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Collective Behavior of Mobile Agents with State-dependent Interactions [★]

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Abstract

In this paper, we develop a novel self-propelled particle model to describe the emergent behavior of a group of mobile agents. Each agent coordinates with its neighbors through a local force accounting for velocity alignment and collision avoidance. The interactions between agents are governed by path loss influence and state-dependent rules, which results in topology changes as well as discontinuities in the local forces. By using differential inclusion technique and algebraic graph theory, we show that collective behavior emerges while collisions between agents can be avoided, if the interaction topology is jointly connected. A trade-off between the path loss influence and connectivity condition to guarantee the collective behavior is discovered and discussed. Numerical simulations are given to validate the theoretical results.

Key words: Multi-agent systems, distributed control, collective motion, topological distance, discontinuous control, continuous time systems

1 Introduction

The collective motion of animals such as fish and birds are examples of large scale self-organization observed in nature. In many cases, cohesive groups are formed, where hundreds or thousands of agents move together in the same direction. In order to reveal the underlying mechanisms, several self-propelled particle models have been proposed and analyzed (Couzin et al., 2002; Cucker & Smale, 2007; Olfati-Saber, 2006; Vicsek & Zafeiris, 2012). Three widely adopted rules behind these models include repulsion, attraction and alignment (Reynolds, 1987).

More recently, collective behavior and self-organization have also attracted attention from engineers with the aim of controlling mobile robots in the context of cooperative control, formation control and so on (Haghighi & Cheah, 2012; Liu et al., 2013).

Besides considerable interests in the numerical or empirical modeling for collective behavior, much attention has been paid to rigorous mathematical analysis. In Jadbabaie et al. (2003) and Ren & Beard (2005), sufficient conditions were given for convergence of a simplified first-order Vicsek model. It was shown that under some joint connectivity conditions on the interaction topologies, all the agents eventually move in the same direction. For the second-order dynamics, the so called C-S model was proposed in Cucker & Smale (2007). Sufficient conditions were established to show that flocking can be achieved asymptotically. Extension to collision avoidance can be seen in Cucker & Dong (2011). One limitation of the C-S model is that each agent must interact with all the others during the motion. A theoretical framework for design and analysis of flocking algorithms with second-order dynamics was presented in Olfati-Saber (2006). Collective motions were obtained theoretically by applying artificial potentials embodying the three rules mentioned previously. In Zhang et al. (2011),

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the authors proposed a self-propelled model with only repulsion and alignment forces. Under a joint connectivity condition, flocks would be assembled in finite time. Distributed coordination of mobile agents with nonlinear interactions was studied in Mei et al. (2013), where only relative positions are needed for each agent. But it is required that the connection pattern between agents be fixed all the time. To tackle the issue of dynamic topology, results of differential inclusions and switched systems were used to examine the stability analysis of time-varying flocking in Tanner et al. (2007), Zavlanos et al. (2009) and Chen & Zhang (2011).

Most of the previous models rely on the aprioristic assumption that agents interact with all those within a fixed range. However, in a recent field study of flocks of starlings (Ballerini et al., 2008a), it is found that interaction is ruled by topological distance rather than metric distance. In other words, the relevant quantity is how many intermediate agents separate two birds, not how far apart they are. This means that the interactions are varying rather than fixed. In this case, whether the collective behavior of mobile agents can be achieved theoretically is still unclear.

As a first step towards this direction, in this paper, we introduce the state-dependent interaction for a self-propelled particle model in the topological sense. With the state-dependent interactions, the connection pattern is no longer fixed, but dynamic. Moreover, the path loss influence depending on relative distances between agents is considered. The main objective is to develop rigorous analysis in a general setting and explore how to achieve the collective motion with collision avoidance. The contributions of this paper can be summarized as follows:

- For the fixed-range interactions, a widely adopted method is to employ the invariance principle to determine the asymptotic stability. In this case, compactness of certain invariant sets follows from the connectivity directly (Chen & Zhang, 2011; Olfati-Saber, 2006; Tanner et al., 2007; Zhang et al., 2011). However, for the state-dependent interactions this is not a trivial task, since no *a priori* information about the boundedness of the state-dependent interactions can be inferred. We develop some novel techniques in terms of nonsmooth analysis coupled with algebraic theory to solve this problem.
- We investigate the impact of the path loss influence on the collective behaviors. Theoretically we show that there is a trade-off between the path loss influence and connectivity condition to guarantee the collective behavior. This is one unique feature of state-dependent interactions, which is not observed in the previous work on fixed-range interactions.

The paper is organized as follows. Section 2 presents the self-propelled particle model with state-dependent interactions. In Section 3, we give some preliminaries about

the solution of the model, followed by the convergence analysis in Section 4. Simulation results are provided in Section 5. Finally, Section 6 concludes the paper.

2 Model formulation

Consider a group of N mobile agents with the dynamics of each agent described by a double integrator

$$\dot{\hat{x}}_i(t) = \hat{v}_i(t), \quad \dot{\hat{v}}_i(t) = \hat{u}_i(t), \quad i = 1, 2, \dots, N, \quad (1)$$

where $\hat{x}_i, \hat{v}_i \in \mathbb{R}^n$ are the position and velocity of agent i , respectively, and $\hat{u}_i \in \mathbb{R}^n$ is the acceleration to be designed.

Numerical and empirical investigations support the idea that the behavior of agents results from local coordination based upon the relative positions and velocities with each other. Motivated by the findings in Ballerini et al. (2008a), we consider that the interaction range is state-dependent by incorporating the topological distance.

Definition 1 *The state-dependent interaction is defined as follows: (i) each agent i interacts with the $N^* \leq N$ closest neighbors $\mathcal{N}^*(\hat{x}_i)$; (ii) if agent j interacts with agent i , then agent i also interacts with agent j .*

Remark 2 *Note that if $N^* = N$, then each agent interacts with all the others and the state-dependent interaction coincides with the all-to-all interaction (Cucker & Dong, 2011; Cucker & Smale, 2007; Gazi & Passino, 2003; Vicsek & Zafeiris, 2012).*

We can model the interaction topology between agents as a dynamic bidirected graph $\mathcal{G}(\hat{x}(t)) = (\mathcal{V}, \mathcal{E}(\hat{x}(t)))$, where $\mathcal{V} = \{1, 2, \dots, N\}$, and $\mathcal{E}(\hat{x}(t)) \subset \mathcal{V} \times \mathcal{V}$ is the set of edges at t . At each time, each agent assesses the position and/or velocity of its neighbors $\mathcal{N}_i(\hat{x}) = \mathcal{N}^*(\hat{x}_i) \cup \{j \notin \mathcal{N}^*(\hat{x}_i) : i \in \mathcal{N}^*(\hat{x}_j)\}$ within two non-overlapping behavioral zones: zone of repulsion and zone of alignment.

In this paper, we propose the following control law for each agent:

$$\begin{aligned} \hat{u}_i &= \underbrace{\phi_{i,rep}}_{\text{repulsion force}} + \underbrace{\psi_{i,al}}_{\text{alignment force}}, \\ \phi_{i,rep} &= \sum_{j \in \mathcal{N}_i(\hat{x})} \phi(\|\hat{x}_{ij}\|^2) \hat{x}_{ij}, \\ \psi_{i,al} &= \sum_{j \in \mathcal{N}_i(\hat{x})} \psi(\|\hat{x}_{ij}\|) \text{SGN}(\hat{v}_{ji}), \end{aligned} \quad (2)$$

where $\hat{x}_{ij} = \hat{x}_i - \hat{x}_j$, $\hat{v}_{ij} = \hat{v}_i - \hat{v}_j$, $\text{SGN}(\hat{v}_{ji}) = \frac{\hat{v}_{ji}}{\|\hat{v}_{ji}\|}$, if $\hat{v}_{ji} \neq 0$ and 0 otherwise; $\phi_{i,rep}$ is the repulsion force

in the zone of repulsion with diameter $r > 0$ corresponding to the hard sphere of agent i (Ballerini et al., 2008b), in which $\phi(s) \geq 0$ is nonincreasing, $\phi(s) = 0$, $\forall s \in [r^2, \infty)$ and $\int_0^\infty \phi(s)ds = \infty$; $\psi_{i,al}$ is the alignment force in a larger zone outside the hard sphere with $\psi(s) \geq 0$ being continuous and nonincreasing. The introduction of $\psi(s)$ can capture the influence or the path loss of communication between neighboring agents, e.g., $\psi(\|\hat{x}_{ij}\|) \propto \frac{1}{\sqrt{1+\|\hat{x}_{ij}(t)\|^\eta}}^1$, where $\eta \geq 0$ denotes the path loss exponent.

Remark 3 A general form of $\phi(\|\hat{x}_{ij}\|^2)$ and $\psi(\|\hat{x}_{ij}\|)$ is adopted in (2). We note that some specific forms of controller have been introduced for applications in Vicsek & Zafeiris (2012) and references therein.

In the following sections, we will examine the conditions under which collective behaviors would emerge and the impact of the path loss influence on the eventual motion.

3 Filippov solution and its properties

Let $\hat{x} = [\hat{x}_1^T, \hat{x}_2^T, \dots, \hat{x}_N^T]^T$, $\hat{v} = [\hat{v}_1^T, \hat{v}_2^T, \dots, \hat{v}_N^T]^T$, $\hat{u} = [\hat{u}_1^T, \hat{u}_2^T, \dots, \hat{u}_N^T]^T$, and the set of discontinuous points of $\hat{u}$ be $\mathcal{S}$. Then $\mathcal{S} \subseteq \mathcal{S}_1 \cup \mathcal{S}_2$, where $\mathcal{S}_1$ is the set of discontinuous points due to switching from one connection pattern to another, $\mathcal{S}_2 = \{[\hat{x}^T, \hat{v}^T]^T \in \mathbb{R}^{2Nn} : \text{there exist } i, j \text{ such that } \hat{v}_i = \hat{v}_j \text{ or } \phi(s) \text{ is discontinuous at } \|\hat{x}_{ij}\|^2\}$. It is clear that $\mathcal{S}_1 \cup \mathcal{S}_2$ has measure zero, and so does $\mathcal{S}$. In this case, Filippov solution is an appropriate choice for such dynamical system (1) with discontinuous right-hand side (2).

Define the center of mass of the group as $x_c = \frac{1}{N} \sum_{i=1}^N \hat{x}_i$ and $v_c = \frac{1}{N} \sum_{i=1}^N \hat{v}_i$. Let $x_i = \hat{x}_i - x_c$ and $v_i = \hat{v}_i - v_c$, then

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{V}, \quad (3)$$

where the acceleration (2) now can be expressed as

$$u_i = \sum_{j \in \mathcal{N}_i(x)} \phi(\|x_{ij}\|^2) x_{ij} + \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) \text{SGN}(v_{ji}). \quad (4)$$

By stacking the position, velocity and acceleration vectors into x , v and u , respectively, we can rewrite (3) as

$$\dot{x}(t) = v(t), \quad \dot{v}(t) \in \mathcal{K}[u](t), \quad (5)$$

where $\mathcal{K}[u] = \bigcap_{\delta > 0} \bigcap_{\mu(\mathcal{M})=0} \overline{\text{co}}\{u(\mathbb{B}(u, \delta) \setminus \mathcal{M})\}$, in which $\mu(\cdot)$ is the Lebesgue measure, $\overline{\text{co}}\{\cdot\}$ denotes the convex closure, $\mathbb{B}(u, \delta)$ the ball centered at u with radius δ and $\mathcal{M} \subset \mathbb{R}^{Nn}$ is any set with measure zero.

¹ This form is inspired by the path loss effect for wireless communications (Goldsmith, 2005). A similar function called communication strength is used in the C-S model (Cucker & Smale, 2007; Vicsek & Zafeiris, 2012).

Before moving onto the main results, we first establish some lemmas that will be useful in the sequel.

Lemma 4 Define $W(z) = \int_z^\infty \phi(s)ds$. Then $W(z)$ is convex on the interval $(0, \infty)$.

PROOF. Note that $\phi(s)$ is piecewise continuous, and $\phi(s) = 0$, $\forall s \geq r^2$, hence $W(z)$ is well-defined and, moreover, continuous on $(0, \infty)$. The proof of convexity is straightforward by examining the definition of convex functions. So we omit the details. $\square$

In the following, we will investigate the stability of (5) and (4) using the Lyapunov theory of nonsmooth systems. We introduce the function in the domain $\mathcal{D} \times \mathbb{R}^{Nn}$,

$$V(x, v) = \|v\|^2 + \frac{1}{2} \sum_{i=1}^N \sum_{j \neq i} W(\|x_{ij}\|^2), \quad (6)$$

where $\mathcal{D} = \{x \in \mathbb{R}^{Nn} : \|x_{ij}\| > 0, \forall j \neq i\}$.

Lemma 5 The function $V(x, v)$ is regular everywhere in its domain $\mathcal{D} \times \mathbb{R}^{Nn}$.

PROOF. First, note that all convex functions are regular (Clarke, 1990). Then by virtue of Lemma 4, $W(\|x_{ij}\|^2)$ is regular with respect to $\|x_{ij}\|^2$ on $(0, \infty)$ whenever $x_i \neq x_j$. Since $\|x_{ij}\|^2$ is a smooth function of both vectors x_i and x_j when $x_i \neq x_j$, it follows that $W(\|x_{ij}\|^2)$ is regular at both x_i and x_j . Therefore, $V(x, v)$ is regular as a finite combination of regular functions $\|v\|^2$ and $W(\|x_{ij}\|^2)$, $\forall x_i \neq x_j$. $\square$

We need an additional concept. The set-valued Lie derivative of $V(x, v)$ with respect to (5) is defined as $\dot{\bar{V}} = \{h \in \mathbb{R} : \text{there exists } \tilde{w} \in \mathcal{K}[[v^T, u^T]^T] \text{ such that } \xi^T \tilde{w} = h, \text{ for all } \xi \in \partial V(x, v)\}$, where ∂V is the Clarke generalized gradient of V (Bacciotti & Ceragioli, 1999).

Lemma 6 The set-valued Lie derivative $\dot{\bar{V}}$ satisfies

$$\dot{\bar{V}} \subset \left\{ -\sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) \|v_{ij}\| \right\}. \quad (7)$$

PROOF. First, we compute $\partial V(x, v)$. From Lemma 4, we know that $W(z)$ is convex, thus locally Lipschitz. It follows from Theorem 1 of Paden & Sastry (1987)

that $\partial W(\|x_{ij}\|^2) = \mathcal{K}[\nabla W(\|x_{ij}\|^2)]$, which yields $\frac{\partial W(\|x_{ij}\|^2)}{\partial x_i} = -2\mathcal{K}[\phi(\|x_{ij}\|^2)]x_{ij}$ and

$$\mathcal{K}[\phi(\|x_{ij}\|^2)] = \begin{cases} \phi(\|x_{ij}\|^2), & \text{if } \phi(s) \text{ is continuous} \\ & \text{at } \|x_{ij}\|^2, \\ I_{ij}^+, & \text{otherwise,} \end{cases} \quad (8)$$

where I_{ij}^+ denotes the interval $[\phi_{ij}^+, \phi_{ij}^-]$, and $\phi_{ij}^+ = \lim_{s \downarrow \|x_{ij}\|^2} \phi(s)$, $\phi_{ij}^- = \lim_{s \uparrow \|x_{ij}\|^2} \phi(s)$. Since $\phi(s)$ is monotone, it has only jump discontinuities. This means that at any discontinuous point $\|x_{ij}^*\|^2$, both $\lim_{s \uparrow \|x_{ij}^*\|^2} \phi(s)$ and $\lim_{s \downarrow \|x_{ij}^*\|^2} \phi(s)$ exist. And thus I_{ij}^+ is well-defined. Hence the regularity of $V(x, v)$ from Lemma 5 ensures that $\partial V(x, v) = [U^T, 2v_1^T, \dots, 2v_N^T]^T$, where $U = [U_1^T, U_2^T, \dots, U_N^T]^T$ and $U_i = \sum_{j \neq i} \frac{\partial W(\|x_{ij}\|^2)}{\partial x_i}$, for all $i = 1, \dots, N$.

We consider the following two cases.

Case I. There exists some $t^* \geq 0$ and a pair $(i_0, j_0) \in \mathcal{E}(\hat{x}(t^*))$ such that $\phi(s)$ is discontinuous at $\|x_{i_0 j_0}(t^*)\|^2$.

Pick arbitrary $\tilde{w} = [v^T, w^T]^T \in \mathcal{K}[[v^T, u^T]^T]$, where $w = [w_1^T, \dots, w_N^T]^T \in \mathcal{K}[u]$, then from (8), we know that for any $\xi \in \partial V(x, v)$, there exists $p_\xi \in I_{i_0 j_0}^+$ satisfying

$$\begin{aligned} \tilde{w}^T \xi = & -2 \sum_{i=1}^N \sum_{j \neq i} \phi(\|x_{ij}\|^2) v_i^T x_{ij} + 2v^T w \\ & + 2(\phi(\|x_{i_0 j_0}\|^2) - p_\xi) v_{i_0 j_0}^T x_{i_0 j_0}. \end{aligned} \quad (9)$$

Moreover, the bidirectional nature of $\mathcal{G}(\hat{x}(t))$ implies

$$\sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) v_i^T v_{ji} = \frac{1}{2} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) v_{ij}^T v_{ji}.$$

Then applying Theorem 1 of Paden & Sastry (1987) and using (8), we have for all $i \in \mathcal{V}$,

$$\begin{aligned} v^T \mathcal{K}[u] \subset & -\frac{1}{2} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) \|v_{ij}\| \\ & + \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \phi(\|x_{ij}\|^2) v_i^T x_{ij} \\ & + (-\phi(\|x_{i_0 j_0}\|^2) + I_{i_0 j_0}^+) v_{i_0 j_0}^T x_{i_0 j_0}. \end{aligned}$$

Since $v^T w \in v^T \mathcal{K}[u]$, there must exist some $p'_\xi \in I_{i_0 j_0}^+$ such that $\tilde{w}^T \xi = -\sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) \|v_{ij}\| +$

$2(-p'_\xi + p'_\xi) v_{i_0 j_0}^T x_{i_0 j_0}$. By the definition of $\dot{\bar{V}}$, it is clear that $\dot{\bar{V}}$ satisfies (7).

Case II. $\phi(s)$ is continuous at all points $\|x_{ij}(t)\|^2$, for all $t \geq 0$ and $(i, j) \in \mathcal{E}(\hat{x}(t))$.

In this case, Eq. (9) can be rewritten as $\tilde{w}^T \xi = -2 \sum_{i=1}^N \sum_{j \neq i} \phi(\|x_{ij}\|^2) v_i^T x_{ij} + 2v^T w$. Following a similar line as in Case I, we can obtain

$$\tilde{w}^T \xi \in \left\{ -\sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \psi(\|x_{ij}\|) \|v_{ij}\| \right\}, \quad \forall \xi \in \partial V(x, v),$$

which shows that the relation (7) also holds. $\square$

We highlight one feature of the solutions to (5), which is critical to ensure collision avoidance for the group of mobile agents.

Theorem 7 *Let (x, v) be a solution to (5) with initial condition $(x(0), v(0))$ satisfying $x_i(0) \neq x_j(0)$, $\forall (i, j) \in \mathcal{E}(\hat{x}(0))$. Then the following estimate of (x, v)*

$$\|v(t)\| \leq \sqrt{V_0}, \quad \|x(t)\| \leq \|x(0)\| + \sqrt{V_0}t, \quad \forall t \geq 0,$$

holds, where V_0 denotes the initial energy $V(x(0), v(0))$. Moreover, there exists a constant $0 < c \leq r$ such that $\|x_{ij}(t)\| \geq c$, $\forall i \neq j$, $t \geq 0$.

PROOF. We divide the proof into two steps.

(i) *Estimate of $\|v(t)\|$:* By Lemma 5, we know that $V(x, v)$ is regular in its domain $\mathcal{D} \times \mathbb{R}^{Nn}$. Thus applying Lemma 1 of Bacciotti & Ceragioli (1999), one finds that $\frac{d}{dt} V(x(t), v(t))$ exists and is contained in $\dot{\bar{V}}$ for almost all $t \geq 0$. Let $\max \dot{\bar{V}}$ be the largest element of $\dot{\bar{V}}$, if it is nonempty, and $\max \dot{\bar{V}} = -\infty$ whenever $\dot{\bar{V}} = \emptyset$. By Lemma 6, we have

$$\frac{d}{dt} V(x(t), v(t)) \leq \max \dot{\bar{V}} \leq 0. \quad (10)$$

Hence, $V(x(t), v(t))$ is a nonincreasing function of t along the solutions of (5), which yields

$$\|v(t)\|^2 + \frac{1}{2} \sum_{i=1}^N \sum_{j \neq i} W(\|x_{ij}\|^2) \leq V_0, \quad \forall t \geq 0. \quad (11)$$

Therefore, we have $\|v(t)\| \leq \sqrt{V_0}$, for all $t \geq 0$.

(ii) *Estimate of $\|x(t)\|$* : From (11), one obtains

$$W(\|x_{ij}\|^2) = \int_{\|x_{ij}(t)\|^2}^{\infty} \phi(s) ds \leq V_0, \quad \forall i \neq j, t \geq 0.$$

Since $\int_0^\infty \phi(s) ds = \infty$, it is necessary that $\|x_{ij}(t)\| \geq c$, $\forall i \neq j, t \geq 0$, for some constant $c > 0$.

We now provide an upper bound of $\|x(t)\|$ via investigating its derivative $\frac{d}{dt}\|x(t)\|$. In view of $\dot{x}(t) = v(t)$, it can be derived that

$$\frac{d}{dt}\|x(t)\| \leq \|v(t)\|, \quad \forall t \geq 0, \quad (12)$$

which by integration and then using the estimate of $\|v(t)\|$ implies $\|x(t)\| = \|x(0)\| + \int_0^t \|v(s)\| ds \leq \|x(0)\| + \sqrt{V_0}t$, $\forall t \geq 0$. This gives the upper bound of $\|x(t)\|$. $\square$

Remark 8 *If the initial energy satisfies $V_0 \leq \int_{\kappa^2}^\infty \phi(s) ds$, for some constant $0 < \kappa < r$, then we can show that $\min_{i \neq j, t \geq 0} \|x_{ij}(t)\| \geq \kappa$. This means that the minimum separation between any two agents during their temporal evolution is greatly related with the initial configuration. As a special case, if the initial conditions are such that $\|x_{ij}(0)\| \geq r$, $\forall i \neq j$ and $v_i(0) = 0$, $\forall i \in \mathcal{V}$, then it is trivial that $\min_{i \neq j, t \geq 0} \|x_{ij}(t)\| \geq r$, i.e. all the agents would not enter the hard sphere of each other.*

4 Convergence analysis

In this section, we will establish the convergence properties of the proposed model (1) with (2).

To enforce the collective behavior, some connectivity conditions must be imposed on the group. It is noted that the connection pattern between agents is time-varying. Assume that there are infinite number of switching instants of graph $\mathcal{G}(\hat{x}(t))$ denoted as $t_1, t_2, \dots$ satisfying $t_{i+1} - t_i \geq \varsigma$ for a given constant $\varsigma > 0$. Obviously, $\mathcal{G}(\hat{x}(t))$ is a fixed graph between t_k and t_{k+1} , and is denoted by $\mathcal{G}_k$. Similarly, we denote $\mathcal{N}_i(\hat{x}(t))$ by $\mathcal{N}_i^k$ on the interval $[t_k, t_{k+1})$, $k = 0, 1, \dots$ with $t_0 = 0$. A typical notion of the connectivity for time-varying $\mathcal{G}(t)$ is the joint connectivity: there exists a subsequence $\{t_{k_1}, t_{k_2}, \dots\}$ such that the union graph $\mathcal{G}[t_{k_i}, t_{k_{i+1}}) = (\mathcal{V}, \bigcup_{t \in [t_{k_i}, t_{k_{i+1}})} \mathcal{E}(t))$ is connected². This means that once we collect all the edges that have appeared between t_{k_i} and $t_{k_{i+1}}$, the graph is connected, although $\mathcal{G}(t)$ might be completely disconnected at some $t \in [t_{k_i}, t_{k_{i+1}})$.

Lemma 9 *Consider any undirected graph $\mathcal{G}$ defined on the set $\{1, 2, \dots, N\}$ with neighboring sets $\mathcal{N}_i$, $1 \leq i \leq N$.*

² A graph is said to be connected if any two vertices i and j are linked by a path.

For each $x = [x_1^T, x_2^T, \dots, x_N^T]^T \in \Omega = \{x \in \mathbb{R}^{Nn} : \sum_{i=1}^N x_i = 0\}$, define $\|x\|_\Omega = \sqrt{\sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \|x_{ij}\|^2}$. If $\mathcal{G}$ is connected, then

$$\sqrt{\frac{2}{Nd(\mathcal{G})}} \|x\| < \|x\|_\Omega \leq \sqrt{2N} \|x\|, \quad (13)$$

where $d(\mathcal{G})$ is the diameter of graph $\mathcal{G}$ ³.

PROOF. For each element x_i , $1 \leq i \leq N$ of x , denote its components as $x_i^1, x_i^2, \dots, x_i^n$. Let i_k be the index of the component with $|x_{i_k}^k| = \max_{1 \leq i \leq N} |x_i^k|$ for each $1 \leq k \leq n$. It follows that $\|x\|^2 \leq N \sum_{k=1}^n (x_{i_k}^k)^2$. By the fact $\sum_{i=1}^N x_i = 0$, we can find a corresponding component $x_{i'_k}^k$ such that $x_{i_k}^k x_{i'_k}^k < 0$ for each $1 \leq k \leq n$. This means that $(x_{i_k}^k - x_{i'_k}^k)^2 > (x_{i_k}^k)^2$, for each $1 \leq k \leq n$.

Consider the two vertices i_k and i'_k of the graph $\mathcal{G}$, the connectivity of $\mathcal{G}$ ensures that there exists a path of length m_k connecting i_k and i'_k , say, $i_k = l_1^k - l_2^k - \dots - l_{m_k+1}^k = i'_k$. This implies that $x_{i_k}^k - x_{i'_k}^k = x_{i_k}^k - x_{l_2^k}^k + x_{l_2^k}^k - x_{l_3^k}^k + \dots + x_{l_{m_k}^k}^k - x_{i'_k}^k$. Applying the Cauchy-Schwarz inequality to $(x_{i_k}^k - x_{i'_k}^k)^2$, we have $m_k \sum_{j=1}^{m_k} (x_{l_j^k}^k - x_{l_{j+1}^k}^k)^2 > (x_{i_k}^k)^2$. Collecting all the edges in $\mathcal{G}$, we can get a no worse result, that is, $d(\mathcal{G}) \sum_{(i,j) \in \mathcal{E}} (x_i^k - x_j^k)^2 \geq m_k \sum_{j=1}^{m_k} (x_{l_j^k}^k - x_{l_{j+1}^k}^k)^2$. Therefore, we have $\|x\|_\Omega^2 = 2 \sum_{(i,j) \in \mathcal{E}} \sum_{k=1}^n (x_i^k - x_j^k)^2 > \frac{2}{Nd(\mathcal{G})} \|x\|^2$, from which the left part follows.

Next, we can obtain $\|x_{ij}\|^2 = \|x_i\|^2 + \|x_j\|^2 - 2 \langle x_i, x_j \rangle$, $\forall 1 \leq i, j \leq N$. The right part of (13) thus follows from $\|x\|_\Omega^2 \leq \sum_{i=1}^N \sum_{j=1}^N \|x_{ij}\|^2 = 2N \|x\|^2$, since $\sum_{i=1}^N x_i = 0$ for each $x \in \Omega$. $\square$

We are now ready to state our main results about the emergence of collective behavior with collision avoidance in the presence of state-dependent interactions.

Theorem 10 *Consider a group of N mobile agents with dynamics (1) steered by the accelerations (2). Suppose that the initial conditions satisfy $\hat{x}_i(0) \neq \hat{x}_j(0)$, $\forall (i, j) \in \mathcal{E}(\hat{x}(0))$, and the graph $\mathcal{G}(\hat{x}(t))$ is jointly connected. If $\psi(s)$ satisfies $\int_0^\infty \psi(s) ds = \infty$, then there exist constants $c > 0$ and $C > 0$ such that $c \leq \|\hat{x}_{ij}(t)\| \leq C$, $\forall i \neq j$, $t \geq 0$. Moreover, all agents move with the same velocity eventually, i.e., $\lim_{t \rightarrow \infty} (\hat{v}_i(t) - \hat{v}_j(t)) = 0$, $\forall i \neq j$.*

³ The diameter of a graph is defined as the greatest length of all paths between any two vertices.

PROOF. We first prove the boundedness of $\|\hat{x}_{ij}\|$. The lower bound is a direct consequence of Theorem 7. It remains to show that $\|x_{ij}\|, \forall (i, j) \in \mathcal{E}(\hat{x}(t))$ is upper bounded. For this purpose, we define another Lyapunov-like function $\tilde{V}(t) = V(x(t), v(t)) + \int_0^{y(t)} \psi(s)ds$, where $y(t) = \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \|x_{ij}(t)\|$. Obviously, $\tilde{V}(t)$ is non-negative for all $t \geq 0$. The time derivative of $\tilde{V}(t)$ along the solutions of (5) can thus be given by $\frac{d}{dt}\tilde{V}(t) = \psi(y(t))\frac{d}{dt}y(t) + \frac{d}{dt}V(x(t), v(t))$. Similar to (12), one has $\frac{d}{dt}\|x_{ij}\| \leq \|v_{ij}\|, \forall i \neq j$. It thus follows from (7) and (10) that

$$\frac{d}{dt}\tilde{V}(t) \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} (\psi(y(t)) - \psi(\|x_{ij}\|)) \|v_{ij}\|.$$

Since $\psi(s)$ is nonincreasing, we have $\psi(y(t)) \leq \psi(\|x_{ij}\|)$, $\forall (i, j) \in \mathcal{E}(\hat{x}(t))$. This implies that $\tilde{V}(t)$ is nonincreasing and thus $\tilde{V}(t) \leq \tilde{V}(0) < \infty, \forall t \geq 0$. By the condition $\int_0^\infty \psi(s)ds = \infty$, we can conclude that $y(t)$ is bounded for all $t \geq 0$. Therefore, there exists a constant $C > 0$ such that $\|x_{ij}(t)\| \leq C, \forall t \geq 0$. By the continuity of $\psi(s)$, we then have $\psi(\|x_{ij}(t)\|) \geq c_\psi, \forall (i, j) \in \mathcal{E}(\hat{x}(t))$, for some constant $c_\psi > 0$.

Next, we show that $\lim_{t \rightarrow \infty} \|v_{ij}(t)\| = 0$. In the following, if there is no confusion, we will write $V(x(t), v(t))$ as $V(t)$. Some idea is borrowed from Ni & Cheng (2010).

It follows from Lemma 6 and the above discussion that

$$\frac{d}{dt}V(t) \leq -c_\psi \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \|v_{ij}(t)\| \leq 0, \quad (14)$$

from which $\lim_{t \rightarrow \infty} V(t)$ exists. Then for arbitrary $\epsilon > 0$, there exists a constant $T > 0$ such that for any $T_2 > T_1 \geq T$, we have $|V(T_2) - V(T_1)| < \epsilon c_\psi$. By integration of (14), we obtain

$$\int_{T_1}^{T_2} \sum_{i=1}^N \sum_{j \in \mathcal{N}_i(x)} \|v_{ij}(\tau)\| d\tau \leq \frac{V(T_1) - V(T_2)}{c_\psi} < \epsilon. \quad (15)$$

Consider the subsequence $\{t_{k_1}, t_{k_2}, \dots\}$, and let the interaction graphs during the interval $[t_{k_m}, t_{k_{m+1}})$ be $\mathcal{G}_{k_m}, \mathcal{G}_{k_m+1}, \dots, \mathcal{G}_{k_{m+1}-1}$. Hence we derive from (15) that $\sum_{l=k_m}^{k_{m+1}-1} \int_{t_l}^{t_{l+1}} \sum_{(i,j) \in \mathcal{E}_l} \|v_{ij}(\tau)\| d\tau < \epsilon/2$, for any integer k_m satisfying $t_{k_m} \geq T$. Recall that $t_{k+1} - t_k \geq \varsigma$ for each integer $k \geq 0$, thus for sufficiently large k_m , one has $\int_{t_l}^{t_{l+1}} \sum_{(i,j) \in \mathcal{E}_l} \|v_{ij}(\tau)\| d\tau < \epsilon/2, \forall l = k_m, \dots, k_{m+1} - 1$. Since $\epsilon > 0$ is arbitrary, it yields $\lim_{t \rightarrow \infty} \int_t^{t+\varsigma} \sum_{(i,j) \in \mathcal{E}_l} \|v_{ij}(\tau)\| d\tau = 0, \forall l =$

$k_m, \dots, k_{m+1} - 1$. Note that there are only finite interaction graphs during the interval $[t_{k_m}, t_{k_{m+1}})$, denote $\mathcal{E}^{k_m} = \bigcup_{l=k_m}^{k_{m+1}-1} \mathcal{E}_l$, it follows that

$$\lim_{t \rightarrow \infty} \int_t^{t+\varsigma} \sum_{(i,j) \in \mathcal{E}^{k_m}} \|v_{ij}(\tau)\| d\tau = 0. \quad (16)$$

It is easy to see that $(\sum_{(i,j) \in \mathcal{E}^{k_m}} \|v_{ij}(\tau)\|)^2 \geq \sum_{(i,j) \in \mathcal{E}^{k_m}} \|v_{ij}(\tau)\|^2$ and $v(\tau) \in \Omega$, for all $\tau \geq 0$. Moreover, the union graph $\mathcal{G}[t_{k_m}, t_{k_{m+1}})$ is connected. Applying Lemma 9, we can obtain $\sum_{(i,j) \in \mathcal{E}^{k_m}} \|v_{ij}(\tau)\|^2 > \sqrt{1/(2Nd_{k_m})} \|v(\tau)\|$, where d_{k_m} is the diameter of graph $\mathcal{G}[t_{k_m}, t_{k_{m+1}})$. This along with (16) implies that

$$\lim_{t \rightarrow \infty} \int_t^{t+\varsigma} \|v(\tau)\| d\tau = 0. \quad (17)$$

Now, we claim that $\lim_{t \rightarrow \infty} \|v(t)\| = 0$. Suppose that $\limsup_{t \rightarrow \infty} \|v(t)\| = \gamma > 0$, then there exists an infinite sequence $\{t_{s_1}, t_{s_2}, \dots\}$ such that $\lim_{k \rightarrow \infty} t_{s_k} = \infty$ and $\|v(t_{s_k})\| \geq \gamma/2$, for sufficiently large k .

Since $v(t)$ is absolutely continuous in any bounded time interval and u_i is essentially locally bounded for all $1 \leq i \leq N$, it follows that $v(t)$ is uniformly continuous. On the other hand, $\|v\|$ is continuous in v . Using Theorem 7, we can show that $\|v(t)\|$ is uniformly continuous in t on the interval $[0, \infty)$. Hence, there exists a positive constant $\delta \leq \varsigma$ such that $\|v(t)\| > \|v(t_{s_k})\| - \gamma/4$, for all $t \in [t_{s_k}, t_{s_k} + \delta]$. Consequently, for large enough k ,

$$\int_{t_{s_k}}^{t_{s_k} + \varsigma} \|v(\tau)\| d\tau \geq \int_{t_{s_k}}^{t_{s_k} + \delta} \|v(\tau)\| d\tau > \frac{\delta\gamma}{4},$$

which is a contradiction to (17). Therefore, we must have $\lim_{t \rightarrow \infty} (\hat{v}_i(t) - v_c(t)) = 0$, for all $i \in \mathcal{V}$. This completes the proof. $\square$

Theorem 10 provides a general result on the emergence of collective behavior under the joint connectivity condition. However, with this rather weak condition, we find that only $\eta \leq 2$ is valid when applying the result to the influence function $\psi \propto \frac{1}{\sqrt{1+\|x_{ij}\|^\eta}}, \eta \geq 0$. This is because

$\int_1^\infty s^{-\eta/2} ds = \infty$ if and only if $\eta \leq 2$. For this influence function, the higher the value of η is, the faster the influence drops with respect to the increase in distance. This reveals that there might be a trade-off between the connectivity condition and the influence strength for the emergence of collective behavior.

The above observation is validated theoretically in the following theorem, where the connectivity condition is strengthened but a wider range of η is incorporated.

Theorem 11 Suppose that the initial configurations of N mobile agents satisfy $\hat{x}_i(0) \neq \hat{x}_j(0)$, $\forall (i, j) \in \mathcal{E}(\hat{x}(0))$ and the influence function is $\psi(\|\hat{x}_{ij}\|) = \sqrt{\frac{H}{1+\|\hat{x}_{ij}\|^\eta}}$, where $\eta > 2$ and $H > 0$ is a constant. If graph $\mathcal{G}(\hat{x}(t))$ is connected for all $t \geq 0$, and

$$V_0^2(1 + \sqrt{2}\|x(0)\|)^{\eta-2} < \frac{8H}{Nd_{\max}(\eta-2)^2}, \quad (18)$$

where $d_{\max}$ is the maximum diameter of all possible connection patterns of the group, then there exist positive constants c and C such that $c \leq \|\hat{x}_{ij}(t)\| \leq C$, $\forall i \neq j$, $t \geq 0$, and further, $\lim_{t \rightarrow +\infty} (\hat{v}_i(t) - \hat{v}_j(t)) = 0$, $\forall i \neq j$.

PROOF. We prove the theorem in two steps.

(i) *Boundedness of $x(t)$* : Note that the method used in the proof of Theorem 10 does not apply for the case $\eta > 2$. We prove the boundedness of $x(t)$ by contradiction. Suppose on the contrary that $\|x(t)\|$ is unbounded, then there exist an integer $k^* \geq 0$, a constant $\delta > 0$ and $\tau^* \in [t_{k^*}, t_{k^*+1})$ such that for any constant $M > \|x(0)\|$, one has $\|x(t)\| \geq M$, $\forall \tau^* - \delta \leq t < \tau^* + \delta$.

By the Cauchy-Schwarz inequality, one has $\|x_{ij}\|^2 \leq 2\|x\|^2$, $\forall 1 \leq i, j \leq N$, which implies $\psi(\|x_{ij}\|) \geq \sqrt{H/(1+2^{\eta/2}\|x\|^\eta)}$ holds for all $j \in \mathcal{N}_i(x)$, $1 \leq i \leq N$. Furthermore, it can be verified that $\sum_{i=1}^N \sum_{j \in \mathcal{N}_i^k} \|v_{ij}\| \geq \|v\|_\Omega$. Since $\sum_{i=1}^N v_i = 0$ and $\mathcal{G}(\hat{x}(t))$ is connected all the time, we then use Lemma 9 to find that on the interval $[t_k, t_{k+1})$,

$$\sum_{i=1}^N \sum_{j \in \mathcal{N}_i^k} \psi(\|x_{ij}\|) \|v_{ij}\| \geq \sqrt{\frac{2H}{Nd(\mathcal{G}^k)}} \frac{\|v\|}{\sqrt{1+2^{\frac{\eta}{2}}\|x\|^\eta}}.$$

Denote $F = \sqrt{2H/(Nd_{\max})}$. Then combining the above inequality with (7) and (10) yields

$$\frac{d}{dt} V(t) \leq -\frac{F\|v(t)\|}{\sqrt{1+2^{\frac{\eta}{2}}\|x(t)\|^\eta}}. \quad (19)$$

Hence, for any two $t_{k_{s1}} < t_{k_{s2}} \in [t_k, t_{k+1})$, by integration and then summing up these inequalities shows

$$\begin{aligned} \int_0^t \frac{\|v(\tau)\|}{\sqrt{1+2^{\frac{\eta}{2}}\|x(\tau)\|^\eta}} d\tau &= \sum_{j=0}^{k-1} \int_{t_j}^{t_{j+1}} \frac{\|v(\tau)\|}{\sqrt{1+2^{\frac{\eta}{2}}\|x(\tau)\|^\eta}} d\tau \\ &\quad + \int_{t_k}^t \frac{\|v(\tau)\|}{\sqrt{1+2^{\frac{\eta}{2}}\|x(\tau)\|^\eta}} d\tau \\ &\leq \frac{V_0 - V(t)}{F}. \quad \forall t_k \leq t < t_{k+1} \end{aligned} \quad (20)$$

Since $V(t)$ is nonnegative, using the relation (12), we conclude that $\int_0^{\tau^*} (1+2^{\frac{\eta}{2}}\|x(\tau)\|^\eta)^{-\frac{1}{2}} d\|x(\tau)\| \leq V_0/F$. By changing the variable $z = \|x(\tau)\|$ and recalling that $\|x(\tau^*)\| \geq M > \|x(0)\|$, we obtain

$$\int_{\|x(0)\|}^M (1+2^{\frac{\eta}{2}}z^\eta)^{-\frac{1}{2}} dz \leq \frac{V_0}{F}. \quad (21)$$

It can be shown that $1+2^{\eta/2}z^\eta \leq (1+\sqrt{2}z)^\eta$ holds for all scalars $z \geq 0$, and $\int_{\|x(0)\|}^M (1+\sqrt{2}z)^{-\eta/2} dz = \frac{2}{2-\eta} [(1+\sqrt{2}M)^{1-\eta/2} - (1+\sqrt{2}\|x(0)\|)^{1-\eta/2}]$. Thus we can derive from (21) that $(1+\sqrt{2}\|x(0)\|)^{1-\frac{\eta}{2}} - (1+\sqrt{2}M)^{1-\frac{\eta}{2}} \leq (\eta-2)V_0/(2F)$. Since M can be arbitrarily large and $\eta > 2$, we further obtain

$$(1+\sqrt{2}\|x(0)\|)^{1-\frac{\eta}{2}} \leq (\eta-2)V_0\sqrt{\frac{d_{\max}N}{8H}},$$

which contradicts with the condition (18). Therefore, we can find a constant $C > 0$ such that $\|x(t)\| \leq C/2$, which implies $\|\hat{x}_{ij}(t)\| \leq 2\|x(t)\| \leq C$, $\forall i \neq j$. The lower bound $\|\hat{x}_{ij}(t)\| \geq c$ has already been established in Theorem 7.

(ii) *Alignment of velocities*: Substituting the upper bound of $\|x(t)\|$ back into (19) gives $\frac{d}{dt} V(t) \leq -\frac{F\|v(t)\|}{\sqrt{1+2^{\frac{\eta}{2}}C^\eta}}$, $\forall t \in [t_k, t_{k+1})$. Then repeating the same argument as in the derivation of (20) shows that

$$\int_0^t \|v(\tau)\| d\tau \leq \frac{\sqrt{1+2^{\frac{\eta}{2}}C^\eta}}{F} V_0 < \infty, \quad \forall t \geq 0. \quad (22)$$

From the proof of Theorem 10, we know that $\|v(t)\|$ is uniformly continuous in t on the interval $[0, \infty)$. Therefore, the generalized Barbalat's lemma (Su & Huang, 2012) gives $\|v(t)\| \rightarrow 0$, as $t \rightarrow \infty$. This guarantees that all the agents asymptotically align their velocities with the mean velocity $v_c(t)$. The proof is thus complete. $\square$

Remark 12 The path loss exponent η is typically between 2 and 6 for wireless communications (Goldsmith, 2005), where $\eta = 2$ is known as the free space model and much different from those with $\eta > 2$. Theorems 10 and 11 show that $\eta = 2$ is also a critical value for collective motion of mobile agents exposing to path loss influence.

Remark 13 Condition (18) relates the initial configuration V_0 , $\|x(0)\|$, the number of agents N , the interaction topology $d_{\max}$ and the environment parameter η in one inequality. Clearly, $d_{\max} \leq N-1$. Thus for a fixed η , we see that V_0 is at most $O(\frac{1}{N})$ so as to enable the emergence of collective motion. The implication is that

Fig. 1. (a) Position trajectories and (b) minimum distance among all 50 agents.

if the initial energy V_0 is large, then collective motion might not be achieved even if the interaction graph is always connected. We also remark that the bound is much conservative, see Fig. 3(b) in the simulation part.

5 Simulation study

In this section, we present some numerical simulations to validate the theoretical results in the previous sections.

The simulations are performed with a group of 50 mobile agents moving in the 3-D open space. The function ϕ_{ij} in the repulsion force (2) used in the simulation is

$$\phi(\|\hat{x}_{ij}\|^2) = \begin{cases} \frac{1}{\|\hat{x}_{ij}\|^2}, & 0 < \|\hat{x}_{ij}\| \leq r, \\ 0, & \|\hat{x}_{ij}\| > r, \end{cases}$$

It can be verified that such ϕ satisfies the properties mentioned in Section 2. Let ψ_{ij} in the alignment force be the one used in Theorem 11, where $H = 10$ and $\eta = 1$ for the first simulation. The initial positions and velocities are all generated randomly in the square $[-20, 20] \times [-20, 20]$ and $[-2, 3] \times [-2, 3]$, respectively, such that the initial graph $\mathcal{G}(\hat{x}(0))$ is connected and all agents are separated. The diameter of the hard sphere is set to $r = 0.4$.

Fig. 1(a) depicts their position trajectories. It is observed that the velocities are asymptotically synchronized. In detail, alignment of velocities can be seen in Fig. 2. Although frequent topology changes produce transients, the overall convergence is evident.

To measure the degree of collision avoidance of the group, we make use of the minimum distance of nearest neighbors $\min_{i \neq j} \|\hat{x}_i - \hat{x}_j\|$ as an indicator. As shown in Fig. 1(b), all the agents are separated at least by 0.6, which is a little greater than the diameter $r = 0.4$ of the hard sphere, thereby forming a collision-free flock. If we use the maximum distance of neighboring agents as the equivalent metric interaction range $R(\hat{x})$ of the group, then we can see from Fig. 3(a) that it is time-varying rather than constant. These figures demonstrate the collective behavior of the group of mobile agents using the state-dependent interactions.

Fig. 2. Alignment of agent velocities.

Fig. 3. (a) Equivalent metric interaction range $R(\hat{x})$. (b) Average error of velocity deviation versus η .

Now, we turn to study the effect of path loss exponent η . To quantify the alignment of velocities of the agents, we calculate the square root of the mean velocity deviation of each agent from the center of mass $\text{Dev} = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\hat{v}_i - v_c\|^2}$. For each simulation, Dev is averaged over the last 50s in order to avoid the transient period. The simulation result is shown in Fig. 3(b), from which we can see that larger η indicates greater disorder among the agents. This is because the higher the value of η is, the less influence between neighbors. Thus it may take more time for the agents to align their velocities, even it can not be aligned any more in some cases. For this simulation, we compute the initial energy V_0 and solve the nonlinear inequality (18). Its approximate solution is around $\eta = 2.5$. However, from Fig. 3(b) we can see that the velocity can be synchronized even for $\eta = 4$. This conservativeness of the bound in (18) stems from two aspects: the lower bound in (13) and the approximation of the integral in (21). To establish some tight bounds and even some necessary and sufficient conditions for the emergence of collective motion could be interesting and are deferred to our future work.

6 Conclusion

A self-propelled particle model has been proposed to mimic the collective behavior of a group of mobile agents with state-dependent interactions. To cope with the dynamic nature of the switching topology and discontinuity of local forces, we use the tools of differential inclusions and nonsmooth analysis. We have given the jointly connected condition to guarantee the velocity alignment and collision avoidance. We have also discussed the path loss influence and initial configurations on the con-

vergence of velocities. Both the theoretical analysis and simulations show that our model can produce desired collective behavior for mobile agents.

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