Characterization of smart PZT transducer and admittance signatures using PZT-structure interaction models for structural health monitoring

Annamdas Venu Gopal Madhav

2007

Annamdas, V. G. M. (2007). Characterization of smart PZT transducer and admittance signatures using PZT-structure interaction models for structural health monitoring. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/12066>

<https://doi.org/10.32657/10356/12066>

Nanyang Technological University

Downloaded on 26 Apr 2023 14:50:08 SGT

**CHARACTERIZATION OF SMART PZT
TRANSDUCER AND ADMITTANCE SIGNATURES USING
PZT- STRUCTURE INTERACTION MODELS FOR
STRUCTURAL HEALTH MONITORING**

ANNAMDAS VENU GOPAL MADHAV

**SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

2007

**Characterization of Smart PZT Transducer and Admittance
Signatures using PZT-Structure Interaction Models for
Structural Health Monitoring**

Annamdas Venu Gopal Madhav

(M.E., Indian Institute of Science-Bangalore)

School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirements for the degree of
Doctor of Philosophy

April 2007

DEDICATED TO

My PhD thesis is dedicated to Mahatma Gandhi, Dr. A. P. J. Abdul Kalam, Mother Teresa, Nelson Mandela and all peace lovers of the world.

Annamdas Venu Gopal Madhav

ACKNOWLEDGEMENTS

I am greatly thankful to my mother (Annamdas Janaki) and father (Annamdas Lakshmi Narayana) for their caring, support and encouragement for all these years of my life.

I am greatly thankful to my supervisor Professor Soh Chee Kiong for his support during these three years. He freely encouraged my ideas, provided me ample resources and guidance, and helped me develop independent research capabilities. His suggestions on my papers were very valuable. I also thank the school of CEE for facilitating to me this research opportunity at NTU. I also thank my research group members especially Prof. Yang Yoawan, Mr. Zhang Lei and Dr. Wang Chao for their suggestions and encouragements during research meetings.

I am thankful to my colleagues, seniors, juniors and final year project students for their valuable discussions and help in my research.

I owe my deep gratitude to my teachers, especially Gopala Krishna Murthy, Ayub Khan and Abidhali Khan whose teachings, guidance and well-wishes were my only inspiration to pursue this degree. I thank all my teachers of Indian Institute of Science, Bangalore especially my M.E supervisor Dr. Chandra Kishan for encouraging me to go for PhD. They teach that character cultivation precedes acquiring any knowledge, and without good character mere technical knowledge can be badly misused. Their perfect example of principle-centered living has helped many students, including me. I am far from being their representative student. Yet, through this work and its results, my only wish is to bring them satisfaction by proper application of their precepts.

My genuine appreciation and thanks are for the lab technicians of the Structures laboratory and the Construction laboratory for their generous support, and ever willingness to offer assistance during my experiments.

I am thankful to my brothers Kiran and Hari, sisters Mamatha and Aditya, and my wife Hema Latha Madhav for providing me excellent moral support and encouragement; I am always indebted to their support.

I thank my childhood classmates and friends, Mr. Arvind, Mr. Malkapuram Srinivas, Dr. Alok Gupta and Mr. Mahajan, for their inspirational mails during last three years. I thank Mrs. Rupali Bhalla for providing me moral support and giving me good wishes, without her, my first year stay in Singapore would have not been comfortable. I also thank my friends in NTU, Manish, Nagi Reddy, Xe Ziquing, Dr. A. Sudhakar Kulkarni, Dr. Gu Xiaoming, Dr. Wan Caiyun and Dr. Tu for there friendship and making my three years of stay (in Singapore) comfortable and wonderful. I also sincerely thank my best friend, guide, philosopher and brother Krishna for his moral support and blessings during crucial periods of my life.

Lastly I remember Mahatma Gandhi, Abdul Kalam (President of India), Mother Teresa and Nelson Mandela (Former President of South Africa) for there contributions to freedom, peace and humanity and their philosophies are always in my mind and always stimulate my ideas.

Annamdas Venu Gopal Madhav

TABLE OF CONTENTS

ACKNOWLEDGEMENTS.....	i
TABLE OF CONTENTS.....	iii
SUMMARY.....	viii
LIST OF FIGURES.....	x
LIST OF TABLES.....	xvi
LIST OF ACRONYMS.....	xviii
LIST OF SYMBOLS.....	xx

CHAPTER 1.....	1
INTRODUCTION.....	1

1.1 Overview of structural damages and failures.....	1
1.2 Structural health monitoring.....	2
1.3 Research motivations.....	3
1.3.1 Requirements of good health monitoring systems.....	3
1.3.2 SHM by electromechanical impedance (EMI) technique.....	4
1.3.3 EMI models based on non-thickness actuations of PZT transducers.....	5
1.3.4 EMI models for surface bonded PZT- structure interaction.....	6
1.3.5 EMI models for single PZT-host structure interaction.....	7
1.3.6 Characterization of PZT transducer and EM admittance signatures.....	7
1.3.7 Model based approach for damage characterization.....	8
1.4 Research objectives.....	8
1.5 Report organization.....	9

CHAPTER 2.....	11
LITERATURE REVIEW ON STRUCTURAL HEALTH MONITORING	11

2.1 Structural health monitoring.....	11
2.1.1 Visual inspection method (VIM).....	12
2.1.2 Low frequency vibration techniques.....	13
2.1.3 Static structural response techniques.....	14
2.1.4 Localized NDE techniques.....	14
2.1.5 Smart material based high frequency techniques.....	14
2.2 History of piezoceramic transducers	16
2.3 State of the art of PZT transducers	16
2.3.1 Choice of transducer constituents	16

2.3.2 Piezoceramic element.....	19
2.4 Basic principle of piezoceramic materials.....	21
2.5 Mechanical impedance of structures	24
2.6 Fabrication techniques of embedded piezoceramic transducers	26
2.7 1D and 2D EMI models based on high frequency excitation	28
2.7.1 1D model of Liang et al., (1994)	28
2.7.2 2D Model of Zhou et al., (1995, 1996)	32
2.7.3 2D model of Bhalla and Soh (2004)	33
2.8 Some of the key developments in field of SHM during last decade	38
2.9 Key comparisons between SHM and, noise and vibration control	41
2.10 Limitations of existing models	43
2.11 Concluding remarks.....	44

CHAPTER 3.....45

EMBEDDED PIEZOELECTRIC CERAMIC TRANSCUDERS IN SANDWICHED BEAMS.....45

3.1 Introduction.....	45
3.2 ASI Model.....	45
3.2.1 Fundamental PZT constitutive equations	47
3.2.2 ASI of actuator	50
3.2.3 Derivation of actuator impedance from stress-strain relationships..... of PZT	50
3.2.4 ASI of host structure	52
3.2.5 Expression for complex EM admittance	54
3.3 Experimental and Numerical Analysis	55
3.3.1 Experimental specimen	55
3.3.2 Experimental initialization.....	57
3.3.3 Determination of structural mechanical impedance using FEM	62
3.3.4 Convergence test	66
3.4 Experimental Results and Discussion.....	69
3.5 Concluding Remarks	72

CHAPTER 4.....73

THREE DIMENSIONAL (3D) ELECTROMECHANICAL IMPEDANCE MODEL FORMULATION OF DIRECTIONAL SUM IMPEDANCE (DSI).....73

4.1 Introduction.....	73
4.2 Dimensions of PZT transducer and their influences on EM admittance signature....	73

4.3	Directional sum impedance (DSI) of PZT-host structure interaction model.....	77
4.3.1	Stress- strain relationship of PZT transducer element subjected to 3D loading.....	82
4.3.2	Solution to stress-strain differential equations of PZT transducer.....	85
4.4	Experimental specimens	86
4.5	Finite element analysis	87
4.6	Numerical evaluation of structural responses and linear impedances.....	90
4.6.1	Stress- strain relationship of PZT transducer element subjected to 3D loading	93
4.7	Admittance formulation.....	94
4.8	Experimental results and discussion.....	97
4.9	Concluding remarks.....	100

CHAPTER 5.....101

CHARACTERIZATION OF ADMITTANCE SIGNATURE AND PZT USING 3D DSI MODEL101

5.1	Introduction.....	101
5.2	Dimensions of PZT transducer and their influences on 3D DSI admittance.....	102
5.3	Characterization of properties of PZT	106
5.4	Effect of property variations on damage indices.....	116
5.5	Frequency Range	118
5.6	Damage description and assessment	118
5.7	PZT efficiency factor.....	134
5.8	Conclusions.....	139

CHAPTER 6.....140

UNIPLEXING AND MULTIPLEXING OF PZT TRANSDUCERS140

6.1	Introduction.....	140
6.2	Experimental investigation	141
6.3	Uniplexing using multiple (>2) PZT transducers.....	143
6.3.1	Uniplexing using thicker transducer	145
6.3.2	Uniplexing using thinner transducers.....	151
6.4	Uniplexing using only two transducers	152
6.4.1	Uniplexing using type $T_{0.3}$ PZT transducers.....	153
6.4.2	Uniplexing using types $T_{0.75}$ and T_1 PZT transducers.....	153
6.5	Multiplexing using multiple PZT transducers	155
6.6	Conclusions.....	158

CHAPTER 7159
3D EMI MODEL FOR MULTIPLE PZT-STRUCTURE INTERACTION159

7.1 Introduction.....	159
7.2 Multiple piezoceramic transducers –structure (MPZT-S) interaction model	160
7.3 Stress-strain relationship of PZT transducer subjected to 3D loading	164
7.4 Three dimensional (3D) differential equation.....	165
7.5 Solution to 3D differential equations.....	166
7.6 Optimal solutions and behaviour of solution in space and time domain.....	167
7.7 Stress-strain relationship in presence of electric field	169
7.8 Experimental specimens	170
7.9 Numerical analysis	172
7.10 Formulations of structural responses and structural linear impedances using numerical analysis as input.....	174
7.11 Admittance formulation	179
7.12 Modal analysis.....	181
7.13 Experimental results and discussion.....	183
7.14 Conclusions.....	187

CHAPTER 8.....188
EXTENDED MPZT-S MODEL FOR MULTIPLE PZT- STRUCTURE
INTERACTION.....188

8.1 Introduction.....	188
8.2 Adhesive thickness influence on EM admittance signature	190
8.3 Extension to MPZT-S (ExMPZT-S) model	191
8.3.1 Linear impedances and PZT forces.....	191
8.3.2 Shear transfer mechanism.....	194
8.3.3 Real dimension method	194
8.3.4 Stresses acting on differential element.....	195
8.4 Case study	196
8.4.1 Modifications to linear impedances and PZT forces.....	197
8.4.2 Numerical Analysis	198
8.4.3 Predicted Admittance Signature.....	200
8.5 Results and discussions.....	201
8.6 Conclusions.....	204

CHAPTER 9	205
CONCLUSIONS AND RECCOMENDATIONS	205
9.1 Research contributions and originality	205
9.1.1 Originality of research.....	205
9.1.2 Research contributions	208
9.2 Characterization of PZT transducers	209
9.3 Characterization of EM admittance signatures	210
9.4 Applications and limitations of the developed interaction models	211
9.4.1 ASI model (SISO based)	211
9.4.2 DSI model (SISO based)	211
9.4.3 MPZT-S model (MISO based).....	212
9.4.4 ExMPZT-S model (MISO based)	212
9.5 Recommendations for Future Work	212
9.5.1 ASI model.....	212
9.5.2 DSI model.....	213
9.5.3 PZT efficiency factor.....	213
9.5.4 Characterizations of PZT transducer.....	213
9.5.5 MPZT-S.....	214
9.5.6 ExMPZT-S.....	214
PUBLICATIONS.....	215
REFERNECS	216

SUMMARY

Structural health monitoring (SHM) and security monitoring (SM) are two important challenges for the modern world in recent time. The monitoring of existing aero, civil and mechanical (ACM) structures has become a regular feature after the world witnessed the various recent deadly failures and damages, due to natural calamities (like earth quakes) or continuous usage of structures causing wear and tear or terror attacks. These include the infamous Tsunami in South East Asia (2004), space shuttle Columbia's explosion (2003) and the September 11, 2001, New York world trade centre terror attack. SHM has attracted the effort of many engineers throughout the world and is fast emerging as a pioneering field. It is helping to improve world economy in two ways; first directly, by providing longevity to all the structures and secondly, by saving millions of dollars via prevention of untimely failure. The field of SHM is very vast, consisting of traditional monitoring methods (such as visual inspection, static/ dynamic structural response methods) as well as smart system based methods.

The last few years have witnessed rapid development in the areas of non-destructive evaluation (NDE) by the emergence of the electromechanical impedance (EMI) technique. This technique employs piezoelectric-ceramic (PZT) transducers for the prediction of structural response known as electromechanical (EM) admittance.

Engineering structures can be classified into two categories based on their stiffness, those which are more stiff than and those which are less stiff than the PZT material. Surface bonded PZT transducers are more efficient when they are stiffer than the host structure, and embedded transducers are more efficient when they are less stiff than the host structure. Both types of PZT transducers are important in the EMI based NDE of the two categories of engineering structures. However, surface bonded PZT transducers have seen more prominent applications in the recent past in SHM. This research developed both surface bonded and embedded PZT- structure interaction models for SHM of existing and future ACM structures.

In addition, most of the existing EMI models are single PZT based interaction models, which has limited area of sensing. These models cannot be

applicable for SHM of huge structures where multiple PZT transducers need to be installing at critical locations of the structure. Thus, the present research developed such multiple PZT- based interaction models. Furthermore PZT material properties are easily influenced by external factors such as the atmosphere and the presence of electric and magnetic fields. Hence, this research made a step by step analysis to understand the PZT properties prior to their usage in the interaction models. Thus, this research presented the characterization of the PZT properties. Last but not least, this research also studied the influencing factors of EM admittance using the developed interaction models; to characterize the EM admittance signature.

LIST OF FIGURES

CHAPTER 1	1
Figure.1 Recent infamous structural damages.	2
(Collapse of Air France Terminal in DeGaul airport – Paris)	2
CHAPTER 2	11
Figure 2.1 Experimental setup	15
Figure 2.2 A 3D piezoelectric material sheet.....	21
Figure 2.3 Representation of harmonic force and velocity by rotating phasors	25
Figure 2.4 Parallel and series combination of a spring-mass-damper system	26
Figure 2.5 Piezoceramic element embedded in carbon-fibre composite.....	27
Figure 2.6 Piezoceramic element embedded in a glass/epoxy laminate.....	28
Figure 2.7 Modelling PZT –structure interaction behaviour	29
Figure 2.8 Modelling of 2D physical coupling by impedance approach.....	33
Figure 2.9 Modelling of interaction of PZT and structure in 2D.....	34
Figure 2.10 Actuations of PZT in presence of electric fields E_1 , E_2 and E_3	43
(a) Uni extensional actuation (b) Bi extensional actuation	43
(c) Extensional, longitudinal and shear actuations	43
CHAPTER 3	46
Figure 3.1 Electromechanical modelling of embedded PZT transducers	48
(a) PZT transducer embedded in sandwiched aluminium beams	48
(b) Electric polarization of the embedded PZT transducer	48
Figure 3.2 Forces acting on the contact area of PZT transducer at any instant of time	49
(a) Expansion of transducer in direction X and shrinkage in direction Z	49
(b) Expansion of transducer in direction Z and shrinkage in direction X.....	49
Figure 3.3 Experimental specimen used in ASI based impedance method	56
(a) Free PZT transducer (b) Embedded PZT transducer.....	56
(c) photo view of specimen 1 and 2.....	56
Figure 3.4 Plots of free-PZT transducer signatures of specimen	61
(a) Conductance signatures (b) Susceptance signatures	61
Figure 3.5 Plots of free-PZT signatures of specimen	62
(a) Conductance signatures (b) Susceptance signatures	62
Figure 3.6 FE mesh of half of the specimen	63
Figure 3.7 Layers used for meshing.....	66

Figure 3.8 Graphical representation of highest modal frequency of specimens 1 and 2.....	69
(a) Mode 28 ($f = 118.982$ kHz) for specimen 1	69
(b) Mode 18 ($f = 119.099$ kHz) for specimen 2.....	69
Figure 3.9 Experimental and analytical signatures of specimen 1.....	70
(a) Conductance (b) Susceptance.....	70
Figure 3.10 Experimental and analytical signatures of specimen 2.....	71
(a) Conductance (b) Susceptance.....	71
CHAPTER 4	73
Figure 4.1 Admittance signatures of four free-free PZT transducers.	76
(a) Conductance signatures (b) Susceptance signatures	76
Figure 4.2 3D distribution of forces and impedances on one-quarter of PZT transducer (a) Embedded PZT transducer.	78
(b) Surface bonded PZT transducer.	78
Figure 4.3 Schematic representation of stresses acting on a differential element.	80
Figure 4.4. Specimens with FE mesh and boundary conditions	88
(a) Embedded PZT specimen and PZT actuations.....	88
(b) FE mesh of embedded PZT specimen.....	88
(c) Surface bonded PZT specimen and PZT actuations.....	89
(d) FE mesh of surface bonded PZT specimen.....	89
Figure 4.5 Admittance signature of embedded transducer specimen	98
(a) Conductance (b) Susceptance.....	98
Figure 4.6 Admittance signature of surface bonded transducer specimen	99
(a) Conductance (b) Susceptance.....	99
CHAPTER 5	101
Figure 5.1 Admittance signature of numerical specimens.....	104
(a) Embedded specimens: conductance of (b) Susceptance	104
(c) Surface bonded specimens: conductance (d) Susceptance.....	105
Figure 5.2 Admittance signature of embedded PZT specimen using PZT-S	112
(a) Conductance (b) Susceptance.....	112
Figure 5.3 Difference of conductance signature of PZT-S for various PZT property variations. (a) Deviations of variations 2, 3, 4 and 7 from variation 1.....	113
(b) Deviations of variations 5 and 6 from variation 1.....	113
Figure 5.4 Elements of FE Analysis of embedded PZT specimen	117
and damages (a) Undamaged state (b) Damage 1 (c) Damage 2.....	117
Figure 5.5 Admittance signatures of pre and post damage cases for variation 1 of PZT-S.....	119
(a) conductance of frequency range 50- 60 kHz.....	119
(b) susceptance of frequency range 50- 60 kHz.....	119
(c) conductance of frequency range 75- 85 kHz.....	119

(d) susceptance of frequency range 75- 85 kHz.....	119
Figure 5.6 Admittance signatures of pre and post damage cases for variation 2 of PZT-S.....	120
(a) conductance of frequency range 50- 60 kHz.....	120
(b) susceptance of frequency range 50- 60 kHz.....	120
(c) conductance of frequency range 75- 85 kHz.....	120
(d) susceptance of frequency range 75- 85 kHz.....	120
Figure 5.7 Admittance signatures of pre and post damage cases for variation 3 of PZT-S.....	121
(a) conductance of frequency range 50- 60 kHz.....	121
(b) susceptance of frequency range 50- 60 kHz.....	121
(c) conductance of frequency range 75- 85 kHz.....	121
(d) susceptance of frequency range 75- 85 kHz.....	121
Figure 5.8 Admittance signatures of pre and post damage cases for variation 4 of PZT-S.....	122
(a) conductance of frequency range 50- 60 kHz.....	122
(b) susceptance of frequency range 50- 60 kHz.....	122
(c) conductance of frequency range 75- 85 kHz.....	122
(d) susceptance of frequency range 75- 85 kHz.....	122
Figure 5.9 Admittance signatures of pre and post damage cases for variation 5 of PZT-S.....	123
(a) conductance of frequency range 50- 60 kHz.....	123
(b) susceptance of frequency range 50- 60 kHz.....	123
(c) conductance of frequency range 75- 85 kHz.....	123
(d) susceptance of frequency range 75- 85 kHz.....	123
Figure 5.10 Admittance signatures of pre and post damage cases for variation 6 of PZT-S.....	124
(a) conductance of frequency range 50- 60 kHz.....	124
(b) susceptance of frequency range 50- 60 kHz.....	124
(c) conductance of frequency range 75- 85 kHz.....	124
(d) susceptance of frequency range 75- 85 kHz.....	124
Figure 5.11 Admittance signatures of pre and post damage cases for variation 7 of PZT-S.....	125
(a) conductance of frequency range 50- 60 kHz.....	125
(b) susceptance of frequency range 50- 60 kHz.....	125
(c) conductance of frequency range 75- 85 kHz.....	125
(d) susceptance of frequency range 75- 85 kHz.....	125
Figure 5.12 Admittance signatures of pre and post damage cases for variation 1 of PZT -R.....	126
(a) conductance of frequency range 50- 60 kHz.....	126
(b) susceptance of frequency range 50- 60 kHz.....	126
(c) conductance of frequency range 75- 85 kHz.....	126
(d) susceptance of frequency range 75- 85 kHz.....	126
Figure 5.13 Admittance signatures of pre and post damage cases for variation 2 of PZT -R.....	127
(a) conductance of frequency range 50- 60 kHz.....	127

(b) susceptance of frequency range 50- 60 kHz.....	127
(c) conductance of frequency range 75- 85 kHz.....	127
(d) susceptance of frequency range 75- 85 kHz.....	127
Figure 5.14 Admittance signatures of pre and post damage cases for variation 3	
of PZT -R	128
(a) conductance of frequency range 50- 60 kHz.....	128
(b) susceptance of frequency range 50- 60 kHz.....	128
(c) conductance of frequency range 75- 85 kHz.....	128
(d) susceptance of frequency range 75- 85 kHz.....	128
Figure 5.15 Admittance signatures of pre and post damage cases for variation 4	
of PZT -R	129
(a) conductance of frequency range 50- 60 kHz.....	129
(b) susceptance of frequency range 50- 60 kHz.....	129
(c) conductance of frequency range 75- 85 kHz.....	129
(d) susceptance of frequency range 75- 85 kHz.....	129
Figure 5.16 Admittance signatures of pre and post damage cases for variation 5	
of PZT -R	130
(a) conductance of frequency range 50- 60 kHz.....	130
(b) susceptance of frequency range 50- 60 kHz.....	130
(c) conductance of frequency range 75- 85 kHz.....	130
(d) susceptance of frequency range 75- 85 kHz.....	130
Figure 5.17 Admittance signatures of pre and post damage cases for variation 6	
of PZT -R	131
(a) conductance of frequency range 50- 60 kHz.....	131
(b) susceptance of frequency range 50- 60 kHz.....	131
(c) conductance of frequency range 75- 85 kHz.....	131
(d) susceptance of frequency range 75- 85 kHz.....	131
Figure 5.18 Admittance signatures of pre and post damage cases for variation 7	
of PZT -R	132
(a) conductance of frequency range 50- 60 kHz.....	132
(b) susceptance of frequency range 50- 60 kHz.....	132
(c) conductance of frequency range 75- 85 kHz.....	132
(d) susceptance of frequency range 75- 85 kHz.....	132
Figure 5.19 RMSD (%) of different variations for damages 1 and 2 of	
specimen 1	133
(a) PZT-S: for frequency range of 50-60 kHz	133
(b) PZT-S: for frequency range of 75-85 kHz	133
(c) PZT- R: for frequency range of 50-60 kHz	133
(d) PZT-R: for frequency range of 75-85 kHz.....	133
Figure 5.20 PZT efficiency factor of different variations with damages 1 and 2	
of specimen 1 (a) RMSD vs mean frequency (b) PZT-S (c) PZT -R.....	136

CHAPTER 6	140
Figure 6.1 Wave distribution in Uniplexing	141
Figure 6.2 Sequence of bonding and locations of benchmark PZT transducer.....	142
Figure 6.3. Mechanical interference due to successive mass additions of benchmark type T_3 PZT at A[1].....	146
(a) Conductance differences between initial and consecutive mass additions....	146
(b) Mechanical interference pattern @ 60 KHz.....	147
Figure 6.4 Conductance differences between initial and consecutive mass additions of benchmark type T_3 PZT at B[3]	149
Figure 6.5 Conductance differences between initial and subsequent mass additions of benchmark type T_3 PZT at C[5]	150
Figure 6.6 Conductance differences between initial and subsequent mass additions of benchmark type $T_{0.3}$ PZT at A[1]	151
Figure 6.7 Conductance differences between initial and subsequent mass additions of benchmark type $T_{0.3}$ PZT at B[3].....	152
Figure 6.8. Conductance differences between initial and varying distances of PZT addition for type $T_{0.3}$ PZT transducer.....	153
Figure 6.9 Conductance differences between initial and varying distances of PZT addition for type $T_{0.75}$ PZT transducer	154
Figure 6.10 Conductance differences between initial and varying distances of PZT addition for type T_1 PZT transducer.....	154
Figure 6.11 Conductance of initial and consecutive mass additions of type T_3 PZT transducer in multiplexing.....	156
Figure 6.12 Conductance of initial and consecutive mass additions of type $T_{0.3}$ PZT transducer in multiplexing	157
 CHAPTER 7	 159
Figure 7.1 Multiple PZT-structure interaction.....	160
Figure 7.2 Schematic representation of stresses acting on a differential element.....	162
Figure 7.3 3D wave propagation of multiple PZT in the presence of sinusoidal voltage (a) Wave 1 in space (b) Wave 2 in space (c) Waves 1 and 2 in time domain	168
Figure 7.4 Experimental specimen and transducers	171
(a) Dimensions and spacing (b) Photo view of specimen.....	171
(c) Photo view of PZT transducer.....	171
Figure 7.5 Kth PZT transducer (a) Overview of aluminium symmetric specimen (case III) (b) Numerical model	173
(c) Distribution of forces along direction X.....	174
Figure 7.6 Experimental and predicted signatures of case I.....	184

(a) Conductance (b) Susceptance.....	184
Figure 7.7 Experimental and predicted signatures of case II.....	185
(a) Conductance (b) Susceptance.....	185
Figure 7.8 Experimental and predicted signatures of case III	186
(a) Conductance (b) Susceptance.....	186
 CHAPTER 8.....	188
Figure 8.1 Epoxy wrapped PZT transducer	189
Figure 8.2 'Free' and epoxy wrapped PZT conductance signatures of PZT	190
Figure 8.3 Conductance signatures of four epoxy wrapped PZT	191
Figure 8.4 ExMPZT-S interaction model	192
Figure 8.5 PZT- structure interaction.....	195
(a) Real dimension method	195
(b) Equivalent dimension method.....	195
Figure 8.6 Schematic representation of stresses acting on a.....	196
differential element	196
Figure 8.7 Faces of one-quarter of PZT specimen.....	197
Figure 8.8 Finite element mesh of one-quarter specimen 1	198
Figure 8.9 Finite element mesh of one quarter specimen 2	199
Figure 8.10 Conductance signatures of experimental and ExMPZT-S Model.....	202
(a) Experimental (b) ExMPZT-S Model.....	202
Figure 8.11 Susceptance signatures of experimental and ExMPZT-S Model.....	203
(a) Experimental (b) ExMPZT-S Model.....	203

LIST OF TABLES

CHAPTER 2	11
Table 2.1 Some varieties of PZT transducers	21
CHAPTER 3	46
Table 3.1 Key properties of Epoxy adhesive (RS 850-940) and Al 6061-T6.....	47
Table 3.2 Key properties of PZT Grade A.....	47
Table 3.3 Details of specimens and PZT transducer.....	57
Table 3.4 Material properties and α_p variations for specimen 1	60
Table 3.5 Mesh sizes used for specimens 1 and 2.....	67
Table 3.6 Model frequencies for different mesh sizes of specimen 1.....	68
CHAPTER 4	73
Table 4.1 Key properties of PZT transducer (Grade B).....	74
Table 4.2 PZT transducer dimensions.....	75
Table 4.3 Dimensions of specimens.....	86
Table 4.4 Details of PZT and mesh sizes of specimens	90
CHAPTER 5	101
Table 5.1 Dimensions of specimens.....	103
Table 5.2 Dimensions of PZT transducers used in specimens.....	103
Table 5.3 Different variations and variations in properties of PZT transducer	110
Table 5.4 Slope and peak influence details of PZT transducer properties.....	114
Table 5.5 Peak heights and PEF for variations of PZT transducer properties	138
CHAPTER 6	140
Table 6.1 Key properties of PZT transducer of grade C	143
Table 6.2 Location of benchmark PZT transducers and their mass additions	145

CHAPTER 7	159
Table 7.1 Modal frequencies of 0.3mm thick multiple PZT transducers.....	182
CHAPTER 8	188
Table 8.1 Dimensions of epoxy adhesive, Al and PZT.....	196
Table 8.2 Mesh sizes of specimens	200

LIST OF ACRONYMS

ACM	Aero, Civil and Mechanical
ASI	Average Sum Impedance
CC	Correlation Coefficient
DSI	Directional Sum Impedance
EDP	Effective Drive Point
EM	Electromechanical
EMI	Electromechanical Impedance
ExMPZT-S	Extended Multiple Piezoelectric Ceramic Transducer –Structure Interaction Model
FEA	Finite Element analysis
FEM	Finite Element method
IDT	Inter Digital transducers
IEEE	Institute of Electrical and Electronics Engineers
ISRO	Indian Space Research Organization
MIMO	Multiple-Input, Multiple-Output
MISO	Multiple-Input, Single-Output
MIT	Mechatronic Impedance Transducer
MPZT-S	Multiple Piezoelectric Ceramic Transducer –Structure Interaction Model
NASA	National Aeronautics and Space Administration
NDE	Non-Destructive Evaluation
PVDF	Polyvinylidene Fluoride

PLZT	Lanthanide-Modified Piezoceramic Transducer
PEF	PZT Efficiency Factor
PZT	Piezoelectric Ceramic Transducer
PZT-AA	Piezoelectric Ceramic Transducer (Active Actuator)
PZT-MA	Piezoelectric Ceramic Transducer (Mass Additives)
PZT-R	Piezoelectric Ceramic Transducer (Rectangle)
PZT-S	Piezoelectric Ceramic Transducer (Square)
PZT-SA	Piezoelectric Ceramic Transducer (Active Sensor)
RC	Reinforced Concrete
RMSD	Root Mean Square Deviation
SHM	Structural Health Monitoring
SISO	Single-Input, Single-Output
SM	Security Monitoring
1D	One- Dimensional
2D	Two Dimensional
3D	Three Dimensional
VIM	Visual Inspection Method
FSM	Full Solution Method
RSM	Reduced Solution Method

LIST OF SYMBOLS

ϵ^T	Dielectric permittivity Tensor
F	Effective Force
F_X, F_Y, F_T and F_B	total forces components acting on face X,Y, Z_{top} and Z_{bottom} of PZT transducer
D	Electric Displacement
S	Strain Tensor
S_1 and S_3	Strain applied on the PZT transducer in directions X and Z respectively
d_{zj}	Piezoelectric strain coefficient
s^E	Elastic Compliance Tensor
T	Stress Tensor
T_1 and T_3	Stresses applied on the PZT transducer in directions X and Z respectively
E_1, E_2, E_3	Applied External Electric Field along X, Y and Z directons
Y	Static Young's Modulus of Elasticity
$\bar{Y}$	Complex Young's Modulus of Elasticity
$\overline{G^E}$	Complex Shear Modulus
ν	Poisson's Ratio
t	Time

ϕ	Phase Angle
Z	Mechanical Impedance
Z_s	Average Sum Impedance of Host Structure
Z_{eq}	Equivalent Mechanical Impedance
Z_a	Short-Circuited Mechanical Impedance
Z_{xx}, Z_{yy}	Direct Impedances
$Z_{xy}, Z_{yx},$	Cross Impedances
Z_{as}	Average Sum Impedance of Actuator
Z_s	Average Sum Impedance
m	Mass
k	Spring Constant
κ	Wave Number,
c	Damper Constant
f	Natural Frequency
ω	Angular Frequency
u	Displacement
ρ	Density
η	Mechanical Loss Factor
δ	Dielectric Loss Factor
α_p	PZT Dependent Factor
I	Electric Current
$\bar{I}$	Instantaneous Electric Current

I_x, I_y, I_z	Inertial forces due to the PZT mass acting along the X, Y and Z directions respectively
G	Conductance
B	Susceptance
G_{Con}	Conductance of Consecutive PZT
$G_{Initial}$	Conductance of initial PZT
G_{free}	Free Conductance of PZT
B_{free}	Free Susceptance of PZT
l	Half -Length
$\hat{n}$	Unit Vector normal to the boundary
u_{eff}	Effective Displacement
δA	Change in the Cross-Sectional Area of the transducer
p_o	Perimeter in the undeformed condition
$\dot{u}_{eff}$	Effective Velocity
$\dot{u}_I$	Total transducer velocity
V_o	Instantaneous Voltage
FP_H	Total Force components acting on the PZT transducer along X direction
FP_T	Total Force components acting on the PZT transducer along Z Direction
F_{TE}, F_{BE}, F_{HE}	Total forces acting on the faces (top, bottom and side) of the PZT transducer

$2L_{Bm}, W_{Bm}, 2H_{Bm}$	Dimensions of beam specimens
A_{XY}	Total Surface Area
A_{0-Free}	Coefficient of Vibration in the absence of Z_s
$\bar{Y}_{at}$	Electric Admittance of PZT Transducer
$\bar{Y}_{Fr-at}$	Complex electromechanical admittance of free PZT transducer
$\bar{Y}_A$	Complex electromechanical admittance of structure
$\bar{Y}^A$	Complex electromechanical admittance of structure with multiple PZT transducers
$\bar{Y}_C$	Electromechanical admittance of complete structure
$\bar{Y}_Q^A$	Quarter admittance of the one-quarter specimen
D_F	Dimensional Factor
MP_F	Material Property Factor
H_{al}	Thickness of the top and bottom aluminium layers
H_{Ep}	Thickness of the epoxy layer
L_{BM}	Half-length of the beam specimen
$[M]$	Mass Matrix
$[K]$	Stiffness Matrix
$[M_i^e]$	Individual element of Mass Matrix
NE	Number of elements
$[K_i^e]$	Individual element Stiffness Matrix

$\{F\}$	Applied Harmonic Force Matrix
$[C]$	Structural Damping Matrix
α_d	Mass Damping Factor
β	Stiffness Damping Factor
M	Material type
$\{F\}$	Applied Harmonic Force Vector
F_R, F_I	Real and Imaginary components of $\{F\}$
$[u]$	Complex displacement vector
u_R, u_I	Real and Imaginary components of the displacement vector
σ	Normal Stress
τ	Shear Stress
σ_X, σ_Y and σ_Z	Normal stresses acting on the PZT transducer along the X, Y and Z directions respectively
τ_{XY}, τ_{YZ} and τ_{ZX}	Shear stresses acting on the PZT transducer along the XY, YZ and ZX directions respectively
ϵ_X, ϵ_Y and ϵ_Z	Normal strains along the X, Y and Z directions of PZT transducer
γ_{XY}, γ_{YZ} and γ_{ZX}	Shear strains along the XY, YZ and ZX directions respectively
Y_R, R and r	Simplification parameters used in 3D DSI model
R_1, R_2 and R_3	Simplification parameters
λ	Response factors
ϵ	Normal Strains

γ	Shear Strains
$\overline{\epsilon_{33}}$	Complex Electric Permittivity
ϵ_{33}	Static Electric Permittivity
C	Means of the frequency range
R	RMSD value
$T_{0.3}, T_{0.75}$ and T_1	PZT Transducer of different thickness
Z_{MS}	Impedance of the host structure bonded with multiple PZT transducers
A_o, C_o, E_o	Vibration coefficients along directions X, Y and Z of PZT transducers
f, f_1, h, h_1, g, g_1	Wave functions in space
ϕ_1, ϕ_2, ϕ_3	Wave functions in space and time
$(D_3)_K$	Charge density of the K^{th} PZT transducer

CHAPTER 1

INTRODUCTION

1.1 Overview of structural damages and failures

Whatever may be the material, engineering structures are bound to undergo damage at some point in their lifetime. The damage could develop because of continuous usage, degradation, environmental factors, earthquakes or terror strikes. Structural health monitoring (SHM) and security monitoring (SM) have emerged as two important areas where the modern world is looking in the recent time, especially after witnessing deadly failures and damage state. Some of the infamous examples are:

- (i) Mid-flight accident of Boeing 737 plane of Aloha Airlines on April 28, 1988, in which entire fuselage panels were ripped apart against air pressure.
- (ii) Terror strikes on the New York World Trade Centre (WTC) on Sept 11, 2001.
- (iii) Mid air crash of the American airbus, A 300-600 (Flight 587) on Nov 12, 2001.
- (iv) NASA space shuttle Columbia's failure due to small undetected damage in its left wing on Feb 1, 2003.
- (v) Partially collapse of Air France Terminal 2E in DeGaul airport in Paris on May 23 2004 as shown in Fig 1.
- (vi) Tsunami in Indonesia on Dec 26, 2004.

Three (i, iii and v) of these damage state occurred due to failure of existing SHM methods in detecting small unseen cracks, which eventually led to catastrophic failures. Thus, every one agrees unanimously that there is need for continuous monitoring of structures (either SHM or SM or both) to check the severity of damage state and to prevent the structures from failing. During the last

couple of decades, the concept of continuous health monitoring, especially using smart materials, is gaining importance (Chang, 2000). SHM, especially using smart piezoceramic material, has emerged as a powerful non destructive evaluation (NDE) tool in the recent times.

Figure.1 Recent infamous structural damages.
(Collapse of Air France Terminal in DeGaul airport – Paris)

1.2 Structural health monitoring

‘Damage’ is a disorder developed in a structure that can adversely affect its performance. ‘Failure’ is the inability of a part or whole of a structure to perform its intended function. ‘Destruction’ refers to the highest degree of (multiple) damage(s). In fact, destruction by nature is complex and always happened in multiple degrees (i.e multiple damages, cracks and failures) resulting in loss of lives and properties. Hence, the most important concern of SHM is to detect damages (or failures) in structures on time. Depending on the damage identification principle, the current SHM techniques can be classified as follows.

- (i) Visual inspection methods,
- (ii) Low frequency vibration techniques,
- (iii) Statistical structural response techniques,
- (iv) Localized NDE techniques,
- (v) Smart system based high frequency vibration techniques.

The limitations of the conventional visual inspection methods, the structural response based methods and the non-smart NDE methods have been highlighted by Bhalla (2001) and Naidu (2003). The smart system based methods are relatively new and more powerful than the conventional methods. Smart systems are those systems in which automated mechanisms takeoff to monitor the structures for damage detection in real time. Sensing technology coupled with smart material properties have emerged as powerful monitoring tools. These smart systems coupled with NDE techniques resulted in more effective NDE methods (Bhalla, 2004).

1.3 Research motivations

The need to address problems of NDE based SHM in a simple yet generic and powerful manner using modern smart technology for monitoring the existing and future aero, civil and mechanical (ACM) structures is the prime motivation for the present research. Additionally, the ‘limitations’ of the existing smart system based models to accomplish the needs of SHM as explained below have motivated the present research.

1.3.1 Requirements of good health monitoring systems

In the field of civil engineering, structures like tall buildings, bridges, etc. are increasing in size to meet the requirements of the increasing population and competitiveness of the world. Similarly, the modern age aerospace shuttles, launch vehicles and satellites are also increasing in size to cater to increasing demand (Ruggiero *et al.*, 2004; Ruggiero and Inman, 2005), especially, for the expedition of space, moon and outer planets (NASA, 2006; ISRO, 2006). Therefore, there is a compelling need to have high quality online structural health monitoring (SHM) of such large, modern structures.

Thus, researchers around the world need to continuously develop new online SHM systems characterized with innovation, quick response, small size and simplicity to suite the existing and future ACM structures. These automated systems should be able to monitor a part or whole of the structure without disturbing its functionality.

Sensing (or actuating) devices are usually either mounted or embedded, depending on the type of sensing (or actuating) material and the type of structure. The materials employed in smart material based SHM systems include piezoelectric, electrostrictive and magnetostrictive elements, electrorheological fluids or solids, shape memory alloys and optical fibres.

1.3.2 SHM by electromechanical impedance (EMI) technique

Over the last few decades, smart materials were extensively used in the fields of noise and vibration controls, and SHM of various ACM structures because of their inherent stimulus-response. These can be surface mounted or embedded in the structures for the diagnosis of damages.

Especially, the piezoelectric-ceramic (PZT) material has emerged as powerful self sensing material, which can excite the structures at high frequencies in the order of kHz. These transducers, when activated at higher modes, facilitate easy detection of minor structural damages (Doebling *et al.*, 1998). The piezoelectric materials belong to the class of dielectrics, which exhibit significant material deformations in response to an applied electric field and conversely produce dielectric polarization in response to mechanical strains (Banks *et al.*, 1996a, 1996b). Even though the piezo-electric crystals (PZT transducers) were first employed in 1940 (Banks *et al.*, 1996b), their application in the electromechanical impedance (EMI) method is one of recent (decade long) developments. In this method, a PZT transducer needs to be permanently bonded on the surface of the monitored structure. The self sensing of PZT transducer enables transduction of electric energy to mechanical energy and vice versa between the PZT transducer and the host structure (Liang *et al.*, 1994) producing structural response known as electromechanical (EM) admittance. Thus, in this method, PZT behaves both as active actuator and active sensor.

Liang *et al.*, (1994) expressed the electrical admittance (inverse of electrical impedance) of the bonded PZT transducer in terms of drive point mechanical impedance of the host structure. The drive point ‘mechanical impedance’ at any point on the structure is the ratio of the driving harmonic force to the resultant

harmonic velocity in the longitudinal direction (along length) of the PZT transducer.

Later, Bhalla and Soh (2004c) expressed the electrical admittance in terms of effective ‘mechanical impedance’ of the host structure. The effective mechanical impedance at any point on a structure is defined as the ratio of the effective force to the resultant effective velocity. Since the ‘mechanical impedance’ of any structure depends on its material properties, structural configuration and boundary conditions, any damage will alter these properties and the alterations are induced in EMI of the structure. These alterations are finally reflected in the EM admittance of the PZT transducer. Thus, at any excited frequency, PZT transducer, either surface bonded or embedded inside the host structure, produces structural response known as the EM ‘admittance signature’. The changes in these signatures are indicative of the presence of structural damages (Sun *et al.*, 1995). Hence, the admittance signature is a function of the stiffness, mass and damping of the host structure (Sun *et al.* 1995), and also the length, width, thickness and orientation of the PZT transducer (Wetherhold *et al.* 2003). However, the existing EMI models (Liang *et al.* 1994; Zhou *et al.* 1996; Bhalla and Soh 2004) have not established ‘complete relationship’ between dimensions of PZT and structure, and EM admittance signature. Especially, the thickness of the PZT was not included in their formulations, which has motivated a part of present research.

1.3.3 EMI models based on non-thickness actuations of PZT transducers

Generally, actuations (Raja *et al.* 2004) of the PZT transducers in the presence of electric fields in the three principle directions can be divided into extensional (along length and width directions), longitudinal (along thickness direction) and shear actuations (in planes connecting length and thickness or width and thickness). However, in the EMI technique, electric field is applied only along the thickness direction, producing extensional actuation along the length and width, and longitudinal actuation along thickness of PZT transducer (For details see next chapter). So far, researchers (Crawley and Luis 1987; Liang *et al.* 1993; Zhou *et al.* 1996; Giurgiutiu *et al.* 1999; Park *et al.* 2003; Bhalla and Soh 2004; Pairs *et al.*

2004) have developed interaction models which are based only on the extensional actuation of PZT transducer ignoring the actuations of the transducer in thickness direction.

Liang *et al.* (1993) developed one dimensional (1D) PZT-host structure interaction model based on the assumption that the PZT transducer behaves as a bar undergoing ‘uni-extensional’ actuation (axial vibration) along the length direction. Zhou *et al.* (1996) extended the 1D model to a 2D model which was later simplified by Bhalla and Soh (2004a). The 2D model assumes that the actuation between the transducer and the host structure is not restricted along the length direction but rather, the actuation is ‘bi-extensional’ extending all over the finite sized PZT transducer along both the length and width directions. In addition, the transducer is assumed to be mechanically and electrically isotropic with plane stress conditions prevailing within the transducer and also the transducer is square in shape and small in thickness. Thus, there is a need for 3D interaction model.

1.3.4 EMI models for surface bonded PZT- structure interaction

ACM structures can be classified into two categories based on their stiffness, those which are stiffer than and those which are less stiff than the PZT material. Surface bonded PZT transducers are more efficient when they are stiffer than the host structure, and embedded transducers are more efficient when they are less stiff than the host structure (Benjeddou *et al.* 2000). Both types of PZT transducers are important in the EMI based NDE of the two categories of ACM structures. In the past, most researchers (Liang *et al.* 1994; Zhou *et al.* 1996; Bhalla and Soh 2004) limited their research to surface bonded PZT – structure interaction models. Moreover, these are valid only for thin PZT transducers, are non-generic in nature, and can not be used for thick PZT transducers. Thus, there is a need to develop EMI models applicable for both surface and embedded PZT; additionally the models should be valid to both thin and thick PZT transducers.

1.3.5 EMI models for single PZT-host structure interaction

As said in section 1.3.1, the sizes of ACM structures are increasing to cater to the need of increasing population and competitiveness of the world. Such large structures require large online structural health monitoring (SHM), and noise and vibration control systems.

The last few decades have seen many ‘single-input, single-output’ (SISO) based single PZT–structure interaction models in SHM (Crawley and Luis 1987; Liang *et al.* 1994; Zhou *et al.* 1996; Park *et al.*, 2003) and, noise and vibration controls (Raja *et al.* 2004; Park *et al.*, 2002). However there is a need to study ‘multiple-input, multiple-output’ (MIMO) or ‘multiple-input, single-output’ (MISO) based multiple PZT–structure interaction models in addition to the SISO models for large structures. Ruggiero *et al.* (2004) outlined MIMO modal analysis technique using smart materials for use in noise and vibration controls. Ruggiero and Inman (2005) stressed the need to analyse such MIMO model to extract modal parameters of large (satellite) structures for use in noise and vibration controls. Thus, similar research is needed in area of SHM of large structures, which has so far been ignored.

Hence, there is a need to develop generic EMI models which suit both single and multiple PZT-structure interaction to cater to the demands of existing and future ACM structures.

1.3.6 Characterization of PZT transducer and EM admittance signatures

The EM admittance signature is a function of the stiffness, mass and damping of the host structure and, length, width, thickness and orientation of the PZT transducer. However, the mechanical and electrical properties of the PZT transducer will also influence the EM admittance signature. Hence, it is important for any numerical or analytical EMI model to characterize the PZT transducer and its host structure to properly predict EM admittance signature. However, the

existing 1D and 2D EMI models could not characterize PZT as they are limited to square, thin and isotropic PZT. Thus, there is a need to develop generic EMI model, which does not have shape (square/rectangle), size (thin/thick) and isotropic (isotropic/non-isotropic) restrictions.

1.3.7 Model based approach for damage characterization

Damage detection and characterization are two important features of any damage analysis. Bhalla (2001) compared many non-parametric indices like root mean square deviation (RMSD) and correlation coefficient (CC), and observed that the RMSD between the signatures is the most suitable non-parametric damage index to quantify structural damage. These indices, although indicate the existence of damages, fail to characterize the damages (Naidu, 2003). Moreover, the structural response (EM admittance signature) changes with any change in frequency of excitation, and the peaks (or valleys) in any admittance signatures are frequency dependent. Thus, the efficiency of any non-parametric indices lies in its ability to indicate the damage using either the changed peaks (or valleys) of signatures and the frequency range of excitation.

However, RMSD or CC uses only the ‘changed peaks (or valleys)’ of signatures without using the frequency range of excitation. Thus, there exists a need to develop a non-parametric index (or factor) to estimate the damage detection capability of PZT using the ‘changed peaks (or valleys)’ and frequency range of excitation.

1.4 Research objectives

In this research, the main objectives are to extend the applicability of the EMI methods to develop generic models which considers actuations of thickness direction along with actuations of length and width directions of the PZT transducer. The specific goals are as given below

1. To develop a generic 2D embedded PZT-structure interaction model considering extensional actuations in the ‘length’ and longitudinal actuations in the ‘thickness’ direction, i.e. a 2D SISO model applicable for embedded PZT (see section 1.3.4).
2. To develop a generic 3D PZT-structure interaction model considering actuations in the length, width and thickness directions for predicting admittance, i.e. a 3D SISO model applicable for both surface and embedded PZT (see section 1.3.3).
3. To characterize PZT transducer and admittance signatures using 3D model and to develop a new non-parametric index using simulated damage propagation (see sections 1.3.6 and 1.3.7).
4. To investigate experimentally the uniplexing and multiplexing based MISO and MIMO models by parallel excitations of PZT transducers (see section 1.3.5).
5. To develop a generic 3D multiple PZT- structure interaction model considering actuations in the length, width and thickness directions for predicting admittance, i.e. a 3D MISO model (see section 1.3.5).
6. To extend the 3D multiple PZT model for adhesively wrapped PZT transducer using experimental and numerical simulations.

In summary, the research aimed to first characterize the PZT transducers using SISO and MISO based 3D PZT-structure interaction models and to later characterize the admittance signatures for successful NDE based SHM of the existing and future ACM structures.

1.5 Report organization

This thesis consists of 9 chapters. In this introductory chapter 1, overview of SHM technology and the need for research in EMI-based SHM are discussed. The research objectives are defined.

In Chapter 2, a review of the topics of SHM using piezo-ceramic transducers are covered, including an overview of current admittance formulations.

The current limitations and challenges in the implementation of existing models on real life structures are discussed.

In Chapter 3, based on concepts outlined in Chapter 2, an embedded generic NDE model for laminated beams is developed and validated, covering the first research objective. The second research objective is presented in Chapter 4, i.e. a generic 3D single PZT-structure interaction model is formulated and experimentally validated.

In Chapter 5, proof-of-concept of the 3D model using simulated damage is presented. A non-parametric index named as PZT efficiency factor (PEF) is presented. Additionally, characterization of PZT is presented.

In Chapter 6, the experimental investigation of uni and multiplexing based MISO and MIMO models are presented. The fifth research objective is presented in Chapter 7, i.e. a generic 3D multiple PZT-structure interaction model is formulated and experimentally validated.

In Chapter 8, the extension of 3D multiple PZT model for adhesively wrapped PZT transducer using experimental and numerical simulations is presented.

Finally in Chapter 9, the conclusions, contributions and future work in the field of SHM and 3D model are presented. This is followed by a list of my publications and a comprehensive list of references cited in this research.

CHAPTER 2

LITERATURE REVIEW ON STRUCTURAL HEALTH

MONITORING

This chapter reviews the existing models in SHM technology relevant to this research work. Following sections include literature about the concepts of SHM, embedded transducers and the existing SHM models.

2.1 Structural health monitoring

This overview of a few recent catastrophic accidents (Chapter 1) clearly highlighted the importance of detection of very small structural damage, from its incipient growth to a threshold leading to catastrophe. Hence, even a minor damage should not be ignored since it carries the potential to grow and cause failure, either leading to wide scale loss of life or property or both.

Structural health monitoring (SHM) is defined in the literature as the “acquisition, validation and analysis of technical data to facilitate life cycle management decisions,”. SHM denotes a reliable system with the ability to detect and interpret adverse ‘changes’ in a structure due to continuous operation or damages (Kessler *et al.*, 2002). Such a system typically consists of sensors, actuators, amplifiers and signal conditioning circuits. While sensors are employed to predict damage, the actuators serve to decelerate or arrest the damage (Bhalla, 2004). Some times, a single transducer serves as both sensor and actuator. Non-destructive evaluation (NDE) based SHM has emerged as pioneering field during the last two decades. In general, the SHM-NDE (Chapter 1) methodologies can be classified, based on their basic principle, as

- 1) Visual inspection method,
- 2) Low frequency vibration techniques,

- 3) Statistical structural response techniques,
- 4) Localized NDE techniques,
- 5) Smart system based high frequency vibration techniques.

2.1.1 Visual inspection method (VIM)

Till today, the VIM using magnifying glass, tap test and some basic (NDE) tests such as dye-penetrant, magnetic particle, etc. has been the most prevalent method of pre-emptive structural inspections in most engineering sectors like aviation structures and mechanical systems, (Bhalla, 2004). Routine physical checkups by trained personnel are a must. However, the procedure is very tedious, time consuming and is also characterised by high implementation costs (Kessler *et al.*, 2002). Some times, it needs dismantling of the critical components (such as the main gear fittings in the case of aircrafts) before inspections and reassembling afterwards (Boller, 2002). This process (dismantling and reassembling) would usually consume up to 45% of the entire inspection time in the aviation sector (Bhalla, 2004). Similarly, in civil-structures, often the critical parts may not be readily accessible and demand the removal of the existing finishes, which makes the process extremely laborious as well as cost intensive. Most of the existing VIM-NDE techniques demand holding the use of a part or whole of structure till the probe is done, which proves impractical for the large-sized civil structures (Kevin *et al.*, 1997). Hence, there is a need for a SHM system which can achieve a significant reduction in the inspection time, effort and cost.

The need to develop this kind of SHM system has attracted a large number of academic and industrial researchers from various disciplines, such as aerospace, civil, and mechanical (ACM) engineering, especially after those deadly damages and failures mentioned in chapter 1. The ultimate goal of all SHM related research is to enable systems and structures to monitor their own integrity while in operation and throughout their design lives, thus facilitating their own prevention against damages, failures and minimizing pre-emptory inspections (Bhalla, 2004).

2.1.2 Low frequency vibration techniques

In these techniques, the structure is subjected to low-frequency excitations ($<100\text{Hz}$), either harmonic or impulse, and the resulting vibration responses like displacements, velocities and accelerations are measured at required locations along the structure. Then, the recorded data is processed to extract the first few mode shapes and the corresponding natural frequencies of the structure, thereby yielding information pertaining to the locations and the severity of the damages (Salawu, 1997; Doebling *et al.*, 1998; Zou *et al.*, 2000; Giurgiutiu and Zagrai, 2002). However, these techniques can capture only large wave lengths and hence variations in global properties. And the basic principle is to compare the pre and post damage changes in the structural properties like stiffness, mass and damping are identified, which eventually lead to changes in the model properties of structure. The vibration techniques can be further divided into time domain and frequency domain (Naidu, 2003). These are also classified based on model updating and direct correlation. This concept was employed even for structural system identification from experimental data analysis (Loh and Tou, 1995).

Researchers proposed many low frequency vibration based SHM methods like the change in curvature mode shapes (Pandey *et al.*, 1991), the change in stiffness (Zimmerman and Kaouk, 1994) and the change in flexibility (Pandey and Biswas, 1994). Many other related publications can be found (Betti and Testa, 1995), reporting the use of similar algorithms, based on the basic principle to identify changes in the modal and the structural parameters (or their derivatives) resulting from damages. The main limitations of the vibration techniques can be summarized as follows

- (i) These techniques fail in locating localized damages (Pandey and Biswas, 1994).
- (ii) Due to low frequency, the techniques are highly susceptible to ambient noise, and tend to be less sensitive even to severe damages (Doebling *et al.*, 1998).
- (iii) These techniques demand expensive hardware and sensors. For a large structure, the overall cost for such sensor systems could easily run into millions of dollars.
- (iv) Often, the performance of these techniques deteriorates in multiple damage scenarios (Wang *et al.*, 1998).

2.1.3 Static structural response techniques

These methods include the static displacement response technique (Banan *et al.*, 1994) and the static strain measurement technique (Sanayei and Saletnik, 1996). These techniques, like the low frequency vibration techniques, essentially aim for structural system identification, but employ static data, such as displacements or strains, instead of the vibration data. For example, the static displacement technique involves applying static forces at specific nodal points on the structure and measuring the corresponding displacements. But measurement of displacements on large structures is a difficult task (Bhalla, 2004). As a first step, it may demand the establishment of a frame of reference. For contact measurement, it would demand the construction of a secondary structure on an independent foundation (Banan *et al.*, 1994; Sanayei and Saletnik, 1996). Besides, the application of large loads to cause measurable deflections (or strains) needs huge machinery and power input making assessment tedious (Bhalla, 2001, 2004).

2.1.4 Localized NDE techniques

Localized NDE techniques include ultrasonic, acoustic, eddy-currents, magnetic field, impact echo testing, thermal field and X-ray analysis (Doherty, 1987). The main limitation of such methods is that a portion or whole of the structure is rendered unavailable during the inspection period (Doebling *et al.*, 1998). The techniques do not lend themselves to autonomous use since experienced technicians are required to interpret the data (Park *et al.*, 2000) and are at the same time uneconomical. A comprehensive coverage of these techniques is presented by Bhalla (2004).

2.1.5 Smart material based high frequency techniques

In these techniques, unlike the low vibration techniques, the host structures are not subjected to global excitations, but a special material, which is surface bonded or embedded into the host structure, is subjected to high frequency excitation. Whereby, the high frequency captures short wave length which is capable of seeing local damage. The material in turn transmits its vibration to the host structure locally. A *mechatronic*

transducer made of piezoceramic (PZT) material (Park, 2000), which converts electrical energy into mechanical energy and vice versa is the main transduction used in these techniques. More about PZT transducers is covered in the sections to follow. A mechatronic transducer, when connected to electromechanical impedance (EMI) analyzer (Figure 2.1) to extract structural EMI responses, is termed as mechatronic impedance transducer (MIT). The extracted EMI responses are utilized for diagnosing the condition of the structures to be monitored and the same transducers plays the dual role as an actuator as well as a sensor. The technique utilizing the PZT based MIT for SHM / NDE has evolved during the last decade and is called as the EMI technique in the literature (details are covered in section to follow).

The EMI technique in detail is as follows. A PZT transducer is bonded to the surface or embedded inside the structure, whose health is to be monitored, using high strength epoxy adhesive. The structural response in the form of electromechanical (EM) admittance signature of the transducer is acquired over a high frequency range in order of kHz. This signature forms the benchmark for assessing the structural health. At any future point of time, when it is desired to assess the health of the structure, the signature is extracted again and compared with the benchmark signature. The signature of the bonded PZT transducer is usually acquired by means of commercially available impedance analyzers, such as the HP 4192A impedance analyzer (HP, 1996) as shown in Figure 2.1. The impedance analyzer imposes an alternating voltage signal of 1 volt rms (root mean square) via 40 channel multiplex N2260 A (Agilent Technologies, 2001) to the bonded PZT transducer over the user specified preset frequency range (Bhalla, 2004).

Figure 2.1. Experimental setup

The magnitude and the phase of the steady state current are directly recorded in the form of conductance and susceptance signatures in the frequency domain, thereby eliminating the requirements of intensive domain transforms. In fact, Sun et al. (1995) reported that higher excitation voltage has no influence on the conductance signature, but may be helpful in amplifying the weaker structural modes. The EM admittance signature is a function of the stiffness, mass and damping of the host structure (Sun et al. 1995), and the length, width, thickness and orientation of the PZT transducer (Wetherhold et al. 2003). Thus, the structure response depends on both PZT transducer and the structural properties.

The major contributions and developments made by various researchers in the area of EMI technique during last decade are summarised by Park *et al.*, (2003b) and Bhalla (2004). However, only 1D and 2D models of Liang *et al.*, (1994) and Bhalla and Soh (2004a), which are relevant to the present research, are discussed in detail in section 2.7.

2.2 History of piezoceramic transducers

Pierre Curie and his brother Jacques Curie discovered the piezoelectricity effect in 1880 in a single crystal of Rochelle salt ($\text{KNaC}_4\text{O}_6/4\text{H}_2\text{O}$) and lithium niobate (LiNbO_3). The origin of the piezoelectric effect is related to an asymmetry in the unit cell of the crystal (such as LiNbO_3 , PZT [$\text{Pb}(\text{Zr}_{1-x}\text{Ti}_x)\text{O}_3$] and PLZT [$(\text{Pb}_{1-x}\text{La}_x)(\text{Zr}_{1-y}\text{Ti}_y)\text{O}_3$]) and the resultant generation of electric dipoles due to the mechanical distortion. But it was only in 1946 when the scientists discovered that barium titanate (BaTiO_3) ceramics could be made piezoelectric by applying electric field. After barium titanate ceramics, scientists discovered a number of piezoceramics and in particular, the lead zirconate titanate (PZT) class in 1956. This class exhibited superior sensitivity even at higher temperature and thus replaced its predecessor BaTiO_3 . In 1958, the terminology, linear piezoelectric formulations and the methods to measure their elastic and electric properties were standardized (Bechmann and Fair, 1958). Later on, new types of piezoelectric materials were developed based on both ceramics and polymers.

Now, these are available in different forms, such as film (Han, *et al.*, 1997; Lee *et al.*, 2000), paint, powder, single fibre and multilayered. They are available in several types, such as polyvinylidene fluoride, PVDF and lanthanide-modified piezoceramic, PLZT or PZT, depending on the chemical composition. The industry is thus growing and more improved varieties are possible to be launched in the market in the near future. Piezoelectrics have found extensive applications in several fields such as noise suppression, vibration control and damage sensing. The main focus of the present research is on utilizing PZT transducers to develop PZT-structure interaction models using EMI technique.

2.3 State of the art of PZT transducers

In the last decade, piezoelectric materials were extensively used in SHM and, noise and vibration controls. Recently, the piezoelectric materials have been employed to produce micro and nano scale systems and wireless inter digital transducers (IDT) using advanced embedded system technologies, which are set to find numerous applications in micro-electronics, bio-medical, and SHM (Varadan *et al.*, 2002; Lynch *et al.*, 2003). However, the applications of PZT transducers for damage detection based SHM are still at the research level (Park *et al.* 2000b, 2003), they are used either as surface bonded or embedded inside the host structure. Several research teams (Liang *et al.*, 1994, 1996; Park *et al.*, 1999, 2001, 2003; Bhalla and Soh, 2004) are currently working on the usage of PZT transducers for the detection of damages. Teams like Paget *et al.*, (2002), Hagood *et al.*, (1988) and Bourasseau *et al.*, (1996) developed customised piezoceramic transducers to best suit their applications, although a few companies offer ready-made PZT transducers. The choice depends on several criteria discussed below.

2.3.1 Choice of transducer constituents

- (a) The piezoceramic transducer, if surface bonded on host structure needs to satisfy the following requirements.

1. PZT are bonded using adhesive, which should be kept thin. The adhesive should be non-reactive with the underlying host structure. Furthermore, life of the adhesive should be equivalent to PZT transducer.
2. Should have negligible stiffness and strength contribution as compared to the host structure (Bhalla, 2004), so that it does not alter the dynamic properties of the host structure.
3. Should be kept away from atmospheric factors like humidity, moisture and temperature.
4. Should be kept away from acidic/ alkaline attacks due to rain or sea water exposures.
5. Should be kept away from the presence of electrical or magnetic zone.

(b) The piezoceramic transducer, if embedded inside a composite structure (or non-homogeneous structure like concrete)

1. Should be non-reactive with the chemical molecules of the host structure. Hence, the embedded PZT transducers must be properly isolated, using inert materials, to make it chemically stable.
2. Should withstand curing pressures and temperatures of the host material.
3. Should have negligible stiffness and strength contribution as compared to the host structure.
4. Should be reliable during electrical and mechanical loading and should withstand the combined mechanical and electric cyclic loading (Mall and Hsu, 2000).
5. The interface between the piezoceramic element and the host structure needs to have a reliable electrical conduction and bonding, hence needs sound interconnector (Hagood, *et al.*, 1988; Paget and Levin, 1999).
6. If host structure is concrete, then the embedded transducer should withstand vibration during the casting process.

Finally, durability and protection from surface finish, vandalism and the environment are the important features of embedment of PZT transducers.

2.3.2 Piezoceramic element

There are some materials like macro fiber composite actuators and active fiber composite actuators but they are extremely thin and were not considered in the present study. However, the most widely used ceramics are PZT, with composition $\text{Pb}(\text{Zr}_{1-x}\text{Ti}_x)\text{O}_3$. Mixing, binding, sintering and poling are the processes involved in manufacturing the ceramic transducers. Solid solution of Lead Zirconate and Lead Titanate, often mixed with other materials to obtain specific properties, when heated to high temperatures of around $800\text{-}1000^\circ\text{C}$, gives perovskite PZT powder. This perovskite powder is mixed with binder and sintered into the desired shape. In the poling process, an electric field is applied across the PZT transducer. The transducer gets elongated in one direction and a permanent dipole moment is induced along that direction. Thus, the material becomes piezoelectrically transversely isotropic in the plane normal to the poling direction and remains mechanically isotropic (Bhalla, 2004). These can be manufactured in any shape, size and thickness.

The physical properties of PZT transducers, as found in the literature, are dependent on the following two factors

1. Manufacturing processes.
2. Commercial supplier.

It is reported in the literature that the properties of piezoceramics transducers vary due to inhomogeneous chemical composition, mechanical differences during formation and the polarization process (Sensor Technology Ltd., 1995) and statistical variations are reported to be very common (Giurgiutia and Zagari, 2000). Also, the suppliers like Mide group (www.mide.com), Piezo Systems Inc., (www.piceramic.de, PI ceramic, 2003), piezo-kinetics group (www.piezo-kinetics.com), etc., provide different set of properties for similar type of PZT transducer. The main characteristic features of PZT are high elastic modulus, low tensile strength and brittleness. Many manufacturing and commercial companies produce a variety of PZT transducers depending on the application type and design, model and the host material.

Surface PZT transducers are mostly adaptable; however, embedded PZT transducers mainly depend on host structure and are manufactured differently for

different host structures (see section 2.6). Basically, embedded PZT transducers are used in the following applications

- 1 Vibration control (Shah *et al.*, 1994; Chandrasekhara and Tenneti, 1995; Chen and Chopra, 1995; Yang and Bian, 1996).
- 2 Damage detection (integrity) of structures (Shah *et al.*, 1994; Islam and Craig 1994; Singh and Vizzini, 1994; Okafor *et al.*, 1996; Mall, 2002).

Many researchers (Yocum *et al.*, 2003; Crawley and Luis, 1987; Shukla and Vizzini, 1996; Mall and Hsu, 2000, Giurgiutiu *et al.*, 2002) studied the effects of using embedded PZT transducers in structures.

Table 2.1 shows some of the varieties of piezo ceramic patches supplied by piezo-kinetics group.

Product	Designed	Application
Navy type-I PKI-402,406,409	To serve as driver where high power and low losses are demanded.	Ultrasonic cleaners, sonars and medical use.
Navy type-II PKI-502,506	For high electromechanical activity and high dielectric constant	Receivers, hydrophones, phono pickups, sound detectors, delay lines, accelerometers, flow detectors and flow meters.
Navy type-III	For extreme driving conditions with low losses.	Medical applications
Navy type –V PKI-532	For low impedance, high dielectric constant and high sensitivity	Sensors.

Table 2.1 Some varieties of PZT transducers

2.4 Basic principle of piezoceramic materials

The basic constitutive relations for piezoelectric materials, under small field condition are (IEEE standard, 1987)

$$D_i = \overline{\varepsilon}_{ij}^T E_j + d_{im}^d T_m \text{ (Direct Effect)} \quad (2.1)$$

$$S_k = d_{jk}^c E_j + \overline{s}_{km}^E T_m \text{ (Converse Effect)} \quad (2.2)$$

In *direct effect*, when a mechanical stress is applied across the PZT transducer, as shown in Figure 2.2, Eq. (2.1) is used for measuring the electrical charge generated. In *converse effect*, when electric field is applied across the PZT as shown in Figure 2.2, Eq. (2.2) is used for deriving the induced mechanical strain. The direct effect is used in sensor applications, where as the converse effect is used in actuator applications. Combinations of both equations are used in all high frequency excitation where most of the time the PZT transducer is used both as an actuator and a sensor. More generally, Eqs. (2.1) and (2.2) can be rewritten in the matrix form as (Sirohi and Chopra, 2000)

$$\begin{bmatrix} D \\ S \end{bmatrix} = \begin{bmatrix} \overline{\varepsilon}^T & d_{im}^d \\ d_{jk}^c & \overline{s}_{km}^E \end{bmatrix} \begin{bmatrix} E \\ T \end{bmatrix} \quad (2.3)$$

where $[D]$ (Coulomb/m²) is the electric displacement vector of size (3 x 1), $[S]$ the dimensionless strain tensor of size (6 x 1), $[E]$ (Volt/m) is the applied external electric field vector of size (3 x 1) and $[T]$ (N/m²) the stress tensor of size (6 x 1).

Figure 2.2 A 3D piezoelectric material sheet.

Accordingly, $[\overline{\varepsilon}^T]$ (F/m) is the dielectric permittivity tensor of size (3 x 3) under constant stress, $[d_{im}^d]$ (C/N) and $[d_{jk}^c]$ (m/V) are the piezoelectric strain coefficient tensors of sizes (3 x 6) and (6 x 3) respectively, and $[\overline{S}^E]$ (m²/N) is the elastic compliance tensor under constant electric field of size (6 x 6). The superscripts 'd' and 'c' indicate the direct and the converse effects respectively. The superscripts 'T' and 'E' indicate the parameter that has been measured at constant stress and electric field respectively. A bar above some parameters indicates that these parameters are measured at dynamic conditions (hence complex in nature). In the absence of mechanical stress, strain per unit electric field is defined as the piezoelectric strain coefficient d_{jk}^c (see Eq. 2.2). Similarly, in the absence of an electric field, the electric displacement per unit stress is given by d_{im}^d (see Eq. 2.1). The two coefficients are numerically equal. In both d_{jk}^c and d_{im}^d , the first subscript denotes the direction of the electric field and the second subscript denotes the direction of the associated mechanical strain.

For a sheet of piezoelectric material (Figure 2.2), the poling direction is usually along the thickness (axis 3). The sheet lies in the plane formed by 1-2 axes and the following points are noticed from Eqs. (2.1) and (2.2)

1. Under static electric field, the crystal is free to deform and will not yield mechanical stress.
2. Under short circuit condition, applied stress will not yield electric field (or surface charge).

The matrix $[d_{jk}^c]$ depends on the crystal structure. For PZT, as reported in the literature, it is given by

$$[d_{jk}^c] = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.4)$$

The coefficients d_{31} , d_{32} and d_{33} are numerically equal to the normal strains in the directions 1, 2 and 3 respectively due to a unit electric field applied along the poling direction 3 under stress free boundary conditions. The coefficient d_{15} relates the shear strain in the 1-3 directional plane to the field E_1 , and d_{24} relates the shear strain in the 2-3 directional plane to the electric field E_2 . It is not possible to produce shear in the 1-2 plane purely by the application of an electric field. Similarly, shear stress in the 1-2 directional plane does not generate any electric response. In all poled piezoelectric materials, d_{31} is negative and d_{33} is positive. For a good sensor, the algebraic sum of d_{31} and d_{33} should be the maximum and at the same time, ϵ_{33} and the mechanical loss factor should be minimum (Kumar, 1991; Bhalla, 2004).

The compliance matrix has the form

$$\overline{s}^E = \begin{bmatrix} \overline{s}_{11}^E & \overline{s}_{12}^E & \overline{s}_{13}^E & 0 & 0 & 0 \\ \overline{s}_{21}^E & \overline{s}_{22}^E & \overline{s}_{23}^E & 0 & 0 & 0 \\ \overline{s}_{31}^E & \overline{s}_{32}^E & \overline{s}_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{s}_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{s}_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{s}_{66}^E \end{bmatrix} \quad (2.5)$$

From energy considerations, the compliance matrix is symmetric. Further, for isotropic materials, there are only two independent coefficients, as expressed below (remaining terms are zero)

$$\overline{s}_{11}^E = \overline{s}_{22}^E = \overline{s}_{33}^E = \frac{1}{Y^E} \quad (2.6)$$

$$\overline{s}_{12}^E = \overline{s}_{13}^E = \overline{s}_{21}^E = \overline{s}_{31}^E = \frac{-\nu}{Y^E} \quad (2.7)$$

$$\overline{s}_{44}^E = \overline{s}_{55}^E = \overline{s}_{66}^E = \frac{1}{G^E} \quad (2.8)$$

where $\overline{Y}^E$ is the complex Young's modulus of elasticity, $\overline{G}^E$ the complex shear modulus, and ν the Poisson's ratio. It may be noted that both Y^E and G^E are related as

$$G^E = \frac{Y^E}{2(1+\nu)} \quad (2.9)$$

For PZT crystal, the electric permittivity matrix can be written as

$$[\overline{\epsilon^T}] = \begin{bmatrix} \overline{\epsilon_{11}^T} & 0 & 0 \\ 0 & \overline{\epsilon_{22}^T} & 0 \\ 0 & 0 & \overline{\epsilon_{33}^T} \end{bmatrix} \quad (2.10)$$

The stress vector is written as

$$T = \begin{bmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{31} \\ T_{12} \end{bmatrix} \quad (2.11)$$

Where i and j in T_{ij} denote the electric field direction (if applied) and stress direction respectively. Eq. (2.3) thus mathematically expresses the electromechanical behaviour of piezoelectric transducers completely.

2.5 Mechanical impedance of structures

As explained by Bhalla and Soh (2004a), a harmonic force, acting upon a structure can be represented by an equivalent rotating phasor of magnitude F_p , on a complex plane as shown in Figure 2.3, rotating anti-clockwise at an angular frequency ω (same as that of harmonic force). The angle between the phasor and the real axis at any time instant 't' is ' ωt '. The instantaneous force, acting upon the structure, is equal to the sum of projections of the phasor on both the real and imaginary axis. Hence, the phasor can be represented using complex notation as

$$F(t) = F_p \cos \omega t + jF_p \sin \omega t = F_p e^{j\omega t} \quad (2.12)$$

The resulting velocity response, $\dot{u}$, at the point of application of the force, is also harmonic in nature, but lags behind the applied force by a phase angle ϕ , due to the 'mechanical impedance' of the structure. Hence, velocity can also be represented as a phasor, as shown in Figure 2.3, and expressed as

$$\dot{u} = \dot{u}_p \cos(\omega t - \phi) + j \dot{u}_p \sin(\omega t - \phi) = \dot{u}_p e^{j(\omega t - \phi)} \quad (2.13)$$

The *mechanical impedance* of the structure at a given point of application of the force is defined as the ratio of the driving harmonic force to the resulting harmonic velocity at that point. Mathematically, the mechanical impedance, Z , can be expressed as

$$Z = \frac{F}{\dot{u}} = \frac{F_p e^{j\omega t}}{\dot{u}_p e^{j(\omega t - \phi)}} = \frac{F_p}{\dot{u}_p} e^{j\phi} \quad (2.14)$$

Figure 2.3 Representation of harmonic force and velocity by rotating phasors
(Bhalla 2004)

Based on this definition, the mechanical impedance of a pure mass ‘ m ’ can be derived as ‘ $m\omega j$ ’ (Hixon, 1988; Bhalla, 2004). Similarly, the mechanical impedance of an ideal spring with spring constant ‘ k ’ and damper with damping constant ‘ c ’ can be obtained as ‘ $-jk/\omega$ ’ and ‘ c ’ respectively. For a parallel and series combination of ‘ n ’ mechanical systems, the equivalent mechanical impedance is respectively given as

$$Z_{eq} = \sum_{i=1}^n Z_i \quad (\text{parallel}) \quad (2.15)$$

$$\frac{1}{Z_{eq}} = \sum_{i=1}^n \frac{1}{Z_i} \quad (\text{series}) \quad (2.16)$$

Thus, for the parallel and series combination of a spring-mass-damper system, such as the one shown in Figure 2.4, the resultant impedances are respectively

$$Z_{eq} = c + j\left(m\omega - \frac{k}{\omega}\right) \quad (2.17)$$

$$Z_{eq} = \frac{m\omega kc}{m\omega k + jc(m\omega^2 - k)} \quad (2.18)$$

Figure 2.4 Parallel and series combination of a spring-mass-damper system

2.6 Fabrication techniques of embedded piezoceramic transducers

Surface bonded PZT transducers are readily available in the international market. The suppliers like Mide group (www.mide.com), Piezo Systems Inc., (www.piceramic.de), piezo-kinetics group (www.piezo-kinetics.com), etc., provide commercial PZT transducers of various shapes (square or rectangular), sizes (thin/thick) and properties (electrical/mechanical). However, ready-made embeddable PZT transducers may not be suitable or adaptable for some embeddable host structures, which may require additional safety features. Thus in the literature two types of configurations are described. The techniques not only concern how to embed the PZT transducer in the host structure but also how to provide proper insulation. Some researchers have chosen to cut the composite plies surrounding the embedded piezoceramic transducer (Hagood *et al.*, 1988; Elspass, *et al.*, 1995; Mall and Hsu, 2000). On the other hand, some researchers directly embedded the PZT transducers, avoided cutting the fibres (Paget and Levin, 1999; Paget *et al.*, 2002. Elspass *et al.*, 1995). They fabricated a piezoceramic transducer by embedding into a carbon-fibre reinforced thermoplastic composite. The material in the interconnectors was the same as the composite. The two interconnectors were placed on each side of the

piezoceramic element. The electrical insulation from the upper and lower interconnectors was achieved by two glass-fibre reinforced thermoplastics (GFRP-ply), as shown in Figure 2.5. Cutouts in the glass and carbon fibre reinforced thermoplastic were made to allow electrical contact between the terminals and the embedded piezoceramic element. Hence, the design and manufacture of such a lay-up was complex.

Figure 2.5 Piezoceramic element embedded in carbon-fibre composite (Elspass *et al.*, 1988)

Hagood *et al.*, (1988), similar to Elspass *et al.*, (1995), used a cutout technique to embed piezoceramic transducers in glass-fibre reinforced polymer (GFRP) laminates. A cutout window had, about the same dimension as the PZT element. Slits were also cut in the plies directly above and below the piezoceramic element to allow the inter connections to be drawn out to the edges, as shown in Figure 2.6. Embedment of the PZT transducer inside the host structure is case specific (Paget., 2002). Depending on the type of composite, the manufacturing technique might vary.

If the layer into which the PZT transducer is to be embedded is an insulator (like epoxy), the special inter connectors can be avoided. If the layer is concrete, then proper inter connection need to be applied to protect the PZT transducer from chemical reactions during curing process and to protect it against vibration during the casting process.

Figure 2.6 Piezoceramic element embedded in a glass/epoxy laminate
(Hagood *et al.*, 1995)

2.7 1D and 2D EMI models based on high frequency excitation

In general, in the EMI technique, the frequency of excitation is in the order of 30-400 kHz.

2.7.1 1D model of Liang *et al.*, (1994)

Assumptions made in idealizing the behaviour of PZT (Liang *et al.*, 1994)

1. The PZT transducer essentially behaves as a thin bar undergoing axial vibration and interacting with the host structure.
2. The PZT transducer-structure system can be modelled as mechanical impedance (due to the host structure) connected to an axially vibrating thin bar, as shown in Figure 2.7. (The transducer expands and contracts in direction '1' when uniform alternating electric field E_3 ($\partial E_3 / \partial x = \partial E_3 / \partial y = 0$) is applied in direction '3'. The transducer has half-length ' l ', width ' w ' and thickness ' h ').
3. The host structure is assumed to be a skeletal structure, that is, it is composed of 1D members with their sectional properties (area and moment of inertia)

lumped along their neutral axes. Therefore, the vibration of the PZT transducer in direction '2' is ignored.

4. The PZT transducers are assumed to be very thin and vibration along direction 3 is also ignored.
5. The vibrating transducer is assumed infinitesimally small and possesses negligible mass and stiffness as compared to the host structure.
6. The two end points of the transducer can therefore be assumed to encounter equal mechanical impedance, Z , from the structure, as shown in Figure 2.7. Thus PZT transducer has zero displacement at the mid-point ($x = 0$) irrespective of the location of the transducer on the host structure.

Figure 2.7 Modelling PZT –structure interaction behaviour

For 1D configuration, the basic piezoelectric Eqs. (2.1) and (2.2) is re-written as

$$D_3 = \overline{\epsilon_{33}^T} E_3 + d_{31} T_1 \quad (2.19)$$

$$S_1 = \frac{T_1}{Y^E} + d_{31} E_3 \quad (2.20)$$

where S_1 is the strain in direction '1', D_3 the electric displacement over the PZT transducer, d_{31} the piezoelectric strain coefficient and T_1 the axial stress in direction '1'.

$\overline{Y^E} = Y^E (1 + \eta j)$ is the complex Young's modulus of elasticity of the PZT transducer at constant electric field and $\overline{\epsilon_{33}^T} = \epsilon_{33}^T (1 - \delta j)$ is the complex electric permittivity (in

direction ‘3’) of the PZT material at constant stress. Here, η and δ denote respectively the mechanical loss factor and the dielectric loss factor of the PZT material.

The 1D vibration of the PZT transducer is governed by the following differential equation (Liang *et al.*, 1994), derived based on dynamic equilibrium of the PZT transducer.

$$\overline{Y}_{11}^E \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \quad (2.21)$$

where u is the displacement at any point on the transducer in direction ‘1’. Solution of the governing differential equation by the method of separation of variables yields

$$u = (A \sin \kappa x + B \cos \kappa x) e^{j\omega t} \quad (2.22)$$

where κ is the wave number, related to the angular frequency of excitation ω , the density ρ and the complex Young’s modulus of elasticity of the transducer by

$$\kappa = \omega \sqrt{\frac{\rho}{\overline{Y}^E}} \quad (2.23)$$

On application of the boundary condition, i.e. at $x = 0$ (mid point of the PZT transducer), $u = 0$ yields $B = 0$.

Hence, the strain in PZT transducer is given by

$$S_1(x) = \frac{\partial u}{\partial x} = A e^{j\omega t} \kappa \cos \kappa x \quad (2.24)$$

and the velocity is given by

$$\dot{u}(x) = \frac{\partial u}{\partial t} = A j \omega e^{j\omega t} \sin \kappa x \quad (2.25)$$

Further, by definition, the mechanical impedance Z of the structure is related to the axial force F in the PZT transducer by

$$F_{(x=l)} = -Z \dot{u}_{(x=l)} \quad (2.26)$$

where the negative sign signifies the fact that a positive velocity causes compressive force in the PZT transducer (Liang *et al.*, 1994). This equation can be further expanded as

$$whT_{1(x=l)} = -Z\dot{u}_{(x=l)} \quad (2.27)$$

Making use of Eq. (2.20) and substituting the expressions for the strain and the velocity from Eqs. (2.24) and (2.25) respectively, the unknown constant A can be derived as

$$A = \frac{Z_a V_o d_{31}}{h_a \kappa \cos(\kappa l_a)(Z + Z_a)} \quad (2.28)$$

where Z_a is the short-circuited mechanical impedance of the PZT transducer, given by

$$Z_a = \frac{\kappa w_a h_a \overline{Y_{11}^E}}{(j\omega) \tan(\kappa l_a)} \quad (2.29)$$

This is defined as the force needed to produce unit velocity in the PZT transducer in short circuited condition (i.e. ignoring the piezoelectric effect) and ignoring the host structure. The electric current, which is the time rate of change of charge, can be obtained as

$$\overline{I} = \iint_A \dot{D}_3 dx dy = j\omega \iint_A D_3 dx dy \quad (2.30)$$

Making use of the PZT constitutive relation (Eq. 2.13), and integrating over the entire surface of the PZT transducer ($-l$ to $+l$), we can obtain an expression for the EM admittance (the inverse of EMI) as

$$\overline{Y} = 2\omega j \frac{wl}{h} \left[(\overline{\epsilon_{33}^T} - d_{31}^2 \overline{Y^E}) + \left(\frac{Z_a}{Z + Z_a} \right) d_{31}^2 \overline{Y^E} \left(\frac{\tan \kappa l}{\kappa l} \right) \right] \quad (2.31)$$

Eq. (2.31) is same as that derived by Liang *et al.* (1994), except that an additional factor of 2 comes into picture. This is due to the fact that Liang *et al.* (1994) considered only one-half of the transducer in their derivation (Bhalla and Soh, 2004a).

Major limitations of this preliminary 1 D model are

1. This model is applicable only for thin PZT transducers.

2. The direction of vibration is restricted to 1D, i.e. along the length of the PZT transducer.

2.7.2 2D Model of Zhou *et al.*, (1995, 1996)

Zhou *et al.* (1995, 1996) extended the 1D impedance method of Liang (1994) to model the interactions of a generic 2D PZT element coupled to a 2D host structure. The analytical model of these researchers is schematically shown in Figure 2.8. In this approach, the structural impedance is represented by direct impedances Z_{xx} and Z_{yy} , and the cross impedances Z_{xy} and Z_{yx} , which are related to the planar forces F_1 and F_2 (in directions 1 and 2 respectively) and the corresponding planar velocities $\dot{u}_1$ and $\dot{u}_2$ by

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = - \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} \quad (2.32)$$

Applying D'Alembert's principle along the two directions and after imposing the boundary conditions, Zhou *et al.* (1995) derived the following expression for the EM admittance across the PZT terminals

$$\bar{Y} = G + Bj = j\omega \frac{wl}{h} \left[\frac{\bar{\epsilon}_{33}^T}{\epsilon_{33}^T} - \frac{2d_{31}^2 \bar{Y}^E}{(1-\nu)} + \frac{d_{31}^2 \bar{Y}^E}{(1-\nu)} \left\{ \frac{\sin \kappa l}{l} \quad \frac{\sin \kappa w}{w} \right\} N^{-1} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \right] \quad (2.33)$$

where κ , the 2D wave number, is given by

$$\kappa = \omega \sqrt{\frac{\rho(1-\nu^2)}{\bar{Y}^E}} \quad (2.34)$$

Figure 2.8 Modelling of 2D physical coupling by impedance approach (Bhalla, 2004)

and N is a 2x2 matrix, given by

$$N = \begin{bmatrix} \kappa \cos(\kappa l) \left\{ 1 - \nu \frac{w}{l} \frac{Z_{xy}}{Z_{axx}} + \frac{Z_{xx}}{Z_{axx}} \right\} & \kappa \cos(\kappa w) \left\{ \frac{l}{w} \frac{Z_{yx}}{Z_{ayy}} - \nu \frac{Z_{yy}}{Z_{ayy}} \right\} \\ \kappa \cos(\kappa l) \left\{ \frac{w}{l} \frac{Z_{xy}}{Z_{axx}} - \nu \frac{Z_{xx}}{Z_{axx}} \right\} & \kappa \cos(\kappa w) \left\{ 1 - \nu \frac{l}{w} \frac{Z_{yx}}{Z_{pyy}} + \frac{Z_{yy}}{Z_{ayy}} \right\} \end{bmatrix} \quad (2.35)$$

Here, Z_{axx} and Z_{ayy} are the two components of the mechanical impedance of the PZT transducer, derived in the same way as 1D impedance approach, in the two principal directions. However, the main limitations of this modelling approach are summarised as

1. The model ignores the PZT transducer vibration in the thickness direction (poling direction), thus it is limited to thin PZT transducers and hence non-generic in nature.
2. Four complex unknowns- Z_{xx} , Z_{yy} , Z_{xy} , Z_{yx} (total eight unknowns, including real and imaginary components) but the number of equations are only two. (Eq. 2.32). Hence, it is not possible to derive complex impedance information of any unknown structure from measurements alone.

2.7.3 2D model of Bhalla and Soh (2004)

This model is based on the concept of ‘effective impedance’ introduced by Bhalla and Soh (2004a). This was the improvement over the 2D model of Zhou *et al.* (1995). The major assumptions made here are:

1. The *mechanical interaction* between the transducer and the host structure is not restricted at the PZT end points alone as assumed by Liang (1994). Rather it extends all over the finite sized PZT transducer. It is assumed that the *force transmission* between the PZT transducer and the structure occurs along the entire boundary of the transducer.
2. *Plane stress* conditions exist within the transducer.

3. The transducer is assumed square shaped and *infinitesimally small* as compared to the host structure, so as to possess negligible mass and stiffness.
4. The *vibration* in the thickness direction is neglected and thus PZT transducer is assumed to be thin.
5. The opposite edges of the transducer therefore encounter *equal dynamic stiffness* from the structure, irrespective of the location of the transducer on the host structure.
6. The PZT transducer is mechanically and electrically *isotropic* in the X-Y plane as shown in Figure 2.9.

Figure 2.9 Modelling of interaction of PZT and structure in 2D(Bhalla, 2004)

Bhalla and Soh (2004a) introduced a new definition of mechanical impedance based on ‘effective velocity’ rather than ‘drive point velocity’. Considering a finite sized square PZT transducer, surface bonded to an unknown host structure (as shown in Figure 2.9), subjected to a spatially uniform electric field undergoing harmonic variation with time. The transducer has half-length equal to ‘ l ’. Its interaction with the structure is represented in the form of boundary traction ‘ f ’ per unit length, varying harmonically with time.

This planar force causes planar deformations in the PZT transducer, leading to variations in its overall area. The ‘effective mechanical impedance’ of the transducer is hereby defined as

$$Z_{a,eff} = \frac{\oint_S \vec{f} \cdot \hat{n} ds}{\dot{u}_{eff}} = \frac{F}{\dot{u}_{eff}} \quad (2.36)$$

where $\hat{n}$ is the unit vector normal to the boundary and ‘F’ represents the overall planar force (or effective force) causing area deformation of the PZT transducer. $u_{eff} = \delta A/p_o$ is defined as ‘effective displacement’ of the PZT transducer, where δA is the change in the cross-sectional area of the transducer and p_o is its perimeter in the undeformed condition. More precisely, p_o is equal to the summation of the lengths of ‘active boundaries’, i.e. the boundaries undergoing mechanical interaction with the host structure. Differentiation with respect to time of the effective displacement yields the effective velocity, $\dot{u}_{eff}$. In order to ensure overall force equilibrium,

$$\oint_S \vec{f} ds = 0 \quad (2.37)$$

The effective drive point (EDP) impedance of the host structure can also be defined on similar lines. However, for determining structural impedance, force needs to be applied on the surface of the host structure along the boundary of the proposed location of the PZT transducer.

Taking advantage of symmetry, it is sufficient to consider the interaction of one quarter of the transducer with the corresponding one-quarter of host structure since it is only the ratio of the two mechanical impedances that will govern the electrical admittance across the terminals of the PZT transducer.

Since the PZT transducer is mechanically and piezoelectrically isotropic in the x-y plane, implies $\overline{Y_{11}^E} = \overline{Y_{22}^E} = \overline{Y^E}$ and $d_{31} = d_{32}$. Therefore, the PZT constitutive relations (Eqs. 2.1 and 2.2) are reduced to

$$D_3 = \overline{\epsilon_{33}^T} E_3 + d_{31}(T_1 + T_2) \quad (2.38)$$

$$S_1 = \frac{T_1 - \nu T_2}{\overline{Y^E}} + d_{31} E_3 \quad (2.39)$$

$$S_2 = \frac{T_2 - \nu T_1}{\overline{Y^E}} + d_{31} E_3 \quad (2.40)$$

where ν is the Poisson's ratio of the PZT transducer. By algebraic manipulation, the following equation is obtained

$$T_1 + T_2 = \frac{(S_1 + S_2 - 2d_{31}E_3)\overline{Y^E}}{1 - \nu} \quad (2.41)$$

If the PZT transducer is in the short-circuited condition (i.e. zero electric field), Eq. (2.41) can be reduced to

$$(T_1 + T_2)_{\text{short-circuited}} = \frac{(S_1 + S_2)\overline{Y^E}}{1 - \nu} \quad (2.42)$$

As derived by Zhou *et al.* (1996), the displacements of the PZT transducer in the two principal directions are given by

$$u_1 = (A_1 \sin \kappa x) e^{j\omega t} \text{ and } u_2 = (A_2 \sin \kappa y) e^{j\omega t} \quad (2.43)$$

where the wave number κ is given by Eq.(2.34), and A_1 and A_2 are constants to be determined from the boundary conditions. The corresponding velocities are obtained by differentiating these equations with respect to time.

$$\dot{u}_1 = \frac{\partial u_1}{\partial t} = (A_1 j\omega \sin \kappa x) e^{j\omega t} \text{ and } \dot{u}_2 = \frac{\partial u_2}{\partial t} = (A_2 j\omega \sin \kappa y) e^{j\omega t} \quad (2.44)$$

Similarly, the corresponding strains can be obtained by differentiation with respect to the two coordinate axes.

$$S_1 = \frac{\partial u_1}{\partial x} = (A_1 \kappa \cos \kappa x) e^{j\omega t} \text{ and } S_2 = \frac{\partial u_2}{\partial y} = (A_2 \kappa \cos \kappa y) e^{j\omega t} \quad (2.45)$$

From Figure 2.9, the effective displacement of the PZT transducer, considering displacements at the active boundaries of one-quarter of the transducer (the boundaries along the nodal axes are 'inactive' boundaries) is given by

$$u_{\text{eff}} = \frac{\delta A}{p_o} = \frac{u_{1o}l + u_{2o}l + u_{1o}u_{2o}}{2l} \approx \frac{u_{1o} + u_{2o}}{2} \quad (2.46)$$

where u_{1o} and u_{2o} are edge displacements, as shown in Figure 2.9.

Differentiating with respect to time, the following effective velocity is obtained

$$\dot{u}_{\text{eff}} = \frac{\dot{u}_{1o} + \dot{u}_{2o}}{2} = \frac{\dot{u}_{1(x=l)} + \dot{u}_{2(y=l)}}{2} \quad (2.47)$$

From Eq. (2.36), the effective impedance of the transducer can be determined as

$$Z_{a,eff} = \frac{(T_{1(x=l)}lh + T_{2(y=l)}lh)_{short-circuited}}{\left(\frac{\dot{u}_{1(x=l)} + \dot{u}_{2(y=l)}}{2}\right)} \quad (2.48)$$

Making use of Eq. (2.42), thus

$$Z_{a,eff} = \frac{(S_{(x=l)}lh + S_{2(y=l)}lh)\overline{Y^E}}{(1-\nu)\left(\frac{\dot{u}_{1(x=l)} + \dot{u}_{2(y=l)}}{2}\right)} \quad (2.49)$$

Substituting the values of the velocities and strains (Eqs. 2.44 and 2.45 respectively) at the two active edges of the PZT transducer, and upon solving, following expression is obtained

$$Z_{a,eff} = \frac{2klh\overline{Y^E}}{j\omega(\tan kl)(1-\nu)} \quad (2.50)$$

The overall planar force (or the effective force) F is related to the EDP impedance of the host structure by

$$F = \oint_S \vec{f} \cdot \hat{n} ds = -Z_{s,eff} \dot{u}_{eff} \quad (2.51)$$

As in the 1D case, negative sign signifies that a positive effective displacement causes compressive force on the transducer (due to reaction from the host structure). Considering a square transducer, Eq. (2.51) can be simplified as

$$T_{1(x=l)}hl + T_{2(y=l)}hl = -Z_{s,eff} \left(\frac{\dot{u}_{1(x=l)} + \dot{u}_{2(y=l)}}{2}\right) \quad (2.52)$$

Making use of Eq. (2.41), the following equation is obtained

$$\frac{(S_{1(x=l)} + S_{2(y=l)} - 2d_{31}E_3)\overline{Y^E}hl}{(1-\nu)} = -Z_{s,eff} \left(\frac{\dot{u}_{1(x=l)} + \dot{u}_{2(y=l)}}{2}\right) \quad (2.53)$$

Substituting the expressions for $(\dot{u}_1 + \dot{u}_2)_{x=y=l}$ and $(S_1 + S_2)_{x=y=l}$ from Eqs. 2.44 and 2.45 respectively, and using $E_3 = (V_o/h)e^{j\omega t}$, where V_o is instantaneous voltage across the PZT transducer, results in following expression is obtained

$$A_1 + A_2 = \frac{2d_{31}V_o Z_a}{(\cos kl)kh(Z_{s,eff} + Z_a)} \quad (2.54)$$

The electric displacement (or the charge density) over the surface of the PZT transducer can then be calculated from Eq. (2.38). Substituting Eq. (2.41) into Eq. (2.38) and substituting $E_3 = (V_o/h) e^{j\omega t}$, following equation is obtained

$$D_3 = \frac{\overline{\epsilon}_{33}^T V_o}{h} e^{j\omega t} + \frac{d_{31} \overline{Y}^E}{(1-\nu)} \left(S_1 + S_2 - 2d_{31} \frac{V_o}{h} e^{j\omega t} \right) \quad (2.55)$$

The instantaneous electric current, which is the time rate of change of charge, can be derived as

$$\bar{I} = \iint_A \dot{D}_3 dx dy = j\omega \iint_A D_3 dx dy \quad (2.56)$$

Substituting D_3 from Eq. (2.55), and S_1 and S_2 from Eq. (2.45), and integrating from $-l$ to $+l$ with respect to both 'x' and 'y', hence

$$\bar{I} = 4\overline{V}\omega j \frac{l^2}{h} \left[\frac{\overline{\epsilon}_{33}^T}{h} - \frac{2d_{31}^2 \overline{Y}^E}{(1-\nu)} + \frac{2d_{31}^2 \overline{Y}^E}{(1-\nu)} \left(\frac{Z_{a,eff}}{Z_{s,eff} + Z_{a,eff}} \right) \left(\frac{\tan \kappa l}{\kappa l} \right) \right] \quad (2.57)$$

The complex electro-mechanical admittance of the PZT transducer is given by

$$\bar{Y} = \frac{\bar{I}}{\bar{V}} = G + Bj = 4\omega j \frac{l^2}{h} \left[\frac{\overline{\epsilon}_{33}^T}{h} - \frac{2d_{31}^2 \overline{Y}^E}{(1-\nu)} + \frac{2d_{31}^2 \overline{Y}^E Z_{a,eff}}{(1-\nu)(Z_{s,eff} + Z_{a,eff})} \left(\frac{\tan \kappa l}{\kappa l} \right) \right] \quad (2.58)$$

which is the desired coupling equation for a square PZT transducer. It should be noted that a factor of 4 is introduced in the final expression, since 'l' here represents half-length of the transducer. However, the limitations of this approach can be summarised as

1. The PZT transducer must be square in shape
2. The PZT transducer assumed to be thin; hence it cannot be applicable for thick PZT transducers.
3. The vibrations in the thickness direction are ignored.
4. The model is suitable for surface bonded PZT transducers only.

2.8 Some of the key developments in field of SHM during last decade

PZT transducers are extensively used in vibration and noise control, and EMI based SHM of various engineering structures. Many researchers (Crawley and Luis 1987; Liang *et al.* 1993; Zhou *et al.* 1996; Bandar and Abdulmalik 2003; Bhalla and Soh 2004) have developed models to describe the interaction between the PZT transducer and the host structure for use in structural health monitoring (SHM). However, the last decade has seen applications of PZT-host structure interaction models in electromechanical impedance (EMI) techniques. The main contributions and developments can be summarised as follows.

- (1) 1D single PZT- structure interaction model was developed by Liang *et al.*, (1994). In which, the PZT transducer was assumed to be a thin bar undergoing axial vibration and interacting with the host structure (as given in section 2.7.1).
- (2) Mayne and Dosch (1994) presented a self sensing actuator system, which was found to act both as an actuator and sensor. The sensor can be used either for measurements of stress or strains. The actuator can be used as a force generator or positioning device and is useful for purposes ranging from dynamic damping to shape control.
- (3) The application of the EMI technique for SHM of a lab sized truss structure was first reported by Sun *et al.* (1995). The study was then extended to a large-scale prototype truss joint by Ayres *et al.* (1998).
- (4) Zhou *et al.* (1995, 1996) extended the 1D impedance method of Liang *et al.*, (1994) to model the interactions of a generic 2D PZT element coupled to a 2D host structure (as given in section 2.7.2).
- (5) Park *et al.* (2000a) reported significant proof of concept applications of the EMI technique on civil-structural components such as composite reinforced masonry walls, steel bridge joints and pipe joints. The technique was found to be very tolerant to mechanical noise and also to small temperature fluctuations.
- (6) Park (2000) extended the EMI technique to high temperature applications (typically $> 500^{\circ}\text{C}$), such as steam pipes and boilers in power plants. Besides, he also developed practical statistical cross-correlation based methodologies for

- temperature compensation. This paved way for the application of the technique to real situations, where the effects of damage and temperature are mixed.
- (7) Soh *et al.* (2000) established the damage detection and localization ability of these transducers for real-life reinforced concrete (RC) structures, by monitoring a 5m span RC bridge during its destructive load testing. Sensor positioning, damage localization and sensor validation criteria were outlined.
 - (8) Inman *et al.* (2001) proposed a novel technique to utilize a single PZT transducer for health monitoring as well as for vibration control
 - (9) Dosch *et al.* (2001) presented a practical implementation of the self- sensing actuator using electrical bridge circuit to measure strain. The bridge circuit is capable of measuring either strain or time rate of strain in the actuator.
 - (10) Abe *et al.* (2002) developed a new stress monitoring technique for thin structural elements (such as strings, bars and plates) by applying wave propagation theory to the EMI measurement data in the moderate frequency range (1-10 kHz).
 - (11) Giurgiutiu *et al.* (2002) combined the EMI technique with wave propagation approach for crack detection in aircraft components. While the EMI technique was employed for near field damage detection, the guided ultrasonic wave propagation technique (pulse echo) was used for far field damage detection.
 - (12) Inman *et al.* (2002) presented the concept of using the same hardware to perform both SHM and control simultaneously. Aircraft panels often require damping treatments in order to reduce fatigue. At the same time, manufacturers are starting to use health monitoring as a basis for maintenance. Here, an example of doing simultaneous health monitoring and control on a sample plate. In this case, smart materials, a piezoceramic patch (PZT) and a fiber optic sensor, are used as the common hardware.
 - (13) Peairs *et al.* (2004a) presented results from proof-of-concept tests on the launch pad's orbiter access arm bolted connection, solid rocket booster support post, mobile launch platform heat shield and crawler transporter bearing. These tests showed that the impedance method can provide a permanent SHM solution to

NASA's (NASA 2006) ground structures. In addition several positive and negative aspects of the impedance method were highlighted.

- (14) Peairs *et al.* (2004b) developed novel low-cost and portable version of impedance analyzer, the major hardware used in the EMI technique, paving way for cost-reduction. Integration of the EMI technique with wireless technology and development of stand-alone sensor cum processor cum transmission units based on MEMS and inter digital transducers (IDT) is also underway (Park *et al.*, 2003b) which would enable large-scale instrumentation and monitoring of civil-structures.
- (15) Bhalla and Soh (2004a) developed 2D plane stress based surface bonded PZT-structure interaction model. This was the improvement over the 2D model of Zhou *et al.* (1995). Later, the applications of the 2D model to civil and aerospace structure were presented by Bhalla and Soh (2004d) (as given in section 2.7.3).
- (16) In the EMI models, usually the PZT transducers are bonded using adhesives. The adhesive forms a finitely thick, permanent interfacial layer between the host structure and the PZT transducer. Hence, the force transmission between the structure and the transducer occurs through the bond layer, via shear mechanism, invariably causing shear lag. The shear lag mechanism for 1D and 2D models were presented by Bhalla and Soh (2004e).
- (17) Cheng and Lin (2005a) developed multiple PZT- structure interaction model considering 'mass of the PZT' in the EMI formulations, which was a 2D model using multiple pairs of transducers bonded on top and bottom of aluminium plate.

2.9 Key comparisons between SHM and, noise and vibration control

Figure 2.10(c) shows the actuations of the PZT transducers based on researchers (Raja *et al* 2004) in the field of noise and vibration controls. In the presence of electric fields (along X, Y and Z directions) the PZT actuations can be divided into extensional (along the X and Y directions), longitudinal (along the Z direction) and shear actuations (in the XZ and YZ planes). Actuations of PZT transducer along the X and Y directions are opposite in nature as compared to actuation along the Z direction. However, in the EMI based SHM models, electric field is applied along the Z direction only; so

actuators in the XZ and YZ planes do not exist, but actuators along the X,Y and Z directions exist.

Figure 2.10 Actuators of PZT in presence of electric fields E_1 , E_2 and E_3

- (a) Uni extensional actuation (b) Bi extensional actuation
- (c) Extensional, longitudinal and shear actuators

As shown in Figure 2.10, the vibrations along the length ($2L$) and width ($2W$) due to electric field E_3 produce extensional actuation where as vibration along thickness ($2H$) of the PZT transducer results in longitudinal actuators. Many researchers of SHM

(Dosch *et al.*, 1991; Liang *et al.*, 1994; Zhou *et al.*, 1996; Giurgiutiu *et al.*, 1999; Inman *et al.*, 2000; Park *et al.*, 2003; Peairs *et al.*, 2004; Bhalla and Soh 2004) had developed interaction models based only on the uni or bi-extensional actuations of PZT transducer ignoring longitudinal actuations (Figure 2.10 a and b). Thus, another key difference between SHM and, noise and vibration control based research is the non utilization of longitudinal actuation.

2.10 Limitations of existing models

Section 2.8 and 2.9 have covered the key limitations and comparisons between EMI based SHM and noise and vibration controls. However, the limitations of existing EMI models are summarized herein as

1. The ACM structures can be classified into two categories based on their stiffness, those which are more stiff than and those which are less stiff than the PZT material. Surface bonded PZT transducers are more efficient when they are stiffer than the host structure, and embedded transducers are more efficient when they are less stiff than the host structure (Benjeddou *et al.* 2000). However all the existing EMI interaction models are based only on the surface bonding approach; moreover, surface bonded PZT transducers are prone to attacks by external factors (see section 2.3.1). Thus, this research developed EMI interaction models (Chapters 4, 7 and 8) which are generic in nature and are applicable for both surface and embedded PZT.
2. Existing EMI models are suitable only for thin (size), square (shape), homogeneous and isotropic PZT transducers (i.e. uniform mechanical and electrical isotropy). However, the present research (Chapters 3, 4, 7 and 8) successfully formulated the models without such limitations.
3. Mechanical and electrical properties of PZT vary due to external factors like acidic/ alkaline attack (see section 2.3.1). Furthermore, structural response (EMI) depends on factors like thickness, length, width and orientation of PZT transducer (Wetherhold *et al* 2003). Thus, the present research performed a

systematic study of variations in PZT properties (i.e. characterization of the PZT properties) in Chapter 5.

4. The existing EMI models do not consider longitudinal actuations in the formulations (see section 2.9) and they are limited to either 1D or 2D. However, the present research developed 3D interaction model considering both extensional and longitudinal actuations of PZT transducers.
5. The practical applications of the SHM require multiple PZT installations at key locations on the ACM structure to be monitored, which is not considered by previous researchers and has been addressed in the present research (Chapters 6, 7 and 8).

2.11 Concluding remarks

This chapter briefly presented some of the existing SHM techniques; and provided review of 1D and 2D EMI models, which provides enough background before introducing new models in later chapters. Additionally, it presented research background necessary for the present thesis. The following key issues were raised during the literature review.

- (1) Requirement for interaction models suitable for both embedded and surface bonded PZT.
- (2) Requirement of multiple PZT interaction model for its use in real life ACM structures
- (3) Requirement for characterization of PZT for its successful implementation in EMI models.

CHAPTER 3

EMBEDDED PIEZOELECTRIC CERAMIC TRANSCUDERS IN SANDWICHED BEAMS

This chapter presents a new EMI model based on ‘embedded’ PZT transducer and its interaction with the host sandwiched beam. This chapter also demonstrates the use of thickness vibration of PZT transducer for the first time in EM admittance formulations.

3.1 Introduction

The embedded PZT-structure interaction model is based on the new concept of ‘Average Sum Impedance’ (ASI). The ASI model considers both the extensional and the longitudinal actuations of the PZT transducer in the formulations. The proposed model is experimentally verified on a system comprised of a PZT transducer embedded inside the epoxy layer of a sandwiched aluminium beam. The major limitation of the existing EMI models (see sections 2.7 and 2.10 of chapter 2) is that they are applicable only for surface bonding PZT type. Moreover they ignore the thickness vibration of the PZT transducer which produces longitudinal actuation and these actuations play a vital role in the thick or confined (embedded) PZT transducers. Hence, the existing models are not applicable to laminated or civil engineering (concrete) structures, where critical zone to be monitored is inside the structure. In general, the inability of the existing models to consider the thickness vibration and its non applicability to embeddable structures had left a gap for development of new ASI model.

3.2 ASI Model

In this section, an analytical model of a PZT transducer embedded in a sandwiched beam is presented, based on the “ASI concept”. The theoretical formulations are experimentally validated later. For this purpose, a sandwiched beam

Chapter 3: Embedded Transducers in Sandwiched Beams

was fabricated using two aluminium beams (Grade Al 6061-T6, Table 3.1) bonded by an high strength epoxy layer (RS 850-940) with a PZT transducer (Grade A, Table 3.2) embedded inside the epoxy layer as shown in Figure 3.1(a).

Property	Epoxy adhesive	Aluminum (Al)
Density (kg / m^3)	1180	2715
Young's modulus (N / m^2)	2×10^9	68.95×10^9
Poisson ratio	0.4	0.33
Damping factor (β_M)	1.5923×10^{-9}	1.5923×10^{-7}

Table 3.1. Key properties of Epoxy adhesive (RS 850-940) and Al 6061-T6

Physical property	Value
Density (kg / m^3)	7800
Young's modulus (N / m^2)	6.667×10^{10}
Poisson ratio	0.33
Electric permittivity, ϵ_{33} (farad/m)	2.124×10^{-8}
Piezoelectric strain coefficient in direction X, d_{31} (m/V)	-2.10×10^{-10}
Piezoelectric strain coefficient in direction Z, d_{33} (m/V)	4.50×10^{-10}
Dielectric loss factor, δ	0.015
Mechanical loss factors η	0.023

Table 3.2. Key properties of PZT Grade A

Figure 3.1. Electromechanical modelling of embedded PZT transducers

(a) PZT transducer embedded in sandwiched aluminium beams

(b) Electric polarization of the embedded PZT transducer

3.2.1 Fundamental PZT constitutive equations

Figure 3.1(b) shows the X , Y and Z directions along the length ($2L$), width (W) and thickness ($2H$) of the embedded PZT transducer respectively.

The derivations covered in this section are based on the following assumptions:

- 1) The PZT transducer material is homogeneous and mechanically isotropic.
- 2) Plane strain conditions exist within the PZT transducer; hence, only X and Z directions are considered. For structures where plane strain conditions may not be applied, the suitable way is to consider extensional actuations along both X and Y directions and longitudinal actuation along Z direction.
- 3) Force transmission between the embedded PZT transducer and the host structure is distributed along both directions X and Z , covering the entire contact area as shown in Figures 3.1(a) and 3.2

Chapter 3: Embedded Transducers in Sandwiched Beams

4) The PZT transducer is infinitesimally small with negligible mass and stiffness as compared to the host structure.

Figure 3.2. Forces acting on the contact area of PZT transducer at any instant of time

- (a) Expansion of transducer in direction X and shrinkage in direction Z
- (b) Expansion of transducer in direction Z and shrinkage in direction X

Under a 1D harmonic electric field (E_3) along Z direction, with an angular frequency ω , the interaction of one half of the transducer with one half of the host sandwiched structure is considered, taking advantage of symmetry about Z direction as shown in figure 3.1(a).

The general 2D stress and strain relationship can be written as

$$\begin{Bmatrix} T_1 \\ T_3 \end{Bmatrix} = \bar{Y} \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu/(1-\nu) \\ \nu/(1-\nu) & 1 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_3 \end{Bmatrix} \quad (3.1)$$

where T_1 and T_3 are the stresses applied on the PZT transducer in directions X and Z respectively, and ν is the Poisson ratio. $\bar{Y}$ is the complex Young's modulus of elasticity of the PZT transducer at zero electric field, and it can be expressed as

$$\bar{Y} = Y(1+\eta j) \quad (3.2)$$

Chapter 3: Embedded Transducers in Sandwiched Beams

where Y is the static Young's modulus of elasticity of the PZT material, and η is the mechanical loss factor.

Equation (3.1) can be rearranged to produce the strain equations, S_1 and S_3 , along X and Z directions respectively as

$$S_1 = \frac{aT_1 + bT_3}{\bar{Y}} \quad (3.3a)$$

$$S_3 = \frac{bT_1 + aT_3}{\bar{Y}} \quad (3.3b)$$

where a and b are two constants such that

$$a = \frac{(1+\nu)(1-2\nu)^2}{(1-\nu)^3} \text{ and } b = \nu \frac{(1+\nu)(2\nu-1)}{(1-\nu)^4} \quad (3.4)$$

Thus, the fundamental relationships of the PZT transducer, in the presence of electric field can be written as

$$S_1 = \frac{aT_1 + bT_3}{\bar{Y}} + d_{31}E_3 \quad (3.5a)$$

$$S_3 = \frac{bT_1 + aT_3}{\bar{Y}} + d_{33}E_3 \quad (3.5b)$$

The electric displacement (or charge density), D_3 , (where subscript 3 denotes the electric field in direction Z) over the surface of PZT transducer can be written as

$$D_3 = \overline{\epsilon_{33}}E_3 + d_{31}T_1 + d_{33}T_3 \quad (3.5c)$$

where d_{zj} represents the piezoelectric strain coefficient of PZT, subscript Z signifies the direction of electric field, and j signifies the direction of the resulting stress (or strain). $\overline{\epsilon_{33}}$ is the complex electric permittivity of the PZT at zero stress, and can be expressed as

$$\overline{\epsilon_{33}} = \epsilon_{33}(1 - \delta j) \quad (3.6)$$

where δ is the dielectric loss factor and ϵ_{33} is the static electric permittivity of the PZT transducer.

3.2.2 ASI of actuator

At any instant of time, the forces developed due to the actuation of the PZT transducer as shown in Figure 3.2, which respectively correspond to expansion in X direction and shrinkage in direction Z or vice versa. The forces take into consideration the alternate signs of d_{31} and d_{33} , as listed in Table 3.2.

Due to the alternate signs, the expansion of transducer in direction X is accompanied by shrinkage in direction Z, and vice versa. The ASI of actuator can be represented mathematically as

$$Z_{as} = \frac{FP_H}{\frac{1}{m} \sum_1^m \dot{u}_{m1}} + \frac{2FP_T}{\frac{1}{n} \sum_1^n \dot{u}_{n3}} = \frac{FP_H}{\dot{u}_{1(X=L)}} + \frac{2FP_T}{\dot{u}_{3(Z=H)}} \quad (3.7)$$

where Z_{as} is the average sum actuator impedance, FP_H and FP_T are the total force components acting on the PZT transducer along X and Z directions respectively, m and n are the finite number of points considered along the boundary (at $X=L$ and $Z=\pm H$ respectively), $\dot{u}_{m3}$ is the velocity of the m^{th} point in Z direction, $\dot{u}_{n1}$ is the velocity of n^{th} point in direction X, $\dot{u}_{1(X=L)}$ is the average velocity in direction X at $X = +L$, and $\dot{u}_{3(Z=H)}$ is the average velocity in Z direction at $Z = \pm H$ of the PZT transducer.

As described in Equation (3.7), the average velocities in X and Z directions are first determined, and the ratios of the force components to the average velocities are then calculated. Finally, these ratios are added to obtain the average sum actuator impedance, hence this is named as the ‘Average Sum Impedance’ (ASI) model. The final value of the actuator impedance is the same irrespective of the direction considered, as shown in Figure 3.2.

3.2.3 Derivation of actuator impedance from stress-strain relationships of PZT

Using strain Equations (3.3a) and (3.3b), in short-circuited condition, i.e., $E_3=0$, the equations for the stresses acting on the PZT transducer can be written as

Chapter 3: Embedded Transducers in Sandwiched Beams

$$T_1 = \bar{Y} \frac{S_1 a - S_3 b}{a^2 - b^2} \quad (3.8a)$$

$$T_3 = \bar{Y}^E \frac{S_3 a - S_1 b}{a^2 - b^2} \quad (3.8b)$$

Let u_1 be the average displacement developed along direction X, and u_3 be the average displacement developed along direction Z. The displacement equations for two directional in-plane vibrations are derived similar to that of Zhou *et al* (1996), which can be expressed as

$$u_1 = (A \sin kx) e^{j\omega t} \quad (3.9a)$$

$$u_3 = (C \sin kz) e^{j\omega t} \quad (3.9b)$$

where A and C are the coefficients of vibration, which are to be determined from the boundary conditions, and k is the wave number given by

$$k = \omega \sqrt{\frac{\rho(1+\nu)(1-2\nu)}{\bar{Y}(1-\nu)}} \quad (3.10)$$

where ω is the angular frequency of vibration, related to natural frequency f as

$$\omega = 2\pi f \quad (3.11)$$

and ρ is the density of the PZT transducer material.

For simplicity, let

$$C = \alpha_p A \quad (3.12)$$

where α_p is a factor dependent on the PZT material properties and dimensions. The physical implication of this approximate is that, the elongation in X direction always has some relationship with contraction of patch in Z direction. The interaction between the embedded PZT transducer and the host structure is not completely characterised by the basic material properties such as Young's modulus, piezoelectric constant, dielectric constant and electromechanical coupling factor (Wang and Shen 1998). Hence the factor α_p is introduced in this model to simplify the formulations. The detailed procedure to determine its value is covered in the section "*Experimental initialization.*"

Chapter 3: Embedded Transducers in Sandwiched Beams

Differentiating Equations (3.9a) and (3.9b) with respect to time, and using Equation (3.12), the velocities in the X and Z directions can be written as

$$\dot{u}_1 = j\omega A(\sin kx)e^{j\omega t} \quad (3.13a)$$

$$\dot{u}_3 = \alpha_p j\omega A(\sin kz)e^{j\omega t} \quad (3.13b)$$

Further, differentiating Equations (3.9a) and (3.9b) with respect to x and z, the strains in the X and Z directions can be written as

$$S_1 = \frac{\partial u_1}{\partial x} = Ak(\cos kx)e^{j\omega t} \quad (3.14a)$$

$$S_3 = \frac{\partial u_3}{\partial z} = \alpha_p Ak(\cos kz)e^{j\omega t} \quad (3.14b)$$

Also, Equation (3.7) can be written as

$$Z_{as} = \frac{T_1 W(2H)}{\dot{u}_{1(X=L)}} + \frac{2T_3 WL}{\dot{u}_{3(Z=H)}} \quad (3.15)$$

where the forces are replaced by the stresses acting on the boundary multiplied by the areas on which the stresses act. Substituting Equations (3.8a), (3.8b) and (3.13) into Equation (3.15), the following expression is derived for Z_{as} .

$$Z_{as} = \frac{W\bar{Y}k}{j\omega(a^2 - b^2)} \left[\frac{H(2a \cos kL - b\alpha_p \cos kH)}{\sin kL} + \frac{L(a\alpha_p \cos kH - 2b \cos kL)}{\alpha_p \sin kH} \right] \quad (3.16)$$

Further, let N be a substitute variable, as given below

$$N = \left[\frac{H(2a \cos kL - b\alpha_p \cos kH)}{\sin kL} + \frac{L(a\alpha_p \cos kH - 2b \cos kL)}{\alpha_p \sin kH} \right] \quad (3.17)$$

With this substitution, the mechanical impedance of actuator (Equation 3.16) can be written in a closed form as

$$Z_{as} = \frac{W\bar{Y} kN}{j\omega(a^2 - b^2)} \quad (3.18)$$

3.2.4 ASI of host structure

In this derivation, the ASI approach described in the previous section is again adopted. The mechanical impedance of the structure is determined in the presence of electric field (i.e. $E_3 \neq 0$). Figure 3.1 shows an embedded PZT transducer in the presence of an electric field, E_3 , where F_{TE} , F_{BE} and F_{HE} are the total forces acting

Chapter 3: Embedded Transducers in Sandwiched Beams

on the respective faces due to actuation in the PZT transducer caused by the presence of E_3 .

The size of the PZT transducer is very small as compared to the host structure. Hence uniform stress distribution prevails along the top and bottom edges of the PZT transducer, and they are assumed to act on equal areas (eventually leading to $F_{TE} = F_{BE} = F$). Thus, the structural impedance can be written as

$$-Z_s = \frac{F_{HE}}{\dot{u}_{1(X=L)}} + \frac{2F}{\dot{u}_{3(Z=H)}} \quad (3.19)$$

The negative sign indicates that a positive average displacement in direction X or Z causes compression in the transducer (due to reaction from the structure).

The electric field is given by

$$E_3 = \frac{V_0}{2H} e^{j\omega t} \quad \text{and} \quad V = V_0 e^{j\omega t} \quad (3.20)$$

where V is applied voltage and V_0 is the instantaneous voltage across the PZT transducer.

Using Equations (3.5a), (3.5b) and (3.5c), the stresses acting along the X and Z directions can be written as

$$T_1 = \frac{(S_1 a - S_3 b) \bar{Y}}{a^2 - b^2} + \frac{\bar{Y} E_3}{a^2 - b^2} (d_{33} b - d_{31} a) \quad (3.21a)$$

$$T_3 = \frac{(S_3 a - S_1 b) \bar{Y}}{a^2 - b^2} + \frac{\bar{Y} E_3}{a^2 - b^2} (d_{31} b - d_{33} a) \quad (3.21b)$$

Substituting Equations (3.16), (3.21a) and (3.21b) into Equation (3.19), the ASI of structure can be written as

$$Z_s + Z_{as} = (-1) \frac{\bar{Y} V_0 W}{A(2H)(a^2 - b^2) j\omega} \left[\frac{2H(d_{33} b - d_{31} a)}{\sin kL} + \frac{2L(d_{31} b - d_{33} a)}{\alpha_p \sin kH} \right] \quad (3.22)$$

$$\text{Let} \quad A_0 = A \left(\frac{V_0}{H} \right) \quad (3.23)$$

Chapter 3: Embedded Transducers in Sandwiched Beams

Substituting Equation (3.21) into Equation (3.22) and after rearranging, the following expression is obtained.

$$A_0 = \frac{\bar{Y} W}{2(a^2 - b^2)j\omega(Z_{as} + Z_s)} \left[\frac{2H(d_{31}a - d_{33}b)}{\sin kL} + \frac{2L(d_{33}a - d_{31}b)}{\alpha_p \sin kH} \right] \quad (3.24)$$

In Equation (3.24), Z_{as} , Z_s and A_0 exist together. Although Z_{as} can be determined from Equation (3.18), there is no close form solutions for Z_s and A_0 . Therefore, Z_s has to be determined using the finite element method (FEM) described in detail in the following section. Then, using Z_s , the coefficient A_0 is calculated, and then coefficient A is calculated using Equation (3.23).

3.2.5 Expression for complex EM admittance

Substituting Equations (3.21a) and (3.21b) into Equation (3.5c), the electric displacement can be written as

$$D_3 = \frac{\bar{Y}}{\varepsilon_{33}} \frac{V_0}{2H} e^{j\omega t} + \frac{d_{31}\bar{Y}}{a^2 - b^2} \left[(S_1a - S_3b) + \frac{V_0}{2H} e^{j\omega t} (d_{33}b - d_{31}a) \right] + \frac{d_{33}\bar{Y}}{a^2 - b^2} \left[(S_3a - S_1b) + \frac{V_0}{2H} e^{j\omega t} (d_{31}b - d_{33}a) \right] \quad (3.25)$$

Electric current is the rate of change of total electric charge over the surface, (either top or bottom surface which share opposite charges) as shown in Figure 3.1(b). Hence, the electric current I can be written as

$$I = \iint_{A_{xy}} \dot{D}_3 dx dy = W \int_{X=-L}^{X=L} \dot{D}_3 dx = 2j\omega W \int_0^L D_3 dx \quad (3.26)$$

where A_{xy} is the total surface area. Here, there is no variation along the Y direction (width, W is constant) but there exists variation along the X direction.

EM admittance is the ratio of the electric current to the applied electrical voltage. The complex EM admittance $\bar{Y}_{at}$ and the applied voltage across the PZT transducer are expressed as

Chapter 3: Embedded Transducers in Sandwiched Beams

$$\bar{Y}_{at} = \frac{I}{V} \quad (3.27)$$

Hence, using Equations (3.26) and (3.27), the final complex EM admittance of the PZT transducer is obtained as

$$\begin{aligned} \bar{Y}_{at} = & \frac{j\omega W}{H} (L\bar{\varepsilon}_{33} + \frac{d_{31}\bar{Y}}{a^2 - b^2} \{(2a \sin kL - b\alpha_p kL \cos kH)A_0 + L(d_{33}b - d_{31}a)\} \\ & + \frac{d_{33}\bar{Y}^E}{a^2 - b^2} \{(\alpha_p a kL \cos kH - 2b \sin kL)A_0 + L(d_{31}b - d_{33}a)\}) \end{aligned} \quad (3.28)$$

The procedure for finding the unknowns α_p and A is described in the later sections. Equation (3.28) is the complete expression for the admittance of the embedded PZT transducer.

3.3 Experimental and Numerical Analysis

This section describes the experimental setup, experimental, initialization, determination of ASI of structure using numerical method, experimental verification and convergence test.

3.3.1 Experimental specimen

Figure 3.3 shows the free PZT transducer and sandwiched beam with embedded PZT transducers used in “experimental initialization” and “experimental verification” respectively. In both the “experimental initialization” and “experimental verification”, the PZT transducer was wired to the impedance analyzer through the switch box.

Chapter 3: Embedded Transducers in Sandwiched Beams

Figure 3.3. Experimental specimen used in ASI based impedance method

(a) Free PZT transducer (b) Embedded PZT transducer (c) photo view of specimen 1 and 2

The experimental EM admittance signature, ie. conductance and susceptance was acquired for the desired frequency range of <200 kHz for experimental initialization and <120 kHz for experimental verification. The highest frequency range, till date used by any researcher (Naidu and Soh 2004) for beams is <60 kHz.

The sandwiched beam specimens (Figure 3.3c) used in the experimental verification were fabricated using aluminium beams, epoxy adhesive and PZT transducer. In order to prepare each specimen, a PZT transducer was first surface bonded at the centre of the bottom aluminium beam using a very thin (negligible) epoxy adhesive. After allowing for initial setting, an epoxy layer of certain thickness (as listed in Table 3.3) was applied over the entire surface of the bottom aluminium beam and the bonded PZT transducer. Over this epoxy layer, another aluminium beam was placed (Figure 3.3) and the whole arrangement was allowed to cure for 24 hours with a nominal pressure applied over the entire arrangement throughout the

Chapter 3: Embedded Transducers in Sandwiched Beams

curing time. The PZT transducer which is sandwiched between the two aluminium beams thus behaved as an embedded transducer in the epoxy layer. The embedded transducer was connected to an impedance analyzer (chapter 2, Figure 2.1) which recorded the admittance signature (structure response). Dimension details of the sandwiched beam specimens and embedded PZT transducers are listed in Table 3.3.

Specimen No.	Layers/ PZT transducer	Length (m)	Width (m)	Height (m)
1	Al (top and bottom)	0.230	0.026	0.0020
	Epoxy (middle)	0.230	0.026	0.0010
	Total specimen dimensions ($2L_{Bm} \times W_{Bm} \times 2H_{Bm}$)	0.230	0.026	0.0050
	PZT ($2L \times W \times 2H$)	0.010	0.010	0.0002
2	Al (top and bottom)	0.140	0.026	0.0020
	Epoxy (middle)	0.140	0.026	0.0012
	Total specimen dimensions ($2L_{Bm} \times W_{Bm} \times 2H_{Bm}$)	0.140	0.026	0.0052
	PZT ($2L \times W \times 2H$)	0.010	0.010	0.0003

Table 3.3. Details of specimens and PZT transducer

3.3.2 Experimental initialization

Before using the electromechanical admittance Equation (3.28) for comparison with the experimental results, it is necessary to determine the ASI of the actuator (Z_{as}) and the host structure (Z_s). To find the impedance of the actuator and the host structure α_p needs to be determined. The electromechanical admittance $\bar{Y}_{at}$ (Equation 3.28) is a complex term, and can be separated into real (conductance) and imaginary (susceptance) parts as given below.

$$\bar{Y}_{at} = G + Bj \quad (3.29)$$

Chapter 3: Embedded Transducers in Sandwiched Beams

The free (unembedded/unbonded) signatures of two PZT transducers were experimentally obtained before embedding the PZT transducers into the specimens, using impedance analyzer and multiplexer (Bhalla and Soh 2004a). The unknown α_p was determined using experimental comparisons as described below.

Factor α_p is an unknown parameter; this is determined by matching of experimental conductance and susceptance signatures of the PZT transducer in ‘free-free’ condition with the analytical free conductance and susceptance signatures respectively. From Equations (3.24) and (3.28), to get the free analytical (PZT transducer in free-free condition) admittance signature, a value of ‘zero’ is substituted for Z_s in Equation (3.24), which resulted in

$$A_{0-Free} = \frac{\bar{Y} W}{2(a^2 - b^2)j\omega(Z_a)} \left[\frac{2H(d_{31}a - d_{33}b)}{\sin kL} + \frac{2L(d_{33}a - d_{31}b)}{\alpha_p \sin kH} \right] \quad (3.30)$$

where A_{0-Free} is the coefficient of vibration in the absence of Z_s .

Substituting Equation (3.30) in Equation (3.28), the complex admittance of the free PZT transducer is obtained as

$$\begin{aligned} \bar{Y}_{Fr-at} = & \frac{j\omega W}{H} (L\bar{\epsilon}_{33}^T + \frac{d_{31}\bar{Y}}{a^2 - b^2} \{(2a \sin kL - b\alpha_p kL \cos kH)A_{0-Free} + L(d_{33}b - d_{31}a)\}) \\ & + \frac{d_{33}\bar{Y}}{a^2 - b^2} \{(\alpha_p a kL \cos kH - 2b \sin kL)A_{0-Free} + L(d_{31}b - d_{33}a)\} \end{aligned} \quad (3.31)$$

where $\bar{Y}_{Fr-at}$ is the complex admittance of free PZT, and can be split into the sum of conductance and susceptance as

$$\bar{Y}_{Fr-at} = G_{Free} + j B_{Free} \quad (3.32)$$

Thus, the derived admittance equation is independent of the host structure but depends on the PZT transducer. Equation (3.31) is used to obtain the G_{Free} and B_{Free} signatures for the free PZT transducer, which correspond to the different trial values of α_p . The particular value of α_p at which the analytical signature and experimental signature match satisfactorily is adopted for that PZT transducer. The values of the α_p obtained for the PZT transducers embedded inside the two sandwiched specimens obtained by trial and error are 0.02 and 0.12 respectively. However, to

Chapter 3: Embedded Transducers in Sandwiched Beams

obtain the trial and error value of α_p for the PZT transducer, it is advisable to begin the initial guess for α_p with a ‘reasonable’ value. This ‘reasonable’ value can be obtained using constant axial strain assumption as explained below.

For a mechanically isotropic bar, the elongation (or compression) is proportional to the overall dimension (length/ width) of the bar. In the absence of electric field, just for the initial ‘reasonable’ guess, a free PZT transducer behaviour is assumed to be similar to a mechanically isotropic bar. Mathematically, taking the ratio of amplitudes of displacements of Equations (3.9a) and (3.9b), and using Equation (3.12), the following equation is obtained.

$$\left| \frac{u_1}{u_3} \right| = \frac{A}{\alpha_p A} = \frac{L}{H} \quad (3.33)$$

For the free PZT transducer, from Figure 3.2, the points of consideration are on the boundary. Hence Equation (3.33) can be written as

$$\alpha_p = \frac{H}{L} = D_F \quad (3.34a)$$

where H and L are the half-height and half-length of the PZT transducer respectively, and D_F is a dimensional factor.

After the initial ‘reasonable’ assumption, many trials were made to predict the final value of α_p . This is done by matching the analytically obtained conductance and susceptance signatures of the free PZT transducer in ‘free-free’ condition using the trial values of α_p with the experimental conductance and susceptance of free PZT transducer in ‘free-free’ condition.

From experiments, it was found that changes in some of the material properties of the PZT transducer changed the predicted value of α_p . Three different α_p values are listed in Table 3.4 for the change in some of the material properties of specimen 1. Other material properties are found not to have changed the α_p values. Hence, α_p is written as

$$\alpha_p = (MP_F) D_F \quad (3.34b)$$

where MP_F is a material property factor.

Chapter 3: Embedded Transducers in Sandwiched Beams

Density (kg / m^3)	7800	8005	8095
Dielectric loss factor, δ	0.015	0.012	0.013
Mechanical loss factors, η	0.023	0.025	0.0205
Electric permittivity, ϵ_{33} (farad/m)	2.125×10^{-8}	2.225×10^{-8}	2.325×10^{-8}
α_p	D_{F1}	$1.8D_{F1}$	$2.0D_{F1}$

Table 3.4. Material properties and α_p variations for specimen 1

Figures 3.4 and 3.5 show the plots of the analytical and experimental admittance signatures for the free PZT transducers of the specimens. Satisfactory agreement of both experiment and analytical conductance and susceptance show that the values obtained for α_p for both the PZT transducers are reliable and can be used in determining the mechanical impedance of the actuator and the structure.

Figure 3.4. Plots of free-PZT transducer signatures of specimen 1

(a) Conductance signatures (b) Susceptance signatures

The additional peaks as shown in Figure 3.5 of the experimental admittance signature are due to the deviation in the shape of the PZT transducer from perfect square shape during manufacturing. This leads to somewhat partly independent peaks corresponding to the two slightly unequal edge lengths. It is reported in the literature that the properties of the piezoceramic transducers vary due to inhomogeneous chemical composition and mechanical differences during the formation and polarization process (Sensor Technology Ltd. 1995), and statistical variation are reported to be common (Giurgiutiu and Zagari 2000).

Figure 3.5. Plots of free-PZT signatures of specimen 2
 (a) Conductance signatures (b) Susceptance signatures

A frequency range of 0-200 kHz was considered, and a peak match of both experimental and analytical conductance and susceptance signatures were obtained at 160 and 170 kHz for specimens 1 and 2 respectively.

3.3.3 Determination of structural mechanical impedance using FEM

Actuator impedance (Z_{as}), as described in the previous section, is determined by substituting the value of α_p into Equation (3.17) and then solve Equation (3.18). Unlike the actuator impedance Z_{as} , a simple close form solution is not available for

Chapter 3: Embedded Transducers in Sandwiched Beams

the structural impedance Z_s . Hence, for complex systems, one needs to rely on FEM as this is the most widely used tool in NDE methods. Therefore in this study, this tool was employed as it has the ability to model real-life complex structures.

Recently, researchers such as Makkonen *et al* (2001) demonstrated that in dynamic analysis problems, excitation of very high frequencies (even up to GHz range) can be modelled with good accuracy using FEM. Bhalla and Soh (2004a) (who excited frequencies of >200 kHz) used FEM to verify their impedance model. The excitation of test specimens with harmonic electric field was compared to linear steady state forced vibration, and the new ASI 2D impedance model was verified as below.

The sandwich specimen was discretised into 2D quadrilateral elements as shown in Figure 3.6. Since the structure was symmetric about the Z direction, only the right half of the structure was modelled. The experimental free-free condition was idealized by using appropriate boundary conditions, and the X component of displacement along the Z direction (i.e., axis of symmetry) and the Z component of displacement up to the end of the transducer along the X direction were set to zero.

Figure 3.6. FE mesh of half of the specimen

As shown in Figure 3.6, H_{al} is the thickness of the top and bottom aluminium layers, H_{ep} is the thickness of the epoxy layer, and L_{BM} is the half-length

Chapter 3: Embedded Transducers in Sandwiched Beams

of the beam specimen. The finite element (FE) analysis was carried out using ANSYS 5.6 (ANSYS, 2000), with a differential element mesh sizes (different sizes for different layers viz., aluminium top and bottom, and epoxy layer). The elements used is the 2D quadrilateral element (solid 42, 4 nodes), with 2 degrees of freedom at each node. Details of the mesh sizes and convergence criteria are discussed in the next section.

The PZT transducer was not discretised since in the actuator impedance Z_{as} (Equation 3.18) and EM admittance (Equation 3.28), the stiffness and damping of PZT transducer were already included (Liang *et al* 1994, Bhalla and Soh 2004a). The following differential equation is employed (ANSYS, 2000).

$$[M][\ddot{u}] + [C][\dot{u}] + [K][u] = \{F\} \quad (3.35)$$

where $[M]$ and $[K]$ are the mass matrix and the stiffness matrix respectively, and are given as

$$[M] = \sum_{i=1}^{NE} [M_i^e] \quad \text{and} \quad [K] = \sum_{i=1}^{NE} [K_i^e] \quad (3.36)$$

$[M_i^e]$ is the individual element mass matrix, NE the number of elements, and $[K_i^e]$ is the individual element stiffness matrix. $\{F\}$ is the applied harmonic force matrix and $[C]$ is the structural damping matrix. In practice, the damping matrix is difficult to determine, since, in structural mechanics, one is more interested in dry friction and hysteretic damping rather than viscous damping. Hence, the structural damping matrix $[C]$ is approximated as Rayleigh damping, as

$$[C] = \alpha_d [M] + \beta [K] \quad (3.37)$$

where α_d and β are the mass damping factor and stiffness damping factor respectively. In most of the cases, the mass damping factor, α_d , is ignored (ANSYS 2000). Hence, the stiffness damping factors β can be written as

$$\beta = \sum_{j=1}^{NMAT} \beta_j [K_j] \quad (3.38)$$

where $[K_j]$ = portion of the stiffness matrix based on material j , and $NMAT$ = number of materials (layers) in the model. In this model, 2 materials and 3 layers

Chapter 3: Embedded Transducers in Sandwiched Beams

(top, middle and bottom) were used, where the top and bottom were aluminium layers and the middle layer was an epoxy layer.

The damping matrix $[C]$ can also be expressed as follows

$$[C] = \left(\frac{\eta}{\omega} \right) [K] \quad (3.39)$$

where η is the mechanical loss factor of the material. Hence, the damping factor β can also be expressed as

$$\beta = \frac{\eta}{\omega} = \frac{\eta}{2\pi f}, \text{ more specifically, } \beta_M = \frac{\eta_M}{2\pi f} \quad (3.40)$$

where the subscript M denotes the material type. The values of the damping factors for aluminium and epoxy used in this model are listed in Table 3.2.

In order to determine the ASI at a particular frequency, an arbitrary harmonic force is applied on three edges of the transducer boundary. Using FEM, dynamic harmonic analysis is performed and the complex displacement responses at the points of force application are obtained for frequency range of 120 kHz. Using the linear sums of interpolation functions of all elements, the required displacements are then obtained. Boundary conditions, both natural and essential, are included in the load vectors and stiffness matrix (Bathe 1996). Equation (3.35) was solved by the solution tool of ANSYS 5.6. The approach employed to determine ASI is described below.

The harmonic load applied on the structure can be expressed as

$$\{F\} = \{F_R + F_I j\} e^{j\omega t} \quad (3.41)$$

where F_R and F_I are the real and imaginary components, respectively, of the applied harmonic force vector $\{F\}$. The resultant harmonic displacement is expressed as

$$[u] = [u_R + u_I j] e^{j\omega t} \quad (3.42)$$

where $[u]$ is the complex displacement vector, and u_R and u_I are the real and imaginary components, respectively, of the displacement vector. The displacement is a complex term, due to the phase lag caused by the impedance of the system.

Substituting Equations (3.41) and (3.42) into Equation (3.35), the following equation is obtained

Chapter 3: Embedded Transducers in Sandwiched Beams

$$\{[K] + j\omega[C] - \omega^2[M]\}[u_R + ju_I] = [F_R + jF_I] \quad (3.43)$$

In matrix form, the above equation can be written as

$$\begin{bmatrix} -\omega^2[M] + [K] & -\omega[C] \\ \omega[C] & -\omega^2[M] + [K] \end{bmatrix} \begin{bmatrix} u_R \\ u_I \end{bmatrix} = \begin{bmatrix} F_R \\ F_I \end{bmatrix} \quad (3.44)$$

The unknown displacements at each load point were obtained from the above equation. Using $\dot{u}_1 = j\omega u_1$, $\dot{u}_3 = j\omega u_3$, (Figure 3.1), the ASI of the structure is given as,

$$Z_s = \frac{F_{HE}}{\frac{j\omega}{m} \sum_1^m (u_{mR} + u_{mI})} + \frac{F_{BE}}{\left(\frac{j\omega}{n\alpha_p} \sum_1^n (u_{nR} + u_{nI}) \right)} + \frac{F_{TE}}{\left(\frac{j\omega}{n\alpha_p} \sum_1^n (u_{nR} + u_{nI}) \right)} \quad (3.45)$$

where the force F subscripts, namely *HE*, *BE* and *TE* indicate the horizontal, bottom and top sides of the PZT transducer. In this model, n and m are 5 and 3 respectively.

The procedure used is the full solution method (FSM). Researchers like Bhalla and Soh (2004a) also used FSM to prove the effectiveness of their impedance method. This is more accurate as compared to the reduced solution method (RSM) used by Makkonen *et al* (2001).

3.3.4 Convergence test

In order for the FE analysis (ANSYS 2000) to produce accurate results, appropriate mesh size is an important criterion. Suitably fine meshing to realistically simulate the transfer of the PZT forces (Liang *et al* 1994) is necessary. Thus in the present research, for the two test specimens used, different sets of mesh sizes were employed till the model frequencies converged.

Table 3.5 list the details of mesh size employed for specimens 1 and 2. The model frequencies for set 2 and set 3 as given in Table 3.5 are found to be in close proximity, thus indicating the convergence of the frequencies. Hence, set 3 was finally chosen for specimen 1. Similar procedure was adopted to select the mesh size for specimen 2 (Table 3.5). Figure 3.7 shows the layers used for meshing the top and bottom aluminium layers, and the two sandwiching epoxy layers.

Chapter 3: Embedded Transducers in Sandwiched Beams

Figure 3.7. Layers used for meshing

Specimen No.	Set No.	Mesh size (m x m) ($10^{-3} \times 10^{-3}$)	Layer description
1	1	2 x 2	Aluminium beam (top)
		2 x 0.8	Epoxy layer (top)
		2 x 0.1	Epoxy layer (bottom)
		2 x 2	Aluminium beam (bottom)
	2	1.25 x 1	Aluminium beam (top)
		1.25 x 0.8	Epoxy layer (top)
		1.25 x 0.1	Epoxy layer (bottom)
		1.25 x 1	Aluminium beam (bottom)
	3	1 x 1	Aluminium beam (top)
		1 x 0.8	Epoxy layer (top)
		1 x 0.1	Epoxy layer (bottom)
		1 x 1	Aluminium beam (bottom)
2	1	1 x 1	Aluminium beam (top)
		1 x 0.9	Epoxy layer (top)
		1 x 0.15	Epoxy layer (bottom)
		1 x 1	Aluminium beam (bottom)

Table 3.5. Mesh sizes used for specimens 1 and 2

Table 3.6 lists the modal frequencies of the mesh size employed for specimen 1 with the mode shape description.

Chapter 3: Embedded Transducers in Sandwiched Beams

Mode	Modal Frequency (kHz)			Mode shape description
	Set 1	Set 2	Set 3	
1	0.330	0.327	0.327	Flexure
2	1.981	1.955	1.954	Flexure
3	5.183	5.104	5.099	Flexure
4	9.342	9.182	9.172	Flexure
5	10.457	10.457	10.456	Axial
6	14.167	13.903	13.883	Flexure
7	19.483	19.090	19.057	Flexure
8	25.226	24.674	24.621	Flexure
9	31.360	30.634	30.554	Flexure
10	31.391	31.362	31.361	Axial + Flexure
11	37.975	36.977	36.858	Flexure
12	45.017	43.712	43.542	Flexure
13	52.251	50.848	50.612	Flexure
14	52.537	52.239	52.233	Axial + Flexure
15	60.542	58.393	58.073	Flexure
16	69.045	66.343	65.918	Flexure
17	73.092	73.052	73.033	Axial + Flexure
18	78.029	74.682	74.132	Flexure
19	87.360	83.341	82.649	Flexure
20	93.797	91.721	91.041	Flexure
21	94.061	93.735	93.697	Axial + Flexure
22	97.379	94.638	94.407	Flexure
23	100.066	98.529	98.294	Flexure
24	107.356	102.909	102.021	Flexure
25	109.756	107.687	107.466	Flexure
26	114.179	112.753	111.629	Flexure
27	119.084	114.138	114.030	Axial + Flexure
28	120.957	119.452	118.982	Flexure

Table 3.6. Model frequencies for different mesh sizes of specimen 1

In order to ensure adequacy of the meshing, modal analysis was additionally performed. The element size should be sufficiently small (typically 3 to 5 nodal points per half-wavelength) to ensure accuracy of solution (Makkonen *et al* 2001, Bhalla and Soh 2004a). All the modes of vibration in the frequency range of interest were analysed, from which the wavelengths of the excited modes were found to be quite large as compared to the element size considered. Figure 3.8 shows mode 28 and mode 18 (highest excited mode), characterised by a natural frequency of 118.982 kHz and 119.009 kHz for specimens 1 and 2 respectively. Hence, the criterion of sufficiently small element size is also clearly satisfied.

Figure 3.8. Graphical representation of highest modal frequency of specimens 1 and 2

(a) Mode 28 ($f = 118.982$ kHz) for specimen 1

(b) Mode 18 ($f = 119.009$ kHz) for specimen 2

3.4 Experimental Results and Discussion

Using the ASI of actuator and structure, obtained by FE analysis as described in the preceding sections, the value for A_0 is obtained from Equation (3.24), which is then substituted into Equation (3.28) to finally derive the admittance. The experimental and analytical conductance and susceptance signatures of specimens 1 and 2 are shown in Figures 3.9 and 3.10

Chapter 3: Embedded Transducers in Sandwiched Beams

(a)

(b)

Figure 3.9. Experimental and analytical signatures of specimen 1

(a) Conductance (b) Susceptance

Chapter 3: Embedded Transducers in Sandwiched Beams

(a)

(b)

Figure 3.10. Experimental and analytical signatures of specimen 2

(a) Conductance (b) Susceptance

Chapter 3: Embedded Transducers in Sandwiched Beams

In the considered range of frequency (<120 kHz), both experimental and analytical peaks were observed. The peaks in signatures are dependent on the type of material of the test specimen.

The peak matches of conductance signatures at 95 kHz for specimen 1 and 50 kHz for specimen 2 are clearly evident. Difficulty in using epoxy to bond the aluminium layers could be one of the reasons for the variations of other peaks, but generally the trends are the same.

3.5 Concluding Remarks

In this chapter, a new ASI concept was introduced. The salient features and applications of ASI model are as follow

1. Longitudinal actuation of PZT transducer, which was previously neglected by other researchers, was successfully employed for the first time in the admittance formulations.
2. The formulations considered both the vibrations in the length and thickness directions of the PZT transducer i.e., extensional and longitudinal actuations.
3. The formulations do not impose constraints like the thickness limitation of the PZT transducer, and can be used for all (thin or thick) PZT transducers.
4. Applicable for 2D embeddable structures.

Thus, this chapter presented a generic 2D (length and thickness) PZT – structure interaction based ASI model. The formulations of the model were validated with two sandwiched beam specimens; they were fabricated using aluminium beams sandwiching an epoxy layer. The ASI based analytical admittance signatures were compared with the experimental signatures, and the trends were found to be in good agreement. Thus where ever possible, it is sensible to employ embedded piezo-impedance transducer and its interaction model instead of the existing surface bonded PZT interaction models.

CHAPTER 4

THREE DIMENSIONAL (3D) ELECTROMECHANICAL IMPEDANCE MODEL:

FORMULATION OF DIRECTIONAL SUM IMPEDANCE (DSI)

The developed ASI model (chapter 3) is applicable only for 2D embeddable structures, i.e., it does not consider extensional actuation along Y (*width*) direction of PZT transducer. As given in the chapter 2 (section 2.9), the interaction of PZT-host structure is governed by the extensional actuations along both length and '*width*', and longitudinal actuations along thickness of the PZT transducer. Hence in this chapter, a new 3D model is formulated and experimentally verified using two specimens.

4.1 Introduction

A new 'Directional Sum Impedance' (DSI) model which is based on 3D interaction of PZT-host structure is presented in the sections to follow. This new model considers the extensional actuations along both *length* and *width* directions and the longitudinal actuations along *thickness* of the transducer. The model do not impose restriction on size (thin or thick) and shape (square or rectangular) of PZT as similar to ASI model, additionally it does not impose restriction on electrical properties (isotropic or anisotropic) of PZT. It is applicable for both the surface bonded and the embedded PZT types. Thus, DSI model contains additional features over the existing PZT-host structure interaction models.

4.2 Dimensions of PZT transducer and their influences on EM admittance signature

The *need* to consider 3D PZT-structure interaction is first demonstrated using experimental investigation in this section before formulating the DSI model.

Similar to ‘experimental initialization’ of chapter 3 (section 3.3.2), the free signatures of four PZT transducers of grade B (Table 4.1) with different dimensions (Table 4.2) were experimentally obtained for the ‘free-free’ boundary condition.

Type	Physical property	PZT
Mechanical	Density (kg / m^3)	7800
	Young’s Modulus (N / m^2) $\times 10^9$	66.67
	Poisson ratio	0.33
	Mechanical loss factor, η	0.023
Electrical	Piezoelectric strain coefficients d_{31} and d_{32} (m/V) $\times 10^{-10}$ (along directions X and Y)	-2.10
	Piezoelectric strain coefficient d_{33} (m/V) $\times 10^{-10}$ (along direction Z)	4.50
	Dielectric loss factor, δ	0.015
	Electric permittivity, ϵ_{33} (farad/m) $\times 10^{-8}$	1.75

Table 4.1. Key properties of PZT transducer (Grade B)

S. No	Notation (Length, width and thickness)	Dimensions (mm^3)
1	$W_1 L_1 T_1$	10 x 10 x 0.5
2	$W_1 L_1 T_2$	10 x 10 x 2
3	$W_2 L_2 T_1$	15 x 15 x 0.5
4	$W_2 L_2 T_2$	15 x 15 x 2

Table 4.2. PZT transducer dimensions

Figure 4.1 shows the experimentally obtained signatures for the four PZT transducers. All the signatures were compared, and different peak and slope shifts were observed. Predominantly horizontal, vertical, and horizontal and vertical peak and slope shifts were observed. It was evident from the figure that the change in thickness of PZT transducer for the same length and width ($W_1 L_1 T_1$ and $W_1 L_1 T_2$ or $W_2 L_2 T_1$ and $W_2 L_2 T_2$) produced vertical peak and slope shift. Change in length and width for the same thickness ($W_1 L_1 T_1$ and $W_2 L_2 T_1$ or $W_1 L_1 T_2$ and $W_2 L_2 T_2$) produced horizontal and vertical peak and slope shifts. Similarly change in length, width and thickness ($W_2 L_2 T_2$ and $W_1 L_1 T_1$ or $W_2 L_2 T_1$ and $W_1 L_1 T_2$) produced horizontal and vertical peak and slope shifts. Thus, for given mechanical and electrical properties of the PZT transducer (Table 4.1) change in length, width and thickness produced peaks and shifts.

In short, the Figure 4.1 illustrates the need for a 3D PZT-host structure interaction model to consider the vibrations along the length, width and thickness of the PZT; that is, the extensional and longitudinal actuations should be considered in the formulation.

Figure 4.1. Admittance signatures of four free-free PZT transducers.

(a) Conductance signatures (b) Susceptance signatures

4.3 Directional sum impedance (DSI) of PZT-host structure interaction model

Figure 4.2 illustrates the PZT-host structure interaction model using embedded and surface bonded PZT transducers. Figure 4.2(a) depicts the embedded PZT transducer interaction with the host structure in the X, Y and Z directions along the length ($2W$), width ($2L$) and thickness ($2H$) of the PZT transducer.

The (distributed) forces are developed due to the two actuation mechanisms, viz. extensional actuations along X and Y directions, and longitudinal actuation along Z direction of the PZT transducer. At any instant of time, the forces developed due to the actuations correspond respectively to expansion in the X and Y directions, and shrinkage in the Z direction; and vice versa. The forces take into consideration the alternate signs of d_{31} (or d_{32}) and d_{33} , as listed in Table 4.1. Due to the opposite signs of d_{31} (or d_{32}) and d_{33} , the expansion of transducer in the X and Y directions is accompanied by shrinkage in the Z direction; and vice versa. Similarly, Figure 4.2(b) illustrates the surface bonded PZT transducer interaction with the host structure.

(a)

(b)

Figure 4.2. 3D distribution of forces and impedances on one-quarter of PZT transducer (a) Embedded PZT transducer (b) Surface bonded PZT transducer.

The DSI of the host structure is defined as the sum of the linear impedances along the X, Y and Z directions, and the cross impedances in the XY, YZ and XZ planes. Hence, impedance of the host structure (Z_s) for a PZT transducer is the sum of the linear and the cross impedances. This can be written as

$$-(Z_s)_{Normal_shear} = (\text{linear impedances})_{Normal} + (\text{cross impedances})_{Shear} \quad (4.1)$$

The subscript *Normal_shear* implies that the structural impedance is dependent on the normal and shear stresses of the PZT transducer. The negative sign indicates a reaction to the applied forces. Impedance of structure is obtained by adding all the impedances acting on all faces of the PZT transducer, as shown in Figure 4.2, which results in the following equation.

$$-Z_s = Z_{s1} + Z_{s2} - Z_{s3} + 2Z_{12} - 2Z_{23} - 2Z_{31} \quad (4.2)$$

Positive and negative signs are in accordance with expansion along the X and Y directions and contraction along the Z direction, and vice versa. Thus, Equation (4.2) expresses the complete linear impedance (CLI) which is deduced based on the normal forces (or stresses), shear forces (or stresses) and velocities obtained by the forces.

The linear impedances are given by

$$Z_{s1} = \frac{F_x}{\dot{u}_1}, \quad Z_{s2} = \frac{F_y}{\dot{u}_2} \quad \text{and} \quad Z_{s3} = \frac{F_z}{\dot{u}_3} = \frac{F_T - F_B}{\dot{u}_{zB} - \dot{u}_{zT}} \quad (4.3)$$

where $F_x = \sigma_x(2LH)$, $F_y = \sigma_y(2WH)$, $F_T = -\sigma_z(LW)$ and $F_B = \sigma_z(LW)$ are the total force components acting on the PZT transducer on face X, face Y, face Z_{Top} and face Z_{Bottom} respectively as shown in Figure 4.2a. F_z represents the resultant force along Z direction, and u represents the displacement. $\dot{u}_{1(X=W)}$ and $\dot{u}_{2(Y=L)}$ are the average velocities of considered points along X and Y directions at $X = +W$ (on face X) and at $Y = L$ (on face Y) respectively. $\dot{u}_{zB}$ and $\dot{u}_{zT}$ are the average velocities of considered points along Z direction on face Z (bottom and top). $\dot{u}_3$ represents the resultant velocity along Z direction. For surface bonded PZT transducer $F_z = -F_B$ and $\dot{u}_3 = \dot{u}_{zB}$ (see Figure 4.2b). Figure 4.3 shows the normal stresses σ_x , σ_y and σ_z

acting on the PZT transducer along the X, Y and Z directions, and τ_{xy} , τ_{yz} and τ_{zx} the shear stresses acting on the planes XY, YZ and ZX respectively.

Figure 4.3. Schematic representation of stresses acting on a differential element.

The cross impedances, which are produced by shear forces (or stresses) acting on the planes XY, YZ and ZX are given by

$$Z_{12} \approx -\frac{Z_{S1}Z_{S2}}{Z_{S1} + Z_{S2} - Z_{S3}}, Z_{23} \approx -\frac{Z_{S2}Z_{S3}}{Z_{S1} + Z_{S2} - Z_{S3}} \text{ and } Z_{31} \approx -\frac{Z_{S3}Z_{S1}}{Z_{S1} + Z_{S2} - Z_{S3}} \quad (4.4)$$

Substituting Equations (4.3) and (4.4) into Equation (4.2) results in

$$-Z_S = Z_{S1} + Z_{S2} - Z_{S3} - 2 \left(\frac{Z_{S1}Z_{S2}}{Z_{S1} + Z_{S2} - Z_{S3}} + \frac{Z_{S2}Z_{S3}}{Z_{S1} + Z_{S2} - Z_{S3}} + \frac{Z_{S3}Z_{S1}}{Z_{S1} + Z_{S2} - Z_{S3}} \right) \quad (4.5)$$

Rearranging Equation (4.5) leads to

$$-Z_S = \frac{(Z_{S1} + Z_{S2} - Z_{S3})^2 - 2(Z_{S1}Z_{S2} + Z_{S2}Z_{S3} - Z_{S3}Z_{S1})}{Z_{S1} + Z_{S2} - Z_{S3}} \quad (4.6)$$

Equation (4.6) is reduced into

$$-Z_S = Z_{S1} \left(\frac{Z_{S1}}{Z_{S1} + Z_{S2} - Z_{S3}} \right) + Z_{S2} \left(\frac{Z_{S2}}{Z_{S1} + Z_{S2} - Z_{S3}} \right) - Z_{S3} \left(\frac{Z_{S3}}{Z_{S1} + Z_{S2} - Z_{S3}} \right) \quad (4.7)$$

Let λ_1, λ_2 and λ_3 be the response factors along the X, Y and Z directions, given by

$$\lambda_1 = \left(\frac{Z_{S1}}{Z_{S1} + Z_{S2} - Z_{S3}} \right), \lambda_2 = \left(\frac{Z_{S2}}{Z_{S1} + Z_{S2} - Z_{S3}} \right) \text{ and } \lambda_3 = \left(\frac{Z_{S3}}{Z_{S1} + Z_{S2} - Z_{S3}} \right) \quad (4.8)$$

Substituting Equation (4.8) into Equation (4.7) results to

$$-Z_S = (Z_{S1}\lambda_1 + Z_{S2}\lambda_2 - Z_{S3}\lambda_3) \quad (4.9)$$

Equation (4.9) contains many unknowns; and moreover, there is no close-form solution to determine the response factors λ_1 , λ_2 and λ_3 and the impedances Z_{S1} , Z_{S2} and Z_{S3} . Hence, a numerical method is used, details of which are presented in the later section.

Equation (4.9) in terms of forces and stresses can be written as

$$-Z_S = \frac{F_X \lambda_1}{\dot{u}_1} + \frac{F_Y \lambda_2}{\dot{u}_2} - \frac{(F_T - F_B) \lambda_3}{\dot{u}_{ZB} - \dot{u}_{ZT}} = \frac{(\sigma_1)2LH}{\dot{u}_1} + \frac{(\sigma_2)2WH}{\dot{u}_2} - \frac{(\sigma_3)LW}{\dot{u}_3} \quad (4.10)$$

where σ_1, σ_2 and σ_3 are the semi-analytical directional stresses given as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}_{Semi_analytical} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_{Numerical} \begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \end{bmatrix}_{Analytical} \quad (4.11)$$

Finally,

$$-(Z_S)_{Directional} = Z_x + Z_y - Z_z \quad (4.12)$$

where Z_x , Z_y and Z_z are the directional structural impedance components along the X, Y and Z directions of the PZT transducer, and Equation (4.12) is termed as the directional sum impedance (DSI) of structure. Thus, Equation (4.2), a function of normal and shear stresses is equivalent to Equation (4.12), a function of directional stresses. Mathematically,

$$-(Z_S)_{Normal_shear} = -(Z_S)_{Directional} \quad (4.13)$$

4.3.1 Stress-strain relationship of PZT transducer element subjected to 3D loading

Considering the coordinate system shown in Figure 4.3, the stress-strain relationship in terms of induced strains is written as

$$\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{bmatrix} = Y_R \begin{bmatrix} 1 & R & R & 0 & 0 & 0 \\ R & 1 & R & 0 & 0 & 0 \\ R & R & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & 0 \\ 0 & 0 & 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{bmatrix} \quad (4.14)$$

where Y_R , R and r are the simplification parameters given as

$$Y_R = \frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)}, R = \frac{\nu}{1-\nu} \text{ and } r = \frac{1-2\nu}{2(1-\nu)} \quad (4.15a)$$

$\bar{Y}$ is the complex Young's modulus of elasticity of the 3D PZT transducer at zero electric field, and it can be expressed as similar to Equation 3.2 (chapter 3) as

$$\bar{Y} = Y(1+\eta j) \quad (4.15b)$$

where Y is the static Young's modulus of elasticity of the PZT material, and η is the mechanical loss factor. ν is the poison's ratio of the PZT transducer, and ε_X , ε_Y and ε_Z are the normal strains along the X, Y and Z directions. τ_{XY} , τ_{YZ} and τ_{ZX} are the shear stresses, and γ_{XY} , γ_{YZ} and γ_{ZX} are the shear strains.

The induced strains in terms of displacements are given as

$$\begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_1}{\partial x} \\ \frac{\partial u_2}{\partial y} \\ \frac{\partial u_3}{\partial z} \\ \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} \\ \frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial x} \\ \frac{\partial u_3}{\partial x} + \frac{\partial u_1}{\partial z} \end{bmatrix} \quad (4.16)$$

Consider the differential element depicted in Figure 4.3, the force equilibrium can be written as

$$\begin{aligned} & \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx \right) dydz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy \right) dx dz + \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial z} dz \right) dx dy \\ & - \sigma_x dydz - \tau_{xy} dx dz - \tau_{xz} dx dy - m_x = 0 \end{aligned} \quad (4.17a)$$

$$\begin{aligned} & \left(\sigma_y + \frac{\partial \sigma_y}{\partial y} dy \right) dx dz + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial x} dx \right) dy dz + \left(\tau_{yz} + \frac{\partial \tau_{yz}}{\partial z} dz \right) dx dy \\ & - \sigma_y dx dz - \tau_{yx} dy dz - \tau_{yz} dx dy - m_y = 0 \end{aligned} \quad (4.17b)$$

$$\begin{aligned} & \left(\sigma_z + \frac{\partial \sigma_z}{\partial z} dz \right) dx dy + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial x} dx \right) dy dz + \left(\tau_{zy} + \frac{\partial \tau_{zy}}{\partial y} dy \right) dx dz \\ & - \sigma_z dx dy - \tau_{zx} dy dz - \tau_{zy} dx dy - m_z = 0 \end{aligned} \quad (4.17c)$$

where m_x , m_y and m_z are the inertial forces of a PZT transducer along the X, Y and Z directions respectively, and are given by

$$\begin{aligned} m_x &= \rho dx dy dz \frac{\partial^2 u_1}{\partial t^2}, \quad m_y = \rho dx dy dz \frac{\partial^2 u_2}{\partial t^2} \\ \text{and } m_z &= \rho dx dy dz \frac{\partial^2 u_3}{\partial t^2} \end{aligned} \quad (4.18)$$

where ρ is the mass density of the element.

Substituting Equation (4.18) into Equation (4.17) results in

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho \frac{\partial^2 u_1}{\partial t^2} \quad (4.19 a)$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = \rho \frac{\partial^2 u_2}{\partial t^2} \quad (4.19 b)$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = \rho \frac{\partial^2 u_3}{\partial t^2} \quad (4.19c)$$

Substituting Equations (4.14) and (4.16) into Equation (4.19a) leads to

$$\begin{aligned}
\frac{\partial}{\partial x}(\sigma_x) + \frac{\partial}{\partial y}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{xz}) = & \frac{\partial}{\partial x} \frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & R & R \end{bmatrix} \begin{bmatrix} \frac{\partial u_1}{\partial x} \\ \frac{\partial u_2}{\partial y} \\ \frac{\partial u_3}{\partial z} \end{bmatrix} \\
& + \frac{\partial}{\partial y} \frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} r \end{bmatrix} \left[\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} \right] \\
& + \frac{\partial}{\partial z} \frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} r \end{bmatrix} \left[\frac{\partial u_3}{\partial x} + \frac{\partial u_1}{\partial z} \right] = \rho \frac{\partial^2 u_1}{\partial t^2} \quad (4.20a)
\end{aligned}$$

Equation (4.20a) can be written as,

$$\begin{aligned}
\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \left\{ \left\langle \frac{\partial^2 u_1}{\partial x^2} \right\rangle + \left\langle R \left[\frac{\partial^2 u_2}{\partial x \partial y} + \frac{\partial^2 u_3}{\partial x \partial z} \right] + r \left[\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} + \frac{\partial^2 u_2}{\partial x \partial y} + \frac{\partial^2 u_3}{\partial x \partial z} \right] \right\rangle \right\} \\
= \rho \frac{\partial^2 u_1}{\partial t^2} \quad (4.20b)
\end{aligned}$$

Equation (4.20b) is a tedious differential equation to solve. Hence, it is split into two equations relating the normal and shear strains, separately as

$$\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \left\langle \frac{\partial^2 u_1}{\partial x^2} \right\rangle = \rho \frac{\partial^2 u_1}{\partial t^2} \quad (4.21a)$$

$$\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \left\langle R \left[\frac{\partial^2 u_2}{\partial x \partial y} + \frac{\partial^2 u_3}{\partial x \partial z} \right] + r \left[\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} + \frac{\partial^2 u_2}{\partial x \partial y} + \frac{\partial^2 u_3}{\partial x \partial z} \right] \right\rangle = 0 \quad (4.21b)$$

Similarly, substituting Equations (4.14) and (4.16) into Equations (4.19b) and (4.19c) results the following differential equations relating the normal and shear strains as

$$\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 u_2}{\partial y^2} = \rho \frac{\partial^2 u_2}{\partial t^2} \quad (4.22a)$$

$$\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \left\langle R \left[\frac{\partial^2 u_1}{\partial x \partial y} + \frac{\partial^2 u_3}{\partial y \partial z} \right] + r \left[\frac{\partial^2 u_1}{\partial x \partial y} + \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial z^2} + \frac{\partial^2 u_3}{\partial y \partial z} \right] \right\rangle = 0 \quad (4.22b)$$

$$\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 u_3}{\partial z^2} = \rho \frac{\partial^2 u_3}{\partial t^2} \quad (4.23a)$$

$$\frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)} \left\langle R \left[\frac{\partial^2 u_1}{\partial x \partial z} + \frac{\partial^2 u_2}{\partial y \partial z} \right] + r \left[\frac{\partial^2 u_1}{\partial x \partial z} + \frac{\partial^2 u_2}{\partial y \partial z} + \frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} \right] \right\rangle = 0 \quad (4.23b)$$

Equations (4.21), (4.22) and (4.23) are the differential equations for 3D normal and shear stress-strain relationships of PZT transducer.

4.3.2 Solution to stress-strain differential equations of PZT transducer

The complete solution to Equations (4.21), (4.22) and (4.23) is given by summation of the normal and shear based solutions. The solutions to the normal stress-strain Equations (4.21a), (4.22a) and (4.23a) are in the form of 3D wave pattern acting along X, Y and Z directions, which are given as

$$u_1 = [A \sin kx + B \cos kx] e^{j\omega t} \quad (4.24a)$$

$$u_2 = [C \sin ky + D \cos ky] e^{j\omega t} \quad (4.24b)$$

and
$$u_3 = [E \sin kz + F \cos kz] e^{j\omega t} \quad (4.24c)$$

A , B , C , D , E and F are the coefficients to be determined, and wave number

$$k = \omega \sqrt{\frac{\rho(1+\nu)(1-2\nu)}{\bar{Y}(1-\nu)}} \quad (4.25)$$

where ω is the angular frequency of excitation. To solve the unknown coefficients A to F , apply the displacement boundary conditions, $u_1 = 0$, $u_2 = 0$ and $u_3 = 0$ at the centre of the PZT transducer (Figure 4.2), which results in

$$B = D = F = 0, \text{ and } A, C \text{ and } E \text{ remain as unknowns} \quad (4.26)$$

The unknown coefficients A , C and E cannot be determined analytically as there are fewer independent equations than there are variables. Hence a numerical method is used, details of which are presented in the later sections.

The solutions to the shear stress-strain Equations (4.21b), (4.22b) and (4.23b) are not in the form of wave pattern, but they are in the form of mixed wave pattern. However, the solutions are required only to u_1 , u_2 and u_3 , which are obtained by solving the normal stress-strain equations. Hence, only the solution relating to the normal stress-strain equations is considered.

4.4 Experimental specimens

Two specimens were used for verification of the 3D model. The first was a sandwiched plate specimen fabricated using aluminium plates (grade A1 6061-T6, Table 3.1 of chapter 3), the PZT transducer (grade B, Table 4.1) and high strength epoxy adhesive (RS 850-940, Table 3.1). A PZT transducer was first surface bonded at the centre of the bottom aluminium plate using a very thin (negligible) layer of epoxy adhesive. After the initial setting, an epoxy layer of certain thickness (as listed in Table 4.3) was applied over the entire surface of the bottom aluminium plate and the bonded PZT transducer.

Specimen No.	Layers / PZT	2 W (mm) (along X)	2L (mm) (along Y)	2H (mm) (along Z)
1	Aluminium (Top and bottom)	50	50	2
	Epoxy (Middle)	50	50	1
	Total specimen (Dimensions)	50	50	5
2	Aluminium	100	100	2

Table 4.3. Dimensions of specimens

Over this epoxy layer, another aluminium plate was placed and the whole arrangement was allowed to cure for 24 hours with a nominal pressure applied over the entire arrangement throughout the curing. The PZT transducer sandwiched between the two aluminium plates thus behaved as an embedded transducer in the epoxy layer.

The second was an aluminium plate specimen with a PZT transducer surface bonded at the centre of the bottom aluminium plate using a very thin (negligible) epoxy adhesive. Details of specimens 1 and 2, and the PZT transducers are listed in Tables 4.3 and 4.4 respectively. Thus, the first specimen behaved as an embedded

transducer specimen and the second as a surface bonded transducer specimen.

4.5 Finite element analysis

In this study, dynamic finite element analysis (FEA) was used as simple close-form solution was not available for the structural impedance Z_s , the coefficients A , C , and E , and the response factors [(Equations (4.13) and (4.26b))].

Figure 4.4 shows the finitely discretized mesh of the specimens used in the experimental verification. Because of symmetry about the X and Y directions, only one quadrant of the specimen was considered. Appropriate boundary conditions were imposed on the planes of symmetry, that is, the x and y components of the displacement were set to zero on the YZ and the ZX planes of symmetry, respectively. In addition, at the centre of the PZT transducer, the z component of the displacement was set to zero as shown in Figure 4.4.

The finite element meshing was carried out using ANSYS 5.6 (ANSYS, 2000). The specimen was discretised into 3D brick elements (solid 45) possessing 3 degrees of freedom at each node, as shown in Figure 4.4, with differential element mesh size (ie, different sizes for different layers viz., aluminium top and bottom, and epoxy layer). Convergence study was performed to decide on the sufficiency of the adopted mesh sizes for both the specimens (Makkonen *et al* 2001; Bhalla and Soh 2004). The details of the mesh sizes are given in Table 4.4.

(a)

(b)

- (a) Embedded PZT specimen and PZT actuations
- (b) FE mesh of embedded PZT specimen
- (c) Surface bonded PZT specimen and PZT actuations
- (d) FE mesh of surface bonded PZT specimen

Specimen No.	Type of PZT	Dimensions of PZT (2W x 2L x 2H) (mm x mm x mm)	Layer	Mesh size (mm x mm x mm)
1	Embedded	10 x 10 x 0.5	Al (top)	1 x 1 x 1
			Epoxy	1 x 1 x 0.25
			Al (bottom)	1 x 1 x 1
2	Surface bonded	10 x 10 x 0.5	Al	1 x 1 x 1

Table 4.4. Details of PZT and mesh sizes of specimens

In order to determine the DSI at a particular frequency, an arbitrary harmonic force was applied on the edges of the transducer. Using FEA, the dynamic harmonic analysis was performed and the complex displacement responses at the points of force application were obtained for frequency range of 100 kHz.

4.6 Numerical evaluation of structural responses and linear impedances

The unknowns A , C and E of Equation (4.26) and, Z_{s1}, Z_{s2}, Z_{s3} , λ_1, λ_2 and λ_3 of Equation (4.9) are determined in this section. Mathematically, the unknowns can be written as

$$p_i = (Z_{s1}, Z_{s2}, Z_{s3}, \lambda_1, \lambda_2, \lambda_3, A, C, E) \quad (4.27)$$

$$\text{Let } A = A_0 W \left(\frac{V_0}{2H} \right), C = C_0 L \left(\frac{V_0}{2H} \right) \text{ and } E = E_0 \left(\frac{V_0}{2H} \right) \quad (4.28)$$

where V_0 is the instantaneous voltage constant.

Substituting Equations (4.28) and (4.26a) into Equation (4.24) results in

$$\dot{u}_1 = j\omega A_0 W \left(\frac{V_0}{2H} \right) \sin kWe^{j\omega x} \quad (4.29a)$$

$$\dot{u}_2 = j\omega C_0 L \left(\frac{V_0}{2H} \right) \sin kL e^{j\omega t} \quad (4.29b)$$

and

$$\dot{u}_3 = j\omega E_0 \left(\frac{V_0}{2H} \right) \sin 2kH e^{j\omega t} \quad (4.29c)$$

Substituting Equations (4.14), (4.16) (4.24), (4.26) (4.28) and (4.29) into Equation (4.3), the linear impedance along X direction in presence of electric field E_3 can be written as

$$Z_{s1} = 2kLH \left[\frac{[A_0 kW \cos kW - d_{31}]}{j\omega A_0 W \sin kW} + \frac{R[C_0 kL \cos kL - d_{32}]}{j\omega A_0 W \sin kW} + \frac{R[E_0 k \cos 2kH - d_{33}]}{j\omega A_0 W \sin kW} \right] \quad (4.30)$$

Let $R_1 = 2kLH$, $R_2 = 2kWH$ and $R_3 = kLW$ (4.31)

Substituting Equation (4.31) into Equation (4.30) and rearranging, yields

$$Z_{s1} j\omega A_0 W \sin kW = R_1 A_0 kW \cos kW + R R_1 C_0 kL \cos kL + R R_1 E_0 k \cos 2kH - R_1 (d_{31} + R(d_{32} + d_{33})) \quad (4.32)$$

Equation (4.32) can be further expressed as

$$A_0 (R_1 kW \cos kW - Z_{s1} j\omega W \sin kW) + C_0 (R R_1 kL \cos kL) + E_0 (R R_1 k \cos 2kH) = R_1 (d_{31} + R(d_{32} + d_{33})) \quad (4.33)$$

Z_{s1} was determined using FEA, as a ratio of sum of the distributed load to the sum of velocities produced at the points/ nodes of consideration. Mathematically

$$Z_{s1} = \frac{\sum_{i=1}^{mx} F_i}{\sum_{X=1}^{mx} \dot{u}_X} = \frac{F_X}{\dot{u}_1} \quad (4.34)$$

where $F_1 = 1+0j$ is the distributed force applied on the PZT transducer on face X, as shown in Figure 4.2. mx is the finite number of points considered on face X (at $X=W$), $\dot{u}_X$ is the velocity of the mx^{th} point in direction X and F_X is the sum of distributed harmonic load of all the considered points.

For simplification, let a_1 , a_2 , a_3 and a_4 be substitution variables given by

$$a_1 = (R_1 kW \cos kW - Z_{s1} j\omega W \sin kW), \quad a_2 = (R R_1 k L \cos kL),$$

$$a_3 = (R R_1 k \cos 2kH) \text{ and } a_4 = R_1 (d_{31} + R(d_{32} + d_{33}))$$

Hence, Equation (4.33) can be written as

$$a_1 A_0 + a_2 C_0 + a_3 E_0 = a_4 \quad (4.35)$$

Similarly, the linear impedances along directions Y and Z (ie, Z_{s2} and Z_{s3})

were determined using FEA as

$$Z_{s2} = \frac{\sum_{Y=1}^{my} F_2}{\sum_{Y=1}^{my} \dot{u}_Y} = \frac{F_Y}{\dot{u}_2} \text{ and } Z_{s3} = - \left[\frac{\sum_{ZB=1}^{mz} F_{3B} - \sum_{ZT=1}^{mz} F_{3T}}{\sum_{ZB=1}^{mz} \dot{u}_{ZB} - \sum_{ZT=1}^{mz} \dot{u}_{ZT}} \right] = \frac{F_Z}{\dot{u}_3} \quad (4.36)$$

F_2, F_{3B} and F_{3T} are the unit forces applied on the PZT transducer on faces Y, Z bottom and Z top respectively, as shown in Figure 4.2. my and mz are the finite number of points of consideration along the boundary on face Y and face Z_{Bottom} . Equal number of points on face Z_{Bottom} and face Z_{Top} are considered. $\dot{u}_Y, \dot{u}_{ZB}$ and $\dot{u}_{ZT}$ are the velocities of the my^{th}, mz^{th} (Z_{Bottom}) and mz^{th} (Z_{Top}) points in directions Y and Z respectively.

Similarly for Z_{s2} , the following Equation results

$$b_1 A_0 + b_2 C_0 + b_3 E_0 = b_4 \quad (4.37)$$

where $b_1 = (R R_2 kW \cos kW), b_2 = (R_2 k L \cos kL - Z_{s2} j\omega L \sin kL),$

$$b_3 = (R R_2 k \cos 2kH) \text{ and } b_4 = R_2 (d_{32} + R(d_{31} + d_{33}))$$

Similarly for Z_{s3} , the following Equation results

$$c_1 A_0 + c_2 C_0 + c_3 E_0 = c_4 \quad (4.38)$$

where $c_1 = (R R_3 kW \cos kW), c_2 = (R R_3 kL \cos kL),$

$$c_3 = (R_3 k \cos 2kH - Z_{s3} j\omega \sin 2kH) \text{ and } c_4 = R_3 (d_{33} + R(d_{31} + d_{32}))$$

Solving Equations (4.35), (4.37) and (4.38) for A_0, C_0 and E_0 results in following expressions

$$E_0 = \frac{(a_4 b_1 - a_1 b_4)(b_2 c_1 - b_1 c_2) - (a_2 b_1 - a_1 b_2)(b_4 c_1 - b_1 c_4)}{(a_3 b_1 - a_1 b_3)(b_2 c_1 - b_1 c_2) - (a_2 b_1 - a_1 b_2)(b_3 c_1 - b_1 c_3)} \quad (4.39)$$

$$C_0 = \frac{(b_4c_1 - b_1c_4) - (b_3c_1 - b_1c_3)E_0}{(b_2c_1 - b_1c_2)} \quad (4.40)$$

$$A_0 = \frac{a_4 - a_2c_0 - a_3E_0}{a_1} \quad (4.41)$$

The unknowns A , C and E were obtained by substituting Equation (4.28) into Equations (4.39), (4.40) and (4.41). Z_{S1} , Z_{S2} and Z_{S3} were determined from Equations (4.34) and (4.36). λ_1 , λ_2 and λ_3 were obtained using Z_{S1} , Z_{S2} and Z_{S3} in Equation (4.8). Thus all the unknowns which were not determined using analytical equations were determined using FEA.

4.6.1 Stress-strain relationship of PZT transducer element subjected to 3D loading

Considering the actuations of PZT as shown in Figure 2.10 (of Chapter 2), The stresses of the transducer in the presence of small electric fields along the X, Y and Z directions were obtained by introducing electric fields and strain displacement coefficients into Equation (4.14). The normal and shear stresses are as given by Sirohi and Chopra (2000)

$$\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{bmatrix}_e = Y_R \begin{bmatrix} 1 & R & R & 0 & 0 & 0 \\ R & 1 & R & 0 & 0 & 0 \\ R & R & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & 0 \\ 0 & 0 & 0 & 0 & 0 & r \end{bmatrix} \begin{bmatrix} \epsilon_X - E_3d_{31} \\ \epsilon_Y - E_3d_{32} \\ \epsilon_Z - E_3d_{33} \\ \gamma_{XY} - 0 \\ \gamma_{YZ} - E_2d_{24} \\ \gamma_{ZX} - E_1d_{15} \end{bmatrix} \quad (4.42)$$

The subscript ‘ e ’ represents the electric field. Electric fields E_1 and E_2 produce shear actuations (d_{15}, d_{24}), and electric field E_3 produces extensional actuations (d_{31}, d_{32}) along X and Y directions and longitudinal actuation (d_{33}) along Z direction. The normal strains obtained using Equations (4.24), (4.26a) and (4.16) can be expressed as

$$\epsilon_x = \frac{\partial u_1}{\partial x} = Ak \cos kx.e^{j\omega t}, \epsilon_y = \frac{\partial u_2}{\partial y} = Ck \cos ky.e^{j\omega t}$$

$$\text{and } \epsilon_z = \frac{\partial u_3}{\partial z} = Ek \cos kz \cdot e^{j\omega t} \quad (4.43)$$

The shear strains can similarly be expressed as

$$\gamma_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} = [Ay \sin kx + Cx \sin ky]e^{j\omega t}, \quad \gamma_{yz} = [Cx \sin ky + Ex \sin kz]e^{j\omega t}$$

$$\text{and } \gamma_{zx} = [Ex \sin kz + Az \sin kx]e^{j\omega t} \quad (4.44)$$

However, in this study, the PZT transducer was only excited to a desired frequency range in the presence of E_3 electric field (along Z direction) as shown in Figure 4.2. This produced extensional actuations along the length and width, and longitudinal actuation along the thickness direction. These actuations produced structural responses in the form of EM admittance signatures, which is the basis for damage detection in SHM. This section deals with the EM admittance formulation as described below.

The electric field E_3 along Z direction is same as the applied electric field in DSI model (chapter 3), which is given as

$$E_3 = \frac{V}{2H} \quad (4.45)$$

where $V = V_0 e^{j\omega t}$ is the voltage applied across the PZT transducer.

Substituting Equation (4.11) into Equation (4.42), the directional stresses can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}_e = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \left\{ Y_R \begin{bmatrix} 1 & R & R \\ R & 1 & R \\ R & R & 1 \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix} - E_3 \begin{bmatrix} d_{31} \\ d_{32} \\ d_{33} \end{bmatrix} \right\} \quad (4.46)$$

4.7 Admittance formulation

The electric displacement (or charge density), D_3 , (where subscript 3 refers to the electric field direction) over the top or bottom surface of the PZT transducer (Figure 4.2) can be written as

$$D_3 = \overline{\epsilon_{33}} E_3 + d_{31} \sigma_1 + d_{32} \sigma_2 + d_{33} \sigma_3 \quad (4.47)$$

$\overline{\epsilon_{33}}$ is the complex electric permittivity of the PZT at zero stress, and can be expressed similar to that of Equation 3.6 given in DSI model as

$$\overline{\epsilon_{33}} = \epsilon_{33} (1 - \delta j) \quad (4.48)$$

where δ is the dielectric loss factor and ϵ_{33} is the static electric permittivity of the PZT transducer.

Equation (4.47) is the DSI based sensor equation, which is semi-analytical in nature, and it forms the basis for obtaining the admittance equation. Substituting Equations (4.41) and (4.46) into Equation (4.47) results in

$$\begin{aligned} D_3 = & \overline{\epsilon_{33}} E_3 + Y_R (d_{31} \lambda_1 \{ [\epsilon_x - E_3 d_{31}] + R[\epsilon_y - E_3 d_{32}] + R[\epsilon_z - E_3 d_{33}] \} \\ & + d_{32} \lambda_2 \{ R[\epsilon_x - E_3 d_{31}] + [\epsilon_y - E_3 d_{32}] + R[\epsilon_z - E_3 d_{33}] \} \\ & + d_{33} \lambda_3 \{ R[\epsilon_x - E_3 d_{31}] + R[\epsilon_y - E_3 d_{32}] + [\epsilon_z - E_3 d_{33}] \}) \end{aligned} \quad (4.49)$$

Substituting Equations (4.43) and (4.44) into Equation (4.49) results in

$$\begin{aligned} D_3 = & \overline{\epsilon_{33}} E_3 + \\ & Y_R (d_{31} \lambda_1 \{ [Ak \cos kx.e^{j\omega t} - E_3 d_{31}] + R[Ck \cos ky.e^{j\omega t} - E_3 d_{32}] + R[Ek \cos kz.e^{j\omega t} - E_3 d_{33}] \} \\ & + d_{32} \lambda_2 \{ R[Ak \cos kx.e^{j\omega t} - E_3 d_{31}] + [Ck \cos ky.e^{j\omega t} - E_3 d_{32}] + R[Ek \cos kz.e^{j\omega t} - E_3 d_{33}] \} \\ & + d_{33} \lambda_3 \{ R[Ak \cos kx.e^{j\omega t} - E_3 d_{31}] + R[Ck \cos ky.e^{j\omega t} - E_3 d_{32}] \\ & + [Ek \cos kz.e^{j\omega t} - E_3 d_{33}] \}) \end{aligned} \quad (4.50)$$

Electric current I is the rate of change of the total electric charge over the surface (area A) of the PZT transducer (either top or bottom, see Figure 4.2).

Mathematically, it can be represented as

$$I = \iint_A \dot{D}_3 dA = j\omega \iint_A D_3 dx dy \quad (4.51)$$

where $\dot{D}_3$ is the time rate of change of charge. Substituting Equation (4.50) into Equation (4.51) results in

$$\begin{aligned} I = & j\omega [LW \overline{\epsilon_{33}} E_3 + Y_R (d_{31} \lambda_1 \{ [LA \sin kx.e^{j\omega t} - LWE_3 d_{31}] + R[WC \sin ky.e^{j\omega t} - LWE_3 d_{32}] \\ & + R[LWEk \cos kz.e^{j\omega t} - LWE_3 d_{33}] \} \\ & + d_{32} \lambda_2 \{ R[LA \sin kx.e^{j\omega t} - LWE_3 d_{31}] + [LWk \sin ky.e^{j\omega t} - LWE_3 d_{32}] \\ & + [LWEk \cos kz.e^{j\omega t} - LWE_3 d_{33}] \}) \end{aligned}$$

$$\begin{aligned}
& + R[LWEk \cos kz.e^{j\omega t} - LWE_3 d_{33}] \Big\} \\
& + d_{33} \lambda_3 \Big\{ R[LA \sin kx.e^{j\omega t} - LWE_3 d_{31}] + R[WC \sin ky.e^{j\omega t} - LWE_3 d_{32}] \\
& + [LWEk \cos kz.e^{j\omega t} - LWE_3 d_{33}] \Big\} \quad (4.52)
\end{aligned}$$

3D EM admittance is the ratio of the resultant electric current to the applied electrical voltage, as similar to 2D EM admittance given in Equation 3.27 (chapter 3)

$$\bar{Y}_{at} = \frac{I}{V} \quad (4.53)$$

where V is the applied voltage across the PZT transducer.

Using Equations (4.45) and (4.52), the following Equation is obtained

$$\begin{aligned}
\bar{Y}_A = j\omega \Bigg[\frac{LW}{2H} \bar{\mathcal{E}}_{33} + Y_R \Bigg[d_{31} \lambda_1 \Big\{ \Big[\frac{LA_0 W}{2H} \sin kW - \frac{LW}{2H} d_{31} \Big] + R \Big[\frac{WC_0 L}{2H} \sin kL - \frac{LW}{2H} d_{32} \Big] \\
+ R \Big[\frac{LW}{2H} E_0 k \cos k2H - \frac{LW}{2H} d_{33} \Big] \Big\} \\
+ d_{32} \lambda_2 \Big\{ R \Big[\frac{LA_0 W}{2H} \sin kW - \frac{LW}{2H} d_{31} \Big] + \Big[\frac{WC_0 L}{2H} \sin kL - \frac{LW}{2H} d_{32} \Big] \\
+ R \Big[\frac{LWE_0}{2H} k \cos k2H - \frac{LW}{2H} d_{33} \Big] \Big\} \\
+ d_{33} \lambda_3 \Big\{ R \Big[\frac{LA_0 W}{2H} \sin kW - \frac{LW}{2H} d_{31} \Big] + R \Big[\frac{WC_0 L}{2H} \sin kL - \frac{LW}{2H} d_{32} \Big] \\
+ \Big[\frac{LWE_0}{2H} k \cos k2H - \frac{LW}{2H} d_{33} \Big] \Big\} \Bigg] \Bigg] \quad (4.54)
\end{aligned}$$

After simplification, the final complex 3D EM admittance of the PZT transducer is given as

$$\begin{aligned}
\bar{Y}_A = \frac{j\omega LW}{2H} \Big[\bar{\mathcal{E}}_{33} + \\
Y_R \Big\{ d_{31} \lambda_1 \Big\{ \Big[A_0 \sin kW - d_{31} \Big] + R \Big[C_0 \sin kL - d_{32} \Big] + R \Big[E_0 k \cos k2H - d_{33} \Big] \Big\} + \\
d_{32} \lambda_2 \Big\{ R \Big[A_0 \sin kW - d_{31} \Big] + \Big[C_0 \sin kL - d_{32} \Big] + R \Big[E_0 k \cos k2H - d_{33} \Big] \Big\} + \\
d_{33} \lambda_3 \Big\{ R \Big[A_0 \sin kW - d_{31} \Big] + R \Big[C_0 \sin kL - d_{32} \Big] + \Big[E_0 k \cos k2H - d_{33} \Big] \Big\} \Big\} \Big] \quad (4.55)
\end{aligned}$$

In the present case, the PZT transducers and the specimens are symmetrical along the XY plane; hence, only one-quarter of the transducer and specimen are considered. The 3D EM admittance $\overline{Y}_C$ of the complete specimen is therefore given as

$$\overline{Y}_C = 4\overline{Y}_A \quad (4.56)$$

4.8 Experimental results and discussion

The experimental and DSI based predicted admittance signatures of specimens 1 and 2 are shown in Figures 4.5 and 4.6. In the range of frequency considered (<100 kHz), both the experimental and semi-analytical peaks matched very well. The peaks in the signatures are dependent on the type of material of the test specimen.

The peak matches of both the conductance and susceptance signatures for both specimens are clearly evident. Thus, a 3D PZT–host structure interaction model, considering both the extensional and longitudinal actuations of PZT transducer, is successfully formulated.

(a)

(b)

Figure 4.5. Admittance signature of embedded transducer specimen

(a) Conductance (b) Susceptance

(a)

(b)

Figure 4.6. Admittance signature of surface bonded transducer specimen

(a) Conductance (b) Susceptance

4.9 Concluding remarks

In this chapter, a new DSI concept was introduced. The salient features and applications of ASI model are as follows

1. The formulations include both the extensional and longitudinal actuations; thus the vibrations of the PZT along the length, width and thickness of the PZT transducer are accorded equal importance.
2. The model does not impose constraints like electric isotropy, thickness (thin or thick), shape (square or rectangle) or size limitation on the PZT transducer.
3. Applicable for both *surface bonded* and *embedded* PZT transducer structures.

Thus, this chapter presented a generic 3D PZT-host structure interaction model, which can be employed to extract the EM admittance signature of any “unknown” 3D host structure. The model has been experimentally validated using embedded and surface bonded transducer specimens. The DSI based semi-analytical admittance signatures were compared with the experimental signatures and the trends were found to be in good agreement. Next chapter focuses on characterization of PZT transducer using 3D DSI model and its implementation in damage analysis.

CHAPTER 5

CHARACTERIZATION OF ADMITTANCE SIGNATURE AND PZT USING 3D DSI MODEL

This chapter investigates and presents the following objectives

1. *Influences of dimensions and type* (surface/ embedded) of PZT on the 3D DSI based conductance and the susceptance signatures.
2. *Influences of mechanical and electrical properties* of the PZT on the 3D DSI conductance and the susceptance signatures. This results in *characterization* of PZT transducer.
3. *Proof-of-concept* of 3D DSI model (chapter 4) in damage analysis, where the damage was numerically simulated for various types of specimens and the predicted signatures were compared.
4. A ‘novel’ *PZT efficiency factor*.
5. Draws attention towards the misleading deviations in the EM admittance signatures due to changes in the PZT properties which involve risk of over or under estimations of *actual* damages during SHM.

5.1 Introduction

The final 3D DSI based equation for complete specimen can be written by substituting Equation 4.55 into Equation 4.56 (of chapter 4) as

$$\begin{aligned} \overline{Y}_A = 2 \frac{j\omega LW}{H} [\overline{\varepsilon}_{33} + \\ Y_R \{ d_{31}\lambda_1 \{ [A_0 \sin kW - d_{31}] + R[C_0 \sin kL - d_{32}] + R[E_0 k \cos k2H - d_{33}] \} + \\ d_{32}\lambda_2 \{ R[A_0 \sin kW - d_{31}] + [C_0 \sin kL - d_{32}] + R[E_0 k \cos k2H - d_{33}] \} + \\ d_{33}\lambda_3 \{ R[A_0 \sin kW - d_{31}] + R[C_0 \sin kL - d_{32}] + [E_0 k \cos k2H - d_{33}] \} \}] \quad (5.1) \end{aligned}$$

In general, the DSI based predicted admittance signatures were obtained using Equation (5.1)

5.2 Dimensions of PZT transducer and their influences on 3D DSI admittance

This section demonstrates the following two cases

- (a) The influence of length/ width (i.e. extensional actuation) for fixed thickness (fixed longitudinal actuation) of *embedded* PZT transducers on DSI admittance signature.
- (b) The influence of thickness (i.e. longitudinal actuation) for fixed length and width (fixed extensional actuation) of *surface bonded* PZT transducers on DSI admittance signature.

For this purpose, two specimen cases (embedded and surface bonded type) as listed in Table 5.1 are considered. In the first specimen case, *two embedded specimens* (PZT-S and PZT-R) and in the second specimen case, *three surface bonded specimens* (T1, T2 and T3) were considered (Table 5.2). Thus, a total of five *numerical* specimens (two embedded and three surface bonded types) were considered to demonstrate two cases.

Throughout the *numerical* investigation, free-free boundary condition of the host structure is assumed. In both these cases, the properties of grade A1 6061-T6 for aluminum plates (Table 3.1 of chapter 3), grade B for PZT transducers (Table 4.1) and RS 850-940 for epoxy adhesive (Table 3.1) were used in the numerical modeling. Finally the predicted signatures are plotted in Figure 5.1.

Thus, this section presents the influence of the extensional and longitudinal actuations for both *embedded* and *surface bonded* PZT in predicting conductance and susceptance signatures, as shown in Figure 5.1.

Chapter 5: Characterization using 3D DSI Model

Specimen cases	Specimen Type	Layers	Length Width Height		
			(mm)	(mm)	(mm)
1	Embedded	Top Al plate	50	50	2
		Epoxy (Middle)	50	50	1
		Bottom Al plate	50	50	2
		Total specimen	50	50	5
2	Surface bonded	Al plate	100	100	2

Table 5.1. Dimensions of specimens

Specimen cases	Description of PZT transducer	Notation	Dimensions of PZT
			(2Wx2Lx2H) (mm x mm x mm)
1	Embedded inside epoxy layer	PZT –S (Square)	10 x 10 x 0.5
		PZT –R (Rectangular)	16 x 10 x 0.5
2	Surface bonded on aluminium layer	T1	10 x 10 x 0.5
		T2	10 x 10 x 1
		T3	10 x 10 x 2

Table 5.2. Dimensions of PZT transducers used in specimens

(a)

(b)

(c)

(d)

Figure 5.1. Admittance signature of numerical specimens
 (a) Embedded specimens: conductance of (b) Susceptance
 (c) Surface bonded specimens: conductance (d) Susceptance

Figures 5.1(a) and 5.1(b) show the variations in DSI based signatures of specimen case1 for two different widths (PZT-S and PZT- R), but with same thickness of PZT transducer. Similarly, Figures 5.1(c) and 5.1(d) depict the variations in the DSI based

signatures of specimen case 2 for various thicknesses (T1, T2 and T3), but with fixed length and width of surface bonded PZT transducer. From Figures 5.1(a) and 5.1(b), it is apparent that longer PZT transducer PZT-R results in taller peaks and higher slopes (anti-clock wise rotation). On the other hand, thicker PZT transducer T2 and T3 results in smaller peak heights and lower slopes as shown in Figures 5.1(c) and 5.1(d). The observations are in accordance with Equation (5.1), where the length and width are present in the numerator and the thickness of PZT is present in the denominator.

5.3 Characterization of properties of PZT

This section presents the study of influence of ‘variations in each and every mechanical and electrical property’ except mass (density) of PZT transducer in predicting the conductance and susceptance signatures. The mass (density, Table 4.1 of chapter 4) influence was not considered because the basic assumption in the DSI formulations is that the PZT is small as compared to the host structure.

In general, the effectiveness of all the EMI models (for example see Equation 5.1) are dependent on the appropriate PZT properties. Any deviations of the predicted from the experimental EM admittance signatures can be attributed to the deviations of PZT properties supplied by the manufacturers (Bhalla and Soh 2004). In order to correctly predict the DSI admittance using Equation (5.1), it is a must to understand the behaviour of each and every property of the PZT transducer.

From Equation (5.1), it is evident that it cannot be split into *PZT dependent* and *structure dependent* equations, as the Equation (5.1) consists implicitly both the PZT and structure properties. Hence, the complete Equation (5.1) was used to study the influence of variations in the PZT properties (listed in Table 4.1 of chapter 4).

The difference in EM admittance due to a change in (any two different) ‘complex electric permittivity’ ($\overline{\epsilon_{33}}$) of PZT but without changing the other properties of the PZT and structure [of Equation (5.1)], can be expressed as

$$(\bar{Y}_c)_2 - (\bar{Y}_c)_1 = j \frac{2\omega LW}{H} [(\bar{\epsilon}_{33})_2 - (\bar{\epsilon}_{33})_1] \quad (5.2)$$

Substituting Equation (4.48) (of chapter 4) into Equation (5.1) and keeping ϵ_{33} as constant but varying δ the following equation can be obtained

$$(\bar{Y}_c)_2 - (\bar{Y}_c)_1 = j \frac{2\omega LW}{H} \epsilon_{33} [1 - (\nabla \delta)j] \quad (5.3)$$

Thus, Equation (5.3) can be written as

$$\nabla \bar{Y}_c = \nabla G + \nabla B j \propto j[1 - (\nabla \delta)j] \propto \nabla \delta \propto \nabla G_\delta \quad (5.4)$$

where ∇ denotes the difference, and subscript ' δ ' denotes the influence of δ in predicting G and B . From Equation (5.4), it is implicit that δ only influences the conductance (G_δ) signature. Let

$$I = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \left(\begin{aligned} & d_{31}\lambda_1 \{ [A_0 \sin kW - d_{31}] + R[C_0 \sin kL - d_{32}] + R[E_0 k \cos k2H - d_{33}] \} + \\ & d_{32}\lambda_2 \{ R[A_0 \sin kW - d_{31}] + [C_0 \sin kL - d_{32}] + R[E_0 k \cos k2H - d_{33}] \} + \\ & d_{33}\lambda_3 \{ R[A_0 \sin kW - d_{31}] + R[C_0 \sin kL - d_{32}] + [E_0 k \cos k2H - d_{33}] \} \end{aligned} \right) \quad (5.5)$$

Substituting Equation (5.5) into Equation (5.1), the difference in electromechanical admittance due to a change in (any two different) 'mechanical loss factor' (η) of PZT but without changing the other properties of PZT and structure [of Equation (5.1)], can be expressed as

$$(\bar{Y}_c)_2 - (\bar{Y}_c)_1 = j \frac{2\omega LW}{H} [Y(\eta_2 - \eta_1)] I j \quad (5.6)$$

Thus, Equation (5.6) can be written as

$$\nabla \bar{Y}_c \propto (j)(\eta_2 - \eta_1)(j) \propto \nabla \eta \propto \nabla G_\eta \quad (5.7)$$

From Equation (5.7), it can be seen that η influences the conductance (G_η) signature.

Substituting Equation (4.48) (of chapter 4) into Equation (5.2) and keeping δ constant but varying ϵ_{33} results in

$$(\bar{Y}_c)_2 - (\bar{Y}_c)_1 = j \frac{2\omega LW}{H} [\{(\epsilon_{33})_2 - (\epsilon_{33})_1\} - \delta\{(\epsilon_{33})_2 - (\epsilon_{33})_1\}j] \quad (5.8)$$

Thus, Equation (5.8) can be written as

$$\nabla \bar{Y}_c \propto j[\nabla(\epsilon_{33}) - \nabla(\epsilon_{33})j] \propto [\nabla(\epsilon_{33}) + \nabla(\epsilon_{33})j] \propto \nabla G_\epsilon + \nabla B_\epsilon j \quad (5.9)$$

From Equation (5.9), it is apparent that ε_{33} influences both the conductance (G_ε) and susceptance (B_ε) signatures.

The difference in electromechanical admittance due to a change in (any two different) ‘Young’s modulus’ (Y) of PZT but without changing the other properties of PZT and structure [of Equation (5.1)] can be expressed as

$$(\bar{Y}_c)_2 - (\bar{Y}_c)_1 = j \frac{2\omega LW}{H} [(Y_2 - Y_1) + \eta (Y_2 - Y_1)j] \quad \text{I} \quad (5.10)$$

Thus, Equation (5.10) can be written as

$$(\bar{Y}_c)_2 - (\bar{Y}_c)_1 \propto (Y_2 - Y_1) + (Y_2 - Y_1)j \propto \nabla Y + \nabla Y j \propto \nabla G_Y + \nabla B_Y j \quad (5.11)$$

From Equation (5.11), it can be seen that Y influences both the conductance (G_Y) and susceptance (B_Y) signatures

From previous chapter, it can be shown that

$$d_{31} \propto A_0, \quad d_{32} \propto C_0 \quad \text{and} \quad d_{33} \propto E_0 \quad (5.12)$$

where A_0, C_0 and E_0 are the vibration coefficients and can be expressed as

$$A_0 = a + j b, \quad C_0 = c + j d \quad \text{and} \quad E_0 = e + j f \quad (5.13)$$

where a, c and e are the real coefficients, and b, d and f are the imaginary coefficients.

The difference in electromechanical admittance due to a change in (any two different) ‘piezo electric strain coefficients’ (d_{31}) of PZT but without changing the other properties of PZT and structure [Equation (5.1)], can be expressed as

$$\begin{aligned} (\bar{Y}_c)_2 - (\bar{Y}_c)_1 = j \frac{2\omega LW}{H} [\nabla d_{31} \lambda_1 \{\nabla A_0 \sin kW - \nabla d_{31}\} + d_{32} \lambda_2 R \{\nabla A_0 \sin kW - \nabla d_{31}\} \\ + d_{33} \lambda_3 R \{\nabla A_0 \sin kW - \nabla d_{31}\}] \end{aligned} \quad (5.14)$$

Equation (5.14) can be further expressed as

$$\nabla \bar{Y}_c \propto j [\{\nabla d_{31} \nabla A_0 - \nabla d_{31} \nabla d_{31}\} + \{\nabla A_0 - \nabla d_{31}\} + \{\nabla A_0 - \nabla d_{31}\}] \quad (5.15)$$

Substituting Equation (5.13) into Equation (5.15) leads to

$$\begin{aligned} \nabla \bar{Y}_c \propto j [\nabla d_{31} \{\nabla a + j \nabla b - \nabla d_{31}\} + \{\nabla a + j \nabla b - \nabla d_{31}\} \\ + \{\nabla a + j \nabla b - \nabla d_{31}\}] \end{aligned} \quad (5.16)$$

Equation (5.16) can be further expressed as

$$\nabla \bar{Y}_C \propto [(-\nabla b \nabla d_{31} - 2 \nabla b) + j (\nabla a - \nabla d_{31}) (\nabla d_{31} + 2)] \quad (5.17)$$

From Equations (5.12), (5.13) and (5.17) the following is obtained

$$\nabla d_{31} \propto (\nabla a + j \nabla b) \text{ and } \nabla \bar{Y}_C \propto (\nabla d_{31} + \nabla d_{31} j) \propto (\nabla G_{d1} + \nabla B_{d1} j) \quad (5.18a)$$

Similarly,

$$\nabla d_{32} \propto (\nabla c + j \nabla d) \text{ and } \nabla \bar{Y}_C \propto (\nabla d_{32} + \nabla d_{32} j) \propto (\nabla G_{d2} + \nabla B_{d2} j) \quad (5.18b)$$

$$\text{and } \nabla d_{33} \propto (\nabla e + j \nabla f) \text{ and } \nabla \bar{Y}_C \propto (\nabla d_{33} + \nabla d_{33} j) \propto (\nabla G_{d3} + \nabla B_{d3} j) \quad (5.18c)$$

From Equation (5.18), it can be observed that any change in d_{31} (or d_{32} or d_{33}) influences both the conductance G_{d1} (or G_{d2} or G_{d3}) and susceptance B_{d1} (or B_{d2} or B_{d3}) signatures.

From Equation (4.15) (of chapter 4) the following can be obtained

$$Y_R = f_v (1 + \eta j) Y = Y f_v + Y \eta j f_v \quad (5.19)$$

The difference in electromechanical admittance due to a change in (any two different) ‘poisson ratio’ (ν) of PZT but without changing the other properties of PZT and structure [Equation (5.1)], and using Equations (5.1) and (5.4) results in

$$(\bar{Y}_C)_2 - (\bar{Y}_C)_1 \propto \{ (f_v)_2 - (f_v)_1 + [(f_v)_2 - (f_v)_1] j \} \propto \nabla f_v + \nabla f_v j \propto \nabla G_v + \nabla B_v j \quad (5.20)$$

From Equation (5.20), it can be seen that ν influences both the conductance (G_v) and susceptance (B_v) signatures

The Equations (5.4), (5.7), (5.9), (5.11), (5.18) and (5.20) show the influences of the variations of the PZT properties in predicting conductance and susceptance signatures. However, the type of signature shifts, that is, either ‘slope’ shift or ‘slope and peak’ shift are yet to be established for the complete characterization of the PZT properties.

For the present study, the embedded transducer specimen PZT-S (see Table 5.2) was considered for frequency range of 0 to 100 kHz. For the considered specimen, without

altering the dimensions and properties of host structure, the DSI based conductance and susceptance signatures for seven different variations (Table 5.3) of PZT transducers were predicted. Each variation comprises of a change in any one mechanical or electrical property (as listed in Table 5.3) while keeping the other properties the same as listed in Table 4.1 (chapter 4). However, variation 1 was unaltered and all of the properties were as listed in Table 4.1.

Variation No.	Changed property	Property type	Adopted value
1	None	-	-
2	Dielectric loss factor δ	Electrical	0.020
3	Mechanical loss factor η	Mechanical	0.013
4	Electric permittivity ϵ_{33} (farad/m)	Electrical	2.1×10^{-8}
5	Young's Modulus (N / m^2)	Mechanical	45×10^9
6	Piezoelectric Strain Coefficients d_{31} , d_{32} and d_{33} (m/V) along directions X, Y and Z	Electrical	-1.50×10^{-10} , -1.50×10^{-10} and 3.0×10^{-10}
7	Poisson ratio	Mechanical	0.43

Table 5.3. Different variations and variations in properties of PZT transducer

Figures 5.2 and 5.3 along with Table 5.4 illustrate the ‘slope’ shifts and ‘slope and peak’ shifts of the conductance and susceptance signatures as explained below. Figure 5.2 shows that there were a total of eight peaks (P1 to P8), where P3 and P7 were the major peaks, and all others were minor peaks. It is evident that there were seven different conductance signatures and only five different susceptance signatures for the seven variations of PZT properties. Figure 5.2(a) shows that the change in mechanical and electrical properties of PZT transducers for variations 2 to 7 produced the vertical shifts in conductance signatures. Figure 5.2 (b) shows that the susceptance signatures of variations 2 and 3 coincide with variation 1. This is in accordance with Equations (5.4) and (5.7); That is, loss factors (mechanical and electrical) only change the conductance signatures without changing the susceptance signatures. Figures 5.3(a) and 5.3(b) show the difference in conductance signatures for various variations (see Table 5.3) from variation 1. The numbers 21, 31, 41, 51, 61 and 71 on the figures represent the deviations of conductance of variations 2, 3, 4, 5, 6 and 7 from variation 1 respectively.

(a)

(b)

Figure 5.2. Admittance signature of embedded PZT specimen using PZT-S

(a) Conductance (b) Susceptance

(a)

(b)

Figure 5.3. Difference of conductance signature of PZT-S for various PZT property variations. (a) Deviations of variations 2, 3, 4 and 7 from variation 1
(b) Deviations of variations 5 and 6 from variation 1

Variation No.	Changed property	Property type	Conductance	Susceptance
2	Dielectric loss factor δ	Electrical	Slope shift	-
3	Mechanical loss factor η	Mechanical	Slope and peak shifts	-
4	Electric permittivity ϵ_{33} (farad/m)	Electrical	Slope shift	Slope shift
5	Young's modulus (N / m^2)	Mechanical	Slope and peak shifts	Slope and peak shifts
6	Piezoelectric strain coefficients d_{31} , d_{32} and d_{33} (m/V) (along directions X, Y and Z)	Electrical	Slope and peak shifts	Slope and peak shifts
7	Poisson ratio	Mechanical	Slope and peak shifts	Slope and peak shifts

Table 5.4. Slope and peak influence details of PZT transducer properties

The difference of admittance for variation in the mechanical loss factor δ was equal to the difference of conductance signature without any difference in susceptance signature [Equation (5.4)]. Number 21 in Figure 5.3(a), which represents the plot of difference in the conductance signature between variation 2 and variation 1, was a 'slope

influencing' factor without any peak influence. Similarly, it can be seen from Figure 5.2(b) that the susceptance signature of variation 2 and variation 1 coincides, thus indicating no change in susceptance signatures between variation 2 and variation 1. It is therefore apparent that the mechanical loss factor (δ) was a conductance 'slope shifting' property.

The difference of admittance for variation in the electrical loss factor η was equal to the difference of conductance signature without any difference in susceptance signature [Equation (5.7)]. Number 31 in Figure 5.3(a) shows that the plot of difference in conductance signature between variation 3 and variation 1 was a sloping straight line with peaks. Similarly, it can be seen from Figure 5.2 (b) that the susceptance of variation 3 and variation 1 coincides, thus indicating no change in susceptance signatures between variation 3 and variation 1. It is therefore apparent that the electrical loss factor (η) was a conductance 'slope and peak shifting' factor.

The difference of admittance for variation in the electric permittivity factor ϵ_{33} was equal to the difference of both conductance and susceptance signatures [Equation (5.9)]. The same is evident from the difference of conductance depicted by Number 41 in Figure 5.3(a) (indicating only slope shift), and of susceptance by Number 4 in Figure 5.2(b). From figures, it is therefore apparent that the ϵ_{33} was a conductance and susceptance 'slope shifting' electrical factor.

The difference of admittance for variation in the Young's modulus Y was equal to the difference of both conductance and susceptance signatures [Equation (5.11)]. The same is evident from the difference of conductance depicted by Number 51 in Figure 5.3(b) (indicating both the slope and peak shifts), and of susceptance by Number 5 in Figure 5.3(b). From these figures, it is therefore apparent that the Y was a conductance and susceptance 'slope and peak shifting' mechanical factor.

The difference of admittance for variation in the piezo electric strain coefficients d_{31} , d_{32} and d_{33} were equal to the difference of conductance and susceptance signatures

[Equation (5.18)]. The same is evident from the difference of conductance depicted by Number 61 in Figure 5.3(b) (indicating slope and peak shift), and of susceptance by Number 6 in Figure 5.2(b). From these figures, it is therefore apparent that the strain coefficients were conductance and susceptance ‘slope and peak shifting’ electrical factor.

The difference of admittance for variation in the Poisson’s ratio ν was equal to the difference of both the conductance and susceptance signatures [Equation (5.20)]. The same is evident from the difference of conductance depicted by Number 71 in Figure 5.3(b) (indicating slope and peak shift), and of susceptance by Number 7 in Figure 5.2(b). From these figures, it is therefore apparent that the strain coefficients were conductance and susceptance ‘slope and peak shifting’ mechanical factor.

Figures 5.2(b) and 5.3 also show that the slope and peak are inversely proportional to each other, that is an increase in slope (anti clock wise rotation) is accompanied by a (vertical) decrease in peak, and vice versa (see variations 5, 6 and 7 of Figure 5.3). Table 5.4 lists the final influence of each and every property on the conductance and susceptance signatures.

5.4 Effect of property variations on damage indices

Embedded PZT (PZT-S and PZT-R) specimens (Tables 5.1 and 5.2) were used to validate the DSI based prediction of admittance signature and damage indices. The specimens and the damages were all simulated using ANSYS 5.6 (ANSYS, 2000). The properties and grades of specimen are considered same as that of chapter 4. This section deals with the signature predictions of the undamaged, damage 1 and damage 2 states (Figure 5.4), and comparisons of the damage indices using a ‘novel’ PZT efficiency factor.

(a)

(b)

Figure 5.4. Elements of FE Analysis of embedded PZT specimen and damages

(a) Undamaged state (b) Damage 1 (c) Damage 2

5.5 Frequency Range

Figure 5.3(a) shows that the conductance signature had eight sharp peaks over the range of 0-100 kHz. Out of these peaks, only P3 and P7 are the major peaks whose presence was felt even in the susceptance signature [Figure 5.3(b)]. Since the major peaks occur in frequency range of 50-60 kHz and 75-85 kHz, these ranges were studied in detail.

5.6 Damage description and assessment

Figure 5.4(a) shows the FE model of one-quarter of the undamaged state specimen, Figures 5.4(b) and 5.4(c) show the simulated damage 1 and damage 2 states respectively. As shown, two damage types were simulated along X- direction. Damage 1 was 20 mm long, 1 mm wide and 1 mm deep, simulated by reducing by 50% the stiffness and mass of the top aluminium surface elements of that region (20 mm x 1 mm x 1 mm) as shown in Figure 5.4(b). Similarly, damage 2 was 10 mm long, 2 mm wide and 1 mm deep, simulated by reducing by 90 % the stiffness and mass of the elements in that region (10 mm x 2 mm x 1 mm) as shown in Figure 5.4 (c)

Figures 5.5 to 5.11 show the undamaged, damage1 and damage 2 states of DSI based prediction of conductance and susceptance signatures for embedded PZT- S specimen for various PZT property variations 1 to 7 (Table 5.4). Similarly, Figures 5.12 to 5.18 show the undamaged, damage1 and damage 2 states of DSI based prediction of signatures for embedded PZT- R specimens for various PZT property variations 1 to 7. The figures also show the undamaged state conductance peak heights, and it was observed that none of the PZT property variation gave the same results. Earlier, it was observed that conductance and susceptance signatures for different variations were different from variation1 to variation 7 (Figure 5.2). Hence, heights of peaks of conductance were different for each different variation for both embedded PZT-S and PZT-R. One more observation was that the peaks of conductance and susceptance signatures of variations 5 and 6 are shorter than the variations 1, 2, 3, 4 and 7 for both the embedded specimens.

Figure 5.5. Admittance signatures of pre and post damage cases for variation 1 of PZT-S

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.6. Admittance signatures of pre and post damage cases for variation 2 of PZT-S

(a) conductance of frequency range 50- 60 kHz

(b) susceptance of frequency range 50- 60 kHz

(c) conductance of frequency range 75- 85 kHz

(d) susceptance of frequency range 75- 85 kHz

Figure 5.7. Admittance signatures of pre and post damage cases for variation 3 of PZT-S

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.8. Admittance signatures of pre and post damage cases for variation 4 of PZT-S

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.9. Admittance signatures of pre and post damage cases for variation 5 of PZT-S

(a) conductance of frequency range 50- 60 kHz

(b) susceptance of frequency range 50- 60 kHz

(c) conductance of frequency range 75- 85 kHz

(d) susceptance of frequency range 75- 85 kHz

Figure 5.10. Admittance signatures of pre and post damage cases for variation 6 of PZT-S

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.11. Admittance signatures of pre and post damage cases for variation 7 of PZT-S

(a) conductance of frequency range 50- 60 kHz

(b) susceptance of frequency range 50- 60 kHz

(c) conductance of frequency range 75- 85 kHz

(d) susceptance of frequency range 75- 85 kHz

Figure 5.12. Admittance signatures of pre and post damage cases for variation 1 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.13. Admittance signatures of pre and post damage cases for variation 2 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.14. Admittance signatures of pre and post damage cases for variation 3 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.15. Admittance signatures of pre and post damage cases for variation 4 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.16. Admittance signatures of pre and post damage cases for variation 5 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.17. Admittance signatures of pre and post damage cases for variation 6 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

Figure 5.18. Admittance signatures of pre and post damage cases for variation 7 of PZT -R

- (a) conductance of frequency range 50- 60 kHz
- (b) susceptance of frequency range 50- 60 kHz
- (c) conductance of frequency range 75- 85 kHz
- (d) susceptance of frequency range 75- 85 kHz

In the EMI based SHM, the prime indicator of damage is the change in the admittance signatures of the embedded/ surface bonded PZT transducer. Bhalla (2001) compared many techniques and observed that the root mean square deviation (RMSD %) between the signatures is the most suitable non parametric damage index to quantify

structural damage. Hence, RMSD was used in the present study as well for damage 1 and 2 states.

Figure 5.19. RMSD (%) of different variations for damages 1 and 2 of specimen 1

- (a) PZT-S: for frequency range of 50-60 kHz
- (b) PZT-S: for frequency range of 75-85 kHz
- (c) PZT-R: for frequency range of 50-60 kHz
- (d) PZT-R: for frequency range of 75-85 kHz

Figures 5.19(a) and 5.19(b) show the plot of RMSD (%) index of different variations for specimen PZT-S for damage 1 and 2 signatures with respect to the undamaged state signature. Similarly, Figures 5.19(c) and 5.19(d) show the plot of RMSD (%) index of different variations for specimen PZT-R for damage 1 and 2 signatures with respect to the undamaged state signature. It was observed that the RMSD were different for different property variations (variation 1 – variation 7), thus indicating the influence of the property variations in the signatures and hence in the RMSD indices.

Thus it is important to note that, even for the same amount of damage (damage 1 or 2), variations (changes) in properties lead to different RMSD values which subsequently leads to either under or over estimation of the damage. This could lead to a unrealistic condition of structural health.

The problem of changes in PZT mechanical and electrical properties arises from deterioration of the material during its life time. The changes may be because of damping due to atmospheric factors like humidity, moisture and temperature. The changes may be because of ‘acidic’ or ‘alkaline’ attacks due to rain or sea water exposure. The changes may also be because of the presence of ‘electrical’ or ‘magnetic’ zone near the PZT. If there is no proper control or protection system for the PZT, there could be misinterpretation of damage even in the absence of any damage; and the misinterpretation could be worsen if the PZT properties ‘change’ continuously during the process of SHM.

Thus, it is crucial to protect the PZT transducer for successful prediction of conductance and susceptance signatures and for its efficient use in non destructive evaluation (NDE) applications.

5.7 PZT efficiency factor

Generally, when the embedded/ surface bonded PZT transducer is excited, it will pick the ‘structural responses’ and express them in the conductance and susceptance signatures. Structural response changes with any change in frequency of excitation, and the

peaks (or valleys) in any conductance or susceptance signatures are frequency dependent. Thus efficiency of any non-parametric indices lies in its ability to indicate the damage using both the changed peaks (and valleys) of signatures and the frequency range of excitation.

However RMSD uses only the ‘changed peaks (or valleys)’ of signatures without using frequency range of excitation. Thus in this study, PZT efficiency factor was formulated to estimate damage detection capability using the ‘changed peaks (or valleys)’ and frequency range of excitation.

The PZT efficiency factor (PEF) for chosen range(s) of frequency, which is aimed, is rationalized as below,

$$\text{PEF} = \frac{\frac{C_1 + C_2 + \dots + C_K + \dots + C_N}{N} \sqrt{R_1^2 + R_2^2 + \dots + R_K^2 + \dots + R_N^2}}{R_1 C_1 + R_2 C_2 + \dots + R_K C_K + \dots + R_N C_N} \quad (5.21)$$

where $C_1, C_2 \dots C_K \dots C_N$ are the means of the frequency range (mean of sum of upper and lower frequency), and subscripts 1, 2, K and N stand for the number of considered frequency ranges as shown in Figure 5.20(a).

R_1, R_2, R_3 and R_4 are the RMSD values. For $N=1$, PEF becomes a ratio of RMSD to RMSD, that is, $\text{PEF} = 1$; thus cannot be used for comparison. Therefore, $N > 1$ is the necessary condition for the application of the PEF.

For our study, two major peaks for ranges 50 - 60 kHz ($C_1=55$) and 75 - 85 kHz ($C_2=80$) were considered. The calculated values of PEF for all the variations for the damage 1 and 2 states were depicted in Figures 5.20(b) and 5.20(c).

Figure 5.20. PZT efficiency factor of different variations with damages 1 and 2 of specimen 1 (a) RMSD vs mean frequency (b) PZT-S (c) PZT-R

PEF values of all the variations for PZT-S for damage 1 and damage 2 were approximately equal to 0.74 and 0.81 respectively. Similarly, PEF values of all the variations for PZT-R for damage 1 and damage 2 were approximately equal to 0.75 and 0.94 respectively. Thus, it indicates that all the variations of properties were almost equally efficient in detecting damage or failure or crack.

Structural response changes with any change in frequency of excitation, and the peaks (or valleys) in any conductance or susceptance signatures are frequency dependent. Hence, what ever may be the PZT property variation, response of the structure for undamaged and damaged states is decided by the peaks of signatures and frequency range of interrogation. This implies that the shorter peak conductance signatures (Figures 5.9, 5.10, 5.16 and 5.17, and Table 5.5) do not mean that they are less capable in detecting damage then the taller peak conductance signatures.

Variation	PZT-S				PZT -R			
	Undamaged peak heights (X 10 ⁻⁴ S)		PEF for damages		Undamaged Peak heights (X 10 ⁻³ S)		PEF for damages	
	50-60 kHz	75-85 kHz	1	2	50-60 kHz	75-85 kHz	1	2
1	0.587	1.45	0.8106	0.7547	0.493	0.616	0.7550	0.9446
2	0.649	1.54	0.8137	0.7402	0.500	0.630	0.7553	0.9454
3	0.615	1.50	0.8125	0.7424	0.490	0.623	0.7575	0.9478
4	0.624	1.49	0.8130	0.7448	0.499	0.624	0.7634	0.9455
5	0.456	1.06	0.8150	0.7358	0.346	0.429	0.7634	0.9468
6	0.369	0.79	0.8170	0.7250	0.232	0.298	0.7575	0.9509
7	0.571	1.43	0.8100	0.7575	0.485	0.605	0.7663	0.9444

Table 5.5 Peak heights and PEF for variations of PZT transducer properties

5.8 Conclusions

This chapter presented the proof-of-concept of DSI based prediction of admittance signatures for square and rectangular PZT embedded and surface bonded PZT specimens, and subsequent application in damage analysis.

This study highlighted the affects of ‘variations’ in PZT properties in the prediction of signatures, and thus characterized the PZT properties. This study demonstrated the importance of maintaining consistent PZT properties throughout the life time of SHM. Factors like, humidity, temperature, acidic, alkaline, electrical and magnetic zones etc, are threat to the consistency of PZT properties. Hence it is necessary to understand the behavioural changes of admittance signatures due to any change in each and every property of the PZT, which has so far been neglected by other EMI researchers. Damage indices for various PZT property variations and the consequences of misleading RMSD values, asserted the need for protection of the PZT transducer for efficient use in SHM.

Further, the study shows that damage indices have limitations in actually finding the efficiency of PZT, and a novel PZT efficiency factor (PEF) was proposed and applied successfully. Thus, this chapter completed the implementation (proof-of-concept) of 3D DSI model and its application in damage analysis.

CHAPTER 6

UNIPLEXING AND MULTIPLEXING OF PZT TRANSDUCERS

In the last few chapters, single PZT- structure interactive models (ASI and DSI) were formulated and successfully verified. These were basically SISO (single-input, single-output) type models. However, researchers are required to study MIMO (multiple-input, multiple-output) or MISO (multiple-input, single-output) type PZT-structure interaction models in addition to the SISO models to effectively deal with SHM of real life ACM structures. This chapter presents the experimental study of such MISO/ MIMO based multiplexing of PZT transducers, together with SISO based uniplexing on a prototype plate structure.

The term ‘uniplexing’ means actuating of a single PZT transducer in the presence of electric field and measuring the resulting structural response by the same PZT (i.e. SISO). While the other bonded PZT transducers behave only as mass additives and they are not subjected to electric field, i.e one PZT transducer is active and the rest are passive. Multiplexing is the process of actuating and sensing of all the PZT transducers in parallel, i.e all the PZT transducers are active. This chapter additionally presents the problems of mechanical interference between transducers during uniplexing/ multiplexing and their influence on EM admittance signatures.

6.1 Introduction

The 2D ASI and 3D DSI model were formulated by neglecting the ‘mass’ of the PZT, because the mass of PZT as compared to the host structure was negligible. However, mass of the PZT becomes significant (Cheng and Lin 2005a) in MISO type multiple PZT- structure interactive models because there is increase in number of PZT transducers. The main objective of the present experimental investigation was to list out the issues involved in multiple PZT- interactive models. Mass, thickness and location of the PZT transducers are the important issues expected in multiple PZT -structure interaction models.

In general, the actuations of any active PZT (say PZT m1) originate from the location of installation and cease at a distance equals to the radius of the sensing zone

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

as shown in Figure 6.1. These actuations, which are in the form of waves (Liang et al., 1994; chapters 4 and 5), are hampered due to the addition of dead loads (mass of other passive PZT, say m_2 to m_{13}) in the sensing zone, and this phenomenon of hampering of PZT waves can be termed as mechanical interference. Such mechanical interference is observed in uniplexing of heavier PZT transducers in the experiments. As only one PZT is actuating in uniplexing there will not be any electrical interference. Electrical interference occurs only if there are at least 2 active PZT transducers in proximity (Cheng and Lin 2005).

The uniplexing and multiplexing are presented in the experimental investigations using four different thicknesses (but same length and width) of PZT transducers. The transducers are used to establish the necessity of 'considering all actuations' (extensional and longitudinal), and the co-relationship between PZT thickness and interference due to PZT transducers.

Figure 6.1 Wave distribution in Uniplexing

6.2 Experimental investigation

The experimental investigation was performed in three stages. The first and the second stages together investigated uniplexing of at least 2 numbers of transducers whereas the third stage investigated multiplexing of at least 2 numbers of transducers. The adopted excitation frequency range was 20 – 100 kHz. In the first stage, uniplexing with multiple (>2 numbers) PZT transducers of types T_3 and $T_{0.3}$ were investigated. Types $T_{0.3}$ and T_3 are transducers with dimensions 10 x 10 x 0.3 mm

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

and 10 x 10 x 3 mm respectively. A total of twenty-four PZT transducers (Table 6.1), twelve for each type of transducers, were surface bonded on two identical 2 mm thick aluminium plates. On each aluminium plate, twelve same type PZT transducers were surface-bonded one after another consecutively at different 'locations' using epoxy adhesive. The numberings shown in Figure 6.2 indicate the sequence of installation (bonding) for each aluminium plate specimen (starting, in order, from 1 to 12). The PZT transducers were excited by sinusoidal voltage applied across the terminals of the transducers.

Figure 6.2 Sequence of bonding and locations of benchmark PZT transducers

In the second stage, uniplexing with only 2 same type PZT transducers was investigated. In this stage, the thicker type T_3 transducer was excluded (reasons are given in a later section) and only the thinner types $T_{0.75}$ (10 x 10 x 0.75 mm), T_1 (10 x 10 x 1 mm) and $T_{0.3}$ were used for investigation. In the final stage, multiplexing with multiple (at least 2) PZT transducers was investigated using thicker T_3 and thinner $T_{0.3}$ types of PZT transducers.

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

The EM admittance signature is effected by the mass, location, length, width, and thickness (Wetherhold et al. 2003) of the PZT transducers. Thus, any change in the EM admittance is also effected by the mass, location and dimensions of the PZT as given below.

$$\Delta Y = \Delta G + j \Delta B = f(\text{Mass, location, dimensions}) \quad (6.1)$$

The change in admittance (ΔY) is the sum of the change in conductance (ΔG) and the change in susceptance (ΔB).

Physical property	Value
<u>Mechanical</u>	
Density (kg / m^3)	7800
Young's Modulus (N / m^2)	66.67×10^9
Poisson ratio	0.33
loss factor, η	0.023
<u>Electrical</u>	
Piezoelectric strain coefficients (m/V) d_{31} , d_{32}	-2.10×10^{-10}
Piezoelectric strain coefficient (m/V) d_{33}	4.50×10^{-10}
Dielectric loss factor, δ	0.015
Electric permittivity, ϵ_{33} (farad/m)	0.98×10^{-8}

Table 6.1: Key properties of PZT transducer of Grade C

6.3 Uniplexing using multiple (>2) PZT transducers

Two types of PZT transducers, one thicker (3 mm, T_3) and other thinner (0.3mm, $T_{0.3}$) as compared to the aluminium plate thickness (2 mm) were selected to study uniplexing.

An initial assumption made was that the *mass* influence on EM admittance is dominant as compared to the location and dimensions of the actuating or non actuating PZT. Hence, the variation of conductance and susceptance is expressed as a function of mass (m) alone, i.e, Equation 6.1 is changed to

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

$$\Delta Y \propto (\Delta G + j \Delta B) \propto f(m) \quad (6.2a)$$

For simplicity, only the representative ΔG is presented in the study, because the conductance is considered to be more significant (Sun et al 1995) than the susceptance signature; however, the ΔB was also found to behave similar to ΔG . Hence, Equation 6.2(a) is simplified to

$$\Delta Y \propto \Delta G \propto f(m) \quad (6.2b)$$

A total of 6 benchmark PZT transducers, representing each of the respective 6 columns (Columns A, B, C, D, E and F at Locations 1, 3, 5, 7, 9 and 11 respectively, as shown in Figure 6.2) were selected as uniplexing locations to illustrate the function (i.e. Equation 6.2b).

At the first uniplexing location, i.e., at A[1] (benchmark PZT in column A at Location 1), a total of 11 sets of conductance difference ($\Delta G = G_{Con_A[1]} - G_{A[1]}$) due to the consecutive mass additions of 11 PZT transducers (at Locations 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12) were recorded (Figure 6.2). Only one PZT at each of the uniplexing location is actuated to produce admittance, with all the other PZT transducers acting only as passive mass additives. $G_{A[1]}$ means initial G at location A[1] and $G_{Con_A[1]}$ means G at A[1] after every consecutive PZT additions.

Similarly, at the other uniplexing locations (B[3], C[5], D[7], E[9] and F[11]), similar sets of conductance difference ($\Delta G = G_{Con} - G_{Initial}$) were recorded and the number of sets of conductance differences and PZT additive locations are as listed in Table 6.2.

The initial and consecutive conductance signatures are given as

$$G_{Initial} = G_{Col[Location]} \text{ and } G_{Con} = G_{Con_Col[Location]} \quad (6.3)$$

where subscripts *Col* and *Location* represents the column and location numbers respectively (i.e benchmark locations of Figure 6.2), and subscript *Con* represents consecutive.

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

Benchmark PZT		Locations of Consecutive Records of Conductance (G_{Con})	No. of Records
Column	Location of Initial Conductance ($G_{Initial}$)		
A	1	A[2], B[3], B[4], C[5], C[6], D[7], D[8], E[9], E[10], F[11] and F[12]	11
B	3	B[4], C[5], C[6], D[7], D[8], E[9], E[10], F[11] and F[12]	9
C	5	C[6], D[7], D[8], E[9], E[10], F[11] and F[12]	7
D	7	D[8], E[9], E[10], F[11] and F[12]	5
E	9	E[10], F[11] and F[12]	3
F	11	F[12]	1

Table 6.2: Location of benchmark PZT transducers and their mass additions

6.3.1 Uniplexing using Thicker Transducer

The variation of conductance (i.e, Equation 6.2) can be quantified by the differences between the initial benchmark reading (see Figure 6.2, before mass addition A[2]) and consecutive mass additions (at various locations) of PZT transducers (i.e. $\Delta G = G_{Con} - G_{Initial}$ as given in Table 6.2).

Figure 6.1 illustrates the definition of mechanical interference, and Figures 6.3, 6.4 and 6.5 presents the experimental results for mass addition at various locations as given in Figure 6.2. The actuations of active PZT transducer are in the form of waves as shown in Figure 6.1 and these waves cease within their sensing radius. However, there exists a local area within the sensing zone which has an additional influence of stiffening on EM admittance signatures. Within the local area, any mass additions (due to passive PZT) will cause more fluctuations in the ΔG than

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

the mass added outside the local area but also within the sensing zone. That is, increase of stiffening ‘near’ the PZT has more influence on EM admittance (as it alters the vibrating waves more) than increase of stiffening ‘far’ from the PZT. Thus mechanical interference is more in the local area and ceases to exist outside the sensing zone.

m2 – m13 are the passive PZT transducers bonded one after the other on either sides of the active PZT, in areas I, II and III as shown in Figure 6.1. Let us consider the local area influence first, i.e area I. Here ΔG due to the addition of m2 and ΔG due to subsequent addition of m3 are observed to have fluctuation (higher difference) even though they are at equal distance from the active PZT (also see Figure 6.3a, locations A[2] and B[3]) but the difference in ΔG reduces later on (also see Figure 6.3b). Even though the distance between pair of m2, m3 and pair of m4, m5 from the active PZT are almost equal, their corresponding ΔG was not equal. This is because the corresponding conductance signature increases gradually with increase in additional masses (Equation 6.2b), which substantially increases the over all mass mounted on the host structure.

(a)

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

Figure 6.3. Mechanical interference due to successive mass additions of benchmark type T_3 PZT at A [1]. (a) Conductance differences between initial and consecutive mass additions (b) Mechanical interference pattern @ 60 KHz.

Now let us consider the region outside the local area but within the sensing zone of PZT, i.e. area II. Here, ΔG due to the addition of m_6 and subsequent addition of m_7 are observed to be very close. Similarly, ΔG due to the addition of m_8 and subsequent addition of m_9 were also observed to be very close. However ΔG due to m_8 and ΔG due to m_7 is not closer even though they are at equal distance from the active PZT. This is because the relative distance between m_7 and its predecessor is less than m_7 and its successor. If m_{7_1} and m_{7_2} are the passive masses added subsequently after m_7 but before m_8 , then the gap between ΔG due to m_8 and ΔG due to m_7 widens. This is because the corresponding conductance signature increases gradually with increase in additional masses. Furthermore, ΔG due to m_{7_1} will be higher than ΔG due to m_7 even though location of m_{7_1} from the active PZT is less than m_7 from the active PZT (mass factor, as m_{7_1} is added later).

Thus, this behaviour of change in signatures in areas I and II are due to mechanical interference and to its function of mass loading, location of added mass and relative distances of added masses. Figure 6.3b elaborates on this, and the location of the passive PZT masses is as shown in Figure 6.2.

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

Specifically, Figure 6.3a illustrates the influence and trend of mass accumulation for the benchmark PZT at A [1] for various locations of consecutive mass additions. The figure shows that the trends of the 11 sets of conductance difference are very consistent with peaks and valleys occurring at the same frequency after consecutive additions of PZT transducers, thereby demonstrating the reliability of the experimental results obtained. However, for the ‘first PZT mass addition’, the conductance difference at Location 2 (coloured pink at A[2]) has a relatively low magnitude (or parallel shifting of signature) as compared to the subsequent mass additions. This is because the PZT transducer is installed at only 50 mm distance away from the benchmark PZT (at A [1]) and the stiffening was not much and could imply possible low mechanical interferences due to installation of the PZT transducer at extremely close proximity to the benchmark PZT. However, this addition is enough to conclude the existence of some mechanical interference due to passive transducer.

Apparently, the initial assumption of the relationship of change in conductance as a function of mass alone, i.e. $\Delta G \propto f(m)$ (Equation 6.2b) is not valid, and the presence of location factor (denoted as x) would have to be included in the function to better reflect about mechanical interference. Thus Equation 6.2(b) is modified as,

$$\text{i.e. } \Delta G \propto f(m, x) \quad (6.4)$$

Additionally, when comparing ΔG of the first and second PZT additions at locations A[2] and B[3] respectively, the mechanical interferences were found to increase rapidly (due to increase in stiffening) when the second PZT was added (at B[3]) at similar distances of 50mm from the benchmark PZT at A[1]. Change in signatures due to the addition of B[3] and B[4] were found to be relatively close [local area effect] but signature differences for A[2] and B[3] were not close even though they both were 50 mm away from the active PZT (A[1]). This is because there exists bigger distance between A[2] and B[3] than A[1] and A[2]. Change in signatures due to mass addition of the pair of C[5] and C[6] was very close, and similar close proximity in ΔG was observed for pairs of D[7] and D[8], E[9] and E[10], and F[11] and F[12]. Distances from A (active PZT location) to F were in increasing order in Figure 6.2; similarly, the masses were added in sequential order from A-F. Hence Figure 6.3a shows the ideal case of increase in ΔG from the first mass addition

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

location A [2] to F [12]. Figure 6.3b is the plot between differences in ΔG at 60 KHz and the added masses from A [2] (or m_2) to F [12] (or m_{12}). The initial few mass additions resulted in curvilinear trend as compared to mass added later on. The region where the curvilinear trend resulted corresponds to the local area of PZT. The figure also represents the nature of mechanical interference, i.e., it gradually increases from the first mass addition to the last mass addition, but in the local area (for initial mass additions) the increase is rapid (curvilinear), i.e from A[2] to D[7]. Mechanical interference may vary from 20-100 KHz but the increasing pattern from mass additions m_2 to m_{12} can be similarly observed in Figure 6.3b.

To substantiate the claim of mechanical interferences, similar analysis of the benchmark PZT at B[3] (this time with a different active PZT) was performed. Figure 6.4 shows the significance of mechanical interference of benchmark PZT at B[3]. Similarly, the first PZT addition (at Location 4 i.e., B[4]) at 50 mm distance from PZT at B[3] also resulted in some mechanical interference. Another observation made was that variations in conductance measurement were even found for mass additions at a distance as far as 158mm (example C [6]) from the benchmark B[3].

Figure 6.4. Conductance differences between initial and consecutive mass additions of benchmark type T_3 PZT at B[3]

Figure 6.5 plots ΔG versus frequency for a benchmark PZT transducer at location C[5]. Other benchmark PZT transducers at D[7], E[9] and F[11] were also observed to give similar results. Similar mechanical interferences were observed for all these benchmark locations. For benchmark PZT at F[11], due to only one record of

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

PZT mass addition, comparison of mass addition is excluded. Even for Figures 6.4 and 6.5, mechanical interference was found to increase in similar pattern to that shown in Figure 6.3b but with different magnitude.

Figure 6.5. Conductance differences between initial and consecutive mass additions of benchmark type T_3 PZT at C[5]

Thus, it can be concluded that for the considered specimen, a distance of about 150 mm (< 200 mm) from benchmark PZT transducer will influence the signature more, and there exists a radius of local area between 150-200mm for 3mm (type T_3) thick transducers.

The above findings confirmed the presence of mechanical interferences (usually a parallel downward shift of signature) for 'PZT mass additions' within proximity of 50mm -150mm from a benchmark PZT transducer (coloured pink, i.e., A[2], B[4] and C[6] in Figures 6.3, 6.4 and 6.5 respectively). This is mainly due to the hindrance on wave propagation near the source of actuation. As such, the aluminium plate, being lighter in density (2715 kg/m³) and thinner (2 mm) than the PZT transducer (7800 kg/m³, 3 mm) would inevitably experience interference. Moreover, the PZT transducer has limited sensing range, hence would result in lower or negligible mechanical interference if the mass addition is located far away from the benchmark PZT transducer. Although the above conclusions made are reliable, more experiments were performed to determine the persistence of interference range for other thinner type $T_{0.3}$ PZT transducers.

6.3.2 Uniplexing using Thinner Transducers

To verify if different dimensions of PZT transducers would have the same behaviour as discussed above, experiments were performed using ten times thinner transducers. Similarly, ΔG were computed for various benchmark locations of PZT transducers. Figure 6.6 plots ΔG versus frequency, showing the influence of mass additions on benchmark PZT at A[1].

Figure 6.6. Conductance differences between initial and consecutive mass additions of benchmark type $T_{0.3}$ PZT at A[1]

The ΔG for consecutive mass additions were found to coincide closely, especially in the lower frequency range. This indicates no mechanical interferences within proximity of 50mm distance from the benchmark PZT at A[1], unlike for the previous 3mm thick transducers. Noticeably, apart from little significance in mechanical interference, the valleys were found to coincide with almost similar magnitude. The rest of the 0.3 mm thick benchmark transducers were also found to follow similar pattern. A typical illustration is shown in Figure 6.7. The experimental results are therefore shown to be consistent throughout the experimental testing.

Figure 6.7. Conductance differences between initial and consecutive mass additions of benchmark type $T_{0.3}$ PZT at B[3]

The experiments reveal that the mechanical interferences mentioned in the previous section are only valid for thicker transducers which are heavier in mass. The aluminium plate (2mm thick) is much thicker than the 0.3mm PZT transducers and thus has no significant impact on the vibrating wave.

Generally, PZT transducers with nominal thickness ($T_{0.3}$) have shown to be consistently less affected by mechanical interference even with the presence of nearby PZT at distance spacing of 50mm. However, possible significant mechanical interference could have been overlooked for cases where PZT are very closely located (< 50 mm) from the benchmark PZT, which might have similar mechanical interferences as experienced by the 3mm thick (T_3) PZT. Hence, second stage of experiments was done, i.e, uniplexing with only 2 PZT transducers in which spacing between transducers was less than 50mm.

6.4 Uniplexing using only two transducers

It was observed in uniplexing of multiple (>2) thicker type T_3 PZT transducers that the first PZT addition is enough to show the existence of mechanical interference if there exists any mechanical interferences due to consecutive additions of PZT. Thus, we concentrated on only two PZT transducers to study the interference between thinner transducers. Three types of PZT transducers ($T_{0.3}$, $T_{0.75}$ and T_1) which are thinner than the aluminium plate were adopted for investigation in this

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

second stage of experimentation. The thicker type T_3 PZT transducers were not considered as the presence and influence of interference between such transducers has been established in the first stage.

6.4.1 Uniplexing using type $T_{0.3}$ PZT Transducers

No visible interference was observed even though they were adjacently placed (at 10mm distance between centres of PZT transducers) as shown in Figure 6.8. This is consistent with the observation made using multiple PZT arrangement.

Figure 6.8. Conductance differences between initial and varying distances of PZT addition for type $T_{0.3}$ PZT transducer

6.4.2 Uniplexing using types $T_{0.75}$ and T_1 PZT Transducers

In this case, the probable presence of interference (if any) for other types of transducers which are also thinner than the aluminium plate was investigated using PZT transducers of thickness 0.75 mm ($T_{0.75}$) and 1.0 mm (T_1). Both were verified to be interference-insignificant as shown in Figures 10 and 11. Hence, if at all there is any interference between thinner transducers, then the radius of such interference would be less than 10mm; i.e., the centre to centre distance between the PZT transducers should be less than 10mm. This is not possible as the length and width of the transducers are already 10mm.

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

Figure 6.9. Conductance differences between initial and varying distances of PZT addition for type $T_{0.75}$ PZT transducer

Figure 6.10. Conductance differences between initial and varying distances of PZT addition for type T_1 PZT transducer

Thus, it can be concluded that, mechanical interferences due to adjacent PZT transducers will be significant if the transducer thickness is more than the thickness of host plate. On other hand, it was observed that thinner transducers do not have significant mechanical interferences even when they were adjacently placed. Due to insignificant interference for 0.3 mm, 0.75 mm and 1 mm thick PZT transducers, a definite relationship between interference and PZT thickness was thus not possible to predict. However, due to the sharp escalation in radius of interference (over 150 mm

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

distance) for the 3 mm thick PZT transducer as compared to that of other radius of interference (less than 10 mm distance) for the thinner PZT transducers, it is only obvious that the relationship is non-linear and most probably follows a high order polynomial series in increasing radius of interference with increasing PZT thickness (also see Figure 6.3b for 3 mm thick PZT transducer).

From uniplexing, it was observed that the thickness /mass of PZT transducer can play an important role in EM admittance. Thickness direction actuation was always neglected by researchers in the past, but Annamdas and Soh (2006) have shown that the thickness actuation can be considered in EMI models, even though it was limited to 2D plane strain. Thus it is necessary to develop 3D EMI models for SHM utilizing extensional actuations along the length and width directions, and longitudinal actuation along the thickness direction of PZT transducer.

6.5 Multiplexing using multiple PZT transducers

In the third stage of experiments, all the PZT transducers were actuated (PZT-AA) in parallel, which resulted in output response from collective sensing (PZT-AS) of transducers. As actuations of all transducers gave 'multiple' input and collective output response gave 'single' output, this multiplexing is thus MISO based. Two types of PZT (T_3 and $T_{0.3}$) transducers were used, one of which is thicker and the other thinner than the thickness of the plate.

For each type of transducer, multiplexing was done 11 times after every consecutive PZT additions (at locations 2 – 12). An initial reading at location 1 (A[1]) was recorded prior to multiplexing, i.e. for single PZT transducer. During uniplexing, other than the benchmark transducer all other transducers behaved as mass additives (PZT-MA); where as during multiplexing, all PZT transducers in this stage behaved simultaneously as PZT-AA, PZT-SA and PZT-MA. Thus here, a situation of electrical interference occurs, which was dealt in detail by Cheng and Lin (2005).

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

Therefore, 11 MISO and 1 SISO results were obtained for each transducer type. A total of 12 outputs were obtained for 12 different inputs, hence this type of combination is referred to as MIMO based.

Multiplexing requires a minimum of 2 PZT transducers. The collective EM admittance signature of multiplexing is the sum of EM admittance of individual PZT transducers, and is given as

$$Y_{Collective} = \sum_{K=1}^{K=N} Y_K \quad (5)$$

where K represents the K^{th} transducer in a total of N multiple transducers.

Figure 6.11 plots the initial conductance taken at location 1 (A[1]), and the other 11 multiplexed conductance signatures recorded after every consecutive installations (at locations 2-12) of the thicker type T_3 transducers. Thus, Figure 6.11 shows a total of 12 conductance signatures recorded from locations 1 to 12. Similarly, Figure 6.12 shows the results of 12 consecutive conductance signatures recorded for the thinner type $T_{0.3}$ transducers.

Figure 6.11. Conductance of initial and consecutive mass additions of type T_3 PZT transducer in multiplexing

Negative values of conductance signature were observed for the thinner type $T_{0.3}$ transducers; hence, it is advisable to limit the frequency of excitations (Figure 6.12). Thus, as the number of PZT transducers increases, the effective frequency range (range of positive values of conductance) was found to decrease, (i.e. in the

Chapter 6: Uniplexing and Multiplexing of PZT Transducers

above case, the effective frequency range for 3 PZT is 60 kHz, but is 40 kHz for 12 PZT). Such behaviour was not observed for the thicker T_3 PZT transducers (Figure 6.11).

Figure 6.12. Conductance of initial and consecutive mass additions of type $T_{0.3}$ PZT transducer in multiplexing

Figures 6.11 and 6.12 show the consistency of peaks and valleys occurring at the same frequency after consecutive additions of PZT transducers, thereby demonstrating the reliability of the experimental results obtained. Counter clock wise and clock wise *Slope shifting* of conductance peaks were observed respectively for the thicker (Figure 6.11) and the thinner (Figure 6.12) transducers with increasing magnitude i.e., with increase in number of PZT transducers. There is a general increase in the amplitude of signatures for both the types of PZT transducers, particularly in the peaks. The opposite nature of slope shifting is due to the differences in sensitivity of the two PZT thicknesses. This is in accordance with the uniplexing results, which also observed opposite behaviour, i.e. the thicker transducers displayed interference as compared to the thinner transducers. Thus, the same argument of using thickness actuation along with extensional actuation (along the length and width directions) in future EMI models holds good. i.e., the thickness / mass of PZT transducer play an important role in EM admittance.

6.6 Conclusions

This chapter presented some of the issues involved in the multiple PZT-structure interaction models. For this purpose, four types of PZT transducers were used to demonstrate the influence of mass, location and thickness of PZT transducers. The keys observations made in this chapter are as follows

1. The thicker and heavier PZT transducers had more interference than the thinner and lighter PZT transducers when placed at the same spacing between the transducers, i.e. the influence of *thickness* and *mass* of PZT on EM admittance was presented. Similarly, it can also be shown that the *lengths* and *widths* of multiple PZT transducers have considerable influence on the EM admittance by considering another *set* of transducers with different lengths and widths.
2. The multiplexing was found to amplify the EM admittance signature and improve the sensing range.
3. Thinner and thicker (as compared to thickness of plate) PZT transducers had respectively clockwise and anti clockwise slope shifting phenomena.
4. The type of multiple PZT transducers to be used and the spacing between them are interlinked.
5. This experimental study on uniplexing and multiplexing with different thickness of PZT transducers also shows that, there exists a need for 3D MISO/ MIMO based analytical models in SHM for effective prediction of EM admittance for NDE applications.

All the observations made in this chapter are used in the next chapter for formulating 3D multiple-PZT structure interaction model.

CHAPTER 7

3D EMI MODEL FOR MULTIPLE PZT-STRUCTURE

INTERACTION

In the previous chapter, it was experimentally demonstrated that the vibrations along the thickness and the *mass* of *multiple* transducers significantly influence the EM admittance signature of MIMO/ MISO models. However, section 4.2 (chapter 4) experimentally demonstrated that length, width and thickness of PZT transducer influences the EM admittance. Moreover, section 2.9 (chapter 2) had shown that in the presence of electric field actuations along length, width and thickness are produced. Thus, this chapter presents a MISO based multiple PZT- structure interaction model developed considering 3D actuations of PZT transducers as similar to DSI. Additionally, it also considered the ‘mass’ influence of multiple transducers.

7.1 Introduction

This MISO based multiple PZT- structure interaction model does not impose any restriction on boundary conditions of the host structure unlike the single PZT- host structure models which were mostly modelled for ‘free-free’ (resting freely) boundary condition of host. Moreover, the model does not impose restriction on the shape (square or rectangular), size (thin or thick) and electrical properties (isotropic or anisotropic) of PZT as similar to DSI model. The developed model is later experimentally verified using over 1 meter long aluminium plate. As this model is generic in nature and the sensing zone can be improved by the using multiple PZT transducers, it can be applicable for all NDE based applications of most ACM structures.

7.2 Multiple piezoceramic transducers –structure (MPZT-S) interaction model

Figure 7.1 depicts the multiple PZT –host structure (MPZT-S) interaction in the X, Y and Z directions, along the length, width and thickness of the PZT transducers. Where L_S , W_S and $2H_S$ are the global dimensions of the host structure. C2 (0,0,0) is the global centre or any reference point of host structure. L_K , W_K and $2H_K$ are the local dimensions of K^{th} PZT transducer.

Figure 7.1. Multiple PZT-structure interaction

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

The (distributed) forces are developed due to the two actuation mechanisms, viz. extensional actuations along directions X and Y, and longitudinal actuation along direction Z of the multiple PZT transducers. At any instant of time, the forces developed due to the actuations correspond respectively to expansion in the X and Y directions, and shrinkage in the Z direction; and vice versa. The forces take into consideration the alternate signs of d_{31} (or d_{32}) and d_{33} . Due to the opposite signs of d_{31} (or d_{32}) and d_{33} , the expansion of transducer in the X and Y directions is accompanied by shrinkage in the Z direction; and vice versa.

The impedance (Z_{MS}) of the host structure bonded with multiple PZT transducers is defined as the sum of the linear impedances along the X, Y and Z directions, and the cross impedances in the XY, YZ and XZ planes of the multiple transducers. Z_{MS} is thus obtained by adding all the impedances acting on all faces, as shown in Figure 7.1, which results in the following equation.

$$-Z_{MS} = Z_{M1} + Z_{M2} - Z_{M3} + 2Z_{M12} - 2Z_{M23} - 2Z_{M13} \quad (7.1)$$

The negative sign indicates reaction to the applied forces. Positive and negative signs are in accordance with expansion along the X and Y directions and contraction along the Z direction, and vice versa.

The linear impedances are given by

$$Z_{M1} = \frac{F_1 - F_3}{\dot{u}_3 - \dot{u}_1} = \frac{F_1 - F_3}{\dot{u}}, \quad Z_{M2} = \frac{F_2 - F_4}{\dot{u}_4 - \dot{u}_2} = \frac{F_2 - F_4}{\dot{v}}$$

and

$$Z_{M3} = \frac{F_5 - F_6}{\dot{u}_6 - \dot{u}_5} = \frac{F_5 - F_6}{\dot{w}} \quad (7.2a)$$

where the total transducers force on face I, $F_I = \sum_{K=1}^N F_{IK}$ and the total transducers

velocity $\dot{u}_I = \sum_{K=1}^N \dot{u}_{IK}$. Subscript K represents the K^{th} PZT transducer, N represents

the total number of PZT transducers, and subscript I represents the face number (4

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

sides + 1 top + 1 bottom = 6 faces) of the PZT transducer as shown in Figure 7.1. F_{IK} and $\dot{u}_{IK}$ are the single transducer force and single transducer velocity of the K^{th} PZT on face I. u , v and w are the total relative displacements of transducers along directions X, Y and Z. Figure 7.2. shows the normal stresses σ_x , σ_y and σ_z acting on the K^{th} PZT transducer along directions X, Y and Z respectively; τ_{xy} , τ_{yz} and τ_{zx} are the shear stresses acting on planes XY, YZ and ZX respectively; and m_x, m_y and m_z are the inertial forces along directions X, Y and Z respectively. Where C1 is the local reference point of differential element, can be taken as 0,0,0.

Figure 7.2. Schematic representation of stresses acting on a differential element

The total transducer forces acting on the faces are given as

$$F_1 = -F_3 = 2\sigma_x \left(\sum_{K=1}^N W_K H \right), F_2 = -F_4 = 2\sigma_y \left(\sum_{K=1}^N L_K H \right)$$

$$F_5 = -F_6 = -\sigma_z \left(\sum_{K=1}^N W_K L_K \right) \quad (7.2b)$$

where L_K , W_K and $2H_K$ are respectively the length, width and thickness of the K^{th} PZT transducer .

The cross impedances due to the shear forces (or stresses) acting on planes XY, YZ and ZX are empirically given by

$$Z_{M12} \approx -\frac{Z_{M1}Z_{M2}}{Z_{M1} + Z_{M2} - Z_{M3}}, \quad Z_{M23} \approx -\frac{Z_{M2}Z_{M3}}{Z_{M1} + Z_{M2} - Z_{M3}}$$

and $Z_{M13} \approx -\frac{Z_{M3}Z_{M1}}{Z_{M1} + Z_{M2} - Z_{M3}}$ (7.3)

Substituting Equations. (7.2a) and (7.3) into Equation. (7.1) results the following

$$-Z_{MS} = (Z_{M1}\lambda_1 + Z_{M2}\lambda_2 - Z_{M3}\lambda_3) \quad (7.4)$$

where λ_1, λ_2 and λ_3 are the response factors along directions X, Y and Z respectively, and are given as

$$\lambda_1 = \left(\frac{Z_{M1}}{Z_{M1} + Z_{M2} - Z_{M3}} \right), \quad \lambda_2 = \left(\frac{Z_{M2}}{Z_{M1} + Z_{M2} - Z_{M3}} \right)$$

and $\lambda_3 = \left(\frac{Z_{M3}}{Z_{M1} + Z_{M2} - Z_{M3}} \right)$ (7.5)

Equation. (7.4) contains all the unknowns; and moreover, there is no close-form solution to determine the response factors λ_1, λ_2 and λ_3 and the impedances Z_{M1}, Z_{M2} and Z_{M3} . Hence, a numerical method is used, details of which are presented in the later section.

Substituting Equation. (7.2) into Equation. (7.4) results in

$$Z_{MS} = \frac{F_1 - F_3}{\dot{u}_3 - \dot{u}_1} \lambda_1 + \frac{F_2 - F_4}{\dot{u}_4 - \dot{u}_2} \lambda_2 - \frac{F_5 - F_6}{\dot{u}_6 - \dot{u}_5} \lambda_3$$

$$= 2 \left(\frac{\sigma_1 \sum W_K 2H_K}{\dot{u}} + \frac{\sigma_2 \sum L_K 2H_K}{\dot{v}} - \frac{\sigma_3 \sum L_K W_K}{\dot{w}} \right) \quad (7.6)$$

where σ_1, σ_2 and σ_3 are the semi-analytical directional stresses given as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}_{Semi_analytical} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_{Numerical} \begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \end{bmatrix}_{Analytical} \quad (7.7)$$

7.3 Stress-strain relationship of PZT transducer subjected to 3D loading

Considering the coordinate system shown in Figure 7.1, the stress-strain relationship in terms of induced strains can be expressed as

$$\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \end{bmatrix} = Y_R \begin{bmatrix} 1 & R & R \\ R & 1 & R \\ R & R & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \end{bmatrix} \text{ and } \begin{bmatrix} \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{bmatrix} = Y_R \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{bmatrix} \quad (7.8)$$

where Y_R , R and r are the simplification parameters given as

$$Y_R = \frac{\bar{Y}(1-\nu)}{(1+\nu)(1-2\nu)}, \quad \bar{Y} = Y(1+\eta j), \quad R = \frac{\nu}{1-\nu} \text{ and } r = \frac{1-2\nu}{2(1-\nu)} \quad (7.9)$$

$\bar{Y}$ is the complex Young's modulus of elasticity of the PZT transducer at zero electric field, Y is the static Young's modulus of elasticity of the PZT material, and η is the mechanical loss factor. ν is the poison's ratio of the PZT transducer, and ε_X , ε_Y and ε_Z are the normal strains along the X, Y and Z directions. τ_{XY} , τ_{YZ} and τ_{ZX} are the shear stresses, and γ_{XY} , γ_{YZ} and γ_{ZX} are the shear strains.

The induced strains in terms of total relative displacements are given as

$$\begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \end{bmatrix} = \begin{bmatrix} D_x u \\ D_y v \\ D_z w \end{bmatrix} \text{ and } \begin{bmatrix} \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{bmatrix} = \begin{bmatrix} D_x v + D_y u \\ D_y w + D_z v \\ D_z u + D_x w \end{bmatrix} \quad (7.10)$$

where $D_x = \frac{\partial}{\partial x}$, $D_y = \frac{\partial}{\partial y}$ and $D_z = \frac{\partial}{\partial z}$

7.4 Three dimensional (3D) differential equation

Considering the differential element of the K^{th} PZT transducer as shown in Figure 7.2 and let ρ be the mass density of the transducer. The force equilibrium along direction X can be written as

$$(\sigma_x + D_x \sigma_x dx) dy dz + (\tau_{xy} + D_y \tau_{xy} dy) dx dz + (\tau_{xz} + D_z \tau_{xz} dz) dx dy - \sigma_x dy dz - \tau_{xy} dx dz - \tau_{xz} dx dy - I_x = 0 \quad (7.11a)$$

$$(\sigma_y + D_y \sigma_y dy) dx dz + (\tau_{yx} + D_x \tau_{yx} dx) dy dz + (\tau_{yz} + D_z \tau_{yz} dz) dx dy - \sigma_y dx dz - \tau_{yx} dy dz - \tau_{yz} dx dy - I_y = 0 \quad (7.11b)$$

$$(\sigma_z + D_z \sigma_z dz) dx dy + (\tau_{zx} + D_x \tau_{zx} dx) dy dz + (\tau_{zy} + D_y \tau_{zy} dy) dx dz - \sigma_z dx dy - \tau_{zx} dy dz - \tau_{zy} dx dz - I_z = 0 \quad (7.11c)$$

where I_x , I_y and I_z are the inertial forces due to the PZT mass acting along the X, Y and Z directions respectively, and are given by

$$I_x = \rho dx dy dz D_{tt} u, \quad I_y = \rho dx dy dz D_{tt} v \quad \text{and} \quad I_z = \rho dx dy dz D_{tt} w \quad (7.12)$$

Equation. (7.11) can be written as

$$D_x \sigma_x + D_y \tau_{xy} + D_z \tau_{xz} = \rho D_{tt} u, \quad D_y \sigma_y + D_x \tau_{xy} + D_z \tau_{yz} = \rho D_{tt} v$$

and $D_z \sigma_z + D_x \tau_{zx} + D_y \tau_{yz} = \rho D_{tt} w$ (7.13)

Substituting Equations. (7.8) and (7.10) into Equation. (7.13) leads to

$$D_x \sigma_x + D_y \tau_{xy} + D_z \tau_{xz} = Y_R \left\{ \langle D_{xx} u \rangle + \langle R [D_{xy} v + D_{xz} w] + r [D_{yy} u + D_{zz} u + D_{xy} v + D_{xz} w] \rangle \right\} = \rho D_{tt} u \quad (7.14)$$

$$Y_R \left\{ \langle D_{yy} v \rangle + R [D_{xy} u + D_{yz} w] + r [D_{xy} u + D_{xx} v + D_{zz} v + D_{yz} w] \right\} = \rho D_{tt} v \quad (7.15)$$

$$Y_R \left\{ \langle D_{zz} w \rangle + R [D_{xz} u + D_{yz} v] + r [D_{xz} u + D_{yz} v + D_{xx} w + D_{yy} w] \right\} = \rho D_{tt} w \quad (7.16)$$

$$\text{where } D_{xx} = \frac{\partial^2}{\partial x^2}, D_{yy} = \frac{\partial^2}{\partial y^2}, D_{xy} = \frac{\partial^2}{\partial x \partial y}, D_{yx} = \frac{\partial^2}{\partial x \partial y}, D_{xz} = \frac{\partial^2}{\partial x \partial z},$$

$$D_{yz} = \frac{\partial^2}{\partial y \partial z} \quad \text{and} \quad D_{tt} = \frac{\partial^2}{\partial t^2}$$

7.5 Solution to 3D differential equations

The exact solution for Equations.(7.14) - (7.16) are difficult to obtain, hence each differential equation is split into two equations relating the approximated relative displacements of transducers (u_A , v_A and w_A) instead of the total relative displacements (u , v and w) as given below.

$$Y_R \{D_{xx} u_A\} = \rho D_{tt} u_A, \quad Y_R \{D_{yy} v_A\} = \rho D_{tt} v_A \quad \text{and} \quad Y_R \{D_{zz} w_A\} = \rho D_{tt} w_A \quad (7.17)$$

$$Y_R \{R(D_{xy} v_A + D_{xz} w_A) + r(D_{yy} u_A + D_{zz} u_A + D_{xy} v_A + D_{xz} w_A)\} = 0 \quad (7.18a)$$

$$Y_R \{R(D_{xy} u_A + D_{yz} w_A) + r(D_{xy} u_A + D_{xx} v_A + D_{zz} v_A + D_{yz} w_A)\} = 0 \quad (7.18b)$$

$$\text{and } Y_R \{R(D_{xz} u_A + D_{yz} v_A) + r(D_{xz} u_A + D_{yz} v_A + D_{xx} w_A + D_{yy} w_A)\} = 0 \quad (7.18c)$$

The solutions to Equation.(7.17) is in the form of 3D wave acting along directions X, Y and Z, which are given as

$$u_A = [A \sin kx + B \cos kx] e^{j\omega t}, \quad v_A = [C \sin ky + D \cos ky] e^{j\omega t}$$

$$\text{and } w_A = [E \sin kz + F \cos kz] e^{j\omega t} \quad (7.19)$$

A , B , C , D , E and F are the coefficients to be determined, and k is the wave number given as

$$k = \omega \sqrt{\frac{\rho(1+\nu)(1-2\nu)}{Y(1-\nu)}} \quad (7.20)$$

where ω is the angular frequency of excitation.

However, the exact total relative displacements, i.e, solutions to Equations. (7.14) to (7.16) can be assumed to be in the form of approximated [Equation. 7.19] solutions as given below

$$u = u_A + f(x, y) - f_1(x, z) + \phi_1(x, t), \quad v = v_A + g(y, x) - g_1(y, z) + \phi_2(y, t)$$

$$\text{and } w = w_A + h(z, x) - h_1(z, y) + \phi_3(z, t) \quad (7.21)$$

where f, f_1, h, h_1, g and g_1 are the unknown wave functions in space, and ϕ_1 , ϕ_2 and ϕ_3 are the unknown wave functions in space and time. The unknown wave

functions can be approximately obtained from the first derivative of total relative displacements [Equation. 7.21] with respect to space and time.

The first derivative of total relative displacements with respect to space (x, y and z) are given as

$$D_x u = [Ak \cos kx] e^{j\omega t} + (f(x, y)f' - f_1(x, z)f'_1) + (\phi_1(x, t)\phi'_1 - e^{j\omega t} Bk \sin kx) \quad (7.22a)$$

$$D_y v = [Ck \cos ky] e^{j\omega t} + (g(y, x)g' - g_1(y, z)g'_1) + (\phi_2(y, t)\phi'_2 - e^{j\omega t} Dk \sin ky) \quad (7.22b)$$

$$D_z w = [Ek \cos kz] e^{j\omega t} + (h(z, x)h' - h_1(z, y)h'_1) + (\phi_3(z, t)\phi'_3 - e^{j\omega t} Fk \sin kz) \quad (7.22c)$$

The first derivatives of total relative displacements with respect to time are given as

$$D_t u = j\omega \left[A \frac{Y_R}{k} \sin kx \right] e^{j\omega t} + j\omega \left[\left[\frac{\dot{\phi}_1}{e^{j\omega t} j\omega} \phi_1(x, t) \right] - \left[A \left(\frac{Y_R}{k} - 1 \right) \sin kx - B \cos kx \right] \right] e^{j\omega t},$$

$$D_t v = j\omega \left[C \frac{Y_R}{k} \sin ky \right] e^{j\omega t} + j\omega \left[\left[\frac{\dot{\phi}_2}{e^{j\omega t} j\omega} \phi_2(y, t) \right] - C \left[\left(\frac{Y_R}{k} - 1 \right) \sin ky - D \cos ky \right] \right] e^{j\omega t}$$

$$\text{and } D_t w = j\omega \left[E \frac{Y_R}{k} \sin kz \right] e^{j\omega t} + j\omega \left[\left[\frac{\dot{\phi}_3}{e^{j\omega t} j\omega} \phi_3(z, t) \right] - E \left[\left(\frac{Y_R}{k} - 1 \right) \sin kz - F \cos kz \right] \right] e^{j\omega t} \quad (7.23)$$

7.6 Optimal solutions and behaviour of solution in space and time domain

There could be many possible solutions to the 3D differential equations [Equations. (7.14) – (7.16)] of total relative displacements of transducers, but in this study, only two solutions are considered by approximations [using Equation. (7.21) with some assumptions]. Both the solutions assumed that the coefficients $A \neq B \neq C \neq D \neq E \neq F \neq 0$ and the propagating waves generated by the applied sinusoidal electric field (along direction Z) are in the form of 3D sine wave pattern in space and time as shown in Figure 7.3.

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

Figure 7.3(a) shows a propagating wave (Wave 1) generated along S_x ($y = z = 0$), S_y ($x = z = 0$) and S_z ($x = y = 0$) for solution 1. Similarly, Figure 7.3(b) shows a propagating wave (Wave 2) generated along S_{xy} ($x = y$), S_{yz} ($y = z$) and S_{zx} ($z = x$) for solution 2.

Figure 7.3. 3D wave propagation of multiple PZT in the presence of sinusoidal voltage (a) Wave 1 in space (b) Wave 2 in space (c) Waves 1 and 2 in time domain

In both the cases, it is also assumed that the wave functions $f = f_1$, $g = g_1$ and $h = h_1$, and the solutions are approximated to have only three coefficients A , C and E . After approximations, the first derivatives of total relative displacements with respective space [Equation. (7.22)] can be modified as

$$D_x u = u' = [Ak \cos kx] e^{j\omega t}, \quad D_y v = v' = [Ck \cos ky] e^{j\omega t}$$

$$\text{and } D_z w = w' = [Ek \cos kz] e^{j\omega t} \quad (7.24)$$

Similarly, Equation. (7.23) is modified as

$$D_t u = \dot{u} = j\omega \left[A \frac{Y_R}{k} \sin kx \right] e^{j\omega t}, \quad D_t v = \dot{v} = j\omega \left[C \frac{Y_R}{k} \sin ky \right] e^{j\omega t}$$

$$\text{and } D_t w = \dot{w} = j\omega \left[E \frac{Y_R}{k} \sin kz \right] e^{j\omega t} \quad (7.25)$$

Finally, the unknown coefficients left to be solved are A, C and E, which cannot be determined analytically. Hence, numerical method is used; the details of which are presented in the later sections. No boundary conditions of host structure are used to extract the coefficients in the analytical model, unlike for the single PZT-structure interaction formulations (Bhalla and Soh 2004a) which are mainly limited to ‘free-free’ boundary conditions. Thus, the presented MPZT-S model is generic and applicable for all boundary conditions of host structure.

7.7 Stress-strain relationship in presence of electric field

The stresses of the transducer in the presence of small electric fields along directions X, Y and Z, considering the actuations shown in Figure 7.1 (also see Figure 2.10 of chapter 2), are obtained by introducing electric fields and strain displacement coefficients into Equation. (7.8) as given by Sirohi and Chopra (2000)

$$\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \end{bmatrix}_e = Y_R \begin{bmatrix} 1 & R & R \\ R & 1 & R \\ R & R & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_X - E_3 d_{31} \\ \varepsilon_Y - E_3 d_{32} \\ \varepsilon_Z - E_3 d_{33} \end{bmatrix}$$

$$\text{and } \begin{bmatrix} \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{bmatrix}_e = Y_R \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} \gamma_{XY} - 0 \\ \gamma_{YZ} - E_2 d_{24} \\ \gamma_{ZX} - E_1 d_{15} \end{bmatrix} \quad (7.26)$$

Subscript e represents the presence of electric fields.

The normal and shear strains are obtained using Equations (7.10) and (7.24).

$$\begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \end{bmatrix} = \begin{bmatrix} D_x u \\ D_y v \\ D_z w \end{bmatrix} = e^{j\omega t} \begin{bmatrix} Ak \cos kx \\ Ck \cos ky \\ Ek \cos kz \end{bmatrix}$$

and

$$\begin{bmatrix} \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{bmatrix} = \begin{bmatrix} D_x v + D_y u \\ D_y w + D_z v \\ D_z u + D_x w \end{bmatrix} = e^{j\omega t} \begin{bmatrix} Ay \sin kx + Cx \sin ky \\ Cx \sin ky + Ex \sin kz \\ Ex \sin kz + Az \sin kx \end{bmatrix} \quad (7.27)$$

However in the present EMI method, all the PZT transducers are excited in parallel at a desired frequency range in the presence of only electric field E_3 (along direction Z) as shown in Figure 7.1. The excitations produce extensional actuations along the length and width, and longitudinal actuation along the thickness of the transducer. These actuations in turn produce structural responses in the form of admittance signatures, which is the basis for damage detection in SHM. The electric field E_3 along direction Z for transducers 1, 2K,.. and N are given in terms of applied voltage (V) and amplitude of voltage (V_0) as

$$(E_3)_K = \frac{V}{2H_K} \text{ and } V = V_0 e^{j\omega t} \quad (7.28)$$

where subscript K represents the K^{th} transducer.

Substituting Equation. (7.26) into Equation. (7.7), the (semi analytical) directional stresses in the presence of electric field can be written as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}_e = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \left(Y_R \begin{bmatrix} 1 & R & R \\ R & 1 & R \\ R & R & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_X - E_3 d_{31} \\ \varepsilon_Y - E_3 d_{32} \\ \varepsilon_Z - E_3 d_{33} \end{Bmatrix} \right) \quad (7.29)$$

The unknowns A , C and E of Equations. (7.24) and (7.25), and Z_{M1} , Z_{M2} , Z_{M3} , λ_1 , λ_2 and λ_3 of Equation. (7.4) will be determined later using numerical analysis.

7.8 Experimental specimen

An aluminium plate specimen of grade A1 6061-T6 (Table 3.1 of chapter 3) and dimensions 1050 mm x 150 mm x 2 mm was used for verification of the MPZT-S

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

interaction model. Three test cases were considered using 4, 8 and 12 PZT transducers (grade C, Table 6.1 of chapter 6). In the first case (I), 4 PZT transducers were surfaced bonded on the aluminium plate using high strength epoxy adhesive of type RS 850-940 (Table 3.1) at locations 1, 2, 3 and 4 as shown in Figure 7.4 (a).

Figure 7.4. Experimental specimen and transducers

- (a) Dimensions and spacing
- (b) Photo view of specimen
- (c) Photo view of PZT transducer

In the second case (II), 4 more PZT transducers were surface bonded at locations 5, 6, 7 and 8; thus 8 transducers were considered. In the final case (III), another 4 more PZT transducers were surface bonded at locations 9, 10, 11 and 12; thus a total of 12 transducers were considered. The dimensions of the PZT

transducers are 10 x 10 x 0.3 mm, and the spacing between the PZT transducers are as shown in Figure 7.4 (a).

7.9 Numerical analysis

The numerical analysis was performed using ANSYS 5.6 (ANYSYS, 2000) to obtain the structural responses (linear impedances and response factors) for use as input to the analytical equations, so as to predict the admittance signature.

Figure 7.4(b) shows the experimental specimen used in this model. The specimen was rested freely on foam; hence the boundary condition for numerical analysis was simulated as free-free condition. In the numerical analysis, the host structure and the PZT transducers were discretized into finite elements, as shown in Figure 7.5(a). The unit distributed loads were then applied along the three principle directions of the transducers in accordance with the extensional and longitudinal actuations of transducers

In all 3 cases, the specimen was symmetric about the X and Y directions, hence, only one quadrant of the specimen was considered for numerical modelling. Thus for case (I), one quarter of the aluminium plate with one PZT surface bonded [at location 1, see Figure 7.4(a)] was modelled. For case (II), one quarter of the plate with 2 PZT transducers surface bonded (at locations 1 and 5) was modelled. Similarly for case (III), one quarter of the plate with 3 PZT transducers surface bonded (at locations 1, 5 and 9) was modelled (Figure 7.5a). Appropriate boundary conditions were imposed on the planes of symmetry for all the 3 cases, that is, the x and y components of the displacement were set to zero on the YZ and the ZX planes of symmetry, respectively.

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

(a)

(b)

Figure 7.5. K^{th} PZT transducer (a) Overview of aluminium symmetric specimen (case III) (b) Numerical model (c) Distribution of forces along direction X

In addition, at the centre of the plate, which is at C2 (see Figure 7.1), the z component of the displacement was set to zero. The finite element meshing was carried out using ANSYS 5.6. The specimen was discretised into 3D brick elements (solid 45) possessing 3 degrees of freedom at each node. Figure 7.5(a) only shows the FE mesh of one PZT transducer, but the specimen is similarly modelled for all 3 different cases. The model does not have any restriction on the type of boundary conditions for the host structure; hence if the structure is non symmetric, the complete structure should be modelled by considering appropriate boundary conditions. However the accuracy of the model depends on adopting suitable mesh size and boundary conditions.

7.10 Formulations of structural responses and structural linear impedances using numerical analysis as input

The following formulations would need input from the numerical analysis described earlier.

$$\text{Let } A = A_0 V_0 \sum_{K=1}^N \frac{L_K}{2H_K}, \quad C = C_0 V_0 \sum_{K=1}^N \frac{W_K}{2H_K} \quad \text{and} \quad E = E_0 V_0 \sum_{K=1}^N \frac{1}{2H_K} \quad (7.30)$$

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

Substituting Equation (7.30) into Equation (7.25), the rate of change of total relative displacements are given as

$$\dot{u} = j\omega \frac{Y_R}{k} A_0 \left\{ V_0 \sum_{K=1}^N \frac{L_K}{2H_K} \sin kL_K \right\} e^{j\omega t}, \quad \dot{v} = j\omega C_0 \frac{Y_R}{k} \left\{ V_0 \sum_{K=1}^N \frac{W_K}{2H_K} \sin kW_K \right\} e^{j\omega t}$$

and $\dot{w} = j\omega E_0 \frac{Y_R}{k} \left\{ V_0 \sum_{K=1}^N \frac{1}{2H_K} \sin 2kH_K \right\} e^{j\omega t}$ (7.31)

Substituting Equations (7.2b), (7.6), (7.27), (7.28), (7.29) and (7.30) into Equation (7.2a) results in the linear impedance along directions X, Y and Z (in the presence of electric field) as

$$Z_{M1} = 2k \left[\frac{\sum_{K=1}^N (W_K \{ (A_0 k L_K \cos kL_K - d_{31}) + R(C_0 kW_K \cos kW_K - d_{32}) \})}{j\omega A_0 \sum_{K=1}^N \frac{L_K}{2H_K} \sin kL_K} \right] + 2k \left[\frac{\sum_{K=1}^N (W_K \{ R(E_0 k \cos 2kH_K - d_{33}) \})}{j\omega A_0 \sum_{K=1}^N \frac{L_K}{2H_K} \sin kL_K} \right] \quad (7.32a)$$

$$Z_{M2} = 2k \left[\frac{\sum_{K=1}^N (L_K \{ R(A_0 k L_K \cos kL_K - d_{31}) + (C_0 kW_K \cos kW_K - d_{32}) \})}{j\omega A_0 \sum_{K=1}^N \frac{W_K}{2H_K} \sin kW_K} \right] + 2k \left[\frac{\sum_{K=1}^N (L_K \{ R(E_0 k \cos 2kH_K - d_{33}) \})}{j\omega A_0 \sum_{K=1}^N \frac{W_K}{2H_K} \sin kW_K} \right] \quad (7.32b)$$

$$Z_{M3} = k \left[\frac{\sum_{K=1}^N \left(\frac{L_K W_K}{H_K} \{ R(A_0 k L_K \cos kL_K - d_{31}) + R(C_0 kW_K \cos kW_K - d_{32}) \} \right)}{j\omega A_0 \sum_{K=1}^N \frac{1}{2H_K} \sin 2kH_K} \right]$$

$$+ k \left[\frac{\sum_{K=1}^N \left(\frac{L_K W_K}{H_K} \{ (E_0 k \cos 2kH_K - d_{33}) \} \right)}{j\omega A_0 \sum_{K=1}^N \frac{1}{2H_K} \sin k2H_K} \right] \quad (7.32c)$$

Rearranging Equation (7.32a) results in

$$A_0 (2k^2 \sum_{K=1}^N W_K L_K \cos kL_K - Z_{M1} \sum_{K=1}^N \frac{L_K}{2H_K} j\omega \sin kL_K) + C_0 (2Rk^2 \sum_{K=1}^N W_K^2 \cos kW_K) \\ + E_0 (2Rk^2 \sum_{K=1}^N W_K \cos 2kH_K) = R_1 (d_{31} + R(d_{32} + d_{33})) \quad (7.33)$$

Let a_1 , a_2 , a_3 and a_4 be substitution variables given by

$$a_1 = (2k^2 \sum_{K=1}^N W_K L_K \cos kL_K - Z_{M1} \sum_{K=1}^N \frac{L_K}{2H_K} j\omega \sin kL_K), \quad a_2 = (2Rk^2 \sum_{K=1}^N W_K^2 \cos kW_K) \\ a_3 = (2Rk^2 \sum_{K=1}^N W_K \cos 2kH_K) \text{ and } a_4 = R_1 (d_{31} + R(d_{32} + d_{33})) \quad (7.34)$$

Substituting Equation (7.34) into Equation (7.33) results in

$$a_1 A_0 + a_2 C_0 + a_3 E_0 = a_4 \quad (7.35)$$

Similarly, the linear impedances (Z_{M2} and Z_{M3}) along directions Y and Z in the presence of electric field are given as

$$b_1 A_0 + b_2 C_0 + b_3 E_0 = b_4 \text{ and } c_1 A_0 + c_2 C_0 + c_3 E_0 = c_4 \quad (7.36)$$

$$\text{where } b_1 = (2Rk^2 \sum_{K=1}^N L_K^2 \cos kL_K)$$

$$b_2 = (2k^2 \sum_{K=1}^N W_K L_K \cos kW_K - Z_{M2} \sum_{K=1}^N \frac{W_K}{2H_K} j\omega \sin kW_K), \quad b_3 = (2Rk^2 \sum_{K=1}^N L_K \cos 2kH_K)$$

$$b_4 = R_1 (d_{31} + R(d_{32} + d_{33})), \quad c_1 = (Rk^2 \sum_{K=1}^N \frac{L_K^2 W_K}{H_K} \cos kL_K), \quad c_2 = (Rk^2 \sum_{K=1}^N \frac{L_K W_K^2}{H_K} \cos kW_K)$$

$$c_3 = \sum_{K=1}^N (k^2 \frac{W_K L_K}{H_K} \cos 2kH_K - Z_{M3} \frac{1}{2H_K} j\omega \sin k2H_K)$$

$$\text{and } c_4 = R_1 (d_{31} + R(d_{32} + d_{33})) \quad (7.37)$$

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

Z_{M1} is determined using ANSYS 5.6, as a ratio of the sum of distributed load to the sum of velocities produced at the points/ nodes of consideration. $F_1 = -F_3$ from Equation (7.2), hence, the multiple linear impedance along direction X is mathematically expressed as

$$Z_{M1} = \frac{F_1 - F_3}{\dot{u}_3 - \dot{u}_1} = \frac{2 \left[\sum_{k=1}^N F_{1k} \right]}{\left[\sum_{k=1}^N \dot{u}_{3k} \right] - \left[\sum_{k=1}^N \dot{u}_{1k} \right]} = \frac{2F_1}{\dot{u}} \quad (7.38)$$

Further, Equation. (7.38) can be reduced as

$$Z_{M1} = \frac{2 \left(\sum_{D=1}^{P(1)} F_{1D} + \dots + \sum_{D=1}^{P(K)} F_{KD} + \dots + \sum_{D=1}^{P(N)} F_{ND} \right)}{\left(\sum_{D=1}^{\alpha(1)} \dot{y}_{1D} + \dots + \sum_{D=1}^{\alpha(K)} \dot{y}_{KD} + \dots + \sum_{D=1}^{\alpha(N)} \dot{y}_{ND} \right) - \left(\sum_{D=1}^{P(1)} \dot{x}_{1D} + \dots + \sum_{D=1}^{P(K)} \dot{x}_{KD} + \dots + \sum_{D=1}^{P(N)} \dot{x}_{ND} \right)} \quad (7.39)$$

where $F_{1D} = \dots = F_{KD} = \dots = F_{ND} = 1 + 0j$, the applied unit forces on all the distributed points of PZT transducers on face 1 as shown in Figure 7.1. (and Figure 7.5). $P(K)$ and $\alpha(K)$ represent the distributed points of the K^{th} PZT transducer on faces 1 and 3 respectively as shown in Figure 7.5(b). The total number of distributed points of PZT transducers on face 1 is the same as on face 3, i.e $P(1) = \alpha(1)$, $P(2) = \alpha(2) \dots P(K) = \alpha(K) \dots P(N) = \alpha(N)$. $\dot{x}_{KD}$ and $\dot{y}_{KD}$ are the obtained velocities of the K^{th} PZT at the D^{th} distributed point of face 1 and face 3 respectively. Similarly $\sum_{D=1}^{P(K)} \dot{x}_{KD}$ and $\sum_{D=1}^{\alpha(K)} \dot{y}_{KD}$ are the single transducer velocities of the K^{th} PZT which are respectively equal to $\dot{u}_{1k}$ and $\dot{u}_{3k}$.

Finally, Equation. (7.39) is reduced as

$$Z_{M1} = 2 \frac{F_{11} + \dots + F_{1K} + \dots + F_{1N}}{(\dot{u}_{31} + \dots + \dot{u}_{3K} + \dots + \dot{u}_{3N}) - (\dot{u}_{11} + \dots + \dot{u}_{1K} + \dots + \dot{u}_{1N})} \quad (7.40)$$

where $\dot{u}_{11}, \dots, \dot{u}_{1K}$, and $\dot{u}_{1N}$ are the sum of velocities on face 1 of PZT 1, ..., K, ..and N. Similarly $\dot{u}_{31}, \dots, \dot{u}_{3K}$, and $\dot{u}_{3N}$ are the sum of velocities on face 3 of PZT 1, ..., K, ..and N. F_{1K} is the total unit force applied (equal to single transducer force) on face 1 of the

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

K^{th} PZT. Similarly, the linear impedances along direction Y (ie, Z_{M2} using faces 2 and 4) and direction Z (ie, Z_{M3} using faces 5 and 6) are determined using ANSYS 5.6 as

$$Z_{M2} = 2 \frac{F_{21} + .. + F_{2K} + .. + F_{2N}}{(\dot{u}_{41} + .. + \dot{u}_{4K} + .. + \dot{u}_{4N}) - (\dot{u}_{21} + .. + \dot{u}_{2K} + .. + \dot{u}_{2N})} \quad (7.41)$$

$$\text{and } Z_{M3} = -2 \frac{F_{51} + .. + F_{5K} + .. + F_{5N}}{(\dot{u}_{61} + .. + \dot{u}_{6K} + .. + \dot{u}_{6N}) - (\dot{u}_{51} + .. + \dot{u}_{5K} + .. + \dot{u}_{5N})} \quad (7.42)$$

where $\dot{u}_{21}, .., \dot{u}_{2K}$, and $\dot{u}_{2N}$ are the sum of velocities on face 2 of PZT 1, 2,..K,..and N. Similarly $\dot{u}_{41}, .., \dot{u}_{4K}$, and $\dot{u}_{4N}$ are the sum of velocities on face 4 of PZT 1,..K,..and N. $\dot{u}_{51}, .., \dot{u}_{5K}$, and $\dot{u}_{5N}$ are the sum of velocities on face 5 of PZT 1,..K,..and N. $\dot{u}_{61}, .., \dot{u}_{6K}$, and $\dot{u}_{6N}$ are the sum of velocities on face 6 of PZT 1,..K,..and N. F_{2K} and F_{5K} are the total unit forces applied on faces 2 and 5 of the K^{th} PZT.

Substitute Equation. (7.40) into Equation. (7.34), Equations. (7.41) and (7.42) into Equation. (7.37), and using Equations. (7.35) and (7.36) for A_0, C_0 and E_0 results the following expressions

$$E_0 = \frac{(a_4 b_1 - a_1 b_4)(b_2 c_1 - b_1 c_2) - (a_2 b_1 - a_1 b_2)(b_4 c_1 - b_1 c_4)}{(a_3 b_1 - a_1 b_3)(b_2 c_1 - b_1 c_2) - (a_2 b_1 - a_1 b_2)(b_3 c_1 - b_1 c_3)},$$

$$C_0 = \frac{(b_4 c_1 - b_1 c_4) - (b_3 c_1 - b_1 c_3)E_0}{(b_2 c_1 - b_1 c_2)} \text{ and } A_0 = \frac{a_4 - a_2 c_0 - a_3 E_0}{a_1} \quad (7.43)$$

The unknowns A , C and E are obtained by substituting Equation. (7.43) into Equation. (7.29). λ_1 , λ_2 and λ_3 are obtained using Z_{M1} [Equation. 7.40], Z_{M2} [Equation. 7.41] and Z_{M3} [Equation. 7.42] in Equation. (7.5). Thus all the unknowns which can not be determined using the analytical equations are determined using numerical analysis.

7.11 Admittance formulation

The ‘sum total’ of electric displacements (or total charge density), D_3 , (where subscript 3 refers to the electric field in direction Z) over the top or bottom surface of the multiple PZT transducers (Figure 7.1.) can be written as

$$D_3 = \sum_{K=1}^N (D_3)_K = \overline{\epsilon_{33}} E_3 + d_{31} \sigma_1 + d_{32} \sigma_2 + d_{33} \sigma_3 \quad (7.44)$$

where $\overline{\epsilon_{33}} = \epsilon_{33}(1 - \delta j)$ is the complex electric permittivity of the PZT at zero stress, δ is the dielectric loss factor, ϵ_{33} is the static electric permittivity of the PZT transducer, and $(D_3)_K$ is the charge density of the K^{th} PZT transducer

Equation. (7.44) is the MPZT-S based sensor equation, which is semi-analytical in nature, and it serves as the basis for obtaining the admittance equation.

Substituting Equation. (7.28) into Equation. (7.44) results the charge density of the K^{th} PZT transducer as

$$\begin{aligned} (D_3)_K = & \overline{\epsilon_{33}} (E_3)_K + Y_R (d_{31} \lambda_1 \{st_1 + R(st_2 + st_3)\} + d_{32} \lambda_2 \{R(st_1 + st_3) + st_2\}) \\ & + Y_R (d_{33} \lambda_3 \{R(st_1 + st_2) + st_3\}) \end{aligned} \quad (7.45a)$$

where $st_1 = [\epsilon_x - (E_3)_K d_{31}]$, $st_2 = [\epsilon_y - (E_3)_K d_{32}]$ and $st_3 = [\epsilon_z - (E_3)_K d_{33}]$.

Substituting Equations (7.26) and (7.27) into Equation. (7.45a) leads to

$$\begin{aligned} (D_3)_K = & \overline{\epsilon_{33}} (E_3)_K + Y_R (d_{31} \lambda_1 \{[Ak \cos kx.e^{j\omega t} - (E_3)_K d_{31}] + R[Ck \cos ky.e^{j\omega t} - (E_3)_K d_{32}] \\ & + R[Ek \cos kz.e^{j\omega t} - (E_3)_K d_{33}]\} + d_{32} \lambda_2 \{R[Ak \cos kx.e^{j\omega t} - (E_3)_K d_{31}] \\ & + [Ck \cos ky.e^{j\omega t} - (E_3)_K d_{32}] + R[Ek \cos kz.e^{j\omega t} - (E_3)_K d_{33}]\} \\ & + d_{33} \lambda_3 \{R[Ak \cos kx.e^{j\omega t} - (E_3)_K d_{31}] + R[Ck \cos ky.e^{j\omega t} - (E_3)_K d_{32}] \\ & + [Ek \cos kz.e^{j\omega t} - (E_3)_K d_{33}]\}) \end{aligned} \quad (7.45b)$$

Electric current I is the rate of change of the total electric charge over the surface area SA ($= \sum_{K=1}^N A_K = \sum_{K=1}^N L_K W_K$) of the multiple PZT transducers (either top or bottom; see Figure 7.1). Mathematically, it can be represented as

$$I = \sum_{K=1}^N I_K = \iint_{SA} \sum_{K=1}^{K=N} (\dot{D}_3)_K dA = j\omega \sum_{K=1}^{K=N} \left[\iint_{A_K} (D_3)_K dx dy \right] \quad (7.46)$$

where $\dot{D}_3$ is the time rate of change of charge, subscript K represents the K^{th} PZT transducer of N multiple transducers, and I_K is the electric charge over the surface area A_K of the K^{th} PZT transducer.

Substituting Equation (7.45b) into Equation (7.46) results the electric charge of the K^{th} PZT transducer as

$$I_K = j\omega [L_K W_K \overline{\varepsilon_{33}} (E_3)_K + Y_R (d_{31} \lambda_1 \{s_A + R(s_C + s_E)\} + d_{32} \lambda_2 \{R(s_A + s_E) + s_C\} + d_{33} \lambda_3 \{R(s_A + s_C) + s_E\})] \quad (7.47)$$

where $s_A = [W_K A \sin kx . e^{j\omega t} - L_K W_K (E_3)_K d_{31}]$,

$s_C = [L_K C \sin ky . e^{j\omega t} - L_K W_K E_3 d_{32}]$ and $s_E = [L_K W_K E k \cos kz . e^{j\omega t} - L_K W_K E_3 d_{33}]$

Electrical admittance ($\bar{Y}^A$) is the ratio of the resultant electric current (I) to the applied instantaneous voltage across the multiple PZT transducers (V), that is

$$\bar{Y}^A = \frac{I}{V} = \frac{1}{V} \sum_{K=1}^N I_K = \sum_{K=1}^N \bar{Y}_K^A . \quad \bar{Y}_K^A \text{ is the admittance of the } K^{\text{th}} \text{ PZT transducer, and is}$$

obtained using Equations. (7.28), (7.30) and (7.47)

$$\begin{aligned} \bar{Y}^A = G + Bj = \sum_{K=1}^N \bar{Y}_K^A &= \sum_{K=1}^N \frac{j\omega L_K W_K}{2H_K} [\overline{\varepsilon_{33}} + Y_R \{ \\ &d_{31} \lambda_1 \{ [A_0 \sin kL_K - d_{31}] + R[C_0 \sin kW_K - d_{32}] + R[E_0 k \cos k2H_K - d_{33}] \} + \\ &d_{32} \lambda_2 \{ R[A_0 \sin kL_K - d_{31}] + [C_0 \sin kW_K - d_{32}] + R[E_0 k \cos k2H_K - d_{33}] \} + \\ &d_{33} \lambda_3 \{ R[A_0 \sin kL_K - d_{31}] + R[C_0 \sin kW_K - d_{32}] + [E_0 k \cos k2H_K - d_{33}] \} \}] \quad (7.48) \end{aligned}$$

where G is the conductance (real part of admittance) and B is the susceptance (imaginary part of admittance).

Equation.(7.48) can be simplified depending on the dimensions of the multiple PZT transducers. For multiple PZT transducers of equal PZT dimensions, that is, lengths of all the PZT transducers are L , widths are W and heights are $2H$. Then the final admittance can be written as

$$\begin{aligned}
\bar{Y}^A = N^2 \frac{j\omega LW}{2H} [\overline{\varepsilon_{33}} + Y_R \{ d_{31} \lambda_1 \{ [NA_0 \sin kL - d_{31}] + R[NC_0 \sin kW - d_{32}] \\
+ R[E_0 k \cos k2H - d_{33}] \} + d_{32} \lambda_2 \{ R[NA_0 \sin kL - d_{31}] + [NC_0 \sin kW - d_{32}] \\
+ R[E_0 k \cos k2H - d_{33}] \} + d_{33} \lambda_3 \{ R[NA_0 \sin kL - d_{31}] \\
+ R[NC_0 \sin kW - d_{32}] + [E_0 k \cos k2H - d_{33}] \} \}] \quad (7.49)
\end{aligned}$$

where N is the total number of PZT transducers. Equation (7.49) can be reduced for single PZT by substituting N = 1. Furthermore, for one-quarter and one-half of PZT, the equation should be multiple by 1/4th and 1/2th respectively, such cases arise for symmetric structures where numerical modelling is required for one-quarter and one-half of the structure respectively. Equation (7.49) can be further reduced for square PZT transducers, that is, where the lengths and widths of the transducers are both equal to L, by changing W to L in the equation. Equation (7.48) serves as the basis for predicting admittance, which is general in nature, and can be used for unlimited number of PZT transducers. Equation (7.49) forms the basis for predicting admittance of PZT transducers with the same dimensions.

7.12 Modal analysis

Let R be the total weight ratio of the PZT transducers to the host structure. Modal analysis was additionally performed to verify the influence of mass increase for three different cases of R, (R = 0.33 %, 0.67 % and 1.00 %). Table. 7.1 shows the result obtained for three cases, which lists all modal frequencies. The table clearly shows the difference in modal frequencies because of difference in the mass of the PZT transducers.

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

Mode Set No.	R=0.33% (4 PZT)	R=0.67% (8 PZT)	R=1.00% (12 PZT)
	Frequency (Hz)	Frequency (Hz)	Frequency (Hz)
1	5.5897	5.5968	5.5958
2	35.682	35.666	35.662
3	101.90	101.82	101.82
4	197.02	196.90	196.84
5	333.04	333.00	332.98
6	486.45	486.45	486.33
7	653.49	653.29	653.14
8	840.47	840.46	840.44
9	1035.4	1035.2	1034.9
10	1114.1	1114.4	1115.0
11	1176.1	1176.7	1177.2
12	1257.4	1258.0	1258.6
13	1336.9	1336.7	1336.9
14	1448.9	1449.6	1450.5
15	1606.8	1607.2	1606.9
16	1682.8	1683.7	1684.6
17	1899.0	1899.9	1901.1
18	1983.4	1983.9	1983.8
19	2186.3	2186.3	2185.1
20	2187.0	2187.3	2187.4
21	2359.0	2359.5	2360.2
22	2523.7	2523.9	2525.9
23	2750.3	2752.3	2751.6
24	2913.5	2914.0	2914.5
25	3150.9	3152.4	3155.1

Table 7.1. Modal frequencies of 0.3mm thick multiple PZT transducers

7.13 Experimental results and discussion

In the present MPZT-S model, vibrations along the length, width and thickness of the PZT transducer were considered in the admittance formulation; thus all the extensional and longitudinal actuations of all the multiple PZT transducers were accorded equal importance. In all the three cases (I, II and III), the PZT transducers were connected to an impedance analyzer via a switch box (Figure 2.1 of chapter 2). They were then subjected to parallel actuations in the presence of sinusoidal voltage, which resulted in the admittance signatures. The MPZT-S based predicted admittance signatures were obtained using Equation (7.49) because the dimensions of all the PZT transducers are 10 x 10 x 0.3 mm. Figures 7.6 - 7.8 compare the experimental and the MPZT-S based (predicted) signatures of the specimen with 4, 8 and 12 surface bonded PZT transducers (at locations 1-4, 1-8 and 1-12). The considerable number of peak (frequency) matches of the conductance signatures for cases (I), (II) and (III) are evident. The specimen tested was over 1 metre long, and the boundary conditions were assumed to be free-free as the specimen was placed on a form (Figure 7.4). The assumption of perfect free-free boundary may not be true as any slight disturbances may disturb the experimental results which might have caused the variations in peaks; but generally the trends are the same. However the accuracy of the model depends on adopting suitable mesh size and appropriate boundary conditions in the numerical analysis. Anyway it is apparent from the figures that there exists reasonable agreement between the experimental and the predicted signatures for the considered range of 30 kHz. The peaks in the signatures are dependent on the type of material of the test specimen.

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

(a)

(b)

Figure 7.6. Experimental and predicted signatures of case I
(a) Conductance (b) Susceptance

Chapter 7: 3D EMI model for Multiple PZT– Structure Interaction

(a)

(b)

Figure 7.7. Experimental and predicted signatures of case II

(a) Conductance (b) Susceptance

(a)

(b)

Figure 7.8. Experimental and predicted signatures of case III

(a) Conductance (b) Susceptance

7.14 Conclusions

This chapter presented a novel MPZT-S interaction model for predicting EM admittance signatures of multiple PZT transducers. The MPZT-S based admittance considered both the extensional and longitudinal actuations, i.e. the vibrations along the length, width and thickness of the PZT transducer were included in the formulations as similar to DSI. The model has been experimentally validated for three cases using surface bonded transducer specimens. These predicted EM admittance signatures were compared with the experimental signatures and the trends were found to be in good agreement.

The MPZT-S formulation does not impose restrictions on the following parameters,

1. Size, shape, thickness and electric isotropy of PZT.
2. The number of PZT.
3. Boundary conditions of host unlike other SISO models.

Thus, this model is very generic in nature. However one more additional feature is added to MPZT-S which is presented in the next chapter

CHAPTER 8

EXTENDED MPZT-S MODEL FOR MULTIPLE PZT- STRUCTURE INTERACTION

This chapter presents extension to the developed MPZT-S model of previous chapter. The extensions were made to include *thickness* and *mass* of epoxy adhesive, which was employed to surface bond the PZT transducers on the host structure. The extended MPZT-S model was later experimentally verified using lab sized aluminium plate for different thickness of adhesive (epoxy) layer.

8.1. Introduction

The basic assumptions made during the formulation of MPZT-S was that, the adhesive used for bonding the PZT transducers on the aluminium plate specimen is negligible in *mass* and *thickness*. However, during the characterization of PZT (chapter 5, section 5.3), it was mentioned that there exists a need to protect PZT from external agents like acidic/alkaline attacks, especially when PZT transducers are embedded inside the concrete (or other embeddable) structure. Furthermore, PZT material is very brittle and it is vulnerable for breakages when embedded and thus it is necessary to protect the PZT. Figure 8.1 shows the photo view of such protected/ wrapped PZT which is ready for embedment. It should be noted that the wrapped PZT is heavier than the free PZT.

Figure 8.1: Epoxy wrapped PZT transducer

Figure 8.2 shows the conductance signature of free PZT transducer (10 mm x 10 mm x 2.0 mm) and wrapped PZT transducer (same PZT after wrapping as shown in Figure 8.1). Both free and wrapped PZT transducers were resting freely (free-free condition) on the foam. It was observed that the peaks were higher in magnitude for free PZT transducer as compared to wrapped PZT transducer. However, the slope (anti clockwise) of wrapped PZT transducer was steeper than the free PZT transducer. The reason for such a difference in signatures is investigated in next section.

Figure 8.2. 'Free' and epoxy wrapped PZT conductance signatures of PZT

8.2 Adhesive thickness influence on EM admittance signature

Section 8.1 and section 5.3 (chapter 5) presented the discussions on the need to wrap or protect PZT transducer. Previous section also presented the plot which shows the conductance difference between free and wrapped PZT transducer transducers. This section presents experimental demonstration the influence of adhesive thickness on EM admittance signature.

PZT 1, PZT 2, PZT 3 and PZT 4 are the four wrapped PZT transducers, where the dimensions (15 mm x 15 mm x 0.5 mm) and grade (Grade B, Table 4.1 of chapter 4) of the four PZT transducers are same. However, the thicknesses of the epoxy (RS 850-940, Table 3.1 of chapter 3) employed for wrapping all the four PZT transducers were different. Figure 8.3 shows the conductance signatures of these four wrapped PZT transducers. The signatures were taken for free-free boundary conditions; the

figure clearly shows the variations in the signatures for different thicknesses of adhesive. Thus, there is need to extend the developed MPZT-S to cater the requirement of including ‘thicknesses’ of adhesives in the model.

Figure 8.3. Conductance signatures of four epoxy wrapped PZT

8.3 Extension to MPZT-S (ExMPT-S) model

8.3.1 Linear impedances and PZT forces

Figure 8.4 depicts the multiple PZT –host structure (ExMPZT-S) interaction in the X, Y and Z directions, along the length, width and thickness of the PZT transducers. Where L_s , W_s and $2H_s$ are the global dimensions of the host structure. C2 (0,0,0) is the global centre or any reference point of host structure. L_K , W_K and $2H_K$ are the local dimensions of K^{th} PZT transducer. $2H_{KE}$ represents thickness of the adhesive layer. The (distributed) forces are developed due to the two actuation mechanisms, viz. extensional actuations along directions X and Y, and longitudinal

actuation along direction Z of the multiple PZT transducers. Furthermore, it is assumed that *mass* of PZT is very less as compared to mass of bonding adhesive. Hence the PZT forces are transferred to the top surface of the bonded epoxy adhesive as shown in Figure 8.4. At any instant of time, the forces developed due to the actuations correspond respectively to expansion in the X and Y directions, and shrinkage in the Z direction; and vice versa.

Figure 8.4. ExMPZT-S interaction model

The impedance (Z_{MS}) of the host structure bonded with adhesives at multiple (PZT) locations was defined earlier (chapter 7) as the sum of the linear impedances along the X, Y and Z directions, and the cross impedances in the XY, YZ and XZ planes of the multiple transducers. In the previous chapter it was seen that PZT had 6 faces (along X, Y, Z, -X, -Y and -Z), but in this model, only forces acting on 5 faces were considered as shown in figure. The PZT transducers were assumed to be thinner and lighter than both the epoxy adhesive and the host structure and hence sixth face (along -Z) was not considered, thus the modified linear impedances (see Equation 7.2a of chapter 7) are given by

$$Z_{M1} = \frac{F_1^T - F_3^T}{\dot{u}_3 - \dot{u}_1} = \frac{F_1^T - F_3^T}{\dot{u}}, \quad Z_{M2} = \frac{F_2^T - F_4^T}{\dot{u}_4 - \dot{u}_2} = \frac{F_2^T - F_4^T}{\dot{v}}$$

$$\text{and } Z_{M3} = \frac{F_5^T}{-\dot{u}_5} = \frac{F_5^T}{\dot{w}} \quad (8.1)$$

where the total transfer force on face I, $F_I^T = \sum_{K=1}^N F_{IK}$ and the total transfer velocity

$\dot{u}_I = \sum_{K=1}^N \dot{u}_{IK}$. Superscript T represents transferred forces due to PZT. Subscript K

represents the K^{th} PZT transducer, N represents the total number of PZT transducers, and subscript I represents the face number (4 sides + 1 top = 5 faces) of the PZT transducer as shown in Figure 8.4 F_{IK} and $\dot{u}_{IK}$ are the single PZT transfer force and single PZT transfer velocity of the K^{th} PZT on face I. u , v and w are the total relative displacements obtained as a result of transfer forces of transducers along directions X, Y and Z. The cross impedances were the same as given in Equation 7.2(b) of chapter 7.

The total transfer forces acting on the faces are given as

$$F_1^T = -F_3^T = \sigma_X \left(\sum_{K=1}^N W_K 2H \right), \quad F_2^T = -F_4^T = \sigma_Y \left(\sum_{K=1}^N L_K 2H \right)$$

$$F_5^T = -\sigma_Z \left(\sum_{K=1}^N W_K L_K \right) \quad (8.2)$$

where L_K , W_K and $2H_K$ be the length, width and thickness of the K^{th} PZT transducer respectively.

The rest of the formulations required for ExMPZT-S were the same as that of MPZT-S.

8.3.2 Shear transfer mechanism

Shear lag and transfer mechanism was demonstrated by Bhalla and Soh (2004c) with different thicknesses of epoxy adhesive employed for surface bonding PZT transducers. It was concluded that a reduced shear is transferred to host in the presence of thick adhesive and thus influences the peaks and troughs in the EM admittance signatures. However, the major limitation of their model was that the PZT need to be square (for limitations see section 2.7 of chapter 2) and moreover, it is a 2D plane stress based SISO model. Hence, a new generic MISO (ExMPZT-S) model based on 3D actuations was proposed to accommodate thicknesses of adhesive.

8.3.3 Real dimension method

Earlier two types of formulation were actually possible for PZT bonded with thicker adhesive (Bhalla and Soh 2004c), first method is by using reduced or equivalent dimensions of PZT (L_K -dl, W_K -dw) as shown in Figure 8.5(b). Second method is by changing “effective mechanical impedance” of the structure by introducing compensation for the ‘shear lag’ caused because of thicker adhesive. However, this ExMPZT-S model employed the *real dimensions* of the adhesive (example L_{KE} , W_{KE} and $2H_{KE}$) and PZT (example L_K , W_K and $2H_K$) in the formulation without introducing any shear lag compensation as shown in Figure 8.5(a).

Figure 8.5. PZT- structure interaction

(a) Real dimension method (b) Equivalent dimension method

8.3.4 Stresses acting on differential element

The normal and shear stresses at the interface of PZT and the epoxy adhesive are used for formulation of 3D differential equation as shown in Figure 8.6. The figure shows the normal stresses σ_x , σ_y and σ_z acting on the K^{th} PZT transducer element along directions X, Y and Z respectively; τ_{xy} , τ_{yz} and τ_{zx} are the shear stresses acting on planes XY, YZ and ZX respectively; and m_x , m_y and m_z are the inertial forces along directions X, Y and Z respectively.

Figure 8.6. Schematic representation of stresses acting on a differential element

8.4 Case study

The MPZT-S and ExMPZT-S are MISO models, but they are also applicable to SISO cases. This section presents a SISO based case study using three aluminium plate specimen cases; same dimensions and type of PZT transducers were surface bonded on the centre of each of the aluminium plate with three different thicknesses of adhesive. The dimensions of the aluminum (Al 6061-T6, Table 3.1 of chapter 3), PZT (Grade B, Table 4.1) and the adhesive (RS 850-940, Table 3.1) were as given in Table 8.1.

Specimen cases	Epoxy adhesive mm ³	Al mm ³	PZT mm ³
1	Negligible	100 x 100 x 2	10 x 10 x 0.3
2	10 x 10 x 0.5		
3	10 x 10 x 1		

Table 8.1. Dimensions of epoxy adhesive, Al and PZT.

8.4.1 Modifications to linear impedances and PZT forces

Section 8.3.1 presented the general MISO based formulations of linear impedances and PZT forces, but this section presents the modified formulations to suit the SISO based case study. The aluminium plate specimen was symmetric along X and Y directions and a free-free boundary conditions was assumed. Because of symmetry along both the directions, only one quadrant of the specimen was considered in the formulation.

Hence, the total transfer forces acting on the faces of considered one quarter specimen (Figure 8.7) are given as

$$\begin{aligned} F_1^T &= \sigma_x ([0.5W_K]2H), F_2^T = \sigma_y ([0.5L_K]2H) \\ F_5^T &= -\sigma_z (0.25W_K L_K) \end{aligned} \quad (8.3)$$

Figure 8.7. Faces of one-quarter of PZT specimen

Forces on faces 3 and 4 are not considered in formulation due to symmetry as shown in Figure 8.7. Similarly linear impedances are modified as below

$$Z_{M1} = \frac{F_1^T}{-\dot{u}_1} = \frac{F_1^T}{\dot{u}}, Z_{M2} = \frac{F_2^T}{-\dot{u}_2} = \frac{F_2^T}{\dot{v}} \text{ and } Z_{M3} = \frac{F_5^T}{-\dot{u}_5} = \frac{F_5^T}{\dot{w}} \quad (8.4)$$

However, rest of the formulation remain the same.

8.4.2 Numerical Analysis

The adopted numerical analysis is similar to that of the chapter 7, dynamic finite element analysis (FEA) was used as simple close-form solution was not available for the structural impedance Z_M , the coefficients A_0 , C_0 , and E_0 , and the response factors.

Figures 8.8 and 8.9 show the finitely discretized mesh of the specimens cases 1 and 2 (Table 8.1). Because of symmetry about the X and Y directions, only one quadrant of the specimen was considered. Appropriate boundary conditions were imposed on the planes of symmetry, that is, the x and y components of the displacement were set to zero on the YZ and the ZX planes of symmetry, respectively. In addition, at the bottom of specimen (C2), the z component of the displacement was set to zero as shown in figures.

Figure 8.8. Finite element mesh of one-quarter specimen case 1

The finite element meshing was carried out using ANSYS 5.6 (ANSYS, 2000). The specimen was discretised into 3D brick elements (solid 45) possessing 3 degrees of freedom at each node, as shown in Figures 8.8 and 8.9, with differential element mesh size (ie, different sizes for different layers viz., aluminium and epoxy layer). The details of the mesh sizes are given in Table 8.2.

Figure 8.9. Finite element mesh of one quarter specimen case 2

In order to determine the impedance at a particular frequency, an arbitrary harmonic force was applied on the edges as shown in Figures 8.8 and 8.9. Using FEA, the dynamic harmonic analysis was performed and the complex displacement responses at the points of force application were obtained for frequency range of 50 kHz. Rest of the procedural steps were similar to that employed in chapter 7.

Specimen cases	Epoxy thickness	Mesh size (mm ³)	
		Epoxy	Al
1	Negligible	-	1 x 1 x 1
2	0.5 mm	1 x 1 x 0.5	
3	1 mm	1 x 1 x 1	

Table 8.2. Mesh sizes of specimens

8.4.3 Formulation of ExMPZT-S based Admittance Signature

Recall Equation 7.49 (of chapter 7), which can be modified for single PZT by substituting $N = 1$ as

$$\begin{aligned} \bar{Y}^A = \frac{j\omega LW}{2H} [& \bar{\mathcal{E}}_{33} + Y_R \{ d_{31}\lambda_1 \{ [NA_0 \sin kL - d_{31}] + R[NC_0 \sin kW - d_{32}] \\ & + R[E_0 k \cos k2H - d_{33}] \} + d_{32}\lambda_2 \{ R[NA_0 \sin kL - d_{31}] + [NC_0 \sin kW - d_{32}] \\ & + R[E_0 k \cos k2H - d_{33}] \} + d_{33}\lambda_3 \{ R[NA_0 \sin kL - d_{31}] \\ & + R[NC_0 \sin kW - d_{32}] + [E_0 k \cos k2H - d_{33}] \} \}] \end{aligned} \quad (8.5)$$

However, in the present case study only ‘one quarter of the specimen (Figure 8.7)’ was modelled, where PZT is also one quarter. Thus the Equation 8.5 is further modified for $N = 1$ and $1/4^{\text{th}}$ of PZT transducer as

$$\bar{Y}_Q^A = 0.25\bar{Y}^A \quad (8.6)$$

where, $\bar{Y}_Q^A$ is the quarter admittance of the considered one-quarter specimen

The unknowns (the coefficients A_0 , C_0 , and E_0 , and the response factors) obtained from numerical analysis were substituted in the Equation (8.5) to obtain $\bar{Y}_Q^A$ from Equation (8.6). However, to obtained complete admittance signature of the complete structure $\bar{Y}_Q^A$ need to be multiplied by 4.

8.5 Results and discussions

In the ExMPZT-S Model, mass of the adhesive (epoxy) was considered in numerical modelling. This feature was added to 3D MPZT-S interactive model developed in chapter 7. Thus, ExMPZT-S model is very generic in nature and applicable for both single and multiple PZT transducers.

In previous chapter vibrations along the length, width and thickness directions of PZT were considered in formulation of MPZT-S model. But adhesive layer was assumed to be negligible and ignored. However in real practice this is not the case, i.e, in order to protect PZT from external factors (section 5.3 of chapter 5), it is important to wrap the PZT with epoxy or any other adhesive which may be much heavier than the PZT. So in real practice *mass* of protective cover will be more as compared to the PZT. Thus, in the present ExMPZT-S model this feature was successfully proposed. The comparisons between the ExMPZT-S and experimental signatures for the considered frequency range (0-50kHz) were as shown in Figure 8.10 and 8.11. It can be noted that there were around 12 peaks in the conductance signatures (Figure 8.10), which were successfully predicted in ExMPZT-S model. The trends and the occurrence of peaks at the same locations in conductance signatures of ExMPZT-S model shows the proof for the successfully implementation of the ExMPZT-S model. Both conductance and susceptance signatures matched satisfactorily.

(a)

(b)

Figure 8.10. Conductance signatures of experimental and ExMPZT-S Model

(a) Experimental (b) ExMPZT-S Model.

(a)

(b)

Figure 8.11. Susceptance signatures of experimental and ExMPZT-S Model

(a) Experimental (b) ExMPZT-S Model.

In the present case study only single PZT transducer was considered, however, the ExMPZT-S model can be extended for heavier PZT transducers and for multiple PZT transducers by making necessary changes in the formulation.

8.6 Conclusions

The ExMPZT-S model has extra feature over the MPZT-S model i.e. the consideration for mass of epoxy. Thus, ExMPZT-S model contains all the features of MZPT-S and the same conclusions mentioned in chapter 7 holds good. Additionally the case study had demonstrated the success of this model. Furthermore, both MPZT-S and ExMPZT-S were developed considering practical issues and requirements, and thus are suitable to real life ACM structures.

CHAPTER 9

CONCLUSIONS AND RECCOMENDATIONS

In this chapter, the research of this thesis is summarized. The originality of the current research and its specific contributions to the field of EMI based SHM are highlighted. The recommendations for improving and extending the models are presented for future research.

9.1 Research contributions and originality

9.1.1 Originality of research

The field of EMI based SHM is very vast and still under intensive research. Damage detection using impedance based NDE is hardly a decade old. Most of the ideas for the present research are inspired by the needs of real life structures, coupled with some ideas inspired by previous researchers. For clear demarcation, the ideas and models that are original to this research are hereby highlighted.

1. Actuators of the PZT transducers (Raja *et al* 2004) in the presence of 3D electric fields can be divided into extensional, longitudinal and shear actuators. However, in the existing EMI (Esteban, 1996; Fairweather, 1998; Park and Inman, 2003a; Bhalla, 2004) models, only 1D electric field is applied resulting in only extensional (along length and width) and longitudinal (along thickness) actuators. One of the major limitations of the existing surface bonded PZT based EMI models are that they ignored the longitudinal actuation of the PZT transducer. Moreover, they are not applicable to laminated or civil engineering structures, where critical zone to be monitored is inside the structure. The longitudinal actuation plays a vital role in the thick or confined (embedded) PZT transducers. In general, the inabilities of the existing models to consider the longitudinal actuation and its non applicability to embeddable structures have left a gap for the development of new ASI model. *The*

originality is in using the longitudinal actuation of PZT transducer in the ASI formulation of EMI model. However, the model does not consider all the actuations of the PZT transducer in the formulation as it was 2D plane strain model.

2. The surface bonding approach employed so far by many EMI researchers were based only on the uni or bi-extensional actuation of the PZT transducer. In addition, only thin patches were modelled, which placed constraints on the substrate/ film surface roughness since sharp edge peaks create high field points that can catastrophically break down the patch/ film (Jason and Andrew, 2002). Hence, in places where high frequency excitation of transducers and high electric field are required, it is better to employ thick PZT patches or films. ***The models developed in this study are applicable for both thick and thin PZT, making it an original contribution.***
3. Any ‘changes’ in the mechanical and electrical properties of PZT influence the predicted admittance signatures. Furthermore, even if there are no changes in the properties of the PZT transducers, any deviations in the predicted admittance from the experimental admittance signatures can be attributed to the deviations of PZT properties supplied by the manufacturers (Bhalla and Soh 2004). Thus, in order to correctly predict the admittance, it is a must to understand the behavior of each and every property of the PZT transducer. In this thesis, characterization of the PZT is presented to efficiently monitor real life structures to avoid misleading variations which may under or over estimate the damages. .
4. In the non-parametric based EMI models, the prime indicator of damage is the change in the admittance signatures of the embedded/ surface bonded PZT transducer. Bhalla (2001) compared many techniques and observed that the RMSD between the signatures is the most suitable non parametric damage index to quantify structural damage. However, admittance signatures change with any change in frequency of excitation, and the peaks (or valleys) in any conductance or susceptance signatures are frequency dependent. Thus, the efficiency of any non-parametric indices lies in its ability to indicate the damage using the changed peaks/ valleys of signatures and the frequency range of excitation. Furthermore, RMSD uses only the ‘changed peaks/ valleys’ of signatures

- without using frequency range of excitation. Even for the same amount of damage, changes in properties lead to different RMSD values which subsequently lead to either under or over estimation of the damage. This could lead to an unrealistic condition of structural health. In this thesis, a novel PZT efficiency factor (PEF) was formulated to estimate damage detection capability using the ‘changed peaks (or valleys)’ and frequency range of excitation. ***The originality is in the use of both frequency range of excitation and changed peaks/ valley in the formulation of PEF.***
5. ASI and DSI models are basically SISO based interaction models. The basic assumption made in the formulations is that the PZT has negligible mass as compared to the host structure. However, for MISO interaction, the increase in the number of PZT transducers increases the overall mass and hence significantly influences the admittance signatures (Cheng and Lin 2005a). Theoretically, it is understood that the influence of mass of PZT exists in the multiple PZT models. ***Hence, the originality is in performing experimental based study using uniplexing and multiplexing to demonstrate the need to consider mass and longitudinal actuation in the MISO model.***
 6. Similarly MPZT-S model for multiple PZT was formulated considering all actuations of PZT. ***Thus, the same originality mentioned earlier holds good; additional feature of this model is that it is applicable for both SISO and MISO interaction.***
 7. The basic assumption made in the formulation of MPZT-S model is that the adhesive (epoxy) thickness used for surface bonding the PZT transducers was negligible. However, this model when extended (ExMPZT-S) is applicable even for thicker adhesive. ***As the ‘mass of the adhesive’ is included in the ExMPZT-S mode, making it an original contribution.*** Thus, the models from ASI to ExMPZT-S are original and generic in nature, which were inspired by the needs of real life structures and from the limitations of previous research.

9.1.2 Research contributions

The research presented in this thesis concentrated mainly on the development of 2D and 3D semi analytical models useful for SHM of existing and future ACM structures. The 3D models developed are applicable for both surface bonded and embedded PZT. Furthermore, they are applicable for single/ multiple PZT-structure interaction. The formulated EMI models were verified by experimental results. The specific research contributions to the existing state of EMI techniques are as follows.

1. The first contribution of this research is the formulation of ASI model which made use of both the extensional actuation along the length and longitudinal actuation along the thickness directions of the PZT transducer. Development of the model consisted of both ‘experimental initialization’ and ‘experimental verification’. A factor dependent on the type of PZT was obtained from lab based experimental initialization of free-PZT, which was later used in the formulation to obtain the predicted admittance signature.
2. The second contribution is in the development of 3D DSI formulations, in which the total mechanical impedance of 3D structure was defined as the sum of linear and cross impedances. The 3D model is semi analytical in nature, and the formulations were verified using surface bonded and embedded PZT specimens. The success of the 3D model eventually led to the development of the MPZT-S and ExMPZT-S models.
3. Characterization of PZT is the fourth contribution of this research, which highlighted many thought provoking ‘variations’ in the PZT properties which may have mislead damage analysis. Step by step analysis of PZT properties was presented. Slope shifting and, slope and peak shifting of conductance and susceptance signatures were resulted from the characterization of PZT.
4. For the same amount of damage, the different variations of PZT properties had yielded different RMSD values, which was certainly an inaccurate index. Thus a new PEF was derived, which is the fifth contribution of this

were research. The inefficiency of RMSD and the efficiency of PEF were demonstrated using damage analysis.

5. One more major (sixth) contribution of this research is the development of 3D MPZT-S model, in which total mechanical impedance of 3D structure was defined as the sum of linear and cross impedances of all the multiple PZT transducers when subjected to parallel excitations. The model was experimentally verified using 1 meter long plate bonded with multiple transducers. Even though the specimen was long and heavy, the PZT transducers were considered to have sufficient mass as compared to the host structure. Thus, this model considered the ‘mass influence’ of PZT transducers in the formulations.
6. The final contribution of this research is the development of the “extended MPZT-S model”, which considered thicker bonding layer. Case study for single PZT bonded to aluminum specimen with different thicknesses of bonding layer was successfully investigated. The model is generic in nature, which considers mass of both PZT and adhesive layer.

9.2 Characterization of PZT transducers

Any ‘changes’ in the mechanical and electrical properties of PZT due to external factors will influence the predicted admittance signatures (chapter 5). The factors are listed as below

1. Deterioration of the PZT material during its life time.
2. Humidity, moisture and heat (due to atmospheric factors).
3. Acidic or alkaline attacks due to rain or sea water exposure
4. Presence of electrical or magnetic zone near the PZT.

A defective PZT will lead to defective admittance signatures, which may lead to over or under estimation of actual damages. Thus, it is a must to protect PZT throughout its implementation in SHM or noise and vibration control for reliable results.

9.3 Characterization of EM admittance signatures

The thesis presents a range of interactive models from SISO to MIMO, which were based on semi-analytical formulations, ie. analytical formulations and numerical analysis. The unique objective of these models was to predict EM admittance signatures. The predicted admittance was later compared with the lab based experimental admittance, and was found to be in satisfactory agreement.

The characteristic features of the EM admittance signatures depend equally on both the PZT and the host structure properties and boundary conditions. In the DSI model (chapter 4), it was shown that the length, width and thickness of PZT transducer influence the EM admittance signatures. It was also observed that the signatures depend on the type of interaction model, i.e. SISO (chapters 3 and 4) or MISO (chapters 6, 7 and 8). The reason was that, multiple PZT predict magnified (summed up) signature (Chapter 6) due to parallel excitations of all transducers as compared to single PZT interaction. The mass of PZT (chapter 7) has been shown to significantly influence the EM admittance signatures. Furthermore, the ExMPZT-S (chapter 8) model even shown that the thickness of the adhesive influences the EM admittance signatures. Thus, the six main characteristic features affecting EM admittance signatures are as follows

1. Host structure dimensions and properties
2. Boundary conditions of host structure
3. PZT dimensions and properties
4. Interaction type. i.e, single/multiple PZT
5. Mass of PZT
6. Adhesive thickness

Hence, to accurately predict admittance signature, it is a must to understand the properties of both the PZT and the host structure.

9.4 Applications and limitations of developed interaction models

This thesis presents a few EMI based SISO and MISO interactive models. Systematic mathematical formulations followed by numerical analysis resulted in predicted admittances. These were compared with experimental admittance signatures and were found to be in suitable agreement. The assumptions, applications and limitations of these models are summarized below

9.4.1 ASI model (SISO based)

Assumptions made

1. PZT is negligible in mass and size as compared to host structure
2. 2D plane strain condition.

Applicable for

1. Thin/ thick embedded PZT transducer
2. Embeddable ACM structures, where critical zone to be monitored is inside the structure.

Not applicable for

1. Surface bonded PZT transducer
2. 3D structures

9.4.2 DSI model (SISO based)

Assumptions

1. PZT is negligible in mass and size as compared to host structure
2. Negligible adhesive thickness used for bonding PZT transducers

Applicable for

1. any thickness (thin/ thick), shape (square/ rectangle) and electrical isotropy (or anisotropy) of the PZT
2. Surface bonded / embedded PZT
3. 3D Embeddable and non embeddable ACM structures.

Not applicable for

1. Multiple PZT- structure interaction
2. Thick adhesive for surface bonding PZT

9.4.3 MPZT-S model (MISO based)

Assumptions

1. PZT mass is non-negligible as compared to host structure
2. Negligible adhesive thickness used for bonding PZT transducers

Applicable for

1. Thin/ thick, square/ rectangle, electrical isotropy/ anisotropy of the PZT
2. Multiple surface bonded / embedded PZT- structure interaction.
3. 3D real life embeddable and non embeddable ACM structures.

Not applicable for

1. Thick adhesive bonding layer.

9.4.4 ExMPZT-S model (MISO based)

Assumptions

1. PZT mass is non-negligible as compared to host structure
2. Non-negligible adhesive thickness used for bonding PZT transducers

Applicable for

1. Thin/ thick, square/ rectangle, electrical isotropy/ anisotropy of PZT
2. Multiple surface bonded / embedded PZT- structure interaction.
3. 3D real life embeddable and non embeddable ACM structures.

Not applicable for

1. Curve-linear host structures like shells and cones.

9.5 Recommendations for Future Work

From the experience gained and from the limitations observed during the research, the recommendations for possible improvements and further investigations in experimental tests and theoretical analyses are as given below.

9.5.1 ASI model

The 2D strain formulations used require a PZT factor α_p , which was obtained during experimental initialization of the free PZT, and later used in

predicting the admittance signatures of the specimen. However, experimental initialization using free PZT requires extreme care (experience) as they may breakdown during experiments. Hence, the formulation should be improved to exclude experimental initialization for which the cross impedance and linear impedance may be included.

9.5.2 DSI model

Simulated damage analysis was performed to investigate variations in the PZT properties which led to formulation of PEF (chapter 5). However, experimental damage analysis need to be done and compared against the predicted signatures of damaged cases.

9.5.3 PZT efficiency factor

Even though PEF and RMSD are very good damage indices they are however ‘frequency range of excitation’ dependent, i.e, if there is any change in the frequency range, the RMSD and PEF value will also change. Furthermore, PEF is more vulnerable as it is also dependent on the number of peaks in the signatures. If there is a change in the frequency range and number of peaks, the PEF value will also change. Thus, standard non-parametric indices which is applicable for any number of peaks and any frequency range of excitation need to be developed.

9.5.4 Characterization of PZT transducer

To effectively use any of the developed EMI models, from ASI to extended MPZT-S, it is a must to characterize the properties of PZT transducers especially when using these interaction models to monitor real life structures. Thus, uniformity in properties of PZT transducers should be maintained throughout the period of SHM; or else, proper remedial models to ‘compensate’ for any variations of PZT properties need to be developed.

9.5.5 MPZT-S

The MPZT-S model is not directly applicable for curve-linear surfaces (shells and cone). However, the model can be extended to curve-linear host surfaces by changing the cross impedances to include torque/ moments.

9.5.6 ExMPZT-S

One assumption made in the MPZT-S model was to neglect the thickness of the adhesive layer. The ExMPZT-S considers the thickness of adhesive; however, the case study was performed for single PZT case where PZT mass was assumed to be negligible as compared to host structure. A few more case studies should be performed for multiple PZT cases to consider both the mass of PZT and mass of adhesive. Here again, the model needs to be extended to real life structures, especially for under ground structures, where protection of PZT is crucial.

PUBLICATIONS

The research effort in the past three years has resulted in the following publications.

JOURNALS

1. Annamdas V. G. M and Soh C. K (2006) "Embedded piezo-impedance transducers in sandwiching beams", Volume 15, Issue 2: 538-549, Smart Materials and Structures [*based on chapter 3 of the thesis*].
2. Annamdas V. G. M and Soh C. K (2007) "Three Dimensional Electromechanical Impedance Model I: Formulation of Directional Sum Impedance ", Volume 20, Issue 1:53-62, Journal of Aerospace Engineering, ASCE [*based on chapter 4*].
3. Annamdas V. G. M and Soh C. K (2007) "Three Dimensional Electromechanical Impedance Model II: Damage analysis and PZT characterization", Volume 20, Issue 1:63-71, Journal of Aerospace Engineering, ASCE [*based on chapter 5*].
4. Madhav A. V. G and Soh C. K (2007) "Multiplexing and uniplexing of PZT transducers for structural health monitoring", Volume 18, Issue 5 (in press), Journal of Intelligent Material Systems and Structures [*based on chapter 6*].
5. Madhav A. V. G and Soh C. K (2006+) "Multiple PZT-Structure Interaction Model", (Under review), Journal of Aerospace Engineering, ASCE [*based on chapter 7*].
6. Madhav A. V. G and Soh C. K (2007) "Electromechanical Impedance Model of Piezoceramic Transducer -Structure in Presence of Thick Adhesive Bonding", Volume 16, Issue 3: 673-686, Smart Materials and Structures [*based on chapter 8*].
7. Madhav A. V. G , Yang .Y and Soh C. K (2007+) " Structural health monitoring of concrete using embedded PZT transducers", (ready for submission) [based on first year PhD report].

CONFERENCES (Presented)

1. Annamdas V. G .M and Soh C. K (2006) "Multiple PZT-Host structure interaction model" SPIE symposium, San Diego California, USA, 26Feb-2Mar. Proc. of SPIE Vol. 6174, 61743G1-G12, [*based on chapter 7*].
2. Annamdas V. G .M, Yaowen Y. and Soh C. K (2007) "Influence of loading on structures actuated with Piezoceramic Transducers (PZT) for use in structural health monitoring " SPIE symposium, University of Adelaide, Australia, 10-13 Dec. 2006 Proc. of SPIE Vol. 6414, 64140X1-X12.

REFERENCES

Abe, M., Park, G. and Inman, D. J. (2002), “Impedance-Based Monitoring of Stress in Thin Structural Members”, Proceeding of 11th International Conference on Adaptive Structures and Technologies, October 23-26, Nagoya, Japan, pp. 285-292.

Agilent Technologies, USA. (2001) Test and Measurement Catalog. pp. 496.

ANSYS Reference Manual; Release 5.6 (2000), ANSYS Inc., Canonsburg, PA, USA.

Ayres, J. W., Lalande, F., Chaudhry, Z. and Rogers, C. A. (1998), “Qualitative Impedance-Based Health Monitoring of Civil Infrastructures”, Smart Materials and Structures, Vol. 7, No. 5, pp. 599-605.

Banan, M. R., Banan, M.R. and Hjelmstad, K. D. (1994), “Parameter Estimation of Structures from Static Response. I. Computational Aspects”, Journal of Structural Engineering, American Society of Civil Engineers, Vol. 120, No. 11, pp. 3243- 3258.

Bandar, A. A., and Abdulmalik A. A. A. (2003). “Review of the mechanics of materials models for one-dimensional surface-bonded piezoelectric actuators”, Smart Materials and Structures, 12, N1-N4.

Banks, H. T., Inman, D. J. and Wang, Y. (1996a), “An Experimentally Validated Detection Theory in Smart Structures”, Journal of Sound and Vibration, Vol. 191, No.5, pp.859-880.

Banks, H. T., Smith, R. C and Wang, Y. (1996b), Smart Material Structures-Modeling, Estimation and Control, Research in Applied Mathematics Series, Masson-John Wiley & sons, Paris, Chichester.

Bathe, K. J. (1996), Finite Element Procedures, Prentice hall, Upper Saddle river, New Jersey (U.S) 07458.

Bechmann, R. and Fair, I. E., (1958) “IRE Standards on Piezoelectric Crystals: Determination of the Elastic, Piezoelectric, and Dielectric constants – The Elctromechanical Coupling Factor,” Proceedings of IRE, 46, PP. 764-778.

Benjeddou, A. Trindade, M. A., and Ohayon, R. (2000). “Piezoelectric actuation mechanisms for intelligent sandwich structures”, Smart Materials and Structures, 9, 328-335.

Betti, R. and Testa, R. B. (1995), “Vibration and Damage Control for Long-Span Bridges”, Smart Materials and Structures, Vol. 4, pp.A91-A100.

Bhalla, S. (2001), “Smart System based Automated Health Monitoring of Structures” M.Engg. Thesis. Nanyang Technological University, Singapore.

Bhalla, S. (2004) “A Mechanical Impedance Approach for Structural Identification, Health Monitoring and Non-Destructive Evaluation Using Piezo-Impedance Transducers” Ph.D Thesis, Nanyang Technological University, Singapore.

Bhalla, S. and Soh, C. K (2004a), “Structural Health Monitoring by Piezo-Impedance Transducers I: Modeling” Journal of Aerospace Engineering, ASCE, Vol. 17, No. 4, pp.154-165.

Bhalla, S. and Soh, C.K. (2004b), “High Frequency Piezoelectric Signatures for Diagnosis of Seismic/Blast induced Structural Damages”, NDT&E International, Vol. 37, pp. 23-33.

Bhalla, S. and Soh, C. K. (2004c), “Structural Impedance Based Damage Diagnosis by Piezo-Transducers”, Earthquake Engineering and Structural Dynamics, Vol. 32, pp. 1897-1916.

Bhalla, S. and Soh, C. K. (2004d), “Structural Health Monitoring by Piezo-Impedance Transducers II: Applications” Journal of Aerospace Engineering, ASCE, Vol. 17, No. 4, pp. 166-175.

Bhalla, S. and Soh, C. K. (2004e), “Electromechanical Impedance Modeling for Adhesively Bonded Piezo-transducers”, Journal of Intelligent Material Systems and Structures, Vol. 15, No. 12, pp. 955-972.

Boller C. (2002), “Structural Health Management of Ageing Aircraft and Other Infrastructure”, Monograph on Structural Health Monitoring, Institute of Smart Structures and Systems (ISSS), pp. 1-59.

Bourasseau, S., Blanquet, P., Coutellier, D., Dupont, M., Pernice, M., Delebarre, C., Thiriot, A and T. Demol, (1996) “Integration of Optical Fibers and Piezoelectric Ceramics in Composite Materials,” 3rd International Conference on Intelligent Materials and 3rd European Conference on Smart Structures and Materials, Pierre-F. P. Gobin, Editor, Proceedings of SPIE, 2779, pp. 180-185.

Chandrashekara, K. and Tenneti, R. (1995), “Thermally Induced Vibration Suppression of Laminated Plates with Piezoelectric Sensors and Actuators”, Smart Materials and Structures, Vol. 4. pp. 281-290.

Chang, F.K. (2000) “A Summary Report on the First International Workshop on Structural Health Monitoring, September 18-20, 1997”, Structural Health Monitoring, Ed. Chang, F-K., Technomic Publication Company, pp. xix-xxix.

Cheng, C. C., and Lin, C. C. 2005a. "An impedance approach for vibration response synthesis using multiple PZT actuators", Sensors and Actuators A, Vol. 118. pp. 116-126.

Cheng, C. C., and Lin, C. C. 2005b. "An impedance approach for determining optimal locations and shapes of multiple induced strain actuators", Smart Materials and Structures, Vol.14. pp. 1120-1126.

Chen Peter C. and Inderjit Chopra, (1995), "Induced Strain Actuation of Composite Beams and Rotor Blades with Embedded Piezoceramic elements", Smart Materials and Structures, Vol. 5, pp. 35-48.

Crawley, E. F., de Luis, J. (1987), "Use of Piezoelectric Actuators as Elements of Intelligent Structures", AIAA Journal, Vol. 25, No. 10, pp. 1373-1385.

Doebbling, S. W., Farrar, C. R. and Prime, M. B. (1998), "A Summary Review of Vibration-based Damage Identification Methods," The Shock and Vibration digest, Vol. 30, No. 2, pp. 91-105.

Doherty, J. E. (1987) "Non-destructive evaluation," Handbook on Experimental Mechanics (A. S. Kobayashi, editor), Society for Experimental Mechanics.

Dosch, J. J., Inman, D. J., and Garcia, E., (1991). "Measurement of Strain and Stress in Piezoelectric Actuator for Collocated Control," Proceedings Active Materials and Adoptive Structures, November pp. 195-198.

"Dosch, J.J., Inman, D. J. and Garcia, E., (2001). "A Self Sensing Piezoelectric Actuator for Collocated Control," in selected papers on Smart Structures for Spacecraft, Das, A and Wada, B, Eds., SPIE Milestones Series, Vol. MS167, pp. 319-338. (A reprint of article originally appearing in Journal of Intelligent Materials Systems and Structures, (1992), Vol. 3, No.1, pp.166-185)".

Elspass, W. J. Kunzmann, J. Flemming, M. and D. Baumann, (1995) "Design, Manufacturing and Verification of Piezoceramics Embedded in Fiber-Reinforced Thermoplastics," Smart Structures and Materials 1995: Smart Structures and Integrated Systems, Proceedings of SPIE, 2443, pp. 327-333.

Esteban, J. (1996) "Analysis of the Sensing Region of a PZT Actuator-Sensor", Ph.D. Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA.

Fairweather, J. A. (1998) "Designing with Active Materials: An Impedance Based Approach", Ph.D. Thesis, Rensselaer Polytechnic Institute, New York.

Giurgiutiu, V., Reynolds, A. and Rogers, C. A. (1999), "Experimental Investigation of Electromechanical Impedance Health Monitoring for Spot-Welded Structural Joints", Journal of Intelligent Material Systems and Structures, Vol. 10, No. 10, pp. 802-812.

Giurgiutiu, V., and Zagrai, A. N. (2000), "Characterization of Piezoelectric Wafer Active Sensors", Journal of Intelligent Material Systems and Structures, Vol.11, No. 12, pp. 959-976.

Giurgiutiu, V., and Zagrai, A. N. (2002), "Embedded Self -Sensing Piezoelectric Active Sensors for On-Line Structural Identification", Journal of Vibration and Acoustics, ASME, Vol. 124, No. 1, pp.116-125.

Giurgiutiu, V., Zagrai, A. N. and Bao, J. J. (2002b), "Embedded Active Sensors for In-Situ Structural Health Monitoring of Thin-Wall Structures", Journal of Pressure Vessel Technology, ASME, Vol. 124, pp. 293-302.

Hagood, N. W. Crawley, E. F., de Luis J. and Anderson, E. H (1988), "Development of Integrated Components for Control of Intelligent Structures," Smart Materials,

Structures and Mathematical Issues, U.S. Army Research Office Workshop, pp. 80-104, Blacksburg, VA, USA, September.

Han, J. H., Rew, K. H. and Lee, I. (1997), “An Experimental Study of Active Vibration Control of Composite Structures with Piezo-ceramic Actuator and a Piezo-film sensor,” Smart Materials and Structures, Vol. 6, No. 5, pp. 549-558.

Hewlett Packard, (1996) HP LF 4192A impedance analyzer Operation manual, Japan.

Hixon, E.L. (1988), Mechanical Impedance, Shock and Vibration Handbook, 3rd ed., Mc Graw Hill Book Co., New York, pp. 10.1-10.46.

IEEE (1987), IEEE Standard on Piezoelectricity, Std. 176, IEEE/ANSI.

Inman, D. J. and Hegewald, Thomas, (2000). “Simultaneous Health Monitoring and Control of Panels,” Proceedings of the 11th International Conference on Adaptive Structures and Technologies, Nagoya, Japan, 23-26 October 2000, pp. 309-316”.

Inman, D. J. (2001), Engineering Vibrations, 2nd ed. Prentice-Hall Inc., New Jersey.

Inman, D. J., Ahmadian, M. and Claus, R. O., (2002). “Simultaneous Active Damping and Health Monitoring of Aircraft Panels,” Journal of Intelligent Materials Systems and Structures, Vol. 12, No. 11, pp.775-785.

Islam, A. S. and Craig, K. C. (1994), “Damage Detection in Composite Structures using Piezoelectric Materials (and neural net)”. Smart Materials and Structures, Vol. 3, pp.318-328.

ISRO (2006), Indian Space and Research Organisation, India, < <http://www.isro.org/> >

Jason Heikenfeld and Andrew J. Steckl, (2002), “Electroluminescent Devices using a High-Temperature Stable GaN-based Phosphor and Thick-Film Dielectric Layer”. IEEE Transactions on electron devices, Vol. 49, No.4, pp. 557-563.

Kessler, S. S., Spearing, S. M., Attala, M. J., Cesnik, C. E. S. and Soutis, C. (2002), “Damage Detection in Composite Materials Using Frequency Response Methods”, Composites, Part B: Engineering, Vol. 33, pp. 87-95.

Kevin L. R., Terry J. W., and F. Wayne K., (1997), “Review of Non-destructive Evaluation Techniques of Civil Infrastructure”. Journal of Performance of Constructed Facilities, Vol. 11, No. 4, pp. 152-160.

Kumar, S. (1991) “Smart Materials for Accoustic or Vibration Control”, Ph.D. Dissertation, Pennsylvania State University, PA.

Lee, E. -Gu., Lee, J. -K., Jang, W. -Y., Lee, J. G. and Kim, Ji-Y, (2000), “Domain Switching Characteristics of Preferentially Oriented lead Zirconate Titanate Thin Films,” Journal of Materials Science Letters, Vol. 19, No. 21, pp. 1917-1919.

Liang, C., Sun, F. P. and Rogers, C. A. (1993), “An Impedance Method for Dynamic Analysis of Active Material Systems”, Proceedings of AIAA/ ASME/ ASCE/ Material Systems, La- Jolla, California, pp. 3587-3599.

Liang, C., Sun, F. P. and Rogers, C. A. (1994), “Coupled Electro-Mechanical Analysis of Adaptive Material Systems - Determination of the Actuator Power Consumption and System Energy Transfer”, Journal of Intelligent Material Systems and Structures, Vol. 5, No.1, pp.12-20.

Liang, C., Sun, F. P., and Rogers, C. A. (1996), "Electro-mechanical impedance modeling of active material systems", Smart Materials and Structures, Vol. 5, pp. 171-186.

Loh, C. H. and Tou, I.C. (1995), "A System Identification Approach to the Detection of Changes in both Linear and Non-linear Structural Parameters", Earthquake Engineering & Structural Dynamics, Vol. 24, No. 1, pp. 85-97.

Lynch, J. P., Partridge, A., Law, K. H., Kenny, T. W., Kiremidjian, A. S. and Carryer, E. (2003), "Design of Piezoresistive MEMS-Based Accelerometer for Integration with Wireless Sensing Unit for Structural Monitoring", Journal of Aerospace Engineering, ASCE, Vol. 16, No. 3, pp. 108-114.

Makkonen, T., Holappa, A., Ella, J. and Salomaa, M. M. (2001), "Finite Element Simulations of Thin-Film Composite BAW Resonators", IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control, Vol. 48, No. 5, pp. 1241-1258.

Mall, S. and Hsu, T. L. (2000), "Electromechanical Fatigue Behaviour of Graphite/Epoxy Laminate Embedded with Piezoelectric Actuator", Materials and Structures, Vol. 9, pp. 78-84.

Mall, S. (2002) "Integrity of Graphite/Epoxy Laminate Embedded with Piezoelectric Sensor/ Actuator under Monotonic and Fatigue Loads", Smart Materials and Structures, Vol. 11 pp.527-533.

Mayne, R. W. and Dosch, J. J., (1994), "Dual Function System having a Piezoelectric Element", Self Sensing Actuator, US Patent #5, 347,870 Sep 20, 1994

Naidu, A. S. K. (2003) “Structural Damage Identification with Admittance Signatures of Smart PZT Transducers”, PhD Thesis. Nanyang Technological University, Singapore.

Naidu, A. S. K. and Soh, C. K., (2004), “Damage Severity and Propagation Characterization with Admittance Signatures of Piezo Transducers.” Smart Materials and Structures. Vol. 13, pp.393–403.

NASA 2006. National aeronautics and space administration, USA, < <http://www.nasa.gov/missions/highlights/index.html> >

Okafor, A. C., Chandrashekara, K and Jiang, Y. P., (1996), “Delamination Prediction in Composite Beams with built-in Piezoelectric Devices using Modal Analysis and Neural Network”, Smart Materials and Structures. Vol. 5, pp.338-347.

Paget, C. A. and Levin, K.(1999) “Structural Integrity of Composites with Embedded Piezoelectric Ceramic Transducer,” Smart Structures and Materials 1999: Smart Structures and Integrated Systems, Proceedings of SPIE, 3668, pp. 306-313.

Paget, A. Christophe , (2002) “Active Health Monitoring of Aerospace Composite Structures by Embedded Piezoceramic Transducers”, Ph.D Thesis, Department of Aeronautics Royal Institute of Technology, SE-100 44 Stockholm, Sweden. Report 2001-25, ISSN 0280-4646.

Paget, C. A. Levin, K. and Delebarre, C. (2002), “Actuation Performance of Embedded Piezoceramic Transducer in Mechanically Loaded Composites,” Smart Materials and Structures. Vol. 11, pp. 886–891.

Pandey A. K. and Biswas, M.(1994), “Damage Detection in Structures using Changes in Flexibility.” Journal of Sound and Vibration, Vol. 169, pp.3-17.

Pandey, A. K., Biswas, M. and Samman, M. M. (1991), "Damage Detection from Changes in Curvature Mode Shapes", Journal of Sound and Vibration, Vol. 145, No. 2, pp. 321-332.

Park, G., Kabeya, K., Cudney, H. H. and Inman, D. J. (1999), "Impedance-Based Structural Health Monitoring for Temperature Varying Applications", JSME International Journal, Vol. 42, No. 2, pp. 249-258

Park, G. (2000), "Assessing Structural Integrity using Mechatronic Impedance Transducers with Applications in Extreme Environments", Ph.D. Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, VA.

Park, G., Cudney, H.H. and Inman, D.J. (2000a), "Impedance-Based Health Monitoring of Civil Structural Components", Journal of Infrastructure Systems, ASCE, Vol. 6, No. 4, pp. 153-160.

Park, G., Cudney, H. H. and Inman, D. J. (2000b), "An Integrated Health Monitoring Technique Using Structural Impedance Sensors", Journal of Intelligent Material Systems and Structures, Vol. 11, pp.448-455.

Park, G., Cudney, H. H. and Inman, D. J. (2001), "Feasibility of Using Impedance-Based Damage Assessment for Pipeline Structures", Earthquake Engineering and Structural Dynamics, Vol. 30, No. 10, pp. 1463-1474.

Park, G., Ruggiero, Eric, J., and Inman, Daniel J. 2002. "Dynamic Testing of an Inflatable Structure Using Smart Materials", Smart Materials and Structures, Vol.11, No.1, pp.147-166.

Park, G., Inman, D. J., Farrar, C. R. (2003a) “Recent Studies in Piezoelectric Impedance-Based Structural health Monitoring”, Proceedings of 4th International Workshop on Structural Health Monitoring: From Diagnostics and Prognostics to Structural Health Management, September 15-17, Stanford University, Stanford, California, DES Tech Publications, Inc., Lancaster, PA, pp. 1423-1430.

Park, G., Sohn, H., Farrar, C. R. and Inman, D. J. (2003b), “Overview of Piezoelectric Impedance-Based Health Monitoring and Path Forward”, The Shock and Vibration Digest, SAGE Publications, Vol. 35, No. 6, pp. 451-463.

Pearis, D. M., Grisso, B. L., Margasahayam, R. N., Page, K. R., and Inman D. J., (2004a). “Proof-of Concept Application of Impedance-Based Health Monitoring on Space Shuttle Ground Structures,” Proceedings, SPIE’s 9th Annual International Symposium on NDE for Structural Health Monitoring, 14-18 March 2004, San Diego, CA, on CD, paper number 5394-15, pp. 99-107.

Peairs, D. M., Park, G., and Inman, D. J., (2004b), “Improving accessibility of the impedance-based structural health monitoring method”, Journal of Intelligent Material Systems and Structure, Vol. 15, No. 2, pp. 129-139.

PI Ceramic (2003), Product Information Catalogue, Lindenstrabe, Germany, <http://www.piceramic.de>.

Raja, S., Sreedeeep, R., and Prathap, G., (2004), “Bending behavior of Hybrid-Piezoelectric Sandwich Beams”, Journal of Intelligent Material Systems and Structures, Vol. 15, No.4, pp. 611-619.

RS Components (2003), Product Information Catalogue, Northants, UK,
<http://www.rs-components.com>.

Ruggiero, Eric, J., Park Gyuhae and Inman, Daniel, J. 2004. “Multi-Output Vibration Testing of an Inflatable Torus”, Mechanical Systems and Signal Processing, Vol. 18, No. 5, pp.1187-1201.

Ruggiero, Eric, J., and Inman, Daniel, J. 2005. “ A Comparison between SISO and MIMO Modal Analysis Techniques on a Membrane Mirror Satellite”, Journal of Intelligent Material Systems and Structures, Vol. 16, No. 3, pp. 273-280.

Salawu, O. S. (1997) “Detection of Structural Damage through Changes in Frequency: A review,” Engineering Structures, Vol.19, No.9, pp. 718-723.

Sanayei, M. and Saletnik, M. J. (1996), “Parameter Estimation of Structures from Static Strain Measurements. I: Formulation”, Journal of Structural Engineering, American Society of Civil Engineers, Vol. 122, No. 5, pp. 555-562.

Sensor Technology Limited (1995), Product Information Catalogue, Collingwood.

Shah, D. K., Chan, W. S. and Joshi, S. P.(1994), “Delamination Detection and Suppression in a Composite Laminate using Piezoceramic layers”. Smart Materials and Structures. Vol. 3, pp.293-301.

Shukla Divyang R. and Vizzini Anthony J, (1996), “Interlacing for Improved Performance of Laminates with Embedded Devices”, Smart Materials and Structures Vol. 5. pp. 225-229.

Singh, D. A. and Vizzini, A. J. (1994), “Structural Integrity of Composite Laminates with Interlaced Actuators”, Smart Materials and Structures, Vol. 3, pp.71-79.

Sirohi, J. and Chopra, I. (2000), "Fundamental Understanding of Piezoelectric Strain Sensors", Journal of Intelligent Material Systems and Structures, Vol.11, No. 4, pp.246-257.

Soh, C. K., Tseng, K. K. H., Bhalla, S., and Gupta, A. (2000), "Performance of Smart Piezoceramic Patches in Health Monitoring of a RC Bridge", Smart Materials and Structures, Vol. 9, No. 4, pp.533-542.

Sun, F. P., Chaudry, Z., Rogers, C. A., Majmundar, M. and Liang, C. (1995), "Automated Real-time Structure Health Monitoring via Signature Pattern Recognition." Proceedings of the SPIE Conference on smart materials and structures, San Diego, California, Vol. 2443: Feb. 27- Mar. 1, pp. 236-47.

Varadan, V. (2000), "Nanotechnology, MEMS and NEMS and their Applications to Smart Systems and Devices", Proceedings of ISSS-SPIE International Conference on Smart Materials, Structures and Systems, edited by B. Dattaguru, S. Gopalakrishnan, S. Mohan, 12-14 December, Bangalore, Micro art Multimedia Solutions (Bangalore) pp. 909-932.

Wang, X., and Shen, Y., (1998a), "On the characterization of piezoelectric actuators attached to structures", Smart Materials and Structures, Vol. 7, pp. 389-395.

Wang, M. L., Heo, G. and Satpathi, D. (1998b), "A Health Monitoring System for Large Structural Systems", Smart Materials and Structures, Vol. 7, No. 5, pp. 606-616.

Wetherhold, R., Messer, M., and Patra, A., (2003), "Optimization of directionally attached piezoelectric actuators", Journal of Engineering Materials and Technology, Vol. 125, pp. 148-152.

Xu, J-F. (2003) “Parameter Identification and Damage Detection using Evolutionary Programming”, PhD Thesis. Nanyang Technological University, Singapore.

Yang, S. M. and Bian, J. J. (1996), “Vibration Suppression Experiments on Composite Laminated Plates using an Embedded Piezoelectric Sensor and Actuator”, Smart Materials and Structures. Vol. 5, pp.501-507.

Yocum M, Aramovich H, Grunwald A and Mall S (2003), “Fully Reversed Electromechanical Fatigue Behaviour of Composite with Embedded Piezoelectric Actuator/Sensor. Smart Materials and Structures. Vol. 12, pp. 556-564.

Zhou, S. W., Liang, C. and Rogers, C. A. (1995), “Integration and Design of Piezoceramic Elements in Intelligent Structures”, Journal of Intelligent Material Systems and Structures, Vol. 6, No. 6, pp.733-743.

Zhou, S. W., Liang, C. and Rogers C. A. (1996), “An Impedance-Based System Modeling Approach for Induced Strain Actuator-Driven Structures”, Journal of Vibrations and Acoustics, ASME, Vol. 118, No. 3, pp. 323-331.

Zimmerman, D. C. and Kaouk, M. (1994), “Structural Damage Detection Using a Minimum Rank Update Theory”, Journal of Vibration and Accoustics, Vol. 116, pp. 222-231.

Zou, Y., Tong, L. and Steven, G. P. (2000), “ Vibration-Based Model-Dependent Damage (Delamination) Identification and Health Monitoring for Composite Structures- A Review,” Journal of Sound and Vibration, Vol. 230, No.2, pp. 357-378.