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Modification of turbulent wake characteristics by two small control cylinders at a subcritical Reynolds number

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In this paper, the flow around a main circular cylinder with two small control cylinders is numerically investigated at a subcritical Reynolds number of 3900. Two small control cylinders with diameter ratio of $d/D = 0.04$ (d and D are the diameters of the control cylinders and the main cylinder, respectively) are symmetrically placed inside the separated shear layers emanating from the main cylinder. The computation is performed with highly resolved direct numerical simulations employing a high-order spectral/hp element method. For validation purposes, the results of the flow over a single cylinder are presented at first, and the current simulation is in good agreement with the existing experiments and large eddy simulations. For the main cylinder with the two control cylinders, the dynamics of shear layers is found to be modified strongly owing to the disturbance from the small control cylinders. As a consequence, the recirculation length is decreased by 89% as compared to that of a single cylinder. The presence of the small control cylinders further leads to a significant increase in the fluctuating lift and the mean drag exerting on the main cylinder. Furthermore, the statistical analysis and turbulent wake visualization imply that the turbulent intensity is amplified in the near wake region and becomes weaker further downstream. This study then suggests that the small control cylinders located in the near wake region of the main cylinder may work differently in the turbulent wake against that in the laminar wake. Published by AIP Publishing. <https://doi.org/10.1063/1.5046447>

I. INTRODUCTION

The flow past bluff bodies has been studied extensively due to its relevance in connection with numerous practical applications in industry, such as airplanes, submarines, automobiles, bridges, and high-rise buildings. Although many different shapes of bluff bodies exist, the circular cylinder is one of the most representative shapes to be investigated with great interest in fluid mechanics. It is well known that the wake dynamics of cylinder flow is a strong function of the Reynolds number (Re), with a comprehensive review presented by Williamson¹ about two decades ago. One of the fundamental features for the flow past a cylinder is that the wake exhibits successive regimes of laminar, transitional, and turbulent with increasing Re . As Re exceeds critical value, the wake becomes unstable and an alternative shedding of vortices into the wake leads to significant increases in the mean drag and lift fluctuations. Three-dimensional effects start to play an important role around $Re \sim 190$, and as Re further increases to $\sim 10^3$, the wake becomes more chaotic and manifests itself

with obvious turbulent properties. In the subcritical regime ($Re < 3.0 \times 10^5$), the boundary layer near the circular cylinder is still laminar, but the wake flow is completely turbulent.

Exploration of the turbulent characteristics of the flow past a single cylinder at subcritical Reynolds numbers has been performed both experimentally and numerically in the past. The flow at $Re = 3900$ was therefore examined extensively as a representative case in the subcritical Reynolds number regime due to the contribution of early particle image velocimetry (PIV) experimental work from Lourenco and Shih.² A number of subsequent experiments were then reported to focus on the statistics of near wake fields in the range of $x/D \leq 10$. For example, Ong and Wallace³ conducted the experiment with hot-wire anemometry (HWA), which provided the mean flow field data at several locations for the range of $3 \leq x/D \leq 10$. Norberg⁴ examined the flow with Re ranged between 1500 and 10 000 using the Laser Doppler Velocimetry (LDV) technique. In addition, Parnaudeau *et al.*⁵ studied both HWA and PIV of the near wake statistics at $Re = 3900$. First- and second-order statistics data were assessed to analyze the accuracy and statistical convergence.

Numerical predictions such as large eddy simulations (LESs) and direct numerical simulations (DNSs) have been

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widely carried out at $Re = 3900$. Over the past two decades, LESs were discussed extensively in the published studies, for example, in Beaudan and Moin,⁶ Mittal and Moin,⁷ Breuer,⁸ Kravchenko and Moin,⁹ Parnaudeau *et al.*,⁵ and Afgan *et al.*¹⁰ Beaudan and Moin⁶ performed the first numerical simulation with the LES method and concluded that the near-wake is highly three-dimensional at this Reynolds number. As a matter of fact, two dimensional calculations are no longer accurate to validate numerical methods at a Reynolds number higher than 250.⁹ Kravchenko and Moin⁹ carried out the computation with LESs based on the B-spline method. They found that the mean flow and turbulence statistics agreed well with the experimental data further downstream. However, the velocity profiles in the very near wake ($x/D \leq 3$) are different from the experiment. Taken together, the DNSs were conducted less for flow over the cylinder in the turbulent regime.^{11,12} Dong *et al.*¹² employed a combined PIV-DNS to investigate the near wake of a cylinder at $Re = 3900, 4000$, and $10\,000$. They focused on the characteristics of turbulence and the instabilities of separating shear layers.

The flow over the cylinder has also been served as a prototype model for study of the vortex suppression method¹³ and energy harvesting techniques.^{14,15} Various active and passive control methods were proposed to modify the wake flows and suppress the vortex shedding in the literature. To name a few, rotary oscillations,^{16,17} rotating cylinders,^{18,19} and blowing and suction^{20,21} are typical examples for the active wake controls. Additionally, surface roughness,²² plates attached to the cylinder,^{22,23} wavy cylinders,²⁴ square cylinders with rounded corners,²⁵ dimples and grooves,^{26–28} and small control cylinders or rods^{29–31} are for the passive controls. Among them, one of the simplest passive control techniques is the placement of small secondary control cylinders in the wake of the main cylinder. There are several researches that focused on the cylinder-control interactions both numerically and experimentally.

Strykowski and Sreenivasan²⁹ found the complete wake suppression behind the main cylinder at a Reynolds number less than 100 when a very small control cylinder with diameter $d/D = 20$ was placed at a proper position in their experiment. Inspired by Strykowski and Sreenivasan,²⁹ a significant number of studies were carried out to investigate the flow around the main cylinder with control cylinders that may suppress the vortex shedding. The simulation of Mittal and Raghuvanshi³² at $Re = 100$ was in accordance with that of Strykowski and Sreenivasan.²⁹ The control cylinder that provided a local favorable pressure gradient in the wake region could stabilize one side of shear layers. The reduction of mean drag and fluid force fluctuations was observed in the simulation of Yildirim *et al.*³³ Kuo *et al.*³⁰ numerically investigated the wake flow pattern of control cylinders behind the main cylinder at Re ranging from 80 to 300. Two small cylinders were placed symmetrically in the wake of the main cylinder with diameter ratio of $d/D = 1/8$. Different from the results of Strykowski and Sreenivasan,²⁹ the vortex street was also observed behind the main cylinder, but the fluctuating lift and the form drag on the main cylinder decrease prominently and monotonously as Re increases. Parezanović and Cadot³⁴ conducted the experiment of flow past a “D”-shaped cylinder with a small control cylinder at

$Re = 13\,000$. They found that a controlled shear layer was split by the small cylinder and a jet was observed flowing into the recirculation region when a small cylinder was placed in the middle of one shear layer.

From the literature review, we can see that most of the previous numerical studies of wake control behind a cylinder focused more on the laminar flow regime at low Re , and the influence of control cylinders on the turbulent wake of the circular cylinder at a subcritical Reynolds number has not been investigated so far. The experiment of Parezanović and Cadot³⁴ presents novel evidence for the control effects of a small cylinder on turbulent wakes, showing quite different features as compared to the laminar flow regime. In their sensitivity analysis, only a single cylinder is placed at one side of the wake behind the “D”-shaped cylinder. So, how the turbulent wake of a circular cylinder is modified by symmetric arrangement of small control cylinders would still be an open question on this issue. Motivated by this, the present work aims to study the turbulent wake of the cylinder flow with the disturbance from placement of small control cylinders in the near wake region. The influence of small control cylinders at a subcritical Reynolds number will be characterized quantitatively and compared with that observed for a single circular cylinder case. The remainder of this paper is organized as follows: The numerical methodology and the physical properties of flow over cylinders employed for their solution are presented in Sec. II. The numerical results of the single cylinder and control cylinders systems are introduced in Secs. III and IV with some discussions, and finally the essential findings and concluding remarks are presented in Sec. V.

II. NUMERICAL METHODOLOGY AND PHYSICAL MODEL

A. Numerical methodology

We consider an incompressible viscous flow governed by the Navier–Stokes equations, which are written in non-dimensional form as

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where $\mathbf{u} \equiv (u, v, w)$ is the velocity vector, t is the time, and p is the static pressure which is assumed to be scaled by the constant fluid density ρ . The non-dimensional parameter Re is the Reynolds number that is defined as $Re = U_\infty D / \nu$, where U_∞ is the uniform speed of upstream flow, D is the diameter of the main cylinder, and ν is the kinematic viscosity.

Computations have been performed by a parallel Fourier spectral/hp element method³⁵ embedded in the open-source framework, **Nektar++**.³⁶ Based on the assumption that the flow variables are homogeneous in the spanwise direction with a periodic length L_z , a Fourier expansion is introduced in the z -direction to the flow variables $\mathbf{u}$ and p as

$$\mathbf{u}(x, t) = \sum_{m=0}^{M-1} \mathbf{u}_m(x, y, t) e^{i\beta m z}, \quad p(x, t) = \sum_{m=0}^{M-1} p_m(x, y, t) e^{i\beta m z}, \quad (3)$$

with $\beta = 2\pi/L_z$. Here, m is the Fourier mode index and M is the number of modes in the Fourier expansion. After applying the Fourier transformation to Eqs. (1) and (2), we obtain a set of uncoupled two-dimensional equations for each mode³⁷

$$\frac{\partial \mathbf{u}_m}{\partial t} + \widehat{N(\mathbf{u})}_m = -\tilde{\nabla} p_m + \nu \tilde{\nabla}^2 \mathbf{u}_m, \quad (4)$$

$$\tilde{\nabla} \cdot \mathbf{u}_m = 0, \quad (5)$$

where $\widehat{N(\mathbf{u})}_m$ represents the Fourier mode of the convective term. In order to avoid formation of convolution sums, the Fourier transformation of the nonlinear terms is applied in physical space. In Eqs. (4) and (5), we defined the differential operators as

$$\tilde{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, i\beta m \right), \quad \tilde{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \beta^2 m^2. \quad (6)$$

The stiffly stable splitting scheme proposed by Karniadakis,³⁸ also referred to as a velocity correction scheme,³⁹ is employed for time integration of the uncoupled equations (4) and (5). For the details of the algorithm, refer to the studies of Karniadakis *et al.*³⁸ and Karniadakis and Sherwin,³⁵ which are stated briefly as follows (we omitted the subscript m for notational convenience):

First, calculate a first intermediate velocity field ($\tilde{\mathbf{u}}$) evaluating the advection term explicitly and combining it with the solution at previous time steps

$$\frac{\tilde{\mathbf{u}} - \sum_{q=0}^{J-1} \frac{\alpha_q}{\gamma_0} \mathbf{u}^{n-q}}{\Delta t} = \sum_{q=0}^{J-1} \frac{\beta_q}{\gamma_0} [-\widehat{N(\mathbf{u})}]^{n-q}, \quad (7)$$

where γ_0 , α_q , and β_q are the stiffly stable time integration coefficients which are given by Karniadakis *et al.*³⁸ J is the order of the time integration for the explicit terms ($J = 2$ in this work). Δt is the time step.

Second, solve a Poisson equation to obtain the pressure solution at the new time level with high-order boundary conditions

$$\Delta p^{n+1} = \left(\frac{\gamma_0}{\Delta t} \right) \nabla \cdot \tilde{\mathbf{u}}. \quad (8)$$

Then, a second intermediate velocity field ($\tilde{\tilde{\mathbf{u}}}$) is calculated by

$$\tilde{\tilde{\mathbf{u}}} - \tilde{\mathbf{u}} = -\left(\frac{\Delta t}{\gamma_0} \right) \nabla p^{n+1}. \quad (9)$$

Finally, obtain the velocity field at the new time level by using the second intermediate velocity field as a forcing term in a Helmholtz equation

$$\left(\Delta - \frac{\gamma_0}{\Delta t \nu} \right) \mathbf{u}^{n+1} = -\frac{\gamma_0}{\Delta t \nu} \tilde{\tilde{\mathbf{u}}}. \quad (10)$$

A spectral-vanishing viscosity technique (SVV), which was developed for the spectral element method by Kirby and Sherwin,⁴⁰ is applied to stabilize the solution. A continuous Galerkin scheme with the modified Jacobi polynomials-based expansions for spatial approximation is adopted in the xy plane.

B. Physical model

As mentioned earlier, two flow configurations of a single cylinder and a main cylinder with two small control cylinders

FIG. 1. Schematic view of the flow past the main cylinder with two small control cylinders for $Re = 3900$. The control cylinders with diameter of $d = 0.04D$ are placed at $(x/D, y/D) = (0.7, \pm 0.7)$.

were investigated numerically at $Re = 3900$. The computational domain is rectangular with a semicircle ahead extending $30D$ downstream and a radius of $20D$ in front of the main cylinder and $40D$ in the cross-flow direction. The spanwise extent of the domain was chosen to be $L_z = \pi D$, which is the same as the size used by most of the previous studies (Refs. 6, 7, and 9). When doubling the length of the z -direction, there was little difference on the results of the statistical data.⁹ In Fig. 1, the center of the main cylinder with diameter D is located at the origin of the flow domain.

As stated earlier, complete suppression was observed in the case of experimental and numerical studies from Strykowski and Sreenivasan.²⁹ When the Reynolds number was equal to 63, the temporal instability was effectively suppressed by a small control cylinder located at $0.7 \leq y/D \leq 1.1$. In order to investigate the mechanisms of possible wake control at subcritical number 3900, two small control cylinders are placed in the shear layers at $(x/D, y/D) = (0.7D, \pm 0.7D)$, which falls in the effective range for the laminar flow regime at $Re < 100$ suggested in the work of Strykowski and Sreenivasan.²⁹

The grid in the near wake region of the cylinders is refined to capture the flow behaviour effectively. For a single cylinder configuration, the spatial discretization in the xy plane consisted of a mesh with 5626 quadrilateral elements, with the solution represented by 6th-order polynomials inside each element. A total of 64 grid points were used in the z direction over the length of πD by a Fourier expansion. For control cylinders system, the spatial discretization in the xy plane consisted of a mesh with 5490 quadrilateral elements and other parameters of mesh are in accordance with the single cylinder. For current simulations, the Kolmogorov length scale η is approximately 0.002 [$\eta = (\nu^3/\epsilon)^{1/4}$, where $\nu = U_\infty D/Re = 1/3900$ and $\epsilon \approx U_\infty^3/D = 1$], while the smallest mesh size next to the cylinder body is $0.0013 < \eta$. There are approximately 88×10^6 cells for the whole domain for both simulations. The total number of grid points is about 5 times higher than that in the simulation of Kravchenko and Moin⁹ (case 2) and 1.8 times higher than that in the high resolution LES (HR LES) of Parnaudeau *et al.*⁵ The views of the two-dimensional mesh

FIG. 2. Computational mesh in the xy plane for the two small control cylinders behind the main cylinder configuration: (a) cross-sectional view and (b) zoom-in view near the cylinders.

for the configuration with the two small control cylinders are shown in Fig. 2.

In both of the simulations, the incoming flow is uniform, and there is no turbulent disturbance at the inlet. The boundary conditions for numerical simulations applied to the physical model are set as follows: the free-stream velocity of $(u, v, w) = (U_\infty, 0, 0)$ was imposed along the inflow boundary; a free-slip condition ($v = 0, \partial u / \partial y = 0, \partial w / \partial y = 0$) was imposed on the far field boundaries; a non-slip and non-impermeability condition of $u = v = w = 0$ was imposed on the wall of the cylinders; and a Neumann boundary condition ($\partial u / \partial x = \partial v / \partial x = \partial w / \partial x = 0$) was implemented at the outlet. For pressure, a high-order Neumann type condition is applied at the inflow and the wall, while a Dirichlet boundary condition is used at the outflow.

The non-dimensional time step is defined as $\Delta t = \Delta T U_\infty / D$, where ΔT is the real time for simulations. The Kolmogorov time scale τ_η is approximately 0.016 [$\tau_\eta = (\frac{\nu}{\epsilon})^{\frac{1}{2}}$]. The convergence is relatively sensitive to the time step and obtained at the time step of $\Delta t = 0.001$ for a single cylinder and $\Delta t = 0.0005$ for the control cylinders system. In the present DNSs, the turbulent flow is considered to be completely established after the loss of initial conditions. The simulations

were initialized from a low Re flow, and a higher Re flow was restarted from the fields obtained at the lower Re one. Each simulation at $Re = 3900$ was computed for 200 non-dimensional time units, and the last 150 time units were preserved to obtain the averages, while the first 50 time units were removed.

III. FLOW PAST SINGLE CYLINDER

In this section, the numerical results of the flow over a single cylinder at $Re = 3900$ are presented. The instantaneous vorticity ω_n ($\omega_n = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2}$, colored by the stream-wise vorticity component ω_x) in the near wake for a single cylinder is illustrated in Fig. 3. It is shown that the large scale structures resemble the classical von Karman vortex street pattern in the wake. Small scale structures are clearly observed far from the cylinder, where the full turbulent effect is fully established at $Re = 3900$. The longitudinal vortices related to the three dimensional motion can be identified here with ω_n . As studied by Parnaudeau *et al.* and Lourenco and Shih,² the first- and second-order statistics are calculated along the

FIG. 3. Isosurface of instantaneous vorticity modules $\omega_n = 6U_\infty / D$ for the flow over a single cylinder at $Re = 3900$ (colored by the longitudinal vorticity ω_x) in (a) top view, (b) side view, and (c) 3-D view.

FIG. 4. Mean longitudinal velocity in the wake centerline of the circular cylinder at $Re = 3900$: (red solid curve) present spectral/hp element; (blue dashed curve) LES of Parnaudeau *et al.*⁵ (brown filled circles) PIV (case 1) of Parnaudeau *et al.*⁵ (black empty circles) experiment of Lourenco and Shih.²

wake centerline and at several typical locations of $x/D = 1.06$, 1.54 , and 2.02 . Breuer suggests that the time period for the statistical analysis should be at least $100D/U_\infty$ time units (22 vortex shedding cycles).⁴¹ Considering computing efficiency and convergence, statistics were accumulated across 31 vortex shedding cycles over $150D/U_\infty$ time units in current simulations. All 64 points in the spanwise direction are used to average the statistics presented in this section.

Figure 4 presents the comparison of the mean streamwise velocities along the wake centerline for the single cylinder between the present simulation and the available studies at $Re = 3900$. The data of current simulation agree well with the LES and the experiments of Parnaudeau *et al.*⁵ The experiment of Lourenco and Shih² and the LES of Parnaudeau *et al.*⁵ both underestimated the minimum velocity value. It is worth noting that the current simulation in the recirculation zone is closer to the experiment of Parnaudeau *et al.*⁵ The recirculation length L_r for current cases is defined as $L_r/D = x_r/D - 0.5$ (x_r is the coordinate value in the wake centerline at which the mean streamwise velocity changes sign from negative to positive). However, the experiment of Lourenco and Shih² underpredicted the size of the recirculation zone possibly due to an earlier transition in the shear layer related to external disturbances in the reference experiments.⁹ The associated isocontour of time-averaged map of $\bar{u}$ is also presented in Fig. 5, showing a good agreement with the LES and the experiment by Parnaudeau *et al.*⁵

Figure 6 shows the variance of streamwise velocity $\overline{u'u'}$ in the wake centerline. Compared with the LES of Parnaudeau *et al.*⁵ current simulation fits the experimental results better at the peak value and further downstream ($2.5 \leq x/D \leq 7$). It is noticed that the variance reaches two peaks in all experiments referred, but both the numerical simulations seem to reach at only one evident peak. Lehmkühl *et al.*⁴² conducted the DNSs for 4000 time units in the case of a single circular cylinder at $Re = 3900$. The existence of low-frequency fluctuations was considered as the results of the wake variation within

FIG. 5. Isocontour maps of the streamwise velocity $\bar{u}$ normalized with U_∞ for a single cylinder at $Re = 3900$.

FIG. 6. Variance of the longitudinal velocity in the wake centerline: (red solid curve) present spectral/hp element; (blue dashed curve) for $Re = 3900$, LES of Parnaudeau *et al.*⁵ (brown filled circles) for $Re = 3900$, experiment of Parnaudeau *et al.*⁵ (black empty diamonds) for $Re = 3000$ and (black filled diamonds) for $Re = 5000$, the experiment of Norberg.⁴

the recirculation zone between two different modes: the high-energy mode (Mode H) corresponded to strong fluctuations in the shear layer and the vortex formation zone; the low-energy mode (Mode L) corresponded to weaker fluctuations in the shear layer. They observed that Mode L is characterized by two-lobed low peaks and Mode H exhibits only one but a higher peak. Furthermore, $\overline{u'u'}$ for the whole time integration period was between the Mode L and Mode H with one peak and an inflection point, which is similar to the current simulations. Therefore, the differences of peaks may be attributed to the selection of time integration falling in different modes.

From Fig. 7, the present mean velocities and fluctuations are discussed in comparison with the results of the experiments of Parnaudeau *et al.*⁵ and Lourenco and Shih² and with the LES of Parnaudeau *et al.*⁵ at three x -locations in the very near

FIG. 7. Mean wake velocity at three locations for a single cylinder at $Re = 3900$. (a) Mean streamwise velocity $\bar{u}/U_\infty$. (b) Mean normal velocity $\bar{v}/U_\infty$. For details, see the caption of Fig. 4.

FIG. 8. Mean Reynolds stress at three locations for a single cylinder at $Re = 3900$. (a) Variance of the streamwise velocity fluctuations $\overline{u'u'}/U_\infty^2$. (b) Variance of the normal velocity fluctuations $\overline{v'v'}/U_\infty^2$. (c) Covariance of the velocity fluctuations $\overline{u'v'}/U_\infty^2$. For details, see the caption of Fig. 4.

FIG. 9. Isocontour maps of the Reynolds stress normalized with U_∞^2 for a single cylinder at $Re = 3900$: (a) $\overline{u'u'}$, (b) $\overline{v'v'}$, and (c) $\overline{u'v'}$.

wake ($x/D \leq 3$). The present simulation consists of both the simulation and the experiment of Parnaudeau *et al.*⁵ It has been commented a lot on the streamwise velocity with U-shaped or V-shaped profiles. Figure 7(a) shows that the experiment of Lourenco and Shih² developed a V-shaped profile, while the rest of the studies showed an U-shape inside the recirculation region close to the cylinder, at $x/D = 1.06$. The sources of such a discrepancy has been referred to in the literature. Kravchenko and Moin⁹ conducted five calculations on the same grid but with different domain sizes in the z -direction and grid resolutions, which demonstrated that insufficient grid resolution and domain size lead to an early transition in the shear layers, and calculated the wrong statistics with a V-shaped profile in the very near wake. All the profiles turn to a V-shape further downstream, at $x/D = 1.54$ and $x/D = 2.02$. It is observed that the present simulation performs better near the centerline than the LES. The mean normal velocity is anti-symmetric about the intersections with the x -axis except for the experiment of Lourenco and Shih,² as depicted in Fig. 7(b). The value of cross-flow velocity is zero in the far field but increases quickly at both sides of the x -axis (where $y = 0$). The current simulation data and the measurements of Parnaudeau *et al.*⁵ almost overlap at $x/D = 1.06$ and 2.02 . On the other hand, both simulations slightly underpredict the velocity at $x/D = 1.54$.

The variance of the streamwise and normal velocity fluctuations ($\overline{u'u'}/U_\infty^2$ and $\overline{v'v'}/U_\infty^2$) together with the covariance of the velocity fluctuations ($\overline{u'v'}/U_\infty^2$) at $Re = 3900$ is illustrated in Fig. 8. The $\overline{u'u'}$ profiles shown in Fig. 8(a) are marked by an obvious double-peak phenomenon, which was mainly caused by the transition of the shear layers symmetrically. The current study has the same shape with the experiment of Parnaudeau *et al.*⁵ at three downstream locations. However, the peak value at $x/D = 1.06$ and 1.54 is slightly lower than the experiment of Parnaudeau *et al.*⁵ (less than 13%). A key factor is the selection of time period as mentioned for the prediction of $\overline{u'u'}$ at the centerline. Such an underestimation was also observed in Mode L by Lehmkuhl *et al.*⁴² (not shown here). In addition, a perfect match is found further downstream at $x/D = 2.02$, where the current simulation is also in agreement with Mode L.⁴² The $\overline{v'v'}$ profiles reach only one peak at the wake centerline ($y/D = 0$) as shown in Fig. 8(b), and the peak value becomes stronger as the wake moves downstream.

Concerning the maximum of variance, the $\overline{v'v'}$ is approximately twice as large as $\overline{u'u'}$. In a similar manner, the current simulation has a slight deviation for peaks due to the same reason as above. Finally, the shear stress $\overline{u'v'}$ is anti-symmetric [see Fig. 8(c)]. There are some differences in both the simulations near the centerline. Nevertheless, compared to the LES, the current simulation is closer to the experimental data far from the x -axis ($0.5 \leq |y/D| \leq 1.0$). As can be seen, a significant deviation from other investigations is observed in the experiment of Lourenco and Shih,² suggesting the influence of the early transition again. At the same time, Afgan *et al.*¹⁰ thought that these experimental data may be not averaged enough. The isocontour maps of the Reynolds stress are shown in Fig. 9. The shear layers are identified in the map of $\overline{u'u'}$. The $\overline{u'u'}$ and $\overline{v'v'}$ are symmetric while $\overline{u'v'}$ is antisymmetric. These numerical maps well match the results of the experiment and the simulation of Parnaudeau *et al.*⁵

On the whole, the present simulation fits well with previous literature and has higher precision than other simulations, especially in the first-order statistics at $Re = 3900$ for a single cylinder. These results will be taken as a reliable reference in Sec. IV to investigate the flow past the main cylinder with two small control cylinders at the same Re .

IV. FLOW PAST THE MAIN CYLINDER WITH TWO CONTROL CYLINDERS

In this section, two small cylinders are symmetrically placed at $(x/D, y/D) = (0.7, \pm 0.7)$, which is located inside the shear layers of the main cylinder. By comparing with the single cylinder, this simulation case is applied to characterize the modification of the turbulent wake on the aspects including instantaneous fields, fluctuating and mean coefficients, power spectra, and mean flow and turbulence statistics, due to the perturbation of two small control cylinders.

A. Instantaneous field

The instantaneous spanwise vorticity ω_z at the bottom ($z = 0$) and half span ($z = \pi/2D$) of the cylinders are shown in Fig. 10. For the two planes, the separating shear layers are clearly visible on both sides of the main cylinder. In the

FIG. 10. Instantaneous spanwise vorticity ω_z normalized by U_∞/D in two symmetry planes in the flow over the circular cylinders at $Re = 3900$: (a) $z = 0$ and (b) $z = \pi D/2$. There are 51 contours from -5 to 5 .

recirculation region, the shear layers interact with each other and roll up to the rear face of the cylinder. Vortices generated from the instabilities of the shear layers and the wake behind the control cylinders accumulate in the near wake at approximately $1 \leq x/D \leq 2$. The large scale structure is observed further downstream in the instantaneous field. However, the vortex is broken down to a great extent because of the rapid dissipation at $x > 2D$. Not surprisingly, small scale still exists far from the main cylinder, which demonstrated the formation of the fully three-dimensional and turbulent wake flow.

Figure 11 shows an isosurface of the instantaneous vorticity modules $\omega_n = 6U_\infty/D$ colored by longitudinal vorticity ω_x for the case with control cylinders. The small scale and the primary vortex formation is clearly identified in the vorticity modules ω_n , but with strong disturbance from the two small cylinders. It is reported that the length of the recirculation region and shear layers are influenced by the external disturbance.⁴³ The additional control cylinders contribute to

narrower shear layers and much shorter recirculation region than the case of the single cylinder. The location of the control cylinders, for the current simulation, resembles the mid-shear (MS) configuration from the experiment by Parezanović and Cadot.³⁴ Instantaneously, the top shear layer in the image is reattached to the upper control cylinder, while the other deviates between the main cylinder and the lower control cylinder. A jet-like flow is implicated towards the recirculation region in the bottom shear, which is also evidenced in the experimental study of Parezanović and Cadot.³⁴ As a whole, the interaction of the control cylinders with the shear layers of the main cylinder significantly modifies the wake topologies and amplifies the near wake fluctuations. A quantitative analysis on the wake characteristics will be presented in Secs. IV B–IV D.

B. Fluctuating and mean coefficients

A comparison of the pressure distribution around the cylinder with and without the control cylinders is shown

FIG. 11. Isosurface of instantaneous vorticity modules $\omega_n = 6U_\infty/D$ for the case with two small control cylinders (colored by the dimensionless longitudinal vorticity ω_x) in (a) top view, (b) side view, and (c) 3-D view.

FIG. 12. Mean C_p profile around the main cylinder. (Red solid curve) Current simulation without the control cylinders; (green dashed curve) current simulation with the control cylinders; (blue dashed curve) LESs of Kravchenko and Moin;⁹ (black empty squares) the experiment of Norberg.⁴⁴

in Fig. 12. For a single cylinder, the current simulation at $\theta < 78^\circ$ agrees well with the LES of Kravchenko and Moin⁹ at $Re = 3900$ and the experiment of Norberg⁴⁴ approximately at $Re \approx 4020$. At $\theta > 78^\circ$, the mean pressure coefficient $\bar{C}_p$ ($\bar{C}_p = 2(\bar{p} - p_0)/U_\infty^2$) for the current simulation is slightly higher than that of the experiment, while the result of LES is slightly lower. It is worth noting that the mean pressure coefficient $\bar{C}_p$ is insensitive to the Reynolds number. When the two small control cylinders are presented, $\bar{C}_p$ along the rear surface is obviously lower than that of a single cylinder. The lift and drag forces per unit length, F_l and F_d , are non-dimensionalized by $\frac{1}{2}\rho U_\infty^2 D$ to give the lift and drag coefficients, C_l and C_d (refer to Ref. 45). Figure 13 shows the time series of the lift coefficient and drag coefficient (both are averaged in the sense of the spanwise direction) exerting on the

FIG. 13. Time series of the drag coefficient C_D and the lift coefficient C_L integrated over the surface of the main cylinder at $Re = 3900$.

main cylinder. Clearly, the mean drag coefficient with control cylinders is higher than that without the control cylinders. Likewise, the rms values of the lift coefficient are also marked with the case with the control cylinders. At the same time, the increase in the mean drag coefficient is consistent with the modified pressure profile along the surface of the main cylinder as depicted in Fig. 12.

C. Frequency spectra

In Fig. 14, the power spectra of the control cylinders are shown along the wake centerline at three locations (where $x/D = 3, 5, 7, 10$) and compared with the case of the single cylinder. The time duration to estimate the power spectra in both simulations is about $\Delta T \approx 130D/U_\infty$, corresponding to 27 vortex shedding cycles for the single cylinder and 48 for the control cylinders. The periodogram technique is recognized as

FIG. 14. A comparison of the power spectra of the velocity fluctuations at four locations: (a) streamwise velocity, (b) normal velocity (red solid curve) with the control cylinders and (black dotted curve) without the control cylinders.

FIG. 15. A comparison of the mean longitudinal velocity in the wake center-line of the circular cylinder at $Re = 3900$: (black solid curve) single cylinder; (red dashed curve) with the control cylinders.

an excellent method to analyze the power spectra. Both frequencies are nondimensionalized by the shedding frequency of the single cylinder, f_{vs} . The power spectra are obtained by using all samples at 64 independent points to average in the spanwise direction.

The spectrum exhibiting an energetic peak at f_{vs} is 0.206 for the main cylinder alone, which is in agreement with the results in the literature (0.208 in Ref. 5 and 0.21 in Ref. 9). The peak at f_{vs} corresponding to the large-scale vortex shedding frequency shown in the power spectra is 0.366 for the control cylinders. This increase in f_{vs} is related to the decrease in the recirculation length, which results in a stronger interaction between the separating shear layers. For v -spectra shown in Fig. 14(b), the two peaks at the first harmonic and second harmonic appear at approximately $f/f_{vs} = 1.78$ and $f/f_{vs} = 5.33$. Concerning u -spectra, the peaks seem less obvious, as shown

in Fig. 14(a). Taking the $-5/3$ slope curve as a reference, the discretization solution in both the current simulations is satisfactory with resolving the flow inside the inertial range and capable of getting enough accurate results.

D. Mean flow and turbulence statistics

The mean flow and turbulence statistics for the case with two small control cylinders are compared to the results of the single cylinder, which are shown as follows. The mean streamwise velocity along the center wake presented in Fig. 15 varies significantly as compared with that of the single cylinder. The gap between the main cylinder and the control cylinders narrows the wake and makes the recirculation length shorter. The recirculation length L_r decreases by 89%, from $L_r = 1.57D$ in the single cylinder to $L_r = 0.17D$ in the case with the control cylinders as depicted in Fig. 15. It is known that energetic fluctuation corresponds to the short recirculation region and weaker fluctuation is accompanied by an enlargement of the recirculation region as a nonlinear effect proposed by Zielinska *et al.*⁴⁶ Therefore, the presence of the control cylinders at this location at $Re = 3900$ is beneficial for the shrinkage of recirculation and contributes more energetic fluctuations as discussed in Sec. IV C. Another obvious difference is the occurrence of the local peak at $x/D = 1.16$, which is not seen in the single cylinder. In Fig. 16(a), jets are observed between the gaps rushing into the wake flow. Compared with the results of the single cylinder in Fig. 5, the recirculation without control seems to be split into two parts by the two jets. The streamwise velocity of the downstream on the center-line reaches a local peak at the end of jets as depicted in Fig. 15. The normal velocity only appears as an apparent change in the near wake region of the control cylinders as depicted in Fig. 16(b). Figure 17 shows the variance of the streamwise and crossflow velocities in the wake center line. Two strong peaks are observed for $\overline{u'u'}/U_\infty^2$ with the control cylinders, in which the higher one occurs at

FIG. 16. Isocontour maps of the velocity normalized with U_∞ for the control cylinders at $Re = 3900$. (a) $\bar{u}/U_\infty$; (b) $\bar{v}/U_\infty$.

FIG. 17. A comparison of the mean variance of the velocity in the wake centerline of the circular cylinder at $Re = 3900$. (a) Variance of the streamwise velocity fluctuations $\overline{u'u'}/U_\infty^2$. (b) Variance of the normal velocity fluctuations $\overline{v'v'}/U_\infty^2$: (black solid curve) single cylinder; (red dashed curve) control cylinders.

the same position of $x/D = 1.54$ with that for the single cylinder, while the secondary peak is observed much earlier at the position of $x/D = 0.74$. The occurrence of the secondary peak is also ascribed to the jet that has the effects to split the shear layers of the main cylinder, which will be discussed further in detail. Similarly, the peak of $\overline{v'v'}/U_\infty^2$ also shifts upstream to the position of $x/D = 0.74$, where it obtained a value as much as 0.87, nearly 2 times that for the single cylinder. This observation therefore suggests a significant decrease in the length of the vortex formation region due to the presence of the control cylinders.

Figure 18(a) sketches the comparison of the mean streamwise velocities at different x/D downstream locations with that in the single cylinder. The U-shape vanishes and changes to a wavy shape without negative velocity occurring at $x/D = 1.06$. The streamwise velocity profile reaches a positive value at

$y/D = \pm 0.61$. As we move further downstream, the V-shapes still exist but the $\bar{u}$ decelerates slowly to a higher value along the wake. There is no negative streamwise velocity at the locations anymore, and the profile at far downstream ($x/D = 3, 5, 10$) resembles that for the wake behind the single cylinder. The mean normal velocity is plotted in Fig. 18(b). As expected, all the $\bar{v}$ profiles for the case with the control cylinders remain antisymmetric at different x -locations. A drastic change in the shape of $\bar{v}$ with extra fluctuations is observed at $x/D = 1.06$. Concerning the profiles of $\bar{v}$ further downstream, there is a marked decline compared with the single cylinder, but at $x/D = 5$ and 10, the curves collapse on that for the wake of the single cylinder, showing a less influenced wake by the control cylinders.

Figure 19 presents the variance of the streamwise and normal velocity fluctuations ($\overline{u'u'}/U_\infty^2$ and $\overline{v'v'}/U_\infty^2$) together

FIG. 18. A comparison of the mean wake velocity at three x -locations $Re = 3900$. (a) Mean streamwise velocity $\bar{u}/U_\infty$. (b) Mean normal velocity $\bar{v}/U_\infty$: (black dashed curve) single cylinder; (red solid curve) control cylinders.

FIG. 19. A comparison of the mean Reynolds stress at three x -locations at $Re = 3900$. (a) Variance of the streamwise velocity fluctuations $\overline{u'u'}/U_\infty^2$. (b) Variance of the normal velocity fluctuations $\overline{v'v'}/U_\infty^2$. (c) Covariance of the velocity fluctuations $\overline{u'v'}/U_\infty^2$: (black solid curve) single cylinder; (red dashed curve) control cylinders.

with the covariance of the velocity fluctuations ($\overline{u'v'}/U_\infty^2$) at $Re = 3900$ for the control cylinders. M-shape is observed for the variation of the streamwise velocity at all three $\overline{u'u'}$ profiles ($x/D = 1.06, 1.54, 2.02$) shown in Fig. 19(a). The major difference between the two simulations occurs at $x/D = 1.06$ with a stronger peak value that appears exactly in the shifted shear layers which will be discussed in Fig. 20(a). As we move downstream, the peak values occurring at the centerline tend to be increasingly smaller for the control cylinders at $Re = 3900$. The mean cross-stream stress reflects a similar decline trend depicted in Fig. 19(b) but goes down faster in its development compared with $\overline{u'u'}/U_\infty^2$ for the control cylinders. On the other hand, the mean shear stress profiles $\overline{u'v'}/U_\infty^2$ shown in Fig. 19(c) are confirmed antisymmetry. At $x/D = 1.06$, the flow between gaps interacts acutely near the end of the recirculation zone. At $x \geq 2.02$, the peaks reach a lower maximum than the

single cylinder. Similar to the mean velocity profile, the impact of the control cylinder is less important on the Reynolds stress in the far downstream wake ($x/D = 3, 5, 10$).

The above comparison of wake statistics at specified distances indicates us to have a full view of wake flows in terms of time-averaged maps. The isocontour maps of the Reynolds stress in the case of the control cylinders are presented in Fig. 20. The view focuses on the flow at $x/D < 3$ considering the variation in the very near wake. In the map of $\overline{u'u'}$, the shear layers separated from the main cylinder are split into two parts because of the presence of the control cylinders. In particular, the destabilization of the main part of shear layers (indicated with a higher value of $\overline{u'u'}$) happened much earlier as compared with the results of the single cylinder shown in Fig. 9(a). On the other hand, the secondary part is originated from the inner side of the control cylinder. This explains

FIG. 20. Isocontour maps of the Reynolds stress normalized with U_∞^2 for the control cylinders at $Re = 3900$: (a) $\overline{u'u'}$, (b) $\overline{v'v'}$, and (c) $\overline{u'v'}$.

the two higher peaks shown in Fig. 19(a). The $\overline{v'v'}$ reaches the peak value in the centerline closer to the main cylinder as compared to the plot in Fig. 9(b). It is also noticed that the core of the $\overline{v'v'}$ shrinks significantly. Influenced by the control cylinders wake, the secondary peaks of $\overline{v'v'}$ at $x/D = 1.06$ shown in Fig. 19(b) is observed locally in this map. The maps of $\overline{u'u'}$ is compared to a “butterfly” by Parnaudeau *et al.*⁵ Due to the presence of the control cylinders, this “butterfly” flies to the main cylinder and “inspires” the near wake flow due to the earlier transitions of shear layers of the main cylinder.

Strykowski and Sreenivasan²⁹ reported that vortex shedding can be suppressed by a properly positioned secondary cylinder over a limited range of Re . They argued that the presence of the control cylinder has the effect of altering local instability of the flow from absolute to convective by diffusing vorticity in the shear layers behind the main cylinder. They further reported that the effective domain becomes very critical with the increase in Re and suppression at the Reynolds number beyond the critical value is not possible. This thereby implicates that the mechanism of wake instability altering may no longer be applicable because the nonlinear effects will dominate the wake flows at higher Re . The diversion of flow into the main wake due to the presence of the control cylinder was observed in the study by Strykowski and Sreenivasan,²⁹ showing a consistent behavior in turbulence condition. The redistribution of turbulent stress shown in the current investigation, however, indicates that the increased entrainment of momentum into the wake connects to an earlier transition of shear layers rather than wake suppression. The experimental investigation by Parezanović and Cadot³⁴ presented the sensitivity of turbulent properties to the position of the control cylinder as a local disturbance at $Re = 13\,000$ and demonstrated the modification of global frequency, base pressure, and spanwise velocity correlation. It is noticed that the wake modification effect of the control cylinders in the current study resembles that of the mid-shear configuration in the study by Parezanović and Cadot.³⁴ Further exploration on the sensitivity of turbulent properties and the role of Reynolds stress may be necessary for the wake modification of circular cylinder flow due to the control cylinders.

V. CONCLUSIONS

In the current investigations, we investigate the flow around a single circular cylinder with and without the two smaller control cylinders in the near wake at a subcritical Reynolds number of $Re = 3900$ by using highly resolved direct numerical simulations based on the high-order spectral/hp element method. Highly refined grids were used to capture the turbulent characteristics of the wake close to the cylinder.

For the case of the single cylinder, mean flow and turbulence statistics inside the recirculation region were presented which show good agreement with the previous experimental and numerical results. In particular, the statistics of mean velocities fit well with the experiment of Parnaudeau *et al.*⁵ and better than other existing numerical studies.^{5,9} These well predicted results can be provided as a benchmark for the study of the control cylinders system. When the smaller control cylinders with the diameter ratio of $d/D = 0.05$ were placed symmetrically at the positions of $(x/D, y/D) = (0.7, \pm 0.7)$, the mean flow and turbulence statistics together with fluctuating and mean quantities were compared with the data of the single cylinder. It was found that the wake flow behind the main cylinder is quite different from the single cylinder, especially in the very near wake. The configuration of the control cylinders resembles the experiment of Parezanović and Cadot³⁴ when a small cylinder is placed in the mid-shear. From an instantaneous snapshot of the wake, while the shear layer from the one side is observed to be reattached on the control cylinder, another one deviates between the gap of the main cylinder and control cylinder, resulting in a highly modified wake topology behind the main cylinder. A jet-like flow is noticed in gap flows into the recirculation region, implying a strong disturbance to the dynamics of shear layers. With earlier destabilization of shear layers and shorter recirculation length, both the mean drag and fluctuating lift forces applied on the cylinder get amplified and the vortex shedding frequency in power spectral density increases from 0.206 in the single cylinder to 0.366. In the mean flow, higher Reynolds stress is obtained in the very near wake, which reflects more energetic turbulence intensity. However, the turbulence

intensity becomes weaker further downstream (for example, at $x/D = 2.02$).

In summary, the existence of the control cylinders at this position greatly modified the turbulent characteristics at $Re = 3900$. This observation is quite different from the previous study at low Reynolds numbers. Our work pays particular attention in the modification of turbulent statistics of a circular cylinder by small control cylinders. Due to the limited computational resources, only one arrangement is considered in the simulations, but it is our future goal to systematically explore the influence of different locations and sizes of control cylinders. This study is believed to fill such a blank at a subcritical Reynolds number and offer a reference database for the flow past circular cylinders in the future work.

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