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MUSIC-like Algorithm for Source Localization in Electrical Impedance Tomography

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Abstract—In electrical impedance tomography (EIT), the noise amplified solution caused during matrix inversion can be avoided with non-parametric spectral based estimation when the conductivity variation is bounded and spatially sparse. Among many spectral based algorithms used in direction-of-arrival (DOA) estimation, an algorithm called multiple signal classification (MUSIC) is one of the most well-known that has super resolution performance. However, its dependency on the model order estimation can lead to performance degradation especially for quasi-static environment like EIT application and this is due to source location changes and conductivity variation. In this paper, the relationship between source position, conductivity variation, ill-conditioned array manifold and eigenvalues of the covariance matrix are explored. An algorithm called MUSIC-like which has high resolution performance comparable to MUSIC is then proposed for EIT application. It is formulated under the beamforming framework, and therefore does not require an estimation of model order from the covariance matrix. Simulation results show that the proposed method is capable of obtaining high resolution performance under various noise levels. An 8-electrode EIT system prototype was built using the proposed method, and experimental results confirm the high resolution performance capability of the proposed method.

Index Terms—Electrical impedance tomography, beamforming, direction-of-arrival, array processing, sensor arrays, inverse problems.

I. INTRODUCTION

Electrical Impedance Tomography (EIT) is a soft field non-invasive imaging technique which is inexpensive and has low complexity in terms of implementation compared to other hard field imaging modalities such as X-ray, Computed Tomography (CT), and Magnetic Resonance Imaging (MRI). An example of how a small system can be implemented compactly is presented in [1] where a real-time system of electrical capacitance tomography (ECT), a close relative to EIT, was implemented based on a field programmable gate array (FPGA) as an image reconstruction unit. With its high temporal resolution, the system is capable of capturing data within milliseconds. This imaging modality has been applied for image reconstruction problem in several fields such as geophysical subsurface imaging, biomedical tomography and industrial pipeline monitoring [2].

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From the conventional direct image reconstruction viewpoint, the reconstructed image can be obtained by solving an underdetermined system of linear equations where the sensitivity matrix is ill-conditioned. The inversion process often leads to a noise amplified solution, and hence the solution needs to be regularized. The regularization framework introduces additional prior knowledge onto the possible solution space which can be interpreted from the statistical viewpoint (under Gaussian noise assumption) as a Maximum A Posterior (MAP) estimate [3]. While such method can reconstruct the whole image pixel-by-pixel, in certain scenarios the objective can be minimized to only a line search for anomalies over a relatively homogeneous background. This notion can be applied to solve problems in various fields such as in biomedical or industrial application where the problem of anomalies detection and non-destructive testing are common.

In biomedical applications such as in brain EIT or tumor localization, the highly active cerebral area or the area where solid tumor is located can be identified by higher blood flow which corresponds to conductivity increase in that particular area. In contrast, the area of the brain affected by ischemic stroke can be identified by a blocking of blood flow which translates to conductivity decrease in that area [2]. For industrial application such as in pipeline monitoring task, the structure of solid contaminants can be specified as small and bounded objects with different electrical characteristic over the background. With all the cases mentioned above, the conductivity variations (anomalies) are structurally bounded and do not occupy large area in the detection space (spatially sparse). Therefore, the objective of reconstructing the whole image can be reduced to a detection and localization of bounded objects with different conductivity value over the background [4]. Multiple data sample (snapshots) can be obtained either in time or frequency domain which could be made possible by using multiple excitation structure developed for fast signal acquisition [5]. With multiple measurement vector (MMV), several source localization algorithms can be applied to uncover the location of the anomalies.

Source localization has long been of a great interest especially in the application of radar, sonar, wireless communication, and target tracking [6] [7] [8] [9]. Direction of arrival (DOA) estimation is one of the method used to localize the target when the geometrical information of the sensors is known. Among many spectral based DOA estimation techniques such as conventional beamforming (CBF), minimum variance distortionless response (MVDR) and its variance such as linearly constrained minimum variance (LCMV) [10], the

multiple signal classification (MUSIC) algorithm is one of the most well-known subspace based algorithms with super resolution (resolvability) performance that is capable of identifying closely spaced sources [11] [12]. With the ability to focus on the desired signal while suppressing interferences and noises, several array processing and beamforming techniques used in DOA estimation have been applied to imaging problem under the localization framework. In [13], it was shown that MUSIC algorithm can be applied as an alternative approach to solve the problem in ill-posed environment of diffuse optical tomography (DOT) which outperforms the conventional compressive sensing technique. The MUSIC algorithm was also successfully applied for solving breast tumor detection and localization problem under quasi-static environment [14]. The LCMV algorithm was proposed for localizing conductivity changes in human head [15] which was later extended to an experimental study in a phantom tank [16].

Although MUSIC algorithm is attractive due to its high resolution performance, its subspace decomposition dependency can lead to performance degradation when perfect decomposition cannot be obtained due to incorrect model order estimation. Such scenarios commonly arise either when the acquired number of sample size is insufficient or when the noise level is high. Specifically in EIT environment where the source signal power can be varied due to multiple factors, this can lead to incorrect model order estimation. In order to retain super resolution performance of the MUSIC algorithm while reducing strong requirement on the model order estimation, the DOA estimation problem was posed under the beamforming framework which does not require model order estimation [17] [18]. Given such robust characteristic, MUSIC-like algorithm is proposed as an alternative approach for DOA estimation in EIT environment.

Five contributions in this paper can be concisely summarized as follows. First, MUSIC-like algorithm is used specifically for EIT application under new formulation of steering vectors based on specific current injection and measurement method. This method avoids subspace decomposition which reduces the reliance on model order estimation, and therefore achieves better performance in the quasi-static environment. Second, the relationship between source location, size, conductivity variation and the eigenvalues extracted from the covariance matrix are explicitly revealed. Third, a thorough analysis on ill-conditioned array manifold and its effects are presented which help to understand the common advantage and disadvantage of the related algorithms. Fourth, performances of the MUSIC-like algorithm are evaluated and compared under different noise levels. Finally, an EIT system prototype comprises 8 electrodes was built, and preliminary experimental study was conducted. The experimental results confirm the high resolution performance of the proposed method as well as the capability to identify sources under close space separation.

II. FORWARD PROBLEM & SIGNAL MODEL

Before moving into the signal model, a forward model and measurement procedure of the EIT system are first introduced as shown in (1) and Fig. 1. An 8-electrode EIT system

Fig. 1. An 8-electrode EIT system with adjacent method for current injection and measurement procedure.

with adjacent data collection method, where two neighbouring electrodes are used for current injection and measurement procedure, is shown in Fig. 1. In order to solve the potential value on the array sensors at the boundary (U_k) as shown in Fig. 1, we follow the complete electrode model presented in [19] as follows,

$$\begin{aligned} \nabla \cdot (\sigma \nabla \phi) &= 0 \quad \text{in } \Omega, \\ (\phi + z_k \sigma \frac{\partial \phi}{\partial \mathbf{n}})|_{\varepsilon} &= U_k \quad \text{for } k = 1, \dots, L, \\ \sigma \frac{\partial \phi}{\partial \mathbf{n}} &= 0 \quad \text{on } \partial \Omega \setminus \bigcup_{k=1}^L \varepsilon_k, \\ \int_{\varepsilon_k} \sigma \frac{\partial \phi}{\partial \mathbf{n}} &= 0 \quad \text{for } k \in \{1, \dots, L\} \setminus \{j, j+1\}, \\ \int_{\varepsilon_j} \sigma \frac{\partial \phi}{\partial \mathbf{n}} ds &= I = - \int_{\varepsilon_{j+1}} \sigma \frac{\partial \phi}{\partial \mathbf{n}} ds. \end{aligned} \quad (1)$$

The potential U_k at electrode ε_k on the boundary can be obtained by solving the Laplace equation with Neumann boundary condition where a pair of electrodes ε_j and ε_{j+1} are injecting a low frequency alternating current from different positions. In the specified formulation above, L is the number of array electrodes and z_k is the contact impedance of electrode ε_k . Since the receiving signal on the boundary are typically small, difference measurements are commonly used ($U_k - U_{k+1}$). By using the adjacent electrodes for both current injection and potential measurements, the measurement vector is defined as

$$\mathbf{v} = [\underbrace{v^{1,1}, \dots, v^{1,L}}_{1^{st} \text{ injection}}, \dots, \underbrace{v^{L,1}, \dots, v^{L,L}}_{L^{th} \text{ injection}}], \quad (2)$$

where $v^{1,1} = U_1 - U_2$ from the first injection, and hence $v^{j,k} = U_k - U_{k+1}$ from the j^{th} injection. Since the current that passes through the injecting electrodes ε_j and ε_{j+1} can change the contact impedance z_j and z_{j+1} , the three measurements related to electrodes ε_j , and ε_{j+1} are commonly omitted. The receiving electrodes ε_k for $k \in \{1, \dots, L\} \setminus \{j, j+1\}$, where $\setminus$ defines exclusion, are used to obtain data. With L number of electrodes, the measurement vector is now with length of $M = (L - 3)L$. Difference potential measurement either in time or frequency domain are commonly used by computing the difference between the perturbed potential on the boundary against the unperturbed or referenced potential. When the conductivity variation is spatially sparse and bounded, the perturbed potential received by the array electrodes can be

considered as an electric field generated by an induced current source [2]. For simplicity, the background conductivity is defined as $\sigma = 1$, the Laplace equation in (1) can be rewritten as

$$\nabla \cdot [(\sigma + \delta\sigma)\nabla(\phi + \delta\phi)] = 0 \rightarrow \nabla \cdot \nabla \delta\phi = \nabla \cdot (\delta\sigma|_p \nabla \phi), \quad (3)$$

where $\sigma \rightarrow \sigma + \delta\sigma$ and $\phi \rightarrow \phi + \delta\phi$. The medium Ω is discretized into small pixels as $\Omega = \bigcup_{i=1}^N \xi_i$ where ξ_i represents each pixel position and N is the number of all pixels in the image. The perturbed potential field $\delta\phi$ is generated by a conductivity variation $\delta\sigma|_p$ at pixel p^{th} . The received signal snapshots (either in time or frequency domain) on the boundary can be modelled under linear approximation as

$$\mathbf{x}(n) = \mathbf{A}(\Xi)\mathbf{s}(n) + \mathbf{n}(n), \quad (4)$$

where $\mathbf{x}(n) \in R^{M \times 1}$ is a snapshot of the measurement vector $\delta\mathbf{v}$, $\mathbf{A}(\Xi) \in R^{M \times N}$ is a sensing matrix (Jacobian matrix), $\mathbf{s}(n) \in R^{N \times 1}$ is a snapshot of conductivity variation (source vector), and $\mathbf{n}(n) \in R^{M \times 1}$ is specified as a combination between white additive noise and unmodelled error. In Fig. 2

Fig. 2. Sparsity structure of the conductivity variations in the medium over different time snapshots.

(left), two sources are specified by the small greyscale circles inside the medium (large circle) for each time snapshot (t_i). The conductivity values are varied over different snapshots which are specified by different greyscale color. The source matrix shown in Fig. 2 (right-top) is considered to have a joint sparse structure as shown in Fig. 2 (right-bottom) where each column shares the same nonzero indices (greyscale cells). The greyscale cells from each column are the conductivity variations correspond to the sources for each time snapshots in Fig. 2 (left). The MMV formulation can now be rewritten as

$$\mathbf{X} = \mathbf{A}(\Xi)\mathbf{S} + \mathbf{N} \rightarrow \mathbf{X} = \mathbf{A}_{\mathcal{P}}(\Xi)\mathbf{S}_{\mathcal{P}} + \mathbf{N}, \quad (5)$$

when $\mathcal{P}$ denotes a set where its members are indices of the nonzero rows of the source matrix $\mathbf{S}$. The sensing matrix $\mathbf{A}(\Xi)$ and the source matrix $\mathbf{S}$ can be rewritten as $\mathbf{A}_{\mathcal{P}}(\Xi) \in R^{M \times \mathcal{P}}$ and $\mathbf{S}_{\mathcal{P}} \in R^{\mathcal{P} \times K}$. The matrix $\mathbf{A}_{\mathcal{P}}(\Xi) = [\mathbf{a}(\xi_1), \dots, \mathbf{a}(\xi_{\mathcal{P}})]$ can now be considered as an array manifold comprises $\mathcal{P}$ column vectors each with length M called steering vector or array response. In [16], the steering vector $\mathbf{a}(\xi_i) \in R^{M \times 1}$ forming the array manifold was computed as an expected potential variation on the electrodes due to conductivity variation at pixel ξ_i^{th} . However, since the signal strength is also a function of source location, the steering vector used in this paper is

compensated by using a normalized potential variation defined as

$$\mathbf{a}(\xi_i) = \underbrace{[a_1^1(\xi_i), \dots, a_{L-3}^1(\xi_i)]}_{1^{st} \text{ Induced Polarity}}, \dots, \underbrace{[a_1^L(\xi_i), \dots, a_{L-3}^L(\xi_i)]}_{L^{th} \text{ Induced Polarity}}^T, \quad (6)$$

where $a_1^j(\xi_i), \dots, a_{L-3}^j(\xi_i)$ are the normalized expected potential variation due to j^{th} induced polarity source, and each component can be computed by solving the forward problem of

$$a_k^j(\xi_i) = \frac{\int_{\xi_i} \nabla \phi^k \nabla \phi^j dr}{\sqrt{\sum_{j=1}^L \sum_{k \notin j-1, j, j+1} |\int_{\xi_i} \nabla \phi^k \nabla \phi^j dr|}}. \quad (7)$$

The steering vector specified in (6) is analogous to the lead field matrix used in active brain source localization, but with all information regarding the source polarity are now incorporated into a single vector due to the rotation of current injection. Assuming infinite number of snapshots with zero mean data matrix $\mathbf{X}$, the array covariance matrix can be expressed as

$$\mathbf{R} = \mathbf{A}_{\mathcal{P}}(\Xi)\mathbf{R}_{ss}\mathbf{A}_{\mathcal{P}}(\Xi)^T + \sigma_n^2\mathbf{I}_M, \quad (8)$$

where $\mathbf{R} = E(\mathbf{X}\mathbf{X}^T)$, $\mathbf{R}_{ss} = E(\mathbf{S}_{\mathcal{P}}\mathbf{S}_{\mathcal{P}}^T)$, $\sigma_n^2\mathbf{I}_M = E(\mathbf{N}\mathbf{N}^T)$, and $E(\cdot)$ denotes the expectation operation. In the next section, several factors that affect the source signal power which in turn influences the source localization performance will be discussed.

III. ANALYSIS OF SIGNAL POWER IN EIT

In this section, several factors affecting the eigenvalues extracted from the covariance matrix as well as the analysis on the ill-conditioned array manifold will be discussed. Two matrix decompositions namely singular value decomposition (SVD) and eigenvalue decomposition (EVD) needed for further analysis are first explicitly defined. First, an $M \times \mathcal{P}$ array manifold can be compactly decomposed by SVD as $\mathbf{A}_{\mathcal{P}}(\Xi) = \mathbf{U}_r\mathbf{\Sigma}_r\mathbf{V}_r^T$, where $\mathbf{U}_r \in R^{M \times r}$ is formed by left singular vectors spanning data range space. Diagonal matrix $\mathbf{\Sigma}_r \in R^{r \times r}$ comprises singular values in a non-increasing order. Matrix $\mathbf{V}_r^T \in R^{r \times \mathcal{P}}$ is formed by right singular vectors which span model range space. Second, an $M \times M$ covariance matrix can be decomposed by EVD as $\mathbf{R} = [\tilde{\mathbf{U}}_r|\tilde{\mathbf{U}}_0][\tilde{\mathbf{\Lambda}}_r|\tilde{\mathbf{\Lambda}}_0][\tilde{\mathbf{U}}_r|\tilde{\mathbf{U}}_0]^T$ where $\tilde{\mathbf{U}}_r \in R^{M \times r}$ is formed by eigenvectors spanning signal subspace which also span the same range space as $\mathbf{U}_r$ specified earlier. Matrix $\tilde{\mathbf{U}}_0 \in R^{M \times M-r}$ is formed by eigenvectors spanning noise subspace. Diagonal matrices $\tilde{\mathbf{\Lambda}}_r$ and $\tilde{\mathbf{\Lambda}}_0$ specified in a non-increasing order consist of eigenvalues associated with the eigenvectors spanning signal and noise subspaces respectively. In general, the source signal power can be varying due to multiple factors such as frequency used, level of current injection, level of conductivity variation, source size, source location, and correlation between steering vectors. In this section, two most general but important factors will first be discussed followed by several scenarios caused by such factors.

First Factor: Source location. The energy from induced source is diffused all over the domain which causes difficulty to localize source further away from the boundary (deep source) due to low sensitivity. Considering two sources with equivalent conductivity variation level (equal signal power σ_s^2), one located close to the boundary while another located further away, the covariance matrix from (8) can be expanded by SVD as

$$\mathbf{R} = [\mathbf{U}_r \mathbf{\Sigma}_r \mathbf{V}_r^T] \sigma_s^2 \mathbf{I}_2 [\mathbf{V}_r \mathbf{\Sigma}_r \mathbf{U}_r^T] + \sigma_n^2 \mathbf{I}_M. \quad (9)$$

The singular values in $\mathbf{\Sigma} = \text{diag}(\sigma_1, \sigma_2)$ are not equivalent ($\sigma_1 > \sigma_2$), and the covariance matrix can be rewritten as

$$\mathbf{R} = \tilde{\mathbf{U}} \begin{bmatrix} (\sigma_1 \sigma_s)^2 + \sigma_n^2 & 0 & 0 & 0 \\ 0 & (\sigma_2 \sigma_s)^2 + \sigma_n^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix} \tilde{\mathbf{U}}^T. \quad (10)$$

Although the conductivity variation from both sources are equivalent, it can be seen in (10) that the second eigenvalue $\lambda_2 = (\sigma_2 \sigma_s)^2 + \sigma_n^2$ is lower than the first.

Second Factor: Correlation between steering vectors. The neighboring pixels often omit similar array response profile which causes dependent column space in the array manifold, and hence $\mathbf{A}_P(\Xi)$ becomes ill-conditioned. Consider three point sources where two located close to one another, and the latter located further away independently. With equivalent distance between all sources to the boundary, the signal powers are comparable $\sigma_1^2 \approx \sigma_2^2 \approx \sigma_3^2$. A snapshot of received signal at the array electrodes can be expanded under SVD as

$$\mathbf{x} = \mathbf{U}_r \begin{bmatrix} \sigma_1 + \delta & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \epsilon \end{bmatrix} \mathbf{V}_r^T \mathbf{s}_P + \mathbf{n}, \quad (11)$$

where ϵ is typically small due to high correlation between two of the steering vectors. The dimension of data range space is now limited to only two with an increasing value in the first singular value. The covariance matrix of the received data in (11) can be expressed as

$$\mathbf{R} = \tilde{\mathbf{U}} \left[\begin{array}{ccc|ccc} \lambda_1 & 0 & 0 & & & \\ 0 & \lambda_2 & 0 & & & \\ 0 & 0 & \lambda_3 & & & \\ \hline & & & 0 & & \\ & & & & \Lambda_0 & \end{array} \right] \tilde{\mathbf{U}}^T, \quad (12)$$

where $\lambda_1 = (\{\sigma_1 + \delta\} \sigma_s)^2 + \sigma_n^2$, $\lambda_2 = (\sigma_2 \sigma_s)^2 + \sigma_n^2$, $\lambda_3 = (\epsilon \sigma_3)^2 + \sigma_n^2$. With small ϵ , λ_3 will now become part of Λ_0 . It should be noted that the ill-conditioning of the array manifold does not cause the covariance matrix to become ill-conditioned since $\lambda_{R, \min}$ is always part of the noise eigenvalues in Λ_0 and will not become singular. However with both factors mentioned above, the level of eigenvalues associated with the signal subspace Λ_r can be greatly varied.

Before exploring three different cases affected by these two factors, first we show that the conductivity variation produces similar signal perturbation upon the sensor array analogous to the scenario of size variation. Four snapshots of conductivity

variations and four snapshots of size variations are shown in Fig. 3 (left) and (right) respectively. Four data snapshots were

Fig. 3. Sources with conductivity variations (left) and size variations (right) between snapshots.

generated according to the conductivity variation ($\delta \mathbf{v}_c(t_i)$) and another four snapshots for the size variation ($\delta \mathbf{v}_s(t_i)$). For an 8-electrode system, the number of measurement obtained from one snapshot is 40 which can be considered as multichannel signal. Data snapshots were averaged over multiple snapshots

Fig. 4. Signal perturbation due to conductivity variations and size variations.

for each scenario. The absolute value of the averaged data are shown in Fig. 4 marked as (*) for conductivity variation and (o) for size variation. It can be seen that both scenarios produce similar signal perturbation intensity especially at channels 1, 6, 11, 17, 24, 30, 35, and 40 where high perturbation occurred. For different conductivity value or different source size, the signal perturbation pattern will be comparable to Fig. 4 (high perturbation on the indicated channels) but with different intensity level. Since conductivity variation and source size variation produce similar effect, only the aspect of conductivity variation will be discussed along with source location for each scenario.

Case I: Assuming there are two scenarios of two independent sources, the first two sources are located closer to the boundary as shown in Fig. 5(a) while the latter are located further away from the boundary as shown in Fig. 5(b).

The signal powers of those closer to the boundary are higher than the latter marked as (o) and (*) respectively in Fig. 6. Additionally, with the same setting but decrease in conductivity variation, the signal powers also decrease accordingly as marked (◇) and (□) in Fig. 6.

Case II: For p sources with q non-coherent sources ($q < p$) and $p - q$ partial coherent sources, a snapshot of received signal

 Fig. 5. Example of independent sources from *Case I*.

Fig. 6. The first ten eigenvalues of the covariance matrix obtained from non coherent case.

can be expressed as

$$\mathbf{x} = \sum_{i=1}^q \sigma_i(\mathbf{v}_i^T \mathbf{s}_p) \mathbf{u}_i + \sum_{j=q+1}^p \sigma_j(\mathbf{v}_j^T \mathbf{s}_p) \mathbf{u}_j + \mathbf{n}. \quad (13)$$

In this case when $\forall \sigma_j \approx \epsilon$ are small, the second and third terms in (13) will be combined when the conductivity variation is low or in the case of deep source. An example of 3 independent

 Fig. 7. Example of partial coherent sources from *Case II*.

sources is shown in Fig. 7(a) whereas 7(b) shows the scenario of two non-coherent sources located along with one partial coherent source. The eigenvalues of both scenarios are shown in Fig. 8 marked as (o) and (*) respectively. It can be seen in Fig. 8 that one of the eigenvalues in the partial coherent case is dropped. Note that there is a distinction between partial coherent source in this case and highly coherent source in the next case. For partial coherent source, the value projected in the first eigenvalue is not increased as marked in Fig. 8 as "Equivalent Signal Powers". However, this is not true in the case of highly coherent source which will be discussed in the following case.

Fig. 8. The first ten eigenvalues of the covariance matrix obtained from partial coherent case.

Case III: An example of three independent sources is shown in Fig. 9(a), and its eigenvalues are plotted in Fig. 10 as (o) accordingly. However, when one of the sources is

 Fig. 9. Example of highly coherent sources from *Case III*.

highly coherent as shown in Fig. 9(b), the signal power of the first eigenvalue is increased (λ_1 in (12)) due to coherency between the two steering vectors as marked in Fig. 10 as "Increased Signal Power", and the eigenvalue marked as "Highly Coherent Source" in Fig. 10 is dropped (λ_3 in (12)) to the same level as noise power. This can also be interpreted as a merging between two highly coherent sources into one distributed source.

Fig. 10. The first ten eigenvalues of the covariance matrix obtained from highly coherent case.

From different cases discussed above, the result of ill-conditioned array manifold can be found embedded in the covariance matrix where most spectral based algorithms use for weight vector training. With high variation in eigenvalues, the performance of spectral based algorithms are affected. The MUSIC-like algorithm, however, does not depend on the model order estimation and has the ability to incorporate prior into the problem formulation. Therefore, it is able to adaptively regulate its performance under different circumstances.

IV. MUSIC-LIKE ALGORITHM FOR EIT

In this section, the formulation of the DOA based MUSIC-like algorithm adapted to EIT application is briefly reviewed. The optimization problem presented in this paper is aligned with those presented in [17] and [18] for radar application while different geometrical structure of array sensors is being used in this paper which includes the adjacent current injection, measurement procedure, and the source information prescribed in steering vector $\mathbf{a}(\xi)$ where ξ is the source position in the discretized model. Multiple snapshots of receiving signal $\mathbf{X}$ generated by p stationary signal sources with additional white Gaussian noise are captured. With the covariance matrix specified in (8), the optimization problem proposed in [17] and [18] can be defined as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^T \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T \mathbf{a}(\xi) \mathbf{a}(\xi)^T \mathbf{w} + \beta \|\mathbf{w}\|_2^2 = c, \end{aligned} \quad (14)$$

where the weight vector $\mathbf{w}$ is the solution from the optimization problem, $\mathbf{a}(\xi)$ is a steering vector satisfying a normalized 2-norm $\|\mathbf{a}(\xi)\|_2 = 1$, β is a control parameter that introduces certain relaxation to the constraint, and c is a constant. The Lagrangian function for (14) can be expressed as

$$L(\mathbf{w}, \lambda) = \mathbf{w}^T \mathbf{R} \mathbf{w} - \lambda (\mathbf{w}^T (\mathbf{a}(\xi) \mathbf{a}(\xi)^T + \beta \mathbf{I}) \mathbf{w} - c). \quad (15)$$

Let $\frac{\partial L(\mathbf{w}, \lambda)}{\partial \mathbf{w}^T} = \mathbf{R} \mathbf{w} - \lambda (\mathbf{a}(\xi) \mathbf{a}(\xi)^T + \beta \mathbf{I}) \mathbf{w} = 0$, the solution $\mathbf{w}$ is the general eigenvector of the matrix pair $(\mathbf{R}, (\mathbf{a}(\xi) \mathbf{a}(\xi)^T + \beta \mathbf{I}))$:

$$\mathbf{R} \mathbf{w} = \lambda (\mathbf{a}(\xi) \mathbf{a}(\xi)^T + \beta \mathbf{I}) \mathbf{w}, \quad (16)$$

which is equivalent to the following expression

$$(\mathbf{a}(\xi) \mathbf{a}(\xi)^T + \beta \mathbf{I})^{-1} \mathbf{R} \mathbf{w} = \lambda \mathbf{w}. \quad (17)$$

The weight solution $\mathbf{w}$ is the eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$, and the spatial spectrum of the MUSIC-like algorithm is defined as

$$P(\xi) = 10 \log_{10} \frac{1}{|\mathbf{w}^T \mathbf{a}(\xi)|^2}. \quad (18)$$

The bound of β was proposed in [18] as

$$\underbrace{\max_{p \in P} \frac{\lambda_{\mathbf{R}, \min}}{(\mathbf{a}(\xi)^T \mathbf{R}^{-1} \mathbf{a}(\xi))^{-1}}}_{\beta_{\min}} < \beta < \underbrace{\min_{p \notin P} \frac{\lambda_{\mathbf{R}, \min}}{(\mathbf{a}(\xi)^T \mathbf{R}^{-1} \mathbf{a}(\xi))^{-1}}}_{\beta_{\max}} \quad (19)$$

with the choice of β to be a value between $\beta_{\min}$ and $\beta_{\max}$ defined as

$$\beta = (1 - \alpha) \beta_{\min} + \alpha \beta_{\max}, \quad (20)$$

where $0 < \alpha < 1$ to be chosen by $\alpha = \beta_{\min} / \beta_{\max}$. Although the objective function of MVDR and MUSIC-like algorithm are alike, the constraint of both algorithms are different. With β parameter introduced, the MUSIC-like algorithm is able to automatically adjust its performance depending on the signal and noise level estimated in $\beta_{\min}$ and $\beta_{\max}$. With higher noise level ($\beta_{\min}$ increases), the value in β will be readjusted accordingly with higher value and vice versa. The result is that the weight vector solution is resided in the noise subspace, and hence the MUSIC-like algorithm is able to produce direction finding function analogous to that of the MUSIC algorithm without estimating the model order. The pseudospectrum of the MVDR algorithm, however, depends on the signal power, and hence approaches to the MUSIC algorithm performance only under high SNR when the weight vector is completely resided in the noise subspace or when infinite snapshots of data were obtained.

V. SIMULATION RESULTS

In this section, different scenarios are simulated in order to exhibit the influence of several factors mentioned in section III to the performance of related algorithms. The forward problem is solved by Finite Element Method (FEM) provided in the platform developed by [20] in MATLAB. Signal from a discretized circular model of 1,600 pixels with 8 sensors on the boundary is generated with Gaussian noise added. Let d represent the diameter of the circular model which is normalized to a unit circle with radius $r = 1$. A measurement data matrix $\mathbf{X}$ of size 40×100 is captured (40 measurements of 100 snapshots). Four algorithms namely One-Step Gauss Newton with Laplace prior [21], MVDR [16], MUSIC [11] [14], and MUSIC-like algorithm [17] [18] are compared. The hyper-parameter of the One Step GN with Laplace prior is chosen based on Generalized Cross-Validation (GCV) from multiple snapshots of data [22]. With high variation of the signal source, the median value is selected for robustness and outlier avoidance. A conductivity variation within area of 1.5% of the whole medium is presented at $r = 0.2$ and $\theta = 0^\circ$ (polar coordinate). The conductivity variation is varied at a maximum of 20% above the background value with additional noise at the level of $\sigma_n = 200 \mu V$. The reconstructed image under regularization approach and pseudospectrum from the three DOA estimation techniques are shown in Fig. 11. It can be seen that the MUSIC-like algorithm is able to achieve high specificity (narrow peak) when compared to the other algorithms. Such high specificity translates into high resolution performance which will be defined later.

The advantage of having high resolution performance can be seen in Fig. 12 when two sources are presented. One of the sources is located at $r = 0$ and $\theta = 0^\circ$ (deep source) while another source is located at $r = 0.4$ and $\theta = 0^\circ$. In this case, the conductivity variation is bounded within the area of 1% over the entire area with maximum variation of 10% above the background value with additional noise at $\sigma_n = 10 \mu V$. It can be seen in Fig. 12 that the regularization based approach is unable to distinctly detect both sources while the MUSIC-like algorithm presents the best resolution performance (two

Fig. 11. A comparison between an image reconstructed by GN-Laplace, MVDR, MUSIC and MUSIC-like algorithm for single source.

Fig. 12. A comparison between an image reconstructed by GN-Laplace, MVDR, MUSIC and MUSIC-like algorithm for two sources.

peaks are well separated) followed by MUSIC and MVDR algorithm. The resolution performance between three DOA estimation techniques are satisfying since both sources can be identified. The resolution performance of the MVDR algorithm is known to be tied with the source signal power, hence in the condition where the obtained SNR is low, the gap between the two peaks is not well separated even though both sources can be identified. The resolution performance of the MUSIC algorithm is usually high, but in a low SNR condition, performance degradation is expected. The MUSIC-like algorithm which does not depend on subspace decomposition can be seen to have a good resolution performance even in such condition.

Next, the two performance parameters viz., resolution and root-mean-squared error (RMSE) of estimated center-of-gravity (CoG) are used to evaluate the performance of each algorithm. After the image is reconstructed (the pseudospectrum will be considered as the reconstructed image as well), the pixel values are normalized within the range of [0,1] which is now denoted as s_{rec} . The reconstructed and normalized image s_{rec} is thresholded at 50% which is denoted as $\hat{s}$. The resolution performance which reflects the compactness of the reconstructed image (spatial resolution) defined in [21] can be calculated as follows:

$$\text{Resolution} = \frac{A_{\hat{s}}}{A_0}, \quad (21)$$

where $A_{\hat{s}}$ is the area of the normalized and thresholded image $\hat{s}$ (related to the source), and A_0 is the total area of the whole image. Since the signal power is varied depending on the source location, each performance parameter is evaluated as followed. At each position from the center $r = 0$ and $\theta = 0^\circ$ to the area closer to the boundary $r = 0.8$ and

$\theta = 0^\circ$ (under normalized unit radius), each performance parameter is evaluated under Monte Carlo simulation of $N = 200$ trials. The performance parameter is then averaged over multiple position under one noise level σ_n . The procedure is repeated under different noise levels which are $\sigma_n = 200\mu V, 150\mu V, 100\mu V, 50\mu V, 10\mu V, 5\mu V$, and $1\mu V$. For a fixed signal power, different noise levels added is equivalent to having different SNR levels. For the resolution performance

Fig. 13. The resolution of each algorithm under different noise levels for the case of one source.

shown in Fig. 13, the lower the value reflects the higher specificity or higher resolution. From Figs. 11 and 12, it is evidently clear that the MUSIC-like algorithm has the highest resolution performance followed by MUSIC, MVDR, and One Step GN-Laplace algorithm as also reflected in Fig. 13. It can also be seen in Fig. 13 that as the noise level decreases (higher SNR), the resolution improves accordingly.

Next, the center of gravity (CoG) of the source in the original image (ground truth) is denoted as ξ_{CoG} while the CoG of the reconstructed image (estimated) is denoted as $\hat{\xi}_{CoG}$. The position error defined in [21] is calculated as $PE(\xi_{CoG}) = \hat{\xi}_{CoG} - \xi_{CoG}$. The RMSE for center of gravity can be calculated as follows:

$$\text{RMSE}(\xi_{CoG}) = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{\xi}_{CoG_i} - \xi_{CoG_i})^2}. \quad (22)$$

In Fig. 14, the RMSE for CoG of the regularization approach has the lowest average error. Among the DOA estimation techniques, the MUSIC-like has the lowest average CoG error followed by MUSIC and MVDR algorithm. As the noise level decreases, the performance of all algorithms improve accordingly.

Additionally, the performance parameters for the case of two sources are further evaluated and presented in Figs. 15 and 16. Each source is bounded within the area of 4% over the entire area with maximum conductivity variation of 20% over the background. One of the sources is located at $r = 0.8$ and $\theta = 0^\circ$ while the other source is varied from $r = 0.8$ with $\theta = 180^\circ$ to $r = 0$ with $\theta = 0^\circ$. Each performance parameter is evaluated under Monte Carlo simulation of $N = 200$ trials over different positions and noise levels. It can be seen in Figs. 15 and 16 that the results follow the same trend as for the case of one source where the MUSIC-like algorithm is able

Fig. 14. The average RMSE for CoG of each algorithm under different noise levels for the case of one source.

Fig. 15. The resolution of each algorithm under different noise levels for the case of two sources.

to achieve the highest resolution performance followed by the MUSIC, MVDR, and regularization method. The regularization approach is however able to achieve the lowest average CoGs error followed by the MUSIC-like, MUSIC, and MVDR algorithm.

Fig. 16. The average RMSE for CoGs of each algorithm under different noise levels for the case of two sources.

VI. PROTOTYPE AND PRELIMINARY EXPERIMENTAL RESULTS

In this section, experimental results are presented in order to show the possibility of applying the MUSIC-like algorithm to EIT application. Further development for real application can be implemented with more realistic 3D model design where the forward solution is solved by FEM with specific mesh implementation for each application. In this experiment, an 8-electrode EIT system prototype was constructed with a polyethylene container of size 25 cm in diameter as shown in Fig. 17 (top figure). After the water was filled in the tank, a

Fig. 17. An 8-electrode EIT system prototype (top figure) with metallic rods used in the experiment (bottom figure).

current of 1 mA at 1 kHz from improved Howland current source was injected with the adjacent injection and measurement method. With 8 electrodes, the measurement vector obtained by each snapshot is a vector of length 40×1 . Several square and round shape metallic rods made by aluminum, copper, and stainless steel shown in Fig. 17 (bottom figure) were used to create different data snapshots. Each rod was used to emulate a conductivity variation found in real application where the conductivity variation occurs over different time or frequencies.

Fig. 18. Three experimental setups with sources located at different positions.

In the experiment, all rods were paired up in a round-robin fashion where the pair $\{1, 2\}$ is not equivalent to the pair $\{2, 1\}$ in our case since they are located at different locations. As

with the norm [23], the number of snapshots required should be more than the number of measurement M (40 in our case). With 13 rods, 156 pairs of rod combination were generated and each pair was used to generate one data snapshot for each position setup (P_1 , P_2 , and P_3) as shown in Fig. 18. The collinearity between each data snapshot contributes to high condition number of the covariance matrix ($\kappa(R) \approx 1000$). In order to mitigate the ill-conditioned problem in the covariance matrix as well as to improve the robustness of the algorithms performance, we introduce diagonal loading term into the original covariance matrix as $\mathbf{R}_{DL} = \mathbf{R} + \epsilon \mathbf{I}$ where we limit the condition number to $\kappa(\mathbf{R}_{DL}) \leq 100$. For $\kappa(\mathbf{R}_{DL}) = 100$, the diagonal loading term ϵ can be calculated by $\frac{\lambda_1 + \epsilon}{\lambda_M + \epsilon} = 100$ where λ_1 and λ_M are the largest and smallest eigenvalues extracted from $\mathbf{R}$. Due to additional noise injection ($\epsilon \mathbf{I}$) introduced into the data covariance matrix which can result in higher noise floor, all of the normalized images are now thresholded at 75%. Images were reconstructed under different position setups by One Step GN with Laplace prior, MVDR, MUSIC, and MUSIC-like algorithm as shown in Figs. 19, 20, and 21. The performance of each algorithm was evaluated and shown in Table I where $PE(\%)$ indicates position error in percentage, RES indicates resolution performance (smaller value indicates higher resolution), and both objects are specified as O_1 and O_2 .

TABLE I
PERFORMANCE EVALUATION OF SIMILAR SOURCE PROFILES WITH DIAGONAL LOADING TERM

		GN-Laplace		MVDR		MUSIC		MUSIC-like	
		O_1	O_2	O_1	O_2	O_1	O_2	O_1	O_2
P_1	$PE(\%)$	1.9	2.1	8.4	9.2	6.5	7.0	6.1	3.1
	$RES(\times 10^{-2})$	2.64	3.55	1.60	2.00	0.67	1.40	0.67	0.74
P_2	$PE(\%)$	10.2	7.1	11.3	9.5	10.6	5.0	6.9	4.1
	$RES(\times 10^{-2})$	5.03	5.55	1.90	1.90	1.30	0.80	1.00	0.93
P_3	$PE(\%)$	n/a	n/a	n/a	n/a	n/a	n/a	6.3	11.3
	$RES(\times 10^{-2})$	n/a	n/a	n/a	n/a	n/a	n/a	0.38	2.10

Fig. 19. Image reconstruction based on GN-Laplace, MVDR, MUSIC, and MUSIC-like algorithm under experimental setup P_1

From Table I, the regularization method is able to obtain the lowest position error for position setup P_1 . However, it also has the poorest resolution performance which prohibits clear identification of both sources in P_2 and P_3 as marked "n/a". For algorithm with high resolution performance such as MUSIC and MUSIC-like algorithm, both sources can be distinctively identified as shown in Fig. 20. From the performance parameters evaluated in Table I, the MUSIC-like algorithm is able to obtain the best overall performance for all experimental

Fig. 20. Image reconstruction based on GN-Laplace, MVDR, MUSIC, and MUSIC-like algorithm under experimental setup P_2

Fig. 21. Image reconstruction based on GN-Laplace, MVDR, MUSIC, and MUSIC-like algorithm under experimental setup P_3

setups (P_1 , P_2 , and P_3) especially the resolution performance of the case P_3 as shown in Fig. 22 which exhibits the 1 dimensional slice cut of Fig. 21 along the blue dashed line. As can be seen in Fig. 22, the MUSIC-like algorithm produces two distinctive peaks (compared with the other algorithms) along the source positions.

Next, the performance parameters of the spectral based algorithms (MVDR, MUSIC, and MUSIC-like) under ill-conditioned covariance matrix are evaluated and shown in Table II. The performance of MVDR and MUSIC algorithm are affected since their performances are directly linked to the eigenvalues of the covariance matrix, and hence distinct identification of both sources could not be obtained for P_2 and P_3 . However, the MUSIC-like algorithm has the ability to automatically adjust its performance through β parameter. Therefore, it is able to achieve better performance compared to MVDR and MUSIC algorithm. The overall performance of the MUSIC-like algorithm without diagonal loading term is slightly better due to the absence of additional noise injection ($\epsilon \mathbf{I}$).

TABLE II
PERFORMANCE EVALUATION OF SIMILAR SOURCE PROFILES WITHOUT DIAGONAL LOADING TERM

		MVDR		MUSIC		MUSIC-like	
		O_1	O_2	O_1	O_2	O_1	O_2
P_1	$PE(\%)$	14.7	12.8	10.0	10.5	3.1	6.9
	$RES(\times 10^{-2})$	5.10	7.30	2.60	5.80	0.03	0.45
P_2	$PE(\%)$	n/a	n/a	n/a	n/a	4.8	2.9
	$RES(\times 10^{-2})$	n/a	n/a	n/a	n/a	0.09	0.29
P_3	$PE(\%)$	n/a	n/a	n/a	n/a	3.0	1.6
	$RES(\times 10^{-2})$	n/a	n/a	n/a	n/a	0.58	0.96

Lastly, the performance parameters of two sources with different conductivity values and variations will be evaluated. The

Fig. 22. Normalized pixel intensity along 1D slice of reconstructed images for case P_3 .

metallic rods as shown in Fig. 17 (bottom figure) are now separated into two groups which are group $A = \{1, 3, 7, 8, 10, 11\}$ and group $B = \{2, 4, 5, 6, 9, 12, 13\}$ where on average the conductivity value and variation of group A is higher than group B . According to Fig. 18, the source O_1 now corresponds to the rods from group A (strong source) and the source O_2 now corresponds to the rods from group B (weak source). Each rod in group A is paired up with all rods in group B to generate each data snapshot, and hence 42 data snapshots were obtained for each position setup (P_1 , P_2 and P_3). Since the number of data snapshot is very limited, only the case where diagonal loading term is introduced will be evaluated.

TABLE III
PERFORMANCE EVALUATION OF DIFFERENT SOURCE PROFILES WITH DIAGONAL LOADING TERM

		GN-Laplace		MVDR		MUSIC		MUSIC-like	
		O_1	O_2	O_1	O_2	O_1	O_2	O_1	O_2
P_1	PE(%)	6.5	3.6	5.0	4.7	15.5	n/a	5.5	3.7
	RES($\times 10^{-2}$)	5.30	4.33	0.80	1.42	1.64	n/a	0.30	0.80
P_2	PE(%)	14.2	12.7	2.7	11.5	11.5	n/a	3.0	10.5
	RES($\times 10^{-2}$)	5.23	0.67	1.12	3.03	2.32	n/a	0.41	2.03
P_3	PE(%)	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
	RES($\times 10^{-2}$)	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a

As can be seen in Table III, the MUSIC algorithm is unable to identify the source O_2 which corresponds to the weak source. All algorithms are unable to identify closely spaced sources (P_3) where the strong source is located next to the weak source. From the result of Table III and Table I, we can see that the performance of MUSIC-like algorithm shares the robust characteristic of the MVDR algorithm and the high resolution performance of the MUSIC algorithm.

VII. CONCLUSION

In this paper, a thorough analysis on the effect of conductivity variation level, size, location, and the ill-conditioned array manifold to the eigenvalues extracted from the covariance matrix is fully explored which help to understand the common

advantage and disadvantages of the DOA based techniques. An alternative algorithm called MUSIC-like which has high resolution performance comparable to the MUSIC algorithm but avoids model order estimation is proposed specifically for the EIT environment. An EIT system working prototype comprises 8 electrodes was built to test the proposed method. Simulation and experimental results using the prototype confirm the advantage of using the MUSIC-like algorithm both from the evaluated performance parameters viewpoint as well as the detectability of sources.

REFERENCES

- [1] M. Meribout and I. M. Saied, "Real-time two-dimensional imaging of solid contaminants in gas pipelines using an electrical capacitance tomography system," *IEEE Trans. Ind. Electron.*, vol. 64, no. 5, pp. 3989–3996, May 2017.
- [2] D. S. Holder, *Electrical impedance tomography: methods, history and applications*. Bristol, U.K.: IOP Publishing, 2005.
- [3] M. Meribout and I. M. Saied, "Electrical impedance tomography: regularized imaging and contrast detection tomography system," *IEEE Trans. Med. Imag.*, vol. 15, no. 2, pp. 170–179, Apr. 1996.
- [4] A. Samorè, M. Guermandi, S. Placati, and R. Guerrieri, "Parametric detection and classification of compact conductivity contrasts with electrical impedance tomography," *IEEE Trans. Instrum. Meas.*, vol. 66, no. 10, pp. 2666–2679, Oct. 2017.
- [5] Z. Fan and R. X. Gao, "Enhancement of measurement efficiency for electrical capacitance tomography," *IEEE Trans. Instrum. Meas.*, vol. 60, no. 5, pp. 1699–1708, May 2011.
- [6] S. J. Kim and B. K. Kim, "Dynamic ultrasonic hybrid localization system for indoor mobile robots," *IEEE Trans. Ind. Electron.*, vol. 60, no. 10, pp. 4562–4573, Oct. 2013.
- [7] B. Wang, S. Zhou, W. Liu, and Y. Mo, "Indoor localization based on curve fitting and location search using received signal strength," *IEEE Trans. Ind. Electron.*, vol. 62, no. 1, pp. 572–582, Jan. 2015.
- [8] J. Wang, Q. Gao, Y. Yu, H. Wang, and M. Jin, "Toward robust indoor localization based on bayesian filter using chirp-spread-spectrum ranging," *IEEE Trans. Ind. Electron.*, vol. 59, no. 3, pp. 1622–1629, Mar. 2012.
- [9] F. Deng, S. Guan, X. Yue, X. Gu, J. Chen, J. Lv, and J. Li, "Energy-based sound source localization with low power consumption in wireless sensor networks," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 4894–4902, Jun. 2017.
- [10] L. C. Godara, *Smart antennas*, ch. 6, pp. 325–338. Boca Raton, FL: CRC press, 2004.
- [11] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, Mar. 1986.
- [12] M. Kaveh and A. Barabell, "The statistical performance of the MUSIC and the Minimum-Norm algorithms in resolving plane waves in noise," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 2, pp. 331–341, Apr. 1986.
- [13] O. Lee, J. Kim, Y. Bresler, and J. C. Ye, "Diffuse optical tomography using generalized MUSIC algorithm," in *Proc. IEEE Int. Symp. Biomed. Imag.*, pp. 1142–1145, Jun. 2011.
- [14] B. Scholz, "Towards virtual electrical breast biopsy: space-frequency music for trans-admittance data," *IEEE Trans. Med. Imag.*, vol. 21, no. 6, pp. 588–595, Jun. 2002.
- [15] M. Fernández-Corazza, N. V. Ellenrieder, and C. H. Muravchik, "Spatial filtering in electrical impedance tomography," *J. Phys. Conf. Ser.*, vol. 407, no. 1, Dec. 2012.
- [16] M. Fernández-Corazza, N. V. Ellenrieder, and C. H. Muravchik, "Linearly constrained minimum variance spatial filtering for localization of conductivity changes in electrical impedance tomography," *Int. J. Numer. Meth. Biomed. Engng.*, vol. 31, no. 2, Feb. 2015.
- [17] Z. Ying and B. P. Ng, "MUSIC-like DOA estimation without estimating the number of sources," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1668–1676, Mar. 2010.
- [18] V. V. Reddy, B. P. Ng, and A. W. H. Khong, "Insights into MUSIC-like algorithm," *IEEE Trans. Signal Process.*, vol. 61, no. 10, pp. 2551–2556, May 2013.
- [19] J. K. Seo and E. J. Woo, *Nonlinear inverse problems in imaging*, ch. 7, p. 209. Chichester, UK: John Wiley & Sons, 2013.

- [20] N. Polydorides and W. R. B. Lionheart, "A matlab toolkit for three-dimensional electrical impedance tomography: a contribution to the electrical impedance and diffuse optical reconstruction software project," *Meas. Sci. Technol.*, vol. 13, no. 12, pp. 1871–1883, Nov. 2002.
- [21] A. Adler *et al.*, "GREIT: a unified approach to 2D linear EIT reconstruction of lung images," *Physiol. Meas.*, vol. 30, no. 6, pp. S35–S55, Jun. 2009.
- [22] P. C. Hansen, "Regularization tools version 4.0 for matlab 7.3," *Numerical algorithms*, vol. 46, no. 2, pp. 189–194, Oct. 2007.
- [23] D. G. Manolakis, V. K. Ingle, and S. M. Kogon, *Statistical and adaptive signal processing: spectral estimation, signal modeling, adaptive filtering, and array processing*, ch. 11, pp. 660–661. Norwood, MA: Artech House, 2005.

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