Numerical studies on 2-D compressional wave propagation in jointed rock masses

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NUMERICAL STUDIES ON 2-D COMPRESSSIONAL WAVE PROPAGATION IN JOINTED ROCK MASSES

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**SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
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Numerical Studies on 2-D Compressional Wave Propagation in Jointed Rock Masses

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SUMMARY

An accidental explosion in an existing tunnel or an explosion in a borehole will generate compressional two-dimensional (2-D) wave that travels through the surrounding rock mass. Stability of the adjacent underground structures is affected by the peak particle velocity or peak particle acceleration induced by the propagated wave. Therefore, the problem of 2-D wave propagation in fractured rock masses due to explosion applied at the boundary of a tunnel or borehole is of practical importance. The current thesis aims to numerically study this problem.

The Universal Distinct Element Code (UDEC) is used in the studies performed. Currently, UDEC is the most commonly used discrete element program in engineering problems related to jointed rock masses. Although UDEC is extensively verified and used in static problems in jointed rock masses, its reliability in dynamic problems has not been thoroughly investigated. Therefore, firstly UDEC is validated in modeling wave propagation problems in both intact rocks and jointed rock masses, and then a study on the factors influencing the accuracy of UDEC results is performed. Based on the study performed, the relationship between the numerical error and mesh ratio (the ratio of mesh size to wavelength of the input wave) is established.

A new method is developed to process UDEC results in terms of amplitude of velocity/stress wave in wave propagation problems in intact rock and jointed rock mass. The new data processing method does not only reduce the time of processing UDEC results, but also leads to more accurate results at any arbitrary points in the model.

As a very important reference for the study on 2-D compressional wave propagation in jointed rock masses, a study on 2-D compressional wave propagation in intact rock is also performed. Based on the results obtained, several charts have been prepared for easy determination of the wave amplitude at any point in the surrounding intact rock.

Compressional wave propagation from a tunnel or borehole in jointed rock masses is a multiple factor-influenced problem. Therefore, a study on characteristics of the transmission ratio under various influence factors is performed, and based on the analysis, research strategies are proposed for this multiple factor-influenced problem. After the research strategies are developed, extensive parametric studies under every influence factor are performed. Based on the results obtained from the parametric studies, prediction models for estimating the transmission ratio are proposed for 2-D compressional wave originating from a tunnel or borehole and traveling through a single joint or multiple parallel joints. Along the special radian direction normal to the joints, prediction models for estimating the maximum wave rebound ratio are also proposed. Using these proposed prediction models and the charts prepared for determination of wave amplitude at any point in intact rock, the amplitude of the wave at the same point in the fractured rock case can be estimated.

The proposed prediction models are applied to estimate the values of peak particle velocity at different points in a field explosion test conducted at Mandai, Singapore. The estimated values of peak particle velocity agree very well with the field-recorded values.

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LIST OF NOTATIONS

a_1	Incident S-wave or P-wave in 1-D wave problem
a_2	Transmitted S-wave or P-wave in 1-D wave problem
A	Amplitude of a wave
$A_{i(\text{analytical})}$	Amplitude of a wave for a point from analytical solutions
A_{intact}	Amplitude of a wave in an intact rock
A_{joint}	Amplitude of a wave in a jointed rock mass
$A_{i(\text{UDEC})}$	Amplitude of a wave for a point from UDEC results
c	Velocity of wave propagation
c_d	Damping matrices
c_p	P-wave velocity
c_s	S-wave velocity
E	Young's modulus
E_e	Young's modulus of the equivalent continuous rock
E_r	Young's modulus of an intact rock
F	Resultant force
g	Gravitational acceleration
G	Shear modulus of a rock
G_e	Shear modulus of the equivalent continuous rock
G_r	Shear modulus of an intact rock
j_{kn}	Normal stiffness of joint
j_{ks}	Shear stiffness of joint
k	Stiffness in general
k^2	Deduced parameter used in the analytical solution for harmonic wave from a cylindrical cavity in an elastic medium
K	Bulk modulus of a rock
K_{eq}	Equivalent stiffness of a zone
K_m	Stiffness matrix
l_r	Mesh ratio
m	Mass

M_m	Mass matrix
n	Number of joints
r	Radius distance
R	Reflection ratio
$ R $	Magnitude of reflection coefficient in 1-D wave problem
R_0	Radius of a cavity
$ R_{a_1 - a_2 n} $	Magnitude of reflection coefficient in 1-D wave problem in general
$Re b$	Rebound ratio
$ R_{p-pl} $	Magnitude of reflected P-wave coefficient for P-wave incidence across single joint in 1-D wave problem
$ R_{p-sl} $	Magnitude of reflected S-wave coefficient for P-wave incidence across single joint in 1-D wave problem
$ R_{s-pl} $	Magnitude of reflected P-wave coefficient for S-wave incidence across single joint in 1-D wave problem
$ R_{s-sl} $	Magnitude of reflected S-wave coefficient for S-wave incidence across single joint in 1-D wave problem
s	Spacing of joints
s	Surface enclosing a given mass lumped at a grid-point (Chapter 2)
S_{flat}	Flat value related to joint spacing in the general curve of transmission ratio versus spacing/wavelength
S_{peak}	Peak value related to joint spacing in the general curve of transmission ratio versus spacing/wavelength
t	Time
t_n	Viscous shear traction
t_s	Viscous normal traction
T	Transmission ratio
$ T $	Magnitude of transmission coefficient in 1-D wave problem
$ T_{a_1 - a_2 n} $	Magnitude of transmission coefficient in 1-D wave problem in general
T_{flat}	Flat value related to transmission ratio in the general curve of transmission ratio versus spacing/wavelength
$ T_{p-pl} $	Magnitude of transmitted P-wave coefficient for P-wave incidence across single joint in 1-D wave problem
$ T_{p-pn} $	Magnitude of transmitted P-wave coefficient for P-wave incidence across multiple joints in 1-D wave problem
$ T_{p-sl} $	Magnitude of transmitted S-wave coefficient for P-wave incidence across single joint in 1-D wave problem

$ T_{p-sn} $	Magnitude of transmitted S-wave coefficient for P-wave incidence across multiple joints in 1-D wave problem
T_{peak}	Peak value related to transmission ratio in the general curve of transmission ratio versus spacing/wavelength
$ T_{s-p1} $	Magnitude of transmitted P-wave coefficient for S-wave incidence across single joint in 1-D wave problem
$ T_{s-pn} $	Magnitude of transmitted P-wave coefficient for S-wave incidence across multiple joints in 1-D wave problem
$ T_{s-s1} $	Magnitude of transmitted S-wave coefficient for S-wave incidence across single joint in 1-D wave problem
$ T_{s-sn} $	Magnitude of transmitted S-wave coefficient for S-wave incidence across multiple joints in 1-D wave problem
u	Displacement
u_{cs}	Critical shear displacement
u_n	Normal displacement
u_s	Shear displacement
u_x	Displacement in x direction
u_y	Displacement in y direction
v	Velocity
v_0	Initial velocity
v_n	Normal component of velocity
v_s	Shear component of velocity
VX	Value of velocity wave in x direction
VY	Value of velocity wave in y direction
z	Seismic impedance
α	Radian angle
α_m	Mass-proportional damping constant
β^2	Deduced parameter used in the analytical solution for harmonic wave from a cylindrical cavity in an elastic medium
β_s	Stiffness-proportional damping constant
ψ	Dilation angle
ϕ	Reflection or transmission angle for S-wave
ϕ_f	Friction angle
θ	Reflection or transmission angle for P-wave
σ	Normal stress

τ	Shear stress
ε	Strain
ξ	Ratio of spacing of joints to wavelength
ξ_{cri}	Critical value of nondimensional fracture spacing
ξ_i	Critical damping ratio
ξ_{thr}	Threshold value of nondimensional fracture spacing
λ, μ	Lamé constants
Φ	Potential function
Δ	Difference
ω	Angular frequency
ω_i	Angular frequency of the system
ρ	Density
∇^2	Laplacian operator
Δz_{min}	Smallest width of an adjoining zone

CHPATER 1

INTRODUCTION

1.1 Background

Rock masses naturally contain geological discontinuities, ranging in size from micro-cracks to joints and faults. The discontinuities (such as rock joints) often occur as nearly parallel groups, characterized by orientation, joint spacing, opening and stiffness (Brown 1981; Zhu and Zhao 2004). Due to rock discontinuities, the engineering behavior of a rock mass is inhomogeneous, anisotropic and nonlinear. As stated by Brown (1987) and Jing (2003), the primary requirement for solving problems in a rock mass is to take the effects of discontinuities into account, either implicitly or explicitly.

Three main categories of theories for considering the effects of discontinuities on wave propagation in the rock mass have been reported in the literature, and related experimental and numerical studies have been presented. The first category is the equivalent medium method or the continuum based method, which has been mainly applied to analyze the effects of pores, impurities or micro-cracks on wave propagation in porous two-phase media or cracked material (Biot 1956a, b; O'connell and Budianski 1974). The second category is the wave scattering theory, which has been widely utilized to study the effects of micro-cracks on wave velocity and attenuation in cracked media (Lax 1952; Garbin and Knopoff 1973; Hudson 1980). The third category is displacement discontinuity theory, which has been applied to study wave propagation in the fractured rock masses with joint spacing comparable to wavelength (Schoenberg 1980; Hopkins *et al.* 1988; Pyrak-Nolte *et al.* 1990b; Cai 2001; Zhao 2005). From the above-mentioned representative literatures, most of the studies based on the above three theories have been done so far for one-dimensional (1-D) cases. Among these previous studies, those related to the theory of displacement discontinuity are of engineering importance. In those 1-D studies, the effects of P-wave and S-wave on the performance of wave

propagation across rock joints were separately considered, and the solutions were obtained based on the method of characteristics.

However, for a practical dynamic problem in a rock mass involving wave propagation from an underground explosion source, the problem becomes a 2-D compressional wave propagation in jointed rock mass. The stability or assessment of damage of an underground structure near the explosion source depends heavily on the peak particle velocity (PPV) or peak particle acceleration (PPA) in the sense of the combined wave.

Review of literature indicates that there are no analytical solutions available for such 2-D wave problems in jointed rock masses. In addition, limited numerical studies were reported in the literature for the analysis of some specific field cases. There are no detailed studies performed to investigate the 2-D wave propagation in jointed rock masses. Furthermore, experimental studies are difficult to conduct due to the potential difficulties, complexities and dangers.

The continuous development in Singapore has required the construction of tall structures and deep rock caverns. The biggest underground structure in rock masses in Singapore has been under construction since August 1999, including several parallel caverns, which were planned for ammunition storage. In the initial planning stage of the project, one of the major concerns was that an accidental explosion in one of the ammunition storage caverns could lead to failure and instability of other adjacent or distant caverns. A research program was thus proposed, regarding the safety problems due to an accidental explosion (Zhou *et al.* 1992; Zhao *et al.* 1995; Zhao *et al.* 1999; Zhou *et al.* 2004). The research work presented in this thesis has been raised from that research program by focusing on study on 2-D compressional wave propagation in the fractured rock mass.

1.2 Objectives and scope of the research work

The main objectives of this research are:

- (1) To numerically investigate the factors influencing 2-D compressional wave propagation in jointed rock masses due to explosion in a tunnel or borehole.

- (2) To develop and verify prediction models to estimate the transmission and rebound ratios in jointed rock masses due to explosion in a tunnel or borehole.

The Universal Distinct Element Code (UDEC) is used in the study. Currently, UDEC is the most commonly used program in analyzing engineering problems in jointed rock masses. The joints in this thesis are assumed to be planar, dry, large in extent and with linearly deformational behavior.

In order to achieve the objectives, the following studies are performed:

- (1) A study on verification of UDEC modeling wave propagation in rock.
- (2) A study on effects of the factors influencing accuracy of UDEC results in 2-D wave propagation.
- (3) Development of a data processing method of comprehensive processing wave amplitude in UDEC modeling.
- (4) A study on 2-D compressional wave propagation from a tunnel or borehole in intact rock.
- (5) A study on the characteristics of the transmission ratio in 2-D compressional wave propagation from a tunnel or borehole in a rock mass.
- (6) A study on effects of single joint on 2-D compressional wave propagation from a tunnel or borehole in a rock mass.
- (7) A study on effects of multiple parallel joints on 2-D compressional wave propagation from a tunnel or borehole in a rock mass.

1.3 Organization of the thesis

The background and the objectives of the thesis are described in Chapter 1, while the previous work by other researchers, related to current research topic, are reviewed in Chapter 2.

In order to verify that UDEC is capable of modeling the wave propagation through joints, with reasonable reliability and accuracy, studies on verification of UDEC in dynamic problems are conducted in Chapter 3.

In order to keep the numerical error consistently below a reasonable magnitude for different modeling conditions and to avoid masking the effects of joints in 2-D wave propagation problems, studies on the effects of the factors influencing accuracy of UDEC results are conducted in Chapter 4.

A methodology for data processing or data fitting is proposed in Chapter 5. A study on 2-D compressional wave propagation in intact rock is also presented in Chapter 5. This study on 2-D compressional wave propagation in intact rock is a very important reference for the studies on 2-D compressional wave propagation in rock masses with joints.

Compressional wave propagation from a tunnel or borehole through single joint and multiple parallel joints in a rock mass is a multiple factor-influenced problem. In Chapter 6, studies on characteristics of the transmission ratio under various influence factors are performed. Throughout the analysis, the major effective factors, which dominate the performance of the transmission ratio, are determined. Therefore, these obtained characteristics are used to simplify the problem by filtering out the factors in subsequent studies, which have negligible effects on the transmission ratio.

Using the research strategies proposed in Chapter 6 and the data processing method developed in Chapter 5, extensive parametric studies on effects of single joint are performed both for the radian direction normal to the joint and for other arbitrary radian directions in Chapter 7. Prediction models for the transmission ratio through single joint are proposed, for both arbitrary directions and the direction normal to the joint.

In Chapter 8, extensive parametric studies on effects of multiple parallel joints are performed both for the radian direction normal to the joints and for other arbitrary radian directions. Prediction models for the transmission ratio through multiple

parallel joints are also proposed in Chapter 8. For the radian direction normal to the joints, a prediction model for the maximum rebound ratio is proposed based on extensive parametric studies as well.

In Chapter 9, the prediction models for wave propagation proposed in Chapters 7 and 8 are applied to a field explosion test, conducted at a quarry in Mandai, Singapore. This application of the proposed models serves as the verification example.

The conclusions, drawn from the current thesis, and recommendations for future research are presented in Chapter 10.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the existing theories and studies on the effects of fractures and cracks on wave propagation are reviewed. The explicit discrete element code, Universal Distinct Element Code (UDEC) is used in the analysis in this thesis. Therefore, a summary of the theoretical background and the features of UDEC, together with its applications in engineering problems, are also presented.

2.1 Overview of theories on effects of fractures and cracks on wave propagation

Equivalent medium theories

Equivalent medium theories are mainly used to analyze the effects of pores, impurities or micro-cracks on wave propagation in porous two-phase media or cracked material (Biot 1956a, b; O'connell and Budianski 1974), and are also used to analyze the effects of macro-fractures of high-density distribution in the rock mass (Crampin 1981; Johnston *et al.* 1979). In these theories, the materials and the contained pores or cracks are approximated as an equivalent continuous, homogenous and anisotropic medium with effective properties. Thus, the wave propagates as in a homogeneous and isotropic medium, if the pored and cracked medium is continuous. The effective properties of the equivalent medium are used to account for the effects of pores and cracks on wave velocity and attenuation.

An early effort to examine the effects of pores on wave propagation in fluid-saturated porous solid was made by Biot (1956a, b). In these two classical papers, a comprehensive theory was developed to examine the effects of parallel cylindrical pores saturated with fluid on P-wave propagation in direction of these cylindrical pores. A major advance in the equivalent medium theories for determining the effective moduli was explored by O'connell and Budianski (1974), through the studies on the interactions between highly-concentrated cracks. They used self-consistent analytical method to determine the elastic moduli of a solid with

randomly oriented ellipsoidal cracks, and to examine the wave propagation velocities in this solid under dry and saturation conditions.

Extensive studies have been conducted based on the equivalent medium theories to determine the effective elastic moduli and examine the effects of pores and micro-cracks on wave propagation (Chatterjee and Mal 1978; Crampin 1984; Zimmerman 1984; Sayers and Kachanov 1991). The methods for calculating the effective constants were mainly based on the geometry, concentration and distribution of pores and cracks, and the liquid contained in the pores and cracks. Various mechanisms have been proposed as the source of wave attenuation mechanisms, such as friction, contact line movement, viscosity, bubble movement, fluid flow and squirting flow (Crampin 1981; Johnston *et al.* 1979).

Wave scattering theories

The wave scattering theories have been widely utilized to study the effects of micro-cracks on wave velocity and attenuation in cracked media. In these theories, the elastic waves propagating through cracks were considered to be uniformly scattered by cracks under the conditions that the crack size was small compared to the wavelength and the crack concentration was dilute enough so that no interactions of wave scattering occurred between cracks.

Foldy (1945) proposed a multiple scattering theory of waves for cracks in a defective solid. The same theory was also developed in the study of Lax (1952). By considering the effects of wave scattering at the crack faces, Garbin and Knopoff (1973) presented techniques for analyzing the wave velocity variation with angle of wave incidence to the cracks and polarization of the waves in a material permeated with a random distributions of void (stress-free) circular cracks and liquid-filled cracks. In their studies, elastic shear and compression moduli were derived for the cracked material.

Hudson (1980) proposed a ‘method of smoothing’ to calculate overall elastic constants and to examine the wave velocity and attenuation in cracked material by taking into account of the scattering effects. His method was found to be accurate to

the second order of crack density. The results also showed the same frequency dependence of attenuation caused by scattering. In addition, his studies also demonstrated that the attenuation coefficient due to wave scattering was proportional to crack density and to the third power of ratio of the mean crack radius to the wavelength. Hudson and Knopoff (1989) and Peacock and Hudson (1990) further extended Hudson's method of analyzing the effects of wave scattering on the cases of various orders in crack density or more sets of cracks aligned in different directions. By modifying the analytical expressions proposed by himself, Hudson (1988) also evaluated the wave propagation across partially saturated ellipsoidal cracks. The results showed that the situation where cracks were partially liquid-filled were very different from either extremely dry or fully liquid-filled cracks.

Laboratory experiments have been carried out by many researchers to measure the wave velocity and attenuation in cracked material. The laboratory measurements were further compared with the theoretical scattering model of Hudson (1980) (Xu and King 1989; Ass'ad *et al.* 1993). The laboratory tests agreed well with the theoretical predictions.

The effects of wave scattering by cracks on wave attenuation have been studied in terms of the reflection and transmission coefficients corresponding to single or a periodic distribution of cracks over a plane. A relatively simple case of wave scattering, with a penny-shaped crack, has been studied by Mal (1970). Bostrom and Eriksson (1993) examined the wave scattering by two penny-shaped cracks with spring boundary conditions. Further efforts to examine the wave scattering in terms of reflection and transmission coefficients of cracks were made for more complicated cases such as: a periodic array of spherical cavities, a doubly periodic array of cracks, a multiple periodic array of cracks, a periodic array of inclined screens and a periodic array of inclined cracks (Angel and Achenbach 1985; Achenbach and Kitahara 1986; Achenbach and Li 1986; Mikata and Achenbach, 1988).

The wave scattering theories are suitable for analyzing the wave attenuation across the micro-cracks in which the size is smaller than the wavelength. The crack shapes,

crack density and distributions are very important parameters in the wave scattering formulations. Apparent attenuation due to the scattering of energy by the cracks is considered as the principal mechanism. This attenuation is evaluated in terms of the reflection and transmission coefficients of a distribution of cracks. If the cracks are filled with liquid, instinct attenuation can be taken into account based on the real energy loss-viscous dissipation by the liquid. A quality factor and wave attenuation coefficient can also be derived in the scattering theories to quantify the wave attenuation due to both energy scattering and dissipation.

Displacement discontinuity theories

Different from the wave scattering theories, displacement discontinuity theories are applicable to analyze the effects of fractures that are of planar shape, large length and small thickness relative to the wavelength. The theories have been analytically, experimentally and numerically studied at the fracture that is treated as a displacement discontinuity of infinitesimal thickness. The stresses across the fracture are continuous, while the particle displacements across the fracture are discontinuous. The fracture is mathematically represented, in the wave equations, by displacement discontinuity boundary conditions between two half-spaces. The displacement theories investigate the effects of fractures on the basis of the analytical model of wave reflection and transmission. The effects of the fractures on wave attenuation are evaluated in terms of magnitudes of reflection coefficient ($|R|$) and transmission coefficient ($|T|$), which are the ratios of the amplitudes of the reflected and transmitted waves to those of the incident wave.

The displacement continuity theories were developed from the traditional studies on wave reflection and transmission at an interface as a boundary between two half infinite spaces with dissimilar media (Macelwane and Sohon 1936). In their studies, the interface was regarded as completely welded, the boundary conditions between two half spaces were expressed by continuous stresses and displacements.

When an elastic wave of either P-wave or S-wave hits an interface between two dissimilar media, it is reflected, transmitted and converted. As an example, for the case of an incident plane P-wave traveling parallel to the X-Y plane and impinging

at arbitrary incident angle θ_1 to a flat interface along the X-Y plane, as shown in Figure 2.1, four waves will be produced: two of them are reflected back and two are transmitted into the second medium. Wave type and propagation direction are changed to:

A reflected P-wave at reflection angle θ_1 .

A reflected S-wave at reflection angle ϕ_1 .

A transmitted P-wave at transmission angle θ_2 .

A transmitted S-wave at transmission angle ϕ_2 .

In the analysis, the left and right sides of the interface are treated as two half infinite spaces with dissimilar media, the interface is then viewed as the boundary between the two half spaces.

Figure 2.1 Reflection and transmission of an incident P-wave at an interface between two different rock materials

A completely welded interface has the bonding that is completely fixed and rigid. Therefore, the stresses and the displacements across the interface are both continuous. Assuming this is a plane strain problem for which all displacement lies in the X-Y plane, the displacement continuity boundary can be expressed as: $u_y^{(1)} = u_y^{(2)}$ and $u_x^{(1)} = u_x^{(2)}$, where u_x and u_y are displacements in x- and y- directions, respectively, and superscripts (1) and (2) refer to medium on either side of the interface. Upon these boundary conditions, the wave motion equation can be solved.

Compared to the completely welded interface, a fracture can be treated as a non-welded interface. The fracture may have closure and slip deformations under normal and shear stresses, resulting in the displacement discontinuity across the fracture. Therefore, the displacement continuity boundary conditions are substituted by different stress-deformation relations for the fracture subject to normal and shear stresses. Studies on the effects of fractures on 1-D wave propagation have been conducted for various conditions of fracture deformational behaviors. These include the studies on effects of single fracture on P-wave or S-wave propagation by considering the linearly deformational behaviors of the fracture (Schoenberg 1980; Myer *et al.* 1985 and 1990; Pyrak-Nolte *et al.* 1987; Pyrak-Nolte *et al.* 1990a; Pyrak-Nolte 1996); the studies on the effects of single fracture by considering the slip rate-dependent shear behavior of the fracture (Miller 1977 and 1978); and the studies on the effects of multiple fractures by considering the linearly deformational behaviors (Hopkins *et al.* 1988; Schoenberg 1983; Pyrak-Nolte *et al.* 1990b; Myer *et al.* 1992; Yi *et al.* 1997; Cai 2001; Zhao 2005).

Schoenberg (1980) investigated the 1-D elastic wave reflection and transmission at a fracture termed as the slip interface, across which the stresses were continuous but displacement were not. The discontinuity in displacements across the fracture was correlated with the stresses by boundary compliance based on the linear stress-deformation relations of the fracture. The wave equation, with the presence of the wave reflection coefficient ($|R|$) and transmission coefficient ($|T|$) of the slip interface, could be solved using the slip interface boundary conditions.

Further analytical and numerical studies have been conducted in a similar procedure presented by Schoenberg (1980), but with additional considerations of fracture conditions (such as liquid-filled or clay-filled fractures) and incident wave types (such as interface waves) (Pyrak-Nolte *et al.* 1987, Pyrak-Nolte 1988; Myer *et al.* 1990; Pyrak-Nolte *et al.* 1990a; Yi *et al.* 1997).

According to Schoenberg (1980), Pyrak-Nolte *et al.* (1987), Pyrak-Nolte (1988) and Pyrak-Nolte *et al.* (1990a), the expressions of $|R_1|$ and $|T_1|$ become simple for the normally incident wave transmission across single dry fracture in an identical rock mass (1-D case):

$$|R_1| = \left[\frac{1}{4\left(\frac{k}{\omega z}\right)^2 + 1} \right]^{\frac{1}{2}} \quad (\text{For P-wave or S-wave}) \quad (2.1)$$

$$|T_1| = \left[\frac{4\left(\frac{k}{\omega z}\right)^2}{4\left(\frac{k}{\omega z}\right)^2 + 1} \right]^{\frac{1}{2}} \quad (\text{For P-wave or S-wave}) \quad (2.2)$$

where $|R_1|$ and $|T_1|$ are the magnitudes of reflection and transmission coefficients of the fracture, ω is the angular wave frequency, k and z depend upon the type of waves. For the compressional wave incidence, k is the normal stiffness of the fracture, and z equivalent to ρc_p is the seismic impedance of the rock for P-waves, c_p denotes the wave velocity for P-wave; while for shear wave incidence, k is the shear stiffness, and z equivalent to ρc_s is the seismic impedance for S-waves, c_s is the wave velocity for S-wave.

The formulae obtained by Schoenberg (1980), Pyrak-Nolte *et al.* (1987), Pyrak-Nolte (1988), Pyrak-Nolte *et al.* (1990a), Cai (2001) and Zhao (2005) show that the magnitudes of reflection and transmission coefficients are dependent on the ratio of the fracture stiffness to the product of seismic impedance and angular wave frequency. The low stiffness of the fracture and the high frequency of the incident waves result in high magnitude of $|R_1|$ and low magnitude of $|T_1|$, namely, high

wave attenuation due to the fracture. From the formulae, it can also be seen that the equations are general representations for the effects of single interface, ranging from welded interfaces to free boundaries on wave propagation. As the fracture stiffness k approaches infinite value, the value of $|R_1|$ is equal to zero and $|T_1|$ is one, representing a welded interface with total wave transmission; whereas as k reaches zero, the value of $|R_1|$ is one and $|T_1|$ is zero, indicating a free boundary with total wave reflection.

Displacement discontinuity theories have been further extended to the complex case of multiple parallel fractures (1-D case), where linearity was usually assumed in the displacement discontinuity boundary conditions defined by the fracture deformational behaviors (Schoenberg 1983; Hopkins *et al.* 1988; Pyrak-Nolte *et al.* 1990b; Myer *et al.* 1992; Yi *et al.* 1997). Traditional methods for analyzing the wave propagation across multiple fractures were based on equivalent medium theories. In studies in equivalent medium theories, the multiple-fractured medium was treated as a static equivalent transversely isotropic medium and the effects of the multiple fractures were lumped into an effective elastic moduli. By contrast, the displacement discontinuity theories modeled the effects of multiple fractures in such a way that the effects of individual fracture were explicitly considered through representing them as discontinuity displacement boundary conditions in the wave equation.

Wave propagation across multiple parallel fractures becomes complicated due to multiple wave reflections among the fractures. It is often difficult, in the displacement discontinuity model, to explicitly analyze the superposition of multiple reflected and transmitted waves generated at the multiple fractures. A simplified method of study is to ignore the multiple reflections as a short-wavelength approximation, i.e. the wavelength is short relative to the fracture spacing. Pyrak-Nolte *et al.* (1990b) have proposed a simple formula for calculating the magnitude of transmission coefficient $|T_n|$ for wave incidence normal to n identical parallel fractures spaced sparsely as: $|T_n| = |T_1|^n$, where $|T_1|$ is the magnitude of transmission coefficient for normally incident wave transmission across single fracture.

However, the multiple reflections have significant effects on wave transmission across multiple fractures, especially when the fractures are closely spaced. It is necessary to consider the multiple reflections in solving the problem of wave attenuation across multiple fractures. Studies on wave transmission across multiple perfectly welded interfaces between layers have shown that the reflections could have significant effects on the amplitudes of the reflected and transmitted waves (Schoenberg and Levin 1974; Bedford and Drumheller 1994). In an experimental study on wave attenuation across a pair of parallel fractures created by thin parallel strips of lead between steel surface, Hopkins *et al.* (1988) observed an increase in the amplitude of transmitted waves across the two closely spaced fractures. This finding implies that the simplified equation proposed by Pyrak-Nolte *et al.* (1990b) may not be applicable if the effects of multiple reflections are significant due to the close spacing among the fractures.

Different from the case of single fracture, multiple wave reflections occur when wave propagates across parallel fractures. The formulation of superposition of the multiple reflected and transmitted waves is a key to investigate the effects of multiple fractures on wave attenuation. Supposing a total wave field results from the superposition of multiple reflected and transmitted wave fields, ignoring the detailed process of wave superposition, and based on the method of characteristics and the displacement discontinuity theory, Cai (2001), Zhao (2005) and Zhao *et al.* (2004) established a set of recurrence equations with respect to particle velocities before and after the multiple fractures for both small and large spacing relative to the wavelength. These equations can be numerically solved. Based on the solutions of particle velocity and displacement, the magnitude of transmission coefficient $|T_n|$ for wave transmission across the multiple parallel fractures can be calculated from the ratio of the amplitude of the transmitted wave to that of the incident wave.

2.2 Overview of numerical methods in rock engineering

A number of powerful and versatile numerical methods are available to engineers today. The advent of fast and friendly personal computers has made it possible to solve complex problems, which, in the past, could not be solved without mainframe

machines and without considerable effort in terms of time and resources. The speed of computation has increased by a factor of ten every five years in the past thirty years. This trend is likely to continue. It is therefore apparent that numerical techniques will play an increasingly crucial role in the solution of engineering problems.

There are mainly three methods, which have been used in the problems of rock mechanics. They are (1) The Finite Element Method (FEM), (2) The Boundary Element Method (BEM) and (3) The Discrete Element Method (DEM). All methods are approximate methods, and all methods have their advantages and disadvantages.

Finite element method (FEM)

Until today, FEM is still one of the most popular methods in engineering fields. It has been applied to a large number of problems in different fields. Its popularity, particularly for load-deformation problems, largely depends on the fact that it is very appealing to engineers. They are able to relate it to a large extent to the background of structural mechanics as the physical meaning of the steps of calculations is relatively transparent. FEM has been extremely popular with geotechnical engineers. Its strength lies in its generality and flexibility to handle all types of load, sequence of construction, installation of supports

The method essentially involves dividing the body into smaller elements of various shapes held together at the nodes, which are corners of elements. In general, the greater the number of elements used to model the problem, the better approximation to the solution is obtained. Displacements at the nodes are treated as unknowns and are calculated. Stresses are calculated at one or more integration points inside each of the elements. Each element can have different material properties.

The major disadvantage of the method is that considerable effort is required in preparing data for a problem. This is particularly crucial in 3-D problems and has led to 'mesh generation' programs. These programs produce the input data required for the Finite Element program. Still considerable effort is needed in starting up the problem. FEM is also expensive in computer time. A large set of simultaneous

equations has to be solved to obtain solutions, especially when stress-strain relationship is nonlinear.

To model the rock fractures, Ngo and Scordelis (1967) proposed a two-node linkage element for rock joint. As an improvement, based on the lumped interface, Goodman *et al.* (1968) proposed a joint element in FEM model. The Goodman joint element is a linear line element suitable for two-dimensional (2-D) analysis with four nodes and no thickness. The stiffness matrix for the joint element is derived in the same way as for the regular finite element. No dilatancy of joints is taken into account and joint stiffness is treated to be constant in this early joint element. Mehtab and Goodman (1970) extended the formulation of the Goodman joint element to three-dimensional solution. Goodman (1976) presented another version of the joint element capable of modeling material non-linearity.

Boundary element method (BEM)

The Boundary Element Method (BEM) is becoming increasingly popular. The main reasons for this growth are (1) reduced set of equations, (2) a smaller amount of data, (3) proper modeling of infinite domains, (4) no interpolation error inside the domain and (5) valuable representation for stress concentration problems. BEM can be of as many different types as the domain methods, ranging from simple techniques such as indirect methods to the more versatile direct formulations.

Kupradeze (1965) established the foundations of the indirect boundary element method adopting the Kelvin fundamental solution to solve elastostatic problems. Watson, applying the same technique, has obtained a numerical solution for a pre-stressed concrete pressure vessel of a nuclear reactor treating the structure as a thick shell. The starting point of the direct formulation is due to Rizzo in 1967. His work, which presented the solution of the 2-D elastostatic problem, was extended to the 3-D case. The direct boundary element methods, which are more reliable than indirect technique, are based on adopting the real physical variables of the problem as the unknown of the system.

In this method only the surface of the rock mass to be analyzed needs to be discretized, i.e. divided into smaller patches. BEM differs from the FEM by the fact that approximations only occur on the boundary of the problem domain. The solution inside the domain will always satisfy the equations of equilibrium and compatibility exactly. For 2-D solutions, line elements at the boundary represent the problem, while for fully 3-D problems, surface elements are required. The data preparation is relatively simple. However, the computer program is not so transparent. Whenever there is a change of material properties, the surface defining the separation has to be discretized. Thus, if there are a number of layers of different materials, data preparation can still become complex. BEM appears to be a very efficient method for homogenous, linear elastic problems, particularly in 3-D problems. For complex nonlinear material laws with a number of sets of materials, advantages of the method are considerably diminished. The matrices of equations arising in this method are not banded and symmetric as for FEM but are fully populated. Thus, though the number of equations to be solved is considerably reduced, computation time does not reduce in the same proportion.

The method makes use of certain closed form relations of what may be called ‘elementary’ problems. These solutions frequently contain trigonometric and logarithmic terms, which slow down the computations.

Recognizing the advantages and disadvantages of the two methods of FEM and BEM, many researchers have combined the two methods. The coupled FEM/BEM method in which for a certain region, close to an opening or some other area of interest, FE discretization is used, while for the remaining area BE discretization is adopted.

Discrete element method (DEM)

The assumption of continuity is widely used as the basis for the idealization of many geomechanical materials including soil and rock mass. Powerful numerical techniques such as FEM, BEM and Finite Difference Method (FDM) are founded on the assumption of continuity and have proved extremely successful tools for the geotechnical engineer. However, there are many solutions, especially in rock

mechanics, when continuity cannot be assumed. DEM originated from such considerations.

DEM is based on treating the rock mass as a discontinuum rather than continuum as in the case of FEM and BEM methods. When loads are applied, the changes in contact forces are traced with time. In the earlier versions of the method rigid spherical balls or discs were used as elements. The equations of dynamic equilibrium for such element are repeatedly solved till the laws of contacts and boundary conditions are satisfied.

In the recent versions of the method, the elements can be of arbitrary shape as in FEM. They can also be deformable. Complex constitutive laws can also be used. The elements can be split up based on the assumed fracture criterion during the calculation process without any external intervention. Thus, the method is extremely powerful.

DEM in rock mechanics has evolved from various disciplines such as, physics of particles, geomechanics and structural engineering (Trollope 1967; Kawai 1977 and 1979). The theoretical background for the method was derived from the fields of aeroelasticity, the Finite Element Method (Bathe and Wison 1976; and Zienkiewicz, 1977), and the Finite Difference Method (Wilkins 1969). DEM's formulations started with the engineering empiricism of Burman (1971), Cundall (1971) and Maini and Cundall (1978) and proceeded to the more rigorous derivations of Kawai (1977) and Belytschko *et al.* (1984).

The formulation and development of DEM have greatly progressed since the initial presentation by Cundall (1971). It was originally created as a 2-D representation of a jointed rock mass for rock mechanics problem, and it has been extended to granular media such as sands and particle flow research. There are several programs in discrete element modeling according to Cundall and Hart (1993). They can be summarized into four classes mainly based on the treatment of the blocks and contact. They are distinct element programs (TRUBAL, UDEC, 3DEC, DIBS, 3DSHEAR), Modal methods (CICE, Williams and Mustoe), discontinuous deformation analysis (DDA) and Momentum-exchange methods.

The most commonly using of explicit DEM code in practice in solving engineering problems in jointed rock masses is the Universal Distinct Element Code (UDEC) (Jing 2003) that has been developed by Itasca group. This code has been implemented in the studies carried out in this thesis.

2.3 Backgrounds of UDEC and the applications in rock engineering

2.3.1 Features of UDEC

The formulation and development of the distinct element method has progressed for a period of over 30 years, beginning with the initial presentation by Cundall (1971). It was originally created as a 2-D representation of a jointed rock mass, but the method has been also extended to applications in particle flow research, studies on microscopic mechanism in granular material and crack development in a rock mass and concrete.

The most recent 2-D code for UDEC has been developed to combine formulations to represent both rigid and deformable bodies separated by deformable joints.

UDEC allows finite displacements and rotations of discrete bodies, including complete detachment, and recognizes new contacts automatically as the calculation progresses. Many finite element, boundary element and Lagrangian finite difference programs have interface elements or slide lines that enable them to model a discontinuous material to some extent. But there may not be an automatic scheme for recognizing new contacts, and the formulation may be limited to small displacements or rotation. So such programs are usually adapted from existing continuum programs.

UDEC has the following features:

- UDEC is ideally suited to study potential modes of failure directly related to the presence of joints.
- Joint generators are built into UDEC to create individual or sets of joints.

- Different representations of joint material behavior are available in UDEC, including the most common Coulomb Slip Model, the Continuously Yielding Model and the Barton-Bandis Joint Model.
- Blocks can be either rigid or deformable. There are seven built-in material models for deformable blocks, which represent the different behaviors for intact rock material among the joints.
- Joints are considered as planar features oriented normal to the plane of analysis.
- The explicit solution algorithm in UDEC permits either dynamic or static analysis. UDEC contains non-reflecting and free-field boundary conditions for dynamic analysis. Both stress (force) and fixed displacement (zero velocity boundary conditions) are available for static analysis.
- Structure element logic is implemented to simulate rock reinforcement and surface support in the model.
- UDEC contains a powerful built-in programming language named FISH, which enables the user to define new variables and functions. FISH makes the command-driven program UDEC to be a very friendly program.
- In UDEC, the information are all stored within a linked-list data structure that corresponds to the topological structure of the physical system. Each physical entity (block corner and contact) is represented by a data element that is linked by pointers to the data structure from a main storage array. Material properties and initial and boundary conditions are also linked to the data structure during model generation. The data can be accessed via FISH.

2.3.2 Theoretical background of UDEC

The theoretical background of UDEC is presented for the sake of completeness in discussion of UDEC analysis. Actually, the theoretical background of UDEC covers a wide range, here only the main aspects are summarized from UDEC manual.

Block deformability

In UDEC, the block can be either rigid or deformable block. For the rigid block, the formulation represents the medium as a set of distinct blocks that do not change their geometry as the results of applied loading. Deformable blocks are internally discretized into finite-difference triangular elements. The complexity of deformation of the blocks depends on the number of elements into which the blocks are divided. The vertices of the triangular elements are grid-points, and the equations of motion

for each grid-point are formulated as follows: $\ddot{u}_i = \frac{\int_s \sigma_{ij} n_j ds + F_i}{m} + g_i$, where u is displacement; s is the surface enclosing the mass m lumped at the grid-point; n_j is the unit normal to s ; F_i is the resultant of all external forces applied to the grid-point (from block contacts or other wise) and g_i is the gravitational acceleration.

Rock joint representation

A rock joint is represented as a contact surface formed between two block edges. Adjacent blocks can touch to each other either along a common edge or at discrete points where a corner meets an edge or another corner. For rigid blocks, a contact in UDEC is treated as each corner interacting with a corner or edge of an opposing block. For deformable ones, point contacts are created at all grid-points located on the block edge in contact. By rounding the corner, adjacent blocks can smoothly slide past one another, when two opposing corners interact.

Contact points are updated automatically when block motion occurs. Contact updating is triggered by significant relative motion within a domain. A friction displacement is accumulated for each domain. When this displacement exceeds a certain tolerance level, an update is triggered. This ensures that contacts are always detected before physical contact is made. During an update, new contacts are created and the old contacts are deleted depending on the relative motion at each contact.

Only two types of contact are needed by the data structure for representing a system of blocks: corner-to-corner contacts and edge-to-corner contacts. These are termed “numerical contacts”. Physically, however, edge-to-edge contact is important,

because it corresponds to the case of a rock joint closed along its entire length. A physical edge-to-edge corresponds to a domain with exactly two numerical contacts in its linked-list

There are four built-in joint models and optional model available to represent the material behavior of discontinuities:

- Point contact-Coulomb slip model
- Joint area contact-Coulomb slip model
- Joint area contact-Coulomb slip with residual strength model
- Continuously yielding model
- Barton-Bandis joint model

Here, the mechanism of the most commonly used Coulomb slip model is summarized.

In the normal direction, the stress-displacement relation is assumed to be linear and governed by the stiffness j_{kn} such that $\Delta\sigma_n = j_{kn} \times \Delta u_n$, where $\Delta\sigma_n$ is the effective normal stress increment, and Δu_n is the normal displacement increment. For the joint, if the tensile strength is exceeded, then $\Delta\sigma_n = 0$. The response is controlled by a constant shear stiffness, j_{ks} . The shear stress τ is limited by a combination of cohesion c_j and frictional angle ϕ_f . Thus, if $|\tau| < c_j + \sigma_n \tan \phi_f = \tau_{\max}$, then $\Delta\tau = -j_{ks} \times \Delta u_s^e$ or else, if $|\tau| > \tau_{\max}$, then $\tau = \text{sign}(\Delta u_s) \tau_{\max}$, where Δu_s^e is the elastic component of the incremental shear displacement, and Δu_s is the total incremental shear displacement. The dilation is governed in the Coulomb slip model by a specified dilation angle ψ . In the model, the dilation is restricted such that, if $|\tau| \leq \tau_{\max}$, then $\psi=0$; and if $|\tau| = \tau_{\max}$ and $|u_s| \geq u_{cs}$ then $\psi=0$, where u_{cs} is the critical shear displacement.

Mechanical damping

Mechanical damping is used in UDEC to solve both the static and the dynamic problems. For static analysis, the approach is conceptually similar to dynamic relaxation. Damping is proportional to velocity and the magnitude of the damping force is proportional to the velocity of the blocks. The form of damping is provided in UDEC, in which the damping force on a node is proportional to the magnitude of the unbalanced force. It is regarded as local damping. The direction of the damping force is such that energy is always dissipated.

For dynamic analysis, Raleigh damping is used in UDEC. Raleigh damping was originally used in the analysis of structures and elastic continua to damp the natural oscillation modes of the system. The equations are, therefore, expressed in matrix form. A damping matrix, C_d , is used, with components proportional to the mass (M_m) and stiffness (K_m) matrices: $C_d = \alpha_m M_m + \beta_s K_m$, where α_m is the mass-proportional damping constant, and β_s is the stiffness-proportional damping constant.

For a multiple degree-of-freedom system, the critical damping ratio, ξ_i , at any angular frequency of the system, ω_i , can be found from (Bathe and Wilson 1976) as

$$\xi_i = \frac{1}{2} \left(\frac{\alpha_m}{\omega_i} + \beta_s \omega_i \right).$$

The critical damping ratio, ξ_i , is also known as the fraction of

critical damping for model i with angular frequency ω_i . Mass-proportional damping is dominant at lower angular frequencies, while stiffness-proportional damping dominates at higher angular frequencies.

Conservation of momentum and energy in the distinct element formulation

The distinct element method can satisfy the conservation laws, by using an approach based on the use of Newton's laws of motion.

Based on Newton's laws of motion, the distinct element method satisfies the conservation of momentum and energy by laws.

Considering two bodies (denoted by subscripts a and b) in contact for a period, T , by Newton's laws, a common force, F , acts in opposite directions on the two bodies, which accelerate in proportion to the forces: $m_a \ddot{u}_a = F$, $m_b \ddot{u}_b = -F$, where the subscripts a and b are two opposite directions, m and u are mass and displacement respectively. Combining the equations and integrating,

$$\int_0^T m_a \ddot{u}_a dt = -\int_0^T m_b \ddot{u}_b dt, \quad (2.3)$$

$$m_a (\dot{u}_a^{(T)} - \dot{u}_a^{(0)}) = -m_b (\dot{u}_b^{(T)} - \dot{u}_b^{(0)}) \quad (2.4)$$

$$m_a \dot{u}_a^{(T)} + m_b \dot{u}_b^{(T)} = m_a \dot{u}_a^{(0)} + m_b \dot{u}_b^{(0)} \quad (2.5)$$

Equation (2.5) indicates that the total momentum at the end of an arbitrary time period is identical to that at the beginning.

Assuming a body with initial velocity v_0 is brought to a final velocity of v in a distance S by a constant force F : $m\dot{v} = F$. Using the identity $\dot{v} = v dv/ds$, $m \int_{v_0}^v v dv = \int_0^S F ds$, assuming m is constant. Hence,

$$\frac{1}{2} m(v^2 - v_0^2) = FS \quad (2.6)$$

The equation expresses the fact that the work done by the force is equal to the change in kinetic energy of the body.

If the force opposing motion is related to the displacement by the equation ($f = -ks$), where k and s donate the spring stiffness and strain respectively, then Equation (2.6) is replaced by $m \int_{v_0}^v v dv = -\int_0^s ks ds$. Hence, $\frac{1}{2} m(v_0^2 - v^2) = \frac{1}{2} ks^2$.

Hence, the decrease in kinetic energy equals the energy stored in the spring. The same argument may be used in reverse to show that the kinetic energy acquired by a body is equal to the decrease in energy stored in a spring. Hence, the kinetic energy of a body after an elastic collision is equal to the kinetic energy before the collision.

Boundary conditions

Either stress (load) or displacement (velocity) may be applied at the boundary of a UDEC model. The condition is applied to the centroid of blocks along the boundary for a rigid block model. For the deformable ones, the displacements are specified in terms of prescribed velocities at given grid-points. At a stress boundary, forces are derived as follows: $F_i = \sigma_{ij}^b n_j \Delta s$, where n_j is the outward normal vector of the boundary segment, and Δs is the length of the boundary segment over which the stress, σ_{ij}^b , acts.

In dynamic problems, viscous boundary developed by Lysmer and Kuhlemeyer (1969) is used in UDEC. It is based on the use of independent dashpots in the normal and shear directions at the model boundaries. The dashpots provide viscous normal and shear traction given by $t_n = -\rho c_p v_n$ and $t_s = -\rho c_s v_s$, where v_n and v_s are the normal and shear components of the velocity at the boundary, ρ is the mass density, and c_p and c_s are the P-wave and S-wave velocity.

Mechanical time-step determination: solution stability

The solution scheme used for the distinct element method is conditionally stable. A limiting time-step is determined to satisfy both the stability criterion for calculation of internal block deformation as well as inter-block relative displacement. The time-step required for the stability of block deformation computation is estimated as

$$\Delta t_n = 2 \min \left(\frac{m_i}{k_i} \right)^{\frac{1}{2}}, \quad (2.7)$$

where m_i is the mass associated with block node i , and k_i is a measure of stiffness of the elements surrounding the node. The stiffness term, k_i , must account for both the stiffness of the intact rock and that of the discontinuities. It is calculated as the sum of the two components.

For calculation of inter-block relative displacement, the limiting time-step is calculated, by analogy to a simple degree-of-freedom system, as:

$$\Delta t_b = 2(\text{frac})\left(\frac{M_{\min}}{K_{\max}}\right)^{\frac{1}{2}} \quad (2.8)$$

where $M_{\min}$ is the mass of the smallest block in the system, and $K_{\max}$ is the maximum contact stiffness. The term *frac* is a user-supplied value that accounts for the fact that single block may be in contact with several other blocks simultaneously. A typical value for *frac* is 0.1.

The controlling time-step for a distinct element analysis is $\Delta t = \min(\Delta t_n, \Delta t_b)$.

2.3.3 Applications of UDEC in rock engineering

UDEC has been widely applied in rock engineering. A large quantity of associated publications has been published. Here only those related to wave propagation (both in 1-D and 2-D problems) or stability of underground excavation subjected to dynamic loads are reviewed.

2.3.3.1 General applications of UDEC modeling in rock engineering

UDEC has been applied most frequently in the studies related to mining engineering and other underground civil engineering problems. Both static analysis and dynamic analysis for deep underground openings have been performed. A most common application is to predict the stability-related performance of an excavation, such as deformation of the underground openings, the induced stress redistribution (Cundall and Lemos 1990; Sakurai and Kitamura 1990; Choi and Coulthard 1990; Chryssanthakis *et al.* 1992; Kulatilake *et al.* 1992; Jing and Stephansson 1991; Rosengren 1993; Senseny and Simons 1994; Barton *et al.* 1994; Bhasin *et al.* 1996; Shen and Barton 1997; Bhasin and Hoeg 1998; Sofianos and Kapanis 1998; Ma and Brady 1999; Chen *et al.* 2000; Chen and Zhao 2002). Fault-slip induced failure around excavations is another example of analysis conducted with UDEC (Rosengren 1993). Blasting effects have been studied by applying dynamic stress or velocity waves at model boundaries (Brady 1990; Rosengren 1993; Souley *et al.* 1995; Song and Kim 1995; Chen and Zhao 1998; Alehossein and Hood 1998; Ma and Brady 1999; Chen *et al.* 2000). Research in the area of fault-slip induced

seismicity has also been conducted by use of the continuously yielding joint model (Cundall and Lemos 1990). Structural elements have been employed to simulate various rock reinforcement systems such as grouted rockbolting and shotcrete (Rosengren 1993; Barton *et al.* 1994; Asef *et al.* 2000). Several researchers conducted research in laboratory test simulation and constitutive model development or in comparing the rock mass response by using different rock models or joint models in UDEC (Jing *et al.* 1993, Jing *et al.* 1994; Souley and Homand 1996; Lanaro *et al.* 1997; Bhasin and Hoeg 1998; Ma and Brady 1999).

2.3.3.2 Applications of UDEC in 1-D wave propagation

Most of the applications of UDEC in seismology and wave attenuation through fractures belong to 1-D wave propagation in a rock mass.

Summary on solutions from the method of characteristics in 1-D wave propagation

The essence of wave attenuation across the fractures is that a decrease in wave amplitude occurs for the transmitted waves across the fractures in comparison with the incident wave amplitude. This decrease may be caused by the real energy loss or by the wave energy scattering or reflecting of the fractures involving no real energy loss. Studies on the effects of fractures on wave propagation have been conducted starting from the investigations on the effects of single joint (Schoenberg 1980; Pyrak-Nolte *et al.* 1987, Pyrak-Nolte 1988; Pyrak-Nolte *et al.* 1990a; Cai 2001; Zhao and Cai 2001; Zhao 2005) to multiple parallel fractures (Schoenberg 1983; Hopkins *et al.* 1988; Pyrak-Nolte *et al.* 1990b; Cai 2001; Zhao 2005; Zhao *et al.* 2004).

Based on displacement discontinuity theories, for the case of normally incident 1-D wave attenuation across multiple parallel joints with linearly deformational behavior, Cai (2001) and Zhao (2005) established a set of recurrence equations and numerically solved. In addition, Zhao (2005) has performed the studies on P-wave attenuation across multiple parallel joints with nonlinearly deformational behavior and S-wave attenuation across multiple parallel joints with the Coulomb slip behavior.

Also based on displacement discontinuity theories, for the case of 1-D obliquely incident wave across single joint and multiple parallel joints with linearly deformational behavior, Zhao (2005) has conducted a study on wave propagation. For the purpose of clear statement of the study, for different input waves, the magnitude of transmission coefficient is termed in a general form as $|T_{a_1-a_2n}|$, and the magnitude of reflection coefficient is termed as $|R_{a_1-a_2n}|$. In these two terms, the first subscript a_1 is either S (for S-wave incidence) or P (for P-wave incidence); the second subscript a_2 is either S (for transmitted S-wave in $|T_{a_1-a_2n}|$ or reflected S-wave in $|R_{a_1-a_2n}|$) or P (for transmitted P-wave in $|T_{a_1-a_2n}|$ or reflected P-wave in $|R_{a_1-a_2n}|$); the third subscript n is the quantity of the joints in the joint set. For the case of single joints, results on $|T_{p-pl}|$, $|T_{p-sl}|$, $|T_{s-sl}|$, $|T_{s-pl}|$, $|R_{p-pl}|$, $|R_{p-sl}|$, $|R_{s-sl}|$ and $|R_{s-pl}|$ as functions of normalized stiffness have been obtained; on the other hand, results on $|T_{p-pl}|$, $|T_{p-sl}|$, $|T_{s-sl}|$, $|T_{s-pl}|$, $|R_{p-pl}|$, $|R_{p-sl}|$, $|R_{s-sl}|$ and $|R_{s-pl}|$ as functions of incidence angle have been obtained as well. For the case of multiple parallel joints, results on $|T_{p-pn}|$, $|T_{s-sn}|$, $|T_{p-sn}|$ and $|T_{s-pn}|$ as functions of ξ (the ratio of joint spacing over the wavelength) for different n have been also obtained. Among these results in 1-D obliquely incident wave propagation in jointed rock masses, the results on $|T_{p-pl}|$ and $|T_{p-sl}|$ as functions of incidence angle will be referred in Chapter 7, therefore these results are briefed.

Figure 2.2 shows $|T_{p-pl}|$ and $|T_{p-sl}|$ as functions of incidence angle for P-wave incidence at normalized stiffness of 2.2, where the incidence angle is defined as the angle between the normal line and incident radial line. From this figure, the following general trends have been reached:

- (1) At incidence angle of 0° (i.e., the case of normally incident 1-D P-wave), the transmitted wave field is only P-wave.

- (2) $|T_{p-sl}|$ slowly increases with increasing incidence angle until it reaches the maximum value of about 85° , and then decreases with increasing incidence angle.
- (3) $|T_{p-pl}|$ does not change much with increasing incidence angle until close to 90° .

Figure 2.2 $|T_{p-pl}|$ and $|T_{p-sl}|$ as functions of incidence angle at normalized stiffness of 2.2

Summary on comparison between the solutions from the method of characteristics and UDEC results

Besides the study based on the method of characteristics on 1-D wave propagation, Cai (2001) and Zhao (2005) have conducted numerical analysis in UDEC on 1-D normally incident wave propagation across multiple parallel joints with linearly deformational behavior. In addition, Zhao (2005) has conducted a numerical study in UDEC on P-wave attenuation across multiple parallel joints with nonlinearly deformational behavior. From the comparison between UDEC results and the solutions from the method of characteristics performed by these two researchers, the general trends of the magnitude of the transmission coefficient $|T_n|$ with ξ for different n from UDEC modeling agree with the solutions from the method of characteristics, while $|T_n|$ from UDEC results is consistently larger than the solutions from the method of characteristics.

Among these comparison studies by Cai (2001) and Zhao (2005), some of the wave phenomena in 1-D normally incident wave propagation across multiple parallel joints will be referred in Chapters 7 and 8, therefore, the UDEC results together with the wave phenomena are reviewed and discussed in details as follows.

Cai (2001) has compared the results from the recurrence equations with UDEC solutions as shown in Figure 2.3, for 1-D normally incident S-wave propagation through n ($n=2, 4, 6, 8$ and 10) parallel joints (at a fixed normalized shear stiffness of 1.6). Cai (2001) also has compared the results from the recurrence equations with UDEC solutions as shown in Figure 2.4, for 1-D normally incident S-wave propagation through 2 parallel joints with different normalized stiffness $k/\omega z$ ($0.47, 0.63, 0.94, 1.6$ and 3.1). It is noticed that in normally incident 1-D case, the relationship between $|T_n|$ and normalized normal stiffness for P-wave is the same as that between $|T_n|$ and normalized shear stiffness for S-wave.

As shown in Figure 2.3 and Figure 2.4, the variation of the magnitude of transmission coefficient $|T_n|$ with the ratio (ξ) of the joint spacing to the wavelength for every level of normalized shear stiffness and every given number of joints has three modes, that is:

- If $\xi \geq \xi_{thr}$, $|T_n|$ is independent of the joint spacing.
- If $\xi_{thr} > \xi > \xi_{cri}$, $|T_n|$ increases as the joint spacing decreases.
- If $\xi \leq \xi_{cri}$, $|T_n|$ decreases with the joint spacing decreases.

Also as shown in Figure 2.3 and Figure 2.4, as two important indices of ξ , the threshold value (ξ_{thr}) and the critical value (ξ_{cri}) are very meaningful in determining how $|T_n|$ is influenced by the normalized stiffness of joints, the joint spacing and joint number. The following general trends have been reached:

- (1) When the normalized stiffness of joint is fixed as shown in Figure 2.3, ξ_{cri} increases as the number of joints decreases.
- (2) When the normalized stiffness of joint is fixed as shown in Figure 2.3, ξ_{thr} increases as the number of joints increases.
- (3) When the number of joints is fixed as shown in Figure 2.4, ξ_{cri} increases as the normalized stiffness of joint increases.
- (4) When the number of joints is fixed as shown in Figure 2.4, ξ_{thr} increases as the normalized stiffness of joint decreases.

Figure 2.3 Comparison between UDEC results and solution of $|T_n|$ from the method of characteristics as a function of ξ of joint spacing to wavelength for $k/(\omega z)=1.6$ and different fracture number n (after Cai 2001)

Figure 2.4 Comparison between UDEC results and the results from the method of characteristics for $|T_2|$ as a function of ratio (ξ) of joint spacing to wavelength for different $k/(z\omega)$ (after Cai 2001)

2.3.3.3 Applications of UDEC in 2-D wave propagation

Most of the applications of UDEC modeling in dynamic analysis in underground openings belong to 2-D wave problems in rock masses.

Applications of UDEC modeling in terms of stability-related performance

Rosengren (1993) conducted a stability analysis of the interaction between two tunnels in Sweden under an assumed blast loading. Four situations (Unreinforced

and static load, unreinforced and dynamic load, reinforced and static load and reinforced dynamic load) were taken into account in the analysis. It was concluded that for the unreinforced case, there were no significant difference in the stability situation between the static and dynamic conditions. It was also concluded that the conventional reinforcement offered very important supporting effects to stabilize the rock when it is subjected to explosive load. Ma and Brady (1999) studied the displacement of an underground excavation subject to a series of mining explosion-induced loading. This research showed progressive accumulation of rock displacement and excavation deformation under successive episodes of dynamic loading. The way that displacement and excavation deformation developed was dependent on the joint model used in UDEC. Both the Coulomb-slip and the continuously yielding joint models were tested in this study. UDEC results reproduced the measured results fairly well, while the continuously yielding model presented the history of deformation more consistent with the practical record than the Coulomb-slip model.

Applications of UDEC modeling in rock support

Barton *et al.* (1994) modeled rock bolt in their UDEC-BB model for the 62 m span Olympic Ice Hockey cavern. Each of the excavation steps was first run to equilibrium without any use of reinforcement, when 100% of the deformation was achieved, the point where approximately 50% of this deformation had occurred was used as the starting point for bolting installation. This was done in an attempt to allow for the elastic deformation that would have already occurred before bolt installation.

Rosengren (1993) modeled two tunnels under both static and dynamic condition, using UDEC. For dynamic loading, there was an accidental blasting assumed in the upper tunnel. Rosengren modeled both reinforced and unreinforced tunnel. His research showed there was a large difference in the results when the reinforcement was considered. The final vertical displacement for a specific point was 1 mm for the case of static load, whereas the corresponding deformation for the dynamic case was 26 mm. The large difference in the deformation also was evident in the large differences in bolt forces and strains between the two cases. The static case

produced very small bolt forces and strain ($F_{\max} = 29.7\text{KN}$, $\varepsilon_{\max} = 0.1\%$ for the monitored point). The dynamic case, however, produced much higher forces and strains ($F_{\max} = 70\text{KN}$, $\varepsilon_{\max} = 9.7\%$).

Fatigue of rock and rock mass

Ma and Brady (1999) studied the displacement of an underground excavation subjected to a series of mining explosion-induced loading by using UDEC. This research showed progressive accumulation of rock displacement and excavation deformation under successive episodes of dynamic loading. This implies that more damage to an underground excavation may occur through a number of seismic events, with relatively small magnitudes. In this aspect, the rock mass under dynamic loading shows fatigue feature. The way that displacement and excavation deformation develop depends on the joint model used in UDEC. Both the Coulomb-slip and continuously yielding joint models were tested in their model. The UDEC results reproduced the measured results fairly well. On the other hand, the continuously yielding model presents the history of deformation more consistent with the practical record than the Coulomb-slip model.

Explosion input in UDEC modeling

In some analytical procedures, dynamic loads are replaced by equivalent static loads, using simple rules of thumb expressed in terms of dynamic contribution. The contribution factor is often between 1 and 2, but it may be greater for explosive loads. However, as stated by Rosengren (1993), it is very difficult to define the dynamic load for the actual event. The load distribution in time, magnitude and extent depends on a number of factors, such as the volume of leaking gas/ liquid, energy content, and the external conditions such as ventilation and time before initiation.

In numerical analysis, the dynamic loading is often treated as a triangular pulse by giving the peak value, rising time and duration. Figure 2.5 shows a typical triangular pulse wave, where $p_{\max}$ is the peak value in terms of velocity (m/s) or pressure (Pa or MPa), t_r is the rising time and t_d is the duration.

Figure 2.5 Triangular pulse wave

Using UDEC, Rosengren (1993) modeled a road tunnel across the existing subway tunnel in Stockholm, Sweden, assuming a scenario in which a truck loaded with explosive gas or liquid starts to leak and then explodes. In his research, the peak value in terms of pressure was 1 MPa, the rising time is 2 milliseconds (ms) and the duration is 100 ms. Senseny and Simons (1994) conducted comparison study between several numerical methods, including UDEC, and calculation approaches in jointed rock mass, under both static and dynamic loading. For dynamic loading, the tunnel was loaded dynamically by a large-amplitude, triangular stress pulse. The pulse amplitude was about 17.5 times the unconfined compressive strength of the intact rock. The pulse rising time was about five times of time that the wave propagates from the center to the boundary of the tunnel, and the pulse duration was longer than ten times of the mentioned time period.

Noting that UDEC cannot model the explosion process, Chen (1999) proposed a coupled method of UDEC and Autodyn to simulate the explosion process in a rock chamber and the explosion wave propagation in a jointed rock mass. Firstly, in Autodyn, the explosion history of the surrounding rock mass near the rock chamber was recorded in terms of particle velocity or pressure. Then the history was converted into the formatted explosion input compatible with UDEC. The converted input was thus applied to UDEC model at the same locations and directions as in Autodyn model.

The explosion input to UDEC in terms of the velocity with history time at the ambient rock surrounding the explosion chamber was computed in a continuum-based equivalent rock mass in Autodyn. In Autodyn modeling, 1-D Euler remapping was used to model the initial explosion process. The surrounding medium could be air, water, or any other appropriate materials. The result of the remapping is then introduced to the model of 2-D or 3-D analysis. The explosion histories of Autodyn modeling were obtained using random time, however, the explosion input is required to input at an even time interval. Therefore, all the explosion histories must be converted into a form suitable for the UDEC modeling.

CHAPTER 3

VERIFICATION OF UDEC MODELING IN DYNAMIC PROBLEMS

3.1 Introduction

The key to modeling discontinuity problems imposed by the presence of joints in the rock mass is to properly present the joints and their behavior in the numerical model. The Universal Distinct Element Code (UDEC) is a two-dimensional (2-D) discrete element program developed by Cundall and co-workers. In UDEC, the blocks can be rigid or deformable, and in the latter case they can be subdivided into internal mesh of finite difference elements. The simulation of the displacement of the joints is implemented through movements of the blocks. The contact forces and displacements at the interfaces (joints) of the stressed assembly of blocks are found through a series of calculations, which trace the movements of the blocks. This is a dynamic process in which the velocity of propagation depends on the physical properties of the discrete block system. For continuous analysis where the rock mass is composed of an intact block without joints, UDEC can be used as an explicit finite difference element program. For discontinuous analysis where the rock mass are composed of multiple blocks separated by joints, due to its inherent discontinuum mechanism formulation, UDEC provides the capability of modeling discontinuity problems of a rock mass containing multiple parallel or intersecting joints as displacement discontinuities, in both static and dynamic solutions.

UDEC has the potential applications to model one-dimensional (1-D) and 2-D (plane-strain) dynamic problems in the fractured rock mass. However, it is necessary to verify UDEC for modeling the wave propagation across joints in the rock mass.

The studies performed in this chapter aim to verify UDEC modeling in dynamic problems, in both 1-D and 2-D wave propagations. There are several cases included in these studies. For verification of UDEC modeling in 1-D wave propagation, the studies cover 1-D wave propagation in an ideal elastic medium, 1-D wave

propagation through single joint and 1-D wave propagation through multiple parallel joints.

It is noted that, for the case of single joint in 1-D wave propagation, similar studies have been performed by several researchers (Chen 1999; Cai 2001; Zhao 2005). For the case of multiple parallel joints, Cai (2001) and Zhao (2005) have compared the solutions from the method of characteristics with UDEC results. However, for completeness of verification of UDEC modeling in dynamic problems in rock masses, verification examples for 1-D wave propagation through single and multiple parallel joints are still included in this thesis. In these examples, the UDEC analysis is independently performed with different joint parameters from those in literatures (Chen 1999; Cai 2001; Zhao 2005).

For verification of UDEC modeling in 2-D wave propagation, the study performed in the present research covers 2-D harmonic wave propagation from cylindrical cavity in an elastic rock. The UDEC results, with respect to displacement and velocity waves, are compared with the analytical solutions given by Hefny (2004). On the other hand, the UDEC results, in terms of radial stress wave, are compared with the analytical solutions given by Graff (1979).

3.2 A study on verification of UDEC in 1-D wave propagation

3.2.1 1-D wave propagation in an elastic medium

According to the elastic wave theory, a 1-D planer elastic wave propagates in an ideally elastic medium without wave attenuation (no material and geometrical damping).

A UDEC model is established as shown in Figure 3.1, where the length in the x-direction is 400 m, and the height in the y-direction is 2 m. The rock within the geometry is treated as single deformable block, which is divided into triangular element meshes. The left and right sides are treated as viscous boundaries preventing wave reflection. The top and lower sides are constrained in the horizontal direction, by setting zero x-velocity to both sides. A sinusoidal shear stress wave with unity amplitude is tangentially applied to the left boundary and

propagates through the model along the x-direction. The properties of the rock material and the velocity of the shear wave in the rock material are shown in Table 3.1. These properties are similar to the Bukit Timah Granite in Singapore (Lee 2002).

According to the elastic wave theory, the shear wave should travel through the model without attenuation. Therefore two measuring points *A* and *B* are positioned in the model at $x=10$ m, $y=1.0$ m and $x=390$ m, $y=1.0$ m respectively, to check the value of the wave amplitude during the wave propagation. Point *A* is close to the wave input boundary while point *B* is close to the right hand boundary.

Figure 3.1 UDEC model for 1-D wave propagation in an intact rock

Table 3.1 Mechanical properties and wave propagation velocity in the rock

Properties	Value
Density (ρ) (kg/m^3)	2650
Bulk Modulus (K) (GPa)	44
Shear Modulus (G) (GPa)	39
Velocity of shear wave (c_s) (m/s)	3830

Regarding the verification of UDEC modeling in 1-D wave propagation, shear waves with different frequencies are tested in this model. The frequencies (Hz) considered in the analysis are: 7, 8, 9, 12, 16, 20, 23, 29, 33, 39, 47, 52, 59, 67, 78, 94, 118, 157, 235 and 470. The mesh size in each case is taken as $\lambda/16$, where λ is the corresponding wavelength.

Figure 3.2 shows the UDEC results for all the considered frequencies. The results are shown in terms of the ratios of the amplitude of the shear stress wave at point *B* to that at point *A* versus frequencies. The straight line represents the analytical solution in terms of the above mentioned ratio. As seen from Figure 3.2, there is a good agreement between the analytical solution and UDEC results.

Figure 3.2 Comparison between UDEC results and analytical solution, in terms of the ratio of amplitude of the stress wave at Point B to that at Point A for the case of intact rock

3.2.2 1-D wave propagation through single joint

When a 1-D wave propagates through single joint, part of the wave will be reflected and part of it will be transmitted. The magnitude of transmission coefficient $|T_1|$ is the ratio of the amplitude of the transmitted wave to that of the incident wave.

The analytical solution for $|T_1|$ in the process of 1-D shear wave propagation is given by Pyrak-Notle *et al.* (1987 and 1990a) as:

$$|T_1| = \left[\frac{4\left(\frac{jks}{\omega z}\right)^2}{4\left(\frac{jks}{\omega z}\right)^2 + 1} \right]^{\frac{1}{2}} \quad (3.1)$$

where jks is the shear stiffness of the joint, ω is the angular frequency, and z is the seismic impedance of the rock for shear wave which is termed as:

$$z = \rho c_s \quad (3.2)$$

where ρ is density of the rock, c_s is the shear wave propagation velocity in the rock. The ratio of $jks/(\omega z)$ in Equation (3.1) is known as the normalized shear stiffness of the joint.

UDEC model shown in Figure 3.3 is used in the case of single joint. In the model, the joint is placed at $x=200$ m. The measuring point A is positioned in the model at $x=201$ m, $y=1$ m where the history of the transmitted wave is recorded.

Table 3.2 shows the properties of the rock material and the properties of shear wave adopted in the analysis. To ensure no frictional damage occurring in the process of wave propagation, the cohesion of the joint is assumed to be of a relatively high value.

Figure 3.3 UDEC model for 1-D shear wave propagation across single joint

Table 3.2 Properties of the rock and the shear wave

Properties	Value
Density (ρ) (kg/m^3)	2650
Bulk modulus (K) (GPa)	44
Shear modulus (G) (GPa)	39
Wave type	Harmonic shear wave
Velocity of shear wave (c_s) (m/s)	3830

A continuous sinusoidal shear stress wave with unity amplitude and a frequency of 50 Hz is tangentially applied to the left side boundary of the model. The analysis for different values of shear stiffness of the joint is performed. The shear stiffness considered are: 0.001G, 0.005G, 0.01G, 0.015G, 0.02G, 0.03G, 0.05G, 0.1G, 0.2G, 0.3G and 0.4G. (It is noted that j_{kn} and j_{ks} are digitally given the values of K or G times a certain factor. The units of j_{kn} and j_{ks} are stress per length, different from the units of K and G). Therefore, the corresponding values of normalized shear stiffness $j_{ks}/(\omega z)$ are: 0.01, 0.06, 0.12, 0.183, 0.243, 0.365, 0.61, 1.22, 2.43, 3.65 and 4.87, respectively.

The steady-state UDEC results (after more than five periods from the beginning of the wave arrival) are obtained for magnitude of transmission coefficient $|T_1|$ (measured at point A) and are shown in Figure 3.4.

Figure 3.4 shows a comparison between the analytical solution and UDEC results for the variation of magnitude of transmission coefficient $|T_1|$ with the normalized shear stiffness $j_{ks}/(\omega z)$. As seen from the figure, there is a good agreement between

the analytical solution and UDEC results, supporting the capability of UDEC in analyzing 1-D wave propagation through single joint.

Figure 3.4 Comparison between UDEC results and analytical solution, in terms of $|T_1|$, for single joint case

3.2.3 1-D wave propagation through multiple parallel joints

Different from the case of single joint, superposition of multiple reflected and transmitted waves between two adjacent joints occurs when a wave propagates across multiple parallel joints. It is difficult to explicitly analyze the superposition process of multiple reflected and transmitted waves. Cai (2001) used the method of characteristics to solve the problem in 1-D wave propagation in terms of the magnitude of transmission coefficient $|T_n|$ for the case of multiple parallel joints. A set of recurrence equations with respect to particle velocities before and after the joints was established. The equations were then numerically solved. Based on the solutions of particle velocity and displacement, the magnitude of transmission coefficient $|T_n|$ was calculated from the ratio of the amplitude of the transmitted wave to that of the incident wave.

Figure 3.5 shows the variation of $|T_n|$, calculated using the method of characteristics given by Cai (2001), with the ratio of joint spacing to wavelength (ξ) for $jks/(\omega z)$ of 1.41 for cases with different numbers of joints ($n=2, 3$ and 5).

As seen from Figure 3.5, for a given n value, $|T_n|$ is influenced by two important indices of ξ , namely ξ_{cri} and ξ_{thr} . If $\xi \geq \xi_{thr}$, $|T_n|$ is independent of joint spacing; if $\xi_{thr} < \xi < \xi_{cri}$, $|T_n|$ increases as the joint spacing decreases; if $\xi \leq \xi_{cri}$, $|T_n|$ decreases as the joint spacing decreases. On the other hand, if $\xi \geq \xi_{thr}$, $|T_n|$ depends strongly on the joint number n ; if $\xi_{thr} < \xi < \xi_{cri}$, $|T_n|$ decreases weakly with n , and if $\xi \leq \xi_{cri}$, the decreasing momentum of $|T_n|$ becomes further weaker and even independent of n . It is found that both ξ_{thr} and ξ_{cri} are dependent on the value of $jks/(\omega z)$ and the number of joints.

Based on the model shown in Figure 3.3, additional joints are subsequently introduced into the model for examining the effects of different numbers of joints. UDEC model for multiple parallel joints is shown in Figure 3.6. All the joints are uniformly spaced and are of the same shear stiffness. For each case with a given number of joints, the joints are evenly spaced at the spacing of $\lambda/50$, $\lambda/20$, $\lambda/14$, $\lambda/10$, $3\lambda/20$, $\lambda/5$, $\lambda/4$ and $3\lambda/10$ respectively, where λ is the wavelength. There are

always two measuring points, point *A* and point *B*. Point *A* is always fixed at $x=10$ m, $y=1$ m, and point *B* is positioned immediately after the last joint in the joint set.

The bulk modulus and shear modulus of the rock are 44 GPa and 39 GPa respectively. The density of the rock is 2650 kg/m^3 , therefore, the shear velocity in this rock is 3830 m/s. The shear stiffness of the joint is 4.5 GPa/m. To ensure no frictional damage occurring in the process of the wave propagation, the cohesion of the joint is assumed to be of a relatively high value.

A sinusoidal shear stress wave with unity amplitude is tangentially applied to the left side. The wave frequency is 50 Hz, giving a corresponding $jks/(\omega z)$ value of 1.41. In present study, the magnitude of the transmission coefficient $|T_n|$ for the case of multiple joints is obtained for $jks/(\omega z)$ of 1.41 for different numbers of joints at $n=2, 3$ and 5.

Figures 3.7 (a), (b) and (c) show comparisons between the results obtained from the method of characteristics and UDEC results in terms of $|T_n|$ for the cases of $n=2, 3$ and 5, respectively.

It can be seen from the figures that UDEC results reasonably agree with those from the method of characteristics, while UDEC results are consistently and lightly larger than those given by Cai (2001). This supports and verifies the use of UDEC in 1-D wave propagation problems through multiple parallel rock joints.

Figure 3.5 Variation of $|T_n|$ with ratio of ξ of joint spacing to wavelength (solutions from the method of characteristics, $n=2, 3$ and 5)

Figure 3.6 UDEC model for 1-D wave propagation across multiple joints

(a) Comparison between results from UDEC and the method of characteristics for $|T_2|$ as a function of ratio of ξ of joint spacing to wavelength

(b) Comparison between results from UDEC and the method of characteristics for $|T_3|$ as a function of ratio of ξ of joint spacing to wavelength

Figure 3.7 Comparison between results from UDEC and the method of characteristics for $|T_2|$, $|T_3|$ and $|T_5|$ as a function of ratio of ξ of joint spacing to wavelength

(c) Comparison between results from UDEC and the method of characteristics for $|T_5|$ as a function of ratio of ξ of joint spacing to wavelength

Figure 3.7 (Continued) Comparison between results from UDEC and the method of characteristics for $|T_2|$, $|T_3|$ and $|T_5|$ as a function of ratio of ξ of joint spacing to wavelength

3.3 A study on verification of UDEC in 2-D wave propagation

This part aims to verify UDEC modeling in 2-D wave propagation. The case of harmonic wave from a cylindrical cavity in an elastic medium is chosen to perform the study. The analytical solution in terms of radial stress wave given by Graff (1979) is compared with UDEC results, and the analytical solutions in terms of displacement and velocity waves given by Hefny (2004) are compared with UDEC results as well.

3.3.1 Harmonic waves from a cylindrical cavity in an elastic medium-background of the analytical solution

Suppose that there is an infinite cylindrical cavity of radius a in an elastic medium, with the axis of the cavity coinciding with the z -axis. If all the loads on the cavity surface are z independent, the problem is two dimensional in terms of x and y in X-Y coordinate system, or r and θ in polar coordinate system. Figure 3.8 shows the problem in polar coordinate system.

Figure 3.8 Problem of harmonic compressional wave from a cylindrical cavity in an elastic medium

The governing scalar wave equation is:

$$\nabla^2 \Phi = \frac{1}{c_p^2} \frac{\partial^2 \Phi}{\partial t^2} \quad (3.3)$$

where Φ , t , c_p and ∇^2 are potential function, time, compressional wave velocity and Laplacian operator respectively.

Suppose a uniform harmonically varying pressure (τ_{rr}) is acting on the cavity surface $r=a$, such that $\tau_{rr}(a) = -p_0 e^{-i\omega t}$, where p_0 is the amplitude of the harmonic wave.

The radial stress wave at any point in the surrounding medium given by Graff (1979) can be expressed as:

$$\tau_{rr} = -\mu(k^2 \beta^2 \Phi + \frac{2}{r} \frac{d\Phi}{dr}) \quad (3.4)$$

where the deduced parameters β^2 and k^2 are expressed as: $\beta^2 = \frac{\omega^2}{c_p^2}$ and

$k^2 = \frac{\lambda + 2\mu}{\mu}$. Φ is the potential function, which can be expressed by the following

equation for this problem:

$$\Phi(r, t) = \frac{p_0}{\mu N(\beta a)} H_0^{(1)}(\beta r) e^{-i\omega t} \quad (3.5)$$

$\frac{d\Phi}{dr}$ can be expressed as:

$$\frac{d\Phi}{dr} = \frac{\beta}{\mu N(\beta a)} p_0 e^{-i\omega t} \left[-H_1^{(1)}(\beta r) \right] \quad (3.6)$$

In Equations (3.5) and (3.6), $N(\beta a)$ is expressed as:

$$N(\beta a) = k^2 \beta^2 H_0^{(1)}(\beta a) - \frac{2\beta}{a} H_1^{(1)}(\beta a) \quad (3.7)$$

where $H_m^{(n)}(x)$ is Hankel function.

The displacement and velocity given by Hefny (2004) are expressed as:

$$u = \frac{d\Phi}{dr} = \frac{\beta}{\mu N(\beta a)} p_0 e^{-i\omega t} \left[-H_1^{(1)}(\beta r) \right] \quad (3.8)$$

$$v = \frac{du}{dt} = \frac{i\omega\beta}{\mu N(\beta a)} p_0 e^{-i\omega t} \left[H_1^{(1)}(\beta r) \right] \quad (3.9)$$

3.3.2 Comparison between analytical and UDEC results

In this section, the results obtained from the analytical solutions are compared with UDEC results for two cases. The problem considered for the two cases, designated as case A and case B, is that of a harmonic wave propagating in an elastic rock from a cylindrical cavity as shown in Figure 3.8.

For case A, a cavity with a radius of 3 m exists in an infinite elastic rock shown in Figure 3.8. A compressional harmonic stress wave with unity amplitude and a frequency of 275 Hz is applied at the boundary of the cavity. The bulk modulus (K) of the rock material is 44 GPa, the shear modulus of the rock (G) is 39 GPa, and density (ρ) is 2650 kg/m³. For case B, a cavity with a radius of 5 m exists in an infinite elastic rock also shown in Figure 3.8. The compressional harmonic stress wave with unity amplitude and different frequencies of 100 Hz, 700 Hz and 1500 Hz is applied at the boundary of the cavity. The bulk modulus of the rock (K) is 40 GPa, the shear modulus of the rock (G) is 26 GPa and density (ρ) is 2650 kg/m³.

The detailed parameters for cases A and B are systematically listed in Table 3.3. These two cases have (1) different radii of the cavities, (2) different frequencies of the input harmonic waves, and (3) different rock materials.

Table 3.3 Parameters for cases A and B

Parameter	Value (Case A)	Value (Case B)
Bulk modulus (K)(GPa)	44	40
Shear modulus (G) (GPa)	39	26
Density (ρ) (kg/m ³)	2650	2650
Wave type	Harmonic sine wave	Harmonic sine wave
Frequency (Hz)	275	100, 700 and 1500
Radius of the cavity (a) (m)	3.0	5.0

3.3.2.1 Analytical results

The analytical results at several points with different radial distances from the center of the wave source, in terms of radial stress wave, displacement wave and velocity wave, are given by Equations (3.4), (3.8) and (3.9) respectively.

A MATLAB file is coded to calculate the analytical radial stress wave, displacement wave and velocity wave. A sample of the obtained results is shown in Table 3.4 for case A, in terms amplitude of displacement, velocity and radial stress waves. Complete results for case A and case B are visually illustrated and compared with UDEC results in Section 3.3.2.3.

Table 3.4 Analytical solution in terms of amplitude of displacement, velocity and radial stress for case A

Point No. i	r_i (m)	Amplitude of displacement wave (m) for Case A	Amplitude of velocity wave (m/s) for Case A	Amplitude of radial stress wave (Pa) for Case A
1	3.0	3.92×10^{-11}	6.59×10^{-8}	1
2	3.2	3.75×10^{-11}	6.31×10^{-8}	0.942
3	4.0	3.24×10^{-11}	5.44×10^{-8}	0.794
4	6.0	2.55×10^{-11}	4.25×10^{-8}	0.631
5	10	1.92×10^{-11}	3.19×10^{-8}	0.492

3.3.2.2 UDEC modeling

Figure 3.9 shows the corresponding UDEC model for cases A and B. The geometry is 30 m $\times$ 30 m. The left and lower boundaries are symmetrical boundaries. The right and upper boundaries are viscous boundaries to ensure no reflection from these boundaries. The cavity, with a 3.0/5.0 meters radius, is located at the lower corner on the left. The mesh size value of 0.2 m is chosen to control the numerical error less than 2% (refer to Section 4.3).

In order to compare the analytical and UDEC results, the corresponding parameters as shown in Table 3.3 are coded into the corresponding UDEC input file for cases A and B, respectively.

For case A, a compressional harmonic stress wave is normally applied on the boundary of the cylindrical cavity, with unity amplitude and a frequency of 275 Hz. The displacement, velocity and radial stress waves for points (3.0, 0), (3.2, 0), (4, 0), (6, 0) and (10, 0) are recorded.

For case B, a compressional harmonic stress wave is normally applied at the boundary of the cylindrical cavity, with unity amplitude and different frequencies at 100 Hz, 700 Hz and 1500 Hz. The radial stress waves at several points in the range (5, 0) to (25, 0) are recorded.

Figure 3.9 Sketch of UDEC model for cases A and B

3.3.2.3 Comparison between the analytical solutions and UDEC results

For case A, Figures 3.10 (a), (b) and (c) show the comparison between the analytical and UDEC results, in terms of displacement, velocity and radial stress waves, respectively. It can be seen that there are good agreements between the analytical and UDEC results.

Figures 3.11 (a), (b) and (c) show the comparison of radial stress amplitude between analytical and UDEC results for all the tested frequencies of 100 Hz, 700 Hz and 1500 Hz. It can be seen that there are good agreements between analytical and UDEC results.

(a) Comparison of amplitude of displacement wave between analytical and UDEC results

Figure 3.10 Comparison between analytical and UDEC results for case A, in terms of amplitude of displacement, velocity and radial stress waves

(b) Comparison of amplitude of velocity wave between analytical and UDEC results

(c) Comparison of amplitude of radial stress wave between analytical and UDEC results

Figure 3.10 (Continued) Comparison between analytical and UDEC results for case A, in terms of amplitude of displacement, velocity and radial stress waves

(a) Comparison of amplitude of radial stress wave between analytical and UDEC results for frequency of the input wave 100 Hz

(b) Comparison of amplitude of radial stress wave between analytical and UDEC results for frequency of the input wave 700 Hz

Figure 3.11 Comparison of amplitude of radial stress wave between analytical and UDEC results for different frequencies of the input wave for Case B

(c) Comparison of amplitude of radial stress wave between analytical and UDEC results for frequency of the input wave 1500 Hz

Figure 3.11 (Continued) Comparison of amplitude of radial stress wave between analytical and UDEC results for different frequencies of the input wave for Case B

In order to quantify the comparison between the analytical and UDEC results, the error in UDEC results for every monitoring point is calculated as:

$$|\text{error}_i| = \left| \frac{A_{i(\text{UDEC})} - A_{i(\text{analytical})}}{A_{i(\text{analytical})}} \right| \times 100\% \quad (3.10)$$

Where parameter A is the amplitude of wave, i is the monitoring point, therefore $A_{i(\text{UDEC})}$ stands for amplitude of a wave for a point from UDEC results; $A_{i(\text{analytical})}$ stands for amplitude of a wave for a point from analytical solutions.

Table 3.5 shows the average error for case A in terms of displacement, velocity and radial stress waves. Table 3.6 shows the average error for case B in terms of radial stress wave for different frequencies.

It can be seen from Table 3.5 and Table 3.6 that the average error in UDEC results is less than 2% for case A and case B.

The good agreement obtained between the analytical and UDEC results for different conditions verifies that UDEC can model 2-D dynamic problems at a high degree of accuracy.

It can be seen from Table 3.6 that the average error in UDEC results is dependent on frequency of the input wave. This study on UDEC modeling accuracy will be performed in Chapter 4.

Table 3.5 Average error (%), in terms of displacement, velocity and radial stress waves for case A

	Error for displacement wave	Error for velocity wave	Error for radial stress wave
Ave. error (%)	0.75	0.85	1.17

Table 3.6 Average error (%), in terms of radial stress wave, for case B

Frequency (Hz)	Average error (%)
100	0.82
700	1.13
1500	1.78

3.4 Summary

This chapter aims to verify UDEC modeling in dynamic problems, both in 1-D and 2-D wave propagations. A series of tasks on verification study are carried out.

For 1-D wave propagation through single joint, the magnitude of transmission coefficient $|T_1|$ obtained from UDEC results is compared with that obtained from the analytical solution given by Pyrak-Nolte *et al.* (1987 and 1990a). For 1-D wave propagation through multiple parallel joints, the magnitude of transmission coefficient $|T_n|$ obtained from UDEC results is compared with that obtained from the method of characteristics (Cai 2001). For all these cases, UDEC results agree well with results from the analytical solutions and the method of characteristics.

For 2-D case, the problem of harmonic wave propagating from a cylindrical cavity in an elastic medium is chosen. UDEC results are compared with the analytical solutions in terms of radial stress wave given by Graff (1979) and in terms of displacement and velocity waves given by Hefny (2004). Good agreements are achieved between the analytical solutions and UDEC results, in terms of displacement, velocity and radial stress waves. The average error in UDEC results is less than 2% for all the cases considered using small mesh size. The effects of mesh size on UDEC results will be studied in detail in Chapter 4.

From all these verification studies in this part, it can be concluded that UDEC is capable of modeling dynamic problems.

CHPATER 4

A STUDY ON UDEC MODELING ACCURACY

In order to keep a consistent numerical error at a reasonable magnitude in UDEC calculation and to avoid masking the effects of joints for different modeling conditions in the process of 2-D wave propagation in a rock mass, the effects of the factors influencing accuracy of UDEC results must be investigated. This investigation is very important in this thesis where extensive parametric studies are performed on effects of joints in the process of 2-D wave propagation under different conditions. Therefore, in this chapter, a study on effects of the factors influencing accuracy of UDEC results is performed.

The factors that may affect the accuracy of UDEC results in modeling 2-D wave propagation are stiffness of fictitious joints and mesh ratio (l_r), which is defined as mesh size to wavelength. The wave frequency is related to the wavelength as: wavelength is equivalent to wave velocity over frequency. Therefore, from the definition of the term of mesh ratio (l_r), it can be seen that the mesh size and frequency of the input wave are involved in UDEC modeling accuracy. In this chapter there are three focuses. (1) The first topic focuses on studying effects of fictitious joints on accuracy of UDEC results, which are performed in Section 4.1. In order to construct a UDEC model for a given geometry, often it is necessary to introduce fictitious joints into the model. Fictitious joints are also necessary in order to be able to change the mesh size in a model. When fictitious joints are introduced in a model, the surrounding blocks must be glued at the fictitious joints to prevent slip and separation. (2) The second topic focuses on studying effects of mesh size and frequency of the input wave on accuracy of UDEC results, which are presented in Section 4.2. In the process of analysis of effects of mesh size and frequency of the input wave, the verification example of 2-D harmonic wave case, presented in Section 3.3, provides a sound basis for the study on UDEC modeling accuracy. The study on UDEC modeling accuracy is performed by comparing the UDEC results with the analytical solutions given by Graff (1979), under different mesh sizes for different frequencies of the input wave. (3) Finally the effects of mesh ratio (l_r) on

accuracy of UDEC results are generalized in Section 4.3. Based on the analysis, a relationship between the numerical error and mesh ratio (l_r) is established.

Based on the analysis results performed in this chapter, values of the factors influencing the accuracy of UDEC results under given conditions are recommended and these values are applied to the corresponding analysis throughout this thesis.

4.1 Effects of fictitious joints

In a UDEC model, for some cases, such as in a model with different zoning mesh size, or a model with a separate void block, it is required to include fictitious joints. When fictitious joints are included in the model, the surrounding blocks must be glued at the fictitious joints to prevent slip and separation. This is accomplished by setting cohesion and tensile strength of the fictitious joints to high values. Shear and normal stiffness of the fictitious joints must also be set to high enough values to preclude differential joint displacement between the adjacent blocks. So, It is tempting to give very high values for stiffness to prevent displacement along the fictitious joints. However, the time-step in UDEC is based upon stiffness of the joints; the response will be very slow for very high stiffness. In order to keep the calculation reasonably fast, it is recommended in UDEC manual to use proper value for the normal stiffness (j_{kn}) and shear stiffness (j_{ks}) of the fictitious joints, as described in Equation (4.1).

$$j_{ks} \ \& \ j_{kn} \leq 10 \times \max \left[\left(K + \frac{4}{3}G \right) / \Delta z_{\min} \right] \quad (4.1)$$

where $\Delta z_{\min}$ is the smallest width of an adjoining zone, K and G are bulk and shear modulus of the rock material. The value $\left[\left(K + \frac{4}{3}G \right) / \Delta z_{\min} \right]$ is called equivalent stiffness of the zone. In this section the equivalent stiffness of a zone is termed as K_{eq} .

Although the recommended value of K_{eq} has set the upper limit of stiffness of the fictitious joint and may keep the calculation reasonably fast, the effects on accuracy

of UDEC results are not known. In this section, the effects of the normal and shear stiffness of the fictitious joint on accuracy of UDEC results are investigated. The lower limit of stiffness of the fictitious joint is recommended to ensure accuracy of UDEC results.

4.1.1 Problem statement and modeling condition

The problem considered in this study is shown in Figure 4.1. A cavity with a radius of 5 m exists in an infinite rock mass with single joint. The joint is supposed to be fictitious joint, parallel to the y-axis with a normal distance of 10 m from the center of the cavity. The lower, right and top boundaries are viscous boundaries, while the left boundary is a symmetrical boundary. The mesh size is 0.4 m. A sinusoidal compressional pulse stress wave with unity amplitude and a frequency of 400 Hz is applied at the boundary of the cavity.

K value of the rock material is 44 GPa, G value is 39 GPa, and density is 2650 kg/m³. These properties are similar to those of the Bukit Timah Granite in Singapore.

Figure 4.1 Sketch of UDEC model for effects of fictitious joint

Different values are given to the normal stiffness of the fictitious joint, while the shear stiffness is kept constant at 70K. The normal stiffness of the joint considered are 0.1K, 0.2K, 0.3K, 0.4K, 0.5K, 0.6K, 0.7K, 0.8K, 0.9K, 1.0K, 2K, 3K, 4K, 5K, 6K, 7K, 8K, 9K, 10K, 15K, 20K, 50K and 70K. (It is noted that j_{kn} and j_{ks} are digitally given the values of K or G times a certain factor. The units of j_{kn} and j_{ks} are stress per length, different from the units of K and G).

4.1.2 Results and discussion

History of the transmitted radial stress wave is monitored at several points along the x-axis, behind the joint to the right boundary of UDEC model with an interval of 1 m.

First, UDEC model without the fictitious joint is run, and the history of radial stress wave for every monitoring point is recorded. And then UDEC model with the fictitious joint is run, and the history of radial stress wave for every monitoring point is also recorded. The amplitude of transmitted radial stress wave from the model with the fictitious joint is compared with that of the radial stress wave in an intact rock at every monitoring point.

The ratio of A_{joint} to A_{intact} is used to evaluate the effects of the fictitious joint, where A_{joint} is amplitude of the radial stress wave at a monitoring point obtained from the model with fictitious joint, and A_{intact} is the amplitude of the radial stress wave at the same point from the model of intact rock.

The average ratios of A_{joint} to A_{intact} for every case with different values of stiffness of the fictitious joint are plotted versus j_{kn}/K_{eq} in Figure 4.2 (a). In order to clearly illustrate the effects of the fictitious joint at small values of j_{kn}/K_{eq} , the results are re-plotted in Figure 4.2 (b) for values of j_{kn}/K_{eq} less than 4.

From the results shown in Figures 4.2 (a) and (b), it can be seen that when the value of j_{kn}/K_{eq} is greater than 2, the ratio reaches the maximum value of 1 and does not change with any further increasing in j_{kn}/K_{eq} . This ratio sets the lower limit of stiffness of the fictitious joint where the accuracy of UDEC results is kept at a reasonable level.

(a) Effects of fictitious joint ($jkn/K_{eq} < 20$)

(b) Effects of fictitious joint ($jkn/K_{eq} < 4$)

Figure 4.2 The effects of fictitious joint for different ranges of jkn/K_{eq}

4.2 Effects of mesh size and frequency of the input wave

It has been widely recognized that the mesh size of the numerical model significantly influences the accuracy of numerical results in the process of wave propagation, whether the model is based on continuum or discontinuum mechanism. Theoretically, a numerical model with a finer mesh size produces more accurate results, because the model is hardened by predefining a function to represent the displacement or stress field in the elements

The time-step calculation in UDEC is based upon stiffness. The response and solution convergence will be very slow if very high stiffness is specified. For too fine mesh element, the equivalent stiffness (K_{eq}) of a zone is too high, resulting in a too long computation time for a dynamic problem. How to achieve a proper balance between the mesh size and the computation accuracy for a dynamic problem in UDEC modeling is a problem. The study in this section is to investigate the effects of the mesh size for several given frequencies in the process of 2-D wave propagation.

In the UDEC analysis presented in the main parts (Chapters 5, 6, 7 and 8) in this thesis, several frequencies of the input 2-D wave are frequently used. For the purpose of comparison of UDEC results under different frequencies of the input wave, consistent numerical error must be ensured. It has been noted that the numerical error is controlled by the combined term mesh ratio (l_r) of mesh size over wavelength in several literatures (Kuhlmeyer and Lysmer 1973; Chen 1999; Cai 2001). However, in order to clearly show the values of mesh size under a given numerical error for different tested frequencies and illustrate the development of the numerical error against the value of mesh size for a specific frequency, the effects of mesh size and frequency of the input wave are presented in an independent section.

4.2.1 Review of the analytical solutions for harmonic wave from a cylindrical cavity in an elastic medium

In order to study the effects of the mesh size and frequency of the input wave, the mesh size is changed for every model to study its effects on accuracy of UDEC

results. The solution from every model with different mesh size is always compared with the analytical solution. So the analytical solution for harmonic waves from a cylindrical cavity in an ideal elastic medium is first reviewed.

The problem of harmonic waves from a cylindrical cavity in an ideal elastic rock is stated in Section 3.3.1. The solution of the problem, in terms of the radial stress at any point in the surrounding rock, is expressed by Equation (3.4). Calculation of the analytical solution in terms of radial stress wave at any point for harmonic wave from a cylindrical cavity in an ideal elastic medium is performed and automated using a MATLAB file.

4.2.2 Problem statement and modeling conditions

The problem considered is shown in Figure 4.3. A cavity with a radius of 5 m exists in an infinite intact rock. A quarter symmetrical model with a dimension of 30 m × 30 m is used to fully make use of the memory of the computer and to reduce the computational time. The top and right boundaries are viscous boundaries, while the left and the lower boundaries are symmetrical boundaries. The compressional harmonic stress wave with unity amplitude is applied at the boundary of the cavity.

K value of the rock material is 40 GPa, G value is 26 GPa, and density is 2650 kg/m³. These are typical values for Bukit Timah granite in Singapore (Lee 2002).

Different frequencies at 200 Hz, 300 Hz, 400 Hz, 500 Hz, 700 Hz, 1000 Hz and 1500 Hz have been tested. For every frequency, mesh size is changed as 0.1 m, 0.2 m, 0.4 m, 0.8 m, 1.2 m, 1.6 m, 2.0 m, 2.5 m, 3.0 m and 5.0 m to study the effects of mesh size and frequency on UDEC modeling accuracy.

Figure 4.3 Sketch of UDEC model for effects of mesh size

4.2.3 Results and discussion

Time histories of the radial stress wave are monitored at several points along x-axis. In the first 1 m, spacing of the monitoring points is 0.2 m. In the next 3 m, the spacing of the monitoring points is 0.5 m. In the last 15 m, the spacing of the monitoring points is 5 m.

The Error is defined as:

$$\text{error}_i = \frac{|A_{i,j} - A_{i \text{ (Analytical)}}|}{|A_{i \text{ (Analytical)}}|} \times 100\% \quad (4.2)$$

where $A_{i,j}$ is the amplitude of the radial stress wave from UDEC results at monitoring point i for mesh size value j , $A_{i \text{ (Analytical)}}$ is the amplitude of the radial stress wave calculated from Equation (3.4) for monitoring point i .

For purpose of illustration only, Figures 4.4 (a), (b) and (c) show the comparisons between the analytical and UDEC results in terms of the amplitude of the radial

stress wave, for three frequencies (200 Hz, 500 Hz and 1000 Hz, respectively) for mesh sizes varying from 0.1 m to 5 m.

As seen from these figures, cases with small mesh sizes give good agreements between the analytical and UDEC results, while UDEC results deviate roughly from the analytical results as the mesh size increases.

For practical UDEC modeling, the mesh size values yielding small average error have more significance. To illustrate the good agreement between the analytical and UDEC results, the comparisons for the cases that give an average error less than 2% are re-plotted in Figures 4.5 (a), (b) and (c) for frequencies of 200 Hz, 500 Hz and 1000 Hz respectively. It is noticed that comparisons between the analytical solution and UDEC results for the remaining tested frequencies are not visually shown in Figure 4.4 and Figure 4.5, however the study on effects on accuracy of UDEC results have been performed together with the above mentioned three frequencies.

(a) Comparison between analytical and numerical results for different mesh size for frequency of the input wave of 200 Hz

Figure 4.4 Comparison between analytical and numerical results for different mesh size for different frequencies of the input wave

(b) Comparison between analytical and numerical results for different mesh size for frequency of the input wave of 500 Hz

(c) Comparison between analytical and numerical results for different mesh size for frequency of the input wave of 1000 Hz

Figure 4.4 (Continued) Comparison between analytical and numerical results for different mesh size for different frequencies of the input wave

(a) Comparison between analytical and numerical results for mesh size with less than 2% average error (frequency of the input wave=200 Hz)

(b) Comparison between analytical and numerical results for mesh size less than 2% average error (frequency of the input wave=500 Hz)

Figure 4.5 Comparison between analytical and numerical results for mesh size values, with less than 2% average error

(c) Comparison between analytical and numerical results for mesh size values less than 2% average error (frequency of the input wave=1000 Hz)

Figure 4.5 (Continued) Comparison between analytical and numerical results, with less than 2% average error

Figure 4.6 shows the average error versus mesh size for all the tested frequencies.

It can be seen from Figure 4.6 that the accuracy of UDEC results are highly dependent on the mesh size adopted, for every given value of wave frequency. In general, as the mesh size increases, the average error increases for every given frequency. With the same mesh size, the average error increases as the frequency increases. For example, Table 4.1 shows the values of average error corresponding to different given values of frequency, while the mesh size is fixed at 2.0 m. It means, to keep the average error in a certain range, for higher frequency, it requires finer mesh size.

In practical UDEC modeling, 5% and 2% are often taken as the acceptable values of average error. Table 4.2 and Table 4.3 show the recommended upper limits of mesh size for different given frequency that yield average errors less than 5% and 2%, respectively.

Although, the finer mesh size can yield accurate UDEC results, it requires a larger computer memory and takes longer computational time. For the mesh size value, within the acceptable average error, the computational time varies much with different values of mesh size. For example, Table 4.4 shows the computational time (CPU speed is 2.0 GHz), for different mesh size values, with less than 2% average error for the case of 700 Hz. From Table 4.4, it can be seen that decreasing the mesh size from 0.4 m to 0.1 m improves the average error from 1.3% to 0.7%, however the computational time dramatically increases from 5 minutes to 313 minutes. Therefore, it is necessary to keep a proper balance between the mesh size and the computational time without much suffer to UDEC results.

Figure 4.6 Effect of frequency for different mesh size for multiple frequencies

Table 4.1 Average error for cases with different frequencies at mesh size of 2.0 m

Frequency (Hz)	200	300	400	500	700	1000	1500
Average error (%)	3.5	5.7	9.1	16.5	21.7	45.8	146.0

Table 4.2 Upper limit of mesh size giving less than 5% average numerical error

Frequency (Hz)	200	300	400	500	700	1000	1500
Upper limit of mesh size (m)	3.5	1.8	1.6	1.2	0.7	0.4	0.4

Table 4.3 Upper limit of mesh size giving less than 2% average numerical error

Frequency (Hz)	200	300	400	500	700	1000	1500
Upper limit of mesh size (m)	0.8	0.8	0.8	0.7	0.5	0.4	0.2

Table 4.4 Computational time and numerical error for different mesh size

	Mesh size=0.1 m	Mesh size=0.2 m	Mesh size=0.4 m
Computation time (min.)	313	40	5
Average error (%)	0.7	0.8	1.3

4.3 Effects of mesh ratio

In 1-D wave propagation, the studies showed that the accuracy of numerical results is governed by the mesh ratio (l_r) which is defined as mesh size over wavelength, (Chen 1999; Cai 2001). In Section 4.2, effects of mesh size and frequency on UDEC modeling accuracy are studied. In this part, the effects of mesh ratio (l_r) on accuracy of UDEC results in the process of 2-D wave propagation are studied.

Figure 4.7 (a) shows the average error versus the mesh ratio (l_r), obtained from the same data shown in Figure 4.6. It is noted that in order to clearly show the relationship between average error and mesh ratio (l_r), Figure 4.7 (a) only shows mesh ratios (l_r) less than 0.5 and the corresponding values of average error, which is already very big in UDEC modeling. From Figure 4.7 (a), it can be seen that small values of mesh ratio (l_r) yield small average error, and the average error increases as the mesh ratio (l_r) increases. It also can be seen that for most cases the average error increases more dramatically as the mesh ratio (l_r) increases above about 0.2. For practical UDEC modeling, small mesh ratio (l_r) is more meaningful. Therefore, the average error against the mesh ratio (l_r) in the range of $l_r < 0.2$ is highlighted.

Figure 4.7 (b) shows the average error against the mesh ratio (l_r) in the range of $l_r < 0.2$. The relationship between average error and mesh ratio (l_r) is best fitted to the straight line:

$$y = 37.24x + 0.51 \quad (4.3)$$

where y is the numerical error and x is mesh ratio (l_r).

(a) Average error against mesh ratio (l_r) ($l_r < 0.5$) (b) Average error against mesh ratio (l_r) ($l_r < 0.2$)Figure 4.7 Average error against mesh ratio (l_r) in different ranges

In 1-D wave propagation problem, Kuhlemyer and Lysmer (1973) recommended a mesh ratio (l_r) as $l_r < 1/10$ to $1/8$. Cai (2001) conducted similar research on effects of mesh ratio (l_r) in 1-D wave propagation, the mesh ratio (l_r) of about $1/10$ was recommended for less than about 5% numerical error.

From Equation (4.3), in 2-D wave propagation problems, a mesh ratio (l_r) of $1/8$ should be adopted to keep the average error below 5%, and a mesh ratio (l_r) of $1/25$

should be considered to keep the average error below 2%. In the research performed in this thesis, the recommended mesh ratio (l_r) of 1/25 is always adopted to keep the average error below 2%.

4.4 Summary

In this chapter, the factors that may affect the accuracy of UDEC results (such as the normal and shear stiffness of fictitious joints, mesh size and wave frequency) in modeling 2-D wave propagation are studied in detail.

For effects of fictitious joints, it is recommended in UDEC manual to use the values for jkn/K_{eq} and jks/K_{eq} less than 10 to ensure that the calculation is reasonably fast. However, the effects of jkn/K_{eq} and jks/K_{eq} on the accuracy of UDEC results have not been studied before. Also the lower limit of these values is not known. The study performed in this chapter on the effects of the stiffness of fictitious joints on the accuracy of UDEC results show that the lower limit of these values is 2. For values less than 2, the error increases as the stiffness ratio of jkn/K_{eq} and jks/K_{eq} decreases. For values of stiffness ratio of jkn/K_{eq} and jks/K_{eq} greater than 2, the computational error is negligible.

The studies presented in this chapter also show that the mesh size and frequency of the input wave greatly affect the accuracy of UDEC results. For the effects of mesh size on the accuracy of UDEC results, it is found that, for a given input wave frequency, the average error increases as the mesh size increases. For the effects of the frequency of the input wave on the accuracy of UDEC results, for the same mesh size, the average error increases as the frequency increases.

It has been found that small values of mesh ratio (l_r), which is defined as mesh size to wavelength, yield small average error, and the average error increases as the mesh ratio (l_r) increases. Average error increases more dramatically as the mesh ratio (l_r) increases above value of 0.2. In the range of mesh ratio (l_r) less than 0.2, the relationship between average error and mesh ratio (l_r) is best fitted by the linear equation: $y=37.24x+0.51$, where y is the numerical error and x is mesh ratio (l_r). From the relationship between average error and mesh ratio (l_r) in 2-D wave

propagation problems, a mesh ratio (l_r) of 1/8 is recommended to keep the average error below 5%, while a mesh ratio (l_r) of 1/25 is also recommended to keep the average error below 2%.

In the studies performed in subsequent chapters, the mesh ratio of 1/25 is always adopted to keep the average numerical error consistently below 2%.

CHAPTER 5

DEVELOPMENT OF A DATA PROCESSING METHOD AND A STUDY ON 2-D COMPRESSIONAL WAVE PROPAGATION IN AN INTACT ROCK

5.1 Introduction

Compressional wave propagation from a tunnel or borehole in a rock mass, with single joint or multiple parallel joints, is a complicated problem. The amplitude corresponding to one point in the surrounding rock mass for a compressional wave originating from the boundary of a tunnel or borehole may be different from the amplitude corresponding to another point, influenced by a set of possible factors. The factors include the position of the point, the parameters of the joint or the joint set, and the size of the tunnel or borehole from which the wave originates. Moreover, even for 2-D compressional wave propagation in an intact rock, the amplitude corresponding to a point depends on the radial distance of the point due to geometry damping and the radius of the cavity from which the wave originates. The amplitude in an intact rock will be often used as reference to study the transmission ratio in 2-D wave propagation in Chapters 6, 7, 8 and 9. These factors make the problem of 2-D compressional wave propagation in a rock mass different from 1-D wave propagation in the rock mass.

This chapter is organized as follows:

Firstly, Section 5.2 includes the fundamental concepts, the transmission ratio (T_n) and radian angle (α) in 2-D wave propagation problem. These two concepts must be clearly defined, because these two concepts provide a sound basis for the analysis of 2-D compressional wave propagation throughout all the main parts in Chapters 6, 7, 8, and 9 in this thesis. Secondly, two technical methodologies in data processing are developed in Section 5.3 and Section 5.4. Specifically proposed for 2-D wave propagation problem in this thesis, however comprehensively suitable for dynamic problems in a rock mass, a novel method of comprehensive processing wave amplitude in UDEC modeling is developed in Section 5.3. The data fitting method

which is comprehensively involved in Chapters 6, 7 and 8 is presented in Section 5.4. Thirdly, as an important reference for the study on 2-D compressional wave propagation in a rock mass with joints, a study on 2-D compressional wave propagation in an intact rock is presented in Section 5.5.

5.2 Fundamental definitions

5.2.1 The transmission ratio T_n

This sub-section provides the definition of the transmission ratio T_n for a wave from a cavity or borehole traveling across a joint or a set of joints in the rock mass. In the remaining parts of the thesis, this definition of the transmission ratio T_n is applied.

Consider the problems shown in Figures 5.1 (a) and (b), in which a 2-D compressional wave applied at the boundary of a tunnel or borehole is passing through single joint and multiple parallel joints respectively. The transmission ratio corresponding to point A, which is located behind the existing single joint (Figure 5.1 (a)) or behind the last joint for multiple parallel joints situation (Figure 5.1 (b)), is defined as:

$$T_n = \frac{A_{\text{joint}}}{A_{\text{intact}}} \quad (5.1)$$

where the term A_{joint} is the amplitude of the transmitted wave at point A; and A_{intact} is the amplitude of the wave corresponding to the same point A in the case of intact rock. Charts will be presented in Section 5.5 for determination of the parameter A_{intact} at any point with any radial distance from the cavity center in the case of intact rock.

The transmission ratio T_n is a very important parameter for the performance measurement in 2-D wave propagation in a rock mass. Based on this definition, simplified models will be proposed for predicting the transmission ratio T_n for both cases of single joint and multiple joints in Chapters 7 and 8, respectively.

Figure 5.1 Sketch of 2-D wave propagation through single joint and multiple parallel joints

5.2.2 Radian angle

The radian angle α corresponding to a point (Such as point A in Figure 5.1 (a) and Figure 5.1 (b)) located behind the existing joint is defined as the angle between the joint (Figure 5.1 (a)) or the multiple parallel joints (Figure 5.1 (b)) and the radial line passing through the point, in clockwise direction. According to this definition, the problems shown in Figure 5.1 (a) and Figure 5.1 (b) would have a range of 0° to 180° . The radial line with radian angle of 90° indicates a line perpendicular to the joint(s).

5.3 Development of a new method of comprehensive processing of wave amplitude in UDEC modeling

5.3.1 Introduction

Although UDEC has strong computational capabilities, its output data processing method is limited, especially for dynamic problems. In this section, combining UDEC, MATLAB and EXCEL, a method for analysis of velocity/stress amplitude in dynamic problems in a jointed rock mass is proposed. This method is comprehensively used in the studies performed on 2-D wave propagation through single joint and multiple parallel joints in a rock mass, in Chapters 6, 7 and 8.

In dynamic problems in UDEC, time history is often recorded by *hist* command. Velocity history and stress history are most commonly recorded. Usually the coordinates of the point at which the history is monitored are pre-defined in the input file. This method has obvious shortcomings.

- For large number of points of interest, it is time consuming and practically impossible to predefine every point.
- The maximum number of time histories available in UDEC is not enough for a practical dynamic problem. There are many history types to record in a dynamic problem, such as strain rate, energy, velocity, stress, displacement, free-field stress/acceleration/displacement/velocity, fluid flow rate, etc. In addition, every history type has several components to be recorded. However, in standard UDEC code, a maximum of 500 time histories can be made in one run. This means in a UDEC run, only several points can be fully recorded with time histories.
- It takes a long time for computation. In conventional method, history and the position are predefined. If only limited number of histories are predefined and recorded for some purposes, UDEC model would be ran again for other purposes, with some other points and histories pre-defined.
- Depending on the mesh ratio (l_r) defined as mesh size to wavelength, the obtained history results could be of limited accuracy. In the conventional method, velocity and stress histories are recoded by *hist* command for any particular point (x,y) of interest. The outcome from *hist* command is not for the exact point (x,y), but for the central point or the grid-point of the zone nearest to the point (x,y).

Recognizing the shortcomings of the output data processing method in standard UDEC code, a new method to record and process velocity history for every grid-point and stress history for every central point of the zone is proposed. The following techniques are involved in this method:

- Modification of UDEC source code to increase the maximum number of histories and tables. In order to transfer the history information to a file, tables covering indexed arrays of number pairs are involved. However, like the maximum quantity of histories, 500 is also the maximum quantity of tables in standard UDEC code. The maximum quantity of tables is also needed to increase.
- Automation of UDEC input file for recording the time history for every grid-point or centroid of a zone
- Processing history data in MATLAB and visually presenting data in EXCEL.

5.3.2 Modification of UDEC source code

In UDEC, the upper limit of history and table number is 500. This is much less than what is needed to record velocity/stress history for every grid-point/central point of the zone even in a small model size. For example, a typical small UDEC model in 2-D wave propagation considered in this thesis, with a dimension of $100 \text{ m} \times 70 \text{ m}$ and a mesh size of 0.79 m , has about 25,000 grid-points. Therefore, it is obvious that the upper limit of history number in standard UDEC code must be increased. In this research, the variables controlling the upper limit of history and table number in UDEC source code are increased to 60,000. Then UDEC source code is recompiled.

5.3.3 Automating time histories in UDEC input file

Figure 5.2 shows a global structure in a UDEC model. Any UDEC model is composed of blocks. Any deformable block is composed of finite-difference zones. Triangular zones are often used in UDEC. Grid-points are associated with corners of the finite-difference zones. There are always three grid-points associated with each zone. A pair of x- and y- coordinates are defined for each grid-point, thus specifying the exact location of the finite difference zones.

There are a set of variables in UDEC to describe every block, zone and grid-point. The variables in UDEC are stored in “linked lists”, which consist of addresses (the first memory location for a particular item, i.e., block, zone or grid-point) and

offsets, which prescribe the memory location of the variable related to the address. Figure 5.3 shows the linked-list arrangement in the main data arrays

Table 5.1 shows the pre-defined variables and functions in UDEC involved in automating histories in UDEC modeling. These functions and commands make it possible to automatically record velocity/stress history for every grid-point/ centroid of a zone by FISH language. Because there are a lot of zones in a practical UDEC model, it requires big memory to store history information in tables, big capacity of hard drive to save the history files and fast processing speed.

With the latest powerful personal computer, it is now possible to automatically record the history information for very large number of grid-point/centroid. Although it still takes a long time in UDEC modeling and processing data in MATLAB using this method, about 2.5 to 12 hours for a case, it is useful to probe this method, considering the fast upgrading speed in computers.

Figure 5.2 Structure of UDEC model (UDEC manual)

Figure 5.3 Data structure in UDEC (UDEC manual)

Table 5.1 Functions and variables involved in the automating file

Command/Function	Description
block_head	Index to list of blocks
b_gp(ib)	Index of grid-point list
b_next(ib)	Next block pointer
b_zone(ib)	Index of zone list
gp_x(gp)	x-coordinate
gp_y(gp)	y-coordinate
gp_xvel(gp)	x-velocity
gp_yvel(gp)	y-velocity
gp_next(gp)	Next grid-point pointer
z_x(zi)	x-coordinate of zone centroid
z_y(zi)	y-coordinate of zone centroid
z_sxx(zi)	xx-stress
z_sxy(zi)	xy-stress
z_syy(zi)	yy-stress
z_next(zi)	Index of next zone in main list
*notes: ib	Number of block
gp	Number of grid-point
zi	Number of zone

In the embedded language FISH in UDEC, a file is coded based on the variables and functions in Table 5.1. The file will produce a set of files covering the following information:

- Position information:
 1. Number of the grid-point/zone ceteroid.
 2. x-, y- coordinates for every grid-points/zone centroids in X-Y coordinate system

3. Radial coordinate and angular coordinate for every grid-point/zone centroid in polar coordinates system

- History information: time, velocity/stress history component values at every grid-point/zone centroid.

After the history information is produced, it is saved to tables and after that saved to files, pending to be processed in MATLAB. The history information is an array, with two columns. One column is for time and the other is for velocity/stress values. These files are processed in MATLAB.

5.3.4 Processing data in MATLAB

Figure 5.4 shows the mechanism of data processing with respect to velocity amplitude for a point from a practical UDEC modeling in this research. For every point, the variations of the values of velocity components in x- direction ($VX_{i,j}$) and y- direction ($VY_{i,j}$) are recorded and saved to the history files, where i is grid-point number from 1 to the total number of grid-point, j is the number of computational time steps, from 1 to total number of steps.

Each file is an array of two columns. The first column contains time $T_{i,j}$ for every step. The second column contains value of velocity component at either x- or y- direction for each time-step. At any grid-point at time $T_{i,j}$, the resultant velocity ($V_{i,j}$) of the velocity in x- direction ($VX_{i,j}$) and the velocity in y- direction ($VY_{i,j}$) is calculated as:

$$V_{i,j} = \sqrt{VX_{i,j}^2 + VY_{i,j}^2} \quad (5.2)$$

For grid-point i , the amplitude A_i of the resultant velocity wave is the maximum value of $V_{i,j}$. The amplitude A_i of the resultant velocity wave at any grid-point is calculated by a searching file, and the corresponding time t_i (at the amplitude occurrence) can be determined. The searching method is programmed in a MATLAB file that can be called directly in MATLAB.

Combining the position information, the amplitude of velocity wave for every grid-point can be obtained. Table 5.2 shows the general format of a final file regarding amplitude of velocity for every grid-point in a UDEC model. The final file covers all the information for analyzing amplitude of velocity for every grid-point in a practical UDEC model. The final results file can be further processed or illustrated in MATLAB or EXCEL. In this thesis, most data are visually shown in EXCEL.

For the amplitude of any component, σ_{xx} , σ_{yy} and τ_{xy} or σ_1 , σ_2 and σ_3 , stress history at centroid of each zone can be calculated by using similar searching method.

Figure 5.4 Velocity and its components

Table 5.2 Format of final result file

i----- grid-point No.	X _i -----x- coordinate	Y _i -----y- coordinate	R _i ----- radial distance	Ang _i -- angle	t _i --time at amplitude occurrence	VX _{i,t_i}	VY _{i,t_i}	A _i
1	X ₁	Y ₁	R ₁	Ang ₁	t ₁	VX _{1,t₁}	VY _{1,t₁}	A ₁
2	X ₂	Y ₂	R ₂	Ang ₂	t ₂	VX _{2,t₂}	VY _{2,t₂}	A ₂
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
n	X _n	Y _n	R _n	Ang _n	t _n	VX _{n,t_n}	VY _{n,t_n}	A _n

5.3.5 Features and advantages of the new method

The new data processing method has the following features: (1) The UDEC source code is modified, so that the information for real-time recording of history data has been significantly increased up to the maximum capacity of the computer memory space. (2) The time history for every grid-point or centroid of a zone is automatically real-time recorded in UDEC input file. (3) The processing of history data using MATLAB and EXCEL is done in real-time. Therefore, this new method is not only able to fully utilize the computing power and memory capacity provided by the latest high speed personal computers, but also able to combine the functions of UDEC, MATLAB and EXCEL in the process of analyzing data. So that these three software tools are collaborated to optimize the function built in different software packages. The proposed method is able to comprehensively process velocity/stress amplitude for every grid-point/centroid of every zone in a UDEC model in rock dynamics, especially with 2-D wave propagation problems. The advantages of this new method are:

- Extending UDEC capacity: Most of the information for velocity/stress amplitude for every grid-point/zone centroid is recorded in the final result file, so it is possible to plot velocity/stress amplitude status for every grid-point/zone centroid. Standard UDEC can show velocity/stress wave status at a certain moment, but it is not the status of velocity/stress amplitude. In dynamic analysis, amplitude information is more meaningful.
- Saving UDEC modeling time: The final result file covers all of information in velocity/stress amplitude analysis, including position, time, amplitude information for every grid-point/centroid in a UDEC model. Data processing is done on the final result file as needed. It does not need to run UDEC model more than one time for the same case, so, saving UDEC modeling time.
- More accurate result: In the method, the position information of the grid-point/zone centroid is the exact position (x,y) of the grid-point/centroid of a zone. By contrast, nearest position of the grid-point/centroid of the zone to the point (x,y) is used by *hist* command in standard UDEC.

- Convenient for analyzing results: It is more convenient to operate on the final result file than to run UDEC model from the very beginning. UDEC modeling result can be studied more carefully and directly.

5.4 Data fitting method

In the following studies, the amplitude of the transmitted wave at any point in the jointed rock mass is often compared with that obtained from the case of intact rock to investigate the effects of joints on the performance of the transmission ratio. However, the inherent zoning mechanism in UDEC makes the grid-point position of the model with the joints different from that in the model of intact rock. Based on the definition of the transmission ratio T_n defined in Equation (5.1), in order to obtain the transmission ratio T_n at any grid-point, a polynomial fitting method is implemented to determine the amplitude of the wave for the case of intact rock at the same locations of grid-points for the case of jointed rock mass.

To illustrate the data fitting method, the problem of a 2-D compressional wave from a cavity passing through single joint in a rock mass is considered as shown in Model B in Figure 5.5. The wave amplitudes for all the grid-points along the radial line perpendicular to the joint obtained from the case of single joint (Model B) are shown in Figure 5.5. The amplitudes of the wave for all the grid-points along the same line obtained from the case of intact rock (Model A) are also shown in the figure. To determine the transmission ratio, the amplitudes of the wave for the two cases (a rock mass with single joint and intact rock) at the same grid-points should be obtained. Figure 5.6 shows the zoom-in plot for the radial distance between 20 m to 25 m as indicated in Figure 5.5. From Figure 5.6, it is clear that the original grid-point positions from the case of intact rock (Model A) are different with those from the case of single joint (Model B).

The original data in terms of amplitude ($A_{a, i}$) versus radial distance ($x_{a, i}$) (the subscript 'a' is for Model A, and i is for grid-point) along the radial line normal to the joint in Model A are assumed to follow the curve which can be presented as:

$$A_a = f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0 \quad (5.3)$$

The serial $[a_n, a_{n-1}, \dots, a_1, a_0]$ is unknown, where n is the root of the polynomial $f(x)$.

Using the original data in Model A $(x_{a, i}, A_{a, i})$, the unknown serial $[a_n, a_{n-1}, \dots, a_1, a_0]$ can be determined in a MATLAB file. Upon the unique serial $[a_n, a_{n-1}, \dots, a_1, a_0]$ is obtained, the function of the curve can be determined.

Submitting the grid-point position in the Model B, denoted as $(x_{b, i})$ into the fitted curve $f(x)$ shown in the Equation (5.3), then the fitted amplitude $A_{a, i}$ in Model A at the corresponding grid-point position $(x_{b, i})$ in Model B can be obtained as:

$$A_{a, i} = f(x_{b, i}) = a_n x_{b, i}^n + a_{n-1} x_{b, i}^{n-1} + \dots + a_1 x_{b, i}^1 + a_0 \quad (5.4)$$

Hence, the accurate the transmission ratio T_n for grid-point i in Model B can be expressed as:

$$T_n = \frac{A_{b, i}}{A_{a, i}} \quad (5.5)$$

Figure 5.5 A comparison of the original amplitude and the fitted amplitude along the radial line perpendicular to the joint

Figure 5.6 A comparison of the original amplitude and the fitted amplitude along the line perpendicular to the joint and crossing the cavity center (zoom-in plot of Figure 5.5)

5.5 A study on 2-D compressional wave propagation in an intact rock

Compressional wave originating from a tunnel or a borehole traveling through joints will be investigated in Chapters 6, 7, 8 and 9. The study focuses on the effects of the joints on the performance of the transmission ratio T_n . According to Equation (5.1), the amplitude at a point from the case with joints is often compared with that at the same point for the case of intact rock. This means the amplitudes corresponding to points in the case of intact rock are used as reference in the study on performance of the transmission ratio in the case of jointed rock mass. However, the amplitude corresponding to a point in the case of intact rock depends on the rock material, properties of the input wave and the radius of the cavity. In this part, the concept of scaled distance for wave from a tunnel or borehole is proposed to eliminate the effects of radius of the cavity on the amplitude corresponding to any point in the surrounding intact rock. So, the performance of compressional wave with a certain frequency in an intact rock is evaluated in terms of wave amplitude versus scaled distance, instead of radial distance from the cavity center. The

performance of compressional wave in an intact rock versus the scaled distance will be used as the reference for the further investigation presented in Chapters 6, 7, 8 and 9.

5.5.1 Effects of scaled distance on 2-D compressional wave propagation in an intact rock

In this sub-section, the performance of 2-D compressional wave in terms of wave amplitude versus radial distance from the cavity center with different radius (R_0) is presented. Based on the analysis, a concept of scaled distance is proposed to eliminate the effects of the radius of the cavity on the wave amplitude corresponding to any point in the surrounding intact rock for 2-D compressional wave.

Problem statement and modeling conditions

The problem and UDEC model are shown in Figure 5.7, where a cavity with radius of R_0 is located in an infinite intact rock. Four cases with different values of radius of the cavity ($R_0=0.4$ m, 1 m, 2 m and 5 m) are considered. For the case with cavity radius of 0.4 m, the corresponding geometry is 80 m $\times$ 80 m, while for the cases with cavity radius of 1 m, 2 m and 5 m, the corresponding geometry is 200 m $\times$ 200 m. The four boundaries of the geometry are viscous boundaries. A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2% for 200 Hz frequency.

Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study. The bulk modulus (K), shear modulus (G), and density (ρ) of the Bukit Timah granite in Singapore were obtained from extensive laboratory tests (Lee 2002). The wave propagation velocities in the rock for P-wave (c_p) and S-wave (c_s) are calculated using Equations (5.6) and (5.7)

$$c_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \quad (5.6)$$

$$c_s = \sqrt{\frac{G}{\rho}} \quad (5.7)$$

The mechanical properties and the wave velocities in the Bukit Timah granite are compressively used in Chapters 6, 7, 8 and 9.

Velocity histories corresponding to all the grid-points, along x-axis from the boundary of the cavity to the right boundary of the model, are recorded for all the cases.

Figure 5.7 Sketch of UDEC model in intact rock

Table 5.3 Mechanical properties and wave propagation velocities in Bukit Timah granite in Singapore

Properties	Value
Density (ρ) (kg/m^3)	2650
Bulk Modules (K) (GPa)	40
Shear Modules (G) (GPa)	26
Velocity of P-wave (c_p) (m/s)	5292
Velocity of S-wave (c_s) (m/s)	3133

Numerical results and discussion

Figure 5.8 shows the amplitude of the velocity wave versus the radial distance (R) at points along x-axis for the 4 cases with different radii of cavity considered in the analysis ($R_0=0.4$ m, 1 m, 2 m and 5 m).

As shown in Figure 5.8, for all cases of different cavity radii (R_0), the amplitude of the propagating wave decreases with radial distance, with its maximum value at the boundary of the cavity ($R=R_0$). At the boundary of the cavity, the amplitude is equal to the amplitude of the input wave (unity amplitude).

It can be seen from Figure 5.8 that the attenuation in wave amplitude with radial distance is faster for small cavities than for large cavities. To further illustrate this observation, the values of wave amplitude at a point of the same radial distance of 5 m from the cavity boundary are obtained and tabulated in Table 5.4, for the four cases with different radii of the cavity. It can be seen from Table 5.4 that when the wave propagates through the same radial distance from the cavity boundary, the maximum amplitude attenuation is 0.89 m/s for the case of $R_0=0.4$ m. The minimum amplitude attenuation is 0.37 m/s obtained from the case of $R_0=5$ m. The amplitude attenuation increases as the cavity radius decreases.

Figure 5.8 Amplitude of velocity wave versus radial distance from the cavity center for different radius

Table 5.4 Momentum of decreasing in amplitude at a point with the same value of $(R-R_0)$ for cases with different values of R_0

R_0 (m)	0.4	1	2	5
Amplitude at boundary $R-R_0=0$ (m/s)	1	1	1	1
Amplitude at the point $R-R_0=5$ m (m/s)	0.11	0.23	0.39	0.63
Amplitude attenuation (m/s)	0.89	0.77	0.61	0.37

Concept of scaled distance

The results of wave amplitude shown in Figure 5.8 are re-plotted against the quantity of $(R - R_0)/R_0^{1.5}$ in Figure 5.9. As seen from Figure 5.9, the amplitude of the wave at any point with the same value of $(R - R_0)/R_0^{1.5}$ is independent of the value of cavity radius.

The quantity $(R - R_0)/R_0^{1.5}$ at any point with radial distance of R from the cavity center is termed as the “scaled distance” for the point.

Figure 5.9 Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$

5.5.2 Variation of amplitude with the scaled distance for some reference cases in an intact rock

In this part, charts for determination of the wave amplitude at any point resulting from propagation of compressional wave applied at the boundary of a cavity in an intact rock are prepared for different input wave frequencies. These charts will be used as references for the calculation of the transmission ratio in Chapters 6, 7, 8 and 9. The amplitude is plotted as function of the scaled distance $(R - R_0)/R_0^{1.5}$.

Problem statement and modeling conditions

The problem and UDEC model are shown in Figure 5.7, where a cavity with radius of R_0 of 0.4 m is located in an infinite intact rock. The geometry is 80 m $\times$ 80 m. The four boundaries of the geometry are viscous boundaries. A sinusoidal compressional pulse velocity wave with unity amplitude and different frequency of 200 Hz, 500 Hz, 700 Hz and 1000 Hz is applied at the boundary of the cavity, respectively. The corresponding mesh size is chosen as 0.79 m, 0.4 m, 0.3 m and 0.2 m to keep the numerical error less than 2% for the above mentioned four frequencies respectively. Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study.

Velocity histories corresponding to all the points, along x-axis from the boundary of the cavity to the right boundary of the model, are recorded.

Variation of amplitude with the scaled distance for different frequencies in an intact rock.

Figures 5.10 (a), (b), (c) and (d) show the amplitude of the compressional velocity wave at points in the surrounding intact rock versus the scaled distance $(R - R_0)/R_0^{1.5}$ for different frequencies of 200 Hz, 500 Hz, 700 Hz and 1000 Hz, respectively. These figures will be used as the references for the analysis in Chapters 6, 7, 8 and 9.

Discussion

From Figures 5.10 (a), (b), (c) and (d), it can be seen that for every frequency of the input wave, at scaled distance $(R - R_0)/R_0^{1.5}$ of 0 (i.e., exactly at the boundary of the cavity), the amplitude is 1 (i.e., equivalent to the amplitude of the input wave). When scaled distance $(R - R_0)/R_0^{1.5}$ is large enough, the amplitude of the wave corresponding to the point in the intact rock is approaching zero. It means for input wave regardless of its frequency, the variation of amplitude at any point in the surrounding intact rock with the scaled distance $(R - R_0)/R_0^{1.5}$ is from the amplitude of the input wave exactly at the boundary to the value approaching to 0 for the points far away from the boundary. So, for the input waves with different frequencies, no difference in amplitude of the propagating wave corresponding to the point at the boundary (equivalent to the amplitude of the input wave) and the points far away from the cavity (approaching to 0).

However, a relatively big difference of the amplitude corresponding to the points near to the boundary in the intact rock exists among the input waves with different frequencies. As an example, Figure 5.11 shows the variation of the amplitude corresponding to points in the surrounding rock with the scaled distance in a small range of $(R - R_0)/R_0^{1.5}$ from 0 to 20 (i.e., near to the cavity), for different frequencies of the input wave as 200 Hz, 500 Hz, 700 Hz and 1000 Hz. It can be seen that there is a relatively big difference among the amplitude for different frequency in this range of scaled distance from 0 to 20. It also can be seen that at the same scaled distance, the wave with higher frequency has higher amplitude.

(a) Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for frequency of 200 Hz

(b) Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for wave frequency of 500 Hz

Figure 5.10 Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for different frequencies of the input wave

(c) Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for wave frequency of 700 Hz

(d) Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for wave frequency of 1000 Hz

Figure 5.10 (Continue) Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for different frequencies of the input wave

Figure 5.11 Amplitude of velocity wave versus scaled distance $(R-R_0)/R_0^{1.5}$ for different frequencies (200 Hz, 500 Hz, 700 Hz and 1000 Hz) of the input wave for the parts near the cavity (with a small range of $(R-R_0)/R_0^{1.5}$)

5.6 Summary

In this chapter, a new method of comprehensive processing of wave amplitude in rock dynamic problems has been developed. This new method is not only able to fully utilize the computing power and memory capacity provided by the latest high speed personal computers, but also able to combine the functions of UDEC, MATLAB and EXCEL in the process of analyzing data. The proposed new data processing method is able to comprehensively process velocity/stress amplitude for every grid-point/centroid of every zone in a UDEC model in rock dynamics more accurately. The advantages of this method are (1) The standard capacity of UDEC model is extended, (2) Time in data processing involved in UDEC results is reduced, and (3) The output numerical results are more accurate.

In this chapter, a study on 2-D compressional wave propagation in intact rock has also been performed. In this study, a very important concept, scaled distance

$(R - R_0)/R_0^{1.5}$ is proposed. It has been found that the amplitude of the wave at any point with the same value of $(R - R_0)/R_0^{1.5}$ is independent of the value of radius of the cavity, from which the compressional wave originates.

The study performed shows that for a given frequency, the amplitude of the wave at any point of radial distance R from the center of the tunnel is function of the deformation parameters and scaled distance. Several charts have been prepared for easy determination of the amplitude at any point in the surrounding intact rock. These charts are also very important references for calculation of the transmission ratio in the case of 2-D compressional wave propagation in a rock mass performed in Chapters 6, 7, 8 and 9.

CHAPTER 6

A STUDY ON CHARACTERISTICS OF THE TRANSMISSION RATIO IN THE PROCESS OF 2-D COMPRESSIONAL WAVE PROPAGATION IN A ROCK MASS

Compressional wave propagation from a tunnel or borehole in a rock mass is a multiple factor-influenced problem. The transmission ratio at a point in the process of 2-D compressional wave propagation through a joint or a joint set may be affected by a set of factors. The factors are stiffness in both normal direction (j_{kn}) and shear direction (j_{ks}), the radian angle, the radial distance from the cavity center, the spacing and the number of the joints in the joint set and the radius of the cavity from which the source wave originates. This chapter focuses on studying the effects of all these possible factors influencing the performance of the transmission ratio in the process of 2-D compressional wave propagation in a rock mass.

The general objective of this chapter is to perform parametric studies to investigate the characteristics of the transmission ratio under various factors. Based on the analysis performed, the effective factors dominating the performance of the transmission ratio can be determined. Therefore, these obtained performance characteristics can be used to simplify the problem by filtering out the factors having negligible effects on the transmission ratio. Based on the analysis, research strategies are proposed for the corresponding problems. This is very important for solving the problems in the following chapters.

6.1 Introduction

Regarding the problems of wave propagation in a rock mass, when the wave passes through single joint or multiple parallel joints, the number of factors influencing the transmission ratio is different. In the multiple joints situation, there are two additional factors including the number of joints and the spacing between joints that must be taken into account. The study on the factors influencing the transmission ratio T_n for multiple parallel joints could be much more complicated than the case of single joint. Therefore, the effects of the factors influencing the performance of

the transmission ratio T_1 in the process of wave propagation through single joint and multiple joints are studied, respectively.

In the case of normally incident 1-D wave propagation through single joint, the analytical solutions for the magnitudes of transmission coefficient $|T_1|$ and the reflection coefficient $|R_1|$ have been obtained by several researchers (Pyrak-Nolte *et al.* 1987 and 1990a). On the other hand, Cai (2001) and Zhao (2004) have studied the problem of 1-D wave transmission through multiple parallel joints using the method of characteristics. From the research work, it has been found that the magnitude of transmission coefficient $|T_1|$ for 1-D wave propagation through single joint is dominated by the normalized stiffness of the joint in terms of $k/\omega z$. In the term of $k/\omega z$, k is the stiffness of the joint (normal stiffness is involved in P-wave case, and shear stiffness is involved in S-wave case, respectively); ω is the angular frequency of the wave; and z is the seismic impedance. The wave impedance z is equal to ρc , where ρ is the density of the rock material and the parameter c is the wave velocity (c_p and c_s are involved in the calculation of the impedance for P-wave and S-wave respectively, where c_p and c_s are the P-wave and S-wave propagation velocities respectively). By contrast, the magnitude of transmission coefficient $|T_n|$ in the case of 1-D wave propagation through the multiple parallel joints is dominated by the normalized stiffness in terms of $k/\omega z$ (normalized normal stiffness and normalized shear stiffness are involved in P-wave and S-wave cases respectively), the number of the joints and the spacing between the joints.

However, when the practical dynamic problem involves the compressional wave propagation from an underground explosion source in a tunnel or borehole in a rock mass, the wave propagation becomes a 2-D problem. In this situation, additional factors might significantly affect the transmission ratio as mentioned before. The additional factors are: (1) The stiffness of the joint, in terms of both normal and shear stiffness (j_{kn} and j_{ks}), (2) The position of the joint for the single joint case and the position of the first joint in a joint set for the multiple joints case and (3) The radius of the tunnel or borehole from which the compressional wave originates. All these additional factors must be taken into account in the process of 2-D compressional wave propagation.

The major research presented in this chapter is focused on the following topics.

- (1) The first focus is on studying the effects of the joint position in the case of 2-D compressional wave propagation through single joint and the effects of the position of the first joint in the joint set in the case of multiple parallel joints. Such studies are important to fully investigate the effects of joints.
- (2) The second focus is on studying the effects of different parameters of the joints on the transmission ratio in the process of 2-D compressional wave propagation in a rock mass. These parameters include the radian angle, the radial distance from the wave source, normalized stiffness of the joint (including normal and shear directions), the number of joints and the spacing between the joints. This study investigates the sensitivities how the transmission ratio is affected by these parameters in different directions, including the radian direction normal to the joint and the other arbitrary directions for single joint and multiple joints, respectively.
- (3) The third focus is on studying the effects of radius of the cavity from which the wave originates on the transmission ratio in the process of 2-D compressional wave propagation through single joint and multiple joints, respectively.
- (4) Finally, based on the analysis, research strategies are proposed for further study on effects of joint in the radian direction of 90° and other arbitrary directions, for both the cases of single joint and multiple parallel joints, respectively. These strategies will be comprehensively used in Chapters 7 and 8 in the corresponding problems.

6.2 Effects of the position of joint

This section aims to study the effects of the position of the joint (single joint case) and the position of the first joint in the joint set (multiple joints case) from the cavity center on wave amplitude at any point located behind the joint (single joint case) and the last joint in the set (multiple joint case) in the rock mass.

6.2.1 Effects of the position of the single joint

Figure 6.1 shows the model for the analysis of the effects of the position of the joint in the process of 2-D compressional wave propagation through single joint. A cavity with a radius of 5 m exists in an infinite rock mass with single joint parallel to the y-axis and located at a variable normal distance from the centre of the cavity. The four boundaries are viscous boundaries. A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study. The stiffness values of the joint in normal and shear directions are assumed to be the same and equal to $0.3K$, where K is the bulk modulus of the rock. (It is noted that j_{kn} and j_{ks} are digitally given the values of K or G times a certain factor. The units of j_{kn} and j_{ks} are stress per length, different from the units of K and G).

The effects of the position of the single joint on the performance of the 2-D compressional wave propagation in the rock mass are studied by measuring the amplitude of the transmitted velocity wave in the radian direction of 90° for 4 joint positions at 10 m, 15 m, 25 m and 60 m from the cavity center.

Figure 6.1 Sketch of UDEC model for effects of the position of the single joint

Figure 6.2 shows the amplitudes of velocity wave corresponding to all the monitoring points along the radian direction of 90° , from the boundary of the cavity to the right boundary of the model, for 4 different joint positions. For the purpose of comparison, the amplitudes of velocity wave corresponding to all the points along the same radial line for the case of intact rock are also included.

From Figure 6.2, when 2-D compressional wave propagates before reaching the joint, for all the cases with the single joint located at different positions, the corresponding amplitude of velocity increases to a certain value, greater than the amplitude of velocity for the case of intact rock. This is due to the wave reflection before the joint. However, when 2-D compressional wave propagates behind the joint, for all the measured cases with the single joint located at different positions, the corresponding amplitude of velocity decreases to a certain value, smaller than the amplitude of velocity for the case of intact rock (curve a-b). This is due to the wave attenuation through the joint.

It can also be seen from Figure 6.2 that for all the cases with different joint positions, the amplitude of the wave before reaching a critical distance before the joint is the same with that in the case of intact rock (curve c-d).

Figure 6.2 Effect of the position of the joint

6.2.2 Effects of the position of the first joint in multiple parallel joints

The geometry of the model for the effects of the first joint position in the case of multiple joints is shown in Figure 6.3, where a cavity with a radius of 5 m exists in an infinite rock mass with a set of 2 joints, in which the first joint is located at a variable normal distance from the centre of the cavity. The 2 joints are parallel to the y-axis. The four boundaries are viscous boundaries. A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study. The stiffness values of the joint in normal and shear directions are assumed to be the same and equal to $0.3K$, where K is the bulk modulus of the rock material.

The effects of the position of the first joint in the set on the performance of the 2-D compressional wave propagation are studied by measuring the amplitude data of velocity wave corresponding to every point along the radian direction of 90° for 4 different positions. The first joint in the set is positioned at 10 m, 15 m, 25 m and 60 m from the centre of the cavity, respectively. The joint spacing is fixed at 0.1 wavelength.

Figure 6.3 Sketch of UDEC model for effects of the first position for the case of multiple parallel joints

Figure 6.4 shows the amplitudes of velocity wave corresponding to all the points along the radian direction of 90° , from the boundary of the cavity to the right boundary of the model, for the cases with 4 different positions at which the first joint is located. It is noted that the amplitudes of velocity wave corresponding to the points along the same radial line for the case of intact rock are also included in Figure 6.4, which are used as reference for comparison.

From Figure 6.4, it can be seen that when the compressional wave propagates before reaching the first joint, for all cases considered, the corresponding amplitude of velocity wave increases to a certain value, greater than the amplitude of velocity wave from the case of intact rock. This is due to wave reflection before the joint. However, when the 2-D compressional wave propagates behind the last joint in the set, for all the measured cases, the corresponding amplitude of velocity wave decreases to a certain value, smaller than the amplitude of velocity wave from the case of intact rock. This is due to wave attenuation through the joints (curve a-b).

It can also be seen from Figure 6.4 that for all the cases with different joint positions, the amplitude of the wave before reaching a critical distance before the first joint is the same with that in the case of intact rock (curve c-d).

Figure 6.4 Effects of position of the first joint in the case of 2 parallel joints

6.2.3 Discussion

For the case of single joint as shown in Figure 6.2, it can be seen that when the distance from the joint to the cavity centre is equal to or greater than 10 m, the amplitudes of velocity wave behind the joint for all the cases considered are almost following the same curve (curve a-b). For example, for the point at radial distance of 65 m from the cavity center, the amplitude of the transmitted velocity wave is the same value of 0.18 m/s for all the cases with different distances from the cavity center to the joint including 10 m, 15 m, 25 m and 60 m. It means that the transmitted wave amplitude is independent of the position of the joint when the joint position is located at a distance more than 10 meters from the cavity center. Therefore, if the distance from the joint to the cavity centre is more than 10 meters, the effects of the position of the single joint on the performance of wave propagation in a rock mass can be ignored. So, in the further study on the effects of single joint in Chapter 7, the normal distance between the joint and the cavity center is set to 15 meters.

For the case of multiple joints as shown in Figure 6.4, it can be seen that when the distance from the first joint in the set to the cavity centre is equal to or more than 10 meters, for all the cases considered, all the amplitudes of velocity wave behind the last joint in the set are following the same curve (curve a-b). It shows that the transmitted velocity amplitude is independent of the position of the first joint in the joint set. Therefore, if the distance from the first joint in the set to the cavity centre is more than 10 meters, the effects of the position of the first joint for the case of multiple joints on wave propagation in a rock mass can be ignored. So, in the further study on the effects of multiple parallel joints in Chapter 8, the normal distance of the first joint in the joint set to the cavity center is set to 15 meters.

6.3 A study on characteristics of T_n under parameters related to single joint or multiple parallel joints

In this section, the characteristics of the transmission ratio under parameters related to single joint or multiple parallel joints are studied. Factors having significant

influence on the transmission ratio and those having negligible influence on the transmission ratio which can be filtered out are determined. Based on the analysis of characteristics of the transmission ratio, the research strategies are proposed and justified, which will be comprehensively used in Chapters 7 and 8 for the further studies on effects of single joint and multiple parallel joints. The investigation for the effects of one particular factor on the performance of the transmission ratio are performed by setting the target factor as a variable while keeping the others at fixed values.

6.3.1 UDEC model and the modeling conditions

UDEC models for both of the single joint case and the multiple joints case together with the modeling conditions are clearly stated.

Figure 6.5 (a) shows UDEC model used for the analysis of the 2-D compressional wave propagation through single joint in a rock mass. Likewise, Figure 6.5 (b) shows UDEC model used for the analysis of the 2-D compressional wave propagation through multiple joints in a rock mass.

As shown in Figures 6.5 (a) and (b), a cavity with a radius of 5 meters exists in an infinite rock mass. The joint (for the case of single joint as shown in Figure 6.5 (a)) and the first joint in the joint set (for the case of multiple parallel joints as shown in Figure 6.5 (b)) are located with a normal distance of 15 meters from the center of the cavity, parallel to y-axis. Both for the cases of single joint as shown in Figure 6.5 (a) and multiple parallel joints as shown in Figure 6.5 (b), the four boundaries are viscous boundaries. Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study.

In Figures 6.5 (a) and (b), α is the radian angle. Point A represents an arbitrary point behind the joint for the case of single joint or behind the last joint for the case of multiple parallel joints.

(a) Sketch of UDEC model for single joint

(b) Sketch of UDEC model for multiple joints

Figure 6.5 Sketch of UDEC models

6.3.2 The transmission ratio along a radial line with the same radian angle

6.3.2.1 T_1 along a radial line with the same radian angle

Parametric studies on effects of single joint on the transmission ratio T_1 at points along a radial line with any given radian angle α are conducted on UDEC model as shown in Figure 6.5 (a). The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The analysis for different

values of normal and shear stiffness are performed. $j_{ks}=j_{kn}=0.03K$, $0.07K$, $0.1K$, $0.2K$ and $0.4K$, where K is the bulk modulus of the rock material listed in Table 5.3.

The amplitudes of the velocity wave corresponding to points along radial lines with different given radian angles α ($\alpha=90^\circ$, 72° and 60°) behind the joint are recorded.

Figures 6.6 (a), (b) and (c) show the calculated transmission ratio T_1 at points along the radian angles of $\alpha=90^\circ$, $\alpha=72^\circ$ and $\alpha=60^\circ$, respectively. It should be noted that the values of radial distance from the cavity center shown in Figures 6.6 (a), (b) and (c) are different from each other, because of the difference of the radian angle.

(a) T_1 along radial line with α of 90°

Figure 6.6 T_1 along three radial lines with α of 90° , 72° and 60°

(b) T_1 along radial line with α of 72°

(c) T_1 along radial line with α of 60°

Figure 6.6 (Continued) T_1 along three radial lines with α of 90° , 72° and 60°

6.3.2.2 T_n along a radial line with the same radian angle

Parametric studies on effects of multiple parallel joints (for cases of 2, 4 and 8 joints) on the transmission ratio T_n at the points along radial lines with different given radian angles are conducted using UDEC model as shown in Figure 6.5 (b). The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The normal and shear stiffness of the joint are set as: $jks=jkn=0.3K$, where K is the bulk modulus of the rock material listed in Table 5.3. The ratio of joint spacing to wavelength (spacing/wavelength) is fixed at 0.05.

In the first group of tests, the radial lines with three different angles of $\alpha=90^\circ$, $\alpha=72^\circ$ and $\alpha=60^\circ$ are chosen. The transmission ratio T_n corresponding to every point, behind the last joint along these three radial lines are investigated respectively to study the characteristics of T_n corresponding to the points along any radial line.

Figures 6.7 (a), (b) and (c) show the transmission ratio T_n corresponding to the points along the radial lines with radian angles $\alpha=90^\circ$, $\alpha=72^\circ$ and $\alpha=60^\circ$ respectively, for the cases of 2 joints, 4 joints and 8 joints spaced at a fixed value of 0.05 wavelength.

(a) T_n along a radial line with $\alpha=90^\circ$ for 2, 4 and 8 joints spaced at 0.05 wavelength

Figure 6.7 T_n along a radial line with different radian angles: 90, 72 and 60 degree, for different number of joints: 2, 4 and 8, spaced at 0.05 wavelength

(b) T_n along a radial line with $\alpha=72^\circ$ for 2, 4 and 8 joints spaced at 0.05 wavelength

(c) T_n along a radial line with $\alpha=60^\circ$ for 2, 4 and 8 joints spaced at 0.05 wavelength

Figure 6.7 (Continued) T_n along a radial line with different radian angles: 90, 72 and 60 degree, for different number of joints: 2, 4 and 8, spaced at 0.05 wavelength

The second group of tests focuses on the characteristics of performance of the transmission ratio T_n corresponding to points on the radial line with a fixed radian angle (here value of 90° is chosen) under different joint spacing. UDEC model used for the tests is shown in Figure 6.5 (b). The normal and shear stiffness of joints are $j_{ks}=j_{kn}=0.3K$, where K is the bulk modulus of the rock material shown in Table 5.3. The input wave is a sinusoidal compressional pulse velocity wave with a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The ratio of spacing/wavelength is changed as: 0.03, 0.05, 0.1, 0.15 and 0.2, respectively.

Figures 6.8 (a), (b) and (c) show the transmission ratio T_2 , T_4 and T_8 corresponding to the points along the radial line with a radian angle of 90° for the cases of 2 joints, 4 joints and 8 joints respectively, with the following ratios of spacing/wavelength: 0.03, 0.05, 0.1, 0.15 and 0.2.

(a) T_2 for different ratios of spacing/wavelength along the radial line with α of 90°

Figure 6.8 T_n ($n=2, 4$ and 8) for different ratios of spacing/wavelength along the radial line with α of 90°

(b) T_4 for different ratios of spacing/wavelength along the radial line with α of 90°

(c) T_8 for different ratios of spacing/wavelength along the radial line with α of 90°

Figure 6.8 (Continued) T_n ($n=2, 4$ and 8) for different ratios of spacing/wavelength along the radial line with α of 90°

6.3.2.3 Discussion

From Figures 6.6 (a), (b) and (c). It can be seen that the transmission ratio T_1 corresponding to the points located behind the joint along a radial line with any given radian angle is approximately independent of the radial distance of the points. This implies that the transmission ratio T_1 corresponding to one single point located

behind the joint can be used to represent the transmission ratio T_1 corresponding to all the points behind the joint on the same radial line.

Likewise, for the multiple joints case as shown in Figures 6.7 (a), (b) and (c) and Figures 6.8 (a), (b) and (c), it can be seen that the transmission ratio T_n at points behind the last joint along a radial line with any given radian angle α is also approximately independent of the radial distance from the cavity center. This observation is applicable regardless of the number of the joints and the spacing of the joints in the joint set. Therefore, this implies that the transmission ratio T_n , corresponding to single point located behind the last joint along the radial line, can be used to represent the transmission ratio T_n corresponding to all the other points behind the last joint on the same radial line.

It can be seen from Figures 6.6, 6.7 and 6.8 that the transmission ratios at the points behind the single joint (Figure 6.6) and the last joint in the joint set (Figures 6.7 and 6.8) slightly fluctuate with the radial distance from the cavity center. Three possible factors may lead to this light oscillation. (1) The numerical errors in the inherent calculation mechanism in UDEC program package. (2) The effects of boundary in UDEC modeling. (3) The difference between the exact radian angle at the grid-points calculated and the radian angle considered. Taking the considered radian angle of 60° for example, usually very few grid-points are located along the radial line with exact radian angle of 60° , or by chance, even no grid-points at all along that radial line. Therefore, in this situation, all the grid-points with radian angles $60^\circ \pm 0.07^\circ$ are calculated.

It is noted that, in Chapters 7 and 8, the prediction models for estimating the transmission ratio will be proposed by using the research strategies, based on this approximation, therefore, the reliability of the prediction models would be affected. However, it is also noted that the reliability of the prediction models will be finally verified by a filed explosion test in Chapter 9.

6.3.3 The transmission ratio along a circle

Recalling the results illustrated in Figure 6.6, Figure 6.7 and Figure 6.8, it can be

seen that the transmission ratio corresponding to any points on the same radial line (both for the cases of single joint and multiple joints) is independent of the radial distance of the points.

Based on this observation, it is reasonable to study the transmission ratio corresponding to all the points along a circle, in which all the points have the same radial distance from the cavity center, regardless of the cases of single joint and multiple parallel joints. Since there are two symmetries on each circle, the study on the effects of single joint or multiple joints along a circle can be done in one or two quadrants corresponding to the ranges of radian angle from either 0° to 90° or 0° to 180° .

This section aims to investigate the performance of the transmission ratio corresponding to the points along a circle, where all the points have the same radial distance from the cavity center, for the cases of single joint and multiple parallel joints.

6.3.3.1 T_1 along a circle

The parametric studies on T_1 corresponding to the points along a circle, where all the points have the same radial distance from the cavity center, are conducted using UDEC model shown in Figure 6.5 (a). The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The normal and shear stiffness of joint are taken as: $j_{ks}=j_{kn}=0.03K$, where K is the bulk modulus of the rock material shown in Table 5.3.

Parametric studies are conducted to determine the transmission ratio T_1 along several circles with different radial distance from the cavity center at 39 m, 50 m, 60 m, 70 m and 80 m. Figure 6.9 shows the effects of the single joint on T_1 corresponding to the points along circles with the above mentioned radial distances from the cavity center.

Figure 6.9 T_1 along different circles with different radial distance from the cavity center (39 m, 50 m, 60 m, 70 m and 80 m) for fixed stiffness ($j_{ks}=j_{kn}=0.03K$)

6.3.3.2 T_n along a circle

The parametric studies on the effects of multiple parallel joints on the transmission ratio T_n along a circle, where all the points have the same radial distance from the cavity center, are also conducted for the cases with different numbers of joints as 2 joints, 4 joints and 8 joints. UDEC model is shown in Figure 6.5 (b). The input wave is a compressional sinusoidal pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The normal and shear stiffness of the joint are set as: $j_{ks}=j_{kn}=0.3K$, where K is the bulk modulus of the rock material shown in Table 5.3. The ratio of spacing/wavelength is fixed at 0.05.

The parametric studies are conducted to determine the transmission ratio T_n along several circles with different radial distances from the cavity center at 41.5m, 50 m, 60 m, 70 m and 80 m for all the cases of 2 joints, 4 joints and 8 joints.

Figures 6.10 (a), (b) and (c) show the transmission ratio T_n corresponding to the points along the circles with different radial distances at 41.5 m, 50 m, 60 m, 70 m and 80 m from the cavity center for 2 joints, 4 joints and 8 joints respectively. The joint stiffness is $j_{kn}=j_{ks}=0.3K$ and the ratio of spacing/wavelength is equal to 0.05.

(a) T_2 along different circles with different radial distances from the cavity center for two joints spaced at 0.05 wavelength for fixed stiffness ($j_{ks}=j_{kn}=0.3K$)

(b) T_4 along different circles with different radial distances from the cavity center for four joints spaced at 0.05 wavelength for fixed stiffness ($j_{ks}=j_{kn}=0.3K$)

Figure 6.10 T_n along different circles with different radial distances from the cavity center for different number of joints spaced at 0.05 wavelength for fixed stiffness ($j_{ks}=j_{kn}=0.3K$)

(c) T_8 along different circles with different radial distances from the cavity center for eight joints spaced at 0.05 wavelength for fixed stiffness ($j_{ks}=j_{kn}=0.3K$)

Figure 6.10 (Continued) T_n along different circles with different radial distances from the cavity center for different number of joints spaced at 0.05 wavelength for fixed stiffness ($j_{ks}=j_{kn}=0.3K$)

6.3.3.3 Discussion

From Figure 6.9, it can be seen that all the curves of the transmission ratio T_1 versus radian angle converge to the same curve, for all the points along different circles with different radial distances from the cavity center (39 m, 50 m, 60 m, 70 m and 80 m). This means that the transmission ratio T_1 is affected by the radian angle, but it is independent of the radial distance from the cavity center. It can be found that the transmission ratio T_1 corresponding to points along single circle, where all the points are behind the joint with the same radial distance from the cavity center, can represent the transmission ratio T_1 for every point behind the single joint. This is a very important research strategy which will be used in Chapter 7 for further study on effects of single joint in the process of 2-D compressional wave propagation for arbitrary directions.

Likewise, for the multiple joints case, it can be seen from Figures 6.10 (a), (b) and (c) that the transmission ratio T_n is affected by the radian angle, but it is independent of radial distance from the center of cavity. The transmission ratio T_n corresponding to the points along single circle, where all the points have the same

radial distance from the cavity center, can represent the transmission ratio T_n for all the other points behind the joints. This is a very important research strategy which will be used in Chapter 8 for further study on effects of multiple parallel joints in the process of 2-D compressional wave propagation for arbitrary directions.

6.3.4 Effects of shear stiffness on the transmission ratio along the radial line with the radian angle of 90°

6.3.4.1 Effects of shear stiffness on T_1 along the radial line with α of 90°

In the process of 2-D compressional wave propagation through single joint, the investigation of the effects of shear stiffness of the joint on the performance of the transmission ratio T_1 for single joint case is a very important and a necessary part in this research project. It is helpful to understand the degree of sensitivity how the transmission ratio T_1 is affected by the shear stiffness of the joint.

The geometry of the model used for the investigation is shown in Figure 6.5 (a). The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. Parametric studies on the effects of shear stiffness on T_1 corresponding to the points along a circle are conducted, where all the points have the same radial distance from the cavity center, including the point with the radian angle of 90° . The shear stiffness value used in the investigation is chosen as $jks=0.03G, 0.07G, 0.1G, 0.2G$, and $0.4G$, respectively, while the normal stiffness is fixed as $jkn= 0.4K$. K and G are bulk modulus and shear modulus of the rock material shown in Table 5.3.

Figure 6.11 illustrates the performance of the transmission ratio T_1 corresponding to the points along a circle where all the points have the same radial distance from the cavity center for different shear stiffness values of $0.03G, 0.07G, 0.1G, 0.2G$ and $0.4G$ and the fixed normal stiffness of $0.4K$.

Figure 6.11 T_1 for different shear stiffness and fixed normal stiffness

6.3.4.2 Effects of shear stiffness on T_n along the radial line with α of 90°

Likewise, for 2-D compressional wave propagation through multiple parallel joints, to investigate the effects of shear stiffness of the joints on the transmission ratio T_n for multiple joints case is also a very important and a necessary part in this research project. It is helpful to understand how the transmission ratio T_n is affected by the shear stiffness of the joints and what degree of the sensitivity is.

The geometry of the model used for the investigation is shown in Figure 6.5 (b). The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The parametric studies focus on the effects of shear stiffness on T_n corresponding to all the points along a circle, where all the points have the same radial distance from the cavity center, including the point with the radian angle $\alpha=90^\circ$. The shear stiffness value used in the investigation is chosen as $jks=0.03G$, $0.07G$, $0.1G$, $0.2G$, and $0.4G$, respectively, while the normal stiffness is fixed at $jkn=0.4K$. K and G are bulk modulus and shear modulus of the rock material shown in Table 5.3. Three cases of 2 joints, 4 joints and 8 joints with the ratio of spacing/wavelength of 0.05 are investigated.

Figures 6.12 (a), (b) and (c) show the performance of the transmission ratio T_2 , T_4 and T_8 corresponding to the points along a circle for different shear stiffness of 0.03G, 0.07G, 0.1G, 0.2G and 0.4G and the fixed normal stiffness of 0.4K, for 2, 4 and 8 parallel joints, with the ratio of spacing/wavelength of 0.05.

(a) T_2 for different shear stiffness and fixed normal stiffness along a circle with the same radial distance from the cavity center for two joints spaced at 0.05 wavelength

(b) T_4 for different shear stiffness and fixed normal stiffness along a circle with the same radial distance from the cavity center for four joints spaced at 0.05 wavelength

Figure 6.12 T_n for different shear stiffness and fixed normal stiffness along a circle with the same radial distance from the cavity center for 2, 4 and 8 joints spaced at 0.05 wavelength

(c) T_8 for different shear stiffness and fixed normal stiffness along a circle with the same radial distance from the cavity center for eight joints spaced at 0.05 wavelength

Figure 6.12 (Continued) T_n for different shear stiffness and fixed normal stiffness along a circle with the same radial distance from the cavity center for 2, 4 and 8 joints spaced at 0.05 wavelength

6.3.4.3 Discussion

From Figure 6.11, it can be seen that in the radian direction of 90° , the effects of shear stiffness of the joint on the transmission ratio T_1 in the process of 2-D compressional wave propagation through single joint are negligible. By contrast, the transmission ratio T_1 corresponding to the points in any other arbitrary radian directions (other than 90°) is sensitive to both the normal stiffness and the shear stiffness of the joint. Therefore, the transmission ratio T_1 can be studied for the case of the radian direction of 90° and for the case of arbitrary radian directions, respectively, because the case of the radian direction of 90° is simpler, hence more analysis of parametric study can be performed. This research strategy is used in Chapter 7 for the further study on effects of single joint.

The difference between the ratios of jks/G and jkn/K is unlikely very large in real joints. Therefore, from the above mentioned observations on the effects of shear stiffness on the transmission ratio T_1 in the radian direction of 90° , it is reasonable to study the transmission ratio T_1 at the points along the radian direction of 90° by

considering single point behind the joint using the same ratio of jks/G and jkn/K . This research strategy will be used in Chapter 7 for further study on effects of single joint in the process of 2-D compressional wave propagation in the radian direction of 90° .

Likewise, for the effects of shear stiffness of the joints on T_n corresponding to the points along the radian direction of 90° for the case of multiple joints, from Figures 6.12 (a), (b) and (c), it can be seen that the effects of shear stiffness of the joints on the transmission ratio T_n are negligible as well. Therefore, the transmission ratio T_n can be studied for the case of the radian direction of 90° and for the case of arbitrary radian directions, respectively, because the case of the radian direction of 90° is simpler and more analysis of parametric study can be performed. This research strategy is used in Chapter 8 for the further study on effects of multiple parallel joints.

It is reasonable to study the transmission ratio T_n corresponding to the points along the radian direction of 90° by just considering single point behind the multiple parallel joints using the same ratio of jks/G and jkn/K . This research strategy will be used in Chapter 8 for further study on effects of multiple parallel joints in the process of 2-D compressional wave propagation in the radian direction of 90° .

6.4 A study on characteristics of the transmission ratio under different radius of the cavity

Different from 1-D wave propagation in a rock mass, the radius of the tunnel or borehole where the wave originates is involved in the process of 2-D compressional wave propagation. This part aims to study the effects of the radius of the tunnel or borehole on the transmission ratio corresponding to the points along a circle where all the points have the same radial distance from the cavity center for the cases of single joint and multiple parallel joints.

6.4.1 T_1 under different radius of the cavity

Using UDEC model shown in Figure 6.5 (a), parametric studies on effects of the

radius of the tunnel or borehole on the transmission ratio T_1 at points along a circle, where all the points have the same radial distance from the cavity center, are conducted. 2 meters and 5 meters are considered for the radius of the tunnel. The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep numerical error less than 2%. The normal and shear stiffness of the joint are fixed as: $j_{kn}=0.4K$, $j_{ks}=0.4G$, where K and G are bulk modulus and shear modulus of the rock material shown in Table 5.3.

Figure 6.13 shows the comparison of the transmission ratio T_1 corresponding to the points along a circle, where all the points have the same radial distance from the cavity center, for the single joint case between two radius values of the tunnel: $R=2$ m and $R=5$ m.

6.4.2 T_n under different radius of the cavity

Using UDEC model in Figure 6.5 (b), parametric studies on effects of the radius of the tunnel or borehole on the transmission ratio T_n at points along a circle where all the points have the same radial distance from the cavity center are conducted. 2 meters and 5 meters are considered for the radius of the tunnel. The input wave is a sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. The normal and shear stiffness of the joint are fixed as: $j_{kn}=0.4K$, $j_{ks}=0.4G$, where K and G are bulk modulus and shear modulus of the rock material shown in Table 5.3.

Figure 6.14 shows the comparison of the transmission ratio T_2 corresponding to the points along a circle, where all the points have the same radial distance from the cavity center, for the case of 2 joints with the ratio of spacing/wavelength of 0.05, between two radius values of the tunnel: $R=2$ m and $R=5$ m;

6.4.3 Discussion

From Figure 6.13 and Figure 6.14, it can be seen that for the two cases considered,

the radius of the tunnel or borehole for the source wave has negligible effects on the transmission ratio in the process of 2-D compressional wave propagation through both single or multiple parallel joints.

Figure 6.13 Comparison of T_1 for 2-D compressional wave from tunnels with different radius

Figure 6.14 Comparison of T_2 for 2 joints spaced at 0.05 wavelength for 2-D compressional wave from tunnels with different radius

6.5 Summary

Three main studies have been performed in this chapter. (1) The effects of the position of the joint for the case of single joint and the first joint for the case of multiple parallel joints are investigated. (2) The characteristics of the transmission ratio under parameters related to single joint or joint set are studied. Based on the analysis, the effective factors dominating the performance of the transmission ratio have been determined. Therefore, these obtained performance characteristics can be used to simplify the problem by filtering out the factors, which have negligible effects on the transmission ratio. Based on this study, research strategies have been established and are comprehensively used in Chapter 7 for the further study on effects of single joint and in Chapter 8 for the further study on effects of multiple parallel joints. (3) Studies on effects of the radius of the cavity, from which the wave originates, on 2-D compressional wave propagation are also performed.

For the effects of the position of the joint for the case of single joint and the first joint in the case of multiple parallel joints, it has been found that if the normal distance from the cavity wall is larger than the radius of the cavity, the effects of the position of the joint or the first joint in the joint set on the transmission ratio can be ignored.

Regarding the most important part in this chapter, the characteristics of the transmission ratio under the parameters related to single joint or joint set, the following findings are obtained:

- In the case of 2-D compressional wave propagation through single joint, the stiffness of the joint significantly affects the transmission ratio T_1 . On the other hand, the transmission ratio T_1 is independent of the radial distance from the center of the wave source for the points located behind the joint. Hence, the transmission ratio T_1 at any single point located behind the joint and on a specific radial line can be used to represent the transmission ratio T_1 at any other points located on the same radial line. The radian angle has been found to have significant effects on the performance of the transmission ratio T_1 . Likewise, the transmission ratios T_1 corresponding to points along a circle,

where all the points have the same radial distance from the cavity center, can be used to represent the transmission ratio T_1 at any other points behind the single joint.

- In the case of 2-D compressional wave propagation through multiple parallel joints, the stiffness, the number of joints and the spacing of the joints significantly influence the transmission ratio T_n . However, the transmission ratio T_n at any point on a specific radial line is independent of the radial distance from the center of wave source for the points behind the last joint in the joint set. Therefore, the transmission ratio T_n corresponding to any single point located behind the last joint in the joint set and on a specific radial line can be used to represent the transmission ratio T_n for all the other points on the same radial line behind the joints. The radian angle has also been found to have significant effects on performance of the transmission ratio T_n , when 2-D compressional wave passes through multiple parallel joints. However, the radial distance from the cavity center does not affect the transmission ratio T_n . Hence, the transmission ratio T_n corresponding to the points along a single circle, where all the points have the same radial distance from the cavity center, can be used to represent the transmission ratio T_n for any other points behind a certain number of joints.
- Based on the studies performed in this chapter, the following research strategies have been proposed and used in Chapter 7 for the further study on effects of single joint and in Chapter 8 for the further study on effects of multiple parallel joints in the process of 2-D compressional wave propagation.
 - (1) The study on 2-D compressional wave propagation for both of the single joint case and the multiple joints case can be done by just using the points along a circle where all the points have the same radial distance from the cavity center.
 - (2) In the radian direction of 90° (radian direction normal to the joint), the transmission ratio in the process of 2-D compressional wave propagation through single joint or multiple joints is dominated by the normal stiffness of

the joint. By contrast, the shear stiffness of the joint has negligible influence. Therefore, it is reasonable for the study on the transmission ratio corresponding to the points along the radian angle of 90° by using one single point behind the joint and keeping the same shear stiffness ratio of j_{ks}/G and the normal stiffness ratio of j_{kn}/K .

(3) Although the shear stiffness of the joints does not significantly affect the transmission ratio in the radian direction of 90° , it significantly affects the transmission ratio in other arbitrary directions, for both the cases of single joint and multiple joints. Therefore, the transmission ratio is further studied for both the cases of the radian direction of 90° and arbitrary radian directions. However, for the case of 90° , which is simpler than the case of arbitrary directions, more parametric studies can be performed.

It has been found that the radius of the tunnel or borehole for the source wave does not affect the transmission ratio in the process of 2-D compressional wave propagation through both single and multiple parallel joints.

It should be noted that the transmission ratio in the context of this study is defined as the ratio of the amplitude of the transmitted wave at a point after the joint (for case of single joint) or the last joint in the joint set (for the case of multiple joints) over the wave amplitude at the same point in the case of intact rock

CHAPTER 7

A STUDY ON EFFECTS OF SINGLE JOINT IN THE PROCESS OF 2-D COMPRESSIONAL WAVE PROPAGATION

7.1 Introduction

In Chapter 6, the following research strategies have been justified for the study on effects of single joint in the process of 2-D compressional wave propagation, which is performed in this chapter.

- (1) The study on 2-D compressional wave propagation for the single joint case can be performed by just using the points along single circle, where all the points have the same radial from the cavity center.
- (2) In the radian direction of 90° , it is reasonable to study the transmission ratio T_1 for single joint, by considering single point behind the joint on the radial line with the radian angle of 90° and using the same ratio of j_{kn}/K and j_{ks}/G .

In this chapter, firstly, a study is performed, focusing on the effects of single joint with different shear stiffness and normal stiffness on the performance of the transmission ratio T_1 in arbitrary radian directions, by using the research strategy as described above (the first item). Secondly, a study is performed, focusing on the effects of single joint on the performance of the transmission ratio T_1 in the radian direction normal to the joint (with the radian angle of 90°), by using the research strategy as described above (the second item).

The parametric studies on the effects of single joint presented in Chapter 6 have mainly focused on the performance of T_1 in the process of 2-D compressional wave propagation, where the normal and shear stiffness are assumed to be the same. By contrast, the parametric studies conducted in this chapter will focus on the effects of the stiffness on the performance of the transmission ratio T_1 in the process of 2-D compressional wave propagation in a rock mass, where normal and shear stiffness are treated as variables. Therefore, the situations being studied in this chapter are more comprehensive and complicated, however more meaningful in engineering

than the situations in Chapter 6. The other advantage of the study in this chapter is that the parametric studies are performed on a model with bigger geometry to enlarge the range of the radian angles.

7.2 A study on effects of single joint

7.2.1 Problem statement and modeling conditions

The problem and UDEC model considered in the study are shown in Figure 7.1. A cavity with a radius of 5 m exists in an infinite rock mass with single joint, parallel to the y-axis with a normal distance of 15 m from the center of the cavity. All the four boundaries are viscous boundaries.

A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2% for a frequency of 200 Hz. Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study in this chapter.

Figure 7.1 Sketch of UDEC model for effects of single joint

The overall objective in this chapter is to study the transmission ratio T_1 for single joint for the combined velocity wave. It means both of the normal and shear stiffness of the joint affect the transmission ratio T_1 . Therefore, both the normal and shear stiffness of the joint are assigned with different values to study the effects of the joint on T_1 in this chapter. The normal stiffness of the joint is chosen as 0.03K, 0.07K, 0.1K, 0.2K and 0.4K, and for each of these given normal stiffness, the shear stiffness of the joint is varied as: 0.03G, 0.07G, 0.1G, 0.2G and 0.4G. K and G are the bulk modulus the shear modulus of the rock material. (It is noted that j_{kn} and j_{ks} are digitally given the values of K or G times a certain factor. The units of j_{kn} and j_{ks} are stress per length, different from the units of K and G).

In order to study the effects of the joint on the transmission ratio T_1 , all the velocity waves at monitoring points along a circle, where all the points have the same radial distance from the cavity center, are comprehensively recorded and processed.

7.2.2 Results and discussion

Figures 7.2 (a) to (e) show the transmission ratio T_1 corresponding to the points behind the joint versus the radian angle for combinations of a given value of normal stiffness (0.03K (a), 0.07K (b), 0.1K (c), 0.2K (d) and 0.4K (e) respectively) and different values of shear stiffness (0.03G, 0.07G, 0.1G, 0.2G and 0.4G).

(a) T_1 for different shear stiffness (0.03G, 0.07G, 0.1G, 0.2G and 0.4G) and the given normal stiffness 0.03K

(b) T_1 for different shear stiffness (0.03G, 0.07G, 0.1G, 0.2G and 0.4G) and the given normal stiffness 0.07K

Figure 7.2 T_1 for different combination of shear and normal stiffness

(c) T_1 for different shear stiffness (0.03G, 0.07G, 0.1G, 0.2G and 0.4G) and the given normal stiffness 0.1K

(d) T_1 for different shear stiffness (0.03G, 0.07G, 0.1G, 0.2G and 0.4G) and the given normal stiffness 0.2 K

Figure 7.2 (Continued) T_1 for different combination of shear and normal stiffness

(e) T_1 for different shear stiffness (0.03G, 0.07G, 0.1G, 0.2G and 0.4G) and the given normal stiffness 0.4K

Figure 7.2 (Continued) T_1 for different combination of shear and normal stiffness

From Figures 7.2 (a), (b), (c), (d) and (e), it can be seen that for every given value among all the five given values of normal stiffness, the transmission ratios T_1 corresponding to the points with the radian angle of 90° are almost the same regardless of the values of shear stiffness. It means that the effects of the shear stiffness on the performance of the transmission ratio T_1 are negligible at the radian angle of 90° . By contrast, the transmission ratio corresponding to the points at other arbitrary radian directions (other than 90°) increases with the value of the shear stiffness, i.e., when a higher value of the shear stiffness is used, a higher value of the transmission ratio T_1 is obtained.

On the other hand, it can also be seen that the shape of the curves of the transmission ratio T_1 versus radian angle is dependent on the combination of normal and shear stiffness. From these curves, there are two obvious trends that for a very low value of normal stiffness of joint, as shown in Figure 7.2 (a) for the given value of normal stiffness of 0.03K, the minimum transmission ratio T_1 always occurs at the radian angle of 90° . By contrast, for a very high value of normal stiffness as shown in Figure 7.2 (e) for a given value of normal stiffness of 0.4K, the maximum transmission ratio T_1 approximately always occurs at the radian angle of 90° .

Regarding these obvious trends, the curves corresponding to two extreme combinations of normal and shear stiffness of the joint are discussed in the following two paragraphs. One extreme combination of the normal and shear stiffness of the joint is that a very low normal stiffness of the joint of $0.03K$ is combined with a very high shear stiffness of the joint of $0.4G$, corresponding to the curve shown in Figure 7.2 (a). The other extreme combination of the normal and shear stiffness of the joint is that a very high normal stiffness of the joint of $0.4K$ is combined with a very low shear stiffness of the joint of $0.03G$, corresponding to the curve shown in Figure 7.2 (e). The curves with other combinations of the normal and shear stiffness of the joint are between these two curves with these two extreme combinations.

The reason for occurrence of the minimum transmission ratio T_1 at the radian angle of 90° for the case of the extreme combination of a very low normal stiffness of $0.03K$ and a very high shear stiffness of $0.4G$ (shown in Figure 7.2 (a)) can be explained in the following two aspects. Firstly, for this extreme combination of normal and shear stiffness, the transmitted combined wave at the radian angle of 90° is mainly the component of P-wave, which is controlled by the normal stiffness of the joint with a very low value of $0.03K$. So that, it results in a very low transmission ratio T_1 . (It is noted that, for the case of 1-D obliquely incident P-wave upon single joint, the phenomenon that the transmitted wave is mainly P-wave at the incidence angle normal to the joint can also be seen in Figure 2.2.) Secondly, when the radian angle changes either increasingly towards 180° or decreasingly towards 0° , the component of the S-wave in the combined transmitted wave increases correspondingly, which is controlled by the shear stiffness with a very high value of $0.4G$. So that, it results in an increasing transmission ratio T_1 , although whether the transmitted combined velocity wave is enhanced or weakened is also influenced by other additional factors, such as the phases and frequencies of the transmitted P- and S-waves. (It is noted that, for the case of 1-D obliquely incident P-wave upon single joint, the phenomenon that the transmitted S-wave gradually increases as the incidence angle deviates from the incidence angle normal to the joint can also be seen on the curve for $|T_p - S_1|$ in Figure 2.2) Therefore, for this

extreme combination of a very low normal stiffness of the joint and a very high shear stiffness of the joint, the minimum transmission ratio T_1 occurs at the radian angle of 90° .

The reason for occurrence of the maximum transmission ratio T_1 at the radian angle of 90° for the case of the extreme combination of a very high normal stiffness of $0.4K$ and a very low shear stiffness of $0.03G$ (shown in Figure 7.2 (e)) can be explained as follows. For this extreme combination of normal and shear stiffness of the joint, the transmitted combined wave at the radian angle of 90° is mainly the component of P-wave, which is controlled by the normal stiffness of the joint with a very high value of $0.4K$. So that it results in a very high transmission ratio T_1 . By contrast, when the radian angle changes either increasingly towards 180° or decreasingly towards 0° , the component of the S-wave in the combined transmitted wave increases correspondingly, which is controlled by the shear stiffness of the joint with a very low value of $0.03G$. So that it results in a decreasing transmission ratio T_1 . So, for this extreme combination of a very high normal stiffness of the joint and a very low shear stiffness of the joint, the maximum transmission ratio T_1 occurs at the radian angle of 90° .

It can be summarized that for the combination of very high value of normal stiffness with other values of shear stiffness, the highest transmission ratio can be achieved at the radian angle of 90° . By contrast, for the combination of very small value of normal stiffness with other values of shear stiffness, the lowest transmission ratio can be achieved at the radian angle of 90° . Hence, the performance of the transmission ratio at the radian angle of 90° is dominated by the values of normal stiffness. However, when the radian angle changes either increasingly towards 180° or decreasingly towards 0° , the effects of the normal stiffness decreases while the effects of the shear stiffness increases correspondingly.

7.3 Normalized normal and shear stiffness

In the research work of Schoenberg (1980) and Pyrak-Nolte *et al.* (1987 and 1990a), it has been found that in 1-D wave propagation in a rock mass, the magnitudes of reflection coefficient $|R|$ and transmission coefficient $|T|$ can be expressed as a

function of $k/\omega z$, which is called the normalized stiffness of joint. The parameter k and the parameter z depend upon the type of waves. For compressional wave incidence, k denotes the normal stiffness of the joint and z equivalent to ρc_p is the seismic impedance of the rock for P-wave, where ρ is the density of the rock and c_p is the P-wave velocity in the rock. On the other hand, for shear wave incidence, k denotes the shear stiffness and z equivalent to ρc_s is the seismic impedance for S-wave, in which ρ is the density of the rock and c_s is the S-wave velocity in the rock.

This section investigates the effects of the normalized stiffness of the joint on the performance of 2-D compressional wave propagation in the rock mass.

The problem and geometry of the model are shown in Figure 7.1. The properties of the rock material are given in Table 5.3. The properties of the input compressional pulse velocity wave and stiffness of the joint for the two cases are as follows:

Case A: the frequency of the compressional pulse velocity wave is 200 Hz, where the normal stiffness j_{kn} is 0.1K and the shear stiffness j_{ks} is 0.1G.

Case B: the frequency of compressional pulse velocity wave is 500 Hz, where the normal stiffness j_{kn} is 0.25K and the shear stiffness j_{ks} is 0.25G.

It can be seen that $j_{kn}/(\omega z)$ and $j_{ks}/(\omega z)$ for case A are the same as those for case B.

Figure 7.3 shows a comparison between case A and case B for the performance of the transmission ratio T_1 corresponding to the points behind the joint along a circle, where all the points have the same radial distance from the center of the cavity.

From Figure 7.3, it can be seen that the transmission ratio T_1 obtained from case A is almost the same as that obtained from case B for all values of radian angles. It indicates that in the process of 2-D compressional wave propagation, the transmission ratio T_1 is a function of the normalized stiffness of the joint.

Figure 7.3 Comparison of T_1 for two different cases (A and B), with the same normalized shear and normal stiffness ($jks/(\omega z)$ and $jkn/(\omega z)$)

7.4 Prediction models for T_1

Based on extensive parametric studies on performance of the transmission ratio T_1 for both the radian direction of 90° and the arbitrary radian directions respectively, the corresponding prediction models for T_1 in the process of 2-D compressional wave propagation through single joint are proposed in this section.

7.4.1 Prediction model for T_1 for the radian direction of 90°

In UDEC model as shown in Figure 7.1, in the process of 2-D compressional wave propagation through single joint, the transmission ratio T_1 corresponding to one point behind the joint is investigated for the following combinations of jkn and jks : (0.03K, 0.03G), (0.07K, 0.07G), (0.1K, 0.1G), (0.2K, 0.2G), (0.3K, 0.3G), (0.4K, 0.4G), (0.5K, 0.5G), (0.6K, 0.6G), (0.7K, 0.7G) and (0.8K, 0.8G).

Figure 7.4 shows the relationship between the transmission ratio T_1 and the normalized normal stiffness $jkn/(\omega z)$ in the process of 2-D compressional wave propagation through single joint in the radian angle of 90° . This curve can be used as the prediction model for the transmission ratio T_1 in the radian direction of 90° , because it comprehensively shows the relationship between the transmission ratio T_1 and its influence factor, normalized normal stiffness of the joint.

Figure 7.4 T_1 as a function of the normalized normal stiffness along the radial line with α of 90°

In the special direction of 90° in the process of 2-D compressional wave propagation through single joint, the effects of shear stiffness of the joint are negligible, while normal stiffness of the joint dominates the transmission ratio. So, for the case of 2-D compressional wave, the factor influencing the transmission ratio T_1 is the same with the factor influencing the magnitude of transmission coefficient $|T_1|$ for normally incident 1-D P-wave propagation through single joint. Thus, in this radian direction, 2-D compressional wave propagation through single joint is comparable with the normally incident 1-D P-wave propagation through single joint.

It can be seen in Figure 7.4 that T_1 at the points along the radian direction of 90° increases with the normalized normal stiffness ranging from a value close to 0, representing a free boundary (i.e., two separate rocks), to a value close to 1, indicating no interface between the rocks (i.e., an intact rock). The trend and the shape of the curve shown in Figure 7.4 is similar to the relationship between the magnitude of transmission coefficient $|T_1|$ and the normalized normal stiffness for the single joint in normally incident 1-D P-wave propagation shown in Figure 3.4. (In normally incident 1-D case, the relationship between $|T_1|$ and $jkn/(\omega z)$ for P-wave is the same with that between $|T_1|$ and $jks/(\omega z)$ for S-wave.)

7.4.2 Prediction model for T_1 for arbitrary directions

Model for T_1 for arbitrary directions and discussion

Figure 7.5 shows the performance of the transmission ratio T_1 in the process of 2-D compressional wave propagation through single joint for different combinations of the given normalized normal stiffness ($jkn/(\omega z) = 0.067, 0.157, 0.224, 0.448$ and 0.897) and the given normalized shear stiffness ($jks/(\omega z) = 0.075, 0.174, 0.249, 0.498$ and 0.997). Figure 7.5 is obtained based on the data shown in Figures 7.2 (a), (b), (c), (d) and (e) using normalized stiffness of the joint. Because all the effects of the three factors influencing T_1 (the normalized normal and shear stiffness and the radian angle α) are comprehensively generalized into Figure 7.5, the curves shown in Figure 7.5 can be used as the prediction model for T_1 for 2-D compressional wave propagation through single joint.

Figure 7.5 T_1 for different normalized shear stiffness (0.075, 0.174, 0.249, 0.498 and 0.997) and different normalized normal stiffness (0.067, 0.157, 0.224, 0.448 and 0.897)

Figure 7.5 illustrates the transmission ratio T_1 versus the radian angle for 5 groups of curves corresponding to 5 given values of normalized normal stiffness, where each group of curves corresponds to different combinations of a given value of normalized normal stiffness and 5 different values of the normalized shear stiffness. From the comparison of these 5 groups of curves, it can be seen that the transmission ratio T_1 in the direction of 90° increases as the value of the normalized normal stiffness increases. This general trend can be used as a criterion to identify the curve groups corresponding to different given values of the normalized normal stiffness.

By contrast, if comparing the effects of the normalized shear stiffness on the transmission ratio T_1 in arbitrary radian angles within each group of curves, the transmission ratio T_1 at a certain arbitrary radian angle (other than 90°) increases significantly as the value of normalized shear stiffness increases. This trend can be used as a criterion to identify the curve for the different normalized shear stiffness within each group of curves corresponding to a given value of normalized normal stiffness of the joint.

When the normalized shear stiffness is fixed, at an arbitrary radian angle, the value of the transmission ratio T_n increases as the normalized normal stiffness value increases.

From the above observations, it can be summarized as follows: (1) When the normalized normal stiffness of the joint is fixed, the transmission ratio T_1 corresponding to radian angles near 90° is dominated by the normalized normal stiffness of the joint, in this scenario, the normalized shear stiffness has negligible influence. (2) When the value of normalized normal stiffness of the joint is very small, the minimum transmission ratio T_1 occurs at the direction of 90° . However, when the value of normalized normal stiffness is high enough, the maximum transmission ratio T_1 also occurs at the radian direction of 90° . (3) Under a given normalized shear stiffness, at an arbitrary radian angle, the value of the transmission ratio T_1 increases as the normalized normal stiffness value increases.

Figure 7.5 shows the variations of the transmission ratio T_1 with radian direction, including the radian angle of 90° , for different combinations of normalized normal and shear stiffness of the joint. For the radian angle of 90° , noticing the effects of normalized shear stiffness of the joint on T_1 are negligible, the variation of the transmission ratio T_1 with the normalized normal stiffness of the joint has been specifically obtained as shown in Figure 7.4. It has been found that, for the radian angle of 90° , the variation of the transmission ratio T_1 with the normalized normal stiffness of the joint implicitly incorporated in Figure 7.5 is compatible with the curve shown in Figure 7.4.

Figure 7.6 shows the transmission ratio T_1 versus the normalized normal stiffness of the joint in the radian angle of 90° . In the figure, the solid curve is obtained using the data presented in Figure 7.4, The diamond points represent the transmission ratio T_1 corresponding to different combinations of normalized normal and shear stiffness of the joint as (0.067, 0.075), (0.157, 0.174), (0.224, 0.249), (0.448, 0.498) and (0.897, 0.997) respectively, obtained from Figure 7.5. It can be seen that the transmission ratio T_1 corresponding to five combinations of normalized normal and shear stiffness of the joint obtained from Figure 7.5 matches well the solid curve presented in Figure 7.4. Noticing that the above data shown in Figure 7.4 and Figure 7.5 are obtained from independent computing processes, so, in the direction of 90° , the two prediction models (Figure 7.4 and Figure 7.5) of the transmission ratio T_1 are in essence the same.

Finally, also From Figure 7.5, for all the radian angles, the range of the transmission ratio T_1 covers the range of around 0.2 to 0.9. It indicates that the combinations of the normalized normal stiffness ($j_{kn}/(\omega z)=0.067, 0.157, 0.224, 0.448$ and 0.897) and normalized shear stiffness ($j_{ks}/(\omega z)=0.075, 0.174, 0.249, 0.498$ and 0.997) of the joint cover the most sensitive and practical range of the normalized normal and shear stiffness of the joint for a 2-D compressional wave problem.

Figure 7.6 Comparison of the curve from Figure 7.4 and the data from Figure 7.5

Steps of estimation of T_1 for arbitrary directions

Based on Figure 7.5, it is possible to predict the transmission ratio T_1 for any practical joint and input frequency in the process of 2-D compressional wave propagation through single joint using linear interpolation. The following flow chart shows the procedures of estimating T_1 for arbitrary directions. This flow chart will be illustrated in Section 7.5.

Figure 7.7 Procedure of estimating T_1 for arbitrary directions

7.5 Sensitivity study on the prediction models for T_1

This part aims to perform sensitivity study on the prediction models proposed in Section 7.4.1 for T_1 corresponding to the points along the radian angle of 90° , and in Section 7.4.2 for T_1 for arbitrary directions in a hypothetical case. The estimated transmission ratio T_1 from these two prediction models for the hypothetical case is compared with UDEC results.

Statement of the hypothetical problem and its UDEC modeling conditions

A cavity exists in an infinite rock mass containing Bukit Timah granite in Singapore (the properties are shown in Table 5.3) with single joint. A compressional pulse wave with unity amplitude and a frequency of 500 Hz is applied at the boundary of the cavity, traveling through the joint, with j_{kn} and j_{ks} of 24.0 Gpa/m and 10.4 Gpa/m, respectively.

The UDEC model for this hypothetical problem is shown in Figure 7.1. A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 500 Hz is input in the cavity. The mesh size of 0.4 m is chosen to keep the numerical error less than 2% for a frequency of 500 Hz. In order to compare the estimated transmission ratio for the hypothetical case with those obtained from UDEC results, in UDEC input file, the values of normal and shear stiffness of the joint are chosen as 24.0 Gpa/m and 10.4 Gpa/m. The rock properties in Table 5.3 are coded in the input file.

Estimating T_1 for the hypothetical problem

For this hypothetical problem, the transmission ratio T_1 in arbitrary directions for 2-D compressional wave propagation through the joint is determined, by using the flow chart as shown in Figure 7.7.

Step 1: Determining the following parameters involved in estimating T_1 for arbitrary directions: (1) $K=40$ Gpa/m, $G=26$ Gpa/m, $\rho=2650$ kg/m³. (2) $j_{kn}=24.0$ Gpa/m, $j_{ks}=10.4$ Gpa/m. (3) Amplitude and angular frequency ω of the input wave are 1 m/s and 3142 radian/s.

Step 2: Calculating the following values: (1) P-wave velocity c_p is calculated and listed in Table 5.3. (2) The seismic impedance for P-wave z is equal to ρc_p . (3) The target values of $jkn/(\omega z)$ and $jks/(\omega z)$ are calculated as 0.54 and 0.4 respectively.

Step 3: For the problem, the transmission ratio for the radian directions 20° , 30° , ... till 90° with an increasing step of 10° will be determined.

Step 4: For every radian angle α , taking 20° for example, the transmission ratio T_1 as a function of normalized normal stiffness is determined for the 5 given values of normalized shear stiffness ($jks/(\omega z)$) = 0.075, 0.174, 0.249, 0.498 and 0.997. Figure 7.8 shows these relations, which are directly obtained from Figure 7.5.

On Figure 7.8, for the target $jks/(\omega z)$ of 0.4 at the radian angle of 20° , T_1 can be obtained (as shown by the coarse line in Figure 7.9) by using the linear interpolation based on the data corresponding to $jks/(\omega z)$ of 0.249 and $jks/(\omega z)$ of 0.498. The estimated T_1 in the radian direction of 20° for $jks/(\omega z)$ of 0.4 and $jkn/(\omega z)$ of 0.54 can be taken from the coarse line in Figure 7.9 as 0.73.

Likewise, the transmission ratio T_1 corresponding to the other radian angles for $jks/(\omega z)$ of 0.4 and $jkn/(\omega z)$ of 0.54 can be estimated using the linear interpolation as well. The obtained values of T_1 at different radian angles for this hypothetical problem are tabulated in Table 7.1 and visually presented in Figure 7.10.

Figure 7.8 T_1 versus normalized normal stiffness for different given levels of normalized shear stiffness in the radian direction of 20°

Figure 7.9 T_1 versus normalized normal stiffness for different given levels of normalized shear stiffness, including the target $jks/(\omega z)$ of 0.4 by linear interpolation, in the radian direction of 20°

Figure 7.10 Estimated T_1 for arbitrary directions for target values of $jkn/(\omega z)$ of 0.54 and $jks/(\omega z)$ of 0.4

Table 7.1 Radian angles and T_1 for the hypothetical case

α ($^\circ$)	14.6	20	30	40	50	60	70	80	90
T_1	0.75	0.73	0.69	0.67	0.65	0.69	0.70	0.71	0.71

Sensitivity studies on the prediction models for T_1 for α of 90° and for arbitrary directions

The estimated transmission ratio T_1 corresponding to the points along the radian direction of 90° can be directly obtained from Figure 7.4 as 0.73.

Figure 7.11 shows the comparison between the estimated T_1 from the prediction model for T_1 for the radian direction of 90° shown in Figure 7.4 and T_1 calculated from UDEC results for the hypothetical case.

Figure 7.12 shows the comparison between the estimated T_1 from the prediction model for T_1 for arbitrary directions and T_1 calculated from UDEC results for the case.

It can be seen from Figure 7.11 and Figure 7.12, the agreements between T_1 calculated from UDEC results and the estimated T_1 from the prediction models proposed in this thesis are very good.

Figure 7.11 Comparison between the estimated and practical numerical results, in terms of transmission, in radian direction of 90°

Figure 7.12 Comparison between the estimated and practical numerical results, in terms of transmission ratio versus radian angle, for $jkn/(\omega z)$ of 0.54 and $jks/(\omega z)$ of 0.4

7.6 Summary

Using the new data processing method proposed in Chapter 5, extensive parametric studies on 2-D compressional wave propagation through single joint are performed, both for the radian direction of 90° and arbitrary directions.

The parametric studies have been performed to investigate the effects of the influence factors (including the normalized normal and shear stiffness and the radian angle) on the performance of the transmission ratio T_1 , in the process of 2-D compressional wave propagation through single joint in the rock mass. Based on the results of the parametric studies, a prediction model to estimate the value of transmission ratio T_1 in arbitrary directions is proposed. For the special direction of 90° , since the effects of the normalized shear stiffness of the joint on the performance of the transmission ratio T_1 are negligible, a more straightforward and simpler prediction model is also proposed.

Comparing the prediction model for T_1 in the direction of 90° with the prediction model for T_1 in arbitrary directions, it is shown that both models give similar estimated T_1 value in the direction of radian angle of 90° . The model specially developed for the radian direction of 90° is simpler and the estimated T_1 can be

directly obtained from the prepared chart. However, the prediction model for the transmission ratio T_1 for arbitrary directions can be used for estimating T_1 for radian directions other than 90° , but the linear interpolation in the data processing is required to obtain the estimated results.

From the prediction model for T_1 in arbitrary directions shown in Figure 7.5, it can be seen that for combinations of a given value of normalized normal stiffness with different values of normalized shear stiffness, the effects of normalized shear stiffness on the performance of T_1 in the direction of radian angle 90° are negligible. However, it has been also found that in the direction of arbitrary radian angles (other than 90°), the effects of the normalized shear stiffness on the transmission ratio T_1 become significant. So, for combinations of a given value of normalized normal stiffness with different values of normalized shear stiffness, the transmission ratios corresponding to a fixed radian angle (other than a narrow angle range around 90°) vary as the value of the normalized shear stiffness changes. Generally, when the normalized shear stiffness increases, the corresponding transmission ratio T_1 increases. Also as shown in Figure 7.5, it can be seen that when the normalized normal stiffness of the joint is small enough, the minimum transmission ratio T_1 always occurs at the radian angle of 90° . On the other hand, when the normalized normal stiffness is high enough, the maximum transmission ratio T_1 occurs at the radian angle of 90° . Under a given normalized shear stiffness, at an arbitrary radian angle, the value of the transmission ratio T_1 increases as the normalized normal stiffness value increases.

Sensitivity studies on the two proposed prediction models for the transmission ratio T_1 are conducted. The agreement between the estimated results and the UDEC results is very good.

CHAPTER 8

A STUDY ON EFFECTS OF MULTIPLE PARALLEL JOINTS IN THE PROCESS OF 2-D COMPRESSIONAL WAVE PROPAGATION

8.1 Introduction

In Chapter 6, the following research strategies have been justified for the study on effects of multiple parallel joints in the process of 2-D compressional wave propagation.

- (1) In the radian direction of 90° , it is reasonable to study the transmission ratio T_n for multiple parallel joints, by considering single point behind the joints on the radial line with the radian angle of 90° and using the same ratio of j_{kn}/K and j_{ks}/G .
- (2) The study on 2-D compressional wave propagation for the case of multiple parallel joints can be performed by considering the points along single circle, where all the points have the same radial distance from the cavity center.

Theses research strategies are followed in the study presented in this chapter.

This chapter includes two main parts. In the first part, extensive parametric studies on the effects of multiple parallel joints on 2-D compressional wave propagation in the direction of 90° are carried out, by using the first research strategy described above. Based on the results obtained, two prediction models, one for predicting the transmission ratio T_n and the other for predicting the maximum rebound ratio, are proposed for the problem of 2-D compressional wave propagation through multiple parallel joints in the radian direction of 90° . These research works are presented in Section 8.2. In the second part, extensive parametric studies on the effects of multiple parallel joints on the performance of the transmission ratio T_n in other arbitrary directions are carried out, by using the second research strategy described above. Based on the results obtained, a prediction model that can be used to

estimate the transmission ratio T_n in arbitrary directions is proposed. These research works are presented in Section 8.3.

The parametric studies on the effects of multiple parallel joints presented in Chapter 6 have mainly focused on the performance of T_n in the process of 2-D compressional wave propagation, where the normal and shear stiffness are assumed to be the same. By contrast, the parametric studies conducted in this chapter focus on the performance of the transmission ratio T_n in the process of 2-D compressional wave propagation in a rock mass, where the normal and shear stiffness have different values. Therefore, the situations being studied in this chapter are more comprehensive and complicated, however more meaningful in engineering than the situations in Chapter 6. The other advantage of this chapter is that the parametric studies are based on a model with bigger geometry to enlarge the range of the radian angles.

8.2 A study on effects of multiple joints along the radian direction of 90°

There are two focuses in this section. The first focus is on performing extensive parametric studies on the effects of multiple parallel joints on the performance of the transmission ratio T_n corresponding to the direction of the radian angle of 90° . Based on the analysis performed, a prediction model for the transmission ratio T_n corresponding to the radian direction of 90° is proposed. A hypothetical case study on the sensitivity of the prediction model is conducted. The second focus is on studying the effects of the multiple parallel joints on the performance of superposition of the multiple reflected and transmitted waves between two adjacent joints along the radian direction of 90° . Based on the analysis, a prediction model that can be used to estimate the maximum rebound ratio is proposed as well.

8.2.1 A study on transmission ratio T_n along the radian angle of 90°

8.2.1.1 Problem statement and modeling conditions

The problem and UDEC model considered in this study are shown in Figure 8.1. A cavity with a radius of 5 m exists in an infinite rock mass with multiple parallel

joints. The joints are parallel to the y-axis with the first joint located at a normal distance of 15 m from the center of the cavity. The top, right and left boundaries are viscous boundaries, while the lower boundary is a symmetrical boundary. A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2%. Properties of Bukit Timah granite of Singapore shown in Table 5.3 are considered for the study.

The performance of the transmission ratio T_n is investigated under the following conditions: (1) 5 groups of different combinations of normal and shear stiffness, i.e., (0.03K, 0.03G), (0.07K, 0.07G), (0.1K, 0.1G), (0.2K, 0.2G) and (0.4K, 0.4G). K and G are bulk modulus and shear modulus of the rock material respectively. (It is noted that j_{kn} and j_{ks} are digitally given the values of K or G times a certain factor. The units of j_{kn} and j_{ks} are stress per length, different from the units of K and G). (2) The ratio of joint spacing to wavelength (spacing/wavelength) is changed in the range from 0.002 to 0.9, mainly depending on the occurrence of the full curve (as the general curve, which will be shown in Figure 8.3) of the transmission ratio T_n versus the ratio of spacing/wavelength for each given value of normalized stiffness, each given number of joints and the limitation of the geometry of UDEC model. (3) The numbers of the joints used in the analysis are 2, 4 and 8.

Figure 8.1 Sketch of UDEC model for effects of multiple parallel joints

8.2.1.2 UDEC results and discussion

Figure 8.2 shows the transmission ratio T_n versus spacing/wavelength in the radian direction of 90° for different given values of $jkn/(\omega z)$; Figures 8.2 (a), (b) and (c) show the performance of T_n for 2 joints, 4 joints and 8 joints, respectively.

From Figures 8.2 (a), (b) and (c), it can be seen that for 2-D compressional wave propagation through multiple parallel joints along the radian direction of 90° , the variation of T_n with the ratio of spacing/wavelength has three different modes. These three modes are similar with those for the case of normally incident 1-D wave as shown in Figure 2.4.

For example, in Figure 8.2 (a) for the curve corresponding to $jkn/(\omega z)$ of 0.897, (1) when spacing/wavelength is bigger than 0.2, T_2 has a value of 0.73 and is independent of spacing/wavelength; (2) when spacing/wavelength decreases from 0.2 to 0.07, T_2 increases from 0.73 to the maximum value of 0.84; (3) when spacing/wavelength decreases below 0.07, T_2 decreases from the maximum value of 0.84 to a certain value.

Also from Figures 8.2 (a), (b) and (c), for the radian direction of 90° , it can be seen that T_n depends not only on the normalized normal stiffness of the joints, but also on spacing/wavelength and the number of joints.

From Figures 8.2 (a), (b) and (c), for each given number of joints and each given value of normalized normal stiffness, the curve of T_n as a function of the ratio of spacing/wavelength can be simplified as the general curve shown in Figure 8.3. As seen from Figure 8.3, there are two critical points ($S_{\text{peak}}, T_{\text{peak}}$) and ($S_{\text{flat}}, T_{\text{flat}}$), where S stands for the ratio of spacing/wavelength, and T stands for the transmission ratio. These two critical points ($S_{\text{peak}}, T_{\text{peak}}$) and ($S_{\text{flat}}, T_{\text{flat}}$) are dependent on two factors, the number of joints and the normalized normal stiffness of the joints.

So, T_n in the radian direction of 90° varies with the ratio of spacing/wavelength in three different modes, which can be described as follows:

- If $\text{spacing/wavelength} \geq S_{\text{flat}}$, T_n is independent of the ratio of spacing/wavelength.
- If $S_{\text{flat}} > \text{spacing/wavelength} \geq S_{\text{peak}}$, T_n increases from T_{flat} to the maximum value T_{peak} as the ratio of spacing/wavelength decreases from S_{flat} to S_{peak} .
- If $\text{spacing/wavelength} \leq S_{\text{peak}}$, T_n decreases from the maximum value T_{peak} as the ratio of spacing/wavelength decreases.

(a) T_2 versus spacing/wavelength along the radian direction of 90° for different values of $jkn/(\omega z)$

(b) T_4 versus spacing/wavelength along the radian direction of 90° for different values of $jkn/(\omega z)$

Figure 8.2 T_2 , T_4 and T_8 versus spacing/wavelength along the radian direction of 90° for different values of $jkn/(\omega z)$

(c) T_8 versus spacing/wavelength along the radian direction of 90° for different values of $jkn/(\omega z)$

Figure 8.2 (Continued) T_2 , T_4 and T_8 versus spacing/wavelength along the radian direction of 90° for different values of $jkn/(\omega z)$

Figure 8.3 General curve for T_n versus spacing/wavelength for 2-D compressional wave passing through multiple parallel joints along the radian direction of 90°

8.2.1.3 Prediction model for T_n for the radian angle of 90°

This part aims to propose a prediction model for the transmission ratio T_n in the radian direction of 90° in the process of 2-D compressional wave passing through multiple parallel joints.

From the discussion in Section 8.2.1.2, along the direction of 90° , in the simplified general curve for T_n versus spacing/wavelength for 2-D compressional wave passing through multiple parallel joints, the two critical points are dependent on two factors (the number of joints and the normalized normal stiffness). If these two critical points are determined for any number of joints and any normalized normal stiffness of the joints, the simplified general curve of the transmission ratio T_n versus the ratio of spacing/wavelength can be determined for any given number of joints and any given value of normalized normal stiffness. The relationship between the two critical points (S_{peak} , T_{peak}) and (S_{flat} , T_{flat}) defined in Figure 8.3 and the two factors (number of the joints and the normalized normal stiffness) can be used to establish a prediction model for T_n along the radian angle of 90° .

Relationship between (S_{peak} , T_{peak}) and (S_{flat} , T_{flat}) and $jkn/(\omega z)$ for given numbers of joints

The relationship between the two critical points (S_{peak} , T_{peak}) and (S_{flat} , T_{flat}) and the normalized normal stiffness for the given numbers of joints (2, 4 and 8) can be directly obtained from Figures 8.2 (a), (b) and (c). Figure 8.4, Figure 8.5, Figure 8.6 and Figure 8.7 respectively show S_{peak} , T_{peak} , S_{flat} and T_{flat} as a function of the normalized normal stiffness for different given numbers of joints: 2, 4 and 8.

From Figure 8.4, it can be observed that, for every given number of joints, S_{peak} increases as the normalized normal stiffness of the joints $jkn/(\omega z)$ increases. From Figure 8.6, it can be observed that, for every given number of joints, S_{flat} decreases as the normalized normal stiffness $jkn/(\omega z)$ increases. These two trends are similar with the general trends in 1-D wave propagation as stated at the end in Section 2.3.3.2 in the third item and the fourth item, respectively.

From Figure 8.5, it can be seen that, for each given number of joints, T_{peak} increases as the normalized normal stiffness increases. From Figure 8.7, it can be seen that for each given number of joints, T_{flat} increases as the normalized normal stiffness increases. It means higher normalized normal stiffness results in higher transmission ratio. This trend is compatible with the common sense, and it can be seen in Figure 2.4 in 1-D wave propagation.

It can be imagined, if the normalized normal stiffness is very big, the maximum value of T_{peak} and T_{flat} would be close to 1 (never bigger than 1), representing an intact rock without joints. By contrast, if the normalized normal stiffness of the joints is very small, the minimum value of T_{peak} and T_{flat} would approach 0, representing several independent rocks without any wave transmission at the joints.

Figure 8.4 S_{peak} as a function of normalized normal stiffness $jkn/(\omega z)$ for different number of joints along the radian direction of 90°

Figure 8.5 T_{peak} as a function of normalized normal stiffness $jkn/(\omega z)$ for different number of joints along the radian direction of 90°

Figure 8.6 S_{flat} as a function of normalized normal stiffness $jkn/(\omega z)$ for different number of joints along the radian direction of 90°

Figure 8.7 T_{flat} as a function of normalized normal stiffness $jkn/(\omega z)$ for different number of joints along the radian direction of 90°

Relationship between (S_{peak}, T_{peak}) and (S_{flat}, T_{flat}) and $jkn/(\omega z)$ for arbitrary numbers of joints

From Figure 8.4, Figure 8.5, Figure 8.6 and Figure 8.7, the two critical points (S_{peak}, T_{peak}) and (S_{flat}, T_{flat}) can be determined for any normalized normal stiffness for 3 given numbers of joints 2, 4 and 8. However, for the prediction model for T_n , it is necessary to determine the relationship between the two critical points (S_{peak}, T_{peak}) and (S_{flat}, T_{flat}) and the normalized normal stiffness for arbitrary numbers of joints. This relationship is implicitly incorporated in Figures 8.2 (a), (b) and (c).

In order to apparently determine the relationship between the two critical points (S_{peak}, T_{peak}) and (S_{flat}, T_{flat}) and the normalized normal stiffness for any number of joints, the parameters S_{peak} , T_{peak} , S_{flat} and T_{flat} are generalized as a function of the number of joints for different given values of normalized normal stiffness.

Figure 8.8, Figure 8.9, Figure 8.10 and Figure 8.11 show S_{peak} , T_{peak} , S_{flat} and T_{flat} versus the number of joints for different given values of normalized normal stiffness $jkn/(\omega z)$. The readings of S_{peak} , T_{peak} , S_{flat} , T_{flat} , for different given numbers of joints

and different given values of the normalized normal stiffness are directly taken from the curves shown in Figures 8.2 (a), (b) and (c).

Figure 8.8 S_{peak} as a function of the number of joints for different normalized normal stiffness $j_{kn}/(\omega z)$ along the radian direction of 90°

Figure 8.9 T_{peak} as a function of the number of joints for different normalized normal stiffness $j_{kn}/(\omega z)$ along the radian direction of 90°

Figure 8.10 S_{flat} as a function of the number of joints for different normalized normal stiffness $j_{kn}/(\omega z)$ along the radian direction of 90°

Figure 8.11 T_{flat} as a function of the number of joints for different normalized normal stiffness $j_{kn}/(\omega z)$ along the radian direction of 90°

From Figure 8.8 it can be observed that, for a given value of normalized normal stiffness, S_{peak} increases as the number of joints decreases. For example, at the given value of the normalized normal stiffness of 0.897, S_{peak} increases from 0.05 to 0.07 as the number of joints decreases from 8 to 2. By contrast, from Figure 8.10 it can be observed that for a given value of normalized normal stiffness, S_{flat} increases as

the number of joints increases. For example, at the given value of the normalized normal stiffness of 0.897, S_{flat} increases from 0.2 to 0.36 as the number of joints increases from 2 to 8. These two trends are similar with the general trends in 1-D wave propagation as stated at the end in Section 2.3.3.2 in the first item and the second item, respectively.

From Figure 8.9, it can be seen that for each given value of normalized normal stiffness, T_{peak} decreases as the number of the joints increases. Likewise, from Figure 8.11, it can also be seen that for each given value of normalized normal stiffness, T_{flat} decreases as the number of joints increases. This is because, for a given value of normalized normal stiffness, heavier wave attenuation occurs in the joint set with more joints.

Prediction model for T_n for the radian angle of 90°

Based on Figure 8.8, Figure 8.9, Figure 8.10 and Figure 8.11, respectively, S_{peak} , T_{peak} , S_{flat} and T_{flat} can be presented as a function of normalized normal stiffness for arbitrary numbers of joints by using linear interpolation or extrapolation. When the number of joints is less than 8, the values of S_{peak} , T_{peak} , S_{flat} and T_{flat} can be obtained by linear interpolation method. Otherwise, these values can be estimated by linear extrapolation when the number of joints is greater than 8.

S_{peak} , T_{peak} , S_{flat} and T_{flat} as a function of normalized normal stiffness $jkn/(\omega z)$ for arbitrary numbers of joints are presented in Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15, respectively. In these figures, the light solid lines are obtained by linear interpolation and the dash lines are obtained by linear extrapolation method based on the data presented in Figure 8.8, Figure 8.9, Figure 8.10 and Figure 8.11, while the coarse solid lines are obtained directly from Figures 8.2 (a), (b) and (c).

$(S_{\text{peak}}, T_{\text{peak}})$ and $(S_{\text{flat}}, T_{\text{flat}})$ defined in Figure 8.3 can be obtained from Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15 for arbitrary numbers of joints for any value of normalized normal stiffness in the radian direction of 90° . In this case, T_n can be presented as a function of spacing/wavelength. Therefore, Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15 can be used as a prediction model for the

transmission ratio T_n along the radian direction of 90° in the process of 2-D compressional wave propagation through multiple parallel joints.

Figure 8.12 S_{peak} as a function of normalized normal stiffness $jkn/(\omega z)$ for arbitrary numbers of joints along the radian direction of 90°

Figure 8.13 T_{peak} as a function of normalized normal stiffness $jkn/(\omega z)$ for arbitrary numbers of joints along the radian direction of 90°

Figure 8.14 S_{flat} as a function of normalized normal stiffness $jkn/(\omega z)$ for arbitrary numbers of joints along the radian direction of 90°

Figure 8.15 T_{flat} as a function of normalized normal stiffness $jkn/(\omega z)$ for arbitrary numbers of joints along the radian direction of 90°

8.2.1.4 Sensitivity study on the prediction model for T_n for the radian angle of 90°

This section focuses on the sensitivity study on the prediction model for T_n along the radian direction of 90° in the process of 2-D compressional wave propagation through multiple parallel joints, which has been proposed in the Section 8.2.1.3. The estimated values of transmission ratios T_n from the prediction model are compared with UDEC results for a hypothetical case study.

Statement of the hypothetical case and its estimated transmission ratios T_n

In the hypothetical case, a circular cavity exists in an infinite rock mass with six parallel joints. A compressional wave applied at the boundary of the cavity is traveling through six parallel joints, with different values of joint spacing. The values of normalized normal and shear stiffness are 0.54 and 0.4, respectively.

Using Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15, the two critical points (S_{peak} , T_{peak}) and (S_{flat} , T_{flat}) defined in Figure 8.3 can be determined for the hypothetical case study. It is found that $S_{\text{peak}}=0.044$, $T_{\text{peak}}=0.62$, $S_{\text{flat}}=0.35$ and $T_{\text{flat}}=0.27$. Using these critical points, the estimated transmission ratio T_6 versus the ratio of spacing/wavelength is shown in Figure 8.16. This curve of the estimated T_6 versus spacing/wavelength will be compared with UDEC results.

Figure 8.16 Estimated T_6 as a function of the ratio spacing/wavelength along radian direction of 90°

UDEC modeling conditions

The UDEC model used for this investigation is shown in Figure 8.1, where the number of joints is 6. The properties of the rock material are as shown in Table 5.3.

A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 500 Hz is applied at the boundary of the cavity. The mesh size of 0.4 m is chosen to keep the numerical error less than 2% for a frequency of 500 Hz. In order to compare the estimated transmission ratios T_6 with those obtained from UDEC results along the radian direction of 90° , the values of normal and shear stiffness of the joint are chosen as 0.6K and 0.4G, giving the normalized normal stiffness of 0.54 and the normalized shear stiffness of 0.4 for frequency of 500 Hz. The ratios of spacing/wavelength are chosen as 0.02, 0.035, 0.05, 0.06, 0.07, 0.13, 0.2, 0.35, 0.4 and 0.5, respectively.

Comparison between the estimated T_n and UDEC results

Figure 8.17 visually shows the comparison between the estimated transmission ratio T_6 and UDEC results for 2-D compressional wave propagation through 6 parallel joints along the radian direction of 90° . The ratios of spacing/wavelength are different (0.02, 0.035, 0.05, 0.06, 0.07, 0.13, 0.2, 0.35, 0.4 and 0.5), with $jkn/(\omega z)$ of 0.54 and $jks/(\omega z)$ of 0.4.

From Figure 8.17, it can be seen that the estimated transmission ratios T_6 agree reasonably well with the transmission ratios T_6 calculated from UDEC results. There is a reasonable difference between the estimated results and UDEC results. Two possible reasons may lead to the difference. One is the simplification (shown in Figure 8.3) of variation of the transmission ratio T_n versus spacing/wavelength. The other possible reason is the assumed linear relationship between the transmission ratio T_n and the influencing factors in the process of establishing the prediction model.

Figure 8.17 Comparison between the estimated and the practical numerical results

8.2.2 A study on maximum rebound ratio for multiple parallel joints along the radian angle of 90°

In the process of 2-D compressional wave propagation through multiple parallel joints, superposition of multiple reflected and transmitted waves occurs between two adjacent joints. This section aims to study the wave amplitude ratio after multiple reflections and transmissions in the process of 2-D compressional wave propagation through multiple parallel joints along the radian direction of 90°. A prediction model for the maximum rebound ratio is obtained based on UDEC results.

8.2.2.1 Introduction

In the process of 2-D compressional wave propagation through multiple joints along the radian direction of 90°, the superposition of the multiple reflected and transmitted waves between two adjacent joints is defined as rebound wave. The rebound ratio is defined as:

$$\text{Reb}_n = \frac{A_{\text{Reb}, n}}{A_{\text{intact}}}$$

where $A_{\text{Reb}, n}$ is the amplitude of the rebound wave (due to multiple reflections and transmissions) corresponding to the point between the $(n-1)^{\text{th}}$ and the n^{th} joints. A_{intact} is the amplitude of the compressional wave at the corresponding point in a model of intact rock. The maximum rebound ratio (Max. Reb_n) is the maximum value of Reb_n corresponding to all grid-points between the $(n-1)^{\text{th}}$ and the n^{th} joints.

In order to compare the definitions of the transmission ratio and the rebound ratio, two cases of 2-D wave propagation in a jointed rock mass are introduced, as shown in Figure 8.18 (a). In case A, a 2-D wave propagates in an infinite rock mass with a joint set. In case B, the same wave propagates only through $(n-1)$ joints, while other conditions are the same as case A. The left plot in Figure 8.18 (a) shows the typical comparison between the rebound ratios Reb_n at the points between the $(n-1)^{\text{th}}$ joint and the n^{th} joint obtained from case A and the transmission ratios T_{n-1} at the same points obtained from case B, where the joint spacing is quite big (It should be noted the transmission ratio is independent of the radial distance from the cavity center, therefore, the transmission ratio T_{n-1} is a flat line). It can be seen, when the point is before the n^{th} joint within the range of a critical distance, the transmission ratio T_{n-1} (line b-d) is always smaller than the rebound ratio Reb_n (curve b-c), because in this range, the wave reflection has significant effects. By contrast, when the point before the n^{th} joint is beyond the range of the critical distance, the rebound ratio Reb_n is approximately equal to the transmission ratio T_{n-1} (line a-b). In this range, the wave reflection has negligible effects. Therefore, Max. Reb_n is always bigger than T_{n-1} . (As shown in Figure 8.18 (b), it should be noted that for a small joint spacing, Reb_n is bigger than T_{n-1} at all the grid-points between the two joints.)

This means that Max. Reb_n is the upper limit of the ratio of the wave amplitude from the model with joints to the wave amplitude from the model of intact rock, while the transmission ratio T_{n-1} is the lower limit of the mentioned ratio.

(a) Comparison between T_{n-1} and $Re b_n$ for the case of big joint spacing obtained from UDEC results (b) Comparison between T_{n-1} and $Re b_n$ for the case of small joint spacing obtained from UDEC resultsFigure 8.18 Comparison between T_{n-1} and $Re b_n$

8.2.2.2 Problem statement and modeling conditions

The investigation of the maximum rebound ratio is based on the geometry shown in Figure 8.1, but the geometry is enlarged to a dimension of $150 \text{ m} \times 250 \text{ m}$, where the length of 200 m is increased to 250 m , while the height is unchanged. A cavity with a radius of 5 m exists in an infinite rock mass with 14 joints, which are parallel to the y-axis with the first joint in the set is located at a normal distance of 15 m from the cavity center to the first joint in the joint set. In the geometry, the top, right and left boundaries are viscous boundaries, while the lower boundary is a symmetrical boundary.

A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2% for the frequency of 200 Hz . Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study. The stiffness of the joints (j_{kn} and j_{ks}) considered in this analysis are the following combinations ($0.03K$, $0.03G$), ($0.07K$, $0.07G$), ($0.1K$, $0.1G$), ($0.2K$, $0.2G$) and ($0.4K$, $0.4G$). The ratios of spacing/wavelength considered in the analysis are 0.04 , 0.07 , 0.1 , 0.13 , 0.17 , 0.20 , 0.25 , 0.30 , 0.40 and 0.50 .

The velocity wave corresponding to all the grid-points within the first and the last joint in the joint set, along the radial line with α of 90° , are recorded and processed for studying the maximum rebound ratio.

8.2.2.3 Discussion of the results and development of a prediction model for Max. Reb_n for the radian angle of 90°

Figures 8.19 (a), (b), (c), (d) and (e) show Max. Reb_n before each joint in the radian direction of 90° for different ratios of spacing/wavelength for the five given values of the normalized normal stiffness.

It can be seen that the effects of all the factors influencing Max. Reb_n (normalized normal stiffness, spacing/wavelength and the joint from which the wave rebounds) are comprehensively generalized into Figures 8.19 (a), (b), (c), (d) and (e). These

figures can be used as a prediction model to estimate the maximum rebound ratio for any joint in the joint set in the radian direction of 90° .

From Figures 8.19 (a), (b), (c), (d) and (e), the following observations can be made:

(1) In the first joint, Max. Reb_1 is controlled by the normalized normal stiffness of the joint along the radian direction of 90° . The value of Max. Reb_1 is independent of the ratio of spacing/wavelength. Max. Reb_1 decreases as the normalized normal stiffness increases. For example, as shown in Figures 8.19 (a), (b), (c), (d) and (e), Max. Reb_1 decreases from 1.88 to 1.34, as the normalized normal stiffness of the joint increases from 0.067 to 0.897.

(2) For the other joints (other than the first one)

(a) Max. Reb_n along the radian direction of 90° is influenced by the normalized normal stiffness, spacing/wavelength and the joint from which the wave rebounds.

(b) Comparing the effects of the given values of normalized normal stiffness that are shown in Figures 8.19 (a), (b), (c), (d) and (e) (fixing other two factors, spacing/wavelength and the joint from which the wave rebounds), it can be seen that Max. Reb_n increases as normalized normal stiffness increases.

(c) On every single figure in Figures 8.19 (a), (b), (c), (d) and (e), for each given value of normalized normal stiffness for any specific joint from which the wave rebounds, Max. Reb_n decreases as spacing/wavelength increases. For example, in Figure 8.19 (a), for $jkn/(\omega z)$ of 0.067, the Max. Reb_3 decreases from 0.92 to 0.18, as spacing/wavelength increases from 0.04 to 0.5.

For a point between two adjacent joints (the n^{th} and the $(n-1)^{\text{th}}$ joints) in a joint set, the maximum rebound ratio Max. Reb_n is the upper limit of the ratio of the wave amplitude from the model with joints to the wave amplitude from the model of intact rock, while the transmission ratio T_{n-1} is the lower limit.

(a) Max. Reb_n before joints for different ratio of spacing/wavelength and fixed $jkn/(\omega z)=0.067$

(b) Max. Reb_n before joints for different ratio of spacing/wavelength and fixed $jkn/(\omega z)=0.157$

Figure 8.19 Max. Reb_n before joints for different ratio of spacing/wavelength and for different $jkn/(\omega z)$

(c) Max. Reb_n before joints for different ratio of spacing/wavelength and fixed $jkn/(\omega z)=0.224$

(d) Max. Reb_n before joints for different ratio of spacing/wavelength and fixed $jkn/(\omega z)=0.448$

Figure 8.19 (Continued) Max. Reb_n before joints for different ratio of spacing/wavelength and for different $jkn/(\omega z)$

(e) Max. Reb_n before joints for different ratio of spacing/wavelength and fixed $jkn/(\omega z)=0.897$

Figure 8.19 (Continued) Max. Reb_n before joints for different ratio of spacing/wavelength and for different $jkn/(\omega z)$

8.2.2.4 Application of the prediction model for Max. Reb_n

A field explosion test will be introduced in Chapter 9. In the field test, there is a set of joints, spaced at 0.33 wavelength, with $jkn/(\omega z)$ of 0.695. In this section, using these parameters for the field test, the procedures of predicting several maximum rebound ratios (Max. Reb₂, Max. Reb₃, Max. Reb₅, Max. Reb₁₀ and Max. Reb₂₁) are introduced to illustrate how to use the prediction model for Max. Reb_n, shown in Figures 8.19 (a), (b), (c), (d) and (e). Among these maximum rebound ratios, the procedure of prediction of Max. Reb₁₀ is introduced in detail.

First, according to the target value of $jkn/(\omega z)$ of 0.695, Figures 8.19 (d) and (e) are chosen for linear interpolation which correspond to $jkn/(\omega z)$ of 0.448 and 0.897, giving a range covering the target value of $jkn/(\omega z)$ of 0.695.

Table 8.1 shows the data drawn from Figures 8.19 (d) and (e), involved in prediction of the maximum rebound ratio before the 10th joint in the joint set spaced at 0.33 wavelength, with $jkn/(\omega z)$ of 0.695, in the radian direction of 90°.

In Table 8.1, fixing number of joints and normalized normal stiffness, treating spacing/wavelength as variable, for the ratio of spacing/wavelength of 0.33, by linear interpolation, Max. Reb₁₀ can be calculated respectively for $jkn/(\omega z)$ of 0.897 and $jkn/(\omega z)$ of 0.448 as 0.498 and 0.375, respectively.

Hence Max. Reb₁₀ for spacing/wavelength of 0.33 and $jkn/(\omega z)$ of 0.695 can be calculated by linear interpolation as 0.443.

Repeating the same procedure for estimating Max. Reb₁₀, other maximum rebound ratios, Max. Reb₂, Max. Reb₃, Max. Reb₅ and Max. Reb₂₁ can be obtained. Table 8.2 shows Max. Reb₂, Max. Reb₃, Max. Reb₅, Max. Reb₁₀ and Max. Reb₂₁ before the 2nd, 3rd, 5th, 10th and 21st joints in the joint set spaced at 0.33 wavelength, with $jkn/(\omega z)$ of 0.695, in the radian direction of 90°.

The estimated Max. Reb_n shown in Table 8.2 will be used to estimate the upper limit of PPVs in the field test in Chapter 9. Therefore, the prediction model for the maximum rebound ratio will be verified by a field test in Chapter 9.

Table 8.1 Max. Reb₁₀ for spacing/wavelength of 0.3, 0.4 and $jkn/(\omega z)$ of 0.897, 0.448

n	Max. Reb ₁₀ for (spacing/wavelength)=0.3 and $jkn/(\omega z)$ =0.897	Max. Reb ₁₀ for (spacing/wavelength)=0.4 and $jkn/(\omega z)$ =0.897	Max. Reb ₁₀ for (spacing/wavelength)=0.3 and $jkn/(\omega z)$ =0.448	Max. Reb ₁₀ for (spacing/wavelength)=0.4 and $jkn/(\omega z)$ =0.448
10	0.518	0.45	0.4	0.315

Table 8.2 Several maximum rebound ratios (*: Reb₂₁ is obtained from extension of the curves)

Rebound ratio	Max. Reb ₂	Max. Reb ₃	Max. Reb ₅	Max. Reb ₁₀	*Max. Reb ₂₁
Value	1.08	0.8	0.63	0.44	0.35

8.3 A study on effects of multiple joints for arbitrary directions

Recalling the research presented in Chapter 6, the transmission ratio T_n in arbitrary radian directions is influenced by both the normal and shear stiffness, while T_n in the radian direction of 90° is dominated by normal stiffness. This means that the performance of the transmission ratio T_n in arbitrary directions may become more complicated since an additional influence factor (shear stiffness) is involved. This section aims to study the effects of multiple parallel joints on the transmission ratio T_n at any point in any arbitrary directions in terms of radian angle, the stiffness of the joints, the number of joints and spacing/wavelength. Based on the obtained results under various conditions, a prediction model is proposed for estimating the transmission ratio T_n in arbitrary directions. Furthermore, sensitivity study on the proposed prediction model is conducted by analyzing a hypothetical case study.

8.3.1 Problem statement and modeling conditions

The problem and UDEC model considered in this study are shown in Figure 8.1. A cavity with a radius of 5 m exists in an infinite rock mass with multiple parallel joints, parallel to the y-axis with a normal distance of 15 m from the center of the cavity. The top, left and right boundaries are viscous boundaries, while the lower boundary is a symmetrical boundary.

A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 200 Hz is applied at the boundary of the cavity. The mesh size of 0.79 m is chosen to keep the numerical error less than 2% for the frequency of 200 Hz. Properties of Bukit Timah granite in Singapore shown in Table 5.3 are considered for the study.

Since this section aims to propose a prediction model for T_n in arbitrary directions, all of the factors influencing T_n in arbitrary directions in the process of 2-D wave propagation through multiple parallel joints must be studied. These factors include normal and shear stiffness, the number of joints and spacing/wavelength and the radian angle. In the following analysis, the normal stiffness of the joints are chosen as 0.03K, 0.07K, 0.1K, 0.2K and 0.4K, while the shear stiffness of the joint are

chosen as $0.03G$, $0.07G$, $0.1G$, $0.2G$ and $0.4G$, respectively. K and G are the bulk modulus and shear modulus of the rock material. The numbers of the joints considered in the analysis are 2, 4 and 8. The ratios of spacing/wavelength are 0.01, 0.05, 0.07, 0.13, 0.20, 0.30 and 0.35.

The velocity wave at the points along a circle, where all the points have the same radial distance from the cavity center, are recorded and processed to investigate the transmission ratio T_n in arbitrary directions. Therefore, the radian angle can be considered as continuous.

8.3.2 UDEC results and discussion

Figures 8.20 (a), (b), (c), (d), (e), (f) and (g) show the transmission ratio T_2 corresponding to points behind 2 parallel joints under a set of conditions, including combination of normalized normal and shear stiffness and spacing/wavelength. In Figures 8.20 (a), (b), (c), (d), (e), (f) and (g), the combinations of normalized normal and shear stiffness are the same. The values of normalized shear stiffness vary as 0.075, 0.174, 0.249, 0.498 and 0.997, while the values of normalized normal stiffness vary as 0.067, 0.157, 0.224, 0.448 and 0.897. The ratio of spacing/wavelength in Figures 8.20 (a), (b), (c), (d), (e), (f) and (g) is 0.05, 0.07, 0.10, 0.13, 0.20, 0.30 and 0.35, respectively.

The conditions in Figure 8.21 and Figure 8.22 are similar to those in Figure 8.20, but for the cases T_4 (4 joints) and T_8 (8 joints), respectively.

It is noted that in Figure 8.20, Figure 8.21 and Figure 8.22, comparing the range of radian angles as shown in every single figure, the lower limit of the radian angle in every figure is lightly different with each other, because of the limitation of the geometry of UDEC model and the difference of spacing/wavelength.

From Figure 8.20, Figure 8.21 and Figure 8.22, the following observations can be obtained:

- 1) Under a given set of conditions (number of joints, ratio of spacing/wavelength), for a given normalized normal stiffness, T_n in the direction of 90° for different

values of normalized shear stiffness are approximately the same. This means that the effects of normalized shear stiffness on the performance of T_n in the radian angle of 90° are negligible.

- 2) There are 5 given values of normalized normal stiffness illustrated in each figure. It can be seen that for all these 5 given values of normalized normal stiffness, T_n in the radian angle of 90° increases as the normalized normal stiffness increases. This trend can be used as a criterion to identify the group of curves corresponding to the given value of the normalized normal stiffness in each figure.
- 3) Under a given set of conditions (number of joints, ratio of spacing/ wavelength and normalized normal stiffness), it can be seen that the value of the transmission ratio T_n increase as the normalized shear stiffness value increases in each group of curves in every figure (except a small range of radian angles around 90°). Therefore, the effect of normalized shear stiffness becomes larger as the radian angle deviates from 90° . Obviously, such a trend can be used as a criterion to identify the cure corresponding to different normalized shear stiffness within each group of curves, under a set of given conditions (normalized normal stiffness, number of joints and ratio of spacing/wavelength).
- 4) Under a given set of conditions (number of joints, ratio of spacing/wavelength and normalized shear stiffness), at an arbitrary radian angle, the value of the transmission ratio T_n increases as the normalized normal stiffness value increases.
- 5) When the value of normalized normal stiffness is very small, the minimum T_n is approximately corresponding to the radian angle of 90° . By contrast, when the value of normalized normal stiffness is quite high, the maximum T_n is also approximately corresponding to the radian angle of 90° . This has been discussed in Section 7.2.2.
- 6) Figure 8.20, Figure 8.21 and Figure 8.22 illustrate the performance of T_n in arbitrary radian directions including the radian angle of 90° for various different

conditions including different numbers of joints, different ratios of spacing/wavelength and different combinations of $j_{kn}/(\omega z)$ and $j_{ks}/(\omega z)$. By contrast, Figures 8.2 (a), (b) and (c) specifically illustrate the relationship of T_n , in the radian direction of 90° , versus the ratio of spacing/wavelength for different values of normalized normal stiffness and different numbers of joints.

In the radian angle of 90° , it has been found that the relationship between T_n and the ratio of spacing/wavelength implicitly incorporated in Figure 8.20, Figure 8.21 and Figure 8.22 is compatible with the corresponding curve shown in Figures 8.2 (a), (b) and (c). It is noted that this conclusion is obtained under the same given conditions such as the number of joints, normalized stiffness and ratio of spacing/wavelength.

As an example, Figure 8.23 shows two groups of T_4 in the direction of radian angle of 90° versus the ratio of spacing/wavelength for 2-D wave propagation through 4 parallel joints. One group denoted by solid line is obtained from Figure 8.2 (b) corresponding to the given value of $j_{kn}/(\omega z)$ of 0.897. The other group denoted by circles is obtained from Figures 8.21 (a), (b), (c), (d), (e), (f) and (g) for T_4 corresponding to seven different given ratios of spacing/wavelength at 0.01, 0.05, 0.07, 0.13, 0.20, 0.30 and 0.35, respectively, where $j_{kn}/(\omega z)$ is 0.897 and $j_{ks}/(\omega z)$ is 0.997. It can be seen that in the radian direction of 90° , the seven values of T_4 corresponding to seven different ratios of spacing/wavelength from Figures 8.21 (a), (b), (c), (d), (e), (f) and (g) match well with the corresponding curve of T_4 in Figure 8.2 (b). It is noted the two groups of T_4 are independently obtained from different grid-points and independent computing processes. The variations of T_n versus the ratio of spacing/wavelength shown in Figures 8.2 (a), (b) and (c) are drawn from the UDEC results, where the effects of shear stiffness on T_n are explicitly regarded as negligible. By contrast, the variation of T_n versus the ratio of spacing/wavelength implicitly incorporated in Figure 8.20, Figure 8.21 and Figure 8.22 are drawn from the UDEC results, where the effects of shear stiffness are taken into account. So, in the radian direction of 90° , under the same conditions including the number of joints and normalized normal stiffness, the variation of T_n versus the ratio of

spacing/wavelength explicitly shown in Figures 8.2 (a), (b) and (c) is in essence the same as that implicitly incorporated in Figure 8.20, Figure 8.21 and Figure 8.22. This is because in the radian angle of 90° , the effects of the shear stiffness of the joints on the transmission ratio T_n are very small.

(a) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.01 wavelength

(b) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.05 wavelength

Figure 8.20 T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints with spacing/wavelength

(c) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.07 wavelength

(d) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.13 wavelength

Figure 8.20 (Continued) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints with different spacing/wavelength

(e) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.20 wavelength

(f) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.30 wavelength

Figure 8.20 (Continued) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints with different spacing/wavelength

(g) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints spaced at 0.35 wavelength

Figure 8.20 (Continued) T_2 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints with different spacing/wavelength

(a) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498, 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448, 0.897) for four joints spaced at 0.01 wavelength

Figure 8.21 T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for two joints with different spacing/wavelength

(b) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498, 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448, 0.897) for four joints spaced at 0.05 wavelength

(c) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498, 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448, 0.897) for four joints spaced at 0.07 wavelength

Figure 8.21 (Continued) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for four joints with different spacing/wavelength

(d) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498, 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448, 0.897) for four joints spaced at 0.13 wavelength

(e) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498, 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448, 0.897) for four joints spaced at 0.20 wavelength

Figure 8.21 (Continued) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for four joints with different spacing/wavelength

(f) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for four joints spaced at 0.30 wavelength

(g) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498, 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448, 0.897) for four joints spaced at 0.35 wavelength

Figure 8.21 (Continued) T_4 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for four joints with different spacing/wavelength

(a) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.01 wavelength

(b) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.05 wavelength

Figure 8.22 T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints with different spacing/wavelength

(c) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.07 wavelength

(d) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.13 wavelength

Figure 8.22 (Continued) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints with different spacing/wavelength

(e) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.20 wavelength

(f) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.30 wavelength

Figure 8.22 (Continued) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints with different spacing/wavelength

(g) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints spaced at 0.35 wavelength

Figure 8.22 (Continued) T_8 for different $jks/(\omega z)$ (0.075, 0.174, 0.249, 0.498 and 0.997) and different $jkn/(\omega z)$ (0.067, 0.157, 0.224, 0.448 and 0.897) for eight joints with different spacing/wavelength

Figure 8.23 Comparison of the curve from Figure 8.2 (b) and the data from Figures 8.21 (a) to (g)

8.3.3 The prediction model for T_n for arbitrary directions

8.3.3.1 Introduction to the prediction model for T_n for arbitrary directions

The performance of 2-D compressional wave propagation through multiple parallel joints is affected by multiple factors. From the previous studies, it has been known that the transmission ratio T_n is sensitive to a set of factors including normal and shear stiffness of the joints, spacing/wavelength, number of the joints and radian angle. It is hard to estimate the relationship between the transmission ratio T_n and the factors. However, if only one individual factor is treated as a variable and the values of the other factors are fixed, the relationship of the transmission ratio T_n versus the variable factor can be reasonably treated as a linear function in a small range. It is noted that the effects of different factors on the transmission ratio T_n are independent to each other. This is the basic assumption for prediction model for the transmission ratio T_n .

Table 8.3 lists the values of the factors influencing the transmission ratio T_n for 2-D compressional wave propagation through multiple parallel joints, incorporated in Figure 8.20, Figure 8.21 and Figure 8.22. For example, treating $jkn/(\omega z)$ in a small range from 0.224 to 0.448 as variable only, while fixing the other factors ($jks/(\omega z)=0.498$, spacing/wavelength=0.20, $\alpha=40^\circ$ and number of joints=2), the relationship between T_2 and the variable factor $jkn/(\omega z)$ ranging from 0.224 to 0.448 can be reasonably regarded as linear. In this scenario, the value of T_2 corresponding to any value of $jkn/(\omega z)$ ranging from 0.224 to 0.448 can be obtained by linear interpolation on the corresponding curves in Figure 8.20 (e).

Table 8.3 Influencing factors and the values

Influencing factors	Values used in Figures 8.20 to 8.22
Normalized normal stiffness	0.067, 0.157, 0.224, 0.448, 0.897
Normalized shear stiffness	0.075, 0.174, 0.249, 0.498, 0.997
Ratio of spacing/wavelength	0.01, 0.05, 0.07, 0.13, 0.2, 0.3, 0.35
Radian angle ($^\circ$)	From radian angle at the boundary of UDEC model shown in Figure 8.1 to 90°
Number of joints	2, 4, 8

In Figure 8.20, Figure 8.21 and Figure 8.22, the normalized normal stiffness is from 0.067 to 0.897, while the normalized shear stiffness is from 0.075 to 0.997. As discussed in Section 7.4.2, these ranges cover the most practical normalized stiffness in practical rock dynamic problems. As an example, the values of normalized normal and shear stiffness in the field test in Chapter 9 are 0.695 and 0.475 respectively, within the above mentioned ranges. On the other hand, the ratio of spacing/wavelength used in the above mentioned figures is 0.01, 0.05, 0.07, 0.13, 0.2, 0.3 and 0.35, respectively. Such a range of the ratio of spacing/wavelength also covers the most practical range of the ratio in rock dynamic problems. As an example, the ratio of spacing/wavelength is 0.33 in the filed explosion test in Chapter 9. The radian angle values used in the above mentioned figures are continuous from the radian angle at the boundary of UDEC model to 90° .

The relationships among the transmission ratio T_n and the influence factors including normalized normal and shear stiffness, ratio of spacing/wavelength and radian angle can be obtained from Figure 8.20, Figure 8.21 and Figure 8.22 using linear interpolation or extrapolation.

All of the factors influencing T_n in the process of 2-D compressional wave propagation through multiple parallel joints for arbitrary directions are comprehensively generalized into Figure 8.20, Figure 8.21 and Figure 8.22. Therefore, these figures can be used as the prediction model for the transmission ratio T_n for 2-D compressional wave propagation through multiple parallel joints for arbitrary directions.

8.3.3.2 Discussion

As discussed in Section 8.3.2, along the special radian direction of 90° , the prediction model for arbitrary directions shown in Figure 8.20, Figure 8.21 and Figure 8.22 is in essence the same as the prediction model for the radian angle of 90° shown in Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15.

The prediction model for T_n in arbitrary directions shown in Figure 8.20, Figure 8.21 and Figure 8.22 can be used to estimate T_n for a large range of radian

directions. However, it needs to perform linear interpolation or extrapolation on the curves to obtain relatively accurate results. By contrast, using the prediction model for T_n for the radian angle of 90° shown in Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15, the estimated transmission ratio T_n can be obtained from the charts more quickly and straightforwardly.

8.3.3.3 Procedures of estimating T_n

Figures 8.20 (a), (b), (c), (d), (e), (f) and (g) to Figures 8.22 (a), (b), (c), (d), (e), (f) and (g) can be used as a prediction model for T_n for 2-D wave propagation through multiple parallel joints. The flow chart in Figure 8.24 shows the procedures of estimating T_n for arbitrary directions. The procedures will be illustrated in the hypothetical case study in Section 8.3.4.

Figure 8.24 Procedures of estimating T_n for arbitrary directions

8.3.4 Sensitivity study on prediction model for T_n for arbitrary directions

In this part, a sensitivity study on the prediction model proposed in Section 8.3.3 is performed on a hypothetical case study. The estimated transmission ratios T_n from the prediction model for T_n for different radian angles are compared with UDEC results.

Statement of the problem of the hypothetical case and its UDEC modeling conditions

A circular cavity exists in an infinite rock mass containing Bukit Timah granite in Singapore (the properties are shown in Table 5.3) with multiple parallel joints. A compressional wave with unity amplitude and a frequency of 500 Hz is applied at the boundary of the cavity, traveling through different number of the parallel joints (2, 4, 6, 8 and 10), along different radial lines with radian angles of 50° , 70° and 90° . The joint spacing is 1.8 m. The normal and shear stiffness of the joints are 24.0 Gpa/m and 10.4 Gpa/m, respectively.

The UDEC model used for the hypothetical problem is shown in Figure 8.1, with different number of joints, 2, 4, 6, 8 and 10 respectively. A sinusoidal compressional pulse velocity wave with unity amplitude and a frequency of 500 Hz is applied at the boundary of the cavity. The mesh size of 0.4 m is chosen to keep consistent numerical error less than 2% for a frequency of 500 Hz. In order to compare the estimated transmission ratios T_n with UDEC results for the hypothetical case, in UDEC input file, the values of normal and shear stiffness of the joint are chosen as 24.0 Gpa/m and 10.4 Gpa/m, while the joint spacing is chosen as 1.8 m. The rock properties in Table 5.3 are coded in the input file.

Estimating T_n for the hypothetical case

For this hypothetical problem, the transmission ratios T_n in radian angles of 50° , 70° and 90° for 2-D compressional wave propagation through different numbers of joints are determined, by using Figure 8.20, Figure 8.21 and Figure 8.22 and following the steps in the flow chart as shown in Figure 8.24.

Step 1: Determining the following parameters involved in estimating T_n for arbitrary directions: (1) $K=40$ Gpa/m, $G=26$ Gpa/m, $\rho=2650$ kg/m³. (2) Numbers of joints are 2, 4, 6, 8 and 10. (3) $jkn=24.0$ Gpa/m, $jks=10.4$ Gpa/m. (4) Joint spacing is 1.8 m. (5) Amplitude and angular frequency ω of the input wave are 1 m/s and 3142 radian/s.

Step 2: Calculating or determining the following target values: (1) P-wave velocity c_p is calculated and listed in Table 5.3. (2) The seismic impedance for P-wave z is equal to ρc_p . (3) The target values of $jkn/(\omega z)$ and $jks/(\omega z)$ are calculated as 0.54 and 0.4 respectively. (4) The target value of spacing/wavelength is calculated as 0.17. (5) The target values of radian angle are 50° , 70° and 90°

Step 3: According the ratio of spacing/wavelength of 0.17, Figures 8.20 (d) and (e) are chosen. These two figures correspond to the ratios of spacing/wavelength of 0.13 and 0.20, giving a range covering the target value of spacing/wavelength of 0.17. On Figures 8.20 (d) and (e), T_2 versus radian angle for the target $jkn/(\omega z)$ of 0.54, $jks/(\omega z)$ of 0.4 are estimated (by using the same procedures step 3 and step 4 as shown in Figure 7.7) for spacing/wavelength of 0.13 and 0.2 respectively. These estimated T_2 are listed in the 2nd and the 3rd rows in Table 8.4.

Therefore, by linear interpolation based on the data shown in the 2nd and the 3rd rows in Table 8.4, T_2 versus radian angle for the target values of $jkn/(\omega z)$ of 0.54, $jks/(\omega z)$ of 0.4 and spacing/wavelength of 0.17 can be estimated, which are tabulated in the 4th row in Table 8.4.

Using the same method, T_4 and T_8 versus radian angle for the target values of $jkn/(\omega z)$ of 0.54, $jks/(\omega z)$ of 0.4 and spacing/wavelength of 0.17 can be estimated, which are tabulated in Table 8.5 together with T_2 for these target values.

Figure 8.25 visually shows T_2 , T_4 and T_8 versus radian angle for the target values of $jkn/(\omega z)$ of 0.54, $jks/(\omega z)$ of 0.4 and spacing/wavelength of 0.17.

Step 4: Figure 8.26 shows the relationship between T_n and the number of joints for three radian angles: 90° , 70° and 50° for the hypothetical case, which are obtained either from Table 8.5 or Figure 8.25.

Table 8.4 Radian angles and T_2 for 2 joints, with $j_{kn}/(\omega z)$ of 0.54 and $j_{ks}/(\omega z)$ of 0.4, for different ratios of spacing/wavelength of 0.13, 0.2 and 0.17

Radian angle ($^{\circ}$)	20	30	40	50	60	70	80	90
Transmission ratio T_2 for spacing/wavelength of 0.13	0.72	0.64	0.60	0.59	0.60	0.62	0.64	0.65
Transmission ratio T_2 for spacing/wavelength of 0.20	0.636	0.520	0.544	0.535	0.54	0.545	0.556	0.56
Transmission ratio T_2 for spacing/wavelength of 0.17	0.672	0.606	0.570	0.559	0.564	0.577	0.590	0.597

Table 8.5 Radian angles and T_n for 2, 4 and 8 joints, with $j_{kn}/(\omega z)$ of 0.54 and $j_{ks}/(\omega z)$ of 0.4 and spacing/wavelength of 0.17

Radian angle ($^{\circ}$)	20	30	40	50	60	70	80	90
Transmission ratio T_2	0.672	0.606	0.570	0.559	0.564	0.577	0.590	0.597
Transmission ratio T_4	0.540	0.484	0.444	0.429	0.443	0.467	0.493	0.499
Transmission ratio T_8	0.446	0.382	0.357	0.358	0.372	0.400	0.431	0.443

Figure 8.25 T_n versus radian angles for different numbers of multiple joints (2, 4 and 8) with the ratio of spacing/wavelength of 0.17 and $j_{kn}/(\omega z)$ of 0.54 $j_{ks}/(\omega z)$ of 0.4

Figure 8.26 Relationship between T_n and the number of joints for multiple joints with the ratio of spacing/wavelength of 0.17 $jkn/(\omega z)$ of 0.54 and $jks/(\omega z)$ of 0.4 for three radian angles: 90°, 70° and 50°

Comparison between the estimated T_n and UDEC results and discussion

The estimated T_n at three different radian angles of 90°, 70° and 50° for different numbers of the joints (2, 4, 6, 8 and 10) for the hypothetical case can be obtained in Figure 8.26. For the case of 10 joints, the transmission ratio T_{10} can be obtained by linear extrapolation, while for the other cases (2, 4, 6 and 8 joints) the transmission ratios T_n can be obtained by linear interpolation or directly taken from the curves in Figure 8.26.

Figures 8.27 (a), (b) and (c) show the comparison between the estimated T_n using the proposed prediction model for T_n and UDEC results.

As seen from Figures 8.27 (a), (b) and (c), the estimated results agree with the UDEC results reasonably well.

It can be seen from Figures 8.27 (a), (b) and (c), there are differences between the estimated results and UDEC results. This may be caused by the error accumulation in the process of linear interpolation and extrapolation.

(a) Comparison between estimated T_n and UDEC results for following conditions: 2, 4, 6, 8 and 10 joints, $j_{kn}/(\omega z)$ of 0.54 and $j_{ks}/(\omega z)$ of 0.4, the ratio of spacing/wavelength of 0.17, radian angle 90°

(b) Comparison between estimated T_n and UDEC results for following conditions: 2, 4, 6, 8 and 10 joints, $j_{kn}/(\omega z)$ of 0.54 and $j_{ks}/(\omega z)$ of 0.4, the ratio of spacing/wavelength of 0.17, radian angle 70°

Figure 8.27 Comparison between estimated T_n and UDEC results for following conditions: 2, 4, 6, 8 and 10 joints, $j_{kn}/(\omega z)$ of 0.54 and $j_{ks}/(\omega z)$ of 0.4, spacing/wavelength of 0.17, at different radian angle

(c) Comparison between estimated T_n and UDEC results for following conditions: 2, 4, 6, 8 and 10 joints, $jkn/(\omega z)$ of 0.54 and $jks/(\omega z)$ of 0.4, the ratio of spacing/wavelength of 0.17, radian angle 50°

Figure 8.27 (Continued) Comparison between estimated T_n and UDEC results for following conditions: 2, 4, 6, 8 and 10 joints, $jkn/(\omega z)$ of 0.54 and $jks/(\omega z)$ of 0.4, spacing/wavelength of 0.17, at different radian angle

8.4 Summary

In this chapter, a study on performance of T_n along the special radian direction of 90° in the process of 2-D compressional wave propagation through multiple joints has been performed. A study on performance of superposition of multiple transmitted and reflected waves between two adjacent joints in a joint set has been performed for the special case of the radian direction of 90° as well. Based on the results obtained, prediction models for the transmission ratio and the maximum rebound ratio are proposed. For arbitrary radian directions, a study on T_n in the process of 2-D compressional wave propagation through multiple joints has been also performed. The following paragraphs summarize the conclusions obtained.

(A) Effects of multiple joints along the special direction of 90°

From the studies on T_n in the process of 2-D compressional wave propagation through multiple parallel joints, along the special radian direction of 90° , the following conclusions have been obtained:

- (1) The transmission ratio T_n is not only affected by the normalized stiffness but also affected by both of the ratio of joint spacing to wavelength (spacing/wavelength) and the number of joints. The variation of T_n with spacing/wavelength can be described by a simplified curve. The curve is determined by two critical points ($S_{\text{peak}}, T_{\text{peak}}$) and ($S_{\text{flat}}, T_{\text{flat}}$), as shown in Figure 8.3, for a given value of normalized normal stiffness and a given number of joints. It has also been found that these two critical points ($S_{\text{peak}}, T_{\text{peak}}$) and ($S_{\text{flat}}, T_{\text{flat}}$) are dependent on two factors including the number of joints and the normalized normal stiffness.
- (2) The transmission ratio T_n versus spacing/wavelength is varying in the following three different modes:
 - (a) If spacing/wavelength $\geq S_{\text{flat}}$, T_n is independent of the ratio of spacing/wavelength.
 - (b) If $S_{\text{flat}} > \text{spacing/wavelength} \geq S_{\text{peak}}$, T_n increases from T_{flat} to the maximum value T_{peak} as the ratio of spacing/wavelength decreases.
 - (c) If spacing/wavelength $\leq S_{\text{peak}}$, T_n decreases from the maximum value T_{peak} as the ratio of spacing/wavelength decreases.
- (3) The relationship between ($S_{\text{peak}}, T_{\text{peak}}$) and ($S_{\text{flat}}, T_{\text{flat}}$) and the normalized normal stiffness $j_{kn}/(\omega z)$ for arbitrary numbers of joints is established. Such relationship is generalized into four charts (shown in Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15).

From the studies on the maximum rebound ratio in the process of 2-D compressional wave propagation through multiple parallel joints, along the radian direction of 90° , the following conclusions have been obtained:

- (1) The maximum rebound ratio from the first joint (Max. Reb_1) is controlled by the normalized normal stiffness. Max. Reb_1 increases as the normalized normal stiffness decreases.

- (2) For other joints (other than the first one) the wave rebounds with the following observations
- (a) Max. Reb_n along the radian direction of 90° is influenced by normalized normal stiffness, spacing/wavelength and the joint from which the wave rebounds.
 - (b) If the two influencing factors, spacing/wavelength and the joint from which the wave rebounds are fixed, it can be seen that Max. Reb_n increases as normalized normal stiffness increases.
 - (b) For each given value of normalized normal stiffness, Max. Reb_n decreases as spacing/wavelength increases.
- (3) For a point between two adjacent joints (the n^{th} and the $(n-1)^{\text{th}}$ joints) in a joint set, the value of wave ratio (i.e., the wave amplitude obtained from the model with joints to that obtained from the model of intact rock) lies between the transmission ratio T_{n-1} (lower bound) and the maximum rebound ratio Max. Reb_n (upper bound), depending on the joint spacing and the position of the point.

(B) Effects of multiple joints on the transmission ratio along arbitrary directions

Extensive parametric studies have been performed on the factors influencing T_n in the process of 2-D compressional wave propagation through multiple parallel joints, in arbitrary directions. The effects of all the factors influencing T_n for arbitrary directions are comprehensively organized into a series of curves shown in Figure 8.20, Figure 8.21 and Figure 8.22. The following conclusions are made:

- (1) Under a given set of conditions (normalized normal stiffness, number of joints and spacing/wavelength), the effects of normalized shear stiffness on the performance of T_n in the radian angle of 90° are negligible.
- (2) Under a given set of conditions (number of joints, ratio of spacing/ wavelength and normalized normal stiffness), at an arbitrary radian angle (except a small

range of radian angles around 90°), the value of the transmission ratio T_n increases as the normalized shear stiffness value increases. Therefore, the effect of normalized shear stiffness becomes larger as the radian angle deviates from 90° .

- (3) Under a given set of conditions (number of joints, ratio of spacing/wavelength and normalized shear stiffness), at an arbitrary radian angle, the value of the transmission ratio T_n increases as the normalized normal stiffness value increases.
- (4) Under a given set of conditions (number of joints and ratio of spacing/wavelength), when the normalized normal stiffness of the joints is very small, the minimum T_n is roughly at the radian angle of 90° . By contrast, when the normalized normal stiffness is quite high, the maximum T_n is roughly at the radian angle of 90° .

Based on the studies performed in this chapter, two prediction models are proposed for estimating the transmission ratio. The first prediction model is based on the extensive parametric studies performed in the special radian direction of 90° . Therefore, it can be used directly and only for estimating the transmission ratio at any point along the direction normal to the joints. The second prediction model can be used to estimate the transmission ratio at any point along any arbitrary radian direction including the normal direction ($\alpha=90^\circ$). It should be noted that the use of the two prediction models for the normal direction case leads to approximately the same results. In addition, a third model for estimating the maximum rebound ratio between any two adjacent joints in a joint set is also proposed.

(C) Limitation of the proposed prediction models

However, cohesion of the joint is assumed to be of a relatively high value in the parametric studies where the prediction models proposed in this chapter and Chapter 7 are developed. Therefore, the proposed prediction models are applicable for the wave field where the wave amplitude is relatively small (such as in the far field in an explosion), because for the wave field where the wave amplitude is very

high (such as the near field in an explosion), breakage and nonlinear deformation in rock material and nonlinear deformation in joints occur.

CHAPTER 9

VERIFICATION OF THE PROPOSED PREDICTION MODELS FOR WAVE PROPAGATION BY AN EXPLOSION TEST

9.1 Introduction

In this chapter, a reported explosion field test (Chen *et al.* 2000) conducted in a quarry site in Mandai, Singapore, in a jointed rock mass is introduced. The Mandai explosion test was a part of a research program to study wave propagation in fractured rock masses and the effects on the existing underground structures. There were 5 monitoring boreholes (BHs 1, 2, 3, 4 and 5) located along a line in the rock mass. For four of the five monitoring boreholes, values of peak particle velocity (PPVs) at the monitoring points were recorded.

The PPVs at the locations of the monitoring points in the field are estimated by the methods proposed in Section 7.4, Section 8.2 and Section 8.3. The estimated PPVs are compared with the field records and two empirical formulae.

9.2 The Mandai explosion test

The site of Mandai test was in the rock mass containing Bukit Timah granite in Singapore, as shown in Figure 9.1 (Chen *et al.* 2000). A vertical charge hole of about 11 m in depth was drilled. As shown in Figure 9.2, the charge chamber was 4.2 m in height and 0.8 m in diameter. The explosive charge was placed at the center of the charge chamber. Before the explosion, the charge chamber was covered by a steel plate and concrete blocks. The explosive charge was 41.5 kg TNT in charge weight equivalent to 20 kg/m³ in loading density (the weight is divided by the chamber volume).

Five vertical instrumentation boreholes of 0.2 m in diameter were drilled along the bearing of 230° from the north. Three of the five boreholes were 14 m deep and the other two were 8.5 m deep. The locations of the boreholes (BHs 1, 2, 3, 4 and 5) were 2.5 m, 5 m, 10 m, 25 m and 50 m from the center of the charge hole. Three

types of instruments, including triaxial accelerometers, biaxial accelerometers and uniaxial accelerometers, were installed in the boreholes as shown in Figure 9.2.

Table 9.1 shows the field records, in terms of PPV, for the five monitoring points.

Table 9.1 Mandai test records (*: Datum not available)

Radial distance from the center of the charge hole (m)	Field records (PPVs) (m/s)
2.5	0.61
5.0	0.468
10	0.192
25	*
50	0.021

Figure 9.1 Configuration of Mandai test (after Chen *et al.* 2000)

Figure 9.2 Instruments in boreholes for monitoring the field explosion

9.3 Explosion load from Autodyn modeling

In order to estimate the PPV at any point in the rock mass due to the explosion, the properties of the compressional wave, generated from the explosion, at the borehole boundary should be known. The finite difference program Autodyn is used to model the explosion test and output the particle velocity wave at the boundary of the charge hole. The calculated particle velocity wave will be used in Section 9.4 to estimate the PPVs at the locations of the monitoring points using the proposed method in Section 7.4, Section 8.2 and Section 8.3.

Chen *et al.* (2000) modeled the same explosion test in Autodyn, using the rock deformation parameters obtained from preliminary test results. After Chen's analysis, extensive laboratory test has been done on the properties of the rock material from the explosion test site. It has been found that the Young's modulus

and Poisson's ratio are 64 GPa and 0.23, respectively (Lee, 2002), instead of 72 GPa and 0.16. These new parameters are used in Autodyn modeling in this chapter.

9.3.1 Estimating properties of the rock mass involved in Autodyn modeling

Autodyn is a 2-D finite difference program for a wide variety of non-linear problems including explosions. It is a continuum-based program, treating the jointed rock mass as an equivalent continuous rock. The Young's modulus of the in-situ equivalent continuous rock modulus E_e is estimated by Rock Mass Rating system proposed by Bieniawski in 1976, and the shear modulus G_e of the equivalent continuous rock is determined correspondingly.

In this sub-section, the properties of the in-situ equivalent continuous rock, explosive and the concrete blocks covering the charge hole involved in Autodyn modeling are estimated and determined.

The in-situ rock mass

The rock mass at the test site (Bukit Timah granite) contains two dominant sub-vertical joint sets, with an average joint spacing of 2.5 m. The uniaxial compression strength of the intact rock material is 180 MPa, with a density of 2650 kg/m³. The Rock Quality Designation (RQD) at the site is about 90%. The rock joint surface is mostly fresh, some stained, closed, rough and undulating. The ratio of the average horizontal principle stress to the vertical principle stress is 3 to 4. The rock mass is mostly dry.

Table 9.2 shows the properties of the intact rock, as determined from extensive laboratory testing program. In this table the velocities of P- and S- waves are calculated according to the rock properties by Equations (5.6) and (5.7).

Table 9.2 Properties of the intact rock

Property	Value
Density (ρ) (kg/m^3)	2650
Young's Modulus (E_r) (GPa)	64
Poisson's Ratio	0.23
Cohesion (MPa)	24
Friction angle ($^\circ$)	54
Tensile strength (MPa)	16
Velocity of compressional wave propagation (c_p) (m/s)	5292
Velocity of shear wave propagation (c_s) (m/s)	3133

From Table 9.2, the shear modulus of the intact rock (G_r) can be calculated as:

$$G_r = \frac{E_r}{2(1+\mu)} = \frac{64}{2(1+0.23)} = 26 \text{ GPa} \quad (9.1)$$

where E_r is the Young's modulus of the intact rock, and μ is Poisson's Ratio.

Properties of the equivalent continuous rock

Autodyn is a continuum-based program that requires the jointed rock mass to be converted into its equivalent continuous rock. Firstly, the RMR value of the in-situ rock mass is calculated based on Rock Mass Rating system (RMR) developed by Bieniawski (1976). And then, Young's modulus (E_e) of the in-situ equivalent continuous rock is estimated by the relationship between E_e and RMR value proposed by Serafim and Pereira (1983). Finally, The shear modulus (G_e) of the in-situ equivalent continuous rock is calculated accordingly.

According to the conditions of the in-situ rock mass, the ratings of strength of the intact rock, RQD, spacing of joints and condition of joints are determined as 12, 17, 25 and 20, respectively (in Appendix A). The RMR value is the sum of the four ratings.

$$\text{RMR} = 12 + 17 + 25 + 20 = 74 \quad (9.2)$$

According to the relationship between E_e and RMR value proposed by Serafim and Pereira (1983), E_e for the in-situ equivalent continuous rock can be calculated as:

$$E_e = 10^{\frac{\text{RMR}-10}{40}} = 40 \text{ GPa} \quad (9.3)$$

Assuming the equivalent continuous rock has the same Poisson's ratio μ of 0.23 as the intact rock, G_e can be calculated as:

$$G_e = \frac{E_e}{2(1+\mu)} = \frac{40}{2(1+0.23)} = 16.3 \text{ GPa} \quad (9.4)$$

The equivalent continuous rock is assumed to obey Mohor-Coulomb strength criteria. The properties of the equivalent continuous rock involved in Autodyn modeling are shown in Table 9.3

Table 9.3 Properties of the equivalent continuous rock

Property	Value
Density (ρ) (kg/m^3)	2650
Young's modulus (E_e) (GPa)	40
Poisson's Ratio	0.23
Cohesion (MPa)	24
Friction angle ($^\circ$)	54
Tensile strength (MPa)	16

Properties of the explosive and the concrete blocks involved in Autodyn modeling

The properties of the explosive (TNT) are obtained from the library in Autodyn as shown in Table 9.4, where TNT is assumed to obey the Jones-Wilkins-Lee state equation.

The concrete blocks covering the charge chamber is treated as a linear continuous medium and assumed to obey the porous equation of state and Mohor-Coulomb strength criteria. The properties are obtained from the material library in Autodyn as shown in Table 9.5. The air in the charge chamber is treated as ideal gas.

Table 9.4 Properties of TNT explosive

Property	Value
Reference density (kg/m^3)	1630
Parameter A (GPa)	373.77
Parameter B (GPa)	3.7471
Parameter R1	4.15
Parameter R2	0.9
Parameter W	0.35
C-J detonation velocity (m/s)	6930
C-J pressure (GPa)	21
Burn on compression fraction	0
Pre-burn bulk modulus (GPa)	0
Adiabatic constant (GPa)	0

Table 9.5 Properties of concrete material

Property	Value
Reference density (kg/m^3)	2454
Sound velocity (m/s)	2693
Density #1 (kg/m^3)	2368
Density #2 (kg/m^3)	2376
Density #3 (kg/m^3)	2412
Density #4 (kg/m^3)	2447
Pressure # 1 (MPa)	0
Pressure # 2 (MPa)	14.8
Pressure # 3 (MPa)	59
Pressure # 4 (MPa)	111.1
Shear modulus (GPa)	4.5
Yield stress #1 (MPa)	23.6
Yield stress #2 (MPa)	73.8
Yield stress #3 (MPa)	270
Yield stress #4 (MPa)	270

9.3.2 Explosion load modeled in Autodyn

As shown in Figure 9.3, the Autodyn model is a 2-D symmetrical model with only half of the domain involved in the model. The ground surface at the top of the model is treated as free boundary. The right and the bottom boundaries are viscous boundaries. The left boundary is a symmetrical boundary. The measuring point is located in the surrounding rock close to the boundary of the charge chamber at the charge level to record the properties of the particle velocity wave.

The Autodyn results show that the amplitude of the compressional velocity wave is 3.07 m/s, with a frequency of 700 Hz.

Figure 9.3 Sketch of the computational model in Autodyn

9.4 Estimating PPVs at the monitoring points in Mandai test

The prediction models for the transmission ratio in compressional wave propagation from a tunnel or borehole for single joint proposed in Section 7.4, and for multiple parallel joints proposed in Section 8.2 and Section 8.3, are applied to the Mandai test. All the estimated PPVs from the prediction models for the transmission ratio corresponding to the monitoring points are compared with the field records and two empirical formulae.

In order to apply the prediction models for the transmission ratio proposed in current research, some relevant parameters must be determined first. For a jointed rock mass, the Young's modulus of the in-situ equivalent continuous rock (E_e) and the intact rock (E_r) and the shear modulus of the in-situ equivalent rock (G_e) and the intact rock (G_r) can be known. Hart (1993) proposed formulae for estimating the stiffness of the joint. The relative position between the monitoring points and the joint set is determined by the site investigation and the configuration of the field test. After the stiffness of the joint and the relative position between the joint set and the monitoring points are identified, other relevant parameters for estimating the transmission ratio can be determined.

9.4.1 Relative position between the monitoring points and the joint set

The joints in the granite were mapped in the quarries and inferred from boreholes using impression packer data. Generally, there are two predominantly sub-vertical joint sets, with strike directions of 140° and 230° respectively, perpendicularly intersecting each other. The strike of the most dominant joint set is 140° (Zhao *et al.* 2005), perpendicular to the line along which the monitoring boreholes are located.

In Mandai test when the 2-D compressional wave from the charge hole travels in the in-situ rock mass, it is reasonably to assume that the peak particle velocities corresponding to the points along the monitoring line with the bearing of 230° is dominated by the joint set with strike of 140° that is perpendicular to this line. The joint set (with strike of 230°) parallel to the monitoring line is assumed to have

negligible effects on the PPVs at the monitoring points. It should be noted that an independent study using UDEC confirmed this assumption.

The relative position of the monitoring points and the joint set with strike of 140° is shown in Figure 9.4.

Figure 9.4 Relative position of the monitoring points to the joint set

9.4.2 The factors involved in estimating T_n

In order to estimate the transmission ratios T_n corresponding to the monitoring points in the field, the values of the factors involved in estimating T_n must be determined. The factors include the ratio of joint spacing to wavelength (spacing/wavelength), the properties of the input compressional wave, the radian angle, the number of the joints to be crossed by the wave before reaching the monitoring point and the stiffness of the joints.

From the site investigation, the spacing of the joints is determined to be 2.5 m. The properties of the input compressional wave have been determined by Autodyn modeling in Section 9.3.2. The amplitude of the wave is determined as 3.07 m/s, and the frequency is 700 Hz. The radian angle for all the monitoring points is 90° , since they lie along the radian direction perpendicular to the joint set (Figure 9.4). The number of the joints to be crossed for every monitoring point can be determined based on the spacing of the joints and the relative position of the monitoring points to the joint set. The stiffness of the joints can be determined by the formula proposed by Hart (1993).

For a rock mass containing uniformly spaced joints, Hart (1993) proposed the following relations to estimate the normal and shear stiffness of the joints:

$$j_{kn} = \frac{E_e E_r}{s(E_r - E_e)} \quad (9.5)$$

$$j_{ks} = \frac{G_e G_r}{s(G_r - G_e)} \quad (9.6)$$

where E_e and E_r are Young's modulus of the in-situ equivalent continuous rock and the intact rock, respectively; G_e and G_r are shear modulus for the in-situ equivalent continuous rock and the intact rock respectively; s is the spacing of the joints.

According the properties of the intact rock, shown in Table 9.2 for E_r and in Equations (9.1), (9.3) and (9.4) for G_r , E_e and G_e respectively, the stiffness of the joints in both normal and shear directions are calculated as:

$$j_{kn} = \frac{E_e E_r}{s(E_r - E_e)} = \frac{40 \times 64}{2.5(64 - 40)} = 42.7 \text{ GPa/m}$$

$$j_{ks} = \frac{G_e G_r}{s(G_r - G_e)} = \frac{26 \times 16.3}{2.5(26 - 16.3)} = 17.5 \text{ GPa/m}$$

For the input compressional velocity wave with a frequency of 700 Hz, the normalized normal and shear stiffness of the joint is:

$$j_{kn}/(\omega z) = 0.695$$

$$j_{ks}/(\omega z) = 0.475$$

For estimation purpose, a sketch of the Mandai test showing the relative position between the field monitoring points and the joints in the rock mass is provided in Figure 9.5. The explosion load is applied to the boundary of the borehole. The monitoring points are supposed to be along the radial line with α of 90° . Table 9.6 shows the numbers of joints that the compressional wave crosses before reaching the monitoring points.

The parameters involved in estimating T_n for the monitoring points are summarized as:

- The properties of the input compressional velocity wave: frequency is 700 Hz, amplitude is 3.07 m/s.
- The normal and shear stiffness of the joint: j_{kn} is 42.7 GPa/m, j_{ks} is 17.5 GPa/m, hence the corresponding values of normalized stiffness of joints are: $j_{kn}/(\omega z) = 0.695$, $j_{ks}/(\omega z) = 0.475$.
- The spacing of the joints is 2.5 m, hence the ratio of spacing/wavelength is 0.33.

Table 9.6 Number of joints the compressional wave crosses before reaching the monitoring points for the Mandai test

Monitoring point	1 st (x=2.5 m)	2 nd (x=5.0 m)	3 rd (x=10.0 m)	4 th (x=25.0 m)	5 th (x=50.0 m)
Number of joints to be crossed	1	2	4	9	20

Figure 9.5 Sketch of the Mandai test for the purpose of estimating T_n

9.4.3 Comparison between the field records and the estimated PPVs

Recalling the research performed in Chapters 7 and 8, the following prediction models have been proposed:

- (1) In the process of 2-D compressional wave propagation through single joint, a prediction model for the transmission ratio T_1 for any arbitrary directions is proposed (Section 7.4.2).
- (2) In the process of 2-D compressional wave propagation through multiple parallel joints, a prediction model for the transmission ratio T_n for any arbitrary directions is proposed (Section 8.3.3).
- (3) For the special radian direction of 90° , more extensive works are performed than for arbitrary radian directions, therefore, prediction models for the transmission ratio for both the cases of single joint (Section 7.4.1) and multiple

parallel joints (Section 8.2.1.3) are proposed for the special radian direction of 90° .

- (4) In the process of 2-D wave propagation through both single joint and multiple parallel joints, in the special radian direction of 90° , the prediction model for the maximum rebound ratio corresponding to any joint in the joint set is proposed (Section 8.2.2.3).

From Figure 9.5, it can be seen, for Mandai test, the monitoring points are located along the radial line with the radian angle of 90° (normal to the joints). Therefore in this section, the transmission ratio and the maximum rebound ratio at every monitoring point are estimated using the prediction models developed for the special radian direction of 90° , mentioned above in the third and the fourth items. Using the estimated transmission ratio and the maximum rebound ratio, a range for the peak particle velocity is calculated and compared with the field records at every monitoring point in Mandai test. On the other hand, the transmission ratio at every monitoring point is also estimated using the prediction models developed for arbitrary directions, mentioned above in the first and the second items.

9.4.3.1 Comparison between the field records and the estimated PPVs from the prediction models for T_n and Max. Reb_n for the radian direction of 90°

In the process of 2-D compressional wave passing through joints, in the special radian direction of 90° , models for prediction of the transmission ratio have been proposed for single joint in Section 7.4.1 and for multiple parallel joints in Section 8.2.1.3. However due to superposition of the multiple transmitted and reflected waves in the process of 2-D compressional wave propagation, the peak particle velocity is the amplitude of the rebound wave. Therefore, the prediction model for the maximum rebound ratio has been proposed in Section 8.2.2.3. This part aims to verify these models by comparing the estimated range of PPVs from the proposed prediction models with the field records at every monitoring point.

Estimated PPVs from the prediction models for T_n for the radian direction of 90°

The first monitoring point in the field is located after the first joint close to the boundary of the charge camber. Therefore, the proposed prediction model for the transmission ratio shown in Figure 7.4 is used to estimate T_1 at the first monitoring point. As mentioned before in Section 9.4.2, the value of the normalized normal stiffness $jkn/(\omega z)$ is 0.695. From Figure 7.4, the value of T_1 is 0.78 for a value of $jkn/(\omega z)$ of 0.695. The estimated peak particle velocity at any monitoring point is equal to the estimated transmission ratio T_n times the amplitude of the wave at the same point from the case of intact rock. The amplitude of the wave at any point for the intact rock case can be determined using Figure 5.10 (c). (It is noted the amplitude from Figure 5.10 (c) is for the input wave with unity amplitude. Therefore, in this chapter, the amplitude value from Figure 5.10 (c) is multiplied by 3.07, i.e. the amplitude of the input wave in Mandai test). At the location of the first monitoring point, the amplitude of the wave for the intact rock case is obtained from Figure 5.10 (c) as 0.768 m/s. This leads to a value of estimated PPV of 0.599 m/s at the first monitoring point.

To calculate the value of T_n for the other monitoring points, spacing/wavelength and the two critical points (S_{peak} , T_{peak}) and (S_{flat} , T_{flat}) as defined in Figure 8.3 for a given value of $jkn/(\omega z)$ have to be determined. The spacing between joints in the field is 2.5 m as discussed in Section 9.4.2, this leads to a value of spacing/wavelength of 0.33. The values of S_{peak} , T_{peak} , S_{flat} and T_{flat} can be determined using Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15, respectively. After calculating S_{peak} , T_{peak} , S_{flat} and T_{flat} , the values of T_n is calculated as follows:

(1) If $S_{\text{flat}} \leq$ the target value of spacing/wavelength, the estimated T_n is obtained as:

$$T_n = T_{\text{flat}} \quad (9.7)$$

(2) If $S_{\text{flat}} >$ the target value of spacing/wavelength ($>S_{\text{peak}}$), the estimated T_n is obtained as:

$$T_n = T_{\text{peak}} - \left(\frac{\text{The target value of spacing/wavelength} - S_{\text{peak}}}{S_{\text{flat}} - S_{\text{peak}}} \right) (T_{\text{peak}} - T_{\text{flat}}) \quad (9.8)$$

Using the target values for the Mandai test (jkn/(ωz) of 0.695, spacing/wavelength of 0.33), in Figure 8.12, Figure 8.13, Figure 8.14 and Figure 8.15, the transmission ratios and magnitudes of PPVs for the five monitoring points can be obtained as shown in Table 9.7.

Table 9.7 Estimated PPVs (including the parameters involved in the calculation) and field records (*: Datum not available)

Monitoring point (Row 1)	1 st (X=2.5m)	2 nd (X=5.0m)	3 rd (X=10m)	4 th (X=25m)	5 th (X=50m)
Amount of the joints to be crossed n (Row 2)	1	2	5	9	20
S _{flat} (Row 3)	-	0.215	0.32	0.39	0.51
T _{flat} (Row 4)	-	0.63	0.43	0.25	0.04
S _{peak} (Row 5)	-	0.061	0.053	0.043	0.021
T _{peak} (Row 6)	-	0.76	0.7	0.665	0.59
Amplitude from model of intact rock (m/s) (Figure 5.10 (c)) (Row 7)	0.768	0.503	0.322	0.166	0.091
Estimated transmission ratio T _n (Row 8)	0.78 (Figure 7.4)	0.63 (Equation 9.7)	0.43 (Equation 9.7)	0.322 (Equation 9.8)	0.214 (Equation 9.8)
Estimated PPV (m/s) (Row 7 times Row 8)	0.599	0.317	0.139	0.053	0.019
Field-record PPV (m/s)	0.61	0.468	0.192	*	0.021

Estimated PPVs from the prediction model for the maximum rebound ratio for the radian direction of 90°

The estimated transmission ratio T_n is the ratio after the wave travels through n joints, assuming there are no more joints afterwards. However, in practical cases, the point of interest might be located at any point between two adjacent joints in the joint set. The PPV corresponding to a point between the n^{th} joint and $(n+1)^{\text{th}}$ joint is the peak particle velocity of the wave after superposition of the transmitted wave and the reflected wave. Depending on spacing/wavelength and the location of the point between the two adjacent joints, the PPV at the point will be between a minimum possible value and a maximum possible value. The minimum possible value for PPV is estimated from the transmission ratio T_n , while the maximum possible value for PPV is estimated from the maximum rebound ratio Max. Reb_{n+1} between the n^{th} joint and the $(n+1)^{\text{th}}$ joint. The minimum PPVs calculated from the values of the transmission ratio T_n have been given in Table 9.7. In this part, the maximum possible PPVs at the monitoring points will be estimated from the proposed prediction model for the maximum rebound ratio.

From Figure 9.5, it can be seen that the 1st, 2nd, 3rd, 4th and 5th monitoring point in Mandai test is located before the 2nd, 3rd, 5th, 10th and 21st joint, respectively, from the boundary of the borehole. The value of the maximum rebound ratio at each location has been obtained from Figures 8.19 (d) and (e), using $jkn/(\omega z)$ of 0.695 and spacing/wavelength of 0.33. These values have been shown in Table 8.2.

The maximum possible peak particle velocity at the monitoring point is equal to the estimated maximum rebound ratio times the amplitude of the compressional wave at the same point in the case of intact rock, which can be determined from Figure 5.10 (c).

Table 9.8 summarizes the minimum possible PPVs calculated from the transmission ratio T_n , the maximum possible PPVs calculated from the maximum rebound ratio Max. Reb_{n+1} and the field records at every monitoring point.

Table 9.8 The minimum possible PPVs, the maximum possible PPVs and the field-recorded PPVs (*: Datum not available)

Monitoring point (Row 1)	1 st (X=2.5m)	2 nd (X=5.0m)	3 rd (X=10m)	4 th (X=25m)	5 th (X=50m)
Number of the joints crossed, n (Row 2)	1	2	4	9	20
Max. Reb _{$n+1$} (From Table 8.2) (Row 3)	Max. Reb ₂ =1.08	Max. Reb ₃ =0.8	Max. Reb ₅ =0.63	Max. Reb ₁₀ =0.44	Max. Reb ₂₁ =0.35
Amplitude from model of intact rock (m/s) (From Figure 5.10 (c)) (Row 4)	0.768	0.503	0.322	0.166	0.091
Maximum possible PPV (m/s) (Row 3 times Row 4)	0.829	0.402	0.203	0.073	0.032
Minimum possible PPV (m/s) (From Table 9.7)	0.599	0.317	0.139	0.053	0.019
Filed-recorded Record PPVs (m/s)	0.61	0.468	0.192	*	0.021

Figure 9.6 compares the minimum possible PPVs using the proposed prediction models for the transmission ratio, the maximum possible PPVs from the prediction model for the maximum rebound ratio and the field-recorded PPVs. The horizontal axis in Figure 9.6 represents the scaled range ($r/w^{1/3}$) (refer to Appendix B), where r is the radial distance in meter from the explosion source, w is the charge weight in kg. The solutions from two empirical formulae are also included in Figure 9.6.

From Figure 9.6, it can be seen that the estimated range of PPVs given by the minimum possible PPVs and the maximum possible PPVs agrees well with the field-recorded PPVs. Three of the four available field-recorded PPVs lie exactly between the estimated range, while the fourth available field-recorded PPV lies close to the maximum estimated value. The very good agreement between the estimated and field-recorded values validates the proposed prediction models for estimating peak particle velocity in a rock mass with multiple parallel joints due to application of a compressional wave at the boundary of a tunnel or a borehole.

It can be seen from Figure 9.6, that results from both of the two available empirical formulae (Wu formula (1975) and Ambraseys and Hendron formula (1968)) match with the field data poorer than results from the prediction models for the transmission ratio and the maximum rebound ratio proposed in this research. This is because all the factors influencing the transmission ratio and the maximum rebound

ratio in the process of 2-D compressional wave propagation in a jointed rock mass are taken into account in this research.

Figure 9.6 Comparison among the estimated PPVs and correlated PPVs from the model corresponding to the special direction $\alpha=90^\circ$, field records and PPVs from two empirical formulae (unit of scaled range is $\text{m/kg}^{1/3}$)

9.4.3.2 Comparison between the field records and the estimated PPVs from the prediction models for T_n for arbitrary directions

It should be noted that the proposed prediction models for the transmission ratio for arbitrary directions can also be applied to the Mandai test, because the arbitrary directions cover the radian angle of 90° . The predictions models for the transmission ratio for arbitrary directions both for cases of single joint (Section 7.4.2) and multiple joints (Section 8.3.3) are applied to Mandai test. Figure 9.7 shows the comparison between the field records and the estimated PPVs at the monitoring points in the field, using the prediction models for the transmission ratio for arbitrary directions (refer to Appendix C for details).

It is found the estimated results by using the prediction models for the transmission ratio for arbitrary directions are almost the same with those by using the prediction models for the transmission ratio for the special direction of 90° (the minimum possible PPVs in Figure 9.6). It indicates, in the special radian direction of 90° , the prediction models for the transmission ratio for arbitrary directions is in essence the same as the prediction models for the transmission ratio for the radian direction of 90° . It also indicates the proposed prediction models for arbitrary directions are also reliable for estimating peak particle velocity in a rock mass with multiple parallel joints due to application of a compressional wave at the boundary of a tunnel or a borehole.

Figure 9.7 Comparison among the estimated PPVs, field records and results from two empirical formulae for the monitoring points along radial line with α of 90° (unit of scaled range is $\text{m/kg}^{1/3}$)

9.5 Summary

In this chapter, the proposed prediction models for 2-D compressional wave propagation in rock masses have been used to estimate the amplitude of the velocity wave at different points in the rock mass in a field explosion test conducted in Mandai, Singapore. Estimated results are compared with the field records and the results calculated from two available empirical formulae.

According to the site investigation, the strike of dominant joint set and the line along which the monitoring points are located in the Mandai explosion test are perpendicular. Therefore, the minimum possible values of PPVs at the monitoring points are estimated using the prediction models for the transmission ratio for the radian direction of 90° for both single joint (proposed in Section 7.4.1) and multiple parallel joints (proposed in Section 8.2.1.3). On the other hand, the maximum possible values of PPVs are estimated by using the prediction model for the maximum rebound ratio along the direction of 90° (Section 8.2.2.3). The proposed prediction models for the transmission ratio for arbitrary directions both for single

joint (Section 7.4.2) and multiple joints (Section 8.3.3) are also applied to the Mandai test.

Comparison of results shows that the two available empirical formulae of Ambraseys and Hendron (1968) Wu (1975) give results that are different from and do not agree with the field results.

By contrast, comparison of results shows a good agreement between the field-recorded PPVs and the estimated PPVs at the monitoring points in Mandai test. Three of the four available field-recorded PPVs lie exactly between the estimated range given by the minimum possible PPVs and the maximum possible PPVs, while the fourth available field-recorded PPV lies very close to the maximum possible PPV. The very good agreement between the estimated and field-recorded values validates the proposed prediction models for estimating peak particle velocity in a rock mass with multiple parallel joints due to application of a compressional wave at the boundary of a tunnel or a borehole.

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

Due to its limited land space, Singapore has started construction of its first series of caverns in rock mass. These caverns will be used for ammunition storage. However, there is a concern that an accidental explosion in one of the caverns or tunnels might lead to instability of other nearby or distant tunnels. Due to this concern, the current study has been initiated to numerically investigate 2-D compressional wave propagation in jointed rock masses due to explosion in a tunnel or borehole.

The numerical study is carried out using the Universal Distinct Element Code, UDEC, which is the most commonly used discrete element program in engineering problems related to jointed rock masses.

In this chapter, the conclusions from the studies performed are summarized, and the recommendations for the future research are presented.

10.1 Conclusions

(A) Although UDEC is extensively verified and used in static problems in jointed rock masses, its use and reliability in dynamic problems has not been thoroughly verified. Therefore, studies on verification of UDEC and the factors affecting the accuracy of UDEC results have been performed in the current research, before performing the main analysis. From the study performed on verification of UDEC in dynamic problems and the study on accuracy of UDEC results, the following conclusions have been drawn:

- (1) For 1-D wave propagation, UDEC results agree well with those obtained from the analytical solution and the method of characteristics for cases of intact rock bar, single joint and multiple joints.
- (2) For 2-D wave propagation, the problem of harmonic wave traveling from a cylindrical cavity in intact rock is analyzed using UDEC. The UDEC results

for the radial stress, displacement and velocity waves are compared with the closed form solutions given by Graff (1979) and Hefny (2004). It has been found that UDEC results agree very well with those obtained from the closed form solutions.

- (3) During the formation of the geometrical model of UDEC, it is often necessary to include fictitious joints in the model. These added fictitious joints must have negligible or no effects on the obtained results. UDEC has recommended the use of fictitious joints with high values of ratio of joint stiffness to equivalent stiffness K_{eq} (jkn/K_{eq} and jks/K_{eq}). A value of less than 10 has been recommended to keep the calculation reasonably fast. However, the lower limit of joint stiffness has not been investigated before. In the current research, a study is carried out to investigate the lower limit of jkn/K_{eq} and jks/K_{eq} of the fictitious joints. Results of the performed study show that a minimum value of jkn/K_{eq} and jks/K_{eq} of 2 must be adopted in order to keep the computational error within negligible values.
- (4) The accuracy of UDEC results is highly dependent on the mesh size. In general, the average error decreases as the mesh size decreases. On the other hand, the accuracy of UDEC results is dependent on the frequency of the input wave. The average error decreases as the frequency of the input wave decreases.
- (5) The accuracy of UDEC results is highly dependent on the mesh ratio (l_r), which is defined as the ratio of mesh size to wavelength. Smaller values of mesh ratio (l_r) yield lower average error, and the average error increases dramatically as the value of mesh ratio (l_r) is above 0.2.
- (6) In 2-D wave propagation problems, a mesh ratio (l_r) of 1/8 is recommended to keep the average error below 5%, while a mesh ratio (l_r) of 1/25 is recommended to keep the average error below 2%. In all the subsequent studies, a mesh ratio (l_r) of 1/25 is adopted in order to keep the average error consistently below 2%.

- (B) After verifying UDEC in dynamic problems and investigating the factors influencing the accuracy of UDEC results, a new data processing method is developed to analyze the stress/velocity amplitude in dynamic problems in intact rock and jointed rock masses by combining UDEC with MATLAB and EXCEL software. The source code of UDEC has been modified to record the time history for every grid-point or centroid of a zone in the model. Then, the obtained time history from UDEC analysis is processed in MATLAB software using 2-D linear interpolation function, instead of the less accurate stepwise function adopted in UDEC. Finally, these processed results are visualized in EXCEL software. This new data processing method does not only reduce the time of processing UDEC results, but also leads to more accurate interpolated results at any arbitrary points in the model.
- (C) After developing the data processing method, a reference study on 2-D wave propagation in intact rock due to explosion at the boundary of a tunnel of radius R_0 is carried out. The study performed shows that for a given frequency, the amplitude of the wave at any point of radial distance R from the center of the tunnel is function of the deformation parameters and scaled distance, which is defined as $(R-R_0)/R_0^{1.5}$. Charts have been prepared for ready determination of the amplitude of the propagated wave at any point in the surrounding intact rock. These charts are used as references for calculating the transmission ratio for the propagated wave in jointed rock masses in the subsequent studies.
- (D) Subsequently, three main studies have been performed to investigate the performance of the transmission ratio in the process of 2-D compressional wave propagation in a rock mass with single joint and a set of parallel joints due to explosion at the boundary of a tunnel of radius R_0 . The transmission ratio (T_n) is defined as the ratio of the amplitude of the transmitted wave at a point after the joints over the wave amplitude at the same point in the case of intact rock.

Firstly, a study on the effects of the position of the joint in the case of single joint and the first joint in the case of multiple parallel joints is performed. It is concluded from the study that, if the normal distance from the cavity wall is

equal or larger than R_0 , the position of the single joint or the first joint in the joint set has no influence on the value of the transmission ratio.

Secondly, a study on the effect of joint properties on the performance of the transmission ratio has been performed.

From the study performed on a rock mass with single joint, the following conclusions are obtained:

- (1) In general, the normal and shear stiffness of the joint significantly affect the transmission ratio T_1 of the wave.
- (2) The transmission ratio T_1 is approximately constant along any radial line, i.e., the transmission ratio T_1 is independent of the radial distance from the cavity center for the points along a specific radial line.
- (3) The transmission ratio T_1 is highly dependent on the radian direction of the point considered.
- (4) For arbitrary directions, the relationship between T_1 and the affecting factors (the normalized normal and shear stiffness and the radian direction) are comprehensively generalized into several charts. These charts are used as the basis of prediction model for estimating the transmission ratio T_1 for arbitrary directions.
- (5) In arbitrary directions (other than the radian direction normal to the joint), for a specific value of normalized normal stiffness, when the normalized shear stiffness increases, the corresponding transmission ratio T_1 increases. On the other hand, for a specific value of normalized shear stiffness, at an arbitrary radian angle, the value of the transmission ratio T_1 increases as the normalized normal stiffness value increases.
- (6) When the normalized normal stiffness of the joint is small enough, the minimum transmission ratio T_1 always occurs at the radian direction normal to the joint. On the other hand, when the normalized normal

stiffness is high enough, the maximum transmission ratio T_1 occurs at the radian direction normal to the joint.

- (7) For the special case of points along the radial line normal to the joint, the transmission ratio T_1 is approximately independent of the shear stiffness of the joint. Only the normal stiffness affects the transmission ratio T_1 .
- (8) Along the radian direction normal to the joint, because the effects of the shear stiffness on T_1 are negligible, the relationship between T_1 and the normalized normal stiffness is established. It is found that in general T_1 increases from about 0 for a joint with very small normalized normal stiffness to close to 1 for a joint with very high normalized normal stiffness.

Likewise, from the study performed on a rock mass with multiple parallel joints, the following conclusions are obtained:

- (1) The stiffness of joints, radian direction, number of joints and ratio of joint spacing to wavelength (spacing/wavelength) significantly affect the value of the transmission ratio T_n .
- (2) The transmission ratio T_n is approximately constant along any radial line for the points behind the joints, i.e., the transmission ratio T_n is independent of the radial distance from the cavity center for the points along a specific radial line.
- (3) The transmission ratio T_n is highly dependent on the radian direction of the point under consideration.
- (4) Under a given set of conditions (number of joints, ratio of spacing/wavelength and normalized normal stiffness), for a specific arbitrary radian direction (except a small range of radian direction normal to the joints), the value of the transmission ratio T_n increases as the normalized shear stiffness value increases. Therefore, the effect of normalized shear stiffness becomes larger as the radian direction deviates from the special radian direction normal to the joints.

- (5) Under a given set of conditions (number of joints, ratio of spacing/wavelength and normalized shear stiffness), for a specific arbitrary radian angle, the value of the transmission ratio T_n increases as the normalized normal stiffness value increases.
- (6) Under a given set of conditions (number of joints, ratio of spacing/wavelength), when the normalized normal stiffness of the joints is very small, the minimum T_n is roughly at the radian direction normal to the joints. By contrast, when the normalized normal stiffness is quite high, the maximum T_n is roughly at the radian direction normal to the joints.
- (7) For the special case of points along the radial line normal to the joints, the transmission ratio T_n is roughly independent of the shear stiffness of the joints. Only the normal stiffness affects the transmission ratio T_n .
- (8) For effects of multiple parallel joints in the radian direction normal to the joints, if the ratio of spacing/wavelength is greater than a critical value S_{flat} , the value of transmission ratio becomes a constant and independent of the ratio of spacing/wavelength. As the ratio of spacing/wavelength decrease from the critical value S_{flat} , the transmission ratio T_n increases and reaches the peak value at another critical value of spacing/wavelength S_{peak} . As the ratio of spacing/wavelength further decreases from S_{peak} , the transmission ratio T_n decreases from its peak value.

Thirdly, a study is carried out on the effects of radius of the tunnel, from which the compressional wave originates. From the study performed, it is concluded that, while the amplitude of the transmission wave is dependent on the radius of the tunnel, the transmission ratio as defined in this research work is independent of the radius of the tunnel.

- (E) For the special radian direction normal to the joints, a study is performed on the maximum rebound ratio (Max. Reb_n) in the process of 2-D compressional wave propagation through multiple joints. From the study performed, the following conclusions are obtained: (It should be noted that the rebound ratio Reb_n at a

point between the $(n-1)^{\text{th}}$ and n^{th} joints is defined as the amplitude of the rebound wave after multiple superposition to the wave amplitude at the same point in the model of intact rock. The maximum rebound ratio (Max. Reb_n) is the maximum value of Reb_n between the $(n-1)^{\text{th}}$ and the n^{th} joints)

- (1) The maximum rebound ratio Max. Reb_n is influenced by normalized normal stiffness, spacing/wavelength and the joint from which the wave rebounds. For a specific ratio of spacing/wavelength, the Max. Reb_n increases as normalized normal stiffness increases.
 - (2) For each given value of normalized normal stiffness, the Max. Reb_n decreases as spacing/wavelength increases.
 - (3) For a point between two adjacent joints (the n^{th} and the $(n-1)^{\text{th}}$ joints), the ratio of wave amplitude to the wave amplitude in intact rock lies between T_{n-1} (as a lower limit) and Max. Reb_n (as an upper limit).
- (F) Based on the results obtained from the studies performed, prediction models for estimating the transmission ratio and the maximum rebound ratio of 2-D compressional wave propagating through a rock mass with single joint or multiple parallel joints when an explosion load is applied at the boundary of a tunnel or borehole are developed. Using these models, the amplitude of the propagating wave at any point in the rock mass can be estimated. The proposed models have been used to estimate the amplitude of the velocity wave at different points in the rock mass in a field explosion test conducted in Mandai, Singapore. The estimated results obtained from the proposed models agree well with the field-recorded values.

10.2 Recommendations

Although considerable research work has already been carried out to study the propagation of waves through rock masses due to explosion in a tunnel or borehole, the following issues are not addressed in the current work and they are recommended for the future research:

- (A) The studies have been performed on wave propagation through a single joint or multiple parallel joints in a joint set. In many cases, more than one joint set may exist in the rock mass and a study of 2-D compressional wave propagation through multiple joint sets would be of practical values.
- (B) The problem in the current research is simplified as a plane strain problem. However, when an explosion occurs only along a short section of the tunnel or borehole, the problem becomes 3-D, and the 2-D representation might give inaccurate results. In this case, a study of 3-D compressional wave propagation would be more representative and worthy of investigation.
- (C) In the study performed, the characteristics of the peak amplitude of the propagated wave have been investigated in detail. However, the effects of the propagated wave on stability of nearby tunnels have not been explored, and it is worthy of investigation.
- (D) In the parametric studies presented in this thesis, a wide range of values for the properties of the rock joints has been chosen to cover the range for natural rock joints. However, in order to improve the reliability and utility of the proposed prediction models, determination of the mechanical properties of the natural rock joints would be worthy of further study.

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Part of Bieniawski's 1976 table for defining Rock Mass Rating (RMR) for jointed rock masses

PARAMETER		RANGE OF VALUES					
Strength of intact rock material	Point-load strength index	>8 MPa	4-8 MPa	2-4 MPa	1-2 MPa	For this low range uniaxial compressive test is preferred	
	Uniaxial compressive strength	>200 MPa	100-200 MPa	50-100 MPa	25-50 MPa	10-25 MPa	3-10 MPa 1-3 MPa
1	Rating	15	12	7	4	2	1 0
	Drill core quality <i>RQD</i>	90%-100%	75%-90%	50%-75%	25%-50%	<25%	
2	Rating	20	17	13	8	3	
3	Spacing of joints	>3 m	1-3 m	0.3-1 m	50-300 mm	<50 mm	
	Rating	30	25	20	10	5	
4	Condition of joints	Very rough surfaces Not continuous No separation Hard joint wall contact	Slightly rough surfaces Separation < 1 mm Hard joint wall contact	Slightly rough surfaces Separation < 1 mm Soft joint wall contact	Slickensided surfaces or Gouge < 5 mm thick or Joints open 1-5 mm Continuous joints	Soft gouge >5 mm thick or Joints open > 5 mm Continuous joints	
	Rating	25	20	12	6	0	

APPENDIX A

ROCK MASS RATING (RMR)

APPENDIX B

AMBRASEYS & HENDRON FORMULA AND WU FORMULA

Two empirical formulae are used to estimate the field explosion test, which are the Ambraseys and Hendron (1968) formula and the Wu (1975) formula.

1. Ambraseys and Hendron formula

Based on some construction cases including small-scale modeling of nuclear explosions, Ambraseys and Hendron (1968) suggested an empirical formula for estimating the PPV in m/s as a function of distance with the following form:

$$\begin{aligned} \text{PPV} &= 11.45(r/w^{1/3})^{-2.8}, \text{ if } r/w^{1/3} < 4.14 \\ &= 2.08(r/w^{1/3})^{-1.6}, \text{ if } r/w^{1/3} \geq 4.14 \end{aligned}$$

where r is the radial distance in m from the explosion source, w is the charge weight in kg.

2. Wu formula

Some field tests were conducted with similar conditions to the field explosion test of this study, but with a different rock type. Based on the test results, an empirical formula with the same form of Wu formula (1975) was suggested as:

$$\text{PPV (m/s)} = 1.8(r/w^{1/3})^{-2.5}$$

Where r is the radial distance in m from the explosion source, w is the charge weight in kg.

APPENDIX C

COMPARISON BETWEEN THE FIELD REOCORDS AND THE ESTIMATED PPVS FROM THE PROPOSED PREDICTION MODELS FOR T_n FOR ARBITRARY DIRECTIONS

In this appendix, the field-recorded PPVs are compared with the estimated PPVs at every monitoring point in the Mandai test in Singapore by using the prediction models for the transmission ratio for arbitrary directions both for single joint (Section 7.4.2) and for multiple parallel joints (Section 8.3).

The parameters involved in estimating T_n for the monitoring points are summarized as: (1) The properties of the input velocity wave: frequency is 700 Hz, amplitude is 3.07 m/s. (2) The normal and shear stiffness of the joint: $j_{kn}=42.7$ GPa/m, $j_{ks}=17.5$ GPa/m, hence the corresponding values of normalized stiffness are: $j_{kn}/(\omega z)$ of 0.695 and $j_{ks}/(\omega z)$ of 0.475. (3) The spacing of the joints is 2.5 m, hence the ratio of joint to wavelength (spacing / wavelength) = 0.33. (4) The radian angle is 90° . (5) The numbers of the joints that the compressional wave crosses before reaching the monitoring points are show in Table C.1.

Table C.1 Number of joints the compressional wave crosses before reaching the monitoring points for the Mandai test

Monitoring point	1 st (x=2.5 m)	2 nd (x=5.0 m)	3 rd (x=10.0 m)	4 th (x=25.0 m)	5 th (x=50.0 m)
Number of joints crossed	1	2	4	9	20

Following the same steps in Section 7.4.2, based on Figure 7.5, the transmission ratio T_1 versus radian angle for single joint with $j_{kn}/(\omega z) = 0.695$ and $j_{ks}/(\omega z) = 0.475$ are listed in Table C.2 and visually shown in Figure C.1.

Table C.2 Radian angles and T_1 for single joint with normalized normal stiffness $j_{kn}/(\omega z) = 0.695$ and normalized shear stiffness $j_{ks}/(\omega z) = 0.475$

Radian angle (°)	14.6	20	30	40	50	60	70	80	90
Transmission ratio T_1	0.82	0.8	0.76	0.74	0.74	0.745	0.75	0.76	0.77

Figure C.1 T_n versus radian angle for single joint with normalized normal and shear stiffness 0.695 and 0.475

For the Mandai test, according to the ratio of spacing/wavelength of 0.33, the Figures 8.20 (f) and (g) are chosen for estimating T_2 by linear interpolation. (Figure 8.20 (f) is for ratio of spacing/wavelength of 0.30, while Figure 8.20 (g) for 0.35, the target ratio of spacing/wavelength 0.33 is covered by the range of 0.30 to 0.35). Figures 8.21 (f) and (g) are chosen for estimating T_4 by linear interpolation, and Figures 8.22 (f) and (g) for T_8 .

Based on Figures 8.20 (f) and (g), Figures 8.21 (f) and (g), Figures 8.22 (f) and (g), using the same method described in Section 8.3.3.3, T_n for different radian angle can be obtained as shown in Table C.3 and Figure C.2 for 2, 4 and 8 joints, with the ratio of spacing/wavelength of 0.33, with $jkn/(\omega z) = 0.695$ and $jks/(\omega z) = 0.475$.

From Table C.3 or Figure C.2, the relationship between the transmission ratio and the number of the joints can be obtained for spacing/wavelength of 0.33, $jkn/(\omega z) = 0.695$ and $jks/(\omega z) = 0.475$ for different radian angles. Figure C.3 shows the relationship between the transmission ratio T_n and the number of joints with the above mentioned conditions of spacing/wavelength and normalized joint stiffness for the radian angle of 90° .

Table C.3 Radian angles and T_n for 2, 4 and 8 joints, with normalized normal and shear stiffness $jkn/(\omega z) = 0.695$ and $jks/(\omega z) = 0.475$ and spacing/wavelength of 0.33

Radian angle ($^\circ$)	20	30	40	50	60	70	80	90
Transmission ratio T_2	0.682	0.633	0.603	0.601	0.607	0.620	0.635	0.637
Transmission ratio T_4	0.502	0.442	0.414	0.409	0.415	0.429	0.444	0.449
Transmission ratio T_8		0.301	0.282	0.283	0.294	0.313	0.349	0.359

Figure C.2 T_n versus radian angle for multiple joints with the ratio of spacing/wavelength=0.33 with normalized normal and shear stiffness 0.695 and 0.475 for 2, 4 and 8 joints

Figure C.3 Relationship between the T_n and the number of joints for multiple joints with the ratio of spacing/wavelength=0.33 with normalized normal and shear stiffness 0.695 and 0.475 for the radian angle of 90°

For every monitoring point in the Mandai test, the estimated PPV is the estimated transmission ratio T_n by the amplitude of the compressional wave corresponding to the monitoring point from a model of intact rock. The amplitudes of the compressional wave in the intact rock corresponding to the monitoring points in Mandai test can be obtained in Figure 5.10 (c).

Referring to Table C.1 for the numbers of the joints that the compressional wave crosses before reaching the monitoring points, from Figure C.1 and Figure C.3, the estimated transmission ratios and PPVs for the monitoring points along the radian angle of 90° are listed in Table C.4

Table C.4 Estimated T_n and PPVs for the monitoring points along radian angle of 90° (Notes +: data not available, *: by extension)

Monitoring point	1 st (x=2.5 m)	2 nd (x=5 m)	3 rd (x=10 m)	4 th (x=25 m)	5 th (x=50 m)
PPV from intact model (m/s) (From Figure 5.10 (c)) (Row 1)	0.768	0.503	0.322	0.166	0.091
Estimated T_n (From Figures C.1 and C.3) (Row 2)	$T_1=0.77$	$T_2=0.64$	$T_4=0.45$	$T_9=0.35^*$	$T_{20}=0.22^*$
Estimated PPVs (m/s) (Row 1 times Row 2)	0.591	0.322	0.145	0.058	0.020
Field records (m/s)	0.610	0.468	0.192	+	0.021

Figure C.4 shows the estimated PPVs versus the scaled ranges for the monitoring points in the field, where the estimated PPVs are from the proposed prediction models for the transmission ratio T_n .

The scaled range is termed as $r/w^{1/3}$ as shown in Appendix B, where r is the radial distance in meter from the explosion source, w is the charge weight in kg. In this figure, the field records and solutions from two empirical formulae, as shown in Appendix B, are also included. Comparing with the field records, it is found the estimated results from the prediction models for the transmission ratio T_n for arbitrary radian angles in 2-D compressional wave propagation agree well with the field records.

Figure C.4 Comparison among the estimated PPVs, field records and results from two empirical formulae for the monitoring points along radial line with $\alpha=90^\circ$ (unit of scaled range is $m/kg^{1/3}$)

APPENDIX D

LIST OF PUBLICATIONS GENERATED FROM THIS THESIS

D1 Journal Papers (Including submitted)

- (1) Lei, W.D., Hefny, A.M., Zhao, J., Zhao, X.B., “Development of a new method of comprehensive processing of wave amplitude in discrete element modeling”, Journal of China University of Mining and Technology (Chinese Edition), 2005, Vol. 34, No. 2, pp. 170-174.
- (2) Lei, W.D., Hefny, A.M., Zhao, J., “Pilot study on two dimensional elastic wave propagation in rocks”, Transactions of Nonferrous Metals Society of China, 2005, Vol. 15, No. 4, pp.949-955.
- (3) Lei, W.D., Hefny, A.M., Zhao, J., Zhao, X.B., “Two-dimensional pulse wave propagation in rocks with single joint”, Chinese Journal of Rock Mechanics and Engineering (in Chinese), 2005, Vol. 24, No. 20. (In press)
- (4) Lei, W.D., Hefny, A.M., Zhao, J., Song, H.W., “Verification of numerical modeling in 2-D wave propagation in rock”, Journal of China University of Mining and Technology (English Edition). 2005, Vol. 15, No. 4. (In press)
- (5) Lei, W.D., Hefny, A.M., Zhao, J., “A study on transmission ratio (T_n) in the radian direction normal to rock joints in 2-D compressional wave propagation in rock masses”, Journal of University of Science and Technology Beijing, 2006, Vol. 12, No. 3. (In press)
- (6) Lei, W.D., Hefny, A.M., Zhao, J., “Characteristics of the transmission ratio for multiple parallel joints (T_n) and the implications”, Chinese Journal of Rock Mechanics and Engineering (in Chinese). (In press)
- (7) Lei, W.D., Hefny, A.M., “Discrete element modeling accuracy in rock dynamics”, Journal of University of Science and Technology Beijing. (Submitted)

- (8) Hefny, A.M., Lei, W.D., Zhao, J., “A numerical study on 2-D wave propagation in rock masses with a set of joint along the radian direction normal to the joints, Part A: Estimating transmission ratio and the maximum rebound ratio”, Computers and Geotechnics. (Submitted)

- (9) Lei, W.D., Hefny, A.M., Zhao, J., “A numerical study on 2-D wave propagation in rock masses with a set of joint along the radian direction normal to the joints, Part B: Application of the prediction models in a field explosion test”, Computers and Geotechnics. (Submitted)

- (10) Lei, W.D., Hefny, A.M., Zhao, J., “A study on the maximum rebound ratio in the process of blasting wave propagation along the radian direction normal to the joints, Part I: Prediction model”, Journal of Central South University of Technology. (Submitted)

- (11) Lei, W.D., Hefny, A.M., Zhao, J., “A study on the maximum rebound ratio in the process of blasting wave propagation along the radian direction normal to the joints, Part II: A case study”, Journal of Central South University of Technology. (Submitted)

Note: Several journal papers are in preparation.

D2 Conference Paper

- (1) Hefny, A.M., Lei, W.D., Zhao, J. “A prediction model for the transmission ratio and its verification in 2-D compressional wave propagation in rock masses”, Proceedings of GeoShanghai International Conference, Shanghai, China, 2006. (Submitted)