Buckling behaviour of steel and composite beams at elevated temperatures

Ronny Budi Dharma

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**BUCKLING BEHAVIOUR OF STEEL AND
COMPOSITE BEAMS AT ELEVATED
TEMPERATURES**

RONNY BUDI DHARMA

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2007

Buckling Behaviour of Steel and Composite Beams at Elevated Temperatures

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School of Civil and Environmental Engineering

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in fulfillment of the requirement for the degree of Doctor of Philosophy

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SUMMARY

The objective of this research was to study the buckling behaviour of steel and composite beams at elevated temperatures. Both global buckling in the form of lateral torsional buckling and local buckling were studied. The first part of the study focused on the lateral torsional buckling behaviour of bare steel beams with an aim to develop design approaches for laterally unrestrained steel beams at elevated temperature. The second part focused on the local buckling behaviour which limits the ductility of beams. Both bare steel beams and composite deck slab with re-entrant steel decking were considered for the second part to investigate the ductility issue related to inelastic behaviour in the hogging moment regions under fire conditions and to propose the model of the moment-rotational relationships.

Numerical analysis using MSC.MARC Mentat and published test results were used to study the lateral torsional buckling of steel beams at elevated temperatures. Subsequently, a general approach, different from the current approach, called an alternative approach was suggested. Besides, a simple analytical approach, based on Rankine principle, was applied to estimate the lateral torsional buckling failure of steel beams in fire. Both proposals were shown to provide a good correlation with the numerical and test results.

The investigation of the local buckling behaviour at elevated temperatures comprised of both experimental and numerical investigation. The experimental investigation consisted of two series of tests, namely, investigation on steel beams as the first series and investigation on composite beams as the second series. The numerical investigation involved fairly extensive parametric studies using the numerical model which had been validated with test results. Finally, the analytical models for the moment-rotational relationships under fire conditions were proposed.

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LIST OF SYMBOLS

The following symbols are used in this report:

A	cross sectional area
A_i	i^{th} elementary area
A_p	area of plastic zone
A_s	reinforcement area
b	width of plate; flange width
c	width of flange outstand
C_b	equivalent moment factor
D	overall depth of I-beam
d	depth of plate; depth of web between flanges
E	elastic modulus
E_i	initial elastic modulus
E_t	tangent modulus
E_T	elastic modulus at elevated temperature
f_c	concrete compressive strength
$f_{c,T}$	concrete ultimate strength at elevated temperature
f_L	limiting stress
$f_{\max}$	maximum longitudinal stress
f_p	proportionality limit
f_{pT}	proportionality limit at elevated temperature
f_{ult}	ultimate strength

LIST OF SYMBOLS

f_y	yield strength
$f_{yf,c}$	compression flange yield strength
$f_{yf,c}^*$	increased compression flange yield strength
$f_{yf,t}$	tension flange yield strength
$f_{yf,t}^*$	increased tension flange yield strength
$f_{yr,T}$	yield strength of reinforcement at elevated temperature
f_{yT}	effective yield strength at elevated temperature
f_{yw}	web yield strength
G	shear modulus
h_c	distance from top flange of the steel section towards the concrete slab
h_r	distance of the reinforcement from the centroid of steel section
h_s	distance between the shear centre of the flanges
I	second moment of inertia
I_f	moment inertia of a flange about the axis perpendicular to the web and pass through its centroid
I_w	warping constant
I_x	major axis moment of inertia
I_y	minor axis moment of inertia
J	torsion constant
K_b	effective length factor with respect to lateral bending
k_b	buckling coefficient
$k_{c,T}$	concrete strength reduction factor at elevated temperature

LIST OF SYMBOLS

$k_{ct,T}$	concrete tensile strength reduction factor at elevated temperature
k_E	elastic modulus reduction factor at elevated temperature
k_p	proportionality limit reduction factor at elevated temperature
K_t	effective length factor with respect to twisting
k_y	yield strength reduction factor at elevated temperature
L	beam length
L_E	effective length
L_p	length of buckled flange
L_i	distance between point load and support
l_0	length at the initial temperature
l_p	length of yield line
l_T	length after temperature rise of ΔT
M	applied moment
M_b	ultimate buckling moment capacity
$M_{b,fi}$	ultimate buckling moment capacity in fire
M_{cr}	elastic buckling moment capacity
M_{cr}^{IE}	inelastic buckling moment capacity
M_{cr}^L	elastic buckling moment capacity for non-uniform moment
M_{cr}^S	elastic buckling moment capacity for non-simply supported beam
M_{cx}	in-plane bending moment capacity
$M_{cx,fi}$	in-plane bending moment capacity in fire
M_E	elastic buckling moment capacity considering major axis curvature

LIST OF SYMBOLS

$M_{E,fi}$	elastic buckling moment capacity considering major axis curvature in fire
M_j	bending moment at j^{th} section
M_L	limiting moment
$M_m, M_{\max}$	maximum moment
M_p	plastic moment capacity
$M_{p,T}$	plastic moment capacity at elevated temperature
M_R	Rankine ultimate moment
M_y	yield moment
p_b	buckling stress
$p_{b,fi}$	buckling stress at elevated temperature
p_{cr}	elastic buckling stress
p_E	elastic buckling stress considering major axis curvature
$p_{E,column}$	elastic buckling stress of column
p_E^T	elastic buckling stress considering major axis curvature at elevated temperature
P	load
P_i	external load
$P_{u,20^\circ C}$	maximum load at 20°C
$P_{u,T}$	maximum load at temperature T
$P(\gamma)_T$	load at a certain value of slip γ and temperature T
r_a	available rotational capacity

LIST OF SYMBOLS

r_y	radius of gyration about the minor axis
S_x	plastic section modulus about the major axis
T	temperature
T_{bf}, T_{tf}	bottom and top flange temperature, respectively
T_g	gas temperature
T_0	initial gas temperature
T_w	web temperature
t	thickness of plate; time
t_c	concrete thickness
t_f, t_w	flange and web thickness, respectively
u	lateral deflection
u_0	initial lack of straightness
W_{ext}	total external virtual work done
W_{int}	total internal virtual work of the mechanism
W_{int}^{reinf}	internal work done by the reinforcement
W_l	work done by yield line
W_z	work done by plastic zone
x_p	plastic zone length of reinforcement
y_i	distance between i^{th} elementary area and neutral axis
Z_{ex}	effective section modulus about major axis
Z_x	elastic section modulus about the major axis
Z_y	elastic section modulus about the minor axis

LIST OF SYMBOLS

δ	central lateral deflection
δ_0	central initial lack of straightness
θ	central twist; rotation
θ_a	available inelastic rotation
θ_e	elastic rotation
θ_m	rotation corresponding to maximum moment
θ_0	central initial twist
θ_p	plastic rotation
θ_u	ultimate rotation
ϕ	twist
ϕ_0	initial twist
α_i	imperfection factor
$\alpha, \alpha_1, \alpha_2$	first, second, and third order of coefficient of thermal expansion
β	severity factor
γ	slip
γ_f	flange yield strength factor
γ_{M1}	material partial safety factor
$\gamma_{M,fi}$	material partial safety factor in fire
$\gamma_{\max}$	slip corresponding to the limit of ductility
γ_o	slip corresponding to the limit of linear behaviour
γ_u	slip corresponding to the maximum resistance
γ_w	web yield strength factor

LIST OF SYMBOLS

σ	stress
σ_b	maximum applied bending stress
σ_{cr}	critical plate buckling stress
$\sigma_{c,T}$	concrete stress at elevated temperature
σ_i	stress of i^{th} elementary area
σ_{ob}	pure bending elastic buckling stress
σ_{os}	pure shear elastic buckling stress
σ_s	applied shear stress
τ_y	yield strength in shear
ε	strain
ε_{cr}	creep strain or time-dependent strain
$\varepsilon_{crush,T}$	strain corresponding to concrete crushing at elevated temperature
$\varepsilon_{c,T}$	concrete strain at elevated temperature
$\varepsilon_{cu,T}$	concrete ultimate strain at elevated temperature
$\varepsilon_{i,j}$	strain of i^{th} elementary area at j^{th} section
ε_{IT}	limiting strain for yield strength at elevated temperature
ε_{pT}	strain corresponding to proportionality limit at elevated temperature
ε_{th}	thermal strain
ε_{uT}	ultimate strain at elevated temperature
ε_{yT}	yield strain at elevated temperature
ε_σ	stress-related strain or instantaneous strain
ν	Poisson's ratio

LIST OF SYMBOLS

λ	slenderness ratio
λ_e	elastic critical load factor
λ_{LT}	lateral torsional buckling slenderness ratio
$\overline{\lambda_{LT}}$	lateral torsional buckling modified slenderness ratio
$\overline{\lambda_{LT,fi}}$	modified slenderness ratio for LTB in fire
λ_o	limiting slenderness
λ_p	plastic collapse load factor
λ_R	Rankine load factor
ψ_{LT}	Normalized slenderness ratio
η	Perry imperfection factor
η_{LT}	Perry imperfection factor for LTB
η_{LT}^T	Perry imperfection factor for LTB at elevated temperature
η_s	degree of shear connection
χ	curvature
χ_j	curvature at j^{th} section
χ_{LT}	strength reduction factor
$\chi_{LT,fi}$	strength reduction factor in fire
ζ	non-linearity factor
Δ_i	displacement
Δl	change in length
ΔT	temperature rise
Δx_j	length at j^{th} section

CHAPTER ONE

INTRODUCTION

1.1 Background

Steel has been widely used to serve the growing construction industry for over a century. Even in today's high technological, complex and fast growing construction industry, steel is still one of the designer's best option because of its inherent advantages. The main advantages include high strength-to-weight ratio, ease and speed of erection, high flexibility and its ductile behaviour. In recent years, steel has also been increasingly used in combination with concrete in steel-framed buildings as composite metal decking system. However, one of the disadvantages of steel is that it has higher thermal conductivity compared to other construction materials. When a steel structure is exposed to fire, its mechanical properties, such as strength and stiffness, reduce; hence leading to possible deformation and failure under service loading. Thus, it is important that steel structures be provided with fire resistance to control the spread of fire or to prevent structural collapse.

Over the past years, fire protection method is used to increase the fire resistance of steel structures by delaying the increase in temperature of steel or limiting the temperature rise in steel to about 550°C within the required time. Fire protection is applied to steel members based on the behaviour of a single element under ideal situations, thus neglecting the real behaviour of structures in fire. Besides being overly conservative, this method slows down the construction process and increases the construction cost. Robinson and Latham (1986) reported that fire protection can contribute to about 30% of the total cost of steel structures. Moreover, additional costs are needed for maintenance of fire protection materials. Due to relatively high construction cost, there

is a demand for a more competitive steel construction from the industry. This explains the current trend towards fire engineering performance-based approach to the design of steel structures. It has the advantage of providing fire safety at a reasonable cost without paying for excessive fire protection materials. In this approach, fire safety is determined from calculation by treating the effect of fire like other types of loading rather than just providing fire protection materials.

There are three basics steps in a performance-based approach, namely, estimation of fire severity, assessing the temperature distribution of critical members and prediction of structural response. Finally, the performance of a fire-resistant structure should accomplish at least one of the following (Milke, 1999):

- Restrict the spread of fire beyond the compartment of fire origin.
- Support a load, despite exposure to a fire.

Thus, performance-based design, also known as fire resistance method of design, depends on the availability of engineering methods to evaluate the fire resistance of structures in terms of their structural and thermal response given a set of loading and fire exposure conditions. Thermal response is analyzed because of the need to assess the heat transmission to the unexposed side of assembly and to provide the input for the analysis of the structural response, whereas structural response is being evaluated to address the impact of degraded materials, changes in material properties and thermally induced stresses on the load capacity of structural members.

During the past thirty years, research activities have been intensified to advance the science of structural fire engineering. As a result, the performance-based approach has recently been adopted in many countries. However, the design approach is still at the stage of infancy and is continuously being updated with the latest development.

1.2 Fire Resistance Assessment

Fire resistance is an important part of any fire safety design although it is just one part of the overall fire design strategy. It is regarded as a form of *passive* fire protection as opposed to *active* fire protection (such as sprinklers). The most common method of assessing fire resistance of a building is by conducting a full-scale fire resistance test. However, such test is only conducted when necessary as it is very expensive. As more research works have been conducted, it is becoming apparent that fire resistance can be calculated. Fire resistance calculation involves two main considerations, namely, thermal and structural response analysis. Firstly, fire exposure conditions are assessed to provide an input for the thermal analysis. Thermal response is then analyzed to assess the heat transmission to the unexposed side of the structure and to provide input for structural response analysis. Finally, structural response analysis utilises the temperature profile within the structure from thermal response analysis to determine the material properties and to ascertain thermally-induced stress or curvature.

The fundamental objective of fire resistance analysis is such that the fire resistance exceeds the fire severity. The comparison is usually carried out using time, temperature or strength domain. In time domain, the time to reach failure must exceed the required fire duration. In temperature domain, the temperature which will cause failure of the element must exceed the maximum temperature attained by that element during fire. Similarly, for strength domain, the minimum load capacity during fire must be greater than the applied load at the start of fire. Thus, the philosophy of fire resistance calculation is to apply the same algebraic equations from ambient temperature analysis with some adjustments of the material properties to consider temperature effect.

There are three essential concepts in the assessment of structural fire resistance (Bresler, 1985):

1. A single standard fire curve that is essentially the same worldwide.

2. Classification of various building elements in the local building codes in terms of required standard fire exposure, which is often referred as required endurance.
3. Ratings of fire resistance for various building elements based on their performance in a standard fire test. This rating is called fire resistance.

1.3 Fire Resistance Tests

Fire resistant test is usually carried out on representative specimen exposed to the standard fire in a typical furnace consisting of a large steel box lined with fire bricks or ceramic fiber blanket. There are three types of furnace to test the fire resistance of materials, namely, full-scale, intermediate-scale and small-scale furnaces. In order to assess fire resistance, a full-scale fire-resistance test is most preferred (Buchanan, 2001). A small-scale furnace is usually used for research and development, while an intermediate-scale furnace is used for standard testing of small items such as fire doors. Even though full-scale tests are expensive, for many years it has been considered essential to test elements of building construction at a large scale. This is because cheaper small-scale tests are not able to assess the effects of potential problems caused by connections, shrinkage and deflections. For fire-resistance testing, many countries require the gas temperature inside furnace to follow the International Standard ISO 834 fire (ISO, 1975). Besides, the furnace should also have a number of burners fuelled by gas or fuel oil to provide a uniform temperature inside the furnace.

A few difficulties are often found in the interpretation of test results, which are caused by:

- Complex behaviour of structures under fire conditions
- Limited specimen size due to size of the furnace.
- Behaviour differences of a specimen under standard ISO 834 fire curve (ISO, 1975) and natural fire curve.
- Difficulties in testing a whole structure (only isolated member can be tested).

1.4 Enclosure Fire Behaviour and Standard Fire Exposure

Generally, fire starts because some combustible materials are ignited by a heat source. Fire development in an enclosure undergoes three stages: growth, burning and decaying stage (Figure 1.1). During the growth stage, the fire is localized and the temperature remains low around 250°C. Besides, the temperature distribution inside the enclosure is highly non-uniform. The combustion reaction initially consumes existing oxygen inside the room, but later from the air coming through the opening. There is a large quantity of smoke and toxic gases produced which poses a threat to life at the early stage. After that, *flashover*, which corresponds to a time when there is a spontaneous combustion of all organic materials within the compartment, would occur. This marks the beginning of the burning stage when the temperature starts to increase sharply. The rate of burning depends on the amount of fresh air that can be supplied; hence, the fire is ventilation-controlled. During this stage, there is a high release rate of heat and the damage to the building structures reaches a maximum. Thus, this stage is the main concern for structural fire engineer. Lastly, when most of the combustible materials are consumed and the burning rate starts to decrease, the fire goes to decaying stage. The fire will eventually die out after all combustible materials have been consumed.

Figure 1.1 Typical Temperature Development in an Enclosure Fire

The fire resistance performance of building elements are usually assessed using full-scale fire resistance test. The time-temperature curve used in this test is called the ‘standard fire’. There are several standard time-temperature curves adopted by different codes. The most commonly used one is based on ISO 834 (ISO, 1975) as shown in Figure 1.2. The fire temperature T_g (°C) is defined by

$$T_g = 345 \log_{10}(8t + 1) + T_0 \quad (1.1)$$

where T_0 = initial gas temperature (usually taken as 20°C);

T_g = gas temperature;

t = time (min).

Figure 1.2 ISO 834 Fire Curve

The ISO standard fire curve rises with time without any decaying stage. Thus, it does not represent the natural fire development. Nevertheless, ISO standard fire curve is still widely used because the standard fire curve has the following advantages (Wang, 2002):

- The standard fire exposure concept has a long history and is well established in fire safety engineering. The fact that building structures rarely fail in fires as a result of inadequate specification of the standard fire resistance time indicates that it is safe to use the standard fire curve.

- A large body of knowledge has been obtained from many years of standard fire resistance tests but few data exist for realistic fires.
- The standard fire curve has only one temperature-time relationship. In design calculations, it is much easier to deal with only one standard fire curve than with an infinite number of real fire curves.

In order to overcome the differences between an expected real fire and a standard fire, the concept of equivalent fire severity is introduced. There are three equivalent fire severity concepts (Buchanan, 2001):

1. Equal Area Concept

This concept is introduced by Ingberg (1928). It assumes that two fires are considered to have an equivalent severity if the area under each curve is equal, above a certain temperature.

2. Maximum Temperature Concept

This concept was developed by Law (1971), Pettersson et al. (1976) and others. They define an equivalent fire severity as the time of exposure to the standard fire that would result in the same maximum temperature in a protected steel member, as would occur in a complete burnout of the fire compartment (ISO 834 fire).

3. Minimum Load Capacity Concept

This is the most realistic concept in which the equivalent fire severity is the time of exposure to the standard fire that would result in the same load bearing capacity as the minimum which will occur in a complete burnout of firecell (ISO 834 fire).

1.5 Structural Response and Design in Fire

Structural response in fire is defined in terms of transient deformation, forces, moments and material degradation. When steel is subjected to fire, its mechanical properties, such as strength and stiffness, deteriorate. Besides that, thermal stresses take place due to an increase in temperature across the section (thermal gradient) and thermal restraint

conditions (statically indeterminate structure). Thus, the magnitude of thermal stress depends on the thermal elongation and gradient, rigidity of the structures and the restraint condition. For the case of a statically determinate structure and in the absence of thermal gradient, there will be no thermal stress. Hence, the member will only experience thermal elongation.

Since the first comprehensive treatment of fire engineering of steel structures by Petterson et al. (1976), there has been a great interest and progress to develop an appropriate design method for steel under fire conditions. Structural design in fire is conceptually similar to that for ambient temperature. The main differences of fire design compared with cold design are that at the time of a fire (Buchanan, 2001):

- The applied loads are less;
- Internal forces may be induced by thermal expansion;
- Strengths and stiffness of material may be reduced by elevated temperatures;
- Smaller safety factors can be used, because of the low likelihood of the event;
- Deflections are not important (unless they affect strength or compartmentation, since large deflection may cause integrity failure and fire spread);
- Different failure mechanisms need to be considered.

A safe structure in fire requires that the applied loads are less than the load capacity of the structure for the duration of the fire design time. The applied loads in the event of fire are most likely to be lower than the applied loads in normal condition; therefore, a different load combination is used. A different strength reduction factor that accounts for the uncertainty in the estimation of material strength is applied. For EC3:1.2 (CEN, 2005) or BS5950:8 (BSI, 2003), the steel material reduction factor of safety in fire is one.

In statically determinate structures, a collapse of one member may cause the collapse of the whole structure. However, in more complex structures, the interaction between

structural elements during fire has given rise to beneficial effects. This is due to the formation of alternative load-path mechanisms such as compressive and tensile membrane action, catenary action, and possible rotational restraint from connections, thereby allowing redistribution of the service loads to the parts that are not much affected by heat. The interaction between structural elements during fire, although mostly beneficial, makes the structural behaviour a complex issue and new research works (Bailey, 1998; Wang, 2000) are continuously being carried out.

1.6 Objective and Scope

The objective of this research was to study the buckling behaviour of I-shaped beams at elevated temperatures. Both global buckling in the form of lateral torsional buckling and local buckling were studied. The aims were

- To propose some design approaches, including general and simple calculation methods, for laterally unrestrained steel beams at elevated temperature;
- To investigate the ductility issue related to inelastic behaviour of both steel and composite beams in the hogging moment regions under fire conditions;
- To propose the model for moment-rotational relationships of both steel and composite beams subjected to hogging moment at elevated temperatures.

The first objective was achieved through a numerical study using a finite element program and published experimental results. This numerical study provides an easy and economical way for understanding the behaviour of laterally unrestrained steel beams in fire. A general approach, different from the current approach, called an alternative approach was then suggested. Besides, a simple analytical approach, based on Rankine principle, was applied to estimate the lateral torsional buckling failure of steel beams in fire. The proposed Rankine approach was based on the interaction between the strength and stability of steel beams under thermal effects.

Both experimental and numerical study using a finite element program were used to achieve the second objective. An extensive experimental programme was conducted to study the effects of temperature on rotational capacity, to identify key parameters which affect the ductility at elevated temperatures as measured by rotational capacity and to observe local buckling failure modes. This experimental programme consisted of two series of tests. In the first series, a total of 9 steel beams were tested, while a total of 4 composite beams with re-entrant decking were tested in the second series. These test results were then used to investigate the feasibility of applying the finite element (FE) method to study the inelastic behaviour of both steel and composite beams. The validated FE models were then used for parametric studies.

Using the results and observations of the experiments and numerical works, the third objective was achieved. An analytical model based on plastic collapse mechanism and moment-curvature relationships was then proposed. The plastic collapse mechanism, consisting of plastic zones and yield lines, is replacing the actual buckling shape. The post-buckling moment-rotation relationship was derived directly from the work equation associated with the proposed plastic collapse mechanism, while the pre-buckling rotation was obtained through the integration of curvature diagram. In addition to this model, a simple design moment-rotation equation was also proposed for steel beams to provide practitioners with a quicker and easier approach. This proposed design equation is obtained based on the statistical study of numerical results.

As a result of this study, the ductility of beams in fire can be based on the concept of member behavioural classes instead of the current EC3:1.2 (CEN, 2005) concept of cross-section classes. In addition, this study shows that plastic theory exercised with due care can also be applied to fire engineering design when it considers the more limited rotational capacity of members under fire conditions.

In addition to the discussions related to the behaviour at elevated temperatures, the discussions on the behaviour and design approach at ambient temperature as the basis for elevated temperature case are also included when necessary. It should be noted that this study only deals with steel and composite beams in which no fire protection is applied to the steel section. Both universal and fabricated steel sections with equal flanges are considered. The study on the composite beams covers only composite beams with re-entrant steel decking which are designed with full shear connection. It should also be highlighted that no lateral restraint is considered in this study since it focuses in situations in which fire occurs only at the outer span of continuous steel beam; hence there is only minimum axial restraint from the adjacent structures. In addition, only member behaviour will be investigated without considering the connection behaviour. Lastly, this study only focuses on the structural response incorporating some thermal effects. Therefore, the thermal analysis, i.e. the heat transfer analysis, is not considered to simplify the analytical and numerical models.

1.7 Organization

The first chapter introduces the importance of fire resistance for a building and the need to move towards performance-based design. Some basic concepts for fire resistance design, such as various fire-resistance tests, time-temperature curves and the concept of equivalent fire severity are also presented. Besides these, thermal effects on steel material properties are briefly discussed.

Chapter 2 presents a review of previous research works related to lateral torsional buckling, local buckling, rotational capacity and inelastic behaviour of beams. Besides, some important backgrounds related to this research at ambient temperature are also presented in this chapter. Finally, the motivations of this research study are highlighted. Subsequently, Chapter 3 discusses the mechanical and thermal properties of both steel and concrete.

The lateral torsional buckling behaviour of laterally unrestrained beams is presented in Chapter 4. This chapter is divided into two parts. The first part which discusses the behaviour at ambient temperature provides a basis for the second part on the behaviour at elevated temperature. This first part starts with the analytical derivations of lateral torsional buckling of beams at ambient temperature. After that, both design approaches from BS5950:1-2000 (BSI, 2001) and EC3:1.1 (CEN, 2005) are derived step-by-step by the author. Some comparisons and comments of design approaches at ambient temperature are also discussed in this chapter. The second part of Chapter 4 firstly discusses both approaches at elevated temperature adopted by BS5950:8 (BSI, 2003) and EC3:1.2 (CEN, 2005). The weakness and the need for alternative approaches for LTB of steel beams in fire are then pointed out. Subsequently, both general method (called alternative approach) and simple calculation method based on Rankine principle are proposed by the author. A numerical model using the commercially available finite element program MSC.MARC Mentat is then developed and used to validate both proposed approaches. Published experimental works by Vila Real et al. (2003) are also extracted and compared with the numerical and proposed approaches in this chapter. Finally, a worked example on predicting the LTB buckling moment in fire is provided.

The local buckling failure and ductility issue at elevated temperatures are presented in Chapter 5, 6 and 7. Chapter 5 reports the detail of the experimental programme and results. In addition, the behaviour, the observed failure modes and the influence of various parameters on rotational capacity are also discussed.

Chapter 6 presents the numerical analysis to study local buckling and ductility of beams using MSC.MARC Mentat. The development of the numerical model is firstly presented. Subsequently, the FE models are validated against the experimental results at both ambient and elevated temperature before they are used for parametric studies. Main key factors affecting rotational capacity and inelastic behaviour of beams subjected to

hogging moment are discussed through these parametric studies. In addition, the failure modes shown by the numerical models are also compared with those of the tests.

Chapter 7 provides some discussions on the modelling of moment-rotational relationships at elevated temperatures. Two methods are proposed to predict moment-rotational relationships and rotational capacity of steel and composite beams at elevated temperatures. It is shown that the proposals are able to predict the moment-rotational response with sufficient accuracy. Finally, some conclusions are drawn and a few recommendations are raised for future research in Chapter 8.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

Beams are possibly the most basic type of member present in a civil engineering structure. Their principal function is the transmission of loads primarily by bending and shearing actions. It is normally desirable to maximize the use of material in order to achieve economical and efficient structural design. This coupled with the need to make connections between members, has led to the popularity of I-shaped beams. Simple considerations of minimum material consumption frequently suggest that some plate elements be made thinner but limits must be imposed if certain potentially undesirable instability problems are to be avoided. Instability occurs when a change in the geometry of a structure or structural component under compression will result in the loss of its ability to resist applied loads (Chen & Lui, 1987). Because instability can lead to a catastrophic failure of a structure, it must be taken into account when one designs a structure.

Two instabilities, global and local instability, can occur in beams. Global instability of beams is called lateral torsional buckling (LTB) whereas local instability is defined as local buckling. LTB occurs when a beam which is bent in the stiffer principal plane buckles out of that plane by deflecting laterally and twisting. LTB is one of the major design concerns for laterally unrestrained I-beams because I-beam is relatively weak in both minor-axis bending and twisting when it is subjected to major-axis bending. Local buckling, on the other hand, involves a considerable distortion of the cross-section that takes place locally in the highest moment region. Local buckling does not result in an overall deformation of beams, unlike lateral torsional buckling. The occurrence of local

buckling should be delayed or prevented since it may inhibit the attainment of the design plastic or yield moment. Besides, local buckling may also limit the rotational capacity or ductility of a plastically designed member.

The development of analytical methods, numerical studies and experimental studies for determining lateral torsional buckling and local buckling of beams at both ambient and elevated temperature are reviewed and discussed in this chapter, together with the needs to study the buckling behaviour of beams at elevated temperature.

2.2 Lateral Torsional Buckling

The analytical aspects of determining lateral torsional buckling strength are quite complex, and closed-form solutions exist only for a few simple cases. However, research in this field has been conducted extensively and current research interest continues at a high level. The simplest configuration of simply supported doubly symmetric elastic beams of constant cross-section subjected to uniform moment has been studied extensively by Bleich (1952) and Timoshenko & Gere (1961). There are three common assumptions in the studies, namely, the stiffness does not change along the length of the member, the shear centre remains in the plane of the web and the cross-section of the steel member remains undistorted. In 1979, Kirby and Nethercot presented an approximate solution to include the effect of in-plane deflection that has been ignored before. If the end moments are unequal, closed-form solutions are not available because the resulting governing differential equation will have variable coefficients. Hence, some classical numerical solutions for the critical loads have been studied by Massonnet (1947), Horne (1954) and Salvadori (1955, 1956). Their results have been verified many times by other researchers using various numerical techniques. Salvadori (1955) found that a simple modifier can be applied to the uniform moment case to take into account the effect of moment gradient along the beam. Nethercot and Rockey (1972) and Nethercot (1983) studied the simply supported beam subjected to

transverse load between the ends of the beam. They found that it is important to consider the location of the applied load with respect to the shear centre if the load is applied at the point without lateral braces. They presented the modifier values to take into account the position of load with respect to the shear centre.

Vlasov (1961) and Nethercot & Rockey (1972) have also studied the effect of end restraint to elastic lateral torsional buckling. Vlasov (1961) introduced the use of two effective length factors to take into account many boundary conditions with respect to lateral bending and twisting. Nethercot and Rockey (1972) presented the modifier for two loading cases, namely, mid-span point loading and uniformly distributed loading, with five different support conditions. In 1983, Lindner and Gietzelt studied the effect of end plates. Besides simply supported beams, continuous beams have also been studied. The main interest is on the effect of end restraint of adjacent segments on the critical segments as this can substantially affect the buckling load (Nethercot, 1983). Considerable research on continuous beams has been conducted by Salvadori (1955), Hartmann (1967), Trahair (1969, 1977a, 1977b), Powell and Klingner (1970), Nethercot (1973) and Vacharajittiphan & Trahair (1975).

The review cited above is for elastic beams, whereas research on inelastic lateral torsional buckling of beams has also been performed vigorously since the 1940s, and research continued into the 1980s, especially on problems related to continuous systems. The first paper on the subject was authored by Neal (1950), and the residual stress effect was first considered by Galambos (1963). Subsequently, the residual stress effect was also studied by many investigators, including Massey (1964a, 1964b), Faella & Mazzolani (1972), Yoshida & Nishida (1972), Nethercot (1972, 1974) and Yoshida (1975). They concluded that the predictions of inelastic buckling moment vary not only with the magnitude and distribution of the residual stresses, but also with the beam cross-section, the yield strength and the assumptions made when calculating the material

tangent modulus. The combined effects of residual stresses and geometrical imperfections on inelastic lateral torsional buckling of beams were considered by Lindner (1974), Vinnakota (1977) and Celigoj (1979).

In 1976, Nethercot & Trahair (1976b) studied the inelastic buckling of simply supported beams with unequal end moments, mid-span point load, symmetrically placed point load and uniformly distributed load. They used several hot-rolled beams with idealized residual stresses. The inelastic beams with combination of transverse loads and end moments have also been studied by Yoshida & Imoto (1973), Yoshida (1977) and Nakamura & Wakabayashi (1981). The results of these studies may be used to approximate the inelastic buckling loads of continuous beams in which major axis continuity between adjacent spans induces major axis end restraining moments, when the effects of continuity on minor axis bending and warping are neglected.

In a statically determinate braced beam, there are interactions between adjacent segments during inelastic buckling. This is similar to the interactions taking place in elastic beams. Nethercot & Trahair (1976b, 1977) developed an extension of their elastic method (Nethercot & Trahair, 1976a; Trahair, 1977a) for approximating the inelastic buckling resistance.

Many tests have also been conducted for laterally unsupported beams. Hechtman (1955) carried out a test on 33 hot-rolled steel I-beams. The beams had two different types of semi-rigid end connections and were loaded at the quarter points. Good correlation was obtained with the theoretical predictions for those beams that failed in the elastic range. He also found that geometrical imperfections affected the inelastic buckling strength of the beams. White (1960) and Lee & Galambos (1962) tested a series of simply supported wide-flange steel beams under equal end moments to determine the optimum length of beams in plastically designed unbraced structures. The test beams were simply

supported with overhanging lengths at both ends. It was found that beams shorter than $45r_y$ (radius of gyration about the minor axis) failed by local buckling, while beams longer than $45r_y$ showed considerable post-buckling strength and were able to form plastic hinges of sufficient rotational capacity. The tests on simply supported beams with central lateral restraints and concentrated loads were carried out by Sawyer (1961). He used a semi-graphical analytical method to take into account the effects of residual stresses, initial imperfections, shear stresses, eccentricity of load and cross-sectional shape on the behaviour of tested beams.

In England, Dibley (1969) tested 30 high-strength universal steel beams. Four-point loading was used so that the centre unsupported span carried a uniform moment. The loads were applied by hydraulic jacks and the beams were restrained laterally at the loading points. The residual stresses in these beams were similar in magnitude and distribution to those occurring in lower strength steel beams and it was concluded that the effect of residual stress is less significant in higher-strength beams. Kitipornchai & Trahair (1975) tested simply supported I-beams with mid-span concentrated loads. A total of six UB 29 beams with spans that varied from 2.4 to 6.1 m were tested. Two of them were carried out on stress-relieved beams. They found that the effect of the experimental residual stresses was not very important. This finding was confirmed by theoretical predictions for the hot-rolled beams, which were not very different from those stress-relieved beams. The geometrical imperfections were found to be significant in reducing the strength of the beams below their theoretical buckling loads.

Two-span continuous beams with point loads were tested by Poowannachaikul & Trahair (1976). The test results are surprisingly low and even lower than the simple beams of the same cross-section (Kitipornchai & Trahair, 1975). Fukumoto et al. (1980) tested 75 hot-rolled steel beams having nominally identical cross-sectional dimensions, material properties and residual stresses. Three different spans were used and each beam

was tested with unbraced concentrated load acting at the centre of the top flange. Subsequently, Fukumoto & Itoh (1981) made similar tests on 68 welded steel beams with two different spans. Tests on welded beams were also conducted by Wakabayashi & Nakamura (1983). They tested 18 simply supported welded beams with two point loads. In 1981, Fukumoto proposed that a numerical data bank be established in which the results of these tests can be kept.

Dux and Kitipornchai (1981) carried out experiments on the inelastic buckling of braced beams. Subsequently, Fukumoto et al. (1982) tested a series of 21 two-span continuous welded beams. He made extensive measurements of the dimensions, material properties, residual stresses and initial crookednesses and twists.

As reviewed above, there have been extensive research of lateral torsional buckling of both elastic and inelastic beams at ambient temperature and it is still being conducted by some researchers up to now to improve the accuracy of predictions. However, the problem of LTB of steel beams in fire has been studied by only few researchers. Bailey et al. (1996) carried out a theoretical study to investigate the LTB temperatures of steel beams. In this study, the temperature distribution in the beams was assumed to be uniform. The effects of different parameters, such as bending moment distribution, location of loading, load ratio and beam slenderness were investigated. They found that LTB failure temperatures of steel beams could be much lower than the limiting temperatures given in BS5950:8 (BSI, 2003). Vila Real et al. (2003) also reported the results of theoretical and experimental studies on LTB of steel beams in fire. In this study, they used computer program SAFIR to calculate the failure temperature of unrestrained steel beams. Only the case of beams exposed to heating on all sides and subjected to uniform bending moment was considered in their study. Besides numerical study, they tested 120 IPE beams with lengths varying from 0.5 to 6.5 m. They conducted the isothermal tests at 5 different temperatures: 200, 300, 400, 500, and 600°C. Based on

these results, they proposed a new approach based on the steel column design approach of Franssen et al. (1995). This new approach has since been adopted to the new Eurocode 3 (EN 1993 EC3:1.2, 2005) replacing the old EC3 method (ENV 1993 EC3:1.2, 1995).

In 2003, Yin & Wang reported numerical parametric studies using commercially available finite element package ABAQUS to investigate the effects of a number of design factors on the lateral torsional buckling resistance of steel beams subjected to non-uniform temperature distribution. These numerical studies were then used to check the predictions of design methods in BS5950:8 (BSI, 2003) and ENV 1993 EC3:1.2 (CEN, 1995). They found that both design methods are not suitable to deal with the effects of non-uniform temperature distribution.

Recently, Vila Real et al. (2004) studied the effects of residual stresses using computer program SAFIR. They found that the buckling resistance of beams is less sensitive to residual stresses when the temperature increases, even though they did not consider heat treatment that should eliminate residual stresses in the numerical simulations. Instead of determining the buckling moment at elevated temperature, Mesquita et. al. (2005) investigated the critical temperature of 15 IPE 100 beams. They found that the critical temperatures of these beams are higher than the predicted critical temperature based on EC3:1.2 (CEN, 2005) simplified method.

As reviewed above, there are relatively fewer research works on the behaviour of lateral torsional buckling of steel beams in fire compared to that at ambient temperature. Hence, the design approach is still in the state of infancy and may be over conservative to avoid unpredicted behaviour and uncertainties. Based on the author's study, the Eurocode3 design approach is very conservative especially for medium slenderness range. Besides, there are two other weaknesses:

- EC3:1.2 approach does not allow beams with very low slenderness ratio to attain their plastic moment, except for fully restrained beams (zero slenderness ratio).
- EC3:1.2 approach is only for elevated temperatures and a different approach is used for ambient temperature (EC3:1.1 approach). Thus, there is a missing link between the approach at ambient temperature and that at elevated temperature.

Based on the above arguments on the weaknesses of the current approach, the author feels the need to study LTB of laterally unrestrained steel beams in fire with a purpose of proposing an alternative approach that gives a safe, robust, economical and yet more accurate results. At the same time, this approach must be able to overcome the weaknesses of EC3 approach. Besides, a simple and quick design approach is also needed to provide an alternative for design engineers as the complexity of LTB of steel beams in fire inevitably results in a very complex design approach.

2.3 Local Buckling

In this section, a review of past research related to local buckling failure and ductility of both steel and composite beams is firstly presented. Subsequently, the inelastic behaviour related to local buckling failure at ambient temperature is briefly discussed, including the difference between cross-section and member behavioural classes, the approach to quantify ductility of members and the provisions of design codes. Lastly, the motivation and importance of this study on the ductility and local buckling failure under fire conditions are highlighted.

2.3.1 Review of Past Research

The local buckling problems are initially treated by separating the flange and the web buckling. The flange, modelled to be restrained by its web portion, is treated as a classical buckling problem with bifurcation of the equilibrium position. Haaijer (1957) and Haaijer & Thurlimann (1958) presented the solution of flange buckling based on the

buckling of an orthotropic plate. They assumed that the material is isotropic and homogeneous in the elastic range and that it changes during the yielding process with yielding taking place in slip bands. Besides, the strains in those bands jump from the value at yield (yield strain) to another at the onset of strain-hardening. Besides, they also reported the test results on I-section beams with constant moment. The flange buckling of the beams occurs in the strain-hardening range, after the yield plateau is exceeded. The problem of the buckling of the web of a wide-flange beam subjected to both axial load and bending moment has also been studied by Haaijer & Thurlimann (1958). They presented a diagram for the allowable depth/thickness ratio of the web of a wide-flange section.

A wide range of experimental work has been performed by Sawyer (1961) in order to establish the influence of moment gradient, strain-hardening and instability phenomena. Besides, he also proposed some equations to obtain the moment-rotation characteristics until the maximum moment (post-critical path was ignored). The moment-rotation curve was obtained by the integration of the moment-curvature equation. In 1965, Kato used the plastic mechanism method for an I-section member to study the local buckling collapse mechanism. He proposed that the flange crumpling was produced by yielding. The main deficiency of Kato's model is that the buckling of plates forming the I-sections develops in a way which is geometrically independent of each other.

In 1966, Adams presented the solution to flange buckling based on the torsional buckling of a restrained rectangular plate. In this approach, local buckling occurs when the average strains in the plate are at the strain-hardening region and sufficient portion of the plate has yielded so that at least one full wave length of the buckle can develop. Adams (1966) obtained the same limiting flange width-to-thickness ratio as Haaijer (1957) although his approach was different.

Lay and Galambos (1967) performed a study of rotational capacity for in-plane bending and proposed an approach to determine when the maximum bending moment occurs. The proposed approach considers that the maximum value of bending moment occurs when the length of the plastic zone reaches the length of one wave of plastic buckling of compression flange. In 1969, McDermott studied the influence of cross-section dimensions and fabrication types. The possibility to determine the ultimate rotation, as the one at which the moment drops below plastic moment, has been shown for the first time by the experimental tests conducted by Lukey & Adams (1969). Their experimental program is the most commonly cited among the researchers and many numerical and analytical works are verified using these experimental results.

Hoglund and Nylander (1970) subsequently performed an extensive theoretical and experimental study. They confirmed the validity of the condition proposed by Lay (1965) to determine the flange slenderness for which no plastic buckling occurs. Their experiments were devoted to determine the range of plastic buckling of flanges and webs and also the influence of steel quality on ductility. Climenhaga & Johnson (1972a) performed 18 tests to study the local buckling in continuous composite beams. Based on the observation of the experimental buckling shape, they proposed a local buckling collapse mechanism model (Climenhaga & Johnson, 1972b). Their model corrected the deficiency of Kato's (1965) model by means of geometrically-compatible yield line models. An important progress in using this methodology was performed by Ivanyi (1979a, 1979b) in the so called *interactive plastic hinge model*. The plastic collapse mechanism curve is constructed based on the principle of virtual work.

Hope-Gill and Johnson (1976) conducted three tests on three-span continuous composite beams to assess the limitations on the use of plastic design method for composite beams. Subsequently, Johnson & Hope-Gill (1976) proposed simple rules of checking the applicability of simple plastic theory to continuous beams. Hamada &

Longworth (1976) also tested three two-span continuous composite beams to investigate ultimate loads and failure modes for continuous composite beams. They also proposed a method to predict the ultimate loads and failure modes. They identified two failure modes, namely, crushing of concrete in a positive moment region and local flange buckling in a negative moment region.

Kemp (1985, 1986) conducted a series of 9 beams to measure the inelastic rotations at the maximum moment. He also rearranged the expressions and modified some hypotheses by Lay & Galambos (1967). In 1986, Kuhlmann conducted the first systematic experimental studies on the influence of geometrical dimensions on the rotational capacity of I-section beams. Besides, he proposed a plastic collapse mechanism that was characterized by a semi-pyramidal shape mechanism for buckled flange and a partial mechanism for compression zone webs. He was also the first to study the rotational capacity numerically by developing special software. A good correlation between numerical predictions and experimental results by Lukey & Adams (1969) was observed.

Schilling (1988) tested three steel girders of very slender web (class 4) with stocky flange (class 1). Each specimen was able to reach a maximum moment close to, but less than, the yield moment. A very low ductility has been observed from the tests. Gioncu et al. (1989) have also tested I-beams when examining the ductility problems. They obtained the same conclusion as Kemp (1985) and Kuhlmann (1986) that the flange and the web slenderness are the leading parameters governing the ductility of beams.

Greschik et al. (1990) have investigated the use of FEM for predicting the rotational capacity. Some differences between the experimental and numerical results are explained by the presence of residual stresses, geometrical imperfections and the idealization of the material stress-strain curve. In order to obtain further information

concerning the influence of flange, web slenderness and steel grade, Spangemacher (1991) performed an extensive experimental program. He identified the possible collapse modes, namely, in-plane, out-of-plane, flange-induced and shear-collapse buckling modes. Besides testing, he also used his experimental results to verify his numerical predictions.

Kemp & Dekker (1991) studied the available rotation capacity in steel and composite beams based on 44 published tests. Their analysis indicated that lateral slenderness ratio is the most significant factor influencing available rotation capacity of steel sections. Besides, it is concluded that there is no clear justification for a difference in the local buckling provisions for steel and composite beams.

Mazzolani and Piluso (1992, 1993) proposed relations for determining the ductility at the maximum moment capacity using the same integration method of moment-curvature as Sawyer (1961), but considering the differences in stress levels for compression and tension flanges. Similar method has also been used by Kato (1989, 1990). In 1993, Boerave et al. tested five different beams to examine if plastic design could be applied to modern steels with high yield strength. Besides, they also performed numerical simulations to determine the rotational capacity using numerical models which had been validated with experimental results.

A detailed analysis of tests performed in Japan concerning the ductility of beam-column members has been performed by Nakashima (1992, 1994). The experimental results obtained from 224 specimens were grouped according to different criteria, namely, definition of ductility, type of loading, level of axial force and member slenderness. The analysis revealed that neither residual stresses nor initial geometrical imperfections are major factors affecting the ductility, but strain-hardening characteristics are mainly responsible for the large variation in experimental data.

Feldmann (1994) continued the works on plastic collapse mechanism by proposing a model characterised by a semi-pyramidal shape mechanism for buckled flanges and elastic behaviour of webs. This model is more appropriate for thin-walled members. Aribert (1994) conducted two tests on two span composite beams to study the effect of local plate buckling on the ultimate strength. The beams were subjected to a point load at each span. He also made a numerical model to simulate the experimental results.

Gioncu & Petcu (1995, 1997) also proposed a plastic collapse mechanism model which was based on Climenhaga-Johnson-Ivanyi's model, but with the mechanism length affected by all cross-section dimensions. Axhag (1995) conducted experimental tests and numerical simulations that were focused on obtaining the maximum plastic moment and the slope of the post-critical curve. A statistical study referring to the maximum moment and the slope of decreasing moment-rotation branch has been performed. Susuki et al. (1997) studied numerically the ductile fracture of beams made from high-strength steel.

Tehami (1997) proposed a plastic collapse mechanism model for continuous composite beams in which the length of mechanism is dependent on the section depth only, without considering the flange width as an influencing parameter. Based on Kemp's studies, Dekker (1998) has proposed an equation for evaluating the buckling strain. Considering the idealised stress-strain variation, it is possible to calculate the value of compression buckling stress. Earls and Galambos (1998) have studied the behaviour of beams made of high-performance steel (HSLA 80). In-plane and out-of-plane plastic buckling modes, constant and varying moment gradients, cross-sectional imperfections and lateral bracing are all considered parameters. In 2001, Gioncu and Petcu improved their previous plastic collapse mechanism model. Their improved model may be considered as a synthesis of all previous models. In this model, both flange width and the web depth are considered as mechanism parameters.

In 1999, Elghazouli & Izzuddin studied the influence of local buckling of steel sections on the response of frame members and sub assemblages under fire using nonlinear analysis program ADAPTIC. They merely used elasto-plastic cubic element with simplified bilinear stress-strain relationship. A simple approach is employed in this study to take into account the local buckling effect by reducing the stress using unloading relationship whenever the compressive strain has exceeded the buckling strain suggested by Fukumoto & Itoh (1984). They found that local buckling has a pronounced effect in frame assemblages. Recently, Fontana & Knobloch (2004) proposed a structural model for steel plates in bending and compression at elevated temperatures. A strain-based formulation for effective widths considering plastic stress distribution and strain-dependent nonlinear material behaviour has been developed to analyse the load-carrying capacity at elevated temperatures of steel plates supported on three or four sides.

As reviewed above, extensive research on local buckling at ambient temperature and its effects on ductility have been performed since 1950's. However, this area is surprisingly not well researched under fire conditions although local buckling has been observed in some fire tests. Only a few researchers conducted a study related to local buckling failure at elevated temperatures as mentioned in the previous paragraph. However, to the best knowledge of the author, no research related to the ductility issue of steel and composite beams at elevated temperatures has been conducted.

2.3.2 Local Buckling Behaviour at Ambient Temperature

The plastic behaviour of a structure depends on the amount of moment redistribution. The attainment of the predicted collapse load is related to the position of plastic hinges, where sections reach the full plastic moment capacity, and to the plastic rotation capacity so that other hinges can develop elsewhere in a frame. In plastic analysis, it is normally assumed that the moment capacity of the member will remain at the level of the plastic

moment until sufficient numbers of hinges have developed to form a mechanism. Therefore, it is necessary that the moment capacity should not be impaired by local or lateral torsional buckling until the required rotation has been achieved.

Local and lateral torsional buckling are not always independent phenomena, however, current European code (EC3:1.1 (CEN, 2005)) and American steel design specification (AISC, 1994) simplify the flexural design of I-shaped beams such that cross-sectional proportions are considered independently from the unbraced length when computing design capacity. Such a simplification is based on the notion that cross-sectional instabilities of component plate elements may be considered independently from global instabilities such as lateral torsional buckling. It is clear that this simplification is made out of necessity, due to the substantial complexity of treating the true interactive nature of local and global modes as outlined by the plastic design commentary of the ASCE (1971):

“Even though local and lateral torsional buckling in the inelastic range are manifestations of the same phenomena, namely, the development of large cross-sectional distortions at large strains, they have been treated as independent problems in the literature dealing with these subjects. This is mainly due to the complexity of the problem.”

Treating local buckling separately from lateral torsional buckling in assessing the member ductility means that the current codes still adopt the concept of cross-section behavioural classes which depends only on the width-to-thickness ratios of flanges and of web. The cross-section behavioural concept contains some shortcomings (Mazzolani and Piluso, 1993):

- a) Independent limitations of flange and web ratios are unreasonable, because the flange is restrained by the web and the web is restrained by the flange. Only the Japanese code takes into account the interaction of flange and web for H-sections.

- b) The local ductility depends not only on the width-to-thickness ratios, but also on the ratio between the width of flange and depth of web, member length, moment gradient, level of axial load, eccentricity of axial load, steel quality, flexural torsional buckling, etc.
- c) As far as the values of width-to-thickness ratios in the different ductility classes are concerned, it can be observed that this subdivision does not correspond to the actual behaviour of beam and beam-column which is a continuous one. Thus, the given limiting discrete values of width-to-thickness ratio seem to be arbitrary.

Beams are usually divided into four classes (as shown in Figure 2.1) based on their rotational capacity:

- Class 1 (plastic section); sections belonging to the first class are characterized by the capability to develop a plastic hinge with high rotation capacity.
- Class 2 (compact section); class two sections are able to attain their full plastic moment capacity, but they have a limited rotation capacity.
- Class 3 (semi-compact section); semi-compact section is able to attain its yield moment capacity, but not its plastic moment capacity.
- Class 4 (slender section); slender section is not able to develop their total flexural resistance due to the premature occurrence of local buckling before the section reaches its yield moment capacity.

As mentioned, the current codes use the concept of cross-section behavioural classes. A more rational way of quantifying the ductility of beams is based on the member behavioural classes that take into account other factors besides the width-to-thickness ratios of flange and web. It is widely accepted that ductility may be quantified most directly by measuring the available inelastic rotation θ_a , as shown in Figure 2.2, over which the moment exceeds its design plastic moment capacity M_p . To reduce the influence of the span-to-depth ratio and relative flexural rigidity of different members,

the term available rotational capacity r_a is used to describe the non-dimensional form of inelastic rotation and defined as

$$r_a = \theta_a / \theta_p \quad (2.1)$$

in which θ_p = plastic rotation (rotation up to the attainment of plastic moment).

Figure 2.1 Section Classifications of Beams

The rotational capacity concept may be used explicitly or implicitly. If it is used explicitly, the required rotation of a structure may be determined from plastic analysis and then compared with the available rotational capacity of the member. If the available rotational capacity is not exceeded anywhere in the structure, then the plastic analysis is considered valid (i.e., the idealised moment-rotation behaviour assumed in the analysis is acceptable). In most design codes, however, the rotation capacity provisions are implicit; if cross-sectional geometry limit is satisfied, and/ or the lateral support requirements are met, the element is deemed to have an available rotational capacity greater than the maximum required rotation that possibly will be demanded from the structures. The implicit concept adopted by design codes is convenient and relatively simple, but it is at odds with experimental evidence showing that other factors, besides

cross-sectional geometry limit, are also a crucial limitation on rotational capacity (Kemp, 1985).

Figure 2.2 Standard Moment-Rotation Curve of Plastic/ Compact Beams

There are differences with the limitations of width-to-thickness ratios for flange and web across different design codes. Besides, their theoretical or experimental backgrounds are not necessarily clear (Kato, 1989). Among many design specifications, only the theoretical backgrounds of two design codes: EC3:1.1 (2005) and BS5950:1 (2001) are discussed subsequently. The comparisons of various other design codes on the width-to-thickness limit for flange and web have been presented by Bild and Kulak (1991).

The provisions in design codes are based on the behaviour of a single plate supported along its edges. The general equations for elastic plate buckling is

$$\sigma_{cr} = k_b \left[\frac{\pi^2 E}{12(1-\nu^2)} \right] \left(\frac{t}{b} \right)^2 \quad (2.2)$$

where σ_{cr} = critical plate buckling stress;

k_b = buckling coefficient;

t, b = thickness and width of plate, respectively;

E = elastic modulus;

ν = Poisson's ratio.

The value of buckling coefficient depends on the loading and boundary condition. A long plate element that is simply supported along both transverse edges and one longitudinal edge, but is free along the other edge (similar to flange outstand), has a value of k_b close to 0.425. In this case, the elastic buckling stress is equal to yield strength f_y when

- For a steel with $E = 205000$ MPa and $\nu = 0.3$:

$$\frac{b}{t} \sqrt{\frac{f_y}{275}} = 16.9 \quad (2.3)$$

- For a steel with $E = 210000$ MPa and $\nu = 0.3$:

$$\frac{b}{t} \sqrt{\frac{f_y}{235}} = 18.5 \quad (2.4)$$

For stocky steel plates for which the calculated elastic buckling stress exceeds the yield stress, strain-hardening modulus should be used instead. Basler (1961) showed that the strain-hardening buckling stress (for flange outstand) is equal to the yield stress when

$$\frac{b}{t} \sqrt{\frac{f_y}{275}} = 7.6 \quad (2.5)$$

or
$$\frac{b}{t} \sqrt{\frac{f_y}{235}} = 8.2 \quad (2.6)$$

A flat plate with depth d and thickness t which is simply supported along all four edges and subjected to bending stress (similar to web plate under bending stress) has elastic buckling stress equal to

$$\sigma_{cr} = k_b \left[\frac{\pi^2 E}{12(1-\nu^2)} \right] \left(\frac{t}{d} \right)^2 \quad (2.7)$$

where for long plates, the value of k_b is close to 23.9. In this case, the elastic buckling stress is equal to yield stress when

- For a steel with $E = 205000$ MPa and $\nu = 0.3$:

$$\frac{d}{t} \sqrt{\frac{f_y}{275}} = 126.9 \quad (2.8)$$

- For a steel with $E = 210000$ MPa and $\nu = 0.3$:

$$\frac{d}{t} \sqrt{\frac{f_y}{235}} = 138.9 \quad (2.9)$$

For a simply supported plate element under pure shear, the value of k_b is close to 5.35 if the plate is long (similar to unstiffened web). Hence, the elastic buckling stress is equal to yield strength in shear $\tau_y = f_y / \sqrt{3}$ when

- For a steel with $E = 205000$ MPa and $\nu = 0.3$:

$$\frac{d}{t} \sqrt{\frac{f_y}{275}} = 79 \quad (2.10)$$

- For a steel with $E = 210000$ MPa and $\nu = 0.3$:

$$\frac{d}{t} \sqrt{\frac{f_y}{235}} = 86.5 \quad (2.11)$$

For a plate under combined shear and bending, the elastic buckling stress can be predicted using an interaction equation:

$$\left(\frac{\sigma_s}{\sigma_{os}} \right)^2 + \left(\frac{\sigma_b}{\sigma_{ob}} \right)^2 = 1 \quad (2.12)$$

where σ_s = applied shear stress;

σ_{os} = pure shear elastic buckling stress;

σ_b = maximum applied bending stress;

σ_{ob} = pure bending elastic buckling stress.

It has been shown that the most severe loading condition for which elastic buckling and yielding occurs simultaneously is that of pure shear and the limit is therefore given in eq. (2.10) or (2.11).

Both limiting width-to-thickness ratios of outstand flange and web (neutral axis at mid depth) from EC3:1.1 (CEN, 2005) and BS5950:1-2000 (BSI, 2001) are presented in Table 2.1. For outstand flange, the limit for class 1 is related to the strain-hardening buckling of eq. (2.6) for EC3:1.1 and eq. (2.5) for BS5950:1. Class 2 section has a limit slightly higher than eq. (2.6) and (2.5) for EC3:1.1 and BS5950:1, respectively, however, the basis is unclear. Class 3 section is able to reach first yield, therefore the limit is related to eq. (2.4) and (2.3) for EC3:1.1 and BS5950:1, respectively. A section with width-to-thickness ratio larger than class 3 limit is considered to be class 4 and unable to reach first yield because buckling occurs prematurely. BS5950:1 differentiates the limit between rolled section and welded section, unlike EC3:1.

For web with the neutral axis at mid depth, class 1 section has a limit related to first yield under pure shear. Thus, the limit is close to eq. (2.11) and (2.10) for EC3:1.1 and BS5950:1, respectively. Class 2 has a limit slightly higher than class 1, however, the basis of this is unclear. Class 3 limit is related to the first yield of plate under pure bending, and is close to eq. (2.9) and (2.8) for EC3:1.1 and BS5950:1, respectively. The differences in the limit given by design codes, and between codes and the ones given by the theory of plate buckling are probably due to the different safety factor that are applied by different design codes and the calibration to member testing results, which indeed more reliable and better represent the actual behaviour. The main criticism of the BS5950:1 and EC3:1.1 classification are that they only consider a very few parameters and are based only on material and cross-section factors, while, the very important member characteristics are ignored. Due to these additional factors, it seems rational that the BS5950 and EC3 concept of cross-section behavioural classes should be replaced by the concept of member behavioural classes.

Table 2.1 Comparison of Limiting Width-To-Thickness Ratio

Limit of Width-to-Thickness Ratio				
Section Description		EC3:1.1 (CEN, 2005)		
		Class 1	Class 2	Class 3
Flange Outstand		9ε	10ε	14ε
Web (neutral axis at mid depth)		72ε	83ε	124ε
		Note: ε = √(235/f _y)		
Section Description		BS5950:1-2000 (BSI, 2001)		
		Class 1	Class 2	Class 3
Flange Outstand	Hot Rolled	9ε	10ε	15ε
	Welded	8ε	9ε	13ε
Web (neutral axis at mid depth)		80ε	100ε	120ε
		Note: ε = √(275/f _y)		

2.3.3 Local Buckling & Ductility in Fire – Importance & Motivations

The ability to redistribute moment among members during fire is the reason that beams which form part of an integral frame structure give much better performance than isolated members. In order to avoid premature failure and to allow sufficient redistribution of moment, members should be designed to have sufficient rotational capacity (ductility) in fire. In addition, redistribution of moment results in more efficient and effective use of beams because it transfers part of the hogging moment, which from elastic analysis is usually higher than sagging moment, to the mid-span regions. It is therefore important that the beam's section near the supports possess certain levels of inelastic rotation to allow some degree of moment redistribution. Thus, the available rotational capacity of a beam should be greater than the required rotational capacity to allow moment redistribution in the event of distress. One of the most important factors affecting the rotational capacity of beams is local buckling. Interaction between local and lateral torsional buckling has also been observed during fire incidents as well.

Simply assuming that members with sufficient ductility at ambient temperature will also possess sufficient ductility under fire conditions is erroneous. As temperature increases, the stress-strain relationship of steel becomes highly non-linear. As a result, yielding occurs at a much higher strain compared to ambient temperature. Due to larger strains required in a member to achieve its yield or plastic moment capacity at elevated temperature, consequently, there is less available inelastic rotation. Insufficient rotational capacity of the member in a plastically designed structure will nullify the conventional assumption that moment capacity of the member remains at the level of the plastic moment until sufficient numbers of hinges have developed to form a mechanism. Thus, before plastic theory can be applied in fire engineering design, inelastic behaviour of beams in fire needs to be investigated and quantified.

The local buckling phenomenon has been observed from tests on unprotected continuous composite beams by Zhao & Kruppa (1995) who showed local buckling failure in the web and the lower flange of the steel section near the support. Another interesting observation of local buckling failure in fire is obtained from the large scale fire test in the UK at the Building Research Establishment (normally called Cardington fire test (British Steel, 1998)) as shown in Figure 2.3(a) to (d). As can be seen, the local buckling failure occurred in almost every beam. The lower flange near the support (subjected to compressive stresses from the negative bending moment) buckled and rotated such that one side of it went up and another side went down. The web above the flange was also buckled severely. Almost the entire depth of the web buckled since almost the entire depth of the web near support region is in compression.

Although local buckling failure is more prominent in fire conditions, the design codes have not addressed it in detail. BS5950:8 (BSI, 2003) does not even provide any cross-sectional classifications. EC3:1.2 (CEN, 2005) classifies the cross-section as for normal temperature design, although the effective yield strength at elevated temperature

is reached for large, non-reversible strain of 2%. The only modification is the introduction of a reduction factor 0.85 such that

$$\varepsilon = 0.85 \sqrt{\frac{235}{f_y}} \quad (2.13)$$

The reduction factor results from the simplification of the reciprocal of the ratio between yield strength and elastic modulus reduction factor at elevated temperature $(1/\sqrt{k_y/k_E})$, which has a value of approximately 0.85. The current treatment of local buckling and section classification at elevated temperature in design codes is very brief due to a serious lack of research and understanding at elevated temperature. Obviously, there is an urgent need for a better understanding of local buckling failure and available ductility of steel and composite beams. Besides, the concept of member behavioural classes should be used instead of the cross-section behavioural classes for a safer and more economical member design in fire.

Due to lack of research of local buckling and ductility under fire conditions and the importance of the member's inelastic behaviour to better understand the behaviour of frame structures during fire incidents, the author feels the urgent need to study the behaviour of local buckling of steel and composite beams under fire conditions. Composite beams (steel beams with re-entrant steel decking slab) were considered after completing the study on bare steel beams, due to increasing popularity of such system in the construction industry. The behaviour of the joints, however, is not part of this research since it focuses on the inelastic behaviour of the member. Hence, this study is applicable to relatively rigid type of connection system such as “fully welded”, “top and bottom plate” and “extended end plate”. For these types of connection, the joint stiffness is much greater than the member stiffness, hence the rotation near the support is largely provided by the member and the ductility is also governed by the member. EC3:1.1 (CEN, 2005) consider these types of connections as “continuous joint” in which the joint behaviour may be assumed to have no effect on structural analysis.

This study is also applicable to continuous beams with semi-rigid joints, such as “flush end plate” and “flange and web angles”, under fire conditions. The reason is that, in the event of fire, the temperatures of the joints are often lower than that of the member due to the shielding effect. This was observed during Cardington fire test (British Steel, 1998); hence the relative stiffness of the connections compared to the member is increased. Consequently, the rotation near the support is largely provided by the member. This analogous to a beam with a rigid joint and the member, instead of the connection, often fails by local flange and web buckling (Fig. 2.3). Thus, in the event of fire, the semi-rigid joints in effect behave like rigid joints since the steel section is relatively weaker due to higher temperatures. Through this study, the author hopes that the level of understanding in the area of local buckling behaviour and available ductility of steel and composite members at elevated temperatures is improved.

CHAPTER THREE

ELEVATED TEMPERATURE MATERIAL PROPERTIES

3.1 Introduction

Mechanical and thermal properties of both steel and concrete deteriorate at elevated temperature. During the last three decades, many tests have been conducted to study these properties. These properties are one of the most important factors in the success of analytical and numerical modelling. The important mechanical properties are elastic modulus, yield strength and stress-strain relationships. Among all the thermal properties, thermal strain is the only concern in this research as it only focuses on the structural response. A review of past works in the modelling of both mechanical and thermal properties of steel and concrete material are presented here, together with the adopted material model used in this research. In addition, the stress-strain relationships of reinforcement and the load-slip relationships of shear stud at elevated temperature are also discussed in this chapter.

3.2 Steel Properties at Elevated Temperature

Steel properties may be divided into two categories: *mechanical* and *thermal* properties. Mechanical properties include stress-strain relationships, yield strength, elastic modulus, and Poisson's ratio, while thermal properties include thermal strain, thermal conductivity, specific heat capacity, etc. When developing an analytical model of steel structure at elevated temperatures, a great amount of data involving stress and deformation characteristics of steel at various temperatures are needed. The properties must be based on reliable and well-controlled tests. There are two main methods of

testing to obtain these data at elevated temperatures, namely, *steady-state* and *transient-state* test. Material properties are related to the test method used. Steady-state test consists of heating to the required temperature before any load is applied. The strain that is measured before load application corresponds to thermal strain. Transient state test is characterized by varying temperature while keeping the load constant. Therefore, the total measured strains consist of mechanical, thermal and creep strain. The thermal strain can only be measured on unloaded specimens.

3.2.1 Stress-Strain Relationships

The relationships between stresses and strains are needed for the modelling of material mechanical behaviour. As steel is heated above 200°C, its stiffness starts to reduce followed by a strength reduction when the temperature reaches beyond 400°C. Beyond 400°C, steel also starts to display a creep phenomenon. The occurrence of creep indicates that the stress and the temperature history have to be taken into account. It is generally agreed that the deformation process of steel at transient temperatures can be described by three strain ε components as proposed by Anderberg and Thelandersson in 1976 (Anderberg, 1988) and is defined by

$$\varepsilon = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) \quad (3.1)$$

where ε_{th} = thermal strain;

ε_{σ} = stress-related strain or instantaneous strain;

ε_{cr} = creep strain or time-dependent strain.

Eq. (3.1) includes explicitly the effect of creep; therefore it is based on the steady-state test result, whereby creep strain can be measured directly. There is another model, called pragmatic model, which includes implicitly the effect of creep. The former is actually preferred, but the implementation is very difficult. According to Twilt (1988), it is not necessary to use a complicated creep model under standard fire conditions since a pragmatic model is sufficient. He has established that a steel member that is exposed to

standard fire conditions will achieve a heating rate, under practical circumstances, between $50^{\circ}\text{C}/\text{min}$ (non-insulated member) and $5^{\circ}\text{C}/\text{min}$ (heavily insulated member). His investigation revealed that the difference in the critical temperature of a simply supported steel beam is less than 40°C between these two extreme conditions. Hence, in terms of fire resistance time, this would mean that due to creep, differences are a few minutes (between 0.8 minutes (if heating rate = $50^{\circ}\text{C}/\text{min}$) and 8 minutes (if heating rate = $5^{\circ}\text{C}/\text{min}$)). Hence, it should be noted that, the effect to fire resistance time due to creep is very little. In view of the other uncertainties involved, such accuracy is meaningless. This conclusion is important since most of fire engineering designs are still based on standard fire curve. As such, the pragmatic model is adopted for the current analytical model. The pragmatic model, which is solely based on transient-state test, is constructed by measuring the total deformation under a constant stress and increasing the temperature until failure occurs. The thermal strain is then subtracted from the total strain to obtain the stress-strain relationships, which includes the creep strain corresponding to a specified heating rate. As a result, a series of temperature-dependent stress-strain relationships are obtained.

For ease of implementation, the experimental stress-strain relationships are usually idealised. However, idealization is difficult if bi-linear model (one line to represent the elastic range and another to represent the plastic range), as shown in Figure 3.1(a), is adopted because the yield point and the limit of proportionality are less defined as the steel temperature increases. Thus, by adopting a bi-linear model, there will be no smooth transition from the elastic range to plastic range. This is in contrary to real steel behaviour at elevated temperature. In order to overcome this problem, a tri-linear model, as shown in Figure 3.1(b), is introduced, in which there is a smoother transition from the elastic range to plastic range. A better defined stress-strain curve consists of two straight lines connected by an elliptic curve and is named as bilinear-elliptic model (Figure 3.2). Rubert & Schumann (1986) and Burgess et al. (1990) have adopted this material model

in their research. Eurocode 3:1.2 (CEN, 2005) model adopted also a bilinear-elliptic model based on the *transient-state* test data; hence it incorporates creep effects implicitly. Since EC3 Pt.1.2 has been widely used and accepted as the main steel specification of the European Communities, its stress-strain curve and thermal properties are both adopted in this research.

Figure 3.1 Idealisation for Stress-Strain Relationship

Figure 3.2 Bilinear-Elliptical Idealisation for Stress-Strain Relationship

Figure 3.3 shows a typical stress-strain curve adopted by Eurocode 3: Part 1.2. The first straight line denotes the elastic range up to a proportionality limit (f_{pT}) with the corresponding strain (ε_{pT}). The elliptic curve denotes the transition from elastic to plastic up to an effective yield strength (f_{yT}) with a corresponding yield strain (ε_{yT}). The horizontal line denotes the plastic range up to a limiting strain for yield strength (ε_{iT}). Finally, there is a declining line down to the ultimate strain (ε_{uT}), where its value

is assumed to be independent of temperature. Table 3.1 lists the stress-strain equations of the stress-strain curves for various temperatures, whereas Table 3.2 gives the associated reduction factors for stress-strain relationship of steel.

Figure 3.3 Basic Formulation of Stress-Strain Relationship of Steel at Elevated Temperatures (CEN, 2005)

Table 3.1 Stress-Strain Relationship Adopted by Eurocode 3: Part 1.2 (CEN, 2005)

Strain Range	Stress σ	Tangent modulus
$\varepsilon \leq \varepsilon_{pT}$	εE_T	E_T
$\varepsilon_{pT} < \varepsilon < \varepsilon_{yT}$	$f_{pT} - c + \frac{b}{a} \sqrt{a^2 - (\varepsilon_{yT} - \varepsilon)^2}$	$\frac{b(\varepsilon_{yT} - \varepsilon)}{a \sqrt{a^2 - (\varepsilon_{yT} - \varepsilon)^2}}$
$\varepsilon_{yT} \leq \varepsilon \leq \varepsilon_{lT}$	f_{yT}	0
$\varepsilon_{lT} \leq \varepsilon \leq \varepsilon_{uT}$	$f_{yT} \left[1 - \frac{\varepsilon - \varepsilon_{lT}}{\varepsilon_{uT} - \varepsilon_{lT}} \right]$	-
$\varepsilon = \varepsilon_{uT}$	0.00	-

in which:

$$a^2 = [\varepsilon_{yT} - \varepsilon_{pT}] [\varepsilon_{yT} - \varepsilon_{pT} + c/E_T] \quad (3.2)$$

$$b^2 = c [\varepsilon_{yT} - \varepsilon_{pT}] E_T + c^2 \quad (3.3)$$

$$c = \frac{[f_{yT} - f_{pT}]^2}{[\varepsilon_{yT} - \varepsilon_{pT}] E_T - 2[f_{yT} - f_{pT}]} \quad (3.4)$$

where f_{yT} = effective yield strength at elevated temperature;

f_{pT} = proportionality limit at elevated temperature;

E_T = elastic modulus at elevated temperature;

E = elastic modulus;

ε_{pT} = strain corresponding to proportionality limit at elevated temperature;

ε_{yT} = yield strain at elevated temperature;

ε_{iT} = limiting strain for yield strength at elevated temperature;

ε_{uT} = ultimate strain at elevated temperature;

T = steel temperature.

Table 3.2 Reduction Factors for EC3 Stress-Strain Relationships of Steel
at Elevated Temperatures (CEN, 2005)

Steel Temperature $T(^{\circ}\text{C})$	Reduction Factor for Effective Yield Strength $k_y = f_{yT} / f_y$	Reduction Factor for Proportional Limit $k_p = f_{pT} / f_y$	Reduction Factor for Elastic Modulus $k_E = E_T / E$
20	1.000	1.000	1.000
100	1.000	1.000	1.000
200	1.000	0.807	0.900
300	1.000	0.613	0.800
400	1.000	0.420	0.700
500	0.780	0.360	0.600
600	0.470	0.180	0.310
700	0.230	0.075	0.130
800	0.110	0.050	0.090
900	0.060	0.0375	0.0675
1000	0.040	0.0250	0.0450
1100	0.020	0.0125	0.0225
1200	0.000	0.000	0.000

EC3:1.2 stress-strain curves that are described above are based on the test data from Kirby and Preston (1988). They tested two steel qualities conforming to BS4360: Grades 43A and 50B (BSI, 1990). These represent approximately 94% of all the structural steel sections. Besides, they have ambient temperature properties similar to those defined in Euronorm 25-72: Grades Fe430 and Fe510. Figure 3.4 shows the stress-strain data from Kirby and Preston (1988) *transient-state* tests for a heating rate of 10°C/min and EC3:1.2 stress-strain relationship at various temperatures. As can be seen, the EC3:1.2 steel material stress-strain curves follow closely the test results. Besides EC3:1.2, BS5950:8 also adopted the test data from Kirby and Preston (1988).

Figure 3.4 Comparison of Test Data (Kirby & Preston, 1988) and EC3:1.2 Stress-Strain Relationships

3.2.2 Yield Strength

As the steel temperature increases, the well-defined yield point of steel vanishes. As a result, the concept of yield stress is invalid. An alternative is to use the concept of *proof stress* (Cooke, 1988) as shown in Figure 3.5(a). A tangent of the curve is drawn at O, and

a parallel line to it (line BC) is drawn at 0.2% strain. The stress at point C is called 0.2% proof stress. In other words, 0.2% proof stress is the stress corresponding to 0.2% permanent strain. However, Twilt (1988) claimed that the use of 0.2% proof stress was less appropriate in fire engineering design, since under fire conditions, the deformations are less critical than at room temperature. He preferred the use of *effective yield stress* concept. The idea of effective yield stress is to truncate the material stress-strain curve at a relatively high strain level, usually 0.5%-1%, to introduce a yield plateau. This concept is important in order to extend elementary plastic theory to elevated temperature analysis. Figure 3.5(b) shows the effective yield stress concept graphically. EC3:1.2 also uses the effective yield stress concept by truncating the stress-strain curve at 2% strain to define the start of yield plateau.

Figure 3.5 Two Different Concepts of Defining Yield Stress

BS5950:8 presents the effective yield strength based on Kirby and Preston (1988) test data in terms of strength reduction factor at three different strain levels, i.e. 0.5%, 1.5% and 2%. The strain level to be used depends on the type of member, the presence of fire protection material and the performance of fire protection material as stipulated in the code. Figure 3.6 shows the comparison of strength reduction factor between EC3:1.2 and BS5950:8.

Figure 3.6 Variation of Strength Reduction Factor with Temperature

3.2.3 Elastic Modulus

Elastic modulus reduces with increasing temperatures. Besides, the tangent modulus is no longer constant and the same as initial tangent modulus at elevated temperature, but it is stress-dependent. The elastic modulus is very important in the calculation of structural deformations at elevated temperatures. However, there is little agreement on the elastic modulus data at elevated temperature. Cooke (1988) discussed the reasons behind the differences. Some of the important reasons are

1. Different tensile testing machines have different range of accuracies for measuring very small strains and for achieving a uniform and accurately measured temperature over the entire gauge length.
2. Inaccuracy in strain measurements will mean that the shape of the stress-strain curve is affected and, of major importance, the shape of the curve near the origin. This makes it very difficult to plot a tangent to the curve and, since the slope of the tangent at the origin is the elastic modulus, inaccuracies in tangent plotting procedure result in inaccuracies in the determination of elastic modulus.
3. Differences in chemical composition of test specimens can affect the strain.

4. Different rates of strain are adopted by researchers. A fast rate of strain in the isothermal test would be expected to result in higher measured stresses than a slow rate.

At present, there is still no generally accepted model for determining the temperature-dependent elastic modulus.

3.2.4 Poisson's Ratio

When a prismatic bar is pulled, the axial elongation is accompanied by a lateral contraction. The ratio of the lateral strain to the axial strain is called Poisson's ratio (ν). According to Cooke (1988), the best estimate of ν for structural steel is between 0.27-0.30. BS 5950: Part 1 (BSI, 2001) recommends a value of 0.30 for the design of steel structures at ambient temperature. At elevated temperature, Clark (1953) has reported the values for mild steels up to 650°C, which are around 0.30. Since no significant variation of ν observed as temperature increases, BS 5950: Part 8 (BSI, 2003) uses a single value of 0.30 for structural steels. Thus, this value will be adopted in the numerical study.

3.2.5 Coefficient of Thermal Expansion

When a solid material is heated, it expands according to a general equation:

$$l_T = l_0 (1 + \alpha \Delta T + \alpha_1 \Delta T^2 + \alpha_2 \Delta T^3) \quad (3.5)$$

where l_T = length after temperature rise of ΔT ;

l_0 = length at the initial temperature;

ΔT = temperature rise;

$\alpha, \alpha_1, \alpha_2$ = first, second, and third order of coefficient of thermal expansion.

For pure metals, such as steel, the constants α, α_1 and α_2 have values of the order of $10^{-5}, 10^{-11}$ and 10^{-14} , respectively. Therefore, it is adequate to only use the first order coefficient α and ignore the second and third order coefficients. The coefficient of

thermal expansion varies with temperature. EC3:1.2 (CEN, 2005) gives the values of thermal strain $\Delta l/l_0 = (l_T - l_0)/l_0$ in three different ranges of temperature:

- for $20^\circ\text{C} \leq T < 750^\circ\text{C}$:

$$\Delta l/l_0 = 1.2 \times 10^{-5} T + 0.4 \times 10^{-8} T^2 - 2.416 \times 10^{-4} \quad (3.6)$$

- for $750^\circ\text{C} \leq T < 860^\circ\text{C}$:

$$\Delta l/l_0 = 1.1 \times 10^{-2} \quad (3.7)$$

- for $860^\circ\text{C} \leq T < 1200^\circ\text{C}$:

$$\Delta l/l_0 = 2 \times 10^{-5} T - 6.2 \times 10^{-3} \quad (3.8)$$

However, it also allows the use of single value of $\alpha = 14 \times 10^{-6} / ^\circ\text{C}$ for simple calculations. Figure 3.7 shows the variation of thermal strain with temperatures using three different equations of $\Delta l/l_0$ and a single equation of $14 \times 10^{-6} \times (T - 20)$. This single value is also the value that is used in BS 5950: Part 8 (BSI, 2003). Thus, this study adopts this value for analysis.

Figure 3.7 Thermal Strain of Steel as a Function of Temperature

3.3 Concrete Properties at Elevated Temperature

The concrete properties at elevated temperatures are more variable and complex than the steel properties. Two properties which are relevant to this study are discussed in this section, namely, the stress-strain relationships and the thermal strains. The test methods for concrete material are generally the same as the ones for steel.

3.3.1 Stress-Strain Relationships

Many tests have been conducted since the 1940s to investigate the mechanical properties of concrete at elevated temperatures. However, there is still variability among these test results as explained by Khoury (1992). A review of the mechanical properties of concrete at elevated temperatures was reported by Phan & Carino (1998). They compared the mechanical properties of high strength concrete to normal strength concrete and EC4:1.2 proposals. They found that EC4:1.2 proposals suit only normal strength concrete. Furthermore, the values given in EC4:1.2 may be regarded as the lower bound values for different test results of normal strength concrete (Wang, 2002). Figure 3.8 shows the Eurocode 4 model of stress-strain relationship for concrete at elevated temperatures. It can be seen that the model consists of two parts, namely the ascending and descending part. The ascending part is given by eq. (3.9).

$$\sigma_{c,T} = f_{c,T} \left\{ 3 \left(\frac{\varepsilon_{c,T}}{\varepsilon_{cu,T}} \right) \left/ \left[2 + \left(\frac{\varepsilon_{c,T}}{\varepsilon_{cu,T}} \right)^3 \right] \right. \right\} \quad (3.9)$$

in which $\sigma_{c,T}$, $\varepsilon_{c,T}$, $f_{c,T}$, $\varepsilon_{cu,T}$ are the concrete stress, strain, ultimate strength and strain at elevated temperature, respectively. The value of $f_{c,T}$ can be obtained as

$$f_{c,T} = k_{c,T} f_c \quad (3.10)$$

where $k_{c,T}$ = concrete strength reduction factor at elevated temperature;

f_c = concrete compressive strength.

The descending part (Fig. 3.8) is a straight line joining the ultimate strength to the point of concrete crush at $\varepsilon_{crush,T}$. Table 3.3 shows the values of $k_{c,T}$, $\varepsilon_{cu,T}$ and $\varepsilon_{crush,T}$ for both normal weight (NC) and light weight (LC) concrete according to Eurocode 4:1.2 (CEN, 2005).

Figure 3.8 Eurocode 4:1.2 Model of Stress-Strain Relationship of Concrete at Elevated Temperatures (CEN, 2005)

Table 3.3 Strength Reduction Factors and Strain Limits of Concrete at Elevated Temperatures for Eurocode 4:1.2 Model (CEN, 2005)

Temperature T (°C)	$k_{c,T}$		$\varepsilon_{cu,T} \times 10^3$	$\varepsilon_{crush,T} \times 10^3$
	NC	LC		
20	1.00	1.00	2.5	20.0
100	0.95	1.00	3.5	22.5
200	0.90	1.00	4.5	25.0
300	0.85	1.00	6.0	27.5
400	0.75	0.88	7.5	30.0
500	0.60	0.76	9.5	32.5
600	0.45	0.64	12.5	35.0
700	0.30	0.52	14.0	37.5
800	0.15	0.40	14.5	40.0
900	0.08	0.28	15.0	42.5
1000	0.04	0.16	15.0	45.0
1100	0.01	0.04	15.0	47.5
1200	0.00	0.00	15.0	50.0

The tensile strength of concrete at elevated temperature may be conservatively assumed to be zero according to EC4:1.2. However, if advanced calculation method is used such as finite element method, the concrete tensile strength reduction factor at elevated temperature $k_{ct,T}$ (eq. (3.11)) given in EC2:1.2 (CEN, 2004) can be applied to the ambient temperature tensile strength.

$$k_{ct,T} = 1.0 \quad \text{for } 20^\circ\text{C} \leq T \leq 100^\circ\text{C}$$

$$k_{ct,T} = 1.0 - 1.0(T - 100)/500 \quad \text{for } 100^\circ\text{C} \leq T \leq 600^\circ\text{C} \quad (3.11)$$

The tensile strength and compressive strength reduction factor for concrete at elevated temperatures is shown in Figure 3.9.

Figure 3.9 EC2 and EC4 Strength Reduction Factors of Concrete

3.3.2 Thermal Strain

The deformation of concrete at elevated temperatures is slightly more complicated than that of steel because of an additional component of strain called *transient strain* (Buchanan, 2001). However, this research only concerns about the stress-related strain as discussed previously and thermal strain. Eurocode 4:1.2 (CEN, 2005) gives the values of the thermal strain for both normal and light weight concrete. The thermal strain of

normal weight concrete is given as

$$\begin{aligned}\varepsilon_{th} &= -1.8 \times 10^{-4} + 9 \times 10^{-6} T + 2.3 \times 10^{-11} T^3 \quad \text{for } 20^\circ C \leq T \leq 700^\circ C \\ \varepsilon_{th} &= 14 \times 10^{-3} \quad \text{for } 700^\circ C \leq T \leq 1200^\circ C \quad (3.12)\end{aligned}$$

However, for simple calculations, the relationship between the thermal strain and concrete temperature may be considered to be linear as given in eq. (3.13).

$$\varepsilon_{th} = 18 \times 10^{-6} (T - 20) \quad (3.13)$$

The thermal strain for light weight concrete is less than that of normal weight concrete and is given by EC4:1.2 as

$$\varepsilon_{th} = 8 \times 10^{-6} (T - 20) \quad (3.14)$$

The variation of the thermal strain for both normal and light weight concrete is shown in Figure 3.10. It can be seen that the simple calculation model for normal weight concrete (NC) is suitable only for temperature up to 800°C.

Figure 3.10 Thermal Strain of Concrete as a Function of Temperature

3.4 Reinforcement Properties at Elevated Temperature

The strength and deformation properties of steel reinforcement at elevated temperature are similar to those of structural steel; hence the model for the stress-strain relationship of steel reinforcement at elevated temperatures given by EC4:1.2 (2005) is the same as

that of structural steel (Fig. 3.3). The reduction factors for the strength and stiffness of structural steel at elevated temperature (Table 3.2) may also be used for hot rolled reinforcement, but slightly different factors are used for cold worked reinforcement as shown in Table 3.4. The thermal properties of steel reinforcement are also the same as those of structural steel.

Table 3.4 Eurocode 4:1.2 Reduction Factors for Cold Worked Reinforcement
at Elevated Temperatures (CEN, 2005)

Steel Temperature $T(^{\circ}\text{C})$	Reduction Factor for Effective Yield Strength $k_y = f_{yT} / f_y$	Reduction Factor for Proportional Limit $k_p = f_{pT} / f_y$	Reduction Factor for Elastic Modulus $k_E = E_T / E$
20	1.00	1.00	1.00
100	1.00	0.96	1.00
200	0.87	0.92	0.87
300	0.72	0.81	0.72
400	0.56	0.63	0.56
500	0.40	0.44	0.40
600	0.24	0.26	0.24
700	0.08	0.08	0.08
800	0.06	0.06	0.06
900	0.05	0.05	0.05
1000	0.03	0.03	0.03
1100	0.02	0.02	0.02
1200	0.00	0.00	0.00

3.5 Load-Slip Relationship of Shear Stud at Elevated Temperature

Zhao and Kruppa (1995) investigated the effects of temperature on shear studs and their load-slip relationships. Push-out tests were conducted on 31 specimens which were fabricated according to EC4:1.1 (CEN, 2004). Two types of connectors (headed stud and pinned angle connector) and two types of concrete slabs (solid slab and composite slab with either re-entrant or trapezoidal decking) were used. The failure of the specimens for the headed studs with solid concrete slab and re-entrant decking was always by the shearing of the studs at the level of the flange, while that for the headed studs with

trapezoidal decking were a combination of the shearing of the studs and the shearing off of the concrete rib. The failure mode of the tested angle connectors is quite similar to the headed studs, but with a much lower slipping value. Based on these push-out test results, a series of numerical and theoretical analyses had been carried out and the evolution of load-slip relationships had been established for different types of connectors. Only the load-slip relationships at elevated temperatures of headed shear stud with re-entrant steel decking are reviewed here since it is the only type of shear connector used in the experiments. Zhao and Kruppa (1995) expressed the load-slip relationships at elevated temperature using a general formula which consisted of linear and non-linear part as given in eq. (3.15). The first one corresponds to the linear behaviour of connectors up to γ_o . The second one represents the non-linear behaviour of the connector up to its maximum resistance at γ_u . After the maximum resistance is reached, a straight descending line can be used up to the limited ductility of connector $\gamma_{\max}$.

$$\begin{aligned} \frac{P(\gamma)_T}{P_{u,T}} &= k \frac{\gamma}{\gamma_o} & \text{for } 0 \leq \gamma \leq \gamma_o \\ \frac{P(\gamma)_T}{P_{u,T}} &= \exp \left[\ln k \left(\frac{\gamma}{\gamma_o} \right)^{1/\ln k} \right] & \text{for } \gamma_o \leq \gamma \leq \gamma_u \end{aligned} \quad (3.15)$$

in which $P(\gamma)_T$ is the load at a certain value of slip γ and temperature T , while $P_{u,T}$ is the maximum load at temperature T .

Table 3.5 shows the values of the parameters for the load-slip relationship model of the headed shear stud in a composite slab with re-entrant steel decking. It can be seen that the ultimate shear capacity starts to reduce above 300°C. Figure 3.11 shows the load-slip relationships at different temperatures up to the maximum load. It can be observed that as temperature increases, the shear stiffness reduces and the ultimate shear capacity is reached at a greater slip. Since EC4:1.2 (CEN, 2005) does not provide any load-slip

relationship model for shear studs at elevated temperatures, the model by Zhao & Kruppa (1995) is used in the numerical simulations.

Table 3.5 Parameters for Load-Slip Relationship Model of Headed Shear Stud in Composite Beams with Re-entrant Steel Decking (Zhao & Kruppa, 1995)

Temperature (°C)	$\frac{P_{u,T}}{P_{u,20^\circ C}}$	k	γ_o (mm)	γ_u (mm)	γ_{max} (mm)
100	1.000	0.42	0.16	3.00	11
200	1.000	0.52	0.50	3.50	12
300	1.000	0.58	0.90	4.50	13
400	0.800	0.68	1.30	5.20	14
500	0.624	0.70	1.45	5.80	15
600	0.376	0.72	1.48	6.25	16
700	0.184	0.74	1.50	6.70	17
800	0.088	0.75	1.50	7.10	18

Figure 3.11 Load-Slip Relationship Model for Headed Shear Stud in Composite Beams with Re-entrant Steel Decking (Zhao & Kruppa, 1995)

CHAPTER FOUR

LATERAL TORSIONAL BUCKLING OF UNRESTRAINED STEEL BEAMS

4.1 Introduction

Beams subjected to flexure have much greater strength and stiffness in the plane in which the loads are applied (major principal axis) than in the plane of the minor principal axis. When a short-span beam is loaded, the ultimate load-carrying capacity is governed by its full plastic moment of resistance. This failure is due to excessive plastic yielding of cross-section, unless local buckling of the compression flange or web occurs prematurely. On the other hand, a laterally unrestrained long-span beam may fail by elastic lateral torsional buckling at a load much lower than its full plastic moment capacity. For a beam of intermediate span between these two limits, failure may occur by inelastic lateral torsional buckling. Thus, lateral torsional buckling can be defined as a limit state of structural usefulness where the deformation of a beam changes from predominantly in-plane deflection to a combination of lateral deflection and twisting while the load capacity remains first constant, before dropping off due to large deflections and yielding (Galambos, 1988).

This chapter discusses the lateral torsional buckling (LTB) of laterally unrestrained steel beams at ambient temperature for the first part and that at elevated temperature for the second part. In the first part, the analytical derivations of lateral torsional buckling of steel beams at ambient temperature are presented step-by-step. At the same time, the important behaviour of LTB of steel beams at ambient temperature is also discussed. Subsequently, both design approaches from BS5950:1-2000 (BSI, 2001) and EC3:1.1

(CEN, 2005) are derived step by step by the author. Some comparisons and comments of the design approaches at ambient temperature are also discussed in this first part. The second part begins with a discussion of two current design approaches at elevated temperatures, namely, BS5950:8 (BSI, 2003) and EC3:1.2 (CEN, 2005). The weaknesses and the need for alternative approaches for LTB of steel beams under fire conditions are pointed out. Subsequently, the author proposes both general method (called alternative approach) and simple calculation method (Rankine approach) to predict the buckling moment.

In order to validate the proposed approaches for LTB of steel beams at elevated temperatures, numerical analysis using MSC.MARC Mentat has been conducted. In addition, published experimental works by Vila Real et al. (2003) are also extracted and compared with numerical results and proposed approaches. Only steel I-beams with equal flanges are considered in this study. Besides, uniform temperature distribution is assumed since most unrestrained steel beams do not support a slab on top; hence producing a fairly uniform temperature distribution across the section.

4.2 Lateral Torsional Buckling at Ambient Temperature

Before elaborating on the LTB of beams at elevated temperatures, it is useful to review the LTB of steel beams at ambient temperature. A widely known elastic buckling moment of perfectly straight beams is firstly reviewed. Subsequently, the maximum allowable moment in crooked beams is presented step-by-step by the author. Comparisons of BS5950:1-2000 (BSI, 2001) and EC3:1.1 (CEN, 2005) approach are then made with reference to the pertinent equations.

4.2.1 Unrestrained Beams: (a) Perfectly Straight and (b) Crooked

A straight elastic beam has no out-of-plane deformations until the applied moment reaches the elastic buckling moment. The elastic buckling moment of a straight beam

loaded by equal and opposite end moments, as shown in Figure 4.1, is given by (Timoshenko & Gere, 1961)

$$M_{cr} = \sqrt{\left(\frac{\pi^2 EI_y}{L^2}\right) \left[GJ + \frac{\pi^2 EI_w}{L^2}\right]} \quad (4.1)$$

where M_{cr} = elastic buckling moment capacity;

I_y = minor axis moment of inertia;

L = beam length;

G = shear modulus;

J = torsion constant;

I_w = warping constant.

Figure 4.1 Buckling of Simply Supported I-Beam

Eq. (4.1) ignores the effect of major axis curvature and produces conservative estimates of the elastic buckling moment. This assumption is justified when the major axis flexural rigidity (EI_x) is much larger than the minor axis flexural rigidity (EI_y) so that in-plane deflection will be negligible compared to out-of-plane deflection. If both rigidities are of the same order of magnitude, the effect of in-plane bending should be considered (Chen & Lui, 1987). An approximate solution that includes the effect of major axis curvature is

given by Kirby and Nethercot (1979) as

$$M_E = \frac{M_{cr}}{\sqrt{(1 - EI_y/EI_x)}} = p_E Z_{ex} \quad (4.2)$$

where M_E = elastic buckling moment capacity considering major axis curvature;

I_x = major axis moment of inertia;

p_E = elastic buckling stress considering major axis curvature;

Z_{ex} = effective section modulus about major axis.

The effective section modulus Z_{ex} is taken as the *plastic* section modulus about the major axis S_x for class 1 and 2 sections, and *elastic* section modulus Z_x for class 3 sections.

In reality, beams are seldom straight due to the presence of initial curvature and twist, which will cause them to bend and rotate about its own axis at the beginning of loading. The following presents the derivation of maximum allowable moment of crooked beams, whereas the more detailed derivations are presented in Appendix A1. Consider a beam that has an initial curvature and twist such that

$$\frac{u_0}{\delta_0} = \frac{\phi_0}{\theta_0} = \sin \frac{\pi z}{L} \quad (4.3)$$

where u_0 = initial lack of straightness;

δ_0 = central initial lack of straightness;

ϕ_0 = initial twist;

θ_0 = central initial twist.

in which the central initial lack of straightness and twist δ_0 and θ_0 are related by

$$\frac{\delta_0}{\theta_0} = \frac{M_{cr}}{\pi^2 EI_y / L^2} \quad (4.4)$$

The solution of out-of-plane bending and torsional equations (eq. (A.3) & (A.4)), which satisfies the boundary conditions (eq. (A.5)) is given by

$$\frac{u}{\delta} = \frac{\phi}{\theta} = \sin \frac{\pi z}{L} \quad (4.5)$$

in which
$$\frac{\delta}{\delta_0} = \frac{\theta}{\theta_0} = \frac{M/M_{cr}}{1 - M/M_{cr}} \quad (4.6)$$

where u = lateral deflection;
 δ = central lateral deflection;
 ϕ = twist;
 θ = central twist.

The maximum longitudinal stress $f_{\max}$ in the beam (eq. (A.8)) can be determined from

$$f_{\max} = \frac{M}{Z_x} - \frac{EI_y}{Z_y} \left(\frac{d^2(u + d_f \phi / 2)}{dz^2} \right)_{L/2} \quad (4.7)$$

where $f_{\max}$ = maximum longitudinal stress;
 M = applied moment;
 Z_x = elastic section modulus about the major axis;
 Z_y = elastic section modulus about the minor axis.

The value of $f_{\max}$ includes the summation of stresses due to major axis bending, minor axis bending and warping. Subsequently, by taking the maximum stress as yield strength, the limiting moment M_L at which a beam without residual stresses attains first yield is

$$\frac{M_L}{M_y} = \left(\frac{1.25 + M_{cr}/M_y}{2} \right) - \sqrt{\left(\frac{1.25 + M_{cr}/M_y}{2} \right)^2 - \frac{M_{cr}}{M_y}} \quad (4.8)$$

in which M_y = yield moment.

The variation of the dimensionless limiting moment M_L/M_y is shown in Figure 4.2, in which the ratio $\sqrt{(M_y/M_{cr})}$ is plotted along the horizontal axis. It can be seen that the

limiting moments of short beams approach the yield moment M_y , while the limiting moments of long beams approach the elastic buckling moment M_{cr} .

Figure 4.2 Buckling and Yielding of I-Beams (Trahair et al., 2001)

Thus far, only simply supported beams with equal and opposite end moments have been considered. The value of lateral torsional buckling moment is also influenced by a number of factors. Among the important ones are the cross-sectional shape, the unbraced length and the support conditions of the beam, the type of the applied load, and the location of the applied loads with respect to the centroidal axis of the cross-section. For the type of loading other than uniform moment along the beam length, closed-form solutions are not available, and only numerical & approximate procedures can be obtained. However, the procedures are quite cumbersome. Salvadori (1955) suggested the use of *equivalent moment factor* C_b to account for the effect of moment gradient. Thus, the elastic buckling moment capacity for non-uniform moment M_{cr}^L can be obtained from

$$M_{cr}^L = C_b M_{cr} \quad (4.9)$$

This equivalent moment factor is subsequently used to take into account the effect of the location of the applied loads with respect to the centroidal axis of the cross-section as well. Many researchers have studied this factor for various loading conditions and load

positions such as Salvadori (1955), Nethercot and Rockey (1972), Kirby & Nethercot (1979) and Nethercot (1983).

Support conditions other than simply supported are widely treated using effective length concept. Two effective length factors are usually used as the restraint conditions may be different with respect to lateral bending and twisting. Vlasov (1961) studied the effective length factor with respect to lateral bending K_b and effective length factor with respect to twisting K_t for many boundary conditions. Incorporating both effective length factors, the critical moment M_{cr}^S is expressed as

$$M_{cr}^S = \sqrt{\left(\frac{\pi^2 EI_y}{K_b L^2}\right)} \sqrt{GJ + \frac{\pi^2 EI_w}{K_t L^2}} \quad (4.10)$$

in which M_{cr}^S = elastic buckling moment capacity for non-simply supported beam.

Besides elastic lateral torsional buckling, beams can also fail in inelastic lateral torsional buckling. In a short span beam, yielding occurs before the ultimate moment is reached, and significant portions of the beams are inelastic when buckling commences. Hence, the effective rigidities of these inelastic portions are reduced by yielding. Consequently, the buckling moment is also reduced (Trahair et al., 2001). For beams with equal and opposite end moments, the distribution of yield across the section does not vary along the beam, and when there is no residual stress, the inelastic buckling moment can be calculated from a modified form of eq. (4.1) as

$$M_{cr}^{IE} = \sqrt{\left(\frac{\pi^2 (EI_y)_e}{L^2}\right)} \sqrt{(GJ)_e + \frac{\pi^2 (EI_w)_e}{L^2}} \quad (4.11)$$

in which M_{cr}^{IE} = inelastic buckling moment capacity.

The subscript ‘e’ denotes the reduced inelastic rigidities which are effective at buckling. Estimates of these rigidities can be obtained using tangent modulus of elasticity.

As reviewed before, many research works have been conducted on lateral torsional buckling at ambient temperature over the past decades (Trahair et al., 2001) and a few of these have been incorporated into design codes. There are some essential differences of the national design guides as there is considerable scatter in the test results of more than 450 test beams that have been reported in six studies reviewed by Nethercot and Trahair (1983). Considerable scatter on the test results, as shown in Figure 4.3, shows that not only the effects of initial crookedness, twist and load eccentricity are important on LTB, but also other factors such as residual stress and lateral continuity between adjacent segments. Due to uncertainty of imperfection, design codes often approximate various imperfections in the form of sinusoidal curve. Despite some simplifications, the LTB design procedure is still overly complicated and lengthy because there is usually an attempt to provide continuity between old code formulae and new requirements.

Figure 4.3 Test Results for Hot-Rolled Beams at Ambient (Trahair et al., 2001)

4.2.2 BS5950-Part 1:2000 Approach for LTB

Having presented the buckling moment for a straight beam as well as a crooked beam, this section outlines their relevance to LTB design approach in BS5950-1:2000 (BSI, 2001). It should be noted that the LTB moment of a hot-rolled equal flanged beam

subjected to uniform bending can be calculated from elastic buckling moment M_E given in eq. (4.2), which considers major axis curvature and in-plane moment capacity M_{cx} given by eq. (4.12).

$$M_{cx} = f_y Z_{ex} \quad (4.12)$$

The following provides only the main steps of the background to LTB in the BS5950 approach. Rearranging eq. (4.2), one obtains

$$p_E = \frac{\pi^2 E}{\frac{\pi^2 E M_{cx}}{f_y M_E}} \quad (4.13)$$

The above expression for elastic buckling stress has the same form as the column buckling stress ($p_{E,column} = \pi^2 E / \lambda^2$), with LTB slenderness ratio defined as

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{f_y}} \sqrt{\frac{M_{cx}}{M_E}} \quad (4.14)$$

where $p_{E,column}$ = elastic buckling stress of column;

λ = slenderness ratio;

λ_{LT} = lateral torsional buckling slenderness ratio.

The author is not aware of any published works that present the detailed derivations of λ_{LT} to the form shown in the code. As such, the author provides the derivations in Appendix A2. The slenderness ratio can be rearranged to a familiar form used in BS5950:

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w} \quad (4.15)$$

in which $u = \left\{ \frac{4S_x^2 (1 - EI_y/EI_x)}{(A^2 h_s^2)} \right\}^{1/4}$ (4.16)

$$v = \left\{ 1 + 0.05(\lambda/x)^2 \right\}^{-1/4} \quad (4.17)$$

$$\lambda = L_E / r_y \quad (4.18)$$

$$\beta_w = Z_{ex} / S_x \quad (4.19)$$

$$x = 0.566 h_s (A/J)^{1/2} \quad (4.20)$$

where S_x = plastic section modulus about the major axis;

A = cross sectional area;

h_s = distance between the shear centre of the flanges;

L_E = effective length;

r_y = radius of gyration about the minor axis.

The buckling resistance moment is calculated from $M_b = p_b Z_{ex}$, and p_b is obtained as follows:

$$p_b = \frac{M_b}{Z_{ex}} = \frac{M_b}{M_{cx}} f_y \quad (4.21)$$

where p_b = buckling stress;

M_b = ultimate buckling moment capacity.

BS 5950 uses eq. (4.8) to calculate M_L of a beam with initial curvature and twist, except that the section capacity M_{cx} is introduced in place of the yield moment M_y , and M_E is used to calculate the elastic buckling moment instead of M_{cr} . The factor 1.25 from eq. (4.8) is replaced by $(1 + \eta_{LT} M_E / M_{cx})$, whereby the initial curvature and twist is defined by

$$\eta_{LT} = 0.007 \sqrt{\frac{\pi^2 E}{f_y}} \left(\sqrt{\frac{M_{cx}}{M_E}} - 0.4 \right) \geq 0 \quad (4.22)$$

in which η_{LT} = Perry imperfection factor for LTB.

Eq. (4.22) can be expressed in the form of Perry imperfection equation

$\eta = 0.001a(\lambda_{LT} - \lambda_o)$, where for LTB, $a = 7$ and $\lambda_o = 0.4\sqrt{\pi^2 E / f_y}$. The factor “ a ”

represents the initial crookedness; a higher value is associated with a greater initial crookedness and lower buckling strength. The other factor λ_o represents the limiting slenderness in which a beam can attain its full in-plane moment capacity.

Rewriting eq. (4.8) incorporating all the modifications, one obtains

$$p_b = \left(\frac{f_y + (1 + \eta_{LT})f_y M_E / M_{cx}}{2} \right) - \sqrt{\left(\frac{f_y + (1 + \eta_{LT})f_y M_E / M_{cx}}{2} \right)^2 - \frac{f_y^2 M_E}{M_{cx}}} \quad (4.23)$$

$$p_b = \left(\frac{f_y + (1 + \eta_{LT})p_E}{2} \right) - \sqrt{\left(\frac{f_y + (1 + \eta_{LT})p_E}{2} \right)^2 - f_y p_E} \quad (4.24)$$

$$\text{Defining } \phi_{LT} = \frac{f_y + (\eta_{LT} + 1)p_E}{2}; \text{ one will obtain a simplified equation:} \quad (4.25)$$

$$p_b = \phi_{LT} - \sqrt{\phi_{LT}^2 - f_y p_E} \times \left(\frac{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_y p_E}}{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_y p_E}} \right) \quad (4.26)$$

$$p_b = \frac{f_y p_E}{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_y p_E}} \quad (4.27)$$

It is noteworthy that BS5950 presents the values of bending strength p_b as a function of design yield strength f_y and slenderness ratio λ_{LT} in tables (BSI, 2001). The effect of load types, load positions, and type of supports are also considered using various factors presented in tables (similar to the approach explained in Section 4.2.1).

4.2.3 Eurocode3-Part 1.1:2005 Approach for LTB

The LTB equations in EC3 can be obtained through eq. (4.27) from BS5950 for buckling stress p_b :

$$p_b = \frac{f_y p_E}{\phi_{LT}^{BS} + \sqrt{\phi_{LT}^{BS^2} - f_y p_E}} \quad (4.27)$$

Substituting $p_E = \frac{\pi^2 E}{\lambda_{LT}^2} = \frac{f_y}{\lambda_{LT}^2}$ from eq. (4.14) to eq. (4.27), one obtains

$$\frac{p_b}{f_y} = \frac{1}{\left(\frac{\phi_{LT}^{BS} \overline{\lambda_{LT}}^2}{f_y} \right) + \sqrt{\left(\frac{\phi_{LT}^{BS} \overline{\lambda_{LT}}^2}{f_y} \right)^2 - \overline{\lambda_{LT}}^2}} \quad (4.28a)$$

$$\text{in which LTB modified slenderness ratio } \overline{\lambda_{LT}} = \sqrt{M_{cx}/M_E}. \quad (4.28b)$$

If we define a term ϕ_{LT}^{EC} to relate $\overline{\lambda_{LT}}^2$ to yield strength,

$$\phi_{LT}^{EC} = \frac{\phi_{LT}^{BS} \overline{\lambda_{LT}}^2}{f_y} = \left(\frac{f_y + (\eta_{LT} + 1)p_{cr}}{2} \right) \times \frac{\overline{\lambda_{LT}}^2}{f_y} \quad (4.29a)$$

Rearranging eq. (4.29a), one obtains

$$\phi_{LT}^{EC} = 0.5 \left(1 + \eta_{LT} + \overline{\lambda_{LT}}^2 \right) \quad (4.29b)$$

Thus, the buckling strength p_b can be expressed in terms of a strength reduction factor χ_{LT} , as follows

$$\frac{p_b}{f_y} = \chi_{LT} = \frac{1}{\phi_{LT}^{EC} + \sqrt{\phi_{LT}^{EC2} - \overline{\lambda_{LT}}^2}} \quad (4.30)$$

in which p_{cr} = elastic buckling stress.

It should be noted that EC3 uses p_{cr} instead of p_E since EC3 does not take into account the effect of in plane bending on LTB. Thus, there is a slight difference in the definition of slenderness ratio between the two codes.

The Perry imperfection factor that is used in EC3 is different from BS5950 ($\lambda_o = 0.4\sqrt{\pi^2 E / f_y}$), in which EC3 uses $\lambda_o = 0.2\sqrt{\pi^2 E / f_y}$. Thus,

$$\eta_{LT}^{EC} = 0.001a \sqrt{\frac{\pi^2 E}{f_y}} \left(\sqrt{\frac{M_{cx}}{M_{cr}}} - 0.2 \right) \geq 0 \quad (4.31)$$

$$\eta_{LT}^{EC} = \alpha_{LT} (\overline{\lambda_{LT}} - 0.2) \geq 0 \quad (4.32)$$

in which $\alpha_{LT} = 0.001a\sqrt{\pi^2 E / f_y}$.

EC3 recommended four different values of α_{LT} corresponding to the type of cross section and D/b (overall depth over width) ratio of the beam. The above approach is addressed as ‘general case’ in the EC3:1.1 (CEN, 2005). An alternative method is also provided with some factors applied to yield greater LTB design moment. As shown in eq. (4.32), EC3 assigns an initial crookedness that is independent of the steel grade unlike BS5950. Finally, the design buckling resistance moment M_b of laterally unrestrained beam is taken as

$$M_b = \chi_{LT} \beta_w S_x f_y / \gamma_{M1} \quad (4.33)$$

in which γ_{M1} = material partial safety factor.

In summary, there are five differences between BS5950 and EC3 approach for lateral torsional buckling as listed down in Table 4.1. The two approaches along with test results are plotted in the same graph (Figure 4.4) for S275 steel with $E = 205$ GPa. As can be seen, BS5950 approach gives more conservative results for most slenderness range compared to EC3 approach. It is noteworthy that BS5950 provides a lower bound to scattered test results while a few of test results within medium slenderness range fall below EC3 curve. It should also be noted that BS5950 has a longer plateau of $\overline{\lambda_{LT}}$ (up to $\overline{\lambda_{LT}} = 0.4$) compared to EC3 (up to $\overline{\lambda_{LT}} = 0.2$). Within this range, a beam will achieve its full moment capacity.

Table 4.1 Comparisons of BS5950 and EC3 Approach for LTB

Comparison	BS 5950:2000	EC3 Part1.1 (general case)
The effect of in plane bending	Taken into account (using p_E)	Ignored (using p_{cr})
Definition of $\overline{\lambda}_{LT}$	$\overline{\lambda}_{LT} = \sqrt{M_{cx} / M_E}$	$\overline{\lambda}_{LT} = \sqrt{M_{cx} / M_{cr}}$
Slenderness limit for full plastic bending strength	$\overline{\lambda}_{LT} = 0.4$	$\overline{\lambda}_{LT} = 0.2$
Value of η_{LT}	$\eta_{LT} = 0.007 \sqrt{\frac{\pi^2 E}{f_y}} \left(\sqrt{\frac{M_{cx}}{M_E}} - 0.4 \right) \geq 0$	$\eta_{LT} = \alpha_{LT} \left(\sqrt{\frac{M_{cx}}{M_E}} - 0.2 \right) \geq 0$
Value of η_{LT} for welded section	Three different expression for three slenderness regions	The same expression as rolled section, except that $\alpha_{LT} = 0.49$ or 0.76

Figure 4.4 Comparisons of BS5950 and EC3 Approach at Ambient Temperature

4.3 Lateral Torsional Buckling at Elevated Temperature

The LTB design equations in BS5950:1 and EC3:1.1 for ambient temperature case has been presented in Section 4.2. This section reviews BS5950:8 (BSI, 2003) and EC3:1.2 (CEN, 2005) approach for LTB at elevated temperature. Through the ensuing discussions, the weaknesses of EC3:1.2 will become apparent. Thus, the author proposes an alternative approach that is safe and more accurate. Subsequently, Rankine approach is introduced to provide a quick but more conservative method for calculating buckling moment under fire conditions.

4.3.1 Review of Various Approaches

BS5950:8 (BSI, 2003) deals with unrestrained beams using limiting temperature method. Firstly, the load ratio is calculated based on the buckling failure at ambient temperature. Subsequently, the limiting temperature, which is a function of load ratio, can be found from corresponding table (Wang, 2002). A similar method is also used by Eurocode3:1.2 based on the degree of utilization. There is a slight difference in the definition of load ratio and degree of utilization as pointed out by Bailey et al. (1996). The load ratio is defined as the ratio of applied moment at the fire limit state to cold design moment of resistance, whereas the degree of utilization refers to the ratio of applied moment at the fire limit state to fire design moment of resistance at time zero. A more accurate method is given in ENV 1993 EC3:1.2 (CEN, 1995) based on bending moment capacity method. This method uses the same calculation as that at ambient temperature but incorporating the strength and stiffness reduction factors at elevated temperature and a factor of 1.2 to consider the non-linear stress-strain relationship for steel. This method has since then been modified based on Vila Real et al. (2003) proposal when Eurocode 3 status was converted from ENV to EN. Using the modified slenderness for fire, the strength reduction factor for LTB is calculated as

$$\chi_{LT,fi} = \frac{1}{\phi_{LT,fi}^{EC} + \sqrt{\phi_{LT,fi}^{EC^2} - \lambda_{LT,fi}^2}} \quad (4.34)$$

in which $\phi_{LT,fi}^{EC} = 0.5 \left(1 + \alpha_i \overline{\lambda_{LT,fi}} + \overline{\lambda_{LT,fi}}^2 \right)$ and $\alpha_i = \beta \varepsilon$ is an imperfection factor with severity factor $\beta = 0.65$ obtained from curve-fitting ($\varepsilon = \sqrt{235 / f_y}$).

The beam slenderness at elevated temperature is given by

$$\overline{\lambda_{LT,fi}} = \overline{\lambda_{LT}} \sqrt{k_y / k_E} \quad (4.35)$$

where $\overline{\lambda_{LT,fi}}$ = modified slenderness ratio for LTB in fire;

k_y = yield strength reduction factor at elevated temperature;

k_E = elastic modulus reduction factor at elevated temperature.

Finally, the beam LTB in fire based on the new EC3:1.2 (CEN, 2005) is calculated using

$$M_{b,fi} = \chi_{LT,fi} \beta_w S_x k_y f_y / \gamma_{M,fi} \quad (4.36)$$

where $M_{b,fi}$ = ultimate buckling moment capacity in fire;

$\chi_{LT,fi}$ = strength reduction factor in fire;

$\gamma_{M,fi}$ = material partial safety factor in fire.

Clearly, in the EC3 approach to LTB design, there is a discontinuity from ambient temperature (EC3:1.1) to elevated temperatures (EC3:1.2). The details are discussed in the following section.

4.3.2 Alternative Approach at Elevated Temperatures

Vila Real et al. (2003) made a proposal to modify ENV 1993 EC3:1.2 (CEN, 1995) approach to eliminate the overestimation and the step change at $\overline{\lambda_{LT,fi}} = 0.4$. This has subsequently been accepted and incorporated in the latest version of EC3 (EN 1993 EC3:1.2). However, this latest EC3 approach is too conservative, especially for temperature less than 500°C as can be seen from Figure 4.5(a) to 4.5(d). The horizontal axis is the modified slenderness ratio for LTB in fire, whereas the vertical axis is the non-dimensional ratio of buckling moment to full plastic moment capacity in fire. The reason for this conservatism may be due to some very low test results obtained by Vila

Real et al. that occurred at 500°C (Figure 4.5(d)) and 600°C (Figure 4.5(e)). There were large differences in the failure loads even for the same beam with the same slenderness. This discrepancy may be due to a large degradation of material properties at that temperature range. A slight error in the measurement of temperature can cause a large difference in the test results. Besides being conservative for medium slenderness range, there are two other weaknesses of EC3 approach. First, the slope of the buckling strength curve for very low slenderness is too steep (Figure 4.5), implying that even a very short beam will not be able to achieve its plastic moment capacity unless the compression flange is fully restrained. That is to say, a slight increase in the unrestrained portion will substantially reduce its load-bearing capacity below its plastic moment capacity. This seems unreasonable and FEA of very short beams which are presented later in this chapter shows that they can achieve buckling moments which are very close to their plastic moment capacities, as can be seen in Figure 4.5(c) to 4.5(e). Second, EC3:1.2 approach is only for elevated temperatures and a slightly different form of equation is used at ambient temperature (EC3:1.1). Thus, there exists a discontinuity between the approach at ambient and at elevated temperature. Ideally, EC3:1.2 should give the same form as EC3:1.1 at 20°C.

Figure 4.5(a) Comparison of Test Results and EC3:1.2 Approach at 200°C

Figure 4.5(b) Comparison of Test Results and EC3:1.2 Approach at 300°C

Figure 4.5(c) Comparison of Test Results, FEA and EC3:1.2 Approach at 400°C

Figure 4.5(d) Comparison of Test Results, FEA and EC3:1.2 Approach at 500°C

Figure 4.5(e) Comparison of Test Results, FEA and EC3:1.2 Approach at 600°C

Based on the above arguments, the author proposes an alternative approach that gives more accurate results and is able to overcome the weaknesses of EC3:1.2. This alternative approach will provide a logical link with the ambient temperature approach because it can also be used at ambient temperature and will give the same result as in BS5950 Part 1. It is noteworthy that the limit slenderness λ_o below which a beam can attain its full plastic moment capacity and the slope of the curve above this limit can vary with temperature.

The alternative approach is based on the BS5950 proposal at ambient temperature. Incorporating the stiffness and strength reduction factors at elevated temperature in eq. (4.22), one obtains Perry imperfection factor for LTB at elevated temperature η_{LT}^T , as follows:

$$\eta_{LT}^T = 0.007 \sqrt{\frac{\pi^2 E}{f_y}} \left(\sqrt{\frac{M_{cx}}{M_E}} - 0.4 \sqrt{\frac{k_E}{k_y}} \right) \geq 0 \quad (4.37)$$

Subsequently, the elastic critical buckling stress considering major axis curvature at elevated temperature p_E^T can be obtained as

$$p_E^T = \frac{\pi^2 k_E E}{\lambda_{LT}^2} = k_E p_E \quad (4.38)$$

It should be noted that eq. (4.38) is related to eq. (4.13) by elastic modulus reduction factor at elevated temperature k_E . The values obtained from eq. (4.37) and (4.38) are then substituted into eq. (4.39) to obtain ϕ_{LT}^T :

$$\phi_{LT}^T = \frac{k_y f_y + (\eta_{LT}^T + 1) p_E^T}{2} \quad (4.39)$$

Finally, the buckling stress at elevated temperature $p_{b,fi}$ can be obtained from eq. (4.40), which is a modification of eq. (4.27) considering the effect of fire conditions.

$$p_{b,fi} = \frac{k_y f_y p_E^T}{\phi_{LT}^T + \sqrt{\phi_{LT}^T{}^2 - k_y f_y p_E^T}} \quad (4.40)$$

It should be noted that slenderness ratio λ_{LT} in eq. (4.38) is a function of the geometry of cross section and beam length only. Therefore λ_{LT} is the same at both ambient and elevated temperature, unlike $\overline{\lambda_{LT}}$ which is a function of both elastic modulus and yield strength for a given temperature (eq. (4.28b)).

Simply applying the material reduction factors at elevated temperature will result in unsafe predictions of the buckling load in the inelastic range of slenderness ratio because the stress-strain relationships at elevated temperature are no longer bilinear elastic-plastic, but highly non-linear resulting in tangent stiffness at a particular temperature much lower than the initial tangent, as shown in Figure 4.6. This affects the behaviour of an unrestrained beam significantly because the buckling load is highly dependent on tangent stiffness (eq. (4.38)). With a greater slenderness, the buckling stress is generally lower and the tangent stiffness is closer to the initial value. For a beam of low slenderness, where material yielding governs instead of buckling, the beam bending resistance is not so sensitive to its tangent stiffness. However, for a beam of

medium slenderness, where the applied stress is substantial and above the limit of proportionality, simply applying the material reduction factors can lead to gross overestimation of LTB resistance in fire. Thus, the elastic modulus should be based on tangent stiffness instead of initial tangent. However, applying the concept of tangent modulus in the calculation of buckling capacity requires the knowledge of stress level; therefore, an iterative process is needed.

Figure 4.6 Different Rates of Degradation of Tangent Modulus

A practical and viable option is to consider material non-linearity as a form of imperfection and incorporating it through Perry imperfection factor. This is possible because the effect of inelastic buckling and yielding is similar to the effect of initial imperfection that makes the curve deviate from the ideal elastic buckling curve (Dharma & Tan, 2005a). It should be noted that although the phenomena are different, the result of a reduction in capacity is similar. Therefore, the author uses the term f_{pT} / f_{yT} to consider the highly nonlinear stress-strain relationship at elevated temperatures. This ratio is in terms of material reduction factor as in eq. (4.41) and is termed as non-linearity factor ζ .

$$\zeta = \frac{f_{pT}}{f_{yT}} = \frac{f_{pT}/f_y}{f_{yT}/f_y} = \frac{k_p}{k_y} \quad (4.41)$$

where f_{pT} = proportionality limit at elevated temperature;

f_{yT} = yield strength at elevated temperature;

k_p = proportionality limit reduction factor at elevated temperature;

k_y = yield strength reduction factor at elevated temperature.

It should be noted that ζ varies with temperature as shown in Figure 4.7.

Figure 4.7 Variation of Non-Linearity Factor with Temperature

The shape of the curve that incorporates the initial crookedness and the limiting slenderness λ_o are adjusted based on the experimental results (Vila Real et al., 2003) and current FE predictions at elevated temperature. Clearly, both the shape of the curve and the limiting slenderness are temperature dependent. As a result, the new form of Perry imperfection factor for this alternative approach is

$$\eta_{LT}^T = 0.007\sqrt{\zeta} \sqrt{\frac{\pi^2 E}{f_y}} \left(\sqrt{\frac{M_{cx}}{M_E}} - 0.4\zeta^{1.5} \sqrt{\frac{k_E}{k_y}} \right) \geq 0 \quad (4.42)$$

The above imperfection factor at ambient temperature is the same as in BS5950 because $\zeta = 1$ at ambient temperature (Fig. 4.7). Figure 4.8 shows the comparisons of the alternative approach, EC3:1.2 (CEN, 2005) approach and FE results for S275 steel at

400°C. It can be observed that the alternative approach agrees better with FE results. Note that the validation of alternative approach is presented later in Section 4.4.3.

Figure 4.8 Comparisons of Various Approaches for S275 at 400°C

4.3.3 Rankine Approach at Elevated Temperatures

The complexity of LTB of beams at ambient temperature has increased significantly at elevated temperature. Hence, the design approach tends to be cumbersome for hand calculations. To provide an easy and quick design approach, the author also introduces a simple interaction formula known as Rankine approach to solve LTB of beams at elevated temperature. The Rankine formula is an empirical formula initially proposed in the nineteenth century (Rankine, 1886). It was later modified in the mid-twentieth century (Merchant, 1954) and has since then been adopted into various design codes (BS5950:1, 2001; EC3:1.1, 2005) for steel frames subjected to increasing load. The same formula is also applicable to steel columns (Horne & Merchant, 1965). Apparently, there has been no attempt to apply the Rankine formula to lateral torsional buckling of steel beams at elevated temperatures. The formula has the following form:

$$\frac{1}{\lambda_R} = \frac{1}{\lambda_p} + \frac{1}{\lambda_e} \quad (4.43)$$

in which λ_R , λ_p and λ_e are the Rankine, plastic collapse and elastic critical load factor, respectively.

Applying the formula to LTB problems, the three load factors can be related to Rankine ultimate moment M_R , plastic moment capacity M_{cx} and elastic buckling moment M_E . As a result, the Rankine formula is expressed as

$$\frac{1}{M_{cx}} + \frac{1}{M_E} = \frac{1}{M_R} \quad (4.44)$$

The above equation can be rearranged into

$$M_R = \frac{M_{cx} M_E}{M_E + M_{cx}} \quad (4.45)$$

$$p_b Z_{ex} = \frac{f_y Z_{ex}}{1 + \frac{M_{cx}}{M_E}} \quad (4.46)$$

Substituting $\lambda_{LT} = \sqrt{\pi^2 E / f_y} \sqrt{M_{cx} / M_E}$ to eq. (4.46), one obtains

$$p_b = \frac{f_y}{1 + \frac{\lambda_{LT}^2}{\pi^2 E / f_y}} \quad (4.47)$$

Subsequently, incorporating the respective strength and stiffness reduction factors at elevated temperature into eq. (4.47), one obtains

$$p_{b,fi} = \frac{k_y f_y}{1 + \frac{\lambda_{LT}^2}{\pi^2 k_E E / (k_y f_y)}} \quad (4.48)$$

in which $p_{b,fi}$ = buckling stress at elevated temperature.

Defining $\frac{\lambda_{LT}^2}{\pi^2 E / f_y} = \psi_{LT}$ (normalized slenderness ratio), eq. (4.48) can be written as

$$\frac{p_{b,fi}}{f_y} = \frac{k_y}{1 + \frac{k_y}{k_E} \psi_{LT}} \quad (4.49)$$

It is noteworthy that the normalized slenderness ratio (ψ_{LT}) is actually a ratio of the plastic moment capacity to elastic buckling moment (M_{cx}/M_E). Eq. (4.49) can be plotted as $p_{b,fi}/f_y$ against ψ_{LT} , as shown in Figure 4.9, since both are non-dimensional parameters. By plotting the graph in this manner, an iterative procedure by guessing the failure temperature as required in EC3:1.2 can be avoided because two parameters that are needed to obtain the failure temperature ($p_{b,fi}/f_y$ and ψ_{LT}) are independent of temperature (Dharma & Tan, 2006a). Interpolation can be used for a point that lies between two available curves.

Figure 4.9 LTB Design Curves at Various Temperatures using Rankine Approach

4.4 Numerical Analysis and Comparisons of Different Approaches at Elevated Temperature

Numerical analyses using commercial finite element program MSC.MARC Mentat have been completed to study the LTB of steel beams at elevated temperatures. The

ability of the numerical model is firstly validated against theoretical bifurcation elastic solution and published experimental results at ambient temperature. The validated numerical models are then used to simulate various section sizes and beam length with mid-span point loading or uniform bending at elevated temperatures. Finally, the proposed alternative and Rankine approach at elevated temperature are compared with the results of finite element analysis (FEA) and published test results (Vila Real et al., 2003). In addition, a worked example is also provided.

4.4.1 Validation of Numerical Model

Four node rectangular thick shell elements from MSC.MARC Mentat are used to model the steel beam. This element has three global displacements and three global rotations as degrees of freedom. The membrane strains and curvatures are obtained from the displacement and rotation field, respectively. The beam is simply supported at both ends so that lateral deflection and twist rotation at the supports are prevented, while the flange ends are free to rotate in horizontal planes so that the beam ends are free to warp. This support system can be achieved using three types of end nodes restraint as shown in Table 4.2. The x -axis is perpendicular to the web, the y -axis is parallel to the web, whereas the z -axis is along the beam, as shown in Figure 4.10, with u and θ are displacement and rotation respectively. Firstly, finite element (FE) results are compared with theoretical bifurcation solutions for elastic beams (Timoshenko & Gere, 1961).

Two loading conditions are considered, namely, uniform moment and mid-span point load acting at the shear centre. The beams are assumed to be perfectly straight and remain elastic. Five different UB sections are analyzed with different slenderness ratio ranging from 100 to 150. The elastic modulus is taken as 205 GPa. The theoretical and FE predictions are shown in Table 4.3 and Table 4.4 for mid-span point loading and uniform moment cases, respectively. It can be seen that the error is almost negligible (less than 1%).

Table 4.2 Boundary Conditions at End Nodes

Type of Nodes Restraint	u_x	u_y	u_z	θ_x	θ_y	θ_z
Left Mid-Web Node	0	0	0	1	1	0
Right Mid-Web Node	0	0	1	1	1	0
All Other End Nodes	0	1	1	1	1	0

Note: 0 = fixed; 1 = free

Figure 4.10 Different Types of Nodes at End Cross-Section

Table 4.3 Comparison between FE Predictions and Elastic Theoretical

Bifurcation Solution for Mid-span Point Load Case

Section size	L (m)	λ_{LT}	Theoretical (kN)	Predicted (kN)	Error (%)
838x292x176UB	12	126.6	280.90	282	0.39
533x210x82UB	8	114.8	154.20	154.7	0.32
406x178x54UB	6	100.6	136.77	136	0.56
305x102x28UB	6	150.1	23.53	23.66	0.55
254x102x28UB	6	130.6	26.78	26.7	0.30

Table 4.4 Comparisons between FE Predictions and Elastic Theoretical

Bifurcation Solution for Uniform Moment Case

Section size	L (m)	λ_{LT}	Theoretical (kNm)	Predicted (kNm)	Error (%)
838x292x176UB	12	147.2	624.22	623	0.20
533x210x82UB	8	133.5	228.44	227.5	0.41
406x178x54UB	6	117.0	151.97	151.1	0.57
305x102x28UB	6	174.6	26.14	26.04	0.38
254x102x28UB	6	151.8	29.75	29.52	0.77

After comparing the elastic solutions with theory, the finite element model is then validated with steel beams undergoing inelastic LTB based on experimental results from Kitipornchai and Trahair (1975). A uniaxial representation of the constitutive law is entered based on the reported tensile coupon test results and Von Mises yield criterion is then used in the analysis. It can be seen in Table 4.5 that the buckling load is affected by the assumed initial imperfection. Four different assumed maximum initial imperfections in a form of half sine curve ranging from perfectly straight beam to the maximum initial imperfection of $L/1000$ are compared with the test results. As can be seen, the assumed maximum initial imperfection of $L/1000$ provides a good lower bound to the test results. Thus, this form of imperfection is subsequently used in the analyses.

Table 4.5 Comparisons between FE Predictions and Experimental Results
by Kitipornchai and Trahair (1975)

Section Size	L (m)	Experimental (kN)	Current FEA (kN)			
			$L/1000$	$L/2000$	$L/3000$	Straight
10UB29	2.44	234.96	235.2	249.2	254.8	268.8
	3.05	185.12	166	176	184	198
	3.66	145.07	120.7	127.5	130.9	141.1
	6.1	56.96	49	50.4	51.8	52.5

4.4.2 Numerical Simulations at Elevated Temperature

The validated model described in the previous section has been extended to elevated temperature conditions. The material stress-strain relationships follow EC3:1.2 formulations for steel at elevated temperatures. In addition, the load is applied at two stages; firstly, the temperature is increased to the desired failure temperature, and the load is applied subsequently until the member fails. Seven different section sizes of both UB and UC section of S275 and S355 steel with various slenderness ratio ranging from 21 to 205 producing a set of results representative of almost the whole range of practical slenderness ratio are considered in the numerical simulations as shown in Table 4.6. Two loading conditions, namely, equal & opposite moment and mid-span point load are applied with five different temperatures: 400, 500, 600, 700 and 800°C. As most

unrestrained beams do not support a slab on top, the temperature distribution across the section is fairly uniform. Thus, only uniform temperature distribution is considered.

In the model, a maximum initial out-of-plane bow of $L/1000$ in the form of half sine curve is assumed. This equivalent initial imperfection sums up all the possible imperfections that may exist in real beams and is representative of the test results. Besides, it has commonly been used by other researchers for code calibration (Yin & Wang, 2003). In accordance with BS476-20 (BSI, 1987), failure is assumed to have occurred when the maximum in-plane deflection exceeds $L/20$, or when the deflection exceeds $L/30$, the beam reaches a maximum rate of deflection $L^2/(9000\text{depth})$. A typical LTB failure observed at the end of the simulation is shown in Figure 4.11. The detail results of the numerical analyses are presented in Appendix A3.

Table 4.6 Various Section Sizes, Lengths and Loadings for Numerical Analyses

Section Size	L (m)	Loading	λ_{LT}	Section Size	L (m)	Loading	λ_{LT}
914x305x201UB	12	UM	145.5	203x102x23UB	9	UM	176.6
457x191x98UB	9	PL	110.7	356x406x287UC	3	PL	21.6
457x191x98UB	12	UM	153.6	356x406x287UC	6	PL	37.0
457x191x98UB	9	UM	129.4	305x305x158UC	9	PL	61.1
305x165x54UB	12	UM	159.9	305x305x158UC	6	PL	47.2
305x165x54UB	9	UM	135.6	254x254x89UC	9	PL	72.9
305x165x54UB	9	PL	116.0	254x254x89UC	12	UM	100.5
203x102x23UB	12	UM	205.5	254x254x89UC	12	PL	86.0

Note: PL = mid-span point load; UM = uniform moment

Figure 4.11 Lateral Torsional Buckling Failure of Numerical Model

4.4.3 Comparisons and Discussions of Various Approaches

Based on test results (Vila Real et al., 2003), statistical comparison has been made for the alternative approach, the Rankine approach and the EC3 method. As can be seen from Table 4.7, the alternative approach gives the best mean value up to 400°C. For 500 and 600°C, the alternative approach gives slightly higher predictions because of slight discrepancies of the test results as discussed in Section 4.3.2. EC3 approach gives very conservative predictions for temperature up to 400°C, with as high as 70% underestimation of strength. Compared to all approaches, the alternative approach gives the lowest standard deviation. Rankine method produces similar results compared to EC3 approach, but gives improved accuracy for temperature less than 400°C. Considering the simplest method among all, the Rankine approach is a useful approach for quick estimation of failure load. More experimental results are needed for temperature above 500°C because of limited range of test results reported by Vila Real et al. (2003). Testing at temperature above 600°C is urgently needed as the steel temperature can exceed 600°C in a normal fire. Besides, a wider range of slenderness ratio is required for a further study on LTB behaviour at elevated temperatures.

Numerical analysis has been conducted for unrestrained steel beams at 400 to 800°C as most beams fail in this temperature region. The statistical comparisons of FE results with various approaches are summarized in Table 4.8. The alternative approach provides very accurate results with a maximum mean error of 10%. EC3 approach provides the most conservative predictions. Rankine approach provides comparatively good results and is conservative with a maximum error of approximately 18%. The graphical comparisons of FE results, experimental results and various approaches can be seen in Figures 4.12 to 4.18. These figures show clearly that the alternative approach matches FE predictions quite well for the range of slenderness ratio and temperature considered while the predicted buckling moment based on Rankine approach is always slightly lower compared to that based on the alternative approach. Out of three approaches, the

EC3 approach gives the lowest prediction of the buckling moment. It is noteworthy that there is practically no difference in the predicted buckling moment for slender section with $\sqrt{M_{cx,fi}/M_{E,fi}} \geq 1.5$.

Table 4.7 Comparison of Test Results (Vila Real et al., 2003) with Various Approaches

		Test/Rankine	Test/EC3	Test/Alt.Approach
<i>T</i>=200°C	Mean	1.637	1.719	1.495
	SD	0.291	0.286	0.266
	COV	0.177	0.166	0.178
<i>T</i>=300°C	Mean	1.640	1.719	1.485
	SD	0.307	0.309	0.268
	COV	0.187	0.180	0.180
<i>T</i>=400°C	Mean	1.559	1.639	1.377
	SD	0.318	0.327	0.266
	COV	0.204	0.199	0.193
<i>T</i>=500°C	Mean	0.983	1.035	0.879
	SD	0.270	0.280	0.232
	COV	0.275	0.271	0.263
<i>T</i>=600°C	Mean	0.727	0.765	0.633
	SD	0.198	0.203	0.158
	COV	0.272	0.266	0.250

Table 4.8 Comparison of FE Results with Various Approaches

		FEA/Rankine	FEA/EC3	FEA/Alt.Approach
<i>T</i>=400°C	Mean	1.166	1.255	1.073
	SD	0.067	0.062	0.066
	COV	0.057	0.050	0.062
<i>T</i>=500°C	Mean	1.185	1.271	1.098
	SD	0.083	0.087	0.080
	COV	0.070	0.068	0.073
<i>T</i>=600°C	Mean	1.155	1.241	1.052
	SD	0.065	0.063	0.070
	COV	0.056	0.051	0.067
<i>T</i>=700°C	Mean	1.182	1.271	1.063
	SD	0.090	0.094	0.102
	COV	0.076	0.074	0.096
<i>T</i>=800°C	Mean	1.154	1.236	1.076
	SD	0.078	0.081	0.078
	COV	0.068	0.065	0.072

Figure 4.12 Comparisons of Test Results with Various Approaches at 200°C

Figure 4.13 Comparisons of Test Results with Various Approaches at 300°C

Figure 4.14 Comparisons of Test & FE Results with Various Approaches at 400°C

Figure 4.15 Comparisons of Test & FE Results with Various Approaches at 500°C

Figure 4.16 Comparisons of Test & FE Results with Various Approaches at 600°C

Figure 4.17 Comparisons of FE Results with Various Approaches at 700°C

Figure 4.18 Comparisons of FE Results with Various Approaches at 800°C

4.4.4 Worked Example

A 6.5 m 356×406 UC 235 beam of S275 steel is simply supported at both ends such that lateral deflection and twist rotation at the supports are prevented, but the beam ends are free to warp. A mid-span point loading is applied at the shear centre. The beam is subjected to a maximum uniform temperature of 700°C during fire. The material and section properties are:

$$p_y = 275 \text{ MPa}; E = 210 \text{ GPa}; G = 80.77 \text{ GPa}; S_x = 4.69 \times 10^6 \text{ mm}^3;$$

$$I_x = 7.91 \times 10^8 \text{ mm}^4; I_y = 3.1 \times 10^8 \text{ mm}^4; J = 8.12 \times 10^6 \text{ mm}^4; I_w = 9.54 \times 10^{12} \text{ mm}^6.$$

At 700°C (based on EC3:1.2): $k_y = 0.23$; $k_E = 0.13$; $k_p = 0.075$.

Alternative Approach:

Step 1. Calculate the value of M_{cx} and M_E .

- $M_{cx} = p_y S_x = 275 \times 4.69 \times 10^6 = 1290 \text{ kNm}$

- $M_{cr} = C_b \sqrt{\left(\frac{\pi^2 E I_y}{L^2} \right) \left(GJ + \frac{\pi^2 E I_w}{L^2} \right)}$; in which $C_b = 1.365$ based on EC3.

$$= 1.365 \sqrt{\left(\frac{\pi^2 \times 210 \times 10^3 \times 3.1 \times 10^8}{6500^2} \right) \times \sqrt{(80.77 \times 10^3 \times 8.12 \times 10^6) + \frac{\pi^2 \times 210 \times 10^3 \times 9.54 \times 10^{12}}{6500^2}}} = 5643 \text{ kNm}$$

- Hence, $M_E = \frac{M_{cr}}{\sqrt{(1 - EI_y/EI_x)}} = \frac{5643}{\sqrt{(1 - (3.1 \times 10^8 / 7.91 \times 10^8))}} = 7236.5 \text{ kNm}$

Step 2. Calculate Perry imperfection factor for LTB at elevated temperature η_{LT}^T .

- $$\eta_{LT}^T = 0.007 \sqrt{\zeta} \sqrt{\frac{\pi^2 E}{p_y}} \left(\sqrt{\frac{M_{cx}}{M_E}} - 0.4 \zeta^{1.5} \sqrt{\frac{k_E}{k_y}} \right)$$

$$= 0.007 \sqrt{0.326} \sqrt{\frac{\pi^2 \times 210 \times 10^3}{275}} \left(\sqrt{\frac{1290}{7236.5}} - 0.4 (0.326)^{1.5} \sqrt{\frac{0.13}{0.23}} \right) = 0.12707$$

Step 3. Calculate the value of p_E^T .

- $$p_E^T = \frac{\pi^2 k_E E}{\lambda_{LT}^2} = \frac{\pi^2 \times 0.13 \times 210 \times 10^3}{\left(\pi^2 \times 210 \times 10^3 / 275 \right) \left(1290 / 7236.5 \right)} = 200.5 \text{ MPa}$$

Step 4. Calculate the value of ϕ_{LT}^T .

- $$\phi_{LT}^T = \frac{k_y p_y + (\eta_{LT}^T + 1) p_E^T}{2} = \frac{0.23 \times 275 + (0.12707 + 1) \times 200.5}{2} = 144.61 \text{ MPa}$$

Step 5. Determine the buckling stress at elevated temperature $p_{b,fi}$.

- $$p_{b,fi} = \frac{k_y p_y p_E^T}{\phi_{LT}^T + \sqrt{\phi_{LT}^{T^2} - k_y p_y p_E^T}}$$

$$= \frac{0.23 \times 275 \times 200.5}{144.61 + \sqrt{144.61^2 - (0.23 \times 275 \times 200.5)}} = 53.88 \text{ MPa}$$

Thus, the ultimate buckling moment capacity in fire $M_{b,fi} = p_{b,fi} S_x = 252.7 \text{ kNm}$.

Rankine Approach:

Step 1. Calculate the LTB slenderness ratio λ_{LT} .

- $$\lambda_{LT} = \sqrt{\pi^2 E / p_y} \sqrt{M_{cx} / M_E} = \sqrt{\pi^2 \times 210 \times 10^3 / 275} \sqrt{1290 / 7236.5} = 36.65$$

Step 2. Determine the normalized slenderness ratio ψ_{LT} .

$$\bullet \psi_{LT} = \frac{\lambda_{LT}^2}{\pi^2 E / p_y} = \frac{36.65^2}{(\pi^2 \times 210 \times 10^3) / 275} = 0.1782$$

Step 3. Determine the buckling stress at elevated temperature $p_{b,fi}$.

$$\bullet p_{b,fi} = \frac{k_y p_y}{1 + (k_y / k_E) \psi_{LT}} = \frac{0.23 \times 275}{1 + ((0.23 / 0.13) \times 0.1782)} = 48.09 \text{ MPa}$$

Thus, the ultimate buckling moment capacity in fire $M_{b,fi} = p_{b,fi} S_x = 225.5 \text{ kNm}$.

EC3:1.2 Approach:

Step 1. Calculate the modified slenderness ratio for LTB in fire $\overline{\lambda}_{LT,fi}$.

$$\bullet \overline{\lambda}_{LT,fi} = \overline{\lambda}_{LT} \sqrt{k_y / k_E} = \sqrt{M_{cx} / M_{cr}} \sqrt{k_y / k_E} = \sqrt{1290 / 5643} \sqrt{0.23 / 0.13} = 0.636$$

Step 2. Calculate the value of $\phi_{LT,fi}^{EC}$.

$$\bullet \alpha = \beta \varepsilon = \beta \sqrt{235 / p_y} = 0.65 \sqrt{235 / 275} = 0.6$$

$$\bullet \text{Hence, } \phi_{LT,fi}^{EC} = 0.5 \left(1 + \alpha \overline{\lambda}_{LT,fi} + \overline{\lambda}_{LT,fi}^2 \right) = 0.5 \left(1 + (0.6 \times 0.636) + 0.636^2 \right) = 0.893$$

Step 3. Determine the strength reduction factor for LTB in fire $\chi_{LT,fi}$.

$$\bullet \chi_{LT,fi} = \frac{1}{\phi_{LT,fi}^{EC} + \sqrt{\phi_{LT,fi}^{EC^2} - \overline{\lambda}_{LT,fi}^2}} = \frac{1}{0.893 + \sqrt{0.893^2 - 0.636^2}} = 0.658$$

Step 4. Determine the ultimate buckling moment capacity in fire $M_{b,fi}$.

$$\bullet M_{b,fi} = \chi_{LT,fi} \beta_w S_x k_y p_y / \gamma_{M,fi} = 0.658 \times 1 \times 4.69 \times 10^6 \times 0.23 \times 275 / 1 = 195.2 \text{ kNm}$$

The FE prediction of buckling moment for this example is 253.6 kNm which is very close to the alternative approach's prediction of 252.7 kNm. The Rankine approach's prediction is 225.5 kNm, which is slightly lower than that of alternative approach. EC3:1.2 approach results in the lowest prediction of buckling moment ($M_{b,fi} = 195.2 \text{ kNm}$), which is only 77% of the FE result. Thus, the over-conservatism of EC3:1.2 approach will result in uneconomical member design in fire.

CHAPTER FIVE

EXPERIMENTAL INVESTIGATION ON LOCAL BUCKLING FAILURE AND DUCTILITY

5.1 Introduction

A fairly extensive experimental programme has been conducted by the author to investigate the local buckling failure and ductility of steel and composite beams under fire conditions. These experiments are needed to support and validate both the numerical and analytical models since, to the best knowledge of the author, no test has ever been conducted to study the local buckling behaviour at elevated temperatures. It consisted of two series of tests, namely, steel beams as the first series and composite beams as the second series.

A total of nine steel I-beams were tested to failure in the first series. Parameters including temperature, flange slenderness, web slenderness and effective length were varied in this first series. In the second series, a total of four composite steel beams with re-entrant steel decking profile were tested under hogging moment. The main objective of the experimental programme is to study the ductility issue related to inelastic behaviour of both steel and composite beams at elevated temperature. The second objective is to use the test results to investigate the feasibility of using a finite element model for parametric study. The third objective is to identify key parameters which affect the rotational capacity.

The test set-up was designed to represent the internal joint of a continuous beam. The segments between the plastic hinge and adjacent point of inflection where hogging

moment occurs were represented by each half of a simply supported beam subjected to a mid-span point load. The specimens were heated to the desired temperature before they were subjected to an increasing point load up to failure (isothermal test). The test observations included readings from linear variable differential transducers (LVDTs) for the rotation and deflection measurements. Besides, thermocouples were used to measure the temperature in the specimens and also within the electrical heating furnace. The results reported herein provide test data and hopefully, to shed light in this important area. Ultimately, a better understanding of inelastic behaviour of both steel and composite beams in fire conditions will result in a safer and economical member design.

5.2 Experimental Programme on Steel Beams

A total of nine steel I-beams were tested to failure. One of them was tested at ambient temperature, while the others were tested at either 415°C (since around this temperature the yield strength starts to reduce) or 615°C (since many beams fail around this temperature). The yield strength starts to reduce for temperature above 400°C, hence 415°C is chosen. Three of the specimens were welded sections, while the rest were UB sections. The main parameters investigated were the effects of temperature, flange slenderness, web slenderness and effective length. The fire conditions were simulated using an electrical heating furnace. The specimens were heated before a static point load was applied through a hydraulic jack mounted on the reaction frame. No axial restraint was applied during the tests; hence the tests simulated the outer span of continuous beam, of which the restraint force is negligible.

Four coupon plates, two from each flange and web, were cut from different types of beams used. Tensile coupon tests were then conducted to obtain the stress-strain relationship of flange and web plates at ambient temperature. In addition, two types of geometric imperfections were measured. One was on the cross-section dimensions measured by digital callipers, and the other was on the longitudinal distance from an

assumed straight line, measured by a mechanical scanning machine.

5.2.1 Design of Beam Specimens

The objective in the test is to determine the available rotational capacity of steel beams in the hogging moment region as part of a continuous beam under fire conditions. However, it is very difficult due to the great number of factors influencing the frame behaviour. Thus, it is important to simplify the analysis by using a simple substitute member with very similar behaviour (Gioncu & Petcu, 1997). The hinges where the largest plastic rotation occurs during the development of the collapse mechanism are frequently located at an internal support in a continuous beam as shown in Figure 5.1(a). These segments are subjected to significant moment gradient, and the portions between the plastic hinge and adjacent point of inflection may be represented by each half of a simply supported beam (Kemp, 1986).

Figure 5.1(a) Simplified Substitute Member for Hogging Moment Region

This substitute member is subjected to a central point load with lateral restraints at the centre and the supports as shown in Figure 5.1(b). Additional restraints between the supports and mid-span can also be provided so that lateral torsional buckling does not occur before local buckling. This member is normally called a “standard beam”, so

defined because both theoretical studies and experimental tests at ambient temperature in the technical literature are performed on this beam. This standard beam arrangement was therefore used in the testing. It should be noted that the behaviour of the joints is not part of this research since it focuses on the inelastic behaviour of the member. Hence, this study is applicable to continuous beams with relatively rigid type of connection system and also continuous beams with semi-rigid joints under fire conditions. As mentioned in Section 2.3.3, the reason is that, in the event of fire, the temperatures of the joints are often lower than that of the member due to the shielding effect, as observed during Cardington fire test (British Steel, 1998). Hence, the relative stiffness of the connections compared to the member is increased. Consequently, the rotation near the support is largely provided by the member. This is analogous to a beam with a rigid joint and the member, instead of the connection, often fails by local flange and web buckling.

Figure 5.1(b) Standard Beam Arrangement

All specimens have the same total length of 3650 mm with the distance between supports as 3450 mm. The length of the tested beams is kept the same due to the restriction imposed by the length of electrical heating furnace. The design of the specimens is shown in Figure 5.2. There are web stiffeners at the supports and at the

point load application (mid-span). Besides, end plates are welded at the beam ends to assist in the rotation measurement using linear variable differential transducer (LVDT). A pair of vertical web stiffeners at the beam mid-span effectively simulates the symmetry boundary condition and also prevents the web bearing failure below the mid-span point loading. There are plates attached to the mid-span stiffener (Figure 5.2) for the purpose of mounting a pair of vertical rollers on each side of the beam. Additional plates for roller are sometimes welded at a position between the mid-span and the support to reduce the effective length of the specimens. The reasons are either to investigate the influence of effective length or to avoid premature lateral torsional buckling, thereby ensuring the achievement of full plastic moment capacity. The roller is rested on specially designed lateral restraint so that the beams can move freely in the vertical direction of loading, but are not able to deflect laterally.

Figure 5.2 General Layout of First Series of Test Specimens

In order to assist in the geometric imperfection measurement, in mounting of thermocouples for temperature measurement and for reference purpose, gridlines were drawn on the beams as shown in Figure 5.3. Table 5.1 presents the details of the specimens. Three different section sizes were used. The specimens have flange

slenderness ratio of either 6.09 or 8.15 and web slenderness ratio of either 34.51 or 47.42. S1 and S2 series have the same section size, but different effective lengths so that such effect can be observed by comparing the two test results. Specimen S3-1, one of the beams in S3 series, was tested at ambient temperature so as to observe the difference in the inelastic behaviour between ambient and elevated temperatures. The effect of flange slenderness on the rotational capacity can be gleaned by comparing S1 and S3 series. Finally, by comparing S4 and S3 series, the effect of web slenderness can be studied. The use of welded section is necessary since there is no available UB section for such range of flange and web slenderness ratios. It should be noted that all specimens used are the ones that are capable of achieving their plastic moment capacity at elevated temperature. Thus, the study does not include the sections that are not able to achieve the full plastic moment capacity at elevated temperature.

Table 5.1 Details of First Series of Test Specimens

Test No.	Size	T (°C)	Coupon Type	L_E (mm)	c/t_f	d/t_w	λ_{LT}^*	$M_{p,T}$ (kNm)
S1-1	305x165UB54	415	C	650	6.09	34.51	13.15	236.11
S1-2	305x165UB54	615	A	650	6.09	34.51	13.15	102.85
S2-1	305x165UB54	415	B	1725	6.09	34.51	27.30	242.86
S2-2	305x165UB54	615	B	1725	6.09	34.51	27.30	106.10
S3-1	Welded**	25	T	1725	8.15	34.51	30.06	146.67
S3-2	Welded**	415	T	605	8.15	34.51	13.45	142.67
S3-3	Welded**	615	T	605	8.15	34.51	13.45	62.33
S4-1	406x178UB54	415	E	650	8.15	47.42	13.36	410.72
S4-2	406x178UB54	615	E	650	8.15	47.42	13.36	179.43

Half-span length (L_i) = 1725mm

$$* \lambda_{LT} = \sqrt{\frac{\pi^2 E}{f_y}} \sqrt{\frac{M_{cx}}{M_E}}$$

** $b = 163\text{mm}$; $d = 276\text{mm}$; $t_f = 10\text{mm}$; $t_w = 8\text{mm}$

Figure 5.3 Reference Gridlines of Test Specimens

5.2.2 Instrumentation

The instrumentation devices included thermocouple wires and LVDTs. Type K mineral insulated thermocouple wires were used to measure the temperatures on the specimen as well as the interior of furnace. The locations of thermocouple wires are presented in Table 5.2 with reference to Figure 5.3. The locations of the thermocouple wires were arranged such that the temperature distribution around the critical region (near mid-span) can be recorded. Besides, additional thermocouple wires were used to indicate whether uniform temperature distribution was achieved along the span. Two thermocouples were used to record the gas temperature above and below the specimen. The temperatures were recorded automatically every 15 second using a data acquisition system.

Table 5.2 Locations of Thermocouple Wires

Reference	Position	Reference	Position
Th.1	Grid 8-top flange	Th.12	Grid 5-top flange
Th.2	Grid 8-b	Th.13	Grid 5-b
Th.3	Grid 8-bottom flange	Th.14	Grid 5-bottom flange
Th.4	Grid 7-top flange	Th.15	Grid 9-top flange
Th.5	Grid 7-a	Th.16	Grid 9-b
Th.6	Grid 7-b	Th.17	Grid 9-bottom flange
Th.7	Grid 7-c	Th.18	Grid 10-top flange
Th.8	Grid 7-bottom flange	Th.19	Grid 10-b
Th.9	Grid 6-top flange	Th.20	Grid 10-bottom flange
Th.10	Grid 6-b	Th.21	Furnace gas temperature-below
Th.11	Grid 6-bottom flange	Th.22	Furnace gas temperature-upper

LVDTs, of which schematic positions are shown in Figure 5.4, were used for deflection measurements and rotation calculations. A total of 11 LVDTs were placed in each specimen. One pair of LVDTs was each placed horizontally at each end to calculate the end rotations. Two other LVDTs were placed at mid-span to measure the vertical deflection, and the rest of them were placed along the beam length to measure the in-plane deflections. Eight 100mm-LVDTs and three 300mm-LVDTs were used. These 300mm-LVDTs were placed at position 5, 6 and 7 (Fig. 5.4) since these three locations had greater displacements compared to the other positions. Direct contact between

LVDTs and the top flange surface at position 7 to 11 was not possible due to high temperature environment. Hence, indirect measurements using silica rods passing through the small holes on the furnace top cover panel were used instead of indirect measurements. The low thermal conductivity of silica rod and the relatively constant temperature during loading stage suggest that the error of displacement measurement was negligible (Leston-Jones, 1997).

Figure 5.4 Schematic Positions of LVDTs

5.2.3 Test Set-Up and Procedure

Figure 5.5 shows the general view of test set-up which consisted of a frame for loading support, an electrical heating furnace to simulate elevated temperature conditions, a 100 ton hydraulic jack (200mm available stroke) to impose the loading, two fork supports to provide a simply supported condition at both ends and two lateral restraint systems. The electrical heating furnace consisted of stainless steel frames supporting the heating elements and ceramic fiber board. The use of ceramic fiber board ensured that there was minimal heat loss and the temperature outside the heating panels was lower than 50°C. There are top cover panels which are not shown in Figure 5.5 so that the furnace interior can be viewed.

The total beam length was 3.65m resting on supports of 3.45m span. There was a compression load cell below the hydraulic jack to measure the point load applied as shown in Figure 5.5. Below the compression load cell, a rocker bearing was used to allow minor out-of-plane rotation. A loading rod was welded below the rocker bearing to

transfer the load to the test specimen. One part of loading rod, which was inside the furnace, was highly insulated with fire protection material. The plan view with the cross-sectional views are shown in Figure 5.6, whereas the front and side views are shown in Figure 5.7.

Figure 5.5 General View of Test Set-Up

A special fork support was used to simulate a simply supported condition. The fork support was designed by the author, as shown in Figure 5.8. There were lateral restraints, which can be adjusted depending on the width of the beam tested. The specimen was rested on top of roller bearings at both ends.

Figure 5.6 Plan View and Section View of Test Set-up

Figure 5.7 Front View and Side View of Test Set-Up

Figure 5.8 Adjustable Fork Support Systems

To assist in restraining the test beams from moving laterally, a pair of vertical rollers was attached to the steel plates as shown in Figure 5.9. The rollers were designed such that they could be easily mounted onto the side plates by bolting.

Figure 5.9 Mid-span Roller Systems

The lateral restraint systems, which were designed by the author, were provided at the centre and the supports to prevent global buckling. During most tests, apart from 3 tests, additional restraints located between the supports and mid-span were provided so that

LTB does not prevent the attainment of full plastic moment capacity. As can be seen from Figure 5.10, the lateral restraint system consisted of one “fixed leg” and two “adjustable legs”. The “fixed leg” was used for mid-span restraint, while the “adjustable legs” were used if additional restraints were required. As it is called, the “adjustable legs” could be easily moved by sliding and subsequently fixing them using bolts. The length of lateral restraint could be adjusted to suit the width of specimens such that the mounted rollers were able to rest on both the “fixed leg” and “adjustable legs” and roll down easily when vertical load was applied. The lateral restraint systems inside the furnace were fire protected using isowool.

Figure 5.10 Lateral Restraint Systems

The test consisted of two stages: heating and loading stage. Firstly, the furnace temperature was preset to a desired temperature with a heating rate of $7^{\circ}\text{C}/\text{min}$. After the desired temperature was reached, the furnace maintained that temperature up to the end of the loading stage. The loading stage was started after the temperature distribution of the specimen was deemed to be fairly uniform (the difference of the temperature across the section and along the beam length is less than 5°C and 10°C , respectively). The load was applied until the specimen failed and reached its unloading state, where full

moment-rotation curve was obtained, or the maximum stroke of hydraulic jack was reached, or the beam deformation was too large and could damage the furnace. Before the commencement of each test, a small load was applied to each specimen to check the test set-up, the instrumentation and the loading system.

5.2.4 Geometric Imperfection Measurements

Digital callipers were used to measure the cross-section dimensions. The width and thickness of top and bottom flange plate were measured at 15 locations from grid 1 to grid 15 (Figure 5.3). The depth and thickness of web plate were also measured at these locations. The depth of the web plate was measured from the inner distance between the top and bottom flange, while the web thickness was obtained indirectly by subtracting the two outstands of flange from its width. Table 5.3 presents the maximum, minimum and average values of measurements.

Mechanical scanning machine was used to measure the longitudinal lateral distance from an assumed straight line. The mechanical scanning machine comprises an LVDT with a roller on its tip and a motor mounted onto a frame, which guides the LVDT to move vertically along the beam length. The beam was placed in a vertical direction and secured to the frame before measurements started. Three positions (P1, P2 and P3) to measure the in-plane longitudinal imperfection and two positions (P4 and P5) to measure the out-of-plane longitudinal imperfection are shown in Figure 5.11. The data was automatically captured by a special data logger. A typical plot of imperfection measurement for one of the beams, S2-1, is illustrated in Figure 5.12 and Figure 5.13 for in-plane and out-of-plane imperfections, respectively. It should be noted that the out-of-plane imperfection plot of the web (referred as P5) consisted of two discontinuous lines since the presence of mid-span stiffener prevented a continuous measurement. A summary of maximum imperfection is presented in Table 5.4. The complete plot of imperfection measurements can be found in Appendix B1.

Table 5.3 Measured Cross-Section Dimensions of First Series of Specimens

Beam No.	$b_{top\ flange}$			$b_{bott\ flange}$			$t_{top\ flange}$			$t_{bott\ flange}$			d			t_w		
	max	min	avg	max	min	avg	max	min	avg	max	min	avg	max	min	avg	max	min	avg
S1-1	167.77	167.16	167.46	167.06	166.17	166.73	13.05	12.67	12.87	13.08	12.50	12.71	286.39	283.77	285.41	8.21	7.42	7.81
S1-2	167.79	167.06	167.45	168.21	167.13	167.72	13.05	12.69	12.78	12.89	12.50	12.67	286.29	284.08	285.35	7.94	7.51	7.72
S2-1	167.51	166.18	167.00	166.80	165.97	166.48	12.86	12.56	12.69	13.79	13.42	13.60	286.64	283.66	285.36	8.34	7.22	7.92
S2-2	167.74	166.03	167.03	167.50	166.68	167.03	14.56	13.21	13.56	14.06	13.64	13.82	286.32	283.96	285.41	8.06	7.24	7.56
S3-1	164.17	163.24	163.81	164.17	163.33	163.79	10.27	9.94	10.06	10.13	9.83	9.94	275.91	272.17	274.38	8.62	7.89	8.23
S3-2	163.75	161.89	162.89	164.01	163.13	163.51	10.46	9.81	10.00	10.79	9.64	9.95	278.86	273.16	275.50	8.45	7.95	8.14
S3-3	163.15	162.13	162.72	164.91	163.12	164.00	10.91	10.04	10.30	10.72	9.74	10.19	279.56	273.08	275.93	8.23	7.65	7.95
S4-1	177.02	176.25	176.70	178.67	177.47	178.26	10.20	9.91	10.08	10.63	10.29	10.46	375.92	386.00	380.76	8.12	7.45	7.83
S4-2	177.97	177.49	177.83	176.92	176.41	176.71	10.39	10.21	10.29	10.59	10.45	10.52	376.90	384.70	380.78	8.22	7.53	7.76

Figure 5.11 Positions of Longitudinal Geometric Imperfection Measurement

Figure 5.12 In-Plane Imperfection of S2-1

Figure 5.13 Out-of-Plane Imperfection of S2-1

Table 5.4 Summary of Maximum Longitudinal Imperfection

Beam No.	Maximum Longitudinal Imperfection				
	P1	P2	P3	P4	P5
S1-1	0.32	0.91	1.51	2.32	0.75
S1-2	2.12	2.43	2.46	1.49	0.72
S2-1	2.05	0.71	0.27	1.24	0.33
S2-2	2.07	1.64	1.80	1.59	0.66
S3-1	3.79	3.30	3.21	2.07	0.45
S3-2	2.46	1.29	1.76	1.74	1.72
S3-3	4.55	3.84	3.69	6.81	1.96
S4-1	3.71	2.85	1.96	3.52	1.08
S4-2	1.35	0.91	0.87	4.84	0.53

5.2.5 Material Properties

Tensile tests were performed to obtain the elastic modulus and yield strength of the specimen materials. Test coupons of the flange and web were cut out from the steel beams. Two flange coupons consisting of top and bottom flange and two web coupons were tested at ambient temperature for different types of beams used. Three sources of readings were obtained from each coupon, viz. two strain gauges and one extensometer. Average results from the tensile tests are summarized in Table 5.5. Typical material stress-strain curves are illustrated in Figure 5.14 and Figure 5.15 for coupon type A and T, respectively. Material test was not conducted at elevated temperature due to the expense of testing, sufficiently extensive data in existence and availability of comprehensive model such as widely-accepted EC3 model describing the degradation of material properties with increasing temperatures, as discussed in Chapter 3.

Table 5.5 Tensile Test Results of First Series of Specimens at Ambient Temperature

Coupon Type	Yield strength (MPa)	Elastic modulus (MPa)	Ultimate strength (MPa)
A (flange)	287.1	203038	473.2
A (web)	304.0	202523	484.1
B (flange)	297.6	203677	483.6
B (web)	307.4	205253	491.4
C (flange)	287.4	205557	475.4
C (web)	307.1	206428	485.1
T (flange)	224.1	201697	392.1
T (web)	277.1	206063	452.0
E (flange)	393.5	205283	545.1
E (web)	449.4	205700	590.3

Figure 5.14 Stress-Strain Relationship of Coupon Type A

Figure 5.15 Stress-Strain Relationship of Coupon Type T

5.3 Results and Discussions on Steel Beam Tests

In this section, detailed experimental results of the nine steel beams are presented. The appraisal of the results covers the observed failure modes, the effects of varied parameters and the performance in terms of rotational capacity. A summary of the test results is presented in Table 5.6. The plastic moment capacity $M_{p,T}$ has been calculated based on the cross-section dimensions, tensile test results in Table 5.5, and EC3:1.2 (CEN, 2005) material reduction factors at elevated temperature. The end rotation was obtained from the relative horizontal movement of two parallel LVDTs divided by their

vertical distance apart. The reported rotation is taken as the average of the two end rotations. The failure modes observed is indicated in the last column of Table 5.6.

Table 5.6 Summary of First Series of Test Results

Test No.	Size	T (°C)	c/t_f	d/t_w	λ_{LT}	$M_{p,T}$ (kNm)	M_m (kNm)	$M_m/M_{p,T}$	r_a	Failure Modes
S1-1	305x165UB54	415	6.09	34.51	13.15	236.11	264.70	1.121	3.73*	LB*
S1-2	305x165UB54	615	6.09	34.51	13.15	102.85	109.62	1.066	2.81*	LB*
S2-1	305x165UB54	415	6.09	34.51	27.30	242.86	262.55	1.081	1.60	LTB
S2-2	305x165UB54	615	6.09	34.51	27.30	106.10	112.04	1.056	0.93	LTB
S3-1	Welded	25	8.15	34.51	30.06	146.67	194.41	1.325	7.57	LB-LTB
S3-2	Welded	415	8.15	34.51	13.45	142.67	156.29	1.095	2.28	LB
S3-3	Welded	615	8.15	34.51	13.45	62.33	68.48	1.099	1.11	LB
S4-1	406x178UB54	415	8.15	47.42	13.36	410.72	451.26	1.099	0.68	LB
S4-2	406x178UB54	615	8.15	47.42	13.36	179.43	186.04	1.037	0.30	LB
<i>Note:</i> * Estimated based on numerical simulation LB: local buckling LTB: lateral torsional buckling LB-LTB: local buckling followed by lateral torsional buckling										

5.3.1 General Observations

For all specimens, no local and global buckling took place before the plastic moment capacity was reached. All specimens, except for the one tested at ambient temperature, started to deviate from linearity before they yielded due to high non-linearity of stress-strain relationship at elevated temperature. Beams tested at 615°C deviated from linearity at an earlier stage compared with those tested at 415°C due to the fact that the ratio of proportionality limit to yield strength (f_{pT}/f_{yT}) is smaller at 615°C. After attaining plastic moment capacity, the beam deflection and rotation started to increase rapidly. Finally, a gradual load reduction occurred without sudden failure. Two failure modes were generally observed: local buckling of the beam flange and web near to mid-span and global buckling in the form of lateral torsional buckling. An example of local buckling, which occurred in S3-2 and S4-1, is illustrated in Figure 5.16. The local buckles of flange occurred in an anti-symmetric mode at equal distance from the restrained mid-span. The local buckles of the web occurred approximately across half depth of the web which is in compression. It should be noted that for a more-or-less uniformly heated specimen, the plastic neutral axis is located at mid-height of the web.

The web buckling was comparatively less obvious compared to flange buckling. The reason is that additional rotation was needed for greater lateral movement of the web. Besides, the range of web slenderness used was comparatively stocky (Table 5.6).

Figure 5.16 Local Buckling in S3-2 and S4-1 Specimen

Lateral torsional buckling, which occurred in S2-1 and S2-2, is shown in Figure 5.17. The lateral movement mostly occurred near mid-span with the top flange at both sides of the mid-span moving laterally in opposite direction. It should be noted that the occurrence of LTB was a result of spread of yielding from the mid-span section after the plastic moment capacity had been reached. It is different from conventional elastic LTB which prevents the attainment of plastic moment capacity. A slight LTB was sometimes observed after flange buckling as this had severely reduced the resistance of the flange in the plastic region to lateral torsional buckling, as illustrated in Figure 5.18. The sequence of failure modes could not be observed from the test since the specimens were tested within the furnace. It can only be predicted based on numerical simulation, which is described in Chapter 6. Additional photos of tested beams are shown in Appendix B2.

The maximum plastic moment achieved for all specimens tested at elevated temperatures is less than 110% of its theoretical value. This is due to EC3:1.2 definition of yield stress, which is at 2% strain, and the disappearance of strain hardening above 2% strain at elevated temperatures. The ratio of experimentally obtained maximum to

theoretical plastic moment capacity for S3-1 is rather high ($M_m/M_{p,T} = 1.33$) due to the material stress-strain characteristic, which has ultimate strength much greater compared to its yield strength ($f_{ult}/f_y = 1.76$). Typically, symmetrical rotation was observed at both ends of the beam as shown in Figure 5.19, which gives the load-rotational response for S2-1 and S2-2. This shows that the test set-up was adequate.

Figure 5.17 Lateral Torsional Buckling in S2-1 and S2-2 Specimen

Figure 5.18 Local and Lateral Torsional Buckling in S3-1 Specimen

Figure 5.19 Load-Rotation Responses of S2-1 and S2-2 Specimen

5.3.2 Temperature Effects

The effects of increasing temperature on the behaviour of steel I-beams in terms of rotational capacity can be observed by comparing the moment-rotation curves of S3-1, S3-2 and S3-3, which were tested at 25°C, 415°C and 615°C, respectively. These three moment-rotation curves, which are plotted in non-dimensional form, are shown in Figure 5.20. The horizontal axis shows the rotation θ over the elastic rotation θ_e (defined as $M_{p,T} L_i / [2E_T I]$). At ambient temperature, for S3-1, considerable rotational capacity was observed ($r_a = 7.57$) and the maximum moment was 33% above its theoretical value. When the temperature was increased to 415°C in S3-2, the moment-rotation curve became non-linear even before the beam yielded due to non-linear stress-strain relationships at elevated temperature. The rotational capacity has also reduced to 2.28 even though additional lateral restraints were added, reducing the effective length and LTB slenderness ratio λ_{LT} (Table 5.6). The influences of effective length are discussed in Section 5.3.5. The considerable reduction of rotational capacity at elevated temperatures is a result of two factors. First, the non-linearity of stress-strain relationship at elevated temperatures causes the beam to attain plastic moment capacity at a much higher

rotation, leaving behind limited inelastic rotation. Second, the disappearance of strain-hardening phenomenon means that the yielded part of the beam has practically no rigidity and local buckling takes place at an earlier stage (Dharma & Tan, 2006b).

Figure 5.20 Moment-Rotation Response of S3 Series

Increasing the temperature further to 615°C, it can be observed that the rotational capacity of S3-3 is further reduced ($r_a = 1.11$) compared to S3-1 and S3-2. A rotational capacity of only 1.11 may be insufficient to allow a redistribution of bending moment in the structure. Less rotational capacity at 615°C compared to 415°C is a general observation for all tested beams (Table 5.6). Thus, a beam which has sufficient rotational capacity to allow moment redistribution at ambient temperature, for example S3-1, may gradually lose its rotational capacity at elevated temperatures, hence preventing the redistribution of moments. The conservatism of design plastic moment capacity has also reduced from up to 30% to a low of 10% as a result of increasing temperature. The reduction in maximum load and stiffness as temperature increases can be observed in Figure 5.21, which shows the load-deflection relationship at mid-span for S3 series.

Figure 5.21 Load-Deflection (mid-span) Response of S3 Series

5.3.3 Flange Slenderness Effects

The importance of flange slenderness can be investigated by comparing S3 and S1 series (Table 5.6). These series only differ from each other by their flange slenderness ratio. The moment-rotation and load-deflection (mid-span) curves are shown in Figure 5.22 and Figure 5.23, respectively. It can be observed that S1-1 and S1-2 beams, which have stockier flanges, had greater rotational capacity compared to S3-2 and S3-3 beams. Due to greater rotation that S1 series of specimens could achieve and the limited stroke of hydraulic jack, the tests had to be stopped before the beams reached their unloading stage. As a result, local buckling could not be observed after the test. However, it was predicted through numerical simulations that local buckling failure caused the unloading of these beams. Based on numerical simulations, a mid-span deflection as large as 300 mm was needed if the beams were to buckle locally in the flange and web. The estimated rotational capacity of S1-1 and S1-2 based on the numerical result, as shown in Table 5.6, are greater than that of S3-2 and S3-3, respectively. It should be mentioned that S1-1 test had been loaded twice. Firstly, the loading process had to be stopped when the load reached 170 kN because the loading rod slipped. After the loading rod problem was

rectified, the loading process was repeated. There was apparently little effect of repeated loading on the behaviour of this beam since the first loading stage was still within the elastic range. It should be noted that the beams with the stockiest flange, S1-1 and S1-2, provide the greatest rotational capacity. Similarly, the beams with the most slender flange, S4-1 and S4-2, have the least rotational capacity.

Figure 5.22 Moment-Rotation Response of S1 Series, S3-2 and S3-3

Figure 5.23 Load-Deflection (mid-span) Response of S1 Series, S3-2 and S3-3

5.3.4 Web Slenderness Effects

Figure 5.24 shows the moment-rotation curves of S3 and S4 series, while Figure 5.25 shows the load-deflection relationships at mid-span. These series differ only in their web slenderness. It can be observed that S4-1 and S4-2 provided less rotational capacity than S3-2 and S3-3, respectively, because they have more slender webs. In addition, S4-2 specimen, which was tested at 615°C, merely reached its plastic moment capacity, with the least rotational capacity (Fig. 5.24). Besides, it should be noted that higher grade of steel (S355) used in S4 series may also contribute to less ductility. Even though flanges, as the primary elements of flexural resistance in I-sections, should be the more significant factor than webs in providing sufficient rotational capacity, it has been shown that web slenderness effect is also dominant. It is because the flange is restrained by the web. Therefore, the stockier the web, the greater the restraints it provided against flange buckling, thus enhancing the ductility of the beam section. It should be highlighted that tested beams with the most slender web, S4-1 and S4-2, have the least rotational capacity.

Figure 5.24 Moment-Rotation Response of S3-2, S3-3 and S4 Series

Figure 5.25 Load-Deflection (mid-span) Response of S3-2, S3-3 and S4 Series

5.3.5 Effective Length Effects

The influence of different effective lengths can be observed by comparing S1 and S2 series, of which moment-rotation curves and load-deflection relationships are shown in Figure 5.26 and Figure 5.27, respectively. As additional lateral restraints are provided for S1 series, hence reducing the effective length to 650 mm, increased rotational capacity was observed. Besides, a slight increase in the maximum moment was also observed when the effective length was reduced from 1725 to 650 mm. As a consequence, the failure mode, changed from lateral buckling for S2 series (with greater effective length) to flange and web buckling for S1 series (with smaller effective length). This finding shows that the ductility of beams is influenced not only by conventional flange and web slenderness parameters, but also by the effective length which is one of the factors governing the lateral torsional buckling capacity. The influence of effective length on rotational capacity is further discussed in Chapter 6. In addition, a method to incorporate the influence of effective length together with other factors in predicting the rotational capacity at elevated temperature is presented in Chapter 7.

Figure 5.26 Moment-Rotation Response of S1 and S2 Series

Figure 5.27 Load-Deflection (mid-span) Response of S1 and S2 Series

5.4 Experimental Programme on Composite Steel Beams

This section presents the results of an experimental programme of composite beams subjected to hogging moment at elevated temperature. The experiments simulated fire situations which occur only on the outer span of a continuous composite beam, hence

there is only minimum axial restraint from adjacent unheated structure. A total of four composite steel beams with re-entrant decking profile were tested to failure at elevated temperature. Three different Universal Beam sections were compositely bonded with profile decking slabs by welded-through shear studs. The number of tests conducted was restricted to four by the resources available. However, it was considered sufficient to fulfill the purpose of using the test results to calibrate the finite element simulations. The test set-up was generally similar to the steel beam test, except that each beam was inverted with the concrete slab facing down. Material testing conducted at ambient temperature, included tensile coupon tests of steel I-beams and reinforcement, compression and tensile tests of concrete, and push-out tests of the shear studs. In addition, cross-section geometric imperfections were also measured by digital callipers. Longitudinal in-plane and out-of-plane imperfection were not measured for these four beams because the mechanical scanning machine was too small to fit in these composite beams. The results aimed to provide test data and to validate the finite element model described in Chapter 6.

5.4.1 Design of Composite Steel Beam Specimens

The composite beam was again designed as a simply supported beam subjected to a central point load to represent the portion between the plastic hinge at support and adjacent point of inflection in the hogging moment region of a continuous beam (Fig. 5.1(a)). The only difference is that the composite beam is inverted such that the decking slab is located on the underside of the steel beam and is subjected to tensile force when the vertical load is applied from the top. It should be noted that for ease of readers, the portion of flange which is connected to decking slab will be referred as top flange in the ensuing discussions.

The total length of composite beam was 3650 mm and the distance between supports was 3450 mm, as with the steel beam specimen. The length of concrete slab is kept at

only 2100 mm; it is less than the steel beam due to restriction imposed by the opening width of the furnace at both ends. A typical configuration of the specimens is illustrated in Figure 5.28, which shows the plan, elevation and end view of specimen C1. Detailed configuration of other specimens can be found in Appendix C2. Secondary reinforcement were added to primary reinforcement to prevent cracking and represent a typical constructional form. Holorib S350 (0.9 mm thickness) re-entrant steel decking was adopted as a permanent formwork and placed in the longitudinal direction. The shear studs were connected to the steel beams and steel decking by through-deck welding.

After 28 days of concrete curing, double layer of fire protection material consisted of 25 mm thick vermiculite and 30 mm thick ceramic blanket named isowool was applied to the exposed external surface of concrete. This fire protection material is needed so that there is no heat transfer through the top surface of concrete, simulating the support conditions, where only the steel beam and the bottom surface of concrete slab are exposed to fire (top surface of concrete is relatively cool). To hold the isowool around the concrete surface and to prevent any concrete spalling from damaging the furnace, a perforated steel sheet is used to wrap around the isowool as shown in Figure 5.29. Furthermore, to ensure that the maximum temperature distribution was reached at mid-span region, isowool was used to wrap around the steel beams, except within 400 mm at both sides of mid-span where point load was applied.

As with steel beam specimens, web stiffeners were welded at the supports and at the point load application (mid-span) to prevent web crushing caused by concentrated load. In addition, end plates are also added at the beam ends to assist in the rotation measurement using linear variable differential transducer (LVDT). Side plates were welded to the mid-span stiffener and a certain distance from mid-span for the purpose of mounting vertical rollers.

Figure 5.29 Composite Beam before Testing

The details of the specimens are presented in Table 5.7. Three different UB sections were used as structural steel. C1 and C2 have the same configuration except for the number of shear stud connectors. It should be noted that all UB sections were chosen such that the composite sections would be able to achieve their full plastic moment capacity at elevated temperature. The effective length L_E was set to a value such that the tested beam would not fail in lateral torsional buckling. The number of shear studs was determined to ensure full shear connection between steel beam and slab.

Table 5.7 Details of Second Series of Test Specimens

Test No.	Structural Steel	Reinforcement	No. of Stud	Stud spacing (mm)	L_E (mm)	t_c (mm)
C1	305x165UB54	4-T10	8	280	563	130
C2	305x165UB54	4-T10	4	650	563	130
C3	305x127UB37	5-T10	10	220	563	130
C4	254x102UB25	5-T10	6	380	469	120
Half-span length (L_i) = 1725 mm						
Slab size (length x width) = 2100 mm x 450 mm						
Reinforcement distance to steel decking = 100 mm						
Anti-crack reinforcement = T8 at 200 mm spacing						
Shear stud connector = 19 mm dia. x 100 mm length						

Note : 4-T10 indicates four bars of type T reinforcement with 10 mm diameter.

5.4.2 Instrumentation

The instrumentation devices were similar to steel beam tests. However, a reduced number of LVDTs were placed in each specimen. Two pairs of horizontal LVDTs were each placed at the beam ends to measure the end rotations, while two other LVDTs were placed at mid-span to measure the vertical deflection. Five other LVDTs were not placed

on top of composite beams since the presence of insulation blanket on the top flange would make the deflection measurement at these positions difficult and inaccurate. In addition to 22 thermocouples placed onto the structural steel section similar to the first series (Table 5.2), eight additional thermocouples were used to monitor the slab, reinforcement and shear stud temperature. Three thermocouples, each for measuring reinforcement & concrete temperature and two thermocouples for measuring shear stud temperature. The thermocouples for assessing reinforcement temperature were secured to the reinforcement surface at mid-span area. Concrete temperatures were assessed at three different locations: 0, 50 and 100 mm above the top flange, while shear stud temperature was assessed at 25 mm above its base. A small hole of approximately 2 mm depth was drilled into the stud to place these thermocouples. These thermocouples were placed before casting of concrete.

5.4.3 Test Set-Up and Procedure

The test set-up was the same as that of the steel beam. The test procedure also consisted of two stages: heating and loading stage. Firstly, the furnace temperature was increased at a rate of between 7 and 10°C/min up to a certain temperature. This selected range of heating rate was equivalent to the average heating rate of the first hour to hour and a half of a real fire (British Steel, 1998). After the preset furnace temperature was reached, that temperature was maintained up to the end of loading stage. In the loading stage, the load was applied until the specimen failed and reached its unloading state, where full moment-rotation curve was obtained. As for steel beam tests, a small load was applied to each specimen before the commencement of each test to check the test set-up, the instrumentation and the loading system.

5.4.4 Geometric Properties Imperfection

Digital callipers were used to measure the cross-section dimensions. The width and thickness of top and bottom flange plate were measured at 15 different locations similar

to the first series of specimens. The depth and thickness of web plate were also measured at these locations. Table 5.8 presents the maximum, minimum and average values of measurements.

Table 5.8 Measured Cross-Section Dimensions of Structural Steel

Beam No.	$b_{top\ flange}$			$b_{bot\ flange}$			$t_{top\ flange}$			$t_{bot\ flange}$			d			t_w		
	max	min	avg.	max	min	avg.	max	min	avg.	max	min	avg.	max	min	avg.	max	min	avg.
C1	167.62	166.56	167.12	167.15	166.25	166.76	13.09	12.53	12.84	13.02	12.42	12.76	286.41	284.36	285.24	8.12	7.45	7.86
C2	167.82	167.11	167.51	168.13	167.32	167.69	13.01	12.64	12.88	12.94	12.21	12.62	286.13	284.86	285.46	7.98	7.61	7.79
C3	124.68	123.62	123.89	124.15	123.24	123.68	11.05	10.56	10.78	10.98	10.42	10.68	284.45	282.13	283.52	7.31	7.12	7.25
C4	102.56	100.85	101.76	102.87	101.12	102.06	8.92	8.31	8.64	9.01	8.12	8.57	242.04	239.68	241.06	5.82	6.45	6.21

5.4.5 Material Properties

Material tests were conducted at ambient temperature to obtain the representative stress-strain curves. These also included tensile tests for steel beams and reinforcement, compressive and tensile tests for concrete, and push-out tests. The results of these tests at ambient temperature were used as inputs to various material models at elevated temperatures, as described in Chapter 3, for the purpose of numerical modelling.

Structural Steel

Test coupons of the flange and web were cut out from the steel beams. Two coupons, each for flange and web plates, were tested at ambient temperature for different types of beams used. Three sources of readings were obtained from each coupon, viz. two strain gauges and one extensometer. Average results from the tensile tests are summarized in Table 5.9. The full material stress-strain curves are shown in Figure 5.30 and Figure 5.31 for the flange and web plate, respectively.

Table 5.9 Tensile Test Results of Second Series Structural Steel

Coupon Plate	Yield strength (MPa)	Elastic modulus (MPa)	Ultimate strength (MPa)
C1 & C2 (flange)	286.1	204775	479.1
C1 & C2 (web)	309.9	207450	492.6
C3 (flange)	308.1	207633	494.8
C3 (web)	335.5	209934	507.7
C4 (flange)	311.1	205734	487.7
C4 (web)	329.8	205900	498.4

Note : C1 & C2 specimens were cut from the same batch of I-section

Figure 5.30 Stress-Strain Relationship of Flange Plate

Figure 5.31 Stress-Strain Relationship of Web Plate

Reinforcement

The reinforcement consisted of standard deformed bar with 10 and 8 mm nominal diameter for primary and secondary reinforcement, respectively. Tensile tests were conducted on three samples of primary reinforcement. The average yield stress and ultimate strength were 535.6 and 604.8 MPa, respectively. The average elastic modulus was determined as 203.6 GPa. The stress-strain relationship shows a clearly defined yield point as illustrated in Figure 5.32.

Figure 5.32 Stress-Strain Relationship of Reinforcement

Concrete

The concrete used was Grade 35 normal weight concrete. Both direct compressive test and indirect tensile splitting test were conducted at 28 and 90 days of curing. The composite beam specimens were tested within this period of curing, hence their concrete

properties at the time of testing should be between the lower bound value at 28 days of curing and the upper bound value at 90 days of curing. The concrete cubes and cylinders were covered in damp hessian and polythene sheeting and stored at the same place as the composite beam specimens. The direct compressive test was conducted on standard cylinders of 150 mm in diameter and 300 mm in length, and standard 150 mm cubes. Furthermore, indirect tensile splitting test was conducted on standard cylinders. Each type of test at certain days of curing was repeated three times to obtain a representative average value. Table 5.10 presents the mean compressive and tensile strength tested on 28 and 90 days of curing, together with the respective average density. The tensile strength of concrete was slightly below 10% of the cylinder compressive strength.

Table 5.10 Concrete Test Results

Age at testing	Compressive Strength (MPa)		Tensile Strength (MPa)	Density kg/m ³
	cube	cylinder		
28 days	41.7	33.8	2.8	2310
90 days	42.5	34.5	3.1	2300

Shear Stud Connector

To obtain the load-slip characteristic of the shear stud connectors, two push-out tests had been conducted. The configuration of the specimen follows EC4:1.1 (CEN, 2004) provisions as shown in Figure 5.33 and Appendix C1, together with a picture of test set-up. The concrete and reinforcement used in these push-out specimens also came from the same batch as the ones used for composite beam specimens. The UC section used in this specimen was initially cut into two halves at mid-web so that concrete slabs can be cast horizontally. At 28 days of curing, these two halves of UC section with their respective slabs were then welded back together. Figure 5.34 presents the load-slip relationship derived from two push-out tests, together with its “best-fit” relationship. The average maximum shear strength was 115 kN per connector. Based on EC4:1.1 provisions, the nominal shear strength of this connector is calculated as 99.7 kN, which is quite accurate and provides a lower bound to actual strength.

Figure 5.33 Configuration of Push-Out Test Specimen and Test Set-Up

Figure 5.34 Load-Slip Relationship of Shear-Stud Connector

5.5 Results and Discussions on Composite Steel Beam Tests

In this section, experimental results of four composite steel beams are presented. The appraisal of the results includes temperature distribution of composite beams, observed failure modes and structural performance in terms of rotational capacity. Some general

observations which are common to all four beams are presented before key findings are highlighted and discussed.

5.5.1 Temperature Developments and Distribution

The furnace temperature was set between 650 and 800°C based on 7 to 10°C/min heating rate. This was to allow the bottom flange temperature to rise to between 500 and 650°C before the loading stage commenced. The evolvment of temperature at a few critical sections, together with the plot of critical temperature distribution across the section at the time of yielding, is shown in Figure 5.35 to 5.38. The evolvment of gas temperature for C1 test, as shown in Figure 5.35, was slightly different from the other three tests. It was because the furnace temperature was increased from the initial preset value of 620°C to 720°C and finally to 800°C in order to achieve the bottom flange temperature of above 600°C. The loading stage for all four specimens was completed within 30 to 45 minutes. It is desirable to conduct the loading stage as quickly as possible so that less temperature variation occurs during this stage. This will make the determination of critical temperature easier for the purpose of subsequent modelling and analyses. It can be observed that the variation of temperature between the start and the end of loading stage for various components of tested beams was within 10% to 15%.

There were always temperature variations across the section, as shown in Figure 5.35 to 5.38, which illustrate the critical temperature distribution when the specimens started to yield. The temperature profiles at the time of yielding are detailed in Table 5.11 with the bottom flange temperature as the reference temperature. As shown in Table 5.11, the top flange temperature varied from 0.83 to 0.90 of the bottom flange temperature, with an average value of 0.86. The lower temperature of the top flange is due to the heat sink and shielding effect of concrete slab. The web temperature varied between the top and bottom flange temperatures, with a higher temperature near to the bottom flange. As shown in the time-temperature curve, the temperature evolvment of lower half of the

web was very close to the bottom flange temperature, with the maximum difference at 2% (Table 5.11). The temperature variation of upper half of the web up to the top flange seemed to be parabolic. The greatest concrete temperature was near to the top flange ($h_c = 0$ mm); the value was generally below 400°C even though the bottom flange temperature had already reached 620°C. The temperature profile of concrete seemed to be parabolic as well. The maximum temperature of the shear stud connector for all beams was 340°C. It had an average temperature of 0.44 of bottom flange temperature. This value was in between the recorded concrete temperature closest to the top flange $h_c = 0$ mm and $h_c = 50$ mm. This was reasonable since the measurement was taken at 25 mm above the base of shear studs. The temperature of reinforcement, which was located at 100 mm above the top flange, was below 115°C for all beams. It can also be observed that the temperature evolution of reinforcement located at 100 mm above the top flange, and the concrete ($h_c = 100$ mm) was very close. The reinforcement temperatures were between 0.18 and 0.22 of bottom flange temperature. It should be highlighted that the relative temperatures of C1 and C4 specimens at the time of yielding were slightly higher compared to C2 and C3 specimens. This is because the loading stage for both C1 and C4 test was started after 160 minutes of heating compared to C2 and C3 test in which the loading stage was started after 125 minutes.

Table 5.11 Relative Temperature Profiles of Tested Composite Beams

Thermocouple Location	C1 Test	C2 Test	C3 Test	C4 Test	Average
bottom flange	1.00	1.00	1.00	1.00	1.00
web (50 mm above bottom flange)	1.02	1.01	1.01	1.00	1.01
web (mid-height)	1.02	1.00	1.00	0.99	1.00
web (50 mm below top flange)	0.97	0.96	0.93	0.94	0.95
top flange	0.90	0.87	0.83	0.85	0.86
concrete (directly above top flange)	0.65	0.39	0.54	0.61	0.55
concrete (50 mm above top flange)	0.36	0.24	0.33	0.38	0.33
concrete (100 mm above top flange)	0.26	0.17	0.22	0.22	0.22
reinforcement	0.18	0.17	0.20	0.22	0.19
shear stud	0.51	0.42	0.37	0.48	0.44

Figure 5.35 Temperature Developments and Distribution of C1

Figure 5.36 Temperature Developments and Distribution of C2

Figure 5.37 Temperature Developments and Distribution of C3

Figure 5.38 Temperature Developments and Distribution of C4

The temperature profile across the section of the test specimens (Figure 5.35 to 5.38) is similar to those observed during Cardington fire test. Figure 5.39 shows the temperature profile across the steel section of PB2 and that across the concrete slab (CB3) during Cardington fire test 2 (plane frame test) after 90 minutes of heating (British Steel, 1998). In this plane frame test (test 2), a narrow strip across the entire width (21 m) supporting the fourth floor was heated by a gas-fired furnace built underneath the fourth floor. It can be seen that across the depth of the concrete slab (Fig. 5.39(a)), the temperature distribution seems to be parabolic, similar to that of the test specimens. The temperature distribution across the steel section (Fig. 5.39(b)) is also similar to that of the test specimens. The top flange temperature is approximately 0.86 of the bottom flange temperature, similar to the average value obtained from C1 to C4 specimen. Two other similarities are that, the temperature of the lower half of the web is also very close to the bottom flange temperature and the temperature variation of the upper half of the web is parabolic as well. Thus, it can be concluded that the temperature distribution of the test specimens is realistic.

Figure 5.39 Temperature Distributions of CB3 and PB2 during Cardington Fire Test

5.5.2 General Observations

A summary of test results is presented in Table 5.12. The plastic moment capacity $M_{p,T}$

has been calculated based on the cross-section dimensions, material test results, critical temperature distribution at the time of yielding, and EC3:1.2 (CEN, 2005) together with EC4:1.2 (CEN, 2005) material reduction factors at elevated temperatures. The rotation was taken as the average of two end rotations. Two important rotations, viz. the rotation at which the applied moment reached plastic moment capacity during loading (θ_p) and the rotation at which the moment started to fall below plastic moment capacity during unloading (θ_u), are shown in Table 5.12. Both rotations are relative to the respective elastic rotation θ_e , where θ_e is defined as $M_{p,T} L_i/[2E_T I]$. It should be noted that the stiffness $E_T I$ is calculated based on the cracked stiffness of concrete slab. It can be seen that all specimens reached their respective plastic moment capacity at a rotation of 1.78 to 2.07 of their respective elastic rotation θ_e .

Table 5.12 Summary of Second Series of Test Results

Test No.	$M_{p,T}$ (kNm)	M_m (kNm)	$M_m/M_{p,T}$	θ_p/θ_e	θ_u/θ_e	r_a
C1	133.89	146.05	1.091	1.783	5.544	2.11
C2	155.33	183.79	1.183	2.067	5.879	1.84
C3	177.62	200.70	1.130	1.860	5.306	1.85
C4	105.19	116.01	1.103	1.910	3.829	1.00

The maximum plastic moment for all specimens tested at elevated temperatures is around 110% of its theoretical value, which is due to EC3:1.2 definition of yield stress set at 2% strain, and the disappearance of strain hardening at elevated temperatures. One exceptional case is C2 specimen, which was able to reach 118% of its plastic moment capacity. The reason for this is that the critical (greatest) temperature occurred at certain distance from mid-span region (approximately 150 mm from mid-span) unlike the other three tested beams; therefore higher mid-span moment could be achieved. This is due to the furnace insulation panel, which was placed too close to the beam flange during C2 test, hence providing an additional thermal shielding to the mid-span area. Out of the four beams, C1 specimen has the greatest rotational capacity of 2.11, while C4 specimen has the lowest rotational capacity of only 1.00. The moment-rotation curves are plotted

in non-dimensional form, as shown in Figure 5.40 to 5.43, with the horizontal axis indicating the rotation θ over elastic rotation θ_e . All specimens started to deviate from linearity before they yielded due to highly non-linear stress-strain relationship at elevated temperature. After attaining plastic moment capacity, the beam deflection and rotation started to increase rapidly. Finally, a gradual load reduction occurred without sudden failure after the attainment of maximum moment.

Figure 5.40 Moment-Rotation Response of C1

Figure 5.41 Moment-Rotation Response of C2

Figure 5.42 Moment-Rotation Response of C3

Figure 5.43 Moment-Rotation Response of C4

For all specimens, no local or global buckling had been observed before the plastic moment capacity was reached. The failure mode observed after the test was mainly local buckling near mid-span, as a result of spread of plasticity near mid-span after the beams reached plastic moment capacity, as shown in Figure 5.44. Local buckling of the bottom flange occurred in an anti-symmetric mode, in which one side of the flange curled up while the other side twisted downwards. Local buckling of the web occurred at almost the full depth of the web since the neutral axis was located within the top

flange, that is, almost the entire beam section was under compression. Thus, there must be significant strain in the bottom flange that caused local buckling. This local buckling failure occurred on only one half of the beam with the other half exhibiting no local buckles. This may be due to the slight differences in temperature distribution between the two halves of the beam and may also be attributed to the presence of initial geometric imperfection (Dharma & Tan, 2006e).

Figure 5.44 Local Buckling Failures of Second Series of Specimens

Concrete cracking was limited to the mid-span region as shown in Figure 5.45 for C3 specimen. In addition, parallel cracks reached the top flange, showing that the whole

concrete slab was subjected to tensile force. No spalling of concrete was observed during the test. The rotation at both ends was observed to be almost symmetrical as shown in Figure 5.46, which gives the load-rotational response for C4 test. Once again, this shows that the test set-up was adequate. A slight discrepancy of initial stiffness was possibly caused by differences in temperature variation between the east and the west side of mid-span.

Figure 5.45 Mid-Span Concrete Cracking

Figure 5.46 Load-Rotation Response of C4

5.5.3 Key Findings and Discussions

The main component of composite beam, which contributed most to the ultimate moment capacity, is the primary reinforcement. The reason is that the temperature of reinforcement was much lower (around 100°C) compared to structural steel components, which had already reached 600°C. Around this temperature, reinforcement still maintains its ambient temperature strength since strength reduction only occurs above 300°C according to EC4:1.2. The low temperature of reinforcement is clearly due to low heat conductivity of concrete. Thus, concrete in hogging moment region acts as an insulator for reinforcement since the former does not contribute to ultimate moment capacity (cracking occurred at very early stage of loading).

A simple analysis to determine the location of neutral axis reveals that the plastic neutral axis is always located within the top flange, or near to the web-top flange junction. The shift of neutral axis to within the top flange is not only due to lower temperature of reinforcement, but also because of the non-uniform temperature distribution within the beam section. As a consequence, the full depth of the web is in compression. This was confirmed with the observed failure mode that shows buckling of web across the full depth (Fig. 5.43). Another consequence is that the top flange does not contribute to the moment capacity since its distance from the neutral axis is practically zero (Dharma & Tan, 2006d).

It can be calculated that the degree of connection was not reduced even when the beam was heated, since the temperature of shear stud was still below 300°C. Previous study on the capacity of shear stud by Zhao & Kruppa (1995) concluded that the maximum shear capacity does not reduce when its temperature is below 300°C. Only a slight reduction in the stiffness of load-slip relationship occurs below 300°C. At 400°C, the shear capacity starts to reduce to 80% of its ambient temperature capacity. Thus, it is unlikely that the degree of shear connection at the hogging moment region will reduce

during fire because of the reduction of shear stud capacity, except may be for a very long fire duration (more than 3 hours).

The apparent increase in ultimate moment capacity for C2 specimen shows that the maximum hogging moment at support can be increased if the critical (highest) temperature distribution occurs at certain distance from support position. Thus, it may be beneficial to provide fire protection material to limited area near the supports so that comparatively lower temperature can be achieved, hence ensuring that premature failure near the support region can be avoided or delayed.

Based on temperature measurements of four tested beams, the temperature distribution across the section of a composite beam is proposed as shown in Figure 5.47. This assumed temperature distribution can be used for temperature input in analytical calculations for determining the plastic moment capacity without conducting heat transfer analysis. However, it should be noted that this temperature distribution was obtained based on bottom flange temperature of 500 to 650°C at 7 to 10°C/min rate of heating. For the bottom half of web, its temperature can be assumed to be constant and equal to bottom flange temperature T_{bf} . For the upper half of web, its temperature may be assumed to vary linearly from T_{bf} to $0.85T_{bf}$ at the web-top flange junction. The temperature of shear stud, concrete and reinforcement can be assumed to be less than 300°C, 400°C and 200°C. Thus, these three components may be assumed to possess their strength and stiffness at ambient temperature.

The failure modes observed in these tests are very similar to the ones observed in Cardington fire test, as shown in Figure 5.48, in which local buckling of lower compression flange and web of most fire-exposed beams was observed (Wang, 2002). It should be mentioned that certain beams in Cardington fire test also exhibited local buckling failure at certain distance from the support region similar to C2 specimen

because of the lower temperature distribution near the support. Additional photos of tested beams can be found in Appendix C3.

Figure 5.47 Proposed Temperature Distributions for Analysis

Figure 5.48 Comparisons of Cardington and Current Test Failure Modes

The quantitative effects of flange and web slenderness, effective length, degree of shear connection and reinforcement ratio cannot be quantified from the test results

since only four tests had been conducted and the temperature distribution for each test was different. The complexity of fire test and limited resources dictate that parametric studies should be conducted numerically. The numerical modelling and validation from test results will be presented in Chapter 6.

A comparison between steel and composite beam test results is summarized in Table 5.13 for S1-2 and C1 test. Only these two beams can be compared qualitatively since the other beams were either constructed from different structural steel sections or subjected to different temperatures. It should be mentioned that there are slight differences of bottom flange temperature and effective length for S1-2 and C1 test (Table 5.13); however, these effects can be considered minimal. The first difference was that C1 reached its plastic moment capacity at a much lower rotation ($\theta_p/\theta_e=1.78$) compared to S1-2 ($\theta_p/\theta_e=3.39$), as shown in the moment-rotation response in Figure 5.49. This was because of additional stiffness provided by the presence of slab reinforcement and the fact that the reinforcement temperature was much lower relative to structural steel temperature. The second difference was that the ultimate rotation θ_u for C1 was also much lower ($\theta_u/\theta_e=5.54$) compared to S1-2, which had yet to reach its ultimate rotation even though $\theta_u/\theta_e=9.52$ when the test was stopped (it was estimated that $\theta_u/\theta_e=13.17$). The apparent loss of ductility in the composite beam is due to the increased depth of web in compression caused by the shifting of neutral axis towards the top flange.

The moment-rotation response of C1 was less non-linear compared to S1-2 (Fig. 5.49). It can be seen that the moment-rotation curve of C1 was linear up to $M/M_{p,T}=0.9$ compared to S1-2 which started to deviate from linearity at $M/M_{p,T}=0.6$. This may be attributed again to the lower temperature of slab reinforcement. Even though the maximum moment capacity was increased when reinforcement was added (C1 specimen), there was no obvious increase in maximum moment relative to the

respective plastic moment capacity ($M_m/M_{p,T}$). The value of rotational capacity depends on both the ultimate and plastic rotation, and seems to be slightly lower in C1 compared to S1-2. That the rotational capacity r_a is only slightly reduced in C1, even though the ultimate rotation θ_u was much lower compared to S1-2, was attributed to the beneficial factor of lower plastic rotation in C1. Further investigation was needed to determine whether, in other cases of composite beams, the benefit of lower plastic rotation can compensate for the detriment of lower ultimate rotation θ_u .

Table 5.13 Comparisons of S1-2 and C1 Test Results

Test No.	$T_{bot. flange}$ (°C)	L_E (mm)	$M_{p,T}$ (kNm)	M_m (kNm)	$M_m/M_{p,T}$	θ_p/θ_e	θ_u/θ_e	r_a
S1-2	615	650	102.85	109.62	1.066	3.389	13.169*	2.81*
C1	623	563	133.89	146.05	1.091	1.783	5.544	2.11

* estimated from numerical result

Figure 5.49 Moment-Rotation Comparisons of S1-2 and C1

CHAPTER SIX

NUMERICAL ANALYSIS ON LOCAL BUCKLING FAILURE AND DUCTILITY

6.1 Introduction

This chapter describes the use of finite element models to study the inelastic behaviour of steel and composite beams, in terms of its rotational capacity, at elevated temperatures. Two main objectives of this study are to investigate the feasibility of applying the finite element method to study moment-rotational relationship at elevated temperature and to investigate the main parameters affecting rotational capacity at elevated temperature through parametric study. The finite element (FE) model was developed using MSC. MARC Mentat (MSC. Marc, 2005). Subsequently, it was validated against test results at both ambient and elevated temperature. It is demonstrated that the finite element analysis gives reasonable accuracy compared to test results, providing an efficient, economical and yet accurate tool to study the rotational capacity of beams in fire. An extensive parametric study was then undertaken using this validated model. A few parameters that were varied included temperature, flange & web slenderness, effective length, steel grade, initial longitudinal imperfection, degree of shear connection and reinforcement area. Finally, the relation between isothermal and transient response is briefly discussed.

6.2 Overview and Validation of Steel Beam's FE Model

In this section, the modelling technique, including the type of elements used, the support conditions, material models and iterative technique, is described. The finite element model is then validated with published test results (Lukey & Adams, 1969) at ambient temperature and test results reported in Chapter 5, including the moment-rotation response, the load-deflection response and the failure modes. It is shown that the FE models agree well with test results with considerable accuracy.

6.2.1 Overview of Numerical Modelling

A commercial finite element software MSC.MARC Mentat is used in this study. A four node rectangular thick shell element, which includes transverse shear effects, is employed to model the I-beam. This element has three global displacements and three global rotations as degrees of freedom. The membrane strains and curvatures are obtained from the displacement and rotation field, respectively; the transverse shear strains are calculated at the median plane and interpolated to the integration points. Integration through the shell thickness is performed using Simpson's rule. The default number of integration points is 11. Due to its simpler formulation compared to standard higher order shell elements, it is more efficient and, therefore very attractive in non-linear analysis. Both geometrical and material nonlinearities are considered in the analysis. Dense meshes of shell elements are used to properly model localised instabilities that occur in combination with global instabilities. It should be noted that a preliminary study on the meshing size has already been conducted and it suggested that a mesh size of approximately $40\text{mm} \times 40\text{mm}$ is sufficient. Therefore, in this finite element model, the mesh size is approximately $40\text{mm} \times 40\text{mm}$ to $45\text{mm} \times 45\text{mm}$. The arc-length iterative strategy has been used in the analyses to properly trace the nonlinear equilibrium path of inelastic I-beams. A uniaxial representation of the constitutive law is given in terms of true stress and logarithmic strain. Von Mises yield criterion is then used in the analysis and the corresponding metal plasticity model is characterised as an associated flow plasticity model.

The beam is simply supported at both ends so that lateral deflection and twist rotation at the supports are prevented. However, the flange ends are free to rotate in horizontal planes so that the beam cross-section is free to warp at the ends. This boundary condition is as shown in Figure 6.1, in which the x -axis is along the beam direction, y -axis is perpendicular to the web, and z -axis is parallel to the web, with u and θ denoting displacement and rotation, respectively. Lateral restraints along y -axis are provided at mid-span, end supports and certain distance from mid-span. Concentrated load is applied directly on top of the mid-span stiffeners. A typical finite element mesh is shown in Figure 6.2. Measured initial geometric imperfection has also been included in the numerical models.

Figure 6.1 Boundary Conditions at Both Supports

Figure 6.2 Typical Finite Element Mesh of a Steel Beam

6.2.2 Validation of Finite Element Model at Ambient Temperature

A total of 7 tests, one conducted by the author and six reported by Lukey & Adams (1969), is used to validate the FE model. Test results by Lukey & Adams (1969) were chosen for validation since they are among the best documented and most cited experimental works on the study of local buckling and rotational capacity of I-beams at ambient temperature. The initial geometric imperfection is introduced for all beams. Measured initial imperfection for the authors' single test is incorporated in the model, whereas a maximum initial out-of-plane bow of $L_t/1000$ in the form of full sine curve is assumed for six tests by Lukey & Adams, due to a lack of such information in their paper. Good agreement is obtained between the FE predictions and test results as shown in Table 6.1. The ratio of the maximum moment to the plastic moment capacity (M_m/M_p) and the rotational capacity observed in the numerical simulations are relatively close to

the test results. As mentioned in Section 2.3.2, the available rotational capacity r_a is defined as the ratio of inelastic rotation θ_a , over which the moment exceeds its design plastic moment resistance M_p , and its plastic rotation θ_p .

Table 6.1 Summary of Validation Results of Steel Beams at Ambient Temperature

Source	Test No.	d (mm)	t_w (mm)	b (mm)	t_f (mm)	L_i (mm)	M_p (kNm)	M_m/M_p		r_a	
								Test	FEA	Test	FEA
Author	S3-1	276.00	8.00	163.00	10.00	1725.00	146.67	1.33	1.26	7.57	8.01
Lukey & Adams (1969)	B-2	189.59	4.45	73.91	5.28	518.16	44.14	1.15	1.14	10.40	7.70
	B-3	189.59	4.45	86.11	5.28	627.38	48.82	1.14	1.13	6.70	6.86
	B-5	189.59	4.45	96.77	5.28	723.90	52.92	1.04	1.12	3.20	4.53
	C-1	239.93	4.60	101.85	5.26	685.80	72.29	1.11	1.10	4.20	4.46
	C-4	239.93	4.60	93.47	5.26	647.70	68.26	1.12	1.08	4.20	3.98
	C-5	239.93	4.60	89.92	5.26	619.76	66.55	1.14	1.12	6.50	5.75

Relatively good comparison is obtained in terms of the moment-rotation response as illustrated in Figures 6.3 and 6.4. The agreement between the test data and the numerical prediction is better in the stable (before the occurrence of local buckling) than in the unstable range (after the occurrence of local buckling). The FE predictions of failure modes and buckling shapes are also very similar to the ones observed after the tests, as illustrated in Figure 6.5 for S3-1. The discrepancies between test and FE results, such as the rotational capacity for specimen B-2 (Table 6.1), can be due to the differences in actual material properties of the experiments and finite element models, the assumed initial imperfection in FEA and slight differences in the simulation of support and lateral restraint. The amount and form of initial imperfections assumed in FEA may not represent actual imperfections. The slight differences in the support conditions and stiffness can also affect the moment-rotation response.

The ratio of the maximum moment to the plastic moment capacity (M_m/M_p) of S3-1 is slightly higher compared to the other six tests due to the difference in the material stress-strain characteristic. The steel material used in S3-1 has ultimate strength much higher compared to its yield strength ($f_{ult}/f_y = 1.76$), while those of the other six beams have f_{ult}/f_y between 1.3 and 1.4 (Lukey & Adams, 1969). It should be noted that by using an assumed maximum initial out-of-plane bow of $L_i/1000$ in the form of full sine curve for the 6 beams tested by Lukey & Adams, reasonable accuracy has been obtained. It is also shown in Section 6.3.1 that this form of imperfection produces conservative

estimation of rotational capacity. Thus, this form of initial imperfection will be used in the parametric studies.

Figure 6.3 Moment-Rotation Comparisons of FEA and S3-1 Test Results

Figure 6.4 Moment-Rotation Comparison of FEA and Tests by Lukey & Adams (1969)

Figure 6.5 Failure Modes Comparison of FEA and Test

6.2.3 Validation of Finite Element Model at Elevated Temperature

The FE model is also validated with 8 test results at elevated temperature conducted by the author. The material stress-strain curve is taken from the EC3:1.2 (CEN, 2005) formulations for steel at elevated temperatures. The initial geometric imperfection used in this FE model is taken from the measured profiles of test specimens. There are two load steps: first, the temperature is increased to the desired failure temperature, and subsequently the load is applied. It should be noted that there is no thermally induced compressive stress in the FE model. The rotational capacities and the ratios of maximum moment to plastic moment are compared with FE predictions in Table 6.2. Unlike the predictions at ambient temperature, the predicted maximum moments at elevated temperature are generally less than 110% of plastic moment at that temperature. This is observed in all the specimens tested at elevated temperature (Table 6.2). This is due to the fact that effective yield strength at elevated temperature is defined at 2% strain according to EC3:1.2 and the characteristic stress-strain relationship of steel above 400°C has very little strain hardening above 2% strain. The rotational capacity is much lower compared to ambient temperature because the plastic rotation θ_p at elevated temperature is much greater due to highly non-linear stress-strain relationship at that temperature.

Table 6.2 Summary of Validation Results for Steel Beams at Elevated Temperature

Test No.	Size	T (°C)	c/t_f	d/t_w	$\lambda_{LT}^{\#}$	$M_{p,T}$ (kNm)	$M_m / M_{p,T}$		r_a	
							Test	FEA	Test	FEA
S1-1	305x165UB54	415	6.09	34.51	13.15	236.11	1.121	1.109	N/A**	3.73
S1-2	305x165UB54	615	6.09	34.51	13.15	102.85	1.066	1.085	N/A**	2.81
S2-1	305x165UB54	415	6.09	34.51	27.30	242.86	1.081	1.071	1.60	1.55
S2-2	305x165UB54	615	6.09	34.51	27.30	106.10	1.056	1.050	0.93	1.08
S3-2	Welded*	415	8.15	34.51	13.45	142.67	1.095	1.107	2.28	2.19
S3-3	Welded*	615	8.15	34.51	13.45	62.33	1.099	1.087	1.11	1.18
S4-1	406x178UB54	415	8.15	47.42	13.36	410.72	1.099	1.077	0.68	0.92
S4-2	406x178UB54	615	8.15	47.42	13.36	179.43	1.037	1.057	0.30	0.61

* $b = 163\text{mm}$; $d = 276\text{mm}$; $t_f = 10\text{mm}$; $t_w = 8\text{mm}$

** Unable to capture the unloading part

Comparisons of FE predictions and test results are presented in Figure 6.6 and 6.7 for moment-rotation and load-deflection (mid-span) response, respectively. It can be seen that close agreement exists between the FE predictions and test results. There is slight difference in the non-linear pre-buckling region for two beams: S1-1 and S3-2, which were tested at 415°C. This discrepancy in response may be attributed to the differences between the stress-strain relationships proposed by EC3:1.2 and the actual stress-strain relationship. However, it should be noted that the difference lies only in certain part of the stress-strain relationship relating to the proportionality limit or commencement of non-linearity. The initial stiffness and maximum capacity still show very good agreement and the predictions of maximum moment are within 2%. This discrepancy does not seem to occur on the beams tested at 615°C, which shows that the stress-strain relationship proposed by EC3:1.2 is indeed very close to the actual stress-strain relationship of the tested beams. Besides the stress-strain relationship, the excellent agreement between test results and FE predictions is also attributed to the accuracy of

initial geometric imperfection, which has been incorporated into the FE models. A numerical study on the effect of initial imperfection presented in subsequent section shows that the initial geometric imperfection influences the initiation of buckling, hence affecting the inelastic rotational capacity of simulated beam.

Besides the loading-unloading response, the observed failure modes between tested specimens and numerical models are also very close, as illustrated in Figure 6.8. The model for S3-2 shows local buckling failure (flange and web buckling), similar to local buckling failure observed after the tests (Fig. 6.8(a)). In addition, lateral torsional buckling failure, which occurred in S2-1 is also predicted in the FE model as shown in Figure 6.8(b)).

(a)

(b)

(c)

(d)

Figure 6.6 Moment-Rotation Comparisons of FEA and Tests at Elevated Temperature

Figure 6.7 Load-Deflection Comparisons of FEA and Tests at Elevated Temperature

Figure 6.8 Failure Modes Comparison of FEA and Tests at Elevated Temperature

6.3 Parametric Study on Rotational Capacity of Steel Beams

In this section, the numerical model described in the previous section is used for parametric studies where four reference beams: RB1 to RB4 are used. The simulation configuration, geometric properties and restraint positions of these beams are shown in Table 6.3. These four beams have flange and web slenderness ratio of plastic or compact sections. The lateral restraints are provided such that the LTB slenderness ratio is around 13. Steel grade S275 (yield strength and elastic modulus at ambient temperature are 275 MPa and 210 GPa, respectively) is used for these reference beams, which are subjected to uniform nodal temperature of 600°C. A few variables, which determine the rotational capacity of steel I-beams, are varied, namely, temperature, flange & web slenderness

ratio, LTB slenderness ratio and steel grade. The influence of increasing temperature will first be studied using RB2 and RB4 beams. Secondly, RB1 and RB2 will be used to study the effect of varying flange slenderness ratio. Thirdly, the web slenderness of RB3 and RB4 beams will be varied to study this effect. Finally, the influence of effective length and steel grade on the rotational capacity will be investigated using RB1 & RB3. In addition, the influences of the assumed initial geometric imperfection on the moment-rotation response are also briefly discussed. The results of these parametric studies and some important observations are discussed in the following sub-sections.

Table 6.3 Summary of Reference Beams for Parametric Studies

Reference Beam	b mm	t_f mm	d mm	t_w mm	c/t_f	d/t_w	L_i mm	L_E mm	λ_{LT}
RB1	80.00	5.80	194.20	5.11	6.90	38.00	800	400	13.15
RB2	160.00	10.00	340.00	7.56	8.00	45.00	1500	750	13.15
RB3	80.00	5.33	194.67	3.50	7.50	55.62	800	400	13.36
RB4	160.00	8.00	342.00	11.00	10.00	31.09	1500	750	13.36

$T = 600^\circ\text{C}$
Steel Grade: S 275

6.3.1 Sensitivity of Initial Geometric Imperfection

In order to study the sensitivity of initial geometric imperfection on the moment-rotation response, several forms of longitudinal out-of-plane imperfection have been applied to RB1, including a perfect beam without any initial imperfection, as summarized in Table 6.4. It can be observed from Figure 6.9 that, as expected, the perfect beam has the greatest rotational capacity. Besides, beams with imperfection in the form of a full sine curve (imperfection 1 & 2) has less rotational capacity compared to those with a half sine curve (imperfection 3 & 4). Lastly, with the same assumed shape of imperfection, a

greater value of imperfection ($L_i/500$) produces slightly less rotational capacity. The assumed initial imperfection is observed to affect only the post-buckling response with no difference observed within the pre-buckling response up to the maximum moment (Figure 6.9). This shows that the initial imperfection affects the initiation of local buckling, which largely determines the rotational capacity. The pre-buckling behaviour is not affected by the initial imperfection since it does not change the moment-curvature relationship. This observation is in line with the results of a sensitivity study on the imperfection magnitude conducted by Greschik, et al. (1993). They also observed that increasing the imperfection magnitudes slightly lowers the maximum load. The imperfection-sensitivity analysis for a structure in the plastic range has also been studied by Hutchinson (1973).

Table 6.4 Various Forms of Initial Imperfection

Type	Shape	Maximum Value
Perfect	N/A	N/A
Imperfection 1	full sine curve	$L_i/1000$
Imperfection 2	full sine curve	$L_i/500$
Imperfection 3	half sine curve	$L_i/1000$
Imperfection 4	half sine curve	$L_i/500$

Note: L_i = half span length

Figure 6.9 Sensitivity of Initial Imperfection on Moment-Rotational Relationship

Within these four assumed imperfection, it can be seen that an assumed maximum initial

out-of-plane bow of $L_i/1000$ in the form of full sine curve (imperfection 1) produces a conservative estimation of rotational capacity (second lowest value of rotational capacity). This is in line with the usual principle that the principal imperfection (with a direct effect on a buckling load) is the component in the associated mode-form. Harding (1985) who studied the interaction between imperfection wavelength and buckling mode has also highlighted that more weakening effects are observed for geometric imperfection forms which are similar to the buckling modes. Initial imperfection measurements conducted on the tested UB sections (Chapter 5) also shows that the values of maximum imperfection are generally around $L_i/1000$. Furthermore, it has been shown previously that by using an assumed maximum initial out-of-plane bow of $L_i/1000$ in the form of full sine curve, 6 beams tested by Lukey & Adams (1969) have been successfully simulated with sufficient accuracy. Thus, this form of initial imperfection will be used in the subsequent parametric studies.

6.3.2 Influence of Temperature

In order to study the effects of different temperatures on the rotational capacity, the nodal temperature of RB2 and RB4 are varied from ambient temperature up to 800°C. Figure 6.10 and 6.11 summarise the moment-rotation response at various isothermal conditions. The moment-rotation response has been plotted in a non-dimensional form, with the vertical axis showing the ratio of applied moment to plastic moment capacity at respective temperature. The horizontal axis indicates the ratio of rotation to elastic rotation based on elastic theory. Similar responses are observed from both RB2 (Figure 6.10) and RB4 beams (Figure 6.11). At ambient temperature, both beams have large inelastic rotation with rotational capacity r_a of 19 and 17, respectively. Besides, both beams possess plastic rotation θ_p which is very close to elastic rotation θ_e and quite high moment capacity of around $1.3 M_p$. After the temperature is increased to 300°C, the rotational capacity reduces significantly. This is due to a reduction in modulus of elasticity ($k_E = 0.8$) and the commencement of non-linearity of stress-strain relationship at around $0.613f_y$ (the yield strength is still the same as that at ambient temperature) that cause the beams to attain its plastic moment capacity at a higher strain and the non-linearity in the pre-buckling moment-rotation response. Besides, the value of M_m/M_p has also decreased due to a reduction in the strain hardening modulus. Similarity

in the value of M_m/M_p (ranging between 1.06 and 1.09) can be observed for cases above 400°C because of the disappearance of strain-hardening in the stress-strain relationship of steel above this temperature. There is also similarity in the shape of the moment-rotation response for other cases above 400°C, except for the length of yield plateau, which determines the ductility of steel member.

Figure 6.10 Effect of Temperature on Moment-Rotational Relationship (RB2)

Figure 6.11 Effect of Temperature on Moment-Rotational Relationship (RB4)

Another interesting observation is that the rotational capacity does always reduce as temperature increases. The general trend of the relationship between rotational capacity

and temperature is illustrated in Figure 6.12. Initially the rotational capacity reduces when the temperature is increased from 300°C to 400°C. Subsequent increase of temperature to 500°C causes the rotational capacity to increase very slightly. The rotational capacity then further reduces when temperature is increased up to 700°C. Finally, the rotational capacity increases significantly when temperature reaches 800°C. The temperature corresponding to the lowest rotational capacity is identified to be somewhere around 700°C. The relationship between rotational capacity and temperature can be attributed to the different material stress-strain characteristics at elevated temperatures (Dharma & Tan, 2006c). One key factor which provides an explanation to the above trend is the ratio of elastic modulus to yield strength at elevated temperature E_T/f_{yT} . This ratio can also be expressed in terms of material reduction factors specified in EC3:1.2 (CEN, 2005):

$$E_T/f_{yT} = k_E E/k_y f_y \quad (6.1)$$

in which k_E = elastic modulus reduction factor at elevated temperature;
 k_y = yield strength reduction factor at elevated temperature;
 E = elastic modulus at ambient temperature;
 f_y = yield strength at ambient temperature.

The ratio of elastic modulus reduction factor to yield strength reduction factor at elevated temperature k_E/k_y represents the reduction in the elastic modulus relative to the reduction in the yield strength at that temperature, while the ratio of elastic modulus to yield strength at ambient temperature E/f_y is a constant for a particular steel grade. The variation of k_E/k_y with temperature is shown in Figure 6.13. It can be observed that the elastic modulus and yield strength neither degrades at the same rate at elevated temperature nor has the same ratio of degradation rate. A quick comparison between Figure 6.12 and Figure 6.13 shows a similar trend, in which a higher k_E/k_y results in a greater rotational capacity.

Figure 6.12 Variation of Rotational Capacity with Temperature

Figure 6.13 Variation of Stress-Strain Relationship Parameter k_E/k_y with Temperature

6.3.3 Influence of Flange Slenderness

The flange slenderness c/t_f of RB1 and RB2 has been varied to study its effect on the rotational capacity. Figure 6.14(a) and (b) show the effect of varying the flange slenderness between 4.0 and 18.2 for RB1, and between 4.0 and 17.4 for RB2. The general trend is that by increasing the flange slenderness ratio, the rotational capacity will be reduced. For RB1, plastic moment capacity can still be reached even when the flange slenderness has been increased to 13.3, although no inelastic rotation is observed at this high flange slenderness. Similarly, RB2 still possesses a minimum rotational capacity of 0.35 when its flange slenderness ratio is increased to 11.4. A further increase

in flange slenderness ratios for both RB1 and RB2 cause these beams to fail prematurely before they attain their plastic moment capacity. No difference is observed in the moment-rotation response in the pre-buckling region and all beams reach their plastic moment capacity when approximately $\theta/\theta_e = 3.4$.

Another important observation is that the maximum moment does not differ substantially by changing the flange slenderness ratio for RB1 and RB2. The general trend is that increasing the flange slenderness ratio causes a reduction in the maximum moment. One exception to this trend is observed when the flange slenderness of RB1 is reduced to 4.0, which causes a very slight reduction in maximum moment. The explanation to this exception is related to the appearance of inelastic failure mode of this beam. After careful examination of results obtained during graphical post-processing, a difference in the failure modes was observed between RB1- $c/t_f=6.9$ and RB1- $c/t_f=4.0$. While local buckling of flange and web occur near the mid-span for RB1- $c/t_f=6.9$ and the rest of the beams with higher c/t_f , RB1- $c/t_f=4.0$ exhibits a coupled local buckling and lateral torsional buckling (global buckling). Thus, it can be concluded that reducing flange slenderness ratio will slightly increase the maximum moment up to the point where the flange becomes too stocky. Subsequently, instead of the occurrence of local buckling, coupled local and global buckling modes occur. This appears to be the reason for a slight reduction in the maximum moment.

(a)

(b)

Figure 6.14 Effect of Flange Slenderness Ratio on Moment-Rotational Relationship

6.3.4 Influence of Web Slenderness

The effects of web slenderness ratio d/t_w on the inelastic behaviour of steel I-beams have been studied by varying this parameter on RB3 and RB4 as shown in Figure 6.15. A general trend is that increasing d/t_w results in lower rotational capacity, which is in good accord with test observations reported in Chapter 5. Besides, the maximum moment is reduced by increasing the web slenderness ratio. It should also be highlighted that plastic moment capacity is still reached, albeit with no rotational capacity, even after d/t_w for RB3 has been increased to 81.1. This shows that even after the web has buckled, a comparatively stocky flange is still stable and plays an important role in sustaining a further increase of moment and rotation.

Another important effect of web slenderness ratio is related to the failure modes. Although the failure mode is still localised flange and web buckling, there is a change in the sequence of buckling modes. When d/t_w of RB3 is 24.3, the failure modes observed are flange buckling followed by web buckling combined with a slight lateral torsional buckling at later stage. Increasing d/t_w of RB3 to 55.6, the beam fails in flange buckling followed by web buckling, but does not exhibit lateral torsional buckling. A further increase of d/t_w to 72.1 and 81.1 shows that web buckling occurs first before flange

buckling. Similar behaviour is also observed for RB4, which shows web buckling failure mode before flange buckling takes place when d/t_w is increased to 85.5.

(a)

(b)

Figure 6.15 Effect of Web Slenderness Ratio on Moment-Rotational Relationship

6.3.5 Influence of Effective Length

The influence of effective length on rotational capacity is addressed by varying the lateral restraint distance of RB1 and RB3. Figure 6.16 shows the moment-rotation response of RB1 and RB3 with different effective lengths. It can be observed that reducing the effective length increases both the maximum moment and rotational capacity. Careful examination of the results obtained during graphical post-processing

shows a change of failure modes when the effective length is reduced, which may account for an increase in rotational capacity. Figure 6.17 gives a comparison of the failure modes which occur in RB3- $L_E=300$ and RB3- $L_E=600$. It can be observed that highly coupled local and lateral torsional buckling occurs in RB3- $L_E=600$, whereas only local buckling failure mode is observed in RB3- $L_E=300$ due to shorter effective length. Thus, it can be concluded that shorter effective length will delay or eliminate the occurrence of lateral torsional buckling (commonly takes place in the post-buckling range), hence increasing the beam's rotational capacity.

An interesting anomaly to the above trend can be observed in RB3 when the effective length is reduced from 400 to 300mm. In this particular case, the moment-rotation response, maximum moment, and rotational capacity do not seem to be affected by the reduction in effective length. The explanation to this behaviour is related to the observed failure modes. A close examination of the failure modes in RB3- $L_E=400$ shows that only local buckling of flange and web occurs without any lateral torsional buckling. Therefore, providing a closer bracing (shorter effective length) than this limiting value will neither enhance the rotational capacity nor change the moment-rotation response. Thus, it should be noted that improvement to the rotational capacity can be achieved by reducing the effective length for beams in which lateral torsional buckling failure mode is observed.

(a)

(b)

Figure 6.16 Effect of Effective Length on Moment-Rotational Relationship

(a)

(b)

Figure 6.17 Comparisons of Failure Mode with Different Effective Length

6.3.6 Influence of Steel Grade

The influence of steel grade on the rotational capacity has been studied by varying the yield strength of RB1. Four steel grades are compared, namely S235, S275, S355 and

S450. It can be seen in Figure 6.18(a) that a higher steel grade results in less rotational capacity. The main reason is due to the differences in the stress-strain relationships among the different steel grades. Another reason is that by using a higher steel grade, a higher plastic moment capacity is obtained; hence plastic rotation will be attained at a higher rotation as shown in Figure 6.18(b). As a result, less inelastic rotation is available for beams using higher steel grade. Unlike Figure 6.18(a) which shows the normalised rotation θ/θ_e in the x-axis, Figure 6.18(b) shows the rotation θ in the x-axis.

(a)

(b)

Figure 6.18 Effect of Steel Grade on Moment-Rotational Relationship

A closer examination of the lateral web and flange displacements indicates that both lateral torsional buckling and flange buckling occur earlier in beams with a higher steel grade as illustrated in Figure 6.19 for RB1-S235 and RB1-S450. This is obvious as while the elastic modulus for higher steel grade is about the same, the yield strength is increased substantially, thus forcing beams to buckle earlier. This earlier loss of stability for beams with higher grade steel contributes to limited inelastic rotation.

Figure 6.19 Moment-Node Displacement Plot of RB1-S235 and RB1-S450

6.4 Overview and Validation of Composite Beam's FE Model

In this section, the modelling technique described in Section 6.3 will be extended to composite beams. The element types to model the concrete slab and reinforcement, together with the spring element to model the shear stud, are described. The finite element model is then validated with four test results, including the moment-rotation response and the failure modes. It is again shown that the FE models agree well with test results with considerable accuracy.

6.4.1 Overview of Numerical Modelling

The structural steel is modelled using the same type of element, which is a four node rectangular thick shell element, as described in Section 6.3. The concrete slab is modelled using a 20-node, iso-parametric solid element, which uses tri-quadratic interpolation functions to represent coordinates and displacements; hence the strains have a linear variation (MSC. Marc, 2005). There are 27 Gaussian integration points

within an element, which are used to form the stiffness matrices. Every node in this element has three global displacements as degrees of freedom. Concrete constitutive relation in tension was described using a tension softening stress-strain approach which means that the stress in the direction of maximum stress does not go immediately to zero; instead the material softens until there is no stress across the crack. This softening behaviour is characterized by a descending branch in the tensile stress-strain diagram. The tensile strength at elevated temperature is determined from the ambient temperature tensile strength and the reduction factor at elevated temperature based on EC2:1.2 (CEN, 2004). The compressive stress-strain relationship for concrete follows EC4:1.2 model (CEN, 2005), as described in Section 3.3.1. Only low density mesh is used to model the concrete slab since it will crack at very early stage of loading and does not contribute to the maximum moment capacity. Besides, reducing the mesh size will also reduce the processing time considerably.

The reinforcement is modelled using an iso-parametric, three-dimensional, 20-node empty solid element, which must be used in conjunction with a concrete element to represent a reinforced concrete element. By combining the rebar and concrete elements, the stress distribution of both reinforcement and concrete can be represented accurately, since separate constitutive theories are used in each element (e.g. concrete cracking and reinforcement yielding). Two layers of reinforcement are provided for each rebar element simulating the layer of primary and secondary reinforcement.

To simulate the composite action between the steel beam and concrete slab, adjacent top flange and concrete nodes are tied together using rigid links in two global displacements (along y and z -axis as shown in Figure 6.20). Tying these adjacent nodes together means that there is no relative displacement of these nodes along the y or z -axis. The relative slip in x -direction between the steel beam and concrete element is governed by the load-slip relationship of shear stud connectors, and is simulated using linear springs. The load-slip relationship of each shear stud has to be converted into spring stiffness-slip relationship by calculating the tangent stiffness of load-slip curve before it can be used as an input parameter for each spring. A typical finite element mesh is shown in Figure 6.20.

Figure 6.20 Typical Finite Element Mesh of a Composite Beam

The arc-length method has been selected to properly trace the nonlinear equilibrium path of inelastic I-beams and the evolution of cracks in concrete slabs. Both geometrical and material nonlinearities are also considered in the analysis. A uniaxial representation of the constitutive law is given in terms of true stress and logarithmic strain. Von Mises yield criterion is then used in the analysis and the corresponding metal plasticity model is characterised as an associated flow plasticity model, with isotropic hardening as the default. Three-dimensional cracking of concrete will occur when the tensile stress exceeds the specified critical cracking stress in the direction perpendicular to the maximum principal stress. After that, the second crack can form in a direction perpendicular to the first. Likewise, a third crack can form in a direction perpendicular to the first two crack planes. The boundary condition simulating simply supported conditions is the same as before (Figure 6.1). Concentrated load is applied directly on top of the mid-span stiffeners. Initial longitudinal out-of-plane imperfection has also been included in the form of full sine curve with maximum value of half span/1000 ($L_i/1000$).

6.4.2 Validation of Finite Element Model

The numerical model is validated against four test results. The material stress-strain models at elevated temperatures follow EC3:1.2 (CEN, 2005) and EC4:1.2 (CEN, 2005) formulations. There are two load steps: first, the temperature is increased to the desired failure temperature, and subsequently the load is applied. The failure temperature is approximated as the temperature when the beams reached their respective plastic moment capacity as reported in Chapter 5. This critical temperature distribution is

summarized again in Table 6.5. Only the value of the critical temperature at the centre is shown for the web and concrete. Good agreement is obtained between the FE predictions and test results as shown in Table 6.5. The ratio of maximum moment to plastic moment capacity ($M_m/M_{p,T}$) and the rotational capacity r_a from numerical simulations are relatively close to the test results. Relatively good comparison is obtained in terms of the moment-rotation response as illustrated in Figure 6.21 for all four composite beam specimens. The predicted moment-rotation curves show no horizontal plateau, instead unloading occurs once the ultimate moments are reached. The predicted maximum moments at elevated temperature generally range from 110% to 114% of plastic moment capacity. This is observed in all tests, except for C2 specimen. The ultimate moment prediction for C2 (FEA-1) is lower than the test result because the critical temperature distribution during testing was at a certain distance away from mid-span, unlike the temperature distribution of FEA. Employing the more accurate temperature distribution for C2 specimen by placing the critical temperature distribution at a distance 150 mm from mid-span, the FE predictions (noted as FEA-2) are closer to the test results as shown in Figure 6.21(b). Thus, FE predictions confirm that the reason for higher ultimate moment in C2 is due to the position of critical temperature distribution, which was not at mid-span.

Table 6.5 Summary of Composite Beam Validation Results

Test No.	Critical Temperature Distribution (°C)						$M_m / M_{p,T}$		r_a	
	flange _{bot}	web	flange _{top}	rebar	concrete	stud	Test	FEA	Test	FEA
C1	623	634	560	113	222	320	1.091	1.129	2.11	1.60
C2	601	603	520	103	144	250	1.183	1.107	1.84	1.10
C3	515	512	427	104	169	188	1.130	1.134	1.85	1.52
C4	542	538	463	121	238	260	1.103	1.115	1.00	0.87

(d)

Figure 6.21 Moment-Rotation Comparisons of Composite Beam FEA and Tests

The agreement between the test results and the numerical predictions is better in the stable (before the occurrence of local buckling) than in the unstable range (after the occurrence of local buckling). The slight discrepancies between test results and FEA simulations may be attributed to differences in actual material properties at elevated temperature and Eurocode's stress-strain relationship formulations, the assumed initial imperfection in FEA, and temperature distributions. The FE predictions of failure modes and buckling shapes are also very similar to the ones observed after the test, as illustrated in Figure 6.22 for C1 and C4 specimen. The buckling failure shown by FEA also occurs at one half of the beam, with the other half exhibiting no local buckles, similar to test behaviour. The reason may be that one half of the beam is slightly weaker due to the presence of initial imperfection. Once the weaker half buckles, the bending moment will be subsequently decreased; hence the other half will not buckle. The mid-span cracking has also been simulated well as shown in Figure 6.23 for C3 specimen. It can be seen that the highest cracking strain in the FE model is concentrated at the mid-span area and minor cracking strain extends up to a certain distance from mid-span, similar to the cracking pattern observed after testing.

Figure 6.22 Failure Modes Comparison between FEA and Tests

Figure 6.23 Comparison of Crack Pattern between FEA and Tests

6.5 Parametric Study on Rotational Capacity of Composite Beams

In this section, the numerical model of composite beams described in Section 6.4 is used for parametric studies where three test specimens (C1 to C3) are referred to. In this parametric study, only one parameter is varied at a time while maintaining the other parameters constant. Firstly, the effect of increasing temperature is studied by varying the temperature of C2 specimen. Secondly, three parameters, namely flange & web

slenderness and reinforcement area are studied using C1 specimen. Finally, C3 is used to study the effect of the number of shear studs and effective length. The results of these parametric studies and some important observations are discussed in the following sub-sections.

6.5.1 Influence of Temperature

In order to study the effects of different temperature distributions on the rotational capacity, the proposed temperature distributions presented in Chapter 5 are used, as shown in Figure 6.24. The reference temperature in the ensuing discussions is the bottom flange temperature T_{bf} . The temperatures of the other parts are relative to the bottom flange temperature. The temperature of concrete slab, reinforcement and shear stud, as shown in Figure 6.24, is taken as the average value obtained from 4 test results reported in Chapter 5.

Figure 6.24 Temperature Distributions for Parametric Studies

It can be observed from Figure 6.25 that as the temperature increases, rotational capacity reduces. At ambient temperature, the beam possesses large inelastic rotation with rotational capacity r_a of 17.6 due to a very low plastic rotation (close to elastic rotation θ_e) and a large ultimate rotation θ_u . Furthermore, the presence of strain hardening at ambient temperature helps to delay the local buckling of flange and web, hence increasing the ductility tremendously. After the temperature is increased to 400°C, the rotational capacity reduces significantly to 2.3 due to a much larger plastic rotation ($\theta_p = 3.3\theta_e$) and a much lower ultimate rotation. The significant reduction in rotational capacity is

attributed to the disappearance of strain hardening of structural steel at elevated temperatures. While there is significant reduction in the rotational capacity, the ratio of the maximum moment to the respective plastic moment capacity $M_m/M_{p,T}$ has only reduced slightly. This is because the reinforcement has only reached 76°C, and has not lost its strength and strain hardening.

Figure 6.25 Influence of Temperature on the Moment-Rotation Response

Increasing the temperature of the bottom flange further up to 700°C, there seems a similar trend in the moment-rotation response. The only difference is the amount of inelastic rotation, which is getting smaller as the temperature increases. This trend is different from steel beam, of which rotational capacity does not always reduce as temperature increases (see Section 6.3.2). The reason is that while the strength and stiffness of the compression and tension parts of bare steel beams reduce at the same rate as temperature increases, the same cannot be said for composite beams. For a composite beam subjected to hogging moment, temperatures of the tension parts (mainly reinforcement) are relatively cooler, while temperatures of the compression parts (web and bottom flange) increase as the gas temperature increases. Consequently, the neutral axis shifts towards the top flange. Therefore, the same curvature produces greater compressive strain at the bottom flange due to the increase in the distance between the bottom flange and the neutral axis. In addition, a greater portion of the web is also in compression. Thus, as the temperature increases, the local buckling of the flange and web occurs at smaller θ/θ_e , thereby producing less rotational capacity.

A close examination of the graphical post-processing shows an interesting difference in the failure modes between ambient and elevated temperature. It is observed that local buckling at ambient temperature occurs over a greater length compared to the one at elevated temperature as shown in Figure 6.26. It can be seen that there is more than one buckling wave at ambient temperature, compared to only one buckling wave at 700°C. The reason is that at ambient temperature, the beam is able to spread the yielding/plasticity over a greater distance due to the presence of strain hardening. At elevated temperature, the disappearance of strain hardening at the bottom flange causes the bottom flange to buckle once a minimum length of plasticity has been reached to enable the bottom flange to form one buckling wave. Once the bottom flange buckles, which is followed by a reduction in the applied moment, there is no further spread of plasticity; hence the formation of additional buckling wave is inhibited.

Figure 6.26 Comparisons of Failure Modes at Different Temperatures

6.5.2 Influence of Flange Slenderness

The flange slenderness c/t_f of C1 specimen has been varied to study its effect on the rotational capacity. Figure 6.27 shows the effect of varying the flange slenderness between 5.0 and 9.0. As with steel beams, the general trend is that, by increasing the flange slenderness ratio, the rotational capacity will be reduced. However, there seems a negligible reduction in the ratio of maximum moment to plastic moment capacity $M_m/M_{p,T}$ as the flange slenderness increases. Even though the rotational capacity reduces

by about one third when the flange slenderness is increased from 5.0 to 9.0, there is only very minimal reduction in the value of $M_m/M_{p,T}$.

By observing the local failure process, it is found that after the maximum moment is reached for $C1-c/t_f=5.0$, a slight buckling of the web occurs which brings about a slight reduction in the moment as shown in Figure 6.27. Afterwards, there is a plateau in the moment-rotation response since the bottom flange has not buckled. Finally, once the bottom flange buckles, the moment starts to reduce. Thus, the considerable length of plateau in the moment-rotation response of $C1-c/t_f=5.0$ is attributed to the relatively stocky bottom flange. For $C1-c/t_f=9.0$, which has the most slender flange, the web and flange buckling occur simultaneously, followed by a rapid reduction in the applied moment.

Figure 6.27 Influence of Flange Slenderness on the Moment-Rotation Response

6.5.3 Influence of Web Slenderness

The effects of web slenderness ratio d/t_w on the inelastic behaviour have been studied by varying this parameter on C1 specimen as shown in Figure 6.28. As expected, increasing the web slenderness d/t_w results in a lower rotational capacity and a lower value of $M_m/M_{p,T}$. In addition, plastic moment capacity is still reached, albeit with limited rotational capacity, even after the web slenderness has been increased to 55.0. It should

be noted that there is no difference in the moment-rotation response up to the attainment of plastic moment capacity. The observation of the sequence of buckling shows that for the stockiest web ($C1-d/t_w=30.0$), the bottom flange and web buckle almost concurrently, while for the most slender web ($C1-d/t_w=55.0$), the web buckles first followed by the bottom flange.

Figure 6.28 Influence of Web Slenderness on the Moment-Rotation Response

6.5.4 Influence of Reinforcement Area

The effects of reinforcement area A_s on the moment-rotation response are studied by varying this parameter on C1 specimen from $A_s=157\text{mm}^2$ to $A_s=550\text{mm}^2$. It seems that the effect of varying the reinforcement area to the rotational capacity is negligible, while only a slight reduction is observed as the reinforcement area increases, as shown in Figure 6.29. While the reinforcement area affects the rotational capacity by shifting the neutral axis, the neutral axis position is found to be only slightly affected by the reinforcement area. As an example, an increase of the area of reinforcement from 157 to 550mm^2 has only shifted the neutral axis of around 23 mm (8% of the depth of web). A more dominant factor in shifting the neutral axis is the faster rate of strength reduction of structural steel compared to the reinforcement as temperature increases. As an example, the neutral axis of C1- $A_s=157$ has shifted by 104 mm (37% of the depth of web) as a result of a temperature rise from 25 to 623°C . Observation of the failure modes in this

series shows that all beams exhibited the same failure modes of flange and web buckling. There is no major difference exhibited among them.

Figure 6.29 Influence of Reinforcement Area on the Moment-Rotation Response

6.5.5 Influence of the Number of Shear Studs

The effects of the number of shear studs, which determines the degree of shear connection η_s , are investigated by varying this parameter on C3 specimen. It should be noted that only full shear connection ($\eta_s \geq 100\%$) is considered. Hence, this parametric study aims to compare the behaviour between a composite beam of which shear connection is overly provided and the one of which shear connection is only sufficiently provided ($\eta_s \approx 100\%$). Figure 6.30 shows the difference in the moment-rotation curve of C3 specimen designed with 10, 6 and 4 studs/span resulting in the degree of shear connection of 273%, 164% and 109%, respectively. It can be seen that using only sufficient number of studs (4 studs/span), both the plastic rotation θ_p and ultimate rotation θ_u are larger compared to the other specimens with over-provided shear studs (6 and 10 studs/span). As a result, there is no significant difference in their rotational capacity. The maximum moment has only slightly reduced by using less number of studs. It can be concluded that by over designing the shear connection, the moment-rotation response is slightly stiffer because of less slip between the structural steel and the concrete slab.

Figure 6.30 Influence of Number of Studs on the Moment-Rotation Response

6.5.6 Influence of Effective Length

The influence of effective length on rotational capacity is addressed by varying the lateral restraint distance of C3 specimen. Figure 6.31 shows the moment-rotation response of C3 specimen with different effective lengths. It can be observed that the effects of increasing the effective length in the composite beams are generally the same as in steel beams with increasing the effective length reduces both the maximum moment and rotational capacity. A change of failure modes also occurs when the effective length is increased. Figure 6.32 gives a comparison of the failure modes which occur in C3- $L_E=563$ and C3- $L_E=1725$. It can be observed that flange and web local buckling occurs in C3- $L_E=563$, whereas lateral torsional buckling without any local buckling is observed in C3- $L_E=1725$.

Figure 6.31 Influence of Effective Length on the Moment-Rotation Response

Figure 6.32 Influence of Effective Length on the Failure Modes

6.6 Isothermal versus Transient Response

In the experimental and numerical investigation, the temperature is held constant before the load is applied incrementally. This sequence follows the design process in which the temperature profile of the structure is first determined through a heat transfer analysis before the structural response is analysed. Even though this sequence is different from that during a fire incident in which the applied load is held constant and the temperature is increased, it provides an easier fire engineering design of structures using hand calculations or a design spreadsheet. In addition, the isothermal moment-rotation curves are in fact a mere simplification of three-dimensional parameters of temperature, applied moment, and rotation. Therefore, these isothermal curves can be translated into transient

curves which show the temperature-rotational response under constant moment. The response of a continuous beam under fire in which the applied load is held constant while the temperature is increased can be explained using this temperature-rotational response.

RB2 specimen of which moment-rotation response is shown in Figure 6.33 is used as an illustration to explain the behaviour of a continuous beam in the hogging moment region under fire. The moment-rotation response can be translated into temperature-rotational response by plotting the rotation produced by a constant moment under different temperatures, as shown in Figure 6.34. It should be noted that a smoother temperature-rotational response can be obtained if more moment-rotation curves at other temperatures are given in Figure 6.33. For an example, this RB2 beam is used to construct a continuous beam and at the event of fire, the applied hogging moment at one of the support is 140 kNm. The probable response at this support during fire is shown in Figure 6.35. Initially the rotation increases from point 1 to 2 as the temperature increases under a constant moment of 140 kNm. Afterwards, there are two probable paths depending whether moment redistribution is possible. If it is not possible to redistribute the support hogging moment, the rotation will increase significantly from point 2 to 3 as the temperature increases slightly and the beam fails once it reaches point 3. If a certain amount of moment redistribution is possible, the hogging moment is redistributed to the nearby structure as the temperature increases further. As a result, the hogging moment is reduced to 100 kNm and the temperature-rotation response follows path 2-4.

A subsequent increase in the temperature requires again a certain amount of moment to be redistributed. If the structural arrangement allows the beam to redistribute its moment, the temperature-rotational response will follow path 4-6, otherwise it will follow path 4-5 and the beam fails at point 5. It can be seen from Figure 6.35 that with moment redistribution, the failure temperature has increased by over 100°C. It should be noted that the possibility of moment redistribution also depends on whether the beam possesses sufficient rotational capacity. Insufficient rotational capacity will cause the beam to fail prematurely and therefore, moment redistribution is not possible in a fire incident. Thus, the required rotational capacity during fire should be assessed and

compared with the available rotational capacity of the member, such that not only the strength requirement is satisfied, but also the ductility requirement.

Figure 6.33 Moment-Rotation Response of RB2 at Various Temperatures

Figure 6.34 Temperature-Rotation Response of RB2 at Various Applied Moments

Figure 6.35 Support's Rotational Response of RB2 during Fire

It is worth noting that there are two aspects of transient test which are different from isothermal test:

- The temperature distribution under transient test may not be uniform, especially if the beam carries a concrete slab on its top flange.
- The effective stress-strain relationship is slightly different due to the difference in the testing strain-rates. The testing strain-rates in transient heating are usually lower than those in isothermal testing.

CHAPTER SEVEN

MODELLING OF MOMENT-ROTATIONAL RELATIONSHIP

7.1 Introduction

The assessment of the ductility of both steel and composite beams subjected to hogging moment involves two main problems: the evaluation of the moment-rotational relationship and the determination of rotational capacity. The methods to obtain answers to these problems are generally classified into three, namely, experimental, theoretical and empirical approach. The experimental method, if conducted properly, is obviously the most reliable among them. However, it costs much time and resources. Besides, the applicability of experimental results is limited by the prohibitive costs associated with testing and the wide range of available sections. Thus, the experimental method is only suitable for research purpose and for calibrating finite element models.

The theoretical method comprises numerical, analytical and plastic collapse mechanism. The numerical analysis usually involves the use of a finite element program to build a representative model. One of the advantages of a numerical analysis is that, it offers a very refined analysis such that detailed aspects concerning the inelastic behaviour can be observed; hence a good correlation with experimental results can be achieved. The only disadvantage of numerical analysis is the computing time as dense meshes of elements are needed to properly model the plastic region and to obtain good agreement with test results. Thus, calibrated FE models are usually used for parametric studies. The analytical approach in the context of predicting the moment-rotational relationship involves the determination of pre-buckling moment-rotation response based on the

moment-curvature equations and a simplified model to predict the rotation at the point of buckling. Therefore, the analytical approach can be used to determine the pre-buckling part of the moment-rotational relationship until buckling occurs. However, the analytical model alone is unable to determine the ultimate rotation, which is needed to ascertain the rotational capacity. Even though the accuracy of buckling prediction is not as accurate as FEA, the solution can be obtained by hand calculations. To obtain the post-buckling moment-rotational relationship, recourse is made to plastic collapse mechanism approach. This method is very useful to determine the ultimate plastic rotation and the rotational capacity. As the name implies, plastic collapse mechanism involves modelling of buckling shape formed by several yield lines and plastic zones. This method provides a rather accurate upper-bound limit of the post-buckling moment-rotational relationship, if a proper model for the plastic mechanism is chosen. Besides, it can be implemented into a spreadsheet although hand calculations are nearly impossible due to a large number of fairly complex equations.

The empirical model usually involves simple curve fitting or a statistical approach based on a set of experimental and numerical results. The statistical approach is more sophisticated than a mere curve fitting method because it also involves some regression analyses to determine the best regression model based on a number of possible parameters. Although the empirical approach is devoid of the theoretical background associated with the buckling process, it is able to consider the main parameters influencing the rotational capacity. Besides, it also provides a quick and easy approach for practitioners.

In this study, all three methods mentioned above have been employed. The experimental method has been conducted and reported in Chapter 5, while the numerical analysis has been discussed in Chapter 6. The remaining methods, namely, the analytical, plastic collapse mechanism and statistical-based method are presented in this Chapter. The

model for the moment-rotational relationship of steel I-beams at elevated temperature is first discussed before it is extended to composite beams. Two models are presented for the case of steel beams. The first model, referred as moment-rotation design model, is based on a combination of an analytical method to find the pre-buckling moment-rotational relationship and a statistical-based method to find the post-buckling (unloading) moment-rotational relationship. The second model, referred as plastic collapse mechanism model, uses the kinematic theory to find the post-buckling relationship. It is used in conjunction with the analytical method to find the pre-buckling moment-rotational relationship. The plastic collapse mechanism model of the steel beams is then extended to composite beams as well.

7.2 Development of Moment-Rotation Design Model for Steel Beams

In this section, the development of design moment-rotational relationship at elevated temperature is described. Since the purpose of the research is to quantify the rotational capacity of steel I-beams to be used in plastic design, the design moment-rotational relationship is only developed for beams which are able to achieve their plastic moment capacity. Slender beams which lose their capacity prematurely are not considered. The moment-rotation design model comprises three parts: a non-linear pre-peak curve, a horizontal plateau at the plastic moment capacity and an unloading curve. The proposed design equations offer a quick, accurate and practical method for practitioners and design engineers to describe the full moment-rotational relationship of steel beams at elevated temperatures.

7.2.1 Non-Linear Pre-Peak Region

The moment-rotational relationship in the pre-peak region before buckling occurs can be evaluated from the moment-curvature relationship of the cross-section. Rotation θ can be obtained from the integration of the curvature diagram:

$$\theta = \int_0^x \chi(x) dx \quad (7.1)$$

in which χ = curvature.

For beams under mid-span point loading at ambient temperature, the moment-rotational relationship is a simple linear relationship and can be easily constructed by finding the plastic rotation θ_p (defined as the rotation in which plastic moment capacity M_p is attained). The plastic rotation θ_p can be accurately approximated as elastic rotation θ_e in eq. (7.2).

$$\theta_p \approx \theta_e = \int_0^{L_i} \frac{M_p (x/L_i)}{EI} dx = \frac{M_p L_i}{2EI} \quad (7.2)$$

in which $x = 0$ for end support section and $x = L_i$ for mid-span; I = second moment of inertia.

However, for beams at elevated temperature, the moment-rotational relationship is non-linear since the stress-strain relationship of steel at elevated temperature is not bi-linear elastic-plastic resulting in tangent stiffness significantly lower than the initial tangent when the applied stress is above the proportionality limit. As a result, the moment-rotational relationship must be obtained numerically by discretizing the cross-section into small elementary areas and dividing the longitudinal length into smaller sections, as shown in Figure 7.1. At a certain load, bending moment at j^{th} section can be calculated. Subsequently, the curvature at j^{th} section has to be evaluated under the hypothesis that cross-section remains plane after bending using both compatibility and equilibrium equations. The compatibility equation is expressed as

$$\varepsilon_{i,j} = \chi_j y_i \quad (7.3)$$

in which $\varepsilon_{i,j}$ is the strain of i^{th} elementary area at j^{th} section, χ_j is the curvature at j^{th} section, and y_i is the distance between i^{th} elementary area and neutral axis. The equilibrium equation is given as

$$\sum_{i=1}^n \sigma_i A_i y_i = M_j \quad (7.4)$$

in which σ_i is the stress of i^{th} elementary area A_i and M_j is the bending moment at j^{th} section. The stress-strain relationship, which relates the compatibility and equilibrium equation, is taken from EC3:1.2 (CEN, 2005). Finally, the rotation at a given load and moment can be obtained from the area of the curvature diagram, and is expressed as

$$\theta = \sum_{j=1}^m \chi_j \Delta x_j \quad (7.5)$$

Figure 7.1 Longitudinal and Cross-Section Discretization

7.2.2 Horizontal Plateau Region

Based on the parametric studies presented in Chapter 6 and experimental results reported in Chapter 5, it is observed that the maximum moment achieved for steel beams with temperature exceeding 400°C is generally less than 10% above their plastic moment capacity, unlike steel beams at ambient temperature. As a result, it is unnecessary to accurately predict the maximum moment since the error by assuming the maximum moment equal to the plastic moment capacity is less than 10%. Thus, the author decided that it is sufficient to use a horizontal line at the plastic moment capacity to connect the non-linear pre-peak curve with the unloading curve as described in the next section. The horizontal line will begin at plastic rotation θ_p up to ultimate rotation θ_u , in which the unloading curve crosses the plastic moment capacity.

7.2.3 Unloading Region

For the unloading portion of moment-rotation response, a multi-linear regression model is used to fit the finite element predictions. Both the ultimate rotation θ_u , which determines the start of unloading curve, and the unloading moment-rotation equation need to be determined. In order to cover a wider range of parameters and meet statistical requirements, additional simulations have been conducted. A total of 71 simulations have been performed in this study with a range of parameters as below:

$$4.0 \leq c/t_f \leq 13.0; \quad 24.0 \leq d/t_w \leq 81.0; \quad 11.0 \leq \lambda_{LT} \leq 29.0; \\ 275 \text{ MPa} \leq f_y \leq 355 \text{ MPa}; \quad 400^\circ \text{C} \leq T \leq 800^\circ \text{C}. \quad (7.6)$$

in which, λ_{LT} is the lateral torsional buckling (LTB) slenderness ratio and T is temperature. The LTB slenderness ratio λ_{LT} is used as one of the main parameters in this study since the author finds that this is an important factor (supported by statistical evidence) which not only accounts for effective length, but also represents many other minor factors that may affect the rotational capacity, such as the type of loading (bending moment diagram), and the ratio of beam width to depth (Dharma & Tan, 2005b). As a result, a high accuracy can still be obtained even with only a few regression parameters.

Two most commonly used regression analyses, namely, stepwise regression analysis and best subsets regression analysis are employed to find the best regression model from possible parameters/ variables for the purpose of predicting the ultimate rotation θ_u . Firstly, stepwise regression analysis is used to add and remove potential variables to the regression model for the purpose of identifying useful subsets of the predictors. This regression analysis is effective in reducing the number of potential variables. Subsequently, a few important parameters that have been identified are analysed using best subsets regression. This best subsets analysis generates possible regression models using maximum R -value criterion by first examining one-predictor regression models, and then selecting the two models giving the largest R -value. This process continues

until the model contains all the essential predictors. Finally, the best regression model is selected based on the following criteria:

- R^2 value, which describes the proportion of variation in the response data explained by the predictors in the regression model;
- R^2_{adj} value, which is a modified version of R^2 value taking into account the number of predictors in the model;
- $C-p$ value for assessing how well the model fits the data; and
- s value, which is the error of standard deviation.

The best model, which has high R^2 and R^2_{adj} values, small s value, and $C-p$ value close to the number of predictors, is selected.

The best regression model to predict the ultimate rotation θ_u as a result of the analyses is

$$\sqrt{\frac{\theta_u}{\theta_e}} = 8.21 - 1.26 \frac{k_y}{k_E} - 0.14 \sqrt{\frac{c}{t_f}} \gamma_f \sqrt{\frac{d}{t_w}} \gamma_w - 0.0518 \lambda_{LT} \geq \sqrt{\frac{\theta_p}{\theta_e}} \quad (7.7)$$

in which, γ_f and γ_w are flange and web yield strength factor respectively ($=\sqrt{f_y/275}$).

The recommended range of use for this regression model is the same as the range of the data (eq. (7.6)). The statistical analysis results for the regression model are shown in Table 7.1. It can be observed that the p -values for the constant and the three predictors are zero, which means that the association between the response and all four terms are statistically significant. The high R^2 value of 95.9% and very low s value of 0.1 show that the model fits the FE data very well. In addition, R^2_{adj} value of 95.7%, which is very close to R^2 value, indicates that excessive number of terms is not the reason why the model fits the data well. This conclusion is supported by $C-p$ value of 4.0, which is close to the number of predictors in the model (eq. (7.7)).

In addition to this statistical analysis, the normal probability plot of the residuals and the plot of residuals versus fitted values are shown in Figure 7.2. It can be observed that the

normal probability plot of the residuals roughly follow a straight line, which indicates that the residuals are normally distributed. In addition, the residuals are scattered randomly about zero in the plot of residuals versus fitted values, which means that the variance is approximately constant. Further investigation on this plot indicates that there is no outlier among the data with the maximum residual of approximately only 0.3.

Table 7.1 Statistical Analysis Results of the Ultimate Rotation Regression Model

Predictor	Coefficient	Standard Error	<i>t</i> -value	<i>p</i> -value
Constant	8.21	0.213500	38.44	0.00
k_y/k_E	-1.26	0.068350	-18.36	0.00
$\sqrt{(c/t_f)}\gamma_f\sqrt{(d/t_w)}\gamma_w$	0.14	0.003816	36.76	0.00
λ_{LT}	-0.0518	0.008853	-5.85	0.00
$s = 0.107260$ $R^2 = 95.9\%$ $R^2_{adj} = 95.7\%$ $C-p = 4.0$

Figure 7.2 Statistical Plots from the Regression Model

Even though the model is obtained by regression analyses, the parameters involved are in accordance with earlier test results and parametric studies. It can be observed that the first parameter k_y/k_E in eq. (7.7) is one of the most significant parameters that takes into account the effect of temperature on ultimate rotation (see Section 6.3.2). The second parameter $\sqrt{\frac{c}{t_f}}\gamma_f\sqrt{\frac{d}{t_w}}\gamma_w$ takes into account both flange and web slenderness ratios which have been modified to include the effect of steel grade as well. It is interesting to

note that combining the flange and web slenderness ratios into one parameter yields better regression model than two separate and independent parameters. This statistical analysis shows that there is considerable interaction between the flange and the web during the process of local buckling, which is in line with the conclusion of preliminary studies (Dharma & Tan, 2005b). The third parameter λ_{LT} shows the effect of lateral torsional buckling, in addition to local buckling, on the ultimate rotation. During the regression analyses process, both the conventional lateral slenderness parameter L_E/r_y (effective length over radius of gyration about minor axis), which has been used by Lay & Galambos (1967) and Kemp (1985), and new parameter λ_{LT} were applied as parameters; however the best regression model was obtained by using λ_{LT} . Thus, it can be concluded that λ_{LT} can better explain the ultimate rotation θ_u .

In addition to the ultimate rotation, which marks the beginning of the unloading curve, the unloading curve itself needs to be determined. In this model, a quadratic equation is used to describe the unloading moment-rotational relationship. The coefficients for this quadratic unloading equation are obtained from the regression of FE's post-buckling curves. The unloading moment-rotation equation is defined as

$$\frac{M}{M_{p,T}} = 1 - 0.12 \left(\frac{\theta}{\theta_e} - \frac{\theta_u}{\theta_e} \right) + 0.023 \left(\frac{\theta}{\theta_e} - \frac{\theta_u}{\theta_e} \right)^2 \quad (7.8)$$

$$\text{in which } \theta_u/\theta_e \leq \theta/\theta_e \leq \theta_u/\theta_e + 2. \quad (7.9)$$

7.2.4 Validation of Design Model

The design model for the moment-rotational relationship at elevated temperatures, as graphically depicted in Figure 7.3, has been validated against the experimental results reported in Chapter 5. The correlation with experimental results is quite good, although the comparison is not as good as the correlation with finite element predictions, as illustrated in Figure 7.4. The non-linear pre-peak curves match the FE predictions very well, as expected, since this is obtained from the moment-curvature relationship, while

the horizontal line at plastic moment capacity provides a lower bound, which is very close to actual relationship. The unloading curve compares reasonably well with both FE predictions and test results considering the simplicity of the model and the complexity of the problem.

Figure 7.3 Moment-Rotation Design Model for Steel Beams at Elevated Temperature

Figure 7.4 Validation of Steel Beam Design Model at Elevated Temperature

7.3 Plastic Collapse Mechanism Model for Steel Beams

The analytical approaches are generally limited in plotting the moment-rotation curve just before the maximum moment, due to the difficulties in describing the post-buckling behaviour. The FEA can provide the complete curve, but with very high costs and consumes much time and resources, therefore it is suitable for research purpose but not for practical design. Another useful method is the use of plastic collapse mechanism, which is based on upper bound theorem. The collapse mechanism method is used to plot the post-buckling curve, as shown in Figure 7.5. Therefore, this method is very useful to determine the ultimate plastic rotation and the available rotational capacity (Gioncu & Mazzolani, 2002).

When a member collapses by local buckling, it can be observed that it develops a spatial local plastic mechanism. The local plastic mechanism composes of both yield lines and plastic zones as illustrated in Figure 7.6. The interaction of flange and web buckling is also considered in the mechanism by means of geometrically compatible yield lines. It is assumed that the deforming material is only found in the plastic zones and yield lines. The choice of the mechanism, together with the yield lines and the plastic zones

positions, are based essentially on experimental observations. There are many forms of possible plastic mechanisms; therefore, it is very important to choose a proper pattern by carefully examining the experimental and numerical results. Since the success of this method depends on the chosen plastic mechanism, careful studies are required to obtain the one that gives the minimum value for moment and ultimate plastic rotation.

Figure 7.5 Post-Critical Curve of Plastic Collapse Mechanism

Figure 7.6 Illustration of Plastic Collapse Mechanism

A few different collapse mechanisms have been proposed since 1965 as reviewed in Chapter 2. The evolution of these collapse mechanisms is briefly summarized as shown in Figure 7.7. Climenhaga & Johnson's (1972b) and Ivanyi's (1979b) model corrected Kato's (1965) method so that the yield lines between the flange and the web are geometrically compatible. However, the model's mechanism length is assumed to be the same as the flange width which contradicts with some experimental evidence. Kuhlmann's model (1986) characterised the buckled flange by a semi-pyramidal shape mechanism consisting of yield lines but without the plastic zone. The deficiency of this model is that, there exists an inconsistency of plastic deformations between the flange and the web. Feldmann's model (1994) possesses a similar plastic buckling mechanism as Kuhlmann's model (1986), but it assumes an elastic behaviour of the web; hence this model is more adequate for thin-walled members. Gioncu & Petcu (1995, 1997) improved the Climenhaga & Johnson's (1972b) model by including all cross-section dimensions when determining the mechanism length. Tehami's model (1997) uses a similar model as Climenhaga & Johnson (1972b), but it relates the mechanism length with the section depth, which is against some experimental evidence. Recently, Gioncu & Petcu (2001) improved their model such that both the flange width and the web depth are considered as mechanism parameters.

A preliminary study conducted by the author showed that the most recent model of Gioncu & Petcu (2001) resulted in a more accurate upper bound value of the unloading moment-rotation response compared to the other earlier models. In addition, this model also manages to overcome the shortcomings of earlier models. Thus, the author decided to extend this model to elevated temperature with a few necessary modifications and improvements. Before that, to provide a better understanding, the details of Gioncu & Petcu's model (2001) for steel beams at ambient temperature are presented and discussed as follows.

Author(s)	Flange	Web
Climenhaga and Johnson (1972) Ivanyi (1979)
Kuhlmann (1986)
Feldmann (1994)
Gioncu and Petcu (1995, 1997)
Tehami (1997)
Gioncu and Petcu (2001)

Figure 7.7 Various Plastic Collapse Mechanism Models (Gioncu & Mazzolani, 2002)

Gioncu & Petcu's model (2001) of plastic collapse mechanism is shown in Figure 7.8. It can be seen that the buckled shape of the compression flange is represented by a double-triangular plastic zone and several yield lines, while that of the web is represented by an upper triangular plastic zone connected by several yield lines to the centre of rotation "12". As rotation occurs, the tension zone (above point "12") remains in plane and experiences simple tensile inelastic deformation. In the compression zone

(below point “12”), the web and flange buckle due to the formation of yield lines and plastic zone. Buckling in the compression flange occurs by simultaneous compression and rotation of plastic zone about the flange centre line, and through compatible displacements of rigid triangular areas. The displacement of rigid triangular area is through the formation of yield lines. Note that the compression flange has been drawn in Figure 7.8 on the top face so that the assumed mechanism is clearly seen.

Figure 7.8 Plastic Collapse Mechanism Model (Gioncu & Petcu, 2001)

At the point of collapse, when the local plastic mechanism is formed, total external virtual work done W_{ext} by external loads is

$$W_{ext} = \sum P_i \Delta_i \quad (7.10)$$

in which, P_i = external load and Δ_i = displacement.

The total internal virtual work of the mechanism W_{int} is

$$W_{int} = \sum_i (W_l)_i + \sum_j (W_z)_j \quad (7.11)$$

where W_l = work done by yield line;

W_z = work done by plastic zone.

Subscript i depends on the number of yield lines, while subscript j depends on the number of plastic zones. For yield lines, only the rotation is involved; while for plastic zones, only axial deformations are considered. Thus, the associated internal works are

$$W_l = \frac{t^2 f_y}{4} \int_{l_p} f(\theta) dl \quad (7.12)$$

$$W_z = t f_y \int_{A_p} f(\varepsilon) dA \quad (7.13)$$

where l_p = length of yield line;

A_p = area of plastic zone;

t = thickness of plate;

f_y = yield strength;

ε = strain.

Both $f(\theta)$ and $f(\varepsilon)$ are nonlinear functions (involving trigonometric function).

Two important assumptions about deformations are considered: they are constant over the plastic zones or along the yield lines. Due to the fact that the rotational capacity is determined at the beginning of mechanism rotation θ , the deformations are small and the functions $f(\theta)$ and $f(\varepsilon)$ can be considered linear. Hence, the internal works of eq. (7.12) and (7.13) become

$$W_l = \frac{l_p t^2}{4} f_y \theta \quad (7.14)$$

$$W_z = A_p t f_y \varepsilon \quad (7.15)$$

Thus, the total internal virtual work of the mechanism can be expressed as

$$W_{int} = \frac{1}{4} \sum_i (l_p t^2 f_y \theta)_i + \sum_j (A_p t f_y \varepsilon)_j \quad (7.16)$$

Finally, equating the external and internal work done, the relationship between external moment M and rotation θ for post-critical curve can be obtained:

$$M = \frac{W_{int}}{\theta} \quad (7.17)$$

The length and rotation of the yield lines, together with the strain and volume of the plastic zones to calculate the internal virtual work of the mechanism W_{int} is summarized in Table 7.2. As shown in Figure 7.8 and Table 7.2, there are a few undetermined parameters, such as the length of plastic mechanism βc (Fig. 7.8(b)), the size of plastic zone of compression flange γc , the centre of rotation of plastic mechanism δd and the asymmetry of plastic mechanism ηc (Fig. 7.8(c)), which are related to the cross-section of the I-section. The length of plastic mechanism is determined from a proposal by Lay (1965), which models the flange buckling problem with a flange plate restrained by a torsional spring. Lay (1965) considered the length of buckled flange L_p as

$$L_p = 2\beta c \quad (7.18)$$

$$\text{in which } \beta = 0.713 \left(\frac{d}{b} \right)^{1/4} \left(\frac{t_f}{t_w} \right)^{3/4} \text{ and } c = \text{width of flange outstand.} \quad (7.19)$$

One can see that this flange buckling length parameter (eq. (7.19)) is dependent on the cross-section characteristics. In addition, the coefficient of 0.713 is a result of several simplifications concerning the steel material properties. Concerning the asymmetry of plastic mechanism, Gioncu & Petcu (2001) related the parameter η to β . They suggested that $\eta = \beta$ and $\eta = 1.5\beta$ for a beam connected to a column without connectivity plates and with connectivity plates, respectively. For other cases, a proposed maximum value of η would be $0.9((\delta d/c) - \gamma)$.

Table 7.2 Summary of Internal Virtual Works of Collapse Mechanism
(Gioncu & Mazzolani, 2002)

Plastic Zones			
Zone (<i>j</i>)	Volume ($A_p t$)	Strain (ϵ)	Stress (f_y)
4-5-7-6	$2\gamma^2 c^2 t_f$	$\theta \delta d / (2\gamma c)$	$f_{yf,c}^*$
5-6-11	$\gamma^2 c^2 t_w$	$\theta \delta d / (2\gamma c)$	f_{yw}
12-13-14	$(1-\delta)^2 d^2 t_w / 2$	θ	f_{yw}
15-16-18-17	$2(1-\delta) c t_f d$	θ	$f_{yf,t}^*$
Yield Lines			
Line (<i>i</i>)	Length (l_p)	Rotation (θ)	Stress (f_y)
2-4; 7-9	$(1-\gamma)c$	$2\left(\frac{\gamma}{\beta\chi}\right)^{1/2} \theta^{1/2}$	$f_{yf,c}$
1-4; 3-4; 7-8; 7-10	$[\beta^2 + (1-\gamma)^2]^{1/2} c$	$\frac{1}{[\beta^2 + (1-\gamma)^2]^{1/2}} \left[\frac{\beta}{1-\gamma} + \frac{\beta^2 - 3\gamma - \gamma\beta + 1}{\gamma(1+\beta-\gamma)} \right] \left(\frac{\beta\gamma}{\chi}\right)^{1/2} \theta^{1/2}$	$f_{yf,c}$
1-5; 3-6; 5-8; 6-10	$[1 + (\beta - \gamma)^2]^{1/2} c$	$\frac{[1 + (\beta - \gamma)^2]^{1/2}}{\gamma(1+\beta-\gamma)} \left(\frac{\beta\gamma}{\chi}\right)^{1/2} \theta^{1/2}$	$f_{yf,c}$
4-5; 4-6; 5-7; 6-7	$2^{1/2} \gamma c$	$2^{1/2} \frac{1}{\gamma(1+\beta-\gamma)} \left(\frac{\beta\gamma}{\chi}\right)^{1/2} \theta^{1/2}$	$f_{yf,c}$
5-11	$2^{1/2} \gamma c$	$\left(\frac{2}{\gamma}\right)^{1/2} \left[\frac{1 - (1-\eta/\gamma)\chi}{1 - (1+\eta/\gamma)\chi} \right]^{1/2} \theta^{1/2}$	f_{yw}
6-11	$2^{1/2} \gamma c$	$\left(\frac{2}{\gamma}\right)^{1/2} \left[\frac{1 - (1+\eta/\gamma)\chi}{1 - (1-\eta/\gamma)\chi} \right]^{1/2} \theta^{1/2}$	f_{yw}
5-12	$[1 + (1 + \eta/\gamma)^2 \chi^2]^{1/2} \delta d$	$\frac{1}{\chi^{1/2}} \left[\frac{1 - (1-\eta/\gamma)\chi}{1 - (1+\eta/\gamma)\chi} \right]^{1/2} [1 + (1 + \eta/\gamma)^2 \chi^2]^{1/2} \theta^{1/2}$	f_{yw}
6-12	$[1 + (1 - \eta/\gamma)^2 \chi^2]^{1/2} \delta d$	$\frac{1}{\chi^{1/2}} \left[\frac{1 - (1+\eta/\gamma)\chi}{1 - (1-\eta/\gamma)\chi} \right]^{1/2} [1 + (1 - \eta/\gamma)^2 \chi^2]^{1/2} \theta^{1/2}$	f_{yw}
11-12	$[(1-\chi)^2 + (\eta\chi/\gamma)^2]^{1/2} \delta d$	$\left[\frac{(1-\chi)^2 + (\eta\chi/\gamma)^2}{(1-\chi)^2 - (\eta\chi/\gamma)^2} \right]^{1/2} \theta^{1/2}$	f_{yw}
15-13-17	$2c$	$\left[\frac{(1-\chi)^2 + (\eta\chi/\gamma)^2}{(1-\chi)^2 - (\eta\chi/\gamma)^2} \right]^{1/2} \theta^{1/2}$	$f_{yf,t}^*$

Note: subscript "f" and "w" denoting the thickness of flange and web, respectively.

subscript "c" and "t" denoting the compression and tension part, respectively.

* increased yield stress due to strain-hardening. $\frac{f_{yf,c}^*}{f_{yf,c}} = \frac{1}{1 - (2\beta c / L)}$

$$\chi = \frac{\gamma c}{\delta d}$$

The other two parameters, γc and δd , are treated as variables which are determined by minimising the unloading moment-rotation curve. An increased yield strength can be considered for two plastic zones taking into account that the member reaches the strain-hardening range. Finally, since the rotation produced by the plastic collapse

mechanism is derived solely from the internal work done by the yield lines and plastic zones neglecting the deformation of the other parts, the plastic rotation θ_p should be added to obtain the ultimate rotation θ_u .

A few comments regarding Gioncu & Petcu's model (2001) are as follows:

- This model represents an in-plane local buckling failure mode which means that no out-of-plane or lateral torsional buckling should be involved. Since the length of the hogging area near the supports is comparatively short for most of the continuous beams, the failure mode is mostly in-plane local buckling. Thus, this plastic collapse mechanism model represents a majority of failure modes.
- The collapse mechanism of this model is such that the flange buckles first followed by the web since the determination of the length of plastic mechanism is based on a model of flange plate restrained by torsional spring (Lay, 1965). Thus, the estimation of the buckled length may not be accurate if the web (represented by the torsional spring) buckles before the flange does.
- The model does not take into account the effect of the steel grade on the moment-rotation response. The material characteristics are only simplified into a coefficient of 0.713 in the parameter β without considering a variation of these material properties if different steel grades are used.
- The length of buckled flange L_p is derived considering that the neutral axis is at the mid-depth of the web.

In order to extend this model to elevated temperature case, some parameters will be modified to take into account the temperature effect. In addition, the accuracy of the model will also be improved by considering the effect of steel grade. However, the basic model of the collapse mechanism is kept the same, since the observation of the failure modes at elevated temperature shows a similar buckling pattern as that at ambient temperature. There are two parameters which may be modified, namely, β and η . The

asymmetry of plastic mechanism parameter η , which determines the centre of the flange buckling, is found to be independent of temperature. Therefore, the only observable difference between the ambient and the elevated temperature buckling shapes is the length of the mechanism, or the length of flange buckling. The length of the mechanism is slightly reduced at elevated temperature because of the disappearance of strain-hardening phenomenon. At ambient temperature, the beam is able to spread the yielding/ plasticity over a greater distance due to the presence of strain hardening, thus producing a greater length of mechanism. At elevated temperature, the disappearance of strain hardening causes limited spread of plasticity and hence, the flange buckles with a smaller mechanism length. In view of this observation, the parameter β which determines the length of mechanism is then modified as

$$\beta_T = 0.713 \sqrt{\frac{275}{f_{yf,c}}} \left(\frac{d}{b}\right)^{1/4} \left(\frac{t_f}{t_w}\right)^{3/4} \left(\frac{k_E/k_y}{0.7}\right) \quad (7.20)$$

As seen in eq. (7.20), the first modification is the introduction of $\sqrt{275/f_{yf,c}}$ to take into account the effect of the steel grade on the length of mechanism and hence, on the rotational capacity. This factor modifies the material coefficient of 0.713 and indicates that, as the steel grade increases the length of mechanism is reduced, which is in line with the experimental and numerical evidence. Yield strength of 275 MPa is used as a reference since the coefficient 0.713 is derived from material properties equivalent to steel grade S275. The second modification is the introduction of empirical factor $((k_E/k_y)/0.7)$ to take into account the effect of temperature on the unloading curve and rotational capacity through the length of mechanism parameter. It has been pointed out in Section 6.3.2 that the trend of the unloading curve and rotational capacity are related to the ratio of elastic modulus reduction factor to yield strength reduction factor at elevated temperature. Therefore, this empirical factor is used to represent the effect of increasing temperature as a result of the difference in the stress-strain characteristics. In this model, the stress-strain relationship of steel including its material reduction factors

at elevated temperature follows EC3:1.2 (CEN, 2005). Finally, it should be pointed out that no increase in the flange yield strength is necessary since there is very little or no strain-hardening at elevated temperature unlike that at ambient temperature.

As there are a number of yield lines and plastic zones involved, a minimisation process is needed. Thus a spreadsheet should be used instead of manual calculations. One must mention that, since the kinematic theory is used, the results of this method form an upper bound to the true solution. The proposed plastic collapse mechanism model can be combined with the analytical solution based on the integration of moment-curvature diagram as discussed in Section 7.2.1 for plotting the first part of moment-rotation curve (pre-buckling portion). To join the pre-buckling and the post-buckling curves, a horizontal plateau can be used since it is a lower bound to actual moment-rotation response.

The unloading moment-rotational relationship based on the proposed plastic collapse mechanism (PCM) method is illustrated in Figure 7.9(a) for S3-2, while the full range of moment-rotational relationship which includes the pre-buckling portion and the horizontal plateau is shown in Figure 7.9(b). It can be seen that the moment-rotational relationship obtained from PCM model provides an accurate upper bound to both the test and the numerical results. The portion of PCM unloading curve which is above the plastic moment capacity (Fig. 7.9(a)) is ignored in the proposed moment-rotation model (Fig. 7.9(b)). Figure 7.10 shows the validation results of this proposed PCM model compared to test results and numerical simulations. It can be seen that the PCM model generally provides an accurate upper bound curve to the unloading part of the moment-rotational relationship. In addition, PCM model produces a descending curve parallel to FEA predictions.

Figure 7.9 PCM based Moment-Rotation Model (S3-2)

(a)

(b)

Figure 7.10 Validation of Proposed Plastic Collapse Mechanism Model

This plastic collapse mechanism is also used to predict the ultimate rotation θ_u of a total of 71 FE simulations. Figure 7.11 shows the comparison of the ultimate rotation relative to the elastic rotation θ_u/θ_e obtained from plastic collapse mechanism (PCM) and finite element analysis (FEA). It can be seen that the PCM results provide an upper bound to the FE results. However, some of the PCM predictions are well above the $(\theta_u/\theta_e)_{PCM}=1.3(\theta_u/\theta_e)_{FEA}$ line (a maximum of 30% overestimation is regarded as an acceptable upper bound limit) because there are extreme cases where the failure modes or mechanisms are different from the proposed plastic collapse mechanism. Firstly, some of them involve lateral torsional buckling failure causing out-of-plane displacement. Since the proposed PCM considers only the local flange and web buckling, its results will be inaccurate and erroneous. After careful examination of the failure modes involved in each simulation, the author proposes a maximum LTB slenderness limit of 20 ($\lambda_{LT} \leq 20$) so that in-plane local flange and web buckling occurs instead of lateral torsional buckling.

Figure 7.11 Comparisons of FEA and PCM Predictions

Secondly, the sequence of buckling modes affects the accuracy of this model since the collapse mechanism assumes flange buckling taking place prior to web buckling. In order to predict the sequence of buckling, the author uses the ratio of web slenderness and flange slenderness. Figure 7.12 shows the accuracy of the PCM results against the ratio of the web and flange slenderness $((d/t_w)\gamma_w/(c/t_f)\gamma_f)$, which has been normalized with their respective yield strength factor. It should be mentioned that the set of data in Fig. 7.12 only includes those with $\lambda_{LT} \leq 20$. It can be seen that when $((d/t_w)\gamma_w/(c/t_f)\gamma_f) > 5.5$, the PCM method provides estimates of ultimate rotation above the acceptable limit of $(\theta_u)_{PCM} = 1.3(\theta_u)_{FEA}$. This is because once the web is comparatively more slender than the flange, it will buckle before the flange does. Hence the web does not provide torsional restraint to the flange, which is contrary to the adopted Lay's model (Lay, 1965). As a result, the estimation of the flange buckled length L_p is not accurate. Thus, the author proposes that the ratio of web to flange slenderness should not exceed 5.5 to avoid web buckling occurring before flange buckling.

Figure 7.12 Plot of Ratio of Web to Flange Slenderness-PCM Predictions

Further investigation suggests that the web slenderness should also be less than 44, as shown in Figure 7.13. It can be seen that the PCM predictions are within the acceptable limit of $(\theta_u)_{PCM} = 1.3(\theta_u)_{FEA}$ for $(d/t_w)\gamma_w \leq 44$, while some data with $(d/t_w)\gamma_w \leq 44$ are above the acceptable limit. Finally, an upper bound value of the flange slenderness is also investigated. Figure 7.14 shows an example of the comparison between PCM predictions and FE results as the flange slenderness increases. It can be observed that when $c/t_f = 6.9$, PCM gives a very accurate upper bound value of the ultimate rotation $((\theta_u)_{PCM}/(\theta_u)_{FEA} = 1.08)$. As the flange slenderness increases, the accuracy of PCM prediction reduces. This is because as flange slenderness increases, the web becomes relatively stockier, hence there is less web buckling. That is to say, the yield lines of the web are only partially developed (incomplete yield lines) since certain parts of the yield lines experience only small rotations due to limited web buckling. As a result, less internal energy is dissipated from the web yield lines and hence less inelastic rotations. The PCM model assumes that the web yield lines are always fully developed (complete yield lines), therefore it predicts a higher ultimate rotation compared to the FEA. Thus, the author proposes an upper flange slenderness limit of 10.0, that is, $(c/t_f)\gamma_f \leq 10.0$.

This also ensures that the beam is able to achieve its plastic moment capacity.

In summary, PCM method is best used within the limits of parameters given in eq. (7.21).

It should be noted that these limits include the limits of the FE data (eq. (7.6)).

$$4.0 \leq (c/t_f)\gamma_f \leq 10.0; \quad 24.0 \leq (d/t_w)\gamma_w \leq 44.0; \quad 11.0 \leq \lambda_{LT} \leq 20.0; \\ ((d/t_w)\gamma_w / (c/t_f)\gamma_f) \leq 5.5; \quad 400^\circ C \leq T \leq 800^\circ C. \quad (7.21)$$

Within these limits (eq. (7.21)), the PCM predictions are well between $0.9(\theta_u)_{FEA}$ and $1.3(\theta_u)_{FEA}$ as shown in Figure 7.15.

Figure 7.13 Plot of Web Slenderness-PCM Predictions

Figure 7.14 FE Results and PCM Predictions at Various Flange Slenderness Ratios

Figure 7.15 Comparisons of FEA and PCM Predictions within Parameter Limits

7.4 Plastic Collapse Mechanism Model for Composite Beams

The PCM method is also applied to composite beams at elevated temperatures. Both the yield lines and the plastic zones follow the model described for the steel beam section.

However, one additional internal work done has to included, namely, the internal work done by the plastic zone of the reinforcement as shown in Figure 7.16. This internal work done by the reinforcement W_{int}^{reinf} can be calculated by integrating the strain energy over the deforming volume (plastic zone):

$$W_{int}^{reinf} = A_s x_p f_{yr,T} \theta \quad (7.22)$$

where A_s = reinforcement area;

x_p = plastic zone length of reinforcement $(=h_r + (0.5 - \delta)d)$;

$f_{yr,T}$ = yield strength of reinforcement at elevated temperature.

Since there is a non-uniform temperature distribution across the section, different temperatures and hence different strength reduction factors are used for different beam sections. The concrete slab does not dissipate energy during the plastic deformation since it has cracked at a very early stage; hence it is ignored in the PCM model. It should also be mentioned that the PCM model assumes full shear connection with no slip between the steel I-section and the concrete slab.

Figure 7.16 Sectional View of Plastic Collapse Mechanism of Composite Beams

As mentioned in Section 7.3, the length of mechanism β was originally derived based on the “flange restrained by torsional spring” model (Lay, 1965), in which the stiffness of torsional spring representing the web was estimated with the neutral axis of the steel section at the mid-depth of the web, as shown in Figure 7.17. That is to say, one half of the web is subjected to compressive stress and the other half tensile stress. In the case of a composite steel beam under fire conditions, the neutral axis is shifted towards the top flange or concrete slab, as discussed in Section 5.5.3. Therefore, a greater portion of the web or possibly the entire depth is in compression (Figure 7.17). As a result, the stiffness of the torsional spring restraining the flange should be reduced and hence the equation for the length of mechanism should also be modified accordingly. Thus, the author introduces an empirical factor k_w to take into account the increased portion of the web in compression such that the length of mechanism for composite beams at elevated temperature β_T^{comp} is defined as

$$\beta_T^{comp} = 0.713k_w \sqrt{\frac{275}{f_{yf,c}}} \left(\frac{d}{b_f} \right)^{1/4} \left(\frac{t_f}{t_w} \right)^{3/4} \left(\frac{k_E/k_y}{0.7} \right) \quad (7.23)$$

in which, $k_w = 0.5d/\alpha d$ and α is the portion of the web in compression (Fig. 7.17).

It can be seen that k_w equals to 1 when the neutral axis is at mid-depth of the web (steel beams). When the neutral axis is within or above the top flange, the whole web is in compression, and the value of α is 1 (maximum value) and $k_w = 0.5$.

Figure 7.17 Difference in Neutral Axis Position between Steel and Composite Beams

The moment-rotational relationship in the pre-peak region before buckling occurs can be evaluated in the same way as for the bare steel beams (Section 7.2.1). The only difference is that the slab reinforcement has to be included in the equilibrium equation (eq. (7.4)). A horizontal plateau can be used as a lower bound to join the pre-buckling and post-buckling moment-rotational relationships. The moment-rotational relationships which are obtained through moment-curvature relationship, a lower bound horizontal line and plastic collapse mechanism are validated against test results, as shown in Figure 7.18. It can be seen that the pre-buckling moment-rotational relationships follow the test results closely, while the post-buckling moment-rotational relationships obtained from PCM provides an accurate upper bound to the test results. In between these two curves, the horizontal line provides a conservative estimate. The upper bound estimates of ultimate rotation θ_u for 4 test results are within 12% as shown in Table 7.3. It can also be seen that the predicted rotational capacity r_a obtained from this model is rather accurate compared to the test results considering the complexity of the problem.

Figure 7.18 Validation of PCM based Moment-Rotation Model

Table 7.3 Summary of Validation Results for Composite Beams' PCM Model

Test No.	θ_u / θ_e		r_a		$(\theta_u)_{PCM}$
	Test	PCM	Test	PCM	$(\theta_u)_{Test}$
C1	5.544	5.971	2.11	2.05	1.077
C2	5.879	6.565	1.84	2.49	1.117
C3	5.306	5.674	1.85	2.39	1.069
C4	3.829	4.272	1.00	1.38	1.116

The predicted moment-rotational relationships obtained from this model as temperature increases are studied using C2 specimen as shown in Figure 7.19. It should be noted that the reference temperature in the ensuing discussions is the bottom flange temperature T_{bf} . It can be observed that as this temperature increases, both the ultimate rotation and the rotational capacity reduce. This trend is in line with FE results as discussed in Section 6.5.1. The predictions of ultimate rotation θ_u are also compared with FE results reported in Section 6.5.1, as shown in Figure 7.19. Clearly, the ultimate rotation θ_u predictions are within the 30% upper limit.

Figure 7.19 Temperature Effect on the Proposed PCM based Moment-Rotation Model

A total of 30 FE simulations have been performed to investigate the accuracy of PCM model for composite beams. The parameters in these simulations are within the limits suggested in eq. (7.21) to verify whether these limits of PCM model for the bare steel beams are also applicable to composite beams. Figure 7.20 shows the comparison of ultimate rotation predictions obtained from FEA and PCM model. It can be seen that the PCM predictions are generally within the lower and upper limit of $0.9(\theta_u/\theta_e)_{FEA}$ and $1.3(\theta_u/\theta_e)_{FEA}$, respectively. Thus, it can be concluded that the PCM model for composite beams predicts θ_u with sufficient accuracy so long as the parameters are within the limits given in eq. (7.21). Since the PCM model is based on kinematic theory, the results of this method yield an upper bound to the true solution. Therefore, to obtain a safe lower bound value for design purpose, a partial safety factor should be applied. The author proposes a value of 1.3 to be used as a partial safety factor since the PCM predictions are generally less than 1.3 of the test results and FE predictions.

Figure 7.20 Comparisons of FEA and PCM Predictions on Ultimate Rotation

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The objective of this research is to study the buckling behaviour of I-shaped beams at elevated temperatures. This has been achieved by considering both global and local buckling. Global buckling in the form of lateral torsional buckling which prevents the attainment of the plastic moment capacity has been studied. As a result, two design approaches, including general and simple calculation methods, for laterally unrestrained steel beams have been proposed to predict the buckling moment at elevated temperatures. Local buckling due to localised distortion of the flange and the web which often limits the ductility of a member has been studied for both steel and composite beams in fire conditions. As a result of this study, the models for moment-rotational relationships of both steel and composite beams subjected to hogging moment at elevated temperatures have been proposed. The following sections highlight several main conclusions as a result of this study.

8.1.1 Elevated Temperature Material Properties

Mechanical and thermal properties of both steel and concrete are temperature-dependent. As temperature increases, the mechanical properties of steel and concrete deteriorate. This reduction of mechanical properties at elevated temperatures combined with the thermal effects such as thermal induced strains, leads to major differences in structural behaviour at elevated temperatures compared to that at ambient temperature.

With increasing temperature, the stress-strain relationship of steel becomes highly non-linear. Above 200°C, the stiffness of steel starts to reduce followed by a strength

reduction when the temperature exceeds 400°C. As a consequence, a bi-linear model (one line to represent the elastic range and another to represent the plastic range) is insufficient to represent the stress-strain relationship of steel at elevated temperature. The most commonly used is bilinear-elliptic model given in EC3:1.2, which also provides several material reduction factors at a specific temperature. This model can also be used for steel reinforcement at elevated temperatures.

The concrete properties at elevated temperatures are much more complex than the steel properties. Whether the concrete is normal or lightweight yields different thermal and mechanical properties at elevated temperatures. Besides, the type of aggregate used also affects the concrete properties at elevated temperatures. A commonly adopted concrete stress-strain relationship in compression is the one proposed by EC4:1.2 which consists of non-linear ascending and linear descending part. However, EC4:1.2 curve is only suited for normal strength concrete. For concrete tensile strength, EC4:1.2 refers to EC2:1.2 which gives reduction factors at elevated temperature.

Similar to steel and concrete, the strength and stiffness of shear stud also reduces as temperature increases. The ultimate shear capacity of headed shear stud starts to reduce above 300°C while the stiffness starts to reduce above 100°C. It has been found that the failure of headed studs with re-entrant decking is always by the shearing of the studs at the level of the flange (Zhao & Kruppa, 1995).

8.1.2 Lateral Torsional Buckling of Unrestrained Steel Beams

Finite element models and published experimental results are used to study the LTB of steel beams at elevated temperatures with a view to propose two design approaches. As a result, three weaknesses in EC3:1.2 approach have been identified. First, EC3:1.2 approach is too conservative for steel beams with medium slenderness leading to uneconomical design. Second, the slope of the buckling strength curve for very low slenderness is too steep, implying that even very short beams will not be able to achieve

the plastic moment capacity. This is in contrary to numerical results. Third, EC3:1.2 approach is only for elevated temperatures and a slightly different form of equation is used at ambient temperature (EC3:1.1); hence a discontinuity exists between the approach at ambient and at elevated temperatures. In order to overcome these weaknesses, a general approach, called an alternative approach has been suggested. The alternative approach gives more accurate results and provides a logical link between the ambient and the elevated temperature approach.

In addition to the alternative approach, the author also proposes an analytical approach based on Rankine principle. This Rankine approach provides an easy and quick design approach to estimate the lateral torsional buckling failure of steel beams in fire in which the complexity of the general approach has increased significantly at elevated temperature. Besides its simplicity, the use of Rankine approach also enables the failure temperature to be determined directly without an iterative procedure by guessing the failure temperature as required in EC3:1.2 approach.

Comparisons with FE predictions show clearly that the alternative approach matches well the FE predictions, while the predicted buckling moment based on Rankine approach is always slightly lower compared to that based on alternative approach. The lowest prediction of the buckling moment is given by the EC3:1.2 approach.

8.1.3 Experimental Investigation on Local Buckling Failure and Ductility

The ductility issue of both steel and composite beams in the hogging moment regions under fire conditions has been addressed by a fairly extensive experimental programme. A total of nine steel I-beams were tested to failure in the first series. Parameters including temperature, flange slenderness, web slenderness and effective length were varied in the first series. In the second series, a total of four composite steel beams with re-entrant steel decking profile were tested under hogging moment.

Steel Beam Tests

Two failure modes were generally observed during the first series of steel beam tests: local buckling of the beam flange and web and global buckling in the form of lateral torsional buckling. The local buckles of flange always occurred in an anti-symmetric mode at equal distance from the restrained mid-span while the local buckles of the web occurred approximately across half depth of the web which is in compression. Lateral torsional buckling mostly occurred near mid-span with the top flange at both sides of the mid-span moving laterally in opposite direction. These local and global buckling modes had been observed after the beam achieved plastic moment capacity.

The effects of temperature, flange slenderness, web slenderness and effective length on the inelastic behaviour of steel beams have been observed through the first series of test results. As temperature increases, the rotational capacity reduces substantially due to two factors. First, the non-linearity of stress-strain relationship at elevated temperatures causes the beam to attain plastic moment capacity at a much greater rotation, leaving behind limited inelastic rotation reserve. Second, the disappearance of strain-hardening phenomenon means that the yielded part of steel beam has practically no rigidity; hence local buckling takes place at an earlier stage. The effects of flange and web slenderness on the rotational capacity of steel beams at elevated temperature are similar to the one at ambient temperature, that is, increasing flange and web slenderness bring about a reduction in the rotational capacity. Although flanges are the primary elements of flexural resistance in I-sections, and should play a dominant role (compared to webs) in providing sufficient rotational capacity, it has been shown that web slenderness effect is also important. This is because the flanges are restrained by the web. Therefore, the stockier the web, the greater torsional restraint it provides against flange buckling, thus enhancing the ductility of the beam. Finally, a reduction in the effective length will increase the rotational capacity. Besides, it also causes a slight increase in the maximum moment and a change in the failure mode from lateral torsional to local buckling. This

finding shows that the ductility of steel beams is influenced not only by the flange and the web slenderness ratio, but also by the effective length.

Composite Beam Tests

In the second series, the failure mode observed was mainly local buckling near mid-span as a result of spread of plasticity after the beams reached plastic moment capacity. Local buckling of the bottom flange occurred in an anti-symmetric mode similar to that of the first series, in which one side of the flange curled up while the other side twisted downwards. Local buckling of the web occurred at almost the full depth of the web since the neutral axis was located within the top flange, that is, almost the entire beam section was under compression. These failure modes are very similar to the ones observed in Cardington fire test in which local buckling of the compression flange and web of most fire-exposed beams was observed near the supports.

A non-uniform temperature distribution across the section was observed in the composite beam tests unlike the steel beam tests. The top flange temperature was lower than the bottom flange temperature, while the web temperature varied between the top and bottom flange temperatures, with a higher temperature near to the bottom flange. Based on temperature measurements of four tested beams, temperature distribution across the section of a composite beam has been proposed. For the bottom half of web, its temperature can be assumed to be constant and equal to bottom flange temperature T_{bf} . For the upper half of web, its temperature may be assumed to vary linearly from T_{bf} to $0.85T_{bf}$ at the web-top flange junction. The temperature of shear stud, concrete and reinforcement can be assumed to be less than 300°C, 400°C, and 200°C, respectively.

It is found that the main component of a composite beam, which contributed most to the ultimate moment capacity, is the primary reinforcement. The reason is that the temperature of reinforcement was much lower compared to structural steel components. The neutral axis has also been found to be located within the top flange, or near to the web-top flange junction due to the much lower temperature of primary reinforcement and the non-uniform temperature distribution within the cross section. As a result, the entire depth of the web is in compression. Another important finding is that the degree of shear connection at the hogging moment region is unlikely to decrease during fire, except may be for a very long duration of fire, say more than 3 hours.

8.1.4 Numerical Analysis on Local Buckling Failure and Ductility

The ductility issue of both steel and composite beams in the hogging moment regions under fire conditions has also been studied by an extensive numerical analysis using finite element method. The finite element models have been successfully applied to study the moment-rotational relationship at elevated temperature and to investigate the main parameters affecting the rotational capacity at elevated temperature through parametric studies. It is demonstrated that the finite element analysis gives reasonable accuracy compared to test results, providing an efficient, economical, and yet accurate tool to study the rotational capacity of beams in fire.

Steel Beams

The influence of several parameters on the inelastic behaviour of steel beams at elevated temperature have been investigated, including the initial geometric imperfection, temperature, flange and web slenderness, effective length and steel grade. The assumed initial imperfection affects only the post-buckling response with no noticeable difference in the pre-buckling response. Besides, it is found that an assumed maximum initial out-of-plane bow of $L_i/1000$ in the form of full sine curve produces a conservative estimate of rotational capacity; hence this form of imperfection is used in the parametric studies. The numerical study on temperature effects reveals an interesting observation

that the rotational capacity does not always reduce as temperature increases. The lowest rotational capacity is somewhere around 700°C. It is found that the relationship between rotational capacity and temperature can be attributed to the different material stress-strain characteristics at elevated temperatures. One key factor of stress-strain characteristics, which is the ratio of elastic modulus reduction factor to yield strength reduction factor at elevated temperature k_E/k_y , is found to have a similar trend with the variation of rotational capacity at elevated temperature.

Numerical studies on the effects of flange and web slenderness have strengthened the conclusions obtained from the test results, that is, by increasing the flange and web slenderness ratio, the rotational capacity and the maximum moment will be reduced.

The study on the effective length concludes that a shorter effective length will delay or eliminate the occurrence of lateral torsional buckling, hence increasing the rotational capacity. It should also be noted that rotational capacity can be enhanced by reducing the effective length of beams if lateral torsional buckling is dominant, otherwise, there is hardly any effect on rotational capacity.

Higher steel grade results in less rotational capacity. One reason is due to the differences in the stress-strain relationships among the different steel grades. The other reason is that by using a higher steel grade, a greater plastic moment capacity is obtained; hence plastic rotation will be attained at a higher rotation, resulting in less inelastic rotation.

Composite Beams

The influence of several parameters on the inelastic behaviour of composite beams at elevated temperature have been investigated, including the temperature, flange and web slenderness, reinforcement area, effective length and number of shear studs. It has been observed that as temperature increases, the rotational capacity of composite beams

reduces. This trend is different from steel beams, in which rotational capacity does not always reduce as temperature increases. Another observation is that local buckling occurs over a greater length at ambient temperature compared to that at elevated temperature.

The effects of increasing flange and web slenderness of composite beams are similar to those of steel beams. As the flange and the web slenderness increase, the rotational capacity reduces. The effects of different reinforcement area on rotational capacity are negligible, with only a slight reduction in the rotational capacity as reinforcement area increases. Parametric studies on the number of shear studs conclude that by over designing the shear connections, the moment-rotation response is slightly stiffer due to less slip between the structural steel and the concrete slab. Finally, the effects of increasing the effective length in the composite beams are generally the same as in steel beams, that is, increasing the effective length decreases both the maximum moment and rotational capacity.

8.1.5 Modelling of Moment-Rotational Relationship

The objective of proposing the model for moment-rotational relationships of both steel and composite beams subjected to hogging moment at elevated temperatures has been achieved. The proposed moment-rotation design model comprises three parts: a non-linear pre-peak curve, a horizontal plateau at the plastic moment capacity and an unloading curve. The moment-rotational relationship for the first part before buckling occurs is evaluated from the moment-curvature relationship of the member cross-section. Since the stress-strain relationship of steel at elevated temperature is highly non-linear, the moment-rotational relationship must be obtained numerically by discretizing the cross-section. A horizontal line at the plastic moment capacity connecting the non-linear pre-peak curve with the unloading curve is deemed sufficient for the second part since the maximum moment is generally less than 10% above the plastic moment capacity.

For the third part of the moment-rotational relationship, two approaches are proposed for the steel beams while only one approach is suggested for the composite beams. The two approaches for steel beams are the statistical based approach and the plastic collapse mechanism approach. While the plastic collapse mechanism model simulates the buckling mechanism quite well, it is rather complex for design purpose. The statistical based approach using multi-linear regression offers a quick, accurate, and practical method for practitioners and design engineers. Only the plastic collapse mechanism approach is proposed for composite beams.

It has been shown that the correlation of the proposed approaches with both the experimental and numerical results is reasonably good. The predicted unloading curve based on the statistical approach is close to the test and numerical results while the one based on plastic collapse mechanism approach is slightly higher providing an upper bound curve.

8.2 Recommendations

The followings are some recommendations for future research:

1. The proposed approaches for predicting the lateral torsional buckling moment at elevated temperatures have been validated with mainly FE predictions since only limited test results are available and the experimental results (Vila Real et al., 2003) are rather inconclusive for certain range of temperature. Hence, it is necessary to conduct additional tests to gain a better understanding of LTB failure at elevated temperatures. The steel beams with temperature above 600°C and $\overline{\lambda_{LT,fi}}$ more than 1.5 should also be considered since they have not been reported so far.
2. The composite beam tests that have been carried out in the present research have been limited to four due to available resources. In addition, only composite beams with full shear connections were considered. Therefore, it is proposed that additional tests be conducted in the future which includes beams with partial shear connections

and beams with other types of decking and shear connectors. Future tests may also include more slender section (Class 3 and 4 sections) so that some parameter limits can be drawn to determine whether a member is able to attain its plastic moment capacity before local buckling occurs.

3. Both the steel and composite beam tests simulate the outer span of continuous beams, of which the axial restraint force is negligible; hence no axial restraint was applied during the tests. As a continuation to this research, it is recommended that the inner span of continuous beam, of which the thermal restraint force is present, be studied.
4. This research has quantified the moment-rotational relationship and ductility of both steel and composite beams at elevated temperatures. As a result, the proposed moment-rotational relationship can be incorporated as a rotational spring at the beam-column joints. Hence, the analysis of the frame behaviour under fire can be conducted using simple beam-column elements instead of shell elements, without neglecting the effects of localised buckling which may occur. Thus, the author recommends that the behaviour of steel and composite frames related to the redistribution of moment during fire be studied utilising the proposed moment-rotational relationship to take into account the effects of local buckling.

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APPENDIX A

APPENDIX A1

Derivation of a Crooked Beam

Consider a beam that has an initial curvature and twist such that

$$\frac{u_0}{\delta_0} = \frac{\phi_0}{\theta_0} = \sin \frac{\pi z}{L} \quad (\text{A.1})$$

in which the central initial lack of straightness δ_0 and θ_0 are related by

$$\frac{\delta_0}{\theta_0} = \frac{M_{cr}}{\pi^2 EI_y / L^2} \quad (\text{A.2})$$

The out-of-plane bending and torsional equations are

$$EI_y \frac{d^2 u}{dz^2} = -M(\phi + \phi_0), \text{ and} \quad (\text{A.3})$$

$$GJ \frac{d\phi}{dz} - EI_w \frac{d^3 \phi}{dz^3} = M \left(\frac{du}{dz} - \frac{du_0}{dz} \right) \quad (\text{A.4})$$

with the boundary conditions:

$$(u)_0 = (u)_L = 0, \text{ and } (\phi)_0 = (\phi)_L = 0 \quad (\text{A.5})$$

The solution of eq. (A.3) and (A.4) which satisfies the boundary conditions (eq. (A.5)) is given by

$$\frac{u}{\delta} = \frac{\phi}{\theta} = \sin \frac{\pi z}{L} \quad (\text{A.6})$$

$$\text{in which } \frac{\delta}{\delta_0} = \frac{\theta}{\theta_0} = \frac{M/M_{cr}}{1 - M/M_{cr}} \quad (\text{A.7})$$

The maximum longitudinal stress in the beam is the sum of the stresses due to major axis bending, minor axis bending and warping, and is equal to

$$f_{\max} = \frac{M}{Z_x} - \frac{EI_y}{Z_y} \left(\frac{d^2 (u + d_f \phi / 2)}{dz^2} \right)_{L/2} \quad (\text{A.8})$$

$$\text{Note: } u + d_f \phi / 2 = \delta_0 \left(\frac{M/M_{cr}}{1 - M/M_{cr}} \right) \sin \frac{\pi z}{L} + \frac{d_f}{2} (\theta_0) \left(\frac{M/M_{cr}}{1 - M/M_{cr}} \right) \sin \frac{\pi z}{L} \quad (\text{A.9})$$

$$\frac{d^2(u + d_f \phi / 2)}{dz^2} = - \left(\frac{M/M_{cr}}{1 - M/M_{cr}} \right) \frac{\pi^2}{L^2} \sin \frac{\pi z}{L} \left(\delta_0 + \frac{d_f}{2} \theta_0 \right) \quad (\text{A.10})$$

If the elastic limit is taken as the yield stress f_y , eq. (A.8) becomes

$$f_y = \frac{M}{Z_x} - \frac{EI_y}{Z_y} \left(- \frac{M/M_{cr}}{1 - M/M_{cr}} \right) \frac{\pi^2}{L^2} \left(\delta_0 + \frac{d_f}{2} \theta_0 \right) \quad (\text{A.11})$$

therefore, the limiting nominal stress $f_L = M_L/Z_x$, for which the above elastic analysis

is valid is given by

$$f_L = f_y - \frac{EI_y}{Z_y} \left(\frac{M_L/M_{cr}}{1 - M_L/M_{cr}} \right) \frac{\pi^2}{L^2} \left(\delta_0 + \frac{d_f}{2} \theta_0 \right) \quad (\text{A.12})$$

$$M_L = M_y - \frac{EI_y \pi^2}{L^2} \left(\frac{M_L/M_{cr}}{1 - M_L/M_{cr}} \right) \frac{Z_x}{Z_y} \left(\delta_0 + \frac{d_f}{2} \theta_0 \right) \quad (\text{A.13})$$

$$\text{Note: } N_{oy} = \pi^2 EI_y / L^2; \quad M_L = f_L Z_x; \quad \text{and} \quad M_y = f_y Z_x \quad (\text{A.14})$$

$$M_L = M_y - \frac{N_{oy}}{M_{cr}} \left(\frac{M_L}{1 - M_L/M_{cr}} \right) \frac{Z_x}{Z_y} \left(\delta_0 + \frac{d_f}{2} \theta_0 \right) \quad (\text{A.15})$$

$$M_L = M_y - \frac{\delta_0 N_{oy}}{M_{cr}} \left(\frac{M_L}{1 - M_L/M_{cr}} \right) \frac{Z_x}{Z_y} \left(1 + \frac{d_f}{2} \frac{N_{oy}}{M_{cr}} \right) \quad (\text{A.16})$$

In the case where the central lack of straightness δ_0 is given by

$$\frac{\delta_0 N_{oy}}{M_{cr}} = \theta_0 = \frac{Z_y/Z_x}{1 + (d_f/2)(N_{oy}/M_{cr})} \frac{M_y}{4M_{cr}} \quad (\text{A.17})$$

$$\text{then, } M_L = M_y - \frac{1}{4} \frac{M_L M_y}{M_{cr} - M_L} \quad (\text{A.18})$$

$$M_L^2 - M_L (M_{cr} + 1.25 M_y) + M_{cr} M_y = 0 \quad (\text{A.19})$$

$$M_L = \frac{M_{cr} + 1.25 M_y \pm \sqrt{(M_{cr} + 1.25 M_y)^2 - 4 M_{cr} M_y}}{2} \quad (\text{A.20})$$

$$\frac{M_L}{M_y} = \left(\frac{1.25 + M_{cr}/M_y}{2} \right) - \sqrt{\left(\frac{1.25 + M_{cr}/M_y}{2} \right)^2 - \frac{M_{cr}}{M_y}} \quad (\text{A.21})$$

APPENDIX A2

Derivation of Slenderness Ratio Expression in BS5950-1 (BSI, 2001)

Original expression of slenderness ratio definition:

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{f_y}} \sqrt{\frac{M_{cx}}{M_E}} \quad (\text{A.22})$$

Rearrange the expression for slenderness ratio in a form that is used in BS 5950:2000,

$$\lambda_{LT} = \sqrt{\frac{\pi^2 E}{f_y}} \sqrt{\frac{M_{cx}}{M_E}} \quad (\text{A.23})$$

$$\text{Substituting } M_E = \frac{M_{cr}}{\sqrt{(1 - EI_y/EI_x)}} = p_E Z_{ex}; \quad M_{cr} = \sqrt{\left(\frac{\pi^2 EI_y}{L^2}\right)} \sqrt{GJ + \frac{\pi^2 EI_w}{L^2}}$$

and $M_{cx} = f_y Z_{ex}$, one obtains

$$\lambda_{LT} = \frac{\sqrt{\frac{\pi^2 E Z_{ex} \left(\sqrt{1 - EI_y/EI_x}\right)}{\frac{\pi^2 EI_y}{L^2} \left(GJ + \frac{\pi^2 EI_w}{L^2}\right)}}}{\sqrt{\frac{\pi^2 EI_y}{L^2} \left(GJ + \frac{\pi^2 EI_w}{L^2}\right)}} \quad (\text{A.24})$$

$$\lambda_{LT} = \frac{\sqrt{\pi^2 E Z_{ex} \left(\sqrt{1 - EI_y/EI_x}\right)} \times \left(\frac{A^2 h_s^2}{A^2 h_s^2}\right)^{1/4} \times \left(\frac{4S_x^2}{4S_x^2}\right)^{1/4}}{\sqrt{\frac{I_y}{L^2} \left(GJ + \frac{\pi^2 EI_w}{L^2}\right)}} \quad (\text{A.25})$$

$$\lambda_{LT} = u \sqrt{\frac{\sqrt{\pi^2 E Z_{ex} A h_s}}{\frac{I_y}{L^2} 4S_x^2 \left(GJ + \frac{\pi^2 EI_w}{L^2}\right)}} \quad (\text{A.26})$$

$$\text{in which, } u = \left\{ \frac{4S_x^2 (1 - EI_y/EI_x)}{A^2 h_s^2} \right\}^{1/4} \quad (\text{A.27})$$

$$\lambda_{LT} = u \sqrt{\beta_w} \sqrt{\frac{\sqrt{\pi^2 E A h_s}}{\frac{I_y}{L^2} 4 \left(GJ + \frac{\pi^2 EI_w}{L^2}\right)}} \quad (\text{A.28})$$

$$\text{in which } \beta_w = Z_{ex}/S_x \quad (\text{A.29})$$

$$\lambda_{LT} = u\lambda\sqrt{\beta_w} \sqrt{\frac{Ah_s r_y^2}{L\sqrt{4I_y\left(\frac{GJ}{\pi^2 E} + \frac{I_w}{L^2}\right)}}} \quad (\text{A.30})$$

$$\text{in which } \lambda = L_E/r_y \quad (\text{A.31})$$

Noting that $G = \frac{E}{2(1+\nu)}$ and $\nu = 0.3$; therefore

$$\lambda_{LT} = u\lambda\sqrt{\beta_w} \left(\frac{Ah_s^2 r_y^2}{4(0.03896JL^2 + I_w)} \right)^{1/4} \quad (\text{A.32})$$

Note: $I_w = I_f h_s^2/2$ and $I_y \approx 2I_f$; therefore $I_w = r_y^2 Ah_s^2/4$

$$\lambda_{LT} = u\lambda\sqrt{\beta_w} \left(\frac{Ah_s^2 r_y^2}{4(0.03896JL^2 + (r_y^2 Ah_s^2/4))} \right)^{1/4} \times \frac{\sqrt[4]{0.32}}{\sqrt[4]{0.32}} \quad (\text{A.33})$$

$$\lambda_{LT} = \frac{1}{\sqrt[4]{1 + 0.05 \left(\frac{L}{0.566r_y h_s \sqrt{A/J}} \right)^2}} u\lambda\sqrt{\beta_w} \quad (\text{A.34})$$

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w} \quad (\text{A.35})$$

in which $v = \left\{ 1 + 0.05(\lambda/x)^2 \right\}^{-1/4}$ and $x = 0.566h_s (A/J)^{1/2}$

APPENDIX A3**Data and Results on the Numerical Analysis of Lateral Torsional Buckling**

Table A.1 FEA Data and Results for S275 at 400°C

GRADE 275

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	UM	145.46	0.216
686x254x125UB	12	UM	161.52	0.199
686x254x125UB	9	UM	129.86	0.256
457x191x98UB	9	PL	110.73	0.344
457x191x98UB	12	UM	153.58	0.211
457x191x98UB	9	UM	129.37	0.274
305x165x54UB	12	UM	159.87	0.203
305x165x54UB	9	UM	135.58	0.256
305x165x54UB	9	PL	116.05	0.315
203x102x23UB	12	UM	205.52	0.139
203x102x23UB	9	UM	176.59	0.175
203x102x23UB	6	UM	141.14	0.229
356x406x287UC	3	UM	21.59	0.979
356x406x287UC	6	UM	36.97	0.881
305x305x158UC	9	UM	61.09	0.691
305x305x158UC	6	UM	47.16	0.804
254x254x89UC	9	UM	72.90	0.602
254x254x89UC	12	UM	100.49	0.407
254x254x89UC	12	PL	86.01	0.505
254x254x89UC	6	UM	56.45	0.716

Table A.2 FEA Data and Results for S355 at 400°C

GRADE 355

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	UM	145.46	0.184
914x305x201UB	12	PL	124.50	0.237
686x254x125UB	12	UM	161.52	0.148
305x165x54UB	9	PL	116.05	0.295
305x165x54UB	9	UM	135.58	0.202
356x406x287UC	6	UM	36.97	0.889
356x406x287UC	3	PL	21.59	0.981
305x305x158UC	9	UM	61.09	0.650
305x305x158UC	6	UM	47.16	0.774
254x254x89UC	12	UM	100.49	0.362
254x254x89UC	12	PL	86.01	0.476
254x254x89UC	9	PL	72.90	0.543

APPENDIX A

Table A.3 FEA Data and Results for S275 at 500°C

GRADE 275

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	UM	145.5	0.236
457x191x98UB	9	PL	110.7	0.383
457x191x98UB	12	UM	153.6	0.246
457x191x98UB	9	UM	129.4	0.292
305x165x54UB	12	UM	159.9	0.217
305x165x54UB	9	UM	135.6	0.293
305x165x54UB	9	PL	116.0	0.362
203x102x23UB	12	UM	205.5	0.161
203x102x23UB	9	UM	176.6	0.193
356x406x287UC	3	PL	21.6	0.971
356x406x287UC	6	PL	37.0	0.903
305x305x158UC	9	PL	61.1	0.734
305x305x158UC	6	PL	47.2	0.816
254x254x89UC	9	PL	72.9	0.642
254x254x89UC	12	UM	100.5	0.443
254x254x89UC	12	PL	86.0	0.552

Table A.4 FEA Data and Results for S355 at 500°C

GRADE 355

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	PL	124.50	0.256
914x305x201UB	12	UM	145.46	0.198
686x254x125UB	12	UM	161.52	0.154
305x165x54UB	9	PL	116.05	0.291
305x165x54UB	9	UM	135.58	0.210
356x406x287UC	6	PL	36.97	0.906
356x406x287UC	3	PL	21.59	0.964
305x305x158UC	9	PL	61.09	0.679
305x305x158UC	6	PL	47.16	0.796
254x254x89UC	12	UM	100.49	0.401
254x254x89UC	12	PL	86.01	0.504
254x254x89UC	9	PL	72.90	0.578

Table A.5 FEA Data and Results for S275 at 600°C

GRADE 275

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	UM	145.46	0.202
457x191x98UB	9	PL	110.73	0.323
457x191x98UB	12	UM	153.58	0.197
457x191x98UB	9	UM	129.37	0.255
305x165x54UB	12	UM	159.87	0.184
305x165x54UB	9	UM	135.58	0.245
305x165x54UB	9	PL	116.05	0.298
203x102x23UB	12	UM	205.52	0.134
203x102x23UB	9	UM	176.59	0.164
356x406x287UC	3	PL	21.59	0.961
356x406x287UC	6	PL	36.97	0.869
305x305x158UC	9	PL	61.09	0.679
305x305x158UC	6	PL	47.16	0.781
254x254x89UC	9	PL	72.90	0.587
254x254x89UC	12	UM	100.49	0.383
254x254x89UC	12	PL	86.01	0.464

Table A.6 FEA Data and Results for S355 at 600°C

GRADE 355

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
356x406x287UC	6	PL	36.97	0.867
356x406x287UC	3	PL	21.59	0.951
305x305x158UC	9	PL	61.09	0.652
305x305x158UC	6	PL	47.16	0.750
254x254x89UC	12	UM	100.49	0.328
254x254x89UC	12	PL	86.01	0.423
254x254x89UC	9	PL	72.90	0.514
914x305x201UB	12	PL	124.50	0.223
914x305x201UB	12	UM	145.46	0.170
686x254x125UB	12	UM	161.52	0.155
305x165x54UB	9	PL	116.05	0.263
305x165x54UB	9	UM	135.58	0.184

Table A.7 FEA Data and Results for S275 at 700°C

GRADE 275

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	UM	145.46	0.179
457x191x98UB	9	PL	110.73	0.289
457x191x98UB	12	UM	153.58	0.168
457x191x98UB	9	UM	129.37	0.208
305x165x54UB	12	UM	159.87	0.189
305x165x54UB	9	UM	135.58	0.232
305x165x54UB	9	PL	116.05	0.263
203x102x23UB	12	UM	205.52	0.116
203x102x23UB	9	UM	176.59	0.155
356x406x287UC	3	PL	21.59	0.937
356x406x287UC	6	PL	36.97	0.848
305x305x158UC	9	PL	61.09	0.689
305x305x158UC	6	PL	47.16	0.787
254x254x89UC	9	PL	72.90	0.548
254x254x89UC	12	UM	100.49	0.328
254x254x89UC	12	PL	86.01	0.434

Table A.8 FEA Data and Results for S355 at 700°C

GRADE 355

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
356x406x287UC	6	PL	36.97	0.840
356x406x287UC	3	PL	21.59	0.939
305x305x158UC	9	PL	61.09	0.589
305x305x158UC	6	PL	47.16	0.725
254x254x89UC	12	UM	100.49	0.303
254x254x89UC	12	PL	86.01	0.384
254x254x89UC	9	PL	72.90	0.470
914x305x201UB	12	PL	124.50	0.193
914x305x201UB	12	UM	145.46	0.159
686x254x125UB	12	UM	161.52	0.130
305x165x54UB	9	PL	116.05	0.229
305x165x54UB	9	UM	135.58	0.195

Table A.9 FEA Data and Results for S275 at 800°C

GRADE 275

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
914x305x201UB	12	UM	145.46	0.255
457x191x98UB	9	PL	110.73	0.384
457x191x98UB	12	UM	153.58	0.243
457x191x98UB	9	UM	129.37	0.318
305x165x54UB	12	UM	159.87	0.237
305x165x54UB	9	UM	135.58	0.285
305x165x54UB	9	PL	116.05	0.370
203x102x23UB	12	UM	205.52	0.178
203x102x23UB	9	UM	176.59	0.216
356x406x287UC	3	PL	21.59	0.977
356x406x287UC	6	PL	36.97	0.937
305x305x158UC	9	PL	61.09	0.742
305x305x158UC	6	PL	47.16	0.844
254x254x89UC	9	PL	72.90	0.657
254x254x89UC	12	UM	100.49	0.461
254x254x89UC	12	PL	86.01	0.557

Table A.10 FEA Data and Results for S355 at 800°C

GRADE 355

Section Size	L (m)	Loading	λ_{LT}	$p_{b,fi}/f_{yT}$
356x406x287UC	6	PL	36.97	0.921
356x406x287UC	3	PL	21.59	0.971
305x305x158UC	9	PL	61.09	0.704
305x305x158UC	6	PL	47.16	0.804
254x254x89UC	12	UM	100.49	0.441
254x254x89UC	12	PL	86.01	0.516
254x254x89UC	9	PL	72.90	0.600
914x305x201UB	12	PL	124.50	0.270
914x305x201UB	12	UM	145.46	0.210
686x254x125UB	12	UM	161.52	0.174
305x165x54UB	9	PL	116.05	0.324
305x165x54UB	9	UM	135.58	0.232

APPENDIX B

APPENDIX B1

Longitudinal Imperfection Measurements

Figure B.1 Set-Up for Longitudinal Imperfection Measurements

Figure B.2 Data Logger

APPENDIX B

Figure B.3 In-Plane Imperfection of S1-1

Figure B.4 Out-of-Plane Imperfection of S1-1

Figure B.5 In-Plane Imperfection of S1-2

APPENDIX B

Figure B.6 Out-of-Plane Imperfection of S1-2

Figure B.7 In-Plane Imperfection of S2-1

Figure B.8 Out-of-Plane Imperfection of S2-1

APPENDIX B

Figure B.9 In-Plane Imperfection of S2-2

Figure B.10 Out-of-Plane Imperfection of S2-2

Figure B.11 In-Plane Imperfection of S3-1

APPENDIX B

Figure B.12 Out-of-Plane Imperfection of S3-1

Figure B.13 In-Plane Imperfection of S3-2

Figure B.14 Out-of-Plane Imperfection of S3-2

APPENDIX B

Figure B.15 In-Plane Imperfection of S3-3

Figure B.16 Out-of-Plane Imperfection of S3-3

Figure B.17 In-Plane Imperfection of S4-1

APPENDIX B

Figure B.18 Out-of-Plane Imperfection of S4-1

Figure B.19 In-Plane Imperfection of S4-2

Figure B.20 Out-of-Plane Imperfection of S4-2

APPENDIX B2

Photographs from First Series of Tests – Steel Beams

S1 Specimens

Figure B.21 Tested S1-1 Specimen – Elevation View

Figure B.22 Tested S1-1 Specimen – 3D View

Figure B.23 Tested S1-2 Specimen – Elevation View

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Figure B.24 Tested S1-2 Specimen – 3D View

S2 Specimens

Figure B.25 Tested S2-1 Specimen – Plan View

Figure B.26 Tested S2-1 Specimen – 3D View

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Figure B.27 Tested S2-1 Specimen – Elevation View

Figure B.28 Tested S2-2 Specimen – Lateral Torsional Buckling

Figure B.29 Tested S2-2 Specimen – 3D View

APPENDIX B

Figure B.30 Tested S2-2 Specimen – Plan View

S3 Specimens

Figure B.31 Tested S3-1 Specimen – Elevation View

Figure B.32 Tested S3-1 Specimen – 3D View

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Figure B.33 Tested S3-1 Specimen – Web Buckling

Figure B.34 Tested S3-2 Specimen – Elevation View

Figure B.35 Tested S3-2 Specimen – 3D View

APPENDIX B

Figure B.36 Tested S3-2 Specimen – Flange Buckling

Figure B.37 Tested S3-3 Specimen – Elevation View

Figure B.38 Tested S3-3 Specimen – Flange Buckling

Figure B.39 Tested S3-3 Specimen – 3D View

S4 Specimens

Figure B.40 Tested S4-1 Specimen – Elevation View

Figure B.41 Tested S4-1 Specimen – 3D View

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Figure B.42 Tested S4-1 Specimen – Flange Buckling

Figure B.43 Tested S4-2 Specimen – 3D View

Figure B.44 Tested S4-2 Specimen – Flange Buckling 1

Figure B.45 Tested S4-2 Specimen – Flange Buckling 2

APPENDIX C

APPENDIX C1

Push-Out Test

Figure C.1 Detailed Configuration of Push-Out Test Specimen (CEN, 2004)

APPENDIX C

Figure C.2 Push-Out Test Specimen

Figure C.3 Compression Machine for Push-Out Test

Figure C.4 Push-Out Test Set-Up

APPENDIX C2

Detailed Design of Composite Beam Specimens

Figure C.5 Layout of C1 Specimen

Figure C.6 Layout of C2 Specimen

Figure C.7 Layout of C3 Specimen

Figure C.8 Layout of C4 Specimen

APPENDIX C3

Photographs from Second Series of Tests – Composite Beams

Figure C.9 Tested C1 Specimen – Elevation View

Figure C.10 Tested C1 Specimen – Plan View

Figure C.11 Tested C2 Specimen – Elevation View

Figure C.12 Tested C2 Specimen – Flange Buckling

Figure C.13 Tested C3 Specimen – Concrete Slab Cracking

Figure C.14 Tested C3 Specimen – Elevation View

Figure C.15 Tested C3 Specimen – Web Buckling

Figure C.16 Tested C4 Specimen – Elevation View

Figure C.17 Tested C4 Specimen – Web Buckling

Figure C.18 Tested C4 Specimen – Flange Buckling

Figure C.19 Tested C4 Specimen – Concrete Slab Cracking