Behaviour of reinforced embankment on soft ground and reliability analysis

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2006

Xu, B. (2006). Behaviour of reinforced embankment on soft ground and reliability analysis.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/12053>

<https://doi.org/10.32657/10356/12053>

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**BEHAVIOUR OF REINFORCED EMBANKMENT
ON SOFT GROUND AND RELIABILITY ANALYSIS**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2006

BEHAVIOUR OF REINFORCED EMBANKMENT ON SOFT GROUND AND RELIABILITY ANALYSIS

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A thesis submitted to Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2006

ABSTRACT

Stability analysis of slopes or embankments is greatly affected by uncertainty of soil properties. The current design practice is based on the factor of safety which cannot directly reflect the input uncertainty. Probabilistic stability analysis is a rational way to incorporate uncertainty in the design procedure. Unfortunately, most geotechnical engineers are reluctant to adopt probabilistic techniques. The objective of this study is to provide a practical approach to integrate probabilistic analysis in the stability evaluation of reinforced embankments on soft ground.

Behaviour studies of the unreinforced and reinforced embankment on soft ground are first carried out by finite element method. The effects of reinforcement on ground surface settlement, lateral displacement, excess pore water pressure and the development of plastic zone in the soft soil foundation are investigated. The conventional limit equilibrium method of slices and finite element method are used to calculate the factor of safety for the embankments. The results from different stability analysis methods are compared.

The effects of different optimization procedures on the location of critical circular or noncircular slip surface for reinforced embankments on soft ground are also investigated based on the extended Spencer's method. The optimization procedures include the grid search method, the simplex method, the genetic algorithm method, BFGS method and pattern search method.

Uncertainties of the critical reinforcement strain are studied by direct Monte Carlo simulation based on finite element method through the proposed procedure based on the response surface method. Two different approaches, the direct autocorrelation approach and the variance reduction approach, are considered to model the spatial

variability of soil properties.

The effects of undrained shear strength, critical reinforcement strain, reinforcement modulus, Bjerrum's vane correction factor, unit weight of the embankment fills, friction angle of the embankment fill, spatial variability of undrained shear strength in the foundation and reinforcement force distribution on the reliability of the reinforced embankment on soft ground are investigated in detail via the first-order reliability method (FORM).

In the last part of this study, a simplified reliability analysis approach is proposed for the embankment on soft ground based on the response surface method. The reliability index is calculated by the ellipsoid method based on first-order reliability method (FORM). The response surface is formed progressively. After obtaining the design point and the trust region parameter, the importance sampling technique is adopted to calculate the failure probability. Only a few calculations are required. It is more efficient than the direct Monte Carlo simulation, which may need a large number of calculations. The sensitivity of the input parameters to the reliability of the embankment can be assessed via the Spearman rank correlation coefficients based on the direct Monte Carlo simulation for the response surface obtained.

ACKNOWLEDGMENTS

I would like to express my great gratitude to my supervisor, Associate Professor Low Bak Kong, for his enthusiastic support, the unique mentoring environment he provided and the invaluable discussions we had.

Special appreciation is extended to School of Civil & Environmental Engineering, Nanyang Technological University (NTU) who has generously provided my scholarship.

Finally, I would like to express thanks to my parents and my wife, for their understanding, support and endless love in the period of my study.

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CHAPTER 1 INTRODUCTION

1.1 Overview

Natural soils are highly variable and heterogeneous. Various uncertainties in geotechnical engineering analysis, planning, and design have long been recognized (Lumb 1966; DeGroot and Baecher 1993; Becker 1996a, 1996b). However, most stability analyses of embankments are based on the deterministic methods in which soil layers are assumed to be uniform and the mean values of soil properties are usually used. In order to consider the variations in soil properties, a factor of safety chosen based on professional experience or some design criteria is often adopted in engineering design (Duncan 1996).

In recent years, there is an increasing awareness of the limitation of deterministic stability analysis methods which do not explicitly account for the variability and uncertainty of the soil parameters (Vanmarcke 1977a; Li and Lumb 1987; Low et al. 1998; Duncan 2000; Baecher and Christian 2003). It is desirable to quantify uncertainties of soil parameters and to include them in the slope stability analysis. Statistical procedures can be used to analyze the uncertainties associated with the soil properties. The introduction of statistical technique into the practice of geotechnical engineering helps to develop a more rational approach to the design and safety evaluation of the earth structure. The most important advantage of adopting a probabilistic approach is that various uncertainties can be considered rationally in the analysis (Chowdhury 1984).

The application of probabilistic stability analysis to geotechnical problems was first introduced in the 1970's by the work of Cornell (1971), Vanmarcke (1977b), Yucemen and Ang (1976), and others. Some 25 years later, probabilistic analysis is now on the "threshold of entering mainstream geotechnical practice" (Baecher 1997).

Probabilistic stability analysis methods for slopes or embankments have been under development over 20 years and there has been increased interest in developing and improving such methods in practice. For example, the U.S. Corps of Engineers now

applies probabilistic cost-benefit analysis in the evaluation of existing water-resource structures for rehabilitation purposes (Wolff 1996).

However, performing meaningful and consistent reliability analysis for slopes or embankments is difficult, especially in practice. The reluctance of practicing engineers to apply probabilistic methods is not uncommon. This is because most engineers lack formal training in statistics and probability theory. Hence, they are less comfortable dealing with probabilistic analysis than they are with the conventional deterministic factors of safety.

In addition, most of the current probabilistic stability analyses do not consider the reliability of embankments constructed over thick cohesive deposits and reinforced with polymeric geosynthetic, such as polyester, polyethylene and polypropylene, in textile or grid form.

Geosynthetic reinforcement has been shown to allow cost-effective construction of embankments on soft clay soils (Humphrey and Holtz 1986; Rowe and Mylleville 1996). Current design methods for reinforced embankments are purely deterministic and usually based on the conventional limit equilibrium methods of slices (Jewell 1982; Ingold 1982; Rowe and Soderman 1985; Leshchinsky 1987; Koerner 1997 and others), the use of plasticity solutions (Jewell 1988; Rowe and Soderman 1987), design charts (Rowe 1984; Hird 1986; Low et al. 1990 and others) and the semi-analytical method (Kaniraj and Abdullah 1993a, 1993b, 1994; Kaniraj 1994, 1996; Low and Tang 1997b).

In general, reinforced embankments are designed considering the four principal failure mechanisms, i.e. foundation bearing capacity failure, lateral sliding of embankment over reinforcement, breakage or pullout of reinforcement and rotational slope failure. Among these, the last failure mechanism governs the design in many cases and thus is the focus in this study.

Extending the current probabilistic stability analysis for slopes or embankments with no reinforcement to reinforced cases requires the consideration of additional factors associated with the reinforcement placed at the interface between the embankment fill

and the clay foundation, such as mobilized strains in the reinforcement. There are only a few published studies in the literature (Cheng and Christopher 1991; Low and Tang 1997b) which illustrated the implementation and benefits of probabilistic analysis for reinforced slopes or embankments on soft ground. Given the advantages of probabilistic slope stability analysis and the demands for probabilistic techniques in practice, there is a need to facilitate the application of probabilistic approaches in practice.

1.2 Objective and scope of the research

It is the main objective of this study to present a general methodology for treating the stability problem of reinforced embankments on soft ground within a probabilistic framework. The detailed objectives of the present study are as follows:

- (1) To study the behaviour of the unreinforced and reinforced embankments on soft ground by finite element method and to investigate the effects of the reinforcement on vertical displacement, excess pore water pressure, lateral displacement and stability of embankment on soft soil foundation.
- (2) To compare different stability analysis methods for the unreinforced and reinforced embankments on soft ground. These methods include limit equilibrium method (Spencer's method), enhanced limit method and strength reduction technique. The effects of different optimization procedures on searching for critical circular or noncircular slip surface for reinforced embankments on soft ground are also studied using the extended Spencer's method.
- (4) To investigate the uncertainties associated with the mobilized strain in the reinforcement of reinforced embankments on soft ground.
- (5) To investigate the effects of spatial variability of soil properties on the results of reliability analysis of embankments on soft ground. Only parameter uncertainty is considered in the analysis. The model uncertainty and human uncertainty are not included into the analysis.

(6) To develop a simplified reliability analysis procedure for reliability analysis of reinforced embankments on soft ground.

1.3 Outline of the thesis

This thesis is organized into nine chapters. Chapter 1 presents the need for this study, the objectives and scope of the research study, and the outline of the thesis.

Chapter 2 contains three parts. The first part presents a review about the conventional limit equilibrium methods of slices. Issues related to the applications of these methods in the stability analysis of reinforced embankments are reviewed and discussed. The second part reviews the applications of finite element method in the analysis of reinforced embankments on soft ground. Two different approaches: enhanced limit method and strength reduction technique, for calculating the factor of safety for embankments based on the finite element analysis, are also reviewed and discussed. Finally, the basic concepts of probabilistic slope stability analysis are reviewed.

Chapter 3 analyzes unreinforced and reinforced embankments on soft ground using the finite element method. The effects of reinforcement on ground surface settlement, excess pore water pressure, lateral displacement and development of plastic zone in the soft soil foundation are investigated. The enhanced limit method is used to calculate the factors of safety for unreinforced and reinforced embankments by directly using stresses computed from the finite element analysis. The results are then compared with those from Spencer's method and strength reduction technique.

Chapter 4 compares different optimization procedures for locating critical circular or noncircular slip surface for reinforced embankments on soft ground based on extended Spencer's method. For circular slip surface, which is commonly used in the stability analysis of reinforced embankments, three different optimization approaches are used: the grid search method, the simplex method and genetic algorithm. For critical noncircular slip surface, two different optimization approaches are used: pattern search method which does not require the evaluation of the gradient of the object function and

Broydon-Fletcher-Goldfarb-Shanno (BFGS) method which is a gradient-based approach.

Chapter 5 investigates the uncertainties associated with the mobilized strain in the reinforcement. The mobilized reinforcement strain used in conjunction with the limit equilibrium method to determine the mobilized reinforcement force and calculate the factor of safety of reinforced embankment is defined as critical reinforcement strain. It is the upper bound of the “working” mobilized reinforcement strain. The variability of this strain is investigated by finite element method through response surface method for reinforced embankment on soft ground based on Monte Carlo simulation.

Chapter 6 studies the effects of spatial variability of soil properties on the reliability of embankments on soft ground. Only parameter uncertainty is considered in the analysis. A simple procedure of considering two-dimensional spatial variability of soil properties is first proposed. The invariant first-order reliability method (FORM) is adopted to calculate the reliability index. The probabilistic critical slip surface is located automatically by using optimization procedure. Then, the difference in the reliability index obtained by two different methods of considering spatial variability: the direct autocorrelation approach and the variance reduction approach, is investigated.

Chapter 7 investigates the factors which affect the reliability of reinforced embankment on soft ground. These factors include undrained shear strength, critical reinforcement strain, reinforcement modulus, Bjerrum’s vane correction factor, unit weight of the embankment fill, friction angle of the embankment fill, spatial variability of undrained shear strength in the foundation and reinforcement force distribution. The difference in the reliability index between the mean-value first-order second-moment method (MVFOSM) and the first-order reliability method (FORM) is also studied.

Chapter 8 proposes a practical reliability analysis approach for reinforced embankments based on the response surface method. A progressive sampling procedure is used to generate the sampling points. The reliability index is obtained by the ellipsoid method based on the first-order reliability method (FORM). The failure

probability is computed by the direct Monte Carlo simulation based on the response surface obtained or by the importance sampling technique.

Chapter 9 contains a summary of the findings of this study, with recommendations for future studies.

CHAPETR 2 LITERATURE REVIEW

2.1 Overview

The literature review contained herein describes various issues related to the stability evaluation of reinforced embankments on soft ground, which include the limit equilibrium analysis and finite element analysis. In the final part of this chapter, basic concepts of probabilistic stability analysis methods are reviewed and discussed.

2.2 Limit equilibrium analysis for reinforced embankment on soft ground

The stability of reinforced embankment on soft ground is usually analyzed using limit equilibrium methods of slices. Most limit equilibrium methods of slices are suitable for the stability evaluation of reinforced embankment after minor modification. These methods include Fellenius's method (Fellenius 1936), Bishop's method (Bishop 1955), Janbu's method (Janbu 1973), Spencer's method (Spencer 1967) and Morgenstern and Price's method (Morgenstern and Price 1965).

2.2.1 Conventional limit equilibrium methods of slices

2.2.1.1 Fellenius's method

Fellenius's method (Fellenius 1936) is the simplest one of all the methods which make use of vertical slices. It is also known as the "Swedish", "Ordinary" or "USB" method. Figure 2.1 shows the region above the assumed circular failure surface divided into vertical slices. Figure 2.2 shows a single slice with all forces acting on it. In Fellenius's method, both the vertical and horizontal inter-slice forces are neglected. The normal force on the base of the slice is calculated by summing forces in a

direction perpendicular to the bottom of the slice. By taking the moments about the center of the slip circle and assuming that every point along the slip surface has the same value of factor of safety, the factor of safety can be calculated as follow:

$$F_s = \frac{\sum [c' \Delta l + (W \cos \alpha - u \Delta l) \tan \phi']}{\sum [W \sin \alpha]} \quad (2-1)$$

where c' and ϕ' are shear strength parameters at the mid-point of the slice base; W is the total weight of the slice; α is the inclination of the base of the slice; u is the pore water pressure at the mid-point of the slice base and Δl is the length of the base of the slice. According to the US Army Corps of Engineers (2003), the equation for Fellenius's method can be also written as:

$$F_s = \frac{\sum [c' \Delta l + (W \cos \alpha - u \Delta l \cos^2 \alpha) \tan \phi']}{\sum [W \sin \alpha]} \quad (2-2)$$

The derivation procedures for Eqs. (2-1) and (2-2) are different. Eq. (2-1) is derived by first solving the force because of the total weight of slice in a direction perpendicular to the base of the slice and then subtracting the force because of pore water pressures. For Eq. (2-2), it is derived by first calculating an “effective” weight of slice by subtracting the uplift force due to pore water pressure from the total weight, and then resolving forces in a direction perpendicular to the slice base. It should be noted that Eq. (2-1) can lead to unrealistically low or negative stresses on the base of slice in effective stress analysis. US Army Corps of Engineers (2003) recommended to use Eq. (2-2) because it could lead to more reasonable results when pore water pressures are considered.

Figure 2.1 Vertical slices within slipping soil mass

Figure 2.2 Forces acting on a single vertical slice

The factor of safety calculated from Fellenius's method may differ by as much as 20% from that from rigorous methods (Whitman and Bailey 1967), especially when the pore water pressures are high. Although the error is generally on the safe side, the error may be so large as to yield uneconomical designs. When $\phi = 0$, this method yields the same factor of safety as most rigorous methods.

2.2.1.2 Bishop's method

Bishop (1955) presented a method in which both the vertical and horizontal inter-slice

forces were taken into account in the analysis. Bishop's rigorous formulation for the factor of safety satisfies both force and moment equilibrium but contains too many unknowns to enable a direct solution to be found. From Bishop's original and now somewhat familiar notation, the expression for the factor of safety against a slip failure is:

$$F_s = \frac{\sum [c'b + ((W - ub) + \Delta X) \tan \varphi' / m_\alpha]}{\sum (W \sin \alpha)} \quad (2-3)$$

where $\Delta X = X_n - X_{n+1}$;

$$m_\alpha = \cos \alpha \left(1 + \frac{\tan \alpha \tan \varphi'}{F_s} \right)$$

b is the slice width

W is the total weight of the slice

c' is the effective cohesion

φ' is the effective angle of friction

u is the pore water pressure acting on the base of the slice

α is the angle of the base of the slice to the horizon

E_n is the inter-slice normal force on the left side of the slice

X_n is the inter-slice shear force on the left side of the slice

E_{n+1} is the inter-slice normal force on the right side of the slice

X_{n+1} is the inter-slice shear force on the right side of the slice

Figure 2.3 shows the forces acting on a typical slice in Bishop's method. In order to obtain determinable solutions from this equation, the assumptions about the vertical inter-slice forces need to be made. Most geotechnical engineers use the simplified or modified version of Bishop's method in which it is assumed that the difference in the

vertical inter-slice forces for any slice can be ignored and considered to be zero.

Figure 2.3 Forces acting on a typical slice in Bishop's method

Whitman and Bailey (1967) looked closely at the performance of the Bishop's simplified method and concluded that the error involved in using this method against the more rigorous stability analysis method is "7% or less, usually only 2% or less". They also suggested that the value of the term m_α be checked. The result should be considered unreliable if

$$m_\alpha \leq 0.2 \quad (2-4)$$

Since Eq. (2-3) cannot be solved directly, an iterative technique must usually be used in order to obtain a solution. In practice an initial value of the factor of safety from Fellenius's method is used to estimate the value of m_α . In most cases, only 2 or 3 iterations are enough to obtain a converged solution with the Bishop's simplified method.

However, it should be noted that only moment equilibrium and vertical force

equilibrium can be satisfied in Bishop's simplified method. Therefore, the errors from this method cannot be overlooked. Ting (1983) examined Bishop's simplified method from three points: the area of each slice, the angle of the base inclination and the numbers of slices used in calculating the factor of safety. It was found that the factor of safety is most in error for deep circles in cohesive slopes and that the error is likely to depend on the method of computing the area of the slice.

2.2.1.3 Janbu's method

From the mid fifties to the early seventies, Janbu developed the generalized procedure of slices. The corresponding simplified method is described in details in Janbu (1973). In the generalized procedure of slices, a line of thrust is assumed and the equations of equilibrium are solved. It was pointed out by Sarma (1979) that this is not a rigorous stability analysis method because the moment equilibrium of the last slice may not be satisfied. In fact, this affects the thrust line but has no great influence on the factor of safety. Janbu (1973) stated that the factor of safety is relatively insensitive to the assumption about the location of the thrust line as long as it is reasonable. According to Janbu (1973), the thrust line should be near one third of the height of the slice for cohesionless soils. It should be below this level in the active zone and above it in the passive zone for cohesive soils.

For Janbu's simplified method, only force equilibrium is satisfied and only one iteration of his "rigorous" method is used to obtain the first estimated normal stresses on the base of the slices and inter-slice forces. The result from this first iteration is then multiplied by a correction factor that may depend on the curvature of the slip surface and the soil parameters. These correction factors are obtained from comparing limited factors of safety calculated for homogeneous slopes with those from Janbu's "rigorous" method. Lumsdaine and Tang (1982) reported that, for more complex

problems, there is very little correlation between the correction factors given by Janbu and the actual correction factors obtained by using Janbu's rigorous method for the same stability problem.

Janbu (1973) himself recommended that the simplified method should only be used when computers were not available and an estimate of the factor of safety was to be obtained by hand calculations.

2.2.1.4 Spencer's method

Spencer (1967) presented an alternative way of considering the inter-slice forces by assuming that the resultant side forces at the two sides of a slice were parallel and all inclined at a constant angle θ . Then Q , the resultant of the two inter-slice forces z_n and z_{n+1} , will also be inclined at an angle θ as shown in Figure 2.4.

For overall equilibrium, the following equations should be satisfied:

$$\sum(Q \cos \theta) = 0 \quad (2-5a)$$

$$\sum(Q \sin \theta) = 0 \quad (2-5b)$$

$$\sum(Q \cos(\alpha - \theta)) = 0 \quad (2-5c)$$

If the angle θ is chosen to be constant for all slices, then Eqs. (2-5a) and (2-5b) reduce to:

$$\sum(Q) = 0 \quad (2-6)$$

Figure 2.4 Dimensions of slip surface and forces on a slice (Spencer 1967)

Spencer (1973) modified his original formulation and allowed the angle θ to vary from slice to slice. In fact, the effect of assuming a constant value for the angle θ on the factor of safety was found to be very small by Spencer (1967).

For any particular value of the angle θ , a factor of safety, F_f can be found to satisfy force equilibrium (Eq. (2-6)) and another F_m , can be found to satisfy moment equilibrium (Eq. (2-5c)). A plot can then be made of F_f and F_m against the angle θ

as shown in Figure 2.5. The required factor of safety is that for which $F_f = F_m = F_1$.

The factor of safety F_1 and the corresponding value of the inter-slice force angle θ_1 can be used to determine all the inter-slice forces and their line of thrust.

Figure 2.5 Variation of F_f and F_m with the angle θ (Spencer 1967)

The difference between the factor of safety obtained by using Spencer's method and that by Bishop's simplified method is not great. Spencer (1967) pointed out that the difference between the two methods exceeded 1% in only 7 of 92 cases that he tried.

2.2.1.5 Morgenstern and price's method

This method is one of the most widely used methods developed for analyzing slopes with non-circular slip surfaces. This method was initially presented by Morgenstern and Price (1965) and all static equilibrium requirements can be satisfied and therefore it is a rigorous method.

Figure 2.6 Forces acting on a single slice in Morgenstern and Price's method

The most important part of this method is to assume a functional relationship between the inter-slice shear force X_n and the inter-slice normal force E_n . The function is referred to as $f(x)$ and most programs implementing the method provide a choice of such functions. A scaling factor λ is adopted to overcome the possibility that choosing such a function may actually overdetermine the stability problem. Therefore, $f(x)$ defines the relative inclination of the inter-slice forces while λ defines their absolute magnitude. The inter-slice forces on the left side of slice n in Figure 2.6 can be expressed by the following equation:

$$X_n = \lambda \cdot f(x) \cdot E_n \quad (2-7)$$

The solution procedure proposed by Morgenstern and Price (1965) differs from that adopted by most researchers in that the stability problem is formulated by using differential equations which are integrated over each slice; thus the slices can be very wide if the base is in one soil type. A limitation of the procedure is that it is very difficult to consider concentrated surface loads.

There can be many functions of $f(x)$ that satisfy equilibrium for any particular

problem; while for some other problems, it may be very difficult to find functions of $f(x)$ to obtain the solutions. Chen and Morgenstern (1983) found that the functions of $f(x)$ which can give admissible solutions will also give factors of safety close to each other and that factors of safety derived from inadmissible solutions can vary widely.

The thrust line of the inter-slice forces, average shear stress on the side of each slice and the normal stress on the base of each slice can be obtained as parts of the solution and usually depend on the assumed function of $f(x)$. Mostyn and Small (1987) mentioned that the following conditions must be satisfied in order to obtain an admissible solution:

- I. The resulting thrust line of the inter-slice forces must be reasonable, which means that the resultant of the inter-slice forces should act about one third of the height up in each inter-slice. This guarantees that no tension happens along the side of each slice.
- II. The failure criteria for the soils cannot be violated and the shear strength between the individual slices should not be exceeded.
- III. The normal stress at the base of each slice cannot be negative and should be compressive only.

More details of the additional admissibility requirements can be found in Chen and Morgenstern (1983), such as the requirements about the forces acting on the endpoints and the value of $\lambda f(x)$ at non-vertical endpoints.

The Morgenstern-Price method is generally carried out on computers. The results always include adequate data to determine the admissibility of each solution. After reviewing numerous stability analysis methods, Li and White (1987a) recommended this method for general use.

2.2.1.6 General comments on the conventional limit equilibrium methods of slices

Among stability analysis methods which can analyze non-circular slip surfaces, only Morgenstern-Price's method (1965) does not need the assumption that the normal force on the base of each slice acts at the centre of the slice. The accuracy of other methods may increase as the slices become thinner and a reasonable number of slices are used. Fredlund and Krahn (1977) made a full comparison of various stability methods by assigning an equivalent λ value to each method. For Morgenstern-Price method, the side force function $f(x)=1$ was adopted. For Bishop's simplified method, λ is equal to zero. For Spencer's method (1967), λ is equal to $\tan \theta$. For Janbu's method, an equivalent λ was determined. It was concluded from the above comparisons that the factor of safety satisfying moment equilibrium yielded results close to each other provided that admissible side force functions were adopted.

Ching and Fredlund (1984) conducted a comprehensive study to compare the factors of safety calculated from different stability analysis methods and reached conclusions similar to the above. For homogeneous cohesionless soils ($c' = 0$), all methods of slices gave identical factors of safety. For frictionless soils ($\phi = 0$), the methods satisfying moment or moment and force equilibrium gave the same solutions. Methods that only satisfied force equilibrium produced factors of safety which were greatly dependent on the assumption of the inter-slice forces. Fredlund (1984) summarized the quantitative relationships between limit equilibrium methods. According to him, Fellenius's method always gives conservative factors of safety. Methods that satisfy all

conditions of equilibrium give essentially the same value for the factor of safety. Differences between various methods should not be more than $\pm 5\%$. Bishop's simplified method, although it does not satisfy horizontal force equilibrium, gives virtually the same factors of safety as methods that satisfy overall equilibrium. For the methods that satisfy force equilibrium only, the factors of safety are within $\pm 15\%$ of those satisfying overall equilibrium conditions.

2.2.2 Semi-analytical method for stability analysis of reinforced embankment on soft ground

Low (1989) presented a simple and convenient semi-analytical procedure to calculate the factor of safety of embankments on soft ground. Low et al. (1990) extended this method to the stability analysis of reinforced embankments on soft ground. The factor of safety against rotational failure is defined in terms of overall moment equilibrium. Low and Tang (1997b) combined this method with a spreadsheet's optimization tool to search for the critical slip surface. This method is reviewed here since it is very useful for the stability analysis of reinforced embankments on soft ground where a rotational failure is common. Due to its closed-form nature, the calculation of factor of safety does not involve any convergence problem as often observed in some rigorous limit equilibrium methods of slices.

Figure 2.7 shows a typical reinforced embankment on soft ground and notations in the semi-analytical method. This method utilizes the following equations:

$$l = H \cot \Omega \quad (2-8)$$

$$\theta = \cos^{-1} \left(\frac{z}{z + D} \right) \quad (2-9)$$

$$\theta_h = \cos^{-1}\left(\frac{z-H+h_c}{z+D}\right) - \cos^{-1}\left(\frac{z}{z+D}\right) \quad (2-10)$$

$$M_o = \left[\frac{x}{2} \times (l-x) - \frac{l^2}{6} + (z+D) \times \left(D + \frac{H}{2}\right) - \frac{1}{2} \left(D + \frac{H}{2}\right)^2 - \frac{H^2}{24} \right] \gamma H \quad (2-11)$$

$$\Delta M_o = \frac{\gamma_w h_w^2}{2} \times \left(z - H + h_c - \frac{h_w}{3} \right) - \frac{\gamma h_c^2}{2} \times \left(z - H + \frac{2}{3} h_c \right) \quad (2-12)$$

$$M_R = R \times (c_a \times 2R\theta) + R \times (c_m + \lambda \gamma H \tan \phi_m) \times \theta_h R \quad (2-13)$$

$$\lambda \cong 0.19 + 0.02 \frac{\cot \Omega}{(D/H)} \quad (2-14)$$

$$M_T = T \times Lever \quad (2-15)$$

where c_m and ϕ_m are the cohesion and friction angle of the embankment, l is the horizontal projection of slope AB, M_o is the overturning moment derived from block ABJKEA, ΔM_o is the additional overturning moment due to the water pressure in the crack minus the overturning moment of the block JKQ, M_R is the available soil resisting moment along block IG EK, M_T is the resisting moment from the reinforcement force and c_a is the equivalent undrained shear strength along arc IGE and is a function of z , D and the depth-dependent c_u profile.

Figure 2.7 Notations for reinforced embankment on soft ground in the semi-analytical method (after Low and Tang 1997b)

Depending on the different assumptions about the orientation of the reinforcement force, the lever arm in Eq. (2-15) can be assumed equal to $(z + D)$, z or $[(z + D) + z]/2$, which corresponds to the tangential orientation, the horizontal orientation or the average between these two directions, respectively. The factor of safety for the reinforced embankment on soft ground can be defined as:

$$F_s = \frac{M_R + M_T}{M_o + \Delta M_o} \quad (2-16)$$

if M_T is the *maximum available* resisting moment from the reinforcement force.

or

$$F_s = \frac{M_R}{M_o + \Delta M_o - M_T} \quad (2-17)$$

if M_T is the mobilized working resisting moment from the reinforcement force.

The original work by Low (1989) was also modified by Kaniraj and Abdullah (1993a) to consider the effect of berms and tension crack on the stability of embankments on soft soils. The analysis results showed that berms and flatter slopes are beneficial only if the slip surface is at a shallow depth. It was also found that tension cracks may or may not reduce stability. The effect depended on the soil parameters for the embankment and the foundation soils. The same authors also extended the semi-analytical method to the stability analysis of narrow-crested reinforced embankments on soft ground (Kaniraj and Abdullah 1993b). Kaniraj (1994) improved the work by Kaniraj and Abdullah (1993a) by incorporating the effect of excavation outside the berm. Similar procedure was used to calculate the maximum required bisectorial reinforcement force for reinforced embankment on soft ground by Kaniraj and Panwar (1994) and to investigate the effects of berms and tension crack on the maximum required reinforcement force by Kaniraj and Abdullah (1994). Three different assumptions about the direction of the reinforcement force: horizontal, tangential and bisector to the horizontal and tangential directions were investigated by Kaniraj (1996). The author found that the factor of safety of the reinforced embankment was very sensitive to the assumption about the direction of the reinforcement force.

Borges and Cardoso (2002) extended the work by Kaniraj and Abdullah (1993a) using the reinforcement force from finite element analysis.

2.2.3 Tension crack

Cracks are often found at the crest of the embankment or slopes. The existence of the cracks indicates that the tensile stress in a certain zone exceeds the tensile strength of the soils. Taylor (1948) wrote: “The questionable action and the questionable depth of the tension zone have considerable bearing on the limited dependability of many stability analyses, and the action within the tension zone is a subject that is worthy of much study.”

The existence of tension cracks is likely to decrease stability and therefore increase the tendency of a slope to fail. Firstly, the length of a potential slip surface over which shear strength can be mobilized is reduced. Secondly, a crack may fill with water which exerts an additional driving force.

Most results of slope stability analyses may depend, to a significant extent, on the way in which the stability analysis method copes with the estimation of tension crack depth and location.

The classical approach to estimate the tension crack depth is based on the Rankine's earth pressure theory and leads to the following well known equation for the depth up to which the earth pressure is negative or tensile:

$$z_c = \frac{2c'}{\gamma} \tan(45^\circ + \frac{\phi'}{2}) \quad (2-18)$$

in which c' and ϕ' are the effective stress shear strength parameters of cohesion and angle of internal friction, respectively, and γ is the soil unit weight. It is assumed that the soil has no tensile strength. This depth can be regarded as the depth of tension

crack D_t . This method is only applicable to homogeneous soil and does not take into consideration the slope geometry, pore pressure and the factor of safety.

Spencer proposed and developed a systematic approach to consider tension cracks in limit equilibrium slope stability analysis (Spencer 1967, 1968, 1973, 1981). The classical equation for depth of tension cracks were modified by him to the following:

$$D_t = \frac{2c'_m}{\gamma(1-r_u)} \tan(45^\circ + \frac{\phi'_m}{2}) \quad (2-19)$$

in which $c'_m = \frac{c'}{F_s}$ is the mobilized cohesion, $\phi'_m = \arctan(\tan \frac{\phi'}{F_s})$ is the mobilized friction angle, F_s is the factor of safety and γ is unit weight of soil.

Spencer (1967) suggested a chart-procedure to estimate the crack depth using the method of parallel inter-slice forces. Through extensive trial and error calculation, the depth of tension cracks is determined for every trial slip surface. If the position of the line of thrust is not reasonable, a tension crack is introduced and a different line of thrust is determined. This process is repeated by increasing D_t until a reasonable line of thrust is obtained. However, this method is still based on the Rankine's theory of earth pressure. The depth of tension cracks is not related to the tensile strength of the soils.

Baker (1981) presented a variation method for tension cracks. But the approach is only suitable for straight and homogeneous slopes without pore pressure or external loading.

Lee et al. (1988) investigated the factors which may influence the depth of tension

cracks developed in the embankment. It was observed that the depth of tension cracks increased with the embankment height H , the ratio of the embankment stiffness to the foundation stiffness E_e/E_f and the ratio of the embankment width to the foundation thickness B/D and decreased with the Poisson's ratio of the embankment.

2.2.4 Considering the effect of reinforcement by conventional limit equilibrium methods of slices

Using conventional limit equilibrium methods of slices for stability analyses of unreinforced slopes or embankments is well accepted in geotechnical engineering practice. Evaluation of the stability of a reinforced embankment on soft ground needs to consider the effect of reinforcement, which is usually placed at the interface between the embankment fill and the clay foundation. While most limit equilibrium methods of slices have been modified for the analysis of reinforcement embankments, there are distinct differences in the details of the modifications.

Christie and El-Hadi (1977) modified Fellenius's method (Ordinary Method of Slices) to analyze the reinforced dams by including a resisting moment due to the reinforcement force in the horizontal direction. The same procedure was used by Brakel et al. (1982) for the stability analysis of a reinforced embankment. Inglood (1982) extended Bishop's simplified method to account for horizontal orientation of reinforcement force, recognizing that the assumption of horizontal orientation was conservative. Rowe and Soderman (1984, 1985) and Low (1985) modified Bishop's simplified method for reinforced embankments by including a resisting moment due to either horizontal or tangential orientation of the reinforcement force.

In the modified Fellenius's method (Ordinary Method of Slices) and the modified Bishop's simplified method, the factor of safety of a reinforced slope or embankment

is expressed as:

$$F_s = \frac{(M_R + M_T)}{M_o} \quad (2-20)$$

where M_R is the resisting moment provided by the soil, M_T is the resisting moment provided by the reinforcement force and M_o is the overturning moment. The definition of the factor of safety may also be considered in terms of the resisting moment from the reinforcement force playing the role of reducing the overturning moment. The factor of safety according to this definition can be expressed as:

$$F_s = \frac{M_R}{M_o - M_T} \quad (2-21)$$

Duncan and Wright (2005) called the method using Eq. (2-21) as “Method A” and the method using Eq. (2-20) as “Method B”. According to them, “Method A” is preferable because the soil strength and the reinforcement force have different sources of uncertainty. Factoring them separately makes it possible to reflect these differences. In “Method A”, only the soil strength is divided by the factor of safety calculated in the stability analysis. This is also consistent with the more general definition of the factor of safety where it is defined as the factor by which the shear strength parameters are reduced in order to bring the stability mechanism into a state of limit equilibrium. Both the soil strength and the reinforcement force are divided by the same factor of safety in “Method B”.

However, only overall moment equilibrium is satisfied in Fellenius’s method. For Bishop’s simplified method, horizontal force equilibrium is not satisfied. Both methods are not rigorous. In recent year, more rigorous method such as Janbu’s generalized

procedure of slices (Janbu 1973), Spencer's method (Spencer 1967) and Morgenstern and Price's method (Morgenstern and Price 1965) have been modified and used to analyze reinforced slopes or embankments.

Sabhahit et al. (1994) modified Janbu's generalized procedure of slices (Janbu 1973) to calculate the factors of safety of reinforced embankments on soft ground. The reinforcement force was included in each slice which intersects the reinforcement. The equations for calculating the factor of safety were reformulated by incorporating the effect of the reinforcement. The effect of orientation of reinforcement force on force and moment equilibrium was also investigated. The critical slip surface was not restricted to the circular shape. The results showed that the factor of safety was significantly smaller than that from stability analysis based on circular slip surface analysis. The parametric study conducted by the authors showed that the orientation of reinforcement force affected the stability of the reinforced embankment considerably.

Tandjiria et al. (2002) extended Chen and Li's approach (Chen and Li 1998), which is based on Chen and Morgenstern's method (Chen and Morgenstern 1983), to analyze the stability of reinforced embankments on soft ground. Three different types of reinforcement force distribution: the linear force distribution, the cubic force distribution and the force distribution governed by the bond or rupture strengths were used in the analysis. Critical noncircular slip surface was located using the constrained optimization tool Solver in Microsoft Excel. The results showed that the factor of safety was insensitive to the assumption made for the reinforcement distribution.

Low and Tang (2001) reformulated and extended Spencer's method to calculate the factor of safety for reinforced embankments on soft ground. Details about this procedure will be described in Chapter 4.

2.3 Finite element analysis for reinforced embankment on soft ground

Finite element method is a powerful technique to investigate the behaviour of reinforced embankments on soft ground. Finite element analysis allows consideration of construction consequence, nonlinear behaviour of soils, strains mobilized in the reinforcement, soil-reinforcement interaction, development of plastic zones in the soft foundation, excess pore water pressures and deformations of reinforced embankments. Numerous researchers have used this method to conduct numerical analyses for reinforced embankments on soft ground. According to the way the excess pore water pressure is dealt with, finite element analyses for reinforced embankments on soft ground can be divided into two main categories: undrained analysis and coupled analysis. Undrained analysis uses undrained parameters in the analysis to predict the behaviour of reinforced embankments on soft ground at the end of construction. Dissipation of excess pore water pressures with the consolidation is not taken into account. Coupled analysis considers the time dependent behaviour of reinforced embankments under partially drained conditions. It is more rational, but this approach demands more computational efforts.

2.3.1 Undrained analysis

The undrained behaviour of reinforced embankments on soft ground has been extensively investigated. Brown and Poulos (1978) analyzed the collapse height of unreinforced and reinforced embankments on stable foundation with an elastic-plastic soil model. The authors found that using reinforcement could increase stability by up to 40 percent for the embankment analyzed. Boutrop and Holtz (1983) incorporated Drucker-Prager soil model into the finite element analysis for the reinforced embankment under undrained and drained conditions. The reinforcement was found to have little effect on the ground surface settlement. Comparing the results from

undrained analysis with those from drained analysis, Boutrop and Holtz (1983) found that the mobilized reinforcement force was significantly less under drained condition.

Rowe et al. (1984) analyzed a geotextile-reinforced embankment on a peat deposit with finite element method. The soil was modeled as a nonlinear elastoplastic material with a Mohr-Coulomb failure criterion and a nonassociated flow rule proposed by Davis (1968). The reinforcement was treated as a structural membrane with axial stiffness but negligible flexural rigidity. Slip in the interface between the soil and the reinforcement can occur when the shear stress in the interface reaches a limiting value based on Mohr-Coulomb criterion. Input parameters for the peat deposit were determined from direct shear tests, triaxial compression tests, uniaxial tension tests and simple shear tests. The results of analysis using different input parameters were compared with each other. It was found that the best agreement between observed and calculated deformations was obtained using parameters from simple shear tests. The results showed that the major effect of geotextile reinforcement was to reduce lateral spreading and to increase stability.

Low (1985) used Duncan-Chang hyperbolic soil model to perform parametric studies for geogrid-mattress reinforced embankments and embankments reinforced with flat-lying reinforcement. The author concluded that: (1) the effectiveness of reinforcement depended upon the magnitude of the mobilized force at working condition. (2) Mobilized reinforcement force increased with stiffness of reinforcement and with foundation depth, while it decreased with strength of foundation soils. (3) Mobilized reinforcement force had the effects of decreasing shear stresses and the development of plastic zone in the foundation. (4) Placing reinforcement closer to the interface between the embankment fill and the foundation was slightly more effective in restraining deformations.

Rowe and Soderman (1987) showed that the failure height for a reinforced embankment was related to the modulus of the reinforcement but was not very sensitive to the modulus of the soil. The development of strain in the reinforcement was also examined. The finite element analysis results indicated that the reinforcement played a relatively small role at low load levels since the soil was essentially elastic. Significant strain in the reinforcement began to develop with increasing plasticity. Most of the strains were developed after a continuous plastic zone was formed in the soil. The authors concluded that the reinforcement strain for a given embankment height largely depended on the height of the embankment relative to the height at which contiguous plasticity occurred, and hence was sensitive to the magnitude and distribution of the actual shear strength in the soil deposit.

Monnet et al. (1989) analyzed ALMERE embankment with and without reinforcement. The soil was assumed to behave as a standard elasto-plastic material with Drucker-Prager yield criteria. The behaviour of the interface between the soil and the reinforcement was modeled by the elasto-plastic model. The plastic yield was considered to be reached when the shearing stress exceeded the limiting value based on Mohr-Coulomb criterion. The results showed that the mobilized strain in the reinforcement increased with the height of the embankment. It was also demonstrated by them that using reinforcement can restrict the development of plastic zone in the foundation and thus increase the height of the embankment.

Hird and Kwok (1989) carried out parametric studies for two reinforced embankments with the aim of gaining an improved understanding of the role of the reinforcement. The embankment soil was modeled by a linearly elastic-perfectly plastic model with a Mohr-Coulomb yield criterion. The interface element devised by Goodman et al. (1968) was used to model the behaviour of the interface between the soil and reinforcement. A very high normal stiffness ($k_n = 10^6 \text{ kN/m}^3$) was specified to prevent overlapping of

the soil and the reinforcement. The foundation soils were modeled using Cam clay model of critical state soil mechanics. The reinforcement was modeled as either a linear or bilinear elastic material. In the finite element analysis, the embankment was kept fully drained while the foundation remained undrained. The results confirmed that the reinforcement can resist extrusion of the foundation. In a fully reinforced embankment, the full strength was mobilized on the lower soil-reinforcement interface. The lateral displacement of the foundation can be reduced by reinforcement. However, for both the embankments analyzed, little beneficial effects were obtained by increasing the reinforcement modulus beyond about 7000 kN/m.

Hird et al. (1990) compared the finite element results with rigorous plasticity solutions for reinforced embankments on soft ground. For the analysis of foundation stability, the loads applied to the subsoil by the reinforced embankment were considered similar to those applied by a footing and the problem treated as one of bearing capacity. In the finite element analyses, both the elastic-perfectly plastic model with a Tresca yield criterion and Modified Cam clay model were used for the soil. The authors showed that the finite element analysis successfully predicted collapse load distribution beneath smooth and rough footings. For both a soil of uniform strength and limited depth and a soil of increasing strength with unlimited depth, the results agreed with theoretical plasticity solutions.

2.3.2 Coupled analysis

During the recent ten years, with the fast development of computer technology, it has become feasible to use finite element models that are sufficiently sophisticated to model the time dependent behaviour of reinforced embankment under a wide range of conditions.

Chai and Bergado (1993) studied the performance of two geogrid reinforced embankments on Muar clay with finite element method. The behaviour of the soft foundation soil was modeled by Modified Cam clay model (Roscoe and Burland 1968). The embankment fill was modeled by hyperbolic constitutive law (Duncan et al., 1980). The consolidation process of the soft clay was simulated by coupled consolidation theory (Biot 1941). Two different soil-reinforcement interaction models: direct shear and pullout were used in the analysis. The parametric studies showed that these two models yielded practically the same mobilized reinforcement tension force. The mobilized tensile forces in the reinforcement were found to increase with increasing fill thickness and during consolidation of the foundation soil.

Bergado et al. (1995) analyzed a reinforced embankment on soft Bangkok clay with finite element method. In the analysis, the large deformation phenomenon was simulated by updating the nodal coordinates. The results showed that large total and differential settlements were caused by the embankment loading. Both reinforcement force and lateral displacements were increased when the degree of consolidation increased. The authors found that the predicted foundation settlements agreed reasonably well with the field data, while the agreement between predicted excess pore water pressures, reinforcement forces and foundation lateral displacements were less perfect. The calculation of excess pore water pressures was strongly influenced by the foundation soil permeability. As pointed out by the authors, there were still no satisfactory methods to correctly determine the values and the variation of the permeability of the soils.

Sharma and Bolton (1996) carried out numerical analysis for reinforced embankments on soft clay with consideration of the anisotropic behaviour of the foundation clay. The results showed that predictions of excess pore water pressures, settlements and lateral deformations of clay foundation were in good agreement with the measured values.

Measured and predicted mobilized forces in the reinforcement were also found to be in close agreement. The reinforcement force was observed to be sensitive to the magnitude and distribution of shear strength of clay foundation. The authors suggested using the constitutive model which can account for the induced anisotropy for the clay foundation in the numerical analysis for reinforced embankment on soft clay.

Varadarajan et al. (1999) conducted finite element analysis for a reinforced embankment on soft ground using a coupled FEM formulation and elasto-plasticity theory. Four important factors affecting the behaviour of the embankment: reinforcement type, clay type, foundation depth and drainage condition were studied. The results showed that the deformation of foundation surface decreased with increase in reinforcement stiffness. Maximum effect of reinforcement on the horizontal displacement was noted near the embankment toe. The effect of reinforcement on vertical displacement was found at the center line and away from the toe. The effect of reinforcement in reducing the displacement was found to be more significant for the stronger clay deposit.

Rowe and Li (1999) studied the behaviour of reinforced embankments constructed on soft cohesive foundations under undrained and partially drained conditions with an elliptical cap soil model. The results showed that reinforcement could substantially increase the stability of embankments on the soft foundations under both undrained and partially drained conditions. Excess pore water pressures developed in the foundation soil at the end of construction were found to be significantly less than that expected under undrained conditions. The effect of reinforcement was greater under partially drained conditions than under undrained conditions. The authors ascribed this partly to the increase of strength near the soil reinforcement interface allowing greater mobilization of the reinforcement force. It was also demonstrated by the authors that partial drainage during construction provides an 11-38% increase in the factor of safety

against failure of reinforced embankments relative to that expected under perfectly undrained conditions.

Borges and Cardoso (2001) carried out parametric studies for a reinforced embankment on soft soil. A hardening elasto-plastic model was adopted for the reinforcement. The viscous behaviour of the reinforcement was also considered by a rheological model based on a series of Kelvin's models (Lopes et al. 1994). The behaviour of the soil-reinforcement interface was simulated by a hardening elasto-plastic model. The results showed that the reinforcement force could decrease when the reinforcement stiffness decreased with time due to geosynthetic viscosity. This resulted in the increase of horizontal displacements and settlements. The embankment width was also found to influence the type of failure surface that could occur. When the embankment width was small, failure could occur without cutting the embankment and the reinforcement. In this case, the reinforcement had no contribution to the overall stability of the embankment. The effect of construction sequence on the settlements was also investigated by the authors. It was found that constructing each embankment layer starting from its extremities could reduce the ground surface settlements and lateral displacements in the foundation.

Rowe and Li (2002) investigated the time-dependent behaviour of embankments constructed on rate-sensitive foundation soils using a finite element program that can model the soil-reinforcement interaction and viscoplastic characteristic of soil materials. The influence factors examined in the analysis included the viscoplastic parameters of soft cohesive soil, reinforcement, construction rate and hydraulic conductivity of soils. The analysis results indicated that the embankment stability after the end of construction was more critical than that at the end of construction. An arbitrary selection of the strain rate for use in the determination of undrained shear strength may potentially lead to post-construction failure of embankments constructed

on rate-sensitive foundation soils. The authors showed that the maximum reinforcement strain mobilized at the end of construction was relatively small due to rate effects on the foundation soils during construction. The increase in the reinforcement strain after construction was around one to three times the maximum strain mobilized at end of construction. Partial consolidation during and shortly after construction contributed to a reduction in the maximum reinforcement strain and creep deformations of the foundation soils. The time-dependent response of excess pore water pressures was found to be a function of the viscoplastic parameters of foundation soils, the magnitude of the external loading, the shearing intensity, embankment geometry, drainage paths, construction rates, and the hydraulic conductivity of the soil. The authors concluded that the use of reinforcement can significantly reduce creep deformations of the foundation soils.

Chai et al. (2002) analyzed both reinforced and unreinforced embankments on soft subsoil which were built to failure using finite element method. The analyses confirmed that reinforcement can increase the factor of safety for the embankment on soft deposits. The magnitude of the effect was found to be a function of the mobilized reinforcement force and the geometry of the embankment. The limit equilibrium analysis results indicated that reinforcement only increased the factor of safety by less than 0.055. No obvious beneficial effects of reinforcement were observed when the embankment had a factor of safety 1.2-1.3 at the working state. The reinforcement was only found to have beneficial effects on subsoil deformation when the unreinforced case approached failure.

Bergado et al. (2002) also analyzed the full-scale test embankment with and without geotextile reinforcement constructed on soft Bangkok clay. Both undrained and consolidation analyses were adopted in the study. The analysis results showed that high-strength geotextile can reduce significantly the plastic deformations in the

underlying foundation. A two-step failure mechanism for high-strength geotextile reinforced embankment on soft ground was suggested by the authors. According to them, primary failure of foundation soils may occur at the same deformation as that of the unreinforced embankment just prior to collapse. Secondary failure or collapse of embankment and foundation as a whole may occur with the rupture or pullout of the reinforcement.

2.3.3 Summary of finite element analysis

(1) Reinforcement placed at the interface between the embankment fill and the foundation clay can reduce the lateral displacement in the clay foundation, restrict the development of plastic zones in the foundation and increase the net height of the embankment.

(2) No significant beneficial effect is found for the reinforcement in reducing the ground surface settlement. However, reinforcement does tend to reduce differential settlements at the top of the embankment.

(3) Mobilized tensile force in the reinforcement increases with the reinforcement stiffness and the height of the embankment. Increasing reinforcement stiffness decreases the extent of plastic failure in the foundation soils.

(4) The beneficial effect of reinforcement is not obvious until the corresponding unreinforced embankment is near the failure condition.

(5) For reinforced embankments constructed on rate-sensitive foundation soils, the maximum reinforcement strain mobilized at the end of construction is relatively small due to rate effects on the foundation soils during construction. The increase in the

reinforcement strain after construction is around one to three times the maximum strain mobilized at the end of construction.

(6) In undrained analysis, the elastic-perfectly plastic Mohr-Coulomb model is generally adopted for both the embankment fill and the soft soil in the foundation. In coupled analysis, Modified Cam clay model is widely used for the soft soil.

(7) For the embankment fill, Rowe and Soderman (1985) advocated to use the stress-dependent elastic modulus because the elastic stiffness may be highest for the lowermost layer and lowest for the topmost layer in the embankment fill. However, Hird and Kwok (1989) and Sharma and Bolton (1995) considered that it was not necessary to introduce such a stress-dependent elastic modulus for the embankment fill since most of the embankment deformations were of a plastic nature and the results were generally insensitive to the elastic modulus.

2.3.4 Stability analysis for reinforced embankment on soft ground by finite element method

Application of finite element method for slope stability analysis has been demonstrated by many researchers (Wright et al. 1973; Dunlop and Duncan 1970; Snitbahn and Chen 1976). In this section, two different stability analysis approaches for slopes or embankments using finite element method, strength reduction technique and enhanced limit method, will be reviewed and discussed.

2.3.4.1 Strength reduction technique

In strength reduction technique, repeated nonlinear numerical analyses are carried out with c and $\tan\phi$ progressively reduced by a factor F_r until failure occurs.

Denoting the reduced parameters by asterisks, and the reduction factor by F_r , the following equations are used in finite element analysis:

$$c^* = \frac{c}{F_r} \quad (2-22)$$

$$\phi^* = \tan^{-1} \left(\frac{\tan \phi}{F_r} \right) \quad (2-23)$$

where $F_r = F_s$, the factor of safety, and c^* and ϕ^* are associated with incipient failure.

This method was first introduced by Zienkiewicz et al. (1971) where a homogeneous slope was analyzed by gradually reducing the soil strength parameters. Recent works include those by Brinkgreve and Bakker (1991), Ugai (1989), Matsui and San (1992), Ugai and Leshchinsky (1995), Dawson et al. (1999) and Griffiths and Lane (1999). Strength reduction technique has the advantage that no assumption need be made about the slip surface in advance. The critical factor of safety and the corresponding failure mechanism emerges naturally from the analysis.

Wong (1984) mentioned some difficulties for finite element method in identifying the failure. Currently, three different failure criteria are commonly used: bulging of slope line (Snitbhan and Chen 1976), limit shear (Duncan and Dunlop 1969) and nonconvergence of the solution (Zienkiewicz 1971). The last failure criterion is widely used in strength reduction technique to calculate the factor of safety. However, as pointed out by Krahn (2003), the difficulty in using nonconvergence as a failure criterion is that many factors can cause numerical convergence problems in finite element analysis, such as incremental load-step size, gravity loading procedures, low confining stresses near the ground surface, cohesionless materials and initial in-situ

stress conditions. Therefore, nonconvergence of the solution is still a somewhat dubious criterion for ascertaining the failure.

Most applications of strength reduction technique by finite element method are for slopes or embankments without reinforcement. The only reinforced case found in the literature is a hypothetical reinforced embankment on soft ground (Brinkgreve and Bakker 1991). The stability of this reinforced embankment was analyzed by strength reduction technique. The authors showed that the failure mechanism of the embankment was significantly altered after the inclusion of the reinforcement between the embankment fill and the clay foundation.

2.3.4.2 Enhanced limit method

Bishop (1952) noted that the stresses from a limit equilibrium method of slices did not agree with the actual stresses within an earth structure. Brown and King (1966) determined critical slip surfaces from a finite element analysis with a linear elastic soil model. Kulhawy (1969) calculated the factor of safety using the normal and shear stresses from finite element analysis. The formulation of Kulhawy (1969) was classified as an “Enhanced limit strength method”. This procedure of calculating the factor of safety directly based on the stresses from finite element analysis was also called “Enhanced limit method” by Naylor (1982).

Wright et al. (1973) showed that the factors of safety calculated by enhanced limit method using linear elastic material properties ranged from 0% to 4.5% higher than those from Bishop’s simplified method. When nonlinear elastic material properties were used, the factors of safety from enhanced limit method became 2% to 8% higher than those calculated by Bishop’s simplified method. It was also shown by them that the local factors of safety along one third of the slip surface were less than the overall

factor of safety.

The following definition of the factor of safety proposed in Kulhawy (1969) is adopted by many researchers (Yamagami and Ueta 1988; Zou et al. 1995; Kim and Lee 1997; Farias and Naylor 1998; Fredlund and Scoular 1999; Pham and Fredlund 2003):

$$FS = \frac{\int_A^B \tau_f dL}{\int_A^B \tau dL} \approx \frac{\sum_{i=1}^{N_s} \tau_{f_i} \Delta L_i}{\sum_{i=1}^{N_s} \tau_i \Delta L_i} \quad (2-24)$$

$$\tau_{f_i} = c + \sigma_{n_i} \tan \phi \quad (2-25)$$

$$\tau_i = 0.5(\sigma_y - \sigma_x) \sin 2\alpha + \tau_{xy} \cos 2\alpha \quad (2-26)$$

$$\sigma_{n_i} = \sigma_x \sin^2 \alpha + \sigma_y \cos^2 \alpha - \tau_{xy} \sin 2\alpha \quad (2-27)$$

where τ_i is the mobilized shear stress at the i^{th} segment along the slip surface AB, τ_{f_i} is the available shear strength at the segment, L is the length of the slip surface, N_s is the number of segments along the slip surface, ΔL_i is the length of the i^{th} segment, c is the cohesion of the soil, ϕ is the friction angle of the soil, σ_{n_i} is the normal stress acting at the i^{th} segment along the slip surface, α is the angle of the segment along the slip surface to the horizontal direction, and σ_x , σ_y , τ_{xy} are the normal and shear stresses acting in the x and y coordinate directions along the slip surface.

The enhanced limit method has the advantage that it does not require a set of repetitious numerical analyses and gives reasonably accurate factor of safety from a

linear elastic analysis (Naylor 1999). Farias and Naylor (1998) provided full details on the implementation of this method. However, it was pointed out by Naylor (1982) that the advantages of this method may be offset by the additional work involved in computing the normal and shear stresses along a potential slip surface.

Usually, there are three schemes to interpolate the stresses from the finite element analysis: direct stress interpolation, nodal average stress interpolation and least squares stress analysis.

Direct stress interpolation scheme calculates the stress at any point from the displacements computed in the finite element analysis. Using this approach, the state of stress at point P is computed as

$$\boldsymbol{\sigma}_P = \mathbf{D}\mathbf{B}\bar{\mathbf{u}} \quad (2-28)$$

where $\mathbf{D}$ is the stiffness matrix and $\mathbf{B}$ is the element strain matrix. Stresses computed using Eq. (2-28) are only continuous within a given element. The stresses across element boundaries can become discontinuous because the shape functions across element boundaries are continuous but not smooth. As a result, the derivatives are discontinuous across element boundaries.

Nodal average stress interpolation scheme calculates the stress at any point using the following equation

$$\boldsymbol{\sigma}_P = \mathbf{N}\bar{\boldsymbol{\sigma}} \quad (2-29)$$

where $\mathbf{N}$ is a matrix of shape functions and $\bar{\boldsymbol{\sigma}}$ is a row matrix of nodal average stresses. This interpolation scheme is widely used due to its simplicity. SLOPE/W

(2001) uses this approach to calculate the factor of safety for slope or embankment using stress results from finite element analysis. Although stresses from this approach is more accurate than those from the direct stress interpolation scheme, discontinuous stresses can still be observed across element boundaries.

Least squares stress analysis (Hinton and Campbell 1974; Hinton 1968; Hinton and Irons 1968) computes the state of stress at a given point by fitting a continuous surface to the stresses at stress points (Gauss points). The number of stress points should be enough to construct a continuous surface. With this approach, a better stress picture can be obtained. Details about this procedure can be found in Zienkiewicz and Taylor (2005). One disadvantage of this approach is that more computational efforts are involved in the interpolation process. Kim and Lee (1997) used this method to interpolate stresses from finite element analysis and calculate the factors of safety for homogeneous slopes and an embankment on soft ground.

The only application of enhanced limit method to stability analysis of reinforced embankment on soft ground is the work by Borges and Cardoso (2002). In their analysis, only circular slip surface was considered. They concluded that stability analysis results using stresses from finite element analysis produced results very close to those from conventional limit equilibrium method.

2.3.5 General comments on stability analysis methods

Generally, methods for performing stability analysis for reinforced embankments on soft ground can be divided into two categories: limit equilibrium methods of slices and finite element method. For limit equilibrium methods of slices, Duncan (1996) gave a detailed review. These methods include Fellenius's method (ordinary method of slices), Bishop's modified method, Janbu's rigorous method, Spencer's method and

Morgenstern and Price's method. Although a review on the application of finite element method to embankments was also given in Duncan (1996), the focus was on the deformation of embankments rather than on stability evaluation. It is well known that limit equilibrium methods require the soil mass to be divided into slices and that the direction of inter-slice forces acting on each slice should be assumed in advance. The assumed direction for inter-slice forces is a key factor in distinguishing one limit equilibrium method from another. Despite the fact that limit equilibrium methods of slices have been used extensively and that engineers gained a lot of experience in using these methods, two main disadvantages of limit equilibrium methods should be noted: the assumption about the shape or location of critical slip surfaces and the assumption about the inter-slice forces and their directions.

Application of finite element method in geotechnical engineering has been widely accepted for many years, for the prediction of settlements and deformations, the calculation of flow quantities due to seepage and the study of transient effects due to consolidation, etc. However, finite element method is seldom used to assess the stability of slopes and embankments although it is widely recognized that assumptions for inter-slice forces are not required in this method. The reluctance to use finite element method in stability analysis may result from the fact that the successful application of finite element method may depend mostly on the quality of soil data available. Small difference in input parameters may result in significantly different solutions in finite element analysis. Griffiths (1999) mentioned this problem and pointed out that geotechnical engineers should "know which kind of problem would benefit from a FE treatment and which would not".

The advantage of finite element method in stability analysis over limit equilibrium method is that no assumption need be made in advance about inter-slice forces and their directions. In particular, the method can be applied to problems with complicated

slope geometry and soil deposits in two or three dimensions. Many more advanced and realistic soil models can be included in the analysis. The equilibrium stresses, strains, and the associated shear strength in the soil can be computed very accurately.

2.4 Basic concepts of probabilistic stability analysis for slopes or embankments

2.4.1 Introduction

The materials comprising most slopes or embankments are inherently quite variable. Therefore, it appears natural that probabilistic methods should be applied to stability analyses. Conventional slope stability analysis methods do not account for uncertainty, thus compromising the adequacy of the obtained predictions. This limitation is well recognized by researchers as well as practitioners.

Arthur Casagrande, in the Terzaghi Lecture of 1964, presented the definition of calculated risk for applications within geotechnical engineering (Whitman 1984). Casagrande emphasized that uncertainties are inherent to any project and the levels of uncertainties should be recognized, and felt that calculated risk needs to be recognized and dealt with in two steps:

- ◆ The use of imperfect knowledge, guided by judgment and experience, to estimate the probable ranges for all pertinent quantities that affect the solution of the problem.
- ◆ The decision on an appropriate level of safety, or degree of risk, taking into account the economic factors and the magnitude of losses that would result from failure.

Bjerrum (1966) and Peck (1967) expressed their concern about the ability of the geotechnical engineers to make reliable forecasts of slope failure events. Wolff (1987), in the context of dam slope design, also pointed out the inadequacy of judging a design based on the numerical value of the factor of safety alone and the need to adopt probabilistic analysis in the slope design.

2.4.2 Spatial variability of soil properties

Soil properties are not single valued. The magnitudes of a soil property at two adjacent points are strongly correlated. The correlation generally decreases with the increase in the distance between two locations. This variability is the result of variations in mineralogical composition, conditions during deposition, stress history, and physical and mechanical decomposition processes (Lacasse and Nadim 1996). Lumb (1975) and Vanmarcke (1977a) pointed out that the spatial variability of soil properties cannot be ignored in probabilistic stability analysis. Therefore, the concept of *spatial variability* is reviewed in the first place.

Li and White (1987b) idealized some common soil profile types. Figure 2.8 shows three typical soil profile types. The solid line in each type indicates the trend or mean component. The data points scattered about the trend lines indicate a random component. The variability of the data about the trend is indicated by the dash lines.

Type I profile has a constant mean, coefficient of variation and variance. In type II, the mean increases linearly with depth and the variation is constant. Type III also has a mean increasing linearly with depth, but the coefficient of variation is constant. Spatial variability is the fluctuation of a soil property around the mean or trend line. Therefore, analyzing the spatial variability of soil properties should start by establishing the trend. This is usually done by linear regression. The least squares method is the most

commonly used technique. The variations about the trend are then treated as spatial variability.

Figure 2.8 Soil profile types: (a) Type I (Fixed mean and variance); (b) Type II (Linearly increasing mean and fixed variance) and (c) Type III (Linearly increasing mean and fixed coefficient of variation) (modified after Li and White 1987b).

A simplified model is commonly used to analyze the spatial variability of soil properties (Vanmarcke 1977a; DeGroot and Baecher 1993). The model divides the measured quantity, x_i , at any location, i , into a deterministic trend component, t_i , and a residual component, ε_i , as shown in Figure 2.9. The trend is evaluated deterministically using regression techniques, while the residuals are assessed statistically. They are assumed to have a zero mean and a constant standard deviation independent of location. Residuals are further divided into two components; random error, ε_{ei} , and inherent variability, ε_{vi} . The model can, therefore, be written as follows:

$$x_i = t_i + \varepsilon_i \quad (2-30)$$

where $\varepsilon_i = \varepsilon_{e_i} + \varepsilon_{v_i}$

Figure 2.9 Model of spatial variability of soil properties

There are two methods to measure the spatial variability of soil properties: autocovariance (Christian et al. 1994; DeGroot 1996; Lacasse and Nadim 1996) and semivariogram (Soulie et al. 1990; Bjerg et al. 1992; Chiasson et al. 1995).

2.4.2.1 Quantifying spatial variability by autocovariance

Similarity between data of the same type separated by a lag distance r can be measured by autocovariance $C_x(r)$. The autocovariance is high at small separation distance and decays gradually with the increase of the lag distance r . The autocovariance is defined by the following equation as the expected value of the residuals at locations r distance apart:

$$C_x(r) = E[(x_i - t_i)(x_{i+r} - t_{i+r})] \quad (2-31)$$

where x_i and t_i are the measured and mean values at location i and x_{i+r} and t_{i+r} are the measured and mean values at location $i+r$. The autocovariance of the measured soil property consists of two components: the autocovariance of random errors $C_e(r)$ and that of the inherent spatial variability $C_v(r)$:

$$C_x(r) = C_e(r) + C_v(r) \quad (2-32)$$

At a zero separation distance, the values of $C_e(r)$ and $C_v(r)$ are referred to as random error variance $V_e[x]$ and variance of inherent variability $V_v[x]$, respectively. Because random errors are assumed to be independent of the location, $C_e(r)$ is equal to zero for all r values except for $r = 0$ where $C_e(0) = V_e[x]$. Figure 2.10 shows the components of autocovariance function.

The exponential and squared exponential analytical decay expressions (Figure 2.11) are usually used to describe the autocovariance function (Vanmarcke 1977a; Christian et al. 1994; DeGroot 1996; Lacasse and Nadim 1996). The distance at which $C_v(r)$ decays to 37% (the inverse of the base of natural logarithms, e) of the value of $C_v(r)$ at $r = 0$ is usually defined as the *autocorrelation distance* r_o in exponential and squared exponential models. It provides a single measure for describing the smoothness of spatial variability. A large autocorrelation distance implies a very uniform material, and small distance implies material whose properties vary over short distances.

Figure 2.10 Components of autocovariance function

Two methods can be used to estimate the autocovariance function of soil properties: the method of moments and the maximum likelihood method. The method of moments is the most commonly used approach (DeGroot and Baecher 1993; DeGroot 1996). In this method, sample moments (e.g., mean, variance, autocovariance) are used as estimators of the unknown population moments. The autocovariance $C_x(r)$ of a set of observations can be expressed as:

$$C_x(r) = \frac{1}{N(r)} \sum_{i=1}^{N(r)} (x_i - t_i)(x_{i+r} - t_{i+r}) \quad (2-33)$$

where $N(r)$ is the number of data pairs separated by distance r ; x_i and t_i are the measured and mean values at location i and x_{i+r} and t_{i+r} are the measured and mean values at location $i+r$. Usually, the form of the autocovariance function is selected by fitting an appropriate analytical function to the experimental data. Since the autocovariance at $r=0$ is equal to the true spatial variability variance, extrapolating the autocovariance function to the origin divides the total variance into

true variability and random error. Christian et al. (1994) provided good examples on the application of the method of moments in modeling soil properties.

Figure 2.11 Autocovariance functions: (a) Exponential; (b) Squared exponential (modified after DeGroot and Baecher 1993).

DeGroot and Baecher (1993) pointed out that the moment estimator of autocovariance is only asymptotically unbiased. A large number of samples are usually required to obtain the unbiased estimate of autocovariance. They also indicated that fitting an analytical function to the experimental autocovariance is often done by inspection. This added an element of subjectivity to the analysis.

The probability of obtaining a particular data value x_i conditioned to a parameter θ is given by the magnitude of the probability density function evaluated at x_i , $f(x_i | \theta)$. The parameter θ could be data variance, for example. The basic concept behind the maximum likelihood method is to estimate the most likely value of θ that will produce the observed data for a set of observations $\{x_1, x_2, \dots, x_n\}$. DeGroot and Baecher (1993) applied this approach to the analysis of the spatial variability of field vane data of the James Bay project. They concluded that this method could simultaneously provide estimates of spatial trends, measurement noise and the autocovariance of residuals about spatial trends. DeGroot (1996) showed that the maximum likelihood method was statistically superior to the method of moments.

However, Li (1994) pointed out that a number of local maxima existed for the solutions from the maximum likelihood method. As a result, the estimator obtained may not necessarily be those associated with the global maximum.

2.4.2.2 Quantifying spatial variability by semivariogram

The semivariogram $\gamma_x(r)$ is an average measure of dissimilarity between data separated by a distance r . The following equation is used to calculate the semivariogram $\gamma_x(r)$, which is defined as half the expected value of the variables $(x_i - x_{i+r})^2$:

$$\gamma_x(r) = \frac{1}{2} E[(x_i - x_{i+r})^2] \quad (2-34)$$

where x_i and x_{i+r} are the measured values at locations i and $i+r$, respectively.

The use of the semivariogram is not very common due to the difficulties in spatial interpolation. It is often limited to cases where the mean can be assumed constant or the trend can be easily removed from the observations. The advantage of the semivariogram over autocovariance is that the assumption on the stationarity of the sampled soil properties is less restrictive. A great deal of experience has been obtained in mining applications. Recently, this method has been explored in geotechnical engineering (Soulie et al. 1990 and Chiasson et al. 1995).

For a set of observations, the experimental semivariogram can be calculated as follows:

$$\gamma_x(r) = \frac{1}{2N(r)} \sum_{i=1}^{N(r)} (x_i - x_{i+r})^2 \quad (2-35)$$

where $N(r)$ is the number of data pairs separated by distance r ; x_i and x_{i+r} are the measured values at locations i and $i+r$, respectively. Similar to the autocovariance, an appropriate analytical function is fitted to the experimental data.

2.4.3 Monte Carlo simulation

The Monte Carlo method was developed in 1949 when John von Neumann and Stanislaw Ulam published a paper, “The Monte Carlo method”. Monte Carlo simulation uses a pseudo random number generator to generate random values for each input variable based on the corresponding probability distribution functions. Currently, Monte Carlo simulation is one possible way to deal with problems that are mathematically complex.

Many researchers applied Monte Carlo simulation to the slope reliability analysis (Kim

et al. 1978; Major et al. 1978; Nguyen and Chowdhury 1985; Malkawi et al. 2000; El-Ramly et al. 2002). The first step in the slope reliability analysis by Monte Carlo Simulation is to select an appropriate deterministic performance function, explicit or non-explicit, and to define the probability distribution functions of input variables. Step two is then to use the generated random values to solve the performance function and calculate the factor of safety. This process can be repeated many times to build up a statistical distribution of the output of the performance function (i.e., factor of safety).

The advantage of Monte Carlo simulation is that it does not require the comprehensive statistical and mathematical background. The shape of the probability density function of the factor of safety and the failure probability can be obtained. However, a large number of iterations are usually needed in Monte Carlo simulation to provide accurate results. In particular, when the factor of safety is calculated by searching for critical noncircular slip surface, using Monte Carlo simulation with a large number of iterations is not practical. Most slope reliability analyses in the literature using Monte Carlo simulation did not search for critical slip surface in each iteration. For example, El-Ramly et al. (2002) adopted Monte Carlo simulation to compute the failure probability of James Bay Dykes. In their analysis, the critical slip surface was fixed in order to reduce the time involved in the calculation. As a result, the failure probability could be underestimated since the factor of safety may be overestimated in some iterations.

Variance reduction methods can significantly reduce the number of sample points in Monte Carlo simulation without losing accuracy. According to Rubinstein (1981), variance reduction technique is nothing more than a means to use known information about the problem. Variance reduction methods for Monte Carlo simulation include importance sampling method, stratified sampling, adaptive sampling and conditional

expectation method. Details for these variance reduction methods can be found in many statistical textbooks. It is worth pointing out that it should not be confused with the same term applied to spatial variability where “variance reduction” means the reduction in variance of uncertainties due to spatial averaging. Note that none of these methods has been applied to slope reliability analysis. The application of importance sampling method to the reliability analysis of embankments based on Monte Carlo simulation will be described in details in Chapter 8.

2.4.4 Point estimate method

The point estimate method is an approximate numerical integration technique which was originally developed by Rosenblueth (1981) and later summarized by Harr (1987). This method requires little knowledge of probability theory and therefore is very easy to use. Due to its simplicity, the point estimate method is widely used in reliability analysis (Lind 1983; Harr 1989 and Li 1992).

For the performance function with three random variables, $g(x_1, x_2, x_3)$, the following equations (Baecher and Christian 2003) are used to calculate $E[g]$ and $E[g^2]$ in the point estimate method:

$$E[g] \approx P_{+++} g_{+++} + P_{++-} g_{++-} + P_{+-+} g_{+-+} + \dots \quad (2-36)$$

$$E[g^2] \approx P_{+++} g_{+++}^2 + P_{++-} g_{++-}^2 + P_{+-+} g_{+-+}^2 + \dots \quad (2-37)$$

$$g_{+++} = g(\mu_{x_1} + \sigma_{x_1}, \mu_{x_2} + \sigma_{x_2}, \mu_{x_3} + \sigma_{x_3}) \quad (2-38)$$

$$g_{++-} = g(\mu_{x_1} + \sigma_{x_1}, \mu_{x_2} + \sigma_{x_2}, \mu_{x_3} - \sigma_{x_3}) \quad (2-39)$$

$$g_{+-+} = g(\mu_{x_1} + \sigma_{x_1}, \mu_{x_2} - \sigma_{x_2}, \mu_{x_3} + \sigma_{x_3}) \quad (2-40)$$

$$P_{+++} = P_{---} = \frac{1}{8}(1 + \rho_{12} + \rho_{23} + \rho_{31}) \quad (2-41)$$

$$P_{++-} = P_{--+} = \frac{1}{8}(1 + \rho_{12} - \rho_{23} - \rho_{31}) \quad (2-42)$$

$$P_{+-+} = P_{-+-} = \frac{1}{8}(1 - \rho_{12} - \rho_{23} + \rho_{31}) \quad (2-43)$$

$$P_{+--} = P_{-++} = \frac{1}{8}(1 - \rho_{12} + \rho_{23} - \rho_{31}) \quad (2-44)$$

where P is the weight, μ_{x_i} and σ_{x_i} are the mean value and standard deviation for random variable x_i and ρ_{ij} is the correlation coefficient between random variables x_i and x_j .

Based on Eqs. (2-36)-(2-44), the reliability index can be easily calculated by the following equation:

$$\beta = \frac{E[g]}{\sqrt{E[g^2] - E[g]^2}} \quad (2-45)$$

This technique was modified and implemented by many researchers for probabilistic slope stability analysis (McGuffey et al. 1982; Nguyen and Chowdhury 1984; Wolff 1987 and Li 1992). Cheng and Christopher (1991) performed probabilistic stability analyses for reinforced slopes using the point estimate method. Thornton (1994) used this method to evaluate the probability of slope failures in which input parameters were assumed to be normally distributed and a model was also developed to estimate

the distribution of the factor of safety.

One attractive feature of the point estimate method is that it can be combined with finite element method in the reliability analysis. Schweiger et al. (2001) conducted probabilistic analyses for a cantilever sheet pile wall and a tunnel excavation by integrating the point estimate method with finite element analysis. However, since the number of evaluation of performance function is 2^n for n random variables, the application of the point estimate method will become very cumbersome and impractical when the number of random variables is large.

Christian and Baecher (1999) provided detailed discussions of the accuracy and the limitations of the point estimate method. It was shown that the method was reasonably robust and was satisfactorily accurate for a range of practical problems, though computational efforts increased rapidly with the number of random variables of interest. The authors suggested that cautions should be used in applying this method to cases where the transformation of random variables severely changed the distribution form, or moments are higher than order two. They also found that the results can be seriously in error when the performance function to be integrated was not well represented by a third-order polynomial and when the coefficient of variation of random variable was large.

2.4.5 Mean-value first-order second-moment method (MVFOSM)

The mean-value first-order second-moment method (MVFOSM) is based on Taylor series expansion of the performance function $f(x_1, x_2, \dots, x_n)$ around its mean value. For simplicity, only the linear terms of the series are usually retained. The mean and variance of the factor of safety are given by the following equations:

$$E[F_s] \approx f(E[x_1], E[x_2], \dots, E[x_n]) \quad (2-46)$$

$$V[F_s] \approx \sum_{i=1}^k \sum_{j=1}^k \left[\frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \right] \cdot C(x_i, x_j) \quad (2-47)$$

where $E[F_s]$ and $V[F_s]$ are the mean and variance of the factor of safety respectively and $C(x_i, x_j)$ is the covariance between input variables x_i and x_j .

Given some performance function and limit state, the reliability index β is the number of standard deviations of the performance function by which the expected value of the performance function exceeds the limit state:

$$\beta = \frac{E[F_s] - 1}{\sqrt{V[F_s]}} \quad (2-48)$$

The mean factor of safety is equal to the factor of safety calculated using mean values of input variables. If the input variables are uncorrelated, $C(x_i, x_j)$ is equal to zero except when $i = j$ where it is reduced to the data variance $V[x_i]$. Due to its simplicity, this approach attracts many users (Cornell 1971; Alonso 1976; Chowdhury and Xu 1992; Chowdhury and Xu 1994; Christian et al. 1994; Duncan 2000; Hassan and Wolff 1999; Liang et al. 1999; Xu and Chowdhury 1999; Malkawi et al. 2000).

However, it should be pointed out that this method lacks invariance for different but mechanically equivalent formulations of the same performance function. If the performance function and limit state can be expressed in more than one equivalent way (e.g., Capacity/Demand=1 or Capacity-Demand=0), the different functions will yield different values for the reliability index. Since the performance function $f(\mathbf{x})$ is

linearized at the mean values of the x_i variables, significant errors may be introduced by neglecting higher order terms when $f(\mathbf{x})$ is highly nonlinear. Moreover, this method also does not use the distribution information about the variables even when it is available.

2.4.6 The Hasofer-Lind approach

Hasofer and Lind (1974) developed a more general definition of the reliability index which is invariant and reduces to the mean-value definition for linear functions. The Taylor series is expanded not about the mean or expected value, but about an unknown point termed the most probable failure point (design point) in their method. An iterative solution is required in the analysis. This method is also called advanced first-order second moment method (AFOSM) in Haldar and Mahadevan (2000).

The first step in the Hasofer-Lind approach is to transform random variables x_i to the standardized variables:

$$x'_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \quad (2-49)$$

where μ_{x_i} and σ_{x_i} are the mean value and standard deviation for random variable x_i . The reliability index formulated in Hasofer and Lind (1974), which is also commonly called Hasofer-Lind index, β_{HL} , is defined as the minimum distance between the transformed origin and the transformed limit state surface $g(\mathbf{x}')$.

Figure 2.12 Schematic illustration of the reliability index β_{HL} in Hasofer-Lind approach

Figure 2.12 is a schematic illustration of the reliability index β_{HL} in the standard space. The calculation of the reliability index β_{HL} requires the solutions to the following constrained optimization problem:

$$\text{Minimize } \beta_{HL} = \sqrt{(\mathbf{x}')^T \mathbf{x}'} \quad (2-50)$$

$$\text{Subject to } g(\mathbf{x}') = 0 \quad (2-51)$$

In addition to the reliability index β_{HL} computed from Eqs. (2-50) and (2-51), the design point (the most probable failure point) $\mathbf{x}'^*$ also can be obtained, which is defined as the point on the limit state surface nearest to the origin of the transformed space. It should be noted that an iterative procedure is usually required to locate the design point (Hasofer and Lind 1974; Ang and Tang 1984; Li and Lumb 1987; Li and

White 1989b; Chowdhury and Xu 1994).

2.4.7 First-order reliability method (FORM)

The Hasofer-Lind method is based on independent, normal variables. First-order reliability method (FORM) considers non-normal and correlated random variables. The reliability index and the corresponding design point are calculated based on the similar procedure in the Hasofer-Lind method.

Rosenblatt transformation approach (Rosenblatt 1952), which is described in Ang and Tang (1990), is usually used to transform non-normal distribution functions into equivalent normal distribution functions. Rackwitz and Fiessler (1978) estimated the parameters of the equivalent normal distribution by imposing the conditions that the cumulative distribution functions and the probability density functions of the actual variables and the equivalent normal variables should be equal at the checking point on the failure surface. This approach is known as the Rackwitz-Fiessler method (Haldar and Mahadevan 2000) and has been used extensively in the literature.

Chen and Lind (1983) and Wu and Wirsching (1987) extended the Rackwitz-Fiessler method by introducing a third parameter in addition to the mean and standard deviation.

Two common approaches are available to transform correlated random variables to uncorrelated variables: Cholesky approach and Eigenvalue and Eigenvector approach (Shinozuka 1983). Low and Tang (1997b, 2004) proposed a very efficient approach procedure using the built-in optimization program *Solver* in Microsoft Excel spreadsheet. The formulation of the Hasofer-Lind index is:

$$\beta_{HL} = \min_{X \in F} \sqrt{\left[\frac{x_i - m_i}{\sigma_i} \right]^T [\mathbf{R}]^{-1} \left[\frac{x_i - m_i}{\sigma_i} \right]} \quad (2-52)$$

where $[\mathbf{R}]$ is the correlation matrix and σ_i is the standard deviation of random variable x_i .

Reliability index and the corresponding design point $\mathbf{x}_D$ is calculated based on the limit state function $G(\mathbf{x})=0$. This can be done in Microsoft Excel using its built-in optimization tool “Solver” to minimize β_{HL} with the constraint $G(\mathbf{x})=0$, and by changing automatically the values of the random variables, x_i . This spreadsheet-based technique (Low and Tang 1997b, 2004), based on an intuitive ellipsoidal perspective in the original space of the random variables, will be referred to as the ellipsoid method herein. Based on the reliability index obtained, the failure probability can be given approximately as:

$$p_f \approx 1 - \Phi(\beta) \quad (2-53)$$

where Φ is the cumulative distribution function (CDF) of the standard normal variable. The value of failure probability from Eq. (2-53) will be exact if the random variables are normally distributed and the limit state surface is planar.

2.5 Summary

This chapter reviewed the issues pertinent to the analysis of reinforced embankments on soft ground from both deterministic and probabilistic viewpoints. The following conclusions can be drawn from the review:

(1) The stability of reinforced embankments on soft ground is mainly analyzed by conventional limit equilibrium methods of slices with some modifications to account for the presence of reinforcement.

(2) Finite element method can be used to study the behaviour of reinforced embankments on soft ground. The ground surface settlements, lateral displacements, excess pore water pressures and mobilized force in the reinforcement can be modeled appropriately. However, the applications of finite element method in the determination of the factor of safety for reinforced embankments are still limited.

(3) The conventional deterministic stability analysis for slopes or embankments cannot take into account the uncertainties involved in soil properties and usually relies on conservative parameters to cope with uncertain conditions. Probabilistic stability analysis, on the other hand, accounts for the uncertainties involved in the analysis. The output of the analysis, failure probability or reliability index, is a more rational measure of the reliability of the slope design. Probabilistic stability analysis, by providing more valuable insight into slope design, enhances engineering judgement and improves the decision-making process.

CHAPTER 3 BEHAVIOUR STUDIES OF UNREINFORCED AND REINFORCED EMBANKMENTS ON SOFT GROUND BY FINITE ELEMENT METHOD AND STABILITY ANALYSIS

3.1 Introduction

This chapter consists of two parts. In the first part, the behaviour of unreinforced and reinforced embankments will be investigated using the finite element program PLAXIS¹. An actual unreinforced embankment on soft ground is first studied. Ground surface settlements, lateral displacements, excess pore water pressures and development of plastic zone within the soft soil foundation are investigated. The effects of reinforcement on the behaviour of embankments on soft ground are then studied for a single layer of reinforcement placed at the interface between the embankment fill and the clay foundation. In the second part, different stability methods are used to calculate the factors of safety for unreinforced and reinforced embankments. These methods include limit equilibrium method of slices (Spencer's method), enhanced limit method and strength reduction technique.

Part 1: Behaviour studies of unreinforced and reinforced embankments on soft ground by finite element method

3.2 Unreinforced embankment on soft ground

Figure 3.1 shows the geometry of an embankment constructed on soft ground in Malaysia in 1989. The clay foundation consists of a 2.0 m thick weathered crust underlain by about 6.4 m of very soft silty clay (Upper clay). Below this layer lies a 10 m thick layer of soft clay (Lower clay) which in turn is underlain by about 0.6 m of peat. A 4.1 m thick sandy clay lies below the peat layer (Chai et al. 1994). This embankment was built on natural ground and rapidly loaded to failure. The embankment fill was compacted in 0.2 m layers at a nominal rate of 0.4 m per week until failure occurred. The fill thickness at failure was 5.4 m according to Brand (1991)

¹ More details at <http://www.plaxis.com/>

and 5.5 m according to Indraratna et al. (1992). The latter value is used in this study. Figure 3.2 shows the plan of instrumentations for this embankment.

This embankment was analyzed by both Indraratna et al. (1992) and Chai et al. (1994) using the finite element method. Although the analysis conducted by Indraratna et al. (1992) covered the issues of vertical and lateral displacements of the embankment, excess pore water pressure induced during the construction, and failure mechanism when the critical height was reached, Tan (1994) pointed out that there was some inconsistency between the compression parameters obtained from oedometer test results and the λ (a compression parameter of Modified Cam-Clay model) adopted by Indraratna et al. (1992) in their finite element analysis. Chai et al. (1994) selected parameters for finite element analysis in a more reasonable way. But their focus was on the effects of partial drainage and settlement on the stability of the embankment.

Figure 3.1 Geometry of the Muar embankment on soft ground (after Brand 1991)

In this section, the Muar embankment is analyzed by the finite element program PLAXIS. Both the undrained and coupled consolidation analyses are adopted in the finite element analysis. The purely undrained analysis is useful for predicting the immediate settlements, whereas the coupled (Biot) consolidation analysis can consider both the immediate and time-dependent deformation. For embankments on soft ground, due to the dissipation of excess pore water pressures within the foundation during the construction period, the coupled analysis is considered to be more realistic.

Figure 3.2 Plan of instrumentations for this test embankment (after Brand 1991)

The embankment fill and the overconsolidated weathered crust are modeled by the elastic-perfectly plastic Mohr-Coulomb model. The soft soil model in PLAXIS (Brinkgreve 2002), based on the modified Cam-Clay model, is adopted for the upper clay, lower clay and sandy clay.

Table 3-1 Soil parameters used in the present finite element analysis (after Brand 1991; Indraratna et al. 1992; Chai et al. 1994)

Parameter	Soil layer				
	Fill	Crust	Upper clay	Lower clay	Sandy clay
γ_{sat} (kN/m ³)	--	16.5	15.5	15.5	15.5
γ_{unsat} (kN/m ³)	20.5	14.5	--	--	--
c' (kPa)	19	20	1	5	10
ϕ' (deg)	26	26	20	22	22
E (kPa)	5200	14000	--	--	--
λ^*	--	--	0.217	0.13	0.05
κ^*	--	--	0.072	0.043	0.015
k_h (m/day)	1.0	2.4×10^{-3}	1.2×10^{-3}	7.8×10^{-4}	1.73×10^{-2}
k_v (m/day)	1.0	1.2×10^{-3}	6.0×10^{-4}	3.9×10^{-4}	5.7×10^{-3}
ν	0.3	0.3	0.15	0.15	0.15
OCR	--	--	1.2	1.2	1.3
Soil model	M-C	M-C	SSM	SSM	SSM

Note: M-C = Mohr-Coulomb soil model; SSM = Soft soil model

The modified compression index λ^* is used in the soft soil model instead of the index λ used in Burland (1965), as follows:

$$\lambda^* = \frac{C_c}{2.3(1+e)} \quad (3-1)$$

where C_c is the compression index from the conventional oedometer test and e is the void ratio. The value of the modified swelling index κ^* is taken to be 0.33 times λ^* , as suggested by Brinkgreve (2002). Table 3-1 summarizes all soil parameters used in the finite element analysis for this embankment.

3.2.1 Ground surface settlement of the unreinforced embankment

Ground surface settlement patterns of the embankment at different fill heights (2, 3, 4, 5 m) are shown in Figures 3.3-3.6, respectively. Both the undrained analysis and coupled analysis are adopted to predict the ground surface settlements. It can be seen that, at relatively lower fill height (up to 2 m), the undrained analysis predicts ground surface settlement better than the coupled analysis, which overestimates the vertical displacement of the ground surface.

Figure 3.3 Ground surface settlement at the embankment height of 2 m

This indicates that the consolidation settlement is negligible during the early stage of construction. When the embankment height is increased from 2 m to 4 m, the coupled analysis tends to give more realistic prediction of the ground surface settlement, as shown in Figures 3.4 and 3.5. This is expected because the contribution from the

consolidation settlements due to the dissipation of excess pore water pressure, which can be considered in the coupled analysis, becomes significant at that time.

Figure 3.4 Ground surface settlement at the embankment height of 3 m

Figure 3.5 Ground surface settlement at the embankment height of 4 m

However, it also can be noticed that both the undrained analysis and the coupled analysis cannot predict the heave in the vicinity of the toe correctly. All calculations tend to overestimate the heave. Indraratna et al. (1992) ascribed this partly to the stress-path sensitivity of soil parameters used in the finite element analysis. Similar trend for the heave prediction was also found in the analysis for this case by Chai et al. (1994), but no comments were provided.

Figure 3.6 shows the ground surface settlement at the embankment height of 5 m. Only results from the coupled analysis and measured data are provided. The results from the undrained analysis are not available in this stage. This is because the factor of safety at the embankment height of 5 m is actually below 1.0 according to the undrained analysis. In contrast, the coupled analysis gives the factor of safety of 1.08 for this embankment at the height of 5 meters based on the strength reduction technique. This indicates that the undrained analysis underestimates the stability of this embankment. This may partly be attributed to the neglect of the increase in the undrained shear strength due to the partial drainage. This factor cannot be taken into account by the undrained analysis.

Figure 3.6 Ground surface settlement at the embankment height of 5 m

Chai et al. (1994) studied the stability of this embankment using Bishop's simplified method with consideration of the effects of partial drainage, and found that the factor of safety was around 1.05 when the embankment height was 5 m. According to a separate stability analysis in Chai et al. (1994), the factor of safety for this embankment was 0.92 when the effect of partial drainage was not considered, which corresponds to the undrained condition in the present analysis.

3.2.2 Lateral displacement

Figure 3.7 shows both the calculated and the observed lateral displacements at location of inclinometer I3 (Figure 3.2) at failure ($H = 5.5$ m). It can be observed that the maximum lateral displacement, as expected, occurs within the upper, very soft clay layer at a depth of around 5 m below the base of the embankment. Below this depth, the lateral displacement tends to decrease as the deeper (stiffer) clay deposits are encountered.

The general pattern of lateral displacement predicted by the coupled analysis agrees well with the observed one. However, the values of the lateral displacement from the finite element analysis are greater than the measured values. As shown in Figure 3.8, the lateral displacement at 4.5 m depth in inclinometer I3 increases with the fill thickness. The coupled analysis by the finite element method predicts a similar trend but overestimates the deformation value. The poor lateral displacement prediction given by the finite element analysis is well known (Tavenas and Leroueil 1980; Chai and Bergado 1993; Chai et al. 1994). This limitation is partly due to the effects of anisotropic behaviour of natural clay, rotation of principal stress direction within the foundation and the choice of Poisson's ratio (Poulos 1972). For this Muar trial embankment which is 90 m long and 55 m wide, the assumption of plane strain condition, which is used in the finite element analysis, can also result in the overestimation of the lateral displacement. Figure 3.8 shows that, when the embankment height is raised from 5 m to 5.5 m, there is a rapid increase in the lateral displacement, indicating the impending failure of the embankment.

Figure 3.7 Variation of lateral displacement at location of inclinometer I3 at failure

Figure 3.8 Lateral displacements versus the fill thickness at 4.5 m depth in inclinometer I3

3.2.3 Excess pore water pressure

For this embankment, five piezometers were installed (Figure 3.2) to measure the pore water pressure variations at different depths beneath the embankment (P2, P3, and P4) and also laterally (P2, P5, and P7). The variation of excess pore water pressure beneath the embankment centerline at the fill height of 4 m is shown in Figure 3.9. It can be seen that the coupled analysis can yield a good prediction of the distribution of excess pore water pressures with depth.

Figure 3.9 Variation of excess pore water pressure beneath the embankment centerline at the fill height of 4 m

The maximum excess pore water pressure develops within the upper, very soft clay layer which lies between the crust and the lower soft clay. Near the ground surface, rapid dissipation of excess pore water pressure occurs due to upward drainage. The finite element analysis indicates little dissipation of excess pore water pressure in the lower part of the upper, very soft clay layer and in the top part of the lower, soft silty clay layer. Both the undrained and coupled analyses give the same results about the excess pore water pressure in these locations. The dissipation begins to increase at the depth of around 9 m below the base of the embankment due to the presence of a sandy clay layer with much higher permeability under the lower soft silty clay. As shown in

Figure 3.9, below that depth, the undrained analysis tends to overestimate the excess pore water pressure, whereas the coupled analysis could give the correct prediction about the distribution pattern of the excess pore water pressure.

Figure 3.10 Variation of excess pore water pressure beneath the embankment centerline at the fill height of 5 m

The variation of excess pore water pressure beneath the embankment centerline at the fill height of 5 m is shown in Figure 3.10. As discussed before, the factor of safety for this embankment by the undrained analysis is already below 1.0. Therefore, only results from the coupled analysis are provided in the figure. It can be seen that the coupled analysis predicts the distribution pattern of excess pore water within the foundation only approximately. The finite element analysis overestimates the excess pore water pressure in the upper, very soft clay layer and underestimates the excess pore water pressure in the lower, soft clay layer. One of the possible explanations for this is the variation of permeability of soils during loading and consolidation. However, as pointed out by Tavenas and Leroueil (1980), there are still no satisfactory methods to solve these problems.

Figures 3.11-3.13 show the variation of excess pore water pressure with the embankment construction at different locations (P2, P5, and P7). Both the results from the coupled analysis and the undrained analysis are plotted and compared with the measured data. Since the excess pore water pressures at the height of 5 m from the undrained analysis are incredibly high at failure condition, it is not meaningful to compare these excess pore water pressures at the failure condition with the measured data.

Figure 3.11 Variation of excess pore water pressure with embankment construction at location P2

Figure 3.12 Variation of excess pore water pressure with embankment construction at location P5

Figure 3.13 Variation of excess pore water pressure with embankment construction at location P7

These figures show that the excess pore water pressure decreases as expected along the lateral direction from the embankment centerline toward the toe. This is due to the decreasing effects of the embankment loading at distances away from the centerline. It can also be seen from Figures 3.11-3.13 that the measured excess pore water pressures are much less than those predicted by the coupled analyses at the early stages of construction. This is because the dissipation of excess pore water pressures is significant due to the overconsolidated state of the foundation soils, especially the crust layer, at the early stage of construction.

Both Figures 3.11 and 3.12 show that the coupled analysis overestimates the excess pore water pressure induced during the embankment construction. This indicated that the soil permeability in the field is likely to be higher than that used in the present analysis. However, Figure 3.13 shows that the coupled analysis slightly underestimated the excess pore water pressure at location P7 during the first 4 m increment of the embankment. The possible explanation for this is that the in-situ permeability of the soft clay may be reduced by the placement of the fill during the construction period.

3.2.4 Failure mechanism of the unreinforced embankment

The stability of this embankment received wide attention in the International Symposium on Trial Embankments on Malaysian Marine Clays held in Malaysia in November 1989. A total of 31 'major' and 'minor' predictors predicted the failure height and failure surface for this embankment. The predicted failure height varied between the wide limits of 2.8 m and 9.5 m according to Brand (1991). Figure 3.14 shows the actual failure mode of this embankment observed in the field. It can be seen that the failure surface in the foundation soil is close to circular and touches the bottom of the upper, very soft clay layer.

Figure 3.14 Failure mode of embankment and foundation (Brand 1991)

Figure 3.15 shows the initial and deformed meshes for this embankment corresponding to the height of 5.5 m, using the input parameters shown in Table 3-1. It is clear from this plot that a failure mechanism similar to that observed in the field is obtained by the finite element analysis. It can be seen that failure only occurs in the crust and the upper, very soft silty clay layer and does not develop into the lower, stiffer clay layer. Figure 3.16 shows the maximum incremental displacements from the finite element analysis for the embankment height of 5.5 m. A clear circular shape of failure mechanism develops in the foundation soil. This approach for the failure mechanism, based on incremental displacement vectors, was also used by Indraratna et al. (1992) to determine the shear surface through the clay foundation.

Figure 3.15 Failure mechanism predicted by the finite element analysis at the embankment height of 5.5 m

Figure 3.16 Maximum incremental displacements by the finite element analysis at the embankment height of 5.5 m

In addition, shear strains (ε_s) developed within the foundation also can be used to judge the critical slip surface, as shown in Figure 3.17. A clear shear band with high shear strains is observed in this figure. Brand (1991) pointed out that there was a failure band for this embankment rather than a single failure surface. Several slip surfaces close to one another can yield practically the same factor of safety. The failure

mechanism in Figure 3.17 obtained by the present analysis indicates a shear band touching the bottom of the upper, very soft silty clay layer, consistent with Brand (1991) regarding the slip surface. Deformation and strain information like these are already beyond the capability of the conventional limit equilibrium methods.

Figure 3.17 Shear strains ε_s by the finite element analysis at the embankment height of 5.5 m

The plastic zone within the foundation soil is also important for identifying and understanding the failure mechanism. Figures 3.18 and 3.19 show the plastic zones within the embankment fill and the foundation soil at the embankment height of 5 and 5.5 m, respectively. The black zones in the figure represent the tensile stress points and the red zones represent the stress points which lie on the surface of Mohr-Coulomb failure envelope. The blue zones in the figure represent stress points in yield surface, which is also called “cap surface” in the soft soil model. A stress point in the cap surface means that the stress state reaches the preconsolidation pressure. Most relevant are the red zones indicating the plastic state where the Mohr-Coulomb failure criterion is reached.

Figure 3.18 Plastic zone obtained by the finite element analysis at the embankment height of 5 m

Figure 3.19 Plastic zone obtained by the finite element analysis at the embankment height of 5.5 m

It can be seen from Figure 3.18 that the plastic zone in the foundation soil, which is developed in the upper, very soft silty clay layer, is still not continuous at the embankment height of 5 m. This is consistent with the fact that the factor of safety is above 1.0 at that stage. When the embankment height is raised to 5.5 m, the plastic zone propagates toward the toe and becomes continuous as shown in Figure 3.19. This indicates that the embankment becomes unstable at the height of 5.5 m. Indraratna et al. (1992), based on their finite element analysis, suggested that the plastic zone within the foundation soil started from the bottom of the upper clay and developed toward the

embankment centerline when failure was imminent. However, the present analysis reveals a reverse trend. The failure of the soils in the foundation develops firstly from the crust and upper, very soft clay layers under the embankment and propagates toward the toe.

Failure development can also be observed from the plots of relative shear ratios at different construction stages, as shown in Figures 3.20-3.24. The relative shear ratio is defined as the ratio between the mobilized shear stress and the available shear strength according to the Mohr-Coulomb failure criterion. Chai et al. (1993, 1994) proposed to adopt the contour of relative shear ratios from finite element analysis to indicate the failure mechanism of the embankment, but found that such contours cannot clearly present the potential slip surface developed in the foundation. Nevertheless, the propagation of the failure inside the foundation as construction proceeds can be observed. Figure 3.20 shows that a relatively higher shear ratio zone is formed under the crest of the embankment in the crust layer when the embankment height is 2 m. The maximum shear ratio in this zone is 0.82. When the embankment height reaches 3 m, there is a zone with relative shear ratio of 1.0, extending from the centerline to the shoulder of the embankment in the crust layer. The shear ratios in the upper, very soft silty soil layer under the embankment crest start to increase (Figure 3.21).

As the embankment height increases from 3 m to 5 m, the zones with large shear ratios both in the crust layer and in the upper, very soft silty soil layer tend to grow as shown in Figures 3.22 and 3.23. Moreover, a high shear ratio zone also appears in the toe region and develops with embankment construction. It is interesting to note that three different zones (the zone in the crust layer under the crest of the embankment, the zone in the upper, very soft clay layer under the crest of the embankment and the zone in the crust layer near the toe region) with high relative shear ratios exist in the foundation soil, as shown in Figure 3.24. At the embankment height of 5 m, these three zones, although extensive, are not contiguous.

Figure 3.20 Distribution of relative shear ratios from finite element analysis at the embankment height of 2 m (maximum relative shear ratio is 0.82)

Figure 3.21 Distribution of relative shear ratios from finite element analysis at the embankment height of 3 m

Figure 3.22 Distribution of relative shear ratios from finite element analysis at the embankment height of 4 m

Figure 3.23 Distribution of relative shear ratios from finite element analysis at the embankment height of 5 m

As the height of the embankment reaches 5.5 m, these three zones join together and become continuous (Figure 3.24). This formation of a continuous zone with shear ratio equal to 1.0 throughout the foundation soil indicates that the embankment is unstable at this stage.

Figure 3.24 Distribution of relative shear ratios from finite element analysis at the embankment height of 5.5 m

3.3 Reinforced embankment on the soft ground

To study the effect of the reinforcement placed at the interface between the embankment fill and the clay foundation on the ground surface settlements, lateral displacements, excess pore water pressures, and the stability of the embankment, a hypothetical reinforced embankment is analyzed numerically by the finite element method in this section.

The conditions of this hypothetical reinforced embankment are similar to those of the trial Muar unreinforced embankment studied in the previous section. The difference is that the reinforcement is now placed at the interface between the embankment fill and

the foundation clay. In line with a common practice, the reinforcement is laid horizontally in the middle of a 0.5 m thick sand blanket, as shown in Figure 3.25.

Figure 3.25 Geometry of the reinforced embankment on soft ground

An elastic-perfectly plastic Mohr-Coulomb model is adopted to model the sand blanket in the analysis. It is assumed that the strength parameters, cohesion c' , and friction angle ϕ' , of the sand blanket are 0 and 38° respectively. The two elastic parameters E' and ν' in the elastic-perfectly plastic Mohr-Coulomb model, are given the nominal values of 20000 kPa and 0.2 respectively.

The linear tension-strain relation is used in the finite element analysis for the reinforcement. The only property for this model is the axial stiffness, expressed as the force per unit width per unit strain. The reinforcement with stiffness of 3000 kN/m is adopted in the present analysis. The interaction between the soil and the reinforcement is modeled by the elastic-plastic interface element. The Mohr-Coulomb criterion is used to distinguish between elastic behaviour, where small displacements can occur within the interface, and plastic interface behaviour (slip). This approach is also adopted in Rowe and Soderman (1987), Rowe and Li (1999, 2002) and Borges and Cardoso (2001).

3.3.1 Effect of reinforcement on the ground surface settlement

The ground surface settlements at the interface between embankment and original ground surface of the reinforced embankment at the fill height of 3 m, 4 m, 5 m, and 5.5 m are plotted in Figures 3.26-3.29, respectively. The settlements of the unreinforced embankment at the different heights are also shown in the figures for comparisons. It can be seen that the inclusion of the reinforcement actually reduces, but not significantly, the ground surface settlements before the embankment reaches the critical height (5.5 m) where the unreinforced embankment loses its stability. The reduction ratios of the settlements under the embankment centerline are below 10%.

Figure 3.26 Ground surface settlement for the reinforced embankment at the height of 3 m

At the critical height (5.5 m), the reinforcement exerts a more significant restraining effect on the ground surface settlements (Figure 3.29). Both the vertical displacements and heaves are reduced by the reinforcement. The reduction for the foundation surface settlement under the centerline is around 20%. According to the numerical analyses by Varadarajan et al. (1999), the reduction in vertical displacement of the embankment

was up to 37% at the maximum height of the unreinforced embankment when the reinforcement with stiffness of 8000 kN/m was used.

Figure 3.27 Ground surface settlement for the reinforced embankment at the height of 4 m

Figure 3.28 Ground surface settlement for the reinforced embankment at the height of 5 m

Figure 3.29 Ground surface settlement for the reinforced embankment at the height of 5.5 m

For the present analysis, the factor of safety is 1.002 for the embankment at the height of 5.5 m when no reinforcement is used. This indicates that the reinforcement tends to reduce the surface settlement more significantly when the embankment is much closer to the failure state. This is consistent with the observation made by Chai et al. (2002) that reinforcement can reduce settlement just before failure, based on field measured data for unreinforced and reinforced embankments constructed on soft subsoil.

3.3.2 Effect of reinforcement on lateral displacement

Figures 3.30 and 3.31 show the variation of lateral displacements with depth for different construction stages at the location of inclinometer I3 (Figure 3.2) and at the toe region, respectively.

Figure 3.30 Lateral displacements along inclinometer I3 for different heights of the embankment

The lateral displacements for the unreinforced embankment are also provided for comparison. It can be seen that the lateral displacements of reinforced embankment are almost identical to those of unreinforced embankment at the early stage of construction. The difference is negligible. However, when the reinforced embankment is built from 3 m to 5.5 m, the difference between the lateral displacements of the unreinforced embankment and the reinforced embankment begins to increase and reaches the maximum value at the height of 5.5 m. This indicates that the inclusion of the

reinforcement can reduce the lateral displacements of the embankment, which helps to increase the stability of the embankment.

Figure 3.31 Variation of lateral displacement under the toe for different heights of the reinforced embankment

The most significant reduction of the lateral displacements occurs in the upper, very soft silty clay layer. The lateral displacements below this layer are not influenced by the existence of the reinforcement. This means that the beneficial effect brought about by the reinforcement is limited to a certain range and can only reach the depth of around 8 m within the foundation. It indicates that the reinforcement can significantly improve the stability of the embankment on the soft ground provided that the depth of the soft soil is not too deep.

Figure 3.32 Variation of lateral displacement under the toe with the depth of the foundation for the reinforcements with different stiffness ($H=5.5$ m)

Figure 3.32 shows the variation of lateral displacements under the embankment toe for reinforcements with different stiffness, when the embankment height is 5.5 m. Although using the reinforcement with higher stiffness can reduce the lateral displacements within the clay foundation, diminishing returns set in when the reinforcement stiffness exceeds 5000 kN/m.

3.3.3 Effect of reinforcement on excess pore water pressures within the embankment foundation

Figures 3.33 and 3.34 show the variations of excess pore water pressure with depth under the embankment centerline and beneath the toe region, respectively.

Figure 3.33 Variation of excess pore water pressure under the embankment centerline for different heights of the reinforced embankment

It can be seen that the inclusion of the reinforcement in the embankment almost has no effect on the excess pore water pressures within the foundation, whether under the embankment centerline or under the toe.

The reinforcement is considered to carry the lateral thrust from the embankment fills and therefore reduce the lateral displacements within the foundation and increase the stability of the embankment. However, the additional vertical stresses resulted from the embankment construction may not be influenced by the reinforcement laid horizontally. Since the excess pore water pressures within the foundation are mainly controlled by the additional vertical loading and the permeability properties of the foundation soils, the effect of reinforcement on excess pore water pressures is negligible.

Figure 3.34 Variation of excess pore water pressure under the toe for different heights of the reinforced embankment

The only reference in the literature about the difference in the excess pore water pressures between unreinforced and reinforced embankments is a case history of built-to-failure embankments at Lian Yun Gang in China analyzed by Chai et al. (2002). However, because the construction sequences for unreinforced and reinforced embankments were different from each other, it was difficult to conduct a meaningful comparison between the excess pore water pressures of unreinforced embankment and those of reinforced embankment. Nevertheless, it can still be inferred from their analysis that the difference in the excess pore water pressures between unreinforced and reinforced embankment was small.

3.3.4 Distribution pattern of the reinforcement force

Figure 3.35 shows the variation of mobilized reinforcement forces with the height of the reinforced embankment. Initially, the maximum mobilized tensile force in the reinforcement occurs under the embankment slope.

Figure 3.35 Distribution pattern of the mobilized reinforcement force at different heights of the reinforced embankment ($J=3000$ kN/m)

With further increase in embankment height, the maximum mobilized tensile force shifts toward the embankment centerline. When the critical embankment height of 5.5 m is reached, the maximum mobilized tensile force develops in the embankment centerline. The same transfer mechanism for the maximum mobilized reinforcement force was also noticed by Chai and Bergado (1993).

Figure 3.36 shows the distribution pattern of the mobilized reinforcement force for different reinforcement stiffness at the embankment height of 5.5 m. The value of the reinforcement force increases with the reinforcement stiffness. The distribution pattern of the reinforcement force is not influenced by the reinforcement stiffness.

Figure 3.36 Distribution pattern of the mobilized force in the reinforcement for different stiffness ($H = 5.5$ m)

3.3.5 Effect of reinforcement on the stability of the embankment

The effect of the reinforcement on the stability of the embankment is investigated in this section. Figure 3.37 shows the variation of the net height with the fill thickness for unreinforced and reinforced embankments on soft ground. The net height is defined as the difference between the actual fill thickness and the maximum ground surface settlement. The net height of the unreinforced embankment increases with the fill thickness. When failure is approached, both net height and the fill thickness reach the maximum value. The situation becomes different for the reinforced embankment. Due to the presence of the reinforcement, the net height increases first with the fill thickness and then decreases. The maximum net height of the reinforced embankment is 6.23 m and the corresponding fill thickness is 8.02 m. Excessive deformations are considered to have occurred when the fill thickness exceeds 8.02 since the net height starts to decrease. In practice, the reinforced embankment can be regarded to have failed when the net height decreases.

Figure 3.37 Net height versus the fill thickness for unreinforced and reinforced embankments on soft ground

For the case in hand, the maximum net height of the embankment is increased from 4.5 m to 6.23 m when reinforcement with stiffness of 3000 kN/m is used. This indicates that the inclusion of the reinforcement improves the stability of the embankment.

Figure 3.38 Plastic zone obtained by finite element analysis for the reinforced embankment at the height of 5.5 m

Figure 3.39 Shading plot of relative shear ratios for the reinforced embankment at the height of 5.5 m

Figure 3.38 shows the plastic zone developed within the foundation at the height of 5.5 m. It can be observed that the plastic zone within the foundation is not continuous when the height of the embankment is 5.5 m. By comparing Figure 3.38 with Figure 3.19, it can be seen that the development of the plastic zone within the foundation is restrained to a certain extent. The same conclusion can also be obtained by comparing Figures 3.24 and 3.39.

Part 2: Stability analysis of unreinforced and reinforced embankments on soft ground

3.4 Stability analysis of the embankments on soft ground

Finite element method is very suitable for the prediction of settlements or deformations, the calculation of flow quantities due to seepage and the study of transient effects due to consolidation, etc. However, finite element method is seldom used to calculate directly the factor of safety of embankment on soft ground. Currently, stability analysis of embankment is still mainly carried out by the conventional limit equilibrium

methods of slices. In this section, three different stability analysis methods are used to calculate the factor of safety and determine the critical slip surfaces for the unreinforced and reinforced embankments on soft ground (Figures 3.1 and 3.25).

3.4.1 The stability analysis methods used

The stability analysis methods employed include effective stress stability analysis method, enhanced limit method and strength reduction technique.

3.4.1.1 Effective stress stability analysis method

Effective stress stability analysis method is commonly used to assess the stability of embankment on natural cohesive deposits during staged construction (Bishop and Morgenstern 1965, Tavenas et al. 1978 and Pilot et al. 1982). Pore water pressures measured in the field are used to determine the effective normal stress distribution along the slip surface. The factor of safety is then calculated based on soil effective strength parameters c' and ϕ' from laboratory CD tests or CU tests with pore pressure measurements. In this study, Spencer's method with pore water pressures interpolated from finite element analysis is used to calculate the factor of safety.

3.4.1.2 Enhanced limit method

A thorough review about enhanced limit analysis method has been presented in Chapter 2. This method can overcome the disadvantages of the conventional limit equilibrium methods of slices. In this study, the normal and shear stresses and pore water pressures calculated from PLAXIS are used to calculate the factor of safety. Two types of triangle elements are available in PLAXIS: 6-noded and 15-noded triangle elements. The 6-noded triangle element is used in the analysis. The computation of the state of stress at the object point in the slip surface is divided into two steps. The first step is to determine which triangle element contains the object point. This is done by calculating the total area of $\Delta P13$, $\Delta P12$ and $\Delta P23$ to determine if the total area of three triangles is equal to the area of $\Delta 123$ (Figure 3.40). The object point P is

considered to be contained in the 6-noded triangle element if the total area of $\Delta P13$, $\Delta P12$ and $\Delta P23$ is equal to the area of $\Delta 123$. If the area is not equal, the object point is out of the triangle element. The area of the 6-noded triangle element can be easily calculated as

$$s_{\Delta 123} = Abs \begin{pmatrix} 1 & x_1 & y_1 \\ \frac{1}{2} & 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix} \quad (3-2)$$

where x_i and y_i are the coordinates of node i , respectively. The total area of $\Delta P13$, $\Delta P12$ and $\Delta P23$ is determined by

$$s_{\Delta P13} + s_{\Delta P12} + s_{\Delta P23} = Abs \begin{pmatrix} 1 & x_p & y_p \\ \frac{1}{2} & 1 & x_1 & y_1 \\ 1 & x_3 & y_3 \end{pmatrix} + Abs \begin{pmatrix} 1 & x_p & y_p \\ \frac{1}{2} & 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{pmatrix} + Abs \begin{pmatrix} 1 & x_p & y_p \\ \frac{1}{2} & 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix} \quad (3-3)$$

The search is continued until the area of the triangle element $\Delta 123$ is found to be equal to the total area of $\Delta P13$, $\Delta P12$ and $\Delta P23$.

Figure 3.40 Locating the 6-noded triangle element containing the object point P

The second step is to interpolate the stresses at the object point in the slip surface. Nodal average stress interpolation scheme is used due to its simplicity. The

interpolation procedure involves projecting the known stresses at the integration points to the nodes and averaging the stresses at each node. With known stresses at the nodes, the stresses at any point within the element can be calculated using the shape function. Advantages and disadvantages of this scheme are discussed in Chapter 2.

The procedure of calculating the factor of safety described above, which is based on the stresses computed from finite element analysis, is validated first by analyzing a homogeneous slope before it is applied to the unreinforced and reinforced embankments on soft ground of Figures 3.1 and 3.25.

Figure 3.41 Critical noncircular slip surfaces from enhanced limit method and Spencer's method for a homogeneous slope

The simple homogeneous slope at 2:1 shown in Figure 3.41 has been studied by many researchers as a standard slope problem. The unit weight of the slope soil is 20 kN/m^3 . The friction angle and soil cohesion are 20° and 10 kPa, respectively. Zienkiewicz et al. (1975) obtained a factor of safety of 1.39 for this slope using strength reduction technique. Zou et al. (1995) analyzed this slope using the generalized limit equilibrium wedge analysis method (GWEDGEM) and FEM-based improved dynamic

programming method (FEM-IDPM). The factor of safety was 1.39 and 1.36, respectively.

The critical noncircular slip surfaces by enhanced limit method and Spencer's method for this slope are also shown in Figure 3.41. In the enhanced limit method, the soil is modeled by the elastic-perfectly plastic Mohr-Coulomb model. BFGS method is adopted to search for critical noncircular slip surface. The searching scheme is described in details in Chapter 4. The corresponding factors of safety are 1.38 and 1.35, respectively. Figure 3.41 shows that the critical slip surfaces from enhanced limit method (denoted by the solid line) and Spencer's method (denoted by the dash line) are very close to each other. The values of the factor of safety from these two different methods are also in good agreement. Table 3-2 summarizes the results for this slope by different methods.

Table 3-2 Stability analysis results for the homogeneous slope by different methods

Researcher	Factor of safety	Method
Zienkiewicz et al. (1975)	1.39	Strength reduction technique
Zou et al. (1995)	1.39	GWEDGEM ¹
	1.36	FEM-IDPM ²
This study	1.38	Spencer's method
	1.35	Enhanced limit method

¹ The generalized limit equilibrium wedge analysis method (Giam 1989)

² FEM-based improved dynamic programming method

Figures 3.42 and 3.43 show the distribution of normal stress and shear stress along the critical slip surface from enhanced limit method and Spencer's method. The sliding mass above the slip surface is equally divided into 20 slices with the leftmost slice as the first slice. Compared with the stress distribution along the slip surface from finite element analysis, the stress distribution from Spencer's method is more continuous. For this simple slope, the normal stress distribution from finite element analysis is in good agreement with that from Spencer's method. However, Figure 3.43 shows that the shear stress distributions from enhanced limit method and Spencer's method are still quite different. Figure 3.44 shows the distribution of the local factor of safety

along the slip surface. The local factor of safety from enhanced limit method varies with the location. In some portions of the slip surface, the local factors of safety are less than that from Spencer's method. The local factors of safety from enhanced limit method are higher in other portions of the slip surface.

Figure 3.42 Normal stresses along the critical slip surface

Figure 3.43 Shear stresses along the critical slip surface

Figure 3.44 Distribution of the local factor of safety along the slip surface

3.4.1.3 Strength reduction technique

Strength reduction technique is an alternative way to determine the factor of safety based on finite element method. It is described and discussed detailedly in Chapter 2. This approach calculates the factor of safety by repeatedly conducting nonlinear finite element analysis with c and $\tan\phi$ progressively reduced by a reduction factor. If failure criterion (nonconvergence or excessive deformations) is met, then the final reduction factor is regarded as the factor of safety for the slope or embankment. It was first proposed by Zienkiewicz (1971) and recently used by Dawson et al. (1999) and Griffiths and Lane (1999). PLAXIS has an option to carry out strength reduction technique for slope stability analysis. This option is used to calculate the factor of safety of embankment in the next section.

3.4.2 Stability analysis for the unreinforced embankment on soft ground

For the unreinforced embankment on soft ground (Figure 3.1), three different methods: effective stress stability analysis method, enhanced limit method and strength

reduction technique will be used to analyze the stability. The MATLAB² language is used to code the program for effective stress stability analysis method and enhanced limit method.

For effective stress stability analysis method, Spencer's method is used and pore water pressures are interpolated from results of finite element analysis. A simple subroutine is included in the program of Spencer's method with the purpose to interpolate pore water pressure for the object point. Since the embankment fill has a cohesion value of 19 kPa, a tension crack is assumed to be present in the embankment in order to eliminate negative normal stress at slice base. According to Eq. (2-18), the depth of tension crack is 2.97 m. In order to conduct a meaningful comparison with the results from strength reduction technique, only noncircular slip surface is used.

Figure 3.45 Critical slip surfaces for unreinforced embankment from different methods

According to effective stress stability analysis based on Spencer's method, the critical factor of safety is 1.047. When enhanced limit method is used, the critical factor of safety becomes 1.03. Figure 3.45 shows the critical slip surfaces from Spencer's method (denoted by the dash line) and enhanced limit method (denoted by the solid line). Strength reduction technique gives a factor of safety of 1.002 for this

² More details at <http://www.mathworks.com/>

unreinforced embankment. These three factors of safety agree quite well with each other (the difference is below 5%) and are consistent with the fact that the embankment failed at this height. It can be seen that the factor of safety from shear strength technique is the one closest to the real condition. By comparing Figure 3.45 with Figure 3.19, it is also very clear that the critical noncircular slip surface from enhanced limit method passes through the plastic zones developed in the clay foundation.

Enhanced limit method predicts a factor of safety of 1.083 for the critical noncircular slip surface obtained by Spencer's method. This value is slightly higher than that (1.047) from Spencer's method. The total normal stresses and shear stresses along this slip surface from Spencer's method and enhanced limit method are also calculated and compared. Figures 3.46 and 3.47 show the total normal stress and shear stress along this slip surface from two different methods. It can be seen that the distributions of normal stress and shear stress from Spencer's method are quite different from those by enhanced limit method in some parts of the slip surface. This explains the difference in the stability analysis results between these two approaches.

Figure 3.46 Total normal stresses along the slip surface from Spencer's method and enhanced limit method.

Figure 3.47 Shear stresses along the slip surface from Spencer's method and enhanced limit method.

Figure 3.48 Distribution of the local factor of safety along the slip surface

Figure 3.48 shows the variation of the local factor of safety along the slip surface. The factor of safety from Spencer's method is also plotted in this figure. Compared with the factor of safety from Spencer's method, the distribution of the local factor of safety

from enhanced limit method is not smooth. The value of the factor of safety oscillates around the value (1.047) from Spencer's method.

3.4.3 Stability analysis for the reinforced embankment on soft ground

One of the very attractive features of enhanced limit method is that the interaction between the soil and the reinforcement in the reinforced embankment can be dealt with in a direct manner. There is no need to try to represent the effect of reinforcement by a horizontal load as in the limit equilibrium analysis. There is also no need to independently determine the magnitude of the reinforcement force. The effect of reinforcement is already included in the finite element analysis, which alters the stress states of the soils in the embankment and foundation. This results in the increase of the degree of safety.

Figure 3.49 Variation of the factor of safety with the reinforcement stiffness from enhanced limit method ($H = 5.5$ m)

In this section, enhanced limit method will be used to calculate the factor of safety of the reinforced embankment on soft ground. Figure 3.49 shows the variation of the factor of safety with the reinforcement stiffness. The factor of safety is increased from

1.03 to 1.083 when the reinforcement stiffness is increased from 0 to 5000 kN/m in the analysis. This indicates that using reinforcement with higher stiffness can lead to increase in safety of the embankment. It is also consistent with the fact that higher reinforcement stiffness can yield higher net height for this reinforced embankment (Figure 3.50). Similar findings were reported by Row and Soderman (1987).

Figure 3.50 Variation of the net height with the fill thickness for different reinforcement stiffness

Since strength reduction technique has exhibited its advantages in determining the factor of safety for the unreinforced embankment, this method is also used to calculate the factor of safety of the reinforced embankment at the height of 5.5 m. When the reinforcement with stiffness of 1000 kN/m is used, the factor of safety is found to be 1.33, which is much higher than that from enhanced limit method. Further analyses carried out by increasing the reinforcement stiffness from 1000 kN/m to 5000 kN/m do not result in any significant increase in the value of the factor of safety. The factor of safety remains almost the same around 1.33.

The 33% increase in the factor of safety for the embankment due to the usage of the reinforcement with stiffness of 1000 kN/m is very high. A complete review of the relevant literature indicates that the factor of safety may not be increased too much

(usually around 1-2%) after using the reinforcement, especially for the embankment constructed over such a deep soft foundation. Moreover, this very high increase in the factor of safety also contradicts the latter results where the factor of safety becomes insensitive to the reinforcement stiffness. All these inconsistent results about the factor of safety calculated from strength reduction technique suggest limitations of strength reduction technique in analyzing the stability of reinforced embankments.

One possible reason for the inconsistent results obtained above could be the nonconvergence criterion used in shear strength reduction to judge failure. The fact that excessive deformations may occur or that the net height of the embankment has already started to decrease cannot be accounted for by using nonconvergence criterion. As a result, the mobilized force in the reinforcement could become extremely high when nonconvergence occurs. The unreasonably high tensile force mobilized in the reinforcement result in a high factor of safety. Therefore, the strength reduction technique may not provide the correct and meaningful prediction of the factor of safety of the reinforced embankment. For the unreinforced embankment, because the net height will not decrease with the fill thickness in the absence of the reinforcement, strength reduction technique works well.

Therefore, if strength reduction technique is applied to the stability analysis of reinforced embankments, the corresponding failure criterion needs to be appropriate for the reinforced embankment. Before that is done, it is hard to conduct rational and meaningful comparisons between the results from strength reduction technique and those from other methods for reinforced embankments on soft ground.

3.5 Summary and conclusions

The results from numerical analyses of unreinforced and reinforced embankment on soft ground have been presented. Both the coupled analysis and undrained analysis are used to predict the behaviour of an unreinforced embankment. It is found that the coupled analysis is able to provide more reasonable prediction of the behaviour of the embankment. The undrained analysis tends to underestimate the stability of the

unreinforced embankment due to neglecting the effect of excess pore water pressure dissipation within the clay foundation.

When the embankment is reinforced by a tensile reinforcement placed at the interface between the embankment fill and the clay foundation, it was found that the effect of reinforcement in reducing ground surface settlement is only significant when the height of the reinforced embankment reaches the critical height at which the unreinforced embankment is close to failure. The reinforcement has some effect on reducing the lateral displacement of soft soils in the foundation. The most significant reduction of the lateral displacements occurs in the upper, very soft silty clay layer. The lateral displacements below this layer are not affected by the reinforcement. The beneficial effect brought about by the reinforcement is limited to a certain range and can only reach the depth of around 8 m within the foundation. This indicates that the reinforcement can significantly improve the stability of the embankment on soft ground provided that the thickness of the soft soil is not too large. Little reduction effect on the excess pore water pressure is observed by the inclusion of horizontal reinforcement at the interface between the embankment fill and the foundation clay.

The numerical analysis of the reinforced embankment shows that the stability of the embankment can be improved by the use of reinforcement. The net height of the embankment can be increased. To some extent, the development of plastic zone in the clay foundation can be restrained by the reinforcement. The formation of a continuous plastic zone in the foundation is observed to be delayed with the use of reinforcement. The failure state can be considered to be already reached for the reinforced embankment when excessive deformations occur in the clay foundation. Usually, the net height of the embankment starts to decrease at that stage.

Different stability analysis methods are used to calculate the factors of safety for the unreinforced and reinforced embankments on soft ground. These methods include effective stress analysis based on Spencer's method, enhanced limit method and strength reduction technique. For the unreinforced embankment studied in this chapter, these three methods yield similar values of the factor of safety. The result from

strength reduction technique for the unreinforced embankment is shown to be the closest one to the real condition. For the reinforced embankment, inconsistent results are observed in the analysis by strength reduction technique. The reason may be the nonconvergence criterion used to judge the failure in strength reduction technique.

CHAPTER 4 LOCATING CRITICAL CIRCULAR OR NONCIRCULAR SLIP SURFACE FOR REINFORCED EMBANKMENTS ON SOFT GROUND

4.1 Introduction

Deterministic stability analysis of slopes or embankments consists of two main parts: (i) calculation of the factor of safety for a specified circular or noncircular slip surface, and (ii) search for the minimum factor of safety and the corresponding critical slip surface. A number of slope stability analysis methods are available for circular or noncircular slip surface, such as Bishop's simplified method (Bishop 1955), Janbu's generalized procedure of slices (Janbu 1973), Spencer's method (Spencer 1967), Morgenstern and Price's method (Morgenstern and Price 1965) and others. These methods differ in the ways they deal with the inter-slice forces. A state-of-the-art review was given in Fredlund (1984).

Although these stability analysis methods were not meant to deal with reinforced embankments originally, they can be easily modified to calculate the factor of safety of reinforced embankment on soft ground. The common approach is to modify Bishop's simplified method to analyze the stability of reinforced embankment (Rowe and Soderman 1985). The reinforcement can be considered to provide an additional beneficial resisting moment. Sabhahit et al. (1994) modified Janbu's generalized procedure of slices to take account of the role of the reinforcement. Low and Tang (1997b) used the semi-analytical method to analyze the stability of reinforced embankments. Spencer's method was reformulated and extended in Low and Tang (2001) for the stability analysis of reinforced embankments. Tandjiria et al. (2002) extended the work by Chen and Li (1998), which is based on Chen and Morgenstern's

method (Chen and Morgenstern 1983), to incorporate the reinforcement. It can be seen that the existing stability methods can be extended to study the stability of reinforced embankments easily. There are plenty of works which can be found in the literature.

Duncan (1996) suggested that the critical slip surface can be assumed to be circular, unless there are geological controls that constrain the slip surface to be noncircular. For reinforced embankments on soft ground, a circular slip surface is usually adopted in the stability analysis. This is because rotational failure is commonly observed in the field for reinforced embankments on soft ground.

For reinforced embankments, the algorithm for locating the critical circular slip surface is not usually discussed in the literature, perhaps due to the belief that there is no special difficulty in locating the critical slip surface with circular shape as only three variables are involved (the x and y coordinates of the slip circle center and the radius).

Grid search method is most commonly used and is included in many commercial stability analysis programs. In addition, the simplex method is also considered to be very robust and efficient for locating the critical circular slip surface (Nguyuen 1985; DeNatale 1992). More recently, Malkawi et al. (2001a) proposed an approach to search for the critical circular slip surface using Monte Carlo technique. Goh (2000) and McCombie and Wilkinson (2002) adopted genetic algorithm to search for the critical slip circle.

The situation becomes more complicated for locating critical noncircular slip surface. Revilla and Castillo (1977) and Baker and Garber (1978) adopted the calculus of variations approach to locate the critical slip surface. However, this approach is only valid for some special conditions. For example, Castillo and Luceno (1982) found that the approach proposed by Revilla and Castillo (1977) was only applicable for slopes

composed of frictional materials. Baker (1980) used dynamic programming to determine the critical noncircular slip surface based on Spencer's method. For this approach, Li and White (1987a) and Chen and Shao (1988) pointed out that coupling the calculation and minimization of the factors of safety could complicate its application. Celestino and Duncan (1981) used alternating variables method to locate the critical noncircular slip surface. Their method was extended later by Li and White (1987a). Bardet and Kapuskar (1989) adopted the simplex method to search for the critical noncircular slip surface. They indicated that the simplex method was a reliable, robust and simple tool for locating the minimum factor of safety corresponding to the critical noncircular slip surface although it may not be the fastest and most efficient optimization technique.

In addition to the deterministic optimization methods, probabilistic method can also be used to search for the critical noncircular slip surface. A random procedure was used by Boutrup and Lovell (1980) to calculate the minimum factor of safety. Chen (1992) combined random search with gradient-based optimization method and adopted the estimated minimum from random search method as the starting point employed by the optimization method. Greco (1996) proposed a Monte Carlo technique of random walking type to locate the critical slip surface. More recently, Malkawi et al. (2001b) presented a new search procedure based on Monte Carlo method of random walk type. As discussed by Bhattacharya (2001), this approach was not very different from that developed by Greco (1996) except that the variables were the rotations of the slip surface segments instead of the displacements of the slip surface vertices used in Greco's approach. Goh (1999) used genetic algorithm to search for critical noncircular slip surface based on the multiple-wedge method.

Cheng (2003) introduced the notion of simulated annealing method to locate critical circular and noncircular slip surfaces. In some respects, this method was similar to the

approach by Malkawi et al. (2001b) since random generation of slip surfaces was required. The advantage is that local minimum can be avoided by using simulated annealing technique.

The procedures to search for critical noncircular slip surface mentioned above can, of course, be used to locate the critical slip surface for reinforced embankments on soft ground. However, only a few attempts have been made so far to search for critical noncircular slip surface for reinforced embankments (Sabhahit et al. 1994 and Tandjiria et al. 2002). Powell's method of unconstrained minimization was adopted by Sabhahit et al. (1994) to locate the critical noncircular slip surface using the penalty function approach to consider the constraints. The work by Tandjiria et al. (2002) was based on the spreadsheet in Microsoft Excel environment where the built-in optimization tool Solver was used (Low et al. 1998).

In this chapter, extensive comparisons will be made for different optimization methods to locate the critical circular and noncircular slip surfaces for reinforced embankments on soft ground. The main purpose is to investigate the effects of different optimization procedures on the minimum factors of safety of reinforced embankments on soft ground. For circular slip surface, which is commonly used in the stability analysis of reinforced embankments, three different optimization approaches will be used: the grid search method, the simplex method and genetic algorithm. The grid search method is adopted herein due to its simplicity and popularity. The simplex method is used because it is very reliable and efficient for circular slip surface. The genetic algorithm presents a new direction of optimization procedure in slope stability analysis. It is interesting to compare the results from this approach with those from more conventional optimization methods.

For the critical noncircular slip surface of reinforced embankments on soft ground, two

different optimization approaches will be used: pattern search method which does not require the evaluation of the gradient of the object function and Broydon-Fletcher-Goldfarb-Shanno (BFGS) method which is a gradient-based approach.

4.2 Extended Spencer's method for stability analysis of reinforced embankments

Spencer's method reformulated and extended by Low and Tang (2001) will be used to calculate the factor of safety of reinforced embankments. The formulation in Low and Tang (2001) considers only circular slip surface for reinforced embankments, but the same procedure can be applied to the noncircular slip surface (Low 2003). The following derivations in this section are based on Low and Tang (2001) and Low (2003).

Figure 4.1 Forces acting on a typical slice of a reinforced embankment on soft ground

Figure 4.1 shows a reinforced embankment on soft ground and a typical slice with acting forces. According to Mohr-Coulomb failure criteria, the mobilized shear force T_i at the base of a slice i can be defined as:

$$T_i = [c'_i l + (P_i - u_i l_i) \tan \phi'_i] / F_s \quad (4-1)$$

where F_s is the factor of safety; c'_i and ϕ'_i are the effective shear strength parameters; u_i is the pore water pressure and l_i is the length of the base of a slice. For vertical equilibrium of the slice shown in Figure 4.1, the following equation should be satisfied:

$$P_i \cos \alpha_i = W_i - \lambda_i E_i + \lambda_{i-1} E_{i-1} - T_i \sin \alpha_i \quad (4-2)$$

Using both λ_i and λ_{i-1} is to allow for the fact that a slice adjacent to a water-filled tension crack would have $\lambda_0 = 0$, different from the λ_i values of the other vertical inter-slice surfaces. This can be expressed as:

$$\lambda_i = \lambda^* f(x) \quad (4-3)$$

where λ^* is the scaling factor and $f(x)$ the variation function of the direction of the inter-slice forces. If tension crack is not taken into account, λ_0 is then equal to $\lambda^* f(x)$. In this study, $f(x) = 1$ is used.

Horizontal equilibrium of the slice requires that:

$$E_i = E_{i-1} + P_i \sin \alpha_i - T_i \cos \alpha_i - dQ_i \quad (4-4)$$

where dQ_i is the differential horizontal reinforcing force acting on a slice. The following equation can be obtained by substituting Eqs. (4-1) and (4-4) into Eq. (4-2):

$$P_i = \frac{W_i - (\lambda_i - \lambda_{i-1})E_{i-1} + \lambda_i dQ_i - \frac{1}{F_s} (c'_i l_i - u_i l_i \tan \phi'_i)(\sin \alpha_i - \lambda_i \cos \alpha_i)}{\lambda_i \sin \alpha_i + \cos \alpha_i + \frac{1}{F_s} \tan \phi'_i (\sin \alpha_i - \lambda_i \cos \alpha_i)} \quad (4-5)$$

The overall horizontal force equilibrium is:

$$\sum (T_i \cos \alpha_i - P_i \sin \alpha_i) + \sum dQ_i - P_w = 0 \quad (4-6)$$

where P_w is the known horizontal water thrust force in a water-filled tension crack.

The overall moment equilibrium is:

$$\sum [(T_i \sin \alpha_i + P_i \cos \alpha_i - W_i) \times L_{x_i} + (T_i \cos \alpha_i - P_i \sin \alpha_i) \times L_{y_i}] + y_c \times \sum dQ_i - M_w = 0 \quad (4-7)$$

where

$$L_{x_i} = x_c - 0.5 \times (x_i + x_{i-1}) \quad (4-8)$$

$$L_{y_i} = y_c - 0.5 \times (y_i + y_{i-1}) \quad (4-9)$$

in which x_c and y_c define an arbitrary center and M_w is the overturning moment due to the water pressure in the tension crack.

It can be seen that F_s and λ^* are two unknowns contained implicitly in Eqs. (4-6) and (4-7). For a specified slip surface, whether circular or noncircular, the factor of safety can be calculated by solving these two equations simultaneously. Usually, an

iterative procedure is required for the solution since these two equations are nonlinear in nature. Spencer (1967) proposed to obtain the solution by computing the variation of the factor of safety with λ^* for the force equilibrium equation (F_f) and moment equilibrium equation (F_m) respectively and regarding the intersection point of the two corresponding curves as the final solution which satisfies both force and moment equilibrium. Low and Tang (2001) and Low (2003) used the Solver optimization tool inside Microsoft Excel spreadsheet to find the solution. In this study, MATLAB is used to code the stability analysis program of the extended Spencer's method described above. The function *fsolve* in the optimization toolbox is adopted to solve the two complicated nonlinear equations. The initial values of F_s and λ^* are selected to be 1 and 0, respectively. Usually, the final solution can be obtained after 4 or 5 iterations.

4.3 Locating critical circular slip surface for reinforced embankments on soft ground

Rotational failure is often observed in the field for reinforced embankments on soft ground and can be considered the most probable failure mechanism. The stability analysis using circular slip surface for reinforced embankments can be found in the works by Jewell (1982), Rowe and Soderman (1985), Hird (1986), Jewell (1988) and Low et al. (1990). In this section, a simple hypothetical reinforced embankment on soft ground will be considered to investigate the effects of optimization procedures on searching for critical circular slip surface.

Figure 4.2 shows the geometry of a reinforced embankment on soft ground. The reinforcement is placed at the interface between the embankment fill and the foundation clay. The mobilized force in the reinforcement is assumed to follow a half-sine distribution pattern with the maximum value of 80 kN/m at the center of the

embankment. A 1-m thick weak soil layer is assumed to be present in the middle of the soft foundation with the purpose to investigate the effectiveness of the optimization procedures. Three methods: the grid search method, the simplex method and genetic algorithm, will be used and compared with each other.

Figure 4.2 Cross section of a reinforced embankment on soft ground.

4.3.1 Grid search method

The grid search method is a widely used minimization technique for circular slip surface in slope stability analysis. Initially, a rectangular block is selected for the centers of the potential slip circles. This block is subdivided into a number of equal-sized elements and the factor of safety is evaluated at each of element corners with different values of radius. The searching grid for the centers of slip circles for the reinforced embankment shown in Figure 4.2 ranges from 8 m to 14 m in the x direction and 3 m to 8 m in the y direction with 30 intervals for each direction. The tangent level for the slip circles varies from 0 m to -5 m with 10 intervals in the foundation. The minimum factor of safety is calculated to be 1.180. Note that the minimum factor of safety from the grid search method does not depend on the initial

slip surface since this approach searches all the possible slip surfaces defined before the calculation.

4.3.2 The simplex method

The simplex method is a very efficient minimization method since no derivative calculation is required. A regular simplex in n dimensions is defined by $n+1$ mutually equidistant points. For circular slip surface, the simplex becomes a regular tetrahedron since only 3 variables are involved. The general search procedure is based on the observation that only one new point is needed to transform an existing simplex into a new one. In the initial simplex, the object function is evaluated at each of the $n+1$ vertices. The simplex is then reflected, expanded or contracted in order to achieve a minimum value.

The minimum factor of safety is 1.459 from the simplex method when the initial slip surface is selected to touch the bottom of the foundation clay. The location of the initial slip circle is defined by the x and y coordinates of the slip circle center and the radius, which are (11, 7) and 12 meters, respectively. As a result, the obtained critical circular slip surface also touches the bottom of the foundation clay.

When the initial slip surface is chosen to be tangential to the bottom of the weak layer within the foundation clay, the minimum factor of safety is 1.186 which is significantly lower than the previous one (1.459). This indicates that the successful location of the critical circular slip surface and the corresponding minimum factor of safety in the simplex method greatly depend on the initial trial slip surface.

4.3.3 Genetic algorithm method

Another alternative to the simplex method is the genetic algorithm method, which is also called GA method. The theoretical basis of GA method is the mechanics of natural selection and natural genetics. This method adopts the notion of evolution by combining the survival of the fittest with a structured and randomized routine. It is considered to be able to give global solution to the optimization problems. In this section, GA method is used to search for critical circular slip surface and the corresponding minimum factor of safety. The Genetic algorithm for optimization toolbox¹ for MATLAB (GAOT) by Houck et al. (1995) is used in this study.

Figure 4.3 Critical slip surfaces from different optimization procedures.

The selected trial slip circle is the one that touches the bottom of the foundation clay, which results in unsuccessful location of critical circular slip surface when the simplex method was used. The critical circular slip surface from GA method is found to reasonably pass through the weak soil layer within the foundation without touching the bottom of the foundation clay. The minimum factor of safety is 1.192, very close to the one obtained by the grid search method. Figure 4.3 shows the critical circular slip

¹ More details at <http://www.ie.ncsu.edu/mirage/GAToolBox/gaot/>

surfaces from different optimization procedures. For the simplex method, the results from two different initial slip surfaces are also plotted in the figure for comparison.

It can be seen that the GA method and the grid search method yield very similar critical circular slip surfaces and minimum factors of safety. For the simplex method, if an initial circular slip surface is wisely selected, the resulting minimum factor of safety is as good as that from the grid search method or the GA method.

GA method can reach the real minimum factor of safety using the initial slip circle touching the bottom of the foundation clay, for which the simplex method is trapped into a local minima. This demonstrates the powerful strength of GA method in locating the critical circular slip surface. However, there is still no evidence that GA can obtain better results than the grid search method from the current study. This finding is at variance with McCombie and Wilkinson (2002) which claimed that the grid search method was not as effective as GA method as the latter could locate more critical factor of safety.

For the simplex method, Nguyen (1985) pointed out that, the advantage of this method was that the number of evaluation of the object functions was greatly reduced when compared with the grid search method. However, the development of the computer technology enables the calculation of the factor of safety for a specified circular slip surface to be executed extremely fast, almost 0.001 second for one slip circle. Hence, this advantage of the simplex method is not important. Moreover, the dependency of the simplex method on the selection of the initial slip circle for this case indicates that this method may not be as robust and reliable as generally perceived.

Greco (2003), in his discussion to Malkawi et al. (2001a)'s paper where a searching procedure based on Monte Carlo technique was proposed to locate the critical circular

slip surface, conveyed his concern about the unjustified use of the optimization method for critical slip surface with a circular shape. According to him, there is no need to adopt a particular optimization procedure in searching for critical circular slip surface and the grid search method can handle this problem quite well. This point of view about the effectiveness of grid search method, although different from those from DeNatale (1992), Malkawi et al. (2001a) and McCombie and Wilkinson (2002) who considered the grid search method inefficient and time consuming, is consistent with the results of this section.

4.4 Locating critical noncircular slip surface for reinforced embankments on soft ground

Locating the noncircular critical surface and the corresponding minimum factor of safety is more difficult since more variables are involved in the analysis. In this section, two different optimization procedures will be adopted to locate the critical noncircular slip surface: Broydon-Fletcher-Goldfarb-Shanno (BFGS) method and pattern search method.

The BFGS method is a representative of the gradient-based optimization methods (Venkataraman 2002). This method adopts BFGS formula to update Hessian matrix in the optimization process which is an extension of Davidon-Fletcher-Powell (DFP) formula. Chen and Shao (1988) noted that the procedure in DFP formula to update the Hessian matrix was not efficient since the values of the Hessian matrix's elements were usually of the order of 0.01-0.00001 for the slope stability problems and the initial Hessian matrix during the optimization process was generally taken as the unit matrix. A procedure to reduce the values of Hessian matrix's elements was proposed by them to serve as the initial estimation in order to converge to the real solution efficiently. In this study, the BFGS method available in the optimization toolbox of

MATLAB is adopted to search for the critical noncircular slip surface.

Different from BFGS method, the pattern search method does not require any information about the gradient of the object function and can be regarded as the direct search method. In the pattern search method, a series of exploratory iterations around the initial point are carried out. Each iteration includes a *search* step and a local *poll* step. Both steps evaluate points on a carefully constructed mesh in an attempt to find an improved mesh point in which the value of the object function is minimum. A set of mesh points are evaluated in the *search* step. Common practices include random search, Latin hypercube search, orthogonal arrays or even genetic algorithm. If this step is unsuccessful in locating an improved mesh point, the *poll* step is invoked to evaluate the neighboring points to the current iteration. If both the search and poll steps cannot obtain the improved mesh point, the current mesh size will be reduced by a mesh size parameter. If an improved mesh point can be found in either step, the current mesh size will be increased by a mesh size parameter. Details about this approach can be found in numerous resources (Torczon 1997; Lewis and Torczon 2000; Audet and Dennis 2003). NOMADm toolbox² based on MATLAB (Abramson 2005) is used in this study to locate the critical noncircular slip surface based on the pattern search method.

4.4.1 Basic strategy to locate the critical noncircular slip surface

Figure 4.4 shows a potential noncircular slip surface which consists of $(n-1)$ straight lines with n vertices. The object function for the factor of safety, corresponding to a slip surface, can be defined mathematically by the following equation:

$$F_s = f(x_{\min}, x_{\max}, y_2, \dots, y_{n-1}) \quad (4-10)$$

² More details at <http://en.afil.edu/ENC/Faculty/MAbramson/NOMADm.html>

where $x_{\min}$ and $x_{\max}$ are minimum and maximum x coordinates of the vertices of the slip surface respectively; y_i is the y coordinate of the slip surface's i vertex and n is the number of the vertices of the slip surface.

Figure 4.4 A typical noncircular slip surface for embankment on soft ground.

The advantage of this definition is that, once $x_{\min}$ and $x_{\max}$ are determined, all other x coordinates of the slip surface's vertices are known since the sliding mass is equally divided into slices. As a result, only y coordinates of the $n - 2$ vertices (from vertices 2 to $n - 2$) need to be considered as the variables. Thus, simultaneous variation of x and y coordinates of the vertices in the minimization process, which usually result in unexpected difficulties, can be avoided. This is different from that used by Baker (1980) where only y coordinates of the vertices varied and the width of the slice was fixed. The entry point, which corresponds to the vertices with $x_{\min}$ and the exit point, corresponding to the vertices with $x_{\max}$, need to be interpolated in the analysis. Note also that, the width of the slice in the present analysis is not fixed and varied from time to time.

According to the above description, locating the critical noncircular slip surface is a

problem of minimizing the object function (Eq. (4-10)) of the factor of safety with respect to the array of $(x_{\min}, x_{\max}, y_2, \dots, y_{n-1})$. In order to keep the slip surface geometrically feasible, some constraints must be placed for the variables. For example, the y coordinate of the vertices cannot exceed the permissible maximum y coordinate or go below the permissible minimum y coordinate. This constraint restricts the potential noncircular slip surfaces to remain within the embankment and foundation. For the variables of $x_{\min}$ and $x_{\max}$, some restrictions can also be put on them depending on the problem at hand. For reinforced embankments on soft ground, $x_{\max}$ usually should be beyond the toe of the embankment and $x_{\min}$ should be behind the crest of the embankment.

The most important constraint, which needs to be considered in the analysis, is that the slip surface should be concave upward. This constraint can guarantee the admissibility of the slip surface and avoid any unexpected shape during the optimization process. Greco (1996) considered that such a constraint was not a priori justified and that the part of the slip surface could be convex due to some stratigraphic conditions. However, according to the writer's experience, noncircular slip surface with an irregular shape can often result in convergence problems for some rigorous stability analysis methods, such as Spencer's method. It is possible that a factor of safety cannot be found to satisfy both force and moment equilibriums at the same time. This actually renders the problem complicated and untraceable. A simple but effective constraint for requiring upward concavity of the slip surface is as follows:

$$y_{i+1} \leq 0.5(y_i + y_{i+2}) \quad (4-11)$$

where $i = 1, \dots, n - 2$. This simple equation is used in the present analysis.

4.4.2 Comparisons with a case from the literature

A hypothetical reinforced embankment on soft ground (Figure 4.5) from the literature will be used to validate the procedure described in the previous section for locating the critical noncircular slip surface. The result will be compared with that reported by other researchers. BFGS method is used to search for the critical noncircular slip surface.

This hypothetical reinforced embankment was analyzed by Tandjiria et al. (2002) which extended Chen and Li's method. When the reinforcement force follows the linear-type distribution pattern with the maximum value of 52 kN/m at the center of the embankment, the factor of safety is 1.345 for the critical noncircular slip surface, which agrees well with that (1.350) from Tandjiria et al. (2002). The obtained critical noncircular slip surface is also shown in Figure 4.5. Results from this example demonstrate that the procedure for locating the critical noncircular slip surface adopted in this study works effectively for reinforced embankment on soft ground.

Figure 4.5 The critical noncircular slip surface for a reinforced embankment on soft ground (Fs=1.345)

4.4.3 Searching for the critical noncircular slip surface of a reinforced embankment on homogeneous soft foundation

To demonstrate the effects of different optimization procedures on the results of stability analysis for reinforced embankments on soft ground with noncircular slip surfaces, two hypothetical reinforced embankments are analyzed in this section. The first one is a reinforced embankment on homogeneous soft clay foundation. The geometry and the soil parameters are the same as the case used in the previous section except that the weak soil layer in the mid-depth of the foundation is not present.

The reinforced embankment with a weak layer in the mid-depth of the foundation is taken as the second case. For each optimization approach, BFGS or pattern search method, two different initial slip surfaces will be adopted to investigate the robustness and reliability of the optimization procedure in locating the critical noncircular slip surface.

Figure 4.6 Initial slip surfaces for locating the critical noncircular slip surfaces of a homogeneous reinforced embankment on soft ground

Figure 4.6 shows two different initial slip surfaces used in the analysis to search for the critical noncircular slip surface of a reinforced embankment on homogeneous soft foundation. One circular slip surface is selected to be tangential to the bottom of the foundation. The other one is much shallower.

The results by BFGS and pattern search methods for the homogeneous reinforced embankment are shown in Figure 4.7. The factors of safety are 1.462 and 1.463 by BFGS method from two different trial initial slip surfaces, respectively. The slip surfaces with solid symbols were located starting from the initial slip surface touching the foundation bottom. Those with hollow symbols were obtained from the shallower initial slip surface. The critical noncircular slip surfaces obtained by BFGS method (Figure 4.7) are also very close to each other except for the small portion within the embankment fill. Considering the fact that the searching process is initiated from two different locations, the application of BFGS method in locating the critical noncircular slip surface and the corresponding minimum factor of safety can be regarded as satisfactory for this reinforced embankment on homogeneous foundation.

Figure 4.7 Critical noncircular slip surfaces from BFGS and pattern search methods for a homogeneous reinforced embankment on soft ground.

For the cases where pattern search method was used, the factors of safety are 1.459 and 1.447, depending on the different initial trial slip surfaces. The factors of safety are slightly smaller than those from BFGS method. Although the difference is only about 0.2% and not significant in practical terms, it is still interesting to note that pattern search method actually gives a more critical factor of safety. This is consistent with the results of analysis in Greco (1988) and Greco (1996).

The critical noncircular slip surfaces obtained by the pattern search method from two different initial slip surfaces are also similar to each other, as shown in Figure 4.7. The convergence of the optimization procedure from two different starting points to the almost identical final location means that the pattern search method is a robust and effective approach to search for the critical slip surface. It should be pointed out that, some difference still exists in the critical noncircular slip surfaces obtained by BFGS method and the pattern search method, especially for the portion of the slip surfaces within the embankment fill.

The critical circular slip surface from grid search method is also plotted for comparison. The corresponding factor of safety is calculated to be 1.606. This indicates that the critical noncircular slip surface and its associated minimum factor of safety can be greatly different from the critical circular slip surface. Using noncircular slip surfaces to analyze the stability of reinforced embankments, Sabhahit et al. (1994) and Tandjiria et al. (2002) pointed out that using only circular slip surface can result in overestimating embankment stability and therefore yield unconservative safety evaluation.

4.4.4 Searching for critical noncircular slip surface for a reinforced embankment on non-homogeneous soft foundation

When a thin weak soil layer is assumed to be present in the mid-depth of the soft soil foundation (Figure 4.2), searching for critical noncircular slip surface becomes more complicated. Figure 4.8 shows the two initial slip surfaces adopted for locating critical noncircular slip surface for a reinforced embankment on non-homogeneous foundation.

One initial slip surface is assumed to touch the bottom of the soil foundation and the other one is selected in such a way that it passes through the weak layer. The purpose of choosing these two different initial slip surfaces is to investigate the robustness and reliability of optimization procedures in locating critical noncircular slip surface for the reinforced embankment on a layered soft soil foundation.

Figure 4.8 Initial slip surfaces for locating critical noncircular slip surfaces of a reinforced embankment on soft ground with a weak layer.

Figure 4.9 shows the critical noncircular slip surfaces and the corresponding minimum factors of safety from BFGS method and pattern search method for different initial slip surfaces. The slip surfaces with solid markers were found starting from the initial slip surface touching the foundation bottom and those with open markers were found from initial slip surfaces passing through the weak soil layer.

It can be seen clearly that, the solutions are highly dependent on the selection of the initial slip surfaces. Optimization procedure, starting from different initial slip surfaces, results in different critical noncircular slip surfaces. Due to the presence of the weak soil layer within the soft soil foundation, the actual critical slip surface should pass through this layer. However, according to the present analyses, both BFGS and pattern search method cannot correctly locate that critical slip surface if the initial slip surfaces were assumed to touch the bottom of the foundation. Both optimization procedures end with noncircular slip surfaces still touching the foundation bottom, as shown in Figure 4.9.

Figure 4.9 Critical noncircular slip surfaces from BFGS and pattern search method for a reinforced embankment on soft ground with a weak layer.

When the initial slip surface is selected to pass through the weak soil layer in the mid-depth of the soft soil foundation, both BFGS and pattern search method locate critical noncircular slip surfaces touching the bottom of the weak soil layer and give more critical values of the factors of safety. For BFGS method, it can be seen that the factor of safety corresponding to the “correct” critical noncircular slip surface is significantly smaller and shallower than that for the “wrong” critical noncircular slip surface. Greco (1996) pointed out that gradient-based optimization method was unreliable and could not find the real minimum as the calculation of the derivatives of the safety factors was required. This was consistent with the results of analysis from BFGS method for the reinforced embankment on the soft foundation with a weak soil layer. Under this circumstance, BFGS method cannot be considered to be a robust and reliable approach to search for critical noncircular slip surface as the solution highly depends on the selection of the trial slip surface. It can be concluded that BFGS method only works effectively and properly for cases where the reinforced embankment sits on homogeneous or smoothly varying soft foundation with no abrupt weak layer.

For pattern search method, the finding from the present studies seems to contradict those in Greco (1996) which claimed that the solution from pattern search method did not depend on the initial trial slip surface. In spite of the fact that pattern search method searches for critical noncircular slip surface more effectively and gives more critical factor of safety, the solution still strongly depends on the initial slip surface used in the optimization procedure. In this sense, pattern search method cannot be regarded as superior to BFGS method as the execution of BFGS method is much faster.

4.4.5 Interpolation of soil strength parameters for slices in the stability analysis of reinforced embankment on soft ground

Usually, soil strength parameters for each slice in the stability analysis are interpolated based on the location of the mid-point of each slice base. This procedure works well when soil parameters are continuous. However, when a thin-weak layer is present in the soft foundation, the interpolation procedure based on the midpoint of each slice base could lead to erroneous results.

Figure 4.10 Critical noncircular slip surfaces if the interpolation procedure is only based on the location of the midpoint of each slice base.

Figure 4.10 shows the critical noncircular slip surfaces and the corresponding factors of safety from BFGS method and pattern search method for the reinforced embankment on soft ground with a weak layer if the interpolation procedure is only based on the location of the mid-point of each slice base.

It is obvious that the factor of safety is greatly underestimated. This occurs because the midpoints of two slice bases are just within the weak layer when the slice bases are across the strong and weak soil layers. Since the undrained shear strength of the weak

layer, which is significantly lower than the clay above and below it, is assigned to the slices, the calculated factor of safety is much smaller. A good way to deal with this situation is to use strength parameters weighted for each slice base.

Figure 4.11 A schematic plot of a slice base across the strong and weak soil layers

4.4.6 Successfully locating the critical noncircular slip surface

Searching for the critical noncircular slip surface is not a simple task. The searching process can always get trapped in a local minimum for the embankment with complex soil conditions. This is because the object function of the factor of safety is non-smooth. The additional constraint conditions, which are usually required to ensure the kinematical admissibility of the noncircular slip surface, further complicate the searching process.

According to the writer's experience, it is not practical to try proposing a method to search for the global minimum factor of safety associated with the critical noncircular slip surface since multiple local minimums may exist. Because acceptable search directions for a global minimum can actually decrease or increase the factor of safety, a complicated and heuristic determination of the search direction should be adopted as part of the searching technique. This means that temporary increase in the factor of

safety should be allowed in the searching process. As a result, the solution can escape the snare of the local minimums. It should be pointed out that, the current existing approaches for locating critical noncircular slip surface, whether gradient-based or non gradient-based, do not allow the factor of safety to be increased during the searching process and therefore, cannot avoid the possibility of being trapped in local minimums. However, an approach that allows transient increase of the factor of safety is likely to involve a large number of iterations. Such an approach will be very time-consuming in practice.

A pragmatic approach to search for the critical noncircular slip surface is to initiate the optimization procedure from the critical circular slip surface. In many cases, the location of the critical circular slip surface is not very different from that of the critical noncircular slip surface in spite of their different shapes. As the grid search method can locate the critical circular slip surface accurately and reliably irrespective of the soil conditions, a rough location of the critical noncircular slip surface can be determined by performing a circular slip surface analysis first. Then, this critical circular slip surface can serve as the initial trial slip surface in searching for the critical noncircular slip surface. This can make the starting point as close as possible to the real critical and thus avoid getting trapped in the local minimum. However, where local minimums are close to the real solution, this procedure may not work well.

4.5 Summary and conclusions

The effects of optimization procedures on the critical circular and noncircular slip surfaces for reinforced embankments on soft ground are investigated in this chapter. For circular slip surface analysis, three different optimization approaches are adopted to search for the critical slip surface. These methods include the grid search method, the simplex method and genetic algorithm. The minimum factors of safety and the

corresponding critical circular slip surfaces obtained from these methods are compared with each other. It is found that the grid search method is the most robust, efficient and reliable approach to locate the critical circular slip surface. The results of stability analysis indicate that solutions from the simplex method depend on the selection of initial trial slip surface. For genetic algorithm, despite the fact that this method tends to give global solution to the stability problems and does not rely on the starting location of the slip surface, there is no evidence that the results from this method are better than those from the grid search method.

For noncircular slip surface analysis, two hypothetical reinforced embankments are used to investigate the effects of optimization procedures on the critical noncircular slip surface and the corresponding minimum factor of safety. Two different optimization methods are employed to locate critical noncircular slip surface: BFGS method and pattern search method. For the homogeneous reinforced embankment examined, both BFGS and pattern search methods give similar and reliable predictions of the location of the critical noncircular slip surface and the minimum factor of safety. The solutions do not depend on the selection of the initial trial slip surface. Consequently, both optimization procedures can be regarded as robust and reliable for the homogeneous reinforced embankment on soft ground.

However, if a weak soil layer is present in the mid-depth of the soil foundation, the solutions from BFGS and pattern search methods are found to depend highly on the initial trial slip surface. Starting from different initial slip surfaces, both procedures can lead to different locations of critical noncircular slip surfaces and the corresponding factors of safety. The stability analysis results also indicate that unsuccessful identification of the critical noncircular slip surface can result in a significant overestimation of the factor of safety and renders the design unconservative.

CHAPTER 5 UNCERTAINTIES ASSOCIATED WITH CRITICAL REINFORCEMENT STRAIN

5.1 Introduction

The main difference between unreinforced and reinforced embankments is the presence of a geosynthetic reinforcement in the latter. Reliability analyses of reinforced embankments need to consider the uncertainties associated with the mobilized force in the reinforcement. The mobilized reinforcement force depends on two factors: mobilized reinforcement strain and reinforcement modulus. The uncertainties of reinforcement modulus result mainly from material variability during manufacture and are relatively easy to quantify.

According to Hodge (1992) and Richardson and Bove (1992), a coefficient of variation of 2~7% seems reasonable for the variability of reinforcement modulus. The more difficult problem is to determine the variability of the mobilized strain in the reinforcement because the mobilized reinforcement strain can only be obtained by field measurement or finite element analysis. Both approaches are very time consuming and expensive. Investigations of the variability of the mobilized reinforcement strain usually require substantial data.

Review of the relevant literature indicates that there have been no field tests or finite element analyses conducted to consider the variability of reinforcement force. In a probabilistic stability analysis of reinforced slopes, Cheng and Christopher (1991) used the tensile strength of geotextile as the reinforcement force and adopted a coefficient of variation of 10% for the tensile strength. Kitch (1994) assumed a coefficient of variation of 25% for the serviceability strain of the reinforcement based on the upper and lower limits of published values. Low and Tang (1997b) adopted a coefficient of

variation of 10% for the mobilized reinforcement force in the reliability analysis for the reinforced Muar embankment. It can be seen that there has been no systematic attempt to determine the variability of the reinforcement force.

In this chapter, the mobilized reinforcement strain used to determine the mobilized reinforcement force and to calculate the factor of safety of reinforced embankment in limit equilibrium analysis is defined as the critical mobilized strain which is the upper bound of the “working” mobilized reinforcement strain. The reason to adopt such a concept for mobilized reinforcement strain is discussed. The variability of this mobilized reinforcement strain is then investigated through response surface method based on Monte Carlo simulation.

5.2 Critical reinforcement strain used in the stability analysis of reinforced embankments on soft ground

Stability evaluation of reinforced embankments requires appropriate determination of reinforcement force. However, there is much debate about the selection of the magnitude of the reinforcement force (Schaefer 1987; Rowe and Mylleville 1994). Generally speaking, three ways can be adopted to select the reinforcement force in the stability analysis: (1) ultimate tensile strength of reinforcement; (2) the strength of the reinforcement-soil bond at the interface and (3) mobilized reinforcement force corresponding to a certain critical strain value.

The first approach assumes that all the materials including soils and reinforcement are considered to be perfectly rigid and fail simultaneously at their maximum strength. Palmira et al. (1998) adopted this approach to back analyze reinforced embankments on soft soils. This procedure, although simple and easy to implement, has the limitation that the factor of safety will be overestimated because reinforcement

strength may not be necessarily mobilized simultaneously with the soil strength. This is very common for reinforced embankments on soft ground where plastic zone within the soft soil foundation could be fully developed before the reinforcement force reaches its tensile strength.

The second approach considers that the reinforcement can fail in a pullout mode due to insufficient anchorage or a direct shear mode due to sliding of the fill along the soil-reinforcement interface. Borges and Cardoso (2002) studied the overall stability of reinforced embankments on soft soils by considering mainly the pullout force which can be provided by the soil reinforcement interface.

Jewell (1982) noted reinforced embankments could fail due to excessive displacements in the soft soil foundation prior to catastrophic collapse, and suggested that the reinforcement strain should be limited to a value under conditions of strain compatibility between the soil and reinforcement, but did not elaborate on how to determine such a reinforcement strain. Rowe and Soderman (1985) proposed an allowable compatible strain to be used with limit equilibrium method to calculate the factor of safety of reinforced embankments on homogeneous soft soil foundation. According to them, the allowable compatible strain is the maximum horizontal strain in the interface between the embankment fill and the clay foundation developed in the corresponding unreinforced embankment at failure. This work was further extended by Hinchberger and Rowe (2003) to the condition where the undrained shear strength increases with depth.

Bonaparte and Berg (1987) noted the difference between serviceability and ultimate limits in the allowable reinforcement forces by considering the fact that, when a slope reaches its serviceability limit, excessive displacements may occur although ultimate failure is not imminent. This concept of serviceability limit of reinforcement force used

in the limit equilibrium method is actually similar to the “allowable compatible strain” proposed by Rowe and Soderman (1985) since both consider that reinforced slopes or embankments can be deemed to have failed when excessive deformations occur. Based on their experience and judgment, Bonaparte and Berg (1987) considered the serviceability reinforcement force to be a serviceability strain of 5% without supporting evidence.

An ideal and efficient stability analysis of reinforced embankments should be able to take into account all possible failure mechanisms, such as rupture of the reinforcement, pullout or sliding of the reinforcement, failure due to the excessive deformations or even bearing capacity failure of the foundation soil. In this thesis, since the main concern is about reinforced embankments on soft ground, excessive displacement within the soft ground is considered to be a dominant factor which influences the stability. The assumption is that the performance of reinforced embankments may become unacceptable due to excessive deformations while the tensile force mobilized in the reinforcement is still far below the ultimate tensile strength. For example, if the value of 5% suggested by Bonaparte and Berg (1987) is adopted for the critical reinforcement strain corresponding to excessive deformations of reinforced embankments, then the mobilized reinforcement force will be equal to 200 kN/m for a very high stiffness of 4000 kN/m. This magnitude of reinforcement force is in fact not very large compared with the tensile strength of most reinforcement materials with relatively high stiffness. This does indicate the importance of accounting for failure due to excessive deformation of reinforced embankments in the foundation on soft ground.

If the critical reinforcement strain is used in the stability analysis of reinforced embankments on soft ground to calculate the factor of safety, the computed factor of safety will be associated with the soil strength parameters, not the ultimate tensile

strength of the reinforcement or soil-reinforcement interface capacity. It is considered that this approach is more suitable for stability analysis of existing reinforced embankments on soft ground since the variety in the reinforcement is quite large and it is relatively easy to select appropriate reinforcement. However, the improvement treatment for the soft soil within the foundation is nontrivial and the costs can be very high.

As described above, in their pioneering work, Rowe and Soderman (1985) proposed a procedure to determine the allowable compatible strain in the reinforcement and applied that in the calculation of the factor of safety. The relevant steps included the determination of the factor of safety for a similar unreinforced embankment with the same geometry and ground condition first, and then obtaining the critical height of this unreinforced embankment by multiplying the given height of the unreinforced embankment by the obtained factor of safety. With the critical height and other geometry and soil strength parameters, the allowable compatible strain in the reinforcement can be obtained from a chart which is based on finite element analyses. Finally, the allowable compatible strain in the reinforcement is used to calculate the factor of safety for the reinforced embankment.

However, as recognized by Rowe and Soderman (1985), it was not strictly valid to approximate the critical height of the corresponding unreinforced embankment by multiplying the original height by the factor of safety due to the nonlinearity between the embankment height and the factor of safety. Another limitation is that, a separate stability analysis should be carried out in advance for the corresponding unreinforced embankment in order to get the critical height of the unreinforced embankment which will be used together with other strength and geometry parameters to determine the allowable compatible reinforcement strain according to the chart.

An extensive review of the literature conducted so far about the stability analysis of reinforced embankments indicated that this approach had not been extensively used in practice since it was proposed in 1985. One possible reason may be the difficulties in determining the allowable compatible strain in the reinforcement for reinforced embankments with complicated soil conditions since the chart provided by Rowe and Soderman (1985) is only suitable for reinforced embankments on homogeneous clay foundation. Moreover, the fact that the effect of the reinforcement force on the factor of safety is not very significant could also, to some extent, discourage the extensive and in-depth investigation of the allowable compatible strain in the reinforcement.

In this chapter, the mobilized reinforcement strain corresponding to excessive deformations and unacceptable performance of reinforced embankments is defined as critical reinforcement strain. An alternative procedure is proposed to obtain the critical reinforcement strain for a given reinforced embankment. The basic assumption is that, the critical reinforcement strain contributing maximum allowable tensile force to the stability of the reinforced embankment on soft ground corresponds to the one where strength parameters of the reinforced embankment is reduced in such a manner that excessive deformations just occur and consequently unacceptable performance is reached. This calculated critical strain is used to determine the potential reinforcement force, which can then be used to compute the factor of safety of the reinforced embankment. It should be noted that the definition of critical reinforcement strain adopted herein is not different from the serviceability reinforcement strain proposed by Bonaparte and Berg (1987) and the allowable compatible reinforcement strain proposed by Rowe and Soderman (1985) in spite of the different procedure used to determine this critical strain.

The critical reinforcement strain is more suitable than the “working” mobilized reinforcement strain in the determination of the factor of safety for reinforced

embankments on soft ground. Using mobilized reinforcement strain from either finite element analysis or field measurements under the working condition in the stability analysis could underestimate the safety of reinforced embankments. It is especially true for a reinforced embankment which is not close to its failure state, i.e. at stable working condition, since at this time, the reinforced embankment can exhibit the same behaviour as the unreinforced one and the enhanced safety may not be noticeable.

A good example which can clearly convey this concern about the use of mobilized reinforcement force under working condition in determining the factor of safety is a FHWA geogrid reinforced test slope E1 (Kitch 1994). By using measured mobilized strain in the reinforcement (around 0.45%) in the stability analysis for this reinforced slope, Kitch (1994) found that the factor of safety calculated was below one. However, for this reinforced slope, neither failure nor any sign of distress was observed in the field. Kitch (1994) ascribed this discrepancy to the inaccurate measurement of the shear strength of the soils and the mobilized reinforcement force in the field. However, in the writer's opinion, this discrepancy could also partly result from the underestimation of the resisting force which can be provided by the reinforcement. If a serviceability reinforcement strain with 5% was used in the stability analysis instead of the measured reinforcement strain with only 0.45%, the factor of safety could be higher.

5.3 Determination of critical reinforcement strain using finite element method

As discussed above, reinforced embankments on soft ground can be regarded as unstable when excessive deformations occur. At that time, the net height of the reinforced embankment could not be increased any more. Any additional loading will only deteriorate the embankment although catastrophic failure may not happen. The critical strain in the reinforcement is the maximum strain mobilized in the

reinforcement under this situation. The magnitude of this critical strain in the reinforcement can be obtained by in-situ measurement or numerical analysis. In this section, the finite element program PLAXIS is used to calculate the critical strain in the reinforcement.

Figure 5.1 A reinforced embankment on soft foundation

The critical strain should represent the maximum horizontal strain which could be mobilized in the soil-reinforcement interface. This critical strain corresponds to the condition where the undrained shear strength within the soft foundation is fully mobilized. Therefore it is more suitable to use the reinforcement force based on this strain in limit equilibrium stability analysis of reinforced embankments on soft ground.

To illustrate the way of determining critical reinforcement strain, numerical analyses will be presented in this section for a hypothetical reinforced embankment on soft ground. Figure 5.1 shows the geometry of the reinforced embankment on soft ground. The soil parameters used in the finite element analysis are listed in Tables 5-1 and 5-2, respectively.

Table 5-1 Embankment fill properties for finite element study

Parameter	Value
c'	0
ϕ'	37°
ψ	0
ν'	0.3
γ	19 kN/m^3
E'	15000 kPa

Table 5-2 Foundation soil properties for finite element study

Parameter	Value
c_u	20 kPa
ϕ_u	0
ν_u	0.49
E_u / c_u	200 kPa
γ	15 kN/m^3

The elastic-perfectly plastic Mohr-Coulomb model is used to model the behaviour of soils in the embankment fill and the foundation clay. The stiffness of the reinforcement is selected to be 3000 kN/m. According to the analysis, this reinforced embankment is very stable and the maximum mobilized strain in the reinforcement is calculated to be around 0.7% under working condition. As discussed before, this mobilized reinforcement strain cannot represent the actual potential resisting force which can be provided by the reinforcement since higher reinforcement force can be mobilized when the strength parameters of soils are reduced or further external loadings are applied.

Figure 5.2 Fill thickness versus net height of the reinforced embankment when the strength parameters are reduced by a factor of 1.3

For example, due to the rainfall or the changes in the ground water level, pore water pressure within the embankment can increase and, as a result, the strength of soils can decrease. Although this may seem to reduce the safety degree of the reinforced embankment, the reinforcement force can be mobilized further. In order to investigate the maximum resisting force which can be provided by the reinforcement, it is therefore reasonable to reduce the strength parameters of soils to such an extent that excessive deformations just occur and a continuous plastic zone is formed within the soft soil foundation. The mobilized strain in the reinforcement under such a circumstance should, then, represent the maximum resisting capacity of the reinforcement. If such a strain is exceeded in the reinforcement, the reinforced embankment should be considered unacceptable due to the excessive deformations.

For the reinforced embankment shown in Figure 5.1, it is found that, the strength parameters of soils could be reduced by a factor of 1.3 to just bring this embankment to the failure state, i.e. excessive deformations occur. The variation of net height with

the fill thickness is plotted in Figure 5.2 as the strength parameters of soils are reduced by a factor of 1.3. It can be seen that the maximum net height of this reinforced embankment is reached at the fill thickness of 4.3 m. A decreasing trend for the net height of this embankment is very obvious at this fill thickness. Figure 5.3 shows the plastic zone developed within the soil foundation when the maximum net height is reached. A very clear continuous plastic zone can be identified. This indicates that the undrained shear strength of foundation soils is already fully mobilized at this state.

Figure 5.3 Plastic zone developed in the reinforced embankment when the strength parameters were reduced by a factor of 1.3

The corresponding maximum reinforcement strain, which is defined as the critical reinforcement strain in this work, is found to be 2%. It is much higher than that (0.7%) obtained under the working condition. This indicates that, the factor of safety for this reinforced embankment could be underestimated if mobilized reinforcement force under the working condition is used in the limit equilibrium stability analysis.

However, it is not easy to determine such a critical strain in the reinforcement for the reinforced embankment on soft ground. The selection of the reduction factor should be very careful. Therefore, it is very tedious to bring the reinforced embankment exactly to the failure state. Although strength reduction technique, which is usually adopted in the finite element analysis to calculate the factor of safety, can be used to gradually reduce the strength parameters in the analysis, this technique is not suitable for the

reinforced embankments on soft ground due to the nonconvergence criterion adopted to judge the failure as discussed in Chapter 3. It is not easy to locate the point where the plastic zones within the foundation just becomes continuous. Considering that critical reinforcement strain corresponds to the full mobilization of undrained shear strength of foundation soils, it is speculated that the maximum horizontal strain mobilized in the interface of the embankment fill and the clay foundation could be close to critical reinforcement strain if the strength parameters of a similar unreinforced embankment with the same foundation condition are reduced in such an extent that failure of the embankment just occurs. It has been demonstrated that strength reduction technique works well for unreinforced embankments on soft ground. This technique can be employed to locate the failure state reliably for unreinforced embankments on soft ground.

Finite element analysis is carried out for the unreinforced embankment which is the same as the reinforced one above except the reinforcement is not present. The factor of safety for this embankment is first determined by strength reduction technique, which is 1.268. This means that the embankment can be brought into failure by applying a reduction factor of 1.268 to the strength parameters. The maximum horizontal strain in the interface between the embankment fill and the foundation clay can be easily calculated with the reduced strength parameters. For this case, the maximum horizontal strain mobilized in the interface between the embankment fill and the foundation clay is calculated to be 2.2% which is very close to the critical reinforcement strain (2.0%) obtained above. According to these results, it can be seen that the critical reinforcement strain actually can be reasonably calculated by performing finite element analysis for a similar unreinforced embankment with reduced strength parameters. The maximum horizontal strain mobilized in the interface between the embankment fill and the foundation clay can be considered close to the magnitude of the critical reinforcement strain for the corresponding reinforced

embankment.

For the case where the unreinforced embankment has a factor of safety less than 1.0, the corresponding critical strain in the reinforcement can be directly regarded as the maximum horizontal strain in the interface between the embankment fill and the foundation clay at collapse.

5.4 Uncertainties of critical reinforcement strain for reinforced embankments on soft ground

From the above analyses, it has been shown that the critical strain in the reinforcement as defined in the foregoing section can be calculated by performing finite element analysis for a similar unreinforced embankment and, therefore, does not depend on the reinforcement stiffness. This desirable feature makes the investigation of uncertainties of critical reinforcement strain, which will be used in the limit equilibrium analysis to calculate the factor of safety for the reinforced embankment, less complicated since there is no correlation between the critical reinforcement strain and the reinforcement stiffness.

As noted from the previous analysis, the obtained critical reinforcement strain 2.0% is within the range of 1.0%-9.0% suggested by Rowe and Soderman (1985) for the allowable compatible reinforcement strain. It is also consistent with the suggestion of Christopher et al. (1990) for the serviceability reinforcement strain ($<5.0\%$). Table 5-3 summarizes some values for the critical reinforcement strain (serviceability reinforcement strain or allowable compatible reinforcement strain) found in the literature. It can be seen from Table 5-3 that the critical reinforcement strain ranges from 1% to 9%. Assuming this range represents the mean value plus and minus three standard deviations according to the well known three-sigma rule, the corresponding

coefficient of variation of the critical reinforcement strain will be about 0.27.

Table 5-3 Values of the critical reinforcement strain found in the literature (extended based on Kitch 1994)

Researchers	Strain	Notes
McGown et al. (1984)	6%	Serviceability reinforcement strain
Rowe and Soderman (1985)	1%-9%	Allowable compatible reinforcement strain
Jones (1986)	2%-6%	Serviceability reinforcement strain
Bonaparte and Berg (1987)	5%	Serviceability reinforcement strain
Christopher et al. (1990)	<5%	Serviceability reinforcement strain

This way to determine the uncertainties of the critical reinforcement strain as used by Kitch (1994) is, however, very crude. A more reasonable approach is to carry out Monte Carlo simulation based on finite element method for a number of sets of input parameters and to see how the variability of soil parameters will influence the magnitude of the critical reinforcement strain. However, it is usually very time consuming and impractical to run the finite element analysis for many times. In view of this difficulty, a practical procedure based on response surface approach is proposed to approximate the critical strain in the reinforcement calculated from finite element analysis for the reinforced embankment on soft ground. In this procedure, a response surface function with explicit polynomial form is to be constructed to represent the relationship between the critical reinforcement strain and input parameters. Once such an explicit response surface is available, the uncertainties associated with the critical reinforcement strain can be easily determined by direct Monte Carlo simulations.

5.4.1 Response surface method

Response surface method is a statistical tool proposed by Box and Wilson (1954) to find the operating conditions of a chemical process at which some response was optimized. The basic concept of this method consists in approximating the unknown implicit limit state function by using a simple and explicit function which is chosen appropriately. The construction of a response surface function is based on the experiments or numerical analyses performed at selected sampling points. After that, a regression analysis can be carried out to determine the coefficients of the response surface function in an explicit polynomial form. Upon obtaining a proper response surface function for the problem considered, the computationally expensive numerical analysis can be replaced by a simple and inexpensive model. The response surface is then used to predict system response over the parameter space at reduced cost, say for uncertainty analysis where the response surface may be evaluated hundreds or thousands of times (Romero et al. 2004).

A very important issue regarding the usage of response surface method is the accuracy of the obtained response surface function. The accuracy of the response surface can only be guaranteed around the range where the sampling points are selected. Beyond that range, the obtained response surface may not be able to predict the results accurately, and sometimes the errors can be very significant. By recognizing this dilemma, the response surface is usually formed near the design point through some iterative procedures (Bucher and Bourgund 1990; Kim and Na 1996). In this study, as the main intention is to construct a response surface to predict the critical strain in the reinforcement in lieu of expensive numerical analysis, the accuracy of the response surface is not restricted to the range just around the design point. An ideal response surface should predict the result correctly and efficiently for any sampling point. This is different from the reliability analysis based on response surface method where the

errors of predicting the response in the region far from the design point would not influence the determination of the reliability index of the system.

Constructing a suitable response surface to predict the response in a wide range is more difficult than locating the design point and calculating the reliability index. It is well known that the response surface works very well around the region of the sampling points. Therefore, a sequential procedure is adopted to construct the response surfaces for estimating the critical strain in the reinforcement. It is not practical to use one single response surface to predict all the results for the input variables in a wide range.

5.4.1.1 Selection of the polynomial response surface function

It is crucial to select a suitable polynomial response surface function. Usually, a complete second-order quadratic polynomial function is adopted since a possible calculation of curvature is available. It can also avoid potential oscillations of higher order polynomials. For r input variables, the approximation $G'(\mathbf{x})$ of the actual unknown function $G(\mathbf{x})$ can be expressed as

$$G'(\mathbf{x}) = l + \sum_{i=1}^r m_i x_i + \sum_{i=1}^r \sum_{j=1}^r n_{ij} x_{ij} \quad (5-1)$$

in which x_i , $i = 1, \dots, r$ are input variables and the parameters l , m_i , n_{ij} are the coefficients to be determined. From the values at sampling points selected for each variable, the value of the function at any point in the design space can be estimated by fitting a polynomial of order two. At least $(r+1)(r+2)/2$ sampling points are required to form the response surface function for a problem involving r variables. In practice, a larger number k of points is taken and the coefficients are determined

by minimizing the least squares $\sum_{l=1}^k \left| (G'(\mathbf{x}))^l - (G(\mathbf{x}))^l \right|^2$. However, it is important to limit the number of the points in a reasonable range so that the procedure is practical. This is because the computational efforts will dramatically increase with the number of sampling points, especially for nonlinear finite element analyses.

It should also be mentioned that although a higher order polynomial can improve the accuracy of the approximation at the expense of additional computation, the equations would become ill-conditioned if the approximate response surface function $G'(\mathbf{x})$ is of much higher degree than the actual performance function $G(\mathbf{x})$. Moreover, higher order polynomial functions can exhibit erratic behaviour in the sub-domains which are not covered by the experiments or numerical analyses. Therefore, it is not recommended to use polynomial function of more than two orders to approximate the performance function.

5.4.1.2 Procedures for sequential construction of the response surface

It is assumed that the critical strain in the reinforcement ε_{crit} can be expressed by the function $g(x)$. The input variables $x_i (i = 1, \dots, r)$ are the major influence factors for mobilized reinforcement strain in finite element analysis, such as undrained shear strength, undrained stiffness of soft soils in the foundation and even unit weight, friction angle and cohesion of the embankment fill. This relationship can be described by the following equation:

$$\varepsilon_{crit} = g(x_i) \quad (5-2)$$

where $lb_i \leq x_i \leq ub_i$; lb_i and ub_i are respectively lower bounds and upper bounds

for the input parameters $x_i (i = 1, \dots, r)$. It is assumed that, for each input variable, h sampling points will be selected in the range of this variable. The magnitude of h usually depends on the effect of this variable on the critical strain in the reinforcement. If small variation in this variable will result in significant change in the magnitude of critical reinforcement strain, then the number of sampling points should be quite large. If the critical strain in the reinforcement is not very sensitive to a variable, then the number of the sampling points can be small.

A proper selection of the number of the sampling points in the range considered is very important since a large number of sampling points means that more computational efforts will be needed, while a small number of sampling points may lead to an ineffective response surface. For each input variable, the number of the sampling points can be described as $h_i (i = 1, \dots, r)$. Therefore, the total number of the input combinations is $\prod_{i=1}^r h_i$ (r is the number of the input variables). Finite element analysis will then be conducted for each input combination to determine the corresponding critical reinforcement strain. Finally, a database containing combinations of input variables and the corresponding values of critical reinforcement strain will be formed. The main function of this database is to serve as a base to construct the response surfaces.

The main steps in the proposed response surface method to predict the critical reinforcement strain can be summarized as follows:

1. For any combination of the input variables, the sampling points are selected around this input combination from the database formed in the previous finite element analyses.

2. With the chosen sampling points and the corresponding values of the critical reinforcement strain, a response surface can be constructed easily based on regression analysis.
3. The critical reinforcement strain can then be determined based on the obtained response surface and the input combination of the variables.
4. Steps 1, 2 and 3 can be repeated in the Monte Carlo simulation for a number of of input variable combinations to predict the corresponding critical reinforcement strain.

Figure 5.4 Flow chart for the proposed procedure based on sequential construction of the response surfaces

For each combination of input variables, one response surface will be constructed. Since only simple calculations are involved in the above steps, investigation of the variability of the critical reinforcement strain due to the uncertainties associated with

the input variables can be carried out very efficiently based on direct Monte Carlo simulations. Figure 5.4 shows the flow chart of the proposed procedure for problems involving r variables.

5.4.1.3 Validity of the proposed procedure based on response surface method

To illustrate the proposed procedure, the explicit multimodal function $y(x_1, x_2)$ below is considered (Romero et al. 2004):

$$y(x_1, x_2) = \left[0.8 \times \sqrt{(x_1)^2 + (x_2)^2} + 0.35 \times \sin \left(2.4\pi \times \frac{\sqrt{(x_1)^2 + (x_2)^2}}{\sqrt{2}} \right) \right] \times \left[1.5 \times \sin \left(1.3 \times \arctan \left(\frac{x_2}{x_1} \right) \right) \right] \quad (5-3)$$

where x_1 and x_2 are both in the domain of $[0, 1]$. Figure 5.5 shows a 3D plot of this target function.

The proposed procedure is adopted to estimate the value of this function. For each variable, equal number of sampling points are selected in the domain of $[0, 1]$. Since only two variables are involved, the complete combination of sampling points is used. This is to say, if n sampling points is used for each variable, then n^2 combinations will be adopted to evaluate the corresponding function values.

A simple measure of the quality of global fit over the parameter space, defined as average absolute error (Romero et al. 2004), is used herein and can be expressed as:

$$\mathcal{E} = \frac{\sum_{i=1}^m |exact_i - predcited_i|}{m} \quad (5-4)$$

where m is the number of sampling points to obtain the exact and predicted values based on the exact function and the proposed procedure, respectively in the domain of the parameters. For this problem, a 21×21 grid points appear to be sufficient to achieve adequate representation of the target function and, therefore, is used in the quality studying of the results from the proposed procedure. For comparison, the number of sampling points for each variable used to form the database in advance is 3, 5, 6 and 7, respectively and the corresponding number of combinations of input variables is 9, 25, 36 and 49, respectively.

Figure 5.5 3D plot of the target function

Average absolute errors from using different number of sampling points for each variable are calculated based on Eq. (5-4). The results are plotted against the number of the sampling points for each variable and shown in Figure 5.6. It can be seen that, when the sampling points is 3 for each variable, 9 runs will be carried out to form the database for the proposed procedure. The average absolute error is as high as about 0.2.

Figure 5.6 Convergence of the proposed procedure according to averaged absolute error

Increasing the number of the sampling points for each variable dramatically decreases the average absolute error. The average absolute error drops below 0.03 when the number of sampling points is 6. However, the accuracy of the proposed procedure cannot be improved further for this problem when the number of sampling points is beyond 6. The small average absolute error for the proposed procedure based on sequential construction of response surfaces indicates that this approach can be used with confidence to approximate the values of the function which has no implicit form. In this study, the primary interest is the magnitude of the critical reinforcement strain from the nonlinear finite element analysis.

5.4.2 Reinforced embankment on homogeneous soft soil foundation

Uncertainties associated with the critical reinforcement strain are investigated in this section for a reinforced embankment on homogeneous soft soil foundation. The

geometry of this reinforced embankment is shown in Figure 5.1. There are many factors influencing the magnitude of the critical strain in the reinforcement of a reinforced embankment on soft ground, such as unit weight, friction angle, cohesion of the embankment fill, undrained shear strength and undrained modulus of foundation clay. Since undrained shear strength and the ratio of undrained modulus to undrained shear strength (undrained modulus ratio) have significant influence on the critical reinforcement strain, they are considered to be the random variables in the present analysis, for simplicity. Hence, the uncertainties associated with the critical reinforcement strain derive only from the variability of undrained shear strength and undrained modulus ratio.

The undrained shear strength is assumed to vary from 10 kPa to 30 kPa. The ratio of undrained modulus to undrained shear strength is assumed to vary from 150 to 500 which is a representative range for the soft soils. Five sampling points are selected for the undrained shear strength. They are 10, 15, 20, 25 and 30 kPa respectively. As for the ratio of the undrained modulus to the undrained shear strength, eight values are chosen, namely 150, 200, 250, 300, 350, 400, 450 and 500 respectively. Hence, 40 calculations of critical reinforcement strain based on finite element method is required to form a database which will be used to predict the critical reinforcement strain for any combination of input variables based on response surface method.

As discussed earlier, a very important issue in the proposed procedure based on response surface method is the validation of the results obtained. The quality of the approximations should be measured since this is the indicator of the effectiveness of the proposed procedure. Another 13 sampling points are randomly selected in the range of the parameters of undrained shear strength and undrained modulus ratio for testing the accuracy of the proposed procedure in estimating the critical reinforcement strain. Figure 5.7 shows the plot of critical reinforcement strains from finite element

analyses versus those from the proposed response surface method procedure. The agreement is reasonably good. This demonstrates that the proposed procedure based on sequential construction of response surfaces can be adopted to predict the critical strain in the reinforcement reliably.

Figure 5.7 Comparison of the critical reinforcement strains from finite element analyses with those from proposed procedure based on response surface method

Once the critical reinforcement strain corresponding to any combination of input variables can be accurately determined by the proposed procedure, the effect of the uncertainties of undrained shear strength and undrained modulus ratio of the foundation soft soils on the variability of critical reinforcement strain can be easily investigated using direct Monte Carlo simulation. A mean value of 20 kPa is assumed for the undrained shear strength. According to Lacasse and Nadim (1996) and Lumb (1974), the coefficient of variation of undrained shear strength ranges from 10~30% for clay. Considering the fact that variance of undrained shear strength can be greatly reduced due to the effect of spatial variability, the range of 0~20% for the coefficient of variation of undrained shear strength will be used in the present parametric analysis.

It will be shown latter that the the upper bound of the coefficient of variation of critical reinforcement strain is not sensitive to the uncertainties of undrained shear strength and mainly depends on those of undrained modulus ratio.

The variability of undrained modulus ratio E_u / s_u is not as straightforward as undrained shear strength since it is seldom considered in geotechnical reliability analysis. There is not much information available in the literature regarding the uncertainties of undrained modulus ratio. It is well recognized that the undrained modulus ratio decreases with increasing plasticity and organic content and also decreases when overconsolidation ratio increases (Ladd et al. 1977 and Duncan and Buchignani 1976). D'Appolonia et al. (1971) suggested using the value between 1000 and 1500 for undrained modulus ratio E_u / s_u of lean inorganic clays with moderate to high sensitivity and significantly lower values of around 100 for highly plastic clays and organic clays. Bjerrum (1972) recommended using a range of 500-1500 for undrained modulus ratio E_u / s_u of clays with the undrained shear strength being based on field vane tests. For the very plastic clays subjected to large load increments, the lower value of undrained modulus ratio should be used. For lean clays with small load increments, the relatively higher value should be adopted. According to Duncan and Buchignani (1976), the undrained modulus ratio E_u / s_u for many soft clays lies in the range of 125 and 500.

The uncertainties of this undrained modulus ratio can be approximated by assuming that the above range ($125 \leq E_u / s_u \leq 500$) represents the mean plus or minus three standard deviations. The coefficient of variation can be roughly estimated to be 20%. This value will be used in the analysis to study the variability of critical reinforcement strain. For the mean value of undrained modulus ratio, a value of 300 is adopted.

Figure 5.8 3D plot of the coefficient of variation of the critical reinforcement strain

For different coefficients of variation of undrained shear strength and undrained modulus ratio, 10000 sampling points are generated based on Latin hypercube sampling technique and then used with the proposed procedure to determine the values of the critical reinforcement strain. No correlation between undrained shear strength and undrained modulus ratio is considered during the process of generating sets of sampling points because undrained modulus ratio only depends on the plasticity index and overconsolidation ratio. For each set of coefficients of variation of undrained shear strength and undrained modulus ratio, 10000 critical reinforcement strains are calculated based on the proposed procedure. Subsequently, the corresponding coefficient of variation of critical reinforcement strain is determined by statistical operations.

Figure 5.8 shows the result of the investigation of uncertainties of the critical

reinforcement strain. The effect of uncertainties of undrained shear strength and undrained modulus on the variability of the critical reinforcement strain is reflected in this 3D plot for values of coefficients of variation. This plotting involves 41×41 calculations of the coefficient of variation for the critical reinforcement strain and in each calculation, 10000 values of critical reinforcement strain are determined using the proposed procedure. This would be impossible if coupling of Monte Carlo simulation with finite element analysis directly is used instead of the proposed procedure based on response surface approach.

It can be seen from Figure 5.8 that, the effect of uncertainties of undrained modulus ratio is much more significant than uncertainties of undrained shear strength. This is not surprising since the critical reinforcement strain mainly depends on the stiffness of soils within the foundation clay. The definition of critical reinforcement strain itself implies that the undrained shear strength within the clay foundation could be always fully mobilized. As a result of that, the effect of undrained shear strength may not be as significant as undrained modulus ratio. Figure 5.8 shows that the coefficient of variation of critical reinforcement strain increases very slowly with the increase in the uncertainties of undrained shear strength at the early stage when the coefficient of variation of undrained modulus is negligible. The value then goes up to about 12% when the coefficient of variation of undrained shear strength reaches 20%.

To show this trend more clearly, the relationship between the coefficients of variation of the critical reinforcement strain and those of undrained shear strength is plotted in Figure 5.9 for different values of undrained modulus ratio.

Figure 5.9 shows that the coefficient of variation of the critical reinforcement strain is more sensitive to the uncertainties associated with undrained shear strength when the undrained modulus ratio E_u/s_u is relatively small. This indicates that the

uncertainties of critical reinforcement strain are more dependent on undrained shear strength when the foundation clay is more compressible.

Figure 5.9 Relationship between the coefficients of variation of the critical reinforcement strain and those of undrained shear strength

It also can be inferred from Figure 5.9 that, although the coefficient of variation of the critical reinforcement strain increases with the coefficient of variation of undrained shear strength, the former value is always below the latter one. When uncertainty of undrained modulus ratio is considered, the coefficient of variation of the critical reinforcement strain at the higher value of the coefficient of variation of undrained modulus ratio almost remains constant regardless of the coefficients of variation of undrained shear strength (Figure 5.8). This actually indicates the importance of the undrained modulus ratio to the critical reinforcement strain.

According to the above analyses about the uncertainties of the critical reinforcement strain for the reinforced embankment on the soft ground, it can be concluded that, a value of 30% for the coefficient of variation for the critical reinforcement strain

represents a reasonable upper bound for this homogeneous reinforced embankment. As described in the first section of this chapter, Kitch (1994) assumed a coefficient of variation of 25% for the serviceability strain in the reinforcement based on the upper and lower limits of the published values. The upper bound of the coefficient of variation of the critical reinforcement strain obtained is slightly higher than that proposed by Kitch (1994). It is considered to reasonably represent the uncertainties of the reinforcement strain used in the conventional limit equilibrium method to study the reliability of reinforced embankments on soft ground.

5.5 Summary and conclusions

Reinforced embankments on soft ground can be regarded as unstable when excessive deformations occur within the clay foundation. The mobilized strain in the reinforcement under such a circumstance is defined as critical reinforcement strain in the present study. It is considered to be more appropriate in limit equilibrium analysis to calculate the factor of safety corresponding to the shear strength of soils. In order to investigate the uncertainties associated with this critical reinforcement strain, a procedure based on sequential construction of response surfaces is proposed. An explicit illustrative function is used to validate the effectiveness of the proposed procedure to approximate the values of a target function. A reinforced embankment on homogeneous soft soil foundation is employed in the studying of uncertainties of critical reinforcement strain. Direct Monte Carlo simulation is used to study the effects of undrained shear strength and undrained modulus ratio on the variability of the critical reinforcement strain by the proposed procedure through the response surface method.

It is found that undrained modulus ratio has a more significant influence on the variability of the critical reinforcement strain. Especially in the cases where the

coefficient of variation of undrained modulus ratio is as high as 20%, the coefficient of variation of the critical reinforcement strain can remain around 30% irrespective of uncertainties of undrained shear strength. Therefore, in the reliability analysis of reinforced embankments on soft ground, an upper bound of 30% can reasonably be adopted for the critical reinforcement strain.

CHAPTER 6 EFFECTS OF SPATIAL VARIABILITY ON THE RELIABILITY OF EMBANKMENTS ON SOFT GROUND

6.1 Introduction

The physical properties of soils vary from place to place within the deposits and are spatially autocorrelated. Usually, the values of soil properties are correlated with those at neighborhood points. This means that it is not appropriate to describe soil properties as single-value random variables. The inherent spatial variability of soil properties has significant influence on the reliability analysis of slopes or embankments. Reliability analysis using single-random-variable approach can greatly overestimate the failure probability if the variance reduction due to spatial averaging is not considered (Li and White 1987b; Li 1992; Mostyn and Li 1993; White 1993). El-Ramly et al. (2002) compared the results from reliability analysis of James Bay dykes by considering variance reduction due to spatial variability with those from simplified analysis which ignored spatial variability and found that the simplified analysis overestimated the failure of probability. However, in order to implement the probabilistic stability analysis effectively and efficiently, which was based on Monte Carlo simulation, El-Ramly et al. (2002) fixed the circular slip surface in such a way that it touched the bottom of the foundation. Babu and Mukesh (2003) applied mean value first-order second-moment method to Bishop's simplified method with 2D spatial variability of soil properties. The reliability index was calculated based on the slip surface with the minimum factor of safety.

Alonso (1976) studied slope reliability by considering autocorrelation among the variables of each slice based on the negative exponential autocorrelation function. In the analyses, Alonso assumed that the autocorrelation in each direction was identical and that the critical probabilistic circular slip surface coincided with the critical

deterministic slip surface.

Low and Tang (1997b) included the spatial variability in their reliability analysis for embankments on soft ground using several values of s_u to characterize the undrained shear strength profile in the soft soil foundation and correlation coefficients obeying the exponential autocorrelation function. First-order reliability method was adopted to calculate the reliability index β_{HL} . This approach obtained the reliability index and the corresponding critical circular slip surface or noncircular slip surface (Low 2003) using the built-in optimization routine in Microsoft Excel spreadsheet. Vertical autocorrelation was modeled, but not horizontal autocorrelation. This chapter considers both vertical and horizontal autocorrelation, and investigates to what extent reliability index is underestimated when only vertical autocorrelation is considered.

In spite of the growing concern about spatial variability in slope reliability analysis, probabilistic stability analyses of slopes or embankments without considering spatial variability are not unusual even in recent years (Hassan and Wolff 1999; Liang et al. 1999; Bhattacharya et al. 2003). This could partly be due to the perceived difficulty in considering spatial variability and calculating reliability index simultaneously.

The first objective in this chapter is to propose a simple procedure of considering two-dimensional spatial variability in reliability analysis of embankments on soft ground. The invariant first-order reliability method is adopted to calculate the reliability index. The probabilistic critical slip surface can be located automatically using optimization procedure. The second objective is to investigate the difference in the reliability index obtained by the direct autocorrelation approach and by the variance reduction approach.

6.2 Two ways of considering spatial variability in slope reliability analysis

Two methods are described in the literature for modeling the spatial variation of random variables in slope reliability analysis.

The first way is to use autocorrelation distance r and autocorrelation function (ACF) to determine the autocorrelation of soil properties at two points separated by a given distance. The commonly used autocorrelation functions are the negative exponential and the negative squared exponential functions, as follows:

$$\rho(\Delta x, \Delta y, \Delta z) = \exp\left(-\frac{\Delta x}{r_x} - \frac{\Delta y}{r_y} - \frac{\Delta z}{r_z}\right) \quad (6-1)$$

and

$$\rho(\Delta x, \Delta y, \Delta z) = \exp\left(-\frac{\Delta x^2}{r_x^2} - \frac{\Delta y^2}{r_y^2} - \frac{\Delta z^2}{r_z^2}\right) \quad (6-2)$$

in which ρ is the autocorrelation coefficient, r_x , r_y and r_z are autocorrelation distances in the x, y, z directions. Δx , Δy and Δz are the distances between two points in the x, y, z directions. Works using this approach include Alonso (1976), Calle (1985), Low and Tang (1997b), Low (2003), and Babu and Mukesh (2003, 2004). Li and Lumb (1987) applied four types of functions, including simple negative exponential, square exponential, second-order autoregressive and cosine exponential functions in the slope reliability analysis and found that the reliability results were not sensitive to the form of autocorrelation function used. Based on that finding, they suggested using simple negative exponential autocorrelation function due to its simplicity.

The second way to model spatial variability is to use the scale of fluctuation δ , as proposed in Vanmarcke (1977b). The basic idea of this approach is to reduce the variance along the slip surface over which the point values of soil properties, such as undrained shear strength, are averaged. This is because the longer the slip surface is, the more the fluctuations of soil properties will tend to be cancelled due to the spatial variability. As a result, the values of variance should be reduced to account for this spatial nature of soil properties. By this approach, variance reduction function is used instead of the autocorrelation function (ACF) described in the preceding paragraph. Vanmarcke (1983) derived two commonly used one-dimensional variance reduction functions, as follows:

$$\gamma(T) = \begin{cases} 1, & T \leq \delta \\ \frac{\delta}{T}, & T > \delta \end{cases} \quad (6-3)$$

or

$$\gamma(T) = \begin{cases} 1, & T \leq \frac{\delta}{2} \\ \frac{\delta}{T} \left(1 - \frac{\delta}{4T}\right), & T > \frac{\delta}{2} \end{cases} \quad (6-4)$$

where T is the averaging length and δ is the scale of fluctuation. For slope reliability problem, T is usually the length of the slip surface. Theoretically, the variance reduction function $\gamma(T)$ is related to the autocorrelation function $\rho(\tau)$, as follows (Vanmarcke 1983):

$$\gamma(T) = \frac{2}{T} \int_0^T \left(1 - \frac{\tau}{T}\right) \rho(\tau) d\tau \quad (6-5)$$

Therefore, for the widely used negative exponential autocorrelation

function $\rho(\tau) = e^{-\tau/r_o}$, where r_o is the autocorrelation distance, the corresponding variance reduction function is

$$\gamma(T) = 2 \cdot \left(\frac{\delta}{2T}\right)^2 \cdot \left(\frac{2T}{\delta} - 1 + e^{-2T/\delta}\right) \quad (6-6)$$

where $\delta = 2r_o$. Figure 6.1 shows the plot of variance reduction against the normalized averaging interval T/δ for several commonly used variance reduction functions. It can be seen that, while the difference in the variance reduction could be significant for small T/δ , both Eqs. (6-3) and (6-4) predict almost identical results for T/δ larger than 5.

Figure 6.1 Variance reduction plotted against the normalized averaging interval T/δ for several commonly used variance reduction functions

Due to its simplicity, Eq. (6-3) is widely used for slope reliability problems

(Vanmarcke 1977b; Yucemen and Al-Homoud 1990; El-Ramly et al. 2002; Al-Homoud and Tanash 2004). Vanmarcke (1983) mentioned some inconsistencies for this variance reduction function (Eq. (6-3)) regarding the definition of the scale of fluctuation.

In this study, the reliability analysis based on autocorrelation distance is called *direct autocorrelation approach* since the autocorrelation function (ACF) (Eq. (6-1) or (6-2)) is directly used in the analysis to determine correlation between variables. The approach based on the scale of fluctuation (Eq. (6-3), (6-4) or (6-6)) is called *variance reduction approach* as the theoretical basis for this method is to reduce the variance due to spatial variability according to Vanmarcke (1977b).

6.3 A simple procedure to consider two-dimensional spatial variability in reliability analysis of embankments

Using the autocorrelation function directly in the reliability analysis appears to be a more natural way than variance reduction approach. This is especially so for reliability analysis based on the limit equilibrium method of slices where the autocorrelation function can be easily incorporated based on existing deterministic stability analysis programs. Low et al. (1998) suggested that it is possible to let each slice have its own mean and standard deviation of random variables which can be autocorrelated in order to consider the spatial variability. The US Army Corps of Engineers (1999) also considered it more refined to consider that each slice in a stability analysis can assume random parametric values which are spatially correlated with adjacent slices.

Figure 6.2 shows a typical cross section of an embankment on soft ground. For any two slices i and j , the horizontal and vertical distances between the midpoints of their bases are $\Delta x_{i,j}$ and $\Delta y_{i,j}$ respectively. Soil properties of each slice are considered to be random variables. These soil properties can include soil cohesion, friction angle,

unit weight and pore pressure ratio. Although the number of random variables depends on the number of slices and may be quite large, modern computing technology can handle optimization calculation involving a large number of variables quite easily. The correlation coefficient between two different locations for any variable can be determined by Eq. (6-1) or (6-2), depending on which autocorrelation function is assumed.

Figure 6.2 A typical cross section of the embankment on soft ground

One of the most important advantages of probabilistic stability analysis over conventional deterministic stability analysis is that the uncertainties in the factor of safety can be assessed. Reliability index is often used to express the degree of uncertainty in the calculated factor of safety. The mean value first-order second-moment method (MVFOSM) is commonly employed due to its simplicity (Alonso 1976; Christian et al., 1994; Hassan and Wolff 1999; Malkawi et al., 2000). It is well recognized that the main deficiency of the MVFOSM method is its lack of invariance for different but mechanically equivalent formulations of the limit state, resulting in different values of reliability index. Moreover, because the performance function is linearized at the mean values of random variables, significant error may be introduced by neglecting higher order terms.

The first-order reliability method (FORM), also called Hasofer-Lind method for normal variables, overcomes the disadvantages of the MVFOSM method. In this method, the Taylor series is expanded not about the mean or expected value, but about the design point which represents the worst combination of random variables. The matrix formulation of the Hasofer-Lind index β_{HL} is (Ditlevsen 1981):

$$\beta_{HL} = \min_{\mathbf{X} \in F} \sqrt{(\mathbf{X} - \mathbf{M})^T \mathbf{C}^{-1} (\mathbf{X} - \mathbf{M})} \quad (6-7)$$

where $\mathbf{X}$ = the vector of random variables; $\mathbf{M}$ = the vector of the mean values of random variables; $\mathbf{C}$ = the covariance matrix of random variables and F = the failure region. According to Eq. (6-7), the Hasofer-Lind index β_{HL} can be regarded as the minimum distance in units of directional standard deviation from the mean value point of the random variables to the boundary of the limit state. In slope stability problems, the limit state is $F_s - 1 = 0$, where F_s is the global factor of safety. Low and Tang (1997b, 2004) used the following equivalent form of Eq. (6-7) in order to show the correlation structure more explicitly:

$$\beta_{HL} = \min_{\mathbf{X} \in F} \sqrt{\left[\frac{x_i - m_i}{\sigma_i} \right]^T [\mathbf{R}]^{-1} \left[\frac{x_i - m_i}{\sigma_i} \right]} \quad (6-8)$$

where σ_i is the standard deviation of random variable x_i , m_i is the mean value and $[\mathbf{R}]$ is the correlation matrix. This equation is also adopted throughout this study to calculate reliability index.

The calculation of reliability index based on FORM method requires an iterative procedure to find the minimum of $\sqrt{\left[\frac{x_i - m_i}{\sigma_i} \right]^T [\mathbf{R}]^{-1} \left[\frac{x_i - m_i}{\sigma_i} \right]}$ according to Eq. (6-8).

The values of random variables $\mathbf{X}$ will be varied around the initial values along such a direction that the minimum can be guaranteed. The final values of random variables on the limit state surface define the design point which is also the “most probable point of failure”. This represents the “worst” combination of the variables considered in the analysis. The US Army Corps of Engineers (1999) ascribed the unpopularity of Hasofer-Lind method in slope reliability analysis to the complexity involved in this method, especially when applied to the implicit performance functions of the slope stability model. In reality, the Hasofer-Lind index β_{HL} can be computed easily via constrained optimization of Eq. (6-8).

Low and Tang (1997b, 2004) and Low (2003) used Microsoft Excel’s built-in Solver optimization routine to determine reliability index as defined by Eq. (6-8), with search for the reliability-based critical slip surface. In this study, MATLAB and its optimization toolbox are used to conduct reliability analysis. Spencer’s method and its alternative formulas for force and moment equilibrium derived in Low (2003) are used.

Figure 6.3 shows the procedures for computing reliability index for the probabilistic critical circular slip surface with two-dimensional spatial variability. For an initial trial slip surface, the correlation matrix $[\mathbf{R}]$ is formed corresponding to the number of slices and the number of random variables assigned to each slice. The correlation coefficient between any two variables is calculated based on the autocorrelation function and the separation distance. The dimension of the correlation matrix depends on the total number of the random variables. For example, if only undrained shear strength in the soft ground is regarded as the random variable for each slice, then a 20×20 correlation matrix will be formed for the case where there are 20 slices with bases in the soft ground. Although it is possible that the effectiveness of the proposed approach may diminish if the number of slices or the number of variables for each

slice is large, it will be shown later that the results of reliability analysis are in fact not sensitive to the number of slices when the slice number exceeds a certain value.

Figure 6.3 Procedures for computing the reliability index for probabilistic critical circular slip surface using MATLAB

After forming the correlation matrix, the reliability index is calculated. In this step, the constrained optimization routine is required to solve Eq. (6-8) with the constraint that the equilibrium conditions should be satisfied and the random variables should be located on the limit state surface $F_s - 1 = 0$. The variables in the constrained optimization procedure include the random variables x_i of Eq. (6-8) and a parameter λ^* , which is used in Spencer's method to define the inter-slice force function (see Eq. (4-3)). The initial values of the random variables x_i are usually selected to be the mean values. For the parameter λ^* , the initial value is set to be 0. The calculation of the reliability index is to vary the random variables x_i and λ^* in such a way that

$$\sqrt{\left[\frac{x_i - m_i}{\sigma_i} \right]^T [\mathbf{R}]^{-1} \left[\frac{x_i - m_i}{\sigma_i} \right]} \text{ is minimum and the constraint conditions described above}$$

are satisfied. The constrained optimization function *fmincon* in the optimization toolbox of MATLAB is used. Thus, the computation of the reliability index based on FORM for a specified circular slip surface is reduced to a simple constrained optimization problem.

Then, the optimization procedure is applied to obtain the reliability index and the corresponding probabilistic critical circular slip surface by varying the center (x_0, y_0) and the radius r_0 of the circular slip surface. The simplex method, in the form of the function *fminsearch* in MATLAB, is used due to its simplicity and effectiveness. This method does not require the evaluation about the partial derivatives of the object function and therefore can handle discontinuity problems. It has been used to search for deterministic critical circular and noncircular slip surfaces (Nguyen 1985 and DeNatale 1992). It should be pointed out that the simplex method is not a global optimization procedure. The searching process could be trapped in local minimums. To avoid this, the searching for probabilistic critical circular slip surface in this study is

carried out from different initial slip circles.

6.4 A hypothetical embankment on homogeneous soft foundation

The hypothetical embankment on homogeneous soft ground shown in Figure 6.4 is analyzed using the proposed approach. To simplify the analysis, only uncertainties associated with the undrained shear strength within the foundation are considered. The undrained shear strength is assumed to be normally distributed with the mean value and the standard deviation of 15 kPa and 4 kPa respectively. According to Mostyn and Li (1993), the typical values of horizontal and vertical autocorrelation distance are usually in the range of 2-30 m and 0.1-5 m respectively. In this study, the negative exponential autocorrelation function (Eq. (6-1)) is adopted. The vertical autocorrelation distance r_y is assumed to be 1 m. The horizontal autocorrelation distance r_x is assumed to be 10 m.

Figure 6.4 Hypothetical embankment on soft ground

6.4.1 Deterministic and probabilistic critical slip surfaces

The deterministic stability analysis for this embankment based on Spencer's method gives a safety of factor of 1.51 for critical circular slip surface. The minimum

reliability index obtained by the proposed approach is 2.38 for 20 slices with bases in the soft clay foundation. The corresponding critical deterministic and probabilistic circular slip surfaces are plotted in Figure 6.5.

Figure 6.5 Deterministic and probabilistic critical circular slip surfaces

It can be seen that, for this simple embankment, the probabilistic critical circular surface is very different from the deterministic critical slip surface. The deterministic critical circular slip surface tends to touch the bottom of the foundation to yield the minimum factor of safety. The probabilistic critical circular slip surface is much shallower. The reliability index corresponding to the deterministic critical circular slip surface is calculated to be 2.78 which is higher than the value of 2.38 based on the probabilistic critical circular slip surface. Had one calculated the factor of safety based on previously determined critical probabilistic circular slip surface, a value of 1.61 would be obtained which is higher than the 1.51 associated with the critical deterministic circular slip surface. The difference in deterministic and probabilistic critical slip surfaces is well recognized. Li and Lumb (1987) first searched for the slip surface with minimum reliability index for a homogeneous slope. Hassan and Wolff (1999) proposed a simple search algorithm for reliability-based critical slip surface; the

parametric studies performed on a two-layer slope demonstrated that the probabilistic critical slip surface could be significantly different from the deterministic critical slip surface.

Different from the calculation of critical factor of safety, the computation of reliability index for an embankment or slope is governed mainly by two factors: uncertainties associated with random variables and the distance between the mean value point and the limit state surface. The first factor determines the initial size of the ellipsoid according to the ellipsoidal perspective of Hasofer-Lind index (Low and Tang 1997b, 2004), and the second factor decides the available space in which the ellipsoid can be expanded to touch the limit state surface at the design point. In the process of locating the probabilistic critical slip surface, either or both factors can affect the results.

For the embankment analyzed above, it is found that the slip surfaces which are tangential to the depths of 3 m, 4 m, 5 m, and 6 m in the soft clay foundation have similar factors of safety which vary in a narrow range from 1.54 to 1.51. This means that the limit state surfaces corresponding to circular slip surfaces with different depths could be, more or less, close to each other. As a result, the first factor, that is the uncertainties of random variables, may dominate the calculation of reliability index. It is conceivable that, as the slip surface extends deeper into the foundation, the length of the slip surface within the foundation will increase and this can, according to the concept of variance reduction (Vanmarcke 1977b), result in the reduction of the variance along the slip surface since the longer the slip surface is, the more the fluctuations of undrained shear strength will be cancelled out. Therefore, the reliability index for the corresponding slip surface will be increased. This is likely to be the reason for the much shallower probabilistic critical slip surface compared with the deterministic critical slip surface.

Alonso (1976) suggested that the probabilistic critical slip surface can be regarded the same as the deterministic critical slip surface. For the case in Figure 6.5, using the deterministic critical slip surface to calculate reliability index could result in unconservative evaluation of reliability.

6.4.2 Effect of the number of slices on reliability index

Figure 6.6 Variation of reliability index with the number of slices used in the analysis

The number of slices may influence the results of reliability analysis since the number of random variables in reliability analysis depends on the number of slices within the clay foundation. Figure 6.6 presents the variation of reliability index with the number of slices used in the reliability analysis. The vertical and horizontal autocorrelation distances are 1 m and 10 m, respectively. It can be seen that, the reliability index increases with the number of slices at first but remains almost constant after the number of slices exceeds 20. This indicates that the reliability index computed by direct autocorrelation approach is insensitive to the number of slices selected in the

analysis when the slice number is greater than a certain value. For different problems, the threshold number of slices might vary due to different slope geometries, soil properties and characteristics of spatial variability. However, it is relatively easy to determine the required number of slices by trial.

6.4.3 Effects of vertical and horizontal autocorrelation distances on the reliability index and the corresponding critical slip surface

Figure 6.7 Probabilistic critical circular slip surfaces for different values of vertical autocorrelation distance for $r_x/r_y = 10$

The effects of vertical and horizontal autocorrelation distances on probabilistic critical slip surface are investigated. Figure 6.7 shows the probabilistic critical circular slip surfaces for three different vertical autocorrelation distances while the ratio of the horizontal autocorrelation distance to vertical autocorrelation distance is kept to 10. It can be observed that the probabilistic critical circular slip surface extends deeper when the vertical autocorrelation distance increases from 1 m to 5 m. This suggests that the

probabilistic critical circular slip surface depends on the vertical autocorrelation distance. A larger vertical autocorrelation distance for the undrained shear strength indicates a more uniform clay foundation in the vertical direction. As a result, the slip surface can be located more deeply without much variance reduction along the slip surface. Probabilistic critical circular slip surfaces for different horizontal autocorrelation distances are also obtained for the vertical autocorrelation distance of 1 m as shown in Figure 6.8. Unlike the influence of vertical autocorrelation distance, the horizontal autocorrelation distance is found to have only minor influence on the probabilistic critical slip surface. The probabilistic critical circular slip surface for horizontal autocorrelation distance exceeding 20 m is practically the same as that of 20 m.

Figure 6.8 Probabilistic critical circular slip surfaces for different values of horizontal autocorrelation distance for $r_y = 1$ m

Figure 6.9 shows the variation of reliability index with the horizontal autocorrelation distance for the vertical autocorrelation distance of 1 m. Each reliability index is

obtained by searching for the probabilistic circular slip surface from different initial circular slip surfaces. It can be seen that reliability index decreases with the horizontal autocorrelation distance initially but approaches a constant value when the horizontal autocorrelation distance is larger than 30 m. Similar findings were reported in Li and Lumb (1987) and Yucemen and Al-Homoud (1990).

Figure 6.9 Variation of reliability index with the horizontal autocorrelation distance for $r_y = 1$ m

6.5 Difference between the direct autocorrelation approach and the variance reduction approach in reliability index

As mentioned earlier, apart from the direct autocorrelation approach, the variance reduction approach can also be used to consider the effect of spatial variability. The theoretical basis of this approach is that the average shear strength of a soil along the slip surface is more important for slope stability problem than the variation of strength from point to point within the layer. Thus, the variance of the local average is used in the analysis instead of the point variance. According to this principle, Vanmarcke

(1977b) introduced a variance reduction factor Γ , which lies between 0 and 1, to relate the variance of the local average to the point variance. The spatial variability is taken into account by reducing the variance along the slip surface according to a certain form of variance reduction function (Eq. (6-3), (6-4), or (6-6)). This is different from the method used in the direct autocorrelation approach where the spatial variability of soil properties is accounted for by considering the correlation coefficients between random variables according to an autocorrelation function (e.g., Eq. (6-1)).

Figure 6.10 A typical cross section of the embankment on soft ground for calculation of equivalent scale of fluctuation

In the variance reduction approach, the scale of fluctuation is used instead of the autocorrelation distance. The relationship between the autocorrelation distance and the scale of fluctuation depends on the form of the autocorrelation function, as elaborated in Vanmarcke (1983). For the simple exponential autocorrelation function, the scale of fluctuation is twice the autocorrelation distance. According to Vanmarcke (1977b), when the spatial variability depends on vertical and horizontal directions, a simple

procedure can be used to determine an equivalent scale of fluctuation by the following equation:

$$\frac{1}{\delta_T} = \frac{(\Delta y_1 + \Delta y_2)}{T \cdot \delta_y} + \frac{\Delta x}{T \cdot \delta_x} \quad (6-9)$$

where δ_T = the equivalent scale of fluctuation; T is the length of the slip surface in which the spatial variability is accounted for and equal to $\Delta y_1 + \Delta y_2 + \Delta x$; δ_y and δ_x are the scales of fluctuation in the vertical and horizontal directions, respectively; $\Delta y_1 + \Delta y_2$ is the approximate equivalent length of the slip surface in the vertical direction; and Δx is the approximate equivalent length of the slip surface in the horizontal direction. For the embankment on soft ground shown in Figure 6.10, the equivalent scale of fluctuation for the undrained shear strength within the clay foundation can be easily derived for a specified circular slip surface with center (x_0, y_0) and radius r_0 , as follows:

$$\delta_T = \frac{(1 + \Omega)}{\left(\frac{1}{\delta_y} + \frac{\Omega}{\delta_x}\right)} \quad (6-10)$$

where $\Omega = \sqrt{\frac{r_0 + y_0}{r_0 - y_0}}$; y_0 is the distance from the center of the slip circle to the top of the soft ground and r_0 is the radius of the slip circle. The length of the slip surface within the clay foundation T is cancelled out during the derivation. The following assumption is made for the derivation of the above equation:

$$\frac{\Delta v_1 + \Delta v_2}{\Delta h} = \frac{\Delta y_1 + \Delta y_2}{\Delta x} \quad (6-11)$$

where $\Delta v_1 + \Delta v_2$ and Δh are the actual projected lengths of the slip surface within the clay foundation in the vertical and horizontal directions, respectively.

It can be seen that the equivalent scale of fluctuation depends not only on the vertical and horizontal scale of fluctuation but also the location of the slip surface. When the horizontal scale of fluctuation δ_x is infinite, the corresponding equivalent scale of fluctuation becomes $\delta_T = (1 + \Omega) \cdot \delta_y$, which is only dependent on the vertical scale of fluctuation δ_y and the location of the slip surface according to Eq. (6-10).

The analysis procedure of adopting variance reduction to consider the effect of spatial variability is greatly different from that based on directly using the autocorrelation function for random variables. The former approach has the advantage that fewer random variables are required since local average is considered instead of the point value. This implies that the optimization procedure for calculating the Hasofer-Lind index can be carried out more efficiently. It is therefore interesting and meaningful to compare the results of reliability analysis from direct autocorrelation approach with those from variance reduction approach.

Calle (1985) compared the reliability results from these two different approaches. The only random variable considered in his analysis was cohesion and only horizontal autocorrelation parallel to the slope was taken into account. The parametric study was undertaken for a slope in a cohesive soil with different overburden loadings. It was found that the calculated probability of failure according to these two different approaches agreed well for most cases except for the case with no overburden loading. The difference had a factor of 4 and no explanation was given.

In this study, variance reduction approach is also employed to analyze probabilistically

the embankment on soft ground shown in Figure 6.4. For comparison, Eq. (6-6) is used as the variance reduction function to compute the variance reduction with T equal to the length of the slip circle in the soft clay foundation. The equivalent scale of fluctuation of undrained shear strength is calculated by Eq. (6-10). Reliability index is determined by searching for the probabilistic critical circular slip surface and a value of 2.48 is obtained which is around 4% higher than the one obtained by the direct autocorrelation approach (2.38). The corresponding probabilistic critical circular slip surface is also plotted in Figure 6.5 and it can be seen that the slip surface is also very different from the deterministic one, but very close to that obtained by the direct autocorrelation approach.

The effect of horizontal scale of fluctuation on the results of reliability analysis is also plotted in Figure 6.9. As expected, reliability index from the variance reduction approach decreases initially with the horizontal autocorrelation distance and then tends to remain constant when the horizontal autocorrelation distance is larger than 30 m which corresponds to a horizontal scale of fluctuation of 60 m.

Figure 6.9 also presents an interesting comparison of results between the direct autocorrelation approach and the variance reduction approach for different values of horizontal autocorrelation distance while the vertical autocorrelation distance is fixed at 1 m. In most cases, the variance reduction approach gives a higher prediction of the reliability index. The difference in the reliability index between the two approaches varies from 0.17% to 10%. Moreover, for large horizontal autocorrelation distance, the difference almost remains constant at a value of around 10%. This indicates that there is some difference between the direct autocorrelation approach and the variance reduction approach in predicting the reliability index for slopes or embankments. This is because these two approaches deal with slope reliability problems differently. The direct autocorrelation approach takes spatial variability into account by correlation

coefficients between random variables whose number is selected to be large enough to describe the domain of interest appropriately. The variance reduction approach considers spatial variability by reducing the variance along the slip surface. Moreover, the variance reduction approach adopts an approximate equivalent scale of fluctuation to deal with the spatial variability of soil properties along the slip surface for the case where the variability depends on directions. Despite the different approaches, it is encouraging that the difference in the calculated reliability indices from the two approaches is not greater than 10%.

Figure 6.11 Comparison of reliability results from direct autocorrelation approach and variance reduction approach for $r_x/r_y = 10$

Figure 6.11 and Figure 6.12 show the variation of reliability index from the two approaches with vertical autocorrelation distance. Two different ratios of the horizontal to the vertical autocorrelation distance, 10 and 30 respectively, are used in the studies. Reliability index is obtained by searching for probabilistic critical circular slip surface in each calculation. It can be observed that reliability index decreases with increase in

the vertical autocorrelation distance in both cases. The trend is similar to the one from the case where the effect of the horizontal autocorrelation distance is of interest as shown in Figure 6.9. For $r_x/r_y = 10$, the difference in the reliability index between the two approaches is found to vary in a narrow range from 2% to 4%. This means that at this r_x/r_y ratio, the reliability index from direct autocorrelation approach agrees very well with that from variance reduction approach. This is also confirmed by the two close probabilistic circular slip surfaces shown in Figure 6.5 for $r_x = 1$ m. When $r_x/r_y = 30$, the difference in the reliability index between these two approaches varies from 3% to 10% as shown in Figure 6.12. The maximum difference is at the vertical autocorrelation distance of 0.5 m. A diminishing difference in the reliability index between the two approaches is also obvious in Figure 6.12 when vertical autocorrelation distance increases.

Figure 6.12 Comparison of reliability results from direct autocorrelation approach and variance reduction approach for $r_x/r_y = 30$

From the above analyses, it can be seen that the results from the direct autocorrelation

approach are not always close to those from the variance reduction approach. The difference in reliability index varies from 0.17% to 10% depending on vertical autocorrelation distance and the ratio of vertical to horizontal autocorrelation distance.

6.6 Further application to the embankment on non-homogeneous soft foundation

The reliability analysis procedure based on the direct autocorrelation approach will be further illustrated by analyzing the Nong Ngoo Hao trial embankment probabilistically. Several trial embankments were constructed on the soft ground near Bangkok city with the purpose to evaluate the subsoil for the future construction of the airport. Among these, embankment No.III was built rapidly to failure with a final height of 3.4 m as shown in Figure 6.13.

Figure 6.13 A typical cross section of NNH embankment used in the present analysis (after Bergado and Chang 1989)

Due to the availability of soil test data and detailed statistical evaluation of undrained shear strength by Bergado and Chang (1989), this embankment is used to illustrate the proposed procedure. Table 6-1 shows the mean and standard deviation of undrained shear strength used in the analysis. The undrained shear strength is assumed to be normally distributed. For the spatial variability of undrained shear strength within the soil foundation, the negative squared exponential autocorrelation function was used to

fit the autocorrelation values at various lag distances. The horizontal and vertical autocorrelation distances were found to be 11.74 m and 0.184 m respectively (Bergado and Chang 1989). Therefore, these two values, together with the negative squared exponential autocorrelation function, are used to describe the spatial variability of undrained shear strength.

Table 6-1 Mean value and standard deviation of undrained shear strength of foundation clay (from Bergado and Chang 1989)

Depth (m)	Mean (kPa)	Standard deviation (kPa)
0 ~ 1	18.2	2.8
1 ~ 2	19.0	2.6
2 ~ 3	15.2	1.2
3 ~ 4	13.6	0.9
4 ~ 5	13.8	1.2
5 ~ 6	14.8	1.3
6 ~ 7	17.4	1.4
7 ~ 8	20.9	1.9

The unit weight of the embankment fills and Bjerrum vane correction factor are also treated as random variables with a normal distribution in the analysis. The mean value and standard deviation of unit weight of the embankment fills are 18.5 kN/m^3 and 1.11 kN/m^3 , respectively. The mean value of Bjerrum's vane correction factor is selected to be 0.9 according to the plasticity index PI of the soft soils within the foundation. The coefficient of variation (standard deviation/mean value) of Bjerrum's vane correction factor is assumed to vary from 10% to 25% in the analysis since no information about its uncertainty is available for this embankment. The friction angle of embankment fills is assumed to have a deterministic value of 37.5° .

Figure 6.14 shows the deterministic and probabilistic critical circular slip surfaces obtained for this embankment when the coefficient of variation of Bjerrum's vane correction factor is 10%. The reliability index is calculated to be 1.6 and the corresponding failure probability is 5.5% if a normal distribution is assumed.

It can be seen that the probabilistic critical circular slip surface is different from deterministic one and located more deeply. The critical factor of safety is found to be 1.20 and the factor of safety will become 1.21 if the probabilistic critical circular slip surface is adopted. The probabilistic critical circular slip surface from the variance reduction approach is also plotted in the figure. The corresponding reliability index is 1.56, which is slightly smaller. It can also be observed that the critical slip surface from variance reduction approach is very close to that from the direct autocorrelation approach.

Figure 6.14 Critical circular slip surfaces from deterministic and probabilistic analyses (unit: meter)

Figures 6.15 and 6.16 show the variation of reliability index and failure probability with the coefficient of variation for Bjerrum's vane correction factor. The failure probability increases rapidly with the increment of coefficient of variation of Bjerrum's vane correction factor for both the direct autocorrelation approach and the variance reduction approach.

This indicates the significance of Bjerrum's vane correction factor to the reliability of this embankment. It is also consistent with the conclusion of El-Ramly et al. (2002) regarding the importance of Bjerrum's vane correction factor in the reliability of James Bay dyke. As the coefficient of variation reaches 25%, the calculated reliability index based on the direct autocorrelation approach is 0.7 and the corresponding failure probability is 23.8%.

Figure 6.15 Variation of reliability index with the coefficient of variation of Bjerrum's vane correction factor

Figure 6.16 Variation of failure probability with the coefficient of variation of Bjerrum's vane correction factor

6.7 Summary and conclusions

A simple procedure for limit equilibrium slope reliability analysis accounting for two-dimensional spatial variability of soil properties is presented in this chapter. The spatial variability is taken into account based on the direct autocorrelation approach which explicitly models the correlation coefficients between the random variables. Reliability index is calculated based on the more rigorous first-order reliability method (FORM) and the probabilistic critical slip surface is located automatically via constrained optimization in MATLAB. The main contribution of the procedure presented is its simplicity in dealing with spatial variability in slope reliability analysis. Correlation between variables is accounted for directly by the correlation matrix in Eq. (6-8). The calculation of reliability index is reduced to a simple constrained optimization problem based on the ellipsoid method by Low and Tang (1997b). Partial derivatives of performance function and iterative searching for the design point are

implicitly included in the constrained optimization. Although only normal random variables are considered in the study, extending to non-normal random variables is very straightforward (Low and Tang 2004).

For a simple hypothetical embankment on homogeneous soft foundation, it is found that the probabilistic critical slip surface is different from the deterministic critical slip surface and depends on the vertical autocorrelation distance. The results also show that the horizontal autocorrelation distance has only a minor influence on the probabilistic critical slip surface. The effect of the number of slices in the clay foundation on reliability index is also investigated since the number of random variables is proportional to the number of slices. The studies indicate that the results of reliability analysis are insensitive to the number of slices when it exceeds a certain value. The effect of spatial variability of undrained shear strength within the clay foundation is found to have significant influence on the reliability index of the embankment. With increase in the vertical or horizontal autocorrelation distance, the reliability index decreases initially, but approaches a constant value at large autocorrelation distance. The studies on the difference in the reliability index between the direct autocorrelation approach and the variance reduction approach show that the difference varies from 0.17% to 10% depending on the vertical and horizontal autocorrelation distances.

The direct autocorrelation approach is further applied to an actual embankment on non-homogeneous soft clay foundation. While the critical factor of safety is 1.20, the reliability index of this embankment decreases from 1.6 to 0.7 when the coefficient of variation of Bjerrum's vane correction factor increases from 10% to 25%.

CHAPTER 7 RELIABILITY ANALYSIS OF REINFORCED EMBANKMENT ON SOFT GROUND

7.1 Introduction

In this chapter, the hypothetical reinforced embankment on soft ground in Chapter 5 will be analyzed probabilistically using the reliability analysis approach proposed in Chapter 6 which can consider two-dimensional spatial variability. The main objective is to investigate various influence factors which affect the reliability of reinforced embankments on soft ground. These factors include undrained shear strength, critical reinforcement strain, reinforcement stiffness, Bjerrum's vane correction factor, unit weight and friction angle of the embankment fill, spatial variability of undrained shear strength in the foundation and reinforcement force distribution pattern. The difference in the reliability index between mean-value first-order second-moment method (MVFOSM) and the more rigorous first-order reliability method (FORM) is also studied.

7.2 Reliability analysis approach

Two different reliability approaches which consider spatial variability of soil properties are introduced in Chapter 6. Advantages and disadvantage of both approaches were discussed in Chapter 6. Due to the requirement for calculating equivalent scale of fluctuation approximately for the case where the spatial variability depends on the directions, variance reduction approach is considered not to be rigorous enough and therefore is not used in this chapter. The reliability analysis approach proposed in Chapter 6 which is based on FORM and can consider two-dimensional spatial variability is therefore employed herein to analyze the reinforced embankment probabilistically.

For any specified circular slip surface, the corresponding reliability index is determined by FORM with consideration of correlation coefficients between random variables. Searching for probabilistic circular slip surface is carried out based on the simplex method which is available in the optimization toolbox of Matlab. As observed from the analyses in Chapter 6, the searching process for the minimum reliability index and the associated critical slip surface can be easily trapped in local minimums. The searching process is carried out from different initial slip surfaces to deal with this situation.

Mean-value first-order second-moment method (MVFOSM) is also used to calculate the reliability index because of its simplicity. The main limitation, as discussed in Chapter 2, is that this method lacks invariant solution to different but mechanically equivalent formulations of the same performance function. Moreover, the performance function is linearized at the mean values of random variables and thus significant errors may be introduced when the performance function is nonlinear. However, since this method is still widely used in the reliability analysis of slopes or embankments, it is necessary to compare its results with those from the more rigorous first-order reliability method (FORM), especially when the spatial variability is of concern.

By taking the performance function and limit state as $F_s - 1 = 0$, the reliability index from MVFOSM can be defined by (Hassan and Wolff 1999)

$$\beta = \frac{F_s(m_{X_i}) - 1}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \frac{\partial F_s}{\partial X_i} \frac{\partial F_s}{\partial X_j} \rho_{X_i, X_j} \sigma_{X_i} \sigma_{X_j}}} \quad (7-1)$$

where n = number of random variables; m_{X_i} = mean value of random variables; σ_{X_i} = standard deviation of random variable X_i ; and ρ_{X_i, X_j} = correlation coefficient

between X_i and X_j . The partial derivative of the factor of safety with respect to each random variable is difficult to calculate theoretically. Numerical approximation is often used:

$$\frac{\partial F_s}{\partial X_i} = \frac{F_s(m_{X_i} + \lambda\sigma_{X_i}) - F_s(m_{X_i} - \lambda\sigma_{X_i})}{2\lambda\sigma_{X_i}} \quad (7-2)$$

where λ is an arbitrary constant, usually 1, according to Hassan and Wolff (1999). The effect of this parameter on the results of reliability index when spatial variability is of concern will be investigated.

Locating critical circular slip surface based on MVFOSM is similar to that based on FORM. The optimization procedure, which is the simplex method in this chapter, searches for the minimum reliability index by changing the location of the slip circle. Compared with reliability analysis based on FORM, only $2n+1$ deterministic stability analyses is needed for the calculation of reliability index corresponding to a specified circular slip surface. For FORM, due to the iteration procedure required to find the design point and the corresponding reliability index, the computations can be lengthy, especially when spatial variability is modeled. Therefore, reliability analysis based on MVFOSM is faster than that based on FORM.

7.3 Random variables

For the reinforced embankment on soft ground analyzed in this chapter (Figure 5.1), the undrained shear strength within the soft soil foundation is considered to have a mean value of 20 kPa. The coefficient of variation is assumed to vary from 10% to 30%. Unit weight γ_f and friction angle ϕ_f of the embankment fill are also modeled as random variables, with mean values 19 kN/m³ and 37°, respectively. The coefficient

of variation of the unit weight of the embankment fill is assumed to be between 1% and 5%. For the friction angle of the embankment fill, the corresponding coefficient of variation is assumed to vary from 5% to 15%.

For the critical reinforcement strain ε of this reinforced embankment, the upper bound of the coefficient of variation is 30% based on the studies in Chapter 5. In the parametric studies, this coefficient of variation is assumed to range from 10% to 30% in order to see how significant this factor is to the reliability of this reinforced embankment. The mean value is 2%. The reinforcement stiffness is also considered as a random variable since it, together with the reinforcement strain, determines the magnitude of the reinforcement force which is crucial to the stability of the reinforced embankment. Based on the review of Chapter 5, a coefficient of variation in the range of 1~10% is used in the parametric analysis. The mean value is 3000 kN/m.

Bjerrum vane correction factor, in which great uncertainties are involved, will have significant influence on the reliability of embankments on soft soil foundation as discussed in Chapter 6. Hence, this factor is modeled as a random variable. A mean value of 0.9 is used, with a coefficient of variation ranging from 10% to 30%.

7.4 Results of reliability analyses incorporating spatial variability

Two different reliability analyses based on MVFOSM and FORM, respectively and accounting for two-dimensional spatial variability are carried out using the direct autocorrelation approach described in Chapter 6. The vertical autocorrelation distance is assumed to range from 1m to 5m. The ratio of the horizontal autocorrelation distance to the vertical autocorrelation distance varies from 10 to 50 in the parametric studies to investigate the effect of the spatial variability on the results of reliability analysis.

7.4.1 Comparison of MVFOSM and FORM

The vertical and horizontal autocorrelation distances are assumed to be 1 m and 10 m, respectively. The coefficients of variation are 30%, 10%, 10%, 5% and 10% for critical mobilized reinforcement strain, reinforcement stiffness, friction angle, unit weight of the embankment fill and Bjerrum's vane correction factor, respectively. The probabilistic critical circular slip surface is located based on the Simplex method from different initial slip circles in both methods. All analyses are based on the extended Spencer's method.

Figure 7.1 Variation of the reliability index with the coefficient of variation of undrained shear strength, for MVFOSM and FORM respectively

Figure 7.1 shows the variation of the reliability index with the coefficient of variation of undrained shear strength by both MVFOSM method and FORM method. It can be seen that the reliability index decreases with the increase in the coefficient of variation of undrained shear strength for both methods of reliability analysis. It appears that the reliability index from MVFOSM method is slightly smaller than that from FORM

method. The difference in the reliability index between the two methods is found to be around 6-7% for the reinforced embankment analyzed in this study.

Figure 7.2 shows the critical circular slip surfaces from MVFOSM and FORM when the coefficient of variation of undrained shear strength is 20%. For comparison, deterministic critical circular slip surface is also plotted. The critical factor of safety corresponding to the deterministic critical circular slip surface is 1.25. The reliability index from is 1.61 using FORM. When MVFOSM is used, the reliability index is 1.52, which is 5.6% smaller than the value from FORM. The difference in the reliability index between MVFOSM and FORM has long been known to researchers. Chowdhury and Xu (1992) showed that the reliability index by FORM is usually greater than that by MVFOSM. Kitch (1994) compared the reliability index from MVFOSM with that from FORM for reinforced slopes and found that the former was almost 12% smaller than the latter.

Figure 7.2 Critical circular slip surfaces from deterministic and probabilistic stability analyses

It can be seen from Figure 7.2 that the probabilistic critical circular slip surfaces from FORM and MVFOSM are very close to each other for the reinforced embankment. The deterministic critical circular slip surface is slightly different from the two probabilistic critical circular slip surfaces. However, all three critical slip surfaces are located deeply and tangent to the bottom of the soft soil foundation.

Figure 7.3 shows the variation of the reliability index with the parameter λ of Eq. (7-2), which determines the points for numerical derivatives. The values of reliability index are calculated by MVFOSM based on different values of λ from 0.1 to 2. As shown in Figure 7.3, the reliability index remains almost constant regardless of the value of the parameter λ . Similar findings were obtained by Hassan and Wolff (1999) for a slope on clay foundation. In their analysis, the reliability index remained unchanged when the λ was increased from 0.5 to 1.5. In the other analyses of this chapter, a λ value of 1.0 will be used in the MVFOSM approach.

Figure 7.3 Variation of reliability index with the constant λ

7.4.2 Effect of critical reinforcement strain on reliability index

Critical reinforcement strain is one of the factors which determine the magnitude of the reinforcement force in the stability analysis. Hence, the reliability index of the reinforced embankment could be influenced by this factor. The coefficients of variation of the unit weight and the friction angle of the embankment fill, reinforcement stiffness and Bjerrum's vane correction factor are 5%, 10%, 10% and 10% in the analysis, respectively. The reliability index is calculated by FORM with search for the critical circular slip surface.

Figure 7.4 Variation of the reliability index with the coefficient of variation of critical reinforcement strain

Figure 7.4 shows the variation of the reliability index with the coefficient of variation of critical reinforcement strain under different coefficients of variation of undrained shear strength. It can be seen that the reliability index slightly decreases when the coefficient of variation of critical reinforcement strain increases from 10% to 30%

under different coefficients of variation of undrained shear strength. It appears that the coefficient of variation of critical reinforcement strain has negligible influence on the reliability of the reinforced embankment. For example, the reliability index for the coefficient of variation of critical reinforcement strain of 10% is only 0.6% higher than that for the coefficient of variation of 30% when the coefficient of variation of undrained shear strength is 20%. This shows the insensitivity of the reliability of the reinforced embankment to the critical reinforcement strain. Moreover, it also indicates that there is in fact no need to spend more efforts in investigating the uncertainties of the critical reinforcement strain.

7.4.3 Effect of reinforcement stiffness on reliability index

Apart from the critical reinforcement strain, reinforcement stiffness also determines the value of the reinforcement force in the reinforced embankment on soft ground. A higher stiffness means that a larger tensile force can be mobilized corresponding to a certain critical reinforcement strain. As a result, the reliability of the reinforced embankment could be influenced by the uncertainties of the reinforcement stiffness. In order to study the effect of the reinforcement stiffness on the results of reliability analysis of the reinforced embankment, the reliability index is calculated for different coefficients of variation of the reinforcement stiffness. Figure 7.5 shows the variation of the reliability index with the coefficient of variation of the reinforcement stiffness.

It can be seen that the reliability index changes very little with the coefficient of variation of the reinforcement stiffness under different coefficients of variation of undrained shear strength. It is obvious that the reliability index is not sensitive to the reinforcement stiffness. This observation is also confirmed by the design point obtained in computing the reliability index by FORM method. It is found that the value of the reinforcement stiffness in the design point is always equal to the mean value.

This indicates the insensitivity of the reliability index to the reinforcement stiffness.

Figure 7.5 Variation of the reliability index with the coefficient of variation of the reinforcement stiffness

7.4.4 Effect of Bjerrum's vane correction factor on reliability index

Figure 7.6 shows the variation of the reliability index with the coefficient of variation of Bjerrum's vane correction factor under different coefficients of variation of undrained shear strength. Compared with the effects of critical reinforcement strain and reinforcement stiffness, Bjerrum's vane correction factor is found to have much more significant influence on the reliability index. The reliability index decreases rapidly with the increase in the coefficient of variation of Bjerrum's vane correction factor. For small values of the coefficient of variation of Bjerrum's vane correction factor, it is apparent that the reliability index decreases with the increase in the coefficient of variation of undrained shear strength. However, when the coefficient of variation of Bjerrum's vane correction factor is relatively high, for example 30%, the

effect of the uncertainties of undrained shear strength on the reliability index will become almost negligible.

Figure 7.6 Variation of the reliability index with the coefficient of variation of Bjerrum's vane correction factor

7.4.5 Effect of unit weight of the embankment fill on reliability index

Figure 7.7 shows the variation of the reliability index with the coefficient of variation of unit weight of the embankment fill under different coefficients of variation of undrained shear strength. It can be seen that the reliability index decreases when the coefficient of variation of unit weight of the embankment fill is increased from 1% to 5%. For example, the reliability index for the coefficient of variation of the unit weight of the embankment fill of 1% is about 6.4% higher than that for the coefficient of variation of 5% when the coefficient of variation of undrained shear strength is 20%. However, the effect of uncertainties of unit weight of the embankment fill on the reliability index is not as significant as Bjerrum's vane correction factor. Alonso (1976) mentioned that the variability of the unit weight accounted for less than 8% of the total

variance of the factor of safety.

Figure 7.7 Variation of the reliability index with the coefficient of variation of unit weight of the embankment fill

7.4.6 Effect of the friction angle of the embankment fill on reliability index

Figure 7.8 shows the variation of the reliability index with the coefficient of variation of friction angle of the embankment fill under different coefficients of variation of undrained shear strength. The coefficient of variation of the friction angle of the embankment fill is increased from 5% to 15% under different coefficients of variation of undrained shear strength. It can be seen that the reliability index remains almost constant irrespective of the change in the coefficient of variation of the friction angle. This indicates that the reliability of the reinforced embankment is in fact not sensitive to the uncertainties of the friction angle of the embankment fill.

Figure 7.8 Variation of the reliability index with the coefficient of variation of friction angle of embankment fills

7.4.7 Effect of spatial variability on reliability index

The effect of spatial variability on the reliability index of the reinforced embankment is investigated in this section. Vertical autocorrelation distance is increased from 1 m to 5 m and the ratio of the horizontal autocorrelation distance to the vertical autocorrelation distance, r_h/r_v , is increased from 10 to 50. The coefficients of variation of undrained shear strength, critical reinforcement strain, reinforcement stiffness, unit weight and friction angle of the embankment fill and Bjerrum's vane correction factor are 20%, 30%, 10%, 5%, 10% and 10%, respectively. Figure 7.9 shows the variation of the reliability index with the vertical autocorrelation distance under different values of r_h/r_v . The reliability index decreases with the increase in the vertical autocorrelation distance when the ratio of the horizontal autocorrelation distance to the vertical autocorrelation distance is kept constant. The value of r_h/r_v also influences the reliability of the reinforced embankment. As the vertical autocorrelation distance

remains constant, the reliability index decreases with the increase in the value of r_h / r_v . This indicates that the reliability index of the reinforced embankment is also affected by the horizontal autocorrelation distance. A larger horizontal autocorrelation distance implies a lower reliability index for a particular vertical autocorrelation distance.

Figure 7.9 Variation of the reliability index with the vertical autocorrelation distance

7.4.8 Effect of reinforcement force distribution on reliability index

It is a common practice that a certain form of distribution pattern is assumed for the reinforcement force in the stability analysis of the reinforced embankment on soft ground. As mentioned before, the reinforcement force is commonly assumed to follow a half-sine distribution pattern in this research work. The reinforcement force reaches maximum in the centerline of the embankment and then decreases to zero at the toe of the embankment. It is well recognized that the actual reinforcement force distribution either from measurements in the field or from the finite element analysis cannot be well represented by a single distribution pattern and is greatly dependent on the

embankment geometry, soil properties, height of the embankment and even drainage conditions. Assuming a simple and analytical distribution pattern for the reinforcement force in the stability analysis is mainly for simplifying the analysis.

Figure 7.10 Variation of the reliability index with the coefficient of variation of undrained shear strength under two different assumptions about the distribution pattern of the reinforcement force

In this section, the effect of reinforcement force distribution on the reliability index of the reinforced embankment is studied. In addition to the half-sine type reinforcement force distribution, a linear type force distribution is also adopted. Figure 7.10 shows the variation of the reliability index with the coefficient of variation of undrained shear strength under two different assumptions on the distribution pattern of the reinforcement force. It can be seen that the reliability index based on the half-sine type reinforcement force distribution is in fact identical to that based on the linear type distribution. The difference in the reliability index between these two different assumptions is negligible. The reliability index is insensitive to the assumptions of the

reinforcement force distribution pattern. This is also consistent with the finding by Tandjiria et al. (2002) that the reinforcement force distribution pattern appears not to influence the factor of safety of the embankment.

7.5 Summary and conclusions

For a hypothetical reinforced embankment on soft ground, both MVFOSM and FORM are adopted to calculate the reliability index. Spatial variability is taken into account based on the direct autocorrelation approach. The results show that the reliability index from MVFOSM is around 6-7% smaller than that from FORM. For MVFOSM, the effect of the parameter λ used in numerical derivatives on the reliability index is also investigated. The probabilistic critical circular slip surface from MVFOSM is found to be very close to that from FORM for the case analyzed in this chapter.

For this reinforced embankment, the effects of undrained shear strength, critical reinforcement strain, reinforcement stiffness, Bjerrum's vane correction factor, unit weight of the embankment fill, friction angle of the embankment fill, spatial variability of undrained shear strength in the foundation and reinforcement force distribution pattern on the reliability index are investigated based on FORM. It is found that Bjerrum's vane correction factor is the most significant factor influencing the reliability of the reinforced embankment. Undrained shear strength also affects the reliability of the reinforced embankment greatly but it is not as significant as Bjerrum's vane correction factor.

Critical reinforcement strain is found to have only minor influence on the reliability index. For reinforcement stiffness, its effect on the reliability is almost negligible. The reinforcement force distribution is also found to have no effect on the reliability of the reinforced embankment. The shear strength of the embankment fill, as characterized by

the friction angle has little effect on the reliability index of the reinforced embankment. It is consistent with their negligible influence in the deterministic analysis of embankments on soft ground where the dominant resistance derives from the foundation soil.

The results of analysis show that the spatial variability of undrained shear strength has significant influence on the reliability of the reinforced embankment. The reliability index decreases with the increase of vertical or horizontal autocorrelation distance. This indicates the importance of properly determining the vertical and horizontal autocorrelation distances in a meaningful reliability analysis of the reinforced embankment on soft ground. The reliability index will be greatly underestimated if spatial variability is not taken into account.

CHAPTER 8 A SIMPLIFIED RELIABILITY ANALYSIS APPROACH BY RESPONSE SURFACE METHOD FOR REINFORCED EMBANKMENTS ON SOFT GROUND

8.1 Introduction

In this chapter, a simplified reliability analysis approach for reinforced embankments is proposed based on the response surface method. A progressive sampling procedure is used to generate the sampling points. The reliability index is obtained by the ellipsoid method based on the first-order reliability method (FORM). The failure probability is computed by the direct Monte Carlo simulation based on the response surface obtained or by the importance sampling technique. This approach enables practical engineers to carry out a first and quick probabilistic stability analysis for embankments or slopes and to identify the factors which significantly influence the stability. Most of the existing deterministic stability analysis programs can be used in combination with the proposed approach.

8.2 Response surface method

In probabilistic stability analysis, the failure probability p_f for a slope can be defined as:

$$p_f = P[G(\mathbf{X}) \leq 0] = \int_{G(\mathbf{X}) \leq 0} f(\mathbf{X}) d\mathbf{x} \quad (8-1)$$

where $G(\mathbf{X})$ is the performance function and can be defined as $G(\mathbf{X}) = F_s(\mathbf{X}) - 1$ for slope stability problems, $F_s(\mathbf{X})$ is the function to calculate the factor of safety of embankments and $f(\mathbf{X})$ is the joint probability density function of the basic variable vector $\mathbf{X}$. This equation is very difficult to evaluate for realistic slope systems because it is difficult to identify the joint density function $f(\mathbf{X})$ and to perform the integration of $f(\mathbf{X})$ over the entire failure domain $G(\mathbf{X}) \leq 0$. One way to evaluate Eq. (8-1) is to

use response surface method, which is a statistical technique proposed by Box and Wilson (1954) to find the operating conditions of a chemical process at which some response was optimized. The basic concept of the response surface method consists in approximating the unknown implicit performance function $G(\mathbf{x})$ by a simple and explicit n th order polynomial function $G'(\mathbf{x})$ with undetermined coefficients. Experiments or numerical analyses are then performed at various sampling points x_i to determine the unknown coefficients in the approximate polynomial function $G'(\mathbf{x})$. This method has been introduced and used in chapter 5 to approximate the critical reinforcement strain from finite element analysis. In this chapter, the main intention of using the response surface method is to calculate the design point through iterations in the reliability analysis.

Wong (1985) adopted a second-order polynomial function without quadratic terms to predict the critical load of the slope. However, the number of the cross terms can be very large when the number of random variables is high. Bucher and Bourgund (1990) used a second-order polynomial function without the cross terms to reduce the number of the sampling points. Their work is followed by many researchers. The reliability analyses using response surface method based on the second-order polynomial function without the cross terms include the works by Liu and Moses (1994), Tandjiria et al. (2000), Das and Zheng (2000), Guan and Melchers (2001) and Li (2000). Rajashekhar and Ellingwood (1993) pointed out that it may be inappropriate to totally neglect the cross terms since there are situations where cross terms are important. The accuracy of the response surface can be improved by inclusion of the cross terms in the polynomial function.

Another way to improve the accuracy of the response surface is to use a higher order polynomial function. However, the response surface function would become ill-conditioned if $G'(\mathbf{x})$ is of much higher degree than $G(\mathbf{x})$. Moreover, higher order polynomial functions can exhibit erratic behaviour in the sub-domains not covered by the experiments or numerical analyses. Therefore, the polynomial function with more than two orders will not be considered in this study.

The number of sampling points required to construct the response surface varies with the different forms of polynomial functions. A first-order polynomial function needs at least $n+1$ sampling points to form a response surface. As for the second-order polynomial function without the cross terms, the least number of sampling points is $2n+1$. The number increases to $(n+1)(n+2)/2$ when a second-order polynomial function with the cross terms is used.

Figure 8.1 Experiment designs for two random variables: (a) star design; (b) two-level factorial design and (c) central composite design (Myers and Montgomery 1995)

The selection of sampling points is very important in obtaining a good response surface, especially when the evaluation process is time consuming, for example, searching for noncircular slip surface to get the minimum factor of safety or

conducting nonlinear finite element analysis. The methodology used for generating sampling points is known as *experiment design* (ED). There are some commonly used schemes of experiment design, such as star design, two-level factorial design and central composite design. Figure 8.1 shows those design schemes for two random variables.

For the star design shown in Figure 8.1, the number of sampling points is $2n+1$ for n random variables. For two-level factorial design, the number of sampling points becomes 2^n . The number of sampling points increases to 2^n+2n+1 for central composite design. This exponential increase of the sampling points with the number of random variables results in extremely high computational efforts for the evaluation of the reliability of reinforced embankments, where a large number of random variables are involved. Selecting the sampling points with a reasonable number is not an easy task. If only a few sampling points are used, the approximate response surface obtained may not be very accurate and cannot represent the actual model. However, if the number of sampling points is too large, the computational efforts will become extremely high and may even be more time consuming than direct Monte Carlo simulation (Bucher and Bourgund 1990). In the proposed approach, the sampling points are selected in a progressive manner. The number of sampling points is increased progressively with the each iteration in a manner that efficiently builds on previous information (such as design point and trust region).

8.3 Algorithm for the proposed method

The calculation procedure for reliability index based on the proposed approach is shown in Figure 8.2. A detailed explanation of the algorithm is as follows:

1. For the problem of slope reliability, it is necessary to identify the random variables to be considered in the analysis. The selected variables should have significant effects on the slope stability. Usually, the random variables include unit weight, friction angle, undrained shear strength and even the depth of soft soil layer in the foundation.

Figure 8.2 Calculation procedures for reliability index based on the proposed approach

For reinforced embankments on soft ground, the reinforcement strain and reinforcement stiffness should also be considered in the analysis since these two factors determine the reinforcement force.

2. The first-order polynomial function is used to form the response surface. Level 1 experiment design (Figure 8.3(a)) is adopted to generate the sampling points. For the first-order polynomial function, at least $n+1$ sampling points are required for n random variables. The sampling points $\mathbf{s}_1$ are chosen as follows:

$$\mathbf{s}_1 = \begin{bmatrix} (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n) \\ (\bar{x}_1 - \theta \cdot k \sigma_1, \bar{x}_2, \dots, \bar{x}_n) \\ (\bar{x}_1, \bar{x}_2 - \theta \cdot k \sigma_2, \dots, \bar{x}_n) \\ \dots \\ (\bar{x}_1, \dots, \bar{x}_{n-1}, \bar{x}_n - \theta \cdot k \sigma_n) \end{bmatrix} \quad (8-2)$$

where $\bar{x}_i$ is the mean value of random variable x_i ; n is the number of random variables; σ_i is the standard deviation of random variable x_i ; k is an arbitrary factor and θ is an indication factor. Note that it is not easy to select a reasonable value for k . Bucher and Bourgund (1990) used a constant value of 3 for k . Rajashekhar and Ellingwood (1993) proposed to gradually reduce the value of k in the iteration in order to improve the response surface. Guan and Melcher (2001) investigated the effect of different values of k on the failure probability and pointed out that no clear guidance can be given to the proper selection of k . In this study, the values of k are selected to be 1, 2 and 3 for each analysis. The effect of k on the reliability index will be investigated.

The determination of θ depends on whether the random variable x_i is a “resistance” or “load” variable. For the “resistance” variable such as friction angle, undrained shear strength and reinforcement force, θ is equal to 1. For the “load” variable such as unit weight, θ is -1. The reason to adopt such an indication factor is to avoid any unnecessary calculation in the safe region and to ensure that the response surface can be constructed in the region close to the failure.

The ellipsoid method (Low and Tang 1997b, 2004) is carried out to obtain the reliability index β and the corresponding design point x^* . The trust region parameter σ^* is introduced herein and defined as:

$$\sigma_i^* = \min(|lb_i - x_i^*|, |ub_i - x_i^*|) / 3 \quad (8-3)$$

where $lb_i = \bar{x}_i - 3\sigma_i$ and $ub_i = \bar{x}_i + 3\sigma_i$ are the lower bound and upper bound of the random variable x_i . This parameter will be used later in the generation of the sampling points. The main purpose of using such a parameter is to ensure that the values of the sampling points are within the reasonable range of soil properties and physically feasible.

3. Second-order polynomial function without the cross terms is then adopted to improve the response surface based on the design point x^* and trust region parameter σ^* obtained in the previous stage. Level 2 experiment design (Figure 8.3(b)) is used. In this stage, at least $2n+1$ sampling points are required. The sampling points s_2 are selected as follows:

$$s_2 = \begin{bmatrix} (x_1^*, x_2^*, \dots, x_n^*) \\ (x_1^* \pm k\sigma_1^*, x_2^*, \dots, x_n^*) \\ (x_1^*, x_2^* \pm k\sigma_2^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^*, x_n^* \pm k\sigma_n^*) \end{bmatrix} \quad (8-4)$$

It can be seen that the sampling points are selected around the design point obtained in the previous stage. This guarantees that the response surface can be formed in the failure region. The ellipsoid method is then conducted to obtain a new reliability index and the corresponding new design point. The trust region parameter as defined before is also updated. Several iterations are used to get a converged reliability index according to the following criterion:

$$|\beta^{j+1} - \beta^j| \leq \varepsilon_\beta \quad (8-5)$$

where β^j is the reliability index obtained in the j th iteration and ε_β is the convergence tolerance for β . In this study, a value of 0.02 is used for ε_β . For slope reliability problem, convergence can be achieved very fast after only a few iterations.

4. Considering the fact that neglecting totally the cross terms may result in some errors, a second-order polynomial function with the cross terms is used to construct the response surface in this stage. The quality of the response surface is improved further by including the cross terms. The least number of sampling points is $(n+1)(n+2)/2$. Level 3 experiment design (Figure 8.3(c)) is used. The sampling points $\mathbf{s}_3$ are chosen to be as following:

$$\mathbf{s}_3 = \begin{bmatrix} (x_1^*, x_2^*, \dots, x_n^*) \\ (x_1^* \pm k\sigma_1^*, x_2^*, \dots, x_n^*) \\ (x_1^*, x_2^* \pm k\sigma_2^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^*, x_n^* \pm k\sigma_n^*) \\ (x_1^* - \theta \cdot k\sigma_1^*, x_2^* - \theta \cdot k\sigma_2^*, x_3^*, \dots, x_n^*) \\ (x_1^*, x_2^* - \theta \cdot k\sigma_2^*, x_3^* - \theta \cdot k\sigma_3^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^* - \theta \cdot k\sigma_{n-1}^*, x_n^* - \theta \cdot k\sigma_n^*) \end{bmatrix} \quad (8-6)$$

An iteration procedure same as the one in the previous step, is adopted to calculate the reliability index and obtain the corresponding design point and trust region parameter.

8.4 Using more sampling points to improve the quality of the response surface

The number of sampling points used in the above steps is the least number required to form the response surface. For example, Level 1 experiment design gives the minimum number of sampling points needed to form a response surface with first-order polynomial function. The response surface obtained under such conditions is called a

“saturated” one since the response surface will pass through all sampling points. Using more sampling points, of course, can improve the quality of the response surface. However, more computational efforts will be required if more sampling points are used. The proposed approach offers an option to use more sampling points in a systematic way to form the response surface based on the previous steps. The main purpose is to see whether the reliability results can be improved by using more sampling points. Level 4 experiment design (Figure 8.3 (d)) is conducted to further improve the response surface obtained in the previous steps. The number of sampling points will increase to two times the number of Level 3 experiment design and become $(n+1)(n+2)-1$. The reason to add sampling points in this stage is because the location of the design point usually becomes quite stable after a few iterations in the previous steps. Therefore, the sampling points added will be located very close to the failure region with high confidence. The sampling points $\mathbf{s}_4$ are chosen to be as follows:

$$\mathbf{s}_4 = \begin{bmatrix} (x_1^*, x_2^*, \dots, x_n^*) \\ (x_1^* \pm k\sigma_1^*, x_2^*, \dots, x_n^*) \\ (x_1^*, x_2^* \pm k\sigma_2^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^*, x_n^* \pm k\sigma_n^*) \\ (x_1^* - \theta \cdot k\sigma_1^*, x_2^* - \theta \cdot k\sigma_2^*, x_3^*, \dots, x_n^*) \\ (x_1^*, x_2^* - \theta \cdot k\sigma_2^*, x_3^* - \theta \cdot k\sigma_3^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^* - \theta \cdot k\sigma_{n-1}^*, x_n^* - \theta \cdot k\sigma_n^*) \\ \\ (x_1^* \pm 0.5 \cdot k\sigma_1^*, x_2^*, \dots, x_n^*) \\ (x_1^*, x_2^* \pm 0.5 \cdot k\sigma_2^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^*, x_n^* \pm 0.5 \cdot k\sigma_n^*) \\ (x_1^* - 0.5 \cdot \theta \cdot k\sigma_1^*, x_2^* - 0.5 \cdot \theta \cdot k\sigma_2^*, x_3^*, \dots, x_n^*) \\ (x_1^*, x_2^* - 0.5 \cdot \theta \cdot k\sigma_2^*, x_3^* - 0.5 \cdot \theta \cdot k\sigma_3^*, \dots, x_n^*) \\ \dots \\ (x_1^*, x_2^*, \dots, x_{n-1}^* - 0.5 \cdot \theta \cdot k\sigma_{n-1}^*, x_n^* - 0.5 \cdot \theta \cdot k\sigma_n^*) \end{bmatrix} \quad (8-7)$$

Since the response surface is not a “saturated” one any more, regression analysis is required to determine the coefficients of the second-order polynomial function with the cross terms. After obtaining the response surface, the ellipsoid method is used to calculate the reliability index and obtain the corresponding design point. Compared with the previous analysis steps where a “saturated” response surface is used, more computational efforts will be involved. However, it will be demonstrated later that no significant improvement will be obtained by using more sampling points for slope reliability problem.

Figure 8.3 Experiment designs adopted in the proposed approach for two random variables: (a) Level 1 ($n+1$ sampling points); (b) Level 2 ($2n+1$ sampling points); (c) Level 3 ($(n+1)(n+2)/2$ sampling points) and (d) Level 4 ($(n+1)(n+2)-1$ sampling points).

8.5 Calculation of the failure probability

The reliability analysis procedure described above only gives the value of reliability index and the corresponding design point. The failure probability cannot be obtained directly. One simple way to estimate the failure probability is to use the following equation:

$$p_f \cong 1 - \Phi(\beta) \quad (8-8)$$

where $\Phi(\)$ is the cumulative distribution function (CDF) of the standard normal variable and β is the reliability index calculated from FORM method, which denotes the distance between the design point and the mean value point in units of directional standard deviation. However, it should be pointed out that Eq. (8-8) is only exact for linear limit state surface and normal distribution.

The second way to calculate the failure probability is to conduct direct Monte Carlo simulation based on the response surface obtained from the proposed approach. Since the response surface is expressed by an explicit polynomial function, Monte Carlo simulation can be efficiently carried out even for a large number of runs. However, since the accuracy of the response surface is only guaranteed in the neighborhood of the design point, the estimation from the response surface may not be very accurate for the region far from the design point. As a result, the failure probability from direct Monte Carlo simulation based on the response surface will be affected.

A more rational and efficient way to calculate the failure probability is to use importance sampling technique (Haldar and Mahadevan 2000). The basic idea of importance sampling is to concentrate the distribution of sampling points in the “region of interest” instead of the whole range of the random variables. The basic definition of failure probability is given by Eq. (8-1). This equation can be rewritten as the following by introducing a simple indicator function $I_g(\mathbf{x})$:

$$p_f = \int_{G(X) \leq 0} I_g(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} \quad (8-9)$$

where

$$I_g(\mathbf{x}) = \begin{cases} 0, & g(\mathbf{x}) > 0 \\ 1, & g(\mathbf{x}) \leq 0 \end{cases} \quad (8-10)$$

A new sampling probability density function $h(\mathbf{x})$ is defined to obtain sampling points in the important region. Therefore, the probability of failure can be given by the following equation:

$$p_f = \int_{G(X) \leq 0} \left[\frac{I_g(\mathbf{x}) f(\mathbf{x})}{h(\mathbf{x})} \right] \cdot h(\mathbf{x}) d\mathbf{x} \quad (8-11)$$

Therefore, the failure probability from Monte Carlo simulation can be calculated by

$$p_f = \frac{1}{N} \sum_{i=1}^N I_g(\mathbf{x}_i) \frac{f(\mathbf{x}_i)}{h(\mathbf{x}_i)} \quad (8-12)$$

where N is the number of simulations.

By using importance sampling technique, it is possible that the number of runs required to obtain an accurate estimation about the failure probability can be dramatically reduced, compared with the direct Monte Carlo simulation. However, two difficulties are involved in applying importance sampling method. The first is that the region of importance should be known in advance before carrying out Monte Carlo simulation based on importance sampling technique. The second one is that the accuracy of the estimation about the failure probability from importance sampling technique depends on the choice of the sampling probability density function $h(\mathbf{x})$.

It is usually not easy to select an appropriate sampling probability density function. Shinozuka (1983) proposed to adopt a uniform distribution for the sampling

probability density function $h(\mathbf{x})$ over a defined region. Harbitz (1986) shifted $f(\mathbf{x})$ to the design point and directly used it as the sampling probability density function $h(\mathbf{x})$. Schueller and Stix (1987) suggested that it is reasonable to use a normal sampling probability density function $h(\mathbf{x})$ with its mean in the design point $\mathbf{x}^*$ for failure domains where only one design point exists. As for the variance, there is still no practical guideline for a proper selection. Schueller and Stix (1987) mentioned that assuming a unit variance in the standard normal space could yield satisfactory results. Melchers (1999) described more advanced ways to select an improved sampling probability density function. In this study, the sampling probability density function $h(\mathbf{x})$ is chosen according to the suggestion of Schueller and Stix (1987) due to its simplicity. Figure 8.4 shows the one-dimensional relationship between the probability density function $f(\mathbf{x})$ and the sampling probability density function $h(\mathbf{x})$. The mean value and the standard deviation of variables following the sampling probability density function $h(\mathbf{x})$ is selected to be the design point $\mathbf{x}^*$ and trust region parameter σ^* obtained from the proposed approach described above.

Figure 8.4 One-dimensional relationship between $f(\mathbf{x})$ and $h(\mathbf{x})$

The usage of the trust region parameter σ^* is to ensure that the points generated based on the sampling probability density function $h(\mathbf{x})$ are within the reasonable range and

physically admissible. This can avoid any unrealistic result from each calculation in Monte Carlo simulation.

8.6 Validation of the proposed approach

8.6.1 A hypothetical two-layer slope

A hypothetical two-layer slope is adopted first to validate the proposed approach. This slope is extracted from a series of parametric studies by Hassan and Wolff (1999). Figure 8.5 shows the geometry of this two-layer slope. The unit weight of the upper layer was not reported in Hassan and Wolff (1999). According to the deterministic back-analysis carried out in this study, a unit weight of 19 kN/m^3 for the upper layer can give a factor of safety similar to that obtained in Hassan and Wolff (1999). As a result, this value is used in the present analysis. Only two random variables are considered for this case: friction angle ϕ' of the upper layer and undrained shear strength c_u in the lower layer.

Figure 8.5 A hypothetical two-layer slope

The semi-analytical stability analysis method (Low 1989; Low and Tang 1997b) is used herein to calculate the factor of safety. Due to its analytical nature, this method can be carried out very fast on the computer, even when the optimization procedure is used to search for critical circular slip surface. This means that it can be combined with the direct Monte Carlo simulation to calculate the failure probability of the slope

efficiently. The result of the failure probability from direct Monte Carlo simulation can thus be used as the reference value to judge the effectiveness of the proposed approach. The critical factor of safety from the semi-analytical method is 1.23 for this slope. For comparison, Spencer's method is also used and the corresponding critical factor of safety is 1.26, which is only slightly higher. Direct Monte Carlo simulation with 1×10^6 runs gives a failure probability of 12.9% with 95% confidence in the range of $12.9\% \pm 0.2\%$. Figure 8.6 shows the histogram of the factors of safety from Monte Carlo simulation. The mean value and the standard deviation of the factors of safety are 1.23 and 0.2, respectively.

Figure 8.6 Histogram of the factors of safety from Monte Carlo simulation (1×10^6 runs)

The reliability index is found to be 1.129 based on first-order reliability method (FORM) by directly searching for the probabilistic critical circular slip surface according to the direct search method (Low and Tang 1997b). The failure probability is 12.95% according to Eq. (8-8), which is almost identical to that (12.9%) obtained from direct Monte Carlo simulation with a large number of runs. This close match indicates that the limit state surface could be almost linear. Figure 8.7 shows the limit state surface of the slope. The limit state surface represented by the solid line is

obtained from the actual slope model. For a fixed value of the friction angle, undrained shear strength corresponding to the factor of safety equal to 1.0 was back-calculated based on the slope model. It can be seen that the nonlinearity of the limit state surface for this simple case is negligible. Low et al. (1998) had a similar finding that the limit state surface was quite linear for a homogeneous slope with c' and ϕ' as the random variables.

Figure 8.7 Limit state surfaces from actual slope model and response surface method

The results from the proposed approach based on the response surface method are presented in Tables 8-1, 8-2 and 8-3 for different values of k . The reliability index calculated in the first iteration, where a first-order linear polynomial function is adopted to form the response surface, is already identical to the solution from the direct search method. In this stage, only three sampling points are required. Using second-order polynomial function without or with the cross terms in the later iteration cannot improve the result. Even when more sampling points are used, the reliability index still remains constant. It also can be seen that different values of k (1, 2 and 3) do not have any effect on the reliability index obtained for this case.

In addition to the reliability index, the failure probability can also be calculated efficiently by direct Monte Carlo simulation based on the response surface obtained in

the last iteration. After 1×10^6 runs, the failure probability is found to be $12.85\% \pm 0.07\%$ which is very close to the 12.9% obtained earlier based on the semi-analytical method. This means that the obtained response surface represents the actual limit state surface very well. By plotting the response surface in Figure 8.7, a near-perfect match can be seen.

Table 8-1 Iteration results from the proposed approach for the two-layer slope ($k=1$)

Iteration	Design points		Trust region σ'		β	Type of the response surface function
	ϕ' (deg)	c_u (kPa)	ϕ' (deg)	c_u (kPa)		
1	29.58	29.72	2.86	4.80	1.129	Linear polynomial function
2	29.58	29.73	2.86	4.80	1.129	Second order polynomial function without the cross terms
3	29.58	29.73	2.86	4.80	1.129	Second order polynomial function with the cross terms
4	29.57	29.73	2.86	4.80	1.129	Adding 5 more sampling points

Table 8-2 Iteration results from the proposed approach for the two-layer slope ($k=2$)

Iteration	Design points		Trust region σ'		β	Type of the response surface function
	ϕ' (deg)	c_u (kPa)	ϕ' (deg)	c_u (kPa)		
1	29.59	29.72	2.86	4.80	1.130	Linear polynomial function
2	29.57	29.73	2.86	4.80	1.129	Second order polynomial function without the cross terms
3	29.57	29.73	2.86	4.80	1.129	Second order polynomial function with the cross terms
4	29.57	29.73	2.86	4.80	1.129	Adding 5 more sampling points

Table 8-3 Iteration results from the proposed approach for the two-layer slope ($k=3$)

Iteration	Design points		Trust region σ'		β	Type of the response surface function
	ϕ' (deg)	c_u (kPa)	ϕ' (deg)	c_u (kPa)		
1	29.60	29.71	2.87	4.80	1.130	Linear polynomial function
2	29.57	29.73	2.86	4.80	1.129	Second order polynomial function without the cross terms
3	29.57	29.73	2.86	4.80	1.129	Second order polynomial function with the cross terms
4	29.57	29.73	2.86	4.80	1.129	Adding 5 more sampling points

A sensitivity analysis can be carried out by calculating the Spearman rank correlation coefficients based on the results of Monte Carlo simulation. Figure 8.8 shows the Spearman rank correlation coefficients calculated for the two random variables of this slope. Undrained shear strength is found to have great influence on the reliability of this slope. Compared with undrained shear strength, the contribution of the friction angle is minor.

Figure 8.8 Sensitivity analysis based on Spearman rank correlation coefficients for the two-layer slope

As described in the previous section, the failure probability can be calculated by using importance sampling technique. The number of sampling points is selected to be 20 which are generated based on the sampling probability density function $h(\mathbf{x})$ as described in the previous section. The intention of using such few simulation points is to demonstrate the advantage of using importance sampling in calculating the failure probability of the embankment. Each simulation is repeated 20 times in order to evaluate the variance of the calculated failure probability. The failure probability from importance sampling is found to have a mean value of 12.82% and a standard deviation of 2%. This close agreement with the real solution (12.9%) indicates that importance sampling technique is useful to calculate the failure probability when combined with the proposed reliability analysis approach based on the response surface method.

Table 8-4 summarizes the reliability analysis results for the two-layer slope. The reliability index calculated by mean-value first-order second-moment method (MVFOSM) by Hassan and Wolff (1999) is also included for comparison purpose. The corresponding failure probability will be about 9.21% if Eq. (8-8) is used.

Table 8-4 Summaries of the reliability analysis results for the two-layer slope

Method	Reliability index	Failure probability		
MVFOSM ¹	1.328 ¹	9.21% ²		
The direct search method	1.129	12.9% ²		
Direct Monte Carlo simulation	--	12.9%		
The proposed approach	1.129	12.9% ²	12.85% ³	12.82% ⁴

¹By Hassan and Wolff (1999)

²By Eq. (8-8)

³By direct Monte Carlo simulation based on the response surface obtained.

⁴By importance sampling method.

It can be observed that the reliability index from the proposed approach is identical to that from the direct search method. For this case, since the limit state surface is quite linear, the failure probabilities calculated by Eq. (8-8), Monte Carlo simulation based on the response surface and importance sampling technique are very close to each other and almost identical to that from direct Monte Carlo simulation with a large

number of runs based on the semi-analytical method. The reliability index obtained by MVFOSM method according to Hassan and Wolff (1999) is around 18% higher than that from the proposed approach, where FORM method is used. The reason for the difference in the reliability index between the MVFOSM and FORM has been already discussed in Chapter 7.

8.6.2 Application to James Bay Dykes

Reliability of James Bay dykes was extensively investigated by many researchers. Soulie et al. (1990) investigated the various sources of uncertainty associated with this case. Christian et al. (1994) and El-Ramly (2001) carried out the probabilistic analyses using MVFOSM method and Monte Carlo simulation, respectively. Figure 8.9 shows the geometry of James Bay dykes. The foundation of this dyke consists of a crust which ranges between 1.0 and 6.0 m thick with an average of 4.0 m. The average undrained shear strength over the crust is estimated to be 41 kPa for the entire thickness by accounting for the potential of fissures reducing the operational strength of the crust. Below the crust is a sensitive marine clay overlying a lacustrine clay. The marine clay is about 8 m thick and the lacustrine clay is about 6.5 m thick. The mean values of undrained shear strength for the marine clay and for the lacustrine clay are about 35 kPa and 31 kPa, respectively. The lacustrine clay is underlain by a very stiff layer of till.

Figure 8.9 Geometry of James Bay Dykes

The proposed approach will be adopted to carry out the reliability analysis for this case. Since the main purpose is to validate the proposed approach by comparing the results with those reported by other researchers, only circular slip surface will be considered in the analysis.

Six random variables for James Bay Dykes are considered and summarized in Table 8-5. These variables include unit weight γ_{fill} and friction angle ϕ_{fill} of the embankment fill, the shear strength of Marine Clay layer s_{uM} and the shear strength of Lacustrine Clay layer s_{uL} , the thickness of the Crust clay layer t_{cr} and the depth of the till layer D_{till} . All variables are assumed to be normally distributed. According to El-Ramly (2001), the standard deviations shown in Table 8-5 were the end result of a detailed statistical analysis by Christian and his colleagues. The analysis included the removal of random measurement errors, estimation of statistical errors due to sample size, accounting for bias due to Bjerrum's vane correction factor and variance reduction due to spatial averaging.

Table 8-5 Input variables and statistical parameters for James Bay Dyke (after Christian et al. 1994; El-Ramly 2001; El-Ramly et al. 2002)

Input parameters	Mean Value	Standard Deviation
Fill unit weight, γ_{fill} (kN/m ³)	20.0	1.1
Fill friction angle, ϕ_{fill} (deg.)	30.0	1.79
Thickness of clay crust, t_{cr} (m)	4.0	0.48
Strength of Marine clay, s_{uM} (kPa)	34.5	3.95
Strength of Lacustrine, s_{uL} (kPa)	31.2	6.31
Depth of till layer, D_{till} (m)	18.5	1.0

The deterministic stability analysis based on Spencer's method gives a factor of safety of 1.477 for the critical circular slip surface. This result is slightly higher than that obtained by Christian et al. (1994) based on Bishop's simplified method (1.453). Tables 8-6, 8-7 and 8-8 show the iteration results from the proposed approach for different values of k . It can be seen that the analysis cannot converge when ε_β is set to 0.02 and k is equal to 1. However, the difference in the reliability index between two consecutive iterations is 0.023 which is not very large. The reliability index will converge to the final solution very fast for $k = 2$ or $k = 3$. The reliability index is found to be 2.63 for $k = 2$ and 2.64 for $k = 3$. The difference is very minor. Adding more sampling points does not improve the result in both cases.

The reliability index reported by Christian et al. (1994) was 2.66 based on the mean value first-order second moment method (MVFOSM). It is slightly higher than that obtained by the proposed approach. El-Ramly (2001) carried out a direct Monte Carlo simulation with 14000 runs based on the fixed critical circular slip surface which is assumed to be tangential to the till layer and pass through a fixed point at the top of the embankment. This approximation is made to reduce the computational efforts involved in searching for the critical slip surface in each run. According to El-Ramly (2001), the critical factor of safety is not influenced much by this assumption. The failure probability found by El-Ramly (2001) was 0.27% from the direct Monte Carlo simulation where Bishop's simplified method was adopted to calculate the factor of safety.

According to the proposed approach, the failure probability is 0.43% if Eq. (8-8) is used. If the direct Monte Carlo simulation with 20000 runs is carried out based on the obtained response surface function, the failure probability is $0.37\% \pm 0.09\%$. Figure 8.10 shows the histogram of the factor of safety. Both results are higher than that (0.27%) obtained by El-Ramly (2001). Importance sampling method is also used to calculate the failure probability of this case with Monte Carlo simulation in an efficient manner. The number of sampling points is selected to be 200. Each simulation is repeated 25 times. The mean value of the failure probability is 0.27% and the standard deviation is 0.05%.

Table 8-6 Iteration results from the proposed approach for James Bay Dykes ($k=1$)

Iteration	Design points						Trust region (σ')					β	Type of the response surface function
	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	t_{cr} (m)	s_{uM} (kPa)	s_{uL} (kPa)	D_{fill} (m)	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	t_{cr} (m)	s_{uM} (kPa)	s_{uL} (kPa)	D_{fill} (m)	
1	20.97	29.49	3.98	32.06	19.18	19.86	0.78	1.62	0.47	3.14	2.30	0.55	First-order linear polynomial function
2	20.85	29.63	3.94	31.94	16.82	19.48	0.82	1.67	0.46	3.10	1.52	0.68	Second-order polynomial function without the cross terms
3	20.80	29.64	3.90	31.94	16.80	19.22	0.83	1.67	0.45	3.10	1.51	0.76	
4	20.83	29.65	3.86	31.97	16.94	19.46	0.82	1.67	0.43	3.11	1.56	0.68	
5	20.80	29.65	3.90	31.92	16.74	19.21	0.83	1.67	0.45	3.09	1.49	0.76	
6	20.83	29.64	3.85	31.91	16.97	19.45	0.82	1.67	0.43	3.09	1.57	0.68	
7	20.80	29.64	3.90	31.95	16.72	19.21	0.83	1.67	0.45	3.10	1.48	0.76	

Table 8-7 Iteration results from the proposed approach for James Bay Dykes ($k=2$)

Iteration	Design points							Trust region (σ')					β	Type of the response surface function
	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	t_{cr} (m)	s_{uM} (kPa)	s_{uL} (kPa)	D_{fill} (m)	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	t_{cr} (m)	s_{uM} (kPa)	s_{uL} (kPa)	D_{fill} (m)		
1	20.97	29.49	3.98	32.06	19.18	19.86	0.78	1.62	0.47	3.14	2.30	0.55	2.59	First-order linear polynomial function
2	20.85	29.64	3.89	31.97	16.56	19.26	0.82	1.67	0.44	3.11	1.43	0.75	2.66	Second-order polynomial function without the cross terms
3	20.80	29.65	3.89	32.21	16.95	19.41	0.83	1.67	0.44	3.18	1.56	0.70	2.62	
4	20.78	29.66	3.89	32.04	17.25	19.47	0.84	1.68	0.44	3.13	1.66	0.68	2.61	
5	20.79	29.64	3.89	31.99	17.13	19.47	0.84	1.67	0.44	3.11	1.62	0.68	2.63	Second-order polynomial function with the cross terms
6	20.79	29.64	3.89	31.99	17.13	19.47	0.84	1.67	0.44	3.11	1.62	0.68	2.63	

Table 8-8 Iteration results from the proposed approach for James Bay Dykes ($k=3$)

Iteration	Design points						Trust region (σ')						β	Type of the response surface function
	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	t_{cr} (m)	s_{uM} (kPa)	s_{uL} (kPa)	D_{till} (m)	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	t_{cr} (m)	s_{uM} (kPa)	s_{uL} (kPa)	D_{till} (m)		
1	21.11	29.37	3.94	31.43	15.98	19.43	0.73	1.58	0.46	2.93	1.24	0.69	2.90	First-order linear polynomial function
2	20.81	29.63	3.88	32.08	16.73	19.37	0.83	1.67	0.44	3.14	1.49	0.71	2.65	Second-order polynomial function without the cross terms
3	20.80	29.65	3.88	32.01	16.94	19.30	0.83	1.67	0.44	3.12	1.56	0.74	2.60	
4	20.82	29.64	3.88	32.17	16.73	19.34	0.83	1.67	0.44	3.17	1.49	0.72	2.64	Second-order polynomial function with the cross terms
5	20.82	29.63	3.88	32.14	16.64	19.31	0.83	1.67	0.44	3.16	1.46	0.73	2.64	
6	20.82	29.65	3.88	32.17	16.69	19.32	0.83	1.67	0.44	3.17	1.47	0.73	2.64

Figure 8.10 Histogram of the factors of safety from Monte Carlo simulation (2×10^4 runs) for James Bay Dykes based on the response surface

Table 8-9 summarizes all the results of reliability analysis for this case. It can be seen that the failure probability from importance sampling method in the proposed approach is identical to that obtained by El-Ramly (2001) based on direct Monte Carlo simulation. This perfect match after only a few iterations is very encouraging.

Table 8-9 Summaries of the reliability analysis results for James Bay dykes

Method	Reliability index	Failure probability		
MVFOSM ¹	2.66	--		
Direct Monte Carlo simulation ²	--	0.27%		
The proposed approach	2.63	0.43% ³	0.37% ⁴	0.27% ⁵

¹By Christian et al. (1994)

²By El-Ramly (2001)

³By Eq. (8-8)

⁴By direct Monte Carlo simulation based on the response surface obtained

⁵By importance sampling method

The good agreement observed in the reliability analysis of James Bay dykes demonstrates the usefulness of importance sampling technique even for low failure probability. Figure 8.11 shows the sensitivity analysis results by calculating Spearman rank correlation coefficients for James Bay Dykes. This is based on the results from direct Monte Carlo simulation in which the response surface obtained from the proposed approach is used. The undrained shear strength of the lacustrine clay is found to be the most important influence factor affecting the reliability of this case. The other two important influence factors are the unit weight of the embankment fill and the depth of the till layer. The thickness of the crust layer has insignificant effect on the reliability of this case. Similar conclusions can be found in the sensitivity analyses by Christian et al. (1994) and El-Ramly (2001).

Figure 8.11 Sensitivity analysis based on Spearman rank correlation coefficients for James Bay Dykes

8.7 Application to the reinforced embankment on soft ground

In this section, the proposed approach will be further illustrated by analyzing the reliability of the reinforced embankment on soft ground shown in Figure 5.1. Table 8-10 lists the input variables and statistical parameters used in the reliability analysis for

this reinforced embankment. Extended Spencer's method as described in Chapter 4 is used to calculate the factor of safety. The vertical and horizontal autocorrelation distances are assumed to be 1 m and 10 m, respectively. The negative exponential autocorrelation function (Eq. (6-1)) is adopted. As a result, the vertical and horizontal scales of fluctuation are 2 m and 20 m, respectively. The spatial variability is taken into account by the variance reduction approach which is described and discussed in Chapter 6. For simplicity, the variance reduction factor for the undrained shear strength is calculated based on the deterministic critical circular slip surface according to Eq. (6-6). This assumption is made based on the observation that the input parameters listed in Table 8-10 have only negligible effect on the critical circular slip surface for this reinforced embankment. The critical circular slip surface always touches the bottom of the soft soil foundation.

Table 8-10 Input variables and statistical parameters for the reinforced embankment on soft ground

Input parameters	Mean Value	Coefficient of variation
Unit weight, γ_{fill} (kN/m ³)	19	5%
Friction angle, ϕ_{fill} (deg.)	37	10%
Critical reinforcement strain, ε_f	2%	30%
Reinforcement stiffness, J (kN/m)	3000	10%
Undrained shear strength, s_u (kPa)	20	20%
Bjerrum's vane correction factor, μ	0.9	10%

Tables 8-11, 8-12 and 8-13 show the iteration results from the proposed approach for the reinforced embankment on soft ground under different values of k . No convergence problem is observed for this case. The reliability index converges to 1.76 after four iterations for $k = 1$, $k = 2$ and $k = 3$. The parameter k has no effect on the reliability index. It is noted that this reliability index (1.76) is about 8.5% higher than that (1.61) obtained from the previous analysis in the Chapter 7.

Table 8-11 Iteration results from the proposed approach for the reinforced embankment on soft ground ($k=1$)

Iteration	Design points						Trust region (σ')					β	Type of the response surface function
	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	ε_f	J (kN/m)	s_u (kPa)	μ	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	ε_f	J (kN/m)	s_u (kPa)	μ	
1	19.63	36.81	1.92%	3000	19.23	0.78	0.74	3.64	0.57%	300	0.81	0.05	First-order linear polynomial function
2	19.61	36.81	1.92%	3000	19.24	0.77	0.75	3.64	0.57%	300	0.82	0.05	Second-order polynomial function without the cross terms
3	19.60	36.81	1.92%	3000	19.25	0.77	0.75	3.64	0.57%	300	0.82	0.05	
4	19.60	36.82	1.92%	3000	19.25	0.77	0.75	3.64	0.57%	300	0.82	0.05	Second-order polynomial function with the cross terms
5	19.63	36.81	1.92%	3000	19.23	0.78	0.74	3.64	0.57%	300	0.81	0.05	

Table 8-12 Iteration results from the proposed approach for the reinforced embankment on soft ground ($k=2$)

Iteration	Design points						Trust region (σ')					β	Type of the response surface function
	γ_{fill} (kN/m^3)	ϕ_{fill} (deg.)	ε_f	J (kN/m)	s_u (kPa)	μ	γ_{fill} (kN/m^3)	ϕ_{fill} (deg.)	ε_f	J (kN/m)	s_u (kPa)	μ	
1	19.60	36.80	1.91%	3000	19.22	0.78	0.75	3.63	0.57%	300	0.81	0.05	First-order linear polynomial function
2	19.61	36.81	1.92%	3000	19.24	0.77	0.75	3.64	0.57%	300	0.82	0.05	Second-order polynomial function without the cross terms
3	19.60	36.81	1.92%	3000	19.25	0.77	0.75	3.64	0.57%	300	0.82	0.05	
4	19.60	36.81	1.92%	3000	19.25	0.77	0.75	3.64	0.57%	300	0.82	0.05	Second-order polynomial function with the cross terms
5	19.60	36.81	1.92%	3000	19.25	0.77	0.75	3.64	0.57%	300	0.82	0.05	

Table 8-13 Iteration results from the proposed approach for the reinforced embankment on soft ground (k=3)

Iteration	Design points						Trust region (σ')						β	Type of the response surface function
	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	ε_f	J (kN/m)	s_u (kPa)	μ	γ_{fill} (kN/m ³)	ϕ_{fill} (deg.)	ε_f	J (kN/m)	s_u (kPa)	μ		
1	19.58	36.79	1.90%	3000	19.21	0.78	0.76	3.63	0.57%	300	0.81	0.05	1.68	First-order linear polynomial function
2	19.61	36.80	1.91%	3000	19.24	0.77	0.75	3.63	0.57%	300	0.82	0.05	1.76	Second-order polynomial function without the cross terms
3	19.60	36.80	1.91%	3000	19.25	0.77	0.75	3.63	0.57%	300	0.82	0.05	1.76	
4	19.60	36.81	1.91%	3000	19.25	0.77	0.75	3.64	0.57%	300	0.82	0.05	1.76	Second-order polynomial function with the cross terms
5	19.58	36.79	1.90%	3000	19.21	0.78	0.76	3.63	0.57%	300	0.81	0.05	1.76	

This is because variance reduction approach is used in the proposed simplified procedure to consider the spatial variability of undrained shear strength within the clay foundation, whereas the direct autocorrelation approach was used in the Chapter 7. As discussed in Chapter 6, there is some difference in the reliability index between these two different approaches of considering spatial variability. Moreover, in order to simplify the analysis, the variation reduction factor is calculated only based on the deterministic critical circular slip surface.

The failure probability of this reinforced embankment is 3.94% if Eq.(8-8) is used. The direct Monte Carlo simulation with 50,000 runs based on the response surface gives a failure probability of $4.24\% \pm 0.18\%$. The mean value and the standard deviation of the factor of safety are 1.251 and 0.150, respectively. Figure 8.12 shows the histogram of the factor of safety from the direct Monte Carlo simulation based on the response surface. Sensitivity analysis is also carried out by calculating Spearman rank correlation coefficients.

Figure 8.12 Histogram of the factors of safety from Monte Carlo simulation (5×10^4 runs) for the reinforced embankment on soft ground

Figure 8.13 shows the results of the sensitivity analysis. It can be seen that the most important factor for this reinforced embankment is Bjerrum's vane correction factor. The other two important influence factors are the unit weight of the embankment fill and the undrained shear strength of soft ground, respectively. The contributions of these two different factors to the reliability of the reinforced embankment are very close to each other. the friction angle of the embankment fill is found to have very minor influence on the reliability. This is also consistent with the finding in Chapter 7 where the sensitivity of the friction angle to the reliability is investigated by calculating the reliability indices for different coefficients of variation of friction angle of the embankment fill. The contribution of the reinforcement stiffness J to the reliability is also insignificant.

Figure 8.13 Sensitivity analysis based on Spearman rank correlation coefficients for the reinforced embankment on soft ground

This also can be inferred from the value of the reinforcement stiffness in the final design point, which is 3000 kN/m and identical to the mean value. The effect of the reinforcement strain on the reliability of the reinforced embankment on soft ground is

not very significant. The value of the reinforcement strain in the final design point is around 1.92% which is only slightly smaller than the mean value of the reinforcement strain. This conclusion is similar to that obtained in Chapter 7.

The failure probability of this reinforced embankment is also calculated by importance sampling based on the information on the design point by the response surface method. The number of sampling points is 100. Each simulation is repeated 30 times. The mean value of the failure probability is 4.04% and the standard deviation is 0.72%. It can be seen that this value is in a good agreement with that (3.94%) based on Eq. (8-8) and that (4.24%) from the Monte Carlo simulation based on the response surface.

8.8 Summary and conclusions

A simplified reliability analysis approach is proposed for the embankment on soft ground based on the response surface method. The reliability index is calculated by the ellipsoid method based on first-order reliability method. The response surface is formed progressively. A linear polynomial function is first used to obtain an initial estimation about the reliability index and the corresponding design point with a few calculations. Then, second-order terms and the cross terms are added to improve the results based on the information obtained previously. In this stage, the iteration procedure is employed to obtain the convergence. Finally, more additional sampling points can be added to further improve the results.

After obtaining the design point and the trust region parameter, the importance sampling technique is adopted to calculate the failure probability of the embankment based on the slope stability analysis model. Only a few calculations are required to obtain a good estimation about the failure probability. It is more efficient than the direct Monte Carlo simulation, which may need a large number of calculations. The sensitivity of the input parameters to the reliability of the embankment can be identified easily by calculating Spearman rank correlation coefficients based on the direct Monte Carlo simulation for the response surface obtained.

The proposed approach has been validated by two case studies: a two-layer slope and the James Bay dykes. The results are compared with those reported in the literature. The comparisons show that the proposed approach can provide reasonable estimation of the reliability of the embankment. One interesting finding is that the reliability index and the corresponding design point are already very close to the final solution after the first iteration where the first-order polynomial function is used. Adding the second-order and cross terms subsequently does not improve the results very much. The parameter k which is used to generate the sampling points in each iteration is found to have negligible influence on the reliability results. The only exception is in the case study of James Bay dykes where convergence cannot be obtained when $k = 1$ and the convergence criterion is set to 2%. For $k = 2$ and $k = 3$, no convergence problem was encountered.

The results of analysis for a reinforced embankment on soft ground show that Bjerrum's vane correction factor is the most important influence factor for the reliability. The critical reinforcement strain and reinforcement stiffness do not have significant effect on the reliability of the reinforced embankment. This indicates that the reliability of the reinforced embankment is mainly governed by the uncertainties of soil parameters. Hence, attention can be focused on the accurate determination of soil properties in the design of reinforced embankments on soft ground.

The application of this approach is not only confined to the conventional limit equilibrium method. More advanced methods such as limit analysis method and finite element method can also be used to calculate the rigorous reliability index based on FORM method. Computing the failure probability can be done by importance sampling technique or the direct Monte Carlo simulation based on the response surface obtained.

CHAPTER 9 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

9.1 Summary and conclusions

A series of studies have been conducted to investigate the behaviour of unreinforced and reinforced embankments on soft ground and to integrate probabilistic analysis into the stability evaluation of reinforced embankments on soft ground. In the following sections, summaries and conclusions of the various tasks and findings of this study are presented.

9.1.1 Effects of reinforcement on the behaviour of embankments on soft ground

Based on the numerical analyses carried out for reinforced embankment on soft ground, the effect of reinforcement in reducing ground surface settlement is found to be significant only when reinforced embankment reaches its critical height at which the unreinforced embankment is close to the failure state.

The reinforcement has some effect on reducing the lateral displacement of soft soils in the foundation. The most significant reduction of the lateral displacements occurs in the upper, very soft silty clay layer. The lateral displacements below this layer are not influenced by the existence of the reinforcement. The beneficial effect brought about by the reinforcement is limited to a certain range and can only reach the depth of around 8 m within the foundation. This indicates that the reinforcement can significantly improve the stability of the embankment on soft ground provided that the thickness of the soft soil is not too large.

Little reduction on the excess pore water pressure is observed by the inclusion of

horizontal reinforcement at the interface between the embankment and the foundation clay.

The numerical analysis also shows that the stability of the embankment can be improved by using the reinforcement. The net height of the embankment can be increased. To some extent, the development of plastic zone in the clay foundation can be restrained by the reinforcement. The formation of a continuous plastic zone in the foundation is observed to be delayed with the use of reinforcement. The failure state can be considered to be already reached for the reinforced embankment when excessive deformations occur in the clay foundation. Usually, the net height of the embankment starts to decrease at that stage.

9.1.2 Determination of the factor of safety for embankments on soft ground

Different stability analysis methods have been used to calculate the factors of safety for unreinforced and reinforced embankments on soft ground. These methods include effective stress analysis based on Spencer's method, enhanced limit method and strength reduction technique using the finite element package PLAXIS.

For the unreinforced embankment studied, these three methods yield quite close prediction of the factor of safety. The result from strength reduction technique for the unreinforced embankment is shown to be the closest to the real condition. For the reinforced embankment, inconsistent results are observed in the analysis by strength reduction technique. The possible reason may be the nonconvergence criterion used to judge the failure in strength reduction technique.

9.1.3 Location of critical circular or noncircular slip surface using different optimization procedures

For circular slip surface analysis, three different optimization approaches are adopted to search for the critical slip surface of reinforced embankment on soft ground. These methods include the grid search method, the simplex method and genetic algorithm. It is found that the grid search method is the most robust, efficient and reliable approach to locate the critical circular slip surface. The stability analysis results indicate that solutions from the simplex method depend on the selection of initial trial slip surface. For genetic algorithm, despite the fact that this method tends to give global solution to the stability problems and does not rely on the starting location of the slip surface, there is still no evidence that the results from this method are better than those from the grid search method.

For noncircular slip surface analysis, both BFGS and pattern search methods give close and reliable predictions of the location of the critical noncircular slip surface and the minimum factor of safety for a reinforced embankment on homogeneous soft ground. The solutions do not depend on the selection of the initial trial slip surface. Consequently, both optimization procedures can be regarded as robust and reliable for the reinforced embankment on homogeneous soft ground.

However, if a weak soil layer is assumed to be present in the mid-depth of the soil foundation, the solutions from BFGS and pattern search methods are found to depend highly on the selected initial trial slip surface. Starting from different slip surfaces, both procedures can lead to different locations of critical noncircular slip surfaces and the corresponding factors of safety. The stability analysis results also indicate that, an unsuccessful identification of the critical noncircular slip surface can result in a significant overestimation of the factor of safety and thus renders the design

unconservative.

9.1.4 Uncertainties of the critical reinforcement strain

Reinforced embankments on soft ground can be regarded to have failed when excessive deformations occur within the clay foundation. The mobilized strain in the reinforcement under such a circumstance, defined as the critical reinforcement strain in the present study, is considered to be more appropriate in limit equilibrium analysis for calculating the factor of safety with respect to the shear strength of soils. A procedure based on sequential construction of response surfaces is proposed to investigate the uncertainties associated with this critical reinforcement strain. Direct Monte Carlo simulation is combined with finite element analysis to study the effects of undrained shear strength and undrained modulus ratio on the variability of the critical reinforcement strain using the proposed procedure based on response surface method.

Undrained modulus ratio is found to have a more significant influence on the variability of the critical reinforcement strain. Especially in the cases where the coefficient of variation of undrained modulus ratio is as high as 20%, the coefficient of variation of the critical reinforcement strain can remain around 30% irrespective of the uncertainties of the undrained shear strength. Therefore, in the reliability analysis of reinforced embankments on soft ground, an upper bound of 30% can be reasonably used for the coefficient of variation of critical reinforcement strain.

9.1.5 Effect of spatial variability of soil properties on reliability analysis

For a simple hypothetical embankment on homogeneous soft foundation, it is found that the probabilistic critical slip surface is different from the deterministic critical slip surface and depends on the vertical autocorrelation distance. The results also show that

the horizontal autocorrelation distance has only minor influence on the probabilistic critical slip surface. The effect of the number of slices in the clay foundation on reliability index is also investigated since the number of random variables is proportional to the number of slices. The studies indicate that the results of reliability analysis are insensitive to the number of slices when it exceeds a certain value. The effect of spatial variability of undrained shear strength within the clay foundation is found to have significant influence on the reliability index of the embankment. With increase in the vertical or horizontal autocorrelation distance, the reliability index decreases initially, but approaches a constant value at large autocorrelation distance. The studies on the difference in the reliability index between the direct autocorrelation approach and the variance reduction approach show that the difference varies from 0.17% to 10% depending on the vertical and horizontal autocorrelation distances.

9.1.6 Various factors influencing the reliability of reinforced embankment on soft ground

For the reinforced embankment on soft ground, the effects of undrained shear strength, critical reinforcement strain, reinforcement modulus, Bjerrum's vane correction factor, unit weight of the embankment fill, friction angle of the embankment fill, spatial variability of undrained shear strength in the foundation and reinforcement force distribution pattern on the reliability index are investigated based on FORM method. It is found that Bjerrum's vane correction factor is the most significant factor influencing the reliability of the reinforced embankment. Undrained shear strength also affects the reliability of the reinforced embankment greatly but it is not as significant as Bjerrum's vane correction factor.

The critical reinforcement strain is found to have only minor influence on the reliability index. For reinforcement modulus, its effect on the reliability is almost

negligible. The reinforcement force distribution only affects the reliability index of the reinforced embankment slightly. The soil properties of the embankment fill: unit weight and friction angle, appear not to have significant effects on the reliability of the reinforced embankment.

Spatial variability of undrained shear strength has significant influence on the reliability of the reinforced embankment. The reliability index decreases with the increase of vertical or horizontal autocorrelation distance. This indicates the importance of properly determining the vertical and horizontal autocorrelation distances in reliability analysis of the reinforced embankment. It also can be seen that the reliability index will be greatly underestimated if spatial variability is not taken into account.

9.1.7 Application of the response surface method in the reliability analysis of embankments

A simplified reliability analysis approach is proposed for embankments on soft ground based on the response surface method. Only a few calculations are required to obtain a good estimation of the failure probability. It is more efficient than the direct Monte Carlo simulation, which may need a large number of calculations. The sensitivity of the input parameters to the reliability of the embankment is identified by calculating the Spearman rank correlation coefficients based on direct Monte Carlo simulation of the response surface obtained.

The results of analysis show that the proposed approach can provide reasonable estimation of the reliability of the embankment. One interesting finding is that the reliability index and the corresponding design point are already very close to the final solution after the first iteration where the first-order polynomial function is used.

Adding the second-order and cross terms subsequently does not improve the result very much. The parameter k which is used to generate the sampling points in each iteration is found to have negligible influence on the reliability results.

The only exception is in the case study of James Bay dykes where the convergence cannot be obtained when $k = 1$ when the convergence criterion is set to 2%. When $k = 2$ and $k = 3$, no convergence problem is observed for the cases considered.

The desirable feature of the proposed procedure is that its application is not restricted to the conventional limit equilibrium method. More advanced methods such as limit analysis method and finite element method can also be used to calculate the rigorous reliability index based on FORM method. Computation of the failure probability can be conducted by importance sampling technique or the direct Monte Carlo simulation based on the response surface obtained.

9.2 Recommendations for future work

The followings are recommended for future studies:

- It has been stated in Chapter 3 that strength reduction technique with nonconvergence failure criterion cannot consider the fact that excessive deformations may occur or that the net height of the embankment has already started to decrease before nonconvergence criterion is reached. A new failure criterion should be adopted by strength reduction technique to judge the failure of the reinforced embankment on soft ground. The failure can be considered to have occurred when the net height of the embankment starts to decrease.
- More advanced stress interpolation scheme, such as least squares stress analysis

method, can be used in the enhanced limit method to better interpolate the stresses. It is interesting to compare the factor of safety with that from nodal average stress interpolation scheme. It will be very encouraging if the factor of safety is found to be insensitive to the stress interpolation scheme used.

- The studies of variability of the critical reinforcement strain are limited to reinforced embankments on homogeneous soft ground. Future work can consider more realistic foundation soil profile.
- The effects of spatial variability of soil properties on reliability results of embankments are investigated by searching for the critical circular slip surface. Future work can extend this study with searching for the critical noncircular slip surface.
- In the simplified reliability analysis method proposed in Chapter 8, the sampling probability density function $h(\mathbf{x})$ is chosen according to the suggestion of Schueller and Stix (1987) due to its simplicity. More advanced methods in the selection of the sampling probability density function as described in Melchers (1999) can be used in future work.

REFERENCE

- Abramson, M. A. (2005). *NOMADm version 3.31 User's Guide*. Department of Mathematics and Statistics, Air Force Institute of Technology.
- Al-Homoud, A. S. and Tanash, N. (2004). "Modeling uncertainty in stability analysis for design of embankment dams on difficult foundations." *Engineering Geology*, 71, pp. 323-342.
- Alonso, E. E. (1976). "Risk analysis of slopes and its application to slopes in Canadian sensitive clays." *Geotechnique*, 26, pp. 453-472.
- Ang, A. H-S., and Tang, W. H. (1984). *Probability concepts in engineering planning and design, Volume II decision, risk and reliability*. John Wiley & Sons, New York.
- Ang, A. H-S., and Tang, W. H. (1990). *Probability concepts in engineering planning and design, Volume II decision, risk and reliability*. John Wiley & Sons, New York.
- Audet, C. and Dennis Jr, J. E. (2003). "Analysis of generalized pattern searches." *SIAM Journal of Optimization*, 13(3), pp. 889-903
- Babu, G. L. S. and Mukesh, M. D. (2003). "Risk analysis of landslides-A case study." *Geotechnical and Geological Engineering*, 21, pp. 113-127.
- Babu, G. L. S. and Mukesh, M. D. (2004). "Effect of soil variability on reliability of soil slopes." *Geotechnique*, 54, pp. 335-337.
- Baecher, G. B. (1997) "Parameters and approximations in geotechnical reliability," in,

B.M. Ayyub, Ed., *Uncertainty Modeling and Analysis in Civil Engineering*, CRC Press, FL.

Baecher, G. B., and Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*, John Wiley & Sons, Inc.

Baker, R. and Garber, M. (1978). "Theoretical analysis of the stability of slopes". *Geotechnique*, 28(4), pp. 395-411.

Baker, R. (1980). "Determination of the critical slip surface in slope stability computations." *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 4, pp. 333-358.

Baker, R. (1981). "Tensile strength, tension cracks, and stability of slopes." *Soils and Foundations*, 21(2), pp.1-17.

Bardet, J. P. and Kapuskar, M. M. (1989). "Simplex analysis of slope stability." *Computers and Geotechnics*, 8(4), pp. 329-348.

Becker, D. E. (1996a). "Eighteenth Canadian geotechnical colloquium: limit state design for foundations. Part I. An overview of the foundation design process". *Canadian Geotechnical Journal*, 33(6), pp. 956-983

Becker, D. E. (1996b). "Eighteenth Canadian geotechnical colloquium: limit state design for foundations. Part II. Development for the national building code of Canada". *Canadian Geotechnical Journal*, 33(6), pp. 984-1007.

Bergado, D. T. and Chang, J. C. (1989). "Application of an inter-active computer-aided

probabilistic slope stability assessment for embankments on soft Bangkok clay.” *Proceedings of International Symposium on computer and physical modeling in geotechnical engineering*, Bangkok, Thailand, pp. 89-109.

Bergado, D. T., Chai, J. C., and Miura, N. (1995). “FE analysis of grid reinforced embankment system on soft Bangkok clay.” *Computers and Geotechnics*, 17, pp. 447-471.

Bergado, D. T., Long, P. V., and Murthy, B. R. S. (2002). “A case study of geotextile-reinforced embankment on soft ground.” *Geotextiles and Geomembranes*, 20, pp. 343-365.

Bhattacharya, G. (2002). “Discussion to ‘Global search method for locating general slip surface using Monte Carlo techniques’.” *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 128(12), pp. 1048.

Bhattacharya, G., Jana, D., Ojha, S., and Chakraborty. (2003). “Direct search for minimum reliability index of earth slopes.” *Computers and Geotechnics*, 30, pp. 455-462.

Biot, M. A. (1941). “General theory of three-dimensional consolidation.” *Journal of Applied Physics*, 12, pp. 155-164.

Bishop, A. W. (1952). *The stability of earth dams*. Ph. D. Thesis. University of London.

Bishop, A. W. (1955). “The use of slip circle in the stability analysis of slopes”. *Geotechnique*, 5(1), pp. 7-17.

Bjerg, P.L., Hinsby, K., Christensen, T.H., and Gravesen, P. (1992). "Spatial variability of hydraulic conductivity of an unconfined sandy aquifer determined by mini slug test." *Journal of Hydrology*, 136, pp. 107-122

Bjerrum, L. (1966). "Mechanism of progressive failure in slopes of overconsolidated plastic clays and clay shales." *The 3rd Terzaghi Lecture, ASCE Structural Engineering Conference, Miami, USA*.

Bjerrum, L. (1972). "Embankments on soft ground." *Proc., ASCE Specialty Conf. on Performance of Earth and Earth-Supported Structures*, Purdue University, West Lafayette, Indiana, Vol. II, pp. 1-54.

Bonaparte, R. and Berg, R. (1987). "Long-term allowable tension for geosynthetic reinforcement." *Geosynthetics'87 Conference Proceedings*, Vol 1, pp: 181-192. New Orleans.

Borges, J. L. and Cardoso, A. S. (2001). "Structural behaviour and parametric study of reinforced embankments on soft clays." *Computers and Geotechnics*, 28, pp. 209-233.

Borges, J. L. and Cardoso, A. S. (2002). "Overall stability of geosynthetic-reinforced embankments on soft soils." *Geotextiles and Geomembranes*, 20, pp. 395-421.

Boutrup, A. W. and Lovell, C. W. (1980). "Search technique in slope stability analysis". *Engineering Geology*, 16, pp. 51-61.

Boutrup, E., and Holtz, R. D. (1983). "Analysis of embankments on soft ground reinforced with geotextiles." *Proceedings, Eighth European Conference on Soil Mechanics and Foundation Engineering*, Helsinki, Vol. 2, pp. 469-472.

Box, G. E. P and Wilson, K. B. (1954). "The exploration and exploitation of response surfaces: some general considerations and examples." *Biometrics*, 10(1), 16-60.

Brakel, J., Coppens, M., Maagdenberg, A. C., and Risseeuw, P. (1982). "Stability of slopes constructed with polyester reinforcing fabric, test section at Almere, Holland, '79." *Proceedings, Second International Conference on Geotextiles, Las Vegas, Nevada*. Vol. 3, pp. 727-732.

Brand, E. W. (1991). "Predicted and observed performance of an embankment built to failure on soft clay." *Geotechnical Engineering, Southeast Asian Geotechnical Society*, 22(1), pp. 23-42.

Brinkgreve, R. B. J. (2002). *PLAXIS User's Manual, Version 8*. A. A. Balkema.

Brinkgreve, R. B. J., and Bakker, H. L. (1991). "Non-linear finite element analysis of safety factors." *Proceeding of Computer Methods and Advances in Geomechanics*, pp. 1117-1122.

Brown, C. B., and King, I. P. (1967). "Automatic embankment analysis equilibrium and instability conditions." *Journal of Soil Mechanics and Foundation Division, ASCE*, 93(4), pp. 209-219.

Brown, B. S., and Poulos, H. G. (1978). "Finite element analysis of reinforced embankments." *Proceedings, Symposium on Soil Reinforcing and Stabilizaing Techniques*. Sydney, Australia, pp. 233-252.

Bucher, C. G. and Bourgund, U. (1990). "A fast and efficient response surface approach for structural reliability problems." *Structural Safety*, 7(1), 57-66.

Burland, J. B. (1965). "The yielding and dilation of clay." *Geotechnique*, 15(2), pp. 211-214.

Calle, E. O. F. (1985). "Probabilistic approach of stability of earth slopes." *Proceedings of 11th International Conference on Soil mechanics and Foundation engineering, San Francisco*, Vol. 2, pp. 809-812.

Castillo, E. and Luceno, A. (1982). "A critical analysis of some variational methods in slope stability analysis". *International Journal of Numerical and Analytical Methods in Geomechanics*, 6, pp. 195-209.

Celestino, T. B. and Duncan, J. M. (1981). "Simplified search for non-circular slip surfaces". *Proceeding of the 10th International Conference on Soil Mechanics and Foundation Engineering*, Stockholm, pp. 391-394.

Chai, J. C. and Bergado, D. T. (1993). "Performance of reinforced embankment on Muar clay deposit." *Soils and Foundations*, 33(4), pp. 1-17.

Chai, J. C., Sakajo, S., and Miura, N. (1994). "Stability analysis of embankment on soft ground (A case study)." *Soils and Foundations*, 34(2), pp. 107-114.

Chai, J. C., Miura, N., and Shen, S. L. (2002). "Performance of embankments with and without reinforcement on soft subsoil." *Canadian Geotechnical Journal*, 39, pp. 838-848.

Chen, X., and Lind, N. C. (1983) "Fast probability integration by three-parameter normal tail approximation." *Structural Safety*, 1, pp. 269-276.

Chen, Z. Y. and Morgenstern, N. R. (1983). "Extension to the generalized method of slices for stability analysis". *Canadian Geotechnical Journal*, 20(1), pp. 104-119.

Chen, Z. Y. and Shao, C. M. (1988). "Evaluation of minimum factor of safety in slope stability analysis". *Canadian Geotechnical Journal*, 25, pp. 735-748.

Chen, Z. Y. (1992). "Random trials used in determining global minimum factors of safety of slopes". *Canadian Geotechnical Journal*, 29, pp. 225-233.

Chen, Z. Y. and Li, S. M. (1998). "Evaluation of active earth pressure by the generalized method of slices." *Canadian Geotechnical Journal* 35, pp. 591-599 .

Cheng, S. C. and Christopher, B. (1991). "A probabilistic review of geotextiles reinforced slope design." *Geosynthetics '91: Conference Proceedings*, Atlanta, GA, 26-28 Feb, Vol. 1. Industrial Fabrics Assoc. Intl., St. Paul, MN, pp. 455-468.

Cheng, Y. M. (2003). "Location of critical failure surface and some further studies on slope stability analysis". *Computers and Geotechnics*, 30, pp. 255-267.

Chiasson, P., Lafleur, J., Soulie, M., and Law, K.T. (1995). "Characterizing spatial variability of a clay by geostatistics." *Canadian Geotechnical Journal*, 32, pp. 1-10.

Chowdhury, R. N. (1984). "Recent developments in landside studies: Probabilistic methods". *Proceedings of the 4th International Symposium on Landslides*, Toronto, Canada, Vol. 1, pp. 209-228.

Chowdhury, R. N. and Xu, D. W. (1992). "Reliability index for slope stability assessment-two methods compared." *Reliability Engineering and System Safety*, 37(2),

pp. 99-108.

Chowdhury, R. N., and Xu, D. W. (1994). "Slope system reliability with general slip surfaces." *Soils and Foundations*, 34(3), pp. 99-105.

Christian, J. T., Ladd, C. C., and Baecher, G. B. (1994). "Reliability applied to slope stability analysis." *Journal of Geotechnical Engineering, ASCE*, 120, pp. 2180-2207.

Christian, J.T., and Baecher, G.B. (1999). "Point estimate method as numerical quadrature." *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 125(9): 779-786.

Christie, I. F. and El-Hadi, K. M. (1977). "some aspects of the design of earth dams reinforced with fabric." *Proceedings, First International Conference on the Use of Fabrics in Geotechnics, Paris*. Vol. 1, pp. 99-103.

Christopher, B. R., Gill, S. A., Giroud, J. P., Juran, I., Mitchell, J. K., Schlosser, F., and Dunncliff, J. (1990). *Reinforced Soil Structures*, FHWA, Washington, DC, U.S.A

Cornell, C. A. (1971). "First order uncertainty analysis of soils deformation and stability." *Proceeding of 1st International Conference on Application of Statistics and Probability to Soil and Structural Engineering*, Hong Kong, September 13-16, pp. 129-144

D'Appolonia, D. J., Poulos, H. G., and Ladd, C. C. (1971). "Initial settlement of structures on clay." *Journal of Soil Mechanics and Foundation Engineering, ASCE*, 97(10), pp. 1359-1377.

Das, P. K. and Zheng, Y. (2000). "Cumulative formation of response surface and its use in reliability analysis." *Probabilistic Engineering Mechanics*, 15, pp. 309-315.

Davis, E. H. (1968). "Theories of plasticity and failure of soil masses." *Chapter 6 in Soil Mechanics-Selected Topics*, Ed. by T. K. Lee, Butterworths, London, pp. 341-380.

Dawson, E. W., Roth, W. H., and Drescher, A. (1999). "Slope stability analysis by strength reduction." *Geotechnique*, 49(6), 835-840.

DeGroot, D. G., Baecher, G. B. (1993), "Estimating autocovariance of in-situ soil properties". *Journal of Geotechnical Engineering, ASCE*, 119(1), pp. 147-166

DeNatale, J. S. (1991). "Rapid identification of critical slope surface: structure". *Journal of Geotechnical Engineering, ASCE*, 117(10), pp. 1568-1589.

Ditlevsen, O. (1981). *Uncertainty modeling with applications to multidimensional civil engineering systems*. McGraw-Hill Book Co., Inc., New York.

Duncan, J. M. and Buchignani, A. L. (1976). "An engineering manual for settlement studies." *Geotechnical Engineering Report*, Department of Civil Engineering, University of California at Berkeley.

Duncan, J. M., Byrne, P., Wong, K. S., and Mabry, P. (1980). "Strength, stress-strain and bulk modulus parameters for finite element analysis of stresses and movements in soil." *Geotechnical Engineering Research Report No. UCB/GT/80-01*, Department of Civil Engineering, University of California, Berkeley, 1980.

Duncan, J. M. (1996). "State of the art: Limit equilibrium and finite-element analysis

of slopes”. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 122(7), pp. 577-596.

Duncan, J.M. (2000). “Factors of safety and reliability in geotechnical engineering.” *Journal of the Geotechnical and Geoenvironmental Engineering, ASCE*, 126(4): 307-316.

Duncan, J. M. and Wright, S. G. (2005). *Soil Strength and Slope Stability*, John Wiley & Sons, Inc.

Dunlop, P., and Duncan, J. M. (1970). “Development of failure around naturally excavated slopes.” *Journal of the Soil Mechanics and Foundations Division, ASCE*, 96(2), pp. 471-493.

El-Ramly, H. (2001). *Probabilistic Analyses of Landslide Hazards and Risks: Bridging Theory and Practice*. Ph. D thesis, University of Alberta, Canada.

El-Ramly, H. E., Morgenstern, N. R., and Cruden, D. M. (2002). “Probabilistic slope stability analysis for practice.” *Canadian Geotechnical Journal*, 39, pp. 665-683.

Farias, M. M., and Naylor, D. J. (1998). “Safety analysis using finite elements.” *Computers and Geotechnics*, 22(2), pp. 165-181.

Fellenius, W. (1936). “Calculation of stability of earth dams.” *Transactions, 2nd Congress Large Dams*, Vol. 4, pp. 445, Washington, D. C.

Fredlund, D. G. and Krahn, J. (1977). “Comparison of slope stability methods of analysis”. *Canadian Geotechnical Journal*, 14(3), pp. 429-439.

Fredlund, D. G. (1984). "Analytical methods for slope analysis". *International Symposium on Landslides*, pp. 229-250.

Fredlund, D. G., and Scoular, R. E. G. (1999). "Using limit equilibrium concepts in finite element slope stability analysis." *Proceeding of Slope Stability Engineering*, Yagi, Japan, pp. 31-47.

Giam, P. S. K. (1989). *Improved methods and computational approaches to geotechnical stability analysis*. Ph.D. thesis, Monash University, Clayton, Australia.

Goh, A. T. C. (1999). "Genetic algorithm search for critical slip surface in multiple-wedge stability analysis." *Canadian Geotechnical Journal*, 36, pp. 382-391.

Goh, A. T. C (2000). "Search for critical slip circle using genetic algorithms". *Civil Engineering and Environmental Systems*, 17(3), pp. 181-211.

Goodman, R. E., Taylor, R. L., and Brekke, T. L. (1968). "A model for the mechanics of jointed rock." *Journal of Soil Mechanics and Foundation Engineering*, ASCE, 94, pp. 637-659.

Greco, V. R. (1988). "Numerical methods for locating the critical slip surface in slope stability." *Proceeding of 6th International Conference Numerical Methods in Geomechanics*, Innsbruck, pp. 1219- 1223.

Greco, V. R. (1996). "Efficient Monte Carlo technique for locating critical slip surface". *Journal of Geotechnical Engineering*, ASCE, 122(7), pp. 517-525.

Greco, V. R. (2003). "Discussion of 'An efficient search method for finding the critical

circular slip surface using the Monte Carlo technique’.” *Canadian Geotechnical Journal*, 40(1), pp. 221-222.

Griffiths, D. V. and Lane, P. A. (1999). “Slope stability analysis by finite elements.” *Geotechnique*, 48(3), 387-403.

Guan, X. L. and Melchers, R. E. (2001). “Effect of response surface parameter variation on structural reliability estimates.” *Structural Safety*, 23, pp. 429-444.

Haldar, A., and Mahadevan, S. (2000). *Probability, Reliability, and Statistical Methods in Engineering Design*. John Wiley & Sons, New York.

Harr, M. E. (1987). *Reliability-based design in civil engineering*. McGraw-Hill Inc, USA.

Harbitz, A. (1986). “An efficient sampling method for probability of failure calculation.” *Structural Safety*, 3(2), pp. 109-115.

Hasofer, A. A., and Lind, A. M. (1974). “Exact and invariant second-moment code format.” *Journal of Engineering Mechanics, ASCE*, 100(1), pp. 111-121.

Hassan, A. M. and Wolff, T. F. (1999). “Search algorithm for minimum reliability index of earth slopes.” *Journal of Geotechnical and Geoenvironmental engineering, ASCE*, 125, pp. 301-308.

Hinchberger, S. D. and Rowe, R. K. (2003). “Geosynthetic reinforced embankments on soft clay foundations: predicting reinforcement strains at failure.” *Geotextiles and Geomembranes*, 21, pp: 151-175.

Hinton, E. (1968). *Least squares analysis using finite elements*. M.Sc. thesis. University of Wales, University College of Swansea.

Hinton, E., and Irons, B. M. (1968). "Least squares smoothing of experimental data using finite elements." *Journal of the British Society for Strain Measurement*, 14, pp. 24-27.

Hinton, E., and Compell, J. S. (1974). "Local and global smoothing of discontinuous finite element functions using a least squares method." *Journal of numerical methods in engineering*, 8, pp. 461-480.

Hird, C. C. (1986). "Stability charts for reinforced embankments on soft ground". *Geotextiles and Geomembranes*, 4(2), pp. 107-127.

Hird, C. C., and Kwok, C. M. (1989). "Finite element studies of interface behaviour in reinforced embankments on soft ground." *Computers and Geotechnics*, 8, pp. 111-131.

Hird, C. C., Pyrah, I. C., and Russell, D. (1990). "Finite element analysis of the collapse of reinforced embankments on soft ground." *Geotechnique*, 40(4), pp. 633-640.

Hodge, J. W. (1992). "Quality control in the manufacture of polyester geotextiles." *MQC/MQA and CQC/CQA of Geosynthetics*, Philadelphia, 10-11 Dec., Geosynthetic Research Inst., Philadelphia, PA, pp. 68-76.

Houck, C. R., Joines, J. A., and Kay, M. G. (1995). "A Genetic Algorithm for Function Optimization: A Matlab Implementation." NCSU-IE TR 95-09.

Humphery, D. N. and Holtz, R. D. (1986). "Reinforced embankments-A review of case histories". *Geotextiles and Geomembranes*, 4(2), pp. 129-144.

Indraratna, B., Balasubramaniam, A. S., and Balachandran, S. (1992). "Performance of test embankment constructed to failure on soft marine clay." *Journal of Geotechnical Engineering, ASCE*, 118(1), pp. 12-33.

Ingold, T. S. (1982). "An analytical study of geotextiles reinforced embankments". *Proceedings, Second International Conference on Geotextiles, Las Vegas, Nevada*, Vol. 3, pp. 683-688.

Janbu, N. (1973). *Slope stability computations*, In "Embankment dam engineering" Casagrande Volume, Wiley, pp. 47-86.

Jewell, R. A. (1982). "A limit equilibrium design method for reinforced embankments on soft foundations". *Proceedings, Second International Conference on Geotextiles, Las Vegas, Nevada*, Vol. 4, pp. 671-676.

Jewell, R. A. (1988). "The mechanics of reinforced embankments on soft soils". *Geotextiles and Geomembranes*, 7, pp. 237-273.

Jones, C. P. D. (1986). "Design parameters for high strength geotextiles." *Proceedings of Third International Conference on Geotextiles, Vienna*, Vol. 2, pp. 325-330.

Kaniraj, S. R. and Abdullah, H. (1993a). "Effect of berms and tension crack on the stability of embankments on soft soils." *Soils and Foundations*, Vol. 33, No. 4, pp. 99-107.

Kaniraj, S. R. and Abdullah, H. (1993b). "Rotational stability of narrow-crested reinforced embankments on soft soils." *Geotextiles and Geomembranes*, 12, pp. 599-641.

Kaniraj, S. R. and Abdullah, H. (1994). "The effect of berms and tension cracks on the maximum reinforcement force in embankments on soft soils." *Geotextiles and Geomembranes*, 13, pp. 101-117.

Kaniraj, S. R. and Panwar, V. K. (1994). "Maximum required bisectorial reinforcement force in embankments on soft ground." *Geotextiles and Geomembranes*, 13, pp. 247-261.

Kaniraj, S. R. (1994). "Rotational stability of unreinforced and reinforced embankments on soft soils." *Geotextiles and Geomembranes*, 13, pp. 707-726.

Kaniraj, S. R. (1996). "Directional dependency of reinforcement force in reinforced embankments on soft soil." *Geotextiles and Geomembranes*, 14, pp. 507-519.

Kim, H-S., Major, G., and Ross-Brown, D. M. (1978). "Applications of Monte Carlo techniques to slope stability analyses." *Proceeding of the 19th US Symposium on Rock Mechanics*, Nevada, USA, pp. 28-39.

Kim, J. Y., and Lee, S. R. (1997). "An improved search strategy for the critical slip surface using finite element stress fields". *Computer and Geotechnics*, 21(4), pp. 295-313.

Kim S. H., and Na S. W. (1997). "Response surface method using vector projected sampling points." *Structural Safety*, 19, pp. 3 - 19.

Kitch, W. A. (1994). *Deterministic and probabilistic based analyses of reinforced soil slopes*. PhD thesis submitted to the University of Texas at Austin.

Koerner, R. M. (1997). *Designing with Geosynthetics*, 4th edition, Prentice-Hall, Englewood Cliffs, New Jersey.

Krahn, J. (2003). "The 2001 R.M. Hardy Lecture: The limits of limit equilibrium analyses." *Canadian Geotechnical Journal*, 40(3), 643-660.

Kulhawy, F. H. (1969). *Finite element analysis of the behaviour of embankments*. Ph. D. Thesis, University of California, at Berkley, California, U.S.A.

Lacasse, S., and Nadim, F. (1996). "Uncertainties in characterizing soil properties." *Uncertainty in the Geologic Environment*, Madison, ASCE, pp. 47-75.

Ladd, C. C., Foott, R., Ishihara, K., Schlosser, F., and Poulos, H. G. (1977). "Stress-deformation and strength characteristics." State-of-the-art Report, *Proceedings of 9th International Conference on Soil Mechanics and Foundation Engineering*, Tokyo, 2, pp. 421-494.

Lee, Fook-Hou., Lo, Kwang-Wei., and Lee, Seng-Lip. (1988). "Tension crack development in soils." *Journal of Geotechnical Engineering, ASCE*, 114(2), pp. 915-929.

Leshchinsky, D. (1987). "Short-term stability of reinforced embankment over clay foundation". *Soils and Foundation*, 27(3), pp. 43-57.

Lewis, R. M. and Torczon, V. (2000). "Pattern search algorithms for linearly

constrained minimization.” *SIAM Journal of Optimization*, 10(3), pp. 917-941.

Li, G. J. (2000). *Soft clay consolidation under reclamation fill and reliability analysis*. Ph.D. thesis, Nanyang Technological University, Singapore.

Li, K. S. and Lumb, P. (1987). “Probabilistic design of slopes.” *Canadian Geotechnical Journal*, 24, pp. 520-535.

Li, K. S. and White, W. (1987a). “Rapid evaluation of the critical slip surface in slope stability problems”. *International Journal for Numerical and Analytical Methods in Geomechanics*, 11(5), pp. 449-473.

Li, K. S. and White, W. (1987b). “Reliability index of slopes.” *Reliability and Risk Analysis in Civil Engineering, ICASP5*, University of British Columbia, Vancouver, B. C., Canada, pp. 755-762.

Li, K. S. (1992). “Some common mistakes in probabilistic analysis of slopes.” *Proceedings of 6th International Symposium on landslides*, Christchurch, New Zealand, pp. 475-480.

Li, K. S. (1994). “Discussion of ‘Estimating autocovariance of in-situ soil properties’.” *Journal of the Geotechnical Engineering Division, ASCE*, 120(8), pp.1454- 1455.

Liang, R. Y., Nusier, O. K., and Malkawi, A. H. (1999). “A reliability based approach for evaluating the slope stability of embankment dams.” *Engineering Geology*, 54, pp. 271-284.

Lind, N. C. (1983). “Modelling uncertainty in discrete dynamical systems.” *Applied*

Mathematical Modelling, 7(3), pp. 146-152.

Liu, Y. W. and Moses, F. (1994). "A sequential response surface method and its application in the reliability analysis of aircraft structural systems." *Structural Safety*, 16(1), pp. 39-46.

Lopes, M. L., Cardoso, A. S., and Yeo, K. C. (1994). "Modelling performance of a sloped reinforced wall using creep function." *Geotextiles and Geomembranes*, 13, pp. 181-197.

Low, B. K. (1985). *Analysis of the behaviour of reinforced embankments on weak foundations*. Ph.D. thesis, University of California, Berkeley.

Low, B. K. (1989). "Stability analysis of embankments on soft ground." *Journal of Geotechnical Engineering, ASCE*, 115(2), pp. 211-227.

Low, B. K., Wong, K. S., Lim, C., and Broms, B. B. (1990). "Slip circle analysis of reinforced embankments on soft ground." *Geotextiles and Geomembranes*, 9, pp. 165-181.

Low, B. K. and Tang, W. H. (1997a). "Efficient reliability evaluation using spreadsheet." *Journal of Engineering Mechanics, ASCE*, 123, pp. 749-752.

Low, B. K. and Tang, W. H. (1997b). "Reliability analysis of reinforced embankments on soft ground." *Canadian Geotechnical Journal*, 34, pp. 672-685.

Low, B. K., Gilbert, R. B., and Wright, S. G. (1998). "Slope reliability analysis using generalized method of slices." *Journal of Geotechnical and Geoenvironmental*

Engineering, ASCE, 124, pp. 350-362.

Low, B. K. and Tang, W. H. (2001). "Reliability of embankments on soft ground using constrained optimization." *Proceedings of the Third International Conference on Soft Soil Engineering*, Hong Kong, pp. 123-128, A.A. Balkema Publishers.

Low, B. K. (2003). "Practical probabilistic slope stability analysis." *Proc. Soils and Rock America, 12th Panamerican Conf. on Soil Mechanics and Geotechnical Engineering and 39th U. S. Rock Mechanics Symp.*, M.I.T., Cambridge, Massachusetts, Vol. 2, 2777-2784.

Low, B. K. and Tang, W. H. (2004). "Reliability analysis using object-oriented constrained optimization." *Structural Safety*, 26, pp. 69-89.

Lumb, P. (1966). "The variability of natural soils." *Canadian Geotechnical Journal*, 3(1), pp. 74-97.

Lumb, P. (1975). "Slope failure in Hong Kong." *Journal of Engineering Geology*, 8, pp. 31-65.

Lumsdaine, R. W. and Tang, K. Y. (1982). "A comparison of slope stability calculations". *Proceeding of the 7th South East Asian Geotechnical Conference*, Hong Kong, pp. 31-38.

Major, G., Ross-Brown, D. M., and Kim, H-S. (1978). "A general probabilistic analysis for three-dimensional wedge failures." *Proceeding of the 19th US Symposium on Rock Mechanics*, Nevada, USA, pp. 45-56.

Malkawi, A. I. H., Hassan, W. F., and Abdulla, F. A. (2000). "Uncertainty and reliability analysis applied to slope stability." *Structural Safety*, 22, pp. 161-187.

Malkawi, A. I. H. (2001a). "An efficient search method for finding the critical circular slip surface using the Monte Carlo technique". *Canadian Geotechnical Journal*, 38(5), pp. 1081-1089.

Malkawi, A. I. H. (2001b). "Global search method for locating general slip surface using Monte Carlo techniques". *Journal of Geotechnical and Geoenvironmental Engineering*, 127(8), pp. 688-698.

Matsui, T., and San, K. C. (1992). "Finite element slope stability analysis by shear strength reduction technique." *Soils and Foundations*. 32(1), pp. 59-70.

McCombie, P., and Wilkison, P. (2002). "The use of the simple genetic algorithm in finding the critical factor of safety in slope stability engineering". *Computers and Geotechnics*, 29(8), pp. 699-714.

McGown, A., Paine, N., and Dubois, D. (1984). "Use of geogrid properties in limit equilibrium analysis." *Polymer Grid Reinforcement: Proceedings of a conference sponsored by the Science and Engineering Research Council and Netlon Ltd*, London, pp. 31-36.

McGuffey, V., Grivas, D., Iori, J., and Kyfor, Z. (1982). "Conventional and probabilistic embankment design." *Journal of Geotechnical Engineering Division, ASCE*, 108(10), pp. 1246-1254.

Melchers, R. E. (1999). *Structural reliability analysis and prediction*. JOHN WILEY &

SONS.

Monnet, J., Galera, I., and Mommessin, M. (1989). "Some theoretical approaches about reinforced embankments on weak soil." *Computers and Geotechnics*, 7, pp. 37-52.

Morgenstern, N. R. and Price, V. E. (1965). "The analysis of the stability of general slip surfaces." *Geotechnique*, 15, pp. 79-93.

Mostyn, G. R., and Small, J. C. (1987). "Methods of stability analysis." *Soil slope instability and stabilization, Proceedings of an Extension Course on Soil Slope Instability and Stabilisation*, Balkema, Sydney, pp. 71-120.

Mostyn, G. R. and Li, K. S. (1993). "Probabilistic slope analysis-State-of-play." *Proceedings of probabilistic methods in geotechnical engineering*, Balkema, Rotterdam, pp. 89-109.

Myers, R. H., and Montgomery, D. C. (1995). *Response surface methodology: process and product optimization using designed experiments*. JOHN WILEY & SONS, INC.

Naylor, D. J. (1982). "Finite elements and slope stability." *Numerical Methods in Geomechanics*, 92, pp. 229-244.

Nguyen, V. U. (1985). "Determination of critical slope failure surfaces". *Journal of Geotechnical Engineering, ASCE*, 111(2), pp. 238-250.

Nguyen, V. U., and Chowdhury, R. N. (1984). "Probabilistic study of spoil pile stability in strip coal mines-two techniques compared." *International Journal of Rock*

Mechanics, Mining Science and Geomechanics, 21(6), pp. 303-312.

Nguyen, V. U., and Chowdhury, R. N. (1985). "Simulation for risk analysis with correlated variables". *Geotechnique*, 35(1), pp. 47-58.

Palmaira, E. M., Pereira, J. H. F., and Da Silva, A. R. L. (1998). "Backanalyses of geosynthetic reinforced embankments on soft soils." *Geotextiles and Geomembranes*, 16, pp. 273-292.

Peck, R. B. (1967). "Stability of natural slopes." *Journal of the Soil Mechanics and Foundation Division, ASCE*, 93(4), pp. 403-417.

Pham, H. T. V., and Fredlund, D. G. (2003). "The application of dynamic programming to slope stability analysis." *Canadian Geotechnical Journal*, 40, pp. 830-847.

Phoon, K. K. and Kulhawy, F. H. (1999). "Characterization of geotechnical variability." *Canadian Geotechnical Journal*, 36(4), pp. 612-624.

Pilot, G., Trak, B., and LaRochelle, P. (1982). "Effective stress analysis of the stability of embankments on soft soils." *Canadian Geotechnical Journal*, 19(4), pp. 433-450.

Poulos, H. G. (1972). "Difficulties in prediction of horizontal deformation in foundations." *Journal of Soil Mechanics and Foundation Division, ASCE*, 98(8), pp. 843-848.

Rackwitz, R., and Fiessler, B. (1978). "Structural reliability under combined load sequences." *Computers and Structures*, 9, pp. 489-494.

Rajashekhar, M. R. and Ellingwood, B. R. (1993). "A new look at the response surface approach for reliability analysis." *Structural Safety*, 12(3), pp. 205-220.

Revilla, J. and Castillo, E. (1977). "Determination of Variations applied to stability of slopes". *Geotechnique*, 17(1), pp. 1-11.

Richardson, G. N. and Bove, J. A. (1992). "Flaws in current quality assurance programs." *MQC/MQA and CQC/CQA of Geosynthetics*, Philadelphia, 10-11 Dec., Geosynthetic Research Inst., Philadelphia, PA, pp. 154-165.

Romero, V.J., Swiler, L.P., and Giunta, A.A. (2004). "Construction of response surfaces based on progressive-lattice-sampling experimental designs with application to uncertainty propagation." *Structural Safety*, 26(2), pp. 201-219.

Roscoe, K. H., and Burland, J. B. (1968). "On the generalized stress-strain behavior of wey clays." *Engineering Plasticity*, Cambridge University, Press, pp. 535-609.

Rosenblueth, E. (1952). "Remarks on a multivariate transformation." *Annals of Mathematical Statistics*, 23(3), pp. 470-472.

Rosenblueth, E. (1981). "Two-point estimates in probability moments." *Applied Mathematical Modeling*, 5, pp. 329-335.

Rowe, R. K. (1984). "Reinforced embankments: analysis and design". *Journal of Geotechnical Engineering, ASCE*, 110(2), pp. 231-246.

Rowe, R. K., MacLean, M. D., and Soderman, K. L. (1984). "Analysis of a geotextile-reinforced embankment constructed on peat." *Canadian Geotechnical*

Journal, 21(3), pp. 563-576.

Rowe, R. K. and Soderman, K. L. (1985). "An approximate method for estimating the stability of geotextiles reinforced embankments". *Canadian Geotechnical Journal*, 22(3), pp. 392-398.

Rowe, R. W. and Soderman, K. L. (1987). "Very soft soil stabilization using high strength geotextiles: The role of finite element analysis." *Geotextiles and Geomembranes*, 6, pp. 53-81.

Rowe, R. W. and Mylleville, B. L. J (1994). Analysis and design of reinforced embankments on soft or weak foundations. *Soil structure interaction: numerical analysis and modeling*. E&FN SPON. Edited by John W. Bull.

Rowe, R. K. and Mylleville, B. L. J. (1996). "A geogrid reinforced embankment on peat over organic silt: a case history". *Canadian Geotechnical Journal*, 33(1), pp.106-122.

Rowe, R. W. and Li, A. L. (1999). "Reinforced embankments over soft foundations under undrained and partially drained conditions." *Geotextiles and Geomembranes*, 17, pp. 129-146.

Rowe, R. K. and Li, A. L. (2002). "Behaviour of reinforced embankments on soft rate-sensitive soils." *Geotechnique*, 52(1), pp. 29-40.

Sabhahit, N., Basudhar, P. K., and Madhav, M. R. (1994). "Generalized stability analysis of reinforced embankments on soft clay." *Geotextiles and Geomembranes*, 13, pp. 765-780.

Sarma, S. K. (1979). "Stability analysis of embankments and slopes." *Journal of Geotechnical Engineering, ASCE*, 105(12), pp. 1511-1524.

Schaefer, V. R. (1987). *Analysis of reinforced embankments and foundations overlying soft soils*. PhD thesis submitted to Virginia Polytechnic Institute and State University.

Schueller, G. I. and Stix, R. (1987). "A critical appraisal of methods to determine failure probabilities." *Structural Safety*, 4, pp. 293-309.

Schweiger, H.F., Thurner, R. and Pöttler, R. (2001). "Reliability Analysis in Geotechnics with Deterministic Finite Elements: Theoretical Concepts and Practical Application." *The International Journal of Geomechanics*, 1(4), pp. 389-413.

Sharma, J. S., and Bolton, M. D. (1995). "Finite element analysis of centrifuge tests on reinforced embankments on soft clay." *Computers and Geotechnics*, 19(1), pp. 1-22.

Shinozuka, M. (1983). "Basic analysis of structural safety." *J. Structural Div., ASCE*, 109(3), pp. 721-740.

SLOPE/W. (2001). *A software package for slope stability analysis*, Ver. 5. GEO-SLOPE International, Calgary, Alta.

Snitbahn, H., and Chen, W. F. (1976). "Finite element analysis of large deformation in slopes." *Proceeding of 2nd International Conference on Numerical Methods in Geomechanics*, Blacksburg, VII.

Soulie, M., Montes, P., and Silvestri, V. (1990). "Modelling spatial variability of soil parameters." *Canadian Geotechnical Journal*, 27, pp. 617-630.

Spencer, E. (1967). "A method of analysis of the stability of embankments assuming parallel inter-slice forces". *Geotechnique*, 17, pp. 11-26.

Spencer, E. (1968). "Effect of tension on stability of embankments". *Journal of the Soil Mechanics and Foundation Division, ASCE*, 94(5), pp. 1159-1173.

Spencer, E. (1973). "Thrust line criterion in embankment stability analysis". *Geotechnique*, 23(1), pp. 85-100.

Spencer, E. (1981). "Slip circles and critical shear planes". *Journal of Geotechnical Engineering, ASCE*, 107(7), pp. 929-942.

Tan, S. A. (1994). "Discussion of 'Performance of test embankment constructed to failure on soft marine clay'." *Journal of Geotechnical Engineering, ASCE*, 119(3), pp. 1323-1326.

Tandjiria, V., Teh, C. I., and Low, B. K. (2000). "Reliability analysis of laterally loaded piles using response surface methods." *Structural Safety*, 22, pp. 335-355.

Tandjiria, V., Low, B. K., and Teh, C. I. (2002). "Effect of reinforcement force distribution on stability of embankments." *Geotextiles and Geomembranes*, 20, pp. 423-443.

Tavenas, F., Blanchet, R., Garneau, R., and Leroueil, S. (1978). "The stability of stage-constructed embankments on soft clays." *Canadian Geotechnical Journal*, 15(2), pp. 283-305.

Tavenas, F. and Leroueil, S. (1980). "The behaviour of embankment on clay

foundation.” *Canadian Geotechnical Journal*, 17(2), pp. 236-260.

Taylor, D. W. (1948). *Fundamentals of Soil Mechanics*, John Wiley & Sons, Inc., New York.

Thornton, S. I. (1994). “Probability calculation for slope stability.” *Proceedings of the 8th International Conference on Computer Method and Advances in Geomechanics*. Vol. 3, pp. 2505-2515.

Ting, J. M. (1983). “Geometric concerns in slope stability analyses”. *Journal of Geotechnical Engineering, ASCE*, 109(11), pp. 1487-1491.

Torczon, V. (1997). “On the convergence of pattern search algorithms.” *SIAM Journal of Optimization*, 7(1), pp. 1-23.

Ugai, K. (1989). “A method of calculation of global safety factor of slopes by elasto-plastic FEM.” *Soils and Foundations*, 29(2), pp. 190-195.

Ugai, K., and Leshchinsky, D. (1995). “Three-dimensional limit equilibrium and finite element analyses: a comparison of results.” *Soils and Foundations*, 35(4), pp. 1-7.

U. S. Army Corps of Engineers document. (1999). “ETL 1110-2-556, Risk-based analysis in geotechnical engineering for support of planning studies.” Appendix A, 11-12.

U. S. Army Corps of Engineers document. (2003). “EM 1110-2-1902, Slope Stability.” Appendix C, 12-14.

Vankataraman, P. (2002). *Applied optimization with MATLAB programming*. John Wiley & Sons, Inc.

Vanmarcke, E. H. (1977a). "Probabilistic modeling of soil profiles." *Journal of Geotechnical Engineering, ASCE*, 103, pp. 1227-1246.

Vanmarcke, E. H. (1977b). "Reliability of earth slopes." *Journal of Geotechnical Engineering, ASCE*, 103(11), pp. 1247-1265.

Vanmarcke, E. H. (1983). *Random fields-analysis and synthesis*. Cambridge, MIT Press.

Varadarajan, A., Sharma, K. G., and Aly, M. A. A. (1999). "Finite element analysis of reinforced embankment foundation." *International Journal for Numerical and Analytical Methods in Geomechanics*, 23, pp. 103-114.

Venkataraman, P. (2002). *Applied optimization with MATLAB programming*. John Wiley & Sons, Inc.

White, W. (1993). "Soil variability: Characterisation and modeling." *Proceedings of probabilistic methods in geotechnical engineering*, Rotterdam, pp. 111-120.

Whitman, R. V. and Bailey, W. A. (1967). "Use of computers for slope stability analyses". *Journal of the Soil Mechanics and Foundations Division, ASCE*, 93(4), pp. 475-498.

Whitman, V. W. (1984). "Evaluating calculated risk in geotechnical engineering". *Journal of Geotechnical Engineering Division, ASCE*, 110(2), pp. 145-188.

Wong, F. S. (1985). "Slope reliability and response surface method." *Journal of Geotechnical Engineering Division, ASCE*, 111(1), 32-53.

Wolff, T. F. (1987). "Slope design for earth dams." *Reliability and Risk Analysis in Civil Engineering, ICASP5*, University of British Columbia, Vancouver, B. C., Canada, pp. 725-732.

Wolff, T. F. (1996). "Probabilistic slope stability in theory and practice". *Uncertainty in the Geologic Environment: from Theory to Practice, ASCE Geotechnical Special Publication*, No. 58, ASCE.

Wong, F. S. (1984). "Uncertainties in FE modelling of slope stability." *Computers and structures*, 19(5/6), pp. 777-791.

Wright, S. G., Kulhawy, F. H., and Duncan, J. M. (1973). "Accuracy of equilibrium slope stability analysis". *Journal of Soil Mechanics and Foundations Division, ASCE*, 99(10), pp. 783-791.

Wu, Y. T., and Wirsching, P. H. (1987). "New algorithm for structural reliability estimation." *Journal of Engineering Mechanics, ASCE*, 113(9), pp. 1319-1336.

Xu, D. W., and Chowdhury, R. (1999). "Probabilistic analysis of structured rock/soil slopes- Several methods compared." *Slope Stability Engineering: proceedings of the International Symposium on Slope Stability Engineering*, Matsuyama, Shikoku, Japan.

Yamagami, T. and Ueta, Y. (1988). "Search for critical slip lines in finite element stress field by dynamic programming." *Proceedings, 6th International Conference on Numerical Methods in Geomechanics*, Innsbruck, Austria. A. A. Balkema, Rotterdam,

1988, pp. 1335-1339.

Yucemen, M. S. and Al-Homoud A. S. (1990). “Probabilistic three-dimensional stability analysis of slopes.” *Structural Safety*, 9, pp. 1-20.

Zienkiewicz, O. C. (1971). *The Finite Element Method in Engineering Science*. New York: McGraw-Hill.

Zienkiewicz, O. C., Humpheson, C., and Lewis, R. W. (1975). “Associated and non-associated visco-plasticity and plasticity in soil mechanics.” *Geotechnique*, 25, pp. 671-689.

Zienkiewicz, O. C., and Taylor, R. L. (2005). *The Finite Element Method*, 5th Edition, McGraw-Hill, New York.

Zou, Jin-Zhang., Williams, David. J., and Xiong, Wen-Lin. (1995). “ Search for critical slip surfaces based on finite element method.” *Canadian Geotechnical Journal*, 32, pp. 233-246.