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Distributed Optimal Tie-Line Power Flow Control for Multiple Interconnected AC Microgrids

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Abstract—In a multi-microgrid system (MMG), the microgrids (MGs) are normally managed by independent operators. Distributed energy trading/scheduling schemes via interactions of these MG operators have been extensively investigated. How to coordinate these MGs to implement the acquired optimal schedule in the real time under constant load fluctuation while guaranteeing operational stability is seldom reported. Due to the intrinsic advantages of scalability, robustness, and fast response in comparison to the centralized scheme, a multi-agent based distributed optimal tie-line power flow control strategy is proposed to achieve this objective, which is facilitated by a regional communication network overlapping each MG and distributed sensors monitoring the tie-line power flows. When the MMG is operated in grid-connected mode, the proposed scheme can maintain the scheduled tie-line power flows among the MGs in the presence of any disturbance by adjusting the real-time power outputs of the distributed energy resources (DERs) proportionally. When the MMG is islanded, frequency recovery can be further achieved via a local frequency feedback mechanism. Convergence of the proposed approach is analytically proved. Simulation results in a 4-MG system modified from the IEEE 34-bus test feeder system validate the effectiveness and efficiency of the proposed approach in both grid-connected and islanded modes.

Index Terms—Consensus, distributed control, frequency regulation, multi-microgrid system, optimal tie-line power flow.

NOMENCLATURE

A. Sets and Indices

S_{MG}/N_{MG}	Set/number of the MGs within the MMG
V/N_B	Set/number of the DERs within the MMG
V_i/N_{Bi}	Set/number of the DERs within MG i

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$\mathcal{E}$	Set of distribution lines within the MMG
$\mathcal{E}_i$	Set of communication links within MG i
$\mathcal{G}_i$	Communication network of MG i
$\mathcal{N}_i$	Set of MGs adjacent to MG i
ij	Index of DER j in MG i
$(ij, \tilde{ij}) \in \mathcal{E}$	Distribution line within MG i
$(ij, \tilde{ij}) \in \mathcal{E}$	Distribution line connecting MGs i and $\tilde{i}$

B. Parameters

$a_{Gij}, b_{Gij}, c_{Gij}$	Cost parameters of DER ij
$P_{Gij}^{\min}/P_{Gij}^{\max}$	Minimum/maximum power of DER ij
A_i/w_{jk}^i	Adjacency matrix of MG i and its (j, k) th entry
β_{Gij}	Participation factor of DER ij
M_{ij}	Rotor inertia of DER ij
$g_{ij, \tilde{ij}}$	Conductance of line $(ij, \tilde{ij})$
$b_{ij, \tilde{ij}}$	Susceptance of line $(ij, \tilde{ij})$
α	Feedback control gain
k_{Pij}/k_{Iij}	Droop/integral gain of DER ij

C. Control and State Variables

θ_{ij}	Phase angle of bus ij
ω_{ij}/ω_0	Angular frequency of bus ij and its nominal value
P_{Gij}	Power output of DER ij
P_{Lij}	Load consumption of bus ij
P_{eij}	Net power injection at bus ij
$P_{ij, \tilde{ij}}$	Power flow from buses ij to $\tilde{ij}$
E_{ij}	Voltage magnitude of bus ij
λ_{ij}	Incremental cost of DER ij
ϵ_i^*	Real-time load forecast error within MG i
ϵ_{ij}	Estimation of ϵ_i^* at DER ij

I. INTRODUCTION

AS an effective way to locally integrate various distributed energy resources (DERs), microgrids (MGs) are rapidly expanding recently, e.g., the generation capacity within MGs is expected to exceed 1.8 GW by 2018 in the U.S. [1]. However, when a MG, especially one with a high penetration of renewable energy operates in islanded mode, it is susceptible to lose stability in the face of severe load fluctuation. Interconnecting multiple MGs to form a multi-MG system (MMG) has been recognized as a solution to enhance stability, reliability, and supply security under disturbances [2].

A MMG is normally operated in a hierarchical way. Over a long timescale, the energy scheduling which aims to optimize the utility of each MG via energy trading should

be achieved based on forecast of renewable energy and load consumption. Extensive work has been done for energy scheduling of MMGs. For instance, the authors of [3] proposed a hierarchical bi-level operation scheme for the interactive energy management of the MMG. In [4], a dual-decomposition based energy scheduling algorithm was designed. It guarantees fair energy trading among the MGs while minimizing their aggregate operational cost. The authors of [5] investigated the optimal operation of MMGs via cooperative energy and reserve scheduling. In [6], a distributed energy management and trading scheme of the MMG was investigated where an economic interpretation is provided to demonstrate that the trading is fair to each individual MG. The above works all aim to minimize the operational cost of the MMG by optimally scheduling its operational point within each energy trading period. How to implement the acquired nominal operational point in the real time under constant disturbance (e.g., load fluctuation) is not considered.

Over a short timescale, power sharing control adjusts the actual power of the DERs to maintain a supply-demand balance in the presence of load fluctuation while complying with the energy schedule. If the MMG operates in islanded mode, a frequency regulation is further required in the transient after disturbance to maintain system stability. Existing works focus more on the power sharing control of independent MGs [7]. For instance, the authors of [8] proposed a distributed optimal resource management scheme for the islanded MG, which can minimize the aggregate generation cost while maintaining the nominal system frequency. On this basis, the authors of [9] further took the power flow constraints into consideration.

Recently, the authors of [10] initiated the research on distributed coordination control for MMGs. The control scheme requires distributed communication among the MG agents. Each MG agent further coordinates its respective DERs in a centralized manner. However, the control objective of reducing energy exchanges among the MGs is not economically efficient for the MMG operation. In a MMG, it is suggested that energy trading can further reduce the operational cost of the MGs [4], [6]. As such, the tie-line power flows should be maintained constant within each dispatch period to comply with the energy schedule. This indicates that the MMG control objective in islanded mode should be similar to that of automatic generation control (AGC) in the transmission network. A few papers have investigated the distributed version of AGC [7]. For example, the authors of [11] combined the functions of economic dispatch and AGC and proposed a completely decentralized solution without the need for communication. The proposed approach simplifies each control area as a single node and requires to measure the derivative of frequency deviations, which can be sensitive to the measurement noise. In [12], the similar control objective was achieved by matching the system dynamics with the primal-dual gradient algorithm. This approach considers details within each control area, but requires distributed communication between any pairs of physically connected nodes. Thus it is not robust to communication interruptions. Moreover, it is undesirable that the uncontrollable load change should be measured within each control area in the real time.

In this paper, the real-time power sharing control is to be investigated for the MMG where each MG consists of multiple DERs and certain amount of load. The DERs may include heterogeneous dispatchable distributed generators and energy storage systems. With the aid of the up-to-date smart grid technologies such as P2P communication and distributed sensors monitoring the tie-line power flows, the MMG is transformed into a cyber-physical system (CPS) where each DER is managed by a distributed control agent (CA). The CAs within each MG form a regional communication network where each CA can exchange information with its neighboring CAs in the real time. Based on this cyber-physical architecture, a novel distributed optimal tie-line power flow (DOTLPPF) control is formulated. The control objective is for the MMG to distributively implement the results of robust energy scheduling, e.g., [6], in the real time. The rule is for each MG to assimilate its own load/renewable energy fluctuation by dynamically adjusting the power of DERs in proportion to their participation factors so as to maintain the scheduled tie-line power flows among the MGs and to prevent disturbance propagation which might lead to a system-wide blackout. Compared to the relevant works on distributed real-time power sharing controls of MGs, the novelties of the DOTLPPF control are as follows:

- 1) *Adaptive mode switch*: The DOTLPPF scheme can adaptively switch between islanded mode and grid-connected mode depending on whether the MMG is connected to the upstream transmission system.
- 2) *Easy implementation*: The DOTLPPF scheme only requires a regional communication network for each MG, and the DERs just need to locally measure the system frequency when the MMG is islanded. Thus, this scheme can be easily implemented with a low cost for control and communication in practice.
- 3) *Robustness and flexibility*: Different from the fixed communication topology required by the primal-dual based distributed control, the regional communication network of the DOTLPPF scheme can be randomly configured, which is plug-and-play for the DERs. Moreover, the topology can be redundantly designed to enhance the robustness to communication interruption.
- 4) *Long-timescale optimality*: Compared to the economic dispatch policy for the real-time power sharing control of MGs, the participation factor policy adopted by the DOTLPPF scheme can simultaneously achieve real-time control and near optimal operation over a long timescale. Thus, it is applicable to inter-temporal optimal operation of the MMG as extensive constraints can be dealt with in the robust energy scheduling process.

The rest of the paper is organized as follows: Section II formulates the problem of optimal tie-line power flow control. To address the problem, Section III introduces the DOTLPPF control approaches in both grid-connected and islanded modes. Simulation results and conclusions are presented in Section IV and V, respectively.

With respect to mathematical notations, any variable x without superscript denotes the real-time value; x^* denotes

Fig. 1. The cyber-physical architecture of the MMG.

its scheduled (optimal) value; Δx denotes the deviation of x from x^* ; $\mathbf{0}/\mathbf{1}$ denotes the all-zero/one vector or matrix of proper sizes; I denotes the identity matrix of proper sizes; and a saturation function is defined as

$$\text{Sat}(x, \min, \max) = \begin{cases} \min & \text{if } x < \min \\ \max & \text{if } x > \max \\ x & \text{otherwise} \end{cases}.$$

II. PROBLEM FORMULATION

In this paper, a CPS refers to a MMG including multiple DERs interconnected by physical distribution networks which are controlled by computer-aided CAs interconnected by communication networks. This architecture, which enjoys the merits of fast computing and control, flexibility, scalability, autonomy, and robustness to single-point-failure/communication interruption [9], [13], is extensively applied for managing complex systems. In this section, the physical layer of the MMG is firstly described. This is then followed by formulation of the main control objectives.

A. Modeling of the MMG

Consider a MMG as an interconnection of N_{MG} MGs, whose set is denoted as $\mathcal{S}_{MG} = \{1, \dots, N_{MG}\}$. These MGs can be further connected to the main grid through a grid-tied switch, as illustrated in Fig. 1. With a slight abuse of notation, the main grid is indexed as MG 0 and is simplified as a single bus 0. When the switch is closed, the MMG operates in grid-connected mode, and thus its system frequency is synchronized to the main grid. When the switch is open, the MMG operates in islanded mode, and thus it should achieve frequency regulation by itself. It is also possible for a certain MG to break away from the MG cluster and operate independently. In this situation, extensive centralized/distributed control schemes have been proposed to maintain its operational optimality and stability, e.g., [8], [9] and the references therein, which are out of the scope of this work.

The MMG can be denoted as an undirected graph $(\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are the sets of buses and distribution lines, respectively. Without loss of generality, it is assumed that each bus is connected with a DER and a local load¹. Any non-DER bus can be eliminated in the analysis following the Kron reduction of networks. See [14] for more details. It

is assumed that there are N_{Bi} buses within MG $i \in \mathcal{S}_{MG}$. The set of buses is denoted as $\mathcal{V}_i$. Thus, there are altogether $N_B = \sum_{i=1}^{N_{MG}} N_{Bi}$ DERs in the MMG. Each of these DER buses can be denoted as $ij \in \mathcal{V}_i$ ($\forall i \in \{1, \dots, N_{MG}\}$), and its dynamics can be equivalently characterized by the classic swing equations [15]:

$$\dot{\theta}_{ij} = \omega_{ij} - \omega_0, \quad (1)$$

$$M_{ij} \dot{\omega}_{ij} = P_{Gij} - P_{Lij} - P_{eij}. \quad (2)$$

P_{eij} can be calculated following the Kirchoff's Law and AC power flow equation as

$$P_{eij} = \underbrace{\sum_{(ij, \tilde{ij}) \in \mathcal{E}} P_{ij, \tilde{ij}}}_{\text{Power flows within MG } i} + \underbrace{\sum_{(ij, \tilde{ij}) \in \mathcal{E}} P_{ij, \tilde{ij}}}_{\text{Power flows among different MGs}}, \quad (3)$$

$$P_{ij, \tilde{ij}} = g_{ij, \tilde{ij}} E_{ij} [E_{ij} - E_{\tilde{ij}} \cos(\theta_{ij} - \theta_{\tilde{ij}})] - b_{ij, \tilde{ij}} E_{ij} E_{\tilde{ij}} \sin(\theta_{ij} - \theta_{\tilde{ij}}). \quad (4)$$

$(ij, \tilde{ij}) \in \mathcal{E}$ indicates that there exists a distribution line between buses ij and $\tilde{ij}$. $i \neq \tilde{i}$ means that the distribution line is a tie-line between MGs i and $\tilde{i}$, otherwise it is an internal line within MG i . The neighboring MG set of MG i is denoted by $\mathcal{N}_i$. A MG $\tilde{i} \in \mathcal{N}_i$ if $\exists j, \tilde{j}$ such that $(ij, \tilde{ij}) \in \mathcal{E}$. MG $0 \in \mathcal{N}_i$ if there exists a j such that $(0, ij) \in \mathcal{E}$. The bus voltage magnitude E_{ij} ($\forall ij$) can be restored to the reference value via reactive power coordination which has been extensively and independently investigated in the literatures. Interested readers may refer to [16] for the basic hierarchical reactive power control which is also implemented in the case studies due to its simplicity, and [17], [18] for the advanced distributed reactive coordination schemes.

B. Control Objective

Based on the energy scheduling approach of MMGs (e.g., [4], [19]–[21]), the optimal scheduling results should satisfy

$$\sum_{j=1}^{N_{Bi}} (P_{Gij}^* - P_{Lij}^*) + \sum_{\substack{(ij, \tilde{ij}) \in \mathcal{E} \\ \tilde{i} \in \mathcal{N}_i}} P_{ij, \tilde{ij}}^* = 0, \forall i \in \mathcal{S}_{MG}, \quad (5)$$

where P_{Gij}^* , P_{Lij}^* and $P_{ij, \tilde{ij}}^*$ are the scheduled values of P_{Gij} , P_{Lij} and $P_{ij, \tilde{ij}}$.

However, because of the time-varying renewable generations and load demand, P_{Lij} may constantly fluctuate around P_{Lij}^* within each dispatch time slot. Consequently, the power of DERs P_{Gij} , $\forall ij \in \mathcal{V}_i$ should be dynamically controlled to make the DERs within each MG absorb its own load fluctuation following

$$\sum_{j=1}^{N_{Bi}} (P_{Gij} - P_{Gij}^*) = \sum_{j=1}^{N_{Bi}} (P_{Lij} - P_{Lij}^*), \quad (6)$$

so that the power exchanges among the MGs are regulated unchanged in the real time, i.e., $P_{ij, \tilde{ij}} = P_{ij, \tilde{ij}}^*$ for all $(ij, \tilde{ij}) \in \mathcal{E}$ with $\tilde{i} \in \mathcal{N}_i$. To satisfy (6), two different power sharing policies can be adopted to adjust the power of DERs.

¹Renewable energy systems, e.g., photovoltaic and wind generators, are viewed as negative loads without loss of generality.

Fig. 2. Comparison of objectives and timescales of the robust energy scheduling [6] and the proposed DOTLPP control.

Economic dispatch policy: The DERs within each MG satisfy

$$\min \text{Generation Cost} = \sum_{j=1}^{N_{Bi}} \text{cost}_{ij}(P_{Gij}), \forall i \in \mathcal{S}_{MG} \quad (7a)$$

subject to

$$\sum_{j=1}^{N_{Bi}} (P_{Gij} - P_{Lij}) + \sum_{\substack{(ij, \tilde{i}\tilde{j}) \in \mathcal{E} \\ \tilde{i} \in \mathcal{N}_i}} P_{ij, \tilde{i}\tilde{j}}^* = 0, \forall i \in \mathcal{S}_{MG}, \quad (7b)$$

$$P_{Gij}^{\min} \leq P_{Gij} \leq P_{Gij}^{\max}, \forall j \in \mathcal{V}_i. \quad (7c)$$

The economic dispatch intends to minimize the total generation cost where the cost of each DER including both the distributed generator and energy storage system within each MG can be uniformly modeled as [22], [23]

$$\text{cost}_{ij}(P_{Gij}) = \frac{1}{2} a_{Gij} P_{Gij}^2 + b_{Gij} P_{Gij} + c_{Gij}. \quad (8)$$

Constraint (7b) indicates the real-time power balance within each MG, which can be derived following (5) and (6). Constraint (7c) is the power limit of each DER.

Participation factor policy [24]: Over a long timescale, the multiple MG operators interact and compete with each other to optimize the expected operational cost of each MG via energy trading using the robust energy scheduling optimization, e.g., [6]. Various operational constraints, e.g., power constraints of DERs, state of charge constraints of energy storage devices, power flow constraints, etc. can be included in this formulation. Specifically, the nominal operational point of each DER (P_{Gij}^*) is calculated to serve the forecast load within the specific time slot. Meanwhile, a non-negative participation factor β_{Gij} is also required to ensure the existence of a feasible solution as long as the load fluctuation is within the forecast uncertainty set. The function of robust energy scheduling in the overall operational scheme of a MMG is shown in Fig. 2. Although extensive works have been done to determine the optimal schedule required by the participation factor policy, how to implement these results effectively and maintain system stability in the real time control of a MMG is seldom report. As such, a DOTLPP control will be designed to dynamically and adaptively adjust the actual power of the DER in proportion to β_{Gij} following

$$P_{Gij} = P_{Gij}^* + \beta_{Gij} \epsilon_i^*, \forall j \in \mathcal{V}_i \quad (9)$$

where $\sum_{j=1}^{N_{Bi}} \beta_{Gij} = 1$ and $\epsilon_i^* = \sum_{j=1}^{N_{Bi}} (P_{Lij} - P_{Lij}^*)$ is the real-time load forecast error within MG i . It should be noted that control law (9) can be applied to both heterogeneous dispatchable distributed generators as well as energy storage devices. Their corresponding operational constraints do not need to be considered in the real-time control stage since they have been included in the robust energy scheduling optimization and will not be violated as long as the actual load fluctuation is within the forecast load band.

Remark 1: According to (9), the participation factor policy complies with the results of the robust energy scheduling approach for MMGs, e.g., [6], which aims to achieve the operational optimality over a long timescale, e.g., one day. However, due to the existence of a time-varying load forecast error, the real-time operational point of the MMG always deviates from the optimal energy schedule slightly. In light of this, the real-time operational point is termed as near optimality over a long timescale. Contrarily, the economic dispatch policy (7) which is adopted extensively in references (e.g., [8], [9], [12]) only optimizes the DER power outputs based on the current load level, thus it can only achieve real-time optimality. In the following section, it is to be illustrated that the two policies are intrinsically the same for control design.

At this point, the objectives of optimal tie-line power flow control can be summarized as follows:

- 1) **Grid-connected mode:** In response to any random load disturbance, the power of DERs can be regulated to a steady state where either the economic dispatch policy or the participation factor policy is satisfied, and the MG power exchanges are regulated to the scheduled values.
- 2) **Islanded mode:** Besides satisfying either of the power sharing policies, the system frequency is further stabilized to the reference value, i.e., $\lim_{t \rightarrow +\infty} \omega_{ij} = \omega_0, \forall ij$.

III. THE DOTLPP CONTROL APPROACHES

This section first introduces the requirement for the communication infrastructure design based on which the DOTLPP control approaches in grid-connected and islanded modes are presented, respectively.

A. Design of the Cyber Layer

To facilitate the DOTLPP control, a regional communication network can be set up for each MG $i \in \mathcal{S}_{MG}$. Communication between the neighboring MGs is not required, as illustrated in Fig. 1. The regional communication network of MG i can be depicted with an undirected graph $\mathcal{G}_i = (\mathcal{V}_i, \mathcal{E}_i, \mathcal{A}_i)$. $\mathcal{V}_i$ which is the set of buses in the physical layer of MG i also refers to the set of MG i 's CAs in the cyber layer. CAs integrate the functions of real-time control, computation, communication, and information sensing. Specifically, the power exchange with any neighboring MG $j \in \mathcal{N}_i$ is required to be measured by at least one CA within MG i . To alleviate the communication burden, it is preferred that the CA at the bus which is directly connected to an adjacent MG should locally measure the corresponding power flow. For example in Fig. 1, CA 12 can locally measure the power exchange

between MGs 1 and 2. Moreover, it is also required that each CA should measure the system frequency of the local bus when islanded operation is detected. $\mathcal{E}_i \subseteq \mathcal{V}_i \times \mathcal{V}_i$ is the set of communication links within MG i . $(ij, \tilde{i}\tilde{j}) \in \mathcal{E}_i$ indicates that the CAs at buses ij and $\tilde{i}\tilde{j}$ can exchange information using two-way communication technology. Unlike [12], the topology of MG i 's regional communication network is not required to coincide with its physical interconnection topology, which can enhance the flexibility and robustness of the proposed control approach. For instance, based on the cooperative control theory, a connected communication topology (i.e., there exists at least one path between every pair of the CAs) is a sufficient condition for the state of each CA to reach a consensus [25]. Therefore, the consensus algorithm can be made robust to communication interruptions by adopting a redundant communication topology [26]. $A_i \in \mathbb{R}^{N_{Bi} \times N_{Bi}}$ is the adjacency matrix corresponding to graph $\mathcal{G}_i$. Its elements are defined as

$$[A_i]_{jk} = [A_i]_{kj} = \begin{cases} w_{jk}^i > 0 & \text{if } (ij, ik) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

B. Control Design of the MMG in Grid-Connected Mode

First of all, the consistency between the economic dispatch policy and participation factor policy in the control design is to be validated. Specifically, the incremental cost (IC) of DER ij in the economic dispatch policy can be defined as

$$\lambda_{ij} = \frac{d\text{cost}_{ij}}{dP_{Gij}} = a_{Gij}P_{Gij} + b_{Gij}. \quad (11)$$

The optimal solution of (7) follows the well-known equal IC criterion [22] as

$$P_{Gij} = \text{Sat}\left(\frac{1}{a_{Gij}}\lambda_i^* - \frac{b_{Gij}}{a_{Gij}}, P_{Gij}^{\min}, P_{Gij}^{\max}\right), \quad (12)$$

where λ_i^* is the optimal IC. It can be observed that (9) and (12) are equivalent in format by assuming $1/a_{Gij} = \beta_{Gij}$, $-b_{Gij}/a_{Gij} = P_{Gij}^*$, $P_{Gij}^{\min} = -\infty$, and $P_{Gij}^{\max} = +\infty$. Without loss of generality, the participation factor policy is adopted hereinafter to illustrate the control design and simulation results.

When the MMG is in grid-connected mode, each DER should operate in grid-feeding mode, i.e., to inject power to the MG following a reference value [27]. Within any of the interconnected MG $i \in \mathcal{N}_{MG}$, the power of the DERs should be controlled following (9) as

$$P_{Gij} = P_{Gij}^* + \beta_{Gij}\epsilon_{ij}, \quad \forall j \in \mathcal{V}_i. \quad (13)$$

Different from (9), (13) replaces ϵ_i^* by ϵ_{ij} which is a local estimation of ϵ_i^* and is expected to track ϵ_i^* in the steady state. To achieve this tracking objective, ϵ_{ij} can be dynamically adjusted based on distributed communication and locally measured power flows. For bus ij which is directly connected to a neighboring MG, i.e., $\exists(ij, \tilde{i}\tilde{j}) \in \mathcal{E}$ with $\tilde{i} \in \mathcal{N}_i$, its ϵ_{ij} can be updated as

$$\dot{\epsilon}_{ij} = \sum_{(ij, ik) \in \mathcal{E}_i} w_{jk}^i(\epsilon_{ik} - \epsilon_{ij}) - \alpha \sum_{\substack{(ij, \tilde{i}\tilde{j}) \in \mathcal{E} \\ \tilde{i} \in \mathcal{N}_i}} \Delta P_{ij, \tilde{i}\tilde{j}}, \quad (14a)$$

otherwise,

$$\dot{\epsilon}_{ij} = \sum_{(ij, ik) \in \mathcal{E}_i} w_{jk}^i(\epsilon_{ik} - \epsilon_{ij}), \quad (14b)$$

where $\alpha > 0$ is a sufficiently small control parameter, and $\Delta P_{ij, \tilde{i}\tilde{j}} = P_{ij, \tilde{i}\tilde{j}} - P_{ij, \tilde{i}\tilde{j}}^*$ is the deviation of MG tie-line power flow from its scheduled value. Intuitively, the term $\sum_{(ij, ik) \in \mathcal{E}_i} w_{jk}^i(\epsilon_{ik} - \epsilon_{ij})$ in (14) intends to synchronize different ϵ_{ij} within MG i to a consensus value based on cooperative control theory [25], while the term $\Delta P_{ij, \tilde{i}\tilde{j}}$ in (14a) functions as a negative feedback to drive the consensus value to the actual ϵ_i^* .

The block diagram of the cyber-physical MMG under the DOTLPF control is shown in Fig. 3. It illustrates the flow of information within each MG. Note that when operating in grid-connected mode, the CAs' output signals are directly sent to the DERs. The convergence property of the proposed control in grid-connected mode can be summarized as

Theorem 1: Assume that the regional communication network of each MG is connected. The power output of each DER ij is determined by (13) where the estimated forecast error ϵ_{ij} is updated following (14) based on distributed communication and local power flow measurement. By selecting a sufficiently small control parameter α , the CPS is locally exponentially stable, and the system operational point can achieve the control objective in grid-connected mode in Subsection II-B.

The proof is detailed in Appendix A.

C. Control Design of the MMG in Islanded Mode

When the MMG is in islanded mode, each DER should operate in grid-supporting mode, i.e., to regulate the system frequency by controlling the active power delivered to the grid [27]. Specifically, within any of the interconnected MG $i \in \mathcal{N}_{MG}$, the power of the DERs should be controlled following the classic droop curve as

$$P_{Gij} = P_{Gij}^{\text{nom}} - k_{Pij}\Delta\omega_{ij}. \quad (15)$$

P_{Gij}^{nom} is the nominal power of DER ij determined by

$$P_{Gij}^{\text{nom}} = P_{Gij}^* + \beta_{Gij}\epsilon_{ij}, \quad \forall j \in \mathcal{V}_i, \quad (16)$$

where ϵ_{ij} should be dynamically controlled to track ϵ_i^* . For bus ij which is directly connected to a neighboring MG, i.e., $\exists(ij, \tilde{i}\tilde{j}) \in \mathcal{E}$ with $\tilde{i} \in \mathcal{N}_i$, its ϵ_{ij} can be updated as

$$\dot{\epsilon}_{ij} = \sum_{(ij, ik) \in \mathcal{E}_i} w_{jk}^i(\epsilon_{ik} - \epsilon_{ij}) - k_{Iij}\Delta\omega_{ij} - \alpha \sum_{\substack{(ij, \tilde{i}\tilde{j}) \in \mathcal{E} \\ \tilde{i} \in \mathcal{N}_i}} \Delta P_{ij, \tilde{i}\tilde{j}}, \quad (17a)$$

Otherwise,

$$\dot{\epsilon}_{ij} = \sum_{(ij, ik) \in \mathcal{E}_i} w_{jk}^i(\epsilon_{ik} - \epsilon_{ij}) - k_{Iij}\Delta\omega_{ij}, \quad (17b)$$

where $k_{Iij} = \beta_{Gij}$ is the integral parameter of DER ij .

Since the dynamics of the DOTLPF control (17) is relatively slow due to the small feedback gains, the transient after disturbance is mainly governed by the droop control (15). Consequently, the immediate response of the MMG is similar to that under the conventional hierarchical control.

Fig. 3. Control block diagram of the cyber-physical MMG.

As time goes on, frequency deviation is gradually corrected via a negative feedback mechanism to recover the reference frequency ω_0 . Specifically, when the total reference power of DERs is less than the total power consumption, the primary frequency control (15) results in $\Delta\omega_{ij} < 0$. This can bring up the DER reference power by increasing its ϵ_{ij} following (17) until the frequency deviation vanishes. The converse is also true.

The MMG in islanded mode under the DOTLPF control is shown in Fig. 3 where all the switches should be in islanded position. The convergence property of the proposed control in islanded mode can be summarized as

Theorem 2: Assume that the regional communication network of each MG is connected. Each of the DERs operates in grid-supporting mode (15). The nominal power output of each DER ij is determined by (16) where ϵ_{ij} is updated following (17) based on distributed communication as well as the local system frequency and power flow measurement. By selecting a sufficiently small control parameters α , the MMG is locally Lyapunov stable, and the system operational point can achieve the control objective in islanded mode in Subsection II-B.

The proof is detailed in Appendix B.

IV. SIMULATION RESULTS

In this section, the DOTLPF control is validated in a 4-MG system modified from the IEEE standard 34-bus test feeder with DlgSILENT PowerFactory. To simulate the scenario of real implementation, the DOTLPF control law is discretized using the trapezoid rule method [28] with a step size of 0.05 second. The single-line diagram is shown in Fig. 4 where MGs 1 and 2 have 3 DERs each, MG 3 has 5 DERs, while MG 4 has 4 DERs. The power capacity of each DER is uniformly set to 0.5 MW. Initially, DER 35 is disconnected from MG 3 to test the plug-and-play capability in Section IV-B. These DERs are coordinated based on the participation factor policy. The nominal power output (P_{Gij}^*) and participation factor (β_{Gij}) of each DER is optimized using the distributed robust energy management scheme proposed in [6]. The operational parameters of the DERs in the dispatch time slot considered

TABLE I
OPERATIONAL PARAMETERS OF THE DERs IN THE 4-MG SYSTEM

DER No.	P_{Gij}^* (MW)	β_{Gij}	k_{Pij} (MW/Hz)	DER No.	P_{Gij}^* (MW)	β_{Gij}	k_{Pij} (MW/Hz)
11	0.288	0.17	0.068	21	0.29	0.17	0.068
12	0.251	0.33	0.132	22	0.253	0.33	0.132
13	0.209	0.5	0.2	23	0.211	0.5	0.2
31	0.309	0.067	0.027	41	0.309	0.1	0.04
32	0.273	0.133	0.053	42	0.273	0.2	0.08
33	0.125	0.2	0.08	43	0.232	0.3	0.12
34	0.187	0.267	0.107	44	0.187	0.4	0.16
35	0.1	0.3	0.12				

TABLE II
LOAD DATA OF THE 4-MG SYSTEM

Load No.	L1	L2	L3	L4	L5	L6	L7	L8	L9
Active Power (MW)	0.5	0.2	0.2	0.2	0.4	0.2	0.3	0.5	0.5
Reactive Power (MVar)	0.5	0.1	0.1	0.1	0.1	0.1	0.5	0.1	0.1

are tabulated in TABLE I. The control parameter α is tuned as 0.05. Loads L1-L9 are connected to the the 4-MG system from different locations as in Fig. 4. The initial load data is tabulated in TABLE II. In the following, the proposed control will be validated in grid-connected mode and islanded mode, respectively. The response of the MMG to mode switch between the two will be investigated as well.

A. Response of the MMG to Load Variations

In the scenario of grid-connected mode, L2 in MG 3 increases by 0.2 MW at $t = 10$ s, and L5 in MG 1 increases by 0.405 MW at $t = 100$ s. The responses of the MMG are depicted in Fig. 5 which indicates that a new steady state can be reached after approximately 300 seconds. Figs. 5(a)-5(d) show the variations of the local estimation of load forecast errors in MGs 1-4, respectively. It can be observed from Figs. 5(a)-5(d) that the responses of load forecast errors estimated

Fig. 4. Single-line diagram of the 4-MG system.

Fig. 5. Responses of the MMG to load change in grid-connected mode.

at different DERs within a same MG are almost identical, which indicates that the power adjustments of the DERs are in proportion to their participation factors within the same MG following (13). In the figures hereinafter, only the forecast error of one DER within each MG will be plotted to represent the consensus load forecast errors of MGs 1 ~ 4 (denoted as $\epsilon_1 \sim \epsilon_4$) unless otherwise stated. Moreover, consistent to the control objective, the DER power outputs within MG 3 and MG 1 increase together with the consensus load forecast errors ϵ_3 and ϵ_1 respectively at $t = 10$ s and $t = 100$ s to counteract their internal load increases, while those of MGs 2 and 4 remain unchanged after a transient caused by its adjacent MGs as no internal load change happens. Fig. 5(e) illustrates the responses of DER power outputs within MG 1. It indicates that the power differences before and after the disturbance for DERs 11-13 are 0.067 MW, 0.133 MW, and 0.203 MW, respectively, which exactly covers the 0.405 MW load increase within MG 1. Due to the tie-line power flow control, the power exchanges among the MGs can be recovered to their respective initial levels, as shown in Fig. 5(f).

In the scenario of islanded mode, L1 in MG 1 decreases by

0.4 MW at $t = 20$ s, and L2 in MG 3 decreases by 0.2 MW at $t = 100$ s. The responses of the MMG are depicted in Fig. 6 which indicates that a new steady state can be reached after approximately 200 seconds. Fig. 6(a) plots the consensus load forecast errors $\epsilon_1 \sim \epsilon_4$. Similar to grid-connected mode, the decreases of ϵ_1 and ϵ_3 lead to decreases of the corresponding DER power outputs, which counteract the load decreases within MG 1 and MG 3. On the other hand, ϵ_2 and ϵ_4 remain unchanged as no internal load change takes place. The function of tie-line power flow recovery can also be achieved in islanded mode, as indicated by Fig. 6(b). Fig. 6(c) shows that the system frequency can be recovered to $\omega_0 = 60$ Hz after the transient caused by the load variations. It also investigates the influence of communication delay on the control performance. Specifically, when a communication delay of 0.4 second which is sufficiently longer than that adopted in [8] is considered, a more severe frequency fluctuation can be observed, but the frequency recovery can still be achieved.

It is also worth mentioning that measurement errors are inevitable in practical implementation of the DOTLPF control [29], which may cause the operational states to fluctuate slightly around the steady states. Nevertheless, this fluctuation is normally small and will only cause a trivial influence to the operation of the MMG.

B. Test of the Plug-and-Play Capability

Next, the plug-and-play capability of the proposed control approach is to be validated. In grid-connected mode, a backup DER 35 is connected to MG 3 at $t = 50$ s and disconnected again at $t = 350$ s. The responses of the MMG are depicted in Fig. 7. Specifically, Fig. 7(a) shows the consensus load forecast errors of MGs 1, 2, and 4, whose steady-state values remain unchanged as no internal disturbances occur. The estimated load forecast errors and power outputs of the DERs within MG 3 are depicted in Figs. 7(b) and 7(c). It can be observed that when DER 35 is plugged in, it gradually picks up load and shares the power together with DERs 31-34. Consequently, $\epsilon_{31} \sim \epsilon_{34}$ will slightly decrease in about 150 seconds. Since DER 31 and DER 32 have lower participation factors than DER 33 and DER 34, their power outputs decrease less in this process. When DER 35 is switched off, it takes approximately 150 seconds again for $\epsilon_{31} \sim \epsilon_{34}$ to recover to the initial level. Fig. 7(d) shows that the power exchanges among the MGs stay unchanged in the steady state.

When the MMG is in islanded mode, a simulation is performed in the same scenario, and the results are shown in Fig. 8. The analysis is similar to that of grid-connected mode except that a frequency fluctuation can be observed before it recovers to 60 Hz. Fig. 8(c) further indicates that when a communication delay of 0.4 second is considered, the frequency fluctuation is only slightly severer.

C. Islanding and Reconnection of the MMG to the Main Grid

Finally, the effect of islanding and reconnection to the main grid is tested. At $t = 50$ s, the MMG is disconnected from the main grid due to a certain main grid fault. After the main grid fault is cleared, the MMG is resynchronized and reconnected

Fig. 6. Responses of the MMG to load change in islanded mode.

Fig. 7. Responses of the MMG to switch on/off of DER 35 in grid-connected mode.

Fig. 8. Responses of the MMG to switch on/off of DER 35 in islanded mode.

to the main grid at $t = 450$ s. The simulation results are depicted in Fig. 9. Fig. 9(a) plots the consensus load forecast errors of the 4 MGs. It can be observed that before islanding of the MMG, MG 1 exports 0.4 MW power to the main grid. After islanding, a frequency fluctuation can be observed in Fig. 9(c) before recovering to 60 Hz after about 300 seconds. After this transient, the consensus load forecast error ϵ_1 decreases to some extent as no power is further exported to the main grid, while the consensus load forecast errors $\epsilon_2 \sim \epsilon_4$ are approximately recovered to their initial levels to maintain the MG power exchanges. After the MMG is reconnected to the main grid, it takes approximately 400 seconds for $\epsilon_1 \sim \epsilon_4$ and the power exchanges among the MGs/the main grid to be

recovered to their initial levels at the steady state.

V. CONCLUSION

In this paper, a distributed optimal tie-line power flow control approach was proposed for the multiple-microgrid system. The proposed approach was implemented in the framework of a multi-agent cyber-physical system based on the regional communication within each microgrid and distributed sensors monitoring the tie-line power flows. The objective is to implement the optimal energy schedule of each time slot in the real time while maintaining system stability. This approach is applicable to the multiple-microgrid system operated in both grid-connected and islanded modes. In grid-connected mode, the proposed control can dynamically regulate the power exchanges to the scheduled values among the MGs, while distributed energy resources within each microgrid can share its internal load fluctuation based on the participation factor policy or economic dispatch policy. In islanded mode, the system frequency can be further regulated to the nominal value. Simulation results based on a 4-microgrid system modified from the IEEE 34-bus test feeder demonstrated the effectiveness of the proposed approach in both operational modes.

One of the future works is to explore how to systematically integrate this real-time control scheme and the robust energy scheduling scheme over a long timescale by efficiently and reliably sharing the same communication infrastructure so that a fully distributed and robust operation architecture can be constructed for the multi-microgrid system.

APPENDIX A PROOF OF THEOREM 1

The proof of Theorem 1 can be proceeded in two steps:
Step 1. Linearization
Linearizing (4) around the optimal operation point leads to

$$\begin{aligned} \Delta P_{ij,\tilde{ij}} &= \beta_{ij,\tilde{ij}}(\Delta\theta_{ij} - \Delta\theta_{\tilde{ij}}) + \epsilon_{ij,\tilde{ij}}^b \Delta E_{ij} + \epsilon_{ij,\tilde{ij}}^e \Delta E_{\tilde{ij}} \\ &= \beta_{ij,\tilde{ij}}(\Delta\theta_{ij} - \Delta\theta_{\tilde{ij}}). \end{aligned} \quad (18)$$

For each of the buses within the MMG, distributed voltage restoration control approach can be applied to recover its voltage to the reference value, i.e., $\Delta E_{ij} \approx 0$ for all ij , which

Fig. 9. Islanding and reconnection of the MMG to the main grid.

leads to the second equality of (18). Parameter $\beta_{ij,\tilde{i}\tilde{j}}$ can be calculated as

$$\beta_{ij,\tilde{i}\tilde{j}} = \underbrace{E_{ij}^* E_{\tilde{i}\tilde{j}}^* g_{ij,\tilde{i}\tilde{j}} \sin(\theta_{ij}^* - \theta_{\tilde{i}\tilde{j}}^*)}_{\beta_{ij,\tilde{i}\tilde{j}}^-} - \underbrace{E_{ij}^* E_{\tilde{i}\tilde{j}}^* b_{ij,\tilde{i}\tilde{j}} \cos(\theta_{ij}^* - \theta_{\tilde{i}\tilde{j}}^*)}_{\beta_{ij,\tilde{i}\tilde{j}}^+}. \quad (19)$$

In an MV or LV distribution network, the line impedance is not purely inductive, but has a non-negligible resistance part. Nevertheless, $\beta_{ij,\tilde{i}\tilde{j}}^-$ which accounts for line loss can generally be ignored in the analysis for the following reasons: i) the output impedance of an inverter-based DER is typically inductive, which makes the inductive part dominate the resistive part, i.e., $|b_{ij,\tilde{i}\tilde{j}}| > |g_{ij,\tilde{i}\tilde{j}}|$ [14]; ii) the voltage angle differences of the distribution lines are normally within 3° [30], thus $\cos(\theta_{ij}^* - \theta_{\tilde{i}\tilde{j}}^*) \gg \sin(\theta_{ij}^* - \theta_{\tilde{i}\tilde{j}}^*)$. Therefore, the MMG can be treated as a lossless system by assuming $\beta_{ij,\tilde{i}\tilde{j}} \approx \beta_{ij,\tilde{i}\tilde{j}}^+$. Based on (18), (3) can be written in a matrix form as

$$\begin{bmatrix} \Delta P_{e0} \\ \Delta P_e \end{bmatrix} = B \begin{bmatrix} \Delta \theta_0 \\ \Delta \theta \end{bmatrix}, \quad (20)$$

where $\Delta P_e = [\Delta P_{e1}^\top, \dots, \Delta P_{eN_{MG}}^\top]^\top$ and $\Delta \theta = [\Delta \theta_1^\top, \dots, \Delta \theta_{N_{MG}}^\top]^\top$; P_{ei} and θ_i can be further expanded as $P_{ei} = [P_{ei1}, \dots, P_{eiN_{Bi}}]^\top$ and $\theta_i = [\theta_{i1}, \dots, \theta_{iN_{Bi}}]^\top$; and matrix $B \in \mathbb{R}^{(N_B+1) \times (N_B+1)}$ is defined as

$$[B]_{ij,\tilde{i}\tilde{j}} = \begin{cases} -\beta_{ij,\tilde{i}\tilde{j}} & \text{if } (ij, \tilde{i}\tilde{j}) \in \mathcal{E} \\ \sum_{(ij,kl) \in \mathcal{E}} \beta_{ij,kl} & \text{if } ij = \tilde{i}\tilde{j} \\ 0 & \text{otherwise} \end{cases}. \quad (21)$$

By selecting the bus at MG 0 as the reference bus satisfying $\Delta \theta_0 = 0$, (20) can be reduced to

$$\Delta P_e = \mathcal{B} \Delta \theta, \quad (22)$$

where $\mathcal{B} \in \mathbb{R}^{N_B \times N_B}$ is equivalent to B excluding the first row and column.

Define $\Delta F_{in} = [\Delta F_{in1}^\top, \dots, \Delta F_{inN_{MG}}^\top]^\top$ with $\Delta F_{ini} = [\Delta F_{ini1}, \dots, \Delta F_{iniN_{Bi}}]^\top$. $\Delta F_{inij} = \sum_{(ij,\tilde{i}\tilde{j}) \in \mathcal{E}} \Delta P_{ij,\tilde{i}\tilde{j}}$ indicates power flowing from bus ij to other buses within MG i . Following from (18), ΔF_{in} can be calculated as

$$\Delta F_{in} = \mathcal{B}_{in} \Delta \theta, \quad (23)$$

where

$$[\mathcal{B}_{in}]_{ij,\tilde{i}\tilde{j}} = \begin{cases} -\beta_{ij,\tilde{i}\tilde{j}} & \text{if } (ij, \tilde{i}\tilde{j}) \in \mathcal{E} \\ \sum_{(ij,il) \in \mathcal{E}} \beta_{ij,il} & \text{if } ij = \tilde{i}\tilde{j} \\ 0 & \text{otherwise} \end{cases}.$$

Similarly, define $\Delta F_{out} = [\Delta F_{out1}^\top, \dots, \Delta F_{outN_{MG}}^\top]^\top$ with $\Delta F_{outi} = [\Delta F_{outi1}, \dots, \Delta F_{outiN_{Bi}}]^\top$. $\Delta F_{outij} = \sum_{\substack{(ij,\tilde{i}\tilde{j}) \in \mathcal{E} \\ i \in N_{\tilde{i}\tilde{j}}}} \Delta P_{ij,\tilde{i}\tilde{j}}$ indicates power flowing from bus ij to other MGs. ΔF_{out} can be calculated as

$$\Delta F_{out} = \mathcal{B}_{out} \Delta \theta, \quad (24)$$

where $\Delta F_{out} = \Delta P_e - \Delta F_{in}$ and $\mathcal{B}_{out} = \mathcal{B} - \mathcal{B}_{in}$.

Step 2. Stability of the Closed-Loop Error System

Define $\Delta \epsilon = [\Delta \epsilon_1^\top, \dots, \Delta \epsilon_{N_{MG}}^\top]^\top$ with $\Delta \epsilon_i = [\Delta \epsilon_{i1}, \dots, \Delta \epsilon_{iN_{Bi}}]^\top$. The updating of forecast error (14) linearized around the optimal operation point can be written in the matrix form as

$$\Delta \dot{\epsilon} = -L \Delta \epsilon - \alpha \Delta F_{out} = -L \Delta \epsilon - \alpha \mathcal{B}_{out} \Delta \theta, \quad (25)$$

where $L = \text{diag}(L_1, \dots, L_{N_{MG}})$ with L_i being the Laplacian matrix of the communication topology corresponding to MG i . The Laplacian matrix is defined as $L_i = D_i - A_i$ with the degree matrix $D_i = \text{diag}(A_i \mathbf{1})$.

With $\Delta \omega_{ij} = 0$ across the MMG in grid-connected mode, it follows from (2) and (22) that

$$\Delta P_G - \Delta P_L = \Delta P_e = \mathcal{B} \Delta \theta, \quad (26)$$

where ΔP_G and ΔP_L are similarly defined as ΔP_e . In the following analysis, it is assumed that $\Delta P_L = \mathbf{0}$ ($\Delta P_L \neq \mathbf{0}$ is equivalent to the shift of ΔP_G in the steady state). It can be derived from (13) that $\Delta P_G = \Lambda \Delta \epsilon$, where Λ is a diagonal matrix with the entry corresponding to bus ij being β_{Gij} . Therefore, considering (25) and (26), the dynamics of the closed-loop error system can be described as

$$\Delta \dot{\epsilon} = -(L + \alpha \mathcal{B}_{out} \mathcal{B}^{-1} \Lambda) \Delta \epsilon. \quad (27)$$

The stability of (27) can be characterized by its system matrix. Define $\tilde{L}(\alpha) = L + \alpha \mathcal{B}_{out} \mathcal{B}^{-1} \Lambda$ which can be viewed as L perturbed by $\alpha \mathcal{B}_{out} \mathcal{B}^{-1} \Lambda$. In the sequel, it is to be illustrated that all the eigenvalues of the system matrix have negative real parts using matrix perturbation theory [31, Lemma 7].

Due to the connectivity of MG i 's regional communication network, its corresponding Laplacian matrix L_i has a simple eigenvalue 0 and $N_{Bi} - 1$ positive eigenvalues [25, Theorem 4.31]. It is straightforward that the eigenvalues of L are

the union of the eigenvalues of L_i ($\forall i \in \mathcal{N}_{MG}$). Therefore, L has N_{MG} eigenvalues 0 and the other eigenvalues are positive. Construct matrices $X = [e_1, \dots, e_{N_{MG}}] \in \mathbb{R}^{N_B \times N_{MG}}$ and $Y = \Gamma X^\top$, where $\Gamma \in \mathbb{R}^{N_{MG} \times N_{MG}}$ is a diagonal matrix with the i th entry being $1/N_{B_i}$, and e_i is a vector with its entries corresponding to the buses in MG i being 1 while other entries being 0. It can be readily verified that $YX = I$. The columns (rows) of X (Y) are the right (left) eigenvectors of eigenvalues 0 of L . Based on the matrix perturbation theory, the positive eigenvalues of L remains positive for $\tilde{L}(\alpha)$ when α is sufficiently small, and the perturbation on the zeros eigenvalues can be characterized by the eigenvalues of

$$\begin{aligned} Y\mathcal{B}_{\text{out}}\mathcal{B}^{-1}\Lambda X &= Y(B - B_{\text{in}})\mathcal{B}^{-1}\Lambda X \\ &= Y\Lambda X - YB_{\text{in}}\mathcal{B}^{-1}\Lambda X = Y\Lambda X. \end{aligned} \quad (28)$$

The third equality holds considering $YB_{\text{in}} = \mathbf{0}$. It can be verified that $Y\Lambda X \in \mathbb{R}^{N_{MG} \times N_{MG}}$ is a diagonal matrix with its i th entry being $\sum_{j=1}^{N_{B_i}} a_{Gij}^{-1}/N_{B_i}$. Therefore, the zero eigenvalues of $\tilde{L}(0)$ become positive for $\tilde{L}(\alpha)$ for a sufficiently small α . Since all the eigenvalues of $-\tilde{L}(\alpha)$ are negative, the error system (27) is stable. This concludes the proof of Theorem 1.

APPENDIX B PROOF OF THEOREM 2

In the optimal steady state, the synchronized motion of $\Delta\theta$ is not unique, i.e., the optimal phase angle vector can be $\theta^* + \mathbf{1}c$ for any $c \in \mathbb{R}$. Rather than picking a reference bus as in the proof of Theorem 1, θ^* is selected to satisfy

$$\mathbf{1}^\top \Delta\theta = \mathbf{1}^\top \theta - \mathbf{1}^\top \theta^* \equiv 0. \quad (29)$$

Hence, $\mathbf{1}^\top \dot{\theta}^* = \mathbf{1}^\top \dot{\theta} = \mathbf{1}^\top \Delta\omega$. To guarantee that $\mathbf{P}_G^* \equiv \mathcal{B}\theta^*$, each entry of θ^* should have the same changing rate, i.e.,

$$\dot{\theta}^* = \frac{1}{N_B} \mathbf{1} \Delta\omega \Rightarrow \Delta\dot{\theta} = (I - \frac{1}{N_B} \mathbf{1}) \Delta\omega. \quad (30)$$

In islanded mode, the dynamics of the closed-loop error system can be characterized by

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{\omega} \\ \Delta\dot{\epsilon} \end{bmatrix} = \begin{bmatrix} I - \frac{1}{N_B} \mathbf{1} & & \\ -M^{-1}\mathcal{B} & -M^{-1}K_P & M^{-1}\Lambda \\ -\alpha\mathcal{B}_{\text{out}} & -\Lambda & -L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta\omega \\ \Delta\epsilon \end{bmatrix}. \quad (31)$$

where $\Delta\omega = [\Delta\omega_1^\top, \dots, \Delta\omega_{N_{MG}}^\top]^\top$ with $\Delta\omega_i = [\Delta\omega_{i1}, \dots, \Delta\omega_{iN_{B_i}}]^\top$; M and K_P are diagonal matrices with the entry corresponding to DER ij being M_{ij} and k_{Pij} , respectively. The dynamics of $\Delta\theta$ follow from (30); the dynamics of $\Delta\omega$ follow from (2), (22), (15) and (16); and the dynamics of $\Delta\epsilon$ follow from (17).

Choose a Lyapunov candidate as

$$\begin{aligned} V &= \frac{1}{\alpha^2} \Delta\theta^\top \mathcal{B} \Delta\theta + \frac{1}{\alpha^2} \Delta\omega^\top M \Delta\omega \\ &\quad + \frac{1}{\alpha^2} \Delta\epsilon^2 - \frac{2}{\alpha} \Delta\epsilon^\top M \Delta\omega + \Delta\theta^\top M \Delta\omega. \end{aligned} \quad (32)$$

Define a state transformation matrix $T_B = [\nu_1, \mathcal{T}_B]^\top$, where $\mathcal{T}_B = [\nu_2, \dots, \nu_{N_B}]$ and ν_i is the left eigenvector corresponding to the i th eigenvalue of $\mathcal{B}$, i.e., $\lambda_i(\mathcal{B})$. Specifically, $\mathcal{B}$ has

a simple eigenvalue $\lambda_1(\mathcal{B}) = 0$ with its corresponding left eigenvector $\nu_1 = \mathbf{1}_q$ since it is a symmetrical and irreducible matrix [25]. Consider (29) and apply the state transformation to $\Delta\theta_G$ as $T_B \Delta\theta = [0, \delta\theta^\top]^\top \Rightarrow \Delta\theta = T_B \delta\theta$, where $\delta\theta \in \mathbb{R}^{N_B-1}$ is the transformed state vector. Thus, (32) can be rewritten as

$$\begin{aligned} V &= \frac{1}{\alpha^2} \delta\theta^\top \mathcal{B}' \delta\theta + \frac{1}{\alpha^2} \Delta\omega^\top M \Delta\omega + \frac{1}{\alpha^2} \Delta\epsilon^2 \\ &\quad + \delta\theta^\top \mathcal{T}_B^\top M \Delta\omega - \frac{2}{\alpha} \Delta\epsilon^\top M \Delta\omega, \end{aligned} \quad (33)$$

where $\mathcal{B}' = \mathcal{T}_B^\top \mathcal{B} \mathcal{T}_B = \text{diag}\{[\lambda_2(\mathcal{B}_r), \dots, \lambda_{N_B}(\mathcal{B}_r)]\}$ is positive definite. When $\alpha \rightarrow 0^+$, the positive definite terms of the state vectors (in the first line of (33)) have higher magnitudes, which can bound the indefinite terms with lower magnitudes (in the second line of (33)). Therefore, the Lyapunov function V is positive definite.

Define another state transformation matrix $T_L = [\mathcal{T}_{L1}, \mathcal{T}_{L2}]^\top$, where $\mathcal{T}_{L1} = [e_1, \dots, e_{N_{MG}}]$ and $\mathcal{T}_{L2} \in \mathbb{R}^{N_B \times (N_B - N_{MG})}$ are the sets of left eigenvectors corresponding to the 0 eigenvalues and positive eigenvalues of L , respectively. The state transformation leads to $T_L \Delta\epsilon = [\delta\epsilon_1^\top, \delta\epsilon_2^\top]^\top \Rightarrow \Delta\epsilon = \mathcal{T}_{L1} \delta\epsilon_1 + \mathcal{T}_{L2} \delta\epsilon_2$, where $\delta\epsilon_1 \in \mathbb{R}^{N_{MG}}$ and $\delta\epsilon_2 \in \mathbb{R}^{N_B - N_{MG}}$ are the transformed states.

The time derivative of the Lyapunov function (32) along the trajectory of (31) can be calculated as

$$\begin{aligned} \dot{V} &= -\Delta\theta^\top \mathcal{B} \Delta\theta - \Delta\omega^\top R \Delta\omega - \Delta\epsilon^\top \left(\frac{2}{\alpha^2} L + \frac{2}{\alpha} \Lambda \right) \Delta\epsilon \\ &\quad + \Delta\theta^\top (2\mathcal{B}_{\text{out}} M - K_P) \Delta\omega + \Delta\theta^\top \left(\frac{2}{\alpha} \mathcal{B}_{\text{in}} + \Lambda \right) \Delta\epsilon \\ &\quad + \Delta\omega^\top \left(\frac{2}{\alpha} K_P + \frac{2}{\alpha} M L \right) \Delta\epsilon \\ &= -\delta\theta^\top \mathcal{B}' \delta\theta - \Delta\omega^\top R \Delta\omega - \frac{2}{\alpha} \delta\epsilon_1^\top \mathcal{T}_{L1}^\top \Lambda \mathcal{T}_{L1} \delta\epsilon_1 \\ &\quad - \delta\epsilon_2^\top \mathcal{T}_{L2}^\top \left(\frac{2}{\alpha^2} L + \frac{2}{\alpha} \Lambda \right) \mathcal{T}_{L2} \delta\epsilon_2 \\ &\quad + \delta\theta^\top \mathcal{T}_B^\top (2\mathcal{B}_{\text{out}} M - K_P) \Delta\omega + \delta\theta^\top \mathcal{T}_B^\top \Lambda \mathcal{T}_{L1} \delta\epsilon_1 \\ &\quad + \delta\theta^\top \mathcal{T}_B^\top \left(\frac{2}{\alpha} \mathcal{B}_{\text{in}} + \Lambda \right) \mathcal{T}_{L2} \delta\epsilon_2 + \frac{2}{\alpha} \Delta\omega^\top K_P \mathcal{T}_{L1} \delta\epsilon_1 \\ &\quad + \frac{2}{\alpha} \Delta\omega^\top (K_P + M L) \mathcal{T}_{L2} \delta\epsilon_2 \end{aligned} \quad (34)$$

where $R = \frac{2}{\alpha^2} K_P - \frac{2}{\alpha} M \Lambda - M(I - \frac{1}{N_B} \mathbf{1})$. The second equality follows from the state transformations by considering that $\mathcal{B}_{\text{in}} \mathcal{T}_{L1} = \mathbf{0}$ and $L \mathcal{T}_{L1} = \mathbf{0}$. When $\alpha \rightarrow 0^+$, the negative definite terms of the state vectors (in the first two lines of the second equality of (34)) have higher magnitudes, which can bound the indefinite terms with lower magnitudes (in the last three lines of the second equality of (34)). Therefore, the Lyapunov function $\dot{V}$ is negative definite, and thus the error system (31) is locally Lyapunov stable. Considering condition (29), $[\Delta\theta^\top, \Delta\omega^\top, \Delta\epsilon^\top]^\top \rightarrow \mathbf{0}$ in the steady state. This concludes the proof of Theorem 2.

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