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Coordination of Steer Angles, Tyre Inflation Pressure, Brake and Drive Torques for Vehicle Dynamics Control

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ABSTRACT

During vehicle operation, the control objectives of stability, handling, energy consumption and comfort have different priorities, which are determined by road conditions and driver behavior. To achieve better operation characteristics of vehicle, coordinated control of vehicle subsystems is actively used. The fact of more active vehicle subsystems in a modern passenger car provides more flexibility for vehicle control and control algorithm development. Since the modern vehicle can be considered as over-actuated system, control allocation is an effective control technique to solve such kind of problem.

This paper describes coordination of frictional brake system, individual-wheel drive electric motors, active front and rear steering, active camber mechanisms and tyre pressure control system. To coordinate vehicle subsystems, optimization-based control allocation with dynamic weights is applied. The influence of different weights (subsystem restriction) on criteria of vehicle dynamics (RMSE of yaw rate, sideslip angle, dynamic tyre load factor) and energy consumption and losses (consumed/recuperated energy during maneuver, longitudinal velocity decline, tyre energy dissipation) were analyzed. Based on this analysis, the optimal solution was selected. The proposed control strategy is based on the switching between optimal criteria related to vehicle safety and energy efficiency during vehicle motion. Dynamic weights were utilized to achieve this switching.

The simulation-based analysis and evaluation of both variants was carried out using a nonlinear vehicle model with detailed models of actuators. The test maneuver is 'Sine with Dwell'. Both variants of control allocation guarantees vehicle stability of motion and good handling. Meanwhile, proposed variant demonstrates slightly higher longitudinal velocity at the end of maneuver and higher amount of recuperated energy up to 15%; however, tyre dissipation energy increased to 5% compared to optimal solution from simulation-based analysis.

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INTRODUCTION

Increasing requirements to vehicle safety, performance and environmental friendliness dictate a development of more complex vehicle subsystems to achieve better operation characteristics. Thus, the amount of active subsystems in modern vehicle is increased, and their integrated / coordinated control becomes more significant year by year.

Main stages of integrated/coordinated control are depicted in [Figure 1](#). The control demand with generalized vehicle forces and torques is obtained from high-level controller of vehicle dynamics control. Next stage is a distribution of control demand between four wheels, taking into account e.g. force capacity of each wheel, normal distribution, power minimization and other objectives. Some solutions for force allocation were proposed [1,2,3]. After force distribution, the stage of subsystem coordination is carried out. Control inputs

of vehicle actuators are shown in [Figure 1](#) into outer ring. Moreover, there are some hypothetical subsystems such as regulation of tyre contact temperature, changing of down force using aerodynamic spoilers, and others. Thereby, the control demand from high-level controller can be realized by many ways according to control tasks in current conditions of vehicle motion. One of most common control technique to distribute control demand for over-actuated system is control allocation. A detailed state-of-the-art of control allocation methods and their application for various fields is represented in [4].

For integrated vehicle control a type of allocation parameters depends on actuator control inputs, desired cost functions, inverse tyre model, number of active subsystems and other factors. Below there are some examples of configurations of allocation parameters:

- longitudinal and lateral tyre forces [5-6]. Only the problem of force allocation is being solved;
- kinematic parameters of the tyre (longitudinal slip and wheel slip angles). Based on tyre inversion model, reference tyre kinematic parameters can be obtained for force allocation [7,8];
- reference control inputs of actuators (brake pressure / torque, drive torque, steering angle) [9]. Both tasks, force allocation and subsystem coordination, can be solved jointly;
- mixed force/actuator output, for instance, longitudinal forces and steering angle [10].

Some of research issues related to control allocation are shown in Figure 2. There are representation of actuator dynamics, approximation of nonlinear allocation, adaptation of control allocation to uncertainties and disturbances, achievement of several objectives. On the right side, some solutions for abovementioned problems are shown.

Figure 1. Control demand distribution

Figure 2. Research issues of control allocation

Multi-objective formulation is the most significant issue in the case of multi-actuators, because a set of control tasks can be considered. For instance, Chen and Wang [17] added power consumption (related to power consumption of all in-wheel motors in different modes) into the cost function, and called the method as “energy-efficient control allocation”. Kang et al. [18] proposed to add into the cost function not only energy consumption, but also energy loss (slip control). The drawbacks of the multi-objective cost function are that it could not be convex; the solution is optimal for all evaluation criteria with their pre-defined weighed coefficients; computational budget can increase as a result of smoothing of the function due to additional cost terms.

Another way to provide coordination between wheels/subsystems for several control tasks is dynamic weighting. Tagesson et al. [19] used a weighting matrix to coordinate brake pressures according to normal force distribution during heavy vehicle braking to obtain better vehicle stability. Yim et al. [20] proposed variable weighting to coordinate longitudinal and lateral forces developed by vehicle subsystems taking into account tyre slip and slip angles. Shyrokau and Wang [21] show that control allocation with dynamic weight scheduling demonstrates lower energy loss without significant impairment of stability of motion and vehicle handling compared to control allocation with fixed weight distribution.

The first aim of this paper is to investigate how vehicle subsystem coordination influences on vehicle operation characteristics. Based on this analysis, the optimal configuration of subsystem weights will be selected. The second aim is to investigate and demonstrate an effect of dynamic weighting compared with optimal solution from the first aim. The configuration of active vehicle subsystems includes active front steering, active rear steering, individual-wheel drive electric motors, electro-hydraulic brake system, active camber mechanisms and tyre inflation pressure system.

The paper is organized as follows: a brief description of vehicle model, which is used for analysis and evaluation, is presented in next section. The control system structure and control laws are discussed in Section 3; analysis of subsystem coordination is in Section 4; proposed strategy of dynamic weights is in Section 5. Thereupon simulation results are shown in Section 6. Conclusions and future work is discussed in Section 7.

VEHICLE MODEL

To analyze and evaluate a control system, original vehicle model for mid-sized passenger car was developed. The detailed mathematical description of vehicle model is described in [22]. The model has 14 DoF (schematically shown in Figure 3).

INTEGRATED CONTROL SYSTEM STRUCTURE

- Control system, depicted in [Figure 5](#), includes state observer, high-level controller for vehicle dynamics, middle-level block for optimization-based control allocation and lower-level individual controllers for each subsystem. The actuator control inputs are sent to full vehicle model with active subsystems, described in previous section.

The model of the electric driveline includes Thevenin battery model and four electric motors, which are represented by look-up tables with data from a real motor. The transient processes in the electric motors are described by the first-order plus time delay transfer function. The half-shaft torsion dynamics is neglected.

The diagram illustrates the control architecture for an electric vehicle. It starts with a **Reference vehicle model** that takes δ_{driver} as input and outputs reference states $V_{ref}, \beta_{ref}, \psi_{ref}$. These references are combined with δ_{driver} and fed into the **vehicle dynamics controller**, which produces $\Delta\beta, \Delta\psi$. These error signals are then processed by the **Control Allocation with Proposed Weight Strategy** block to generate control signals: u_{NPS}^{CA} , γ^N , δ^{N1} , δ^{N2} , M_{C1}^{CA} , M_{C2}^{CA} , and u_{BPS} . These signals are sent to the **Computer model for electric vehicle with active subsystems**, which contains five controllers: Inflation Pressure Controller, Camber angle Controller, Active Steering Controller, Electric Motor Controller, and Brake System Controller. The computer model also receives target signals $S^{Nref}, M^{Nref}, \delta^{Nref1}, \delta^{Nref2}, p_{Bref1}, p_{Bref2}, u_{Nref}, u_{\gamma ref}, u_{dref1}, u_{dref2}, u_{Cref1}, u_{Cref2}, u_{Bref1}$. It outputs estimated signals $S^{Nest}, M^{Nest}, \delta^{Nest1}, \delta^{Nest2}, p_{Best1}, p_{Best2}, u_{Nest}, u_{\gamma est}, u_{dest1}, u_{dest2}, u_{Cest1}, u_{Cest2}, u_{Best1}$. An **Extended Kalman Filter** at the bottom receives the reference control signals and outputs the estimated control signals $u_{Nest}, u_{\gamma est}, u_{dest1}, u_{dest2}, u_{Cest1}, u_{Cest2}, u_{Best1}$ back to the computer model. The vehicle dynamics controller also receives the estimated states $V_{ref}, \beta_{ref}, \psi_{ref}$ and outputs the vehicle velocity v .

Figure 5. Vehicle model verification.

Vehicle Dynamics and Actuators Control

The high-level control consists of reference vehicle model and vehicle dynamics controller. Generalized longitudinal and lateral forces and yaw torque are determined according to control errors. The reference longitudinal acceleration is calculated based on the measured pedal force [10]. For lateral dynamics, the reference vehicle model is based on classical bicycle vehicle model [24]. The reference yaw rate is constrained according to adhesion limits [25]. The reference sideslip angle is equal to zero.

The developed vehicle model is realized in Matlab/Simulink. The verification of the developed model is carried out by the comparison of the developed vehicle behavior with the multibody vehicle model in the commercial software under vehicle motion without control. The simulation results of ‘Sine with Dwell’ test at initial velocity of 80 km/h and maximal amplitude of steering wheel angle of 100, 140 and 160 deg are shown in [Figure 4](#). The developed model demonstrates close behavior to multibody model.

(1)

Proportional and integration gains are taken according to recommendations from [9]:

$$K_i = 20 \text{diag}([0.2m_a \quad 0.7m_a \quad 1.5I_{zz}])$$

$$K_p = \frac{2}{3}m_a \sqrt{K_i \text{diag}\left(\left[\frac{16}{m_a} \quad \frac{1}{m_a} \quad \frac{1}{I_{zz}}\right]\right)} \quad (2)$$

Actuator control should achieve the following targets:

- to guarantee precise tracking to reference control signals obtained from the middle level;
- to estimate boundary conditions for control allocation taking into account subsystem behavior and skid/slip control.

The control signals for front and rear steering, camber mechanisms and tyre inflation pressure system are additional steer angles, camber angles and tyre inflation pressures from middle-level control allocation accordingly.

Assuming that real torque of electric motor is precisely estimated, PWM signals are founded using PI motor control:

$$\begin{cases} u_{em,i}^c = K_p^{drive} e_{em,i} + K_i^{drive} \int e_{em,i} dt \\ e_{em,i} = M_{em,i}^{CA} - \hat{M}_{em,i}^{est} \end{cases} \quad (3)$$

Control signals for inlet and outlet valves of frictional EHB system are formulated as:

$$\begin{cases} u_{br,i}^c = K_p^{brake} e_{br,i} + K_i^{brake} \int e_{br,i} dt + K_d^{brake} \frac{de_{br,i}}{dt}; \\ e_{br,i} = p_{br,i}^{ref} - p_{br,i}^{mes} \end{cases}$$

$$u_{i,ideal}^{inlet} = \begin{cases} 1, & e_{br,i} \geq 0 \& |u_{br,i}^c| \geq 1 \\ |u_{br,i}^c|, & e_{br,i} \geq 0 \& |u_{br,i}^c| < 1 \\ 0, & e_{br,i} < 0 \end{cases}$$

$$u_{i,ideal}^{outlet} = \begin{cases} 0, & e_{br,i} \geq 0 \\ |u_{br,i}^c|, & e_{br,i} < 0 \& |u_{br,i}^c| < 1 \\ 1, & e_{br,i} < 0 \& |u_{br,i}^c| \geq 1 \end{cases} \quad (4)$$

The position limits of electric motors and brake system are varied to reach wheel skid/slip minimization and to correct limits according to subsystem behavior. For instance, torque limit of electric motor is corrected by weighed coefficients, which are related to electric motor temperature, state-of-the-charge, longitudinal vehicle velocity, under and overvoltage protection and fault control [22]:

$$M_i^{em,lim} = f(u_{em,i}^c, w_m) \prod w_i(T_i, SOC, V_{xa}, U_i, fault) \quad (5)$$

The constructive parameters of brake mechanism and the level of wheel slip restrict the torque limit of frictional

brakes. The correction taking into account wheel slip is carried out based on the threshold of wheel slip. The algorithm is rule-based and similar to control of anti-lock brake system [26].

Optimization-Based Control Allocation

The relationship between generalized forces and yaw torque from control demand and control inputs is nonlinear. To approximate this relationship into linear case, the feedback linearization by cancelling of nonlinear term is used [10]:

$$v - f(x) = v^* = \frac{\partial g(x, u)}{\partial u} u \approx B_u u \quad (6)$$

Tyre forces can be written in the following form:

$$F_{xwi} = \underbrace{\varepsilon_{Fxi}}_{system} + \underbrace{\frac{1}{\eta_{Fxi}} T_i^{em} + \frac{1}{\eta_{Fxi}} T_i^{br}}_{actuators} + \tilde{C}_{p_{ws}} \Delta p_{wi}$$

$$F_{ywi} = \underbrace{\tilde{C}_{ywi} \left(\tan \frac{V_{yi}}{V_{xi}} - \delta_{wi}^{w/o corr} \right)}_{system} + \underbrace{\tilde{C}_{ywi} \delta_{afs} + \tilde{C}_{ywi} \delta_{ars} + \tilde{C}_{\gamma_i} \gamma_i + \tilde{C}_{p_{wy}} \Delta p_{wi}}_{actuators} \quad (7)$$

Vehicle velocities projections V_{xi} and V_{yi} are calculated based on information from state observer. The wheel steer angles without corrections from active front and rear steering systems are:

$$\delta_{wi}^{w/o corr} = \delta_{wi} - [\delta_{afs} \quad \delta_{ars} \quad \delta_{ars} \quad \delta_{ars}]^T \quad (8)$$

To describe wheel dynamics in braking mode, the coefficients ε_{Fxi} and η_{Fxi} , are founded as [27]:

$$\eta_{Fxi} = \frac{J_w V_{xw}}{\tilde{C}_{xwi} r_w t_s}; \quad \varepsilon_{Fxi} = \left(1 - \frac{r_w}{\eta_{Fxi}} \right) \hat{F}_{xwi} \quad (9)$$

Since the maximal drive torque is lower than brake one, in driving mode wheel dynamics is neglected.

Tyre longitudinal stiffness $\tilde{C}_{xwi}$, cornering stiffness $\tilde{C}_{ywi}$, camber stiffness $\tilde{C}_{\gamma_i}$ and coefficient $\tilde{C}_{p_{ws}}$, related to changing of longitudinal force depends on inflation pressure, are numerically computed as partial derivatives of longitudinal and lateral tyre forces from Pacejka model by Taylor series explanation with sampling time of 1ms.

To find nonlinear term $f(x)$, only tyre forces related to system behavior is used. Tyre projections along the vehicle axes F_{xi}^{sys} and F_{yi}^{sys} are related to tyre forces along wheel axes:

$$\begin{bmatrix} F_{xi}^{sys} \\ F_{yi}^{sys} \end{bmatrix} = \begin{bmatrix} \cos \delta_{wi} & -\sin \delta_{wi} \\ \sin \delta_{wi} & \cos \delta_{wi} \end{bmatrix} \begin{bmatrix} F_{xwi}^{system} \\ F_{ywi}^{system} \end{bmatrix} \quad (10)$$

The mathematical model of vehicle motion is:

$$f(x) = \begin{bmatrix} m_a V_{ya} \dot{\psi} + \sum_{i=1}^4 F_{xi}^{sys} \\ -m_a V_{xa} \dot{\psi} + \sum_{i=1}^4 F_{yi}^{sys} \\ a \sum_{i=1}^2 F_{yi}^{sys} - b \sum_{i=3}^4 F_{yi}^{sys} + \sum_{i=f,r} t_i \frac{-F_{x,il}^{sys} + F_{x,ir}^{sys}}{2} \end{bmatrix} \quad (11)$$

Since the six vehicle subsystems are considered, and electric motors can develop positive (traction) and negative (brake) torques, the control input vector is given as:

$$u^{CA} = [\delta_{afs}^{CA}, \delta_{ars}^{CA}, M_{em,i}^{CA,pos}, M_{em,i}^{CA,neg}, M_{br,i}^{CA}, \gamma_i^{CA}, \Delta p_{wi}^{CA}]^T \quad (12)$$

The control effectiveness matrix B_u is equal to:

$$B_u = \begin{bmatrix} B_{steer} & B_{emotor}^{pos} & B_{emotor}^{neg} & B_{brake} & B_{\gamma} & B_{pw} \end{bmatrix} \quad (13)$$

The parameters of control effectiveness matrix B_{steer} is founded as:

$$B_{steer} = \begin{bmatrix} -\sum_{i=1}^2 \tilde{C}_{ywi} \sin \delta_i & -\sum_{i=3}^4 \tilde{C}_{ywi} \sin \delta_i \\ \sum_{i=1}^2 \tilde{C}_{ywi} \cos \delta_i & \sum_{i=3}^4 \tilde{C}_{ywi} \cos \delta_i \\ B_{steer}^{31} & B_{steer}^{32} \end{bmatrix} \quad (14)$$

$$B_{steer}^{31} = 0.5t_f \sum_{i=1}^2 (-1)^{i+1} \tilde{C}_{ywi} \sin \delta_i + a \sum_{i=1}^2 \tilde{C}_{ywi} \cos \delta_i;$$

$$B_{steer}^{32} = 0.5t_r \sum_{i=3}^4 (-1)^{i+1} \tilde{C}_{ywi} \sin \delta_i - b \sum_{i=3}^4 \tilde{C}_{ywi} \cos \delta_i$$

Control effectiveness matrix for brake system B_{brake} can be written:

$$B_{brake} = \begin{bmatrix} \eta_{fl}^{-1} \cos \delta_{fl} & \eta_{fr}^{-1} \cos \delta_{fr} & \eta_{rl}^{-1} \cos \delta_{rl} & \eta_{rr}^{-1} \cos \delta_{rr} \\ \eta_{fl}^{-1} \sin \delta_{fl} & \eta_{fr}^{-1} \sin \delta_{fr} & \eta_{rl}^{-1} \sin \delta_{rl} & \eta_{rr}^{-1} \sin \delta_{rr} \\ B_x^{31} \eta_{fl}^{-1} & B_x^{32} \eta_{fr}^{-1} & B_x^{33} \eta_{rl}^{-1} & B_x^{34} \eta_{rr}^{-1} \end{bmatrix} \quad (15)$$

$$B_x^{31} = -0.5t_f \cos \delta_{fl} + a \sin \delta_{fl}; B_x^{32} = 0.5t_f \cos \delta_{fr} + a \sin \delta_{fr};$$

$$B_x^{33} = -0.5t_r \cos \delta_{rl} - b \sin \delta_{rl}; B_x^{34} = 0.5t_r \cos \delta_{rr} - b \sin \delta_{rr};$$

The control effectiveness matrices for electric motors and brake system are equal:

$$B_{emotor}^{pos} = B_{emotor}^{neg} = B_{brake} \quad (16)$$

Control effectiveness matrix for camber mechanisms B_{γ} can be calculated as:

$$B_{\gamma} = \begin{bmatrix} \tilde{C}_{\gamma fl} \sin \delta_{fl} & \tilde{C}_{\gamma fr} \sin \delta_{fr} & \tilde{C}_{\gamma rl} \sin \delta_{rl} & \tilde{C}_{\gamma rr} \sin \delta_{rr} \\ \tilde{C}_{\gamma fl} \cos \delta_{fl} & \tilde{C}_{\gamma fr} \cos \delta_{fr} & \tilde{C}_{\gamma rl} \cos \delta_{rl} & \tilde{C}_{\gamma rr} \cos \delta_{rr} \\ B_y^{31} \tilde{C}_{\gamma fl} & B_y^{32} \tilde{C}_{\gamma fr} & B_y^{33} \tilde{C}_{\gamma rl} & B_y^{34} \tilde{C}_{\gamma rr} \end{bmatrix} \quad (17)$$

$$B_y^{31} = -0.5t_f \sin \delta_{fl} + a \cos \delta_{fl}; B_y^{32} = 0.5t_f \sin \delta_{fr} + a \cos \delta_{fr};$$

$$B_y^{33} = -0.5t_r \sin \delta_{rl} - b \cos \delta_{rl}; B_y^{34} = 0.5t_r \sin \delta_{rr} - b \cos \delta_{rr}$$

Although tyre inflation pressure influences on lateral force, we consider, the aim of inflation pressure allocation is to increase longitudinal forces. Therefore, control effectiveness matrix for tyre inflation pressure system B_{pw} is defined as:

$$B_{pw} = \begin{bmatrix} \tilde{C}_{p_{wx}}^{fl} \cos \delta_{fl} & \tilde{C}_{p_{wx}}^{fr} \cos \delta_{fr} & \tilde{C}_{p_{wx}}^{rl} \cos \delta_{rl} & \tilde{C}_{p_{wx}}^{rr} \cos \delta_{rr} \\ \tilde{C}_{p_{wx}}^{fl} \sin \delta_{fl} & \tilde{C}_{p_{wx}}^{fr} \sin \delta_{fr} & \tilde{C}_{p_{wx}}^{rl} \sin \delta_{rl} & \tilde{C}_{p_{wx}}^{rr} \sin \delta_{rr} \\ B_x^{31} \tilde{C}_{p_{wx}}^{fl} & B_x^{32} \tilde{C}_{p_{wx}}^{fr} & B_x^{33} \tilde{C}_{p_{wx}}^{rl} & B_x^{34} \tilde{C}_{p_{wx}}^{rr} \end{bmatrix} \quad (18)$$

The l_2 -optimal control allocation problem is formulated as a minimization of allocation error $(B_u u^{CA} - v^*)$ and control actuations u^{CA} , taking into account actuator constraints [11]:

$$u_{CA} = \arg \min_{u_{lim}^{low} \leq u_{CA} \leq u_{lim}^{up}} \left(\|W_v (B_u u_{CA} - v^*)\|_2^2 + \zeta \|W_u u_{CA}\|_2^2 \right) \quad (19)$$

The parameter ζ defines a significance of actuation minimization and equal to 0.1. Matrix W_v is used to set up a priority among generalized forces and yaw torque. In this paper, longitudinal generalized force is omitted, because there is no aim to support longitudinal velocity at given value. Hence, $W_v = \text{diag}([1e-9 \ 1 \ 1])$. A description of weighting matrix W_u to prioritize actuators is described below. The fixed-point method is suitable for the application in the real-time systems to solve optimization problem [8].

Since tyres have own force capacity (tyre workload), the tyre limitation are related to longitudinal tyre forces for electric motor and frictional brake system. It is assumed that tyre pressure variation has no strong impact and can be neglected for tyre limitation:

$$u_{min}^{tyre} = \begin{bmatrix} 0 & -F_{xw}^{lim} r_w - M_{br,i} & -F_{xw}^{lim} r_w \end{bmatrix}$$

$$u_{max}^{tyre} = \begin{bmatrix} F_{xw}^{lim} r_w & 0 & 0 \end{bmatrix}$$

$$(20)$$

The limit of longitudinal force is calculated from friction ellipse based on normal and lateral force, and friction coefficient.

The actuator position and rates limits are used as optimization constraints, because actuator dynamics are limited. Their values are shown on Table 1. Since the powertrain architecture has near-wheel motors, the rate limit of electric motor is less compared with friction brake system due to low natural frequency of the shaft-shafts. In the case of the application of in-wheel motor, the rate of electric motor will be higher [28].

Table 1. Position and rate limits of actuators

Subsystem	Position limit	Rate limit
AFS	$-25^0 \dots 25^0$	$45^0/\text{sec}$
ARS	$-15^0 \dots 15^0$	$20^0/\text{sec}$
Camber angle	$-10^0 \dots 10^0$	$10^0/\text{sec}$
Electric motor	vary	3000 Nm/sec
Brake system	vary	reduction / build-up $8000 / 12000 \text{ Nm/sec}$
Inflation pressure	0.1 MPa	0.1 MPa/sec

The final constraints for optimization can be written as:

$$\begin{aligned} u_{\lim}^{\text{low}} &= \max(u_{\min}^{\text{position}}, u(t - t_s) - \dot{u}_{\text{rate}} t_s, u_{\min}^{\text{tyre}}) \\ u_{\lim}^{\text{up}} &= \min(u_{\max}^{\text{position}}, u(t - t_s) + \dot{u}_{\text{rate}} t_s, u_{\max}^{\text{tyre}}) \end{aligned} \quad (21)$$

ANALYSIS OF SUBSYSTEM COORDINATION

As noted in previous section, weight coefficients in matrix W_u restrict a participation of vehicle subsystem to generate control demand. The higher value of weight coefficient means higher restriction or lower priority of vehicle subsystem. To analyze subsystem coordination, weight coefficients will be varied within feasible range. The vehicle maneuver is 'Sine with Dwell' at initial velocity of 80 km/h and maximal steering amplitude of 150 deg. The maneuver is repeated for each configuration of weight coefficients. Each simulation is evaluated using the following criteria:

- root mean square error of yaw rate, deg/s:

$$RMSE\dot{\psi} = \sqrt{\frac{1}{T} \int_0^T (\dot{\psi}_{\text{ref}} - \dot{\psi}_{\text{mes}})^2 dt} \quad (22)$$

- root mean square of sideslip angle, deg:

$$RMS\beta_{\text{mes}} = \sqrt{\frac{1}{T} \int_0^T \beta_{\text{mes}}^2 dt} \quad (23)$$

- loss of longitudinal velocity after maneuver, km/h:

$$\Delta V_{xa}^{\text{loss}} = V_{xa}^{\text{init}} - V_{xa} \quad (24)$$

- total tyre energy dissipation, kJ (only for Figure 6 and 8):

$$E_{\text{tyre}}^{\text{total}} = 10^{-3} \sum_{i=1}^4 \int_0^T (F_{xwi} V_{swxi} + F_{ywi} V_{swyi}) dt \quad (25)$$

- total energy consumption, kJ (only for Figure 7 and 8):

$$E_{\text{em}}^{\text{total}} = 10^{-3} \sum_{i=1}^4 \int_0^T P_{\text{em},i} dt \quad (26)$$

- dynamic tyre load factor [29] (only for Figure 8):

$$n_R = 0.25 \sum_{i=1}^4 \sqrt{\frac{1}{T} \int_0^T (F_{zi} - F_{zi}^{\text{static}})^2 dt} \quad (27)$$

Subsystem coordination will be analyzed in three stages:

1) Coordination between AFS, ARS and camber angle variation is considered. Other subsystems are deactivated. The weight coefficients r_i are changed from 0 (no restriction) to 1 (max. restriction). Assuming that subsystem restrictions can be defined by the following rule:

$$r_1 + r_2 + r_3 = 1; \quad 0 \leq r_i \leq 1 \quad (28)$$

the matrix W_u is:

$$W_u = \text{diag}([r_1, r_2, 0^{1 \times 4}, 0^{1 \times 4}, 0^{1 \times 4}, r_3^{1 \times 4}, 0^{1 \times 4}]) \quad (29)$$

This assumption allows to present graphically three subsystems together using ternary plots. Simulation results of coordination between AFS, ARS and camber angle variation with other deactivated subsystems are shown in Figure 6. RMS error of yaw rate is changed from 4.0 to 6 deg/s, RMS of sideslip angle - from 2.0 to 3.2 deg, loss of longitudinal velocity - from 5.4 to 6.6 km/h, and total tyre energy dissipation for all wheels - from 38 to 50 kJ.

For the case of subsystem restriction formulated by eq. (28), AFS and ARS provide better characteristics compared with camber angle variation (in the top of triangles). The explanation of this fact is lower capacity to change lateral force due to camber stiffness and rate of camber angle changing compared with AFS and ARS. However, in this paper coordination of camber angle was carried out without the correction related to normal force distribution (for instance, based on [19]). Normal force distribution has strong impact on camber angle coordination. In future work, the authors will investigate this aspect in details. As a result,

selected weights are 0.1 for AFS, 0.1 for ARS and 0.8 for camber angle variation.

2) Next stage considers coordination between frictional brake torque, brake torque from electric motor and tyre inflation pressure. Drive torque from electric motor, AFS, ARS and camber angle are deactivated. The matrix W_u is defined as:

$$W_u = \text{diag} \left(\begin{bmatrix} 0, 0, 0^{1 \times 4}, r_1^{1 \times 4}, r_2^{1 \times 4}, 0^{1 \times 4}, r_3^{1 \times 4} \end{bmatrix} \right) \quad (30)$$

Figure 6. Coordination of AFS, ARS and camber angle.

Simulation results are shown in Figure 7. Instead of tyre energy dissipation, the energy of recuperation is used. RMS error of yaw rate is changed from 2.5 to 7.0 deg/s, RMS of sideslip angle - from 1.5 to 4 deg, recuperated energy - 2 to 20 kJ, and loss of longitudinal velocity - from 8 to 11 km/h. Minimal RMSE of yaw rate and RMS of sideslip angle is obtained at minimal restriction of brake torque from brake system and electric motor; however, it induces higher losses of longitudinal velocity. Meanwhile, low restriction of tyre pressure variation causes higher errors of yaw rate and sideslip angle. Total recuperated energy is calculated as potential energy from electric motor, assuming there are no rate constraints for energy accumulator. From point of vehicle safety, yaw rate and sideslip angle are more significant, therefore, the selected weights are 0.1 for frictional brake system, 0.1 for brake torque generated by electric motor and 0.8 for tyre inflation pressure variation.

Figure 7. Coordination of brake torques and tyre pressure.

3) Coordination between all vehicle subsystems is considered on the last stage. The relations obtained in previous stages are used to define proportions of subsystem participation. The matrix W_u has the following view:

$$W_u = \text{diag} \left(\begin{bmatrix} 0.1r_1, 0.1r_1, r_3^{1 \times 4}, 0.1r_2^{1 \times 4}, 0.1r_2^{1 \times 4}, 0.8r_1^{1 \times 4}, 0.8r_2^{1 \times 4} \end{bmatrix} \right)$$

Hence, **gr#1** is related to AFS, ARS and camber angle variation, and **gr#2** with frictional brake system, brake torque from electric motor and tyre inflation pressure variation. The simulation results are shown in Figure 8.

RMS error of yaw rate is changed from 2.2 to 3.2 deg/s, RMS of sideslip angle - from 1.2 to 1.6 deg, energy consumption - from -20 to 15 kJ (negative sign means recuperated energy), loss of longitudinal velocity - from 5.5 to 11 km/h, total tyre energy dissipation for all wheels - from 29 to 34 kJ, and dynamic tyre load factor from 0.262 to 0.268.

Higher restriction of group **gr#1**, related to AFS, ARS and camber angle variation, improves RMS error of yaw rate, RMS of sideslip angle and tyre energy dissipation up to 0.6, meanwhile loss of longitudinal velocity increases. The increasing of the group **gr#2** restriction should be in the range between 0.1-0.3. This weight interval guarantees stability criteria and higher recuperated energy. Based on this information, the weights are 0.2 for drive torque, 0.2 for **gr#2** and 0.6 for **gr#1**. The graph of dynamic tyre load factor shows that influence of all subsystems on dynamic load transfer exists; however, it is varied in small range. For future investigation to change and improve dynamic tyre load

factor, the suspension control (variable stiffness, active damping and anti-roll bars) will be analyzed.

Figure 8. Coordination of all vehicle subsystems.

PROPOSED CONTROL STRATEGY

To demonstrate the main idea of proposed strategy, the following evaluated criteria are formulated:

- criterion of vehicle stability and handling (“safety” criterion). Since the vehicle moves on the road with high friction coefficient, error of yaw rate should have high priority. For ice, the weight factors will be conversely:

$$C_{s\&h} = 0.7 \frac{RMSE\psi}{\max(RMSE\psi)} + 0.3 \frac{RMS\beta_{mes}}{\max(RMS\beta_{mes})} \quad (32)$$

- criterion of energy consumption and energy losses (“energy” criterion). Energy losses should be minimized with the highest priority:

$$C_{en} = 0.1 \frac{E_{em}^{total}}{\max(E_{em}^{total})} + 0.7 \frac{\Delta V_{xa}^{loss}}{\max(\Delta V_{xa}^{loss})} + 0.2 \frac{E_{tyre}^{total}}{\max(E_{tyre}^{total})} \quad (33)$$

- complex criterion of stability, handling, energy consumption and energy losses. Stability of vehicle motion is more important compared for energy consumption and energy losses:

$$C_{total} = 0.7C_{s\&h} + 0.3C_{en} \quad (34)$$

The criteria obtained are shown in Figure 9. The left figure is complex criterion, the right figure presents “safety” criterion (blue color) and “energy” criterion (red color).

Figure 9. Evaluated criteria.

It should be noted, that minimum of complex criterion is obtained at above-mentioned weights. Meanwhile, the minimums of “safety” and “energy” criteria are in contradictory positions. The idea of proposed control strategy is the switching between “safety” and “energy” criteria during vehicle motion. Thereby, the initial weights will be defined around “energy” criterion, and, with the increasing of control errors the weights will be changed to the minimum of “safety” criterion. We suppose the similar effect can be obtained by multi-objective cost function [17,18]. Then the weight coefficients, which define significance of energy consumption and energy losses, will be changed during vehicle motion.

To investigate proposed strategy, the weights will be changed according to error of yaw rate. For gr#1 the weights will be changed from 0.50 to 0.65, for gr#2 - from 0.1 to 0.15 and for drive - from 0.4 to 0.20. The changing is in direct ratio with yaw rate error. Coordination between frictional brake system and tyre inflation system is changed: for frictional brake system is from 0.5 to 0.1, for tyre inflation system - from 0.4 to 0.8 proportionally to yaw rate error. The weight for brake torque of electric motor is constant. The results are presented in the next section.

SIMULATION RESULTS

Simulation maneuver is the same as noted above, at initial velocity of 80 km/h and maximal steering amplitude of 150

deg. The simulation results are shown in Figure 10. The comparison between proposed strategy and control allocation with fixed weights at the minimum of complex criterion are carried out. The both control allocations demonstrate good tracking of reference yaw rate and sideslip angle don't exceed 3 deg. Meanwhile, the longitudinal velocity for proposed strategy is slightly higher at the end of maneuver compared with fixed weights. The amount of recuperated energy is higher up to 15% for proposed strategy.

However, the tyre energy dissipation for proposed strategy is higher due to more frequent use of subsystems related to changing of lateral force. Since the weights of subsystem restriction are changed intensively, one of potential problem can be chatter of subsystem actuation. To solve this problem, the minimum time of subsystem operation can be used.

Figure 10. Characteristics of vehicle motion.

Subsystem actuations are shown in Figure 11. The left graphs represent control allocation with fixed weights, the right one - proposed control strategy. The effect of increasing of vehicle velocity is due to the reduction of brake torques from frictional brake system. The energy recuperation is higher due to lower drive torques and the application of other subsystems. The control demands (generalized lateral force and yaw torque) from high-level controller, control allocation and actuators are shown in Figure 12.

Figure 11. Subsystem actuations.

The difference between the demands from high-level controller and control allocation is caused by the second term into the cost function (19) related to actuation minimization. The difference between demands from control allocation and actuators is defined by actuator dynamics. In the paper the actuator dynamics was considered as static in control allocation.

Figure 12. Control and realized demands.

To explain the effect obtained using proposed strategy, the following variants are considered: var#1 - static weights from the minimum of complex criteria; var#2 - static weights related to “energy” criteria, when control error is zero; var#3 - static weights related to “safety” criteria, when control error is maximal; var#4 - dynamic weights.

Simulation results are on Table 2. The parameters obtained lie between points var#2 and var#3.

Table 2. Simulation results

Parameter	var#1	var#2	var#3	var#4
RMSE $\dot{\psi}$, deg/s	2.21	2.69	2.20	2.50
RMS β , deg	1.18	1.42	1.17	1.30
$V_{x,final}$, km/h	69.1	70.1	68.8	70.0
$E_{em,total}$, kJ	-13.6	-16.2	-14.4	-16.1
$E_{tyre,total}$, kJ	29.1	31.9	28.9	30.7

The effect of proposed strategy is quite small. The increasing of final velocity is up to 1 km/h and amount of recuperated energy is 2.5 kJ. Firstly, it should be noted, the effect was obtained in the framework of one maneuver. The total amount of recuperated energy and the reduction of velocity loss will be higher, if the cumulative effect for longer period will be considered. Secondly, main aim was related to guarantee vehicle stability, energy efficiency was the second issue with lower priority. Thirdly, proposed strategy is compared with optimal solution based on simulation-based analysis, which provides high stability and energy efficiency. In our previous work, the control allocation with dynamic weight scheduling was considered in comparison with the control allocation where each subsystem has equal participation [21]. Thirdly, the changing law was selected as proportional and it is connected only with error of yaw rate. In future works, the authors will analyze different laws of weight changing and investigate proposed control strategy using hardware-in-the-loop test rig with hardware

components of friction brake system, steering and dynamic tyre pressure control [30].

SUMMARY/CONCLUSIONS

Coordination between frictional brake system, individual-wheel drive electric motors, active front and rear steering, active camber mechanisms and tyre pressure control system was considered in the paper. To coordinate vehicle subsystems, optimization-based control allocation is applied. Besides control allocation block, the control system includes vehicle motion control based on PI control with anti-windup based on back calculation, state observer and lower-level controllers for each of vehicle subsystem.

The influence of different weights (subsystem restriction) on criteria of vehicle dynamics (RMSE of yaw rate, sideslip angle, dynamic tyre load factor) and energy consumption and losses (consumed/recuperated energy during maneuver, longitudinal velocity decline, tyre energy dissipation) were analyzed. Using results of simulation-based analysis, the optimal solution was selected.

The control strategy based on the switching between optimal criteria related with vehicle safety and energy efficiency during vehicle motion was proposed. Dynamic weights were applied for subsystem coordination / prioritization.

To analyze and evaluate both variants, a nonlinear vehicle model with detailed models of actuators is used. The simulation-based analysis and evaluation was carried out using ‘Sine with Dwell’ test. Both variants of control allocation guarantees vehicle stability of motion and good handling. Meanwhile, proposed variant demonstrates slightly higher longitudinal velocity at the end of maneuver and higher amount of recuperated energy up to 15%, but tyre dissipation energy increased to 5% compared with optimal solution from simulation-based analysis.

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DEFINITIONS/ABBREVIATIONS

AFS - active front steering

ARS - active rear steering

EHB - electro-hydraulic brake

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