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2017

Hasdeo, E. H., & Song, J. C. W. (2017). Long-lived domain wall plasmons in gapped bilayer graphene. *Nano Letters*, 17(12), 7252-7257. doi:10.1021/acs.nanolett.7b02584

<https://hdl.handle.net/10356/105435>

<https://doi.org/10.1021/acs.nanolett.7b02584>

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Long-lived domain wall plasmons in gapped bilayer graphene

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Topological domain walls in dual-gated gapped bilayer graphene host edge states that are gate-tunable and valley polarized. Here we predict that plasmonic collective modes can propagate along these topological domain walls even at zero bulk density, and possess a markedly different character from that of bulk plasmons. Strikingly, domain wall plasmons are extremely long-lived, with plasmon lifetimes that can be orders of magnitude larger than the transport scattering time in the bulk at low temperatures. Importantly, long domain wall plasmon lifetimes persist even at room temperature with values up to a few picoseconds. Domain wall plasmons possess a rich phenomenology including a wide range of frequencies (up to the mid-infrared), tunable sub-wavelength electro-magnetic confinement lengths, as well as a valley polarization for forward/backward propagating modes. Its unusual features render them a new tool for realizing low-dissipation plasmonics that transcend the restrictions of the bulk.

Keywords: topological domain wall, bilayer graphene, plasmon

Edge states are a hallmark of the peculiar twisting of crystal wavefunctions in topological materials [1–5], and host a fermiology that departs from that of its parent bulk [6–8]. Domain wall states (DWS) in gapped bilayer graphene are a particularly interesting example. Arising when the sign of the local gap in gapped bilayer graphene flips in real space [9–19], DWS manifest in a number of different settings, e.g., at stacking faults (AB- and BA-) [11–14] or in a split dual-gate geometry wherein perpendicular applied electric field in adjacent regions have opposite signs [14–17]. Domain walls host counter-propagating one dimensional (1D) gapless DWS living in separate K and K' valleys [13–15], with valley filtered currents that are robust to disorder [18]. In contrast to helical edge states in intrinsic topological insulators [1–3], DWS in gapped bilayer graphene enjoy large and tunable bulk gaps up to 200 meV [20] allowing their unusual behavior to manifest even at room temperature.

Here we show that the collective motion of carriers in DWS manifest unusual plasmon modes — domain wall plasmons (DWPs) — whose characteristics are distinct from conventional bulk plasmons (Fig. 1). Arising from collective charge density oscillations of carriers in DWS (Fig. 1a), DWPs can exist even at zero bulk charge density (no doping) with a tunable frequency from the terahertz up to the mid-infrared (~ 200 meV) (Fig. 1b,c) and disperse linearly at small wavevectors in contrast to that expected from conventional 2D bulk plasmons [21].

Importantly, DWPs are long-lived and possess an insensitivity to bulk long-range disorder. While conventional plasmon lifetimes are limited by bulk transport scattering [22–24], DWPs at low temperature transcend the restrictions of bulk transport scattering exhibiting DWP lifetimes orders of magnitude larger than the bulk transport scattering time (Fig. 2). As we argue below, these long lifetimes persist to high temperatures and

can reach values of a few picoseconds at room temperature (for corresponding bulk transport scattering times of ~ 0.5 ps).

The topological DWS that host DWPs are intimately locked to the difference of valley Chern number on either side of the domain wall [13–15]; DWPs possess valley polarization with backward/forward modes predominantly propagating in K/K' valleys (Fig. 3). As we explain below, in addition to currents in the domain walls, DWP propagation also induces bulk undergap valley current flow, which renormalize the frequency of collective oscillations in the domain wall states. Control of the latter (e.g. via screening from a dielectric background) grants an unconventional knob to tune a myriad of DWP characteristics that range from its velocity and confinement, to the degree of DWP valley polarization.

We expect DWPs to manifest in experimentally available gapped bilayer graphene systems [11–18] such as along AB/BA stacking faults in globally gapped bilayer graphene, as well as electrostatically defined domain walls in split-dual-gate geometries. Indeed, both these methods have been recently employed to study topological domain walls experimentally [11, 12, 16, 17]. DWPs also feature subwavelength confinement of light, and can be probed by a variety of techniques that include gratings, and scanning near-field optical microscopy [25, 26].

Domain wall states and collective dynamics — We begin by considering domain walls in gapped bilayer graphene. These domain walls can be created in a number of ways, for e.g., (i) defined electrostatically where split-dual gates in bilayer graphene are biased to yield adjacent regions with layer potential of opposite signs [14–17], and (ii) at AB/BA stacking faults where the bilayer graphene is globally gapped [11–14].

We account for both these types of domain walls phenomenologically by describing gapped bilayer graphene with a spatially varying band gap: $\tilde{\Delta}(x)$ reversing its sign at $x = 0$ (see Fig. 1a). Domain walls occur at the zero node of $\tilde{\Delta}(x)$ and host gapless DWS (Fig. 1a). We note,

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Figure 1: **a** Domain wall states (DWS) localized at $x = 0$ emerge when the effective band gap of bilayer graphene (see text) in adjacent regions have opposite signs: -2Δ on the left, and $+2\Delta$ on the right. DWS are valley-helical states located inside the bulk band gap: backward/forward propagating correspond to K (red lines) and K' (green lines). **b** Collective modes of carriers in the domain wall states manifest as DWPs, which are propagating charge density waves. DWP current at K (K') valley predominantly propagates along the $-y$ ($+y$) direction. **c** DWP dispersion for small wavevectors q with $\kappa = 1$ (solid line) and $\kappa = 20$ (dashed line), see Eq. (8) with $f_q = 1$. Wavevector q shown is plotted in units of $(\hbar v_0/\Delta)^{-1}$. Purple and orange bars show contributions from DWS and bulk undergap valley Hall motion respectively. Parameter values used: $\sigma_H/v_0 = 2.0$, $2\Delta = 120$ meV. For these parameter values $\hbar v_0/\Delta = 7.7$ nm.

parenthetically, that the qualitative form of DWS is insensitive to the specific $\tilde{\Delta}(x)$ profile used since DWS, arising from band inversion, is locked to its zero nodes. For electric field defined domain walls, $\tilde{\Delta}(x)$ directly correlates with the layer potential difference. For domain walls at stacking faults, however, the physical band gap (layer potential difference) does not flip in real space. Instead, the chirality (in each valley) in AB and BA stacking regions are opposite, leading to opposite signs of the valley specific Berry curvature and Chern number [13–15]. We absorb this (chirality) sign into an effective $\tilde{\Delta}(x)$. DWS are valley-helical states located inside the bulk band gap with backward (forward) moving DWS locked to the valley index K (K') (Fig. 1a) [13–15]. For each valley, there are four gapless domain wall states with the same helicity propagating along $\hat{y}$ stemming from layer and spin degrees of freedom [13–16].

In order to describe the dynamics of carriers in DWS, we separate out the density into bulk, $\rho_b(\mathbf{r})$ as well as DWS density $\rho_e(\mathbf{r})$ (centered at $x = 0$) via

$$\rho^\nu(\mathbf{r}, t) = \rho_b^\nu(\mathbf{r}, t) + \rho_e^\nu(\mathbf{r}, t), \quad \mathbf{j}^\nu(\mathbf{r}, t) = \mathbf{j}_b^\nu(\mathbf{r}, t) + \mathbf{j}_e^\nu(\mathbf{r}, t) \quad (1)$$

where ν is the valley index ± 1 for K or K' valley, respectively. ρ_b is the density in the valence and conduction band (black and gray bands in lower panel of Fig. 1a) while ρ_e is the density in the gapless helical DWS (red and green bands in the lower panel of Fig. 1a). Here domain wall current $\mathbf{j}_e^\nu$ [Eq. (1)] in each of the valleys arises from the helicity of DWS: $\mathbf{j}_e^\nu(\mathbf{r}) = -\nu v_0 \rho_e^\nu(\mathbf{r}) \hat{y}$ where DWS in valley K and K' possess effective chiral velocity $-v_0 \hat{y}$ and $v_0 \hat{y}$, respectively [13–15, 27][45].

In the bulk, charge density evolves dynamically as

$$\partial_t \delta \rho_b^\nu(\mathbf{r}, t) + \nabla \cdot \mathbf{j}_b^\nu(\mathbf{r}, t) = 0, \quad \mathbf{j}_b^\nu(\mathbf{r}, t) = \boldsymbol{\sigma}^\nu [-\nabla \phi(\mathbf{r}, t)] \quad (2)$$

where $-\nabla \phi(\mathbf{r}, t)$ is the electric field, and $\boldsymbol{\sigma}^\nu$ is the bulk conductivity tensor. Here $\delta \rho(\mathbf{r}, t) = \rho(\mathbf{r}, t) - \rho^{(0)}$ where

$\rho^{(0)}$ is the equilibrium charge density. We note that $\boldsymbol{\sigma}^\nu$ contains both diagonal, σ_{xx} , as well as off-diagonal components, σ_{xy}^ν . In gapped bilayer graphene, the latter arises from valley Hall currents [28–30] and as we will see below, plays an integral role in DWP dynamics. Valley dependent Hall motion is characterized by the sign of the gap as well as the valley index; here we model $\sigma_{xy}^\nu(x) = \nu \text{sign}(x) \sigma_H$, $\sigma_H = 4e^2/h$ where the factor 4 corresponds with the number of DWS in each valley [46].

We next proceed to analyze the dynamics of carriers in the DWS. In so doing, we note that DWS and as a result, $\rho_e^\nu(\mathbf{r})$, can possess a small non-zero spatial extent $2\lambda_0$ (along x). Crucially, the dynamics of DWPs that we are interested in are dominated by their self-generated electric fields which can have large spatial extent determined by the DWP wavevector, q . As we will see, for long wavelength plasmons these self-generated electric fields have a far larger spatial extent than the typical lateral widths of DWS localized close to the domain wall. As a result, in describing the dynamics of long wavelength DWPs where $|q|\lambda_0 \ll 1$, DWS charge density profiles can be well-approximated by a Dirac delta function, $\delta \rho_e^\nu \propto \delta(x)$.

For clarity, we first explicitly discuss the behavior of long wavelength DWPs in this section, adopting $\delta \rho_e^\nu \propto \delta(x)$. We note that the finite width of DWS can be explicitly accounted for, and will be discussed in the next section; see also Supporting Information (SI) [27] for detailed analysis of DWP dynamics in finite width DWS. As we discuss below, DWPs arise for both profiles and possess qualitatively similar characteristics.

For long wavelength DWPs, the dynamics of the domain wall charge density can be discerned by applying the continuity relation to Eq. (1) and matching the $\delta(x)$ functions [47]. We obtain

$$(\partial_t - \nu v_0 \partial_y) \delta \rho_e^\nu - 2\nu \sigma_H \partial_y \phi|_{x=0} = \gamma_v (\delta \rho_e^{-\nu} - \delta \rho_e^\nu), \quad (3)$$

where we have used $\partial_x \text{sign}(x) = 2\delta(x)$ and $(\partial_x \partial_y -$

$\partial_y \partial_x \phi(\mathbf{r}, t) = 0$; here we have adopted $\delta \rho'_e \propto \delta(x)$. The first term in Eq. (3) describes dynamics arising from current flow within the DWS, while the second term arises from the bulk valley Hall currents impinging into the DWS. Valley relaxation is accounted for via a phenomenological inter-valley scattering rate γ_v . The bulk contribution to domain wall carrier dynamics naturally arise from the spectral flow encoded in σ_{xy} similar to that in quantum Hall systems [32–34]. We note that longitudinal conductivity σ_{xx} is continuous across DWS and consequently does not directly contribute to the domain wall carrier dynamics in Eq. (3).

Collective modes of the domain wall states emerge as self-sustained density oscillations of Eq. (1-3), and electric potential obeying

$$\phi(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \delta \rho(\mathbf{r}', t), \quad U(\mathbf{r}, \mathbf{r}') = \frac{1}{\kappa |\mathbf{r} - \mathbf{r}'|}, \quad (4)$$

in the non-retarded limit. Here $U(\mathbf{r}, \mathbf{r}')$ is the Coulomb interaction. Since the system is translationally invariant along the domain wall (y direction), potential and density oscillate as $\phi(\mathbf{r}, t) = \tilde{\phi}_q(x, z) e^{i(qy - \omega t)}$ and $\delta \rho(\mathbf{r}, t) = \delta \tilde{\rho}_q(x) \delta(z) e^{i(qy - \omega t)}$, respectively. We note that the spatial extent of $\phi(\mathbf{r}, t)$ along z is typically about $\sim 1/q$. Hereafter, we concentrate on the fields $\tilde{\phi}, \delta \tilde{\rho}$ at $z = 0$.

In what follows, we will describe collective modes along the domain wall compactly in terms of ϕ , by eliminating $\delta \rho$ from the dynamical equations. To do so, we first note that charge density localized on the domain wall, $\delta \rho_{q,e}$, produces a jump in the electric field as

$$\partial_x \tilde{\phi}_q|_{0+} - \partial_x \tilde{\phi}_q|_{0-} = (\partial_x U_q|_{0+} - \partial_x U_q|_{0-}) \delta \tilde{\rho}_{q,e}. \quad (5)$$

where $U_q(x) = \int dk e^{ikx} (q^2 + k^2)^{-1/2} / \kappa$ is the effective one dimensional (1D) Coulomb kernel. In obtaining Eq. (5) we have taken the derivative of Eq. (4), using the plane-wave forms of $\delta \rho, \phi$ and Eq. (1) above. Importantly, $\delta \rho_e = \delta \rho_e^K + \delta \rho_e^{K'}$ in Eq. (5) can be directly related to the electric potential by inverting Eq. (3):

$$\delta \tilde{\rho}_{q,e} = \frac{2\sigma_H (\mathcal{M}^{K'} - \mathcal{M}^K) \partial_y \tilde{\phi}_q|_{x=0}}{\mathcal{M}^K \mathcal{M}^{K'} - \gamma_v^2}, \quad (6)$$

where $\mathcal{M}^\nu = \partial_t + \gamma_v - \nu v_0 \partial_y$ is an operator that acts on $\phi(\mathbf{r}, t)$ and $\nu = \pm 1$ for K (K') valley. See SI [27] for a general derivation.

In addition to continuity of $\tilde{\phi}_q(x)$ and jump in electric field discussed above, electric potential of the plasmon, $\phi(\mathbf{r}, t)$ also satisfies Eq. (4); this yields $\phi(\mathbf{r}, t)$ as a solution to a non-local integro-differential problem. Instead, here we adopt a simplified Coulomb kernel $\tilde{U}_q(x) = \frac{1}{\kappa} \int dk 2q e^{ikx} / (2q^2 + k^2)$ [48] which captures the essential long wavelength features of $U_q(x)$ [35, 36]. Using simplified $\tilde{U}_q(x)$, we find $\tilde{\phi}_q(x)$ as follows

$$(\partial_x^2 - 2q^2) \tilde{\phi}_q(x) = \frac{-4\pi}{\kappa} |q| \delta \tilde{\rho}_q(x). \quad (7)$$

Since Eq. (7) is local, $\tilde{\phi}_q(x)$ profile can be obtained in a straight-forward fashion as described below.

We first discuss the dispersive features of DWPs, focussing on the case $\gamma_v = 0$ and Fermi energy inside the gap and $T = 0$ so that no bulk carriers are excited; see below for a detailed discussion of the role of γ_v and σ_{xx} . In this situation, $\sigma_{xx} = 0$, $\delta \rho_b = 0$ in the bulk, and the solution of Eq. (7) is $\tilde{\phi}_q(x) = \phi_0 e^{-\sqrt{2}|q|x}$. Plugging this $\tilde{\phi}_q(x)$ profile into Eqs. (5) and (6), we obtain the DWP dispersion (Fig. 1c):

$$\omega = v_0 |q| \sqrt{1 + \eta / f_q}, \quad \eta = 4\sqrt{2}\pi\sigma_H / v_0\kappa, \quad (8)$$

where f_q is a form factor. For the Dirac delta DWS profile considered in this section, $f_q = 1$. A more general formulation that considers finite width DWS profiles also produces Eq. (8) but with a different form factor [27]. For example, using an exponential DWS profile, $\delta \tilde{\rho}_{q,e}(x) \propto e^{-|x|/\lambda_0}$ (characteristic of topological DWS), we obtain $f_q = 1 + \sqrt{2}|q|\lambda_0$ (at $T = 0$), see SI [27]. As a result, in the long wavelength limit, $|q|\lambda_0 \ll 1$, this profile reduces to the same dispersion with the Dirac delta DWS profile (see Fig. 2a). As discussed above, this arises since the potential profile $\tilde{\phi}_q(x)$ in Eq. (7) is much more delocalized than DWS profile for long wavelengths (small $|q|$), where $|q|\lambda_0 \ll 1$. Hence, the Dirac delta profile for ρ_e serves as a good approximation in this limit.

At long wavelengths, DWPs possess a linear dispersion (Fig. 1c), markedly distinct from the $\sqrt{q}$ dispersion of 2D bulk plasmon. This mimics the plasmon dispersion in one-dimensional nanowire systems [21]. The first term of Eq. (8) inside the square root comes from the helical group velocity of DWS, whereas η captures collective bulk valley Hall motion that moves along the DWP. We note that for $\hbar\omega \geq 2\Delta$, DWP enters the single particle continuum where particle-hole excitations damp the plasmon and destroy its coherence.

Strikingly, bulk valley Hall currents renormalize the collective mode velocity of DWS in Eq. (8). Choosing $2\Delta = 120$ meV, $\sigma_H = 4e^2/h = 1.4 \times 10^6$ m/s and $\kappa = 1$, we estimate that the valley Hall contribution can be 35 times larger than the single particle DWS contribution (see orange bar vs purple bar in Fig. 1c). As a result, DWP group velocity can be six times larger than v_0 .

Low plasmon velocities yield tight confinement of light when the plasmon is hybridized to form plasmon-polaritons. Indeed, taking $2\Delta = 120$ meV we find a plasmon confinement of about 70 times smaller than free-space wavelength. For example, for $\hbar\omega = 60$ meV, this gives a confinement length as small as 290 nm (c.f. free-space wavelength for the same frequency of 20 μ m). Importantly, since η depends strongly on background κ , screening can dramatically reduce η and DWP velocity, further enhancing the confinement of DWP (dashed line in Fig. 1c; here $\kappa = 20$).

Domain wall plasmon lifetime – The dynamics of (thermally activated) bulk charge as well as inter-

Figure 2: DWP dispersion ω for $T = 50$ K (blue) and $T = 300$ K (red) obtained using exponential profile of DWS $\delta\rho_e \propto e^{-|x|/\lambda_0}$ (solid lines) with width $2\lambda_0 = 50$ nm and the Dirac delta profile $\delta\rho_e \propto \delta(x)$ (dashed lines). Bulk plasmon dispersion ω_b at $T = 300$ K is shown for comparison (red dotted line). Note that for $T = 50$ K, ω_b is negligible. (top) Gray shaded region indicates the single particle continuum (SPC). **b** DWP lifetime τ_p as a function of wave vector q and temperature T for exponential DWS profile exhibit large values exceeding the transport scattering time ($\tau_{tr} = 0.5$ ps) by orders of magnitude at low temperature. The white region (right bottom) delineates the region where $\omega \lesssim 2\omega_b$. The color bar is in a logarithmic scale. **c** DWP lifetime τ_p (in log scale) as a function of temperature for $\hbar\omega = 60$ meV obtained numerically from Eq. (10) for exponential DWS profile (solid lines) and the Dirac delta profile (dashed lines). Bulk transport scattering time $\tau_{tr} = 0.5$ ps is shown as a black dotted line. [Inset] DWP lifetime τ_p as a function of transport lifetime τ_{tr} at 300 K for several excitation energies. We have used parameters $\tau_{tr} = 0.5$ ps, $\tau_v = 100$ ps, $\kappa = 1$, and $2\Delta = 120$ meV.

valley scattering can contribute to the decay and damping of DWP. Employing Eq. (2) we find bulk charge dynamics: $-i\tilde{\omega} \delta\tilde{\rho}_{q,b} + \sigma_{xx} (-\partial_x^2 + q^2) \tilde{\phi}_q = 0$, where we model the bulk conductivity via a Drude model: $\sigma_{xx} = D(\theta)/(\gamma_{tr} - i\tilde{\omega})$, $\gamma_{tr} = 1/\tau_{tr}$ is the carrier scattering rate and $D(\theta)$ is the Drude weight [27] that depends on $\theta = k_B T/\Delta$. At a finite temperature, the density of thermal carriers (encoded in $D(\theta)$) is smaller for larger Δ . Here we have used complex $\tilde{\omega}$ to capture both plasmon oscillations ($\text{Re}(\tilde{\omega})$ denotes the plasmon frequency) as well as decay dynamics ($\text{Im}(\tilde{\omega})$ denotes its inverse lifetime). Using these, $\tilde{\phi}_q(x)$ in the bulk takes the form [27]

$$\tilde{\phi}_q(x) = \phi_0 e^{-\sqrt{2}\alpha(\tilde{\omega})|qx|}, \quad \alpha(\tilde{\omega}) = \left(\frac{\tilde{\omega}^2 - \omega_b^2 + i\gamma_{tr}\tilde{\omega}}{\tilde{\omega}^2 - 2\omega_b^2 + i\gamma_{tr}\tilde{\omega}} \right)^{1/2}, \quad (9)$$

where $\omega_b^2 = 2\pi D(\theta)|q|/\kappa$ is the bulk plasmon frequency.

Substituting $\delta\tilde{\rho}_{q,e}$ from Eqs. (5) and (6) and using the potential profile in Eq. (9) we obtain a complex DWP $\tilde{\omega}$ obeying:

$$\tilde{\omega}^2 - (v_0 q)^2 \left(1 + \frac{\eta\beta(\tilde{\omega})}{\alpha(\tilde{\omega})g_q} \right) + 2i\tilde{\omega}\gamma_v = 0, \quad (10)$$

where $\beta(\tilde{\omega}) = \frac{\tilde{\omega}^2 + i\gamma_{tr}\tilde{\omega}}{\tilde{\omega}^2 + i\gamma_{tr}\tilde{\omega} - 2\omega_b^2}$. For the Dirac delta DWS profile discussed above, the form factor $g_q = 1$. More general DWS profiles can also be considered, see SI [27], and do not alter the structure of Eq. (10). For example, using an exponential DWS profile, $\delta\tilde{\rho}_{q,e}(x) \propto e^{-|x|/\lambda_0}$, we obtain $g_q = 1 + \sqrt{2}\alpha(\tilde{\omega})|q|\lambda_0$.

The plasmon dispersion and lifetime can be discerned from Eq. (10) by writing $\tilde{\omega}(q) = \omega(q) - i/\tau_p(q)$, where $\tau_p(q)$ is the DWP lifetime. Solving Eq. (10) numerically, we plot the plasmon frequency (ω) and lifetime (τ_p) in Figs. 2 (a) and (b,c) respectively; in these, we have used parameters $2\Delta = 120$ meV, and $\kappa = 1$ as well as transport scattering time $\tau_{tr} = 0.5$ ps which corresponds to a relatively high mobility $50,000 \text{ cm}^2/\text{Vs}$ that can be realized in hBN-encapsulated bilayer graphene samples [16, 30]. Here we use a temperature independent τ_{tr} for illustrative purposes. We have also used the intervalley scattering lifetime $\tau_v = 1/\gamma_v = 100$ ps as estimated by Ref. [18].

In Fig. 2a, we show DWP frequency ω at low temperature (blue lines) and room temperature (red lines). We have displayed ω obtained from DWS density with a delta function profile (dashed lines) and DWS density with an exponential profile of width $2\lambda_0$ (solid lines). We note that the width can depend on size of the bandgap induced, as well as the smoothness of the domain wall profile [49]. For illustrative purposes, in Fig. 2 we use $2\lambda_0 = 50$ nm; other values of $2\lambda_0$ chosen do not qualitatively alter our results. Indeed, both delta and exponential profiles show good agreement at small q , and only display deviations at large q . Naturally tighter $2\lambda_0$ widths yield solid line dispersions that adhere close to the dashed lines for a larger range of q .

DWP dispersions in Fig. 2a at room temperature (300 K) and low temperature (50 K) are fairly similar due to the slow increase of the Drude weight with temperature [27]. The bulk plasmon frequency ω_b is negligible at low temperature. While ω_b increases at room temperature (red dotted line), it is still smaller than the DWP frequency for most values of q . At very small q and large temperatures, ω_b becomes comparable to ω (Fig. 2b). When $\omega = a\omega_b$, kinematics allow DWPs to rapidly decay into bulk plasmons, when a is of order unity. While a detailed analysis of DWP to bulk plasmon decay is beyond the scope of this work, we delineate this regime in Fig. 2b with regions $\omega \lesssim a\omega_b$ shown in white. As an illustration, we set $a = 2$. See SI [27] for a detailed comparison of ω to ω_b .

Importantly, as shown in Fig. 2(b), DWPs can exhibit very long lifetimes ~ 3 ps even at room temperature ex-

ceeding reported plasmon lifetimes (~ 0.5 ps) in hBN-encapsulated graphene [38]. Lifetimes in Fig. 2(b) were obtained numerically from Eq. (10) using the exponential DWS profile. Strikingly, τ_p exceeds the bulk transport scattering time of $\tau_{tr} = 0.5$ ps (dotted black line, Fig. 2c), and clearly demonstrates how DWP τ_p can transcend the conventional limit set by bulk transport scattering [24, 38]. With even higher mobilities and larger τ_{tr} (e.g., by using hBN-encapsulation), even longer τ_p can be achieved (see inset of Fig. 2c)[39].

Enhanced lifetimes arises due to a suppression of bulk carrier density that provides a pathway for DWPs to decay. Even though bulk carrier density in the gapped bulk is low (yielding small ω_b), DWP ω can reach sizable values due to the collective motion in the gapless DWS. This large mismatch in frequencies (ω/ω_b) enables long DWP lifetimes. Indeed, for larger ω we find longer DWP lifetime (see inset Fig. 2c). Similarly, for larger gap sizes, ω_b becomes further suppressed enabling even longer lifetime DWPs at high temperature. We emphasize that long DWP lifetimes persist even for wide exponential DWS density profiles (see solid lines Fig. 2c, as well as color plot in Fig. 2b) underscoring the robustness ω/ω_b suppression of DWP decay pathways in the bulk.

This is also borne out in DWP lifetime distinctive temperature dependence. In Fig. 2c, we show τ_p as a function of temperature for a fixed $\hbar\omega = 60$ meV. At high temperature, since bulk Drude weight is thermally activated, τ_p displays an exponential temperature dependence (Fig. 2c) sharply increasing as temperature drops. However, at low temperature ω_b vanishes. As a result, in this regime, intervalley scattering dominates DWP lifetime cutting the exponential rise of DWP lifetime $\tau_p \rightarrow \tau_v$ (see Fig. 2c).

Due to the valley-helical nature of DWS, τ_v can in principle be very large. Indeed, Ref. [18] reported that the 1D channel is insensitive to backscattering and long range disorder giving a mean free path of about $100 \mu\text{m}$ corresponding to τ_v as high as 100 ps. We note that recent transport experiments along both electric field and stacking fault domain walls report shorter τ_v of about a few hundred fs [11, 16]. Shorter τ_v in electric field domain walls may arise from short-ranged disorder that can scatter between valleys such as grain boundaries [11], as well as wide electrostatic profile used to create $\tilde{\Delta}(x)$ in electric field defined domain walls. In the latter, the electrostatic profile is characterized by a finite effective width L_0 of the domain wall [16]. Although domain walls and DWS arise whenever $\tilde{\Delta}(x)$ flips sign, broad L_0 allow additional non-chiral (non-topological) states that can mediate scattering between DWS in separate valleys and consequently reduce τ_v [16]. We note that the typical width used by Ref. [16] was about $L_0 \approx 70 - 100$ nm. With reduced L_0 and smooth potential profile, τ_v may reach long ballistic timescales characteristic of topological domain wall states.

Valley polarization – Single-particle carrier trans-

Figure 3: **a** Valley polarization $\tilde{\rho}_e^\nu = \delta\rho_e^\nu/\delta\rho_e$ at low temperatures where $\nu = K$ (red lines) and K' (green lines) for the exponential DWS profile (solid lines) and the Dirac delta DWS profile (dashed lines). Dotted lines indicate perfect filtering. Here we have used $\kappa = 20$ and all other parameters are the same as Fig. 2. **b** Schematic of mixed contribution of $\tilde{\rho}_e^\nu$ for forward/backward propagating DWPs.

port within DWS are completely filtered by valley index [14]: at $K(K')$ valley, carriers in DWS propagate in the $-y$ ($+y$) direction. In contrast, the collective modes of DWP experience a mixture of both valley contributions since Coulomb interactions are long-ranged and do not discriminate between valleys. In order to quantify how much each valley contributes to the collective motion of DWP, we analyze Eq. (3). For simplicity, we specialize to the limit $\gamma_{tr} = 0, \gamma_v = 0$ and low temperatures.

Fig. 3b shows that DWP valley polarization $\tilde{\rho}_e^\nu = \delta\rho_e^\nu/\delta\rho_e$ are π out-of-phase with each other (i.e. $\delta\rho_e^\nu$ have opposite signs) and have different amplitudes (Fig. 3a,b). Non-zero amplitude $\tilde{\rho}_e^\nu$ in both valleys for DWPs (and partial valley polarization) is a direct consequence of collective motion of bulk valley Hall currents; this departs from the perfect valley polarization regime (dotted lines of Fig. 3a). This contrasts with the $\eta = 0$ case in Eq. (8) where DWPs traveling along $q < 0$ ($q > 0$) are fully K (K') valley polarized. We note that at smaller κ , $\tilde{\rho}_e^\nu$ will deviate even further from perfect filtering (dotted lines of Fig. 3a) due to a stronger Coulomb potential.

DWPs are long-lived and possess decay times that surpass conventional plasmon decay restrictions wherein plasmon lifetime is limited by the bulk's transport scattering time. This property is unusual and stems from the distinct origin of DWPs: collective oscillations of carriers in the domain wall states. Indeed, the domain wall states enable large quality factors for DWP oscillations which can range from about 10^2 (at room temperature) up to 10^4 (at low temperatures) [27]. Surprisingly, DWPs' long lifetime and high quality manifest without sacrificing subwavelength electro-magnetic confinement. A tantalizing prospect for utilizing DWPs are gate-defining topological domain walls in gapped bilayer graphene for plasmonic waveguides. Together with high quality and long-lived DWPs, gate-defined domain walls provide a means for patterning low-dissipation (and valley polarized) plasmonic circuits [23, 40].

Associated Content — Supporting Information: Discussion of the sign of domain wall current, DWP dispersion for finite width DWS profile, Drude weight for gapped bilayer graphene, ω/ω_b ratio and quality factor.

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Notes

The authors declare no competing financial interest.

Acknowledgements — We are grateful for useful conversations with Frank Koppens, Niels Hesp, and Mike Schechter. This work was supported by the Singapore National Research Foundation (NRF) under NRF fellowship award NRF-NRFF2016-05.

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- [44] Wang, W.; Apell, P.; Kinaret, J. *Phys. Rev. B* **2011**, 84, 085423.
- [45] Ref. [15] estimates $v_0 = 4v_F(\sqrt{2}\Delta/t_1)^{1/2}/3$ at zero Fermi energy, where $v_F = 10^6$ m/s is the monolayer Fermi velocity, $t_1 = 0.3$ eV is the interlayer hopping parameter. For $2\Delta = 120$ meV, $v_0 = 0.7v_F$.
- [46] Here we have noted that there are two branches of DWS per valley/spin [14, 31].
- [47] Delineation of ρ_b and ρ_e dynamics arise from general considerations [27]. In particular, direct scattering of carriers between bulk sub-gap states and domain wall in-gap states can only occur via inelastic processes which are extremely slow scattering rate τ_{inter}^{-1} . As a result, for the finite frequency DWP that we treat here $\omega\tau_{\text{inter}} \gg 1$ decoupling their dynamics.
- [48] $U_q(x)$ yields highly non-local integro-differential equation Eq. (4). We note there are other methods to (numerically) analyze integro-differential problem, for example, by the Wiener-Hopf (WH) method [41, 42] or multipole expansion [43, 44]. To illustrate the essential features of DWPs, here we adopt a simplified Coulomb kernel $\tilde{U}_q(x)$ whose Fourier transform matches that of $U_q(x)$ up to leading order in k/q . We note that the WH method may lead to logarithmic dispersion $\omega \propto |q| \ln(1/|q|)$ of DWP [41] that may depart slightly from Eq. (8) at $q = 0$.
- [49] We note that soliton-like domain wall profiles in bilayer AB/BA stacking faults as small as ~ 10 nm have been observed in Ref. [37]; for electric field domain walls, channel widths as small as 70 nm have been fabricated [16].

Supporting Information for “Long-lived domain wall plasmons in gapped bilayer graphene”

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Sign of domain wall current

Domain wall current $\mathbf{j}_e^\nu(\mathbf{r}, t)$ in Eq. (1) of the main text arises from single particle motion in the domain wall states (DWS). We determine the direction $\mathbf{j}_e^\nu(\mathbf{r}, t)$ through bulk-boundary correspondence: valley-helical DWS $\mathbf{j}_e^\nu(\mathbf{r}, t)$ propagates in the same direction as the bulk undergap valley Hall current $\mathbf{j}_b^\nu(\mathbf{r}, t)$ close to the domain wall.

For clarity, we focus on electric field domain walls where $\tilde{\Delta}(x)$ reflect the layer potential difference. We first note that near the domain wall, a confining potential $\mathcal{V}(x)$ creates electric field $-\partial_x \mathcal{V} \hat{\mathbf{x}}$ and the Hall current $\mathbf{j}_b^\nu(\mathbf{r}, t) = -\partial_x \mathcal{V} \hat{\mathbf{x}} \times \sigma_{xy}^\nu \hat{\mathbf{z}}$. Similarly, close to a domain wall where $\tilde{\Delta}(x)$ flips sign, a layer dependent potential $\mathcal{V}_{b,t}(x)$ exhibits a profile near the domain wall (see solid lines of Fig. S-1), acting on carriers to produce a bulk $\mathbf{j}_b^\nu(\mathbf{r}, t)$ in the valence band; for $k_B T \ll \Delta$ valence band carriers dominate the anomalous Hall current in each of the valleys. Note that since $\tilde{\Delta}(x)$ flips sign, valence band carriers reside in different layers on either side of $x = 0$ [1, 2] and experience different $\mathcal{V}_{b,t}$ potential profiles. In $x < 0$ ($x > 0$) region, $-\partial_x \mathcal{V}_b \hat{\mathbf{x}}$ ($-\partial_x \mathcal{V}_t \hat{\mathbf{x}}$) is pointing along $-x$ ($+x$) as shown in red (blue) arrow. Noting $\sigma_{xy}^\nu(x) = \nu \text{sign}(x) \sigma_H$, $\mathbf{j}_b^\nu$ in both regions are directed in the $-\nu \hat{\mathbf{y}}$ direction where $\nu = \pm 1$ for K or K' , respectively. Matching the directions of $\mathbf{j}_b^\nu$ and $\mathbf{j}_e^\nu$, we write $\mathbf{j}_e^\nu = -\nu \nu_0 \rho_e^\nu \hat{\mathbf{y}}$. We expect a similar reasoning also applies for domain walls at stacking faults where the opposite chirality in AB and BA stacking regions flips the sign of effective $\tilde{\Delta}(x)$.

DWP dispersion for finite width DWS profile

In the main text, we analyzed DWPs using a Dirac delta function profile to approximate the density profile of DWS which is valid in the long wavelength limit. In this section, we discuss how DWPs emerge using a more realistic density profile for the DWS that have a *finite extent*. We begin by separating the density and current into the bulk and domain wall contributions as

$$\rho^\nu(\mathbf{r}, t) = \rho_b^\nu(\mathbf{r}, t) + \rho_e^\nu(\mathbf{r}, t), \quad \mathbf{j}^\nu(\mathbf{r}, t) = \mathbf{j}_b^\nu(\mathbf{r}, t) + \mathbf{j}_e^\nu(\mathbf{r}, t), \quad (\text{S-1})$$

Figure S-1: Sketch of layer dependent potential profile for top (t) and bottom (b) layers: $\mathcal{V}_{b,t}(x)$ (blue line for top layer potential $\mathcal{V}_t$ and red line for bottom layer potential $\mathcal{V}_b$) flips the sign at the domain wall. Valence (conduction) band is highlighted in solid (dashed) lines. Valence band carriers in the left (right) side of the domain wall reside at the bottom (top) layer experiencing the electric field $-\partial_x \mathcal{V}_b \hat{\mathbf{x}}$ ($-\partial_x \mathcal{V}_t \hat{\mathbf{x}}$) directed to $-x$ ($+x$) direction.

where b and e subscripts denote contributions for carriers in the bulk bands, and carriers in the topological DWS. We will discuss the profile of ρ_e explicitly below.

The motion of carriers within the bulk bands of the bulk system is governed by the longitudinal conductivity σ_{xx} , and follows Eq. (2) of the main text, reproduced here for convenience of the reader:

$$-i\omega \delta \tilde{\rho}_{q,b}^\nu(x) + \sigma_{xx}(-\partial_x^2 + q^2) \tilde{\phi}_q(x) = 0, \quad (\text{S-2})$$

where we have used $(\partial_x \partial_y - \partial_y \partial_x) \phi(\mathbf{r}, t) = 0$.

To proceed, we focus on the limit of bulk band spectrum that are fully gapped, i.e. only DWS that are localized close to the boundary between two regions of opposite stacking/layer asymmetry are gapless. This is naturally realized in systems with an AB/BA stacking fault domain wall by applying homogeneous bias voltage in the whole sample [3, 4]. Alternatively, for an electric field domain wall realized in a dual split-gate geometry, this fully gapped limit is realized when the distance between two adjacent gates is small so that the intervening region between adjacent gates possess a bulk band gap [5]. This situation enables the clear identification of gapless DWS as the only mobile carriers within the gap. As a result, the valley Hall conductivity can be simply written as $\sigma_{xy}^\nu(x) = \nu \text{sign}(x) \sigma_H$.

We now turn to analyze the carrier dynamics within the DWS. As described in the main text, valley filtered DWS possess a chiral velocity flow that depends on the carrier density distribution within the DWS. Additionally, undergap currents intimately tied to the valley hall

conductivity σ_{xy}^ν also contribute to its carrier dynamics. This can be seen from considering spectral flow from bulk to the domain wall [6–9]. Including both these contributions yield the carrier dynamics for the carriers in the DWS as

$$(\partial_t - \nu v_0 \partial_y) \delta \rho_e^\nu - (\partial_x \sigma_{xy}(x)) (\partial_y \phi(\mathbf{r}, t)) = \gamma_v (\delta \rho_e^{-\nu} - \delta \rho_e^\nu). \quad (\text{S-3})$$

The first term in Eq. (S-3) describes dynamics arising from domain wall current flow within the DWS, while the second term arises from the bulk valley Hall currents impinging into the DWS through the spectral flow. Here we have also included valley relaxation accounted for via a phenomenological inter-valley scattering rate γ_v . Eq. (S-1)–(S-3) apply generically for any ρ_e profile.

Collective modes emerge as the self-sustained density oscillations Eq. (S-2) & (S-3) and self-generated electric potential, $\phi(\mathbf{r}, t)$. The latter can be obtained from analyzing the Poisson equation relating electric potential and charge density in the system. Adopting the same procedure and simplified Coulomb kernel as in the main text (e.g., see Eq. (7) of the main text), and using the plane wave form $\phi(\mathbf{r}, t) = \tilde{\phi}_q(x) e^{i(qy - \omega t)}$ we obtain:

$$(\partial_x^2 - 2q^2) \tilde{\phi}_q(x) = \frac{-4\pi}{\kappa} |q| (\delta \tilde{\rho}_{q,b}(x) + \delta \tilde{\rho}_{q,e}(x)), \quad (\text{S-4})$$

where $\delta \rho_{q,(b,e)}(\mathbf{r}, t) = \delta \tilde{\rho}_{q,(b,e)}(x) e^{i(qy - \omega t)}$ are Fourier modes, and $\delta \tilde{\rho}_{q,e}(x) = \sum_\nu \delta \tilde{\rho}_{q,e}^\nu(x)$.

Recalling $\sigma_{xx} = D(\theta)/(\gamma_{\text{tr}} - i\omega)$ and bulk plasmon frequency $\omega_b^2 = 2\pi D(\theta)|q|/\kappa$, we can rewrite σ_{xx} in terms of ω_b . Substituting $\delta \tilde{\rho}_{q,b}$ in Eq. (S-2) with Eq. (S-4) and rearranging the $\tilde{\phi}_q$ terms into the left hand side, we obtain:

$$(\partial_x^2 - [\alpha(\omega)]^2 q^2) \tilde{\phi}_q(x) = \frac{-4\pi}{\kappa} |q| \delta \tilde{\rho}_{q,e}(x) \beta(\omega), \quad (\text{S-5})$$

where

$$\alpha(\omega) = \left(\frac{\omega^2 - \omega_b^2 + i\gamma_{\text{tr}}\omega}{\omega^2 - 2\omega_b^2 + i\gamma_{\text{tr}}\omega} \right)^{1/2}, \quad \beta(\omega) = \frac{\omega^2 + i\gamma_{\text{tr}}\omega}{\omega^2 + i\gamma_{\text{tr}}\omega - 2\omega_b^2} \quad (\text{S-6})$$

For an *arbitrary* profile of $\delta \tilde{\rho}_{q,e}(x)$, $\tilde{\phi}_q(x)$ can be obtained directly using the Green function of Eq. (S-5):

$$\tilde{\phi}_q(x) = -\frac{4\pi|q|\beta(\omega)}{\kappa} \int_{-\infty}^{\infty} dx' G(x - x') \delta \tilde{\rho}_{q,e}(x'), \quad (\text{S-7})$$

where $G(x) = C_0 e^{-\sqrt{2}\alpha(\omega)|qx|}$ is the Green function, and $C_0 = -(2\sqrt{2}\alpha(\omega)|q|)^{-1}$.

While Eq. (S-1)–(S-5) apply generically for any $\rho_e(x)$ profile, for concreteness and to illustrate DWPs along domain walls of gapped bilayer graphene, we will focus on exponential profiles (characteristic of topological edge states) for the DWS density

$$\delta \rho_e^\nu(\mathbf{r}, t) = \delta \tilde{\rho}_e(x) e^{iqy - i\omega t}, \quad \delta \tilde{\rho}_e(x) = \rho_0^\nu e^{-|x|/\lambda_0} \quad (\text{S-8})$$

that correspond to the quantum mechanical Jackiw-Rebbi type DWS wavefunctions localized close to the domain wall. Here $2\lambda_0$ determines the DWS width, and ρ_0 determines its amplitude.

In what follows, we describe the collective modes along the domain wall compactly in terms of $\delta \tilde{\rho}_{q,e}$, eliminating ϕ from the dynamical equations. In so doing, we first note that ρ_0 can be obtained directly from Eq. (S-3) by integrating over x . Summing $\rho_0 = \sum_\nu \rho_0^\nu$, we have

$$2\lambda_0 \rho_0 = \frac{2\sigma_H(\mathcal{M}^{K'} - \mathcal{M}^K)(iq)\tilde{\phi}|_{x=0}}{\mathcal{M}^K \mathcal{M}^{K'} - \gamma_v^2}, \quad (\text{S-9})$$

where the $\mathcal{M}^\nu = (-i\omega) + \gamma_v - \nu v_0(iq)$. In arriving at Eq. (S-9), we performed integration by parts noting that $\phi(\pm\infty) = 0$.

These dynamics can be compared directly with the self-generated electric potential by directly integrating Eq. (S-7) (using Eq. (S-8)) to obtain $\tilde{\phi}(x=0)$. Comparing this with Eq. S-9, we obtain the DWP dispersion/governing equations in Eq. (10) of the main text. This is reproduced here for convenience, namely

$$\tilde{\omega}^2 - (v_0 q)^2 \left(1 + \frac{\eta\beta(\tilde{\omega})}{\alpha(\tilde{\omega})g_q} \right) + 2i\tilde{\omega}\gamma_v = 0, \quad (\text{S-10})$$

where $g_q = 1 + \sqrt{2}\alpha(\tilde{\omega})|q|\lambda_0$ and we have replaced $\omega \rightarrow \tilde{\omega} = \omega - i/\tau_p$. We note that at $T = 0$ K, where bulk carrier density vanishes, $\omega_b = 0$ and setting $\gamma_{\text{tr}} = \gamma_v = 0$ (giving $\beta = 1$, and $\alpha = 1$), Eq. S-10 yields the DWP dispersion in Eq. (8) of the main text.

Drude weight for gapped bilayer graphene

The Drude weight for gapped bilayer graphene can be obtained semiclassically via

$$D = Ne^2 \int \frac{d^2\mathbf{k}}{(2\pi)^2} v_{\mathbf{k}}^2 \cos^2 \varphi_{\mathbf{k}} \left(-\frac{\partial f(\epsilon)}{\partial \epsilon} \right), \quad (\text{S-11})$$

where $N = 4$ accounts for spin and valley degeneracy, e is electron's charge, $v_{\mathbf{k}} = \partial\epsilon(\mathbf{k})/\partial(\hbar\mathbf{k})$ is electron's group velocity, $\cos \varphi_{\mathbf{k}}$ accounts only for longitudinal motion and $f(\epsilon)$ is the Fermi-Dirac distribution. We adopt a simple two-band model of gapped bilayer graphene $\epsilon_{\pm}(k) = \pm\Delta[1 + (k/k_0)^4]^{1/2}$ [1, 2], where Δ is the half gap, $k_0 = \sqrt{\Delta t_1}/\hbar v_F$, t_1 is the interlayer hopping and v_F is the Fermi velocity of monolayer graphene. Using the form of $\epsilon_{\pm}(k)$ above, we change integration variables in Eq. (S-11) from k to ϵ_+ yielding

$$D = \frac{2Ne^2}{2\pi\hbar^2} \int_{\Delta}^{\infty} d\epsilon_+ \frac{\epsilon_+^2 - \Delta^2}{\epsilon_+} \left(-\frac{\partial f(\epsilon_+)}{\partial \epsilon_+} \right), \quad (\text{S-12})$$

where the factor of 2 in the numerator accounts for equal contributions of electrons in the conduction band and

Figure S-2: **a** Ratio of DWP frequency ω to bulk plasmon frequency ω_b and **b** Q factor using exponential DWS profile with $2\lambda_0 = 50$ nm as a function of wave vector and temperature. White region (right bottom) indicate $\omega \lesssim 2\omega_b$ regions. The color bars are in logarithmic scale. We have used parameters $\tau_{tr} = 0.5$ ps, $\tau_v = 100$ ps, $\kappa = 1$ and $2\Delta = 120$ meV.

holes in the valence band. Integrating by parts, recalling $f(\infty) = 0$, and making the integrand dimensionless, $\xi = \epsilon_+/\Delta$, we obtain

$$D(\theta) = \frac{Ne^2\Delta}{\pi\hbar^2} \mathcal{F}(\theta), \quad \mathcal{F}(\theta) = \int_1^\infty d\xi \left(1 + \frac{1}{\xi^2}\right) \frac{1}{1 + e^{\xi/\theta}}, \quad (\text{S-13})$$

where $\theta = k_B T/\Delta$, k_B is the Boltzmann constant and we have written $f(\epsilon_+)$ explicitly. For all plots in the main text and the supplement, Eq. (S-13) was integrated numerically.

ω/ω_b ratio

The small ratio between bulk plasmon frequency ω_b and DWP frequency ω suppresses the role of τ_{tr} on DWP lifetime. In Fig. S-2a, we compare DWP frequency to the frequency of bulk plasmons ω/ω_b as a function of q and T using Eq. (10) of the main text. Strikingly, at low temperature, DWP frequency is about four orders of magnitude higher than ω_b since there are very few carriers in the bulk, resulting a very soft bulk plasmon mode. Since $\omega_b \ll \omega$ at low temperatures, decay dynamics from bulk transport scattering is completely quenched. Instead, damping from intervalley scattering dominates, and $\tau_p \rightarrow \tau_v$ [Fig. 2c in the main text].

As temperature increases, the Drude weight in Eq. (S-13) also increases making the bulk plasmon stiffer. As a result, the difference between ω and ω_b shrinks and

the ratio of ω/ω_b drops. Nevertheless, even at room temperature, DWP can still exceed the bulk plasmon frequency (see red solid and dotted lines of Fig. 2a in the main text). At high temperature, scattering from the bulk dominates the τ_p because γ_{tr} is about two orders of magnitude larger than γ_v .

Quality factor

The quality factor or inverse loss function, $Q = \text{Re}\tilde{\omega}/\text{Im}\tilde{\omega} = \omega\tau_p$, is a dimensionless quantity describing the number of plasmon oscillations performed before decay. We plot Q for DWPs in Fig. S-2b using a numerical solution of Eq. (10) in the main text showing large Q factors. At room temperature, Q for DWPs can be several hundreds and increases exponentially to about 10^4 at low temperatures (Fig. S-2b). This size of Q is large and particularly arresting when compared to conventional bulk plasmon Q factors in graphene that have been experimentally observed (~ 20) [11] and theoretically predicted (~ 100) [12].

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