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2017

Feroskhan, M., & Go, T. H. (2018). Control strategy of sideslip perching maneuver under dynamic stall influence. *Aerospace Science and Technology*, 72, 150-163.
doi:10.1016/j.ast.2017.11.002

<https://hdl.handle.net/10356/105447>

<https://doi.org/10.1016/j.ast.2017.11.002>

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Control Strategy of Sideslip Perching Maneuver under Dynamic Stall Influence

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This paper presents a sliding control strategy for executing a sideslip perching maneuver under dynamic stall influence. Perching is a type of aggressive maneuver performed by the avian species to acquire rapid deceleration, which is frequently followed by a precision landing. Due to the high turn rates and rapid changes in aerodynamic angles in the three-dimensional domain, robustness to state perturbations and model inaccuracies is mandatory as it is impossible to develop a completely accurate aerodynamic model for perching that involves many dynamic phenomena and different flow regimes. Based on the optimal trajectory solutions generated through an optimization procedure, sliding mode control is proposed to address the need for robust tracking in the nonlinear, unsteady region of flight dynamics with the inclusion of dynamic stall delay. Performance of the controller is validated under different scenarios involving varying initial spatial and state conditions and state perturbations throughout the course of the perching maneuver.

Keywords

Perching maneuver, Sideslip perching, Sliding control, Aggressive maneuvers, High angle of attack landing

1. Introduction

Learning from nature, an act that ranges from drawing inspirations to mimicking its concepts as closely as possible, has paved the way for the generation of new exquisite drone designs, leading to the surge of a new trend in biomimicry. Nature flight characteristics that include insect cruise-to-hover transition maneuver [1], bio-inspired morphing wing [2] and flapping wing flight, transcend the limits of conventional designs and traditional flight schemes. These have been successfully actualized in the recent years. One of the flight-related techniques that predominates current conventional landing mechanics for its sheer energy efficiency is the perching maneuver [3, 4].

The perching maneuver can be described as a rapidly decelerating approach executed by birds to arrive at their target point subtly and precisely. The typical aircraft perching technique that is covered in most literature is two-dimensional, where the whole trajectory lies only in the vertical plane [5, 6]. As shown in Fig. 1, the two-dimensional (2D) perching maneuver features the dive phase and climb phase that results in the unfavorable undershoot. The undershoot is generally viewed as an inefficient and undesirable characteristic especially in scenarios where there is no allowance for vertical displacement. This leads to the inclusion of the third spatial dimension to accommodate a three-dimensional (3D) perching maneuver, with inspirations drawn from the fruit bats' crabbed turn and a spiral-like avian perch landing. The overall drag subjected to the aircraft in a 3D perching maneuver can be synergized via coupling of the resistive drags from both the longitudinal and lateral planes to precipitate a prompt landing [7]. Fig. 2 describes the optimal 3D sideslip perching maneuver that can be executed by an aircraft with conventional wing-tail configuration which has been analysed and elaborated in Ref. [8]. The undershoot characteristic is eliminated in the overall 3D trajectory due to the absence of the climb phase.

Understanding that online-generation of optimal trajectories is still computationally challenging and therefore impractical, perching control strategy should make use of offline-optimal trajectories. However, using the associated open-loop optimal control inputs is not possible. Robustness to model inaccuracies is mandatory as it is already impossible to develop a completely accurate aerodynamic model for perching that involves many dynamic phenomena and different flow regimes. Rejection of perpetual state disturbances due to wind gusts is crucial for a successful execution of perching maneuver in an outdoor environment. Under open loop controls, unmodeled uncertainties and state perturbations would certainly deviate the aircraft away from its optimal

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trajectory. If deviations are not too large, contraction to the nominal state perching solutions can still be guaranteed given the appropriate control scheme. Here, the optimal perching maneuver solutions can be regarded as unstable due to the aircraft's inability to sustain itself in the nominal perching trajectory. Being an unstable mode of maneuver, perching, given the slightest state perturbations, will deviate from its nominal trajectory readily. Perching using an open loop control input is not reliable and often times, unrealizable under real-time circumstances. Hence, closing the loop of the perching maneuver with reference to the nominal path of perching maneuver must be considered. The design of such a controller is incredibly challenging as the 3D perching maneuver is an unsteady maneuvering scheme that involves 6-DOF rapid translational and angular motions. The controller should also provide the solution to the problem of variations in the initial conditions since the optimal trajectory is only generated based on specific initial conditions. Replicating these exact initial conditions in a practical scenario is difficult and could not be achieved consistently due to factors such as model uncertainties and state perturbations. Convergence to the offline-generated optimal trajectory in the presence of uncertain initial state and varying spatial conditions should therefore be a primary feature of the perching controller.

Choe and Hovakimyan developed an $\mathcal{L}_1$ adaptive controller to augment the open-loop control inputs with the feedback of the optimal 2D perching state trajectories generated through the use of DIDO optimization scheme [9]. Fast estimation scheme (FES) in the controller provides estimates of the uncertainties inherent in the plant which are subsequently used to generate the additional control to be combined with the nominal control. It is shown to improve the tracking performance reasonably when tested in the presence of uncertainties such as time-delay in the state feedback, erroneous control commands, and variation in the aerodynamic stability derivatives. To address the perching's sensitivity to initial state and spatial perturbations, Rao and Go devised a boundary-fixed sliding mode control technique that successfully tracks the reference 2D perching trajectory given initial state displacements [10]. However, the robustness of the controller in response to state perturbations during the perching maneuver and to the effect of dynamic stalls during rapid angle of attack changes are not investigated. Moore and Tedrake proposed a feedback controller for perching of a simple glider based on the concept of Linear Quadratic Regulator (LQR)-Trees, which generates a family of neighboring optimal alternative perching trajectories computed offline [11]. The region of initial conditions around the reference trajectory, in which the perching trajectories can still reach the target within some tolerance, is identified. The visual presentation of the region for the overall perching maneuver interestingly resembles a funnel that depicts the region of attraction to optimal trajectory. This leads to the motivation in this paper to employ a robust controller that can take advantage of the region of contraction that exists around an optimal perching trajectory. Although the LQR-Tree controller ensures contraction of the trajectories over a range of initial flight conditions, the control technique presented does not cater to handling disturbances and model inaccuracies during perching. Most of the perching works reported in the literature have overlooked the investigation of the controller's ability to handle state perturbations and model inaccuracies during the course of the maneuver. Examining the controller's response to initial perturbations only validates its effectiveness in the pre-stall regime. Furthermore, as the aircraft undergoes the maneuver, the time window for the control to respond effectively decreases and the effect of control input diminishes due to its low velocity and nonlinearity in the post-stall region. As uncertainties in the aerodynamic modeling is unavoidable with respect to the chaotic behaviour of aerodynamic coefficients at high angles of attack, the controller's robustness becomes a necessary requirement. Most of control applications in perching have been limited to 2D perching maneuvers. In the 3D domain, the perching controller has to deal with the demanding task of tracking a larger number of variables with more sources of perturbations. Therefore, on top of the small time window given, a high level of precision is also required to achieve a successful landing, which makes control development in the 3D perching domain even more challenging.

To handle such a highly nonlinear system while simultaneously dominating the state perturbations and model inaccuracies in the 3D domain, this paper adopts the sliding mode control strategy that is recognized for its robustness. The sliding mode control scheme has been covered in many nonlinear aircraft control literature in which its stability robustness and effectiveness in trajectory tracking under parameter variations and unmodeled dynamics are highlighted [12]. Although the objective of the controller is to track the desired optimal trajectory of the sideslip perching maneuver, it is not feasible to directly track all the 9 body-axes state variables $[u, v, w, p, q, r, \phi, \theta, \psi]$, excluding spatial and internal state variables, since the number of control inputs is fewer than the number of states. Therefore, the 4 tracking state variables, which correspond to the 4 available control inputs $(\delta_e, \delta_a, \delta_r, T)$, must be carefully selected to ensure the system's overall convergence to the optimal state maneuvers. Contraction theory is formulated in this paper as a tool to streamline the selection of the 4 variables to be tracked [13]. Through the tracking of those 4 state variables and some correction factors in the inertial position axes (x_e, y_e, z_e) , the stability of the perching maneuver is ensured. Based on a reference perching trajectory, the controller will be able to determine the control inputs for the aircraft to execute the nominal sideslip perching trajectory and correct any state perturbations and spatial deviations along the way. The state-space design methodology provides a convenient way to formulate the control algorithm since the aircraft's nonlinear equations

of motion are already in a state-space form. Subsequently, the performance of the controller would be validated under 3 different scenarios involving varying initial spatial and state conditions and state perturbations throughout the course of the perching maneuver.

Fig. 1. Typical 2D perching (left) compared to 3D perching trajectory (right, side view) [8]

Fig. 2. Sideslip perching trajectory in multiple views (*aircraft model not drawn to scale*) [8]

2. Mathematical Model

A propeller driven aircraft model with the conventional wing–tail configuration in Ref. [8] is considered in the maneuver simulation. Thrust generation from the propeller is in parallel alignment relative to the chord lines of wing and elevator. The thrust force is adaptable but restricted to thrust-to-weight (T/W) ratio of less than 1. The aircraft has a cylindrical fuselage with an elliptical cross-section with a ratio (major/minor axes) of 2.5. The wing and the tail employ the NACA0012 airfoil and are positioned along the fuselage reference line which implies that the angle of attack, α , of the fuselage is equivalent to the corresponding angle of attack of the wing and elevator. To maintain pragmatic relevance and fundamental conceptualization, the model has deliberately been simplified

and given precedence before implementation of other unconventional advanced features such as aerodynamic and thrust vectoring.

CFD methods essentially require significant amounts of time and computational power which are not suitable for this application. An analytical method that provides sufficient accuracy in incorporating the post-stall aerodynamic characteristics into the flight dynamics model of the aircraft is required. The aerodynamic modeling approach has been detailed in Ref. [8] in which a mathematical model presented by Goman and Khrabrov [14] is used to model the dynamic stall delay that results in the significant undershooting or overshooting of the aerodynamic coefficients as compared to the corresponding coefficients in the static case, as shown in Fig. 3. The instantaneous position of the separation point can be described as a non-dimensional internal state variable, $\bar{x} \in [0,1]$, and be included in a first order mathematical expression in Eq. (1) that depends heavily on the rate of angle of attack, $\dot{\alpha}$. Both time constants τ_1 and τ_2 are proportional to the characteristic time scale $(c_w/2)/(\text{sqrt}(u^2 + w^2))$, where c_w refers to the wing chord and u and w are the longitudinal body velocities. $\bar{x}_o$ represents the separation parameter profile in the static case.

$$\tau_1 \frac{d\bar{x}}{dt} + \bar{x} = \bar{x}_o(\alpha - \tau_2 \dot{\alpha}) \quad (1)$$

The non-dimensional internal state variable, $\bar{x}$, also known as the separation parameter, is used to describe the force contributions via additive effects of attached flow and separated flow. As follows, the effective force and moment coefficients at any angle of attack can be expressed as:

$$C_F(\alpha, \dot{\alpha}) = C_{F,att}(\alpha) \cdot \bar{x}(\alpha, \dot{\alpha}) + C_{F,sep}(\alpha) \cdot (1 - \bar{x}(\alpha, \dot{\alpha})) \quad (2)$$

where C_F defines the force coefficient and subscripts *att* and *sep* refer to attached flow and separated flow respectively.

Fig. 3. Nonlinear lift coefficient variation under dynamic stall

This approach is applied to compute the aerodynamic coefficients for both the horizontal stabilizer and vertical tail. The aerodynamic coefficient derivations for the vertical tail are however referenced from the lateral plane and the analytical sideslip angle, β_a . As the effect of high $\dot{\alpha}$ on the lift coefficient of NACA 0012 airfoil is large (as shown in Fig. 3), its influence on the stability of the optimal trajectory solutions is significant [8]. Its inclusion in the flight model is therefore necessary for better controller performance and analysis. In this paper, elevator downwash, asymmetric flows and propeller slipstream effects on downstream surfaces such as wing, fuselage and tail are neglected to simplify the model.

A six-degree-of-freedom (DOF) nonlinear flight model that includes both longitudinal and lateral body-axes components is adopted for the perching maneuver. The aerodynamic stability derivatives are computed using Digital Datcom program [15] and later verified using Tornado [16]. The flight model features a system of 15 first-order differential equations that include the dynamic equations of the 3 internal state variables, $\bar{x}_w$, $\bar{x}_t$, $\bar{x}_v$ (separation parameters of the wing, horizontal stabilizer and vertical tail) as follows:

$$\dot{u} = \frac{F_x}{m} - (qw - rv) \quad (3)$$

$$\dot{v} = \frac{F_y}{m} - (ru - pw) \quad (4)$$

$$\dot{w} = \frac{F_z}{m} - (pv - qu) \quad (5)$$

$$\begin{aligned} \dot{p} = & \left[I_{zz}M_x(\beta, \dot{\beta}, p, r, \delta_a, \delta_r) + I_{xz}M_z(\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \right] \\ & - qr[I_{xz}^2 + I_{zz}^2 - I_{yy}I_{zz}] + pqI_{xz}[I_{xx} - I_{yy} + I_{zz}] / I_{xx}I_{zz} - I_{xz}^2 \end{aligned} \quad (6)$$

$$\dot{q} = [M_y(\alpha, \dot{\alpha}, q, \delta_e) - rp[I_{xx} - I_{zz}] - [p^2 - r^2]I_{xz}] / I_{yy} \quad (7)$$

$$\begin{aligned} \dot{r} = & \left[I_{xx}M_z(\beta, \dot{\beta}, p, r, \delta_a, \delta_r) + I_{xz}M_x(\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \right] \\ & - qrI_{xz}[I_{xx} - I_{yy} + I_{zz}] + pq[I_{xz}^2 - I_{xx}I_{yy} + I_{xx}^2] / I_{xx}I_{zz} - I_{xz}^2 \end{aligned} \quad (8)$$

$$\dot{\phi} = p + q \tan\theta \sin\phi + r \tan\theta \cos\phi \quad (9)$$

$$\dot{\theta} = q \cos\phi - r \sin\phi \quad (10)$$

$$\dot{\psi} = r \cos\phi \sec\theta + q \sin\phi \sec\theta \quad (11)$$

$$\begin{aligned} \dot{x}_e = & (\cos\theta \cos\psi)u + (\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi)v \\ & + (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{y}_e = & (\cos\theta \sin\psi)u + (\sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi)v + \\ & (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)w \end{aligned} \quad (13)$$

$$\dot{z}_e = (-\sin\theta)u + (\sin\phi \cos\theta)v + (\cos\phi \cos\theta)w \quad (14)$$

$$\dot{\bar{x}}_w = [-\bar{x}_w + \bar{x}_{w,o}(\alpha - \tau_2\dot{\alpha})] / \tau_1 \quad (15)$$

$$\dot{\bar{x}}_t = [-\bar{x}_t + \bar{x}_{t,o}(\alpha - \tau_4\dot{\alpha})] / \tau_3 \quad (16)$$

$$\dot{\bar{x}}_v = [-\bar{x}_v + \bar{x}_{v,o}(\beta - \tau_6\dot{\beta})] / \tau_5 \quad (17)$$

where u, v, w refer to the body axes velocities, p, q, r are the roll, pitch and yaw rates and ϕ, θ, ψ describe the angles of roll, pitch and yaw. C_x, C_y, C_z are the force coefficients in body axes. I_{xx}, I_{yy}, I_{zz} and I_{xz} represent the mass moments and cross product of inertia about the body axes respectively. F_x, F_y and F_z are the total applied forces that can expressed as follows:

$$F_x = \bar{q}SC_x(\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \delta_e) - mg \sin\theta + T \quad (18)$$

$$F_y = \bar{q}SC_y(\alpha, \dot{\alpha}, \beta, \dot{\beta}, p, r, \delta_r) + mg \sin\phi \cos\theta \quad (19)$$

$$F_z = \bar{q}SC_z(\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \delta_e) + mg \cos\phi \cos\theta \quad (20)$$

The described 6-DOF flight dynamics model with the mandatory inclusion of the internal separation variables is subsequently passed onto a dynamic optimization procedure via dynopt, that utilizes pseudospectral optimal control methods to transcribe the ODE problem to a nonlinear programming problem (NLP) that can be conveniently solved [17]. The sideslip perching maneuver is structured as a trajectory optimization problem between two fixed boundary points: the initial point at the coordinate (-30,3) and the target endpoint at (0,0) origin. The two boundary points are thus described as constraints that are explicitly specified in the optimization procedure. The initial point is characterized as a trimmed steady state at a velocity of 10 m/s and the endpoint is described as velocity- reduced perched state (at least 70% of velocity reduction). A detailed formulation of the optimization model has been described in Ref. [8]. The established NLP problem was solved using fmincon based on interior point algorithm to minimize the total perching distance required to generate an optimal sideslip perching trajectory, as depicted in Fig. 2, and its corresponding open-loop control inputs. This sideslip perching trajectory is used as the reference trajectory for the following sections that include the contraction analysis and control implementation.

3. Contraction Analysis

The usual technique for assessing the stability of the optimal trajectory solutions at each point along a flight trajectory is no longer feasible for the perching maneuver. This is because perching is basically an unsteady maneuver, and most of the stability concepts only apply to the dynamics around a steady equilibrium condition. For this reason, the stability of the optimal trajectory solutions can be investigated using ‘contraction theory’ [13]. In the context of stability to a nominal motion, stability can be perceived as a system behavior where effects of initial conditions or perturbations are removed gradually and convergence to nominal state trajectories can

ultimately be achieved without prior knowledge of the optimal solutions itself. Based on this definition, contraction theory suggests that stability can be analyzed differentially. Hence, stability about non-equilibrium points along a solution trajectory can be determined through an analysis of the virtual displacement $\delta\bar{X}$ behavior over time, at any given fixed time in the system. The use of such differential forms describes the behavior of neighboring trajectories and assesses whether convergence to a single trajectory can be achieved. In the case of perturbations, positive conclusions of the contraction theory suggest that exponential convergence to the nominal solution trajectory is guaranteed.

Rao and Go have conducted the above mentioned contraction analysis on the 2D perching maneuver and concluded that the 2D perching's optimal solutions are largely unstable, especially in the region of high angles of attack [10]. State perturbation simulation studies have shown that the 3D perching's open loop trajectories can deviate easily given the slightest perturbations, which suggest the instability of the optimal solutions. This research however implements the contraction analysis as a tool to identify the state variables that make the system not convergent in the vicinity of neighboring trajectories. A control strategy can therefore be developed around the tracking of these variables. The use of contraction analysis for tracking state variable identification is presented in the following.

Given a nonlinear system

$$\dot{\bar{X}} = \bar{F}(\bar{X}, t) \quad (21)$$

$\bar{X}$ and $\bar{F}$ are $n \times 1$ vectors in which the latter is in the form of nonlinear functions. With the assumption that $\bar{F}(\bar{X}, t)$ is continuously differentiable, at a fixed time, t , the exact differential can be expressed as:

$$\delta\dot{\bar{X}} = \frac{\partial \bar{F}}{\partial \bar{X}}(\bar{X}, t) \delta\bar{X} \quad (22)$$

By considering a virtual displacement $\delta\bar{X}$ of infinitesimal magnitude that is manifested between 2 neighboring trajectories, the rate of change of its square distance is described by:

$$\frac{d}{dt}(\delta\bar{X}^T \delta\bar{X}) = 2\delta\bar{X}^T \frac{\partial \bar{F}}{\partial \bar{X}} \delta\bar{X} \quad (23)$$

By introducing λ_{max} as the largest eigenvalue of the symmetric part of Jacobian $\frac{\partial \bar{F}}{\partial \bar{X}}$, Eq. (23) becomes:

$$\frac{d}{dt}(\delta\bar{X}^T \delta\bar{X}) \leq 2\lambda_{max} \delta\bar{X}^T \delta\bar{X} \quad (24)$$

The computation of λ_{max} can be carried out by identifying the largest eigenvalue of the matrix $\frac{1}{2} \left(\frac{\partial \bar{F}}{\partial \bar{X}} + \frac{\partial \bar{F}^T}{\partial \bar{X}} \right)$. Therefore, if $\lambda_{max}(\bar{X}, t)$ is uniformly strictly negative, exponential convergence of $\|\delta\bar{X}\|$ to zero is guaranteed which also implies that stability of the nominal trajectory can then be concluded. Conversely, by examining the eigenvalues of the symmetric part of the Jacobian matrix along the trajectory of the perching maneuver, non-contracting variables that lead to the system having a positive eigenvalue profile can subsequently be identified.

The flight dynamics model of the aircraft undergoing the 3D perching maneuver is dependent solely on the dynamic state variables, $[u \ v \ w \ p \ q \ r \ \phi \ \theta \ \psi]$ in which the kinematic variables are excluded as they are more catered to the determination of the aircraft's spatial representation. The dynamic state variable vector consists of translational dynamic variables $[u \ v \ w]$ and rotational dynamic variables $[p \ q \ r \ \phi \ \theta \ \psi]$. Hence, this motivates the following contraction analysis that is based on aircraft's translational and rotational dynamics.

A. Translational Dynamics

In accordance with Eq. (22), the equations for translational dynamics can be expressed as follows:

$$\delta\dot{\bar{X}}_{tr} = \frac{\partial \bar{F}_{tr}}{\partial \bar{X}_{tr}}(\bar{X}_{tr}, t) \delta\bar{X}_{tr} \quad (25)$$

$$\begin{bmatrix} \delta\dot{u} \\ \delta\dot{v} \\ \delta\dot{w} \end{bmatrix} = \begin{bmatrix} \frac{\partial \delta\dot{u}}{\partial \delta u} & \frac{\partial \delta\dot{u}}{\partial \delta v} & \frac{\partial \delta\dot{u}}{\partial \delta w} \\ \frac{\partial \delta\dot{v}}{\partial \delta u} & \frac{\partial \delta\dot{v}}{\partial \delta v} & \frac{\partial \delta\dot{v}}{\partial \delta w} \\ \frac{\partial \delta\dot{w}}{\partial \delta u} & \frac{\partial \delta\dot{w}}{\partial \delta v} & \frac{\partial \delta\dot{w}}{\partial \delta w} \end{bmatrix} \begin{bmatrix} \delta u \\ \delta v \\ \delta w \end{bmatrix} \quad (26)$$

$$\begin{bmatrix} \delta \dot{p} \\ \delta \dot{q} \\ \delta \dot{r} \\ \delta \dot{\phi} \\ \delta \dot{\theta} \\ \delta \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{\partial \delta \dot{p}}{\partial \delta p} & \dots & \frac{\partial \delta \dot{p}}{\partial \delta \psi} \\ \frac{\partial \delta \dot{q}}{\partial \delta p} & \dots & \frac{\partial \delta \dot{q}}{\partial \delta \psi} \\ \frac{\partial \delta \dot{r}}{\partial \delta p} & \dots & \frac{\partial \delta \dot{r}}{\partial \delta \psi} \\ \frac{\partial \delta \dot{\phi}}{\partial \delta p} & \dots & \frac{\partial \delta \dot{\phi}}{\partial \delta \psi} \\ \frac{\partial \delta \dot{\theta}}{\partial \delta p} & \dots & \frac{\partial \delta \dot{\theta}}{\partial \delta \psi} \\ \frac{\partial \delta \dot{\psi}}{\partial \delta p} & \dots & \frac{\partial \delta \dot{\psi}}{\partial \delta \psi} \end{bmatrix} \begin{bmatrix} \delta p \\ \delta q \\ \delta r \\ \delta \phi \\ \delta \theta \\ \delta \psi \end{bmatrix} \quad (27)$$

$\bar{X}_{tr} = [u \ v \ w]$ and $\bar{F}_{tr}$ represents the vector field from the set of Eqs. (3) to (5). Through numerical computation of the eigenvalues λ_{tr} of the symmetric part of the Jacobian $\frac{\partial \bar{F}_{tr}}{\partial \bar{X}_{tr}}$ for the entire optimized sideslip perching maneuver described in Fig. 2, a plot of the eigenvalue trajectories against the axial horizontal distance can be obtained, as illustrated below.

Fig. 4. Eigenvalue λ_{tr} history of translational dynamics of sideslip perching maneuver

Based on Fig. 4, the three eigenvalue trajectories associated with the translational dynamics of the sideslip maneuver are observed to be negative. This suggests that the translational dynamics are naturally contracting and stable. Exponential convergence of $\delta \bar{X}_{tr}$ to zero is assured. Convergence of translational variable trajectories under state perturbations is also implied. However, as the perching maneuver ensues, this contraction behavior seems to diminish due to the decreasing velocity of the aircraft as a slower stable response is evident with decreasing velocity. The contracting behavior reduces to a point where one of the eigenvalues becomes slightly positive. This indicates non-contracting behavior in translational dynamics near the end of the maneuver. With the exception of the terminal stage of the sideslip perching maneuver where instability of the trajectory is expected due to sluggish aircraft response when body velocities are low, translational dynamics of the sideslip perching maneuver can be concluded as largely contracting. Therefore, direct control for stabilization of perching's translational dynamics may not be necessary. Tracking of the translational state variables is not required.

B. Rotational Dynamics

With the open-loop translational dynamics being largely contracting, coupled with the prior knowledge of the open-loop instability of the perching maneuver, contraction analysis of rotational dynamics can be expected to uncover the non-contracting nature of the rotational dynamics. Similar to the previous section, with reference to Eq. (22), the state-space representation for the perching's rotational dynamics can be described as in Eqs. (27) and (28):

$$\delta \dot{\bar{X}}_{rot} = \frac{\partial \bar{F}_{rot}}{\partial \bar{X}_{rot}}(\bar{X}_{rot}, t) \delta \bar{X}_{rot} \quad (28)$$

In Eq. (27), $\bar{X}_{rot}$ refers to the rotational state variables $[p \ q \ r \ \phi \ \theta \ \psi]$ and $\frac{\partial \bar{F}_{rot}}{\partial \bar{X}_{rot}}$ represents the Jacobian matrix in Eq. (28), based on the set of nonlinear Eqs. (6) to (11). Based on rotational Jacobian matrix of the open-loop sideslip perching trajectory, the rotational eigenvalues λ_{rot} are numerically computed and plotted in the Fig. 5 below.

Fig. 5. Eigenvalue history λ_{rot} of rotational dynamics of sideslip perching maneuver

As observed, three of the 6 rotational eigenvalue trajectories initialize with a positive value and remain positive for nearly the entire maneuver. The Jacobian $\frac{\partial \bar{F}_{rot}}{\partial \bar{x}_{rot}}$ is concluded to be not strictly negative-definite. As such, the aircraft's rotational dynamics are non-contracting and the natural convergence of the rotational-variable trajectories under state perturbations is not assured. Its diverging behavior seems to gradually heighten throughout the maneuver. Since the rotational dynamics are not naturally contracting, the stabilization of perching's rotational variable trajectories is therefore required. This implies tracking of certain rotational variables is necessary for the overall convergence of the aircraft's state trajectories under external disturbances.

Using hierarchical nature in the turn rate and Euler angle dynamics, the rotational dynamic system in Eq. (27) can be partitioned and written as:

$$\frac{d}{dt} \begin{pmatrix} \delta \bar{Z}_1 \\ \delta \bar{Z}_2 \end{pmatrix} = \begin{pmatrix} \bar{F}_{11} & 0 \\ \bar{F}_{21} & \bar{F}_{22} \end{pmatrix} \begin{pmatrix} \delta \bar{Z}_1 \\ \delta \bar{Z}_2 \end{pmatrix} \quad (29)$$

$\delta \bar{Z}_1 = [p \ q \ r]$ and $\delta \bar{Z}_2 = [\phi \ \theta \ \psi]$. Submatrices $\bar{F}_{11}$, $\bar{F}_{21}$ and $\bar{F}_{22}$ are nonlinear vector partial derivatives linking the variables to their corresponding differential velocity vectors. As observed, the hierarchical nature in the rotational dynamics suggests that the first equation in $\delta \bar{Z}_1$ does not depend on the second equation in $\delta \bar{Z}_2$. For the first equation, the eigenvalues of the $\bar{F}_{11}$ matrix are examined along the entire perching maneuver. Fig. 6 shows the eigenvalue profiles that correspond to the symmetric part of $\bar{F}_{11}$ defined in Eq. (30).

$$\bar{F}_{11} = \begin{bmatrix} \frac{\partial \delta \dot{p}}{\partial \delta p} & \frac{\partial \delta \dot{p}}{\partial \delta q} & \frac{\partial \delta \dot{p}}{\partial \delta r} \\ \frac{\partial \delta \dot{q}}{\partial \delta p} & \frac{\partial \delta \dot{q}}{\partial \delta q} & \frac{\partial \delta \dot{q}}{\partial \delta r} \\ \frac{\partial \delta \dot{r}}{\partial \delta p} & \frac{\partial \delta \dot{r}}{\partial \delta q} & \frac{\partial \delta \dot{r}}{\partial \delta r} \end{bmatrix} \quad (30)$$

Fig. 6. Eigenvalue history of $\bar{Z}_1$ rotational dynamics

As illustrated in Fig. 6, exponential convergence of $\delta \bar{Z}_1$ to zero is achieved as $\lambda_{max}(\bar{Z}_1, t)$ is uniformly strictly negative for the entire sideslip perching maneuver. Natural contraction in the p, q and r variable trajectories can

be concluded. For the second equation, since the submatrix $\bar{F}_{21}$ is bounded, reduction of $\bar{F}_{21}\delta\bar{Z}_1$ to zero is also guaranteed due to the exponential convergence of $\delta\bar{Z}_1$. Hence, it can be observed that uniform negative definiteness of $\bar{F}_{22}$, that guarantees the $\delta\bar{Z}_1$ convergence to zero, is required to ensure the contraction in the overall rotational dynamics. However, as $\bar{F}_{22}$ is a positive-definite matrix (as inferred in Fig. 5), $\delta\bar{Z}_2$ convergence is not achieved. Therefore, this is identified as the source of trajectory divergence in the rotational dynamics of the perching maneuver. In order to enforce convergence of $\delta\bar{Z}_2$ to zero, $[\phi \ \theta \ \psi]$ are selected to be the state variables to be tracked by the controller. This methodology successfully identify the appropriate variables to track that will successfully lead to the contraction of rotational state space and subsequently, the overall stability of the sideslip perching trajectory.

However, the availability of 4 control inputs ($\bar{U} \in \mathbb{R}^4$) paves the way for the separation of the pitch angle, θ into angle of attack, α and flight path angle, γ in the flight model. This will lead to a total of 4 state variables $[\alpha, \phi, \psi, \gamma]$ that are required to be tracked to ensure the system's overall convergence. The definition of tracking variables in the aircraft's stability axes motivates the control scheme to be devised using the flight dynamics model based on stability axes. For convenience, the nonlinear equations of motion in Eqs. (31) to (34) are replaced with Eqs. (3) to (5) and Eq. (10) to yield a system of 15 first-order differential equations based on the aircraft's stability axes:

$$\dot{V} = \left(\frac{F_x}{m} - (qw - rv)\right) \cos\alpha \cos\beta + \left(\frac{F_y}{m} - (ru - pw)\right) \sin\beta + \left(\frac{F_z}{m} - (pv - qu)\right) \sin\alpha \cos\beta \quad (31)$$

$$\dot{\alpha} = \frac{\left(\left(\frac{F_z}{m} - (pv - qu)\right) \cos\alpha - \left(\frac{F_y}{m} - (ru - pw)\right) \sin\alpha\right)}{V \cos\beta} \quad (32)$$

$$\dot{\beta} = \left(-\left(\frac{F_x}{m} - (qw - rv)\right) \cos\alpha \sin\beta + \left(\frac{F_y}{m} - (ru - pw)\right) \cos\beta - \left(\frac{F_z}{m} - (pv - qu)\right) \sin\alpha \sin\beta\right) / V \quad (33)$$

$$\dot{\gamma} = -\frac{\left(\left(\frac{F_z}{m} - (pv - qu)\right) \cos\alpha - \left(\frac{F_x}{m} - (qw - rv)\right) \sin\alpha\right)}{V \cos\beta + q \cos\phi - r \sin\phi} \quad (34)$$

The full state vector becomes $\bar{X} = [V, \alpha, \beta, \gamma, p, q, r, \phi, \psi, x_e, y_e, z_e]^T$.

4. Sliding Control Formulation

Sliding control consists of 2 phases: a reaching phase followed by a sliding phase. The sliding control technique is based on the simplification of an n th order system to an equivalent 1st order problem by defining a suitable set of functions, $\bar{s}$. This $\bar{s}$ vector can be termed as a set of sliding surfaces that correspond to the desired dynamics during the sliding phase. A reaching condition is next established to ensure that the sliding surfaces remain attractive to the system trajectories. Conceptually, using the sliding control technique, state trajectories from any initial condition that lie outside their respective sliding surfaces, $\bar{s} = \bar{0}$, would undergo a natural convergence towards the respective time-varying surfaces due to the established reaching condition. This gives the controller its inherent robust characteristics that is very ideal for the perching maneuver. Once the state trajectories reach the corresponding surfaces, it marks the end of reaching phase as the system trajectories stay on the surfaces. The sliding phase then begins with the state trajectories sliding along the surface towards the desired state $\bar{X}_d$ exponentially. Consider the 3D sideslip perching problem that is described by Eq. (35):

$$\dot{\bar{X}} = \bar{F}(\bar{X}, \bar{U}) + \Delta\bar{F}(\bar{X}, \bar{U}) \quad (35)$$

where $\bar{X} = [V, \alpha, \beta, \gamma, p, q, r, \phi, \psi, x_e, y_e, z_e]^T$

$$\bar{U} = [\delta_e, \delta_a, \delta_r, T]^T$$

$\Delta\bar{F}(\bar{X})$ represents the uncertain dynamics in the nonlinear system caused by unmodeled time histories of τ in the internal state variable equations concerning $\bar{x}_w$, $\bar{x}_t$, $\bar{x}_v$ of dynamic stall model. Another requirement in sliding

control is for the system to be affine in control. This is achieved by linearizing the system locally about its instantaneous control input as follows:

$$\bar{F}(\bar{X}, \bar{U} + \Delta\bar{U}) = \bar{F}(\bar{X}, \bar{U}) + \left(\frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial \bar{U}} \right)_{\bar{F}, \bar{U}} \Delta\bar{U} \quad (36)$$

The 4 variables $[\alpha, \phi, \psi, \gamma]$ that are chosen to be tracked would correspond to the following set of 4 sliding functions:

$$\bar{s} = \begin{bmatrix} (\dot{\alpha} - \dot{\alpha}_d) + \lambda_1(\alpha - \alpha_d) \\ (\dot{\phi} - \dot{\phi}_d) + \lambda_2(\phi - \phi_d) \\ (\dot{\psi} - \dot{\psi}_d) + \lambda_3((\psi - \psi_d) - y_s) \\ (\dot{\gamma} - \dot{\gamma}_d) + \lambda_4((\gamma - \gamma_d) - z_s) \end{bmatrix} \quad (37)$$

where $y_s = \tan^{-1} \left(\frac{y_{e_d} - y_e}{\rho_1} \right)$
 $z_s = \tan^{-1} \left(\frac{z_e - z_{e_d}}{\rho_2} \right)$
 $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \rho_1, \rho_2 > 0$

Convergence solely to dynamic state trajectories such as orientation angle and turn rate profiles does not guarantee convergence of spatial trajectory of the perching maneuver in 3D space domain. Therefore, to ensure that perching trajectory in spatial domain is also part of the invariant set, the terms y_s and z_s are correction factors introduced into the sliding functions. Trajectories initializing at the different point can hence be tracked back to desired optimal trajectory. Contraction behavior of both the state and spatial variable profiles to the respective optimal profiles are guaranteed. Next, a practical form of a constant rate reaching law is used for the reaching phase [18]:

$$\dot{\bar{s}} = -\bar{H} \operatorname{sgn}(\bar{s}) \quad (38)$$

where $\bar{H} = \operatorname{diag}[h_1, \dots, h_4]$, $h_i > 0$ (positive definite)

sgn refers to the sign function (applied to the elements of the matrix)

This form of reaching law is chosen due to its simplicity as it ensures that the state trajectories are constrained to approach the sliding surfaces at a constant rate in which the reaching phase would be faster when $\bar{H}$ is large. Without consideration of $\Delta\bar{F}(\bar{X}, \bar{U})$ in the system, by using the reaching law in Eq. (38), contraction to the sliding surface, $\bar{s} = \bar{0}$ is guaranteed and can be shown using the following Lyapunov function candidate:

$$\bar{V}(\bar{s}) = \frac{1}{2} \bar{s}^T \bar{s} \quad (39)$$

$$\begin{aligned} \dot{\bar{V}}(\bar{s}) &= \frac{1}{2} (\dot{\bar{s}}^T \bar{s} + \bar{s}^T \dot{\bar{s}}) \\ &= \bar{s}^T \dot{\bar{s}} \\ &= -\bar{s}^T \bar{H} \operatorname{sgn}(\bar{s}) < 0 \end{aligned} \quad (40)$$

$\bar{V}(\bar{s})$ is positive definite

$\dot{\bar{V}}(\bar{s})$ is negative definite

The expression in Eq. (36) provides convenience for the derivation of the state-space sliding control approach. Based on the reaching law in Eq. (38), the free-order switching control law can be derived as:

$$\dot{\bar{s}} = \frac{\partial \bar{s}}{\partial \bar{X}} \dot{\bar{X}} + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d \quad (41)$$

$$\begin{aligned} &\Rightarrow \frac{\partial \bar{s}}{\partial \bar{X}} \left(\bar{F}(\bar{X}, \bar{U}) + \left(\frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial u} \right)_{x,u} \Delta\bar{U} + \Delta\bar{F}(\bar{X}, \bar{U}) \right) + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d = -\bar{H} \operatorname{sgn}(\bar{s}) \\ \Delta\bar{U} &= - \left(\frac{\partial \bar{s}}{\partial \bar{X}} \frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial u} \right)_{x,u}^{-1} \left(\frac{\partial \bar{s}}{\partial \bar{X}} \bar{F}(\bar{X}, \bar{U}) + \bar{H} \operatorname{sgn}(\bar{s}) + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d + \Delta\hat{\bar{F}}(\bar{X}, \bar{U}) \right) \end{aligned} \quad (42)$$

By using $\Delta\hat{\bar{F}}$ to handle the unknown $\Delta\bar{F}$, the reaching law equation will have an additional term: (This done by substituting Eq. (42) into Eq. (41))

$$\dot{\bar{s}} = -\bar{H} \operatorname{sgn}(\bar{s}) - \left(\Delta\hat{\bar{F}}(\bar{X}, \bar{U}) - \Delta\bar{F}(\bar{X}, \bar{U}) \right) \quad (43)$$

Since the bounds on $|\Delta\bar{F}(\bar{X}, \bar{U})|$ is known:

$$-0.1 F_i \leq \Delta F_i \leq 0.1 F_i \quad (44)$$

where
$$\Delta \bar{F}(\bar{X}, \bar{U}) = \begin{bmatrix} \Delta F_1 \\ \Delta F_2 \\ \Delta F_3 \\ \Delta F_4 \end{bmatrix}$$

$\bar{H}$ must be chosen in way it can always dominate $(\Delta \hat{F}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U}))$ to ensure the reaching condition. Based on Eq. (44), it can be shown that:

$$|\Delta \hat{F}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U})| \leq 0.2 \bar{F}(\bar{X}, \bar{U}) \quad (45)$$

The $\text{sgn}(\bar{s})$ in Eq. (42) would be replaced with $\text{sat}\left(\frac{\bar{s}}{\bar{\varphi}}\right)$ to smoothen out the high frequency chattering in the control inputs. φ represents a thin boundary layer around each sliding surface and its width is crucial for the success of the sliding controller. A boundary layer that is larger than required would lead to a lowered tracking accuracy while a boundary later that is too narrow would not be able to completely eliminate chattering. In this paper, the thickness of the boundary layer is actively adjusted to match the intensity of the dynamic stall uncertainties in the system. With the presence of a time-varying boundary layer and to ensure that the boundary layer is reachable for the trajectory in finite time, the rate of shrinking of the boundary layer should be slower than the rate of approaching the boundary layer. With consideration of $\Delta \bar{F}(\bar{X}, \bar{U})$ in Eq. (43), for the region outside the boundary layer, the derivative of Lyapunov function candidate in Eq. (40) should hence satisfy the following condition in Eq. (46):

$$\dot{V}(\bar{s}) = \bar{s}^T \dot{\bar{s}} \quad (46)$$

$$= -\bar{s}^T \bar{H} \text{sgn}(\bar{s}) - \bar{s}^T (\Delta \hat{F}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U})) \leq |\bar{s}^T|(\text{diag}(\dot{\bar{\varphi}}) - \bar{\eta})$$

$$\therefore \text{Let } h_{ij} = (k_i - \dot{\varphi}_i) \delta_{ij} \Rightarrow \bar{H} = \bar{K} - \dot{\bar{\varphi}} \quad (47)$$

where
$$\bar{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix}, \eta_i > 0$$

$$k_i = 0.2 F_i + \eta_i, k_i > 0$$

$$\delta_{ij} \text{ is the kronecker delta}$$

$$\bar{K} = \text{diag}[k_1, \dots, k_4]$$

$$\bar{\varphi} = \text{diag}[\varphi_1, \dots, \varphi_4]$$

Using the tuning method suggested by Slotine and Li [12] for the variation of the boundary layer:

$$\dot{\bar{\varphi}} = \bar{K}_{\bar{x}_d} - \bar{\lambda} \bar{\varphi} \quad (48)$$

The thickness of the boundary layer is allowed to vary based on the dynamics of the unmodeled uncertainty. Substituting Eq. (48) into Eq. (47), $\bar{H}$ can subsequently be expressed as:

$$\bar{H} = \bar{K} - \bar{K}_{\bar{x}_d} - \bar{\lambda} \bar{\varphi} \quad (49)$$

where $\bar{\lambda} = \text{diag}[\lambda_1, \dots, \lambda_4], \lambda_i > 0$

Putting Eq. (49) into Eq. (42), a modified control law in Eq. (50) is obtained that incorporates the variation of the boundary layer and elimination of chattering in control inputs. The control input required to rectify the perturbations can then be conveniently computed using Eq. (51).

$$\Delta \bar{U} = - \left(\frac{\partial \bar{s}}{\partial \bar{X}} \frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial \bar{U}} \right)_{x,u}^{-1} \left(\frac{\partial \bar{s}}{\partial \bar{X}} \bar{F}(\bar{X}, \bar{U}) + (\bar{K} - \bar{K}_{\bar{x}_d} - \bar{\lambda} \bar{\varphi}) \text{sat}\left(\frac{\bar{s}}{\bar{\varphi}}\right) + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d \right) \quad (50)$$

$$\bar{U}_{required} = \bar{U} + \Delta \bar{U} \quad (51)$$

Throughout the maneuver, the controller bounds in Table 1 are imposed to preserve the practicality of the maneuver, whose values have also been validated with current actuator specifications in the market.

Table 1
Sliding control bounds

Lower bound	Control variable	Upper bound	Max rate
-40 deg	δ_e, δ_r	40 deg	70 deg/s
-15 deg	δ_a	15 deg	70 deg/s

0	T/W	0.8	1.5 N/s
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5. Sliding Control Performance Analysis

The controller is tasked to track the optimal sideslip perching maneuver (Fig. 2), which has been initiated from an altitude of 3 m, horizontal distance of 30 m away from the origin and a speed of 10 m/s in a trimmed cruise condition. The performance of the controller is evaluated under different scenarios: varying initial spatial and state deviations and full state perturbations during the course of the maneuver. All the constants in the $\bar{K}$ and $\bar{\lambda}$ matrices have been initially determined via manual iteration until the desirable control performance is achieved. Control goals would be deemed achieved if the aircraft is able to land within 0.10 m lateral and vertical proximity of the target location (0.3% with respect to the horizontal perching distance of 30 m) and 10% of the reference terminal states of the orientation angles and velocity, which is challenging considering the small time window and the aggressiveness of the perching maneuver. The pitch angle of the aircraft at the landing point is also restricted not to exceed 90 degrees. In the case where a reference state terminally ends at zero, 10% of the maximum value that the corresponding state reaches during the perching maneuver would be selected as the tolerance.

A. Variation in Initial Position

Variations in the initial launch conditions have a significant impact on the success of the perching maneuver [19]. Deviations such as in the initial launch position in yz plane are inevitable in practical applications of perching. Four challenging scenarios are introduced in which the aircraft's initial locations have been significantly displaced to illustrate the effectiveness of the controller. Scenarios A and B depict two cases with vertical spatial displacement of +2 m and -2 m respectively (deviation of about 66.6% of the initial 3m altitude) while scenarios C and D describe two cases of lateral spatial deviations of -2 m and +2 m respectively (deviation of more than 50% of the maximum lateral sideslip displacement).

As shown in Fig. 7, successful convergence of the trajectories of scenarios A and B (vertically displaced) to the reference perching trajectory is achieved in time. The corresponding side view of both the perching trajectories illustrates a seemingly gradual contraction of the longitudinal paths to the respective nominal vertical projection in which their convergence is only accomplished after the -10 m mark. However, it can be highlighted that the contraction of the z variable to the corresponding nominal profile is rather rapid when viewed in the perspective of the available response time of only 4.5 seconds long. Moreover, the contraction of the vertical spatial domain has to be carried out while sustaining the convergence of the lateral xy deviation simultaneously. As such, the rate of convergence cannot be significantly made more rapid without compromising the tracking of the other 10 state variables. Having all the state variables initiated at the reference conditions except for the initial altitude, simulation of scenarios A and B allows for the control and state responses to the initial vertical spatial deviations to be distinctively identified and understood. As observed, the state profiles for α, β, ϕ and ψ variables in Fig. 8 do not depict significant variation from their corresponding nominal profiles. Only the γ profile for both scenarios undergoes significant corrections that are necessary for the overall convergence of the respective perching trajectories. A more positive trend in the γ profile for scenario B is attributed to the upward flight trajectory as the aircraft approaches the nominal trajectory on the underside. This validates the inclusion of the correction factor z_s in Eq. (37) to guarantee convergence of spatial trajectory of the perching maneuver in the vertical domain. The presence of a vertical deviation from the nominal trajectory directly triggers the reaching phase in the 4th sliding function which subsequently induces an immediate response in the thrust control input as shown in Fig. 9. Within the control limits set in Table 1, the significant variation in the resulting thrust input for both scenarios suggests the required modifications of the propelling force to steer the aircraft back to its reference trajectory. The other 3 control histories for the elevator, aileron and rudder only show slight alterations from the respective optimal solutions to sustain the tracking of the remaining variables while accommodating the changes in the γ state trajectory. Due to thrust control adjustments, the velocity profiles for both scenarios differ from the reference velocity profile but merge into a singular unit as the aircraft approaches the final third of the perching maneuver. Therefore, the velocity reduction requirement is successfully achieved along with the terminal convergence of the state variables within 10% of the reference values.

Similar to the observations drawn in scenario A and B, the perching maneuver trajectories for scenario C and D (y -axial displacement of ± 2 m) effectively converge to the optimal trajectory while accomplishing landing at

orientation angles and terminal velocity within the set tolerances. The convergence of the lateral component of the perching trajectory is steadily achieved within the final third of the maneuver. The integration of the correction factor y_s in the 3rd sliding function of Eq. (37) is therefore justified. While smoothly overcoming the enormous initial lateral deviation, the side view of the perching trajectories (Fig. 7) as well as the profiles for α, γ and ϕ angles (Fig. 8) indicate only small differences from the corresponding nominal values. On the contrary, substantial deviations in ψ and β angle profiles can be observed that subsequently reflect their heavy involvement in producing the required lateral spatial convergence. Since the correction factor y_s is incorporated in the 3rd sliding function Eq. (37), the rudder control input is solely responsible in handling the lateral spatial deviations. Thus, in for scenario C in Fig. 9, a considerable positive deviation of the rudder input from the optimal rudder control solutions can be observed in order to produce more yawing motion in the anticlockwise direction.

In order to quantify the effectiveness of the sliding controller, an iterative test is conducted to trace the outline of the convergence area of the perching trajectory based on the varying initial position in the yz plane. A lower altitude limit of 1 m in the z -direction is specified during the test to preserve the practicality of the maneuver as the aircraft will be too close to the ground if being initiated below the 1 m mark. The shaded region includes the perching initiation coordinates that produce perching trajectories that terminates favorably based on success criteria. As illustrated in Fig. 10, the convergence area is rather vast, stretching out to magnitudes of 15 m and 4.5 m in the y and z axes, respectively. The initial points of the 4 scenarios, labelled A, B, C and D previously, fall within the convergence area as anticipated. The convergence area of the perching maneuver is apparently asymmetric as the influence of an initial displacement about the starting point due to the nonlinear nature of the perching dynamics is not equal. Although the convergence area presents an impressive amount of coverage, it is more meaningful and important to specify the convergence region in a circular or elliptical form about the starting point (0,3). This provides a more relevant measure of the convergence area in the aspect of operation and practical application of the perching maneuver. The operational area, expressed as $\frac{x^2}{4.4^2} + \frac{(y-3)^2}{2^2} \leq 1$, is shown in Fig. 10.

Fig. 7. Tracking of sideslip perching trajectories based on varying initial spatial displacements

Fig. 8. Tracking of velocity, aerodynamic angle and orientation angle profiles based on varying initial spatial displacements

Fig. 9. Control input profiles for varying initial spatial displacements

Fig. 10. Convergence area (shaded) based on varying initial starting location

B. Variation in Initial State and Spatial Displacements

The performance of the sliding controller is further evaluated in 2 scenarios that incorporate random initial displacements of state variables that include $V, \alpha, \beta, \gamma, p, q, r, \phi, \psi$ into an already spatially displaced flight condition.

Scenario 1: $\Delta V = 1$, $\Delta \alpha = 4^\circ$, $\Delta \beta = -3^\circ$, $\Delta \gamma = 3^\circ$, $\Delta p = 0.03$, $\Delta q = -0.05$, $\Delta r = 0.03$, $\Delta \phi = -2^\circ$, $\Delta \psi = 2^\circ$, $\Delta y = -3$, $\Delta z = 1$

Scenario 2: $\Delta V = -1$, $\Delta \alpha = -3^\circ$, $\Delta \beta = -2^\circ$, $\Delta \gamma = 1^\circ$, $\Delta p = -0.05$, $\Delta q = 0.04$, $\Delta r = -0.03$, $\Delta \phi = 3^\circ$, $\Delta \psi = 2^\circ$, $\Delta y = +2$, $\Delta z = -1$

The velocity, angular and turn rate perturbations in both scenarios 1 and 2 are regarded as significant with respect to the resultant impact of the perturbed forces and moments on the sideslip perching trajectory. Their magnitudes measure at least within 10% of their respective nominal values or operational range. For instance, Scenario 1 consists of a velocity perturbation of +1m/s which represents a significant 10% of its reference initial value. Also, the orientation angles such as roll and heading angles are perturbed at about 5% of their corresponding maximum operating angles. In addition, a perturbation of +4 deg and -3 deg to the angle of attack and sideslip angle, respectively, are massive since they represent aerodynamic angle displacements of more than 10% of their

pre-stall angle ranges. As such, in the aspect initial spatial and aerodynamic angle deviations, scenario 1 serves as a slightly severe case in comparison to scenario 2. As observed in Fig. 11, the tracking performance of the controller is excellent as convergence of the perturbed sideslip perching trajectories for both scenarios is effectively achieved. The final landing point for both scenarios managed to fall within the 0.1 m distance in the vertical (Δz) and lateral (Δy) directions. Success of the controller can also be seen in Fig. 12 and 13, where the convergence of the desired state values are attained within 10% accuracy. The independent application of both y_s and z_s as correction terms in their corresponding sliding functions of Eq. (37) successfully allows simultaneous contraction of both the lateral and vertical spatial deviations.

In Fig. 11, the smooth contraction of spatial $[x_e, y_e, z_e]$ trajectories to the respective reference spatial profiles in the 3D space domain is seen to be relatively gradual, similar to the results obtained in the previous section. As elaborated, this behavior is expected as the tracking of spatial coordinates is carried out in the post-stall sideslip region, which is then followed by the post-stall angle of attack region. This causes the tracking response to be sluggish than when applied in the linear aerodynamic region due to the lowered sensitivity of the forces due to the change of aerodynamic angle. Convergence to the nominal y -coordinate trajectory is acquired around the second-third of the maneuver and is more rapid as compared to the convergence of the z -coordinate. This can be explained by examining the response of Scenario 1, in which the maneuver is initiated at a higher vertical altitude. This, coupled with the positive velocity perturbation, significantly adds the amount of potential and kinetic energy that is required to be dissipated by the maneuver. Such dissipation could hardly be carried out instantaneously while undergoing a pitching up maneuver and a translational lateral motion simultaneously.

Direct control of the selected 4 state variables $[\alpha, \phi, \psi, \gamma]$ through the independent use of the aircraft's control actuators $[\delta_e, \delta_a, \delta_r, T]$ has also successfully led to the convergence of the rest of system state variables, which include velocity, sideslip angle and turn rates (see Fig. 12 and 13). This observation verifies the use of contraction theory methodology to select the required non-contracting tracking variables to guarantee stability of the perching maneuver in its entirety. Contraction of translational dynamics in the perching maneuver can be concluded as the perturbed velocity profiles of the two scenarios converge to the reference velocity profile. This ensures that the required terminal velocity reduction is still being achieved. The convergence of turn rate variables, heading angle and sideslip angle is relatively accomplished only at about -15 m, which is about halfway in the maneuver. While two of the controlled state variable profiles namely, angle of attack α and roll angle ϕ are observed to converge rapidly within the first one-third of the maneuver, the flight path γ and heading angle ψ trajectories converge at a very much slower rate. This is because of the presence of correction factors y_s and z_s in sliding functions for γ and ψ that temporarily compromise the contractions of γ and ψ profiles for the corrections of the initial y_e and z_e spatial deviations. This means that the overall convergence of the ψ and y_e trajectories as well as the γ and z_e trajectories to their respective optimal solutions can only be done concurrently and gradually.

Fig. 14 depicts no control-input chattering occurrence during the tracking of the reference perching trajectory, due to the time-varying boundary layer $\bar{\varphi}$. For the first half of the maneuver, computed control profiles for scenario 1 and 2, especially the thrust control input, seem to greatly diverge from the optimal control inputs in order to implement the necessary perturbation corrections. However, beyond the halfway mark, it is interesting to note that the control input profiles for δ_e , δ_a and δ_r have reasonably converged to their corresponding reference inputs, signifying the convergence of the corresponding tracking variables at the same time. This is not entirely the case for the thrust control profiles. Major differences in the thrust profile for both scenarios 1 and 2 are evident as corrections for the contraction of the γ and z_e trajectories are still required. Overall, the analysis has clearly illustrated the feasibility of the sliding control technique to deal with the varying initial conditions.

Fig. 11. Sideslip perching trajectories with varying initial spatial and state displacements

Fig. 12. Velocity, aerodynamic angle and orientation angle profiles for Scenario 1 and 2

Fig. 13. Turn rate profiles for Scenario 1 and 2

Fig. 14. Control inputs for tracking in Scenario 1 and 2

C. Perturbation during the Course of the Maneuver

As the sideslip perching maneuver is a rapid and highly agile maneuver, robustness to aerodynamic perturbations that include V , α and β variables is crucial. External disturbances such as wind gusts in an outdoor environment can be conveniently associated as aerodynamic perturbations that are unpredictable in nature. In this analysis, these aerodynamic disturbances are modeled as pulse perturbations introduced during the maneuver. Respective initial displacements for V , α and β variables from Scenario 1 and 2 are utilized:

At -30m mark,

Scenario 1: $\Delta V = 1$, $\Delta\alpha = 4^\circ$, $\Delta\beta = -3^\circ$, $\Delta y = -3$, $\Delta z = +1$

Scenario 2: $\Delta V = -1$, $\Delta\alpha = -3^\circ$, $\Delta\beta = -2^\circ$, $\Delta y = +2$, $\Delta z = -1$

Subsequently, perturbations along the course of a perching trajectory, or *in-maneuver* perturbations are randomly introduced at three different locations $x_e = -20\text{ m}$, -15 m and -10 m , as shown in Table 2. These three locations are chosen as they are centered about the critical mid-point region of the maneuver where the aircraft navigates itself vigorously around the bend. Aerodynamic perturbations are introduced at three locations, instead of a single location at -15 m mark, as it is more representative of the outdoor perching maneuver in which it is subjected to unpredictable aerodynamic perturbations. Also, the perturbations are not introduced at the -5 m mark, taking into account that since the aircraft is less than a second away from the target location, the window for control action is too small for any significant control impact. The velocity perturbations are randomly specified for at least 10% of their nominal terminal values. The angular perturbations are implemented in at least 10% of their aerodynamic stall angles.

Table 2

Aerodynamic perturbations

Perturbations	-20m	-15m	-10m
ΔV	-0.4 m/s	$+0.3\text{ m/s}$	-0.3 m/s
$\Delta\alpha$	-2 deg	$+3\text{ deg}$	$+2\text{ deg}$
$\Delta\beta$	-3 deg	$+2\text{ deg}$	$+2\text{ deg}$

The control responses including the resulting perching trajectories and V , α and β state histories are plotted in the following Fig. 15 and 16. Subjected to the bounds in Table 1, the control strategy proves to be sufficient in overcoming those aerodynamic perturbations. Fig. 15 demonstrates two cases of successful tracking of the optimal perching trajectory, bearing close resemblance to the trajectory convergence plot in Fig. 11. The perturbations have graphically been marked on the plots in Fig. 16. It should be noted that the convergence of the aerodynamic angle profiles is not a priority as their corresponding variations might be necessary to produce the required forces to navigate the aircraft towards the reference path. As such, the aerodynamic angle's convergence criteria have not been included in the perching success definition. In spite of this, as observed, the perturbations have quickly been subdued as reasonable convergence in α and β trajectories to their respective optimal solutions are still being obtained. The control input plots depict more control activity as compared to the control plots in Fig. 9 and 14 where these respective scenarios do not include any perturbations during the maneuver.

Fig. 15. Trajectory convergence in response to aerodynamic perturbations

Fig. 16. Aerodynamic variable and control input profiles

Fig. 17. Convergence area (shaded) given aerodynamic perturbations at $x_e = -30, -20, -15, -10 \text{ m}$

Based on the initial aerodynamic conditions in scenario 1, the highly asymmetric outline of the convergence area of the perching maneuver, subjected to the aerodynamic perturbations prescribed in Table 2, is plotted in Fig. 17. The controller has successfully shown its capability in providing trajectory stability with a sufficient operational area of 1 m radial margin. It is important to highlight here that the convergence area is seen to have

reduced by a factor of 3.5 when compared to the convergence area in Fig. 10 in which the aerodynamic perturbations are not included. The large reduction of the convergence area describes the significant impact of in-maneuver perturbations on the overall perching's convergence area. The controller's large convergence area for the case of initial perturbations only validates its effectiveness in responding to these deviations as the controller is able to exploit the linearity in the aerodynamics of the pre-stall regime. Furthermore, as the associated state and spatial displacements are encountered only at the start of the maneuver, it gives the controller ample time to steadily rectify the discrepancies. However, subjected to large in-maneuver perturbations, the time-span for the controller to respond decreases greatly that leads to the reduction in convergence area.

6. Conclusions

This paper describes the sliding control strategy adopted to address the stability of the optimal perching maneuver solutions when subjected to spatial and state perturbations. Contraction analysis is carried out along a reference sideslip perching maneuver and has identified that the state variables associated with the translational dynamics of the sideslip perching maneuver are naturally contracting and stable. However, its rotational dynamics are shown to be not contracting. Consequently, based on this methodology, the roll angle ϕ , angle of attack α , flight path angle γ and the heading angle ψ are identified to be the state variables that make the system not convergent in the presence of perturbations. By controlling these variables, the proposed sliding controller successfully demonstrated in the perching simulations that the contraction of rotational state space and the overall stability of the sideslip perching trajectory has been achieved. Favorable results indicate the sliding control's excellent tracking capability under varying initial spatial and state deviations as it achieves a large elliptical operational area where convergence of deviated trajectories are guaranteed. When subjected to large in-maneuver perturbations, the controller is still able to provide convergence to the optimal maneuver but with a reduced operational area.

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