A Risk-Averse Stochastic Dynamic Programming Approach to Energy Hub Optimal Dispatch

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Abstract—This paper studies the optimal operation problem of an energy hub with multiple energy sources to serve stochastic electricity and heat loads in the presence of uncertain prices as well as operational constraints such as minimum uptime and downtime requirements. Price and demand uncertainties are modeled by stochastic processes. The goal is to minimize some risk functional of the energy hub operational cost. A stochastic dynamic optimization formulation is introduced for the problem. An approximate dynamic programming framework, based on cost function approximation, is proposed to obtain dynamic dispatch policies. The approach enables a risk-sensitive energy cost function approximation, is proposed to obtain dynamic dispatch policies. The performance of the approach for the energy hub dispatch problem and characteristics of the storage levels are numerically investigated.

Index Terms—energy hub, stochastic optimization, approximate dynamic programming, risk

NOMENCLATURE

I. INTRODUCTION

Growing interest in renewable power sources and emerging new types of stochastic loads such as plug-in electric vehicles pose difficulties on power system operation and control. In order to cope with these challenges, the optimal utilization of the existing infrastructure is an effective strategy from both economic and environmental perspectives. Typical energy infrastructures (e.g., electricity, natural gas, and district heating) have been examined independently, but the new trend is toward an integrated view of energy systems, to use the existing infrastructure more efficiently [1]–[3].

To facilitate the study of multiple energy-carrier flows and their interactions, the concept of a node/bus in individual systems was extended to the generalized notion of energy hub by [4]–[5]. An energy hub is an interface between multiple energy loads and multiple primary energy sources. Variety of converters and energy storage devices can be considered in an energy hub [6]. Since converters in the hub have different characteristics and each energy source or carrier has its own cost, the hub owner should strategically decide on a dispatch control policy to determine the optimal contribution of each energy source and carrier to meet the required energy demands.

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The energy hub optimal dispatch is a multi-stage optimization problem (with \( T \) stages) which specifies, at each point in time, optimal purchases of the considered energy sources/carriers, their dispatch among converters, and optimal operation of the storage devices in the hub. Under the assumption that accurate forecasts for the loads and prices are available to the energy hub controller, this problem is formulated as a deterministic optimization problem, e.g., mixed-integer linear or nonlinear programming (MILP/MINLP). Hence, in such deterministic optimization-based approaches, uncertainty and stochastic nature of these inputs are not addressed.

When loads and prices are subject to uncertainty over the planning time horizon, the energy hub optimal dispatch problem lends itself to a stochastic dynamic optimization formulation. In contrast to deterministic optimization, dynamic optimization furnishes decisions for any possible state for the entire time horizon; i.e., decisions at each stage depend on the information observed up to that time step. Therefore, decisions are capable to incorporate and adapt to the most recent information observed about prices and loads. For a review on dynamic optimization and its difference with alternative multi-stage optimization methods, see [7]–[8]. Built upon the previous work by the authors [9]–[11], a novel approximate dynamic programming approach for an energy hub operation management is developed which is amenable to handling risk-averse objectives and operational constraints. Multiple energy resources are considered to serve stochastic electricity and heat loads in the presence of uncertain prices.

Optimal power dispatch problem within an energy hub or an integrated energy system of multiple energy hubs have been the topic of a number of previous studies (e.g., see [6]–[15] and the references therein). However, the literature on the problem frequently sets up the minimization problem of the energy hub operational cost within a deterministic framework for loads and prices and aim to minimize the cost [14]–[21]. The optimal operation of residential energy hubs based on certain price and loads to minimize the total cost is studied in [18], where the problem is formulated as a mixed-integer linear optimization. Similar optimization method is adopted in [20] for the optimal design and operation of energy hubs to minimize the cost of serving loads while satisfying the required level of reliability for different load types using linearized models for reliability constraints.

Linear network optimization is used in [21] to obtain a network of transmission lines and gas pipelines to minimize the cost of interconnecting energy hubs from a given set of feasible paths. In [22], the problem of determining the optimal energy hub configuration (number and size of the required components of the hub) in a region with natural gas and electricity is studied. The goal here is to maximize the net present value during a planning time horizon, given deterministic forecasts for prices and energy sale capacity. An energy hub with multiple energy carriers is considered in [23] to determine a least-cost capacity expansion plan to serve the hub electricity and heat loads over a time horizon. This problem is modeled and solved by deterministic optimization where expected values are used for electricity prices and loads for all time periods. In [24], a model predictive control approach is employed to determine the cost optimal control strategy in an energy hub, assuming perfect forecasts for energy prices, load profiles, and renewable inputs. Uncertainty in pool prices and the electricity demand is taken into account in [25]. Here, a scenario-based two-stage stochastic optimization is applied to determine static retail prices (first-stage decisions) for an energy hub manager. Parameter uncertainty in operation management of an energy hub with two storage devices and five converters is addressed in [16], where a robust optimization approach is adopted to minimizing the energy expenses for satisfying the energy hub output time-varying demands.

To handle uncertainty in optimization problems, the robust optimization approach models parameter uncertainties deterministically through given uncertainty sets and aims to find a solution with the best worst performance. However, a robust optimization solution is not adapted to the information, such as actual prices and loads, observed over time. In contrast, the present paper focuses on the energy hub optimal dispatch problem when the prices and loads are subject to stochastic uncertainty, and develops a dynamic programming solution capable to adapt to the information set (such as prices and loads) observed over time. Given that the prices and loads are driven by random events, the energy hub optimal dispatch problem is stochastic in nature. Decisions about power flows at every time period are made at specific points in time and affect the future system operations. Hence the problem falls into the class of stochastic dynamic optimization problems.

The energy hub system considered in this paper is supplied by natural gas and electricity as the energy source and carrier, with uncertain prices to serve stochastic electricity and heat demands. The system is equipped with battery storage, heat storage, hydrogen storage, and their corresponding converters. Each storage device has its own specifications such as capacity limitations, conversion losses, maximum rates of conversion, and energy loss rates over time. The goal is to minimize some risk functional of the total operational cost over a finite time horizon. The uncertain prices and loads are modeled by stochastic processes. We seek to develop a stochastic dynamic programming formulation. Modeling some of the operational constraints over time, such as MUT and MDT requirements, within the proposed optimization framework is a challenging task. One of the novelities in this paper is to overcome this modeling complexity.

To calculate a dynamic energy hub dispatch solution, an approximate dynamic programming approach is developed. The method relies on direct policy search and cost function approximation in [8]–[10]. This novel policy function approximation within the direct policy search framework enables the handling of various operational and physical constraints as well as a risk-averse objective function. The importance of considering a risk-averse dispatch policy for systems exposed to highly volatile electricity prices has been previously shown by the authors in [11]. Numerical results are presented to demonstrate the performance of the proposed approach.
The model and developed solution approach can be extended to energy hub models where energy sources are also subject to uncertainty, such as intermittent renewables. The methodology can be employed to better assess the economic value of energy hubs with multiple energy storage technologies, thereby helping industry to move towards efficient use of energy storage systems.

The remainder of the paper is structured as follows. Section II presents the energy hub model and its operational constraints. Section III discusses the stochastic models for prices and loads. Section IV describes the stochastic dynamic programming formulation. Section V explains the algorithmic strategy. Section VI presents the computational results. Section VII concludes and outlines future research directions.

II. THE ENERGY HUB MODEL

The configuration of the considered energy hub is illustrated in Fig. 1. The energy hub operates by electricity and natural gas, to serve demands for electricity and heat. Four converters are present: Combined Heat and Power (CHP), Hydrogen Production Plant (HPP), Fuel Cell (FC), and Boiler (B). This system includes three storage devices: hydrogen storage (H), heat storage (h), and battery storage (b).

The primary goal of the energy hub operations optimization is to determine the optimal contributions of each converter and storage devices inside the hub, with respect to the operational constraints and energy loads, with the objective of minimizing total costs. Although the energy hub in Fig. 1 and the approach in this paper can handle deterministic electricity and gas prices through long-term contracts or derivatives with the corresponding energy markets, we consider a more comprehensive scenario where the energy hub is exposed to real-time stochastic prices. We model the problem through a discrete time system over a finite time horizon $T$.

In the system depicted in Fig. 1, the battery storage unit is used only for price arbitrage. In contrast, the role of the heat storage unit in this system is to enable the electricity production through CHP and the consumption of natural gas to serve the demand and to avoid high power prices.

A power flow at any time step $t$ is expressed by a column vector $P_t$ with 13 elements:

$$P_t \triangleq \left( P_t^{e-bu}, P_t^{e-sell}, P_t^g, P_t^{HPP}, P_t^{CHP}, P_t^{FC}, P_t^B, P_t^{ch-H}, P_t^{dis-H}, P_t^{ch-b}, P_t^{dis-b}, P_t^{ch-h}, P_t^{dis-h} \right).$$  (1)

The following constraints characterize admissible power flow vectors $P_t$.

A. Constraints defining feasible flows

The power flows have to be nonnegative:

$$P_t \geq 0.$$  (2)

A.1. Demand Constraints. Power and heat balance constraints ensure that the loads are satisfied:

$$P_t^{e-bu} - P_t^{HPP} - P_t^{ch-b} + P_t^{dis-b} + P_t^{CHP} + P_t^{FC} - P_t^{e-sell} = L_t^e,$$  (3)

$$\eta_{CHP} P_t^{CHP} + P_t^B = P_t^g.$$

All efficiency parameters take values between 0 and 1. The constraint (3) is satisfied for any value of $L_t^e$, since it is always possible to directly buy power from the grid to serve the load. However, the constraint (4) can cause infeasibility for large values of $L_t^h$ due to the constraints on the converters HPP, CHP, Boiler and the heat storage, as there is no direct access to a heat market to buy and serve the excess heat load.

A.2. Power Balance Constraints. The following constraints must hold for the natural gas and hydrogen:

$$\frac{1}{\eta_{CHP}} P_t^{CHP} + \frac{1}{\eta_B} P_t^B = P_t^g,$$

$$\eta_{HPP} P_t^{HPP} - P_t^{ch-H} + P_t^{dis-H} = \frac{1}{\eta_{FC}} P_t^{FC}.$$  (6)

A.3. Storage Capacity Constraints. Storage levels should remain within their admissible range:

$$R_t^{s \min} \leq R_t^s \leq R_t^{s \max}, \quad \forall s \in \{H, h, b\}.$$  (7)

A.4. Power Capacity Constraints. Due to operational considerations, the converters often cannot operate below or above certain power levels. The auxiliary binary decision variables $\beta^J_t$ are used to constrain the input or output power of each component in the energy hub:

$$\beta^J_{t \min} P^J_t \leq P^J_t \leq \beta^J_{t \max} P^J_t, \quad \forall J \in \{HPP, FC, CHP, B\}.$$  (8)

This constraint implies that a nonzero $P_t^J$ should be in $[\beta^J_{t \min}, \beta^J_{t \max}]$, and $P_t^J = 0$ if and only if $\beta^J_t = 0$, for a positive $\beta^J_{t \min}$. Similarly, the power charged to or discharged from the storage device $s$ is controlled by

$$P_t^{ch-s} b_t^{s \min} \leq P_t^{ch-s} \leq P_t^{ch-s} b_t^{s \max}, \quad \forall s \in \{H, h, b\},$$

$$P_t^{dis-s} b_t^{s \min} \leq P_t^{dis-s} \leq P_t^{dis-s} b_t^{s \max}, \quad \forall s \in \{H, h, b\}.$$  (9)

A.5. Startup Costs. The following constraint ensures that the variable $\delta^J_{t-S}$ is 1 when the status of plant $J$ changes from offline ($b_{t-1}^J = 0$) to online ($b_t^J = 1$) between $t-1$ and $t$,

$$b_t^J - b_{t-1}^J \leq \delta^J_{t-S}, \quad \forall J \in \{HPP, FC, CHP, B\}.$$  (10)

Fig. 1. Configuration of the energy hub.
A.6. Shutdown Costs. When the status of the plant $J$ changes from online ($b_{t-1}^J = 1$) to offline ($b_{t-1}^J = 0$), the binary variable $\delta_t^{J-E} \in \{0, 1\}$ must be 1. This is captured by the constraint,

$$b_{t-1}^J - b_{t}^J \leq \delta_t^{J-E}, \quad \forall J \in \{\text{HPP, FC, CHP, B}\}. \quad (12)$$

A.7. HPP Startup and Shutdown Constraints. Starting up and shutting down the hydrogen production plant require some time to avoid damage due to shifts of temperature and pressure [26]. Therefore, there exist minimum uptime and downtime periods. Let $\text{MUT} \geq 1$ and $\text{MDT} \geq 1$ denote the recommended minimum uptime and downtime, respectively.

We introduce two integer-valued state variables:

$$U_t := \text{number of time steps from the last startup until (the end of) time step } t, \text{ if HPP has been on. If at time } t, \text{ HPP has been off, we have } U_t = 0.$$

$$D_t := \text{number of time steps from the last shutdown until time step } t, \text{ if HPP has been off. If at time } t \text{ HPP has been on, we have } D_t = 0.$$

At any time $t$, either ($U_t = 0$ and $D_t > 0$), or ($U_t > 0$ and $D_t = 0$). Given these state variables, HPP startup and shutdown constraints on $P_{\text{HPP}}$ are:

- [I]: If $U_{t-1} = 0$ (i.e., HPP is off at time step $t-1$) and $D_{t-1} > \text{MDT}$, there is no constraint on $P_{\text{HPP}}$.
- [II]: If $D_{t-1} = 0$ and $U_{t-1} \geq \text{MDT}$, then no specific constraint on $P_{\text{HPP}}$ is imposed.
- [III]: If $U_{t-1} = 0$ and $D_{t-1} < \text{MDT}$, we must have $P_{\text{HPP}} = 0$.
- [IV]: If $D_{t-1} = 0$ and $U_{t-1} < \text{MDT}$, then $P_{\text{HPP}} > 0$.

The following linear constraints ensure the requirements [I]-[IV] on $P_{\text{HPP}}$ hold:

$$\text{(MDT} - D_{t-1})_t P_{\text{HPP}} \leq P_{\text{HPP}} \max U_{t-1} - \text{MDT}, \quad (13)$$

$$P_{\text{HPP}} \min (\text{MDT} - U_{t-1} - D_{t-1} - \text{MUT}) \leq P_{\text{HPP}}. \quad (14)$$

A.8. Mutually Exclusively Activity Constraints. For every time step $t$ and for a storage device, simultaneous charging and discharging is prohibited. This is expressed by the auxiliary binary decision variables:

$$b_t^{h-s} + b_t^{d-s} \leq 1, \quad \forall s \in \{H, h, b\}. \quad (15)$$

In addition, realistic system limitations exclude concurrent operations of HPP and fuel cell plants. Thus, the following constraint on $b_{\text{HPP}}$ and $b_{\text{FC}}$ is imposed:

$$b_{\text{HPP}} + b_{\text{FC}} \leq 1. \quad (16)$$

Simultaneous startup and shutdown of a plant is prohibited by

$$\delta_t^S + \delta_t^E \leq 1, \quad \forall J \in \{\text{HPP, FC, CHP, B}\}. \quad (17)$$

B. Decision Variables

The control variables at time step $t$ include the power flow vector $P_t \in \mathbb{R}_+^{13}$ in equation (1) and the auxiliary decision vectors $b_t \in \{0, 1\}^{10}$ and $\delta_t \in \{0, 1\}^8$,

$$b_t \overset{\text{def}}{=} \left( b_{t}^{h-H}, b_{t}^{h-h}, b_{t}^{d-H}, b_{t}^{d-h}, b_{t}^{\text{HPP}}, b_{t}^{\text{FC}}, b_{t}^{\text{CHP}}, b_{t}^{B} \right),$$

$$\delta_t \overset{\text{def}}{=} \left( \delta_t^{\text{HPP-S}}, \delta_t^{\text{FC-S}}, \delta_t^{\text{CHP-S}}, \delta_t^{\text{HPP-E}}, \delta_t^{\text{FC-E}}, \delta_t^{\text{CHP-E}}, \delta_t^{B} \right).$$

All the decision variables are collected into a single vector $x_t \overset{\text{def}}{=} (P_t, b_t, \delta_t)$. The set of all $x_t$ satisfying constraints (2)-(17) is denoted by $X_t$.

C. Stage Cost

Let the column vector $c_{\theta} \in \mathbb{R}^4_+$ collect the startup and shutdown costs of the plants,

$$c_{\theta} \overset{\text{def}}{=} \left( c_{\text{HPP-S}}, c_{\text{FC-S}}, c_{\text{CHP-S}}, c_{\text{HPP-E}}, c_{\text{FC-E}}, c_{\text{CHP-E}}, c_{B} \right)^T.$$

Let $\Gamma_t^b \geq 0$ and $\Gamma_t^s \geq 0$ denote the added transaction costs from buying and selling at time $t$. At time step $t$, taking the action $x_t$ incurs a cost

$$C_t \overset{\text{def}}{=} (c_{\theta} + \Gamma_t^b) P_{t}^{\text{buy}} + c_{\theta} P_{t}^{\text{sell}} + \Gamma_t^s \delta_t - (c_{\theta} - \Gamma_t^s) P_{t}^{\text{sell}}. \quad (18)$$

III. Stochastic Models for Prices and Loads

Electricity Price. The electricity price $c_t^e$ follows a stochastic process with accounting for intraday, weekly, and annual seasonality, given by

$$c_t^e = \exp (c_{\text{hour}}^e + c_{\text{day}}^e + c_{\text{month}}^e + c_{\text{year}}) \quad (19)$$

The seasonality terms $c_{\text{hour}}, c_{\text{day}}, c_{\text{month}}, c_{\text{year}}$ are deterministic and depend on $t$. The deseasonalized log-price $\tilde{y}_t^e$ follows a mean-reverting jump-diffusion process

$$\tilde{y}_t^e = \mu c + (\tilde{y}_{t-1}^e - \mu c) e^{-\lambda \Delta t} + \sigma c \epsilon_t^e + \sum_{i=1}^{\tilde{q}_t} \tilde{j}_{t,i}^e. \quad (20)$$

The mean-reversion rate $\lambda c$, the mean $\mu c$, the volatility $\sigma c$, and the initial value $\tilde{y}_0^e$ are given. Here, $c_{\epsilon_t}^e$ is a standard normal random variable, $\tilde{j}_t$ is the normally distributed jump size with the mean $\mu J$ and the standard deviation $\sigma J$, $q_t$ follows a Poisson process with intensity $\lambda J$, and $\Delta$ denotes the length of each time period.

Electricity and Heat Loads. The electricity and heat demands at time $t$ are expressed by the stochastic processes

$$L_t^e = L_{\text{hour}}^e + L_{\text{day}}^e + L_{\text{month}}^e + \tilde{y}_t^e, \quad (21)$$

for electricity ($\ell = e$) and heat ($\ell = h$). Here, $L_{\text{hour}}^e, L_{\text{day}}^e$ and $L_{\text{month}}^e$ are deterministic seasonal components. The deseasonalized loads $\tilde{y}_t^e$ and $\tilde{y}_t^h$ follow the AR(1) models

$$\tilde{y}_t^{\ell} = \mu^{\ell} \tilde{y}_{t-1}^{\ell} + \sigma^{\ell} \epsilon_t^{\ell}, \quad (21)$$

where $\epsilon_t^{\ell} \sim N(0,1)$. Thus, $\tilde{y}_t^{\ell} = \phi^{\ell} \tilde{y}_{t-1}^{\ell}$. Similar stochastic demand models have been considered in the literature; e.g., see [27]–[31]. Alternative models for electricity price dynamics can be found in [32]–[35]. For other models for heat demand, see [36]–[37] and the references therein. The methodology in this paper can handle other types of stochastic processes for prices and loads.

IV. Dynamic Optimization Formulation

The information required at each time step $t$ to control the energy hub is summarized in the state variable $S_t$:

$$S_t \overset{\text{def}}{=} \left( W_t, R_t, U_{t-1}, D_{t-1}, b_{t-1}^J \right), \quad (22)$$

for $s \in \{H, h, b\}$ and $J \in \{\text{HPP, FC, CHP, B}\}$. Here, the exogenous information observed at time $t$ is

$$W_t \overset{\text{def}}{=} \left( c_t^e, \tilde{L}_t^e, \tilde{L}_t^h \right). \quad (23)$$
For each storage device $s$, the initial charge level is given, e.g., $R_0^s = 0$. We also set $D_{-1} = \text{MDT}$, $U_{-1} = 0$, and $b_{-1} = 0$.

Each decision affects the context in which future decisions are to be made. After taking the action $x_t$, given $S_t$, the state variable $S_{t+1}$ is computed by the state transition functions described below. For each storage device $s$,

$$R_t^s = (1 - y_{\text{loss}}^s \Delta t) R_{t-1}^s + \left( \eta_{\text{ch}}^s P_t^{ch,s} - \frac{1}{\eta_{\text{dis}}^s} P_t^{\text{dis},s} \right).$$ (24)

When $\delta^s_{\text{HPP-E}} + \delta^s_{\text{HPP-S}} = 1$, the state variables are set to

$$(U_t, D_t) = \begin{cases} (0,1) & \text{if } \delta^s_{\text{HPP-E}} = 1 \text{ and } \delta^s_{\text{HPP-S}} = 0, \\ (1,0) & \text{if } \delta^s_{\text{HPP-E}} = 0 \text{ and } \delta^s_{\text{HPP-S}} = 1. \end{cases}$$ (25)

When $\delta^s_{\text{HPP-E}} = \delta^s_{\text{HPP-S}} = 0$, we have

$$(U_t, D_t) = \begin{cases} (U_{t-1} + 1,0) & \text{if } U_{t-1} > 0, \\ (0,D_{t-1}+1) & \text{if } D_{t-1} > 0. \end{cases}$$ (26)

The dynamic optimization problem of the energy hub under consideration is written as

$$\min_{\pi \in \Pi} \rho \left[ \sum_{t=0}^{T-1} C_t(S_t, X_t^s(S_t)) \mid S_0 \right].$$ (27)

where $\rho$ is a risk functional of the accumulated operational cost of the energy hub. The optimization is limited to the class $\Pi$ of deterministic Markov policies $\pi = (\pi_1, \ldots, \pi_{T-1})$, where each decision rule $\pi_t$ is a function from $S_t$ to $X_t$, i.e., for any time $t$ the energy hub schedule $x_t$ is a function of the state variable $S_t$. Here, $S_t$ is the space of all possible states $S$ in (22). We write $X_t^s(S_t)$ to indicate that the decision at period $t$ is a function of the state $S_t$ as determined by the policy $\pi$.

The expectation $\mathbb{E}$ often plays the role of the risk functional $\rho$ in the dynamic programming literature. Alternative popular functions $\rho$ for measuring risk are variance, Value-at-Risk, Conditional Value-at-Risk, and their weighted sums. Value-at-Risk (VaR) is the quantile of the loss distribution, i.e., maximum possible total loss after excluding worse outcomes whose probability is less than $1 - \alpha$, and Conditional Value-at-Risk (CVaR), called expected shortfall or average Value-at-Risk, measures the expected loss exceeding VaR. For the importance of risk considerations in the management of energy systems, see [38]-[40].

The approaches in dynamic programming (see [7], [8]) to optimize the expected cost typically relies on the Bellman equation, which relates the optimal value of the objective function in period $t$ to its optimal value in period $t+1$. We adopt an algorithmic strategy based on direct policy search [8] and a novel cost function approximation policy, proposed in [10], to solve problem (27).

V. APPROXIMATION DYNAMIC PROGRAMMING

Cost function approximation policies within a direct policy search method work with a parameterized family of policies motivated by approximations of the (derivatives of the) value function. A set of parameter values are then sought to achieve improved objective values. Various closed-form parameterized approximators (also called, policy function approximation in [8]) are discussed in [41]. Nevertheless, optimally tuning such closed-form policies, e.g., affine policies, becomes challenging as the number of constraints on actions grows [9].

To address these challenges, in [10], a policy function approximation $X_t^s(S_t(\theta_t))$ using an arg-min operator is proposed:

$$X_t^s(S_t(\theta_t)) = \arg \min_{x_t \in K} \{ C_t(S_t, x_t) + \mathcal{K}_0(S_t, x_t) \},$$ (28)

for $t = 0, \ldots, T - 1$. Here, the functions $\mathcal{K}_0(S_t, x_t)$ are expressed as linear architectures of the general form

$$\mathcal{K}_0(S_t, x_t) = \sum_{k=1}^{K} \theta_{k,t} \phi_k(S_t, x_t),$$ (29)

where $\theta_{k,t}$ are the components of the parameter vector $\theta_t$, and $\phi_k(S_t, x_t)$ are fixed and easily computable functions, referred to as basis functions. Most approximate dynamic programming methods make use of approximation architectures as in (29), see Section 3.1 in [42] and Chapter 7 in [8]. Often the basis functions are used to approximate the value function, while in the approach described in [10], they are used to express the policy function. The arg-min operator in (28) ensures that the action $X_t^s(S_t(\theta_t))$ corresponding to a parameter vector $\theta_t$ and a state $S_t$ is feasible. The idea here is that $\mathcal{K}_0(S_t, x_t)$ captures some important aspects of the functional form of the optimal value function. However, $\mathcal{K}_0(S_t, x_t)$ intervenes in an argmin expression over the action and can thus be defined up to a function of the state only. For further technical details about the correction function and its relationship with the value function in dynamic programming, see [10].

The parameters $\{\theta_k\}_{k=1}^{K}$ in the policy are tuned by solving:

$$\min_{(\theta_1, \theta_2, \ldots, \theta_K)} \rho \left[ \sum_{t=0}^{T-1} C_t(S_t, X_t^s(S_t(\theta))) \right].$$ (30)

A gradient-based optimization technique is proposed in [9] to solve problem (30) for affine policy function approximations and coherent risk measures including CVaR [43]. In [10], a simulation-based optimization method, capable to handle policy approximators of type (28) is proposed and its effectiveness in the presence of risk considerations and operational constraints was discussed in detail. Solving the resulting problem (30) relies on a derivative-free optimization approach [44] where the function evaluations are performed by Monte Carlo simulation. An overview of the approach is presented in Fig. 2. For details on updating process for $\theta$, see Fig. 2 in [10] and the discussion in Section 3.3. For an optimality analysis of this method for a general dynamic programming problem, the reader is referred to Propositions 1 and 2 in [10]. Detailed computational investigation of the dimensionality reduction procedure is also reported in [10]. This approach is employed to solve problem (30) for the energy hub optimal dispatch problem with the policy approximation functions (28).

For the correction function $\mathcal{K}_0(S_t, x_t)$ expressed by linear basis functions, as here for our energy hub optimal dispatch problem, solving the minimization problem (28) involves solving a mixed-integer linear programming (MILP) problem. Their optimal values are then aggregated for evaluations of the objective function (30) over optimization iterations.
Fig. 2. Flow-diagram of the Approximate Dynamic Programming Method.

Five basis functions are considered in this study,
\[
\begin{align*}
\phi_1(s_t, x_t) &= \left[ E_t[c_{t+1}^H] + \gamma_{t+1}^{e-b} - (\gamma_{t}^{e-b} + \gamma_{t+1}^{b-h})\right] P_t^{ch-b}, \\
\phi_2(s_t, x_t) &= \left[ E_t[L_t^e] - L_t^e \right] P_t^{ch-h}, \\
\phi_3(s_t, x_t) &= \left[ E_t[L_t^{e-h}] - L_t^{e-h} \right] P_t^{ch-b}, \\
\phi_4(s_t, x_t) &= \left[ E_t[c_{t+1}^{e-h}] + \gamma_{t+1}^{e-h} - (\gamma_{t}^{e-h} + \gamma_{t+1}^{b-h})\right] P_t^{ch-H}, \\
\phi_5(s_t, x_t) &= \left[ E_t[L_t^{e-h}] - L_t^{e-h} \right] P_t^{ch-h}.
\end{align*}
\]

Here, \( E_t[\cdot] = E[\cdot|S_t = s_t] \). These basis functions aim to compare the current levels of prices and loads \((\gamma_{t}^{e-b}, L_t^e, L_t^h)\) with the expected prices and loads in the subsequent time steps \((\gamma_{t+1}^{e-b}, c_{t+1}^H, L_t^{e-h})\) for storage units that are affected by these variables. This structure for the basis functions is motivated by the two-threshold policy structure appeared in the single-storage operation optimization problem under some conditions, e.g., see Proposition 3 in [10]. The use of linear basis functions is typical in the literature on approximate dynamic programming. The approximate dynamic programming framework outlined in Section V can be applied for any number and choice of basis functions. Admittedly, for all approximate dynamic programming methods, the performance of the algorithm is sensitive to the selection of basis functions. Thus, selecting the basis functions is an important undertaking in approximate dynamic programming implementations.

VI. COMPUTATIONAL RESULTS

A time horizon \( T = 24 \) hours is considered. The length of each time step is \( \Delta t = 1 \) hour. The model parameter values characterizing the energy hub components and operational limits are given in Tables I-IV. The parameter settings reflected in these tables are typical values, and are inspired by real-world projects in Canada and in the US [45]-[46]. The self-discharge rates \([\text{kWh/h}]\) are set to \( \gamma_{loss}^h = 0.001 \), \( \gamma_{loss}^b = 0.08 \), and \( \gamma_{loss}^b = 0.04 \). The HPP operation time limit parameters are \( \text{MDT} = 1 \) and \( \text{MUT} = 3 \) time steps. Transaction costs are \( \Gamma_{e-b}^b = 0.05 \) and \( \Gamma_{e-h}^b = 0.10 \) per MWh. Our implementation of the method explained in Section V uses the convergence tolerance \( \gamma_{tol} = 10^{-4} \) and \( M = 1000 \) Monte Carlo scenarios.

The parameters defining the electricity load stochastic process is as in [11] calibrated to real data. The parameters of the heat load process is set to 60% of those in the electricity load process. The price model (19) is estimated using the day-ahead (hourly) prices for the NYC zone in 2016 from NYISO. The estimated parameters are \( \lambda_{e} = 0.0251, \mu_{e} = 0.0398, \sigma_{e} = 0.3863, \ell = 0.24, \) and \( J_t \) is normally distributed with mean \( \mu_J = 0.0012 \) and the standard deviation \( \sigma_J = 0.3051 \). Figure 3 illustrates the mean, min, max, and a sample path for loads and electricity price. For comparison purposes, the results are presented for two different gas price levels $32 and $10 per MWh.

Table V reports the computed values of the tuning parameters \( \theta \) for the expected cost minimization dynamic policy as well as the risk-sensitive dynamic energy hub dispatch policy. Here, the Value-at-Risk [47] with the confidence level 95% is considered as the risk measure. Their associated costs and risks are also presented in Table V.

A nonzero coefficient for a basis function shows its relevance for determining the dynamic energy hub dispatch policy. For a high gas price level, such as $32/MWh, all of the policy parameters \( \theta_k \) are nonzero. However, for a lower gas price, such as $10/MWh, fewer policy parameters are nonzero; i.e., some basis functions do not contribute to reducing the expected energy hub operational cost. A comparison between the costs
from the two gas price levels indicates that the expected energy hub operational cost, at the gas price level $32/MWh, is only about twice the expected operational cost of the energy hub system when the gas price equals $10/MWh. When the gas price is high, there are more nonzero policy parameters for the expected cost minimization dispatch policy than those for the risk-sensitive dispatch policy. This fact suggests that for a risk-neutral energy hub controller there are more opportunities to dynamically adapt the dispatch policy to the price and load information observed over the decision time horizon, and thus to reduce the expected cost, than for a risk-sensitive controller in this case. That is expected, because the risk-sensitive dispatch policy is generally more conservative.

Table VI presents the expected cost and risk of two energy hub dispatch policies, namely MILP and MPC, to minimize the expected energy hub operation cost. The MILP refers to the dispatch schedule obtained at the beginning of the time horizon assuming perfect future knowledge using the expected prices and loads. The MPC policy involves decisions of the first step of the solution computed by the deterministic problem from current time \( t \) to the end of the time horizon. This approach solves the deterministic problem over a rolling horizon at any time and state, implements the first step, and reruns for the subsequent time steps. For further details on MPC and its relation to approximate dynamic programming, see [48]. Table VI indicates that for both gas price levels, MPC improves the expected cost of MILP by 2.51% for gas price $32/MWh and 0.82% for gas price $10/MWh. However, the cost variability (VaR\(_{95\%}\)) of the MILP is lower than that of MPC. Using the row, labeled mean, of Table V and the results in Table VI, we observe that for the higher gas price level, the ADP approach improves the expected cost by 8.02% compared to MILP and by 5.37% in comparison to MPC. For the low gas price level, the improvement achieved by the ADP method is less significant and is about 4.96% and 4.10%, for MILP and MPC, respectively.

![Stochastic models for Electricity Price and Demands.](image)

**Fig. 3.** Stochastic models for Electricity Price and Demands.
plots illustrate that, for both gas price levels, the distribution of the energy hub operation cost per hour further deviates from its median during on-peak hours, when higher prices are experienced.

![Box Plot of the Costs](image)

**Fig. 4.** Box Plot of the Costs $[S]$ of the Dynamic Policies over Hours.

Figure 5 demonstrates the storage charge levels over time steps corresponding to one simulated scenario for the electricity price and loads at the two natural gas price levels, when the computed dynamic energy hub dispatch policies are implemented. Note that since the policy adapts to the scenarios, the storage profiles vary over different realizations of the price and loads.

![Charge Levels](image)

**Fig. 5.** Charge Levels [kWh] on a Sample Path (Dash-dot Line: $\hat{c}_t^H = $10/MWh, Dashed Line: $\hat{c}_t^H = $32/MWh).

These plots show that at low gas price levels, the heat storage is utilized more frequently, as it is more cost-effective to serve the electricity demand using natural gas-based power production instead of buying directly from the power market. This conversion leads to a level of heat production greater than the heat demand, which is then stored in the heat storage unit. At more expensive natural gas prices, the hydrogen storage unit is more effective and the heat storage is not used as often. The pattern in the hydrogen storage is due to the on-off operation problem, and solving the problem using a recently proposed approximate dynamic programming framework. As in other approximate dynamic programming methods, the performance of the approach is sensitive to choices such as the set of basis functions employed. It remains for future work to explore the effect of including more basis functions and those of higher orders on the costs and risks. Investigating dynamic policies for the operation of an energy hub under additional path-dependent constraints, such as battery aging considerations in the spirit of [49], is another direction for future research. Studying optimal operation schedules of an energy hub participating in transactive energy markets, such as [50], is interesting.

**VII. Conclusions**

This paper studies optimal operation of an energy hub system, incorporating various operational constraints. In particular, requirements on the minimum downtime and uptime for HPP are modeled during decision time steps. An optimization framework based on stochastic dynamic programming has been developed for the energy hub to simultaneously address uncertainties in loads and prices as well as risk consideration and energy hub operational constraints. The presented numerical results verify the tractability and effectiveness of the developed dynamic energy hub dispatch policy. Different energy storage technologies in the energy hub take distinct roles under different realizations for uncertain parameters. A dynamic policy can adapt to the variations in scenarios and provide more accurate information about component and system valuation (i.e., total benefit over a time horizon) leading to more reliable energy hub planning and operation studies.

The proposed framework shows promising characteristics to be included in the future generations of control platforms for energy storage devices, building energy management systems, and small distribution systems or microgrids. In the future research, we will focus on an integrated energy system composed of several energy hubs, where grid constraints are also incorporated. In the aforementioned framework, we will explore the addition of district cooling as well as grid-to-vehicle (G2V) and vehicle-to-grid (V2G) concepts, which add further dimensions of complexity and stochasticity to the optimization problem. This paper has been focused on developing a dynamic programming formulation (e.g., specifying state variables and transition functions) for the energy hub operation problem, and solving the problem using a recently proposed approximate dynamic programming framework.


