Heave response of a floating vertical cylinder with built-in tuned mass damper

Adi Kurniawan

2006

Adi, K. (2006). Heave response of a floating vertical cylinder with built-in tuned mass damper. Master's thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/12069>

<https://doi.org/10.32657/10356/12069>

Nanyang Technological University

Downloaded on 26 Apr 2023 21:20:34 SGT

HEAVE RESPONSE OF A FLOATING VERTICAL CYLINDER WITH BUILT-IN TUNED MASS DAMPER

ADI KURNIAWAN

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

NANYANG TECHNOLOGICAL UNIVERSITY

2006

Heave Response of a Floating Vertical Cylinder with Built-in Tuned Mass Damper

Adi Kurniawan

School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Master of Engineering

2006

Glory be to God the Trinity,
who has revealed Himself in creation
so that we may know Him.

He has made the earth by His power,
He has established the world by His wisdom,
And has stretched out the heavens at His discretion. (Jeremiah 10:12)
They continue this day according to Your ordinances,
For all *are* Your servants. (Psalm 119:91)

Acknowledgements

The author would like to express his gratitude to Dr. Gho Wie Min for his kind guidance and supervision during the research process. The author would also like to thank Dr. Arthur Lim Teck Bin, Mr. Francis of Chye Guan Silkscreen, Mr. Phua Kok Soon, Mr. Jan Henrik Berg-Jensen of DNV, Mr. Ong Muk Chen, Mr. Huang Guoxing, Miss Selina, Mr. Chia Teck Boon, Mr. Ong Yean-Chau, Mr. Michael Sien Hay Wah, Mr. Farrand Hey Chunn Kiat, Mr. Lim Yee Yan, Mr. Ferdinan Widjaja, and others who have contributed directly or indirectly to this research and without whom this research would have been less enriching.

Research scholarship awarded by Nanyang Technological University Singapore and research grant by Maritime and Port Authority Singapore are gratefully acknowledged.

Table of Contents

Acknowledgements	iii
Table of Contents	iv
Summary	viii
List of Tables	ix
List of Figures	x
List of Symbols	xvi
Chapter 1 Introduction	1
1.1 Background	1
1.2 Scope of study	3
1.3 Objectives	3
1.4 Outline of thesis	4
1.5 Originality and contributions of current research	4
Chapter 2 Literature Review	6
2.1 Various floating structure concepts	6
2.1.1 Ship-shaped vessels	6
2.1.2 Column stabilised semi-submersible platforms	7
2.1.3 Tension leg platforms (TLP)	10
2.1.4 Spar platforms	11
2.1.5 Very Large Floating Structures (VLFS)	12
2.1.6 Summary of floating structure concepts	14
2.2 Motion reduction mechanisms	15
2.3 Tuned Mass Damper (TMD)	20
2.4 Important concepts in hydrodynamics of floating bodies	23
2.4.1 Regimes of flow	23
2.4.2 Linear and nonlinear waves	24
2.4.3 Diffraction and radiation theory	25
2.5 Methods of hydrodynamic analysis of floating bodies	26
2.6 Diffraction and radiation problem of floating vertical circular cylinders	28

Chapter 3	Theoretical Formulation	32
3.1	Hydrostatic stability	32
3.2	Preliminary study on simplified 2-DOF model	33
3.2.1	Free vibration	34
3.2.2	Forced vibration	36
3.2.3	Effectiveness of TMD	40
3.2.4	Amplitude ratio of TMD over main mass	43
3.2.5	Effectiveness of TMD for extreme g	46
3.3	Diffraction and radiation problem	48
3.3.1	Boundary value problem	48
3.3.2	Derivation of incident wave potential	50
3.3.3	Derivation of diffraction potential	53
3.3.4	Derivation of radiation potential	60
3.3.5	Calculation of hydrodynamic forces	71
3.3.6	Matrix equation of motion	76
3.3.7	Computational procedure	77
Chapter 4	Laboratory Experiments	78
4.1	Model design	78
4.1.1	Froude scaling law	78
4.1.2	Fabrication and configuration of scaled model	79
4.1.3	Eight cases of configuration	80
4.2	Experimental setup	87
4.3	6-DOF tracking function and calculations of rotation angles	91
4.4	Experimental run	93
4.5	Experimental results	95
4.5.1	Decay tests	95
4.5.2	Experimental heave response	97
Chapter 5	Numerical Simulations	104
5.1	Numerical modelling	104
5.2	Convergence tests	106
5.3	Numerical results	110

5.3.1	Floating cylinder without TMD	110
5.3.2	Floating cylinder with TMD	110
Chapter 6	Comparison of Results and Discussions	119
6.1	Comparison of results	119
6.2	Parametric study	139
Chapter 7	Conclusions and Recommendations	143
7.1	Conclusions	143
7.2	Recommendations	144
References		146
Appendix A	Formulas for calculating moment of inertias	153
Appendix B	MATLAB routine for computing eigenvalues	155
Appendix C	MATLAB routine for computing diffraction and radiation theory results	156
Appendix D	MATLAB routine for plotting diffraction and radiation theory results	175
Appendix E	MATLAB routine for reading wave time history from .DAF files	177
Appendix F	MATLAB routine for reading heave time history from .TSV files	179
Appendix G	Prefem command input file to generate panel model with normal water plane	181
Appendix H	Prefem command input file to generate panel model with large water plane	182
Appendix I	MATLAB routine for reading numerical simulation results from .LIS files	183
Appendix J	MATLAB routine for reading numerical simulation results for dynamic analysis from .LIS files	187
Appendix K	MATLAB routine for plotting heave RAO from numerical simulation	191

Appendix L	MATLAB routine for plotting heave RRF from numerical simulation	192
Appendix M	Publications	193

Summary

Novel concepts of floating structures continue to be proposed with applications ranging from hydrocarbons exploitation to the possible inhabitation of the ocean. As floating structures move freely in 6 degree-of-freedom, reducing their motions in response to environmental forces is a challenging task. Among the various motion reduction mechanisms, the tuned mass damper (TMD) still has limited use in floating structures.

In the current research, the feasibility of TMD to reduce the heave response of a freely floating vertical circular cylinder is investigated. The TMD is designed and positioned inside the cylinder without significantly altering the hydrostatic stability of the structure. A combination of theoretical, experimental, and numerical studies is employed in the investigation. A preliminary study is first conducted by simplifying the problem into a 2-DOF problem, assuming the added mass and radiation damping as constants. The full linear diffraction and radiation of water waves by the floating cylinder under harmonic wave loadings is then solved semi-analytically, utilising the eigenfunction expansion method, to obtain the hydrodynamic forces and hence, the heave response of the cylinder. Laboratory experiments are conducted with a 1/50 scaled model in a towing tank for eight different model configurations. Numerical simulations utilising the panel method are also performed. Comparisons of results are made for the heave response amplitude operator (RAO) and the response reduction factor (*RRF*). It is found that the heave RAOs from the experiments agree quite well with the numerical and the semi-analytical results. Interestingly, a significant agreement is found between the *RRF* obtained from the simplified 2-DOF model and the numerical as well as the semi-analytical results. Finally, from the parametric study it is shown that, for a mass ratio as small as 2%, by proper selection of TMD frequency and damping, the TMD is capable of reducing the heave response at resonance to as much as half of that of the structure without TMD.

List of Tables

Table 2.1	Critical motions, natural periods, and $\overline{GM}$ of various floating structures	15
Table 2.2	Motion reduction mechanisms	21
Table 4.1	Froude scaling	78
Table 4.2	Eight cases of configuration	85
Table 4.3	Eight cases of configuration with principal dimensions (prototype values)	86
Table 4.4	Test runs (prototype values)	94
Table 4.5	Heave natural periods and frequencies (prototype values) of main structure obtained from decay tests	96
Table 4.6	Heave damping ratios, critical damping coefficients, and damping coefficients (prototype values) of main structure obtained from decay tests	96
Table 6.1	Comparison of natural frequencies (prototype values) of main structure obtained from experiment and numerical simulation	119
Table 6.2	Comparison of damping coefficients (prototype values) of main structure obtained from experiment and numerical simulation	120
Table 6.3	Added mass and damping coefficients used in 2-DOF calculations	121

List of Figures

Figure 2.1	Ship-shaped vessel	7
Figure 2.2	Semi-submersible	8
Figure 2.3	Tension leg platform	10
Figure 2.4	Spar platform	12
Figure 3.1	Hydrostatic stability of floating vertical circular cylinder	32
Figure 3.2	2-DOF system	33
Figure 3.3	U_1/U_{st} for various c_2/c_{c2} ($f = 1, \mu = 1/20, c_1/c_{c1} = 0.1$)	38
Figure 3.4	U_1/U_{st} for various f ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$)	39
Figure 3.5	RRF for various c_2/c_{c2} ($f = 1, \mu = 1/20, c_1/c_{c1} = 0.1$)	42
Figure 3.6	RRF for various f ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$)	42
Figure 3.7	U_2/U_1 for various c_2/c_{c2} ($f = 1$)	44
Figure 3.8	U_2/U_1 for various f ($c_2/c_{c2} = 0.1$)	45
Figure 3.9	RRF for various g ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$)	47
Figure 3.10	RRF for various g ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.02$)	47
Figure 3.11	Problem definition	49
Figure 4.1a	Profile and general configuration of model with box TMD casing (all dimensions in mm)	79
Figure 4.1b	Profile and general configuration of model with tube TMD casing (all dimensions in mm)	80
Figure 4.1c	Profile and general configuration of larger circular cylindrical outer structure (all dimensions in mm)	81
Figure 4.2	TMD masses	81
Figure 4.3	Box TMD casing with TMD	82
Figure 4.4	Tube TMD casing with TMD	82
Figure 4.5	Disassembled scaled model with box TMD casing	83
Figure 4.6	Assembled scaled model with box TMD casing	83
Figure 4.7	Assembled scaled model with tube TMD casing	84

Figure 4.8	Assembled scaled model with tube TMD casing and larger circular cylindrical outer structure	84
Figure 4.9	Towing tank	87
Figure 4.10	Towing tank and model setup	88
Figure 4.11	Piston type wave generator	89
Figure 4.12	Wave generator control room	89
Figure 4.13	Experimental setup	90
Figure 4.14	Heave RAOs of cylinder without TMD obtained from experiments	99
Figure 4.15a	Heave RAO obtained from experiments for <i>box full ballast</i>	100
Figure 4.15b	Heave RAO obtained from experiments for <i>tube full ballast</i> ($m = 75,250\text{kg}$)	100
Figure 4.15c	Heave RAO obtained from experiments for <i>tube full ballast</i> ($m = 100,125\text{kg}$ and $m = 162,750\text{kg}$)	100
Figure 4.15d	Heave RAO obtained from experiments for <i>tube full ballast with water</i>	101
Figure 4.15e	Heave RAO obtained from experiments for <i>tube bottom & top ballast</i> ($m = 75,250\text{kg}$)	101
Figure 4.15f	Heave RAO obtained from experiments for <i>tube bottom & top ballast</i> ($m = 100,125\text{kg}$)	101
Figure 4.15g	Heave RAO obtained from experiments for <i>tube bottom & top ballast</i> ($m = 162,750\text{kg}$ and $m = 198,500\text{kg}$)	102
Figure 4.15h	Heave RAO obtained from experiments for <i>tube bottom & top ballast increased water plane</i>	102
Figure 4.15i	Heave RAO obtained from experiments for <i>tube bottom & top ballast increased water plane with water</i>	102
Figure 4.15j	Heave RAO obtained from experiments for <i>tube bottom ballast increased water plane</i>	103
Figure 4.15k	Heave RAO obtained from experiments for <i>tube top ballast increased water plane</i>	103
Figure 5.1	<i>Tube full ballast</i> panel model (544 panels)	105

Figure 5.2	<i>Tube bottom & top ballast increased water plane panel model</i> (576 panels)	105
Figure 5.3	Convergence of heave RAO (<i>box full ballast</i> without TMD)	107
Figure 5.4	Convergence of heave-heave added mass (<i>box full ballast</i> without TMD)	108
Figure 5.5	Convergence of heave-heave damping (<i>box full ballast</i> without TMD)	109
Figure 5.6	Numerical plots of heave RAO for cylinder without TMD	111
Figure 5.7	Numerical plots of heave-heave added mass for cylinder without TMD	112
Figure 5.8	Numerical plots of heave-heave damping for cylinder without TMD	113
Figure 5.9a	Numerical plots of heave RAO for <i>box full ballast</i>	114
Figure 5.9b	Numerical plots of heave RAO for <i>tube full ballast</i> ($m = 75,250\text{kg}$)	114
Figure 5.9c	Numerical plots of heave RAO for <i>tube full ballast</i> ($m = 100,125\text{kg}$)	115
Figure 5.9d	Numerical plots of heave RAO for <i>tube full ballast</i> ($m = 162,750\text{kg}$)	115
Figure 5.9e	Numerical plots of heave RAO for <i>tube full ballast with water</i> ($m = 75,250\text{kg}$ and $m = 100,125\text{kg}$)	115
Figure 5.9f	Numerical plots of heave RAO for <i>tube full ballast with water</i> ($m = 162,750\text{kg}$ and $m = 198,500\text{kg}$)	116
Figure 5.9g	Numerical plots of heave RAO for <i>tube bottom & top ballast</i> ($m = 75,250\text{kg}$)	116
Figure 5.9h	Numerical plots of heave RAO for <i>tube bottom & top ballast</i> ($m = 100,125\text{kg}$)	117
Figure 5.9i	Numerical plots of heave RAO for <i>tube bottom & top ballast</i> ($m = 162,750\text{kg}$ and $m = 198,500\text{kg}$)	117
Figure 5.9j	Numerical plots of heave RAO for <i>tube bottom & top ballast</i> <i>increased water plane</i>	117

Figure 5.9k	Numerical plots of heave RAO for <i>tube bottom & top ballast increased water plane with water</i>	118
Figure 5.9l	Numerical plots of heave RAO for <i>tube bottom ballast increased water plane</i>	118
Figure 5.9m	Numerical plots of heave RAO for <i>tube top ballast increased water plane</i>	118
Figure 6.1a	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>box full ballast</i>	122
Figure 6.1b	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube full ballast</i>	123
Figure 6.1c	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube full ballast with water</i>	124
Figure 6.1d	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube bottom & top ballast</i>	125
Figure 6.1e	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube bottom & top ballast increased water plane</i>	126
Figure 6.1f	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube bottom & top ballast increased water plane with water</i>	126
Figure 6.1g	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube bottom ballast increased water plane</i>	127
Figure 6.1h	Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for <i>tube top ballast increased water plane</i>	127
Figure 6.2a	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>box full ballast</i>	128

Figure 6.2b	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube full ballast</i>	129
Figure 6.2c	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube full ballast with water</i>	130
Figure 6.2d	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube bottom & top ballast</i>	131
Figure 6.2e	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube bottom & top ballast increased water plane</i>	132
Figure 6.2f	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube bottom & top ballast increased water plane with water</i>	132
Figure 6.2g	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube bottom ballast increased water plane</i>	133
Figure 6.2h	Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for <i>tube top ballast increased water plane</i>	133
Figure 6.2i	Irregular frequency in semi-analytical heave RAO for <i>tube top ballast increased water plane</i> without TMD	133
Figure 6.3a	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>box full ballast</i>	134
Figure 6.3b	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube full ballast</i>	135
Figure 6.3c	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube full ballast with water</i>	136

Figure 6.3d	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube bottom & top ballast</i>	137
Figure 6.3e	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube bottom & top ballast increased water plane</i>	138
Figure 6.3f	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube bottom & top ballast increased water plane with water</i>	138
Figure 6.3g	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube bottom ballast increased water plane</i>	139
Figure 6.3h	Comparison of heave <i>RRFs</i> obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for <i>tube top ballast increased water plane</i>	139
Figure 6.4a	Heave <i>RRF</i> for $f_a = 0.132Hz$ and $c_2 = 16,017 kg/s$	140
Figure 6.4b	Heave RAO for $f_a = 0.132Hz$ and $c_2 = 16,017 kg/s$	141
Figure 6.5	Heave RAOs for various f_a ($\mu = 0.02$, $c_2 = 16,017 kg/s$)	141
Figure 6.6	Heave RAOs for various c_2 ($\mu = 0.02$, $f_a = 0.129Hz$)	142

List of Symbols

A	wave amplitude
A	water plane area
$\mathbf{A}$	added mass matrix
A_{ij}	coefficient of added mass matrix
a	half length of rectangular block dimension parallel to x -axis
B	centre of buoyancy
$\overline{BM}$	vertical position of metacentre above centre of buoyancy
b	half length of rectangular block dimension parallel to y -axis
$\mathbf{C}$	damping matrix
C_{ij}	coefficient of damping matrix
c	half length of rectangular block dimension parallel to z -axis
c	viscous damping coefficient
c	wave celerity
c_c	critical viscous damping constant, $2m\omega_n$
D	diameter of cylinder
d	draft
e	base of natural logarithm
F	force amplitude
$\mathbf{F}$	force vector
$\overline{F}$	complex force amplitude
Fr	Froude number
f	ratio of natural frequencies, $\frac{\omega_a}{\omega_n}$
f_a	natural frequency of TMD
f_n	natural frequency of main mass
G	centre of gravity
$G(\mathbf{x}; \xi)$	Green function

$\overline{GM}$	vertical position of metacentre above centre of gravity, or metacentric height
g	acceleration of gravity
g	forced frequency ratio, $\frac{\omega}{\omega_n}$
H	wave height
$H_m^{(1)}$	Hankel function of the first kind of order m
h	water depth
I_{11}, I_{22}, I_{33}	mass moment of inertia of body about x -, y -, and z -axis passing through motion reference point
I_A	second moment of area of water plane
I_m	modified Bessel function of the first kind of order m
I_{xx}, I_{yy}, I_{zz}	mass moment of inertia about principal x -, y -, and z -axis
i	imaginary number, $\sqrt{-1}$
$\text{Im}\{ \}$	imaginary part of complex quantity
J_m	Bessel function of the first kind of order m
$\mathbf{K}$	hydrostatic restoring matrix
K_{ij}	coefficient of hydrostatic restoring matrix
K_m	modified Bessel function of the second kind of order m
$\overline{KB}$	vertical position of centre of buoyancy above keel
KC	Keulegan-Carpenter number
$\overline{KG}$	vertical position of centre of gravity above keel
k	spring stiffness
k	wave number, $\frac{2\pi}{L}$
L	wavelength
M	metacentre
$\mathbf{M}$	mass matrix
M_{ij}	coefficient of mass matrix
m	mass

$\mathbf{n}$	unit normal vector, pointing out of fluid domain
n_j	j -component of unit normal vector
$\bar{P}_{global}$	position vector of a point in global coordinate system
$\bar{P}_{local}$	position vector of a point in local coordinate system
$\bar{P}_{origin}$	position vector of origin of local coordinate system in global coordinate system
p	pressure
R	radius of cylinder
R_1	inner radius
R_2	outer radius
$\bar{R}$	rotation matrix
$\bar{R}_x, \bar{R}_y, \bar{R}_z$	individual rotation matrices
r	radius of a point
$\text{Re}\{ \}$	real part of complex quantity
RRF	response reduction factor
S_b	submerged surface of body
t	time
U	displacement amplitude
$\mathbf{U}$	displacement vector
$\bar{U}_j$	complex amplitude of displacement in each degree-of-freedom considered
U_{st}	static deflection of main mass, $\frac{F_0}{k_1}$
$u, \dot{u}, \ddot{u}$	displacement, velocity, acceleration
$\mathbf{x}$	position vector of a point on body
$\mathbf{x}_G$	coordinates of centre of gravity
z_B	z -coordinate at cylinder bottom
z_b	z -coordinate of centre of buoyancy
Δ	displacement, $\rho_w \nabla$

$\delta_{j\tau}$	Kronecker delta
ε	phase angle
ε_m	Neumann's symbol
ζ	damping ratio, $\frac{c}{c_c}$
θ	angle
θ	roll rotation angle
λ_j	eigenvalue
μ	mass ratio (TMD mass over main mass), $\frac{m_2}{m_1}$
ξ	position vector of a point on body
π	pi
ρ_A, ρ_S, ρ_W	density of acrylic, steel, and water
σ	source strength
Φ	time dependent velocity potential
ϕ	complex velocity potential
ϕ	pitch rotation angle
$\phi_D, \phi_I, \phi_S, \phi_R$	complex diffraction, incident wave, scattered, and radiation potentials
ϕ_j	unit-amplitude radiation potentials (unit-amplitude refers to unit-amplitude linear or angular velocity of rigid body motion)
ψ	yaw rotation angle
ω	angular frequency of regular incident wave
ω_a	natural angular frequency of TMD, $\sqrt{\frac{k_2}{m_2}}$
ω_n	natural angular frequency of main mass, $\sqrt{\frac{k_1}{m_1}}$
∇	displaced volume
∇^2	Laplace operator

Chapter 1

Introduction

1.1 Background

The development of floating structures has been progressing at a tremendous pace in recent years. The search for hydrocarbons and other mineral resources into deeper waters has led to the innovative design of various types of floating structures. These floating structures are generally classified into ship shaped vessels, column stabilised semi-submersible, tension leg (TLP), and spar platforms, each of which has a wide range of designs. Novel design of floating structures continue to be proposed to better cope with harsh environmental conditions, such as in locations with significant wave heights above 15m. Moan (2003) recently reported that new technology for water depths beyond 3km needs to be qualified and developed.

Apart from the utilisation of floating structures for exploration and exploitation of hydrocarbons and minerals, research has extensively been conducted on Very Large Floating Structure (VLFS), which could be a promising solution for land scarcity problems. VLFS is basically a huge floating platform with dimensions in the order of hundreds of meters to kilometres. It has major advantages over traditional land reclamation. It is cost effective, environmentally friendly, easily constructed, removed, and expanded, as well as protected from seismic shocks (Watanabe et al., 2004a; 2004b). Existing and future applications of the system range from floating storage facilities to floating airports. In 1998, a 1-km long floating runway was built in the Tokyo Bay as a test model. Series of feasibility studies with takeoff and landing experiments of small airplanes have been conducted. With the development of VLFS technology, it is not impossible that floating cities may become a reality in the near future. Research has also been conducted on the Mobile Offshore Base (MOB) concept proposed by the US Office of Naval Research. It is a VLFS which will be used for military purposes.

In addition to providing energy from hydrocarbons located beneath the seabed, the sea also provides energy from wave, current, wind, as well as thermal difference in the water. Various wave energy devices related to the concept of floating structures have been proposed to transform wave energy into electricity.

Another area of applications of floating structures is the floating breakwaters, which provide an alternative to conventional fixed breakwaters. These floating breakwaters are preferred in relatively low wave energy environments or when the use of bottom-founded structures is impossible. Furthermore, floating breakwaters are preferred in certain applications where it is required that the breakwater does not pierce the free surface and extend down to the seabed due to aesthetic or water circulation considerations (Williams et al., 2000a).

These few examples alone show how important floating structures are and how they will find increasingly wider applications in the near future.

However, designing a floating structure is challenging as floating structures move freely in 6 degree-of-freedom (6 DOFs), whereas it is desirable that they have limited motion response under wave excitation. Excessive movement causes discomfort and danger to people, danger in cargo handling and device damage, and increase in maintenance costs (Kano et al., 2000). The variety of present and future applications of floating structures therefore calls for innovative concepts for attaining better motion performance.

There have been many motion reduction mechanisms proposed and already in use in floating structures. However, the use of Tuned Mass Damper (TMD) to reduce the heave response of a floating cylindrical structure has not been intensively studied. In the current research, therefore, the feasibility of TMD to reduce the heave response of a floating cylindrical structure is investigated.

1.2 Scope of study

This study is conducted on a freely floating vertical circular cylinder with built-in TMD. The TMD is designed and positioned inside the cylinder without significantly altering the hydrostatic stability of the cylinder. A preliminary calculation is done on a 2-DOF system moving vertically with the main mass undergoing harmonic excitation. The 2-DOF system can be perceived as a simplified model of the cylinder with built-in TMD oscillating in heave.

The hydrodynamics of the floating cylinder is first mathematically modelled. The radiation and diffraction problem is solved using a semi-analytical method, utilising the eigenfunction expansion method to obtain the velocity potentials and hence the hydrodynamic excitation and reaction forces on the cylinder. After the forces are obtained, the dynamic problem is formulated and the heave response of the cylinder is plotted as Response Amplitude Operator (RAO). The mathematical solution is experimentally verified with the use of 1/50 scaled model tested in a towing tank which is housed in the Hydraulics Laboratory, Nanyang Technological University. The scaled model resembles a prototype structure of 15-m diameter. The model is allowed to float freely in the towing tank and excited with regular waves generated by a wave generator. The motion response of the cylinder is captured via infrared cameras and reflective markers. Numerical simulations are also performed for verification. Comparisons of the heave RAO and the response reduction factor (RRF) between the theoretical, experimental, and numerical results are subsequently made. A parametric study is also conducted to obtain the optimum TMD parameters (frequency and damping ratios) to minimize response.

1.3 Objectives

The objective of this study is to provide understanding on the dynamic behaviour, particularly the heave response, of a floating vertical circular cylinder with built-in TMD, and to investigate the effectiveness of TMD in reducing the heave response of the structure.

1.4 Outline of thesis

The scope of investigation in the current study is divided into theoretical, experimental, and numerical. A brief review on the various concepts of floating structures and motion reduction mechanisms is first presented in Chapter 2. Various methods to solve the hydrodynamic problem of a floating structure is also presented in this chapter, focusing on the use of the eigenfunction expansion method to solve the diffraction and radiation of water waves by a floating vertical circular cylinder. In Chapter 3, a simplified 2-DOF model is used as a preliminary study of the dynamic behaviour of the floating cylinder-TMD system. The effectiveness of the TMD is measured using the response reduction factor (*RRF*). Solution to the full boundary value problem of diffraction and radiation of water waves by a floating vertical circular cylinder is also derived in this chapter, obtaining the velocity potentials and hence, the wave excitation and reaction forces. These are used to form the 7-DOF equation of motion of the floating cylinder-TMD system, which is then solved for the displacements, particularly the heave response of the cylinder. The experimental study is elaborated in Chapter 4, describing the model design, the experimental setup, and the experimental run. The results from the experiment are also presented. The numerical simulation is discussed in Chapter 5. In Chapter 6, the comparison of results from the theory, the experiments, and the numerical simulations is presented, together with the parametric study to obtain the optimum TMD frequency and damping to reduce heave at resonance. Finally, conclusions and recommendations for future work are given in Chapter 7.

1.5 Originality and contributions of current research

The use of TMD to reduce the heave response of floating structures is proposed in the current research. TMDs have been used to reduce the vibrations of land structures, but their applications in floating structures have been limited, and there has been no intensive study on the application of TMD to reduce the heave response of these structures. This research will be useful for understanding the effects of TMD in floating structures in general, and thus for possible applications of TMD to reduce heave in these structures. The floating vertical cylindrical structure with

built-in TMD can also serve as a building block for large floating structures, breakwaters, or wave energy devices.

The research methodology comprises a combination of different methods, namely, semi-analytical calculations utilising the eigenfunction expansion approach, experimental studies with a scaled model, and numerical simulations utilising the panel method, to serve as a reliable way to cross-check the results. Originality and independence of inquiry have been pursued in the experiments, especially in the design of the scaled model with TMD, as well as in some of the derivations of the theoretical results, namely the solution to the equation of motion of a simplified 2-DOF model where both masses are damped and the solution to the diffraction and radiation problem. The analytical solution to the linear diffraction and radiation of water waves by a floating vertical cylinder has actually been available for decades, although not presented in a complete manner. In this research, the treatment of the problem anew has resulted in computer routines for quick plotting of the variations of wave excitation forces, added masses, radiation dampings, and heave responses with or without TMD, for any given geometrical and environmental parameters. The simplification of the analysis into a 2-DOF model not only gives a preliminary understanding into the behaviour of the system, but also, together with the available results on the heave response of a structure without TMD, seems promising as an alternative and easier method to obtain the heave response of the floating structure-TMD system.

Chapter 2

Literature Review

2.1 Various floating structure concepts

This section presents an overview of various floating structure concepts that are available today, namely the ship-shaped vessels, the column stabilised semi-submersible platforms, the tension leg platforms, the spar platforms, and the Very Large Floating Structures (VLFS). Their dynamic characteristics, advantages and disadvantages, present and possible future usages, as well as important research areas relating to them are briefly reviewed. A summary of the critical motions, the typical range of natural period of the critical motions, and the typical $\overline{GM}$ values of the floating structures is also given.

2.1.1 Ship-shaped vessels

A wide range of ship-shaped vessels are used for exploration and production of oil and gas. These vessels include drillships, floating storage units (FSUs), and floating production storage and offloading units (FPSOs). Ship-shaped vessels are characterised by their small beam and large length. As a result, forces and motion responses are strongly dependent on the direction of the environmental forces. Wind, waves, and current on the heading direction of the vessel result in smallest forces, while environmental forces on the beam of the vessel lead to very large resonant rolling motions. A typical ship-shaped vessel is shown in Figure 2.1.

The ship-shaped vessels provide ample work area, deck load and storage capacity, structural strength, mobility, and economic construction. However, they have poor sea motion characteristics because of large water plane area to displacement ratio. The vessel follows the wave with relatively higher heave and roll amplitudes than those of a column stabilised semi-submersible (Clauss et al., 1992).

Station-keeping is one of the complexities associated with ship-shaped vessels. Excessive environmental forces on the vessel are generated due to incident waves on the beam. Thus, alternative methods of mooring, such as catenary anchor leg mooring (CALM), single anchor leg mooring (SALM), articulated towers, and soft yoke systems have been developed with the objective of permitting the ship to rotate according to the direction of the external forces. Thrusters are sometimes provided to reduce force and motions. Integral turret moorings have also been developed to allow the vessel to weathervane as well as to provide higher levels of structural safety and opportunities for more flow and control lines to be utilised (Barltrop, 1998).

Figure 2.1 Ship-shaped vessel

2.1.2 Column stabilised semi-submersible platforms

The semi-submersible concept developed from the idea of making the vessel 'transparent' to the wave forces. Conventional ship-shaped vessels, with their large water plane area and displacement near the water surface, are subject to the full force of surface waves. The principal design feature of the semi-submersible is the major buoyancy members located well below the water level at operating draft. As the wave particle motion decays exponentially with increasing depth below the water surface, less force is imparted on the buoyant members and hence the motions of the structure are reduced (Kuo, 1986).

The dynamic behaviour of semi-submersibles is defined by the displacement of columns and pontoons, as well as by their draft, cross-sectional shape, and spacing (Clauss et al., 1992). Due to the small water plane area and large submerged hull or pontoons, semi-submersibles have small wave frequency responses and quite long natural periods. The small water plane area contributed by the vertical columns results in long heave, pitch, and roll natural periods. The heave natural period is the critical parameter for semi-submersible drilling units. The heave, pitch, and roll hydrodynamic forces at the dominant wave period can be minimised by careful selection of pontoon volume and water plane area. This leads to good response characteristics in typical operating weather conditions. A typical semi-submersible is illustrated in Figure 2.2.

Figure 2.2 Semi-submersible

At a certain wave frequency, the counteracting inertia forces on submerged hulls and surface-piercing columns cancel each other out. This is called the cancellation frequency. As a result, the platform only experiences minimal heave motions due to the considerably lower drag forces. At neighbouring wave frequencies, the counteracting inertia forces on hulls and columns are partly compensated. Depending on the difference between these forces, only a small resultant excitation force acts on the structure and hence its motions in all degree-of-freedom are small.

Semi-submersibles are designed with a fairly low metacentric height, $\overline{GM}$, partly to reduce cost, and because a small $\overline{GM}$ tends to reduce pitch and roll motions. Most semi-submersibles are designed to operate with $\overline{GM}$ values of 1.5-2m and 1-

1.5m for normal operations and severe storm conditions, respectively (Barltrop, 1998).

Although semi-submersibles have excellent motion characteristics, their small water plane area and high centre of gravity lead to low static stability, and thus limited variable deck load. Furthermore, they have higher initial and daily operating costs compared to ship-shaped vessels. However, they are still the ideal platform for exploratory drilling operations in arbitrary irregular seas. They may also be built or converted for permanent oil production.

The station keeping of semi-submersibles is achieved primarily by chain or wire mooring systems. A number of semi-submersibles are also fitted with azimuthing thrusters to relieve loads in the mooring system and to assist in transit. Some of them are also equipped with dynamic positioning systems for deeper waters operations where moorings are impractical. These dynamic positioning systems consist of computer controlled thrusters to control the motion of the semi-submersibles in response to displacements or accelerations (Barltrop, 1998).

Recent design trend of semi-submersibles is to reduce the numbers of columns, to provide larger buoyancy members, and to eliminate braces in the wave zone. The number of displacement hulls or pontoons as well as columns, struts, and nodal joints has been reduced, and connections between columns, pontoons, and braces have been simplified. The deck also consists of fewer modules. Like catamarans, modern semi-submersibles consist of two longitudinal streamlined pontoons or hulls with four, six, or eight columns supporting the main deck. Transverse connections, i.e. horizontal braces, joining the hulls together at the column base level are designed to absorb large counteracting horizontal forces on both sides of the structure, occurring at certain wavelengths. Without transverse struts, high alternating loads can occur at the connections between the columns and the deck. This will result in great structural stress and associated fatigue damage of the structure (Clauss et al., 1992).

2.1.3 Tension leg platforms (TLP)

Although similar in some respects to semi-submersibles, TLPs tend to have more buoyancy in the columns (i.e. greater water plane area), and are distinguished by taut vertical mooring tethers. The excess buoyancy maintains tension in the mooring system (Chakrabarti, 1987). TLPs typically have three to six vertical surface piercing columns. Unlike a semi-submersible, which has two streamlined pontoons, a TLP has a complete ring of pontoons since they are not moved from location to location. The complete ring of pontoons can serve to add more buoyancy. A typical TLP is shown in Figure 2.3.

Figure 2.3 Tension leg platform

TLP is a compliant structure which allows lateral movements of surge, sway, and yaw but restrains heave, pitch, and roll. The surge motion is similar to that of an inverted pendulum where the excess buoyancy, rather than gravity, provides the restoring force. As the vertical movement of the restrained structure is a function only of the horizontal displacement, the low-frequency rolling and pitching motions do not occur. TLPs have a sensitivity to payload due to its effect on tether tensions.

The amount of excess buoyancy is designed such that the tension legs neither go slack nor become overstressed under the worst combination of payload and environmental forces. Excessive deck load may result in slack tether conditions in large waves.

The vertical tethers are designed to avoid resonances by putting the natural periods in heave, pitch, and roll below and those in surge, sway, and yaw well above the range of wave period. Both high-frequency and low-frequency resonance phenomena are important (Clauss et al., 1992). The contribution from the high frequency second, and perhaps higher, order forces can be significant for TLPs, which, due to the high axial stiffness of the tethers, have lightly damped natural periods of about 2 to 4 seconds in heave, roll, and pitch. These are excited by longer period waves and both fatigue life and extreme tether tensions can be adversely affected. The high-frequency motion is often referred to as ‘ringing’ and ‘springing’. Ringing is associated with transient effects while springing with steady-state oscillations (Faltinsen, 1990). The practical calculation of the high frequency second order forces is still an area of research.

2.1.4 Spar platforms

Spars are characterised by their vertical circular cylindrical geometry. A spar can be seen as an almost submerged body. Its stability primarily originates from the centre of gravity being below the centre of buoyancy. This is achieved by having a fixed ballast concentrated at the bottom. As a result, spars have large $\overline{GM}$ value, and hence good stability. The large draft significantly reduces heave response and creates space for oil storage. However, because of its length, the overturning moments due to wind, offloading forces, and current can be significant for a spar (Barltrop, 1998). As strong currents can affect the spar, it may have to be fitted with spiral strakes to suppress vortex-induced vibration. Spars are kept on station by lateral catenary anchor lines, which may be of the semi-taut configuration as opposed to the full catenary (Downie et al., 2000). A typical spar platform can be seen in Figure 2.4.

A variation on the classic spar is the truss spar. The two types have similar overall dimensions, but the lower section of a truss spar is a truss structure. Horizontal steel plates are fitted across the truss bays to act as vertical mass traps for the water. This arrangement increases the effective vertical mass of the structure and also provides heave damping for the structure. A truss spar has less space for oil storage but it can be constructed more quickly and economically than a comparable classic spar. Moreover, it has a much lower drag area than a classic spar so that the current and associated mooring loads are reduced. A truss spar is also less affected by vortex-induced vibration because its buoyant cylindrical section is shorter compared to that of a classic spar.

Figure 2.4 Spar platform

2.1.5 Very Large Floating Structures (VLFS)

According to Dr. Riggs from the University of Hawaii at Manoa (2002), “A VLFS refers to a structure that is typically so large that it cannot be considered a rigid

body, like a ship or an offshore oil platform. ... It is too big to be considered rationally a rigid body and ... the structural dynamics, elasticity, and the flexibility of the body have to be taken into account.”

Watanabe et al. (2004a, 2004b) classified VLFSs under two broad categories, namely the pontoon- and the semi-submersible-type. The pontoon-type is a simple flat box structure and features high stability, low manufacturing cost, easy maintenance and repair, but is only suitable for use in calm waters. The semi-submersible-type, on the other hand, is capable to be located in open seas where the wave heights are relatively large.

Watanabe et al. (2004a) carried out a literature review on the pontoon-type VLFS. For a pontoon-type VLFS, elastic response is more important than rigid body motions. Thus, hydroelastic analysis takes the centre stage in the analysis of pontoon-type VLFS. As also highlighted by Shuku et al. (2001), elastic response is the key feature to pontoon-type VLFS in strength and performance. Depending on the water surface conditions of the sea area, a breakwater may need to be built to reduce the wave forces impacting the VLFS.

The semi-submersible-type of VLFS is used in the Mobile Offshore Base (MOB) concept. MOB is a self-propelled, floating, pre-positioned base that accepts cargo from aircraft and container ships and discharges resources to the shore via surface vessels and aircraft. An MOB platform ranges from a single, 300-m long, module to multiple modules serially aligned to form a runway up to 2-km long. Each module is a semi-submersible unit and can be connected or disconnected (Taylor and Palo, 2000). Wilson (2003) described a proposed concept of military Joint Mobile Offshore Base (JMOB) and highlighted that the big challenge lies in the connection between modules and the long-term station keeping during severe weather conditions.

Important research areas for VLFS include response suppression mechanisms or anti-motion devices, joining mechanisms of several floating structure modules, environmental assessment, safety and reliability analysis, effect of nonlinear waves, alternative concepts, station keeping (mooring and dynamic positioning), as well as

simplified methods of analysis and models for design. Applications of VLFS range from floating entertainment facilities, storage facilities, emergency bases, plants, terminals, airports, and MOB.

Another application of VLFS known as the eco-platform was proposed by Hosoda et al. (2000). It is a large floating platform with facilities to dispose waste and sewage from land area, to reproduce heat, electric energy, industrial resources and materials, and to purify eutrophic sea water and contaminated sea bottom sediment. The concept was proposed to give a better solution for various environmental problems associated with coastal areas.

A recently proposed concept of VLFS is a floating structure supported partially by air cushions. This concept was proposed as a possible solution to the problem associated with large wave-induced loads incurred by VLFS. Lee and Newman (2000) analysed the motions of a large rectangular barge ($1500m \times 150m \times 32m$) with one pressurized air chamber. A uniform pressure in the air chamber was assumed and the results showed that the wave-induced moments and structural loads were substantially reduced in comparison to that of a conventional hull of the same dimensions. Cheung et al. (2000) numerically modelled the dynamic response of a pneumatic floating platform composed of an array of open-bottom vertical cylinders trapping pressurized air that displaces the water. They also conducted laboratory experiments and parametric study to verify the numerical results. In their study, the compressibility and the displacement of the trapped air were modelled by a spring and float assembly. The disadvantage of this air cushion concept was that the motion response of the structure increased with the reduction of wave-induced loads on the structure. A damping mechanism was therefore needed in these structures to reduce the resonant modes to acceptable levels.

2.1.6 Summary of floating structure concepts

It has been shown that each floating structure has a range of period where critical motions occur. If the wave loading has a period within this range, the structure will undergoes excessive motions. Therefore it is necessary to reduce these motions.

Various motion reduction mechanisms will be discussed in the next section. A summary of the various types of floating structures with their critical motions, typical range of natural period of the critical motions, and typical $\overline{GM}$ values is presented in Table 2.1.

Type of floating structure	Critical motions	Natural period of motion (s)	$\overline{GM}$ (m)
Ship-shaped vessel	Roll	12-14	2-5
Semi-submersible	Heave	17-24	1.5-2.5
Tension leg platform	Surge, sway	50-100	1.5-2.5
	Yaw	40-80	
	Heave, pitch, roll	1.5-4	
Spar platform	Heave	25-30	3-10
	Pitch	45-60	
VLFS	Hydroelasticity	-	3

Table 2.1 Critical motions, natural periods, and $\overline{GM}$ of various floating structures

2.2 Motion reduction mechanisms

As mentioned in Chapter 1, the main characteristic of floating structures is their dynamic response to environmental forces. Many types of floating platforms move too much in large waves such that the drilling and on-site production operations may have to be suspended during storms (Barltrop, 1998). Faltinsen (1990) commented that accelerations determine loads on cargo and equipment, and are an important reason for seasickness. Large roll angles also create difficulties for the crew to do a proper job. In addition to interfering with activities on board, excessive motions can reduce the combat readiness of naval vessels, cause the loss of containers on cargo vessels, and force suspension of operations in some circumstances. The latter has been responsible for undesirable delays in unloading cargo offshore (Treake III et al., 2000). On the other hand, the general effectiveness, efficiency, and safety on board increase with the reduction of vessel motions (Kuo, 1986).

It is well-known that the response of a structure will be large if it undergoes periodic excitation at its natural frequencies. There are several measures available to resolve the problem of excessive response as mentioned by Rao (2004):

- a. controlling natural frequencies of the system to avoid resonance under external excitation,
- b. preventing excessive response of the system, even at resonance by introducing damping or energy-dissipating mechanism,
- c. reducing transmission of excitation forces from one part of the system to another by the use of vibration isolators,
- d. reducing response of the system, by incorporating auxiliary mass neutralizer or vibration control device.

It is important to note that the term vibration usually refers to high frequency periodic motion. Rigid body motions of floating structures, on the other hand, are low frequency oscillations. However, the abovementioned measures to reduce excessive response are also applicable for reducing periodic motions of floating structures.

There are several ways to implement the abovementioned four measures. The first measure, i.e. controlling the natural frequency of the structure, is mainly implemented during the design stage. Since natural periods are related to the geometry of the structure, water plane area to displacement ratios can be selected to minimise pitch and heave responses at predominant wave periods. Natural period is often engineered to be above the wave period range as for catenary moored semi-submersible or below as for TLP heave. Thus, designing to control the motions requires a general understanding of wave loading and its interaction with the structural geometry. Leite et al. (1992) presented the work performed for designing the hull of a production semi-submersible, starting from a simple geometry defined in a preliminary parametric study until the final shape with the minimum heave is obtained. This was accomplished by fixing the displacement of the rig and varying its geometrical parameters while preserving the stability performance and the structural integrity.

The natural frequencies of the structure can also be altered by structural modifications. Brown et al. (2004) performed experimental tests with a semi-submersible model utilising Motion Control Tanks (MCTs). These tanks were open

at their base and attached around each of the four columns of the semi-submersible. Each tank enclosed a water column to still water level and an air column above it. Each tank was segmented by vertical bulkheads to form typically four compartments with low pressure valves fitted to the upper part of each tank. The valves could be closed to increase the effective water plane area. Altering the vessel water plane by closing the valves reduced the wave induced motions in commonly occurring sea states. The reason for this was that the vertical wave force cancellations were changed because of the tanks. When the valves were closed, the MCTs gave motion reductions of up to 67% in pitch and 43% in roll. The deck acceleration were also reduced similarly. The MCTs could also protect the vessel columns from accidental flooding, reduce deck wetting in storm conditions, and improve the structural integrity of the vessel.

The second measure is to provide means of introducing viscous damping. This is done by little modification to the structure such as the installation of keels. Bilge keels are common to reduce synchronous rolling at the expense of speed (Clauss et al., 1992). The damping is due both to the resistance of the bilge keel through the water and the effect of bilge keels on the pressure distribution around the body (Faltinsen, 1990).

Downie et al. (2000) proposed the use of perforated plates as damping devices for truss spars. From experiments, they have shown that plates extending beyond the platform of the bays significantly improved the behaviour of the spar. Moreover, the perforated plates offer opportunities for fine tuning the response of the spars and for distributing the fluid loading from the plates to the truss structural members. Tao and Cai (2004) investigated the effect of a heave disk attached to the keel of a TLP column or a spar on the hydrodynamic properties of the structure. They suggested that the disk extension be at least four times the typical heave amplitude to achieve optimum damping.

For immersed long flexible cylindrical elements, such as conductors, risers, TLP tethers, and classic spar platforms, vortex excitation due to current and waves is important. When the vortex shedding frequency is close, e.g. $\pm 30\%$, to a natural

frequency of a structural element, the vortex shedding locks in to the natural frequency of the structure. This may lead to severe vibrations and high dynamic stresses critical for the strength and fatigue of the structure. Increasing the diameter of the elements or installing strakes are some options to mitigate vortex-induced vibrations (Barltrop, 1998).

The third measure is the implementation of compensation devices in the floating structure. For a drilling rig mounted on a floating platform moving in the seaway, the vertical relative motion between the stationary well and the platform can be compensated with the use of a number of hydraulic devices, such as the heave compensator, the riser tensioner, and the guideline tensioner (Clauss et al., 1992). However, there are limits to the compensation (Faltinsen, 1990). For a semi-submersible, although the motions are considerably smaller than those of a ship-shaped structure, they are still too large to permit rigid risers to be continuously connected in all weather conditions (Barltrop, 1998).

The fourth measure is implemented by the installation of mechanical devices, which add additional degree-of-freedom to the structure. As has been discussed in the beginning of this chapter, floating structures utilise buoyancy units or hulls to remain afloat. Since these buoyancy units or hulls are basically hollow, it is possible to install these mechanical devices internally, as well as externally, to reduce motions. Two examples of these devices are the anti-roll tank (ART) and the tuned mass damper (TMD). The TMD will be discussed in the next section.

ARTs have been installed on many ships. These are usually known as Tuned Liquid Dampers (TLDs) in buildings. There are two main types of ART, namely, the U-tube and the free-surface tank. There is also a hybrid of these two types of tank known as the partitioned or baffled tank. By adjusting the water level in the tanks, it is possible to change the resonance period of the free-surface tanks. Thus, this device is well suited for ships operating with a broad spectrum of metacentric height. In order for the ART to work satisfactorily, it is necessary for the natural period of the water motion in the tank to be as close as possible to that of the roll motion. The reason is that the tank moment is closely in phase with the roll velocity

and has maximum damping effect when the tank is forced to oscillate in roll at the natural period of the tank motion. The maximum moment due to the fluid motion occurs when the hydraulic jump is at the mid-point, i.e. when the roll velocity is maximum. However, a major problem with ARTs is saturation. As the ship motions increase, the water motion may hit the tank top, and this reduces the effectiveness of the system (Faltinsen, 1990).

Kano et al. (2000) tested the free-surface tanks. The damping factor for water motion in the tank used in their study could be adjusted to achieve an optimum value by changing the quantity of additional resistance. Webster et al. (2003) explored the use of artificial intelligence to control a free-surface ART. The proposed tank system was able to account for changing loading and operational conditions by constantly re-tuning the configuration of the tank. This overcame the shortcomings of the existing free-surface tanks. Treacle III et al. (2000) evaluated the use of actively controlled moving weights on board ships to reduce roll motion. The weights could simulate in principle anti-roll tank systems, or they could be considered as a possible anti-roll device in their own. It was predicted that the moving-weight system could be an excellent roll-suppressing device when the moving weight was as small as 1% of the displacement of the ship and the maximum distance the weight moves was as small as 15% of the half-beam.

It is also possible to implement combinations of the abovementioned four measures to reduce motions. The Small Water plane Area with Fin (SWAF) concept (Kano et al., 2000), for example, combined the first and second measures. This concept comprised a reduced water plane area and a bottom fin along its entire length. The good motion characteristics of SWAF were due to a combination of small water plane area and damping due to the fin. Another example was the use of lateral keels on the pontoons and blisters on the columns of a semi-submersible (Nishimoto and Leite, 1993). It was found that this combination could be advantageous especially in the conversion of drilling platforms into production units. The motion performance was improved without significant displacement increases or deterioration of stability performance. The damping tank system (SLO-ROL) concept (Kano et al., 2000) consisted of tanks placed alongside the main hull. These tanks were open to

the sea at the bottom and the port and starboard tanks were connected to air ducts. These concepts radically altered the roll performance of the hull due to reduced $\overline{GM}$. The damping effect of the SLO-ROL tanks could be changed by adjusting the orifice of the air duct. The optimized damping reduced the peak values of the roll response amplitude at resonance. The anti-motion device for VLFS proposed by Takagi et al. (2000) could be seen as a modification in design as well as a kind of motion compensation device. The device was a box-shaped body attached to one edge of the VLFS. It was shown in oblique sea that the device reduced the motion of the VLFS. It was also demonstrated that the device reduced the deformation as well as the shearing force and moment of the platform.

The various motion reduction mechanisms being used and proposed for different types of floating structures are summarised in Table 2.2. As a summary, motion reductions can be achieved in the design stage by careful geometric design to control the natural frequencies to avoid resonance. Motions can also be reduced by little modification to the structure to increase damping, such as the installation of keels or fins. Installation of devices to isolate or compensate motions is another option, which allows some parts of the structure to move while minimizes the motion of the other parts. An example to this is the use of the heave compensator. Lastly, motion reduction devices such as anti-roll tank and TMD can be installed in the main structure to reduce its motion.

2.3 Tuned Mass Damper (TMD)

A TMD is a vibration control device consisting of a spring-mass or spring-mass-damper system which is attached to the main structure. The basic theory of TMD is available in literatures, such as given by Rao (2004) and Živanović et al. (2005). In the current study, a TMD refers to a passive TMD, i.e. it does not require external power to adjust the dynamic behaviour of the structure, as opposed to an active TMD.

Adding a spring-mass system to a single-degree-of-freedom (SDOF) system forms a new 2-degree-of-freedom (2-DOF) system. When the main structure is excited at its

natural frequency, the response of the main structure will be completely eliminated if the natural frequency of the spring-mass system is the same as the excitation frequency. Adding damping to the spring-mass system makes it efficient not only at a single frequency but also over a frequency range.

Type of floating structure	Motion reduction mechanism	Suppressed motion
Ship-shaped vessel	Anti-roll tank Fin TMD Actively controlled moving weights Bilge keel	Roll
Semi-submersible	Geometric design MCT Lateral keels Blisters	Improved performance*
Tension leg platform	Heave compensator, riser tensioner, guideline tensioner	Relative vertical motion between stationary well and platform
	Heave disk Tethers design	Heave Springing, ringing
Spar platform	Geometric design Strakes	Vortex-induced vibration
	Heave disk Perforated plates	Heave
VLFS	Box-shaped body	Improved performance*

*Overall motion performance of the structure is somewhat improved

Table 2.2 Motion reduction mechanisms

The effectiveness of a TMD is frequency-dependent. A TMD has an operating range of excitation frequencies, within which the TMD will reduce the vibration. For a 2-DOF system comprising of an undamped main mass and a damped TMD, Den Hartog (1956) derived the optimum frequency and damping of the TMD, which would give the maximum reduction of the structural response at resonance, both as functions of a chosen ratio μ of the TMD and the main structure masses:

$$f_{optimum} = \frac{1}{1 + \mu} \quad (2.1)$$

$$\zeta_{optimum}^2 = \frac{3\mu}{8(1 + \mu)^3} \quad (2.2)$$

TMDs have widely been used to reduce dynamic responses of land structures, such as tall buildings and bridges, due to wind or an earthquake. Živanović et al. (2005) reported that equations (2.1) and (2.2) were still applicable for the design of a TMD where the main structure is lightly damped (the damping is below 2%). Bachman and Weber (as cited in Živanović et al., 2005) showed that a mass ratio of 2% seems to be a solution to successfully reduce the structural response and also keep the absorber mass movement within reasonable limits (Živanović et al., 2005). For the 2-DOF system comprising of an undamped main mass and a damped TMD, the respective vibration amplitudes U_1 and U_2 can be expressed as

$$\frac{U_1}{U_{st}} = \left[\frac{(2\zeta g)^2 + (g^2 - f^2)^2}{(2\zeta g)^2 (g^2 - 1 + \mu g^2)^2 + \{\mu f^2 g^2 - (g^2 - 1)(g^2 - f^2)\}^2} \right]^{\frac{1}{2}} \quad (2.3)$$

$$\frac{U_2}{U_{st}} = \left[\frac{(2\zeta g)^2 + f^4}{(2\zeta g)^2 (g^2 - 1 + \mu g^2)^2 + \{\mu f^2 g^2 - (g^2 - 1)(g^2 - f^2)\}^2} \right]^{\frac{1}{2}} \quad (2.4)$$

It is not likely, however, that equations (2.1) to (2.4) are applicable in the design of TMD for floating structure, due to the presence of hydrodynamic damping on the main structure. Therefore, in section 3.2 the damping of the main mass is taken into account in deriving the vibration amplitudes.

In the design, sufficient space should be provided to accommodate the amplitude of TMD oscillation. Rao (2004) has pointed out that the amplitude of the TMD is expected to be large, and therefore the TMD spring need to be designed from a fatigue point of view.

In the area of floating structures, there have been a few studies on TMD. Kano et al. (2000) investigated the effectiveness of TMD to reduce roll motion. Dimentberg et al. (1996) have conducted a study on TMD in the form of submerged spring-loaded flaps, where the stiffnesses of the springs were adjusted for tuning to any specific (rigid-body) mode of the platform, whereas the design of the flaps provided the desired amount of drag-induced damping. It was found that the efficiency of the TMD increases with the ratio of the TMD and the main structure masses, and it

might be of some practical importance even at a ratio as small as 1%. Korde (1999) proposed a floating wave energy device for optimum energy absorption, the basic operating principle of which is similar to that of a TMD in that it uses a spring-mass system. However, instead of attempting to reduce the floating hull-motion to zero, it seeks to achieve stationarity of a platform (block) aboard the hull. The relative motion between the oscillating hull and the stationary platform is utilised for energy absorption. It can be seen that none of these studies dealt with the use of TMD to reduce heave motion. The aim of the current research is therefore to investigate the feasibility and effectiveness of TMD in reducing the heave motion of floating structures, in particular a floating vertical circular cylinder.

2.4 Important concepts in hydrodynamics of floating bodies

2.4.1 Regimes of flow

In hydrodynamic analyses, both viscous effects and potential flow effects may be important in determining the wave-induced forces and motions of floating bodies. The relative importance of these effects varies depending on the characteristic body dimension, wavelength, and wave height, and determines which theory should be employed in the analysis. Verley (as cited in Coates, 1982) summarized the regimes of flow as follows:

$L/D < 1$	pure reflection
$L/D \leq 5$	diffraction effects predominate
$L/D < 3$	inertia increasingly dominant
$H/D = 1.7$	incipience of drag (and lift)
$H/D > 3$	drag increasingly dominant

Diffraction is the phenomenon of an incident wave pattern being modified by an obstacle. It is generally accepted that the diffraction effect becomes important when the ratio D/L is larger than 0.2. This number is often used as a limit between what is usually referred to as small bodies, or hydrodynamically transparent structures, and large bodies, or hydrodynamically compact structures (Clauss et al., 1992). For

small bodies ($D/L < 0.2$), the diffraction effects are small and therefore can be neglected. The problem can be analysed by Morison's equation, which assumes that the pressure field is undisturbed by the structure and therefore the forces can be determined based on the incident wave properties. It also assumes that the values calculated at the position of the component axis are representative of the respective cross-section of the structural element as a whole. This is an assumption which is only acceptable for $D/L < 0.2$. For large bodies ($D/L > 0.2$), the incident wave is significantly disturbed by the structure, and therefore diffraction effects must be taken into account. The problem must be analysed by potential theory.

In literatures (Mei, 1989; Patel, 2001) it is usually suggested that for the combination of large H/D and small D/L , Morison's equation is used, while for the combination of small H/D and large D/L , potential theory is used. However, the best procedure to determine the wave-induced forces for the combination of both H/D and D/L being small is less clear. Sarpkaya and Isaacson (1981) mentioned that when both KC and D/L are sufficiently small (KC is the Keulegan-Carpenter number, an alternative to H/D), neither flow separation nor wave diffraction should dominate the loading, and the force is then inertial. Patel (2001) suggested that for $D/L < 0.2$ and $KC < 5$, either potential theory or Morison's equation where drag is negligible can be used to compute the wave forces. In the current study, potential theory is employed for the analysis of the cases where both H/D and D/L are small.

2.4.2 Linear and nonlinear waves

Real sea surface is irregular and nonlinear. The sea surface is described mathematically by the kinematic free-surface boundary condition and the dynamic free-surface boundary condition. In linear theory, both the nonlinear free-surface and body boundary conditions are linearised. This is done by transferring the conditions from the instantaneous free-surface and body position to the mean free-surface and body position, respectively, by Taylor expansion and keeping the linear terms. Linear theory is valid only if the wave amplitude is small relative to the

wavelength and the characteristic body dimension. It is a simplification, since the water surface profile is actually not harmonic as in the case of linear theory. The surface profile is actually trochoidal, with shorter and steeper crests than the troughs. Nevertheless, for small wave amplitude, the difference between a trochoidal and a harmonic surface is not obvious. By employing linear theory, the velocity potential and hence the wave-induced motion and load amplitudes are proportional to the wave amplitude. A useful consequence of linear theory is that results in irregular waves can be obtained by superposition of results from regular waves of different amplitudes, wavelengths, and propagation directions. Since it is possible to obtain results in irregular seas by linearly superposing results from regular wave components, it is sufficient from a hydrodynamic point of view to analyse a structure in incident regular sinusoidal waves of small wave steepness. On the other hand, in nonlinear theory the nonlinearities of the free-surface and body boundary conditions are taken into account. Nonlinear effects are important and should be considered in severe sea states and in describing horizontal motions of moored structures. In the current research, linear theory of water waves is employed in the analysis.

2.4.3 Diffraction and radiation theory

A general introduction to the theory of diffraction and radiation of water waves by floating bodies can be found in literatures, such as given by Barltrop (1998), Chakrabarti (1987), Clauss et al. (1992), and Newman (1980). Assuming that the fluid is incompressible, inviscid, and the flow is irrotational, there exists a velocity potential satisfying Laplace equation in the fluid domain. Due to the linearity of Laplace equation, the hydrodynamics of floating structures can be analysed by separating the problem into diffraction and radiation subproblems. In the diffraction subproblem, the structure is assumed to be restrained from any oscillation in the presence of incident waves. The hydrodynamic forces are the wave excitation forces. In the radiation subproblem, the structure is assumed to oscillate with the wave excitation frequency in any degree-of-freedom, in the absence of incident waves. The hydrodynamic forces are identified as the added mass, damping, and restoring

terms. The forces obtained in diffraction and radiation subproblems give the total hydrodynamic forces.

The added mass or the hydrodynamic mass force acting on a body is obtained by integration of the pressure field arising from the relative acceleration between the body and the fluid over the wetted surface. As the added mass force results from the pressure distribution integrated over the wetted surface, the added mass depends on the shape of the body and the direction of acceleration. It also depends on the frequency of the harmonic motion of the body. The added mass is often considerably greater than the mass of water displaced by the body (Clauss et al., 1992). On the other hand, the radiation damping, is associated with the energy given away by the oscillating body (Mei et al., 2005).

In the current study, linear diffraction and radiation theory is applied. The mathematical model is derived by assuming linear wave theory, which is based on linearised free-surface and body boundary conditions.

2.5 Methods of hydrodynamic analysis of floating bodies

Hydrodynamic problems, including diffraction and radiation problems, can be analysed using various methods such as Boundary Element Method (BEM) or Boundary Integral Method (BIM), Finite Element Method (FEM), Finite Difference Method (FDM), and some analytical or semi-analytical methods (Zheng et al., 2005; Maeda, 2001).

Perhaps the most widely used method in recent years is the BEM or BIM, also known as panel method, which is based on the integral equation along the boundaries with Green functions, $G(\mathbf{x}; \boldsymbol{\xi})$. The strength of the Green function on the boundary is determined by satisfying the boundary conditions and initial conditions. The integral equation is discretised and transformed into simultaneous linear equations. The strength of the Green function only on the boundaries gives all information on the fluid velocities and pressures in the entire fluid region (Maeda, 2001).

Lee (1995) and Lee and Newman (2004) gave an overview of the abovementioned numerical technique. The fundamental basis for this method is a form of Green's theorem where the velocity potential at any point in the fluid is represented by surface distributions of singularities over the boundary surfaces. The principal numerical tasks to be performed in the panel method include:

- a. representation of geometry,
- b. representation of singularity distributions and velocity potential on body surface,
- c. numerical integration of singularity distributions (influence functions),
- d. solution of linear system,
- e. post-processing to evaluate desired hydrodynamic parameters such as integrated forces or local pressures and velocities.

There are low- and higher-order panel methods. The development of higher-order methods was motivated by the need for better efficiency and accuracy, especially in second-order problems. In higher-order methods the geometry and solution are represented by polynomials or other basis functions, such as B-splines.

There are potential and source formulations of panel method. In the potential formulation, Green's theorem is used to represent the velocity potential ϕ in the form

$$\left(\frac{2\pi}{4\pi}\right)\phi(\mathbf{x}) + \iint_{S_b} \phi G_{n_\xi} dS_\xi = \iint_{S_b} \phi_n G dS_\xi \quad (2.5)$$

where the factors 2π and 4π apply for points $\mathbf{x}$ on S_b and in the interior domain of the fluid, respectively. On the other hand, in the source formulation the velocity potential is expressed by a source distribution or single-layer distribution, in the form of

$$\phi(\mathbf{x}) = \iint_{S_b} \sigma G dS_\xi \quad (2.6)$$

where the source strength σ is unknown. Evaluating the normal derivative on S_b leads to the integral equation for σ ,

$$\phi_n(\mathbf{x}) = 2\pi\sigma(\mathbf{x}) + \iint_{S_b} \sigma G_{n_x} dS_{\xi} \quad (2.7)$$

where $\phi_n(\mathbf{x})$ is evaluated from the body boundary condition.

Numerical methods are suitable for structures of quite general geometrical form, but their computational procedure is complex. Furthermore, their efficiency and accuracy are relatively lower compared with those of analytical or semi-analytical methods. However, analytical methods are only applicable to some particular problems, with restrictions on body geometry. There are only a few geometries in the theory of water wave diffraction where exact analytical solutions have been found (Mei et al., 2005), one such geometry being the floating vertical circular cylinder. Among the various analytical and semi-analytical methods that have been applied to solve the hydrodynamics of floating structures are the matched eigenfunction expansion, multipole expansion, and multiple scattering technique (Zheng et al., 2005). In the theoretical part of the current study the eigenfunction expansion method is utilised in solving the boundary value problem. The method is mainly based on the series expansion of sinusoidal functions (Fourier series), orthogonal functions, or eigenfunctions. The problem is to determine the coefficients of the series of the complete set of orthogonal functions, which forms the simultaneous linear equations, considering the boundary or initial conditions (Maeda, 2001).

2.6 Diffraction and radiation problem of floating vertical circular cylinders

A number of authors have studied the diffraction and radiation of water waves by floating vertical circular cylinders. Earlier studies were directed towards diffraction of water waves by fixed bodies. The formulation of linear diffraction potential upon a fixed vertical circular cylinder was given by Havelock (1940). He studied the diffraction of plane water waves by a stationary obstacle with vertical sides, in relation to the ship problem. Miles and Gilbert (1968) calculated the scattering of a gravity wave by a floating vertical circular cylinder, through a variational

approximation. The fluid domain was divided into interior and exterior domains. Garrett (1971) investigated the same problem since the work by Miles and Gilbert (1968) contained errors. He used matched eigenfunction expansion approach to determine the horizontal and vertical forces and torque on the cylinder. Fenton (1978) solved for the scattered wave forces on an axisymmetric body which is diffracting water waves, using a one-dimensional integral equation which was set up from a Green's function. Sample results were given for a truncated circular cylinder fixed to the bed.

Among the earlier studies on the radiation of water waves by an oscillating body was the study by Black et al. (1971). Yeung (1981) derived a set of theoretical added masses and wave damping data for a floating circular cylinder in finite-depth water. The hydrodynamic problem was solved by matching eigenfunctions of the interior and exterior problems. Sabuncu and Calisal (1981) similarly obtained hydrodynamic coefficients for vertical circular cylinders at finite water depth for different depth-to-radius and draft-to-radius ratios. A matching technique was used to satisfy the continuity of pressure and normal velocities. Recently, Bhatta and Rahman (2003) derived the wave loadings due to scattering and radiation on a floating vertical circular cylinder in water of finite depth. The usual subdivision of the fluid region into interior and exterior regions was applied.

Extensions to the studies mentioned above have been made in the area of second-order solution as in the study by Moubayed and Williams (1995). They derived an exact, second-order solution for the hydrodynamic loads and associated wave-induced motions of a freely floating circular cylinder in bichromatic waves. They also presented the solution to the first-order problem which was determined semi-analytically by an eigenfunction expansion approach.

Other than that, studies have also been conducted on the hydrodynamics of two concentric cylinders, having its application as wave energy devices. A study of this kind was done by Berggren and Johansson (1992) who utilised the method of matched eigenfunction expansions. The study was further extended by Eidsmoen (1995). Recently, Zheng et al. (2005) analysed the full diffraction and radiation

problem including the sway and roll motions, which had not been considered by Berggren and Johansson (1992) and Eidsmoen (1995).

Various problems involving different configurations of a floating vertical circular cylinder have also been the subject of research by many authors. Linton and Evans (1992) studied the radiation and scattering of surface gravity waves by a vertical circular cylinder placed on the centreline of a channel. The cylinder was either extending from the bottom through the free surface or truncated so as to fill only part of the depth. Calisal and Sabuncu (1993) conducted a study on a discontinuous vertical cylinder, i.e. cylinder having different radii at different depth. Rahman and Bhatta (1993) studied the problem of a large vertical surface-piercing circular cylinder extending to the seabed and undergoing horizontal oscillations. Hwang (1993) utilised FEM for the calculation of hydrodynamic coefficients, wave exciting forces, and associated motion response for a floating vertical circular cylinder in water of finite depth over a moderately sloped seabed. McIver (1994) derived low-frequency asymptotics of hydrodynamic forces on fixed and floating structures by the method of matched asymptotic expansions. The method was applied on single-element structures, including axisymmetric surface-piercing structures in finite depth water. Wu et al. (2004) studied the diffraction and radiation of water waves by a cylinder over a caisson in water of finite depth. Williams et al. (2000b) investigated theoretically the interaction of water waves with a freely floating circular cylinder possessing a sidewall that is porous over a portion of its draft. Drobyshevski (2004) applied the methodology of matched asymptotic expansions to obtain the hydrodynamic properties of a truncated circular cylinder in extremely shallow water.

The diffraction and radiation of water waves by a floating vertical circular cylinder investigated in the current research is similar to that considered by Bhatta and Rahman (2003). The derivations of the solutions in the current research will mainly be based on their methodology, although in some parts the details will actually differ from theirs. The classic works by Garrett (1971) and Yeung (1981) on the diffraction and the radiation subproblem, respectively, as well as the recent work by Zheng et al. (2005) on the diffraction and radiation problem of two concentric

cylinders provide useful references for checking the correctness of the approach taken in the current research.

Chapter 3

Theoretical Formulation

3.1 Hydrostatic stability

The positioning of the TMD in the cylinder needs to take into consideration the hydrostatic stability of the whole structure. The measure of stability is the metacentric height $\overline{GM}$. It is assumed that rotation is small such that the water plane remains constant. The $\overline{GM}$ can then be approximated by (see Figure 3.1)

$$\begin{aligned}\overline{GM} &= (\overline{KB} + \overline{BM} - \overline{KG}) \\ &= (\overline{KB} + \frac{I_A}{\nabla} - \overline{KG})\end{aligned}\quad (3.1)$$

where I_A is the second moment of area of the water plane and ∇ is the displaced volume. If the TMD mass and the ballast are taken as constants, the displacement is constant. Thus, the only variable in equation (3.1) is $\overline{KG}$, i.e. the position of the centre of gravity of the cylinder, which depends on the position of the TMD mass, as it oscillates up and down. The minimum and maximum $\overline{GM}$ are obtained when the TMD mass is at its highest and lowest position, respectively. Since the TMD mass is small relative to the main structure, the variation of $\overline{GM}$ is also small. To maintain a large $\overline{GM}$, the position of the centre of gravity need to be as low as possible. Therefore, in the design, the ballast is put at the bottom of the cylinder. The equilibrium position of the TMD mass also need to be as low as possible.

Figure 3.1 Hydrostatic stability of floating vertical circular cylinder

3.2 Preliminary study on simplified 2-DOF model

As a preliminary study, the dynamics of the floating vertical circular cylinder with TMD is analysed by simplifying the problem into a 2-DOF model. Several assumptions are necessary:

- (1) Only a single mode of motion (heave) is considered. The coupling of other modes is neglected.
- (2) The TMD mass moves in vertical direction only.
- (3) The added mass, the radiation damping, and the amplitude of the wave excitation force are constant (independent of frequency).
- (4) The hydrodynamic restoring and damping forces are modelled as linear spring and viscous damper, respectively.

Based on these assumptions, the dynamic behaviour of the floating cylinder can be approximated by a 2-DOF system as shown in Figure 3.2. The TMD is positioned inside the main structure. This kind of 2-DOF system, where both masses are damped, was treated briefly by Weaver et al. (1990). The problem is treated more thoroughly here.

From Figure 3.2, the equations of motion for m_1 and m_2 can be stated as follows:

$$F(t) + k_1(-u_1) + k_2(u_2 - u_1) + c_1(-\dot{u}_1) + c_2(\dot{u}_2 - \dot{u}_1) = m_1\ddot{u}_1 \quad (3.2)$$

$$k_2(u_1 - u_2) + c_2(\dot{u}_1 - \dot{u}_2) = m_2\ddot{u}_2 \quad (3.3)$$

Figure 3.2 2-DOF system

In matrix notations, the equations can be written as

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} \quad (3.4)$$

where

$$\mathbf{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad (3.5)$$

$$\mathbf{C} = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{pmatrix} \quad (3.6)$$

$$\mathbf{K} = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix} \quad (3.7)$$

$$\mathbf{U} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (3.8)$$

$$\mathbf{F} = \begin{pmatrix} F(t) \\ 0 \end{pmatrix} \quad (3.9)$$

3.2.1 Free vibration

For free vibration, $F(t)$ is set to zero. The solutions are in the general form

$$\mathbf{U} = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} e^{st} \quad (3.10)$$

Substituting (3.10) and its derivatives into equation (3.4),

$$(s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}) \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3.11)$$

For nontrivial solutions,

$$|s^2\mathbf{M} + s\mathbf{C} + \mathbf{K}| = 0 \quad (3.12)$$

Hence, the characteristic equation is

$$s^4 m_1 m_2 + s^3 (m_1 c_2 + m_2 (c_1 + c_2)) + s^2 (m_1 k_2 + c_1 c_2 + m_2 (k_1 + k_2)) + s (c_1 k_2 + c_2 k_1) + k_1 k_2 = 0 \quad (3.13)$$

By introducing the following notations,

$$\mu = m_2 / m_1 \quad (\text{mass ratio})$$

$$\omega_a^2 = k_2 / m_2 \quad (\text{natural frequency of TMD})$$

$$\omega_n^2 = k_1 / m_1 \quad (\text{natural frequency of main system})$$

$$f = \omega_a / \omega_n \quad (\text{natural frequency ratio})$$

$$c_{c1} = 2m_1 \omega_n \quad (\text{critical damping of main mass})$$

$$c_{c2} = 2m_2 \omega_n \quad (\text{critical damping of TMD})$$

equation (3.13) can be transformed into

$$s^4 + 2\omega_n \left(\frac{c_2}{c_{c2}} (1 + \mu) + \frac{c_1}{c_{c1}} \right) s^3 + \omega_n^2 \left(f^2 (1 + \mu) + 4 \frac{c_1}{c_{c1}} \frac{c_2}{c_{c2}} + 1 \right) s^2 + 2\omega_n^3 \left(\frac{c_1}{c_{c1}} f^2 + \frac{c_2}{c_{c2}} \right) s + f^2 \omega_n^4 = 0 \quad (3.14)$$

Since all of the coefficients in (3.14) are positive, nonzero roots of the fourth degree polynomial may be either real and negative or complex with negative real parts. Three sets of roots are possible, namely, four complex roots with negative real parts, four negative real roots, and two negative real roots and a complex conjugate pair (Fertis, 1995). For small damping, the four roots are complex with negative real parts, and they can be written as follows:

$$\begin{aligned} s_1 &= -a_1 + ib_1 \\ s_2 &= -a_1 - ib_1 \\ s_3 &= -a_2 + ib_2 \\ s_4 &= -a_2 - ib_2 \end{aligned} \quad (3.15)$$

In the above four equations, the quantities a_1, a_2, b_1, b_2 are positive; b_1 and b_2 provide the values of the two frequencies of vibration of the 2-DOF system. By means of numerical methods the four complex roots can be solved, and hence the two frequencies of vibration of the 2-DOF system. As an example, for the definite system: $f = 1, \mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1, \omega_n = 1$, the values $b_1 = 1.1$ and $b_2 = 0.9$ can be obtained.

3.2.2 Forced vibration

The behaviour of the 2-DOF system under harmonic excitation force is analysed following the method of Den Hartog (1956) for the analysis of damped vibration absorber, the difference being now that the main mass is also damped. All oscillatory quantities can be expressed in a complex notation, since a time-harmonic dependence applies. The harmonic force $F(t)$ can then be expressed as $\text{Re}\{\bar{F}_0 e^{i\omega t}\}$. Assuming that the solution u is in the form of $\text{Re}\{\bar{U} e^{i\omega t}\}$ and substituting into equation (3.4),

$$(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}) \begin{pmatrix} \bar{U}_1 \\ \bar{U}_2 \end{pmatrix} = \begin{pmatrix} \bar{F}_0 \\ 0 \end{pmatrix} \quad (3.16)$$

Starting from equation (3.16) and expanding,

$$\begin{aligned} -m_1 \omega^2 \bar{U}_1 + k_1 \bar{U}_1 + i\omega c_1 \bar{U}_1 + k_2 (\bar{U}_1 - \bar{U}_2) + i\omega c_2 (\bar{U}_1 - \bar{U}_2) &= \bar{F}_0 \\ -m_2 \omega^2 \bar{U}_2 + k_2 (\bar{U}_2 - \bar{U}_1) + i\omega c_2 (\bar{U}_2 - \bar{U}_1) &= 0 \end{aligned} \quad (3.17)$$

Bringing the terms with $\bar{U}_1$ and $\bar{U}_2$ together,

$$\begin{aligned} (-m_1 \omega^2 + k_1 + i\omega c_1 + k_2 + i\omega c_2) \bar{U}_1 - (k_2 + i\omega c_2) \bar{U}_2 &= \bar{F}_0 \\ -(k_2 + i\omega c_2) \bar{U}_1 + (-m_2 \omega^2 + k_2 + i\omega c_2) \bar{U}_2 &= 0 \end{aligned} \quad (3.18)$$

Expressing $\bar{U}_2$ in terms of $\bar{U}_1$ by means of the second equation of (3.18),

$$\bar{U}_2 = \frac{k_2 + i\omega c_2}{-m_2 \omega^2 + k_2 + i\omega c_2} \bar{U}_1 \quad (3.19)$$

and substituting into the first one give

$$\bar{U}_1 = \bar{F}_0 \frac{A + iB}{C + iD} \quad (3.20)$$

where

$$\begin{aligned} A &= k_2 - m_2 \omega^2 \\ B &= \omega c_2 \\ C &= (-m_1 \omega^2 + k_1)(-m_2 \omega^2 + k_2) - m_2 \omega^2 k_2 - \omega^2 c_1 c_2 \\ D &= \omega [c_2(-m_1 \omega^2 + k_1 - m_2 \omega^2) - c_1 m_2 \omega^2 + c_1 k_2] \end{aligned}$$

Equation (3.20) can be transformed as follows:

$$\begin{aligned} \bar{U}_1 &= \bar{F}_0 \frac{(A + iB)(C - iD)}{(C + iD)(C - iD)} \\ &= \bar{F}_0 \frac{(AC + BD) + i(BC - AD)}{C^2 + D^2} \end{aligned} \quad (3.21)$$

The modulus of $\bar{U}_1$ is the displacement amplitude of the main mass.

$$\begin{aligned} \frac{U_1}{F_0} &= \sqrt{\left(\frac{AC + BD}{C^2 + D^2}\right)^2 + \left(\frac{BC - AD}{C^2 + D^2}\right)^2} \\ &= \sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \end{aligned} \quad (3.22)$$

In equation (3.22), U_1 is a function of eight variables: $F_0, \omega, c_1, c_2, k_1, k_2, m_1, m_2$. It is useful to write the equation in a dimensionless form, for which purpose the notations introduced in section 3.2.1 and the following notations are used:

$$g = \omega / \omega_n \quad (\text{forced frequency ratio})$$

$$U_{st} = F_0 / k_1 \quad (\text{static deflection of main mass})$$

After performing some algebra, equation (3.22) is transformed into

$$\frac{U_1}{U_{st}} = \sqrt{\frac{E^2 + F^2}{G^2 + H^2}} \quad (3.23)$$

where

$$\begin{aligned} E &= 2 \frac{c_2}{c_{c2}} g \\ F &= g^2 - f^2 \\ G &= (2g) \left[\frac{c_2}{c_{c2}} (g^2 - 1 + \mu g^2) + \frac{c_1}{c_{c1}} (g^2 - f^2) \right] \\ H &= \mu f^2 g^2 - (g^2 - 1)(g^2 - f^2) + 4 \frac{c_1}{c_{c1}} \frac{c_2}{c_{c2}} g^2 \end{aligned}$$

Equation (3.23) is the amplitude ratio U_1/U_{st} of the main mass as a function of five essential variables: $\mu, c_1/c_{c1}, c_2/c_{c2}, f, g$. Figure 3.3 shows a plot of U_1/U_{st} as a function of the forced frequency ratio g for the definite system: $f = 1, \mu = 1/20, c_1/c_{c1} = 0.1$ and for various values of TMD damping c_2/c_{c2} . In other words, the figure describes the behaviour of a system in which the main mass is 20 times as great as the TMD mass, the frequency of the TMD is equal to that of the main mass, and with a main mass damping ratio of 0.1.

Figure 3.3 U_1/U_{st} for various c_2/c_{c2} ($f = 1, \mu = 1/20, c_1/c_{c1} = 0.1$)

The work done by the damping force is given by the force times the displacement through which it operates. Here the displacement is equal to the relative motion between the two masses or the extension of the damper spring. There is a damping for which the product of damping force and displacement becomes a maximum, and so the resonant amplitude will be small. For the case where the main mass is undamped, a remarkable peculiarity is observed, i.e. all curves intersect at the same two points, regardless of the value of the TMD damping (Den Hartog, 1956). However, from Figure 3.3 it is observed that unlike the 2-DOF case with undamped main mass, in the current model, where both masses are damped, all curves do not pass through the same two points. Therefore the method used by Den Hartog (1956) cannot be utilised to derive the formulas for optimum TMD frequency and damping. But as seen from Figure 3.4, by changing the natural frequency ratio $f = \omega_a / \omega_n$ of the TMD with respect to the main system, the two peaks can be shifted up and down the curve (see for example the curves of $f = 0.9$ and $f = 1$ in Figure 3.4). It appears that the most favorable case is such that by a proper choice of f and c_2/c_{c2} the two peaks are adjusted to equal heights and the curve is adjusted to pass with a horizontal tangent through the peaks. It seems just that these values of f and c_2/c_{c2} can no longer be derived using exact formulas.

Figure 3.4 U_1/U_{st} for various f ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$)

Another observation is that the g values at the peaks of the curves correspond to the frequencies of free vibration of the 2-DOF system. For example, from Figure 3.4, it can be seen that for $f = 1$ the values of g at the two peaks are $g_1 = 1.1$ and $g_2 = 0.9$, which correspond to the values of b_1 and b_2 obtained in section 3.2.1.

3.2.3 Effectiveness of TMD

To assess the effectiveness of the TMD, the amplitude ratio U_1/U_{st} of the main mass without TMD (SDOF system) is first derived. The effectiveness of the TMD is represented by the response reduction factor (RRF), which is defined as the ratio of U_1/U_{st} with TMD over U_1/U_{st} without TMD.

The equation of motion of the main mass without TMD is

$$F(t) - k_1 u_1 - c_1 \dot{u}_1 = m_1 \ddot{u}_1 \quad (3.24)$$

Considering harmonic excitation force and solution as before,

$$(-m_1 \omega^2 + k_1 + i \omega c_1) \bar{U}_1 = \bar{F}_0 \quad (3.25)$$

Hence,

$$\frac{U_1}{F_0} = \sqrt{\frac{1}{(-m_1 \omega^2 + k_1)^2 + \omega^2 c_1^2}} \quad (3.26)$$

Writing in dimensionless form as before, the above equation is transformed into

$$\frac{U_1}{U_{st}} = \sqrt{\frac{1}{(1 - g^2)^2 + \left(2 \frac{c_1}{c_{c1}} g\right)^2}} \quad (3.27)$$

It is observed that when either the TMD stiffness and damping are zero ($k_2 = c_2 = 0$) or the TMD mass is zero ($m = 0$), the 2-DOF system revert to an SDOF system without TMD. This can be shown by substituting $k_2 = c_2 = 0$ into

equation (3.23) to yield equation (3.27). Similarly, starting from equation (3.23) and substituting $m = 0$, which means $\mu = 0$ and $c_2/c_{c2} = \infty$, equation (3.27) is obtained.

The response reduction factor is thus equal to equation (3.23) divided by equation (3.27).

$$RRF = \sqrt{\frac{(E^2 + F^2) \left[(1 - g^2)^2 + \left(2 \frac{c_1}{c_{c1}} g \right)^2 \right]}{G^2 + H^2}} \quad (3.28)$$

with E, F, G, H defined as in section 3.2.2.

Figure 3.5 shows plots of U_1/U_{st} without TMD, U_1/U_{st} with TMD, and RRF as functions of the forced frequency ratio g for the definite system: $f = 1, \mu = 1/20, c_1/c_{c1} = 0.1$ and for various values of TMD damping c_2/c_{c2} . Figure 3.6 shows plots of U_1/U_{st} without TMD, U_1/U_{st} with TMD, and RRF as functions of the forced frequency ratio g for the definite system: $\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$ and for various values of natural frequency ratio f . For the definite system: $\mu = 1/20, c_1/c_{c1} = 0.1$ it can be observed from the two figures that $c_2/c_{c2} \cong 0.1$ and $f \cong 0.9$ give an optimum RRF throughout the range of excitation frequencies, with RRF as low as 0.5 in the vicinity of the wave excitation frequency. In general, for a given mass ratio, by proper selection of spring stiffness and TMD damping, optimum response reduction can be achieved. For a selected mass ratio μ as small as 0.05, it is shown that TMD is capable of reducing the heave motion of the structure up to half as low as that of the structure without TMD.

Figure 3.5 RRF for various c_2/c_{c2} ($f = 1, \mu = 1/20, c_1/c_{c1} = 0.1$)

Figure 3.6 RRF for various f ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$)

Extending our analysis further, it is found that when either the TMD damping or stiffness is infinite ($c_2 = \infty$ or $k_2 = \infty$), the system revert to an SDOF system where the mass is equal to the main mass in the case of 2-DOF system plus the TMD mass. In other words, the two masses are now lumped together. This can be shown as follows. Starting from equation (3.23) and substituting $c_2 = \infty$,

$$\frac{U_1}{U_{st}} = \sqrt{\frac{1}{(g^2 - 1 + \mu g^2)^2 + \left(2 \frac{c_1}{c_{c1}} g\right)^2}} \quad (3.29)$$

Similarly, starting from equation (3.23) and substituting $k_2 = \infty$, which means that $f = \infty$, equation (3.29) is obtained. When the main mass of the 1-DOF system is equal to $(1 + \mu)m_1$, equation (3.26) becomes

$$\frac{U_1}{F_0} = \sqrt{\frac{1}{(-(1 + \mu)m_1 \omega^2 + k_1)^2 + \omega^2 c_1^2}} \quad (3.30)$$

Nondimensionalising using the same notations as before, equation (3.29) is again obtained.

3.2.4 Amplitude ratio of TMD over main mass

From equation (3.19), the complex displacement ratio of the TMD over the main mass is

$$\frac{\bar{U}_2}{\bar{U}_1} = \frac{k_2 + i\omega c_2}{-m_2 \omega^2 + k_2 + i\omega c_2} \quad (3.31)$$

Hence, the amplitude ratio of TMD over main mass is

$$\frac{U_2}{U_1} = \sqrt{\frac{k_2^2 + \omega^2 c_2^2}{(-m_2 \omega^2 + k_2)^2 + \omega^2 c_2^2}} \quad (3.32)$$

which can be written in nondimensional form as

$$\frac{U_2}{U_1} = \sqrt{\frac{f^4 + \left(2 \frac{c_2}{c_{c2}} g\right)^2}{(f^2 - g^2)^2 + \left(2 \frac{c_2}{c_{c2}} g\right)^2}} \quad (3.33)$$

From equation (3.33) it is found that U_2/U_1 is a function of three essential variables: $c_2/c_{c2}, f, g$. It does not depend on μ and c_1/c_{c1} . Figure 3.7 shows a plot of U_2/U_1 as a function of the forced frequency ratio g for $f=1$, and for various values of TMD damping c_2/c_{c2} . Figure 3.8 shows plots of U_2/U_1 as a function of the forced frequency ratio g for $c_2/c_{c2} = 0.1$, and for various values of natural frequency ratio f .

Figure 3.7 U_2/U_1 for various c_2/c_{c2} ($f=1$)

Figure 3.8 U_2/U_1 for various f ($c_2/c_{c2} = 0.1$)

It can be seen from Figure 3.7 that the peak of U_2/U_1 increases as TMD damping c_2/c_{c2} reduces. When $c_2/c_{c2} = 0$, the peak of U_2/U_1 is equal to infinity, whereas when $c_2/c_{c2} = \infty$, U_2/U_1 is equal to 1, regardless of g . The first can be explained by observing in Figure 3.3 that U_1/U_{st} is zero at $g = f$, and therefore U_2/U_1 is infinite. The latter can be explained by recalling that when $c_2/c_{c2} = \infty$, the two masses are lumped together so that their relative displacement is zero, and therefore U_2/U_1 will be 1 for all g . It is also observed that for a given f , all curves intersect at the same two points, independent of TMD damping. The first point is at $g = 0$, and it can be shown that the second point is at $g = \sqrt{2}f$. The ordinate at these two points is 1. From Figure 3.8, it is observed that the peak of U_2/U_1 decreases and moves to the left as f decreases. When $f = \infty$, the peak is actually at $g = \infty$, but as shown previously, this practically means that the TMD and the main mass are lumped together, such that their relative displacement is zero, and therefore $U_2/U_1 = 1$ for all g .

3.2.5 Effectiveness of TMD for extreme g

So far our attention has been limited to the behaviour of the system at the vicinity of the heave natural frequency of the structure, i.e. at the vicinity of $g = 1$. Since the heave natural frequency of the floating vertical cylinder depends on its geometry, and since the wave excitation frequency varies, it may not necessarily be the case that the wave excitation frequency falls at the vicinity of the heave natural frequency. Therefore, it is necessary to consider the effectiveness of the TMD for extreme g , i.e. for wave excitation frequencies which do not lie at the vicinity of the heave natural frequency of the structure.

For this purpose, equation (3.30), which gives the RRF of the TMD, is used. Figure 3.9 shows the plots of RRF as a function of the forced frequency ratio g for various natural frequency ratio f , for the definite system: $\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$. Figure 3.10 shows plots of RRF as a function of the forced frequency ratio g for various natural frequency ratio f , for the definite system: $\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.02$. From the two figures it is observed that for extreme wave excitation frequencies it is more effective to design the natural frequency of the TMD to be close to the excitation frequency. For $g > 1$, a value of f slightly greater than g gives an optimum response reduction, while for $g < 1$, a value of f slightly less than g gives an optimum response reduction. The range of g where $RRF < 1$ is broader for larger TMD damping. The RRF curve is also flatter for larger TMD damping. Once g happens to be larger (less) than the designed TMD natural frequency f for $g > 1$ ($g < 1$), the lower the TMD damping the more detrimental effect it has on the motion response of the structure. However, the lower the TMD damping the better response reduction it gives at values of g close to f .

Figure 3.9 RRF for various g ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.1$)

Figure 3.10 RRF for various g ($\mu = 1/20, c_1/c_{c1} = 0.1, c_2/c_{c2} = 0.02$)

3.3 Diffraction and radiation problem

Many assumptions are made in simplifying the diffraction and radiation problem into a 2-DOF model. In order to closely model the actual phenomena, the coupling with other modes of motion and, as mentioned in section 2.4.3, the frequency dependency of the added mass, radiation damping, and the amplitude of the wave excitation force need to be accounted for in the theoretical model. Therefore, in order to take into consideration the dependency of those hydrodynamic forces on frequency, potential theory is needed.

3.3.1 Boundary value problem

The diffraction and radiation problem to be solved in the current study is shown in Figure 3.11. The floating vertical circular cylinder is considered rigid, and only wave loads are taken into account; wind and current loads are neglected. The cylindrical coordinate system $\mathbf{x} = (r, \theta, z)$ is adopted, with $z = 0$ representing the plane of the undisturbed free surface and $z < 0$ the fluid domain. The origin of the coordinate system is the intersection of the cylinder axis and $z = 0$. The incident wave is a simple harmonic wave propagating in the positive x -direction. The surface tension of the water is neglected and the atmospheric pressure at the water surface is uniform. It is assumed that the fluid is homogeneous, incompressible, inviscid, and the flow is irrotational. Therefore, the fluid velocity at time t can be represented by the gradient of the velocity potential $\Phi(\mathbf{x}, t)$ satisfying the Laplace equation $\nabla^2 \Phi = 0$ in the fluid domain. The detailed derivation of the Laplace equation as the governing equation for the problem is available in literatures. Since the incident wave is a simple harmonic wave, the velocity potential is expected to be harmonic in t , and it can be expressed as

$$\Phi(\mathbf{x}, t) = \text{Re} \{ \phi(\mathbf{x}) e^{-i\omega t} \} \quad (3.34)$$

where ϕ is the complex time independent velocity potential.

Figure 3.11 Problem definition

The vertical component of the fluid velocity is zero at the bottom boundary surface $z = -h$, which is assumed to be horizontal everywhere, smooth, and impermeable. The submerged surface S_b of the structure is assumed to be impermeable, with its normal velocity equal to the normal component of the fluid velocity. The wave amplitude and motions relative to the wavelength are assumed to be small. Hence, the free-surface boundary condition can be linearised about $z = 0$, and the body boundary condition can be linearised about its mean position. As discussed in section 2.4.3, since Laplace equation is linear and homogeneous, the principle of superposition applies on the velocity potential,

$$\begin{aligned}\phi &= \phi_D + \phi_R \\ &= \phi_I + \phi_S + \phi_R\end{aligned}\tag{3.35}$$

where ϕ_D is the complex diffraction potential, ϕ_R is the complex radiation potential, ϕ_I is the complex incident wave potential, and ϕ_S is the complex scattered potential. Except for the incident wave potential ϕ_I , all components of the potential satisfy the radiation condition of outgoing waves in the far field. Brebbia and Walker (1979) derived the formulation of the radiation condition as

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial \phi}{\partial r} - ik\phi \right) = 0\tag{3.36}$$

3.3.2 Derivation of incident wave potential

Derivations of the incident wave potential are given in literatures (Lamb, 1932; Wehausen and Laitone, 1960; Newman, 1980; Sarpkaya and Isaacson, 1981; Dean and Dalrymple, 1991). The derivation of the incident wave potential is presented below, combining approaches of different authors with some modifications.

The incident wave propagates in the positive x -direction. It is assumed that the wave contour does not change in the y -direction. Therefore the problem can be described in terms of two-dimensional Laplace equation:

$$\frac{\partial^2 \Phi_I}{\partial x^2} + \frac{\partial^2 \Phi_I}{\partial z^2} = 0, \text{ in the fluid domain} \quad (3.37)$$

with bottom boundary condition:

$$\frac{\partial \Phi_I}{\partial z} = 0, \text{ at } z = -h \quad (3.38)$$

free-surface boundary conditions:

$$\frac{\partial \Phi_I}{\partial z} - \frac{\partial \eta}{\partial t} = 0, \text{ at } z = 0 \quad (3.39)$$

$$\eta = -\frac{1}{g} \frac{\partial \Phi_I}{\partial t}, \text{ at } z = 0 \quad (3.40)$$

which may be combined to give

$$\frac{\partial^2 \Phi_I}{\partial t^2} + g \frac{\partial \Phi_I}{\partial z} = 0, \text{ at } z = 0 \quad (3.41)$$

and periodicity condition:

$$\Phi_I(x, z, t) = \Phi_I(x - ct, z) \quad (3.42)$$

which describes a progressive wave with celerity c . By separation of variables, the velocity potential can be expressed as

$$\Phi_I(x, z, t) = Y(x - ct)Z(z) \quad (3.43)$$

Substituting into the Laplace equation (3.37) gives

$$Y''Z + YZ'' = 0 \quad (3.44)$$

where the primes denote differentiation with respect to the argument. From (3.44) two equations are obtained:

$$Y'' - \lambda Y = 0 \quad (3.45)$$

$$Z'' + \lambda Z = 0 \quad (3.46)$$

Three cases of λ are considered. If $\lambda = 0$, there is no periodicity in Y , therefore $\lambda \neq 0$. If $\lambda > 0$, $Y = C_1 \cosh \sqrt{\lambda}(x - ct) + C_2 \sinh \sqrt{\lambda}(x - ct)$ and again there is no periodicity in Y , therefore $\lambda < 0$. If $\lambda = -k^2 < 0$,

$$Y = C_1 \cos k(x - ct) + C_2 \sin k(x - ct) \quad (3.47)$$

$$Z = C_3 \cosh kz + C_4 \sinh kz \quad (3.48)$$

Substituting (3.38) into (3.48),

$$C_4 = C_3 \tanh kh \quad (3.49)$$

Therefore,

$$Z = C_3 \frac{\cosh k(z + h)}{\cosh kh} \quad (3.50)$$

and

$$\Phi_I = C_3 (C_1 \cos k(x - ct) + C_2 \sin k(x - ct)) \frac{\cosh k(z + h)}{\cosh kh} \quad (3.51)$$

Equation (3.51) can be written as

$$\Phi_I = \text{Re} \left\{ \frac{\cosh k(z + h)}{\cosh kh} C e^{ik(x - ct)} \right\} \quad (3.52)$$

where $C = C_3(C_1 - iC_2)$.

From (3.40),

$$\eta = \text{Re}\{De^{ik(x-ct)}\} \quad (3.53)$$

where $D = i \frac{Ckc}{g}$, which can be written as $D = \frac{Ekc}{g} e^{i\varepsilon}$, where E is real.

Putting the expression of D back into (3.53),

$$\eta = \text{Re}\left\{\frac{Ekc}{g} e^{i(k(x-ct)+\varepsilon)}\right\} \quad (3.54)$$

It can be seen that the simplest solution is that the surface elevation is harmonic in the x -direction with a wavelength $L = 2\pi/k$ and in time with a wave period $T = 2\pi/kc$, which is our condition in 3.3.1, i.e. that the incident wave is a simple harmonic wave propagating in the positive x -direction (the most general case can be derived from this solution by superposition, as discussed in section 2.4.2). The constant k is the wave number, while $kc = \omega$, the wave angular frequency. The phase angle ε is arbitrary and can be set to zero by a suitable choice of origin $x = 0$. The wave amplitude is obtained as $A = E\omega/g$.

Therefore, the incident wave potential is

$$\Phi_I = \text{Re}\left\{\frac{-igA}{\omega} \frac{\cosh k(z+h)}{\cosh kh} e^{i(kx-\omega t)}\right\} \quad (3.55)$$

where, following the notation in (3.34), the complex incident wave potential is

$$\phi_I = \frac{-igA}{\omega} \frac{\cosh k(z+h)}{\cosh kh} e^{ikx} \quad (3.56)$$

Lastly, substituting (3.54) and (3.55) into (3.39), or (3.55) into (3.41), gives

$$\omega^2 = gk \tanh kh \quad (3.57)$$

Equation (3.57) is called the linear dispersion relation, which determines the wavelength for given values of water depth h and frequency ω .

3.3.3 Derivation of diffraction potential

The diffraction potential satisfies Laplace equation in cylindrical coordinate system,

$$\nabla^2 \phi_D = \frac{\partial^2 \phi_D}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_D}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_D}{\partial \theta^2} + \frac{\partial^2 \phi_D}{\partial z^2} = 0, \text{ in the fluid domain} \quad (3.58)$$

By separation of variables, a product solution is assumed:

$$\phi_D(r, \theta, z) = \sum_{m=0}^{\infty} Z(z) R(r) \cos m\theta \quad (3.59)$$

Substituting into (3.58) gives

$$\frac{R''(r)}{R(r)} + \frac{1}{r} \frac{R'(r)}{R(r)} - \frac{m^2}{r^2} + k^2 = 0 \quad (3.60)$$

$$Z''(z) - k^2 Z(z) = 0 \quad (3.61)$$

where the primes denote differentiation with respect to the argument.

A physically acceptable solution is obtained if $k^2 > 0$ or $k^2 < 0$. If $k^2 > 0$, then

$$Z = C_1 \cosh kz + C_2 \sinh kz \quad (3.62)$$

If $k^2 < 0$, then

$$Z = C_3 \cos kz + C_4 \sin kz \quad (3.63)$$

The diffraction potential can be decomposed into interior and exterior solutions. The interior region is defined in $-h \leq z \leq -d$ and $0 \leq r \leq R$, while the exterior region is defined in $-h \leq z \leq 0$ and $r \geq R$. For the interior region, the following boundary conditions apply:

$$\frac{\partial \phi_D^i}{\partial z} = 0, \text{ at } z = -d \text{ and } 0 \leq r \leq R \quad (3.64)$$

$$\frac{\partial \phi_D^i}{\partial z} = 0, \text{ at } z = -h \text{ and } 0 \leq r \leq R \quad (3.65)$$

Applying the bottom boundary condition (3.65), the following is obtained from (3.62):

$$C_2 = C_1 \tanh kh \quad (3.66)$$

Therefore,

$$Z = C_1 \frac{\cosh k(z+h)}{\cosh kh} \quad (3.67)$$

Applying the condition at the cylinder bottom (3.64), the following is obtained from (3.67):

$$k_0 = 0 \quad (3.68)$$

Similarly, applying the bottom boundary condition (3.65), the following is obtained from (3.63):

$$Z = C_3 \frac{\cos k(z+h)}{\cos kh} \quad (3.69)$$

Applying (3.64), the following is obtained from (3.69):

$$k_n = \frac{n\pi}{h-d}, \quad n = 0, 1, 2, \dots \quad (3.70)$$

Equation (3.60) is a Bessel equation of order m with the parameter k . For $k^2 = 0$, this equation becomes an Euler-Cauchy differential equation. For a physically acceptable solution, it must be bounded at $r = 0$, therefore,

$$R(r) = C_5 r^m \quad (3.71)$$

For $k^2 < 0$, a physically acceptable solution for the Bessel equation (3.60) which requires boundedness at $r = 0$ is the modified Bessel function of the first kind I_m . Therefore, a general solution for the interior region is

$$\phi_D^i = \sum_{m=0}^{\infty} \left(\frac{\beta_{0m}}{2} \left(\frac{r}{R} \right)^m + \sum_{n=1}^{\infty} \beta_{nm} \frac{I_m(k_n r)}{I_m(k_n R)} \cos k_n (z + h) \right) \cos m\theta \quad (3.72)$$

Defining

$$\zeta_m^i = \frac{\beta_{0m}}{2} \left(\frac{r}{R} \right)^m + \sum_{n=1}^{\infty} \beta_{nm} \frac{I_m(k_n r)}{I_m(k_n R)} \cos k_n (z + h) \quad (3.73)$$

at $r = R$, the expression becomes a half-range Fourier cosine series expansion defined in $-h \leq z \leq -d$. The Fourier coefficients are obtained from

$$\beta_{nm} = \frac{2}{h-d} \int_{-h}^{-d} \zeta_m^i(R, z) \cos k_n (z + h) dz, \quad n = 0, 1, 2, \dots \quad (3.74)$$

For the exterior region, the following boundary conditions apply:

$$-\omega^2 \phi_D^e + g \frac{\partial \phi_D^e}{\partial z} = 0, \text{ at } z = 0 \text{ and } r \geq R \quad (3.75)$$

$$\frac{\partial \phi_D^e}{\partial z} = 0, \text{ at } z = -h \text{ and } r \geq R \quad (3.76)$$

$$\frac{\partial \phi_D^e}{\partial r} = 0, \text{ at } r = R \text{ and } -d \leq z \leq 0 \quad (3.77)$$

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial \phi_s}{\partial r} - ik \phi_s \right) = 0 \quad (3.78)$$

The incident wave potential has been solved in section 3.3.2. What remains is solving for the scattered potential. Applying the bottom boundary condition (3.76), equation (3.67) is obtained from (3.62), and equation (3.69) from (3.63).

Applying the free-surface boundary condition (3.75), the linear dispersion relation (3.57) is obtained from (3.67). The linear dispersion relation is written as

$$\omega^2 = g\lambda_0 \tanh \lambda_0 h \quad (3.79)$$

This equation has only one eigenfunction (3.67) corresponding to the eigenvalue λ_0 . From the orthonormal property of eigenfunctions, equation (3.67) can be expressed as

$$Z_0(z) = N_0^{-1/2} \cosh \lambda_0(z+h) \quad (3.80)$$

where

$$N_0 = \frac{h}{2} \left(1 + \frac{\sinh 2\lambda_0 h}{2\lambda_0 h} \right) \quad (3.81)$$

Applying the free-surface boundary condition (3.75), the following is obtained from (3.69):

$$\omega^2 = -gk \tan kh \quad (3.82)$$

This equation has an infinite number of eigenvalues k . Equation (3.82) is written as

$$\omega^2 = -g\lambda_j \tan \lambda_j h, \quad j = 1, 2, \dots \quad (3.83)$$

Corresponding to the eigenvalue λ_j , there is an eigenfunction (3.69), which, from the orthonormal property of eigenfunctions, can be expressed as

$$Z_j(z) = N_j^{-1/2} \cos \lambda_j(z+h), \quad j = 1, 2, \dots \quad (3.84)$$

where

$$N_j = \frac{h}{2} \left(1 + \frac{\sin 2\lambda_j h}{2\lambda_j h} \right), \quad j = 1, 2, \dots \quad (3.85)$$

The set of functions

$$Z_j(z) = \begin{cases} N_0^{-1/2} \cosh \lambda_0(z+h), & j = 0 \\ N_j^{-1/2} \cos \lambda_j(z+h), & j = 1, 2, \dots \end{cases} \quad (3.86)$$

where

$$N_j = \begin{cases} \frac{h}{2} \left(1 + \frac{\sinh 2\lambda_0 h}{2\lambda_0 h} \right), & j = 0 \\ \frac{h}{2} \left(1 + \frac{\sin 2\lambda_j h}{2\lambda_j h} \right), & j = 1, 2, \dots \end{cases} \quad (3.87)$$

forms an orthonormal set defined in the interval $-h \leq z \leq 0$.

For $k^2 > 0$, a general solution for the propagating mode that satisfies the far field radiation condition is

$$\phi_s = \sum_{m=0}^{\infty} C_{0m} Z_0(z) H_m^{(1)}(\lambda_0 r) \cos m\theta \quad (3.88)$$

where $H_m^{(1)}$ is the Hankel function of the first kind. For $k^2 < 0$, a general solution for the evanescent mode that satisfies the far field radiation condition is

$$\phi_s = \sum_{m=0}^{\infty} \sum_{j=1}^{\infty} C_{jm} Z_j(z) K_m(\lambda_j r) \cos m\theta \quad (3.89)$$

where K_m is the modified Bessel function of the second kind.

The incident potential (3.56) can be written in terms of polar coordinates, while writing k as λ_0 ,

$$\phi_I = \frac{-igA}{\omega} \frac{\cosh \lambda_0(z+h)}{\cosh \lambda_0 h} e^{i\lambda_0 r \cos \theta} \quad (3.90)$$

which can be further expressed as

$$\phi_I = \frac{-igA}{\omega} \frac{\cosh \lambda_0(z+h)}{\cosh \lambda_0 h} \sum_{m=0}^{\infty} \varepsilon_m i^m J_m(\lambda_0 r) \cos m\theta \quad (3.91)$$

where

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m = 1, 2, \dots \end{cases} \quad (3.92)$$

and J_m is the Bessel function of the first kind.

Combining (3.88), (3.89), and (3.91), the diffraction potential in the exterior region can be expressed as

$$\begin{aligned} \phi_D^e = & \sum_{m=0}^{\infty} \left(\left(J_m(\lambda_0 r) + \alpha_{0m} \frac{H_m^{(1)}(\lambda_0 r)}{H_m^{(1)}(\lambda_0 R)} \right) \frac{Z_0(z)}{Z_0(0)} \right. \\ & \left. + \sum_{j=1}^{\infty} \alpha_{jm} \frac{Z_j(z)}{Z_j(0)} \frac{K_m(\lambda_j r)}{K_m(\lambda_j R)} \right) \frac{(-igA)}{\omega} \varepsilon_m i^m \cos m\theta \end{aligned} \quad (3.93)$$

For continuity at $r = R$, the following conditions are required:

$$\phi_D^i = \phi_D^e, \text{ at } r = R \text{ and } -h \leq z \leq -d \quad (3.94)$$

$$\frac{\partial \phi_D^i}{\partial r} = \frac{\partial \phi_D^e}{\partial r}, \text{ at } r = R \text{ and } -h \leq z \leq -d \quad (3.95)$$

From (3.74) and (3.94),

$$\begin{aligned} \beta_{nm} = & \frac{2}{h-d} \frac{(-igA)}{\omega} \varepsilon_m i^m \left(\frac{J_m(\lambda_0 R)}{Z_0(0)} \int_{-h}^{-d} Z_0(z) \cos k_n(z+h) dz \right. \\ & \left. + \sum_{j=0}^{\infty} \frac{\alpha_{jm}}{Z_j(0)} \int_{-h}^{-d} Z_j(z) \cos k_n(z+h) dz \right) \end{aligned} \quad , n = 0, 1, 2, \dots \quad (3.96)$$

Defining

$$\mathfrak{I}_{nj} = \frac{1}{(h-d)Z_j(0)} \int_{-h}^{-d} Z_j(z) \cos k_n(z+h) dz \quad (3.97)$$

then (3.96) can be rewritten as

$$\beta_{nm} = 2 \frac{(-igA)}{\omega} \varepsilon_m i^m \left(J_m(\lambda_0 R) \mathfrak{I}_{n0} + \sum_{j=0}^{\infty} \alpha_{jm} \mathfrak{I}_{nj} \right) \quad (3.98)$$

Integrating (3.97) twice by parts gives

$$\mathfrak{I}_{nj} = \begin{cases} \frac{(-1)^n (h-d) \lambda_0 \sinh \lambda_0 (h-d)}{\left((h-d)^2 \lambda_0^2 + n^2 \pi^2\right) \cosh \lambda_0 h}, j=0 \\ \frac{(-1)^n (h-d) \lambda_j \sinh \lambda_j (h-d)}{\left((h-d)^2 \lambda_j^2 - n^2 \pi^2\right) \cosh \lambda_j h}, j=1,2,\dots \end{cases}, n=0,1,2,\dots \quad (3.99)$$

Multiplying equations (3.77) and (3.95) by $Z_\tau(z)$, $\tau=0,1,2,\dots$, and integrating with respect to z over their respective region of validity and then adding the resulting two equations together,

$$\int_{-h}^{-d} \frac{\partial \phi_D^i}{\partial r} \Big|_{r=R} Z_\tau(z) dz = \int_{-h}^0 \frac{\partial \phi_D^e}{\partial r} \Big|_{r=R} Z_\tau(z) dz \quad (3.100)$$

Expanding,

$$\begin{aligned} & \frac{(-igA)}{\omega} \varepsilon_m i^m \frac{\lambda_0 J'_m(\lambda_0 R)}{Z_0(0)} \delta_{0\tau} \\ &= \frac{(-igA)}{\omega} \varepsilon_m i^m \left(-\alpha_{0m} \frac{\lambda_0 H_m^{(1)}(\lambda_0 R)}{H_m^{(1)}(\lambda_0 R) Z_0(0)} \delta_{0\tau} - \sum_{j=1}^{\infty} \alpha_{jm} \frac{\lambda_j K'_m(\lambda_j R)}{K_m(\lambda_j R) Z_j(0)} \delta_{j\tau} \right) \\ &+ \frac{m\beta_{0m}}{2R} (h-d) Z_\tau(0) \mathfrak{I}_{0\tau} + \sum_{n=1}^{\infty} \beta_{nm} \frac{k_n I'_m(k_n R)}{I_m(k_n R)} (h-d) Z_\tau(0) \mathfrak{I}_{n\tau} \end{aligned} \quad (3.101)$$

where $\delta_{j\tau}$ is the Kronecker delta.

Substituting (3.98) into (3.101) gives

$$\begin{aligned} & \frac{\lambda_0 J'_m(\lambda_0 R)}{Z_0(0)} \delta_{0\tau} - J_m(\lambda_0 R) (h-d) Z_\tau(0) \left(\frac{m}{R} \mathfrak{I}_{00} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I'_m(k_n R)}{I_m(k_n R)} \mathfrak{I}_{n0} \mathfrak{I}_{n\tau} \right) \\ &= \sum_{j=0}^{\infty} \left(-\aleph_{jm} \delta_{j\tau} + (h-d) Z_\tau(0) \left(\frac{m}{R} \mathfrak{I}_{0j} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I'_m(k_n R)}{I_m(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \right) \right) \alpha_{jm} \end{aligned} \quad (3.102)$$

where

$$\aleph_{jm} = \begin{cases} \frac{\lambda_0 H_m^{(1)}(\lambda_0 R)}{H_m^{(1)}(\lambda_0 R) Z_0(0)}, j=0 \\ \frac{\lambda_j K'_m(\lambda_j R)}{K_m(\lambda_j R) Z_j(0)}, j=1,2,\dots \end{cases} \quad (3.103)$$

Equation (3.102) can be written in the form of

$$\sum_{j=0}^{\infty} D_{j\tau} \alpha_{jm} = A_{\tau}, \quad \tau = 0, 1, 2, \dots \quad (3.104)$$

where

$$D_{j\tau} = -\mathfrak{N}_{jm} \delta_{j\tau} + (h-d) Z_{\tau}(0) \left(\frac{m}{R} \mathfrak{I}_{0j} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I'_m(k_n R)}{I_m(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \right) \quad (3.105)$$

$$A_{\tau} = \frac{\lambda_0 J'_m(\lambda_0 R)}{Z_0(0)} \delta_{0\tau} - J_m(\lambda_0 R) (h-d) Z_{\tau}(0) \left(\frac{m}{R} \mathfrak{I}_{00} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I'_m(k_n R)}{I_m(k_n R)} \mathfrak{I}_{n0} \mathfrak{I}_{n\tau} \right) \quad (3.106)$$

Equation (3.104) is a complex matrix equation. The unknowns are the coefficients α_{jm} 's. The equation can be solved numerically by truncating the terms at certain $j = \tau$. Once these coefficients are known the diffraction potential is known.

3.3.4 Derivation of radiation potential

The radiation potential also satisfies Laplace equation in cylindrical coordinate system as in equation (3.58).

$$\nabla^2 \phi_R = 0, \text{ in the fluid domain} \quad (3.107)$$

The radiation potential can be expressed in the form of

$$\phi_R = -i\omega \sum_j \bar{U}_j \phi_j \quad (3.108)$$

for a single rigid body with 6 DOFs, $j = 1, 2, \dots, 6$ corresponding to surge, sway, heave, roll, pitch, and yaw, respectively. The general kinematic body boundary conditions are

$$\frac{\partial \phi_j}{\partial n} = n_j, \text{ on } S_b \quad (3.109)$$

where $(n_1, n_2, n_3) = \mathbf{n}$, $(n_4, n_5, n_6) = \mathbf{x} \times \mathbf{n}$

For a floating vertical circular cylinder under plane wave loading in the positive x -direction, the sway, roll, and yaw radiation potentials are zero. A product solution is assumed in the form of

$$\phi_j(r, \theta, z) = \begin{cases} Z(z)R(r)\cos\theta, & j = 1, 5 \\ Z(z)R(r), & j = 3 \end{cases} \quad (3.110)$$

Surge motion

As in the derivation of the diffraction potential, the radiation potential can be decomposed into interior and exterior solutions. For the interior region, the following boundary conditions apply:

$$\frac{\partial \phi_1^i}{\partial z} = 0, \text{ at } z = -d \text{ and } 0 \leq r \leq R \quad (3.111)$$

$$\frac{\partial \phi_1^i}{\partial z} = 0, \text{ at } z = -h \text{ and } 0 \leq r \leq R \quad (3.112)$$

Following the steps given in section 3.3.3 for the derivation of the interior solution for the diffraction potential, and employing the same notations, the following is obtained for the interior region:

$$\phi_1^i = \left(\frac{\beta_0}{2} \frac{r}{R} + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} \cos k_n(z+h) \right) \cos \theta \quad (3.113)$$

where I_1 is the modified Bessel function of the first kind of order 1.

Defining

$$\zeta_1^i = \frac{\beta_0}{2} \frac{r}{R} + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} \cos k_n(z+h) \quad (3.114)$$

the Fourier coefficients are obtained from

$$\beta_n = \frac{2}{h-d} \int_{-h}^{-d} \zeta_1^i(R, z) \cos k_n(z+h) dz, \quad n = 0, 1, 2, \dots \quad (3.115)$$

For the exterior region, the boundary conditions are

$$-\omega^2 \phi_1^e + g \frac{\partial \phi_1^e}{\partial z} = 0, \quad \text{at } z = 0 \text{ and } r \geq R \quad (3.116)$$

$$\frac{\partial \phi_1^e}{\partial z} = 0, \quad \text{at } z = -h \text{ and } r \geq R \quad (3.117)$$

$$\frac{\partial \phi_1^e}{\partial r} = \cos \theta, \quad \text{at } r = R \text{ and } -d \leq z \leq 0 \quad (3.118)$$

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial \phi_1^e}{\partial r} - ik \phi_1^e \right) = 0 \quad (3.119)$$

Applying the free-surface boundary condition (3.116) and the bottom boundary condition (3.117),

$$\phi_1^e = \left(\alpha_0 \frac{H_1^{(1)}(\lambda_0 r)}{H_1^{(1)}(\lambda_0 R)} \frac{Z_0(z)}{Z_0(0)} + \sum_{j=1}^{\infty} \alpha_j \frac{K_1(\lambda_j r)}{K_1(\lambda_j R)} \frac{Z_j(z)}{Z_j(0)} \right) \cos \theta \quad (3.120)$$

where $Z_j(z)$ and N_j are defined in (3.86) and (3.87), respectively, and $H_1^{(1)}$ and K_1 are the Hankel function of the first kind of order 1 and the modified Bessel function of the second kind of order 1, respectively.

For continuity at $r = R$, the following conditions are required:

$$\phi_1^i = \phi_1^e, \quad \text{at } r = R \text{ and } -h \leq z \leq -d \quad (3.121)$$

$$\frac{\partial \phi_1^i}{\partial r} = \frac{\partial \phi_1^e}{\partial r}, \quad \text{at } r = R \text{ and } -h \leq z \leq -d \quad (3.122)$$

From (3.115) and (3.121),

$$\beta_n = 2 \sum_{j=0}^{\infty} \alpha_j \mathfrak{T}_{nj} \quad (3.123)$$

where $\mathfrak{I}_{nj}$ is defined in (3.97) and (3.99).

Multiplying equations (3.118) and (3.122) by $Z_\tau(z)$, $\tau = 0, 1, 2, \dots$, and integrating with respect to z over their respective region of validity and then adding the resulting two equations together,

$$\int_{-h}^{-d} \frac{\partial \phi_1^i}{\partial r} \bigg|_{r=R} Z_\tau(z) dz + \int_{-d}^0 \cos \theta Z_\tau(z) dz = \int_{-h}^0 \frac{\partial \phi_1^e}{\partial r} \bigg|_{r=R} Z_\tau(z) dz \quad (3.124)$$

Expanding,

$$\begin{aligned} \sum_{j=0}^{\infty} \alpha_j \mathfrak{I}_j \delta_{j\tau} &= \frac{\beta_0}{2R} (h-d) Z_\tau(0) \mathfrak{I}_{0\tau} + \sum_{n=1}^{\infty} \beta_n \frac{k_n I_1'(k_n R)}{I_1(k_n R)} (h-d) Z_\tau(0) \mathfrak{I}_{n\tau} \\ &\quad + \int_{-d}^0 Z_\tau(z) dz \end{aligned} \quad (3.125)$$

where $\mathfrak{I}_j$ is equal to $\mathfrak{I}_{jm}$ in (3.103) for $m = 1$.

Substituting (3.123) into (3.125),

$$\begin{aligned} \sum_{j=0}^{\infty} \left(\mathfrak{I}_j \delta_{j\tau} - (h-d) Z_\tau(0) \left(\frac{1}{R} \mathfrak{I}_{0j} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I_1'(k_n R)}{I_1(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \right) \right) \alpha_j \\ = \int_{-d}^0 Z_\tau(z) dz \end{aligned} \quad (3.126)$$

Equation (3.126) can be written in the form of equation (3.104), where α_{jm} is now α_j , and

$$D_{j\tau} = \mathfrak{I}_j \delta_{j\tau} - (h-d) Z_\tau(0) \left(\frac{1}{R} \mathfrak{I}_{0j} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I_1'(k_n R)}{I_1(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \right) \quad (3.127)$$

$$A_\tau = \int_{-d}^0 Z_\tau(z) dz = \begin{cases} \frac{N_0^{-1/2}}{\lambda_0} (\sinh \lambda_0 h - \sinh \lambda_0 (h-d)), \tau = 0 \\ \frac{N_\tau^{-1/2}}{\lambda_\tau} (\sin \lambda_\tau h - \sin \lambda_\tau (h-d)), \tau = 1, 2, \dots \end{cases} \quad (3.128)$$

Once the coefficients α_j 's are known the surge radiation potential is known.

Heave motion

For the interior region, the following boundary conditions apply:

$$\frac{\partial \phi_3^i}{\partial z} = 1, \text{ at } z = -d \text{ and } 0 \leq r \leq R \quad (3.129)$$

$$\frac{\partial \phi_3^i}{\partial z} = 0, \text{ at } z = -h \text{ and } 0 \leq r \leq R \quad (3.130)$$

Decomposing ϕ_3^i into homogeneous and nonhomogeneous parts, two boundary problems are constructed. For the homogeneous part, by separation of variables the following is obtained:

$$\phi_{3h}^i = \frac{\beta_0}{2} + \sum_{n=1}^{\infty} \beta_n \frac{I_0(k_n r)}{I_0(k_n R)} \cos k_n(z+h) \quad (3.131)$$

where I_0 is the modified Bessel function of the first kind of order 0. The particular solution can be written as

$$\phi_{3p}^i = \frac{1}{2(h-d)} \left((z+h)^2 - \frac{r^2}{2} \right) \quad (3.132)$$

Hence,

$$\phi_3^i = \frac{\beta_0}{2} + \sum_{n=1}^{\infty} \beta_n \frac{I_0(k_n r)}{I_0(k_n R)} \cos k_n(z+h) + \frac{1}{2(h-d)} \left((z+h)^2 - \frac{r^2}{2} \right) \quad (3.133)$$

The coefficients β_n are obtained from

$$\beta_n = \frac{2}{h-d} \int_{-h}^{-d} \phi_3^i(R, z) \cos k_n(z+h) dz - I_{n3}, \quad n = 0, 1, 2, \dots \quad (3.134)$$

where

$$I_{n3} = \begin{cases} \frac{(h-d)}{3} - \frac{R^2}{2(h-d)}, n=0 \\ \frac{2(-1)^n(h-d)}{n^2\pi^2}, n=1,2,\dots \end{cases} \quad (3.135)$$

For the exterior region, the boundary conditions are

$$-\omega^2\phi_3^e + g \frac{\partial\phi_3^e}{\partial z} = 0, \text{ at } z=0 \text{ and } r \geq R \quad (3.136)$$

$$\frac{\partial\phi_3^e}{\partial z} = 0, \text{ at } z=-h \text{ and } r \geq R \quad (3.137)$$

$$\frac{\partial\phi_3^e}{\partial r} = 0, \text{ at } r=R \text{ and } -d \leq z \leq 0 \quad (3.138)$$

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial\phi_3^e}{\partial r} - ik\phi_3^e \right) = 0 \quad (3.139)$$

Applying the free-surface boundary condition (3.136) and the bottom boundary condition (3.137),

$$\phi_3^e = \alpha_0 \frac{H_0^{(1)}(\lambda_0 r) Z_0(z)}{H_0^{(1)}(\lambda_0 R) Z_0(0)} + \sum_{j=1}^{\infty} \alpha_j \frac{K_0(\lambda_j r) Z_j(z)}{K_0(\lambda_j R) Z_j(0)} \quad (3.140)$$

where $Z_j(z)$ and N_j are defined in (3.86) and (3.87), respectively, and $H_0^{(1)}$ and K_0 are the Hankel function of the first kind of order 0 and the modified Bessel function of the second kind of order 0, respectively.

For continuity at $r=R$, the following conditions are required:

$$\phi_3^i = \phi_3^e, \text{ at } r=R \text{ and } -h \leq z \leq -d \quad (3.141)$$

$$\frac{\partial\phi_3^i}{\partial r} = \frac{\partial\phi_3^e}{\partial r}, \text{ at } r=R \text{ and } -h \leq z \leq -d \quad (3.142)$$

From (3.134) and (3.141),

$$\beta_n = 2 \sum_{j=0}^{\infty} \alpha_j \mathfrak{I}_{nj} - I_{n3} \quad (3.143)$$

where $\mathfrak{I}_{nj}$ is defined in (3.97) and (3.99).

Multiplying equations (3.138) and (3.142) by $Z_\tau(z)$, $\tau = 0, 1, 2, \dots$, and integrating with respect to z over their respective region of validity and then adding the resulting two equations together,

$$\int_{-h}^{-d} \frac{\partial \phi_3^i}{\partial r} \bigg|_{r=R} Z_\tau(z) dz = \int_{-h}^0 \frac{\partial \phi_3^e}{\partial r} \bigg|_{r=R} Z_\tau(z) dz \quad (3.144)$$

Expanding,

$$\sum_{j=0}^{\infty} \alpha_j \mathfrak{S}_j \delta_{j\tau} = \sum_{n=1}^{\infty} \beta_n \frac{k_n I'_0(k_n R)}{I_0(k_n R)} (h-d) Z_\tau(0) \mathfrak{I}_{n\tau} - \frac{R}{2(h-d)} \int_{-h}^{-d} Z_\tau(z) dz \quad (3.145)$$

where $\mathfrak{S}_j$ is equal to $\mathfrak{S}_{jm}$ in (3.103) for $m = 0$.

Substituting (3.143) into (3.145),

$$\begin{aligned} & \sum_{j=0}^{\infty} \left(\mathfrak{S}_j \delta_{j\tau} - 2(h-d) Z_\tau(0) \sum_{n=1}^{\infty} \frac{k_n I'_0(k_n R)}{I_0(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \right) \alpha_j \\ &= - \left((h-d) Z_\tau(0) \sum_{n=1}^{\infty} \frac{k_n I'_0(k_n R)}{I_0(k_n R)} \mathfrak{I}_{n\tau} I_{n3} + \frac{R}{2(h-d)} \int_{-h}^{-d} Z_\tau(z) dz \right) \end{aligned} \quad (3.146)$$

Equation (3.146) can be written in the form of equation (3.104), where α_{jm} is now α_j , and

$$D_{j\tau} = \mathfrak{S}_j \delta_{j\tau} - 2(h-d) Z_\tau(0) \sum_{n=1}^{\infty} \frac{k_n I'_0(k_n R)}{I_0(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \quad (3.147)$$

$$A_\tau = - \left((h-d) Z_\tau(0) \sum_{n=1}^{\infty} \frac{k_n I'_0(k_n R)}{I_0(k_n R)} \mathfrak{I}_{n\tau} I_{n3} + \frac{R}{2(h-d)} \int_{-h}^{-d} Z_\tau(z) dz \right) \quad (3.148)$$

where

$$\int_{-h}^{-d} Z_{\tau}(z) dz = \begin{cases} \frac{N_0^{-1/2}}{\lambda_0} \sinh \lambda_0 (h-d), \tau = 0 \\ \frac{N_{\tau}^{-1/2}}{\lambda_{\tau}} \sin \lambda_{\tau} (h-d), \tau = 1, 2, \dots \end{cases} \quad (3.149)$$

Once the coefficients α_j 's are known the heave radiation potential is known.

Pitch motion

It is assumed that the pitch motion is about the motion reference point, i.e. the point where a vertical line through the model's centre of gravity crosses the waterline, or the coordinates (0,0,0). For the interior region, the following boundary conditions apply:

$$\frac{\partial \phi_5^i}{\partial z} = -r \cos \theta, \text{ at } z = -d \text{ and } 0 \leq r \leq R \quad (3.150)$$

$$\frac{\partial \phi_5^i}{\partial z} = 0, \text{ at } z = -h \text{ and } 0 \leq r \leq R \quad (3.151)$$

Decomposing ϕ_5^i into homogeneous and nonhomogeneous parts, two boundary problems are constructed. For the homogeneous part, by separation of variables the following is obtained:

$$\phi_{5h}^i = \left(\frac{\beta_0}{2} \frac{r}{R} + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} \cos k_n (z+h) \right) \cos \theta \quad (3.152)$$

where I_1 is the modified Bessel function of the first kind of order 1. The particular solution can be written as

$$\phi_{5p}^i = \frac{-1}{2(h-d)} \left(r(z+h)^2 - \frac{r^3}{4} \right) \cos \theta \quad (3.153)$$

Hence,

$$\phi_5^i = \left(\frac{\beta_0}{2} \frac{r}{R} + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} \cos k_n (z+h) - \frac{1}{2(h-d)} \left(r(z+h)^2 - \frac{r^3}{4} \right) \right) \cos \theta \quad (3.154)$$

Defining

$$\zeta_5^i = \frac{\beta_0}{2} \frac{r}{R} + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} \cos k_n (z+h) - \frac{1}{2(h-d)} \left(r(z+h)^2 - \frac{r^3}{4} \right) \quad (3.155)$$

the coefficients β_n are obtained from

$$\beta_n = \frac{2}{h-d} \int_{-h}^{-d} \zeta_5^i(R, z) \cos k_n (z+h) dz - I_{n5}, \quad n = 0, 1, 2, \dots \quad (3.156)$$

where

$$I_{n5} = \begin{cases} \frac{-(h-d)R}{3} + \frac{R^3}{4(h-d)}, & n = 0 \\ \frac{2(-1)^{n+1}(h-d)R}{n^2 \pi^2}, & n = 1, 2, \dots \end{cases} \quad (3.157)$$

For the exterior region, the boundary conditions are

$$-\omega^2 \phi_5^e + g \frac{\partial \phi_5^e}{\partial z} = 0, \quad \text{at } z = 0 \text{ and } r \geq R \quad (3.158)$$

$$\frac{\partial \phi_5^e}{\partial z} = 0, \quad \text{at } z = -h \text{ and } r \geq R \quad (3.159)$$

$$\frac{\partial \phi_5^e}{\partial r} = -z \cos \theta, \quad \text{at } r = R \text{ and } -d \leq z \leq 0 \quad (3.160)$$

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial \phi_5^e}{\partial r} - ik \phi_5^e \right) = 0 \quad (3.161)$$

Applying the free-surface boundary condition (3.158) and the bottom boundary condition (3.159),

$$\phi_5^e = \left(\alpha_0 \frac{H_1^{(1)}(\lambda_0 r)}{H_1^{(1)}(\lambda_0 R)} \frac{Z_0(z)}{Z_0(0)} + \sum_{j=1}^{\infty} \alpha_j \frac{K_1(\lambda_j r)}{K_1(\lambda_j R)} \frac{Z_j(z)}{Z_j(0)} \right) \cos \theta \quad (3.162)$$

where $Z_j(z)$ and N_j are defined in (3.86) and (3.87), respectively, and $H_1^{(1)}$ and K_1 are the Hankel function of the first kind of order 1 and the modified Bessel function of the second kind of order 1, respectively.

For continuity at $r = R$, the following conditions are required:

$$\phi_5^i = \phi_5^e, \text{ at } r = R \text{ and } -h \leq z \leq -d \quad (3.163)$$

$$\frac{\partial \phi_5^i}{\partial r} = \frac{\partial \phi_5^e}{\partial r}, \text{ at } r = R \text{ and } -h \leq z \leq -d \quad (3.164)$$

From (3.156) and (3.163),

$$\beta_n = 2 \sum_{j=0}^{\infty} \alpha_j \mathfrak{I}_{nj} - I_{n5} \quad (3.165)$$

where $\mathfrak{I}_{nj}$ is defined in (3.97) and (3.99).

Multiplying equations (3.160) and (3.164) by $Z_\tau(z)$, $\tau = 0, 1, 2, \dots$, and integrating with respect to z over their respective region of validity and then adding the resulting two equations together,

$$\int_{-h}^{-d} \frac{\partial \phi_5^i}{\partial r} \bigg|_{r=R} Z_\tau(z) dz - \int_{-d}^0 z \cos \theta Z_\tau(z) dz = \int_{-h}^0 \frac{\partial \phi_5^e}{\partial r} \bigg|_{r=R} Z_\tau(z) dz \quad (3.166)$$

Expanding,

$$\begin{aligned} \sum_{j=0}^{\infty} \alpha_j \mathfrak{N}_j \delta_{j\tau} &= \frac{\beta_0}{2R} (h-d) Z_\tau(0) \mathfrak{I}_{0\tau} + \sum_{n=1}^{\infty} \beta_n \frac{k_n I_1'(k_n R)}{I_1(k_n R)} (h-d) Z_\tau(0) \mathfrak{I}_{n\tau} \\ &\quad - \frac{1}{2(h-d)} \int_{-h}^{-d} \left((z+h)^2 - \frac{3}{4} R^2 \right) Z_\tau(z) dz - \int_{-d}^0 z Z_\tau(z) dz \end{aligned} \quad (3.167)$$

where $\mathfrak{N}_j$ is equal to $\mathfrak{N}_{jm}$ in (3.103) for $m = 1$.

Substituting (3.165) into (3.167),

$$\begin{aligned}
 & \sum_{j=0}^{\infty} \left(\aleph_j \delta_{j\tau} - (h-d) Z_{\tau}(0) \left(\frac{1}{R} \mathfrak{I}_{0j} \mathfrak{I}_{0\tau} + 2 \sum_{n=1}^{\infty} \frac{k_n I_1'(k_n R)}{I_1(k_n R)} \mathfrak{I}_{nj} \mathfrak{I}_{n\tau} \right) \right) \alpha_j \\
 &= -(h-d) Z_{\tau}(0) \left(\frac{I_{05}}{2R} \mathfrak{I}_{0\tau} + \sum_{n=1}^{\infty} I_{n5} \frac{k_n I_1'(k_n R)}{I_1(k_n R)} \mathfrak{I}_{n\tau} \right) \\
 & - \frac{1}{2(h-d)} \int_{-h}^{-d} \left((z+h)^2 - \frac{3}{4} R^2 \right) Z_{\tau}(z) dz - \int_{-d}^0 z Z_{\tau}(z) dz
 \end{aligned} \tag{3.168}$$

Equation (3.168) can be written in the form of equation (3.104), where α_{jm} is now α_j , $D_{j\tau}$ is as in (3.127), and

$$\begin{aligned}
 A_{\tau} &= -(h-d) Z_{\tau}(0) \left(\frac{I_{05}}{2R} \mathfrak{I}_{0\tau} + \sum_{n=1}^{\infty} I_{n5} \frac{k_n I_1'(k_n R)}{I_1(k_n R)} \mathfrak{I}_{n\tau} \right) \\
 & - \frac{1}{2(h-d)} \int_{-h}^{-d} \left((z+h)^2 - \frac{3}{4} R^2 \right) Z_{\tau}(z) dz - \int_{-d}^0 z Z_{\tau}(z) dz
 \end{aligned} \tag{3.169}$$

where

$$\begin{aligned}
 & \frac{1}{2(h-d)} \int_{-h}^{-d} \left((z+h)^2 - \frac{3}{4} R^2 \right) Z_{\tau}(z) dz \\
 &= \begin{cases} \left(\left(\frac{(h-d)}{2\lambda_0} + \frac{1}{\lambda_0^3(h-d)} - \frac{3R^2}{8\lambda_0(h-d)} \right) \sinh \lambda_0(h-d) \right. \\ \left. - \frac{\cosh \lambda_0(h-d)}{\lambda_0^2} \right) N_0^{-1/2}, \tau = 0 \\ \left(\left(\frac{(h-d)}{2\lambda_{\tau}} - \frac{1}{\lambda_{\tau}^3(h-d)} - \frac{3R^2}{8\lambda_{\tau}(h-d)} \right) \sin \lambda_{\tau}(h-d) \right. \\ \left. + \frac{\cos \lambda_{\tau}(h-d)}{\lambda_{\tau}^2} \right) N_{\tau}^{-1/2}, \tau = 1, 2, \dots \end{cases} \tag{3.170}
 \end{aligned}$$

and

$$\int_{-d}^0 z Z_{\tau}(z) dz = \begin{cases} \frac{N_0^{-1/2}}{\lambda_0^2} (d\lambda_0 \sinh \lambda_0(h-d) - \cosh \lambda_0 h + \cosh \lambda_0(h-d)), \tau = 0 \\ \frac{N_{\tau}^{-1/2}}{\lambda_{\tau}^2} (d\lambda_{\tau} \sin \lambda_{\tau}(h-d) + \cos \lambda_{\tau} h - \cos \lambda_{\tau}(h-d)), \tau = 1, 2, \dots \end{cases} \tag{3.171}$$

Once the coefficients α_j 's are known the pitch radiation potential is known.

3.3.5 Calculation of hydrodynamic forces

The i^{th} component of the generalised hydrodynamic force on the body is

$$F_i = \iint_{S_b} p n_i dS = \text{Re} \{ \bar{F}_i e^{-i\omega t} \} \quad (3.172)$$

where

$$\begin{aligned} \bar{F}_i &= i\rho\omega \iint_{S_b} \phi n_i dS \\ &= i\rho\omega \iint_{S_b} \phi_D n_i dS + \sum_j \rho\omega^2 \bar{U}_j \iint_{S_b} \phi_j n_i dS \end{aligned} \quad (3.173)$$

Denoting

$$\begin{aligned} \bar{F}_{Di} &= i\rho\omega \iint_{S_b} \phi_D n_i dS \\ \bar{F}_{ij} &= i\rho\omega \iint_{S_b} \phi_j n_i dS \end{aligned} \quad (3.174)$$

equation (3.173) can be written as

$$\bar{F}_i = \bar{F}_{Di} - i\omega \sum_j \bar{U}_j \bar{F}_{ij} \quad (3.175)$$

$\{\bar{F}_{Di}\}$ is the diffraction or wave excitation force vector, while $(\bar{F}_{ij})$ is the radiation or reaction force matrix.

The reaction force due to j mode is

$$\begin{aligned} &\text{Re} \{ -i\omega \bar{U}_j \bar{F}_{ij} e^{-i\omega t} \} \\ &= \text{Re} \{ (\text{Re} \{ \bar{F}_{ij} \} + i \text{Im} \{ \bar{F}_{ij} \}) (-i\omega \bar{U}_j e^{-i\omega t}) \} \\ &= \text{Re} \left\{ \left(i\rho\omega \iint_{S_b} \text{Re} \{ \phi_j \} n_i dS - \rho\omega \iint_{S_b} \text{Im} \{ \phi_j \} n_i dS \right) (-i\omega \bar{U}_j e^{-i\omega t}) \right\} \\ &= - \left(\rho \iint_{S_b} \text{Re} \{ \phi_j \} n_i dS \right) \text{Re} \{ -\omega^2 \bar{U}_j e^{-i\omega t} \} \\ &\quad - \left(\rho\omega \iint_{S_b} \text{Im} \{ \phi_j \} n_i dS \right) \text{Re} \{ -i\omega \bar{U}_j e^{-i\omega t} \} \\ &= - \left(\rho \iint_{S_b} \text{Re} \{ \phi_j \} n_i dS \right) \ddot{u}_j - \left(\rho\omega \iint_{S_b} \text{Im} \{ \phi_j \} n_i dS \right) \dot{u}_j \end{aligned} \quad (3.176)$$

The first integral in (3.176) is proportional to the body acceleration and is actually the added mass (see section 2.4.3). The added mass matrix is

$$\mathbf{A} : A_{ij} = \rho \iint_{S_b} \operatorname{Re}\{\phi_j\} n_i dS = \frac{1}{\omega} \operatorname{Im}\{\bar{F}_{ij}\} \quad (3.177)$$

The second integral in (3.176) is proportional to the body velocity and is actually the radiation damping (see section 2.4.3). The radiation damping matrix is

$$\mathbf{C} : C_{ij} = \rho \omega \iint_{S_b} \operatorname{Im}\{\phi_j\} n_i dS = -\operatorname{Re}\{\bar{F}_{ij}\} \quad (3.178)$$

Wave excitation forces

From the first equation in (3.174), the wave excitation force in positive x -direction is

$$\begin{aligned} \bar{F}_{D1} &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \phi_D^e(R, \theta, z) \cos \theta d\theta dz \\ &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \sum_{m=0}^{\infty} \left((J_m(\lambda_0 R) + \alpha_{0m}) \frac{Z_0(z)}{Z_0(0)} \right. \\ &\quad \left. + \sum_{j=1}^{\infty} \alpha_{jm} \frac{Z_j(z)}{Z_j(0)} \right) \frac{(-igA)}{\omega} \varepsilon_m i^m \cos m\theta \cos \theta d\theta dz \\ &= -2\pi i \rho g A R \int_{-d}^0 \left((J_1(\lambda_0 R) + \alpha_{01}) \frac{Z_0(z)}{Z_0(0)} + \sum_{j=1}^{\infty} \alpha_{j1} \frac{Z_j(z)}{Z_j(0)} \right) dz \\ &= -2\pi i \rho g A R \left((J_1(\lambda_0 R) + \alpha_{01}) \frac{\sinh \lambda_0 h - \sinh \lambda_0 (h-d)}{\lambda_0 \cosh \lambda_0 h} \right. \\ &\quad \left. + \sum_{j=1}^{\infty} \alpha_{j1} \frac{\sin \lambda_j h - \sin \lambda_j (h-d)}{\lambda_j \cos \lambda_j h} \right) \end{aligned} \quad (3.179)$$

where the coefficients α_{j1} 's are obtained from equation (3.104).

The wave excitation force in positive z -direction is

$$\begin{aligned}
 \bar{F}_{D3} &= i\rho\omega \int_0^R \int_0^{2\pi} \phi_D^i(r, \theta, -d) r d\theta dr \\
 &= i\rho\omega \int_0^R \int_0^{2\pi} \sum_{m=0}^{\infty} \left(\frac{\beta_{0m}}{2} \left(\frac{r}{R} \right)^m + \sum_{n=1}^{\infty} \beta_{nm} \frac{I_m(k_n r)}{I_m(k_n R)} (-1)^n \right) \cos m\theta d\theta dr \\
 &= 2\pi i\rho\omega \int_0^R \left(\frac{\beta_{00}}{2} + \sum_{n=1}^{\infty} \beta_{n0} \frac{I_0(k_n r)}{I_0(k_n R)} (-1)^n \right) r dr \\
 &= 2\pi i\rho\omega \left(\frac{\beta_{00} R^2}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n \beta_{n0}}{I_0(k_n R)} \int_0^R r I_0(k_n r) dr \right) \\
 &= 2\pi i\rho\omega \left(\frac{\beta_{00} R^2}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n \beta_{n0} R I_1(k_n R)}{k_n I_0(k_n R)} \right)
 \end{aligned} \tag{3.180}$$

where the coefficients β_{n0} 's are obtained from equations (3.98) and (3.104).

The wave excitation moment about positive y-axis is

$$\begin{aligned}
 \bar{F}_{D5} &= -i\rho\omega \int_0^{2\pi} \left(R \int_{-d}^0 \phi_D^e(R, \theta, z) z dz + \int_0^R \phi_D^i(r, \theta, -d) r^2 dr \right) \cos \theta d\theta \\
 &= -2\pi i\rho g A R \int_{-d}^0 \left((J_1(\lambda_0 R) + \alpha_{01}) \frac{Z_0(z)}{Z_0(0)} + \sum_{j=1}^{\infty} \alpha_{j1} \frac{Z_j(z)}{Z_j(0)} \right) z dz \\
 &\quad - \pi i\rho\omega \int_0^R \left(\frac{\beta_{01} r}{2R} + \sum_{n=1}^{\infty} \beta_{n1} \frac{I_1(k_n r)}{I_1(k_n R)} (-1)^n \right) r^2 dr \\
 &= -2\pi i\rho g A R \left(\frac{(J_1(\lambda_0 R) + \alpha_{01})}{Z_0(0)} \int_{-d}^0 z Z_0(z) dz + \sum_{j=1}^{\infty} \frac{\alpha_{j1}}{Z_j(0)} \int_{-d}^0 z Z_j(z) dz \right) \\
 &\quad - \pi i\rho\omega \left(\frac{\beta_{01} R^3}{8} + \sum_{n=1}^{\infty} \frac{(-1)^n \beta_{n1} R^2 I_2(k_n R)}{k_n I_1(k_n R)} \right)
 \end{aligned} \tag{3.181}$$

where the coefficients α_{j1} 's and β_{n1} 's are obtained from equations (3.98) and (3.104), and $\int_{-d}^0 z Z_j(z) dz$, $j = 0, 1, 2, \dots$ is obtained from (3.171).

Reaction forces

The nonzero components of the reaction force matrix are calculated as follows:

$$\begin{aligned}
 \bar{F}_{11} &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \phi_1^e(R, \theta, z) \cos \theta d\theta dz \\
 &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \sum_{j=0}^{\infty} \alpha_j \frac{Z_j(z)}{Z_j(0)} \cos^2 \theta d\theta dz \\
 &= -\pi i\rho\omega R \int_{-d}^0 \sum_{j=0}^{\infty} \alpha_j \frac{Z_j(z)}{Z_j(0)} dz \\
 &= -\pi i\rho\omega R \left(\alpha_0 \frac{\sinh \lambda_0 h - \sinh \lambda_0 (h-d)}{\lambda_0 \cosh \lambda_0 h} + \sum_{j=1}^{\infty} \alpha_j \frac{\sin \lambda_j h - \sin \lambda_j (h-d)}{\lambda_j \cos \lambda_j h} \right)
 \end{aligned} \tag{3.182}$$

where the coefficients α_j 's are obtained from equation (3.126).

$$\begin{aligned}
 \bar{F}_{15} &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \phi_5^e(R, \theta, z) \cos \theta d\theta dz \\
 &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \sum_{j=0}^{\infty} \alpha_j \frac{Z_j(z)}{Z_j(0)} \cos^2 \theta d\theta dz \\
 &= -\pi i\rho\omega R \left(\alpha_0 \frac{\sinh \lambda_0 h - \sinh \lambda_0 (h-d)}{\lambda_0 \cosh \lambda_0 h} + \sum_{j=1}^{\infty} \alpha_j \frac{\sin \lambda_j h - \sin \lambda_j (h-d)}{\lambda_j \cos \lambda_j h} \right)
 \end{aligned} \tag{3.183}$$

where the coefficients α_j 's are obtained from equation (3.168).

$$\begin{aligned}
 \bar{F}_{33} &= i\rho\omega \int_0^R \int_0^{2\pi} \phi_3^i(r, \theta, -d) r d\theta dr \\
 &= i\rho\omega \int_0^R \int_0^{2\pi} \left(\frac{\beta_0}{2} + \sum_{n=1}^{\infty} \beta_n \frac{I_0(k_n r)}{I_0(k_n R)} (-1)^n + \frac{(h-d)}{2} - \frac{r^2}{4(h-d)} \right) d\theta r dr \\
 &= 2\pi i\rho\omega \int_0^R \left(\frac{\beta_0}{2} + \sum_{n=1}^{\infty} \beta_n \frac{I_0(k_n r)}{I_0(k_n R)} (-1)^n + \frac{(h-d)}{2} - \frac{r^2}{4(h-d)} \right) r dr \\
 &= 2\pi i\rho\omega \left(\frac{\beta_0 R^2}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n \beta_n R I_1(k_n R)}{k_n I_0(k_n R)} + \frac{(h-d)R^2}{4} - \frac{R^4}{16(h-d)} \right)
 \end{aligned} \tag{3.184}$$

where the coefficients β_n 's are obtained from equations (3.143) and (3.146).

$$\begin{aligned}
 \bar{F}_{51} &= -i\rho\omega \int_0^{2\pi} \left(R \int_{-d}^0 \phi_1^e(R, \theta, z) z dz + \int_0^R \phi_1^i(r, \theta, -d) r^2 dr \right) \cos \theta d\theta \\
 &= -\pi i \rho \omega R \int_{-d}^0 \sum_{j=0}^{\infty} \alpha_j \frac{Z_j(z)}{Z_j(0)} z dz \\
 &\quad - \pi i \rho \omega \int_0^R \left(\frac{\beta_0 r}{2R} + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} (-1)^n \right) r^2 dr \\
 &= -\pi i \rho \omega R \sum_{j=0}^{\infty} \frac{\alpha_j}{Z_j(0)} \int_{-d}^0 z Z_j(z) dz \\
 &\quad - \pi i \rho \omega \left(\frac{\beta_0 R^3}{8} + \sum_{n=1}^{\infty} \frac{(-1)^n \beta_n R^2 I_2(k_n R)}{k_n I_1(k_n R)} \right)
 \end{aligned} \tag{3.185}$$

where the coefficients α_j 's and β_n 's are obtained from equations (3.123) and (3.126), and $\int_{-d}^0 z Z_j(z) dz$, $j = 0, 1, 2, \dots$ is obtained from (3.171).

$$\begin{aligned}
 \bar{F}_{55} &= -i\rho\omega \int_0^{2\pi} \left(R \int_{-d}^0 \phi_5^e(R, \theta, z) z dz + \int_0^R \phi_5^i(r, \theta, -d) r^2 dr \right) \cos \theta d\theta \\
 &= -i\rho\omega R \int_{-d}^0 \int_0^{2\pi} \sum_{j=0}^{\infty} \alpha_j \frac{Z_j(z)}{Z_j(0)} \cos^2 \theta d\theta z dz - i\rho\omega \int_0^R \int_0^{2\pi} \left(\frac{\beta_0 r}{2R} \right. \\
 &\quad \left. + \sum_{n=1}^{\infty} \beta_n \frac{I_1(k_n r)}{I_1(k_n R)} (-1)^n - \frac{r(h-d)}{2} + \frac{r^3}{8(h-d)} \right) \cos^2 \theta d\theta r^2 dr \\
 &= -\pi i \rho \omega R \sum_{j=0}^{\infty} \frac{\alpha_j}{Z_j(0)} \int_{-d}^0 z Z_j(z) dz \\
 &\quad - \pi i \rho \omega \left(\frac{\beta_0 R^3}{8} + \sum_{n=1}^{\infty} \frac{(-1)^n \beta_n R^2 I_2(k_n R)}{k_n I_1(k_n R)} - \frac{R^4(h-d)}{8} + \frac{R^6}{48(h-d)} \right)
 \end{aligned} \tag{3.186}$$

where the coefficients α_j 's and β_n 's are obtained from equations (3.165) and (3.168), and $\int_{-d}^0 z Z_j(z) dz$, $j = 0, 1, 2, \dots$ is obtained from (3.171).

The added mass and radiation damping matrices can be obtained from the reaction forces by the relations (3.177) and (3.178). Once these matrices are obtained, they are input into the matrix equation of motion of the floating vertical circular cylinder, which is discussed in the following section.

3.3.6 Matrix equation of motion

In the derivation of the equation of motion of the floating vertical circular cylinder with TMD, several assumptions are necessary:

- (1) The structure undergoes only small translations and rotations.
- (2) The structure is axisymmetric.
- (3) The TMD mass moves in vertical direction only.
- (4) The effect of the TMD oscillations to variations of the moment of inertia is neglected.

The detailed derivation of the equation of motion from the conservation of linear and angular momentum for a 6-DOF rigid body is presented by Mei et al. (2005). The equation of motion of the main structure and the TMD, with 7 DOFs (6 DOFs from the main structure and 1 additional DOF from the TMD), is given as

$$\{-\omega^2(\mathbf{M} + \mathbf{A}) - i\omega\mathbf{C} + \mathbf{K}\}\mathbf{U} = \mathbf{F} \quad (3.189)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{A}$ is the added mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the restoring matrix, $\mathbf{U}$ is the displacement vector, and $\mathbf{F}$ is the excitation force vector. The nonzero components of the mass matrix $\mathbf{M}$ are given as follows:

$$\begin{aligned} M_{11} &= M_{22} = M_{33} = m_1 \\ M_{15} &= M_{51} = m_1 z_G \\ M_{24} &= M_{42} = -m_1 z_G \\ M_{44} &= I_{11} \\ M_{55} &= I_{22} \\ M_{66} &= I_{33} \\ M_{77} &= m_2 \end{aligned} \quad (3.190)$$

where I_{11} , I_{22} , and I_{33} are the mass moment of inertias of the floating cylinder about the x -, y -, and z -axis, respectively, passing through the motion reference point.

The nonzero components of the restoring matrix $\mathbf{K}$ are given as follows:

$$\begin{aligned}
 K_{33} &= \rho g A + k_2 \\
 K_{37} &= K_{73} = -k_2 \\
 K_{44} &= K_{55} = \rho g I_A + m_1 g (z_b - z_G) \\
 K_{77} &= k_2
 \end{aligned} \tag{3.191}$$

The contribution of the TMD to be added into the damping matrix is

$$\begin{aligned}
 C_{33} &= C_{77} = c_2 \\
 C_{37} &= C_{73} = -c_2
 \end{aligned} \tag{3.192}$$

The matrix equation of motion (3.189) can be solved for the displacements $\mathbf{U}$, and hence the heave response of the floating cylinder U_3 , once the hydrodynamic forces (the added mass matrix $\mathbf{A}$, the radiation damping matrix $\mathbf{C}$, and the wave excitation force vector $\mathbf{F}$) are obtained.

3.3.7 Computational procedure

A MATLAB routine (see Appendix B) is first created to compute the eigenvalues numerically via graphical means. These eigenvalues are then used in solving the linear simultaneous equations, which are put in matrix form, for the coefficients α 's and β 's of the eigenfunction expansion of the diffraction and radiation potentials. Once these coefficients are known, the hydrodynamic forces can be obtained. The bulk computation of these coefficients is performed by another MATLAB routine (see Appendix C). The number of terms used in the computation is $j = 20$. The routine not only computes the coefficients but also the hydrodynamic forces and the displacements of the floating cylinder with and without TMD. Another MATLAB routine (see Appendix D) is created to plot the variations of the hydrodynamic forces (wave excitation forces, added masses, and radiation dampings), the heave response of the cylinder with and without TMD, and the response reduction factor RRF over a range of predetermined wave excitation frequencies. These results will be compared in Chapter 6 with those from the 2-DOF model, the experiments, and the numerical simulations.

Chapter 4

Laboratory Experiments

4.1 Model design

This section gives an overview of the design of the scaled model used in the laboratory experiments. The modelling principles and the profile and general configuration of the model are described.

4.1.1 Froude scaling law

The modelling follows Froude scaling law. It is applicable in the case of water flow with a free surface, where the gravitational effect predominates and the effect of other factors, such as viscosity, surface tension, roughness, etc., is neglected. Froude scaling law is given as

$$Fr = \frac{\dot{u}_p^2}{gl_p} = \frac{\dot{u}_m^2}{gl_m} \quad (4.1)$$

where the subscripts p and m indicate prototype and model values, respectively. The Froude scale ratio is given in Table 4.1. Difference in water densities is neglected.

Physical quantity	Scale ratio
Length (l)	λ
Velocity ($\dot{u}$)	$\sqrt{\lambda}$
Mass (m)	λ^3
Force (F)	λ^3
Stiffness (k)	λ^2
Period (T)	$\sqrt{\lambda}$
Acceleration ($\ddot{u}$)	1

Table 4.1 Froude scaling

4.1.2 Fabrication and configuration of scaled model

The model is constructed to a scale of 1/50. It consists of a circular cylindrical outer structure and an inner TMD casing where the TMD is. A larger circular cylindrical outer structure is also constructed for the model with larger water plane area. The outer structure and the TMD casing are all made of acrylic. The joining method is by glue joint. Four circular steel disks weighing 6.474kg, 5.968kg, 6.014kg, and 3.145kg are used as ballast at the bottom compartment of the structure. The movement of ballast due to model motion is prevented. The profile and general configuration of the model with all its major dimensions are shown in Figures 4.1a, 4.1b, and 4.1c.

Figure 4.1a Profile and general configuration of model with box TMD casing
(all dimensions in mm)

The TMD mass is a circular cylinder made of steel. Five TMD masses with different weights (0.614kg, 0.602kg, 0.801kg, 1.302kg, and 1.588kg) and a number of tension springs with different stiffnesses (0.03 N/mm , 0.04 N/mm ,

0.045 N/mm , 0.055 N/mm , 0.06 N/mm , 0.09 N/mm , 0.11 N/mm , 0.12 N/mm , and 0.19 N/mm) are used. Four of the TMD masses are shown in Figure 4.2. The TMD casing can be detached from the outer structure and the TMD mass and spring can be assembled into and disassembled from the casing. For the box TMD casing, steel rods are provided to guide the movement of the TMD mass in vertical direction and to prevent it from hitting the inner wall of the TMD casing. The box TMD casing is shown in Figure 4.3, while the tube TMD casing is shown in Figure 4.4. Figure 4.5 shows the disassembled scaled model.

Figure 4.1b Profile and general configuration of model with tube TMD casing
(all dimensions in mm)

4.1.3 Eight cases of configuration

Eight cases of configuration of the scaled model are tested, as described in Table 4.2. Some of these configurations are shown in Figure 4.6 to Figure 4.8. The filling of water into the tube TMD casing for Case 3 and 6 is to simulate an added TMD damping. The displacement, centre of gravity, moment of inertia, and metacentric

height of the structure are calculated from geometry and weighed values. The formulas to calculate the moment of inertias are given in Appendix A. In the calculation of the moment of inertias, the structure is assumed to be axisymmetric for all cases. For Case 1, the model is not actually axisymmetric since the TMD casing is a box. However, the variations of moment of inertias about its principal transverse axes are small, thus the model can be assumed axisymmetric.

Figure 4.1c Profile and general configuration of larger circular cylindrical outer structure (all dimensions in mm)

Figure 4.2 TMD masses

The movement of the TMD mass only slightly alter the position of centre of gravity and metacentric height of the structure, and therefore these variations can be neglected. The principal dimensions (prototype values) of the eight different cases are presented in Table 4.3.

Figure 4.3 Box TMD casing with TMD

Figure 4.4 Tube TMD casing with TMD

Figure 4.5 Disassembled scaled model with box TMD casing

Figure 4.6 Assembled scaled model with box TMD casing

Figure 4.7 Assembled scaled model with tube TMD casing

Figure 4.8 Assembled scaled model with tube TMD casing and larger circular cylindrical outer structure

Case	Name	Description
1	<i>Box full ballast</i>	Box TMD casing; all four disks used as ballast
2	<i>Tube full ballast</i>	Tube TMD casing; all four disks used as ballast
3	<i>Tube full ballast with water</i>	Tube TMD casing; all four disks used as ballast; tube filled with water
4	<i>Tube bottom & top ballast</i>	Tube TMD casing; disks weighing 6.474kg and 3.145kg used as ballast
5	<i>Tube bottom & top ballast increased water plane</i>	Tube TMD casing; disks weighing 6.474kg and 3.145kg used as ballast; larger circular cylindrical outer structure
6	<i>Tube bottom & top ballast increased water plane with water</i>	Tube TMD casing; disks weighing 6.474kg and 3.145kg used as ballast; larger circular cylindrical outer structure; tube filled with water
7	<i>Tube bottom ballast increased water plane</i>	Tube TMD casing; disk weighing 6.474kg used as ballast; larger circular cylindrical outer structure
8	<i>Tube top ballast increased water plane</i>	Tube TMD casing; disk weighing 3.145kg used as ballast; larger circular cylindrical outer structure

Table 4.2 Eight cases of configuration

Particulars	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
Outer diameter (m)	15	15	15	15	23.5	23.5	23.5	23.5
Draft without TMD (m)	22.1	20.6	21.2	12.1	6.7	7.0	5.8	4.9
Displacement without TMD (tonnes)	3899.5	3636.9	3743.5	2139.1	2915.4	3022.0	2522.3	2106.1
Ballast (tonnes)	2700.1	2700.1	2700.1	1202.4	1202.4	1202.4	809.3	393.1
$I_{xx} = I_{yy}$ about motion reference point (kgm^2)	1.49×10^9	1.08×10^9	1.16×10^9	2.72×10^8	2.79×10^8	3.00×10^8	2.74×10^8	2.76×10^8
I_{zz} about motion reference point (kgm^2)	6.27×10^7	5.80×10^7	5.81×10^7	3.33×10^7	6.64×10^7	6.65×10^7	5.99×10^7	5.30×10^7
$\overline{KB}$ (m)	11.0	10.3	10.6	6.1	3.4	3.5	2.9	2.4
$\overline{KG}$ (m)	4.3	5.4	5.7	7.0	6.7	7.1	7.4	8.4
$\overline{BM}$ (m)	0.6	0.7	0.7	1.2	5.1	5.0	5.9	7.1
$\overline{GM}$ (m)	7.4	5.6	5.5	0.3	1.8	1.3	1.5	1.1

Table 4.3 Eight cases of configuration with principal dimensions (prototype values)

4.2 Experimental setup

The experiments are conducted in a $42m$ (length) $\times$ $1.55m$ (width) $\times$ $1.55m$ (depth) towing tank (Figure 4.9). The water depth is set at $0.8m$. The interference effects from the tank walls are neglected based on the fact that the model diameter is less than $1/5$ the tank width (for model with smaller water plane) and that the generated wave height is small (Chakrabarti, 1994). A beach (perforated slope) is located at the far end of the tank to minimize wave reflections from the far end.

Figure 4.9 Towing tank

The model is positioned at the centre line of the tank and let to float freely without any constraints (see Figure 4.10). This is consistent with the mathematical model in which constraining forces are not taken into account.

Regular waves acting on the model are created by a piston type wave generator (Figure 4.11). The piston is double-ended, ensuring symmetrical performance whether the wave paddle moves back or forth. There are limits to the maximum wave heights that can be generated. The wave generator system determines these maximum values as function of the wave period for given water depths, based on the linear wave theory and empirical wave breaking theory (*DHI Power Pack and Servo Actuator*, 1999). The wave generator is also equipped with an active wave

absorption control system (AWACS) to absorb the reflected waves by means of digital filters and a servo system (*DHI AWACS*, 1994). The AWACS is calibrated in calm water condition each time before starting a series of experimental runs. A wave synthesizer is used to control the wave generator (*DHI Wave Synthesizer*, 1999). The wave generator control room is shown in Figure 4.12, showing the AWACS and the wave meter.

Figure 4.10 Towing tank and model setup

The wave environment is monitored with a wave meter. The input to the wave meter comes from wave probes installed along the tank centreline. Four wave probes are installed ahead of the cylinder while one wave probe is installed after the cylinder (see Figure 4.13). The wave probe comprises two thin parallel stainless steel electrodes. When immersed in water, the meter measures the conductivity of the instantaneous water volume between the two electrodes. The conductivity changes proportionally to changes of the water surface elevation between the electrodes. The voltage is displayed on the wave meter panel. The sampling

frequency of the wave meter is set at 25 Hz. The probes are calibrated in calm water condition each time before starting a series of experimental runs.

Figure 4.11 Piston type wave generator

Figure 4.12 Wave generator control room

The motion response of the scaled model is captured using an optical tracking system which consists of a real-time processing unit, a host PC, infrared-emitting cameras, and small reflective markers placed on the scaled model. The camera flashes infrared light and the markers reflect it back to the camera. The camera

measures the 2-dimensional position of the centroid of each marker using the camera's internal signal processing algorithm. By combining the 2-D data from the two cameras, together with the information about camera locations, the 3-D position of the markers is calculated, and hence the real-time 6-DOF data of the model. The 6-DOF time history records are stored for analysis.

Figure 4.13 Experimental setup

Two infrared-emitting cameras are used. The sampling frequency of the camera system is set at 50 Hz. The measurement volume which covers the movement range of the model is determined first. It is defined by the cross-section of the cameras' fields of view. The cameras are positioned next to the towing tank (see Figure 4.13), keeping the model and the markers in the line of sight all the time within the measurement volume. The focus and aperture of the cameras are adjusted before conducting the tests. The camera system is also calibrated each time before a series of motion capture runs takes place. The calibration is to determine the position and orientation of the global coordinate system and the cameras. With the information about the orientation and position of each camera, the software will be able to track and perform calculations on the 2D data into 3D data. The calibration is done by moving a calibration object, which consists of two markers located a fixed distance from each other, to cover the entire measurement volume. The calibration

algorithms extract each camera's position and orientation by evaluating the camera's view of the object during the calibration (*Qualisys Track Manager*, 2004).

For the 6-DOF tracking function to work, the rigid body, i.e. the scaled model, must be defined by at least four points, i.e. four reflective markers (the 6-DOF computation of a rigid body is described in the next section). In the current study five markers are used. The markers cannot all be in the same plane and the configuration should not be symmetrical. The size of the markers is 19 mm for all markers. The configuration of the markers can be seen from the previous Figures 4.5 to 4.8. The origin of the local coordinate system is set to be at the motion reference point, i.e. the point where a vertical line through the model's centre of gravity crosses the waterline.

4.3 6-DOF tracking function and calculations of rotation angles

In each frame captured by the cameras, the 6-DOF tracking function locates the position and orientation of the rigid body in the measurement volume. The coordinates x, y, z define the position of the origin of the rigid body's local coordinate system relative to the origin of the global coordinate system. The rotation angles θ (roll), ϕ (pitch), and ψ (yaw) define the rotation about the x -, y -, and z -axis, respectively, of the local coordinate system relative to the orientation of the global coordinate system, where positive rotation is clockwise when looking in the direction of the axis (right hand rule). The function uses the rigid body definition, which is defined by the reflective markers, to compute $\vec{P}_{origin}$, the position vector of the origin of the local coordinate system in the global coordinate system, and $\vec{R}$, the rotation matrix which describes the rotation of the rigid body. A position ($\vec{P}_{local}$) in the local coordinate system is related to a position ($\vec{P}_{global}$) in the global coordinate system by the following equation:

$$\vec{P}_{global} = \vec{R} \cdot \vec{P}_{local} + \vec{P}_{origin} \quad (4.2)$$

The rotation angles are calculated from the rotation matrix ($\bar{R}$). The rotations are described with individual rotation matrices: $\bar{R}_x, \bar{R}_y, \bar{R}_z$. The three rotation matrices are:

$$\bar{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \quad (4.3)$$

$$\bar{R}_y = \begin{pmatrix} \cos \phi & 0 & \sin \phi \\ 0 & 1 & 0 \\ -\sin \phi & 0 & \cos \phi \end{pmatrix} \quad (4.4)$$

$$\bar{R}_z = \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.5)$$

The rotation matrix ($\bar{R}$) is calculated by multiplying the three rotation matrices.

$$\bar{R} = \bar{R}_x \cdot \bar{R}_y \cdot \bar{R}_z = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \quad (4.6)$$

where

$$\begin{aligned} r_{11} &= \cos \phi \cos \psi \\ r_{12} &= -\cos \phi \sin \psi \\ r_{13} &= \sin \phi \\ r_{21} &= \cos \theta \sin \psi + \cos \psi \sin \theta \sin \phi \\ r_{22} &= \cos \theta \cos \psi - \sin \theta \sin \phi \sin \psi \\ r_{23} &= -\cos \phi \sin \theta \\ r_{31} &= \sin \theta \sin \psi - \cos \theta \cos \psi \sin \phi \\ r_{32} &= \cos \psi \sin \theta + \cos \theta \sin \phi \sin \psi \\ r_{33} &= \cos \theta \cos \phi \end{aligned}$$

The rotation angles can therefore be obtained from the rotation matrix using the following equations:

$$\begin{aligned}
\phi &= \arcsin(r_{13}) \\
\theta &= \arccos\left(\frac{r_{33}}{\cos\phi}\right) \\
\psi &= \arccos\left(\frac{r_{11}}{\cos\phi}\right)
\end{aligned} \tag{4.7}$$

4.4 Experimental run

Free motion tests (also known as decay tests) are carried out in still water to estimate the heave natural periods and damping ratios of the main structure (the cylinder without TMD). The cylinder is pushed down symmetrically at a certain draft and then released to perform free oscillations in heave. The motions are captured by the motion tracking system and stored as time history records.

An experimental run consists of subjecting the free floating cylinder to a regular plane wave each time with a different period, and capturing its heave response. For each run, waves are generated by the wave generator and the information of the water surface elevation from the five wave probes installed along the towing tank is captured by the wave meter and stored as time history records in .DAF files. At the same time, the 6-DOF displacements of the cylinder are captured by the motion tracking system and stored as time history records in .TSV files.

The range of wave periods is from 3 to 20s (prototype value). The wave height used for most of the runs is 0.5m (prototype value) and it may be varied to up to 2m (prototype value). Given a water depth of 0.8m, the value of H/D is found not larger than 0.133 for all cases, therefore drag is not an issue. However, the value of D/L can be as small as 0.05 for large period waves. In fact, D/L is greater than 0.2 only for waves with period less than 7s. For periods larger than 7s, where both H/D and D/L are small, as discussed in section 2.4.1 the potential theory is assumed applicable. The durations of the wave generator run, the wave meter capture, and the tracking system capture vary in the range of 30 to 90s.

Two sets of runs are conducted for each case of configuration described in section 4.1.3: one for the cylinder without TMD, and the other for the cylinder with TMD. For the cylinder with TMD, the parameters to be varied are the spring stiffness and damping ratio. The spring stiffness is varied each time by using a tension spring with different stiffness. For a given stiffness and mass, the TMD natural frequency f_a can be obtained. There are only two variations of TMD damping: when the tube TMD casing is filled with water (Case 3 and 6) and when it is not. The various test runs are summarised in Table 4.4.

Case	m (kg)	k (kN/m)	f_a (Hz)
1: <i>Box full ballast</i>	76,750	225	0.272503572
		475	0.39593851
2: <i>Tube full ballast</i>	75,250	75	0.158890346
		112.5	0.194600137
		225	0.275206152
	100,125	100	0.159055564
		150	0.194802487
		275	0.263763814
	162,750	300	0.216082831
3: <i>Tube full ballast with water</i>	75,250	75	0.158890346
	100,125	100	0.159055564
	162,750	225	0.187133221
	198,500	300	0.195659304
4: <i>Tube bottom & top ballast</i>	75,250	75	0.158890346
		112.5	0.194600137
		150	0.224704882
		225	0.275206152
	100,125	100	0.159055564
		137.5	0.186509181
	162,750	225	0.187133221
		300	0.216082831
5: <i>Tube bottom & top ballast increased water plane</i>	100,125	300	0.195659304
6: <i>Tube bottom & top ballast increased water plane with water</i>	100,125	100	0.159055564
	162,750	225	0.187133221
	198,500	300	0.195659304
7: <i>Tube bottom ballast increased water plane</i>	100,125	100	0.159055564
	162,750	225	0.187133221
8: <i>Tube top ballast increased water plane</i>	100,125	100	0.159055564

Table 4.4 Test runs (prototype values)

4.5 Experimental results

MATLAB routines are created for reading the wave time history from .DAF files (see Appendix C) and for reading the heave time history from .TSV files (see Appendix D).

4.5.1 Decay tests

From decay tests the natural periods as well as the damping ratios in heave of the main structure are obtained. Since radiation damping is dependent on frequency, the damping ratios obtained from the decay tests are those at the natural frequencies only.

The natural period is estimated by taking the average period over a number of oscillations from the time history record. The estimated natural period is further verified with the frequency at the peak of the Fast Fourier Transform (FFT) of the time history record. Assuming small damping, the damping ratio is estimated by curve fitting a logarithmic decay function $F = C_1 e^{-C_2 t}$ enveloping the heave time history record. The damping ratio is estimated from C_2 / ω_n . The heave natural periods and frequencies (prototype values) of the main structure obtained from decay tests are shown in Table 4.5. The heave damping ratios, critical damping coefficients, and damping coefficients (prototype values) of the main structure are shown in Table 4.6. The critical damping coefficients are obtained from

$$c_{c1} = 2m_1\omega_n \quad (4.8)$$

where ω_n is obtained from the decay tests, while m_1 is equal to the mass of the cylinder without TMD plus the added mass, which is estimated as the mass of water contained in a hemisphere with a diameter equal to that of the cylinder. For cases with smaller water plane, the added mass is 883,572.9kg, while for cases with larger water plane, the added mass is 3,397,599.7kg. This estimation is in agreement with the numerical results presented in the following chapter.

Case	Natural period (s)	Natural frequency (Hz)
1: Box full ballast	10.459	0.096
2: Tube full ballast	10.168	0.098
3: Tube full ballast with water	10.257	0.097
4: Tube bottom & top ballast	8.130	0.123
5: Tube bottom & top ballast increased water plane	7.265	0.138
6: Tube bottom & top ballast increased water plane with water	7.234	0.138
7: Tube bottom ballast increased water plane	6.925	0.144
8: Tube top ballast increased water plane	6.378	0.157

Table 4.5 Heave natural periods and frequencies (prototype values) of main structure obtained from decay tests

Case	Damping ratio	Critical damping coefficient (kg/s)	Damping coefficient (kg/s)
1: Box full ballast	0.028	5.75×10^6	1.60×10^5
2: Tube full ballast	0.026	5.59×10^6	1.48×10^5
3: Tube full ballast with water	0.025	5.67×10^6	1.43×10^5
4: Tube bottom & top ballast	0.037	4.67×10^6	1.73×10^5
5: Tube bottom & top ballast increased water plane	0.060	1.09×10^7	6.55×10^5
6: Tube bottom & top ballast increased water plane with water	0.079	1.12×10^7	8.80×10^5
7: Tube bottom ballast increased water plane	0.086	1.07×10^7	9.19×10^5
8: Tube top ballast increased water plane	0.104	1.08×10^7	1.13×10^6

Table 4.6 Heave damping ratios, critical damping coefficients, and damping coefficients (prototype values) of main structure obtained from decay tests

The damping coefficients can then be obtained from

$$c_1 = \zeta_1 c_{c1} \quad (4.9)$$

These damping coefficients will be compared in the following chapter with those from numerical simulations.

As can be seen in Table 4.5, for the same radius, shallow-draft cylinders have higher natural periods than deeper ones. It is also observed from Table 4.6 that for the same radius, shallow-draft cylinders have more damping than deeper ones.

4.5.2 Experimental heave response

From linear wave theory, the heave response is linearly proportional to the wave height, as long as the wave heights are small. Therefore, the heave response can be normalised with respect to the wave height, giving the Response Amplitude Operator (RAO), which is independent of the wave height. The RAO is obtained by first taking the root mean square (RMS) of the wave time history and of the heave displacement time history. The RAO is the ratio of the heave RMS over the wave RMS. The RAO is plotted against wave frequency to give the heave response variation over a range of wave frequencies. Figure 4.14 shows the plots of heave RAOs of the cylinder without TMD for the eight different cases. The heave RAOs of the cylinder with TMD for all of the cases are shown in Figures 4.15a to 4.15k.

The plots appear to show a typical pattern which is also observed in the 2-DOF plots of the amplitude ratio U_1/U_{st} in Section 3.2. They have a peak at resonance, and for smaller wave excitation frequencies (left hand side of the peak) they go asymptotically to 1, while for larger wave excitation frequencies (right hand side of the peak) they go asymptotically to zero. Indeed, the behaviour of the RAO curve is similar to the amplitude ratio U_1/U_{st} curve due to the fact that U_{st} is the static displacement of the structure. In the case of a floating structure excited by wave forces, U_{st} is equivalent to the wave amplitude since as the wave frequency goes to zero the structure moves together with the wave, giving a vertical displacement equal to the wave amplitude.

From Figure 4.14, it is observed that for the first three cases the peak heave RAO is about 7. For Case 4 the peak reduces to about 4, while for Case 5 to 8 the peak reduces further to about 2.5, with Case 8 having the lowest peak at 2. This is in agreement with the damping ratios obtained from decay tests, as seen in Table 4.5. The higher the damping, the peak is expected to be lower. Case 1 to 3 have similar

damping ratios, Case 4 has a higher damping ratio, while Case 5 to 8 have even higher damping ratios. The natural frequencies of the cylinder without TMD obtained from the heave time history are also in agreement with the frequencies of the peaks as shown in Figure 4.14.

From Figure 4.15a, the effect of the TMD is hardly noticed. This is expected, since the natural frequencies of the TMD ($f_a = 0.273Hz$ and $f_a = 0.396Hz$) is significantly higher than the natural frequency of the main structure. It does not mean that there is no response reduction, but as the response is already very small at these frequencies, the reduction is hardly observable. As shown in the preliminary study of a simplified 2-DOF model, the effect of the TMD is observable when the TMD natural frequency is close enough to the natural frequency of the main structure. From Figure 4.15b, although the effect of the TMD is still not too evident, it is observed that to the right hand side of the peak the response of the cylinder with TMD is lower than that of the cylinder without TMD, while to the left hand side of the peak it is slightly higher. The same observation can be made from Figure 4.15c to Figure 4.15g. Generally, for the same f_a , these effects are more pronounced as the TMD mass is increased.

From Figures 4.15h to 4.15k (cases with larger water plane), the natural frequency of the main structure is increased and thus, the TMD natural frequency f_a is closer to, even if it is still higher than, the natural frequency of the main structure. Therefore, it is expected that the response reduction is more evident, as shown in those figures at frequencies near $0.15Hz$.

Figure 4.14 Heave RAOs of cylinder without TMD obtained from experiments

Figure 4.15a Heave RAO obtained from experiments for *box full ballast*

Figure 4.15b Heave RAO obtained from experiments for *tube full ballast*
 ($m = 75,250\text{kg}$)

Figure 4.15c Heave RAO obtained from experiments for *tube full ballast*
 ($m = 100,125\text{kg}$ and $m = 162,750\text{kg}$)

Figure 4.15d Heave RAO obtained from experiments for *tube full ballast with water*

Figure 4.15e Heave RAO obtained from experiments for *tube bottom & top ballast*
($m = 75,250\text{kg}$)

Figure 4.15f Heave RAO obtained from experiments for *tube bottom & top ballast*
($m = 100,125\text{kg}$)

Figure 4.15g Heave RAO obtained from experiments for *tube bottom & top ballast*
($m = 162,750\text{kg}$ and $m = 198,500\text{kg}$)

Figure 4.15h Heave RAO obtained from experiments for *tube bottom & top ballast*
increased water plane

Figure 4.15i Heave RAO obtained from experiments for *tube bottom & top ballast*
increased water plane with water

Figure 4.15j Heave RAO obtained from experiments for *tube bottom ballast increased water plane*

Figure 4.15k Heave RAO obtained from experiments for *tube top ballast increased water plane*

Chapter 5

Numerical Simulations

5.1 Numerical modelling

The numerical analysis is performed using a commercial software SESAM (*SESAM: HydroD*, 2004; *SESAM: Manager*, 2002; *SESAM: Prefem*, 2003; *SESAM: Wadam*, 2004). It utilizes 3D panel method to evaluate velocity potentials and hydrodynamic coefficients. The radiation and diffraction velocity potentials on the submerged body surface are determined from the solution of an integral equation obtained by using Green's theorem with the free-surface source potentials as the Green's functions. The source strengths are evaluated based on the source distribution method using the same source potentials. The integral equation is discretised into a set of algebraic equations by approximating the body surface with a number of plane quadrilateral panels. The source strengths are assumed to be constant over each panel. The solution of the algebraic equation system provides the strength of the sources on the panels (*SESAM: Wadam*, 2004).

In the analysis, one-node Gauss quadrature is used for numerical integration of the Green's function and its derivatives, while logarithmic singularities are integrated analytically. Panel dimensions are based on the maximum diagonal of the panels. Irregular frequencies are removed from the radiation-diffraction solution. This is achieved by a modified integral equation obtained by including a panel model of the internal water plane, which is automatically created by the software. Finite depth formulation is used in the Green's functions. The linear system of equations for the velocity potentials are solved by direct Gauss elimination.

The inputs for the analysis include a panel model for calculation of hydrodynamic loads from potential theory, a mass model for dynamic analysis of the floating body, and environmental data for description of boundary conditions and wave properties.

The panel model is created using a SESAM module, Prefem (SESAM: Prefem, 2003). The geometry is first defined and the meshing is done automatically based on the defined geometry. Quadrilateral flat thin shell element type is used for all mesh elements. The Prefem command input file to generate the model is given in Appendix E. For the subsequent hydrodynamic analysis, the wet surface of a panel model is identified by defining a dummy load on the panel model. This is achieved by applying a hydro pressure load as load case number 1, pointing from the fluid onto the model. The completed panel model is stored in the Input Interface File for transfer to the hydrodynamic analysis module. In the analysis, the panels are automatically cut at the water plane. Figure 5.1 and Figure 5.2 show the typical panel models, which have been cut at the water plane, one for Case 2 (*tube full ballast*) and the other for Case 5 (*tube bottom & top ballast increased water plane*).

Figure 5.1 *Tube full ballast panel model (544 panels)*

Figure 5.2 *Tube bottom & top ballast increased water plane panel model (576 panels)*

In addition to the panel model, the hydrodynamic analysis of floating bodies needs a mass model as an input. The mass is used both in the hydrostatic calculations to report imbalances between weight and buoyancy of the structure and in the equation of motion. The global mass matrix is established by direct input specification, which comprises specifying the total mass of the structure together with the centre of gravity and the radii of gyration. These are input manually based on Table 4.3.

The environmental data are specified as in the experiments, neglecting the effects of the tank walls. Among the environmental data to be defined are the hydro location, frequency set, and direction set. The hydro location contains the gravity, water density, water kinematic viscosity, and water depth. Under hydro location, the frequency domain condition is defined. It contains the data used to discretize the frequency and heading domain in a frequency domain wave load analysis, namely the frequency set and the direction set. The frequency set is a list of periods, wavelengths, or wave angular frequencies. Direction set defines a list of wave propagation angles. In the current analysis, the direction is set to be 0 degree, meaning that the waves propagate along the x -axis in the positive x -axis direction.

From these inputs (panel model, mass model, and environmental data) the body added mass, damping, and restoring matrices as well as exciting forces and moments used in the equations of motion can be calculated, and hence the motion responses of the floating body for a set of wave frequencies and heading angles. All results are reported in a .LIS file.

5.2 Convergence tests

Numerical simulations with different number of panels are conducted to check the convergence of the numerical results. Convergence tests are carried out for Case 1 (*box full ballast*) without TMD. A MATLAB routine is written to read and plot the heave RAOs, heave-added mass, and heave-damping from the .LIS files (see Appendix G). Figure 5.3, 5.4, and 5.5 show the convergence of the heave RAO, heave-added mass, and heave-damping, respectively. From the figures it is shown that the results converge well.

Figure 5.3 Convergence of heave RAO (*box full ballast without TMD*)

Figure 5.4 Convergence of heave-added mass (*box full ballast without TMD*)

Figure 5.5 Convergence of heave-heave damping (*box full ballast without TMD*)

5.3 Numerical results

5.3.1 Floating cylinder without TMD

Figures 5.6, 5.7, and 5.8 show the numerical plots of heave RAO, heave-added mass, and heave-damping for the cylinder without TMD for the eight different cases. The number of panels of the panel models and the natural frequencies obtained numerically are shown in Figure 5.6.

From Figure 5.6 it is observed that the plots for Cases 1 to 3 have about the same peak, i.e. about 15. For Case 4, the peak is lower, i.e. about 8. For Case 5 to 8 the peak is even lower, i.e. about 2, with Case 8 having the lowest peak. From Figures 5.7 and 5.8, it can be seen that the heave-added mass and the heave-damping vary with wave frequency. From Figure 5.7 it is shown that the added mass is higher for the cylinder with larger water plane, while from Figure 5.8 it can be seen that the damping ratio is higher for the cylinder with shallower draft.

5.3.2 Floating cylinder with TMD

The software can only analyze the hydrodynamics of the cylinder without TMD. To obtain the heave response of the cylinder with TMD, the cylinder without TMD is first analysed and the 6×6 added mass and damping matrices are extracted from the analysis results. The matrices are input into the equation of motion (3.105), combining them with the contributions from the TMD. The equation of motion can then be solved to obtain the heave response of the cylinder with TMD. This is done over a range of wave frequencies for each experimental case. MATLAB routines are created for this purpose (see Appendix H and I). The contributions from the TMD are the TMD mass m_2 , stiffness k_2 , and damping coefficient c_2 . The value of c_2 is taken to be 200kg/s for Case 3 and 6, and 100kg/s for the other cases.

Figure 5.6 Numerical plots of heave RAO for cylinder without TMD

Figure 5.7 Numerical plots of heave-heave added mass for cylinder without TMD

Figure 5.8 Numerical plots of heave-heave damping for cylinder without TMD

The numerical plots of heave RAO for the structure with TMD for the different cases are shown in Figures 5.9a to 5.9m. Similar to the experimental results, and as has also been predicted from the 2-DOF theory, the effects of the TMD are not evident when the natural frequency of the TMD, f_a , is relatively far from that of the main structure, f_n . The effects are hardly noticeable in Case 1, since the natural frequency of the TMD is more than about three times that of the main structure. The effects become noticeable when the natural frequency of the TMD is close enough to that of the main structure, as in Cases 5 to 8 (Figures 5.9j to 5.9m).

Figure 5.9a Numerical plots of heave RAO for *box full ballast*

Figure 5.9b Numerical plots of heave RAO for *tube full ballast* ($m = 75,250\text{kg}$)

Figure 5.9c Numerical plots of heave RAO for *tube full ballast* ($m = 100,125\text{kg}$)

Figure 5.9d Numerical plots of heave RAO for *tube full ballast* ($m = 162,750\text{kg}$)

Figure 5.9e Numerical plots of heave RAO for *tube full ballast with water*
($m = 75,250\text{kg}$ and $m = 100,125\text{kg}$)

Figure 5.9f Numerical plots of heave RAO for *tube full ballast with water*
($m = 162,750\text{kg}$ and $m = 198,500\text{kg}$)

Figure 5.9g Numerical plots of heave RAO for *tube bottom & top ballast*
($m = 75,250\text{kg}$)

Figure 5.9h Numerical plots of heave RAO for *tube bottom & top ballast*
($m = 100,125\text{kg}$)

Figure 5.9i Numerical plots of heave RAO for *tube bottom & top ballast*
($m = 162,750\text{kg}$ and $m = 198,500\text{kg}$)

Figure 5.9j Numerical plots of heave RAO for *tube bottom & top ballast increased water plane*

Figure 5.9k Numerical plots of heave RAO for *tube bottom & top ballast increased water plane with water*

Figure 5.9l Numerical plots of heave RAO for *tube bottom ballast increased water plane*

Figure 5.9m Numerical plots of heave RAO for *tube top ballast increased water plane*

Chapter 6

Comparison of Results and Discussions

6.1 Comparison of results

The comparison of natural frequencies of the main structure obtained from the experiment and the numerical simulation is shown in Table 6.1. The natural frequencies obtained from the experiment agree very well with those obtained from the numerical simulation. The differences are within 3% of the numerical values, except for Case 8 (*tube top ballast increased water plane*), which differs 7.5% from the numerical value. The relatively high difference for this case is probably due to the fact that the structure has the shallowest draft, which leads to difficulties in performing the decay test.

Case	Natural frequency from experiment (Hz)	Natural frequency from numerical simulation (Hz)
1: <i>Box full ballast</i>	0.096	0.096
2: <i>Tube full ballast</i>	0.098	0.099
3: <i>Tube full ballast with water</i>	0.097	0.098
4: <i>Tube bottom & top ballast</i>	0.123	0.122
5: <i>Tube bottom & top ballast increased water plane</i>	0.138	0.135
6: <i>Tube bottom & top ballast increased water plane with water</i>	0.138	0.134
7: <i>Tube bottom ballast increased water plane</i>	0.144	0.140
8: <i>Tube top ballast increased water plane</i>	0.157	0.146

Table 6.1 Comparison of natural frequencies (prototype values) of main structure obtained from experiment and numerical simulation

Table 6.2 shows the comparison of damping coefficients of the main structure obtained from the experiment and the numerical simulation (the damping

coefficients from the numerical simulation are taken as constants). The damping coefficients obtained from the experiment are found higher than those obtained from the numerical simulation. This is probably due to viscous effects, which are not taken into account in the numerical simulations.

Case	Damping coefficient from experiment (kg/s)	Damping coefficient from numerical simulation (kg/s)
1: <i>Box full ballast</i>	1.60×10^5	6×10^4
2: <i>Tube full ballast</i>	1.48×10^5	6×10^4
3: <i>Tube full ballast with water</i>	1.43×10^5	6×10^4
4: <i>Tube bottom & top ballast</i>	1.73×10^5	8×10^4
5: <i>Tube bottom & top ballast increased water plane</i>	6.55×10^5	6×10^5
6: <i>Tube bottom & top ballast increased water plane with water</i>	8.80×10^5	6×10^5
7: <i>Tube bottom ballast increased water plane</i>	9.19×10^5	6×10^5
8: <i>Tube top ballast increased water plane</i>	1.13×10^6	7×10^5

Table 6.2 Comparison of damping coefficients (prototype values) of main structure obtained from experiment and numerical simulation

The comparison of heave RAOs obtained from the experiments, the numerical simulations, and the 2-DOF theory for all the cases is presented in Figures 6.1a to 6.1h. For the 2-DOF calculations, constant values of added mass and main mass damping ratio are used for each case. These constant values of added mass and damping are approximated from the numerical plots of added mass and damping (Figure 5.7 and 5.8), and are listed in Table 6.3. The mass ratio for each case is the ratio between the TMD mass and the combination of the mass of the main structure and the added mass. The natural frequency used in the 2-DOF calculations for each case is also obtained from the numerical simulation result.

As can be seen from Figures 6.1a to 6.1h, the given damping used as input into the 2-DOF model seems to be too small. It produces peaks about three times higher than those of the numerical plots. When damping is increased three times, the peaks

fall to about the same heights as those of the numerical plots. However, the 2-DOF curves are always flatter than the numerical ones, regardless of damping. This difference in the behaviour of the 2-DOF and the numerical RAO curves can be explained from the fact that the variation of wave excitation forces is neglected in the 2-DOF model. For the same wave height, waves with higher frequencies will actually have smaller excitation forces on the structure and thus, the response is expected to be smaller than that obtained by assuming the wave excitation forces to be constant across all frequencies. For a clearer comparison, the heave RAOs from the experiments and the numerical simulations as well as the semi-analytical theory are plotted with the 2-DOF results omitted. These are shown in Figures 6.2a to 6.2h.

Case	Added mass (kg)	Damping coefficient (kg/s)
1: <i>Box full ballast</i>	8.4×10^5	6×10^4
2: <i>Tube full ballast</i>	8.4×10^5	6×10^4
3: <i>Tube full ballast with water</i>	8.4×10^5	6×10^4
4: <i>Tube bottom & top ballast</i>	8×10^5	8×10^4
5: <i>Tube bottom & top ballast increased water plane</i>	2.7×10^6	6×10^5
6: <i>Tube bottom & top ballast increased water plane with water</i>	2.7×10^6	6×10^5
7: <i>Tube bottom ballast increased water plane</i>	2.6×10^6	6×10^5
8: <i>Tube top ballast increased water plane</i>	2.5×10^6	7×10^5

Table 6.3 Added mass and damping coefficients used in 2-DOF calculations

Overall, good agreement is found between the experiment, numerical, and semi-analytical theory results. For Cases 1 to 4 (smaller water plane), the peaks of the experimental heave RAOs are found lower than those of the numerical and semi-analytical ones. This is probably due to viscous damping which is not considered in the numerical simulations and the semi-analytical theory. The cylinders in Cases 1 to 4 have larger drafts compared to those in Cases 5 to 8 and thus it is expected that viscous effects will be larger for the cylinders in Cases 1 to 4. Comparing damping coefficients from the decay tests and those from the numerical simulations, it can be

seen that the differences relative to the decay test results are indeed greater for Cases 1 to 4 than those for Cases 5 to 8.

Figure 6.1a Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *box full ballast*

For Cases 5 to 8 (larger water plane), while the peaks are well predicted, a peculiarity is observed, i.e. just to the right hand side of the peak the experimental results seem to form a narrow trough. This is similar to cancellation frequencies observed in semi-submersibles. This phenomenon is not observed in the numerical results, however. A possible explanation for the phenomenon is the occurrence of Benjamin-Feir instability in the wave tank. It is a hydrodynamic mechanism which leads to the appearance of subharmonic waves with frequencies lying between 0.7 to 0.9 times the old frequency, and the occurrence of a clearly recognized beating or amplitude variation of the wave train (Huang et al., 1982). This beating has indeed been observed in the water surface elevation time history from the experiments. As the phenomenon is only observed in Cases 5 to 8, another possible explanation is the interference effects from the tank walls, which may be significant when the cylinder diameter is larger than 1/5 the tank width. It is also possible that the

phenomenon is a kind of cancellation effect not predicted by the numerical simulations and the semi-analytical theory.

Figure 6.1b Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube full ballast*

Figure 6.1c Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube full ballast with water*

Figure 6.1d Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube bottom & top ballast*

Figure 6.1e Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube bottom & top ballast increased water plane*

Figure 6.1f Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube bottom & top ballast increased water plane with water*

Figure 6.1g Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube bottom ballast increased water plane*

Figure 6.1h Comparison of heave RAOs obtained from experiments, numerical simulations, and 2-DOF theory, for *tube top ballast increased water plane*

From Figures 6.2a to 6.2h, it can be seen that the semi-analytical curves are located slightly to the right of the experiment and the numerical curves. This is probably due to a lower value of main mass used in the semi-analytical calculations, which is expected to give a higher heave natural frequency of the floating cylinder. Another observation that can be made is that the peaks of the semi-analytical curves tend to be lower than those of the numerical ones. This could be due to the different approaches employed in the semi-analytical and the numerical calculations. As to how far agreement can be found between the two results (such as by refinement of the panel model or adding the number of terms in the eigenfunction expansion of the potentials) have not been explored further in this research. It has been shown however that the curves from the two methods exhibit similar trend. Furthermore, irregular frequencies are observed in the semi-analytical heave RAOs for Cases 4 to 8, with Case 8 exhibiting the most pronounced resonance at the irregular frequency. The data points exhibiting irregularities have been removed from the figures. The irregular frequency before removal for Case 8 can be seen in Figure 6.2i.

Figure 6.2a Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *box full ballast*

Figure 6.2b Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube full ballast*

Figure 6.2c Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube full ballast with water*

Figure 6.2d Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube bottom & top ballast*

Figure 6.2e Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube bottom & top ballast increased water plane*

Figure 6.2f Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube bottom & top ballast increased water plane with water*

Figure 6.2g Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube bottom ballast increased water plane*

Figure 6.2h Comparison of heave RAOs obtained from experiments, numerical simulations, and semi-analytical theory, for *tube top ballast increased water plane*

Figure 6.2i Irregular frequency in semi-analytical heave RAO for *tube top ballast increased water plane* without TMD

The comparison of heave RRF s obtained from the numerical simulations, the 2-DOF theory, and the semi-analytical theory is shown in Figures 6.3a to 6.3h. The heave RRF s from the numerical simulations are plotted using a MATLAB routine as shown in Appendix J. The numerical heave RRF is obtained by dividing the heave RAO with TMD, obtained as in section 5.3.2, by the heave RAO without TMD obtained from the numerical simulations. It is found that the numerical, the 2-DOF, and the semi-analytical RRF s match very closely, especially for Cases 1 to 4. Therefore, while the 2-DOF theory tend to overestimate the heave RAO, it gives a very good approximation for the heave RRF . The feasibility of using the simplified 2-DOF RRF formula to predict the behaviour of the structure will be discussed in the next section.

From Figures 6.3a to 6.3h it is observed that when the natural frequency of the TMD is further from that of the main structure, the range of $RRF < 1$ becomes wider. However, except in the vicinities of the natural frequencies of the TMD and the main structure, the RRF in this range increases. On the other hand, when the natural frequency of the TMD is closer to the natural frequency of the main structure, the range of $RRF < 1$ becomes narrower, but the RRF in this range reduces. It is also observed that the higher the TMD mass, the lower is the RRF , i.e. the higher is the effectiveness of the TMD in reducing heave. Also, the higher the damping of the TMD, the flatter is the RRF curve.

Figure 6.3a Comparison of heave RRF s obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *box full ballast*

Figure 6.3b Comparison of heave *RRFs* obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube full ballast*

Figure 6.3c Comparison of heave *RRFs* obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube full ballast with water*

Figure 6.3d Comparison of heave *RRFs* obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube bottom & top ballast*

Figure 6.3e Comparison of heave *RRFs* obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube bottom & top ballast increased water plane*

Figure 6.3f Comparison of heave *RRFs* obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube bottom & top ballast increased water plane with water*

Figure 6.3g Comparison of heave $RRFs$ obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube bottom ballast increased water plane*

Figure 6.3h Comparison of heave $RRFs$ obtained from numerical simulations, 2-DOF theory, and semi-analytical theory, for *tube top ballast increased water plane*

6.2 Parametric study

The parametric study is carried out to determine the optimum TMD parameters (stiffness and damping) which give the maximum reduction of the heave response at resonance. Case 5 (*tube bottom & top ballast increased water plane*) is chosen for this study. The natural frequency of the main structure is $0.135Hz$. As a trial, the optimum frequency and damping of the TMD derived by Den Hartog (1956), as shown in equations (2.1) and (2.2), are used. Setting $\mu = 0.02$, where μ is the ratio of the TMD mass over the effective main mass (mass of the structure and the added mass), the equations yield $f_{optimum} = 0.98$, giving $f_a = 0.132Hz$, and

$\zeta_{optimum} = 0.084$, giving $c_2 = 16,017 \text{ kg/s}$. The heave *RRF* is plotted in Figure 6.4a, from which it is found that the formulas for optimum TMD parameters given by Den Hartog (1956) give good response reduction. At the vicinity of resonance, the heave response of the structure with TMD is reduced to as low as half of that without TMD, while at the other frequencies the *RRF* curve is maintained flat. The corresponding heave RAO is plotted in Figure 6.4b.

It is noted in section 6.1 that the 2-DOF *RRF* formula gives a very good approximation for the heave *RRF*. Therefore, the heave RAO with TMD can be alternatively obtained by multiplying the 2-DOF *RRF* formula with the heave RAO without TMD from numerical simulations. This gives a reasonably good approximation for the heave RAO with TMD, as shown in Figure 6.4b. This finding can be used as an alternative and easier way to obtain the heave RAO of a floating structure with TMD since setting up and solving the matrix equation of motion is unnecessary.

Figure 6.4a Heave *RRF* for $f_a = 0.132 \text{ Hz}$ and $c_2 = 16,017 \text{ kg/s}$

Figure 6.4b Heave RAO for $f_a = 0.132\text{Hz}$ and $c_2 = 16,017\text{kg/s}$

The TMD frequency and damping from Den Hartog's formula can be further optimised. Starting from Den Hartog's formula, the TMD frequency can be varied to get the two peaks of the RAO curve to lie at approximately the same height, while keeping the other TMD parameters constant ($\mu = 0.02$, $c_2 = 16,017\text{kg/s}$). The numerical result is shown in Figure 6.5. The optimum TMD frequency is found to be $f_a = 0.129\text{Hz}$. Then, keeping the TMD frequency at $f_a = 0.129\text{Hz}$, the TMD damping coefficient is varied. The result is shown in Figure 6.6. It is found that for the given TMD mass and frequency, a value of damping between $16,000\text{kg/s}$ to $22,000\text{kg/s}$ will give optimum response reduction.

Figure 6.5 Heave RAOs for various f_a ($\mu = 0.02$, $c_2 = 16,017\text{kg/s}$)

Figure 6.6 Heave RAOs for various c_2 ($\mu = 0.02$, $f_a = 0.129 \text{ Hz}$)

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

It has been shown from the literature review that every floating structure has its own range of natural periods where critical motions may occur. The degree of motions of these structures cause concerns over seasickness, operation efficiency, and even safety. In the current study, the use of tuned mass damper (TMD) is proposed to reduce the heave response of a floating structure. The feasibility study is focused on a freely floating vertical circular cylinder.

A simplified 2-DOF model is used as a preliminary study on the dynamic behaviour of a floating cylinder-TMD system. Using the simplified 2-DOF model, for a selected mass ratio μ as small as 0.05, it is shown that the TMD is capable of reducing the heave motion to as much as half of that of the structure without TMD. The effectiveness of the TMD is measured using the response reduction factor (*RRF*), which is defined as the ratio between the heave RAO with TMD and that without TMD.

The solution of the diffraction and radiation of water waves by a floating vertical circular cylinder has been derived. The computation of the hydrodynamic forces is performed by MATLAB routines, which also plots the variation of the wave excitation forces, the added masses, the radiation dampings, and the heave RAOs of the floating cylinder with and without TMD, for a given set of input parameters: water depth, draft, radius, main mass, TMD mass, TMD stiffness, TMD damping, *z*-coordinate of the centre of gravity, and mass moment of inertia about *x*-, *y*-, and *z*-axis, respectively, passing through the motion reference point.

From the laboratory experiment results, for cases with smaller water plane the effects of the TMD are not clearly seen, since the difference between the TMD frequency and the natural frequency of the main structure is relatively large. It does

not mean that there is no reduction, but since at frequencies higher than the natural frequency of the main structure the response is already small, the reduction is hardly observable from the heave RAO plots. For cases with larger water plane, since the natural frequency of the main structure is closer to that of the TMD, response reductions are seen more clearly.

In general, the numerical simulations and the semi-analytical theory give good predictions of the dynamic behaviour of the structure. Good agreement exists between numerical, semi-analytical, and experimental heave RAOs, although the numerical simulations tend to overpredict the peaks of the experimental heave RAOs for cases with smaller water plane. This is probably due to viscous damping neglected in the simulations. It is also found that while the simplified 2-DOF model overpredicts the heave response of the floating cylinder (due to constant excitation force assumed in the 2-DOF model), the *RRF* derived from the simplified 2-DOF model agrees very well with that obtained from the numerical simulations and the semi-analytical theory. It has been shown that this finding can be used as an alternative and easier way to obtain the heave RAO of a floating structure with TMD.

From the parametric study, it is found that the formulas for optimum TMD parameters given by Den Hartog (1956) give good response reduction at resonance. For a given mass ratio as small as 2%, the heave response is reduced to as much as half of that of the cylinder without TMD at the vicinity of resonance. By further variations of the values from Den Hartog's formulas, optimum TMD parameters which give better response reduction can be obtained.

7.2 Recommendations

In the laboratory experiments, due to the limitations of the scaled model, the natural frequencies of the TMD are all higher than the heave natural frequency of the floating cylinder. They are not close enough to the natural frequency of the cylinder so as to observe significant response reduction at resonance. It is therefore recommended that laboratory experiments are conducted with a TMD having a

natural frequency slightly lower than that of the cylinder, in order to observe the effect of the TMD at resonance. For this purpose, the scaled model should be able to contain a larger elongation of spring and a larger amplitude of the TMD movement. The results from these will be useful in providing further verification for the numerical and the semi-analytical results obtained in the current research.

The current study focuses on the effects of TMD on the heave motion only. Further investigations can be made on the effects of TMD on the other modes of motion. It may also be useful to improve the efficiency of the MATLAB routines created in the current study. Possible extensions of this research can be made on the heave response of a floating cylinder with TMD under irregular wave loadings. Similar studies on the effect of TMD can also be conducted on a moored floating cylinder, or an array of floating cylinders. The practical aspect of design of the TMD in floating structures is also an important area to be investigated.

References

- Barltrop, N. D. P. (Ed.). (1998). *Floating Structures: a guide for design and analysis*. The Centre for Marine and Petroleum Technology.
- Berggren, L. and Johansson, M. (1992). Hydrodynamic coefficients of a wave energy device consisting of a buoy and a submerged plate. *Applied Ocean Research*, 14, 51-58.
- Bhatta, D. D. and Rahman, M. (2003). On scattering and radiation problem for a cylinder in water of finite depth. *International Journal of Engineering Science*, 41, 931-967.
- Black, J. L., Mei, C. C., and Bray, C. G. (1971). Radiation and scattering of water waves by rigid bodies. *Journal of Fluid Mechanics*, 46, 151-164.
- Brebbia, C. A. and Walker, S. (1979). *Dynamic analysis of offshore structures*. Boston: Newnes-Butterworths.
- Brown, D. T., Patel, M. H., Kitney, N., and Ketting, H. (2004). Wave induced motions and deck loads of unbraced semi-submersibles. *International Journal of Maritime Engineering*. The Royal Institution of Naval Architects.
- Calisal, S. M. and Sabuncu, T. (1993). A generalized method for the calculation of hydrodynamic forces on discontinuous vertical cylinders. *Ocean Engineering*, 20, 1-18.
- Chakrabarti, S. K. (1987). *Hydrodynamics of offshore structures*. Southampton: Computational Mechanics.
- Chakrabarti, S. K. (1994). *Offshore structure modeling*. Singapore: World Scientific.
- Cheung, K. F., Phadke, A. C., Smith, D. A., Lee, S. K., and Seidl, L. H. (2000). Hydrodynamic response of a pneumatic floating platform. *Ocean Engineering*, 27, 1407-1440.

Clauss, G., Lehmann, E., and Østergaard, C. (1992). *Offshore Structures, Vol 1 Conceptual Design and Hydromechanics*. Berlin: Springer-Verlag. (Translated by M. J. Shields).

Coates, L. E. (1982). *Diffraction of waves by a vertical circular cylinder: a numerical and experimental study*. Ph.D. thesis, City University.

Dean, R. G. and Dalrymple, R. A. (1991). *Water Wave Mechanics for Engineers and Scientists*. Singapore: World Scientific.

Den Hartog, J. P. (1956). *Mechanical Vibrations*. Fourth Edition. McGraw-Hill.

DHI AWACS Active Wave Absorption Control System Type 420 User Manual. (1994). Danish Hydraulic Institute.

DHI Hydraulic Power Pack Type 301/30 NI, DHI Servo Actuator Type 63/50-710 User Manual. (1999). Danish Hydraulic Institute.

DHI Wave Synthesizer For Windows NT Version 1.0 User Manual. (1999). Danish Hydraulic Institute.

Dimentberg, M., Chen, S., Hou, Z., and Noori, M. (1996). Tuned vibration absorbers for floating bodies. *Journal of Vibration and Control*, 2, 415-429.

Downie, M. J., Graham, J. M. R., Hall, C., Incecik, A., and Nygaard, I. (2000). An experimental investigation of motion control devices for truss spars. *Marine Structures*, 13, 75-90.

Drobyshevski, Y. (2004). Hydrodynamic coefficients of a floating, truncated vertical cylinder in shallow water. *Ocean Engineering*, 31, 269-304.

Eidsmoen, H. (1995). Hydrodynamic parameters for a two-body axisymmetric system. *Applied Ocean Research*, 17, 103-115.

Faltinsen, O. M. (1990). *Sea Loads on Ships and Offshore Structures*. Cambridge University Press.

Fenton, J. D. (1978). Wave forces on vertical bodies of revolution. *Journal of Fluid Mechanics*, 85, 241-255.

Fertis, D. G. (1995). *Mechanical and Structural Vibrations*. Wiley.

Garrett, C. J. R. (1971). Wave forces on a circular dock. *Journal of Fluid Mechanics*, 46, 129-139.

Havelock, T. H. (1940). The pressure of water waves upon a fixed obstacle. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 175, No. 963, pp. 409-421.

Hosoda, R., Nomura, T., Yamaguchi, K., and Murata, B. (2000). Concept of eco-platform. *Proceedings of 23rd Annual UJNR/MFP Meeting*.

Huang, X., Hoff, J. R., and Naess, A. (1982). On the Behavior of Semisubmersible Platforms at Large Angles. *Proceedings of 14th Annual Offshore Technology Conference, Houston, Texas, 3-6 May 1982*, pp. 193-203.

Hwang, Y.-L. (1993). Dynamic response of floating structures in water of variable depth. In *International Conference on Offshore Mechanics and Arctic Engineering, Vol. I*, pp. 191-198. New York: ASME.

Kano, T., Makabe, T., Wada, K., Matsuura, M., Tamura, K., Kihara, K., Sasajima, K., and Nishigaki, M. (2000). Study on motion suppression systems applied to floating structures for coastal development. *Proceedings of 23rd Annual UJNR/MFP Meeting*.

Korde, U. A. (1999). On providing a reaction for efficient wave energy absorption by floating devices. *Applied Ocean Research*, 21, 235-248.

Kuo, C. (1986). *Stationing and stability of semi-submersibles* (Advances in underwater technology, ocean science and offshore engineering, Vol. 9).

Lamb, H. (1993). *Hydrodynamics*, 6th ed. Cambridge: Cambridge University Press.

Lee, C.-H. (1995). *WAMIT theory manual*, MIT Report 95-2. Departement of Ocean Engineering, Massachusetts Institute of Technology.

Lee, C.-H. and Newman, J. N. (2000). Wave effects on large floating structures with air cushions. *Marine Structures*, 13, 315-330.

Lee, C.-H. and Newman, J. N. (2004). Computation of wave effects using the panel method. In S. Chakrabarti (Ed.), *Numerical models in fluid-structure interaction*, Preprint. Southampton: WIT Press.

Leite, A. J. P., Nishimoto, K., Aranha, J. A. P., and Morooka, C. K. (1992). Minimization of vertical wave exciting force and heave motion of a production semisubmersible with rigid risers. In *International Conference on Offshore Mechanics and Arctic Engineering*, Vol. I-A, pp. 215-222. New York: ASME.

Linton, C. M. and Evans, D. V. (1992). The Radiation and Scattering of Surface Waves by a Vertical Circular Cylinder in a Channel. *Philosophical Transactions: Physical Sciences and Engineering*, 338, 325-357.

Maeda, H. (2001). Marine Hydrodynamics and Dynamics of a Floating Structure. In F. El-Hawary (Ed.), *The Ocean Engineering Handbook*. Boca Raton: CRC Press LLC.

McIver, P. (1994). Low-frequency asymptotics of hydrodynamic forces on fixed and floating structures. In M. Rahman (Ed.), *Ocean Waves Engineering*. Boston: Computational Mechanics Publications.

Mei, C. C. (1989). *The Applied Dynamics of Ocean Surface Waves*. World Scientific.

Mei, C. C., Stiassnie, M., and Yue, D. K.-P. (2005). *Theory and Applications of Ocean Surface Waves*. Singapore: World Scientific.

Miles, J. and Gilbert, F. (1968). Scattering of gravity waves by a circular dock. *Journal of Fluid Mechanics*, 34, 783-793.

Moan, T. (2003). Marine Structures for The Future. *Centre for Offshore Research and Engineering Report No. 2003-01*. National University of Singapore.

Moubayed, W. I. and Williams, A. N. (1995). The sum- and difference-frequency wave loads and motions of a freely floating circular cylinder. *Journal of Fluids and Structures*, 9, 323-355.

Newman, J. N. (1980). *Marine Hydrodynamics*. Third printing. The MIT Press.

Nishimoto, K. and Leite, A. J. P. (1993). Effect of lateral keel and blisters on semi-submersibles for the minimization of heave motion. *Proceedings of the Third International Offshore and Polar Engineering Conference, Singapore, 6-11 June 1993*, pp. 621-628. International Society of Offshore and Polar Engineers.

Patel, M. (2001). Fluid loading on offshore structures. In V. Li and D. Patel (Eds.), *Advanced Offshore Engineering* (pp. 6.1-6.60). London: Bentham Press.

Qualisys Track Manager User Manual. (2004). Qualisys Medical AB.

Rahman, M. and Bhatta, D. D. (1993). Evaluation of added mass and damping coefficient of an oscillating circular cylinder. *Appl. Math. Modelling*, 17, 70-79.

Rao, S. S. (2004). *Mechanical Vibrations*. Fourth Edition. Prentice-Hall.

Sabuncu, T. and Calisal, S. (1981). Hydrodynamic coefficients for vertical circular cylinders at finite depth. *Ocean Engineering*, 8, 25-63.

Sarpkaya, T. and Isaacson, M. (1981). *Mechanics of Wave Forces on Offshore Structures*. New York: Van Nostrand Reinhold Company.

SESAM User Manual: HydroD (Wave load analysis of fixed and floating structures). (2004). Det Norske Veritas.

SESAM User Manual: Manager (Managing SESAM Programs and Files). (2002). Det Norske Veritas.

SESAM User Manual: Prefem (Preprocessor for Generation of Finite Element Models). (2003). Det Norske Veritas.

SESAM User Manual: Wadam (Wave Analysis by Diffraction and Morison Theory). (2004). Det Norske Veritas.

Shuku, M., Inoue, S., Horiba, S., Kobayashi, E., and Simamune, S. (2001). Overview of Mega-Float and its utilization. *Mitsubishi Heavy Industries, Ltd. Technical Review Vol. 38 No. 2*.

Takagi, K., Shimada, K., and Ikebuchi, T. (2000). An anti-motion device for a very large floating structure. *Marine Structures, 13*, 421-436.

Tao, L. and Cai, S. (2004). Heave motion suppression of a Spar with a heave plate. *Ocean Engineering, 31*, 669-692.

Taylor, R. and Palo, P. (2000). U. S. Mobile Offshore Base Technological Report. *Proceedings of 23rd Annual UJNR/MFP Meeting*.

Treacle III, T. W., Mook, D. T., Liapis, S. I., and Nayfeh, A. H. (2000). A time-domain method to evaluate the use of moving weights to reduce the roll motion of a ship. *Ocean Engineering, 27*, 1321-1343.

University of Hawaii at Manoa College of Engineering. (2002). Riding the Wave of the Future. Retrieved February 8, 2005, from <http://xtreme.eng.hawaii.edu/research-projects/vlfs/>

Watanabe, E., Utsunomiya, T., and Wang, C. M. (2004a). Hydroelastic analysis of pontoon-type VLFS: a literature survey. *Engineering Structures, 26*, 245-256.

Watanabe, E., Wang, C. M., Utsunomiya, T., and Moan, T. (2004b). Very large floating structures: applications, analysis and design. *Centre for Offshore Research and Engineering Report No. 2004-02*. National University of Singapore.

Weaver, Jr., W., Timoshenko, S. P., and Young, D. H. (1990). *Vibration problems in engineering*. New York: Wiley.

Webster, B. N., Birmingham, R. W., Jones, E. B., and Roskilly, A. P. (2003). The application of artificial intelligence to roll stabilisation for a range of loading and operating conditions. *International Journal of Maritime Engineering*. The Royal Institution of Naval Architects.

Wehausen, J. V. and Laitone, E. V. (1960). Surface Waves. In S. Flugge (Ed.), *Encyclopedia of Physics, Vol. 9, Fluid Dynamics III*. Berlin: Springer-Verlag.

Williams, A. N., Lee, H. S., and Huang, Z. (2000a). Floating pontoon breakwaters. *Ocean Engineering*, 27, 221-240.

Williams, A. N., Li, W., and Wang, K.-H. (2000b). Water wave interaction with a floating porous cylinder. *Ocean Engineering*, 27, 1-28.

Wilson, J. (2003). Battle island: Military Joint Mobile Offshore Base. *Popular Mechanics*, April 2003. Retrieved 14 February 2005, from <http://www.popularmechanics.com/science/defense/1281531.html>

Wu, B. J., Zheng, Y. H., You, Y. G., Sun, X. Y., and Chen, Y. (2004). On diffraction and radiation problem for a cylinder over a caisson in water of finite depth. *International Journal of Engineering Science*, 42, 1193-1213.

Yeung, R. W. (1981). Added mass and damping of a vertical cylinder in finite-depth waters. *Applied Ocean Research*, 3, 119-133.

Zheng, Y. H., Shen, Y. M., You, Y. G., Wu, B. J., and Rong, L. (2005). Hydrodynamic properties of two vertical truncated cylinders in waves. *Ocean Engineering*, 32, 241-271.

Živanović, S., Pavic, A., and Reynolds, P. (2005). Vibration serviceability of footbridges under human-induced excitation: a literature review. *Journal of Sound and Vibration*, 279, 1-74.

Appendix A

Formulas for calculating moment of inertias

The formulas for calculating moment of inertias from geometry are given below.

The density of acrylic, water, and steel are taken as $\rho_A = 1,190 \text{ kg/m}^3$, $\rho_W = 1,000 \text{ kg/m}^3$, and $\rho_S = 7,850 \text{ kg/m}^3$, respectively.

It can be shown that the moment of inertia of a hollow cylinder about its principal transverse axes is

$$I_{xx} = I_{yy} = \frac{1}{12} \pi h \rho (R_2^2 - R_1^2) (h^2 + 3R_1^2 + 3R_2^2), \quad (\text{A.1})$$

while the moment of inertia of a solid cylinder about its principal transverse axes is

$$I_{xx} = I_{yy} = \frac{1}{12} \pi h \rho R^2 (h^2 + 3R^2), \quad (\text{A.2})$$

and the moment of inertia of a rectangular block about its principal x-axis is

$$I_{xx} = \rho \left(\frac{8}{3} abc (b^2 + c^2) \right). \quad (\text{A.3})$$

The moment of inertia of a rectangular block about its principal y- and z-axis are obtained similarly.

The moment of inertia of a solid cylinder about its principal z-axis is

$$I_{zz} = \frac{1}{2} \pi h \rho R^4 \quad (\text{A.4})$$

while the moment of inertia of a hollow cylinder about its principal z-axis is

$$I_{zz} = \frac{1}{2} \pi h \rho (R_2^2 - R_1^2)^2 \quad (\text{A.5})$$

The moment of inertia of each element of the structure about x - or y -axis passing through the centre of gravity of the structure is calculated based on the parallel axis theorem. Taking the summation of the moment of inertias of all the elements, the moment of inertia of the structure about x -, y -, or z -axis passing through its centre of gravity is obtained.

The moment of inertia of the structure about its motion reference point, i.e. the point where a vertical line through the centre of gravity crosses the waterline, is obtained similarly.

Appendix B

MATLAB routine for computing eigenvalues

```

function m = findeigenval(h,omega,incr)
%find m_0 to m_N, for N = 20
%h = water depth
%omega = angular frequency
%incr = increment used in the algorithm

g = 9.81;
nu = omega.^2.*h/g;
N = 20;

%preallocate vector of m (eigenvalues*h)
m = zeros(N+1,1);

%create vector of m (eigenvalues*h)
for k = 1:N+1
    if k == 1
        m(k) = abs(fzero(@(m) m.*tanh(m)-nu, 1));
    else
        m(k) = fzero(@(m) i*m.*tanh(i*m)-nu, (k-1)*pi);
        n = 1;
        while ((round(100000*(k-1.5)*pi) >= round(100000*m(k)))...
            | (round(100000*m(k)) >= round(100000*(k-0.5)*pi)))
            m(k) = fzero(@(m) i*m.*tanh(i*m)-nu, (k-1.5+...
                incr*n)*pi);
            if ((round(100000*(k-1.5)*pi) < round(100000*m(k)))...
                & (round(100000*m(k)) < round(100000*...
                    (k-0.5)*pi)))
                break
            end
            n = n+1;
            if n == 1/incr
                disp('Change incr value and repeat command');
                break
            end
        end
    end
end

format long, m;

```


Appendix C

MATLAB routine for computing diffraction and radiation theory results

```
function [FD1,FD3,FD5,...
    A11,A15,A33,A51,A55,...
    C11,C15,C33,C51,C55,...
    U3,UTMD3] = ...
    diffrad(h,d,R,omega,m1,m2,k2,c2,zG,I11,I22,I33)

%Function to compute diffraction forces, added masses, dampings,
%and displacements for predetermined omegas (angular frequencies):
%0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.53 0.55 0.58
%0.6 0.63 0.65 0.68 0.7 0.73 0.75 0.78 0.8 0.83 0.85 0.88
%0.9 0.93 0.95 0.98 1 1.03 1.05 1.08 1.1 1.13 1.15 1.18
%1.2 1.23 1.25 1.28 1.3 1.33 1.35 1.38 1.4 1.45 1.5 1.55
%1.6 1.65 1.7 1.75 1.8 1.85 1.9 1.95 2 2.05 2.1 2.15 2.2
%2.25 2.3 2.35 2.4 2.45 2.5 2.6 2.7 2.8 2.9 3 3.5 4 4.5 5
%-----
%h = water depth
%d = draft
%R = radius of cylinder
%omega = angular frequency
%m1 = main mass
%m2 = TMD mass
%k2 = TMD stiffness
%c2 = TMD damping
%zG = z-coordinate of centre of gravity
%I11 = Ixx about motion reference point
%I22 = Iyy about motion reference point
%I33 = Izz about motion reference point

%rho (water density)
rho = 1000;
%g (acceleration of gravity)
g = 9.81;
%wpa (water plane area)
wpa = pi*R^2;
%Iwpa (second moment of area of water plane)
Iwpa = pi/4*R^4;
%zb (z-coordinate of centre of buoyancy)
zb = -d/2;
%ome (predetermined omegas)
ome = [0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.53 0.55 0.58...
    0.6 0.63 0.65 0.68 0.7 0.73 0.75 0.78 0.8 0.83 0.85 0.88...
    0.9 0.93 0.95 0.98 1 1.03 1.05 1.08 1.1 1.13 1.15 1.18...
    1.2 1.23 1.25 1.28 1.3 1.33 1.35 1.38 1.4 1.45 1.5 1.55...
    1.6 1.65 1.7 1.75 1.8 1.85 1.9 1.95 2 2.05 2.1 2.15 2.2...
    2.25 2.3 2.35 2.4 2.45 2.5 2.6 2.7 2.8 2.9 3 3.5 4 4.5 5];

%set number of terms
N = 20;
```

```

%-----
%obtain set of eigenvalues for a given omega
%-----

%Input the m values from excel file
%The m values (eigenvalues*h) were computed separately using the
%function findeigenval(h,omega,incr)
m = zeros(N+1,76);

m(:,1:5) = ...
[
0.203310071 0.307601307 0.415167091 0.527289031 0.645420431
3.12856033 3.112121907 3.088838997 3.058461085 3.020699139
6.276689184 6.268550893 6.257125103 6.242383494 6.224295404
9.420449668 9.415033948 9.40744248 9.397667024 9.385698756
12.56312504 12.55906581 12.55337892 12.5460609 12.53710807
15.70536705 15.70212061 15.69757357 15.69172416 15.6845705
18.84739251 18.84468758 18.8408995 18.83602725 18.83006975
21.98929428 21.98697599 21.98372964 21.97955459 21.97445017
25.13111875 25.12909037 25.12625015 25.12259764 25.1181324
28.2728917 28.27108877 28.26856433 28.26531806 28.26134965
31.41462858 31.413006 31.41073413 31.40781275 31.40424162
34.55633924 34.5548642 34.55279894 34.55014332 34.54689715
37.69803023 37.69667813 37.69478504 37.69235084 37.68937539
40.83970609 40.83845801 40.83671059 40.83446372 40.8317173
43.98137006 43.98021114 43.97858857 43.97650226 43.97395212
47.12302452 47.12194288 47.1204285 47.11848132 47.11610128
50.26467126 50.26365722 50.26223751 50.26041207 50.25818084
53.40631163 53.40535725 53.40402106 53.40230303 53.4002031
56.5479467 56.54704534 56.5457834 56.54416084 56.54217761
59.68957731 59.68872339 59.68752788 59.68599073 59.68411192
62.83120412 62.8303929 62.82925717 62.8277969 62.82601205
];

m(:,6:10) = ...
[
0.771237078 0.906686548 1.054025365 1.215823379 1.321010106
2.975262613 2.921919406 2.860587579 2.791465041 2.746494723
6.202832524 6.177975013 6.149719127 6.11808642 6.097507978
9.371529648 9.3551542 9.336571538 9.315787878 9.302267405
12.52651709 12.5142857 12.50041357 12.48490328 12.47481336
15.67611089 15.66634419 15.65527023 15.64289034 15.63483685
18.82302601 18.81489539 18.80567778 18.79537396 18.78867103
21.96841574 21.96145091 21.95355562 21.94473034 21.93898924
25.11285402 25.10676222 25.09985696 25.09213856 25.08711753
28.25665882 28.25124536 28.24510925 28.23825071 28.23378905
31.40002055 31.39514939 31.38962811 31.38345689 31.37944234
34.54306026 34.53863257 34.53361405 34.52800483 34.52435588
37.68585857 37.68180029 37.67720056 37.67205945 37.66871502
40.82847123 40.82472546 40.82047996 40.81573481 40.81264797
43.97093808 43.96746009 43.96351815 43.9591123 43.95624618
47.1132883 47.11004236 47.10636343 47.10225158 47.09957672
50.25554378 50.25250084 50.24905202 50.24519737 50.24268982
53.39772123 53.3948574 53.39161159 53.38798385 53.3856239
56.53983369 56.53712904 56.53406366 56.53063759 56.52840883
59.6818914 59.67932917 59.67642522 59.67317956 59.67106818
62.8239026 62.82146853 62.81870985 62.81562656 62.8136208

```

];

m(:,11:15) = ...

[

```

1.39490246 1.511858727 1.594170078 1.724564078 1.816325723
2.715191936 2.666485106 2.633013985 2.581653169 2.546874894
6.083134322 6.060611046 6.044967679 6.020590105 6.003750381
9.292819338 9.278000948 9.267695042 9.251603826 9.240460434
12.46776151 12.45669808 12.44900029 12.43697368 12.42863834
15.62920789 15.62037561 15.614229 15.60462315 15.59796314
18.78398586 18.77663394 18.77151701 18.7635191 18.75797285
21.93497629 21.92867893 21.92429569 21.91744395 21.91269196
25.08360784 25.07810008 25.07426628 25.06827305 25.06411616
28.23067033 28.22577602 28.22236913 28.21704306 28.21334872
31.37663614 31.37223221 31.36916661 31.36437395 31.36104947
34.52180522 34.51780228 34.51501578 34.51065935 34.5076374
37.66637722 37.66270831 37.6601543 37.65616128 37.65339136
40.81049021 40.80710384 40.80474649 40.8010609 40.7985042
43.95424271 43.95109846 43.94890965 43.94548751 43.94311353
47.09770693 47.09477248 47.09272969 47.08953585 47.08732022
50.24093698 50.23818607 50.23627105 50.23327693 50.23119984
53.38397425 53.38138526 53.37958295 53.37676505 53.37481019
56.52685088 56.52440581 56.52270368 56.5200424 56.51819619
59.66959228 59.66727596 59.66566346 59.6631423 59.66139329
62.81221873 62.81001828 62.80848644 62.80609139 62.80442985
];

```

m(:,16:20) = ...

[

```

1.961519042 2.063479826 2.224308773 2.336817809 2.513489056
2.494338287 2.459337431 2.407338018 2.373268769 2.323471939
5.977649624 5.959715792 5.932068503 5.913174203 5.884204762
9.223132892 9.211180633 9.192666633 9.179943651 9.160308176
12.41566364 12.40670259 12.39280046 12.38322956 12.36842715
15.58759137 15.58042397 15.56929677 15.56163005 15.54976126
18.74933337 18.74336125 18.73408624 18.72769291 18.71779035
21.90528859 21.90017002 21.89221884 21.88673661 21.87824263
25.0576393 25.05316078 25.04620284 25.04140464 25.03396898
28.20759218 28.20361141 28.19742619 28.19316036 28.18654881
31.35586902 31.35228642 31.34671947 31.34287973 31.33692801
34.50292818 34.49967133 34.4946103 34.49111931 34.48570775
37.64907478 37.64608938 37.64144998 37.63824968 37.63328847
40.79451981 40.79176408 40.78748148 40.78452719 40.77994717
43.93941385 43.93685499 43.93287822 43.93013484 43.92588163
47.08386726 47.081479 47.07776731 47.07520673 47.07123682
50.22796275 50.22572378 50.22224404 50.21984342 50.21612144
53.37176357 53.36965631 53.36638123 53.36412177 53.36061858
56.51531886 56.51332868 56.51023553 56.50810155 56.50479287
59.65866743 59.656782 59.65385163 59.65182993 59.6486953
62.80184031 62.80004915 62.79726529 62.79534465 62.79236667
];

```

m(:,21:25) = ...

[

```

2.636485727 2.828656398 2.961780606 3.168783131 3.311544795
2.291353257 2.245082605 2.215632722 2.173696376 2.147274984
5.864515265 5.834494019 5.814203491 5.783440149 5.762765555
9.146863183 9.126187673 9.112080664 9.090463761 9.075766253

```

```

12.35826687 12.34259863 12.33187452 12.31538288 12.30412604
15.54160567 15.52901293 15.52038161 15.50708682 15.49799588
18.71098185 18.70046196 18.69324591 18.6821214 18.67450719
21.87240054 21.86337019 21.85717305 21.84761436 21.84106811
25.02885364 25.02094457 25.01551531 25.00713821 25.00139903
28.18199971 28.17496485 28.17013474 28.16268038 28.15757206
31.33283244 31.32649815 31.32214841 31.3154343 31.31083243
34.4819836 34.47622318 34.47226709 34.46615988 34.46197342
37.62987402 37.62459226 37.62096462 37.61536392 37.61152427
40.77679491 40.77191846 40.76856898 40.76339736 40.7598516
43.92295419 43.91842533 43.91531445 43.91051096 43.90721737
47.06850429 47.06427681 47.06137283 47.05688861 47.05381378
50.2135595 50.20959582 50.20687296 50.20266826 50.19978498
53.35820719 53.35447635 53.35191336 53.34795542 53.34524126
56.50251532 56.49899149 56.49657066 56.49283215 56.49026838
59.64653753 59.64319895 59.64090535 59.63736323 59.63493409
62.79031671 62.7871449 62.78496582 62.78160052 62.77929258
];

```

```

m(:,26:30) = ...
[
3.532635951 3.684561175 3.919094106 4.079804737 4.327298448
2.109964049 2.086619317 2.05382803 2.033395605 2.004776795
5.731597584 5.71076944 5.67954628 5.658797137 5.627862042
9.053323735 9.038118421 9.014982167 8.99936187 8.975677767
12.28686192 12.27510922 12.25713237 12.2449266 12.22630557
15.4840256 15.47449439 15.45988014 15.44993141 15.43470999
18.66279358 18.65479254 18.64250838 18.63413394 18.62130104
21.8309909 21.82410271 21.81351878 21.80629723 21.79522051
24.99256051 24.98651623 24.97722423 24.97088065 24.9611446
28.14970283 28.14431971 28.13604123 28.1303874 28.12170631
31.30374191 31.2988904 31.29142757 31.2863294 31.27849913
34.45552197 34.45110699 34.4443144 34.43967315 34.43254309
37.6056066 37.60155639 37.59532412 37.59106509 37.58452107
40.75438637 40.75064545 40.74488848 40.74095379 40.73490732
43.90214048 43.89866511 43.89331632 43.88966026 43.88404138
47.04907381 47.04582888 47.0408344 47.03742026 47.03217272
50.19534009 50.192297 50.18761293 50.18441077 50.17948872
53.34105691 53.33819208 53.33378217 53.33076729 53.32613283
56.48631578 56.48360951 56.47944352 56.47659526 56.47221674
59.63118894 59.62862463 59.62467704 59.621978 59.61782869
62.77573421 62.77329773 62.76954682 62.76698217 62.76303933
];

```

```

m(:,31:35) = ...
[
4.496530265 4.756665911 4.934252046 5.206836556 5.392679933
1.986978733 1.962075067 1.94659308 1.924924634 1.91144454
5.607414187 5.577086193 5.557140309 5.527698869 5.508425195
8.959743686 8.935668472 8.919527756 8.895224952 8.878987738
12.21369527 12.19450703 12.18154618 12.16187543 12.14862281
15.42437003 15.40858355 15.39788223 15.38157812 15.37054881
18.61256888 18.59921249 18.59014061 18.57628951 18.56689831
21.78767564 21.77612237 21.76826578 21.75625461 21.74809959
24.95450851 24.94433943 24.93741874 24.92682936 24.91963313
28.11578658 28.10671071 28.10053069 28.09106908 28.08463523
31.27315784 31.26496587 31.25938557 31.25083854 31.245024
34.42767825 34.42021504 34.41512971 34.40733838 34.40203617

```

```

37.58005527 37.57320284 37.56853268 37.56137574 37.55650401
40.73078046 40.72444713 40.72013001 40.71351289 40.70900772
43.88020592 43.87431906 43.87030574 43.86415336 43.85996395
47.02859042 47.02309156 47.01934235 47.01359417 47.0096795
50.17612836 50.17096976 50.16745223 50.16205873 50.15838523
53.32296862 53.31811079 53.31479811 53.30971831 53.30625816
56.46922711 56.46463705 56.46150677 56.45670635 56.45343627
59.61499544 59.61064527 59.60767844 59.60312839 59.60002868
62.76034696 62.75621292 62.75339336 62.74906898 62.74612285
];

```

```

m(:,36:40) = ...
[
5.677604916 5.871652901 6.168861418 6.371087221 6.680551118
1.892557473 1.880791628 1.864280132 1.853975992 1.839488539
5.480099428 5.461632405 5.434595859 5.417032469 5.391403193
8.854622911 8.838399033 8.814134769 8.798030406 8.774021394
12.12856083 12.11507914 12.09472231 12.08107708 12.06052507
15.35377966 15.34245903 15.32528217 15.3137099 15.29618688
18.55258489 18.54289714 18.52815734 18.51819815 18.5030713
21.73565167 21.72721317 21.71435224 21.70564693 21.69239928
24.90863797 24.90117651 24.88979195 24.88207682 24.87032116
28.07479825 28.06811794 28.05791733 28.05099888 28.04044785
31.23612957 31.23008623 31.22085308 31.2145871 31.20502504
34.39392253 34.38840756 34.3799782 34.37425517 34.36551755
37.54904705 37.54397696 37.5362251 37.53096029 37.52291934
40.70211036 40.69741968 40.69024614 40.68537282 40.67792771
43.85354891 43.84918545 43.84251102 43.83797582 43.83104571
47.00368433 46.99960585 46.99336636 46.98912598 46.9826452
50.15275876 50.14893065 50.14307342 50.13909227 50.13300679
53.30095798 53.2973515 53.29183279 53.28808131 53.28234617
56.44842683 56.44501791 56.43980102 56.43625437 56.43083181
59.59527993 59.59204817 59.58710203 59.58373916 59.57859715
62.74160911 62.73853711 62.73383517 62.73063811 62.72574926
];

```

```

m(:,41:45) = ...
[
6.890941878 7.212648003 7.431197878 7.76514041 7.991846884
1.830429611 1.817667119 1.809670261 1.798380721 1.791292046
5.374804536 5.350649384 5.335044618 5.312386338 5.297777954
8.758135931 8.734524568 8.718947928 8.695860773 8.680671393
12.04678336 12.02613713 12.01236614 11.99172578 11.97799141
15.28440523 15.26660107 15.25465435 15.23663666 15.22457056
18.49286799 18.47739664 18.46697854 18.45120806 18.44060629
21.68344554 21.66983998 21.66065784 21.64672561 21.63733665
24.86236521 24.85025869 24.84207605 24.82964082 24.82124689
28.03330053 28.02241375 28.01504786 28.0038415 27.99626833
31.19854333 31.18866331 31.18197355 31.1717877 31.16489839
34.35959172 34.35055421 34.34443148 34.33510339 34.32879027
37.51746392 37.50914042 37.50349898 37.49490021 37.48907787
40.67287502 40.66516352 40.65993511 40.65196303 40.6456299
43.82634141 43.81915981 43.81428937 43.80686096 43.80182766
46.97824507 46.97152642 46.96696895 46.96001628 46.95530419
50.12887439 50.12256349 50.11828184 50.11174871 50.10732006
53.27845117 53.27250198 53.26846514 53.26230456 53.25812776
56.42714869 56.42152246 56.41770429 56.41187664 56.40792502
59.57510425 59.56976807 59.56614636 59.56061795 59.55686878

```

```
62.72242807 62.71735377 62.71390949 62.70865142 62.70508522
];
```

```
m(:,46:50) = ...
[
8.572885425 9.174312124 9.796126462 10.43832825 11.10091744
1.774954607 1.76037275 1.747307217 1.735556792 1.724951708
5.26297313 5.230556087 5.200422796 5.172449984 5.146503713
8.643480259 8.607515551 8.572885328 8.539666139 8.507905608
11.94381945 11.9100016 11.87667741 11.84397241 11.81199648
15.19426947 15.16386709 15.13347793 15.10321317 15.07317889
18.4138316 18.38673782 18.35941029 18.33193525 18.30439869
21.61353847 21.58932306 21.564753 21.53989308 21.51480964
24.79991837 24.77813406 24.75594022 24.73338554 24.71052074
27.97699192 27.95725104 27.93708046 27.91651718 27.89560022
31.14734047 31.12932441 31.11087683 31.09202625 31.07280305
34.31268554 34.29613635 34.27916344 34.26178915 34.24403745
37.47421428 37.45892325 37.44322119 37.42712587 37.41065643
40.63276957 40.6185669 40.60396818 40.58898772 40.57364103
43.78896517 43.77571166 43.76207788 43.74807553 43.7337173
46.94325805 46.93083858 46.91805462 46.90491586 46.89143278
50.09599508 50.0843136 50.07228299 50.05991132 50.04720739
53.24744411 53.23641982 53.22506108 53.2133747 53.20136811
56.39781518 56.38737954 56.37662336 56.36555244 56.35417308
59.54727518 59.53736963 59.52715665 59.51664118 59.50582866
62.6959584 62.68653255 62.67681156 62.6667997 62.65650166
];
```

```
m(:,51:55) = ...
[
11.78389399 12.4872579 13.21100917 13.95514781 14.7196738
1.715348203 1.706624054 1.698674928 1.691411403 1.684756524
5.122445931 5.100139246 5.079450241 5.060251637 5.042423593
8.477625775 8.44882682 8.421490869 8.395585652 8.371067862
11.78084298 11.75058845 11.72129296 11.69300086 11.66574201
15.0434745 15.01419148 14.98541231 14.95720984 14.92964687
18.27688518 18.24947683 18.22225218 18.1952853 18.16864503
21.48956988 21.46424112 21.43889005 21.41358194 21.38838
24.68739822 24.66407154 24.64059501 24.61702308 24.59340989
27.87437043 27.85287018 27.83114309 27.80923368 27.78718704
31.05323932 31.03336871 31.01322625 30.99284815 30.97227156
34.2259338 34.20750516 34.18877982 34.16978729 34.15055814
37.39383336 37.37667844 37.35921469 37.3414663 37.32345854
40.55794475 40.54191668 40.52557573 40.50894186 40.49203608
43.71901688 43.70398895 43.68864917 43.67301417 43.65710151
46.87761678 46.86348007 46.84903575 46.83429777 46.8192809
50.03418073 50.02084161 50.00720108 49.99327092 49.97906368
53.18904935 53.17642716 53.16351089 53.15031061 53.13683701
56.34249216 56.33051713 56.318256 56.3057174 56.29291053
59.49472497 59.48333654 59.47167027 59.4597336 59.44753449
62.64592254 62.63506789 62.62394371 62.61255646 62.60091306
];
```

```
m(:,56:60) = ...
[
15.50458716 16.30988787 17.13557594 17.98165138 18.84811417
1.678643804 1.673015578 1.66782165 1.663018163 1.658566672
5.025854369 5.010440528 4.996086844 4.982705999 4.970218157
```

```

8.347886115 8.325983494 8.305299657 8.285772549 8.26733976
11.63953313 11.61437938 11.590276 11.56720986 11.54516096
14.90277606 14.87664015 14.8512723 14.82669672 14.80292937
18.14239428 18.11658967 18.09128112 18.06651184 18.04231828
21.36334467 21.33853305 21.31399841 21.28978977 21.26595164
24.56980873 24.54627148 24.52284822 24.49958673 24.47653213
27.76504838 27.74286273 27.72067451 27.69852716 27.6764628
30.9515343 30.93067458 30.90973076 30.88874099 30.86774297
34.13112386 34.11151661 34.09176909 34.07191426 34.05198516
37.30521763 37.28677061 37.26814521 37.24936971 37.23047274
40.47488029 40.4574973 40.43991062 40.42214444 40.40422348
43.64092961 43.62451775 43.60788593 43.59105488 43.5740459
46.80400075 46.78847368 46.7727168 46.75674789 46.74058534
49.96459263 49.94987176 49.93491573 49.91973989 49.90436014
53.12310147 53.109116 53.09489326 53.08044652 53.0657896
56.27984519 56.26653177 56.25298122 56.23920508 56.22521542
59.43508144 59.42238347 59.40945013 59.39629148 59.38291811
62.58902091 62.57688791 62.5645224 62.55193322 62.53912966
];

```

```
m(:,61:65) = ...
```

```

[
19.73496432 20.64220183 21.56982671 22.51783894 23.48623853
1.654433361 1.650588393 1.647005359 1.64366082 1.640533912
4.958550473 4.947636573 4.937416027 4.927833838 4.918839961
8.24993956 8.233511681 8.217997879 8.203342313 8.189491797
11.52410386 11.50400888 11.48484325 11.466572 11.44915883
14.77997874 14.75784668 14.73652928 14.71601767 14.69629882
18.01873033 17.99577158 17.97345968 17.95180679 17.93082004
21.24252377 21.21954111 21.19703377 21.17502711 21.15354189
24.45372656 24.43120891 24.40901464 24.38717565 24.36572022
27.65452189 27.63274293 27.61116224 27.58981369 27.56872858
30.84677368 30.82586904 30.80506372 30.7843909 30.76388204
34.03201465 34.0120352 33.9920787 33.97217619 33.9523577
37.21148311 37.19242968 37.17334109 37.15424568 37.13517124
40.38617283 40.36801786 40.34978404 40.33149682 40.31318147
43.5568808 43.53958177 43.52217133 43.50467213 43.4871069
46.7242481 46.7077556 46.69112763 46.67438429 46.6575459
49.88879299 49.87305543 49.85716487 49.84113912 49.82499626
53.05093691 53.03590332 53.02070419 53.00535527 52.98987266
56.21102483 56.19664641 56.18209368 56.16738061 56.15252153
59.36934107 59.35557189 59.34162256 59.32750544 59.31323333
62.52612148 62.51291887 62.49953246 62.48597326 62.47225266
];

```

```
m(:,66:70) = ...
```

```

[
24.47502548 25.4841998 27.5637105 29.72477064 31.96738022
1.637606013 1.634860463 1.629858134 1.625424409 1.621475715
4.910388841 4.902438999 4.887895305 4.874944641 4.863366571
8.17639593 8.164007151 8.141174701 8.120669309 8.102206897
11.4325667 11.41675844 11.38734661 11.36063731 11.33636145
14.67735631 14.65917095 14.62498477 14.59355317 14.56467683
17.91050203 17.89085137 17.85352938 17.8187812 17.78649955
21.13259452 21.11219725 21.0730833 21.03622749 21.00161174
24.34467299 24.32405506 24.28417418 24.24617959 24.2101261
27.54793551 27.52746025 27.48755231 27.44915508 27.41237594
30.74356674 30.72347258 30.68404729 30.64578318 30.60882131

```

```

33.93265206 33.91308675 33.87447941 33.83672352 33.79997868
37.11614489 37.09719288 37.05961191 37.02261641 36.98637336
40.29486293 40.27656568 40.24012991 40.20405588 40.16851028
43.46949829 43.45186876 43.4166352 43.38157778 43.34685802
46.64063286 46.62366559 46.58964942 46.55565698 46.52184167
49.8087546 49.79243258 49.75962139 49.72670968 49.69384119
52.97427274 52.9585721 52.92693578 52.89509823 52.86319294
56.13753107 56.12242418 56.09192178 56.06113696 56.03019291
59.29881931 59.28427681 59.25486122 59.22509809 59.19510066
62.45838241 62.44437457 62.41599569 62.38721735 62.35814338
];

```

```
m(:,71:75) = ...
```

```

[
34.29153925 36.69724771 49.9490316 65.23955148 82.56880734
1.617943361 1.614770455 1.602875541 1.595243548 1.590051234
4.852976472 4.843619055 4.80835856 4.785612297 4.770096001
8.085540416 8.070455549 8.01305082 7.97562945 7.949968705
11.31427091 11.29413991 11.21646825 11.16507184 11.12955806
14.53815423 14.51378864 14.418186 14.35373232 14.30875775
17.75655469 17.72880406 17.61785249 17.54142458 17.48746812
20.96918276 20.93886143 20.81519624 20.72798667 20.66559757
24.1760283 24.14386793 24.01002594 23.91328314 23.84306367
27.37728285 27.34390912 27.20222517 27.09720571 27.01979387
30.57326763 30.53919516 30.39174352 30.27967282 30.19572598
33.76437514 33.73001404 33.57858571 33.46062818 33.3708082
36.95102638 36.91669472 36.76280032 36.64003859 36.54499904
40.13364202 40.09958042 39.94446895 39.81789129 39.71826684
43.31262445 43.27901041 43.12369667 42.99419112 42.8905892
46.48834789 46.45530866 46.30060389 46.1689576 46.06195229
49.66115384 49.6287774 49.47531978 49.34222217 49.23234996
52.83135002 52.79969409 52.64797716 52.51402561 52.40178294
55.99921135 55.96831061 55.81870864 55.68441573 55.57025794
59.16498212 59.1348539 58.98764389 58.85344535 58.73778679
62.32887861 62.29952739 62.15490768 62.02117063 61.9043856
];

```

```
m(:,76) = ...
```

```

[
101.9367992
1.586357235
4.759041301
7.93163451
11.10407747
14.27631254
17.4482844
20.61994071
23.79123246
26.96211449
30.13254569
33.30248925
36.4719128
39.64078838
42.80909243
45.97680567
49.14391292
52.31040288
55.47626789

```



```

58.64150363
61.80610883
];

for n = 1:76
    if omega == ome(n)
        m = m(:,n);
    end
end

%preallocate eigenvalues
lambda = zeros(N+1,1);

%obtain eigenvalues
lambda = m/h;

%-----
%obtain wave excitation forces
%-----

%preallocate vector Nj
Nj = zeros(N+1,1);

%create vector Nj
for j = 0
    Nj(j+1) = h/2*(1+sinh(2*lambda(j+1)*h)/(2*lambda(j+1)*h));
end
for j = 1:N
    Nj(j+1) = h/2*(1+sin(2*lambda(j+1)*h)/(2*lambda(j+1)*h));
end

%preallocate vector Z0
Z = zeros(N+1,1);

%create vector Z0
for j = 0
    Z0(j+1) = Nj(j+1)^(-1/2)*cosh(lambda(j+1)*h);
end
for j = 1:N
    Z0(j+1) = Nj(j+1)^(-1/2)*cos(lambda(j+1)*h);
end

%preallocate matrix J
J = zeros(N+1);

%create matrix J
for n = 0:N
    for j = 0
        J(n+1,j+1) = (-1)^n*(h-d)*lambda(j+1)*sinh(lambda(j+1)*...
            (h-d))/(((h-d)^2*lambda(j+1)^2+n^2*pi^2)*cosh...
            (lambda(j+1)*h));
    end
    for j = 1:N
        J(n+1,j+1) = (-1)^n*(h-d)*lambda(j+1)*sinh(lambda(j+1)*...
            (h-d))/(((h-d)^2*lambda(j+1)^2-n^2*pi^2)*cosh...
            (lambda(j+1)*h));
    end
end

```

```

end



```

%preallocate vectors H0 and H1
H0 = zeros(N+1,1);
H1 = zeros(N+1,1);

%create vectors H0 and H1
for j = 0
 H0(j+1) = lambda(j+1)*exp(pi*i)*besselh(1,1,lambda(j+1)*R)/...
 (besselh(0,1,lambda(j+1)*R)*Z0(j+1));
 H1(j+1) = lambda(j+1)*(besselh(0,1,lambda(j+1)*R)-...
 besselh(1,1,lambda(j+1)*R)/(lambda(j+1)*R))/...
 (besselh(1,1,lambda(j+1)*R)*Z0(j+1));
end
for j = 1:N
 H0(j+1) = lambda(j+1)*(-besselk(1,lambda(j+1)*R))/...
 (besselk(0,lambda(j+1)*R)*Z0(j+1));
 H1(j+1) = lambda(j+1)*(besselk(0,lambda(j+1)*R)-exp(pi*i)*...
 besselk(1,lambda(j+1)*R)/(lambda(j+1)*R))...
 /exp(pi*i)/(besselk(1,lambda(j+1)*R)*Z0(j+1));
end

%preallocate vector k
k = zeros(N+1,1);

%create vector k
for n = 0:N
 k(n+1) = n*pi/(h-d);
end

%preallocate matrices D0, D1, and vectors A0, A1
D0 = zeros(N+1);
D1 = zeros(N+1);
A0 = zeros(N+1,1);
A1 = zeros(N+1,1);

%create matrices D0, D1, and vectors A0, A1
for tau = 0:N
 for j = 0:N
 sum_D0 = 0;
 sum_D1 = 0;
 for n = 1:N
 sum_D0 = k(n+1)*besseli(1,k(n+1)*R)/besseli(0,...
 k(n+1)*R)*J(n+1,j+1)*J(n+1,tau+1)+sum_D0;
 sum_D1 = k(n+1)*((k(n+1)*R*besseli(0,k(n+1)*R)-...
 besseli(1,k(n+1)*R))/k(n+1)*R)/besseli(1,...
 k(n+1)*R)*J(n+1,j+1)*J(n+1,tau+1)+sum_D1;
 end
 if j == tau
 D0(j+1,tau+1) = -H0(j+1)+(h-d)*Z0(tau+1)*2*sum_D0;
 D1(j+1,tau+1) = -H1(j+1)+(h-d)*Z0(tau+1)*(1/R*...
 J(1,j+1)*J(1,tau+1)+2*sum_D1);
 else
 D0(j+1,tau+1) = (h-d)*Z0(tau+1)*2*sum_D0;
 D1(j+1,tau+1) = (h-d)*Z0(tau+1)*(1/R*J(1,j+1)*...
 J(1,tau+1)+2*sum_D1);
 end
 end
end

```


```

```

sum_A0 = 0;
sum_A1 = 0;
for n = 1:N
    sum_A0 = k(n+1)*besseli(1,k(n+1)*R)/besseli(0,k(n+1)*R)*...
        J(n+1,1)*J(n+1,tau+1)+sum_A0;
    sum_A1 = k(n+1)*((k(n+1)*R*besseli(0,k(n+1)*R)-...
        besseli(1,k(n+1)*R))/k(n+1)*R)/besseli(1,k(n+1)*R)*...
        J(n+1,1)*J(n+1,tau+1)+sum_A1;
end
if tau == 0
    A0(tau+1) = lambda(1)*(-besselj(1,lambda(1)*R))/...
        Z0(1)-besselj(0,lambda(1)*R)*(h-d)*Z0(tau+1)*2*sum_A0;
    A1(tau+1) = lambda(1)*((lambda(1)*R*besselj(0,...
        lambda(1)*R)-besselj(1,lambda(1)*R))/lambda(1)*R)...
        /Z0(1)-besselj(1,lambda(1)*R)*(h-d)*Z0(tau+1)*...
        (1/R*J(1,1)*J(1,tau+1)+2*sum_A1);
else
    A0(tau+1) = -besselj(0,lambda(1)*R)*(h-d)*Z0(tau+1)*...
        2*sum_A0;
    A1(tau+1) = -besselj(1,lambda(1)*R)*(h-d)*Z0(tau+1)*...
        (1/R*J(1,1)*J(1,tau+1)+2*sum_A1);
end
end

%preallocate vectors alpha0, alpha1
alpha0 = zeros(N+1,1);
alpha1 = zeros(N+1,1);

%create vectors alpha0, alpha1
alpha0 = D0\A0;
alpha1 = D1\A1;

%preallocate vectors beta0, beta1
beta0 = zeros(N+1,1);
beta1 = zeros(N+1,1);

%create vectors beta0, beta1
for n = 0:N
    sum_beta0 = 0;
    sum_beta1 = 0;
    for j = 0:N
        sum_beta0 = alpha0(j+1)*J(n+1,j+1)+sum_beta0;
        sum_beta1 = alpha1(j+1)*J(n+1,j+1)+sum_beta1;
    end
    beta0(n+1) = 2*(-i)*g/omega*(besselj(0,lambda(1)*R)*J(n+1,...
        1)+sum_beta0);
    beta1(n+1) = 2*(-i)*g/omega*2*i*(besselj(1,lambda(1)*R)*...
        J(n+1,1)+sum_beta1);
end

%preallocate vector intZ
intZ = zeros(N+1,1);

%create vector intZ
for j = 0
    intZ(j+1) = Nj(j+1)^(-1/2)/lambda(1)^2*(d*lambda(1)*...
        sinh(lambda(1)*(h-d))-cosh(lambda(1)*h)+cosh...
        (lambda(1)*(h-d)));
end

```

```

end
for j = 1:N
    intZ(j+1) = Nj(j+1)^(-1/2)/lambda(j+1)^2*(d*lambda(j+1)*...
        sin(lambda(j+1)*(h-d))+cos(lambda(j+1)*h)-cos...
        (lambda(j+1)*(h-d)));
end

%obtain wave excitation force in x-direction
sum_FD1 = 0;
for j = 1:N
    sum_FD1 = alphas(j+1)*(sin(lambda(j+1)*h)-sin(lambda(j+1)*...
        (h-d)))/(lambda(j+1)*cos(lambda(j+1)*h))+sum_FD1;
end

FD1 = -2*pi*i*rho*g*R*((besselj(1,lambda(1)*R)+alpha(1))*...
    ((sinh(lambda(1)*h)-sinh(lambda(1)*(h-d)))/(lambda(1)*...
    cosh(lambda(1)*h)))+sum_FD1);

%obtain wave excitation force in z-direction
sum_FD3 = 0;
for n = 1:N
    sum_FD3 = (-1)^n*beta0(n+1)*R*besseli(1,k(n+1)*R)/(k(n+1)*...
        besseli(0,k(n+1)*R))+sum_FD3;
end

FD3 = 2*pi*i*rho*omega*(beta0(1)*R^2/4+sum_FD3);

%obtain wave excitation moment about y-axis
sum_intZ = 0;
sum_FD5 = 0;
for n = 1:N
    sum_intZ = alphas(n+1)/Z0(n+1)*intZ(n+1)+sum_intZ;
    sum_FD5 = (-1)^n*beta1(n+1)*R^2*besseli(2,k(n+1)*R)/...
        (k(n+1)*besseli(1,k(n+1)*R))+sum_FD5;
end

FD5 = -2*pi*i*rho*g*R*((besselj(1,lambda(1)*R)+alpha(1))/...
    Z0(1)*intZ(1)+sum_intZ)-pi*i*rho*omega*(beta1(1)*R^3/8+...
    sum_FD5);

%preallocate vector FD (wave excitation force vector)
FD = zeros(6,1);

%create vector FD
FD(1) = FD1;
FD(3) = FD3;
FD(5) = FD5;

%-----
%obtain added mass and radiation damping matrices
%-----

%preallocate matrices A (added mass matrix), C (radiation damping
matrix)
A = zeros(6);
C = zeros(6);

```

```

%-----
%preallocate matrix Ds and vector As
Ds = zeros(N+1);
As = zeros(N+1,1);

%create matrix Ds and vector As
Hs = H1;
for tau = 0:N
    for j = 0:N
        sum_Ds = 0;
        for n = 1:N
            sum_Ds = k(n+1)*((k(n+1)*R*besseli(0,k(n+1)*R)-...
                besseli(1,k(n+1)*R))/k(n+1)*R)/besseli(1,...
                k(n+1)*R)*J(n+1,j+1)*J(n+1,tau+1)+sum_Ds;
        end
        if j == tau
            Ds(j+1,tau+1) = Hs(j+1)-(h-d)*Z0(tau+1)*(1/R*...
                J(1,j+1)*J(1,tau+1)+2*sum_Ds);
        else
            Ds(j+1,tau+1) = -(h-d)*Z0(tau+1)*(1/R*J(1,j+1)*...
                J(1,tau+1)+2*sum_Ds);
        end
    end
    if tau == 0
        As(tau+1) = Nj(tau+1)^(-1/2)/lambda(1)*(sinh...
            (lambda(1)*h)-sinh(lambda(1)*(h-d)));
    else
        As(tau+1) = Nj(tau+1)^(-1/2)/lambda(tau+1)*...
            (sin(lambda(tau+1)*h)-sin(lambda(tau+1)*(h-d)));
    end
end

%preallocate vector alphas
alphas = zeros(N+1,1);

%create vector alphas
alphas = Ds\As;

%obtain F11
sum_F11 = 0;
for j = 1:N
    sum_F11 = alphas(j+1)*(sin(lambda(j+1)*h)-sin(lambda(j+1)*...
        (h-d)))/(lambda(j+1)*cos(lambda(j+1)*h))+sum_F11;
end

F11 = -pi*i*rho*omega*R*(alphas(1)*(sinh(lambda(1)*h)-...
    sinh(lambda(1)*(h-d)))/(lambda(1)*cosh(lambda(1)*h))+sum_F11);

%obtain A11 and C11
A11 = 1/omega*imag(F11);
C11 = -real(F11);

%-----
%preallocate matrix Dp and vector Ap
Dp = zeros(N+1);
Ap = zeros(N+1,1);

```

```


```

%preallocate vector I5
I5 = zeros(N+1,1);

%create vector I5
for n = 0
 I5(n+1) = -(h-d)*R/3+R^3/(4*(h-d));
end
for n = 1:N
 I5(n+1) = 2*(-1)^(n+1)*(h-d)*R/(n*pi)^2;
end

%preallocate vector intp
intp = zeros(N+1,1);

%create vector intp
for tau = 0
 intp(tau+1) = (((h-d)/(2*lambda(1))+1/(lambda(1)^3*(h-d))-...
 3*R^2/(8*lambda(1)*(h-d)))*sinh(lambda(1)*(h-d))-...
 cosh(lambda(1)*(h-d))/lambda(1)^2)*Nj(1)^(-1/2);
end
for tau = 1:N
 intp(tau+1) = (((h-d)/(2*lambda(tau+1))-1/(lambda(tau+1)^3*...
 (h-d))-3*R^2/(8*lambda(tau+1)*(h-d)))*sin(lambda...
 (tau+1)*(h-d))+cos(lambda(tau+1)*(h-d))/lambda...
 (tau+1)^2)*Nj(tau+1)^(-1/2);
end

%create matrix Dp and vector Ap
Hp = H1;
Dp = Ds;

for tau = 0:N
 sum_Ap = 0;
 for n = 1:N
 sum_Ap = I5(n+1)*k(n+1)*((k(n+1)*R*besseli(0,k(n+1)*R)-...
 besseli(1,k(n+1)*R))/k(n+1)*R)/besseli(1,k(n+1)*R)*...
 J(n+1,tau+1)+sum_Ap;
 end
 Ap(tau+1) = -(h-d)*Z0(tau+1)*(I5(1)/(2*R)*J(1,tau+1)+sum_Ap)-
 ...
 intp(tau+1)-intZ(tau+1);
end

%preallocate vector alphap
alphap = zeros(N+1,1);

%create vector alphap
alphap = Dp\Ap;

%obtain F15
sum_F15 = 0;
for j = 1:N
 sum_F15 = alphap(j+1)*(sin(lambda(j+1)*h)-sin(lambda(j+1)*...
 (h-d)))/(lambda(j+1)*cos(lambda(j+1)*h))+sum_F15;
end

F15 = -pi*i*rho*omega*R*(alphap(1)*(sinh(lambda(1)*h)-...

```


```

```

sinh(lambda(1)*(h-d))/(lambda(1)*cosh(lambda(1)*h))+sum_F15);

%obtain A15 and C15
A15 = 1/omega*imag(F15);
C15 = -real(F15);

%-----
%preallocate matrix Dh and vector Ah
Dh = zeros(N+1);
Ah = zeros(N+1,1);

%preallocate vector I3
I3 = zeros(N+1,1);

%create vector I3
for n = 0
    I3(n+1) = (h-d)/3-R^2/(2*(h-d));
end
for n = 1:N
    I3(n+1) = 2*(-1)^n*(h-d)/(n*pi)^2;
end

%preallocate vector intz
intz = zeros(N+1,1);

%create vector intz
for j = 0
    intz(j+1) = Nj(j+1)^(-1/2)/lambda(1)*sinh(lambda(1)*(h-d));
end
for j = 1:N
    intz(j+1) = Nj(j+1)^(-1/2)/lambda(j+1)*sin(lambda(j+1)*(h-d));
end

%create matrix Dh and vector Ah
Hh = H0;

for tau = 0:N
    for j = 0:N
        sum_Dh = 0;
        for n = 1:N
            sum_Dh = k(n+1)*besseli(1,k(n+1)*R)/besseli(0,...
                k(n+1)*R)*J(n+1,j+1)*J(n+1,tau+1)+sum_D0;
        end
        if j == tau
            Dh(j+1,tau+1) = Hh(j+1)-2*(h-d)*Z0(tau+1)*sum_Dh;
        else
            Dh(j+1,tau+1) = -2*(h-d)*Z0(tau+1)*sum_Dh;
        end
    end
    sum_Ah = 0;
    for n = 1:N
        sum_Ah = k(n+1)*besseli(1,k(n+1)*R)/besseli(0,k(n+1)*R)*...
            J(n+1,tau+1)*I3(n+1)+sum_Ah;
    end
    Ah(tau+1) = -((h-d)*Z0(tau+1)*sum_Ah+R/(2*(h-d))*intz(tau+1));
end

```

```


```

%preallocate vector alphah
alphah = zeros(N+1,1);

%create vector alphah
alphah = Dh\Ah;

%preallocate vector betah
betah = zeros(N+1,1);

%create vector betah
for n = 0:N
 sum_betah = 0;
 for j = 0:N
 sum_betah = alphah(j+1)*J(n+1,j+1)+sum_betah;
 end
 betah(n+1) = 2*sum_betah-I3(n+1);
end

%obtain F33
sum_F33 = 0;
for n = 1:N
 sum_F33 = (-1)^n*betah(n+1)*R*besseli(1,k(n+1)*R)/(k(n+1)*...
 besseli(0,k(n+1)*R))+sum_F33;
end

F33 = 2*pi*i*rho*omega*(betah(1)*R^2/4+sum_F33+(h-d)*R^2/4-...
 R^4/(16*(h-d)));

%obtain A33 and C33
A33 = 1/omega*imag(F33);
C33 = -real(F33);

%-----
%preallocate vector betas
betas = zeros(N+1,1);

%create vector betas
for n = 0:N
 sum_betas = 0;
 for j = 0:N
 sum_betas = alphas(j+1)*J(n+1,j+1)+sum_betas;
 end
 betas(n+1) = 2*sum_betas;
end

%obtain F51
sum_ints = 0;
for j = 0:N
 sum_ints = alphas(j+1)/Z0(j+1)*intZ(j+1)+sum_ints;
end

sum_F51 = 0;
for n = 1:N
 sum_F51 = (-1)^n*betas(n+1)*R^2*besseli(2,k(n+1)*R)/...
 (k(n+1)*besseli(1,k(n+1)*R))+sum_F51;
end

```


```



```

F51 = -pi*i*rho*omega*R*sum_ints-pi*i*rho*omega*(betas(1)*...
      R^3/8+sum_F51);

%obtain A51 and C51
A51 = 1/omega*imag(F51);
C51 = -real(F51);

%-----
%preallocate vector betap
betap = zeros(N+1,1);

%create vector betap
for n = 0:N
    sum_betap = 0;
    for j = 0:N
        sum_betap = alphap(j+1)*J(n+1,j+1)+sum_betap;
    end
    betap(n+1) = 2*sum_betap-I5(n+1);
end

%obtain F55
sum_intp = 0;
for j = 0:N
    sum_intp = alphap(j+1)/Z0(j+1)*intZ(j+1)+sum_intp;
end

sum_F55 = 0;
for n = 1:N
    sum_F55 = (-1)^n*betap(n+1)*R^2*besseli(2,k(n+1)*R)/...
              (k(n+1)*besseli(1,k(n+1)*R))+sum_F55;
end

F55 = -pi*i*rho*omega*R*sum_intp-pi*i*rho*omega*(betap(1)*...
      R^3/8+sum_F55-R^4*(h-d)/8+R^6/(48*(h-d)));

%obtain A55 and C55
A55 = 1/omega*imag(F55);
C55 = -real(F55);

%create matrices A, C
A(1,1) = A11;
A(1,5) = A15;
A(3,3) = A33;
A(5,1) = A51;
A(5,5) = A55;
C(1,1) = C11;
C(1,5) = C15;
C(3,3) = C33;
C(5,1) = C51;
C(5,5) = C55;

%-----
%create matrices M (mass matrix), K (restoring matrix)
%-----

%preallocate matrices M, K
M = zeros(6);

```

```

K = zeros(6);

%create matrix M
M(1,1) = m1;
M(2,2) = m1;
M(3,3) = m1;
M(1,5) = m1*zG;
M(5,1) = m1*zG;
M(2,4) = -m1*zG;
M(4,2) = -m1*zG;
M(4,4) = I11;
M(5,5) = I22;
M(6,6) = I33;

%create matrix K
K(3,3) = rho*g*wpA;
K(4,4) = rho*g*Iwpa+m1*g*(zb-zG);
K(5,5) = rho*g*Iwpa+m1*g*(zb-zG);

%-----
%solve equation of motion
%-----

%obtain vector U (displacement without TMD)
U = (-omega^2*(M+A)-i*omega*C+K)\FD;

%preallocate matrices ME, CE, KE, FE
ME = zeros(7);
CE = zeros(7);
KE = zeros(7);
FE = zeros(7);

%create matrix ME
ME(1:6,1:6) = M+A;
ME(7,7) = m2;

%create matrix CE
CE(1:6,1:6) = C;
CE(3,3) = C(3,3) + c2;
CE(7,3) = -c2;
CE(3,7) = -c2;
CE(7,7) = c2;

%create matrix KE
KE(1:6,1:6) = K;
KE(3,3) = K(3,3) + k2;
KE(7,3) = -k2;
KE(3,7) = -k2;
KE(7,7) = k2;

%create matrix FE
FE(1:6) = FD;

%obtain vector UTMD (displacement with TMD)
UTMD = (-omega^2*ME-i*omega*CE+KE)\FE;

%obtain U3 (heave RAO without TMD) and UTMD3 (heave RAO with TMD)

```

```
U3 = U(3);  
UTMD3 = UTMD(3);
```

Appendix D

MATLAB routine for plotting diffraction and radiation theory results

```
function plotdiffrad =
plotdiffrad(h,d,R,m1,m2,k2,c2,zG,I11,I22,I33)

%Function to plot the variation of diffraction forces,
%added masses, dampings, heave RAOs, and RRFs
%over a given range of wave frequencies

ome = [0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.53 0.55 0.58...
        0.6 0.63 0.65 0.68 0.7 0.73 0.75 0.78 0.8 0.83 0.85 0.88...
        0.9 0.93 0.95 0.98 1 1.03 1.05 1.08 1.1 1.13 1.15 1.18...
        1.2 1.23 1.25 1.28 1.3 1.33 1.35 1.38 1.4 1.45 1.5 1.55...
        1.6 1.65 1.7 1.75 1.8 1.85 1.9 1.95 2 2.05 2.1 2.15 2.2...
        2.25 2.3 2.35 2.4 2.45 2.5 2.6 2.7 2.8 2.9 3 3.5 4 4.5 5];
f = ome./(2*pi);

FD1 = zeros(76,1);
FD3 = zeros(76,1);
FD5 = zeros(76,1);
A11 = zeros(76,1);
A15 = zeros(76,1);
A33 = zeros(76,1);
A51 = zeros(76,1);
A55 = zeros(76,1);
C11 = zeros(76,1);
C15 = zeros(76,1);
C33 = zeros(76,1);
C51 = zeros(76,1);
C55 = zeros(76,1);
U3 = zeros(76,1);
UTMD3 = zeros(76,1);

for n = 1:76
    [FD1(n,1),FD3(n,1),FD5(n,1),...
     A11(n,1),A15(n,1),A33(n,1),A51(n,1),A55(n,1),...
     C11(n,1),C15(n,1),C33(n,1),C51(n,1),C55(n,1),...
     U3(n,1),UTMD3(n,1)] = ...
    diffrad(h,d,R,ome(n),m1,m2,k2,c2,zG,I11,I22,I33)
end

%plot diffraction forces
subplot(3,1,1), plot(f, abs(FD1), 'b-'), ylabel('force (N)');
subplot(3,1,2), plot(f, abs(FD3), 'r-'), ylabel('force (N)');
subplot(3,1,3), plot(f, abs(FD5), 'm-'), ylabel('moment (Nm)');
xlabel('wave frequency (Hz)');

%plot added mass and radiation damping
figure
subplot(2,1,1), plot(f,A11, 'b-',f,A15, 'b--',f,A33, 'r-',...
```

```
f,A51,'b:',f,A55,'b-.'), ylabel('added mass (kg)');
grid on
subplot(2,1,2), plot(f,C11,'b-',f,C15,'b--',f,C33,'r-',...
    f,C51,'b:',f,C55,'b-.'), ylabel('radiation damping (kg/s)');
xlabel('wave frequency (Hz)');
grid on

%plot heave RAO with and without TMD
figure
plot(f,abs(U3),'b.',f,abs(UTMD3),'bx');
xlabel('wave frequency (Hz)');
ylabel('heave RAO');
legend('without TMD');
grid on

%plot RRF
RRF = abs(UTMD3)./abs(U3);

figure
plot(f,RRF,'bx');
xlabel('wave frequency (Hz)');
ylabel('RRF');
AXIS([0 0.4 0 2]);
grid on
```

Appendix E

MATLAB routine for reading wave time history from .DAF files

```
%function to read .DAF files and obtain:
%- plots of water surface elevation time history from each wave
% probe
%- plots of water surface elevation Fast Fourier Transform (FFT)
% from each wave probe
%- peak frequency for each FFT curve
%- wave RMS
function waverms = readdaf(namefile)

%open file
namefile = ['C:\Adi\Research\' namefile '.daf'];
fid = fopen(namefile);
C = textscan(fid, '%f%f%f%f%f%f', 'headerLines', 2, ...
'delimiter', ';');
fclose(fid);

%sampling frequency (Hz)
fs = 25;

%get time series (seconds)
t = C{1};

%output total number of frames
length(t)

%preallocate matrix of water surface level, where wse(:,i) is a
%series of water surface elevations at probe i
wse = zeros(length(t),5);

%get water surface elevation (mm) at each probe, where mm/volt =
%20/0.4
for i = 1:5
    wse(:,i) = 20/0.4*C{i+1};
end

%input starting and ending frame number to be used
start = input('Input starting frame number (odd number): ');
stop = input('Input last frame number (even number): ');
t = t(start:stop);

%determine actual number of frames used
N = length(t);

wse = wse(start:stop,:);
```

```

%plot water surface level time history
for i = 1:5
    subplot(5,2,2*i-1), plot(t,wse(:,i)), ...
        axis([t(1) t(end) -20 20]);
end

%preallocate matrix of y and val
y = zeros(N,5);
val = zeros(N,5);

%plot FFT of water surface level at each probe
for i = 1:5
    y(:,i) = fft(wse(:,i));
    val(:,i) = abs(y(:,i));
    freq = fs*(0:(N/2))/N;
    subplot(5,2,2*i), plot(freq,val(1:(N/2+1),i));
end

%filter offset
y(1,:) = [0];
for i = 1:5
    wse(:,i) = real(ifft(y(:,i)));
end

%preallocate peak frequency for FFT curve
actfreq = 0;

%plot filtered water surface level time history and FFT
figure
for i = 1:5
    subplot(5,2,2*i-1), plot(t,wse(:,i)), ...
        axis([t(1) t(end) -20 20]);
    y(:,i) = fft(wse(:,i));
    val(:,i) = abs(y(:,i));
    freq = fs*(0:(N/2))/N;
    subplot(5,2,2*i), plot(freq,val(1:(N/2+1),i));
    %get peak frequency for each FFT curve
    [Y,I] = max(val(:,i));
    actfreq(i) = freq(I);
end

%output peak frequency for FFT curve
actfreq

%preallocate matrix of wave RMS
rms = zeros(1,4);

%get wave RMS for probes 1-4
for i = 1:4
    rms(1,i) = norm(wse(:,i))/sqrt(N);
end

%output wave RMS
rms

%get average wave RMS
waverms = sum(rms(1,1:4))/4;

```

Appendix F

MATLAB routine for reading heave time history from .TSV files

```
%function to read .TSV files and obtain:
%- plots of heave time history
%- plots of heave FFT
%- heave RMS
function heaverms = readtsvheave(namefile)

%open file
namefile = ['C:\Adi\Research\' namefile '.tsv'];
fid = fopen(namefile);
C = textscan(fid, '%*f%f%f%*[\n]', 'headerLines', 4);
fclose(fid);

%sampling frequency (Hz)
fs = 50;

%get heave displacements (mm)
z = C{1};

%output total number of frames
frames = length(z)

%input starting and ending frame number to be used
start = input('Input starting frame number (odd number): ');
stop = input('Input last frame number (even number): ');
z = z(start:stop);

%determine actual number of frames used
N = length(z);

%set new zero for t
t = (0:N-1)./fs;

%plot heave time history
subplot(2,1,1), plot(t,z), grid on;

%plot FFT of heave
y = fft(z);
val = abs(y);
freq = fs*(0:(N/2))/N;
subplot(2,1,2), plot(freq,val(1:(N/2+1))), grid on;

%filter offset
y(1:ceil(0.1*N/fs)) = [0];
y(N-ceil(0.1*N/fs):N) = [0];
z = real(ifft(y));
```



```
%plot filtered heave time history
figure
subplot(2,1,1), plot(t,z), grid on;

%plot FFT of filtered heave
y = fft(z);
val = abs(y);
freq = fs*(0:(N/2))/N;
subplot(2,1,2), plot(freq,val(1:(N/2+1))), grid on;

%get heave RMS
heaverms = norm(z)/sqrt(N);
```

Appendix G

Prefem command input file to generate panel model with normal water plane

```
GENERATE SURFACE A 1 2 1 6 1 5 1 8 END  
CYLINDRICAL 0.0 0.0 0.0 0.0 0.0 0.0 1.0 1.0 0.0 0.0  
0.0 0.0 0.0  
7.5 0.0 0.0 END 0.0 90.0 0.0 END
```

```
GENERATE SURFACE B 1 5 1 8 1 2 1 18 END  
CYLINDRICAL 0.0 0.0 0.0 0.0 0.0 0.0 1.0 1.0 0.0 0.0  
7.5 0.0 0.0  
0.0 90 0 END 0.0 0 30 END
```

```
SET ELEMENT-TYPE SURFACE ALL-SURFACES-INCLUDED SHELL-4NODES END  
END  
MESH ALL
```

```
DELETE MESH ALL  
END  
CHANGE ROTATION-OF-SURFACE ( AU12 AU11 AU13 AU14 )  
END  
MESH ALL
```

```
PROPERTY LOAD 1 HYDRO-PRESSURE ALL-SURFACES-INCLUDED OUTSIDE  
OUTSIDE-SURFACE  
END  
END
```

Appendix H

Prefem command input file to generate panel model with large water plane

```
GENERATE SURFACE A 1 2 1 10 1 5 1 12 END  
CYLINDRICAL 0.0 0.0 0.0 0.0 0.0 0.0 1.0 1.0 0.0 0.0  
0.0 0.0 0.0  
11.75 0.0 0.0 END 0.0 90.0 0.0 END
```

```
GENERATE SURFACE B 1 5 1 12 1 2 1 20 END  
CYLINDRICAL 0.0 0.0 0.0 0.0 0.0 0.0 1.0 1.0 0.0 0.0  
11.75 0.0 0.0  
0.0 90 0 END 0.0 0 30 END
```

```
SET ELEMENT-TYPE SURFACE ALL-SURFACES-INCLUDED SHELL-4NODES END  
END  
MESH ALL
```

```
DELETE MESH ALL  
END  
CHANGE ROTATION-OF-SURFACE ( AU12 AU11 AU13 AU14 )  
END  
MESH ALL
```

```
PROPERTY LOAD 1 HYDRO-PRESSURE ALL-SURFACES-INCLUDED OUTSIDE  
OUTSIDE-SURFACE  
END  
END
```

Appendix I

MATLAB routine for reading numerical simulation results from .LIS files

```
%function to read .LIS files and obtain:
%- number of panels
%- mass of the structure
%- metacentric height
%- wave period
%- nondimensionalizing factors
%- added mass matrix
%- damping matrix
%- heave displacements
%- heave natural frequency
%- plots of heave-heave added mass and damping
%- plot of heave RAO
function heave = readlis(namefile)

%open file
namefile=['C:\Adi\Research\' namefile '.lis'];
fid = fopen(namefile);

%obtain number of panels
begin = 539; %input manually number of headerlines neglected
panels = textscan(fid, '%*s*s*s*s*s*s*f', 1, ...
'headerLines', begin);

panels = panels{1};
disp(['Number of panels = ' num2str(panels) '.']);

fclose(fid);
fid = fopen(namefile);

%obtain mass of the structure
mass = textscan(fid, '%*s*s*s*s*s*s*s*f', 1, ...
'headerLines', begin+12);

mass = mass{1};
disp(['Mass of the structure = ' num2str(mass) ' kg.']);

fclose(fid);
fid = fopen(namefile);

%obtain metacentric height
GM = textscan(fid, '%*s*s*s*s*s*s*f', 1, ...
'headerLines', begin+38);

GM = GM{1};
disp(['Metacentric height = ' num2str(GM) ' m.']);

fclose(fid);
```

```

fid = fopen(namefile);

%obtain wave period
T = textscan(fid, '%*f%f%f%f%f', 25, 'headerLines', begin+75);

T = T{1};

fclose(fid);
fid = fopen(namefile);

%obtain nondimensionalizing factors
start = 3788; %input manually number of headerlines neglected
NDF = textscan(fid, '%*s*s%f', 5, 'headerLines', start);

NDF = NDF{1};

RO = NDF(1);
disp(['Density of the fluid = ' num2str(RO) ' kg/m^3.']);
G = NDF(2);
disp(['Acceleration of gravity = ' num2str(G) ' m/s^2.']);
VOL = NDF(3);
disp(['Displaced volume = ' num2str(VOL) ' m^3.']);
L = NDF(4);
disp(['Characteristic length = ' num2str(L) ' m.']);
WA = NDF(5);
disp(['Wave amplitude = ' num2str(WA) ' m.']);

fclose(fid);
fid = fopen(namefile);

%obtain frequency dependent matrices, numeric index represents
%frequency case, with case 1: T = 3s, and case 25: T = 15s
MA = cell(25,1);
C = cell(25,1);
U = cell(25,1);

%obtain added mass matrix arranged in a cell array
for i = 1:25
    MA{i} = textscan(fid, '%*f%f%f%f%f%f', 6, ...
        'headerLines', start+107+(i-1)*140);

    MA{i} = cell2mat(MA{i});

    MA{i}(1:3,1:3) = MA{i}(1:3,1:3).*RO*VOL;
    MA{i}(1:3,4:6) = MA{i}(1:3,4:6).*RO*VOL*L;
    MA{i}(4:6,1:3) = MA{i}(4:6,1:3).*RO*VOL*L;
    MA{i}(4:6,4:6) = MA{i}(4:6,4:6).*RO*VOL*L*L;

    fclose(fid);
    fid = fopen(namefile);
end

%obtain damping matrix arranged in a cell array
for i = 1:25
    C{i} = textscan(fid, '%*f%f%f%f%f%f', 6, ...
        'headerLines', start+128+(i-1)*140);

```

```

C{i} = cell2mat(C{i});

C{i}(1:3,1:3) = C{i}(1:3,1:3).*RO*VOL*sqrt(G/L);
C{i}(1:3,4:6) = C{i}(1:3,4:6).*RO*VOL*sqrt(G*L);
C{i}(4:6,1:3) = C{i}(4:6,1:3).*RO*VOL*sqrt(G*L);
C{i}(4:6,4:6) = C{i}(4:6,4:6).*RO*VOL*L*sqrt(G*L);

fclose(fid);
fid = fopen(namefile);
end

%obtain heave displacements arranged in a cell array
for i = 1:25
    U{i} = textscan(fid, '%*s%f%f%f*f', 1, ...
        'headerLines', start+193+(i-1)*140);

    U{i} = cell2mat(U{i});

    fclose(fid);
    fid = fopen(namefile);
end

%obtain heave natural frequency
for i = 1:25
    naturalT{i} = textscan(fid, '%*f%f*f*f*f*f*f*f*f*f', ...
        1, 'headerLines', start+147+(i-1)*140);

    naturalT{i} = cell2mat(naturalT{i});

    fclose(fid);
    fid = fopen(namefile);
end

naturalT = cell2mat(naturalT);
naturalf = mean(1./naturalT);
disp(['Heave natural frequency = ' num2str(naturalf) ' Hz.']);

fclose(fid); %end of reading file

%plot heave RAO
U = cell2mat(U);
f = 1./T;
figure(1)
plot(f, U, '.');
xlabel('wave frequency (Hz)');
ylabel('heave RAO');
grid on
hold on

%plot MA33
MA33 = zeros(25,1);
for i = 1:25
    MA33(i) = MA{i}(3,3);
end
figure(2)

```

```
plot(f, MA33, '.');
xlabel('wave frequency (Hz)');
ylabel('heave-heave added mass (kg)');
grid on
hold on

%plot C33
C33 = zeros(25,1);
for i = 1:25
    C33(i) = C{i}(3,3);
end
figure(3)
plot(f, C33, '.');
xlabel('wave frequency (Hz)');
ylabel('heave-heave damping (kg/s)');
grid on
hold on
```

Appendix J

MATLAB routine for reading numerical simulation results for dynamic analysis from .LIS files

```
%function to read .LIS files and obtain:
%- mass inertia coefficient matrix
%- hydrostatic restoring coefficient matrix
%- added mass matrix
%- damping matrix
%- exciting force vector
%- heave RAO without TMD
%- heave RAO with TMD
function [f, U3, U3TMD] = readlisdyn(namefile,m,c,k)

%open file
namefile=['C:\Adi\Research\' namefile '.lis'];
fid = fopen(namefile);

%obtain wave period
begin = 507; %input manually number of headerlines neglected
T = textscan(fid, '%*f%f%f%f%f%f', 25, 'headerLines', begin+75);

T = T{1};

fclose(fid);
fid = fopen(namefile);

%obtain nondimensionalizing factors
start = 3742; %input manually number of headerlines neglected
NDF = textscan(fid, '%*s*s%f', 5, 'headerLines', start);

NDF = NDF{1};

RO = NDF(1);
disp(['Density of the fluid = ' num2str(RO) ' kg/m^3.']);
G = NDF(2);
disp(['Acceleration of gravity = ' num2str(G) ' m/s^2.']);
VOL = NDF(3);
disp(['Displaced volume = ' num2str(VOL) ' m^3.']);
L = NDF(4);
disp(['Characteristic length = ' num2str(L) ' m.']);
WA = NDF(5);
disp(['Wave amplitude = ' num2str(WA) ' m.']);

fclose(fid);
fid = fopen(namefile);

%obtain mass inertia coefficient matrix
M = textscan(fid, '%*f%f%f%f%f%f%f', 6, 'headerLines', start+64);

M = cell2mat(M);
```



```

M(1:3,1:3) = M(1:3,1:3).*RO*VOL;
M(1:3,4:6) = M(1:3,4:6).*RO*VOL*L;
M(4:6,1:3) = M(4:6,1:3).*RO*VOL*L;
M(4:6,4:6) = M(4:6,4:6).*RO*VOL*L*L;

fclose(fid);
fid = fopen(namefile);

%obtain hydrostatic restoring coefficient matrix
K = textscan(fid, '%*f%f%f%f%f%f', 6, 'headerLines', start+76);

K = cell2mat(K);

K(1:3,1:3) = K(1:3,1:3).*RO*VOL*G/L;
K(1:3,4:6) = K(1:3,4:6).*RO*VOL*G;
K(4:6,1:3) = K(4:6,1:3).*RO*VOL*G;
K(4:6,4:6) = K(4:6,4:6).*RO*VOL*G*L;

fclose(fid);
fid = fopen(namefile);

%obtain frequency dependent matrices
MA = cell(25,1);
C = cell(25,1);
F = cell(25,1);
for i = 1:25
    F{i} = cell(6,1);
end

%obtain added mass matrix arranged in a cell array
for i = 1:25
    MA{i} = textscan(fid, '%*f%f%f%f%f%f', 6, ...
        'headerLines', start+107+(i-1)*115);

    MA{i} = cell2mat(MA{i});

    MA{i}(1:3,1:3) = MA{i}(1:3,1:3).*RO*VOL;
    MA{i}(1:3,4:6) = MA{i}(1:3,4:6).*RO*VOL*L;
    MA{i}(4:6,1:3) = MA{i}(4:6,1:3).*RO*VOL*L;
    MA{i}(4:6,4:6) = MA{i}(4:6,4:6).*RO*VOL*L*L;

    fclose(fid);
    fid = fopen(namefile);
end

%obtain damping matrix arranged in a cell array
for i = 1:25
    C{i} = textscan(fid, '%*f%f%f%f%f%f', 6, ...
        'headerLines', start+128+(i-1)*115);

    C{i} = cell2mat(C{i});

    C{i}(1:3,1:3) = C{i}(1:3,1:3).*RO*VOL*sqrt(G/L);
    C{i}(1:3,4:6) = C{i}(1:3,4:6).*RO*VOL*sqrt(G*L);
    C{i}(4:6,1:3) = C{i}(4:6,1:3).*RO*VOL*sqrt(G*L);

```

```

C{i}(4:6,4:6) = C{i}(4:6,4:6).*RO*VOL*L*sqrt(G*L);

fclose(fid);
fid = fopen(namefile);
end

%obtain exciting force vector arranged in a cell array
for i = 1:25
    for j = 1:6
        F{i}{j} = textscan(fid, '%*s%f%f%f*f', 1, ...
            'headerLines', start+171+(j-1)*2+(i-1)*115);
        fclose(fid);
        fid = fopen(namefile);
        F{i}{j} = cell2mat(F{i}{j});
    end

    F{i} = cell2mat(F{i});

    F{i}(1:3,:) = F{i}(1:3,:).*RO*VOL*G/L;
    F{i}(4:6,:) = F{i}(4:6,:).*RO*VOL*G;
end

f = cell(25,1);
for i = 1:25
    f{i} = F{i}(:,1) + sqrt(-1)*F{i}(:,2);
end
F = f;

fclose(fid); %end of reading file

D = cell(25,1);
U = cell(25,1);
U3 = zeros(25,1);

%obtain heave RAO without TMD
for i = 1:25
    w = 2*pi/T(i);
    D{i} = -w^2*(M+MA{i}) + sqrt(-1)*w*C{i} + K;
    U{i} = D{i}\F{i};
    U{i} = abs(U{i});
    U3(i) = U{i}(3);
end

f = 1./T;

ME = cell(25,1);
for i = 1:25
    ME{i} = zeros(7);
end

CE = cell(25,1);
for i = 1:25
    CE{i} = zeros(7);
end

KE = zeros(7);

```

```
FE = cell(25,1);
for i = 1:25
    FE{i} = zeros(7,1);
end

%obtain heave RAO with TMD
for i = 1:25
    ME{i}(1:6,1:6) = M + MA{i};
    ME{i}(7,7) = m;

    CE{i}(1:6,1:6) = C{i};
    CE{i}(3,3) = C{i}(3,3) + c;
    CE{i}(7,3) = -c;
    CE{i}(3,7) = -c;
    CE{i}(7,7) = c;

    KE(1:6,1:6) = K;
    KE(3,3) = K(3,3) + k;
    KE(7,3) = -k;
    KE(3,7) = -k;
    KE(7,7) = k;

    w = 2*pi/T(i);
    D{i} = -w^2*(ME{i}) + sqrt(-1)*w*CE{i} + KE;
    FE{i}(1:6) = F{i};
    U{i} = D{i}\FE{i};
    U{i} = abs(U{i});
    U3TMD(i) = U{i}(3);
end
```

Appendix K

MATLAB routine for plotting heave RAO from numerical simulation

```
%function to obtain:
%- plot of heave RAO without TMD and heave RAO with TMD
function [freq, HeaveRAO, HeaveRAOTMD] = ...
plotdyn(namefile1,namefile2,m,c,k)

freq = zeros(50,1);
heaveRAO = zeros(50,1);
heaveRAOTMD = zeros(50,1);

[freq(1:25), heaveRAO(1:25), heaveRAOTMD(1:25)] = ...
readlisdyn(namefile1,m,c,k);

[freq(26:50), heaveRAO(26:50), heaveRAOTMD(26:50)] = ...
readlisdyn(namefile2,m,c,k);

[freq,IX] = sort(freq);

for i = 1:50
    HeaveRAO(i) = heaveRAO(IX(i));
    HeaveRAOTMD(i) = heaveRAOTMD(IX(i));
end

figure
plot(freq,HeaveRAO,'b',freq,HeaveRAOTMD,'r:');
xlabel('wave frequency (Hz)');
ylabel('heave RAO');
grid on
```

Appendix L

MATLAB routine for plotting heave *RRF* from numerical simulation

```
%function to obtain:
%- plot of heave RRF
function [freq, HeaveRAO, HeaveRAOTMD] = ...
plotRRFdyn(namefile1,namefile2,m,c,k)

freq = zeros(50,1);
heaveRAO = zeros(50,1);
heaveRAOTMD = zeros(50,1);

[freq(1:25), heaveRAO(1:25), heaveRAOTMD(1:25)] = ...
readlisdyn(namefile1,m,c,k);

[freq(26:50), heaveRAO(26:50), heaveRAOTMD(26:50)] = ...
readlisdyn(namefile2,m,c,k);

[freq,IX] = sort(freq);

for i = 1:50
    HeaveRAO(i) = heaveRAO(IX(i));
    HeaveRAOTMD(i) = heaveRAOTMD(IX(i));
end

figure
plot(freq,HeaveRAOTMD./HeaveRAO,'b. ');
xlabel('wave frequency (Hz)');
ylabel('RRF');
axis([0 0.4 0 2]);
grid on
```

Appendix M

Publications

- 1) Gho, W. M. and Kurniawan, A. (2005). Effectiveness of Tuned Mass Damper to Reduce Heave Motion of Floating Vertical Cylinders. *Proceedings of 19th TEAM Conference, Singapore, 5-8 December 2005*, pp. 264-273.
- 2) Kurniawan, A. and Gho, W. M. (2006). Feasibility of tuned mass damper to reduce the heave response of a floating vertical circular cylinder. *Singapore Maritime and Port Journal* (accepted for publication).