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Vision-Based Flexible Leader-Follower Formation Tracking of Multiple Nonholonomic Mobile Robots in Unknown Obstacle Environments

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Abstract—This brief investigates the flexible leader-follower formation tracking problem for a group of nonholonomic mobile robots, while most of formation control related work in the literature focuses on rigid formation. The flexible formation discussed in this brief is defined in curvilinear coordinates in terms of longitudinal separations between robots along the reference trajectory and lateral deviations with respect to this trajectory. Unlike the previous studies on flexible formation control, this brief is under a more challenging assumption that global position and orientation measurements are not available. To obtain the relative pose relationships amongst robots, a stereo camera is mounted on each follower. In consideration of the fact that visual observations are noise-corrupted and intermittently available, a particle filter based relative pose estimation approach is employed to estimate the position and orientation of the leader in the local reference frame of the follower using the polluted and discontinuous information. Also, to form a flexible formation, the leader historical trajectory is reconstructed with respect to the current local frame attached on the follower, based on which a reference point is generated. In addition, this brief considers the situation where robots operate in unknown obstacle environments. To ensure robot safety in such environments, a multi-objective control law is proposed to balance reference tracking and collision avoidance in different situations. Simulation and real robot experiment have been performed to demonstrate the efficacy of the proposed method.

Index Terms—Flexible formation, formation control, leader-follower, multi-robot system, collision avoidance.

I. INTRODUCTION

Multi-robot system has attracted numerous attentions from both research and industrial communities in recent years due to its extensive applications in both civilian and military fields. Among the current research hotspots in this area, formation control, which is to form a certain geometric pattern with or without a group reference [1], is one of the most attractive topics which has been investigated extensively in the past decade (see [2] for a recent survey). As a result, a large variety of solutions have been proposed. Formation tracking, which is formation control with a group reference, is more preferable in practical applications such as formation flying, coordinated target tracking, mobile sensor networks and unknown environment exploration. Existing approaches on formation tracking can be classified into two categories,

leader-follower [3]–[6] and virtual-structure [7]–[9]. In leader-follower formation tracking, the leader acts as the group reference, while in virtual-structure formation tracking, a virtual leader is introduced to play the role of group reference.

Even though formation tracking has been investigated in depth in the past decade, it is obvious that almost all of the existing approaches aim at forming a rigid formation, which is indeed advantageous in some applications such as cooperative transportation of objects, robot formation parade and so forth. However, in many other applications where geometric shape of the formation is not strictly required, rigid formation may suffer from certain limitations especially when motion constraints of the robots are taken into consideration. Reference [10] points out that to maintain a perfect square using four differentially driven mobile robots, the only possible motion is in a straight line. Then, a more flexible approach is proposed that is to maintain the formation in curvilinear coordinates rather than in the usual rectilinear coordinate system, which is the embryo of flexible formation. Since then, some interesting results along this idea have been published [11]–[14]. By introducing a hierarchical formation structure, [11] proposes a vector field based target tracking control law such that the decentralized and flexible formation tracking can be achieved. Reference [12] proposes an adaptive and predictive formation control scheme to achieve highly accurate relative positioning of the formation whose desired shape is defined with respect to the formation reference path, in terms of curvilinear distances between robots along the reference path and lateral deviations with respect to this trajectory. Reference [13] investigates the flexible leader-follower formation tracking control problem for nonholonomic mobile robots. The method proposed in [13] consists of two parts, curvilinear-based real-time formation reference generator and single-robot trajectory tracking controller. Reference [14] analyzes the maneuverability of the flexible formation by taking the motion constraints of the robots into account. In spite of the references introduced above, there are also several other types of flexible formation which can be found in the literature like the non-rigid distance-based formation in [15], [16].

Although there are already a few research results on flexible formation tracking, several problems still remain unsolved. First of all, most of the existing approaches which address the flexible formation tracking problem like [12] and [13] assume that a global positioning system (GPS) receiver is installed on each robot and the reference trajectory of the leader is previously learned or received online by the followers. How-

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ever, in many practical applications, especially those in indoor environments, it is infeasible to obtain global position and orientation measurements of multiple robots with respect to a common coordinate system. Therefore, the study of flexible formation tracking under such situations is of great significance. Secondly, when put into practice, it is often required that the formation would work in an obstacle environment without map knowledge. In this case, safety becomes a critical issue and the robots should have the ability to avoid obstacles encountered. Besides, the obstacle avoidance reaction may cause collisions among neighboring robots, which adds complexity to the problem. Therefore, a control scheme that accounts for formation tracking, obstacle avoidance and inter-robot collision avoidance should be designed.

In this brief, the flexible formation tracking problem for multiple nonholonomic mobile robots is studied. Due to the lack of global pose measurements, a stereo camera is installed on each follower to detect the relative position and orientation relationship with the leader. In consideration of the noise and intermittent interruption in visual observations, a particle filter based relative pose estimation approach is proposed to estimate the instantaneous position and orientation of the leader in the local frame attached on each follower. To form a flexible formation, a trajectory reconstruction method is employed to transform the leader historical trajectory to the current local coordinate system of the follower, then the reference point is generated for the follower to track. To guarantee robot safety in unknown obstacle environments, a multi-objective control law is proposed for the follower to achieve and coordinate reference tracking, obstacle avoidance and inter-robot collision avoidance. Finally, simulation and real robot experiment are performed to validate the effectiveness of the proposed method.

The contributions of this brief are twofolds. First of all, a flexible leader-follower formation tracking control approach is proposed which can work in unknown obstacle environments without global pose measurements. Secondly, real robot experiment has been performed to demonstrate the efficacy of the proposed method.

The remainder of this brief is organized as follows. In Section II, the problem is mathematically formulated. Section III provides the main results of this brief. Simulation and experimental results are given in Section IV, while final conclusions and future works are stated in Section V.

II. PROBLEM FORMULATION

In this section, mathematical models of leader-follower robot pair and flexible formation will be provided, based on which the problem will be formulated. Before proceeding further, some definitions about coordinate systems are given. Global frame, which can also be called the inertial frame, is represented as $\{G\} = \{\vec{x}_G, \vec{y}_G\}$. Local frame is attached on and moving with each robot. For example, the local frame of robot i is represented as $\{i\} = \{\vec{x}_i, \vec{y}_i\}$ with $\vec{x}_i$ pointing towards its motion direction. The pre-superscript is used to describe the coordinate frame in which the corresponding variable is expressed. For example, ${}^i p_j$ represents the position

of robot j with respect to $\{i\}$. For a variable without pre-superscript, it means that this variable is defined with respect to $\{G\}$.

A. Leader-Follower Pair

This brief considers the differential-drive wheeled mobile robot, whose kinematic model is given as follows.

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i, \mathbf{u}_i^w, \mathbf{w}_i), \quad (1)$$

where $i \in \{l, f_j\}$, $j \in \{1, 2, \dots, n-1\}$, $\mathbf{x}_i = [x_i, y_i, \theta_i]^T$, $\mathbf{u}_i^w = [v_i^l, v_i^r]^T$ and $\mathbf{w}_i = [w_i^l, w_i^r]^T$. The function $f(\cdot)$ is defined as $f(\mathbf{x}_i, \mathbf{u}_i^w, \mathbf{w}_i) = A_i \mathbf{u}_i$, where $\mathbf{u}_i = B(\mathbf{u}_i^w + \mathbf{w}_i)$,

$$\mathbf{u}_i = [v_i, \omega_i]^T, A_i = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{d} & \frac{1}{d} \end{bmatrix}.$$

The physical meanings of the variables listed above are illustrated as follows, $\mathbf{p}_i = [x_i, y_i]^T$ and θ_i are the position and orientation of robot i with respect to $\{G\}$, respectively, v_i and ω_i are the linear velocity and angular velocity of robot i in $\{i\}$, respectively, d is the distance along the axle between two wheel centers, v_i^l and v_i^r are the linear velocities of the left and right wheels, respectively, w_i^l and w_i^r are the mutually independent random variables representing additive process noise to left and right wheels, respectively. It is assumed that robot l , which has the knowledge of the desired trajectory needed to complete the task, acts as the leader of the whole formation, while robot f_j , $j \in \{1, 2, \dots, n-1\}$, act as the followers. The class of nonholonomic mobile robots considered in this brief is shown in Fig. 1, where the notations given above are also depicted.

Fig. 1. Flexible formation

Furthermore, the linear velocity and angular velocity of a robot should satisfy certain constraints due to electromechanical limitations. In detail, robots are subject to the following kinematic constraints.

$$v_{\min} \leq v_i \leq v_{\max}, |\omega_i| \leq \omega_{\max}, \quad (2)$$

where $v_{\min}$ and $v_{\max}$ are the lower and upper bounds of v_i , respectively, while $\omega_{\max}$ is the bound of ω_i . For ground mobile robots, reverse movement is generally allowed. Therefore, $v_{\min} < 0$, $v_{\max} > 0$ and $\omega_{\max} > 0$.

To obtain relative pose measurements of the leader with respect to the follower, a RGBD camera is used which can provide accurate range, bearing and orientation observations. Thus, the measurement equation can be written as follows.

$$\mathbf{y}_{f_j} = h(f_j \mathbf{x}_l, \varepsilon_{f_j}), \quad (3)$$

where $\mathbf{y}_{f_j} = [\rho_{f_j}, \alpha_{f_j}, f_j \theta_l]^T$, $\varepsilon_{f_j} = [\varepsilon_{\rho_{f_j}}, \varepsilon_{\alpha_{f_j}}]^T$, $f_j \mathbf{x}_l = R(\theta_{f_j})(\mathbf{x}_l - \mathbf{x}_{f_j})$ and $R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$. In addition, function $h(\cdot)$ is defined as $\rho_{f_j} = \|f_j \mathbf{p}_l\|_2 + \varepsilon_{\rho_{f_j}}$, $\alpha_{f_j} = \text{atan2}(f_j y_l, f_j x_l) + \varepsilon_{\alpha_{f_j}}$ and $f_j \theta_l = \theta_l - \theta_{f_j}$. ρ_{f_j} and α_{f_j} are the range and bearing measurements of the leader with respect to $\{f_j\}$, respectively, while $\varepsilon_{\rho_{f_j}}$ and $\varepsilon_{\alpha_{f_j}}$ are the independent random measurement noise.

B. Flexible Formation

In this brief, flexible formation is defined in curvilinear coordinates rather than in rectangular coordinates as in rigid formation, as is defined in [10]–[12]. A flexible formation containing three robots is shown in Fig. 1. The local position of robot f_j with respect to $\{l\}$ in curvilinear coordinates is ${}^l \mathbf{p}_{f_j} = [p_{ff_j}, q_{ff_j}]^T$, where p_{ff_j} represents the longitudinal curvilinear separation and q_{ff_j} is the lateral deviation.

Next, the position and orientation of robot f_j will be determined. First, define the trajectory of robot l with respect to time t as $\mathbf{t}_l(t) = [x_l(t), y_l(t), \theta_l(t)]^T$, where $t \in [0, t_f]$. Then, the trajectory set of robot l can be given as $\Gamma = \{\mathbf{t}_l(t) | t \in [0, t_f]\}$. Similarly, with a slight abuse of notation, define the trajectory of robot l with respect to arc length s as $\mathbf{t}_l(s) = [x_l(s), y_l(s), \theta_l(s)]^T$, where $s \in [0, s_f]$.

Define s_i as the curvilinear coordinate of robot i along Γ . Obviously, $s_l(t) = \int_0^t v_l(\tau) d\tau$ and $s_{f_j} = s_l + p_{ff_j}$. Consequently, the position of robot f_j can be obtained as

$$\mathbf{p}_{f_j} = \mathbf{p}_l(s_{f_j}) + {}^G R_l(\theta_l(s_{f_j})) {}^l \mathbf{p}'_{f_j}, \quad (4)$$

where ${}^l \mathbf{p}'_{f_j} = [0, q_{ff_j}]^T$. The orientation of robot f_j is

$$\theta_{f_j} = \theta_l(s_{f_j}). \quad (5)$$

C. Problem Statement

In order to form a flexible formation which is defined with respect to the reference trajectory in terms of curvilinear separations between robots along the reference path and lateral deviations from this trajectory, the historical trajectory information of the leader should be available to the followers, from which the corresponding reference poses can be determined for the followers. In addition, since there does not exist global pose feedback, relative pose measurements are the necessary information to update the leader trajectory, which should be further processed as they are corrupted with noise. Finally, the followers should track the determined reference poses with the capability of collision avoidance to ensure safety in unknown environments.

Based on the discussions above, the flexible leader-follower formation tracking problem studied in this brief can be divided into the following four subproblems. 1) (*Vision-based*

pose estimation). Design a filtering algorithm to estimate the relative pose of the leader with respect to the follower. 2) (*Leader trajectory reconstruction*). Design a method to obtain the leader historical trajectory expressed in the current local frame of the follower. 3) (*Reference generation*). Design a strategy to determine the reference pose with respect to the leader historical trajectory for the follower. 4) (*Multi-objective Control*). Design a control scheme for the follower such that it can track its reference pose, while obstacle and inter-robot collision avoidance can be guaranteed for all the time.

III. MAIN RESULTS

This section proposes a flexible formation tracking control scheme without global pose measurements, which is described in Fig. 2. The proposed control scheme consists of four parts, vision-based pose estimation, leader trajectory reconstruction, reference generator and multi-objective control law, which will be illustrated in the following subsections.

Fig. 2. Block diagram of the proposed flexible formation control scheme

A. Vision-Based Pose Estimation

To implement leader-follower formation maneuvering, relative pose of the leader is of interest for estimation by the follower robot. In this work, a particle filter based relative pose estimation approach is employed to estimate the local position and orientation of the leader with respect to the follower, due to the nonlinearities and non-Gaussian noise in the process and observation models.

In practice, control inputs are imposed on the robot discretely, while measurements are also acquired periodically. Therefore, before going into further details, the discrete-time system and measurement equations should be given as follows.

$$\begin{aligned} \mathbf{x}_{i,k} &= f_{k-1}(\mathbf{x}_{i,k-1}, \mathbf{u}_{i,k-1}^w, \mathbf{w}_{i,k-1}), \\ \mathbf{y}_{f_j,k} &= h_{k-1}(f_j \mathbf{x}_{l,k}, \varepsilon_{f_j,k}), \end{aligned} \quad (6)$$

where $f_{k-1}(\cdot)$ and $h_{k-1}(\cdot)$ are the discretized functions. In particular, $f_{k-1}(\mathbf{x}_{i,k-1}, \mathbf{u}_{i,k-1}^w, \mathbf{w}_{i,k-1}) = \mathbf{x}_{i,k-1} + \Delta \mathbf{x}_{i,k-1}$,

where

$$\begin{aligned}\Delta \mathbf{x}_{i,k-1} &= [\Delta x_{i,k-1}, \Delta y_{i,k-1}, \Delta \theta_{i,k-1}]^T, \\ \Delta x_{i,k-1} &= \begin{cases} T_s v_{i,k-1} \cos \theta_{i,k-1} & \omega_{i,k-1} = 0 \\ \frac{v_{i,k-1}}{\omega_{i,k-1}} [\sin \theta_{i,k} - \sin \theta_{i,k-1}] & \omega_{i,k-1} \neq 0 \end{cases}, \\ \Delta y_{i,k-1} &= \begin{cases} T_s v_{i,k-1} \sin \theta_{i,k-1} & \omega_{i,k-1} = 0 \\ \frac{v_{i,k-1}}{\omega_{i,k-1}} [\cos \theta_{i,k-1} - \cos \theta_{i,k}] & \omega_{i,k-1} \neq 0 \end{cases}, \\ \Delta \theta_{i,k-1} &= T_s \omega_{i,k-1}.\end{aligned}$$

Assume that the probability distribution function (pdf) of the initial state $f_j \mathbf{x}_{l,0}$, which is the initial position and orientation of the leader l with respect to $\{f_j\}$, is already known. The relative pose of the leader with respect to the follower can be estimated recursively by a particle filter based algorithm, which is presented in Algorithm 1.

To initialize, L initial particles are randomly generated on the basis of the pdf of $f_j \mathbf{x}_{l,0}$, which are approximated by a set of particles and associated weights, $\{f_j \mathbf{x}_{l,0}^{m,+}, q_k^m\}_{m=1}^L$. The proposed estimation algorithm consists of two stages, time update stage and measurement update stage. The time update stage predicts the relative pose of the leader in the local frame of the follower, which can be expressed as follows. As the pose estimation algorithm is carried out in the follower, control inputs of the leader, $\mathbf{u}_{l,k-1}^w$, is assumed to be known by the follower in real time via wireless communication.

$$\begin{aligned}f_j \mathbf{x}_{l,k}^{m,\sim} &= f_{k-1}(f_j \mathbf{x}_{l,k-1}^{m,+}, \mathbf{u}_{l,k-1}^w, \mathbf{w}_{l,k-1}), \\ f_j \mathbf{x}_{f_j,k}^{m,\sim} &= f_{k-1}(\mathbf{0}, \mathbf{u}_{f_j,k-1}^w, \mathbf{w}_{f_j,k-1}), \\ f_j \mathbf{x}_{l,k}^{m,-} &= R(f_j \theta_{f_j,k}^{m,\sim})(f_j \mathbf{x}_{l,k}^{m,\sim} - f_j \mathbf{x}_{f_j,k}^{m,\sim}).\end{aligned}\quad (7)$$

where $\forall m \in \{1, 2, \dots, L\}$ and $R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$. In addition, $\mathbf{w}_{i,k-1}$ is randomly generated based on its pdf which is assumed to be known. In the measurement update stage, the weight q_k^m is updated for each particle $f_j \mathbf{x}_{l,k}^{m,-}$ conditioned on the measurement $\mathbf{y}_{f_j,k}$, which can be obtained by evaluating the likelihood function $p(\mathbf{y}_{f_j,k} | f_j \mathbf{x}_{l,k}^{m,-})$ according to the measurement equation (6) and the assumed known pdf of $\varepsilon_{f_j,k}$. Resampling is employed in the algorithm when the effective particles quantity $\hat{N}_{eff} = \frac{1}{\sum_{m=1}^L (q_k^m)^2}$ is smaller than a threshold N_{thr} to prevent weight degeneracy.

B. Leader Trajectory Reconstruction

To form a flexible formation, the follower should keep the historical trajectory information of the leader. Technically, define a sliding most-recent time window $T_w = k_w T_s$, where k_w is a positive integer. The follower f_j keeps the trajectory of the leader l from a historical time step $k - k_w + 1$ to the current time step k . As there does not exist a global coordinate frame, the stored historical trajectory of l is with respect to $\{f_j\}$. Mathematically, the stored historical leader trajectory at time step k can be written as $f_j \hat{\mathbf{x}}_{l,k-k_w+1:k}$. It should be noted that the historical leader trajectory at time step $k - 1$, $f_j \hat{\mathbf{x}}_{l,k-k_w:k-1}$, is with respect to $\{f_j\}$ at $k - 1$, however, when being updated to $f_j \hat{\mathbf{x}}_{l,k-k_w+1:k}$ at k , it should be transformed to $\{f_j\}$ at k as well. To perform the transformation, the pose

Algorithm 1 Vision-based pose estimation algorithm

Input:

$\mathbf{y}_{f_j,k}$, pdfs of $f_j \mathbf{x}_{l,0}$, $\mathbf{w}_{i,k}$ and $\varepsilon_{f_j,k}$

Output:

$f_j \hat{\mathbf{x}}_{l,k}$

- 1: Generate a set of L particles $\{f_j \mathbf{x}_{l,0}^{m,+}, q_k^m\}_{m=1}^L$ according to the pdf of $f_j \mathbf{x}_{l,0}$;
- 2: **for** $k = 1, 2, \dots$ **do**
- 3: Perform the time update step to obtain a set of *a priori* particles $f_j \mathbf{x}_{l,k}^{m,-}$ using (7);
- 4: **if** $\mathbf{y}_{f_j,k}$ exists **then**
- 5: Update the weight q_k^m of each particle $f_j \mathbf{x}_{l,k}^{m,-}$ conditioned on $\mathbf{y}_{f_j,k}$;
- 6: **else**
- 7: $\{q_k^m\}_{m=1}^L \leftarrow \{q_{k-1}^m\}_{m=1}^L$;
- 8: **end if**
- 9: Normalization: $q_k^m = \frac{q_k^m}{\sum_{m=1}^L q_k^m}, \forall m \in \{1, 2, \dots, L\}$;
- 10: **if** $\hat{N}_{eff} < N_{thr}$ **then**
- 11: Resampling: generate a set of a posteriori particles $f_j \mathbf{x}_{l,k}^{m,+}$ based on $\{q_k^m\}_{m=1}^L$;
- 12: $q_k^m = \frac{1}{L}, \forall m \in \{1, 2, \dots, L\}$;
- 13: **end if**
- 14: Estimation: $f_j \hat{\mathbf{x}}_{l,k} = \sum_{m=1}^L q_k^m f_j \mathbf{x}_{l,k}^{m,+}$;
- 15: **end for**

variation of f_j should first be determined, then the coordinates of the historical trajectory of the leader are transformed, which can be illustrated as

$$\begin{aligned}f_j \tilde{\mathbf{x}}_{f_j,k} &= f_{k-1}(\mathbf{0}, \hat{\mathbf{u}}_{f_j,k-1}^w, \mathbf{0}), \\ f_j \hat{\mathbf{x}}_{l,t} &= R(f_j \theta_{f_j,k}^{m,\sim})(f_j \hat{\mathbf{x}}_{l,t} - f_j \tilde{\mathbf{x}}_{f_j,k}),\end{aligned}\quad (8)$$

for $t = k - k_w + 1$ to k , where $\hat{\mathbf{u}}_{f_j,k-1}^w$ is the measurement of real $\mathbf{u}_{f_j,k-1}^w$ by encoders. Based on the discussions above, the leader trajectory reconstruction algorithm is presented in Algorithm 2.

Algorithm 2 Leader trajectory reconstruction algorithm

Input:

$f_j \hat{\mathbf{x}}_{l,k-k_w:k-1}$, $f_j \hat{\mathbf{x}}_{l,k}$, $\hat{\mathbf{u}}_{f_j,k-1}^w$

Output:

$f_j \hat{\mathbf{x}}_{l,k-k_w+1:k}$

- 1: Marginalization:
 $f_j \hat{\mathbf{x}}_{l,k-k_w+1:k-1} \leftarrow f_j \hat{\mathbf{x}}_{l,k-k_w:k-1} \setminus f_j \hat{\mathbf{x}}_{l,k-k_w}$;
- 2: Coordinate Transformation: (8);
- 3: Augmentation:
 $f_j \hat{\mathbf{x}}_{l,k-k_w+1:k} \leftarrow f_j \hat{\mathbf{x}}_{l,k-k_w+1:k-1} \cup f_j \hat{\mathbf{x}}_{l,k}$;

C. Reference Generator

In this subsection, a reference generation algorithm is designed to find the reference state for the follower f_j from the historical trajectory of the leader, $f_j \hat{\mathbf{x}}_{l,k-k_w+1:k}$, obtained above. The proposed reference generator is presented in Algorithm 3. In terms of the obtained historical trajectory, given a sufficiently small sampling time interval, the incremental

trajectory between two points can be approximated as a straight line. Thus, the trajectory length between $f_j \hat{\mathbf{x}}_{l,t+1}$ and $f_j \hat{\mathbf{x}}_{l,t}$, $t \in \{t \in \mathbb{N} | k - k_w + 1 \leq t \leq k - 1\}$, is given as $\Delta d_{t,t+1} = \|f_j \hat{\mathbf{p}}_{l,t} - f_j \hat{\mathbf{p}}_{l,t+1}\|_2$.

Algorithm 3 Reference generation algorithm

Input:

$$f_j \hat{\mathbf{x}}_{l,k-k_w+1:k}, p_{ffj}, q_{ffj}, {}^l v_{l,k}$$

Output:

$$f_j \mathbf{x}_{r,k}, f_j v_{r,k}, f_j \omega_{r,k}$$

- 1: Solve p according to $\arg \min_p \left| \sum_{t=p}^{k-1} \Delta d_{t,t+1} - p_{ffj} \right|$;
 - 2: Compute curvature $f_j \hat{c}_{l,p} = \frac{f_j \hat{\theta}_{l,t+1} - f_j \hat{\theta}_{l,t}}{\Delta d_{t,t+1}}$;
 - 3: Compute reference pose:
 $f_j \mathbf{p}_{r,k} = f_j \hat{\mathbf{p}}_{l,p} + {}^G R_l(f_j \hat{\theta}_{l,p}) {}^l \mathbf{p}'_{f_j}$, $f_j \theta_{r,k} = f_j \hat{\theta}_{l,p}$;
 - 4: Compute reference linear and angular velocities:
 $f_j v_{r,k} = {}^l v_{l,k}(1 - q_{ffj} f_j \hat{c}_{l,p})$, $f_j \omega_{r,k} = f_j v_{r,k} f_j \hat{c}_{l,p}$;
-

D. Multi-Objective Control Scheme

As the reference states of the followers have been obtained, the followers should track their own reference states such that the flexible formation can be formed. In this brief, a linear trajectory tracking control law is employed to produce the control input $\mathbf{u}_{f_j,k}^w$ for the follower f_j at time step k , which is given as follows.

$$\mathbf{u}_{f_j,k}^w = B^{-1} \mathbf{u}_{f_j,k}, \mathbf{u}_{f_j,k} = \mathbf{u}_{Ff_j,k} + K_{s,k} f_j \mathbf{x}_{r,k}, \quad (9)$$

where $\mathbf{u}_{Ff_j,k} = \begin{bmatrix} f_j v_{r,k} \cos f_j \theta_{r,k} & f_j \omega_{r,k} \end{bmatrix}^T$, $K_{s,k} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \text{sgn}(f_j v_{r,k}) & k_3 \end{bmatrix}$, $k_1 = k_3 = 2\zeta \sqrt{f_j \omega_{r,k}^2 + b f_j v_{r,k}^2}$, $k_2 = b |f_j v_{r,k}|$, $\zeta \in (0, 1)$ is a damping coefficient, and $b > 0$ is a designed parameter. Interested readers can refer to [17] for details of this controller.

Using controller (9), a flexible formation can be formed. However, in cluttered environments, the follower may collide with obstacles located on or close to the reference trajectory. Thus, the follower should have obstacle avoidance capability. Furthermore, obstacle avoidance reaction may lead the follower to collide with other robots nearby. Therefore, to guarantee safety of the formation, a multi-objective control law should be designed for the follower such that when there are no potential collisions, the reference trajectory can be tracked accurately, but when there are obstacles or other robots nearby, obstacle and inter-robot collision avoidance should be performed.

To deal with obstacle avoidance, the nearest obstacle point to the robot, $\mathbf{p}_o$, is considered. To facilitate avoidance control design, some regions are defined, which are given as follows. $\Phi_{f_j}^O \triangleq \{\mathbf{p}_{f_j} | d_{f_j o} \geq r_o + \delta_o\}$, $\Omega_{f_j}^O \triangleq \{\mathbf{p}_{f_j} | r_o \leq d_{f_j o} < r_o + \delta_o\}$, $\Lambda_{f_j}^O \triangleq \{\mathbf{p}_{f_j} | r_o + \delta_o - \mu_o \leq d_{f_j o} < r_o + \delta_o\}$, where $d_{f_j o} = \|\mathbf{p}_{f_j} - \mathbf{p}_o\|_2$, r_o is the safe distance between robot and obstacle, δ_o is a buffer distance, and μ_o is a transition distance. Based on the regions defined above, the following

two functions are designed.

$$G_o(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j o} < r_o \\ -\frac{1}{\delta_o^2} (d_{f_j o} - r_o)^2 & d_{f_j o} \geq r_o \text{ and } d_{f_j o} < r_o + \delta_o \\ +\frac{2}{\delta_o} (d_{f_j o} - r_o) & d_{f_j o} \geq r_o + \delta_o \end{cases},$$

$$W_o(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j o} < r_o + \delta_o - \mu_o \\ -\frac{1}{\mu_o^2} (d_{f_j o} - r_o - \delta_o + \mu_o)^2 & d_{f_j o} < r_o + \delta_o \text{ and } d_{f_j o} \geq r_o + \delta_o - \mu_o \\ +\frac{2}{\mu_o} (d_{f_j o} - r_o - \delta_o + \mu_o) & d_{f_j o} \geq r_o + \delta_o \end{cases} \quad (10)$$

where $G_o(\mathbf{p}_{f_j})$ is the potential function and $W_o(\mathbf{p}_{f_j})$ is the transition function.

To address inter-robot collision avoidance, the following regions are also defined. $\Phi_{f_j}^N \triangleq \bigcup_{n_j \in N(f_j)} \Phi_{f_j n_j}^N$, $\Omega_{f_j}^N \triangleq \bigcup_{n_j \in N(f_j)} \Omega_{f_j n_j}^N$, $\Lambda_{f_j}^N \triangleq \bigcup_{n_j \in N(f_j)} \Lambda_{f_j n_j}^N$, where $\Phi_{f_j n_j}^N \triangleq \{\mathbf{p}_{f_j} | d_{f_j n_j} \geq r_n + \delta_n\}$, $\Omega_{f_j n_j}^N \triangleq \{\mathbf{p}_{f_j} | r_n \leq d_{f_j n_j} < r_n + \delta_n\}$, $\Lambda_{f_j n_j}^N \triangleq \{\mathbf{p}_{f_j} | r_n + \delta_n - \mu_n \leq d_{f_j n_j} < r_n + \delta_n\}$. In addition, $d_{f_j n_j} = \|\mathbf{p}_{f_j} - \mathbf{p}_{n_j}\|_2$, $n_j \in N(f_j)$, $N(f_j)$ is the set of neighboring robots of f_j , r_n is the safe distance among robots, δ_n is a buffer distance, and μ_n is a transition distance. Then, the potential function $G_n(\mathbf{p}_{f_j})$ and transition function $W_n(\mathbf{p}_{f_j})$ can be expressed as

$$G_n(\mathbf{p}_{f_j}) = \prod_{n_j \in N(f_j)} g_{n_j}^n(\mathbf{p}_{f_j}), W_n(\mathbf{p}_{f_j}) = \prod_{n_j \in N(f_j)} w_{n_j}^n(\mathbf{p}_{f_j}), \quad (11)$$

where

$$g_{n_j}^n(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j n_j} < r_n \\ -\frac{1}{\delta_n^2} (d_{f_j n_j} - r_n)^2 & d_{f_j n_j} \geq r_n \text{ and } d_{f_j n_j} < r_n + \delta_n \\ +\frac{2}{\delta_n} (d_{f_j n_j} - r_n) & d_{f_j n_j} \geq r_n + \delta_n \end{cases}$$

and

$$w_{n_j}^n(\mathbf{p}_{f_j}) = \begin{cases} 0 & d_{f_j n_j} < r_n + \delta_n - \mu_n \\ -\frac{1}{\mu_n^2} (d_{f_j n_j} - r_n - \delta_n + \mu_n)^2 & d_{f_j n_j} < r_n + \delta_n \text{ and } d_{f_j n_j} \geq r_n + \delta_n - \mu_n \\ +\frac{2}{\mu_n} (d_{f_j n_j} - r_n - \delta_n + \mu_n) & d_{f_j n_j} \geq r_n + \delta_n \end{cases}$$

From the discussions above, it can be deduced that to guarantee safety, the following constraint must be satisfied.

$$G(\mathbf{p}_{f_j}) = G_o(\mathbf{p}_{f_j}) G_n(\mathbf{p}_{f_j}) > 0. \quad (12)$$

Then several regions are defined in the following definition based on the regions introduced above.

Definition 1. (*Safe Region, Dangerous Region, Transition Region, Critical Region*). Safe region of robot f_j is defined as $\Phi_{f_j} \triangleq \{\mathbf{p}_{f_j} | \mathbf{p}_{f_j} \in \Phi_{f_j}^O \cap \Phi_{f_j}^N\}$. Transition region of robot f_j is defined as $\Lambda_{f_j} \triangleq \{\mathbf{p}_{f_j} | \mathbf{p}_{f_j} \in \Lambda_{f_j}^O \cup \Lambda_{f_j}^N\}$. Dangerous region of robot f_j is defined as $\Omega_{f_j} \triangleq \{\mathbf{p}_{f_j} | \mathbf{p}_{f_j} \in \Omega_{f_j}^O \cup \Omega_{f_j}^N, \mathbf{p}_{f_j} \notin \Lambda_{f_j}\}$.

When robot f_j is located in safe region Φ_{f_j} , where there are no obstacles and neighboring robots nearby, the reference tracking controller (9) is imposed on the robot such that the desired flexible formation can be formed. However, when f_j is inside transition region Λ_{f_j} and dangerous region Ω_{f_j} , where some obstacles or neighboring robots are very close to it, the robot must be driven to avoid collisions with them. To facilitate the avoidance controller design, the following potential function is designed.

$$\varphi_{f_j} = \frac{\gamma}{(\gamma^\alpha + G(\mathbf{p}_{f_j}))^{\frac{1}{\alpha}}}, \quad (13)$$

where γ and α are positive parameters. From (10), (11) and (12), it can be found that $\varphi_{f_j} \in [\frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}, 1]$. Based on (13), a multi-objective controller is proposed as follows.

$$\mathbf{u}_{f_j} = R'(\theta_{f_j})(\mathbf{u}_{f_j}^t + \mathbf{u}_{f_j}^a), \quad (14)$$

where $R'(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, $\mathbf{u}_{f_j}^t = \dot{\mathbf{p}}_{f_j}^d + \rho(\mathbf{p}_{f_j}^d - \mathbf{p}_{f_j})$ and $\mathbf{u}_{f_j}^a = -(1 - W(\mathbf{p}_{f_j}))\mathbf{u}_{f_j}^t - k_{f_j} \nabla \varphi_{f_j}$ are the tracking part and avoidance part, respectively, $\mathbf{p}_{f_j}^d$ is the reference position, $W(\mathbf{p}_{f_j}) = W_o(\mathbf{p}_{f_j})W_n(\mathbf{p}_{f_j})$, ρ and k_{f_j} are positive parameters. The following theorem proves that using the proposed controller (14), collision avoidance can be guaranteed.

Theorem 1. Consider robot f_j described as (1) and the reference position trajectory $\mathbf{p}_{f_j}^d$. Assume that the initial position of f_j satisfies (12), i.e., $G(\mathbf{p}_{f_j}(0)) > 0$. When the controller (14) is applied, collision avoidance can be maintained for all the time, i.e., $G(\mathbf{p}_{f_j}(t)) > 0, \forall t > 0$.

Proof: First of all, we will prove that for $\dot{\mathbf{p}}_{f_j} = \mathbf{u}_{f_j}^t + \mathbf{u}_{f_j}^a$, $G(\mathbf{p}_{f_j}(t)) > 0$ for $\forall t > 0$. Inside Ω_{f_j} , according to (10), (11) and (14), it can be found that the tracking part $\mathbf{u}_{f_j}^t$ is neutralized such that

$$\dot{\mathbf{p}}_{f_j} = -k_{f_j} \nabla \varphi_{f_j}. \quad (15)$$

Based on (13), it can be derived that $\nabla \varphi_{f_j} = -\frac{\gamma \nabla G(\mathbf{p}_{f_j})}{\alpha(\gamma^\alpha + G(\mathbf{p}_{f_j}))^{\frac{1}{\alpha} + 1}}$, where $\nabla G(\mathbf{p}_{f_j}) = G(\mathbf{p}_{f_j})(\frac{\nabla G_o(\mathbf{p}_{f_j})}{G_o(\mathbf{p}_{f_j})} + \sum_{n_j \in N(f_j)} \frac{\nabla g_{n_j}^d(\mathbf{p}_{f_j})}{g_{n_j}^d(\mathbf{p}_{f_j})})$.

Consider the Lyapunov function candidate, $V = \varphi_{f_j} - \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}$. By taking the derivative of V with respect to time and combining with (15), it can be obtained that

$$\dot{V} = -k_{f_j}(\nabla \varphi_{f_j})^T \nabla \varphi_{f_j} \leq 0. \quad (16)$$

As $G(\mathbf{p}_{f_j}) > 0$ implies that $0 \leq V < 1 - \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}$ and the fact that $V = 1 - \frac{\gamma}{(\gamma^\alpha + 1)^{\frac{1}{\alpha}}}$ if and only if $G(\mathbf{p}_{f_j}) = 0$, combining with (16) and $G(\mathbf{p}_{f_j}(0)) > 0$, it can be derived that $G(\mathbf{p}_{f_j}(t)) > 0$ for $\forall t > 0$.

According to Theorem 2 in [18], it can be concluded that inside Ω_{f_j} , control law (14) designed for model in (1) is convergent with the same attraction region as $\dot{\mathbf{p}}_{f_j} = \mathbf{u}_{f_j}^t + \mathbf{u}_{f_j}^a$. Therefore, using control law (14), $G(\mathbf{p}_{f_j}(t)) > 0$ for $\forall t > 0$. ■

It is obvious that the tracking part $\mathbf{u}_{f_j}^t$ is actually a simple PD tracking controller, which controls the robot to track its

reference position $\mathbf{p}_{f_j}^d$. From the proposed controller (14), it can be observed that $\mathbf{u}_{f_j}^t$ works in transition region Λ_{f_j} such that the overall control is the combination of reference tracking and collision avoidance, which is beneficial in that even though potential collisions are tried to be avoided, the proposed controller can drive the robot to the reference position roughly as well. This special design is helpful to reduce the possibility of loss of leader detection while avoiding potential collisions.

As in this brief global pose measurements do not exist, the proposed control law (14) should be modified such that all the variables are in the local frame $\{f_j\}$. In addition, for real robot implementations, the control law should be in discrete time domain. Therefore, the control law can be modified as follows.

$$\mathbf{u}_{f_j,k} = \mathbf{u}_{f_j,k}^t + \mathbf{u}_{f_j,k}^a, \quad (17)$$

where $\mathbf{u}_{f_j,k}^t = f_j \hat{\mathbf{p}}_{r,k} + \rho^{f_j} \mathbf{p}_{r,k}$ and $\mathbf{u}_{f_j,k}^a = -(1 - W(\mathbf{p}_{f_j,k}))\mathbf{u}_{f_j,k}^t - k_{f_j} \nabla \varphi_{f_j,k}$. $f_j \hat{\mathbf{p}}_{r,k} = (f_j \mathbf{p}_{r,k} - f_j \mathbf{p}_{r,k-1})/T_s$ is the estimation of $f_j \dot{\mathbf{p}}_{r,k}$.

In summary, the proposed multi-objective control scheme can be presented as follows. When the robot f_j is located in safe region Φ_{f_j} , (9) is employed, and when f_j is located in transition region Λ_{f_j} and dangerous region Ω_{f_j} , (17) is applied.

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

To illustrate the effectiveness of the proposed method, a numerical simulation was performed in MATLAB using one leader and two followers. In the simulation, the leader followed a reference path, while two followers tracked the leader aiming at keeping a flexible formation. The parameters of the simulation platform were set as follows. The simulation time duration was 170s. The sampling time interval was set as 33.3ms. For vision detection, the maximum and minimum detection ranges were set as 5m and 1m, respectively, while the camera horizontal field-of-view was set as 100°. Kinematic process noise for each wheel was formulated as $\mathcal{N}(0, 0.05^2)$ m/s, while inter-robot visual observation noise in range, bearing and orientation were modeled by Student's t-distributions, $St(0, -4444, 1)$ m, $St(0, 3283, 3)^\circ$ and $St(0, 3283, 3)^\circ$, respectively. In addition, the parameters of the relative pose estimation algorithm was set as $L = 3000$ and $N_{thr} = 1500$. The desired flexible formation configuration were set as ${}^l\mathbf{p}_{f_1} = [-1.5, 0.5]^T$ and ${}^l\mathbf{p}_{f_2} = [-3.0, -0.5]^T$. The control parameters were designed as $\zeta = 0.9$, $b = 1$, $r_o = r_n = 0.5$ m, $\delta_o = \delta_n = 0.3$ m, $\mu_o = \mu_n = 0.2$ m, $\gamma = \alpha = 1$, $\rho = 2$ and $k_a = 0.8$.

In the simulation, the leader followed a predesignated path in an obstacle environment. The followers tracked the leader in a flexible formation while performing obstacle and inter-robot collision avoidance simultaneously. Fig. 3(a) depicts the paths of the three robots, from which it can be observed that when there were no obstacles nearby, the robots can form the desired flexible formation successfully. When the formation were approaching the three round obstacles and at each turning, the robots would react to avoid collisions with obstacles and neighboring robots while still attempting to keep

Fig. 3. Simulation results

the desired formation. Fig. 3(b) shows the formation error of the two followers during operation, from which it can be found that there are two major fluctuations of f_1 , at 30s ~ 40s and 120s ~ 140s, respectively. Combined with Fig. 3(a), it can be inferred that these two major fluctuations were caused by the first and the third round obstacles, respectively. As the robot should react to avoid collisions with obstacles and neighboring robots, the formation objective was temporarily less prioritized. After the avoidance reaction, the robot continued to form the desired formation. Besides the two major fluctuations, there are two slight fluctuations of f_1 at around 50s and 110s, respectively. From Fig. 3(a), it can be deduced that these small fluctuations were caused by the turning movement at the second and the fourth walls, as follower f_1 was at the inner side at each of the two turns which was close to the walls. Similar phenomena can be found on follower f_2 during operation. Fig. 3(c) and 3(d) describe the collision avoidance performance of two followers. Fig. 3(c) depicts the distances from the two followers to their corresponding nearest obstacle point, respectively, while Fig. 3(d) shows the inter-robot distances among three robots. From these two figures, it can be found that the distances are constantly kept larger than 0.5m, the designed safe distance, which demonstrates the efficacy of the proposed multi-objective control scheme.

B. Experimental Results

To validate the proposed method in a real environment, an experiment was carried out at the Robotics I Laboratory, Nanyang Technological University, Singapore. The proposed algorithm was implemented on a multi-robot system composed of two Pioneer 3-AT mobile robots as shown in Fig. 4. The leader was equipped with a Dell Precision M2800 Mobile Workstation, while the follower was equipped with a ZOTAC ZBOX-VR7N70 mobile PC and a ZED stereo camera. Both the leader and follower used a Hokuyo UTM-30LX Scanning Laser Rangefinder only for obstacle avoidance purpose.

The algorithm was programmed using C++ language under ROS, Kinetic release. In the experiment, the leader moved autonomously and the proposed approach was tested on the follower. The control period was set as 33.3ms. It was assumed that the kinematic noise for each wheel of both robots was described by $\mathcal{N}(0, 0.06^2)$ m/s. In addition, the measurement noise in range, bearing and orientation were modeled by Student's t-distributions, $St(0, -4444, 1)$ m, $St(0, 3283, 3)^\circ$ and $St(0, 3283, 3)^\circ$, respectively, according to real noise samples.

The parameters of the relative pose estimation algorithm was set as $L = 5000$ and $N_{thr} = 2500$. The desired flexible formation configuration was set as ${}^l p_{f_1} = [-1.5, -0.5]^T$. The control parameters were designed as $\zeta = 0.9$, $b = 3$, $r_o = 0.4$ m, $r_n = 0.5$ m, $\delta_o = \delta_n = 0.3$ m, $\mu_o = \mu_n = 0.2$ m, $\gamma = \alpha = 1$, $\rho = 1$ and $k_a = 0.8$.

In the experiment, the leader moved from a room to another one through a narrow door autonomously, while the follower tracked the leader trying to form a flexible formation with guaranteed collision avoidance at the same time. The map of the environment and the paths of two robots are both described in Fig. 5(a). Fig. 5(b) depicts the formation error of the follower, from which it can be observed that although starting with a relatively large formation error, the follower achieved a satisfactory formation accuracy before 20s. It can be revealed from Fig. 5(b) that the formation error experienced dramatic fluctuations between time points 50s and 80s, during which the follower was avoiding nearby obstacles, as illustrated in Fig. 5(d). At around time point 90s, as revealed in Fig. 5(b) and 5(c), the follower lost visual detection of the leader for about 10s, which degraded the relative pose estimation accuracy, and thus increased the formation error. Despite the detrimental effects brought by obstacle avoidance and visual measurement interruption, the amplitude of the formation error was bounded by 0.25m at most of the time, which is acceptable in practical applications. After the leader detection was recovered, the formation error converged to the neighborhood of zero again. This demonstrates the robustness of our proposed algorithm against measurement interruption. It should be noted that without measurements, uncertainties in the estimation will increase as time goes, resulting in an increasing estimation error. The performance of the algorithm in leader loss situations depends on the kinematic noise of the robot wheels. Large noise covariance will result in large estimation error under such situations, which may even cause task failure that the follower cannot detect the leader forever. Fig. 5(d) depicts the distance from the follower to the nearest obstacle, measured by the onboard laser range finder. The distance is found always larger than 0.6m, which also indicates that both obstacle avoidance and inter robot collision avoidance were guaranteed.

In real implementations, several factors including wireless communication, image processing, pose estimation, trajectory reconstruction, etc. may cause time delay, which would have a negative impact on the performance of our proposed algorithm. According to the experimental results, our algorithm can be executed in real time, which means that the execution time of the proposed algorithm is less than the sampling period, 33.3ms. For the ground robot formation whose speed is not very fast in general, this algorithm is qualified. However, for

high-speed applications like unmanned aerial vehicle formation flying, time delay will be a challenging issue.

Fig. 4. Experimental setup

V. CONCLUSIONS AND FUTURE WORKS

This brief has addressed the flexible leader-follower formation control problem of multiple nonholonomic mobile robots while global pose measurements are not available. To achieve relative localization between the leader and the follower, a particle filter based relative pose estimation algorithm is employed aiming at estimating the position and orientation of the leader with respect to the local frame of the follower using the noise corrupted and intermittently interrupted visual observations. By using a trajectory reconstruction approach, the leader trajectory is transformed to the current local coordinate system of the follower, from which the reference point is generated. To track the reference point, a multi-objective control law is proposed which can work in unknown obstacle environments with guaranteed collision avoidance. Simulation and real robot experiment were conducted to validate the efficacy of the proposed method. Future work will investigate the formation reconfiguration problem according to the surroundings in order to perform more flexible and safer maneuvers in cluttered environments.

Fig. 5. Experimental results

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