Behaviour of reinforced concrete short columns subjected to multi-directional loading

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2005

Nguyen, X. H. (2005). Behaviour of reinforced concrete short columns subjected to multi-directional loading. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/11958>

<https://doi.org/10.32657/10356/11958>

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Figure 5.12 Strain in strain gauge Y3 of column B30

Figure 5.13 Strain in strain gauge X3 of column B60

5.2.3 Strain in longitudinal reinforcement

Strain of main reinforcing bar at a corner of columns U0, U90, B30 and B60 is plotted in Figure 5.14 to Figure 5.17 at different displacement in pulling direction.

Strain along the rebar D of column U0 is shown in Figure 5.14. It is obvious that, when ultimate shear capacity was reached, strain changed from negative to positive, i.e. longitudinal reinforcement buckled at the locations about 200mm from two ends of the column, where shear reinforcement yielded extensively. The buckling of main reinforcement of column U0 is illustrated in Figure 5.18. The buckling of main reinforcement at the peak load suggests that lateral load carrying capacity of the column was lost due to crushing of concrete at locations 200mm from two ends of the column.

Figure 5.15 shows strain distribution along rebar F of column U90. Strain of the rebar in compression was well below yield strain when the column reached

Chapter 5 Experimental Results

maximum shear force. Compression yielding of main reinforcement occurred at displacement 8.2mm, when shear capacity of the column decreased about 20%.

Strain along the rebar F of column B30 is presented in Figure 5.16. Strain in the rebar at the end of the column was very close to yield strain when peak shear force was reached and strain was much larger than yield strain when displacement was 5.5mm. It suggests that main reinforcement yielded in compression right after the peak load. As seen from Figure 5.17, longitudinal reinforcement of column B60 did not yield at all.

Figure 5.14 Strain of longitudinal reinforcement at a corner of column U0

Figure 5.15 Strain of longitudinal reinforcement at a corner of column U90

Chapter 5 Experimental Results

Figure 5.16 Strain of longitudinal reinforcement at a corner of column B30

Figure 5.17 Strain of longitudinal reinforcement at a corner of column B60

Figure 5.18 Buckling of longitudinal reinforcement of column U0

Strain of longitudinal reinforcement on the cross sections of column U0 is shown in Figure 5.19. The figure reveals that strain distribution is nonlinear. It means Bernoulli's assumption of a cross-section is not valid for this column, particularly after cracking.

Figure 5.20 shows strain of longitudinal reinforcement on different cross-sections of column U90. Strain distribution of longitudinal reinforcement in each cross section of column is approximately linear, particular at low displacement. It suggests that Bernoulli's assumption may be used for column U90. This condition is different from column U0 because shear span to depth ratio of column U0 is smaller and its shear deformation dominates over flexural deformation.

Figure 5.19 and Figure 5.20 show that at almost all cross sections, the whole cross section was under compression except a few cross sections, where strain of some reinforcing bars was slightly positive because the bars were crossed by some cracks as discussed in next section.

Figure 5.19 Strain of longitudinal reinforcement on the cross-sections of column

Chapter 5 Experimental Results

Figure 5.20 Strain of longitudinal reinforcement on cross-sections of column U90

5.2.4 Crack development and failure mode

The crack pattern at displacement of 9Δ (drift ratio of 0.9%), where column U0 was assumed to lose its shear capacity, is given in Figure 5.21. The crack patterns of side 1 and side 2 were comparable to those of side 3 and side 4, respectively. On side 1 and side 3, the first diagonal cracks appeared when shear force was 450kN in the first cycle toward displacement 4Δ (drift ratio of 0.4%) in both pull and push directions. More diagonal cracks took place as displacement level was increased. Diagonal cracks had relatively similar crack width. As crack spacing was small, diagonal cracks could be considered smeared. Concrete started spalling after shear force reached maximum value. Cracks in pull and push directions were symmetrical and diagonal crack angle were about 25° with respect to axis of the column. On sides 2 and 4, horizontal flexural shear cracks appeared at locations away from two ends of columns and vertical cracks developed in the middle of column when shear force reached maximum value. There were very few cracks on these sides even at failure of the column. The diagonal crack orientation demonstrates that diagonally compressive force could be transmitted from top to bottom of the column and shear resistance of the column involved arch mechanism. The column reached maximum shear force at crushing of diagonally compressed concrete after yielding of shear reinforcement. The mode of failure is shear yielding compression (SYCF).

As seen in Figure 5.21, crushing on side 1 and side 3 initiated at location 200mm away from bottom of the column. As bending moment increases from zero at section at the middle of the column to maximum value at sections at the ends, principal compressive stress is greatest at the ends of the column. However, because of high confinement of stubs, effective compressive strength of concrete is smallest at the middle of the column. Thus, crushing initiated at an intermediate location within shear span, where principal compressive stress was relatively high and effective compressive strength of concrete was relatively low.

Figure 5.22 shows cracks of column U90 at displacement 12Δ (drift ratio of 1.2%). The integers marked in this figure present the development of cracks at displacement 6Δ , 11Δ and 12Δ . Crack pattern on the opposite sides of the column was similar. On sides 1 and 3, in the first cycle to displacement 6Δ (drift ratio of

Chapter 5 Experimental Results

0.6%), the joint between the column and its stubs started to open when pulling and pushing forces were +360kN and -320kN, respectively. In the second cycle, horizontal flexural shear crack first appeared near two ends of the column when pulling force was 320kN. On sides 2 and 4, first diagonal cracks appeared when shear strength reached peak value, at displacement 6Δ . Diagonal crack development of column U90 is clearly shown in Figure 5.23. At displacement 6Δ , there were very few largely spaced diagonal cracks so that they were considered localised. As displacement of the column was increased, the first cracks became major cracks while new cracks were visible. At displacement as large as 11Δ , crack spacing became small and diagonal cracks could be considered as smeared. The increase of number of diagonal cracks and crack width took place rapidly in the first cycle at displacement level 11Δ . In this cycle, more horizontal cracks, near the middle of the column, and vertical cracks appeared on sides 1 and 3.

The major diagonal crack angles in pull or push direction was about 25° at the middle of the column and decreased near two ends of column. After first diagonal cracking at displacement 6Δ , when displacement was further increased, brittle shear sliding occurred along the major cracks and shear resistance of the column reduced sharply. Shear reinforcement yielded in the post peak range, when diagonal cracks were extensively opened. The column failed in shear tension (STF) mode and shear strength corresponded to cracking strength.

In unidirectional shear test (Verma, 1993; Priestley et al., 1994; Sezen, 2002) of short columns with shear span ratio equal or more than two, initial flexural shear crack appeared at both ends of the columns. At higher displacement, the flexural shear crack became steeper and shear cracks took place in the middle of the columns. This development of crack was not observed in columns U0 and U90. Flexural cracks did not occur at the ends of the columns, where compression depth covered the whole cross section. Horizontal cracks took place at distances from the ends of the columns because of flexure and shear. The horizontal cracks appeared at the middle of column were due to shear because bending moment at this section was small. It is widely known that shear force causes tension in longitudinal reinforcement, which then transmits tension to its surrounding concrete. The diagonal compression field model (Nielsen, 1999; Collins et al., 1996) can explain

Chapter 5 Experimental Results

shear induced tension in addition to flexural tension in longitudinal direction. At section at the ends of the columns, because diagonal compression strut could transfer shear force to supports, shear induced tension was negligible. As shear span ratios of columns U0 and U90 were equal to 0.875 and 1.75, which were very short, shear behaviour was more significant compared to flexural behaviour. Longitudinal tensile stress in concrete at section at the ends of the columns was less than that caused by both shear and flexure at other sections. This could be the reason why horizontal cracks occurred at distances from both ends of the columns.

Also, because of short shear span ratio, there was no clear distinction between the regions at the ends and at the middle of the columns. After initiation of diagonal crack, cracks could appear over the whole length of columns U0 and U90.

The distance between two legs of stirrups is a factor that affects behaviour of columns. To assure RC members will not fail at cracking load in sudden way, minimum shear reinforcement is proposed (ACI-318M-02 and CSA-A23.3-94). However, column U90 failed at first diagonal cracking though its shear reinforcement ratio is more than the minimum value. Because the distance between two legs of stirrups was large, shear reinforcement did not fully participate in shear resisting mechanism and it did not yield at maximum shear force. The crack pattern on sides 1 and 3 of column U90 shows a tendency of punching of concrete in the middle of the column due to diagonal compression. Thus, though shear reinforcement ratio is more than the minimum value, premature failure of column U90 occurred because of large distance between two legs of stirrups. Anderson and Ramirez (1989) also found that this factor makes shear capacity of beams lower.

Figure 5.24 shows the crack pattern when column B30 lost its shear capacity. The first diagonal crack appeared on sides 1 and 3 right before peak load, when pulling force was 450kN and displacement was 3.5mm (drift ratio of 0.5%). On these sides, at displacement 3Δ (drift ratio of 0.6%), diagonal cracks were rather well distributed with small crack spacing. On sides 2 and 4, diagonal cracks first occurred at peak load when displacement was 3Δ . As displacement increased, more cracks appeared on all sides of the column and some cracks opened largely.

After transverse reinforcement yielded, column B30 reached maximum shear force at crushing of concrete when displacement was 3Δ . In the second cycle at the

Chapter 5 Experimental Results

same displacement, shear strength decreased more than 20%. Column B30 failed in shear yielding compression (SYCF) mode in brittle manner.

The crack pattern of column B60 at displacement of 8Δ (drift ratio of 0.8%) was given in Figure 5.25. The first cracks occurred on all sides of the column at displacement 5Δ (drift ratio of 0.5%) in pulling direction, when the column attained maximum shear force. When displacement was further increased, more cracks were formed. On longer sides (sides 1 and 3) of the column, cracks were localised even when column has lost its shear carrying capacity. The crack pattern on surface of the column indicates some major crack surfaces. Maximum shear force was reached at first diagonal cracking. Shear reinforcement yielded right after peak load. As cracking became more severe, sliding occurred along the major cracks and column B60 failed in shear tension (STF) mode.

In column B30 and B60, concrete started spalling after shear force reached maximum value. The development of cracks on shorter sides (sides 2 and 4) of the columns was similar to that of column U90. At low displacement, cracks were localised with large crack spacing. More cracks were visible on shorter sides of the columns and crack spacing became small, as displacement became large.

For columns under unidirectional shear, U0 and U90, diagonal crack surfaces were planes passing through cross section of the column. These planes were perpendicular to one principal plane of cross section of columns. In contrast, in columns under bi-directional shear, B30 and B60, each crack surface was not a plane but have irregular shape and varying orientations over cross section of the columns. Diagonal crack angles on all sides of columns B30 and B60 were tabulated in Table 5-2 and Table 5-3.

Chapter 5 Experimental Results

Figure 5.21 Crack pattern of column U0 at displacement 9Δ (drift ratio of 0.9%)

Chapter 5 Experimental Results

Figure 5.22 Crack pattern of column U90 at displacement 12 Δ (drift ratio of 1.2%)

Figure 5.23 Cracks on side 4 of column U90 at displacement 12Δ (drift ratio of 1.2%)

Chapter 5 Experimental Results

Side 1

Side 2

Side 3

Side 4

Figure 5.24 Crack pattern of column B30 when it totally lost its shear capacity

Chapter 5 Experimental Results

Figure 5.25 Crack pattern of column B60 at displacement 8Δ (drift ratio of 0.8%)

Table 5-2 Diagonal crack angles of column B30

Load direction	Side1	Side2	Side3	Side4
Pulling	30 ⁰	20 ⁰	30 ⁰	10 ⁰
Pushing	35 ⁰	20 ⁰	30 ⁰	30 ⁰

Table 5-3 Diagonal crack angles of column B60

Load direction	Side1	Side2	Side3	Side4
Pulling	20 ⁰	15 ⁰	40 ⁰	15 ⁰
Pushing	40 ⁰	25 ⁰	20 ⁰	15 ⁰

5.2.5 Behaviour of the columns in two principal directions

Curvature of columns U0 and U90 is plotted in Figure 5.26 and Figure 5.27. It is shown that curvature distribution is symmetrical. Rotations of the top stub of column U0 in plane of loading and the orthogonal plane are given in Figure 5.28. Obviously, the rotation is small, i.e. the test set up is able to simulate double curvature bending. As stiffness of column U0 and B30 was higher than other two columns, their top stubs underwent certain deformation with cracking as shown in Figure 5.29. This suggests that the boundary condition of these two columns was not exactly like the double curvature bending.

Generally, columns U0, U90, B30 and B60 could undergo deformations in loading direction and out of loading plane. These deformations are plotted against each other in Figure 5.30 to Figure 5.33. As seen in Figure 5.30 and Figure 5.31, columns U0 and U90 mainly deform in a principal direction, deformation in other principal direction is negligible. On the other hand, Figure 5.32 and Figure 5.33 show that columns B30 and B60 deform in two orthogonal directions.

The resultant deformation of columns B30 and B60 and its components are illustrated in Figure 5.34. During cyclic loading, displacement in loading direction was increased to the next level after three reversals. Angle between resultant displacement and loading direction of a column at maximum displacement in the first cycle could be representative angle for each level of displacement. This angle

Chapter 5 Experimental Results

is plotted versus displacement levels for columns B30 and B60 in Figure 5.35. The average angles for various displacement levels are 30° and 25° for columns B30 and B60, respectively. These angles are shown in Figure 5.32, Figure 5.33 and Figure 5.34. Thus, columns B30 and B60 moved in directions forming angles about 30° and 25° , respectively, to the loading direction.

Shear force applied to columns B30 and B60 could be resolved in two principal directions. Relationship of load-displacement in each principal direction of columns B30 and B60 is plotted in Figure 5.36 and Figure 5.37. Load-displacement envelopes in two principal directions of the four columns are plotted in Figure 5.38. The figure shows the interaction of behaviour of columns B30 and B60 in two principal directions. Displacement in weak direction does not affect behaviour of the two columns in strong direction unless it exceeds the displacement corresponding to maximum shear strength of column U90. This is similar to test result of square columns of Maruyama et al. (1984).

In contrast, displacement in strong direction, even smaller than that corresponding to maximum shear strength of column U0, could reduce stiffness of the column in weak direction. The difference in how displacement in one principal direction affects behaviour of the columns in another principal direction could be due to the difference in degree of damage. Significant damage could occur at small deformation in strong direction while damage only becomes noticeable beyond displacement at peak in weak direction as indicated by crack development of column U0 and column U90, respectively.

Based on measurement of displacement transducers attached to surface of columns, flexural and shear deformations of columns could be calculated according to Verma (1993). Deformation of columns is summation of shear and flexural deformations. Calculated deformation of columns U0 and U90 is drawn against experimental value in Figure 5.39 and Figure 5.40. As seen from Figure 5.39, shear deformation is a significant part of total deformation of column U0. This part increases with the total deformation. In contrast, Figure 5.40 shows that flexural deformation is the major part of total deformation of column U90. Though shear deformation increases with total deformation, it only becomes large near the failure of column U90. Calculated deformation of column U90 is a good match of

Chapter 5 Experimental Results

experimental value while that does not hold for column U0. One reason could be that the Bernoulli's assumption of plane section, which is used to estimate flexural deformation, is not suitable for columns under high shear.

In each principal direction, total and shear deformation of columns B30 and B60 are calculated. The ratio between shear deformation to total deformation in each principal direction of these columns is plotted in Figure 5.41 and Figure 5.42. The figures indicate a general trend. Shear deformation increases with total deformation. Shear deformation is a major part of total deformation in the strong direction of columns but is minor part of total deformation in the weak direction of columns. Therefore, as shear span decreases shear deformation is more significant.

Figure 5.26 Curvature distribution of column U0

Chapter 5 Experimental Results

Figure 5.27 Curvature distribution of column U90

Figure 5.28 Rotations of top stub of column U0 in two orthogonal planes

Chapter 5 Experimental Results

Figure 5.29 Cracking of top stub of columns U0 and B30

Figure 5.30 Deformations of column U0 in two orthogonal directions

Chapter 5 Experimental Results

Figure 5.31 Deformations of column U90 in two orthogonal directions

Figure 5.32 Deformations of column B30 in two orthogonal directions

Figure 5.33 Deformations of column B60 in two orthogonal directions

Chapter 5 Experimental Results

Figure 5.34 Resultant displacement of columns B30 and B60 and its components

Figure 5.35 Angle between resultant displacement and loading direction of columns B30 and B60 at various displacement levels

Chapter 5 Experimental Results

Figure 5.36 Load-displacement hysteretic curves of column B30 in two principal directions

Chapter 5 Experimental Results

Figure 5.37 Load-displacement hysteretic curves of column B60 in two principal directions

Chapter 5 Experimental Results

Figure 5.38 Load-displacement envelopes in two principal directions of columns U0, U90, B30 and B60

Figure 5.39 Deformation of column U0

Chapter 5 Experimental Results

Figure 5.40 Deformation of column U90

Figure 5.41 Ratio between shear deformation and total deformation of column B30

Figure 5.42 Ratio between shear deformation and total deformation of column B60

5.2.6 Net displacement of the columns

At the intersection between column and bottom stub, there were local deformations. Columns may immerse into bottom stub on one side and crack may open on the opposite side. These local deformations caused rotation at the base of columns. This rotation can be computed from readings of four transducers numbered 19-22 positioned as shown in the last chapter. Displacement at top of columns due to rotation at base is equal to the rotation multiplied by column's height. Subsequently, net displacement of column is equal to total displacement of column minuses displacement due to rotation at base.

Figure 5.43 and Figure 5.44 show load-net displacement curves of columns U0 and U90. In Figure 5.43, load-net displacement of column U0 is plotted up to failure, where maximum shear force decreased 20%. The response beyond failure is not of interest.

Net displacement of columns B30 and B60 in a principal direction is equal to total displacement in that direction subtracts the corresponding displacement due to rotation at base. Figure 5.45 and Figure 5.46 show relationship of load-net displacement in each principal direction of columns B30 and B60. Net displacement in two principal axes of columns B30 and B60 is plotted against each other in Figure 5.47 and Figure 5.48.

The resultant net displacement and its components in two principal directions of columns B30 and B60 are illustrated in Figure 5.49. Like the case of resultant of total displacement, angle between resultant net displacement and weak direction of a column at maximum displacement in the first cycle could be representative for each level of displacement. This angle is plotted in Figure 5.50 versus displacement levels for columns B30 and B60. The average angles for various displacement levels are 41° and 13° for columns B30 and B60, respectively. These angles are shown in Figure 5.47, Figure 5.48 and Figure 5.49.

Chapter 5 Experimental Results

Figure 5.43 Load-net displacement hysteretic curve of column U0

Figure 5.44 Load-net displacement hysteretic curve of column U90

Chapter 5 Experimental Results

Figure 5.45 Load-net displacement hysteretic curves of column B30 in two principal directions

Chapter 5 Experimental Results

Figure 5.46 Load-net displacement hysteretic curves of column B60 in two principal directions

Figure 5.47 Net displacement of column B30 in two principal directions

Chapter 5 Experimental Results

Figure 5.48 Net displacement of column B60 in two principal directions

Figure 5.49 Resultant net displacement of columns B30 and B60 and its components

Figure 5.50 Angle between resultant net displacement and weak direction of columns B30 and B60 at various displacement levels

5.2.7 Measured axial compression

Axial load applied on column U0, U90, B30 and B60 is plotted versus displacement in loading direction in Figure 5.51 to Figure 5.54. The figures disclose that axial compression reduced when loading proceeded. Axial load varied between 5% greater and 15% smaller the desire value of 2000kN. The reduction of compression was due to shortening of the columns. Except for column U0, axial compression was about 1920kN when the columns reached maximum shear strength. Because pre-stress was reapplied on column U0 as described previously, when it attained maximum shear strength, axial load was about 2000kN.

Figure 5.51 Axial load-displacement in loading direction of columns U0

Figure 5.52 Axial load-displacement in loading direction of columns U90

Figure 5.53 Axial load-displacement in loading direction of columns B30

Figure 5.54 Axial load-displacement in loading direction of columns B60

5.3 Columns UT0, UT90, BT30 and BT60 under shear combined with torsion

5.3.1 Hysteretic behaviour

The four columns were loaded by cyclic lateral force in displacement control mode with displacement increment of 0.7mm. Load-displacement hysteretic loop of column UT0 is shown in Figure 5.55. The column attained maximum shear of 501kN at displacement 3.5mm (drift ratio of 0.5%) in pulling direction. At displacement 3.5mm in pushing direction, shear force was 475kN, which was less than the peak value in pulling direction. Shear force decreased when displacement increased further. Testing was stopped at displacement 4.9mm (drift ratio of 0.7%) when shear strength reduced more than 20%.

Load-displacement hysteretic loop of column UT90 is given in Figure 5.56. Maximum shear of 348kN was reached at displacement 4.2mm (drift ratio of 0.6%) in pulling direction. At displacement 4.2mm in pushing direction, shear force was 282kN. Beyond this displacement level, shear strength decreased. Testing was

Chapter 5 Experimental Results

stopped at displacement 5.6mm (drift ratio of 0.8%) when shear force was less than 80% of maximum shear force.

Figure 5.57 shows load-displacement hysteretic loop of column BT30. At displacement 3.5mm (drift ratio of 0.5%) in pulling direction, the column attained maximum shear of 446kN. At displacement 3.5mm in pushing direction, shear force was 432kN. In the first cycle at displacement 4.9mm (drift ratio of 0.7%) in pushing direction, diagonal crack quickly opened, displacement increased rapidly with a substantial reduction in shear force and testing was stopped.

Figure 5.58 presents load-displacement hysteretic loop of column BT60. At displacement 3.5mm (drift ratio of 0.5%) in pulling direction, the column attained maximum shear of 351kN. At the same displacement in pushing direction, shear force was 343kN. Testing was stopped at displacement 5.6mm (drift ratio of 0.8%) when shear strength reduced more than 20%.

Figure 5.55 to Figure 5.58 show that these columns reached shear capacity and failed at very limited drift ratios. The columns behaved in the same manner in pull and push directions. However, at the same displacement level, pulling and pushing forces were slightly unequal. Peak loads at the same displacement level decreased from the first cycle to the third cycle. Shear failure of columns was exhibited by pinching effect. First diagonal cracking and yielding of transverse reinforcement are also marked in the hysteretic curves of the four columns in Figure 5.55 to Figure 5.58.

Because of intentional eccentricity, shear was accompanied by torsion. The torque was not exactly equal to shear force times the given eccentricity. During the test, the columns underwent lateral displacement and twist, which made the axis of the hydraulic jack formed an angle to its original axis (Figure 5.59). The applied load could be resolved into one load acting on the designated loading axis and another load in the perpendicular direction. The latter resulted in a counter torque to that created by the former in case of pulling force or an adding torque in case of pushing force. Hence, with the same shear force the actual torque applied to columns in pushing direction was greater than that in pulling one. This was more obvious after the columns obtained maximum shear. The unequal torque applied to columns may explain the unsymmetrical part of the load-displacement curve,

torque-twist curve and several phenomenons. The torque-twist curves of columns UT0, UT90, BT30 and BT60 are given in Figure 5.60 to Figure 5.63.

Figure 5.55 Load-displacement hysteretic curve of column UT0

Figure 5.56 Load-displacement hysteretic curve of column UT90

Chapter 5 Experimental Results

Figure 5.57 Load-displacement hysteretic curve of column BT30

Figure 5.58 Load-displacement hysteretic curve of column BT60

Chapter 5 Experimental Results

Figure 5.59 Top view of shear test with torsion

Figure 5.60 Torque-twist curve of column UT0

Figure 5.61 Torque-twist curve of column UT90

Figure 5.62 Torque-twist curve of column BT30

Figure 5.63 Torque-twist curve of column BT60

5.3.2 Strain in transverse reinforcement

Figure 5.64 to Figure 5.67 show strain in transverse reinforcement located at bottom half of columns UT0, UT90, BT30 and BT60. Strain at various displacement levels of the columns in the first cycle is plotted. The figures show that strain in shear reinforcement was relatively low before the formation of the first diagonal crack. Strain in one strain gauge attached to transverse reinforcement of columns UT0, UT90, BT30 and BT60 is given in Figure 5.68 to Figure 5.71.

As seen in Figure 5.64 and Figure 5.68, in column UT0, transverse reinforcement first yielded in the first cycle at displacement 2.1mm (drift ratio of 0.3%) in pushing direction. Strain of transverse reinforcement at location 200mm away from the base of the column was largest along the height of the column. This indicates large crack width at this location.

Figure 5.65 and Figure 5.69 reveals that transverse reinforcement of column UT90 first yielded in the first cycle at displacement 4.2mm (drift ratio of 0.6%) in pushing direction.

From Figure 5.66 and Figure 5.70, first yielding of transverse reinforcement of column BT30 was found in first cycle at displacement 2.8mm (drift ratio of 0.4%) in pulling direction. First yielding of transverse reinforcement of column BT60 took place in first cycle at displacement 3.5mm (drift ratio of 0.5%) in pushing direction as seen in Figure 5.67 and Figure 5.71.

Thus, transverse reinforcement of column UT0, BT30 yielded before the columns attained maximum shear and transverse reinforcement of columns UT90, BT60 yielded after peak load.

The displacement of the columns at first yielding of transverse reinforcement is compared with that of the corresponding columns under shear force in Table 5-4. The table indicates that with the presence of torsion, shear reinforcement yielded at lower displacement. First yielding of transverse reinforcement of the four columns U0, U90, B30 and B60 took place in pulling direction while it happened in columns UT0, UT90 and BT60 in pushing direction. Higher torsion in pushing direction is likely the cause of this difference.

Chapter 5 Experimental Results

Figure 5.64 Strain of transverse reinforcement of column UT0 at various displacement levels in pushing direction

Figure 5.65 Strain of transverse reinforcement of column UT90 at various displacement levels in pushing direction

Chapter 5 Experimental Results

Figure 5.66 Strain of transverse reinforcement of column BT30 at various displacement levels in pulling direction

Figure 5.67 Strain of transverse reinforcement of column BT60 at various displacement levels in pushing direction

Figure 5.68 Strain in strain gauge X1 attached to transverse reinforcement of column UT0

Figure 5.69 Strain in strain gauge Y2 attached to transverse reinforcement of column UT90

Figure 5.70 Strain in strain gauge W3 attached to transverse reinforcement of column BT30

Figure 5.71 Strain in strain gauge W2 attached to transverse reinforcement of column BT60

Table 5-4 Displacement at yielding of transverse reinforcement

Loading	Shear				Shear combined with torsion			
Column	U0	U90	B30	B60	UT0	UT90	BT30	BT60
Disp. (mm)	2.8	8.2	4.8	3.5	2.1	4.2	2.8	3.5
Drift ratio	0.4%	1.2%	0.7%	0.5%	0.3%	0.6%	0.4%	0.5%

5.3.3 Strain in longitudinal reinforcement

Strain of main reinforcement bar at a corner of columns UT0, UT90, BT30 and BT60 is shown in Figure 5.72 to Figure 5.75 at different peak displacement. Strain of longitudinal reinforcement of these four columns did not reach yielding strain at all. Theoretically, torsion creates an additional tensile stress in longitudinal reinforcement. This causes a constant tensile strain along the length of a rebar. Strain in longitudinal reinforcement of these columns is equal to tensile strain caused by shear force plus that caused by torsion. As tensile strain resulted from torsion is a constant along the length of a rebar, the shape of tensile strain distribution in main reinforcement of columns subjected to shear combined with torsion is similar to that of corresponding columns subjected to shear force alone.

Figure 5.72 Strain of longitudinal reinforcement at a corner of column UT0

Chapter 5 Experimental Results

Figure 5.73 Strain of longitudinal reinforcement at a corner of column UT90

Figure 5.74 Strain of longitudinal reinforcement at a corner of column BT30

Figure 5.75 Strain of longitudinal reinforcement at a corner of column BT60

5.3.4 Crack development and failure mode

The crack pattern of column UT0 at displacement of 7Δ (drift ratio of 0.7%) is given in Figure 5.76. First diagonal cracks appeared on side 1 in the first cycle toward displacement 3Δ (drift ratio of 0.3%) in pushing direction when loading was 331kN. At displacement 4Δ , smeared crack was developed with small crack spacing. Concrete started spalling on side 1 when column attained maximum shear force. Diagonal cracks first occurred on side 3 in the third cycle at displacement 3Δ . As seen in Figure 5.76, at displacement 3Δ , cracks on side 3 were localised with large spacing. New cracks developed as displacement of the column increased. Cracks only became smeared when displacement was 6Δ . Generally, cracks on side 1 were more extensive than on side 3. On side 2 and 4, very few diagonal cracks first occurred at displacement 5Δ . At displacement was as large as 7Δ , when a significant number of new cracks took place, crack distribution became uniform with small crack spacing. On sides 1, 2 and 4, cracks in pushing direction were more severe than cracks in pulling direction due to the higher torque in pushing direction as explained before. Transverse reinforcement of the column yielded before peak load when diagonal cracks opened widely. After that, effective compressive strength of concrete decreased and column failed because of concrete crushing. Like column U0, location of crushing is on side 1 at a distance away from the end of column UT0 as seen in Figure 5.76. The mode of failure of the column is shear yielding compression.

Figure 5.77 shows the crack pattern of column UT90 at displacement of 8Δ (drift ratio of 0.8%). First cracks appeared on side 2, 3, and 4 in the first cycle at displacement 4.2mm in pulling direction when shear force was 330kN. At displacement 8Δ , more diagonal cracks were observed on side 4 than on side 2. Cracks were denser on side 4 than on side 2. On other two sides, several diagonal cracks appeared at large displacement. Some flexural cracks took place on side 1 at large displacement. The severity of cracks in pushing direction over those in pulling direction was not seen. This could be due to torsion in both directions were almost equal as seen in Figure 5.61. Shear capacity of the column was attained at first

Chapter 5 Experimental Results

diagonal crack followed by yielding of transverse reinforcement. The column failed in shear tension mode.

The crack pattern of column BT30 at displacement of 7Δ (drift ratio of 0.7%) is shown in Figure 5.78. The first crack appeared on side 1 in the first cycle at displacement 2.1mm in pushing direction. Transverse reinforcement yielded before the column reached maximum shear capacity, which corresponded to crushing of concrete. Column failed in shear yielding compression mode.

Figure 5.79 indicates crack pattern of column BT60 at displacement 8Δ (drift ratio of 0.8%). First diagonal cracks were visible at displacement 3.5mm, when maximum shear force was reached. Transverse reinforcement yielded after peak load, at displacement 3.5mm in pushing direction. As displacement increased further diagonal cracks developed extensively, shear slip with sliding surface passing through a corner of column took place (see side 2 in Figure 5.79). The failure mode of the column was shear tension.

For columns BT30 and BT60, cracking on side 1 and 2 was more severe with smaller crack spacing than that on the other two sides. At low displacement of the columns, crack spacing on sides 3 and 4 was large. It became small when displacement of the columns was large. On side 1 and 2, cracks in pushing direction were slightly dominant over those in pulling direction. Spalling started at displacement 4.2mm (drift ratio of 0.6%).

Unlike columns U0 and U90, diagonal crack surfaces of columns UT0 and UT90 were no longer flat planes passing through cross section of the columns. Diagonal crack angles of column UT0, UT90, BT30, and BT60 are given in Table 5-5 to Table 5-8.

Chapter 5 Experimental Results

Figure 5.76 Crack pattern of column UT0 at displacement 7Δ (drift ratio of 0.7%)

Chapter 5 Experimental Results

Figure 5.77 Crack pattern of column UT90 at displacement 8Δ (drift ratio of 0.8%)

Chapter 5 Experimental Results

Figure 5.78 Crack pattern of column BT30 at displacement 7Δ (drift ratio of 0.7%)

Chapter 5 Experimental Results

Figure 5.79 Crack pattern of column BT60 at displacement 8Δ (drift ratio of 0.8%)

Chapter 5 Experimental Results

Table 5-5 Diagonal crack angles of column UT0

Load direction	Side1	Side2	Side3	Side4
Pulling	25 ⁰	90 ⁰	25 ⁰	25 ⁰
Pushing	30 ⁰	30 ⁰	20 ⁰	30 ⁰

Table 5-6 Diagonal crack angles of column UT90

Load direction	Side1	Side2	Side3	Side4
Pulling	30 ⁰	20 ⁰	20 ⁰	25 ⁰
Pushing	20 ⁰	25 ⁰	30 ⁰	25 ⁰

Table 5-7 Diagonal crack angles of column BT30

Load direction	Side1	Side2	Side3	Side4
Pulling	30 ⁰	30 ⁰	10 ⁰	25 ⁰
Pushing	30 ⁰	10 ⁰	0 ⁰	90 ⁰

Table 5-8 Diagonal crack angles of column BT60

Load direction	Side1	Side2	Side3	Side4
Pulling	20 ⁰	10 ⁰	0 ⁰	20 ⁰
Pushing	25 ⁰	20 ⁰	0 ⁰	10 ⁰

5.3.5 Behaviour of the columns in two principal directions

Deformations of columns UT0, UT90, BT30 and BT60 in two principal directions are plotted against each other in Figure 5.80 to Figure 5.83. As seen in Figure 5.80 and Figure 5.81, columns UT0 and UT90 mainly deform in a principal direction, deformation in other principal direction is negligible. On the other hand, Figure 5.82 and Figure 5.83 show that columns BT30 and BT60 deform in two orthogonal directions.

Angle between resultant displacement and loading direction of column BT30 and BT60 at maximum displacement in the first cycle is plotted in Figure 5.84 versus the displacement. The average angles at various displacements are 33⁰ and 22⁰ for columns BT30 and BT60, respectively. These angles are shown in Figure

Chapter 5 Experimental Results

5.82 and Figure 5.83. The resultant deformation of columns BT30 and BT60 and its components are illustrated in Figure 5.85.

Shear force applied to these columns could be resolved in two principal directions. Relationship of load-displacement in each principal direction of columns BT30 and BT60 is plotted in Figure 5.86 and Figure 5.87. Load-displacement envelopes in two principal directions of the four columns are plotted in Figure 5.88. The figure shows that the effect of shear force in one principal direction to behaviour of the columns in the other principal direction is similar to the cases of columns B30 and B60. Displacement in weak direction does not affect behaviour of the two columns in strong direction unless it exceeds the displacement corresponding to unidirectional shear capacity. In contrast, displacement in strong direction, even smaller than that corresponding to unidirectional shear capacity, could reduce stiffness of the column in weak direction.

Figure 5.80 Deformations of column UT0 in two principal directions

Chapter 5 Experimental Results

Figure 5.81 Deformations of column UT90 in two principal directions

Figure 5.82 Deformations of column BT30 in two orthogonal directions

Figure 5.83 Deformations of column BT60 in two orthogonal directions

Chapter 5 Experimental Results

Figure 5.84 Angle between resultant displacement and loading direction of columns BT30 and BT60 at various displacement levels

Figure 5.85 Resultant displacement of columns BT30 and BT60 and its components

Chapter 5 Experimental Results

Figure 5.86 Load-displacement hysteretic curves of column BT30 in two principal directions

Chapter 5 Experimental Results

Figure 5.87 Load-displacement hysteretic curves of column BT60 in two principal directions

Chapter 5 Experimental Results

Figure 5.88 Load-displacement envelopes in two principal directions of columns UT0, UT90, BT30 and BT60

5.3.6 Net displacement of columns

The same method used to compute net displacement of columns under shear force is used for columns under shear combined with torsion. Figure 5.89 and Figure 5.90 show load-net displacement curves of columns UT0 and UT90. Relationship of load-net displacement in each principal direction of columns BT30 and BT60 is plotted in Figure 5.91 and Figure 5.92, respectively.

Net displacement in two principal axes of columns BT30 and BT60 is plotted against each other in Figure 5.93 and Figure 5.94. The resultant net displacement and its components in two principal directions of columns B30 and B60 are illustrated in Figure 5.95. Angle between resultant net displacement and weak direction of a column at maximum displacement in the first cycle could be representative for each level of displacement. This angle is plotted in Figure 5.96 versus displacement levels for columns BT30 and BT60. The average angles are 27°

Chapter 5 Experimental Results

and 12^0 for columns BT30 and BT60, respectively. These angles are shown in Figure 5.93, Figure 5.94 and Figure 5.95.

Figure 5.89 Load-net displacement hysteretic curve of column UT0

Figure 5.90 Load-net displacement hysteretic curve of column UT90

Chapter 5 Experimental Results

Figure 5.91 Load-net displacement hysteretic curves of column BT30 in two principal directions

Chapter 5 Experimental Results

Figure 5.92 Load-net displacement hysteretic curves of column BT60 in two principal directions

Figure 5.93 Net displacement of column BT30 in two principal directions

Chapter 5 Experimental Results

Figure 5.94 Net displacement of column BT60 in two principal directions

Figure 5.95 Resultant net displacement of columns BT30 and BT60 and its components

Figure 5.96 Angle between resultant net displacement and weak direction of columns BT30 and BT60 at various displacement levels

5.3.7 Measured axial compression

Axial load applied on column UT0, UT90, BT30 and BT60 is plotted versus displacement in loading direction in Figure 5.97 to Figure 5.100. As seen in these figures, axial compression reduced when loading proceeded. Compared to the desire value of 2000kN, axial compression was about 5% greater initially and about 15% smaller at the end of the test. Axial compression reduced because of shortening of the columns.

Figure 5.97 Axial load-displacement in loading direction of columns UT0

Figure 5.98 Axial load-displacement in loading direction of columns UT90

Chapter 5 Experimental Results

Figure 5.99 Axial load-displacement in loading direction of columns BT30

Figure 5.100 Axial load-displacement in loading direction of columns BT60

4 Column T

The torque-twist relationship of column T is presented in Figure 5.101. The response is symmetrical and shows pinching effect. The first cracks took place on side 1 and side 3 in both directions of loading at a torque about 60kNm with the corresponding twist of about 5×10^{-3} radian. Before the occurrence of the first crack, column behaved almost elastically. Generally, a horizontal plateau may exist at racking torque because of a delay in mobilizing stirrups. However, this plateau was not seen in case of column T. After one cycle at twist angle 5×10^{-3} radian, the column was loaded in displacement control mode with increment of twist angle $\theta = 1 \times 10^{-3}$ radian. Torsion capacity of 70.8kNm was reached at twist of 10×10^{-3} radian. This value is significantly higher than torsional strength estimated in the last chapter based on spaced truss analogy. Concrete spalling occurred in the first cycle at a twist angle of 20×10^{-3} radian. Severe damage was found in the middle of column,

Chapter 5 Experimental Results

where concrete at the four corners was about to fall down at a twist of 30×10^{-3} radian. At this point, maximum torque decreased more than 20%.

Figure 5.60 to Figure 5.63 in comparison to Figure 5.101, show that shear force combined with torsion reduces initial torsional stiffness and torsional strength of columns.

The distribution of cracks of column T is depicted in Figure 5.102. The numbers marked in this figure present the development of cracks at twist angles $26^\circ, 36^\circ$, so on. Diagonal cracks first appeared on sides 2 and 4 at twist 26° . The number of intersecting cracks and the crack width increased with the twist of the column. Diagonal cracks were formed on all faces of columns with crack angle about 30° . Vertical cracking appeared near the two ends of column on side 1 and side 3 at twist angle 46° .

The cracks on all sides of the column were well distributed and crack spacing was small. Unlike slender members, where cracks on adjacent faces are continuous helices, cracks of short column are confined within individual face. The crack orientation on the shorter faces shows truss action. Crack orientation on the longer faces clearly shows arch action because diagonally compressive force could be transmitted from top to bottom of the column.

Figure 5.103 plots strain in long and short legs of stirrups. As seen in the figure, strain of transverse reinforcement at ultimate load was close to yield strain. It reached yield strain after peak load at a twist angle of 15×10^{-3} radian. Yielding of transverse reinforcement was observed at the same time on both short and long sides of the column. Along the height of column, strain of the stirrup at the middle of column is the largest. Column is confined at two ends by the stubs and damage is concentrated in its middle part. The yielding of transverse reinforcement after peak load demonstrates that the column failed because of diagonal compression. First cracking and yielding of transverse reinforcement are marked in Figure 5.101.

Strain in longitudinal reinforcement was well below yield value as shown in Figure 5.104. Torsion resulted in additional tension, which slightly reduced compression in longitudinal reinforcement. Figure 5.105 shows axial compression versus twist of column T. Initial axial load was 5% greater than 2000kN. When

Chapter 5 Experimental Results

loading proceeded, the column was shortened and axial compression decreased. It was 10% smaller than 2000kN at the end of loading process.

Figure 5.101 Torque-twist curve of column T

Chapter 5 Experimental Results

Side 1

Side 2

Side 3

Side 4

Figure 5.102 Crack pattern of column T at failure

Chapter 5 Experimental Results

a) Strain in long leg of stirrups

b) Strain in short leg of stirrups

Figure 5.103 Strain in long and short legs of stirrups of column T

Figure 5.104 Strain of the main reinforcement at corners of column T

Figure 5.105 Axial load versus twist angle of column T

5.5 Summary and discussions

5.5.1 Load-displacement response

The envelopes of the load-displacement curves in the pulling direction of eight columns tested under shear force with or without torsion are given in Figure 5.106. On the ascending branch, the responses of columns U0 and U90 are the upper and lower bounds of the response of columns under shear force without torsion. Similarly, those of columns UT0 and UT90 are the upper and lower bounds of the response of columns under shear combined with torsion.

Comparison of columns under shear force and their counterparts under shear force combined with torsion reveals that torsion reduces shear strength and stiffness of column. On the ascending branch, as shear force increases, the reduction of shear strength and stiffness due to torsion is obvious. This reduction is even more pronounced on descending branch for columns under unidirectional shear, i.e. torsion accelerates strength and stiffness degradation. However, for those under bi-directional shear, the descending branch is nearly unaffected.

Figure 5.106 reveals that the displacement in loading direction of the columns at maximum shear force is almost insensitive to torsion. However, torsion affects the displacement at failure of some columns. Columns UT0 and UT90 failed at much lower drift ratios compared to columns U0 and U90, respectively. The drift ratios of columns BT30 and BT60 at failure are nearly equal to those of column B30 and B60, respectively. Hence, torsion reduces ductility of columns under unidirectional shear but does not seem to affect that of columns under bi-directional

Figure 5.105 Axial load versus twist angle of column T

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Figure 5.106 Envelopes of the load-displacement curves in pulling direction

shear. In case of bi-directional shear, because axial compression seems to govern the post-peak behaviour, torsion has no distinctive influence on the descending part.

Under shear with or without torsion, the first eight columns failed in either shear tension mode or shear yielding compression mode. The column under torsion failed in compression mode. In all nine columns, spalling always occurred after columns reached ultimate capacity.

Failure mode of columns is not only dependent on amount of reinforcement but also on axial compression. For columns subjected to pure torsion, compression failure only occurs when amount of longitudinal and transverse reinforcement is large. However, when column T was under high axial compression, even with transverse reinforcement as low as 0.25% the column failed in torsion compression.

5.5.2 Bi-directional shear

Shear capacity of columns decreases significantly when angle between shear force and the strong direction of the columns changes from 0° to 30° and 60° . Figure 5.107 shows the interaction of shear strength V_x and V_y in two principal directions of columns. The data points of columns B30 and B60 are very close to the elliptical curve passing to the data points of columns U0 and U90. Therefore, bi-directional shear strength of HSC columns under high axial compression follows the

Chapter 5 Experimental Results

interaction curve, which is applicable for NSC columns under lower axial compression (Umebara and Jirsa, 1984).

5.3 Interaction between shear and torsion

The interaction curve proposed by Ersoy and Ferguson (1968) is plotted in figure 5.108 together with the experimental data. The figure reveals that the data points are very close to the circular interaction curve. Because the eccentricity is small, the applied torsion is not more than 35% of torsional capacity of the columns and shear strength decreases only about 10%. It is suggested that further experiment for larger torsion is necessary to verify the whole interaction curve.

5.4 Crack development of the columns

Crack development is one of crucial factor related to behaviour of short columns. In RC columns lightly reinforced in transverse direction, crack development is dependent on strain gradient. Mohammadi (2001) emphasized that without strain gradient, members well reinforced in one direction will develop smeared cracks. However, under strain gradient, members lightly reinforced in one direction will have crack localisation at small displacement. As displacement increases, further cracks could occur. Cracks become well distributed when displacement is large. The distinction between these two behaviours is very important in finite element analysis because tension stiffening model is used for the first case, where cracks are smeared, while transient tension model (Mohammadi, 2001) is used for the second case. These tension models are discussed in the next chapter.

Effect of strain gradient on crack development is also observed in the present study. Crack development in each column is examined here. Each column could be divided into zones with and without stiffening effect of transverse reinforcement as shown in Figure 5.109. Shear reinforcement was only effective within outer layers but not effective in core concrete. As there was no shear reinforcement in core concrete, very few cracks with large crack spacing would develop there. Since shear

Chapter 5 Experimental Results

reinforcement ratio in outer layers was small, crack development there was related strain gradient.

In column U0, flexural cracks in two shorter outer layers were localised. Because shear span ratio of column U0 is less than one, its flexural behaviour is significant compared to shear behaviour. As a result, smeared cracks developed in two longer outer layers of the column. The outer layers having localised cracks and smeared cracks are labelled localised crack and smeared crack, respectively, in figure 5.109.

Figure 5.107 Biaxial shears and the interaction curve

Figure 5.108 Interaction between shear and torsion

Chapter 5 Experimental Results

In column U90, flexural cracks in two longer outer layers were localised. As shear span ratio of column U90 is greater than one, strain gradient is significant. At small displacement, diagonal cracks in shorter outer layers were localised. When displacement increased, bond between reinforcement and concrete in the outer layers became effective. Because reinforcement could transmit tensile stress to cracked concrete, several new diagonal cracks appeared in these layers. Cracks only became well distributed when displacement of the column was large. The outer layers, which had localised cracks at small displacement and smeared cracks at large displacement, are also labelled localised crack in Figure 5.109. Thus, diagonal crack development in column U0 and U90 was different. The cause of this difference is bending moment relative to shear force or shear span ratio.

In columns B30, and B60, crack development in shorter outer layers was similar to that in shorter outer layers of column U90, i.e. cracks were localised at the beginning and may become smeared when displacement of columns was large. Longer outer layers of column B60 had localised cracks while cracks in longer outer layers of column B30 were nearly smeared because behaviour of this column was closer to behaviour of column U0.

Without effect of strain gradient, diagonal cracks on all faces of column T were distributed regularly. In columns subjected to torsion and shear force, UT0, UT90, BT30 and BT60, shear stresses caused by shear force and torsion were additive on particular side(s) and were subtractive on the opposite. Because torsion increased shear stress on particular side(s), reduced shear stress on the opposite side(s), it indirectly decreased or increased effect of bending relative to shear. By that way, torsion changed crack development on different sides of these columns. In an outer layer, if effect of bending is negligible compared to effect of shear, cracks are smeared with small crack spacing. Else, cracks are localised with large crack spacing at the beginning. They may become smeared when displacement of columns is large. In comparison to corresponding columns under shear force, on outer layers of columns under shear force combined with torsion, when shear stresses resulted from shear force and torsion are additive, cracks are distributed more uniformly whereas it is reverse on opposite side(s) (Figure 5.109).

Chapter 5 Experimental Results

Figure 5.109 Reinforcement effective zone and crack distribution

While bending affects crack development and their distribution, axial load strongly affects crack orientation. Figure 5.110 shows stresses in column just before cracking. Initial crack angle with respect to vertical axis of columns can be computed from elastic theory as:

Chapter 5 Experimental Results

$$\tan 2\phi = \tau / \sigma_0 = \frac{f_t}{\sigma_0} \sqrt{1 + \sigma_0 / f_t} \quad (5-1)$$

The lower bound tensile strength of concrete, computed according to Equation (4-2), is about 2MPa. Substitution of this value into equation (5-1) gives crack angle about 27° , which is very close to experimental diagonal crack angle on wider faces of columns U0, UT0, on shorter faces of columns U90, UT90 and on all faces of column T. Crack angle on these faces does not depend on whether shear stress is caused by shear force or torsion. Diagonal crack angle at the middle of columns is independent from shear span to depth ratio when comparing U0 to U90 and UT0 to UT90. Thus, equation (5-1) and experiment confirm that axial compression has decisive effect on crack angle. The similarity of diagonal crack angle in these columns is due to the same axial compression.

Besides, bending in companion with shear force caused additional axial compression, which reduced crack angle with respect to vertical axis of columns. Under bending, axial stress was not a constant and crack angle varied according to moment distribution along column height. Bending moment is small at the middle of column. Because bending moment becomes large near two ends of column diagonal crack angle decreases at these locations. Crack pattern of column U90 clearly shows this phenomenon.

Crack angle of column T was 30° , which was 5° greater than crack angle of column U0. Thus, bending in column U0 has slightly reduced diagonal crack angle. In this column, the reduction in crack angle was marginal because shear behaviour dominated over flexural behaviour.

Because torsion indirectly decreased effect of bending on particular side(s) or increased that effect on opposite side(s), as discussed earlier, it changed crack orientation of columns under shear combined with torsion. Diagonal crack angle on the side(s), where bending was significant compared to shear was slightly less than that on the opposite side(s).

In columns under torsion or combined shear and torsion, continuous helical cracks were not formed because columns are short. Thus, estimation of torsional capacity based on spaced truss analogy is rather inaccurate.

Chapter 5 Experimental Results

(a) Assumed external stresses (b) Mohr's circle

Figure 5.110 Stresses in column just before cracking (Ichinose, 1992)

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

CHAPTER 6 THREE-DIMENSIONAL MODELLING OF CRACKED CONCRETE

6.1 Introduction

Constitutive model of concrete is one of the most crucial parts of the finite element analysis. Before cracking, the stiffening effect of reinforcement on concrete is negligible and concrete is assumed to follow the elasto-plastic and fracture model. In this study, the three-dimensional elasto-plastic and fracture model (Hauke, 1998) is adopted. Once cracks appeared in an element volume, smeared crack concrete model is used.

The two-dimensional fixed crack model and the three-dimensional model of cracked concrete (Maekawa et al., 2003) have been reviewed in Chapter 3. Having analyzed the shortcoming of the model, appropriate modifications are proposed in this chapter.

6.2 Modifications to the two-dimensional fixed crack model

As discussed in Chapter 3, the three-dimensional model of Maekawa et al. (2003) can predict response of RC columns or beams under monotonic unidirectional shear, bi-directional shear, and torsion up to peak load. The descending branch is not predicted well. In post peak region, the cracked concrete model plays an important role and it is most probably the source of deficiency.

The three-dimensional cracked concrete model was formulated from the two-dimensional cracked concrete model. In this section, the imperfection of the two-dimensional cracked concrete model is discussed and modifications are proposed. Modification to the formulation of the three-dimensional cracked concrete model is presented in the next section.

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

6.2.1 Deficiency in the original two-dimensional cracked concrete model

This section shows the verification of the original two-dimensional cracked concrete model with tested panels given in Table 6-1. The analysis of these panels was conducted by Maekawa et al. (2003). The compression model can be verified with panels PV21, SP-1.7-0, SE8, SE9 and SE10, whose behaviour is mainly affected by this model. The shear model can be verified with panels PV12 and PV19. Under pure shear stress, behaviour of these two panels having significantly different reinforcement ratios in two orthogonal directions is governed by shear transfer model.

Figure 6.1 shows computational and experimental responses of panels PV21, SP-1.7-0, SE8, SE9 and SE10 including compressive strength reduction factor. As seen in the figure, the analysis of PV21 overestimated its shear capacity when the original compression model was used. Computation would agree with experiment if a residual compressive strength of 45% uniaxial compressive strength was used instead of the 60% as proposed in the original model.

Table 6-1 Tested panels

Specimen	Loading	Reinforcing bar				Concrete	
		ρ_x (%)	f_{yx} (MPa)	ρ_y (%)	f_{yy} (MPa)	f'_c (MPa)	f_t (*) (MPa)
$\tau : \sigma_x : \sigma_y$						
PV21	1:0:0	1.79	458	1.3	302	19.5	2.2
SP-1.7-0	1:0:0	1.75	444.4	1.75	444.4	20.9	1.96
SE8	1:0:0	2.93	492	0.98	479	37.0	1.9
SE9	1:0:0	2.93	422	2.93	422	44.2	2.2
SE10	1:1/3:1/3	2.93	492	0.98	479	34.0	2.2
PV12	1:0:0	1.79	469	0.45	269	16.0	1.6
PV19	1:0:0	1.79	458	0.71	299	19.0	1.9

(*) tensile strength used in analysis

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Analysis of panels SP-1.7-0, SE8 and SE9 under cyclic pure shear also overestimated shear capacity. Analytical capacity of panels SP-1.7-0, SE8 and SE9 could match experimental result if residual compressive strength was 45%, 30% and 40% of uniaxial compressive strength, respectively.

Furthermore, computational capacity of panel SE10 was higher than the experimental capacity. Agreement could be reached if residual compressive strength was reduced to 50% uniaxial compressive strength.

Figure 6.1 Shear stress-shear strain curves of panels PV21, SP-1.7-0, SE8, SE9 and SE10 (Maekawa et al., 2003)

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.2 Shear stress-shear strain curves of panels PV12 and PV19 (Maekawa et al., 2003)

Figure 6.2 compares analytical response with experimental response of panels PV12 and PV19. It is clear that the computational shear strength is significantly higher than the experimental shear strength for both panels.

Therefore, the original two-dimensional cracked concrete model overestimates load carrying capacity of the panels, whose behaviour is governed by compression and/or shear models. Maekawa et al. (2003) supposed that the overestimation is due to the fact that concrete spalling has not been incorporated in the cracked concrete model whereas spalling might occur in these panels. However, the main reason may lie on the deficiencies in the compression and shear models of cracked concrete as discussed in following sections.

6.2.2 The modified compressive strength reduction factor

Compressive stress parallel to crack is affected by tensile strain in orthogonal direction. As tensile strain is more than 0.005, residual compressive stress is assumed to be 60% of uniaxial compressive strength (Maekawa et al., 2003). However, this residual compressive stress level leads to the overestimation of shear strength of panels as discussed earlier.

Belarbi and Hsu (1995) implemented bi-directional compression-tension test on large panels. The compression softening factor, proposed by several researchers, as a function of tensile strain, is plotted in Figure 6.3 against experimental result. In

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

this figure, the compressive strength reduction factor of Maekawa et al. (2003) is presented by the model of Miyahara et al. (1987). The figure clearly demonstrates that compression softening factor of Maekawa et al. (2003) is too high at tensile strain greater than 0.005.

Considering the overestimation of shear capacity of panels and the models of other researchers (Figure 6.3), a residual compressive strength of 30% uniaxial compressive strength is proposed for 3D finite element analysis in this study. The modified compressive strength reduction factor is plotted in Figure 6.4.

Figure 6.3 Models of compressive strength reduction factor
(Belarbi and Hsu, 1995)

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.4 Proposed model of compressive strength reduction factor

6.2.3 The modified shear model for cracked concrete

6.2.3.1 Model of shear transfer along crack surface

Shear transfer model based on contact density of Maekawa et al. (2003) exhibits deficiencies in certain cases. The original shear model equations reviewed in Chapter 3 are rewritten here:

$$\tau_{st} = f_{st} \frac{\beta^2}{1 + \beta^2} \quad (6-1)$$

$$\beta = \frac{slip}{w} = \frac{\gamma_{cr}}{\epsilon_t} \quad (6-2)$$

$$f_{st} = 18 f_c'^{1/3} \quad (f_c' : \text{kgf} / \text{cm}^2) \quad (6-3)$$

From these equations, shear transfer stiffness-shear strain relationship can be obtained. Figure 6.5 presents normalised shear transfer stiffness-normalised shear strain relationship for constant tensile strain equal to two times and four times of cracking tensile strain of concrete having compressive strength of 60MPa. Although in actual structures, tensile strain is not constant but increases with shear strain, Figure 6.5 illustratively indicates that there are certain cases that shear transfer

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

stiffness is larger than shear modulus of concrete. This is unreasonable because shear transfer stiffness must be smaller than shear modulus of concrete. It demonstrates that shear transfer model of Maekawa et al. (2003), equations (3-17) to (3-19), does not always accurately reflect shear transfer behaviour.

Many researchers (Cervenka, 1985; Vecchio and Collins, 1986; Balakrishnan and Murray, 1988; Barzegar, 1989; Yamada and Krauthammer, 1997; Elmsori et al., 1998) proposed shear transfer model of cracked concrete as a function of crack width or tensile strain in direction perpendicular to crack surface.

In contrast, Maekawa et al. (2003) did not directly take into account crack width in equation (3-17). Instead, they expressed shear transfer stress as a function of normalised shear strain, which is the ratio between shear slip and crack width. It suggests that the ignorance of the direct dependency of shear transfer stress on crack width in the model of Maekawa et al. (2003) is the reason of deficiency.

The strong dependence of shear transfer stress on crack width has been found in an experiment of Al-Mahaidi et al. (1998). This relationship was explicitly given by Reinhardt and Walraven (1982) in their formula of shear transfer stress as follows:

$$\tau = -\frac{f'_{cc}}{30} + \left[1.8w^{-0.80} + (0.234w^{-0.707} - 0.20)f'_{cc} \right] \Delta \quad (6-4)$$

Figure 6.5 Normalised shear transfer stiffness computed according to shear model of Maekawa et al. (2003)

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

where:

τ = shear transfer stress, MPa

f'_{cc} = cube compressive strength of concrete

Δ = shear slip

Crack width is closely related to compression perpendicular to crack surface, which is caused by reinforcement across the crack (Reinhardt and Walraven, 1982). If the reinforcement ratio is higher, the crack width is smaller and shear transfer stress is higher. Thus, shear transfer stress can be formulated as a function of amount and yield strength of reinforcement across crack instead of crack width. Shear transfer capacity has been expressed in term of reinforcement index ρf_y in shear friction equation (ACI-318M-02):

$$\tau = \rho f_y \mu \quad (6-5)$$

μ = friction coefficient, = 1.4

Alternatively, ACI code recommended that shear transfer capacity can be computed with consideration of a cohesion term:

$$\tau = 2.8 + 0.8 \rho f_y \quad (6-6)$$

One of the most accurate models of shear transfer capacity, equation (6-7), was proposed by Walraven et al. (1994) from data of push-off tests.

$$\tau = C_1 (\rho f_y)^{C_2} \quad (6-7)$$

where:

$$C_1 = 0.822 f_{cc}^{0.406}$$

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

$$C_2 = 0.159 f_{cc}'^{0.303}$$

Therefore, shear transfer model need to be expressed in term of crack width or alternatively, reinforcement crossing crack.

Shear transfer capacity, equation (3-19), is expressed purely in term of concrete compressive strength. Testing result proves that the equation could overestimate shear transfer strength. Figure 6.6 shows typical push-off tests while Figure 6.7 shows shear transfer capacity of pre-cracked specimens from push-off test of several researchers (Mansur et al., 1998; Mattock, 2001; Kahn and Mitchell, 2002; Mattock and Hawkins, 1972; and Walraven et al., 1987). Tested values can be found in Appendix A. Figure 6.7 excludes test result of initially uncracked specimens because only pre-cracked specimens present shear transfer behaviour. The uncracked specimens demonstrate different behaviour (Hsu et al., 1987).

Shear transfer capacity, equation (3-19), is also plotted in Figure 6.7. The figure shows that the equation significantly overestimates shear transfer capacity in many cases, particularly for HSC. A detailed examination reveals that the estimation is close to experimental data, when reinforcement index ρf_y is high. Thus, shear transfer capacity, equation (3-19), is the source of deficiency in shear transfer model of Maekawa et al. (2003). The model may provide good prediction when reinforcement index is high but it is not precise in general. The precision could be achieved if the effect of concrete compressive strength and particularly, reinforcement index are properly included in shear transfer capacity equation.

Figure 6.6 Typical set up of push-off tests (Kahn and Mitchell, 2002)

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Shear transfer capacity, equations (3-19), (6-5) to (6-7), is plotted versus reinforcement ratio and concrete compressive strength in Figure 6.8 and Figure 6.9, respectively. As seen in these figures, the two models of ACI-318M-02 provide the lowest shear transfer capacity. This is logical because conservative solution is normally proposed for design purpose. The model of Maekawa et al. (2003) always gives the highest shear transfer capacity. As shown in Figure 6.8, for concrete compressive strength of 60MPa, shear transfer capacity of Walraven et al. (1994) is as low as 30% of that proposed by Maekawa et al. (2003) for low reinforcement ratio. The two models provide comparable result when reinforcement ratio is as high as 2%. Shear transfer capacity of Walraven et al. (1994), equation (6-7) is adopted in this study because of its higher reliability. Thus, shear transfer capacity will be expressed by equation (6-7) instead of equation (3-19) in the modified model of shear transfer along crack.

Figure 6.7 Shear transfer capacity of push-off tests

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.8 Shear transfer capacity versus reinforcement ratio

Figure 6.9 Shear transfer capacity versus concrete compressive strength

In analysis, the ratio of reinforcement across crack is approximated by effective transverse reinforcement ratio of RC columns. When shear force is applied in a principal direction of columns, the effective transverse reinforcement ratio is equal to shear reinforcement ratio in loading direction. If loading is applied in a non-principal direction (Figure 6.10), the effective transverse reinforcement ratio will be interpolated from shear reinforcement ratios in two principal directions as follows:

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

$$\rho_{eff} = \rho_x \sin^3 \psi + \rho_y \cos^3 \psi \quad (6-8)$$

where:

ρ_{eff} = effective transverse reinforcement ratio

ρ_x, ρ_y = shear reinforcement ratios in strong and weak directions of member, respectively

ψ = angle between resultant displacement and the weak direction of member

Equation (6-8) is plotted in Figure 6.11 with $\rho_x = 0.005$ and $\rho_y = 0.0025$, which are shear reinforcement ratios in two principal directions of the columns tested in this study.

In columns subjected to torsion, cracks mainly occur in outer layers not in core concrete (Figure 6.12). Because transverse reinforcement confines cracks within outer layers, the effective transverse reinforcement ratio should be obtained by smearing transverse reinforcement over the outer layer only. Thus, the effective transverse reinforcement ratio is normally greater than shear reinforcement ratios in both principal directions of columns. In analysis, the effective transverse reinforcement ratio can be approximately assumed equal to the greater of the two shear reinforcement ratios. This approximation is equivalent to assuming that the thickness of outer layers is equal to half of the smaller size of cross section of columns.

When columns are subjected to shear force combined with torsion, cracks develop in outer layers and in core concrete as well. Transverse reinforcement confines crack surfaces spreading over entire cross section of columns. Thus, the effective transverse reinforcement ratio should be defined from shear reinforcement ratios similar to the columns under shear force alone.

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.10 Cross section of a member under loading in non-principal direction

Figure 6.11 Interpolation of effective transverse reinforcement ratio

Figure 6.12 Cross section of a member under torsion

6.2.3.2 Shear stiffness of element with one-way crack

The expression of shear stiffness of elements with one-way crack (Mackawa et al., 2003) reviewed in Chapter 3 is rewritten here:

$$G = \frac{1}{1/G_c + 1/G_{cr}} \quad (6-9)$$

Originally, when cracks close, shear stiffness of the elements is assumed equal to shear modulus of uncracked concrete by assigning shear stiffness of cracks, G_{cr} , in equation (6-9), a very large value compared to shear modulus of concrete. This treatment may not be accurate because G_{cr} and shear stiffness of an element are always less than shear modulus of concrete. Substantial overestimation in load carrying capacity could be seen when a member is subjected to high axial load

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

because, in this case, the cracks will be easily closed during unloading and not easily open during reloading.

This assumption may not lead to severe error for normal strength concrete, where crack surface is rough and shear stiffness of cracks G_{cr} is large. Nevertheless, for HSC, the assumption results in significant overestimation because crack surface is smooth and shear stiffness of cracks G_{cr} is smaller. Maekawa et al. (2003) managed to avoid the overestimation by reducing shear stress by half for HSC as seen in Chapter 3. Reducing shear stress means decreasing shear stiffness of finite element in equation (6-9). This is equivalent to reducing shear stiffness of both cracked and uncracked parts in equation (6-9). The treatment is inappropriate because for HSC, only shear stiffness of cracks G_{cr} should be reduced, not shear stiffness of the uncracked part.

For both NSC and HSC, equation (6-9) would give reliable result if G_{cr} is accurately modelled in both cases: crack closure and crack opening. When a crack opens, G_{cr} is computed from shear transfer model proposed in previous section. When crack closes, G_{cr} is assumed equal to shear modulus of concrete multiplied by a factor related to concrete strength as:

$$G_{cr} = \mu \cdot G_c \quad (6-10)$$

μ = shear transfer factor, $\mu \leq 1$

Shear transfer factor will be high for NSC and low for HSC because of the different roughness of crack surface. Here, if concrete compressive strength is more than 50MPa, HSC will be assumed. When crack closes, shear transfer factor is tentatively assumed equal to 0.5 and 0.3 for NSC and HSC, respectively.

Substitution of G_{cr} in equation (6-10) into equation (6-9) yields shear stiffness of element, which is equal to $0.33 G_c$ and $0.23 G_c$ for NSC and HSC, respectively.

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

6.2.3.3 Shear stiffness of element with two-way cracks

Shear stiffness of an element with two-way cracks (Maekawa et al., 2003) discussed in Chapter 3 is rewritten here:

$$G = \frac{1}{1/G_c + 1/G_{cr1} + 1/G_{cr2}} \quad (6-11)$$

Like the case of one-way crack, because the assumption that shear stiffness of the finite element is equal to shear modulus of uncracked concrete when both cracks close (Maekawa et al., 2003) is unrealistic, modifications are proposed here.

When at least one crack closes, for example crack 2, shear stiffness of crack 2 G_{cr2} is substantially greater than shear stiffness of crack 1 G_{cr1} . Because the term $1/G_{cr2}$ becomes negligible, it is discarded and equation (6-11) takes the form similar to equation (6-9) as:

$$G = \frac{1}{1/G_c + 1/G_{cr1}} \quad (6-12)$$

If the other crack (crack 1) also closes, shear stiffness of that crack, G_{cr1} in equation (6-12), is computed according to equation (6-10).

When a crack open, its shear stiffness will be calculated according to the previously proposed shear transfer model.

6.3 Modifications to the three-dimensional model of cracked concrete

6.3.1 The selective composition of normal stresses

The decomposition of strain and composition of stress (Maekawa et al., 2003) employ the available two-dimensional constitutive material model in three-dimensional analysis. According to the method, each shear stress is computed independently in one 2D plane whereas each normal stress is the average of two corresponding normal stresses computed in a couple of 2D planes. While result of

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

shear stresses seems to be reasonably accurate, the averaging of normal stresses could lead to inaccuracy of the cracked concrete model. Considering the drawback of the composition method, modification is proposed accordingly.

The shortcoming of the normal stress averaging method is examined in case of a rectangular RC column under loading in a principal plane. Figure 6.13 shows the RC column and the decomposition of the 3D concrete element A taken from the column. Within the finite element, if crack appears, a local orthogonal coordinate system, of which the first axis is perpendicular to the crack plane, is selected. This local coordinate system will not change in subsequent loading. Global strains of the element are transformed into the local coordinate system. Stresses of the finite element will be calculated with respect to the local coordinate system.

This 3D element is subjected to normal tensile stress in direction (1) and normal compressive stress in direction (3). There is tensile strain in direction (2) perpendicular to loading plane due to Poisson's effect. Under this effect, concrete tends to expand. If there is reinforcement in this direction, reinforcement tends to confine the expansion of concrete leading to tension in reinforcement and compression in concrete. Though shear stress is zero at the moment of cracking, it generally differs from zero when loading proceeds because the principal stress does not coincide with crack direction.

Stresses in each plane are computed from strains based on two-dimensional concrete model. In plane (1,3), $\sigma_{11}^{(1,3)}$ follows tension softening/stiffening model, $\sigma_{33}^{(1,3)}$ complies with compression softening model whereas $\sigma_{13}^{(1,3)}$ obeys shear model.

Because crack may not exist in plane (2,3), elasto-plastic and fracture (EPF) model is applied to compute compressive stresses $\sigma_{22}^{(2,3)}$ and $\sigma_{33}^{(2,3)}$. Compressive stress $\sigma_{22}^{(2,3)}$ represents the confinement behaviour. Compressive stress $\sigma_{33}^{(2,3)}$, computed based on EPF model, is normally much higher than the corresponding stress $\sigma_{33}^{(1,3)}$ computed according to compression softening model.

In plane (1,2), tensile stress $\sigma_{11}^{(1,2)}$ is computed from tensile strain according to tension softening/stiffening model. Since the element is considered crack, $\sigma_{22}^{(1,2)}$ is miscalculated by tension softening/stiffening model.

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

The two normal stresses $\sigma_{11}^{(1,3)}$, $\sigma_{33}^{(1,3)}$ computed in plane (1,3) and $\sigma_{22}^{(2,3)}$ are taken to represent three normal stresses σ_{11} , σ_{33} and σ_{22} of the 3D element. Thus,

$$\sigma_{11} = \sigma_{11}^{(13)} \quad (6-13)$$

$$\sigma_{22} = \sigma_{22}^{(23)} \quad (6-14)$$

$$\sigma_{33} = \sigma_{33}^{(13)} \quad (6-15)$$

However, according to the averaging method, each normal stress of the 3D concrete element is calculated as the average of two corresponding stresses in a couple of planes as:

$$\sigma_{11} = (\sigma_{11}^{(12)} + \sigma_{11}^{(13)}) / 2 \quad (6-16)$$

$$\sigma_{22} = (\sigma_{22}^{(12)} + \sigma_{22}^{(23)}) / 2 \quad (6-17)$$

$$\sigma_{33} = (\sigma_{33}^{(13)} + \sigma_{33}^{(23)}) / 2 \quad (6-18)$$

Due to the reason mentioned earlier, the averaging of normal stresses will sometimes provide inaccurate solution in directions (2) and (3). Because this leads to overestimation of shear strength of columns, the averaging method is eliminated in the modified three-dimensional cracked concrete model. In the modified model, normal stresses of the 3D element are selected from those computed in three planes as follows:

$$\begin{aligned} \sigma_{ii} &= \min(\sigma_{ii}^{(i,j)}, \sigma_{ii}^{(i,k)}) && \text{if } \sigma_{ii}^{(i,j)} * \sigma_{ii}^{(i,k)} < 0 \\ \sigma_{ii} &= \min(\sigma_{ii}^{(i,j)}, \sigma_{ii}^{(i,k)}) && \text{if } \sigma_{ii}^{(i,j)} > 0 \text{ and } \sigma_{ii}^{(i,k)} > 0 \\ \sigma_{ii} &= -\min(|\sigma_{ii}^{(i,j)}|, |\sigma_{ii}^{(i,k)}|) && \text{if } \sigma_{ii}^{(i,j)} < 0 \text{ and } \sigma_{ii}^{(i,k)} < 0 \\ \sigma_{ii} &= \sigma_{ii}^{(i,j)} && \text{if } \sigma_{ii}^{(i,j)} \neq 0 \text{ and } \sigma_{ii}^{(i,k)} = 0 \end{aligned} \quad (6-19)$$

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

This method is called selective composition method of normal stresses. When this method is applied to the element A, its 3D normal stresses will be $\sigma_{11}^{(1,3)}$, $\sigma_{33}^{(1,3)}$ and $\sigma_{22}^{(2,3)}$ as expressed by equations (6-13) to (6-15).

The selective composition method of normal stresses is formulated on the basis that crack plane is perpendicular to tensile stress $\sigma_{11}^{(1,3)}$ and parallel to compressive stress $\sigma_{33}^{(1,3)}$. For cyclic loading, the crack may close and new crack may open (Figure 6.13). The new crack plane remains perpendicular to the loading plane (1,3). Though the new crack plane could be neither perpendicular to axis (1) nor parallel to axis (3) and stresses $\sigma_{11}^{(1,3)}$ and $\sigma_{33}^{(1,3)}$ could change sign alternatively, the two normal stresses in plane (1,3) still present 3D normal stresses σ_{11} and σ_{33} . Additionally, the occurrence of the new crack, which is computationally considered in plane (1,3), does not change the characteristics of normal stress $\sigma_{22}^{(2,3)}$ computed in plane (2,3). $\sigma_{22}^{(2,3)}$ still presents 3D normal stress σ_{22} . Therefore, the 3D normal stresses still can be expressed by equations (6-13) to (6-15) i.e., the selective composition method of normal stresses is also valid for cyclic loading.

The selective composition method of normal stresses, which is formulated for rectangular columns subjected to loading in a principal plane, is also applicable to rectangular columns under loading in a plane other than two principal planes. A finite element B taken from a column subjected to loading in non-principal plane is shown in Figure 6.14. Once crack starts, strains of the element are transformed into the local coordinate system selected in connection with the first crack plane. Similar to element A, element B has normal tensile stress in direction (1) that is perpendicular to the first crack plane, normal compressive stress in direction (3) and compressive stress in direction (2) caused by confinement. Different from element A, element B has two more shear stresses in planes (1,2) and (2,3). As seen in Figure 6.14, unlike element A, the first crack plane of element B is not perpendicular to any principal planes of the column. However, these differences do not affect the selective composition of normal stresses.

Following strain transformation, strains of the 3D element are decomposed into three orthogonal planes (Figure 6.14). Like the case of element A, according to the

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

selective composition of normal stresses, three normal stresses of the element B will be $\sigma_{11}^{(1,3)}$, $\sigma_{33}^{(1,3)}$ and compressive normal stress $\sigma_{22}^{(2,3)}$.

When loading proceeds, other crack plane could form within element B. If this new crack plane is not perpendicular to plane (1,3) the validity of the strain decomposition and stress composition method needs to be considered. Nevertheless, for columns loaded in one plane, the new crack plane could be approximately assumed perpendicular to plane (1,3). Thus, the situation is similar to columns under cyclic loading in a principal plane. In other words, the selective composition of normal stresses is valid for columns under cyclic loading in an arbitrary plane.

When a column is subjected to torsion, crack may occur at outer layer while core concrete is uncracked. The space truss model is widely accepted for torsional members because each face of the space truss behaves like a member under plane loading. Since behaviour of cracked concrete in outer layer of the column is similar to that of concrete in the column under loading in a principal plane, the selective composition can be applicable for columns under torsion.

If a column is under torsion combined with shear force, cracks may happen in core concrete mainly due to shear force. Though shear stress of concrete is summation of those caused by torsion and shear force, concrete in outer layer behaves like that in a column subjected to loading in a principal plane. The application of the selective composition method to concrete at outer layer is rational. The same application to core concrete is valid like the case of a column under shear force. Thus, the selective composition method is valid for columns subjected to combined shear force and torsion.

In addition, the validity of the selective composition method is verified for a column subjected to sequential shear force in two principal planes. First, the column is subjected to loading in one principal plane. The first crack takes place and a local coordinate system is selected like that in Figure 6.13. This coordinate system will not change when loading proceeds. After that, shear force is applied in the other principal plane of the column and new crack occurs. Decomposition of element C taken from the column is given in Figure 6.15. Element C has initial crack plane, which is similar to element A in Figure 6.13, and the subsequent crack plane, which is not perpendicular to any planes of the local coordinate system. In comparison to

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

element A, element C has two more shear stresses in planes (1,2) and (2,3) and the compressive stress $\sigma_{22}^{(2,3)}$ in element A could be tensile stress in element C. Though the subsequent crack plane does not perpendicular to planes (1,2) and (2,3), two-dimensional cracked model is still applied in these planes. This is an approximation in the original decomposition method of Maekawa et al. (2003) and is also adopted in this study.

Regarding normal stresses, the element C is under two tensile stresses σ_{11}, σ_{22} and one compressive stress σ_{33} . From the view of compression softening, this compressive stress could be softened by both tensile strain in directions 1 and 2. Two corresponding normal compressive stresses $\sigma_{33}^{(2,3)}, \sigma_{33}^{(1,3)}$ are different because they are softened by two different tensile strains in direction 1 and 2. Additionally, two corresponding normal tensile stresses $\sigma_{11}^{(1,2)}$ and $\sigma_{11}^{(1,3)}$ or $\sigma_{22}^{(1,2)}$ and $\sigma_{22}^{(2,3)}$ are not equal in general. As load carrying capacity of a column is governed by the minimum allowable stress, the original averaging of normal stresses could overestimate load carrying capacity. On contrary, the selective composition method selects normal stress with minimum absolute value between two corresponding normal stresses. The applicability of the selective composition of normal stresses has been proved for subsequent loading in one direction. Obviously, the method is also valid for subsequent loading in reverse direction. In other words, the method is applicable to reverse cyclic loading.

In short, the selective composition of normal stresses is proposed to replace the averaging method. The new method is versatile and applicable for complex loading conditions including unidirectional shear, bi-directional shear, sequential shear force in two principal planes, torsion and torsion combined with shear force.

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.13 Application of decomposition of strain and composition of stresses to a column under loading in a principal plane

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.14 Application of decomposition of strain and composition of stresses to a column under loading in non-principal plane

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

Figure 6.15 Application of decomposition of strain and composition of stresses to a column under subsequent loading in two principal planes

Chapter 6 Three-Dimensional Modelling of Cracked Concrete

6.3.2 Tension softening model for members lightly reinforced in transverse direction

This section discusses application of tension softening models to members with low amount of transverse reinforcement. After reviewing the original two-dimensional and three-dimensional tension modelling, an appropriate tension softening models with suitable tension softening parameter are recommended for three-dimensional analysis.

The two-dimensional tension modelling is reviewed in case of a shear wall as seen in Figure 6.16a. In the shear wall, longitudinal reinforcement ratio and transverse reinforcement ratios are more and less than critical value in respective directions. According to the RC zone method (An, 1996), the whole cross section of the wall is considered as RC zone. The tension stiffening model with stiffening parameter, $c=0.4$ will be applied to concrete despite amount of transverse reinforcement.

Figure 6.16 Shear wall lightly reinforced in transverse direction