Model updating and structural assessment using vibration data with artificial intelligence algorithms

Tu, Zhenguo

2005

Tu, Z. G. (2005). Model updating and structural assessment using vibration data with artificial intelligence algorithms. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/12039>

<https://doi.org/10.32657/10356/12039>

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MODEL UPDATING AND STRUCTURAL ASSESSMENT USING VIBRATION DATA WITH ARTIFICIAL INTELLIGENCE ALGORITHMS

TU ZHENGUO

**SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

2005

Model Updating and Structural Assessment Using Vibration Data with Artificial Intelligence Algorithms

Tu Zhenguo

School of Civil & Environmental Engineering

**A thesis submitted to Nanyang Technological University
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy**

2005

ACKNOWLEDGEMENTS

I would like to express my first and foremost gratitude to my supervisor, Associate Professor, Lu Yong, for his warm encouragement, patient guidance, and stimulus during the entire period of my research study. His unwavering enthusiasm and interest in research is rare, for which I am most grateful. It is a great benefit and honor to work with him.

I would like to acknowledge and thank my colleagues for their helpful discussions and suggestions. I wish to deliver special thanks to my friends Gu Xiaoming, Gao Feng and Xu Jianfeng for their kind helps.

I also would like to thank technical staff in the PE laboratory, School of Civil and Environmental Engineering, Nanyang Technological University, for their help and support.

Finally, I would like very much to thank my family for their help, support and endurance. In particular, I am very grateful to my wife, Yanqiu, for her love, understanding and encouragement, all of which made this effort possible.

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SUMMARY

Catastrophic structural failures have been observed in bridges, buildings and offshore platforms in many parts of the world. Many of such incidents, however, share the same origin that dramatic changes of effective structural parameters, in terms of material properties, geometric characteristics and boundary conditions, have taken place under the particular service environment. The prevention of such structural failures requires timely evaluation of the structural conditions throughout their life span.

The FE model updating techniques have evolved for purpose to provide a realistic analytical model for a given structure so that its present condition can be assessed more reliably and the remaining service life may also be predicted under foreseeable loading conditions. However, the conventional model updating methods have difficulties in practical applications due to the following three main reasons: 1) high demand on the form and amount of the measurement data, 2) likelihood of being trapped to local optima due to some inherent limitations of the optimization algorithm and, 3) vulnerability to measurement noises.

This research programme aims at improving FE model updating techniques in these crucial aspects. In addition, appropriate forms of the measurement data and effective expansion of the response dataset are also investigated, in conjunction with appropriate algorithms, to ensure a more reliable updating outcome. The main developments and contributions of this study may be summarized in the following four aspects.

i) A methodology to employ GA in conjunction with the eigensensitivity approach is developed for FE model updating based on a limited amount of modal data, for example the lowest three natural frequencies and the first mode shape. The incorporation of the traditional gradient-based eigensensitivity method effectively

compliments the GA in tackling difficulties arisen from the low sensitivity of such response data to some structural parameters. In case a certain level of noises (errors) exists in the measured response data, a sensitivity analysis scheme is proposed to identify the required order of modal data to achieve a specified degree of accuracy for a given number of parameters to update.

ii) A two-level neural network is proposed to effectively update an FE model involving parameters of different nature, for example stiffness parameters and damping parameters, at two stages. To mobilize the noise-resistance ability of ANN, the noise-injection learning strategy is adopted in training the networks and an implementation scheme is proposed. It is proved that marked noise-resisting ability can be acquired through such a training process. In updating the parameters related to damping, the integrals of frequency response functions are introduced as parameter-sensitive response information to enable an effective updating of such parameters.

iii) To improve the search efficiency of GA in handling large-size problems, a stochastic genetic algorithm (StGA) with a unique stochastic coding scheme is further developed in a systematic manner. The special feature of this coding strategy is that each chromosome actually represents a region which is defined in a stochastic manner, and in the genetic search process the stochastic regions evolve towards the most promising area where the optimum is located, thus increases the efficiency.

iv) To tackle another major issue in FE model updating with the limited knowledge space (response dataset), the inclusion of the modal frequencies of a structure under artificial boundary conditions (referred to as “ABC frequencies”) is considered. Numerous ABC frequencies can be made available from a typical modal test for various configurations of the artificial boundaries. A method based on binary genetic algorithm is proposed to optimize the artificial boundary configuration such that the resulting ABC frequencies are most effective for updating. It is proved that ABC frequencies are effective candidate response data for FE model updating.

In addition to various numerical examples to demonstrate the implementation and the general effectiveness of the proposed schemes, experimental studies are also conducted to explore some practical aspects in the FE model updating.

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LIST OF SYMBOLS

$\{ \}^T, [ ]^T, [ ]^H$	Transposed vector, matrix, and transpose of complex conjugate
$[ ]^{-1}, [ ]^+$	Inverse, pseudo-inverse matrix
$\mathbf{I}$	Unity matrix
J	Objective function
$\{z_e\}$	Measured quantity vector
$\{z\}_j$	Current estimate of the analytical response vector
$\{\theta\}_j$	Parameter vector in the current iteration step
$\mathbf{W}_{\varepsilon\varepsilon}$	Positive-definite weighting matrix for response data
$\mathbf{W}_{\theta\theta}$	Positive-definite weighting matrix for structural parameters
$\mathbf{V}_\varepsilon$	Variance matrix of the measurement noise
$\mathbf{D}_j$	Correlation matrix for the j th parameter estimate and the measurement noise
$\mathbf{D}_j$	Correlation matrix between the j^{th} parameter estimate and the measurement noise
$\{\phi_a\}_i$	The i^{th} mode shape measured at the master DOFs
$\{\phi_b\}_i$	The i^{th} mode shape measured at the slave DOFs
$\mathbf{M}$	Global mass matrix
$\mathbf{C}$	Global viscous damping matrix
$\mathbf{K}$	Global stiffness matrix
$\mathbf{H}$	Frequency Response Function matrix
$\mathbf{H}^m$	Frequency Response Function matrix for a full-order model of a structure

---

$\bar{\mathbf{H}}^m$	Measured Frequency Response Function matrix
$\mathbf{Z}$	Impedance matrix
$\mathbf{S}_n$	Normalized sensitivity matrix
$\mathbf{G}_n$	Normalized gain matrix
$\mathbf{B}$	Dynamic stiffness matrix
$\mathbf{I}_B$	Integration of FRF matrix
$\mathbf{A}$	Fisher Information Matrix
Φ	Reduced and truncated modal matrix
EI	Bending stiffness
W_ω	Weighting factor assigned to the natural frequencies
W_ϕ	Weighting factor assigned to the mode shapes
J_ω	Penalty function associated with the natural frequencies
J_ϕ	Penalty function associated with the mode shapes
$q_{\max}$	The selection proportions assigned to the fittest chromosome
$q_{\min}$	The selection proportions assigned to the weakest chromosome
T_n	Tournament size for the genetic selection
$\mathbf{x}_m$	A row of the input data matrix of ANN
$\mathbf{x}_m^N$	A row of the normalized input data matrix of ANN
$\{\eta_b\}$	Left eigenvectors
v_j	A noise item
$\tilde{R}_j$	j^{th} response component after the noise injection
R_j	j^{th} response component
$\{\delta R\}_i$	Response error patterns
$\{\delta p\}_i$	Parameter error patterns

ζ_i	i^{th} modal damping ratio
λ_b^a	Antiresonance eigenvalue
$\bar{B}_l$	Lower bound vector of the optimization variables
$\bar{B}_u$	Upper bound vector of the optimization variables
M_j	Mean vector in a chromosome
V_j	Variance vector in a chromosome
c_{ji}	Asexually produced children from the chromosome j
c_j^*	The best of c_{ji}
V_i^R	Prescribed range for the initialization of the variance associated with the i^{th} variable
δ_i	Adaptation step of the variance associated with the i^{th} variable
μ_i	Mean value of the best fit individuals from all 50 runs
μ_{NG}	Finally evolved optimum from StGA
σ_{NG}	Standard deviation of the best fit individuals from the last generation
α	Individual frequency response function (FRF)
$P_{xx}(\omega)$	Auto-power spectral density of the excitation
$P_{xy}(\omega)$	Cross-power spectral density of the excitation and the response
$H_1(\omega)$	H_1 estimator of FRF

CHAPTER 1

INTRODUCTION

1.1 Background

Many structures worldwide constructed tens or even hundreds of years ago are still in service today. Their possible failure could be catastrophic not only in terms of losses of lives and economy but also concerning the social and psychological impacts. These structures are more or less in a deficient state with certain abnormal behavior owing to some complicated factors such as failure of the structural materials, loosening of assembled parts, damage of effective components due to flaws and cracks, and violation of modern design standards, among others. The 2001 ReportCard for America's Infrastructure pointed out that nearly 1/3 to 1/2 of the nation's infrastructure (roads, bridges, school buildings, etc.) are structurally deficient and need urgent repair (ASCE News 2001). Incidents have occurred where structures actually collapsed due to the lack of timely inspections and retrofits, causing injuries and deaths. For examples, two sections of a bridge in Taiwan broke away and dived into the river below on August 27th, 2000, injuring 22 people. On March 4th, 2001, a bridge in northern Portugal collapsed into a swollen river, killing dozens of people.

Structural deterioration has become a challenge to civil engineering community worldwide. To confront this issue, one of the focused areas in civil engineering research in recent years has been the damage diagnosis and identification, which aims at providing a reasonable assessment of a structure's condition in terms of the presence of damage, the location of damage, and the extent of damage (Doebeling et al. 1996). Such assessment is expected to lead to decisions as whether and to what extent repairing/retrofitting work needs [to](#) be performed on a particular structure. However, it

could be very costly in practice to scrutinize an entire structure in a conventional way for possible damages; and moreover, some local defects may not have significant structural effect. In this regard, it would be more desirable to have a reliable mathematical model (the FE model in particular) of the structure so that the relative importance of different structural changes can be assessed more reliably, not only in terms of their influence on the structural integrity in the present state, but also in terms of their influences on the performance of the structure under any foreseeable future loading conditions. The usefulness of such a validated structural FE model can be extended to the evaluation of the effectiveness of different repairing/strengthening schemes.

To establish a rigorous model for an existing structure, the FE model-updating techniques started to emerge in the 1970's. Generally speaking, the updating of a representative FE model is achieved by matching the analytical response to the actually measured response data. In this process, the physical properties (or their equivalent parameters) of the structure are identified. Thus, the updated model can be used to rate the soundness of a structure in terms of its current condition as well as the future performances under certain prescribed loading and boundary conditions.

The conventional model updating schemes can be classified into two categories: direct updating and iterative-based updating (Friswell and Mottershead 1995). The direct updating methods aim at achieving refined mathematical model by directly modifying the structural matrices, i.e. the stiffness matrix, mass matrix and damping matrix, such that the experimental data can be reproduced. The iterative-based updating methods, on the other hand, update the model by making physically meaningful adjustments on uncertain structural parameters until a satisfactory match between the analytical and experimental data is reached. With the updated structural parameters, this model would allow analytical simulations of the structure with changed boundary conditions and/or loading configurations, and hence can also be used to predict the structure performance subject to the above changes. In fact, the iterative-based updating methods have become dominant schemes in FE model updating applications.

Despite some reported successful applications in the past (Morassi and Rovere 1997; Mottershead and Mares 2000; Jones 2000; Brownjohn et al. 2003; Wu and Law 2004), the conventional iterative-based updating methods bear serious limitations due to three main constraints: a) the available measured data are usually incomplete, b) there exist random measurement noise and errors, and c) there often exist many vertices and valleys in the error function used for optimization. To deal with the lack of completeness of the measured information, either a reduction of the analytical model or an expansion of the measurement data is required in order to perform updating with the iterative-based methods. In this process, additional errors may be introduced. The inevitable measurement errors may lead to significant errors on some parameters being updated because of the inherent sensitivity to the measured data; whereas the existence of multiple plateaux and valleys on the error surface could snag the iterative-based updating process, which usually makes use of the first order derivative information of a predefined objective function, into a particular local optimum rather than a desired global solution.

The recent development in artificial intelligence algorithms provides some seemingly powerful alternatives to the traditional iterative-based optimization approaches for carrying out FE model updating. These algorithms operate on a mechanism that is capable of avoiding or minimizing the problems mentioned above. Among other algorithms, the Genetic Algorithms (GA) and Artificial Neural Networks (ANN) appear to exhibit the most attractive features in solving model updating related problems. For instance, both GA- and ANN-based model updating do not require model reduction or data expansion as commonly utilized in the traditional iterative-based updating schemes, thus avoiding the errors pertaining to such operations.

GAs originate from the analogy of the evolutionary process of biology following the “survival of the best” principle, and they perform a robust global optimization by starting with a population of potential solution candidates and iteratively evolving better individuals from generation to generation via genetic operations while exploring a given solution space (Holland 1962, 1975). GAs have been proven to be effective

and robust in solving a broad spectrum of real-world problems (Goldberg 1987; Jenkins 1992; Zimmerman 1993; Keane 1995; Dunn 1997; Rejeb and AbuElhaij 2000; Spalding and MacNish 2003; Ozcan and Onbasiloglu 2004). Recently, the applications of GAs have been extended to the damage identification and FE model updating fields (Mares and Surace 1996; Friswell and Penny 1998; Jung-Huai et al. 2001; Sazonov et al. 2002). However, according to the survey of literature conducted in the present study, the development of GA-based model updating techniques is still at a very basic stage, and much remains to be done to advance the exploration towards a comprehensive application. For example, no studies have been carried out to demonstrate the effectiveness of GAs in FE model updating from a systematic view point, except a few special numerical case studies. Furthermore, the potential advantage and efficiency that may result from a combination of GAs with conventional iterative-based updating methods are yet to be explored. Besides, the general low efficiency of the regular GAs concerning computational cost remains a major obstacle for their applications in dealing with practical situations. To improve the computational efficiency requires the consideration of more efficient and effective encoding strategies and proper methods for performing the corresponding genetic operations. The stochastic coding in GAs, in this regard, can be a competent candidate, and it still needs to be explored systematically

ANN, as another type of artificial intelligence algorithm, is inspired by the way the densely interconnected, parallel structure of the mammalian brain processes information. In essence, an ANN is embodied as a collection of mathematical models that emulate some of the observed properties of biological nervous systems and draw on the analogies of adaptive biological learning. Their exclusively attractive features include 1) generalization ability with a fast and accurate response to previously unseen input data, 2) error tolerance capability, and 3) ability to simulate natural phenomena with vaguely understood relation. These advantageous aspects enable ANNs to be applied successfully in a wide variety of areas (Sabourin et al. 1992; Cichocki et al. 1993; Szewczyk et al. 1993; Connor et al. 1994; Kim and Calise 1997; Spooner and Passino 1999; Lei and Meng 2003). The recent introduction of ANNs to damage detection and model updating applications has brought new hope in tackling the

measurement noise (error) problem, as demonstrated in several reported studies (Masri et al. 1995; Atalla et al. 1998; Chung-Bang et al. 2001). In their work, however, efforts were made mainly to explore the ANNs with some special applications. The inherent noise-resistance ability of ANNs has not been looked upon in a systematic manner. Furthermore, the structural systems being investigated were quite simple, and the results do not necessarily represent what could happen in more realistic engineering structures. Another desirable feature of ANNs is that they work directly on the input-output data pairs and do not require special properties such as the differentiability of the inputs and outputs. This, in conjunction with the generalization and the potential noise-resisting abilities, makes ANN a desirable choice in dealing with complicated model updating and damage identification problems. In chapter 4 of this thesis, the above-mentioned favorable features are explored by applying ANN to identify structural damping properties using FRF integrals as the response data.

Although artificial intelligence algorithms (GAs and ANNs) are robust for model updating in many different aspects, their success, however, still relies on the suitability of the available response data with respect to the unknown parameters. Where a large number of parameters need to be identified for a real structural system, a reliable updating would require the availability of sufficient and high quality response data. However, the quantity and quality of available response data are subjected to many practical constraints. Therefore, appropriate selection of the data to be measured plays also a crucial role in the application of good model updating algorithms. In the vibration-based model updating, the accuracy of the mode shape measurement is often questionable. A recent development of the artificial boundary method (ABM) has brought a potentially effective substitute for mode shape data for model updating. Via the ABM, the modal frequencies of a structural system under different artificial boundary conditions can be obtained from the normal modal testing without the need of physical modification to the actual structure. This means a huge number of additional dynamic frequency data can be made available to expand the response dataset. The measurement accuracy of such frequencies can be as good as the natural frequencies. So far, only a few exploratory applications of ABM for model updating have been

reported (Gordis 1999; Jones and Turcotte 2002), in which the selection of the artificial boundaries was made according to experience and judgment. To make this scheme more effective in the model updating applications, a rigorous approach for optimizing the configurations of the artificial boundaries is needed.

1.2 Research Objectives

Motivated by the understandings outlined above, this study aims to carry out a comprehensive investigation of the application of ANNs and GAs for model updating and structural assessment, and develop novel schemes to improve the efficiency and robustness of the algorithms in dealing with the FE model updating problems. Besides, the effectiveness of using extra response data, particularly the modal frequencies under artificial boundary conditions, is also investigated.

More specifically, the main objectives of this research can be summarized as follows:

- 1) To evaluate the effectiveness and limitations of using GA to carry out general FE model updating tasks. Particular attention is paid to the performance of the algorithm under practical restrictions, such as limited order of measured modal data, and the influence of noise on the updating results.
- 2) To develop a combined GA and eigensensitivity approach for improved updating results with limited modal data. In this approach, GA is used for the search of an approximate global solution while the traditional gradient-based updating scheme is incorporated to perform further local hill-climbing. In this way, the advantages of both algorithms can be exploited.
- 3) To improve the general efficiency of GAs so that large size updating problems may be handled using GA-based optimization schemes. For this purpose, a GA with

stochastic coding (referred to as “StGA”) will be studied and a generic procedure for the implementation of StGA will be developed.

- 4) To investigate the application of ANNs in performing FE modal updating, with particular interests in their noise resistance abilities. A methodology will be developed to implement the noise-injection learning algorithm in the training of ANN and subsequently evaluate the actual noise-resisting ability of the trained network.
- 5) To explore the effectiveness of using extra modal frequency information under artificial boundary conditions of the structure for model updating, especially for cases where less sensitive parameters are involved, and develop a GA-aided procedure for the optimal selection of the artificial boundaries.

1.3 Thesis Organization

This thesis consists of eight chapters. Chapter 1 gives a brief introduction of the background and the research objectives. Chapter 2 presents a literature review of existing FE model updating techniques.

In Chapter 3, an outline of the GA paradigm and a review and discussion of some previous work on GA-based damage identification and FE model updating are presented first. A methodology is then developed, in which GA is applied to yield a global solution, while the conventional eigensensitivity approach is incorporated to perform local hill-climbing for further refinement of the updated model. Both numerical and experimental studies are carried out to demonstrate the implementation of the procedure and the accuracy of the updating results.

Chapter 4 starts with a general introduction to the ANN algorithms and a review and discussion of their applications in FE model updating. A two-level updating network scheme is then developed for FE model updating involving also the damping parameters. The measurement data considered include anti-resonance frequencies, as well as frequency response function data.

Chapter 5 presents a stochastic genetic algorithm (StGA) for large-size optimization problems. StGA employs a stochastic coding technique such that the optimum is evolved through stochastically defined regions rather than single points as in usual GAs. The performance of StGA is compared with a number of other global optimization methods through the optimization of a broad range of test functions. The comparison results show that StGA is superior over other techniques in handling large-size problems.

Chapter 6 presents a GA-aided procedure of using modal frequencies under artificial boundary conditions for FE model updating. A method based on binary genetic algorithm is proposed to optimize the selection of artificial boundaries for the generation of such frequencies. Numerical studies are carried out to verify the effectiveness of the proposed method.

An experimental investigation of the model updating on two generic frames is presented in Chapter 7. The laboratory modal testing, data acquisition and modal analysis are described. The experimental modal data are employed in the updating of FE models for the frames. Some practical considerations, such as an appropriate choice of the parameters to identify and the simplification of joints for model updating, are discussed and incorporated in the updating process. Finally, Chapter 8 provides a summary of the main conclusions from this study.

CHAPTER 2

REVIEW OF CLASSICAL FE MODEL UPDATING TECHNIQUES

2.1 Introduction

The finite element method (FEM) has become the predominant tool of analyzing the general behaviors and dynamics of modern structures. It allows a complex continuous structure to be mathematically approximated as a discrete system made up of mass, stiffness and damping matrices.

However, real world structures have very complicated geometry, which require hundreds of thousands of degree of freedoms (DOF) to accurately model them. Even for relatively simple structures, there may exist different kinds of joint connections among members such as bolted connections, welded connections, etc, which are very difficult to model without resorting to fine FE mesh. However, the computational burden in analyzing such large order problems can be intractable. Therefore, a reasonable way out is to make rational simplifying assumptions on structural geometry and connections so as to keep the order of the model manageable.

When modeling structures using FEM, three basic types of errors can exist (Mottershead and Friswell, 1993): (i) *model structure errors* - these errors occur when the model doesn't represent the physical behavior of the prototype. Typical examples of this type of errors can be omission of important physical relationships, erroneous

modeling of boundary conditions, mismodeling of joints, a non-linear structure assumed to be linear, and wrongly connected elements. Some recent works in connection with this aspect can be found in (Gordis 1996; Freund and Ben-Haim 1995; Gladwell 1997). (ii) *model order errors* - these errors occur while the FE mesh is not sufficiently fine for a proper representation of the characteristics of the structure. (iii) *Model parameter errors* - these errors will tend to govern when the aforesaid two sources of errors are resolved, but the numerical value of the physical parameters of the structure, i.e., geometrical and material parameters are incorrect.

The first two types of errors can generally be reduced to an acceptable degree by engineering judgment in conjunction with a trial-and-error method. The third type of error, i.e., the model parameter errors, is difficult to be corrected without a systematic methodology. It is, in fact, the central issue of the finite element model updating methods.

The FE model updating techniques began to appear in the early 1970s as a development of system identification and has since the 1990s become a subject of considerable interests in diverse fields such as the design, construction and maintenance of mechanical systems and aerospace structures. An early review paper regarding FE model updating appeared in 1979 (Ibanez 1979), and it gave a detailed account of pseudo-inverse techniques including singular value decomposition (SVD). Link (1986) presented comparisons among some specific applications in aerospace and structural engineering. Natke (1988) produced a review paper addressing various special aspects about updating. Ibrahim (1988) discussed model reduction and eigenvector expansion in connection with sensitivity techniques and Lagrange multiplier approaches used for such response data as modal data and frequency response function (FRF). Imregun and Visser (1991) discussed error matrix methods. A recent comprehensive review paper on FE model updating was reported by Mottershead and Friswell (1993).

FE model updating has become a widely adopted means to correct the errors in FE models by matching the response outputs predicted from a FE model with the

experimental counterpart. The model updating techniques are implemented by modifying the structural mass, stiffness, and damping parameters of the FE model until a satisfactory agreement between analytical and experimental modal data is reached. Certainly, system identification techniques can also produce a mathematical model to accurately reproduce the measured data. However, such “representational” model due to the lack of physical interpretation is inadequate for predicting the behavior of the system under different loading, boundary conditions, or configurations (Friswell et al. 1995). In contrast, a physically interpretable FE model can be achieved with some FE model updating techniques since in these methods the close match between the analytical and the measured data is obtained by making changes to physically meaningful model parameters. Therefore, such an updated model can be used to evaluate the condition of the structure at the present state and to predict the performance and possibly even the remaining service life of the structure under foreseeable future loading conditions. Hence, it is a vital part of monitoring the integrity of structural systems during their service life.

2.2 Model Updating Methods

To date, a variety of methods for FE model-updating have been proposed (Mottershead and Friswell 1993). From the viewpoint of modification of system matrices or estimation techniques, model updating techniques may be classified into two categories: 1) Direct techniques, and 2) Iterative techniques.

2.2.1 Direct Techniques

Direct methods are of one-step procedure and they may be classified as global methods. This type of methods directly reconstructs the updated global system matrices, i.e., stiffness matrix, mass matrix, and damping matrix from the reference data (usually the

measured modal data including natural frequencies and mode shapes). Such techniques possess two great advantages compared to iterative methods. First, they do not need iteration and therefore the possibilities of divergence and excessive computation are eliminated. Second, they can exactly reproduce the experimental data, which cannot be achieved by iterative methods. Despite these apparent advantages, this category of techniques is rarely adopted in practical model updating due to the following two main reasons: 1) the updated results depend largely on the mode shape data; however, the measurement accuracy for mode shapes is often questionable, and 2) the updated system matrices lose their physical meaning, which implies a loss of representation of the actual connectivity of the nodes, and the favorable characteristics of the system matrices such as symmetry, positive-definiteness and sparseness.

Minas & Inman (1990) and Smith & Beattie (1991) developed iterative methods to avoid the connectivity problem. In their methods, quasi-Newton methods were considered for stiffness updating and structural connectivity can thus be preserved. However, interpreting the results is further complicated because measurements are lower frequency modes while the higher modes contribute most to the stiffness matrix. Lam & Inman (1995) and Starek and Inman (1995) have tried to address the problems of symmetry and positive-definiteness.

Some commonly used updating methods of this class are summarized below:

- a) Reference basis methods. These methods were introduced by Baruch (1978, 1982 and 1984) and Berman (1979a, 1979b and 1983) in the late 1970s and early 1980s. Their working mechanism is simply to minimize a function subject to exact constraints on the independent variables. A large subset of methods in this category have so far been devised and they contributed to either improve the updated model or extend the method applicability (Baruch and Bar Itzhack 1978; Caesar 1986; Berman and Nagy 1983). However, the updated model from these algorithms could exhibit spurious modes in the frequency range of interest and therefore the subsequent use of the model may be unreliable.

- b) *Matrix mixing methods*. This category of schemes (Caesar 1987; Link et al. 1987) is a development of methods of Thoren (1972) and Ross (1971). The matrix mixing approaches use the data from the finite element model to fill in the gaps in the measured data so that the full order complete modal information is made available for further application. The updated system matrices, however, are generally fully populated and hence bear little relation to the physical connectivity in the structure. To et al. [13] and Neidbal et al. [14] advanced the methods by enforcing the orthogonality with respect to the measured modal vectors, which has the advantage of preserving the physical connectivity of the updated FE model.
- c) *Eigenstructure assignment methods*. These methods were pioneered by Minas and Inman (1988; 1989; 1990). The procedure of eigenstructure assignment was developed by Srinathkumar (1978) and adapted to mechanical structures by Andry et al (1983). Its operational mechanism is based on the design of a fictitious controller, which would minimize the modal force error. This strategy is limited to only update structural stiffness and damping matrices, while the mass matrix is assumed to be accurate. The considerable virtue of these methods is that they require the number of measured eigenvectors to be less than that of DOFs since rotational and internal degrees of freedom, which are present in finite element models, are mostly unmeasurable.

2.2.2 Iterative Techniques

Iterative-based model updating methods are generally based on sensitivity analysis aiming at reproducing the measured response data, such as frequencies and mode shapes, from the updated FE model. These methods are also called sensitivity-based methods, which allow a wide choice of parameters to update. Different categories of measured data may be used with different approaches. The measured data can be

generally grouped into two categories: a) dynamic data, and b) static and other forms of system data.

In what follows, a commonly used iterative updating method called “Penalty Function Methods” is described in detail. Two other types of iterative methods, namely, Minimum Variance Methods and Perturbed Boundary Condition Testing Methods, are then discussed briefly.

2.2.2.1 Penalty Function Methods

Penalty function methods generally use a truncated Taylor series expansion of a certain type of response data as a function of the unknown parameters subject to updating. This expansion is often limited to the first two terms to produce a linear approximation. The parameter increments in the iteration procedure are determined by a minimization of a predefined penalty function.

A) Penalty Function Method Using Dynamic Data

Dynamic or vibration data (and possibly its derivative information as well) are most commonly used data for FE model updating because of the advantage that they depend only on the structural inherent properties regardless of what excitation is applied. The basic premise is that the FE modeling errors in stiffness, mass or energy dissipation properties of a system will affect sensibly the predicted dynamic response of that system, and hence, by matching the measured dynamic response data, the errors in the model properties can be corrected.

The system dynamic feature can be described either in modal domain or frequency domain. Correspondingly, two types of penalty function methods exist.

i) Modal Domain Dynamic Data

An objective function involving modal data including the natural frequencies and the mode shapes is minimized. The commonly used objective functions or error functions include (Xia 2000):

- a) Average value of weighted relative difference between predicted and measured frequencies,

$$E_{fr} = \frac{1}{N} \sum_{i=1}^N C_{R_i} \frac{\Delta f_i}{f_i}$$

where N , C_{R_i} , Δf_i and f_i respectively denotes the number of frequencies, the weighting factor assigned to the i^{th} frequency, the difference between the i^{th} predicted and the measured frequency, and the i^{th} measured frequency.

- b) Average value of weighted margin of the MAC (Modal Assurance Criterion) values selected as responses,

$$E_{MAC} = \frac{1}{C_R} \sum_{i=1}^N (1 - C_{R_i} MAC_i) \quad C_R = \sum_{i=1}^N C_{R_i}$$

- c) Average value of weighted relative difference between predicted and measured mode shape components,

$$E_{f\phi} = \frac{1}{C_R} \sum_{i=1}^{N_m} \sum_{j=1}^{N_{nd}} C_{R_{ij}} \frac{\Delta \phi_{ij}}{\phi_{ij}} \quad C_R = \sum_{i=1}^{N_m} \sum_{j=1}^{N_{nd}} C_{R_{ij}}$$

where N_m denotes the total number of modes selected, $N_{nd} = N_n \times N_d$, N_n is the number of nodes selected and N_d is the number of DOF selected at each node.

d) Combination of all the above-mentioned error functions

$$E = E_{fr} + E_{MAC} + E_{f\phi}$$

It has to be pointed out that these objective functions are often nonlinear functions of the structural parameters being updated, and hence may cause convergence problems. Besides, they may possess numerous local optima that can trap the optimization procedure. Nevertheless, iterative methods are attractive as they allow a large number of physical parameters to be updated simultaneously.

In formulating the minimization problem, the FE modal data needs to be expressed as a function of the selected structural physical parameters using 1st order Taylor series expansion:

$$\{\delta\mathbf{z}\} = \mathbf{S}_j \{\delta\boldsymbol{\theta}\} \quad (2-1)$$

where $\{\delta\boldsymbol{\theta}\} = \{\boldsymbol{\theta}\} - \{\boldsymbol{\theta}\}_j$, $\{\boldsymbol{\theta}\}_j$ is the parameter vector in the current iteration step, $\{\boldsymbol{\theta}\}$ is the initial estimated parameter vector. $\{\delta\mathbf{z}\}$ is the difference between the measured and current analytical responses (modal data), $\mathbf{S}_j$ is the sensitivity matrix containing the 1st order derivative of the modal data with respect to the unknown parameters, evaluated at the current parameter estimate $\{\boldsymbol{\theta}\}_j$.

Depending on the number of parameters to identify as compared to the number of responses used in the updating procedure, there may be two categories of systems: (i) Overdetermined Systems, where the number of parameters is less than that of

responses; (ii) Underdetermined Systems, where the number of parameters is more than that of responses.

For both systems, the penalty function can be expressed as

$$J(\{\delta\theta\}) = \{\varepsilon\}^T \{\varepsilon\} \quad (2-2)$$

where $\{\varepsilon\} = \{\delta z\} - \mathbf{S}_j \{\delta\theta\}$ is the error in the predicted responses based on the current parameter estimate. Substituting the expression for ε into equation (2.2) gives

$$J(\{\delta\theta\}) = \{\delta z\}^T \{\delta z\} - 2\{\delta\theta\}^T \mathbf{S}_j^T \{\delta z\} + \{\delta\theta\}^T \mathbf{S}_j^T \mathbf{S}_j \{\delta\theta\} \quad (2-3)$$

Minimizing J involves differentiating it with respect to each entity in the parameter vector $\{\theta\}$ and setting the expressions equal to zero.

For overdetermined Systems, the result from Eq. (2-3) is given by

$$\{\delta\theta\} = [\mathbf{S}_j^T \mathbf{S}_j]^{-1} \mathbf{S}_j^T \{\delta z\} \quad (2-4)$$

Or

$$\{\theta\}_{j+1} = \{\theta\}_j + [\mathbf{S}_j^T \mathbf{S}_j]^{-1} \mathbf{S}_j^T \{\delta z\} \quad (2-5)$$

Considering the fact that measurement errors always exist, and high order modal data are usually prone to higher measurement errors, an improvement to equation (2.5) is to impose distinct weighting matrices so as to compensate for these experimental differences. The weighted penalty function can be written as

$$J(\{\delta\theta\}) = \{\varepsilon\}^T \mathbf{W}_{\varepsilon\varepsilon} \{\varepsilon\} \quad (2-6)$$

where $\mathbf{W}_{\varepsilon\varepsilon}$ is a positive-definite weighting matrix, usually in the form of a diagonal matrix whose elements are given by the corresponding reciprocal of the variance of the individual measurements. Solving the minimization problem (2-6) yields

$$\{\theta\}_{i+1} = \{\theta\}_i + [\mathbf{S}_j \mathbf{W}_{\varepsilon\varepsilon}^T \mathbf{S}_j]^{-1} \mathbf{S}_j^T \mathbf{W}_{\varepsilon\varepsilon} \{\delta z\} \quad (2-7)$$

Because the system is over-determined, the matrix $[\mathbf{S}_j^T \mathbf{W}_{\varepsilon\varepsilon} \mathbf{S}_j]$ is square and probably of full rank. If a rank deficiency occurs, a singular value decomposition technique (SVD) may be adopted to solve the system of equations.

When the system is underdetermined, $\mathbf{S}^T \mathbf{S}$ becomes rank deficient. The set of equations is underdetermined, leading to multiple possible solutions. Under this circumstance, the one that represents the smallest change relative to the values of the initial parameter guess is often chosen as the most desirable solution. Therefore, the optimization problem is now constrained and can be stated as

$$J(\{\delta\theta\}) = \{\delta\theta\}^T \mathbf{W}_{\theta\theta} \{\delta\theta\}, \quad \text{s.t.} \quad \{\delta z\} = \mathbf{S} \{\delta\theta\} \quad (2-8)$$

The corresponding solution to this problem is obtained as

$$\{\theta\}_{j+1} = \{\theta\}_j + \mathbf{W}_{\theta\theta}^{-1} \mathbf{S}_j^T [\mathbf{S}_j \mathbf{W}_{\theta\theta}^{-1} \mathbf{S}_j^T]^{-1} \{\delta z\} \quad (2-9)$$

It should be pointed out that better updating results can generally be achieved by normalizing the selected responses and the updating parameters because of the improved conditioning with the sensitivity matrix $\mathbf{S}$.

Numerous applications of using modal domain data for damage identification and parameters estimation can be found in the literature. Kiddy and Pines (1997) assigned a sensitivity-based updating technique to identify damages of a rotating composite

beam in a vacuum. The modal residuals of the structural response were used as the response data in the updating procedure. Elemental sensitivities were derived including an enhanced stiffness sensitivity arising from centrifugal loading of the composite beam. This methodology was analytically shown to be capable of detecting changes of both mass and stiffness in the rotating beam.

Morassi and Rovere (1997) applied a sensitivity-based updating algorithm for detecting a notch in a five-storey steel frame structure using only frequency data of the structure before and after damage. Torkamani and Ahmadi (1998) reviewed four least-squares methods to solve a sensitivity-based equation, which was derived from minimizing the difference between analytical and actual eigendata, and developed a new statistical technique to identify stiffness change. Numerical analysis of two frame structures demonstrated that the statistical method provided more accurate parameter estimation when eigenvalues and eigenvectors were combined.

Ara_ujo dos Santos J.V et al. (2000) presented the use of orthogonality conditions sensitivities of mode shapes in the updating procedure to detect damages in composite structures. The damage was directly related to the stiffness reduction of the damaged element. A numerical example on a laminated rectangular plate showed good efficiency and stability of the proposed method on the identification of damage in multiple elements. The method was also shown to be applicable with noise-contaminated modal data. In addition, the orthogonality condition sensitivities were demonstrated to be more effective measures as compared to the eigenvector sensitivities.

Brownjohn and Xia (2000) tuned some uncertain parameters of substructures of a curved cable-stayed bridge to obtain an improved model based on measured dynamic data. Mottershead and Mares (2000) validated an aluminum space frame model using a sensitivity-based iterative technique, in which over twenty parameters were updated simultaneously. Jones (2000) used an iterative FE model updating technique to tune a flexible truss structure model by taking the joint rigid link parameters as unknown

quantities and incorporating anti-resonance into the modal data to make the problem well-conditioned.

Brownjohn et al. (2003) applied a sensitivity-based model updating procedure to validate the FE model of a highway bridge before and after upgrading. Measurements were taken to extract the structural modal data. The updating exercise successfully identified the girder stiffness in the two directions and the effects of two major structural changes from upgrading, namely an enhanced rotational constraint at the abutment and the addition of large guardrails. It was concluded that a procedure using non-invasive full-scale dynamic testing allied with FE model updating can assist bridge managers assess their structures by providing validated structural models.

ii) Frequency Domain Dynamic Data

The basic premise of a successful application of modal data based iterative methods is that the modal information, i.e., natural frequencies and mode shapes, must be extracted with satisfactory accuracy. However, it is often difficult to extract sufficient and accurate modal data from a structure with high modal density. To evade this difficulty, using frequency response function (FRF) data in the objective function is deemed to be a more flexible alternative.

Two different measures of error are applicable in the objective function using frequency domain data, namely equation error and output error.

Taking the Laplace transformation of the FE equation of motion (EOM) one obtains the expression of EOM in frequency domain, given by

$$[\mathbf{K} + j\omega\mathbf{C} - \omega^2\mathbf{M}]\{x(\omega)\} = \{f(\omega)\} \quad (2-10)$$

or

$$\mathbf{B}(\omega)\{x(\omega)\} = \{f(\omega)\} \quad (2-11)$$

where $\mathbf{B}(\omega) = [\mathbf{K} + j\omega\mathbf{C} - \omega^2\mathbf{M}]$ is the dynamic stiffness matrix.

The measure of equation error is calculated by (Friswell and Mottershead 1995)

$$\{\varepsilon_E\} = \{f(\omega)\} - \mathbf{B}(\omega)\{x(\omega)\} \quad (2-12)$$

The simplest form of objective function is to minimize the Euclidean norm of the equation error, which is given by

$$J(\{\theta\}) = \|\{\varepsilon_E\}\|^2 = \sum_{i=1}^n \sum_{j=1}^m \left| \{f(\omega_j)\}_i - (\mathbf{B}(\theta, \omega_j)\{x(\omega_j)\})_i \right|^2 \quad (2-13)$$

where n is the number of DOF and m is the number of measured frequencies. The minimization of $J(\{\theta\})$ is equivalent to solving the following linear equation,

$$\begin{bmatrix} \mathbf{B}_1(\omega_1)\{x(\omega_1)\} & \mathbf{B}_2(\omega_1)\{x(\omega_1)\} & \cdots & \mathbf{B}_p(\omega_m)\{x(\omega_m)\} \\ \mathbf{B}_1(\omega_2)\{x(\omega_2)\} & \mathbf{B}_2(\omega_2)\{x(\omega_2)\} & \cdots & \mathbf{B}_p(\omega_2)\{x(\omega_2)\} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{B}_1(\omega_m)\{x(\omega_m)\} & \mathbf{B}_2(\omega_m)\{x(\omega_m)\} & \cdots & \mathbf{B}_p(\omega_m)\{x(\omega_m)\} \end{bmatrix} \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \vdots \\ \delta\theta_p \end{bmatrix} = \begin{bmatrix} \{f(\omega_1)\} - \mathbf{B}_0(\omega_1)\{x(\omega_1)\} \\ \{f(\omega_2)\} - \mathbf{B}_0(\omega_2)\{x(\omega_2)\} \\ \vdots \\ \{f(\omega_m)\} - \mathbf{B}_0(\omega_m)\{x(\omega_m)\} \end{bmatrix}$$

or

$$\mathbf{A}\{\delta\theta\} = \{b\} \quad (2-14)$$

where A and b are real components of the matrices above. By taking the pseudo-inverse of A , one can solve for $\delta\theta$ yielding

$$\{\delta\theta\} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \{b\} \quad (2-15)$$

In case of singularity, some advanced techniques such as singular value decomposition (SVD) are required to be applied in order to solve for $\{\delta\theta\}$ in equation (2-15).

The output error is given by

$$\{\varepsilon_o\} = \mathbf{B}^{-1}(\omega)\{f(\omega)\} - \{x(\omega)\} \quad (2-16)$$

The advantages of this approach lie in that it minimizes the difference between measured and analytical data, and that it is not necessary to measure all DOF. The drawback is that the output error loses its good characteristics having linear relationship with the entries in $\mathbf{B}(\omega)$, and instead, a highly nonlinear phenomenon emerges.

Similarly, the objective function is constructed to minimize the Euclidean norm of the output error

$$J(\{\theta\}) = \|\{\varepsilon_o\}\|^2 = \sum_{i=1}^r \sum_{j=1}^q \sum_{k=1}^m \left| \left[\mathbf{a}_m(\omega_k) - \mathbf{D}\mathbf{B}^{-1}(\{\theta\}, \omega_k)\{f(\omega_k)\} \right]_{ij} \right|^2 \quad (2-17)$$

where $\mathbf{D}$ is the transducer location matrix and $\mathbf{a}_m$ is the experimental receptance. This is a highly nonlinear equation. In order to obtain the optimal solution of (2-17), the 1st order derivative of the inverse of the dynamic stiffness matrix $\mathbf{B}^{-1}$ with respect to the parameters must be calculated beforehand, which can be fulfilled by differentiating $\mathbf{B}\mathbf{B}^{-1} = \mathbf{I}$

$$\frac{\partial(\mathbf{B}\mathbf{B}^{-1})}{\partial\theta_j} = \mathbf{0} \Rightarrow \frac{\partial\mathbf{B}^{-1}}{\partial\theta_j} = -\mathbf{B}^{-1} \frac{\partial\mathbf{B}}{\partial\theta_j} \mathbf{B}^{-1} \quad (2-18)$$

Some numerical methods require the 2nd-order derivative of $\mathbf{B}^{-1}$, which can be got via differentiating (2-18) with respect to θ_k ,

$$\frac{\partial^2 \mathbf{B}^{-1}}{\partial \theta_j \partial \theta_k} = - \left(\frac{\partial \mathbf{B}^{-1}}{\partial \theta_k} \frac{\partial \mathbf{B}}{\partial \theta_j} \mathbf{B}^{-1} + \mathbf{B}^{-1} \frac{\partial^2 \mathbf{B}}{\partial \theta_j \partial \theta_k} \mathbf{B}^{-1} + \mathbf{B}^{-1} \frac{\partial \mathbf{B}}{\partial \theta_j} \frac{\partial \mathbf{B}^{-1}}{\partial \theta_k} \right) \quad (2-19)$$

B) Penalty Function Methods using Static and Other Forms of System Data

The methods described above are based on structure dynamic information either in modal domain or in frequency domain. In fact, static data can also be effective in some situations, with a similar updating procedure.

Sanayei and Onipede (1991) proposed a parameter identification method in which the error function is defined as the applied static force minus the structural stiffness matrix multiplied by the measured displacements. They implemented the method in the numerical analysis of a truss and a frame structure.

Farehat and Hemez (1993) applied the iterative-based method to update a FE model via mode expansion by which the connectivity of the model with the initial one is preserved. The modal force error is defined as a function of the non-measured mode shapes (expanded) and the elemental parameters. In their study, the mass matrix was also updated. Hemez and Farhat (1995) combined the modal force error and the static force error in the error function to identify structural damage. Experiments on the NASA ten-bay truss demonstrated the difficulties in assessing damage in some cases.

Sanayei and Saletnik (1996a) used a strain error function, defined as the difference between the analytical strains and measured ones, to replace the static force error function. In the companion paper, Sanayei and Saletnik (1996b) applied this method on a truss and a frame to perform parameter estimation.

It should be mentioned that the above described updating methods, regardless the type of measured data, are all based on deterministic strategies. There are of some statistical model updating schemes available in the literature. Examples include minimum variance estimation, perturbation methods, simulation schemes, and Bayesian

frameworks. Detailed information regarding these algorithms can be found in Friswell (1995) and will not be discussed in detail here.

2.2.2.2 Perturbed Boundary Condition Testing Method

These methods were motivated by the fact that in general the measured data is not available in enough quantity to enable a unique solution for the parameters to be identified. Increasing the available measured dataset is therefore desirable. The perturbed boundary condition testing (Chen et al. 1993 and Lammens et al. 1993), as its title implies, was developed to perform model updating by including the combination of the response data of the same structure under slightly different configurations or with masses or springs added to the structure (Nalitoela et al. 1990 and 1992). A sensitivity matrix S_i is calculated with respect to each different configuration of the structure to form the final S for the updating.

The central issue is how to choose adequately the coordinates to perturb, i.e., where to exert extra masses and where to impose additional springs. It is possible that extra data obtained from some configurations do not at all improve the conditioning of the updating problem while increasing computational burden. The so called condition number of the sensitivity matrix S is generally used as a measure of the quality of the response data from the selected structural configuration.

Despite the appealing merits of such an approach, it has to be realized that its application may experience some difficulties. For example, the masses added must be significant to effect a non-linear change of frequencies with respect to the structural parameters; and when introducing extra inertia to the translational degree of freedom, the inertia of the coincident rotary degree of freedom may also be altered in an appreciable manner, and vice versa. Besides, these methods are costly in practice since multiple tests are required. For some structures, it could be difficult to mount lumped masses or spring elements due to their particular design and serving environment as well. In these regards, the use of antiresonance data is considered to be

a better alternative to the perturbed boundary condition testing methods. The acquisition of antiresonance data does not require physical changes to the structure and a large amount of frequency data can be obtained by exerting fictitious constraints on the structure. In this thesis, the incorporation of antiresonant frequencies in FE model updating is investigated in Chapter 6 and a GA-aided scheme is proposed to find an optimal configuration of the fictitious or artificial constraints.

2.2.2.3 Minimum Variance Methods

The above described updating methods, regardless of the type of measured data, are all based on deterministic strategies. Minimum variance methods are statistical model updating schemes and may be treated as penalty function methods in which the weighting matrices vary in a particular way over iterations. The methods originate from Bayesian estimation with the only difference being the need to iterate in model updating as the modal model is a non-linear function of the uncertain structural parameters. Statistical techniques are known to be most suitable for handling large amount of data, though they are considered to be appealing when used to deal with the model updating problem in which the quantity of measured data is small and limited. This is because the approach offers a rational way to weight the measured and analytical data and meanwhile provides a measure of the quality of the updated parameters via their estimated variance.

Collins et al. (1972 and 1974) introduced the minimum variance method on the assumption that the parameter estimates and the measured data are statistically independent. This assumption, however, becomes untrue after the first iteration. Friswell (1989) has overcome this issue by calculating the correlation between the measurements and the updated parameter estimates at each iteration. This correlation matrix is then used to compute the next estimate of the parameter vector based on the argument similar to that of Collins et al. (1972 and 1974).

The utmost objective of the minimum variance methods is to seek the parameter estimate with minimum variance. In the solution procedure, the assumption is made on the measured data that its errors have zero mean and variance matrix $\mathbf{V}_\varepsilon$, i.e.,

$$E[\{\varepsilon\}] = \{0\} \text{ and } Var[\{\varepsilon\}] = E[\{\varepsilon\}\{\varepsilon\}^T] = \mathbf{V}_\varepsilon \quad (2-20)$$

where E denotes the expected value. Let $\{\theta_0\}$ be the original estimate of the uncertain parameters. The mean and variance matrices of this estimate are

$$E[\{\theta_0\}] = \{\theta\} \quad Var(\{\theta_0\}) = \mathbf{V}_0 \quad (2-21)$$

Considering that the parameter estimates after the 1st iteration will not be independent from the measurement error $\{\varepsilon\}$, a non-zero correlation matrix $\mathbf{D}_j$ is used for the j th parameter estimate and the measurement noise,

$$E[\{\theta\}_j \{\varepsilon\}^T] = \mathbf{D}_j \text{ and assume } \mathbf{D}_0 = \mathbf{0} \quad (2-22)$$

Thus, the updated parameter estimate is given by

$$\{\theta\}_{j+1} = \{\theta\}_j + (\mathbf{V}_j \mathbf{S}_j^T - \mathbf{D}_j) \mathbf{V}_{zj}^{-1} (\{z\}_m - \{z\}_j) \quad (2-23)$$

where $\mathbf{V}_{zj}$ is the self-correlation matrix for the difference vector between the measurement data and the FE data corresponding to the j^{th} parameter estimate and is given by

$$\mathbf{V}_{zj} = E[(\{z\}_m - \{z\}_j)(\{z\}_m - \{z\}_j)^T] = \mathbf{S}_j \mathbf{V}_j \mathbf{S}_j^T - \mathbf{S}_j \mathbf{D}_j - \mathbf{D}_j^T \mathbf{S}_j^T + \mathbf{V}_\varepsilon \quad (2.24)$$

The updates of matrices $\mathbf{V}_j$ and $\mathbf{D}_j$ through iterations are implemented according to the following two equations, respectively

$$\mathbf{V}_{j+1} = \mathbf{V}_j - [\mathbf{V}_j \mathbf{S}_j^T - \mathbf{D}_j] \mathbf{V}_{zj}^{-1} [\mathbf{V}_j \mathbf{S}_j^T - \mathbf{D}_j]^T \quad (2-25)$$

$$\mathbf{D}_{j+1} = \mathbf{D}_j + (\mathbf{V}_j \mathbf{S}_j^T - \mathbf{D}_j) \mathbf{V}_{zj}^{-1} (\mathbf{S}_j \mathbf{D}_j - \mathbf{V}_\varepsilon) \quad (2-26)$$

It is noteworthy that for overdetermined systems (less number of parameters than measurements), the variance of measurements cannot be set to zero; otherwise, the iteration process will break down and the parameter estimates will diverge simply because there is no sufficient freedom for the parameters to reproduce the measurements.

2.3 Model Reduction/ Expansion Algorithms

Owing to the fact that measurements are usually limited to some selected DOFs (normally translational DOFs), the model reduction or data expansion techniques must be incorporated during updating, i.e., either the analytical model is reduced to be compatible with experimental DOFs, or the measured data are expanded to match the DOFs in the analytical model.

2.3.1 Model Reduction Algorithms

Most reduction techniques employ a transformation matrix, $\mathbf{T}$, between the measured or master DOFs (represented by subscript a) and the total DOFs of the analytical model (denoted by subscript n), as expressed by the following equation:

$$\{\phi_n\}_i = \begin{Bmatrix} \phi_a \\ \phi_b \end{Bmatrix}_i = \mathbf{T} \{\phi_a\}_i \quad (2-27)$$

where subscript b indicates the unmeasured DOFs, or slavery DOFs. The corresponding stiffness and mass matrices can be correlated through the transformation matrix as follows:

$$\mathbf{K}_a = \mathbf{T}^T \mathbf{K}_n \mathbf{T} \quad \mathbf{M}_a = \mathbf{T}^T \mathbf{M}_n \mathbf{T} \quad (2-28)$$

Depending on different forms of the transformation matrix, $\mathbf{T}$, several model reduction methods have been proposed as follows.

■ Guyan Reduction

Guyan reduction (Guyan 1965) is essentially a static condensation and is especially suitable for large analytical models. When ignoring inertia and damping terms, the modal domain equation of motion (EOM) of a free vibration system can be expressed by

$$\begin{bmatrix} \mathbf{k}_{aa} & \mathbf{k}_{ab} \\ \mathbf{k}_{ba} & \mathbf{k}_{bb} \end{bmatrix} \begin{Bmatrix} \phi_a \\ \phi_b \end{Bmatrix} = \{0\} \quad (2-29)$$

From this equation, the second term ϕ_b is solved as

$$\{\phi_b\} = -\mathbf{k}_{bb}^{-1} \mathbf{k}_{ba} \{\phi_a\} \quad (2-30)$$

Therefore, the transformation matrix becomes

$$\mathbf{T} = \begin{Bmatrix} \mathbf{I} \\ \mathbf{t} \end{Bmatrix} = \begin{Bmatrix} \mathbf{I} \\ -\mathbf{k}_{bb}^{-1} \mathbf{k}_{ba} \end{Bmatrix} \quad (2-31)$$

The negligence of inertia terms leads to the heavy dependence of this method on the quality of the chosen measured DOFs, and a poor selection will yield inaccurate models (O'Callahan 1989). A general guideline is to select those DOFs where the mass terms (or inertia) are sufficiently small relative to the stiffness terms.

■ Improved Reduced System (IRS)

O'Callahan (1989) developed an improved reduced system (IRS) on the basis of Guyan reduction. In this technique, the mass effect was considered by incorporating a complement in eq. (2.31). The transformation matrix $\mathbf{T}$ is designed as

$$\mathbf{T} = \begin{Bmatrix} \mathbf{I} \\ \mathbf{t}_i \end{Bmatrix} \quad (2-32)$$

where

$$\mathbf{t}_i = \mathbf{t} + \mathbf{k}_{bb}^{-1}(\mathbf{m}_{ba} + \mathbf{m}_{bb}\mathbf{t})(\mathbf{t}^T \mathbf{m}_n \mathbf{t})^{-1}(\mathbf{t}^T \mathbf{k}_n \mathbf{t}) \quad (2-33)$$

The analysis of a cantilever beam showed the IRS procedure can result in an improvement on the reduced model.

■ System Equivalent Reduction Expansion Process (SEREP)

O'Callahan et al. (1989) proposed the SEREP. It has some distinguished advantages over other methods, namely, allowing arbitrary selection of modes and master DOFs, preserving the frequencies and mode shapes, and the reduction/expansion process is reversible.

Denoting the number of measured modes and DOFs by m and a , respectively,

$$\Phi_{nm} = \begin{bmatrix} \Phi_{am} \\ \Phi_{bm} \end{bmatrix} \quad (2-34)$$

and the transformation matrix is taken as a generalized inverse of the modal matrix,

$$\mathbf{T} = [\Phi_{nm}][\Phi_{am}]^+ \quad (2-35)$$

where

$$[\Phi_{am}]^+ = ([\Phi_{am}]^T [\Phi_{am}])^{-1} [\Phi_{am}]^T \quad (2-36)$$

2.3.2 Data Expansion Algorithms

Model reduction techniques result in matrices in which the connectivity property of the original FE model is lost, and so is the physical meaning of the model (Farhat and Hemez, 1993). An alternative is to expand the measured mode shapes to the analytical model size by approximating the unmeasured DOFs. The methods used for the data expansion is closely related to the model reduction techniques. Similarly a transformation matrix $\mathbf{T}$ is used to link the master DOFs (“ a ”) and the full FE model DOFs (“ b ”),

$$\Phi_n = \begin{bmatrix} \Phi_a \\ \Phi_b \end{bmatrix} = \mathbf{T} \Phi_a \quad (2-37)$$

■ Guyan Expansion

This expansion technique is the inverse process of the static condensation method described before. It is easily proved that the transformation matrix remains the same, i.e.,

$$\mathbf{T} = \begin{Bmatrix} \mathbf{I} \\ \mathbf{t}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{I} \\ -\mathbf{k}_{bb}^{-1}\mathbf{k}_{ba} \end{Bmatrix} \quad (2-38)$$

■ Eigenvector Mixing

The implementation of the eigenvector mixing method is straightforward in the sense that the unmeasured or missing DOFs in the experimental data are filled in by the corresponding analytical counterpart,

$$\boldsymbol{\Phi}_n = \begin{bmatrix} \boldsymbol{\Phi}_a^E \\ \boldsymbol{\Phi}_b^A \end{bmatrix} \quad (2-39)$$

■ Dynamic Expansion

This technique takes the structural frequencies λ_i into account by using the dynamic stiffness matrix for an undamped structure. One can partition the eigenvalue function according to DOFs “a” and “b” as

$$\begin{bmatrix} \mathbf{k}_{aa} & \mathbf{k}_{ab} \\ \mathbf{k}_{ba} & \mathbf{k}_{bb} \end{bmatrix} \begin{Bmatrix} \phi_a \\ \phi_b \end{Bmatrix}_i = \lambda_i \begin{bmatrix} \mathbf{m}_{aa} & \mathbf{m}_{ab} \\ \mathbf{m}_{ba} & \mathbf{m}_{bb} \end{bmatrix} \begin{Bmatrix} \phi_a \\ \phi_b \end{Bmatrix}_i \quad (2-40)$$

Expanding the expression gives

$$\begin{aligned} \mathbf{k}_{aa}\{\phi_{ai}\} + \mathbf{k}_{ab}\{\phi_{bi}\} &= \lambda_i (\mathbf{m}_{aa}\{\phi_{ai}\} + \mathbf{m}_{ab}\{\phi_{bi}\}) \\ \mathbf{k}_{ba}\{\phi_{ai}\} + \mathbf{k}_{bb}\{\phi_{bi}\} &= \lambda_i (\mathbf{m}_{ba}\{\phi_{ai}\} + \mathbf{m}_{bb}\{\phi_{bi}\}) \end{aligned} \quad (2-41)$$

ϕ_{bi} can be solved from both equations. From the first equation, one gets

$$\{\phi_{bi}\} = -(\mathbf{k}_{ab} - \lambda_i \mathbf{m}_{ab})^+ (\mathbf{k}_{aa} - \lambda_i \mathbf{m}_{aa}) \{\phi_{ai}\} \quad (2-42)$$

and similarly from the second equation,

$$\{\phi_{bi}\} = -(\mathbf{k}_{bb} - \lambda_i \mathbf{m}_{bb})^+ (\mathbf{k}_{ba} - \lambda_i \mathbf{m}_{ba}) \{\phi_{ai}\} \quad (2-43)$$

Or from the combination of the two equations,

$$\{\phi_{bi}\} = - \begin{bmatrix} \mathbf{k}_{ab} & -\lambda_i \mathbf{m}_{ab} \\ \mathbf{k}_{bb} & -\lambda_i \mathbf{m}_{bb} \end{bmatrix}^+ \begin{bmatrix} \mathbf{k}_{aa} & -\lambda_i \mathbf{m}_{aa} \\ \mathbf{k}_{ba} & -\lambda_i \mathbf{m}_{ba} \end{bmatrix} \{\phi_{ai}\} \quad (2-44)$$

Eqs. (2-42) to (2-44) indicate various expansion algorithms. Eq. (2-42) often gives bad results due to the ill-conditioning of the inverse matrix caused by the fact that the number of measured DOFs is usually less than that of unmeasured ones. To overcome this issue, Gysin (1990) included all the DOFs in the inverse process as in Eq. (2-43).

■ SEREP Expansion

This method manifests itself in many forms in terms of the transformation matrix $\mathbf{T}$.

1) FE model based:

$$\mathbf{T} = \Phi^A (\Phi_a^A)^+ \quad (2-45)$$

2) Experimental data based:

$$\mathbf{T} = \Phi^A (\Phi_a^E)^+ \quad (2-46)$$

3) Mixing/FE based:

$$\mathbf{T} = \begin{Bmatrix} \Phi_a^E \\ \Phi_b^A \end{Bmatrix} \left(\Phi_a^A \right)^+ \quad (2-47)$$

4) Mixing/Experimental data based:

$$\mathbf{T} = \begin{Bmatrix} \Phi_a^E \\ \Phi_b^A \end{Bmatrix} \left(\Phi_a^E \right)^+ \quad (2-48)$$

Gysin (1990) conducted a comparison study on the accuracy of different expansion methods with a spring-mass structure. Both the expansion error and the damage localization capacity were evaluated for each technique. Numerical results demonstrated that the dynamic expansion method performed best amongst all these techniques.

2.4 Some Critical Analyses

2.4.1 Correlation Analysis

In model updating, it is crucial that analytical and experimental quantities are paired correctly before comparison.

Correlation, including modal correlation and spatial correlation, is a technique to examine quantitatively and qualitatively the correspondency and difference between analytical and experimental modal parameters.

a) Frequency Pair

In general, the correlation between the analytical and the experimental frequencies is examined by their relative difference. A small relative difference indicates better correlation. The relative frequency difference is defined by

$$\Delta_f = \frac{f_{FEA} - f_{EMA}}{f_{EMA}} \times 100\% \quad (2-49)$$

where f_{FEA} , f_{EMA} represent analytical and experimental natural frequencies respectively. During the course of updating, if some reference modal data bear a relative frequency difference that is higher than a prescribed threshold, these data should be given up.

b) Mode Shape Pair

There exist several methods to make comparison between the experimental and the analytical mode shapes. The most effective way is visual inspection, which, however, is very costly in practice. Numerical correlation indices such as the modal assurance criterion (MAC) (Allemang and Brown, 1982) is usually used to automatically check all possible mode shape pairs and give a quantity embodying the level of correlation. MAC is defined based on a matrix with entries calculated by

$$MAC_{ij} = \frac{|\phi_{ei}^H \phi_{aj}|^2}{(\phi_{ei}^H \phi_{ei})(\phi_{aj}^H \phi_{aj})} \quad (2-50)$$

where ϕ_{ei} represents the i^{th} experimental mode shape, ϕ_{aj} indicates the j^{th} analytical mode shape and the superscript H means the transpose of the complex conjugate. The value of MAC_{ij} falls within the interval 0 and 1. The closer the value is to 1 the more similar the two mode shapes are.

MAC provides only global correlation information. The correlation information of the mode shapes in certain zones of the model may be a valuable guidance for the localization of the source of the observed errors. So, it is important to be able to rank DOFs in terms of how they impact the modal assurance criterion (MAC) results. For this purpose, coordinate modal assurance criterion (COMAC) (Lieven and Ewins, 1988) was developed and it is related to the degree of freedom of the structure rather than the whole mode shape vector; hence it correlates the amplitude of the two matched set of mode shapes at a single measured point.

c) Spatial Correlation or Node-Point Pair

Two models defined in the same coordinate system can have several grid points at identical positions or within a given tolerance distance. The investigation of how many of these grid locations in each of the models are within such a given tolerance distance is an operation referred to as *spatial correlation*. The experimental point and analytical node with the shortest spatial distance are taken as a pair. If the two models topologically do not coincide or coordinate systems do not match, transformation of either the FE model or the test model must be done beforehand.

2.4.2 Sensitivity Analysis

Sensitivity analysis is a technique to provide an analyst with an approximate indication as to how the responses of a structural FE model are influenced by the modification of structural physical properties such as boundary conditions, material stiffness, geometry, etc. The basic element with this technique is the sensitivity matrix S , which has been discussed extensively in section 2.2.2. Depending on the specific applications, the sensitivity matrix may be treated differently. In the present study, this technique will be applied to predict the parameter errors due to the measurement noise. The detailed

operation of the sensitivity analysis will be presented in Chapter 3. Some popular example applications of sensitivity analysis techniques are listed below:

- a) What-If analysis – To study the effect of modeling assumptions, for example the rigid boundary conditions instead of elastic ones, on the modal parameters or other response quantities.
- b) Design optimization – To find the optimal locations to modify the structure in order to shift modal parameter values.
- c) Identification of sensitive and insensitive areas of a structure for a given response and parameter combinations - This will help the analyst to decide which parameters and responses to include in the selection for model updating.
- d) Pretest analysis - Sensitivity analysis can also be used in pretest analysis applications, for instance studying the effect of transducer mass loadings on the modal parameters.

2.4.3 Error Localization

It is generally understood that even for moderate-size structures, such as offshore platforms, there may exist numerous parameters that are deemed uncertain while the amount of useful information from measurements are limited and unable to adjust all the parameters in doubt to a satisfactory degree. This conflicting problem has been the drive of considerable research effort made in locating the most inaccurate model parameters prior to model updating. This area of work has become known as *error localization*. Via error localization techniques, the parameters that are most effective in producing a genuine improvement in the modeling of a structure are found and are to be included as a compact set in the updating, thus improving the numerical conditioning of the updating problem. Methods used for the error localization are often closely related to those for updating, and may similarly be influenced by the incompleteness of measurements. Existing techniques regarding error localization

include a) balancing the eigenvalue equation, b) substructure energy functions, and c) best subspace method (Friswell and Mottershead 1995). Studies in this area can also be found in some reported articles (Zhang and Lallement 1987; Sidhu and Ewins 1984; Lallement and Piranda 1990; Gysin 1990; Link and Santiago 1991).

2.4.4 Regularization

Despite the rapid development on the measurement techniques, the portion of the knowledge space of the structural behavior that can be measured by experiment is small. However, the volume of unknown parameters requiring correction, even after error localization, can still be very large. Consequently, the system of equations become strongly under-determined. To achieve an over-determined problem, regularization techniques are generally applied. Many techniques regarding the regularization have so far been proposed. Lallement and Cogan (1992) enlarged the knowledge space by using anti-resonance data. Wada, Kuo and Glaser (1986) used multiple tests with varying boundary conditions and Nalitoela, Penny and Friswell (1990) made use of added masses. Ben-Haim and Prells (1993) used adaptive excitation to enable parameter updating in subsets, which is usually called selective sensitivity technique. Zhang and Natke (1991) applied component mode synthesis to achieve a two-level updating procedure, which has the effect of reducing the number of parameters to be estimated at each level.

2.5 Summary

FE model updating techniques aim to produce an improved structural FE model by matching the analytical data with the measured data. The methods used in model updating can be classified into local updating (iterative-based) and global updating (direct updating). Global updating methods can reproduce the measured data by updating the system matrices (stiffness, mass and damping matrices) directly, but the

resulting FE models do not have physical meaning. In contrast, local updating methods are able to give physically meaningful and interpretable updated FE models because the match between the analytical data and the experimental counterparts is achieved by modifying the structural physical parameters with genuine modeling errors. Furthermore, desirable properties such as positive-definiteness, symmetry and sparseness of the stiffness matrix of the original FE model are preserved by the local updating procedure. Therefore, it is widely used for structure design, evaluation of structural integrity and structural serviceability estimation, etc, and is also one of the principal interests of the present study.

The selection of reference data is crucial for successful updating. Reference response must be clean and reliable, which can be dynamic and/or static. Since dynamic modal data have some unique attractive features such as cost-effectiveness for measurement, reflecting inherent characteristics of the structure, allowing for updating both structural stiffness and mass matrices, etc, they are preferred in most updating applications. In cases where the modal data cannot be extracted with an acceptable accuracy, frequency domain data such as FRFs may be used in model updating as reference responses. One has to bear in mind that computational cost and convergence issues may arise with these techniques.

The selection of physical parameters is another pivotal step in model updating. The selected parameters must be uncertain and they should show close relation to the reference responses used. For problems where a large number of parameters can be candidates for updating, error localization techniques should be used to pick out those parameters that make most contribution to the discrepancy between analytical and experimental data. Regularization techniques are generally required to realize a unique solution for an under-determined problem.

Reliable model updating results represent a good and consistent correlation. For the updating of a complex structure, multiple trials may be required before a satisfactory updated model can be attained. The unsatisfactory results are due primarily to the improper choice of the structure parameters for updating. For real complex structures,

this problem could be difficult to overcome even after the application of all the relevant techniques mentioned in this chapter. In this respect, engineering insight plays an important role by analyzing the response correlation results and making apt decisions. For example, if the errors on all responses are the same magnitude and have the same sign, probably a systematic shift of structural stiffness or mass is sufficient to get convergence. If the distribution of errors is not even, i.e. small differences for some responses and great discrepancies on other responses, many more local parameters of different types should be used in order to simultaneously compensate for these errors.

It should be noted that the FE model updating methods reviewed in this chapter do not include artificial intelligence techniques. Existing applications of genetic algorithms (GA) and artificial neural networks (ANN) in FE model updating and damage identification will be reviewed and discussed in association with the development of the relevant methodologies in the chapters that follow.

CHAPTER 3

DYNAMIC MODEL UPDATING USING COMBINED GENETIC-EIGENSENSITIVITY ALGORITHM

3.1 Introduction

As a general principle, a desirable model updating is to make physically meaningful adjustments to structural parameters so that the computed responses correlate satisfactorily with the measured data. The FE model so updated can then be used to assist in structure design and assess the structural integrity and serviceability under a variety of possible loading conditions.

Generally speaking, a sound updating process depends on three basic factors: 1) a proper selection of variable structural parameters, 2) a good decision on the measurement information, and 3) an effective algorithm for optimization in performing the inverse procedure. To adequately determine uncertain parameters for updating is a structure-specific matter, in which acute engineering insight and a sufficient understanding towards the underlying structural system are required. Applicable candidates of experimental data may span a broad spectrum, amongst which modal or vibration data are widely used as discussed in Chapter 2.

To develop and apply robust schemes for carrying out the optimization in the model updating procedure is one of the primary objectives of this study. The current model updating techniques employ gradient-based schemes to perform minimization to achieve a desired match between analytical and measured data. Despite a wide success in a number of reported applications (Mottershead et al. 2000; Jones 2000), such iterative-based deterministic optimization methods bear the limitation that the

searching procedure may be trapped into a particular local optimum rather than the global actual solution; and this problem may become more serious in situations where the objective functions are high-dimensional complex surfaces containing many plateaux and valleys. Unfortunately, this is often the case in the FE model updating domain (Mottershead and Friswell 1993; Doebling et al. 1996).

To the delight of researchers and engineers, the recent development in artificial intelligence algorithms has brought in seemingly powerful alternatives to the gradient-based optimization approaches in searching for an optimum. Among other algorithms, genetic algorithms (GAs) appear to be particularly attractive because they operate to globally optimize a given objective function and hence are capable of finding a global solution. GAs follow a guided stochastic process and have distinguished features; they search for the candidate solution from a large population of points rather than a single point like in traditional optimization methods, and they do not require the continuity and differentiability of the objective function being optimized. Thus, constructing an error function becomes very flexible.

Recently, some researchers applied GAs in model updating and damage detection problems (Friswell et al. 1998, Chou and Ghaboussi 2001). In those studies, variables are encoded with binary bits. Such coding scheme may cause the computation to be very intensive and even intractable if the number of parameters being optimized is very large such as in realistic civil engineering structures. In contrast, real-number coding GAs, which encode each parameter using only one real number in the chromosome, can substantially reduce the computational effort. In addition, there will be no loss of precision due to the real number representation as in the binary-coding GAs (Wright, 1991). These features make real coding GAs more desirable in a robust model updating procedure.

In this chapter, a procedure using real-coding GA to perform dynamic model updating is presented. An important consideration of this study is to restrict the required set of modal data within a practically affordable limit, for example the first 3 natural frequencies and the first mode shape for a sway-type frame. An objective function is

constructed to include both modal frequency and mode shape errors. To deal with the inherent sensitivity problem which could arise when the number of variables is increased with the same set of available modal data, an eigensensitivity-based updating method is incorporated to perform further hill-climbing on the basis of the GA-updated model to improve the accuracy. The procedure is illustrated using representative multi-storey sway-type frame structures taking the inter-storey stiffness as variables for updating. The combined approach proves to be very effective so that accurate updating results can be achieved for a sufficiently large number of parameters to be updated with a limited amount of modal data.

The investigation is then extended to the ability of a GA-based approach to find an optimal solution with modal data containing certain level of measurement errors. In such cases, the inherent sensitivity problem dictates that higher order modal information will be necessary for a satisfactory updating of a similar number of parameters (An alternative to this is to reduce the number of parameters to update while keeping the same set of modal data. This will involve other considerations which are beyond the scope of the present study.) A sensitivity analysis is hence proposed to first determine the required order of modal data for a targeted degree of accuracy, and GA is then employed to perform the updating using the “noisy” modal data. It is shown that GA still can perform to the anticipated satisfaction under a noisy measurement condition.

To illustrate the effectiveness of the proposed approach in a real structural and measurement environment, an RC frame tested on an earthquake simulator is subjected to the model updating, based on the measured natural frequencies and the first mode shape. The updated model is then used to predict the seismic response and the prediction is compared with the actually measured response to verify the adequacy of the updated model.

3.2 Overview of Genetic Algorithms

The searching procedure of genetic algorithms derives from the process of natural selection and evolution originally observed and documented by Charles Darwin. This cross-fertilization from one field of science to another has demonstrated extensive and fruitful applications in numerous problems from different domains, including optimization, automatic programming, machine learning, economics, operations research, ecology, population genetics, studies of evolution and learning, and social systems etc. Prominent features of GAs lie in its inherent advantage that following the philosophy of “survival of the fittest” it can arrive at the global optimal solution.

GAs differs from calculus-based search procedures in several ways (Goldberg 1989): a) They search from a population of points in parallel, not a single point; b) They make use of information about the objective function itself, not derivatives or other auxiliary knowledge; and c) They adopt probabilistic transition rules, not deterministic ones. Besides, GA works by encoding the pheno-space decision variables of the optimization problem using finite length string chromosomes” in geno-space, rather than by searching for the solution directly within a given domain. Furthermore, the ways to decode the strings in geno-space and thus to evaluate their fitness can be very flexible, which makes it possible for GAs to optimize problems that cannot be explicitly expressed mathematically.

The basic elements in GAs are “strings”, or in genetic terminology “chromosomes”, coding the information about the phenotype of variables. “Genes” denote components in chromosomes. An individual chromosome is composed of a number of “genes”. Different numbers of genes may be used to represent different variables. To represent the actual problems in the form of “strings” in GAs can be very flexible and is also subjected to the nature of underlying problems, examples of which are widely used binary coding, integer representation (Goldberg 1989) and real-number encoding (Michalewicz 1994). The choice of a proper form of genes for a particular type of problem can be vital to the success of the optimization. For example, when dealing with discrete problems, binary genes may be preferred, while for continuous problems

(e.g., the FE model updating problems concerned in the present study) the use of real number genes is favored for efficiency consideration (Goldberg 1989). For each chromosome, a quantity called “fitness” is defined to measure the quality of the solution represented by this chromosome. The “fitness” of chromosomes refers to the corresponding values of the defined objective function. Following the defined coding form and the objective (fitness) function, genetic optimization procedure proceeds step-by-step. After the initialization stage, the genetic operations, i.e., selection, crossover and mutation, are to be activated repeatedly. The sequence of these operators is detailed as follows.

Initialization First, GA is initialized by establishing the starting population, which can be generated either randomly or heuristically. The choice of the population size, i.e., the number of strings, is a trade-off decision between computing time and the requirement on the precision of the optimal solution. In general, larger population size can result in better solution. As an illustration, assuming the underlying problem has four variables, the initial population with real number genes can take the following form:

Chromosome 1:	0.43	0.61	0.85	0.92
Chromosome 2:	0.52	0.67	0.28	0.52
Chromosome 3:	0.36	0.49	0.65	0.47
Chromosome 4:	0.97	0.57	0.63	0.59

In practice, the population size could be far larger than the above. After the initial population is established, a new population is to be produced through the genetic operations, as discussed in what follows.

Selection In this process, a decision is made on the number of copies for each chromosome into the next generation. Typical selection strategies include roulette wheel selection and ranking selection. Roulette wheel selection, developed by Holland (1975), was the first selection method. It operates by applying a probability, p_i , for the survival of each individual to the next generation,

$$p_i = \frac{F_i}{\sum_{j=1}^{PopSize} F_j} \quad (3-1)$$

where F_i, F_j and $PopSize$ denote the fitness of chromosome i, j and the population size, respectively. The roulette wheel selection, due to its full mapping scheme (maps the solutions to a fully ordered set of values on $\mathfrak{R}^+$, which is the space representing the positive rational numbers), limits GA to only perform a maximization problem. In order to handle minimization or negative fitness problems extensions such as windowing and scaling techniques must be incorporated.

In contrast, ranking selection methods, which include linear ranking selection and normalized geometric ranking selection (Joines and Houck, 1994), only require the objective function to map the solutions to a partially ordered set, thus allowing for minimization and negativity. Furthermore, fitness scaling can also be fulfilled simultaneously. These schemes work by assigning a selection probability p_i for each individual based on the rank of its fitness. Linear ranking selection first requires the population to be sorted according to its fitness values in a descending order and then individual strings are assigned offspring counts based on their rankings. A string with count number “one” is the best candidate and with the increase of the count number much weaker chromosomes are expected. Thus, the selection proportion assigned to the k^{th} sorted string is defined by

$$p_k = q_{\max} - \left(\frac{k-1}{n-1} (q_{\max} - q_{\min}) \right) \quad (3-2)$$

where, $q_{\max}$ and $q_{\min}$ indicate the selection proportions assigned to the fittest and the weakest strings, respectively, and n denotes the population size as in Eq. (3-1). Since the population size is fixed through generations in the entire genetic search process, the summation of individual selection proportions should be equal to one, i.e.,

$$\sum_{k=1}^n p_k = \sum_{k=1}^n \left[q_{\max} - \frac{k-1}{n-1} (q_{\max} - q_{\min}) \right] = 1$$

From the above equation, the relationship between $q_{\max}$ and $q_{\min}$ is obtained as

$$q_{\max} + q_{\min} = \frac{2}{n} \quad (3-3)$$

Thus, once $q_{\max}$ is set, $q_{\min}$ becomes a dependent variable and should be determined from Eq. (3-3).

For normalized geometric ranking, the operation process is the same as described above except that a different method for defining p_k should be adopted as

$$p_k = q'(1-q)^{r-1} \quad \text{and} \quad q' = \frac{q}{1-(1-q)^P} \quad (3-4)$$

where, q represents the probability of selecting the best individual, r is the rank of the individual chromosome, and P denotes the predefined population size. There is not a uniform conclusion regarding which of the two ranking selection schemes performs better in a GA application; rather, a judgment may have to be made based on the nature of the particular problem under consideration. In the present study, the linear ranking selection is adopted. Via selection operation, the number of schemata with fitness over the average will increase exponentially in future generations.

Crossover This operation reproduces the offspring population by recombining the information from two parent chromosomes. The fact that the application of genetic operators and their derivatives totally depends on the chromosome representation (also called encoding) used forces one to bind the discussion of genetic operators and coding scheme together. The genetic operators designed for real coding scheme, which is applied in this research study, include the following:

- a) The simplest type of crossover is single point crossover. First, two parent chromosomes are selected randomly and then two offspring are produced by crossing over their genes at a randomly chosen position. For example, assume that

the two parents consist of m genes and they are represented by $x = [x_1 \ x_2 \ \cdots \ x_m]$ and $y = [y_1 \ y_2 \ \cdots \ y_m]$. If the crossover point is chosen at the k^{th} position counting from the left side, then the resulting two offspring become

$$\begin{aligned} x' &= [x_1 \ x_2 \ \cdots \ x_k \ y_{k+1} \cdots y_m] \\ y' &= [y_1 \ y_2 \ \cdots \ y_k \ x_{k+1} \cdots x_m] \end{aligned} \quad (3-5)$$

For the simple case mentioned earlier, if chromosome 1 and chromosome 3 are chosen as parents and the crossover point is randomly selected at the 2nd position, the two descendants from the crossover would be

Offspring 1: 0.43 0.61 0.65 0.47

Offspring 2: 0.36 0.49 0.85 0.92

This type of crossover can be equally applied for other coding schemes.

- b) Arithmetic crossover. It produces two offspring, which are the complimentary linear combinations of the two parents as

$$\begin{cases} X' = r_1 X + r_2 Y \\ Y' = r_1 Y + r_2 X \end{cases} \quad (3-6)$$

where $r_1 + r_2 = c$. c is a positive number, which, generally, equals one.

- c) Heuristic or direction-based crossover. The resulting descendants based on the above described two types of crossover methods may be worse than their parents. Heuristic crossover method can guarantee the occurrence of better offspring by taking the fitness information into consideration. Assuming that the two parents X and Y are chosen and X is better than Y in term of fitness, then two offspring, X' and Y' , are generated by

$$\begin{cases} X' = X + r(X - Y) \\ Y' = X \end{cases} \quad \text{feasibility} = \begin{cases} 1 & \text{if } a_i \leq x'_i \leq b_i, \forall i \\ 0, & \text{Otherwise} \end{cases} \quad (3-7)$$

Despite its appealing advantage, heuristic crossover may require extra computation effort to achieve feasible offspring because the newborn descendants might violate the prescribed bounds.

Mutation This final operator is motivated to help further increase the genetic diversity and improve the global nature of the search. It simply alters one gene (representing a variable) in a chromosome based on a certain rule. Each gene in the population may be mutated with a probability p_m . Mutation methods that work with the real number coding method are presented below (assuming that a gene x_i is randomly selected for mutation within a predefined bound $[a_i, b_i]$).

- a) Uniform mutation. It simply replaces x_i by a random number uniformly selected within $[a_i, b_i]$, and thus the mutated gene, x'_i , becomes $x'_i = U(a_i, b_i)$.
- b) Boundary mutation. As the name implies, it operates to set x_i equal to either of its two boundary values, which is controlled by a random number r within 0 and 1. For example, a random number, r , less than 0.5 indicates the use of a_i , or else b_i .
- c) Direction-based mutation. This mutation method follows the same principle as in the heuristic crossover scheme. As a result, the string, at which mutation is performed, becomes fitter than before. Similarly, it is assumed that a certain gene, x_i , of a string, x , is selected for mutation. Thus, the mutated gene, x'_i , is defined by

$$x'_i = x_i + rd$$

where r is chosen as a random number within $[0,1]$ such that the feasibility of x'_i is satisfied. d takes an ascent direction of the objective function, i.e. the orientation along the gradient $\nabla f(x_1, \dots, x_n)$. Since GAs do not require differentiability of the fitness function, the components of ∇f are estimated by difference quotients

$$\frac{\partial f}{\partial x_i} = \frac{f(x_1, \dots, x_i + \Delta x_i, \dots, x_n) - f(x_1, \dots, x_i, \dots, x_n)}{\Delta x_i} \quad (3-8)$$

For practicality, direction-based mutation would not be preferable because its inherent advantage described above is overridden by the largely increased computation cost that arises from the calculation of reference response at each mutation operation. For the numerical investigations presented in this chapter, the uniform mutation proves to be able to work satisfactorily and is thus adopted.

Following the selection, crossover and mutation operations, a new generation is produced. Typically, the process will iterate for many generations till the convergence criterion is satisfied, which may be a maximum number of generations allowed, or when there is no increase in the fitness over a few generations.

The above procedure constitutes the basis for most applications of GAs. As regards the encoding forms, recently there is a trend towards using integer encoding for combinatorial problems and real number coding for continuous optimization problems as encountered in this research study. Moreover, after crossover and mutation operations, better solutions may not be guaranteed to emerge in offspring generations and some useful information in the parent generation may be lost. In this regard, some replacement strategies such as the elitist strategy (Goldberg 1989) as used in the present study are necessary to be incorporated into GAs operations. The elitist strategy operates to replace a small portion (replacement rate) of the chromosomes in the offspring generation that are evaluated to be worst with the best members (referred to as “elites”) in the parent generation. This is to ensure the best members are retained from one generation to the next.

The unique operating mechanism as described above gains GAs its ability in finding a global optimal solution for a given objective function. Since FE model updating can be ultimately formulated into a numerical optimization problem defined by an objective function, it is reasonable to expect that the application of GA can result in a more robust updating procedure. A review of the current development in GA-based model updating techniques follows.

3.3 Review of GA-Based Structural Assessment Applications

Many publications can be found in the recent literature discussing GA's application on damage detection. Application of GA for FE model updating problems, however, is less touched on. FE model updating differs from damage identification in that it aims at determining actual structural parameters rather than the relative changes at a few damage locations, and hence imposes different demands on the responses and analysis algorithm. Current GA-based model updating studies basically remain at the stage of studying the relevant algorithms (GAs) and limited individual applications for special cases. A systematic evaluation of the applicability of the method and development of a comprehensive implementation procedure taking into account the fundamental model-updating theories are still lacked.

Larson and Zimmerman (1993) presented a model-refinement approach by using GA. Background information was given related to the model refinement and genetic algorithms together with their applications. GA was used to determine changes in the structural physical parameters such that analytical and experimental modal data are satisfactorily matched. A simple numerical example, where only a few variable parameters are assumed to be refined, was given to show the applicability of GA in performing a model refinement. The results demonstrated that a satisfactory refining (updating) was achieved. However, this study did not investigate what would happen if a large number of variable parameters is required for refinement.

Hemez et al. (1995) applied a genetic solver rather than a gradient-based method to locate and quantify damage in truss structures using both simulated and experimental data. The genetic updating was compared with results obtained from the conventional Element-By-Element updating method that is based on minimizing the out-of-balance forces of the model. It was found that similar results could be obtained with GA, indicating that GA could become a potential alternative to carry out model updating problems.

Mares and Surace (1996) investigate the detection of macroscopic structural damage using GA based on modal data, implemented through the residual force method. A special objective function for the genetic search optimization procedure was proposed. During GA optimization, the binary-encoding scheme was adopted. Such coding strategy, however, may require considerable amount of computation effort when dealing with a relatively large number of variable parameters under optimization. The proposed method was tested on two numerical examples, where the number of damage sites up to three was required to locate and quantify their degrees of damage. An appropriate response setting was obtained by a trial-and-error procedure in which a number of GA runs were carried out, which was quite time-consuming. The results indicated that damages could be detected with good accuracy through the application of GA. The limitation with the proposed scheme was that the number of damage locations was assumed to be very small (maximum up to 3), and furthermore the costly trial-and-error method in determining the proper response information may fail when tackling complex problems.

Friswell et al. (1998) also investigated a damage detection problem by using GA. The objective was to locate single or multiple damage sites and furthermore to estimate the degree of damage. The pheno-type variables for GA were the locations where damage might occur, while the reference response consisted of the structural modal information (natural frequencies and mode shapes). A standard eigensensitivity method is incorporated into the detection procedure to quantify the damage after detection of the damage site from GA. To demonstrate the proposed scheme, numerical simulations were carried out on a cantilevered beam. Four damage episodes were assumed and the lowest five natural frequencies were used with equal weights as

responses. A successful identification was achieved with all the four cases. Besides, the method was also applied on an experimental case study (cantilevered steel plate), in which only one element suffered damage and the first 12 frequencies were measured as response. This case study was also shown to be successful. The limitation of the application was that the number of damage sites was assumed to be known; otherwise, the algorithm would not be able to proceed. Moreover, the presumed robustness of the method in resisting systematic errors was not supported by any systematic evaluation evidence.

Xia and Hao (2001) used a genetic algorithm with real number encoding to identify the structural damage with vibration data. The objective function was minimized by directly comparing the changes in the measured modal data before and after damage in an attempt to neutralize some system errors. Three different response settings were considered, namely the frequency changes, the mode shape changes and their combination. It was observed that the weights assigned to the individual types of response greatly influenced the identification results. It was also found that the response with the combination of the two types of modal data resulted in the best detection. To demonstrate the proposed approach, a laboratory-tested cantilever beam and a model steel frame were used. It was shown that all the damaged elements would be detected accurately by genetic algorithm. The method requires the availability of the modal data from both undamaged and damaged states of the structure.

Chou and Ghaboussi (2001) formulated the damage identification as an optimization problem, which is then solved by using “binary-encoding” GA. Reference responses included static measurements of displacements at a few DOFs to identify the changes of the characteristic properties of structural members such as the Young's modulus and cross-sectional area. To avoid structural analyses in the fitness evaluation, the displacements at unmeasured DOFs were also determined by GA. A comparison was performed between the detections using the implicit redundancy string representation (IRR) GA and usual string representation GA. It was observed that IRR GA outperforms simple GA in reducing the computational effort due to the introduction of redundant segments in the strings. The identified results from both methods were found to be acceptable. This study, however, did not present the condition (response

information) under which GA could successfully implement the given task. Besides, the approach is limited to work on small-scale problems because of the inefficient coding method.

3.4 FE Model Updating Using GA and Combined GA-Eigensensitivity Approach

3.4.1 Definition of Objective Function

Model updating by nature is to make adjustment on the variable structural parameters so that the predicted response by the model best matches the measured counterparts. Therefore, the updating procedure can be ascribed to performing optimization on an error function. In the present study, the available measurements are assumed to be the modal data i.e. the resonance frequencies and mode shapes. Subsequently, the objective or error function is defined to consist of two parts; one relating to the error in the natural frequency, J_ω , and the other relating to the error in mode shape, J_ϕ . These two types of errors are weighted to give a total error of

$$J = W_\omega J_\omega + W_\phi J_\phi \quad (3-9)$$

where W_ω and W_ϕ are the weighting factors. It is noted that the objective function herein is defined on the basis of modal frequencies and mode shapes, without inclusion of damping. This implies that 1) the effect of damping on the modal frequencies and mode shapes is regarded to be negligible, which should be valid for most lightly-damped (proportional or not) structures as is the case of typical civil engineering structures; and 2) the damping coefficient itself will require a separate procedure to update, if needed. If the damping effect is to be considered in a more

robust manner, the objective function will have to be defined accordingly, and GA can still be applied as a search tool

Another point worth mentioning is that the representation of the structural parameters identified through the updating procedure to the real physical parameters depends upon the overall adequacy of the FE model for the structure. Where an FE model is a much simplified form of the actual structure, the parameters identified should be interpreted as an equivalent measure of the actual physical properties.

The objective function for frequency errors may be defined as a weighted sum of squares of the relative difference between measured and predicted frequencies. Any uncertain frequencies that cannot be paired with sufficient confidence are simply excluded from the objective function. Thus,

$$J_{\omega} = \sum_{i=1}^{N_f} W_{\omega i} \left(\frac{\omega_{mi} - \omega_{ai}}{\omega_{mi}} \right)^2 \quad (3-10)$$

where N_f denotes the number of selected modes, ω_{mi} and ω_{ai} indicate the i th pair of the measured and predicted frequencies. The adoption of the individual weighting factor $W_{\omega i}$ enables the use of smaller weights on less-accurately measured modes, such as higher modes. A variant of Eq. (3-10) is to replace the sum-squared errors by the average absolute relative difference, as

$$J_{\omega} = \frac{1}{N_f} \sum_{i=1}^{N_f} W_{\omega i} \left| \frac{\omega_{mi} - \omega_{ai}}{\omega_{mi}} \right| \quad (3-11)$$

Similarly, the mode shape error function may be defined as

$$J_{\phi} = \sum_{i=1}^{N_m} W_{\phi i} (\phi_{mi} - \phi_{ai})^T (\phi_{mi} - \phi_{ai}) \quad (3-12)$$

where ϕ_{mi} and ϕ_{ai} are the i th pair of measured and analytical mode shapes, N_m is the number of selected modes and $W_{\phi i}$ is the weighting factor for the i th mode. An alternative form of (3-12) is

$$J_{\phi} = \frac{1}{N_s} \sum_{i=1}^{N_m} W_{\phi i} \sum_{j=1}^{N_{nd}} |\phi_{ij}^m - \phi_{ij}^a| \quad (3-13)$$

where N_s denotes the total number of mode shape displacements, $N_s = N_m \times N_n \times N_d$, in which N_n is the number of nodes selected in each mode and N_d is the number of DOFs at each node.

In GA-based approach, the selection of the frequencies and mode shape data to be included in the above-mentioned objective functions is very flexible. For example, when only partial mode shape data is available for a particular mode, there is no need to perform mode shape expansion and one can simply include those measured DOFs in the objective function. It is essential, however, that both the measured and analytical mode shapes are normalized in a consistent manner and paired correctly. In this study, the mode shapes are normalized such that the maximum absolute mode shape displacement is equal to unity.

Combining the modal frequency and mode shape errors, in this study the objective function is defined as

$$J = -\frac{1}{N_f} \sum_{i=1}^{N_f} W_{\omega i} \left| \frac{\omega_i^m - \omega_i^a}{\omega_i^m} \right| - \frac{1}{N_s} \sum_{i=1}^{N_m} W_{\phi i} \sum_{j=1}^{N_{nd}} |\phi_{ij}^m - \phi_{ij}^a| \quad (3-14)$$

The reason for using absolute rather than relative mode shape error is because the relative differences of a few mode shape components with close-to-zero values (near the mode shape nodal points) may dominate the entire objective function. The use of the negative sign in Eq. (3-14) is to turn the problem into a maximization operation.

It has to be pointed out that the allocation of weights has considerable influence on the updating process and the results by GA. There is no unique method for the selection of weighting factors W_{ω_i} and $W_{\phi_{ij}}$. A general rule is that the weighting factors may be chosen as inversely proportional to the variance of the corresponding measurements. It is worth pointing out that a scrutiny on the fitness function (3-14) sheds light on the use of the ranking selection method in the present study in that Eq. (3-14) always holds a negative value, which will be troublesome with roulette-wheel types of methods.

3.4.2 Implementation Scheme of GA in FE Model Updating

Having specified the objective function, a searching space for the individual variables needs to be defined before GA can start. In a model updating problem, the variables are the structural parameters to be updated and the searching space for GA should be defined according to the possible variation range of each parameter. The evaluation of the fitness involves a structural analysis procedure to calculate the eigendata. Figure 3-1 shows the flowchart of a GA-based model updating procedure.

Figure 3-1 Flowchart of GA-based dynamic model updating procedure

In the actual implementation, it is necessary to choose a proper configuration for GA. Studies by DeJong (1975), Grefenstette (1986) and Schaffer (1989) indicate that, for binary-encoding GAs, a population size of 20-30 chromosomes, a crossover rate of 0.6-0.9 and a mutation rate 0.01-0.02 perform adequately in most cases. However, for a real-coding GA, theoretically a lower crossover rate and a higher mutation probability should be considered. In the present study, the configuration of GA is chosen following a preliminary trial procedure, in which the mutation rate was selected between 0.05 and 0.15, and 10 runs with different seeds were performed. The running average error upon the final GA solution and the variance value are used to gauge the quality of the updating. The chosen configuration details are shown in Table 3-1.

Table 3-1 GA configuration

Encoding Scheme	Population Size	Termination Criterion	Selection Method
Real number coding	80	Maximum generations 200	Linear ranking
Crossover	Mutation	Replacement Strategy	Fitness Scaling
Heuristic crossover probability $p_c = 0.80$	Uniform mutation probability $p_m = 0.12$	Elitist strategy replacement rate $\lambda = 0.10$	No

3.4.3 Eigensensitivity Algorithm for Dynamic Model Updating

The eigensensitivity-based model updating method has been well developed over the past several decades as described in Chapter 2. In the present study, it is used to fine-tune the global solution established from GA, in view of the fact that performing a local climbing can help overcome the difficulties arising from inadequate sensitivity of the modal data to the model parameters. The approach performs the iteration process based on equation (2-7), where the weighting matrix, $\mathbf{W}_{\varepsilon\varepsilon}$ is determined according to the error margin of the measured modal data. Figure 3-2 depicts a general procedure for the eigen-sensitivity based model updating.

Figure 3-2 Flowchart of eigensensitivity-based model updating procedure

3.5 Numerical Investigation

Numerical studies are conducted to demonstrate the effectiveness and efficiency of the above-mentioned combined GA and eigensensitivity approach in structural model updating. The influences of the possible measurement errors on the model updating results are also investigated by perturbing the numerically simulated (exact) modal properties within a prescribed margin of errors.

The example structures are chosen to be multi-storey frames which are commonly seen in low-to-medium-rise buildings. Assuming a pure-sway mode of deformation (rigid floors), the frames can be simplified as lumped-mass cantilever models as shown in Figure 3-3, allowing only lateral sway DOFs. Thus, the total number of DOFs is equal to the number of storeys of the frame. All the effective masses are lumped to the nodal points at the floor levels.

Figure 3-3 Multi-storey sway-type RC frame structure and its equivalent lumped mass model allowing only sway DOFs

The bending stiffness of the equivalent columns, which represents the inter-storey stiffness of the frame, are considered as structural parameters to be updated. In the current GA the stiffness variables are represented by the stiffness modification factors (SMFs) defined as the ratio between the actual stiffness and a reference stiffness (for

example the uncracked stiffness for RC structures), such that the range of SMFs fall within zero and one. It should be pointed out that the choice of the reference stiffness is not critical in the model updating by GA since the updated stiffness will be the product of the reference stiffness and the corresponding SMF. Any inaccuracy in the reference stiffness is automatically compensated in the updating of SMFs.

3.5.1 Model Updating with Exact Modal Data

The natural frequencies and mode shapes computed from the numerical simulation represents the exact measurement without errors or noises. These data are used to test the ability of GA in obtaining the correct model parameters through the updating procedure. For practical consideration, only the very basic set of modal data, i.e., the lowest three modal frequencies and the first mode shape, are employed for the model updating. Three structural cases are numerically simulated, with the number of storeys respectively equal to 6, 12 and 24 to represent a varying number of DOFs, which is equal to the number of unknown parameters here. The reference stiffness is the lateral stiffness corresponding to the uncracked gross-section stiffness of columns. For each arbitrarily chosen “actual” stiffness distribution along the structure height, the modal properties are obtained from the structural analysis and then supplied to GA.

i) Scenario 1: 6-Storey Frame

As depicted in Figure 3-3, the 6-storey frame model has the number of DOFs equal to 6. Table 3-2(a) lists the physical properties of the frame. The “actual” stiffness distribution is simulated by assigning each individual SMF a value randomly chosen from a broad range between 0.35 and 0.85 (see Table 3-3). The “measured” modal data are then simulated from the structural analysis using the above stiffness properties. In the objective function (Eq. (3-14)), weights W_{oi} ($i = 1, 2, 3$) are assigned equal to 1.0 and $W_{\phi i}$ ($i = 1$) equal to 0.40 based on the consideration that in the actual measurement the mode shape data may be less accurate than the frequencies. The operating parameters of the GA are given in Table 3-1. In order to gauge the

performance of GA, the so-called online and the offline performances are adopted. The online performance is the average fitness values of all individuals in each generation whereas the offline performance is a running average of the fitness values of those best individuals in the present and past generations.

Table 3-2 Properties of RC frames considered in the numerical investigation

a) 6-storey frame

Storey	Storey 1	Storey 2	Storey 3	Storey 4	Storey 5	Storey 6
Height (m)	5.0	3.0	3.0	3.0	3.0	3.0
Column cross section (m ²)	0.6 × 0.6	0.5 × 0.5	0.45 × 0.45	0.45 × 0.45	0.35 × 0.35	0.35 × 0.35
Lumped mass (kg)	47500	42000	40500	39500	37000	35500
Modulus	$E = 2.6 \times 10^{10} \text{ Pa}$					

b) 12-storey frame

Storey	1~3	4~6	7~9	10~12
Height (m)	5.0	3.0	3.0	3.0
Column cross section (m ²)	0.8 × 0.8	0.65 × 0.65	0.5 × 0.5	0.4 × 0.4
Lumped mass (kg)	58000	47000	42000	40000
Modulus	$E = 2.6 \times 10^{10} \text{ Pa}$			

c) 24-storey frame

Storey	1~4	5~8	9~12	13~16	17~20	21~24
Height (m)	5	3	3	3	3	3
Column cross section (m ²)	1.2 × 1.2	0.85 × 0.85	0.7 × 0.7	0.52 × 0.52	0.45 × 0.45	0.38 × 0.38
Lumped mass (kg)	76000	65500	60000	56000	46000	40000
Modulus	$E = 2.6 \times 10^{10} \text{ Pa}$					

For the 6-storey frame case, the online and offline performances of GA in updating the model using the above mentioned modal data are depicted in Figure 3-4. Table 3-3 compares the updated and actual (simulated) SMFs. As can be seen, the model is updated successfully. The maximum error in the updated SMFs is less than 1.5 percent.

The fact that the online and offline performance curves become almost superposed suggests that convergence is already achieved in the GA.

Table 3-3 Comparison between actual SMFs and updated results from GA

	β_1	β_2	β_3	β_4	β_5	β_6
Actual SMFs	0.5414	0.7230	0.5595	0.4857	0.4150	0.5133
Updated SMFs	0.5428	0.7137	0.5614	0.4895	0.4152	0.5111
Errors(%)	0.2528	-1.2841	0.3416	0.7909	0.0561	-0.4310

Figure 3-4 Performance curves of GA

ii) Scenario 2: 12-Storey Frame

Similarly, all the 12 SMFs in this frame case are selected as variables for updating. The physical properties of the frame are listed in Table 3-2(b). The parameter settings for GA as well as the weighting factors imposed on the modal data are the same as in

Scenario 1. Prior to updating, a sensitivity analysis (the procedure is similar to that described in the next section) is performed and it reveals that the given set of modal data are not very sensitive to the stiffness changes in some storeys. This implies that larger updating errors could occur in the respective SMFs.

Figure 3-5 compares the actual SMFs and the GA updated results. The updated SMFs are generally of a good accuracy except for a few storeys such as the 11th and the 12th storeys where the errors reach 5 ~ 10% . This result is not surprising given the prediction from the aforementioned sensitivity analysis. Apparently, with the increase of the number of variables to be updated, the inherent sensitivity problem becomes significant with the same set of modal data. To get rid of the problem, one way is to increase the size of the modal data set. But for practical reasons, it would be more desirable if better updating results can be achieved without increasing the demand on modal data. To this end, here the conventional eigensensitivity-based model updating method is incorporated to perform further local climbing on the basis of the global solution from GA. In fact, this combined approach proves to work out very successfully, as will be demonstrated with the next structure case.

Figure 3-5 Comparison between actual and the updated SMFs by GA for 12-storey frame

iii) Scenario 3: 24-Storey Frame

In this case, all the 24 SMFs are unknown variables for updating. The reference modal data set is still limited to including only the lowest 3 frequencies and the first mode shape. The physical properties of the structure are given in Table 3-2(c). The basic GA configurations remain the same.

(a) Updated by GA only

(b) Updated by GA and eigensensitivity method

Figure 3-6 Comparison between actual and the updated SMFs for 24-storey frame

For a randomly selected set of SMFs, the updated results by GA are depicted in Figure 3-6(a). Most of the SMFs still show a good accuracy, but large errors up to $\pm 20\%$ are observed in a few storeys such as 11th, 21st, 23rd and 24th storeys.

The eigensensitivity-based updating scheme is then applied to perform local-climbing on the GA updated results. Since the GA global solution has provided a very good initial state for the eigensensitivity-based updating procedure to start with, the local climbing procedure converges very quickly with just a few iterations. The final SMFs after the local climbing procedure are depicted in Figure 3-6(b). Marked improvement can be observed comparing the results before and after the local climbing operation, especially at the aforementioned few storeys.

3.5.2 Model Updating with Modal Data Containing Errors

The above examples have demonstrated that, by using the combined GA and eigensensitivity approach, almost perfect model updating results can be achieved with a very basic set of modal data, provided that these modal data are free of measurement errors or noise. In actual applications, however, it is difficult to achieve very accurate modal data which may be considered totally error free. As a norm for civil engineering structures, errors in the range of $\pm 1\%$ and $\pm 10\%$ may be expected in the measured natural frequencies and mode shapes, respectively (Friswell and Mottershead 1995). Under this circumstance, it is more rational to figure out a possible solution band within which the actual parameters should fall.

Because of the inherent sensitivity problem, it can be expected that the solution band will be wide when the modal data set is small. To improve the updating results, a practical option is to reduce the number of parameters to identify. This would involve other considerations and judgment which are beyond the scope of the present study. As far as the capability of the proposed updating algorithm is concerned, the algorithm is expected to also work with an increased data set so that the required number of parameters is updated to a satisfactory accuracy. The increase of the dataset may be

achieved by using higher-order modal data, incorporating time-domain information, considering structural local features, among others. For illustrative purposes concerning the application of GA, here we suppose that higher-order modal data are available, and the measurement errors are within $\pm 2\%$ for all modal frequencies and $\pm 5\%$ for mode shape data.

To assist identifying the required order of modal information to achieve a prescribed accuracy in the updated results with the contaminated modal data, a sensitivity analysis can be performed. First the analytical modal quantities are expanded using a 1st order Taylor series at the point of the nominal SMFs. Thus, the relationship between modal data changes, δR , and SMFs variations, δP , can be established via sensitivity matrix S as follows

$$\{\delta R\} = S\{\delta P\} \quad (3-15)$$

To avoid ill-conditioning of the sensitivity matrix, the normalized version of the sensitivity matrix S_n is utilized here. S_n represents the percentage change of the response values for one percent change of the parameter values and is defined as

$$[S_n]_{ij} = \frac{\partial R_i}{\partial P_j} \frac{P_j}{R_i} \quad (3-16)$$

For over-determined systems under consideration, the inverse gain matrix G_n of the normalized sensitivity matrix S_n can be expressed as (Friswell and Mottershead 1995)

$$G_n = [S_n^T S_n]^{-1} S_n^T \quad (3-17)$$

The gain matrix G_n is calculated based on the updated FE model. Thus, the normalized modal data increment vector, $\{\delta\bar{R}\}$, is related to the SMFs increment vector, $\{\delta\bar{P}\}$, as

$$\{\delta\bar{P}\} = \mathbf{G}_n \{\delta\bar{R}\} \quad (3-18)$$

A series of perturbation vectors $(\delta\bar{R}_i, i = 1, 2, \dots, m)$ of the modal data within a specified error range are generated randomly. Subsequently, the m resulting vectors of structural parameter changes corresponding to the m $\delta\bar{R}_i$ can be obtained from Eq. (3-18).

Take the 6-storey and 12-storey frames as examples: Figure 3-7 show the trend of decreasing parameter errors with increasing order of modal information. (It should be noted that for different stiffness values the sensitivity results can differ, and the results shown in Figure 3-7 represents a typical trend). As can be seen, for the 6-storey frame to achieve a stiffness parameter error within $\pm 10\%$ (average about $\pm 6\%$), the first three modes (frequencies and mode shapes) will be required; while for the 12-storey frame to achieve a similar degree of accuracy, modal data up to the 6th mode will be necessary.

Figure 3-7 Variation of nominal errors in the updated parameters (SMFs) with increasing order of available modal data based on sensitivity analysis

i) 6-Storey Frame

GA is applied to actually perform the updating for the 6-storey frame based on the lowest 3 modes containing the above-mentioned level of measurement errors. For an arbitrary selected combination of SMFs in the range of 0.35~0.85, the modal frequencies and mode shapes are first obtained from structural analysis as mentioned in the previous cases. These modal data are regarded as the measured nominal values for which an error of $\pm 2\%$ in the frequencies and $\pm 5\%$ in the mode shapes existed with a uniform probability distribution. 30 sets of reference modal data are then generated via random sampling, and GA is applied to perform model updating based on each of these modal data sets.

Table 3-4 summarizes the updated SMF results, together with the theoretically anticipated solution bandwidth based on the sensitivity analysis. As can be seen, the GA updated results are very satisfactory in that all 6 SMFs fall within the theoretical margin, whereas the maximum parameter error is less than $\pm 6\%$. This indicates that GA as an algorithm is capable of achieving satisfactory updating results even under a noisy modal data environment.

Table 3-4 Summary of GA updating results for 6-storey frame with modal data containing errors (first 3 modes, $\pm 2\%$ error in frequencies and $\pm 5\%$ error in mode shape)

	Actual SMFs	Updated Results from GA		Sensitivity analysis
		SMFs Range	Error range (%)	Anticipated error (%)
β_1	0.5856	0.5588 ~ 0.6132	-4.584~ 4.7104	-4.608~ 4.810
β_2	0.4164	0.3932 ~ 0.4348	-5.557~4.4420	-5.572~5.490
β_3	0.6906	0.6607 ~ 0.7227	-4.325~ 0.4650	-5.370~ 0.651
β_4	0.5168	0.5029 ~ 0.5346	-2.696~ 3.442	-4.063~ 3.546
β_5	0.7081	0.6705 ~ 0.7506	-5.316~ 5.9943	-5.393~ 6.064
β_6	0.7355	0.7194 ~ 0.7603	-2.200~ 3.374	-2.200~ 3.690

ii) 12-Storey Frame

For the 12-storey frame, the sensitivity analysis (Figure 3-7) suggests that, with the aforementioned level of errors in the modal data, the lowest 6 modes will be required in order to achieve an updating accuracy within $\pm 10\%$ for all 12 SMFs. Following a similar procedure as described in the 6-storey frame case, GA is applied to update the 12-storey frame model for a total of 30 sets of randomly sampled modal data within the given margin of errors. Table 3-5 summarizes the updated SMF results. The comparison once again shows a good consistency between the GA solution bandwidth and the theoretically anticipated errors.

Table 3-5 Summary of GA updating results for 12-storey frame with modal data containing errors (first 6 modes, $\pm 2\%$ error in frequencies and $\pm 5\%$ error in mode shape)

	Updated Results from GA			Sensitivity Analysis
	Actual SMFs	SMFs Range	Error Range (%)	Anticipated error (%)
β_1	0.6913	0.6802 ~ 0.7160	-1.631 ~ 3.576	-3.192~4.065
β_2	0.6325	0.5909 ~ 0.6531	-6.503 ~ 3.255	-8.364~7.953
β_3	0.3513	0.3644 ~ 0.3742	3.729~ 6.112	-5.165-6.016
β_4	0.7951	0.8189 ~ 0.8690	2.987 ~ 9.282	-4.562~10.660
β_5	0.3847	0.3977 ~ 0.4143	3.375 ~ 7.684	-7.953~8.056
β_6	0.4027	0.4239 ~ 0.4389	5.258 ~ 8.899	-7.982~9.652
β_7	0.3850	0.3940 ~ 0.4109	2.328 ~ 6.712	-6.325~7.152
β_8	0.5266	0.5257 ~ 0.5444	-0.173 ~ 3.372	-3.562~3.657
β_9	0.6836	0.6874 ~ 0.7163	0.553 ~ 4.778	-5.236~5.601
β_{10}	0.4875	0.4973 ~ 0.5152	2.002 ~ 5.675	-5.706~6.320
β_{11}	0.6584	0.6597 ~ 0.6886	0.195 ~ 4.589	-5.023~4.953
β_{12}	0.6778	0.6736 ~ 0.6994	-0.616 ~ 3.186	-4.029~4.268

The above numerical examples demonstrated clearly the superior performance of the combined algorithm. In lieu of a rigorous mathematical proof on the improved performance, a sound rationale can be provided, as follows. Despite its robustness, GA generally can only find an approximate solution to a complex mathematical problem. In most cases this may suffice. However, in model updating problems where a large number of variables need to be updated while only a limited set of low-order modal data is available, the sensitivity can become an outstanding issue. The consequence is that small errors in the objective function may project to large inaccuracies on some of the structural parameters being updated. Where higher-order modal data are not available or not to a required measurement accuracy standard as is the case in many practical situations, one would have to resort to local method to refine the updated results from GA. The local-climbing or gradient-based mathematical methods are known to be powerful at finding accurate local solutions, but they require a proper starting solution. Now that GA has provided an approximate global solution, the incorporation of the gradient-based methods, in particular the eigensensitivity algorithm, becomes an obvious choice for improving the GA solution.

3.6 Application in Seismic-Response Prediction – Experimental Study

Actual structures have large number of unmeasured DOFs, along with various other structural and measurement uncertainties. It has been a major challenge for theoretical algorithms which are verified numerically to be applied for real structural and measurement conditions. As a matter of fact, very few real applications of sophisticated model updating schemes have been reported in the literature. In this section, a six-storey RC frame tested under simulated earthquakes will be used as a case study to demonstrate the effectiveness and usefulness of the proposed model updating scheme in real applications.

At this juncture, it may be appropriate to point out that the FE model updating method proposed in the present study is not just limited to the identification of structures at their intact states and it can be equally applied after a structure experiences a damaging

load. When a structure experiences a damaging load, there will be nonlinear response during the loading process and after that the structure rests to a new state, for example with reduced stiffness in some members. The aim of the model updating is to identify the (stiffness) parameters of the current state of the structure, whether it is damaged or not, using modal data which may be obtained by testing on the (damaged) structure with small amplitude vibration (modal test). The linear response restriction actually applies to the modal test phase in acquiring the modal parameters of the structure; it does not mean the structure itself has to remain at its original elastic state during all past events.

Depending on the intended use of the analytical model and the available measurement data set for model updating, a structure may be idealized in different ways. In the present case study, the frame will be simplified into a pure-sway lumped mass cantilever, thus each storey is represented by a single mass and an equivalent storey stiffness parameter.

The selected test frame was 1:5.5 scaled version of a 6-storey RC frame (Lu 2002). For convenience, all the quantities presented here are converted back to the full-size scale. Figure 3-8 shows the geometry and the dimensions of the frame. The total lumped masses at the storey levels are: 47.5, 42.0, 39.5, 37.0 and 35.5 tons for storeys 1 to 6, respectively.

Before the real earthquake simulation tests, the frame was subjected to a series of pre-test operations, namely the installation onto the simulator platform, fixing of the added masses, and some small amplitude tests for system checking and identification of the dynamic properties of the frame. Besides, construction joints and other imperfections also existed, so the state of the frame prior to the main tests was unclear. Nevertheless, the lowest few natural frequencies and the first mode shape were available from a complementary random vibration test, making it possible to perform a model updating procedure to identify the equivalent structural parameters. With the updated model, the state of the frame can be assessed and its response to the subsequent earthquake excitation can be predicted more accurately.

Figure 3-8 Geometry of the test frame and test setup

Figure 3-9(a) shows typical frequency response functions from the random vibration test. The lowest 3 natural frequencies were identified to be 1.46, 3.61, and 6.74 Hz, while the first mode shape was recovered as shown in Figure 3-9(b). Simplifying the frame into a 6-DOF lumped-mass cantilever, and assuming the effective lumped masses at floor levels remain unchanged, the parameters that need to be updated in order to complete the model are the 6 stiffness parameters, or the stiffness modification factors (SMFs) as defined earlier. The reference stiffness is defined from the uncracked gross-section stiffness of the columns. With the above modal dataset the GA-eigensensitivity updating procedure is carried out, and the updating results are summarized in Table 3-6. The SMFs vary between 0.356 to 0.530 in different storeys.

Figure 3-9 Measured frequency and mode-shape data

Table 3-6 Updating results for the 6-storey test frame

Storey	1	2	3	4	5	6
Lateral Stiffness (KN/mm)	107.8	240.7	158.0	158.0	57.8	57.8
Updated SMFs using measured modal data	0.485	0.356	0.491	0.530	0.436	0.498
Updated SMFs using calculated modal data for original frame	0.566	0.371	0.591	0.518	0.753	0.787

It should be pointed out that the above SMFs refer to the uncracked storey stiffness with rigid floors. Since the actual frame involves also rotational DOFs at beam-column joints, the equivalent lateral storey stiffness is certainly smaller than that with rigid floors even if all members remain in the initial uncracked state. Therefore, it is necessary to evaluate the equivalent lateral stiffness of the undamaged frame before the above SMF values can be correctly interpreted. For this purpose, the frame is modeled with a refined beam-column model assuming uncracked stiffness for all members, and the dynamic properties are computed by applying a standard structural analysis procedure. Using the lowest 3 natural frequencies and the first mode shape

from the above frame analysis, the equivalent SMFs of the 6-DOF model are obtained using GA. The results are also listed in Table 6 for a comparison. Figure 3-10 shows a comparison among the undamaged lateral storey stiffness assuming rigid floors, the equivalent undamaged storey stiffness of the frame, and the equivalent storey stiffness of the actual test frame. As can be seen, the lateral stiffness of the undamaged frame is already significantly smaller than that assuming a rigid floor. On average, the SMFs with the undamaged frame are about 0.6. Comparing these undamaged SMFs with the SMFs identified from the measured modal data, it can be concluded that a certain degree of damage, corresponding to a stiffness degradation up to 17% in storeys 1~4 and as much as 40% in storeys 5~6, has already occurred in the test frame before the major tests.

Figure 3-10 Comparison of storey lateral stiffness

With the updated SMFs, the 6-DOF lumped-mass cantilever model is expected to represent well the actual test frame in terms of its basic dynamic properties and hence can be used as a basis for the dynamic response prediction. As a verification of such

capacity, the updated 6-DOF model is used to predict the response to an earthquake excitation as experienced by the actual frame. The earthquake excitation to which the test frame was subjected at the stage under consideration was a scaled El Centro type ground motion to have the peak ground acceleration (PGA) equal to 0.1g. It should be noted that the model updating within the scope of this paper is limited to a particular state of the structure at which the modal data are measured. The updated model provides a good initial state for subsequent predictions but future nonlinear response will depend upon the non-linear behavioral parameters which are not covered by the current model updating. The test frame under 0.1g excitation remained basically at the linear range, so it is suitable for use to verify the current updating results.

Figure 3-11 shows the actual base accelerogram and the comparison between the measured and predicted displacement time histories using the updated 6-DOF model. The damping coefficient is assumed to be 6% as estimated for the actual test frame during the experiment. For comparison, the predicted response using the 6-DOF model but with a rather arbitrarily assigned value of 0.7, which could still be regarded as reasonable without an updating procedure, are also shown in the figure.

As can be seen, the predicted displacement time histories using the updated 6-DOF model match almost perfectly the measured responses, both in terms of the response amplitudes and the detailed history. On the contrary, the response calculated using the arbitrarily assigned stiffness shows a poor comparison with the measured response.

The above example demonstrates that the proposed model updating scheme is applicable in a real structural and measurement environment. The updated computational model resembles well the actual dynamic system and hence provides a good basis for the dynamic response prediction and the assessment of the physical state of the structure.

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Figure 3-11 Comparison of measured and predicted response time histories:
Left - using updated stiffness values; Right - using arbitrarily
assigned stiffness equal to 70% of the uncracked column stiffness

3.7 Conclusion

In this chapter, a combined GA and eigensensitivity approach is presented for dynamic model updating based on measurable dynamic properties. The ability of GA in performing global optimization renders the updating procedure a desired robustness to handle problems with parameters varying in a wide range, as may be the case of building structures after experiencing a damaging incident such as an earthquake. The effectiveness of the approach is investigated with both numerically simulated and physically tested scenarios.

With a limited amount of modal data, namely the lowest 3 natural frequencies and the first mode shape, GA is shown to be able to perform satisfactorily model updating for cases involving a relatively small number of unknown parameters, such as a 6-storey sway frame. With the increase in the number of storeys, however, the GA updated results tend to become less satisfactory due to the inherently low sensitivity of such modal dataset to the variable stiffness in some storeys. To tackle this problem, the eigensensitivity based updating method is incorporated to perform further local climbing on the basis of the GA global solution. As a result, an almost exact solution is achieved with only a few iterations. The combined approach proves to be effective for the updating of a large number of parameters with a relatively small dataset.

In case modal data contain a certain level of measurement errors, e.g., in modal frequencies and in mode shapes, a sensitivity analysis indicates that higher order modal data will be necessary in order to achieve satisfactory updating results. The required order of modal data can be estimated through a sensitivity analysis. Provided that such a modal data set is available, GA is shown to be able to update the model to the degree of accuracy as can be expected from the sensitivity analysis.

The application of the proposed approach on an actual frame specimen demonstrates that the procedure is workable under a real structural and measurement environment. The updated model resembles well the actual dynamic system and hence provides a good basis for the dynamic response prediction and the assessment of the physical state of the structure.

CHAPTER 4

A TWO-LEVEL NEURAL NETWORK APPROACH FOR DYNAMIC FE MODEL UPDATING INCLUDING DAMPING

4.1 Introduction

A major source for inaccurate model updating comes from errors contained in the experimental data. This has been shown in the numerical investigation described in Chapter 3. Another source of errors in model updating using the conventional optimization scheme is the mode shape reduction or expansion process for purposes of maintaining consistency between the two sets of data (analytical and experimental mode shapes). In addition, the conventional optimization methods might result in a misleading solution due to its local searching mechanism. In these respects, the recent development in artificial intelligence algorithms – the Artificial Neural Networks (ANN) – provides a potentially powerful alternative to the traditional iterative-based method because of the particular working mechanism with which ANN operates, i.e., the learning phenomenon.

The appealing features of ANN concerning the FE model updating include a) it performs a function approximation of any complexity via a learning process based on the given discrete training patterns; and once trained, new updating can be carried out readily using the network (i.e., “generalization” in ANN terminology), in other words, ANNs, unlike other updating techniques including GA, represent the general solution

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to the underlying inverse problem; b) it has the potential to resist the influence of noises that are contained in the input data (structural response); and c) it can provide a global solution if properly trained. Furthermore, similar to GA, ANN itself does not require specific form or completeness of the response data; hence, the definition of the objective function can be extremely flexible.

In recent years, some researchers have applied ANN for structural damage detection related applications (Szewczyk and Hajela 1994; Masri et al. 1996; Levin and Lieven 1998; Yun et al. 2001). Much experience has been accumulated through these exploratory investigations. However, there appears to be lacking a generic procedure to apply ANN in FE model updating concerning general structural parameters, especially when damping is involved.

In the present study, the Multi-layer Feed-forward (MLP) neural network is employed to perform a general FE model-updating task including the updating of the damping parameters. To reduce the computational demand, and considering the fact that the influence of damping on the resonant and antiresonant frequencies of a lightly-damped system is negligible, the updating of the FE model parameters is divided into two levels using two separate neural networks. The first level network is trained to update the structural parameters (stiffness in particular) using the natural and antiresonant frequencies as the response data without considering the damping effect. With the updated structural parameters from the above first level updating, the second level network then deals with updating the variable damping ratios. For the updating of the damping ratios, the integrals of frequency response function (FRF) are used as the reference responses because of their inherent relationship with the damping factors. In order for the identification of a proper response configuration, a sensitivity analysis scheme is proposed, taking into account the carry-over error during the first-level

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updating in addition to the anticipated measurement error (“noise”) in the response data.

A numerical example is given to illustrate the implementation of the proposed scheme and demonstrate the effectiveness of the procedure; and in particular, the noise-resisting ability of the neural network trained using the noise-injection strategy is highlighted.

4.2 Overview of Artificial Neural Networks (ANN)

Artificial neural networks, or simply neural networks, are essentially computational structures that mimic the operation of biological neurons of mammalian brains. Such structures encapsulate a variety of simple processing units (artificial neurons), interconnected with each other. Each neuron receives a number of inputs and produces one output as shown in Figure 4-1(a); however, there exist a few different types of neurons associated with different schemes in manipulating the input information, $\mathbf{x}$, to result in the output, $\mathbf{y}$.

Among many different forms of network topologies (interconnection style of neurons), the MLP network (Fahlman 1989; Bishop 1995) (see Figure 4-1(b)), which possesses a layered structure and allows only connections from neurons in one layer to those in layers of its forward direction, has been applied widely due to its effectiveness and simplicity. This special topology form is mainly designed for approximating an unknown function relation, and the ability for the network to do so is realized through a learning process on the provided data patterns (training data), whereby the interconnection weights, w , are continuously adjusted until a predefined error criterion is reached.

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In the present study, the widely-used Levenburg-Marquardt (LM) back-propagation training algorithm is adopted to carry out the neural network learning towards minimizing a predefined error function, which is generally formulated as a mean-square-error between the network outputs and the actual values corresponding to the given set of input vectors. As a result, an optimal set of weights is obtained.

a) A simple neuron (processing unit): $O = \phi\left(\sum_{i=1}^n w_i h_i + b\right)$,
 ϕ : Log-sigmoid transfer function

b) An l -layered MLP network

Figure 4-1 Schematic illustration of the neuron (processing unit) and typical network structure

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In general, the design of a MLP network needs to decide on the following: the number of hidden layers; the associated number of neurons in each hidden layer; and the interconnection patterns among the neurons. The identification of a true optimal combination can be very time consuming and some novel methods have been proposed in an attempt to tackle this problem (Harp 1990; Funabiki 1997). At the present stage, a trial procedure is still commonly practiced in the design of ANN topology for engineering applications. In fact, it has also been shown that a network with only one hidden layer suffices to approximate a large spectrum of complex functions (Hornik et al. 1989; Funahashi, 1989; Cybenko 1989; Hartman et al. 1990); therefore, the required effort in identifying a desirable network design can be considerably reduced by using just one or two hidden layers. Based on these considerations and following some trial exercises, in the present study the neural network structure is unified to contain two hidden layers and both hidden layers are to have an equal number of hidden neurons, leaving only the number of hidden neurons to be determined in the network topology design, for which a trial process is effective.

For the improvement of the network performance, a data scaling process is usually required. This is because the compiled raw training data, such as the modal frequencies of a structure used in this study, can vary significantly in their original values. When such data are directly used in the training procedure, the network could exhibit ill-conditioning and possibly does not learn at all. Besides, the application of a network trained in this way may also bear the risk that some input components are in fact ignored; consequently, the network would no longer represent the underlying system. This problem can be avoided by a proper scaling on the raw input data patterns, such that the input data are normalized to fall within a prescribed bound, for example in the range of [-1,1]. For this particular value range, the transformation can be of a linear form,

$$\mathbf{x}_m^N = 2 \frac{\mathbf{x}_m - \min(\mathbf{x}_m)}{\max(\mathbf{x}_m) - \min(\mathbf{x}_m)} - 1 \quad (4-1)$$

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where $\mathbf{x}_m$ is a row of the input data matrix, in which each column represents one given data pattern for training, and $\mathbf{x}_m^N$ is the normalized quantity.

It is well recognized that the applicability of traditional updating methods is often restricted due to the measurement errors that exist in practically all measured response data. In this regard, the neural network technique is particularly appealing because of its potential noise-resisting capability. This capability can be acquired through the so-called noise-injection learning. Usually the possible margin of errors in the measured structural response information is assessable from relevant past experiences. In the implementation of the noise-injection learning algorithm for the neural network training, a similar level of random noise simulating the actual measurement errors or noises is injected into the network input data (responses) while their correct output counterparts are retained. Such noise-injection operation is straightforward and it can be expressed as

$$\tilde{R}_j = R_j(1 + v_j) \quad (4-2)$$

where R_j and $\tilde{R}_j$ represent the noise-free (calculated) and the noise-injected response components, respectively, and v_j is a noise item simulating anticipated noise or errors in the measured response data. At this juncture, it also becomes obvious that the success of the noise-injection learning in real applications is subject to the adequacy of the noise model chosen in representing the actual noises for the particular problem under consideration.

4.3 Review of ANN-Based Structural Assessment Applications

As described in the previous sections, the artificial neural network as a non-parametric approach is capable of implicitly approximating any function with finite discontinuities in a parallel manner and it has been applied successfully in many fields, such as nonlinear regression, classification, prediction, signal processing, data conceptualization and automatic control. This section focuses on the applications of neural networks for the damage detection and FE model updating problems.

Szewczyk and Hajela (1994) formulated the detection of damage in structural systems as an inverse problem, which was solved by using feature-sensitive neural network or the so-called “counterpropagation neural network (CPN)”. In this approach, possible damage and its degree were identified by relating the changes in structural static displacements directly to degradation of structural components in terms of elastic moduli. Numerical studies indicated that such networks can be trained with a realistic number of patterns, and that they were capable of making correct diagnosis based solely on the measurements of static displacement responses at a few points under a given load configuration. In addition, preliminary results of several numerical simulations showed that the performance of networks deteriorated gradually in the case of noisy and incomplete input data, while the resultant outputs from the CPN were still acceptable.

Masri et al. (1996) explored a methodology based on a neural network to detect parameter changes in either linear or nonlinear systems. The restoring force associated with an element or substructure of the system was selected as response. Demonstrative examples, obtained by randomly exciting linear as well as nonlinear systems with a wide range of parameter changes, with and without noise pollution, were presented to gauge the sensitivity of the approach to small perturbations in the example system parameters. It was shown that the existence of nonlinearity and/or noise may degrade the accuracy of the trained networks; however, the proposed scheme is regarded as robust for detecting relatively small changes in the dominant structural system parameters.

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Levin and Lieven (1998) used a radial basis function (RBF) neural network to perform FE model updating on a simple cantilevered beam. In their study, twenty variable parameters were selected for updating, while the response data used were the first three bending and first two extensional modes. The input vectors for the network included 55 components. To determine the appropriate centers for the RBF network, a trial-and-error procedure was employed to construct the hidden layer (centers) by introducing one center each time, to result in the minimum mean-square-error (MSE) on the training patterns. The actual updating process was carried out iteratively using a sequence of networks to approach the expected solution, where each network was trained with the training data that are generated based on the updated model from its preceding network. In addition, the noise-resistance capability of the neural network was investigated via simulated noisy measurements, in which both the errors of FRFs data and noise caused by experimental modal analysis (EMA) process were considered. The results indicated that the proposed updating technique can successfully tackle the FE model updating problem, even in the presence of realistic level of noise in the measurements. However, the proof of the noise-resisting ability of the ANN by an observation that the updating errors fell within a predefined threshold seems not sufficiently convincing.

Lopes and Andrade (1998) utilized a MLP neural network to simulate the inverse of the sensitivity matrix of the structural response components with respect to the variable parameters. The input data for the network is an alteration of the structural modal quantities (natural frequencies and mode shapes) and the corresponding outputs represent the variations of discrete masses or physical boundary conditions. Once trained, the network can predict the changes of the variable parameters due to the changes of measurements. Furthermore, the approach can simulate the terms of the sensitivity matrices both for small and large parameter variations. The proposed methodology was evaluated by updating a jacket type offshore spatial structure and it was proved to be successful. However the article did not touch on the potential influence of measurements errors on the final updating results from the neural network. Moreover, the requirement on the response in order to guarantee a successful

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parameter updating was not discussed. The difficulties that could arise when dealing with complex structures was not mentioned either.

Ziemianski et al. (1999) presented the application of multi-layer feed-forward neural network for updating of a mathematical model of the structure based on dynamic data. The selected responses were the frequency response functions (FRFs). A data compression technique based on the neural network was adopted to extract effective characteristic FRF points among large amount of available choices, so as to reduce the dimension of the input layer to improve the computational efficiency. The proposed method was verified on a 12-storey frame structure consisting of twelve concentrated masses coupled by springs. Two updating scenarios were simulated with assumed stiffness reduction at only one storey and mass alteration at two storeys, respectively. The updating results for the two cases were of good accuracy, which proved the effectiveness of the FRF data compression technique. In addition, by an incomplete trial-and-error procedure, it was concluded that only one FRF was enough to realize a successful updating on the cases mentioned above. This research work has similar limitations as mentioned in the comments on the previous article reviewed.

Yun et al. (2001) proposed a neural network technique to estimate the structural joint damage from modal data. The joint fixity factors were defined to represent the damage extent of a joint and these were subjected to identification by the neural network. The authors employed a substructural identification scheme for local damage assessment. In the training of the neural network, the so called “noise-injection” learning, whereby the exact training patterns are injected with noise of a realistic level, was used, and it was proved to be very effective in case that the measured data is severely corrupted with errors. The proposed technique was then applied for a numerical simulation of a two-bay and ten-storey steel frame case and also for an experimental study on a two-storey frame. The identified results were found to be satisfactory in terms of the predefined performance criterion. The applicability of the proposed approach is limited by the assumption that the available response information is sufficient and sensitive enough with respect to the variable parameters (fixity factors), whereas no guideline was given as how to assess the suitability of the measured responses.

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From the above review, it can be observed that at present the ANN-based model updating technique is mostly limited to the exploration of proper response data required for ANN to achieve satisfactory performance. The methods involved are primarily of trial-and-error nature. Moreover, there appears to be lacking a generic procedure for the application of ANN in FE model updating concerning general structural parameters, especially when damping is considered. In the sections that follow, a comprehensive two-level ANN-based model updating involving damping will be presented.

4.4 Basic Considerations for the Two-Level Neural Network Updating Scheme

4.4.1 Antiresonant Frequencies

As the damping effect is ignored at the first-level updating on structural parameters in the two-level network scheme, it is deemed appropriate not to consider mode-shape data as responses because the mode shapes are more sensitive to damping. This, however, will result in a drastic reduction in the size of the available measured data set. To compensate for this, the antiresonant frequencies are considered together with the natural frequencies to enlarge the response data set.

The antiresonant frequencies are defined as the frequencies at which the magnitude of the frequency response at a measured DOF approaches zero (Peyt 1990), as depicted in Figure 4-2(a). Lallement and Cogan (1992) introduced the concept of using antiresonant frequencies to update FE models. The reason is that these antiresonant frequencies can be easily and accurately measured in a similar way as for the natural frequencies. Furthermore, a system can have much greater number of antiresonant frequencies than natural frequencies because every different frequency response function (FRF) between an actuator and a sensor contains another set of antiresonant frequencies. Lallement and Cogan referred to this increased amount of data as an “enlargement of the knowledge space” (Lallement and Cogan 1992). Mottershead

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(1998) showed that the antiresonance sensitivities to structural parameters can be expressed as a linear combination of natural frequency and mode shape sensitivities, and furthermore that the dominating contributors to the antiresonance sensitivities are the sensitivities of the nearest frequencies and corresponding mode shapes. Therefore, Mottershead concluded that the antiresonant frequencies can be a preferred alternative to mode shape data.

a) Definition of antiresonant frequencies

b) Integration intervals of FRF for damping ratio updating

Figure 4-2 Definition of antiresonant frequencies and integrals of FRF curve

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To calculate antiresonant frequencies of a dynamic system, He and Li (1995) developed an accurate and efficient method for undamped systems. Consider the eigen-equation of a dynamic system given by

$$(\mathbf{K} - \omega^2 \mathbf{M})\{\phi\} = 0 \quad (4-3)$$

where matrices $\mathbf{K}$, $\mathbf{M}$, vector ϕ and scalar ω denote structural stiffness matrix, mass matrix, mode shape vector and eigen-frequency, respectively. Generally, the FRF matrix $\mathbf{H}(\omega)$ is defined as

$$\mathbf{H}(\omega) = (\mathbf{K} - \omega^2 \mathbf{M})^{-1} \quad (4-4)$$

The FRF for a sensor at DOF p and an actuator at DOF q should be the pq^{th} element of $\mathbf{H}(\omega)$

$$H_{pq}(\omega) = (\mathbf{K} - \omega^2 \mathbf{M})_{pq}^{-1} = \frac{adj(\mathbf{K} - \omega^2 \mathbf{M})_{pq}}{Det(\mathbf{K} - \omega^2 \mathbf{M})} = (-1)^{p+q} \frac{Det(\mathbf{K}_{pq} - \omega^2 \mathbf{M}_{pq})}{Det(\mathbf{K} - \omega^2 \mathbf{M})} \quad (4-5)$$

where $\mathbf{K}_{pq}$ and $\mathbf{M}_{pq}$ indicate that row p and column q are deleted from the matrices. Therefore, the antiresonant frequencies ω_a for the FRF between p and q are the positive roots of the following equation

$$Det(\mathbf{K}_{pq} - \omega_a^2 \mathbf{M}_{pq}) = 0 \quad (4-6)$$

For systems involving the damping effect, equation (4-3) should be modified to incorporate damping in order to calculate its antiresonant natural frequencies, which in this case are complex quantities. It is worth noting that matrices $\mathbf{K}_{pq}$ and $\mathbf{M}_{pq}$ in

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equation (4-6) remain symmetric in case of collocated sensor and actuator, i.e., $p = q$, and therefore the corresponding FRFs will always contain real antiresonant frequencies; otherwise, antiresonant frequencies of complex conjugates can be expected. In this research, only those antiresonant frequencies obtained from collocated FRFs are considered.

4.4.2 Damping Model

The identification of damping in structural systems is extremely important if a model is to predict reliably the transient responses, transmissibility, decay times or other characteristics in design and analysis that are dominated by energy dissipation. However, unlike the overall stiffness and mass matrices, the damping matrix $\mathbf{C}$ cannot be constructed from the element damping matrix (Bathe 1996). In the present study, it is not intended to carry out an in-depth investigation on the most appropriate damping model for a given system; but rather, the focus is placed on the identification of the damping values under a preselected damping model. For this purpose, the commonly used Rayleigh damping model in lightly damped systems is employed, which can be expressed as:

$$\mathbf{C} = \mathbf{M} \sum_{k=0}^{r-1} \alpha_k [\mathbf{M}^{-1} \mathbf{K}]^k \quad (4-7)$$

where r is the number of damping ratios used to approximate the structural damping effect and coefficients α_k ($k = 0, 1, \dots, r-1$) are obtained from the r number of simultaneous equations

$$\zeta_i = \frac{1}{2} \left(\frac{a_0}{\omega_i} + a_1 \omega_i + \dots + \alpha_{r-1} \omega_i^{2r-3} \right) = \frac{1}{2} \sum_{k=1}^r \alpha_{k-1} \omega_i^{2k-3} \quad (i = 1, 2, \dots, r) \quad (4-8)$$

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where ζ_i denotes the i^{th} modal damping ratio. The expression (4-7) for the damping matrix, allowing for the orthogonality of mode shapes, provides a very convenient way for incorporation into the calculation of structural dynamic responses.

The damping model represented by (4-7) implies that the total damping in the structure is the sum of the individual damping in each mode. Thus, the ability to measure values for the damping ratio ζ_i , and hence the damping behavior of the complete structure system, is an important consideration in practice. It is noted that, with $r = 2$, equation (4-7) reduces to the Rayleigh damping which is frequently called the “proportional damping”. However, the Rayleigh damping model obviously damps the higher modes considerably more than the lower modes. Hence, more damping ratios would be desirable to better simulate the true damping behavior of a structural system.

Although the existing modal-analysis theory can identify damping parameters based on structural FRF data, the results can be highly susceptible to the measurement errors. In the present study, an ANN-based method is proposed for identifying the damping ratios under the above-mentioned damping model. The reference responses are taken from the integrals of FRFs over a specified small frequency range in the vicinity of the natural resonance frequencies, as shown in Figure 4-2(b). In this way, the identified damping ratios can be of better accuracy because the noise components on the FRF curves are somehow “neutralized” through the integration process. It is noted that integrals of FRF curves can also be used as conditioned frequency domain data for structural parameter updating as proposed in previous work (Atalla 1996; Inman 1998). In the present study, the integrals of FRFs are employed in its capacity for updating structural damping ratios.

4.4.3 Sensitivity Analysis for Selection of Response Configuration and Evaluation of Network Performance

In the context of ANN-based FE model updating, the sensitivity analysis can be engaged for two purposes. One is to assist in the determination of a desired response configuration (i.e., the composition of the response vector). The proposed model updating herein uses the modal frequencies and FRF integrals as the response data for the updating of the FE model parameters. Since different antiresonant frequencies and FRF curves can be obtained by placing the actuator and sensor at different locations (DOFs) of the structure, it is possible to have different combinations of the response components within a practical limit of the modal order. Based on the sensitivity analysis, a response configuration that yields a smaller variation of the FE parameters with a given level of perturbation to the response data is considered more desirable. Another purpose of the sensitivity analysis is to provide a margin of anticipated errors in the FE parameters in case the response data contain a certain level of measurement errors. This estimate can be compared with the actual neural network prediction errors to evaluate the noise-resisting ability of the trained neural network.

As the proposed updating scheme involves two levels of neural networks and each performs a separate task based on different types of response data, separate sensitivity analysis is carried out for the two updating processes. Because the second network proceeds on the basis of the updated structural parameters from the first network, the errors in the structural parameters will result in errors in the computed FRFs to be used for the training of the second network. Such “carry-over” errors should be considered in conjunction with the errors that can be anticipated in the actual measured FRFs for the sensitivity analysis concerning the damping ratios.

4.4.3.1 Sensitivity Analysis Concerning Structural Parameters (First-Level Network)

In this sensitivity analysis, both natural and antiresonant frequencies are involved. Equation (4-3) can be rewritten for the antiresonance eigenproblem as

$$\left(\mathbf{K}_{pq} - \lambda^a \mathbf{M}_{pq}\right) \{\phi_b\} = 0 \quad (4-9)$$

Differentiating equation (4-9) with respect to a structure parameter p_j yields

$$\left(\mathbf{K}_{pq} - \lambda_b^a \mathbf{M}_{pq}\right) \frac{\partial \{\phi_b\}}{\partial p_j} + \left[\frac{\partial \mathbf{K}_{pq}}{\partial p_j} - \lambda_b^a \frac{\partial \mathbf{M}_{pq}}{\partial p_j} - \frac{\partial \lambda_b^a}{\partial p_j} \mathbf{M}_{pq} \right] \{\phi_b\} = 0 \quad (4-10)$$

where λ_b^a is an antiresonance eigenvalue, p_j denotes a structural parameter, and the subscript pq denotes that row p and column q have been removed from matrices $\mathbf{M}$ and $\mathbf{K}$. Now consider a different but related eigenproblem

$$\left(\mathbf{K}_{pq} - \lambda_b^a \mathbf{M}_{pq}\right)^T \{\eta_b\} = 0 \quad (4-11)$$

Since the eigenvalues of a matrix are invariant with respect to the transpose operation, the eigenvalues from (4-11) will be the same as those from (4-9). However, the eigenvectors $\{\eta_b\}$ will definitely differ from $\{\phi_b\}$. Transposing (4-11) gives

$$\{\eta_b\}^T \left(\mathbf{K}_{pq} - \lambda_b^a \mathbf{M}_{pq}\right) = 0 \quad (4-12)$$

The eigenvector $\{\eta_b\}$ is called the *left eigenvector* because it pre-multiplies $\left(\mathbf{K}_{pq} - \lambda_b^a \mathbf{M}_{pq}\right)$, whereas the standard eigenvector post-multiplies $\left(\mathbf{K}_{pq} - \lambda_b^a \mathbf{M}_{pq}\right)$.

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Thus, pre-multiplying (4-10) by $\{\eta_b\}^T$ and considering (4-12) give the antiresonance sensitivity

$$\frac{\partial \lambda_b^a}{\partial p_j} = \frac{\{\eta_b\}^T \left(\frac{\partial \mathbf{K}_{pq}}{\partial p_j} - \lambda_b^a \frac{\partial \mathbf{M}_{pq}}{\partial p_j} \right) \{\phi_b\}}{\{\eta_b\}^T \mathbf{M}_{pq} \{\phi_b\}} \quad (4-13)$$

Notice that if $\mathbf{M}_{pq}$ and $\mathbf{K}_{pq}$ are equally symmetric matrices, then $\{\eta_b\} = \{\phi_b\}$, and hence (4-13) reduces to the symmetric eigenvalue sensitivity problem as in solving for the sensitivity of natural frequencies.

Denote the parameters to be updated in a vector form as $\{p\}$ and the responses used as $\{R\}$ including natural and antiresonance frequencies to a practically affordable order. The relationship between a small perturbation of responses, $\delta\{R\}$, and the corresponding variation of parameters, $\delta\{p\}$, can be expressed as

$$\delta\{R\} = \mathbf{S} \delta\{p\} \quad (4-14)$$

where the sensitivity matrix $\mathbf{S}$ consists of two types of sensitivity components, namely natural frequency sensitivity and antiresonance sensitivity as

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$$\mathbf{S} = \frac{1}{2\pi} \begin{bmatrix} \frac{\partial \lambda_1}{\partial p_1} (2\omega_1)^{-1} & \frac{\partial \lambda_1}{\partial p_2} (2\omega_1)^{-1} & \dots & \frac{\partial \lambda_1}{\partial p_m} (2\omega_1)^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \lambda_n}{\partial p_1} (2\omega_n)^{-1} & \frac{\partial \lambda_n}{\partial p_2} (2\omega_n)^{-1} & \ddots & \frac{\partial \lambda_n}{\partial p_m} (2\omega_n)^{-1} \\ \frac{\partial (\lambda_b^a)_1}{\partial p_1} [2(\omega_b^a)_1]^{-1} & \frac{\partial (\lambda_b^a)_1}{\partial p_2} [2(\omega_b^a)_1]^{-1} & \dots & \frac{\partial (\lambda_b^a)_1}{\partial p_m} [2(\omega_b^a)_1]^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial (\lambda_b^a)_l}{\partial p_1} [2(\omega_b^a)_l]^{-1} & \frac{\partial (\lambda_b^a)_l}{\partial p_2} [2(\omega_b^a)_l]^{-1} & \dots & \frac{\partial (\lambda_b^a)_l}{\partial p_m} [2(\omega_b^a)_l]^{-1} \end{bmatrix} \quad (4-15)$$

where $(\omega_b^a)_i = \sqrt{(\lambda_b^a)_i}$ is the antiresonant eigen-frequency. Inverting equation (4-14) gives the operational formula

$$\delta\{p\} = \mathbf{S}^{-1} \delta\{R\} = \mathbf{G} \delta\{R\} \quad (4-16)$$

which gives the variation of the physical parameters as a result of a small perturbation of the responses. Such sensitivity results can be used in judging the adequacy of a particular response configuration. On the other hand, when the response perturbation is set to represent an anticipated margin of errors in the measured response, the parameter variation as expressed in Eq. (4-16) will indicate the errors that can be expected in the updated parameters. This error estimation can then be used as a norm to evaluate the performance of the trained neural network.

4.4.3.2 Sensitivity Analysis Concerning Damping Parameters (Second-Level Network)

The second neural network is used to update structural damping ratios using the integrals of FRFs as reference responses. The sensitivity thus refers to the variation of

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damping ratios to small perturbation of FRF integrals. Rewriting equation (4-8) in matrix form,

$$\{\zeta\}_{r \times 1} = \mathbf{A}_{r \times r} \{\alpha\}_{r \times 1} \quad (4-17)$$

where $A_{ij} = \frac{1}{2} \omega_i^{2j-3}$, with $\omega_i (i = 1, 2, \dots, r)$ are the circular frequencies. Thus,

$$\{\alpha\}_{r \times 1} = \mathbf{A}^{-1}_{r \times r} \{\zeta\}_{r \times 1} \quad (4-18)$$

The individual response components, i.e., the integral of FRFs, $\mathbf{I}_B$, are obtained from

$$\mathbf{I}_B = \int_{\omega_1(\zeta_i)}^{\omega_2(\zeta_i)} \mathbf{B}(\omega, \{\zeta\}, \{p\}) d\omega \quad (4-19)$$

where $\mathbf{B}$ denotes the dynamic stiffness matrix, $\mathbf{B}(\omega, \{\zeta\}, \{p\}) = (\mathbf{K} - \lambda \mathbf{M} + j\omega \mathbf{C})^{-1}$, $\{\zeta\}$ is the damping ratio vector, $\{\zeta\} = [\zeta_1 \ \zeta_2 \ \dots \ \zeta_r]^T$, and $\{p\}$ is the structural parameter vector, $\{p\} = [p_1 \ p_2 \ \dots \ p_r]^T$. The integration bounds ω_1 and ω_2 are determined by a proportion (say 0.9 and 1.1, respectively) of the individual eigen-frequency ω_i . Since the influence of small damping on ω_i is negligible, the sensitivity of $\mathbf{I}_B$ with respect to ζ_i can be obtained by

$$\frac{\partial \mathbf{I}_B}{\partial \zeta_i} = \int_{\omega_1}^{\omega_2} \frac{\partial \mathbf{B}(\omega, \{\zeta\}, \{p\})}{\partial \zeta_i} d\omega \quad (4-20)$$

In the above expression, the parameter vector $\{p\}$ consists of known values from the output of the first network, hence

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$$\frac{\partial \mathbf{B}}{\partial \zeta_i} = -\mathbf{B} \left[j\omega \frac{\partial \mathbf{C}}{\partial \zeta_i} \right] \mathbf{C}, \text{ where } \frac{\partial \mathbf{C}}{\partial \zeta_i} = \mathbf{M} \sum_{k=0}^{r-1} \frac{\partial \alpha_k}{\partial \zeta_i} [\mathbf{M}^{-1} \mathbf{K}]^k \quad (4-21)$$

Substituting (4-21) into (4-20),

$$\frac{\partial \mathbf{I}_B}{\partial \zeta_i} = \int_{\omega_1}^{\omega_2} -\mathbf{B} \left(j\omega \mathbf{M} \sum_{k=0}^{r-1} \frac{\partial \alpha_k}{\partial \zeta_i} [\mathbf{M}^{-1} \mathbf{K}]^k \right) \mathbf{B} d\omega \quad (4-22)$$

Hence, the relationship between a small variation of the damping ratios and the corresponding variation of integrals of FRFs can be established via the sensitivity matrix $\mathbf{S}_d$ with entries calculated from equation (4-22),

$$\delta \{I\} = \mathbf{S}_d \delta \{\zeta\} \quad (4-23)$$

Inverting (4-23),

$$\delta \{\zeta\} = \mathbf{G}_d \delta \{I\}, \quad (4-24)$$

where the gain matrix $\mathbf{G}_d = \mathbf{S}_d^{-1}$, and the vector $\{I\}$ consists of entries from matrix $\mathbf{I}_B$.

It has to be pointed out that the structural parameters ($\{p\}$) used to compute the FRFs and thereby the integral $\mathbf{I}_B$ for training of the second neural network are the output from the first neural network, and these parameters contain errors in themselves already. Hence, the error vector $\delta \{I\}$ in equation (4-24) should consist of two parts; one due to the above carry-over error ($\delta \{I\}^c$) which takes effect during the neural network training, and another due to the (random) errors in the actual measured FRFs

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$(\delta\{I\}^M)$ when subsequently applying the trained ANN for actual damping updating.

Thus,

$$\delta\{I\} = \delta\{I\}^M + \delta\{I\}^C \quad (4-25)$$

$\delta\{I\}^C$ can be evaluated based on the parameter error vector $\delta\{p\}$ from network-1.

From Eq. (4-19),

$$\frac{\partial \mathbf{I}_B}{\partial p_i} = \int_{\omega_1(\{p\})}^{\omega_2(\{p\})} \frac{\partial \mathbf{B}(\omega, \{\zeta\}, \{p\})}{\partial p_i} d\omega \quad (4-26)$$

where $\frac{\partial \mathbf{B}(\omega, \{\zeta\}, \{p\})}{\partial p_i} = -\mathbf{B} \frac{\partial [\mathbf{K} - \omega^2 \mathbf{M} + j\omega \mathbf{C}]}{\partial p_i} \mathbf{B}$. In actual applications, eq. (4-26) is

generally approximated by a finite-difference solution,

$$\left. \frac{\partial \mathbf{I}_B}{\partial p_i} \right|_{\{p^*\}} \approx \left. \frac{\Delta \mathbf{I}_B}{\Delta p_i} \right|_{\{p^*\}} = \left(\int_{\omega_1(p_i + \Delta p_i)}^{\omega_2(p_i + \Delta p_i)} \mathbf{B}(p_i + \Delta p_i) d\omega - \int_{\omega_1(p_i)}^{\omega_2(p_i)} \mathbf{B}(p_i) d\omega \right) / \Delta p_i \quad (4-27)$$

where vector $\{p^*\}$ represents the nominal state of structure parameters at which sensitivity analysis is conducted and Δp_i denotes a small incremental quantity corresponding to a particular structural parameter p_i . Thus, equation (4-27) forms the entries of the sensitivity matrix $\mathbf{S}_d$. Subsequently, the errors in the FRF integrals induced by the inaccuracy in the parameters $\{p\}$ can be computed in a forward manner,

$$\delta\{I\}^C = \mathbf{S} \delta\{p\} \quad (4-28)$$

4.5 Design of Neural Networks and Training

An ANN-based model updating scheme generally consists of the following stages: 1) generating the training data, 2) training the neural network, 3) testing the neural network. The trained neural network can subsequently be used to update the FE model parameters when fed with measured response data from the actual structure.

The preparation of proper training data plays a crucial role for a successful updating since their quality directly affects the “expressing power” (the generalization capacity) of the network. The training data actually comprises a number of paired vectors (called “training pairs”), and each pair includes a parameter vector (network output) and the corresponding response vector (network input). Before the generation of the training pairs, a proper response configuration must be determined. This can be carried out using the aforementioned sensitivity analysis scheme. It has to be pointed out that the results of a sensitivity analysis are dependent upon the chosen nominal state around which the perturbation takes place. Therefore, in cases where the intended coverage of the structure states is wide (as presumed in the present study), the sensitivity analysis concerning the response configuration selection should be conducted on a bunch of randomly selected structural states, and the final judgment should be made based on the collective trend from the sensitivity results.

Once the response configuration is decided, the generation of the training pairs becomes straightforward: For n number of randomly sampled parameter vectors within the targeted variation range, $\{p\}_j$ ($j = 1, 2, \dots, n$), n number of response vectors, $\{R\}_j$ ($j = 1, 2, \dots, n$), are calculated by a forward FE analysis. Thus, n number of training pairs (or “training patterns”) are readily available for the training of the neural network.

The next decision is on the network topology, i.e., the number of hidden layers and the number of neurons (processing units) in each hidden layer. Note that the number of neurons in the input and output layers are already determined upon the selection of the response and structural parameter configuration. Since in this study the neural network

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is chosen to have two hidden layers with an equal number of neurons in each hidden layer, only a desired number of neurons needs to be decided. This can be achieved by performing a trial procedure. After the number of neurons is determined, the Levenberg-Marquardt training algorithm is employed to train the network. The termination of the training is decided based on the cross-validation output.

For the testing of the generalization ability during the network training, a group of data patterns (called “test patterns”) not included in the training patterns is utilized for the purpose of assessing the network performance throughout the training process. A network is regarded as a good candidate if it could result in small errors not only on the training data but also on the test data.

4.6 Numerical Example

A numerical example is given to illustrate the implementation of the above-described procedure. For demonstrative purposes, again the simple multi-storey building frame used in the numerical investigation of Chapter 3 is chosen. All the assumptions remain unchanged. The inter-storey sway stiffness of the frame at all individual storeys is considered as the structural parameters to be updated, along with the damping ratios for the first four natural modes.

4.6.1 Updating of the Structural Parameters (First-Level Network)

For the purpose of better network conditioning, in the first network the stiffness variables are represented by stiffness modification factors (SMFs) (see the definition in Chapter 3). In the present example, the target SMF variation range is set to (0.35, 1.0) to cover cases ranging from the undamaged to a heavily damaged state. For the six-storey frame considered in this example, six SMFs are subject to updating. The basic properties of the frame are the same as those given in Table 3-2(a) of Chapter 3.

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i) Selection of Response Configuration

Several combinations of natural frequencies and antiresonant frequencies are configured for selection. For practicality concerns, the response components are restricted within the lowest few modes. Table 4-1 lists eight such response configurations, where a higher designation number roughly represents a higher demand on the measured data.

Table 4-1 Eight candidate response vector settings for Network-1

Setting No.	1	2	3	4	5	6	7	8
Number of natural frequencies	1	2	3	4	4	4	4	4
Number of anti-resonance frequencies*	5	4	3	2	4	2,2	3,3	4,4

* Single numbers refer to anti-resonances frequencies from FRF(1,1), while double numbers separated by a comma refer to anti-resonance frequencies respectively obtained from FRF(1,1) and FRF(2,2). FRF(i,j) denotes the frequency response at DOF i due to the unit force of flat spectrum applied at DOF j.

As the targeted variation range of the structural states is broad (SMFs within 0.35~1.0), a total of 20 nominal states of the structure are chosen in a random manner within the above range. Sensitivity analysis is then conducted with respect to each of these nominal models for all the candidate response configurations using an automated procedure. The basic routine is as follows. For any particular nominal state defined by a set of SMFs, an FE analysis is first performed to compute the required modal response, from which the response data for the i^{th} response configuration are extracted. A perturbation is imposed on the extracted response data with a perturbation factor assumed to follow a Gaussian distribution with zero mean and 1% variance (Note that other distribution patterns, if deemed appropriate, can be applied in a similar way.) Thus, by random sampling a sufficient number (say 1000 as in this example) of the response perturbation vectors, $\delta\{R\}$, are generated. Subsequently, the corresponding parameter variation (error) vectors, $\delta\{p\}$, are calculated according to equation (4-16). These parameter error vectors are then examined on a statistical basis and the mean-

square-error (MSE) is used to represent the overall sensitivity of the parameters for the particular response configuration. Besides the MSE, the detailed distribution of the error for individual parameters can also be examined to further assess the adequacy of the response configuration. Figure 4-3 summarizes the sensitivity analysis results in the form of scatter plot of the MSEs of the structural parameters against all the response configurations being considered.

Figure 4-3 Mean square error (MSE) of SMFs at various nominal states versus different response configurations from sensitivity analysis

It can be observed that four response configurations, namely, $\{R\}^{(5)}$, $\{R\}^{(6)}$, $\{R\}^{(7)}$ and $\{R\}^{(8)}$, exhibit a quite stable error margin in terms of the overall MSE. Further inspection of the error distributions of the individual SMFs, as shown in Figure 4-4 for one particular nominal state, reveals that the error band of SMFs reduces from above $\pm 10\%$ to within $\pm 6\%$ when $\{R\}^{(7)}$ is used instead of $\{R\}^{(5)}$. Slight further improvement of the results can be achieved when using $\{R\}^{(8)}$, which however includes two more response components. Therefore, the response configuration $\{R\}^{(7)}$ is chosen for the subsequent preparation of the training pairs and the neural network training.

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Figure 4-4 Error distribution of SMFs resulted from different response settings according to sensitivity analysis

ii) Training of the Neural Network

After selecting the response configuration, standard FE calculations can proceed to generate the parameter-response data pairs for the network training. The required number of the training data pairs is determined based on two factors; the dimension of the input layer (response vector) and the value ranges of the physical parameters (the “SMFs” herein) within which the network is expected to generalize. Now that the response vector has been selected as $\{R\}^{(7)}$ which contains 10 components, the dimension of the network input layer is thus 10. On the other hand, the variation range of the structural parameters, i.e., SMFs, is targeted to cover 0.35~1.0. Thus, according to Vapnik-Chervonenkis dimension theory (Vapnik and Chervonenkis 1971), it is found that 1500 training pairs are appropriate for the network training. The numbers of testing and cross-validation data patterns are subsequently chosen to be 800 and 600, respectively. All these required data patterns are then generated from the FE analysis, and they are used as “noiseless” data patterns for the network training.

Figure 4-5(a) illustrates the network performance in terms of the overall MSE on the training and testing data for 10 different networks with different number of neurons in their hidden layers. It can be seen that the MSE generally decreases with increasing number of neurons in the hidden layers (i.e., larger networks). As expected, all networks perform better on the training patterns than on the testing patterns. In conjunction with an analysis of the percentage error distributions of the individual SMFs from the network output for the test patterns, it is found that the neural network with 12 neurons in each of the two hidden layers can already achieve an error band within $\pm 3\%$ for all individual SMFs, given noiseless input data. Therefore, this network topology is selected.

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a) MSE from ANN with different number of hidden units in its two hidden layers

b) Typical network performance curves with ongoing training process

Figure 4-5 Performance of neural networks with different topology settings

During the network training process, the development of the network performance is continuously monitored. Figure 4-5(b) depicts the progressive network performance in terms of the MSE during the training iterations. As can be seen, the training procedure stops at epoch 35 since there is no further improvement of the network performance on the cross-validation data patterns in nearly 10 epochs (from epoch 25 to 35).

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The above neural network training is based on the “noiseless” data patterns directly obtained from the FE analysis. The so-trained neural network is designated as Network-1A. To produce a noise-resisting neural network, a noise-injection learning strategy is implemented to repeat the training procedure with the same network topology setting. In this process, however, the 1500 training data patterns are treated by injecting noise (representing the measurement errors) to the response side (network input) according to equation (4-2). The “noise” component, v_j , is obtained by random sampling from a prescribed probability distribution. Since only frequency data are used here, it is assumed that the error in the measured frequency data follows a Gaussian distribution with a zero mean and 1% variance. The output side of the training data, i.e., the SMFs, remains unchanged and the original pairing is also retained. The network trained using the above noise-injected training patterns is designated as Network-1B.

To evaluate the performance of the above two networks (Network-1A and Network-1B) under a noisy measurement data environment, four arbitrary structure states are subjected to updating using these two networks. Table 4-2 lists the values of the SMFs for the four structure states. For each state, the exact response data are first calculated using the FE analysis, and they are then “polluted” by adding random error to generate 1000 sets of noisy measurement responses. These noisy response data are then fed into the two networks one by one to perform the updating. Figure 4-6 compares the overall MSE of the output parameters from the two networks. The distributions of the percentage error for the individual parameters (SMFs) from the two networks are compared in Figure 4-7. As can be clearly observed, Network-1B exhibits a significantly improved performance under the simulated noisy measurement data. The maximum percentage error for the individual parameters is reduced to within $\pm 4\%$ as compared to $\pm 8\%$ from Network-1A. The latter is comparable with the prediction of the parameter errors from the sensitivity analysis.

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Table 4-2 Four model scenarios used for performance comparison between Network-1A (noiseless data trained) and Network-1B (noise-injection trained)

Scenario No.	Storey 1	Storey 2	Storey 3	Storey 4	Storey 5	Storey 6
1	0.734	0.867	0.762	0.699	0.786	0.767
2	0.821	0.812	0.856	0.733	0.902	0.881
3	0.781	0.947	0.780	0.718	0.798	0.948
4	0.761	0.897	0.962	0.863	0.866	0.751

Figure 4-6 Comparison of MSE of outputs from Network-1A (noiseless data trained) and Network-1B (noise-injection trained) when fed with noisy measurement data

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Fig. 4-7 Comparison of percentage errors in the updated parameters from Network-1A and Network-1B

4.6.2 Updating of Structural Damping Ratios (Second-Level Network)

In this example, four damping ratios, $\zeta_i (i = 1, 2, \dots, 4)$, corresponding respectively to the lowest four natural modes, are considered for updating.

i) Selection of Response Configuration

The responses used for the damping updating are the integrals of frequency response functions (FRFs) around the natural frequencies (see Figure 4-2(b)). For a multi-degree-of-freedom system, it is possible to measure a number of FRFs, so there could be different response configurations to consider for the sake of a better network performance. The identification of a desired configuration beforehand also helps the planning of the test programme in real applications. For an illustrative purpose, eight candidate response configurations, namely $\{I\}^{(1)}, \{I\}^{(2)}, \dots, \{I\}^{(8)}$ as shown in Table 4-3, are subjected to the sensitivity analysis in a way similar to that described in Network-1. Note that in Table 4-3 the number in parentheses, (i, j) , denotes the frequency response function with force (actuator) applied along the DOF j and the response (sensor) measured at the DOF i . The four lowest natural modes are considered, thus with three arbitrary FRFs each response vector will consist of 12 components (4 modes $\times$ 3 FRFs). Each integral is obtained by integrating the FRF over a frequency interval from $0.9f_k$ to $1.1f_k$, with f_k being the k^{th} natural frequency up to the 4th mode.

Table 4-3 Eight candidate FRFs as responses for Network-2

Setting No.	1	2	3	4	5	6	7	8
FRF components	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)
	(2,1)	(2,1)	(2,1)	(3,1)	(3,1)	(4,1)	(4,1)	(5,1)
	(3,1)	(5,1)	(6,1)	(5,1)	(6,1)	(5,1)	(6,1)	(6,1)

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(a) Mean square error of damping ratios with different response configurations

(b) Error distributions of damping ratios from response setting $\{I\}^{(8)}$

Figure 4-8 Predicted errors of damping ratios under different response settings according to sensitivity analysis

A sensitivity analysis is conducted to calculate the variation of the damping parameters, $\delta\{\zeta\}^{(i)}$, corresponding to a perturbation the FRF integrals included in a particular configuration $\delta\{I\}^{(i)}$. As mentioned in section 4.4.3.2, the perturbation on FRFs is considered to represent two parts of errors, one is the so-called carry-over that is associated with the errors from the first level neural network, $\delta\{I\}^{(i)C}$, and the other

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represents the errors in the actual measured FRF integrals, $\delta\{I\}^{(i)M}$, which is assumed to also follow a Gaussian noise with zero mean and 1% variance. The overall error vector, $\delta\{I\}^{(i)}$, is obtained as the sum of $\delta\{I\}^{(i)M}$ and $\delta\{I\}^{(i)C}$. Subsequently, the error in the damping ratios, $\delta\{\zeta\}^{(i)}$, can be calculated according to equation (4-24). Figure 4-8(a) depicts the overall mean-square-error (MSE) in $\delta\{\zeta\}^{(i)}$ from the above sensitivity analysis for 20 arbitrary chosen nominal states of the structure, for which both the SMFs and the damping ratios are chosen in an arbitrary manner within their respective range. It can be seen that the 8th response configuration exhibits the best outcome and so it is selected for the Network 2 training. The distribution of the errors corresponding to one particular state of the structure is illustrated in Figure 4-8(b), which indicates that the maximum anticipated damping updating error is about $\pm 10\%$ with the assumed margin of error in the measured FRF integrals.

ii) Training of the Neural Network

The number of training pairs for Network-2 is chosen to be 900 according to the Vapnik-Chervonenkis dimensional analysis. The numbers of testing and cross-validation patterns are chosen to be 600 and 500, respectively. After a trial procedure, it is found that a neural network with 9 neurons in each of the two hidden layers is adequate.

The training data as well as the testing and cross-validation data are generated so that a damping variation range from 0.5% to 10% is covered. The neural network, designated as Network-2B, is then trained following the noise-injection learning strategy in a similar way as for the previous Network-1B.

To examine the generalization and noise-resisting ability of the above trained network, an arbitrary model with assumed parameters shown in Table 4-4 is subjected to updating. For 1000 sets of error-polluted FRF integral data ($\{I\}_j^8, j = 1, \dots, 1000$), considering a Gaussian error distribution with zero mean and 1% variance, Figure 4-8

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shows the error distributions in the updated 4 damping ratios as compared with the theoretically predicted error from the sensitivity analysis. Once again, a remarkable reduction of the error margin is achieved by implementing the noise-injection learning strategy in the neural-network training. A summary of the updated damping ratios for ζ_1 to ζ_4 , along with the updated structural stiffness parameters using Network-1B, is given in Table 4-4. Of course, it should be mentioned again that the realization of the noise resisting capacity of a neural network in real applications is subject to the adequacy of the noise model used in the neural network training in representing the actual noise or error in the measurement data. Inconsistency in the noise description could otherwise contribute to erroneous updating results.

Table 4-4 An assumed model scenario and the updating results from the two-level networks

	Stiffness Modification factors (SMFs) (p_i)						Damping ratios (ζ_i)			
	p_1	p_2	p_3	p_4	p_5	p_6	ζ_1	ζ_2	ζ_3	ζ_4
Model scenario	0.65	0.78	0.80	0.59	0.72	0.70	0.01	0.014	0.032	0.056
Updating results	0.65	0.772	0.813	0.578	0.714	0.689	0.0098	0.0142	0.0330	0.0545
Updating errors (%)	2.3	-1.0	1.7	-2.0	-0.8	-1.6	-2.5	1.6	3.3	-2.6

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Figure 4-9 Comparison of percentage errors in the updated damping ratios from Network-2B with the predicted errors from sensitivity analysis

4.7 Conclusions

This chapter presents a comprehensive procedure to use artificial neural networks for FE model updating including both structural and damping parameters. The methodology involves a series of sensitivity analyses for the selection of a desired response configuration as well as for the evaluation of the post-trained network performance; it also covers the selection of the network topology through trial training and the incorporation of a noise-injection learning strategy. A two-level neural network FE model updating scheme is developed so that the structural parameters (SMFs) and the damping ratios (ζ_i) can be updated using two separate networks. A numerical example is given to demonstrate the effectiveness and efficiency of the proposed method.

The two-neural network scheme proves to work out successfully in updating the structural parameters and the damping ratios. Apart from the satisfactory accuracy that can be achieved with noiseless response data, the numerical example also shows that the noise-injection learning strategy can result in a neural network with substantially enhanced noise-resisting ability. In the example shown, the error margin in the updated structural parameters and damping ratios from the noise-injection learning is reduced by more than 50% as compared to the results from the noiseless-data trained neural work when fed with noisy data. For a given level of noise with zero mean and 1% variance in the frequency response data, the neural network is able to identify the structural parameters as well as the damping ratios within an error of $\pm 4\%$.

It should be pointed out that the realization of the noise-resisting ability in real applications is subject to the adequacy of the noise model used in the network training in describing the actual noise in the measured data. The applicability of the updated damping ratios using the proposed damping model also depends on the adequacy of such damping model in representing the actual damping mechanism of the system under consideration. Should other noise models or damping models be deemed more adequate for a particular structure, they can be implemented in the neural network training in a similar way as described in this chapter.

CHAPTER 5

A ROBUST STOCHASTIC GENETIC ALGORITHM (STGA) FOR GLOBAL NUMERICAL OPTIMIZATION

5.1 Introduction

In Chapter 3, GA has been shown to perform satisfactory updating, and the global optimal solution can generally be achieved with a reasonable number of fitness evaluations. However, the observation on the efficiency was made from cases where the number of variables subjected to optimization is very limited (e.g., up to 24). When the number of variables is very large as could be experienced in modeling real civil engineering structures, the amount of time for GA to evolve the global optima may become unaffordable. So far, most of the successful applications of GAs as well as other paradigms of evolutionary algorithms (EAs) are limited to problems with dimensions below 30 (Goldberg 1989; Hager et al. 1994; Michalewicz 1996; Yao et al. 1997). To promote broader applications of the algorithms and reduce the risk of getting a misguided solution, a more effective and efficient GA is needed.

Back in 1995, Krishnakumar et al. (1995) proposed the use of stochastic coding in binary GA (abbreviated as “StGA”) with the intention to improve the efficiency. However, no evidence of significant improvement was reported. Since then, little has been done to further pursue the potential merits of this profound idea except a few applications (for example, Mulgund et al. applied StGA for air combat tactics

optimization (Mulgund et al. 1998)), and no comparison has been reported to assess the actual performance of the algorithm. In fact, there still lacks a rigorous procedure for incorporating this novel algorithm. In view of the above, this study is conducted to advance the idea of StGA with the following two main objectives: 1) to develop a generic procedure for the implementation of StGA, and 2) to demonstrate the effectiveness and efficiency of StGA as compared to other algorithms and to explore its ability in tackling large-dimension problems.

In this chapter, the mechanism of the stochastic coding scheme is described first. The corresponding genetic operations, namely, selection, crossover and mutation, are discussed in detail. For the performance enhancement, a replacement strategy similar to that applied in the conventional GA is adopted. Necessary operational details and general guidelines on the actual coding are provided. The performance of the proposed StGA is assessed by carrying out optimization on 20 test functions of moderate dimensions (up to 30). Comparing to some well-known global optimization algorithms such as ES, EP and SA, StGA is shown to be able to achieve more accurate results and yet with a much reduced computational effort. Finally, the capability of StGA in solving large dimension problems is examined through optimizing two functions of dimension as high as 100. Comparing with the observations reported in the literature, StGA maintains a superior efficiency and effectiveness for such large-scale problems. It is therefore concluded that StGA is a promising technique in performing global optimization for practical applications.

5.2 Stochastic Genetic Algorithm

The operation of StGA stems from a totally different concept from the usual GA, particularly in terms of the coding technique. The exclusive features of StGA include:

1) each chromosome represents a stochastic region defined by a normal distribution, 2) these regions are dynamically adapted towards the most promising region, 3) no region in the search space is absolutely forsaken, and 4) the search region is not explicitly restrained.

In this study, the following minimization problem with fixed boundaries is considered:

$$\min_{\bar{x}} f(\bar{x}) \quad \text{subject to} \quad \bar{B}_l \leq \bar{x} \leq \bar{B}_u \quad (5-1)$$

where $\bar{x} = (x_1, x_2, \dots, x_m)$ is the variable vector in $\Re^M$, $f(\bar{x})$ denotes the objective function, and $\bar{B}_l = (B_{l1}, B_{l2}, \dots, B_{lm})$, $\bar{B}_u = (B_{u1}, B_{u2}, \dots, B_{um})$ represent respectively the lower and the upper bound of the variables (i.e., a predefined feasible solution space) such that the meaningful range of x_i is $[B_{li}, B_{ui}]$.

5.2.1 Stochastic Coding Mechanism

Most paradigms of GA, regardless of what coding strategy is used, interpret each coded genotype “chromosome” (or “string”) as one possible candidate solution. Hence GA actually explores the solution space for optimum in a point-by-point manner. This approach could be quite inefficient because in the early stage of evolution enormous effort may thus be wasted in evaluating the least significant digits of gene that contribute little towards locating the most essential genes (Nicol and Richard 1992). Although the later developed real-coding GA achieves a faster convergence rate, the computational efficiency remains a major constraint. To facilitate the salutation of large-scale real-life problems, some special techniques have emerged, such as the Approximate Function Evaluation (Grefenstette and Fritzpatrick 1990), the Messy

Floating Point GA (Deb 1991), and the Dynamic Coding GA (Nicol and Richard 1992). The applicability of these methods, unfortunately, is subjected to a variety of strict prerequisites on the physical problem itself.

The main purpose of the present StGA is to achieve a highly efficient evolution with necessary robustness by incorporating an innovative stochastic coding method. StGA codes each chromosome as a representative of a stochastic region described by a multivariate Gaussian distribution rather than a single candidate solution as in the conventional GA. In StGA, a chromosome, C_j , comprises a binary string M_j representing the mean vector of the Gaussian distribution and a real number vector V_j representing the corresponding variable variances (see Figure 5-1). The whole binary string is divided into m substrings ($M_{ji}, i=1,2,\dots,m$). Each substring indicates the geno-space coding of one particular variable x_i , and is associated with a variance V_{ji} . Thus,

$$C_j = \left[(M_{j1}, V_{j1}) \quad (M_{j2}, V_{j2}) \quad \dots \quad (M_{jm}, V_{jm}) \right] \quad (5-2)$$

Figure 5-1 Chromosome model for StGA

Figure 5-1 illustrates schematically the definition of stochastic regions decoded from C_j for a simple case involving two variables ($m = 2$). Coding the physical problem in this way facilitates StGA to perform efficient search by dynamically shifting emphasis to different favorable regions in the feasible space without abdicating any portion of the region. As such, StGA effectively avoids the kind of failure as could be experienced in the normal dynamic coding scheme due to constantly throwing away potential regions regarded as “unpromising” in terms of their fitness.

Figure 5-2 Schematic illustration of stochastic regions in StGA

As binary bits are used to code the variable mean values, a decision has to be made on the substring length for each variable. In conventional GA, the substring length (which represents the variable precision) is required to be long enough to describe the continuous space in order to achieve a desired accuracy. However, a longer substring length implies a substantial increase of the computational effort for GA, which can become prohibitive as the problem dimension escalates. StGA provides a possibility to reduce the computational demand by allowing a somewhat coarse division of the variable space without compromising the accuracy of the final solution. This is made possible by the fact that those points that are not covered by the binary string could

also be approached by StGA through numerical sampling within the stochastic regions. Despite this advantage, each substring still needs to have a sufficient number of bits to prevent blind genetic search. The choice of an appropriate number of bits will be discussed further in association with the actual coding for the numerical experiment in Section 5.3.

5.2.2 Initialization of Population

Two initialization processes are required in StGA, one for the mean vectors M_j , and the other for the variance vectors V_j . Generally speaking, M_j is initialized within the predefined searching space, $[\bar{B}_l, \bar{B}_u]$, in a random manner. In case a priori knowledge about the potential solution is available, M_j may be initialized so that its values are close to the guessed solution to put the algorithm at a good starting point. The initialization of the variance vector V_j , however, is not so straightforward. The initialization of V_j bears great significance as it affects the explorative space in StGA's evolution process. It is not possible though to derive a universal rule for the determination of adequate values for V_{ji} . Section 5.3 will provide some general guideline on an empirical basis. Generally speaking, a larger space would require bigger V_{ji} to be efficient.

5.2.3 Selection Operation

The selection process is to pick up individuals from groups of parents and children in the preceding generation to form the mating pool for the next generation to evolve. Differing from other paradigms of GA, in StGA an additional selection called "local selection" is required prior to the normal genetic selection.

a) Local selection   Local selection essentially serves two purposes; one is to assess the fitness of the stochastic regions represented by each chromosome in the population, and the other is to implement the adaptation of the variance value V_{ji} . It operates as follows. First, within each region R_j (represented by chromosome C_j), N number of children ($c_{ji}, i = 1, \dots, N$) are produced asexually through random sampling according to the predefined normal distribution (see Figure 5-2). With a fitness evaluation, the best child c_j^* is chosen to actually represent this particular region and the corresponding fitness value is regarded as that of chromosome C_j . The same c_j^* is then employed to redo the coding to supersede the parent mean vector, M_j . Such a fitness evaluation scheme implies that in StGA the local selection of each generation is based on the fitness of individuals of its immediate foregoing generation. Meanwhile, the variance values V_{ji} is adapted following the $(1/5)^{th}$ principle such that if at least one out of five asexually generated children (c_{ji}) results in improved fitness as compared with that of the mean vector, M_j , the individual variance values are decreased; otherwise, they are increased. This operation is detailed in section 5.3.2.

It should be noted that in the above process some of the values in c_{ji} may violate the prescribed boundaries. Possible ways to tackle this problem include: resampling until the breach of boundary constraints disappears; assigning such c_{ji} a very small fitness value; or replacing any violating value in c_{ji} with a random number drawn within its associated boundary. The present study applies a different strategy such that the out-of-boundary variable values are forced to equal their closest boundary values. This approach is deemed to be more appropriate for StGA, because in StGA the individual stochastic regions (R_j) are small with respect to the whole search space; thus, the occurrence of boundary violation signifies that StGA is exploring the boundary of the respective variables and so it is rational to adopt the nearest boundary values to substitute those out-of-boundary values. Resampling, however, may be frustrated for large dimension cases because of the prohibitively large number of trials. The method of allocating small fitness appears to be unreasonable as the chromosomes in question

might also be good schemata.

b) Global genetic selection Genetic selection is usually made within the parent population based on the fitness values of the individuals; an individual with better fitness value is more likely to survive into the mating pool. It is worth pointing out that global selection in the StGA is performed on the population resulting from the local selection.

A number of selection methods exist, including the roulette wheel selection, the stochastic universal selection and the ranking selection, among others (Goldberg 1989). The present study employs the so-called tournament selection, which is generally adopted in evolutionary programming (EP) (Fogel 1992). This selection method allows for a control of the selection pressure (the ratio of selection probability between the fittest and the least fit individuals). In general, the tournament selection produces one individual each time for the mating pool. It operates by first randomly picking T_n number of individuals from the parent population, then ranking them, and the best one is sent into the mating pool. The above procedure is repeated till the mating pool is full. It is noteworthy that T_n essentially plays the role of controlling the selection pressure in the sense that increasing T_n strengthens the selection pressure, and vice versa. Larger T_n promotes the GA convergence process, but at the risk of leading to premature behavior; smaller T_n tends to reduce such risk, but at the cost of increased computational effort. Therefore, some kind of trade-off decision is required.

5.2.4 Crossover Operation

As one of the major genetic operators, crossover is designed to produce offspring in

the hope that better fitness is achieved through exchanging partial genetic information (the coding segment of the chromosome) of two parents. Before crossover, two parents should be prepared, say C_j and C_k (see Figure 5-3). The detailed operation process differs slightly under different crossover schemes. The present study adopts the one point crossover scheme. For StGA, the variance terms V_{ji} should also undergo crossover because they are bonded together with the mean values M_{ji} to define the stochastic regions.

For the crossover, a cut site s is first randomly chosen within the length of M_j to determine which portion of binary bits should be exchanged. s could take place between two adjacent substrings M_{jp} and $M_{j(p+1)}$ (see s_2 in Figure 5-3); in this case, we simply exchange all items (binary substrings and variance values) at the right hand side of s_2 between parent C_j and C_k to generate the two descendants $\bar{C}_j$ and $\bar{C}_k$,

$$\begin{aligned}\bar{C}_j &= [(M_{j1}, V_{j1}) \cdots (M_{jp}, V_{jp}) (M_{k(p+1)}, V_{k(p+1)}) \cdots (M_{km}, V_{km})] \\ \bar{C}_k &= [(M_{k1}, V_{k1}) \cdots (M_{kp}, V_{kp}) (M_{j(p+1)}, V_{j(p+1)}) \cdots (M_{jm}, V_{jm})]\end{aligned}\quad (5-3)$$

However, it is more likely that s falls within a particular substring, e.g., the p^{th} substring M_{jp} and M_{kp} , as indicated by s_1 in Figure 5-3. The crossover site thus splits M_{jp} and M_{kp} into four substrings, M_{jp}^L , M_{jp}^R , M_{kp}^L and M_{kp}^R . As such, the crossover on the corresponding variance terms V_{jp} and V_{kp} cannot be performed in a straightforward manner. For simplicity, this study proposes a linear interpolation to produce two new offspring variance terms as,

$$V_{jp}^* = rV_{jp} + (1-r)V_{kp}$$

$$V_{kp}^* = rV_{kp} + (1-r)V_{jp} \quad (5-4)$$

where r is a random number between 0 and 1. By the above interpolation one is more likely to retain the favorable stochastic regions discovered up to this step by StGA. Thus, the two descendants $\bar{C}_j$ and $\bar{C}_k$ generated from the crossover can be expressed as

$$\begin{aligned} \bar{C}_j &= [(M_{j1}, V_{j1}) \cdots (M_{jp}^L + M_{kp}^R, V_{jp}^*) (M_{k(p+1)}, V_{k(p+1)}) \cdots (M_{km}, V_{km})] \\ \bar{C}_k &= [(M_{k1}, V_{k1}) \cdots (M_{kp}^L + M_{jp}^R, V_{kp}^*) (M_{j(p+1)}, V_{j(p+1)}) \cdots (M_{jm}, V_{jm})] \end{aligned} \quad (5-5)$$

In general, each parent chromosome in the population undergoes crossover only with certain probability p_c . For the binary coding, Dejong (1975) suggested taking the value of p_c between 0.7 and 0.9. Trial analysis in the experimental phase of this study tends to support a use of $p_c = 0.85$.

Figure 5-3 Possible scenarios of crossover operation in StGA:

s_1 : cutting through substrings;

s_2 : cutting between adjacent substrings

5.2.5 Mutation Operation

Aimed at increasing the diversity of the population so as to enhance the chance for GA to escape from local optima, mutation in StGA is implemented following a similar procedure as in the conventional GA. The actual process for mutation depends on the coding scheme. In StGA, only the binary strings, which code the mean values of the stochastic regions, undergo mutation while the variance terms are not subject to mutation because they are adapted in connection with the properties of the stochastic regions. And therefore, binary mutation is used with StGA and it only performs one bit flip, i.e., the bit value changes from 0 to 1 or from 1 to 0. For the binary coding, the mutation probability p_m should be kept small, otherwise, the favorable building blocks (schemata) discovered so far by GA will be exhaustively destroyed, which implies failure. Based on the suggestion of Dejong (1975) and the experience from trial analyses, p_m is taken in the range of [0.01, 0.025] for the various optimization cases described later in this chapter.

5.2.6 Replacement Operation and Termination of Evolution

As in the traditional GA, replacement strategy is also adopted in StGA to further improve the search effectiveness. It operates according to the so-called elitism strategy such that a small portion of the top ranking individuals from the parent generation is taken to substitute the same number of the least fit individuals in the offspring generation. As a result, the performance curve (fitness vs. generation) of StGA becomes monotonically increasing. The termination of evolution may be decided by certain criteria, e.g., a preset fitness value for the best individual or a prescribed maximum number of generations, depending on the nature of the underlying problems.

The general sequence of StGA is summarized by a flowchart in Figure 5-4. Further details on the actual operations will be given in the next section.

Figure 5-4 General flowchart of StGA

5.3 Implementation of StGA and Numerical Experiment

5.3.1 Test Functions

Numerical experiments are conducted to test the effectiveness and efficiency of StGA. 20 test functions from three categories (Yao et al. 1999) are selected, covering a broader range than in some other relevant studies for purposes of demonstrating the robustness and reliability of the present algorithm.

Table 5-1 lists the 20 test functions and their key properties. The detailed descriptions of functions 12 to 15 and 18 to 20 are given in the appendix. These functions can be divided into three categories of different complexities. f_1 to f_7 are unimodal functions, which are relatively easy to optimize, but the difficulty increases as the problem dimension goes high. f_8 to f_{13} are multimodal functions with many local optima and they represent the most difficult class of problems for many optimization algorithms. As an example, Figure 5-5 shows the surface landscapes of f_8 and f_9 when the dimension is set to 2. f_{14} through f_{20} are likewise multimodal functions, but they only contain a few local optima. It is interesting to note that some functions possess rather unique features. For instance, f_6 is a discontinuous step function having a single optimum; f_7 is a noisy quartic function involving a uniformly distributed random variable within $[0, 1]$.

Generally speaking, for unimodal functions the convergence rates are of main interest as optimizing such functions to a satisfactory accuracy is not a major issue. For multimodal functions, however, the quality of the final results is more crucial since it reflects StGA's ability in escaping from local deceptive optima and locating the desired near-global solution.

Chapter 5 A robust stochastic genetic algorithm (StGA) for global numerical optimization

Table 5-1 List of 20 test functions (n = problem dimension, $f_{\min}$ = minimum function value, SD = prescribed search domain)

Test functions	n	SD	$f_{\min}$
$f_1(x) = \sum_{i=1}^n x_i^2$	30	$[-100,100]^n$	0
$f_2(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	30	$[-10,10]^n$	0
$f_3(x) = \sum_{i=1}^n (\sum_{j=1}^i x_j)^2$	30	$[-100,100]^n$	0
$f_4(x) = \max_i (x_i , 1 \leq i \leq n)$	30	$[-100,100]^n$	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	$[-30,30]^n$	0
$f_6(x) = \sum_{i=1}^n (\lfloor x_i + 0.5 \rfloor)^2$	30	$[-100,100]^n$	0
$f_7(x) = \sum_{i=1}^n -ix_i^4 + \text{random}[0,1]$	30	$[-1.28,1.28]^n$	0
$f_8(x) = \sum_{i=1}^n -x_i \sin(\sqrt{x_i})$	30	$[-500,500]^n$	-12569.5
$f_9(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	30	$[-5.12,5.12]^n$	0
$f_{10}(x) = -20 \exp(-0.2 \sqrt{1/30 \sum_{i=1}^n x_i^2}) - \exp(1/30 \sum_{i=1}^n \cos 2\pi x_i)$	30	$[-32,32]^n$	0
$f_{11}(x) = 1/4000 \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos(x_i/\sqrt{i}) + 1$	30	$[-600,600]^n$	0
$f_{12}(x) = \pi/30 \{ 10 \sin^2(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] + (y_n - 1)^2 \} + \sum_{i=1}^n u(x_i, 10, 100, 4)$	30	$[-50,50]^n$	0
$f_{13}(x) = 1/10 \{ 10 \sin^2(3\pi x_1) + \sum_{i=1}^{n-1} (x_i - 1)^2 [1 + \sin^2(3\pi x_{i+1})] + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)] \} + \sum_{i=1}^n u(x_i, 5, 100, 4)$	30	$[-50,50]^n$	0
$f_{14}(x) = [1/500. + \sum_{j=1}^{25} (j + \sum_{i=1}^2 (x_i - a_{ij})^6)^{-1}]^{-1}$	2	$[-65.536, 65.536]^n$	1
$f_{15}(x) = \sum_{i=1}^{11} [a_i - x_1(b_i^2 + b_i x_2)] / b_i^2 + b_i x_3 + x_4]^2$	4	$[-5,5]^n$	$3.075e^{-4}$
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + 1/3 x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	$[-5,5]^n$	-1.0316285
$f_{17}(x) = (x_2 - 5.1/4\pi^2 x_1^2 + 5/\pi x_1 - 6)^2 + 10(1 - 1/8\pi) \cos(x_1) + 10$	2	$[-5,10] \times [0,15]$	0.398
$f_{18}(x) = -\sum_{i=1}^5 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0,10]^n$	-10.1422
$f_{19}(x) = -\sum_{i=1}^7 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0,10]^n$	-10.3909
$f_{20}(x) = -\sum_{i=1}^{10} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0,10]^n$	-10.5300

$$f_8(x_1, x_2)$$

$$f_9(x_1, x_2)$$

Figure 5-5 Graphs of f_8 and f_9 with a dimension of 2

5.3.2 Numerical Implementation of StGA

When implementing StGA, a proper set-up of the key parameters is required. First of all, the variable resolutions used in the coding process should be kept reasonably high so that the nearby region around the actual global solution can be approached by StGA. A judgment on this may be reached with the help of a trial-and-error procedure. The next decision is on two important StGA-specific parameters, namely, the initial variance values V_{ji} and the adaptation step values δ_i for each variance term. In the present study, V_{ji} is initialized through a uniform random draw within a prescribed range, denoted by R_i^V , while δ_i is determined with reference to R_i^V . It is worth pointing out that the adaptation of V_{ji} is basically a linear process in that the adaptation step δ_i vary only within a preset small range in the evolution procedure. Theoretically speaking, too large a δ_i could degrade the StGA performance because there will be little region exploitation in the later generations due to the small variance values that remained, whereas too small a δ_i would also slow down the evolution process due to the fact that in early generations much effort is spent exploring some unpromising regions. In this respect, a series of preliminary experimental studies have been conducted so as to provide a rough guideline for the choice of R_i^V and δ_i . In general, the following empirical formulae may be considered in setting R_i^V and δ_i :

$$R_i^V = \left(\frac{1}{120} \sim \frac{1}{80} \right) [B_{ui} - B_{li}] \quad (5-6)$$

$$\delta_i = \left(\frac{2}{100} \sim \frac{5}{100} \right) V_i^R \quad (5-7)$$

where V_i^R takes a random value within R_i^V following a uniform distribution. It should be pointed out that the above proposal is not meant to be universally applicable and adjustment may be necessary in dealing with some particular problems.

To further demonstrate the implementation of the above procedure, the operation for

the optimization of function f_8 is detailed in the following. Due to its unknown surface feature, each variable of the function is represented using as many as 14 binary bits in coding the variable mean, resulting in a division length of 0.10, which is deemed fine enough for the solution of this particular problem based on a preliminary analysis. The population size is set to 20 and it is initialized with the variance terms being generated uniformly within R_i^V of [8.5, 12.0] while the adaptation steps of variances $\delta_i (i = 1, 2, \dots, 30)$ take values from [0.20, 0.40] in a random manner. These settings are consistent with the aforementioned empirical formulae. Thus, a chromosome C_k in the initial population may look like this:

$$\begin{array}{ccccccc} 11100101101011(M_{k1}) & 00110011001010(M_{k2}) & \dots\dots\dots & 11110011011111(M_{k30}) \\ 10.22(V_{k1}) & 11.32(V_{k2}) & \dots\dots & 9.15(V_{k30}) \end{array}$$

For the StGA to start, other pertinent parameters are set as follows: Tournament size in the global selection = 2; Replacement rate = 10%; Probability for crossover = 0.85; Probability for mutation = 0.02.

It should also be mentioned that a proper choice of variable division length (grid resolution) will depend on the landscape of the function (i.e., the sensitivity of the function with respect to each variable) as well as the size of the search domain, and the desirable resolution may vary among different variables. For simplicity, the present study adopts a uniform resolution for all variables involved in each test function. Figure 5-6 shows the number of binary bits used in coding each variable in optimizing these test functions.

Figure 5-6 Number of binary bits used in coding the variables of the test functions

5.4 Performance Assessment of StGA and Comparison with Other Algorithms

The performance of StGA is evaluated based on the optimization results on the 20 test functions as compared to some existing global optimization algorithms. For each test function, 50 runs with different seeds from the random number generator are performed to observe the consistency of the outcome. At each generation, the mean of the function values represented by the best fit individuals from all 50 runs, denoted by μ_i ($i=1, \dots, NG$, NG is the maximum number of generation), is computed to plot the evolution curve, whereas the standard deviation of the function values corresponding to the best fit individuals from the last generation of the 50 runs, σ_{NG} , is used to indicate the consistency of the algorithm. Generally speaking, a small σ_{NG} will signify a good consistency while a large σ_{NG} may imply certain deficiency. The last mean value μ_{NG} represents the finally evolved optimum from StGA, and hence is

used together with σ_{NG} to represent the optimization results in the comparison.

Table 5-2 summarizes the key parameter settings of StGA for each test function, including the population size, the number of asexually generated children in the local selection, and the number of generations. From these parameters, the number of function evaluations, which serves as a measure of the computational effort in this study, is calculated and they are also shown in Table 5-2.

Table 5-2 StGA parameter settings and estimated computational effort (number of function evaluations)

TF	NP	NS	NG	MNFE
f_1	30	5	200	30,000
f_2	22	5	160	17,600
f_3	40	5	115	23,000
f_4	40	5	160	32,000
f_5	50	5	180	45,000
f_6	20	5	15	1,500
f_7	30	5	170	25,500
f_8	20	5	15	1,500
f_9	30	5	190	28,500
f_{10}	40	5	50	10,000
f_{11}	50	5	210	52,500
f_{12}	20	5	80	8,000
f_{13}	20	5	160	16,000
f_{14}	20	5	8	800
f_{15}	40	5	150	30,000
f_{16}	20	5	40	4,000
f_{17}	20	5	50	5,000
f_{18}	20	5	100	10,000
f_{19}	20	5	48	4,800
f_{20}	20	5	85	8,500

TF: Test function   **NP:** Population size

NS: Number of asexually produced children

NG: Number of generations   **MNFE:** Mean number of function evaluations

5.4.1 Existing Algorithms for Comparison

There exist a number of global optimization algorithms suitable for continuous problems. For the present comparison, the following well-known algorithms are considered, and they will be applied for all or some of the 20 selected test functions depending on their specialized purposes:

- 1) Conventional Evolutionary Programming (CEP) with different mutation operators (Chellapilla 1998), namely, a) Gaussian Mutation Operator (CEP/GMO), designed for fast convergence on convex function optimization; b) Cauchy Mutation Operator (CEP/CMO), aimed for effective escape from the local optima; and c) Mean Mutation Operator (CEP/MMO), which is a linear combination of Gaussian mutation and Cauchy mutation.
- 2) Fast Evolutionary Programming (FEP) (Yao et al. 1999): FEP essentially employs a Cauchy mutation operator, but incorporates the Gaussian mutation operator in an effective way.
- 3) Fast Evolution Strategy (FES) (Yao et al. 1997): FES applies Cauchy mutation in the evolution strategies to generate each new generation.
- 4) Evolutionary Optimization (EO) (Angeline 1998): EO uses a mutation operator and a selection scheme to evolve a population.
- 5) Particle Swarm Optimization (PSO) (Angeline 1998): PSO is a new evolutionary computing scheme; it explores the insect swarm behavior.

Among these algorithms, of particular interest for present comparison purposes is the

FEP proposed by Yao et al. (1999). The invention of the earlier CEP was dedicated to the global optimization of continuous problems and it proved to be quite successful. FEP further enhances the capacity of CEP. Hence, a comparison with FEP will effectively demonstrate the global optimization capability of the present StGA.

5.4.2 Comparison between StGA and Other Algorithms

a) Comparison with FEP

Table 5-3 presents the optimization results obtained by StGA in comparison with those from FEP. As can be seen, StGA is able to locate the near-optimal solutions for all the 20 test functions with relatively small variance, indicating that the algorithm is both effective and statistically stable. Comparing with FEP, StGA achieves generally much better optimization accuracy while the required computational effort is reduced considerably.

For unimodal functions f_1 to f_7 , StGA is able to obtain practically perfect optimization results, while FEP has difficulty with functions f_4 and f_5 and the accuracy for the remaining functions is also less good than StGA. From the evolution curves (the best individual vs. number. of generations) shown in Figure 5-7, it can be observed that StGA can quickly converge towards the optima (zero value in these cases) within a relatively small number of generations. Taking the case of f_1 as an example, several orders of reduction of the function value, from 10^6 to 10^0 , takes about 100 generations; whereas with FEP 800 generations are required to achieve a similar result. It is interesting to note that for function f_6 the convergence is extremely fast and the correct optimum is found with only about 12 generations. In contrast, FEP requires

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1500 generations to converge for this particular function. In this case, StGA reduces the computational effort by more than 100 times.

Table 5-3 Comparison of optimization results and computational effort between StGA and FEP

TF		f_1	f_2	f_3	f_4	f_5
Computational effort (MNFE)	StGA	30,000	17,600	23,000	32,000	45,000
	FEP	150,000	200,000	500,000	500,000	2×10^6
Optimization results: Mean best μ_{NG} (Variance σ_{NG})	StGA	2.45×10^{-15} (5.25×10^{-16})	2.03×10^{-7} (2.95×10^{-8})	9.98×10^{-29} (6.9×10^{-29})	2.01×10^{-8} (3.42×10^{-9})	0.04435 (0)
	FEP	5.7×10^{-4} (1.3×10^{-4})	8.1×10^{-3} (7.7×10^{-4})	1.6×10^{-2} (1.4×10^{-2})	0.30 (0.50)	5.06 (5.87)
TF		f_6	f_7	f_8	f_9	f_{10}
Computational effort (MNFE)	StGA	1,500	25,500	1,500	28,500	10,000
	FEP	150,000	300,000	900,000	500,000	150,000
Optimization results: Mean best μ_{NG} (Variance σ_{NG})	StGA	0.0 (0.0)	8.4×10^{-4} (1.0×10^{-3})	-12569.5 (0.0)	4.42×10^{-13} (1.14×10^{-13})	3.52×10^{-8} (3.51×10^{-9})
	FEP	0.0 (0.0)	7.6×10^{-3} (2.6×10^{-3})	-12554.5 (52.6)	4.6×10^{-2} (1.2×10^{-2})	1.8×10^{-2} (2.1×10^{-3})
TF		f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
Computational effort (MNFE)	StGA	52,500	8,000	16,000	800	30,000
	FEP	200,000	150,000	150,000	10,000	400,000
Optimization results: Mean best μ_{NG} (Variance σ_{NG})	StGA	2.44×10^{-17} (4.54×10^{-17})	8.03×10^{-7} 1.96×10^{-14}	1.13×10^{-5} (4.62×10^{-13})	1.0 (0.0)	3.18×10^{-4} (4.73×10^{-6})
	FEP	1.6×10^{-2} (2.2×10^{-2})	9.2×10^{-6} (3.6×10^{-6})	1.6×10^{-4} (7.3×10^{-5})	1.22 (0.56)	5.06 (5.87)
TF		f_{16}	f_{17}	f_{18}	f_{19}	f_{20}
Computational effort (MNFE)	StGA	4,000	5,000	10,000	4,800	8,500
	FEP	10,000	10,000	10,000	10,000	10,000
Optimization results: Mean best μ_{NG} (Variance σ_{NG})	StGA	-1.03034 (1.00×10^{-3})	0.3986 (6.00×10^{-4})	-9.828 (0.287)	-10.40 (0.0)	-10.450 (0.037)
	FEP	-1.0300 (4.9×10^{-7})	0.3980 (1.5×10^{-7})	-5.52 (1.59)	-5.52 (2.12)	-6.57 (3.14)

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Individual performance curves in 50 runs

Mean performance curves

(Abcissa = number of generations; Vertical axis = function value)

Figure 5-7 Evolution curves of StGA on functions f_1 , f_5 and f_6

It is noted from Figures 5-7 and 5-9 that there appears to be a jump to a low function value on the evolution curves of functions f_1 , f_5 , f_6 and f_{10} . In fact, there are two different scenarios here; one represents a true jump which is due to the property of the function itself such as f_6 , as will be discussed later; another is not exactly a “jump” but appears so because of the use of the logarithm scale for the function value while

the actual optimum is zero. In such a case, when the evolution is getting closer to the optimum, such as for f_1 from $9.54E-3$ to $2.31e-15$, it appears as a big “jump” on the log scale plot.

From a more general perspective, certain degree of “jump” on a GA evolution curve is not uncommon when the replacement method is applied, due to the fact that GA could sometimes experience a few generations without achieving a better solution. Such situation can be diagnosed from the evolving variable values before and after the jump, as illustrated in Figure 5-8(a) for function f_5 . This function has the true optimum equal to zero. At a step prior to the “jump”, most variables (25 out of the total 30) are already very close to their optimal values. After a couple of more generations, StGA also locates the near-optimal values of these last few variables and the function value thus decreases from 1.65 to 0.044. Similar situations also happen to the optimization of functions f_1 and f_{10} . Function f_6 belongs to a different category in that it possesses a solution region instead of a single optimal point. All variables have the same optimal region of $[-1.5, 0.5]$, as shown in Figure 5-8(b) between the two thick dashed lines. It can be seen that before the jump all the 30 variables except the 7th variable are already in their optimal region; and when the optimal region of this variable is also located, the function value jumps from 1 to 0.

Figure 5-8 Typical best variable sets evolved during StGA optimization around a “jump” of function values

For functions f_8 to f_{13} , which feature numerous local optima, the results shown in Table 5-3 clearly indicate that StGA can identify the actual optima of these functions with almost perfect accuracy. Meanwhile, the efficiency as compared to FEP increases by 4 to 600 times in terms of the number of function evaluations. Figure 5-9 shows typical evolution curves for functions f_8 , f_{10} and f_{12} , which demonstrates that StGA behaves in a very stable manner over the 50 runs, despite the numerous local optima in these functions.

For the 7 multimodal functions with fewer number of local optima (f_{14} to f_{20}), generally speaking StGA also exhibits a superior performance over FEP. Figure 5-10 depicts the evolution curves for functions f_{15} , f_{17} and f_{20} . Particularly noteworthy is the case of the function family f_{18} to f_{20} , which differ only in the number of terms in the summation. Whereas FEP appears to be unable to approach the optima for these functions, StGA maintains a satisfactory performance.

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Individual performance curves in 50 runs

Mean performance curves

(Abcissa = number of generations; Vertical axis = function value)

Figure 5-9 Evolution curves of StGA on functions f_8 , f_{10} and f_{12}

Figure 5-10 Evolution curves of StGA on functions f_{15} , f_{17} and f_{20}

b) Comparison with CEP, FES, EO and PSO

The performance of StGA is further compared with some other well-established algorithms such as CEP, FES, EO and PSO. Since from the literature the optimization results using these algorithms are available only for some of the 20 test functions, the comparison will be made accordingly. The comparison results are summarized in Table

5-4 to Table 5-6.

Table 5-4 Comparison of performance between StGA and FES

TF	MNFE		Mean best μ_{NG} (Variance σ_{NG})	
	StGA	FES	StGA	FES
f_8	1,500	900,030	-12569.5 (0.0)	-12556.4 (32.53)
f_9	28,500	500,030	4.42×10^{-13} (1.14×10^{-13})	0.16 (0.33)
f_{10}	10,000	150,030	3.52×10^{-8} (3.51×10^{-9})	1.2×10^{-2} (1.8×10^{-3})
f_{11}	52,500	200,030	2.44×10^{-17} (4.54×10^{-17})	3.7×10^{-2} (5.0×10^{-2})
f_{12}	8,000	150,030	8.03×10^{-7} (1.96×10^{-14})	2.8×10^{-6} (8.1×10^{-7})
f_{13}	16,000	150,030	1.13×10^{-5} (4.62×10^{-13})	4.7×10^{-5} (1.5×10^{-5})

Table 5-5 Performance comparison among StGA, PSO and EO

TF	MNFE			Mean best μ_{NG} (Variance σ_{NG})		
	StGA	PSO	EO	StGA	PSO	EO
f_1	30,000	250,000	250,000	2.45×10^{-15} (5.25×10^{-16})	11.175 (1.3208)	9.8808 (0.9444)
f_5	45,000	250,000	250,000	0.04435 (0)	1911.598 (374.2935)	1610.39 (293.5783)
f_9	28,500	250,000	250,000	4.42×10^{-13} (1.14×10^{-13})	47.1354 (1.8782)	46.4689 (2.4545)
f_{11}	52,500	250,000	250,000	2.44×10^{-17} (4.54×10^{-17})	0.4498 (0.0566)	0.4033 (0.0436)

Table 5-6: Performance comparison among StGA, GMO, GMO and MMO

TF	MNFE				Mean best μ_{NG} (Variance σ_{NG})			
	StGA	CEP/GMO	CEP/CMO	CEP/MMO	StGA	CEP/GMO	CEP/GMO	CEP/MMO
f_1	30,000	1500,00	1500,00	1500,00	2.45×10^{-15} (5.25×10^{-16})	3.09×10^{-7}	3.07×10^{-6}	9.81×10^{-7}
f_2	17,600	250,000	250,000	250,000	2.03×10^{-7} (2.95×10^{-8})	1.99×10^{-3}	5.87×10^{-3}	3.23×10^{-3}
f_3	23,000	250,000	250,000	250,000	9.98×10^{-29} (6.9×10^{-29})	17.60	5.78	11.80
f_4	32,000	250,000	250,000	250,000	2.01×10^{-8} (3.42×10^{-9})	5.18	0.66	1.88
f_5	45,000	250,000	250,000	250,000	0.04435 (0)	86.70	114.0	63.8
f_7	25,500	250,000	250,000	250,000	0.008412 (0.001023)	12.20	9.42	9.53
f_9	28,500	250,000	250,000	250,000	4.42×10^{-13} (1.14×10^{-13})	120.0	4.73	9.52
f_{10}	10,000	1500,00	1500,00	1500,00	3.52×10^{-8} (3.51×10^{-9})	9.10	1.3×10^{-3}	7.49×10^{-4}
f_{11}	52,500	250,000	250,000	250,000	2.44×10^{-17} (4.54×10^{-17})	2.52×10^{-7}	2.2×10^{-6}	6.99×10^{-7}

From Table 5-4, it can be seen that FES generally can achieve satisfactory optimization results for the listed functions (except f_9), but StGA exhibits more accurate results while the required computational effort is only about one-tenth of that required by FES. Results shown in Table 5-5 and Table 5-6 indicate that, while StGA maintains a consistent and satisfactory performance, PSO, EO and CEP appear to be unable to approach the optima for most of the listed functions, even after spending a considerable computational effort.

5.4.3 Performance of StGA in Solving Large-Scale Optimization Problems

In the preceding sections, the superior performance of StGA as compared to other algorithms has been demonstrated by optimizing both unimodal and complex multimodal functions up to moderate dimensions (up to 30). In order to examine the performance of StGA in handling large scale problems, in this section the algorithm is used to perform optimization for two functions having a dimension as high as 100. The first function is an expanded version of f_5 with dimension increased from 30 to 100, denoted as f_5^{100} . The global minimum of this function remains zero. The other function takes the form

$$g(x) = 1/m \sum_{i=1}^m (x_i^4 - 16x_i^2 + 5x_i) \quad \text{s.t.} \quad -10 \leq x_i \leq 10 \quad (5-8)$$

For any positive integer m , the global minimum of $g(x)$ is -78.3323 , and it occurs at point $X_{\min} = (-2.9035, -2.9035, \dots, -2.9035)^T$.

The optimization of f_5^{100} has been reported in (Zhang and Xu 1999) using an Efficient Evolutionary Programming (EEP) algorithm while the optimization of $g(x)$ has been performed in (Siarry et al. 1997) using Enhanced Simulated Annealing algorithm (ESA). These previous results are used so as to compare them with StGA.

The key parameters used in StGA for optimizing the above two functions are listed in Table 5-7. Table 5-8 compares the optimization results from StGA with those from ESA and EEP. Once again, StGA exhibits a dramatic improvement in the optimization performance. For the function $g(x)$, the optimal value identified by StGA is within 0.1% of the true optimum, while the EEP error is about 2%. For f_5^{100} , the

superiority of StGA over ESA is more obvious. Besides, the computational cost using StGA is only $1/8$ to $1/4$ of that using ESA or EEP. Figure 5-11 shows the evolution curves of StGA in optimizing f_5^{100} and $g(x)$.

Table 5-7 StGA parameter setting and required computational effort for optimizing f_5^{100} and $g(x)$

TF	NP	NS	NG	MNFE
f_5^{100}	30	5	240	36,000
$g(x)$	22	5	130	14,300

Table 5-8 Comparison of optimization results and computational effort among StGA, EEP and ESA

TF	MNFE		Mean best μ_{NG} (Variance σ_{NG})	
	StGA	ESA ⁽¹⁾ /EEP ⁽²⁾	StGA	ESA ⁽¹⁾ /EEP ⁽²⁾
f_5^{100}	36,000	150,000 ⁽¹⁾	1.01 (1.1959)	17.10 ⁽¹⁾ (NA)
$g(x)$	14,300	122,000 ⁽²⁾	-78.29368 (0.03)	-76.782 ⁽²⁾ (NA)

(Abcissa = number of generations; Vertical axis = function value)

Figure 5-11 Evolution curves of StGA on functions f_5^{100} and $g(x)$

5.5 Conclusion

A stochastic genetic algorithm (StGA) is presented to deal with global optimization problems with continuous variables. The methodology involves a novel coding mechanism in which the search space is dynamically divided into stochastic regions represented by a mean vector (coded in binary strings) and a variance vector. An effective crossover scheme is proposed such that when the cut site of crossover happens to fall within a substring representing a mean phenotype variable value, an interpolated variance is produced for crossover of the variance term. To further enhance the StGA performance, a similar replacement scheme as in the conventional

genetic algorithm is incorporated into the operation of StGA.

The algorithm is tested on 20 functions of moderate dimensions from three different categories. Results obtained from 50 trials for each function show that StGA is able to find the near-global solution for all these test functions; and, moreover, the behavior of the algorithm is very stable as indicated by a small variance among the 50 trial runs. Comparison of the StGA outcome with those from several other global optimization algorithms demonstrates that StGA outperforms the other techniques with a dramatic improvement in terms of effectiveness as well as efficiency. In general, the accuracy of StGA increases by several orders of magnitude. For those functions where other algorithms experience difficulties in approaching the optima, StGA still exhibits a satisfactory performance. On average, the number of function evaluations required by StGA is about one order less than that required by other algorithms. In some function cases, StGA reduces the computational effort by as much as 100 to 600 times.

StGA is also shown to be capable of solving large-dimension problems with good efficiency.

CHAPTER 6

FE MODEL UPDATING USING ARTIFICIAL BOUNDARY CONDITIONS WITH GA

6.1 Introduction

A typical FE model may be defined by a large number (order of $10^2 - 10^3$) of physical parameters. However, a typical modal test on a structure may yield only a small number (order of 10^1) of modal parameters that can be used to guide the adjustment of the FE model parameters (Gordis 1999). This disparity in the number of known parameters (measured modal data) versus the number of physical variables to be reconciled leads to an underdetermined problem. In this regard, there exists a well-recognized need to enlarge the known parameter dataset.

For dynamic FE model updating, resonance frequency data are usually included because of their significance with respect to the structural parameters and their good measurement accuracy. In fact, some pioneering studies attempted to locate the damage solely using natural frequencies (Cawley and Adams, 1979; Hearn and Testa, 1991). However, this method bears serious limitations due primarily to two problems: 1) some local damages require the use of very high order frequencies, which is beyond the practical measurability; and 2) the frequency data alone are not capable of telling apart parameters of a symmetric structural system. To supplement the frequency data, mode shape data have been extensively used. Such data, however, are known to be prone to large measurement errors, which can easily reach 20% in some cases, leading to its limited applicability. To expand the response dataset, some researchers have tried to obtain different sets of resonance frequencies by conducting a series of independent

tests on the same structure, but with different boundaries applied physically on the structure for each test. For example, Li et al. (1995) described procedures called ‘perturbed boundary condition (PBC)’ testing, in which additional configurations of the structure are independently tested. These configurations may include different boundary conditions and additional masses at selected points of the structure. This method, however, requires that physical modifications be made to the structure during the tests, and a separate test is needed for each different configuration. It is difficult to implement in practice.

Recently, a method called “artificial boundary method (ABM)” (Gordis 1996b, 1999) has been developed. Via ABM, a large number of additional and distinct mode frequencies can be easily identified from the same modal test performed to identify the natural frequencies of a structure, without the need of any physical modification to the structure. The only information that is required is the incomplete FRF matrix corresponding to those DOFs where the artificial boundaries are applied. These additional and distinct mode frequencies correspond exactly to those natural frequencies found when certain combinations of measured coordinates are restrained to the ground. This means that with one modal test on a particular structure, many sets of natural frequencies can be obtained, and each set corresponds to the same state of the structure but under different boundary conditions with extra artificial pinned supports. Theoretically speaking, the number of the extra pins is not restricted as there exist an infinite number of DOFs for a continuous structure; however, due to some practical limitations (for example, the number of measuring sensors) and the inaccessibility of certain parts of the structure, a proper choice of the number of extra pins has to be considered.

The artificial boundary method is applicable in practice since the additional boundary conditions are only artificial boundary conditions (ABCs) and the actual structure is not altered. In addition, this method is effective as it produces extra frequency data which can be as accurate as the natural frequencies. A typical example is with the antiresonant frequencies from a driving-point frequency response function (FRF) (see Chapter 4). The driving-point antiresonances correspond to the natural frequencies of

the structure with the driving-point DOF pinned to the ground. In other words, the antiresonance data is a special case of the ABM and it is associated with only one artificial pin.

Despite the potential benefits that ABM can provide for FE model updating, as with other measurement data, it is always an important consideration how to utilize the extended capability in expanding the data set effectively. The underlying issue concerns the sensitivity. In the present study it is found that the application of different configurations for the ABCs can significantly affect the effectiveness of the corresponding frequencies for model updating. For example, a good configuration of ABCs may result in an average error of 4% in the updated parameters, whereas a 20% error could be associated with a poor setting of ABCs. Therefore, it is necessary to have a systematic approach that can be used to determine the proper configurations for ABCs before updating. So far, only a few exploratory works have been reported on the application of ABM for FE model updating (Gordis 1999; Dambrogio and Fregolent 2000; Jones and Turcotte 2002). In these works, the ABCs were chosen according to general engineering judgment.

In the present study, the artificial boundary method is used to provide extra frequency data (abbreviated as “ABC frequencies” in the subsequent descriptions) and these data are combined with the structural natural frequencies to improve the updating. In the proposed procedure, the mode shape data are not directly used in the updating; they are used only for pairing the analytical and experimental modal frequencies. For this purpose, the requirement on the quality of the mode shape data is considerably relaxed and measurements at a few selected DOFs may suffice. The selection of DOFs for measuring the mode-shape data is based on the effective independence distribution vector method (Kammer 1991 and 1992). To determine a suitable configuration for the ABCs, a method using binary GA is proposed. The search domain of GA consists of all the accessible DOFs of a structure. Each chromosome represents an arbitrary boundary configuration. The fitness of a chromosome is evaluated according to the effectiveness of the ABC frequencies, which is assessed via a sensitivity analysis. The effectiveness of both the proposed GA-based ABC selection procedure and the

updating process using the combined natural and ABC frequency data is demonstrated numerically with examples as portal frames.

6.2 Theory of Artificial Boundary Method (ABM)

In Chapter 4, a basic use of antiresonance data has been explored. Antiresonant frequencies can be extracted from a direct point FRF of a structure and they can be interpreted as the natural frequencies of the structure when a single artificial pinned constraint (abbreviated as “APC”) is added at a particular DOF. Generally speaking, if a number of APCs are present, the resonant frequencies from the structure can be more effective for FE model updating from the sensitivity point of view. This consideration has led to the development of the artificial boundary method (ABM) (Gordis 1992, 1994 and 1996b). In a real test, to obtain the frequencies associated with a particular configuration of the artificial boundary condition (ABC), the ABM requires only the measurements of the incomplete FRF matrix that corresponds to the DOFs at which APCs are exerted. The theory of ABM will be briefly introduced in what follows.

For a structure, the application of the artificial pinned constraints of any configuration essentially defines an omitted coordinate set system (or OCS) (Gordis 1992, 1996b). This system is defined by the set of all unmeasured DOFs, which is of infinite dimension for a continuous test system. To understand the OCS, the fundamental relations in model reduction that define the OCS are first described. The similar relations in the frequency domain are then given, which will enable the identification of ABC configuration natural frequencies (ABC frequencies).

6.2.1 OCS and Model Reduction

The equation of motion of the forced vibration of a linear structural system at a forcing frequency ω (rad/s) can be written as

$$\begin{bmatrix} \mathbf{k}_{aa} & \mathbf{k}_{ao} \\ \mathbf{k}_{oa} & \mathbf{k}_{oo} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{m}_{aa} & \mathbf{m}_{ao} \\ \mathbf{m}_{oa} & \mathbf{m}_{oo} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_a \\ \mathbf{x}_o \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_a \\ \mathbf{f}_o \end{Bmatrix} \quad (6-1)$$

where $\mathbf{k}$ and $\mathbf{m}$ are stiffness and mass matrices, $\mathbf{x}$ and $\mathbf{f}$ are vectors of generalized response and excitation amplitudes, respectively. The subscript 'a' represents measured coordinates or DOFs ('analysis coordinate set') and the subscript 'o' refers to the DOFs unmeasured ('omitted coordinate set'). Eq. (6-1) can be rearranged as

$$\begin{bmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ao} \\ \mathbf{Z}_{oa} & \mathbf{Z}_{oo} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_a \\ \mathbf{x}_o \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_a \\ \mathbf{f}_o \end{Bmatrix} \quad (6-2)$$

where $\mathbf{Z}$ is the impedance matrix, $\mathbf{Z} = \mathbf{k} - \omega^2 \mathbf{m}$. Assuming that no excitations are imposed on the omitted coordinates, by a simple manipulation of Eq. (6-2) the exact relationship between the omitted coordinate set and the analysis coordinate set can be found,

$$\{\mathbf{x}_o\} = \left[\mathbf{I} - \omega^2 \mathbf{k}_{oo}^{-1} \mathbf{m}_{oo} \right]^{-1} \left[-\mathbf{k}_{oo}^{-1} \mathbf{k}_{oa} + \omega^2 \mathbf{k}_{oo}^{-1} \mathbf{m}_{oa} \right] \{\mathbf{x}_a\} \quad (6-3)$$

The origin of the omitted coordinate set system is reflected by the first bracketed inverse term $\left[\mathbf{I} - \omega^2 \mathbf{k}_{oo}^{-1} \mathbf{m}_{oo} \right]^{-1}$ in Eq. (6-3). This term will be singular at the frequencies which satisfy $\text{Det} \left[\mathbf{I} - \omega^2 \mathbf{k}_{oo}^{-1} \mathbf{m}_{oo} \right] = 0$. It is obvious that these frequencies are the eigenvalues of the system defined by $\mathbf{k}_{oo}$ and $\mathbf{m}_{oo}$, i.e., the omitted coordinate set system. This system is obtained by fully restraining to the ground all coordinates in the analysis coordinate set.

6.2.2 OCS and Frequency Response Function Matrices (FRF Matrices)

With a limited instrumentation of sensors and actuators, a reduced-order model can be defined and the impedance of the reduced-order model depends nonlinearly on the impedance of the full-order model (Berman 1984). According to Berman, if the exact full-order FRF model of a structure is considered, a FRF matrix of infinite dimension for a full-order model of a structure system can be given by

$$\mathbf{H}^m = \begin{bmatrix} \mathbf{H}_{aa} & \mathbf{H}_{ao} \\ \mathbf{H}_{oa} & \mathbf{H}_{oo} \end{bmatrix} \quad (6-4)$$

where the superscript ‘m’ denotes an experimental quantity and the number of coordinates in the omitted coordinate set is infinite. Then, the incomplete FRF matrix measured in a test is seen to be the matrix partition $\mathbf{H}_{aa}$ of the full-order FRF matrix $\mathbf{H}^m$, i.e.

$$\overline{\mathbf{H}}^m = \mathbf{H}_{aa} \quad (6-5)$$

where the overbar indicates a reduced model. From the identity relation $\mathbf{ZH} = \mathbf{I}$, the measured FRF matrix $\overline{\mathbf{H}}^m$ can be expressed in terms of the partitioned impedance matrices by

$$\overline{\mathbf{H}}^m = \mathbf{H}_{aa} = \left(\mathbf{Z}_{aa} - \mathbf{Z}_{ao} \mathbf{Z}_{oo}^{-1} \mathbf{Z}_{oa} \right)^{-1} \quad (6-6)$$

For a linear dynamic system, the reduced-order force-displacement relation in modal domain obtained from the exact dynamic reduction is (Gordis 1996b)

$$(\mathbf{f}_a) = \left(\mathbf{Z}_{aa} - \mathbf{Z}_{ao} \mathbf{Z}_{oo}^{-1} \mathbf{Z}_{oa} \right) (\mathbf{x}_a) \quad (6-7)$$

The common terms in Eqs. (6-6) and (6-7) signify that a spatially incomplete FRF matrix actually represents a dynamically reduced model.

Applying inverse operation on Eq. (6-6) gives,

$$\left(\overline{\mathbf{H}}^m\right)^{-1} = \mathbf{Z}_{aa} - \mathbf{Z}_{ao}\mathbf{Z}_{oo}^{-1}\mathbf{Z}_{oa} \quad (6-8)$$

From Eq. (6-8), it can be easily identified that the elements of $\left(\overline{\mathbf{H}}^m\right)^{-1}$ will also be singular (or ‘large’ for a damped system) when the matrix $\mathbf{Z}_{oo}^{-1}$ becomes singular, i.e., $\text{Det}(\mathbf{Z}_{oo}^{-1}) = 0$. Recall that $\mathbf{Z}_{oo} = \mathbf{k}_{oo} - \omega^2\mathbf{m}_{oo}$ is the impedance matrix of the omitted coordinate set system, so its inverse $\mathbf{Z}_{oo}^{-1}$ becomes singular only at the natural frequencies of the omitted coordinate set system. Therefore, by identifying the frequencies at which the elements in $\left(\overline{\mathbf{H}}^m\right)^{-1}$ are singular, the natural frequencies of the OCS (or ABC frequencies) can be obtained. This is the core operation of the artificial boundary method. The numerical solution for the ABC frequencies is straightforward by considering the artificial boundaries in the system matrices of the actual structure.

6.2.3 Examples of ABC Configuration Frequencies

In this section, an example is provided to illustrate the process of identifying the ABC frequencies for a structural system. The structure used for the illustration is the same as that used in section 6.3 and it is a 3-storey-one-bay portal frame as shown in Fig 6-1.

Two artificial pinned constraints are assumed and their corresponding DOF numbers are denoted by n_1 and n_2 , respectively, as indicated in Figure 6-1. The calculation of ABC frequencies for the current configuration of ABCs is done as per Eq. (6-8). In this case, the spatially incomplete FRF matrix $\overline{\mathbf{H}}^m$ has a dimension of 2×2 ,

$$\overline{\mathbf{H}}^m = \begin{bmatrix} \alpha_{n_1 n_1} & \alpha_{n_1 n_2} \\ \alpha_{n_2 n_1} & \alpha_{n_2 n_2} \end{bmatrix} \quad (6-9)$$

where α represents the individual frequency response function (FRF). During a test, a column of the matrix $\bar{\mathbf{H}}^m$ can be obtained by applying the excitation (for example from an impact hammer) on one of these DOFs and measuring the accelerations (from accelerometers) at all these DOFs (in the present case n_1 and n_2). Thus, by moving the excitation among different locations of the artificial pins, the whole $\bar{\mathbf{H}}^m$ is obtained.

Figure 6-1 3-storey-1-bay frame with two APCs

In this illustrative example, the solution of $\bar{\mathbf{H}}^m$ is simulated via the FE model of the structure in which the excitation is modeled by an impact pulse and the acceleration responses are computed accordingly. Inverting the $\bar{\mathbf{H}}^m$ at each frequency results in the impedance matrix $(\bar{\mathbf{H}}^m)^{-1}$:

$$(\bar{\mathbf{H}}^m)^{-1} = \begin{bmatrix} z_{n_1 n_1} & z_{n_1 n_2} \\ z_{n_2 n_1} & z_{n_2 n_2} \end{bmatrix} \quad (6-10)$$

The magnitude of $z_{n_1 n_1}$ is plotted in Figure 6-2 (thin line). For comparison, a particular direct-point FRF $\alpha_{n_3 n_3}^{ABCs}$ of the ABC system (calculated at DOF n_3 as shown in Figure 6-1) is also plotted in Figure 6-2 (thick line), in which the peaks correspond to the natural frequencies of the ABC system. It is evident that all the peaks of $z_{n_1 n_1}$ coincide

with those of the direct-point FRF of the ABC system except two peaks (circled in Figure 6-2), which are missing from $\alpha_{n_3 n_3}^{ABCs}$. This is because DOF n_3 becomes a stationary point for these two modes. By solving the FE eigenvalue problem, the natural frequencies for the ABC system are obtained and listed in Table 6-1. These analytical frequencies are exactly matched by the frequencies predicted by $z_{n_1 n_1}$.

Figure 6-2 Comparison between frequencies from a driving point FRF of the ABC system and those corresponding to the peaks of an element of the impedance matrix $H^{-1}_{aa}(2,2)$

Table 6-1 Natural frequencies calculated from the ABC system

Modes	1	2	3	4	5	6	7	8	9	10
Frequencies	13.8	48.6	59.2	63.9	83.6	120.0	156.34	172.5	187.0	207.5

The above example shows the case of two artificial pins for the artificial boundary method. A similar procedure can be applied for any configuration of artificial pins.

6.3 The Necessity for the Selection of Artificial Boundaries

Reliable updating results depend very much on the quality and the richness of the response information. The accuracy of the ABC frequencies is satisfactory since they can be measured as accurately as the natural frequencies. Moreover, there exists a huge amount of such data that can be readily included in the FE updating procedure. This means it is rich also. However, it is not necessarily true that more response data always results in a better updating outcome. Data that are non-sensitive or less sensitive to the physical variables should not be used because they are worsening the conditioning of the updating. Therefore, a proper selection of the data to be measured and used in the model updating is important.

Some methods have been developed for the optimal selection of mode shapes. For example, Sanayei et al (1996b) developed a Best-In-Worst-Out method to pick out a subset of static force and strain measurements that have the least sensitivity to the measurement noise. The available measurement that has the smallest effect on the parameter estimation is removed one by one until the output error becomes too large. Some other selection methods concerning mode shapes can be found in Doebling (1997b), Shi et al. (2000), and Xia and Hao (2000).

However, there has been no previously reported work on the optimal choice of artificial boundaries for the generation of ABC frequencies for FE model updating. In some applications, the selection was simply based on engineering experience or judgment (Gordis 1999; Jones and Turcotte 2002). In fact, as will be shown in the following illustrative example by means of a sensitivity analysis, that the ABC frequencies from different artificial boundary configurations can result in very different updating results.

For this purpose, a 3-storey, 1-span portal frame, shown in Figure 6-3, is selected. Six stiffness parameters are supposed to be identified in the form of the stiffness modification factors (SMFs), including 3 beam SMFs and 3 column SMFs (considering one SMF for each individual beam and one SMF for all columns in the

same storey). The measurements are assumed to be composed of the first six natural frequencies, plus six ABC frequencies associated with a one-point pinned artificial boundary. Two different sets of the ABC frequencies are considered, corresponding respectively to two different artificial boundary configurations, to examine their effectiveness (in terms of sensitivity) for the updating of the six SMFs. The first artificial boundary is chosen in an arbitrary manner, while the second one is obtained from using binary GA, as will be described later. The SMF errors are predicted by sensitivity analyses, in which 1% uniformly distributed random errors are assumed to exist in the response data. Figure 6-3 shows the configurations of the two different boundaries. Figures 6-4 and 6-5 show the resulting SMF error bandwidths.

Figure 6-3 Two different boundary configurations used for the computation of ABC frequencies

As can be seen, the use of the arbitrarily chosen artificial boundary (AB1) results in as large as 30% errors in SMF 5-6 and around 10% for other SMFs. Striking improvement is achieved with the optimized AB2; the errors for all the six variables are less than 8%. This clearly indicates the importance of a proper choice of the artificial boundary for the inclusion of the ABC frequencies into the response set. A number of additional cases are also examined and similar trends are observed. In the

present study, a binary GA-based method is proposed to automate the selection of artificial boundaries.

Figure 6-4 SMF error bandwidths corresponding to AB1

Figure 6-5 SMF error bandwidths corresponding to AB2

Another point worth mentioning is that, for a geometrically symmetric structural system, the FE model updating procedure by solely matching natural frequencies may converge to a false solution because the mirrored version of the target structure will exhibit exactly the same resonant frequencies as the actual structure. Usually this

problem can be resolved by incorporating appropriate mode shape data to reflect the spatial distribution of the physical state. In the case of using ABC frequencies, the problem is readily avoided. This is because the ABC frequencies actually represent the resonant frequencies of an altered system with extra artificial pinned constraints; the introduction of these constraints breaks the original symmetry and leads effectively to an unsymmetric system.

6.4 Binary GA for the Optimal Selection of Artificial Boundary

6.4.1 Selection Procedure

In this section, a method using binary GA for the optimal configuration of artificial boundaries will be presented. For a real structure, there may exist numerous candidate artificial boundary configurations with different number of pinned constraints. An artificial boundary is considered good if the resulting ABC frequencies exhibit high sensitivities with respect to the updating variables.

There are two different methods to evaluate the “sensitivity”; one is to directly assess the sensitivity matrix in terms of a certain measure (the “direct method”), and the other is by evaluating the statistics of the parameter errors from different error patterns of the response data (the “indirect method”). For the direct method, some researchers use the mean of all components in the sensitivity matrix as an overall measure of sensitivity for the mode-shape selection (Xia and Hao 2000). However, as can be seen from the example given in section 6.3, the mean values of two very different sensitivity matrices can be quite close, e.g., 0.0391 vs. 0.0394 (0.8% difference). This implies that the overall mean value alone may not indicate adequately the goodness of a chosen response dataset. The condition number of a sensitivity matrix may better serve the purpose, but its value lacks physical meaning. For these reasons, the present study

adopts the indirect method. It is noted that the indirect method does not cause significant increase of the computational effort since it only involves a matrix inversion operation and a few matrix multiplications. It has, however, the advantages of high reliability and is a physically explainable indicator.

Before starting the GA selection process, a search domain is prepared first. Since the objective of the GA optimization is to find the effective locations for the artificial pins, the feasible domain should be those degrees of freedom of a FE model that are accessible for measurements (called the “DOF library”). If any special feature of the structural system is known beforehand such that some DOFs can be judged as poor candidate locations to be involved in the artificial boundaries, the size of the DOF library can be further reduced by excluding those DOFs. In the program implementation, the DOF library is arranged in the form of a row vector, which consists of the numbers of the candidate DOFs in the search domain. For example, if a cantilever beam is modeled using four 2D beam elements as shown in Figure 6-6, then the DOF library will include 6 translational DOFs, i.e., DOFs 1, 2, 4, 5, 7, and 8 and is represented by a row vector, [1, 2, 4, 5, 7, 8].

Figure 6-6 An illustrative example of DOF library

The phenotype variables for GA are integer numbers which identify the DOFs in the DOF library. Since only a fraction of the total DOFs are in the DOF library, it is more efficient to identify the DOFs according to their sequential number in the DOF library instead of using their actual DOF number. For each optimization process, the total number of pinned constraints is predefined, and the procedure then determines the optimal locations of these pins. The procedure can be repeated if different numbers of pinned constraints need to be tried.

In the GA chromosome, the integer variables identifying the DOFs are coded using binary bits and the number of binary bits depends on the size of the DOF library. For the above beam example, 3 bits are sufficient to represent totally the 6 candidate DOFs.

For the genetic operations, the single point crossover is used and mutation is implemented by bit flip, i.e., “0” to “1” or “1” to “0”, which is called binary mutation. These operations have been described in Chapter 3 and are briefly presented as follows. Assuming two artificial pins are to be applied on the beam structure, a GA chromosome will then involve two integer variables. Using 3 binary bits for each variable leads to a chromosome length of 6. Let’s assume that the following two chromosomes are selected for crossover and a crossover point is chosen after the third bit:

Chromosome A   010 • 101

Chromosome B   011 • 001

After crossover, two new chromosomes are generated as below

Chromosome A₁   010 • 001

Chromosome B₁   011 • 101

Let’s further assume that mutation happens to occur at the first bits of chromosome A₁, then this chromosome becomes,

Chromosome A₂   110 • 001

Chromosome B₁ remains unchanged.

The decoding process is quite straightforward. The binary chromosomes are first converted back to integer numbers, from which the corresponding DOFs in the DOF library are located, thus the artificial boundary represented by this chromosome is obtained. For the above example, the two evolved chromosomes A₂ and B₁ are decoded as [6, 1] and [3, 5], respectively. By referring back to the DOF library, which is [1, 2, 4, 5, 7, 8], the locations of two artificial pins represented by each of the two

chromosomes are obtained as [DOF8, DOF1] and [DOF4, DOF 7] for chromosome A_2 and chromosome B_1 , respectively. They are schematically shown in Figure 6-7.

Figure 6-7 The artificial pins decoded from two example chromosomes

Thus, a set of ABC frequencies corresponding to the system with the artificial pinned constraints represented by a chromosome can be calculated. The fitness of a chromosome is then evaluated by means of a sensitivity analysis based on the set of ABC frequencies (indirect method). The details of evaluating the fitness using sensitivity analysis have been presented in Chapter 3. A brief description in relation to the ABC frequencies is given below.

A prescribed NEP number of response error patterns $\{\delta R\}_i$ are generated first. The response vector $\{R\}$ is composed of a predefined number of ABC frequencies associated with one particular configuration of AB. For the generation of error patterns $\{\delta R\}_i$, 1% uniformly distributed random errors are used based on the consideration that the ABC frequencies can be obtained with a similar accuracy as the natural frequencies. A same number of parameter error patterns $\{\delta p\}_i (i = 1, \dots, \text{NEP})$ are obtained by multiplying $\{\delta R\}_i$ by the gain matrix $\mathbf{G}$. A statistic analysis is then performed on $\{\delta p\}_i$. The error pattern for one particular parameter p_j is a vector and denoted by $\{\delta p_j\}$, which consists of NEP number of scalar components. For each $\{\delta p_j\}$, an independent statistic analysis is conducted to calculate its mean value μ_j and the standard deviation value σ_j . Generally μ_j is close to zero. Hence, σ_j actually represents the parameter error and a larger value of σ_j signifies a larger parameter error. Thus, n number of σ_j are obtained and they form a vector designated as SD,

$$SD = [\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_n] \quad (6-11)$$

The assessment of the overall error from the n different parameters and thereby the chromosome fitness is implemented by

$$\text{Fitness} = -(\mu_{SD} + \sigma_{SD}/\mu_{SD}) \quad (6-12)$$

where μ_{SD} denotes the mean of SD, which is always a non-negative value, and σ_{SD} is the standard deviation of SD. The inclusion of the second term in Eq. (6-12) is to take care of special situations where the mean μ_{SD} is already quite small but there still exist a few large σ_j in SD. Such a chromosome should not be regarded as a good solution since generally speaking all physical parameters subjected to updating are considered equally important, and hence a desired good response set should result in a relatively uniform distribution of errors among all parameters.

To illustrate the above-mentioned concept for fitness evaluation, a simple example involving 6 variables $p_i (i=1, \dots, 6)$ and 8 response components $R_i (i=1, \dots, 8)$ is furnished. For simplicity, 3 response error patterns $\{\delta R\}_i (i=1, 2, 3)$ are used (NEP=3). Thus, 3 parameter error patterns $\{\delta p\}_i (i=1, 2, 3)$ are produced according to the following formula,

$$[\{\delta p\}_1 \quad \{\delta p\}_2 \quad \{\delta p\}_3] = [\mathbf{G}] \times [\{\delta R\}_1 \quad \{\delta R\}_2 \quad \{\delta R\}_3] \quad (6-13)$$

Table 6-2 lists the 3 response error patterns $\{\delta R\}_i$ and the resulting 3 parameter error patterns $\{\delta p\}_i$. To obtain SD, for each $\{\delta p_j\} (j=1, 2, \dots, 6)$ the corresponding standard deviation σ_j is computed as listed in the shaded area of Table 6-2. Thus,

$$SD = [3.11 \quad 0.18 \quad 0.60 \quad 3.19 \quad 2.28 \quad 0.31] \quad (6-14)$$

The mean μ_{SD} and the standard deviation σ_{SD} of SD are then calculated,

$$\mu_{SD} = 1.61 \text{ and } \sigma_{SD} = 1.41 \quad (6-15)$$

From these two quantities, the chromosome fitness is obtained as

$$\text{Fitness} = -(\mu_{SD} + \sigma_{SD} / \mu_{SD}) = -2.49 \quad (6-16)$$

Table 6-2 Simulated response error patterns and the output of parameter errors

Response Error Patterns (%)								
	δR_1	δR_2	δR_3	δR_4	δR_5	δR_6	δR_7	δR_8
$\{\delta \mathbf{R}\}_1$	-0.63	-0.16	0.98	-0.39	-0.20	-0.83	-0.45	-0.57
$\{\delta \mathbf{R}\}_2$	-0.61	0.50	0.97	-0.63	0.32	-0.86	0.50	-0.72
$\{\delta \mathbf{R}\}_3$	0.83	0.51	-0.31	-0.12	-0.80	0.14	0.90	0.52
Parameter Error Patterns (%)								
	δp_1	δp_2	δp_3	δp_4	δp_5	δp_6		
$\{\delta \mathbf{p}\}_1$	2.56	0.09	-1.89	-1.43	-3.91	0.18		
$\{\delta \mathbf{p}\}_2$	-1.29	0.42	-1.59	2.65	0.54	-0.11		
$\{\delta \mathbf{p}\}_3^*$	-3.60	0.13	-0.73	4.86	-0.83	0.50		
STD	3.11	0.18	0.60	3.19	2.28	0.31		

* Refers to standard deviation.

6.4.2 Repair of Invalid Chromosomes

It should be pointed out that, within any GA chromosome, there should be no repeated DOFs since it is meaningless to impose the same artificial pinned constraint on the same DOF for multiple times. If repeated integer values exist in a chromosome, the simplest way to handle it is to assign a very small fitness value for this chromosome

and let the evolution continue. However, such operation tends to reduce the diversity of the GA population and thus degrades the GA performance. This study proposes a “repair” strategy to revamp the irrational chromosomes so that no repeated integer value exists in a chromosome. Actually, the repair process may be performed in many different ways. In this study, an invalid chromosome is repaired by replacing each repeated value (DOF) with a new DOF randomly drawn from the GA search domain (the DOF library) excluding the DOFs that are already present in the chromosome.

Consider again the beam example given in section 6.4.1 but assume the use of 5 artificial pinned constraints. An invalid chromosome as shown in Figure 6-8 is to be repaired. As can be observed, the integers 2 and 3 are repeated values, and they need to be replaced. Since the DOF library has 6 numbers, excluding the 3 numbers already included in the chromosome, only three numbers are valid substitutes for the two repeated values. The numbers “6” and “4” are randomly selected and the repaired chromosome is also given in Figure 6-8.

<u>Invalid chromosome</u>					
Genotype:	010	001	011	010	011
Phenotype:	2	1	3	2	3
<u>Repaired chromosome</u>					
Phenotype:	2	1	3	6	4
Genotype:	010	001	011	110	100

Figure 6-8 An example of invalid chromosome and its repaired version

6.5 Modal Frequency Pairing

In this study, the modal data used in the updating are assumed to be within a limited number of lower modes for practicality concerns. Furthermore, only the frequency data, including both resonant and ABC frequencies, are used, while the mode shapes are not directly involved in the objective function. The mode-shape information is only used to pair the analytical and experimental resonant frequencies. For the purpose of pairing, the requirement on the quality of the measured mode shapes is much relaxed because the MAC values determining the mode-shape pairs are not very sensitive to measurement errors. Therefore, the primary objective concerning the pairing is to use as small a number of measurement points as possible while guaranteeing a reliable pairing result.

A number of methods have been devised in the past to determine the optimal measurement set for modal testing based on active vibration control theories. Lim (1992) developed a method to select optimal actuator and sensor locations based on the degree of effectiveness/versatility of pairs of actuators and sensors. Kammer (1991) presented a sensor placement method for modal identification and correlation according to the contribution of each candidate sensor location to the linear independence of the corresponding target modes. Breitfeld (1996) concluded that the optimal setting for measurement locations must preserve the orthogonality of the eigenvectors to avoid spatial aliasing.

In the present study, the method developed by Kammer (1991, 1992), called “effective independence distribution vector method” (EIDVM), is used to determine the sensor locations or measurement points so that a dependable matching result can be achieved within the frequency range of interest. A brief description of this method follows. The chief objective of this scheme is to select measurement locations which make the mode shapes of interest as linearly independent as possible, while retaining as much as possible the information about the selected modal response in the measurement data. It begins with a FE model of the system. The very first step is to eliminate all coordinates that are not measurable, such as rotational and inaccessible DOFs. The effective

independence procedure is then started by forming the Fisher Information Matrix $\mathbf{A}$, given by

$$\mathbf{A} = \Phi^T \Phi \quad (6-17)$$

where Φ is the reduced and truncated modal matrix following the DOF removal and the order reduction. The matrix $\mathbf{E}$ is then formed as

$$\mathbf{E} = \Phi \mathbf{A}^{-1} \Phi^T \quad (6-18)$$

Matrix $\mathbf{E}$ is an idempotent matrix, satisfying

$$\mathbf{E}^2 = \mathbf{E} \quad (6-19)$$

Matrix $\mathbf{E}$ has the property that its trace equals its rank. Thus the terms on the diagonal of $\mathbf{E}$ represent the fractional contribution of each measurement location to the rank of $\mathbf{E}$. $\mathbf{E}$ will be full rank only if all the modes of interest are linearly independent. The selection procedure is to examine the elements of the diagonal of $\mathbf{E}$. The smallest element is removed since it relates to the coordinate that contributes least to the independence of the chosen modes. The Matrix $\mathbf{E}$ is then updated by applying the new mode-shape matrix with one DOF removed and this process is repeated, removing coordinates iteratively. Upon termination of this process, the remaining coordinates serve as the measurement locations.

The number of measurement locations resulting from the above process is desired to be small and meanwhile the inter-independence of the mode shapes has to be assured. In this regard, during the iteration process the quality of the resulting measurement locations from each step should be assessed in terms of a predefined criterion. Among a few criteria, the commonly used is MAC (Modal Assurance Criterion, see Chapter 2 for its definition). Iteration stops when the current set of measurement locations is evaluated to violate the criterion.

6.6 Numerical Examples

In Chapters 3 and 4, a lumped-mass-stick model was used to test the updating methods. This simplified model assumes no rotational DOF at beam-column connections and, hence, represents only a special class of structures. A more realistic structural system would involve rotational DOFs, which affect the structural dynamic characteristics, and therefore should generally be considered in a FE model. In the present study, the rotational DOFs are incorporated.

The structures used in the numerical examples are reduced-scale concrete portal frames, which can be viewed as the scaled representative of a residential building. Figure 6-9 shows a typical model frame with 2 storeys and 2 bays.

Figure 6-9 An illustrative 2-storey-and-2-bay concrete frame model (Unit: mm)

In all cases, the cross-section of the beams is assumed to be $60 \times 50 \text{ mm}^2$, and that of the columns is $60 \times 60 \text{ mm}^2$. The mass density for columns is $2.50 \times 10^3 \text{ kg/m}^3$. Considering the floor mass, the equivalent density of beams is set to $2.25 \times 10^4 \text{ kg/m}^3$. The Young's modulus for concrete is $2.6 \times 10^{10} \text{ N/m}^2$. The span length of the frames is

uniformly 0.8m, the first storey height is 1.0m, and the height of the other storeys is 0.6m. Only in-plane vibration is considered and 3 DOFs are used for each FE node.

The bending stiffness (EI) of the beams and the columns about z axis are considered to be the unknown variables for updating. The Stiffness Modification Factor (SMF) is used as the variable parameters in the updating process. Each beam-column member is assumed to be known in terms of its EI . Thus, the number of SMFs subjected to updating is equal to the number of beam-column members. For example, for the 2-storey-2-bay frame shown in Figure 6-9, 10 SMFs are to be updated. Two scenarios are investigated. In both scenarios, an independent SMF is considered for each individual beam. In the first scenario, however, all columns of the same storey are assumed to have an identical SMF, while in the second scenario, different columns have different SMFs. In the following numerical investigations, the variation range of the SMFs is defined by a large interval of $[0.2, 1.0]$, which covers practically all ranges of damage, from very light to extremely severe.

Because of the involvement of unknown parameters associated with beams, the global lateral vibration modes will tend to be insufficient to result in a well-conditioned updating process due to their insensitivity to the beam bending stiffness. Additional modal information, such as that from some local modes, becomes necessary. This is particularly so for the second scenario where independent SMFs are assumed for different columns of the same storey. It should be pointed out that, although the local modes may be regarded as higher order modes in view of the whole structure, they still belong to lower modes from the perspective of individual members.

Considering the possible measurement errors, for each updating case a number of response datasets are generated by adding into the exact response data, which are numerically simulated based on the assumed parameter values, with errors sampled from the prescribed error distribution. Since only the frequency data are used in the updating, the error is assumed to be within 1% with a uniform distribution. A separate model updating is performed based on each sample set of the response data, and the

updating results from all the sample datasets are finally evaluated from a statistical point of view.

It should also be noted that, as mentioned in Chapter 4, the results of a sensitivity analysis are generally dependent on the nominal state assumed for the sensitivity analysis. However, trial analyses have indicated that the parameter error distributions predicted from the sensitivity analyses with different states usually exhibit a similar trend. Therefore, in the numerical cases only the prediction from one particular sensitivity analysis is presented.

6.6.1 Scenario A: Individual Beam SMFs and Common Column SMF Per Storey

Case 1: 2-storey-2-bay (2S2B) frame

In this case, 6 SMFs are to be identified, including 4 for the four beams and 2 for columns in the two storeys, respectively. An arbitrarily assumed state of the frame is simulated, with SMFs equal to [0.80, 0.75, 0.70, 0.60, 0.90, 0.50]. “Measurements” are to be generated from numerical simulation of this state.

To achieve a satisfactory updating accuracy for all these six parameters, a suitable set of measurements are required. For frames, the lowest global vibration frequencies are known to be sensitive to the change of column bending stiffness. So, the first two global natural frequencies are chosen. Four natural frequencies corresponding to some local vibration modes are included in order to identify the four beam SMFs. Thus, 6 natural frequencies are used in the updating procedure. As will be described later, some pertinent ABC frequencies are incorporated and the artificial boundaries used for their generation are determined by binary GA in an optimization procedure as described earlier.

Figure 6-10 plots the first 6 mode shapes of the 2S2B frame. It can be easily identified that the first two are global vibration modes and they correspond to the lowest natural frequencies, 6.55 Hz and 28.38 Hz, respectively. For these two modes, the flexural vibration magnitude of the 4 beams is seen significantly smaller than that of the 6 columns. From the mode shapes it can be reasonably deduced that the lowest global vibration frequencies are not sensitive to the bending stiffness of beams but they are very sensitive to the column bending stiffness. In contrast, the 4 higher modes (mode 3 to mode 6) are clearly associated with the flexural vibration of the 4 beams, whereas the participation of the column vibration is not significant. Therefore, these four modes are included to facilitate the updating of the beam SMFs.

To determine the appropriate measurement locations for measuring these modes and pairing them with the corresponding analytical frequencies, the EIDVM (effective independence distribution vector method) mentioned in section 6.5 is used. To do this, the mode shapes of interest (the first 6 modes for the present frame case) are calculated from the initial state of the frame and they form a modal matrix Φ as

$$\Phi_{n \times 6} = [\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4 \quad \phi_5 \quad \phi_6]_{n \times 6} \quad (6-20)$$

where n is the number of FE DOFs and ϕ_i ($i = 1, 2, \dots, 6$) indicate the individual mode shapes. Then, the DOFs that are meaningful for measurements, i.e., DOFs being important and accessible, are picked out. For the frame, since only flexural vibration is considered, the meaningful DOFs are chosen to be those that are translational and associated only with the flexural deformation of frame members. That is to say, for columns the DOFs in the horizontal direction are selected, whereas for beams the DOFs in the vertical direction are chosen. A vector $\mathbf{D}_c$ is used to represent these chosen DOFs. By considering only $\mathbf{D}_c$, the initial modal matrix Φ reduces to $\Phi^{\mathbf{D}_c}$. The modal matrix $\Phi^{\mathbf{D}_c}$ is then used to generate the idempotent matrix $\mathbf{E}$ according to Eq. (6-18).

(1) The 1st mode (6.55 Hz)

(2) The 2nd mode (28.38 Hz)

(3) The 3rd mode (46.92 Hz)

(4) The 4th mode (55.18 Hz)

(5) The 5th mode (60.83 Hz)

(6) The 6th mode (65.83 Hz)

Figure 6-10 The first 6 mode shapes of the 2S2B frame

One DOF is removed from $\mathbf{D}_c$ by an examination of the diagonal elements of the matrix $\mathbf{E}$. A new idempotent matrix $\mathbf{E}'$ is obtained by considering only the reduced $\mathbf{D}_c$ in the modal matrix $\Phi^{\mathbf{D}_c}$. The DOF removal is then performed based on $\mathbf{E}'$. This process repeats until the resulting DOF set is evaluated to be unsatisfactory.

The suitability of the resulting DOFs should be assessed after each iteration. The assessment is based on the MAC matrix. Since the measurement is simulated numerically here, the MAC matrix actually becomes an auto-MAC matrix, which is computed from the analytical mode shapes as

$$\mathbf{MAC}_F(i, j) = \frac{(\boldsymbol{\phi}_i^T \boldsymbol{\phi}_j)^2}{\boldsymbol{\phi}_i^T \boldsymbol{\phi}_i \boldsymbol{\phi}_j^T \boldsymbol{\phi}_j} \quad (6-21)$$

where $\boldsymbol{\phi}_i$ and $\boldsymbol{\phi}_j$ are the i^{th} and j^{th} reduced mode shape, respectively, and they contain only the DOFs retained by the current iteration. It is noteworthy that the diagonal terms of this auto-MAC matrix are always equal to one, while the off-diagonal elements are expected to be small with the use of proper DOFs and can be large (even close to one) if the DOFs are poorly chosen. In general, if the maximum value of the off-diagonal terms exceeds a prescribed threshold, say 0.6, the corresponding DOFs are considered to be unsatisfactory and the EIDVM will terminate its iteration process and output the final results.

Figure 6-11 shows the 6 measurement points from the optimal choice of EIDVM. Figure 6-12 plots the auto-MAC matrix ($\mathbf{MAC}_F$) in which mode shapes $\boldsymbol{\phi}_i$ include only the selected 6 DOFs. It is obvious that the MAC matrix has unity values along its diagonal and very small values for the off-diagonal terms, which indicates that the 6 modes of interest are already linearly independent from each other based on just 6 measurement DOFs optimally chosen. And hence, in the updating process the “measured” frequencies can be paired correctly with the corresponding analytical frequencies by using mode shapes including merely the optimal DOF set.

Measurement points
 Artificial boundary

Figure 6-11 Measurement locations and artificial boundaries for the 2S2B frame

Figure 6-12 Auto-MAC matrix from the initial model of the 2S2B frame
(FEA: Analyzed model)

The sensitivity analysis predicts that if the response data are chosen to be the first 6 frequencies, averagely around 10% errors are expected for the first 5 SMFs while as high as 30% is likely to happen for the 6th SMF as shown in Figure 6-13(a). This prediction suggests that more pertinent measurement information is necessary in order to increase the accuracy of the updated SMFs.

a) Using natural frequencies only

b) Using natural frequencies combined with ABC frequencies

Figure 6-13 SMF variation ranges predicted by sensitivity analysis

ABC frequencies are therefore considered as additional data. One set of 6 ABC frequencies is obtained from the use of 2 pinned constraints on the frame. The optimal locations for the application of these 2 pins are found by binary GA, whose settings are given in Table 6-3. The evolutionary curves for the binary GA are plotted in Figure 6-14. Although GA does not fully converge when the evolution process is terminated, the optimum is considered to have been reached because no better solution evolves in nearly 10 consecutive generations. The evolved pin locations are shown with triangles in Figure 6-11. Figure 6-13(b) shows the variation ranges of the 6 variables resulting from the sensitivity analysis with all 12 frequency data. Marked improvement in the accuracy of the SMFs is achieved due to the addition of the ABC frequency data; the maximum error in SMF 1, 3 and 4 is about 8% and it is only 4% for the remaining SMFs. From a practical viewpoint, such error level is acceptable.

Table 6-3 Binary GA settings

Para.	Coding	Pop Size	Selection	X-over	Mutation	Repla.	Termi.
Setting	Binary	20	Nor. Geo	One-point	Binary	“Elitism”	Max. Gen
			10% (max.)	70%	10%	10%	20

Figure 6-14 Performance curves of the binary GA

The actual updating of the model uses 12 response components including 6 natural frequencies and 6 ABC frequencies. The updating is performed by real coding GA and its parameter settings are given in Table 6-4. To reflect the influence of the measurement errors (1% uniform distribution), 6 sample response datasets are generated on the basis of the exact response data that are computed for the arbitrarily assumed state of the structure. Table 6-5 lists the six sets of the updated SMFs using respectively the 6 sample response datasets. The corresponding errors are given in Table 6-6.

As can be seen, the errors in the updated SMFs are less than 5%, which is consistent with the prediction of the sensitivity analysis. In this regard, the updating can be considered successful. Another indication of a satisfactory updating can be viewed from the response correlation between the “measured” and updated models. Figure 6-15 shows the pairing of natural frequencies and ABC frequencies between the updated (FEA) and measured (EMA) models where errors are shown as departures from a diagonal line with unit slope. Very small errors are observed for all the response components. Figure 6-16 shows the MAC matrix which demonstrates clearly a correct pairing between the analytical and “measured” mode shapes.

Table 6-4 Configurations of the real coding GA

Para.	Coding	Pop Size	Selection	X-over	Mutation	Repla.	Termi.
Setting	Real	50	Nor. Geo 5% (max.)	Heuristic 80%	Uniform 10%	“Elitism” 10%	Max. Gen 60

Table 6-5 SMFs updated from the 6 different updating scenarios by GA

Sampled Response set	Updated SMFs					
	1	2	3	4	5	6
1	0.759	0.699	0.582	0.784	0.917	0.488
2	0.814	0.686	0.587	0.766	0.912	0.480
3	0.767	0.699	0.577	0.781	0.910	0.493
4	0.818	0.695	0.604	0.754	0.898	0.490
5	0.815	0.708	0.5840	0.748	0.910	0.509
6	0.788	0.716	0.591	0.761	0.896	0.512

Table 6-6: Percentage errors of the updated results

Updating Scenario	SMF percentage errors (%)					
	1	2	3	4	5	6
1	-5.154	-0.199	-3.066	4.544	1.893	-2.440
2	1.640	-1.943	-2.154	2.107	1.365	-4.092
3	-4.160	-0.177	-3.805	4.184	1.132	-1.380
4	2.296	-0.754	0.622	0.507	-0.180	-2.087
5	1.904	1.064	-2.675	-0.308	1.115	1.780
6	-1.4480	1.637	-1.464	1.415	-0.399	2.466

a) Frequency pair

b) ABC frequency pair

Figure 6-16 MAC matrix between the measured and updated model.
(EMA: Measured model)

Case 2: 3-storey-and-3-bay (3S3B) frame

In this case study, a more difficult updating problem is examined. The target frame consists of 3 storeys and 3 bays. 12 SMF variables are to be identified, as indicated in Figure 6-17. An arbitrary state of the frame is randomly selected for updating, with SMFs equal to $[0.75, 0.89, 0.70, 0.85, 0.69, 0.72, 0.81, 0.84, 0.96, 0.86, 0.84, 0.78]$.

The response dataset is composed of the first 12 natural frequencies, including 3 lower global vibration modes and 9 local modes associated mainly with the 9 beams, as well as an adequate number of ABC frequencies. The measurement points for the natural frequencies are obtained again by EIDVM, following a similar procedure as in the previous case study, and they are shown in Figure 6-17. The auto-MAC matrix is schematically shown in Figure 6-18. The suitability of the chosen measurement points is demonstrated by the satisfactory MAC matrix, which has very small off-diagonal elements.

Fig. 6-17 Measurement points of the 3S3B frame for natural frequencies

For each artificial boundary configuration, only the first 10 ABC frequencies are considered. These modes involve primarily the first bending mode of each individual beam, and hence are presumed to have good measurement accuracy. It is found that using only one set of ABC frequency data as in Case 1 is no longer sufficient for the current scenario with 12 variables. Two more artificial boundaries are thus employed to make available 20 extra ABC frequencies for the response dataset. The configurations of the artificial boundaries found by GA including respectively 2, 3 and 4 pinned constraints, are shown in Figure 6-19. It is noted that to obtain the ABC frequencies, the impedance matrix in Eq. (6-10) will be a matrix of, 2×2 , 3×3 , and 4×4 , respectively, for the cases of 2, 3 and 4 artificial pins.

The error bandwidths for the 12 SMFs from the sensitivity analysis are plotted in Figure 6-20. For most of the variables, the maximum error is around 6%, and for the remaining SMFs, e.g., SMF 3, 10 and 11, a slightly increased error of 9% is expected. The average error for all variables is less than 4%.

Figure 6-20 SMF error bandwidths predicted from sensitivity analysis

Table 6-7 SMF percentage errors resulting from the 10 model updating scenarios

Sampled Response set	Percentage errors of the updated SMFs (%)											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1.03	-0.81	2.71	-3.33	2.34	-2.95	3.72	4.39	-4.52	-2.77	5.32	-0.36
2	-2.74	0.06	-2.32	5.79	2.09	-1.80	-2.30	5.61	-5.60	1.06	0.30	6.39
3	-6.01	1.40	1.55	0.68	1.28	-0.75	0.21	-0.42	-0.18	1.45	3.59	-1.19
4	0.05	-1.70	1.75	1.41	-1.41	0.07	-0.25	2.98	-0.18	1.14	-2.12	-0.56
5	-5.62	0.20	-1.23	0.75	3.46	-3.01	2.37	2.86	-1.04	2.72	-3.88	-2.16
6	-2.64	-0.44	1.94	-0.01	0.54	0.63	-2.32	-0.01	-0.32	-3.35	2.38	2.02
7	1.31	0.67	0.29	1.50	0.45	-1.02	-2.87	0.28	0.22	0.76	-1.63	1.12
8	2.91	0.04	-1.78	2.78	-2.78	1.26	1.49	-4.30	4.76	-0.95	-3.38	-0.00
9	-2.05	-2.32	4.33	1.31	-0.12	2.95	-2.71	-5.98	4.40	-4.83	-1.44	1.90
10	-2.07	0.91	2.56	0.49	1.44	-0.34	-1.88	-3.72	2.39	0.55	-0.88	-2.55

Similar to the previous example in considering the measurement errors, 10 sample sets of response data are generated on the basis of the simulated exact response data. Subsequently, 10 sets of updated SMFs are obtained. Table 6-7 gives the percentage errors of the updated SMFs from the 10 sample response datasets. Once again, a successful updating is achieved as indicated by the small errors associated with the updated SMFs. All the updating errors are consistent with the theoretically predicted error bandwidths from the sensitivity analysis.

6.6.2 Scenario B: Individual Beam SMFs and Individual Column SMFs

In this scenario, one independent SMF is used for each individual column during updating as opposed to Scenario A where all columns in the same storey were assumed to share the same SMF. This poses a much more complicated updating problem because of the low sensitivity of the response data with respect to the individual column stiffness.

An elaboration of the reduced sensitivity when individual column SMFs are considered is given as follows. A modal frequency, f_i , may be related to the unknown variables p_i , as

$$f_i = F_i(p_1, p_2, \dots, p_n) \quad (6-22)$$

where F_i represents a certain function relation between f_i and $\{p_i\}$ and the subscript n denotes the number of variables. For the current problem, $\{p_i\}$ consists of the beam SMFs and the column SMFs. Let us assume that m number of variables, $p_i, p_{i+1}, \dots, p_{i+m-1}$, change uniformly and thus are considered as one variable $\bar{p}_i$. In calculating the sensitivity of f_i with respect to $\bar{p}_i$, other variables become constants and thus f_i can be rewritten as

$$f_i = F_i(\bar{p}_i) = F_i(p_i, p_{i+1}, \dots, p_{i+m-1}) \quad (6-23)$$

A first-order Taylor series expansion of f_i on $\bar{p}_i$ gives

$$F_i(\bar{p}_i, \bar{p}_i, \dots, \bar{p}_i) = F_i(0, 0, \dots, 0) + \left(\frac{\partial F_i}{\partial p_i} + \frac{\partial F_i}{\partial p_{i+1}} + \dots + \frac{\partial F_i}{\partial p_{i+m-1}} \right) \times \bar{p}_i \quad (6-24)$$

It follows that

$$\frac{\partial f_i}{\partial \bar{p}_i} = \lim_{\bar{p}_i \rightarrow 0} \frac{F_i(\bar{p}_i, \bar{p}_i, \dots, \bar{p}_i) - F_i(0, 0, \dots, 0)}{\bar{p}_i} = \frac{\partial F_i}{\partial p_i} + \frac{\partial F_i}{\partial p_{i+1}} + \dots + \frac{\partial F_i}{\partial p_{i+m-1}} = \sum_{k=i}^{i+m-1} \frac{\partial F_i}{\partial p_k} \quad (6-25)$$

Eq. (6-25) tells that the sensitivity of f_i with respect to a collective variable $\bar{p}_i$ is equal to the sum of individual sensitivities of f_i with respect to each constituent variable p_k .

Based on the above analysis, it can be understood that a separate consideration of individual column SMFs might reduce considerably the sensitivity of a frequency.

To compensate the reduced sensitivity, it is necessary to determine and subsequently supply additional response data that exhibit desirable sensitivity to individual column SMFs. Adequately configured ABC frequencies prove to be a good candidate for this purpose.

Case 1: 1-Storey-2-Bay Frame

In this example, the 3 columns in the same storey have different SMFs. Hence, the parameters for updating consist of 5 SMFs, as indicated in Figure 6-21. An arbitrary state of the frame is randomly chosen, with SMFs equal to [0.85, 0.70, 0.90, 0.87, 0.65].

Figure 6-21 One-story-and-2-bay frame

To demonstrate the change of sensitivity as expressed in Eq. (6-25), Figure 6-22 shows a comparison of the two sensitivity matrices, $\mathbf{S}_A$ and $\mathbf{S}_B$, with the same set of response data composed of the first 5 natural frequencies. $\mathbf{S}_A$, shown in Figure 6-22(a), involves 3 SMFs in which the three columns share the same $\overline{\text{SMF1}}$, while $\mathbf{S}_B$, shown in Figure 6-22(b), includes 5 SMFs with one independent SMF for each column. It can be seen that the sensitivity of the combined $\overline{\text{SMF1}}$ is approximately equal to the sum of the sensitivities of the separate SMF1~SMF3 for each natural frequency. This is consistent with Eq. (6-25), and it shows that the sensitivity in the case of the combined $\overline{\text{SMF1}}$ is distributed among three sensitivity values when a separate SMF is considered for each column. It is noted that the sensitivities with regard to the two beams remain unchanged in both situations.

		SMFs		
		SMF1	SMF4	SMF5
Natural Frequencies	f_1	0.3699	0.0675	0.0621
	f_2	0.1920	0.1060	0.1964
	f_3	0.1087	0.2180	0.1504
	f_4	0.2172	0.0890	0.1864
	f_5	0.2691	0.1340	0.0750

a) Sensitivity matrix S_A with 3 SMFS

		SMFs				
		SMF1	SMF2	SMF3	SMF4	SMF5
Natural Frequencies	f_1	0.1121	0.1598	0.1021	0.0675	0.0621
	f_2	0.0398	0.0844	0.0643	0.1060	0.1964
	f_3	0.0654	0.0042	0.0372	0.2180	0.1504
	f_4	0.0392	0.0988	0.0826	0.0890	0.1864
	f_5	0.1322	0.0176	0.1153	0.1340	0.0750

b) Sensitivity matrix S_B with 5 SMFS

Figure 6-22 Two sensitivity matrices using 5 natural frequencies and 3 and 5 SMFs, respectively

Three different response settings, as shown in Table 6-8, are compared with regard to their effectiveness in updating the five parameters. The effectiveness is evaluated by examining the error bandwidth of SMFs based on the sensitivity analysis.

Table 6-8 Three response settings under comparison

Response data set	Response setting
1	The first 5 natural frequencies
2	The first 5 natural frequencies and 5 ABC frequencies from one artificial boundary configuration optimized by GA
3	The first 15 natural frequencies

Figure 6-23 compares the SMF error bandwidths from the use of the 3 different response data settings. As much as 80% error in the SMFs is possible if only using the first 5 natural frequencies. Due to the addition of 5 ABC frequencies from a GA optimized artificial boundary configuration, the SMF errors are reduced dramatically, to an average error around 5%. For the response setting 3, which consists of up to 15th natural mode frequencies, a similar level of SMF error as from the response setting 2 is achieved. This result indicates that lower-order ABC frequencies (the first 5 in this case) can be effective alternatives to higher-order natural frequencies for FE model updating.

Figure 6-23 SMF error bandwidths from 3 different response settings

The response setting 2 is considered in the actual updating. Figure 6-24 shows the performance curves of the binary GA for locating the optimal configuration of the artificial boundary used in obtaining the 5 ABC frequencies. A very fast convergence of the binary GA is observed and the optimal solution is found within 8 generations. Figure 6-25 shows the optimal artificial boundary configuration involving two pinned constraints.

Figure 6-24 Evolutionary curves of the binary GA

Figure 6-25 Artificial pins selected by GA

Table 6-9 lists the updated SMF errors corresponding to 10 sampled response datasets for the assumed state of the structure.

Table 6-9 SMF percentage errors for the 1S2B frame

Sampled Response set	Percentage errors of the updated SMFs (%)				
	SMF1	SMF 2	SMF 3	SMF 4	SMF 5
1	-2.653	-2.482	2.704	-0.766	-0.725
2	0.904	3.287	-3.190	-2.528	2.137
3	2.944	0.559	-3.963	-0.025	2.852
4	-4.714	1.108	0.092	3.407	-2.109
5	6.251	3.544	0.121	-3.396	-1.379
6	2.618	3.731	-1.539	-0.804	0.189
7	1.358	-1.016	-2.424	-0.424	1.437
8	2.043	1.860	1.098	-0.544	-1.250
9	4.733	3.196	0.279	-3.316	0.457
10	0.786	2.419	2.120	-0.080	-0.224

Once again, the updating results are satisfactory as evidenced by the small updating errors, which are also in good agreement with the anticipated errors from the sensitivity analysis.

Case 2: 3-Storey-and-1-Bay Frame

In this frame, 9 SMFs, as indicated in Figure 6-26, are to be identified. An arbitrary state of the structure is simulated, with SMFs equal to [0.90 0.60 0.55 0.87 0.92 0.48 0.76 0.95 0.77].

Figure 6-26 3-storey-and-1-bay frame

Table 6-10 shows the sensitivity values of the first 9 natural frequencies with respect to the 9 SMF variables. It can be observed that all the 9 natural frequencies are not sensitive to SMF 4, 5, 7 and 8, whereas at least one frequency exhibits large sensitivity to the remaining SMFs (see the shaded areas). Therefore, ABC frequencies are considered to supplement the natural frequencies.

Table 6-10 Sensitivities of the frequencies w.r.t the 9 SMFs

SMFs f_i	SMF1	SMF2	SMF3	SMF4	SMF5	SMF6	SMF7	SMF8	SMF9
f_1	0.122	0.088	0.150	0.012	0.011	0.069	0.031	0.009	0.022
f_2	0.087	0.075	0.000	0.053	0.035	0.092	0.050	0.027	0.082
f_3	0.044	0.044	0.033	0.095	0.063	0.001	0.067	0.072	0.062
f_4	0.009	0.007	0.042	0.026	0.030	0.295	0.083	0.033	0.030
f_5	0.018	0.010	0.245	0.021	0.033	0.089	0.029	0.009	0.029
f_6	0.005	0.002	0.072	0.007	0.011	0.009	0.044	0.059	0.240
f_7	0.021	0.151	0.135	0.023	0.058	0.043	0.017	0.013	0.028
f_8	0.000	0.000	0.024	0.011	0.011	0.261	0.034	0.012	0.025
f_9	0.056	0.217	0.052	0.017	0.017	0.034	0.010	0.015	0.065

Via GA, two artificial boundaries are chosen as shown in Figure 6-27. The first 9 ABC frequencies from each configuration are considered. Thus, the response data set is expanded to contain 27 components including 9 natural frequencies and 18 ABC frequencies from the two artificial boundary configurations.

Figure 6-27 Two artificial boundaries of the 3S1B frame selected by GA

Figure 6-28 SMF bandwidths from sensitivity analysis

Figure 6-28 shows the 9 SMF error bandwidths predicted by the sensitivity analysis with the above set of response data. The error bands for the 9 SMFs are in the range of $\pm 3\%$ (SMF 6) to $\pm 8\%$ (SMF 1, 4, 5, 7 and 8).

Five sampled response datasets are used in the updating. Table 6-11 summarizes the corresponding updating errors. The average error for the 9 SMFs is of the order of 5%, which is consistent with the anticipation from the sensitivity analysis.

Table 6-11 SMF percentage errors from the 5 updating scenarios

Updating Scenario	SMF percentage errors from GA updating (%)								
	1	2	3	4	5	6	7	8	9
1	-2.394	-0.898	-0.457	-4.143	8.521	1.725	-0.805	-4.262	0.600
2	-4.021	0.811	-2.260	3.334	-1.372	0.999	-1.207	2.507	-0.673
3	-7.896	1.473	-1.993	7.029	1.722	1.560	-6.606	-4.840	-0.248
4	0.205	-0.204	-0.877	-1.320	7.692	0.449	0.636	-3.572	-0.449
5	4.365	-3.538	0.341	-2.839	4.084	0.810	-0.312	-6.362	2.564

6.7 Summary and Conclusions

This chapter presents a GA based FE model updating method incorporating response frequency data under artificial boundary conditions. The basic theory for the acquisition of such frequency data is provided. These frequencies correspond exactly to the natural frequencies of the structure with extra artificial pinned constraints (APC), and in actual modal testing they can be extracted by identifying the peaks of an element in the measured impedance matrix. Because of the flexibility of applying APCs on a structural system, numerous ABC frequencies can be made available to supplement the response dataset for the FE model updating.

Numerical investigations demonstrate that the effectiveness of ABC frequencies for FE model updating depends on the configurations of the artificial boundaries. A binary GA based scheme is thus proposed to optimize the locations of APCs such that the resulting ABC frequencies are most effective for the FE model updating.

A series of numerical examples are provided, in which the binary GA is employed to optimize the configurations for acquiring the ABC frequencies, while the real-coding GA is employed to perform the actual FE model updating. The results show that satisfactory updating outcome can be achieved by using a response dataset composed of lower-order natural frequencies and an adequate number of the ABC frequencies from the optimized artificial boundary configuration(s). It is also found that using the ABC frequencies from configurations involving just a few APCs is usually sufficient to achieve a satisfactory updating result for a frame model.

It can be generally concluded that the ABC frequencies can be used as an effective substitute for the mode-shape data. In conjunction with the application of GA, it is possible to update more complex FE models with satisfactory results.

CHAPTER 7

EXPERIMENTAL STUDY

7.1 Introduction

In the preceding chapters, it has been demonstrated that GA is an effective tool for FE model updating. The examples presented so far, however, are mostly from numerical simulations (or numerical experiments), except for one real test scenario where a largely simplified FE model was considered. In this chapter, the laboratory experiments on two physical model structures will be presented and GA is applied to identify the structural parameters for the FE model based on actually measured vibration data.

It is generally understood that, even without any defects and damages, an FE model without validation may not represent the actual dynamics of the structure due to inaccurate modeling of the structural joints and components. Thus, an FE model for the intact state of the structure also requires updating. In fact, a good initial FE model is necessary in the application of many damage identification algorithms to quantify subsequent structural damages. Although in some other methods a sound initial FE model may not be necessary and the damage identification procedure may be based on the difference between the damaged and undamaged response data, it is not always possible to maintain the required consistency in the response data for the intact structure and for future damaged states of the structure. A validated FE model for the intact state of a structure is always desirable.

7.2 General Considerations about Modeling of Joints

A successful FE model updating depends very much on the suitability of the selected structural parameters to be corrected. If some critical variable parameters are not included in the updating process, the updated model can lose its physical representation of the actual structure and the subsequent use of this model will become futile. Appropriate engineering judgments have to be exercised in this regard.

Structural joints are commonly regarded as a difficult subject for model updating. The various types of joints existing in actual structures may have different effects on the dynamic behavior of a structure. Some experimental results (Beards 1986) on assembled structures have shown that much of the flexibility and up to 90% of the damping are attributable to joints. Therefore, appropriate modeling of such joints can be a key to the success of an updating procedure. A brief review of the classical identification methods for joints can be found in Ren (1992).

The present study is not intended to delve into the general subject of identification of joints. The emphasis in this respect is placed on the feasibility and effectiveness of using GA for updating a real structural model including parameters associated with joints. For this purpose, in the FE model the joints are modeled using a few simple and yet physically meaningful parameters so that the joint effect on the structural dynamic response of interest is properly accounted for.

For welded joints as in one of the test frames which will be described later, a straightforward modeling method is to use a series of translational and rotational springs, leaving their stiffness coefficients to be identified. However, to update these stiffness coefficients can be difficult because of the generally low sensitivity of the structural modal data to such coefficients (Mottershead 1996). The application of offset

parameters (Friswell 1998) has attracted wide attention recently in the identification of this category of joints owing to the fact that these offset parameters are more sensitively related to the structural modal properties. In the present study, the offset parameters are employed in modeling the frame joints.

The so-called offset parameters are the variables that describe the dimension of a rigid area. When using offset parameters to represent the behavior of welded joints in the FE model, the entire joint is considered as a rigid zone such that the nodal displacements of all FE nodes on the border of this rigid zone are inter-dependent. This is because once a particular master node is defined, the displacement quantities of all other nodes within the rigid zone can be expressed by multiplying the nodal displacements of the master node by some link matrices. The mass and rotary inertia of the rigid area are lumped at the selected master node.

Figure 7-1 shows the modeling of a typical welded joint using two offset parameters, w and h . It is noted that any one of the three nodes, a, b and c, can be defined as the master node. Suppose node a is selected to be the master node, nodes b and c thus become the slave nodes. Figure 7-2 shows the two link matrices, $\mathbf{T}_1$ and $\mathbf{T}_2$, calculated based on the two offset parameters. The physical meaning of the offset parameters can be interpreted as such that the shaded (rigid) joint region in Figure 7-2 expands or contracts as the dimensions w and h are increased or reduced by the updating. The mass matrix, however, does not need to be dependent on w and h .

- FE node
- Euler-Bernoulli finite element
- └ Rigid zone   └ Rigid zone

NL-B: “Net” length of the beam

NL-1C: “Net length of the 1st story columns

NL-2C: “Net length of the 2nd story columns

Figure 7-1 Finite element model of the two-storey, one-bay test frame

$$\begin{Bmatrix} u_b \\ v_b \\ \theta_b \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & w \\ 0 & 0 & 1 \end{bmatrix} \times \begin{Bmatrix} u_a \\ v_a \\ \theta_a \end{Bmatrix} = \mathbf{T}_1 \times \mathbf{U}_a$$

$$\begin{Bmatrix} u_c \\ v_c \\ \theta_c \end{Bmatrix} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & w \\ 0 & 0 & 1 \end{bmatrix} \times \begin{Bmatrix} u_a \\ v_a \\ \theta_a \end{Bmatrix} = \mathbf{T}_2 \times \mathbf{U}_a$$

Figure 7-2 Link matrices between the master node and slave nodes

7.3 Test Structures and Their FE models

The test structures are portal frames with different configurations. The frames are fixed at the base by welding the bottom ends of the columns onto a thick steel plate. The base plate is then fastened to the ground floor by bolts as shown in Figure 7-3. All material properties and geometric parameters of the beams and columns, including boundary conditions, are presumably known.

Figure 7-3 A two-storey-two-bay steel frame model

7.3.1 Test Frame 1 (Welded Joints)

Figure 7-4 shows the two-storey, one-bay steel frame. The beams and columns are connected via welded joints. All members of the frame have the same cross-sectional dimension of $20 \times 6 \text{ mm}^2$. The Young's modulus is $2.0 \times 10^9 \text{ N/m}^2$ and the mass density is $7.8 \times 10^3 \text{ kg/m}^3$.

In the FE model, the frame members are modeled using the Euler-Bernoulli beam element (line element). Each FE node has 3 degrees of freedom (DOFs) including two translational DOFs, u_x and u_y , and a rotational DOF, θ_z . The FE mesh of the frame can be seen from Figure 7-1(a). The storey heights and the span used in the FE model are measured from center to center, as shown by the dash lines in Figure 7-4.

Figure 7-4 Test steel frame 1 (2-storey-and-one-bay)

The offset parameters introduced in the previous section are used in the modeling of the joints. For the current frame, considering the similar geometric dimension and joint details, all the four welded joints are assumed to have the same set of offset parameters in the updating. Thus, two offset parameters, w and h , are to be identified through updating.

Another important variable to be identified is the equivalent “net” length of the frame members (beams and columns). Because of the existence of irregular welding details at

the base supports and at the joints, the actual “net” length of the individual members to be used in the FE model are difficult to determine. As a matter of fact, inaccurate member length in the FE model can result in appreciable errors in the analytical modal properties. Since in the present study the joint details are treated as a rigid zone, the “net” length of the frame members should not include the rigid region. Figure 7-1(a) shows the “net” length of the columns and beams (NL_1C, NL_2C and NL_B). For simplicity and considering the symmetric setting of the test frame, in the updating procedure the same length correction factor, LCF, is assumed for all the beams and columns in the frame.

Apart from the above-mentioned joint offset parameters and the member length parameter, two general parameters, namely, a stiffness modification factor (GSMF) and a density modification factor (DMF), are also included for updating. The general SMF is applied on the individual reference bending stiffness (flexural rigidity) of all frame members to account for the geometric errors upon manufacturing. The mass density modification factor (DMF) accounts for errors in the specified density.

Thus, the final updating of the FE model for test frame 1 includes five parameters to identify, namely, w , h , GSMF, DMF and LCF.

7.3.2 Test Frame 2 (Angle-Plate Enhanced Screwed Joints)

Plain welded joints are weak in nature. Some engineering structures, however, require that joints be particularly solid to assure safety. A strong joint can be achieved through enhancement using angles, plates, gussets and so on. Fasteners used to make such connections can be bolts, welds, nails, and screws.

This test structure is a steel frame with 2 storeys and 2 bays and the frame is assembled together by screwing the regular beams and columns onto angle parts. Figure 7-5 shows the detailed configuration of the frame. Such a test model, to some extent, represents those steel structures with significant enhancement at the joints, posing an interesting model-updating problem with unknown joint mass as well as joint stiffness.

As in the modeling of the first test frame, the members of the present frame are also modeled using Euler-Bernoulli line elements. The frame members include both regular members and combined sections (see Figure 7-6(c)). Figure 7-7 shows the FE mesh of a single bay of the frame and other bays are meshed in a similar manner.

a) Geometry of the 2-story, 2-bay test frame: Unit (mm)

b) Drawing of an angle bar component: Unit (mm)

c) Joint configurations: Unit (mm)

Figure 7-5 Configuration of the 2-story-and-2-bay frame

Figure 7-6 Modeling of joints using offset parameters

NL: "Net Length"   RM: Regular Member   CS: Combined Section

Figure 7-7 FE mesh of the frame and an illustration of the member "net" length

In representing the joint connections, the offset parameters are employed. Figure 7-6 shows the offset parameters applied for the three different types of joints in the current frame (JT1, JT2 and JT3). The blank areas represent rigid-body parts whose effects are reflected by the offset parameters. The mass of the rigid area is lumped at the master node. To simplify the updating problem, during the updating procedure one global variable, called "global offset modification factor (GOMF)", is used for all joints to correct the dimension of the offset parameters. This means all rigid areas expand or contract by a same factor.

In the FE modeling, the overlapping parts of regular members and angle bar components (combined sections) are treated as beam elements having a combined dimension of the cross-section. Considering that the combined section using bolts is somewhat weaker than a single cross-section of the combined dimension, a SMF (≤ 1) denoted by GSMF_CS is used in the updating procedure to correct the bending stiffness in these regions.

Based on a similar consideration as in the parameter selection of the first test frame, a same variable called “length correction factor for regular members” (LCF_RM) is used to correct the “net” length of all the 10 regular frame members. Similarly, a variable called “length correction factor for combined sections (LCF_CS)” is employed to correct the “net” length of all the combined sections. Figure 7-7 shows the definition of “net” length of the regular members (NL_RM) and combined sections (NL_CS).

Besides, a global stiffness modification factor for regular members (GSMF_RM), and a density modification factor (DMF), are also considered in the updating.

Thus, for the present test frame 6 parameters are to be identified in the updating process, namely, 1) the global offset modification factor (GOMF), 2) the global stiffness modification factor for combined sections (GSMF_CS), 3) the length correction factor for regular members (LCF_RM), 4) the length correction factor for combined sections (LCF_CS), 5) the global stiffness modification factor for regular members (GSMF_RM), and 6) the density modification factor (DMF).

7.4 Testing and Data Acquisition

In this experiment, forced-vibration testing (FVT) is adopted. The FVT technique refers to the situation where the excitation or force function acting on the test structure is controlled. There exist a few force functions for the excitation. In this test, the hammer impact is used to initiate vibration on the test structures. Impact testing bears a number of benefits such as easy implementation, minimum requirement on hardware, and shorter measurement time, etc.

A force transducer is embedded behind the tip of the hammer. Since the force is an

impulse, the amount of the energy imparted to the structure is a function of the mass and the velocity of the impact. It is difficult to control the velocity, so the force level is usually controlled by varying the mass. The frequency content of the energy applied to the structure is a function of the stiffness of the hammer tip and, to a lesser extent, the mass of the hammer. The harder the tip, the shorter the pulse duration, and thus the higher the frequency content. Therefore, a harder tip is necessary to excite structural higher modes. Additional masses can also be attached to the back of the hammer head to increase the excitation force. More detailed considerations on impact hammer can be seen in Corelli and Brown (1984). In the present study, a Dytran 5801A5 hammer with a sensitivity of 5mV/lbf. is used throughout the test. It comes with three tips of different hardness such that the frequencies within the bandwidth of interest for the frames can be excited.

Accelerometers are used to measure the structural acceleration response. The measured acceleration signals and the impact force signal are then processed to obtain the required modal data. There are many types of accelerometers in terms of such properties as working mechanism, sensitivity and mass. Accelerometers must be chosen carefully to suit an intended application. For the current application, PiezoBEAM type accelerometers of the model Kistler 8636C50 are used. These accelerometers are designed to have a magnetic mounting base, which facilitates easy installation on metal structures like the steel frame used in the present test. The weight of accelerometers used in the experiment is 5 grams and has a sensitivity of 100mV/g ($g = 9.8\text{m/s}^2$).

Figure 7-8 shows the forced vibration test (FVT) system used in the present study. For a general dynamic testing, multiple accelerometers can be used to acquire multiple response measurements for each excitation. In the present study, considering that the extra masses of the accelerometers can cause systematic errors on the measured data

for a relatively light test structure, only one accelerometer is used for each excitation test and this accelerometer is kept on a fixed position throughout the test while impacts rove among different locations. From the roving hammer test, partial FRF components in one particular row of the FRF matrix are obtained and then processed to obtain the required modal data (natural frequencies and mode shapes).

Figure 7-8 FVT system using hammer excitation

A signal conditioning unit and an anti-aliasing filter are used to ensure reliable and high-quality signals. The recorded excitation force and acceleration responses from the data-acquisition system are then used to determine the frequency response function (FRF), which is defined as the ratio of the Fourier transforms of the response and the excitation. For different types of responses, the FRFs are respectively called the Inertance (acceleration divided by force), the Mobility (velocity divided by force), and the Receptance (displacement divided by force). Figure 7-9 shows the test equipment

system. Standard modal analysis software is used to extract the natural frequencies and mode shapes from the measured data. In order to reduce the effects of nonlinearities and to improve the statistical reliability of the measured data, several time records are averaged before entering the normal modal analysis procedure.

Figure 7-9 Setup of the experiment

Generally speaking, impact testing has two potential signal-processing problems associated with it. The first one is related to noise that can be present in either the force or response signal, especially when the record time is long. The second is about leakage that could happen with response signals of too short record time. Both problems can be compensated by the windowing technique. A force window is usually applied to the raw force signal to eliminate the noise after the pulse, while an

exponential window is used to force the raw response data to decay out within the record time to avoid leakage. In addition, overloading can be a difficult problem to overcome in impact testing. When it occurs, the force or response signal is clipped, causing distortions in the measurements. The pulse duration is very short, typically less than one millisecond, such that the distortions may not be detected. During the present experiment, overloads were carefully monitored for each and every impact throughout the test.

During the test, the frames were excited to vibrate within their plane. The response of the beams are measured in the vertical direction, while the response of the columns are measured in the horizontal direction. Figure 7-10 shows the location of the fixed accelerometer and the impact locations of the roving hammer for test frame 1.

Figure 7-10 Instrumentation of the test frame

7.5 Modal Analysis and FE Model Updating of Two Test Frames

In this section, GA will be used to perform the actual model updating to correct those uncertainties in the FE model.

In the actual GA model updating of the two tested frames, only natural frequencies are used in the response dataset. This is because the number of uncertain parameters in the test models is relatively small (up to six actually). Hence, using lower-order natural frequencies alone is already sufficient. The measured mode shapes are considered only for the purpose of pairing the frequencies in the GA search. Although the mode-shape data could also be used, it is not necessary. The modal analysis from the recorded raw signals during the FVT tests is presented first and the updating results are then given.

7.5.1 Test Frame 1

7.5.1.1 Test Results

The recorded time histories of the impacting force input and the acceleration output of one typical measurement for test frame 1 are shown in Figure 7-11 and Figure 7-12. The sampling rate was set to 1000 Hz, which is sufficient to cover up to 12 modes as predicted from the FE analysis. The block size for FFT was chosen to be 8192 data points such that the frequency resolution is 0.1221 Hz. The diagrams of the auto-power spectral density of the input $P_{xx}(\omega)$, cross-power spectral density of input and output $P_{xy}(\omega)$, and the FRF estimate ($H_1(\omega) = \frac{P_{xy}(\omega)}{P_{xx}(\omega)}$) are obtained from a modal analysis procedure. The amplitude spectra are illustrated in Figures 7-13 to 7-15,

respectively. The extracted natural frequencies are listed in the second column of Table 7-1.

Figure 7-11 Impacting force time history

Figure 7-12 Recorded acceleration time history

Figure 7-13 Auto-power spectral density of the impacting force

Figure 7-14 Magnitude of cross-power spectral density

Figure 7-15 Frequency response function (amplitude)

Table 7-1 Measured frequencies and frequencies predicted from two analytical models

Mode no.	f_{EMA} (Hz)	f_{FEA}^1 (Hz)	Δf^1 (%)	f_{FEA}^2 (Hz)	Δf^2 (%)
(1)	(2)	(3)	(4)	(5)	(6)
1	11.17	10.65	-4.65	11.11	-0.57
2	48.48	46.32	-4.45	48.58	0.21
3	79.64	77.39	-2.83	79.83	0.24
4	100.82	94.40	-6.36	99.96	-0.85
5	124.45	121.13	-2.67	126.02	1.26
6	130.99	124.57	-4.90	131.87	0.67
7	226.2	219.00	-3.31	224.76	-0.77
8	266.6	253.24	-5.01	264.57	-0.76
9	288.1	274.56	-4.70	286.09	-0.70
10	340.2	326.95	-3.90	346.58	1.88
11	370.8	344.86	-7.00	365.16	-1.52
12	405.9	381.45	-6.03	408.12	0.54

f_{EMA} : measured frequencies f_{FEA}^1 : predicted frequencies without updating

f_{FEA}^2 : predicted frequencies from the updated model

For a comparison, the analytical frequencies corresponding to the nominal properties of the frame without correction in the FE model are also calculated and listed in column 3 of Table 7-1. Figure 7-16(a) shows the pairing of the two sets of frequencies and Figure 7-16(b) plots the percentage errors of the analytical frequencies. It can be observed that the analytical frequencies exhibit a poor correlation with the measured frequencies and the average error is about 4.5%, which indicates that the FE model apparently deviates from the actual model and needs validation.

Figure 7-16 Correlation between frequencies measured and those calculated

7.5.1.2 Updating of the FE Model

As mentioned in Section 7.3.1, five parameters are to be corrected in the FE model for this test frame using the first six measured frequencies as listed in the second column of Table 7-1.

GA is applied to perform the updating. Table 7-2 gives the parameter settings of GA. Figure 7-17 depicts the evolution history of GA. It can be seen that for the present problem the convergence is very fast and is achieved in less than 15 generations.

Table 7-2 GA settings

Para.	Coding	Pop Size	Selection	X-over	Mutation	Repla.	Termi.
Setting	real	50	Nor. Geo	Heuristic	Uniform	“Elitism”	Max. Gen
			6% (max.)	70%	10%	10%	50

Figure 7-17 GA performance curves

Table 7-3 presents the updated results from GA. The offset parameters, w and h , describing the rigid zones are identified to be 3.0mm and 7.91mm, respectively. The updated member “net” length exhibits a reduction of about 2% relative to the nominal length. The GSMF is very close to 1.0, indicating that the flexural rigidity of the regular members does not deviate notably from that calculated according to the nominal cross-section properties. The density is reduced by around 4.5%, leading to a change of density from 7800kg/m^3 to 7465kg/m^3 . This result is consistent with the actually calculated density from the sample steel strips.

Table 7-3 GA updated results

Parameters	w (mm)	h (mm)	LCF (mm)	GSMF	DMF
Updated results	3.000	7.910	-8.266	0.995	0.957

The natural frequencies of the updated model are listed in column 5 of Table 7-1, and column 6 lists the corresponding percentage errors. Figure 7-18(a) shows a comparison of prediction errors of the natural frequencies from the initial and the updated FE models. It can be seen that the prediction errors are reduced remarkably based on the updated model as opposed to the initial model without correction. Moreover, a good match between the measured frequencies and the predicted counterpart from the updated model is achieved, with the error being within 1% for all modes except the 5th mode for which the error is 1.26%.

The quality of the updated model is further verified by examining the comparison between the measured and analytical frequencies for some higher modes which are not included in the updating procedure. Table 7-1 also lists the measured and analytical frequencies for such modes as 7 to 12. As can be seen, the prediction errors for these modes are also very small, which confirms the adequacy of the updated FE model. Figure 7-18(b) compares the frequency errors of modes 7 to 12 predicted from the initial model and the updated one. It is clear that a markedly improved correlation with the measured modal data is achieved by the updated model.

a) Comparison of prediction errors of the first 6 modes

b) Comparison of prediction errors of mode 7 to 12

Figure 7-18 Comparison of prediction errors of the first 12 modes from the initial and the updated FE models

7.5.2 Test Frame 2

7.5.2.1 Test results

The dynamic testing of the present frame was performed following a similar procedure as for frame 1. Preliminary analysis indicates that the maximum frequency of interest is around 200 Hz and hence the sampling rate was set to 500 Hz. The block size for FFT was still chosen to be 8192 data points, giving a frequency resolution of 0.06105

Hz. The measured natural frequencies are given in column 2 of Table 7-4.

Table 7-4 Measured frequencies and predicted frequencies
from two analytical FE models

Mode no.	f_{EMA} (Hz)	f_{FEA}^1 (Hz)	Δf^1 (%)	f_{FEA}^2 (Hz)	Δf^2 (%)
(1)	(2)	(3)	(4)	(5)	(6)
1	5.38	6.04	12.26	5.44	1.07
2	16.12	18.20	13.75	16.39	1.65
3	39.3	42.59	8.38	39.07	-0.58
4	51.02	55.38	8.54	50.38	-1.26
5	57.40	63.28	10.24	57.43	0.06
6	68.85	75.76	10.04	68.58	-0.39
7	72.50	80.44	10.96	72.53	0.04
8	97.30	109.19	12.22	97.65	0.36
9	99.20	111.90	12.80	99.83	0.63
10	153.0	165.32	8.05	150.93	-1.36
11	177.0	190.16	7.44	172.58	-2.49

f_{EMA} : measured frequencies f_{FEA}^1 : predicted frequencies without updating
 f_{FEA}^2 : predicted frequencies from the updated model

Column 3 of Table 7-4 lists the natural frequencies predicted from the initial FE model using the nominal properties of the test frame, while the corresponding errors as compared with the measured frequencies are given in column 4. As can be seen, the average error is more than 10%, indicating that the initial FE model is very poor and needs correction.

7.5.2.2 Updating of the FE Model

As described in section 7.3.2, 6 parameters are to be identified for the FE model of the present model through updating, namely, GOMF, GSMF_CS, LCF_RM, LCF_CS,

GSMF_RM, and DMF.

In the updating procedure, the first 6 natural frequencies are used as the response data. GA is applied to perform the updating. Figure 7-19 shows the performance curves of GA. The convergence is achieved in less than 15 generations.

Figure 7-19 GA performance curves

The 6 updated parameters are presented in Table 7-5. The flexural rigidity of the regular members exhibits a reduction of about 12.5% from that calculated using the nominal dimension of the member cross-section. On the other hand, the flexural rigidity of the combined sections in the joint regions is identified to be 35.3% of the value calculated as if they were single sections of the combined size. This significant reduction appears to be reasonable as the actual bonding of two pieces is provided by screws. It is noteworthy, however, that the updated bending stiffness of the combined sections is still about 6 times as large as that of the regular members.

Table 7-5 Updated results from GA

Parameters	GSMF_RM	GSMF_CS	LCF_RM	LCF_CS	GOMF	DMF
Updated results	0.875	0.353	1.038	0.988	1.134	0.986

The updated “net” lengths of the regular members and the combined sections exhibit an increase of 3.8% and a reduction of 1.12%, respectively. The global offset modification factor, GOMF, is identified to be equal to 1.134, indicating that the updated offset dimension increases by 13.4% as compared to the nominal reference values. The density is not changed notably, with a reduction of 1.4%.

Table 7-4 (column 2 and 5) compares the measured 6 frequencies and the predicted results using the updated FE model. The predicted frequency errors are plotted in Figure 7-20(a). It can be observed that the 6 frequencies predicted from the updated FE model match very well with the measured frequencies. The comparison for a few higher modes (modes 7-11) that are not involved in the updating procedure also shows a satisfactory result (see Figure 7-20(b)).

a) Frequency errors of modes 1 to 6

b) Frequency errors of modes 7 to 11

Figure 7-20 Percentage errors of natural frequencies between measured and updated

7.6 Conclusions

A real coding GA has been applied in this chapter to identify the FE model of two laboratory tested portal frames using measured frequency data. Forced vibration testing is used to determine the modal properties of the test frames. It is recognized that the parameters associated with the joint region are crucial parameters influencing the accuracy of the FE model. In the present study, the so-called offset parameters are adopted to model the effect of the joints. Besides, the net length and the flexural rigidity are also corrected through the updating procedure.

Two test frames, one with welded joints and another with angle-plate enhanced screwed joints, are tested, and their FE models are updated using the measured natural frequencies with GA. The updating on the FE model proves to be satisfactory. The natural frequencies predicted by the updated FE models compare very well with the measured results. The validity of the identified models is further demonstrated by their capability of predicting satisfactorily the natural frequencies beyond those used in the updating.

This experimental study confirms that using GA as the search engine, and with adequate settings of the FE model, it is possible to update the FE model for real structures to a satisfactory degree based on a suitable set of vibration data.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

Conventional FE model updating methods have difficulties in practical applications due to their high demands on the measurement data, their general susceptibility to measurement noises, and the possibility of being snagged into local optima. The artificial intelligence algorithms, in particular GAs and ANNs, offer desirable solutions to these problems in general. This study aims to develop general methodologies and implementation schemes of using these algorithms for FE model updating, and investigate into their effectiveness. Major contributions of this study include the incorporation of GA and eigensensitivity algorithms to tackle the problem arisen from inadequate sensitivity with a limited response dataset; the development of a two-level ANN approach for FE model updating including the known damping factors; the general evaluation of the noise-resisting abilities of using ANN for model updating; the development of the general coding strategy for stochastic GA to improve the computational efficiency for general optimization applications; as well as the development of a GA-based scheme for inclusion of the artificial boundary condition frequencies in the FE model updating.

Detailed conclusions on each aspect of this investigation have been provided in the individual chapters of this thesis. The following summarizes the major conclusions and observations

A. Model Updating using GA

- i) GA can be applied effectively in performing FE model updating. In general, the updating accuracy using GA can be as good as what may be anticipated from a sensitivity analysis. However, inherent sensitivity problem dictates that when only a limited amount of response data is available, for example only the lowest 3 natural frequencies and the first mode shape, the application of GA alone is not able to achieve satisfactory updating results if the number of parameters to identify exceeds a certain limit (e.g., 6 in the cases examined). With the incorporation of the conventional eigensensitivity method to perform further local climbing based on the GA global solution, it was demonstrated that accurate updating results can be achieved for a large number of parameters, without the need of increasing the order of modal data.
- ii) In case the available modal data contain a certain level of measurement errors, e.g., $\pm 2\%$ in modal frequencies and $\pm 5\%$ in mode shapes, the solution accuracy generally deteriorates. Where the parameter set to be identified is specified, a sensitivity analysis as suggested in this thesis can be performed to determine the required order of modal data for a target updating accuracy. Provided that such modal data set is available, GA is able to successfully update the model to the anticipated accuracy.
- iii) Through the application on laboratory tested RC and steel frame models, the proposed GA-based FE model updating procedure proves to be workable under real test and structural conditions.

B. ANN-Based Model Updating

- i) A two-level ANN-based FE model updating algorithm, as proposed in this thesis, is effective in updating a combination of structural parameters and the damping ratios. By incorporating the noise-injection learning strategy, the trained ANN can

acquire marked noise-resisting ability. Based on the numerical investigation, more than 50% reduction of the error margin in the updated structural parameters can be achieved with the noise-injection learning strategy.

- ii) The performance of a neural network for FE model updating is observed to depend largely on the network topology. An adequate size of the network is viable. The proposed scheme for response selection proves to be effective in safeguarding a successful updating.
- iii) Antiresonance information can play a constructive role in model updating. The antiresonant frequencies supplement the response data and hence can reduce the reliance on mode shape data for which the measurement accuracy is usually much lower than the frequency data. It is also shown that the integral of FRFs can serve as effective response data for the updating of structural damping properties.

C. Model Updating Using Natural Frequencies and ABC Frequencies

- i) ABC frequencies prove to be effective data to supplement the response dataset for FE model updating. These frequency data correspond physically to the natural frequencies of a structure with artificial pinned constraints (APC). Because of the flexibility of applying APCs on a structural system, numerous ABC frequencies can be made available with good accuracy to supplement response dataset for the FE model updating.
- ii) ABC frequencies from different APC configurations can affect significantly the updating results. The proposed binary GA based strategy with a specially defined fitness function proves to be able to work out successfully in finding an optimal setting of APCs so that the resulting ABC frequencies are most effective for model updating.

- iii) With the enhancement of ABC frequencies, a satisfactory FE model updating can be achieved by using a response dataset composed of natural frequencies and an adequate number of ABC frequencies. Thus, ABC frequencies can generally be used as an effective substitute for the mode shape data.

D. Stochastic Genetic Algorithm (StGA)

- i) The proposed StGA proves to be an effective and efficient technique in dealing with global optimization problems with continuous variables. The superiority of StGA is due to its novel coding mechanism in which the global optimum is evolved through stochastically defined dynamic regions, not single points as in the usual GAs.
- ii) The adaptation step δ_i of the predefined variance for the variables subjected to optimization is an important factor in determining the performance of StGA. Too large a δ_i could degrade the StGA performance because there will be little region exploitation in the later generations due to the small variance values remained, whereas too small a δ_i would also slow down the evolution process due to the fact that in early generations much effort is spent exploring some unpromising regions. A proper setting of δ_i may be found via a series of preliminary analyses.
- iii) StGA is capable of finding the near-global solution for a wide range of functions, irrespective of their complexity, and the behavior of the proposed algorithm is very stable. StGA is also capable of solving large dimension problems with a good efficiency. On average, the number of function evaluations required by StGA is about one order less than other comparable algorithms.

8.2 Recommendations on Future Research

Several areas have also been identified to be worthy of further study for purpose of promoting a broader applicability of the analytical work developed in this thesis.

In the present study, relatively simple FE models have been considered for algorithm verification purposes. However, it has to be recognized that as the structural system and its FE model become more complicated, additional challenges on the updating algorithms and the effective response data collection will arise, although the general ideas and schemes developed and explored in this study shall still apply. In this regard, further study is required to extend the application of the proposed schemes to more complex FE model updating situations. Some purposely-designed real tests are also expected to carry out so as to assess the performance of the proposed methods from different aspects.

StGA needs further explorations, especially its parameter configurations. Necessary investigations may include the influences of different parameter settings on the performance of StGA, and provision of analytical guidance on the choice of these critical parameters. Subsequently, the theoretical findings and analytical developments shall be verified on real applications representing large-size complex FE model updating problems, from which the advantage of the improved efficiency with the proposed StGA may also be exploited.

In this thesis, the feed-forward type of neural network trained using the back-propagation algorithm was employed and the network topology was determined based on a trial analyses. When the training dataset becomes large, as would be the case in model updating of complex real-life structure systems, and a considerable number of connecting weights are to be optimized, trial analyses can be impractical due to the high demand on computational effort. In this regard, it is desirable to have a methodology that can effectively automate the selection or at least provide instructive guidance for the selection process to avoid blind trial procedure. Another possible option is to use radial basis function neural network (RBF-NN). RBF-NN bears the advantages that the training process is very fast and the computational effort required

is reduced remarkably as compared to feed-forward network. However, the performance of RBF-NN depends largely on the adequate choices of parameters associated with the basis functions; however, at the present stage, such critical variables are still determined by using some simple schemes, which generally require empirical data to be input. So, it is considered to be a worth research topic to come up with a systematic methodology for the design of an effective RBF-NN that is capable of achieving satisfactory updating of real structures.

A more comprehensive integration of the various techniques investigated in this thesis will be very promising towards a more powerful solution tool for the FE model updating in real-life applications. Continued efforts will need to be made in this direction.

PUBLICATIONS

Journal Papers

1. Yong Lu and Zhenguo Tu, (2004), “A two-level neural network approach for dynamic FE model updating including damping”, Journal of Sound and Vibration, 275(3-5), 931-952.
2. Zhenguo Tu and Yong Lu, (2004), “A robust stochastic genetic algorithm (StGA) for global numerical optimization”, IEEE Transactions on Evolutionary Computation, 8(5), 456 – 470.
3. Yong Lu and Zhenguo Tu (2004), “Dynamic model updating using combined genetic-eigensensitivity algorithm and application in seismic response prediction”, Earthquake Engineering and Structural Dynamics, 34(9), 1149-1170.

Conference Papers

1. Zhenguo Tu and Yong Lu, (2003), “Global optimization of continuous problems using stochastic genetic algorithm”, Congress on Evolutionary Computation, CEC '03, December 8-12, Canberra, Australia, 2, 1230 – 1236.
2. Lu, Y. and Tu, Z. “Structural damage assessment based on dynamic data using genetic algorithms.” Proceedings of the Eighth International Conference on Recent Advances in Structural Dynamics, Southampton, UK, 14-16 July 2003. (CD-ROM, Paper No. 95)

REFERENCES

Ahmadian, H., Mottershead, J. E., and Friswell, M. I., (1998), “Regularization Methods for Finite Element Model Updating”, *Mechanical Systems and Signal Processing*, 12(1), 47-64.

Andry, A. N., Shapiro, E. Y. and Chung, J. C., (1983), “Eigenstructure Assignment for Linear Systems”, *IEEE Transaction on Aerospace and Electronic Systems*, AES-19(5), 711-729.

Angeline, P. J., (1998), “Evolutionary Optimization versus Particle Swarm Optimization: Philosophy and Performance Differences”, in *Proc. Evol. Prog.* VII, V. W. Porto, N. Saravanan, D. Waagen, and A. E. Eiben, Eds. Berlin, Germany: Springer-Verlag, 601-610.

Araujo dos Santos, J. V., Mota Soares, C. M., Mota Soares, C. A., and Pina, H. L. G., (2000), “A Damage Identification Numerical Model Based On the Sensitivity of Orthogonality Conditions and Least Squares Techniques”, *Computers and Structures*, 78(1), 283-291.

Atalla, M. J. and Inman, D. J. (1998). “On Model Updating Using Neural Networks”, *Mechanical systems and signal processing*, 12(1), 135-161.

Baruch, M. and Bar Itzhack, I. Y., (1978), “Optimal Weighted Orthogonalization of Measured Modes”, *AIAA Journal*, 16(4), 346-351.

Baruch, M., (1978), “Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Vibration Tests”, *AIAA Journal*, 16(11), 1208-1210.

Baruch, M., (1982), “Optimal Correction of Mass and Stiffness Matrices Using Measured Modes”, *AIAA Journal*, 20(11), 1623-1626.

Baruch, M., (1984), “Methods of Reference Basis for Identification of Linear Dynamic Structures”, *AIAA Journal*, 22 (4), 561-564.

Bathe, K-J., (1996), *Finite Element Procedures*, New Jersey: Prentice-Hall, Inc.

Beards, C. F., (1986), “Structural Damping by Slip in Joints”, *Shock and Vibration Dig.*, 17(11), 17-19.

Berman, A., (1979a), “Mass Matrix Correction Using an Incomplete Set of Measured Modes”, *AIAA Journal*, 17, 1147-1148.

Berman, A., (1979b), “Optimal Weighted Orthogonalization of Measured Modes”, *AIAA Journal*, 17, 927-928.

Berman, A. and Nagy. E. J., (1983), “Improvement of A Large Analytical Model Using Test Data”, *AIAA Journal*, 21(8), 1168-1173.

Berman, A., (1984), “System Identification of Structural Dynamic Model—Theoretical and Practical Bounds”, *Proceedings of the AIAA/ASME/ASCE/AHS 25th Structures, Structural Dynamics, & Materials Conference*, 123-128.

Ben-Haim, Y. and Prells, U., (1993), “Selective Sensitivity in the Frequency Domain, Part 1: Theory”, *Mechanical Systems and Signal Processing*, 7, 461-475.

Bishop, C.M., (1995), *Neural Networks for Pattern Recognition*, Oxford: Oxford University Press.

Breitfeld, T., (1996), "A Method for Identification of a Set of Optimal Measurement Points for Experimental Modal Analysis", Modal Analysis: the International Journal of Analytical and Experimental Modal Analysis, 11(1), 1-9.

Brownjohn, J. M. and Xia, P-Q., (2000), "Dynamic Assessment of Curved Cable-Stayed Bridge by Model Updating", Journal of Structural Engineering, ASCE, 126(2), 252-260.

Brownjohn, J. M. W., Moyo, P., Omenzetter, P. and LU, Y., (2003), "Assessment of Highway Bridge Upgrading by Dynamic Testing and Finite-Element Model Updating", Journal of Bridge Engineering, ASCE, 8(3), 162-172.

Caesar, B., (1986), "Update and identification of dynamic mathematical models", 4th IMAC, 394-401.

Caesar, B., (1987), "Updating System Matrices Using Modal Test Data", 5th IMAC, 453-459.

Calif, Y. X., (1991), "Evolution of Connectionist Networks", Preprints of the International Symposium on AI, Reasoning and Creativity (T. Dartnall, ed.), (Queensland, Australia), Griffith University, 49-52.

Cawley, P. and Adams, R. D., (1979), "The Location of Defects in Structures from Measurements of Natural Frequencies", Journal of Strain Analysis, 14, 49-57.

Chellapilla, K. (1998), "Combining Mutation Operators in Evolutionary Programming", IEEE Trans. Evol. Comput., 2, 91-96.

Chen, K., Brown, D. L. and Nicolas, V. T., (1993), "Perturbed Boundary Condition Model Updating." 11th International Modal Analysis Conference, Kissimmee, Florida, February 1993, 661-667.

- Chen, G. and Ewins, D. J., (2001), "Verification of FE Models for Structural Dynamics", *Proceedings of 19th International Modal Analysis Conference*, Kissimmee, Florida, Feb. 5-8, 385-391.
- Chou, J. H. and Ghaboussi, J., (2001), "Genetic Algorithm in Structural Damage Identification", *Computer and structures*, 79, 1335-1353.
- Chou, J. H. and Ghaboussi, J., (2001), "Genetic Algorithm in Structural Damage Detection", *Computer and Structures*, 79(14), 1335-1353.
- Collins, J. D., Young, J. and Kiefling, L., (1972), "Methods and Applications of System Identification in Shock and Vibration", *System Identification of Vibrating Structures*, Presented at the 1972 Winter Annual Meeting of the ASME, 45-71.
- Collins, J. D., Hart, G. C., Hasselman, T. K. and Kennedy, B., (1974), "Statistical Identification of Structures", *AIAA Journal*, 12(2), 185-190.
- Corelli, D., and Brown, D. L., (1984), "Impact Testing Considerations", *Proceedings of 2nd International Modal Analysis Conference*, SEM, Orland, Florida, 735-742.
- Cybenko, G., (1989), "Approximation by Superpositions of a Sigmoidal Function", *Mathematics of Control, Signals and Systems*, 2(4), 303-314.
- Dambrogio, W. and Fregolent, A., (2000), "The Use of Antiresonances for Robust model Updating", *Journal of Sound and Vibration*, 236(2), 227-243.
- Deb, K., (1991), "Binary and Floating-point Function Optimization Using Messy Genetic Algorithms", *Ph.D. Dissertation*, Dept. of Engineering Mechanics, The University of Alabama, Tuscaloosa.
- Dejong, K. A., (1975), "Analysis of the Behavior of a Class of Genetic Adaptive Systems", *Ph.D. Dissertation*, University of Michigan, Ann Arbor, MI.

- Doebling, S. W., Hemez, F. M., Peterson, L. D. and Farhat, C., (1997b), "Improved Damage Location Accuracy Using Strain Energy-Based Mode Selection Criteria", *AIAA Journal*, 35(4), 693-699.
- Dunn, S. A., (1997), "Modified Genetic Algorithm for the Identification of Aircraft Structures", *Journal of Aircraft*, 34(2), 251-253.
- Dunn, S. A., (1998), "Use of Genetic Algorithms and Stochastic H-Climbing in Dynamic Finite Element Model Identification", *Computer and Structures*, 66(4), 489-497.
- Fahlman, S. E., (1989), "Faster-Learning Variations on Back-Propagation: An Empirical Study", *Proceedings of the 1988 Connectionist Models Summer School*, Morgan Kaufmann, 38-51.
- Farhat, C. and Hemez, F. M., (1993), "Updating Finite Element Dynamic Models Using an Element-by-Element Sensitivity Methodology", *AIAA Journal*, 31(9), 1720-1711.
- FEMtools User's Manual*, (2002), Version 2.1.2, Dynamic Design solution (DDS), Belgium.
- Fogel, D. B., (1992), "Evolving Artificial Intelligence", *Ph.D Thesis*, University of California, San Diego.
- Freund, Y. and Ben-Haim, Y., (1995), "Selectively Sensitive Identification of Connectivity Matrices in Linear Elastic Systems", *Proceedings of the 13th IMAC*, 1474-1478.

Friswell, M. I., (1989), "The Adjustment of Structural Parameters Using a Minimum Variance Estimator." *Journal of Mechanical Systems and Signal Processing*, 3(2), 143-155.

Friswell, M. I. and Mottershead. J. E., (1995), *Finite Element Model Updating in Structural Dynamics*, Netherland: Kluwer academic publishers.

Friswell, M. I., Penny, J. E. T. and Garvey, S. D., (1998), "A Combined Genetic and Eigensensitivity Algorithm for the Location of Damage in Structures." *Computers and structures*, 69, 547-556.

Funabiki, N., Kitamichi, J. and Nishikawa, S., (1997), "Evolutionary neural network algorithm for max cut problems", *Proc., the 1007 IEEE International Conference on Neural Networks. Part 2, Piscataway, NJ, USA, IEEE Press*, 1260-1265.

Funahashi, K. I., (1989), "On the Approximate Realization of Continuous Mappings by Neural Networks", *Neural Networks*, 2(3), 183-192.

Gladwell, G. M. L., (1997), "Inverse Vibration Problems for Finite Element Models." *Inverse Problems*, 13, 311-322.

Gordis, J. H., (1992), "Spatial, Frequency Domain Updating of Linear, Structural Dynamic Models", *Proceeding of the 34th AIAA/ASME/ASCE/AHS/ACS Structures, Structural dynamics, and Materials Conference*, 3050-3058.

Gordis, J. H., (1994), "An Analysis of the Improved Reduced System Model Reduction Procedure", *Modal Analysis*, 9, 269-285.

Gordis, J. H., (1996a), "On the Analytical Disassembly of Structural Matrices", *Modal Analysis*, 11, 39-48.

Gordis, J. H., (1996b), "Omitted Coordinate Systems and Artificial Constraints in Spatially Incomplete Identification", *Modal Analysis*, 11, 83-95.

Gordis, J. H., (1999), “Artificial Boundary Conditions for Model Updating and Damage Detection”, Mechanical Systems and Signal Processing, 13(3), 437-448.

Goldberg, D. E., (1989), Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley Longman, Reading, Mass.

Greenwood, G. W., Lang, C. and Hurley, S., (1995), “Scheduling Tasks in Real-Time Systems Using Evolutionary Strategies”, Proceedings of the Third Workshop on Parallel and Distributed Real-Time Systems, 195-196.

Grefenstette, J. and Fritzpatrick, M., (1990), Genetic Search with Approximate Function Evaluations, Department of Computer Science, Vanderbilt University.

Gysin, H., (1990), “Comparison of Expansion Methods for F.E. Modeling Error Localization”, 8th IMAC, 195-204.

Gysin, H., (1990), “Comparison of Expansion Methods for FE Modeling Error Localization”, Proceedings of 8th International Modal Analysis Conference, 195-204.

Hager, W. W., Hearn, W. and Pardalos, P. M., (1994), Eds., Large Scale Optimization: State of the Art, Norwell, MA: Kluwer

Hao, H. and Xia, Y., (2002), “Vibration-based Damage Detection of Structures by Genetic Algorithm”, Journal of computing in civil engineering, 16(3), 222-229.

HARP, S. A., SAMAD, T. and GUHA, A., (1990), “Designing application-specific neural networks using the genetic algorithm”, Proc., IEEE conference on Neural Information Processing Systems, San Mateo, 2, 447-454.

Hartman, E. J., Keeler, J. D. and Kowalski, J. M., (1990), "Layered Neural Networks with Gaussian Hidden Units as Universal Approximations", *Neural Computation*, 2(2), 210-215.

He, J. and Li, Y. Q., (1995), "Relocation of Anti-Resonances of a Vibratory System by Local Structural Changes", *The International Journal of Analytical and Experimental Modal Analysis*, 10(4), 224-235.

Hearn, G. and Testa, R. B., (1991), "Modal Analysis for Damage Detection in Structures", *Journal of Structural Engineering*, 117, 3042-3063.

Hemez, F. M. and Farhat, C., (1995), "Structural Damage Detection via a Finite Element Model Updating Methodology", *Modal Analysis: The International Journal of Analytical and Experimental Modal Analysis*, 10(3), 152-166.

Hemez, F. M., Farhat, C., Bacher, E. and Vallat, S., (1995), "On the Efficiency of Model Updating via Genetic Algorithm for Structural Damage Detection", *Proceedings of the 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and AIAA/ASME Adaptive Structures Forum*, Part 4 (of 5), New Orleans, LA, USA, 2792-2801.

Holland, J. H., (1975), *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, MI.

Hornik, K., Stinchcombe, M. and White, H., (1989), "Multilayer Feedforward Networks Are Universal Approximators", *Neural Networks*, 2(5), 359-366.

Imregun, M., and Visser, D. J., (1991) "A Review of Model Updating Techniques." *Shock and Vibration Digest*, 23(1), 9-20.

- Joines, J. and Houck, C., (1994), "On the Use of Non-Stationary Penalty Functions to Solve Constrained Optimization Problems with Genetic Algorithms", *In 1994 IEEE International Symposium Evolutionary Computation*, Orlando, FI, 579-584.
- Jones, K. W., (2000), "Finite Element Model Updating Using Antiresonance Frequencies", *MS thesis*, AFIT/GA/ENY/00-M08. Air Force Institute of Technology (AU), Wright-Patterson AFB OH.
- Judd, J. S., (1990), *Neural Network Design and the Complexity of Learning*, MIT Press-Bradford Book, Cambridge, MA.
- Kabe, A. M., (1985), "Stiffness matrix adjustment using mode data", *AIAA Journal*, 23(9), 1431-1436.
- Kammer, D. C., (1991), "Sensor Placement for On-orbit Modal Identification and Correlation of Large Space Structures", *Journal of Guidance, Control, and Dynamics*, 14(2), 251-259.
- Kiddy, J. S. and Pines, D. J., (1997), "Damage Detection of Main Rotor Faults Using a Sensitivity-Based Approach", *Smart Structures and Materials*, 3041, 611-618.
- Kim, B. S. and Calise, A. J., (1997), "Nonlinear Flight Control Using Neural Networks", *Journal of Guidance, Control and Dynamics*, 20(1), 26-33.
- Krishnakumar, K., Swaminathan, R., Garg, S. and Narayanaswamy, S., (1995), "Solving Large Parameter Optimization Problems Using Genetic Algorithms", *Proceedings of the Guidance, Navigation, and Control Conference*, Baltimore, MD.
- Ladislav, S and Inman. D. J., (1995), "A Symmetric Positive Definite Inverse Vibration Problem with Underdamped Modes", *In 1995 Design Engineering Technical Conference*, volume 3C, 1089-1094, Boston.

Lallement, G. and Piranda, J., (1990), "Localization Methods for Parameter Updating of Finite Element Models in Elastodynamics", 8th IMAC, 579-585.

Lallement, G. and Cogan, S., (1992), "Reconciliation between Measured and Calculated Dynamic Behaviors: Enlargement of Knowledge Space", 10th IMAC, 487-493.

Lam, H. F., Ko, J. M. and Wong, C. W., (1998), "Localization of Damaged Structural Connections Based on Experimental Modal and Sensitivity Analysis", Journal of Sound and Vibration, 210(1), 91-115.

Lam, M. J. and Inman, D. J., (1995), "Methods of Preserving Symmetry in Model Updating", Journal of Vibration and Acoustics, 117, 349-354.

Lammens, S., Heylen, W., Sas, P. and Brown, D., (1993), "Model Updating and Perturbed Boundary Condition Testing." 11th International Modal Analysis Conference, Kissimmee, Florida, February 1993, 449-455.

Larson, C. B. and Zimmerman, D. C., (1993), "Structural Model Refinement Using a Genetic Algorithm Approach", Proceedings of the 11th International Modal Analysis Conference, Kissimmee, FL, USA, 1095-1101.

Lei, M. H. and Meng, C. H., (2003), "Application of Four-Layer Neural Network on Information Extraction", Proceedings of the International Joint Conference on Neural Networks, July 20-24, 3, 2146 - 2151.

Levin, R. I. And Lieven, N. A. J., (1998), "Dynamic Finite Element Model Updating Using Neural Networks", Journal of Sound and Vibration, 210 (5), 593-607.

Li, S., Shelley S. and Brown, D., (1995), "Perturbed Boundary Condition Testing", Proceedings of the 13th International Modal Analysis Conference, 1, 902-907.

- Lieven, N. A. J. and Ewins, D. J., (1998), "Spatial Correlation of Mode Shapes, the Coordinate Modal Assurance Criterion (COMAC)", *Proceedings of 6th International Modal Analysis Conference*, SEM, Kissimmee, Florida, 690-695.
- Lin, H. J. and Vitter, J. S., (1990), "Complexity Issues in Learning by Neural Networks", *Technical Report TR-CS-90-01*, Department of Computer Science, Brown University, Providence, RI.
- Lim, K. B., (1992), "Method for Optimal Actuator and Sensor Placement for Large Flexible Structures", *Journal of Guidance, Control, and Dynamics*, 15(1), 49-57.
- Link, M., (1986), "Survey on Selected Updating Procedures." *Proceedings of the 11th International Modal Analysis Seminar*, SEM, K. U. Leuven.
- Link, M., Weiland, M. and Barragan, J. M., (1987), "Direct Physical Matrix Identification as Compared to Phase Resonance Testing: An Assessment Based on Practical Application", *5th IMAC*, 804-811.
- Link, M. and Santiago, O. F., (1991), "Updating and Localizing Structural Errors Based on Minimization of Equation Errors", *International Conference on Spacecraft Structures and Mechanical Testing*, ESA.ESTEC, Noordwijk, Holland, 503-510.
- Lu, Y., Tassios, T. P., Zhang, G. F., and Vintzileou, E., (1999), "Seismic Response of RC Frames with Strength and Stiffness Irregularities", *ACI Structural Journal*, 96(2): 221-229.
- Mares, C. and Surace, C., (1996), "Application of Genetic Algorithms to Identify Damage in Elastic Structures", *Journal of Sound and Vibration*, 195(2), 195-215.
- Masri, S. F., Nakamura, M., Chassiakos, A. G. and Caughey, T. K., (1996), "Neural Network Approach to Detection of Changes in Structural Parameter", *Journal of Engineering Mechanics*, 122(4), 350-360.

- Minas, C. and Inman, D. J., (1988), "Correcting Finite Element Models with Measured Modal Results Using Eigenstructure Assignment Methods", 6th IMAC, 583-587.
- Minas, C. and Inman, D. J., (1989), "Model Improvement by Pole Methods", Vibration Analysis Techniques and Applications (T. S. Sankar, editor), ASME, New York, 179-185.
- Minas, C. and Inman, D. J., (1990), "Matching Finite Element Methods to Modal Data", Transactions of the ASME, Journal of Vibration and Acoustics, 112(1), 84-92.
- Michalewicz, Z., (1994), Genetic Algorithms + Data Structures = Evolution Programs, AI series. Springer-Verlag, New York.
- Morassi, A. and Rovere, N., (1997), "Localizing a Notch in a Frame from Frequency Measurements", Journal of Engineering Mechanics, ASCE, 123(5), 422-432.
- Mottershead, J. E. and M. I. Friswell, (1993), "Model Updating in Structural Dynamics: A Survey", Journal of Sound and Vibration, 167(2) 347-375.
- Mottershead, J. E., Friswell, M. I., Ng, G. H. T. and Brandon, J. A, (1996), "Geometric Parameters for Finite Element Model Updating of Joints and Constraints", Mechanical Systems and Signal Processing, 10(2), 171-182.
- Mottershead, J. E., (1998), "On the Zeros of Structural Frequency Response Functions and Their Application to Model Assessment and Updating", Proceedings of the 16th International Modal Analysis Conference, 500-503.
- Mottershead, J. E., and Mares, C. (2000), "Selection and Updating of Parameters for an Aluminium S-Frame Model", Mechanical Systems & Signal Processing, 14(6), 923-944.

Natke, H. G., (1988), "Updating Computational Models in the Frequency Domain Based on Measured Data: A Survey." *Probabilistic Engineering Mechanics*, 3 (1), 28-35.

Mulgund, S., Harper, K., Krishnakumar, K. and Zacharias. G., (1998), "Air Combat Tactics Optimization Using Stochastic Genetic Algorithms", *Proceedings of 1998 IEEE International Conference on Systems, Man, and Cybernetics*, San Diego, CA, 3136-3141.

Nalitolela, N., Penny, J. E. T. and Friswell. M. I., (1990), "Updating Structural Parameters of a Finite Element Model by Adding Mass and Stiffness to the System", *8th IMAC*, 836-842.

Nalitolela, N. G., Penny, J. E. T. and Friswell, M. I., (1992), "A Mass or Stiffness Addition Technique for Structural Parameter Updating." *International Journal of Analytical and Experimental Modal Analysis*, 7(3), 157-168.

Neidbal, N., Klusowski, E., and Lubner, W., (1990), "Updating a Finite Element Model by Means of Normal Mode Parameters", *8th IMAC*, 586-592.

Nicol, N. S. and Richard, K. B., (1992), "Dynamic Parameter Encoding for Genetic Algorithms," *UCSD Technical Report*, CS 90-175.

O'Callahan, J., (1989), "A Processing for an Improved Reduction System (IRS) Model", *Proceedings of 7th International Modal Analysis Conference*, 17-21.

O'Callahan, J., Avitabile, P. and Riemer, R., (1989), "System Equivalent Reduction Expansion Process (SEREP)", *Proceedings of 7th International Modal Analysis Conference*, 29-37.

Ozcan, E. and Onbasioglu E., (2004), “Genetic Algorithms for Parallel Code Optimization”, *Congress on Evolutionary Computation, CEC '04*, Portland, Oregon, USA, June 19-23, 2, 1375 – 1381.

Peyt, M., (1990), *Introduction to Finite Element Analysis*, New York: Cambridge University Press.

Piedras Lopes, T. A. and Andrade, O. P., (1998), “A Neural Network Approach for the Evaluation of Modal Sensitivity Matrix”, *The NECI Scientific Literature Digital Library*.

Rejeb, J. and AbuElhaij, M., (2000), “New Gender Genetic Algorithm for Solving Graph Partitioning Problems”, *Proceedings of the 43rd IEEE Midwest Symposium on Circuits and Systems*, 1, 444 – 446.

Ren, Y., (1992), “The Analysis and Identification of Friction Joint Parameters in the Dynamic Response of Structures”, *PhD Thesis*, Imperial College of Science Technology and Medicine, London University, London.

Ross, R. G., (1971), “Synthesis of Stiffness and Mass Matrices”, *SAE Conference Paper* 710787.

Sanayei, M. and Onipede, O., (1991), “Damage Assessment of Structures Using Static Test Data”, *AIAA Journal*, 29(7), 1174-1179.

Sanayei, M. and Saletnik, M. J., (1996a), “Parameter Estimation of Structures from Static Strain Measurements, Part I: Formulation”, *Journal of Structural Engineering, ASCE*, 122(5), 555-562.

Sanayei, M. and Saletnik, M. J., (1996b), “Parameter Estimation of Structures from Static Strain Measurements, Part II: Error Sensitivity Analysis”, *Journal of Structural Engineering, ASCE*, 122(5), 563-572.

Sathyanarayan, S. R., Birru, H. K. and Chellapilla, K., (1999), “Evolving Nonlinear Time-Series Models Using Evolutionary Programming”, *Proceedings of the 1999 Congress on Evolutionary Computation*, 1, 236-243.

Sazonov, E. S., Klinkhachorn, P., and Halabe, U. B., (2002), “Genetic Algorithms-Based Parameter Optimization of a Non-Destructive Damage Detection Method”, *Proceedings of the Thirty-Fourth Southeastern Symposium on System Theory*, March 18-19, 152 – 156.

Sidhu, J. and Ewins, D. J., (1984), “Correlation of Finite Element and Modal Test Studies of a Practical Structure”, *2nd IMAC*, 756-762.

Seywald, H., Kumar, R. R. and Deshpande, S. M., (1995), “Genetic Algorithm Approach for Optimal Control Problems with Linearly Appearing Controls”, *Journal of Guidance, Control, and Dynamics*, 18(1), 177-182.

Shi, Z. Y., Law, S. S. and Zhang, L. M., (2000), “Optimum Sensor Placement for Structural Damage Detection”, *Journal of Engineering Mechanics, ASCE*, 126(11), 1173-1179.

Siarry, P., Berthiau, G., Durbin, F. and Haussy, J., (1997), “Enhanced Simulated Annealing for Globally Minimizing Functions of Many-Continuous Variables”, *ACM Trans. Math. Software*, 23, 209-228.

Smith, S. W. and Beattie, C. A., (1991), “Secant-Method Adjustment to Structural Model”, *AIAA Journal*, 29(1), 119-126.

Spooner, J. T. and Passino, K. M., (1999), “Decentralized Adaptive Control of Nonlinear Systems Using Radial Basis Neural Networks”, *IEEE Trans. Autom. Contr.*, 44(11) 2050-2057.

Spalding, J. D. and MacNish, C., (2003), “A Genetic Algorithm to Sequence DNA Using Sequencing Hybridisation Experimental Data”, Congress on Evolutionary Computation, CEC '03, Canberra, Australia, Dec. 8-12, 2, 1466 – 1471.

Srinathkumar, S., (1978), “Eigenvalue/Eigenvector Assignment Using Output Feedback”, IEEE Transaction on Automatic Control, AC-23, 79-81.

Szewczyk, Z. P. and Hajela, P (1994), “Damage Detection in Structures Based on Feature-Sensitive Neural Networks,” Journal of computing in civil engineering, ASCE, 8(2), 163-177.

Thoren, A. R., (1972), “Derivation of Mass and Stiffness Matrices from Dynamic Test Data”, AIAA conference paper, 72-346.

To, W. M., Lin, R. M. and Ewins, D. J., (1990), “A Criterion for the Localization of Structural Modification Sites Using Modal Data”, 8th IMAC, 961-967.

Torkamani, M. A. M. and Ahmadi, A. K., (1998), “Stiffness Identification of Frame Using Simulated Ground Excitation”, Journal of Engineering Mechanics, ASCE, 114(5), 753-776.

Tuccillo, R. and Senatore, A., (1995), “A Genetic Algorithm Based Approach to Radial Flow Impeller Design”, Computational Fluid Dynamics in Aeropropulsion ASME, 49, 51-62.

Vapnik, V. N. and Chervonenkis, A. Y. A., (1971), “On the Uniform Convergence of Relative Frequencies of Events to Their Probabilities”, Theory Prob Applic, 16(2), 264-280.

Wada, B. K., Kuo, C. P. and Glaser, R. J., (1986), “Multiple Boundary Condition Tests (MBCT) for Verification of Large Space Structure”, AIAA Paper, 86-0905.

Wollam, J. D., (2000), “Research Engineering of Foreign Missile via Genetic Algorithm”, *AFIT, AY2000 Research*, Air Force Institute of Technology (AU), Wright-Patterson AFB OH.

Xia, P. Q., (2000), “Condition Assessment of Bridges by Dynamic Measurements and Finite Element Model Updating”, *PhD Dissertation*, School of Civil and Structural Engineering, Nanyang Technological University, Singapore.

Xia, Y. and Hao, H., (2000), “Measurement Selection for Vibration Based Structural Damage Identification”, *Journal of Sound and Vibration*, 236(1), 89-104.

Wright, A. H., (1991), “Genetic Algorithms for Real Parameter Optimization.” *Foundation of Genetic Algorithms*, G.J.E. Rawlins, ed., Morgan Kaufmann Publishers, San Mateo.

Yao, X., Liu, Y. and Lin, G. M., (1997), “Fast Evolutionary Strategy“, in *Evolutionary Programming VI*, P. J. Angeline, R. Reynolds, J. McDonnell, and R. Eberhart, Eds. Berlin, Germany: Sprintger-Verlag, 151-161.

Yao, X., Liu, Y. and Lin, G. M., (1999), “Evolutionary Programming Made Faster”, *IEEE Transaction on Evolutionary Computation*, 3(2), 82-102.

Yun, C. B., Yi, J. H. and Bahng, E. Y. (2001), “Joint Damage Assessment of Framed Structures Using a Neural Networks Technique,” *Engineering Structures*, 23, 425-435.

Zhang, J. H. and Natke, H. G., (1991), “A Two-Level Updating Procedure of the Component Mode Synthesis Model”, *Mechanical Systems and Signal Processing*, 5(6), 501-514.

Zhang, J. H. and Xu, X. H., (1999), “An Efficient Evolutionary Programming Algorithm”, *Computers & Operations Research*, 26, 645-663.

Zhang, Q. and Lallement, G., (1987), “Dominant Error Localization in a Finite Element Model of a Mechanical Structure”, *Mechanical Systems and Signal Processing*, 1(2), 141-149.

Ziaei-Rad, S., (1997), Methods for Updating Numerical Models in Structural Dynamics, *Ph.D. Thesis*, Imperial College, University of London.

Ziemianski, L., Miller, B. and Piatkowski, G., (1998), “Dynamic Model Updating by Neural Networks”, *The NECI Scientific Literature Digital Library*.

APPENDIX

Detailed Description of Some Test Functions Used in Chapter 5

A) f_{12} and f_{13} (generalized penalized functions)

$$\begin{aligned} f_{12}(x) = & \pi/30 \left\{ 10 \sin^2(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 [1 + 10 \sin^2(\pi y_{i+1})] \right. \\ & \left. + (y_n - 1)^2 \right\} + \sum_{i=1}^n u(x_i, 10, 100, 4) \\ & - 50 \leq x_i \leq 50, \quad \min(f_{12}) = f_{12}(-1, -1, \dots, -1) \end{aligned}$$

$$\begin{aligned} f_{13}(x) = & 1/10 \left\{ 10 \sin^2(3\pi x_1) + \sum_{i=1}^{n-1} (x_i - 1)^2 [1 + \sin^2(3\pi x_{i+1})] \right. \\ & \left. + (x_n - 1)^2 [1 + \sin^2(2\pi x_n)] \right\} + \sum_{i=1}^n u(x_i, 5, 100, 4) \\ & - 50 \leq x_i \leq 50, \quad \min(f_{13}) = f_{13}(-1, -1, \dots, -1) \end{aligned}$$

where

$$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a, \\ 0, & -a \leq x_i \leq a \\ k(-x_i - a)^m, & x_i < -a \end{cases}$$

$$y_i = 1 + 1/4(x_i + 1)$$

B) f_{14} (Shekel's Foxholes Function)

$$\begin{aligned} f_{14}(x) = & \left[1/500 + \sum_{j=1}^{25} (j + \sum_{i=1}^2 (x_i - a_{ij})^6)^{-1} \right]^{-1} \\ & - 65.536 \leq x_i \leq 65.536, \quad \min(f_{14}) = f_{14}(-32, -32) \approx 1 \end{aligned}$$

where

$$\left(a_{ij} = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & \cdots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & \cdots & 32 & 32 & 32 \end{bmatrix} \right)$$

C) f_{15} (**Kowalik's Function**)

$$f_{15}(x) = \sum_{i=1}^{11} \left[a_i - x_1 (b_i^2 + b_i x_2) / b_i^2 + b_i x_3 + x_4 \right]^2$$

$$-5 \leq x_i \leq 5, \quad \min(f_{15}) = f_{15}(0.1928, 0.1908, 0.1231, 0.1358) = 0.0003075$$

Table-A1 Coefficients appearing in function f_{15}

i	1	2	3	4	5	6	7	8	9	10	11
a_i	0.1957	0.1947	0.1735	0.1600	0.0844	0.0627	0.0456	0.0342	0.0323	0.0235	0.0246
b_i^{-1}	0.25	0.5	1	2	4	6	8	10	12	14	16

D) f_{18} to f_{20} (**Shekel's Family**)

$$f(x) = - \sum_{i=1}^m \left[(\mathbf{x} - \mathbf{a}_i)(\mathbf{x} - \mathbf{a}_i)^T + c_i \right]^{-1}$$

where

$$\mathbf{x} = (x_1, x_2, x_3, x_4)^T \quad 0 \leq x_j \leq 10 \quad j = 1, 2, \dots, 4$$

$f(x)$ with m equal to 5, 7 and 10, respectively, become f_{18} , f_{19} and f_{20} . These three functions have 5, 7 and 10 local minima, respectively

Table-A2 Coefficients in functions, f_{18} to f_{20}

i	$a_{ij}, j = 1, 2, \dots, 4$				c_i
1	4	4	4	4	0.1
2	1	1	1	1	0.2
3	8	8	8	8	0.2
4	6	6	6	6	0.4
5	3	7	3	7	0.4
6	2	9	2	9	0.6
7	5	5	3	3	0.3
8	8	1	8	1	0.7
9	6	2	6	2	0.5
10	7	3.6	7	3.6	0.5