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A study of energetic large-scale structures in turbulent boundary layer

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This study established the connection in turbulent boundary layers between the first two dominant proper orthogonal decomposition (POD) modes and the instantaneous large-scale turbulence structures. The velocity fields consistent with the signature velocity fields of the hairpin vortex packets in two-dimensional PIV (particle image velocimetry) measurement planes are observed as the major contributors to the first two POD modes. Another kind of equally important turbulence structure is the large region of Q4 vectors, which may possibly be obtained by slicing the outskirts of the three-dimensional structure of the hairpin vortex packet by PIV planes. The streamwise Reynolds normal stress, Reynolds shear stress, and the length scales of the two-point velocity correlation coefficients ρ_{uu} and ρ_{uv} are noticeably decreased without those large-scale turbulence structures contributing significantly to the first POD mode. Similarity of these results is observed at a higher Reynolds number.

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I. INTRODUCTION

Since introduced by Lumley,¹ proper orthogonal decomposition (POD) has become one of the major tools to study the coherent structures in turbulence. Turbulent structures refer to those turbulent flow motions with temporal and spatial coherence, which were also termed eddies or coherent structures in the past.^{2–4} Bakewell and Lumley⁵ measured the streamwise fluctuating velocities in a pipe flow at a Reynolds number (based on the bulk velocity and pipe diameter) of 8700 using hot-films and their POD analysis revealed that randomly distributed counter-rotating eddy pairs elongated in the streamwise direction are the dominant large-scale structures in the buffer layer. In a direct numerical simulation of a channel flow at $Re_\tau = 180$ (based on skin friction velocity u_τ and half channel height h), Moin and Moser⁶ found that the characteristic eddies extracted from POD consisted of an ejection straddled by weak streamwise vortices who are inclined at 10° from the wall in the near-wall region but at much higher angles further away from the wall. These vortices accounted for about 76% of the kinetic energy but they were less than 100 wall units long in the streamwise direction near the wall. Although these streamwise vortices were found in pairs through POD, instantaneous fields were observed by the authors to contain mostly solitary inclined vortices. The occurrences of paired vortices were simply the results of symmetries in the statistics and the techniques used to determine the phases.

Using experimentally measured turbulent channel flows at Reynolds numbers of $Re_h = 5378$ and 29 935, Liu *et al.*⁷ observed that the large-scale structures obtained from projecting sample velocity fields onto the dominant modes resemble the signature of a hairpin vortex. In the streamwise–wall-normal plane, these large-scale structures illustrated an inclined shear layer separating a region of second quadrant vectors and a region of fourth quadrant vectors. In turbulent boundary layers over both a smooth wall and a realistic rough surface at $Re_\theta \approx 13\,000$, Wu and Christensen⁸ decomposed the instantaneous fluctuating velocities into large-scale and small-scale fields using POD modes in

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both streamwise–wall-normal and streamwise-spanwise planes. They found that the large-scale fields could reflect the general features of the hairpin vortex packet as depicted in the work of Adrian⁴ while the small-scale fields illustrated swirling motions of individual vortex heads and intermittent intense ejections and sweeps associated with these vortices.

However, the link is weak between the turbulence structures inferred from POD analysis and those observed in the instantaneous flow fields. As claimed by Moin and Moser,⁶ the relationship between the characteristic eddies in the channel flow and the instantaneous structures was unclear. The available connection was mostly based on the qualitative visual similarities. For example, Baltzer and Adrian⁹ stated that it was only *likely* that the hairpin vortex packets in wall turbulence contributed strongly to the dominant POD modes since they all have sizable extents in both wall-normal and streamwise directions. Another issue is that although the first POD mode was found to occupy the most amount of kinetic energy in both channel flows⁷ and boundary layers,^{8,10} it was only represented by a relatively uniform flow field of second or fourth quadrant vectors.^{7,10} The second (or fourth) quadrant vectors refer to those velocity vectors with $u < 0$ and $v > 0$ (or $u > 0$ and $v < 0$). It is still elusive as to what are the instantaneous turbulence structures that are the major contributors to the first POD mode.

In this study, the coefficients of the POD mode were used to identify the instantaneous turbulence structures contributing significantly to the first and second POD modes in turbulent boundary layers, thus establishing, to some extents, the quantitative relationship between them.

II. PROPER ORTHOGONAL DECOMPOSITION

The principle and equations of POD can be found in many places in the literature and they are only summarized below. In this study, well-established snapshot POD was used on the PIV (particle image velocimetry) measured velocity fields in boundary layers in the streamwise–wall-normal plane. In POD, any instantaneous fluctuating velocity field $u(x, t)$ can be decomposed into the form of

$$u(x, t) = \sum_{n=1}^{\infty} a_n(t) \phi_n(x), \quad (1)$$

where $\phi_n(x)$ is deterministic spatial POD modes, and $a_n(t)$ is random temporal coefficients. In snapshot POD, the coefficients $a_n(t)$ are obtained first by solving the eigenvalue problem with a positive definite Hermitian kernel of the form

$$\lambda_n a_n(t) = \int_T \left(\int_{\Omega} u(x, t) u(x, t') dx \right) a_n(t') dt', \quad (2)$$

where the integrations are over the spatial domain Ω and a time interval T . For an ensemble of PIV measured velocity fields, the spatial domain could be the whole or part of the PIV field of view and the time domain represents the ensemble or the collection of samples of the velocity fields. The eigenvalues λ_n are real and positive and form a decreasing and convergent series. The POD modes can then be computed through the equation

$$\phi_n(x) = \frac{\int_T u a_n dt}{\int_T a_n^2 dt}. \quad (3)$$

The turbulent kinetic energy is equal to half of the summation of the eigenvalues, i.e.,

$$k = \frac{1}{2} \langle \mathbf{u} \cdot \mathbf{u} \rangle = \frac{1}{2} \sum \lambda_n. \quad (4)$$

Since the POD modes are usually normalized to be orthonormal (i.e., $\int_{\Omega} \phi_n^2 dx = 1$), the POD coefficients are related to the eigenvalues by

$$\int_T [a_n(t)]^2 dt = \lambda_n, \quad (5)$$

TABLE I. Fractional energy contributions of the first 10 POD modes.

Mode n	1	2	3	4	5	6	7	8	9	10
	20.6%	8.9%	4.8%	4.4%	2.6%	2.4%	2.1%	1.6%	1.4%	1.4%

which dictates that the instantaneous fluctuating velocity field with a larger value of a_n^2 contributes more to the eigenvalue λ_n . Therefore, Eq. (5) provides a possible means to identify what are the instantaneous fluctuating flow structures that contribute significantly to the first few dominant POD modes. It is this equation that the current study is based on to establish the connection between the large-scale instantaneous turbulence structures, most importantly the hairpin vortex packets as shown in the results below, and the first two POD modes. The turbulence structures contributing significantly to other POD modes were not sought in this work, however, since each of the rest modes accounts for a much smaller amount of kinetic energy. Since the contribution to the eigenvalue λ_n comes from a_n^2 , instantaneous velocity fields with either positive a_n or negative a_n contain equivalently important information on the flow structures relevant to the n th POD mode.

III. EXPERIMENTAL DATA

Two-dimensional PIV measurements were performed in the streamwise-wall-normal (x - y) plane at the spanwise center of the test section of an Eiffel-type, open circuit, boundary-layer wind tunnel with a turbulence intensity of 0.16%. The flow field was illuminated with a 500 μm thick laser sheet produced by a pair of Nd:YAD lasers with an energy of 190 mJ per pulse. The images of the seeding particles were recorded with a $4\text{k} \times 2.8\text{k}$, 12-bit CCD camera over a field of view of about $1.3\delta \times \delta$ ($x \times y$), where δ is the boundary layer thickness. The PIV images were interrogated using a recursive cross-correlation method. The resulting velocity fields were then validated and low-pass filtered to remove erroneous vectors and to remove high frequency noises. The uncertainty in instantaneous velocity was estimated to be approximately 1%. Details of the experiments were documented in other places.^{8,10,11}

Data sets at two Reynolds numbers, based on the freestream velocity and momentum thickness, of $\text{Re}_\theta = 8200$ and 12 000 were available for this study. Results presented here were mainly obtained from the data at $\text{Re}_\theta = 8200$ while the data at $\text{Re}_\theta = 12\,000$ were used for the discussion of the Reynolds number effect. An ensemble of 2000 instantaneous fluctuating velocity fields from each data set was used for the POD analysis.

IV. POD MODES ϕ_1, ϕ_2

The fractional contributions of the POD modes to the total turbulent kinetic energy, $E_n = \frac{\lambda_n}{\sum \lambda_n}$, for the first ten modes are presented in Table I for the data set at $\text{Re}_\theta = 8200$. The first mode accounts for more than 1/5 of the energy while the first two modes together contribute almost 30%. The energy contribution from any of the other modes is significantly smaller. Similar energy contributions of POD modes were also observed in other studies.^{7,8,12} This study focuses on the turbulence structures contributing strongly to the first two POD modes and therefore these structures are large-scale and energy-containing structures.

The vector fields of the first two POD modes, ϕ_1 and ϕ_2 , are presented in Figure 1. The first mode, as shown in Figure 1(a), illustrates a large-scale Q2 event that spans almost the whole field of view of $1.3\delta \times \delta$. As commonly accepted in the quadrant analysis of turbulence,^{13–15} a Q2 event refers to a region of second-quadrant velocity vectors, i.e., $u < 0$, and $v > 0$. Similarly, a Q4 event mentioned below refers to a region of fourth-quadrant velocity vectors with $u > 0$ and $v < 0$. This Q2 event is stronger below $y = 0.6\delta$ as illustrated by longer vector lengths. For the second POD mode, Figure 1(b) shows a long shear layer across the entire 1.3δ in the streamwise direction and extends from $y \approx 0.1\delta$ to $y \approx 0.5\delta$. This shear layer separates the upstream Q4 event from the downstream Q2 event and its inclination angle is about 18° from the wall.

FIG. 1. Vector fields of (a) the first POD mode ϕ_1 , and (b) the second POD mode ϕ_2 at $Re_\theta = 8200$. Not every vector is shown for clarity.

V. POD COEFFICIENT a_1

The scatter plot and the histogram of the first POD coefficient, a_1 , which are normalized by its root-mean-square (RMS) value, σ_{a_1} , are presented in Figure 2. An instantaneous fluctuating velocity field with a positive (or negative) value of a_1 means that its reconstruction using the first POD mode will represent a Q2 (or Q4) event. If a_1 is large for a particular field, the fluctuating velocities tend to illustrate an extended region of strong Q2 (or Q4) event, as shown in Figures 3 and 4. Figure 2(a) shows that a_1 values are clustered within ± 1.5 times its RMS value and there are only a small amount of velocity fields with a_1 's larger than twice of its RMS value. However, since the contribution to the first eigenvalue comes from a_1^2 , as shown in Eq. (5), those few velocity fields with large a_1 values carry a significant weight, in one-to-one comparison with other fields, in defining the first POD mode. Therefore, the instantaneous turbulence structures in these velocity fields are the dominant structures associated with the first mode. It can also be observed from the histogram shown in Figure 2 that the distribution of a_1 is approximately symmetric between positive and negative values, indicating roughly equal contributions from Q2 and Q4 events to the first POD mode.

VI. DOMINANT TURBULENCE STRUCTURES FOR ϕ_1

An instantaneous fluctuating velocity field with a relatively large positive coefficient for the first POD mode ($a_1 = 2.06\sigma_{a_1}$) is shown in Figure 3. The main feature of this field is a large-scale Q2 event which extends beyond 0.6δ in the wall-normal direction and is at least 1.3δ long in the streamwise direction. The Q2 vectors are qualitatively similar to the vectors shown by the first POD

FIG. 2. (a) Scatter plots of the normalized (by its RMS value σ_{a_1}) first POD coefficient, a_1 , of all the instantaneous velocity fields. (b) Histogram of normalized a_1 .

FIG. 3. An instantaneous fluctuating velocity field with a large positive POD coefficient of $a_1 = 2.06\sigma_{a1}$ for the first POD mode. The color contours represent the correlation between swirling strength and vorticity to illustrate the cores of the prograde spanwise vortices. Only every other vector is shown.

mode in Figure 1(a). What is interesting in this particular field is that many Q2 vectors are associated with two hairpin-vortex-packet-like structures⁴ which are circled in the figure. One vortex packet is larger and is located further away from the wall while the other is smaller and is located closer to the wall. The vortex cores of the heads of the individual hairpins within each packet are illustrated by the

FIG. 4. An instantaneous fluctuating velocity field with a large negative POD coefficient of $a_1 = -2.06\sigma_{a1}$ for the first POD mode. Only every other vector is shown.

color contours of the correlation of swirling strength and vorticity. Ren and Wu¹⁶ have shown that the correlation of swirling strength and vorticity in the PIV measured plane has higher signal-to-noise ratio and therefore is better than just the swirling strength to identify the strong vortices. A few vortices at the upper side of the larger vortex packet are not identified by the chosen contour levels since they are quite weak. However, the velocity vector fields show that they do appear to belong to that packet. In general, the vortices in the outer larger vortex packet are weaker and are further spaced between each other, compared to those in the inner smaller vortex packet. These structures are very similar to the hierarchy of hairpin vortex packets described by Adrian.⁴

All the instantaneous fluctuating velocity fields whose coefficients a_1 are positive and larger than twice its RMS value, σ_{a1} , are inspected carefully. The common feature is a large-scale Q2 event whose upper side is ridden by several prograde spanwise vortices whose rotations are in the same sense as the mean shear.^{17–20} The hairpin vortex packet thus appears to be the best structure to describe those instantaneous flows that contribute significantly to the first POD mode with positive coefficients.

Figure 4 presents an instantaneous fluctuating velocity field with a negative coefficient of $a_1 = -2.06\sigma_{a1}$. This field illustrates a large-scale Q4 event from approximately the edge of the boundary layer to close to the wall. There are some small-scale spanwise vortices within this Q4 event, but these vortices do not appear to be organized into large-scale structures like hairpin vortex packets. However, on the right side of the field of view at around $y = 0.6\delta$, a few prograde spanwise vortices (who are circled in the figure) are seen to seem to organize into a small vortex packet. At a different advection velocity, they do illustrate the velocity signature of a hairpin vortex packet, which means there exists a region of collective Q2 vectors beneath the hairpin vortex heads. But in the fluctuating velocity field shown in Figure 4, this packet is observed to have Q1 vectors above the vortices. After examining all the fields with large negative a_1 values ($|a_1| > 2\sigma_{a1}$), it is found that the large-scale Q4 is indeed the common structure that appears in these instantaneous fluctuating velocities.

It is not clear from the current two-dimensional PIV measurements whether the large-scale Q2 event is related with the large-scale Q4 event so that they actually belong to a single large-scale structure. However, the model of hairpin vortex packet could produce both a large-scale Q2 event collectively induced inside the hairpin vortex legs and large-scale Q4 events outside the legs of the vortices. The distribution of a_1 shown in Figure 2(b) is approximately symmetric, even for large values, and it may indicate that these large-scale Q2 and Q4 events are PIV samplings of the same large-scale turbulence structure which is best described by the hairpin vortex packet⁴ so far. However, further studies using three-dimensional velocity measurement technique such as tomographic PIV^{21–23} are needed to investigate the three-dimensional topology of these large-scale structures.

VII. CONTRIBUTION TO TURBULENCE STATISTICS

In order to study how these large-scale structures contributing significantly to the first POD mode affect the turbulence single- and two-point statistics, a new ensemble of fluctuating velocity fields is obtained by removing the small number of fields (about 4%) whose $|a_1| > 2\sigma_{a1}$ from the original ensemble of 2000 fields. This new ensemble is referred to as “Ensemble-w/o-A1” hereafter. Turbulence statistics are then computed and compared between these two ensembles.

Figure 5 presents the comparison of the profiles of Reynolds normal and shear stresses with and without those fluctuating velocity fields which are the dominant contributors to the first POD mode. When the dominant structures are removed, the streamwise Reynolds normal stress $\langle u^2 \rangle$ from close to the wall up to $y = 0.6\delta$ is reduced by about 3.5%–7%, as observed from Figure 5(a). This region of decreased $\langle u^2 \rangle$ at $y < 0.6\delta$ coincides with the region where the vectors of the first POD mode, ϕ_1 , are relatively large, as well as with the region where the instantaneous large-scale Q2 or Q4 event is strong in those velocity fields removed from the original ensemble. However, Figure 5(b) illustrates that the wall-normal Reynolds normal stress profile is not affected. Therefore, the velocity fields with those large-scale structures contributing strongly to the first mode possess significantly large amounts of kinetic energy from the u velocity component ($\langle u^2 \rangle$), but not from the v component ($\langle v^2 \rangle$). Wu and Christensen⁸ also found that a larger amount of $\langle u^2 \rangle$ is contributed from reconstructed velocity fields with low-order POD modes while a majority of $\langle v^2 \rangle$ is carried by fields reconstructed using

FIG. 5. Comparison of Reynolds stresses between the original ensemble of fluctuating velocity fields (Original Ensemble) and the ensemble without those velocity fields whose $|a_1| > 2\sigma_{a1}$ (Ensemble-w/o-A1). (a) Streamwise Reynolds normal stress $\langle u^2 \rangle$; (b) wall-normal Reynolds normal stress $\langle v^2 \rangle$; and (c) Reynolds shear stress $-\langle uv \rangle$. Not every data point is shown for clarity.

high-order modes. Figure 5(c) illustrates that the Reynolds shear stress was also reduced between $y \approx 0.05$ and 0.6δ . Without those large-scale velocity fields, the shear stresses in the wall-normal range of $0.1\text{--}0.3\delta$ are significantly decreased by as much as about 10%. This observation of the pronounced reduction of $-\langle uv \rangle$ may not be very surprising since the large-scale structures removed from the original ensemble are large-scale Q2 and Q4 events which are important Reynolds shear stress contributors. Liu *et al.*⁷ also observed in turbulent channel flows that the large-scale POD modes containing half of the turbulence kinetic energy contains more than 2/3 of the total Reynolds shear stress in the outer region of the channel.

Figure 6 presents the comparison, at a representative wall-normal location of $y_{ref} = 0.15\delta$, of two-dimensional two-point velocity correlation coefficients given by

$$\rho_{ij}(\Delta x, y; y_{ref}) = \frac{\langle u_i(x, y_{ref})u_j(x + \Delta x, y) \rangle}{\sigma_i(y_{ref})\sigma_j(y)}, \quad (6)$$

FIG. 6. Comparison of two-point velocity correlation coefficients between the original ensemble of fluctuating velocity fields (Original Ensemble) and the ensemble without those velocity fields whose $|a_1| > 2\sigma_{a1}$ (Ensemble-w/o-A1). (a) ρ_{uu} ; (b) ρ_{vv} ; and (c) ρ_{uv} . Dashed lines are for “Original Ensemble” and solid lines are for “Ensemble-w/o-A1.”

where Δx is the spatial separation in the streamwise direction, y_{ref} is the wall-normal reference location, and σ is the root-mean-square. Figure 6(a) shows that although the general shapes of the autocorrelations of the streamwise velocity component, ρ_{uu} , are similar between the original ensemble and “Ensemble-w/o-A1,” the length scales in both streamwise and wall-normal directions are obviously reduced when the velocity fields contributing strongly to the first POD mode are taken out. The absolute differences of the length scales are larger for smaller correlation levels which represent larger-scale structures. However, the inclination angles of ρ_{uu} are about the same between these two ensembles of velocity fields. For the autocorrelations of the wall-normal velocity component, ρ_{vv} , little difference is observed for correlation levels higher than 0.1, as shown in Figure 6(b). Since the cross correlation coefficients ρ_{uv} and ρ_{vu} are nearly anti-symmetric in turbulent boundary layers,⁷ only ρ_{uv} is presented in Figure 6(c). The difference of ρ_{uv} is similar to that of ρ_{uu} in that the length scale computed from “Ensemble-w/o-A1” is dramatically smaller than that computed from the original ensemble, particularly for low correlation levels. Figure 6 illustrates the importance of those small number (about 4% of total fields) of fluctuating velocity fields in contributing to the long length scales, in both streamwise and wall-normal directions, of ρ_{uu} and ρ_{vv} . However, the inclination angle of ρ_{uu} does not appear to be affected by these fields.

VIII. DOMINANT TURBULENCE STRUCTURES FOR ϕ_2

Figures 7 and 8 present examples of instantaneous flow structures that contribute dominantly to the second POD mode which illustrates a long shear layer separating large-scale Q2 and Q4 vectors. The flow field shown in Figure 7 has a positive value of coefficient a_2 and therefore has velocity vectors similar to those in ϕ_2 . In this field, there exists a large region of Q2 event in the lower part of the boundary layer, as well as a weaker Q4 region coming from the edge of the boundary layer. Between these large-scale Q2 and Q4 vectors is a region with much smaller scale flow structures. At the upper interface of the Q2 event, a few strong prograde spanwise vortices can be observed

FIG. 7. An instantaneous fluctuating velocity field with a large positive POD coefficient of $a_2 = 2.10\sigma_{a1}$ for the second POD mode. Only every other vector is shown.

FIG. 8. An instantaneous fluctuating velocity field with a large negative POD coefficient of $a_2 = -2.88\sigma_{a1}$ for the second POD mode. Only every other vector is shown.

from both the contours of the swirling-strength–vorticity correlation and the velocity vectors. These vortices, together with the strong Q2 vectors underneath, are consistent with the PIV signatures of a hairpin vortex packet. Along the lower interface of the Q4 event exists a series of weaker prograde spanwise vortices. However, no obvious collective induction of strong Q2 vectors is found under them even though other convection velocities have been tried. Therefore, these vortices may not fit in the model of a hairpin vortex packet. Rather, they are more like the vortices generated by an instable shear layer. Other instantaneous velocity fields whose a_2 are positive and larger than $2\sigma_{a2}$ are qualitatively similar to the one presented in Figure 7. The hairpin vortex packet illustrated in Figure 7 may just be part of a much larger ramp-like packet,⁴ which is observed to be as long as several boundary layer thickness in the streamwise direction.

The flow field, shown in Figure 8, which is representative of those having a large negative coefficient ($|a_2| > 2\sigma_{a2}$), illustrates a separation of a Q4 event which is located closer to the wall and a Q2 event at the outer region of the boundary layer. The region of large-scale Q2 vectors has a train of weak prograde spanwise vortices on top, which is similar to that induced by the hairpin vortex packet, except that this Q2 event ends at the interface between Q2 and Q4 events and does not extend down close to the wall as in the classical PIV signature of the vortex packet. However, this velocity field may be generated if the upper packet is shifted in the spanwise direction with respect to the lower vortex packet and when the PIV plane is located inside the legs of the upper packet but at the out-skirt of the lower packet. Three-dimensional velocity measurement with a relatively large coverage in the spanwise direction is needed to reveal its true flow topology.

Since the second POD mode contributes much smaller to the turbulence kinetic energy than the first mode, the instantaneous velocity fields who are major contributors to the second mode therefore have only small effects on turbulence statistics (figures are not shown for brevity).

IX. THRESHOLD EFFECT

While about 4% of the instantaneous velocity fields measured here have $|a_1|$ values larger than $2\sigma_{a1}$ (twice the root-mean square value of a_1), the percentage of flow fields with $|a_1| > 1.5\sigma_{a1}$ is around 13% and that with $|a_1| > 1.0\sigma_{a1}$ is approximately 33% or third of the total number of

FIG. 9. Comparison of Reynolds stresses between ensembles without instantaneous fluctuating velocity fields whose a_1 values are higher than different thresholds. (a) Streamwise Reynolds normal stress $\langle u^2 \rangle$; (b) Reynolds shear stress $-\langle uv \rangle$. Every third data point is shown for clarity.

velocity fields. It is expected that with smaller POD coefficient a_1 , the instantaneous velocity field is less similar to the vector field of the first POD mode and more smaller flow structures tend to exist in that field. What is to be studied in this section is how the turbulence statistics will be changed if the instantaneous velocity fields whose $|a_1|$ values larger than different thresholds are taken out from the ensembles. The effects on turbulence statistics by using different thresholds of the second POD coefficient a_2 are much smaller and therefore not presented here.

Figure 9 shows the comparison of streamwise Reynolds normal stress $\langle u^2 \rangle$ and Reynolds shear stress $-\langle uv \rangle$ computed from ensembles at different thresholds of a_1 . Results of wall-normal Reynolds normal stress $\langle v^2 \rangle$ are not presented since almost no difference was observed. It is found from Figure 9(a) that $\langle u^2 \rangle$ decreases with lower thresholds within the inner 60% of the boundary layer, which is expected since more instantaneous velocity fields with large a_1 values are excluded in the ensembles and these missing fields are large contributors to the turbulent kinetic energy. The thresholds appear to have little effects beyond $y = 0.6\delta$. Note that the changes in the turbulence statistics presented in Figure 9 as well as in the following figures are not due to the decreased numbers of velocity fields in different ensembles.

Reynolds shear stress is observed from Figure 9(b) to decrease with lower thresholds, too, except at the outer edge of the boundary layer. It is because the large-scale turbulence structures are also Reynolds shear stress carriers. The threshold effects appear to be quite small very close to the wall. What is interesting is the lack of the plateau in shear stress profile between $y = 0.05$ and 0.15δ when thresholds are lower than $1.5\sigma_{a1}$. Instead of having a approximately constant stress region as for the original ensemble of velocity fields, the shear stress profiles at the two lowest thresholds reach a peak at the wall-normal location of slightly higher than $y = 0.05\delta$ and then drop sharply. That is, those 13% of the instantaneous velocity fields whose a_1 values are larger than $1.5\sigma_{a1}$ are important for maintaining a constant-stress region in shear stress profile of the turbulent boundary layer at $Re_\theta = 8200$.

The comparison of integral length scales of ρ_{uu} at different thresholds in both streamwise and wall-normal directions is presented in Figure 10. Since little effects are observed on inclination angles of ρ_{uu} and length scales of ρ_{vv} , these results with varying thresholds are not needed to be presented here. The integral length scale in the streamwise direction is computed as

$$L_{uu}^x(y_{ref}) = \int_{\Delta x=0}^{\Delta x=\delta} \rho_{uu}(\Delta x, y = y_{ref}; y_{ref}) d(\Delta x), \quad (7)$$

FIG. 10. Comparison of integral length scales of streamwise velocity in both streamwise (L_{uu}^x) and wall-normal (L_{uu}^y) directions between ensembles without instantaneous fluctuating velocity fields whose a_1 values are higher than different thresholds.

where ρ_{uu} is the autocorrelation coefficient at the wall-normal location of y_{ref} . Due to the limited field of view, the integration is from $\Delta x = 0$ to one boundary layer thickness δ in the streamwise direction rather than to infinity as formally defined for the integral length scale. Similarly, the integral length scale in the wall-normal direction is given by

$$L_{uu}^y(y_{ref}) = \int_{y=0}^{y=\delta} \rho_{uu}(\Delta x = 0, y; y_{ref}) dy. \quad (8)$$

Figure 10(a) shows that the integral length scales of streamwise velocity in both streamwise (L_{uu}^x) and wall-normal (L_{uu}^y) directions are not affected by different thresholds of a_1 beyond $y = 0.8\delta$. The major differences in these length scales occur around 0.3δ , and both L_{uu}^x and L_{uu}^y decrease with smaller thresholds. Starting from the threshold of $1.5\sigma_{a1}$, there exist two peaks of L_{uu}^x , one peak is at around $y = 0.075\delta$ and the other is at 0.8δ , and the shortest lengths scales of L_{uu}^x occur between $y = 0.2$ and 0.4δ . At threshold of $1.0\sigma_{a1}$, length scale peak at the boundary layer edge at 0.8δ is even higher than the peak close to the wall at 0.075δ , and L_{uu}^x at the valley of $y \approx 0.3\delta$ is less than half of the integral length scale of the original velocity ensemble at the same wall-normal location. The results indicate that the instantaneous velocity fields significant for the first POD mode play more important roles in maintaining a long length scale of ρ_{uu} in the middle of the boundary layer than at inner and outer regions of the boundary layer.

X. REYNOLDS NUMBER EFFECT

The vectors of the first and second POD modes at the higher Reynolds number of $Re_\theta = 12000$ are found to be little different from those at the lower Reynolds number. The histogram of the first POD coefficient a_1 normalized by its own RMS value at $Re_\theta = 12000$ is also similar to that at the lower Re shown in Figure 2. In addition, the instantaneous fluctuating velocity fields contributing significantly to the first two POD modes illustrate similar turbulence structures as described above at $Re_\theta = 8200$. Hairpin vortex packet, as well as large-scale Q4 events, can still be identified to be the major structures as the most important contributors to the first POD mode. Since these results are very similar to the figures shown above for the lower Reynolds number, the similar plots at the higher Re are therefore not repeated here. These similarities substantiate the findings of Liu *et al.*²⁴ who

FIG. 11. Comparison of Reynolds shear stress between ensembles without instantaneous fluctuating velocity fields whose a_1 values are higher than different thresholds at the higher Reynolds number of $Re_\theta = 12\,000$. Only every third data point is shown.

observed outer similarity of one-dimensional POD modes for channel flows of different Reynolds numbers, as well as the similarity of eigenvalue spectra of channel flows and boundary layers.

While the threshold effects of the contribution of large-scale structures on other turbulence statistics presented in Sec. IX are similar, the change of Reynolds shear stresses with a_1 thresholds at $Re_\theta = 12\,000$, as presented in Figure 11, has some interesting differences from that at $Re_\theta = 8200$. Comparing the shear stress profiles at thresholds of $|a_1| = 1.5$ and $1.0\sigma_{a_1}$ between two Reynolds numbers in Figures 9(b) and 11, it can be observed that rather than totally disappearing at $Re_\theta = 8200$, the constant-pressure region at $Re_\theta = 12\,000$ still remains, but becomes shorter with lower a_1 thresholds.

XI. CONCLUSIONS

Through the values of the POD coefficients, this study physically established the connection in turbulent boundary layers between the first two POD modes and the instantaneous large-scale turbulence structures which are the major contributors to these modes. The results presented here confirmed the conjecture of Baltzer and Adrian⁹ that hairpin vortex packets contributed strongly to the dominant POD modes since fluctuating velocity fields consistent with the description of the packets were indeed observed in the instantaneous flow fields contributing significantly to the first two POD modes. It was also found that the velocity fields matching the classic signature of the hairpin vortex packet, i.e., a large region of strong Q2 vectors with spanwise vortices above, were not the only major turbulence structure for the first POD mode. Equally important are the large-scale Q4 vectors. However, it is possible that a large region of Q4 vectors is produced at the outskirts of the hairpin vortex packet, given the fact that the distribution of positive and negative values of the first POD coefficient is nearly symmetric. If that were the case, then the hairpin vortex packet would be the single most important turbulence structure to contribute to the first POD mode which accounts for a dominant portion of the turbulence kinetic energy. But this conjecture needs to be verified in the future using volumetric three-dimensional velocity fields.

Without those large-scale turbulence structures contributing largely to the first POD mode, both streamwise Reynolds normal stress and Reynolds shear stress below $y = 0.6\delta$ are noticeably reduced, and the length scales of two-point velocity correlation coefficients, ρ_{uu} and ρ_{uv} in both streamwise and wall-normal directions are significantly decreased. However, the wall-normal Reynolds normal stress and ρ_{vv} are negligibly affected. The general trend is the further reduction of Reynolds stresses

($\langle u^2 \rangle$ and $\langle uv \rangle$) and integral length scales of ρ_{iii} when more major first-POD-mode-contributing flow fields are taken out from the ensembles for computing statistics.

The results regarding the instantaneous structures for the first two POD modes and their effects on statistics are similar for a higher Reynolds number. One interesting observation on the Reynolds number effect is that while the constant-stress region disappears in the profiles of the Reynolds shear stress at the lower Re, it still exists at the higher Re, but becomes shorter when the thresholds of a_1 , the first POD coefficient, are low.

In the future, the three-dimensional topology and the dynamics of the instantaneous large-scale turbulence structures contributing significantly to the first one and two POD modes can be studied using time-resolved volumetric three-dimensional velocity measurement techniques such as tomographic PIV. The wall pressure and shear stress signatures of these energetic large-scale structures are worth investigating, too.

- ¹ J. L. Lumley, "The structures of inhomogeneous turbulent flow," in *Proceedings of the International Colloquium on the Fine Scale Structure of the Atmosphere and its Influence on Radio Wave Propagation*, edited by A. M. Yaglom and V. I. Tararsky (Nauka, Moscow, 1967), pp. 166–178.
- ² A. A. Townsend, *The Structure of Turbulent Shear Flow*, 2nd ed. (Cambridge University Press, 1976).
- ³ B. Cantwell, "Organized motion in turbulent flow," *Annu. Rev. Fluid Mech.* **13**, 457 (1981).
- ⁴ R. J. Adrian, "Hairpin vortex organization in wall turbulence," *Phys. Fluids* **19**, 041301 (2007).
- ⁵ H. P. Bakewell and J. L. Lumley, "Viscous sublayer and adjacent wall region in turbulent pipe flow," *Phys. Fluids* **10**, 1880–1889 (1967).
- ⁶ P. Moin and R. D. Moser, "Characteristic-eddy decomposition of turbulence in a channel," *J. Fluid Mech.* **200**, 471–509 (1989).
- ⁷ Z. Liu, R. Adrian, and T. Hanratty, "Large-scale modes of turbulent channel flow: Transport and structure," *J. Fluid Mech.* **448**, 53–80 (2001).
- ⁸ Y. Wu and K. T. Christensen, "Spatial structure of a turbulent boundary layer with irregular surface roughness," *J. Fluid Mech.* **655**, 380–418 (2010).
- ⁹ J. Baltzer and R. Adrian, "Structure, scaling, and synthesis of proper orthogonal decomposition modes of inhomogeneous turbulence," *Phys. Fluids* **23**, 015107 (2011).
- ¹⁰ Y. Wu, "Experimental investigation of highly irregular roughness effects in wall turbulence," Ph.D. thesis (University of Illinois at Urbana-Champaign, 2008).
- ¹¹ Y. Wu and K. T. Christensen, "Outer-layer similarity in the presence of a practical rough-wall topography," *Phys. Fluids* **19**, 085108 (2007).
- ¹² P. Johansson and H. Andersson, "Generation of inflow data for inhomogeneous turbulence," *Theoret. Comput. Fluid Dynamics* **18**, 371–389 (2004).
- ¹³ J. M. Wallace, H. Eckelmann, and R. S. Brodkey, "The wall region in turbulent shear flow," *J. Fluid Mech.* **54**, 39–48 (1972).
- ¹⁴ S. S. Lu and W. W. Willmarth, "Measurements of the structure of the Reynolds stress in a turbulent boundary layer," *J. Fluid Mech.* **60**, 481–511 (1973).
- ¹⁵ Y. Wu and K. T. Christensen, "Reynolds-stress enhancement associated with a short fetch of roughness in wall turbulence," *AIAA J.* **44**, 3098 (2006).
- ¹⁶ H. Ren and Y. Wu, "Turbulent boundary layers over smooth and rough forward-facing steps," *Phys. Fluids* **23**, 045102 (2011).
- ¹⁷ R. J. Adrian, C. D. Meinhart, and C. D. Tomkins, "Vortex organization in the outer region of the turbulent boundary layer," *J. Fluid Mech.* **422**, 1–54 (2000).
- ¹⁸ K. Christensen and Y. Wu, "Visualization and characterization of small-scale spanwise vortices in turbulent channel flow," *J. Visualization* **8**(2), 177–185 (2005).
- ¹⁹ Y. Wu and K. T. Christensen, "Population trends of spanwise vortices in wall turbulence," *J. Fluid Mech.* **568**, 55–76 (2006).
- ²⁰ V. K. Natrajan, Y. Wu, and K. T. Christensen, "Spatial signatures of retrograde spanwise vortices in wall turbulence," *J. Fluid Mech.* **574**, 155–167 (2007).
- ²¹ G. Elsinga, F. Scarano, B. Wieneke, and B. Oudheusden, "Tomographic particle image velocimetry," *Exp. Fluids* **41**, 933–947 (2006).
- ²² H. Lee and Y. Wu, "Experimental study of rotational effect on stalling," *Chin. Phys. Lett.* **30**, 064703 (2013).
- ²³ H. Lee and Y. Wu, "An experimental study of stall delay on the blade of a horizontal-axis wind turbine using tomographic particle image velocimetry," *J. Wind Eng. Ind. Aerodyn.* **123**, 56–68 (2013).
- ²⁴ Z.-C. Liu, R. Adrian, and T. Hanratty, "Reynolds number similarity of orthogonal decomposition of the outer layer of turbulent wall flow," *Phys. Fluids* **6**, 2815–2819 (1994).