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2014

Liu, Z., Guan, Y. L., & Chen, H.-H. (2015). Fractional-delay-resilient receiver design for interference-free MC-CDMA communications based on complete complementary codes. IEEE transactions on wireless communications, 14(3), 1226-1236.

<https://hdl.handle.net/10356/107459>

<https://doi.org/10.1109/TWC.2014.2365467>

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Fractional-Delay-Resilient Receiver Design for Interference-Free MC-CDMA Communications Based on Complete Complementary Codes

Zilong Liu, Yong Liang Guan, Hsiao-Hwa Chen

Abstract

Complete complementary codes (CCC) refer to a set of two-dimensional matrices which have zero non-trivial aperiodic auto- and cross- correlation sums. A modern application of CCC is in interference-free multicarrier code-division multiple-access (MC-CDMA) communications. In this paper, we first show that in asynchronous “fractional-delay” uplink channels, CCC-MC-CDMA systems suffer from orthogonality loss which may lead to huge interference increase when a conventional correlator based receiver is deployed. Then, by exploiting the correlation properties of CCC, we present a fractional-delay-resilient receiver which is comprised of a chip-spaced correlating array. Analysis and simulations validate the interference-free achievability of the proposed CSCA receiver in strong interference scenarios.

Index Terms

Complete Complementary Codes (CCC), Multi-Carrier CDMA (MC-CDMA), Interference-Free Performance, Fractional-Delay-Resilient.

I. INTRODUCTION

In the late 1940s, in the study of infrared spectrometry [1], [2], Golay introduced the concept of “complementary pair” which is defined as a pair of sequences whose out-of-phase aperiodic auto-correlation sums are zero, now known as Golay complementary pair (GCP) in the literature. Mathematical properties of GCPs are given in [3]. The idea of GCP was generalized by Tseng and Liu to “complementary codes” (each with two or more constituent sequences), and further to mutually orthogonal complementary code set (MOCCS) which features zero aperiodic cross-correlation sums, in addition to their zero aperiodic auto-correlation sums [4]. Denoting the set size and the number of constituent sequences of an MOCCS by K and M , respectively, it is known that $K \leq M$ [5]. In particular, an MOCCS is called a set of complete complementary codes (CCC) if $K = M$ [6]. Algebraic constructions of CCC are given in [7]–[9]. Since the transmission of a complementary code requires a “multi-channel” system where the constituent sequences are sent and received in distinct non-interfering channels, each complementary code can be written as a two-dimensional matrix by vertically stacking M ($M \geq 2$) constituent sequences row by row, therefore a set of CCC can be viewed as a set of M two-dimensional matrices where each matrix has M row sequences (i.e., constituent sequences). For ease of presentation, in this paper, a complementary code is sometimes called a complementary matrix, and vice versa.

Owing to their zero non-trivial aperiodic auto- and cross- correlation sum property, complementary codes have found a number of modern applications in engineering, e.g., peak-to-mean power control in code-keying orthogonal frequency-division multiplexing (OFDM) systems [10]–[12], intersymbol interference (ISI) channel estimation [13], [14], radar waveform design [15], [16], etc. In particular, CCC have been

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The work of Z. Liu and Y. L. Guan was supported by the Advanced Communications Research Program DSOC106271, a research grant from the Defense Research and Technology Office (DRTech), Ministry of Defence, Singapore. The work of H.-H. Chen was supported by the research grant 102-2221-E-006-008-MY3 from the ministry of Science and Technology of Taiwan.

applied in asynchronous multi-carrier code-division multiple access (MC-CDMA) communications to support interference-free multi-user communication [17]–[19]. In a CCC-MC-CDMA transmitter, every data symbol of a specific user is spread by a complementary matrix by simultaneously sending out all of its row sequences (i.e., constituent sequences) over a number of non-interfering subcarrier channels. At the receiver, when a conventional correlator¹ is deployed, de-spreadings are performed separately in each subcarrier channel, followed by summing the correlator outputs of all the subcarrier channels to obtain a correlation sum.

A traditional understanding of asynchronous CCC-MC-CDMA communications is that the conventional correlator based receiver described above is sufficient to achieve “interference-free” performance [19]. The logic behind is that, since CCC have zero aperiodic auto- and cross- correlation sums for all non-trivial time-shifts, the pairwise orthogonality could be maintained for any inter-user delay. In fact, this understanding overlooked the effect of multicarrier modulation and demodulation. It is known from the Fourier transform that a delay in the time domain gives rise to a phase rotation in the frequency domain, but the phase rotation is different at different carrier frequencies even if the time delay is the same. Based on this relationship, given a non-zero inter-user delay (between the desired user and an interfering user), each subcarrier channel will have a different phase gain (after the multicarrier demodulation and despreading in a conventional correlator based receiver), where the phase is linearly-proportional to the subcarrier index and the propagation delay. Because of this, the pairwise aperiodic cross-correlation sum is a “weighted” sum of terms modulated by the resultant phase-gain vector. In this paper, we will show that in practical asynchronous CCC-MC-CDMA systems, the orthogonality of complementary codes will be lost (due to non-zero pairwise aperiodic cross-correlation “weighted” sum) if any inter-user delay takes on a fraction of half chip-duration. Such a problem is referred to as the “fractional-delay” problem in this paper. Since in any real-time asynchronous CDMA channel, unless a global timing control system is present², the chance that all inter-user delays take on integer multiples of half chip-duration is rare, such a “fractional-delay” problem will exist, almost surely. As a result, “interference-free” performance cannot be guaranteed for conventional correlator based receiver in CCC-MC-CDMA, leading to the notorious “near-far effect” with a high probability. The main objective of this paper is to present a new receiver design which is capable of achieving (or almost achieving) the “single-user BER” performance in CCC-MC-CDMA, by fully exploiting the zero aperiodic sum property of CCC.

The key principle behind our proposed receiver is that in the received CCC-MC-CDMA signals, there exists an interference space (consisting of interferers’ data patterns only) which is separable from the desired signal space. The main structure of this receiver is an array of correlators, where each correlator in this array uses a distinct cyclically-shifted version of all (or some) complementary matrices. The proposed receiver is therefore called chip-spaced correlating array (CSCA) receiver. Since the correlating array generates a set of “projections” in interference space, our proposed CSCA receiver is able to detect interferers’ data patterns symbol-by-symbol, which enables interference cancellation. Provided that there are sufficient non-zero “projections” and the interference is strong enough, our proposed CSCA receiver is capable of recovering then cancelling all interferers’ data patterns correctly, thus giving rise to the single-user BER (i.e., interference-free) performance.

The rest of this paper is organized as follows. In Section II, we define CCC, and give a description on the system model (e.g., the CCC-MC-CDMA transmitter, the asynchronous CDMA channel, and the conventional correlator based receiver, etc). In Section III, we shall first reveal the “fractional-delay” problem, then we introduce the proposed CSCA receiver. Some numerical simulation results will be given in Section IV. Finally, this work is concluded in Section V.

¹i.e., a matched filter.

²Note that building a global timing control system (which is used to adjust every user’s timing in a close-loop) will consume more system resources. It also requires a central control-node, which is difficult in infrastructureless wireless networks.

II. PRELIMINARIES

A. Complete Complementary Codes (CCC)

Given two length- N complex-valued sequences $\mathbf{a}$ (or $\{a_t\}$) and $\mathbf{b}$ (or $\{b_t\}$), their aperiodic correlation function of time-shift τ is defined as

$$R(\mathbf{a}, \mathbf{b})(\tau) = \begin{cases} \sum_{t=0}^{N-1-\tau} a_t (b_{t+\tau})^*, & 0 \leq \tau \leq (N-1); \\ \sum_{t=0}^{N-1+\tau} a_{t-\tau} (b_t)^*, & -(N-1) \leq \tau \leq -1; \\ 0, & |\tau| \geq N. \end{cases} \quad (1)$$

When $\mathbf{a} \neq \mathbf{b}$, $R(\mathbf{a}, \mathbf{b})(\tau)$ is called the aperiodic cross-correlation function (ACCF); Otherwise, it is called the aperiodic auto-correlation function (AACF). For simplicity, the AACF of $\mathbf{a}$ will be sometimes written as $R(\mathbf{a})(\tau)$.

Let $\mathcal{C} = \{\mathbf{C}^0, \mathbf{C}^1, \dots, \mathbf{C}^{K-1}\}$ be a set of K matrices, each of size $M \times N$, i.e.,

$$\mathbf{C}^k = \begin{bmatrix} \mathbf{c}_0^k \\ \mathbf{c}_1^k \\ \vdots \\ \mathbf{c}_{M-1}^k \end{bmatrix}_{M \times N}, \quad 0 \leq k \leq K-1 \quad (2)$$

where

$$\mathbf{c}_p^k = (c_{p,0}^k, c_{p,1}^k, \dots, c_{p,N-1}^k), \quad 0 \leq p \leq M-1$$

is the p th row sequence (i.e., constituent sequence) of $\mathbf{C}^k$. The aperiodic correlation function of $\mathcal{C}$, which is in the form of aperiodic correlation sum, is defined as

$$R(\mathbf{C}^{k_1}, \mathbf{C}^{k_2})(\tau) = \sum_{m=0}^{M-1} R(\mathbf{c}_m^{k_1}, \mathbf{c}_m^{k_2})(\tau), \quad (3)$$

where $0 \leq k_1, k_2 \leq K-1$. $\mathcal{C}$ is said to be a mutually orthogonal complementary code set (MOCCS) if the following condition is satisfied for any $k_1 \neq k_2$ or $k_1 = k_2, \tau \neq 0$, i.e.,

$$R(\mathbf{C}^{k_1}, \mathbf{C}^{k_2})(\tau) = 0. \quad (4)$$

In particular, when $M = 2$, each complementary matrix reduces to a GCP [3]. It is noted that $K \leq M$ [5]. When $K = M$, $\mathcal{C}$ is said to be a set of CCC. CCC are attractive because for a given M , each set of CCC is capable of supporting the maximum number of mobile users. In a CCC-MC-CDMA system, each complementary matrix, e.g., $\mathbf{C}^k$, is used by a specific user to spread every data symbol in time- and frequency- domains. The time- and frequency- spreadings for complementary matrix $\mathbf{C}^k$ is shown in Fig. 1.

B. System Model

Consider an uplink BPSK CCC-MC-CDMA system with M non-overlapping subcarriers and N chip durations. By the set size of CCC, this system is able to support at most $K = M$ mobile users. Suppose that user 0 is of interest to the receiver, and hence user 1 to user $K-1$ are the interferers whose signals are to be mitigated or suppressed. We need the following notations to describe the system.

Fig. 1. Time- and frequency- spreading for complementary matrix $\mathbf{C}^k$ (where $0 \leq k \leq K-1$) in a CCC-MC-CDMA system.

- T_s The symbol duration;
 T_c The chip duration with $T_s = NT_c$;
 A_k The signal amplitude of user k , where $k \in \{0, 1, \dots, K-1\}$. Assume $A_0 = 1$;
 θ_k the initial carrier phase of user k ;
 τ_k The inter-user delay from user k to user 0. Clearly, we have $\tau_0 = 0$. Furthermore, assume $0 \leq \tau_1 \leq \tau_2 < \dots < \tau_{K-1} < T_s$.
 w_c The RF center carrier.
 Δw The carrier spacing, where $\Delta w = \frac{4\pi}{T_c}$ for non-overlapping subcarriers.
 w_m The m th subcarrier, where $w_m = w_c + m\Delta w$ and $m \in \{0, 1, \dots, K-1\}$;
 $w_T(t)$ Rectangle waveform function, where $w_T(t) = \begin{cases} 1 & 0 \leq t < T; \\ 0 & \text{otherwise.} \end{cases}$
 $b_k[p]$ The p th transmitted symbol of user k , where $b_k[p] \in \{-1, 1\}$;
 $b_k(t)$ The transmitted waveform at time t of user k . Clearly, $b_k(t) = \sum_{p=-\infty}^{+\infty} b_k[p]w_{T_s}(t - pT_s)$;
 $\mathbf{C}^k$ The complementary matrix of user k for frequency- and time- spreadings;
 $\mathbf{C}^k(t)$ The M -dimensional transmitted signal at time t of user k , where $\mathbf{C}^k(t) = [\mathbf{c}_0^k(t), \mathbf{c}_1^k(t), \dots, \mathbf{c}_{M-1}^k(t)]^T$ and $\mathbf{c}_m^k(t) = \sum_{p=-\infty}^{+\infty} w_{T_s}(t - pT_s) \cdot \left[\sum_{n=0}^{N-1} c_{m,n}^k w_{T_c}(t - nT_c - pT_s) \right]$.

Based on the above notations, let $\mathbf{s}_k(t)$ be the M -dimensional transmitted signal (at time t) of user k . For simplicity, suppose that each complementary matrix is real-valued. Also, let $\text{diag}[\mathbf{a}]$ denote a diagonal matrix with the main diagonal vector equals to vector $\mathbf{a}$. Then,

$$\mathbf{s}_k(t) = A_k b_k(t) \cdot \text{Re} \left\{ \mathbf{E}(t) \exp(\sqrt{-1}\theta_k) \right\} \cdot \mathbf{C}^k(t), \quad (5)$$

Fig. 2. Conventional correlator based receiver.

where $\text{Re}\{\cdot\}$ denotes the real-part of a complex-valued data and

$$\mathbf{E}(t) = \text{diag} \left[\exp(\sqrt{-1}w_0t), \exp(\sqrt{-1}w_1t), \dots, \exp(\sqrt{-1}w_{M-1}t) \right]$$

denotes a diagonal matrix which is due to the carrier modulation at the k th transmitter.

Also, let

$$\mathbf{E}_k = \text{diag} \left[\exp(\sqrt{-1}w_0\tau_k), \exp(\sqrt{-1}w_1\tau_k), \dots, \exp(\sqrt{-1}w_{M-1}\tau_k) \right]. \quad (6)$$

Thus, due to the channel delay, we have

$$\mathbf{s}_k(t - \tau_k) = A_k b_k(t - \tau_k) \cdot \text{Re} \left\{ \mathbf{E}(t) \mathbf{E}_k^* \exp(\sqrt{-1}\theta_k) \right\} \cdot \mathbf{C}^k(t - \tau_k). \quad (7)$$

Taking into account the multicarrier demodulation (i.e., carrier demodulation and the low-pass filtering) at the receiver, let us remove $\mathbf{E}(t)$ from $\mathbf{s}_k(t - \tau_k)$. Then, by summing up all users' signals³, the total M -dimensional received signal, denoted by $\mathbf{x}(t) = [x_0(t), x_1(t), \dots, x_{M-1}(t)]^T$, can be written as

$$\begin{aligned} \mathbf{x}(t) &= b_0(t) \mathbf{C}^0(t) + \mathbf{n}(t) + \\ &\underbrace{\sum_{k=1}^{K-1} A_k b_k(t - \tau_k) \cos(\theta_k - \theta_0) \cdot \text{Re}\{\mathbf{E}_k^*\} \cdot \mathbf{C}^k(t - \tau_k)}_{\text{MAI}}, \end{aligned} \quad (8)$$

where $\mathbf{n}(t) = [n_0(t), n_1(t), \dots, n_{M-1}(t)]^T$ is an M -dimensional i.i.d.⁴ additive Gaussian noise vector with $n_m(t) \sim \mathcal{N}(0, \sigma^2/2)$ ($m \in \{0, 1, \dots, M-1\}$). As a matter of fact, this M -dimensional signal, i.e., $\mathbf{x}(t)$, will sum up to a scalar, denoted by $r(t)$, over the wireless channel. However, since the M subcarrier channels are non-overlapping, it is therefore equivalent (and more convenient) to express the received signal after multicarrier demodulation as an M -dimensional vector. See Fig. 2.

Applying the conventional correlator based receiver, the received signal for the 0th symbol of user 0 can be written as

$$\begin{aligned} z &= \int_0^{T_s} \langle \mathbf{x}(t), \mathbf{C}^0(t) \rangle dt \\ &= b_0[0] \cdot E_b + n + \sum_{k=1}^{K-1} A_k \cos(\theta_k - \theta_0) \cdot \sum_{m=0}^{M-1} \cos(w_m \tau_k) [b_k[-1] R_m^{k,0}(\tau_k) + b_k[0] \check{R}_m^{k,0}(\tau_k)], \end{aligned} \quad (9)$$

³From now on, we will drop off the scalar value of 2, which is caused by the carrier demodulation.

⁴i.e., independent identical distributed.

where $E_b = MT_s$, $n \sim \mathcal{N}(0, MT_s\sigma^2/2)$, and

$$\begin{aligned} R_m^{k,0}(\tau_k) &= \int_0^{\tau_k} \mathbf{c}_m^k(t - \tau_k) (\mathbf{c}_m^0(t))^* dt, \\ \check{R}_m^{k,0}(\tau_k) &= \int_{\tau_k}^{T_s} \mathbf{c}_m^k(t - \tau_k) (\mathbf{c}_m^0(t))^* dt, \end{aligned} \quad (10)$$

are the continuous aperiodic cross-correlations. It is noted that for

$$0 \leq m_k T_c \leq \tau_k = m_k T_c + \epsilon_k < (m_k + 1) T_c \leq T_s,$$

$R_m^{k,0}(\tau_k)$ and $\check{R}_m^{k,0}(\tau_k)$ can be expressed as the sum of the discrete-versions of the aperiodic cross-correlations, i.e.,

$$\begin{aligned} R_m^{k,0}(\tau_k) &= T_c \cdot R_{\mathbf{c}_m^k, \mathbf{c}_m^0}(m_k - N) + \epsilon_k \cdot [R_{\mathbf{c}_m^k, \mathbf{c}_m^0}(m_k + 1 - N) - R_{\mathbf{c}_m^k, \mathbf{c}_m^0}(m_k - N)], \\ \check{R}_m^{k,0}(\tau_k) &= T_c \cdot R_{\mathbf{c}_m^k, \mathbf{c}_m^0}(m_k) + \epsilon_k \cdot [R_{\mathbf{c}_m^k, \mathbf{c}_m^0}(m_k + 1) - R_{\mathbf{c}_m^k, \mathbf{c}_m^0}(m_k)], \end{aligned} \quad (11)$$

respectively.

To simplify the writing in (9), let

$$\begin{aligned} \mathbf{A} &= \text{diag}[A_1, A_1, A_2, A_2, \dots, A_{K-1}, A_{K-1}], \\ \Theta &= \text{diag}[\cos(\theta_1 - \theta_0), \cos(\theta_1 - \theta_0), \cos(\theta_2 - \theta_0), \\ &\quad \cos(\theta_2 - \theta_0), \dots, \cos(\theta_{K-1} - \theta_0), \cos(\theta_{K-1} - \theta_0)], \\ \mathbf{b} &= [b_1[-1], b_1[0], b_2[-1], b_2[0], \dots, b_{K-1}[-1], b_{K-1}[0]]^T, \\ \mathbf{c}_k &= [\cos(w_0 \tau_k), \cos(w_1 \tau_k), \dots, \cos(w_{M-1} \tau_k)]^T, \\ &\quad \text{for } 1 \leq k \leq K-1 \\ \mathbf{P} &= [\mathbf{c}_1^T \mathbf{R}^{1,0}, \mathbf{c}_2^T \mathbf{R}^{2,0}, \dots, \mathbf{c}_{K-1}^T \mathbf{R}^{K-1,0}], \end{aligned} \quad (12)$$

where $\mathbf{b}$ is a vector of all interferers' data patterns, and

$$\mathbf{R}^{k,0} = \begin{bmatrix} R_0^{k,0}(\tau_k) & R_1^{k,0}(\tau_k) & \dots & R_{M-1}^{k,0}(\tau_k) \\ \check{R}_0^{k,0}(\tau_k) & \check{R}_1^{k,0}(\tau_k) & \dots & \check{R}_{M-1}^{k,0}(\tau_k) \end{bmatrix}^T. \quad (13)$$

Writing $b = b_0[0]$, (9) can be rewritten as

$$z = b \cdot E_b + \underbrace{\mathbf{P} \mathbf{A} \Theta \mathbf{b}}_{\text{MAI}} + n. \quad (14)$$

See Fig. 2 for the multicarrier demodulation and the conventional correlator processing. After the calculation of z , the receiver performs the BPSK detection based on $\text{sgn}(\text{Re}\{z\})$, where $\text{sgn}(x)$ denotes the sign function which equals to -1 if x is negative, and 1 otherwise.

Denote by $\mathbf{P}_e^{\text{SUD}}$ the BER of the conventional correlator based receiver, conditioned on $\mathbf{P}$, $\mathbf{A}$ and Θ . By (14), we have

$$\mathbf{P}_e^{\text{SUD}} = Q \left(\sqrt{\frac{2\gamma_s}{1 + \frac{\mathbf{P} \mathbf{A}^2 \Theta^2 \mathbf{P}^T}{N_0}}} \right) \quad (15)$$

where $\gamma_s = \frac{E_b}{N_0}$, $N_0 = MT_s \delta^2$ and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt. \quad (16)$$

III. PROPOSED FRACTIONAL-DELAY-RESILIENT CHIP-SPACED CORRELATING ARRAY (CSCA) RECEIVER

A. “Fractional-Delay” Problem

Remark 1: Ideally, we wish that the MAI term in (14) is 0, i.e., $\mathbf{PA}\Theta\mathbf{b} = 0$. However, this can only be guaranteed when all inter-user delays are integer multiples of the half chip-duration.

To understand this, let us consider a simple 2-user example with $K = M = 2$, $\theta_1 = \theta_0$, and $w_0 = 0$. In this case, the MAI term can be simplified to

$$\begin{aligned} \text{MAI} &= A_1 \sum_{m=0}^1 \cos(w_m \tau_1) [b_1[-1] R_m^{1,0}(\tau_1) + b_1[0] \check{R}_m^{1,0}(\tau_1)] \\ &= b_1[-1] \cdot A_1 [R_0^{1,0}(\tau_1) + \cos(\Delta w \tau_1) R_1^{1,0}(\tau_1)] \\ &\quad + b_1[0] \cdot A_1 [\check{R}_0^{1,0}(\tau_1) + \cos(\Delta w \tau_1) \check{R}_1^{1,0}(\tau_1)], \end{aligned}$$

where $\Delta w = w_1 - w_0 = \frac{4\pi}{T_c}$.

By (11) and the fact that $\mathbf{C}^1$ and $\mathbf{C}^0$ have zero aperiodic cross-correlation sums for any time-shift, we assert that

$$[R_0^{1,0}(\tau_1) + \cos(\Delta w \tau_1) R_1^{1,0}(\tau_1)] = 0$$

can only be guaranteed if $\cos(\Delta w \tau_1) = 1$. This is equivalent to the condition that τ_1 should be an integer multiple of the half chip-duration, because only in this condition, we have

$$\Delta w \tau_1 = \frac{4\pi \tau_1}{T_c} = 2\pi \frac{\tau_1}{T_c/2} \equiv 0, \pmod{2\pi}, \quad (17)$$

leading to $\cos(\Delta w \tau_1) = 1$. The same can be said for

$$[\check{R}_0^{1,0}(\tau_1) + \cos(\Delta w \tau_1) \check{R}_1^{1,0}(\tau_1)] = 0.$$

If (17) is not met, the fractional-delay problem arises and the MAI will not be zero.

B. Proposed CSCA Receiver

It is shown in (9) that the received signal in conventional correlator based receiver consists of three parts, i.e., the desired signal term, the MAI term, and the noise term. To mitigate the MAI term in (9), our idea is to generate a set of linear equations, each of which represents a projection in the interference space. Intuitively, provided that there are sufficient independent interference projections, the detection performance may be improved by interference cancellation.

Note that a precondition for this idea to work is that the interference space should be separable from the desired signal space. This can be ensured by the zero aperiodic correlation sum property (auto- and cross- correlation) of CCC. A detailed description of the proposed chip-spaced correlating array (CSCA) receiver will be presented below.

First, let us generate a chip-spaced correlating signal set, denoted by $\mathcal{S}$, which is shown in (18).

Note that there are $KN - 1$ chip-spaced signals in $\mathcal{S}$. Also note that for each chip-spaced signal in $\mathcal{S}$, e.g., $\mathbf{D}^{uN+g}(t)$, its discrete version is $T^g \mathbf{C}^u$ and is orthogonal with $\mathbf{C}^0$, i.e.,

$$\langle T^g \mathbf{C}^u, \mathbf{C}^0 \rangle = 0, \quad (19)$$

$$\mathcal{S} = \left\{ \mathbf{D}^{uN+g}(t) = \begin{bmatrix} \mathbf{d}_0^{uN+g} \\ \mathbf{d}_1^{uN+g} \\ \vdots \\ \mathbf{d}_{M-1}^{uN+g} \end{bmatrix} = \mathbf{C}^u(t - gT_c) \left| \begin{array}{l} u = 0, 1 \leq g \leq N-1, \\ \text{or } 1 \leq u \leq K-1, 0 \leq g \leq N-1 \end{array} \right. \right\}. \quad (18)$$

$$\begin{aligned} z_g^u &= \int_0^{T_s} \langle \mathbf{x}(t), \mathbf{C}^u(t - gT_c) \rangle dt \\ &= \underbrace{\sum_{k=1}^{K-1} A_k \cos(\theta_k - \theta_0) \sum_{m=0}^{M-1} \cos(w_m \tau_k) \left[b_k[-1] \underline{R}_m^{k,u}(gT_c, \tau_k) + b_k[0] \check{\underline{R}}_m^{k,u}(gT_c, \tau_k) \right]}_{\text{MAI}} + n_g^u, \end{aligned} \quad (21)$$

due to the zero aperiodic sum properties of the CCC. Thus, it is easy to see that each chip-spaced signal (continuous version) in $\mathcal{S}$ is also orthogonal with the continuous version of $\mathbf{C}^0$, i.e.,

$$\int_0^{T_s} \langle \mathbf{C}^u(t - gT_c), \mathbf{C}^0(t) \rangle dt = 0. \quad (20)$$

Applying $\mathbf{C}^u(t - gT_c)$ as the de-spreading matrix, by (20), we have (21), where $n_g^u \sim \mathcal{N}(0, MT_s \sigma^2/2)$, and

$$\begin{aligned} \underline{R}_m^{k,u}(gT_c, \tau_k) &= \int_{-gT_c}^{\tau_k - gT_c} \mathbf{c}_m^k(t - \tau_k + gT_c) (\mathbf{c}_m^u(t))^* dt, \\ \check{\underline{R}}_m^{k,u}(gT_c, \tau_k) &= \int_{\tau_k - gT_c}^{T_s - gT_c} \mathbf{c}_m^k(t - \tau_k + gT_c) (\mathbf{c}_m^u(t))^* dt. \end{aligned} \quad (22)$$

Recalling the continuous aperiodic cross-correlations in (10), we have

$$\underline{R}_m^{k,u}(0, \tau_k) = R_m^{k,u}(\tau_k), \quad \check{\underline{R}}_m^{k,u}(0, \tau_k) = \check{R}_m^{k,u}(\tau_k). \quad (23)$$

Hence, $\underline{R}_m^{k,u}(gT_c, \tau_k)$ and $\check{\underline{R}}_m^{k,u}(gT_c, \tau_k)$ may be regarded as the generalized continuous aperiodic cross-correlations of $R_m^{k,u}(\tau_k)$ and $\check{R}_m^{k,u}(\tau_k)$, respectively. Similar to $\mathbf{P}$ and $\mathbf{R}^{k,0}$, let

$$\begin{aligned} \mathbf{P}_g^u &= [\mathbf{c}_1^T \mathbf{R}_g^{1,u}, \mathbf{c}_2^T \mathbf{R}_g^{2,u}, \dots, \mathbf{c}_{K-1}^T \mathbf{R}_g^{K-1,u}], \\ \mathbf{R}_g^{k,u} &= \begin{bmatrix} \underline{R}_0^{k,u}(gT_c, \tau_k), \underline{R}_1^{k,u}(gT_c, \tau_k), \dots, \underline{R}_{M-1}^{k,u}(gT_c, \tau_k) \\ \check{\underline{R}}_0^{k,u}(gT_c, \tau_k), \check{\underline{R}}_1^{k,u}(gT_c, \tau_k), \dots, \check{\underline{R}}_{M-1}^{k,u}(gT_c, \tau_k) \end{bmatrix}^T. \end{aligned} \quad (24)$$

Therefore, (21) can be rewritten in the following matrix form.

$$z_g^u = \underbrace{\mathbf{P}_g^u \mathbf{A} \Theta \mathbf{b}}_{\text{MAI}} + n_g^u. \quad (25)$$

Let

$$\begin{aligned} \mathbf{z}_{(KN-1) \times 1} &= [\mathbf{z}^0, \mathbf{z}^1, \dots, \mathbf{z}^{K-1}]^T, \\ \mathbf{Q}_{(KN-1) \times (2K-2)} &= [\mathbf{Q}^0, \mathbf{Q}^1, \dots, \mathbf{Q}^{K-1}]^T, \\ \mathbf{n}_{(2K-2) \times 1} &= [\mathbf{n}^0, \mathbf{n}^1, \dots, \mathbf{n}^{K-1}]^T, \end{aligned} \quad (26)$$

where

$$\begin{aligned}
\mathbf{z}^0 &= [z_1^0, z_2^0, \dots, z_{N-1}^0], \\
\mathbf{Q}^0 &= [\mathbf{P}_1^0, \mathbf{P}_2^0, \dots, \mathbf{P}_{N-1}^0], \\
\mathbf{n}^0 &= [n_1^0, n_2^0, \dots, n_{N-1}^0], \\
\mathbf{z}^k &= [z_0^k, z_1^k, \dots, z_{N-1}^k], \quad 1 \leq k \leq K-1, \\
\mathbf{Q}^k &= [\mathbf{P}_0^k, \mathbf{P}_1^0, \dots, \mathbf{P}_{N-1}^k], \quad 1 \leq k \leq K-1, \\
\mathbf{n}^k &= [n_0^k, n_1^k, \dots, n_{N-1}^k], \quad 1 \leq k \leq K-1.
\end{aligned} \tag{27}$$

By (14), (25), (26) and (27), we have

$$\mathbf{y} = \mathbf{H}\mathcal{I} + \mathbf{w}, \tag{28}$$

where

$$\mathbf{y} = \begin{bmatrix} z \\ \mathbf{z} \end{bmatrix}, \quad \mathcal{I} = \begin{bmatrix} b \\ \mathbf{b} \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} n \\ \mathbf{n} \end{bmatrix} \tag{29}$$

and

$$\mathbf{H} = \begin{bmatrix} E_b & \mathbf{P}\mathbf{A}\Theta \\ \mathbf{0} & \mathbf{Q}\mathbf{A}\bar{\Theta} \end{bmatrix}_{KN \times (2K-1)} \tag{30}$$

with $\mathbf{0}$ a column vector of $KN - 1$ zeros. Note that the noise term $\mathbf{w}$ in (28) is a KN -dimensional i.i.d. additive Gaussian noise vector, each with variance $MT_s\sigma^2/2$. Also, except for the first top linear equation in (28), the remaining equations consist of the MAI and noise terms only, i.e.,

$$\mathbf{z} = \mathbf{Q}\mathbf{A}\Theta\mathbf{b} + \mathbf{n}. \tag{31}$$

Thus, (31) may be interpreted as a set of projections in interference space, thanks to the intrinsic ideal correlation properties of CCC.

Next, we consider the following detection methods for (28).

1) Maximum-Likelihood (ML) Joint detector: the receiver finds $\hat{\mathcal{I}}$ which satisfies

$$\hat{\mathcal{I}} = \begin{bmatrix} \hat{b} \\ \hat{\mathbf{b}} \end{bmatrix} = \arg \min_{\mathcal{I}^* \in \{1, -1\}^{2K-1}} \|\mathbf{y} - \mathbf{H}\mathcal{I}^*\|, \tag{32}$$

where $\hat{b}$ and $\hat{\mathbf{b}}$ are the desired user's data and the interferers' data, respectively.

Let $\epsilon = b - \hat{b}$ and $\mathbf{e} = \mathbf{b} - \hat{\mathbf{b}}$. The maximum likelihood (ML) detection in (32) is equivalent to finding $\hat{\mathcal{I}}$ such that

$$\|E_b\epsilon + \mathbf{P}\mathbf{A}\Theta\mathbf{e} + n\|^2 + \|\mathbf{Q}\mathbf{A}\Theta\mathbf{e} + \mathbf{n}\|^2 \tag{33}$$

is minimized. Clearly, when a correct detection is made, i.e., when $\epsilon = 0$, $\hat{\mathbf{b}}$ minimizes

$$\|\mathbf{P}\mathbf{A}\Theta\mathbf{e} + n\|^2 + \|\mathbf{Q}\mathbf{A}\Theta\mathbf{e} + \mathbf{n}\|^2. \tag{34}$$

2) ML Interference cancelation (IC) detector: the receiver first finds $\hat{\mathbf{b}}$ which satisfies

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}^* \in \{1, -1\}^{2K-2}} \|\mathbf{z} - \mathbf{Q}\mathbf{A}\Theta\mathbf{b}^*\|. \tag{35}$$

Whenever $\hat{\mathbf{b}}$ is obtained, the receiver carries out IC by subtracting the estimated MAI (i.e., $\mathbf{P}\mathbf{A}\Theta\hat{\mathbf{b}}$) from z , the output of the conventional correlator. After this, the receiver finds $\hat{b}$ which minimizes

$$\|E_b\epsilon + \mathbf{P}\mathbf{A}\Theta\mathbf{e} + n\|^2, \tag{36}$$

subject to (35). Clearly, when a correct detection is made, i.e., when $\epsilon = 0$, $\hat{\mathbf{b}}$ minimizes

$$\|\mathbf{Q}\mathbf{A}\Theta\mathbf{e} + \mathbf{n}\|^2. \tag{37}$$

Fig. 3. The proposed CSCA receiver structure using ML IC detector, where $\mathbf{D}^i(t)$ (for $i \in \{1, 2, \dots, KN-1\}$), defined in (18), represents the de-spreading waveform for the i -th chip-spaced correlator.

See Fig. 3 for the proposed CSCA receiver using ML IC detector, where $\mathbf{D}^i(t)$ [cf. (18)] represents the despreading waveform for the i -th chip-spaced correlator.

We note that the BER of the ML joint detector, denoted by P_e^J , is upper bounded by the BER of the ML IC detector, denoted by P_e^{IC} , i.e.,

$$P_e^J \leq P_e^{IC}. \quad (38)$$

This is ensured by the fact that the constraint for $\hat{\mathbf{b}}$ in (34) is more stringent than that in (37). Since the derivation of P_e^J is intractable⁵, we instead work on that of P_e^{IC} . This will shed some light on how the “single-user BER” is achieved by the proposed CSCA receiver.

Specifically, we have

$$\begin{aligned} P_e^{IC} &= \Pr[b = 1] \cdot \Pr[z - \mathbf{P} \mathbf{A} \Theta \hat{\mathbf{b}} < 0 | b = 1] \\ &\quad + \Pr[b = -1] \cdot \Pr[z - \mathbf{P} \mathbf{A} \Theta \hat{\mathbf{b}} > 0 | b = -1] \\ &= \sum_{\mathbf{b}, \hat{\mathbf{b}}} \Pr[z - \mathbf{P} \mathbf{A} \Theta \hat{\mathbf{b}} < 0 | b = 1, \hat{\mathbf{b}}] \cdot \Pr[\hat{\mathbf{b}} | \mathbf{b}] \cdot \Pr[\mathbf{b}] \\ &= 2^{2-2K} \cdot \sum_{\mathbf{b}, \hat{\mathbf{b}}} \Pr[z - \mathbf{P} \mathbf{A} \Theta \hat{\mathbf{b}} < 0 | b = 1, \hat{\mathbf{b}}] \cdot \Pr[\hat{\mathbf{b}} | \mathbf{b}], \end{aligned} \quad (39)$$

⁵Though a loose union bound may be derived, we seek a tighter upper bound.

where

$$\begin{aligned}
& \Pr \left[z - \mathbf{PA}\Theta\hat{\mathbf{b}} < 0 | b = 1, \hat{\mathbf{b}} \right] \\
&= \Pr \left[n + E_b + \mathbf{PA}\Theta(\mathbf{b} - \hat{\mathbf{b}}) < 0 \right] \\
&= \Pr \left[n < -E_b - \mathbf{PA}\Theta(\mathbf{b} - \hat{\mathbf{b}}) \right] \\
&= Q \left(\sqrt{2\gamma_s + \frac{\mathbf{PA}\Theta(\mathbf{b} - \hat{\mathbf{b}})}{N_0/2}} \right).
\end{aligned} \tag{40}$$

By (40), P_e^{IC} in (39) can be expressed as

$$\begin{aligned}
P_e^{\text{IC}} &= Q \left(\sqrt{2\gamma_s} \right) \Pr \left[\hat{\mathbf{b}} = \mathbf{b} | \mathbf{b} \right] \\
&+ 2^{2-2K} \cdot \sum_{\mathbf{b} \neq \hat{\mathbf{b}}} Q \left(\sqrt{2\gamma_s + \frac{\mathbf{PA}\Theta(\mathbf{b} - \hat{\mathbf{b}})}{N_0/2}} \right) \cdot \Pr \left[\hat{\mathbf{b}} \neq \mathbf{b} | \mathbf{b} \right].
\end{aligned} \tag{41}$$

Note that the “single-user BER” performance is achieved if and only if $P_e^{\text{IC}} = Q \left(\sqrt{2\gamma_s} \right)$. Ideally, one wishes that $\Pr[\hat{\mathbf{b}} = \mathbf{b} | \mathbf{b}] = 1$, and in this case, surely $\Pr[\hat{\mathbf{b}} \neq \mathbf{b} | \mathbf{b}] = 0$ (which implies that the second term in (41) is 0). To this end, we shall next derive $\Pr[\hat{\mathbf{b}} = \mathbf{b} | \mathbf{b}]$.

First, consider the complete set of the interferers’ vectors, i.e.,

$$\{\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_{2^{2K-2}-1}\}.$$

Without loss of generality, let the transmitted interferers’ vector $\mathbf{b} = \mathbf{b}_0$. Also, let

$$\mathbf{e}_k = \mathbf{b}_0 - \mathbf{b}_k, \quad \text{for } 1 \leq k \leq 2^{2K-2} - 1.$$

Obviously, each $\|\mathbf{e}_k\|^2 > 0$. $\Pr[\hat{\mathbf{b}} = \mathbf{b} | \mathbf{b}]$ denotes the probability of the following event:

$$\begin{aligned}
& \text{Event A} \\
& \triangleq \text{Event} \left\{ \|\mathbf{z} - \mathcal{M}\mathbf{b}_0\| < \|\mathbf{z} - \mathcal{M}\mathbf{b}_k\|, \quad \text{for all } k \neq 0 \right\},
\end{aligned} \tag{42}$$

where $\mathcal{M} = \mathbf{QA}\Theta$. Substituting $\mathbf{z} = \mathcal{M}\mathbf{b}_0 + \mathbf{n}$ into (42), it is easy to show that Event A is equivalent to

$$\begin{aligned}
& \text{Event B} \\
& \triangleq \text{Event} \left\{ (u_1 > v_1, u_2 > v_2, \dots, u_{2^{2K-2}-1} > v_{2^{2K-2}-1}) \right\},
\end{aligned} \tag{43}$$

where

$$u_k = \mathbf{e}_k^T \mathcal{M}^T \mathbf{n}, \quad \text{and} \quad v_k = -\frac{\|\mathcal{M}\mathbf{e}_k\|^2}{2}. \tag{44}$$

Let

$$\begin{aligned}
\mathbf{u} &= (u_1, u_2, \dots, u_{2^{2K-2}-1})^T, \\
\Xi &= [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_{2^{2K-2}-1}], \\
\Omega &= \frac{\Xi^T \mathcal{M}^T \mathcal{M} \Xi N_0}{2}.
\end{aligned} \tag{45}$$

One can show that the probability density function (pdf) of $\mathbf{u}$, denoted by $f(\mathbf{u})$, is a multivariate Gaussian distribution with zero mean vector and covariance matrix Ω , i.e.,

$$f(\mathbf{u}) = \frac{1}{(2\pi)^{\frac{2^{2K-2}-1}{2}} \det[\Omega]} \cdot \exp \left(-\frac{1}{2} \mathbf{u}^T \Omega^{-1} \mathbf{u} \right), \tag{46}$$

where $\det[\Omega]$ denotes the determinant of the matrix Ω . By (43) and (46), we obtain

$$\begin{aligned} & \Pr[\hat{\mathbf{b}} = \mathbf{b} | \mathbf{b}] \\ &= \int_{v_1}^{+\infty} \int_{v_2}^{+\infty} \cdots \int_{v_{2^{2K-2}-1}}^{+\infty} f(\mathbf{u}) du_1 du_2 \cdots du_{2^{2K-2}-1}, \end{aligned} \quad (47)$$

where

$$v_k = -\frac{\|\mathcal{M}\mathbf{e}_k\|^2}{2} = -\frac{\mathbf{e}_k^T \mathcal{M}^T \mathcal{M} \mathbf{e}_k}{2} = -\frac{\mathbf{e}_k^T \Theta \mathbf{A} \mathbf{Q}^T \mathbf{Q} \mathbf{A} \Theta \mathbf{e}_k}{2}. \quad (48)$$

For matrix $\mathbf{Q}$ with size $(KN - 1) \times (2K - 2)$, $\mathbf{Q}^T \mathbf{Q}$ in (48) approaches a matrix which has strong diagonal elements and weak off-diagonal ones when N becomes sufficiently large, because the coupling between the column vectors of $\mathbf{Q}$, when viewed as a random matrix, weakens for larger N .

By enumerating all possible $\mathbf{e}_k$, we assert that $\Pr[\hat{\mathbf{b}} = \mathbf{b} | \mathbf{b}]$ approaches to 1 if and only if all v_k ($1 \leq k \leq 2^{2K-2} - 1$) tend to negative infinity. Thus, by (38) and (41), we have the following remark.

Remark 2: The proposed CSCA receiver using ML joint detector or ML IC detector tends to achieve single-user BER performance if $\Theta \mathbf{A} \mathbf{Q}^T \mathbf{Q} \mathbf{A} \Theta$ tends to a diagonal matrix with dominant diagonal elements.

For low-complexity receiver design, we consider linear minimum-mean-squared-error (LMMSE) detection followed by interference cancelation, called LMMSE IC detector (as opposed to ML IC detector). Specifically, the receiver finds the interference data pattern $\hat{\mathbf{b}}$ based on $\mathcal{L}\mathbf{z}$, which is a linear transform of the interference vector $\mathbf{z}$, such that $E\{(\mathbf{b} - \hat{\mathbf{b}})^T(\mathbf{b} - \hat{\mathbf{b}})\}$ is minimized. By [20], it can be shown that

$$\mathcal{L}_{\text{LMMSE IC}} = \mathcal{M}^T \left(\mathcal{M} \mathcal{M}^T + \frac{N_0 \mathbf{I}_{KN-1}}{2} \right)^{-1}, \quad (49)$$

where $\mathcal{M} = \mathbf{Q} \mathbf{A} \Theta$, $N_0 = MT_s \sigma^2$ and $\mathbf{I}_{KN-1}$ is an identity matrix of order $KN - 1$. Then, the receiver carries out IC by subtracting away the estimated MAI (i.e., $\mathbf{P} \mathbf{A} \Theta \hat{\mathbf{b}}$) from the conventional correlator output z .

It is also possible to consider LMMSE joint detector (as opposed to ML joint detector) by minimizing $E\{(\mathcal{I} - \hat{\mathcal{I}})^T(\mathcal{I} - \hat{\mathcal{I}})\}$ based on matrix $\mathbf{H}$ in (28). The corresponding linear transform is

$$\mathcal{L}_{\text{LMMSE joint}} = \mathbf{H}^T \left(\mathbf{H} \mathbf{H}^T + \frac{N_0 \mathbf{I}_{KN}}{2} \right)^{-1}. \quad (50)$$

However, as will be shown in Section IV, the LMMSE joint detector performs worse (than the LMMSE IC detector) because it aims to minimize the MMSE of the vector $\mathcal{I}$ that consists of the desired user data b and the interference data pattern $\mathbf{b}$. In such a detector, the desired user data is in the same priority level as that of an interference data. This violates the fact that the desired user data has the highest priority.

There are some implementation considerations for the proposed CSCA receiver.

- 1) Channel estimation in the receiver for the signal amplitudes (A_k), initial carrier phases (θ_k), and inter-user delays (τ_k) is necessary. This can be accomplished by inserting pilots into the transmitted signals.
- 2) For the proposed CSCA receiver in Fig. 3, there are $KN - 1$ chip-spaced correlators in total, where all correlating signals from $\mathcal{S}$ [defined in (18)] are used. Hence, the detector complexity is linear with the number of CDMA users, denoted by K . This is in contrast to the huge complexity of traditional multi-user detectors, which is linear with K times the message length [21]–[23]. Complexity reduction of our proposed CSCA receiver is possible by using G chip-spaced correlators, where $G < KN - 1$, to form a matrix $\mathbf{Q}_G$ of order $G \times (2K - 2)$. Although the single-user BER may not be achieved after this complexity reduction, it is still of interest for some communications

with lower QoS requirements. For instance, a $\text{BER} \approx 10^{-3}$ is sufficient for voice communication. Therefore, (28) can be modified to the following form.

$$\begin{bmatrix} z \\ \mathbf{z}_G \end{bmatrix}_{(G+1) \times 1} = \begin{bmatrix} E_b & \mathbf{PA}\Theta \\ \mathbf{0}_G & \mathbf{Q}_G \mathbf{A} \bar{\Theta} \end{bmatrix}_{(G+1) \times (2K-1)} \cdot \begin{bmatrix} b \\ \mathbf{b} \end{bmatrix}_{(2K-1) \times 1} + \begin{bmatrix} n \\ \mathbf{n}_G \end{bmatrix}_{(G+1) \times 1}, \quad (51)$$

where $\mathbf{0}_G$ is a column vector of G zeros. For a fixed G , the detection performance may vary depending on how these G chip-spaced correlators are chosen. In general, a matrix $\mathbf{Q}_G$ where $\mathbf{Q}_G^T \mathbf{Q}_G$ is full rank (see *Remark 2*) and with dominant diagonal elements is preferred.

- 3) Suppose that the l th element in the vector $\mathbf{PA}\Theta$ has a very small magnitude. This means that the l th binary data in the interference vector $\mathbf{b}$ will make a small contribution to the MAI term (i.e., $\mathbf{PA}\Theta \mathbf{b}$) and thus can be neglected. As a result, the corresponding l th column of the matrix $\mathbf{Q}_G \mathbf{A} \bar{\Theta}$ and the l th data in $\mathbf{b}$ may be removed from (51). Therefore, by deleting the $(l+1)$ -th column vector of the following matrix in (51),

$$\begin{bmatrix} E_b & \mathbf{PA}\Theta \\ \mathbf{0} & \mathbf{Q}_G \mathbf{A} \bar{\Theta} \end{bmatrix}_{(G+1) \times (2K-1)}, \quad (52)$$

one obtains another matrix with size $(G+1) \times (2K-2)$. Such a column-reduction can be repeated if there are more negligible elements in the vector $\mathbf{PA}\Theta$, leaving only the effective columns in the matrix shown in (52). This may save more computation during the detection.

IV. NUMERICAL RESULTS

In this section, we present numerical results for the BERs of the proposed CSCA receiver in “fractional-delay” transmission scenarios. The goal of this section is three-fold:

- 1) To show that “single-user BER” performance cannot be achieved by the conventional correlator based receiver in CCC-MC-CDMA systems;
- 2) To show that “single-user BER” performance can be achieved by the proposed CSCA receiver in CCC-MC-CDMA systems with strong interference;
- 3) To show that “single-user BER” performance cannot be attained by the proposed CSCA receiver if complementary matrices are not used.

With the above, we prove that it is indeed the unique correlation properties of CCC together with the proposed CSCA receiver structure which makes a difference. In the simulation, random matrices generated “on the fly” are used to represent non-complementary matrices. Note that the proposed ML IC detector and LMMSE IC detector cannot be applied to MC-CDMA systems with random matrices because the zero vector (which is a vector of $KN - 1$ zeros) in $\mathbf{H}$ of (30) does not exist for this case, which prevents the interference space from separating from the desired signal space.

We simulate a 2-user and a 4-user MC-CDMA systems for BER comparisons from different perspectives. To simulate the worst interference case in “fractional-delay” transmission⁶, in either MC-CDMA system, the carrier phase matrix Θ is set to be an identity matrix. Also, identical signal-to-noise ratio γ_s of 6dB is set for both systems. The other simulation parameters for both systems are shown below.

- 1) For the 2-user MC-CDMA, $\tau_1 = 1.75T_c$, $K = M = 2$ and $N = 8$. The two complementary matrices are

$$\mathbf{C}^0 = \begin{bmatrix} + & + & + & - & + & - & + & + \\ + & + & - & + & + & - & - & - \end{bmatrix},$$

$$\mathbf{C}^1 = \begin{bmatrix} - & - & - & + & + & - & + & + \\ - & - & + & - & + & - & - & - \end{bmatrix}.$$

⁶such that $\mathbf{P}_e^{\text{SUD}}$ is the worst.

- 2) For the 4-user MC-CDMA, $(\tau_1, \tau_2, \tau_3) = (2.25, 3.25, 6.75)T_c$, $K = M = 4$ and $N = 8$. The four complementary matrices are

$$\begin{aligned} \mathbf{C}^0 &= \begin{bmatrix} + & + & + & - & + & - & + & + \\ + & + & - & + & + & - & - & - \\ - & - & - & + & + & - & + & + \\ - & - & + & - & + & - & - & - \end{bmatrix}, \\ \mathbf{C}^1 &= \begin{bmatrix} + & + & + & - & + & - & + & + \\ + & + & - & + & + & - & - & - \\ + & + & + & - & - & + & - & - \\ + & + & - & + & - & + & + & + \end{bmatrix}, \\ \mathbf{C}^2 &= \begin{bmatrix} - & - & - & + & + & - & + & + \\ - & - & + & - & + & - & - & - \\ + & + & + & - & + & - & + & + \\ + & + & - & + & + & - & - & - \end{bmatrix}, \\ \mathbf{C}^3 &= \begin{bmatrix} - & - & - & + & + & - & + & + \\ - & - & + & - & + & - & - & - \\ - & - & - & + & - & + & - & - \\ - & - & + & - & - & + & + & + \end{bmatrix}. \end{aligned}$$

Fig. 4 shows the BER (for the desired user) versus the interference-to-signal amplitude ratio for 2-user MC-CDMA systems with different detectors. It is shown that the BER curves of conventional correlators (marked by circles and stars for CCC and random matrices, respectively) grow unacceptably high for increasing A_1/A_0 , indicating that conventional correlator fails in “fractional-delay” transmission scenarios. For BER curves associated to CCC, we have the following observations: 1, the BER curves of the ML joint detector and ML IC detector approach to the single-user BER when MAI is strong (i.e., when A_1/A_0 reaches about 10dB or higher); 2, the ML joint detector BER curve is always upper bounded by the ML IC detector BER, thus (38) is validated; 3, When A_1/A_0 is small (say, less than -10dB), the MAI is negligible and thus the BER curves for all three detectors (ML joint detector, ML IC detector, and conventional correlator) are close to the single-user BER. Interestingly, in this region, the BER curve of conventional correlator matches very well with that of ML joint detector, but is slightly better than that of ML IC detector. The latter is because detection of the interference data pattern using the ML IC detector is more error prone over this weak interference region; 4, As expected, in an A_1/A_0 region around 0dB, the BER curves of ML joint- and ML IC- detectors display the largest distances away from the single-user BER. For the 2-user MC-CDMA system using random matrices, since the interference space is inseparable from the desired signal space, only BER of ML joint detector (marked by triangle symbols) is shown. One can see that this BER curve is almost flat (but away from the single-user BER) with small BER increase for A_1/A_0 around 0dB. Moreover, when A_1/A_0 exceeds 10dB, the BER curve for the random matrices still displays a gap from the single-user BER, showing that the proposed CSCA receiver does not owe its effectiveness to random matrices.

Fig. 5 shows BER comparisons for the 4-user case using ML- and LMMSE detectors. For presentation convenience, all interferers’ amplitudes are set to be identical, i.e., $A_1 = A_2 = A_3$. One can see that the BER curve of LMMSE IC detector almost matches with that of ML IC detector, showing that the low-complexity LMMSE IC detector also features almost optimum BER performance. In comparison, the BER of LMMSE joint detector tends to a saturated flat curve which is far away from that of LMMSE ML detector. This is consistent with our analysis for LMMSE joint detector in Section III. On the other hand, the BER behavior of ML joint detector (or ML IC detector) in Fig. 5 is similar to that in Fig. 4. Note that in the A_1/A_0 region around 0dB, the BER curve of ML joint detector (or ML IC detector) in Fig. 5 displays even larger distance from the single-user BER than that in Fig. 4. This is because the

Fig. 4. BER versus the interference-to-signal amplitude ratio (A_1/A_0) for a 2-user MC-CDMA system using CCC and random matrices, where $\gamma_s = 6\text{dB}$ and $\tau_1 = 1.75T_c$.

Fig. 5. BER versus the interference-to-signal amplitude ratio ($A_1/A_0 = A_2/A_0 = A_3/A_0$) for a 4-user MC-CDMA system using CCC, where $\gamma_s = 6\text{dB}$ and $(\tau_1, \tau_2, \tau_3) = (2.25, 3.25, 6.75)T_c$.

interference vector length for the 4-user case is almost twice of the 2-user case. Thus, in this sensitive region, a decision mistake may lead to a higher amount of MAI residue.

In either Fig. 4 or Fig. 5, a total of $KN - 1$ chip-spaced correlators are employed for each joint- or IC-detector. However, this is not necessary. As mentioned before, $G < KN - 1$ chip-spaced correlators may be used for complexity reduction. The BER versus γ_s for ML joint detector for $G \in \{3, 6, 9, 12, 15\}$ in the 2-user CCC-MC-CDMA system are shown in Fig. 6. For fixed γ_s , as G grows, the BER of ML joint detector decreases and converges to that of $G = KN - 1$. When $A_1/A_0 = 1$ (i.e., $A_1/A_0 = 0\text{dB}$), as shown in the left of Fig. 6, the ML joint detector with $G = 9$ chip-spaced correlators is sufficient to achieve the BER of the receiver with $KN - 1 = 15$ correlators. On the other hand, when $A_1/A_0 = 3$ (i.e., $A_1/A_0 \approx 9.5\text{dB}$), as shown in the right of Fig. 6, the ML joint detector with $G = 6$ chip-spaced correlators is sufficient to achieve the single-user BER. In both cases ($A_1/A_0 = 1$ and $A_1/A_0 = 3$), the BER obtained by conventional correlator is always worse than that by the ML joint detector with $G \in \{3, 6, 9, 12, 15\}$ correlators. Finally, the BER of conventional correlator in the right of Fig. 6 displays very high error floor due to the strong interference for $A_1/A_0 = 3$, further showing the failure of conventional correlator.

Fig. 6. ML joint detector BER versus γ_s for a 2-user CCC-MC-CDMA system using G correlators, where $\tau_1 = 1.75T_c$ and $G_r = \frac{G}{KN-1}$.

V. CONCLUSIONS

For asynchronous MC-CDMA communications based on complete complementary codes (CCC), a traditional understanding is that a conventional correlator based receiver is sufficient to achieve the “interference-free” performance. However, we have shown that this understanding is not applicable in practical scenarios where fractional-valued inter-user delays (with respect to the half chip-duration) commonly exist, which resulting in the orthogonality loss of CCC and therefore interference increase.

Motivated by this practical problem, we have developed a “fractional-delay-resilient” receiver which features interference-free achievability in asynchronous MC-CDMA channels with strong interference. The proposed receiver, called the chip-spaced correlating array (CSCA) receiver, consists of a correlator array that uses all (or some) cyclically-shifted versions of complementary codes as the despreading sequences. The key finding behind the proposed CSCA receiver is that the interference space is separable from the desired signal space which enables interference cancelation, thanks to the unique correlation properties of CCC. For low-complexity receiver design, we have designed linear-MMSE interference cancelation (IC) detector which is capable of achieving almost optimum BER performance, as shown in Fig. 5. The significance of our proposed CSCA receiver is that it retains interference-free performance in practical “fractional-delay” asynchronous CDMA channels, bringing CCC-MC-CDMA a step towards practical usage.

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