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Enhancing Image Denoising by Controlling Noise Incursion in Learned Dictionaries

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***Abstract*—Existing image denoising frameworks via sparse representation using learned dictionaries have an weakness that the dictionary, trained from noisy image, suffers from noise incursion. This paper analyzes this noise incursion, explicitly derives the noise component in the dictionary update step, and provides a simple remedy for a desired signal to noise ratio. The remedy is shown to perform better both in objective and subjective measures for lesser computation, and complements the framework of image denoising.**

***Index Terms*—image denoising, sparse representation, dictionary training, SGK, K-SVD.**

I. Introduction

The objective of image denoising is to estimate the original image $\mathbf{X} \in \mathcal{R}^{\sqrt{N}\times\sqrt{N}}$ from the corrupt image $\mathbf{Y} = \mathbf{X} + \mathbf{V}$, where N is the number of pixels in the image. The assumption is that $\mathbf{V}$ is Additive White Gaussian Noise (AWGN). Recently proposed image denoising framework via sparse and redundant representations using learned dictionary has gained popularity among the researchers, and motivate many extensions [1]. The prior imposed in this denoising framework is that the signal of interest will have a sparse representation on a dictionary containing signal prototypes (or atoms), whereas the additive noise can not have sparse representation in any dictionary. Thus it extracts small overlapping image blocks of size $\sqrt{n} \times \sqrt{n}$ from $\mathbf{Y}$, where n is a square (or any other appropriate block size), and estimates the sparse representation α_{ij} for each image block $\mathbf{y}_{ij} = \{\mathbf{R}_{ij}\mathbf{Y}\} \in \mathcal{R}^n$ on a dictionary $\mathbf{D} \in \mathcal{R}^{n\times k}$. That is

$$\forall_{ij}\alpha_{ij} = \arg\min_{\alpha} \|\alpha\|_0 \text{ such that } \|\mathbf{y}_{ij} - \mathbf{D}\alpha\|_2^2 \leq \epsilon^2,$$

where the estimation error bound ϵ^2 depends on the noise variance σ^2. $\mathbf{R}_{ij}$ is an $n \times N$ matrix that extracts a $\sqrt{n} \times \sqrt{n}$ block $\mathbf{y}_{ij}$ from the columnized image $\mathbf{Y}$ starting from its 2D coordinate (i, j). In order to reconstruct the global image, these individual sparse representations are aggregated as a closed form solution of a MAP formulation,

$$\hat{\mathbf{X}} = \left(\lambda\mathbf{I} + \sum_{ij}\mathbf{R}_{ij}^T\mathbf{R}_{ij}\right)^{-1}\left(\lambda\mathbf{Y} + \sum_{ij}\mathbf{R}_{ij}^T\mathbf{D}\alpha_{ij}\right).$$

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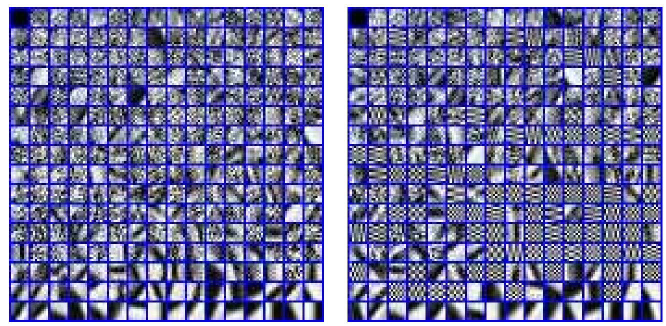

(a) K-SVD (b) controlled-K-SVD

Fig. 1. The dictionaries trained on the sample obtained from the 'Lena' image at $\sigma = 35$ using (a) standard K-SVD, and (b) controlled-K-SVD.

For the details of the image denoising framework, we suggest the reader to refer to [1].

However, a priori probability of sparsity is greatly influenced by the dictionary, and the dictionary that accurately defines the signal with fewer coefficients performs a better denoising. A dictionary trained on a sample set of signals will produce more accurate prototypes than standard parametric dictionaries like DCT, Wavelet, etc. We don't have any access to the noise free image to obtain these accurate image block prototypes. Thus the use of K-SVD [2] to train a dictionary on a set of blocks extracted from the noisy image $\mathbf{Y}$ is considered to be one of the main contributions of [1], which was shown performing better denoising in comparison to both overcomplete DCT and the globally trained dictionary. However, there exists a good chance of noise being incurred into the dictionary while training on noisy signals, which remained unnoticed. The evidence of noise incursion can be seen from the first two rows of the atoms shown in Fig. 7 of [1]. To give a more subjective view of noise incursion, another trained dictionary's atoms are shown in Fig. 1 (a).

A. Dictionary Training on the Corrupted Image

Let's denote the set of noisy image blocks extracted for dictionary training as $\mathcal{Z} = \{\mathbf{y}_j\}_{j=1}^{M}$ [1]. Each noisy image block $\mathbf{y}_j = \mathbf{x}_j + \mathbf{v}_j$, where $\mathbf{x}_j$ is the original noise free image block and $\mathbf{v}_j$ is the associated noise block. Starting with an initial dictionary, most of the dictionary training algorithms iterate between the *Sparse Coding Stage* and *Dictionary Update Stage* [3].

[1] We kept the notations same as section III of [1]

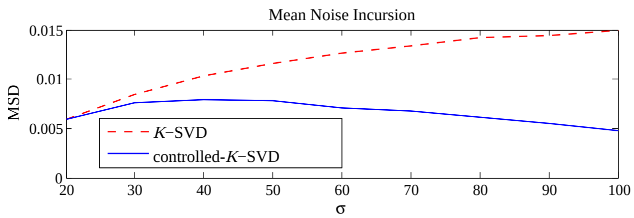

Fig. 2. Plot of the estimated noise MSD of the dictionary trained on noisy image at various noise level σ. The plot is the average over 6 standard images.

- *Sparse Coding Stage*: Using any pursuit algorithm the representation vector α_j for each block $\mathbf{y}_j$ is estimated, i.e.

$$\forall_j \alpha_j = \arg\min_{\alpha} \|\alpha\|_0 \text{ such that } \|\mathbf{y}_j - \mathbf{D}\alpha\|_2^2 \leq \epsilon^2.$$

- *Dictionary Update Stage*: In the case of sequential update algorithms, each column/atom ($\mathbf{d}_l \in \mathcal{R}^n$) for $l = 1, 2, \ldots, k$ in $\mathbf{D}$ is updated one after another as the following.
  - Find the blocks that use the atom $\mathbf{d}_l$, i.e. $\omega_l = \{j|\alpha_j(l) \neq 0\}$.
  - Compute the representation error without the contribution of the atom $\mathbf{d}_l$ for each $j \in \omega_l$, i.e.

$$\mathbf{e}_j^l = \mathbf{y}_j - \sum_{m \neq l} \mathbf{d}_m \alpha_j(m). \tag{1}$$

  - Stack all $\{\mathbf{e}_j^l\}_{j\in\omega_l}$ to form a matrix $\mathbf{E}_l. \in \mathcal{R}^{n\times|\omega_l|}$, and also form a row vector $\mathbf{a}_l \in \mathcal{R}^{1\times\omega_l}$ containing all corresponding $\{\alpha_j(l)\}_{j\in\omega_l}$.
  - Update the dictionary column $\mathbf{d}_l$ to minimize $\|\mathbf{E}_l - \mathbf{d}_l\mathbf{a}_l\|_2^2$.

K-SVD [2] updates both $\mathbf{d}_l$ and $\mathbf{a}_l$ using SVD that solves

$$\{\bar{\mathbf{d}}_l, \bar{\mathbf{a}}_l\} = \arg\min_{\mathbf{d}_l,\mathbf{a}_l} \|\mathbf{E}_l - \mathbf{d}_l\mathbf{a}_l\|_2^2, \tag{2}$$

whereas SGK [3] only updates $\mathbf{d}_l$ as the least square solution of

$$\bar{\mathbf{d}}_l = \arg\min_{\mathbf{d}_l} \|\mathbf{E}_l - \mathbf{d}_l\mathbf{a}_l\|_2^2. \tag{3}$$

As a result SGK executes faster than K-SVD, with an equally efficient convergence [3]. The comparison of the denoising performance of both the dictionary training algorithms is presented in [4], which shows the advantage of SGK over K-SVD. Inherently, there exists a problem of noise incursion in all such update algorithms, which we will explain in the next section. Then we will illustrate one way to avoid noise incursion, that improves the denoising by complementing the framework of [1].

B. Measuring Noise Incursion

To give a quantitative view of noise incursion, we consider estimating the noise Standard Deviation (SD) in the trained dictionary atoms by treating them as $\sqrt{n} \times \sqrt{n}$ image blocks. The most simple and effective noise SD estimation techniques are based on sparsifying orthonormal transforms such as wavelet or discrete cosine transforms. The noise characteristics are preserved by these transforms, whereas the underlying image energy is compacted to few coefficients. Thus robust statistics (most notably median) on the insignificant coefficients produce an estimate of the SD of the noise [5]. One of the most popular algorithms is the median absolute deviation (MAD) of the wavelet detail coefficients of the image [6]. Since we know from the JPEG compression that DCT is a better decorrelating transform for small image blocks, we take MAD of the $n/4$ smallest magnitude DCT coefficients as the estimate of noise SD. Fig. 2 plots the mean SD (MSD) of the trained dictionary atoms versus the noise level (σ) of the noisy image, where the dictionaries are trained in the same manner as [1], taking overcomplete DCT as the starting dictionary, and $\sqrt{n} = 8$. It can be seen from the dashed line plot that the MSD is increasing with the noise level σ.

II. Noise Incursion

To make our presentation simpler, we will analyze the dictionary noise incursion problem in the case of SGK. However, this analysis can also be perceived from the prospective of K-SVD. In order to minimize $\|\mathbf{E}_l - \mathbf{d}_l\mathbf{a}_l\|_2^2$, SGK updates each atom as follows,

$$\bar{\mathbf{d}}_l = \mathbf{E}_l\mathbf{a}_l^T\left(\mathbf{a}_l\mathbf{a}_l^T\right)^{-1} = \frac{\sum_{j\in\omega_l} \mathbf{e}_j^l \alpha_j(l)}{\sum_{j\in\omega_l} \alpha_j^2(l)}. \tag{4}$$

A. Analysis of Noise Incursion

The expression of $\mathbf{e}_j^l$ in equation (1) can be rewritten as

$$\begin{aligned}\mathbf{e}_j^l &= \mathbf{y}_j - \sum_{m=1}^{k} \mathbf{d}_m \alpha_j(m) + \mathbf{d}_l \alpha_j(l) \\ &= \mathbf{v}_j + \mathbf{r}_j + \mathbf{d}_l \alpha_j(l),\end{aligned} \tag{5}$$

where $\mathbf{r}_j = \mathbf{x}_j - \sum_{m=1}^{k} \mathbf{d}_m \alpha_j(m)$ is the original signal residue that could not be approximated. Using the above expression the atom update equation (4) can be restated as

$$\begin{aligned}\bar{\mathbf{d}}_l &= \frac{\sum_{j\in\omega_l} \mathbf{v}_j\alpha_j(l) + \sum_{j\in\omega_l} \mathbf{r}_j\alpha_j(l) + \mathbf{d}_l \sum_{j\in\omega_l} \alpha_j^2(l)}{\sum_{j\in\omega_l} \alpha_j^2(l)} \\ &= \underbrace{\frac{\sum_{j\in\omega_l} \mathbf{v}_j\alpha_j(l)}{\sum_{j\in\omega_l} \alpha_j^2(l)}}_{\text{noise component } \varepsilon_l} + \underbrace{\frac{\sum_{j\in\omega_l} \mathbf{r}_j\alpha_j(l)}{\sum_{j\in\omega_l} \alpha_j^2(l)}}_{\text{signal component } \delta_l} + \mathbf{d}_l.\end{aligned} \tag{6}$$

It can be seen that even if we start with noiseless dictionary atoms $\forall_m\{\mathbf{d}_m\}$, the noise component $\varepsilon_l = \frac{\sum_{j\in\omega_l} \mathbf{v}_j\alpha_j(l)}{\sum_{j\in\omega_l} \alpha_j^2(l)}$ is being added to them in the update step. Moreover, the noise incurred in the previous atoms may propagate to the next atom to be updated. It is because the signal component $\delta_l = \frac{\sum_{j\in\omega_l} \mathbf{r}_j\alpha_j(l)}{\sum_{j\in\omega_l} \alpha_j^2(l)}$ needs the residue to be computed based on the previously updated atoms $\mathbf{r}_j = \mathbf{x}_j - \sum_{m=1}^{k} \mathbf{d}_m\alpha_j(m)$. Similarly, the noise will keep propagating from iteration to iteration.

It is clear that we can not avoid noise incursion while training the dictionary on noisy samples. However, we can minimize the effect of noise incursion by introducing some

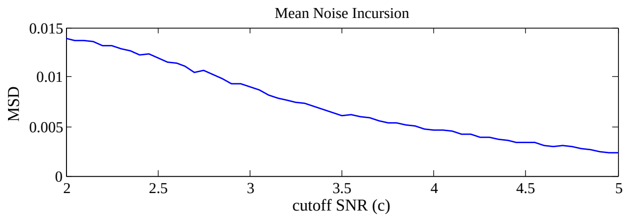

Fig. 3. Plot of the estimated noise MSD of the dictionary trained on noisy image using controlled-K-SVD for various SNR threshold, the noise level in the image is $\sigma = 75$. The plot is the average over 6 standard images.

measures. It is known that ε_l is the weighted sum of AWGN (i.e $\forall_{j\in\omega_l}\{\mathbf{v}_j\}$). Hence we can state that

$$\mathbb{E}\left[\varepsilon_l(i)\right] = \frac{\sum_{j\in\omega_l}\mathbb{E}\left[\mathbf{v}_j(i)\right]\alpha_j(l)}{\sum_{j\in\omega_l}\alpha_j^2(l)} = 0,$$
$$\mathbb{E}\left[\varepsilon_l^2(i)\right] = \frac{\sum_{j\in\omega_l}\mathbb{E}\left[\mathbf{v}_j^2(i)\right]\alpha_j^2(l)}{(\sum_{j\in\omega_l}\alpha_j^2(l))^2} = \frac{\sigma^2}{\sum_{j\in\omega_l}\alpha_j^2(l)}, \quad (7)$$

where $\mathbb{E}\left[\mathbf{v}_j^2(i)\right] = \sigma^2$ is the variance of the additive noise. Thus the expected value of the incurred noise energy is

$$\mathbb{E}\left[\|\varepsilon_l\|_2^2\right] = \sum_{i=1}^{n}\mathbb{E}\left[\varepsilon_l^2(i)\right] = \frac{n\sigma^2}{\sum_{j\in\omega_l}\alpha_j^2(l)}. \quad (8)$$

It shows that the energy of the incurred noise $\mathbb{E}\left[\|\varepsilon_l\|_2^2\right]$ depends on the magnitude of $\mathbf{a}_l$, that is $\|\mathbf{a}_l\|_2^2 = \sum_{j\in\omega_l}\alpha_j^2(l)$.

B. Controlled Dictionary Training Alleviating Noise Incursion

The best way to check noise incursion is by maintaining a good Signal to Noise Ratio (SNR) in an updated atom $\bar{\mathbf{d}}_l = \mathbf{d}_l + \delta_l + \varepsilon_l$. That is

$$\text{SNR} = \mathbb{E}\left[\|\mathbf{d}_l + \delta_l\|_2^2\right] / \mathbb{E}\left[\|\varepsilon_l\|_2^2\right]$$

Therefore, we only update a particular atom $\mathbf{d}_l$ if the SNR is above a cutoff c. That is

$$\text{SNR} \geq \text{c} \implies \mathbb{E}\left[\|\mathbf{d}_l + \delta_l\|_2^2\right] \geq \text{c}\mathbb{E}\left[\|\varepsilon_l\|_2^2\right], \quad (9)$$

where c is a predefined minimum value of SNR. Prior to any atom update we can obtain the value of $\mathbb{E}\left[\|\varepsilon_l\|_2^2\right]$ using equation (8). However, the value of $\mathbb{E}\left[\|\mathbf{d}_l + \delta_l\|_2^2\right]$ is impossible to obtain. Since we try to obtain a close approximation of the signal in the sparse coding stage, we can assume that the norm of the updated atom $\mathbb{E}\left[\|\mathbf{d}_l + \delta_l\|_2^2\right] \sim \|\mathbf{d}_l\|_2^2$, because $\|\delta_l\|_2^2 \ll \|\mathbf{d}_l\|_2^2$. Thus we can restate the condition (9) as

$$\|\mathbf{d}_l\|_2^2 \geq \text{c}\mathbb{E}\left[\|\varepsilon_l\|_2^2\right] \implies \|\mathbf{d}_l\|_2^2 \sum_{j\in\omega_l}\alpha_j^2(l) \geq \text{c}n\sigma^2. \quad (10)$$

Now we can check the above criterion for each atom before updating it. If the criterion is satisfied, we will update the atom, otherwise the atom will not be updated to avoid noise incursion. The update criterion (10) can be applied in general to any sequential algorithm aiming to minimize $\|\mathbf{E}_l - \mathbf{d}_l\mathbf{a}_l\|_2^2$ in the update step, e.g K-SVD, SGK, and trained dictionary based image restoration framework like LSC [7].

We should note that the update criterion (9) is only dependent on the sparse representation coefficients, which allows us to obtain the set of indices to be updated prior to the dictionary update stage. The proposed control mechanism will add a computational cost of Nm operations to the dictionary training algorithms, where N is the total number of training samples and each sample is assumed to have m number of sparse representation coefficients. Considering computational complexity of the dictionary training algorithms, Nm additional operations are very insignificant. In contrast, introducing the update criterion will reduce the unwanted update of some atoms, as a result of which the computational burden of the dictionary training algorithm will be reduced. However, when the noise is very low, or all the atoms are satisfying the update criterion, a minor increase in computation time will be noticed.

III. Experiment

To illustrate the phenomenon of noise incursion, we have applied the proposed control mechanism in [1], where K-SVD is used. Therefore, we suggest the reader to refer to [1] for the detailed implementation of image denoising via sparse and redundant representation over learned dictionaries. For the experiment, 6 standard images are taken, that is Lena, Barbra, Boat, Finger Print, House and Pepper.

In order to visually illustrate the noise incursion phenomenon, two dictionaries are trained on the sample obtained from the 'Lena' image at $\sigma = 35$, one using standard K-SVD and another using controlled-K-SVD (taking c $= 3.85$). A visual comparison between trained dictionaries is made in Fig. 1, which reveals more noisy atoms in standard K-SVD compared to controlled-K-SVD. In order have a quantitative view, noise MSD are obtained for the dictionaries trained at various noise levels, and plotted in Fig. 2. The plot shows the reduction in noise MSD due to controlled-K-SVD, which is the average over 6 standard images.[2]

In order to observe the effect of the cutoff SNR c, the dictionaries are trained using controlled-K-SVD for various c. It is expected that high value of c will result in small noise MSD in the updated dictionary. To verify it, the MSD vs. c is plotted in Fig. 3. To have a better visualization the noise level of the corrupted image is kept at $\sigma = 75$. Along with the reduction in noise, a high value of c also compromises the dictionary adaptation, because it may happen that none of the atoms are being updated for not satisfying criterion (10). Therefore, we have chosen an intermediate value of c $= 3.85$ for our experiments.

In order to show the benefit of a better dictionary, the denoising performances are compared by reproducing the results of [1] using both standard K-SVD and controlled-K-SVD. The problem of noise incursion in the trained dictionary is mostly visible at higher noise levels. The performance (in PSNR and SSIM) of the proposed controlled dictionary training, and the standard dictionary training is indifferent for lower values of σ. If we closely look at Fig. 2 for $\sigma < 25$, it

[2]MSD for controlled-K-SVD decreases when σ is high, which may appear a little counter intuitive. We should note that MSD is computed by averaging the noise SD of these few updated atoms along with not updated atoms (with almost 0 noise SD). Therefore when σ is high, noise will get incurred into more number of atoms, and only allow a few atoms to be updated. This results in more number of atoms with almost 0 noise SD causing the mean to drop.

TABLE I
COMPARISON OF THE DENOISING PSNR RESULTS IN DECIBEL. IN EACH CELL TWO DENOISING RESULTS ARE REPORTED. LEFT: USING STANDARD K-SVD LEARNED DICTIONARY [1]. RIGHT: USING CONTROLLED-K-SVD. ALL NUMBERS ARE AN AVERAGE OVER FIVE TRIALS. THE LAST ROW PRESENTS THE AVERAGE RESULT OVER ALL IMAGES.

σ/PSNR	25/21.15		50/14.15		75/10.64		100/8.13	
Lena	30.84	**30.85**	27.32	**27.47**	25.28	**25.49**	23.94	**24.20**
Barbara	**29.23**	29.22	25.19	**25.24**	22.76	**22.85**	21.59	**21.72**
Boats	29.05	29.05	25.67	**25.73**	23.68	**23.83**	22.50	**22.68**
Fgrpt	27.02	27.02	23.02	**23.04**	19.87	**19.94**	18.25	**18.34**
House	**32.12**	32.09	28.02	**28.16**	25.31	**25.50**	23.62	**23.81**
Peppers	29.72	29.72	26.13	**26.17**	23.61	**23.72**	21.78	**21.91**
Average	29.66	29.66	25.89	**25.97**	23.42	**23.55**	21.95	**22.11**

TABLE II
COMPARISON OF THE DENOISING SSIM INDEXES. IN EACH CELL TWO DENOISING RESULTS ARE REPORTED. LEFT: USING K-SVD LEARNED DICTIONARY [1]. RIGHT: USING CONTROLLED-K-SVD. ALL NUMBERS ARE AN AVERAGE OVER FIVE TRIALS. THE LAST ROW PRESENTS THE AVERAGE RESULT OVER ALL IMAGES.

σ/SSIM	25/0.584		50/0.378		75/0.267		100/0.196	
Lena	0.906	**0.907**	0.812	**0.816**	0.726	**0.732**	0.653	**0.661**
Barbara	0.914	0.914	0.805	**0.806**	0.698	**0.700**	0.626	**0.630**
Boats	0.880	0.880	0.765	**0.767**	0.667	**0.671**	0.594	**0.600**
Fgrpt	0.953	0.953	0.844	0.844	0.679	**0.680**	0.536	**0.538**
House	0.846	0.846	0.763	**0.769**	0.679	**0.691**	0.611	**0.626**
Peppers	0.857	0.857	0.773	**0.776**	0.689	**0.697**	0.626	**0.637**
Average	0.893	0.893	0.794	**0.796**	0.690	**0.695**	0.608	**0.615**

TABLE III
COMPARISON OF THE DICTIONARY TRAINING TIME. IN EACH CELL TWO DENOISING RESULTS ARE REPORTED. LEFT: USING K-SVD LEARNED DICTIONARY [1]. RIGHT: USING CONTROLLED-K-SVD. ALL NUMBERS ARE AN AVERAGE OVER FIVE TRIALS. THE LAST ROW PRESENTS THE AVERAGE RESULT OVER ALL IMAGES.

σ/PSNR	25/21.15		50/14.15		75/10.64		100/8.13	
Lena	1.473	**1.261**	1.345	**0.562**	1.312	**0.395**	1.323	**0.285**
Barbara	1.788	**1.781**	1.418	**1.155**	1.370	**0.790**	1.377	**0.484**
Boats	1.655	**1.652**	1.389	**0.855**	1.344	**0.464**	1.359	**0.344**
Fgrpt	2.394	**2.380**	1.695	**1.576**	1.451	**1.052**	1.406	**0.742**
House	1.484	**1.220**	1.349	**0.568**	1.288	**0.381**	1.304	**0.315**
Peppers	**1.659**	1.677	1.395	**1.005**	1.326	**0.593**	1.348	**0.389**
Average	1.742	**1.662**	1.432	**0.953**	1.348	**0.613**	1.353	**0.426**

can be found that the noise MSD of K-SVD and controlled-K-SVD are almost the same. However, the computation of controlled-K-SVD is reduced even at this range. Therefore results are presented only for the higher values of σ. The improvement in Peak Signal-to-Noise Ratio (PSNR) is shown in Table I, where PSNR $= 20\log_{10}\left(\frac{255}{\text{RMSE}}\right)$, and RMSE is the root mean square error of the reconstructed image $\hat{\mathbf{X}}$ with respect to the original image $\mathbf{X}$. Similarly, improvement in Structural SIMilarity (SSIM) index using controlled-K-SVD is shown in Table II. The SSIM index is a method for measuring the similarity between two images. The SSIM index can be viewed as a quality measure of one of the images being compared, provided the other image is regarded as of perfect quality [8]. The codes to reproduce the results can be obtained from https://sites.google.com/site/sujitkusahoo/codes.

As discussed in section II-B, a reduction in computation due to controlled dictionary training can observed from Table III. It can be also be observed that there is a minor increase in the execution time for controlled-K-SVD in the case of Pepper at noise level $\sigma = 25$. In the case of Pepper, when the noise is very low, all the atoms may be satisfying the update criterion. Therefor it takes slightly more time for checking the criterion in addition to the computation of K-SVD. In [4], it is shown that by replacing K-SVD with SGK, we can achieve a similar denoising performance with reduced computation. Therefore, results for controlled-SGK can be obtained similar to controlled-K-SVD. Likewise, the controlled dictionary training can be extended to any sparse signal processing framework where a dictionary is needed to be trained on noisy samples.

IV. DISCUSSION

Noise incursion is an inherent problem of dictionary training on noisy samples, which we have analyzed in the case of sequential updates, and proposed a controlled update procedure to overcome it. We have shown that the proposed controlled dictionary training is complementing the framework of image denoising via sparse representation, both in terms of quality and execution time.

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