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Connectionist agent-based learning in bank-run decision making

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It is of utter importance for the policy makers, bankers, and investors to thoroughly understand the probability of bank-run (PBR) which was often neglected in the classical models. Bank-run is not merely due to miscoordination (Diamond and Dybvig, 1983) or deterioration of bank assets (Allen and Gale, 1998) but various factors. This paper presents the simulation results of the nonlinear dynamic probabilities of bank runs based on the global games approach, with the distinct assumption that heterogeneous agents hold highly correlated but unidentical beliefs about the true payoffs. The specific technique used in the simulation is to let agents have an integrated cognitive-affective network. It is observed that, even when the economy is good, agents are significantly affected by the cognitive-affective network to react to bad news which might lead to bank-run. Both the rise of the late payoffs, R , and the early payoffs, r , will decrease the effect of the affective process. The increased risk sharing might or might not increase PBR, and the increase in late payoff is beneficial for preventing the bank run. This paper is one of the pioneers that links agent-based computational economics and behavioral economics. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5022222>

We have witnessed a new wave of bank runs across the world during the recent financial turmoil. Examples include the runs on Northern Rock in September 2007, on Busan II Savings Bank in February 2011, on Greek banks in June 2015, on Home Capital Group in April 2017. There are, broadly speaking, two opposing views about the cause of bank runs, that is, bank run is due to miscoordination or the deterioration of the quality of the bank's assets. However, real-world bank runs tend to involve various factors, which make it difficult to conclude the cause. Besides, there are no probabilities attached to these possibilities in the classical models. In contemporary behavioral economic theories, the well-known dual-system theory proposes that human behavior comes from the interaction of cognitive and affective processes. With the development of agent-based modeling, agents' heterogeneity in their cognitive capacity can be simulated by Artificial Neural Networks (ANNs), which is a solution for the shortcoming of behavioral economic mechanism. This paper applies an ANNs nonlinear dynamic model with the global game framework to figure out the integrated cognitive-affective process in bank-run decision making. Therefore, fully capturing the heterogeneous nature of human behavior, this internal "mental model" figures out the dynamic process for the bank-run phenomenon.

at the same time. In other words, bank runs are the result of pure coordination failure. The bank run model in DD has two self-fulfilling symmetric pure-strategy Nash equilibria. In one equilibrium, depositors choose to withdraw only when they need liquidity. In the other equilibrium, all depositors "run" to the bank to withdraw money triggered by fear that the bank will not be able to repay them. When depositors make a run on a bank, they make withdrawals that can overwhelm the bank and make the situation worse, which forces the bank to fire sell its long-term investment and end up as a self-fulfilling prophecy. When this happens, even banks with healthy assets may be subject to bank runs because depositors believe they are, or might become, insolvent. This paper establishes the theoretical possibility that non-fundamental factors could be responsible for crisis outcomes.

The competing view (represented by Allen and Gale 1998, hereafter AG) is that bank runs are caused by adverse information about the quality of the bank's assets. They believed that bank runs arise endogenously as a consequence of the standard deposit contract in a world with aggregate uncertainty about asset returns. While from DD perspective, bank runs can be pure sunspots, not necessarily related to changes in the real economy, bank runs are a natural outflow of the business cycle from AG angle.

However, to empirically test the competing theories of bank runs is challenging. Real-world bank runs tend to involve various factors, which make it difficult to conclude whether the bank run is due to miscoordination or the deterioration of the quality of the bank's assets. Besides, one problem of classical model is that there are no probabilities attached to these possibilities, i.e., such models predict that panic-based bank runs are possible, but do not address the probability of a bank run. However, the probability of a bank run is of obvious interest for policy or investment analysis. It is therefore desirable to have a model that removes the multiplicity of equilibria or at least one that allows us to identify

I. INTRODUCTION

Diamond and Dybvig (1983) (hereafter DD) proposed an influential analysis of bank runs. The driving force for a bank run is the fact that in a fractional reserve system, a bank does not hold enough liquid assets to serve all its depositors, should they all decide to withdraw their deposits

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an economically plausible equilibrium from the different equilibria. The probability of a bank run can then be inferred from this equilibrium.

The theory of global games (Carlsson and van Damme, 1993) provides such a framework (see Morris and Shin, 2003 for an overview). The main difference is that classic panic-based bank run models assume that everyone knows with certainty the return properties of the deposit (the fundamental state of the banking system). However, in reality, depositors typically do not know for certain which payoffs will result. Instead, they hold highly correlated but unidentical beliefs about the true payoffs. The global games approach models this situation by assuming that every agent receives a noisy signal that is highly informative with respect to the true fundamental state of the economy but does not allow agents to determine the exact value. In such an environment, agents are expected to play the so-called threshold strategies, i.e., they withdraw their deposits if their signal is above a certain threshold and leave their money in the bank if the signal is below a certain threshold. The intuition for this behavior is that based on their own signal, agents form beliefs about signals that the other players have received. Whereas for some intermediate signal, given the beliefs about the other players' actions, leaving the money in the bank and withdrawing the money are equally attractive. This signal defines a threshold strategy equilibrium if the threshold for each player is the best response to the threshold of the other players.

A further advantage of the global games approach is the theoretical integration of the panic-based and the fundamental view of bank runs, two related, but different, views found in the literature on financial stability. The panic-based view stresses that bank runs can be pure sunspots, not necessarily related to changes in the real economy. Fundamentals may be good enough so that the prevention of a bank run is desirable, but runs still occur due to strategic uncertainty about others' beliefs.

As described earlier, one of the very attractive feature of the global games approach is that it allows the calculation of the probability of a bank run. It can, therefore, be used for policy analysis in the sense that a policy maker can change the parameters in the bank run model, such as increasing partial deposit insurance or changing the expected return of bank deposit, and calculate the predicted change in the probability of a bank run, thus to make corresponding policies. However, such an inference should only be drawn if the global games approach provides a good approximation of behavior.

Although contemporary behavioral economic theories try to incorporate cognitive and emotional factors to better describe human economic decisions, they still cannot fully capture the heterogeneous nature of human behavior. For example, the prospect theory from Kahneman and Tversky (1979) describes the pattern observed from the majority of their subjects, giving no explanation concerning the behavior of the minority (14%–35% of their subjects). Recent experimental studies strongly suggest that it is no longer appropriate to assume homogeneity in either risk preference or time preference. For example, both Benjamin and Shapiro (2005)

and Christelis *et al.* (2010) showed that people with higher cognitive abilities participate more in financial markets. If this is the case, together with the finding by Frederick (2005) that the risk attitudes of the subjects with higher cognitive ability are exactly the opposite of the prospect theory from Kahneman and Tversky (1979), the foundation of modeling financial market participants according to prospect theory might not be well grounded.

The solution for the shortcoming in behavioral economic theories is to make artificial agents heterogeneous in their cognitive capacity by agent-based modeling. Computational agents can be as rational (or irrational) as real people, with learning models that range from simple reactive reinforcement learning ("If this happens, what should I do?") to sophisticated anticipatory learning ("If I do this, what might happen?"). To be specific, there are several types of agent cognition: reflexive agents, utility-based agents, and adaptive agents (Russell and Norvig 1995). Often, these types of cognition are thought of the types with increased complexity, among which reflexive agents being the simplest, and adaptive agents the most complex.

In order to present the agents' learning process clearly, we should combine agent-based modeling with machine learning, which is inspired by the work done by Herbert Simon on bounded rationality and satisfying in "Administrative Behavior" and "Sciences of the Artificial" (Barnard and Simon, 1947; Simon, 1955; 1959; 1969). Simon is the first to apply machine learning techniques to economic decision-making problems, and he is also one of the founders of modern-day Artificial Intelligence (AI). Artificial Neural Networks (ANNs) use the machine learning to simulate computational parallels of biological neurons, which seem only appropriate that in applying ANNs to decision making problems. We rely on various aspects of Simon's path-breaking work in this study.

The well-known dual-system theory (e.g., Epstein, 1994; Stanovich and West, 2000; Peters and Slovic, 2000; Kahneman and Frederick, 2002; and Loewenstein and O'Donoghue, 2004) typically incorporates the bounded rationality into economic behavior. It proposed that a person's behavior is determined by an interaction between cognitive processes that access options with a broad, goal-based perspective, and affective process that encompasses emotions and other motivational states. A single integrated system approach has been advocated by a smaller number of theorists (e.g., Damasio *et al.*, 1994; Mellers *et al.*, 1997; and Gray 2004). According to this view, emotions provide dynamic signals that feed into and help guide the cognitive system over time for making decisions. To describe exactly how this temporal integration and interaction occurs is a major challenge for this viewpoint. Busemeyer *et al.* (2007) extend their decision field theory (Busemeyer and Townsend, 1993 and Busemeyer *et al.*, 2002), which has been successfully used to explain data in traditional decision-making tasks, to provide dynamic mechanisms for integrating fast emotional signals with slower cognition information to guide decisions. Their model assumes that emotions, motivations, and cognitions interact to produce a decision, as opposed to being processed independently and in parallel. Emotions are temporary states reflecting changes in motivational levels. Affect is an evaluation of an emotional state according to a positive

(approach) or negative (avoidance) feeling (movement tendency). The dynamic nature of emotions presents challenges to traditional static theories of decision making, and the model attempts to address these dynamic characteristics formally.

Based on [Weihsong and Qiao \(2016\)](#), the so-called animal spirits of depositors can cause the tipping point of bank runs, and here we try to figure out the dynamic psychological foundation for this phenomenon. Therefore, this paper applies an ANNs nonlinear dynamic model with the global game framework to figure out the integrated cognitive and affective process in bank-run decision making. Under different levels of uncertainty, the effect of cognitive/affective process in decision making is concerned, and the effect of changing of early/late payoffs will be tested.

The rest of the paper is organized as follows: In Sec. II, we discuss the theoretical bases of the format and process of decision making in bank runs. The global game of bank runs shows the logic of the decision making, and the nonlinear dynamic decision making of bank runs is the ANNs agent-based modeling for the decision process of heterogeneous depositors. Section III reveals the specific parametrization and design, and then analyzes the results of the simulation. In Sec. IV, we will come to a conclusion.

II. GLOBAL GAME OF BANK RUNS

Table I shows the global game of bank runs. It is assumed that all N agents are patient agents. Although human beings' rationality is bounded, which will lead to the bias, however, it does not equal to irrational. To some degree, all the depositors are "strategic" players in the market, so we set all the depositors patient. If the number of withdrawers e exceeds the threshold number of withdrawers $\hat{e} = \frac{N}{r}$, the bank will not have enough money to pay every withdrawer the promised rate r . Therefore, the bank runs will occur if $e \geq \hat{e}$.

As DD model (1983) shows, banks can offer a demand-deposit contract to achieve risk sharing. To simplify it, the logic of contract is in the following. First, agents deposit their endowment in bank in period n . Then, they will make a decision in period $n+1$, withdrawing the money or waiting until period $n+2$. The early payoffs is r , and the late payoffs is $\frac{N-r \times e}{N-e}R$. If there are no early withdraws, late payoffs will become R .

In period $n+1$, when there is no bank run (number of withdrawers is less than the threshold number of withdrawers), the payoff for all agents is r , which is the promised fixed return, and most of the agents will withdraw their money in the next period. However, when there is bank run

(number of withdrawers equals to or exceeds the threshold number of withdrawers), there is some probability that agents will still be paid r , and some probability that agents will be paid nothing. When bank run happens, all agents will go to withdraw; therefore, when $e \geq \hat{e}$, it will result as $e = N$. The probability of getting early payoffs r equals to $\frac{\hat{e}}{e} = \frac{N/r}{N} = \frac{1}{r}$.

In period $n+2$, when there was bank run in period $n+1$, the payoff for agents will be zero. When there is no bank run in period $n+1$, agents will receive the payoffs of $\frac{N-r \times e}{N-e}R$ with probability of θ , where the interval for θ is $(0, 1)$ ([Goldstein and Pauzner, 2005](#)).

In the case of no bank run in period $n+1$, the likelihood of being paid nothing still exists, that is, $\theta = 0$. The probability of θ is introduced to represent the fundamental of economy. In the situation that the fundamental of economy is good, θ approaches 1, that is, the environment is clear, and the chance of occurrence of bank run is certainly slim. On the other hand, when θ becomes smaller, which means the depositors are likely to get nothing in this period, this will cause panic and ultimately bank run. Therefore, if θ is sufficiently small (large), all players have a dominant strategy to withdraw early (late). However, when θ is in indeterminacy region, the strategy becomes uncertainty.

At any point in time, each agent will make their own evaluation on this two-stage demand-deposit contract. In the following simulating test, r and R are fixed in each of simulation; however, the factor θ , which reflects the fundamental of economy, might be changed. It is assumed that agents repeatedly make evaluation and choices whether to withdraw in period $n+1$ or in period $n+2$. This can be further elaborated that agents study the outcome and make choices between these two periods in period $n+1$. And this behavior will repeatedly happen over infinite periods of time. Therefore, the iteration of the model includes both of these two stages.

Additionally, the announcements in the market will impact on agents. When making decision, the agents cannot precisely predict θ ; thus, they will concern on any information they can receive from the market. Based on the information they have, they try their best to make the optimal decision. However, this information might also be harmful to make a rational decision.

Generally, as θ is unknown, the agents make their decision to run on the bank or not based on two factors: the fundamental of economy that they already have general idea on and announcements from market which will have impact on their decision making. To find out the detailed influences from these two factors, a connectionist model with dual processes is proposed in Sec. III.

III. DUAL-PROCESS DECISION MAKING OF BANK RUNS

Agents can never make a "so called" optimal decision, only a satisfactory decision can be achieved. This concept of "bounded rationality" is introduced by Simon (1947), and he assumes that the human being as a system of behavior is quite simple. The apparent complexity of our behavior is

TABLE I. Global game of bank runs. N = numbers of agents; e = number of withdrawer; $\hat{e}$ = threshold number of withdrawers; r = early payoffs; R = late payoff; θ = probability of getting late payoffs.

Payoffs withdrawal	Payoffs	
	No run ($e < \hat{e}$)	Bank run ($e \geq \hat{e} \equiv \frac{N}{r}$) Payoffs
in Period $n+1$	r	r (with probability of $\frac{\hat{e}}{e} = \frac{1}{r}$) 0 (with probability of $1 - \frac{\hat{e}}{e}$)
in Period $n+2$	$\frac{N-r \times e}{N-e}R$ (with probability of θ) 0 (with probability of $1-\theta$)	0

largely a reflection of the complexity of the environment in which we live in. Thus, [Simon \(1955\)](#) proposed to use the “thinking machine,” that is, an artificial intelligence model, to simulate our internal system. It is not necessarily related to biological functioning, but it helps to understand the process of our decision making. [Simon \(1961\)](#) further concludes that the central motivation of the artificial neural network (ANN) is to investigate human processes of decision making. In other words, ANN can be loosely defined as artificial intelligence models inspired by analogy with the brain and realizable in computer programs.

To incorporate the bounded rationality into economic behavior, the dual-system theory has been accepted by the mainstream scholars. Now, it is time to revisit Simon’s spirit that simulates our internal system with this frontier theory. Therefore, to better understand the actual human decision making in bank-run phenomenon, a “connectionist” artificial neural network model capturing the dual-system theory will be applied with the global game framework.

It is clear that when the environment is certainly good or bad, depositors will easily choose to “run” or “no run.” However, the world we are living in is much more complicated than we thought. It is impossible for us to capture and analyze all the emerging information, let alone to make an optimal decision. When we are making a decision, it is not only captured by our cognitive processes in our mind, but also by affective processes, which happens simultaneously. Cognition by itself cannot produce action to influence behavior, and the cognitive processes must operate via the affective processes ([Camerer et al., 2005](#)). In most complex situation, the affective process is even with greater impact on our decision making.

Decision making in bank-run emerges from a continuous interplay of two broad types of decision processes, which capture the difference between deliberative/cognitive thinking and impulsive/affective decision. In contrast to most formal models in economics, such processes codetermine the decision making at the intra-individual level ([Alos-Ferrer and Strack 2014](#)). The interactions between the cognitive and affective process can be captured by one integrated system (Fig. 1).

For cognitive process, agents will make an evaluation for the payoffs. First, the early payoffs $\mathbf{M}_e$ and late payoffs $\mathbf{M}_l$ will be disclosed by banks, and there are two scenarios which are run on banks ($\mathbf{M}_{er}$ and $\mathbf{M}_{lr}$) or no-run on banks ($\mathbf{M}_{enr}$ and $\mathbf{M}_{lnr}$). Agents run on banks when they find that it is unlikely to get their promised returns. Thus, for both run and no-run scenarios, they will evaluate the weight/probability to get their payoffs at early ($\mathbf{W}_{er}$ and $\mathbf{W}_{lr}$) or late period ($\mathbf{W}_{enr}$ and $\mathbf{W}_{lnr}$). A comparison will be made based on the evaluation, which will result as $\mathbf{C}$, and the cognitive process $\mathbf{O}$ will then be formed.

Besides, any information in market will have an impact on agents’ decision, and most of the time they find that the information is ambiguous. This is when the affective process works. $\mathbf{G}$ represents the announcements from market, and $\mathbf{F}$ indicates whether agents follow the announcements. The impact of announcements is denoted as $\mathbf{D} = \mathbf{G} \cdot \mathbf{F}$.

With the influence by the announcements, the emotion and emotional state of agents will become different. Thus, the affective magnitude $\mathbf{T}$ is introduced to measure the emotion and the emotional state. The two components ($\mathbf{D}$ and $\mathbf{T}$) form the affective process $\mathbf{A}$, which, together with $\mathbf{O}$, will achieve the decision making probability of bank-run ($\mathbf{PBR}$). When $\mathbf{PBR}$ is larger than 0.5, it means that the bank-run phenomenon probably exists in the market, and it is updated and becomes historical feedback in the following period.

At any moment in time, the choice of “run” or “no run” is determined by the Probability of Bank Run (PBR). A new state of $\mathbf{PBR}(t+1)$ is formed at each moment from the previous state $\mathbf{PBR}(t)$ and the new input cognitive vector $\mathbf{O}(t+1)$ and affective vector $\mathbf{A}(t+1)$, according to the following stochastic difference equation:

$$\mathbf{PBR}(t+1) = \mathbf{A}(t+1) \circ \mathbf{PBR}(t) + \mathbf{O}(t+1), \quad (1)$$

where $\circ$ means Hadamard product.

Based on this simple updating equation, the PBR for “run” or “no run” form a PBR state matrix, denoted as $\mathbf{PBR}(t) = [\mathbf{PBR}_r(t) \quad \mathbf{PBR}_{nr}(t)]$. The dynamic probability at time t , denoted $\mathbf{PBR}_i(t)$, represents the integration of cognitive and affective processes considered for choice $i = r$ or nr up to that point in time. Cognitive process is a stochastic dynamic mechanism, while the affective one is nonlinear dynamic. Generally, the integration is one kind of dynamic learning in bank-run decision making.

A. Cognitive process

At any moment in time, each “run” or “no run” choice is associated with a cognitive process value. The cognitive process for option i at time t , denoted $\mathbf{O}_i(t)$, represents the momentary advantage or disadvantage of option i when compared with other option on payoffs under consideration. The ordered set of cognitive process for “run” or “no run” choice forms a cognitive vector, denoted $\mathbf{O}(t) = [\mathbf{o}_r(t) \quad \mathbf{o}_{nr}(t)]'$, and it is formed by the following matrix product:

$$\mathbf{O}(t) = \mathbf{C} \times \mathbf{M}(t) \circ \mathbf{W}(t). \quad (2a)$$

This vector is determined by three different components.

The first component of cognitive process is the personal evaluation of early and late payoffs, denoted by the vector

$$\mathbf{M}(t) = [\mathbf{m}_r(t) \quad \mathbf{m}_{nr}(t)]'. \quad (2b)$$

If the early payoff is higher than the late payoff, that is, $\mathbf{m}_r(t) > \mathbf{m}_{nr}(t)$, depositors will choose to run on the bank. In no run situation, agents must receive payoffs $r > 1$, but in run situation they will receive 0 with $1 - 1/r$ probability.

The second component is personal evaluation of weight attached to early and late payoff, denoted by the vector

$$\mathbf{W}(t) = [\mathbf{w}_r(t) \quad \mathbf{w}_{nr}(t)]'. \quad (2c)$$

With a high level of certainty, the depositor can easily choose to run or no run. For example, with the no-run situation considered, it will become $\mathbf{w}_{lnr}(t) = 0$ or $\mathbf{w}_{lnr}(t) = 1$ when the fundamental of economy is extremely bad or good.

FIG. 1. Connectionist interpretation of dual-process bank run's decision making. **M**: payoffs; **W**: weight; **C**: contrast; **O**: cognitive processes; **G**: announcement; **F**: following of announcement; **T**: affective magnitude; **A**: affective processes; **PBR**: probability of bank run.

The matrix product of probability and payoffs, $\mathbf{M}(t)\mathbf{W}(t)$, determines the weighted payoffs of run or no-run outcomes at each time point.

The third and final component used to determine the cognitive process of each option is the comparison process that contrasts the weighted payoff of each option. This comparison process is needed to determine the relative advantage or disadvantage of each choice on the payoffs being considered at that moment. The cognitive process for each option is produced by contrasting the weighted payoffs of “no run” against “run.” This comparison process can be represented by a simple contrast matrix

$$\mathbf{C} = \begin{bmatrix} 1 & -1/2 \\ -1/2 & 1 \end{bmatrix}. \quad (2d)$$

B. Affective processes

At any moment in time, each “run” or “no run” choice is also associated with an affective process value. The affective process for option i at time t , denoted by $A_i(t)$, represents the

effect of pessimistic/optimistic announcements with different affective magnitudes. The ordered set of affective process for “run” or “no run” choice forms an affective vector, denoted by $\mathbf{A}(t) = [a_r(t) \ a_{nr}(t)]'$, and it is formed by the matrix product

$$\mathbf{A}(t) = \mathbf{D}(t) \circ \mathbf{T}(t). \quad (3a)$$

The first component $\mathbf{D}(t)$ is the impact of announcements, which is the public information that will impact on agents but not on the fundamental of economy. The component of depositors reacting from diverse information follows an exogenous random process. This component consists of two parts. The first part is the announcements (or called information) that is denoted by vector $\mathbf{G}(t) = [g_r(t) \ g_{nr}(t)]'$. The second part is reaction (or called decision) from the announcements denoted by vector $\mathbf{F}(t) = [f_r(t) \ f_{nr}(t)]'$. The agents are heterogeneous, so different agents receive diverse information in the market resulting in different levels of impact.

Therefore, the impact of announcements for “run” or “no run” outcome is denoted by

$$\mathbf{D}(t) = \mathbf{G}(t) - \mathbf{F}(t) = [g_r(t) - f_r(t) \ g_{nr}(t) - f_{nr}(t)]. \quad (3b)$$

In our assumption, the announcements, but not the fundamental of economy, may cause the bank-run phenomenon as well. When it is in economically good times, normally the pessimistic announcements are rare, and the bank runs are not likely to happen in this market. However, there are times then rumors are spread and lots of pessimistic announcements exist in a strong market. If most of agents think the information is reliable, they might still choose to run on banks even in an economically sound market. To find out the effect of panic, it is assumed that there is 80% probability to get the pessimistic announcements and 20% probability for the optimistic ones. This assumption is based on the fact that no matter how good the market is, bad news will be filled with the whole market once it is formed. By this assumption, it is easy for us to find out the clear effect of pessimistic announcements, that is, when the non-fundamental factors could be responsible for bank runs.

The famous Reference-Dependent Model (Tversky and Kahneman 1991) proposed that losses loom larger than corresponding gains, which means that the bad announcements have bigger impact than good announcements. Therefore, it is assumed that the depositor will accept the pessimistic announcements with 80% probability and 20% for the optimistic one.

The second component is the affective magnitude. By human nature, the change of our affective magnitude is nonlinear. To guarantee a late withdrawal, the agents who choose to wait at least have to receive a higher payoff than those who choose to withdraw, that is, $\frac{N-r \times e}{N-e} \times R(\theta) > r$. If θ is sufficiently small, all players have a dominant strategy to withdraw early. However, when θ is in indeterminacy region or the pessimistic announcements are too strong, the strategy becomes uncertainty. The effect of θ is nonlinear, and it is defined as $\theta^{-\gamma}$, where $0 \leq \gamma \leq 1$. Here, we simplify this effect as $\theta^{-0.6}$ where $\gamma = 0.6$. Considering the effect of θ , to equalize the payoffs associated with “withdraw” and “wait”, that is, $r = \frac{N-r \times e}{N-e} \times R \times \theta^{-0.6}$, we can get the threshold withdrawal fraction to avoid the occurrence of bank-run, that is, $\frac{e}{N} = \frac{R \times \theta^{-0.6} - r}{r \times (R \times \theta^{-0.6} - 1)}$. Besides, the panic will be spread with different degree when the amount of the agent changes, and this nonlinear effect can be denoted as $PD = \sqrt[4]{N}$, where $\delta > 1$. Here, we simplify this effect as $PD = \sqrt[4]{N}$, where $\delta = 4$. Therefore, combining the effect from the threshold withdrawal fraction to avoid bank-run and the amount of agents, we define the affective magnitude as

$$AM = \frac{e}{N} \times \sqrt[4]{N} = \frac{(R \times \theta^{-0.6} - r) \times \sqrt[4]{N}}{r \times (R \times \theta^{-0.6} - 1)}.$$

Thus, the vector is denoted by

$$\mathbf{T} = \left[\frac{(R \times \theta^{-0.6} - r) \times \sqrt[4]{N}}{r \times (R \times \theta^{-0.6} - 1)} \quad 1 - \frac{(R \times \theta^{-0.6} - r) \times \sqrt[4]{N}}{r \times (R \times \theta^{-0.6} - 1)} \right]^T. \quad (3c)$$

In each period t , $t = 100$, the following sequence of events takes place:

1. The evaluation on the value and the probability of each payoffs, and the contrast of each decision will form the cognitive process all at once.
 - (a) Each agent can receive the early and late payoffs from the market, and then they should estimate the probability of getting each payoff.
 - (b) First, they should guess whether the bank run would happen. If there is no run, they must get r payoffs in early period. If there is run, they can get r with the probability of $1/r$.
 - (c) Second, if no run, they should figure out the probability θ of getting the payoff $\frac{N-r \times e}{N-e} R$ in the late period.
 - (d) There is a deliberative contrast in our brain, and then form the cognitive process.
2. The impact of announcements and the affective magnitude will influence the affective process together.
 - (a) The announcements will shift the decision of bank run. There is 80% probability to get the pessimistic announcements and 20% probability for the optimistic one.
 - (b) Each agent makes different or unique decisions even when they receive the same information. The depositor will accept the pessimistic announcements with 80% probability and 20% for the optimistic one.
 - (c) In the affective process, the panic of the bank run will come from two parts after the announcements, how much they can get from each payoff and how many depositors withdraw when the bank run will occur.

Integrating the effect of the cognitive process and the affective process, PBR is updated and becomes historical feedback for each agent in the following period.

IV. SIMULATION

A. Parametrization and design

To examine the behavior of the model and its robustness, simulations are conducted for a large number of parameter values in Table II.

The probability of getting late payoffs θ might be set randomly within different intervals to check the result with different levels of uncertainty, e.g., θ is in indeterminacy region, such as between 0.3 and 0.7. In the timeline, the sequent θ has 0.8 probability to keep the same rate, and changes with 0.2 probability. If the agents are completely rational, the runs only occur when θ is relatively low. However, the impact of announcements in the market will have a great impact on the bank run phenomenon, such as the announcements before the Northern Rock in September 2007. It shows that the noise in the market will lead to the bank run, even when the bank and market are good enough.

B. Simulating Results

In Sec. IV, we analyze the simulating results. First, the comparison is made between single cognitive process and cognitive-affective process in bank-run decision making. Second, the details of cognitive-affective process are figured out. Last but not the least, the effect of increased risk sharing (r), the greater late payoffs (R), and the recovering of fundamental economics (θ) will be tested. In the beginning, the benchmark initial value is set as $r = 1.2$, $R = 2$, and $n = 100$.

TABLE II. Parameter values.

Symbol	Assignment	Description
T	100	Length of a simulation
N	10, 100, and 1000	Number of agents
r	1.20, 1.32, 1.45, 1.60	The early payoffs with 10% incremental ratio
R	2.0, 2.20, 2.42, 2.66	The late payoffs with 10% incremental ratio
θ	Between 0 and 1	The probability of getting late payoffs, fixed or randomly to test the fundamental economics
G	Between 0 and 10	Announcements, 0 means extremely optimistic and 10 means extremely pessimistic
F	Between 0 and 10	Reaction on announcements, which is the difference with G

If the agents can precisely analyze the whole picture of market, bank-run only occurs when the fundamental of economy is terrible. From the left-hand side of Fig. 2, the result of single cognitive processes is plotted. When the agents have complete rationality, they will switch their strategy with the changing of real economy. When θ equal to 0.6, the probability of bank run (PBR) is around 0.5. The agents will fully consider the risk about the payoffs, and they will have 50% chance to choose to run or no run. Therefore, the risk of bank-run can be clearly figured out by real economy (shown in the lower left of Fig. 2), and it is impossible to occur if the bank works well. However, the stories that the depositors run on the well-run banks are not rare. Therefore, the introduction of integrated cognitive-affective processes is crucial, and the results are plotted on the right-hand side of Fig. 2, which is obvious that the PBR is fluctuating within a large scale while the fundamental of economy is stable. Influenced by our emotion, the announcements in the market will have dramatic impact on our decision making. Even when the

bank works well, PBR might jump to a high level as well (shown in the lower right of Fig. 2).

To check the general effect from fundamental of economy, θ varying from 0 to 1 with 1000 different values is simulated (Fig. 3). The prediction is made 1000 times at each time point, and then the average of PBR is taken. As the result, the fundamental of economy will decide the average level of PBR, and there is a clear linear negative relationship between θ and PBR.

Besides, the fluctuation of the PBR comes from affective process, and it is combined by two components: the impact of announcements and affective magnitude. The results are listed in three diagrams in Fig. 4.

For the first component impact of announcements, there are ten levels about the signal from the market, that is, 0.1 means the most optimistic (0 can be ignored) and 1 means the most pessimistic [Fig. 4(b)]. The level 0 is neglected, as it is impossible there is no impact from market. The market is complex, even in the best/worst period, there is still some

FIG. 2. Comparison of cognitive and cognitive-affective processes.

FIG. 3. Average-diminishing PBR and its corresponding θ .

bad/good news in the market. Therefore, absolutely good or bad information is rare, which means it is hard to achieve the lowest or highest level of impact from the announcement. Most of values distribute between 0.4 and 0.9 located at a higher level of PBR, and this shows that the pessimistic announcements might increase PBR significantly.

The second component is the affective magnitude, and it has a positive relationship with PBR [Fig. 4(c)]. One

explanation for this is that while the fundamental of economy goes down, depositors get panic easily, and their decision making will be much more greatly impacted by their emotion in the short term. After the agents receive the pessimistic information from the market, they become more scared when they feel the environment is terrible.

For the whole affective process effect, the pessimistic announcements will lead to the dramatic jump of PBR [Fig. 4(a)]. The graph shows that while the announcements become more pessimistic (higher value of announcements impact), the affective processes have greater impact on the higher level of PBR rather than the lower level. Therefore, the line of affective processes becomes flatter when the value increases. This means that in the same level of announcements, the affective-process effect is stronger in the high level of PBR.

Based on the whole analysis above, Fig. 5 reveals the result that the average level of PBR is shifted by the fundamental of economy, but whether the bank-run phenomenon occurs is significantly impacted by the cognitive-affective process. The former shows AG's opinion that the bank run is caused by the deterioration of the quality of the bank's

FIG. 4. PBR and affective factors.

FIG. 5. PBR and its corresponding cognitive-affective processes $\theta = 0.0 \sim 1.0$; $R = 2.0$; $r = 1.2$.

assets, and the latter explains DD's perspective that the bank run is due to miscoordination. If the economy is really terrible, there is no doubt that the bank runs are bound to happen as human beings have the cognitive capability. While the affective effect is dramatically increasing, the bank-run still occurs even the fundamental of economy is quite good. In a nutshell, with the theoretical framework of behavioral economics, the cognitive-affective process here shows a psychological basic for both DD perspective and AG perspective.

Based on the common sense, the increase in risk sharing should increase PBR; the greater late payoffs should decrease PBR; the recovering of fundamental of economy should decrease PBR as well. In the following, the effect of increased risk sharing (r), greater late payoffs (R), and recovering of fundamental economics (θ) are tested. In all scenarios, both the increased risk sharing (r) and late payoffs (R) will decrease the effect of affective processes, and increase the effect of cognitive processes (see the Appendix). This illustrates that the adjustment of both early and late payoffs is a strong long-term signal influencing the bank-run phenomenon. However, the direction and magnitude of influence are obviously different.

With a high level of uncertainty, that is, when θ is very low (smaller than 0.3) and the fundamental of economy is bad, the adjustment of both r and R is useless at this scenario (see the Appendix Figs. 7 and 8). At this time, the environment is so clear that the agents are bound to go to run, so the affective effect can be ignored. Although the increase in R will decrease PBR by a little, the value is still significantly high.

With an intermediate uncertainty, when θ is between 0.3 and 0.7, the increase in R will reduce PBR as the effect of affective processes will be decreased even negligible. When R is large enough, the effect of cognitive process will dominate PBR (see the Appendix Fig. 9), and AG is correct at this moment. At this time, the increased risk sharing will shift PBR up, and the effect of affective process still impacts PBR (see

the Appendix Fig. 10). At this time, the increased risk sharing is a disaster to the market.

With a low level of uncertainty, when θ is between 0.7 and 0.9, the increase in R can eliminate the bank run phenomenon (see the Appendix Fig. 11). The increase in r will decrease the effect of affective process, so the fluctuation of PBR become small, and this effect will shift PBR down. On the other hand, the increased risk sharing will move the average of PBR up (see the Appendix Fig. 12). Therefore, the comprehensive effect is quite unclear, and the reaction of the increased risk sharing is small proved by the experiment result (Klos and Sträter 2012).

The simulating result is in line with DD's perspective that bank runs still occur even the fundamental of economy is good. Figure 6 is an example of low level of uncertainty, that is, θ is between 0.7 and 0.9. Most of the points locate at either very low or high level, which shows the dynamic version of DD's model, indicating that there are two self-fulfilling symmetric pure-strategy Nash equilibria. At this scenario, the fundamental of economy is relatively good, yet there exist lots of pessimistic announcements in the market. It is concluded from the stimulation that if PBR is smaller than 0.23 (the red line in the upper panel), it is quite stable in the low level. However, if PBR is larger than 0.23, PBR dramatically jumps to a high level.

The tipping point effect of the bank-run phenomenon proposed in our previous work (Weihong Huang and Qiao Huang 2016) explained the above. It refers to one dramatic moment in the bank-run phenomenon when everything can change all at once. Our recent work finds that this effect exists at both low and middle levels of uncertainty. While, in the high level of uncertainty, the bank run is likely to happen, this kind of effect is neglected. Generally, this is a dynamic process that the historical results will influence the updating PBR. However, the affective processes are temporary states that impact by the announcements. Therefore, there exists high probability that PBR will tip to a high level in two conditions: (1) the impact

FIG. 6. PBR and its corresponding cognitive-affective processes $\theta = 0.7 \sim 0.9$; $R = 2.0$; $r = 1.2$.

of affective processes increases to a high level; (2) PBR passes the tipping point. If the bank-run has been formed and the pessimistic announcements remain over a long period time in the market, PBR will stay in a very high level. At this time, as we proposed earlier, both the increasing of late payoffs value and the recovery of fundamental of economy are useful to solve this disaster. The best is to take the intervention ahead of PBR passing the tipping point.

V. CONCLUSION

This paper focuses on certain probability of bank runs that is neglected by the classical model. The global game of the bank runs shows the logic of the decision making, and the heterogeneous agents hold highly correlated but unidentical beliefs about the true payoffs. The dynamic decision process integrates the well-known dual-system theory into a ANNs agent-based modeling system.

The pessimistic announcements will cause the bank-run even when the fundamental of economy is good, because the affective process plays a significant role in decision making. Both the rise of R and r will decrease the effect of affective process. However, with the change of r , the announcements in the market still dramatically switch the depositors' strategies despite the lower level of uncertainty. The increased risk sharing might or might not increase PBR, which depends on the level of uncertainty. The increase in late payoff is beneficial for preventing the bank run if the environment is not extremely terrible, even it can prevent the bank-run

when the fundamental of economy is good. This simulation result is robust for different number of agents. Therefore, when the bank-run phenomenon happened, banks should take strategy that increase the late payoffs and government must recover fundamental of economy as soon as possible. For the policy makers and financial analysts, to find out the tipping point of bank run is a creative and practical solution in the more and more complicated banking system.

This paper is one of pioneers that links agent-based computational economics to behavioral economics, and the findings in this research may shed some light on investor's analysis, banker's adjustment, central bank's monetary policy, and government's economic policy. The future research will further explore the tipping point of bank-runs when r or R changes with different levels of θ . With different tipping points, the effect of cognitive-affective process will be varied, and the relationship with PBR should be figured out.

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APPENDIX: DIFFERENT R/r WITH DIFFERENT LEVEL OF UNCERTAINTY

FIG. 7. Different R with high level of uncertainty. FIG. 8. Different r with high level of uncertainty.

FIG. 9. Different R with middle level of uncertainty. FIG. 10. Different r with middle level of uncertainty.

FIG. 11. Different R with low level of uncertainty. FIG. 12. Different r with low level of uncertainty.

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