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2014

Zuo, C., Chen, Q., & Asundi, A. (2014). Phase space representation of transport of intensity phase retrieval for partially coherent fields. Proceedings SPIE 9234, International Conference on Experimental Mechanics 2013 and Twelfth Asian Conference on Experimental Mechanics.

<https://hdl.handle.net/10356/104867>

<https://doi.org/10.1117/12.2054320>

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Phase space representation of transport of intensity phase retrieval for partially coherent fields

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ABSTRACT

The concept of phase-space representation of transport of intensity phase retrieval is developed, in particular for partially coherent fields. It can be used to interpret the physical meaning of the phase recovered from the TIE, investigate the effect of partially coherent illumination and finite aperture of imaging systems on phase retrieval.

Keywords: transport-of-intensity equation; phase retrieval; phase space; partially coherent fields

1. INTRODUCTION

The transport of intensity equation (TIE) was originally derived by Teague [1] as a non-iterative propagation-based phase retrieval method for a completely coherent field, in which context the definition of phase is clear. However, as we know no currently physically realizable sources are intrinsically perfectly coherent, and one must consider the effect of partial coherence on phase retrieval. The fact that TIE is not restricted to the purely coherent regime but works also with a partially coherent source seems to be very well known, and a wide range of applications in x-ray diffraction [2], electron-beam microscopy [3], and quantitative optical phase microscopy [4, 5] have been reported. However, partially coherent field does not have a well-defined phase since the field experiences statistical fluctuations over time, which brings up an important question about what “phase” is being measured in such scenarios. The aim of this paper to extend the theory of phase-space optics to reinterpret TIE particularly for a partially coherent field. The mutual intensity function is conveniently replaced by the Wigner distribution function, yielding an elegant description and intuitive understanding of issues related to TIE and optical systems under partially coherent illumination. With this tool, we interpret the physical meaning of the phase recovered from the TIE, investigate the effect of partially coherent illumination and finite aperture of imaging systems on phase retrieval.

2. THEORY

Let us consider the partially coherent stochastic field $u(\mathbf{x})$ to be paraxial, temporally stationary, and quasi-monochromatic, which can be defined by the Wigner distribution function (WDF) [6]

$$W(\mathbf{x}, \mathbf{u}) = \int \Gamma\left(\mathbf{x} + \frac{\mathbf{x}'}{2}, \mathbf{x} - \frac{\mathbf{x}'}{2}\right) \exp(-i2\pi\mathbf{u}\mathbf{x}') d\mathbf{x}', \quad (1)$$

where $\mathbf{x} = (x, y)$ and $\mathbf{u} = (u, v)$ are the two-dimensional spatial and spatial frequency vectors, respectively, Γ is the mutual intensity related to the ensemble average $\Gamma(\mathbf{x}_1, \mathbf{x}_2) = \langle u(\mathbf{x}_1)u^*(\mathbf{x}_2) \rangle$. The intensity of the wave field is given by the values on the diagonal of the mutual intensity, or equivalently, a projection of the WDF along the $\mathbf{u}$ axis

$$I(\mathbf{x}) = \Gamma(\mathbf{x}, \mathbf{x}) = \int W(\mathbf{x}, \mathbf{u}) d\mathbf{u}. \quad (2)$$

The paraxial propagation of WDF obeys a so-called transport equation [6], whose solution takes the form

$$W_z(\mathbf{x}, \mathbf{u}) = W_0(\mathbf{x} - \lambda z \mathbf{u}, \mathbf{u}) \quad (3)$$

where z is the propagation distance and λ is the wavelength of the quasi-monochromatic illumination. Combining the definition of intensity given in Eq. (2), we may find that

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\lambda \nabla_{\mathbf{x}} \cdot \int \mathbf{u} W(\mathbf{x}, \mathbf{u}) d\mathbf{u}, \quad (4)$$

where $\nabla_{\mathbf{x}}$ is the 2D gradient operator over $\mathbf{x}$. Equation (4) can be regarded as TIE for a paraxial partially coherent field which reveals that the transport of intensity along optical axis gives the spatial divergence of first-order local moment of WDF with respect to the frequency variable. In the limit of the completely coherent wave, the field $u(\mathbf{x})$ then becomes deterministic and can be fully described by the complex amplitude function. Without loss of generality, considering the case that a thin sample of transmittance $T(\mathbf{x})$ located at the plane $z = 0$ illuminated with a plane monochromatic wave with uniform unit intensity, the complex field immediately after passing through the sample can be represented by $u(\mathbf{x}) = T(\mathbf{x}) = \sqrt{\tau(\mathbf{x})} \exp[i\phi(\mathbf{x})]$, where $\tau(\mathbf{x})$ and $\phi(\mathbf{x})$ are the amplitude and the phase functions of the specimen. Then the mutual intensity $\Gamma(\mathbf{x} + \mathbf{x}'/2, \mathbf{x} - \mathbf{x}'/2)$ in Eq. (1) is equal to the product $T(\mathbf{x} + \mathbf{x}'/2)T^*(\mathbf{x} - \mathbf{x}'/2)$, where the asterisk denotes complex conjugation. The WDF of the coherent field thus takes the form

$$W_T(\mathbf{x}, \mathbf{u}) = \int T\left(\mathbf{x} + \frac{\mathbf{x}'}{2}\right) T^*\left(\mathbf{x} - \frac{\mathbf{x}'}{2}\right) \exp(-i2\pi \mathbf{u} \mathbf{x}') d\mathbf{x}'. \quad (5)$$

The normalized first order local frequency moment (or center of gravity) of the WDF is associated with instantaneous frequency [7], which is defined as the transverse derivative of the phase of the complex field:

$$\frac{\int \mathbf{u} W_T(\mathbf{x}, \mathbf{u}) d\mathbf{u}}{\int W_T(\mathbf{x}, \mathbf{u}) d\mathbf{u}} = \frac{1}{2\pi} \nabla_{\mathbf{x}} \phi(\mathbf{x}). \quad (6)$$

Substitution of Eq. (6) into Eq. (4) yields the well-known Teague's TIE for coherent field [1]

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\frac{1}{k} \nabla_{\mathbf{x}} \cdot [I(\mathbf{x}) \nabla_{\mathbf{x}} \phi(\mathbf{x})]. \quad (7)$$

Comparing partially coherent TIE [Eq. (4)] with Teague's coherent TIE [Eq. (7)], one may find that ***the “phase” measured by Teague's TIE is a scalar potential whose gradient yields the (irrotational component of) normalized first order local frequency moment of WDF of the partially coherent field.*** Under geometrical optics approximation, the normalized first order local moment of WDF describes the normalized ensemble-averaged transverse energy flux density (Poynting vector). The discrepancy of the phase retrieved by Teague's TIE and the true phase of the specimen raise one important question - how to retrieve the phase of the object when it is illuminated by partially coherent light? Consider a thin sample illuminated by the mutual intensity $\Gamma_{in}(\mathbf{x}_1, \mathbf{x}_2)$, immediately after the sample, the WDF of the wave field can be viewed as the windowed WDF of the object transmittance, which has the form of a mere multiplication in the $\mathbf{x}$ -direction and a convolution in the $\mathbf{u}$ -direction of the WDFs of the incidence illumination and the object transmittance

$$W_{obj}(\mathbf{x}, \mathbf{u}) = W_T(\mathbf{x}, \mathbf{u}) \otimes_{\mathbf{u}} W_{in}(\mathbf{x}, \mathbf{u}) = \int W_T(\mathbf{x}, \xi) W_{in}(\mathbf{x}, \mathbf{u} - \xi) d\xi. \quad (8)$$

The above equation reveals that the partially coherent incident illumination blurs the object WDF along the frequency dimension. It is then readily to find ***the normalized first order local frequency moment of the resultant WDF can be expanded as the sum of their respective normalized first order local frequency moments***

$$\frac{\int \mathbf{u} W_{obj}(\mathbf{x}, \mathbf{u}) \otimes_{\mathbf{u}} W_{in}(\mathbf{x}, \mathbf{u}) d\mathbf{u}}{\int W_{obj}(\mathbf{x}, \mathbf{u}) \otimes_{\mathbf{u}} W_{in}(\mathbf{x}, \mathbf{u}) d\mathbf{u}} = \frac{\int \mathbf{u} W_{obj}(\mathbf{x}, \mathbf{u}) d\mathbf{u}}{\int W_{obj} d\mathbf{u}} + \frac{\int \mathbf{u} W_{in}(\mathbf{x}, \mathbf{u}) d\mathbf{u}}{\int W_{in} d\mathbf{u}}. \quad (9)$$

Since phase derivative retrieved by TIE is equal to the first order local frequency moments of the total WDF, to ensure that the phase retrieved by TIE is equal to the phase delay the object induced under coherent illumination [Eq. (6)], the WDF of the incidence illumination should satisfy

$$\int \mathbf{u} W_{in}(\mathbf{x}, \mathbf{u}) d\mathbf{u} = 0. \quad (10)$$

Equation (10) reveals that under certain illumination conditions – the first order local frequency moment of the illumination WDF vanishes, one can directly apply Teague's TIE to recover the phase of the object $\phi(\mathbf{x})$ even the illumination is not fully coherent.

Another important assumption in TIE is perfect imaging which is not fulfilled in a practical imaging system, like the microscope. In fact, what we measure is the phase of the wavefront in the image plane, which is not exactly the phase of the object itself. The WDF of the wave-field passing through the imaging system in the image plane can be

$$W_{image}(\mathbf{x}, \mathbf{u}) = W_{obj}(\mathbf{x}, \mathbf{u}) \otimes_{\mathbf{x}} W_{psf}(\mathbf{x}, \mathbf{u}) = W_T(\mathbf{x}, \mathbf{u}) \otimes_{\mathbf{u}} W_{in}(\mathbf{x}, \mathbf{u}) \otimes_{\mathbf{x}} W_{psf}(\mathbf{x}, \mathbf{u}). \quad (11)$$

It can be seen that the WDF of the field passing through the imaging system is equivalent to a convolution along $\mathbf{u}$ -direction of the WDF of imaging point spread function (PSF). The imaging PSF will blur the phase since TIE retrieves the normalized first order local moment of WDF along frequency dimension. In general, due to the bilinear nature of image formation in partially coherent systems, there is no direct method to compensate this phase blurring. But for a weakly scattering specimen $u(\mathbf{x}) \approx 1 + i\phi(\mathbf{x}) + \eta(\mathbf{x})$ ($\eta(\mathbf{x}) = \frac{1}{2} \ln \tau(\mathbf{x})$), the system then becomes linear and can be described in terms of weak object optical transfer function [4, 8]. Thus the defocus phase transfer function can be applied to retrieve the phase or correct the phase blurring caused by the imaging PSF.

3. CONCLUSION

We have presented a phase-space representation of transport of intensity phase retrieval. These results extend the theory of deterministic TIE to partially-coherent fields and may be instructive for developing phase-retrieval algorithms that properly account for the partially coherent sources and the PSF of the imaging system.

4. ACKNOWLEDGEMENT

This project was supported by the Research Fund for the Doctoral Program of Ministry of Education of China (No. 20123219110016), and the Research and Innovation Plan for Graduate Students of Jiangsu Higher Education Institutions, China (No. CXZZ11_0237). C. Zuo gratefully acknowledges the financial support from China Scholarship Council (No. 201206840009).

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