over a broad frequency range at a time by forcing an oscillating flow with multiple tones. Comparing the numerical results with those obtained from the theoretical models, large eddy simulation, and experimental measurements, good agreement is observed. © 2014 Acoustical Society of America.

[<http://dx.doi.org/10.1121/1.4876376>]

PACS number(s): 43.28.Py, 43.28.Ra, 43.50.Gf, 43.55.Ev [JWP]

Pages: 3243–3251

I. INTRODUCTION

Thermoacoustic instability is generated by the interaction between unsteady heat release and acoustic waves.^{1–7} Under lean premixed conditions, a small change in the flow rate of inlet air or fuel may cause unsteady heat release. Unsteady heat release is an efficient monopole-like sound source and generates acoustic waves. These pressure waves propagate within a combustor. At the boundaries, due to the impedance change, the acoustic waves are reflected back to the combustion zone, causing more unsteady heat release.⁶ This feedback can result in the acoustic oscillation amplitude successively increasing. Eventually, some non-linearity in the combustion system limits the amplitude of the acoustic oscillations.

To mitigate thermoacoustic instabilities, the coupling between unsteady heat release and acoustic waves must somehow be interrupted.^{8–11} Perforated liners along the bounding walls of a combustor are widely used as acoustic dampers to dissipate the acoustic waves. They are usually metal sheets, which have tiny orifices in them. In practice, a cooling air flow through the orifices (known as bias flow) is needed to prevent the liners from being damaged due to the extremely high temperature. Over the last few decades, acoustic liners have been the subject of intense research activity, aiming to better understand and predict their damping performance. The main mechanism of such liners involves vortex shedding generated over the rims of the

orifices. When fluids flow through them, an unsteady “jet” is generated and undergoes subsequent viscous dissipation to convert acoustic fluctuations into non-radiating vortical fluctuations.

The need to improve acoustic liner design with current low-emission engines and “quiet” heating, ventilation, and air conditioning systems leads to a resurgence of liner focused research.¹¹ Both numerical and experimental investigations are conducted. However, experimental investigations focus on measuring the liner impedance or power absorption coefficient,^{12,13} since they are easier to measure in comparison with the flow field near the tiny orifice, typically around 1 mm in diameter. Ingård and Labate¹⁴ experimentally visualized that the incident sound amplitude, frequency, the orifice diameter, and thickness affected the induced motion of the fluid near the orifice. Hughes and Dowling¹⁵ showed that the sound incident on a perforated liner with a bias flow might be completely absorbed, if the flow speed and the liner geometry were chosen properly. Jing and Sun’s experiments¹⁶ confirmed that the orifice thickness and the bias flow Mach number play important roles in affecting the liners damping performance.

In order to shed light on their acoustic damping mechanism, numerical investigations of perforated orifices are widely conducted. The majority modeling work has been carried out in frequency domain. Howe¹⁷ used Rayleigh conductivity to model the acoustic energy dissipated by the periodic shedding of vorticity for a single orifice at a high Reynolds number. Eldredge and Dowling¹⁸ developed a one-dimensional lined duct model in frequency domain 10 yrs ago to simulate the absorption of the axial plane wave by using a

^{a)}Author to whom correspondence should be addressed. Electronic mail: zhaodan@ntu.edu.sg

double-layer liner with a bias flow present. The damping mechanism of vortex shedding was embodied by using a homogeneous compliance adapted from Rayleigh conductivity.

In comparison with frequency-domain modeling,^{18,19} time-domain simulations,^{20–24} become more and more popular. This is most likely due to the availability of high performance computers and the development of more efficient computational methods. Tam *et al.* carried out direct numerical simulation (DNS)^{22,24} of a single aperture, showing that vortex shedding was the dominant damping mechanism for large-amplitude incident waves. Mendez and Eldredge²³ conducted compressible large-eddy simulations (LESs) on single and multiple orifices to study the flow characteristics. Zhang and Bodony²¹ used DNS to investigate the acoustics behavior of a honeycomb liner. They found that the orifice boundary layer played a critical role in affecting the nonlinearity.

The numerical simulations described above attempt to solve the Navier-Stokes (NS) equations by using the finite volume (FV) or finite difference (FD) method. As an alternative computational tool, the lattice Boltzmann method (LBM) could be used to model and simulate complex physics in fluids.²⁵ Unlike the conventional numerical schemes, the LBM is thought to be one of the particle techniques. It determines the time evolution of fluid fields based on a time-space discretization of the Boltzmann equation, known as the lattice Boltzmann equation (LBE).^{26,27} The LBE controls the particles associated with collision and propagation over a discrete lattice mesh. The conserved flow variables such as density, momentum, and internal energy are obtained by performing a local integration of the particle distribution. Compared with conventional NS solvers, the LBM is easier to implement and code, is more successful in dealing with complex boundaries, and has the potential for parallelization.²⁵ The method has been successfully applied to study not only single- and multiple-phase flows but also aeroacoustics, such as jet,²⁸

cavity,²⁹ and airfoil³⁰ noise. Much of the previous works has confirmed that the LBM possesses the required accuracy to capture the weak acoustic pressure fluctuations.^{28–31}

In this work, a three-dimensional (3D) lattice Boltzmann investigation is conducted to simulate the vorticity-involved acoustic damping mechanism of an in-duct circular orifice, and to characterize its damping effect. The simulations are conducted in time domain, as described in Sec. II. The 3D numerical scheme and configuration are illustrated. To characterize the acoustic damping of the in-duct orifice, Rayleigh conductivity and power absorption coefficient are used. This is described in Sec. III. In Sec. IV, time evolution of vortex shedding at the orifice rim is simulated. In addition, its damping effect is estimated. A comparison is then made between the present numerical results, experimental, and theoretical ones. Finally, in Sec. V, the main findings of the present work are discussed and summarized.

II. NUMERICAL METHOD AND CONFIGURATION OF INTEREST

The LBM that originated from lattice gas automata²⁵ is based on mesoscopic models and kinetic equations. Simplified kinetic models are developed. It is assumed that the fluid flow composes of a collection of pseudo-particles. They are represented by a set of density distribution functions. These particles are associated with collision and propagation over a discrete lattice mesh. In order to predict the particles motions, the fluid domain is discretized into a specific group by a series of nodes and lattices, depending on the models. Figure 1 illustrates a three-dimension, nineteen-velocity (D3Q19) lattice model used in the present work. It shows that a particle has 19 feasible discrete propagation directions in each node and the motion in three dimensions.

The discrete velocity vector $\mathbf{e}_i$ of the particles is defined as

$$\mathbf{e}_i = \begin{cases} (0, 0, 0), & i = 0, \\ (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1), & i = 1, 2, 3, 4, 5, 6, \\ (\pm 1, \pm 1, 0), (\pm 1, 0, \pm 1), (0, \pm 1, \pm 1), & i = 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. \end{cases} \quad (1)$$

The propagation and collision of the particles over the lattice are governed by the discretized Boltzmann equation in both space and time (known as LBE) as

$$f_i(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau}(f_i(\mathbf{x}, t) - f_i^{\text{eq}}(\mathbf{x}, t)), \quad (2)$$

where f_i is the distribution function associated with the propagation direction i at the spatial coordinate $\mathbf{x}$ and time t . τ is a relaxation parameter. It defines the particle collision and is related to the kinematic viscosity of the fluid as given as $\nu = (2\tau - 1)/6$. f_i^{eq} is the equilibrium distribution function,

which is the second order truncated expansion of the Maxwell-Boltzmann equilibrium function as given as

$$f_i^{\text{eq}} = \rho w_i \left(1 + \frac{3}{c_l^2} \mathbf{e}_i \cdot \mathbf{u} + \frac{9}{2c_l^4} (\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2c_l^2} \mathbf{u} \cdot \mathbf{u} \right), \quad (3)$$

where w_i is the weighting factor and it is given as

$$w_i = \begin{cases} \frac{1}{3}, & i = 0, \\ \frac{1}{18}, & i = 1, 2, 3, 4, 5, 6, \\ \frac{1}{36}, & i = 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. \end{cases} \quad (4)$$

FIG. 1. D3Q19 lattice model.

The left-hand side of Eq. (2) describes the propagation operator and it determines the diffusion of the distribution functions over the lattices. The right-hand side of Eq. (2), known as the Bhatnagar-Gross-Krook³² collision operator, is a simplification of the collision function with the relaxation time τ in the i th direction involved. The macroscopic flow variables ρ and $\mathbf{u}$ at each lattice point are obtained in terms of the local distribution function f_i by

$$\rho(\mathbf{x}, t) = \sum_{i=0}^{18} f_i(\mathbf{x}, t), \quad \rho \mathbf{u}(\mathbf{x}, t) = \sum_{i=0}^{18} \mathbf{e}_i f_i(\mathbf{x}, t). \quad (5)$$

The state equation is assumed to hold and $p = \rho c_s^2$. Here c_s is the dimensionless sound speed and it is set to $c_s = 1/\sqrt{3}$. By using the Chapman-Enskog expansion, the LBE can be recovered to the compressible NS equations at the hydrodynamic limit.³³

To simulate turbulence when the Reynolds number, Re , is large, it is a general practice to combine the Smagorinsky subgrid model³⁴ with the collision term of the LBE. The implementation involves two steps:

(1) Evaluating the local stress tensor

$$\Pi = \sum_{m,n} \mathbf{e}_{i,m} \mathbf{e}_{i,n} (f_i - f_i^{\text{eq}}), \quad (6)$$

where $(m, n) \in \{x, y\} \times \{x, y\}$.

(2) Computing the enhanced relaxation time

$$\tau_e = 3(\nu + C^2 S) + 0.5, \quad (7)$$

where $S = (\sqrt{\nu^2 + 18C^2 \sqrt{\Pi_{m,n} \Pi_{m,n}}} - \nu)/(6C^2)$ and C is a Smagorinsky constant. And it is set to 0.18 in the present model.

In practice, perforate liners may consist of thousands of orifices. However, if the open area ratio is small enough, it can be assumed that there is no interaction of the jet flows through neighboring orifices. Thus the damping behaviors of a single orifice can be used to characterize the liner damping performance.^{21,23,24} This is one of the common hypotheses made to study acoustic liner damping behaviors.

The geometric configuration of interest is based on the experimental test rig,³⁵ as illustrated in Fig. 2. An acoustically excited flow with a mean value $\bar{u}$ and a fluctuating part u' is forced to pass through a circular orifice with diameter $2a$ in a duct with width d . The orifice thickness is denoted by T . When the fluid is injected through the orifice, a bias flow is formed with a mean velocity $\bar{U} > \bar{u}$, and the Reynolds number can be defined as $Re = 2\rho \bar{U} a / \nu$. The inlet is located upstream at l mm from the orifice, and the outlet is at L mm downstream. Here, L is chosen large enough to avoid spurious interactions between the outlet condition and the zone of computational interest.

A no-slip boundary condition is applied at the wall by applying the bounce-back scheme.³⁶ Due to the assumptions of spatial periodicity of the perforated orifices and no jet flows interaction, the periodic boundary condition is applied in both directions tangential to the perforated plate to reduce the computation domain, as shown in Fig. 2. The inlet and outlet are treated with an absorbing boundary condition (ABC) scheme.³⁷ The concept of the ABC technique is similar to the perfectly matched layer method (PML) commonly used in computational aeroacoustics (CAA). It involves using a “buffering zone” between the fluid region and the outlet boundary to create an asymptotic transition toward a target flow in terms of a target distribution function f_i^T . This is done by adding an extra term to Eq. (2) as given as

$$f_i(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = -\frac{1}{\tau} [f_i(\mathbf{x}, t) - f_i^{\text{eq}}(\mathbf{x}, t)] - \sigma [f_i^{\text{eq}}(\mathbf{x}, t) - f_i^T(\mathbf{x}, t)], \quad (8)$$

where $\sigma = \sigma_T (\eta/\beta)^2$ is an absorption coefficient, $\sigma_T = 0.3$ is a constant, η describes the distance from the beginning of the buffer zone, and β is the thickness of the buffer. f_i^T is

FIG. 2. Geometry of an in-duct circular orifice.

obtained in the same way as f_i^{eq} by using Eq. (3). The non-reflection condition is achieved by setting $\mathbf{u}_T = \mathbf{u}_0$ and $\rho_T = \rho_0$, where $\mathbf{u}_0$ and ρ_0 are undisturbed velocity and density at the boundary. The thickness of the ABC buffer β used in the presented work is set to 20 cells.³⁷ It is worth noting that the parameters involved in the lattice Boltzmann model are generally dimensionless ones. However, the dimensional parameters can be easily obtained from the dimensionless ones by using Eq. (6) described in the previous work.³⁸ In addition, interested readers can refer to Ref. 25 (pp. 117–118).

III. DEFINITION OF SOUND ABSORPTION COEFFICIENT

In order to characterize the orifice damping performance and to simulate the round jet generation, the governing equations described above are now solved by assuming harmonic flow disturbances with radian frequency $\omega = 2\pi f$. The fluctuating pressure $p'(t)$ and volume flow rate $q'(t)$ are then given as

$$\begin{aligned} p'(t) &= \Re\{\hat{p}(\omega)\exp(-j\omega t)\}, \\ q'(t) &= \Re\{\hat{q}(\omega)\exp(-j\omega t)\}, \end{aligned} \quad (9)$$

where $j = \sqrt{-1}$. $\hat{p}(\omega)$ and $\hat{q}(\omega)$ denote Fourier transform of the pressure $p'(t)$ and the volume flow rate fluctuation $q'(t)$, respectively. It has been shown¹⁷ that the volume flow rate is related to the pressure fluctuation via Rayleigh conductivity and it is defined as

$$K_R^o = \frac{j\rho_0\omega\hat{q}(\omega)}{\Delta\hat{p}(\omega)}, \quad (10)$$

where ρ_0 is the mean fluid density and $\Delta\hat{p}(\omega) = \hat{p}_d(\omega) - \hat{p}_u(\omega)$. $\hat{p}_d(\omega)$ and $\hat{p}_u(\omega)$ are the pressure fluctuations in frequency domain at the downstream and upstream of the orifice, respectively.

The Rayleigh conductivity of a circular aperture with a low Mach number bias flow has been studied by Howe.¹⁷ The unsteady flow physics are characterized by solving the Poisson equations with the Kutta conditions imposed to obtain the vorticity. Since the incident sound is linearly dependent on the resulting vorticity, Rayleigh conductivity is uncorrelated to the sound amplitude. He then obtained an analytical expression for a cylindrical vortex sheet of radius a and a constant convection velocity $\bar{U}$, as given as

$$K_R^o = 2a[\gamma(\text{St}) - j\delta(\text{St})], \quad (11)$$

where γ and δ depend on Strouhal number $\text{St} = \omega a / \bar{U}$, and they are given as

$$\gamma - j\delta = 1 + \frac{\frac{\pi}{2}I_1(\text{St})e^{-\text{St}} - jK_1(\text{St})\sinh(\text{St})}{\text{St}\left[\frac{\pi}{2}I_1(\text{St})e^{-\text{St}} + jK_1(\text{St})\cosh(\text{St})\right]}, \quad (12)$$

where I_1 and K_1 are modified Bessel functions. To account for the orifice thickness effect, an extension of the above

model is achieved by adding an extra term representing the effect of the orifice thickness T . The revised Rayleigh conductivity²³ is then given as

$$K_R = 2a\left(\frac{1}{\gamma - j\delta} + \frac{2T}{\pi a}\right)^{-1}. \quad (13)$$

If Rayleigh conductivity is estimated, then the acoustic damping effect of the orifice can be characterized by determining power absorption coefficient Δ . It describes the fraction of incident waves being absorbed. It is defined as

$$\Delta(\omega) = 1 - |R(\omega)|^2, \quad (14)$$

where $R(\omega)$ is the reflection coefficient given as

$$R(\omega) = \frac{z_t(\omega) + 1}{z_t(\omega) - 1} = |\hat{R}(\omega)|e^{j\Phi}, \quad (15)$$

where $|\hat{R}(\omega)|$ and Φ are the amplitude and phase of the reflection coefficient. $z_t(\omega)$ is the normalized acoustic impedance of the system. It is given as the sum of the perforated orifice impedance z_p and the cavity impedance z_c as

$$z_t = z_p + z_c. \quad (16)$$

If plane waves are propagating between the perforated orifice and the back wall, then the cavity impedance can be shown as: $z_c = -j\cot(k\mathcal{L})$, where $k = \omega/c$ is the acoustic wave number and $\mathcal{L}$ is the cavity depth. The impedance of the perforated orifice is related to Rayleigh conductivity as: $z_p(\omega) = [\hat{p}_d(\omega) - \hat{p}_u(\omega)]/\hat{u}_0(\omega) = j\omega d^2/(cK_R)$, where c is the sound speed. Thus the total impedance of the system is then obtained:

$$z_t(\omega) = z_p(\omega) + z_c(\omega) = \frac{j\omega d^2}{cK_R} - j\cot(k\mathcal{L}). \quad (17)$$

By substituting Eqs. (15) and (17) into Eq. (14), the power absorption coefficient Δ can then be calculated.

IV. RESULTS AND DISCUSSION

A. Experimental validation of the numerical model

Before applying the numerical model to shed light on the vorticity-involved damping mechanism, it needs to be validated. For this, we choose two experimental measurements as bench tests. One is the measured round jet by O'Neill *et al.*³⁹ An axisymmetric jet through a circular orifice with diameter d_0 is generated by the discharge. The discharge is generated by a piston driven by a computer-controlled stepper motor. The piston is located in a cylinder, 700 mm long with a 50 mm diameter bore. Figures 3(a) and 3(b) show the measured and predicted velocity profile in the downstream region, as the Reynolds number is set to $\text{Re} = 680$. It can be seen that good agreement is obtained between the measured velocity profiles of the jet flow and the predicted ones by using the present model.

The other benchmark test we choose to further validate our model is the experiment conducted by Jing and Sun.⁴⁰ A

FIG. 3. Comparison between the measured and predicted velocity profile in the downstream region at $Re = 680$ (a) $x = 3d_0$, (b) $x = 12d_0$. u_x denotes the mean velocity component along the jet direction (x axis), u_m is the maximum mean velocity, and y is the axis perpendicular to the jet direction.

steel plate with a single orifice of radius $R = 4$ mm at its center is placed at the pipe downstream end. An anechoic chamber is connected to the impedance tube just behind the orifice plate to reduce the sound being reflected back to the circular orifice. In order to measure its acoustic impedance, the classical two-microphone technique^{20,41} is used. The measured acoustic impedance $z(\omega)$ is given as $z(\omega) = z_r(\omega) + jz_x(\omega)$. Here $z_r(\omega)$ and $z_x(\omega)$ describe the acoustic resistance and reactance, respectively. Figure 4 shows the numerically predicted $z_r(\omega)$ and $z_x(\omega)$ and the experimentally measured ones. It can be seen that z_r and z_x depend on both the amplitude and frequency of the incident sound waves. Furthermore, the numerical results agree well with the experimental measurement. This indicates that the present model can be used to simulate the acoustics damping behaviors of the perforated orifice. Note that in our simulations, constant amplitude of acoustic excitation is used over a broad frequency range. Thus, the normalized sound pressure can be assumed to be a function of the forcing frequency. In addition, to compare with the experimental results, the specific acoustic impedance of the in-duct orifice is normalized by $\rho\omega R$, and the sound pressure amplitude is normalized by $\rho\omega^2 R^2$.

B. Vorticity-involved sound absorption mechanism

Now the model is used to gain insights on the acoustic damping physics of the orifice, as the sound waves propagate

FIG. 4. Comparison of the normalized specific acoustic impedance between numerical and experimental results: (a) Resistance and (b) reactance.

through it. The numerical parameters are set as: $T = 2$ mm, $2a = 6$ mm, and $d = 75$ mm. Both l and L are set to 200 mm, since vortex shedding from either side of the orifice needs to be considered. Cartesian mesh is used and the cell space is set to $\Delta x = \Delta y = \Delta z = 0.1$ mm. This is to ensure that the cell size meets the criterion⁴² that a minimum of 12 cells per wavelength is needed so that the phase speed error is less than 1%. The simulation is conducted by adding a periodic flow fluctuation at upstream with no mean flow ($\bar{u} = 0$) as given as

$$p'(t) = |\hat{p}| \sin(2\pi ft), \quad (18)$$

where $|\hat{p}|$ and f are the amplitude and frequency of the acoustic wave, respectively. $|\hat{p}|$ is set to 0.1% of the atmospheric pressure and f is set to 397.89 Hz. Note that the in-duct orifice damping mechanism is studied by considering the incident acoustic fluctuation at one frequency only. This is different from characterizing the damping effect of the orifice as described in Sec. IV C, where the incident acoustic waves consist of multiple tones.

Figure 5 illustrates time evolution of 3D (left column) and two-dimensional (2D) (right column) vortex shedding generated from the orifice during one period $t_p = 1/f$. It can be seen that there are five stages [(a)–(e) or (f)–(j)]:

- (1) At $t = t_0$, a round jet flow is formed due to the pressure gradient. And two axisymmetric vorticities are generated at the orifice lips;
- (2) at $t = t_0 + t_p/4$, the vorticities in side B reach their maximum strength and are ready for shedding;
- (3) at $t = t_0 + 2t_p/4$, the vorticities are completely shed in side B. The direction of the local flow through the orifice reverses and new axisymmetric vorticities are generated in side A;
- (4) at $t = t_0 + 3t_p/4$, vortex in side B propagates toward the downstream and the new vorticities in side A reach their maximum strength;
- (5) at $t = t_0 + 4t_p/4$, the vorticities in side A are completely shed as a new vortex and flow reverse again. A periodic vortex shedding is accomplished.

It is also found that the vortices are fully dissipated after three periods due to the viscosity of the fluid. And the sound field is periodically evolved, especially near the vicinity of the orifice rim due to the vortex shedding present. However, in the far field (far downstream), the sound propagates like plane waves. This indicates that vorticity is quickly

FIG. 5. (Color online) Time evolution of vortex shedding generated from a circular orifice, as the incident sound wave is set to $f = 397.89$ Hz. (a)–(e) Iso-surfaces of vorticity magnitude at constant value of 14.0. (f)–(j) 2D contours of vorticity magnitude.

dissipated, before it travels far downstream of the orifice. This is consistent with the observation of the vorticity evolution reported by Leung *et al.*,⁴³ who used the high-order FD scheme to conduct 2D simulations on an in-duct orifice.

In order to understand the relation between the incident sound wave and vortex shedding, the spectra of the vorticity and pressure fluctuation are calculated, as shown in Fig. 6. It can be seen that the vortex shedding frequency is coincident with the incident sound frequency. This indicates that the vortex shedding is caused by the incident sound, and together with subsequent viscous dissipation provides an effective sound damping mechanism. Furthermore, harmonic peaks are observed. Their presence indicates the nonlinearity. However, it has a negligible influence on the Rayleigh conductivity when flow reversal does not occur,⁴⁴ as discussed in the present work. The vortex shedding induced by the interaction between the orifice rim and acoustic waves gives rise to the conversion from acoustical to kinetic energy, and it is one of the main mechanisms of sound absorption of the orifice, as revealed from the present simulations. The vorticity generation mechanism due to the flow

separation is consistent with the DNS simulation⁴³ and the experimental measurements,¹⁴ although flow visualization and measurement is very challenging to be conducted on such a tiny orifice.

FIG. 6. Spectra of vorticity (a) and pressure fluctuation (b), as the incident acoustic excitation consists of only one tone at $f = 397.89$ Hz.

FIG. 7. Frequency spectrum of the pressure fluctuations calculated downstream of the circular orifice by using the LBM.

It is worth noting that the damping mechanism of the in-duct orifice is investigated by using the 3D LBM in the present work. This approach is different from the conventional NS equation solvers by using high order FV- or FD-based numerical methods.⁴³ However, a similar damping mechanism is revealed.

C. Characterization of in-duct orifice damping effect

Numerical simulations are now conducted to estimate the power absorption coefficient and Rayleigh conductivity, which characterize the damping effect of the perforated orifice. This is achieved by considering the numerical configuration as described in Fig. 2. And the geometric parameters are set as: $T = 1.5$ mm, $2a = 6$ mm, $d = 35$ mm, $l = 95$ mm, and $L = 285$ mm.

Now the acoustically excited flow consists of multiple tones as given as

$$u_{in}(t) = \bar{u} + u' = 0.115 + 0.005 \sum_{k=1}^{28} \sin(2\pi f_k t), \quad (19)$$

where f_k is the forcing frequency. It can be seen that the acoustically excited flow covers a broad frequency range (31.83 to 891.27 Hz) with an incremental step of 31.83 Hz, and so the estimation of the damping performance. This is different from frequency-domain simulations, of which the orifice damping effect is characterized one frequency at a

FIG. 8. Variation of calculated Rayleigh conductivity with Strouhal number St .

time. The frequency range of interest corresponds to Strouhal numbers, St , varied from 0.12 to 3.36.

The spectrum of the pressure fluctuations downstream of the orifice is shown in Fig. 7. It can be seen that there are multiple tones present due to the sinusoidal forcing. And sound pressure level of the multiple tones are different, although the incident sound levels are set to be the same, as described in Eq. (19). It indicates that the orifice damping effect depends on the incident sound frequency.

Rayleigh conductivity is then estimated by using these recorded pressure fluctuations, as shown in Fig. 8. It can be seen that Rayleigh conductivity is varied with increased Strouhal number. Comparing our results with the modified Howe's model (MHM),²³ as described in Eq. (13) reveals that good agreement is obtained.

Figure 9(a) shows the variation of the estimated power absorption coefficient Δ with the forcing frequency f . It can be seen that the sound absorption is maximized at approximately $f \approx 350$ Hz. Near this frequency, almost all the incident acoustic waves are absorbed, i.e., $\Delta = 1.0$. A comparison is then made between the results from the present model and those from Mendez and Eldredge,²³ Bellucci *et al.*,³⁵ Jing and Sun,⁴⁵ Howe's model (HM), and MHM. It can be seen that good agreement is observed at $f \leq 350$ Hz. However, a larger damping effect is predicted by Bellucci *et al.*³⁵ and HM neglecting the plate thickness. In addition, the present results agree well with those from MHM, which considers the orifice thickness effect. This indicates that the plate thickness plays an important role in affecting the orifice's damping performance. Figure 9(b) illustrates the phase

FIG. 9. Comparison between the numerical results [LBM and LES by Mendez and Eldredge (Ref. 23)] and those from the experimental measurements by Jing and Sun (Ref. 45) and Bellucci *et al.* (Ref. 35) and the theoretical ones from HM (Ref. 17) and MHM (Ref. 23): (a) The sound absorption coefficient Δ ; (b) the phase of the reflection coefficient $\Phi(R)$.

of the reflection coefficient as defined in Eq. (15). A quantitatively good agreement is also observed.

V. DISCUSSION AND CONCLUSIONS

In this work, a 3D numerical investigation is performed by using the LBM to assess the acoustic damping effect of an in-duct circular orifice. The LBM is intended to solve the discrete LBE, which controls the particles associated with collision and propagation over a discrete lattice mesh. Time evolution of acoustic variables such as pressure, velocity, and density is determined by performing a local integration of the particle distribution at each time step. This is different from the conventional numerical schemes attempting to solve NS equations by using high order FD or FV methods. Compared with the conventional numerical NS solvers, the LBM as an alternative computational tool,²⁵ is faster (for a given dispersion error) and less dissipative³¹ than the conventional NS schemes. For a tolerated dispersion error of 0.1%, the LBM is found³¹ to be about 37% less expensive in computation than the classical second order NS schemes. However, it is approximately 9% less expensive in comparison with optimized third order NS schemes. A similar finding on reduced computational cost in terms of the central processing unit time is reported by Geller *et al.*⁴⁶ Thus the LBM has great potential to be applied to study CAA problems.

In order to illustrate the orifice sound absorption mechanism, sinusoidal pressure fluctuation with a single tone is applied first. It is shown clearly that an unsteady round jet is generated and undergoes subsequent viscous dissipation to convert acoustic fluctuations into non-radiating vortical ones. Periodic vortex shedding is also found to occur at the edges of the orifice. However, its shedding frequency is coincident with the frequency of the incident sound wave. Furthermore, harmonics are found to be present in the frequency spectra of the vorticity and pressure, indicating the nonlinearity. The acoustic damping effect of the in-duct circular orifice in the presence of a mean (bias) flow is then characterized by applying sinusoidal pressure fluctuations with multiple tones via calculating the power absorption and reflection coefficients. It is shown that the damping effect depends on the incident sound frequency, the orifice thickness, and the bias flow Mach number. Comparing our numerical results with those obtained from the theoretical models and the experimental measurements, a quantitatively good agreement is observed. This confirms that the present particle-based model can be used to study the acoustic damping behavior of the perforated orifices in time domain.

ACKNOWLEDGMENTS

The work is supported by Singapore Ministry of Education AcR-Tier1-RG91/13-M4011228.050. This financial support is gratefully acknowledged.

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