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Global Task-Space Adaptive Control of Robot [★]

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Abstract

Task-space feedback information such as visual feedback is used in many modern robot control systems as it improves robustness to model uncertainty. However, existing task-space feedback control schemes are only valid locally in a finite task space within a limited sensing zone where the singularity of the Jacobian matrix can be avoided. The global stability problem of task-space control system has not been systemically solved. In this paper, we introduce a novel regional feedback method for robot task-space control. Each feedback information is employed in a local region, and the combination of regional information ensures the global convergence of robot motion. The transition from one feedback information to another is embedded in the controllers without using any hard or discontinuous switching. Using the regional feedback, a new task-space control method is proposed, which consists of a reaching task variable that drives the robot from one task space to another and a desired task variable to move the robot to the desired position at the ending stage. We shall show that the proposed regional feedback control method is a united formulation to address various open issues in task-space control problems such as singularity problem and limited sensing zone. This is the first result in task-space control that the global dynamic stability can be guaranteed with the consideration of singularity issues and limited sensing zones.

Key words: Robotic Manipulators; Adaptive Control; Regional Feedback; Global Stability.

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1 Introduction

Various task-space controllers which either formulate the control problem in Cartesian space or sensory task space have been proposed for robot manipulator. Takegaki and Arimoto [Takegaki and Arimoto, 1981] proposed the first Cartesian-space regulator, and inspired by the original work, much progress has been achieved in understanding the task-space regulation problems [Arimoto, 1996, Kelly, 1999]. To deal with tracking control, Slotine and Li [Slotine and Li, 1987] proposed a Cartesian-space adaptive controller, which contained a PD feedback and a full dynamics compensation.

Recent advances in sensing technology led to the research and development of sensory task-space feedback control laws for robot systems. While the sensory information is important to improve the endpoint accuracy in the presence of uncertainty, most sensory control schemes require the exact knowledge of the Jacobian matrix from joint space to task space. Unfortunately, it is difficult to derive the Jacobian matrix accurately espe-

cially when the robot manipulates a tool with unknown length. Several approximate Jacobian controllers for set-point control of robots with uncertainties in both kinematics and dynamics [Cheah *et al.*, 2003; Dixon, 2007; Ozawa and Oobayashi, 2009] have been proposed. To deal with tracking control problem with uncertain kinematics, an adaptive Jacobian controller was proposed in [Cheah *et al.*, 2006]. In [Wang and Xie, 2009], an adaptive controller was proposed for free-floating space manipulator with uncertainties in both kinematics and dynamics. To avoid singularities associated with the Euler angles representation, an adaptive Jacobian tracking controller based on the unit quaternion representation was developed [Braganza *et al.*, 2008]. For the vision based control, it is also difficult to derive the exact image Jacobian matrix [Weiss *et al.* 1987; Espiau *et al.*, 1992] due to the uncertainty in the depth information. Using a depth-independent interaction matrix, Liu *et al.*, [Liu *et al.*, 2006] introduced a vision based controller to cope with the uncertain depth problem. Cheah *et al.* [Cheah *et al.*, 2010] presented an adaptive Jacobian setpoint controller with concurrent adaptation to uncertain depth information and kinematics.

One common assumption of task-space sensory feedback control is that the position of the end effector must stay within the sensing zone for the entire task.

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In addition, it is also assumed in task-space control methods that the Jacobian matrix is non-singular. A large number of research efforts have been devoted to singularity avoidance by exploiting the kinematic redundancy [Nakamura, 1991, Chiaverini, 1997], but those methods are essentially singularity avoidance techniques and are only feasible for redundant robots. In addition, the damped least-square inverse Jacobian matrix [Nakamura, 1991] and the Jacobian transpose [Sciavicco and Siciliano, 1988] were proposed to overcome the problem of singularity, but it is not possible to prove the stability of closed-loop system in task-space tracking control since neither the damped inverse Jacobian nor the Jacobian transpose is the true inverse of the Jacobian matrix. Some work has been performed to analyze the stability with the damped inverse Jacobian [Caccavale *et al.*, 1997, Nenchev *et al.*, 2000]. However, unlike task-space control, the stability analysis of these methods is limited to kinematic control, without considering the effects of the robot dynamics.

Therefore, existing task-space controllers are only valid locally in a finite task space within a limited sensing zone where the singularity of the Jacobian matrix is avoided, and the global dynamic stability problem of task-space control system has not been systemically solved. In this paper, we present a regional feedback method for robot task-space control. Each feedback information is employed within a local region, and the combination of the local feedback covers the entire workspace and thus guarantees the global stability. The main contributions of this paper are summarized as follows: i) This is the first result that solves the open issues on the global dynamic stability of task-space robot control system with consideration of singularity of Jacobian matrix and limited sensing zone. The effects of nonlinearity and uncertainty of robot dynamics are taken into consideration in the stability analysis. ii) The results provide a new formulation that allows the problems of singularity and limited sensing zone to be addressed in a unified way. iii) The proposed method allows a smooth transition between regional feedback and hence eliminates the problem of discontinuous switching that causes chattering or vibration during the movement. Instead of designing multiple controllers in different regions and switching between them, we present a new task-space control strategy that allows the use of dual task-space information in a single controller. Experimental results are presented to illustrate the performance of the proposed controller.

2 Problem Statement

Current task-space control methods are only valid locally in a finite task space and thus the stability of the closed-loop system cannot be ensured globally. In this section, we first review some problems in robot task-space control and then propose a unified formulation to address these issues.

Fig. 1. Singularity problem: the singular regions are not reachable.

Singularity occurs when the Jacobian matrix is not full rank and it is commonly assumed that the robot is operating in a finite task space such that the singularity problem can be avoided. This limits the potential workspace of the robot when the task-space control is employed. For example, Fig. 1 shows the feasible workspace of a 3 d-o-f manipulator where singularities are avoided.

Fig. 2. Problem of limited field of view: the end effector cannot leave the field of view during the course of movement.

For vision based control, since the camera has limited field of view as illustrated in Fig. 2, the end effector of the robot must start within the field of view and cannot leave the field of view during the course of movement. A variety of approaches have been proposed in the literature of visual servoing to keep the image features within the field of view [Chesi and Hashimoto 2010; Gans and Hutchinson 2007; Malis 2004; Cowan *et al.*, 2002; Mezouar and Chaumette, 2002; Garcia-aracil *et al.*, 2005; Gans *et al.*, 2011], but these methods assume that the image features are observable at the beginning stage and do not consider image occlusion. In addition, these visual servoing techniques are formulated as kinematic control methods without the consideration of the effects of robot dynamics.

In micromanipulation tasks, there is always an inherent trade-off between the resolution and the field of view. A high-resolution microscope is required to improve the end point accuracy of manipulation tasks but it has very limited field of view. A coarse-to-fine mechanism using mixed cameras is introduced in [Ralis *et al.*, 2000]. However, the stability of the closed-loop system is not considered, and the transition among different sensory feedback is not smooth.

To develop the regional task-space feedback controller, we consider the dynamics of the robotic manipulator as: [Arimoto, 1996]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \left[\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})\right]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \quad (1)$$

where $\mathbf{q} = [q_1, \dots, q_n]^T \in \mathbb{R}^n$ represents the joint angles, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$ is an inertia matrix which is symmetric and positive definite, $\mathbf{S}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$ is a skew-symmetric matrix, $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^n$ denotes a vector of gravitational force, and $\boldsymbol{\tau} \in \mathbb{R}^n$ denotes a vector of control inputs. The dynamic model as described by equation (1) is linear in a set of physical parameters $\boldsymbol{\theta}_d = [\theta_{d1}, \dots, \theta_{dk}]^T$ as:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \left[\frac{1}{2}\dot{\mathbf{M}}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})\right]\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\theta}_d, \quad (2)$$

where $\mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in \mathbb{R}^{n \times k}$ is a known dynamic regressor matrix.

In task-space control, the position of the end effector is specified in task space directly, and the velocity of the end effector is related to the joint velocity as:

$$\dot{\mathbf{x}} = \mathbf{J}_x(\mathbf{q})\dot{\mathbf{q}}, \quad (3)$$

where $\mathbf{x} = [\mathbf{x}_1, \dots, \mathbf{x}_m]^T$ are the positions of features in task space, and $\mathbf{x}_i = [x_{i1}, \dots, x_{in_i}]^T \in \mathbb{R}^{n_i}$ denote feature points, $i = 1, 2, \dots, m$, and m is the total number of features. The matrix $\mathbf{J}_x(\mathbf{q}) \in \mathbb{R}^{\sum_{i=1}^m n_i \times n}$ is a Jacobian matrix from joint space to the corresponding task space. We consider $\mathbf{x}$ as a desired task variable in which the desired task is specified. However, the task-space feedback information is not always accessible due to limited sensing zone or singularity issue.

Given a desired trajectory $\mathbf{x}_d$, our main objective is to formulate and solve a task-space control problem such that the global stability can be guaranteed. To achieve that, we introduce a reaching task variable $\mathbf{r} \in \mathbb{R}^{n_r}$, which specifies the position of the end effector in a different coordinate space with the available feedback information as:

$$\dot{\mathbf{r}} = \mathbf{J}_r(\mathbf{q})\dot{\mathbf{q}}, \quad (4)$$

where $\mathbf{J}_r(\mathbf{q}) \in \mathbb{R}^{n_r \times n}$ is the Jacobian matrix from joint space to the corresponding task space. The variable $\mathbf{r}$ is used to drive the end effector into the working range of $\mathbf{x}$. By using these two task variables, we propose a unified control methodology which integrates regional feedback information smoothly into one controller. The regional feedback information consists of the reaching task variable $\mathbf{r}$ which drives the end effector to approach the desired task region, and the desired task variable $\mathbf{x}$ which ensures the convergence to the desired trajectory.

3 Task-Space Regions

We divide the robot workspace into external regions and internal regions as illustrated in Fig. 3. In these regions, the desired task variable $\mathbf{x}$ is not available, and hence the reaching task variable $\mathbf{r}$ is employed. According to specific robot tasks, the reaching task variable $\mathbf{r}$ and the desired task variable $\mathbf{x}$ can be formulated in different task coordinates.

In the following development, we firstly give several examples of the feedback variables $\mathbf{x}$ and $\mathbf{r}$ and the corresponding task-space regions in section 3.1. After that, a general and systematic formulation of region functions is proposed in section 3.2. The potential energy functions and the regional feedback errors by using the region functions are introduced in section 3.3.

Fig. 3. An illustration of external and internal task-space regions of the 3 d-o-f robot shown in Fig. 1. The feedback information $\mathbf{x}$ is not feasible in these regions (dark regions), and the feedback information $\mathbf{r}$ is activated to move the end effector outside these regions.

3.1 Examples of Regional Feedback

3.1.1 Singularity Problem

In Cartesian-space control, the desired task variable $\mathbf{x}$ is specified in Cartesian space as:

$$\mathbf{x} = \mathbf{p} = [\mathbf{p}_1, \dots, \mathbf{p}_m]^T, \quad (5)$$

where $\mathbf{p}_i$ denote feature points in Cartesian space. The Jacobian matrix $\mathbf{J}_x(\mathbf{q})$ denotes the mapping from joint space to Cartesian space.

As discussed in section 2, the desired task feedback $\mathbf{x}$ is not feasible near the singular positions where the inverse Jacobian matrix becomes singular. To solve the problem, the joint position is specified as the reaching task variable $\mathbf{r}$ and employed near or at the singular positions. The variable $\mathbf{r}$ is thus specified in joint space as:

$$\mathbf{r} = \mathbf{q} = [q_1, \dots, q_n]^T, \quad (6)$$

and the Jacobian matrix $\mathbf{J}_r(\mathbf{q})$ becomes an identity matrix: $\mathbf{J}_r(\mathbf{q}) = \mathbf{I}$ that is invertible.

In general, the manipulator singularities can be divided into external singularity and internal singularity. The external singularity occurs at the external workspace boundary, and the internal singularity includes the internal boundary singularity and interior singularity. [Cheah and Li, 2011].

To illustrate the concept, let us consider a 2 d-o-f planar manipulator. The Jacobian matrix of the manipulator is singular when $\det[\mathbf{J}(\mathbf{q})] = l_1 l_2 \sin(q_2) = 0$, where l_1 and l_2 are the lengths of the first and the second link respectively, and q_2 denotes the second joint angle. Therefore, the singular joint configurations are: $q_2 = 0$ (external boundary singularity), and $q_2 = \pi$ (internal boundary singularity).

Based on the singular configurations, two regions can be specified as:

$$h_{E_1}(\mathbf{q}) = b_{E_1}^2 - (q_2)^2 \geq 0, \quad (7)$$

$$h_{I_{11}}(\mathbf{q}) = (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0, \quad (8)$$

where b_{E_1} and $b_{I_{11}}$ are positive constants, and $h_{E_1}(\mathbf{q}) \geq 0$ is the external region which covers the external boundary singularity, and $h_{I_{11}}(\mathbf{q}) \leq 0$ is the internal region which covers the internal boundary singularity. The joint information is used where $h_{E_1}(\mathbf{q}) \geq 0$ or $h_{I_{11}}(\mathbf{q}) \leq 0$ to drive the end effector away the singular configurations. After the end effector leaves the singular regions, the Cartesian-space feedback is activated so as to move the end effector towards the desired position.

For the 3 d-o-f manipulator illustrated in Fig. 1, the Jacobian matrix of manipulator is singular when:

$$\det[\mathbf{J}(\mathbf{q})] = [l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)] l_3 l_2 \sin(q_3) = 0, \quad (9)$$

where q_2 and q_3 denotes the second and the third joint angles, and l_2 and l_3 are the lengths of the second and the third link respectively. Therefore, the singularities occur when the end effector is near the positions where $q_3 = 0$, $q_3 = \pi$ and $[l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)] = 0$. Three regions can be specified in joint space as:

$$h_{E_1}(\mathbf{q}) = b_{E_1}^2 - (q_3)^2 \geq 0, \quad (10)$$

$$h_{I_{11}}(\mathbf{q}) = (q_3 - \pi)^2 - b_{I_{11}}^2 \leq 0, \quad (11)$$

$$h_{I_{12}}(\mathbf{q}) = [l_2 \cos(q_2) + l_3 \cos(q_2 + q_3)]^2 - b_{I_{12}}^2 \leq 0, \quad (12)$$

where $b_{I_{12}}$ is also a positive constant, and $h_{E_1}(\mathbf{q}) \geq 0$ is the external region which covers the external boundary singularity, and $h_{I_{11}}(\mathbf{q}) \leq 0$ and $h_{I_{12}}(\mathbf{q}) \leq 0$ are the internal regions which cover the internal boundary singularity and the interior singularity.

3.1.2 Limited Field of View

In vision-based control, the desired task variable $\mathbf{x}$ is specified in image space as:

$$\mathbf{x} = \mathbf{x}_I = [\mathbf{x}_{I1}, \dots, \mathbf{x}_{Im}]^T, \quad (13)$$

where m is the number of image features, and $\mathbf{x}_{Ii} = [x_{i_h}, x_{i_v}]^T$ denotes an image feature point, and x_{i_h} represents the horizontal coordinate, and x_{i_v} is the vertical coordinate. The Jacobian matrix $\mathbf{J}_x(\mathbf{q})$ represents the mapping from joint space to image space.

Due to the limited field of view or image occlusion, the visual feedback $\mathbf{x}$ cannot cover the entire robot workspace as shown in Fig. 2. To solve the problem, the reaching task variable $\mathbf{r}$ is specified in Cartesian space as:

$$\mathbf{r} = \mathbf{p}, \quad (14)$$

and the Jacobian matrix $\mathbf{J}_r(\mathbf{q})$ denotes the mapping from joint space to Cartesian space.

To illustrate the concept, let us consider a standard camera with a rectangular field of view. To cover those regions that are outside the field of view, an external region can be specified in Cartesian space as [Cheah and Li, 2010]:

$$\mathbf{h}_{E_1}(\mathbf{p}) = \begin{bmatrix} \frac{(p_{11} - p_{r11})^2}{(p_{b11} - p_{r11})^2} - 1 \\ \frac{(p_{12} - p_{r12})^2}{(p_{b12} - p_{r12})^2} - 1 \\ \frac{(p_{13} - p_{r13})^2}{(p_{b13} - p_{r13})^2} - 1 \end{bmatrix} \geq \mathbf{0}, \quad (15)$$

where the vector $\mathbf{p}_{b1} = [p_{b11}, p_{b12}, p_{b13}]^T$ represents a set of boundary positions, and the vector $\mathbf{p}_{r1} = [p_{r11}, p_{r12}, p_{r13}]^T$ denotes static reference positions. When the function $\mathbf{h}_{E_1}(\mathbf{p})$ is mapped from Cartesian space to image space, it corresponds to a rectangle which can match the field of view, and the Cartesian-space feedback is employed where $\mathbf{h}_{E_1}(\mathbf{p}) \geq \mathbf{0}$ to drive the end effector towards the field of view. Since the objective is to bring the end effector into the field of view, only the position information of the end effector is usually sufficient.

3.1.3 Micromanipulation

In the micromanipulation system, the desired task variable $\mathbf{x}$ represents the position of the end effector measured by the microscope. Therefore, the variable $\mathbf{x}$ is specified in the image space of microscope as:

$$\mathbf{x} = \mathbf{x}_z = [\mathbf{x}_{z1}, \dots, \mathbf{x}_{zm}]^T, \quad (16)$$

where $\mathbf{x}_{zi} = [x_{zi_h}, x_{zi_v}]^T \in \mathbb{R}^2$ denotes an image feature point. Another camera with a wide angle can be used to

control the movement of the end effector at the beginning stage. Therefore, the reaching task variable $\mathbf{r}$ is defined as the vision information obtained from the wide-angle camera as:

$$\mathbf{r} = \mathbf{x}_w = [\mathbf{x}_{w1}, \dots, \mathbf{x}_{wm}]^T. \quad (17)$$

where $\mathbf{x}_{wi} = [x_{wi_h}, x_{wi_v}]^T \in \mathbb{R}^2$ denotes a feature point in the image space of the wide-angle camera. Using regional feedback, the transition from the feedback $\mathbf{r}$ to the feedback $\mathbf{x}$ is smooth, which prevents the vibration during the course of micromanipulation.

Remark 1. The orientation of the end effector can be described by Euler angles or the unit quaternion. If Euler angles are used to represent the end effector's posture, only a local description of the orientation can be obtained for task-space robot control because of the representation singularity. This problem can be solved by introducing internal regions to enclose the joint configurations which lead to the representation singularity. Inside the internal regions, only the reaching task variable $\mathbf{r}$ is employed to move the robot outside the internal regions. The representation singularity can also be eliminated if the orientation of the end effector is represented by the unit quaternion. The corresponding region functions with the unit quaternion can be formulated by using the unit quaternion tracking error [Braganza *et al.*, 2008].

3.2 Region Functions

3.2.1 Region Functions with Reaching Task Feedback

As seen from the previous section, there exist some regions in the robot workspace, where the desired task feedback $\mathbf{x}$ is not feasible due to singularities, limited sensing zone or image occlusion. Therefore, we formulate several functions in the coordinates of $\mathbf{r}$ to represent those regions, and use the reaching task feedback $\mathbf{r}$ to drive the end effector out of these regions. The regions formulated in the coordinates of $\mathbf{r}$ are classified as two categories: external regions and internal regions.

The external regions are specified to cover the positions that are beyond the external boundaries of the desired task variable $\mathbf{x}$, such as the external boundary singularity and the field of view. In general, the external region functions are specified in the coordinates of $\mathbf{r}$ as:

$$\mathbf{h}_E(\mathbf{r}) = [h_{E_1}(\mathbf{r}), h_{E_2}(\mathbf{r}), \dots, h_{E_m}(\mathbf{r})]^T \geq \mathbf{0}, \quad (18)$$

where m is the total number of external regions. The robot employs the feedback $\mathbf{r}$ inside the external regions where $\mathbf{h}_E(\mathbf{r}) \geq \mathbf{0}$. The vector inequality $\mathbf{h}_E(\mathbf{r}) \geq \mathbf{0}$ implies that $h_{E_i}(\mathbf{r}) \geq 0$ for all $i = 1, \dots, m$.

The specific forms of $h_{E_i}(\mathbf{r})$ depend on the robot tasks. For example, consider the singularity problem of a 2 d-o-f manipulator in section 3.1.1, the reaching task variable

$\mathbf{r}$ is specified in joint space (i.e. $\mathbf{r} = \mathbf{q}$), and the external boundary singularity occurs when the end effector is near the position $q_2 = 0$. Therefore, the external region is specified as: $h_{E_1}(\mathbf{q}) = b_{E_1}^2 - (q_2)^2 \geq 0$ to cover the external singular positions. The constant b_{E_1} represents the size of external region and it can be set to ensure that the manipulator is sufficiently far away the singular configuration when the end effector leaves the external region.

Similarly, the internal regions are specified to cover the positions that cannot be reached by the desired task variable $\mathbf{x}$ within the external boundaries, such as the internal boundary singularity, the interior singularity and the image occluded areas. Each external region $h_{E_i}(\mathbf{r})$ may be accompanied with several internal regions $h_{I_{i1}}(\mathbf{r}), h_{I_{i2}}(\mathbf{r}), \dots, h_{I_{iN_i}}(\mathbf{r})$. For example, in Fig. 1, there are two internal singular regions inside a single robot workspace. In Fig. 2, there are two occluded regions within a field of view. Therefore, the internal region functions are specified in general as:

$$\mathbf{h}_I(\mathbf{r}) = [\underbrace{h_{I_{11}}(\mathbf{r}), \dots, h_{I_{1N_1}}(\mathbf{r})}_{N_1}, \dots, \underbrace{h_{I_{i1}}(\mathbf{r}), \dots, h_{I_{iN_i}}(\mathbf{r})}_{N_i}, \dots, \underbrace{h_{I_{m1}}(\mathbf{r}), \dots, h_{I_{mN_m}}(\mathbf{r})}_{N_m}]^T \leq \mathbf{0}, \quad (19)$$

where N_i is the number of internal regions corresponding to the i^{th} external region $h_{E_i}(\mathbf{r})$. The robot employs the feedback $\mathbf{r}$ where $\mathbf{h}_I(\mathbf{r}) \leq \mathbf{0}$. The vector inequality $\mathbf{h}_I(\mathbf{r}) \leq \mathbf{0}$ implies that $h_{I_{ij}}(\mathbf{r}) \leq 0$ for all $i = 1, \dots, m$.

The specific forms of $h_{I_{ij}}(\mathbf{r})$ also vary according to different robot tasks. Consider again the singularity problem of 2 d-o-f manipulator, since the internal boundary singularity occurs when the end effector is near the position $q_2 = \pi$, the internal region is specified as: $h_{I_{11}}(\mathbf{q}) = (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0$ to cover the internal singular positions. The constant $b_{I_{11}}$ represents the size of internal region, and it is also set to ensure that the manipulator is sufficiently far away the singular configuration when the end effector leaves the internal region.

The feedback information $\mathbf{r}$ can be specified in different coordinates such as Cartesian space, image space or joint space, and it is employed inside the external regions $\mathbf{h}_E(\mathbf{r}) \geq \mathbf{0}$ or the internal regions $\mathbf{h}_I(\mathbf{r}) \leq \mathbf{0}$.

3.2.2 Region Functions with Desired Task Feedback

The end effector is driven by the desired task variable $\mathbf{x}$ when it leaves the external and internal regions in the coordinates of $\mathbf{r}$. To ensure a smooth transition from the feedback $\mathbf{r}$ to the feedback $\mathbf{x}$, another sets of external and internal regions are formulated in the coordinates of $\mathbf{x}$, to represent the remaining workspace where the desired task feedback $\mathbf{x}$ is feasible.

The external regions in the coordinates of $\mathbf{x}$ are formulated to include the regions where the desired task feedback is feasible, and the region functions are specified as:

$$\mathbf{f}_E(\mathbf{x}) = [f_{E_1}(\mathbf{x}_1), f_{E_2}(\mathbf{x}_2), \dots, f_{E_m}(\mathbf{x}_m)]^T \leq \mathbf{0}. \quad (20)$$

The robot employs the feedback $\mathbf{x}$ where $\mathbf{f}_E(\mathbf{x}) \leq \mathbf{0}$.

Similarly, the internal regions in the coordinates of $\mathbf{x}$ are formulated to exclude the regions where the desired task feedback is not feasible, and the region functions are specified as:

$$\mathbf{f}_I(\mathbf{x}) = \left[\overbrace{f_{I_{11}}(\mathbf{x}_1), \dots, f_{I_{1N_1}}(\mathbf{x}_1)}^{N_1}, \dots, \overbrace{f_{I_{m1}}(\mathbf{x}_m), \dots, f_{I_{mN_m}}(\mathbf{x}_m)}^{N_m} \right]^T \geq \mathbf{0}. \quad (21)$$

The robot employs the feedback $\mathbf{x}$ where $\mathbf{f}_I(\mathbf{x}) \geq \mathbf{0}$.

The functions of $f_{E_i}(\mathbf{x}_i)$ in equation (20) can be specified as follows:

$$f_{E_i}(\mathbf{x}_i) = \frac{(x_{i1} - x_{Ei1})^{n_{Ei}}}{a_{i1}^{n_{Ei}}} + \dots + \frac{(x_{in_i} - x_{Ein_i})^{n_{Ei}}}{a_{in_i}^{n_{Ei}}} - 1 \leq 0 \quad (22)$$

where $\mathbf{x}_{Ei} = [x_{Ei1}, \dots, x_{Ein_i}]^T \in \mathbb{R}^{n_i}$ represent a set of reference positions, and $a_{i1}, \dots, a_{in_i}$ are positive constants and n_{Ei} are the orders of region functions which are even integers.

The functions of $f_{I_{ij}}(\mathbf{x}_i)$ in equation (21) can be specified as follows:

$$f_{I_{ij}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{Iij1})^{n_{Iij}}}{b_{ij1}^{n_{Iij}}} + \dots + \frac{(x_{in_i} - x_{Iijn_i})^{n_{Iij}}}{b_{ijn_i}^{n_{Iij}}} - 1 \geq 0 \quad (23)$$

where $\mathbf{x}_{Iij} = [x_{Iij1}, \dots, x_{Iijn_i}]^T \in \mathbb{R}^{n_i}$ are a set of reference positions and $b_{ij1}, \dots, b_{ijn_i}$ are positive constants, and n_{Iij} are the orders of region functions which are also even integers.

Both the functions $f_{E_i}(\mathbf{x}_i)$ and $f_{I_{ij}}(\mathbf{x}_i)$ denote superellipses, and the position and shape of superellipse can be varied by adjusting the reference positions $\mathbf{x}_{Ei}$, $\mathbf{x}_{Iij}$ and the orders n_{Ei} , n_{Iij} respectively. The reference positions and orders of $f_{E_i}(\mathbf{x}_i)$ and $f_{I_{ij}}(\mathbf{x}_i)$ should be chosen so that the external regions $f_{E_i}(\mathbf{x}_i) \leq 0$ include the task space where $\mathbf{x}$ is available and the internal regions $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ exclude the task space where $\mathbf{x}$ is not feasible.

Therefore, the region functions $f_{E_i}(\mathbf{x}_i)$ and $f_{I_{ij}}(\mathbf{x}_i)$ denote the external boundary and internal boundary for the feedback $\mathbf{x}$ respectively. It is also possible that there

is no internal boundary for $\mathbf{x}$ in some cases, such as no vision occlusion within the field of view. In that case, it is not required to specify $f_{I_{ij}}(\mathbf{x}_i)$.

The external regions $f_{E_i}(\mathbf{x}_i) \leq 0$ and $h_{E_i}(\mathbf{r}) \geq 0$ should match each other, to allow the robot employs the feedback $\mathbf{x}$ within the external boundaries of $\mathbf{x}$ and uses the feedback $\mathbf{r}$ outside the external boundaries. Similarly, the two internal regions $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ and $h_{I_{ij}}(\mathbf{r}) \leq 0$ should also match each other, to allow the robot uses the feedback $\mathbf{r}$ in the regions where the feedback $\mathbf{x}$ is not feasible and uses the feedback $\mathbf{x}$ after it leaves the regions. The combination of the regions in different coordinates covers the entire workspace and ensures the global movement of robot.

For example, consider the planar 2 d-o-f manipulator illustrated in section 3.1.1, the desired task feedback $\mathbf{x}$ is specified in Cartesian space (i.e. $\mathbf{x} = \mathbf{p}$), and it is activated after the end effector leaves the singular regions. The singular joint configuration $q_2 = 0$ corresponds to a circle in Cartesian space where the manipulator is fully stretched out, and the boundary of the external region $h_{E_1}(\mathbf{q}) = b_{E_1}^2 - (q_2)^2 \geq 0$ which covers $q_2 = 0$ also corresponds to a circle in Cartesian space. To match $h_{E_1}(\mathbf{q}) \geq 0$, the external region function $f_{E_1}(\mathbf{p}_1)$ in Cartesian space is specified as:

$$f_{E_1}(\mathbf{p}_1) = \frac{(p_{11} - p_{E11})^2}{a^2} + \frac{(p_{12} - p_{E12})^2}{a^2} - 1 \leq 0, \quad (24)$$

where $a_{11} = a_{12} = a$, and the order n_{E1} is set as $n_{E1} = 2$ so that the superellipse denoted by $f_{E_1}(\mathbf{p}_1)$ is specified as a circle, and the reference position $\mathbf{p}_{E1} = [p_{E11}, p_{E12}]^T$ is the center of the circle which is set as the origin of the Cartesian coordinates in this case. The constant a denotes the radius and it is set to ensure a slight overlap between $h_{E_1}(\mathbf{q}) \geq 0$ and $f_{E_1}(\mathbf{p}_1) \leq 0$.

Similarly, the joint configuration $q_2 = \pi$ also corresponds to an inner circle in Cartesian space where the manipulator is fully folded back, and the boundaries of the internal region $h_{I_{11}}(\mathbf{q}) = (q_2 - \pi)^2 - b_{I_{11}}^2 \leq 0$ which covers $q_2 = \pi$ corresponds to a circle in Cartesian space as well. To match $h_{I_{11}}(\mathbf{q}) \leq 0$, the internal region function $f_{I_{11}}(\mathbf{p}_1)$ in Cartesian space is specified as:

$$f_{I_{11}}(\mathbf{p}_1) = \frac{(p_{11} - p_{I11})^2}{b^2} + \frac{(p_{12} - p_{I12})^2}{b^2} - 1 \geq 0, \quad (25)$$

where the order n_{I11} , reference position $\mathbf{p}_{I11}$ and radius b are set in a similar argument.

For the problem of limited field of view illustrated in section 3.1.2, the image-space regions are also specified to match the Cartesian-space regions, so that the end effector can transit smoothly from one region to another. For a rectangular field of view of a standard camera, the

order n_{E_i} in equation (22) can be set high (e.g. $n_{E_i} = 20$) to form a rectangular region with rounded corners, and the reference position $\mathbf{x}_{E_i}$ can be set as the center of the field of view.

Next, a set of task-oriented regions are introduced within the external regions so as to move the end effector towards the desired trajectory:

$$\mathbf{f}_T(\mathbf{x}) = [f_{T_1}(\mathbf{x}_1), f_{T_2}(\mathbf{x}_2), \dots, f_{T_m}(\mathbf{x}_m)]^T \leq \mathbf{0}. \quad (26)$$

Let $\mathbf{x}_d = [\mathbf{x}_{d1}, \dots, \mathbf{x}_{dm}]^T \in \mathbb{R}^{\sum_{i=1}^m n_i}$ be the desired trajectory, where $\mathbf{x}_{di} = [x_{di1}, \dots, x_{din_i}]^T \in \mathbb{R}^{n_i}$ is the desired trajectory for the i^{th} feature point. The task-oriented regions are defined to enclose the desired trajectory as:

$$f_{T_i}(\mathbf{x}_i) = \frac{(x_{i1} - x_{di1})^2}{(x_{bi1} - x_{di1})^2} + \dots + \frac{(x_{in_i} - x_{din_i})^2}{(x_{bin_i} - x_{din_i})^2} - 1 \leq 0 \quad (27)$$

where $\mathbf{x}_{bi} = [x_{bi1}, \dots, x_{bin_i}]^T \in \mathbb{R}^{n_i}$ denote the boundary positions. Since the desired position $\mathbf{x}_{di}(t)$ is time-varying and not necessary the geometric center, the boundary positions are divided into several parts.

Remark 2. All the regions specified in the coordinates of $\mathbf{r}$ are static, hence the construction of region functions is a one-time setup that does not vary with the desired motion. In addition, the regions specified in the coordinates of $\mathbf{r}$ are slightly overlapped with the regions in the coordinates of $\mathbf{x}$, so that the end effector does not get stuck when it transits from the feedback $\mathbf{r}$ to $\mathbf{x}$.

3.3 Potential Energy

Using the region functions in coordinates of $\mathbf{x}$ and $\mathbf{r}$, we can now propose the potential energy functions.

3.3.1 Potential Energy with Reaching Task Feedback

Using the external regions in equation (18), potential energy functions are introduced as:

$$P_{E_i}(\mathbf{r}) = \frac{k_{rE_i}}{N} [\max(0, h_{E_i}(\mathbf{r}))]^N, \quad (28)$$

where k_{rE_i} are positive constants, N is an even integer so that the potential energy $\in C^2$. Similarly, the potential energy functions $P_{I_{ij}}(\mathbf{r})$ for the internal regions in equation (19) are introduced as:

$$P_{I_{ij}}(\mathbf{r}) = \frac{k_{rI_{ij}}}{N} [\min(0, h_{I_{ij}}(\mathbf{r}))]^N, \quad (29)$$

where $k_{rI_{ij}}$ are positive constants. Note that $P_{E_i}(\mathbf{r})$ and $P_{I_{ij}}(\mathbf{r})$ are given as:

$$P_{E_i}(\mathbf{r}) = \begin{cases} 0, & h_{E_i}(\mathbf{r}) \leq 0, \\ \frac{k_{rE_i}}{N} [h_{E_i}(\mathbf{r})]^N, & h_{E_i}(\mathbf{r}) > 0, \end{cases} \quad (30)$$

$$P_{I_{ij}}(\mathbf{r}) = \begin{cases} 0, & h_{I_{ij}}(\mathbf{r}) \geq 0, \\ \frac{k_{rI_{ij}}}{N} [h_{I_{ij}}(\mathbf{r})]^N, & h_{I_{ij}}(\mathbf{r}) < 0. \end{cases} \quad (31)$$

Therefore, the potential energy is smooth and lower bounded by zero. It is zero when the position of the end effector is outside the internal or external regions.

The overall potential energy with the reaching task variable $\mathbf{r}$ is the summation of $P_{E_i}(\mathbf{r})$ and $P_{I_{ij}}(\mathbf{r})$ as:

$$P_R(\mathbf{r}) = k_p \alpha_r \sum_{i=1}^m [P_{E_i}(\mathbf{r}) + \sum_{j=1}^{N_i} P_{I_{ij}}(\mathbf{r})], \quad (32)$$

where k_p and α_r are positive constants. Partial differentiating $P_R(\mathbf{r})$ with respect to $\mathbf{r}$ yields:

$$\begin{aligned} \left(\frac{\partial P_R(\mathbf{r})}{\partial \mathbf{r}} \right)^T &= k_p \alpha_r \sum_{i=1}^m \{ k_{rE_i} [\max(0, h_{E_i}(\mathbf{r}))]^{N-1} \left(\frac{\partial h_{E_i}(\mathbf{r})}{\partial \mathbf{r}} \right)^T \\ &+ \sum_{j=1}^{N_i} k_{rI_{ij}} [\min(0, h_{I_{ij}}(\mathbf{r}))]^{N-1} \left(\frac{\partial h_{I_{ij}}(\mathbf{r})}{\partial \mathbf{r}} \right)^T \} \triangleq k_p \alpha_r \Delta \boldsymbol{\varepsilon}_r, \end{aligned} \quad (33)$$

where $\Delta \boldsymbol{\varepsilon}_r$ denotes the reaching regional feedback error which drives the end effector to leave the external and internal regions. From equation (33), note that $\Delta \boldsymbol{\varepsilon}_r = 0$ when $h_{E_i}(\mathbf{r}) \leq 0$ and $h_{I_{ij}}(\mathbf{r}) \geq 0$, which indicates that the regional feedback naturally reduces to zero after the end effector leaves those regions.

3.3.2 Potential Energy with Desired Task Feedback

For the construction of the potential energy in the coordinates of $\mathbf{x}$, two reference regions are formulated for the external region functions $f_{E_i}(\mathbf{x}_i)$ and the internal region functions $f_{I_{ij}}(\mathbf{x}_i)$ respectively.

First, a set of reference regions inside $f_{E_i}(\mathbf{x}_i) \leq 0$ are introduced as:

$$f_{Er_i}(\mathbf{x}_i) = \frac{(x_{i1} - x_{Ei1})^{n_{Ei}}}{(\kappa_{Ei} a_{i1})^{n_{Ei}}} + \dots + \frac{(x_{in_i} - x_{Ein_i})^{n_{Ei}}}{(\kappa_{Ei} a_{in_i})^{n_{Ei}}} - 1 \leq 0 \quad (34)$$

where κ_{E_i} are positive constants and $\kappa_{E_i} < 1$.

By using $f_{E_i}(\mathbf{x}_i)$ in equation (22) and $f_{Er_i}(\mathbf{x}_i)$ in equation (34), the potential energy functions $P_{E_i}(\mathbf{x}_i)$ are in-

troduced as:

$$P_{E_i}(\mathbf{x}_i) = \frac{k_{xE_i}}{N^2} \{ \min[0, [\min(0, f_{E_i}(\mathbf{x}_i))]^N - (\kappa_{E_i}^{n_{E_i}} - 1)^N] \}^N, \quad (35)$$

which can also be written as:

$$P_{E_i}(\mathbf{x}_i) = \begin{cases} 0, & f_{E_i}(\mathbf{x}_i) \leq 0, \\ \frac{k_{xE_i}}{N^2} \{ [f_{E_i}(\mathbf{x}_i)]^N - (\kappa_{E_i}^{n_{E_i}} - 1)^N \}, & f_{E_i}(\mathbf{x}_i) < 0, f_{E_i}(\mathbf{x}_i) > 0, \\ \frac{k_{xE_i}}{N^2} (\kappa_{E_i}^{n_{E_i}} - 1)^N, & f_{E_i}(\mathbf{x}_i) \geq 0, \end{cases} \quad (36)$$

where k_{xE_i} are positive constants. An illustration of $P_{E_i}(\mathbf{x}_i)$ in a 2-D space is shown in Fig. 4. From equation (36), it is seen that $P_{E_i}(\mathbf{x}_i)$ are smooth and lower bounded by zero. The potential energy $P_{E_i}(\mathbf{x}_i)$ is defined to drive the end effector from $f_{E_i}(\mathbf{x}_i) \leq 0$ towards $f_{E_i}(\mathbf{x}_i) \leq 0$ and hence $f_{T_i}(\mathbf{x}_i) \leq 0$.

Fig. 4. Example of potential energy of $P_{E_i}(\mathbf{x}_i)$ with $n_{E_i} = 2$ in 2-D space. The top contour corresponds to $f_{E_i}(\mathbf{x}_i)$ while the bottom contour corresponds to $f_{E_i}(\mathbf{x}_i)$.

Similarly, to introduce the potential energy functions for $f_{I_{ij}}(\mathbf{x}_i)$, we define another set of reference regions enclosing $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ as:

$$f_{I_{ij}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij}1})^{n_{I_{ij}}}}{(\kappa_{I_{ij}} b_{ij1})^{n_{I_{ij}}}} + \dots + \frac{(x_{in_i} - x_{I_{ij}n_i})^{n_{I_{ij}}}}{(\kappa_{I_{ij}} b_{ijn_i})^{n_{I_{ij}}}} - 1 \geq 0 \quad (37)$$

where $\kappa_{I_{ij}} > 1$ are constants.

Using the internal region functions and the reference region functions in equations (23) and (37), the potential energy functions $P_{I_{ij}}(\mathbf{x}_i)$ are proposed as:

$$P_{I_{ij}}(\mathbf{x}_i) = \frac{k_{xI_{ij}}}{N^2} \{ \min[0, [\min(0, f_{I_{ij}}(\mathbf{x}_i))]^N - (\frac{1}{\kappa_{I_{ij}}^{n_{I_{ij}}} - 1)^N] \}^N. \quad (38)$$

where $k_{xI_{ij}}$ are positive constants. An illustration of the potential energy in a 2-D space is shown in Fig. 5. From equation (38), it is seen that $P_{I_{ij}}(\mathbf{x}_i)$ are also smooth and lower bounded by zero. The $P_{I_{ij}}(\mathbf{x}_i)$ will be used

to construct an energy function to avoid the use of the feedback $\mathbf{x}$ when it is not feasible.

Fig. 5. Example of potential energy $P_{I_{ij}}(\mathbf{x}_i)$ with $n_{I_{ij}} = 8$ in 2-D space. The top contour corresponds to $f_{I_{ij}}(\mathbf{x}_i)$ while the bottom contour corresponds to $f_{I_{ij}}(\mathbf{x}_i)$.

By using the task-oriented regions in equation (27), the corresponding potential energy functions $P_{T_i}(\mathbf{x}_i)$ are specified as follows:

$$P_{T_i}(\mathbf{x}_i) = \frac{k_{T_i}}{N} \{ 1 - [\min(0, f_{T_i}(\mathbf{x}_i))]^N \}, \quad (39)$$

where k_{T_i} are positive constants. An illustration of $P_{T_i}(\mathbf{x}_i)$ in a 2-D space is shown in Fig. 6(b), where the bottom of $P_{T_i}(\mathbf{x}_i)$ corresponds to the desired position, and the potential field ensures the convergence after the end effector enters the task-oriented regions.

The overall potential energy $P_D(\mathbf{x})$ with the desired task variable $\mathbf{x}$ is specified as the combination of $P_{E_i}(\mathbf{x}_i)$, $P_{I_{ij}}(\mathbf{x}_i)$ and $P_{T_i}(\mathbf{x}_i)$ as follows:

$$P_D(\mathbf{x}) = k_p \alpha_x \sum_{i=1}^m \{ P_i(\mathbf{x}_i) \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \sum_{j=1}^{N_i} k_{c_{ij}} [\frac{k_{xI_{ij}}}{N^2} (\frac{1}{\kappa_{I_{ij}}^{n_{I_{ij}}} - 1)^N - P_{I_{ij}}(\mathbf{x}_i)] \}, \quad (40)$$

where α_x is a positive constant and $k_{c_{ij}}$ are positive constants, and $P_i(\mathbf{x}_i) = P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)$. An illustration of the combination is shown in Fig. 6, and the rationale for the combination of potential energy functions in equation (40) is summarized as follows:

(i) The top contour of potential energy $P_{E_i}(\mathbf{x}_i)$ (see Fig. 6(a)) is fixed to match $h_{E_i}(\mathbf{r})$, but its bottom is flat. Whereas the potential energy $P_{T_i}(\mathbf{x}_i)$ (see Fig. 6(b)) drives the end effector towards the desired position but its top contour does not match $h_{E_i}(\mathbf{r})$. To combine both features, $P_i(\mathbf{x}_i)$ (see Fig. 6(c)) is defined as the summation of $P_{E_i}(\mathbf{x}_i)$ and $P_{T_i}(\mathbf{x}_i)$. Therefore, as $\mathbf{x}_{di}$ varies, the bottom of potential energy $P_i(\mathbf{x}_i)$ changes while the top contour remains the same (see Fig. 6(c)). Since the top contour of $P_i(\mathbf{x}_i)$ corresponds to the external region functions $f_{E_i}(\mathbf{x}_i)$ and its bottom is the desired position, the potential field of $P_i(\mathbf{x}_i)$ enables the end effector to

move towards the desired position after it enters the external regions.

(ii) The potential energy $P_i(\mathbf{x}_i)$ is further multiplied with the potential energy $P_{I_{ij}}(\mathbf{x}_i)$ (see Fig. 6(d)). The multiplied potential energy is flattened at $f_{I_{ij}}(\mathbf{x}_i) < 0$ where the gradient reduces to zero. It avoids the use of feedback $\mathbf{x}$ where $f_{I_{ij}}(\mathbf{x}_i) < 0$, and the reaching task feedback $\mathbf{r}$ is employed in those regions.

(iii) After that, the potential energy obtained in step (ii) is offset inside the regions $f_{I_{ij}}(\mathbf{x}_i) < 0$ by adding the term $\sum_{j=1}^{N_i} k_{c_{ij}} [\frac{k_{x_{I_{ij}}}}{N^2} (\frac{1}{\kappa_{I_{ij}}} - 1) N^2 - P_{I_{ij}}(\mathbf{x}_i)]$ (see Fig. 6(e)). Note that the potential energy function $P_{I_{ij}}(\mathbf{x}_i)$ is zero when $f_{I_{ij}}(\mathbf{x}_i) \leq 0$ (see Fig. 6(d)). The offset term is therefore introduced to raise the potential energy so as to enable the end effector to move away from the internal regions after leaving them. By adjusting the value of $k_{c_{ij}}$ in the offset term, the energy level can be varied to allow the end effector to pass through the internal regions or avoid them.

The aim of this combination is to ensure the convergence of tracking errors in the presence of regions where the desired feedback $\mathbf{x}$ is not feasible.

Partial differentiating $P_D(\mathbf{x})$ with respect to $\mathbf{x}$ yields:

$$\begin{aligned} \left(\frac{\partial P_D(\mathbf{x})}{\partial \mathbf{x}}\right)^T &= k_p \alpha_x \sum_{i=1}^m \left\{ \left(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + \right. \\ &P_i(\mathbf{x}_i) \sum_{j=1}^{N_i} \left[\left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \prod_{k \neq j}^{N_i} P_{I_{ik}}(\mathbf{x}_i) \right] \\ &\left. - \sum_{j=1}^{N_i} k_{c_{ij}} \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T \right\} \triangleq k_p \alpha_x \Delta \varepsilon_x, \end{aligned} \quad (41)$$

where $\Delta \varepsilon_x$ is the desired regional feedback error which drives the end effector toward the desired trajectory, and it is activated automatically after the end effector is inside the external and internal regions in the coordinates of $\mathbf{x}$. From equation (38), $\left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ in equation (41) is given as:

$$\begin{aligned} \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T &= k_{x_{I_{ij}}} \{ \min\{0, [\min(0, f_{I_{r_{ij}}}(\mathbf{x}_i))]\}^N \\ &- (\frac{1}{\kappa_{I_{ij}}} - 1)^N \} \}^{N-1} [\min(0, f_{I_{r_{ij}}}(\mathbf{x}_i))]^{N-1} \\ &\times \left(\frac{\partial f_{I_{r_{ij}}}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T. \end{aligned} \quad (42)$$

In addition, $\left(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ in equation (41) is given as:

$$\left(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T + \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T, \quad (43)$$

Fig. 6. An illustration of the combination of potential energy functions in 2-D space.

where

$$\begin{aligned} \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T &= k_{x_{E_i}} \{ \min\{0, [\min(0, f_{E_i}(\mathbf{x}_i))]\}^N - \\ &(\kappa_{E_i}^{n_{E_i}} - 1)^N \} \}^{N-1} [\min(0, f_{E_i}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T, \end{aligned} \quad (44)$$

and

$$\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = -k_{T_i} [\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} \left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T, \quad (45)$$

where the partial derivative $\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T$ is the gradient of potential energy $P_{T_i}(\mathbf{x}_i)$ for the task-oriented regions. From equation (45), if the end effector is outside the task-oriented regions where $f_{T_i}(\mathbf{x}_i) > 0$, the gradient of $P_{T_i}(\mathbf{x}_i)$ reduces to zero. After the end effector enters the task-oriented regions, $f_{T_i}(\mathbf{x}_i) \leq 0$, and the gradient of $P_{T_i}(\mathbf{x}_i)$ becomes:

$$\left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = -k_{T_i} [f_{T_i}(\mathbf{x}_i)]^{N-1} \left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T,$$

where $\left(\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}\right)^T = [0, \dots, \frac{2(x_{i1} - x_{di1})}{(x_{bi1} - x_{di1})^2}, \dots, \frac{2(x_{in_i} - x_{din_i})}{(x_{bin_i} - x_{din_i})^2}, \dots, 0]^T \in \Re^{\sum_{i=1}^m n_i}$, so it is nonzero until the end effector reaches the desired position.

Remark 3. The potential energy $P_D(\mathbf{x})$ in equation (40) can be simplified if there is no occlusion within the field

of view [Cheah and Li, 2010] or no internal singularity inside the external task-space boundary:

$$P_D(\mathbf{x}) = k_p \alpha_x \sum_{i=1}^m P_i(\mathbf{x}_i) = k_p \alpha_x \sum_{i=1}^m [P_{E_i}(\mathbf{x}_i) + P_{T_i}(\mathbf{x}_i)]. \quad (46)$$

Remark 4. The reference regions of the external and internal regions are used to introduce differences in the energy levels and note that the internal regions are within each external region. The potential energy $P_{E_i}(\mathbf{x}_i)$ is expressed in terms of the external region $f_{E_i}(\mathbf{x}_i)$ so that $f_{E_i}(\mathbf{x}_i)$ corresponds to the higher energy level of $P_{E_i}(\mathbf{x}_i)$ as illustrated in Fig. 6(a). The potential energy $P_{I_{ij}}(\mathbf{x}_i)$ is expressed in terms of the reference region $f_{I_{ij}}(\mathbf{x}_i)$ so that $f_{I_{ij}}(\mathbf{x}_i)$ corresponds to the lower energy level of the offset energy term as illustrated in Fig. 6(e). In both cases, the regions and their reference regions correspond to the higher levels and the lower levels of the energy functions respectively.

4 Adaptive Controller with Regional Feedback

Since the desired position is only specified in the region functions $f_{T_i}(\mathbf{x}_i)$, the desired velocity is specified as:

$$\dot{\mathbf{x}}_{di} = \mathbf{0}, \quad \text{if } f_{E_i}(\mathbf{x}_i) \geq 0 \text{ or } f_{I_{ij}}(\mathbf{x}_i) \leq 0. \quad (47)$$

That is, the desired velocity is zero if the end effector is outside the external and internal reference regions in the coordinates of $\mathbf{x}$. Equation (47) can be satisfied by using the weight factors [Cheah and Li, 2011] in the desired trajectory. In general, the desired trajectory $\mathbf{x}_{di}$ is consisted of a time-varying part and a constant part, which is denoted as:

$$\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + \mathbf{x}_{vi}(t), \quad (48)$$

where $\mathbf{x}_{ci} = [x_{ci1}, \dots, x_{cin_i}]^T$ is the constant part, and $\mathbf{x}_{vi}(t) = [x_{vi1}, \dots, x_{vin_i}]^T$ is the time-varying part. By using the weight factors, $\mathbf{x}_{di}$ is revised as:

$$\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + w_i \mathbf{x}_{vi}(t), \quad (49)$$

where w_i are weight factors which are defined so that $w_i = 0$ when $f_{E_i}(\mathbf{x}_i) \geq 0$ or $f_{I_{ij}}(\mathbf{x}_i) \leq 0$, and w_i smoothly transits to 1 when $f_{E_i}(\mathbf{x}_i) < 0$ and $f_{I_{ij}}(\mathbf{x}_i) > 0$. The details are given in Appendix A. Therefore, when the end effector is outside the external and internal reference regions, $w_i = 0$ and thus $\mathbf{x}_{di}(t) = \mathbf{x}_{ci}$ and $\dot{\mathbf{x}}_{di} = \mathbf{0}$, then equation (47) is satisfied. When the end effector enters the external and internal reference regions, w_i smoothly increase to 1, and hence $\mathbf{x}_{di}(t) = \mathbf{x}_{ci} + \mathbf{x}_{vi}(t)$ which is the actual trajectory.

Next, a reference vector $\dot{\mathbf{x}}_a = [\dot{\mathbf{x}}_{a1}, \dots, \dot{\mathbf{x}}_{am}]^T \in$

$\mathbb{R}^{\sum_{i=1}^m n_i}$ is introduced, where $\dot{\mathbf{x}}_{ai}$ are specified as:

$$\dot{\mathbf{x}}_{ai} = [\dot{x}_{di1} \frac{x_{bi1} - x_{i1}}{x_{bi1} - x_{di1}}, \dots, \dot{x}_{din_i} \frac{x_{bin_i} - x_{in_i}}{x_{bin_i} - x_{din_i}}]^T. \quad (50)$$

The reference vector $\dot{\mathbf{x}}_a$ is derived from the time derivative of $P_{T_i}(\mathbf{x}_i)$ as shown in equation (B.3) in Appendix B.

From equation (49), note that the reference vectors $\dot{\mathbf{x}}_{ai}$ described by equation (50) can be expressed as:

$$\dot{\mathbf{x}}_{ai} = \mathbf{A}_i \dot{\mathbf{x}}_i + \dot{\mathbf{x}}_{fi}, \quad (51)$$

where

$$\mathbf{A}_i = \begin{bmatrix} \frac{\partial w_i}{\partial x_{i1}} (x_{vi1} \frac{x_{bi1} - x_{i1}}{x_{bi1} - x_{di1}}) & \cdots & \frac{\partial w_i}{\partial x_{in_i}} (x_{vin_i} \frac{x_{bin_i} - x_{in_i}}{x_{bin_i} - x_{din_i}}) \\ \vdots & \ddots & \vdots \\ \frac{\partial w_i}{\partial x_{i1}} (x_{vin_i} \frac{x_{bin_i} - x_{in_i}}{x_{bin_i} - x_{din_i}}) & \cdots & \frac{\partial w_i}{\partial x_{in_i}} (x_{vin_i} \frac{x_{bin_i} - x_{in_i}}{x_{bin_i} - x_{din_i}}) \end{bmatrix},$$

$$\text{and } \dot{\mathbf{x}}_{fi} = [w_i \dot{x}_{vi1} \frac{x_{bi1} - x_{i1}}{x_{bi1} - x_{di1}}, \dots, w_i \dot{x}_{vin_i} \frac{x_{bin_i} - x_{in_i}}{x_{bin_i} - x_{din_i}}]^T.$$

Next, a compound matrix is introduced as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{A}_m \end{bmatrix}, \quad (52)$$

where the matrix $\mathbf{A}$ is introduced to avoid the use of joint acceleration in the proposed controller.

Using equation (52), a sliding vector is defined as:

$$\mathbf{s} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r = \dot{\mathbf{q}} - [\mathbf{I}_n - \mathbf{J}_x^+(q) \mathbf{A} \mathbf{J}_x(q)]^{-1} \times [\mathbf{J}_x^+(q) \dot{\mathbf{q}}_f - \alpha_x \mathbf{J}_x^T(q) \Delta \boldsymbol{\varepsilon}_x - \alpha_r \mathbf{J}_r^T(q) \Delta \boldsymbol{\varepsilon}_r], \quad (53)$$

where $\mathbf{I}_n \in \mathbb{R}^{n \times n}$ is an identity matrix, $\dot{\mathbf{q}}_r$ is a reference vector, and $\dot{\mathbf{x}}_f = [\dot{\mathbf{x}}_{f1}, \dots, \dot{\mathbf{x}}_{fm}]^T$.

Using the regional feedback errors $\Delta \boldsymbol{\varepsilon}_x$ and $\Delta \boldsymbol{\varepsilon}_r$, a novel task-space tracking controller is proposed as:

$$\boldsymbol{\tau} = -[\mathbf{I}_n - \mathbf{J}_x^+(q) \mathbf{A} \mathbf{J}_x(q)]^T [k_p \alpha_x \mathbf{J}_x^T(q) \Delta \boldsymbol{\varepsilon}_x + k_p \alpha_r \mathbf{J}_r^T(q) \Delta \boldsymbol{\varepsilon}_r] - \mathbf{K}_s \mathbf{s} + \mathbf{Y}_d(q, \dot{q}, \ddot{q}, \ddot{q}_r) \hat{\boldsymbol{\theta}}_d, \quad (54)$$

where $\mathbf{K}_s$ is a positive definite matrix. When the end effector reaches the regions where the feedback $\mathbf{x}$ is not feasible due to singularities, occlusion or limited sensing zone, the reaching regional feedback $\Delta \boldsymbol{\varepsilon}_r$ enables the end effector to pass through those regions. After the end effector leaves the regions, $\Delta \boldsymbol{\varepsilon}_r$ reduces to zero, and the desired regional feedback $\Delta \boldsymbol{\varepsilon}_x$ is activated to drive the end effector to the desired trajectory. Note that the

proposed controller is continuous since the region errors $\Delta\epsilon_x$ and $\Delta\epsilon_r$ are continuous.

The estimated parameters $\hat{\theta}_d$ are updated by the following update law:

$$\dot{\hat{\theta}}_d = -L_d Y_d^T(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r) \mathbf{s}, \quad (55)$$

where L_d is a positive definite matrix.

Using equation (53) and the properties of robot dynamics, the dynamic equation is written as:

$$M(\mathbf{q})\dot{\mathbf{s}} + [\frac{1}{2}\dot{M}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{s} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r)\boldsymbol{\theta}_d = \boldsymbol{\tau}. \quad (56)$$

The closed-loop equation of the system is obtained by substituting equation (54) into equation (56) to give:

$$\begin{aligned} M(\mathbf{q})\dot{\mathbf{s}} + [\frac{1}{2}\dot{M}(\mathbf{q}) + \mathbf{S}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{s} + [\mathbf{I}_n - \mathbf{J}_x^+(\mathbf{q})\mathbf{A}\mathbf{J}_x(\mathbf{q})]^T \\ \times [k_p\alpha_x\mathbf{J}_x^T(\mathbf{q})\Delta\epsilon_x + k_p\alpha_r\mathbf{J}_r^T(\mathbf{q})\Delta\epsilon_r] + \mathbf{K}_s\mathbf{s} \\ + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r)\Delta\boldsymbol{\theta}_d = \mathbf{0}, \end{aligned} \quad (57)$$

where $\Delta\boldsymbol{\theta}_d = \boldsymbol{\theta}_d - \hat{\boldsymbol{\theta}}_d$.

To prove the stability, a Lyapunov-like function is proposed as:

$$V = \frac{1}{2}\mathbf{s}^T M(\mathbf{q})\mathbf{s} + P_D(\mathbf{x}) + P_R(\mathbf{r}) + \frac{1}{2}\Delta\boldsymbol{\theta}_d^T L_d^{-1}\Delta\boldsymbol{\theta}_d. \quad (58)$$

Differentiating equation (58) and using equations (47), (50) (53), (55) and (57), we have:

$$\begin{aligned} \dot{V} = -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\epsilon_x + \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\epsilon_r]^T \\ \times [\alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\epsilon_x + \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\epsilon_r]. \end{aligned} \quad (59)$$

The details of the derivations from equation (58) to equation (59) are given in Appendix B.

We can now state the following theorem:

Theorem: *The task-space regional feedback control law (54), the update law (55) for the robot system (1) guarantee the global convergence of the tracking errors. That is $\mathbf{x}(t) \rightarrow \mathbf{x}_d(t)$, $\dot{\mathbf{x}}(t) \rightarrow \dot{\mathbf{x}}_d(t)$ as $t \rightarrow \infty$.*

Proof: Since $V > 0$ and $\dot{V} \leq 0$, V is bounded. Hence, $\mathbf{s}$, $\Delta\boldsymbol{\theta}_d$, $P_R(\mathbf{r})$ and $P_D(\mathbf{x})$ are bounded. Since $P_R(\mathbf{r})$ is bounded, $P_{E_i}(\mathbf{r})$ and $P_{I_{ij}}(\mathbf{r})$ are bounded. The boundedness of $P_{E_i}(\mathbf{r})$ and $P_{I_{ij}}(\mathbf{r})$ ensures the boundedness of $h_{E_i}(\mathbf{r})$ and $h_{I_{ij}}(\mathbf{r})$. In addition, $P_{E_i}(\mathbf{x}_i)$, $P_{I_{ij}}(\mathbf{x}_i)$ and $P_{T_i}(\mathbf{x}_i)$ are also bounded since $P_D(\mathbf{x})$ is bounded. The

boundedness of $P_{E_i}(\mathbf{x}_i)$, $P_{I_{ij}}(\mathbf{x}_i)$ and $P_{T_i}(\mathbf{x}_i)$ ensures the boundedness of $f_{E_i}(\mathbf{x}_i)$, $f_{I_{ij}}(\mathbf{x}_i)$ and $f_{T_i}(\mathbf{x}_i)$. Since the region functions are bounded, the reaching task variable $\mathbf{r}$ and the desired task variable $\mathbf{x}$ are bounded. Hence, $\frac{\partial h_{E_i}(\mathbf{r})}{\partial \mathbf{r}}$, $\frac{\partial h_{I_{ij}}(\mathbf{r})}{\partial \mathbf{r}}$, $\frac{\partial f_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}}$, $\frac{\partial f_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}}$ and $\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}}$ are also bounded. Therefore, $\Delta\epsilon_r$ and $\Delta\epsilon_x$ are bounded. Since $\mathbf{x}$ is bounded, $\dot{\mathbf{x}}_a$ is bounded if $\dot{\mathbf{x}}_d$ is bounded. Hence, $\dot{\mathbf{q}}_r$ is bounded. From equation (53), $\dot{\mathbf{q}}$ is bounded because $\mathbf{s}$ is bounded. The boundedness of $\dot{\mathbf{q}}$ guarantees the boundedness of $\dot{\mathbf{x}}$ and $\dot{\mathbf{r}}$ since both $\mathbf{J}_x(\mathbf{q})$ and $\mathbf{J}_r(\mathbf{q})$ are trigonometric functions of $\mathbf{q}$ or constant. Therefore, $\Delta\dot{\epsilon}_r$ and $\Delta\dot{\epsilon}_x$ are bounded. Then $\ddot{\mathbf{q}}_r$ is bounded if $\ddot{\mathbf{x}}_d$ is bounded. From the closed-loop equation (57), we can conclude that $\dot{\mathbf{s}}$ is bounded. Thus, $\ddot{V}$ is bounded since $\dot{\mathbf{s}}$, $\mathbf{s}$, $\Delta\epsilon_r$, $\Delta\epsilon_x$, $\Delta\dot{\epsilon}_r$, $\Delta\dot{\epsilon}_x$ are bounded. Therefore, $\ddot{V}$ is uniformly continuous. Applying Barbalat's lemma [Slotine and Li, 1991], we have $\dot{V} \rightarrow 0$ which also indicates:

$$\begin{aligned} \alpha_x \mathbf{J}_x^T(\mathbf{q})\Delta\epsilon_x + \alpha_r \mathbf{J}_r^T(\mathbf{q})\Delta\epsilon_r \rightarrow \mathbf{0}, \\ \mathbf{s} \rightarrow \mathbf{0}. \end{aligned} \quad (60)$$

If the end effector is located where $f_{E_i}(\mathbf{x}_i) \geq 0$ or $f_{I_{ij}}(\mathbf{x}_i) \leq 0$, $\Delta\epsilon_x = \mathbf{0}$, $\Delta\epsilon_r \neq \mathbf{0}$, which contracts with equation (60) since $\mathbf{J}_r(\mathbf{q})$ is non-singular. Therefore, the end effector can only settle down where $h_{E_i}(\mathbf{r}) \leq 0$ and $h_{I_{ij}}(\mathbf{r}) \geq 0$, and hence $\Delta\epsilon_r = \mathbf{0}$. Since $\mathbf{J}_x(\mathbf{q})$ is also non-singular, from equation (60) $\Delta\epsilon_x = \mathbf{0}$. From equation (41), $\Delta\epsilon_x = \mathbf{0}$ can only be satisfied where $f_{T_i}(\mathbf{x}_i) \leq 0$, $f_{E_{r_i}}(\mathbf{x}_i) \leq 0$ and $f_{I_{r_{ij}}}(\mathbf{x}_i) \geq 0$. Hence $\Delta\epsilon_x = \mathbf{0}$ means that $\frac{\partial f_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} = \mathbf{0}$. That is, $\mathbf{x} \rightarrow \mathbf{x}_d$ as $t \rightarrow \infty$. From the definition of $\dot{\mathbf{x}}_a$ in equation (50), $\mathbf{x} \rightarrow \mathbf{x}_d$ indicate that $\dot{\mathbf{x}}_a \rightarrow \dot{\mathbf{x}}_d$. Then from the definition of $\mathbf{s}$ in equation (53), $\dot{\mathbf{x}}_a \rightarrow \dot{\mathbf{x}}_d$, $\Delta\epsilon_r \rightarrow \mathbf{0}$, $\Delta\epsilon_x \rightarrow \mathbf{0}$ and $\mathbf{s} \rightarrow \mathbf{0}$ implies $\dot{\mathbf{x}} \rightarrow \dot{\mathbf{x}}_d$ as $t \rightarrow \infty$. $\triangle\triangle\triangle$

Remark 5. The combination of potential energies does not result in local minimum for the tracking control task. This can be shown by analyzing the partial derivatives of the potential energy in equations (33) and (41) and the proposed controller in equation (54).

(i) If the end effector is located where $f_{E_i}(\mathbf{x}_i) \leq 0$, $f_{E_{r_i}}(\mathbf{x}_i) > 0$ and $f_{I_{r_{ij}}}(\mathbf{x}_i) \geq 0$, the gradient of $P_i(\mathbf{x}_i)$ is nonzero, and the gradient of $P_{I_{ij}}(\mathbf{x}_i)$ reduces to zero.

Therefore, $\Delta\epsilon_x = \sum_{i=1}^m [(\frac{\partial P_i(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)]$, which is nonzero until the end effector is inside the external reference regions such that $f_{E_{r_i}}(\mathbf{x}_i) \leq 0$.

(ii) After the end effector enters the external reference regions, it is inside the task-oriented regions such that $f_{T_i}(\mathbf{x}_i) \leq 0$. If the end effector is located where $f_{T_i}(\mathbf{x}_i) \leq 0$ and $f_{I_{r_{ij}}}(\mathbf{x}_i) \geq 0$, then both the gradient of $P_{E_i}(\mathbf{x}_i)$ and $P_{I_{ij}}(\mathbf{x}_i)$ are zero, and $P_i(\mathbf{x}_i) = P_{T_i}(\mathbf{x}_i)$. Therefore,

$$\Delta\epsilon_x = \sum_{i=1}^m [(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i)], \text{ which is nonzero until}$$

the end effector reaches the desired position.

(iii) When the end effector starts outside the external or internal regions such that $f_{E_i}(\mathbf{x}_i) > 0$ or $f_{I_{ij}}(\mathbf{x}_i) < 0$, the reaching task feedback $\mathbf{r}$ is activated to enable the end effector to leave those regions.

(iv) However, there is one saddle point between each internal region and internal reference region where $f_{I_{ij}}(\mathbf{x}_i) \geq 0$ and $f_{I_{r_{ij}}}(\mathbf{x}_i) < 0$, and the gradient of potential energy $(\frac{\partial P_D(\mathbf{x})}{\partial \mathbf{x}})^T$ at the saddle point is zero. The saddle point exists in the presence of each internal region. This is because that the potential energy $P_i(\mathbf{x}_i)$ is firstly multiplied by $P_{I_{ij}}(\mathbf{x}_i)$ and then added with the offset term. If the end effector reaches the saddle position, both $\Delta \varepsilon_x$ and $\Delta \varepsilon_r$ reduce to zero. However, both $\mathbf{s}$ and $\boldsymbol{\tau}$ are nonzero because the velocity control term $\dot{\mathbf{q}}$ is not zero. When the system starts from rest at the saddle point with $\dot{\mathbf{x}}_d = \mathbf{0}$, the initial estimates of the uncertain parameters $\hat{\boldsymbol{\theta}}_d$ can be set as nonzero values so that $\mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r) \hat{\boldsymbol{\theta}}_d$ is nonzero and thus result in a non-zero initial control input. This enables the end effector to leave the saddle point. If all elements in the regressor matrix reduce to zero, the initial control input may reduce to zero, and an over-parameterization of dynamic model is needed to move the end effector out of the saddle point.

Remark 6. The end effector does not avoid the regions where the singularities or occlusion may occur, and it can start from those regions. Once the end effector is outside the external and internal regions in the coordinates of $\mathbf{x}$ such that $f_{E_i}(\mathbf{x}_i) \geq 0$ or $f_{I_{ij}}(\mathbf{x}_i) \leq 0$, only the reaching task feedback $\mathbf{r}$ is employed. From equation (41), the desired regional feedback error $\Delta \varepsilon_x = \mathbf{0}$. From the definition of the weight factors in Appendix A, $w_i = 0$, and hence equation (47) is satisfied, $\dot{\mathbf{x}}_{di} = \mathbf{0}$. From equations (50), (51) and (52), $\dot{\mathbf{x}}_f = \mathbf{0}$, and $\mathbf{A}$ reduces to a zero matrix. Therefore, $[\mathbf{I}_n - \mathbf{J}^+(\mathbf{q})\mathbf{A}\mathbf{J}(\mathbf{q})]^{-1} = \mathbf{I}_n$, the controller becomes:

$$\boldsymbol{\tau} = -k_p \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r - \mathbf{K}_s \mathbf{s} + \mathbf{Y}_d(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_r) \hat{\boldsymbol{\theta}}_d,$$

where $\mathbf{s} = \dot{\mathbf{q}} + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \varepsilon_r$, which drives the end effector to leave the regions where the singularities or occlusion may occur, without the pseudo inverse matrix $\mathbf{J}_x^+(\mathbf{q})$.

After the end effector leaves the regions such that $h_{E_i}(\mathbf{r}) < 0$ and $h_{I_{ij}}(\mathbf{r}) > 0$, from equation (33) the reaching regional feedback error $\Delta \varepsilon_r$ naturally reduces to zero, and the desired regional feedback error $\Delta \varepsilon_x$ is activated. In addition, the weight factors w_i smoothly increase to 1. In the case that $w_i = 1$, $\dot{\mathbf{x}}_{di} \neq \mathbf{0}$.

5 Experiment

The experimental setup [Cheah, 2008] consists of a Sony SCARA robot and a PSD camera (C5949) manufactured

by Hamamatsu. The proposed controllers were implemented on the first two links of the robot.

5.1 Singularity Problem

In the first experiment, the reaching task variable $\mathbf{r}$ is specified in joint space, and the desired task variable $\mathbf{x}$ is specified in Cartesian space. The external singularity occurs where $q_2 = 0$, and the internal singularity occurs where $q_2 = \pi$. The external region $h_{E_1}(\mathbf{q}) \geq 0$ and the internal region $h_{I_{11}}(\mathbf{q}) \leq 0$ are formulated in joint space to cover those singular configurations with the boundary of $\pi/15$ and $\pi/10$ respectively.

Both the external and internal singularity correspond to circular paths in Cartesian space. Therefore, for the purpose of matching, another external region $f_{E_1}(\mathbf{p}_1) \leq 0$ and internal region $f_{I_{11}}(\mathbf{p}_1) \geq 0$ are formulated in Cartesian space, and the parameters in equations (22), (23), (34) and (37) are set as: $\mathbf{p}_{E_1} = \mathbf{p}_{I_{11}} = [0, 0]^T$, $a_{1_1} = a_{1_2} = 0.718 \text{ m}$, $\kappa_{E_1} = 0.65$, $n_{E_1} = 2$, $b_{1_{11}} = b_{1_{12}} = 0.112 \text{ m}$, $\kappa_{I_{11}} = 1.5$, $n_{I_{11}} = 2$. The parameters of task-oriented region function $f_{T_1}(\mathbf{p}_1)$ in equation (27) are set as: $p_{b_{1_1}} = -0.51 \text{ m}$ if $p_{1_1} \leq p_{d_{1_1}}$, else $p_{b_{1_1}} = 0.51 \text{ m}$, and $p_{b_{1_2}} = -0.51 \text{ m}$ if $p_{1_2} \leq p_{d_{1_2}}$, else $p_{b_{1_2}} = 0.51 \text{ m}$.

The end effector is required to start from an initial position at $(-0.05 \text{ m}, 0.72 \text{ m})$ where q_2 is near the singular position $q_2 = 0$, and track a time-varying trajectory specified in Cartesian space as: $p_{d_{1_1}} = -0.35 + 0.1 \cos(0.4t - \frac{\pi}{2})w$, $p_{d_{1_2}} = 0.13 + 0.1 \sin(0.4t - \frac{\pi}{2})w$ where the w is the weight factor. The order of potential energy in equations (28), (29), (35), (38), (39) is $N = 4$, and the control parameters in equation (54) and (55) were set as: $k_{r_{E_1}} = 200$, $k_{T_1} = 0.3$, $k_{x_{E_1}} = 6000$, $k_{x_{I_{11}}} = 200000$, $\mathbf{K}_s = \text{diag}\{0.0005, 0.0005\}$, $k_p = 1$, $\alpha_x = 1$, $\alpha_r = 1$, $\mathbf{L}_d = \text{diag}\{0.001, 0.001\}$. The experimental results are shown in Fig. 7(a)-7(c). From Fig. 7, it is seen that the end effector can start from the singular configurations and transit smoothly from joint space to Cartesian space. After it enters the task-oriented region $f_{T_1}(\mathbf{p}_1) \leq 0$, the position of the end effector converges to the desired trajectory.

5.2 Limited Field of View

In the second experiment, the PSD camera is used to measure the position of the end effector. The reaching task variable $\mathbf{r}$ is specified in Cartesian space, and the desired task variable $\mathbf{x}$ is specified in image space.

The external region $f_{E_1}(\mathbf{x}_{I_1}) \leq 0$ in image space is used to specify the size of the field of view. Since the shape of the field of view is rectangular, the parameters of region functions in equation (22) and (34) are set as: $\mathbf{x}_{E_1} = [-0.22 \text{ V}, 0.55 \text{ V}]^T$, $a_{1_1} = a_{1_2} = 1.85 \text{ V}$, $\kappa_{E_1} = 0.81$, $n_{E_1} = 10$. The external region $h_{E_1}(\mathbf{p}_1) \geq 0$ in Cartesian

Fig. 7. The end effector starts from an initial position at $(-0.05 \text{ m}, 0.72 \text{ m})$ near the external singular configuration.

space enables the robot to reach the field of view, and the parameters of region function in equation (15) are set as: $\mathbf{p}_{r1} = [-0.42 \text{ m}, 0.13 \text{ m}]^T$, and $p_{b11} = -0.48 \text{ m}$ if $p_{11} \leq p_{r11}$, else $p_{b11} = -0.36 \text{ m}$, and $p_{b12} = 0.06 \text{ m}$ if $p_{12} \leq p_{r12}$, else $p_{b12} = 0.19 \text{ m}$. The parameters of the task-oriented region function $f_{T1}(\mathbf{x}_{I1})$ in equations (27) are set as: $x_{b1h} = -1.72 \text{ V}$ if $x_{1h} \leq x_{d1h}$, else $x_{b1h} = 1.78 \text{ V}$, and $x_{b1v} = -1.45 \text{ V}$ if $x_{1v} \leq x_{d1v}$, else $x_{b1v} = 2.25 \text{ V}$. Next, suppose that there is an occluded area locating near the position $(-0.22 \text{ V}, 1.75 \text{ V})$ within the field of view. To enclose the occluded area, an internal region $f_{I11}(\mathbf{x}_{I1}) \geq 0$ is introduced and the parameters of region function in equation (23) and (37) are set as: $\mathbf{x}_{I11} = [-0.22 \text{ V}, 1.75 \text{ V}]^T$, $b_{111} = 1.12 \text{ V}$, $b_{112} = 0.2 \text{ V}$, $\kappa_{I11} = 1.25$, $n_{I11} = 2$.

The end effector is controlled to start from an initial position at $(-0.10 \text{ m}, 0.61 \text{ m})$ which is outside the field of view, and track the desired trajectory in image space as: $x_{d1h} = -0.22 + 0.7\cos(0.4t)w$, $x_{d1v} = 0.55 + 0.7\sin(0.4t)w$, where the w is the weight factor. The control parameters were set as: $k_{rE1} = 3 \times 10^{-7}$, $k_{T1} = 0.1$, $k_{xE1} = 0.01$, $\mathbf{K}_s = \text{diag}\{0.0005, 0.0005\}$, $k_p = 1$, $\alpha_x = 1$, $\alpha_r = 1$, $\mathbf{L}_d = \text{diag}\{0.001, 0.001\}$. The experimental results are shown in Fig. 8(a)-8(d). As seen from Fig. 8(a)-8(d), the end effector can transit smoothly from outside to inside the field of view. In the presence of image occlusion, the end effector is able to pass through the occluded area with the Cartesian-space feedback and complete the tracking task after it leaves the occluded area.

6 Conclusion

In this paper, a novel task-space control method with regional feedback is proposed. The regional feedback consists of a reaching task variable in the initial stage and a desired task variable in the ending stage. With the regional feedback, the robot can transit smoothly from one task space to another, and converge to the desired trajectory in the end. Using the regional feedback, the problem of global stability is formulated and solved. A global task-space controller with regional feedback is proposed, and experimental results have shown that the

proposed controller can efficiently solve the problem of limited field of view and the singularity problem. The results can also be extended to a multiple regional feedback controller [Li and Cheah, 2012].

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Fig. 8. In the presence of image occlusion, the robot can pass through the occluded area with Cartesian-space feedback, and converge to the desired trajectory in the end.

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Appendix

A Weight Factor

To satisfy equation (47), weight factors are introduced as:

$$w_i(\mathbf{x}_i) = w_{E_i}(\mathbf{x}_i) \prod_{j=1}^{N_i} w_{I_{ij}}(\mathbf{x}_i), \quad (\text{A.1})$$

where $w_{E_i}(\mathbf{x}_i)$ and $w_{I_{ij}}(\mathbf{x}_i)$ are the weight factors corresponding to the i^{th} external region and the ij^{th} internal region respectively. Both $w_{E_i}(\mathbf{x}_i)$ and $w_{I_{ij}}(\mathbf{x}_i)$ must be continuous and smooth between 0 and 1.

To construct $w_{E_i}(\mathbf{x}_i)$, two regions inside the $f_{E_i}(\mathbf{x}_i)$ are defined as:

$$f_{w_{E_i}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{E_{i1}})^{n_{E_i}}}{c_{i1}^{n_{E_i}}} + \dots + \frac{(x_{in_i} - x_{E_{in_i}})^{n_{E_i}}}{c_{in_i}^{n_{E_i}}} - 1 \leq 0,$$

$$f_{w_{E_i}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{E_{i1}})^{n_{E_i}}}{(\kappa_{w_i} c_{i1})^{n_{E_i}}} + \dots + \frac{(x_{in_i} - x_{E_{in_i}})^{n_{E_i}}}{(\kappa_{w_i} c_{in_i})^{n_{E_i}}} - 1 \leq 0,$$

where $c_{i1}, \dots, c_{in_i}$ are positive constants, and $\kappa_{w_i} < 1$ are also positive constants. Based on the $f_{w_{E_i}}(\mathbf{x}_i)$ and $f_{w_{E_i}}(\mathbf{x}_i)$, the weight factors $w_{E_i}(\mathbf{x}_i)$ are proposed as:

$$w_{E_i}(\mathbf{x}_i) = \begin{cases} 0, f_{w_{E_i}}(\mathbf{x}_i) \geq 0, \\ 1 - \frac{\{[f_{w_{E_i}}(\mathbf{x}_i)]^4 - (\kappa_{w_i}^{n_{E_i}} - 1)^4\}^4}{(\kappa_{w_i}^{n_{E_i}} - 1)^{16}}, f_{w_{E_i}}(\mathbf{x}_i) > 0, f_{w_{E_i}}(\mathbf{x}_i) < 0, \\ 1, f_{w_{E_i}}(\mathbf{x}_i) \leq 0. \end{cases} \quad (\text{A.2})$$

Similarly, to construct $w_{I_{ij}}(\mathbf{x}_i)$, two regions are defined as:

$$f_{w_{I_{ij}}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij1}})^{n_{I_{ij}}}}{d_{ij1}^{n_{I_{ij}}}} + \dots + \frac{(x_{in_i} - x_{I_{ijn_i}})^{n_{I_{ij}}}}{d_{ijn_i}^{n_{I_{ij}}}} - 1 \geq 0,$$

$$f_{w_{I_{ij}}}(\mathbf{x}_i) = \frac{(x_{i1} - x_{I_{ij1}})^{n_{I_{ij}}}}{(\kappa_{w_{ij}} d_{ij1})^{n_{I_{ij}}}} + \dots + \frac{(x_{in_i} - x_{I_{ijn_i}})^{n_{I_{ij}}}}{(\kappa_{w_{ij}} d_{ijn_i})^{n_{I_{ij}}}} - 1 \geq 0,$$

where $d_{ij1}, \dots, d_{ijn_i}$ are positive constants, and $\kappa_{w_{ij}} < 1$ are also positive constants. Based on the $f_{w_{I_{ij}}}(\mathbf{x}_i)$ and $f_{w_{I_{ij}}}(\mathbf{x}_i)$, the weight factors $w_{I_{ij}}(\mathbf{x}_i)$ are proposed as:

$$w_{I_{ij}}(\mathbf{x}_i) = \begin{cases} 0, f_{w_{I_{ij}}}(\mathbf{x}_i) \leq 0, \\ \frac{\{[f_{w_{I_{ij}}}(\mathbf{x}_i)]^4 - (\kappa_{w_{ij}}^{n_{I_{ij}}} - 1)^4\}^4}{(\kappa_{w_{ij}}^{n_{I_{ij}}} - 1)^{16}}, f_{w_{I_{ij}}}(\mathbf{x}_i) > 0, f_{w_{I_{ij}}}(\mathbf{x}_i) < 0, \\ 1, f_{w_{I_{ij}}}(\mathbf{x}_i) \geq 0. \end{cases} \quad (\text{A.3})$$

B Lyapunov-like Analysis

The Lyapunov-like function is proposed in equation (58) as:

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{M}(\mathbf{q}) \mathbf{s} + P_D(\mathbf{x}) + P_R(\mathbf{r}) + \frac{1}{2} \Delta \boldsymbol{\theta}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d. \quad (\text{B.1})$$

Next, note that the time derivative of $P_D(\mathbf{x})$ in equation (40) is given as:

$$\dot{P}_D(\mathbf{x}) = k_p \alpha_x \sum_{i=1}^m \left\{ \dot{P}_i(\mathbf{x}_i) \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) + P_i(\mathbf{x}_i) \sum_{j=1}^{N_i} [\dot{P}_{I_{ij}}(\mathbf{x}_i) \prod_{k \neq j} P_{I_{ik}}(\mathbf{x}_i)] - \sum_{j=1}^{N_i} k_{c_{ij}} \dot{P}_{I_{ij}}(\mathbf{x}_i) \right\}. \quad (\text{B.2})$$

where $\dot{P}_i(\mathbf{x}_i) = \dot{P}_E(\mathbf{x}_i) + \dot{P}_T(\mathbf{x}_i)$. From equation (39), the time derivative of $P_{T_i}(\mathbf{x}_i)$ is given as:

$$\begin{aligned} \dot{P}_{T_i}(\mathbf{x}_i) &= -k_{T_i} [\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} \dot{f}_{T_i}(\mathbf{x}_i) \\ &= \left[\dot{x}_{i1} - \frac{\dot{x}_{di1}(x_{bi1} - x_{i1})}{(x_{bi1} - x_{di1})}, \dots, \dot{x}_{in_i} - \frac{\dot{x}_{din_i}(x_{bin_i} - x_{in_i})}{(x_{bin_i} - x_{din_i})} \right] \\ &\quad \times \left[-\frac{2k_{T_i} [\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} (x_{i1} - x_{di1})}{(x_{bi1} - x_{di1})^2}, \dots, \right. \\ &\quad \left. -\frac{2k_{T_i} [\min(0, f_{T_i}(\mathbf{x}_i))]^{N-1} (x_{in_i} - x_{din_i})}{(x_{bin_i} - x_{din_i})^2} \right]^T \\ &= (\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_{ai})^T \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}_i} \right)^T = (\dot{\mathbf{x}} - \dot{\mathbf{x}}_a)^T \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T, \quad (\text{B.3}) \end{aligned}$$

where the corresponding terms in $\dot{\mathbf{x}} - \dot{\mathbf{x}}_a$ are multiplied by zero in $(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T$.

Differentiating equation (B.1) with respect to time and substituting equations (B.2) and (B.3) into it, we have:

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{s}} + \frac{1}{2} \mathbf{s}^T \dot{\mathbf{M}}(\mathbf{q}) \mathbf{s} + k_p \alpha_x \dot{\mathbf{x}}^T \Delta \boldsymbol{\varepsilon}_x + k_p \alpha_r \dot{\mathbf{r}}^T \Delta \boldsymbol{\varepsilon}_r \\ &\quad - k_p \alpha_x \sum_{i=1}^m \left[\dot{\mathbf{x}}_a^T \left(\frac{\partial P_{T_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) \right] - \dot{\boldsymbol{\theta}}_d^T \mathbf{L}_d^{-1} \Delta \boldsymbol{\theta}_d. \quad (\text{B.4}) \end{aligned}$$

Substituting equations (53), (55) and (57) into equation (B.4), we obtain:

$$\begin{aligned} \dot{V} &= -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]^T \times \\ &\quad [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r] + k_p \alpha_r \dot{\mathbf{x}}_a^T \mathbf{J}_x^T(\mathbf{q}) \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r + \\ &\quad k_p \alpha_x \sum_{i=1}^m \left\{ \dot{\mathbf{x}}_a^T \left(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T \prod_{j=1}^{N_i} P_{I_{ij}}(\mathbf{x}_i) - \sum_{j=1}^{N_i} [k_{c_{ij}} \dot{\mathbf{x}}_a^T \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T] \right. \\ &\quad \left. + [P_{T_i}(\mathbf{x}_i) + P_{E_i}(\mathbf{x}_i)] \sum_{j=1}^{N_i} [\dot{\mathbf{x}}_a^T \left(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}} \right)^T \prod_{k \neq j} P_{I_{ik}}(\mathbf{x}_i)] \right\}. \quad (\text{B.5}) \end{aligned}$$

From equations (47) and (50), we have:

$$\dot{\mathbf{x}}_{ai} = \mathbf{0}, \quad \text{if } f_{E_i}(\mathbf{x}_i) \geq 0 \text{ or } f_{I_{ij}}(\mathbf{x}_i) \leq 0. \quad (\text{B.6})$$

The condition indicates that $f_{E_i}(\mathbf{x}_i) \geq 0$ or $f_{I_{ij}}(\mathbf{x}_i) \leq 0$ includes that $h_{E_i}(\mathbf{r}) \geq 0$ or $h_{I_{ij}}(\mathbf{r}) \leq 0$ where the reaching regional feedback $\Delta \boldsymbol{\varepsilon}_r$ is nonzero, thus $\mathbf{x}_{ai}$ and $\Delta \boldsymbol{\varepsilon}_r$ cannot be nonzero at the same time.

Equation (B.6) also indicates that $\dot{\mathbf{x}}_a$ is nonzero at $f_{E_i}(\mathbf{x}_i) < 0$ and $f_{I_{ij}}(\mathbf{x}_i) > 0$ where the gradient of $P_{E_i}(\mathbf{x}_i)$ and $P_{I_{ij}}(\mathbf{x}_i)$ reduces to zero. Thus, $\mathbf{x}_{ai}$ and $(\frac{\partial P_{E_i}(\mathbf{x}_i)}{\partial \mathbf{x}})^T$, $\mathbf{x}_{ai}$ and $(\frac{\partial P_{I_{ij}}(\mathbf{x}_i)}{\partial \mathbf{x}})^T$ cannot be nonzero at the same time. Therefore, the last three terms in above equation (B.5) are always zero so that:

$$\begin{aligned} \dot{V} &= -\mathbf{s}^T \mathbf{K}_s \mathbf{s} - k_p [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]^T \\ &\quad \times [\alpha_x \mathbf{J}_x^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_x + \alpha_r \mathbf{J}_r^T(\mathbf{q}) \Delta \boldsymbol{\varepsilon}_r]. \quad (\text{B.7}) \end{aligned}$$