Turbulence-induced secondary flows in straight open channel with imposed transverse bed non-uniformities

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TURBULENCE-INDUCED SECONDARY FLOWS IN STRAIGHT OPEN CHANNEL WITH IMPOSED TRANSVERSE BED NON-UNIFORMITIES

WANG ZHIQIAN

**SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

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STRAIGHT OPEN CHANNEL WITH IMPOSED
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SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING

A thesis submitted to the Nanyang Technological University
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Doctor of Philosophy

2006

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ABSTRACT

Characteristics of straight open channel flows can be modified in the presence of secondary flows. This study aims to investigate, both experimentally and analytically, time-mean characteristics of straight open channel flows subject to large-scale longitudinal vorticity, which are generated by longitudinal bedforms.

Experiments were conducted in a tilting, rectangular flume. Six different longitudinal bed configurations, which included alternate bed strips with different roughness heights and bed ridges in wavy and rectangular shapes, were employed to artificially generate cross-flow secondary motions. Flow velocities were measured using a two-dimensional Laser Doppler Anemometer (LDA) and a one-dimensional Ultrasonic Doppler Velocimeter (UDV). Preliminary tests of suspended sediment were also conducted with sediment concentration being measured using an OBS-3 system.

The results presented in this report largely consist of three parts, i.e. characteristics of secondary flows, characteristics of the primary flow subjected to secondary flows, and implications for suspended sediment transport. First, the experimental results demonstrate that secondary flows generated by the longitudinal bedforms generally appear as pairs of counter-rotating circulations across the primary flow. Generally, the downflow occurs over the rough strip or the bed trough whereas the upflow occurs over the smooth strip or the ridge. Relevant analyses show that the cellular secondary motion can be reasonably described by a stream function that varies sinusoidally in the transverse and vertical directions. Comparisons with the experimental data show that the derived formulations work generally correct for the organized flow cells, the latter being generated in open channel flows with alternate roughness strips or wavy bed ridges. On the other hand, the experimental observations also show that the structure of the secondary flows developed over the rectangular ridges is more complex, which is closely related to the way of the connection between the ridge and trough.

Because of cellular secondary flows, the vertical profile of the streamwise velocity generally deviates from the log-law and the distribution of the Reynolds

shear stress appears to be non-linear. However, the modified streamwise velocity can be well represented by a function proposed in the log-wake form if lateral variations in the velocity gradient, zero-velocity bed level and wake strength are properly considered. Regardless of disturbances caused by the cellular secondary motion, the primary flow according to its velocity distribution can still be divided into the inner region and outer region. For the outer region, the wake term is positive in the upflow zone and negative in the downflow zone, but becomes vanishingly small for the velocity profile measured at the location of the circulation centre.

The thesis also demonstrates that the flow field can be linearized so that a flow quantity can be generally decomposed into two components, one being related to the average reference flow and the other symbolizing the perturbation caused by the bed configuration. This perturbation-based approach provides an analytical description of the altered Reynolds shear stress distribution that differs from the linear profile used for two dimensional open channel flows.

Finally, the influences of secondary flow advection and three-dimensional diffusion processes on sediment concentration distribution are discussed. A major implication is that fine sediment load may tend to be homogenized within the retention zone, which is partially evidenced by the experiment results.

Keywords: Open channel flow; Secondary flow; Velocity distribution; Reynolds shear stress; Bed shear stress; Longitudinal bedform; Log-wake law; Suspended sediment transport.

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LIST OF SYMBOLS

A	=	cross-sectional area with a lateral dimension, λ (m^2);
a	=	vertical distance of reference position in Rouse equation (m);
a_l, a_w	=	variation parameters for velocity gradient and wake strength of U -profiles, respectively (-);
a_{sl}, a_{sw}	=	variation parameters for stress gradient and wake strength of $-\overline{u'v'}$ -profiles, respectively (-);
B	=	channel width (m);
B_r	=	integration constant in log-law (-);
B_l	=	constant in the criterion for dividing log-layer and buffer layer, say $yu^*/\nu > B_l$ for log-layer, (-);
b	=	bed elevation (m);
C	=	suspended sediment concentration (kg m^{-3});
C_0	=	suspended sediment concentration for the reference flow (kg m^{-3});
C_a	=	suspended load concentration at the reference position (kg m^{-3});
C_m	=	bulk-average suspended sediment concentration (kg m^{-3});
C_{sound}	=	sound speed of the ultrasonic wave in the fluid (m s^{-1});
c_1	=	$a_l u_{*0}$ (m s^{-1});
c_2	=	$a_w u_{*0}$ (m s^{-1});
c_3	=	$a_{sl} u_{*0}^2$ ($\text{m}^2 \text{s}^{-2}$);
c_4	=	$a_{sw} u_{*0}^2$ ($\text{m}^2 \text{s}^{-2}$);
d_s	=	distance from the top of sediment bed at which the mean velocity is zero (m);
Fr	=	Froude number (-);
f_B	=	friction coefficient of sediment (-);

f_d	=	Doppler frequency shift (-);
f_e	=	emission frequency of ultrasound (-);
g	=	gravitational accelerate (m s^{-2});
H	=	elevation of free surface (m);
h	=	flow depth (m);
h_0	=	flow depth for the reference flow (m);
h_{rg}	=	ridge height (m);
J	=	mean flow energy slope (-);
k	=	dimensionless lateral wave number (-);
k_s	=	representative roughness height (m);
L_r	=	representative length (m);
l	=	mixing length (m);
m	=	integers (-);
N	=	sampling number (-);
n	=	Manning coefficient ($\text{m}^{-1/6}$);
$\mathbf{n}$	=	unit vector outward normal to sediment path, (-);
n_e	=	$\kappa U_m / u_{*m}$, exponent for power law of velocity profile (m s^{-1});
P	=	mean flow pressure (N/m^2);
po	=	porosity of sediment (-);
p_i	=	volumetric ratio of i -th fraction sediment to the total bed materials in the surface layer (-);
p_B	=	bedload pick-up probability density (-);
Q	=	flow discharge ($\text{m}^3 \text{s}^{-1}$);
q_B	=	bedload flux per unit channel width ($\text{kg m}^{-1} \text{s}^{-1}$);
q_{Bz}	=	lateral bedload flux per unit channel width ($\text{kg m}^{-1} \text{s}^{-1}$);

R	=	hypothesized zero-velocity level, or hydrodynamic roughness length (m);
Re	=	Reynolds number (-);
$R_{0(J)}$	=	hypothesized zero-velocity level for the reference flow corresponding to $u^*_{0(J)}$ (m);
$R_{0(v)}$	=	hypothesized zero-velocity level for the reference flow obtained from the velocity profile at the circulation centre $u^*_{0(v)}$ (m);
s	=	unit vector tangential to sediment path, (-);
S	=	particle location from the UDV probe
t	=	time (s);
T_d	=	delay time between the emitted burst and the echo issued from the particle (s);
T_{prf}	=	time interval between two ultrasound emissions (-);
t	=	transit time of seeding particles (s);
U, V, W	=	time-averaged flow velocities in streamwise, vertical and spanwise directions, respectively ($m\ s^{-1}$);
U_d	=	velocity deviation due to the non-uniformity of bed shear stress in the Odgaard's (1984) model ($m\ s^{-1}$);
U_m	=	mean streamwise velocity in the central region of the flume ($m\ s^{-1}$);
U_{ms}	=	mean streamwise velocity over the entire cross section of the flume ($m\ s^{-1}$);
U_s, V_s, W_s	=	sediment particle velocity in the x, y, z directions, respectively ($m\ s^{-1}$);
U^+	=	U/u^* , dimensionless velocity ($m\ s^{-1}$);
U_f	=	vector of fluid velocity ($m\ s^{-1}$);
U_s	=	vector of sediment particle velocity ($m\ s^{-1}$);
u^*	=	friction velocity ($m\ s^{-1}$);
u^*_{m}	=	average friction velocity ($m\ s^{-1}$);

$u_{*0(J)}$	=	$\sqrt{gh_m J}$, friction velocity for the reference flow obtained from energy slope (m s^{-1});
$u_{*0(v)}$	=	friction velocity for the reference flow obtained from velocity profile at circulation centre (m s^{-1});
$u_{*0(\tau)}$	=	friction velocity for the reference flow obtained from Reynolds shear stress profile at circulation centre (m s^{-1});
u', v', w'	=	fluctuating flow velocities in streamwise, vertical and spanwise directions, respectively (m s^{-1});
$-\overline{u_i' u_j'}$	=	Reynolds stress ($\text{m}^2 \text{s}^{-2}$);
V_r	=	representative velocity (m s^{-1});
V_o	=	voltage output (V);
V_{real}	=	velocity component in the direction of ultrasonic beam (m s^{-1});
V_{us}	=	real flow velocity (m s^{-1});
x, y, z	=	Cartesian coordinates in streamwise, vertical and spanwise directions, respectively (m);
y_c, z_c	=	locations of flow circulation centre in vertical and spanwise directions, respectively (-);
Y	=	transformed vertical coordinate, of which the mean streamwise velocity is zero at the origin (-);
y^+	=	yu_*/ν for smooth bed or y/k_s for rough bed, dimensionless vertical coordinate (m);
Z	=	$\omega/(\beta \kappa u_*)$, exponential value in Rouse equation (-);
α	=	angular velocity of vortex (1/s);
β	=	proportionality constant for sediment diffusion to turbulent diffusion (-);
δ_B	=	thickness of the bedload layer (m);
δ_b	=	amplitudes for bed shear stress (-);
δ_d	=	phase shift of the received ultrasound echo (-);

$\delta_l, \delta_r, \delta_w$	=	amplitudes for a_l, R and a_w , respectively (-);
δ_{sl}, δ_{sw}	=	amplitudes for a_{sl} and a_{sw} , respectively (-);
$\Delta\tau_{bx}$	=	amplitude of the lateral variation in the streamwise bed shear stress (N m^{-2});
ε	=	turbulent diffusion coefficients of suspended sediment ($\text{m}^2 \text{s}^{-1}$);
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	=	turbulent diffusion coefficients of suspended load in the x, y, z directions, respectively ($\text{m}^2 \text{s}^{-1}$);
ε_{y0}	=	the vertical diffusion coefficient for the reference flow ($\text{m}^2 \text{s}^{-1}$);
ϕ_s	=	curves normal to the sediment path (-);
γ	=	specific weight of fluid (N m^{-3});
γ_s	=	specific weight of sediment (N m^{-3});
η	=	$(y-b)/h$, relative vertical coordinate (-);
ϕ_y, ϕ_z	=	original phase in the vertical and transverse directions (-);
κ	=	von Karman constant (-);
Λ	=	mean step length of bedload particle (m);
λ	=	average longitudinal bedform width or mean secondary flow cell breadth (m);
λ_r, λ_s	=	widths of rough and smooth strips, respectively (m);
$\lambda_{rg}, \lambda_{tr}$	=	widths of ridges and troughs, respectively (m);
$\lambda_{dn}, \lambda_{up}$	=	breadths of the downflow and upflow zones, respectively (m);
ν	=	kinematic viscosity of fluid ($\text{m}^2 \text{s}^{-1}$);
ν_t	=	eddy viscosity ($\text{m}^2 \text{s}^{-1}$);
ν_{t0}	=	eddy viscosity for the reference flow ($\text{m}^2 \text{s}^{-1}$);
Π	=	Cole's wake strength parameter (-);
θ	=	bed slope angle (-);
θ_f	=	$W/U _{y=0}$, angle of deflection of the flow direction (-);

θ_p	=	angle between the trajectory of the particle and the axis of the ultrasonic beam (-);
θ_z	=	side-slope angle of ridges (-);
ρ	=	liquid particle density (kg m^{-3});
ρ_s	=	sediment particle density (kg m^{-3});
τ	=	streamwise flow shear stress in two dimensional flow (N m^{-2});
τ_b	=	bed shear stress in two dimensional flow (N m^{-2});
τ_b^*	=	$\tau_b / (\gamma_s - \gamma) d$, dimensionless bed shear stress (-);
τ_c^*	=	$\tau_c / (\gamma_s - \gamma) d$, dimensionless critical bed shear stress (-);
τ_{bx}, τ_{by}	=	bed shear stress in the x, y directions, respectively (N m^{-2});
τ_x, τ_y, τ_z	=	flow shear stress in the x, y, z directions, respectively (N m^{-2});
ϖ	=	$t_i / \sum_{j=0}^{N-1} t_j$, weighting factor for calculation of velocity moments from LDA data (-);
$\Omega_x, \Omega_y, \Omega_z$	=	vorticity components in x, y, z directions (s^{-1}).
ω	=	settling velocity of sediment particle (m s^{-1});
ω_0	=	settling velocity of sediment particle in still fluid (m s^{-1});
$\boldsymbol{\omega}$	=	vector of settling velocity of sediment particle (m s^{-1});
$\boldsymbol{\omega}_0$	=	vector of settling velocity of sediment particle in still fluid (m s^{-1});
ω_{D50}	=	settling velocity of sediment particles with median size (m s^{-1});
ω_{F50}	=	settling velocity of flocs of median size (m s^{-1});
ψ	=	stream function of fluid ($\text{m}^2 \text{s}^{-1}$);
ψ_s	=	stream function of sediment ($\text{m}^2 \text{s}^{-1}$);
ζ	=	z/λ , relative transverse coordinate (-).

Subscripts/superscripts

m	mean values;
max	maximum values;
min	minimum values;
0	quantities for reference flow;
1	amplitude of perturbations from reference flow.

Abbreviations

LDA	Laser Doppler Anemometer;
LES	Large Eddy Simulation;
DNS	Direct Numerical Simulation;
OBS	Optical Backscatter;
UDV	Ultrasonic Doppler Velocimeter.

Chapter

1

INTRODUCTION

1.1 BACKGROUND

Open channel flows are three-dimensional. The primary flow in the streamwise direction can be affected by the secondary flow components in the vertical and transverse directions, and vice versa. Such interactions are very complex when secondary flows are unsteady and random. However, relevant fundamental information could be featured by considering three dimensional flows with the simplest boundary conditions. One of such examples is the turbulent flows in straight, wide open channels with longitudinal bedforms, where secondary flows could be generated by lateral perturbations either in the bed elevation or roughness. These secondary flows usually appear as pairs of counter-rotating flow cells elongated longitudinally, being called cellular secondary flows.

The existence of cellular secondary flows in rivers was first inferred from indirect flow measurements (Nezu and Nakagawa, 1993). It is with the advent of measurement technologies from 1970's that accurate measurements of secondary flows became possible. Many laboratory experiments and field observations have been conducted to explore cellular secondary flows (Müller and Studerus, 1979; McLean, 1981; Ikeda, 1981; Nakagawa et al., 1981; Studerus, 1982; Nezu and Nakagawa, 1984; Nezu et al., 1993; McLelland, et al., 1999; Wang, et al., 2003; and

Wang, et al., 2004). However, it is noted that these previous investigations were mainly aimed to clarify the existence of cellular secondary flows, which are generally limited to qualitative descriptions of relevant phenomena.

Characteristics of the primary flow are modified in the presence of secondary flows. However, the influence of cellular secondary flows on the primary flow was often ignored in the previous studies. For example, regardless of the secondary flows, the vertical profile of the streamwise velocity was simply compared to the log-law and the distribution of the Reynolds shear stress was assumed to be linear. The correctness of these applications of conventional theories for uniform open channel flows is doubtful. Unfortunately, there has been no systematic investigation conducted so far.

Another important question is sediment transport in the presence of secondary flows. It has been often observed in natural rivers that suspended sediment concentration is non-uniform in spanwise direction (Vanoni, 1946; Chiu and Hsiung, 1981). This could also be attributed to the existence of secondary flows. However, relevant investigations are also very limited.

All the above-mentioned limitations pertaining to the previous studies on cellular secondary flows over longitudinal bedforms indicate the necessity of further research for the relevant topics.

1.2 OBJECTIVES

The goal of this work was to explore the turbulent flows in straight open channels with various longitudinal bedforms. Specific objectives are

- (1) To provide detailed information of a systematic experimental study on this topic; and
- (2) To find analytical models to describe time-mean secondary velocities, primary velocity, Reynolds shear stress and bed shear stress.
- (3) To discuss the possible effects of secondary flows on suspended sediment concentration distribution.

1.3 LAYOUT OF THE THESIS

This thesis consists of seven Chapters. Chapter 1 is an introductory chapter which presents the background and the objectives of this study.

In Chapter 2, a review of the previous studies pertaining to the cellular secondary flows is given. It includes: classification and mechanisms of secondary flows in straight open channels; interactions between cellular secondary flows and longitudinal bedforms; general features of time-mean flow structures in the presence of cellular secondary motions; and influence of cellular secondary flows on sediment transport.

Chapter 3 describes the details of the experimental setups and procedures.

In Chapter 4, the experimental results of secondary flows over various longitudinal bedforms are presented, which are then followed by analytical considerations.

Chapter 5 is concerned with the characteristics of the primary flow affected by cellular secondary flows. A theoretical analysis is also conducted for the distributions of the streamwise velocity, Reynolds shear stress and bed shear stress. The analytical models are then applied with the experimental data.

Chapter 6 provides some implications for suspended sediment transport, which includes a qualitative analysis of the settling behavior and concentration distribution of suspended sediment in the presence of cellular secondary flows.

Chapter 7 is a concluding chapter, which summarizes this thesis.

Chapter

2

LITERATURE REVIEW

2.1 INTRODUCTION

In comparison with numerous achievements on the primary flow characteristics in open channels, the knowledge of secondary flows is rather limited in the literature. This chapter includes a review on the previous studies pertaining to cellular secondary flows.

First, the classifications of secondary flows occurring in open channels, in particular for those appearing as streamwise circulations, are presented. Then, the mechanisms and characteristics of turbulence-induced secondary flows are described. Following that, the knowledge of longitudinal bedform and its interaction with cellular secondary flows is presented. Next, previous results on the mean flow structure with cellular secondary motions are summarized. Finally, the topic of sediment transport subject to cellular secondary flows is also addressed.

2.2 OPEN CHANNEL FLOWS WITH STREAMWISE CIRCULATION

Open-channel flows are three-dimensional. Secondary flows refer to vertical and transverse flow components that are imposed on the longitudinal primary flow. As a common phenomenon in open channel flows with a generally flat bed, secondary flows often appear as streamwise circulation (with longitudinal mean vorticity), the streamlines exhibiting a spiral shape.

2.2.1 CLASSIFICATION OF SECONDARY FLOWS

According to Prandtl (1952), there are two mechanisms that are responsible for generating secondary flows in the plane normal to the longitudinal primary flow. The first mechanism is associated with the skewing of the mean flow, which leads to the so-called Prandtl's first kind of secondary flow. For example, for the mean flow that is non-uniform in the streamwise direction, such as in curved channels or meandering rivers, vortices may be stretched and amalgamated in the transverse direction, eventually resulting in streamwise circulation. The generation of this kind of secondary flow is an essentially inviscid process either driven by centrifugal or transverse pressure gradient. Therefore, it can occur in both laminar and turbulent flows.

On the other hand, in turbulent channel flows, streamwise circulation can only be generated by the non-homogeneity and anisotropy of turbulence. This is called Prandtl's second kind of secondary flow, or turbulence-induced (or sometimes stress-induced) secondary flow. Since it only occurs in turbulent flows, no counterpart can be found in laminar flows. In the following, the review is limited to the case of the turbulence-induced secondary flow.

2.2.2 MECHANISM OF TURBULENCE-INDUCED SECONDARY FLOW

For the steady turbulent flow in a straight open channel, the distribution of the mean velocity components is governed by the following equations,

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 \quad (2-1)$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = g \sin \theta - \frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} + \nu \nabla^2 U \quad (2-2)$$

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} = -g \cos \theta - \frac{1}{\rho} \frac{\partial P}{\partial y} - \frac{\partial \overline{v'u'}}{\partial x} - \frac{\partial \overline{v'v'}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z} + \nu \nabla^2 V \quad (2-3)$$

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\partial \overline{w'u'}}{\partial x} - \frac{\partial \overline{w'v'}}{\partial y} - \frac{\partial \overline{w'w'}}{\partial z} + \nu \nabla^2 W \quad (2-4)$$

in which x, y, z = rectangular Cartesian coordinates in the streamwise, vertical and spanwise directions, respectively (see Fig. 2-1); U, V, W = time-averaged velocity components in the x, y, z direction, respectively; and u', v', w' = x, y and z components of the turbulent velocity fluctuations; $-\overline{u'u'}, -\overline{u'v'}, -\overline{u'w'}$ and other similar terms are the components of Reynolds stress tensor, respectively; θ = slope of channel bed; g = gravitational acceleration; ρ = fluid density; P = mean pressure; ν = kinematic viscosity; and ∇^2 denotes Laplacian operator. For fully developed turbulent flows, the last terms in Eqs. (2-2) to (2-4) can be ignored; and for uniform flows, the gradients with respect to x equal zero, i.e. $\partial/\partial x = 0$.

Fig. 2-1. Coordinates used for open channel flows.

To understand the mechanism of the turbulence-induced secondary flow, it is convenient to consider the streamwise vorticity, Ω_x , which can be derived from Eqs. (2-1) to (2-4) with the pressure terms cancelled by cross differentiating. The exact streamwise mean vorticity equation is (Perkins, 1970)

$$\begin{aligned}
 U \frac{\partial \Omega_x}{\partial x} + V \frac{\partial \Omega_x}{\partial y} + W \frac{\partial \Omega_x}{\partial z} = & \left(\Omega_x \frac{\partial U}{\partial x} + \Omega_y \frac{\partial U}{\partial y} + \Omega_z \frac{\partial U}{\partial z} \right) + v \nabla^2 \Omega_x \\
 & + \frac{\partial^2}{\partial y \partial z} (\overline{v'^2} - \overline{w'^2}) + \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) (-\overline{v'w'}) + \frac{\partial}{\partial x} \left[\frac{\partial}{\partial y} (-\overline{u'w'}) - \frac{\partial}{\partial y} (-\overline{u'v'}) \right]
 \end{aligned} \quad (2-5)$$

where the three components of the vorticity are defined as

$$\Omega_x = \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z}, \quad \Omega_y = \frac{\partial U}{\partial z} - \frac{\partial W}{\partial x}, \quad \Omega_z = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \quad (2-6)$$

In Eq. (2-5), the first term on the right hand side describes the interaction between shear and vortex, which is responsible for the vortex-stretching and thus the generation of the ‘skew-induced’ streamwise vorticity. It is noted that this term is only related to the gradients of the mean flow velocities. Therefore, secondary flows so generated can occur in curved channel either for laminar or turbulent flows. For a straight uniform flow, Eq. (2-5) can be simplified as follows (Nezu and Nakagawa, 1993)

$$\underbrace{V \frac{\partial \Omega_x}{\partial y} + W \frac{\partial \Omega_x}{\partial z}}_A = \underbrace{\frac{\partial^2}{\partial y \partial z} (\overline{v'^2} - \overline{w'^2})}_B + \underbrace{\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) (-\overline{v'w'})}_C + \underbrace{v \nabla^2 \Omega_x}_D \quad (2-7)$$

In Eq. (2-7), the term *A* indicates the advection of streamwise vorticity; the term *B* promotes the secondary flow while the term *C* suppresses it; and the viscous term *D* is negligible except for the near wall region. This explanation was first given by Einstein and Li (1958). Subsequently, it was also verified experimentally (Nezu and Nakagawa, 1984) and numerically (Demuren and Rodi, 1984). Gerard (1978) pointed out that turbulence-induced secondary flow is due to the cross-flow imbalance of the Reynolds normal stresses. Speziale (1982, 1987) showed that it is the essential difference in normal stresses that dominates the development of longitudinal vortices. These illustrations have been further summarized by Nezu and Nakagawa (1993) and Nezu (2002), as shown in Fig. 2-2.

Fig. 2-2. Mechanism of turbulence-induced secondary flows (after Nezu, 2002)

Fig. 2-2 shows that the imbalance between $\overline{v'v'}$ and $\overline{w'w'}$ caused by the non-uniformity of boundary conditions first leads to the secondary shear stress, $-\overline{v'w'}$, which generates the streamwise vorticity. It can be also seen that as a feedback system, the secondary flow also affects the streamwise bed stress τ_b , the primary velocity U and the streamwise Reynolds shear stresses, $-\overline{u'v'}$ and $-\overline{u'w'}$.

2.3 CORNER FLOWS IN NARROW OPEN CHANNELS AND NONCIRCULAR DUCTS

Since Prandtl's time, secondary flows have been known to occur in noncircular ducts if the sidewall effect is strong. As demonstrated in Fig. 2-3(a), near the sidewall two identical streamwise circulations appear symmetrically with respect to the corner bisector. Such secondary flows are usually called 'corner flows' because they are directed from the duct core towards the corner. Many experiments (e.g. Leutheusser, 1963; Gessner, 1973; Melling and Whitelaw, 1976) were conducted to measure corner flows in square or rectangular ducts using hot-wire anemometers or Laser Doppler Anemometer. Relevant numerical calculations (e.g. Naot and Rodi, 1982) have also been successfully performed.

Fig. 2-3. Secondary flow streamlines for square duct and narrow open channel (Naot and Rodi, 1982).

In contrast, secondary flows in open channels were less studied until the 1980's. Naot and Rodi (1982) have conducted a notable numerical study to simulate the secondary motions in open channel flows using an algebraic stress model, predicting a 3D flow structure near the open-channel corner. It consists of a strong vortex near the free surface and a weak one that is squeezed into the corner region. The upper vortex is called 'free-surface vortex' and the lower one 'bottom vortex'.

A calculated example is shown in Fig. 2-3(b), in which the channel aspect ratio of width to depth is 2. Such secondary flow structure is obviously different from that in the duct [Fig. 2-3(a)]. This is due to the free surface effects. In order to verify this prediction, Nezu and Rodi (1985) conducted a LDA measurement, and the measured results agreed well with the prediction.

The prediction of Naot and Rodi (1982) and the experiment of Nezu and Rodi (1985) have also shown that the corner flow structure depends on the aspect ratio. As demonstrated in Fig. 2-4, as the aspect ratio increases, the upper vortex grows in dimension, while the lower vortex is relatively suppressed. It is also noted that the limit width of the upper vortex seems to be twice the flow depth, e.g., in the cases of $B/h = 4$ and $B/h = 6$, where B is the channel width and h the flow depth. In other words, the corner vortex starts to be dampened gradually beyond the sidewall region that is $2h$ wide. Similarly, Nezu et al. (1985) also found that for wide closed ducts with boundaries, either completely smooth or homogeneously rough, 2D flows were available only in the central region.

In fact, the above findings also serve as the basis for the classification of the narrow and wide open channels. For example, Nezu and Rodi (1985), and Nezu et al. (1985) have proposed a classification criterion as follows

$$\text{Narrow open channel if } B/h \leq 5 \quad (2-8)$$

$$\text{Wide open channel if } B/h > 5 \quad (2-9)$$

In the above-mentioned cases, the secondary flows, i.e. corner flows, are particularly related to the sidewall effect prevailing in narrow channel flows. The following section will be devoted to the discussion of secondary flows that are associated with longitudinal bedforms in the wide open channels, which are often referred to as cellular secondary flows.

Fig. 2-4. Calculated secondary flow streamlines in open channels with various aspect ratios (Naot and Rodi, 1982)

2.4 CELLULAR SECONDARY FLOWS AND LONGITUDINAL BEDFORMS IN WIDE OPEN CHANNELS

2.4.1 CELLULAR SECONDARY FLOWS

Secondary flows in wide open channels often appear as pairing, counter-rotating flow cells on the cross section plane, as sketched in Fig. 2-5. Typically, the flow cells, occurring in pairs, separate and reattach alternatively in the streamwise direction. Therefore, such secondary flows are called cellular secondary flows. The transverse breadth of two neighboring flow cells is defined as the secondary flow wavelength, which is usually twice the secondary flow depth. The fluid particle in each cell follows a spiral path of a pitch measured commonly 50-150 times the cell width or thickness (Allen, 1984). Hence the deviation angle of a limiting streamline from the direction of the mean flow may be less than $10-15^\circ$.

Fig. 2-5. Idealized 3D flow with longitudinal vortices in straight channels (after Allen, 1984).

The existence of cellular secondary flows in rivers was first inferred from sediment concentration distributions (instead of direct velocity measurements) by geologists and river engineers. Vanoni (1946) reported that free surface velocity and suspended load concentration varied periodically in the spanwise direction, as shown in Fig. 2-6, which was attributed to the existence of cellular secondary flows in the river. Similarly, Kinoshita (1967) discovered fascinating light-and-dark strip patterns on the water surface of a river in flood by aerial stereoscopic surveys. The light zones were shining because of foam, indicating the parallel convergence of

surface flows; the dark zones constituted boils with a high concentration of sediment. The flows in light zones were 10-20% faster than those in the dark zones. The spanwise spacing of these strips was found to be approximately twice the flow depth. He therefore deduced that cellular secondary flows in the form of longitudinal spiral vortices might exist in straight and wide rivers.

Fig. 2-6. Cross-sectional flow circulation, River Po in Italy (Vanoni, 1946)

The precise field measurement of cellular secondary flows was successfully conducted only after the advent of electromagnetic current meter (EM) and acoustic Doppler velocimeter (ADV). Nezu et al. (1993) measured secondary flows using two sets of EM in a river of 17.5m wide and 2.2m deep. Fig. 2-7 shows the measured vector description of secondary flows. Cellular secondary flows can be recognized in the similar manner as those sketched in Fig. 2-5.

Fig. 2-7. Vector description of secondary flows (Nezu et al, 1993)

Usually, the generation of cellular secondary flows is related to the lateral variation of bed configurations (Casey, 1935; Karcz, 1966; Culbertson, 1967; Ikeda, 1981; Allen, 1984). Nezu and Nakagawa (1993) considered that, if the bed shear stress varies periodically in the spanwise direction caused by lateral non-uniformity of bed configuration, streamwise vorticity would continuously take place and then stable cellular secondary flows could be observed at all cross sections. In particular, bed elevation change or roughness discontinuity in the spanwise direction is one of possible reasons to induce spanwise variations in the bed shear stress. For example, Karcz (1966) proposed a pattern of cellular secondary flows over sand ribbons in a wide fluvial channel, as shown in Fig. 2-8.

Fig. 2-8. Schematic patterns of cellular secondary flows and sand ribbons in wide river (Karcz, 1966)

Experimental studies that investigate cellular secondary flows related to longitudinal bedforms are due to Müller and Studerus (1979), McLean (1981), Studerus (1982), Nezu and Nakagawa (1984), Wang, et al. (2003), and Wang, et al. (2004). In particular, Nezu and Nakagawa (1984) carried out an exploratory experiment in an open channel flow over artificial longitudinal ridges of 45° trapezoidal cross section. The ridge elements were 8 m long, 5 mm thick and 20 mm wide. The spacing between ridge elements was 8 cm that was twice the flow depth. The velocity was measured using X-type hot-film anemometers. As shown in Fig. 2-9, a pair of counter-rotating flow cells can be easily recognized. The upflow occurs over the ridge and down flow over the trough. The flow cells appear a circular motion with a diameter of the flow depth.

Fig. 2-9. Velocity vectors of secondary flows measured over the ridges of trapezoidal section in the experiment by Nezu and Nakagawa (1984).

Other contributions in this aspect include analytical considerations by Colombini (1993) and numerical simulations by Hayashi et al. (2002) and Falcomer and Armenio (2002). It is found that similar characteristics of secondary flow cells that are observed in laboratory test and actual rivers could be also reproduced numerically. Fig. 2-10 shows an example provided by Hayashi et al. (2002). A fully-developed turbulent flow in a straight open channel with longitudinal wavy ridges was simulated using the technique of Direct Numerical Simulation (DNS). The flow field used for computation had a length of $6.4h$ and a span of $4h$ [Fig. 2-10(a)]. The computed pairs of secondary flow cells are similar to those shown in

Fig. 2-9. Falcomer and Armenio (2002) computed flows over the trapezoidal ridges using the technique of Large Eddy Simulation (LES), which also reproduced secondary flow cells.

Fig. 2-10. DNS of secondary flow cells in an open-channel flow over wavy ridges (Hayashi et al., 2002). (a) Computational zone; and (b) velocity vectors of secondary flows.

Since longitudinal bedforms can be observed in various environments (McLelland, et al. 1999), cellular secondary flows might be ubiquitous in nature. To better understand cellular secondary flows, it would be essential to be familiar with features of longitudinal bedforms. The latter will be discussed in the following section.

2.4.2 LONGITUDINAL BEDFORMS

Longitudinal bedforms have been intensively studied by geologists and hydraulic engineers since 1930s. They appear in various patterns in different natural environments. Generally, bedforms that elongate parallel with the mean flow

motion include longitudinal desert dunes, sand ribbons observed in shallow seas and rivers, and various small bedforms apparently dependent on wave-action at the strand (Allen, 1984). In the fluvial environment, longitudinal bedforms usually possess a dimension smaller than that of transverse bedforms, because they are best visible at, or close to, transition to upper plane-bed regime. These longitudinal bedforms appear with periodic, spanwise variations in bed texture (roughness) and/or bed elevation, but tending to be remarkably uniform in the streamwise direction. It is also noted that longitudinal bedforms have been observed in the streams over a clay-bed, sand-bed or gravel-bed.

For convenience, the longitudinal bedforms are classified as ‘strips’ and ‘ridges’ in this study. The ‘strips’ are the longitudinal bedforms characterized with dominant bed roughness variation in the transverse direction, while the ‘ridges’ (or ‘ridges/troughs’) are those with significant lateral variation in bed elevation. This classification is further illustrated in Fig. 2-11. It should be mentioned that several other terms are also used in the literature. For example, ‘ribbons’ are sometimes used either for ‘strips’ or ‘ridges’. The scoured troughs are often called ‘furrows’ or ‘grooves’. Furthermore, ‘streaks’ refer to the earlier stage of the formation of longitudinal bedforms.

Fig. 2-11. Schematics of the two typical types of longitudinal bedforms.

Numerous longitudinal bedforms are recorded in streams or ebbing tidal flows (Allen, 1984). A field example of sand strips formed by the ebbing tide over a gravel bed is shown in Fig. 2-12. The wavelength of these sand strips is in the order of 0.1-0.3 m. Apparently, the strips of fine sand are interwoven with those of coarse

gravel; they are substantially heterogeneous in size. Another field example of ridges parallel with the tidal currents is shown in Fig. 2-13, where the bedforms 300 m long were characterized with a wavelength of 2 m and bed elevation difference of 0.35 m.

Fig. 2-12. Sand strips developed by ebbing tide over a gravel bed. (a) General view; and (b) close-up. A trowel of 0.28 m long points in current direction (see Allen, 1984).

Fig. 2-13. Longitudinal ridges and furrows on intertidal mud flat. A trowel stuck upright into mud is 0.28 m long. The ebbing tide flows from upper right to lower left. (see Allen, 1984).

In addition to the longitudinal bedforms in the geophysical flows, they have been also observed in laboratory flume experiments. Casey (1935) conducted a sediment transport experiment using mixed grades of sand in a straight flume. He noticed that the finer particles moved over the coarser in the form of longitudinal strips, with a spanwise wavelength of roughly twice the flow depth. Similar experimental observations were also made by Vanoni (1946), Allen (1966), Günter (1971), Ikeda (1981), Hirano and Ohmoto (1988), Nezu and Nakagawa (1989) and McLelland, et al. (1999). Fig. 2-14 shows an example of the longitudinal sand strips formed in a flume, which consists of coarse and fine sand bands as a result of lateral sorting processes.

Fig. 2-14. Longitudinal strips (Günter, 1971). (a) General pattern of alternating coarse and fine sand bands; (b) a close-up of the bed showing the effect of sorting. Flow is from bottom to top.

Ikeda (1981) studied the size and the shape of self-formed strips with uniform non-cohesive sands. His experimental result indicated that longitudinal sand ridges could be formed over the entire movable bed, as shown in Fig. 2-15, of which the crests were several millimeters high. He observed that some of the ridges were very stable and persisted almost throughout the run time, and the distance between the neighboring crests was roughly twice the depth.

Fig. 2-15. View of longitudinal sand ridges. Arrows indicate ridges formed over movable sediment bed (Ikeda, 1981).

2.4.3 INTERACTIONS BETWEEN LONGITUDINAL BEDFORMS AND CELLULAR SECONDARY FLOWS

Cellular secondary flows and the associated longitudinal bedforms are the coupling phenomena, which can be observed in wide open channels with movable beds. Cellular secondary flows may be initiated by longitudinal bed forms, and in turn they can enhance, maintain or depress the bedforms. Therefore, an interactive relationship exists between cellular secondary flows and longitudinal bedforms.

Fig. 2-16. Inferred cellular secondary flows in a straight, wide river (after Nezu and Nakagawa, 1993).

Nezu and Nakagawa (1984, 1993) have postulated an evolution model for illustrating the interactive relationship (Fig. 2-16). In this model, it was supposed that the process started with the presence of the corner vortex, which was due to the sidewall effect. With the corner vortex, a lateral variation of bed shear stress appeared. As a result, bedload transport also varied laterally. This further led to the formation of a sand ridge. Following that, the bed shear stress varied further in the lateral direction, together with the generation of more vortices. If this process progressed repeatedly, sand ridges and cellular secondary currents would eventually occur throughout the entire cross-section.

Nezu and Nakagawa (1989) also reported that regular patterns of ridges and troughs could occur in the central region of the flow channel even when the first ridge near the sidewall did not sufficiently develop. This observation implies that the sidewall effect is not the necessary cause for the generation of longitudinal bedforms and cellular secondary flows. Ikeda (1981), Colombini (1993) and McLelland, et al. (1999) argued that the initiation of cellular secondary flows and sand ridges is rather an instability-related process. Small disturbances either from bed surface or flow itself are able to induce streamwise vortices, which then yields the lateral sediment transport and sorting process. With the same feed-back mechanism as suggested by Nezu and Nakagawa (1984), the secondary motions would enhance the initial bed disturbance, and then ridges or strips could be generated.

In particular, lateral sorting by cellular secondary flows may take place in a stream with poorly-graded bed material. Tsujimoto (1989) stated that the secondary flow might be initiated by the instability induced by bed-surface composition. Since the secondary flow causes lateral bedload transport from the rough to smooth zones, the difference in bed-surface composition may be amplified, and then bed deformation occurs. The longitudinal strips formed at the early stage are more characterized by the lateral variation in roughness rather than the lateral undulation of the bed. For example, McLelland, et al. (1999) have reported the elevation of finer grain strips is at most 1 mm above the adjacent coarser grain strips, with the bed elevation difference being about 1% of the flow depth. Allen (1984) considered that secondary flows in an alluvial channel would sweep out the finer sediment from the troughs and deposit it onto the ridges, making coarser sediment be exposed to the flow in the troughs. The finer sediment deposited on the ridges might be intermittently lifted up toward the free surface by strong upward secondary flows. Thus, the longitudinal bed forms are generally characterized by lateral variations in bed elevation as well as in bed roughness.

If the bed material is fine-grained, homogeneous sediment, longitudinal sand ridges might rapidly develop in the absence of sediment sorting. Karcz (1973) described the evolution of sand ridges and troughs from the pre-existing transverse ripples. The deformation process started with the appearance of a number of straight dislocation lines, which would further induce streamwise vortices. Gradually, a thread of scour appeared along the line, the sediment was removed laterally by transverse flows when accumulation occurred, and the remnant ripple crests and troughs were progressively leveled and uprounded. This sequence of events finally led to a very regular pattern of ridges with intervening troughs. Some other studies also show the development of longitudinal ridges in unimodal heterogeneous sediments without apparent lateral sorting processes by secondary flows (Hirano and Ohmoto, 1988 ; Nezu et al., 1988).

In addition to the qualitative descriptions or conceptual models, numerical simulations were also performed on bedform development due to cellular secondary flows. Colombini (1993) simulated the formation of sand ridges from a bed with uniform sands by presetting sinuous perturbations in the bed elevation. The result

showed that the longitudinal sand ridge formed was controlled by a delicate balance between the stabilizing gravity force and the destabilizing spanwise bed shear stress. Colombini and Parker (1995) proposed a model to predict the formation of sand ribbons that included various elevation and roughness variations. Being different from the results of Colombini (1993), the latter model predicted that a slight degree of sediment heterogeneity may result in an instability leading to the ribbon formation.

As the interactions between the flows and bedforms reach an equilibrium state, the mean secondary flows and bedforms will become stable and independent of time. The dimension of flow cells and bedform spacing are largely controlled by the scale of flow depth. Data on longitudinal bedform morphology highlight that the spanwise wavelength of longitudinal bedforms ranges from 1 to 4 times of flow depth (McLelland, et al., 1999). Most frequently, the wavelength observed in laboratory and field is near twice the flow depth (Wolman and Brush, 1961; Kinoshita, 1967; Ikeda, 1981; Hirano and Ohmoto, 1988). The underlying mechanism for this specific feature is not well-understood, which requires further investigations.

2.5 CHARACTERISTICS OF MEAN FLOW WITH CELLULAR SECONDARY MOTIONS

2.5.1 2D OPEN CHANNEL FLOWS

For comparison purposes, a summary of the characteristics of two-dimensional open channel flows is first given here. For open channel flows without lateral variations, i.e. $W = 0$ and $\partial/\partial z = 0$, Eqs. (2-1) to (2-4) can be reduced as follows

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (2-10)$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = g \sin \theta - \frac{\partial}{\partial x} \left(\frac{P}{\rho} \right) + \frac{\partial(-\overline{u'^2})}{\partial x} + \frac{\partial(-\overline{u'v'})}{\partial y} + \nu \nabla^2 U \quad (2-11)$$

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -g \cos \theta - \frac{\partial}{\partial y} \left(\frac{P}{\rho} \right) + \frac{\partial(-\overline{u'v'})}{\partial x} + \frac{\partial(-\overline{v'^2})}{\partial y} + \nu \nabla^2 V \quad (2-12)$$

Furthermore, for 2D uniform flows, the conditions of $V = 0$ and $\partial/\partial x = 0$ also applies. With these, integration of Eq. (2-12) from $y = h$ to $y = y$ yields

$$\frac{P}{\rho} = (h - y) g \cos \theta + \left(\overline{v'^2} \right)_s - \overline{v'^2} \quad (2-13)$$

where h = the flow depth and $\left(\overline{v'^2} \right)_s$ = the magnitudes of the vertical turbulence intensity at the free surface, which usually is zero.

By substituting Eq. (2-13) into Eq. (2-11) and taking integration, one obtains

$$\frac{\tau_x}{\rho} = -\overline{u'v'} + \nu \frac{\partial U}{\partial y} = u_*^2 \left(1 - \frac{y}{h} \right) \quad (2-14)$$

and in particular at $y = 0$,

$$\frac{\tau_{bx}}{\rho} = u_*^2 = gh \sin \theta = ghJ \quad (2-15)$$

where τ_x = streamwise shear stress; τ_{bx} = streamwise bed shear stress; u_* = the friction velocity; and $J = \sin\theta$ = the bed slope. Eq. (2-14) indicates that the total shear stress, which comprises viscous shear stress and Reynolds shear stress, decreases linearly from the value τ_{bx} at the bed to zero at the free surface. In this study, it will be shown that cellular secondary flows can cause significant deviations of τ_x -profiles from the linear distribution.

Generally, for uniform open channel flows, the flow field can be roughly divided into two sub-regions, inner layer ($0 \leq y/h \leq 0.15 \sim 0.2$) and outer layer ($50 < yu_*/\nu$), as shown in Fig. 2-17. In the outer layer, the turbulence structure is dominated by the outer variables, i.e., flow depth h and maximum streamwise velocity $U_{\max}$, whereas the turbulence structure in the inner layer is dominated by the inner variables, i.e., friction velocity u_* and inner length scale ν/u_* . In the overlap region for inner layer and outer layer, both inner variables and outer variables influence the turbulent structure. The velocity distribution in this overlap region obeys the log-law, and thus it is often referred to as log-layer. Over a smooth bed [Fig. 2-17(a)], the inner layer can be further divided into viscous sublayer ($0 \leq yu_*/\nu \leq 5$), buffer layer and log-layer. According to Raupach, et al (1991) and Nikora, et al (2001), for a rough bed [Fig. 2-17(b)], the inner layer also includes a roughness layer ($0 \leq y/d \leq 1 \sim 4$, in which d is the size of roughness element and the upper limit depends on the properties of roughness) where the flow is strongly influenced by individual roughness element. The roughness layer can be further divided into a form-induced sublayer and a interfacial sublayer.

Fig. 2-17. Subdivisions of uniform open channel flows.

For uniform 2D open channel flows, the log-law states

$$U^+ = \frac{1}{\kappa} \ln y^+ + B_r \quad (2-16)$$

where $U^+ = U/u_*$, the dimensionless velocity; $y^+ = yu_*/\nu$ for smooth bed and $y^+ = y/k_s$ for rough bed, the dimensionless vertical distance; and B_r = the additive constant. The Karman constant, $\kappa \approx 0.4$, is believed to be universal. The additive constant B_r is determined by the no-slip boundary condition at the bed but, because the log-law is only valid for $y^+ \gg 1$, its value also depends on the details of the buffer and viscous layers. For smooth walls, its experimental value is about 5.2.

A composite velocity profile valid also in the outer region is often written as

$$U^+ = \frac{1}{\kappa} \ln y^+ + B_r + w\left(\frac{y}{h}\right) \quad (2-17)$$

where the ‘wake’ component $w(y/h)$ represents the effect of the outer layer dynamics. An appropriate wake function proposed empirically by Coles (1956) is given by

$$w\left(\frac{y}{h}\right) = \frac{2\Pi}{\kappa} \sin^2\left(\frac{\pi y}{h}\right) \quad (2-18)$$

where Π = Coles' wake strength parameter. The wake function measures the contribution of the outer-layer structures to the mean velocity profile, while B_r measures the influence of the near-wall layer. The velocity defect law derived from Eqs. (2-16), (2-17) and (2-18) reads

$$U_{\max}^+ - U^+ = -\frac{1}{\kappa} \ln\left(\frac{y}{h}\right) + \frac{2\Pi}{\kappa} \cos^2\left(\frac{\pi y}{h}\right) \quad (2-19)$$

It will be shown later in this report that the velocity distribution in the presence of cellular secondary flows can also be formulated in the form of log-wake law, but with some modifications made for delineating effects of secondary flows.

2.5.2 STRUCTURE OF CELLULAR SECONDARY FLOW

Cellular secondary flows are basically characterized by its structure of pairing circulations. This has been verified experimentally by previous investigations, which are often limited to phenomenological descriptions. Theoretically, the cellular secondary motion can be effectively described with the vorticity equation, Eq. (2-7), but it is mathematically unsolvable. As an alternative approach, Ikeda (1981) proposed an analytical model for cellular secondary flows based on several assumptions as detailed subsequently. The first assumption made is that the convection term A and the viscous term D in Eq. (2-7) can be neglected, which leads to Eq. (2-7) to be reduced to

$$\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)(-\overline{v'w'}) = \frac{\partial^2}{\partial y \partial z}(\overline{w'^2} - \overline{v'^2}) \quad (2-20)$$

This assumption is considered to be acceptable, as verified experimentally by Nezu and Nakagawa (1984). To solve Eq. (2-20), Ikeda (1981) further assumed that the Reynolds shear stress could be written as

$$-\overline{v'w'} = \nu_t \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right) \quad (2-21)$$

where ν_t = the turbulent viscosity. It was taken to be a constant computed as

$$v_t = \frac{\kappa u_{*m} h}{6} \quad (2-22)$$

where u_{*m} = the average bed shear velocity. In terms of the stream function ψ , V and W are expressed as

$$V = -\frac{\partial \psi}{\partial z}, \quad W = \frac{\partial \psi}{\partial y} \quad (2-23)$$

Then, Eq. (2-21) can be rewritten in the form

$$-\overline{v'w'} = v_t L \psi \quad (2-24)$$

in which $L = \partial^2 / \partial y^2 - \partial^2 / \partial z^2$. Next, Ikeda (1981) assumed the term $(\overline{w'^2} - \overline{v'^2})$ to vary vertically as follows

$$\frac{(\overline{w'^2} - \overline{v'^2})}{u_{*m}^2} = a_1 \left[1 + a_2 \cos(k \frac{y}{h}) \right] \left(1 - \frac{y}{h} \right) \quad (2-25)$$

in which a_1 = coefficient shown to be approximately unity by Perkins (1970); a_2 = amplitude of the perturbation, much smaller than unity; and k = dimensionless lateral wave number. Similarly, the stream function ψ was taken as

$$\psi = f\left(\frac{y}{h}\right) \sin k \frac{z}{h} \quad (2-26)$$

in which $f(y/h)$ = dimensionless vertical distribution of the stream function, and it was approximated by

$$f\left(\frac{y}{h}\right) = 1 - \frac{y}{h} \quad (2-27)$$

Finally, with Eqs. (2-24) to (2-27), the solution to Eq. (2-20) was given by

$$\frac{\psi}{u_* h} = \frac{6a_2}{\kappa k^3} \sin\left(\frac{kz}{h}\right) \left[\left(\frac{2y}{h} - 1\right) \cos\left(\frac{ky}{h}\right) + 1 \right] \text{ for } k = (2m-1)\pi \quad (2-28)$$

$$\frac{\psi}{u_* h} = \frac{6a_2}{\kappa k^3} \sin\left(\frac{kz}{h}\right) \left[1 - \cos\left(\frac{ky}{h}\right) \right] \text{ for } k = 2m\pi \quad (2-29)$$

in which $m = 1, 2, 3$, and so on. Ikeda (1981) argued that at $k = \pi$ the circulation cell appeared to be largest. Thus, the velocity components for the secondary flow motion are expressed as

$$\frac{V}{u_*} = -\frac{6a_2}{\kappa\pi^2} \cos\left(\frac{\pi z}{h}\right) \left[\left(\frac{2y}{h} - 1\right) \cos\left(\frac{\pi y}{h}\right) + 1 \right] \quad (2-30)$$

$$\frac{W}{u_*} = \frac{6a_2}{\kappa\pi^2} \sin\left(\frac{\pi z}{h}\right) \left[\frac{2}{\pi} \cos\left(\frac{\pi y}{h}\right) - \left(\frac{2y}{h} - 1\right) \sin\left(\frac{\pi y}{h}\right) \right] \quad (2-31)$$

Although the above derivation is based on some assumptions, they are quite suggestive for describing cellular secondary flows. For example, it demonstrates that the variations of cellular secondary flows in the transverse and vertical directions could be reasonably described by the sinusoidal functions. However, it should be noted that because of the approximations involved, the model proposed has its inherent weakness. An example is that with k being taken to be π , this model is applicable only for particular secondary flow cells, which are of identical width and height. In fact, the observed aspect ratio of height to breadth for a flow cell may range from 1 ~ 4, and the circulation centre may not necessarily be the same as the geometrical centre. On the other hand, this model can be considered to be handicapped due to the use of the apparent viscosity [Eq. (2-21)] for the description of the secondary Reynolds shear stresses. This is because any turbulence model used for secondary shear stress, if being based on the eddy viscosity concept, will fail to reproduce the turbulence-induced secondary flow motion (Naot and Rodi, 1982; Colombini, 1993).

Other than the work done by Ikeda (1981), there is no other analytical model proposed so far for investigating the secondary flow cells. This is in contrast to several efforts that have been made for laboratory and field measurements and numerical simulation. In Chapter 4, Ikeda's idea will be further explored on the basis of the experimental observations made in this study.

2.5.3 STRUCTURE OF PRIMARY FLOW MODIFIED BY CELLULAR SECONDARY MOTION

The primary flow structure can be significantly modified due to the existence of cellular secondary flows. For example, the near-bed streamwise velocity above the rough strips is apparently less than that above the smooth strips (Wang et al, 2003). In contrast, the flow in the upper portion exhibits higher velocity above the rough strips and smaller velocity above the smooth strips. However, this variation is either simply ignored or underestimated in the previous studies. This may be because cellular secondary flows are usually much weaker than primary flows. For practical applications, it was often assumed that the velocity and shear stress distributions that are developed for 2D primary flows remained unchanged in the presence of secondary flow cells. For example, the primary velocity in the inner region was often fitted to the log-law for evaluating the bed shear stress (Studerus, 1982; Nezu and Rodi, 1985), and the vertical distribution of the streamwise Reynolds shear stress was approximated to be linear (Perkins, 1970; Ikeda, 1981). It should be noted that these assumptions may be reasonable for certain cases but were made without any validation.

Odgaard (1984) seems to have been the first to explore effects of cellular secondary flows on the distribution of the primary flow velocity and shear stress. Based on experimental measurements, Odgaard proposed a parabolic function for the description of the streamwise Reynolds shear stress, which is given by

$$-\overline{u'v'} = u_{*m}^2 \left(1 - \frac{y}{h}\right) + 3(u_{*m}^2 - u_*^2) \left(\frac{y}{h}\right) \left(1 - \frac{y}{h}\right) \quad (2-32)$$

where u_* = local shear velocity and u_{*m} = average shear velocity. Eq. (2-32) shows that it is the local variation of bed shear stress that causes the $-\overline{u'v'}$ profile to generally deviate from the linear distribution. Odgaard (1984) also assumed that U was given by

$$U = U_0 + U_d \quad (2-33)$$

in which U_0 = velocity distribution for the corresponding uniform open channel flow; and U_d = the velocity deviation due to the non-uniformity of bed shear stress. He chose the power law to describe U_0

$$\frac{U_0}{U_m} = \frac{n_e + 1}{n_e} \left(\frac{y}{h} \right)^{1/n_e} \quad (2-34)$$

in which U_m = the depth-averaged velocity; and $n_e = \kappa U_m / u_{*m}$. Based on some reasoning, the velocity deviation, U_d , was assumed to follow a parabolic distribution given by

$$\frac{U_d}{U_m} = \frac{u_* y}{\kappa U_m h} \left(\frac{3}{2} - \frac{9}{4} \frac{y}{h} \right) \quad (2-35)$$

It should be mentioned that the parabolic functions used in the above derivation are completely empirical without any theoretical basis. In fact, some experimental results of $-\overline{u'v'}$ can be better represented with a third-order polynomial given as (Wang and Cheng, 2003)

$$-\overline{u'v'} = u_{*m}^2 \left(1 - \frac{y}{h} \right) + (u_{*m}^2 - u_*^2) \left(\frac{y}{h} \right) \left[5 \left(\frac{y}{h} \right) - 4 \left(\frac{y}{h} \right)^2 \right] \quad (2-36)$$

It is also noted that several other assumptions, which could be questionable, were also made in Odgaard's (1984) work. However, his work is commendable in presenting the important concept that the primary flow may be linearly divided into two components, a basic 2D flow and an additive deviation flow caused by the cellular secondary motion. This concept is also applicable for the cases concerned in this study, of which the detail is demonstrated later in Chapter 5 of this report and also in author's recent publication (Wang and Cheng, 2005).

2.6 EFFECTS OF CELLULAR SECONDARY FLOWS ON SEDIMENT TRANSPORT

In the presence of secondary flows, sediment transport will become more complicated than that in 2D flows. However, very few studies have been conducted to identify effects of secondary flows on sediment transport.

2.6.1 BEDLOAD TRANSPORT

Because bedload transport is generally acknowledged to be a function of bed shear stress, spatial variations in bed shear stress that are related to secondary flows could be represented in terms of local discharge of bedload. Such variations were observed in field by Powell et al. (2000) in investigating the cross-stream variation in bedload transport in two coarse-grained ephemeral channels. Their observation shows that on average, transport rates at the center of the channels were about twice the amount of those recorded at the channel margins. They have attributed these local variations in bed load to local difference in bed shear stresses.

In the presence of secondary flows, the bed shear stress τ_b can be resolved into a streamwise component τ_{bx} and a transverse component τ_{bz} . Generally, τ_{bz} is a small fraction of τ_{bx} , and their ratio is a measure of the strength of secondary flows. When sediment grains are subjected to transverse bed shear stress τ_z above the threshold for bedload transport, they would move laterally. A bed ridge may thus form as long as the τ_{bz} acting upslope on each moving grain exceeds the downslope gravitational force. A limiting and equilibrium side-slope θ_z of the bed ridge would appear when the two forces are equal (Allen, 1984). With this consideration, the limiting slope can be given by equating the transverse shear stress and gravity forces in the form of

$$\sin \theta_z = \frac{3\tau_{bz}}{2d(\rho_s - \rho)g} \quad (2-37)$$

in which θ_z = side-slope angle of ridges; τ_{bz} = transverse component of bed shear stress; d = sediment particle diameter (assumed spherical); ρ_s = sediment particle

density; ρ = fluid density; and g = gravitational acceleration. Eq. (2-37) indicates that the side slope increases with increasing lateral bed shear stress, but decreases with increasing particle size and submerged density.

To predict evolution of sediment strips, Tsujimoto and Kitamura (1996) proposed a sediment transport model in combination with an algebraic stress model for secondary flows in a channel without sidewalls. Their bedload transport model incorporated the pick-up rate and step length for each grain size of graded bed material. The lateral bedload flux of the i -th sand fraction is given as follows

$$q_{Bzi} = q_{Bi} \tan \theta_i \quad (2-38)$$

where q_{Bi} is the fractional bedload transport rate due to primary flow and θ_i is the angle of bedload motion with respect to streamwise direction. They are further expressed as

$$q_{Bi} = (1 - po) \delta_B p_i p_{Bi} \Lambda_i \quad (2-39)$$

$$\tan \theta_i = -\sqrt{\frac{1}{f_B}} \sqrt{\frac{\tau_{ci}^*}{\tau_{bi}^*}} \tan \theta_z + \tan \theta_f \quad (2-40)$$

in which po = porosity of sediment; δ_B = thickness of the bedload layer; p_i = volumetric ratio of i -th fraction sediment to the total bed materials in the bed surface layer; p_{Bi} = pick-up probability density of i -th fraction sediment; Λ_i = mean step length of i -th fraction sediment; f_B = friction coefficient of sediment; $\tau_{bi}^* = \tau_b / (\gamma_s - \gamma) d_i$ = dimensionless bed shear stress with respect to i -th fraction sediment; τ_{ci}^* = dimensionless critical bed shear stress; θ_z = lateral slope angle; and $\theta_f = W/U|_{y=0}$, angle of deflection of the flow direction. Using this model, the bedload behavior, as determined by secondary flow and selective motion due to sediment size, was simulated and the subsequent change of bed-surface composition was also predicted. In their study, Tsujimoto and Kitamura (1996) did not mention any spatial variation in the sediment transport rate over rough and smooth strips. However, this problem was approached by McLelland, et al. (1999). They conducted an experiment with a mean bed shear stress exceeding the threshold value for initial motion by 30%. Spatial variations in local bedload transport rates

were then examined. Their experimental results show that the bedload transport rate over the central coarse-grained strip was 21% greater than the transport rate over the fine-grained strip. Similar information has also been presented by Hirano and Ohmoto (1988) and Nezu and Nakagawa (1989) .

2.6.2 SUSPENDED LOAD TRANSPORT

As the limited studies on bedload transport, the studies on suspended load transport in this respect are also very preliminary. Vanoni (1946) might have been the first to link the cellular secondary flow with suspended load transport. He attributed the creation of cellular secondary flow to sediment concentration gradients that were locally unstable in the spanwise direction. However his conceptual idea has not been verified yet. Powell (1946) also believed that the cellular secondary flow was a cause of the unequal distribution of suspended load, rather than a result of it. Allen (1984) considered that the best explanation seemed to be the transverse variation of the bed roughness, which gave rise to a secondary motion and then variations in sediment concentration. It is noted that since Vanoni (1946), only few investigations on this topic have been conducted. This might be because of the difficulties encountered in theoretical considerations, and also in the simultaneous measurements of suspended load concentration and relatively weak secondary flow.

In the context of open channel hydraulics, the investigation done by Chiu and Hsiung (1981) is particularly related to the problem considered here. They attempted to link the distribution of suspended load concentration to the varying primary velocity, secondary flow, and shear stress distribution using a mathematical modeling technique. Their basic consideration was that secondary flow modified the Reynolds shear stress and then the turbulent diffusion processes. Regardless of its importance, the effect of secondary flows is often ignored in the conventional studies of sediment suspension. Hence, the inclusion of secondary flow should receive an emphasis in future.

Other relevant studies are due to Stommel (1949), Tooby et al. (1977), Nielsen (1992), and Farmer and Li (1994), who have studied the trapping mechanism of longitudinal vortices. They have considered that the most important

flow structure in connection with suspended load is that of a vortex with horizontal axis because such vortices are able to trap sediment particles and carry them along over considerable distances. When the fluid acceleration is negligible compared to the acceleration of gravity, the relative velocity between sand and water is everywhere equal to the still water settling velocity ω_0 (Nielsen, 1992). That is, the sediment particle velocity can be written in the form

$$\mathbf{U}_s = \mathbf{U}_f + \boldsymbol{\omega}_0 \quad (2-41)$$

where $\mathbf{U}_s$ = fluid velocity vector; $\mathbf{U}_f$ = sediment velocity vector; and $\boldsymbol{\omega}_0$ = still water sediment settling velocity vector, and its magnitude is ω_0 . Consider the fluid motion of a forced vortex (the velocity is directly proportional to the radius of the vortex-the closer the center, the slower the velocity) with the velocity field on the cross section plane

$$\mathbf{U}_f = \begin{pmatrix} W \\ V \end{pmatrix} = \alpha \begin{pmatrix} -y \\ z \end{pmatrix} \quad (2-42)$$

which is analogous to a rigid body rotating around the origin with angular velocity α , see Fig. 2-18. Substituting Eq.(2-42) into Eq.(2-41) yields the approximate sediment particle velocity

$$\mathbf{U}_s = \mathbf{U}_f + \boldsymbol{\omega}_0 = \alpha \begin{pmatrix} -y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_0 \end{pmatrix} = \alpha \begin{pmatrix} -y \\ z - \omega_0 / \alpha \end{pmatrix} \quad (2-43)$$

Hence, the sediment paths are analogous to the circular fluid paths except for a horizontal shift of ω_0/α , as shown in Fig. 2-18. The closed sediment path indicates a retention zone where sediment particles may be trapped inside if there is no lateral exchange of particles across the boundary of the zone (Stommel, 1949; Nielsen, 1992; and Farmer and Li, 1994). Obviously, such a distribution pattern of suspended particle in the presence of longitudinal vortex is different from that in uniform open channel flows, in which sediment concentration generally increases monotonously from the free surface to the bed.

Although the trapping mechanism was first studied for the longitudinal vortices induced by wave or Langmuir circulation induced by wind in marine environments, it is very heuristic for the present study. Based on the concepts and methods introduced above, preliminary theoretical analysis will be conducted in Chapter 6 for suspended sediment concentration in open channel flows with cellular secondary flows.

Fig. 2-18. Sediment particle with settling velocity ω_0 moving in a force vortex with velocity field U_f given by Eq.(2-42) (after Nielsen, 1992).

2.7 SUMMARY

In open channel flows with generally flat bed, secondary flows often occur due to many causes. Prandtl's first kind of secondary flow refers to the circulation induced by the skewing of mean flow, for example, in curved channels or meandering rivers. Prandtl's second kind of secondary flow refers to the circulation induced by non-homogeneity and anisotropy of turbulence, which often occurs in wide straight open channels. The generation of skew-induced secondary flow is an essentially inviscid process driven by either centrifugal or transverse pressure gradient, while the turbulence-induced secondary flow is mainly generated by the cross-flow imbalance of the Reynolds normal stresses.

The turbulence-induced secondary flow includes the corner flows generated by sidewall and the cellular secondary flow generated by longitudinal bedforms. It is shown that the corner flow structure depends on the aspect ratio of channel width to flow depth, B/h . Based on the experimental findings, Nezu and Rodi (1985) and Nezu et al. (1985) have proposed a criterion for the classification of the narrow and wide open channels. A channel is considered wide (e.g., $B/h > 5$) when its central region is free from secondary flows generated by sidewalls.

Cellular secondary flows can be generated in wide open channel flows by longitudinal bedform. Since longitudinal bedforms are often observed in natural streams, cellular secondary flows would be very common for many flow situations. However, relevant investigations are very limited. Most of them are preliminary, only with the aim of detecting the existence of secondary flow cells. This leads to that the results obtained are largely limited to phenomenological descriptions. In addition, theoretical considerations on the structure of cellular secondary motions and its effect on the primary flow are also very lacking. Regardless of its weakness, the pioneering work done by Ikeda (1981), which is particularly related to this study, can be considered to be indicative for the description of cellular secondary velocity components. In Chapter 4, Ikeda's (1981) ideas will be further explored and improved, which leads to analytical formulations to be developed for the description of the cellular secondary flows for various bed configurations.

In the previous studies, some variations of primary flow structure, which are due to cellular secondary flows, were often simply ignored or misunderstood. Odgaard (1984) appeared to be the first to explore the variation in the primary flows by including effects of secondary flows. However, his model is rather empirical, which has not been convincingly validated by experiment data. In Chapter 5 of this study, a theoretical attempt will be made to propose a new model for the primary flow structure, which is subject to cellular secondary flows.

Because of secondary flows, mechanism of sediment transport is modified, which is more complicated than that developed for 2D flows. The relevant studies in this aspect are very limited, in particular, for sediment suspension. In this study, a framework is presented in Chapter 6, where concentration distributions of suspended sediment in the presence of cellular secondary flows are investigated.

Chapter

3

EXPERIMENTAL FACILITIES AND PROCEDURES

3.1 INTRODUCTION

All experiments were performed in a straight flume with a rectangular cross section at the Hydraulics Laboratory of Nanyang Technological University. Steady turbulent flows over six sets of artificial longitudinal bedforms were investigated. Similar flow conditions were adopted for all experiments. In particular, the flow depth for each experiment was taken to be about the average width of bedforms. This was to minimize variations in the bulk flow conditions so that possible effects of various bedforms on secondary flow structures can be highlighted. A two-dimensional Laser Doppler Anemometer (LDA) was used to measure the streamwise and vertical components of flow velocity, and a one-dimensional Ultrasonic Doppler Velocimeter (UDV) was used to measure the spanwise velocity. The concentration of suspended sediment (refined Kaolin) was detected using an optical backscatterance sensor (OBS-3).

The information presented in this chapter includes descriptions of the flow system, the artificial bedforms, the measuring techniques, the general experimental procedures and the data collection approaches.

3.2 FLUME SYSTEM

The flume system is schematically shown in Fig. 3-1. It consists of the flume itself, pipelines and pumps.

3.2.1 FLUME

The tilting flume used in the study was 14 m long, 0.6 m wide and 0.6 m deep. Its photograph is shown in Fig. 3-2. It had two glass sidewalls and a steel bottom, strengthened together by a cast-iron frame. The slope of the flume was adjustable through jacks. Guiding and stabilizing devices were used at the entrance section to minimize entrance effects on flows at the section tested.

3.2.2 WATER CIRCULATION

Water was circulated by a water pump, passing through the pipelines, head and tail tanks. Flow depth was adjusted by the tailgate. The flow rate was monitored with the flow meter and controlled using the two valves, one being operated manually and the other controlled electronically.

3.2.3 PUMPS

The water pump provided flow rates up to 300 m³/h. An inverter was used to control the pump speed, and thus the flow rate in the open channel. For experiments with sediment transport, sediment particles settled down in the collection tank, and the mixture of sediment and water was then circulated by the sediment pump with a delivery capacity up to 30 m³/h.

Fig. 3-1. Schematics of the flume system.

Fig. 3-2. Photograph of the flume

3.3 ARTIFICIAL BEDFORMS

Six sets of artificial longitudinal bedforms were used for generating cellular secondary flows in the open channel. These bedforms can be categorized into bed strips and bed ridges. The bed strips, being characterized by lateral periodic variations in the bed roughness, comprised of smooth and sediment-roughened plates, of which the surfaces were set at the same level across the channel. In contrast, the bed ridges were characterized by lateral periodic variations in the bed elevation only. These bedforms are further described in the following subsections.

3.3.1 CASE S75: ROUGH/SMOOTH STRIPS (WIDTH RATIO = 1:1)

This bedform consisted of five rough strips and four smooth strips, as shown in Fig. 3-3. They were arranged in an alternate fashion. The two strips near the sidewalls were rough, 37.5 mm in width. The other strips were all 75 mm wide. The total widths of the rough strips and smooth strips were identical. The bed configuration was symmetric with a rough strip placed at the centerline of the flume.

The rough strips were prepared by densely-packed sediment particles. The size distribution of the sediment is plotted in Fig. 3-4. The sediment was generally uniform with a median diameter of 2.55 mm. The PVC plates were glued to the flume bed to form the smooth strips. The rough strip surface was carefully leveled up, being set at the same elevation as that of the smooth strip surface.

Fig. 3-3. Longitudinal bedforms for Case S75: (a) cross section; (b) plan view; and (c) photograph.

Fig. 3-4. Size distribution of sediment particles used for rough strips.

3.3.2 CASE S50: ROUGH/SMOOTH STRIPS (WIDTH RATIO = 2:1)

This bedform is similar to that used for Case R75. The only difference is that the width of each rough strip in the center zone of the flume was increased to 100 mm and that of each smooth strip was decreased to 50 mm, as shown in Fig. 3-5. This yielded the width ratio of rough to smooth strips to be 2:1. The sediment used was also the same as that shown in Fig. 3-4. This case was designed to explore possible changes in the structure of the secondary flow with the varied width ratio in comparison with Case S75.

Fig. 3-5. Longitudinal bedforms for Case S50: (a) cross section; (b) plan view; and (c) photograph.

3.3.3 CASE R75-10: RECTANGULAR RIDGES (Dimensions = 75×10 mm; Width ratio of ridge to trough = 1:1)

The bedform for Case R75-10 consisted of four ridges and five troughs, as shown in Fig. 3-6. Each ridge element was made of PVC plates, 75 mm in width and 10 mm in height. The plates were glued longitudinally to the flume bed with a spacing of 75 mm. The two troughs close to the side walls were set to be 37.5 mm wide so that the total widths of ridges and troughs were identical. The bed configuration was symmetric with respect to the centerline of the flume.

Fig. 3-6. Longitudinal bedforms for Case R75-10: (a) cross section; (b) plan view; and (c) photograph.

3.3.4 CASE R75-5: RECTANGULAR RIDGES (Dimensions = 75×5 mm; Width ratio of ridge to trough = 1:1)

This case was the same as Case R75-10 except that the height of the ridge elements was reduced to 5 mm (see Fig. 3-7). The ridge elements were made of iron plates. The ratio of the ridge to the trough width was 1:1 and the geometry of the bed was symmetrical across the flume. By comparing experimental results obtained for Case R75-10 to those for Case R75-5, we could explore how secondary flows vary with the change in the ridge height.

Fig. 3-7. Longitudinal bedforms for Case R75-5: (a) cross section; and (b) plan view.

3.3.5 CASE R50-10: RECTANGULAR RIDGES (Dimensions = 50×10 mm; Width ratio of ridge to trough = 1:2)

For this case, the ridge was 50 mm wide and 10 mm high. The trough width was increased to 100 mm, and therefore the ratio of ridge to trough widths changed to be 1:2. Fig. 3-8 shows the schematic and photo of this bedform. The ridges were still prepared by PVC plates with 10 mm height. This case was designed for investigating the change in the secondary flow structure, which is subject to different width ratios, when being compared to Case R75-10.

Fig. 3-8. Longitudinal bedforms for Case R50-10: (a) cross section; (b) plan view; and (c) photograph.

3.3.6 CASE WR: WAVY RIDGES (11.7 mm in height)

The bedform used for Case WR consisted of wave-shaped bed ridges, as shown in Fig. 3-9. The flume bed was covered by laterally corrugated iron sheets with painted surfaces. The wavy bedform was used to simulate longitudinal ridges which could develop from a flat sediment bed (see Ohmoto and Hirano, 1988). It is noted that the bed elevation varied gradually in the lateral direction, which is different from the cases with rectangular ridges, i.e., Cases R75-10, R75-5 and R50-10. The

spacing between two neighboring ridge cusps was set to be 150 mm and the ridge height was 11.7 mm.

Fig. 3-9. Longitudinal bedforms for Case WR: (a) cross section; (b) plan view; and (c) photograph.

The ridge shape can be approximated by the following function

$$y/\lambda = 0.157736(z/\lambda)^2 \quad (3-1)$$

where λ = half of the spacing between two neighboring ridge cusps = 75 mm.

Fig. 3-10. Cross section of the wavy bedform. The solid line is computed using Eq. (3-1).

3.4 MEASURING TECHNIQUES

3.4.1 LASER DOPPLER ANEMOMETER (LDA)

A Dantec LDA system, FlowLite 2D, was employed to measure the vertical and longitudinal components of the velocity. As shown in Fig. 3-11, the system comprises of an integrated four-beam, two-component laser-optics unit, laser generator and signal processor, and a traversing system, together with a PC installed with supporting application software, BSA 1.6 (Dantec, 2000).

The light source for the system was a 10 mW HeNe laser generator and a 10 to 200 mW diode-pumped frequency doubled Nd:YAG laser generator. A red HeNe laser beam (632.8 nm) and a green laser beam (532 nm) were generated, each being further split into two separate beams. To detect the flow direction, the frequency of the split beam was shifted by 40 MHz. Four beams were then transmitted into the flow through a 5 m-fiber-optical cable and a probe of 275 mm long and 60 mm in diameter (Flowlite model 60×63). A lens with 400 mm focal length was placed in front of the probe to focus the four parallel beams to a common point. The sampling volume was thus formed at that point, which measured $0.119 \times 0.119 \times 2.51$ mm in the downstream (x), vertical (y), and lateral (z) directions, respectively. The backscattered light was transmitted through the receiving cable and then the light was separated and passed into two photo-multiplier tubes. These tubes acted to amplify the scattered light and converted them into electronic signals. Next, the signals were transmitted to a digital burst correlator, which converted the frequency of the intensity fluctuations into velocity. The velocity was then sent to the PC for further analysis along with accompanying information of particle arrival time and transit time. The data analysis yielded mean velocity and other statistical properties, such as RMS values. With two velocity components being measured simultaneously, the analysis also yielded cross moments, allowing the calculation of Reynolds stresses.

The LDA probe was mounted on a automated traversing system [see Fig. 3-11(a)] that could move within ± 0.1 mm in the three orthogonal directions. With

the software package, data collection was scheduled as required. To facilitate data collected at a satisfied sampling rate with good quality, seeding particles [Titanium (IV) oxide TiO_2 power, less than $5\ \mu\text{m}$ in diameter, see Fig. 3-11(c)] was used to trace flows.

Fig. 3-11. LDA system (Dantec, FlowLite 2D). (a) Optical probe unit and traversing system; (b) Laser generator, signal processor, PC and traversing controller; and (c) seedings.

3.4.2 ULTRASONIC DOPPLER VELOCIMETER (UDV)

A DOP2000 ultrasonic pulsed Doppler velocimeter (model 2125, see Fig. 3-12) manufactured by Signal Processing (Lausanne, Switzerland) was used to measure the transverse velocity. This instrument was able to perform instantaneous velocity measurement at a number of points along the ultrasonic beam simultaneously. As shown in Fig. 3-12, it mainly included a display device with built-in digital

ultrasonic synthesizer and processor, and a series of transducers with various emission frequencies.

Fig. 3-12. UDV system (DOP2000, model 2125). (a) Display unit with built-in digital ultrasonic synthesizer and processor; and (b) four standard transducers with emission frequencies of 1, 2, 4, and 8 MHz.

This instrument was operated largely with an emitter and a receiver (Signal Processing SA, 2002). Its operation principle can be further explained in the following. The emitter shoots periodically ultrasonic beams with a Pulse Repetition Frequency (PRF), while the receiver collects continuously echoes being issued from the seeding particles, which are present in the path of the ultrasonic beam. By sampling the incoming echoes, the displacement of seeding particles are measured, as illustrated by Fig. 3-13, where a particle is assumed to be present along the ultrasonic beam.

Fig. 3-13. Principles of pulsed Doppler ultrasound.

The velocity is thus measured by computing the variation in the particle location between two emissions

$$V_i = \frac{S_2 - S_1}{T_{prf} \cos \theta_p} \quad (3-2)$$

where V_i = the instantaneous velocity of the fluid at the measured location; S_1, S_2 = the particle location from the probe at first emission and second emission, respectively; T_{prf} = the time interval between two emissions; and θ_p = the angle between the trajectory of the particle and the axis of the ultrasonic beam.

On the other hand, from the delay time T_d between the emitted burst and the echo issued from the particle, the particle location can be computed by

$$S = \frac{C_{sound} T_d}{2} \quad (3-3)$$

where C_{sound} = the sound velocity of the ultrasonic wave in the fluid. Substituting Eq. (3-3) into Eq. (3-2) yields

$$V_i = \frac{C_{sound} (T_{d2} - T_{d1})}{2T_{prf} \cos \theta_p} \quad (3-4)$$

Furthermore, the delay time difference, δ_d , can be related to a measurement of the phase shift of the received echo in the form,

$$\delta_d = 2\pi f_e (T_{d2} - T_{d1}) \quad (3-5)$$

where f_e = emission frequency. Therefore, Eq. (3-4) can be re-written as

$$V_i = \frac{C_{sound}}{2f_e \cos \theta_p} \frac{\delta_d}{2\pi T_{prf}} = \frac{C_{sound}}{2f_e \cos \theta_p} f_d \quad (3-6)$$

where $f_d = \frac{\delta_d}{2\pi T_{prf}}$ = the Doppler frequency shift. Eq. (3-6) illustrates that the

velocity can be measured by the Doppler frequency shift.

3.4.3 OPTICAL TURBIDITY MONITOR (OBS-3)

The suspended load concentration in the experiment was measured by an optical turbidity monitor, OBS-3, which was manufactured by D&A Instrument Company. The OBS-3 sensor consisted of a high intensity infrared emitting diode (IRED), a detector (four photodiodes), and a linear, solid state temperature transducer (D&A, 1991). The IRED produced an infrared beam with a wavelength 875 nm. As shown in Fig. 3-14, the infrared beam was emitted radially from a point located on the probe side face that was 11 mm away from the end. The half-power points of the beam were at 50° in the axial plane of the sensor and 30° in the radial plane. The infrared beam emitted into the fluid flow were scattered by suspended matters, and thus the turbidity or suspended solids concentration was measured by the optical backscatter (OBS) detector. The visible light incident on the sensor was absorbed by a filter. The sensor was connected to a CR510 data logger through a 5 m-long cable. With accessory software, the data was downloaded from the data logger to the PC in ASCII format. The measurement range of the OBS-3 was 0~5000 mg/l with an accuracy ± 0.1 mg/s for mud, or 0~50 g/l with an accuracy ± 0.1 g/l for sand, and the sampling rate was 7 Hz.

Fig. 3-14. The OBS sensor and the beam pattern (D&A, 1991).

3.5 EXPERIMENTAL PROCEDURES

Altogether six series of experiments were completed in this study. They were conducted under similar flow conditions but with different bed configuration as mentioned in Section 3.3. Each experiment was started with a general preparation, followed by velocity measurements using the LDA and UDV for the condition of clear-water flow. Then, sediment-concentration measurements were made with the OBS-3 for the sediment-laden flow.

3.5.1 GENERAL PROCEDURE

The procedure described subsequently was generally followed for performing each experiment. First, ridge or strip elements were glued to the flume bed with planned alignment. In particular, for the sediment-roughened bed strips, special care was needed to ensure that the surface of the rough strip was flat with the elevation the same as that of the smooth strip surface.

Then, water was introduced into the flume. After that, the flow discharge and flume slope were adjusted to ensure a steady, uniform flow with desired flow depth. The surface fluctuations were also monitored. If necessary, extra flow guider was used to minimize the free surface fluctuations. The water surface level was measured with a point gauge movable along the flume, from which the average flow slope was obtained. The flow discharge was recorded from the electronic flow meter.

Next, all the instruments involved were examined. The traversing system for positioning the probe was arranged to the desired position. Seeding powders were added into the flow for achieving satisfactory sampling rates. Flow velocities were collected using the LDA and UDV. After that, a suitable amount of Kaolin powder was gradually added into the flow until very slight deposition was observed on the channel bed. Sediment concentration distribution was then measured using the OBS-3.

3.5.2 USE OF LDA AND UDV SYSTEMS FOR CLEAR-WATER FLOW CONDITIONS

Tap water was used for the experiment. Instantaneous velocity was measured for clear-water flow conditions. The test section was located approximately 8.5 m downstream from the channel entrance, where the flow was considered fully developed. The longitudinal and vertical velocities were measured by the LDA system, while the transverse velocity was measured by the UDV system.

3.5.2.1 LDA MEASUREMENT

The LDA velocity measurements were conducted over a half of the cross-section because of the symmetrical bed configuration. As shown later in this report, such experimental results are helpful in identifying the central region of flow that was free from the sidewall effects. This is because the sidewall can also generate secondary flows that are different from those produced, for example, by bed strips.

Two types of measurements were completed using two sets of sampling meshes. The first type (2D) was designed for measuring streamwise and vertical velocities simultaneously, while the second (1D) was to sample streamwise velocity only.

The measuring meshes designed for the 2D measurement for all experiments are schematized in Fig. 3-15. The sampling meshes for the runs S50 and R50-10 comprised of 492 measuring points. These points were located along 31 vertical lines that were evenly spaced 10 mm apart. For the other experiments, S75, WR, R75-10 and R75-5, the sampling meshes had 658 points. The points were located along 41 vertical lines evenly spaced 7.5 mm apart. The measurable region along different vertical line varied, and more points were taken along the vertical lines close to the sidewall, as illustrated in Fig. 3-15. In the central part of the channel, the measurable region ranged approximately from $y = 10$ to 60 mm, and the sampling points was vertically spaced 5 mm apart. More points were added to the meshes for the lines near the sidewall, and 2 or 3 mm vertical spacing was used for the additional points.

Fig. 3-15. Sampling points on the test section for 2D measurement by LDA, where the flow is into the page.

A minimum of 5000 realizations for each point is usually suggested for LDA's users (Dantec, 2000). However, the sampling rate was variable point by point. For most points, it ranged within 100~300 Hz and altogether 20000 sets of data were recorded for each point. However, for the points at lowest positions with smaller velocity and less passing seeding particles, the sampling rate might be only several decades. In such a case, the data collection automatically terminated once the sampling duration reached 300s.

The 1D measurement for the streamwise velocity was conducted along the same vertical lines as indicated in Fig. 3-15. Each line contained more points than those for the 2D measurement, especially in the region near the bed. The vertical spacing between neighboring points varied with the distance from the bed. Smaller spacing was adopted near the bed because of the large velocity gradient, and the finest resolution was about 0.1 mm. The flow conditions and LDA setting parameters remained the same as those for the 2D measurement. 20000 sets of data were also recorded at most of the positions, or the data collection automatically terminated at the sampling duration of 300 s. The sampling rate was about 100~300 Hz, but it became much lower for the points very close to the bed.

The measured instantaneous velocities were analyzed together with the information of seeding particle arrival time and transit time. The data analysis yielded the mean velocities and other statistical properties. For each burst the arrival time and the transit time of the seeding particle were recorded along with the velocity components (u_i, v_i). The formulas for computing the time-mean streamwise and vertical velocities and their cross-moment are listed below

$$\text{Mean velocities} \quad U = \sum_{i=0}^{N-1} \varpi_i u_i, \quad V = \sum_{i=0}^{N-1} \varpi_i v_i \quad (3-7)$$

$$\text{Cross-moment} \quad \overline{(u-U)(v-V)} = \sum_{i=0}^{N-1} \varpi_i (u_i - U)(v_i - V) \quad (3-8)$$

where U and V = the time-mean streamwise and vertical velocity; ϖ_i = the weighting factor; and N = the sampling number. Normally, the weighting factor was computed based on the transit time as follows

$$\varpi_i = t_i / \sum_{j=0}^{N-1} t_j \quad (3-9)$$

With a constant transit time, the weighting factor reduces to $1/N$.

In plotting contours of a particular quantity measured at the cross section, the sampling meshes (Fig. 3-15) were extended to a complete rectangular size, in which unknown values at additional points were extrapolated from the measured data using kriging (McLelland, et al., 1999; Wang, et al., 2003).

3.5.2.2 UDV MEASUREMENT

The measurement of transverse velocity using the UDV was conducted for all cases except for Case R75-5. This is because Case R75-5 was completed when the instrument was not available. Being different from the LDA system, the UDV was used to measure the velocity component which was in the same direction of the axis of ultrasonic beam. It could detect velocities of a great number of points along the emitted ultrasound path simultaneously. Thus, the transverse velocity was measured by simply placing the UDV transducer perpendicular to the outer surface of the glass sidewall of the flume without any intrusion into the flow. An example of horizontal sampling lines along which the transverse velocity was measured by the UDV is shown in Fig. 3-16. These lines crossed over the test section. They were spaced 5 mm apart vertically, which was in accordance with the LDA sampling mesh. The lowest line was just 1 mm above the top of the bed surface and the highest line was 2 or 3 mm below the free surface.

Fig. 3-16. Sampling lines along which the transverse velocity was measured for Case R50-10. They are indicated by the dashed lines. The flow is into the page.

Many efforts were made to ensure that the transducer was placed strictly perpendicular to the sidewall. To exclude air from the interface between the transducer front and the sidewall, the interface was also sealed by gel. Nearly five thousands instantaneous velocity profiles were recorded at each elevation. For each profile, the spatial resolution, i.e. the spacing between two neighboring points along the line was 0.75 mm. Thus, approximately 800 sampling points were taken for each line. The sampling rate was about 18.5 Hz. The formulae for calculating the time-mean velocity, W , is similar to Eq. (3-7). Because the UDV has a constant sampling rate, the weighting factor is identical to $1/N$ for all UDV data.

Near the sidewall, the emitted ultrasound fields were seriously affected by the refraction and the reflection by glass-water interfaces, which brought considerable noises to the measured results. Fig. 3-17 shows an example of the entire transverse profile of W for Case S75, in which apparent noises can be clearly seen in the region close to the both sidewalls. However, the noise was not considerable for the central region of the flow, say, $1.0h$ away from the sidewall.

Fig. 3-17. An example of entire transverse profile of W for Case S75. The circles denote the regions affected by the noise.

3.5.2.3 COMPARISON OF UDV AND LDA MEASUREMENTS

To compare the measurement accuracies of the two techniques, the UDV was also employed to measure flow velocity in the longitudinal direction. The comparison of the primary velocity rather than the vertical velocity was chosen due to the limitation for UDV to measure the latter for the present cases. To measure the vertical velocity, the probe of UDV should be aligned with a line passing through the flume bottom. If doing so, the solid-water interface will cause serious noises in a

range about 10cm, as indicated in Fig. 3-17. The situation would be even worse by noting that the vertical velocity is usually of very small magnitude. Moreover, the flow depth is only around 7.5cm, which is in the badly-affected range. Therefore, the UDV is considered inapplicable for the measurement of the vertical velocity. On the other hand, the longitudinal velocity in the central region of the channel can be reliably measured, as explained in the following.

Considering that the UDV measures the velocity component in the direction of the ultrasonic beam, the real flow velocity can be computed based on the Doppler angle formed by the axis of the ultrasonic beam and the direction of the real velocity vector. This is illustrated in Fig. 3-18. Obviously, the real velocity, V_{real} , can be related to the velocity component measured in the direction of the ultrasonic beam, V_{us} , in the form,

$$V_{real} = \frac{V_{us}}{\cos \theta_p} \quad (3-10)$$

where θ_p = Doppler angle.

Fig. 3-18. Relationship between the measured component of velocity and the module of the real velocity.

The comparison was made by using LDA and UDV to measure longitudinal velocity distributions at different elevations for Case R50-10. The flow conditions involved are given in Table 3-3. The results are plotted in Fig. 3-19. It shows that the two kinds of velocity measurements are very close to each other in the central region of the flow. Significant differences can be observed only near the side wall because of the difficulty inherent in the UDV technique as mentioned earlier.

Fig. 3-19. Comparisons of primary flow velocities measured by LDA (scattered points) and those by UDV (solid curves). U_{ms} is the section-averaged velocity of the primary flow.

3.5.3 USE OF OBS-3 FOR MEASURING SEDIMENT CONCENTRATION

Preliminary tests were also conducted in this study to measure concentration distributions of suspended sediment that was subject to cellular secondary flows. Refined Kaolin was chosen as the suspended sediment because of the flow conditions considered and also the availability of sediment in the lab. To have a considerable suspension of sediment, the ratio of settling velocity to friction velocity should be small enough, e.g. $\omega/\kappa u_* < 0.5$. The flow conditions adopted in the experiments (Table 3-3) indicates that the suitable settling velocity should be less than 5mm/s, which requires the size of the sediment particles to be less than 100 μm . Kaolin was the only available sediment with such small size in the lab. Since the tests were limited to the condition of low sediment concentration, the flow condition remained the same as that used for the flow measurements for individual cases. The amount of Kaolin to be added into the flow was decided by trial and error so that no significant deposition was observed on the channel bed. The concentration measurements were conducted for all cases except for Case R75-5.

The size distribution of Kaolin is shown in Fig. 3-20, which indicates $d_{10} = 1.2 \mu\text{m}$, $d_{50} = 7.62 \mu\text{m}$, and $d_{90} = 19.46 \mu\text{m}$. The specific gravity of the sediment, γ_s , is 2.379. Its settling velocity for median particles at 30°C is 0.0542 mm/s if computed with the Stroke's equation,

$$\omega_{d_{50}} = \frac{gd_{50}^2(\gamma_s - 1)}{18\nu} \quad (3-11)$$

where ν = the kinematic viscosity of fluid.

The calibration of the OBS-3 system was done before the experiments by preparing some samples with known ratios of water to Kaolin. The concentrations of these samples measured using the OBS-3 were correlated with corresponding voltage outputs, yielding the relationship as shown in Fig. 3-21. It can also be described by the following equation

$$C = -0.00035(V_o)^2 + 5.3113(V_o) + 3.5 \quad (3-12)$$

where C is the concentration measured in mini-liter and V_o is the voltage output in mV. Eq. (3-12) was used to convert the measured voltage outputs to the concentration during the experiments.

Fig. 3-20. Particle size distribution of refined Kaolin.

Fig. 3-21. Calibration of OBS-3 Kaolin-monitoring sensor.

Being different from usual approaches, OBS-3 in fact measured the average concentration of a cone-shaped volume in front of the sensor submerged in flow. The size of the measured volume was determined by the distance that the emitted infrared light could reach and the radial angle. However, the exact volume size was unknown. In this study, the sensor was placed 10 cm downstream from sampling points. Before sampling, the OBS-3 was also used to monitor concentration fluctuations. The recording started provided that the fluctuation became stationary. It was also observed that the average sediment concentration remained unchanged before and after the sampling for each experiment.

An example of the sampling positions is shown in Fig. 3-22. These positions were arranged in the left half of the cross section. They were spaced 5 mm and 25 mm apart, respectively, in the vertical and lateral directions. The lowest points were about 18 mm above the bed ridge.

Fig. 3-22. The measuring positions for the OBS-3 (Case R50-10), which are indicated by small circles. The flow is into the page.

3.6 SUMMARY OF EXPERIMENTAL CONDITIONS

The geometrical characteristics of various artificial bedforms used in the experiments are summarized in Table 3-1. The sampling parameters used for the LDA, UDV and OBS-3 systems, including the number of sampling points and lines, and average sampling rates, are listed in Table 3-2.

Table 3-1. Geometrical Characteristics of Bedforms

Case No.	Bedform Type	λ_r (mm)	λ_s (mm)	λ_{rg} (mm)	λ_{tr} (mm)	λ_r / λ_s or $\lambda_{rg} / \lambda_{tr}$	h_{rg} (mm)
S75	Rough/smooth strips	75	75	—	—	1:1	—
S50	Rough/smooth strips	100	50	—	—	2:1	—
R75-10	Rectangular ridges	—	—	75	75	1:1	10
R50-10	Rectangular ridges	—	—	100	50	2:1	10
R75-5	Rectangular ridges	—	—	75	75	1:1	5
WR	Wavy ridges	—	—	—	—	—	11.7

Notes: λ_r = width of rough strip; λ_s = width of smooth strip; λ_{rg} = width of ridge; λ_{tr} = width of trough; h_{rg} = height of ridge.

Table 3-2. Sampling Information

Case No.	LDA				UDV		OBS-3	
	2D		1D					
	Sampling points	Sampling rate (Hz)	Sampling points	Sampling rate (Hz)	Sampling lines	Sampling rate (Hz)	Sampling points	Sampling rate (Hz)
S75	658	250	1107	262	16	18.5	45	7
S50	492	128	806	151	16	18.5	60	7
R75-10	658	184	1148	242	16	18.5	54	7
R50-10	492	145	806	157	16	18.5	60	7
R75-10	658	118	1107	116	—	—	—	—
WR	658	220	902	164	15	18.5	39	7

Note: the sampling rates listed here are the average rates for all sampled points or lines.

The flow conditions are summarized in Table 3-3. The flow depth was chosen to be nearly the same as the average width of strips or ridges. This was determined based on typical values of the ratio of flow depth to wavelength of longitudinal bedforms, which were reported in previous experimental and field observations (McLelland, et al., 1999). The bed slope also varied within a small range. The conditions adopted aimed to minimize variations in the bulk flow conditions, so that effects of the different bedforms on secondary flow structures can be highlighted.

The maximum flow depth h_{max} , minimum flow depth h_{min} , and mean flow depth h_m are sketched in Fig. 3-23. For the cases of rough/smooth strips, there are no variations in the flow depth, both laterally and longitudinally. For the case of ridges, the flow depth varied, with the maximum appearing over the trough bottom and minimum at the ridge cusp. Generally, the average flow depth, h_m , can be defined as

$$h_m = A/\lambda \quad (3-13)$$

where A = the cross-sectional area with a lateral dimension, λ , as shown in Fig. 3-23; and λ = the average ridge or strip width (or half of the bedform wavelength).

Fig. 3-23. Schematics of the flow depth.

The section-averaged flow velocity, U_{ms} , was calculated based on the flow discharge and cross section area; while the average flow velocity for the central zone of flow, U_m , was calculated with the LDA velocity measurements for the area of the central region, A , indicated in Fig. 3-23. The latter will be used for the data analysis conducted subsequently.

Three friction velocities were evaluated in this study. The average friction velocity denoted by $u_{*(J)}$ was calculated from the bed slope, J , as

$$u_{*(J)} = \sqrt{gJh_m} \quad (3-14)$$

In addition, the average friction velocity denoted by $u_{*(\tau)}$ was calculated by extrapolating the Reynolds shear stress profiles with the equations proposed in this study. The third average shear velocity, $u_{*(v)}$, was obtained by fitting measured velocity profiles to the formulas derived. These evaluations were made based on the measurements for the central region as shaded in Fig. 3-23. The procedures for performing the calculations are discussed in detail in Chapter 5.

The Froude number, Fr , is calculated as

$$Fr = \frac{U_m}{\sqrt{gh_m}} \quad (3-15)$$

The Reynolds number, Re , is given by

$$Re = \frac{U_m h_m}{\nu} \quad (3-16)$$

Table 3-3. Flow Conditions

Case No.	Maximum flow depth $h_{\max}$ (m) (1)	Minimum flow depth $h_{\min}$ (m) (2)	Mean flow depth h_m (m) (3)	Flow discharge Q (m^3/s) (5)	Section-averaged flow velocity U_{ms} (m/s) (6)	Central-region-averaged flow velocity U_m (m/s) (7)	Mean energy slope J (‰) (8)	Mean friction velocity (m/s) (9)			Froude number Fr (-) (10)	Reynolds number Re (-) (11)
								from bed slope $u_{*0(D)}$	from stress profile $u_{*0(\tau)}$	from velocity profile $u_{*0(v)}$		
S75	0.0750	0.0750	0.0750	0.021111	0.4691	0.4739	1.2	0.0297	0.0310	0.0339	0.5528	44206
S50	0.0750	0.0750	0.0750	0.018333	0.4074	0.4147	1.0	0.0271	0.0260	0.0335	0.4837	38684
R75-10	0.085	0.075	0.0800	0.022778	0.4745	0.5218	0.6	0.0217	—	—	0.6320	45149
R50-10	0.085	0.075	0.0800	0.024444	0.4989	0.5549	0.7	0.0237	—	—	0.6652	49013
R75-5	0.08	0.075	0.0775	0.022778	0.4898	0.5228	0.9	0.0261	—	—	0.6220	46881
WR	0.081	0.069	0.0782	0.023333	0.5078	0.5167	0.8	0.0248	0.0252	0.0275	0.6016	48358

Chapter

4

STRUCTURE OF SECONDARY FLOWS

4.1 INTRODUCTION

In this chapter, an in-depth description is given of the structure of secondary flows developed over various longitudinal bedforms, which includes phenomenological features of steady longitudinal circulations and distributions of mean vertical and spanwise velocities.

The measured results of the secondary flows are first presented for Case S75, which is considered relatively simple because its rough and smooth bed strips are of the identical width. Based on the experimental observations and kinematic considerations, an analytical model is then proposed to describe velocity distributions of secondary flow cells. .

Similar information is also provided for the other five cases (S50, WR, R75-10, R50-10 and R75-5). It demonstrates how the different bed configurations affect secondary flows and their velocity distributions in the spanwise and vertical directions. Finally, an empirical relationship between maximum vertical velocity and bed configuration is proposed based on the experimental data.

4.2 SECONDARY FLOW OVER LONGITUDINAL STRIPS OF EQUAL WIDTH (CASE S75)

As discussed previously in Chapter 2, longitudinal bedforms in natural streams could appear as longitudinal bed strips with varying roughness heights, which depend on characteristics of flow and bed sediment and lateral sorting processes.

For Case S75, the widths of the rough and smooth bed strips were specifically set to be equal. As demonstrated in this section, this simple configuration can generate secondary flow cells that are almost symmetrical with respect to the circulation centre. In the subsequent sections, secondary flow cells will be shown to be modified by varying strip widths or bed elevation.

4.2.1 EXPERIMENTAL RESULTS

Fig. 4-1 shows the contour map of V for Case S75 in the central region of $-2 < z/\lambda < 0$, where z is the transverse coordinate with $z = 0$ taken at the centreline of the flow channel; and λ is the average width of strips, which is equal to the flow depth h for this case.

Fig. 4-1. Contour plot of V in the central zone for Case S75. U_m is the average velocity in the zone.

The distribution pattern of V varies with the bed configuration. Downflow occurs over the rough strips while upflow over the smooth strips. The cores of the contours are roughly located at the middle of the flow depth above the centreline of each strip. The vertical division between the downflow zone and the upflow zone is just located at the strip interface. The maximum vertical velocity is about 2.0% of the average velocity of the primary flow in the central zone of the channel.

In contrast, the distribution of V is gradually skewed while approaching the sidewall ($-4 < z/\lambda < -2$). This can be seen from Fig. 4-2, in particular, for the region immediate to the sidewall ($-4 < z/\lambda < -3.5$).

Fig. 4-2. Contour plot of V near the sidewall for Case S75.

The above results suggest that the sidewall effect is significant only for the zone which extends about $2h$ from the sidewall. In other words, the occurrence of the secondary motions in the central zone is only related to the presence of the bed strips. Therefore, in the subsequent analysis only the data collected in the central zone are used to examine the characteristics of the cellular secondary flows. Meanwhile, the flow information in the near-sidewall region is used to ensure the correct definition of the central zone for all cases investigated.

Because of the difficulties encountered in measuring the transverse velocity W using the two-dimensional LDA, it was obtained instead by the Ultrasonic

Doppler Velocimeter (UDV). Fig. 4-3 plots the transverse profiles of W at 16 elevations in the central region of the flow channel ($-2 < z/\lambda < 2$). Similar to the vertical velocity, W also varies with the bed configuration in the central region. It undulates transversely with a wavelength identical to that of the bed strips. From the channel bottom to the middle of the flow depth, the amplitude of W decreases gradually to zero, and then it increases while approaching the free surface. It should be mentioned that the W -variations at the lower and upper portions have different phases (e.g., the two profiles measured at $y = 1$ mm and $y = 70$ mm). In Fig. 4-3, the positive W -value indicates the motion from the left to right side, while the negative W -value means the motion from the right to left side.

Fig. 4-3. Transverse profiles of W at various elevations for Case S75. The arrows denote the transverse flow directions.

Fig. 4-3. (continued).

Fig. 4-3. (continued).

Using the measured V and W , the velocity vectors at the test section are plotted in Fig. 4-4. A pair of counter-rotating flow circulations can be easily recognized. Conventionally, they are called secondary flow cells or cellular secondary flows. The width of the flow cells is nearly equal to the strip width, and the height of the flow cells is the same as the flow depth. The mean magnitude of the velocity vectors is about $0.01U_m$ and the maximum magnitude is about $0.02U_m$. Fig. 4-4 shows that two lateral flows meet near the smooth strip and then go upwards, separating near the free surface. The opposite phenomenon can be observed over the rough strip.

Fig. 4-4. Velocity vectors of secondary flows measured for Case S75.

4.2.2 ANALYTICAL CONSIDERATION FOR CASE S75

The above experimental results demonstrate that steady secondary flows can be generated by longitudinal bed strips. The generation mechanism for the secondary flow cells could be explained as follows. First, the lateral variation in the bed roughness definitely induces perturbations to the streamwise bed shear stress τ_b . It could also cause the distributions of other quantities, such as the Reynolds stresses, to be non-uniform in the transverse direction. The imbalance in the disturbed Reynolds stresses then yields lateral motion of the flow. Due to the periodic bed configuration, pairs of counter-rotating circular flows are finally formed. Obviously, this will further leads to variations in the characteristics of the primary flow.

It is already known that evident secondary flow can be generated over longitudinal bed strips. The time-mean secondary flows appear as pairing cells in the cross-sectional plane, as shown in Fig. 4-4, Fig. 4-9, and Fig. 4-15. Such results have also been reported by Müller and Studerus (1979) and Studerus (1982). Secondary flow cells have a vertical dimension same as the flow depth and a transverse dimension same as the mean strip width. Flow cells appear circle-like shapes and secondary flow streamlines should be closed for continuity condition.

Thus, we may find simple stream function $\psi(y, z)$ for secondary flows based on kinematic considerations.

The recurrence of cellular secondary flows in the transverse direction suggests that $\psi(y, z)$ is a periodic function with respect to the transverse coordinate z . For Case S75, the widths of upwelling and downwelling portions are same owing to the equal width of smooth and rough strips. The transverse variation generally possesses a simple wavy pattern like sinusoid, as illustrated by the transverse profiles of W in Fig. 4-3. Therefore, $\sin(\pi z/\lambda + \phi_z)$ is proposed here for describing the transverse variation, where λ is the average strip width (half of the wavelength of bedforms) and ϕ_z is the initial phase in the spanwise direction dependant on the adopted coordinates. In fact, sinusoidal function has been empirically used for cellular secondary flows in previous studies (Ikeda, 1981; Nezu and Nakagawa, 1993; Colombini, 1993; Wang and Cheng, 2005).

Since secondary flows circulate in the cross-sectional plane, they may vary similarly in all directions. Fig. 4-4 shows that secondary flow cells appear to be symmetric with respect to circulation center. Generally, the circulation centre locates over the interface of rough and smooth strips and at the middle of the flow depth. The exact vertical position for circulation centre can not be theoretically determined. The measurement indicates that the circulation centre is nearly at $y/h = 0.5$, where y is the vertical coordinate and h is the flow depth. Small shift of the circulation centre can be observed for real cases. But here an idealized case is considered for simplicity, for which the circulation center is exactly located at $y/h = 0.5$. The discrepancy caused by this simplification will be examined later. Therefore, it is supposed the vertical variation can also be described by a sinusoidal function, say, $\sin(\pi y/h + \phi_y)$, where ϕ_y = the initial phase in the vertical direction. By combining the transverse and vertical functions, the stream function for the secondary flow may be written as

$$\psi = \frac{-V_r L_r}{\pi} \sin(\pi \eta + \phi_y) \sin(\pi \zeta + \phi_z) \quad (4-1)$$

in which V_r, L_r = typical velocity and length scales, respectively; $\eta = y/h$; $\zeta = z/\lambda$. By differentiating the stream function, V and W , can be obtained

$$V = \frac{\partial \psi}{\partial z} = -\frac{V_r L_r}{\lambda} \sin(\pi\eta + \phi_y) \cos(\pi\zeta + \phi_z) \quad (4-2)$$

$$W = -\frac{\partial \psi}{\partial y} = \frac{V_r L_r}{h} \cos(\pi\eta + \phi_y) \sin(\pi\zeta + \phi_z) \quad (4-3)$$

With the idealized conditions that $V(\zeta = 0.0, \eta = 0.5) = V_{\max}$; $V(\zeta = 0.0, \eta = 0.0) = 0$; $W(\eta = 0.5, \zeta = 1.0) = W_{\max}$; and $W(\eta = 0.0, \zeta = 0.0) = 0$, it can be obtained $V_r = V_{\max}$, $L_r = \lambda$ and $\phi_z = \phi_y = 0$. Therefore the specific expressions for stream function and secondary velocities are

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin(\pi\eta) \sin(\pi\zeta) \quad (4-4)$$

$$V = \frac{\partial \psi}{\partial z} = -V_{\max} \sin(\pi\eta) \cos(\pi\zeta) \quad (4-5)$$

$$W = -\frac{\partial \psi}{\partial y} = \frac{\lambda V_{\max}}{h} \cos(\pi\eta) \sin(\pi\zeta) \quad (4-6)$$

Plotted in Fig. 4-5 is an example of the idealized circulation cells, which are computed using Eq. (4-4) for the condition of $h = \lambda = 0.075$ m and $V_{\max} = 0.01$ m/s. The streamlines appear to be two sets of concentric circles, of which the centres are located at the middle of the flow depth above the strip interface. The computed streamlines is consistent with the velocity vectors shown in Fig. 4-4.

Furthermore, with $V_{\max} = 0.02U_m$ known from the measurement, W and V are computed using Eqs. (4-5) and (4-6). Fig. 4-6 shows the comparison between the measured and predicted results of W and V . In the figure, the computed results are denoted by solid lines, and the crosses represent the measured values. All the velocities are normalized by $0.02U_m$. It can be seen that in spite of the simple mathematical formulation proposed for the stream function, the predicted velocity distributions are reasonably in good agreement with the experimental results. In the subsequent sections, the formulation will be further modified for characterizing other irregular secondary flow structures.

Fig. 4-5. Simplified secondary flow cells over rough and smooth strips with equal width. The closed curves are ψ -contours computed using Eq. (4-4).

Fig. 4-6. Comparison between measured and predicted results of V and W for Case S75. The measured values are denoted by crosses and the computed results are denoted by solid lines. The relative velocities, $V/(0.02U_m)$ and $W/(0.02U_m)$, are plotted by scaling with the distance between two neighboring vertical dashed lines, which is taken to be 1.0.

4.2.3 DISCUSSION

The analytical consideration presented in the foregoing section provides an alternative to the Ikeda model. In comparison with the present model, the Ikeda model is subject to several assumptions that are not justified. For example, one of the assumptions is that the transverse Reynolds shear stress could be expressed in terms of secondary velocity gradients, i.e. $-\overline{vw} = \nu_t \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right)$ (Eq. 2-20).

However, as pointed out by Naot and Rodi (1982) and Colombini (1993), any turbulence model based on the eddy viscosity concept must fail to reproduce the turbulence-driven secondary flows. The applicability of the Ikeda model is also limited by the difficulty of determining its two parameters in advance, the mean friction velocity and the variation amplitude of the friction velocity. Usually, it is impossible to directly measure the two parameters, particularly the latter. In addition, it is noted that the Ikeda model was developed only to describe the regular secondary flow cells that have identical width and height. Therefore, it cannot be used for Case S50 and Case WR investigated in this study. Based on these considerations, no attempts are made to quantitatively compare the present analysis with the Ikeda model.

The present model is semi-theoretical, which is derived based only on the kinematic considerations. This is because as mentioned in Section 2.5.2, the equation for longitudinal vorticity, Eq. (2-7) is mathematically unsolvable. However, as shown in the last section, the present model is practically simple but generally correct for describing the secondary flow structure. In comparison, the dynamic model, such as the one by Ikeda (1981) discussed in Section 2.5.2, may involve unjustified assumptions related to Reynolds stresses. As will be demonstrated in Chapter 5 and Chapter 6, the present model for secondary flow structure is also very suggestive for seeking analytical models for primary flow structure and concentration distribution of suspended sediment.

Because of the difficulties encountered in describing the secondary flow velocity in the proximity of the bed, the non-slip condition for W is not considered in the model. However, experimental results show that the model can well predict

the distribution trend of W even immediately above the bed. Note that the lowest and second lowest measuring positions are 1 mm and 5 mm from the bed, respectively. From the practical viewpoint, this model is validate for the major flow region (e.g. $0.05 < y/h < 1$). The understanding of the secondary flow in the small region near the bed requires further investigations.

The local deviations of the measurement from the model prediction shown in Fig. 4-6 (and similar) should mostly result from the limitation of the UDV. It is known that ultrasonic beam of UDV gradually expands along the beam path, which means that sampling volume will increase in size with distance from the transducer. For the transducer used in the experiments that had an emitting frequency of 8 MHz and a Piezo diameter of 5mm, the sampling volume height even reached about 6mm at the middle of the channel. The potential consequence of the beam expansion was that the magnitude of the measured W component could be reduced or amplified since velocity in the proximity was also read. This beam expansion presented most serious problems either near the bed or near the water surface where echoes from the stationary boundary were likely to bias the velocity measurement. That is why there is systematic discrepancy at the two areas. In comparison, the discrepancy between the LDA data and the predicted velocities are smaller and more randomly since the LDA sampling volume was of a tiny size of 0.1mm scale. Moreover, the present model derived based on simple kinematic consideration only provides semi-theoretical but practically easy expressions for secondary flow velocities, rather than gives an exact theoretical solution. Theoretical limitations still exists.

The above arguments are also applicable for other experimental cases.

4.3 SECONDARY FLOW OVER LONGITUDINAL STRIPS OF UNEQUAL WIDTH (CASE S50)

In the preceding case, the width of the strip, whether rough or smooth, are set to be equal. This bed configuration generates regular secondary flow cells, each of them being symmetrical. However, for real longitudinal bedforms observed in laboratory or natural streams, the width of fine-grained strips may not be the same as that of rough-grained strips. The naturally-developed strips depend on bed material composition as well as flow conditions. For example, with less fine grains available on the channel bed, the fine-grained strip may be narrower than the coarse-grained strip, and vice versa. To investigate possible effects of the strip width on secondary flows, examined in this section are the experimental results collected for Case S50, for which different widths are used for the smooth and rough strips, respectively.

4.3.1 EXPERIMENTAL RESULTS

Fig. 4-7 shows the contours of V in the central region ($-2 < z/\lambda < 0$) for Case S50, in which the rough strip is twice the width of the smooth strip. It is clear that the V -distribution is different from that given in Fig. 4-1 for Case S75 but still varies laterally with the changes in the bed configuration. For Case S50, the upflow zone is narrower than the downflow zone, which causes that the magnitude of upwelling velocity becomes slightly larger than that of downwelling velocity. The maximum upwelling velocity is about 2.5% of the mean primary velocity, while the maximum downwelling velocity decreases to about 1.5%. The maximum upwelling and downwelling velocities occur both approximately at $y/h = 0.5$, but above the centrelines of smooth and rough strips respectively. The vertical velocity reduces to zero along the vertical line located at the strip interface.

Fig. 4-8 plots the transverse profiles of W measured at 16 different elevations for Case S50, which are similar to those given in Fig. 4-3 for Case S75. In the central region, these profiles vary periodically in the transverse direction, but the variations are not as regular as those for Case S75. The maximum- and minimum-values of W occur nearly at the strip interface.

Fig. 4-7. Contour plot of V in the central zone for Case S50.

Fig. 4-8. Transverse profiles of W at various elevations for Case S50.

Fig. 4-8. (continued).

Fig. 4-8. (continued).

Using the measured V and W , the velocity vectors at the test section are plotted in Fig. 4-9(a), in which the mean magnitude of the velocity vectors is about $0.011U_m$ and the largest magnitude is about $0.025U_m$. For comparison, Fig. 4-9(b) shows the secondary velocity vectors reported by Studerus (1982), in which the width ratio between the smooth strip and the rough strip is 5:3. For both cases, secondary flow cells can be clearly recognized, and the circulation centre is nearly located at $y/h = 0.5$ over the strip interface. Apparently, the upflow zone is narrower than the downflow zone, and the secondary flow cell is asymmetrical with respect to the circulation centre.

In comparison with Case S75, the results presented for Case S50 imply that secondary flow cells can also be generated regardless of the unequal strip width. Obviously, their structures vary with the width ratio of the strips. However, stable secondary flow cells may not remain if the width of the smooth strip is further reduced so that the upflow is squeezed into a very small zone. Similar phenomenon may also occur by narrowing the rough strip. Such extreme cases are not investigated in this study.

Fig. 4-9. (a) Velocity vectors measured for Case S50; and (b) similar results reported by Studerus (1982).

4.3.2 ANALYTICAL CONSIDERATION FOR CASE S50

As discussed previously for Case S75, owing to the sudden change in the bed roughness, the strongest transverse flow occurs at the interface between the rough and smooth strips. This leads to the observed circulation centre of each flow cell being located above the strip interface. The formed flow cell extends from the centre of the rough strip to that of the smooth strip, but its circulation center is not the same as its geometrical centre because of the unequal strip width. Therefore, the periodic functions, $\sin(\pi\zeta)$ and $\cos(\pi\zeta)$, used for delineating the transverse

variations of V and W for Case S75 [see Eqs. (4-5) and (4-6)] are not applicable for the present case. However, it is also noted that regardless of the lateral shift of the centre of flow cells, the vertical location of the circulation centre is almost fixed at the middle of the flow depth. Thus, the sine function, $\sin(\pi\eta)$, remains unchanged for Case S50.

For the transverse variation, it is assumed here that the variation within each strip takes place still in the sine or cosine fashion, but with the period depending on the strip width. Therefore, the term $\sin(\pi\zeta)$ included in the stream function Eq. (4-4) can be replaced by the following function for Case S50

$$PS_s(\zeta) = \begin{cases} \frac{\lambda_s}{\lambda} \sin \left[\frac{(\zeta - 2m)\lambda}{\lambda_r} \pi \right] & 2m - \frac{\lambda_r}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_r}{2\lambda} \\ \frac{\lambda_s}{\lambda} \sin \left[\frac{(\zeta - 2m - \frac{\lambda_r}{2\lambda})\lambda}{\lambda_s} \pi + \frac{\pi}{2} \right] & 2m + \frac{\lambda_r}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_r}{2\lambda} + \frac{\lambda_s}{\lambda} \end{cases} \quad (4-7)$$

where $m = 0, \pm 1, \pm 2, \dots$; λ_r = the width of the rough strip; λ_s = the width of the smooth strip; and $\lambda = (\lambda_r + \lambda_s)/2$ = the average strip width. Apparently, Eq. (4-7) reduces to $\sin(\pi\zeta)$ when $\lambda_r = \lambda_s$. Fig. 4-10 shows the lateral variation of $PS_s(\zeta)$ computed for the case of $\lambda_r / \lambda_s = 2$.

Fig. 4-10. $PS_s(\zeta)$ for $\lambda_r / \lambda_s = 2$.

With Eqs. (4-4) and (4-7), the stream function for the secondary flow observed in Case S50 can be expressed as

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin(\pi\eta) PS_s(\zeta) \quad (4-8)$$

Using Eq. (4-8) with $h = \lambda = 0.075$ m and $V_{\max} = 0.01$ m/s, two examples of the idealized flow cells are plotted in Fig. 4-11. For Fig. 4-11(a), $\lambda_r = 0.1$ m and $\lambda_s = 0.05$ m, and for Fig. 4-11(b), $\lambda_r = 0.05$ m and $\lambda_s = 0.1$ m.

Fig. 4-11. Simplified secondary flow cells over rough and smooth strips with unequal width: (a) $\lambda_r = 0.1$ m and $\lambda_s = 0.5\lambda_r$; (b) $\lambda_r = 0.01$ m and $\lambda_s = 2\lambda_r$. The closed curves are ψ -contours computed using Eq. (4-8).

By differentiating the modified stream function, V and W are thus derived as

$$\frac{V}{V_{\max}} = -\sin(\pi\eta)PC_s(\zeta) \quad (4-9)$$

$$\frac{W}{V_{\max}} = \frac{\lambda}{h} \cos(\pi\eta)PS_s(\zeta) \quad (4-10)$$

where $PC_s(\zeta)$ is given by

$$PC_s(\zeta) = \begin{cases} \frac{\lambda_s}{\lambda_r} \cos \left[\frac{(\zeta - 2m)\lambda}{\lambda_r} \pi \right] & 2m - \frac{\lambda_r}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_r}{2\lambda} \\ \cos \left[\frac{(\zeta - 2m - \frac{\lambda_r}{2\lambda})\lambda}{\lambda_s} \pi + \frac{\pi}{2} \right] & 2m + \frac{\lambda_r}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_r}{2\lambda} + \frac{\lambda_s}{\lambda} \end{cases} \quad (4-11)$$

It is noted that the integrations of $PS_s(\zeta)$ and $PC_s(\zeta)$ for a region covering two neighboring strips, say, for $0 \leq \zeta \leq 2\lambda$, are equal to zero. This satisfies the condition of mass conservation for any elevation in the flow, i.e. the total flux induced by the upflow being equal to that by the downflow and the total flux induced by the right-directed flow being equal to that by the left-directed flow.

With $V_{\max} = 0.025U_m$ known from the measurement, W and V for Case S50 can be predicted using Eqs. (4-9) and (4-10). Fig. 4-12 shows the comparison between the measured and predicted results.

Fig. 4-12. Comparison between measured and predicted results of V and W for Case S50. The measured values are denoted by crosses and the computed results are denoted by solid lines. The relative velocities, $V/(0.025U_m)$ and $W/(0.025U_m)$, are plotted by scaling with the distance between two neighboring vertical dashed lines, which is taken to be 1.3.

In the preceding analysis, it is assumed that the circulation centre is located exactly above the strip interface. However, this may not hold for some cases. For example, the circulation centre tends to shift slightly from the strip interface towards the geometrical center of the circulation cell (see Fig. 4-9). This might be due to the reason that a stable circulating cell tends to be geometrically symmetrical. Therefore, λ_r and λ_s may not represent the exact widths of the downflow and upflow zones, respectively. Moreover, as mentioned before, there may be no clear dividing lines between the coarse-grained and fine-grained strips due to the lateral sorting process in natural streams, and thus λ_r and λ_s are even unknown. Given the above reasons, in applying Eqs. (4-7) and (4-11) for describing the periodic transverse variation of the flow field, it is more reasonable to replace the widths of the strips with the lateral dimensions of the upflow and downflow zones. Therefore,

$PS_s(\zeta)$ and $PC_s(\zeta)$ can be replaced by $PS(\zeta)$ and $PC(\zeta)$, respectively, which are given by

$$PS(\zeta) = \begin{cases} \frac{\lambda_{up}}{\lambda} \sin \left[\frac{(\zeta - 2m)\lambda}{\lambda_{dn}} \pi \right] & 2m - \frac{\lambda_{dn}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{dn}}{2\lambda} \\ \frac{\lambda_{up}}{\lambda} \sin \left[\frac{(\zeta - 2m - \frac{\lambda_{dn}}{2\lambda})\lambda}{\lambda_{up}} \pi + \frac{\pi}{2} \right] & 2m + \frac{\lambda_{dn}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{dn}}{2\lambda} + \frac{\lambda_{up}}{\lambda} \end{cases} \quad (4-12)$$

$$PC(\zeta) = \begin{cases} \frac{\lambda_{up}}{\lambda_{dn}} \cos \left[\frac{(\zeta - 2m)\lambda}{\lambda_{dn}} \pi \right] & 2m - \frac{\lambda_{dn}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{dn}}{2\lambda} \\ \cos \left[\frac{(\zeta - 2m - \frac{\lambda_{dn}}{2\lambda})\lambda}{\lambda_{up}} \pi + \frac{\pi}{2} \right] & 2m + \frac{\lambda_{dn}}{2\lambda} \leq \zeta \leq 2m + \frac{\lambda_{dn}}{2\lambda} + \frac{\lambda_{up}}{\lambda} \end{cases} \quad (4-13)$$

where λ_{dn} , λ_{up} = breadths of the downflow and upflow zones, respectively; and λ = the average width of strips, which is the same as the average breadth of downflow and upflow zones, i.e., $\lambda = (\lambda_{dn} + \lambda_{up})/2$.

Then, ψ , W and V can be re-written respectively to be

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin(\pi\eta) PS(\zeta) \quad (4-14)$$

$$\frac{V}{V_{\max}} = -\sin(\pi\eta) PC(\zeta) \quad (4-15)$$

$$\frac{W}{V_{\max}} = \frac{\lambda}{h} \cos(\pi\eta) PS(\zeta) \quad (4-16)$$

4.4 SECONDARY FLOWS OVER LONGITUDINAL RIDGES OF WAVY SHAPE (CASE WR)

In the last two sections, we have examined the structures of the secondary flows induced by lateral variations in the bed roughness. Since longitudinal bedforms may also be characterized by the lateral undulation in the bed elevation, which is usually called sand ridge, its effect on secondary flows is investigated in this section. To simulate possible lateral variations in the bed surface elevation, separate bed ridges with a trapezoidal section was earlier used by Nezu and Nakagawa (1984) in their exploratory experiment. It should be mentioned that the trapezoidal section may be quite different from real observations. In this study, the lateral variation of the bed elevation is modelled to be wave-shaped with small amplitude. This is to be consistent with the fact that the bed surface elevation in general increases gradually from trough to ridge. The wavy shape of the sand ridges used in the experiment was made largely based on the laboratory observation of the sand ridges developed from a flat bed (Ohmoto and Hirano, 1988).

4.4.1 EXPERIMENTAL RESULTS

Fig. 4-13 shows the contour map of V for Case WR. The distribution pattern is very similar to those obtained for Cases S75 and S50. The difference is that for Case WR, the downflow occurs over the trough while the upflow over the crest. The upflow zone is more concentrated around the ridge cusp and thus the upflow is nearly twice stronger than the downflow. The alternate appearance of upflow and downflow zones signifies the existence of secondary flow cells. The maximum upwelling velocity is about 1.6% of the average velocity of the primary flow.

Fig. 4-14 plots the transverse profiles of W measured at 15 different elevations. It shows that W undulates in the transverse direction, with a fluctuation period the same as the wavelength of bedform. In Fig. 4-14, the positive velocity indicates the left-to-right motion, while the negative value indicates the right-to-left motion. In the lower portion of the flow the transverse motion is directed from the trough to the crest, while in the upper portion the transverse motion occurs in the opposite direction.

Fig. 4-13. Contour plot of V in the central zone for Case WR.

Fig. 4-14. Transverse profiles of W at various elevations for Case WR.

Fig. 4-14. (continued).

Fig. 4-14. (continued).

Being derived from the measured V and W , a plot of cross-sectional velocity vectors is shown in Fig. 4-15(a). The counter-rotating secondary flow cells are very similar to those measured for Cases S75 and S50. The near-bed flow separates over the bed trough but unites over the bed ridge, while the opposite situation occurs near the free surface. Obviously, these flow cells measure as wide as the ridge spacing and as high as the flow depth. The mean magnitude of the velocity vectors shown in Fig. 4-15(a) is $0.007U_m$ and the largest magnitude is about $0.016U_m$.

In the experiment of Nezu and Nakagawa (1984), cellular secondary flows were generated by separate bed ridges. The ridge used by them had a 45° trapezoidal cross section, which was 5 mm thick and 20 mm wide, and placed with a spacing of twice the flow depth of 80 mm. Secondary flows were measured using X-type hot-film anemometers, and the results are reproduced in Fig. 4-15(b). It can be seen that the pair of secondary cells is similar to those observed over the wavy ridges.

Fig. 4-15. (a) Velocity vectors of secondary flows measured for Case WR; and (b) similar results for the case of the ridge of trapezoidal section reported by Nezu and Nakagawa (1984).

4.4.2 ANALYTICAL CONSIDERATION FOR CASE WR

The above-mentioned results demonstrate that cellular secondary flows can also be generated by the wavy ridge without lateral roughness variations. The cellular structure is very similar to that related to the alternate bed strips with different roughness heights. By comparing the two conditions, one may infer that the bed trough serves equivalently as the rough strip, while the bed ridge is comparable to the smooth strip. This further implies that there exist similar generation mechanisms of secondary flows for both cases. This can be explained by considering the fact that lateral bed perturbations either in bed roughness or elevation always cause

lateral gradient of the bed shear stress. The heterogeneous distribution of the bed shear stress then causes lateral imbalance of turbulent stresses, which eventually generates the streamwise vortex.

It should be noted that the side slope of ridges or the transition between crest and trough plays a very important role in producing secondary flow cells. Since the lateral flow near the bed tends to move from the trough to the crest, the side slope should not be too steep. Otherwise, the lateral flow would be obstructed and thus the whole secondary flow structure could be quite different. Such cases will be examined in the next section.

Fig. 4-15(a) shows that for Case WR, the circulation centre is nearly located at the mid-depth, but its transverse location apparently deviates from the midpoint between the crest and the trough. The upflow is concentrated around the ridge crest, and the width ratio of downflow to upflow is about 3/2. The concentrated upflow phenomena can also be observed for the case of trapezoidal ridges, as shown in Fig. 4-15(b), where the circulation center occurs above the side slope of the ridge elements.

Given the similarity between the roughness-induced and elevation-induced secondary flows, it is assumed that the circulation cells over the wavy ridges can also be described by Eq. (4-14). Considering the variation in the bed elevation, however, the following transformation should be made for the vertical coordinate, i.e. by taking

$$\eta = \frac{y - b}{h} \quad (4-17)$$

where b = the local bed surface elevation measured from the datum; $y - b$ = the local vertical distance from the bed surface; and h = the local flow depth. The transverse coordinate, z , is still normalised using λ . Here, it should be noted that λ , which was formerly defined as the average strip width, is the half spacing between two adjacent ridge cusps. Therefore, the stream function for the wavy ridge case is given by

$$\psi = -\frac{\lambda V_{\max}}{\pi} \sin\left(\pi \frac{y-b}{h}\right) PS(\zeta) \quad (4-18)$$

where $PS(\zeta)$ is defined by Eq. (4-12).

Fig. 4-16 shows the idealized flow cells over two types of ridges, of which the streamlines are computed using Eq. (4-18) with $h_{\max} = \lambda = 0.075$ m and $V_{\max} = 0.01$ m/s. In Fig. 4-16(a), the ridge used is similar to that employed for Case WR, and the computed results demonstrate that the width ratio of the downflow to upflow, $\lambda_{dn}/\lambda_{up}$, equals 1.5. In comparison, the ridge used in Fig. 4-16(b) is similar to that given by Nezu and Nakagawa (1984), which leads to the flow width ratio being increased to 4.0.

Using the modified stream function given by Eq. (4-18), the velocities, V and W , can be derived as

$$\frac{V}{V_{\max}} = -\sin(\pi\eta) PC(\zeta) - \frac{\lambda(1-\eta)}{h} \frac{\partial h}{\partial z} \cos(\pi\eta) PS(\zeta) \quad (4-19)$$

$$\frac{W}{V_{\max}} = \frac{\lambda}{h} \cos(\pi\eta) PS(\zeta) \quad (4-20)$$

where $PC(\zeta)$ is defined by Eq. (4-13). Eq. (4-19) indicates that the vertical velocity is also related to the lateral gradient of the bed, i.e., $\partial h/\partial z$.

To compare with the experimental data, V and W are computed using Eqs. (4-19) and (4-20) with $V_{\max} = 0.016U_m$ and $\lambda_{dn}/\lambda_{up} = 1.5$ known from the measurement. The results are plotted in Fig. 4-17, which demonstrate that the computation is in good agreement with the measurements.

Fig. 4-16. Simplified secondary flow cells over (a) wavy ridges and (b) trapezoidal ridges.

Fig. 4-17. Comparison between measured and predicted results of V and W for Case WR. The measured values are denoted by crosses and the computed results are denoted by solid lines. The relative velocities, $V/(0.016U_m)$ and $W/(0.016U_m)$, are plotted by scaling with the distance between two neighboring vertical dashed lines, which is taken to be 1.0.

4.5 SECONDARY FLOWS OVER LONGITUDINAL RIDGES OF RECTANGULAR SHAPE (CASES R75-10, R50-10, R75-5)

From section 4.4, it is known that cellular secondary flows can be generated by the wave-shaped longitudinal ridges. As mentioned earlier, the wavy ridge provides a gradually-varied lateral bed slope, which serves as a smooth transition to direct the transverse flow from the trough bottom to the ridge top. This is actually a basic condition for the formation of a complete flow cell, which extends laterally from the trough bottom to the ridge top. In this section, we explore what happens to the induced secondary flows if longitudinal bed ridges are prepared without gradually-varied bed surface. An extreme condition considered here is that longitudinal ridges were prepared with a rectangular cross section. Such conditions are quite artificial, but they can be also observed in field, for example, for ridges formed by scoured cohesive bed sediment (see Fig. 2-13). The relevant experiments conducted in this study include Cases R75-10, R50-10 and R75-5.

With the channel bed covered by rectangular ridges, the transverse flow near the bed would be impeded by the vertical side of the rectangle that connects ridge and trough. Moreover, the vertical connection may also induce upwelling flows, which further affects the near-bed flow field. Therefore, vortices formed near the bed may not be easily extended from the trough bottom to the ridge top. The flow cells, if they can be generated, may have a breadth shorter than the width of the ridge. The large-scale secondary flow structure is thus different from that over the wavy or trapezoidal ridges. However, there is almost no experimental evidence reported to date. This section will treat the secondary flows over such rectangular ridges based on the experimental results obtained for Cases R75-10, R50-10 and R75-5.

4.5.1 EXPERIMENTAL RESULTS

Fig. 4-18 shows the vertical velocity contours over three types of ridge elements, of which the width and height are 75×10 mm (R75-10), 50×10 mm (R50-10), and 75×5 mm (R75-5), respectively.

Fig. 4-18. Contour plot of V in the central zones for Cases R75-10, R50-10 and R75-5.

The vertical velocity magnitudes for Cases R75-10 and R50-10 are almost the same since the ridge height for the two cases are identical. The maximum upwelling velocity is about 1.5% of the average velocity of primary flow, and the maximum downwelling velocity is about 0.5%. The vertical velocity for R75-5 is relatively smaller since the ridge height is smaller. The relative maximum upwelling velocity is about 1.0%, and the relative maximum downwelling velocity is about 0.4%.

Fig. 4-19 and Fig. 4-20 show that the transverse velocity profiles measured for Cases R75-10 and R50-10, respectively. Compared with the W -profiles for the preceding cases, the W -profiles in the presence of the rectangular ridges exhibit more fluctuations with smaller amplitudes. Apparent transverse velocity occurs only in the lower portion of the flow. Generally, W varies with the lateral changes in the bed surface elevation. It appears to be largest at the location above the transition from the trough to the crest, and becomes zero above the midpoints of the ridge and trough zones. When approaching the free surface, the variation amplitude decreases gradually. Small random variations are observed near the free surface, which may be due to the free-surface fluctuations.

Using the data of V and W discussed above, the velocity vectors for the tested cross section for Cases R75-10 and R50-10 are plotted in Fig. 4-21. It can be seen that the relatively strong upflow occurs over a small region near the vertical side of the ridge, while the downflow occurs over the crest as well as the trough. The maximum secondary velocity appears above the side of the ridge. When compared with those induced by the wavy boundary and roughness variations, the secondary flows related to the rectangular ridges are much weaker and less organised. They appear in pairs above the ridge zone as well as the trough zone. The vertical dimension of the flow cells is shorter than the flow depth; for the upper portion of the flow, the secondary flow becomes insignificant.

From the three cases discussed in this section, it follows that the secondary flow structure is closely related to the placement and size of the ridge elements and the flow depth. The transverse dimension of the flow cell is generally determined by the width of the ridge or the trough. If the ridge width is too small, there may be no

stable cells observed [see Fig. 4-21(b) for Case R50-10]. This is possibly because the lateral space is not large enough so that a complete flow cell can develop. On the other hand, it is the ridge height that controls the maximum velocity magnitude, and the flow depth that limits the maximum height of flow cells.

Fig. 4-19. Transverse profiles of W at various elevations for Case R75-10.

Fig. 4-19. (continued)

Fig. 4-20. Transverse profiles of W at various elevations for Case R50-10.

Fig. 4-20. (continued)

Fig. 4-21. Velocity vectors of secondary flows measured for (a) Case R75-10 and (b) Case R50-10.

4.5.2 DISCUSSION

The experimental results presented above have demonstrated that the secondary flow structure associated with the rectangular ridges has its own features compared with those over roughness strips and wavy ridges. First, there exist two pairs of flow cells within the region covering one ridge and one trough. In contrast, there is only one pair of flow cells over the same lateral distance for the cases of roughness strips and wavy ridge. Second, the flow cells associated with the rectangular bedforms are much weaker and less organized.

These differences arise largely from the existence of the vertical side of the rectangular ridges, where the flow is subject to the sudden change in the bed elevation between the ridge and trough. The change definitely causes the imbalance of turbulent stresses between the ridge zone and trough zone, and thus induces transverse flows near the bed. However, the development of the transverse flows is impeded locally by the transition. In fact, the secondary flow so generated is similar to that observed in a compound open channel, where the ridge and trough are compared to floodplain and main channel, respectively. However, it should be noted that the lateral change in the bed elevation for the compound channel flow is usually much larger.

Due to the complexity of the secondary flow structure developed over the rectangular ridges, as sketched in Fig. 4-22, it is impossible to simply extend the mathematical model proposed in section 4.1 for the description of the secondary flow velocities. Therefore, no analytical formulations of the velocity distributions for Cases R75-10, R50-10 and R75-5 are provided here.

Fig. 4-22. Schematics of secondary flow cells over rectangular ridges.

4.6 SECONDARY FLOW INDEX AND MAXIMUM VERTICAL VELOCITY

The models proposed in Sections 4.2 to 4.4 require the maximum vertical velocity $V_{\max}$ to be determined in advance. This section is to seek an empirical relationship between $V_{\max}$ and bed configurations.

It is already known that lateral variations either in bed elevation or bed roughness will cause perturbations to streamwise bed shear stress τ_{bx} . Simultaneously, the lateral imbalance of turbulent stresses in the flow field is induced, which will consequently generate cellular secondary flows. One can expect that the intensity of secondary flows is closely related to the maximum difference in bed elevation and/or roughness height, i.e. their variation amplitudes. The secondary flow intensity should be also affected by the primary flow since the overall turbulence strength is determined by the bulk flow properties. To quantify the secondary flow intensity, we introduce a secondary flow index, i.e., the spatially-averaged magnitude of vertical velocity

$$|V|_m = \frac{\iint |V| dydz}{\iint dydz} \quad (4-21)$$

The index $|V|_m$ should depend on the following variables,

$$|V|_m = f(\Delta b, \Delta(n^2), h_m, n_m^2, U_m) \quad (4-22)$$

where $\Delta b = b_{\max} - b_{\min}$, the amplitude of varying bed elevation, in which $b_{\max}$ is the highest bed elevation and $b_{\min}$ is the lowest bed elevation; $\Delta(n^2) = n_{\max}^2 - n_{\min}^2$, the amplitude of varying square Manning coefficient that represents the variation of bed roughness; $n_{\max}$ is the maximum Manning coefficient; $n_{\min}$ is the minimum Manning coefficient; h_m = the mean flow depth; n_m = mean Manning coefficient; and U_m = the mean primary velocity. The selection of n^2 rather than n is due to the following reason.

In this relationship, the time-mean properties of primary flow are described by three independent factors, h_m , n_m and U_m , from which the other factors, such as the mean bed shear stress τ_{bxm} , can be estimated using conventional hydraulic equations. The bed roughness is represented by $\Delta(n^2)$ because the amplitude of bed shear stress $\Delta\tau_{bx}$ is more likely proportional to it rather than Δn if the following equation is supposed to be retained

$$\tau_{bx} = \frac{\gamma U_m^2}{h^{1/3}} n^2 \quad (4-23)$$

where τ_{bx} = the bed shear stress in the x direction; U_m = the depth-averaged velocity for uniform open channel flows. Meanwhile, $\Delta\tau_{bx}$ may be linearly related to Δb if the following relationship is assumed to be generally correct

$$\tau_{bx} = \gamma J h = \gamma J (H - b) \quad (4-24)$$

where γ = the weight of the fluid; J = mean flow slope; h = flow depth; H = the elevation of free surface, and b = the bed elevation.

In a dimensionless form, Eq. (4-22) may be expressed as

$$\frac{|V|_m}{U_m} = f\left(\frac{\Delta b}{h_m}, \frac{\Delta(n^2)}{n_m^2}\right) \quad (4-25)$$

On the other hand, substituting Eq. (4-15) into Eq.(4-21) yields

$$|V|_m = \frac{\int_0^\lambda \int_0^h \left| -\frac{V_{\max}}{\pi} \sin(\pi\eta) PC(\zeta) \right| dy dz}{\int_0^\lambda \int_0^h dy dz} = \frac{\frac{4\lambda_{up} h V_{\max}}{\pi^2}}{\lambda h} = \frac{\lambda_{up}}{\lambda} \frac{4V_{\max}}{\pi^2} \quad (4-26)$$

where λ_{up} = the width of the upflow zone in a flow cell; and λ = the average width of strips. This equation indicates that $V_{\max}$ linearly varies with the ratio between upflow and downflow zones of secondary flow cells,

$$V_{\max} \propto \frac{\lambda}{\lambda_{up}} |V|_m \quad (4-27)$$

Such a relationship is physically reasonable. For a flow cell with the same intensity, the narrower zone for upflow will lead to a greater maximum upward velocity for the continuity condition, and vice versa. Therefore, from Eqs. (4-25) and (4-27), it is possible to express $V_{\max}$ as

$$\frac{V_{\max}}{U_m} \frac{\lambda_{up}}{\lambda} = f\left(\frac{\Delta b}{h_m}, \frac{\Delta(n^2)}{n_m^2}\right) \quad (4-28)$$

In the following, Eq. (4-28) is substantiated using the experimental data. The Manning coefficient for the smooth PVC bed strip used for the experiment is taken as 0.01. For the rough bed strips, it can be estimated based on the relevant roughness size with the Strickler-type relationship (Strickler, 1923), which states

$$n = \frac{d_{50}^{1/6}}{K} \quad (4-29)$$

where $K = 21.1$. The average Manning coefficient n_m can be calculated using Eq. (4-23) given the energy slope, J , and the average flow velocity, U_m .

Fig. 4-23 shows the empirical relationship of $V_{\max}$ and $\Delta b / h_m + \Delta(n^2) / n_m^2$. The experimental data used are listed in Table 4-1. They include the results presented in this study and those given by Nezu and Nakagawa (1984). For the latter data, U_m is roughly taken as $U_{\max} / 1.15$ since U_m is not provided. In the upper range of the available, the relationship seems to be a linear line that intercepts with a positive value. However, the intensity of secondary flows should reduce to zero if the perturbation of the lateral variation in the bed roughness and surface elevation is vanishingly small. It is simply supposed to follow an exponential reducing form $1 - e^{-x}$ while approaching zero. Combining the two features and fitting with the experimental data, an empirical relationship is then obtained

$$\frac{V_{\max}}{U_m} \frac{\lambda_{up}}{\lambda} = (0.0067E + 0.0122)[1 - \exp(-15.4E)] \quad (4-30)$$

where $E = \Delta b / h_m + \Delta(n^2) / n_m^2$. Fig. 4-23 indicates that $V_{\max}$ increases rapidly with E for $E < 0.2$, whereas it increases moderately with E in a linear fashion for $E > 0.2$.

Table 4-1. Maximum Vertical Velocity and Bed Conditions

Case No.	$\Delta b / h_m$	$\Delta(n^2) / n_m^2$	$\frac{V_{\max}}{U_m} \frac{\lambda_{up}}{\lambda}$
S75	–	1.136811	0.02
S50	–	1.044629	0.016667
Wang and Cheng (2005)	–	1.749476	0.025
Müller and Studerus (1979)	–	1.754217	0.0242
WR	0.153453	–	0.0128
S75-10	0.125	–	0.012
S75-5	0.0645	–	0.0056
S50-10	0.122	–	0.012
Nezu and Nakagawa (1984) (Case K)	0.128205	–	0.011213
(Case I)	0.217391	–	0.012938
(Case J)	0.128205	–	0.012

Fig. 4-23. Relationship between maximum vertical velocity and variation magnitude of bed roughness or elevation.

4.7 SUMMARY

Various secondary flow structures developed over six different longitudinal bedforms are investigated in this chapter. The information presented includes the phenomenological descriptions of steady longitudinal circulations and the distributions of mean vertical and transverse velocities.

The experimental results demonstrate that in wide open channels secondary flows could be definitely generated by the longitudinal bedforms. With bed roughness or elevation that undulates in the lateral direction, the induced secondary flows appeared as a series of counter-rotating circulations. The results are summarized as follows.

(1) Experiments with different roughness strips (Cases S75 and S50)

For the two cases, the open channel flows over longitudinal strips with alternate roughness heights were experimentally observed. The results show that the downflow occurs over the rough strip while the upflow occurs over the smooth strip. Near the bed, the lateral flow is directed from the rough strip to smooth strip. The strongest lateral flow usually occurs over the interface between the rough and smooth strips due to the sudden change in the bed roughness. The opposite flow situation can be observed near the free surface. A complete flow cell in the dimension of the flow depth extends laterally from the centre of the rough strip to that of the smooth strip.

The circulation centre of flow cells is nearly located at the mid-depth above the strip interface. For Case S75, because of its equal strip width, the formed flow cell appears to be axisymmetrical. For Case S50, the rough strip is twice wider than the smooth strip so that the flow cell is skewed, of which the upflow zone is narrower than the down flow zone.

(2) Experiments with wavy bed ridges (Case WR)

For the open channel flows over the wave-shaped bed ridges, the generated secondary flow is very similar to those observed for the case of roughness strips.

The results show that in the sense of cellular circulation, the bed trough is analogous to the rough strip while the ridge crest is comparable to the smooth strip. The flow near the bed is laterally directed from the bed trough to the crest. The circulation centre is located nearly at the mid-depth, but its transverse location deviates from the midpoint between the crest and trough bottom. The lateral slope of the bed ridge, or the gradual variation in the bed elevation between crest and trough, plays a very important role in producing secondary flow cells. It enables the fluid to move smoothly from the trough bottom to ridge top.

(3) Experiments with rectangular bed ridges (Cases R75-10, R75-5, R50-10)

The experimental results demonstrate that the secondary flow structure developed over rectangular ridges has different features, in comparison with those observed for the cases of roughness strips and wavy bed ridges. First, there are two pairs, rather than one pair, of flow cells formed in a lateral region with a width of a bedform wavelength. Second, the flow cells are much weaker and less organized. Such differences arise from the existence of the vertical connection between the ridge and trough.

(4) Analytical formulations of secondary flow structure

In this study, a simple stream function is proposed for the description of the cellular secondary flows. The formulation is first made for Case S75, where the secondary flow cell appears symmetrically circular, by considering periodic variations both in the transverse and vertical directions. This formulation is then modified for applying to Case S50, where the circulation is skewed laterally. For the case with the wavy bed ridges, the same stream function is used but the vertical coordinate is shifted by considering local variations in the bed elevation.

(5) Maximum vertical velocity

The maximum vertical velocity $V_{\max}$ is essential for the proposed models of secondary flows. Generally, it depends on the primary flow properties and the amplitude of the variations in bed elevation and roughness. A dimensionless relationship is derived based on the experimental data, which shows that $V_{\max}$, when

scaled with the average velocity of the primary flow, is proportional to the relative bed elevation variation and the relative difference of squared Manning roughness coefficient.

Chapter

5

CHARACTERISTICS OF PRIMARY FLOW AFFECTED BY CELLULAR SECONDARY MOTION

5.1 INTRODUCTION

This chapter is focused on the structure of the primary flow that is considerably subject to the presence of the secondary flows. Data analyses are performed for Cases S75, S50 and WR where apparent organized secondary flow cells were observed.

The experimental results basically demonstrate that the longitudinal velocity of the primary flow varies significantly in the transverse direction, but some similarities can still be observed when compared to that observed for usual uniform open channel flows. By applying linear approximation together with the conventional theory developed for uniform open channel flow, analytical expressions are proposed for describing the distributions of streamwise velocity, Reynolds shear stress and bed shear stress, which are modified by the cellular secondary motion. The analytical results are then compared with the experimental data collected in this study.

5.2 SUBDIVISION OF PRIMARY FLOW

Fig. 5-1 plots the general distribution of measured primary velocity, U , for Cases S75, S50 and WR. Similar to the secondary velocity distribution presented in Chapter 4, the primary velocity also varies sinusoidally in the transverse direction. This implies that the primary flow structure is modified in the presence of secondary flows.

Fig. 5-1. Contour plots of U/U_m for Cases S75, S50 and WR, superimposed with idealized secondary flow cells.

To facilitate the comparison of various velocity profiles, it is better to measure the vertical distance locally from the bed level. Therefore, the following transformation for the vertical coordinate is made (see Fig. 5-2),

$$Y = y - b + d_s \quad (5-1)$$

where y = the original vertical coordinate with the origin set at the lowest point of bed surface; b = the elevation of local bed ($b \equiv 0$ for the case of bed strips, whereas b varies laterally for the case of ridges); and d_s = the vertical displacement measured downward from the top of bed particles. The d_s -value is set so that at the location of $y = d_s$, the mean velocity reduces to zero. Obviously $d_s = 0$ for smooth bed, but for rough bed $d_s \approx 0.25k_s$ (Hinze, 1975), where k_s is the representative roughness length.

Fig. 5-2. Sketch of coordinate system. Here x , y and z are the longitudinal, vertical and transverse coordinates, respectively; and Y is the transformed vertical coordinate ($= y - b + d_s$).

With the transformed coordinate Y , the vertical profiles of U measured in the central region of the channel ($z/\lambda = -1 \sim 0$) for Cases S75, S50 and WR are plotted in Fig. 5-3. Again, it is demonstrated that the velocity profiles vary for the different transverse positions. The two vertical profiles, one measured at the centerline of the rough strip and the other at the centerline of the smooth strip, generally encompass all other profiles. In other words, they define upper and lower velocity limits for the varying velocity distributions. In comparison, the vertical profile measured at the strip interface or at the location passing through the flow circulation centre (not geometrical centre) appears to be straight for the region above the buffer layer. This

is clearly different from those measured at the other locations. Fig. 5-3 also shows that for Cases S75 and S50, the velocity near the bed is relatively smaller over the rough strip due to the direct effects of the bed roughness, while in the upper flow portion the velocity over the rough strip is larger than that over the smooth strip. Similarly, for Case WR, the velocity in the upper flow portion is larger over the trough than over the ridge. These results are generally consistent with the observations given in previous studies (McLean, 1981; Nezu et al, 1981; Studerus, 1982; Wang et al, 2003; and Wang et al, 2004).

In spite of the variations, Fig. 5-3 also shows that the vertical structure of the primary flow is almost the same as that observed in uniform open channel flow. In the vertical direction, the flow according to its properties can be generally characterized by different sub-regions. As reviewed in Chapter 2, the uniform open channel flow are usually divided into inner region ($y/h < 0.2$), outer region [$Y > 50\nu/u_*$ for smooth bed or $Y > (1.0\sim 4.0) d$ for rough bed] and the overlap region, the log-layer. The inner region for the smooth bed can be further divided into viscosity sublayer, buffer layer, and log-layer, while for the rough bed it can be divided into roughness layer and log-layer (Raupach, et al., 1991; Nikora, et al, 2001).

Fig. 5-3. Vertical distributions of U for Cases S75, S50 and WR.

Fig. 5-3. (Continued).

Similar sub-regions also exist in the presence of secondary flows. As shown in Fig. 5-3, in the region of $65\nu/u_* < Y < 0.2h$ for the smooth strip or $1.0d_{50} < Y < 0.2h$ for the rough bed, the velocity distribution is logarithmic, and thus the region

can be referred to as log-layer as usual. The only difference is that the division between the log-layer and buffer layer is given approximately at $Y = 65\nu/u_*$, which is slightly larger than that given in the conventional theory $Y = 50\nu/u_*$. This might be due to the presence of the secondary flow. On the other hand, in the outer region, the velocity profile generally deviates from the logarithmic distribution. It can be also seen that the deviation varies with the velocity profile, bending upward in the upflow region (similar to the so-called wake effect) but downward in the downflow region. In the following, for convenience, the deviations (whether positive or negative) are all assumed to be ‘wake’-related.

Below the log-layer, i.e. $Y < 65\nu/u_*$ or $Y < 1.0d_{50}$, the distribution of U is more complex. Generally, the velocity gradient dU/dy within the roughness layer over the rough bed is less than that in the log-layer, which agrees with previous laboratory studies (Raupach, et al., 1991); while dU/dy within the buffer layer over the smooth bed is larger than that in the log-layer, as described by the traditional consideration without secondary flows.

It should be mentioned that, in the subsequent analysis, we will only consider the primary flow above the buffer layer or roughness layer, i.e. $Y > 65\nu/u_*$ or $Y > 1.0d_{50}$. This is because inclusion of the buffer layer will make the problem more complex. Moreover, the roughness elements used for rough strips are irregular sediment particles and thus understanding of the flow within the roughness layer requires additional theoretical efforts (Nikora, et al, 2001).

5.3 ANALYTICAL CONSIDERATIONS

In this section, analytical efforts are made for formulating the distributions of streamwise velocity and Reynolds shear stress in the presence of cellular secondary flows.

5.3.1 FLOW LINEARIZATION

We consider a steady turbulent flow in a wide, straight open channel, of which the bed roughness or elevation varies periodically in the lateral direction. For analysis purposes, it is assumed that the variations in the bed elevation or roughness are small but serve as an effective disturbance to generate cellular secondary flows. Obviously, the mean flow so induced is three-dimensional (3D), and therefore the streamwise components of bed shear stress, velocity and Reynolds shear stress would also vary laterally.

First, we define a ‘reference flow’, which is a two-dimensional (2D) uniform flow over a flat bed. For the reference flow associated with a particular case, the longitudinal bed slope and mean flow depth would remain unchanged. Obviously, the mean velocity profile for the reference flow follows the log-law, and the vertical distribution of the shear stress is linear. The typical flows with secondary flow cells and the corresponding reference flow are sketched in Fig. 5-4.

Then, it is assumed that the 3D flow under consideration results from the perturbation to the reference flow due to the lateral bed variations. If the amplitude of perturbation is small enough, the flow can be assumed as a linear combination of the reference flow and a first-order perturbation to the flow. This assumption is acceptable by noting that the cellular secondary flow is usually much weaker than the primary flow. As mentioned earlier, the maximum velocity magnitude for the cellular secondary flow is commonly less than 5% of the average velocity of the primary flow. In fact, linear decomposition of flow field in terms of flow velocity has long been used in other studies. For example, in the conventional theory of turbulent boundary layer, the mean velocity profile can be successfully represented by a linear combination of two universal functions (Coles, 1956), the well-

established law of the wall and the law of the wake (or Coles-law). The latter can be associated with variations in the pressure gradient in the streamwise direction.

Fig. 5-4. Schematics of 3D flows over longitudinal bedforms and corresponding reference flow with same bulk flow properties.

Based on the above consideration, a flow quantity for the 3D flow can be expressed as

$$F(Y, z) = f_0(Y) + F_1(Y, z) \quad (5-2)$$

where F = a time-mean quantity related to the disturbed flow; f_0 = the corresponding time-mean quantity related to the reference flow; and F_1 = the first-

order perturbation. Furthermore, the experimental results obtained in this study have demonstrated the periodic feature in the lateral direction. This suggests a periodic function that could be included in the perturbation term in Eq. (5-2), for example, in the following form

$$F(Y, z) = f_0(Y) + f_1(Y)G(z) \quad (5-3)$$

where $f_1(Y)$ = the amplitude of the first-order perturbation; and $G(z)$ = the periodic function. As will be demonstrated later, the periodic formulation to be employed is consistent with those for the secondary flow cells that are discussed in Chapter 4. It should be mentioned that the decomposition given by Eq. (5-3) is basically the same as those adopted earlier by Odgaard (1984) and Colombini (1993).

5.3.2 STREAMWISE VELOCITY DISTRIBUTION

First, the log-law is used to represent the profile of the streamwise velocity for the reference flow,

$$\frac{U_0(Y)}{u_{*0}} = \frac{1}{\kappa} \ln \frac{Y}{R_0} \quad (5-4)$$

where U_0 = the streamwise velocity of the reference flow; u_{*0} = the friction velocity of the reference flow; κ = the von Karman coefficient; R_0 = hypothesized zero-velocity level (or hydrodynamic roughness length); and Y = the elevation measured from the datum as shown in Fig. 5-2. Based on the considerations made in subsection 5.3.1, the friction velocity (u_{*0}) used here is taken to be the same as the average friction velocity obtained for the 3D flow.

In the presence of secondary flow cells, the streamwise velocity U for the 3D flow varies laterally and vertically in comparison with $U_0(Y)$. Following the approximation given by Eq. (5-3), U can be generally written as

$$U(Y, z) = U_0(Y) + U_1(Y)G(z) \quad (5-5)$$

where U_1 = the amplitude of the first-order perturbation for the streamwise velocity. Obviously, the first term on the right hand side of Eq. (5-5) is related to the

reference flow and can be evaluated using Eq. (5-4), while the second term is due to the secondary current.

To determine the unknown function, $U_1(Y)$, we may start with a discussion on the well-known log-wake law, which was developed for evaluating the mean velocity profile for boundary layers. First, we note that the log-wake law is also given by a linear combination of a log-term with a wake-term (Coles, 1956) as

$$\frac{U}{u_{*0}} = \frac{1}{\kappa} \ln \frac{Y}{R_0} + \frac{2\Pi}{\kappa} \sin^2 \frac{\pi Y}{2h} \quad (5-6)$$

where Π = the wake strength parameter. The wake term is used to take into account the deviation from the log-law in the outer flow region. The mechanism for the appearance of the wake-term is not theoretically clear but is generally considered to be associated with bulk flow characteristics such as those induced by the longitudinal pressure gradient dP/dx , for example, in boundary layers. For uniform open channel flows, the pressure gradient disappears and therefore the wake term would not exist if reasoning based on the pressure gradient. However, laboratory measurements reveal that the deviation from the log-law often exists in the upper flow portion (Nezu and Nakagawa, 1993), and the corresponding wake term obtained by curve fitting is usually very small but seldom vanishes. A possible explanation for this phenomenon is that the wake effect may be related to large scale flow structures, particularly, for the free-surface region where the bed shear effect is not significant. Examples of such structures are bursting-related boils occurring at the free surface. However, as to the flows considered in this study, the large scale structure which may significantly modify the properties of the primary flow could be dominated by the deliberately-generated cellular secondary flows. Therefore, it is expected that the dominant large scale structure would play an important role in formulating the wake term, either empirically or theoretically. Actually, this contemplation also has its physical grounds from the experimental observations.

As discussed in the preceding section, the experimental results obtained show that being modified by the secondary flows, the velocity profile generally appears to be of the log-wake type, i.e. logarithmic in the inner region (above the

buffer layer) and wake-like in the outer region. In particular, as shown in Fig. 5-3, the velocity gradient generally varies for different lateral locations even within the log-layer, which may further imply the existence of different zero-velocity levels.

With the above considerations and also being analogous to Eq. (5-6), the modified velocity profile for the primary flow can be expressed in the form

$$\frac{U(Y, z)}{u_{*0}} = \frac{a_l}{\kappa} \ln \frac{Y}{R} + \frac{a_w}{\kappa} \sin^2 \frac{\pi Y}{2h} \quad (5-7)$$

where R = local zero-velocity level for a particular transverse position; a_l and a_w = z -dependent coefficients related to the logarithmic term and wake term, respectively; and h = local flow depth. For the reference flow, Eq. (5-7) reduces to Eq. (5-4), suggesting that $a_l = 1$ and $a_w = 0$. It should be emphasized that the second term on the right hand side of Eq. (5-7) is analogous to the wake term given in Eq. (5-6) to account for the deviation from the log-law for the outer flow region.

To compare Eq. (5-7) with Eq. (5-4) [together with Eq. (5-5)], Eq. (5-7) can also be rearranged as

$$\frac{U(Y, z)}{u_{*0}} = \frac{1}{\kappa} \ln \frac{Y}{R_0} + \frac{1}{\kappa} \left[(a_l - 1) \ln \frac{Y}{R_0} + a_l \ln \frac{R_0}{R} + a_w \sin^2 \frac{\pi Y}{2h} \right] \quad (5-8)$$

Therefore, the perturbation term included in Eq. (5-5) can be defined by

$$\frac{U_1(Y)G(z)}{u_{*0}} = \frac{1}{\kappa} \left[(a_l - 1) \ln \frac{Y}{R_0} + a_l \ln \frac{R_0}{R} + a_w \sin^2 \frac{\pi Y}{2h} \right] \quad (5-9)$$

It is noted that on the right hand side of Eq. (5-9), $(a_l - 1)$, a_w and $a_l \ln(R_0/R)$ are all z -dependent. As a first approximation, we may assume that they are all proportional to the periodic function, $G(z)$, i.e.

$$a_l - 1 = \delta_l G(z), \quad a_w = \delta_w G(z), \quad a_l \ln \frac{R_0}{R} = \delta_r G(z) \quad (5-10)$$

where δ_l , δ_w , δ_r = amplitude parameters for the corresponding terms. Substituting Eq. (5-10) into Eq. (5-9), we get

$$\frac{U_1(Y)}{u_{*0}} = \frac{1}{\kappa} \left(\delta_l \ln \frac{Y}{R_0} + \delta_r + \delta_w \sin^2 \frac{\pi Y}{2h} \right) \quad (5-11)$$

5.3.3 STREAMWISE REYNOLDS SHEAR STRESS DISTRIBUTION

For the reference flow, the streamwise Reynolds shear stress τ_{x0} varies linearly, i.e.

$$\frac{\tau_{x0}}{\rho} = (-\overline{u'v'})_0 = u_{*0}^2 \left(1 - \frac{Y}{h} \right) \quad (5-12)$$

The relevant concepts of the eddy viscosity and mixing length can be mathematically presented as

$$(-\overline{u'v'})_0 = \nu_{t0} \frac{\partial U_0}{\partial Y} \quad (5-13)$$

$$\nu_{t0} = l_0^2 \frac{\partial U_0}{\partial Y} \quad (5-14)$$

where ν_{t0} and l_0 = the eddy viscosity and mixing length, respectively, for the reference flow. By manipulating with Eqs. (5-4), (5-12), (5-13) and (5-14), it is easy to get

$$\nu_{t0} = \kappa u_{*0} Y (1 - Y/h) \quad (5-15)$$

$$l_0 = \kappa Y \sqrt{1 - Y/h} \quad (5-16)$$

With the perturbation due to the bed variations, the streamwise Reynolds shear stress of the disturbed flow, τ_x , deviates from the linear distribution. By extending the concept of the eddy viscosity for the case under consideration, we have

$$\frac{\tau_x}{\rho} = -\overline{u'v'} = \nu_t \frac{\partial U}{\partial Y} \quad (5-17)$$

in which ν_t = the eddy viscosity for the disturbed flow that takes the form as

$$v_t = l^2 \frac{\partial U}{\partial Y} \quad (5-18)$$

where l = the mixing length for the disturbed flow.

By applying the decomposition technique given in Eq. (5-3), v_t and l can be also expressed in the form of a linear combination, respectively,

$$v_t = v_{t0} + v_{t1}G(z) \quad (5-19)$$

$$l = l_0 + l_1G(z) \quad (5-20)$$

where v_{t1} and l_1 = the corresponding perturbation amplitudes for the eddy viscosity and mixing length, respectively. Substituting Eqs. (5-5) and (5-20) into Eq. (5-18) and ignoring the second-order perturbation terms yields

$$\frac{v_{t1}}{v_{t0}} = \frac{\partial U_1 / \partial Y}{\partial U_0 / \partial Y} + 2 \frac{l_1}{l_0} \quad (5-21)$$

Again, substituting Eqs. (5-11) and (5-19) into Eq. (5-17) and ignoring the second-order perturbation, one can get

$$-\overline{u'v'} = v_{t0} \frac{\partial U_0}{\partial Y} + \left(v_{t0} \frac{\partial U_1}{\partial Y} + v_{t1} \frac{\partial U_0}{\partial Y} \right) G(z) \quad (5-22)$$

Eq. (5-22), in the similar form of Eq. (5-3), indicates that how $-\overline{u'v'}$ can vary with the perturbation in the primary velocity. With Eqs. (5-7), (5-21) and (5-22), the modified Reynolds shear stress can be further expressed as

$$-\overline{u'v'} = u_{*0}^2 \left(1 - \frac{Y}{h} \right) + u_{*0}^2 \left(1 - \frac{Y}{h} \right) \left(\delta_{sl} + \delta_{sw} \frac{Y}{h} \sin \frac{\pi Y}{h} \right) G(z) \quad (5-23)$$

where $\delta_{sl} = 2\delta_l + 2l_1/l_0$ and $\delta_{sw} = \pi\delta_w$. From Eq. (5-23), it follows that the vertical distribution of the Reynolds shear stress is generally nonlinear in the presence of the secondary currents.

For the convenience of the subsequent data analysis, Eq. (5-23) is rearranged into

$$\frac{-\overline{u'v'}}{u_{*0}^2} = a_{sl} \left(1 - \frac{Y}{h}\right) + a_{sw} \left(1 - \frac{Y}{h}\right) \frac{Y}{h} \sin \frac{\pi Y}{h} \quad (5-24)$$

in which

$$a_{sl} = 1 + \delta_{sl} G(z), \quad a_{sw} = \delta_{sw} G(z) \quad (5-25)$$

Apparently, Eq. (5-24) is the counterpart of velocity formula Eq. (5-7) for the Reynolds shear stress distribution.

5.3.4 STREAMWISE BED SHEAR STRESS

The streamwise bed shear stress at different transverse locations will be estimated by extrapolating the $-\overline{u'v'}$ profile to the bed surface. By taking that $Y = 0$ in Eq. (5-24), the lateral variation of the bed shear stress may be expressed using the periodic function as

$$\tau_b = \rho u_*^2 = \rho u_{*0}^2 [1 + \delta_b G(z)] \quad (5-26)$$

where δ_b denotes the amplitude parameter for the bed shear stress. It should be pointed out that based on the above reasoning, δ_b should be the same as δ_{sl} included in Eq. (5-23) if the present method is used. The new symbol δ_b is used because bed shear stress can also be evaluated using other methods. In fact, a similar formula has been previously proposed by Perkins (1970) and Ikeda (1981), which describes the bed shear stress varying in a cosine form, i.e., $G(z) = \cos(\pi z/\lambda)$ where λ is the average strip width.

More discussions on calculation of bed shear stress will be given in Section 5.5

5.4 APPLICATIONS

5.4.1 PROCEDURE

In this section, the analytical formulations proposed are used to represent the experimental results obtained for Cases S75, S50 and WR. To substantiate the analyses, several parameters and coefficients involved in the formulations given in the foregoing section should be determined first.

The first two parameters to be evaluated are u_{*0} and R_0 , which are used for defining the reference flow. Three different approaches will be used, which are based on the measured energy slope, flow velocity and Reynolds shear stress, respectively. Using the measured energy slope, the friction velocity for the reference flow can be computed as follows

$$u_{*0(J)} = \sqrt{gh_m J} \quad (5-27)$$

where the subscript (J) in $u_{*0(J)}$ indicates the value obtained from the energy slope, and h_m = average flow depth. Then, by assuming that the flow discharges for the reference flow and disturbed flow are nearly identical, the zero-velocity bed level, R_0 , can be determined by integrating the log-law. From Eq. (5-4) and by noting $R_0 \ll h_m$, the depth-averaged velocity can be expressed as

$$U_{0m} = \frac{u_{*0}}{h_m} \int_{R_0}^{h_m} U_0(Y) dY = \frac{u_{*0}}{\kappa} \left(\ln \frac{h_m}{R_0} - 1 \right) \quad (5-28)$$

where U_{0m} = depth-averaged velocity for the reference flow. By replacing U_{0m} with U_m measured in the central zone of the open channel for the disturbed flow, Eq. (5-28) can be rewritten as

$$R_{0(J)} \approx h_m \exp \left[-\frac{\kappa U_m}{u_{*0(J)}} - 1 \right] \quad (5-29)$$

where the subscript (J) indicates the value computed based on $u_{*0(J)}$. Obviously, the values of $u_{*0(J)}$ and $R_{0(J)}$ so computed are subject to accuracy of the measured energy slope.

The other two methods for evaluating R_0 and u_{*0} are associated with the velocity profile and Reynolds shear stress distribution measured at certain locations, which are proposed based on the following considerations.

First, it is noted from the experimental results that $V = 0$ and $\partial W / \partial z = 0$ at certain z -values, for example, at the strip interface ($z/\lambda = -0.5$) for Case S75. This suggests that for these particular locations, the effect of the secondary flow on the vertical profile of the primary flow velocity would be insignificant. The corresponding evidence is actually given in Fig. 5-3, which shows that the velocity profile measured at $z/\lambda = -0.5$ for Case S75 appears to be linear above the buffer layer when plotted with the log-linear coordinates. Similar phenomena can also be observed for Cases S50 and WR in the same figure. Furthermore, it is also noted that the velocity distributions measured at the other locations vary, but around the linear line. As a result, it may be reasonable to assume that the linear velocity profile measured at the transitional location approximately represents the respective reference flow.

With this assumption, we may obtain the values of the friction velocity (denoted by $u_{*0(v)}$) and zero-velocity level (denoted by $R_{0(v)}$) by fitting the velocity profile measured at the strip interface, say, $z/\lambda = -0.5$ for Case S75, to Eq. (5-4). It should be mentioned that for Case WR, the so-called transitional location cannot be simply identified from the bed configuration. Rather, it is selected as the vertical line passing through the center of the flow circulation, where the condition of $\partial W / \partial z = 0$ exists and the vertical velocity is almost negligible.

In addition to those evaluated from the measured velocity profiles, the values of the friction velocity can be also estimated by linearly extrapolating the Reynolds shear stress profile measured at the transitional location to the channel bed surface with Eq. (5-12). The values so obtained are denoted by $u_{*0(\tau)}$.

All values of the friction velocity and zero-velocity bed level, which are calculated using the above three approaches, are listed in Table 5-1. They vary within an acceptable range considering the experimental uncertainties. However, to avoid any further doubts that may be caused by the friction velocity for the reference flow, the two respective u_{*0} -values will be employed when examining lateral variations in the flow velocity and Reynolds shear stress in the subsequent analyses.

Table 5-1. Friction Velocity and Zero-velocity Bed Level Estimated for the Reference flow

Case No.	Friction velocity (m/s)			Zero-velocity bed level (m)	
	Calculated from bed slope $u_{*0(J)}$	Estimated from stress profile $u_{*0(\tau)}$	Estimated from velocity profile $u_{*0(v)}$	$R_{0(J)}$	$R_{0(v)}$
S75	0.0297	0.0310	0.0339	4.66×10^{-5}	1.30×10^{-4}
S50	0.0271	0.0260	0.0334	6.08×10^{-5}	2.38×10^{-4}
WR	0.0248	0.0252	0.0275	6.82×10^{-6}	1.34×10^{-5}

To explore how the velocity profile varies laterally, Eq. (5-7) is rearranged in the following form for fitting to the measured velocity profiles

$$U(Y, z) = \frac{c_1}{\kappa} \ln \frac{Y}{R} + \frac{c_2}{\kappa} \sin^2 \frac{\pi Y}{2h} \quad (5-30)$$

where $c_1 = a_l u_{*0(v)}$; $c_2 = a_w u_{*0(v)}$ and $\kappa = 0.4$. Here, the three parameters, c_1 , c_2 and R , are unknowns and to be optimized through curve fitting. As mentioned previously, only the measured data above the buffer layer are used.

Similarly, the profiles of the Reynolds shear stress are examined with Eq. (5-24), which is also rewritten as

$$-\overline{u'v'} = c_3 \left(1 - \frac{Y}{h} \right) + c_4 \left(1 - \frac{Y}{h} \right) \frac{Y}{h} \sin \frac{\pi Y}{h} \quad (5-31)$$

where $c_3 = a_{sl} u_{*0(\tau)}^2$ and $c_4 = a_{sw} u_{*0(\tau)}^2$. The two parameters, c_3 and c_4 , are optimized by fitting Eq. (5-31) to the measured $-\overline{u'v'}$ profiles.

5.4.2 CASE S75

Fig. 5-5 plots the measured streamwise velocity profiles for Case S75 at eleven locations, $z/\lambda = 0.0, -0.1, -0.2 \dots -1.0$, in the central region of the channel flow. For comparison, two corresponding profiles are plotted together, one being over the rough strip and the other over the smooth strip. They are located at the same distance from the strip interface ($z/\lambda = -0.5$). The two profiles show different distributions; the one over the rough strip turns downward but the other over the smooth strip curves up. However, the one at the strip interface or transitional location is nearly a straight line beyond the buffer layer.

Fig. 5-5 also shows the fitted velocity profiles using Eq. (5-30) with optimized values of c_1 , c_2 and R . Clearly, the measured and fitted profiles agree well with each other. The shear velocity $u_{*0(v)}$ and zero-velocity level $R_{0(v)}$ for the reference flow are determined from the profile at the strip interface ($z/\lambda = -0.5$). The curve fitting gives that $u_{*0(v)} = 0.0339$ m/s and $R_{0(v)} = 1.30 \times 10^{-4}$ m. Then, the values of a_l and a_w for each profiles are calculated by noting $c_1 = a_l u_{*0(v)}$ and $c_2 = a_w u_{*0(v)}$. Fig. 5-6 shows the values of the parameters, a_l , a_w and R , obtained for the different profiles, which vary sinusoidally in the transverse direction. The variations can be described, respectively, as follows

$$a_l = 1 + \delta_l \cos(\pi z / \lambda), \quad a_w = \delta_w \cos(\pi z / \lambda), \quad a_l \ln \frac{R_{0(v)}}{R} = \delta_r \cos(\pi z / \lambda) \quad (5-32)$$

where $\delta_l = 0.5$, $\delta_w = -0.95$ and $\delta_r = -2.1$.

Fig. 5-5. Vertical profiles of U for Case S75. Scatter points denote experimental data; solid lines are fitted profiles.

Fig. 5-6. Velocity profile parameters, a_l , a_w and $a_l \ln(R_{0(v)}/R)$, for Case S75.

Inherently, Eq. (5-32) can also be obtained by simply substituting the periodic function $G(z/\lambda)$ [i.e., $\cos(\pi z/\lambda)$] into Eq. (5-10), which implies that the deviation of velocity profiles from the log-law for the reference flow is closely related to the secondary flow, especially its vertical component.

Fig. 5-7 plots the measured streamwise Reynolds shear stress at the same locations ($z/\lambda = 0.0, -0.1, -0.2 \dots -1.0$). Similarly, two profiles are plotted together for comparison purpose, i.e., one over the rough strip and the other over the smooth strip with the same distance from the strip interface ($z/\lambda = -0.5$). It can be seen that most profiles deviate from the linear distribution significantly. However, the deviations of these profiles vary over the different bed strips. Higher shear stress occurs over the smooth strip in the upper flow portion, say, $Y/h > 0.3$. In the region near the bed, however, the shear stress tends to be smaller over the smooth strip. In comparison, the profile at the strip interface nearly follows a straight line. The bed shear stress for the reference flow is obtained by extrapolating this profile onto the bed surface, which yields $u_{*0(\tau)} = 0.031$ m/s.

The other $-\overline{u'v'}$ profiles are also fitted by Eq. (5-31). Fig. 5-7 demonstrates that the measured shear stress profiles can be well represented by Eq. (5-31) with optimized values of c_3 and c_4 . The parameters, a_{sl} and a_{sw} , for each profiles are then obtained from the relations, $c_3 = a_{sl}u_{*0(\tau)}^2$ and $c_4 = a_{sw}u_{*0(\tau)}^2$. Their values are plotted in Fig. 5-8, showing that the variations of the two parameters can be approximated by the cosine relationships, respectively, i.e.

$$a_{sl} = 1 + \delta_{sl} \cos(\pi z / \lambda), \quad a_{sw} = \delta_{sw} \cos(\pi z / \lambda) \quad (5-33)$$

with $\delta_{sl} = 0.27$ and $\delta_{sw} = -1.22$. This shows that the profile of $-\overline{u'v'}$ can also be described using the same transverse periodic function as that used for V , i.e. $G(z/\lambda) = \cos(\pi z/\lambda)$. This is consistent with the above derivation that U and $-\overline{u'v'}$ have the same periodic properties.

Fig. 5-7. Distributions of streamwise Reynolds shear stress for Case S75. Scatter points denote experimental data; solid lines are fitted profiles.

Moreover, the streamwise bed shear stresses at different locations can be obtained by extrapolating the corresponding $-\overline{u'v'}$ profiles to the bed surface. From Eq. (5-24), it is known $u_*^2 = a_{sl} u_{*0}^2$. Fig. 5-9 plots the bed shear stress distribution, showing that it can be described by the following cosine function

$$\frac{u_*^2}{u_{*0(\tau)}^2} = 1 + \delta_b \cos(\pi z / \lambda) \quad (5-34)$$

where $\delta_b = 0.27$.

The above application for Case S75 indicates that the proposed analytical model works well for describing the mean primary flow structure over the rough/smooth strips. In the following, similar analyses will be performed for Cases S50 and WR.

Fig. 5-8. Parameters used for Reynolds shear stress profile, a_{sl} and a_{sw} , for Case S75.

Fig. 5-9. Bed shear stress distribution for Case S75.

5.4.3 CASE S50

The basic procedure for applying the analytical model for Case S50 is the same as that used for Case S75. Fig. 5-10 plots the streamwise velocity profiles for Case S50 at 9 locations, $z/\lambda = 0.0, -2/15, -4/15 \dots -16/15$, in the central region of the channel flow. Being similar to those for Case S75, the profiles over the rough strip bend down and those over the smooth strip curve up. However, the one at the strip interface ($z/\lambda = -2/3$) remains as a straight line beyond the buffer layer. Fig. 5-10

also shows the velocity profiles fitted using Eq. (5-30) with optimized values of c_1 , c_2 and R , being in good agreement with the measurements.

Fig. 5-10. Vertical profiles of U for Case S50. Scatter points denote experimental data; solid lines are fitted profiles.

The values of $u_{*0(v)}$ and $R_{0(v)}$ are determined from the profile measured at the strip interface (i.e., $z/\lambda = -2/3$), which gives $u_{*0(v)} = 0.0334$ m/s and $R_{0(v)} = 2.38 \times 10^{-4}$ m. The variations of the parameters, a_l , a_w and R , are shown in Fig. 5-11. It demonstrates that, with the same periodic function, $PC_s(z/\lambda)$, which is used for delineating the vertical velocity, the three parameters can be expressed respectively as follows

$$a_l = 1 + \delta_l PC_s(z/\lambda), \quad a_w = \delta_w PC_s(z/\lambda), \quad a_l \ln \frac{R_{0(v)}}{R} = \delta_r PC_s(z/\lambda) \quad (5-35)$$

where $\delta_l = 0.58$, $\delta_w = -1.2$ and $\delta_r = -2.2$. The periodic function is given by

$$PC_s(z/\lambda) = \begin{cases} \frac{\lambda_s \pi}{\lambda_r} \cos \left[\frac{(\zeta - 2n)\lambda}{\lambda_r} \pi \right] & 2n - \frac{\lambda_r}{2\lambda} \leq \zeta \leq 2n + \frac{\lambda_r}{2\lambda} \\ \pi \cos \left[\frac{(\zeta - 2n - \frac{\lambda_r}{2\lambda})\lambda}{\lambda_s} \pi + \frac{\pi}{2} \right] & 2n + \frac{\lambda_r}{2\lambda} \leq \zeta \leq 2n + \frac{\lambda_r}{2\lambda} + \frac{\lambda_s}{\lambda} \end{cases} \quad (5-36)$$

where $n = 0, \pm 1, \pm 2, \dots$; λ_r = the width of the rough strip; λ_s = the width of the smooth strip; and $\lambda = (\lambda_r + \lambda_s)/2$ = the average strip width.

Fig. 5-11. Velocity profile parameters, a_l , a_w and $a_l \ln(R_{0(v)}/R)$, for Case 50.

Fig. 5-12 plots the $-\overline{u'v'}$ profiles measured at the locations of $z/\lambda = 0.0, -2/15, -4/15 \dots -16/15$. These profiles generally deviate from the linear distribution, being similar to those obtained for Case S75. In the upper flow portion, say, $Y/h >$

0.3, the shear stress is higher over the smooth strip than that over the rough strip, and vice versa in the lower flow portion, say, $Y/h < 0.3$. The profile at the strip interface is used to determine the bed shear stress for the reference flow by extrapolating it onto the bed surface, which yields $u_{*0(\tau)} = 0.026$ m/s.

Fig. 5-12. Distributions of streamwise Reynolds shear stress for Case S50. Scatter points denote experimental data; solid lines are fitted profiles

The other $-\overline{u'v'}$ profiles are also fitted by Eq. (5-31) with optimized c_3 and c_4 . As plotted in Fig. 5-12, they can be well described by Eq. (5-31). The parameters, a_{sl} and a_{sw} are plotted in Fig. 5-13, and their lateral variation can be also approximated using the periodic function $PC_s(z/\lambda)$, i.e.

$$a_{sl} = 1 + \delta_{sl} PC_s(z/\lambda), \quad a_{sw} = \delta_{sw} PC_s(z/\lambda) \quad (5-37)$$

in which $\delta_{sl} = 0.26$ and $\delta_{sw} = -0.8$.

Fig. 5-13. Parameters used for Reynolds shear stress profile, a_{sl} and a_{sw} , for Case S50.

The streamwise bed shear stresses at different locations are obtained by extrapolating the corresponding $-\overline{u'v'}$ profiles to the bed surface. Fig. 5-14 shows that the calculated results of the relative value $u_*^2 / u_{*0(\tau)}^2$ can be represented by the function

$$\frac{u_*^2}{u_{*0(\tau)}^2} = 1 + \delta_b PC_s(z/\lambda) \quad (5-38)$$

where $\delta_b = 0.17$.

Fig. 5-14. Bed shear stress distribution for Case S50.

5.4.4 CASE WR

The measured U profiles for Case WR are first presented in Fig. 5-15, which includes the profiles measured at eleven locations in the central region ($z/\lambda = 0.0, -0.1, -0.2 \dots -1.0$). As mentioned earlier, for this case, the transitional location where the flow simulates the reference flow is selected as the vertical line passing through the center of the circulation cell. From the information of secondary flows for Case WR, as presented in Section 4.4, the circulation center appearing in the concerned area ($0 < z/\lambda < -1.0$) is nearly located at $z/\lambda = -0.6$. Therefore, we may define $0 < z/\lambda < -0.6$ as ‘trough zone’ and $-0.6 < z/\lambda < -1.0$ as ‘ridge zone’. By comparing to the strip-related cases, S75 and S50, the U profiles measured over the trough and ridge for Case WR resemble those over the rough and smooth strips, respectively. In particular, as shown in Fig. 5-15, the profiles over the trough zone bend down and the profiles over the ridge zone curve up, while the one over the transitional position ($z/\lambda = -0.6$) nearly appears as a straight line. These profiles are then fitted using Eq. (5-30) with optimized c_1 , c_2 and R , which are also shown in Fig. 5-15.

The values of $u_{*0(v)}$ and $R_{0(v)}$ are estimated from the profile at the transitional location ($z/\lambda = -0.6$), which yields $u_{*0(v)} = 0.0275$ m/s and $R_0 = 1.34 \times 10^{-5}$ m. The obtained values of a_l , a_w and R , as shown in Fig. 5-16, can be described respectively by

$$a_l = 1 + \delta_l PC(z/\lambda), \quad a_l \ln \frac{R_{0(v)}}{R} = \delta_r PC(z/\lambda), \quad a_w = \delta_w PC(z/\lambda) \quad (5-39)$$

in which $\delta_l = 0.32$; $\delta_w = -0.9$; $\delta_r = -1.8$; and the periodic function is given by

$$PC(\zeta) = \begin{cases} \frac{\pi \lambda_{up}}{\lambda} \cos \left[\frac{(\zeta - 2n)\lambda}{\lambda_{dn}} \pi \right] & 2n - \frac{\lambda_{dn}}{2\lambda} \leq \zeta \leq 2n + \frac{\lambda_{dn}}{2\lambda} \\ \pi \cos \left[\frac{(\zeta - 2n - \frac{\lambda_{up}}{2\lambda})\lambda}{\lambda_{up}} \pi + \frac{\pi}{2} \right] & 2n + \frac{\lambda_{dn}}{2\lambda} \leq \zeta \leq 2n + \frac{\lambda_{dn}}{2\lambda} + \frac{\lambda_{up}}{\lambda} \end{cases} \quad (5-40)$$

where λ_{dn} , λ_{up} = the widths of the downflow and upflow zones, respectively; and λ = the average width of ridge and trough that is exactly equal to the average width of downflow and downflow zones, i.e. $\lambda = (\lambda_{dn} + \lambda_{up})/2$. It is noted that $PC(z/\lambda)$ used here also describes the major transverse variation of the vertical velocity for Case WR.

Fig. 5-15. Vertical profiles of U for Case WR. Scatter points denote experimental data; solid lines are fitted profiles.

Fig. 5-16. Velocity profile parameters, a_l , a_w and $a_l \ln(R_{0(v)}/R)$, for Case WR.

Fig. 5-17 plots the measured $-\overline{u'v'}$ profiles, which deviate from the linear distribution to some extent. Certain similarities can be observed when comparing them with those measured for the former strip cases, S75 and S50. In the upper flow portion of $Y/h > 0.3$, the shear stress is higher over the ridge than that over the trough, and vice versa in the lower flow portion of $Y/h < 0.3$. The profile at the location of the circulation centre is used to determine the bed shear stress for the reference flow by extrapolating it onto the bed surface with Eq. (5-31), which yields $u_{*0(\tau)} = 0.0252$ m/s.

The other $-\overline{u'v'}$ profiles are also fitted by Eq. (5-31) with optimized c_3 and c_4 . As plotted in Fig. 5-17, they can be well described by Eq. (5-31). The lateral variations of a_{sl} and a_{sw} are plotted in Fig. 5-18. They can be approximated using the periodic function $PC(z/\lambda)$, respectively, i.e.

$$a_{sl} = 1 + \delta_{sl} PC(z/\lambda), \quad a_{sw} = \delta_{sw} PC(z/\lambda) \quad (5-41)$$

where $\delta_{sl} = 0.22$ and $\delta_{sw} = -1.25$.

Fig. 5-17. Distributions of streamwise Reynolds shear stress for Case WR. Scatter points denote experimental data; solid lines are fitted profiles.

The streamwise bed shear stresses at different locations are shown in Fig. 5-19, being described by

$$\frac{u_*^2}{u_{*0(\tau)}^2} = 1 + \delta_b PC(z/\lambda) \quad (5-42)$$

where $\delta_b = 0.22$.

Fig. 5-18. Parameters used for Reynolds shear stress profile, a_{sl} and a_{sw} , for Case WR.

Fig. 5-19. Bed shear stress distribution for Case WR.

5.5 DISCUSSION

In the preceding analysis, the bed shear stress is calculated by extrapolating Reynolds shear stress profile to the bed surface. This is because

$$\frac{\tau_{bx}}{\rho} = -\overline{u'v'} + \nu \frac{dU}{dY} \Big|_{Y=0} \approx -\overline{u'v'} \Big|_{Y \rightarrow 0} \quad (5-43)$$

in which the viscous term is negligible considering that turbulence related shear stress dominates in major flow portion. The similar consideration could be made also for three-dimensional flows.

In the presence of secondary circulations, if Eq. (5-31) is used, the local bed shear stress can be expressed as

$$\frac{\tau_{bx}}{\rho} = a_{sl} u_{*0}^2 = [1 + \delta_{sl} G(z)] u_{*0}^2 \quad (5-44)$$

On the other hand, we note that for 2D open channel flows the friction velocity u_* (and thus the bed shear stress $\tau_{bx} = \rho u_*^2$) can be estimated by fitting the measured primary velocity profile to the log-law. However, the feasibility of applying this method to the present cases is not evident since the velocity profile is described by Eq. (5-7). In Eq. (5-7), the quantity $a_l u_{*0}$ can be regarded as the counterpart of the friction velocity u_* included in the conventional log-law. Thus, the square of $a_l u_{*0}$, i.e.,

$$(a_l u_{*0})^2 = [1 + \delta_l G(z)]^2 u_{*0}^2 \approx [1 + 2\delta_l G(z)] u_{*0}^2 \quad (5-45)$$

corresponds to the bed shear stress τ_{bx} / ρ . In the last step, the second-order perturbation is ignored as in the earlier derivations. If $(a_l u_{*0})^2$ is equal to the local bed shear stress, we can obtain $2\delta_l = \delta_{sl}$ by equating (5-44) and (5-45). However, the experimental results show that the computed values of $2\delta_l$ is significantly greater than those of δ_{sl} , as listed in Table 5-2. The large discrepancy cannot be simply explained from measurement uncertainties. It could be however due to the

fact that the velocity gradient near the bed ($Y/h < 0.2$) is also affected by secondary flows as well as the local bed shear stress. The slope of the profile of U/u_* vs. $\ln(Y/R)$ might be amplified or reduced in comparison to the slope $1/\kappa$ for the log-law. With a larger variation amplitude $2\delta_l$ relative to δ_{sl} , the method may overestimate the local bed shear stress (or friction velocity) in the downflow zone where $G(z) > 0$, or underestimate it in the upflow zone where $G(z) < 0$. Only at the transitional location where no vertical flow exist and $G(z) = 0$, the friction velocity may be estimated correctly by this method. This is consistent with the earlier assumption that the log-law holds at the transitional locations. Therefore, further investigations are required to justify the method related to the velocity profile.

Table 5-2. Comparison of the Values of $2\delta_l$ and δ_{sl}

Case No.	$2\delta_l$	δ_{sl}
S75	1.00	0.27
S50	1.16	0.26
WR	0.64	0.22

It should also be mentioned that, in this study, the term ‘log’ in log-wake-law is used to describe the general logarithmic distribution within the layer of $Y/h < 0.2$. It does not imply that the velocity profiles still obey the classic log-law there, for which the slope of U/u_* vs. $\ln(Y/R)$ is constantly equal to $1/\kappa$.

5.6 SUMMARY

The distributions of the streamwise velocity and Reynolds shear stress are modified in the presence of secondary flow cells. They vary periodically in the transverse direction for the bed configurations considered. The vertical profile of the streamwise velocity generally deviates from the logarithmic distribution, and most of the Reynolds shear stress profiles are no longer in linear distribution. The deviation is considerable, in particular, for the region where the upflow or downflow prevails. However, on the other hand, the vertical structure of the primary flow in terms of velocity distribution is still similar to that of uniform open channel flow. Several sub-regions of the flow can be identified based on similar criteria. The log-layer exists in the inner region, and in the outer region the velocity distribution can be approximated as a log-wake function. This result suggests that the existing theories for 2D open channel flows are suggestive to the cases concerned in this study provided that the effects of secondary flows are properly considered.

Since the secondary flow is usually much weaker than the primary flow, the flow field can be linearized so that a flow quantity can be reasonably decomposed into two components, one being related to the reference flow and the other symbolizing the first-order perturbation caused by the bed variations. The application of this approach, together with the existing theory for 2D open channel flow, leads to an analytical formulation of the streamwise velocity. It resembles the conventional log-wake type, but the velocity gradient, zero-velocity bed level and wake strength all vary with the transverse positions. It can be considered as a more general form for describing the streamwise velocity distribution in open channel flow. Other formulae derived include those for formulating the distributions of streamwise Reynolds shear stress and bed shear stress. It has been shown that the proposed analytical models represent well the experimental results. The spanwise variations of the parameters in those formulae can be described by a periodical function same as that used for the vertical velocity. This implies an inherent relationship between the primary flow structure and the secondary flow structure.

Chapter

6

IMPLICATIONS FOR SUSPENDED SEDIMENT TRANSPORT

6.1 INTRODUCTION

In addition to the experimental and analytical investigations of flow structures presented in the last two chapters, preliminary experimental study and qualitative analysis are also conducted in this study to explore possible effects of secondary flows on suspended sediment transport. In this chapter, the measured distribution of suspended sediment concentration is first presented. Then, the settling behavior of sediment particles in the cellular flows is analyzed. The influences of secondary flow advection and 3D diffusion process on the sediment concentration distribution are discussed. Some implications are also provided to qualitatively explain the experimental results and the field measurement available in literature.

6.2 EXPERIMENTAL RESULTS

Fig. 6-1 plots the contour maps of Kaolin concentration distribution measured for Cases S75, S50 and WR, while Fig. 6-2 shows the contour maps for Cases R75-10 and R50-10. Basically, both figures show that the concentrations, as usual, increase from the free surface to the bed. However, the concentration distributions shown in Fig. 6-1 are apparently more uniform in the vertical direction than those in Fig. 6-2. For the same vertical range (about $0.25 < y/h < 0.77$), the relative concentration, C/C_m , varies from 0.9 to 1.05 for the former three cases included in Fig. 6-1, while $C/C_m = 0.8\sim 1.15$ for the other two cases in Fig. 6-2. Here, C_m is the average concentration determined by measuring the suspension samples taken from the tail tank. In the transverse direction, considerable variations are also observed for $-3.7 < z/\lambda < 0$ for the latter two cases. Furthermore, for Cases R75-10 and R50-10, the highest concentration is found above the ridge connection where the upflow occurs (see Fig. 4-21).

Fig. 6-3 demonstrates that the laterally-averaged concentration distributions for Cases S75, S50 and WR are more uniform than those for Cases R75-10 and R50-10. It should be noted that the bulk flow conditions are almost the same for all these cases, but the organized secondary flow cells are much stronger in the former three cases. For example, the magnitude of $V_{\max}$ for Cases S75, S50 and WR is much greater than that of the sediment settling velocity, and therefore sediment particles could be uplifted by secondary flows. These results imply that secondary flow cells could play an important role in homogenizing the concentration distribution. This mechanism will be analyzed theoretically later in this chapter.

For comparison purposes, Fig. 6-3 is also superimposed with the computed concentration profiles. They serve as a good reference for demonstrating various deviations, which are induced by the different secondary flows. It can be observed that significant deviations are associated with Cases S75, S50 and WR. The relevant computations were performed base on the Rouse equation given by

$$\frac{C}{C_a} = \left(\frac{h-y}{y} \frac{a}{h-a} \right)^z \quad (6-1)$$

where $Z = \omega/(\beta \kappa u_*)$; u_* = the shear velocity; C_a = a reference concentration of the suspension at a distance ' a ' above the bed; and β = proportionality constant to be determined experimentally. It has been reported that $\beta = 1$ for fine sediment particles, say, smaller than 0.1 mm in diameter, and $\beta < 1$ for coarse sediment (Chien and Wan, 1998). In this study, the reference height a is taken to be $0.05h$, and C_a is estimated from following relationship

$$\frac{C_a}{0.95h} \int_{0.05h}^h \left(\frac{h-y}{y} \frac{a}{h-a} \right)^Z dy \approx C_m \quad (6-2)$$

where C_m = average concentration. The C_a -values estimated for the different cases are included in Table 6-1, together with the values of $u_{*(J)}$ (computed from the energy slope), Z and C_m . In addition, it should be noted that the real settling velocity of Kaolin particles in the flows are much larger than that for individual particles because of the effect of flocculation. Flocculation usually occurs with the inclusion of certain amount of particles finer than 0.01 mm (Chien and Wan, 1998). Migniot (1968) reported that the flocculation caused the settling velocity to be increased by factor of F defined as $\omega_{F50} / \omega_{D50}$, where ω_{F50} = the settling velocity of a floc and ω_{D50} = the settling velocity of a basic sediment particle both being represented by their median values. With reference to empirical values suggested in previous studies, F is taken to be 20 in this study, which yields that $\omega_{F50} = 1.084$ mm/s.

Table 6-1. Parameters Used for Computing Concentration Profiles

Case No.	$u_{*(J)}$ (m/s)	Z (-)	C_m (g/l)	C_a (g/l)
S75	0.029698	0.09125	2.131	2.813
S50	0.027111	0.09996	1.804	2.442
WR	0.024758	0.109459	1.903	2.648
R75-10	0.021689	0.12495	1.756	2.555
R50-10	0.023669	0.114495	1.832	2.587

Fig. 6-1. Kaolin concentration distribution for Cases S75, S5 and WR. C_m is the mean concentration.

Fig. 6-2. Kaolin concentration distribution for Cases R75-10 and R50-10. C_m is the mean concentration.

Fig. 6-3. Sediment concentration distribution, where h_m = the averaged flow depth. The experimental results (denoted by small black squares) are the laterally-averaged values. The theoretical profiles (denoted by solid lines) are calculated using the Rouse equation with Z estimated from sediment property and bulk flow conditions.

6.3 IMPLICATIONS FOR SETTLING BEHAVIOUR OF SEDIMENT PARTICLES SUBJECT TO CELLULAR FLOWS

To predict sediment concentration distribution, it is essential to know settling behaviour of suspended particles. The settling velocity is conventionally determined from the falling speed of sediment particles in still fluid. The settling velocity so defined is applicable for describing sediment suspension for 2D flows with low concentration if turbulence effect is negligible. However the presence of secondary motions, which includes time-mean fluid flow in the vertical and lateral direction, may considerably modify the settling behaviour of suspended sediment. Such modifications are explored in this section for the condition of cellular secondary flows.

If assuming that the relative velocity between sediment and fluid flow is everywhere equal to the still-fluid settling velocity ω_0 , the settling velocity modified by cellular secondary flow can be defined by

$$\boldsymbol{\omega} = \boldsymbol{U}_f + \boldsymbol{\omega}_0 \quad (6-3)$$

where $\boldsymbol{\omega}$ = the resultant settling velocity vector; $\boldsymbol{U}_f$ = the fluid velocity vector of cellular flow; and $\boldsymbol{\omega}_0$ = the settling velocity vector in still fluid given by

$$\boldsymbol{\omega}_0 = -\omega_0 \mathbf{j} \quad (6-4)$$

in which $\mathbf{j}$ = unit vector in the y-direction.

The cellular flows here refer to the circulating motions in the cross-sectional plane with closed streamlines, which can be described by the models for cellular secondary flows proposed in Chapter 4. Taking the simplest case (the model for Case S75) as an example, the flow velocity vector is given by

$$\boldsymbol{U}_f = V\mathbf{j} + W\mathbf{k} = -V_{\max} \sin(\pi \frac{y}{h}) \cos(\pi \frac{z}{\lambda}) \mathbf{j} + \frac{\lambda V_{\max}}{h} \cos(\pi \frac{y}{h}) \sin(\pi \frac{z}{\lambda}) \mathbf{k} \quad (6-5)$$

in which V_{max} = the maximum upwelling velocity; y = the vertical coordinate; h = the flow depth; z = the transverse coordinate; λ = the average bedform width; and $\mathbf{k}$ = unit vector in the z -direction. Substituting Eqs. (6-4) and (6-5) into Eq. (6-3) yields

$$\boldsymbol{\omega} = \left[-V_{max} \sin\left(\pi \frac{y}{h}\right) \cos\left(\pi \frac{z}{\lambda}\right) - \omega_0 \right] \mathbf{j} + \frac{\lambda V_{max}}{h} \cos\left(\pi \frac{y}{h}\right) \sin\left(\pi \frac{z}{\lambda}\right) \mathbf{k} \quad (6-6)$$

Eq. (6-6) can be used to calculate possible trajectories of settling sediment particles in the cross-sectional plane if V_{max} and ω_0 are given. Examples are given in Fig. 6-4. Note that Eq. (6-6) reduces to $-\omega_0 \mathbf{j}$ if $V_{max} = 0$ (without secondary fluid motion), implying that sediment particles would settle down vertically. Relevant calculations show that the ratio ω_0/V_{max} determinates the shapes of cross-sectional trajectories. If $\omega_0/V_{max} > 1$, sediment particles will fall down along curved paths [Fig. 6-4(a)], whereas if $\omega_0/V_{max} < 1$, some trajectories appear to be closed paths [Fig. 6-4(b)].

Fig. 6-4. Trajectories of settling sediment particle computed using Eq. (6-6) for cases (a) $\omega_0/V_{\max} = 2.0$ and (b) $\omega_0/V_{\max} = 0.2$.

6.4 IMPLICATIONS FOR CONCENTRATION DISTRIBUTION

For a steady, longitudinally uniform flow, the mean sediment concentration is described by the following convection-diffusion equation

$$V_s \frac{\partial C}{\partial y} + W_s \frac{\partial C}{\partial z} = \frac{\partial}{\partial y} \left(\varepsilon_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(\varepsilon_z \frac{\partial C}{\partial z} \right) \quad (6-7)$$

in which V_s = the vertical component of time-mean particle velocity; W_s = the transverse component of time-mean particle velocity; C = the time-mean sediment concentration; and $\varepsilon_y, \varepsilon_z$ = the sediment diffusion coefficients in the y and z directions, respectively.

Without losing much generality, we still take Case S75 as an example for the following discussion. Then, the sediment velocity vector $V_s \mathbf{i} + W_s \mathbf{j}$ is supposed to be the same as the settling velocity vector in the corresponding cellular flows, which is already given by Eq. (6-5). Therefore, Eq. (6-7) is rewritten as

$$\begin{aligned} & \left[-V_{\max} \sin\left(\pi \frac{y}{h}\right) \cos\left(\pi \frac{z}{\lambda_c}\right) - \omega_0 \right] \frac{\partial C}{\partial y} + \left[V_{\max} \left(\frac{\lambda_c}{h} \right) \cos\left(\pi \frac{y}{h}\right) \sin\left(\pi \frac{z}{\lambda_c}\right) \right] \frac{\partial C}{\partial z} \\ & = \frac{\partial}{\partial y} \left(\varepsilon_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(\varepsilon_z \frac{\partial C}{\partial z} \right) \end{aligned} \quad (6-8)$$

On the other hand, the concentration equation for the 2D reference flow is

$$\frac{\partial}{\partial y} \left(\varepsilon_{y0} \frac{\partial C}{\partial y} \right) + \omega_0 \frac{\partial C}{\partial y} = 0 \quad (6-9)$$

where ε_{y0} = the vertical diffusion coefficient for the reference flow. The comparison between Eq. (6-8) and Eq. (6-9) suggests that sediment concentration distribution in the presence of secondary flows will be affected in two distinctive aspects:

- 1) **Advection by time-mean secondary flows; and**
- 2) **Turbulent diffusion in vertical and spanwise directions.**

It is difficult to theoretically solve Eq. (6-8). Alternatively, qualitative analysis will be conducted, and some implications will be provided for understanding the

sediment concentration distribution modified secondary flows. The two aspects will be considered individually.

6.4.1 ADVECTION BY SECONDARY FLOWS

In this subsection, we mainly deal with possible influence of the advection by time-mean secondary flows on the concentration distribution. For simplicity, sediment diffusion coefficients are set as a constant ε_s , i.e., $\varepsilon_y = \varepsilon_z = \varepsilon_s$, and then Eq. (6-8) becomes

$$\begin{aligned} & \left[-V_{\max} \sin\left(\pi \frac{y}{h}\right) \cos\left(\pi \frac{z}{\lambda_c}\right) - \omega_0 \right] \frac{\partial C}{\partial y} + \left[V_{\max} \left(\frac{\lambda_c}{h}\right) \cos\left(\pi \frac{y}{h}\right) \sin\left(\pi \frac{z}{\lambda_c}\right) \right] \frac{\partial C}{\partial z} \\ & = \varepsilon_s \left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \end{aligned} \quad (6-10)$$

As mentioned in Section 6.3 the settling behaviour of sediment particles is controlled by the ratio of $\omega_0/V_{\max}$. The settling pattern for $\omega_0/V_{\max} > 1$ is completely different from that for $\omega_0/V_{\max} < 1$. The two cases are thus analyzed separately.

6.4.1.1 CASE I ($\omega_0/V_{\max} < 1$)

If $\omega_0/V_{\max} < 1$, sediment particles near the circulation core travel along closed circles in the cross-sectional plane, as shown in Fig. 6-5. In this way, secondary flow cells act as a trap for sediment particles, lasting as long as the circulation is sustained. This concept was first enunciated by Stommel (1949). Tooby et al (1977) also experimentally showed that small sediment particles in a vortex follow circular paths very closely. A review in this respect was given by Nielsen (1992). The zone, in which sediment particles can be trapped, is often referred to as retention zone (Stommel, 1949). Fig. 6-5 shows an example of retention zones that are specified by the closed 2D sediment streamlines in the cross-sectional plane. Each retention zone consists of two counter-rotating sub-regions. The area of retention zone increases with decreasing $\omega_0/V_{\max}$, and thus lighter sediment particles will be more likely to be trapped in a larger zone, and vice versa.

Fig. 6-5. The retention zone of sediment particles that is identified by the closed sediment streamlines on the cross-sectional plane. The ratio between the still-fluid settling velocity and maximum upwelling velocity, $\omega_0/V_{max} = 0.4$.

The existence of retention zone suggests that the concentration distribution may differ from that for 2D open channel flows. One of the related findings in previous investigations shows that a passive scalar can be homogenized within the retention zone bounded by closed streamlines of fluid (Batchelor, 1956; Rhines and Young, 1983). It is thus natural to enquire if this phenomenon still happens to suspended sediment load. In comparison, for sediment particles of density greater than fluid, the retention zone is specified by sediment streamlines rather than by fluid streamlines. Moreover, the sediment diffusion process should be quite different from that for passive scalars, making the advection more complicated. In the following, we will demonstrate that for the simplified case of homogeneous diffusion, sediment concentration distributes uniformly in the retention zone.

First, we introduce a function ψ_s for describing the sediment streamlines projected on the cross-sectional plane

$$\frac{\partial \psi_s}{\partial z} = -V_s = -(V - \omega_0), \quad \frac{\partial \psi_s}{\partial y} = W_s = W \quad (6-11)$$

It is easy to show that

$$\Psi_s = \Psi + \omega_0 z \quad (6-12)$$

where Ψ = the stream function for secondary flow cells.

Second, we adopt (Ψ_s, ϕ_s) as orthogonal curvilinear coordinates, where the lines of $\Psi_s = \text{constant}$ are used to define the projected sediment streamlines in the cross-sectional plane; and the lines of $\phi_s = \text{constant}$ are everywhere normal to these sediment streamline in the same plane. Note that ϕ_s also has a dimension of the product of velocity and length $[UL]$ as Ψ_s , but it is generally not the potential function of U_s when the latter is rotational.

Then, $C = f(x, y)$ is converted to be a function, $C = f(\Psi_s, \phi_s)$. The problem now turns to be to verify if C does not vary with Ψ_s and ϕ_s in the retention zone.

Along a closed sediment streamline (Ψ_s is constant, but ϕ_s varies), C tends to be homogenized. Consider a small circular tube with sediment-laden flow circulating inside. Sediment concentration in the tube should not monotonously increase or decrease in one direction since there is no starting point or end point. In case there is a location with higher concentration, the concentration will be gradually averaged along the circulating path. In an equilibrium state, C will have a constant concentration along the small circular tube. Suppose this feature holds for the closed 2D sediment streamlines, i.e., C does not change with ϕ_s . Then, the function $C = f(\Psi_s, \phi_s)$ reduces to $C = f(\Psi_s)$. Therefore, we only need to prove that C also does not vary with Ψ_s so that the result of $C(\Psi_s, \phi_s) = \text{constant}$ is finally obtained.

Taking an area integral of Eq. (6-10) along a certain closed streamline of Ψ_s yields

$$\iint V_s \frac{\partial C}{\partial y} + W_s \frac{\partial C}{\partial z} dydz = \varepsilon_s \iint \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} dydz \quad (6-13)$$

According to the Gauss' divergence theorem in a 2D space, the integral of the terms on the right hand side of Eq. (6-13) gives

$$\varepsilon_s \iint \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} dydz = \varepsilon_s \iint \nabla \bullet \nabla C dydz = \varepsilon_s \oint \nabla C \bullet \mathbf{n} ds \quad (6-14)$$

where the line integral follows the closed sediment streamlines; $\mathbf{n}$ = the unit vector outward normal to the sediment streamlines; and ds = the line element along the sediment streamlines. Considering $C = f(\psi_s) = f(\psi_s(x, y))$ and applying the chain rule, we get

$$\nabla C = \frac{dC}{d\psi_s} \nabla \psi_s \quad (6-15)$$

Using this equation and noting that $\nabla \psi_s = |\mathbf{U}_s| \mathbf{n}$, Eq. (6-14) can be rewritten as

$$\varepsilon_s \oint \nabla C \bullet \mathbf{n} ds = \varepsilon_s \frac{dC}{d\psi_s} \oint |\mathbf{U}_s| \mathbf{n} \bullet \mathbf{n} ds = \varepsilon_s \frac{dC}{d\psi_s} \oint |\mathbf{U}_s| \mathbf{s} \bullet \mathbf{s} ds = \varepsilon_s \frac{dC}{d\psi_s} \oint \mathbf{U}_s \bullet d\mathbf{s} \quad (6-16)$$

On the other hand, the terms on the left hand side of Eq. (6-13) is

$$\iint V_s \frac{\partial C}{\partial y} + W_s \frac{\partial C}{\partial z} dydz = \iint \mathbf{U}_s \bullet \nabla C dydz = 0 \quad (6-17)$$

because $\mathbf{U}_s$ and ∇C are perpendicular to each other. Substituting Eqs. (6-14), (6-16) and (6-17) into Eq. (6-13) yields

$$\frac{dC}{d\psi_s} \oint \mathbf{U}_s \bullet d\mathbf{l} = 0 \quad (6-18)$$

Because $\oint \mathbf{U}_s \bullet d\mathbf{l}$ is the sediment velocity integral along a certain closed streamline and must be non-zero, we finally have

$$\frac{dC}{d\psi_s} = 0 \quad (6-19)$$

Eq. (6-19) shows that C does not vary with ψ_s . Noting that C also does not vary with ϕ_s , we thus conclude that C is actually constant, independent of ψ_s and ϕ_s .

Therefore, the concentration appears to be homogenized within the retention zone provided that sediment diffusion coefficient is constant everywhere. In fact,

similar conclusion has been drawn by Farmer and Li (1994) in investigating oil dispersion in Langmuir circulations, for which oil drops however have an upward buoyancy velocity.

6.4.1.2 CASE II ($\omega_0/V_{\max} > 1$)

If $\omega_0/V_{\max} > 1$, settling sediment particles will travel only along open arcs, as shown in Fig. 6-6. Basically, the sediment concentration should decrease gradually from the bed to the free surface because sediment particles tend to settle down. Moreover, it would become non-uniform in the spanwise direction.

With the simplified condition that no lateral diffusion appears along the sediment motion direction, the contour lines of sediment concentration might be perpendicular to the sediment streamlines. Fig. 6-6 shows the conceptual contour lines of sediment concentration that are sketched perpendicular to the sediment streamlines for the condition $\omega_0/V_{\max} = 2$. It implies that the sediment concentration varies wavyly in the transverse direction. The larger concentration may occur in the downflow zone (over rough strips) near the free surface, whereas the larger concentration may occur in the upflow zone (over smooth strip) near the bed. Generally, the sediment concentration at the middle flow portion is more uniform in the spanwise direction.

Fig. 6-6. Sediment streamlines (solid lines) in the presence of secondary flow cells for $\omega_0/V_{\max} = 2$, and the conceptual concentration contour lines (dashed lines) for the condition of no lateral diffusion appearing along the sediment streamlines.

Even though the above conjecture is made only by considering the advection by secondary flows, the similar distribution pattern of sediment concentration shown in Fig. 6-6 has actually been observed in natural streams. Fig. 6-7 shows an example of the primary velocity and suspended sediment concentration measured at a cross section of the River Po, as reproduced from the paper of Vanoni (1946). The secondary flows inferred by Vanoni are also plotted in Fig. 6-7(b). It shows that, for example, the higher concentration in the upper flow portion appears at $z = 170$ m, where downflow occurs. In contrast, the lower concentration in the lower flow portion is also located at that distance. Because the information related to the field measurement is limited, the comparison is qualitative. However, this example evidences that sediment concentration could be distributed wavy in the spanwise direction due to the effect of cellular secondary flows.

Fig. 6-7. Flow characteristics and sediment concentration distribution, River Po in Italy reported by Vanoni (1946), where the dashed circles illustrated in (b) are the inferred secondary flows.

Moreover, Fig. 6-6 and Fig. 6-7 also suggest that the vertical profile of sediment concentration is more uniform in the downflow region than in the upflow

region, as sketched in Fig. 6-8(a). This may be quite contradicted to the instinctive expectation that downflow would enhance the settling of sediment particles and thus the concentration gradient would be larger than that estimated from the two-dimensional convection-diffusion theory. However, due to the three-dimensionality of the sediment transport, the lateral advection plays an important role in the redistribution of sediment concentration. The conceptual distribution pattern of sediment concentration along a flow cell is schematized in Fig. 6-8(b), where the numbers 1, 2, 3 and 4 represents the consequence of sediment concentration from low to high magnitude. The largest and smallest concentrations both occur in the upflow region. The distribution along each side could be roughly explained in the following. For example, for the downflow region, the concentration at point 2 could be increased due to the incoming convection by W_s , while the concentration at point 3 could be reduced because of the outgoing convection by W_s . If such lateral convections are strong, the vertical concentration distribution in the downflow zone would become more uniform. The reverse process may occur in the upflow zone, resulting in a greater vertical concentration gradient.

Fig. 6-8. (a) Vertical profiles of sediment concentration in upflow region and downflow region; and (b) possible distribution pattern of sediment concentration along a flow cell, where the numbers represent the consequence of sediment concentration from low to high.

6.4.2 MODIFICATION OF SEDIMENT DIFFUSION MECHANISM

In the preceding subsection, the advection influence is assessed by taking the diffusion coefficients to be constant. However, in turbulent flows the sediment diffusion coefficients generally vary with positions in space. Conventionally, the sediment diffusion coefficients are estimated from the momentum diffusion

coefficients (or eddy viscosities). For example, the vertical diffusion coefficient ε_y can be simply related to the vertical eddy viscosity ν_t by the following relationship

$$\varepsilon_y = \beta \nu_t \quad (6-20)$$

in which β = the proportional factor that depends on the properties of sediment. Experiments have shown that ε_y is roughly the same as ν_t for fine sediment, but may be less than ν_t for coarse sediment due to the sediment inertia greater than the fluid. The eddy viscosity ν_t is a measure of the flow turbulence, and can be defined by the following formula

$$\nu_t = \frac{\tau_x}{\rho \frac{dU}{dy}} \quad (6-21)$$

where τ_x = the streamwise Reynolds shear stress.

For the ‘reference 2D flow’ (see the definition in Chapter 5), the Reynolds shear stress has a linear distribution and the velocity profile obeys the log-law. The eddy viscosity ν_{t0} and the vertical diffusion coefficient ε_{y0} for the reference flow are

$$\nu_{t0} = \kappa u_{*0} y \frac{h_0 - y}{h_0} \quad (6-22)$$

$$\varepsilon_{y0} = \beta \kappa u_{*0} y \frac{h_0 - y}{h_0} \quad (6-23)$$

where κ = von Karman constant; u_{*0} = the bed shear stress for the reference flow; and h_0 = the flow depth for the reference flow.

In the presence of cellular secondary flows, the eddy viscosity and sediment diffusion coefficient will be modified. In Section 5.3.3 of Chapter 5, it has been proposed that

$$\nu_t = \nu_{t0} + \nu_{t1} G(z) \quad (6-24)$$

$$\frac{v_{t1}}{v_{t0}} = \frac{\partial U_1 / \partial y}{\partial U_0 / \partial y} + 2 \frac{l_1}{l_0} \quad (6-25)$$

where U_0 = the primary velocity for the reference flow; l_0 = the mixing length for the reference flow; U_1 = the perturbation amplitude of the primary velocity; v_{t1} = the perturbation amplitudes for the eddy viscosity; l_1 = the perturbation amplitude for the mixing length; and $G(z)$ = the periodic function. With the expressions for U_0 and U_1 , i.e. Eq. (5-4) and Eq. (5-11), Eq. (6-25) can be rewritten as

$$\frac{v_{t1}}{v_{t0}} = (\delta_{sl} - \delta_l) + \frac{\delta_{sw}}{2} \frac{y}{h} \sin \frac{\pi y}{h} \quad (6-26)$$

where δ_l = the parameter for the log-term in the proposed U formula; δ_{sl} ($= 2\delta_l + 2l_1/l_0$) is the parameter for the linear term in the proposed Reynolds shear stress formula; and δ_{sw} ($= \pi\delta_l$) is the parameter for the wake term in the Reynolds shear stress formula. Assuming that the relationship, Eq. (6-20), is still applicable, one can obtain from Eqs. (6-24) and (6-26) the sediment diffusion coefficient subject to cellular secondary flows

$$\varepsilon_y = \left(\beta \kappa u_{*0} y \frac{h_0 - y}{h_0} \right) \left\{ 1 + \left[(\delta_{sl} - \delta_l) + \frac{\delta_{sw}}{2} \frac{y}{h} \sin \frac{\pi y}{h} \right] G(z) \right\} \quad (6-27)$$

The parameters in this equation can be determined experimentally from the measured velocity profiles and Reynolds shear stress profiles, respectively. Table 6-2 lists the calculated values of $\delta_{sl} - \delta_l$ and $\delta_{sw}/2$. Note that the spanwise origin for the adopted coordinates is located at the center of the downflow zone, and $G(z)$ is positive in the downflow zone and negative in the upflow zone. Thus, the negative values of $\delta_{sl} - \delta_l$ and $\delta_{sw}/2$ mean that ε_y is decreased compared with ε_{y0} in the downflow zone and increased in the upflow zone. In particular, Fig. 6-9 shows an example of the contour plots of ε_y for Case S75, for which $G(z) = \cos(\pi z/\lambda_c)$ and $\beta = 1$. It clearly demonstrates that greater ε_y occurs in the upflow zone (over the smooth strips), especially in the upper flow portion.

Table 6-2. Measured Values of the Parameters in Eq. (6-27)

Case No.	$\delta_{sl} - \delta_l$	$\delta_{sw}/2$
S75	-0.23	-0.61
S50	-0.32	-0.65
WR	-0.10	-0.63

Fig. 6-9. Contours of dimensionless diffusivity, $\varepsilon_y/(\kappa hu^*_0)$ for Case S75.

To assess the general influence due to the modified ε_y , it is convenient to consider the direct changes to the sediment concentration of the reference flow C_0 when ε_y is modified. The original distribution C_0 is of Rouse type for which higher concentration is located near the bed. Since ε_y measures the diffusion strength, one can thus expect that in the upflow zone more suspended sediment will be diffused upward with the same initial concentration gradient. Therefore, the sediment concentration in the upflow zone may be re-distributed to be more uniform in the vertical direction relative to C_0 , and vice versa in the downflow zone. Interestingly, such an influence on concentration distribution due to the modified ε_y is just reverse to the effect of secondary flow advection for the case of $\omega_0/V_{\max} > 1$, as shown in Fig. 6-6. Therefore, the overall effect by secondary flow depends on which effect will overwhelm the other. The real case shown in Fig. 6-7 may result from the stronger lateral convection of suspended sediment.

The above discussion is focused on the vertical diffusion that generally dominates the sediment diffusion process. However, in 3D sediment-laden flows spanwise diffusion also plays a role, as indicated by Eqs. (6-7) and (6-8). Such a spanwise diffusion is induced by the lateral concentration non-uniformity, which could be caused by secondary flow advection or lateral non-uniformity of ε_y . Spanwise diffusion should lead to a transport of sediment from the high-concentration location to the low-concentration location and thus a reduced spanwise concentration gradient. As vertical diffusion process, spanwise diffusion is controlled by the lateral concentration gradient and the spanwise diffusion coefficient ε_z . The coefficient ε_z is even more difficult to be estimated than ε_y . However, one can expect that the general effect of spanwise diffusion is to diminish the lateral non-uniformity of concentration.

6.4.3 DISCUSSION ON THE EXPERIMENTAL RESULTS

In this experimental study, cellular secondary flows have been observed in the runs of Cases S75, S50 and WR. Because $\omega_0/V_{\max} \approx 0.1$ for these cases, sediment particles in the retention zones may move forward in spiraling paths, i.e., they may be captured by the secondary flow cells. Due to the advection effect as discussed, the sediment concentration may tend to be homogenized in the retention zone. At the same time, the diffusion effect should be also considered. Theoretically, a net diffusion only occurs in the direction along which a concentration gradient exists. Diffusion process usually helps to homogenize the concentration in that direction. In this sense, the uniform distribution in retention zone is an equilibrium state, and sediment concentration can be gradually homogenized. This trend has been experimentally demonstrated. As shown in Fig. 6-3, the Kaolin concentration profiles for Cases S75, S50 and WR are more uniform than those for Cases R75-10 and R50-10. It should be noted that the bulk flow conditions for all the five experiment cases were similar (see Table 3-3), but the evident difference is that there were no apparent cellular secondary flows observed for the latter two cases. Therefore, the secondary flow cells might play a role in making the sediment concentration more uniform.

However, small vertical concentration gradient still exists. The discrepancy between the measured results and the theoretical prediction can be due to the deviations of the real sediment trajectories from the theoretical sediment streamlines. The assumption $\mathbf{U}_s = \mathbf{U}_f + \boldsymbol{\omega}_0$ is generally correct, but it is only strictly valid for the flow fully steady and uniform (Nielsen, 1992), which is however not the case for the turbulence-driven secondary flows. At the same time, the 3D diffusion process may make the real trajectories more complicated than the idealized sediment paths. It is also possible that the homogenizing process for the uniform distribution might take a long time so that the equilibrium state was not fully developed. Therefore, it is more like a trend that if $\omega_0/V_{\max} < 1$ sediment concentration in the retention zone will be gradually distributed uniformly during the transport process, but a fully uniform state may not be reached. Such a trend is demonstrated by the present experiment. However, much further investigations are required to examine relevant details.

6.5 SUMMARY

The trajectory of settling sediment (or sediment streamline) subjected to secondary flows could be basically divided into two types according to the ratio $\omega_0/V_{\max}$. If projected onto the cross-sectional plane, the 2D sediment streamlines appears as open curves when $\omega_0/V_{\max} > 1$, or as closed circles when $\omega_0/V_{\max} < 1$. The advection process by secondary flows should differ for the two cases. Their possible sediment concentration distributions have been discussed separately with the simplified conditions. On the other hand, it is also highlighted that the vertical sediment diffusion coefficient is modified and spanwise diffusion also plays a role in sediment concentration distribution.

For the case of $\omega_0/V_{\max} < 1$, sediment concentration may have a trend to be homogenized in the retention zone by considering both the advection and diffusion effects, which is partially evidenced by the present experimental results.

Chapter

7

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

Characteristics of open channel flows are modified in the presence of secondary motions, which is different from those observed for the condition of uniform two-dimensional flows. This study focused largely on time-mean structures of the open channel flows subject to cellular secondary motions, which were generated by various longitudinal bedforms. This study also aimed to provide some implications for suspended sediment transport affected by secondary flows.

A series of laboratory experiments was conducted for open channel flows with six artificial longitudinal bedforms. The bedforms included alternate bed strips with different roughness heights and bed ridges of wavy and rectangular shapes. Detailed flow measurements were conducted using a two-component laser Doppler anemometer (LDA) and a one dimensional ultrasonic Doppler velocimeter (UDV).

The experimental results clearly demonstrate that cellular secondary flows could be generated by the longitudinal bedforms. For the cases of roughness strips and wavy ridges, the generated secondary flows appeared as pairs of counter-

rotating vortices or cells across the section. The vertical and spanwise dimensions of flow cells were observed to be the same as the flow depth and the average width of bed ribbons, respectively. Downflow occurred over the rough strip or the bed trough whereas upflow occurred over the smooth strip or the ridge. Near the bed, the lateral flows were directed from the rough strip (or trough bottom) to smooth strip (or ridge cusp). Opposite flows were also observed near the free surface. The strongest lateral flow usually occurred over the interface between the rough and smooth strips due to the sudden change in the bed roughness, and thus the circulation centre of flow cells was nearly located above the strip interface. The formed flow cell appeared to be symmetrical for the case of strips of equal widths, but skewed for the other cases of unequal strips. In comparison, for the case of rectangular ridges, the secondary flows developed in the different fashion due to the existence of the vertical connection between the ridge and trough, appearing to be much weaker and less organized.

The intensity of cellular secondary flows or maximum vertical velocity are closely associated with the perturbation amplitude of the bed roughness and/or bed surface elevation. When scaled with the average velocity of the primary flow, the dimensionless maximum vertical velocity is shown to be proportional to the summation of the relative bed elevation variation and the relative difference of squared roughness in terms of the Manning coefficient. In addition to the experimental observations, the time-mean secondary flows are also described analytically by a stream function that varies sinusoidally in the transverse and vertical directions.

The experimental results also show that the primary flow can be significantly modified due to the presence of cellular secondary motions. Both the streamwise velocity and Reynolds shear stress varied periodically with the changes in the bed roughness or elevation in the transverse direction. The spanwise variations can be described by the same periodical functions as those used for describing the vertical velocity of the secondary flow, which further implies an inherent relationship between the primary and secondary flows. The vertical profile of the streamwise velocity can be well represented by a function proposed in the log-wake form. The wake term for the outer region is positive in the upflow zone

and negative in the downflow zone, but nearly vanishes at the location of the circulation centre. The sub-regions of the flow in terms of streamwise velocity distribution, i.e. the inner and outer regions, can be identified based on the criteria developed for uniform open channel flow.

Considering that the flow is disturbed by small variations in the bed, the flow field is linearized so that a flow quantity is decomposed into two components, one being related to the average reference flow and the other symbolizing the first-order perturbation caused by the bed configuration. This perturbation approach is used in this study to explore the modified flow characteristics. For example, the modified distribution of the Reynolds shear stress is analytically formulated based on the concept of the mixing length.

Finally, some implications for suspended sediment transport are also provided. A major influence of secondary flow on suspended sediment is that the secondary flow modifies the cross-flow sediment trajectories or sediment streamlines. For fine sediment, the 2D streamlines projected on the cross sectional plane may appear to be closed circles. The sediment particles can be trapped by the flow cells and the concentration tends to be homogenised within the retention zones. Similar phenomenon has been also observed in the preliminary experiment conducted in this study. The results obtained also show that the vertical profile of sediment concentration becomes more uniform in the presence of cellular secondary flows.

7.2 RECOMMENDATIONS FOR FUTURE WORK

In this study, instantaneous secondary flow structures were not investigated due to the apparatus limitation that the vertical and spanwise velocities can not be measured simultaneously. However, such information is vital for understanding the evolution process of cellular secondary flow cells and relevant mechanisms. For research to be continued in this respect, it is suggested to employ a 3D Stereoscopic Particle Image Velocimeter (SPIV) for conducting 3D instantaneous velocity measurements, which is advantageous particularly in investigating cross-flow structures.

Since cellular secondary flow is driven by imbalance of Reynolds stresses, turbulence characteristics should be known for clarifying the generation mechanisms of cellular secondary flows. On the other hand, secondary flows could in turn modify the distribution patterns of turbulence characteristics. Therefore, a further investigation of turbulence characteristics related to cellular secondary flows is also necessary.

In this study, all experiments were conducted for similar flow conditions, where the flow depth was set to be almost the same as the average strip width. Such simplified conditions represent well those observed in natural rivers and laboratory flumes, but it would be also interesting to explore how the flow structure is to be modified if the flow depth is much higher or smaller than the average strip width, as may be encountered in deep oceans or shallow streams.

Due to the experimental limitations, such as the flume dimension and instrument resolution, the experiment related to suspended sediment was conducted with fine sediment only and the result is considered to be preliminary. Similar experiments can be made by considering transport of coarse sediment in the presence of secondary flows.

The experimental data collected in this study can be further used to validate relevant numerical simulations such as LES and DNS. In comparison with laboratory experiments, computer simulation may facilitate in-depth understanding

of instantaneous flow structures in particular in the near-bed region because it allows easy and precise control of the bed configurations and flow conditions.

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