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**DEVELOPMENT OF DISCONTINUOUS DEFORMATION
ANALYSIS FOR DYNAMIC ANALYSIS OF ROCK MASS**

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2008

DEVELOPMENT OF DISCONTINUOUS DEFORMATION ANALYSIS FOR DYNAMIC ANALYSIS OF ROCK MASS

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SUMMARY

The response of rock mass subjected to a blast is an important topic in underground construction and design of protective structures. Dynamic loadings from the blast are usually classified as impulsive loads with high-amplitude and short duration. Due to the complex nature of the blast load and limited experimental data, dynamic behaviors of rocks are relatively difficult to predict. Adequately detailed numerical simulations thus become desirable to provide an alternative tool of scientific investigation, instead of carrying out expensive and time-consuming or even dangerous experiments in laboratories or on sites.

In this study, a relatively new numerical method called discontinuous deformation analysis (DDA) is selected to model the response of rock mass subjected to dynamic loads. The DDA treat a discontinuous domain as a system of individually deformable blocks with constant stress and strain which move independently without interpenetration. The unique features of the DDA method make it suitable to solve the rock mass problems. In this thesis, some new subroutines and modifications have been added to the original DDA code to extend its applications.

The linear displacement function in the DDA facilitates the contact judgment but generates constant stress field within a block, which can not effectively describe the stress variation within a block or across the block interface. To improve the stress accuracy over the block-interface, a stress recovery procedure is proposed based on the concept of glutting small blocks together. Two cases are simulated. The results indicate that the proposed recovery method can provide a more accurate stress state at the corner than the average method, especially for the corner surrounded by irregular blocks.

Numerical simulation of the response of rock mass under blast loads is constructed in this study. The deformation of the surrounding rock mass and the blast wave propagation are investigated. It indicates that the current DDA program is able to predict and to provide some valuable results on the rock medium under dynamic loads. The numerical simulations on the laboratory scaled blast experiments have also been investigated. To consider the influence factors that control rock fracturing, studies with regard to the different material properties and borehole pressure waveforms are carried out. Some preliminary understanding and interpretation to the simulation results are given.

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LIST OF SYMBOLS

ϕ	Friction angle of rock joint
c	Cohesion of rock joint
t	Tensile strength of rock joint
τ	Shear Stress
HE	High explosives
S_v	Shear waves
ρ	Density of rock mass
P_b	Maximum blast pressure
r_0	Rotation angle of the block
Π	Potential energy
E	Young's modulus
K	Global stiffness matrix
F	Global loading matrix
D	Deformation variables
ε	Strain
σ	Stress
M	Mass weight
p	Stiffness of springs
B	Strain matrix
A	Block area
$r^{(i)}$	Inter-block force
T	Stress tensor
ν	Poisson's ratio
W	Weight of the effective explosive
Δt	Time step interval
m	Coefficient of uniformity
n	Exponential coefficient of the stress attenuation
P	Compressive wave
PP	Reflected wave
μ	Mean value

CHAPTER 1 INTRODUCTION

1.1 BACKGROUND

Rock masses are brittle and heterogeneous materials extensively utilized in civil engineering, including special protective structures. The jointed rock mass can have very complex mechanical behavior, such as anisotropy, heterogeneity, dilatancy, irreversible strain and strongly path-dependent stress-strain relationship, which is generally associated with the existence of a great deal of cracks and their propagation. The presence of the discontinuities is the most important influence factor to govern the rock mass behavior in rock engineering.

Explosives are often used to break rock. The blast holes are drilled into the rock mass and the explosives are placed in. The detonation pressurizes the borehole wall and generates a very strong shock wave that is transmitted into the rock. The response of rock mass under dynamic loadings is by far, the important concern in geotechnical engineering worldwide which is not well understood. Analyzing the behavior of rock mass under dynamic loads and understanding the mechanisms of dynamic fracture caused by rock blasting have been the subjects in this study.

Numerical simulations provide an alternative tool of scientific investigation, instead of carrying out expensive, time-consuming or even dangerous experiments in laboratories or on site. Sometimes it is also more useful than the traditional experimental methods in terms of providing insightful and complete information that can not be directly measured or observed, or difficult to acquire via other means.

In 1988, a new numerical method called the discontinuous deformation analysis (DDA) was first proposed by Dr. Shi (Shi, 1988). Due to its unique features for solving the discontinuous problems, it is very suitable for rock mass analysis. Many studies on the DDA have been carried out over the last two decades, and its

CHAPTER 1 INTRODUCTION

applications have been extended to many rock engineering areas (Yin et al, 2002; MacLaughlin et al., 2003; Hatzor et al., 2004). However, the studies on the rock mass behavior and fracture propagation process under dynamic loads by using the DDA method are still limited.

1.2 OBJECTIVES OF THIS THESIS

The purpose of this thesis is to extend the original DDA code and make it suitable for the numerical analysis on the rock mass behavior under the blast loads.

The study focuses on numerical investigation of rock mass response under blast loading, fracture initiation and propagation process under blast loads. The scopes are outlined as follows:

- To carry out a comprehensive review on the theory of DDA and rock mass behavior under static and dynamic loading.
- To implement some subroutines to the original DDA code to improve its ability to simulate the rock mass response and extend its applications.
- To improve the stress state by developing a stress recovery procedure based on the error minimization.
- To study the response of rock mass to underground explosion with a blast load transfer model.
- To carry out numerical simulations of laboratory scaled blast tests, to study the fracture processes, the final fracture patterns and implement a rock mass heterogeneity model into the DDA code.

In summary, throughout the study, we extend the DDA's ability on the applications of the rock mass under dynamic loads and provide some insights and knowledge on rock fracturing process.

1.3 ORGANIZATION OF THIS THESIS

This thesis is divided into eight chapters. Chapter 1 introduces the background and the objectives of this research work. In Chapter 2, a comprehensive review will be

given to the behavior of jointed rock mass and its response under the blast loads. The methods of modeling blast loads are also introduced. Chapter 3 presents a brief review on the current numerical methods in rock mechanics and rock engineering. The discontinuous deformation analysis (DDA) is compared with the other methods for its unique features on solving the discontinuous problems. The basic concepts and theories of the DDA are also described. In Chapter 4, the supplements and modifications are implemented into the original DDA code to improve its ability of simulating the rock mass problems. Chapter 5 proposes a stress recovery procedure for DDA to improve its nodal stresses and inter-block stresses. Based on the above chapters, two numerical simulations are performed for analyzing the rock mass under dynamic loads. The response of rock mass with an underground explosion is analyzed in Chapter 6 and Numerical simulation of scaled blast tests is given in Chapter 7. Finally, Chapter 8 draws the conclusions and gives recommendations for the future work.

CHAPTER 2 MECHANICAL BEHAVIOUR OF JOINTED ROCK MASS

2.1 OVERVIEW OF ROCK MASS

2.1.1 Rock Material and Rock Mass

Rock has been used as a natural material for a long history in various manners such as a construction material of buildings and other civil structures, or as a foundation base of surface structures, or as a surrounding medium of underground caverns. Natural rock that differs from most other engineering materials consists of rock material and a lot of discontinuities (Fig. 2.1). Rock material is termed to describe the intact rock between discontinuities while the rock mass is the total in-situ medium containing discontinuities. The rock material has relatively high stiffness and strength. Similar to soil and concrete, the rock material can often be treated as a continuous medium in the theoretical analysis and numerical modeling. However, the presence of the discontinuities, which produce most of the deformations in rock mass and have a dominant effect on the rock mass response (Brady and Brown, 1993), makes rock mass discontinuous, inhomogeneous and anisotropic media and greatly governs the rock engineering behavior depending on their strength, orientation, persistence, aperture, spacing and filling materials, roughness and matching of the surfaces. In the case of wave propagation induced by dynamic loads, the discontinuities give more attenuation than rock material to the wave propagation and also control the initiated fracture orientation and fracture generation patterns. In the engineering view, rock mass can be treated to be composed of rock materials conglomerated by the discontinuities such as rock interfaces, faults, joints and innumerable/infinite microcracks depending on the research scale size.

Fig. 2.1 Typical jointed rock mass

2.1.2 Discontinuities of Rock Mass

Discontinuities in a rock mass were formed under different geological conditions and have undergone a long geological history. According to their formations and geological characteristics, they can be classified into several types: rock interface, fault, joint and microcrack.

Rock interface is originally formed together with the formation of rocks. A typical rock interface is the bedding plane existing in the sedimentary rocks. It divides sedimentary rocks into beds or strata and represents interruptions in the course of deposition of the rock mass. It is often perfectly-bonded between two opposite sides. Both compression and tension can be transmitted across the interface, and no sliding displacement occurs. The interface is actually a discontinuity of media properties and makes the rock masses a stratified anisotropic medium.

Faults are fractures on which identifiable shear displacement takes place. They may

be recognized by the relative displacement of the rock on opposite sides of the fault plane and a wide fractured or squeezing zone could be observed due to the strong geological activities.

Joints and microcracks are the most common and generally the most significant discontinuities in the rock mass. They are of geological origin along which there has been no visible displacement. Joints may be saturated or filled with other materials. Microcracks can be considered as a kind of joints which have imperfectly-bonded interfaces in which certain tension strength and sliding displacements may also occur. So joints and microcracks are displacement discontinuities and lead the rock mass to be a fractured or jointed medium.

As commonly done in rock mechanics, the term “joint” is used to represent a collective term for all discontinuities in rock mass.

2.2 STATIC BEHAVIOR OF JOINTED ROCK MASS

The mechanical behavior of jointed rock mass is governed by the geomechanical properties, including Young’s modulus, Poisson’s ratio, compressive and tensile strength, joint orientation, spacing, friction and roughness et al.

2.2.1 Rock Behavior

Rock material has compressive strength as well as tensile strength depending on the rock type. The tensile strength of rock material is usually much lower than its corresponding compressive strength (e.g. one tenth).

2.2.2 Joint Behavior

The joint is usually described by Mohr-Coulomb slip criterion together with a tensile cut-off.

- **Mohr-Coulomb Slip Criterion**

The Mohr-Coulomb slip criterion is expressed as (Zheng and Gong, 2002):

$$\tau = c + \sigma_n \tan \varphi \quad (2-1)$$

where c is the cohesion, φ is the friction angle and σ_n is the normal stress along the oblique face.

When the shear exceeds this value, the slip will take place.

- **Tensile Cutoff**

When the tensile stress exceeds a given value, the tensile resistance will be zero and the joint is open.

2.2.3 Rock Mass Behavior

Actual rock mass is often fractured by a lot of joints and become anisotropic media. Therefore, the response of jointed rock mass is governed by the stress distribution and properties of joint mechanical properties.

Numerical modeling of the mechanical behavior of jointed rock mass is a difficult task as the discontinuities not only require special modeling consideration, but also require different treatments depending on the scale of the problem involved. The usual way to treat the discontinuity is described in the following. The discontinuities such as joints or faults are divided into two groups. The primary discontinuity set is the one with a wider spacing and dominates the kinematics of the system, which is modeled explicitly as physical discontinuities. The secondary discontinuity set, characterized by high density and small spacing, is modeled as an equivalent continuum by incorporating joint compliance and strength characteristics (Lin and Ku, 2006).

2.3 DYNAMIC RESPONSE OF JOINTED ROCK MASS

Static behaviors of rocks are relatively mature to trace with the development of rock mechanics. The compressive and tensile properties of rock material have been acknowledged and employed in the design of engineering constructions. However, dynamic behaviors of rocks are relatively difficult to be obtained because of the short duration of high strain rate loadings. Modeling the dynamic response of rock materials is a challenging area of research.

Dynamic loadings are usually concerned about high-amplitude, short duration impulsive loads. All the seismic, earthquake, impact, explosion and blasting belong to dynamic loads. It is of great importance to investigate the dynamic behaviors of rock-like materials such as the dynamic strength and fracture process. The study and research focus on the fracture mechanics and fragmentation process for rock-like materials under high strain rate loadings. With the development of digital image technique and the use of the high speed camera, rock fracture mechanism under dynamic loadings is on the way of being understood.

Different from the static loads effect, the rock mass response to the dynamic loads becomes more complicated depending on the loading rate.

2.3.1 Rock Response

Similar to the other continuous materials, rock material also possesses geometric damping and material damping to the wave propagation. The wave, which is subjected to geometric spreading, decreases in amplitude and frequency with distance away from the wave source where the energy is distributed over. During the wave propagation, the energy dissipates in the host medium of rock material, due to the temperature exchange with surrounding media (Chen, 1999).

The dynamic behavior of rock material is often described by its strength (compressive and tensile strength), elastic modulus and stress-strain relationship and strain rate. Generally, the strength of rock material increases with increasing strain rate, but its amount varies with the rock types. Brace and Jones (1971) summarized the experimental results under different strain rates. It is shown that the strengths of igneous rocks increases about 10% with the strain rate increasing of three orders of magnitude (10^3 s^{-1}), while the strength of limestone increases 18%. Zhao (1998) reviewed the experimental results on rock strength at different strain rates and found that rock strength increases with increasing strain rate. Rocks which have higher strength are less sensitive to the strain rate effect. A schematic relation between compressive strength and strain rate was suggested by Zhao and Zhao (1998) as shown in Fig. 2.2. It was also found by Yang and Li (1992) that the elastic modulus increases with the increase in strain rate. The dynamic modulus can be greater than static modulus by up to 300% depending on the effect of the deformation of the closed microcracks in the rock material.

Fig. 2.2 Schematic relation between compressive strength and strain rate

2.3.2 Joint Response

It is commonly accepted that the joints and the microcracks dominate the response of jointed rock mass under loading (Goodman, 1976; Bandis et al, 1983). But relatively few research studies on the dynamic effect on the jointed rock have been

conducted either theoretically or experimentally. When an incident wave hits a jointed interface, the reflection and transmission waves will occur as shown in Fig. 2.3 (Cook, 1992). P is compression wave; S_v is plane shear waves polarized in the x - z plane, respectively. ϕ_i and θ_i ($i = 1, 2$) are incident, transmitted or reflected angles. Only material damping is considered and represented by the transmission coefficient, reflection coefficient and energy absorption coefficient. The geometric damping is often neglected because the thickness of joints is too small relative to the model size.

Fig. 2.3 Wave transmission and reflection at a rock joint (Cook, 1992)

2.3.3 Rock Mass Response

The natural rock mass that waves of the dynamic loads go through is often a composition of rock material and joints. So its response is related to both rock material and joints. A wave going through rock material encounters material damping and geometric damping. When the wave hits a joint, the wave will be partially transmitted, reflected and lose some energy. For rock mass with joint sets or microcracks, multiple wave reflections occur. The superposition and interaction of the waves including wave transmission and reflection between the joints cause

very complicated wave propagation in multi-jointed rock mass. In general, the poorer rock mass with more joints cause more wave attenuation, which is also joint orientation dependent (Cook, 1992).

2.4 INTRODUCTION OF BLAST EFFECT

For the consideration of dynamic load induced by the explosives, the most significant feature of an explosion is the huge release of energy to the atmosphere at an instant moment which results in a pressure transient, or blast wave. With the initiation, the explosive generates a mass of high temperature and pressured gas, and the peak velocity can reach 2000-8000 m/s. The energy of 1 Kg standard explosive can be 2000-6000 kJ and the temperature can reach 2000-4000 °C; while up to hundreds of thousands times of atmospheric pressures are generated in the microseconds. The blast wave propagates outward in all directions from the source at supersonic or sonic speed. The magnitude and shape of the blast wave depends on the nature of the energy release and on the distance from the explosion epicenter.

When the explosive is detonated in the blasthole, the surrounding medium will be subjected to an abrupt released energy. The detonated explosive can be looked as a shock epicenter. In the wave front, it first creates a huge shock wave transmitting in all directions from the blasthole with a very high pressure. It has a sudden, almost instantaneous rise in pressure above ambient atmospheric conditions to a peak free field overpressure. The blast wave attenuates while it propagates outward from the explosion epicenter. The peak overpressure and impulse decrease with the distance while the duration tends to increase. The gas generated during the explosion acts on the surroundings rock after the shock wave has passed. The blast then becomes the co-effect of the shock wave and explosion gas. Because the velocity of explosion gas is lower than that of the shock wave, they will separate with the subsequent process. The rock blocks around the blast will be mainly affected by the destroy

effect from the shock wave (Qi et al., 1995) and the fracture propagation due to the sustained pressure from the explosion gases.

2.5 MODELING OF BLAST LOADS

2.5.1 State Equation of High Explosive

High explosives (HE) are chemical substances which, when subjected to tensile stimuli, react chemically to give a very rapid (of microseconds order) release of energy. TNT is often used as the standard or reference explosive for equating the effects of various explosives. The free-air equivalent weight of a particular explosive is the weight of the standard explosive TNT required to produce a selected shock wave parameter of magnitude equal to that produced by a unit weight of the explosive in question (TM5-855-1, 1984).

- **Chapman-Jouget (CJ) Point**

In the hydrodynamic theory of detonation, this very rapid time interval is shrunk to zero and a detonation wave is assumed to be a discontinuity which propagates through the unreacted material instantaneously, liberating energy and transforming the explosive into detonating products (Gour, 1990). The Rankine-Hugoniot equations may be written as:

$$D^2 = v_0^2(p - p_0)/(v_0 - v) \quad (2-2)$$

$$u^2 = (p - p_0)(v_0 - v) \quad (2-3)$$

$$e - e_0 = Q + \frac{1}{2}(p + p_0)(v_0 - v) \quad (2-4)$$

where p is the pressure, v is the specific volume, e is the specific internal energy, u is the particle velocity, Q is the specific chemical energy and D is the detonation velocity. For a given initial pressure p_0 , initial density is defined as $\rho_0 = 1/v_0$.

Eq. (2-2) describes a straight line in the (p, v) plane, known as the Rayleigh line,

defining the locus of all (p, v) states consistent with the prescribed detonation velocity and conservation of mass and momentum. Eq. (2-4) is a purely thermodynamic equation which defines a concave downward curve in the (p, v) plane, known as the Hugoniot curve, which is the locus of all (p, v) states attainable from (p_0, v_0) by a discontinuous transition with energy conservation. The Hugoniot curve for a shock wave in inert material passes through the point (p_0, v_0) but for a detonation wave the exothermic energy term Q in the energy conservation equation results in the Hugoniot curve being displaced upwards in the (p, v) plane. The points of intersection of the Rayleigh line with the Hugoniot curve define material states consistent with all the conservation relations and the prescribed propagation velocity (Fig. 2.4).

Fig. 2.4 CJ Point

The Hugoniot curve as defined by Eqs. (2-2) to (2-4) divides the p - v space into an upper branch with $p > p_0$, $v < v_0$, a lower branch with $p < p_0$, $v > v_0$ and a region in between where $p > p_0$, $v > v_0$, which has no physical meaning since it defines an imaginary propagation velocity. The upper branch of the Hugoniot curve corresponding to higher velocities is appropriate to detonations while the lower branch corresponds to deflagrations. It can be seen from Fig. 2.4 that the intersection of the Rayleigh line and the Hugoniot curve define the existence of any detonation velocity above a minimum value and each value is consistent with two

possible end values for the detonation products. Chapman and Jouget independently proposed that the velocity is the minimum velocity consistent with the Rankine-Hugoniot relations and this postulate is known as the Chapman-Jouget (CJ) hypothesis. It is seen in Fig. 2.4 that the CJ hypothesis corresponds to tangency of the Rayleigh line with the Hugoniot curve, we denote this condition at the tangent point as the CJ point (AUTODYN Theory Manual).

Because the detonation wave is assumed to be infinitesimally thin and the detonation process to be instantaneous we are concerned only with changes of state from the CJ point in the (p, v) plane. The detonation products may either be subjected to increasing pressure from shocks or compression waves or decrease in pressure through adiabatic expansion. Following expansion they may suffer an increase in pressure from subsequent compression waves or shocks. However because the detonation products are very hot it is difficult to change their entropy significantly so both for pressures above and below the CJ pressure the achievable states lie close to the adiabat passing through the CJ point (known usually as the CJ adiabat). An equation of state of a Gruneisen forms and the reference curve based on the CJ adiabat promises to be a good representation of the behavior of the detonation products. Attention has been therefore concentrated on developing good analytic forms for the CJ adiabat.

Based on this concept, several forms of explosive equations of state have been proposed evolutionarily since the 1939 such as Ideal gas form, Constant beta form, Wilkins form and the Jones–Wilkins–Lee (JWL) form. The Jones-Wilkins-Lee (JWL) equation of state models the pressure generated by the expansion of the detonation product of the chemical explosive, and it has been widely used in engineering calculations (Lee, 1968). This form of equation of state is employed in the current study.

- **Jones-Wilkins-Lee (JWL) Equation of State**

This integrated experiment form is the modification of the original Wilkins form. Providing a stringent test of the low pressure behavior of detonation when it expands along the CJ adiabat, this form is the expansion of a hollow metal cylinder filled with explosive and detonated across the explosive face at one end. The detonation wave proceeds axially along the cylinder and the expansion of the metal can be accurately measured. This experiment is called the “cylinder test”.

The Wilkins form of equation is capable of predicting the motion correctly until the pressure in the products falls below about 5 kbar but then becomes insufficiently energetic.

To fit this data, it was proposed (Lee, 1968) that the reference adiabat should be further modified by replacing the first term (the power law term) in the Wilkins equation by a second exponential term to give:

$$p = C_1 e^{-r_1 v} + C_2 e^{-r_2 v} + B v^{-(1+w)} \quad (2-5)$$

Giving an energy equation form:

$$P = C_1 \left(1 - \frac{w}{R_1 v} \right) e^{(-r_1 v)} + C_2 \left(1 - \frac{w}{R_2 v} \right) e^{(-r_2 v)} + \frac{w e}{v} \quad (2-6)$$

where v is the specific volume, e is the specific energy.

This formulation is known as the “Jones-Wilkins-Lee” (JWL) equation of state and is currently in favor for hydrodynamic calculations of detonation product expansions. The values of constants C_1 , R_1 , C_2 , R_2 , w for many common explosives have been determined from many dynamic experiments which are obtained by fitting Eq. (2-6) to the CJ isentropic calculated using an ideal detonation

thermodynamics code. The values of these constants should be considered as a set of interdependent parameters and one constant can not be changed unilaterally without considering the effect of this change on the other parameters (Fig. 2.5).

Fig. 2.5 Pressure with specific volume in JWL equation

Table 2.1 JWL parameters used for modeling TNT

C_1 (GPa)	C_2 (GPa)	r_1	r_2	ω	e (MJm ⁻³)	VOD (ms ⁻¹)	ρ_0 (kgm ⁻³)
373.8	3.747	4.15	0.9	0.35	6000	6930	1630

The JWL equations of state and the reaction rates have been fitted to one and two dimensional shock initiation and detonation data for four explosives: PBX-9404, RX-03-BB, PETN, and cast TNT. A relevant table for JWL parameters for modeling TNT in this study is listed in Table 2.1.

2.5.2 Pressure Waveform

- **Peak Blast Pressure**

The Jones-Wilkins-Lee (JWL) equation of state models the pressure generated by the expansion of the detonation product of the chemical explosive, and it has been widely used in engineering calculations which can be written in the form of Eq. (2-6) (Lee, 1968).

The blast wave pressure can also be derived from the detonation fluid dynamics. For a fully coupled blasthole, the detonation wave front pressure is

$$P = \frac{\rho_0 D^2}{4} \quad (2-7)$$

where, ρ_0 is the density of the explosive, D is the detonation velocity.

Then, based on the theory of transmission and reflection, the transmitted pressure acting on the interface between explosion and rock can be derived as

$$P_m = \frac{1+N}{1+N\rho_0 D / \rho v} P \quad (2-8)$$

where, ρ is the density of rock, v is P wave propagation velocity in rock, ρv is the wave impedance of rock, and

$$N = \frac{\rho_0 D}{\rho_1(D' + u_1)} P \quad (2-9)$$

where, $\rho_0 D$ is the wave impedance of the explosive, $\rho_1(D' + u_1)$ is the wave impedance of detonation product.

According to the approximated theory of acoustics, Eq. (2-8) can be simplified as

$$P_m = \frac{2\rho v}{\rho v + \rho_0 D} P \quad (2-10)$$

Eq. (2-10) indicates that when the same explosive is initiated, the value of the pressure acting on the surrounding rock will vary in different rock materials. The larger the wave impedance ρv , the larger the maximum P_m will be.

- **Peak Pressure on the Fractured Zone**

When an explosive is detonated in the rock, a crushed zone will be generated in the immediate vicinity of the borehole if the blast load peak pressure exceeds the dynamic compressive strength of the rock. Following the form of the crushed zone, radial fractures also occur when the stress exceeds the tensile strength of the rock. The dimension of crushed zone is usually between 2D to 7D, where D is the diameter of the blasthole (Thornton et al., 2001). In the current study, a proper crushed zone is assumed. Then, the peak pressure, which is determined by JWL equation with the specific volume v , is applied to the boundary where the crushed zone ends.

- **Pressure Time History of Blast Loads**

The blast induced pressure wave usually decays exponentially. The waveform of the pressure time function can be expressed in the following form (Cho and Kaneko, 2004)

$$P = P_b \xi \left[e^{-\alpha t} - e^{-\beta t} \right] \quad (2-11)$$

where P is the pressure at time t , P_b denotes the maximum blast pressure and α and β are constants and

$$\xi = \frac{1}{e^{-\alpha t_0} - e^{-\beta t_0}} \quad (2-12)$$

$$t_0 = [1/(\beta - \alpha)] \ln(\beta / \alpha) \quad (2-13)$$

where the values of the parameters used in this study are listed as follows: $P_b = 30\text{MPa}$, $\beta/\alpha = 1.5$, t_0 can be taken as 10, 100 and 1000 μs , which form three different

pressure time loading curves.

2.6 ROCK RESPONSE UNDER BLAST LOADS

2.6.1 Introduction

Rocks are brittle and heterogeneous materials, and rock mass is characterized by the existence of discontinuities such as joints, faults and fractures. Most rock structures (e.g. underground caverns, tunnels, hydro-power stations) are often subjected to dynamic loading, such as seismic wave, impact, blasting and ground shock. So the mechanisms of dynamic fracture caused by rock blasting have been the subject of study in their applications on designing mining excavations, recovering oil from oil shale, and other fields (Haghighi et al, 1988; Fournery, 1993). In rock blasting, both the stress waves and gas pressurization make significant contributions to rock fracture and fragmentation.

2.6.2 Rock Fragmentation Mechanism under Blast Loads

In the fragmentation of rocks with explosives, two breakage mechanisms, namely crushed zone and cracked zone are involved, illustrated in Fig. 2.6.

- **Crushed Zone**

In the first instants of detonation, the surrounding rock media will undertake the intense shock by the generated compressive stress wave. It expands in cylindrical form, can reach even hundreds and thousands atmospheric pressure that well exceeds the dynamic compressive strength of any rock, provoking the destruction of the rock mass surrounding the charge and forming a crushed zone (shown in Fig. 2.6).

With high strength explosives in porous rocks crushed zone might reach a radius of up to $8D$, but it is normally between $2D$ and $7D$, where D is the diameter of the

blasthole (Thornton et al, 2001). According to Hagan (Hagan, 1979), this breakage mechanism consumes almost 30% of the energy transported by the stress wave. This part of energy only contributes a very small volume of fragmentation, which is around 0.1% (this value may be larger when near the ground surface because of charge location dependence) of the total broken rock volume (Carlos et al, 1995).

Fig. 2.6 Illustration of blast induced crushed zone and cracked zone

- **Cracked Zone**

The cracked zone is formed due to both the radial fracture and the circular fracture surrounding the charge (shown in Fig. 2.7).

Radial Fracturing

During propagation of the stress wave, the rock surrounding the blasthole is subjected to an intense radial compression which induces tensile components in the tangential planes of the wave front. As we know, rock belongs to the brittle material, and its tensile strength is much smaller than its compressive strength (about 1/15-1/20 compressive strength). When the tangential stresses exceed the dynamic

tensile strength of the rock, the formation of a dense area of radial cracks around the crushed zone that surrounds the blasthole is initiated. This kind of crack will extend until the tensile stress is less than the tensile strength of the rock.

When the rock has natural fractures, the extension of the cracks is closely related to these weak parts. If the explosive columns are intersected lengthwise by a pre-existing crack, these will open with the effect of the stress wave and limit the development of radial cracks in other directions.

If some natural fractures are parallel to the blasthole, they will interrupt the propagation of the radial cracks. While the shock wave propagates through the rock mass, it will be reflected and refracted at the discontinuities. Some of the energy carried by the shock wave will then be consumed by the deformation of the discontinuities, and therefore, the shock wave will attenuate rapidly.

Fig. 2.7 Rock fragmentation process under blast load

Gas Extended Fractures

After the stress wave passes, the gas expands rapidly and causes a quasi-static stress field around the blasthole. During or after the formation of radial cracks by the

tangential tensile component of the wave, the gas starts to expand and penetrate into the fractures. The radial cracks are prolonged under the influence of the stress concentrations at their tips.

Circular Fracturing by Release-Of-Load

After the compressive wave has passed and a state of quasi-static equilibrium has been produced by a subsequent fall of pressure in the blasthole as the gases escape through the stemming through the radial cracks, the cracked rock will have some displacement outwards. The domain near the charge will gradually form a reduced stress zone. Those original rocks which are a little further from the charge can not move to push the rock outside them, the stored stress energy will be released, the rock mass will move towards the blasthole. During this process, the tangential compressive stress will induce tensile stress. When this tensile stress exceeds the tensile strength of the rock, the inwards rock will be snapped and form some circular cracks around the charge.

Reflection Breakage or Spalling

When the stress wave reaches a free surface, two waves (a tensile wave and a shear wave) are reflected. The compressive wave is reflected as tensile wave which forms tensile stress acting on the rock towards the center of the charge. When the reflected stress exceeds the tensile strength of the rock, it will form a set of tensile cracks along the free surface. This occurs when the radial cracks have not propagated further than one third the distance between the charge and the free face. This kind of cracks will intersect with the cracks around the charge. The broken rock will be cast out from the free surface with the effect of the expanding gas. We as usual call it “Blast Funnel” (Qi et al, 1995).

As depicted in Fig. 2.7, both the shock wave and the effect of expanding gas exist in

the process of rock blast, but the duration of gas is longer than that of shock wave.

2.6.3 Effect of Shock Wave and Gas in Rock Fracturing

Whether the major cause of fragmentation is due to the large amplitude stress waves that travel through the rock, or due to the very large gases that expand with quasi-static stress field or both, has been discussed for the last 50 years and is still debatable. Many formal publications attempt to offer the evidence that either stress waves or gas pressure is responsible for the fragments. Essentially, this discussion has been divided into three groups of theories. The first establishes the shock pressure as the dominant factor in fracture (Preece et al, 1994; Mortazavi and Katsabanis, 1998), while the second contends that the gas pressure generated by the explosive is the primary agent in the process (Mchugh, 1983; Fournery et al, 1993). The third tend to support the view that both the stress wave and gas pressurization contribute to rock fragmentation (Brinkman, 1987; Wilson, 1987). The stress waves generated by the detonation of an explosive charge are responsible for the development of a major fractured zone in the rock mass and the subsequent fragment size distribution, while the explosion gases are important in extending the fractures that are formed after passage of the stress wave, and in throwing the fragments.

To rocks with different properties, these two actions have different influence on the destruction of the rock mass. Usually in the floppy terrain such as soft rock, its failure is mainly because of the effect of expanding gas; while in most stiff materials like the granite, the blast shock will dominate the failure of the rock mass.

CHAPTER 3 NUMERICAL MODELING OF ROCK MASS

3.1 INTRODUCTION

Numerical simulation using computers or computational simulation has increasingly become a very important approach for solving complex practical problems in engineering and science. Numerical simulation translates important aspects of a physical problem into a discrete form of mathematical description, recreates and solves the problem on a computer, and reveals phenomena virtually according to the requirements of the analysts. Rather than adopting the traditional theoretical practice of constructing layers of assumptions and approximations, this modern numerical approach attacks the original problems in all its detail without making too many assumptions, with the help of the increasing computer power.

Numerical simulation provides an alternative tool of scientific investigation, instead of carrying out expensive, time-consuming or even dangerous experiments in laboratories or on site. The numerical tools are often more useful than the traditional experimental methods in terms of providing insightful and complete information that can not be directly measured or observed, or difficult to acquire via other means.

Numerical simulation with computers also plays a valuable role in theory validation, offers insights to the experimental results and assists in the interpretation or even the discovery of new phenomena. It acts also as a bridge between the experimental models and the theoretical predictions.

Because there are much more limitations for theoretical or experimental approaches, numerical methods have been a preference in analysis of underground structures built in rock mass. However, the presence of discontinuities in the rock mass, which usually appear in the form of faults, joints or bedding planes, is quite a challenge for numerical simulation. One of the main tasks of numerical modeling is to represent

such discontinuities physically in the computer model. And, to simulate the mechanical behaviors of rock mass such as the discontinuous deformation, crack propagation and fragmentation, as well as the large-scale displacement, is still a long way to go.

In this chapter, a brief introduction of the numerical methods for rock mass will be given first. Comparison of the continuum method with the discontinuum method is also discussed. Then, the basic concepts and theories of Discontinuous Deformation Analysis (DDA) will be given. The block displacement functions, global simultaneous equilibrium equations, contact representation and detection and loading or constraints on block systems will be introduced respectively. After that, some considerations in the DDA program on solving the dynamic problems will be given. The framework of the DDA is listed. Finally, concluding remarks on how to model rock mass under dynamic loadings by the DDA is given.

3.2 NUMERICAL METHODS IN ROCK MECHANICS AND ROCK ENGINEERING

3.2.1 Special Nature of Rock Mass

The rock mass is largely Discontinuous, Inhomogeneous, Anisotropic and Not-Elastic (DIANE) (Harrison, 2000). The fractures are mostly clustered in certain directions resulting from their geological modes and formation history.

One of the main tasks of numerical modeling in rock mechanics is to be able to characterize such mechanical discontinuities in a computer model either explicitly or implicitly in the so called “material conceptualization”. Additionally, the interaction between the rock mass and the engineering structure has to be incorporated in the modeling procedure for design and the consequences of the construction process have also to be characterized.

3.2.2 Difficulties in Capturing Rock Reality

The extent to which these features (DIANE) can actually be incorporated into a computer model will depend on the physical processes involved and the modeling techniques used. Hence, both the modeling and any subsequent rock engineering design will contain subjective judgments (Jing, 2003).

The reason for the general difficulty in modeling rock mass, by whatever numerical method, is that rock is a natural geological material and the physical or engineering properties have to be established, rather than defined through a manufacturing process. Rock mass is under stress and continuously loaded by dynamic movements of the upper crust of the Earth, such as tectonic movements, earthquakes, land uplifting or subsidence, glaciation cycles and tides. A rock mass is also a fractured porous medium containing fluids in either liquid or gas phases, e.g. water, oil, natural gases and air, under complex in situ conditions of stresses, temperature and fluid pressures.

The complex combination of constituents and its long history of formation make rock mass a difficult material for mathematical representation via numerical modeling. Until now, it is still hard for us to adequately represent the rock mass in computational models, capturing such fracturing and the complete DIANE nature of the rock mass, plus the consequences of engineering.

Of course as we know the numerical model does not have to be complete and perfect, it only needs to be adequate for the purpose. Thus, the challenge is to know how to develop an adequate model for our current problems. To some extent, rock mechanics modeling and rock engineering design are both a science and an art.

3.2.3 Numerical Methods in Rock Mechanics

The fractured rock mass is a discrete system. Closed-form solution does not exist for such problems and numerical methods must be used for solving practical problems. There are mainly two types of methods, namely continuum methods

(such as Finite Difference Method (FDM), the Finite Element Method (FEM), the Boundary Element Method (BEM)) and discontinuum methods (such as the Distinct Element Method (DEM) (Cundall, 1971), Discontinuous Deformation Analysis (DDA) (Shi, 1988)).

For continuum methods, the problem domain is usually divided into a finite number of sub-domains whose behavior is approximated by simpler mathematical descriptions with certain DOFs. These sub-domains, usually treated as continuous, must satisfy both the governing differential equations of the problem and the continuity condition at their interfaces. Numerical approaches with the above-mentioned background are termed as continuum-based numerical methods. Different discretization leads to various continuum methods.

The FDM is a direct approximation of the governing PDEs by replacing partial derivatives with differences at grids imposed over problem domains. Thus, the FDM transfers the original PDEs into a system of algebraic equations in terms of unknowns at grid points. By imposing the initial and boundary conditions, the solution of the system equation can be solved. The FEM divides the problem domain into a collection of elements of smaller size and standard shapes with fixed number of nodes. Trial functions, usually polynomial, are used to approximate the behavior of PDEs at the element level and generate the local algebraic equations. The local elemental equations are then assembled into a global equation according to the topologic relations between nodes and elements. By imposing the initial and boundary conditions, the global equation can be solved. The BEM, on the other hand, requires discretization at the boundary of the solution domain only, thus reducing the problem dimensions by one and greatly simplifying the input requirements.

The discrete element methods represent the fractured medium as assemblages of blocks and solve the equations of motion of these blocks through continuous detection and treatment of contacts between blocks. The blocks can be rigid or deformable. Large displacements caused by rigid body motion of individual blocks,

including block rotation, fracture opening and complete detachments. The discontinuum methods include the Distinct Element Method (DEM) (Cundall, 1971), Discontinuous Deformation Analysis (DDA) (Shi, 1988) and so on.

Numerical modeling of the mechanical behavior of jointed rock mass is a difficult task as the discontinuities not only require special modeling consideration, but also require different treatments depending on the scale of the problem involved. The usual way to treat the discontinuity is described in the following. The discontinuities such as joints or faults are divided into two groups. The primary discontinuity set is the one with a wider spacing and dominates the kinematics of the system, which is modeled explicitly as physical discontinuities. The secondary discontinuity set, characterized by high density and small spacing, is modeled as an equivalent continuum by incorporating joint compliance and strength characteristics (Lin and Ku, 2006).

The choice of continuum or discrete methods depends on problem scale and fracture geometry. Continuum methods should be used for rock masses with no fractures or only with secondary fractures, the behavior of the latter being described through equivalent properties established by a homogenization process. The continuum approach can be used only if a few fractures are present and no fracture opening and no complete block detachment is possible. The discrete approach is most suitable for the rock mass with primary fractures where the number of fractures too large for continuum-with-fracture-elements approach, or where large scale displacement of individual blocks are possible, for which the continuum method is not applicable at all (Jing, 2003).

Though the discontinuum methods (such as DEM, and DDA) are suitable for the jointed rock mass problems, it also has several limitations. Their block system identification depends on the reconstruction of the fracture system. However, the real fracture system is hidden inside rock masses and can never be fully assessed by measurements. The usual procedure is to establish probabilistic density functions of the fracture parameters (orientation, spacing, trace length, etc.) and then use the

Monte-Carlo method to re-generate a system of synthetic fracture system which shares the same statistical geometric properties of the sampled fracture population from the borehole logging. The reliability of such a model is therefore strongly dependent on the quality and quantity of the data available (Jing, 2003).

3.3 THE DISCONTINUOUS DEFORMATION ANALYSIS (DDA)

3.3.1 Overview of DDA

In 1988, a new numerical method “Discontinuous Deformation Analysis (DDA)” was introduced by Shi (Shi, 1988; Shi and Goodman, 1988). It is a new discrete element method originated from backward analysis to find a best fit to the deformed configuration of a block system from measured displacements and deformations (Shi and Goodman, 1989).

From a theoretical point of view, the DDA is a generalization of the FEM and the DEM. Its block system seems to resemble the DEM, and it more closely parallels the FEM in the whole problem solving procedure. It is similar to the DEM because it also considers the rock mass as an assemblage of discrete blocks. But it differs from the DEM in that it is displacement-based, implicit method. It acts the same way as FEM does: chooses the displacements as variables; minimizes the total potential energy to establish the equilibrium equations; adds stiffness, mass and loading submatrices to the coefficient matrices of the simultaneous equations. The unique features of the DDA can be generalized as follows: 1) complete block kinematics and its numerical realization; 2) perfect first order displacement approximation; 3) strict postulate of equilibrium; 4) correct energy consumption (frictional loss along contacts is the sole source of energy consumption without adding any artificial damping; (5) easy incorporate parameters and mechanical relations (Ke, 1993).

Compared with the explicit DEM, the DDA method has four basic advantages:

- 1) The equilibrium condition is automatically satisfied for quasi-static problem without using excessive iteration cycles;

- 2) The time interval can be larger without numerical instability problem;
- 3) Analytical integration method is adopted instead of numerical integration method, such as Gauss quadrature techniques;
- 4) It is easy to programming by converting an existing FEM code into a DDA code.

3.3.2 Recent Development of DDA

The recent developments of the DDA method mainly focus on three aspects: 1) Improvement of the contact algorithm; 2) Rock fracturing simulation; 3) Improvement of the stress field in each block. The penalty method was originally used in the DDA method to enforce the contact constraints between blocks. However, this method highly depends on the choice of the penalty value and satisfies the contact constraints only approximately. Cai and Liang (1996) adopted the Lagrange Multiplier Method to fulfill the contact constraints of no penetration and no tension. Lin et al. (1996) implemented an Augmented Lagrangian Method into the DDA code, which allows block-to-block contacts to be enforced more precisely and block contact forces to be determined more accurately.

In the original DDA code, the fracturing within a block is not considered. However, fracturing simulation is essential for rock engineering application. Ke (1993) extended the DDA by incorporating artificial joints to numerically simulate fracture propagation, in which a block is divided into several sub-blocks and the fracture initiate and propagate along the artificial joints between sub-blocks. Lin et al. (1996) used a three-parameter Mohr-Coulomb criterion to allow intact rock to be broken into smaller blocks (both tensile and shear failure) and fractures can propagate in sub-blocks either in Mode I or Mode II.

The original DDA simulates each discrete block as a simple deformable body with only six degrees of freedom (six variables in each block, which are three rigid body motion and three constant strain components) in a 2D case and is not suitable for irregularly shaped blocks. Ke (1993) and Lin et al. (1996) discretized each large block into several sub-blocks to capture the variance of stress in the original large

block. Koo and Chern (1996) and Hsiung (2001) developed the second and third order displacement functions to improve the accuracy of the stress field in each block. Some other researchers (Shyu, 1993; Chang, 1994; Jing, 1998) focus on the full internal discretization of blocks by triangular or four-noded FEM elements. These improvements make the DDA more suitable for arbitrary shaped deformable blocks. Al-Zaharani (2000) presented a hybrid numerical model that combined the DDA with the BEM, in which the DDA was used to numerically simulate the near field behavior and the BEM was employed to model the behavior of the far field.

There are also some other improvements, such as 3-D DDA (Shi, 2001; Liu and Kong, 2003), coupled flow-stress analysis (Kim et al., 2000; Jing et al., 2001), user-friendly code structure and environment (Chen et al., 1996; Chen, 1998) and so on.

The DDA method has also been extended to apply in a wide variety of geomechanical problems. Ke (1995) developed a DDA scheme to solve a rigid particle system consisting of circular discs. Yeung and Goodman (1995) presented the simulation of several rock slope failure problems. Chen et al. (1995) applied DDA to study the performance of a tunnel due to slope instability. Onhishi et al. (1996) described the application of DDA using a linear displacement function and a post-correction technique in rock fall analysis. Lin and Chen (1997) presented applications to the soil slope stability problems. Shyu et al. (1997) refined the DDA program and presented the analysis of deformable particles. Koo and Chern (1998) presented a rigid body version of the DDA method and its application in rock fall simulation. Zhang and Lu (1998) added more comprehensive representation of the fractures. Some research works have also been carried out on the dynamic problems by using DDA methods (Hatzor and Feintuch, 2001; Hatzor et al., 2002).

3.3.3 Representation of Blocks

In the 2-dimensional discontinuous deformation analysis, each element represents a block which can move, rotate, deform and all block interfaces are discontinuous.

The complete first order approximation of block displacements has the following forms (Shi, 1988):

$$\begin{aligned} u &= a_1 + a_2x + a_3y \\ v &= b_1 + b_2x + b_3y \end{aligned} \quad (3-1)$$

where (u, v) are the displacements of point (x, y) . At the point (x_0, y_0) the displacements are (u_0, v_0) :

$$\begin{aligned} u_0 &= a_1 + a_2x_0 + a_3y_0 \\ v_0 &= b_1 + b_2x_0 + b_3y_0 \end{aligned} \quad (3-2)$$

Subtracting Eq. (3-2) from Eq. (3-1):

$$\begin{aligned} u &= a_2(x - x_0) + a_3(y - y_0) + u_0 \\ v &= b_2(x - x_0) + b_3(y - y_0) + v_0 \end{aligned} \quad (3-3)$$

In Eq. (3-3), (a_2, a_3, b_2, b_3) can be obtained from the simple geometrical equation and rotation relationship:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = a_2 \\ \varepsilon_y &= \frac{\partial v}{\partial y} = b_3 \end{aligned} \quad (3-4)$$

$$\begin{aligned} \frac{1}{2}\gamma_{xy} &= \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \frac{1}{2}(b_2 + a_3) \\ r_0 &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(b_2 - a_3) \end{aligned} \quad (3-5)$$

Substituting Eq. (3-4) and Eq. (3-5) into Eq. (3-3), then,

$$\begin{aligned} u &= \varepsilon_x(x - x_0) + \left(\frac{1}{2}\gamma_{xy} - r_0\right)(y - y_0) + u_0 \\ v &= \left(\frac{1}{2}\gamma_{xy} + r_0\right)(x - x_0) + \varepsilon_y(y - y_0) + v_0 \end{aligned} \quad (3-6)$$

Assuming each block has constant stresses and constant strain throughout, the displacement (u, v) of any point (x, y) of a block can be represented as:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 & 0 & -(y-y_0) & (x-x_0) & 0 & \frac{(y-y_0)}{2} \\ 0 & 1 & (x-x_0) & 0 & (y-y_0) & \frac{(x-x_0)}{2} \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ r_0 \\ \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} \quad (3-7)$$

Eq. (3-7) forms the total block displacement function which is the accumulation of displacements induced by six variables ($u_0, v_0, r_0, \varepsilon_x, \varepsilon_y, \gamma_{xy}$). (u_0, v_0) is the rigid body translation of a specific point (x_0, y_0) within the block, r_0 is the rotation angle (given in radians) of the block with the rotation center at (x_0, y_0), ($\varepsilon_x, \varepsilon_y, \gamma_{xy}$) are the normal and shear strains of this block.

Figs. 3.1 to 3.5 show the basic movement/deformation of a typical block.

Fig. 3.1 Parallel translation

Fig. 3.2 Rotation

Fig. 3.3 Normal strain

Fig. 3.4 Shear strain

Fig. 3.5 General deformation

Eq. (3-7) can be simplified as:

$$\begin{pmatrix} u \\ v \end{pmatrix} = [T_i] \{D_i\} = \begin{pmatrix} t_{11} & t_{12} & t_{13} & t_{14} & t_{15} & t_{16} \\ t_{21} & t_{22} & t_{23} & t_{24} & t_{25} & t_{26} \end{pmatrix} \begin{pmatrix} d_{1i} \\ d_{2i} \\ d_{3i} \\ d_{4i} \\ d_{5i} \\ d_{6i} \end{pmatrix} \quad (3-8)$$

where $\{D_i\}$ represents the displacement variables of the i th block.

Eq. (3-1) presents a complete 2-dimensional first order Taylor series in the coordinate (x, y) and can be rewritten in the matrix form as

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 & 0 & x & 0 & y & 0 \\ 0 & 1 & 0 & x & 0 & y \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \\ a_3 \\ b_3 \end{pmatrix} \quad (3-9)$$

If Eq. (3-9) is generalized in the series form, the displacements of any point (x, y) can be represented in general by a two dimensional series approximation as

$$\begin{aligned} u &= \sum_{j=1}^m a_j f_j(x, y) \\ v &= \sum_{j=1}^m b_j f_j(x, y) \end{aligned} \quad (3-10)$$

This can be extended to the higher order displacement formulation to describe the stress field within a block more accurately.

3.3.4 Equilibrium Equation

- **Potential energy of the i th block**

By the second law of thermodynamics, a mechanical system under loading (external and/or internal) must move or deform in a direction which produces the minimum total energy of the whole system. For a block system, the total energy Π_i consists of the potential energy due to different mechanisms like external loads, block deformation, system constraints, kinetic and strain energy of the blocks and the dissipated irreversible energy.

$$\Pi_i = \{f\}^T \{D\} + \frac{1}{2} \{\varepsilon\}^T \{\sigma\} dA \quad (3-11)$$

where $\{f\}$ is the loading vector on the i th block distributed to the six deformation variables, $\{D\}$ is the displacement vector of the i th block, $\{\varepsilon\}$ and $\{\sigma\}$ are the strain and stress vector of the i th block, respectively.

The minimization of the system energy will produce an equation of motion for the block system, the same as that used in the FEM. Here we minimize the whole potential energy by first order differentiation with respect to the all six deformation variables:

$$\frac{\partial \Pi_i}{\partial d_{ri}} = 0, \quad r = 1, \dots, 6 \quad (3-12)$$

where d_{ri} is the deformation variable of the i th block.

For the i th block equations

$$\frac{\partial \Pi_i}{\partial u_0} = 0 \text{ and } \frac{\partial \Pi_i}{\partial v_0} = 0 \quad (3-13)$$

represent the equilibrium of all the loads and contact forces acting on the i th block along x and y directions.

$$\frac{\partial \Pi_i}{\partial r_0} = 0 \quad (3-14)$$

represents the moment equilibrium of all the loads and contact forces acting on the i th block.

$$\frac{\partial \Pi_i}{\partial \varepsilon_x} = 0, \quad \frac{\partial \Pi_i}{\partial \varepsilon_y} = 0 \text{ and } \frac{\partial \Pi_i}{\partial \gamma_{xy}} = 0 \quad (3-15)$$

represent the equilibrium of all external forces and stresses on the i th block along x , y directions.

• Global Simultaneous Equilibrium Equations

Individual blocks are connected and a block system is formed by contacts and displacement constraints between blocks. The process of deriving the global equations is very similar to the FEM. Firstly, all the potential energies of elastic strains, initial stresses, point loading, line loading, volume loading, viscosity and inertia force are computed. Secondly, the derivatives of individual potential energy with respect to deformation variables are computed and the corresponding submatrices are formed separately. Thirdly, the global equations are obtained by adding the submatrices to the matrices of the global equation at the corresponding position. Details of this process will be introduced in the following sections.

Assuming there are n blocks in a defined block system, the simultaneous equilibrium equations have the form:

$$\begin{pmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{1n} \\ K_{21} & K_{22} & K_{23} & \cdots & K_{2n} \\ K_{31} & K_{32} & K_{33} & \cdots & K_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & K_{n3} & \cdots & K_{nn} \end{pmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{pmatrix} \quad (3-16)$$

where $\{D_i\}$ and $\{F_i\}$ are 6×1 submatrices, $\{D_i\}$ represents the deformation variables of the i th block, $\{F_i\}$ is the loading distributed to the six deformation variables, $[K_{ij}]$ is in itself a 6×6 sub-matrix since each block has six degrees of

freedom defined by components of $\{D_i\}$, $[K_{ii}]$ depends on the material properties while $[K_{ij}]$ ($i \neq j$) is defined by the contact between the i th and the j th block, $[K_{ii}]$ represents the sum of contributing submatrices for the i th block, $[K_{ij}]$ represents the sum of contributing submatrices of contacts between the i th and the j th blocks and other inter-block actions like bolting, the matrix $[K]$ can also be called the global stiffness matrix.

In Eq. (3-16), $[K_{ij}]$ comes from the differentiations of Eq. (3-11):

$$\frac{\partial^2 \Pi}{\partial d_{ri} \partial d_{sj}}, \quad r, s = 1, \dots, 6 \quad (3-17)$$

where d_{ri} is the deformation variable of the i th block, d_{sj} is the deformation variable of the j th block.

$\{F_i\}$ comes from the differentiations of Eq. (3-11):

$$-\frac{\partial \Pi(0)}{\partial d_{ri}}, \quad r = 1, \dots, 6 \quad (3-18)$$

Therefore all terms of Eq. (3-17) form a 6×6 submatrix, which is the submatrix $[K_{ij}]$ in the global Eq. (3-16), while all terms of Eq. (3-18) form a 6×1 submatrix, which is added to the submatrix $\{F_i\}$ in the global Eq. (3-16). Eq. (3-12) represents the i th row of Eq. (3-16).

Fig. 3.6 Three blocks system with block numbers and node numbers

Fig. 3.7 Matrix structure of the DDA method for the three-block system (Jing, 2003)

Because the 'elements' of the 'mesh' are formed by the blocks which imply discontinuity at block boundaries. The 'mesh' generation is different from the traditional FEM. Each element will have different node number and be added into the global stiffness matrix respectively. Figs. 3.6 and 3.7 provide a simple

illustration of the matrix structure of a three-block system (Jing, 2003).

3.3.5 Sub-matrix of a Single Block

The movements and deformations of a block are determined by the given displacement functions. The total potential energy of elastic stresses, point loading, line loading, area loading, initial stresses, viscosity and inertia force are computed. The derivatives with respect to the deformation variables of individual potential energies are computed and the corresponding submatrices are formed separately. The global simultaneous equations are established by adding the individual submatrices to the matrices of the global equations (Zienkiewicz, 1977).

Because the submatrices for each loading and stress are computed and added to the global equations separately for each loading increment or time increment, more advanced stress-strain relationships can be introduced with this method by simply changing the elastic submatrices. Also, more complex loadings can be introduced by deriving the corresponding submatrices. Here the submatrices induced by different mechanism are introduced in details (Shi, 1988).

- **Sub-matrix of Elastic strain**

The condition of every block can be plane stress or plane strain. For each displacement step, assuming the material behaviors of the blocks are linearly elastic, the relationship of stress and strain can be expressed as

$$\{\sigma_i\} = [E]\{\varepsilon_i\} \quad (3-19)$$

For plane stress problem,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3-20)$$

For plane strain problem

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{pmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (3-21)$$

For the i th block, the rigid body motion terms do not induce strain, so, $\{\varepsilon_i\}$ can be replaced by $\{D_i\}$, and the matrix $[E]$ can be extended to a 6×6 matrices, $[E_i]$ with the form of $[E_i] = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{E} \end{bmatrix}$.

The strain energy Π_e produced by the elastic stress of the i th block is:

$$\Pi_e = \iint \frac{1}{2} (\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}) \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} dx dy \quad (3-22)$$

Substituting Eq. (3-19) into Eq. (3-22), the strain energy can be represented in terms of the block deformation parameters as

$$\begin{aligned} \Pi_e &= \iint \frac{1}{2} (\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}) [E] \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} dx dy \\ &= \frac{1}{2} \iint \{D_i\}^T [E_i] \{D_i\} dx dy \\ &= \frac{S}{2} \{D_i\}^T [E_i] \{D_i\} \end{aligned} \quad (3-23)$$

where S is the area of the i th block.

The derivatives are computed to minimize the strain energy Π_e :

$$[k_{rs}] = \frac{\partial^2 \Pi_e}{\partial d_{ri} \partial d_{si}} = \frac{S}{2} \frac{\partial^2}{\partial d_{ri} \partial d_{si}} (\{D_i\}^T [E_i] \{D_i\}) = S [E_i] \rightarrow [K_{ii}], \quad r, s=1, \dots, 6 \quad (3-24)$$

where $[k_{rs}]$ forms a 6×6 submatrix $[K_{ii}]$, which is added to the global Eq. (3-16).

- **Submatrices of Initial stress**

In the DDA program, the i th block can have its initial constant stresses input $(\sigma_x^0 \ \sigma_y^0 \ \tau_{xy}^0)$ as the load conditions from the measured record. Its potential energy can be represented as:

$$\begin{aligned} \Pi_\sigma &= -\iint (\varepsilon_x \sigma_x^0 + \varepsilon_y \sigma_y^0 + \gamma_{xy} \tau_{xy}^0) dx dy \\ &= -S(\varepsilon_x \sigma_x^0 + \varepsilon_y \sigma_y^0 + \gamma_{xy} \tau_{xy}^0) = -S\{D_i\}^T \begin{pmatrix} 0 \\ 0 \\ 0 \\ \sigma_x^0 \\ \sigma_y^0 \\ \tau_{xy}^0 \end{pmatrix} = -S\{D_i\}^T \{\sigma_0\} \end{aligned} \quad (3-25)$$

By taking the derivatives, we minimize Π_σ :

$$\{f_r\} = -\frac{\partial \Pi_\sigma}{\partial d_{ri}} = S \frac{\partial (\{D_i\}^T \{\sigma_0\})}{\partial d_{ri}} = S\{\sigma_0\} \rightarrow \{F_i\} \quad (3-26)$$

where $\{f_r\}$ forms a submatrix $\{F_i\}$ which is added into the global Eq. (3-16).

- **Submatrix of Loading**

There are three kinds of loading acting on the i th block, namely point loading, line loading and volume loading.

Point Loading

Assuming the point loading acting on point (x, y) of i th block is (F_x, F_y) , the potential energy of the point loading is simply expressed as:

$$\Pi_p = -(F_x u + F_y v) = -(u \ v) \begin{pmatrix} F_x \\ F_y \end{pmatrix} = -\{D_i\}^T [T_i(x, y)]^T \begin{pmatrix} F_x \\ F_y \end{pmatrix} \quad (3-27)$$

The derivatives can be computed by minimizing Π_p :

$$\{f_r\} = -\frac{\partial \Pi_p(0)}{\partial d_{ri}} = \frac{\partial}{\partial d_{ri}} \{D_i\}^T [T_i(x, y)]^T \begin{pmatrix} F_x \\ F_y \end{pmatrix} = F_x \{t_{1r}\} + F_y \{t_{2r}\}$$

$$= \begin{pmatrix} t_{11} & t_{21} \\ t_{12} & t_{22} \\ t_{13} & t_{23} \\ t_{14} & t_{24} \\ t_{15} & t_{25} \\ t_{16} & t_{26} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \end{pmatrix} \rightarrow \{F_i\}, \quad r=1, \dots, 6 \quad (3-28)$$

which are the products a 6×2 matrix and a 2×1 matrix. The resulting 6×1 submatrix $\{F_i\}$ is added into the global Eq. (3-16).

Line Loading

Line loading can be used at where the object is under the distributed force. When consider the blast pressure due to the underground explosion, the loading around the surrounding block mass can be also considered as the line loading along the interface. The application and optimization of line loading have been implemented in the current DDA program. Details will be given in Chapter 4.

Volume Loading

Assume that (x_0, y_0) is the center of gravity of this block, then:

$$x_0 = \frac{\int x dx dy}{\int_s dx dy} = \frac{S_x}{S}$$

$$y_0 = \frac{\int y dx dy}{\int_s dx dy} = \frac{S_y}{S} \quad (3-29)$$

If the body force loading (f_x, f_y) is constant, the potential energy is:

$$\begin{aligned}
 \Pi_v &= -\iint (f_x u + f_y v) dx dy \\
 &= -\iint (u \quad v) \begin{pmatrix} f_x \\ f_y \end{pmatrix} dx dy \\
 &= -\{D_i\}^T \iint [T_i]^T dx dy \begin{pmatrix} f_x \\ f_y \end{pmatrix}
 \end{aligned} \tag{3-30}$$

where,

$$\iint [T_i]^T dx dy = \begin{pmatrix} S & 0 \\ 0 & S \\ -S_y + y_0 S & S_x - x_0 S \\ S_x - x_0 S & 0 \\ 0 & S_y - y_0 S \\ (S_y - y_0 S)/2 & (S_x - x_0 S)/2 \end{pmatrix} = \begin{pmatrix} S & 0 \\ 0 & S \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{3-31}$$

Because (x_0, y_0) is the center of gravity of this block. The last 4 rows of Eq. (3-31) are zero.

The derivatives can be computed by minimizing Π_v :

$$\{f_r\} = -\frac{\partial \Pi_v(0)}{\partial d_{ri}} = \frac{\partial}{\partial d_{ri}} (\{D_i\}^T \begin{pmatrix} f_x S \\ f_y S \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}) = \begin{pmatrix} f_x S \\ f_y S \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \{F_i\} \tag{3-32}$$

$\{f_r\}$ forms a submatrix $\{F_i\}$, which will be added into the global Eq. (3-16).

• Submatrices of Bolting System

Bolting system (bolts or bars) is often considered in the rock excavation engineering for the stability of the weak rocks surrounding the structures. Bolting system can have the following applications:

- 1) Lockup the moveable rock mass with those stable rocks in the deep depth;
- 2) Lockup several rocks together to integrate like a girder to the tensile constrains

between rocks;

- 3) Supply a supplementary strength to the surrounding rocks when integrating them.

The basic application of the bolting system has been implemented in the current DDA program. Detailed theories and supplements will be given in Chapter 4.

- **Submatrix of Inertia Force**

The force of inertia per unit area of the block is the products of its time dependent acceleration and mass weight. Using the $(u(t), v(t))$ as the time dependent displacement of any point (x, y) of the i th block and M as the mass per unit area, it can be represented as:

$$\begin{pmatrix} f_x \\ f_y \end{pmatrix} = -M \begin{pmatrix} \frac{\partial^2 u(t)}{\partial t^2} \\ \frac{\partial^2 v(t)}{\partial t^2} \end{pmatrix} \quad (3-33)$$

The potential energy of the inertia force of the i th block is:

$$\begin{aligned} \Pi_i &= -\iint (u \ v) \begin{pmatrix} f_x \\ f_y \end{pmatrix} dx dy = \iint M (u \ v) \begin{pmatrix} \frac{\partial^2 u(t)}{\partial t^2} \\ \frac{\partial^2 v(t)}{\partial t^2} \end{pmatrix} dx dy \\ &= \iint M \{D_i\}^T [T_i]^T [T_i] \frac{\partial^2 \{D(t)\}}{\partial t^2} dx dy \end{aligned} \quad (3-34)$$

The numerical model in the current DDA program considers both static and dynamic analyses in a dynamic set-up environment. The only difference between them is that the static computation assumes the velocity as zero at the beginning of each time step while the dynamics computation inherits the velocity of the previous time step.

Assume $\{D_i(t)\}$ as the block displacement at the beginning of the time step, Δ as the time interval of this step and $\{D_i\}$ as the block displacements at the end of the

time step. Using the time integration, $\{D_i\}$ is represented as:

$$\{D_i\} = \frac{\Delta^2}{2} \frac{\partial^2 \{D(t)\}}{\partial t^2} + \Delta \frac{\partial \{D(t)\}}{\partial t} \quad (3-35)$$

$$\frac{\partial^2 \{D(t)\}}{\partial t^2} = \frac{2}{\Delta^2} \{D_i\} - \frac{2}{\Delta} \frac{\partial \{D(t)\}}{\partial t} \quad (3-36)$$

In the above equations, $\frac{\partial \{D(t)\}}{\partial t}$ is the deformation velocity at the beginning of the time step, denoted as $\{V_0\}$. At the end of a time step, the new block velocity is saved as $\{V_1\}$ and used at the beginning of the next time step. Meanwhile, the new geometry of deformed blocks, block stresses and contact forces use the same method in every time step. Incremental displacements and deformations are accumulated to result in possibly large displacements and deformations.

Therefore we have:

$$\Pi_i = \{D_i\}^T \iint [T_i]^T [T_i] dx dy \left(\frac{2M}{\Delta^2} \{D_i\} - \frac{2M}{\Delta} \{V_0\} \right) \quad (3-37)$$

Π_i is minimized with respect to $\{D_i\}$ to reach equilibrium:

$$fr = -\frac{\partial \Pi_i}{\partial d_{ri}} = \frac{\partial}{\partial d_{ri}} \{D_i\}^T \iint [T_i]^T [T_i] dx dy \left(\frac{2M}{\Delta^2} \{D_i\} - \frac{2M}{\Delta} \{V_0\} \right) \quad (3-38)$$

In Eq. (3-37), integration $\iint [T_i]^T [T_i] dx dy$ can be replaced by:

$$\iint [T_i]^T [T_i] dx dy = \begin{pmatrix} S & 0 & 0 & 0 & 0 & 0 \\ 0 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & S_1 + S_2 & -S_3 & S_3 & (S_1 - S_2)/2 \\ 0 & 0 & -S_3 & S_1 & 0 & S_3/2 \\ 0 & 0 & S_3 & 0 & S_2 & S_3/2 \\ 0 & 0 & (S_1 - S_2)/2 & S_3/2 & S_3/2 & (S_1 + S_2)/4 \end{pmatrix} \quad (3-39)$$

Further details are given in Shi (1988).

Eq. (3-38) can be transferred to the following two equations:

$$\begin{aligned} \frac{2M}{\Delta^2} \iint [T_i]^T [T_i] dx dy &\rightarrow [K_{ii}] \\ \frac{2M}{\Delta} (\iint [T_i]^T [T_i] dx dy) \{V_0\} &\rightarrow \{F_i\} \end{aligned} \quad (3-40)$$

which form submatrices $[K_{ii}]$, and $\{F_i\}$ in the global Eq. (3-16).

- **Submatrix of Displacement Constrains**

In the current DDA program, the boundary condition includes two situations: specific points in some of the blocks have existing displacement δ along a direction; some of the blocks are fixed at specific points in specified directions.

Submatrix of Displacement Constraints in a Direction

The measured displacements can be applied to the block system by using a very stiff spring which has the measured displacement as pre-tension distance.

Its direction along which the displacement δ exists can be represented as (l_x, l_y) and $l_x^2 + l_y^2 = 1$. The spring displacement is $d = (\delta - (l_x u + l_y v))$. Denote the spring stiffness as p , the spring force is:

$$f = -pd = -p(\delta - (l_x u + l_y v)) \quad (3-41)$$

Because p is independent of the computation result, it should be set large enough ($10E$ to $1000E$, where E is the material Young's Modulus) to guarantee the displacement of the spring can be neglected (10^{-3} to 10^{-4} times the total displacement).

The strain energy of this spring is:

$$\begin{aligned}
 \Pi_c &= \frac{p}{2} d^2 = \frac{p}{2} ((l_x u + l_y v) - \delta)^2 = \frac{p}{2} \left((u \ v) \begin{pmatrix} l_x \\ l_y \end{pmatrix} - \delta \right)^2 \\
 &= \frac{p}{2} (u \ v) \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) \begin{pmatrix} u \\ v \end{pmatrix} - p\delta (u \ v) \begin{pmatrix} l_x \\ l_y \end{pmatrix} + \frac{p}{2} \delta^2 \\
 &= \frac{p}{2} \{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \{D_i\} - p\delta \{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} + \frac{p}{2} \delta^2
 \end{aligned} \tag{3-42}$$

which are divided into two parts:

$$[k_{rs}] = \frac{\partial^2 \Pi_c}{\partial d_{ri} \partial d_{si}} = p [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \rightarrow [K_{ii}] \tag{3-43}$$

$$\{f_r\} = -\frac{\partial \Pi_c(0)}{\partial d_{ri}} = p\delta [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} \rightarrow \{F_i\} \tag{3-44}$$

which form submatrixes $[K_{ii}]$, and $\{F_i\}$ in the global Eq. (3-16).

Submatrixes of Displacement Constraints at a Point

When some of the blocks are fixed at specific points, their constraints can be applied to the block system by using two very stiff springs in x and y direction. Assume the fixed point is (x, y) of i th block, with the displacements $(u(x, y), v(x, y)) = (0, 0)$.

There are two springs which are along the x and y directions respectively. The stiffness of the springs is the same as p , and the spring forces are:

$$\begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} -pu \\ -pv \end{pmatrix} \tag{3-45}$$

The strain energy of this spring is:

$$\Pi_m = \frac{p}{2} \left((u \ v) \begin{pmatrix} u \\ v \end{pmatrix} \right) = \frac{p}{2} \{D_i\}^T [T_i]^T [T_i] \{D_i\} \tag{3-46}$$

The derivatives are computed to minimize the spring strain energy Π_m :

$$[k_{rs}] = \frac{\partial^2 \Pi_m(0)}{\partial d_{ri} \partial d_{si}} = p [T_i]^T [T_i] \rightarrow [K_{ii}] \quad (3-47)$$

$[k_{rs}]$ forms a 6×6 submatrix $[K_{ii}]$ which is added to the global Eq. (3-16).

3.3.6 Block System Analysis

When connecting the individual blocks into a block system, we extend the single block into the rock mass formulation. We begin to consider the analysis of block system kinematics. For block system movements, no penetration can be allowed between blocks. If two blocks are in contact, very stiff contact springs will be applied between the two blocks. However, the number, locations and directions of the contact springs depend on the contact type and state detected. This section will introduce the equations of constraints and how to impose these equations on the global equilibrium equations to finish the whole mechanic analysis.

When the block system moves or deforms, the blocks are in contact only along the boundary and the non-penetration inequalities can be transformed into the equations when two blocks are in contact.

The equations can be imposed on the global equation by adding very stiff springs or penalties to lock the movement in one or two directions which are the compressive and shear contact force. If a tensile contact force occurs between two blocks which has not any tensile strength, they will separate after the removal of the blocks. Then the non-tension constraining inequalities can be reduced to locking or unlocking of the locks. The global equations are solved while selecting the lock or constraining positions repeatedly. Furthermore, the block contact force in the forward analysis can be computed precisely by the selection of lock positions a few times for a block system with tension and penetration.

- **Distance between Blocks**

When we execute the program, the block positions, block shapes and block contacts change following the loading steps or time steps. Before the next calculation time

step, the program needs to get all the pairs of blocks which could possibly meet together next. Any two blocks will meet at the next time step only if they are close to each other during the current step. Here the program needs to make a judgment by comparing the nearest distance between two blocks and the predefined maximum displacement ρ . Their relationship can be represented as: $\eta_{ij} < 2\rho$ (two blocks will meet at the next time step).

The nearest distance between the i th block and the j th block can be represented as:

$$\eta_{ij} = \min \left\{ \sqrt{(x_1 - x_1)^2 + (y_2 - y_1)^2}, \forall (x_1, y_1) \in B_i, \forall (x_2, y_2) \in B_j \right\} \quad (3-48)$$

where $P_1=(x_1, y_1)$ and $P_2=(x_2, y_2)$ is any point pair of the i th and the j th block respectively and B_i and B_j represent all points of the i th and the j th block. Because of the infinite number of points in a block, it is impossible to compute the distance between two blocks by using Eq. (3-48). So the points are limited to the boundary segments of the i th and the j th block and this simplification has not any precision loss in case two blocks do not overlap.

The problem of minimum distance between two blocks simplify to the minimum distance from a point P_1 of the i th block to the points of the line segment P_2P_3 of the j th block which we denote as P_{23} . The distance is:

$$\bar{\eta} = \sqrt{(\bar{x} - x_1)^2 + (\bar{y} - y_1)^2} = \frac{|P_2P_3 \times P_2P_1|}{\sqrt{P_2P_3 \times P_2P_3}} \quad (3-49)$$

where P_{23} is represented as:

$$\begin{aligned} \bar{x} &= x_2 + (x_3 - x_2)t \\ \bar{y} &= y_2 + (y_3 - y_2)t, \quad (0 < t < 1) \end{aligned} \quad (3-50)$$

Further details are given in Shi's PhD thesis (Shi, 1988).

• Contact Representation and Detection

There are three different kinds of contacts between two adjacent blocks: edge-to-edge, angle-to-edge, and angle-to-angle, as shown in Fig. 3.8. In fact, an

edge-to-edge contact can be transformed into two angle-to-edge contacts. The angle-to-angle contact can be divided into convex-angle to convex-angle and convex-angle to concave-angle contact.

Fig. 3.8 Three contacts between two adjacent blocks

When an angle-to-edge contact occurs, a normal spring should be added, while whether a shear spring should be added is determined by the Mohr-Coulomb criterion (Fig. 3.9). When a convex-angle to convex-angle contact occurs, a normal spring should be added. When a convex-angle to concave-angle occurs, one or two normal springs should be added.

Fig. 3.9 Normal and shear contact spring

When point P_1 passes the reference line P_2P_3 after deformation, at the next time step there is a spring between P_1 and P_2P_3 to push P_1 back from inter penetration to the line P_2P_3 , the distance d from them should be zero after the displacement increment is applied. The distance d is calculated as:

$$d = \frac{\Delta}{l} = \frac{1}{l} \begin{vmatrix} 1 & x_1 + u_1 & y_1 + v_1 \\ 1 & x_2 + u_2 & y_2 + v_2 \\ 1 & x_3 + u_3 & y_3 + v_3 \end{vmatrix} \quad (3-51)$$

$$= \frac{1}{l} \left(\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & u_1 & y_1 \\ 1 & u_2 & y_2 \\ 1 & u_3 & y_3 \end{vmatrix} + \begin{vmatrix} 1 & x_1 & v_1 \\ 1 & x_2 & v_2 \\ 1 & x_3 & v_3 \end{vmatrix} + \begin{vmatrix} 1 & u_1 & v_1 \\ 1 & u_2 & v_2 \\ 1 & u_3 & v_3 \end{vmatrix} \right)$$

where (x_i, y_i) and (u_i, v_i) are denoted as the coordinates and displacement increment

of point P_i respectively, $i=1, 2, 3$; l is the distance of P_2P_3 . The fourth form of Eq. (3-51) is a second order infinitesimal which can be neglected. Then from Eq. (3-51), we have:

$$d = \frac{\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}}{l} + \frac{(y_2 - y_3 \quad x_3 - x_2) \begin{pmatrix} u_1 \\ v_1 \end{pmatrix}}{l} \quad (3-52)$$

$$+ \frac{(y_3 - y_1 \quad x_1 - x_3) \begin{pmatrix} u_2 \\ v_2 \end{pmatrix}}{l} + \frac{(y_1 - y_2 \quad x_2 - x_1) \begin{pmatrix} u_3 \\ v_3 \end{pmatrix}}{l}$$

$$\begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = [T_i(x_1, y_1)] \{D_i\}$$

$$\begin{pmatrix} u_2 \\ v_2 \end{pmatrix} = [T_j(x_2, y_2)] \{D_j\} \quad (3-53)$$

$$\begin{pmatrix} u_3 \\ v_3 \end{pmatrix} = [T_j(x_3, y_3)] \{D_k\}$$

Then the strain energy of the contact spring is:

$$\Pi_k = \frac{p}{2} d^2 \quad (3-54)$$

Substitute Eq. (3-52) into Eq. (3-54) and minimize Π_k by taking the derivatives, four 6×6 and two 6×1 submatrices $[K_{ii}]$, $[K_{ij}]$, $[K_{ji}]$, $[K_{jj}]$, $[F_i]$, $[F_j]$ can be determined, which will be added to the global Eq. (3-16), respectively.

After all the possible contacts are detected and constrained by springs applied between the blocks in contact, the submatrices of contact springs are computed and added to the global system equations at the corresponding position, then Eq. (3-16) are solved. According to the results, whether the requirements of no-penetration and no-tension are satisfied is judged. If the requirements are satisfied, the computation of this time step is finished; otherwise the springs should be added where penetrations occur, and the ones where tensions occur should be removed. The iteration is repeated until all the contact springs are applied correctly.

3.4 DYNAMIC MODELING USING DDA

Compared to static problems, there are more difficulties for the DDA modeling of dynamic problems. The numerical accuracy depends much on the setting of computational model and utilization of algorithm, including model simplification, model size, block size, initial condition, boundary condition, material modal, time intervals, maximum displacement ratio and damping for energy lose. Here, boundary condition, time step setting and damping consideration will be reviewed and discussed.

3.4.1 Boundary Conditions

In the numerical modeling of underground structures, only the zone of interest around the force source is taken as the computational model. The actual computational model is always limited within a finite zone by cutting the infinite region. The induced artificial boundaries shall be reflected as the real boundary conditions. Rolling boundary can be used to save the block numbers for the symmetrical model for both static and dynamic problem. The fixed or elastic displacement boundary conditions can be realistically applied at the artificial boundaries in the static problem but not in dynamic problem because of the reflection of the outward propagating waves. There are several ways now we could use to avoid and decrease the influence of the reflection. One is to set a relative greater distance away from the analysis domain; the other is to set the analysis time before the reflecting wave come. Another approval is to employ non-reflecting boundary, such as using the dashpot to absorb all the elastic energy of the incident wave (Mike Ross, 2004). There is a lack of development in this idea for the DDA.

3.4.2 Time Step Setting

Time step setting in the DDA is related with both the maximum displacement ratio and block size. To maintain the computation of implicit algorithm stable, the time interval varies for different kinds of problems. For a discontinuous problem, the

time step should meet both the requirement of the internal block system calculations and the calculation of contacts. The internal block calculations of blocks require that no information is transmitted throughout the same block in less than one time step. The time step interval can be estimated as:

$$\Delta t^a = l_{\min} / c_p \quad (3-55)$$

where c_p is the P-wave velocity and $l_{\min}$ is the minimum length of the block.

For the calculation of contacts of a rigid block system, the time step interval can be obtained by analogy to a single degree of freedom system (Last and Harper, 1990) as:

$$\Delta t^b = 2b\sqrt{(M_{\min} / K_{\max})} \quad (3-56)$$

where $M_{\min}$ is the mass of the smallest block in the system; $K_{\max}$ is the maximum contact stiffness; b is user-supplied factor in order to account for the situation that a block may be in contact with several blocks.

Thus, the time step interval is chosen as:

$$\Delta t = \min(\Delta t^a, \Delta t^b) \quad (3-57)$$

3.4.3 Damping Consideration

In a homogeneous linear elastic material, stress waves travel indefinitely without change in amplitude. This type of behavior can not occur, however, in real conditions. The amplitudes of stress waves in real materials, such as those that comprise the earth or rock mass, attenuate with distance. This attenuation can be attributed to two sources of energy dissipation: one involves the material through which the waves travel, and the other concerns on the geometry of the wave propagation problem.

The type of damping described in our research range is called material damping,

because the material absorbs the energy of travel during wave propagation. The reduction in energy per unit volume causes the amplitude of the wave to decrease with increasing displacement.

In the DDA, damping consideration is used to the dynamic problem. A dynamic coefficient value from 0-1 can be set to account for the different extent of the energy dissipation during the dynamic loads passing through the blocks. For the next time step, the start displacement velocity v_1 will be the product of the velocity at the end of this time step and the dynamic coefficient value.

3.5 DDA PROGRAM FRAMEWORK

The key part of the DDA code includes two programs: DC and DF. DC is a pre-processor for program DF. It forms the block system configuration. Program DF is for static and dynamic analysis. It reads from the output file “blk” of DC and cooperate its input file to execute together. Fig. 3.10 shows the DDA program flowchart.

(a)

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(b)

Fig. 3.10 Framework of DDA: (a) DC; (b) DF

3.6 CONCLUDING REMARKS

DDA originated from a back analysis algorithm for determining a best fit to a deformed configuration of a block system from measured displacements and deformations (Shi and Goodman, 1985). It was later further developed to perform complete deformation analysis of a block system. For the current DDA program, many improvements have been made, including the arbitrarily shaped deformable blocks, contact detecting and rigid block system, user-friendly code structure and environment.

In general, the DDA belongs to the implicit discrete element method (DEM). Compared with the explicit approach of the DEM, the DDA method has several unique advantages over the explicit DEM:

- 1) The equilibrium condition is automatically satisfied for quasi-static problems without using excessive iteration cycles. Closed-form integrations for the element and block stiffness matrices can be performed without the need for Gaussian quadrature techniques. Its block system seems to resemble the distinct element method but it also more closely parallels to the finite element method in the whole problem solving procedure. As a displacement-based implicit method, it can provide satisfying accuracy.
- 2) It is easy to convert an existing FEM code into a DDA code and include many mature FEM techniques without inheriting the limitations of the ordinary FEM, such as small deformation, continuous material geometry, and reduced efficiency for dynamic analysis. On the other hand, the advanced techniques for the formation of the matrix and solving of the global simultaneous equation are the same as the FEM.
- 3) The blocks forming the “elements” of the “mesh” can be any shape or size, even contain holes, which are all independent with each other. The nodes on the interface of the blocks are originally assigned double nodes and there is no need to insert zero-thickness interface elements with double nodes along each line of the block system as commonly used in the FEM.
- 4) The whole block system can be considered as a continuous field before block

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separation appears, just like in a typical FEM analysis. All blocks are glued together during the analysis when the inter-block stress is less than the joint strength. Block relative movements start to occur when the inter-block stress exceeds the joint property (i.e. opening when the normal stress exceeds the tensile strength, and sliding when the shear stress exceeds the shear strength based on Coulomb's friction law).

These advantages show that the DDA method has emerged as an attractive model for geo-mechanical problems because its advantages cannot be replaced by continuum-based methods or explicit DEM formulations.

Everything has its two sides. The developing DDA still has some key drawbacks such as the accuracy of the contact force, stress state in the single block, factor influence and computing efficiency, which leaves us much work to do in the future.

CHAPTER 4 IMPROVEMENTS OF THE DDA PROGRAM

4.1 INTRODUCTION

A discontinuous deformation analysis computer code is developed in our research group based on original discontinuous deformation analysis code version 96 (Yeung, 1996). In the current DDA code, several improvements have been made to improve the DDA and to extend its applications, which are discussed in the following sections in details.

4.2 LINE LOADING

4.2.1 Theory of Line Loading

Assume the loading is distributed on a straight line from point (x_1, y_1) to point (x_2, y_2) (shown in Fig. 4.1), the equation of the loading line can be expressed as:

$$\begin{aligned} x &= (x_2 - x_1)t + x_1 \\ y &= (y_2 - y_1)t + y_1 \end{aligned} \quad (4-1)$$

The length of this line segment is:

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (4-2)$$

The loading is:

$$\begin{aligned} F_x &= F_x(t) \\ F_y &= F_y(t) \quad 0 \leq t \leq 1 \end{aligned} \quad (4-3)$$

which are variants along a loading line. The potential energy of the line loading $(F_x(t), F_y(t))$ is

$$\begin{aligned}
 \Pi_l &= -\int_0^1 (u \ v) \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} l dt \\
 &= -\{D_i\}^T \int_0^1 [T_i]^T \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} l dt \\
 &= -(d_{1i} \ d_{2i} \ d_{3i} \ d_{4i} \ d_{5i} \ d_{6i}) \int_0^1 [T_i]^T \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} l dt
 \end{aligned} \tag{4-4}$$

Minimizing the potential energy Π_l , the derivatives can be represented as:

$$\begin{aligned}
 \{f_r\} &= -\frac{\partial^2 \Pi_l}{\partial d_{ri}} = \frac{\partial}{\partial d_{ri}} (\{D_i\}^T \int_0^1 [T_i]^T \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} l dt) \\
 &= \int_0^1 [T_i]^T \begin{pmatrix} F_x(t) \\ F_y(t) \end{pmatrix} l dt \rightarrow \{F_i\}
 \end{aligned} \tag{4-5}$$

which are the products a 6×2 and a 2×1 matrix. This 6×1 sub-matrix is added to the submatrix $\{F_i\}$ in the global equation.

Fig. 4.1 Illustration of a line loading

When the line loading $(F_x(t), F_y(t))$ is constant (F_x, F_y) , the matrix integration of Eq. (4-3) has the analytical formula, which is derived in the following

$$\int_0^1 [T_i]^T \begin{pmatrix} F_x \\ F_y \end{pmatrix} l dt = \left(\int_0^1 [T_i]^T l dt \right) \begin{pmatrix} F_x \\ F_y \end{pmatrix} \tag{4-6}$$

The integrations of the elements have to be computed in order to compute integration of matrix $[T_i]$:

$$\begin{aligned}\int_0^1 (x - x_0) l dt &= \int_0^1 ((x_2 - x_1)t + (x_1 - x_0)) l dt \\ &= \frac{l}{2} (x_2 + x_1 - 2x_0) \\ \int_0^1 (y - y_0) l dt &= \int_0^1 ((y_2 - y_1)t + (y_1 - y_0)) l dt \\ &= \frac{l}{2} (y_2 + y_1 - 2y_0)\end{aligned}\quad (4-7)$$

Therefore,

$$\left(\int_0^1 [T_i]^T l dt \right) \begin{pmatrix} F_x \\ F_y \end{pmatrix} = l \begin{pmatrix} F_x \\ F_y \\ -\frac{1}{2}(y_2 + y_1 - 2y_0)F_x + \frac{1}{2}(x_2 + x_1 - 2x_0)F_y \\ \frac{1}{2}(x_2 + x_1 - 2x_0)F_x \\ \frac{1}{2}(y_2 + y_1 - 2y_0)F_y \\ \frac{1}{4}(y_2 + y_1 - 2y_0)F_x + \frac{1}{4}(x_2 + x_1 - 2x_0)F_y \end{pmatrix} \rightarrow \{F_i\} \quad (4-8)$$

This 6×1 submatrix is added to the submatrix $\{F_i\}$ in the global equation.

When the line loading $(F_x(t), F_y(t))$ is linear (F_x, F_y) along the straight line, we set $F_x(0)=F_{1x}$ and $F_y(0)=F_{1y}$, $F_x(1)=F_{2x}$ and $F_y(1)=F_{2y}$. Then the matrix integration Eq. (4-3) can be evaluated analytically as follows

$$\begin{aligned}\int_0^1 (x - x_0) (F_{1x} + (F_{2x} - F_{1x})t) l dt \\ = \int_0^1 ((x_2 - x_1)t + (x_1 - x_0)) (F_{1x} + (F_{2x} - F_{1x})t) l dt \\ = \frac{l}{2} (x_1 + x_2 - 2x_0) F_{1x} + \frac{l}{6} (x_1 + 2x_2 - 3x_0) (F_{2x} - F_{1x})\end{aligned}\quad (4-9)$$

$$\begin{aligned}\int_0^1 (y - y_0) (F_{1y} + (F_{2y} - F_{1y})t) l dt \\ = \int_0^1 ((y_2 - y_1)t + (y_1 - y_0)) (F_{1y} + (F_{2y} - F_{1y})t) l dt \\ = \frac{l}{2} (y_1 + y_2 - 2y_0) F_{1y} + \frac{l}{6} (y_1 + 2y_2 - 3y_0) (F_{2y} - F_{1y})\end{aligned}\quad (4-10)$$

Therefore

$$\begin{aligned}
 & \left(\int_0^l [T_i]^T l dt \right) \begin{pmatrix} F_x \\ F_y \end{pmatrix} = \\
 & l \begin{pmatrix} F_x \\ F_y \\ -\frac{1}{2}(y_1 + y_2 - 2y_0)F_{1y} + \frac{1}{6}(y_1 + 2y_2 - 3y_0)(F_{2y} - F_{1y}) + \left(\frac{1}{2}(x_1 + x_2 - 2x_0)F_{1x} + \frac{1}{6}(x_1 + 2x_2 - 3x_0)(F_{2x} - F_{1x}) \right) \\ \frac{1}{2}(x_1 + x_2 - 2x_0)F_{1x} + \frac{1}{6}(x_1 + 2x_2 - 3x_0)(F_{2x} - F_{1x}) \\ \frac{1}{2}(y_1 + y_2 - 2y_0)F_{1y} + \frac{1}{6}(y_1 + 2y_2 - 3y_0)(F_{2y} - F_{1y}) \\ \frac{1}{2} \left(\frac{1}{2}(y_1 + y_2 - 2y_0)F_{1y} + \frac{1}{6}(y_1 + 2y_2 - 3y_0)(F_{2y} - F_{1y}) \right) + \frac{1}{2} \left(\frac{1}{2}(x_1 + x_2 - 2x_0)F_{1x} + \frac{1}{6}(x_1 + 2x_2 - 3x_0)(F_{2x} - F_{1x}) \right) \end{pmatrix} \rightarrow \{F_i\}
 \end{aligned}
 \tag{4-11}$$

This 6×1 submatrix is added to the submatrix $\{F_i\}$ in the global equation.

4.2.2 Fulfillment of Line Loading in DDA

Different from the traditional FEM methods which transfer the line loading to the element along the line and then to the nodal force, the line loading in the DDA program needs to find all blocks under the line loading firstly, and then apply the loadings to the gravity center of the block (Fig. 4.2).

Fig. 4.2 Line loading on blocks with different value and loading area

In the input data file of the DC, one line (L6) is added to input the number of the linear loading nll and there will also be one L12 for each of the nll loading points. Each L12 will have four parameters of point (x_1, y_1) and (x_2, y_2) on a straight line where the loading is distributed. Then in the input data file of the DF, there should be one L9 for each of the interpolation nodes to input loading data after those for fixed points. For each loading point, time-dependent loads are defined by loads at specific times. Different Loading values can be permitted in the two points. Loads

at times between the specified times are obtained by interpolating linearly. In the program, computer will automatically check those blocks under the line loadings and calculate the interpolated values and positions of the loadings on the blocks at each time step.

4.3 BOLTING SYSTEM

4.3.1 Introduction

Rockbolts have been used for many years for the support of underground excavations and a wide variety of bolt types have been developed to meet different needs which arise in mining and civil engineering. Bolts belong to the active supports for the underground structures. It could be used to reduce displacements along joints, increase in ground strength and reduce the development of new fractures.

Fig. 4.3 Components of an anchored rockbolt with provision of grouting

Rockbolts generally consist of plain steel rods with a mechanical anchor at one end and a face plate and nut at the other. Many types and styles have been invented or expanded, such as grouted bolts, self drilling bolts, friction bolts and expandable friction bolt, but the basic principle of operation is the same in all of these anchors. As shown in Fig. 4.3, the components of a typical bolting bar are a tapered cone with an internal thread and a pair of wedges held in place by a bail. The cone is screwed onto the threaded end of the bolt and the entire assembly is inserted into the hole that

has been drilled to receive the bar. The length of the hole should be at least 100 mm longer than the bolt otherwise the bail will be dislodged by being forced against the end of the hole. Once the assembly is in place, a sharp pull on the end of the bolt will seat the anchor.

In the current study, the bolt system is simplified as a 1-D element connecting two blocks. After each time step, the displacements of its two ends are obtained. Then, the forces applied to the two blocks are calculated based on its length change and the stiffness. Frictional resistance along the bars and the effect of grout materials are not taken into account.

4.3.2 Theory of Bolting System

Consider a bolt or bar is used to connect the i th block and the j th block. The displacements of the two end points, (x_1, y_1) (x_2, y_2) are

$$\begin{aligned} d_{x1} = u_1 = u(x_1, y_1); \quad d_{y1} = v_1 = v(x_1, y_1) \\ d_{x2} = u_2 = u(x_2, y_2); \quad d_{y2} = v_2 = v(x_2, y_2) \end{aligned} \quad (4-12)$$

The length of the bolt is calculated as

$$l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (4-13)$$

The length change of the bolt is

$$dl = \left[(u_1 \ v_1) \begin{pmatrix} l_x \\ l_y \end{pmatrix} - (u_2 \ v_2) \begin{pmatrix} l_x \\ l_y \end{pmatrix} \right] = \left(\{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} - \{D_j\}^T [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} \right) \quad (4-14)$$

where l_x, l_y are the direction cosines of the bolt

$$\begin{aligned} l_x &= \frac{(x_1 - x_2)}{l} \\ l_y &= \frac{(y_1 - y_2)}{l} \end{aligned} \quad (4-15)$$

Here, we consider the bolt force comes from its strain, we denote the stiffness as s :

$$f = -s \frac{dl}{l} \quad (4-16)$$

The strain energy of the bolt is

$$\begin{aligned} \Pi_b &= -\frac{1}{2} f dl = \frac{s}{2l} dl^2 = \frac{s}{2l} (\{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} - \{D_j\}^T [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix})^2 \\ &= \frac{s}{2l} \{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \{D_i\} \\ &\quad - \frac{s}{l} \{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_j] \{D_j\} \\ &\quad + \frac{s}{2l} \{D_j\}^T [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_j] \{D_j\} \end{aligned} \quad (4-17)$$

which can be derived into four parts:

$$\begin{aligned} [k_{rs}] &= \frac{\partial^2 \Pi_b}{\partial d_{ri} \partial d_{si}} = \frac{s}{2l} \frac{\partial^2}{\partial d_{ri} \partial d_{si}} \{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \{D_i\} \\ &= \frac{s}{l} [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \rightarrow [K_{ii}] \end{aligned} \quad (4-18a)$$

$$\begin{aligned} [k_{rs}] &= \frac{\partial^2 \Pi_b}{\partial d_{ri} \partial d_{sj}} = \frac{s}{l} \frac{\partial^2}{\partial d_{ri} \partial d_{sj}} \{D_i\}^T [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_j] \{D_j\} \\ &= \frac{s}{l} [T_i]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_j] \rightarrow [K_{ij}] \end{aligned} \quad (4-18b)$$

$$\begin{aligned} [k_{rs}] &= \frac{\partial^2 \Pi_b}{\partial d_{rj} \partial d_{si}} = \frac{s}{l} \frac{\partial^2}{\partial d_{rj} \partial d_{si}} \{D_j\}^T [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \{D_i\} \\ &= \frac{s}{l} [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_i] \rightarrow [K_{ji}] \end{aligned} \quad (4-18c)$$

$$\begin{aligned} [k_{rs}] &= \frac{\partial^2 \Pi_b}{\partial d_{rj} \partial d_{sj}} = \frac{s}{2l} \frac{\partial^2}{\partial d_{rj} \partial d_{sj}} \{D_j\}^T [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_j] \{D_j\} \\ &= \frac{s}{l} [T_j]^T \begin{pmatrix} l_x \\ l_y \end{pmatrix} (l_x \ l_y) [T_j] \rightarrow [K_{jj}] \end{aligned} \quad (4-18d)$$

which form submatrixes $[K_{ii}]$, $[K_{ij}]$, $[K_{ji}]$, $[K_{jj}]$ in the global Eq. (3-16).

4.3.3 Fulfillment of Bolting System in DDA

An illustration of the bolt system in tunnel is shown in Fig. 4.4. Based on the discussion of the bolt system theory, a subroutine has been implemented into the original DDA code.

Fig. 4.4 Illustration of bolt system used in tunnel

Several modifications have also been done for the DC and DF files of the DDA program. In DC file, the number of bolt elements should be filled in the line 4 (L4) and the corresponding coordinates of the bolting point need to be provided after the coordinate of material lines in L10 with the format of $(x_1, y_1), (x_2, y_2)$. In the DF file, one additional line with the stiffness of the rock-bolt S is added between L7 and L8 in the original program.

An illustration example with a two-block system is shown in Fig. 4.5. The top block is fixed. There is a bolt connecting top block and bottom block in Fig. 4.5 (b), while no bolt in Fig. 4.5 (a). After 10 time steps, it can be found that the bottom block drops down under gravitational loading for the case without bolt. The result indicates that the implemented bolt system works well.

(a)

(b)

Fig. 4.5 Block displacement under gravity load:

(a) without bolt; (b) with bolt

4.4 MODIFICATION OF PRE-PROCESSOR AND POST-PROCESSOR

The original version of the DDA is not user-friendly. For its pre-processor, the user needs to input all the parameters in a TXT file. It is not very convenient and it is easy to make some typing errors. So, a windows interface has been developed for the preprocessor as shown in Fig. 4.6.

CHAPTER 4 IMPROVEMENTS OF THE DDA PROGRAM

Fig. 4.6 New windows interface for pre-processor

The new windows interface uses the Visual C++ language (based on the Windows programs of MFC) as the developing language. It faces to the object, realizes the consistent graphical interfaces and visual document. The language can also leave the independent module in the developing terrace for the user and later developer to replenish and improve. Three menus, called Input preparation, Execution and Results, can be seen to be added in the mainframe of the interface.

In the menu of Input preparation, one independent module is designed for calculating the blast loading history (details will be given in Chapter 6) and two independent modules (dialog window) for input preparation: Geometry and Execution initialization are copied respectively for the input data of DC and DF. The

user can easily finish the block coordinates input and fill in each parameter under the guidance. The problem often met in the original program that the number of line input coordinates is not corresponding to the actual coordinates input will be prevented by a warning message. Then after the user closes the dialog window, the program will automatically save the file as the input data form for DC and DF programs.

By using the dynamic linking command function, the executable DC and DF programs are added in the new interface as the sub-menu items which can be directly clicked to execute. A module of “Graph” has been implemented in the DC programs to form the block configuration. This can be used to check correctness of the blocks system and apply the measured points to the required blocks before the execution of DF programs.

For the post-processor, the current program can not only provide the final block system deformation after loading but also output the displacement, stress and velocity of the blocks.

Geometry Information Input

Please input the geometry information for Block production

(1)minimum edge length parameter e_0 =

[2]geometry lines coordinate input

[3]material lines coordinate input

[4]coordinate input for loadings

[5]fixed lines coordinate input

[6]measured point coordinate input

[7]hole point coordinate input

OK Cancel

Execution Initialization Input

(1)control parameter for type of analysis = [0 for static analysis; 1 for dynamic anlaysis]

(2)number of time steps =

(3)number of block materials = number of joint materials =

(4)assumed maximum displacement ratio =

(5)upper limit of time interval used in each time step =

(6)stiffness of contact spring [F/L] =

(8)qq = factor of voer-relaxation used by SOR iteration method; (1.4)

Fig. 4.7 Two dialogue windows for input data

4.5 BOUNDARY CONDITIONS

In topology, the boundary of a subset S of a topological space X is the set of points which can be approached both from S and from the outside of S . More formally, it is the set of points in the closure of S , not belonging to the interior of S . An element of the boundary of S is called a boundary point of S (Wikipedia Library). On the field of engineering, boundary can be simply evaluated as the dividing line of two different material media or the terminated line when some part of the material is fixed in the 2-dimensional problem. Hence, we need an “artificial” boundary condition to simulate a model without any finite boundary (Mike, 2004).

Different boundary conditions are listed as following:

Free boundary: an interface between the air and the solid material or other media. The reason for the "free" designation arises from the large difference in the densities of the gas and solid. No constraint exists in any direction.

Rolling boundary: material can move freely in x or y direction but has constraint in the other direction. Due to the axi-symmetry, some problems can be simplified to be

half of its size domain for modeling. The rolling boundary can also be called as the axi-symmetry axis boundary.

Fixed boundary: If the physical process concentrates in the center of the simulation region and causes very little influence on the boundary, and at the same time, the surrounding environment of the simulation region is stable, then we can set the boundary to fixed boundary conditions. Points on the fixed boundary have no movement in either direction.

Fixed boundary with time-dependent displacements: this is the special case of above condition which can be used when displacements of some part of domain is known or measured in a direction in advance.

No-reflecting boundary: which can also be called absorbing boundary. For problems involving wave propagation analysis, the usual finite boundary of the finite element model will cause the elastic waves to be reflected and superimposed with the progressing waves. Therefore, we wish to create a boundary, which is perfectly radiating to outgoing waves (no spurious reflections) and transparent to incoming waves. In addition, we would like the boundary to be as close to the finite structure as possible for computational efficiency.

In the current programs, points on the free surface boundary can move along any direction freely; points on the fixed boundary have constraints in both x and y directions; for the situation of the fixed boundary with time-dependent displacements, the measured displacements of some points can be applied to the block system by using a very stiff spring.

Some modifications have been done to model the media response better in the boundary. Rolling boundary has been extracted by detaching the submatrixes of the fixed points. For example, when only x direction is constrained, we have rolling boundary in the y direction. The stiffness spring is only applied to the x direction. The spring force can be expressed as:

$$f_x = -pu \quad (4-19)$$

Submatrix $[K_{ii}]$ will only have one form:

$$p[T_1]^T [T_1]. \text{ (for } [T_1] = (t_{11}, t_{12}, t_{13}, t_{14}, t_{15}, t_{16}) \text{ in } x \text{ direction)}$$

Submatrix $\{F_i\}$ will only have one form:

$$p\delta[T_1]^T [l_x]. \text{ (for } \delta \text{ is the existing displacement in } x \text{ direction)}$$

The same deduction can be applied when y direction is constrained.

The constraints in the original programs are only dots dark in the relative points which don't satisfy the block configuration requirements now. New constraints expressions have been added to the programs to denote the fixed points and rolling points respectively, shown in Fig. 4.8.

Fig. 4.8 Expressions of different boundary conditions in DDA program: (a) fixed in two directions; (b) fixed in y direction; (c) fixed in x direction

4.6 BLAST LOADING

Blast loading is hard to measure and model. In this study, an independent module has been supplemented into the original program. A series of steps have been carried out for the transfer of the actual TNT explosive blast to the numerical loading. Several schemes can be referred for modeling the loading on the detonated wall surface such as TM5 and JWL state equation. The blast pressure attenuates quickly with the increasing distance from the charge, as seen in Fig. 2.5.

4.6.1 Blast Cave Shape Confirmation

For simplification, the shape of blast charge is assumed as a sphere. *TNT* explosive

is selected as the blast material in the numerical modeling. Its density is 1630kg/m^3 . For example, 50kg TNT has radius of 0.194m . Because the current programs are 2-dimensional version, the blasthole domain we set in the model is a circle (radius= 0.194m) which is full coupled explosive. Details are shown in Table 4.1.

Table 4.1 Dimension of TNT for different weight with the shape of sphere

Weight (kg)	1	5	10	50	80	500	1000	10000
Radius (m)	0.053	0.090	0.114	0.194	0.227	0.418	0.527	1.136

4.6.2 Blast Pressure Rising Time and Maximum Blast Pressure

The peak blast pressure is expressed using the JWL state of equation (Eq. (2-6)). The detailed parameters for the TNT explosive are expressed in Table 2.1. Because the peak blast pressure is high enough to crush the rock mass and form a crushed zone near the blast charge, the current DDA programs can only calculate the elastic deformation and response of rock mass outside the crushed zone. The pressure-loading history we use on the interface wall of cave and rock mass is related to both the peak pressure and rock mass properties. The relationship of the pressure rising time and constants ratio β/α can be obtained from the Eq. (2-11). Here we neglect the time that the shock wave propagates to the crushed zone because the time required for such high velocity blast wave is very short.

4.6.3 Pressure-Time History of Blast Loading

The blast induced pressure wave usually decays exponentially. The waveform of the pressure time function used here is expressed as in Eq. (2-9).

A subroutine program added into pre-processor menu could calculate the blast pressure loading on the interface of charge and rock mass. By using this module, we can not only initialize the relationship of crushed zone size and the charge magnitude but also obtain the peak pressure and maximum pressure as well as the rising and decaying loading time. It saves the user's time to confirm the pressure history and loading time. Some assumptions and simplifications have been used for transferring the explosive pressure to the final pressure history (Fig. 4.9 and 4.10).

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Fig. 4.9 Comparison of pressure-time curves for applied waveforms under different pressure rising time t_0 (Maximum pressure=100MPa)

Fig. 4.10 Blast loading history calculation

CHAPTER 5 STRESS RECOVERY PROCEDURE FOR DISCONTINUOUS DEFORMATION ANALYSIS

5.1 INTRODUCTION

One crucial numerical analysis in rock engineering is the fracture propagation in rock mass under either static or dynamic loads. Many natural materials (like rock mass) contains weak layers (such as joints), one method in fracture propagation simulation is to consider each mesh line as a potential crack (applied by several researchers (Ngo et al., 1967; Aminpour et al., 1991; Wu, 1994; Sluys et al., 1998; Carol et al., 2001), and to determine the crack opening or sliding based on appropriate strength criteria. Although the cracks can only develop along the initial mesh lines, these methods at least eliminate completely the need of re-meshing. As those mesh lines are often treated as double noded interfaces, an accurate inter-element stress is required to determine the failure mode (Camacho et al., 1996). However, the inter-element stress is not as accurate as the stress field inside the element, it is essential to use some post-processing techniques to improve the stress along the inter-element boundary.

A number of nodal stress recovery (or smoothing) procedures have been proposed for the FEM, including the averaging method (Bramble et al., 1977), the local or global projections (Hinton et al., 1974; Silva et al, 2004) and super-convergence patch recovery (Zhu et al., 1990; Zienkiewicz et al., 1992; Heimsund et al., 2000). Most of those methods used in FEM usually require some kinds of recovery procedure based on the more reliable stresses obtained at the Gaussian points, and require nearly the same size element around the generic node (Stein et al., 1974). Some methods are applicable to a part of the mesh because of their computational

requirements (Boroomand et al., 1997), otherwise it may require a computational effort higher than that needed for the FEM analysis itself.

As DDA's blocks are all independent with each other, the nodes on the interface of the blocks are originally assigned double nodes, there is no need to insert zero-thickness interface elements with double nodes along each line of the block system as commonly used in the FEM. Therefore, the block model in the DDA makes it ideal to use the block boundary as a potential crack. These interfaces are assigned a relative elastic stiffness which may be interpreted as the penalty coefficient necessary to evaluate stress traction transmitted across their surface (Gens et al., 1988). Thus, an accurate inter-block stress evaluation will become essential in the crack propagation analysis (Lin et al., 1996).

In this chapter, the concept of glutting the blocks as continua is adopted to simulate stress field before the blocks separate and slide. A stress recovery procedure adopting a least-square formulation involving the residuals of the equilibrium equations is developed for improving inter-block stress evaluation. Two numerical examples are presented. The resulting stress fields and inter-block forces are compared with those from the traditional stress average smoothing method and the theoretical solutions.

5.2 CONCEPT OF GLUTTING BLOCKS IN DDA

DDA's block displacement function is proved to be equivalent to the first order approximation of the displacement in the FEM, but the blocks used in the DDA can assume any given geometry as opposed to the simple geometry of the FEM elements. The choice of the linear displacement function in the DDA is to ease the complex contact determination between the blocks, so that the DDA can model the object material as a system of individually deformable blocks that move

independently without interpenetration.

The whole block system can be considered as a continuous field before block separation appears. All blocks are glutted together during the analysis when the inter-block stress is less than the joint strength. Block relative movements start to occur when the inter-block stress exceeds the joint property (i.e. joint opening when the normal stress exceeds the tensile strength, and joint sliding when the shear stress exceeds the shear strength based on Coulomb's friction law). With strong joint properties (large friction angle, cohesion, and tensile strength), the glutted blocks just behave like the continuous finite elements. To apply the DDA for the fracture analysis, an accurate inter-block stress evaluation is the first important step towards to block separation analysis.

However, the linear displacement function in the DDA generates constant stress field within a block, which can not effectively model the stress variation within a block and across the block interface. One way to improve the stress accuracy over the block-interface is to use small blocks glutting together for the area with large stress concentration, but too many blocks will lead to a high computing cost. In the following section, a stress recovery procedure is developed for better interface stress evaluation. The proposed procedure follows a recent paper on the FEM stress recovery (Ciancio et al., 2006), with revised formulas suitable for DDA implementation.

5.3 INTER-BLOCK FORCES AND STRESSES

5.3.1 Inter-block Forces

Equilibrium state of node k is shown in Fig. 5.1. Nodal force vector, $\mathbf{f}^{(k)}$ can be expressed as

$$\mathbf{f} = \int_A \mathbf{B}^T \boldsymbol{\sigma} dA \quad (5-1)$$

where $\mathbf{B}$ is the strain matrix, $\boldsymbol{\sigma}$ is the stress field of the block and A is the block area.

Fig. 5.1 Inter-block force ($\mathbf{r}$) and nodal force ($\mathbf{f}$)

Fig. 5.2 Forces equilibrium at nodes

$\mathbf{r}^{(k-1)}$ is the interaction force between node k and its adjacent node $k-1$ in its left hand

side, $\mathbf{r}^{(k)}$ is the interaction force between node k and its adjacent node $k+1$ in its right hand side. Both $\mathbf{r}^{(k-1)}$ and $\mathbf{r}^{(k)}$ are unknowns and can not be determined by the one equilibrium equation node k . So, additional equation is required (Ciancio et al., 2006).

In Fig. 5.2, there are totally N blocks around one node. $\mathbf{f}^{(k)}$ ($k=1, \dots, N$) is the nodal force vector, which can be determined by using Eq. (5-1), $\mathbf{r}^{(k)}$ ($k=1, \dots, N$) is the inter-block forces, $\mathbf{g}^{(k)}$ ($k=1, \dots, N$) is the gravitational loading of block at the node, which can be calculated as $\mathbf{g}^{(k)} = \frac{1}{3} \cdot \mathbf{g} \cdot A$ for a triangular block, where $\mathbf{g}$ is the acceleration vector, A is the area of block k . Then, the equilibrium equation for each node in the vector format can be written as following

$$\begin{cases} \mathbf{f}^{(1)} - \mathbf{r}^{(N)} + \mathbf{r}^{(1)} - \mathbf{g}^{(1)} = 0 \\ \mathbf{f}^{(2)} - \mathbf{r}^{(1)} + \mathbf{r}^{(2)} - \mathbf{g}^{(2)} = 0 \\ \mathbf{f}^{(3)} - \mathbf{r}^{(2)} + \mathbf{r}^{(3)} - \mathbf{g}^{(3)} = 0 \\ \vdots \\ \mathbf{f}^{(N)} - \mathbf{r}^{(N-1)} + \mathbf{r}^{(N)} - \mathbf{g}^{(N)} = 0 \end{cases} \quad (5-2)$$

Eq. (5-2) appears to have N equations in vector format and N unknown vectors ($\mathbf{r}^{(k)}$, $k=1, 2, \dots, N$). However, these equations are not independent. In fact, we have N unknown vectors ($\mathbf{r}^{(k)}$, $k=1, 2, \dots, N$), but only $N-1$ independent equations in vector format. So, one additional equation in vector format is needed.

Assuming $\bar{\mathbf{r}} = \mathbf{r}^{(1)}$, then, other unknown force vectors can be represented by $\bar{\mathbf{r}}$ as following:

$$\begin{cases} \mathbf{r}^{(2)} = \bar{\mathbf{r}} - \mathbf{f}^{(2)} + \mathbf{g}^{(2)} \\ \mathbf{r}^{(3)} = \bar{\mathbf{r}} - \mathbf{f}^{(3)} - \mathbf{f}^{(2)} + \mathbf{g}^{(3)} + \mathbf{g}^{(2)} \\ \vdots \\ \mathbf{r}^{(N)} = \bar{\mathbf{r}} - \mathbf{f}^{(N)} - \mathbf{f}^{(N-1)} - \dots - \mathbf{f}^{(2)} + \mathbf{g}^{(N)} + \mathbf{g}^{(N-1)} + \dots + \mathbf{g}^{(2)} \end{cases} \quad (5-3)$$

In the expanded form, each equation becomes:

$$\begin{pmatrix} r_x^{(k)} \\ r_y^{(k)} \end{pmatrix} = \begin{pmatrix} \bar{r}_x \\ \bar{r}_y \end{pmatrix} - \begin{pmatrix} h_x^{(k)} \\ h_y^{(k)} \end{pmatrix} \quad (k=1, \dots, N) \quad (5-4)$$

where

$$\begin{pmatrix} h_x^{(k)} \\ h_y^{(k)} \end{pmatrix} = \sum_{i=2}^k \left(\begin{pmatrix} f_x^{(i)} \\ f_y^{(i)} \end{pmatrix} - \begin{pmatrix} g_x^{(i)} \\ g_y^{(i)} \end{pmatrix} \right) \quad \text{and} \quad \begin{pmatrix} h_x^{(1)} \\ h_y^{(1)} \end{pmatrix} = \sum_{i=2}^k \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5-5)$$

In the following sections, one additional equation in vector format will be found out to solve the unknowns in Eq. (5-4).

5.3.2 Inter-block Stresses

The Inter-block stress is defined as the stress along the surface (or line in 2D) shared by two blocks. Then, the inter-block force can be calculated by inter-block stress times half area of inter-surface.

Fig. 5.3 Plane element in global and local coordinates

Assuming the area of the inclined face is ΔA , then, the areas for vertical surface and horizontal surface will be $\Delta A \cos \theta$ and $\Delta A \sin \theta$, respectively. The equilibrium equations along the x' and y' directions are (Ferdinand et al., 2002):

$$\begin{aligned} \sum F_{x'} = 0: & \quad \sigma_{x'} \Delta A - \sigma_x (\Delta A \cos \theta) \cos \theta - \tau_{xy} (\Delta A \cos \theta) \sin \theta \\ & \quad - \sigma_y (\Delta A \sin \theta) \sin \theta - \tau_{xy} (\Delta A \sin \theta) \cos \theta = 0 \\ \sum F_{y'} = 0: & \quad \tau_{y'} \Delta A + \sigma_x (\Delta A \cos \theta) \sin \theta - \tau_{xy} (\Delta A \cos \theta) \cos \theta \\ & \quad - \sigma_y (\Delta A \sin \theta) \cos \theta - \tau_{xy} (\Delta A \sin \theta) \sin \theta = 0 \end{aligned} \quad (5-6)$$

Thus, we get:

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ \tau_{y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \end{aligned} \quad (5-7)$$

where $\sigma_{x'}$ and $\tau_{y'}$ are normal and shear stresses of the inclined area.

Then, the relationship between unknown interaction forces r_x and r_y in the global coordinates (x, y) and the nodal stress at the local coordinates (x', y') can be expressed as (Ciancio et al., 2006):

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma \\ \tau \end{pmatrix} = \frac{1}{\Omega} \mathbf{Q} \begin{pmatrix} r_x \\ r_y \end{pmatrix} \quad (5-8)$$

For two-node edge, the nodal area Ω equals to half of inclined surface area. $\mathbf{Q}$ is the transformation matrix from the global (x, y) to the local (x', y') coordinate system, expressed as $\mathbf{Q} = \begin{bmatrix} \mathbf{n} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$.

The stress tensor $\mathbf{T}$ in matrix form can be expressed as: $\mathbf{T} = \begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix}$.

We can also express the global stress in the following form:

$$\begin{aligned}\mathbf{nTn}^T &= (\cos \theta \quad \sin \theta) \begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ \mathbf{tTn}^T &= (-\sin \theta \quad \cos \theta) \begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta\end{aligned}\quad (5-9)$$

which are the same as the $\sigma_{x'}$ and $\tau_{y'}$ derived in Eq. (5-7).

5.4 STRESS RECOVERY PROCEDURE

If we substitute the expression of $\mathbf{r}^{(k)}$ (Eq. (5-4)) into the equation of the stress traction vector (Eq. (5-8)), the value of the traction vector $\boldsymbol{\sigma}^{(k)}$ for each plane k can be computed as a function of $\bar{\mathbf{r}}$. Due to the arbitrary value of $\bar{\mathbf{r}}$, the coordinates of $\boldsymbol{\sigma}^{(k)}$ on Mohr's space will be representative of points that in general do not belong to any single circle. For getting the definite value of $\bar{\mathbf{r}}$, an intrinsic error is defined as the distance between the points $\boldsymbol{\sigma}^{(k)} = (\sigma^{(k)}, \tau^{(k)})$ and the Mohr's circle. This error is calculated as the distance between the stress vector $\boldsymbol{\sigma}^{(k)}(\bar{\mathbf{r}})$ (which is a function of $\bar{\mathbf{r}}$) and the projection of $\mathbf{T}$ onto plane k . Borrowing the concepts from microplane model, a global equation for that error may be expressed in the following form (Carol et al., 1992; Ciancio et al., 2006):

$$\phi = \sum_{k=1}^N \left((\sigma^{(k)}(\bar{\mathbf{r}}) - \mathbf{n}^{(k)} \mathbf{T} \mathbf{n}^{(k)T})^2 + (\tau^{(k)}(\bar{\mathbf{r}}) - \mathbf{t}^{(k)} \mathbf{T} \mathbf{n}^{(k)T})^2 \right) \quad (5-10)$$

In Eq. (5-10), the only unknown variables are the components of the stress tensor $\mathbf{T}$, if the value of $\boldsymbol{\sigma}^{(k)}(\bar{\mathbf{r}})$ is known. Then minimization of ϕ with respect to $\mathbf{T}$ ($\partial\phi/\partial\sigma_x, \partial\phi/\partial\sigma_y$ and $\partial\phi/\partial\tau_{xy} = 0$), leads to the following matrix equations:

$$\mathbf{A}\boldsymbol{\sigma} = \mathbf{c} \quad \text{or} \quad \boldsymbol{\sigma} = \mathbf{A}^{-1}\mathbf{c} \quad (5-11)$$

where

$$\mathbf{A} = \sum_{k=1}^N \begin{pmatrix} \cos^2 \theta_k & 0 & \sin \theta_k \cos \theta_k \\ 0 & \sin^2 \theta_k & \sin \theta_k \cos \theta_k \\ \sin \theta_k \cos \theta_k & \sin \theta_k \cos \theta_k & 1 \end{pmatrix} \quad (5-12)$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} \text{ and } \mathbf{c} = \sum_{k=1}^N \begin{pmatrix} \cos^2 \theta_k \sigma^k - \sin \theta_k \cos \theta_k \tau^k \\ \sin^2 \theta_k \sigma^k + \sin \theta_k \cos \theta_k \tau^k \\ 2\sigma^k \sin \theta_k \cos \theta_k + \tau^k (\cos^2 \theta_k - \sin^2 \theta_k) \end{pmatrix} \quad (5-13)$$

where N is the number of the blocks around a corner node.

We can easily get the relationship between $\boldsymbol{\sigma}$ and $\bar{\mathbf{r}}$ from Eq. (5-11). Now setting free $\bar{\mathbf{r}}$ and inserting Eq. (5-11) into Eq. (5-10), we obtain the function only with the arbitrary value of $\bar{\mathbf{r}}$. For the re-arranged Eq. (5-10), we minimize it again, this time with respect to $\bar{\mathbf{r}}$ ($\partial\phi/\partial\bar{r}_x$ and $\partial\phi/\partial\bar{r}_y = 0$), finally we obtain two equations with the unknown variables $\bar{r}_x$ and $\bar{r}_y$. Once $\bar{\mathbf{r}}$ is obtained, the stress tensor $\mathbf{T}$ can be computed expediently.

The stress recovery procedure can be summarized as follows (Equations can be obtained by formula management software Mathematica as shown in the appendix):

- 1) Determine the Inter-block force and stress expression by Eqs. (5-4) and (5-8).
- 2) Determine the global equation for intrinsic error using Eq. (5-10).
- 3) Minimize Eq. (5-10) with respect to $\mathbf{T}$ to obtain the relationship between $\boldsymbol{\sigma}$ and $\bar{\mathbf{r}}$ by $\partial\phi/\partial\sigma_x = 0, \partial\phi/\partial\sigma_y = 0$ and $\partial\phi/\partial\tau_{xy} = 0$.
- 4) Minimize Eq. (5-10) by $\partial\phi/\partial\bar{r}_x = 0$ and $\partial\phi/\partial\bar{r}_y = 0$ to get $\bar{\mathbf{r}}$.
- 5) Calculate inter-block force on the oblique plane by Eq. (5-4).
- 6) Obtain the recovered stress value by Eq. (5-8).

5.5 VERIFICATION EXAMPLES

In this section, two verification examples are presented to verify the efficiency of the proposed stress recovery procedure in improving the inter-block stresses and

nodal stresses. The results will be compared with both theoretical solutions and those from averaging method, in which the nodal stress is just averaged stress of surrounding blocks.

5.5.1 Inter-block Stresses of a Squared Plate

In this example, a square domain of $1 \times 1 \text{ m}^2$ dimension with elastic modulus $E = 10 \text{ GPa}$, and Poisson ratio $\nu = 0.2$, shown in Fig. 5.4. Each node of the triangle block is subjected to one third of gravity load of the corresponding block. The plane strain condition is assumed. The specific mass is 1 kg/m^2 and the gravity load in y direction is 10 N/m^2 .

Fig. 5.4 A square plate under gravity load

An irregular block system which consists of 12 arbitrary blocks is used in the DDA model. The node A (0.5 m below the top free surface) has 7 blocks around it, while the node B (0.8 m below the top free surface) connects 4 surrounding blocks. The bottom side is fixed at y direction while the other three surfaces are free. The stress state at these two nodes and their tractions along the interfaces converging at those points are calculated using the proposed stress recovery procedure and are represented in Table 5.1. For comparison purpose, the results from the averaging

method are also given.

Table 5.1 Comparison of σ_y calculated by stress recovery procedure, averaging method, and theoretical solutions (Unit: Pa)

Node A			Node B		
Method	σ_y	Error (%)	Method	σ_y	Error (%)
Theoretical	-5	-	Theoretical	-8	-
Averaged	-4.82	3.6	Averaged	-7.83	2.1
Recovered	-5.09	1.8	Recovered	-8.07	0.9

Theoretically, it is known that the stress state in y direction at nodes A and B should be $\sigma_{y(A)} = -5.0\text{Pa}$ and $\sigma_{y(B)} = -8.0\text{Pa}$. From Table 5.1, it can be observed that the proposed recovery procedure has improved the stress accuracy substantially, and the relative error for both nodes has been reduced by about 50%. This shows that for the irregular finite blocks system, the recovered method is better than the average smoothing method.

5.5.2 A Circular Tunnel in an Infinite Elastic Domain

In this example, the DDA is used to model a typical circular tunnel in an infinite elastic medium subjected to uniform far-field stress σ_∞ . Due to symmetry, only one quarter of the problem domain is modeled, and the problem domain is selected as $10R$ by $10R$, where R is the radius of the tunnel.

The analytical solution of this problem gives the stress state along the vertical direction as (Timoshenko, 1951):

$$\sigma_y = \frac{1}{2} \left(2 + \left(\frac{1}{r} \right)^2 + 3 \left(\frac{1}{r} \right)^4 \right) \quad (5-14)$$

where r is the distance from the measured point to the center of the hole, which gives the maximum stress $\sigma_{y\max} = 3\sigma_\infty$.

From the above description, the parameters used in DDA analysis are as following: domain size is 10m×10m with the radius of the hole as 1m, the far-field stress in the y direction is modeled as a normal tensile traction 1MPa loaded on the upper surface; the material is isotropic with $E=200\text{GPa}$ and $\nu=0.3$. The model contains 384 triangular blocks as shown in Fig. 5.5. Rolling boundaries are set on the symmetrical axes, and the upper and right boundaries are free surfaces.

Fig. 5.5 A circular tunnel under vertical load

Totally five measure nodes are selected in the model. The node 2 is surrounded by irregular blocks, while the other 4 nodes are surrounded by regular blocks, as shown in Fig. 5.6.

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Fig. 5.6 Enlarged area A of Fig. 5.5

Fig. 5.7 Comparison of σ_y at different distance from the center

Table 5.2 Comparison of the average and the recovered methods (σ_y , MPa)

Method Node	Analytical	Averaging	Error	Recovered	Error
Node 1 (r=1.090m)	2.484	2.552	2.7%	2.548	2.6%
Node 2 (r=1.268m)	1.892	1.822	3.7%	1.863	1.5%
Node 3 (r=1.650m)	1.386	1.447	4.4%	1.445	4.3%
Node 4 (r=2.021m)	1.212	1.243	2.5%	1.239	2.2%
Node 5 (r=2.593m)	1.108	1.109	0.1%	1.106	0.2%

Fig. 5.8 Mohr's circle for stress state of measure node 2

Fig. 5.7 shows the recovered results of the measured nodes from the averaging method and the recovery procedure. Both are in good agreement with the analytical solution. It can be seen from Table 5.2 that for the nodes surrounded by the regular blocks, the accuracy by the recovery method is more or less the same as the average method. For the node surrounded by the irregular blocks (Node 2), the recovery method has an evident advantage. The relative error for Node 2 has been reduced from 3.7% by the average method to 1.5%. Fig. 5.8 shows the Mohr's circle for Node 2. It can be seen that the unrevised points (which are obtained from the DDA analysis) have quite large distances from the analytical Mohr's circle and the recovered stresses at the four blocks are much closer to the Mohr's circle.

From this example, it shows that the recovered method can provide better results than the averaging method, especially for the case with irregular blocks or coarse block system. This is a very useful property as rock mass is often arbitrarily partitioned into irregular blocks to model its heterogeneous material properties.

5.6 CONCLUSIONS

This chapter presents a post-processing procedure called the stress recovery method to improve the nodal stresses and inter-block stresses. By glutting the blocks together, the DDA is adopted to model the continuum media, A minimization procedure is developed to adjust the stresses of the surrounding blocks to match the Mohr's circle at the nodes. Two examples are presented. The results are compared with theoretical solutions and the averaging method. The averaging method is good only when with regular mesh and fine mesh. In other words, the accuracy of the averaging method is strongly mesh dependent. However, the stress recovery method can provide better results than the averaging method, especially for the cases with irregular mesh and coarse mesh. The stress recovery method is convenient to be used for both regular and irregular blocks and permits any number of blocks around one node. The recovery method presented constructs a good foundation for future work on the fracture propagation analysis. Now, the stress recovery method is only used as the post-processor. In the future, this method will be implemented into the DDA code to improve the stress state after each time step.

CHAPTER 6 ROCK MASS RESPONSE SUBJECTED TO UNDERGROUND BLAST LOADING

6.1 INTRODUCTION

A numerical model is constructed to test the response of the rock mass under dynamic loading. Fig. 6.1 sketches the test domain, in which the explosive is placed several meters below the surface, and the weight of the explosive decides the charge size. The properties of rock mass and the weight of explosive together decide the crushed zone region and final blast cavity surface. Because the response of an underground structure depends on the input load which is transmitted from the rock mass and be measured in terms of ground shock or stress wave (Lu et al., 2005), it is important to first examine the prediction of the stress wave in the free-field rock mass before we go into the coupled analysis of rock mass-structure. In the current study, only the scenarios without a buried structure will be considered.

Referred from some trial analyses, the computational domain is chosen to be on the order of 36m wide and 20m deep. The explosive charge is chosen to be 80kg (TNT equivalent) which is embedded at about 5.0m deep. For a spherical shape explosive, the radius of the charge is about 0.2271m. Take the portrait plane, the problem is considered as a plane strain problem. The Poisson's ratio and Young's modulus of the jointed rock mass are assumed to be 0.4 and 80GPa, while the rock density is assumed to be 2650kg/m³. The compressive strength of the rock mass is assumed to be 120MPa. Self-weight of the rock mass is neglected due to its little effect compared with blast loadings.

Fig. 6.1 Test domain of the numerical modeling

6.2 MESH SIZE AND TIME INTERVAL

The accuracy of the numerical solutions for shock load analysis depends on not only the numerical integration scheme, but also the mesh size and the integration time step. In general, a small mesh size with a small time step will provide a good accuracy, but this will at the same time require an extensive computing time. Striking a balance between the accuracy and the efficiency is therefore essential for the transient dynamic analysis.

The calculations for the blast loading are carried out by using the modified DDA program. From the calculation module as discussed in Section 4.6, the radius of the crushed zone is about 1m. The blast cavity surface is considered as the interface between the blast charge and rock mass. The explosive effect can be simplified as a pressure time history and applied on the cavity wall as a line loading. Assuming the pressure rising time of the blast loads is about 2ms, the maximum pressure is calculated to be about 100MPa and total sustained loading time is 4ms. For such transient dynamic shock load, the Fourier decomposition method can be used to express the shock load as a series of harmonic loads, and obtain the highest frequency in the series which contribute significantly to the

loading magnitude. In this problem, its harmonic loading frequency is about 125 Hz. A cut-off frequency is chosen, in which the modes having frequencies higher than the cut-off one are assumed to be insignificant. For our problem, the cut-off frequency is 500 Hz, about 4 times that of the loading frequency.

The wave length and the period corresponding to this cut-off frequency can be obtained as:

$$L_w = c / f_{co} \quad (6-1)$$

$$T_w = 1 / f_{co} \quad (6-2)$$

where c is the slowest wave speed of the material; L_w is the wave length and T_w is the wave period. In our problem, the wave length is about 6 m and wave period is about 2 ms.

The size of the mesh should be sufficiently small so that the wave pattern can be correctly represented by the nodal points of the blocks. The numerical experiments indicated that the block length should be within one eighth of the wave length L_w . The maximum integration time step Δt is recommended to be at least one tenth of the wave period T_w . It should be noted that above empirical rules are based on some simple test cases under harmonic loading, which may not be totally suitable for shock loads. Based on the above rules, the block size in our problem should be less than 0.6m and the time interval should be less than 0.2 ms.

Due to axi-symmetry, only half of the 2-D domain is modeled. The top is free, while the bottom and right sides are fixed by using a large rock mass with a small Young's modulus to absorb the stress wave and decrease the reflected wave at the computational boundary, which is equivalent to the non-reflecting boundary. The left side is the symmetrical axis, rolling boundary is applied which is constrained in the x direction and free along the y axis. The jointed block size for the whole

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region ranges from $0.13 \times 0.13 \text{ m}^2$ to $0.5 \times 0.5 \text{ m}^2$, forming 1569 blocks. Fig. 6.2 shows the final rock mass configuration. Some parameters used in the models for the rock mass and the blast charge are list in Table 6.1 based on existing literature and trials (Pan and Zhao, 1993).

Fig. 6.2 Rock mass configuration by DC

Table 6.1 Parameters for the DDA analysis

Code for type of analysis: 0 for static; 1 for dynamics	1
Initial time interval (ms)	<0.2
Total time steps	400
Maximum displacement ratio	0.001
Maximum pressure loading (MPa)	100
Dynamic loading history time (ms)	4
Stiffness of contact normal springs (penalty)	automatic
Block unit mass, ρ (kg/m^3)	2650
Block Young's modulus, E (GPa)	80
Block Poisson's ratio, ν	0.4
Compressive strength of rock mass (MPa)	120
Φ (friction angle), c (Cohesion MPa) and T_0 (Tensile strength, MPa) for joint set	(Case 01) 0, 0, 0
	(Case 02) 45, 6.6, 1.1
	(Case 03) 45, 66, 11

Three cases are analyzed with the same block properties but different joint properties:

- 1) Without any joint strength, corresponding to completely jointed rock mass;
- 2) Joint strength equal to 10% of the rock mass strength, corresponding to the real joints;
- 3) Joint strength same with the rock mass strength to represent the continuous media.

Several measure points are arranged along the radial direction (at a embedded depth of 5.0m) with the different distances away from the charge center to record the stress attenuation in the rock mass. Several other measure points are arranged on the vertical line with different distances from the charge to get the stress and displacements in the targeted rock mass.

6.3 NUMERICAL RESULTS AND DISCUSSIONS

The following issues will be discussed based on the numerical analysis:

- 1) Pressure time curves at the different distances away from the charge center
- 2) Pressure and acceleration attenuation
- 3) Pressure profile along the vertical lines
- 4) Deformation of the rock cover and blast funnel formation

6.3.1 Pressure Time Curves

The pressure time curves (before the stress wave reflected back) at four selected points are shown in Fig. 6.3. The four points are at an embedded depth of 5m with the horizontal distances of 2m, 3m, 6m and 9m away from the blast center. In the current study, we take the stress positive in compression and negative in tension. It can be seen that the pressure at the points is very low before the wave arrives, and then a peak value appears just after the wave front reaches. The response rapidly

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decreases after the peak wave passes and changes into a small tensile stress.

(a)

(b)

(c)

Fig. 6.3 Pressure time curves at different distances
from the blast charge center: (a) Case 1; (b) Case 2; (c) Case 3

The arrival time for each measured point for the highest stress level in the three

cases is nearly identical. Comparing the stress values of the three cases, Case 1 has a slightly larger stress value than the other two cases. This may be due to the fact that the real joint will attenuate the strength rock mass and co-act in the relative low stress region when the stress field has exceed the strength of the joints.

We can also find that in these three cases, a very small upwards fluctuation appears just before the maximum peak stress occurs. This is probably caused by the program's own strict equilibrium of the potential energy among the whole blocks system at the first several time steps (contact spring needs to adjust its stiffness to an optimized value) before the maximum pressure loading arrives.

It should be noted that the stress time history shown is only the first part of the record (about 0.007s) which is 2 times that of the dynamic loading. In the late stage, the stress at certain points may jump to a bigger value because of the huge disturbance caused by the waves reflected from the fixed right hand boundary, and those results have been disregarded. In the future, no-reflection boundary should be supplemented in the program to solve this inconvenience.

6.3.2 Maximum Pressure and Velocity Attenuation

Fig. 6.4 shows the maximum pressure versus the distance along a horizontal line at the same height of the blast center. The maximum pressure attenuates along the distances from the blast center. For comparison, the maximum pressure (PP) in the rock domain generated by a contained explosion can generally be expressed as (TM5-855-1, 1984):

$$PP = 15 \left(\frac{R}{\sqrt[3]{W}} \right)^{-1.3} \quad (\text{MPa}) \quad (6-3)$$

Fig. 6.4 Maximum pressure versus $R/W^{1/3}$ with empirical equation

The three cases are in the same order and tendency with the TM5 as shown in Fig. 6.4. Take the Case 2 for example, we can find that the maximum pressures in near region conform well to the TM5 formula which is derived from the test data. The maximum pressures in these points have little disturbance from the surrounding rock. While the results of the points far from charge center have a little difference because of the disturbance from the reflection.

For the maximum particle velocity (PPV), we can get it from the relationship of the displacements of blocks and every step time. The velocity is found to reach the maximum value when the block reaches its maximum pressure and attenuate quickly to the constants. A respective empirical equation recommended is also compared (Murrell, 1996) with the simulation results:

$$PPV = 5 \times \left(\frac{R}{\sqrt[3]{W}} \right)^{-1.5} \quad (m/s) \quad (6-4)$$

Fig. 6.5 Maximum particle velocity versus $R/W^{1/3}$ with empirical equation

The numerical results as shown in Fig. 6.5 give similar trends as the empirical formula for the points near the blast center. Difference starts to appear farther away from the center, probably due to the identical DDA material model (elastic without damping).

6.3.3 Pressure Profile along the Vertical Lines

The pressure profiles at three vertical lines (2m from the surface to the 8m depth) are shown in Fig. 6.6 for the Case 3. The distances between the vertical lines and the explosive center are 3m, 6m and 9m respectively.

It can be seen from the results that the highest pressure always appears in the middle of the observed line (the same horizontal level as the blast center). For a given distance, the maximum pressure variation along the depth is much larger in the near region. While at the distance of 9m, the profile of maximum pressure has a nearly uniform distribution along the depth.

Fig. 6.6 Pressure profile on the vertical lines (case 3)

6.3.4 Deformation of the Blocks and Blast Filter Formation

Figs. 6.8 are the deformation configuration of rock mass after the 150 time steps calculation for the three cases.

Fig. 6.7 Deformation configuration of rock mass (Time step:150; about 0.02s)

Fig. 6.7 (a) and (b) show a clear hump on the rock mass above the blast charge while the response in Fig. 6.7 (c) is not very distinct. In Fig. 6.7 (a), when there is no joint strength between the blocks, the adjacent blocks will detach when they

have different deformation. While in Fig. 6.7 (b), the blocks deform differently, the strength of the real joints will constrain the blocks to slide or detach by its friction angle, cohesion and tensile strength. When the tensile force or the force in the tangential direction is larger than the strength of the joints, the blocks will detach or slide. And in the next time step, the strength of the joints becomes zero even if they contact again.

Some blocks on the blast wall have fallen down towards to the blast center, caused by the tensile pressure after dynamic loading returns back from the free surface and fixed boundary. This conforms to the site observations that when the charge is intensive strong, the rock mass near the blast charge will have a distinct movement towards the direction of the blast center (Liu, 1980).

In Figs. 6.7 (a) and (b), we find that there has an inverse triangle shape of the final blast filter in the upper part of rock mass and it has a tendency in the 45° direction to move from the minimum free surface. This is accordant with the results of small experimental tests as we call the formation of the Blast Filter (Qi et al., 1995).

Actually, the block system is continuously moving outwards. The additional simulations have been done for more time steps as shown in Fig. 6.8.

Fig. 6.8 Deformation configuration of rock mass (Time step:600; about 0.06s)

6.4 CONCLUSIONS

In this chapter, the response of the rock mass subjected to underground explosion is investigated using the 2-dimensional DDA numerical models. The general response characteristics are discussed and compared with other empirical equations. The numerical analysis of the explosive test provides essential information on the rock mass response due to the blast loads. Based on the numerical analysis results, some observations can be made as follows:

- 1) The rock mass response due to blast loads has the behavior of $1/R$ attenuation which extent is slower than that of the soil materials.
- 2) The rock mass with real joints strength in case 2 generates a smaller pressure in the region near the blast center and a bigger pressure variation far from the blast center compared the case 1 and 3. The strength of rock mass is attenuated due to the existence of weak strength of the joints when the high stress field has exceeded strength ability of the joints. In the relative low stress region, both the rock mass and joints will co-act to resist outside pressure.
- 3) The velocity and acceleration responses are found to reach the maximum value when the block meets its maximum pressure and attenuate quickly to the constants.
- 4) The results of displacements are more reasonable than that of the pressure. This is mainly because that there are no variations of stress and strain in a block which cause the considerable precision error if the block size is not small enough while in the current DDA program the displacement state can be linearly changeable. That is why some pressure value may not be very accurate.

CHAPTER 7 NUMERICAL SIMULATION OF SCALED BLAST TEST

7.1 INTRODUCTION

Drilled boreholes are widely used for many engineering applications, including drilling and blasting, stimulating oil and gas wells, hydraulic fracturing for geothermal reservoirs, breaking apart rocks with an expansive demolition agent (Sang, 2003). Although borehole pressure has been well known by researchers for decades for resource recovery and construction, the mechanism by which rock is broken apart under pressure in a borehole is still not understood well, especially when the borehole is under dynamic loading like blasting. The laboratory scaled blast tests have provided some important guidelines for field practice through the observation of the behavior of down-scaled specimens (Rossmanith et al., 2005), also valuable theoretical study is proceeded based on the theory of elastic wave propagation and fracture mechanics (Rossmanith and Uenishi, 2006). But until now, the numerical simulation for modeling the total fracture process is still very limited.

The purpose of this chapter is to discuss how the borehole pressure waveform in rock affects dynamic fracture propagation and patterns in the scaled blast test. As rock is an inhomogeneous material, fracturing can be treated as a stochastic process. The fracture propagation process and the final fracture pattern are stress loading rate dependent. If the stress loading rate is high, intense crushing with large number of microcracks of the rock near the borehole occurs with little or no fracturing extension; if the loading rate is low, the fractures produce a single extensive hydraulic fracture similar to that brought about by static pressurization, only a few larger microcracks form under the applied tensile stress field. Recently, the effect of

the blast pressure waveform on the fracture process was studied using a numerical model based on discrete elements (Donze et al., 1997). Although their results agreed well with observations and theoretical expectations, the fundamental mechanisms of fracture propagation and the patterns associated with the applied stress field were not considered.

In the present chapter, blast loading induced rock fracture will be simulated using the DDA program. The influences of the loading conditions (e.g. loading rate) and boundary conditions (i.e. free boundary, rigid boundary et al.) on the rock fracture patterns will be studied. The heterogeneity effect of the rock mass is considered by assigning joints with different properties conforming to Weibull distribution. The post-processor of the DDA is also modified. The openings between blocks are marked out by using black lines in the gray background of the specimen to represent the cracks.

7.2 NUMERICAL MODELS

A numerical model with borehole is constructed by the DDA. It consists of a blast hole in rock with a free or fixed outer boundary, as shown in Fig. 7.1. The radius of the model and borehole are 0.4m and 0.03m, respectively. The model is divided into 1800 quadrangle blocks with separate joint surfaces illustrated in Fig. 7.2. Several measured points have been set along the positive direction of x coordinate. The parameters and calculation conditions are listed in Table 7.1.

Fig. 7.1 Geometry of the analysis model for fracture process simulation

Fig. 7.2 DDA blocks system layout for the analysis model

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Table 7.1 Parameters for the DDA analysis

Parameter	Value
Problem type	Plane strain
Density (kg/m ²)	2650
Poisson ratio ν	0.3
Elastic modulus E (GPa)	60
Friction angle of joint (°)	30
Cohesion of joint (MPa)	66
Average Tensile strength of joint (MPa)	15
Time step Δt (μs)	0.5-2
Total analysis time	2-5 times of loading times, till results stable
Factor of successive over-relaxation iteration	1.6
Coefficient of uniformity factor, m	2, 5, 20 and ∞

To generate the various applied borehole pressure waveforms, a general form of the applied pressure pulse function introduced in Chapter 2 was used:

$$P = P_b \xi \left[e^{-\alpha t} - e^{-\beta t} \right] \quad (7-1)$$

Fig. 7.3 Pressure-time curves when $\beta/\alpha = 1.5, 15$ and 30

Fig. 7.3 shows the applied pressure waveforms with peak values at time t_0 when $\beta/\alpha = 1.5, 15$ and 30 .

7.3 SIMULATION OF ROCK FRACTURE PROPAGATION PROCESS

The rising time t_0 varies between 10, 50, 250 and 500 μ s with $\beta/\alpha=1.5$ and 30 to investigate the dynamic fracture processes induced by the applied pressure waveforms. The β/α is used to analyze the effect of the decaying time which is simply equivalent to the ratio of loading rising time to decaying time. For $\beta/\alpha=1.5$, decaying time is two times that of rising time while 6 times that of rising time if $\beta/\alpha=30$. The maximum pressure value of 30MPa is considered (in Section 7.4.2, 100MPa is used). Note that from a practical point of view, a short pulse has higher amplitude than a long pulse. However, to compare the fracture processes for different stress-loading rates, it is necessary to consider the same maximum induced-stress level for all of the analysis models.

The fracture processes for the maximum principal stress field of the time steps for each rising time with $\beta/\alpha=1.5$ are compared in Fig. 7.4. Although the same maximum pressure is applied to all of the models, the stress field in the rock model varied significantly with increases in the rising time. The different stress fields result in different fracture processes for each model. When $t_0=10\mu$ s and 50 μ s, the stress fields generate a compressive fracture zone around the borehole with radial tensile cracks in its vicinity which arrest quickly. When $t_0=250\mu$ s and 500 μ s, radial tensile cracks develop in the vicinity of the borehole and propagate continuously to form long cracks.

Crack extensions corresponding to tensile fractures increase with the rising time. When $t_0=250\mu$ s and 500 μ s, the applied pressure induces a quasi-static stress field during the early stages of the load application, which leads to a reduction in the number of radial tensile cracks and longer cracks from the borehole. The increase in the number of cracks is associated with the stress-loading rates in the borehole. The number of cracks emanating from the borehole wall decreases as the stress-loading

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rate increases. Note that the applied stress-loading rates are approximately 3, 0.6, 0.12 and 0.06 MPa/us at $t_0=10, 50, 250$ and $500\mu s$, respectively.

Fig. 7.4 Crack initiation and propagation in scaled blast simulation when $\beta/\alpha=1.5$

Fig. 7.5 shows the fracture processes for $\beta/\alpha=30$. A higher stress level is found throughout stress fields in the models when compared with those in the case when

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$\beta/\alpha=1.5$, which ultimately results the differences in the patterns and extension of the fracture zone.

Fig. 7.5 Crack initiation and propagation in scaled blast simulation when $\beta/\alpha=30$

The influence of the decaying time on the fracture pattern is also examined from the final fracture patterns as shown in Figs. 7.4 and 7.5. The fracture zone increases as the decaying time increased when $t_0=10\mu s$ and $50\mu s$. However, when $t_0=250\mu s$ and

500 μ s, there are little significant differences in the fracture zone; only the length of cracks increases as the decaying time increases when $t_0 = 500\mu$ s. These results indicate that the fracture patterns are affected to a greater extent by the rising time than the decaying time. Therefore, it is possible that if the rising time becomes significantly long, then the maximum extension of the tensile radial cracks will not be very dependent on the increase in the decaying time. This result agrees well with the known fracture processes of work obtained using static borehole pressures. For static borehole pressures, the crack extension only depends on the maximum value of the applied pressure (Sang, 2003).

7.4 FURTHER DISCUSSIONS

7.4.1 Influence of Stress Loading Rate on Fracture Patterns

The length and shape of the wave pulse are the most important and decisive control parameters with respect to the development of the fracture network and fragmentation (Rossmanith and Uenishi, 2006)

The analysis in Section 7.3 shows that the fracture processes are significantly affected by the rising time of the pressure that is applied to the borehole wall. This implies that the fracture processes are closely associated with initial pressure loading rates applied on the borehole. In Fig. 7.4, the extensions of fractures and fracture patterns when $t_0=10\mu$ s and 250μ s differ significantly. To examine the fracture processes at different stress-loading rates, the processes in Figs.7.4 (a) and (c) corresponding to approximately 3 and 0.12MPa/ μ s respectively, are illustrated in Figs.7.6 and 7.7 in details.

Fig. 7.6 shows radial crack propagation around the borehole at about 5 μ s intervals, which corresponds to Fig. 7.4 (a). The radial tensile fractures start from the vicinity

of the compressive failure zone at about 12 μ s. After an additional 5 μ s, the tensile stress field causes the radial cracks to run, which is followed by stress releases around the running cracks. At time 40 μ s, all the cracks arrest and the final fracture pattern forms. The differences in the crack growth velocities increase with calculation time. This is because the cracking is affected by the different spatial microscopic strengths for an applied stress level. The crack length increases continuously with the applied stress, while at the same time the differences in the crack growth lengths also increase. The stress released around the leading cracks disperses the concentrated stresses near the tips of the following cracks. This means that if a crack slows down and lags behind the neighboring crack tips, the energy of the stress field ahead of the crack will be made available to the neighboring cracks which will absorb the energy and propagate at the expense of the slower crack. From Fig. 7.6, it can be found that with intense short cracks around the vicinity, several cracks propagate longer. From this case, it is shown that a high stress-loading rate increases the number of radial cracks and leads to intense stress release around the running cracks. The fast stress release affects fracture propagation and results in shorter cracks.

Fig. 7.7 shows the fracture processes in rock for a lower stress-loading rate, corresponding to Fig. 7.4 (c), at 40 μ s intervals. The cracks initiate at about 120 μ s. Preferential cracks propagate with the radiating tensile stresses after each additional 40 μ s. As the preferential cracks propagate, stresses are radiated and released from around the cracks. As compared to the fracture processes shown in Fig. 7.6, the interval of the cracks is larger and stress released between the cracks has little effect on the growth of adjacent cracks. This indicates that the stress release decreases, (i.e., strain energy release decreases), as the stress-loading rate decreases. Ultimately, this results in longer crack extensions.

Fig. 7.6 Fracture initiation and propagation for $t_0=10\mu s$

Fig. 7.7 Fracture initiation and propagation for $t_0=250\mu s$

Comparison of the fracture processes and final fracture patterns between Fig. 7.6 and Fig. 7.7 indicates that when the loading rate is high (3MPa/us in Fig. 7.6), a lot of fractures will initiate around the blasthole but with a very limited extension. However, when the loading rate is low (0.12MPa/us in Fig. 7.7), less fracture will be generated around the blasthole but with a much longer length. This result agrees well with the observations of earlier researchers (Cho and Kaneko, 2004).

7.4.2 Influence of Boundary Conditions on Fracture Patterns

For the discussion in the above sections, the boundary condition effect is not considered because the maximum pressure is relatively small and the wave transmitted to the outside boundary is small, which will have little influence on the fracture pattern. However, when the maximum pressure becomes large enough to cause rock mass failure when it is reflected back, the boundary conditions must be taken into account. In this section, free and rigid (or fixed) boundary conditions are considered. The appropriate conditions must be met: stress-free in free boundary ($\sigma(r = R_{\text{out}}/2; t) = 0$) and deformation-free in inextensible boundary ($u(r = R_{\text{out}}/2; t) = 0$). The maximum pressure level considered on the boundary conditions is changed to 100MPa. All other parameters are same as the case in Fig. 7.4 (b).

Upon detonation, a radial stress wave front P expands from the bore-hole wall. For times $t < t_b$ (t_b =the time of the P wave hit the outer boundary), the incident dynamic wave fields for both cases are identical as the wave front has not experienced the different boundary conditions. At the time $t=t_b$, there occurs reflection of the outgoing dispersive (compressive wave front) P wave at the outer boundary (free in case A and rigid in case B, a converging reflected (tensile wave front) PP stress wave reaches the out boundary and focuses towards the center of the borehole. After time t_b , the total wave fields become different because the reflected wave fields are different. In the current study, plane wave is assumed. The reflected waves have the

same shape with the incident wave but different sign of the amplitudes (free boundary-Tensile *PP* pulse; rigid boundary-Compressive *PP* pulse). Mathematically, the condition for stress free or deformation free can be achieved by the superposition of appropriate converging stress fields which converge to the center in such a way that, at the out boundary, the total stresses or displacements, respectively, are zero for all times. However, the presence of dispersion with the expansion of circularly crested elastic stress waves makes it all different. The structure of the wave changes with radial position, the wave pulse flattens out and becomes progressively wider. From an analysis point of view, this is not an easy task because the converging wave field itself is dispersive and the resulting wave fields are very complicated. That is also a very important reason why we need to use numerical modeling to help us understand this complicated process.

The fracture propagation processes for the two cases with free boundary and rigid boundary are compared in Fig. 7.8. It can be found that different boundary conditions result in different fracture behavior of the specimens.

In the first case (free boundary) as shown in Fig. 7.8 (a), a reflected sign-inverted *PP* stress wave converges back to the centre $t > t_b$. By doing so, the tensile *PP* wave increases its amplitude. Spalling is possible in the vicinity of the outer boundary, if the resulting reflected stress wave amplitude is stronger than the tensile strength of the rock. The converging leading tensile *PP* pulse will also create a strong radial tensile field in the center region around the blast hole. This will give rise to circumferential cracking by initiation of radial emerging blast-hole cracks and the formation of pie-shaped fragments. This could be seen in the simulation after waveform has reflected back. And the number of cracks is increased because of the effect of overlapped wave.

Fig. 7.8 Fracture propagation in different boundary conditions:

(a) free boundary; (b) rigid boundary

In the second case (rigid boundary) as shown in Fig. 7.8 (b), if the boundary is a rigid one or equivalently, inextensible in the circumferential direction, the length of the circumference is constant and this is equivalent with the fact, that there is no radial expansion at position $r = D/2$. Hence, wave reflection at a rigid boundary leaves the sign of the stress wave unaltered and an impinging compressive P blast wave front will be reflected as a compressive PP wave front. Hence, for reasonably

long wave pulses, the outgoing P wave and the converging PP wave will superimpose and thereby create a strong compressive stress field in the vicinity of the blast-hole boundary, thereby reducing the chances for the development of additional fracture and damage. In this numerical simulation, the number of the borehole cracks becomes less and the maximum superposition occurs near the outer boundary because of the relative short wave pulse.

The boundary conditions at the outside of the model have a significant influence on the dynamic stress field in the specimen and, therefore, control the development of the fracture network and associated fragmentation when the wave pulse is large. Its dynamics effect is entirely controlled by the ratio of the length of the outgoing wave pulse over the diameter of the specimen. When P and PP waves meet in the different (free or rigid boundary) conditions, it will increase or decrease the stress field at the region of the superposition.

7.4.3 Wave Propagation Attenuation

The formation of the rock mass fracture is also a process that the rock mass goes through from continuous to discontinuous media. As this discontinuous property makes the wave transmit, reflect and refract in the material, the fracture process becomes much more complex. When we only want to study the wave propagation attenuation in the material, the concept introduced in Chapter 5 that glutting all the blocks together as a continua is applied. The joint boundaries between the blocks have been assigned same strength with the rock material and the total domain is treated as a continuous media.

In this study, the cylindrical wave propagation is considered. The cylindrical wave propagation corresponds to the case in which an infinite linear explosive charge detonates instantaneously along its entire length, i.e. with infinite velocity of

detonation. Assume the wave field is generated due to a pressure pulse $p(t)$ which is suddenly applied to the cylindrical blast hole boundary. The governing equation for the cylindrical charge implies that the amplitude of the wave front decays proportionally to the inverse square-root of the increasing distance from the center of the detonation, expressed in the form as (Rossmanith et al., 2005):

$$P_r = P_m \left(\frac{R}{R_0} \right)^n \quad (7-2)$$

where P_r is the maximum stress at radius R , P_m is the initial shock pressure on the borehole wall, R_0 means the original borehole radius and n is the exponential coefficient of the peak stress attenuation, a negative number, always treated to be -1.5 in general for commercial explosives loaded in most rocks.

Fig. 7.9 Stress attenuation from the borehole center along the x direction

The maximum pressures from the ten measured points along the positive x coordinate are compared with the theoretical solution as shown in Fig. 7.9. We can see that the results from the DDA agree well with the theoretical solutions. This verifies again the validity of DDA on the stress state of the continuous media when large number of blocks is used as discussed in Chapter 5.

7.5 INFLUENCE OF HETEROGENEITY OF ROCK MASS ON FRACTURE PATTERNS

All the analysis described above assumes that the rock mass material is uniform. That is why all the fracture patterns shown above are symmetrical. For theoretical analysis, above study is helpful for us to understand the mechanism of the fracturing process. But in the real world, perfect symmetry is never achieved due to the presence of all kinds of imperfection. As a geological material, heterogeneity is the most intrinsic character of rock mass which can be tested in the mechanics experiments. The results show that the rock sample has very large dispersion on its strength behavior due to the heterogeneity. To consider this material heterogeneity, random number satisfying Weibull distribution is generated to give the spatial distribution of the microscopic properties in the analysis model.

Although the heterogeneity of rock mass is characterized in both the strength and elastic modulus, it is difficult to determine both heterogeneities experimentally. Therefore, it is reasonable to consider the random distribution of the joint strength while keeping a constant elastic modulus to account for the material heterogeneity of rock mass.

The best approach to model the material heterogeneity is to obtain the full information of material. However, it is not realistic in practice. Here, Weibull distribution is adopted to characterize the heterogeneity of material.

The two-parameter Weibull distribution can be expressed as

$$f(T) = \frac{m}{\mu} \left(\frac{T}{\mu} \right)^{m-1} \exp \left(- \left(\frac{T}{\mu} \right)^m \right) \quad (7-3)$$

where μ is the mean value, m is the shape parameter describing the scattering of T .

The mathematical expectation and dispersion of the Weibull distribution are

$$E(T) = \mu \cdot \Gamma\left(1 + \frac{1}{m}\right) \quad (7-4)$$

$$D(T) = \mu^2 \left[\Gamma\left(1 + \frac{2}{m}\right) - \Gamma^2\left(1 + \frac{1}{m}\right) \right] \quad (7-5)$$

Fig. 7.10 Probability density of Weibull distribution

Here, the tensile strength of rock joint conforms to the Weibull distribution. Fig. 7.10 shows the Weibull distribution with the shape parameter m and the mean value $\mu=15\text{MPa}$. From this figure, it can be seen that the presence of the shape parameter m allows Weibull function to take a wide variety of shape. The shape parameter m can be defined as the heterogeneity index. A smaller m implies more heterogeneous and $m=\infty$ denotes that material is homogeneous.

Because the Weibull cumulative distribution is non-monotone, the cumulative distribution function is derived as

$$Q(T) = \int_0^T f(T) dT = 1 - \exp\left(-\left(\frac{T}{\mu}\right)^m\right) \quad (7-6)$$

The Weibull cumulative distribution with different index is shown in Fig. 7.11.

Fig. 7.11 Cumulative distribution function of Weibull distribution

The disorder of the probability distribution for joint friction angles can be achieved by the Monte Carlo method. One simple way is to generate a series of random data which have a uniform distribution between 0 and 1. Then, a series of desired parameters, which conform to Weibull distribution, can be obtained as

$$T = \mu (\ln(U(0,1)))^{1/m} \quad (7-7)$$

where $U(0,1)$ represents the random data between 0 and 1 generated by Monte Carlo method.

In this study, three additional analysis were performed for $m=2$, 5 and 20 to examine the influence of rock heterogeneity on fracture patterns. The results are shown in Fig. 7.12. Except for the rock heterogeneity, all other parameter conditions are the same as the case in Section 7.2.

DEVELOPMENT OF DDA FOR DYNAMIC ANALYSIS OF ROCK MASS

Fig. 7.12 Final fracture patterns with different heterogeneity parameter, m

Compared with the original homogeneous specimen, Fig. 7.12 shows that the heterogeneity of rock mass can strongly affect the fracture patterns. The final fracture patterns are not unique and this sufficiently account for the effect of the extent of the material dispersion. The rising time have the same influence on the fracture patterns. The longer rising time, the less but longer cracks are generated. There have no uniform extensions of the radial cracks around the borehole generated. It is mainly because that the growth velocities of the radiating crack are different in an earlier stages of the applied pressure. The velocity differences led to an increase in crack arrest due to stress released from adjacent cracks. When $m=20$, the number and length of the cracks become similar as the previous simulation when $m=\infty$ in Sections 7.2 and 7.3, only the shape and position are different. The number of the cracks increases much and fracture patterns become very chaotic when m becomes small. This is mainly due to the reason that, with the decrease of the m value, the material becomes more heterogeneous around its domain which would weak its strength ability (This has been considered in Eq. (7-4). that the actual average value decreases when shape parameter m decreases).

7.6 CONCLUSIONS

The proposed dynamic fracture process analysis is performed by various applied pressure forms, boundary conditions and material heterogeneity to investigate their influences on the dynamic fracture processes in rock.

The influence of the blast pressure waveforms has been numerically investigated. Numerical results indicates that tensile stress induced crack extensions increase with the rising time, and fracture patterns are more affected by the rising time than the decaying time. The effect of the stress-loading rate on fracture processes is also investigated. A higher stress-loading rate increases the number of radial cracks, thereby release stress levels around the running cracks. The stress released from

adjacent cracks affects crack extension and results in shorter crack propagation lengths. At lower stress-loading rates, the number of cracks and crack arrest caused by stress released at adjacent cracks are reduced which leads to longer crack extension. These numerical results demonstrate that when preferential cracks develop earlier, the crack extension becomes longer.

When the maximum pressure becomes large, different boundary conditions will play an important role on the final fracture patterns for its reflection effect. For the free boundary, the converging leading tensile *PP* pulse will also create a strong radial tensile field in the center region around the blast hole which increases the number of the cracks; while for the rigid boundary, the reflected converging compressive *PP* pulse may induce a compressive stress field in the immediate vicinity of the blast hole which reduces the probability of blast hole cracking or, in some extent suppresses the blast hole breakdown.

Wave propagation attenuation is also studied. In the assumption of intact material, the simulated domain is divided and glued together as continua. The results of DDA agree well with the analytical solution. This verifies the conclusion in Chapter 5 that DDA could provide accurate stress state in the continuous media when blocks number is large.

Finally, the numerical simulation considers the heterogeneity of the rock mass by using different shape parameter m conforming to the Weibull distribution to define the joint strength of the rock mass. The results show that the heterogeneity of rock mass can strongly affect fracture processes. The number of the cracks increases as the shape parameter m decreases while the length of the cracks increased as the shape parameter m increases.

CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

This thesis aims to develop an efficient discontinuous deformation analysis program suitable to dynamic rock mass analysis. Comprehensive reviews on the basic concepts and theory of DDA and rock mass behavior under static/dynamic loading have been organized to support the feasibility of the DDA Program. Some modifications on a preliminary DDA code have been carried out to simulate the rock mass better. A stress recovery procedure has been developed for improving inter-block stress evaluation. Two numerical models have been simulated to study the rock mass response under the blast loads and fracture pattern analysis on scaled blast tests. The DDA program is testified to be an efficient numerical tool for modeling the rock mass response and its applications have been extended.

The main conclusions and contributions drawn from this thesis are summarized as follows:

- 1) A stress recovery procedure is developed to improve inter-block stress evaluation for the DDA analysis. The formulation of a minimization procedure for the calculation of inter-block forces and tractions as well as stress state at nodes as a post-processing procedure has been presented. With the new concept to partition the domain into blocks and conglomerate them together with a strong joint strength, it is able to use the DDA for continuous as well as discontinuous analyses. Numerical examples are presented to study the stress accuracy of the proposed method, and the results verify that the proposed recovery method can provide a more accurate stress state than the average method. It can be used to determine contacts more accurate in the future and thus constructs a good

foundation for future work on the fracture propagation analysis.

- 2) The response of rock mass subjected to underground blasting load is analyzed by the DDA program with a blast load transfer model implemented. Both the pressure wave propagation and deformation variations of the blocks are analyzed and interpreted. It shows that the DDA method can predict some valuable results and the blast wave propagation in the rock medium. The numerical results indicate that the response of rock mass due to blast loads has the behavior of $1/R$ attenuation. This extent is slower than that of the soil material. The strength of rock mass is attenuated due to the existence of weak strength of the joints when the high stress field has exceeded strength ability of the joints. In the region where stress state is low, the rock mass and joints will endure the outside effect together. The velocity and acceleration of the blocks are found to reach the maximum value when the block reaches its maximum pressure and attenuate very quickly. The final deformation of the rock mass surrounding the explosive is accordant well with the observations from the field experiment tests.
- 3) Another numerical simulation is performed by the developed DDA for studying the fracture processes on a laboratory scaled blast test. The analysis considers various applied pressure forms and material heterogeneity to investigate their influences on the dynamic fracture processes in the rock specimen. The fracture patterns under different conditions are analyzed and compared with the experiment findings. It shows that the fracture patterns are more affected by the rise time than the decay time. A higher stress-loading rate increased the number of radial cracks, but the rapid stress released from adjacent cracks affects crack extension and results in shorter cracks. At lower stress-loading rate, few cracks initiate but propagate longer. When the maximum pressure becomes large,

different boundary conditions will play an important role on the final fracture patterns for its reflection effect. The reflected *PP* pulse will create a tensile or compressive stress field due to the free or rigid boundary, which will increase the number of cracks or reduce the probability of blast hole cracking or, in some extent suppress blasthole breakdown. Wave propagation attenuation is also studied. Based on the concept of glutting blocks together as continua, the results of the attenuated stress state from the DDA agree well with the analytical solution. Finally, the numerical modeling investigates the heterogeneity of rock mass by a new proposed model which assigns joints different properties conforming to Weibull distribution. Different shape parameter m is used to define the tensile strength of the joint boundary. The results demonstrate that the heterogeneity of rock mass can strongly affect fracture processes. The number of the cracks increases as the shape parameter m decreases while the length of the cracks increases as the shape parameter m increases.

8.2 RECOMMENTATIONS

The attempt to use current DDA programs to model the rock mass response under the dynamic loads depicts that the DDA method is able to predict some valuable results on the rock mass response under dynamic loads and help us understand well the fracture propagation process in the rock medium. But still some deficiencies limit our application and efficiency on the practical problems. Based on the present study, further researches are recommended to study various aspects outlined below:

- 1) Due to the complexity of the blast reaction, its application as pressure time history acting on the blocks seems relatively simple in the current DDA. The explosion process should be generated in the DDA program. In the current model, the explosion load is treated as a triangular pulse based on experience or transferred from the output data of the other program such as AUTODYN. The former seems too simple while the second requires more works and may cause

errors because different treatments for the rock mass from continuous media to discontinuous. This should be investigated and extended in the future.

- 2) Chapter 5 introduces a stress recovery method for advancing the stress accuracy of the DDA results. In the current study, it can be only applied to improve the DDA's final output stress value. It is still not able to adjust the stress state during the DDA calculation process for the next time step. Studies on how to couple this method with the contact determination would be performed in the future. Based on this development, the analysis of rock mass fracture initiation and propagation will be more accurate.
- 3) The scaled blast test simulation is a good supplement for the field practice and theoretical investigation. In many cases, the gaps between the numerical simulation and field results cannot be easily bridged, the interpretation and transfer of the numerical simulations to the field results are not straightforward. In this study, only several influences are compared and analyzed, still many issues remain unsolved such as specimen size effect, more boundary conditions and etc. Therefore, to investigate the influence factors as many as possible will help us gain more knowledge and understanding from the numerical simulation and apply to the practical work.
- 4) So far, all the simulations are based on the 2-D model. As 3-D modeling would be better for explosion problems because it could consider the actual wave geometric spreading, the relation between 2-D and 3-D models should be investigated. If their correlation can be soundly estimated, the 2-D modeling results can be mapped to that of the actual 3-D dimension. This will avoid the gap of space effect and make 2-D modeling more useful.

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APPENDIX I: FLOWCHART OF THE STRESS RECOVERY PROCEDURE IN MATHEMATICA

APPENDIX II: SOME MODIFIED PROGRAM CODES

Bolting system

```

/*****
/* dfbolt: submatrix of bolting */
*****/
void dfBolt()
{
    for (i=1; i<= n4; i++)
    {
        i0 = (long)h[i][5];           //the first bolting point
        x  = h[i][1];
        y  = h[i][2];
        dspl();
        for (j=1; j<= 6; j++)
        {
            for (l=1; l<= 2; l++)
            {
                E1[j][l] = t[l][j];
            } /* l */
        } /* j */
        i0 = (long)h[i][6];           //the second bolting point
        x  = h[i][3];
        y  = h[i][4];
        dspl();
        for (j=1; j<= 6; j++)
        {
            for (l=1; l<= 2; l++)
            {
                E2[j][l] = t[l][j];
            } /* l */
        } /* j */
        double al,ax1,ay1;
        al = h[i][1]-h[i][3]*(h[i][1]-h[i][3])+(h[i][2]-h[i][4])*(h[i][2]-h[i][4]);
        al = sqrt(al);               // length of the bolting line
        ax1 = (h[i][1]-h[i][3])/al;
        ay1 = (h[i][2]-h[i][4])/al;

        for (j=1; j<= 6; j++)

```

APPENDIX II SOME MODIFIED PROGRAM CODES

```

{
    E1[j][1] = E1[j][1]*ax1+E1[j][2]*ay1;
} /* j */
for (j=1; j<= 6; j++)
{
    E2[j][1] = E2[j][1]*ax1+E2[j][2]*ay1;
} /* j */

long ii1,ii2,ii3,ii4;
ii1=(long)h[i][5]; // [kii]
ii2=k2[ii1];
if (pp==0) i3=n[ii2][1]+n[ii2][2]-1;
if (pp==1) i3=n[ii2][1];
for (j=1; j<= 6; j++)
{
    for (l=1; l<= 6; l++)
    {
        ji=6*(j-1)+1;
        a[i3][ji] += E1[j][1]*E1[l][1]*S0/al; //add to the [kii]
    } /* l */
} /* j */
ii3=(long)h[i][6]; // [kjj]
ii4=k2[ii3];
if (pp==0) i3=n[ii4][1]+n[ii4][2]-1;
if (pp==1) i3=n[ii4][1];
for (j=1; j<= 6; j++)
{
    for (l=1; l<= 6; l++)
    {
        ji=6*(j-1)+1;
        a[i3][ji] += E2[j][1]*E2[l][1]*S0/al; //add to the [kjj]
    } /* l */
} /* j */

    if (pp==0 && ii2<ii4) goto b101; //add to the [kij]
    if (pp==1 && ii2>ii4) goto b101;
    for (j=n[ii2][1]; j<= n[ii2][1]+n[ii2][2]-1; j++)
    {
        i3=j;
        if (k[j]==ii4) goto b102;
    } /* j */
    /*-----*/
b102::
    for (j=1; j<= 6; j++)

```

APPENDIX II SOME MODIFIED PROGRAM CODES

```

        {
            for (l=1; l<= 6; l++)
            {
                ji=6*(j-1)+1;
                a[i3][ji] += (-1)*E1[j][1]*E2[l][1]*S0/al;
            } /* l */
        } /* j */
        goto b104;
b101;;
        for (j=n[ii4][1]; j<= n[ii4][1]+n[ii4][2]-1; j++)
        {
            i3=j;
            if (k[j]==ii2) goto b103;
        } /* j */
        /*-----*/
b103;;
        for (j=1; j<= 6; j++)
        {
            for (l=1; l<= 6; l++)
            {
                ji=6*(j-1)+1;
                a[i3][ji] += (-1)*E2[j][1]*E1[l][1]*S0/al;
            } /* l */
        } /* j */
b104;;
    } /* i */
}

```