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Investigating the Effects of Gyroscopic Coupling and Countertorque in a Fixed-Wing Nano Air Vehicle

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For the nonlinear equations of motion of the nano air vehicle, including the gyroscopic coupling and countertorque generated by the propulsion system, a linear state-space model is developed for analyzing the effects of gyroscopic coupling and countertorque on the dynamic behavior of the nano air vehicle. By analyzing the trajectories of eigenvector components obtained by increasing the gyroscopic coupling, it is shown that the different modes of the nano air vehicle influence both the lateral and longitudinal variables, thereby losing the conventional characteristics of these modes. The investigation shows that the stability of the coupled phugoid and coupled Dutch roll modes are improved, whereas the stability of the coupled spiral, coupled roll subsidence, and coupled short-period modes are degraded with the enhancement of gyroscopic coupling. Concurrently, increasing the gyroscopic coupling increases the damping ratio of the coupled Dutch roll and coupled phugoid modes. In contrast to this, the gyroscopic coupling reduces the damping ratio of the coupled short-period mode. The frequency of the coupled phugoid and coupled short-period modes is also increased with the increment in the gyroscopic coupling. The analysis also indicates that the countertorque destabilizes the coupled spiral mode.

Nomenclature

A_{coupled}	= stability matrix of coupled state-space model
A_{Lat}	= stability matrix of uncoupled lateral state-space model
A_{Long}	= stability matrix of uncoupled longitudinal state-space model
$A_{\text{Lat}}^{\text{Long}}$	= lateral coupling block of A_{coupled}
$A_{\text{Long}}^{\text{Lat}}$	= longitudinal coupling block of A_{coupled}
B_{Lat}	= lateral control matrix
B_{Long}	= longitudinal control matrix
$B_{\text{Lat}}^{\text{Long}}$	= lateral coupling block of B_{coupled}
$B_{\text{Long}}^{\text{Lat}}$	= longitudinal coupling block of B_{coupled}
b	= wingspan
C_D, C_L, C_Y	= aerodynamic force coefficients
$C_{D_q}, C_{L_q}, C_{M_q}$	= change in C_D, C_L , and C_M with respect to q
C_l, C_m, C_n	= aerodynamic moment coefficients
$C_{l_p}, C_{n_p}, C_{Y_p}$	= change in C_l, C_n , and C_Y with respect to p
$C_{l_r}, C_{n_r}, C_{Y_r}$	= change in C_l, C_n , and C_Y with respect to r
c	= wing chord
F_{Tx}	= dynamic thrust
$h_{x_{rp}}^b$	= body-fixed angular momentum of the rotor/propeller of the nano air vehicle (X^b component)
J_{xx}, J_{yy}, J_{zz}	= body-fixed moment of inertia components of the nano air vehicle
J_{xz}	= body-fixed product of inertia component of the nano air vehicle
$J_{xx_{rp}}^b$	= body-fixed moment of inertia of rotor/propeller of the nano air vehicle

L_A, M_A, N_A	= aerodynamic moments (roll, pitch, and yaw components)
L_T	= rolling moment due to thrust (countertorque)
M_T	= pitching moment due to thrust
p, q, r	= body-fixed angular rates (roll, pitch, and yaw components)
$\bar{p}, \bar{q}, \bar{r}$	= linearized p, q , and r
S	= reference area
U_{con}	= control vector of coupled state-space model ($[\bar{\delta}_e, \bar{\delta}_r]^T$)
u, v, w	= body-fixed translational velocities (X^b, Y^b , and Z^b components)
$\bar{u}, \bar{v}, \bar{w}$	= linearized u, v , and w
V_a	= airspeed
X_A, Y_A, Z_A	= aerodynamic forces (X^b, Y^b , and Z^b components)
X^b, Y^b, Z^b	= body-fixed coordinates (forward, right, and downward components)
$\bar{x}$	= state vector of coupled state-space model ($[\bar{u}, \bar{w}, \bar{q}, \bar{\theta} \bar{p}, \bar{r}, \bar{\phi}]^T$)
$\bar{x}_{\text{Lat}}$	= lateral state vector ($[\bar{v}, \bar{p}, \bar{r}, \bar{\phi}]^T$)
$\bar{x}_{\text{Long}}$	= longitudinal state vector ($[\bar{u}, \bar{w}, \bar{q}, \bar{\theta}]^T$)
Z_{VT}	= vertical distance between center of gravity and thrust line
α, β	= angle of attack and sideslip angle
δ_e, δ_r	= elevator and rudder deflections
δ_T	= propeller speed
$\bar{\delta}_e, \bar{\delta}_r$	= linearized δ_e and δ_r
$\bar{\delta}_{\text{Lat}}$	= lateral control vector ($[\bar{\delta}_r]$)
$\bar{\delta}_{\text{Long}}$	= longitudinal control vector ($[\bar{\delta}_e, \bar{\delta}_T]^T$)
$\bar{\delta}_T$	= linearized δ_T
ρ_a	= air density
ϕ, θ	= Euler angles (roll angle and pitch angle)
$\bar{\phi}, \bar{\theta}$	= linearized ϕ and θ
$\Omega_{x_{rp}}^b$	= angular velocity of the rotor/propeller (X^b component)

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I. Introduction

A NANO AIR VEHICLE (NAV) is a small micro air vehicle with overall dimensions of less than or equal to 7.5 cm and with a

gross takeoff weight of 10 to 20 g [1,2]. A NAV is mainly designed to perform intelligence, surveillance, and reconnaissance missions at an affordable cost. The challenges faced in the design and development of a NAV include aerodynamic uncertainty at low Reynolds numbers (less than 100,000) and sensitivity of the NAV to the atmospheric turbulence [3,4].

The flight dynamics of a NAV are influenced by the effects that are usually neglected in large aircraft and unmanned aerial vehicles (UAVs). The most important effect is coupling between lateral and longitudinal dynamics, which is due to inertial, gyroscopic, and aerodynamic effects. The inertial coupling occurs if the aircraft has asymmetric mass distribution. The aerodynamic coupling results mainly from asymmetric flight conditions that are due to the nonzero sideslip angle, roll angle, or angular rate. A single-propeller aircraft usually has asymmetric flight conditions unless the aircraft is employed with counter-rotating propellers, or it should have an even number of propellers, symmetrically distributed spanwise [5]. Most of the micro air vehicle and NAVs are driven by a single engine/propeller combination. The aerodynamic coupling in the NAV was investigated in [6]. Another source of coupling is the gyroscopic effect. The gyroscopic coupling occurs as the result of a net angular momentum associated with rotors that spin relative to the body-fixed coordinate frame. In [5], the effects of gyroscopic coupling on the dynamics of a larger aircraft were studied.

As mentioned in [5], gyroscopic coupling induces considerable coupling between the lateral and longitudinal dynamics if the mass of the propulsion system is a significant fraction of the total mass of a aircraft. Generally, the mass of the propulsion system of a NAV provides notable contributions to the total mass [2]. This results in a substantial rotor/propeller moment of inertia. Hence, the effects of the gyroscopic coupling on the dynamics of a NAV cannot be neglected. At the same time, most of the NAVs are equipped with a single rotating motor/propeller that generates counter torque. This counter torque also induces coupling between the lateral and longitudinal dynamics due to an appreciable value of the product of inertia and a small value of the moment of inertia of a NAV. Hence, while developing a NAV, it is important to understand the effects of the gyroscopic coupling and counter torque on the dynamics of a NAV. Therefore, the main objective of this paper is to examine the coupling effects on the linear dynamics of the fixed-wing NAV due to the gyroscopic coupling and counter torque by conducting a parametric study.

The remainder of the paper is arranged as follows. Section II describes the nonlinear equations of motion of the NAV, including the gyroscopic coupling and counter torque. The linear equations of motion are given in Sec. III. Section IV discusses the analysis of

coupling in the NAV. The flight-test results are presented in Sec. V. Section VI contains concluding remarks.

II. Nonlinear Equations of Motion of NAV Including Gyroscopic Coupling and Counter torque

The 7.5 cm NAV shown in Fig. 1a is a horizontal tailless aircraft with a high-wing configuration and two control surfaces, namely, the elevator and rudder. The high-wing configuration with a center of gravity (CG) below the wing camber line is used to enhance the lateral stability of the NAV, thereby reducing the effect of the counter torque produced by the motor/propeller combination. In addition to that, the lateral stability is enhanced by the keel effect (dihedral effect or pendulum effect). The keel effect is the result of the rolling moment due to side force. The side-force-producing surfaces above the CG will increase the dihedral effect, whereas the side-force-producing surface below the CG will decrease the dihedral effect. Therefore, the CG of the NAV is placed far below the wing, as shown in Fig. 1b. To get enough pitching moment, the value of Z_{VT} is selected such that the thrust also provides an adequate pitchup moment. The Z_{VT} is shown in Fig. 1b. The wing planform of the NAV is rectangular. The airfoil for the wing is an Eppler 59. The modified J5012 is used as the airfoil for the vertical fin. The NAV is propelled using a single motor/propeller combination. The propeller used is a GWS 3 × 2 for which the diameter is 7.62 cm with a pitch of 5.08 cm. The mass and mechanical properties of the vehicle are shown in Table 1.

The conventional nonlinear force and moment equations of motion of a rigid aircraft were presented in [7]. While deriving the rotational equations shown in [7], the angular momentum of the rotor/propeller combination is neglected with the assumption that the aircraft has negligible cross-coupling from spinning bodies like the turbines, propellers, and compressors. The more accurate and realistic rotational equations of motion were derived in [8], considering the angular momentum of spinning bodies. Based on the rotational equations of motion given in [8], three rotational equations of motion of the NAV, including the angular momentum of the rotor/propeller combination, are written as

$$\begin{aligned} \dot{p} = & \frac{J_{xz}}{J_D} [N_A + h_{x_{ip}}^b q + (J_{xx}p - J_{xz}r)q - J_{yy}pq] \\ & + \frac{J_{zz}}{J_D} [L_A + L_T + (J_{xz}p - J_{zz}r)q + J_{yy}qr] \end{aligned} \quad (1)$$

Fig. 1 Geometric diagram (left) and forces and moments acting on the NAV (right).

Table 1 Mass and mechanical properties of the NAV

Parameter	Value
Weight, g	19.40
Wing chord, cm	6.00
Wingspan, cm	7.50
Wing area, cm ²	45
Vertical fin chord, cm	7.50
Vertical fin span, cm	7.00
Vertical fin area, cm ²	48
Elevator chord, cm	1.50
Elevator span, cm	7.50
Elevator area, cm ²	11
Rudder chord, cm	1.50
Rudder span, cm	2.50
Rudder area, cm ²	3.75
Wing aspect ratio	1.25
J_{xx} , g · cm ²	98.90
J_{yy} , g · cm ²	186.22
J_{zz} , g · cm ²	104.26
J_{xz} , g · cm ²	44.64
$J_{x\bar{x}p}$, g · cm ²	5.18
le , cm	5.08
lr , cm	5.58
lv , cm	0.53
zr , cm	1.50
zv , cm	2.00
zw , cm	4.20
Z_{VT} , cm	1.40

$$\dot{q} = \frac{1}{J_{yy}} \left[M_A + M_T - h_{x\bar{p}}^b r - (J_{xz}p - J_{zz}r)p - (J_{xx}p - J_{xz}r)r \right] \quad (2)$$

$$\begin{aligned} \dot{r} = & \frac{J_{xz}}{J_D} [L_A + L_T + (J_{xz}p - J_{zz}r)q + J_{yy}qr] \\ & + \frac{J_{xx}}{J_D} [N_A + h_{x\bar{p}}^b q + (J_{xx}p - J_{xz}r)q - J_{yy}pq] \end{aligned} \quad (3)$$

where $J_D = J_{xx}J_{zz} - J_{xz}^2$, $h_{x\bar{p}}^b = J_{x\bar{x}p}^b \Omega_{x\bar{p}}^b$, and $\Omega_{x\bar{p}}^b = 2\pi\delta_T$.

In the translational equations of motion described in [7] and the rotational equations of motion shown in Eqs. (1–3), the externally applied aerodynamic forces (X_A , Y_A , and Z_A) and moments (M_A , L_A , and N_A) acting on the NAV along the body axis are given by Eqs. (4–9):

$$\begin{aligned} X_A = & (0.5\rho_a V_a^2 S) \left[C_L(\alpha, \beta, \delta_e) \sin(\alpha) - C_D(\alpha, \beta, \delta_e) \cos(\alpha) \right. \\ & \left. + (C_{Lq} \sin(\alpha) - C_{Dq} \cos(\alpha)) \left(\frac{0.5\bar{c}}{V_a} \right) q \right] \end{aligned} \quad (4)$$

$$\begin{aligned} Z_A = & (0.5\rho_a V_a^2 S) \left[-C_L(\alpha, \beta, \delta_e) \cos(\alpha) - C_D(\alpha, \beta, \delta_e) \sin(\alpha) \right. \\ & \left. - (C_{Lq} \cos(\alpha) + C_{Dq} \sin(\alpha)) \left(\frac{0.5\bar{c}}{V_a} \right) q \right] \end{aligned} \quad (5)$$

$$Y_A = (0.5\rho_a V_a^2 S) \left[C_Y(\alpha, \beta, \delta_r) + C_{y_p} \left(\frac{0.5\bar{c}}{V_a} \right) p + C_{y_r} \left(\frac{0.5\bar{c}}{V_a} \right) r \right] \quad (6)$$

$$M_A = (0.5\rho_a V_a^2 \bar{c} S) \left[C_M(\alpha, \beta, \delta_e) + C_{M_q} \left(\frac{0.5\bar{c}}{V_a} \right) q \right] \quad (7)$$

$$L_A = (0.5\rho_a V_a^2 b S) \left[C_l(\alpha, \beta, \delta_r) + C_{l_p} \left(\frac{0.5b}{V_a} \right) p + C_{l_r} \left(\frac{0.5b}{V_a} \right) r \right] \quad (8)$$

$$N_A = (0.5\rho_a V_a^2 b S) \left[C_n(\alpha, \beta, \delta_r) + C_{n_p} \left(\frac{0.5b}{V_a} \right) p + C_{n_r} \left(\frac{0.5b}{V_a} \right) r \right] \quad (9)$$

The externally applied moments due to thrust (M_T and L_T) are given in Eqs. (10) and (11):

$$M_T = F_{Tx} Z_{VT} \quad (10)$$

$$L_T = -\frac{F_{Tx} u}{2\pi(\delta_T)} \quad (11)$$

The aerodynamic coefficients C_L , C_D , and C_M given in Eqs. (4), (5), and (7) depend on α , β , and δ_e . Generally, the effect of β on the longitudinal aerodynamic coefficients of normal aircraft (with symmetric flight conditions) is neglected because aerodynamic cross-coupling is negligibly small. However, in the case of the NAV, the influence of sideslip is significant because the NAV has asymmetric flight conditions due to notable aerodynamic cross-coupling. The aerodynamic forces and moments are obtained from wind-tunnel tests. All dynamic derivatives are computed by applying the theoretical and empirical formulas mentioned in [9,10]. The complete equations of motion of the NAV are defined by the translational equations of motion as expressed in [7], the rotational equations of motion expressed in Eqs. (1–3), and the three kinematic equations and one navigational equation as given in [11]. These equations of motion are used for trim calculations.

The values of the trim variables at a flight velocity of 8 m/s are shown in Table 2 for straight and constant altitude flight conditions. The nonzero trim values of β and ϕ shown in Table 2 are due to the asymmetrical components of the forces and moments in the dynamics of the NAV, which is driven by a single propeller/motor combination. For the NAV with symmetrical flight conditions, the trim values of β and ϕ are nearly zero.

III. Linear Equations of Motion

The nonlinear equations of motion are used to predict the motion of the NAV. As established mathematical tools are available for the analysis and the control system design of linear systems, linearized models are obtained with the assumption that they approximately capture the behavior of the nonlinear model in the neighborhood of

Table 2 Trim solution for a straight and constant altitude flights at 8 m/s and $Re = 33,000$

Trim variables	Trim value
α , rad	0.2950
δ_e , rad	-0.1924
δ_T , rps	175.8746
Thrust, N	0.0870
Lift, N	0.1627
Drag, N	0.0760
δ_r , rad	-0.1146
β , rad	-0.0366
θ , rad	0.2950
ϕ , rad	-0.0236
u , m/s	7.6497
w , m/s	2.3229
v , m/s	-0.2928
p , rad/s	0.0000
q , rad/s	0.0000
r , rad/s	0.0000

the equilibrium/trim point. The linearized equations of motion of the NAV are found by taking the Jacobian of the translational equations of motion expressed in [7], the rotational equations of motion shown in Eqs. (1–3), and the kinematic and navigational equations as given in [11]. The conventional linear decoupled longitudinal and lateral state-space models are expressed in Eqs. (12) and (13), respectively, as

$$\dot{\bar{\mathbf{x}}}_{\text{Long}} = A_{\text{Long}} \bar{\mathbf{x}}_{\text{Long}} + B_{\text{Long}} \bar{\delta}_{\text{Long}} \quad (12)$$

$$\dot{\bar{\mathbf{x}}}_{\text{Lat}} = A_{\text{Lat}} \bar{\mathbf{x}}_{\text{Lat}} + B_{\text{Lat}} \bar{\delta}_{\text{Lat}} \quad (13)$$

Generally, decoupled lateral and longitudinal equations of motion exist under the absence of rotor gyroscopic effects, aerodynamic cross-coupling, and inertial coupling [12]. However, with the presence of significant cross-coupling results in coupled linear equations of motion, as given in Eqs. (14) and (15),

$$\dot{\bar{\mathbf{x}}} = A_{\text{coupled}} \bar{\mathbf{x}} + B_{\text{coupled}} \mathbf{U}_{\text{con}} \quad (14)$$

$$B_{\text{coupled}} = \begin{bmatrix} -4.86 & 0.13 & 0 \\ -11.81 & 0 & 0 \\ -533.80 & 2.68 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4.30 \\ 0 & -1.74 & -237.30 \\ 0 & -0.75 & -226.20 \\ 0 & 0 & 0 \end{bmatrix} \quad (18)$$

IV. Analysis of Coupling in the NAV

The eigenvalues and the eigenvectors of the decoupled longitudinal and lateral models expressed in Eqs. (12) and (13), respectively, are shown in Tables 3–5. These tables indicate that the eigenstructure of the different modes of the NAV is similar to the eigenstructure of a conventional aircraft. The eigenstructure of the

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} X_u & X_w & X_q & -gc\theta & X_v & X_p & X_r & 0 \\ Z_u & Z_w & Z_q & -gc\phi s\theta & Z_v & Z_p & Z_r & -gs\phi c\theta \\ M_u & M_w & M_q & 0 & M_v & M_p & M_r & 0 \\ 0 & 0 & c\phi & 0 & 0 & 0 & -s\phi & -qs\phi - rs\phi \\ Y_u & Y_w & Y_q & -gs\phi s\theta & Y_v & Y_p & Y_r & gc\theta c\phi \\ L_u & L_w & L_q & 0 & L_v & L_p & L_r & 0 \\ N_u & N_w & N_q & 0 & N_v & N_p & N_r & 0 \\ 0 & 0 & s\phi t\theta & \frac{qs\phi + rc\phi}{c\theta^2} & 0 & 1 & c\phi t\theta & qt\theta c\phi - rt\theta s\phi \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{w} \\ \bar{q} \\ \bar{\theta} \\ \bar{v} \\ \bar{p} \\ \bar{r} \\ \bar{\phi} \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_T} & 0 \\ Z_{\delta_e} & 0 & 0 \\ M_{\delta_e} & M_{\delta_T} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Y_{\delta_r} \\ 0 & L_{\delta_T} & L_{\delta_r} \\ 0 & N_{\delta_T} & N_{\delta_r} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\delta}_e \\ \bar{\delta}_T \\ \bar{\delta}_r \end{bmatrix} \quad (15)$$

The state-space model shown in Eq. (15) can be partitioned into longitudinal and lateral component forms as

$$\begin{bmatrix} \dot{\bar{\mathbf{x}}}_{\text{Long}} \\ \dot{\bar{\mathbf{x}}}_{\text{Lat}} \end{bmatrix} = \begin{bmatrix} A_{\text{Long}} & A_{\text{Long}}^{\text{Lat}} \\ A_{\text{Lat}}^{\text{Long}} & A_{\text{Lat}} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{x}}_{\text{Long}} \\ \bar{\mathbf{x}}_{\text{Lat}} \end{bmatrix} + \begin{bmatrix} B_{\text{Long}} & B_{\text{Long}}^{\text{Lat}} \\ B_{\text{Lat}}^{\text{Long}} & B_{\text{Lat}} \end{bmatrix} \begin{bmatrix} \bar{\delta}_{\text{Long}} \\ \bar{\delta}_{\text{Lat}} \end{bmatrix} \quad (16)$$

When the influence of lateral variables on the longitudinal dynamics as well as the influence of longitudinal variables on the lateral dynamics are negligibly small, then the elements of $A_{\text{Long}}^{\text{Lat}}$, $A_{\text{Lat}}^{\text{Long}}$, $B_{\text{Long}}^{\text{Lat}}$, and $B_{\text{Lat}}^{\text{Long}}$ will be negligible. This results in decoupled longitudinal and lateral equations of motion of the NAV as expressed in Eqs. (12) and (13).

The system matrix of the coupled linear model at the straight and constant altitude flight condition with a flight speed of 8 m/s is given in Eq. (17):

$$A_{\text{coupled}} = \begin{bmatrix} -1.72 & 0.27 & -2.30 & -9.38 & -0.65 & 0 & -0.29 & 0 \\ -1.64 & -2.69 & 7.58 & -2.85 & -0.11 & 0.29 & 0 & 0.22 \\ -23.39 & -84.55 & -2.52 & 0 & -0.055 & 0 & -30.75 & 0 \\ 0 & 0 & 0.99 & 0 & 0 & 0 & 0.024 & 0 \\ -0.02 & -0.016 & 0 & 0.07 & -2.21 & 2.20 & -7.59 & 9.37 \\ 23.55 & -1.84 & 30.73 & 0 & -177.17 & -7.98 & 3.94 & 0 \\ 15.92 & -15.03 & 68.08 & 0 & -36.23 & -3.79 & -0.19 & 0 \\ 0 & 0 & -0.01 & 0 & 0 & 1 & 0.30 & 0 \end{bmatrix} \quad (17)$$

Table 3 Eigenvalues of decoupled system

Modes	Values
<i>Longitudinal eigenvalues ($\lambda_{1,2}$ and $\lambda_{3,4}$)</i>	
Short period	$-2.73 \pm 24.2i$
Phugoid	$-1, -0.464$
<i>Lateral eigenvalues ($\lambda_{5,6}$, λ_7, and λ_8)</i>	
Dutch roll	$3.80 \pm 16.4i$
Roll subsidence	-17.9
Spiral	-0.112

Table 6 Eigenvalues of coupled model

Eigenvalues	Values
$\lambda_{1,2}$	$-2.63 \pm 49.7i$
$\lambda_{3,4}$	$-1.41 \pm 0.862i$
$\lambda_{5,6}$	$-0.330 \pm 18.8i$
λ_7	9.52
λ_8	0.951

coupled system is shown in Tables 6–8. These tables indicate that the eigenstructure of the coupled system differs vastly from the eigenstructure given in Tables 3–5. To study solely the impact of gyroscopic coupling and counter torque on the dynamics of the NAV, a linear model of the NAV at the straight and constant altitude flight condition is taken into account. For a straight and constant altitude flight condition, the aerodynamic coupling is negligible. To examine the character of the coupling between longitudinal and lateral dynamics as a result of gyroscopic coupling and counter torque effects, a parametric study is conducted about the variation of eigenstructure attributable to the varying degrees of coupling within the system matrix. To understand the effects of gyroscopic coupling alone, one must first determine the elements in A_{coupled} related to the gyroscopic coupling. In that respect, the partial derivative of Eq. (2) with respect to r and the partial derivative of Eqs. (1) and (3) with respect to q are taken; then, the conditions of straight and constant altitude flight are applied to these partial derivatives to obtain Eqs. (19–21):

$$\frac{\partial \dot{q}}{\partial r} = \frac{-h_{x_{rp}}^b}{J_{yy}} = \frac{-2\pi\delta_T J_{xx_{rp}}^b}{J_{yy}} = M_r \quad (19)$$

$$\frac{\partial \dot{p}}{\partial q} = \frac{J_{xz}}{J_D} h_{x_{rp}}^b = 2\pi\delta_T J_{xx_{rp}}^b \frac{J_{xz}}{J_D} = L_q \quad (20)$$

$$\frac{\partial \dot{r}}{\partial q} = \frac{J_{xx}}{J_D} h_{x_{rp}}^b = 2\pi\delta_T J_{xx_{rp}}^b \frac{J_{xx}}{J_D} = N_q \quad (21)$$

For the linear model of the NAV, the elements M_r , L_q , and N_q in Eqs. (19–21) depend upon $h_{x_{rp}}^b$ through the δ_T and $J_{xx_{rp}}^b$, indicating that they embody gyroscopic effects. A new system matrix A_{coupledy} is obtained by setting every other cross-coupling term of A_{coupled} to

Table 7 Eigenvectors of coupled model

Eigenvector components	$\lambda_{1,2}$		$\lambda_{5,6}$	
	Amplitude	Phase, rad	Amplitude	Phase, rad
$\bar{u}$	0.031	± 2.92	0.0105	∓ 0.029
$\bar{w}$	0.096	± 0.004	0.031	± 2.532
$\bar{q}$	0.619	± 1.58	0.063	∓ 1.4
$\bar{\theta}$	0.014	∓ 0.080	0.0032	∓ 2.99
$\bar{v}$	0.119	± 1.588	0.126	∓ 1.89
$\bar{p}$	0.061	∓ 2.11	0.987	0
$\bar{r}$	0.767	0	0.061	± 1.34
$\bar{\phi}$	0.004	∓ 1.91	0.0527	∓ 1.57

zero to examine coupling effects only due to M_r , L_q , and N_q . By varying the $J_{xx_{rp}}^b$ or δ_T , it is possible to vary $h_{x_{rp}}^b$, thereby varying the gyroscopic coupling. When either $J_{xx_{rp}}^b$ or δ_T increases, gyroscopic coupling also increases because $h_{x_{rp}}^b$ is proportional to $J_{xx_{rp}}^b$ and δ_T . From Eqs. (19–21), it is understood that, when $J_{xx_{rp}}^b$ or δ_T is zero, the elements M_r , L_q , and N_q are also zero. Then, the eigenvalues of A_{coupledy} are equivalent to the eigenvalues of system matrices of the decoupled lateral and longitudinal models of the NAV. Therefore, the trajectories of eigenvalues of A_{coupledy} start from the eigenvalues of decoupled system matrices. The trajectories of eigenvalues of A_{coupledy} for the analysis are obtained by varying δ_T , with $J_{xx_{rp}}^b$ kept as constant.

The study of the gyroscopic effect by varying δ_T is also useful in understanding the effect of counter torque. This is because the effect of counter torque on the dynamics of the NAV can be better understood by a comparative study of the trajectories of eigenvalues of A_{coupledy} and the system matrix with only the cross-coupling elements related to gyroscopic coupling and counter torque. If any difference is found while comparing these trajectories, it can be attributed to the effect of counter torque. As the elements of gyroscopic coupling and counter torque depend on δ_T , the trajectories of the eigenvalues of the system matrix are obtained by varying δ_T , which is a common variable to gyroscopic coupling as well as counter torque terms.

Table 4 Eigenvectors of longitudinal model

Eigenvector components	$\lambda_{1,2}$		$\lambda_{3,4}$			
	Amplitude	Phase, rad	Amplitude	Phase, rad	Amplitude	Phase, rad
$\bar{u}$	0.089	± 1.39	0.952	0	0.950	0
$\bar{w}$	0.297	∓ 1.57	0.265	0	0.265	0
$\bar{q}$	0.950	0.00	0.107	0	0.070	0
$\bar{\theta}$	0.039	∓ 1.68	0.106	0	0.152	0

Table 5 Eigenvectors of lateral model

Eigenvector components	$\lambda_{5,6}$		λ_7		λ_8	
	Amplitude	Phase, rad	Amplitude	Phase, rad	Amplitude	Phase, rad
$\bar{v}$	0.106	0.961	0.029	0.00	0.059	0
$\bar{p}$	0.971	0	0.287	0.00	0.942	0
$\bar{r}$	0.2061	-2.467	0.707	0.00	0.325	0
$\bar{\phi}$	0.0605	1.837	0.646	0.00	0.0583	0

Table 8 Eigenvectors of coupled model

Eigenvector components	λ_7		$\lambda_{3,4}$		λ_8	
	Amplitude	Phase, rad	Amplitude	Phase, rad	Amplitude	Phase, rad
$\bar{u}$	0.137	0.0	0.177	± 3.069	0.457	0
$\bar{w}$	0.0630	0.00	0.099	∓ 3.039	0.3428	0
$\bar{q}$	0.0243	0.00	0.025	∓ 2.264	0.1364	0
$\bar{\theta}$	0.003	0	0.01	± 1.49	0.1283	0
$\bar{v}$	0.0189	0.00	0.0129	± 0.719	0.0419	0
$\bar{p}$	0.977	0.00	0.8102	± 3.14	0.2436	0
$\bar{r}$	0.1684	0.00	0.348	± 0.739	0.6105	0
$\bar{\phi}$	0.108	0.00	0.450	± 0.44	0.4528	0

The eigenvectors of A_{coupled} are shown in Tables 7 and 8. It is obvious from these tables that each mode of the coupled model of NAV influences both lateral and longitudinal variables; hence, it is difficult to relate these modes to the conventional modes of the aircraft. However, by observing the trajectories of eigenvalues of the coupled model, it is easy to identify the eigenvalues associated with different modes of the NAV based on the continuation of standard eigenvalues of decoupled system. The migration of eigenvalues of $A_{\text{coupledgy}}$ due to the change in gyroscopic coupling elements by varying δ_T from 0 to 400 rps is shown in Figs. 2a–2d.

Figure 2a shows the trajectories of eigenvalues $\lambda_{1gy,2gy}$ and $\lambda_{5gy,6gy}$. The trajectories of eigenvalues $\lambda_{1gy,2gy}$ and $\lambda_{5gy,6gy}$ begin from the eigenvalues of short-period and Dutch roll modes of the decoupled model, respectively. To understand the effect of gyroscopic coupling on the eigenvalues in a better way, the trajectories of eigenvector components of corresponding eigenvalues are studied. Figures 3a–3d show the variation of the key eigenvector components of eigenvectors corresponding to eigenvalues $\lambda_{1gy,2gy}$ and $\lambda_{5gy,6gy}$.

Figures 3a–3d show that, when the δ_T is low (up to 50 rps), the influence of $\lambda_{1gy,2gy}$ on the longitudinal variables $\bar{q}$ and $\bar{w}$ are greater than the lateral variables $\bar{r}$ and $\bar{v}$. This suggests that, when the gyroscopic coupling is low, eigenvalues $\lambda_{1gy,2gy}$ exhibit characteristics

of a short-period mode. However, when the gyroscopic coupling increases by increasing the δ_T , the influence of modes of $\lambda_{1gy,2gy}$ on the lateral variables $\bar{r}$ and $\bar{v}$ increases. This is evident from Figs. 3c and 3d. From Fig. 3c, it can be seen that the influence of $\lambda_{5gy,6gy}$ on the variable $\bar{r}$ reduces with the increase of δ_T , i.e., with the increase in gyroscopic coupling.

The gyroscopic coupling also changes the damping ratio and the frequency of eigenvalues $\lambda_{1gy,2gy}$ and $\lambda_{5gy,6gy}$. The variation of the damping ratio and the frequency of $\lambda_{1gy,2gy}$ and $\lambda_{5gy,6gy}$ with respect to δ_T are shown in Figs. 4a and 4b, respectively. As shown in Fig. 4b, the gyroscopic coupling considerably increases the frequency of $\lambda_{1gy,2gy}$. However, the gyroscopic coupling slightly increases the frequency of $\lambda_{5gy,6gy}$. It is obvious from Fig. 4a that the gyroscopic coupling considerably increases the damping ratio of $\lambda_{5gy,6gy}$, whereas the damping ratio of $\lambda_{1gy,2gy}$ reduces. The gyroscopic coupling also affects the stability characteristics of different modes of the NAV, as is observed from Fig. 2a, where the increase in the gyroscopic coupling forces $\lambda_{5gy,6gy}$ to migrate from initial unstable region to stable left half-region of the complex plane. Nevertheless, the stability of $\lambda_{1gy,2gy}$ reduces as shown in Fig. 2a.

It can be seen from Table 8 that eigenvectors of eigenvalues $\lambda_{3gy,4gy}$, λ_{7gy} , and λ_{8gy} are mostly influenced by eigenvector components

Fig. 2 Trajectories of eigenvalues of $A_{\text{coupledgy}}$ obtained by varying δ_T from 0 to 400 rps.

Fig. 3 Trajectories of eigenvector components of eigenvalues $\lambda_{1gy,2gy}$ and $\lambda_{5gy,6gy}$ obtained by varying δ_T from 0 to 400 rps.

corresponding to the variables $\bar{u}$, $\bar{w}$, $\bar{q}$, $\bar{\theta}$, $\bar{p}$, $\bar{r}$, and $\bar{\phi}$. Hence, these eigenvalues are analyzed together. Figure 2d shows that the trajectory of eigenvalue λ_{7gy} starts from the roll subsidence mode eigenvalue of the decoupled system and moves toward the origin of the complex plane when δ_T increases. Similarly, Fig. 2c shows that the trajectory of eigenvalue λ_{8gy} starts from the spiral mode eigenvalue of the decoupled system and moves toward the origin of the complex plane as δ_T increases. When δ_T increases, the eigenvalues $\lambda_{4gy,4gy}$ begin migration from the real eigenvalues of the phugoid mode of the decoupled system. As seen from Fig. 2b, at a very low value of the δ_T corresponding to a low gyroscopic coupling, the eigenvalues $\lambda_{3gy,4gy}$ remain real. When the δ_T is increased further, the $\lambda_{3gy,4gy}$ break from the real axis and become a complex conjugate pair. This is shown in Fig. 2b. The influence on the lateral-longitudinal variables by $\lambda_{3gy,4gy}$,

λ_{7gy} , and λ_{8gy} can be understood by analyzing the trajectories of the eigenvector components obtained by varying the δ_T . The trajectories of eigenvector components associated with the eigenvalues $\lambda_{3gy,4gy}$, λ_{7gy} , and λ_{8gy} are shown in Figs. 5–8.

Figure 5a shows the variation in the $\bar{u}$ component of eigenvectors corresponding to the eigenvalues $\lambda_{3gy,4gy}$, λ_{7gy} , and λ_{8gy} . From this figure, it is clear that the influence of the modes corresponding to $\lambda_{3gy,4gy}$ and λ_{8gy} on the longitudinal variable $\bar{u}$ vary considerably with the increment in the δ_T . Up to 50 rps, oscillation in $\bar{u}$ is mainly caused by $\lambda_{3gy,4gy}$. Above 50 rps, the influence of λ_{8gy} on $\bar{u}$ is considerably larger than $\lambda_{3gy,4gy}$. This is depicted in Fig. 5a. The trajectories of the $\bar{w}$ component of eigenvectors related to $\lambda_{3gy,4gy}$, λ_{7gy} , and λ_{8gy} are shown in Fig. 5b. It is apparent from Fig. 5b that, up to 25 rps, the influence of $\lambda_{3gy,4gy}$ on the longitudinal variable $\bar{w}$ is more than the influence of

Fig. 4 Variation of the damping ratio and the frequency of eigenvalues of $A_{coupledgy}$ with respect to the variation of δ_T from 0 to 400 rps.

a) Variation in $\bar{u}$ component of eigenvectors associated with eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy}

b) Variation in $\bar{w}$ component of eigenvectors associated with eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy}

Fig. 5 Trajectories of eigenvector components of eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy} obtained by varying δ_T from 0 to 400 rps.

a) Variation in $\bar{q}$ component of eigenvectors associated with the eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy}

b) Variation in $\bar{\theta}$ component of eigenvectors associated with the eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy}

Fig. 6 Trajectories of eigenvector components of eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy} obtained by varying δ_T from 0 to 400 rps.

λ_{8gy} . However, when the level of gyroscopic coupling is high, λ_{8gy} becomes dominant in $\bar{w}$ than $\lambda_{3gy, 4gy}$. The variations in the $\bar{q}$ component of eigenvectors associated with eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy} are shown in Fig. 6a. Figure 6a shows that, when the gyroscopic coupling is increased by means of increasing the δ_T , the influence of $\lambda_{3gy, 4gy}$ on the longitudinal variable $\bar{q}$ reduces; whereas the influence of λ_{7gy} increases. At a higher level of gyroscopic coupling, the influence of λ_{7gy} and $\lambda_{3gy, 4gy}$ on $\bar{q}$ is nearly the same. It is also visible from Fig. 6a that the influence of λ_{8gy} on $\bar{q}$ is negligibly small. Figure 6b depicts the trajectory of the $\bar{\theta}$ component of

eigenvectors associated with $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy} . Beyond 50 rps, the domination of eigenvalue λ_{8gy} on the longitudinal variable $\bar{\theta}$ is higher than the eigenvalues $\lambda_{3gy, 4gy}$, as shown in Fig. 6b. Figure 6b shows that the effect of λ_{7gy} on $\bar{\theta}$ is almost negligible.

Figures 7a and 7b show the variation in the $\bar{p}$ and $\bar{r}$ components of the eigenvectors associated with $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy} with respect to δ_T . When δ_T is below 25 rps, the influence of the eigenvalue λ_{8gy} is more dominant in the lateral variable $\bar{p}$ than the influence of the eigenvalues $\lambda_{3gy, 4gy}$. However, the influence of the eigenvalues $\lambda_{3gy, 4gy}$ is higher than λ_{8gy} when δ_T increases above 25 rps. Similarly, Fig. 7b

a) Variation in $\bar{p}$ component of eigenvectors associated with eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy}

b) Variation in $\bar{r}$ component of eigenvectors associated with eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy}

Fig. 7 Trajectories of eigenvector components of eigenvalues $\lambda_{3gy, 4gy}$, λ_{7gy} , and λ_{8gy} obtained by varying δ_T from 0 to 400 rps.

Fig. 8 Variation in $\bar{\phi}$ component of eigenvectors associated with eigenvalues $\lambda_{3_{gy},4_{gy}}$, $\lambda_{7_{gy}}$, and $\lambda_{8_{gy}}$ with respect to the variation in δ_T from 0 to 400 rps.

Fig. 9 Variation of damping ratio and frequency of eigenvalues $\lambda_{3_{gy},4_{gy}}$ with respect to the variation of δ_T from 0 to 400 rps.

reveals that the influence of $\lambda_{7_{gy}}$ and $\lambda_{8_{gy}}$ on the lateral variable $\bar{r}$ reduce with the increase of δ_T . In contrast to this, the influence of the eigenvalues $\lambda_{3_{gy},4_{gy}}$ on $\bar{r}$ increases with δ_T . It can be seen from Fig. 7b that, when δ_T is above 100 rps, the influence of $\lambda_{3_{gy},4_{gy}}$ is more dominant in $\bar{r}$ than the $\lambda_{7_{gy}}$ and $\lambda_{8_{gy}}$.

As shown in Fig. 8, the influence of $\lambda_{8_{gy}}$ is dominant in $\bar{\phi}$ up to 50 rps. When δ_T increases above 50 rps, the influence of $\lambda_{3_{gy},4_{gy}}$ on $\bar{\phi}$ is higher than both the $\lambda_{7_{gy}}$ and the $\lambda_{8_{gy}}$. This indicates that, when the gyroscopic coupling is at a higher level, the oscillation in $\bar{p}$ is mainly due to $\lambda_{3_{gy},4_{gy}}$. It is important to note that the influences of the eigenvalue $\lambda_{7_{gy}}$ on the longitudinal variables $\bar{u}$, $\bar{w}$, and $\bar{\theta}$ are small when compared with the influence of the $\lambda_{3_{gy},4_{gy}}$ and the $\lambda_{8_{gy}}$. The only longitudinal variable that has dominant influence from the $\lambda_{7_{gy}}$ is $\bar{q}$. In spite of that dominance, it is seen from Fig. 6a that the influence of $\lambda_{7_{gy}}$ on longitudinal variable $\bar{q}$ is small.

The outcome of this analysis indicates that, when the gyroscopic coupling is low (when the δ_T is below 25 rps), the eigenvalues $\lambda_{1_{gy},2_{gy}}$,

$\lambda_{5_{gy},6_{gy}}$, $\lambda_{7_{gy}}$, $\lambda_{3_{gy},4_{gy}}$, and $\lambda_{8_{gy}}$ and their corresponding eigenvectors display the characteristic of the short-period, Dutch roll, roll subsidence, phugoid, and spiral modes. However, when the gyroscopic coupling is increased by incrementing the δ_T , these modes shed their conventional characteristics because the longitudinal and the lateral variables get influence from each of the modes. Therefore, the eigenvalues $\lambda_{1_{gy},2_{gy}}$, $\lambda_{3_{gy},4_{gy}}$, $\lambda_{5_{gy},6_{gy}}$, $\lambda_{7_{gy}}$, and $\lambda_{8_{gy}}$ and their corresponding eigenvectors are called coupled short-period, coupled phugoid, coupled Dutch roll, coupled roll subsidence, and coupled spiral modes. The study also indicates that the influence of eigenvalue $\lambda_{7_{gy}}$ on the longitudinal variables is negligibly small as compared to the influence of other lateral eigenvalues. Hence, gyroscopic coupling does not induce significant coupling between the coupled roll subsidence mode and the longitudinal variables.

The increasing gyroscopic coupling also affects the damping ratio and the frequency of $\lambda_{3_{gy},4_{gy}}$. The damping ratio and the frequency of $\lambda_{3_{gy},4_{gy}}$ for different δ_T are shown in Fig. 9. Because the eigenvalues $\lambda_{3_{gy},4_{gy}}$ are real up to 30 rps, the damping ratios of these eigenvalues are unity. When the δ_T increases above 30 rps, the damping ratio of $\lambda_{3_{gy},4_{gy}}$ begins to decrease. However, above 100 rps, the damping ratio begins to increase. The frequency of $\lambda_{3_{gy},4_{gy}}$ increases with the increase in the δ_T . Therefore, the increasing gyroscopic coupling increases the frequency of $\lambda_{3_{gy},4_{gy}}$. In addition to the damping ratio and the frequency, the effect of increasing gyroscopic coupling alters the stability characteristic of the eigenvalues $\lambda_{3_{gy},4_{gy}}$, $\lambda_{7_{gy}}$, and $\lambda_{8_{gy}}$. From Fig. 2a, it is understood that the stability of eigenvalues $\lambda_{3_{gy},4_{gy}}$ is improved when the gyroscopic coupling is increased by incrementing δ_T . However, Figs. 2b and 2c show that the relative stability of the system corresponding to eigenvalues $\lambda_{7_{gy}}$ and $\lambda_{8_{gy}}$ reduces when gyroscopic coupling increases. The effects of gyroscopic coupling on the linear dynamics of the NAV are summarized in Tables 9 and 10.

To give readers a better understanding of the severity of gyroscopic coupling in the NAV, the percentage change in the real and the imaginary parts of eigenvalues from the decoupled model to the coupled model (including only gyroscopic coupling) is compared with the corresponding percentage change in the real and the imaginary parts of the eigenvalue of the Spitfire Mark XVIII [5] is shown in Table 11. The coupling considered in [5] is both aerodynamic and gyroscopic coupling. Table 11 shows that, even with only the gyroscopic coupling, the percentage change in eigenvalues of short-period, spiral, roll subsidence, and phugoid modes from the decoupled model to the coupled model of the NAV is large as compared with that of the Spitfire Mark XVIII. The percentage change in Dutch roll eigenvalues shown in Table 11 indicates that there is a large contrast between the eigenvalues of the decoupled model and the coupled model of the NAV. However, the percentage change in the real part of the Dutch roll eigenvalues of the NAV is smaller than that of the Spitfire Mark XVIII, whereas the imaginary parts of the Dutch roll eigenvalues of the NAV and Spitfire Mark XVIII are nearly the same. Therefore, the gyroscopic coupling itself has a large influence on the dynamics of the NAV.

To study the effect of counter torque on the dynamics of the NAV, the system matrix $A_{\text{coupledgy}}$ is modified by including the elements associated with the counter torque. The effect of counter torque enters into the linear dynamics of the NAV through L_u and N_u . The elements L_u and N_u also consist of aerodynamic effects. The equation of L_u and N_u for the straight and constant altitude flight condition is given as follows:

Table 9 Variation in the characteristics of the eigenvalues of the NAV with the increment of the gyroscopic coupling

Eigenvalues	Stability	Frequency	Damping ratio	Nature of eigenvalues of $A_{\text{coupledgy}}$	
				With increment in gyroscopic coupling	With null gyroscopic coupling
$\lambda_{1_{gy},2_{gy}}$	Reducing	Increasing	Reducing	Remains complex	Complex
$\lambda_{3_{gy},4_{gy}}$	Improving	Reducing	Increasing	Migrates to complex	Real
$\lambda_{5_{gy},6_{gy}}$	Improving	Increasing slightly	Increasing	Remains complex	Complex
$\lambda_{7_{gy}}$	Reducing	—	—	Remains real	Real
$\lambda_{8_{gy}}$	Reducing	—	—	Remains real	Real

$$L_u = L_{u_A} + L_{u_T} \quad (22)$$

$$L_{u_A} = \frac{J_{xz}}{J_D} \frac{\partial N_A}{\partial u} + \frac{J_{zz}}{J_D} \frac{\partial L_A}{\partial u} \quad (23)$$

$$L_{u_T} = \frac{J_{zz}}{J_D} \frac{\partial L_T}{\partial u} = \frac{J_{zz}}{J_D} \frac{1}{2\pi\delta_T} \frac{\partial F_{Tx}u}{\partial u} \quad (24)$$

$$N_u = N_{u_A} + N_{u_T} \quad (25)$$

$$N_{u_A} = \frac{J_{xx}}{J_D} \frac{\partial N_A}{\partial u} + \frac{J_{xz}}{J_D} \frac{\partial L_A}{\partial u} \quad (26)$$

$$N_{u_T} = \frac{J_{xz}}{J_D} \frac{\partial L_T}{\partial u} = \frac{J_{xz}}{J_D} \frac{1}{2\pi\delta_T} \frac{\partial F_{Tx}u}{\partial u} \quad (27)$$

In the [recedomg equations, L_{u_A} and N_{u_A} are due to the aerodynamic moments, whereas L_{u_T} and N_{u_T} are due to countertorque. To study the effect of countertorque, terms that are influenced by the countertorque in L_u and N_u are retained in $A_{\text{coupledgy}}$, where the terms that are influenced by aerodynamics are neglected.

A new system matrix is formed by including the elements associated with the gyroscopic effects and the countertorque. The behavior of the trajectories of the eigenvalues of this system matrix are compared with the trajectory of the eigenvalues of $A_{\text{coupledgy}}$, and the differences in these trajectories of eigenvalues are due to countertorque only. The new system matrix is formed by making all coupling elements zero except M_r , N_q , L_q , L_{u_T} , and N_{u_T} .

In the matrix $A_{\text{coupledgyct}}$, the elements L_q , N_q , and M_r are proportional to δ_T . The trajectories of the eigenvalues of $A_{\text{coupledgyct}}$ are obtained by varying δ_T from 80 to 400 rps. As L_{u_T} and N_{u_T} are inversely proportional to δ_T , at a low value of δ_T , the behavior of the trajectories of eigenvalues will be governed by countertorque through L_{u_T} and N_{u_T} . At a large value of δ_T , the trajectories of the eigenvalues are shaped mainly by L_q , N_q , and M_r . If any deviation is found in the

Fig. 10 Variation of λ_{8gy} and λ_{8gyct} with respect to the variation of δ_T from 80 to 400 rps.

trajectories of the eigenvalues of $A_{\text{coupledgyct}}$ when it is compared with the trajectories of the eigenvalues of $A_{\text{coupledgy}}$ at a lower value of δ_T , it is due to L_{u_T} and N_{u_T} , and thereby the countertorque.

After comparing the trajectories of the eigenvalues of $A_{\text{coupledgy}}$ and $A_{\text{coupledgyct}}$, a significant deviation has only occurred between the trajectories of λ_{8gy} and λ_{8gyct} . The trajectories of λ_{8gy} and λ_{8gyct} are shown in Fig. 10. A vast difference is observed from the beginning of the trajectories of λ_{8gy} and λ_{8gyct} . The trajectory of eigenvalue λ_{8gyct} begins from the right side of the complex plane and migrates toward the left side of the complex plane. Contrary to this, the trajectory of eigenvalue λ_{8gy} starts from the left side of the complex plane. When the δ_T increases, the effect of the countertorque decreases. Therefore, eigenvalue λ_{8gyct} migrates toward the left side of the complex plane. This suggests that the countertorque considerably effect the stability of λ_{8gyct} .

V. Flight-Test Results

For the flight test, the NAV and the lightweight autopilot, which weighs 1.8 g, are fabricated. The dimensions of the autopilot are 3.0 cm in length, 2.2 cm in width, and 0.34 cm in height. Figures 11a and 11b show the fabricated NAV and the lightweight autopilot. The microcontroller used in this autopilot is model STM32F405. For the gyroscope, accelerometer, and magnetometer, a single chip solution (MPU-9150) is used. To test the gyroscopic coupling and the unstable characteristics of the NAV, the vehicle is hand launched after rotating the propeller at 175.87 rps (trim propeller speed). At the time of launch, the elevator and rudder control surfaces are also maintained at trim conditions. After that, a positive step δ_T input is given for a pitchup. With the positive step δ_T input excitation, it is observed that the vehicle pitches up suddenly. Then, the NAV yaws and rolls to the right side, accomplishing an unstable spiral dive to right. The gyroscope measurements are shown in Figs. 12a–12c.

After analyzing the gyroscope measurements, it is established that the unstable spiral dive is due to the unstable coupled spiral mode. This is explained as follows. In the flight test, power to the motor is provided at the zeroth second. The NAV is hand launched after 5 s,

Table 10 Variation in the nature of various modes of the NAV with the increment of the gyroscopic coupling

Eigenvalues	Mode	
	Low gyroscopic coupling ($\delta_T \leq 25$ rps)	High gyroscopic coupling ($\delta_T > 25$ rps)
$\lambda_{1gy}, \lambda_{2gy}$	Short period	Unrecognizable
$\lambda_{3gy}, \lambda_{4gy}$	Phugoid	Unrecognizable
$\lambda_{5gy}, \lambda_{6gy}$	Dutch roll	Unrecognizable
λ_{7gy}	Roll subsidence	Roll subsidence
λ_{8gy}	Spiral	Unrecognizable

Table 11 Percentage change in eigenvalues of the NAV and Spitfire Mark XVIII due to cross-coupling

Mode	NAV		Spitfire	
	Change in real part of eigenvalues, %	Change in imaginary part of eigenvalues, %	Change in real part of eigenvalues, %	Change in imaginary part of eigenvalues, %
Short period	41.76	105.79	36.27	17.58
Dutch roll	141.84	11.59	178.46	9.88
Roll subsidence	51.4	—	0	—
Spiral	91.96	—	3.33	—
Phugoid	41,203.8	—	0.75	6.05

Fig. 11 Fabricated NAV and autopilot.

Fig. 12 Measured angular rates of NAV.

and the positive step δ_T input is applied at 6.7 s. When the positive step δ_T is applied, the propeller speed increases from the trim value. This increases the gyroscopic coupling, thereby reducing the relative stability of the coupled spiral mode as shown in Fig. 2c. Due to the severe gyroscopic coupling, when the positive step δ_T input is given to the NAV, the vehicle begins to pitch up as the thrust axis is below the CG (shown in Fig. 1b). The pitching up of the NAV results in a positive pitch rate. Thereafter, the NAV yaws toward the right side, followed by a roll into the right side. The outcomes of these are the positive yaw and roll rate, as shown in Figs. 12a and 12c.

When the δ_T increases, the thrust and u also increase. The increment in the thrust and u increases the counter torque. The accretion of the gyroscopic coupling and counter torque with the application of the positive step δ_T results in the unstable coupled spiral mode, as discussed in the previous section. This instability of the coupled spiral mode is indicated by the diverging angular rates shown in Figs. 12a–12c. The unstable coupled spiral mode causes the NAV to execute the unstable spiral dive toward the ground (with negative pitch rate) to the right side (with respect to the rear side of the NAV). The unstable spiral dive is too fast because the unstable spiral mode pole at 0.95 yields a time to double of 0.7 s. The quickly

diverging spiral dive makes it extremely arduous for the pilot to stabilize the NAV.

VI. Conclusions

The objective of this paper is to study the effect of gyroscopic coupling and counter torque on the dynamics of the fixed-wing NAV. From the outcomes of this investigation, it is possible to conclude that the dynamics of the NAV is dominated by the gyroscopic coupling and counter torque. The detailed parametric study shows that the gyroscopic effect and counter torque produce stronger coupling between the longitudinal and lateral dynamics such that there is a large discrepancy between the coupled and decoupled models in terms of their eigenvalues and eigenvectors, unlike in the case of weaker coupling in conventional aircraft and UAVs. In the presence of the coupling, the nature of the modes of the coupled model is vastly different from the nature of the classical modes of the aircraft. When the gyroscopic coupling is low, the nature of the modes of the NAV resembles the nature of the classical modes of the aircraft. However, when the gyroscopic coupling becomes high, the modes of the NAV shed the characteristics of the classical modes. The investigation also

found that the gyroscopic coupling and counter torque significantly affected the stability and the performance of various modes of the NAV. The increase in the magnitude of the gyroscopic coupling terms degraded the stability of the coupled short-period, coupled roll subsidence, and coupled spiral modes; whereas the stability of the coupled Dutch roll and coupled phugoid modes were improved. The increase in the gyroscopic coupling deteriorated the damping ratio of the coupled short-period mode, whereas the damping ratio of the Dutch roll mode was enhanced. At a higher level of gyroscopic coupling, the damping ratio of the coupled phugoid mode was improved. The frequency of the coupled short-period and coupled phugoid modes increased with increasing gyroscopic coupling. The significant effect of counter torque is to make the coupled spiral mode unstable. The analysis is also helpful in choosing or designing an appropriate propulsion system for the NAV.

The NAV and the lightweight autopilot are fabricated for the flight test. The outcomes of the flight test substantiate the findings of the study. The analysis of the gyroscope measurements indicates that the coupled spiral mode of the NAV is unstable due to the gyroscopic coupling and counter torque. The NAV requires a stability augmentation system because the unstable coupled spiral mode is too fast for the pilot to control. The classical control system design techniques for the flight control system design are based on the assumption that the longitudinal and lateral dynamics are decoupled. However, if the aircraft dynamics are heavily coupled like that of the NAV, the flight control system design using classical control may mostly fail to achieve desired objectives. An alternate solution is to use modern multivariable control techniques to design the flight control system. As the NAV has unfavorable couplings, the multivariable control system design technique, like the eigenstructure assignment, is best suited for the flight control system design. While using eigenstructure assignment technique, this study will be useful in selecting the desired closed-loop eigenvalues and the desired entries of corresponding eigenvectors.

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