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A State-Space Thermal Model Incorporating Humidity and Thermal Comfort for Model Predictive Control in Buildings

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Abstract

A major challenge in applying Model Predictive Control (MPC) to building automation and control (BAC) is the development of a simplified mathematical model of the building for real-time control with fast response times. However, building models are highly complex due to nonlinearities in heat and mass transfer processes of the building itself and the accompanying air-conditioning and mechanical ventilation systems. This paper proposes a method to develop an integrated state-space model (SSM) for indoor air temperature, radiant temperature, humidity and Predicted Mean Vote (PMV) index suitable for fast real-time multiple objectives optimization. Using the model, a multi-objective MPC controller is developed and its performance is evaluated through a case study on the BCA SkyLab test bed facility in Singapore. The runtime of the MPC controller is less than 0.1 s per optimization, which is suitable for real-time BAC applications. Compared to the conventional ON/OFF control, the MPC controller can achieve up to 19.4% energy savings while keeping the PMV index within the acceptable comfort range. When the MPC controller is adjusted to be thermal-comfort-dominant that achieves a neutral PMV index at most office hours, the system can still bring about 6% in energy savings as compared to the conventional ON/OFF control.

Key words: Model predictive control; state-space model; optimization; thermal comfort; building automation and control; linearization.

1. Introduction

Energy consumption by buildings has been increasing steadily over the years and constitutes 20% to 40% of total energy use in developed countries (Pérez-Lombard et al., 2008). In Singapore, this number is as high as 38% (Building and Construction Authority, 2014) and more than half is consumed by air-conditioning and mechanical ventilation (ACMV) systems to maintain acceptable indoor conditions. This large proportion of energy consumption by buildings calls for improvements in building energy efficiency through technological development. As an advanced and well-established method of process control, model predictive control (MPC) has recently been gaining popularity in building automation and control (BAC) for indoor climate regulation and energy optimization (Wang and Ma, 2008, Afram et al., 2014). Many recent studies as summarized by Kwadzogah et al. (2013), Afram et al. (2014), Hilliard et al. (2016) and Mirakhorli et al. (2016) have demonstrated the energy-saving potential of MPC. The human thermal comfort improvement potential of MPC is also presented by several studies (Freire et al., 2008, Castilla et al., 2014, Hazyuk et al., 2014, Chen et al., 2015, Ascione et al., 2016, Yaser et al., 2017).

Developing a simplified building mathematical model has been a major challenge of MPC implementation for BAC (Cigler et al., 2013). Typically, MPC requires an appropriate model of building physics for the prediction of future responses in order to optimize the control inputs of various systems (ACMV, lighting, etc.). As such, the mathematical model of the building must be sufficiently accurate in predicting building dynamics and computationally efficient for real-time control and optimization. Various building energy simulation tools such as EnergyPlus (DoE, 2016), and TRNSYS (Klein, 2010) can be used to construct and solve for the physical model of the building. However, these models, and their accompanying solutions, contain numerous nonlinearity that complicates and is prohibitive to real-time control and optimization (Cigler et al., 2013). An alternative would be to develop simplified physical models of buildings in linearized state-space form that is of medium to high fidelity and is computationally more efficient (Cigler et al., 2013).

Generally, models used for calculating building thermal processes can be classified into three categories: black-box, gray-box and white-box model (Cigler et al., 2013, Killian, Kozek, 2016). In a black-box model, the structure and parameters are identified directly from input/output data (Wesley et al., 2014, Dehkordi & Candanedo, 2016), which normally is obtained through experimental results or from building energy performance simulation tools.

However, large amounts of input and output data are required for the black-box model that makes it very difficult to obtain sufficient data to construct an accurate model, especially for existing buildings that are already operational. In contrast to black-box model, a gray-box model relies on a thermal resistance-capacitance (RC) network to describe building thermal dynamics (Sirok' et al., 2011, He et al., 2016). It is an analogy to electric circuitry where temperature gradients and heat fluxes are represented by electrical potentials and currents, respectively. A gray-box model retains the physical meaning of building thermodynamics, but the R and C parameters are identified by data obtained from real building measurements or building energy simulation tools. Similar to the black-box model, the gray-box model also requires sufficient input/output data from real buildings in order to identify model parameters.

A white-box model, on the other hand, is a physics-based model that also relies on a thermal RC network. However, the R and C parameters are derived directly from detailed physical geometry and material properties of the building. As the white-box model can be created based on the as-built drawings without supporting data from experiments, it is applicable for different buildings and purposes. The white-box model has been applied to build both full-scale (Sturzenegger et al., 2012) and reduced-order (Nielsen, 2015) building thermal models. A white-box can also be used to train a gray/black-box model (Kim et al., 2012, Hu, Karava, 2014) or simplified with model reduction techniques (Gouda et al., 2002).

In many previous studies, the focus had been on the prediction of indoor temperature or cooling loads while latent loads and human thermal comfort are seldom covered (Kramer et al., 2012). In fact, it has been shown that latent loads can take up a significant portion of annual energy consumption (up to 60%) by ACMV systems in commercial buildings situated in the tropics (Qi et al., 2012). Humidity is also a key factor in the predicted mean vote (PMV) thermal comfort index (Fanger, 1970). Therefore, in order to effectively achieve optimal building energy savings and indoor thermal comfort using MPC, it is necessary to include both humidity and thermal comfort index in the prediction model.

In this work, a mathematical building model, represented in state-space form is developed based on Building Construction Authority (BCA) SkyLab Test Bed facility to predict indoor temperature, humidity, and thermal comfort. Thermal and humidity dynamic models are created using second-order RC network based on the as-built drawings. A linear

approximation method is then used to linearize the ACMV cooling coil model and PMV calculation model. The proposed model, including room temperature and humidity ratio, is then calibrated and validated against experimental data obtained in BCA SkyLab. Finally, based on the proposed model, a MPC controller is proposed to control the non-linear ACMV system of a virtual test bed in MATLAB/Simulink environment. Energy savings and thermal comfort improvement potentials of the proposed MPC controller are demonstrated through the simulations.

2. Mathematical modeling

The test building in this work is the Test Cell of the BCA SkyLab located in the BCA Academy in Singapore. BCA SkyLab has two side-by-side, identically equipped experimental cells with full-height window façade on one side, as shown in Figure 1. The two cells are furnished to mimic the interior of typical office environment. The Test cell was used to develop the dynamic model of the building with the properties of the envelope summarized in Table 1. The model considers the partition walls between the cells and the control room to be adiabatic on the assumption that the temperature differences between these three compartments are negligible.

The internal heat load in the Test Cell was physically simulated by four heating dummies and two humidifying devices, as shown in Figure 2 and summarized in Table 2. There were four typical office workstations in the cell. Each heating dummy, housing a light bulb of 75 W inside, was fabricated according to EN 14240 (2004) to represent sensible heat load from human occupants, following ASHRAE (2009) for an office worker with moderating duties. A heating dummy was put at the seating position of each workstation. The sensible heat from plug load was set at 180 W per workstation following the recommended load factor for an office space of medium/heavy load density in ASHRAE (2009). Out of the 180 W, 60 W comes from under the desk. The remaining 120 W of sensible heat were placed on top of the desks to represent other office equipment such as monitors and printers. The plug loads were simulated by putting light bulbs with corresponding wattage in empty desktop computer casings. Moisture, equivalent to a total of 220 W latent heat, was generated by the two humidifying devices to simulate the latent load of four occupants.

A fan coil unit (FCU) ACMV system was installed in the Test Cell, as shown in Figure 3. The FCU operates at constant fan speed (air volume), as shown in Table 2, and is controlled by a

thermostat that is linked to the actuator that regulates the chilled water flow rate through the cooling coil in the FCU. The thermostat controlled the chilled water flow rate according to room temperature set point that is determined by a proportional–integral–derivative (PID) control. The conditioned air is then supplied to the cell through four 4-way spread type ceiling air diffusers and subsequently returned through the six slot type diffusers on the ceiling. The lighting was regulated by a Digital Addressable Lighting Interface (DALI) automated dimming system according to the illuminance measured by DALI sensor which faces the exterior window. The blind angles were adjusted by an in-house developed control algorithm according to indoor Daylight Glare Probability (DGP) and daylight penetration. Details can be found in Lamano et al., (2018) and Zhou et al., (2017).

In developing the mathematical model, the room space and ACMV system are considered as a complex open system with air-moisture mixture gas flow and vapor-water phase change. In the ACMV system, as shown in Figure 3, the fresh air mixes with the return air before entering the cooling coil in the FCU. Both the sensible and latent loads of the mixed air are dealt with by the cooling coil and part of the moisture in the mixed air stream will leave the ACMV system at the cooling coil due to the condensation. The ACMV system supplies the conditioned air (supply air) to the room space. Inside the room space, there are internal heat and moisture sources including human, lighting, and equipment. The room space also exchanges heat with the surroundings through the building envelope, as illustrated in Figure 4.

The moisture and heat balance inside the room space are described by Equations (1) and (2), respectively.

$$m_{air,z} \frac{d\psi_z}{dt} = \dot{m}_{occ} + \dot{m}_{ACMV}, \quad (1)$$

$$m_{air,z} C_{air} \frac{dT_z}{dt} = Q_{inte,sen} + Q_{env} + Q_{ACMV}, \quad (2)$$

where Q is heat flow rate (W), $\dot{m}$ is mass flow rate (kg/s), m is mass (kg), T is temperature (K) and ψ is humidity ratio (kg/kg). The subscript *inte* refers to internal, *z* refers to thermal zone, *sen* refers to sensible and *env* refers to envelope. $\dot{m}_{occ}$ is the moisture load from human occupants and $\dot{m}_{ACMV}$ is moisture gain from ACMV system. Q_{inte} , Q_{ACMV} , and Q_{env} are internal sensible heat gain, heat brought by ACMV system and heat gain from building envelope.

2.1 Heat gain through building envelope

RC model is adopted to represent the building elements with various heat conduction and thermal storage. As suggested by Gouda et al., (2002), the second-order RC model is capable of simulating both lightweight and heavyweight buildings with acceptable accuracy, and will be used in this study to describe heat transfer processes in the building envelope.

Figure 5 shows the lumped parameter and RC representations of a building wall model. The wall is virtually split into two aggregates and only heat conduction in the normal direction is considered. In each of the aggregates, temperature is assumed uniform. The RC model is a 5R2C model which includes two thermal capacitances of the two aggregates, three thermal conduction resistances of aggregates and two surface thermal resistances between surface and air (outer and inner surfaces). The same model treatment is applied to the roof and the floor.

Based on the RC network representation, the state parameters representing the temperatures of the two aggregates can be derived as follows,

$$\frac{dT_1}{dt} = - \left[\frac{1}{C_{the,1}(R_{sur,o} + R_{sur,1})} + \frac{1}{C_{the,1}R_{1,2}} \right] T_1 + \frac{1}{C_{the,1}R_{1,2}} T_2 + \frac{1}{C_{the,1}(R_{sur,o} + R_{sur,1})} T_o + \frac{R_{sur,o}R_{sur,1}}{C_{the,1}(R_{sur,o} + R_{sur,1})} q_{sol} \alpha_{sur,o}, \quad (3)$$

$$\frac{dT_2}{dt} = - \left[\frac{1}{C_{the,2}(R_{sur,z} + R_{2,sur})} + \frac{1}{C_{the,2}R_{1,2}} \right] T_2 + \frac{1}{C_{the,2}R_{1,2}} T_1 + \frac{1}{C_{the,2}(R_{sur,z} + R_{2,sur})} T_z + \frac{R_{sur,z}R_{2,sur}}{C_{the,2}(R_{sur,z} + R_{2,sur})} \frac{q_{sol,tra} A_{win}}{A_{int}}, \quad (4)$$

where the symbols R , C , α and q refer to thermal resistance (K-m/W), thermal capacity (J/K), absorptance and heat flux (W/m²), respectively. Subscripts *sur*, *o*, *1*, *the*, *sol*, *z*, *tra*, *int* and *win* refer to surface, outside, aggregate number, thermal, solar radiation, thermal zone, transmitted, interior and window, respectively. The last term in Equations (3) and (4) represents the effect of the solar energy absorbed by exterior and interior surfaces.

The interior surface temperature can be modelled by,

$$T_{sur,z} = (T_2 R_{sur,z} + T_z R_{2,sur}) / (R_{2,sur} + R_{sur,z}) + R_{2,sur} R_{sur,z} q_{sol,tra} A_{win} / A_{env} (R_{2,sur} + R_{sur,z}). \quad (5)$$

Based on the temperature of all the interior surfaces, the mean radiant temperature T_{mr} can be calculated by,

$$T_{mr} = (T_{sur,z}^1 + T_{sur,z}^2 + \dots T_{sur,z}^N) / (A^1 + A^2 + \dots A^N), \quad (6)$$

where $T_{sur,z}^N$ and A^N are the surface temperature (K) of surface and area (m^2) of the interior surface N , respectively. In the equations, the R and C of aggregates are determined by the materials' thermal properties. The parameter R between surface and air is determined by convective and long-wave radiation heat transfer. The convection heat transfer can be described by a linear equation,

$$q_{conv} = h_{conv}(T_{sur} - T_{air}), \quad (7)$$

where the convection heat transfer coefficient ($W/K\cdot m^2$), h_{conv} , for outer surface can be estimated by average ambient velocity (Palyvos, 2008). h_{conv} for inner surface can be assumed to be 4.2 and 3.8 for horizontal and vertical surfaces, respectively (Walton, 1983).

The long-wave radiation heat transfer is calculated by a nonlinear equation,

$$q_{rad} = \varepsilon\sigma(T_{sur}^4 - T_{air}^4). \quad (8)$$

To obtain the constant parameter R in (5), the radiation heat transfer equation is linearized as,

$$q_{rad} = \varepsilon\sigma(T_{sur} + T_{air})(T_{sur}^2 + T_{air}^2)(T_{sur} - T_{air}) = h_{rad}(T_{sur} - T_{air}), \quad (9)$$

where the symbols ε , σ and h refer to surface emittance, Stefan-Boltzmann constant and heat transfer coefficient, respectively. Subscripts *air* and *rad* refer to indoor or outdoor air and radiation, respectively. The radiation heat transfer coefficient h_{rad} is considered constant since its variation is usually very small compared to its absolute value (Park, 2013) in normal indoor conditions.

2.2 Heat gain from double-pane glazing window and automated blinds system

The Test Cell of BCA SkyLab has a full-height double-pane glazing window and two sets of indoor Venetian blinds installed immediately behind the window, as shown in Figure 2. The daylight redirecting upper blinds covered the top quarter height of the window and the glare-control lower blinds covered the remaining three-quarter height. The daylight reflectances of the slats of the upper and lower blinds were 0.91 and 0.56, respectively. The heat gain from the window-blind system consisted of two components, 1) conduction heat gain due to temperature difference between indoor and outdoor as well as the 2) transmitted solar heat gain. The first component affects the indoor air directly while the second component was assumed to distribute on all the interior surfaces evenly. The latter is for simplicity, with the assumption that solar heat absorbed by any interior surface would quickly distribute to other interior surfaces by radiation.

The double-pane glazing window model is shown in Figure 6. The heat storage of the glass layers are assumed to have negligible effect on room air temperature since the glass layers are thin (6 mm in this case) (Winkelmann, 2001). Winkelmann (2001) also suggested that the glass layers can be assumed to be opaque to long-wave radiation and is valid for most glass products. In addition, the heat transfer coefficient of one glass layer is $166.7 \text{ W/(K-m}^2\text{)}$, which is far greater than the surface convective/radiative heat transfer coefficient. Thus, the conductive thermal resistance of the glass layers can also be neglected. In Figure 6, the subscripts *abs* and *ref* refer to absorption and reflection, respectively.

The model for heat gain through the window can be derived based on the RC model as

$$q_{win} = \tau_1 \tau_2 q_{sor} + \frac{[R_{sur,o} q_{sur,abs,1} + (R_{12} + R_{sur,o}) q_{sul,abs,2}]}{R_{sur,o} + R_{1,2} + R_{sur,z}} + (T_o - T_z) / (R_{sur,o} + R_{1,2} + R_{sur,z}), \quad (10)$$

where subscripts *abs* and *ref* refer to absorbed and reflected. The first two terms on the right hand side of Equation (10) are solar heat gain terms and the third term is conduction heat gain. Parameter *R* for glass surface can be calculated in a similar way to the wall, as shown in Equation (9). The *R* for air gap can be modelled as (Winkelmann, 2001)

$$q_{rad,1,2} = \frac{\sigma \varepsilon_1 \varepsilon_2}{1 - (1 - \varepsilon_1)(1 - \varepsilon_2)} (T_1^4 - T_2^4) = h_{rad} (T_1 - T_2), \quad (11)$$

$$R_{1,2} = R_{cond,gap} + 1/h_{rad}. \quad (12)$$

Subsequently, the solar heat gain coefficient (SHGC) can be calculated as,

$$SHGC = [\tau_1 \tau_2 q_{sor} + [R_{sur,o} q_{sur,abs,1} + (R_{12} + R_{sur,o}) q_{sul,abs,2}] / (R_{sur,o} + R_{1,2} + R_{sur,z})] / q_{sol}. \quad (13)$$

For control purpose of this study, a simplified blind model based on the interior attenuation coefficient (IAC) table (ASHRAE, 2009) is adopted to evaluate the thermal performance of a blind.

IAC is the effect of shading and it is defined as,

$$IAC = SHGC_{sys} / SHGC_{glz}, \quad (14)$$

where the subscripts *sys*, *glz*, refer to the window-blind system and glazing, respectively. Data selected from the IAC table according to the daylight reflectance of the upper and lower blinds are summarized in Table 3. IAC for diffuse solar radiation for three blind angles can be

read directly from Table 3 and IAC for solar direct beam radiation IAC_{beam} can be calculated by,

$$IAC_{beam} = IAC_0 + (IAC_{60} - IAC_0) \cdot \min(1, 0.02\Omega) , \quad (15)$$

where Ω refers to solar profile angle.

Based on Table 3, linear equations are fitted to describe the IAC of the two sets of blinds in the test building. For the upper blinds,

$$IAC_{dif} = -0.003\theta + 0.875 , \quad (16)$$

$$IAC_{beam} = -0.0052\theta + 0.9683 + (0.0027\theta - 0.2367) * \min(1, 0.02\Omega), \quad (17)$$

where θ and Ω refer to blind slat opening angle ($^\circ$) and solar profile angle ($^\circ$). For the lower blinds,

$$IAC_{dif} = -0.002\theta + 0.92 , \quad (18)$$

$$IAC_{beam} = -0.0034\theta + 0.975 + (0.0017\theta - 0.1483) * \min(1, 0.02\Omega) . \quad (19)$$

The solar heat gain from the window-blind system could be modelled by,

$$q_{win,bli} = SHGC \cdot IAC_{beam} I_{beam,inc} + SHGC \cdot IAC_{dif} I_{dif,inc} , \quad (20)$$

where I refers to solar radiation (W/m^2). Subscript *bli* refers to blinds, *inc* refers to incident, *if* refers to diffuse solar radiation and *beam* refers to solar beam radiation. The model for solar heat gain through the window-blind system is nonlinear, it would not be possible to be integrated within the SSM. As such, the solar heat gain will be an input parameter for SSM and the constraints are expressed as,

$$q_{win,bli}(Pos_{bli} = closed) \leq q_{win,bli} \leq q_{win,bli}(Pos_{bli} = 100\% open), \quad (21)$$

where Pos_{bli} refers to blind slat angle. With this model, the optimal blind angle can be determined by the optimal solar heat gain and Equation (21). However, in this study, the blind angle followed a pre-defined slat angle schedule that is based on an in-house developed control algorithm implemented in BCA SkyLab, as described by Lamano et al., (2018).

2.3 ACMV system

When the ACMV system is in operation, the supply air relative humidity (RH) is assumed to be 100% and the heat/mass transfer between the mixed air and the cooling coil can be described by,

$$Q_{cc} = \dot{m}_{ma}(C_{air} + C_{vap}\psi_{ma})(T_{ma} - T_{sa}) + \dot{m}_{ma}L_{vap}(\psi_{ma} - \psi_{sa}), \quad (22)$$

$$\psi_{sa} = 0.62198p_{vap,sat}/(p_z - p_{vap,sat}), \quad (23)$$

$$p_{vap,sat} = e^{(77.345+0.0057T_z-7235/T_z)/T_z^{8.2}}, \quad (24)$$

where the subscripts *ma*, *vap*, *sat* and *cc* refer to the mixed air in FCU, water vapor, saturation and cooling coil, respectively. *L* refers to the specific latent heat (J/kg) of water condensation.

The mass flow rate, temperature and humidity ratio of the mixed air can be modelled by,

$$\dot{m}_{ma} = \dot{m}_{ra} + \dot{m}_{fa}, \quad (25)$$

$$T_{ma} = (H_{fa} + H_{ra})/[\dot{m}_{ma}(C_{air} + C_{vap}\psi_{ma})], \quad (26)$$

$$\psi_{ma} = (\dot{m}_{ra}\psi_{ra} + \dot{m}_{fa}\psi_{fa})/\dot{m}_{ma}. \quad (27)$$

Then the moisture gain from ACMV system can be modelled by,

$$\dot{m}_{acmv} = \dot{m}_{sa}(\psi_{sa} - \psi_{ra}). \quad (28)$$

The heat gain from ACMV system is modelled by,

$$Q_{ACMV} = \dot{m}_{sa}(C_{air} + C_{vap}\psi_{sa})T_{sa} - \dot{m}_{sa}(C_{air} + C_{vap}\psi_{ra})T_{ra}. \quad (29)$$

Given that Equations (22) to (29) are nonlinear, which need numerical methods to solve, they are not applicable for SSM since the solution will take too long for real-time optimization and control. However, Equations (23) and (24) can be approximated by the linear regression method and for temperatures between 283.15K to 293.15K (covering the typical range of supply air temperatures), they can be approximated by

$$\psi_{sa} = 7.014 \times 10^{-4}T_{sa} - 0.1913. \quad (30)$$

The approximated results using Equation (30) are compared to those obtained from solving the nonlinear equations in Equations (22 – 29). As shown in Figure 7, the linear approximation is in good agreement with results from the nonlinear equations.

Given that the mass flow rate of water vapor is small as compared to air, the sensible enthalpy of water vapor is assumed negligible. With this assumption, Equations (22 – 29) can be simplified to,

$$\dot{m}_{ACMV} = 2.578 \times 10^{-4}\dot{m}_{fa}T_{fa} + 2.578 \times 10^{-4}\dot{m}_{ra}T_{ra} - 2.565 \times 10^7 Q_{cc} + 0.6325\dot{m}_{fa}\psi_{fa} - (\dot{m}_{fa} + 0.3675\dot{m}_{ra})\psi_{ra} - 0.07031\dot{m}_{fa} - 0.07031\dot{m}_{ra}, \quad (31)$$

$$Q_{ACMV} = 369.4\dot{m}_{fa}T_{fa} - (1005\dot{m}_{fa} + 635.6\dot{m}_{ra})T_{ra} - 0.3675Q_{cc} + 9.062 \times 10^5\dot{m}_{fa}\psi_{fa} + 1.734 \times 10^5\dot{m}_{fa} + 1.734 \times 10^5\dot{m}_{ra}, \quad (32)$$

Which are bilinear. According to the assumptions made above, the equations are valid under the conditions of $RH_{sa} = 100\%$, $283.15K < T_{sa} < 293.15K$.

When the supply and fresh air flow rates are constant, like the ACMV system in the test building, Equations (31) and (32) become linear and can be employed by SSM. In a case where the supply and fresh air flow rates are not constant, the bi-linear model can still be used for MPC with sequential quadratic programming.

2.4 Human thermal comfort

A popular method to evaluate thermal comfort condition is the Predicted Mean Vote (PMV) index proposed by Fanger (1970). This index is based on the energy balance of the human body with its surrounding environment (ISO, 1994, Alamin et al., 2017).

The PMV index has six key parameters including metabolic rate, clothing insulation, air temperature, mean radiant temperature, air velocity and air relative humidity (ASHRAE, 2010, ISO, 1994). The PMV index is calculated by

$$PMV = (0.303e^{-0.036M} + 0.028)Q_{diff}. \quad (33)$$

The difference between the internal heat production and loss, Q_{diff} , that occurs in a human body is calculated by,

$$Q_{diff} = M - Q_{work} - Q_{res} - Q_{sens} - Q_{evap}, \quad (34)$$

where

$$Q_{res} = 0.0014M(307.15 - T_{air}) + 1.72 * 10^{-5}M(5867 - p_{vap}), \quad (35)$$

$$Q_{sens} = 39.6 * 10^{-9}f_{clo}(T_{clo}^4 - T_{mr}^4) + f_{clo}h_{conv}(T_{clo} - T_{air}), \quad (36)$$

$$Q_{evap} = 0.42(M - Q_{work} - 58.15) + 3 * 10^{-3}[5733 - 6.99(M - Q_{work}) - p_{vap}]. \quad (37)$$

The cloth surface temperature can be estimated by

$$T_{clo} = T_{skin} - R_{clo}[f_{clo}h_{conv}(T_{clo} - T_{air})] - Ins_{clo}[39.6 * 10^{-9}f_{clo}(T_{clo}^4 - T_{mr}^4)] \quad (38)$$

In Equations (33 - 38), M is the metabolic rate of a human being (W), p is air pressure (Pa), f_{clo} is clothing factor, and Ins_{clo} is clothing insulation (1 clo = 0.155 m²-K/W). The subscripts clo , mr , vap , $conv$, $sens$, $evap$, res , $skin$ and $work$ refer to clothing, mean radiant, water vapor, convection, sensible, evaporation from occupant skin, respiration of occupant, skin surface, and external work, respectively.

There are two nonlinear items, radiative heat transfer $39.6 * 10^{-9} f_{clo} (T_{clo}^4 - T_{mr}^4)$ and water vapour pressure, p_{vap} , in the PMV model. The radiative heat transfer term can be linearized by,

$$\begin{aligned} Q_{rad} &= 39.6 * 10^{-9} f_{clo} [(T_{clo})^4 - (T_{mr})^4] \\ &= 39.6 * 10^{-9} f_{clo} (T_{clo} + T_{mr})(T_{clo}^2 + T_{mr}^2)(T_{clo} - T_{mr}) \\ &= h_{rad} f_{clo} (T_{clo} - T_{mr}). \end{aligned} \quad (39)$$

Then the cloth surface temperature and sensible heat loss can be simplified and re-written as

$$T_{clo} = [T_{skin} + R_{clo} f_{clo} (h_{conv} T_{air} + h_{rad} T_{mr})] / [1 + Ins_{clo} f_{clo} (h_{conv} + h_{rad})], \quad (40)$$

$$Q_{sens} = f_{clo} h_{rad} (T_{clo} - T_{mr}) + f_{clo} h_{conv} (T_{clo} - T_{air}). \quad (41)$$

The water vapor pressure can also be calculated by the following equation, for air temperatures within 293.15 K to 303.15 K covering the range of typical room temperatures,

$$p_{vap} = \psi_z p_z / (\psi_z + 0.622) \cong 1.598 \times 10^5 \psi_z. \quad (42)$$

Then the heat loss of respiratory heat and evaporation can be simplified and re-written as

$$Q_{res} = 0.0014M(307.15 - T_{air}) + 1.72 * 10^{-5}M(5867 - 1.598 \times 10^5 \psi_z), \quad (43)$$

$$\begin{aligned} Q_{evap} &= 0.42(M - Q_{work} - 58.15) + 3 \times 10^{-3} [5733 - 6.99(M - Q_{work}) - \\ &1.598 \times 10^5 \psi_z]. \end{aligned} \quad (44)$$

In a scenario where the cloth factor, metabolic rate of the occupants, external work of the occupants and room pressure can be assumed constant, likely so in a typical office environment, Equations (33), (40), (41), (43) and (44) become linear. The PMV equation can be reduced to

$$\begin{aligned} PMV &= \\ &\left[\frac{(0.6835h_{conv} + 0.005136h_{rad} + 0.06025)T_{air} + 0.6784h_{rad}T_{mr} +}{(35.71h_{conv} + 35.71h_{rad} + 418.9)\psi_z + 7.308 - 207.6h_{rad} - 207.6h_{conv}} \right] / \\ &(h_{conv} + h_{rad} + 11.73). \end{aligned} \quad (45)$$

The convective heat transfer coefficient can be assigned a constant value when there is no significant variation of air velocity and it can be determined by

$$h_{conv} = \begin{cases} 2.38(T_{clo} - T_{air})^{0.25}, & 2.38(T_{clo} - T_{air})^{0.25} > 12.1v_{air}^{0.5} \\ 12.1v_{air}^{0.5}, & 2.38(T_{clo} - T_{air})^{0.25} < 12.1v_{air}^{0.5} \end{cases}. \quad (46)$$

A representative air velocity in the occupied zone was obtained by measurement conducted in the test building at its design operation conditions, as shown in Table 2. Air velocity measurement was made at a point indicated in Figure 1 at 1.4 m above the floor by an omni-directional anemometer (Swema 03+ Draught Probe) at 1-minute intervals. The mean velocity of 0.136 m/s from this measurement was adopted as the representative air velocity

for subsequent PMV calculation. The PMV index in the test building can be calculated by

$$PMV = 0.153T_z + 0.142T_{mr} + 35.71\psi_z - 88.34. \quad (47)$$

3. Model calibration and validation

The simplified RC building model, presented in section 2, was calibrated and refined using measurement data obtained in the test building. Ten days (10th to 19th October 2017) worth of experiments were conducted to measure the room temperature and humidity responses to different conditions, as shown in Figures 8 and 9. Subsequently, the accuracy of the calibrated model was validated against the experimental data.

The test conditions in test building are shown in Table 4. From 10th to 13th October, the ACMV system and internal loads were running at their designed operation conditions, as shown in Table 2. The data obtained in these 4 days were used for model calibration. From 14th to 19th October, the data obtained in this period were used for model validation. Within this period, two days (16th and 17th) were operated in the same way as that in the calibration period. Another three days (14th, 15th, 18th) were having both the ACMV system and internal loads switched off to obtain the free responses of room air temperature and humidity to outdoor weather conditions. One day (19th) was having the ACMV system on during office hours but without internal loads. This experimental arrangement allows validation under different sets of conditions.

Weather, room conditions, and ACMV system parameters were measured in the test building, as summarised in Table 5. The locations of these sensors are indicated in Figures 1 and 3. Sensor readings were recorded by the data acquisition system in BCA SkyLab at one-minute intervals. For the ACMV system, parameters of the air loop (dry bulb temperature, humidity and flow rates) and the chilled water loop (temperature, flow rate) were measured for calculating cooling power supply as the input for the simplified RC building model. The weather conditions were also measured by sensors installed on the rooftop and were used as inputs for the RC model. The room conditions including air temperature and humidity were measured as the benchmark for subsequent model calibration and validation.

In RC building models, the parameters R and C of building components are the key factors that affect the simulation performance. Thus, the R and C parameters are tuned in this calibration procedure. For building envelope, the R and C can be calculated by thermal

properties of its material as the initial values. Other indoor thermal mass such as furniture, ducts, etc. is represented by a 1R1C node which is only connected to room air temperature node. The R and C are assigned an estimated value and bounded between 0 and positive infinity. The Trust-Region-Reflective Least Squares optimization method (Coleman & Li, 1996) is then employed to tune the RC model parameters in MATLAB environment. The objective is to minimize the sum squared error of room temperature described by

$$J = \text{Minimize}(\sum_{k=0}^N (T_{z,est} - T_{z,exp})^2), \quad (48)$$

where $T_{z,est}$ and $T_{z,exp}$ refer to simulated room temperature (K) and measured room temperature (K), respectively.

Good agreement between the simulated room temperature (using the calibrated RC model) and the measured room temperature is found, as shown in Figure 9. The mean absolute percentage error (MAPE) of room temperature is 1.55% in the validation days. The air humidity ratio response of the calibrated RC model is also validated. Figure 9 shows the good agreement between the simulated and measured humidity ratios during office hours and the MAPE is 4.93%. Since the RC model does not consider the moisture penetration from the ambient through building cracks, the simulated humidity ratio result during the non-office hours (ACMV off) is constant, which deviates from the measurement data. However, the mismatch in non-off hours does not affect the MPC performance in office hours since the MPC is a closed loop control.

4. Control scenarios

The 1st order dynamic equations (1)-(4) can be written as,

$$dx_i/dt = \sum_{j=1}^{n_x} A_{i,j}x_j + \sum_{k=1}^{n_u} B_{i,k}u_k, \quad (49)$$

In Equation (49), $\mathbf{x}$ is a state vector including indoor air temperature, humidity ratio as well as the aggregate temperature of building envelopes. The symbol $\mathbf{u}$ is an input vector consisting of the weather conditions, internal gains and ACMV system control variables. $\mathbf{A}$ and $\mathbf{B}$ are state and input matrices for calculating state varying ratio ($\dot{\mathbf{x}}$). The symbol n refers to number of variables and the subscripts i , j and k are indices for states and inputs. For instance, in Equation (3), T_1 and T_2 are state parameters, and T_o and q_{sol} are input parameters which can be measured.

The output parameters, indoor air temperature, indoor humidity ratio and PMV, are calculated

by 1st order algebraic equations based on the input $\mathbf{u}$ and state $\mathbf{x}$ as shown in Equation (50).

$$\mathbf{y}_l = \sum_{j=1}^{n_x} \mathbf{C}_{l,j} \mathbf{x}_j + \sum_{k=1}^{n_u} \mathbf{D}_{l,k} \mathbf{u}_k, \quad (50)$$

where $\mathbf{y}$, $\mathbf{C}$ and $\mathbf{D}$ are output vector, output matrix and feedthrough matrix, respectively. The subscript l is an index for outputs.

With that, the mathematic model of the test building becomes state-space form. The input, state and output vectors, and matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are summarized in Tables 6 - 10. Since the outputs T_z , and ψ_z are also state parameters, there is no feedthrough from inputs. For the output PMV (calculated by Equations (5), (6) and (43)), it requires feedthrough from measured inputs parameters T_{fa} , Q_{win} and constant 1. Thus, the developed SSM is adequate for building MPC controllers.

In order to test the implementation and performance of the proposed MPC controller, a nonlinear model with 97 states of BCA SkyLab was built in MATLAB/Simulink to represent the true building as a virtual test bed. The virtual test bed includes all the three rooms of BCA SkyLab, the interactions between different rooms as well as air leakage. In contrast to the SSM, the nonlinear virtual test bed model was solved by finite difference method (FDM) to obtain the heat conduction in the envelope and theoretical nonlinear equations are used to represent the ACMV system and thermal comfort.

For testing the MPC, the optimal supplied cooling power generated by the MPC controller is sent to the FCU of the ACMV system in the virtual testbed to achieve the optimized thermal comfort and energy saving, as shown in Figure 10. The objective function of the MPC controller is defined by

$$J = \text{Minimize}(\sum_{k=0}^N \hat{Q}_{t+k|t}^2 + \sum_{k=0}^N (W_{PMV} \widehat{PMV}_{t+k|t})^2 + \sum_{k=0}^N W_{\epsilon} (\epsilon_{t+k|t})^2). \quad (51)$$

The objective function seeks to minimize the cooling power as represented by the first term on the right hand side of Equation (50). The PMV index deviation from thermal neutrality is represented by the second term. In the equation, ϵ , W_{PMV} and W_{ϵ} are the slack variable, weighting factor of the significance of indoor thermal comfort and constraint violation penalty weighting factor, respectively. $\hat{Q}$ and $\widehat{PMV}$ are normalized cooling energy and PMV index, respectively, where

$$\hat{Q} = Q / (Q_{max} - Q_{min}), \quad (52)$$

$$\widehat{PMV} = PMV / (PMV_{max} - PMV_{min}). \quad (53)$$

PMV_{max} and PMV_{min} are the upper and lower limits of PMV index. Q_{max} and Q_{min} are the upper and lower limits of FCU cooling capacity. Specifically, the constraints of PMV index are set $-0.5 < PMV < 0.5$ for general thermal comfort, according to ASHARE (2010) and of FCU cooling capacity are between 0 W and 4000 W during office hours according to the cooling coil capacity. The room air temperature is not constrained since the constraints of PMV already ensure the indoor thermal comfort. The MPC controller is implemented in MATLAB/Simulink environment. To avoid non-solution in the optimization routine, soft constraints are adopted to allow the PMV to exceed the constraints within a certain tolerance range. The tolerance range is defined by as

$$\widehat{PMV}_{ll} - \epsilon \cdot ECR_{ll} < \widehat{PMV} < \widehat{PMV}_{ul} + \epsilon \cdot ECR_{ul} \quad (54)$$

where ECR is the Equal Concern for the Relaxation in MATLAB (Bemporad et al. 2017). In Equation (54), subscripts *ll* and *ul* are the lower limit and lower limit for the constraint of PMV index, respectively. The MPC controller seeks to compromise between the soft constraints and the objective function to solve the optimal slack variable. If the optimal slack variable is 0, the MPC controller will not violate the constraints. In this study, the constraint violation penalty weighting factor and ECR are 100000 and 1. The combination represents small allowable violations according to Bemporad (2017). The control interval and prediction horizon for the MPC controller are 3 minutes and 10 control intervals, respectively.

Three simulation scenarios are considered in this study:

1. ON/OFF control of FCU cooling power, representing conventional control method. The cooling power by FCU is maximum (ON) when the room air temperature reaches 25°C and reduced to 0 W (OFF) when the room air temperature decreases to 23°C. This range of room air temperature is set according to the recommended comfort temperature in Singapore Standard 554 (Building and Construction Standards Committee, 2009).
2. MPC control with a low weighting factor (W_{PMV}) of 1 is used in the objective function making the energy saving term to dominate in the optimization.
3. MPC control with a high weighting factor of 4 is used in the objective function making the thermal comfort term to dominate in the optimization.

During office hours (9 am to 6 pm, Monday to Friday), the blinds follow the schedule as described by Lamano et al., (2018). The fan in FCU is on to provide the design supply air and fresh air flow rates, as described in Table 2. The fractions of lighting and equipment load with

respect to their respective full load are 1 (representing switched on). During non-office hours, the blinds are set at open position. FCU fan, lighting and equipment are switched off. The weather data of Singapore from International Weather for Energy Calculations (ASHRAE, 2001) is used to represent a typical year. Simulations of all the three scenarios are conducted for January, which is during the monsoon season of Singapore when the weather conditions can exhibit rapid changes. The simulation studies are performed in MATLAB/Simulink which is installed in a computer with Intel(R) Core(TM) i7-7500U (2.7 - 2.9 GHz) CPU and 24 GB memory.

5. Results and discussion

The running time per optimization in the MPC controller was 0.082 s for Scenario 2 and are 0.077 s for Scenario 3. Each optimization included solving the SSM to obtain the optimal control signal. The running time is very small compared to the control interval (180 s). This result indicates that the MPC controller developed in this study is computationally efficient for real-time BAC applications.

Figure 11 presents the simulation results (the indoor PMV, air temperature and normalized total cooling power supplied by FCU) of the three scenarios of a sunny day (2nd January) in the simulation period. Looking at the results during office hours, the PMV, room air temperature and cooling power of the MPC scenarios (Scenarios 2 & 3) have much less variations than scenario 1 with ON/OFF control. It indicates that possible disturbance to occupants due to variation of indoor air condition was much better controlled by MPC than the conventional on/off type control. The PMV of Scenario 2 and 3 always approach 0.5 and 0, respectively (with certain tolerance due to soft constraints). Since the MPC controller is a multiple objectives controller, the controller always seek to compromise between minimizing energy consumption and minimizing PMV deviation from 0 depending on the preset weighting factor. For Scenario 2, the MPC controller always seeks for least energy consumption while PMV is still within the tolerance from the upper limit, during office hours. For Scenario 3, the PMV is kept around 0 during office hours, indicating the best thermal comfort out of the three scenarios. For Scenario 1, the PMV and room air temperature exhibit fluctuating patterns during office hours because of the fluctuation of cooling power due to the ON/OFF actions. The ACMV system and internal loads are off during non-office hours.

Figure 12 presents the simulation results of the three scenarios of a rainy day (18th January).

Similar to the results shown in Figure 11 for sunny day, the PMV, room air temperature and cooling power of the MPC scenarios have much less variations than Scenario 1 for office hours. Figure 12a shows that the PMV index is kept close to 0.5 throughout the office hours in this day but the index is kept slightly below 0.5 instead of being kept slightly above 0.5 as observed in a sunny day (Figure 11a). This is due to the lesser cooling power demand in the rainy day than the sunny day as shown in Figure 12 c). Similar to that observed in a sunny day, Scenario 3 has the best thermal comfort, approaching 0 throughout office hours, among the three scenarios in a rainy day. Figure 11 and 12 show that MPC is able to maintain very stable indoor air conditions and thermal comfort for occupants despite weather conditions change.

The PMV and normalized cooling power distributions during office hours in the month of January are plotted in box-and-whisker plots and histograms, as shown in Figure 13 and Figure 14, respectively. Figure 13 exhibits the outlier, maximum, 3rd quartile, median, mean, 1st quartile and minimum values of the PMV and normalized cooling power and Figure 14 shows the percentage of time of PMV, and normalized cooling power at indicated values during office hours. The daily PMV MAD from 0 and daily mean normalized cooling power during office hours are also plotted in Figure 15 for workdays.

Figure 13 a) shows that more than 3 quarters ($> 75\%$) of the office hours are having negative PMV values in Scenario 1. It indicates that the ON/OFF control could overcool the room and lead to extra energy consumption for the majority of the time. This echoes Figure 14 b) that the normalized cooling power is running at maximum in nearly 80% of the time during office hours and Figure 15 b) that the normalized cooling power is always the largest among the three scenarios. Table 11 shows that Scenario 2 consumes least cooling energy among the three scenarios but the mean PMV index is the highest, slightly beyond the upper limit of 0.5. However, Scenario 2 achieves the most stable indoor environment as indicated by having the lowest standard deviation of PMV during office hour of 0.092 among the three scenarios. The stable indoor environment can improve human thermal comfort according to ASHRAE (2010). The overall energy savings for Scenario 2 compared to Scenario 1 is 19.4%. Scenario 3 achieves the best thermal comfort among the three scenarios, with near 0 median PMV. Over 90% of the office hours are having PMV values within -0.09 to 0.12 and the daily mean PMV deviation from 0 during office hours is below 0.1 in 20 workdays out of the total 21 workdays, which indicates that the indoor environment is very close to thermal neutrality.

With this high level of thermal comfort, Scenario 3 still achieves 6% of energy savings compared to Scenario 1, as shown in Table 11. The energy savings are estimated based on the electricity consumption calculated using an assumed chiller COP (coefficient of performance) of 5.86 from the specification of the chiller used in BCA SkyLab. The percentage of energy savings by the MPC scenarios are calculated by

$$\% \text{ energy savings} = 100\% \times (EC_{ON/OFF} - EC_{MPC})/EC_{ON/OFF}, \quad (55)$$

where EC refers to energy consumption.

5 Conclusion

This paper proposes a method to construct a state-space model of a building, which can be used to predict indoor temperature, humidity and thermal comfort for indoor environment control with MPC. The RC model is adopted to represent the integrated heat and moisture dynamic in the building to reduce the number of states. The heat transfer and dehumidification processes in the ACMV system and PMV index calculation model are linearized by linear approximation of the nonlinear items in the theoretical models. The method allows the development of SSM based on as-built drawing with physical meaning directly, instead of training a black or gray model by measuring a large number of input/output data. A simplified SSM having 11 states based on the Test Cell of the BCA SkyLab testing facility is developed using as-built drawing data. The SSM is calibrated and validated with experimental data obtained in BCA SkyLab.

An MPC controller is developed with two objectives: 1) minimize the total cooling energy consumption of FCU; 2) minimize the deviation of PMV index from 0 to achieve best thermal comfort. Simulations are conducted for three scenarios including an ON/OFF scenario representing conventional control and two MPC control scenarios with one in energy-saving-dominant setting and another one in thermal-comfort-dominant setting. The simulations test the proposed SSM in a nonlinear virtual test bed having 97 states in the MATLAB/Simulink environment. The proposed SSM is shown to be suitable for real-time indoor environment control with MPC. The proposed MPC controller using the simplified SSM requires less than 0.1 s for solving the optimization problem, suggesting that the controller is suitable for real-time BAC applications. By operating at the thermal-comfort-dominant setting, the MPC controller can maintain excellent thermal comfort at close to thermal neutrality while still able to achieve 6% energy savings as

compared to the conventional on/off control. Switching to the energy-saving-dominant setting, the MPC controller pushes the energy savings to 19.4% while keeping the PMV value at around 0.5 that is at the upper bound of the comfortable range.

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Figure 4 Heat and moisture balance inside the Test Cell of BCA SkyLab

Symbols T , q , A , C , α and R refer to temperature, heat flux, surface area, thermal capacity, absorptance and thermal resistance ;
Subscripts sur , sol , the , z , win , int , tra and o refer to surface, solar radiation, thermal, thermal zone, window, interior, transmission and outside.

Figure 5. Model of building envelope of BCA SkyLab

Symbols T , q , C , and R refer to temperature, heat flux, thermal resistance and thermal capacity ;
Subscripts sur , sol , the , z , tra , abs , ref and o refer to surface, solar radiation, thermal, thermal zone, transmitted, absorbed, reflected and outside.

Figure 6. Double-pane low-E glazing window model and its RC representation

Figure 7 Comparison of nonlinear equations and linear equation

Figure 8. Test condition for model calibration and validation

Figure 9. Comparison of simulated a) room air temperature and b) humidity ratio with measured data

Figure 10. Schematic diagram of the MPC controller test using the virtual test bed

Figure 11. Indoor a) PMV, b) room air temperature and c) normalized cooling power supplied by FCU. Comparing results from three scenarios on 2nd January

Figure 12. Indoor a) PMV, room b) air temperature and c) normalized cooling power supplied by FCU. Comparing results from three scenarios on 18th January

Figure 13. Indoor a) PMV and b) normalized cooling power distributions of three scenarios during January

Figure 14. Percentage of time indoor PMV (a) and normalised cooling power (b) are at the indicated values during office hours of January

Figure 15. (a) Daily MAD of PMV from 0, and, (b) daily mean normalized cooling power during office hours of workdays in January

Table 1. Building envelope properties of BCA SkyLab

Envelop	External wall	Internal wall	Roof	Floor	Window
Layers	<ul style="list-style-type: none"> Aluminium composite panel Air gap Fiber cement board Gypsum plasterboard Rockwool insulation Plasterboard Rockwool insulation Plasterboard Rockwool insulation Gypsum plasterboard 	<ul style="list-style-type: none"> Gypsum plasterboard Rockwool insulation Plasterboard Rockwool insulation Plasterboard Rockwool insulation Gypsum plasterboard 	<ul style="list-style-type: none"> Screed board Aluminium composite panel Air gap Aluminium tile 	<ul style="list-style-type: none"> Rotating platform Air gap Closed cell insulation Air gap Floor steel panel Finishes 	<ul style="list-style-type: none"> Low E glass Air gap Clear glass
U value	0.22 W/m ² K	0.2 W/m ² K	0.245 W/m ² K	0.25 W/m ² K	1.56 W/m ² K

Table 2 Operation schedule of the BCA SkyLab Test Cell

Parameter	Value
Occupancy rate	0.092 person/m ² floor area
Internal heat loads	
• Equipment	16 W/m ² floor area
• People (Sensible heat)	75 W/person
• People (Latent heat)	55 W/person
• Lighting	8.22 W/m ² floor area
Conditioned air supply flow rate	539.4 L/s
Fresh air supply flow rate	26.2 L/s
Office hours	9 am – 6 pm of weekday

Table 3. Data extracted from ASHRAE IAC table (ASHRAE, 2009)

	IAC _{dif}		IAC _{beam,0}		IAC _{beam,60}	
Blind angle	Upper blind	Lower blind	Upper blind	Lower blind	Upper blind	Lower blind
0°	0.86	0.91	0.98	0.98	0.74	0.83
45°	0.77	0.85	0.71	0.81	0.6	0.74
90°	0.59	0.73	0.51	0.67	0.5	0.67

Table 4 Experimental conditons in BCA SkyLab for model calibration and validation

Date in October	10	11	12	13	14	15	16	17	18	19
Purpose	Model calibration				Model validation					
ACMV system	On	On	On	On	Off	Off	On	On	Off	On
Internal loads	On	On	On	On	Off	Off	On	On	Off	Off

Table 5 Sensors installed in BCA SkyLab Test Cell

Category	Sensors	Brand	Model	Range	Resolution	Accuracy
Weather	Global and Diffuse	AT	SPN1 &	0 -	0.6 W/m ²	10 W/m ²
	Irradiance sensor	Delta-T	SPN1/BP	2000		
		Devices		W/m ²		
	Outdoor air RH	Eyc tech	THS 307	0 - 100%	0.10%	1.50%
	Outdoor air dry bulb temp	Kimo	TH210	-20 – 80°C	0.1°C	0.25°C
Room	Interior globe	Kimo	TM 110	-20 - 80°C	0.1°C	0.5°C
conditions	Interior dry bulb temps	Precon	ST-S3EW-XPA	-40 - 105°C	0.1°C	0.2°C
	Interior relative humidity	Vaisala	HMW90	0 - 100%	0.10%	1.70%
	Omnidirectional air	Swema	Swema 03	0.05 - 10 m/s	-	0.03
	velocity sensor					
ACMV	In Line Electromagnetic	ABB	Water Master	0 - 20	-	0.20%
	Flow Meter		Series	m ³ /h		
	Duct Temperature Sensor	ACI	A/AN-D	-40 -150°C	0.1°C	0.2
	Air Flow Rate Sensor	ACI	ESF-35-2	0 – 16 m/s	-	5%
	Normal Type Immersion	ACI	A/AN-I	-40 -150°C	0.1°C	0.2°C
	Temperature Sensor					

Table 6. Outputs, inputs and states of SSM

Inputs	Description	States	Description	Outputs	Description
T_{fa}	Fresh air temperature (K)	T_{im}	Temperature of indoor thermal mass (K)	T_z	Room air temperature (K)
ψ_{fa}	Fresh air humidity ratio(kg/kg)	T_z	Room air temperature (K)	ψ_z	Indoor humidity ratio(kg/kg)
Q_{cc}	Cooling power of cooling coil (W)	ψ_z	Indoor humidity ratio(kg/kg)	PMV	Thermal comfort index
$Q_{inte,sen}$	Internal sensible heat gain (W)	$T_{ew,1}$	Temperature of exterior wall aggregate 1(K)		
$Q_{inte,lat}$	Internal latent heat gain (W)	$T_{ew,2}$	Temperature of exterior wall aggregate 2(K)		
Q_{win}	Heat gain from window-blind system (W)	$T_{iw,1}$	Temperature of interior wall aggregate 1(K)		
$q_{sor,ew}$	Incident solar irradiance on outer surface of exterior wall (W/m ²)	$T_{iw,2}$	Temperature of interior wall aggregate 2(K)		
$q_{sor,roof}$	Incident solar irradiance on outer surface of roof (W/m ²)	$T_{flo,1}$	Temperature of floor aggregate 1(K)		
I	Constant 1	$T_{flo,2}$	Temperature of floor aggregate 2(K)		
		$T_{roof,1}$	Temperature of roof aggregate 1(K)		
		$T_{roof,2}$	Temperature of roof aggregate 2 (K)		

Table 7 State matrix **A** of SSM

	T_{tm}	T_z	ψ_z	$T_{ew,1}$	$T_{ew,2}$	$T_{iw,1}$	$T_{iw,2}$	$T_{flo,1}$	$T_{flo,2}$	$T_{roof,1}$	$T_{roof,2}$
T_{tm}	-4.34 E-04	4.34E -04	0	0	0	0	0	0	0	0	0
T_z	9.11E -03	-1.21 E-02	2.04	0	1.68E -04	0	2.20E -04	0	2.14E -04	0	5.85E -04
ψ_z	0	6.78E -07	-9.55 E-04	0	0	0	0	0	0	0	0
$T_{ew,1}$	0	0	0	-3.16 E-05	1.08E -05	0	0	0	0	0	0
$T_{ew,2}$	0	1.95E -05	0	1.08E -05	-3.03 E-05	0	0	0	0	0	0
$T_{iw,1}$	0	0	0	0	0	-1.08 E-05	1.08E -05	0	0	0	0
$T_{iw,2}$	0	1.95E -05	0	0	0	1.08E -05	-3.03 E-05	0	0	0	0
$T_{flo,1}$	0	0	0	0	0	0	0	-6.25 E-05	2.31E -05	0	0
$T_{flo,2}$	0	3.94E -05	0	0	0	0	0	2.31E -05	-6.25 E-05	0	0
$T_{roof,1}$	0	0	0	0	0	0	0	0	0	-9.14 E-05	3.53E -05
$T_{roof,2}$	0	4.21E -05	0	0	0	0	0	0	0	3.53E -05	-7.74 E-05

Table 8 Input matrix **B** of SSM

	T_{fa}	ψ_{fa}	Q_{cc}	$Q_{inte,sen}$	$Q_{inte,lat}$	$Q_{win,bli}$	$q_{sor,ew}$	$q_{sor,roof}$	I
T_{im}	0	0	0	0	0	0	0	0	0
T_z	1.35E-04	0.104261	-1.3E-06	3.84E-06	0	3.17E-06	0	0	0.45
ψ_z	-1.7E-08	7.86E-05	-1E-09	0	1.57E-09	0	0	0	-1.82E-04
$T_{ew,1}$	2.08E-05	0	0	0	0	0	6.43E-07	0	0
$T_{ew,2}$	0	0	0	0	0	1.1E-08	0	0	0
$T_{iw,1}$	0	0	0	0	0	0	0	0	0
$T_{iw,2}$	0	0	0	0	0	1.1E-08	0	0	0
$T_{flo,1}$	3.94E-05	0	0	0	0	0	0	0	0
$T_{flo,2}$	0	0	0	0	0	2.23E-08	0	0	0
$T_{roof,1}$	5.61E-05	0	0	0	0	0	0	1.74E-06	0
$T_{roof,2}$	0	0	0	0	0	2.39E-08	0	0	0

Table 9 Output matrix **C** of SSM

	T_{im}	T_z	ψ_z	$T_{ew,1}$	$T_{ew,2}$	$T_{iw,1}$	$T_{iw,2}$	$T_{flo,1}$	$T_{flo,2}$	$T_{roof,1}$	$T_{roof,2}$
T_z	0	1	0	0	0	0	0	0	0	0	0
ψ_z	0	0	1	0	0	0	0	0	0	0	0
PMV	0	0.27	35.71	0	3.28E-03	0	4.3E-03	0	4.18E-03	0	1.14E-2

Table 10 Feedthrough matrix **D** of SSM

	T_{fa}	ψ_{fa}	Q_{cc}	$Q_{inte,sen}$	$Q_{inte,lat}$	$Q_{win,bli}$	$q_{sor,ew}$	$q_{sor,roof}$	I
T_z	0	0	0	0	0	0	0	0	0
ψ_z	0	0	0	0	0	0	0	0	0
PMV	1.45E-04	0	0	0	0	6.18E-05	0	0	-88.34

Table 11. Summary of simulation results of January using Singapore weather data

Scenario	Electric energy consumption (kWh)	Electric energy saving ref. Scenario 1 (%)	SD of normalized cooling power during office hours (kW)	MAD of PMV from 0 during office hours	SD of PMV during office hours
1	100.70	-	0.412	0.173	0.190
2	81.15	19.4	0.070	0.556	0.092
3	94.71	6.0	0.052	0.065	0.136

Key: SD - standard deviation, MAD - mean absolute deviation.