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Performance of Sideslip Perching Maneuver with an Adaptive Gain Control Feature

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The avian-inspired sideslip perching maneuver is an aggressive flight scenario that involves high turn rates and high rates of change of aerodynamic angles to decelerate and descend the aircraft rapidly for a precision landing. The occurrence of aerodynamic stall delay is unpredictable. Besides dynamic stall effects, state perturbations and model uncertainties heavily influence the accuracy of the landing and evidently, the success of the maneuver. This paper seeks to extend the convergence area related to the initial perching launch points and enhance the stability of the sideslip perching maneuver by formulating an adaptive gain sliding mode control technique. Based on a reference optimal sideslip trajectory generated by an optimization model, a comparative study between the proposed adaptive controller and a nominal sliding mode controller is carried out to evaluate their performances under different scenarios such as varying initial spatial and state conditions with induced state perturbations along the perching maneuver and dynamic stall uncertainties.

Keywords

Perching maneuver, Deep stall landing, Perching control, Adaptive Gain, Dynamic stall.

1. Introduction

Aircraft perching is one of the aggressive maneuvering techniques that has been recently explored to expand the flight envelope of aircrafts and offer new functional possibilities, especially in the field of intelligence, surveillance and reconnaissance (ISR) [1-4]. The fixed-wing aircraft's perching maneuver closely mimics the avian agile landing procedure to acquire rapid deceleration by efficiently utilizing gravity and the separation drag at elevated angles of attack [5, 6]. Consequently, the high angle of attack technique allows the aircraft to achieve landing with low impact and high precision, eliminating the need for excessive landing distance [7, 8].

A three-dimensional (3D) variant of the perching maneuver, namely, the sideslip perching maneuver differs significantly from the typical longitudinal aircraft perching technique in its overall trajectory and drag mechanisms [9]. Most of the literature commonly covers the longitudinal perching maneuver, which is essentially a two-dimensional (2D) flight trajectory that involves the dive and climb motions only in the vertical plane, as shown in Fig. 1. The dive phase provides the aircraft with momentum for the subsequent climb phase where the aircraft undergoes rapid deceleration due to the separation drag from high angles of attack and gravity. The necessary presence of the dive and climb phases produces an undershoot, which is a highly undesirable characteristic that renders some intended applications such as landing on the ground to be impractical [10, 11]. On the other hand, by accommodating the lateral spatial dimension, the sideslip perching maneuver eliminates the climb phase requirement and consequently, the unfavourable undershoot. Fig. 2 depicts the 3D sideslip perching maneuver performed by a conventional fixed-wing aircraft that is generated by an optimization procedure in Ref. [9]. Through a methodical approach in utilizing the drags from both the longitudinal and lateral planes, the aircraft undergoes 2 phases; the yaw phase followed by the pitch up phase that allows the aircraft to experience rapid deceleration while lowering its altitude towards the target landing point.

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Fig. 1. Typical 2D perching maneuver.

As perching is an unstable agile maneuver, the perching aircraft requires an effective closed loop control strategy to ensure that the aircraft is able to sustain itself in its optimal trajectory while overcoming perturbations and model inaccuracies. Furthermore, perching is a relatively short maneuver and hence, the controller only has a small time-window to overcome perturbations while ensuring the required tracking for a precision landing. Since perching involves different flow regimes due to the high variations of speed and angle of attack, developing an accurate aerodynamic model is difficult. These uncertainties are further aggrandized with the inevitable occurrence of dynamic stall, whose effects on the optimal perching trajectory are crucial for attaining the intended trajectory. With the dependence on high maneuverability during perching, aerodynamic coefficients under rapid turning motions would deviate from their corresponding static values. Therefore, robust characteristics of the controller to model inaccuracies become a crucial factor to be considered in developing the perching control. The controller's robustness has to be tested in the presence of dynamic stall delay uncertainties, contributed by the unknown time histories of the τ constants given in the flow separation parameter equation (based on the Goman and Khrabov's first order lag model) [12]. The perching control problem differentiates itself from the typical trajectory tracking problem as it not only requires the tracking of the maneuver in 3D domain, but also has a strict terminal velocity requirement and terminal orientation constraints for landing. As such, the controller algorithm should ensure the convergence of all the spatial trajectory, attitude and velocity parameters for the underactuated perching problem. Their convergence to the respective reference states should be guaranteed even in the presence of variations in the initial perching conditions such as velocity, attitude and errors in altitude, which are typically encountered. As the perching maneuver is sensitive to both the variations in initial state conditions and perturbations during the course of the maneuver, investigations of the controller's ability to overcome these factors are emphasized in this paper.

Control applications in perching have mostly been discussed in the context of 2D perching maneuvers. To cope with the nonlinearity of the post-stall flow regimes, Moore and Tedrake devised a feedback controller that uses the Linear Quadratic Regulator (LQR)-Trees algorithm to execute 2D perching of a simple glider in presence of variations in initial positions [13]. The region of attraction to an optimal, offline-generated trajectory is firstly identified using the sum of squares (SOS) optimization with an S-matrix parameterization. The region consists of trajectories with varying initial conditions. Its respective visual projection represents a funnel that will lead to the final landing point. For the sample initial point that leads to the landing point within the specified tolerance, an optimal trajectory is generated for that point and the process of identifying the funnel is repeated and added to the tree-library. The control technique however does not address model inaccuracies and perturbations during the perching maneuver. Choe and Hovakimyan handled model uncertainties in perching such as parametric uncertainties in altitude and velocity, input gain uncertainties and time delay in control commands using an adaptive $\mathcal{L}_1$ controller [14]. The control input implemented is a combination of the open-loop control input and the control input that is computed based on the plant's uncertainty estimates. Tracking the offline-generated perching trajectory was successfully shown to be better using the adaptive $\mathcal{L}_1$ controller than the nominal controller. Robustness of the controller in response to state perturbations during the perching maneuver was not investigated. Ref. [15] highlighted that the capability of sliding mode control in ensuring trajectory convergence was significantly reduced when the perching maneuver is subjected to aerodynamic perturbations. The convergence area of the controller, that depicts the region of perching starting points that results in a successful landing, decreased significantly by a factor of 3.5. Perching performance under these conditions would be deemed insufficient and should be improved. The controller's performance in the presence of rotational state perturbations and under dynamic stall uncertainty were not evaluated. As such, most perching work which are only limited to the typical 2D perching maneuvers, have overlooked the investigation of the controller's ability to handle state perturbations during the course of perching and have not considered the impact of dynamic stall uncertainty contributed by high rates of angles of attack.

The sideslip perching problem offers the challenging task of controlling all the 9 state variables $[u, v, w, p, q, r, \phi, \theta, \psi]$ in the highly nonlinear system. To handle the state perturbations and model inaccuracies in a 3D domain, a sliding mode control technique augmented with an adaptive gain is implemented by taking advantage of sliding mode control's inbuilt robustness. The proposed adaptive gain feature focuses on enhancing the contraction of the perching maneuver to the desired trajectory in spatial domain since the convergence to the optimal perching trajectory is usually a stricter requirement than the convergence to the reference attitude variables. Hence, the proposed controller serves to extend the region of contraction where trajectories with initial spatial and state displacements are ensured to converge. The formulation of the control law for the underactuated problem is presented based on a state-space framework that involves the tracking of appropriate state variables that correspond to the 4 available control inputs $(\delta_e, \delta_a, \delta_r, T)$. The performance of the controller is subsequently compared to the nominal sliding mode control (without the adaptive gain feature) under 3 different scenarios involving aerodynamic perturbations and full-state perturbations and model uncertainties due to dynamic stall.

Fig. 2. Sideslip perching trajectory in multiple views (aircraft model not drawn to scale) [9].

2. Flight Model

The perching aircraft adopts a conventional fixed-wing design in Ref. [9]. The aircraft's wings, elevators and vertical tail are characterized by the rectangular planform with NACA0012 airfoil cross-section. The fuselage is a cylindrical structure containing an elliptical cross-section with a ratio (major/minor axes) of 2.5. The propeller-driven model has a maximum thrust-to-weight (T/W) ratio of less than 1. The dynamic stall model for the aircraft aerodynamics model is taken from Goman and Khrabov [12]. As verified in Refs. [12, 16], the first order lag model is able to successfully capture the unsteady aerodynamic coefficients at high angles of attack including the undershooting and overshooting of the stall lift coefficient. In the longitudinal frame, this model is inter-dependent on both the rate of angle of attack, $\dot{\alpha}$ and the instantaneous location of flow separation, $\bar{x} \in [0,1]$, as seen in Eq. (1). The location of the flow separation over the chord is normalized and taken as a non-dimensional internal state variable. In this way, the mathematical expression in Eq. (1) can conveniently be integrated into the flight model by retaining the state-space representation.

$$\tau_1 \frac{d\bar{x}}{dt} + \bar{x} = \bar{x}_o (\alpha - \tau_2 \dot{\alpha}) \quad (1)$$

$\bar{x}_o$ represents the profile of the flow separation location for the static case of the nonlinear lift coefficient. Both time constants τ_1 and τ_2 correspond to the characteristic time scale $(c_w/2)/(\text{sqrt}(u^2 + w^2))$, in which c_w is the wing chord and u and w are the longitudinal body velocities. To model the nonlinear aerodynamic coefficient, $\bar{x}$ is used as a mixing parameter to describe the total lift contributions of the attached flow and separated flow. At any angle of attack, the total lift coefficient C_L can be defined as follows:

$$C_L(\alpha, \dot{\alpha}) = C_{L,att}(\alpha) \cdot \bar{x}(\alpha, \dot{\alpha}) + C_{L,sep}(\alpha) \cdot [1 - \bar{x}(\alpha, \dot{\alpha})] \quad (2)$$

where subscripts *att* and *sep* refer to attached flow and separated flow respectively. The same approach can be repeated to derive the drag and moment coefficients for the wing and the horizontal stabilizer. Based on the NACA0012 airfoil, the impact of $\dot{\alpha}$ on the lift coefficient is found to be significant, as depicted in Fig. 3. It describes the overshooting and undershooting behavior of the maximum lift coefficient due to high rates of angle of attack with reference to the static lift coefficient. The variations in the aerodynamic coefficients with dynamic stall have a large effect on the control inputs required for the perching trajectory. As shown in Fig. 3, should the control inputs generated for the trajectory without dynamic stall consideration be applied in the presence of dynamic stall, the perching trajectory would significantly deviate leading to a failed maneuver. The dynamic characteristic of the aerodynamic coefficients should therefore be included in the flight model. Furthermore, the control system's robustness to the corresponding dynamic stall model inaccuracies should also be ensured for the perching maneuver. A detailed description on the aerodynamic modeling approach for the aircraft, including the fuselage, has been elaborated in Ref. [9, 17].

Fig. 3. Variation of lift coefficient and perching trajectory due to rate of angle of attack.

The nonlinear flight model is developed based on the six degree of freedom (DOF) stability axes which constitutes the full state vector to be $\bar{X} = [V, \alpha, \beta, \gamma, p, q, r, \phi, \psi, x_e, y_e, z_e, \dot{x}_w, \dot{x}_t, \dot{x}_v]^T$, where V refers to the aircraft's true airspeed, α and β are the aerodynamic angles, γ is the flight path angle, p, q and r are the roll, pitch and yaw rates, ϕ and ψ are the roll and yaw angles, x_e, y_e and z_e correspond to inertial position axes, and $\dot{x}_w, \dot{x}_t, \dot{x}_v$ are the 3 separation parameters of the wing, horizontal stabilizer and vertical tail respectively. The model comprises of 15 first-order nonlinear differential equations listed from Eqs. (3) to (17). The system of equations highlights the internal state dynamic equations of the separation parameters in $\dot{x}_w, \dot{x}_t, \dot{x}_v$. The aerodynamic stability derivatives are computed using Digital Datcom program [18] and later verified using Tornado [19]. I_{xx}, I_{yy}, I_{zz} and I_{xz} are the mass moments and cross product of inertia about the body axes respectively. m refers to the aircraft's mass. C_L, C_D and C_y define the

total lift, drag and side force coefficients respectively. C_m , C_l and C_n describe the pitch, roll and yaw moment coefficients respectively. b and S indicate the wingspan and its corresponding area. c is the mean aerodynamic chord and ρ refers to the free stream air density.

$$\dot{V} = \frac{1}{m} \left[T \cos \alpha \cos \beta - \frac{1}{2} \rho V^2 SC_D (\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \delta_e) \cos \beta + \frac{1}{2} \rho V^2 SC_Y (\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \sin \beta \right] - g \sin \gamma \quad (3)$$

$$\dot{\alpha} = \frac{1}{Vm \cos \beta} \left[-\frac{1}{2} \rho V^2 SC_L (\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \delta_e) + mg \cos \alpha \cos \phi \cos (\alpha + \gamma) \right] + \frac{g}{V \cos \beta} \sin \alpha \sin (\alpha + \gamma) - \frac{T \sin \alpha}{Vm \cos \beta} + q - \tan \beta [p \cos \alpha + r \sin \alpha] \quad (4)$$

$$\dot{\beta} = \frac{1}{mV} \left[\frac{1}{2} C_D (\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \delta_e) \rho V^2 S \sin \beta + \frac{1}{2} \rho V^2 SC_Y (\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \cos \beta \right] - \frac{T}{mV} \cos \alpha \sin \beta + \frac{g}{V} [\cos \alpha \sin \beta \sin (\alpha + \gamma) + \cos \beta \sin \phi \cos (\alpha + \gamma)] - \frac{g}{V} \sin \alpha \sin \beta \cos \phi \cos (\alpha + \gamma) + p \sin \alpha - r \cos \alpha \quad (5)$$

$$\dot{\gamma} = -\frac{1}{Vm \cos \beta} \left[-\frac{1}{2} \rho V^2 SC_L (\alpha, \dot{\alpha}, \beta, \dot{\beta}, q, \delta_e) + mg \cos \alpha \cos \phi \cos (\alpha + \gamma) \right] - \frac{g}{V \cos \beta} \sin \alpha \sin (\alpha + \gamma) + \frac{T \sin \alpha}{Vm \cos \beta} - q + \tan \beta [p \cos \alpha + r \sin \alpha] + q \cos \phi - r \sin \phi \quad (6)$$

$$\dot{p} = \frac{1}{I_{xx} I_{zz} - I_{xz}^2} \left[\frac{1}{2} \rho V^2 S b \left[I_{zz} C_l (\beta, \dot{\beta}, p, r, \delta_a, \delta_r) + I_{xz} C_n (\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \right] \right] - \frac{1}{I_{xx} I_{zz} - I_{xz}^2} \left[q r [I_{xz}^2 + I_{zz}^2 - I_{yy} I_{zz}] - p q I_{xz} [I_{xx} - I_{yy} + I_{zz}] \right] \quad (7)$$

$$\dot{q} = \frac{1}{I_{yy}} \left[\frac{1}{2} \rho V^2 S c C_m (\alpha, \dot{\alpha}, q, \delta_e) - r p [I_{xx} - I_{zz}] - [p^2 - r^2] I_{xz} \right] \quad (8)$$

$$\dot{r} = \frac{1}{I_{xx} I_{zz} - I_{xz}^2} \left[\frac{1}{2} \rho V^2 S b \left[I_{xx} C_n (\beta, \dot{\beta}, p, r, \delta_a, \delta_r) + I_{xz} C_l (\beta, \dot{\beta}, p, r, \delta_a, \delta_r) \right] \right] - \frac{1}{I_{xx} I_{zz} - I_{xz}^2} \left[q r I_{xz} [I_{xx} - I_{yy} + I_{zz}] - p q [I_{xz}^2 - I_{xx} I_{yy} + I_{xx}^2] \right] \quad (9)$$

$$\dot{\phi} = p + q \tan (\alpha + \gamma) \sin \phi + r \tan (\alpha + \gamma) \cos \phi \quad (10)$$

$$\dot{\psi} = r \cos \phi \sec (\alpha + \gamma) + q \sin \phi \sec (\alpha + \gamma) \quad (11)$$

$$\dot{x}_e = V \cos \alpha \cos \beta \cos (\alpha + \gamma) \cos \psi + V \sin \beta [\sin \phi \sin (\alpha + \gamma) \cos \psi - \cos \phi \sin \psi] + V \sin \alpha \cos \beta [\cos \phi \sin (\alpha + \gamma) \cos \psi + \sin \phi \sin \psi] \quad (12)$$

$$\dot{y}_e = V \cos \alpha \cos \beta \cos (\alpha + \gamma) \sin \psi + V \sin \beta [\cos \phi \cos \psi + \sin \phi \sin (\alpha + \gamma) \sin \psi] + V \sin \alpha \cos \beta [\cos \phi \sin (\alpha + \gamma) \sin \psi - \sin \phi \cos \psi] \quad (13)$$

$$\dot{z}_e = -V [\cos \alpha \cos \beta \sin (\alpha + \gamma) - \sin \beta \sin \phi \cos (\alpha + \gamma) - \sin \alpha \cos \beta \cos \phi \cos (\alpha + \gamma)] \quad (14)$$

$$\dot{\bar{x}}_w = \left[-\bar{x}_w + \bar{x}_{w,o} (\alpha - \tau_2 \dot{\alpha}) \right] / \tau_1 \quad (15)$$

$$\dot{\bar{x}}_t = \left[-\bar{x}_t + \bar{x}_{t,o} (\alpha - \tau_4 \dot{\alpha}) \right] / \tau_3 \quad (16)$$

$$\dot{\bar{x}}_v = \left[-\bar{x}_v + \bar{x}_{v,o} (\beta - \tau_6 \dot{\beta}) \right] / \tau_5 \quad (17)$$

At this moment, online-generation of optimal perching trajectories for a highly agile and dynamically elaborate maneuver is still computationally challenging. Its infeasible implementation obliges the use of offline-optimal trajectories where perching control strategy should be centered upon. The reference perching trajectory is generated using a dynamic optimization procedure that has been detailed in Ref. [9]. The sideslip perching maneuver is firstly modeled as a trajectory optimization problem between two fixed boundary points with the aim of minimizing the total perching distance between the boundary points. The initial boundary point is chosen to be 30 m away from the landing point and at an altitude of 3 m, where the aircraft will be travelling at 10 m/s and at the trimmed cruise steady state. The endpoint is located at the origin, where the aircraft should be at a perch-landing state where its velocity is significantly reduced and its state variables such as p, r, ϕ, ψ and the aircraft's side velocity are approximately zero. A terminal velocity with at least 70% velocity reduction is set as one of the constraints. Based on pre-specified control and state bounds and maneuver constraints, the trajectory optimization problem is transcribed into a nonlinear programming problem (NLP) using orthogonal collocation on finite elements method [20]. It is subsequently solved using fmincon based on interior point algorithm. The result is an optimal sideslip perching trajectory depicted in Fig. 2 which is used as the reference trajectory for control simulations.

3. Adaptive Gain Sliding Mode Control Formulation

Consider the 3D sideslip perching problem:

$$\dot{\bar{X}} = \bar{F}(\bar{X}, \bar{U}) + \Delta \bar{F}(\bar{X}, \bar{U}) \quad (18)$$

where $\bar{X} = [V, \alpha, \beta, \gamma, p, q, r, \phi, \psi, x_e, y_e, z_e]^T$ and $\bar{U} = [\delta_e, \delta_a, \delta_r, T]^T$. $\Delta \bar{F}(\bar{X})$ represents the uncertain dynamics in the nonlinear system which are contributed by the unmodeled time histories of the τ constants in the internal separation parameter equations (Eq. (1)). Using the sliding mode control, a perturbed state in the perching dynamics would naturally be driven towards a predefined sliding surface, s . The natural contraction towards the time-varying sliding surface is driven by a reaching law that inherently provides the system with robustness that is useful for perching [21, 22]. A constant rate reaching law is defined as follows:

$$\dot{\bar{s}} = -\bar{H} \operatorname{sgn}(\bar{s}) \quad (19)$$

$\bar{s}$ vector is a set of 4 sliding surfaces that correspond to the 4 available control inputs. $\bar{H}$ is a positive definite matrix in the form of $\operatorname{diag}[h_1, \dots, h_4]$, where $h_i > 0$, and sgn refers to the sign function. The constant rate reaching law indicates that the perturbed state will converge towards the sliding surface at a constant rate that is dictated by the magnitude of h_i . Once on the sliding surface, the perturbed state will slide exponentially towards its desired value to complete the maneuver tracking. The set of 4 sliding surfaces $[s_1, \dots, s_4]$, which are functions of α, ϕ, ψ and γ variables respectively, are expressed in Eq. (20). The 4 variables, α, ϕ, ψ and γ are selected out of the 9 available state variables by using contraction analysis [23]. This technique successfully identifies those 4 system variables that are required to be tracked to overall convergence of the system in the presence of external disturbances [15].

$$\bar{s} = \begin{bmatrix} (\dot{\alpha} - \dot{\alpha}_d) + \lambda_1 (\alpha - \alpha_d) \\ (\dot{\phi} - \dot{\phi}_d) + \lambda_2 (\phi - \phi_d) \\ (\dot{\psi} - \dot{\psi}_d) + \lambda_3 ((\psi - \psi_d) - y_s) \\ (\dot{\gamma} - \dot{\gamma}_d) + \lambda_4 ((\gamma - \gamma_d) - z_s) \end{bmatrix} \quad (20)$$

$y_s = \tan^{-1} \left(\frac{y_{ed} - y_e}{\rho_1} \right)$ and $z_s = \tan^{-1} \left(\frac{z_e - z_{ed}}{\rho_2} \right)$ are necessary spatial correction factors that are included in the sliding functions to ensure that the perching trajectory in the spatial 3D domain is also part of the invariant set. As

such, perching trajectories that deviate due to external disturbances or when initialized at different positions can track back to the desired optimal trajectory. Contraction behavior of both the state and spatial variable profiles to the respective optimal profiles are guaranteed. $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \rho_1$ and ρ_2 are positive constants. To illustrate the stability of controller, the following Lyapunov function $\bar{V}$ is assigned:

$$\bar{V}(\bar{s}) = \frac{1}{2} \bar{s}^T \bar{s} \quad (21)$$

$$\begin{aligned} \dot{\bar{V}}(\bar{s}) &= \frac{1}{2} (\dot{\bar{s}}^T \bar{s} + \bar{s}^T \dot{\bar{s}}) \\ &= \bar{s}^T \dot{\bar{s}} \\ &= -\bar{s}^T \bar{H} \operatorname{sgn}(\bar{s}) < 0 \end{aligned} \quad (22)$$

Since $\bar{V}(\bar{s})$ is positive definite and $\dot{\bar{V}}(\bar{s})$ is negative definite, the contraction to the sliding surface $s = 0$ for the nominal controller is guaranteed, that is, $\lim_{t \rightarrow \infty} \bar{s}(t) = 0$. Once on the sliding surface s_1 , based on the transfer function of s_1 and α that is proper and exponentially stable, $\lim_{t \rightarrow \infty} (\alpha - \alpha_d) = \lim_{t \rightarrow \infty} (\dot{\alpha} - \dot{\alpha}_d) = 0$. This applies for the 3 other sliding surfaces too. For the derivation of the state-space sliding mode control approach, the system is first established to be control-affine form by linearizing the system locally about its instantaneous control input.

$$\bar{F}(\bar{X}, \bar{U} + \Delta \bar{U}) = \bar{F}(\bar{X}, \bar{U}) + \left(\frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial \bar{U}} \right)_{\bar{F}, \bar{U}} \Delta \bar{U} \quad (23)$$

The free-order switching control law can be derived as:

$$\dot{\bar{s}} = \frac{\partial \bar{s}}{\partial \bar{X}} \dot{\bar{X}} + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d \quad (24)$$

Combining the reaching law with Eqs. (23) and (24), it yields

$$\begin{aligned} &\Rightarrow \frac{\partial \bar{s}}{\partial \bar{X}} \left(\bar{F}(\bar{X}, \bar{U}) + \left(\frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial u} \right)_{x,u} \Delta \bar{U} + \Delta \bar{F}(\bar{X}, \bar{U}) \right) + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d = -\bar{H} \operatorname{sgn}(\bar{s}) \\ \Delta \bar{U} &= - \left(\frac{\partial \bar{s}}{\partial \bar{X}} \frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial u} \right)_{x,u}^{-1} \left(\frac{\partial \bar{s}}{\partial \bar{X}} \bar{F}(\bar{X}, \bar{U}) + \bar{H} \operatorname{sgn}(\bar{s}) + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d + \Delta \hat{\bar{F}}(\bar{X}, \bar{U}) \right) \end{aligned} \quad (25)$$

By using $\Delta \hat{\bar{F}}$ to handle the unknown $\Delta \bar{F}$ and substituting Eq. (25) into Eq. (24), the reaching law equation will have an additional term as shown below.

$$\dot{\bar{s}} = -\bar{H} \operatorname{sgn}(\bar{s}) - \left(\Delta \hat{\bar{F}}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U}) \right) \quad (26)$$

As the bounds on $|\Delta \bar{F}(\bar{X}, \bar{U})|$ are known to be $-0.1 F_i \leq \Delta F_i \leq 0.1 F_i$, $\bar{H}$ should be defined to always dominate $(\Delta \hat{\bar{F}}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U}))$ in order to guarantee Eq. (22):

$$\left| \Delta \hat{\bar{F}}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U}) \right| \leq 0.2 \bar{F}(\bar{X}, \bar{U}) \leq \bar{H} \quad (27)$$

The use of $\operatorname{sgn}(\bar{s})$ in Eq. (25) would introduce the high frequency chattering in the control inputs. To eliminate this, $\operatorname{sgn}(\bar{s})$ would be replaced with $\operatorname{sat}\left(\frac{\bar{s}}{\varphi}\right)$ where φ represents a thin boundary layer around each sliding surface. The thickness of the boundary layer should be optimal as a boundary layer that is too large would lead to a lowered tracking accuracy. A narrow boundary layer however, would not be successful in smoothening out the chattering. A time-varying boundary layer that can be actively adjusted to match the intensity of the dynamic stall uncertainties is used. To guarantee that the boundary layer is reachable for the deviated state in finite time, the shrinking rate of the

boundary layer should be lower than the approaching rate of the boundary layer. This is done by satisfying the Lyapunov condition in Eq. (28) in which $\bar{H}$ is to be defined as Eq. (29).

$$\dot{\bar{V}}(\bar{s}) = -\bar{s}^T \bar{H} \text{sgn}(\bar{s}) - \bar{s}^T \left(\Delta \bar{F}(\bar{X}, \bar{U}) - \Delta \bar{F}(\bar{X}, \bar{U}) \right) \leq |\bar{s}^T| \left(\text{diag}(\dot{\bar{\varphi}}) - \bar{\eta} \right) \quad (28)$$

$$\bar{H} = \bar{K} - \dot{\bar{\varphi}} \quad (29)$$

where $\bar{K} = \text{diag}[k_1, \dots, k_4]$, $\bar{\varphi} = \text{diag}[\varphi_1, \dots, \varphi_4]$, $h_{ij} = (k_i - \dot{\varphi}_i) \delta_{ij}$, $k_i = 0.2 F_i + \eta_i$, $k_i > 0$, δ_{ij} is the Kronecker delta and $\bar{\eta} = [\eta_1, \eta_2, \eta_3, \eta_4]^T$, $\eta_i > 0$. The variation of the boundary layer, $\dot{\bar{\varphi}}$ can be expressed as [24]:

$$\dot{\bar{\varphi}} = \bar{K}_{\bar{X}_d} - \bar{\lambda} \bar{\varphi} \quad (30)$$

Here, the thickness of the boundary layer is allowed to vary based on the dynamics of the unmodeled uncertainty. By substituting Eq. (30) into Eq. (29),

$$\bar{H} = \bar{K} - (\bar{K}_{\bar{X}_d} - \bar{\lambda} \bar{\varphi}) \quad (31)$$

$\bar{\lambda}$ is a diagonal matrix where $\lambda_i > 0$. The control law in Eq. (25) becomes:

$$\Delta \bar{U} = - \left(\frac{\partial \bar{s}}{\partial \bar{X}} \frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial u} \right)_{x,u}^{-1} \left(\frac{\partial \bar{s}}{\partial \bar{X}} \bar{F}(\bar{X}, \bar{U}) + (\bar{K} - (\bar{K}_{\bar{X}_d} - \bar{\lambda} \bar{\varphi})) \text{sat} \left(\frac{\bar{s}}{\bar{\varphi}} \right) + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d \right) \quad (32)$$

Eq. (32) forms the nominal sliding mode controller that has an unadaptable contraction rate for deviated trajectories. By augmenting the controller with an adaptive gain feature, a more powerful reaching law can be acquired. The proposed control law can be expressed as follows:

$$\Delta \bar{U} = - \left(\frac{\partial \bar{s}}{\partial \bar{X}} \frac{\partial \bar{F}(\bar{X}, \bar{U})}{\partial u} \right)_{x,u}^{-1} \left(\frac{\partial \bar{s}}{\partial \bar{X}} \bar{F}(\bar{X}, \bar{U}) + (\bar{K} - (\bar{K}_{\bar{X}_d} - \bar{\lambda} \bar{\varphi})) \text{sat} \left(\frac{\bar{s}}{\bar{\varphi}} \right) + \bar{Q} \bar{s}_{\text{spatial}} + \frac{\partial \bar{s}}{\partial \bar{X}_d} \dot{\bar{X}}_d \right) \quad (33)$$

$$\bar{U}_{\text{required}} = \bar{U}_{\text{adp}} + \Delta \bar{U}_{\text{adp}} \quad (34)$$

$$\text{where } \bar{s}_{\text{spatial}} = \begin{bmatrix} (\dot{\alpha} - \dot{\alpha}_d) + \lambda_1 (\alpha - \alpha_d) \\ (\dot{\phi} - \dot{\phi}_d) + \lambda_2 (\phi - \phi_d) \\ \tan^{-1}(y_e - y_{e_d}) \\ \tan^{-1}(z_{e_d} - z_e) \end{bmatrix}$$

The adaptation law:

$$\dot{\bar{Q}} = \bar{\kappa} |\bar{s}_{\text{spatial}}| \text{sign}(|\bar{s}_{\text{spatial}}| - \bar{\varepsilon}) \quad (35)$$

The lower limit condition:

$$\bar{Q}_{\min} = \bar{\mu} \quad (36)$$

where $\bar{Q} = \text{diag}[q_1, \dots, q_4]$, $q_i > 0$ are adaptable gain constants and $\bar{\kappa}$, $\bar{\varepsilon}$, $\bar{\mu}$: diagonal constant matrices with positive elements.

The addition of the $\bar{Q} \bar{s}_{\text{spatial}}$ correction term transforms the original constant rate reaching law into a proportional rate reaching law. For s_1 and s_2 sliding functions in Eq. (20), the $\bar{Q} \bar{s}_{\text{spatial}}$ term ensures exponential convergence to the sliding surfaces during the reaching phase of the sliding mode control. Thus, the rate of contraction of $\bar{s}$ trajectories to the respective sliding surfaces ($\bar{s} = 0$) would be hastened. Furthermore, the $\bar{Q} \bar{s}_{\text{spatial}}$ term acts as a correction term that involves $\bar{Q}$ being adaptively tuned based on the $\bar{s}_{\text{spatial}}$ term and the $\bar{\kappa}$ term. The $\bar{\kappa}$ term dictates

the rate of adaptation in the control law. The adaptive feature is unique as the original $\bar{s}$ term is modified to $\bar{s}_{spatial}$ term to give more priority and add more emphasis to spatial convergence. Spatial convergence should be given precedence over the corresponding orientation angle's convergence because it is relatively easier to adjust the orientation angle of the aircraft once spatial convergence has been somewhat achieved.

Without the $\bar{Q}\bar{s}_{spatial}$ term, the sliding functions s_3 and s_4 in Eq. (20) are dominated by the convergence of heading angle ψ and flight path angle γ respectively. This is because, as the trajectories spatially converge, the sensitivity of the sliding functions s_3 and s_4 is very high with respect to small angular deviations and low with respect to the spatial correction terms y_s and z_s (for the given values of ρ_1 and ρ_2). Moreover, the impact of the y_s and z_s terms near the end of the maneuver is low without the initial impetus provided by the large deviations at the start of the maneuver. Although the thickness of the boundary layer $\bar{\varphi}$ as well as the gain terms ρ_1 and ρ_2 can be reduced to magnify the effect of y_s and z_s near the sliding planes, this would only destabilize the system and introduce chattering. Increasing the constants λ_i in Eq. (20) for a higher rate of contraction in the sliding phase would lead to the undesired excitation of high frequency dynamics. Therefore, when subjected to perturbations, sluggishness in the contraction of the deviated trajectory can be observed when it is spatially close to the reference trajectory. In addition, if the deviation of the heading angle ψ and lateral displacement in y_s destructively interfere with each other, it leads to a delay in the convergence of the states to the desired reference values. In the worst case scenario, if in s_3 and s_4 respectively, $(\psi - \psi_d) = y_s \neq 0$ and/or $(\gamma - \gamma_d) = z_s \neq 0$, a false interpretation of convergence to the sliding plane can occur, which is a special case of delay caused by the destructive interference.

This ineffectiveness is prevented by the $\bar{Q}\bar{s}_{spatial}$ term that will constantly drive the spatial trajectories towards the desired profiles. In the presence of destructive interference in the s_3 and s_4 terms, the $\bar{Q}\bar{s}_{spatial}$ term generates the required control inputs that will prioritize the convergence of the spatial variables. The control gain matrix $\bar{Q}$ will be adaptively tuned until desired control gain values in $\bar{Q}$ are acquired, leading to spatial convergence. In this perspective, a more powerful reaching law is generated that ensures a faster contraction rate for perturbed variables and better performance in executing perching maneuvers.

The adaptive gain feature allows the controller to be used in systems with models that have uncertainties which are bounded but unknown [25]. This is advantageous for the case of perching maneuvers where model uncertainty due to dynamic stall can be unknown yet bounded. Due to the adaptation law, the adaptive gain constants $\bar{Q}$ will increase to a value sufficient for counteracting the associated state deviations. The system can therefore handle a larger upper bound of system parameter uncertainty.

The adaptation process in Eq. (35), which drives the magnitude of control gains in $\bar{Q}$, reverses when $|\bar{s}_{spatial}| < \bar{\epsilon}$ [26]. A boundary layer $\bar{\epsilon}$ ensures that $\bar{Q}$ stops increasing once intended spatial tolerance is achieved and subsequently allows the gain $\bar{Q}$ to decline, while $|\bar{s}_{spatial}| < \bar{\epsilon}$. This regulates the gains in $\bar{Q}$ to be at the lowest possible level required to maintain the given spatial accuracy and acquire higher efficiency. This allows the controller to adjust its gain $\bar{Q}$ and find the appropriate values to compensate for the current uncertainties and perturbations. However, minimum values for $\bar{Q}$ are assigned to be $\bar{\mu}$ to ensure only positive values of $\bar{Q}$ are used (Eq. (36)). The controller bounds which meet the specifications of the currently available actuators in the market are specified in Table 1.

Table 1 Control bounds.

Lower bound	Control variable	Upper bound	Max rate
-40 deg	δ_e, δ_r	40 deg	70 deg/s
-15 deg	δ_a	15 deg	70 deg/s
0	T/W	0.8	1.5 N/s

4. Control Performance Analysis

The controller is tasked to provide stability to the optimal perching trajectory under different scenarios such as variations in initial spatial and state deviations given aerodynamic perturbations, full state perturbations and dynamic stall inaccuracies in the maneuver. Its tracking performance in the perching simulations is analyzed and compared to the nominal controller. The $\bar{K}$ and $\bar{\lambda}$ matrices have been initially determined via manual iteration until the desirable control performance is achieved. Control goals for the perching performance are listed as follows: 1) Lands within 0.1

m lateral and vertical proximity of the target location (0.3% of the 30 m horizontal perching distance) 2) Terminal states of the orientation angles and velocity should be within 10% of the respective reference states. 3) Terminal pitch angle should not exceed 90 degrees. 4) For the reference state that ends at zero, 10% of the maximum value that the state reaches during the maneuver would be selected as the tolerance. Considering the small time window and the aggressiveness of the perching maneuver, these goals are rather challenging but useful in identifying the effectiveness of the controller.

A. Aerodynamic Perturbations

External disturbances such as wind gusts can be treated as aerodynamic perturbations, that include V, α and β variables, which are unpredictable in nature. In this analysis, these aerodynamic disturbances are modeled as random pulse perturbations introduced maneuver, as shown in Table 2. The velocity perturbations are randomly specified for at least 10% of their nominal terminal values. The angular perturbations are implemented in at least 10% of their aerodynamic stall angles. The three locations at $x_e = -20$ m, -15 m and -10 m, are specifically chosen as they represent the critical region where the aircraft has to maneuver precisely around the aggressive bend. Few locations instead of a single location at -15 m mark are selected as it is more representative of intermittent aerodynamic gusts in the outdoor environment. These perturbations are not introduced after the -5 m mark because the window for control action would be too small for any significant control impact, since the aircraft is less than a second away from the target location.

Table 2 Aerodynamic perturbations.

Perturbations	-30m	-20m	-15m	-10m
ΔV	1 m/s	-0.4 m/s	+0.3 m/s	-0.3 m/s
$\Delta \alpha$	4 deg	-2 deg	+3 deg	+2 deg
$\Delta \beta$	-3 deg	-3 deg	+2 deg	+2 deg

To effectively compare the performance of the adaptive gain sliding mode controller with the nominal sliding mode controller in handling aerodynamic perturbations, an iterative test is conducted to trace the outline of the convergence area of the perching trajectory based on the varying initial position in the yz plane. Fig. 4 bears the comparison of the convergence area between the sliding mode controller with the adaptive gain feature (black outline with blue-shaded region) and without the adaptive gain feature (dashed outline with grey-shaded region). The sliding mode control with the adaptive gain feature shows superior performance in terms of convergence area as it extends relatively 2.3 times larger than the nominal controller. The symmetric operational area of the modified controller, marked by the red dashed line, can be expressed as $\frac{y^2}{2.85^2} + \frac{(z-3)^2}{2^2} \leq 1$. The effectiveness of the controller is further validated by its operational area that is about 5.7 times larger than the operational area of the nominal controller, which sizes up to a circular area of only 1m radius (portrayed by blue dashed line).

The simple addition of the adaptive gain component $\bar{Q}\bar{s}_{spatial}$ is able to provide a more powerful reaching law when required, such as in the presence of perturbations. The strength of the reaching law is adaptable, which is driven by the $\bar{s}_{spatial}$ term in the adaptation law in Eq. (35). Unlike in the nominal controller, the reaching law is rigid and provides limited rate of contraction. Hence, perturbed trajectories initiated far from the optimal starting point do not converge fast enough and are not able to meet the control goals. This results in the convergence area of the nominal controller to diminish substantially.

Fig. 4. Convergence area comparison between the nominal and adaptive gain sliding mode controllers given aerodynamic perturbations at $x_e = -30, -20, -15, -10$ meters.

A test point (2.5,3.75) that lies within the convergence area of the adaptive gain sliding mode control but outside the convergence area of the nominal sliding mode control, is selected to illustrate the effect of the adaptable reaching law. Subjected to the aerodynamic perturbations in Table 2, the results of the perching maneuver initiating at the test point are tabulated in Table 3 and depicted in Figs. 5 to 7. Fig.5 highlights the perching trajectory of the sliding mode control with the adaptive gain (in solid line) being successfully guided back the optimal path while satisfying the specified control goals. With lowered delay in contraction, terminal states using the adaptive gain sliding mode control are achieved within desired tolerances, where the deviations are -0.0272 m and -0.0366 m in the y-axial and z-axial direction respectively. The perching trajectory executed using the nominal controller did not achieve sufficient contraction and ended with terminal deviations of 0.1334 m (33.4%) and 0.2602 m (160%) in the lateral and vertical directions. Table 3 indicates the error comparison of the state variables specific to the landing criteria. The state variables V , ϕ , ψ and θ for both control simulations are found to be within the landing tolerances specified. This is supported by Fig. 6 which shows the temporary divergence of the ϕ , ψ and γ trajectories to accommodate the corrections required for the spatial and aerodynamic perturbations, yet is still able to converge back to the respective reference trajectories in time. Therefore, the cause of the landing failure for the nominal controller is solely due to the lack of spatial convergence that has been addressed by the adaptive gain feature, as marked in Table 3.

Table 3 Convergence criteria comparison subjected to aerodynamic perturbations.

Deviation	Nominal	With Adaptive Gain	Tolerance
Δy (m)	0.1334	-0.0272	0.1
Δz (m)	0.2602	-0.0366	0.1
ΔV (m/s)	0.26	0.24	0.3
$\Delta \phi$ (deg)	2.34	0.24	6.3
$\Delta \psi$ (deg)	0.69	-0.29	5.1
θ (deg)	66.84	69.01	90

Fig. 5. Trajectory convergence comparison between 2 the sliding mode controllers in response to aerodynamic perturbations.

Fig. 6. Aerodynamic variable profile comparison between the 2 sliding mode controllers in response to aerodynamic perturbations.

The side view in Fig. 5 illustrates the rapid vertical contraction of the trajectory in the last third of the maneuver provided by the adaptive gain component. As the control gain q_4 of $\bar{Q}$ in Fig. 8 increases continuously due to the initial sustained vertical deviation, the added contraction allows the aircraft to nullify its vertical deviation effectively. However, the trajectory of the control without the adaptive gain component remains relaxed and deviated due to limited contraction strength. While the vertical trajectory profile is being swiftly corrected, the aircraft simultaneously manages to maintain its lateral convergence due to the large adaptive gain accumulated by q_3 . The adaptive gain feature mitigates the effect of ‘false’ convergence to the s_3 sliding plane. However, for the nominal controller, the final moments of the corresponding perching trajectory show that the aircraft stagnates at the deviated lateral position while the aircraft tries to recover its heading angle ψ . As highlighted, the correction of the lateral deviation has not been prioritized by the controller due to destructive addition of the positive heading angle deviation and negative lateral deviation. The integration of the adaptive gain feature leads to different control inputs as plotted in Fig. 7. No control-input chattering is observed due to the implementation of the time-varying boundary layer $\bar{\varphi}$.

Fig. 7. Turn rate and control input comparison between the 2 sliding mode controllers in response to aerodynamic perturbations.

Fig. 8. Control gain matrix Q in response to aerodynamic perturbations.

B. Full –State Perturbations

To further illustrate that the reliability of the controller, it is tested to handle a more challenging environment in which unstructured full-state perturbations are introduced along the perching maneuver. Building on the case of aerodynamic perturbations from Table 2, perturbations to the remaining variables $\gamma, p, q, r, \phi, \psi$ of normalized magnitude are integrated to form the full-state perturbation scenario. Therefore, full-state perturbations hereby imply deviations to the $V, \alpha, \beta, \gamma, p, q, r, \phi, \psi$ state variables. These full state perturbations will be introduced in two scenarios, namely Scenario 1 and 2. These scenarios are characterized with initial state and spatial displacements as shown below. Their magnitudes measure at least within 10% of their respective nominal values or operational range.

Scenario 1: $\Delta V = 1$, $\Delta \alpha = 4^\circ$, $\Delta \beta = -3^\circ$, $\Delta \gamma = 3^\circ$, $\Delta p = 0.03$, $\Delta q = -0.05$, $\Delta r = 0.03$, $\Delta \phi = -2^\circ$, $\Delta \psi = 2^\circ$, $\Delta y = -3$, $\Delta z = +1$

Scenario 2: $\Delta V = -1$, $\Delta \alpha = -3^\circ$, $\Delta \beta = -2^\circ$, $\Delta \gamma = 1^\circ$, $\Delta p = -0.05$, $\Delta q = 0.04$, $\Delta r = -0.03$, $\Delta \phi = 3^\circ$, $\Delta \psi = 2^\circ$, $\Delta y = +2$, $\Delta z = -1$

As observed in Table 4, a set of $V, \alpha, \beta, \gamma, p, q, r, \phi, \psi$ perturbations are specified for each location. The magnitude of the perturbations at each location is standardized and differs randomly only in sign. Table 4 represents the ‘basic’ form of perturbations in which perturbations for velocity and orientation angles are specified within 5% of the landing velocity and maximum operating value respectively. The aerodynamic perturbations are at least 5% of their respective stall angles. As such, all angular perturbations are fixed at 1.5 degrees. Angular turn rates are set at 0.04 rad/s which are within 5% of their maximum operating values.

This standardization provides a congenial way to represent disturbances of higher intensities by multiplying the archetypal perturbation set in Table 4 by a scalar factor k_p . Other sets of perturbations of larger magnitude can be derived. In this way, $k_p = 2$ would imply perturbations consisting of $\Delta V = 0.3$, $\Delta \alpha = -3.0^\circ$ and $\Delta p = 0.08$ rad/s and so on. Hence, based on the different values of k_p , a parametric analysis on the reliability and robustness of the controller in handling different intensities of perturbation can be carried out.

Table 4 Full-state perturbations.

Perturbations		-20m	-15m	-10m
$k_p = 1$	ΔV	-0.15 m/s	+0.15 m/s	-0.15 m/s
	$\Delta \alpha$	-1.5 deg	+1.5 deg	+1.5 deg
	$\Delta \beta$	-1.5 deg	+1.5 deg	+1.5 deg
	$\Delta \gamma$	+1.5 deg	+1.5 deg	+1.5 deg
	Δp	-0.04 rad/s	+0.04 rad/s	-0.04 rad/s
	Δq	+0.04 rad/s	+0.04 rad/s	+0.04 rad/s
	Δr	-0.04 rad/s	-0.04 rad/s	-0.04 rad/s
	$\Delta \phi$	+1.5 deg	+1.5 deg	-1.5 deg
	$\Delta \psi$	-1.5 deg	+1.5 deg	+1.5 deg

Based on the Scenario 1 and 2 and the standardized full-state perturbation scheme in Table 4, the performance of the adaptive gain sliding mode controller in handling full-state perturbations is analyzed by testing different k_p values. The spatial deviations of the simulation results are presented in Table 5 below.

Table 5 Terminal Trajectory deviations comparison based on full-state perturbations.

Scenario		Dev (m)	$k_p = 1.0$	$k_p = 1.5$	$k_p = 2.0$	$k_p = 2.5$	$k_p = 3.0$
NOMINAL	1	Δy	-0.0856	-0.1504	-0.1671	-0.2398	-0.3183
		Δz	0.0881	0.0837	0.1140	0.1257	0.1367
	2	Δy	-0.0340	-0.0683	-0.1477	-0.1382	-0.2786
		Δz	-0.0387	-0.0318	-0.0098	0.0595	0.3492
ADAPTIVE GAIN	1	Δy	-0.0167	-0.0490	-0.0747	-0.0527	0.0102
		Δz	-0.0214	-0.0570	-0.0106	-0.0709	0.0937
	2	Δy	-0.0190	-0.0758	-0.0668	-0.0531	0.9385
		Δz	-0.0206	-0.0060	0.0306	-0.0252	-0.4621

Table 5 highlights the enhancement to the system’s robustness characteristics by using the sliding mode control with adaptive gain feature. Based on the k_p level in Scenario 1, the adaptive gain feature makes the system more robust by at least a factor of 3 as the maximum k_p that the aircraft is able to handle increases from 1.0 to 3.0. The Δy and Δz deviations, that are observed to increase as k_p rises, are dealt with successfully as they are shown to have reduced values instead. In Scenario 2, improvement of k_p from 1.5 to 2.5 is obtained. Overall, noting that these are substantial perturbations prescribed to all the dynamic and orientation state variables, this result signifies an explicit upgrade in robustness acquired through the integration of the adaptive gain feature in the control system.

To further illustrate the improvement of the control strategy, the results of the perching maneuver for Scenario 1 at $k_p = 2.5$ are tabulated in Table 6 accompanied with Figs. 9 to 12. As illustrated by the blue line in Fig. 10, the

sliding mode control with the adaptive gains allows a faster rate of convergence in the lateral direction. Although this compromises the vertical trajectory's convergence slightly, the vertical deviation is further worsened by the perturbations introduced into the maneuver. However, due to the built-up of required gains, the resulting contraction of the vertical trajectory allows the aircraft to recover rapidly and converge back to its optimal path. The steady build-up of the vertical gain is shown by q_4 , the rising red dashed line in Fig. 9.

Observing the y -axial deviation profile based on the top view in Fig. 11, the perching trajectory manages to relatively converge to the reference solution between the -15 m to -10 m region despite the perturbations at -20 m and -15 m mark. The built-up of the lateral gain is denoted by rising blue line in control gain q_3 . As the aircraft passes the -16.88 m mark, the q_3 line in Fig. 9 is observed to decline. This is due to the presence of the ε_3 boundary condition in the adaptation process of Eq. (35) that ensures q_3 stops increasing and starts declining once 0.1 m spatial tolerance is achieved. However, the large perturbation introduced at the -10m mark causes a sudden veer in the different direction. This deviation can be noticed for both trajectories executed by the 2 sliding mode controllers. The influence of the adaptive gain is clearly shown here as q_3 starts increasing again once the trajectory is out of the tolerance zone which allows the trajectory to swiftly navigate back the optimal path and satisfy the lateral convergence criteria.

Immediate response in the corresponding control gains can be observed in Fig. 9 as the q_1 and q_2 profiles go through sudden changes at the occurrence of perturbations. Tighter ε_1 and ε_2 boundary conditions specified led to q_1 and q_2 profiles to decline rather slowly when $|s_1| < \varepsilon_1$ and $|s_2| < \varepsilon_2$. By tracking the 4 variables $\alpha, \phi, \psi, \gamma$, the control inputs of the adaptive gain sliding mode controller have ensured the contraction in the entirety of the required and other state profiles of the sideslip perching maneuver.

Table 6 Convergence criteria comparison subjected to full-state perturbations.

Deviation	Nominal	With Adaptive Gain	Tolerance
Δy (m)	-0.2398	-0.0527	0.1
Δz (m)	0.1257	-0.0709	0.1
ΔV (m/s)	0.14	0.27	0.3
$\Delta \phi$ (deg)	0.13	0.09	6.3
$\Delta \psi$ (deg)	-1.42	0.53	5.1
θ (deg)	67.95	67.80	90

Fig. 9. Control gain matrix Q of the modified controller in response to full-state perturbations.

Fig. 10. Trajectory convergence comparison between the 2 sliding mode controllers in response to full-state perturbations $k_p = 2.5$.

Fig. 11. Aerodynamic variable profile comparison between the 2 sliding mode controllers in response to full-state perturbations $k_p = 2.5$.

Fig. 12. Turn rate and control input comparison between the 2 sliding mode controllers in response to full-state perturbations $k_p = 2.5$.

C. Dynamic Stall Model Inaccuracies

The effect of the dynamic stall on the optimal perching trajectory is significant. Therefore, sliding mode control's robustness has to be tested in the presence of dynamic stall delay uncertainties which are contributed by the unknown time history of τ given in the flow separation parameter equation of Eq. (1). The dynamic stall delay phenomenon for the wing, horizontal and vertical tail is subsequently modeled using three separate equations that in total feature six τ constants (τ_1 to τ_6).

The constants τ_1 to τ_4 are scaled by a factor of $(c/2)/(\text{sqrt}(u^2 + w^2))$ while τ_5 and τ_6 are scaled by $(c_v/2)/(\text{sqrt}(u^2 + v^2))$. However, τ is an unknown variable and its trend is unpredictable. Though τ can be determined via experiment, these constants are dependent specifically on the rate of pitch motion the surface is experiencing. However, perching maneuver involves a multitude of pitching motions, that makes assigning a constant value for τ , not feasible. Handling this uncertainty becomes important as these constants indirectly dictate the magnitude of the aerodynamic forces and moments which greatly affect the overall sideslip trajectory, as depicted in Fig. 13 below. Let the current set of τ_1 to τ_6 constants used for the reference perching trajectory be $\hat{\tau}$. Other perching trajectories in Fig. 13 based on multiples of these $\hat{\tau}$ values are generated using the same optimal control inputs to simulate cases where $\hat{\tau}$ values are modeled inaccurately. Hence, a reasonable range of 0.25 to 2.5 has been selected

for the analysis. As observed, divergence in the perching trajectories and their corresponding desired states, that are caused by the different $\hat{\tau}$ values, illustrates the importance of the controller's robustness in handling τ deviations. Table 7 indicates the lateral and vertical divergences for each $\hat{\tau}$ case that are highly deviated from the tolerance values specified (shaded cells are not within 0.1 m tolerance). The exponential sliding mode controller with the adaptive gain is subsequently tested to analyze its capability in overcoming the significant effect of τ model inaccuracies.

Table 7 Spatial divergence comparison subjected to varying τ values.

Deviation	$0.25 \hat{\tau}$	$0.5 \hat{\tau}$	$1.5 \hat{\tau}$	$2.0 \hat{\tau}$	$2.5 \hat{\tau}$	Tolerance
Δy (m)	-4.325	-0.2699	-0.8210	-2.750	-5.0327	0.1
Δz (m)	-2.1138	-0.010	0.0407	-0.4051	-3.5634	0.1

Table 8 Spatial divergence under sliding mode control based on varying τ constants.

Deviation	$0.25 \hat{\tau}$	$0.5 \hat{\tau}$	$1.5 \hat{\tau}$	$2.0 \hat{\tau}$	$2.5 \hat{\tau}$	Tolerance
Δy (m)	-0.0536	-0.0042	-0.0175	-0.0519	-0.1302	0.1
Δz (m)	-0.0133	-0.0112	-0.0140	-0.0180	-0.0565	0.1

Fig. 13. 3D perching trajectory comparison under different time constants based on optimal control inputs.

Fig. 14. 3D perching trajectory comparison under different time constants based on sliding mode control with adaptive gain.

The simulation results of the sliding mode control in presence of the varying set of τ values are plotted in Fig. 14. As observed, the trajectories successfully maintained their convergence when subjected up to a set of τ values between

0.25 $\hat{t}$ to 2.0 $\hat{t}$. In this range, the terminal spatial deviations are small as indicated in Table 8. The sliding mode controller is not successful in coping with the dynamic stall inaccuracy at about 2.5 $\hat{t}$ and above as its terminal lateral deviation exceeds the 0.1 m allowance. Nonetheless, the sliding mode controller still manages to correct its vertical deviation entirely and reduce its 97.4% of the lateral deviation and comes within close range to the 0.1 m tolerance. The corresponding desired state variables are also achieved within 10% of the nominal values. These results have further validated the effectiveness of the sliding mode control with the adaptive gain strategy in handling the dynamic stall model uncertainties.

5. Conclusions

This paper presents a sliding mode control strategy with an adaptive gain feature that provides stability to the optimal sideslip perching maneuver solutions when subjected to varying initial spatial and state conditions, state deviations in the course of perching and model inaccuracies associated with the dynamic stall delay. By controlling the roll angle, ϕ , angle of attack, α , flight path angle, γ and the heading angle, ψ , the proposed sliding mode controller successfully provides the required contraction of the deviated trajectories to the optimal path and the overall convergence of all the state variables in the system. The nominal sliding mode control (without the adaptive gain) offers only a constant rate reaching law which limits the rate of contraction of the trajectories. It is found that when the nominal sliding mode control is employed, convergence of deviated trajectories is not attained fast enough before the end of the maneuver. The adaptive gain feature provides an explicit upgrade in robustness by providing an adaptable reaching law that can induce a faster contraction rate in the presence of these perturbations. The augmented sliding mode controller acquires a convergence area 2.3 times larger than the nominal sliding mode control and is able to sustain higher intensity of perturbations by at least 2 fold. The controller is also successful in handling dynamic stall model uncertainties of a considerable range, which includes handling 0.25 to 2.0 times the actual delay time constants.

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