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A Semidefinite Relaxation Approach for Beamforming in Cooperative Clustered Multicell Systems with Novel Limited Feedback Scheme

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Abstract—In this paper, we consider sub-optimal cooperative downlink beamforming strategies to maximize the user sum rate in a clustered multicell system with limited feedback. We decouple the problem into multiple independent sub-problems through the uplink-downlink duality theorem. The semidefinite relaxation is applied to transform the original nonconvex problem into a convex semidefinite programming problem, which can be solved efficiently in polynomial time. We propose a novel limited feedback scheme based on compressive sensing technique that can yield high quality channel state information (CSI) feedback. We also consider a limited feedback scenario where base stations within one cluster have different estimates of the same CSI knowledge. We investigate the channel quantization criterion by choosing the codeword vector that has the minimum norm of the difference between the codeword and normalized channel. Numerical results demonstrate the effectiveness of the proposed beamforming algorithm and limited feedback schemes.

Index Terms—Channel state information, compressive sensing, convex optimization, limited feedback, multicell systems, semidefinite programming relaxation.

I. INTRODUCTION

PERFORMANCE of wireless communication systems can be greatly improved by multiple-input multiple-output (MIMO) techniques. Information theoretical results of sum-rate capacity of MIMO downlink channels have been well studied in [1]. It has been shown that the dirty paper coding (DPC) technique achieves the capacity region of broadcast channels and is the theoretical optimal precoding technique. However, DPC is difficult to be realized in practical multicell systems due to its highly nonlinear nature. Linear precoding scheme, such as beamforming, has been proposed and studied extensively because of its simplicity and suboptimality with respect to the DPC approach. Transmit beamforming is a precoding technique that can realize the transmit diversity [2]. Nevertheless, the design of linear processing schemes in multi-user multi-antenna systems is still an open issue.

Recently, base station (BS) cooperation has been shown to be an effective approach to dramatically improve throughput by enhancing the desired signal power and mitigating the interference simultaneously [3]–[5]. The cooperation can be performed at two levels. Firstly, BSs are coordinated only at the beamforming level. In this situation, the system is termed as a “multicell system” [3]. Secondly, BS cooperation takes

place at both signal and beamforming levels. By sharing the user data among BSs, the whole network forms a virtual “network MIMO system” [5], where users are served by the cooperative BSs simultaneously.

The performance optimization problem in the downlink multi-antenna systems has been solved in various ways. In [6], the uplink-downlink duality was established, and the downlink beamforming problem was transformed to a corresponding uplink problem. In [7], the authors approximated the sum rate formulation into a Rayleigh quotient and the optimization problem was solved through the generalized eigenvalue decomposition. In [8], the authors proposed an iterative algorithm based on the zero-gradient conditions. In [9], the authors proposed a linear precoded cooperative transmission protocol that implements a nonorthogonal relaying strategy for multicell multiple-input single-output (MISO) broadcast channels. Recently, the convex optimization based beamforming technique has drawn great attentions [10], [11]. The downlink optimization problems are often formulated as a quadratically constrained quadratic program (QCQP) or a fractional QCQP. A convex optimization technique known as semidefinite programming (SDP) relaxation has been applied to solve the downlink optimization problems [12]. It has been shown that the semidefinite relaxation (SDR) approach can provide either a global optimum solution or a feasible solution [13].

Most of the existing works consider global BS cooperation. However, global cooperation leads to a heavy backhaul burden as there is a large amount of data/control traffic among BSs. Furthermore, the network is susceptible to central processor breakdown due to its centralized configuration. Clustering is a good way to deal with this problem [4]. In this work, we consider a clustered multicell system where the network is divided into some clusters and BSs in the same cluster work cooperatively to design their beamforming strategies. We also assume that the MISO case with single active user per cell and uniform transmit power allocation is applied [14]–[15].

The knowledge of channel state information at transmitter (CSIT) can be acquired by sending quantized CSI over finite-rate (or limited) feedback links from the mobile users (MUs) to BSs [16]. Vector quantization (VQ) is used to convey accurate CSI. A quantization codebook is assumed to be known at both the transmitter and receiver. The conventional approach to design the codebook is based on random vector quantization (RVQ) [17].

The main contributions of this paper are summarized as follows:

- 1) We investigate a clustered cooperative multicell system, and the achievable rate region is derived. As the objective function needs to be jointly optimized explicitly, we adopt sub-optimal uplink-downlink duality to decouple the original problem into independent problems that could be solved locally in a similar way as in [5]. Then we use the SDR to relax and transform the original nonconvex optimization problem into a convex SDP.
- 2) We propose a realistic limited feedback model in terms of the varying distance between BS and MUs. The size of the quantization codebook is determined by the user's location. A channel quantization criterion with respect to minimum vector difference is adopted, and this new criterion is better than the conventional one as it does not suffer from the phase confusion problem.
- 3) Lastly, we propose a novel limited feedback scheme with low computational complexity based on the compressive sensing technique. The CSI will first be compressed through linear measurement and is then fed back to BSs. The feedback link is modelled as a quantization procedure where the feedback accuracy is related to quantization noise. At BSs, the CSIT can be recovered by solving an l_1 -norm minimization problem.

Notations: Boldface lower case $\mathbf{x}$ denotes the column vector, and upper case $\mathbf{X}$ denotes a matrix. The inverse, transpose, and conjugate transpose of $\mathbf{X}$ are given by $\mathbf{X}^{-1}$, $\mathbf{X}^T$, and $\mathbf{X}^H$, respectively. $\mathbf{X} \succeq \mathbf{0}$ means that $\mathbf{X}$ is positive semidefinite. The distribution of a circularly symmetric complex Gaussian (CSCG) vector with zero mean and covariance matrix $\mathbf{Q}$ is denoted as $\mathcal{CN}(\mathbf{0}, \mathbf{Q})$. $\|\cdot\|$ stands for the Euclidean norm.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Downlink Model

We consider a downlink multicell cellular network with M base stations where each BS is equipped with N_t transmit antennas, and each MU has a single receive antenna. The whole network is divided into some clusters. The size of each cluster is V . The number of adjacent clusters that generate interference is I . The system model is illustrated in Fig. 1. In each cluster there are K active mobile users. Based on the physical location of users, they could be divided into two types: cluster-interior users and cluster-edge users. We assume that all the users within the same cell are orthogonal to each other through intra-cell time-division multiple access (TDMA) [7]; therefore, there is no intra-cell interference. The system can be seen as a multicell interference channel model [18]. We also assume that $K \leq V$, which means in a particular time slot, at most one user per cell is active. As we apply universal frequency reuse where all the BSs in the system operate on the same frequency band, cluster-edge users may suffer severe interference from both adjacent clusters and their own cluster. The received signal of user k in cluster c is given by

Fig. 1. Clustered cooperative multicell cellular system with cluster size $V = 3$. Solid lines denote transmission of desired signals while dotted lines represent interference from adjacent clusters.

$$y_{k,c} = \sum_{j=1}^V \mathbf{h}_{jk,c}^H \mathbf{x}_{j,c} + \underbrace{\sum_{\hat{c}=1, \hat{c} \neq c}^I \sum_{i=1}^V \mathbf{h}_{ik,\hat{c} \rightarrow c}^H \mathbf{x}_{i,\hat{c}}}_{\text{inter-cluster interference}} + n_k \quad (1)$$

where $\mathbf{h}_{jk,c} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{jk})$ represents the channel from BS $_j$ to MU $_k$ in cluster c , and $\mathbf{h}_{ik,\hat{c} \rightarrow c}$ represents the interfering channel from BS $_i$ in adjacent cluster $\hat{c}$ to MU $_k$ in cluster c . The propagation channel is assumed to be slowly-varying, flat Rayleigh fading. $\mathbf{x}_{j,c} \in \mathbb{C}^{N_t \times 1}$ is the transmit signal at BS $_j$ in cluster c . $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ denotes the independent complex additive white Gaussian noise (AWGN) with zero mean and variance of σ_k^2 . $\mathbf{x}_{j,c}$ is given by

$$\mathbf{x}_{j,c} = \sum_{k=1}^K \mathbf{w}_{jk} s_k \quad (2)$$

where $\mathbf{w}_{jk}$ is the beamforming vector, and the transmission power is defined in terms of $\|\mathbf{w}_{jk}\|^2 = p_{jk}$. Let $s_k \sim \mathcal{CN}(0, 1)$ be the data symbol intended for MU $_k$, where $\mathbb{E}\{|s_k|^2\}$ is normalized to one. Each base station is subject to an individual transmit power constraint; that is, $\mathbb{E}\{\|\mathbf{x}_{j,c}\|^2\} = \sum_{k=1}^K p_{jk} \leq P_j$. As we adopt no power control strategy, we assume all the data streams have uniform power allocation, i.e., $p_{jk} = p_{j\hat{k}} = p$, for any $\hat{k} \neq k$. This is a common assumption to simplify the problem when we focus on the design of beamformers' directions [14], [15]. We also assume that $N_t \geq K$. Substituting (2) into (1), we get:

$$y_{k,c} = \underbrace{\sum_{j=1}^V \mathbf{h}_{jk,c}^H \mathbf{w}_{jk} s_k}_{\text{desired signal}} + \underbrace{\sum_{\hat{k}=1, \hat{k} \neq k}^K \sum_{j=1}^V \mathbf{h}_{jk,c}^H \mathbf{w}_{j\hat{k}} s_{\hat{k}}}_{\text{intra-cluster interference}} + \underbrace{\sum_{\hat{c}=1, \hat{c} \neq c}^I \sum_{i=1}^V \mathbf{h}_{ik,\hat{c} \rightarrow c}^H \mathbf{x}_{i,\hat{c}}}_{\text{inter-cluster interference}} + n_k. \quad (3)$$

For cluster-interior users, their received signals only contain the first two terms of (3). We assume that the data symbols intended for all users within the same cluster are shared by the BSs in that cluster; the BSs are connected via backhaul links which are assumed to be error-free and the BSs are well synchronized. This enables a cooperative transmission

technique to be employed where each receiver is served by several BSs simultaneously. On the other hand, BSs may also share the CSIT via backhaul links if needed. BSs will design their beamforming vectors $\mathbf{w}_{jk}$ based on either local full CSI or imperfect CSI through limited feedback.

B. Problem Formulation

The signal-to-interference-plus-noise ratio (SINR) at MU_k is given by

$$\text{SINR}_k = \frac{\left| \sum_{j=1}^V \mathbf{h}_{jk,c}^H \mathbf{w}_{jk} \right|^2}{\sum_{\substack{\hat{k}=1 \\ \hat{k} \neq k}}^K \left| \sum_{j=1}^V \mathbf{h}_{jk,c}^H \mathbf{w}_{j\hat{k}} \right|^2 + \sum_{\substack{\hat{c}=1 \\ \hat{c} \neq c}}^I \sum_{i=1}^V \left| \sum_{m=1}^K \mathbf{h}_{ik,\hat{c} \rightarrow c}^H \mathbf{w}_{im} \right|^2 + \sigma_k^2} \quad (4)$$

for $k = 1, 2, \dots, K$. The maximal achievable instantaneous transmission rate at MU_k is $R_k = \log_2(1 + \text{SINR}_k)$. The sum rate of all the users in one cluster is expressed as $R_s = \sum_k \log_2(1 + \text{SINR}_k)$. For the cooperative multi-antenna systems, there are several typical optimization goals. One design criterion that has been widely used is to minimize the transmit power at base stations [19]. While in the max-min-fair beamforming problem, the goal is to maximize the user with the worst SINR [20], [21]. In some existing works [8], [22], the authors tried to maximize the weighted sum rate (WSR). In this work, we aim at designing a beamforming scheme to maximize the sum rate. The problem is formulated as follows:

$$\max_{\mathbf{w}_{jk} \in \mathbb{C}^{N_t \times 1}} \sum_{k=1}^{K_t} R_k, \quad \text{s.t.} \quad \|\mathbf{w}_{jk}\|^2 = p_{jk}, \quad \forall j, k \quad (5)$$

where K_t is the total number of MUs in one cluster. The fixed uniform power allocation is chosen in a way that it satisfies the per-BS power constraints. The dependency of SINR_k on different beamformers $\mathbf{w}_{jk}$ implies that a joint optimization of all the users within the multicell system is required to maximize the sum rate. In the next section, we will remove this dependency through the sub-optimal uplink-downlink duality and solve the problem in a distributed manner.

C. Achievable Rate Region

We derive the achievable rate region for the clustered multicell scenario. From the information-theoretical point of view, the channel capacity is the maximum mutual information that can be obtained during the communications. The rate achieved by a cluster-edge user k is given by

$$R_k = \log \frac{\left| \sigma_k^2 \mathbf{I} + \mathbf{h}_{k,c} \mathbf{T}_c \mathbf{h}_{k,c}^H + \sum_{\hat{c} \neq c} \mathbf{h}_{k,\hat{c} \rightarrow c} \mathbf{T}_{\hat{c}} \mathbf{h}_{k,\hat{c} \rightarrow c}^H \right|}{\left| \sigma_k^2 \mathbf{I} + \sum_{\hat{k} \neq k} \mathbf{h}_{k,c} \mathbf{W}_{\hat{k}} \mathbf{h}_{k,c}^H + \sum_{\hat{c} \neq c} \mathbf{h}_{k,\hat{c} \rightarrow c} \mathbf{T}_{\hat{c}} \mathbf{h}_{k,\hat{c} \rightarrow c}^H \right|} \quad (6)$$

where $\mathbf{h}_{k,c} = [\mathbf{h}_{1k,c}; \dots; \mathbf{h}_{V_k,c}]$ is a column vector that includes the local channels, $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ with $\mathbf{w}_k = [\mathbf{w}_{1k}; \dots; \mathbf{w}_{V_k}]$ being a column vector that includes the individual beamformer, $\mathbf{T}_c = \mathbb{E}\{\mathbf{x}_{all,c} \mathbf{x}_{all,c}^H\}$ with $\mathbf{x}_{all,c} = [\mathbf{x}_{1,c}; \dots; \mathbf{x}_{V,c}]$. The rate

achieved by a cluster-interior user is the same as (6) except that there is no inter-cluster interference. The achievable sum-rate capacity of one cluster is defined as the convex union of all the user rates given by

$$\mathfrak{R} = \bigcup_{\substack{\mathbf{w}_{jk} \in \mathbb{C}^{N_t \times 1}, \|\mathbf{w}_{jk}\|^2 = p_{jk}, \forall j, k}}^{K_t} (R_1, \dots, R_{K_t}). \quad (7)$$

Each cluster is a network MIMO system, which can be seen as a superposition of multiple broadcast channels. The rate tuples $(R_1, \dots, R_{K_t})$ are achievable with feasible precoding strategies. The outer boundary of $\mathfrak{R}$ is known as the Pareto boundary, and rate tuples on the boundary are Pareto optimal. We look for proper beamformers to approach this boundary.

III. SDR BASED ALGORITHM WITH FULL CSI

The nonconvex optimization problem in (5) requires a complicated joint optimization across MUs as the SINR depends on multiple beamformers. We first apply the sub-optimal uplink-downlink duality to decouple various beamformers and transform the global optimization problem into a local one. Then, we reformulate the problem as a convex SDP problem by dropping the rank-one constraint and solve it accordingly. We assume full CSIT is available at all BSs.

The downlink is a network MIMO system. The direct dual of it is still in the form of network MIMO, which means the uplink dual problem is a non-convex optimization problem that is intractable even if the global CSI is available. Instead, we seek an approximate design criterion by decomposing the network MIMO into a superposition of multiple broadcast channels and applying the uplink-downlink duality on the individual broadcast channel in a similar way as in [5]. We will use a sub-optimal uplink-downlink duality to the downlink network MIMO to obtain a distributed optimization criterion [20], [21], [23]. Our goal is to achieve the performance close to the Pareto boundary. The interdependency among multiple beamformers can be removed with the aid of sub-optimal uplink-downlink duality, and the optimal beamforming vector can be obtained by the following theorem:

Theorem 1. *Representing the downlink network MIMO as the superposition of V broadcast channels in each cluster and considering full uniform power allocation for beamformers, the Pareto optimal rate tuple can be approximately achieved by choosing beamforming vectors as*

$$\mathbf{w}_{jk} = \arg \max_{\|\mathbf{w}\|^2 = p} \frac{q_{jk} \mathbf{w}^H \mathbf{G}_{jk} \mathbf{w}}{\sum_{\hat{k} \neq k} q_{j\hat{k}} \mathbf{w}^H \mathbf{G}_{j\hat{k}} \mathbf{w} + \frac{\sigma_k^2}{P_j}}, \quad (8)$$

where $\mathbf{G}_{jk} = \mathbf{h}_{jk,c} \mathbf{h}_{jk,c}^H$ for $1 \leq k \leq K$. Duality variables q_{jk} can be interpreted as the transmit power of each user in the virtual uplink, and L is the number of channels originating from BS _{j} to users in the own cluster and adjacent clusters.

Proof. First, we decompose the downlink network MIMO into multiple broadcast channels. In the dual uplink multiple access channel, we treat each BS as an individual receiver rather than

treating the cooperative transmit antenna array as a whole receiver. Let $\rho_k = \log_2(1 + \text{SINR}_k^*)$ be an arbitrary Pareto optimal rate tuple. The rate tuple is achieved by choosing beamformers that solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{w}_{jk} \in \mathbb{C}^{N_t \times 1}, p_{jk} \geq 0} \min_k \frac{R_k}{\rho_k} \\ \text{s.t. } \|\mathbf{w}_{jk}\|^2 = p_{jk}, \forall j \text{ and } k, \end{aligned} \quad (9)$$

since the rate R_k cannot be increased without decreasing the rate of any other user. According to Lemma 1 in [6], the optimal solution to (9) yields the same R_k/ρ_k for all users. Here $\rho_1, \rho_2, \dots, \rho_K$ represent the target rate tuples of different users, which can be viewed as the weighting factors to consider heterogeneous user requirements. Larger ρ_k corresponds to higher rate R_k and vice versa. Each user's rate depends on the choice of beamformers and transmit power. The Pareto optimum can be achieved by maximizing the minimum weighted user rate with respect to the individual broadcast channel. Furthermore, as the user rate is monotonically increasing with the SINR, maximizing the minimum weighted R_k is equivalent to max-min of weighted SINR_k given by

$$\begin{aligned} \max_{\mathbf{w}_{jk} \in \mathbb{C}^{N_t \times 1}} \min_k \frac{\text{SINR}_k^{DL}}{\gamma_k} \\ \text{s.t. } \|\mathbf{w}_{jk}\|^2 = p_{jk}, \forall j \text{ and } k, \end{aligned} \quad (10)$$

where γ_k is the SINR weighting factor. Based on [21], the above downlink max-min SINR problem is dual to the following virtual uplink min-max optimization problem:

$$\begin{aligned} \min_{\nu} \max_{\mathbf{q}, \mathbf{w}, \alpha} \alpha \\ \text{s.t. } \alpha \gamma_k \leq \text{SINR}_k^{UL} \\ q_{jk} \geq 0, \nu_{jk} \geq 0, \|\mathbf{w}_{jk}\|^2 = p_{jk} \\ \sum_{k=1}^K q_{jk} \leq \sum_{k=1}^K p_{jk}, \sum_{k=1}^K \nu_{jk} p_{jk} \leq \sum_{k=1}^K p_{jk} \end{aligned} \quad (11)$$

where α is a slack variable and

$$\text{SINR}_k^{UL} = \frac{q_{jk} \mathbf{w}_{jk}^H \mathbf{G}_{jk} \mathbf{w}_{jk}}{\sum_{k \neq j}^L q_{jk} \mathbf{w}_{jk}^H \mathbf{G}_{jk} \mathbf{w}_{jk} + \frac{\nu_{jk} \sigma_k^2}{P_j}} \quad (12)$$

is the virtual uplink SINR. The duality variables $\{q_{jk}\}_{k=1}^K$ are interpreted as the transmit powers of each user in the virtual uplink, and $\{\nu_{jk}\}_{k=1}^K$ are the virtual noise variances at the BS. If we set $\nu_{jk} = 1$ for $k = 1, \dots, K$ and apply the uniform power allocation, (11) can be further transformed into

$$\begin{aligned} \max_{\mathbf{w}, \alpha} \alpha \\ \text{s.t. } \alpha \gamma_k \leq \text{SINR}_k^{UL} \\ \|\mathbf{w}_{jk}\|^2 = p, \forall j \text{ and } k, \end{aligned} \quad (13)$$

where

$$\text{SINR}_k^{UL} = \frac{q_{jk} \mathbf{w}_{jk}^H \mathbf{G}_{jk} \mathbf{w}_{jk}}{\sum_{k \neq j}^L q_{jk} \mathbf{w}_{jk}^H \mathbf{G}_{jk} \mathbf{w}_{jk} + \frac{\sigma_k^2}{P_j}}. \quad (14)$$

Consequently, the optimal beamforming vectors for the downlink problem can be found in a distributed way by solving the problem given in (13). When the optimum of (11) is

achieved, ν_{jk} are usually not equal to 1 for $k = 1, \dots, K$ and the non-uniform power allocation should be applied. Thus, fixing the values of ν_{jk} and q_{jk} leads to a sub-optimal uplink-downlink duality. The optimal beamformers should also maximize the virtual uplink SINR in (8) for each user. $\square$

As for the distributed optimization criterion, it is much easier to obtain the sub-optimal beamformer as compared with solving the joint optimization problem. We have thus transformed the original joint optimization problem given in (5) into a distributed problem given in (8) which could be solved locally. The convex optimization technique can provide a local optimum which is also the global optimum [10]. The SDP has been applied to solve the downlink optimization problem [11], [12]. The beamforming problem is often in the form of fractional QCQP which is nonconvex and NP-hard. SDR is a computationally efficient approximation to QCQP or fractional QCQP. The SDR makes use of the following two fundamental observations:

$$\mathbf{W} = \mathbf{w} \mathbf{w}^H \iff \mathbf{W} \succeq 0, \text{rank}(\mathbf{W}) = 1 \quad (15)$$

$$\mathbf{w}^H \mathbf{G} \mathbf{w} = \text{tr}(\mathbf{w}^H \mathbf{G} \mathbf{w}) = \text{tr}(\mathbf{G} \mathbf{w} \mathbf{w}^H) = \text{tr}(\mathbf{G} \mathbf{W}). \quad (16)$$

By defining $\mathbf{W} = \mathbf{w} \mathbf{w}^H$, we rewrite (8) as

$$\begin{cases} \mathbf{W}^* = \arg \max_{\mathbf{W} \in \mathbb{C}^{N_t \times N_t}} \frac{\text{tr}(\mathbf{G} \mathbf{W})}{\text{tr}(\mathbf{Q} \mathbf{W})} \\ \text{s.t. } \text{tr}(\mathbf{W}) \leq P_{SDR} \\ \mathbf{W} \succeq 0 \\ \text{rank}(\mathbf{W}) = 1 \end{cases} \quad \begin{aligned} (17a) \\ (17b) \\ (17c) \\ (17d) \end{aligned}$$

where $\mathbf{Q} = \sum_{k \neq j}^L \mathbf{G}_{jk} + \mathbf{I}_{N_t} \cdot K \sigma_k^2 / P_j^2$, $P_{SDR} = P_j / K$, and we select the coefficient $q_{jk} = 1$ to account for strong interference. Problem **P1** explicitly reveals and isolates the nonconvex part of the problem. It can be observed from **P1** that the rank-one constraint in (17d) is not convex and is difficult to handle. The basic idea behind SDR is to approximate **P1** by removing the rank-one constraint while keeping the positive semidefinite constraint. In this way, we end up with the following semidefinite program

$$\begin{cases} \mathbf{W}^* = \arg \max_{\mathbf{W} \in \mathbb{C}^{N_t \times N_t}} \frac{\text{tr}(\mathbf{G} \mathbf{W})}{\text{tr}(\mathbf{Q} \mathbf{W})} \\ \text{s.t. } \text{tr}(\mathbf{W}) \leq P_{SDR} \\ \mathbf{W} \succeq 0 \end{cases} \quad \begin{aligned} (18a) \\ (18b) \\ (18c) \end{aligned}$$

where $\mathbf{W}^*$ represents a global optimum solution of problem **P2**, which can be obtained by existing numerical algorithms in polynomial time. It has been shown that the solution to **P2** after dropping the rank-one constraint is very close to the optimal solution to **P1** [13]. Note that (18) is a quasi-convex optimization problem. The general approach to solve this type of problem is to transform it to the corresponding epigraph form by introducing a new variable as suggested in [10] and [24]; the optimal solution can be obtained by the classical bisection method [25] in which a sequence of SDP problems need to be solved. Fortunately, we are able to obtain the global optimal solution to problem **P2** by solving just one SDP. By considering that the denominator of (18a) is always positive,

we can convert the fractional SDP to an equivalent SDP. Precisely, let us define a new variable $\mathbf{Z} = s\mathbf{W}$, where $s > 0$ satisfies $\text{tr}(\mathbf{Q}s\mathbf{W}) = 1$. Then multiplying by s the numerator and the denominator of (18a), the fractional SDR problem **P2** can be turned into a convex SDP as follows:

$$\mathbf{P3} \begin{cases} \mathbf{Z}^* = \arg \max_{\mathbf{Z} \in \mathbb{C}^{N_t \times N_t}, s} \text{tr}(\mathbf{G}\mathbf{Z}) & (19a) \\ \text{s.t. } \text{tr}(\mathbf{Q}\mathbf{Z}) = 1 & (19b) \\ \text{tr}(\mathbf{Z}) \leq s \cdot P_{SDR} & (19c) \\ \mathbf{Z} \succeq 0, s > 0. & (19d) \end{cases}$$

Proposition 1. *The fractional quasi-convex problem **P2** has the same optimal objective value as the convex SDP in **P3**. Furthermore, if $\mathbf{Z}^*$ solves **P2**, $(\frac{\mathbf{Z}^*}{\text{tr}(\mathbf{Q}\mathbf{Z}^*)}, \frac{1}{\text{tr}(\mathbf{Q}\mathbf{Z}^*)})$ solves **P3**; if $(\mathbf{Z}^*, s^*)$ solves **P3**, then $\mathbf{Z}^*/s^*$ solves **P2**.*

Proof. Suppose that $\mathbf{W}^*$ is the optimal solution of **P2**, $\text{opt}(\mathbf{P2})$ is the optimal value of **P2** and $\text{opt}(\mathbf{P3})$ is the optimal value of **P3**. It can be shown that $(\mathbf{W}^*/\text{tr}(\mathbf{Q}\mathbf{W}^*), s^*)$ with $s^* = 1/\text{tr}(\mathbf{Q}\mathbf{W}^*)$ is feasible for **P3**, and the objective function value at the feasible point is $\text{tr}(\mathbf{G}\mathbf{W}^*)/\text{tr}(\mathbf{Q}\mathbf{W}^*) = \text{opt}(\mathbf{P2})$. Thus, we have $\text{opt}(\mathbf{P3}) \geq \text{opt}(\mathbf{P2})$. On the other hand, if $(\mathbf{Z}^*, s^*)$ is the optimal solution of **P3** with $s^* > 0$, then $\mathbf{Z}^*/s^*$ is feasible for **P2**. The objective function value of **P2** is given by $\text{tr}(\mathbf{G}\mathbf{Z}^*)/\text{tr}(\mathbf{Q}\mathbf{Z}^*) = \text{tr}(\mathbf{G}\mathbf{Z}^*) = \text{opt}(\mathbf{P3})$. Thus, we have $\text{opt}(\mathbf{P2}) \geq \text{opt}(\mathbf{P3})$; combined with the previous conclusion, it yields $\text{opt}(\mathbf{P2}) = \text{opt}(\mathbf{P3})$. $\square$

The objective function and constraints in **P3** are linear in $\mathbf{Z}$, and the optimum solution can be obtained by using an interior-point algorithm [25]. Once the solution $\mathbf{Z}^*$ is obtained, we need to check its rank. If $\mathbf{Z}^*$ is of rank one, the solution is the global optimum of the fractional SDP of **P1** since **P2** is a relaxation of **P1** by dropping the rank-one constraint. The optimal beamformer can then be obtained by performing the eigenvalue decomposition on $\mathbf{Z}^*$. If the solution is of more than rank one, we use Gaussian randomization [13] to construct an approximate solution to (8). The idea of Gaussian randomization is to pick a random vector from $\mathcal{N}(\mathbf{0}, \mathbf{Z}^*)$ and project it to the feasible region. The specific procedure is

$$\mathbf{w}^* = \arg \max_{\xi_l, l=1, \dots, L} \frac{\xi_l^H \mathbf{R} \xi_l}{\xi_l^H \mathbf{Q} \xi_l}. \quad (20)$$

The optimal transmit beamformer will enhance the desired signal by the beam pattern main lobe and suppress the interfering signals by beam pattern nulls. A generalized eigen decomposition approach has been proposed to solve this problem [26]. However, the solution to (8) is not unique due to the phase shifts in the beamformers $\mathbf{w}$. The solution is invariant to an angular rotation [7]. It is possible that the beamformers are selected such that the signals arriving at a given MU from multiple BSs will sum up destructively. Our proposed SDR-based algorithm solves the problem efficiently in the polynomial time in $\mathcal{O}(N_t^2 \log(1/\epsilon))$ with ϵ being an accuracy parameter [25], which is comparable to the computational complexity of implementing eigenvalue decomposition. Furthermore, the SDR-based algorithm solves the problem in a different way from the eigen-beamforming approach in [26]. A global optimal solution can be obtained

when a rank-one solution is achieved.

IV. BEAMFORMING WITH LIMITED FEEDBACK

In a frequency-division duplexing (FDD) system, it is difficult to obtain accurate CSI at the BS because of the finite quantization effects over the limited feedback channels. Furthermore, CSIT exchange between BSs will degrade system performance due to the backhaul latency effects. In this section, we propose a dynamic channel quantization model. Then we propose a novel limited feedback scheme based on the compressive sensing technique.

A. Dynamic Quantization Codebook Model

We consider a more realistic limited feedback model as compared to the case where the codebook size is fixed. Cluster-interior users are assumed to broadcast CSI to the BS within their own cluster, while the edge users broadcast CSI to BSs within own cluster and adjacent interfering clusters. It is likely that MUs that are closer to some BSs are able to convey more precise CSI as compared to those MUs who are far away from BSs. Thus, each BS will have CSI estimates of different quality. This scenario can be realized by varying the size of the codebooks. We refer to this model as the *dynamic quantization codebook (DQC)* model. Since the MUs keep moving, the distances between them and the BSs are variant.

As codebook design for the multicell systems is still an ongoing research topic, we use RVQ to facilitate analysis [16]. In RVQ, each codeword is randomly and independently generated from the N_t dimensional complex unit sphere. CSI consists of channel direction information (CDI) and channel quality information (CQI). In this paper, the CQI is defined as the channel gain $\|\mathbf{h}_{jk}\|$. We assume the CQI is directly fed back without quantization. Additionally, users need to quantize CDI and send the corresponding codeword index back to BS. Quantization error is the major source of channel uncertainties that leads to sum-rate loss. The codeword selection criterion is based on the sum-rate loss minimization. We consider the mean loss in sum rate of interior users as follows:

$$\begin{aligned} \mathbb{E}\{R_{loss}\} &= \mathbb{E}\{R_{s,full} - R_{s,LF}\} \\ &= \sum_k \mathbb{E} \left\{ \log \frac{|\mathbf{h}_{k,c}^H \mathbf{w}_k|^2}{|\mathbf{h}_{k,c}^H \hat{\mathbf{w}}_k|^2} \right\} + \sum_k \mathbb{E} \left(\log \frac{\sum_{\hat{k}=1, \hat{k} \neq k}^K |\mathbf{h}_{k,c}^H \hat{\mathbf{w}}_{\hat{k}}|^2 + \sigma_k^2}{\sum_{\hat{k}=1, \hat{k} \neq k}^K |\mathbf{h}_{k,c}^H \mathbf{w}_{\hat{k}}|^2 + \sigma_k^2} \right) \end{aligned} \quad (21)$$

where $\mathbf{h}_{k,c} = [\mathbf{h}_{1k,c}; \dots; \mathbf{h}_{V_k,c}]$ and $\mathbf{w}_k = [\mathbf{w}_{1k}; \dots; \mathbf{w}_{V_k}]$. The mean capacity loss is upper bounded by

$$\begin{aligned} \mathbb{E}\{R_{loss}\} &\leq \sum_k \mathbb{E} \left\{ \log \left(|\mathbf{h}_{k,c}^H \hat{\mathbf{h}}_{k,c}|^2 \right) \right\} + \\ &\quad \mathbb{E} \left\{ \log \left(\sigma_k^2 + N_t(K-1 - |\mathbf{h}_{k,c}^H \hat{\mathbf{h}}_{k,c}|^2) \right) \right\} \end{aligned} \quad (22)$$

under the assumption that high SINR approximation is applied. The direction of channel $\mathbf{h}_{jk}$ is given by $\hat{\mathbf{h}}_{jk} = \mathbf{h}_{jk}/\|\mathbf{h}_{jk}\|$, $\hat{\mathbf{h}}_{k,c} = [\hat{\mathbf{h}}_{1k}; \hat{\mathbf{h}}_{2k}; \dots; \hat{\mathbf{h}}_{V_k}]$, $\hat{\mathbf{h}}_{k,c}$ is the corresponding quantized channel, and $\hat{\mathbf{w}}_{k,c}$ is the beamformer obtained through quantized channels. The proof of reachability of this upper

Fig. 2. Channel direction information and codeword. A comparison between maximum inner product criterion and minimum norm of difference criterion.

bound can be found in [7]. A quantization criterion is used to minimize the mean loss upper bound in (22), and the quantized CDI is selected as

$$\hat{\mathbf{h}}_{jk} = \arg \max_{\mathbf{f}_i \in \mathcal{F}_k} |\tilde{\mathbf{h}}_{jk}^H \mathbf{f}_i|^2 \quad (23)$$

where $\mathcal{F}_k$ is the quantization codebook, $\mathbf{f}_i$ is codeword vector with $i = 1, \dots, 2^B$ and B is the number of feedback bits. By searching the whole codebook, a codeword is selected as the quantized channel $\hat{\mathbf{h}}_{jk}$ if it maximizes the inner product between the channel direction and itself. Obviously, our objective is to find a codeword vector with the smallest angular separation from channel direction $\tilde{\mathbf{h}}_{jk}$. However, this conventional criterion suffers from a phase problem. As illustrated in Fig. 2, $\mathbf{f}_i$ is a certain codeword. Suppose the angle between $\mathbf{f}_i$ and $\tilde{\mathbf{h}}_{jk}$ is $\varphi = \pi - \theta$. Using the fact that $\cos(\theta) = -\cos(\pi - \theta)$, we have $\cos^2(\theta) = \cos^2(\pi - \theta) = \cos^2(\varphi)$, and thus $|\tilde{\mathbf{h}}_{jk}^H \mathbf{f}_i|^2 = |\tilde{\mathbf{h}}_{jk}^H \mathbf{f}'_i|^2$. If $|\tilde{\mathbf{h}}_{jk}^H \mathbf{f}_i|^2$ happens to be the maximum, all the other codewords with angle larger than θ and smaller than φ will be dropped. In this way, although it minimizes the mean loss in sum rate, we select a poor quantized CDI of low quality.

To overcome the phase confusion problem, we resort to another quantization scheme where the quantized channel is instead obtained in the optimum norm sense given by

$$\hat{\mathbf{h}}_{jk} = \arg \min_{\mathbf{f}_i \in \mathcal{F}_k} \|\mathbf{f}_i - \tilde{\mathbf{h}}_{jk}\|. \quad (24)$$

This scheme is called *minimum norm of difference (MND)* criterion that is similar to the approach in [15]. In MND, receiver will find a codeword that has the minimum norm of difference between itself and the CDI. As shown in Fig. 2, the norm of $(\mathbf{f}_i - \tilde{\mathbf{h}}_{jk})$ is smaller than the norm of $(\mathbf{f}'_i - \tilde{\mathbf{h}}_{jk})$, despite $|\tilde{\mathbf{h}}_{jk}^H \mathbf{f}_i|^2 = |\tilde{\mathbf{h}}_{jk}^H \mathbf{f}'_i|^2$. MND has nothing to do with the phase between vectors; it indeed guarantees the selection of proper codeword that is absolutely close to the CDI in terms of vector direction. The beamformers based on quantized channels with MND will be more suitable for mitigating the interference, and the mean loss in sum rate can be minimized.

B. Application of CS in Limited Feedback

1) *Background Information*: Compressive sensing exploits a sparse structure in the signal to reduce the sampling rate. The basic idea of CS is that if we capture the non-zero components of the sparse signal through very few measurements, we are able to recover the exact signal. The CS involves three major steps: sparse representation, measurements taking and signal recovery via the l_1 -norm minimization.

Consider an N -dimensional signal $\mathbf{x}$. It can be represented by an orthonormal basis as $\mathbf{x} = \sum_{i=1}^N u_i \boldsymbol{\psi}_i = \boldsymbol{\Psi} \mathbf{u}$. $\mathbf{x}$ is said to

be S -sparse if the number of non-zero elements of $\mathbf{u}$ is S . The orthonormal basis $\boldsymbol{\Psi}$ is called the dictionary basis. Consider an $M \times N$ measurement matrix $\boldsymbol{\Phi} = [\phi_1, \phi_2, \dots, \phi_N]$ with $M \ll N$ and $M > S$. The measurement can be obtained by

$$\mathbf{f} = \boldsymbol{\Phi} \boldsymbol{\Psi} \mathbf{u} + \mathbf{e} = \boldsymbol{\Theta} \mathbf{u} + \mathbf{e} \quad (25)$$

where $\boldsymbol{\Theta} = \boldsymbol{\Phi} \boldsymbol{\Psi}$ is the compression matrix, and $\mathbf{e}$ is the noise. If $\boldsymbol{\Theta}$ satisfies the Restricted Isometry Property (RIP) [27], $\mathbf{u}$ can be recovered with high accuracy from the incomplete measurements $\mathbf{f}$. We will use the Gaussian random matrix as $\boldsymbol{\Phi}$, because it is universal in the sense that $\boldsymbol{\Theta}$ will be i.i.d. Gaussian and have the RIP for any orthonormal basis $\boldsymbol{\Psi}$ [28]. If the number of measurement satisfies

$$M \geq CS \log(N/S) \quad (26)$$

where C is a small positive constant, the Gaussian measurement matrix will have optimal restricted isometry behavior. Assuming that the power of the error is upper bounded by $\|\mathbf{e}\|^2 \leq \varepsilon^2$, it has been shown in [27] that the solution $\hat{\mathbf{u}}$ to the convex programming

$$\min_{\mathbf{u}} \|\mathbf{u}\|_{l_1} \quad \text{s.t.} \quad \|\boldsymbol{\Phi} \boldsymbol{\Psi} \mathbf{u} - \mathbf{f}\|^2 \leq \varepsilon^2 \quad (27)$$

is a good approximation to signal $\mathbf{u}$ with $\|\mathbf{u} - \hat{\mathbf{u}}\|^2 \leq D\varepsilon^2$, where D is a constant and l_1 -norm is defined as $\|\mathbf{u}\|_{l_1} = \sum_i |u_i|$. Convex programming algorithms such as interior-point methods [25] show good stability in the presence of small perturbations.

2) *CS-Based Limited Feedback Scheme*: We propose a novel limited feedback scheme based on CS theory. We assume that the MUs have perfect channel estimation. Since we have very limited feedback rate on the reverse links, the feedback CSI accuracy will be determined by the feedback rate. We model the feedback process as a quantization effect. The received signal at BSs will be contaminated by quantization noise. The CS-based limited feedback scheme and the MND approach are suitable for high SINR region since the mean rate loss in (21) is based on high SINR approximation.

Firstly, we need to obtain the sparse representation of CSI at MUs. Since the entries of channel $\mathbf{h}_{jk,c}$ are i.i.d. Gaussian distributed, $\mathbf{h}_{jk,c}$ does not have sparsity naturally. We may apply the Kahunen-Loève Transform (KLT) as the dictionary base [29]. KLT is optimal in the sense that it can decorrelate the signal and maximally compress the information. Given that we have the instantaneous channel correlation matrix $\mathbf{G}_{jk} = \mathbf{h}_{jk,c} \mathbf{h}_{jk,c}^H$, the KLT dictionary basis $\boldsymbol{\Psi}$ can be computed by eigenvalue decomposition as

$$\mathbf{G}_{jk} = \boldsymbol{\Psi} \boldsymbol{\Lambda} \boldsymbol{\Psi}^T \quad (28)$$

where $\boldsymbol{\Psi}$ is comprised of eigenvectors of $\mathbf{G}_{jk}$, and the diagonal entries of $\boldsymbol{\Lambda}$ are the corresponding eigenvalues. It has been shown that KLT provides the optimal sparse representation with only one non-zero element ($S = 1$) [29]. The sparse channel vector can be obtained as: $\mathbf{u}_{jk,c} = \boldsymbol{\Psi}^H \mathbf{h}_{jk,c}$. Then we use Gaussian matrix to take measurements. A Gaussian random matrix $\boldsymbol{\Phi} \in \mathbb{R}^{M \times N_t}$, with each entry generated according to $\mathcal{N}(0, 1)$, is assumed to be known at both MU and BS. The dimension of $\boldsymbol{\Phi}$ should satisfy (26).

Next, the compressed CSI will be sent back to BS through the reverse link. We model the feedback link with a quantization effect, where the quantization noise is determined by the feedback rate B . The received noisy CSI measurements at BS_j are given by

$$\mathbf{f}_{jk,c} = \mathcal{Q}(\Theta \mathbf{u}_{jk,c}) = \Theta \mathbf{u}_{jk,c} + \mathbf{e} \quad (29)$$

for $j = 1, \dots, V$ and $k = 1, \dots, K$. $\mathcal{Q}(\cdot)$ is a uniform quantizer, and $\mathbf{e} \in \mathbb{C}^{N_t \times 1}$ is i.i.d. Gaussian noise. The energy of measurement noise is upper limited as $\|\mathbf{e}\|^2 \leq \varepsilon^2$, where ε^2 is the quantization mean-square error. In the rate-distortion theory, ε^2 can also be taken as the distortion rate of the uniform quantizer. Define the quantization interval to be $\Delta = L/2^B$, where L is the interval of magnitude of quantizer input. Let $R = \log_2(L/\Delta) = B$ be the rate of the quantizer codebook, the distortion rate $D(R)$ can be approximated as [30]

$$D(R) \approx \frac{M\Delta^2}{12} = \frac{ML^2}{12} 2^{-2R}. \quad (30)$$

The quantizer input $\Theta \mathbf{u}_{jk,c}$ is the product of two independent Gaussian vectors. It is shown that the distribution of the product of two independent complex Gaussian variables $X \sim \mathcal{N}(0, \sigma_x^2)$ and $Y \sim \mathcal{N}(0, \sigma_y^2)$ is given by [31]

$$f_{XY}(z) = \frac{1}{\pi \sigma_x^2 \sigma_y^2} K_0\left(\frac{z}{\sigma_x \sigma_y}\right) \quad (31)$$

where $K_0(r)$ is the modified Bessel function of the second kind. Since almost all values of $K_0(r)$ lie within the range $[-4, 4]$, the magnitude interval of quantizer input is $L = 8\sigma_x \sigma_y$. After obtaining the noisy feedback, BS recovers $\mathbf{u}_{jk,c}$ via solving the following l_1 minimization problem:

$$\hat{\mathbf{u}}_{jk,c} = \arg \min_{\mathbf{u}_{jk,c}} \|\mathbf{u}_{jk,c}\|_{l_1} \quad \text{s.t.} \quad \|\Theta \mathbf{u}_{jk,c} - \mathbf{f}_{jk,c}\|^2 \leq \varepsilon^2 \quad (32)$$

with $\varepsilon^2 = ML^2 2^{-2R}/12$. BSs should be aware of the dictionary basis Ψ so that the channel can be obtained as $\hat{\mathbf{h}}_{jk,c} = \Psi \hat{\mathbf{u}}_{jk,c}$. As Ψ comes from the eigen decomposition of channel, it is “signal-dependent”. We will adopt a dynamic feedback strategy to provide BSs with the knowledge of Ψ . We have assumed that the channel is slowly-varying fading, where the channel condition varies slightly within a fading block. At the beginning of each fading block, MU will feed back the dominant eigenvalue and eigenvector of (28) and the CS measurement to the own BS. In this way, BS can obtain the dictionary basis by performing eigen decomposition. For the subsequent sessions, MUs only feed back the CS measurements, and BSs can use the same Ψ to perform the transformation. The dictionary basis Ψ will be updated at the beginning of the next fading block. Hence, CSIT can then be obtained to design the transmit beamformers.

The proposed CS-based limited feedback scheme is fundamentally different from the VQ-based approach. For RVQ, the quantized CSI is obtained indirectly by choosing a codeword. In the CS-based scheme, we directly handle the channel vectors through compression and measurement. Furthermore, the codebook size of RVQ is $N_t \times 2^B$. If the antenna number N_t becomes large, the computational complexity will be very high. In contrast, the CS limited

Fig. 3. Sum rate per cluster for different SNR with $N_t = 4$, $K_t = 3$.

feedback scheme has much lower computational complexity in $\mathcal{O}(N_t^2)$. Particularly, it is suitable for a massive multi-antenna system by remarkably compressing the channel based on (26). If we need more precise CSIT, we may increase the number of measurements accordingly. However, only a small number of measurements is enough for the recovery.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present some numerical results to evaluate the performance of our proposed algorithm. In all simulations, spatially uncorrelated Rayleigh-fading channels are assumed. We consider a simple case of 3-BS cooperation. The system SNR is the average SNR an MU receives from a BS, without considering the co-channel interference. The optimal upper bound is generated by grouping all the BSs into one cluster and applying the DPC approach. The quantization codebook is randomly generated as in [17]. As for the proposed DQC model, we assume that BSs can always keep track of the position of MUs so that the codebook size can be assigned accordingly. The SDP problem **P3** is solved by the optimization solver CVX [32]. There are two levels of dynamic codebook; the number of bits for them are B_{L1} and B_{L2} , respectively.

Figure 3 shows the sum rate of a clustered system with $V = 3$. The number of antennas at each BS is $N_t = 4$. The number of neighboring clusters that generate interference is $I = 4$. The number of bits for quantization codebook is $B_{L1} = 9$ and $B_{L2} = 8$ for the desired channels within the own cluster, and $B_{L1} = 6$ and $B_{L2} = 5$ for the interfering channels from adjacent clusters. For simplicity, there are three active MUs in each cluster with two of them cluster interior users and the other one the edge user. Sum rates of the proposed SDR based beamforming, eigen-beamforming approach in [26], zero-forcing beamforming (ZFBF), maximum ratio transmission (MRT) and different limited feedback schemes are compared. It can be observed that the proposed SDR-based beamforming scheme achieves almost the same performance as that of the eigen-beamforming approach, and it outperforms ZFBF and MRT at all SNRs. There is some performance loss compared with sum-rate maximizing precoding due to the

Fig. 4. Sum rate per cluster versus feedback rates with $V = 3$, $N_t = 8$, $K_t = 3$, and SNR = 15 dB.

distributed optimization goal. The CS-based limited feedback is better than the RVQ, especially at the high SNR region. RVQ with MND is better than the conventional criterion.

In Fig. 4, different limited feedback schemes are compared by varying the number of feedback bits. The number of feedback bits for each MU is set to be equal. The transmit antenna number is set to $N_t = 8$ and SNR is fixed at 15 dB. It is observed that when the feedback rate is higher, there is a large gap between the CS limited feedback and RVQ schemes. When B increases, the quantization error in (30) decreases rapidly, so the recovered CSI at BS is closer to the original CSI. The recovery accuracy can be improved by increasing the number of measurements in (26). However, the sum rates under RVQ schemes increase slowly. To obtain more precise CSI, we need a larger amount of feedback bits, which however makes the codebook design very complicated.

Figure 5 illustrates the sum rate of the proposed SDR based beamforming scheme and ZFBF scheme with perfect CSIT for different N_t with cluster size of $V = 3$. With more transmit antennas, the system degree of freedom (DoF) is increased, so is the sum-rate capacity. The proposed SDR-based scheme outperforms the ZFBF scheme consistently. The simulation result accords with our analysis.

In Fig. 6, we compare the performance of the two quantization criteria in the system where $V = 3$, $N_t = 5$, and $K_t = 15$. For scheme 1 of the DQC model, the numbers of bits for feedback are $B_{L1} = 9$ and $B_{L2} = 8$ for the desired channels within the own cluster, and $B_{L1} = 6$ and $B_{L2} = 5$ for the interfering channels from neighboring clusters. For scheme 2, we set $B_{L1} = 11$ and $B_{L2} = 10$ for the desired channels and $B_{L1} = 8$ and $B_{L2} = 7$ for the interfering channels. We can observe that with a larger codebook size, MUs are more likely to feedback precise channel information, and the beamforming scheme based on the quantized channels is more effective. Moreover, the quantization accuracy becomes a dominant constraint in the high SINR region.

VI. CONCLUSIONS

We have considered beamforming schemes in a cooperative clustered multicell multi-antenna system. The sum-rate

Fig. 5. Sum rate per cluster of the proposed algorithm with different N_t for $V = 3$ and $K_t = 3$.

Fig. 6. Comparison of quantization criteria with $V = 3$, $N_t = 5$, $K_t = 15$.

maximization problem was transformed into a convex SDP by applying a sub-optimal uplink-downlink duality and SDR approach. We have also proposed a dynamic quantization codebook model, with users selecting quantized CSI based on the minimum norm of vector difference. Lastly, we have proposed a novel limited feedback strategy based on the compressive sensing technique. Numerical results have shown that the proposed SDR-based beamforming scheme can achieve significant sum-rate improvement, the CS limited feedback scheme has lower computational complexity and better performance, and the MND criterion is better than the conventional one.

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