PUNCHING SHEAR STRENGTH OF SLAB-COLUMN CONNECTIONS IN FLAT PLATE SYSTEM

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2005

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A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Master of Engineering

2005

Statement of Originality

I hereby certify that the content of this thesis is the result of work done by me and has not been submitted for a higher degree to any other university or Institution.

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Date

ACKNOWLEDGEMENTS

I would like to express my utmost gratitude and appreciation to my supervisor, Assistant Professor Paulus Irawan, from the School of Civil and Environmental Engineering of Nanyang Technological University, for his interest, continuous support, intellectual and invaluable suggestions and encouragement throughout this study. His explanation, understanding and invaluable helps make me more eager to pursue my research work.

According to Myanmar traditional ideas, an excellent teacher should have good will, kindness, sacrificing attitude, fair to all students, like to teach and give guidance. Professor Paulus Irawan is such an excellent teacher. He helps me to master my research field. So, admiration and thanks to him are full in my heart. I am satisfied and feel honoured to obtain a chance of doing research under the supervision of such an excellent teacher.

I would like to thank my family, who has supported me whenever I need special food and medicine. And I also would like to thank my friends Naing Naing Maw, Naing Naing Aung, Sithu Htun, Khin Kyu Kyu Win, and Phyu Phyu Thant who take care of me when I am sick and make sure that I have a good health throughout the period of this research work.

I would like to extend my gratitude to Professor Er Meng Hwa - Deputy President 2 of Nanyang Technological University - and Minister U Thaung from Ministry of Science and Technology, Myanmar, Deputy Minister 2 U Nyi Hla Nge, and Dr. Htin Aung - Professor and head of Civil Engineering Department in Yangon Technological University -, for their support for me to study in Nanyang Technological University.

Lastly, I would also like to acknowledge the financial support that has been granted by Nanyang Technological University, Singapore.

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ABSTRACT

Nowadays the flat plate floor system, which is directly supported on the top of columns without drop panels and column capitals, becomes more popular and is used in apartment buildings, car park buildings and office buildings because of the benefits achieved by using this floor system.

However, the disadvantage of this system is the vulnerability of the punching shear failure around the region of column. This failure can occur when the strength around the region of column is not enough to resist the stress concentrated in that region. The high stresses can concentrate around the slab-column connections due to shear force, unbalanced moment, and combination of shear and unbalanced moment. Some of the sources of these forces are the very high gravity forces, unsymmetrical layout of structure, wind and earthquake loading. When the slabs have openings near the column, uneven shear stresses that cause the punching shear failure can exist in the vicinity of column.

Due to the fact that the region of opening is weak against the punching shear failure and can reduce the punching shear capacity of the slab-column connection, the reduced capacity due to opening needs to be recovered. Shear reinforcement can be used for this purpose. There are many types of shear reinforcements. The shear stud reinforcements (SSR) are commonly used because they can be easily installed.

In this study, in order to obtain better understanding on the punching shear capacity of this floor system, an experimental work involving 6 slab specimens were tested to investigate the behavior of slab-column connections with opening and shear reinforcement. The columns are rectangular shapes. The opening in three specimens is located at the corner of column while the opening in the other three specimens is located at the shorter side of column width. Except the two control specimens, all specimens have different amount and arrangement of shear stud reinforcement. The specimens were loaded to fail under monotonic uniformly distributed loading.

From test results, it can be seen that the punching shear failure is more vulnerable when the opening is located at the shorter side of column. The strength against the punching shear is different according to the location of opening around the column. The capacity regained by shear stud reinforcement is also different according to the amount and arrangement of shear studs. This experimental study shows the more effective arrangement of shear reinforcement.

As a theoretical research work, a model to predict the upper bound punching shear capacity of square and rectangular slab-column connections with or without opening and with or without shear stud reinforcement is proposed based on the theory of plasticity. First, punching shear failure criterion of concrete is established. Second, the collapse mechanism for different slab-column connections is presented. Third, the plastic strain along the punching crack is derived. Fourth, the dissipation energy in the punching shear failure surface is described. Fifth, by virtual work principle and variational calculus, the upper bound solution for punching shear capacity of slab-column connections is achieved.

To take into account the softening effect of concrete, an effectiveness factor on the compressive strength of concrete, which depends on slab thickness, aspect ratio of longer and shorter column widths and compressive strength of concrete, is proposed. In addition, an effectiveness factor on the tensile strength contributed by shear stud reinforcement is also recommended because all shear studs may not yield at the punching failure of slab.

The proposed plasticity model with the proposed effectiveness factors on the compressive strength and tensile strength contributed by SSR give the punching shear capacities which are in good agreement with test results for slab-column connections with square and rectangular columns, with or without opening and with or without shear stud reinforcement.

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LIST OF SYMBOLS

$c_{i1} \text{ \& } c_{i2}$	= incremental length of failure surface parallel to shorter column width and longer column width, respectively, in mm
c	= cohesion of concrete material
f_A	= separation resistance of material in MPa
f_{c1}	= principal tensile stress in MPa
f_{c2}	= principal compressive stress in MPa
f_c'	= cylinder compressive strength of concrete in MPa
f_t	= tensile strength of slab in MPa
f_{ct}	= tensile strength of concrete in MPa
f_{st}	= tensile strength of shear reinforcement in MPa
r	= radius of failure surface in mm
r_h	= radius of failure surface at the tension face of the slab in mm
r_0'	= $r'(0)$ = tangent value of failure surface at the compression face of the slab
r_h'	= $r'(h)$ = tangent value of failure surface at the tension face of the slab
r_{cr}	= critical radius of failure surface at the tension face of slab that gives the least punching shear capacity of slab in mm
$A, B \text{ \& } C$	= constants related to radius of failure surface
$A_{cr}, B_{cr} \text{ \& } C_{cr}$	= constants related to critical radius of failure surface
$\bar{A}_f$	= net area of failure surface of slab without including ineffective area of failure surface A_{op} due to opening in mm^2
A_f	= total area of failure surface of slab without opening after punching in mm^2
A_s	= total area of shear studs passing through the failure surface in mm^2
A_{op}	= ineffective area of failure surface due to opening in mm^2
$C_1 \text{ \& } C_2$	= shorter and longer column width, respectively, in mm
P	= punching shear capacity of slab-column connection without opening and without shear reinforcement in kN

P_C	= punching shear capacity for curved portion of failure surface from the depth h_0 to the bottom of slab in kN
P_L	= punching shear capacity for straight portion of failure surface from the top of slab to the depth h_0 in kN
P_{CS}	= punching shear capacity of slab-column connection without opening and with shear reinforcement for curve portion of failure surface in kN
P_{LS}	= punching shear capacity of slab-column connection without opening and with shear reinforcement for straight portion of failure surface in kN
	= punching shear capacity of slab-column connection with opening in kN
$\bar{P}_{op}$	= ineffective punching shear capacity of slab-column connection due to opening in kN
P_{op+S}	= punching shear capacity of slab-column connection with opening and with shear reinforcement in kN
P_S	= punching shear capacity of the slab-column connection without opening and with shear reinforcement in kN
$\bar{P}_S$	= punching shear capacity of slab-column connection without opening and with shear reinforcement in kN based on the ratio of shear stud reinforcements passing through the net area of failure surface $\bar{A}_f$ without including the ineffective area of failure surface A , due to opening
W_A	= dissipation energy per unit area of failure surface
W_V	= dissipation energy per one unit volume of yield region
τ	= shear stress in the sliding surface in MPa
σ	= normal stress in the separation surface in MPa
μ	= tangent value of internal friction angle of concrete material
φ	= angle of internal friction angle of concrete
a	= slope of failure surface
α_{avg}	= average slope of failure surface
ε_1 & ε_2	= principal positive and negative strain, respectively

$\varepsilon^+ \text{ \& } \varepsilon^-$	= summation of positive principal strain and summation of negative principal strain, respectively
ρ_s	= ratio of shear stud reinforcement passing through the total area of failure surface A_f
$\overline{\rho}_s$	= ratio of shear stud reinforcement passing through the net area of failure surface $\overline{A}_f$ of slab without including of ineffective area of failure surface A , due to opening
$\Delta_1 \text{ \& } \Delta_2$	= components of circumferential length of failure surface parallel to shorter and longer column width, respectively, in mm

CHAPTER 1

INTRODUCTION

Chapter 1

Introduction

1.1 Background

A flat plate floor is essentially a flat slab structure without drop panels and column capitals. The floor systems do not use beams or walls to support the slab, but instead of these, they are directly supported on the top of the columns only. This floor system becomes more popular and is nowadays used in the buildings such as apartment buildings, car park buildings and office buildings. By using this flat plate floor system, such structures can reduce the story heights; the formwork is simple because there are no beams; it is economical on formwork and labor; the flexibility of planning can be increased and the appearance is pleasant. Some types of slabs are shown in Figure 1.1.

Figure 1.1 Some types of slabs

(a) Flat plate

(b) Flat slab with drop panel

However, the undesirable thing of the flat plate structures is that it may fail with brittle punching shear failure mode. This is due to the fact that in flat plate structures, high stresses are concentrated at the area near the slab-column connection where the

column punches through the slab as shown in Figure 1.2. This failure can lead to a catastrophic damage without warning. The high shearing force can be produced by high gravity load. The unbalanced bending moment may exist due to unsymmetrical geometry layout of columns or lateral forces due to wind and earthquake loads. The high shear stress due to the combination of high shearing force and unbalanced bending moment can cause severe failure around the slab-column connection. Therefore, it must be ensured that there is enough strength at the slab-column connections, and that the column does not punch through the slab.

The punching shear failure surface observed in practice is curved that extends from the compression side of the slab at the face of the column to the tension side of the slab at some distance from the column. The slope of failure surface at the compression side of slab is usually between 25 and 30 degrees. The distance of failure surface from the column face at the tension side of slab is usually between $1.5d$ and $4d$, where d is the effective depth of slab.

Figure 1.2 Sketch of punching shear failure of a reinforced concrete slab

There are three types of slab-column connections: interior, edge and corner slab-column connections. In the interior slab-column connections, if the arrangement of columns is symmetrical and there is no lateral force, there will be no unbalanced bending moment. Thus, the typical punching shear failure as the truncated cone surrounding the column will occur due to high shear stress from the shearing force only. In the case of the edge and corner slab-column connections, the unbalanced bending moment will occur and the failure occurs at the side with the highest shear stress due to the combination of shearing force and unbalanced bending moment. The punching region is confined to the area near the heavily loaded face of column. The

plan views of three typical punching shear failures are shown in Figure 1.3. It is required that reasonable and reliable shear capacity of the slab-column connections be predicted accurately to avoid the brittle punching shear failure mode. Thus, to achieve the reasonable and reliable shear capacity of the slab-columns connections, there is a need to better understand the mechanism, behavior and failure mode occurring in the slab-column connections at ultimate load.

In flat plate structure, the holes or openings are often perforated in the vicinity of columns. Such openings are required for sanitary reasons, ventilation, heating, air-conditioning, and electrical ducts, etc. The vicinity of the column is a critical zone where the concentrated shear stresses are the largest. Therefore, location of opening in this vicinity of column decreases the shear capacity of the flat plate floor system. The use of shear reinforcement can recover the shear capacity of the slabs to some extent. Many researchers such as Di Stasio and Van Buren (1960), Moe (1961), Hognestad, Elstner, and Hanson (1964), Mowrer and Vanderbilt (1967), Regan (1974), Broms (1990), EI-Salakawy, Polak, and Soliman (1999), and Kuang (2002) have done the study on the effect of openings in decreasing the punching shear capacity of the interior slab-column connections and the effect of using various types of shear reinforcement in increasing the punching shear strength. However, the method that can accurately predict the punching shear capacity of slab-column connections with opening and shear stud reinforcement is rare.

Figure 1.3 Typical punching shear failures by Alexander and Simmonds (1987)

Among the various type of shear reinforcement, the shear studs become more popular due to easy installation and less congestion with flexural reinforcement. Accordingly, research works to better understand on the behavior of slab-column connections with

opening near column and shear stud reinforcement are needed and should be conducted. To fulfil such needs, a total of six interior slab-column specimens with rectangular column, with opening near column and with shear stud reinforcement in various amount and arrangements were tested to investigate the behavior of slab-column connections with opening and shear stud reinforcements.

There are many design codes to calculate the punching shear strength of slab-column connections with openings, ACI building code ACI 318-02, Australian Standard 3600-2001, British Standard BS8110-97, Canadian Standard A23.3-94, CEB/FIP model code 90, and Eurocode2 EC2, etc. Generally, these codes are experimental based and the mechanisms of forces to resist punching shear are approximated. These codes give the reduction of punching shear capacity of the slabs with openings in the vicinity of column. By simplifying the equations used in the design, the capacity of slab can be predicted to some degree, but the behavior of the slabs with openings is still needed to be studied for various locations of openings and for the use of various amounts and arrangement of shear stud reinforcement. Complex stress states occur in the vicinity of slab-column region, therefore, it is necessary to obtain better understanding the mechanism of slab-column connections, especially for slab with opening.

In addition, there are many analytical approaches to predict the punching shear capacity of slab-column connection which have been presented by many researchers in the literature; flexural capacity approach (Long, 1975), plasticity theory (Braestrup, 1976), mechanical model (Kinnunen and Nylander, 1960), truss model (Alexander and Simmonds, 1987), and analytical model (Menbtrey, 1996), etc. These models can predict the punching shear capacity of slab-column connections to some degree. However, the shape of failure surface considered in some approaches is not consistent with the experimental results. Among these models, the plasticity model can predict the upper bound punching shear capacity of slab-column connection with circular column and give the possible failure surface closely observed in the experiment. Plasticity model is based on the punching shear failure criteria assuming that the concrete material is rigid-plastic modified Coulomb's material. Equilibrium and strain compatibility conditions are also consistent in plasticity model.

A theoretical solution to predict the punching shear capacity of slabs by theory of plasticity was developed by Braestrup in 1976. The concrete was considered rigid-plastic material. Modified Mohr-Coulomb failure criterion for concrete in straight line envelop with slope of 37 degrees was established. In 1986, Jiang and Shen proposed a theoretical solution similar to Braestrup's plastic solution for the punching of concrete slab based on plasticity theory. The difference between Braestrup's and their plasticity models was that the modified Mohr-Coulomb failure criterion in second degree parabolic for concrete was assumed. Due to this assumption, their plasticity model can be used to predict the punching load of slab for any value of punching angle at the compression side of slab.

The important thing in the plasticity model that influences the accuracy of results is the consideration on effect of softening of concrete. As the development regarding to the softening of concrete in plasticity model, Bortolotti in 1990 presented the plasticity model for the punching shear strength of concrete slab by taking into account the softening process in tension and compression of concrete visualized by failure surface profile. Based on this visualization, the modifications were made in the form of modified Mohr-Coulomb failure criterion possible due to softening from tension and compression of concrete.

When the slab is laterally restrained, the punching shear capacity of slab can enhance due to compressive membrane stresses which are developed. In 1993, Kuang and Morley presented a plastic theoretical model for the punching shear failure of laterally restrained concrete slabs, in which a parabolic Mohr-Coulomb failure criterion for concrete was adopted and the effect of compressive membrane action due to lateral restraint was calculated by a membrane-modified flexural theory of elasto-plasticity developed by Braestrup (1980).

The plasticity models mentioned above are based on the shear and normal stress in the failure surface presented by the theoretically established failure criterion of concrete. In the crack surface, the shear stress and its accompanying compressive stress may be induced by the aggregate interlocking. The plasticity model for the punching shear strength of slab which was based on shear and compressive stress empirically

presented from aggregate interlocking in the surface of established punching shear crack was developed by Yankelevsky and Leibowitz in 1996.

All plasticity models mentioned above are able to predict the punching shear capacity for the slab with circular column simply supported or laterally restrained along the circumferential edge of the slab and can give reasonably good results compared to the experiments. However, the effect of flexural reinforcement is neglected and contribution of shear reinforcement in punching shear capacity can not be presented. When the slab has opening near the column, the punching shear capacity of slab may decrease due to opening. Due to opening, some parts of failure surface can be considered ineffective in resisting punching load. One of the methods to recover the punching shear capacity lost due to opening is the use of shear reinforcement. Plasticity model needs to be able to predict the punching shear capacity of slab-column connections with opening and shear reinforcement.

In this study, rigid-plastic theoretical model for the punching shear failure of simply supported slab-column connection with square or rectangular column, with or without opening near the column and with or without shear reinforcement under concentric punching load is proposed. By establishing the punching shear failure criteria with the assumption of rigid-plastic modified Coulomb's material as the plasticity model by Braestrup (1976), the plastic solutions are derived to predict the punching shear capacity of slab-column connection with square or rectangular column, with or without opening near the column and with or without shear reinforcement. The softening effect of concrete is also considered in plasticity model of this study. In the proposed plasticity model, recommendations are made on the effectiveness factor of concrete and effectiveness factor of tensile strength of shear stud reinforcement.

1.2 Objectives and scope

The first objective of this study is to investigate the behavior of slab-column connections with opening and shear stud reinforcement. To fulfill this objective, six interior rectangular slab-column specimens with openings and shear stud

reinforcement were tested. There were two types of location of opening and four different arrangements of shear stud reinforcement in the specimens.

The second objective is to propose a plasticity model extended from Braestrup's plasticity model (1976). By using the extended plasticity model, the punching shear capacity of slab with square and rectangular columns, with opening near the column and with shear reinforcement can be predicted. The punching shear capacities of interior slab-column specimens collected from the literatures were predicted by the extended plasticity model.

1.3 Organization

Chapter one introduces the background, objective and scope of this study. Chapter two reviews the previous research works on slab-column connections.

Chapter three consists an experimental program for interior rectangular slab-column connections with opening near column and with shear stud reinforcement.

In chapter four, discussions on the experimental results from the experimental program described in chapter three are made.

Chapter five presents the plasticity model to predict the punching shear capacity of slab-column connection under concentric loading.

In chapter six, the results and discussions are made based on the comparisons of the punching shear capacity of slab-column connection predicted by plasticity model and the experimental results.

In chapter seven, summary and conclusion of this study are presented and future research works in the plasticity model are suggested.

Finally, three appendices; A, B and C, are presented. Appendix A is for the data from experimental program described in chapter three. It presents the applied loads to the

slabs, deflections on the selected positions on the top and bottom of slabs measured by LVDTs (Linear variable transducer), and the strains occurred in the selected flexural reinforcements and all shear studs used in the slabs measured by strain gauges.

Appendix B is for the detail description of plasticity model proposed in chapter five. It presents the detailed derivation for punching shear capacity of slab-column connections under concentric punching load based on the theory of plasticity.

In appendix C, the calculation program prepared by Matlab software to predict the punching shear capacity of square and rectangular slab-column connections with openings and with shear studs by the plasticity model proposed in chapter five is presented.

CHAPTER 2

LITERATURE REVIEWS

Chapter 2

Literature reviews

2.1 Introduction

The flat plate buildings become more widely used and more popular because of their advantages as previously described in chapter one. The vulnerability of the failure mode is brittle punching shear failure in the vicinity of slab-column connections of these buildings. This type of failure mode does not usually give the warning. It leads to a catastrophic damage. Unbalanced moment cannot be avoided in exterior and corner slab-column connections due to unsymmetrical geometry. When lateral loads due to wind or earthquake are present, there will also be unbalanced moment in interior slab-column connections.

The adequate shear strength against both shear force and unbalanced moment is needed in slab-column connections in the flat plate structures. The shear reinforcement can increase the punching shear capacity of slab-column connections to some extent. There are many methods to predict the punching shear capacity of slab-column connections which have been presented in the literature.

This chapter reviews the methods from previous research works to predict the punching shear capacity of slab-column connections. They are (1) plasticity models, (2) other analytical models, (3) experimental works related to slabs with openings, (4) experimental works related to shear reinforcement, and (5) methods specified by different codes.

2.2 Plasticity models

Some previous research works based on the theory of plasticity to obtain the upper bound solutions of the punching shear capacity for slab in the literature are reviewed. Generally, in the beginning plasticity models primarily use the modified Mohr-

Coulomb failure criterion with straight line envelope (Braestrup, 1976). The concrete is assumed as perfectly plastic material. Later, modified Mohr-Coulomb failure criterion is modified to parabolic shaped failure criterion (Jiang and Shen, 1985). The collapse mechanism as shown in Figure 2.1 is considered that the portion confined by failure surface was punched out of slab and the remaining was rigid. Based on the related established failure criterion and collapse mechanism, the principal stresses and plastic strains on the yield surface are determined and so the dissipation energy is obtained. Finally, by using the virtual work principle and variational calculus, and the punching shear failure surface satisfies the most extreme condition, the upper bound punching shear capacity of slab under concentric punching load is achieved.

Figure 2.1 Collapse mechanism in punching shear failure of slab considered in plasticity models

2.2.1 Plasticity model by Braestrup (1976)

In 1976, Braestrup found a solution to predict the punching shear capacity of slab-column connections under concentric punching load by applying the theory of plasticity. The slab-column connection with circular column simply supported along the circular edge of slab was considered. A collapse mechanism due to punching failure was considered as shown in Figure 2.1. The effect of flexural reinforcement was neglected. The failure criterion including both the sliding and separation failure was established considering that concrete was the perfectly plastic material and shown in Figure 2.2. The failure criteria established by Braestrup was

Sliding failure:

$$|\tau| = c - \mu\sigma \quad (2.1)$$

Separation failure:

$$\sigma = f_A \quad (2.2)$$

where τ = shear stress in the sliding surface
 σ = normal stress in the separation surface
 c = cohesion of concrete
 μ = tangent value of internal friction angle for concrete
 f_A = separation resistance of concrete

**Figure 2.2 Modified Coulomb's failure criterion for concrete
(Braestrup, 1976)**

By applying the virtual work principle and calculus of variation, Braestrup derived the upper bound punching shear capacity of slab and its failure surface curvature in such a way that the external virtual work and the dissipation energy sustained in the failure surface considered were equalized and radius of failure surface satisfying the most extreme condition for punching shear capacity of slab was obtained. In this model, the slope of failure surface at the compression side of the slab was restricted not more than 37 degree of internal friction angle of concrete. When the slope of failure surface at the compression side of the slab was equal or greater than 37 degree, the shape of failure surface was assumed as cantenary curve. When the slope of failure surface at the compression side of the slab was less than 37 degree, the shape of failure surface was assumed as the combination of a straight line and a cantenary curve as shown in Figure 2.3.

For the slope of failure surface at the compression side of the slab was equal or greater than 37 degree, the punching shear capacity P was derived as

$$P = \frac{\pi}{2} f'_c L C h + L \left(\left(\frac{d_1}{2} \sqrt{\left(\frac{d_1}{2} \right)^2 - C^2} - A B \right) - M \left(\left(\frac{d_1}{2} \right)^2 - A^2 \right) \right) \quad (2.3)$$

For the slope of failure surface at the compression side of the slab was less than 37 degree, the punching shear capacity P was derived as

$$P = \frac{\pi}{2} f'_c \left(\left(\frac{(d \cos \varphi + h_o \sin \varphi)(1 - \sin \varphi)}{\cos^2 \varphi} \right) + \left[LC(h - h_o) + L \left(\frac{d_1}{2} \sqrt{\left(\frac{d_1}{2} \right)^2 - C^2} - AB \right) - M \left(\left(\frac{d_1}{2} \right)^2 - A^2 \right) \right] \right) \quad (2.4)$$

where f'_c = cylinder compressive strength of concrete

h = thickness of slab

h_o = vertical distance from the top of slab to intersection point of straight and curved failure surface shown in Figure 2.3

$A, B \& C$ = constants related to punching shear failure surface

d = punch diameter or diameter of column

d_1 = diameter of failure surface at the tension side of slab

$$L = 1 - 2 \frac{f_t}{f'_c} \frac{\sin \varphi}{1 - \sin \varphi} \quad (2.5)$$

$$M = 1 - 2 \frac{f_t}{f'_c} \frac{1}{1 - \sin \varphi} \quad (2.6)$$

φ = internal friction angle of concrete

f_t = tensile strength of concrete

**Figure 2.3 Failure surface of circular slab-column connection considered
(Braestrup, 1976)**

One of the most important factors on the plasticity model was the way to take into account the softening effect of concrete. The effectiveness factor v_c on the compressive strength of concrete was considered as the softening effect of concrete.

By applying this effectiveness factor, the plasticity model will give the optimum results on the punching shear capacity of simply supported slab-column connection with circular column under concentric loading with the tensile strength of concrete was considered zero. The effectiveness factor v_c was considered as

$$v_c = \frac{4.22}{\sqrt{f'_c}} \quad (2.7)$$

However, the optimum results on the punching shear capacity predicted by Breastup's plasticity model (1976) using the above effectiveness factor was still large scatter. Sigurdsson in 1991 modified the effectiveness factor on the compressive strength of concrete by taking into account the scale effect and different flexural reinforcement ratios in two perpendicular directions. The modified effectiveness factor proposed by Sigurdsson was

$$v_c = \frac{1.47}{\sqrt{f'_c}} \left(1 + \frac{0.48}{\sqrt{h}} \right) (1 + 0.125r) < 1 \quad (2.8)$$

where $r = \sqrt{r_x r_y}$ = flexural reinforcement ratio

r_x & r_y = flexural reinforcement ratio in x and y direction, respectively

h = slab thickness

2.2.2 Plasticity model by Jiang and Shen (1986)

In 1986, Jiang and Shen presented a solution for punching shear capacity of annularly supported concrete slab under circular punching load. In contrast to Braestrup's plasticity model, they considered that the punching shear failure might occur at the smaller punch angle or the initial slope of failure surface at the compression side of slab can be less than the internal friction angle of concrete. Therefore, they adopted a failure criterion for the concrete based on the second-degree parabola for the modified Coulomb's material as shown in Figure 2.4. By establishing the failure criterion based on second-degree parabola, the corresponding principal tensile and compressive stresses and related dissipation energy in the failure surface were determined. And, by applying virtual work principal and variational calculus, they derived the upper bound solution for the punching shear capacity of slab. The established failure criterion is:

$$\frac{\sigma_n}{f_t'} + \frac{1}{K} \left(\frac{\tau_{nt}}{f_c'} \right)^2 = 1 \tag{2.9}$$

$$K = \frac{1}{4} \left[m + 2(1 - \sqrt{m+1}) \right] \tag{2.10}$$

$$m = \frac{f_c'}{f_t'} = \frac{v_c f_c}{v_t f_t} \tag{2.11}$$

- where σ_n & τ_{nt} = normal and shear stress on the yield surface, respectively
- f_c' & f_t' = effective compressive and tensile strength of concrete, respectively
- f_c & f_t = compressive and tensile strength of concrete measured from standard tests, respectively
- v_c & v_t = factors on the compressive and tensile strength of concrete, respectively

And the upper bound solution of the punching shear capacity of slab derived as:

$$P = \pi f_t' \left(\frac{d_1^2}{4} - \frac{d^2}{4} + \frac{2Kh^2}{\ln d_1 - \ln d} \right) \tag{2.12}$$

- where d = punch diameter or diameter of column
- d_1 = diameter of failure surface at the tension side of slab
- h = slab thickness
- φ = internal friction angle of concrete

Figure 2.4 Parabolic Mohr failure criteria for concrete
(Jiang and Shen, 1986)

2.2.3 Plasticity model by Bortolotti (1990)

In 1990, Bortolotti proposed a theoretical solution for the punching shear strength of concrete slab by using theory of plasticity. The material was assumed rigid plastic. The Coulomb yield criterion for concrete was modified. The pure shear circle was considered as the intersection element between two failure modes: sliding and separation. In pure shear case, both sliding and separation might occur. In his pure tension and pure torsion tests, failure occurred by separation with shear failure surfaces normal to the first principal direction of stress. Based on this, he supposed that such failure involved at least all stress states. The separation failure condition was considered as $\sigma_1 = f_t$ where f_t was tensile strength of concrete, while beyond this circle and toward the compressions, the sliding failure was believed to occur. The modified Coulomb's yield criterion was expressed in normal and principal reference system as shown in Figure 2.5.

Figure 2.5 Modified failure criterion for concrete (Bortolotti, 1990)

$$|\tau| + \sigma \tan \bar{\varphi} - \frac{f_t}{\cos \bar{\varphi}} = 0 \quad (2.13)$$

$$\sigma_1(1 + \sin \bar{\varphi}) - \sigma_3(1 - \sin \bar{\varphi}) - 2f_t = 0 \quad (2.14)$$

with the limitation $\sigma_1 = f_t$ for $-f_t \leq \sigma_3 \leq f_t$ (2.15)

$$f_c = \frac{2.6}{1 - \sin \bar{\varphi}} \quad (2.16)$$

$$f_t = f_c \frac{1 - \sin \bar{\varphi}}{2} \quad (2.17)$$

$$K = \frac{f_t}{f_c} = \frac{2}{1 - \sin \bar{\varphi}} \quad (2.18)$$

where σ & τ = normal and shear stress, respectively

σ_1 & σ_3 = principal tensile and compressive stress, respectively

f_t & f_c = tensile and cubic compressive strength of concrete, respectively

$\bar{\varphi}$ = internal friction angle of concrete related to its compressive strength

He considered that the angle at each point of failure surface may be different from the internal friction angle $\bar{\varphi}$ because the plastic strains cause a softening effect in concrete. Due to this, the internal friction angle at each point of failure surface may change. Thus, he considered the effect of strain softening in compression and tension for the form of failure surface as shown in Figure 2.6. It could be seen that the instantaneous value of internal friction angle φ along the failure surface might be lower than $\bar{\varphi}$ due to softening in compression and larger than $\bar{\varphi}$ due to tension softening. Based on above modified Coulomb's yield criterion accompanied by pure shear Mohr's circle and the effect of strain softening in compression and tension, failure criteria due to softening from compression and tension were considered as shown in Figures 2.7a and 2.7b, respectively.

Figure 2.6 Proposed failure surface (Bortolotti, 1990)

If the stress value below the punch disk σ_p was less than f_c , softening in tension only was considered. The punching shear capacity of slab was derived as

$$P = \frac{\pi f_c}{2} \left(r_h^2 \sqrt{1 - \left(\frac{r_0}{r_h} \cos \bar{\varphi} \right)^2} - r_0^2 \sin \bar{\varphi} + r_0^2 \cos^2 \bar{\varphi} \ln \left(r_h \left(1 + \sqrt{1 - \left(\frac{r_0}{r_h} \cos \bar{\varphi} \right)^2} \right) / r_0 (1 + \sin \bar{\varphi}) - (r_h^2 - r_0^2) \right) \right) \quad (2.19)$$

If the stress value below the punch disk σ_p was greater than f_c , softening in both tension and compression was considered. The punching shear capacity of slab was obtained as

$$P = \frac{\pi f_c}{2} \left(r_h^2 \sqrt{1 - \left(\frac{r_0}{r_h} \right)^2} + r_0^2 \ln \left(\frac{r_h}{r_0} \left(1 + \sqrt{1 - \left(\frac{r_0}{r_h} \right)^2} \right) \right) \right) - \sin \bar{\varphi} \left(\bar{r}^2 \sqrt{1 - \left(\frac{r_0}{\bar{r}} \right)^2} + r_0^2 \ln \left(\frac{\bar{r}}{r_0} \left(1 + \sqrt{1 - \left(\frac{r_0}{\bar{r}} \right)^2} \right) \right) \right) - (r_h^2 - \bar{r}^2) \quad (2.20)$$

where $\sigma_p = \frac{k_m P_0}{\pi d_0^2 / 4}$ - stress below the punch disk

$P_0 = \pi h f_t (d_0 + h)$ = theoretical punch load

d_0 & r_0 = punch disk diameter and radius, respectively

k_m = corrective load factor by Bortolotti (1990)

$\bar{r}$ & r_h = radius of punch cone at depth $\bar{h}$ and h , respectively

h = slab thickness

(a) Failure criterion due to softening in compression

(b) Failure criterion due to softening in tension

Figure 2.7 Failure criterion (Bortolotti,1990)

2.2.4 Plasticity model by Kuang and Morley (1992)

In 1992, Kuang and Morley developed a plastically theoretical model for the punching shear failure of laterally restrained concrete slab. A parabolic Mohr failure criterion for concrete early shown in Figure 2.4 was used. The effect of compressive membrane forces was taken into account in their model by using a membrane-modified flexural theory of elasto-plasticity. The failure mechanism including the effect of compressive membrane action was considered as a function of $r = r(y)$ as shown in Figure 2.8. The relative displacement u was inclined at an angle β to the vertical. In the internal virtual work, the contribution for the effect of compressive membrane forces was superimposed. By the aid of virtual work principle and variational calculus, the punching shear capacity of laterally restrained slab was achieved.

Parabolic Mohr failure criterion:

$$\frac{\sigma}{f'_f} + \left(\frac{\tau}{c_k f_t} \right)^2 = 1 \quad (2.21)$$

where τ & σ = shear and normal stresses on an arbitrary plane, respectively

f_c & f_t = concrete uniaxial compressive and tensile strength, respectively

c_k = a parameter related to the ratio of compressive to tensile strength of

$$\text{concrete} = \sqrt{1 + \frac{f_c}{f_t}} - 1$$

External work,

$$W_E = P u \cos \beta \quad (2.22)$$

Internal virtual work,

$$W_A = \int_0^h D_A 2\pi r \sqrt{1 + (r')^2} dy + N_{rs} u \sin \beta \quad (2.23)$$

Then, the punching shear capacity of the slab was obtained as

$$\begin{aligned} P = 2\pi f_t \left(\frac{C_1^2}{2} (e^{2C_2 h} - 1) - \frac{2C_1}{C_2} (e^{C_2 h} - 1) \tan \beta + \frac{h}{C_2} \tan^2 \beta \right) \\ + \pi f_t \frac{c_k^2}{2} \left(\frac{h}{C_2} + \frac{2C_1}{C_2} (e^{C_2 h} - 1) \tan \beta + \frac{C_1^2 \tan^2 \beta}{2} (e^{2C_2 h} - 1) + \frac{\tan^4 \beta}{2C_2^2} \right) + N_{rs} \tan \beta \end{aligned} \quad (2.24)$$

where N_{rs} = radial compressive membrane force

C_1 & C_2 = constants

h = slab thickness

f_t = effective tensile strength of concrete

Figure 2.8 Failure mechanism of laterally restrained slab subjected to concentric punching load (Kuang and Morley, 1992)

The compressive membrane force N_{rs} was determined by mean of membrane-modified flexural theory of plasticity for compressive membrane action in circular reinforced concrete slabs, where an elasto-plastic analysis for arching action in restrained slabs had been developed by Braestrup and Morley in 1980. The elasto-plastic restrained slab was treated by the flow theory, lumping the support stiffness and in-plane slab stiffness into a single boundary spring, and assuming membrane action to start an initial elastic deformation. A clamped circular slab with depth h and radius a was considered. When the concentrated load P was applied to the center of slab, a conical mechanism was assumed to develop with central deflection w_0 , rotation θ , and middle surface extension Λ as shown in Figure 2.9. Strain rates and stresses of yielding slab section were considered as shown in Figure 2.10. By using geometrical condition of compatibility, its corresponding kinematic condition, flow rule, and considering the horizontal equilibrium of the slab, the radial compressive membrane forces were derived.

$$n_a = k \exp\left(\frac{w_0}{h\phi}\right) - \left(n_0 + \frac{1}{4}\phi\right) + \frac{1}{4} \frac{w_0}{h} \quad (2.25)$$

$$n_r = n_a + \frac{w_0}{2h} \left(1 - \frac{r}{a}\right) \quad (2.26)$$

$$N_{rs} = 2\pi f_c \int_0^h \left(n_a + \frac{1}{2} \frac{w_0}{h} \left(1 - \frac{r}{a}\right) r \right) dy \quad (2.27)$$

- where $k = \left(n_0 + \frac{1}{4} \phi - \frac{1}{4} \frac{w_i}{h} \right) \exp \left(\frac{w_i}{h \phi} \right)$
- w_i = initial deflection at which membrane action starts, taken empirically as $0.03 h$
- $n_0 = \frac{1}{h f_c} \left(\frac{1}{2} h f_c - A_{sh} f_y \right)$
- $\phi = \frac{a f_c}{2 h S}$ = flexible factor
- $\frac{1}{S} = \frac{a R}{0.8 E_c A_b + E_s A_{sb}} + \frac{a}{E_c h}$ = restraining stiffness
- R = radius of edge beam
- $E_c = 4730 \sqrt{f'_c}$ = elastic modulus of concrete
- f'_c = cylinder compressive strength of concrete
- $f_c = 0.85 f'_c$ = compressive strength of concrete
- E_s = elastic modulus of steel
- A_b & A_{sb} = the sectional areas of restraining ring beam and reinforcement, respectively
- A_{sh} = sectional area of steel reinforcement per unit length

(a) Conical failure mechanism

(b) Deformation of radial segment of slab

Figure 2.9 Conical failure mechanism for centrally loaded restrained circular slab
(Kuang and Morley, 1992)

**Figure 2.10 Strain rates and stresses of yielding slab section
(Kuang and Morley, 1992)**

Finally, the plasticity model by Kuang and Morley gave the punching shear capacity of laterally restrained reinforced slabs in good agreement with a wide range of experimental results when they use Equation 2.7 as the plastic effectiveness factor for compressive strength and the tensile strength of concrete is equal to 1% of effective concrete compressive strength.

2.2.5 Plasticity model by Yankelevsky and Leibowitz (1996)

In 1996, Yankelevsky and Leibowitz proposed a model that predicted (1) the punching shear capacity of simply supported circular slab-column connections, (2) shear and normal stresses along the punching cracked surface and their distribution, (3) variation of these stresses with the axial displacement, and (4) variation of resisting force as a function of the increasing vertical displacement of slab. This proposed model was based on the theory of plasticity and rigid post-fracture behavior, utilizing the post-fracture properties of concrete at rough crack interfaces that was developed.

They considered an axi-symmetric slab simply supported along its circumference with an axi-symmetric circular column which was subjected to an axial force. The final failure stage of the slab-column system was considered as the resistance at this stage was concentrated along the failure surface. The assumptions used in their model were:

- (1) A single cracked surface was already in the slab. The cracked surface was continuous and started along the column circumference. The surface geometry was presented by the function $r(y)$ as shown in Figure 2.11.

- (2) Concrete was ideally rigid and the cracked surface subdivided the slab into rigid bodies, the punched wedge and the outer slab.
- (3) The contribution of reinforcement was disregarded.
- (4) The deformation and the accompanied resistance were concentrated along the cracked surface.
- (5) The cracked failure was rough and a mechanism of aggregate interlock was developed.

(a) Kinematics of punched slab

(b) Shear and normal stress along the cracked surface

Figure 2.11 Final failure stage of punched slab

(Yankelevsky and Leibowitz, 1996)

Based on the test data which had been presented by Reinhardt and Walraven in 1981, these shear and compressive stresses along the cracked surface due to aggregate interlocking developed at the final failure stage and resisted the punching, were empirically expressed as

$$\tau = \frac{b}{1+2a} w^{-1.35} \Delta, \quad 0 \leq \tau \leq 0.5 \text{ N mm}^{-2} \quad (2.28a)$$

$$\tau = -a + b w^{-1.35} \Delta, \quad 0.5 \leq \tau \leq 10.0 \text{ N mm}^{-2} \quad (2.28b)$$

$$\sigma = -0.5 + \frac{A(\Delta + 0.05)}{(0.5 + A)w + 0.05B} \quad 0 \leq \sigma \leq 0.5 \text{ N mm}^{-2} \quad (2.29a)$$

$$\sigma = -A + B w^{-1.0} \Delta \quad 0.5 \leq \sigma \leq 10.0 \text{ N mm}^{-2} \quad (2.29b)$$

where σ = compressive stress along the cracked surface

τ = shear stress along the cracked surface

$$a = 0.5224 + 0.0260 f_c$$

$$b = 0.9424 + 0.0635 f_c$$

$$A = 0.969 + 0.054 f_c$$

$$B = 0.3786 f_c^{0.65}$$

f_c = cylinder compressive strength of concrete

Δ = crack shear displacement

w = crack width

In the above expressions, the crack shear displacement Δ was less than 2 mm and the crack width was narrower than 2 mm and wider than 0.03 mm. The punching shear capacity of slab was derived by summing up the contributions in resistance to punching of shear and compressive stresses along the cracked surface.

$$P = 2\pi \int (\tau - \sigma r') r(y) dy \quad (2.30)$$

where P = punching shear capacity of simply supported circular slab-column connections

r' = tangent value of radius of failure surface

2.2.6 Plasticity model by Salim and Sebastian (2002)

In 2002, Salim and Sebastian developed the way to predict the punching shear capacity of slabs without shear reinforcement and without in-plane restraint by employing the upper bound theory of plasticity. To take into account the variation in the angle of friction of concrete with related stress state, they adopted the parabolic Mohr failure criterion with the material being assumed rigid-perfectly plastic as previously shown in Figure 2.4 from plasticity model by Jiang and Shen (1986).

Based on the parabolic Mohr failure criterion and assumed failure mechanism, they

established dissipation energy sustained in the failure surface. By using virtual work principle and variational calculus, the upper bound solution of slab for punching load was obtained as:

$$P = \frac{\pi}{4} f_t \left(d_1^2 - d^2 + \frac{2 c_k^2 h^2}{\ln(d_1/d)} \right) \quad (2.31)$$

where c_k = a parameter related to the ratio of compressive to tensile strength of

$$\text{concrete} = \sqrt{1 + \frac{f_c}{f_t}} - 1$$

f_c = uniaxial compressive cylinder strength of concrete

f_t = uniaxial tensile cylinder strength of concrete

d = punch diameter or diameter of column

d_1 = diameter of failure surface at the tension side of slab

h = thickness of slab

Moreover, they showed that the Sigurdsson's effectiveness factor v_c as in Equation 2.8 for the softening effect on the compressive strength of concrete and the ratio of effective compressive strength and tensile of concrete taking equal to 400 as the effectiveness factor for the softening effect on the tensile strength of concrete gave the punching shear capacity of slab predicted by plasticity model in good agreement with the test.

2.2.7 Summary, comparison and discussion

In order to get better understanding regarding to the plasticity models described above, a comparison between the previous plasticity models is prepared in a table as shown in Table 2.1. And, the summary, comparison and discussion on these plasticity models are summarized.

For simply supported circular slab-column connections under concentric punching load, the upper bound punching shear capacity using theory of plasticity have been developed by Braestrup (1976). The concrete is assumed as perfectly plastic material.

And then, the punching shear failure is considered that it is a curve shape failure surface and a combined failure mode of sliding and separation failure based on the observation from the experiments. In order to derive the plastic strains at yield surface, Mob-Coulomb failure criterion with straight line envelope is established. Then, a reasonable collapse mechanism generated due to the concentric load during punching shear failure is assumed that the portion confined by failure surface is punched out of slab and the remaining is rigid. Based on the related established failure criterion and assumed collapse mechanism, the principal stresses and plastic strains on the yield surface are determined and so the dissipation energy is derived. In this case, the plastic strains are limited at the apex on the yield surface so that both sliding and separation failure are included. Finally, by using the virtual work principle and variational calculus, and the punching shear failure surface satisfies the most extreme condition, the upper bound punching shear capacity of slab under concentric punching load is achieved.

However, the Braestrup's plasticity model was valid to predict the punching shear capacity of slab only when the slope of failure surface at the compression face was equal or greater than 37 degree. Due to this, Mohr-Coulomb failure criterion was modified to parabolic shaped failure criterion by Jiang and Shen (1986). The benefit obtained from the modification of parabolic shape failure criterion by Jiang and Shen is that the plasticity model could predict the punching shear capacity of slab for such case that the initial slope of failure surface at the compression face was less than 37 degree.

Furthermore, due to the fact that the accuracy of prediction on the punching shear capacity by plasticity model depend on the consideration of softening effect which is taken into account on the compressive and tensile strength of concrete. Accordingly, based on the consideration that the failure surface generatrix can visualize the softening effect of concrete as shown in Figure 2.6, the modified Mohr's Coulomb failure criterion was modified by taking into account the softening effect on the failure criterion using the pure shear case, which can hold sliding failure as well as separation based on the experiment, by Bortolotti (1990). In his modification to the failure criteria, he used the instantaneous tensile strength, which was the decreased tensile strength due to softening effect of concrete, and the instantaneous compressive

strength, which was the decreased compressive strength due to softening effect of concrete. The instantaneous angle of internal friction may increase from that equal to the internal friction angle of concrete to 90 degree as the tensile strength decreases due to the softening of concrete while the instantaneous angle of internal friction may decrease from that equal to the internal friction angle of concrete to zero as the compressive strength decreases due to softening effect of concrete. His established failure criterion including softening effect of concrete can be seen in Figures 2.7a and 2.7b. These figures are combined into the same figure as shown in Table 2.1 so that it can be seen clearly the softening effect of concrete on the compressive and tensile strength.

Due to the fact that when the slab is laterally restrained, the punching shear capacity of the slab can be enhanced by the membrane forces produced by restrained action, a plastic theoretical model for the punching shear failure of laterally restrained concrete slab was developed by Kuang and Morley (1992). A parabolic Mohr failure criterion for concrete shown in Figure 2.4 was used. The effect of compressive membrane forces was taken into account in their model by using a membrane-modified flexural theory of elasto-plasticity. The effectiveness factors used in their model for the softening effect of concrete on the compressive and tensile strength of concrete can be seen in Table 2.1.

In the previous models, the principal tensile and compressive stresses on the yield surface are derived by mean of the plastic strain based on the established failure criterion. However, these models did not consider the aggregate interlocking in the crack interface. Therefore, in 1996, Yankelevsky and Leibowitz proposed a plasticity model to predict the punching shear capacity of the slab based on consideration that the shear and compressive stresses along the cracked surface due to aggregate interlocking developed at the final failure stage and these stress resisted the punching. This proposed model was based on the rigid post-fracture behavior, utilizing the post-fracture properties of concrete at rough crack interfaces that was developed. Based on the test data which had been presented by Reinhardt and Walraven in 1981, the shear and normal stress were empirically derived. However, in their model, the softening effect of concrete is not considered.

In 2002, Salim and Sebastian developed a plasticity model to predict the punching shear capacity of slabs without shear reinforcement and without in-plane restraint. To take into account the variation in the angle of friction of concrete with related stress state, they adopted the parabolic Mohr failure criterion with the material being assumed rigid-perfectly plastic. Based on the parabolic Mohr failure criterion and assumed failure mechanism, they established dissipation energy sustained in the failure surface. By using virtual work principle and variational calculus, the upper bound solution of slab for punching load was obtained. Moreover, they showed that the Sigurdsson's effectiveness factor v_c as in Equation 2.8 for the softening effect on the compressive strength of concrete and the ratio of effective compressive strength and tensile of concrete taken equal to 400 as the effectiveness factor for the softening effect on the tensile strength of concrete gave the punching shear capacity of slab predicted by plasticity model in good agreement with the test.

However, all plasticity models previously developed could cover only to predict the punching shear strength of slab-column connections with circular column shape. In addition, they did not consider the punching shear strength of slab-column connections when the slab has rectangular column shape, opening and shear reinforcement. Thus, there are needs to propose the plasticity model to be able to predict the punching shear capacity of slab-column connections when the slab has square or rectangular column, opening, and shear reinforcement.

Table 2.1 Summary and comparison for previously developed plasticity models

Author	Braestrup (1976)	Jiang & Shen (1986)	Bortolotti (1990)	Kuang & Morley (1992)	Yankelovsky & Leibowitz (1996)	Salim & Sebastain (2002)
Type of slab-column connection	Simply supported circular slab-column connection					
	Laterally restrained circular slab-column connection					
Material & Collapse mechanism	Simply supported circular slab-column connection					
	Laterally restrained circular slab-column connection					
Established failure criterion	Modified Mohr's coulomb failure criterion	Parabolic Mohr's failure criterion	Modified Mohr's Coulomb failure criterion with pure shear circle	Parabolic Mohr's failure criterion	Rigid post-fracture behavior	Parabolic Mohr's failure criterion

	$ \tau = c - \mu\sigma$ Sliding failure $f_A = f_t$ Separation failure	$M(\sigma_n, \tau_m)$ α ϕ u σ_n τ_m f_c' f_t	σ_c σ_t σ τ $\phi < \phi_0$ $\phi = 0$ $\phi > \phi_0$	$M(\sigma_n, \tau_m)$ α ϕ u σ_n τ_m f_c' f_t	τ and σ are from Eq. 2.28 and 2.29, respectively.	$M(\sigma_n, \tau_m)$ α ϕ u σ_n τ_m f_c' f_t
	$4.22 \frac{f_c'}{\sqrt{f_c'}}$	$0.35 f_c'$	A_1 A_2 A_3 r_0 r_h $\phi < \phi_0$ $\phi = \phi_0$ $\phi > \phi_0$	$4.22 \frac{0.85 f_c'}{\sqrt{0.85 f_c'}}$	-	Sigurdsson's effectiveness factor (Eq. 2.8)
Consideration for softening effect of concrete	0	$6.7 \% \text{ of } f_{et}$				
f_{ec}	0	$6.7 \% \text{ of } f_{et}$				
f_{et}	0	$6.7 \% \text{ of } f_{et}$				

f_{ec} = effective compressive strength of concrete
 f_{et} = effective tensile strength of concrete

2.3 Other analytical models

2.3.1 Mechanical model by Kinnunen and Nylander (1960)

In 1960, Kinnunen and Nylander proposed a mechanical model to explain the punching shear phenomenon and to predict the failure load. A polar symmetrical slab supported on a column was used. Shear reinforcements were not included. In this model the assumptions were (1) the portion of slab outside of the failure shear cracks rotated as a rigid body; (2) the slab portion outside shear cracks was carried by a compressed conical shell which developed from the column to the root of the shear cracks; and (3) at the time that tangential concrete strains at the bottom surface of the slab reached a critical value, the failure was assumed to occur.

The equations to calculate the punching shear strength of slabs were derived by considering static vertical equilibrium and moment equilibrium on the forces acting on a segmental part of the slab, which was outside of failure shear cracks and between the radial cracks. The forces acting on the segmental part of the slab were shown in Figure 2.12.

2.3.2 Truss model by Alexander and Simmonds (1987)

In 1987, Alexander and Simmonds proposed the three-dimensional truss model to determine the ultimate strength of slab-column connections without shear reinforcement as shown in Figure 2.13. In this model, the flows of forces within the connection that is consistent with known cracking are described and the contribution of flexural steel to punching strength is accounted. The reliability of this model was evaluated by predicting the ultimate capacities and failure modes of 43 slab-column connections without shear reinforcement reported in the literature. This model can predict the ultimate capacity of slab-column connections under combination of shear and moment transferred.

Figure 2.12 Mechanical model (Kinnunen and Nylander, 1960)

Figure 2.13 Truss model for interior slab-column connection
(Alexander and Simmonds, 1987)

By using this model, the shear moment interaction diagram is drawn to calculate the shearing capacity of the slab connection. On the shear moment interaction diagram, the point corresponding to the observed value of shear and moment at failure is plotted and connected to the origin by the straight line. The predicted failure values of shear and moment are obtained by taking the intersection point where this line meets with the interaction diagram.

2.3.3 Mechanical model by Mentrety (1996)

In 1996, Mentrety proposed a mechanical model to calculate the punching strength of the reinforced concrete slabs. The numerical simulations were based on finite element method. The results obtained from numerical simulation of the failure mechanism indicate that the punching strength can be computed by integrating the vertical tensile stresses around the punching crack shown in Figure 2.14. The vertical tensile stresses are assumed to be constant. The punching crack is assumed to be truncated cone in shape. His model takes into account the geometry of slab and support, concrete strength, percentage of flexural reinforcement, size effect, radius of punching crack initiation, contribution from the dowel effect, and the influence of stirrups, shear heads, and prestressing tendons. The punching strength of a general slab is expressed as summation of all the vertical tensile strength components crossing the punching cracks as follows:

$$V_{pun} = V_c + V_{dow} + V_{sti} + V_{prt} \quad (2.32)$$

where V_{pun} = punching strength of a general slab

V_c = concrete strength

V_{dow} = resistance dowel force due to the horizontal reinforcement

V_{prt} = vertical projection of the tendon's strength

The dowel force of the horizontal reinforcement is derived as the following,

$$V_{dow} = \frac{1}{2} \sum^{bars} \phi_s^2 \sqrt{f_c f_s (1 - \xi^2)} \sin \alpha \quad (2.33)$$

where ϕ_s = diameter of corresponding bars

f_c = concrete cylinder compressive strength

f_s = steel yield strength

- ξ = σ_s/f_s
 σ_s = axial tensile stress in the reinforcing bars
 α = inclination of punching crack with horizontal

Figure 2.14 Components of forces contributing to the punching strength (Menétrey ,1996)

The effect of shear reinforcement contributed by stirrups and shear heads is calculated by the following equation,

$$V_{sti} = \frac{1}{2} \sum^{stirrups} A_{sw} f_s \sin \beta_s \tag{2.34}$$

- where A_{sw} = cross-sectional area of one stirrup
 β_s = inclination of the stirrups with the plane of the slab

The vertical projection of the tendon strength is expressed as,

$$V_{prt} = \sum^{tendons} A_p f_p \sin \beta_p \tag{2.35}$$

- where A_p = area of the prestressing tendons
 f_p = tensile strength of prestressing steel
 β_p = inclination of the tendons at the intersection of the punching crack with the plane of the slab

This model shows that the predictions give great consistency with the experimental results. However, the assumption that the vertical tensile stress is uniformly distributed on the failure surface causes doubts because the inclination of the punching crack is curved.

2.4 Previous experimental works related to slabs with openings

2.4.1 Study by Di Stasio and Buren (1960)

In 1960, Di Stasio and Van Buren presented a method to calculate the maximum unit shearing stresses, measuring both diagonal tension and punching shear, due to combined gravity load and bending in flat plate floor slabs about interior columns. First, they considered that the critical section for diagonal tension was along the perimeter at a distance $(t/4)$ inch from the column face, where t is slab thickness. Second, at the critical section for diagonal tension, the unbalanced moment, eccentric vertical load and torsional moment were considered to contribute the total moment at the slab-column connection. The loading condition, critical perimeter and distribution of shear stresses for interior slab-column connections are shown in Figure 2.15. Finally, by taking equilibrium of moments at the slab-column connection, the maximum shear stress at the extreme fiber of the critical section was obtained as follows.

$$v_2 = \frac{8t}{7d} \left(\frac{V_u}{A} + \frac{(M_u - (M_1 + m) - (M_2 - m))}{J_c} x_2 \right) \quad (2.36)$$

When the unit shear stresses have the low permissible values in flat plate construction, considerable additional resistance can be developed through the dowel action of reinforcing bars crossing the section, and it may be reasonable to increase the assumed area of critical section and property of assumed critical section by the factor $1 + (n - 1)\rho$. On this basis,

$$A = A_c [1 + (n - 1)\rho] \quad (2.37a)$$

$$J = J_c [1 + (n - 1)\rho] \quad (2.37b)$$

where A = effective area of peripheral section

A_c = assumed area of critical section

J = property of assumed critical section analogous to polar moment of inertia

$M, M_1, \& M_2$ = maximum working stress design resisting moments over the width

M_u = unbalanced moment at center of column

- V_u = total shear force around the critical section
- d = effective depth of slab
- e = distance of centroid of critical section from the center of column
- m = residual balanced floor moments on width of critical section
- n = modular ratio
- x_1 & x_2 = distances from centroid of critical section to extreme fibers
- ρ = percentage of total (top and bottom) slab steel crossing the perimeter of the peripheral section

Figure 2.15 Interior slab-column connection (Di Stasio and Van Buren, 1960)

Furthermore, they also made a provision for the effect of openings in the vicinity of column. They provided two cases. First, any opening or series of openings of total width Σw at the distance x (less than $\Sigma w/2$) outside an edge of the critical section, should be considered to reduce the effective length of that edge by an amount $O_1 = \Sigma w - 2x$. Second, for opening extending inside the critical section, any point on the periphery at $t - 1\frac{1}{2}$ in from the column face, which can not be connected by a straight line not crossing the opening with a point on the long axis of the column within a distance $(C_2 - C_1)/2$ from the column centroid, where C_1 and C_2 , and t are shorter and longer column width, and slab thickness, respectively, should be excluded from the effective critical shear section. It can be seen in Figure 2.16.

Figure 2.16 Opening in slab adjacent to column (Di Stasio and Buren, 1960)

2.4.2 Study by Moe (1961)

In 1961, Moe reported an investigation of shearing strength in slabs and used 15 specimens to study the effect of openings. One specimen was without opening. The remaining specimens were with openings of different pattern adjacent to the column. Pattern of holes in test specimens and details of test specimen are shown in Figure 2.17. All specimens had 10x10x10 in column stubs and 6 in thickness. The specimens were supported along four edges by a steel frame. The concentrated load was applied to the specimens from the top of column stubs. The main variables were number, size, and position of holes in the slabs. The equation of ultimate shear strength for slabs was developed as

$$\frac{v}{\sqrt{f'_c}} = \frac{V}{bd\sqrt{f'_c}} = \frac{15\left(1 - 0.075\frac{r}{d}\right)}{1 + 5.25\frac{bd\sqrt{f'_c}}{V_{flex}}} \quad (2.38)$$

where b = width of critical section in shear

- (1) For square openings adjacent to the column, b was taken as total periphery minus the sum of the width of the holes, measured along the periphery.
- (2) For circular openings adjacent to the column, b was measured along the lines connecting the comers of the column to the periphery of the openings.
- (3) When openings were placed at some distance from the column, b was measured along the straight lines from the comers of the

column stubs to the nearest points on the periphery of the openings.

- d = effective depth of slabs
- f_c' = concrete cylinder strength in compression
- r = side length of loaded area of square shape
- V_{flex} = shear force at which flexure failure takes place

(a) Pattern of holes in test specimens

(b) Detail of test specimen

Figure 2.17 Interior slab-column specimen with openings (Moe, 1961)

The following facts were also indicated from his study,

- (1) The critical section to predict the ultimate strength of slabs should be the perimeter of loaded area. If the slabs have openings, the critical section should be based on the net width of perimeter of loaded area as above specification of b.
- (2) The shear strength of slabs depends to some extent on the flexural strength of the slabs.
- (3) The tri-axial state of stress in the compression zone at the critical section greatly influences the shear strength of that section.

- (4) The unit shearing stress is highest when the column size is small compared to the slab thickness.
- (5) Although the concentration of the bending reinforcement across the column increases the flexural rigidity of slabs, it was not effective on increasing the shearing strength.

2.4.3 Study by Hognestad, Elstner, and Hanson (1964)

In 1964, Hognestad, Elstner, and Hanson tested nine slab specimens to investigate the shear strength of slabs with lightweight concrete. Three slab specimens were made with normal weight concrete similar to Moe's specimens (1961) and six were made with two different lightweight concretes. Two types of square and rectangular columns were used. Only one square slab had four numbers of 5 in square openings adjacent to the column. The sizes of square and rectangular columns were 5x5 in and 6x8 in, respectively. All slabs were 6 ft square and 6 in thickness and supported along four edges with comers free to lift. The load was applied through the column stubs. They modified the equation of shear stress by Moe (1961) for rectangular column shapes as the following and also recommended to be calculated for each side of column.

$$v_u = \frac{V_u}{bd} = \left(15 \left(1 - 0.075 \frac{r_1^2 + r_2^2}{(r_1 + r_2)} \right) - 5.25 \phi_0 \right) \sqrt{f'_c} \quad (2.39a)$$

- where V_u = ultimate shear force
 b = peripheral of critical shear section in slabs
 d = effective depth to tension reinforcement
 r_1, r_2 = longest and shortest size of the column
 ϕ_0 = ratio of V_{test} or V_u to V_{flex}
 V_{flex} = computed ultimate load controlled by flexure

They concluded that the Moe's equation (1961) could not give reliable prediction of the shear strength of reinforced concrete slabs with lightweight concrete by using concrete cylinder compressive strength. By using splitting tensile strength, assuming $f_{sp} = 6.7\sqrt{f'_c}$, in Moe's equation, the reliable prediction was obtained as

$$v_u = \frac{V_u}{bd} = \frac{2.24 \left(1 - 0.075 \frac{r}{d} \right) f_{sp}}{1 + \left(0.784 \frac{f_{sp}}{V_{flex}} \right)} \quad (2.39b)$$

in which for rectangular column sections, the term r/d should be change with $\frac{r_1^2 + r_2^2}{(r_1 + r_2)d}$.

2.4.4 Study by Mowrer and Vanderbilt (1967)

In 1967, Mowrer and Vanderbilt tested 51 reinforced concrete interior slab-column specimens to provide more information on the shear strength of lightweight plates. Test specimens were divided into two series. In first series, 25 specimens were tested. The following variables were considered; the holes pattern around the column, amount of reinforcement, and concrete strength. The 17 specimens were for lightweight and 8 were for normal weight. The specimens were 36 in square, 3 in thickness, and had a 6 in square column stubs. In this series, all specimens were simply supported. The loads were applied through the column stubs. In the second series, 26 lightweight specimens were tested and the variables considered were the ratio of column width to effective depth, r/d , the amount of reinforcement and the edge fixity. The specimens were 48 in square and had a 5 in column stubs. Nine specimens were tested with two or four edge fixed while 17 were tested with simply supported edge with comers free to lift. Loads were applied through the column stubs.

They modified the Moe's equation (1961) for shear strength to give better prediction of results as follows,

$$\frac{V}{bd\sqrt{f'_c}} = \frac{9.7 \left(1 + \frac{d}{r} \right)}{1 + 5.25 \frac{bd\sqrt{f'_c}}{V_{flex}}} \quad (2.40)$$

They also suggested that the use of compressive strength gave better agreement between the computed and measured strength than the splitting strength, f_{sp} .

2.4.5 Study by Ivy, Ivey and Buth (1969)

In 1969, Ivy, Ivey and Buth tested 14 specimens of full-scale lightweight concrete slabs. The variables were slab size, slab geometry and aggregate type. Ten of these were similar to Hognestad's slabs H_1 and H_6 slabs (1964) and had 6 ft square and 6 in thickness with 10 in square column stubs. The H_6 slabs had 5x5 in openings on each side of the column. The other four slabs were larger scale and simply supported at the boundaries and had 24 in square column and 7 in thickness. Two slabs had 6 ft square and 10 ft square boundaries. The other two slabs were 10 ft square and had octagonal boundaries.

The Moe's equation and Hognestad's modified equation were used and it was found that the test data showed good prediction. Regarding the openings, they found that the ratio of V_{test} to V_{calc} were higher for slabs with openings than that without openings because of the stress concentration at the corners of the column stubs and the nominal ultimate stress tended to increase as the perimeter of the critical shear area decreased.

2.4.6 Study by Regan (1974)

Because the opening reduces the concrete areas against punching shear stress, the shear stress at the critical shear perimeter at a distance from the column is critically higher. Therefore, the reduced critical shear area due to opening needs to be used in checking the average shear stress at the limited critical shear parameter around the column. In 1974, Regan proposed two methods to reduce the critical shear perimeter at a distance of $1.5h$ from the column face when the slab had opening near the column, where h was slab thickness. By these methods, the critical shear perimeter of slab-column connections was reduced to take into account the effect of opening in determining the average shear stress at the critical section. The proposed methods are explained by Figure 2.18.

**Figure 2.18 Projection of width of openings on to the critical perimeter
(Regan, 1974)**

Method 1 projects the widths of opening on the critical shear perimeter, two parallel projection lines are used and pass through two corners of opening, which can be the largest width in reducing the critical shear perimeter, while in method 2, two projection lines propagate from the center of column through two corners of opening, which can be the largest width in reducing the critical shear perimeter.

He reported that while method 1 was more realistic but did not produce safe predictions when compared to the test results from the literature, methods 2 was over conservative. He also suggested that openings near edge columns should be avoided, but if they could not be avoided it might be possible to replace the lost area of concrete by shear reinforcement placed at sides of the openings.

2.4.6 Study by Broms (1990)

In 1990, Broms investigated different types and arrangements of shear reinforcement at interior columns with symmetrical loadings. Eight specimens were tested. All slabs were 2600 mm square, 180 mm thickness and simply supported at eight points as shown in Figure 2.19a. Although all specimens were of the same dimensions, different reinforcement arrangements were used. Only one specimen had openings around the column as shown in Figure 2.19b. One specimen had no shear reinforcement. Three specimens had multiple U-stirrups as shear reinforcement. Two

specimens and one specimen with openings were provided a combination of bent bars and multiple U-stirrups as shear reinforcement. The load was applied from below of the column stubs by means of hydraulic jack.

Figure 2.19 Test specimen (Broms, 1990)

He found that (1) openings in the slab inside the indicated critical perimeter as shown in Figure 2.19b affected neither the strength nor the stiffness of the slab, provided that there was space enough for the bent bars to pass within the column width, (2) ordinary shear reinforcements in the form of stirrups enclosing only the flexural reinforcement were not effective enough to give the desired ductility, and (3) a combination of bent bars and multiple U-stirrups enclosing the bottom compression reinforcement gave the specimens excellent performance.

And he recommended about the bent bars that (1) the bent bars should be distributed within the column width, (2) the inner bent of bars should be placed at the column face, (3) the inner bent bars should be given a moderate inclination so that the outer bent bars are placed at an average distance of $2d$ from the column face, where d is the effective depth of the slab, and (4) in order to give the full development length, the bent bars should extend at least the distance $0.25 l_n$ from the face of column, where l_n is clear span measured centre-to-centre of supports.

2.4.7 Study by El-Salakawy, Polak, and Soliman (1999)

In 1999, El-Salakawy, Polak, and Soliman studied the effect of openings on the punching shear behavior of edge slab-column connections. They tested six full-scale reinforced concrete flat plate edge connections with an opening in the vicinity of column. The main parameters of these tests were the locations and the sizes of openings. One specimen had no opening and the remaining five had various arrangements for openings around the column. The openings in the specimens were square, with sides parallel to the sides of the column. All specimens were made with normal density concrete with approximately compressive strength of 32 MPa. Two sizes of openings were used. One was the same size as column. The other was 60% of the column size. The specimens were subjected to the vertical load at the end of upper column stubs and the unbalanced moment by applying the two equal and opposite horizontal forces H near the column ends. The specimens were simply supported along the three edges. The details of the specimen are shown in Figure 2.20.

Figure 2.20 Test specimen (Salakawy, Polak, and Soliman, 1999)

From the tests results, they gave the following conclusions:

- (a) The opening located at the front of the column decreased the shear capacity of the connection more than the same size of opening located at the side of column.
- (b) The distance of openings from the face of column influenced the capacity of connection. The closer the distance of opening from the face of column, the smaller the strength is.

- (c) The size of openings was significant in the reduction of stiffness. The smaller size of openings will give much smaller reduction in stiffness than the larger one.
- (d) The CSA A233-M94 and ACI 318-95 gave more conservative strength prediction when the slabs had a larger opening.

2.4.8 Study by Kuang (2001)

In 2001, Kuang investigated the punching shear behavior of flat plate with opening. A total of 20 full-scale reinforced concrete slabs were tested. Four slab specimens had square columns and the remaining had rectangular columns. The main parameters of tests were different locations of opening adjacent to the column, column rectangularity, and different loading ratio. No shear reinforcements were included in all tested specimens.

Based on these parameters considered on the tested slab specimens, the slab specimens were grouped into 5 series each of which had one related control slab specimen. Series 1 was representative for square slab-column connections while series 2 and 3 were for rectangular slab-column connections with ratios of longer to shorter column width; 3 and 5, respectively. Series 4 and 5 were representative for rectangular slab-column connections with the same ratio of longer to shorter column width of 3 and with different loading ratios of x-direction to y-direction; 1.6 and 0.63, respectively, where x- and y-direction were parallel to longer and shorter column width, respectively.

There were five kinds of opening locations in the slab specimens. They were (1) opening location adjacent to the corner of column, (2) opening location parallel and adjacent to the shorter side of column, (3) opening location perpendicular and adjacent to shorter side of column, (4) opening location parallel and adjacent to middle of longer side of column and (5) opening location perpendicular and adjacent to the end of longer column side. The dimensions of opening were 200mm x 400mm. The opening locations included in his slab specimens can be seen in Figure 2.21.

The important conclusions that can be made based on the observation of his test results are as follows.

- (1) When the openings are located at adjacent to the column in the slabs, the punching shear capacity of such slabs with opening can decrease due to opening.
- (2) Among all the slabs with opening, the decrease in the ultimate punching shear capacity due to opening for the slabs with opening located adjacent to the middle of longer column side was the least as compared to other slabs with opening located elsewhere adjacent to the column.
- (3) However, the location of opening parallel and adjacent to shorter side of column had the most serious effect in reducing the punching shear capacity of slab as compared to the other slabs with opening located elsewhere adjacent to the column.
- (4) The punching shear capacity of slabs with rectangular column is more than that of slab with square column.
- (5) The punching shear capacity of the slab with rectangular column having ratio 3 of longer to shorter column width increase about 34% more than that of the slab with square column. While The punching shear capacity of the slab with rectangular column having ratio 5 of longer to shorter column width increase about 53% more than that of the slab with square column. Therefore, the more rectangularity of the column can increase more the punching shear capacity of slab.

He proposed an extension to BS8110-97's formula in order to predict the punching shear capacity of slabs with rectangular column and with or without opening more accurately that modification was made to the critical punching shear perimeter u of BS8110-97's formula. His modification was that the critical shear perimeter at a distance of $1.5d$ from the column face between two projection lines, which propagated from the center point of the column edge section through two comers of opening, was ineffective. His proposed extension to critical shear perimeter presented by BS8110-97 together with his slab specimens is shown in Figure 2.21.

Figure 2.21 Test specimens and modified method of two projection lines for BS8110-97 (Kuang, 2001)

2.5 Experimental works related to slabs with shear reinforcement

One of the ways to increase the shear strength of slab-column connections is the use of shear reinforcement such as shear stud, stirrup, and shear heads. When the slab has openings, the shear reinforcement may be used to recover the loss of shear capacity of slab due to opening. When the slab-column connections are subjected to high shear force or combination of shear and unbalanced moment, the shear reinforcement may be used to increase the shear capacity of the connections. In design, when the factor shear force exceeds the available design shear strength without shear reinforcement, the shear reinforcement can be used.

2.5.1 Study by Islam and Park (1976)

In 1976, Islam and Park investigated the effect of various types of shear reinforcement on the strength and ductility of slab-column connections with shear and unbalanced moment. They tested eight half scale models of reinforced concrete interior flat plate column specimens under combined gravity and seismic type loading. Three of the specimens were without shear reinforcement. The remaining five specimens were with various arrangement of shear reinforcement. The type of shear reinforcements was used; cranked (bent up) bars, shear head reinforcement in the form of pairs of channel sections and vertical closed stirrups. The details of the specimen are shown in Figure 2.23. The four specimens were cyclic loaded, which caused reversals unbalanced bending moment simulating earthquake effects. The other specimens were loaded monotonically with increasing unbalanced moment.

They concluded that (1) the slab-column connections without any shear reinforcement had little ductility and failure occurred suddenly, (2) although the cranked bent-up bars could increase the shear strength, they did not increase the ductility, (3) the structural shear head reinforcement increased the shear strength, but gave only a limited increase in the ductility of the connection, and (4) the closed stirrups resulted in increase in the strength and a substantial increase in the ductility of the slab-column connections.

Figure 2.22 Test specimen (Islam and Park, 1976)

2.5.2 Study by Elgabry and Ghali (1987)

In 1987, Elgabry and Ghali investigated the shear strength of interior slab-column connections reinforced with shear studs and suggested code provisions for the design of this type of shear reinforcement. They tested five full-scale reinforced concrete flat plate connections with interior columns subjected to shear-moment transfer. One specimen had no shear reinforcement and the remaining four consisted of various arrangement of shear stud reinforcement. The detail of the test specimens are shown in Figure 2.23. The test results indicated that the well-anchored stud-shear reinforcements are effective in increasing the shear strength and ductility of slab-column connections subjected to shear force and unbalanced moment. They recommended the following:

- (1) The distance s_0 between the first row of shear studs and the column face should not be smaller than $d/4$.
- (2) The upper limits for s_0 and for spacing s between the rows should be based on the value V_u at a critical section at $d/2$ from the column face.

$$\left. \begin{array}{l} s_0 \leq d/2 \\ s \leq 3d/4 \end{array} \right\} \text{ when } 0.33\sqrt{f'_c} < \frac{V_u}{\phi} \leq 0.5\sqrt{f'_c} \tag{2.41a}$$

2.5.3 Study by Ghali (1989)

In 1989, Ghali studied the shear stud reinforcement (SSR) to increase the strength and the ductility of slab-column connections and discussed its advantages. SSR was composed of vertical bars with anchor heads at their tops, welded to a steel strip at the bottom. He also discussed the disadvantages of stirrups. In practice, stirrups or structural shear heads are often used to increase the shear capacity because they are less expensive. However, some disadvantages of stirrups that he discussed are:

1. The tensile forces, which resist shear, in the vertical portions of the stirrups can not develop unless the vertical branches are anchored effectively near the top and bottom surface of the slab.
2. It is difficult to install.
3. The vertical portions of the stirrups typically is bent at the top and bottom of the slab and thus can not be fully effective in controlling the width of shear cracks close to the top and bottom surfaces.
4. It decreases the effective depth of the slab. Thus, more flexural reinforcements are required.

Some advantages of SSR are:

1. They are easy to install and control. Once the strips are placed on chair and secured to the forms, the spacing, the verticality the studs, and the concrete covers at the top and bottom are insured.
2. There is no interference of the shear reinforcement with the flexural reinforcement.
3. They are more effective in controlling the width of shear cracks that intersect the studs close to the top and bottom surfaces, because they are straight vertical and are anchored as closely as possible to the top and bottom surface of the slab.
4. The cost of materials can be reduced because the amount of steel used for the studs and strips is small compared to the amount of steel in conventional stirrups
5. The specified concrete cover can be maintained because the cover above the anchored heads is the same as the cover for the top flexural reinforcement.

6. Higher ultimate strength and more ductile behavior of slab can be achieved due to more efficient anchorage of SSR compared to conventional stirrups.

2.5.4 Study by Mortin and Ghali (1991)

In 1991 Mortin and Ghali verified the effectiveness of shear stud reinforcement in improving shear strength and ductility. Shear stud reinforcement is a vertical stem mechanically anchored at top and bottom. They tested six full-scale reinforced concrete flat plate connections with edge columns subjected to shear-moment transferred. Two specimens had no shear reinforcement and the remaining four had various arrangement of stud-shear reinforcement. The specimens were rectangular, simply supported on three sides, with the column stubs located at the center of unsupported side. Test variables were the width of the stud strip, placement of the studs within the spandrel strip, the number of row of studs, the spacing of the studs, and the size of the shear reinforced zone. The details of specimens are shown in Figure 2.25.

Their results showed that shear stud reinforcement can substantially improve the ductility and shear strength of the thin, reinforced concrete, edge slab-column connections. They recommended the design rules suggested by Dilger and Ghali (1981) which were: (1) The maximum allowable shear stress in critical section at a distance $d/2$ from the column face should be increased from $6\sqrt{f'_c}$ psi (allowed by ACI 318-89) to $8\sqrt{f'_c}$ psi due to the improved anchorage provided by shear studs, (2) The strength resisted by concrete within the shear reinforced zone reduced from $3\sqrt{f'_c}$ psi to $2\sqrt{f'_c}$ psi, and (3) In a critical section at $d/2$ from the outermost row of shear studs, the allowable shear stress should be a function of distance from the column. This function was equal to $4\sqrt{f'_c}$ psi and $2\sqrt{f'_c}$ psi at a distance from the column face $\leq d$ and $\geq 4d$, respectively, with linear variation between the two zones. They also suggested that at least two strips of studs should be placed in the spandrel strip. One of these should be adjacent to the free edge and the other along the inner face of the column to the free edge.

Figure 2.24 Test specimen (Mortin and Ghali, 1991)

2.6 Reviews on punching shear strength by codes

2.6.1 Background

There are many design codes to calculate the shear strength of the slab-column connections. In this chapter, six design codes are reviewed. They are ACI 318-02 and British Standard BS 8110-97. In different codes, different critical perimeter at some distance from the vicinity of column face is assumed as a control parameter. The punching shear strength is calculated based on a nominal shear stress on a critical section area. The critical sectional area is located along a critical perimeter and has a depth equal to effective depth of slabs.

If the slab-column connection includes holes or openings, the shearing strength is calculated based on the reduced critical shear perimeter due to openings. The codes reduce the critical shear perimeter by several methods.

2.6.2 Design requirements

The basic equation for shear design is usually stated as:

$$v_u \leq \phi v_n \quad (2.45)$$

$$\phi v_n = \frac{\phi(V_c + V_s)}{b_o d} \quad (2.46)$$

$$v_u = v_c b_o d \quad (2.47)$$

- where b_o = critical shear perimeter
 V_u = factored shear force due to loads
 V_c = shear force contributed by concrete
 V_s = shear force contributed by shear reinforcement if any
 v_u = maximum shear stress due to factor shear force or combination of factored shear force and unbalanced moment
 v_n = nominal shear strength of the slabs
 v_c = shear strength contributed by concrete
 v_s = shear strength contributed by shear reinforcement if any
 ϕ = strength reduction factor, equals to 0.85 for shear

The shear strength contributed by concrete is derived from the nominal shear stress on the critical section located at some distance from column face according to the design codes, which will be discussed in the following sections.

2.6.3 American Concrete Institute ACI 318-02

2.6.3.1 Shear force only

ACI 318-02 recommends that the shear strength of slabs without shear reinforcement can be calculated based on some variables. They are concrete strength, ratio of the size of loaded area and slab thickness, loaded area aspect ratio (shape of loaded area), perimeter aspect ratio, and shear to moment ratio at slab-column connections. By considering these variables, the shear strength is given by the smallest of the following:

$$v_c = \frac{1}{6} \times \left(1 + \frac{2}{\beta_c}\right) \times \sqrt{f'_c} \quad (2.48)$$

$$v_c = \frac{1}{6} \times \left(1 + \frac{\alpha_s}{2\beta_0}\right) \times \sqrt{f'_c} \quad (2.49)$$

$$v_c = \frac{1}{3} \times \sqrt{f'_c} \quad (2.50)$$

- where b_o = critical shear perimeter shown in Figure 2.25 for square and rectangular column type and $\pi(D+d)$ for circular columns
- D = diameter of column
- v_c = nominal shear strength for normal weight concrete and $0.75 \times V_c$ for lightweight concrete
- β_c = loaded area aspect ratio (longer size of loaded area to shorter size of loaded area)
- α_s = constant, 40 for interior column
30 for edge columns
20 for corner columns
- β_o = perimeter aspect ratio, b_o/d

Figure 2.25 Critical shear perimeter of critical section (ACI 318-02)

2.6.3.2 Presence of unbalanced moment

ACI 318-02 recommends that the maximum factored shear stress on the critical section due to factored shear force and the unbalanced moment be calculated based on the assumption that this shear stress varies linearly on the critical peripheral line from

the column faces as shown in Figure 2.26, 2.27, and 2.28. The equation to calculate the combined shear stress is as follows:

$$v_u = \frac{V_u}{A_c} \pm \frac{\gamma_{vx} M_{ux} y}{J_x} \pm \frac{\gamma_{vy} M_{uy} x}{J_y} \quad (2.51)$$

where A_c = critical sectional area, $b_o d$

M_{ux}, M_{uy} = unbalanced moment in x and y directions, respectively

J_x, J_y = polar moment of inertia of the shear perimeter about the axis x and y axes, respectively

x, y = coordinates of the point at which v_u is calculated with respect to centroidal axes of the assumed critical sections

γ_{vx} & γ_{vy} = fraction of unbalanced moment transferred by shear in x and y directions, respectively

The polar moment of inertia of the shear perimeter about the x and y axes are given as.

For interior column

$$J_x = d \left(\frac{l_y^3}{6} + \frac{l_x l_y^2}{2} \right) + \frac{l_y d^3}{6} \quad (2.52)$$

$$J_y = d \left(\frac{l_x^3}{6} + \frac{l_y l_x^2}{2} \right) + \frac{l_x d^3}{6} \quad (2.53)$$

For edge column

$$J_x = d \left(\frac{l_y^3}{12} + \frac{l_x l_y^2}{2} \right) + \frac{l_y d^3}{12} \quad (2.54)$$

$$J_y = \frac{l_x d}{6} (d^2 + l_x^2) + 2 d l_x \left(\frac{l_x}{2} - c_{BC} \right)^2 + l_y c_{BC}^2 \quad (2.55)$$

where $c_{BC} = \frac{l_x^2}{2l_x + l_y}$

= distance of centroidal axis of critical section parallel to x-axis from the face BC shown in Figure 2.27

For corner column

$$J_x = \frac{l_y d}{6} (d^2 + l_y^2) + 2 d l_y \left(\frac{l_y}{2} - c_{AB} \right)^2 + d l_x c_{AB}^2 \quad (2.56)$$

$$J_y = \frac{l_x d}{12} (d^2 + l_x^2) + d l_x \left(\frac{l_x}{2} - c_{BC} \right)^2 + d l_y c_{BC}^2 \quad (2.57)$$

where $c_{AB} = c_{BC} = \frac{l_x l_y}{2(l_x + l_y)}$
 = distance of centroidal axis of critical section parallel to x-axis and y-axis from the face AB and BC, respectively shown in Figure 2.28.

And the fraction of unbalanced moment transferred by shear in x and y directions are given by the equations:

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\frac{l_y}{l_x}}} \quad (2.58a)$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\frac{l_x}{l_y}}} \quad (2.56b)$$

Figure 2.26 Shear stress distribution at the critical section in the vicinity of interior column (ACI 318-02)

Figure 2.27 Shear stress distributions at the critical section in the vicinity of edge column (ACI 318-02)

Figure 2.28 Shear stress distribution at the critical section in the vicinity of corner column (ACI 318-02)

2.6.3.3 Opening in slabs

When the openings in slabs are located at a distance closer than 10 times the slab thickness from the column, the shear strength needs to be reduced by considering the portion of the critical shear area bounded by the two straight lines projecting from the

column centroid to the edges of the openings ineffective. The ineffective portion of the critical shear is shown in Figure 2.30.

Figure 2.29 Ineffective portion of the slab due to openings (ACI 318-02)

2.6.4 British Standard BS 8110-97

2.6.4.1 Shear force only

In BS8110-97, the critical shear perimeter is located at 1.5d distance from the column face as shown in Figure 2.30. The design punching strength of slabs without shear reinforcement is based on the shear strength of the beam which is given by:

$$v_c = \frac{0.79 \rho^{\frac{1}{3}} (f_{cu} / 20)^{\frac{1}{3}} (400 / d)^{\frac{1}{4}}}{\gamma_m} \quad (2.59)$$

- where
- d = average effective depth of flexural reinforcement ≤ 400 mm
 - f_{cu} = characteristic cube strength ≤ 40 N/mm²
 - v_c = design punching strength of the slabs for normal weight concrete
(For lightweight concrete grades of 20 or more, v_c is multiplied by 0.8)
 - ρ = flexural reinforcement ratio, $100A_s/(bd)$ along the critical section (a width of slab equal to that of loaded area plus 1.5d to either side of it) ≤ 3 %
 - γ_m = material safety factor for concrete in shear

The punching shear capacity, V , due to a concentrated load from the column can be obtained as follows:

$$V = v_c u d \quad (2.60)$$

where u = critical perimeter

$$4(D+3d) \quad (\text{For circular column})$$

$$4(c+3d) \quad (\text{For square column})$$

D = diameter of circular column

c = side of square column

Figure 2.30 Location of shear perimeter (BS 8110-97)

2.6.4.2 Presence of unbalanced moment

When the unbalanced moment is transferred from slab to column, the design effective shear force in the vicinity of the column at the perimeter, V_{eff} , should be taken as:

For interior slab-column connections,

$$V_{eff} = V_t \left(1 + \frac{1.5 M_t}{V_t x} \right) \quad (2.61)$$

For corner and edge slab-column connections where the axis of the unbalanced moment parallel to the free edge,

$$V_{eff} = 1.25 V_t \quad (2.62)$$

For edge slab-column connections where the axis of the unbalanced moment perpendicular to the free edge,

$$V_{eff} = V_t \left(1.25 + \frac{1.5M_t}{V_t x} \right) \tag{2.63}$$

where

- x = length of the side of the perimeter considered parallel to the axis of bending
- M_t = design moment transferred from the slab to the column at the connection
- V_t = shear force at the perimeter which is calculated on the assumption that the maximum design load is applied to all panels adjacent to the column at the connection

2.6.4.3 Opening in slabs

When opening in slabs are located at a distance less than six times the effective depth of the slabs from the edge of the concentrated load, then that part of the perimeter which is enclosed by the two radial lines projecting from the centroid of the loaded area to the edge of opening is considered as ineffective as shown in Figure 2.31.

Figure 2.31 Shear perimeter of slabs with opening (BS 8110-97)

2.6.5 Summary

When a slab has one or more openings in the vicinity of column region, the shear strength of the slab will be reduced by the presence of these openings. Several methods were developed and proposed in the design codes to predict the shear strength of the slab-column connections empirically. In these methods, due to the effect of openings the critical shear perimeter around the column is reduced by various ways to obtain the shear strength of the slabs with openings.

In many design codes reviewed in the previous sub-chapters, the critical perimeter at some distance from the vicinity of column face is assumed as a control parameter. The punching shear strength is calculated based on a nominal shear stress on a critical sectional area. The critical sectional area is located along a critical perimeter and has a depth equal to effective depth of slabs. Code equations are based empirically on test results and do not give the actual mechanism of slab-column connections. Regarding to the slab-column connections with holes or openings, the shearing strength is calculated based on the reduced critical shear perimeter due to openings. The effect of reinforcement that is reduced due to the presence of opening is not taken into consideration.

Many researches have been done to study the shear strength of interior slab-column connections with openings. However, data are still needed, particularly for slab-column connections with rectangular columns, with opening and shear reinforcement. The critical shear area around the column plays in very important role resisting the punching shear capacity of slab. Due to present of opening, some parts of critical shear area around the column may decrease or are ineffective in resisting the punching load. Therefore, it is necessary to define the critical shear area against punching correctly to predict the shear strength more reasonably. The use of suitable shear reinforcements can recover the shear strength of slab-column connections with opening.

As reviewed previously, there are many models that predict the punching shear capacity of slab-column connection. Among these models, the model based on theory

of plasticity can give the actual collapse mechanism and nature of failure surface due to punching observed in practice. In practice, the punching shear failure surface is in curve shape. The softening effect of concrete and the variation of normal and shear stress along the failure surface play in important role in plasticity model. Previously reviewed plasticity models are for punching shear capacity of slab-connection with circular column. Thus, there are needs to extend the plasticity model for slab-column connection with square or rectangular column, with or without shear reinforcement, and with or without opening.

CHAPTER 3

EXPERIMENTAL PROGRAMME

Chapter 3

Experimental programme

3.1 Introduction

This experimental study is a part of research collaboration programme to study the behavior of flat plate floor systems with openings and was carried out together with two final year students, at the Nanyang Technological University, Singapore. A total of 6 normal strength concrete slabs with opening and different arrangement of shear stud reinforcement were tested. The specimens were tested to failure under monotonic loading in order to study the effectiveness of shear stud reinforcement under ultimate load conditions.

Mainly, this program aims to study (1) the behavior of slab-column connections of flat plate floor system with openings in the vicinity of column, (2) the effectiveness of shear stud reinforcement in recovering the punching strength of slab-column connections due to openings, and (3) the effectiveness on the arrangement of shear studs. To fulfill this aim, the structural behavior of specimens are examined in term of the ultimate capacity, the first cracking load, failure patterns of cracks, and the deflections of slabs.

In this chapter, the details of tested specimens are firstly presented. The arrangements of shear stud reinforcements used in the specimens are shown. The experimental set-up and procedures of testing are followed.

3.2 Slab-column specimens

A total of six interior slab specimens supported on rectangular columns were tested. Two types of locations for openings in slabs were considered. The slab specimens were classified as two series, A and B as shown in Figure 3.1. The slabs in A-series

had opening located at the corner of the column stub while the slabs in B-series had opening positioned at the shorter side of the column.

Figure 3.1 Plan views of slab specimens

3.2.1 Slab notations

The slab specimens were labeled as "A1", "A2", "A3", "B1", "B2" and "B3". The first alphabet letter was to classify the slabs into the series. The second letter 1", "2" and "3" was to distinguish between the first, second and third specimen of each series.

3.2.2 Slab details

All the slab specimens were designed as full-scale models. Each slab had the size of 2650 mm by 3400 mm, thickness of 200 mm and a rectangular column of 1000 mm by 250 mm cross-section with 250 mm depth. Each of all slabs had four free edges. The sizes of the openings in all the slab specimens were 200 mm by 400 mm and positioned at the corner and at the shorter side of column in series A and B, respectively. The slab "A1" and "B1" were control specimens without shear reinforcement for each series. Shear studs of different amount and arrangement were placed in the remaining specimens in order to investigate their effectiveness in recovering the punching shear capacity lost due to openings.

3.2.2.1 Flexural reinforcing steel

In each slab, two layers of flexural reinforcement were placed at the top and bottom of the slab to prevent flexural failure from occurring. At the top layer, the high strength deformed T20 steel bars were placed at 125 mm centre-to-centre spacing in both directions. At the bottom layer, the high strength deformed T13 steel bars were placed at 200 mm centre-to-centre spacing in both directions. The top and bottom clear concrete cover of the slab was 20 mm. The top and bottom layers of flexural reinforcement equivalent to that interrupted by the opening were compensated on the sides of the opening. Details of the flexural reinforcement used in the slab specimens for each of the series are shown in Figures 3.2. In the column, the ten numbers of T20 high strength deformed bars were evenly spaced with T10 links of 100 mm centre-to-centre spacing. The clear concrete cover of the column was 40 mm. The flexural reinforcement provided were more than the required amount based on the predicted ultimate loads in order to ensure punching shear failure to occur before the yielding of the longitudinal reinforcement.

3.2.2.2 Shear studs

All specimens had the shear stud reinforcements except the control specimens, A1 and B1. The high strength deformed T10 vertical bars were used as shear stud reinforcement in this experimental study. One end of each shear stud had an anchorage head of 30 mm diameter while the other end was welded to a common rail of steel plate of 20 mm by 5 mm to keep them at right position. Overall height of shear studs including the thickness of the anchorage head was 160 mm. The center-to-center spacing between rows of shear studs was $0.75d$. The "d" is effective depth of slab. The details of shear studs reinforcement in the specimens are shown in Table 3.1 and Figure 3.3. The arrangements of shear stud reinforcements are shown in Figure 3.4.

(a) A series specimens

(b) B series specimens

(c) Typical cross-section of all slab specimens

Figure 3.2 Details of flexural reinforcement in slab specimens

Table 3.1 Details of shear studs reinforcement in slab specimens

Specimen	Number of rails of shear stud reinforcement	Number of shear studs per rail	Center to center spacing of shear studs
A2	5	8	0.75d
A3	6	5	0.75d
B2	6	8	0.75d
B3	6	5	0.75d

Figure 3.3 Typical structural details for the shear studs

In each series, different arrangements of shear stud reinforcement were used. The two control specimens of A1 and A2 had no shear stud reinforcement. In specimens A2, A3, B2, and B3, different amount and arrangement of shear stud reinforcements were used. The arrangements of shear studs for each specimen can be seen in Figure 3.4. The following describes the quantities of shear studs used and arranged, and the reason these arrangements were chosen.

- (1) In specimen A2, five numbers of rails in which each rail consisted of 8 shear studs were used. All shear studs reinforcements were placed near the opening so that the loss in shear capacity at the lower half due to the presence of the opening might be regained and it might prevent the failure crack from intercepting the opening.
- (2) In specimen A3, six numbers of rails in which each rail consisted of the minimum requirement of 5 shear studs were used. Three rails were placed around the one shorter column width and not in the middle region of the longer column widths so that the shorter column width where the stress concentrations occurred might be to strengthen. Other three rails in which two rails were placed beside the opening were placed around the remaining shorter column width.

Figure 3.4 Arrangement of shear stud reinforcements in the slab specimens

- (3) In specimen B2, six numbers of rails in which each rail consisted of 8 shear studs were used. All the rails were placed in the region of the slab near the opening. Like specimen A2, the using longer rail was to prevent the failure crack from intercepting the opening. The tilting of two rails with 45 degrees near the opening was to provide additional stiffness in the region near the opening.

(4) In specimen B3, six numbers of rails in which each rail consisted of the minimum requirement of 5 shear stud were used. Four rails were placed around the shorter column width to strengthen. The tilting of two rails with 45 degrees near the opening was to provide additional stiffness in the region near the opening like specimen B2.

3.2.2.3 Material properties

Properties of concrete

The design concrete specifications for all specimens are as follows:

- (a) Cement : Ordinary Portland Cement,
- (b) Concrete characteristic strength : 40 N/mm² at 28 days in accordance to BS 5328,
- (c) Maximum size of aggregate : 20mm, and
- (d) Slump of concrete : 100 ±25mm.

Using two batches of concrete, all specimens were prepared. Each series of specimen was cast using concrete from each batch. From each batch of concrete, the total twelve numbers of concrete cylinder and cubes were prepared and cured alongside the slab specimens. All cylinders and cubes were tested in compression to obtain the compressive cylinder strength (f_c') and the compressive cube strength (f_{cu}). The Young's modulus of concrete (E_c) was obtained by taking the initial slope of stress strain curve. The average f_c' , f_{cu} , and E_c are shown in Table 3.2.

Table 3.2 Test results of compressive concrete cylinder and cube strengths

Series of Specimen	Batch number	Average f_c' (N/mm ²)	Average f_{cu} (N/mm ²)	Average E_c (GPa)
A	1	38.2	47.9	28.5
B	2	36.3	44.8	22.4

Properties of steel

In all specimens, high strength deformed steel bars (T-bars) with 20, 13, and 10 mm diameter were used as top and bottom flexural reinforcements and shear stud reinforcement, respectively. Tensile tests on two steel bars from each diameter size

were done in Universal Testing Machine to obtain the yield strength (f_y) of these steel bars. The 0.2% off-set method was used to obtain the yield strength of steel bars. The yield strength (f_y) and Young's Modulus (E_c) of each diameter size of steel bars are shown in Table 3.3.

Table 3.3 Steel bar tensile test results

Type of bars	Bar Marking	Average f_y	Average E_s
Top reinforcement	T20	580	192.3
Bottom reinforcement	T13	600	212.4
Shear studs	T10	540	184.7

3.3 Experimental set-up

The experimental set-up for testing of the slab specimens is shown in Figure 3.5. In each specimen, 4 numbers of holes with 80mm diameter that were positioned in the middle of each edge were perforated to place the four pull-down prestressing rods vertically, which are shown in Figure 3.6. Plan view of testing set-up for slab is shown in Figure 3.7. The column stubs of the slab specimens were placed on top of a big steel plate which was anchored to the laboratory strong floor. Therefore, the test specimens were only supported on column while all edges were free.

The four spreader beams were located on the slab, which were parallel with the edge of the slabs. Each of the spreader beams was supported at their ends by two bearing supports rested on the slab. Four pull-down prestressing rods passed through the four holes of slab and the holes in the center of each spreader beam. The diameter of pull-down prestressing rod was 40 mm and it transferred the loads from a hydraulic jack to a spreader beam. Each load cells was installed on top of each spreader beam by a number of bolts and nuts through the pull-down prestressing rod. Below the slab, each hydraulic jack was individually supported on the laboratory strong floor by a number of holding down bolts and connected to a central controlling valve to control the total load acting on the slab specimens.

The four 1000 kN capacity hydraulic jacks were used to apply the vertical loads to the slab specimens through the four spreader beams. Each spreader beam distributed the applied loads to the slabs at two ends of spreader beam through the bearing plates. By this method, eight point loads were applied to the slab specimens. By placing the bearing supports under the spreader beams at equal distances away from the edge of the column, a simply supported system was performed with the spreader beams in order to transfer the loads. Besides, the load cells controlled the total applied load, which was connected directly with the prestressing rods and hydraulic jack. This load cell had a load measuring capacity of up to 1000 kN.

Figure 3.5 Testing set-ups

Figure 3.6 Testing set-up complete with spreader beams and LVDTs

3.4 Instrumentation

The instrumentation consists of

- (1) four 1000 kN load cells,
- (2) a series of Linear Variable Differential Transducer (LVDT),
- (3) strain gauges, and
- (4) a set of data logging equipment.

Each of them serves their functions described as in the following sections.

3.4.1 Load cell

The function of the load cells was to measure the applied loads. Four number of load cells were used and each load cell was placed on top of a spreader beam which was directly in contact with the pre-stressing rod on the top side of the slab specimen to measure the pull-down system of the hydraulic jack. The spreader beams transferred the loads from the hydraulic jacks to the slabs.

3.4.2 Linear variable differential transducer (LVDT)

In order to observe and measure the deflection of the tested slab-column specimens, 17 numbers of linear variable transducers were installed. The 12 numbers of LVDTs (D1 to D12) were situated at 0.5d, 3.5d and 7d from the column face on top side of slab while 4 numbers of LVDTs (D13 to D16) were placed at the corners of the perimeter 7d from the column face below the slab. The remaining One LVDT (D17) was located directly above the center of column to record the vertical displacement of the column itself so that the relative deflection for the rest of the locations could be calculated. In series B, LVDTs number D4 and D8 were placed at 0.5d and 3.5d from the edge of the opening while in series A, at 0.5d and 3.5d from the face of shorter column width. All transducers were connected to the data logging equipment via electrical cables to record the data. The locations of LVDTs for all slab-column specimens can be seen Figure 3.8.

Figure 3.8 LVDT locations on slab specimens

3.4.3 Strain gauges

In order to measure the strains of the flexural and shear stud reinforcement and to conclude that whether the punching shear occurred before or after the flexural reinforcement had yielded, the 5 mm electrical resistance strain gauges were used. Also, the strains recorded with the strain gauges could be used to investigate the stresses at different locations of the slab. These electrical resistance strain gauges were attached to the specified positions on the surface of selected flexural reinforcement and at mid height of all shear stud reinforcement. To smooth the surface, grinding was done at designated positions on selected flexural reinforcements and at the mid-height of all shear stud reinforcements. A layer of silicon gel was applied to protect all strain gauges. The strain gauges on slab specimens were positioned at distance of 1d and 2d from column face due to the consideration that most of the punching shear failures might occur in the vicinity of the column at a distance of 1d and 2d from the column face. Also, some strain gauges were installed at the critical perimeter of 0.5d from the outermost peripheral of shear studs. The "d" is the effective depth of slab. The number of strain gauges installed in each specimen is shown in Table 3.4 and their locations on shear studs in the slabs can be seen at the same location of shear studs shown in Figure 3.4 and on flexural reinforcement shown in Figure 3.9.

Table 3.4 Distribution of strain gauges on flexural and shear stud reinforcement for slab specimens

Slab Specimen	Number of strain gauges	
	on tension reinforcements	on shear stud reinforcements
A1	18	0
A2	21	40
A3	24	30
B1	12	0
B2	18	48
B3	21	30

Figure 3.9 Strain gauge locations on flexural reinforcements
in slab-column specimens

3.4.4 Data logging equipment

In the data logging equipment, a data logger, a personal computer and a 50-channel switch box were utilized. The wires were used to connect all measuring instruments to the data logger through the switch boxes. The personal computer was used with a visual basic program called the Laboratory Windows in order to record the data of the load, deflection and strain continuously throughout the test.

3.5 Procedure of testing

When the specimen was ready to be loaded, the 50 kN load was applied and then released to almost zero to stabilize the entire set-up. Then all specimens were loaded at eight points with equal loads of 5 kN each on top side of the slab. Initially the load was applied with increment of 20 kN until it reached 200 kN. After that, the load was increased with 40 kN increment up to the failure of the specimen. The first crack in all slab specimens was carefully observed. The crack patterns were drawn at every 200 kN load from the start of the test until failure. The crack widths were also measured at every load increment.

In order to observe the post-punching behavior of slabs, the loading was continued even after it reached the failure load. When the crack pattern could be seen clearly, the loading was stopped.

CHAPTER 4

EXPERIMENTAL RESULTS AND DISCUSSION

Chapter 4

Experimental results and discussion

4.1 Introduction

As previously presented in chapter three, in order to get better understanding on the behavior of the rectangular slab-column connections with opening and to investigate the effectiveness of shear stud reinforcement in improving and recovering the punching shear capacity of the slab-column connections lost due to opening, a total of six slab-column specimens with opening as comprised in two series A and B were tested. The location of opening in the slab-column specimens of series A was at the corner of the column while the location of opening in the slab-column specimens of series B was at the shorter column width. Except the control specimen A1 and B1, all slab-column specimens had shear stud reinforcements (SSR) with different quantities and arrangements. The locations of opening and arrangements of shear stud reinforcements in the slab-column specimens according to north-south and east-west directions are shown in Figure 4.1.

Figure 4.1 Location of opening and arrangement of SSR in the slabs

In this chapter the ultimate punching shear capacities of slab-column specimens obtained by testing are discussed first. Secondly, the serviceability load, crack patterns, and deflections of the slab-column specimens observed from testing are discussed. Thirdly, the discussion is made based on the analysis of strain gauge measurements recorded on the flexural and shear reinforcements of the slab-column specimens during testing. Finally, summary and conclusion are drawn.

4.2 Ultimate punching shear capacity

The ultimate punching shear capacity is defined as the maximum applied punching load at which the slab fails in punching shear failure. All slab-column specimens failed suddenly before the longitudinal bars yield. It indicates that all the slab-column specimens had failed due to punching shear failure. The ultimate punching shear capacities of the slab-column specimens obtained by testing are tabulated together with their percentage increases compared to the respective control specimens and the quantities of shear studs reinforcements used in the slabs in Table 4.1.

Table 4.1 Test results for the ultimate punching capacities of the slabs

Specimen		P (kN)	Difference between ultimate punching shear capacities of A1 and B1 (%)	Increase in ultimate punching shear capacity (%)	Quantity of shear stud reinforcements
A Series	A1	740	+21.6	-	No
	A2	908	-	22.7	5×8 studs per rail
	A3	948	-	28.1	6×5 studs per rail
B Series	B1	580	-21.6	-	No
	B2	848	-	46.2	6×8 studs per rail
	B3	692	-	19.3	6×5 studs per rail

P = Ultimate punching shear capacity

From Table 4.1, it can be seen that the ultimate punching shear capacity of the control slab-column specimen A1 is 21.6 % larger than that of the control slab-column specimen B1. It may be due to that the shear stresses concentrate highly at the shorter column width and vary in decreasing rate from the shorter column width to the longer column width along the peripheral around the column. As such, we can point out that (1) the placing of opening near the column can decrease the punching shear capacity of the slab; (2) the decrease in the punching shear capacity due to opening can differ according to location of openings itself; and (3) the location of opening at the shorter column width can affect more than the location of opening at the comer of column.

About the shear stud reinforcements, on the comparison the ultimate punching shear capacities of the slab-column specimens with their respective control slab-column

specimens, the punching shear capacities of all slab-column specimens with shear stud reinforcements were observed greater than that of their respective control slab-column specimens. It indicates that the use of shear stud reinforcements are significantly effective in recovering the punching shear capacity of the slab lost due to opening.

4.2.1 Slab-column specimens for series A

The slab-column specimen A2 had 5x8 studs per rail which were placed near the region around opening, while the slab-column specimen A3 had 6x5 studs per rail which were placed near the region around the opening and around the shorter column width where opening was not located. The arrangement of shear stud reinforcements used in the slab-column specimens of series A can be seen in Figure 4.1.

From Figure 4.1, it can be seen that the increases in the ultimate punching shear capacity of slab-column specimen A2 and A3 compared to their control slab-column specimen A1 are 22.7% and 28.1%, respectively. And so, the increase in the ultimate punching shear capacity of slab-column specimens A3 is about 6% greater than the increase in the ultimate punching shear capacity of slab-column specimen A2. Therefore, it can be concluded that the arrangement of shear stud reinforcements in the slab-column specimens A3 is more effective in increasing and recovering the punching shear capacity than that in the slab-column specimen A2.

4.2.2 Slab-column specimens for series B

The slab-column specimen B2 had 6x8 studs per rail in which two rails were placed near the region around the opening by tilting 45° and other four rails were placed from the column end having opening approximately to the middle of longer column width. The slab-column specimen B3 had 6x5 studs per rail in which two rails were placed near the region around the opening by tilting with 45° and other four rails were placed around the shorter column width where the opening was not located. The arrangement of shear studs reinforcements in slab-column specimens of series B can be seen in Figure 4.1.

From Table 4.1, it can be seen that the increase in the ultimate punching shear capacity of slab-column specimens B2 and B3 compared to their control slab-column specimen B1 are 46.2% and 19.3%, respectively. The increase in the ultimate punching shear capacity of slab-column specimens B2 is about 27% greater than the increase in the ultimate punching shear capacity of slab-column specimen B3. Therefore, for series B, it is evident that the arrangement and quantity of shear stud reinforcements in slab-column specimens B2 was significantly more effective in increasing the punching shear capacity than that in the slab-column specimen B3. But the quantity of shear stud reinforcements in slab-column specimen B2 is greater than that slab-column specimen B3. Therefore, if same quantity of shear stud reinforcement in series B is used, we can not certainly conclude whether the arrangement of shear reinforcement in slab-column specimen B2 may be effective more than that of slab-column specimen B3.

4.2.3 Findings

From the analysis based on the ultimate punching shear capacities of the slab-column specimens above, the findings can be summarized as the following.

- (1) The location of opening at the shorter column width could affect the punching shear capacity of slab-column connection about 22% more severe than that at the corner of column.
- (2) For slabs with opening at the corner of column, the arrangement and quantity of shear stud reinforcements in slab A2 and A3 increased the ultimate punching shear capacity of slab 22.7% and 28.1%, respectively, while for slabs with opening at the shorter column width, the arrangement and quantity of shear stud reinforcements in slab B2 and B3 increased the ultimate punching shear capacity of slab 46.2% and 19.3%, respectively. Therefore, the use of shear stud reinforcements are significantly effective in increasing and recovering the punching shear capacity of the slab lost due to opening and effectiveness of shear studs depends on their arrangement and quantity.

4.3 Serviceability

In this section, the serviceability conditions of tested slab-column specimens from the previous chapter are discussed based on the first cracking load, crack patterns and deflection of slabs due to the applied loads.

4.3.1 First cracking load

The serviceability load is defined as the load at which the first flexural crack occurs. The first cracking loads of all slab-column specimens are shown in Table 4.2. All slab-column specimens reached their first cracking load about 11%-22% of the ultimate punching shear capacity of each slab. The first cracking load of the control specimen A1 is about 38% more than that in specimen B1. Therefore, the location of opening adjacent to shorter column width affect the serviceability of the slab more than that of the location of opening at the comer of column.

Table 4.2 Test results of first cracking load

Spec.		First cracking load (kN)	Percentage of first cracking load on ultimate punching shear capacity (%)	Difference between first cracking loads of A 1 and B1 (%)	Difference of first cracking load between the respective specimens and their control specimen (%)
A Series	A1	160	21.6	+37.5	
	A2	120	13.2		-25
	A3	140	14.8	-	-12.5
B Series	B1	100	17.2	-37.5	
	B2	140	16.5		+40
	B3	80	11.6	-	-20

4.3.1.1 Slab-column specimens for series A

In series A, the first cracking loads of slab-column specimen A2 and A3 are about 25% and 13% less than that of the control slab-column specimens A1, respectively. It indicates that the use of shear stud reinforcements in this series A is not actually

effective in improving the serviceability load. However, it can be seen that the first cracking loads of the slab-column specimen A2 is about 12% greater than that of the slab-column specimen A3.

4.3.1.2 Slab-column specimens for series B

In series B, the first cracking load of slab-column specimen B2 is 40% more than that of the control slab-column specimen B1 while the first cracking load of slab-column specimen B3 is 20% less than that of the control slab-column specimen B1. Because the quantity of shear stud reinforcements used in slab-column specimen B2 is more than that in slab-column specimen B3, it can not clearly indicate that the use of quantity and arrangement of shear stud reinforcement in this series B is effective in improving the serviceability load.

4.3.1.3 Findings

From the analysis of serviceability loads of slab-column specimens, the following findings can be made.

- (1) All slab-column specimens reached their first cracking load about 11%-22% of the ultimate punching shear capacity of each slab. The first cracking load of the control specimen A1 is about 38% more than that of specimen B1. Therefore, the location of opening adjacent to shorter column width affects the serviceability of the slab more than the location of opening at the corner of column.
- (2) The first cracking load of slab A2, A3 and B3 are about 25%, 13% and 20% less than that of their control slab while the first cracking load of slab B2 is 40% more than that of its control slab. As known the use of shear reinforcement is to resist shear only, therefore, the use of shear stud reinforcements is not effective in improving the serviceability load.

4.3.2 Crack patterns

For all slab-column specimens during testing, the crack patterns developed on the tension face of the slab and seen from the inside of opening due to the applied loading were marked at every total applied load starting from the first cracking load that the first crack occurred, followed by 200 N, 400 kN, 600 kN to a load just before the ultimate punching load for each slab-column specimens was reached. The crack patterns viewed from the tension face of all slab-column specimens are shown in Figure 4.2. And the inclined crack patterns seen from the inside of opening are shown in Figures 4.5 and 4.6. The information recorded on crack patterns for all slab-column specimens are summarized in Table 4.3.

During the application of punching load to the slabs, the similar sequences of crack pattern developed for the slab-column specimens were observed. There were three kinds of crack observed in the crack patterns of the slabs after punching: radial, circumferential and inclined cracks. The radial and circumferential cracks formed on the tension face of the slabs can be seen in Figures 4.2 to 4.4. The inclined crack patterns formed through the slab thickness, which can be viewed from the inside of opening, can be seen in Figure 4.5 and 4.6. First, the radial cracks started to develop from the weakest column corner where the opening was located. While these radial cracks were continuing to extend to the slab edge, the circumferential cracks between the radial cracks gradually appeared around the region near the shorter column widths and so they extended along the perimeter near the region around the column face. While the radial cracks were extending to the slab edges and the circumferential cracks were spreading to some distance from the column face, the inclined cracks were gradually developed into the thickness of slab. Gradually, the inclined cracks proceeded to extend into the compression zone of the slab. Finally, the circumferentially and radially major inclined crack caused the punching shear failure in the slab.

The initial cracks for all specimens started between about 11% and 22% of the ultimate capacity of the specimens. In all slab-column specimens, the initial radial cracks started from the corner of column where opening was located. It indicates that the stresses at shorter side of rectangular column face near opening were more critical

than other places when the slab had no opening. And then, it can be concluded that the opening located near the region around the shorter column width is a source which causes the first crack in the slab.

At failure, the failure cracks extended from about $2d$ to greater than about $3d$ from the column face and created a small failure perimeter near the region around the opening in all specimens. It is pointed out that the region near the openings is the weakest portion in the slabs regardless whether the slabs are reinforced with shear reinforcement. Therefore, it can be suggested that the shear reinforcement placed at the region near the opening should be arranged to effectively resist the punching shear. Therefore, placement of shear studs with longer or shorter rails near the region around the opening in series A slabs and the tilting of shear studs with longer or shorter rails near the region around the opening may become the reason to investigate whether it may be able to protect the cracks not to cross the opening and if it may effectively contribute in resisting punching shear stress that is critical in that region.

Figures 4.4 to 4.6 show the major punching cracks formed in all slab-column specimens and the inclined crack formed in the slab specimens that can be seen from the inside of the opening. From these figures, it can be seen that in Series A, the initial inclined cracks near the bottom of the slabs were about 22, 25, and 27 degree measured from the bottom surface of slab for slab-column specimens A1, A2, and A3, respectively, while in Series B, the initial inclined cracks near the bottom of the slabs were about 40, 45, and 53 degree measured from the bottom surface of the slab for slab B1, B2, and B3, respectively. As early concluded, because the location of opening at the shorter side of column cause severely more critical stress near the opening than that at the corner of the column, the slope of initial inclined cracks for slab in series B were larger than that for slabs in series. Therefore, the slope of the initial inclined crack can be steeper as the critical stresses in the slab are higher.

Alexander and Sirmonds (1987) stated that the inclined crack of the punching cone is between 25 and 30 degrees for slabs without openings. Therefore, it is shown that although their statement on the initial inclined crack is satisfied for the slab with opening located at the corner of the column, when the slabs have openings at the shorter side of column, their statement on the slope of the initial inclined crack is no longer accurate.

4.3.2.1 Slab-column specimens for series A

From the crack patterns of slab-column specimen A1 shown in Figures 4.2 and 4.3, it can be seen that the failure crack was characterized by the major circumferential crack and the one major circumferential crack was formed at a distance of about $3d$ around the opening in south-west direction of the slab, where d is the effective depth of the slab. It can be also observed that the major circumferential crack was formed by the inclined crack which had propagated from the vertical faces in west and south direction of opening in the slab.

In crack pattern of slab A2 from the Figures 4.2, 4.3 and 4.5, it can be seen that the major circumferential cracks were formed in both north and south directions of the slab. In north direction of slab, the failure crack was characterized by a half circle major circumferential crack around the shorter column width at a distance of about $2.5d$ from the column face. In southern direction of the slab, the failure crack was characterized by the radial crack extended to a distance of about $2.5d$ from the column face, which was formed by the inclined crack that had propagated from the vertical face in the south direction of opening.

It can be seen from the crack pattern of slab A3 that the failure cracks were characterized by three major radial cracks extended to a distance of about $4d$ from the column face. These three major radial cracks were formed by the inclined cracks which had propagated from the vertical faces in west and south direction of opening. One of the cracks started from the corner of opening, where the north and west direction of opening intersected, while the other cracks started from the south direction of opening.

When the crack patterns of slab A2 and A3 are compared, due to that slab A3 had the shear stud reinforcements on the shorter rails near the region around the opening and around the shorter column width in the north direction of the slab while the slab A2 had the shear stud reinforcements on the longer rails near the region around the opening only, it can be seen that in the north direction of slab A3, there was no major circumferential crack that characterized the failure crack.

4.3.2.2 Slab-column specimens for series B

For slab B1 from Figures 4.2, 4.4 and 4.6, it can be seen that all failure cracks occurred around the opening only. It indicates that when the rectangular slab-column connection has no opening, the regions around the shorter column widths are the weakest strength zone. When the slab has opening, the region around the opening in the slab is the weakest strength zone. In the figures of crack pattern for slab B1, there were six major cracks which were three major radial cracks and three major circumferential cracks. Two radial cracks were formed from the inclined cracks which had developed in the vertical face of southern direction of opening. The remaining one radial crack and other two circumferential cracks propagated from the inclined crack that had developed in the west vertical face of opening. The remaining one circumferential crack was formed by the inclined crack which propagated from the vertical face in the eastern direction of the opening. All failure cracks extended from the vertical face of opening to about $3d$ from the column face.

As for slab B2 from Figures 4.2 and 4.6, it is observed that the failure cracks were formed in two directions: west and south direction of opening. The failure crack in the southern direction of opening was from the column face to a distance of about $3d$ while the failure crack in the western direction was from the column face to a distance of about $4d$. Mainly, six major radial cracks propagated from the inclined cracks developed in the vertical faces of opening.

As for slab B3 from Figure 4.2 and 4.5, it can be seen that the failure cracks occurred around the opening. There were four major radial cracks, which characterized the failure cracks, propagated from the inclined cracks that had developed at vertical faces in east, south and west direction of opening. They extended to a distance of about $3d$.

The failure crack in slab B2 is narrower than in slab B3. This could be due to that the longer rail of shear studs used in slab B2 has more effectiveness than the shorter rail of shear studs used in slab B3.

4.3.2.3 Findings

From the crack patterns marked on the tension face of the slabs and in the vertical faces of openings, the findings are summarized as the following.

- (1) The opening located near the region around the shorter column width is a source which causes the first crack in the slab.
- (2) The slope of the initial inclined crack can be steeper as the critical stresses in the slab are higher. For slabs A1, A2 and A3 with opening at the corner of column, the inclined cracks near the bottom of the slabs were about 22, 25, and 27 degree measured from the bottom surface of slab, respectively. For slabs B1, B2 and B3 with opening at the shorter column width, the inclined cracks near the bottom of the slabs were about 40, 45, and 53 degree measured from the bottom surface of the slab, respectively.
- (3) When the crack patterns of slab-column specimens of each series are compared with their respective control specimen, it can be seen evidently that when the slab has shear stud reinforcement, the characterization of failure crack changes to major radial crack from major circumferential crack.
- (4) When the control slab had no shear stud reinforcements, the failure cracks were occurred at a distance about $3d$ from the column face. For other slabs with shear stud reinforcement, the failure cracks were occurred at the distance of about $2.5d$ to greater than $3d$.
- (5) The failure crack in slab B2 is narrower than the failure crack in slab B3. Therefore, the longer rails of shear stud used in slab B2 is more effective than the shorter rails of shear stud used in slab B3.

Table 4.3 Summary of recorded information on crack patterns for slab-column specimens

Series	Spec:	SSR		Number of major cracks						Failure crack			Slope of initial inclined crack near the bottom of the slab from bottom surface of the slab
				Circumferential crack			Radial crack						
		Around the shorter column width	Around the opening	Around the shorter column width	Distance from the column face	Around the shorter column width	Around the opening	Extended distance from the column face	Around the shorter column width in northern direction of slab	Around the opening in southern direction of slab	Distance of failure crack from the column face		
A	A1				1	$\approx 3d$			$\approx 3d$		✓	$\approx 3d$	22°
	A2		✓	1		$\approx 2.5d$		1	$\approx 2.5d$	✓	✓	$\approx 2.5d$	25°
	A3	✓	✓					3	$> 3d$		✓	$> 3d$	27°
B	B1	-	-	-	3	$< 3d$	-	4	$< 3d$	-	✓	$< 3d$	40°
	B2	-	✓	-	1	$> 3d$	-	3	$> 3d$	-	✓	$> 3d$	45°
	B3	✓	✓	-	-	-	-	4	$\approx 3d$	-	✓	$\approx 3d$	53°

d = effective depth of the slab

SSR = shear stud reinforcement

Figure 4.2 Top views of slab-column specimens at failure

Figure 4.3 Crack pattern on horizontal surface of slab-column specimens for series A

Figure 4.4 Crack pattern on horizontal surface of slab-column specimens for series B

Figure 4.5 Inclined crack patterns inside the opening of slab-column specimens for series A

Figure 4.6 Inclined crack patterns inside the opening of slab-column specimens for series B

4.3.3 Deflection

In order to investigate the effectiveness of shear studs used in the slab in enhancing and recovering the stiffness of the slabs lost due to presence of opening, the deflections of the slab specimens were measured by using the 17 number of LVDTs (D1 to D17) (Linear Variable Differential Transducer) placed at various locations at the top and bottom of the slabs. The relative deflections were calculated by taking the difference between the readings of LVDTs that were placed at top and bottom of the slab and the reading of LVDT (D17) that was placed directly at the center of column. Using these relative deflections, the load-deflection curves were plotted and so the deflection-distance curves for series A and B of slab according to the east-west and north-south direction of the slab were plotted corresponding to the applied load values of 360 kN and 680kN and 360 kN and 560 kN, respectively. Load-deflection and deflection-distance curves for each slab are shown in Figures 4.7 to 4.12 and Figures 4.13 to 4.16, respectively.

From Figures 4.7 to 4.16, the similar observations on the deflection for all slab-column specimens are as follows.

From load-deflection curves of Figures 4.7 to 4.12, first similar observation is that the deflections of slabs happened in all slab-column specimens tend to increase away from the column face. The deflection-distance curves from Figures 4.13 to 4.16 also show such observation that the deflection increases as the distance is further away from the column face. It is due to the fact that the slab is stiffer near the region around the column face than in the region away from the column face.

Second similar observation from the load-deflection curves of Figures 4.7 to 4.12 is that all slab-column specimens underwent lesser deflection in east-west direction i.e. direction of longer column width than in north-south deflection i.e. direction of shorter column width. This observation indicates that the region around the shorter column width in north-south direction of the slab is not stiffer than the region around the longer column width in east-west direction of the slab. Therefore, based on the first and second similar observation, it can be concluded that if the opening in the slab

is needed, it is better to place it near the face of column. The location of opening at the longer column width in the slab is better than the location of opening at the shorter column width in the slab.

From the load-deflection curves from Figures 4.7 to 4.12, third similar observation is that in each slabs with shear studs, the slope of load-deflection curves gets smaller at about 35% of the ultimate load. It is due to the fact that the stiffness of the slabs decreased when the inclined cracks occurred and the flexural cracks increased more. After reaching the ultimate punching capacity of the slabs, the slope deflection curve of the specimens with shear stud reinforcements were more gradual than that of the specimens without shear stud reinforcement. It indicates that the slabs with shear stud reinforcements behaved in more ductile manner than slabs without shear stud reinforcements.

Last similar observation from Figures 4.13 to 4.16 is that the north-south direction of the slabs had larger deflection than the east-west direction of the slabs, especially the deflection in south direction of the slabs were the largest in all direction of the slabs. It indicates that the south direction of the slab had the lowest stiffness in all directions of the slabs due to the presence of opening at the shorter column width and at the corner of column in south direction of the slab.

Then, the different observations on the deflections of the slabs from series A and B are discussed as follows.

4.3.3.1 Slab-column specimens for series A

The deflection profiles of slabs for series A in east-west and north-south directions are shown in Figures 4.13 and 4.14, respectively. From these figures, the following discussions are made on how the location of opening and the shear studs affect on the stiffness of the respective slabs from series A.

For control slab A1, it is observed from Figure 4.13a that the deflections of the control slab A1 in east and west direction are approximately equal and not

significantly large. It is pointed out that location of opening at the corner of slab in series A did not affect the stiffness of the slab in east-west direction remarkably. Figure 4.13b shows that the control slab A1 had larger deflection in south direction than in north direction. It indicates that the control slab A1 was not stiffer in south direction than in north direction because the opening in slab A1 was located at the corner of column in south direction.

As for slab A2, Figure 4.13a and 4.13b show that the deflection of slab A2 in east direction was larger than that of control slab A1. It indicates that the east direction of slab A2 was not as stiff as even the control slab A1. Therefore, the placement of longer rails A21 and A22 were not effective in improving the stiffness of the slabs in east direction. Then, it can be observed from Figure 4.14b is that the slab A2 had a little larger deflection in north-south direction than the control slab A1. It indicates that the stud rail A 23 in slab A2 was not effective on the stiffness of the slab decreased due to opening. Furthermore, it is observed from Figure 4.13a and 4.13b that the deflection of slab A2 in west direction was less than that of control slab A1 and then the slab A2 underwent lesser deflection in west direction than in east direction. It indicates that the west direction of slab A2 was stiffer than the control slab A1 and then the slab A2 was stiffer in west direction than in east direction even if the opening was located at the corner of the column in south-west direction of the slab. It is evident that the presence of the longer rails A24 and A25 in slab A2, which were placed at the side of longer column width near the opening in west direction, could significantly improve the stiffness of slab in addition to recover the stiffness decreased due to opening.

As for slab A3, it is observed from Figures 4.13 and 4.14 that the slab A3 underwent the larger deflection than the control slab A1 in all directions. It indicates that the slab A3 was not as stiff as even the control slab A1. Figure 4.13a and 4.13b show that slab A3 had larger deflection in west direction than in east direction at larger load 680 kN although the deflection of slab A3 had not much difference in east and west direction at the smaller load 360 kN. Thus, the shorter rails A32, A33, A35 and A36 placed in east-west direction of slab A3 had no effectiveness in improving the stiffness of the slab. This may be due to that the lengths of these stud rails were not enough to be

effective in improving the stiffness of the slab decreased due to opening. Figures 4.14a and 4.14b show that the slab A3 had larger deflection in south direction than in north direction. It indicates that the south direction of slab A3 was not stiffer than the north direction of slab A3. This is due to the opening located at the corner of column in south direction of slab A3. Thus, the shorter rails A31 and A34 in north-south direction of slab A3 were not effective in recovering the stiffness decreased due to opening.

Furthermore, Figures 4.13 and 4.14 show that the east direction of slab A2 had larger deflection than in that direction of slab A3 and the deflection of slab A2 in north direction was approximately equal to the deflection of slab A3 in that direction. Thus, in east direction, the longer rails A21 and A22 in slab A2 were more effective in improving the stiffness of the slab than the shorter rails A32 and A33 in slab A3 although the longer rails A21 and A22 were concluded not to be effective in recovering the stiffness of the slab decreased do to opening as compared to the control slab A1 previously discussed. Figures 4.13 and 4.14 show that in west and south direction, the slab A2 had smaller deflection than the slab A3. It indicates that in south direction, the longer rail A23 in slab A2 was more effective than the shorter rail A34 in slab A. This is because of the longer length of rail A23 than rail A34.

4.3.3.2 Slab-column specimens for series B

The deflection profiles of slabs for series B in east-west and north-south directions with the applied loads 360 kN and 560 kN are shown in Figures 4.15 and 4.16, respectively. From these figures, the following discussions are made on how the location of opening and the shear studs affect the stiffness of the respective slabs of series B.

For control slab B1, like the control slab A1, it is similarly observed from the deflection-distance graphs in Figure 4.15 that the deflections in west and east direction are approximately equal and not significantly large. It is pointed out that location of opening at the shorter column width in series B did not affect the stiffness of the slab in east-west direction remarkably. Figure 4.16 show that the control slab

B1 had larger deflection in south direction than in north direction. It indicates that the control slab B1 was not stiffer in south direction than in north direction because the opening in slab B1 was located at the shorter column width in south direction.

As for slab B2, it can be seen from Figures 4.15 that in east-west direction, the slab B2 had smaller deflection than the control slab B1 although the slab B2 had approximately equal deflection in east direction to the control slab B1 at the lower load of 360 kN. It indicates that the slab B2 was stiffer in east-west direction than the control slab B1. This is due to the fact that the presence of longer stud rails B21, B22, B25 and B26 was effective in improving the stiffness of the slab in that direction. Figure 4.16 show that the slab B2 had smaller deflection in south direction than the control slab B1 while the slab B2 had a little larger deflection in north direction than the control slab B1. It indicates that the longer inclined rails B23 and B24 were effective in improving the stiffness of the slab decreased due to opening.

As for slab B3, Figures 4.15 shows that in east direction, the slab B3 had larger deflection than the control slab B1 while in west direction, the slab B3 had a little smaller deflection at the larger load 560 kN than the control slab B1. It indicates that in east direction, the slab B3 was not stiffer than the control slab B1. Therefore, the presence of shorter rail B33 and B36 in east-west direction of slab B3 was not effective in improving the stiffness of the slab decreased due to opening. And, Figure 4.16 shows that the slab B3 had lesser deflection in south direction than the control slab B1 while the slab B3 had larger deflection in north direction than the control slab B1. It indicates that the shorter inclined rails B34 and B35 were effective in improving the stiffness of the slab decreased due to opening while the shorter rails B31 and B32 were not effective in improving the stiffness of the slab decreased due to opening.

Figures 4.15 and 4.16 also show that the slab B2 had lesser deflection in all directions than the slab B3. It indicates that the amount and arrangement of shear studs in slab B2 was more effective in recovering the stiffness of the slab decreased due to opening than the amount and arrangement of shear studs in the slab B2.

4.3.3.3 Findings

Based on the discussion regarding to the deflection of the slabs due to the applied load are summarized as follows.

- (1) Both locations of opening at the corner of column and at the shorter column width in the slab can affect significantly the stiffness of the slab in the direction where opening is located but they do not affect remarkably the stiffness of the slab in the longer column width direction of slab where opening is not located.
- (2) The slab is stiffer near the region around the column face than in the region away from the column face.
- (3) The slabs with shear stud reinforcements could be able to behave in more ductile manner than slabs without shear stud reinforcements.
- (4) The longer rails A24 and A25 in west direction of slab A2, which were placed at the side of longer column width near the opening, significantly improved the stiffness of slab in addition to recover the stiffness decreased due to opening.
- (5) All shorter rails placed in slab A3 had no effectiveness in improving the stiffness of the slab. This may be due to that these rails had not enough length to be effective in improving the stiffness of the slab decreased due to opening.
- (6) All longer rails placed in slab B2 were effective in improving the stiffness of the slab in east-west direction and north direction. However, due to the absence of rail in north direction of the slab B2, the north direction of the slab B2 was not as stiff as even the control slab B1 in that direction. The amount and arrangement of shear studs placed in slab B2 could give more effective in improving the stiffness of the slab than the amount and arrangement of shear studs placed in slab B3.

Figure 4.9 Load vs displacement curve for slab A3

Figure 4.10 Load vs displacement curve for slab B1

Figure 4.11 Load vs displacement curve for slab B2

Figure 4.12 Load vs displacement curve for slab B3

(a) Deflection vs distance graph for A series in east-west direction at 360 kN load

(b) Deflection vs distance graph for A series in east-west direction at 680 kN load

Figure 4.13 Deflection vs distance graphs for A series in east-west direction

(a) Deflection vs distance graph for A series in north-south direction at 360 kN load

(b) Deflection vs distance graph for A series in north-south direction at 680 kN load

Figure 4.14 Deflection vs distance graph for A series in north-south direction

(a) Deflection vs distance graph for B series in east-west direction at 360 kN load

(b) Deflection vs distance graph for B series in east-west direction at 560 kN load

Figure 4.15 Deflection vs distance graph for B series in east-west direction

(a) Deflection vs distance graph for B series in north-south direction at 360 kN load

(b) Deflection vs distance graph for B series in north-south direction at 560 kN load

Figure 4.16 Deflection vs distance graph for B series in north-south direction

4.4 Analyzing on the strain gauge measurements

In order to classify the failure of the slab-column specimens occurred due to applied load was flexural failure or punching shear failure and to investigate the effectiveness of shear studs with various amount and arrangement used in the slabs in enhancing and recovering the punching shear strength of the slabs, the strains in the flexural reinforcements and shear stud reinforcements due to the applied load need to be known.

In order to measure the strains in the flexural and shear stud reinforcements throughout the test, strain gauges were installed on all shear stud reinforcements and the selected flexural reinforcements at the distance of $1d$ and $2d$ from the column face and at the critical perimeter of $0.5d$ from the outermost peripheral of shear studs, where d is the effective depth of the slab. Around the opening, six strain gauges were installed at distance of $1d$ and $2d$ from the opening. The strain gauge readings recorded during testing were tabulated as shown in appendix A. Besides that, the load-strain curves for tension bars and shear studs, and strain-displacements curves for shear studs were plotted. In this case, because the strains recorded on the tension bars cut due to opening were relatively small as compared to the strains on the other bars, no load-strain curves about the tension bars cut due to opening were plotted. For tension bars placed around the column face and shear studs, the plotted load-strain curves and strain-displacements curves are shown in Figure 4.17 to 4.24.

4.4.1 Analysis of strains for flexural reinforcements

From the load-strain curves for flexural reinforcements shown in Figures 4.17 to 4.19 and Figures 4.21 to 4.24, the similar observations about the strains recorded on the flexural reinforcements in all slabs are as follows:

The first similar observations from the load-strain curves for flexural reinforcements shown in Figures 4.17 to 4.19 and Figures 4.21 to 4.24 is that the strain gauge readings of the flexural bars in all slabs increased a little only with the load increment up to 11% to 22% of the applied load to the slabs. Second similar observation is that after this, the increasing rate in all the strain gauge readings became larger with the

load increment up to about 35% of the ultimate punching capacity of the slab. These two similar observations indicate that because after the flexural cracks begin to appear at 11% to 22% of the ultimate punching shear capacity of the slab and the flexural cracks continue to increase up to reach about 35 % of the ultimate punching shear capacity of the slab, the strains in the flexural reinforcements increased.

Third similar observation is that after reaching about 35% of the ultimate punching capacity of the slab, the increasing rate of the strains in the flexural reinforcement became larger up to the ultimate punching capacity of the slab. It indicates that after the inclined shear cracks appeared at about 35% of the ultimate punching shear capacity of the slab, the flexural and inclined shear cracks gradually increased again up to the ultimate punching shear capacity of the slabs. Fourth similar observation is that after reaching the ultimate punching shear capacity of the slabs, there was a sudden drop in the value of the applied load prior to the flexural reinforcement reaching the yielding strain. It indicates that all the slab-column specimens had failed by punching shear failure.

From the load-strain curves shown in Figures 4.17 to 4.19 and Figures 4.21 to 4.24 for slabs of series A and B, respectively, fifth similar observation can be seen that the strains observed near the region around the column perimeter tend to be larger than the strains observed at the region further away from the column perimeter. It indicates that the higher stresses concentrated near the region around the column perimeter than further away from the column perimeter. Sixth similar observation which can be seen from these figures is that the strains observed in the flexural bars of the slabs A2 and A3, and B2 and B3 with shear stud reinforcement were larger than the strains observed in the flexural bars of their respective control slab A1 and B1 without shear stud reinforcement, respectively. The use of shear studs in the slabs is really effective in decreasing the stresses produced from the applied punching load.

There is no discussion about the strains recorded on the flexural bars cut due to opening because it can be seen from the strain gauge readings recorded during testing shown in Tables A1, A2, A3, B1, B2 and B3 in appendix A that the strains recorded on the flexural bars cut due to opening were relatively small as compared to the strains on the other bars.

4.4.1.1 Slab-column specimens for series A

In this series, the slab A1 is the control slab without shear stud reinforcements. From the load-strain curves for slab A1 shown in Figure 4.17, it can be observed that the strain of the longitudinal bar in the south direction of the slab around the opening were larger than the strain of longitudinal bars in the north direction of the slab. This indicates that due to opening, the stresses at region near the opening in the south direction of the slab A1 became higher than in the north direction of the slab A1 where the opening was not located. However, as in the east-west direction of the slab A1, the strains recorded on the transverse bars placed near the opening were not so large and not also as large as the strains recorded on the transverse bars away from opening. Therefore, it indicates that due to opening located at the corner of the column, the uneven stress distribution in the east-west direction of the slab could not be occurred due to opening in addition to that the higher stress concentration could not be occurred near the opening itself in the west direction of the slab.

For slab A2 with longer stud rails, it can be seen from Figure 4.18 that smaller strains were observed on the flexural bars in south direction of the slab than in north direction of the slab. This is due to the fact that the presence of longer stud rail A23 in south direction of the slab could decrease the stresses due to opening near the region around the shorter column width in that direction of the slab. As in the east-west direction of the slab away from opening, the strains observed on the flexural bar in west direction of the slab were smaller than that observed on the flexural bar in east direction of the slab. But as in the east-west direction of the slab near the opening, the strains observed on the flexural bar in west direction of the slab were larger than that observed on the flexural bar in east direction of the slab. It indicates that independent on the shear reinforcements those were placed near the region around the opening, the failure cracks would occur just near the opening. The strains in the nearest transverse flexural bars with opening in west direction of the slab would be still larger than that in other flexural bars away from opening.

As for slab A3 with shorter stud rails than slab A2, it can be seen from Figure 4.19 that the smaller strain on the flexural bar at strain gauge V4 in south direction of the

slab than at strain gauge V3 in north direction of the slab was observed. It indicates that the shorter stud rail A34 was effective in reducing the stress occurred due to opening. But the interesting thing is that the strains on the flexural bar in south direction of the slab were tend to be larger away from the column face. This is due to the fact that although the stud rail A34 placed in south direction of slab A3 was effective in reducing the stress due to opening, the stud rail A34 had not enough length to be able to reduce the stress occurred due to opening.

However, it can be seen from Figures 4.18 and 4.19 that the strains recorded on the flexural bars at V3 and V4 in south direction of slab A2 were larger than the strains recorded on the flexural bars at V4 and V5 in that direction of slab A3, respectively. Similarly, it can be seen that the strains recorded on the flexural bars at V1 and V2 in north direction of slab A2 were larger than the strains recorded on the flexural bars at V2 and V3 in that direction of slab A3, respectively. These facts indicate that although the shorter rail A34 placed in south direction of slab A3 had not enough length to be able to reduce the stress occurred due to opening as discussed early, the shorter rail A34 placed in south direction of slab A3 was more effective in reducing the stresses occurred due to opening than the longer rail A23 placed in that direction of slab A3. It can be due to the fact that the shorter rail A31 placed in the north direction of slab A3 contributed in reducing the stresses by the shorter rail A34 placed in south direction of slab A3.

4.4.1.2 Slab-column specimens for series B

For this series of slabs, the slab B1 is the control slab without shear studs. From the load-strain curves of Figure 4.21 for slab B1, it can be seen that the larger strains were observed in the north direction of the slab than in the south direction of the slab. This is due to the fact that the distance of the strain gauges in the north direction of the slab is closer from the column face than that in the south direction of the slab. Furthermore, the longitudinal bars in the south direction of the slab were discontinued due to the presence of opening. As a result, the discontinued longitudinal bars in the south direction of the slab were not able to take much load. In the east-west direction of the slab, the strains observed on the transverse bars at the column end located near

the opening were smaller than that at the other end. It indicates that the opening had not caused the stress concentration to occur in the region around the opening.

As for the slab B2 with longer shear stud rails tilting near the opening, the load-strain curves for slab B2 from the Figure 4.22 show that the strains observed on the longitudinal bars in the south direction of the slab B2 was smaller than the strains observed on the longitudinal bars in the north direction of the slab B2. It could be due to the same reason previously mentioned in the slab B1. In east-west direction of the slab B2, the strains recorded on the transverse bars at the column end away from opening were relatively larger than that at the other end of column close to opening. It indicates that the tilted longer rails B23 and B24 placed around the opening and other longer rails B21, B22, B25 and B26 placed on the longer column widths toward the opening were effective in reducing the stresses occurred due to the opening.

As for the slab B3 with shorter shear stud rails tilting near the opening, it can be seen from the Figure 4.23 that the strains observed on the longitudinal flexural bars get smaller in south direction of the slab than in the north direction of the slab. It also could be due to the same reason previously mentioned in the slab B1. The strains observed on the transverse bars near the column face in east-west direction of the slab, where the opening was not located, were larger than those on other flexural bars in that direction of the slab. Besides, such strains were the largest as compared to the strains observed on other flexural bars in slabs B3 and as well as in the slab B1 and B2. It indicates that the placement of shorter stud rail B33 and B36 in east-west direction of the slab B3 was not effective in reducing the higher stress and then the placement of the tilted stud rails B 34 and B35 around the opening were effective in reducing the higher stress. Otherwise, it may be due to that the opening had not caused the stress concentration to occur in the region around the opening as mentioned earlier in slab B1. Moreover, as compared to the strains observed on the longitudinal flexural bars in the north direction of the slab B1 and B2, the strains observed in the longitudinal bars in the north direction of the slab B3 were the smallest. It can be concluded that the presence of the shorter stud rail B31 and B32 at the region in the north direction of the slab B3 were significantly effective in reducing the stresses in that direction.

Figure 4.17 Load vs strain curve for tension bars used in slab A1

Figure 4.18 Load vs strain curve for tension bars used in slab A2

Figure 4.19 Load vs strain curve for tension bars used in slab A3

Figure 4.20 Strain vs distance from column face for tension bars for A series slabs

Figure 4.21 Load vs strain curve for tension bar for slab B1

Figure 4.22 Load vs strain curve for tension bars used in slab B2

Figure 4.23 Load vs strain curve for tension bars used in slab B3

Figure 4.24 Strain vs distance from column face for tension bars for B series slabs

4.4.2 Analysis of strains for shear stud reinforcements

In the slabs of series A and B, the shear stud reinforcements with various amount and arrangement were used to improve and recover the strength in resisting punching shear capacity lost due to opening. The distance of shear stud from the column face and centre-to-centre spacing of shear studs were $0.5d$ and $0.75d$, respectively, where d is the effective depth of the slab. In order to investigate the effectiveness of shear stud reinforcements related to their positions arranged in the slab, the strains occurred in the shear studs due to the applied load were measured by strain gauges installed on all shear studs.

For each shear stud placed in each slab, the strains observed during testing were tabulated and are shown in appendix A. And then, the load-strain curves were plotted. In order to better understand the effectiveness of shear stud reinforcements, the curves of strain-distance from the column face were plotted corresponding to a load just slightly smaller than the ultimate load. The arrangement of shear studs used in each slab, load-strain curves and strain-displacement curves are shown in Figures 4.25 to 4.34. In the figures for arrangements of shear studs, the shear studs are addressed as a sequence of a, b, c ... starting from the outermost shear stud from the column face to the shear stud near the column face as shown in Figures 4.25 and 4.26. The labels addressing the shear studs on the load-strain curves refer to the point that the ultimate load reached during testing.

It can be evidently seen from the load-strain curves of Figures 4.27, 4.28, 4.31 and 4.32 that the most shear studs had undergone the positive strain and these positive strains were observed to be significant in some shear studs only, and that each shear stud with significant positive strain had a strain plateau around the ultimate load. The positive strains recorded on shear studs mean that such shear studs were being pulled in tension. The main purpose of using shear studs in slabs is to increase the punching shear capacity. When the punching shear failure is preceded by the vertical separation of the slab, the shear studs crossed by the inclined cracks will be in tension due to separation. The shear studs of the slabs with significant positive strains were effective in resisting separation due to punching.

From strain-distance curves of Figures 4.29, 4.30, 4.33 and 4.34, for each strain profile, there was an optimum strain point observed on each stud rail of the slab at a particular distance from the column face. Therefore, the shear stud with the optimum strain point on that rail was the most effective in resisting punching among other shear studs on that rail. The distance of that shear stud from the column face at which the shear stud rail had the optimum strain point might be the nearest point with the circumferential failure crack on the slab surface and had been crossed by the major inclined cracks to occur the punching shear failure of the slab.

4.4.2.1 Slab-column specimens for series A

For this series, the arrangement of shear studs used in each slab is shown in Figure 4.25, and load-strain curves and strain-displacement curves are shown in Figures 4.27 to 4.30. From the load-strain curves, it can be seen that all of the shear studs in the slab could not reach their yield strain or near their yield strain at the ultimate punching load. Such similar observation also can be obtained from the strain-distance curves of Figures 4.29 and 4.30 for this series A. These figures also show that such shear studs reaching yield strain or nearly reaching yield strain were positioned on rails adjacent to the opening. This means that the inclined cracks in this series A were more severe around the opening and the location of shear studs in this region would be effective in resisting the vertical separation of the inclined cracks.

Figure 4.25 Arrangement of shear studs used in series A slabs

Figure 4.29 for slab A2 shows that the strains observed in the shear studs located on rails A23 and A24 near the opening in the west and south directions of the slab, respectively, were larger than that observed in the shear studs on rail A21, A22, and A25. Similarly, Figure 4.30 for slab A3 show that the strain observed in the shear studs on rail A34 and A35 near the opening in the west and south directions of the slab were larger than that observed in the shear studs on rail A31, A32, A33 and A36. Therefore, the most effective stud rails in resisting the vertical separation of inclined crack causing punching shear failure in slab A2 and A3 were rail A23 and A24, and A34 and A35, respectively.

Figures 4.29 and 4.30 strain-distance curves also show that when the longer stud rail A23 in slab A2 and the shorter stud rail A34 are compared, the longer stud rail A23 in slab A2 had larger strain than in the shorter stud rail A34 in slab A3. Therefore, the use of more number of shear studs on longer rail like A23 is more effective in resisting the vertical separation of inclined cracks than the less number of shear studs on shorter rail like A34 as the larger separation of inclined cracks occurred in the south direction of the slab A2. Smaller vertical separation in the south direction of the slab A3 could be due to the presence of the shear studs on rail A31 in the north direction of the slab A3. The stud rail A31 in the south direction of the slab A3 had significant strain in only one shear stud (1e) at the distance of $0.5d$ from the column face while the shear studs on rails A32 and A35 of slab A3 had very small strain. Therefore, the rail A32 and A35 in slab A3 were not much effective in resisting punching because the region in which the stud rails A32 and A35 were located was not the severe stress concentration region.

Furthermore, it can be seen from the Figure 4.29 of strain-distance curves that the optimum point of strain profile in the rails A23 and A24, which were previously shown to be effective, occurred in the shear studs at the distance between d and $2d$. And the observation for slab A3 is that the optimum point of strain profile for the effective rails A34 and A35 described previously for slab A3 occurred in the shear studs at the distance between d and $3d$. Therefore, the arrangement of longer stud rails as in the slab A2 were not as effective as the shorter stud rail as in slab A3 when the slab had opening at the corner of column.

4.4.2.2 Slab-column specimens for series B

For the slabs of this series, the arrangement of shear studs used in each slab is shown in Figure 4.26, and load-strain curves and strain-displacement curves are shown in Figures 4.31 and 4.32, and 4.33 and 4.34, respectively. From these figures, it can be seen that the shear studs tilting around the openings had larger strains than the shear studs in other locations of the slabs. Therefore, the tilted stud rails near the region around the opening were more effective in resisting the vertical separation of inclined cracks, which were more severe in the region around the opening, than the shear stud rails which are not placed around the opening.

Figure 4.26 Arrangement of shear studs used in series B slabs

It can be seen from Figures 4.33 and 4.34 that the strains recorded on the rails B21, B22, B25 and B26 in east-west direction of the slab B2 were larger than the strains recorded on the rails B33 and B36 in that direction of the slab B3. Moreover, the strains recorded on the longer rails 23 and B24 tilting near the region around the opening in the slab B2 were larger than the strains recorded on the shorter rail B35 and B36 in the slab B3 which were in the same location like in slab B2. Therefore, in contrast to longer shear stud rail in series A, the use of longer shear stud rails tilting near the region around the opening which was positioned at the shorter column width,

could be able to give a significantly more effectiveness in resisting the vertical separation of the inclined cracks due to the applied load.

From the Figures 4.33 and 4.34, it can also be seen that the strains recorded on the tilted rails B23, B24 in slab B2 and B34 and B35 in slab B3 were observed to be significant in the shear studs even at a distance of greater than $3d$ from the column face. This observation indicated that the tilted rails had to be able to intercept many inclined cracks, which had extended in radial direction around the opening region from the column face, thereby resulting in the majority of shear studs being effective. Furthermore, from this observation, it can be suggested that the major inclined cracks might have intercepted the opening. It can be observed that the largest strain on the rails B23 and B24 in slab B2 had occurred in the shear studs at the distance within $2d$ and $3d$ from the column face while the largest strain on the rails B34 and B35 in slab B3 had occurred in the shear studs at a distance within d and $2d$ from the column face.

4.4.3 Findings

The followings are the summary of important findings obtained from analysis of strains on the flexural reinforcements and shear stud reinforcements in the slabs of series A and B.

4.4.3.1 Slab-column specimens for series A

- (i) Referring to slab A1 as shown in Figure 4.17, for the slab without shear stud reinforcements, due to location of opening in south direction of the slab, the stresses near the opening in that direction of the slab were higher than the stresses in the direction of the slab where the opening was not located. And the uneven stress distribution in the east-west direction of the slab could not occur due to opening in addition to that the higher stress concentration could not occur near the opening in the west direction of the slab.
- (ii) The most effective stud rails in resisting the vertical separation of inclined crack causing punching shear failure in slab A2 were longer stud rails A23 and A24. The most effective stud rails in resisting the vertical separation of inclined crack

causing punching shear failure in slab A3 were longer rails A34 and A35. Therefore, the shear studs on rails nearest with the opening were more effective than shear studs on rails located elsewhere in the slabs.

- (v) The use of longer rails of shear studs failed to prevent the major inclined crack from intercepting opening.
- (vi) Shear studs closer to the column face tend to be more effective.

4.4.3.2 Slab-column specimens for series B

- (i) Referring to the slabs as shown in Figure 4.26, the shear studs tilting around the openings had larger strains than the shear studs in other locations of the slabs. Therefore, the tilted stud rails near the region around the opening were more effective in resisting the vertical separation of inclined cracks, which were more severe in the region around the opening, than the shear stud rails which were not placed around the opening.
- (ii) The use of longer shear stud rails tilting near the region around the opening which was positioned at the shorter column width could be able to give a significantly more effectiveness in resisting the vertical separation of the inclined cracks due to the applied load.
- (iii) The tilted rails had to be able to intercept many inclined cracks, which had extended in radial direction around the opening region from the column face.
- (iv) The largest strain on the rails B23 and B24 in slab B2 had occurred in the shear studs at the distance within $2d$ and $3d$ from the column face.
- (v) The largest strain on the rails B34 and B35 in slab B3 had occurred in the shear studs at a distance within d and $2d$ from the column face.
- (vi) The use of longer rails of shear studs failed to prevent the major inclined crack from intercepting opening.

Figure 4.27 Load vs strain for shear studs used in slab A2

Figure 4.28 Load vs strain for shear studs used in slab A3

Figure 4.29 Strain Vs distance from column for shear studs used in slab A2

Figure 4.30 Strain Vs distance from column for shear studs used in slab A3

Figure 4.31 Load vs strain for shear studs used in slab B2

Figure 4.32 Load vs strain for shear studs used in slab B3

Figure 4.33 Strain Vs distance from column for shear studs used in slab B2

Figure 4.34 Strain Vs distance from column for shear studs used in slab B3

4.5 Summary and conclusions

The openings are constructed in the vicinity of the slab-column connections because of the need to provide sanitary and electrical and mechanical ducts. When the slabs have openings around the column, the shear capacity will be reduced. To regain the loss of punching shear capacity of slab-column connections due to opening, the shear stud reinforcement can be used. Thus, it is required to study the arrangement of shear stud reinforcement in increasing the punching shear strength of the slabs.

The six number of normal strength concrete slabs with different arrangement of shear stud reinforcement were tested. Two different opening locations were considered. One was at the adjacent corner of column while the other one was at the shorter side of column. The different amount and arrangement of shear stud reinforcements in the specimens were considered. The specimens were tested to failure under monotonic loading to study the effectiveness of the shear stud reinforcement under ultimate load condition. The structural behavior of the specimens were examined in term of the ultimate capacity, the first cracking loads, failure patterns of cracks, and the deflections of slabs.

From this experimental study, the following conclusions are drawn.

- (1) If possible, the openings should be located at the longer side of column than the shorter side of column when the openings around the column are needed.
- (2) When the rectangular slab-column connection consists of opening near the column, the region near the opening is the weakest portion in the slab regardless the slab is reinforced with shear reinforcements.
- (3) The distances of critical perimeter of the slab-column connection with rectangular column may range from $2d$ to $4d$ when the slabs have openings, where d is the effective depth of slabs.
- (4) The use of shear stud reinforcements is significantly effective in recovering the punching shear capacity of the slabs lost due to openings.
- (5) The effectiveness of the shear stud reinforcements in increasing the punching shear capacity of the slabs can be different depending on their arrangement in

the slab. Therefore, the various arrangements of shear stud reinforcements accompanying with various locations of openings should be studied to obtain the more reliable data.

- (6) When the opening is located at the corner of column in interior slab-column connection with rectangular column, the use of more number of stud rails with shorter lengths is more effective in increasing punching shear capacity than the use of lesser number of stud rails with longer lengths.
- (7) However, when the opening is located at the shorter column width in the interior slab-column connection with rectangular column, the use of longer rails and tilting the rails near the opening is more effective in increasing the punching shear capacity than the use of shorter rails and tilting the rails near the opening.
- (8) Tilting of longer stud rails near the region around the opening is significantly effective in increasing the punching shear capacity and stiffness of the slab with opening.
- (9) The location of opening at the shorter column width affect the serviceability load of the slab more than that of the location of opening at the corner of longer column width.
- (10) The angle of inclined cracks becomes steeper if the slabs have openings at the shorter side of column than at the corner of column. The A-series specimens have the angle of inclined cracks ranged from 22 to 27 degrees while B-series specimens have the angle of inclined cracks ranged from 40 to 53 degrees.

CHAPTER 5

PUNCHING SHEAR CAPACITY OF SLAB-COLUMN CONNECTION BY THEORY OF PLASTICITY

Chapter 5

Punching shear capacity of slab-column connection by theory of plasticity

5.1 Introduction

In designing slab-column connections, punching shear capacity needs to be checked because the slabs surrounding the column may fail in brittle punching shear failure mode when high shear forces are applied to the slab-column connections. The punching shear failure surface occurred around the column observed in practice is a curve and extends from the face of column in compression to the tension face of slab (Braestrup, 1976). The normal and shear stresses along the failure surface work against the punching load. The punching shear failure of slab is a two-dimensional plane strain problem and is usually a combined failure mode of sliding and separation. The punching shear resistance may decrease when the slab has opening. The punching shear strength can be recovered by using shear reinforcement.

As reviewed in chapter two, there are various models previously developed to predict the punching shear capacity of slab-column connections such as plasticity model by Braestrup (1976), mechanical model by Kinnunen and Nylander (1960), truss model by Alexander and Simrnonds (1987), mechanical model by Menétrey (1996). All these models could give accurate punching shear capacity of slab-column connections to some degree.

The theory of plasticity has been used to predict the upper bound punching shear capacity for circular slab-column connections simply supported around the column under concentric load by Braestrup (1976). He assumed a failure mechanism that part of slab bounded by the yield surface was punched out in the direction of concentric load and part of slab outside the yield surface remained as rigid. The concrete was considered as a modified Coulomb's material. Both the sliding and separation failure

modes were considered in his model. He also considered the softening effect of concrete on the compressive strength by applying an effectiveness factor, which depends linearly on the square root of compressive strength of concrete.

The Braestrup's plasticity model (1976) had been modified by Jiang and Shen (1986), Bortolotti (1990), Kuang and Morley (1992), Yankelevsky and Leibowitz (1996), and Salim and Sebastian (2002) in the consideration of failure criterion for concrete and the softening of concrete on the compressive and tensile strength of concrete. However, all plasticity models previously developed could only predict the punching shear strength of slab-column connections with circular column shape. In addition, they did not consider the punching shear strength of slab-column connections when the slab has rectangular column shape, opening and shear reinforcement. Thus, there are needs to extend the plasticity model to be able to predict the punching shear capacity of slab-column connections when the slab has rectangular column, opening, and shear reinforcement.

Many design codes such as ACI 318-02, British Standard BS 8110-97, and CEB/FIP Model Code 90 use the average shear stress at the control cylindrical failure surface in a distance from the face of column to check the punching shear capacity of slab-column connections. This average shear stress on the control cylindrical surface must not exceed a particular design strength that is proportional to the tensile strength of concrete. This does not mean that the punching is associated with splitting failure in the control cylindrical failure surface, rather the control surface is assumed to represent a truncated cone or pyramid (Regan and Braestrup, 1985). In addition, the control cylindrical failure surface differs from the actual failure surface observed in practice.

The square slab-column connections have greater punching shear strength than the rectangular slab-column connections if the areas of column section are same. The punching shear strength of slab-column connections can increase as the aspect ratio and the area of column section increase. However, the punching shear resistance may decrease when the slab has opening near the column. These facts can be seen clearly in research work based on punching shear strength of slabs with openings and supported on rectangular columns by Kuang in 2001. One of the ways to increase the punching

shear strength of slab-column connections is the use of shear reinforcement. Therefore, when the slab-column connection is subjected to high shear force and has opening, the shear reinforcement should be used to recover and improve the punching shear capacity of the slab-column connections. As mentioned in literature reviews from chapter two, there are many types of shear reinforcements such as shear stud, stirrup, and shear heads. The shear stud reinforcement becomes more popular to improve and recover the punching shear capacity of slab-column connections due to its advantages than other types of shear reinforcements. In addition, from the experimental results and discussion described in chapter 4 based on the experimental program presented in chapter 3, it can also be seen that the shear studs can effectively recover the reduced punching shear strength of slab-column connections due to opening.

According to the facts described above, it can be concluded that there is no reliable plasticity model to predict the punching shear capacity of the slab-column connections with rectangular column, opening and shear reinforcement. Therefore, a plasticity model that can predict the punching shear strength of slab-column connections with rectangular column, opening and shear reinforcement should be developed. Regarding to the shear reinforcement, the shear studs can effectively recover the reduced punching shear strength of the slab-column connections due to opening. The shear stud reinforcements will be used in the development of plasticity model in this study.

In this chapter, Braestrup's plasticity model (1976) is extended to predict the upper bound punching shear capacity of rectangular slab-column connections with or without opening and with or without shear stud reinforcement. Extensive mathematical derivation of the theory of plasticity is made by author itself. First, punching shear failure criterion of concrete is established. Second, the collapse mechanism of rectangular slab-column connections is presented. Third, the principle strain along the punching crack is derived. Fourth, the dissipation energy in punching shear failure surface is described. Fifth, the upper bound solution for punching shear capacity of square or rectangular slab-column connections is presented. And finally, the flow chart is prepared to explain the procedure to predict the punching shear capacity of slab-column connections by the proposed plasticity model. Detailed derivations are presented in appendix B.

5.2 Punching shear failure criteria of concrete

The punching shear failure is a combination of sliding and separation failure modes in concrete slab-column connections. When sliding failure mode occurs, there is motion parallel to the failure surface, while motion at the separation failure mode is perpendicular to the failure surface. When the shear stress $|\tau|$ in the section exceeds the sliding resistance, sliding failure mode may occur. The sliding resistance is contributed by the cohesion and the internal friction of the material. The internal friction is a fraction μ of the normal stress σ in the section. When the sliding failure occurs in a material, this material complies with the following sliding failure condition,

$$|\tau| = c - \mu\sigma \tag{5.1}$$

- where τ = shear stress in the sliding surface
 σ = normal stress in the separation surface
 c = cohesion of concrete material
 μ = tangent value of internal friction angle of concrete material

The material that complies with sliding failure condition is called a Coulomb's material. The stress state presented by Mohr's circle at the sliding failure is shown in Figure 5.1.

Figure 5.1 Mohr's circle when sliding failure occurs

When the tensile stress σ in the material exceeds its separation resistance, separation failure mode may occur. The stress state presented by Mohr's circle at separation failure mode is shown in Figure 5.2.

Figure 5.2 Mohr's circle when separation failure occurs

In separation failure mode, the material complies with the following separation failure condition.

$$\sigma = f_A \quad (5.2)$$

where the f_A is the separation resistance of material.

The material that complies with both sliding and separation failure condition is called a modified Coulomb's material. In this study, the concrete is considered as modified Coulomb's material. The Mohr's circle of principal stresses in concrete corresponding to the stress field at punching shear failure can be presented by σ and τ coordinate system as shown in Figure 5.3. The f_{c1} and f_{c2} are principal tensile and compressive stresses in concrete, respectively. The f_c' is the cylinder compressive strength of concrete and f_t is the tensile strength of concrete. And ϕ is angle of internal friction angle of concrete. The failure condition for punching shear failure is derived as follows.

From Figure 5.3,

$$\begin{aligned} b = \text{radius of Mohr's circle} - a &= \frac{f_{c1} - f_{c2}}{2} - \left(-\frac{f_{c1} + f_{c2}}{2} \sin \phi \right) \\ \therefore b &= \frac{f_{c1} - f_{c2}}{2} + \frac{f_{c1} + f_{c2}}{2} \sin \phi \end{aligned} \quad (5.3)$$

**Figure 5.3 Failure condition with Mohr's circle of principal stresses
in concrete at punching shear failure**

However, $b = c \cos \alpha$

$$\therefore \frac{f_{c1} - f_{c2}}{2} + \frac{f_{c1} + f_{c2}}{2} \sin \varphi = c \cos \varphi \quad (5.4)$$

$$f_{c1} - f_{c2} + f_{c1} \sin \varphi + f_{c2} \sin \varphi = 2 c \cos \varphi \quad (5.5)$$

$$f_{c1}(1 + \sin \varphi) - f_{c2}(1 - \sin \varphi) = 2c \cos \varphi \quad (5.6)$$

$$f_{c1} \left(\frac{1 + \sin \varphi}{1 - \sin \varphi} \right) - f_{c2} = 2c \left(\frac{\cos \varphi}{1 - \sin \varphi} \right) \quad (5.7)$$

$$k = \frac{1 + \sin \varphi}{1 - \sin \varphi} \quad (5.8)$$

$$\sqrt{k} = \frac{\cos \varphi}{1 - \sin \varphi} \quad (5.9)$$

After substituting k and $\sqrt{k}$;

$$kf_{c1} - f_{c2} = 2c\sqrt{k} \quad (5.10)$$

The cohesion c of concrete can be determined by the simple compression case as shown in Figure 5.4. In simple compression case, $f_{c1} = 0$ and $f_{c2} = -f'_c$. The sliding failure condition for simple compression is

$$f'_c = 2c\sqrt{k} \quad (5.11)$$

$$k = \left(\frac{f'_c}{2c} \right)^2 \quad (5.11a)$$

$$c = \frac{f'_c}{2\sqrt{k}} \quad (5.12)$$

Figure 5.4 Sliding failure of simple compression case

After the cohesion c is substituted in Equation 5.10, the sliding failure condition becomes

$$k f_{c1} - f_{c2} = f'_c \tag{5.13}$$

The simple tension case is shown in Figure 5.5. In simple tension case, $f_{c1} = f_t$ and $f_{c2} = 0$. The sliding failure for simple tension case is

$$k f_t = f'_c \tag{5.14a}$$

$$f_t = \frac{f'_c}{k} \tag{5.14b}$$

$$\frac{f_t}{f'_c} = \frac{1}{k} \tag{5.14c}$$

Therefore, the tensile failure is a sliding failure when the situation is satisfied as follows:

$$\frac{f_t}{f'_c} \geq \frac{1}{k} \tag{5.15}$$

Figure 5.5 Separation failure of simple tension case

And the separation failure for simple tension case is

$$f_{c1} = f_A \quad (5.16)$$

Therefore, the tensile failure is a separation failure when the situation is satisfied as follows:

$$\frac{f_t}{f_c'} < \frac{1}{k} \quad (5.17)$$

For modified Coulomb's material, according to sliding and separation failure conditions of Equations 5.13 and 5.16, the yield condition drawn in a principal stress coordinate system is shown in Figure 5.6. By considering the failure condition at the apex from Figure 5.6, the punching shear failure condition is:

$$k f_{c1} - f_{c2} - f_c' = f_{c1} - f_t \quad (5.18)$$

$$(k-1) f_{c1} - f_{c2} = f_c' - f_t \quad (5.19)$$

Figure 5.6 Yield condition for modified Coulomb's material in plane strain

5.3 Collapse mechanism

A simply supported slab-column connection is considered. The column is square or rectangular shape. The slab is reinforced with flexural steel in both directions in such a way that the flexural failure is prevented. The vertical concentric punching load P is applied through the column center. Due to concentric punching load P , the slab fail in punching shear failure.

In punching shear failure, a collapse mechanism is considered as shown in Figure 5.7. During punching shear failure, a failure mechanism consisting in punching out of a solid revolution is generated. As such, finally, the part I of slab is punched out vertically to a vertical displacement u . The remaining part II of slab outside the failure surface is considered rigid.

The failure surface generatrix after punching shear failure is symmetric and shown in Figure 5.7a. The least slope of failure surface in the compression face of slab should not be less than 37° because the internal friction angle for concrete is at least 37° when sliding failure occurs. Thus, the radius of failure surface at the tension face of slab r_h must be equal or greater than $h \tan 37^\circ$, where the h is slab thickness.

The flexural reinforcement is assumed to be effective only in its axial direction. Due to the fact that the relative displacement of slab due to concentric load is perpendicular to the flexural reinforcement, no contribution is considered by the flexural reinforcement in the punching shear capacity. Dowel action of flexural reinforcement is neglected.

5.4 Principal strains along the punching crack

According to the assumed collapse mechanism shown in Figure 5.7, a yield zone with small thickness δ , at which the strains increase without increasing the stresses due to the concentric punching load P , is constituted around the column in the slab. The yield zone is shown with shaded portion in Figure 5.7a. The discontinuity occurs

along the yield zone. The vertical deformation u is considered constant for all points along the failure surface. The strain in the circumferential direction is zero.

Figure 5.7 Collapse mechanism in punching shear failure

The deformation u is resolved into two components, tangential component Δ_t and normal component Δ_n along the failure surface. From Figure 5.7,

$$\varepsilon_n = \frac{\Delta_n}{\delta} = \frac{u \sin \alpha}{\delta} \quad (5.20)$$

$$\varepsilon_t = 0 \quad (5.21)$$

$$\gamma_{nt} = \frac{\Delta_t}{\delta} = \frac{u \cos \alpha}{\delta} \quad (5.22)$$

However,

$$\varepsilon_1 \text{ or } \varepsilon_2 = \frac{\varepsilon_n + \varepsilon_t}{2} \pm \sqrt{\left(\frac{\varepsilon_n - \varepsilon_t}{2}\right)^2 + \left(\frac{\gamma_{nt}}{2}\right)^2} \quad (5.23)$$

By substituting $\varepsilon_n, \varepsilon_t$ and γ_{nt} ,

$$\varepsilon_1 \text{ or } \varepsilon_2 = \frac{u}{2\delta} (\sin \alpha \pm 1) \quad (5.24)$$

5.5 Dissipation energy in failure surface

Concrete is considered as modified Coulomb's material as described in the previous section. Accordingly, both sliding and separation failure are involved when the punching shear failure occur. To ascertain that both sliding and separation failure are involved, the plastic strain is defined at the apex of failure criterion for concrete shown in Figure 5.8.

Figure 5.8 Yield condition with flow rule for modified Coulomb's material

The respective dissipation energy for the sliding failure, separation failure and combined failure of sliding and separation when they occur is derived as follows.

For sliding yield surface, by using the flow rule and the sliding failure condition of Equation 5.13, the plastic strains along the sliding yield surface are:

$$\varepsilon_1 = \lambda k \quad \varepsilon_2 = -\lambda \quad (5.25)$$

$$\varepsilon^+ = \varepsilon_1 = \lambda k \quad (5.26)$$

$$\lambda = \frac{\varepsilon^+}{k} \quad (5.27)$$

$$\varepsilon^- = \varepsilon_2 = -\lambda \quad (5.28)$$

$$\frac{\varepsilon_1}{|\varepsilon_2|} = k \quad \text{or} \quad \frac{\varepsilon^+}{|\varepsilon^-|} = k \quad \text{or} \quad \varepsilon^+ = k |\varepsilon^-| \quad (5.29)$$

However, $k = \frac{1 + \sin \varphi}{1 - \sin \varphi} \geq 1$, then

$$\frac{\varepsilon^+}{|\varepsilon^-|} \geq 1 \quad (5.30)$$

where ε_1 & ε_2 = principal positive and negative strains, respectively

ε^+ & ε^- = summation of positive principal strain and summation of negative principal strain, respectively

For sliding failure to occur alone, the ratio between the positive principal strain and absolute value of negative principal strain must be equal or greater than one. From Eqs 5.8, 5.24, 5.29 and 5.30, we can show as follows:

$$\sin \varphi = \sin \alpha = \frac{k-1}{k+1} \quad (3.30a)$$

$$\alpha = \begin{cases} \varphi \\ \pi - \varphi \end{cases} \quad (3.30b)$$

Therefore, the angle of internal friction and slope of failure surface are equal at the compression zone. The initial slope at the compression face of the slab should not be less than the internal friction angle of concrete when punching shear failure occurs.

After the plastic principal strains are defined, the dissipation energy in sliding yield surface can be obtained as the following.

$$W_V = f_{c1} \varepsilon_1 + f_{c2} \varepsilon_2 \quad (5.31)$$

By substituting ε_1 and ε_2 from Equation 5.25,

$$W_V = f_{c1} \lambda k + f_{c2} (-\lambda) \quad (5.32a)$$

$$W_V = \lambda (k f_{c1} - f_{c2}) \quad (5.32b)$$

$$W_V = \lambda f_c' \quad (5.33)$$

$$W_V = \frac{2\alpha}{k} \varepsilon^+ \quad \text{or} \quad W_V = |\varepsilon^-| f_c' \quad (5.34)$$

where W_V is dissipation energy per one unit volume of yield region. Equation 5.34 gives the dissipation energy when only the sliding failure occurs. For separation yield surface, by using the flow rule and the separation failure condition of Equation 5.16, the plastic strains along the separation yield surface are:

$$\varepsilon_1 = \lambda \quad (5.35a)$$

$$\varepsilon_2 = 0 \quad (5.35b)$$

For combination of sliding and separation failure, the plastic strains are considered at the apex of yield surface in Figure 5.8. The plastic strains at the apex of yield surface are:

$$\varepsilon_1 = \lambda k + \lambda = \lambda(k+1) \quad (5.36a)$$

$$\varepsilon_2 = -\lambda \quad (5.36b)$$

From the plastic strains at the apex of yield surface,

$$\frac{\varepsilon_1}{|\varepsilon_2|} = k+1 \quad (5.37)$$

$$\frac{\varepsilon^+}{|\varepsilon^-|} = k+1 \quad (5.38)$$

$$\varepsilon^+ = |\varepsilon^-| (k+1) \quad (5.39)$$

$$|\varepsilon^-| = \varepsilon^+ - k |\varepsilon^-| \quad (5.40)$$

$$\lambda = |\varepsilon^-| = \varepsilon^+ - k |\varepsilon^-| \quad (5.41)$$

From Equations 5.29 and 5.38, we can see that (1) when the ratio between the positive principal strain and absolute value of negative principal strain is equal or greater than one, only sliding failure occurs and (2) when it is equal or greater than two, both separation and sliding failure modes occur.

Then, the dissipation energy at the apex of yield surface is

$$W_v = f_{c1} \lambda(k+1) + f_{c2} (-\lambda) \quad (5.42)$$

$$W_v = \lambda(k f_{c1} - f_{c2}) + \lambda f_{c1} \quad (5.43)$$

$$W_v = \lambda f_c' + \lambda f_t \quad (5.44)$$

$$W_v = \underbrace{|\varepsilon^-| f_c'}_{\text{Sliding failure}} + \underbrace{(\varepsilon^+ - k |\varepsilon^-|) f_t}_{\text{Separation failure}} \quad (5.45)$$

This is dissipation energy for combination of sliding and separation failure modes. We can see that the first part of the dissipation energy is for sliding failure and the second part for separation failure. In this case, to ascertain that both sliding and separation failure modes occur simultaneously, the ratio between the positive principal strain and absolute value of negative principal strain must be equal or greater than two. The dissipation energy per unit area of failure surface is $W_A = \delta W_v$.

By using Equation 5.45,

$$W_A = \delta (|\varepsilon^-| f_c' + (\varepsilon^+ - k |\varepsilon^-|) f_t) \quad (5.46)$$

However, ε_1 or $\varepsilon_2 = \frac{u}{2\delta}(\sin\alpha \pm 1)$ from Equation 5.24,

Therefore,

$$\varepsilon^+ \text{ or } |\varepsilon^-| = \frac{u}{2\delta}(\sin\alpha \pm 1) \quad (5.47)$$

By substituting ε^+ and $|\varepsilon^-|$ into Equation 5.46, the dissipation energy per unit area of failure surface is obtained as a function of the compressive strength, tensile strength and angle of internal friction of concrete.

$$W_A = \frac{1}{2} f_c' u (L - M \sin\alpha) \quad (5.48)$$

$$L = 1 - 2 \frac{f_t'}{f_c'} \frac{\sin\varphi}{1 - \sin\varphi} = 1 - \frac{f_t'}{f_c'} (k - 1) \quad (5.49)$$

$$M = 1 - 2 \frac{f_t'}{f_c'} \frac{1}{1 - \sin\varphi} = 1 - \frac{f_t'}{f_c'} (k + 1) \quad (5.50)$$

where $k = \frac{1 + \sin\varphi}{1 - \sin\varphi}$ and φ = angle of internal friction of concrete.

5.6 Upper bound punching shear capacity

5.6.1 Slab-column connection with rectangular column

The simply supported rectangular slab-column connection is considered under concentric load applied through the centre of column and is shown in Figure 5.9. The collapse mechanism considered is shown in Figure 5.10. The least upper bound value of punching shear capacity for slab-column connection is achieved as follows.

Figure 5.9 Simply supported slab-column connection under concentric load and punching shear failure surface

Chapter 5 Punching shear capacity of slab-column connection by theory of plasticity

Figure 5.10 Punching shear failure surface for simply supported slab-column connection

The total dissipation of energy in failure surface is

$$W_T = \int_A W_A dA \tag{5.51}$$

where W_A is dissipation energy per unit area of failure surface.

By substituting W_A from Equation 5.48,

$$W_T = \frac{1}{2} f'_c u \int_A (L - M \sin \alpha) dA \tag{5.52}$$

The external virtual work, W_E , is the product of applied punching load P and the displacement u in the applied punching load direction. According to virtual work principal, by equating the external virtual work and the total dissipation energy in failure surface,

$$P = \frac{1}{2} f_c' \int_A (L - M \sin \alpha) dA \quad (5.53)$$

where
$$dA = 4 \left(\frac{1}{2} \pi r + \Delta \right) \frac{dy}{\cos \alpha} \quad (5.54)$$

By substituting dA , the punching shear capacity is obtained as a function of $r(y)$.

$$P = 2 f_c' \int_0^h \frac{(L - M \sin \alpha)}{\cos \alpha} \left(\frac{1}{2} \pi r + \Delta \right) dy \quad (5.55)$$

$$\Delta = \Delta_1 + \Delta_2 \quad (5.56)$$

$$\Delta_1 = \frac{C_1}{2} - \left(\frac{C_2}{2} + r_h \right) \tan \frac{\beta_1}{2} \quad (5.56a)$$

$$\Delta_2 = \frac{C_2}{2} - \left(\frac{C_1}{2} + r_h \right) \tan \frac{\beta_2}{2} \quad (5.56b)$$

$$\beta_1 = 2 \tan^{-1} \left(\frac{c_{i1}}{C_2 + 2r_h} \right) \quad (5.57)$$

$$\beta_2 = 2 \tan^{-1} \left(\frac{c_{i2}}{C_1 + 2r_h} \right) \quad (5.58)$$

where C_1 & C_2 = shorter and longer column width, respectively

r_h = radius of failure surface at tension face of slab

c_{i1} & c_{i2} = incremental length of failure surface shown in Figure 5.10

From Equation 5.55,

$$P = 2 f_c' \int_0^h F(r) dy \quad (5.59)$$

$$(5.60)$$

and,
$$r' = \frac{dr}{dx} = \tan \alpha \quad (5.61a)$$

$$dx = \frac{dr}{r'} \quad (5.61b)$$

From the above relation, $\sin \alpha = \frac{r'}{\sqrt{1+(r')^2}}$ (5.62)

$$\cos \alpha = \frac{1}{\sqrt{1+(r')^2}} \quad (5.63)$$

By substituting Equation 5.60, the function $F(r)$ becomes $F(r, r')$ as

$$F(r, r') = \frac{1}{2} (\pi r + 2\Delta) (L \sqrt{1+(r')^2} - M r') \quad (5.64)$$

Equation 5.55 becomes,

$$P = f_c' \int_0^h (\pi r + 2\Delta) (L \sqrt{1+(r')^2} - M r') dy \quad (5.65)$$

The shape of failure surface $r(y)$ will give the minimum value of P. Therefore, the functional equation 5.65 is optimized with $r(y)$ and after using the following Euler's equation of $F - r' F_{r'} = C1$,

$$\frac{L (\pi r + 2\Delta)}{2\sqrt{1+(r')^2}} = C1 \quad (5.66)$$

and, by replacing $A = \frac{L}{2 C1}$,

$$\frac{A (\pi r + 2\Delta)}{\sqrt{1+(r')^2}} = 1 \quad (5.67)$$

After solving Equation 5.67,

$$1 + (r'(y))^2 = A^2 (\pi r + 2\Delta)^2 \quad (5.68)$$

Using Equations 5.61a, 5.61b and 5.68, the useful relations between $r(y)$ and $r'(y)$, dy and $r(y)$, and dy and $r'(y)$ to derive the punching shear capacity of slab by using Equation 5.65 are obtained as follows:

$$r(y) = \frac{1}{\pi} \left(\frac{\sqrt{1+(r'(y))^2}}{A} - 2\Delta \right) \quad (5.69a)$$

$$r'(y) = \sqrt{A^2 (2\Delta + \pi r(y))^2 - 1} \quad (5.69b)$$

$$dy = \frac{1}{\sqrt{A^2 (2\Delta + \pi r(y))^2 - 1}} dr(y) \quad (5.69c)$$

$$dy = \frac{dr'(y)}{A^2 \pi \left(\sqrt{1+(r'(y))^2} \right)} \quad (5.69d)$$

Then, by differentiating Equation 5.68 with y , the second order differential equation of $r(y)$ is

$$r''(y) = A^2 \pi (nr(y) + 2\Delta) \quad (5.70)$$

Solving the above second order differential equation, the shape of punching shear failure surface is obtained as a catenary curve as follows:

$$r(y) = -\frac{2\Delta}{\pi} + B e^{A\pi y} + C e^{-A\pi y} \quad (5.71)$$

$$r'(y) = A\pi(B e^{A\pi y} - C e^{-A\pi y}) \quad (5.72)$$

where A , B & C are constants.

By substituting $r(y)$ and $r'(y)$ from equations 5.71 and 5.72 into equation 5.68 and solving it, the relation between A , B and C is obtained.

$$A = \frac{1}{2\pi\sqrt{BC}} \quad (5.73)$$

We have the following boundary conditions,

$$(1) \ y = 0, \quad r(0) = 0 \quad (5.74a)$$

$$(2) \ y = h, \quad r(h) = r_h \quad (5.74b)$$

By substituting boundary condition (1) of Equation 5.74a into Equations 5.71,

$$B = \frac{2\Delta}{\pi} - C \quad (5.75)$$

And by substituting $y = 0$ into Equation 5.72,

$$r'(0) = A\pi(B - C) \quad (5.76)$$

By substituting boundary condition (2) of Equation 5.74b into Equations 5.72,

$$r_h = r(h) = -\frac{2\Delta}{\pi} + B e^{A\pi h} + C e^{-A\pi h} \quad (5.77)$$

and by substituting $y = h$ into Equation 5.72,

$$r'_h = r'(h) = A\pi(B e^{A\pi h} - C e^{-A\pi h}) \quad (5.78)$$

Using trial and error method, the constants A , B and C and r_h are determined to satisfy Equations 5.71, 5.73, 5.75 and 5.77.

When the concrete is cracked, the cracking reduces the concrete strength. Therefore, the friction angle of concrete is at least equal to the friction angle valid for the uncracked concrete. Due to concrete cracking, the cohesion of concrete is strongly reduced to about 50% of the cohesion of cement paste in which friction angle is zero and k is one (Nielsen, 1998). It can be seen from Eq. 5.11a that the concrete, which is assumed as modified Coulomb's material, therefore has at least a value of angle of friction of 37 degree when punching shear failure occurs. This means that the initial slope of failure surface at compression face of slab should be equal or greater than 37 degree. Thus, the analysis assuming modified Coulomb's material is valid only for $r_h \geq h \tan \alpha_0$, where α_0 is initial slope of failure surface at the compression side of slab. Therefore, punching shear capacity for slab-column connection under concentric loading is considered for two cases. First, the punching shear capacity when the initial slope of failure surface at $y = 0$ is equal or greater than 37 degree and the second, when it is less than 37 degree.

5.6.1.1 Initial slope of failure surface at the compression face of slab is equal or greater than 37°

When the initial slope of failure surface at the top of slab is equal or greater than 37 degree, the failure surface of slab is considered as a catenary curve only. The punching shear capacity of slab-column for this case can be obtained by the equation 5.65.

Figure 5.11 Punching shear failure surface in curve shape for simply supported slab-column connection under concentric load

By substituting Equations 5.69a and 5.69c into Equation 5.65,

$$P = \frac{f_c}{A \pi} \int_{r'(0)}^{r'(h)} \frac{\sqrt{1 + (r')^2} \left(L \sqrt{1 + (r')^2} - M r' \right)}{\sqrt{1 + (r')^2}} dr' \quad (5.79)$$

By solving it,

$$P = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1 + (r_h')^2} \right) - \left(\sinh^{-1}(r_0') + r_0' \sqrt{1 + (r_0')^2} \right) \right) - M \left((r_h')^2 - (r_0')^2 \right) \right) \quad (5.80)$$

where r_0' and r_h' are $r'(0)$ and $r'(h)$ from Equations 5.76 and 5.78, respectively.

In addition, by substituting Equations 5.69a, 5.69c and 5.69d into Equation 5.65,

$$P = f_c' \int_0^{r_h} (\pi r(y) + 2\Delta) \left(L \sqrt{A^2 + (\pi r(y) + 2\Delta)^2} - M \sqrt{-1 + A^2 + (\pi r(y) + 2\Delta)^2} \right) dr(y) \quad (5.81)$$

By integrating and inserting the limits, the punching shear capacity of slab-column connection is obtained as function of r_h .

$$P = \frac{f_c'}{2A^2\pi} \left(L \left(-2A\Delta \sqrt{-1 + 4A^2\Delta^2} - \ln(2A\Delta + \sqrt{-1 + 4A^2\Delta^2}) \right) + \ln \left(\sqrt{A^2(-2\Delta - \pi r_h)^2} + \sqrt{-1 + A^2(-2\Delta - \pi r_h)^2} \right) - \sqrt{A^2(-2\Delta - \pi r_h)^2} \sqrt{-1 + A^2(-2\Delta - \pi r_h)^2} \right) + M \left(-4A^2\pi\Delta r_h - A^2\pi^2 r_h^2 \right) \quad (5.82)$$

where r_h is obtained from Equation 5.77.

Thus, Equations 5.80 or 5.82 give the punching shear capacity for slab-column connection without opening and without shear reinforcement when the shape of failure surface is a catenary curve. Whether the shape of failure surface is a catenary curve is checked as follows.

Limitation to check if the shape of failure surface is a catenary curve

When the slope of failure surface at the compression face of slab is equal or greater than 37° , the tangent value of failure surface at $y = 0$ becomes

$$A\pi(B - C) \geq \tan 37^\circ = \frac{3}{4} \quad (5.83)$$

By substituting $B = \frac{2A}{\pi} - C$ from Equation 5.75,

$$C \leq \frac{8A\Delta - 3}{8A\pi} \quad (5.84)$$

By substituting $B = \frac{2A}{\pi} - C$ into Equation 5.77,

$$r_h = r(h) = \frac{2}{\pi} \left(\Delta(1 - e^{A\pi h}) + C\pi \sinh(A\pi h) \right) \quad (5.85)$$

By solving the above equation, the constant C will be obtained as,

$$C = \frac{2\Delta(1 - e^{A\pi h}) + \pi r_h}{2\pi \sinh(A\pi h)} \quad (5.86)$$

By substituting Equation 5.84 into Equation 5.86, and solving it, the radius of failure surface at the tension face of slab is

$$r_h \leq \frac{8A\Delta(1 + \cosh(A\pi h)) + 3\sinh(A\pi h)}{4A\pi} \quad (5.87)$$

Therefore,
$$\frac{3}{4}h \leq r_h \leq \frac{8A\Delta(1 + \cosh(A\pi h)) + 3\sinh(A\pi h)}{4A\pi} \quad (5.88)$$

Thus, when the radius of failure surface at the bottom of slab satisfies Equation 5.88, the equation of failure surface for $0 \leq y \leq h$ is a catenary curve.

5.6.1.2 Initial slope of failure surface at the compression face of slab is less than 37°

When the initial slope of failure surface at $y = 0$ is less than 37° , the failure surface of slab is considered as a combination of two portions; (a) straight portion with slope of 37° and (b) curved portion. The straight portion is from compression face of slab to h_0 at which straight portion intersects with curved portion. The curved portion is from h_0 to the tension face of slab. The h_0 is the vertical distance from the top of slab to the intersection of straight portion and curved portion of failure surface shown in Figure 5.12.

Figure 5.12 Punching shear failure surface composed with straight and curve portion for simply supported slab-column connection under concentric load

(a) For the straight portion of failure surface from $y = 0$ to h_0

For the straight portion of failure surface,

$$r(y) = \frac{3}{4}y \quad \text{for } 0 \leq y \leq h_0 \quad (5.89)$$

$$r'(y) = \frac{3}{4} \quad \text{for } 0 \leq y \leq h_0 \quad (5.90)$$

By substituting Equations 5.89 and 5.90 into Equation 5.65 and inserting the limit from 0 to h_0 ,

$$P_L = f_c' \int_0^{h_0} \left(2\Delta + \frac{3}{4}\pi y \right) \left(L \sqrt{1 + \left(\frac{3}{4} \right)^2} - \frac{3}{4}M \right) dy \quad (5.91)$$

And the punching shear capacity for straight portion of failure surface from the top of slab to the depth h_0 ,

$$P_L = \frac{1}{32} (5L - 3M) f_c' h_0 (16\Delta + 3\pi h_0) \quad (5.92)$$

(b) For the curved portion of failure surface from $y = h_0$ to h

For the curved portion of failure surface,

$$r(y) = -\frac{2\Delta}{\pi} + B e^{A\pi y} + C e^{-A\pi y} \quad \text{for } h_0 \leq y \leq h \quad (5.93)$$

$$r'(y) = A\pi (B e^{A\pi y} - C e^{-A\pi y}) \quad \text{for } h_0 \leq y \leq h \quad (5.94)$$

By equating the radii of failure surface by two Equations 5.89 and 5.93 at $y = h_0$

and substituting $B = \frac{2\Delta}{\pi} - C$, we get

$$\frac{3}{4}h_0 = -\frac{2\Delta}{\pi} + \left(\frac{2\Delta}{\pi} - C\right)e^{A\pi h_0} + C e^{-A\pi h_0} \quad (5.95)$$

$$h_0 = \frac{4}{3} \left(-\frac{2\Delta}{\pi} + \left(\frac{2\Delta}{\pi} - C\right)e^{A\pi h_0} + C e^{-A\pi h_0} \right) \quad (5.96)$$

From the above equations, the following new constant values for curved portion of failure surface are obtained.

$$C = \frac{-8\Delta(-1 + e^{A\pi h_0}) + 3\pi h_0}{4\pi(e^{-A\pi h_0} - e^{A\pi h_0})} \quad (5.97)$$

$$B = \frac{2\Delta}{\pi} - C \quad (5.98)$$

$$A = \frac{1}{2\pi\sqrt{BC}} \quad (5.99)$$

$$r_h = r(h) = -\frac{2\Delta}{\pi} + B e^{A\pi h} + C e^{-A\pi h} \quad (5.100)$$

By using the new constant values for curved portion of failure surface, the $r'(h_0)$ and new $r'(h)$ values can be obtained as,

$$r_{h_0}' = r'(h_0) = A\pi(B e^{A\pi h_0} - C e^{-A\pi h_0}) \quad (5.101)$$

$$r_h' = r'(h) = A\pi(B e^{A\pi h} - C e^{-A\pi h}) \quad (5.102)$$

The above Equations 5.101 and 5.102 are substituted in Equation 5.80 instead of $r'(0)$ and $r'(h)$, respectively. The punching shear capacity for curved portion of failure surface is

$$P_C = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1 + (r_h')^2} \right) - \left(\sinh^{-1}(r_{h_0}') + r_{h_0}' \sqrt{1 + (r_{h_0}')^2} \right) \right) - M \left((r_h')^2 - (r_{h_0}')^2 \right) \right) \quad (5.103)$$

The punching shear capacity P for slab-column connection when the initial slope at $y = 0$ of failure surface is less than 37 degree can be obtained by summing up the Equations 5.92 and 5.103 as:

$$P = P_L + P_C \quad (5.104)$$

Thus, Equation 5.104 gives the punching shear capacity for slab-column connection when the shape of failure surface is a combined shape of a straight line and a curve. Whether the shape of failure surface is a combined shape of a straight line and a curve is checked as follows.

Limitation to check if the shape of failure surface is a combination of a straight line and a curve

The slope of failure surface at the compression face of slab is

$$r'(0) < \frac{3}{4} \quad (5.105)$$

By solving in the same way, the radius of failure surface at the tension face of slab is

$$r_h > \frac{8 A \Delta (1 + \cosh(A \pi h)) + 3 \sinh(A \pi h)}{4 A \pi} \quad (5.106)$$

Thus, when the radius of failure surface at the tension face of slab r_h is beyond the right hand side of equation 5.106, the failure surface is a combination of a straight line and a curve.

5.6.2 Slab-column connection with opening

When the slab-column connection has opening adjacent to the column, the punching shear capacity of slab-column connection decreases due to opening. The area of failure surface is reduced due to opening. Moreover, some parts of failure surface around the opening are considered ineffective to resist punching due to opening.

We need to know which part of failure surface is ineffective due to opening. The ineffective failure surface due to opening has been limited by several ways according to codes described in chapter 2. The ineffective failure surface due to opening should be the most possible one to give the punching shear capacity lost due to opening. The punching shear capacity sustained in the ineffective area of failure surface due to opening is the amount of punching shear capacity lost due to opening. Thus, the punching shear capacity of slab-column connection with opening can be obtained by subtracting the punching shear capacity lost in ineffective area of failure surface due to opening from the punching shear capacity of slab-column connection without opening described previously.

The ineffective area of failure surface due to opening is defined as the area under the plan area of failure surface bounded by two projection lines which propagate from the inside of column section through two corners of opening and intersect at two points on the circumference of failure surface. Regarding to these two projection lines that confine the ineffective area of failure surface due to opening, there are two methods in

propagation of two projection lines considered in this study. They are classified as methods AI and AII depending on their starting points inside the column section. In method AI, the projection lines start from the center of column while in method AII, they start from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest with the opening. These methods have been suggested by Kuang (2001). For example, for the slab with opening located adjacent to the shorter column width, the ineffective areas of failure surface due to opening confined by two projection lines using by methods AI and AII are presented by the shaded areas shown in Figure 5.13.

Figure 5.13 Ineffective area of failure surface in the slab due to opening confined by two projection lines

Accordingly, the punching shear capacity sustained by the ineffective area of failure surface due to opening is defined as the punching shear capacity of slab-column connection lost due to opening. Thus, the punching shear capacity of slab-column connection with opening can be achieved as follows.

$$P_{op} = P - \bar{P}_{op} \tag{5.107}$$

where

P = punching shear capacity of slab-column connection without opening and without shear reinforcement

P_{op} = punching shear capacity of slab-column connection with opening

$\bar{P}_{op}$ = punching shear capacity of slab-column connection lost due to opening

The punching shear capacity of slab-column connection lost due to opening can be determined as the following.

$$\bar{P}_{op} \times u = \int_{op} W_A dA_{op} \quad (5.108)$$

where A_{op} = ineffective area of failure surface due to opening

W_A = dissipation energy per unit area of failure surface from Equation 5.48

After substituting W_A ,

$$\bar{P}_{op} = \frac{1}{2} f_c' \int_{A_{op}} (L - M \sin \alpha) dA_{op} \quad (5.109)$$

$$dA_{op} = L_{op} \frac{dy}{\cos \alpha} \quad (5.110)$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (5.111)$$

where L_{op} is the ineffective circumferential length of failure surface lost due to opening and η_1 & η_2 may vary depending on the opening locations. For several types of opening location, L_{op} , η_1 & η_2 are presented in the sub-sections.

From Equation 5.109,

$$\bar{P}_{op} = \frac{f_c'}{2} (\eta_1 P_{\eta_1} + \eta_2 P_{\eta_2}) \quad (5.112)$$

$$P_{\eta_1} = \frac{1}{A\pi} \left(L(r_h' - r_0') + M \left(-\sqrt{1 + (r_h')^2} + \sqrt{1 + (r_0')^2} \right) \right) \quad (5.113)$$

$$P_{\eta_2} = \frac{1}{2A^2\pi^2} \left(L \left(\sin^{-1}(r_h') - \sin^{-1}(r_0') + r_h' \sqrt{1 + (r_h')^2} - r_0' \sqrt{1 + (r_0')^2} + 4A\Delta(r_0' - r_h') \right) \right. \\ \left. + M \left((r_0')^2 - (r_h')^2 + 4A\Delta \left(\sqrt{1 + (r_h')^2} - \sqrt{1 + (r_0')^2} \right) \right) \right) \quad (5.114)$$

The ineffective areas of failure surface due to openings are different from each other depending on the locations of opening near the column. Therefore, even for the same slab-column connections, if the locations of openings are different, the punching shear capacities lost due to openings may be different. It is due to the fact that the concentration of shear stresses around the column perimeter varies along the column perimeter. The concentration of shear stresses is more critical near the region around

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the shorter column width than near the region around the longer column width. This fact has been proved by the experimental results in chapter 4 for the experimental program presented in chapter 3 that the punching shear capacity of slab with opening located adjacent to the shorter column width is less than that of slab with opening located at the corner of column. Therefore, it is reasonable that the L_{op} , η_1 and η_2 from Equations 5.111 and 5.112 may vary depending on the location of opening.

Accordingly, for several locations of opening located in the slab, the η_1 and η_2 terms included in Equation 5.111 and 5.112 need to be derived. There are five kinds of opening locations considered in this study. They are (a) opening location at adjacent corner of column, (b) opening location parallel and adjacent to the shorter side of column, (c) opening location perpendicular and adjacent to the longer side of column end, (d) opening location parallel and adjacent to the middle of longer side of column, and (e) opening location perpendicular and adjacent to the shorter side of column. All opening locations considered in this study are shown in Figure 5.14.

Figure 5.14 Different kinds of opening locations

The η_1 and η_2 from Equation 5.111 and 5.112 differ depending on the different locations of opening in the slab. Moreover, even for the slabs with the same types of opening location, the terms including in η_1 and η_2 may differ again according to different types of ineffective area of failure surface. This means that for the slabs with opening located at the same position near the column face, the ineffective area of failure surface due to opening may be different depending on the size of opening and radius of failure surface at the tension face of slab because the intersection points of two projection lines with the circumference of failure surface can be different. The η_1 and η_2 terms from Equation 5.111 and 5.112 related to five kind of opening locations considered are presented as follows. Detailed derivations of L_{op} , η_1 , η_2 and their related terms are presented in appendix B.

5.6.2.1 Ineffective area of failure surface due to opening by using method AI

The ineffective area of failure surface due to opening by using method **AI** is that the area under the plan area of failure surface bounded by two projection lines which propagated from the center of column through two comers of opening and extend up to intersect the circumference of failure surface. For five kinds of opening locations considered in this study, the η_1 and η_2 terms for Equation 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AI are as follows:

(a) Opening location at adjacent corner of column

Opening in the slab is located at adjacent corner of column. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure 5.15. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two comers of opening at B and G and they intersect the circumference of failure surface at the tension face of the slab at D and E.

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Figure 5.15 Ineffective area of failure surface due to opening location (a) by using method AI

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AI are:

$$\eta_1 = O_{e2} \tan \beta' + O_{e1} \tan \beta'' \quad (5.115a)$$

$$\eta_2 = \frac{\pi}{2} - \theta' - \theta'' - \tan \beta' - \tan \beta'' + \tan \theta' + \tan \theta'' \quad (5.115b)$$

(b) Opening location parallel and adjacent to the shorter side of column

Opening in the slab is located parallel and adjacent to the shorter side of column. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure 5.16. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two corners of opening at B and G and they intersect the circumference of failure surface at the tension face of the slab at C and F.

Figure 5.16 Ineffective area of failure surface due to opening location (b) by using method AI

The η_1 and η_2 terms for Equations 5.1 11 and 5.1 12 related to this ineffective area of failure surface due to opening by using method **AI** are:

$$\eta_1 = 2r_{11} + C_2 - C_1 \tan \beta' - 2r_{12} \tan \theta' \quad (5.116a)$$

$$\eta_2 = (\pi - 2\theta') + 2(-\tan \beta' + \tan \theta') \quad (5.116b)$$

(c) Opening location perpendicular and adjacent to the longer side of column end

For the type of opening location (c), according to where two projection lines propagate from the center of column through two corners of opening and intersect the circumference of failure surface depending on opening size and location, there may have two possible cases regarding to the ineffective area of failure surface due to opening.

Case (a)

Opening in the slab is located perpendicular and adjacent to the longer side of column end. The ineffective area of failure surface due to this location of opening by using method **AI** is shown in Figure 5.17. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two corners of opening at A and F and they intersect the circumference of

failure surface at the tension face of the slab at C and E. The intersection point C of one projection line with the circumference of failure surface is at the quadrant of failure surface parallel to longer column width while the intersection point E of the remaining projection line with the circumference of failure surface is at the quadrant of failure surface between the lines projected from the corners of column along the shorter and longer column widths as shown in Figure 5.17.

The η_1 and η_2 terms from Equation 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AI are:

$$\eta_1 = \frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \tag{5.117a}$$

$$\eta_2 = \frac{\pi}{2} - \beta' - \tan \beta'' \tag{5.117b}$$

Figure 5.17 Ineffective area of failure surface due to opening location (c) by using method AI for case (a)

Case (b)

Opening in the slab is located perpendicular and adjacent to the longer side of column end. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure 5.18. To confine the ineffective area of failure surface due to opening, two projection lines propagate from the center of the column through two corners of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at D and E. In contrast to case (a) for this type of opening location (c), both intersection points of two projection lines are at the same quadrant of failure surface as shown in Figure 5.18..

Figure 5.18 Ineffective area of failure surface due to opening location (c) by using method AI for case (b)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AI are:

$$\eta_1 = \frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \quad (5.118a)$$

$$\eta_2 = \frac{\pi}{2} - \beta' + \tan \theta'' - \tan \beta'' \quad (5.118b)$$

(d) Opening location parallel and adjacent to the middle of longer side of column

Similar to opening location (c), as for this type of opening location (d), according to where two projection lines those propagate from the center of column through two comers of opening and intersect the circumference of failure surface depending on opening size and location, there may have two possible cases regarding to the ineffective area of failure surface due to opening.

Case (a)

Opening in the slab is located parallel and adjacent to the middle of longer side of column. The ineffective area of failure surface due to this location of opening by using method A is shown in Figure 5.19. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two corners of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E each of which is at each quadrant of failure surface between the lines projected from the comers of column along the shorter and longer column widths in each end of longer column width close to opening as shown in Figure 5.19.

Figure 5.19 Ineffective area of failure surface due to opening location (d) by using method A I for case (a)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AI are:

$$\eta_1 = O_{c2} \quad (5.119a)$$

$$\eta_2 = 2 \left(\theta' - \sin \theta' + \tan \left(\frac{\pi}{2} - \beta' \right) \right) \quad (5.119b)$$

Case (b)

Opening in the slab is located parallel and adjacent to the middle of longer side of column. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure 5.20. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two corners of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E. In contrast to case (a) of this type of opening location, both intersection points of two projection lines with the circumference of failure surface are at the same quadrant of failure surface parallel to the longer column width close to opening as shown in Figure 5.20.

**Figure 5.20 Ineffective area of failure surface due to opening location (d)
by using method AI for case (b)**

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method **AI** are:

$$\eta_1 = O_{c2} \quad (5.120a)$$

$$\eta_2 = 2 \left(\frac{\pi}{2} - \beta' \right) \quad (5.120b)$$

(e) Opening location perpendicular and adjacent to the shorter side of column

Opening in the slab is located perpendicular and adjacent to the shorter side of column. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure 5.21. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two comers of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E.

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method **AI** are:

$$\eta_1 = C_1 \quad (5.121a)$$

$$\eta_2 = 2 \beta' \quad (5.121b)$$

Figure 5.21 Ineffective area of failure surface due to opening location (e) by using method AI

5.6.2.2 Ineffective area of failure surface due to opening by using method AII

The ineffective area of failure surface due to opening by using method AII is that the area under the plan area of failure surface bounded by two projection lines which start from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest with the opening. For five kinds of opening locations considered in this study, the η_1 and η_2 terms from Equation 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are as follows:

(a) Opening position at adjacent corner of column

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are same as Equation 5.115a and 5.115b, respectively.

Figure 5.22 Ineffective area of failure surface due to opening location (a) by using method AII

(b) Opening: location parallel and adjacent to the shorter side of column

The η_1 and η_2 terms from Equation 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are same as Equation 5.116a and 5.116b, respectively.

**Figure 5.23 Ineffective area of failure due to opening location (b)
by using method AII**

(c) Opening location perpendicular and adjacent to longer side of column end

Case (a)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are:

$$\eta_1 = O_{c2} \quad (5.122a)$$

$$\eta_2 = \left(\frac{\pi}{2} - \beta'\right) + \tan\left(\frac{\pi}{2} - \beta''\right) \quad (5.122b)$$

Figure 5.24 Ineffective area of failure surface due to opening location (c) by using method AII for case (a)

Case (b)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are:

$$\eta_1 = O_{c2} \tag{5.123a}$$

$$\eta_2 = \left(\frac{\pi}{2} - \beta'\right) + \theta' - \tan \theta' + \tan\left(\frac{\pi}{2} - \beta''\right) \tag{5.123b}$$

Figure 5.25 Ineffective area of failure surface due to opening location (c) by using method AII for case (b)

(d) Opening: location parallel and adjacent to the middle of longer side of column

Case (a)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are:

$$\eta_1 = O_{c2} \quad (5.124a)$$

$$\eta_2 = \theta' + \theta'' - \tan \theta' - \tan \theta'' + \tan\left(\frac{\pi}{2} - \beta'\right) + \tan\left(\frac{\pi}{2} - \beta''\right) \quad (5.124b)$$

Figure 5.26 Ineffective area of failure surface due to opening location (d) by using method AII for case (a)

Case (b)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are:

$$\eta_1 = \frac{5}{6}C_2 - \frac{C_1}{2} \tan \beta' - \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \quad (5.125a)$$

$$\eta_2 = \theta'' - \tan \theta'' - \tan \beta' + \tan\left(\frac{\pi}{2} - \beta''\right) \quad (5.125b)$$

Figure 5.27 Ineffective area of failure surface due to opening location (d) by using method AII for case (b)

Case (c)

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are:

$$\eta_1 = \frac{5}{6} C_2 - \frac{C_1}{2} \tan \beta' - \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \tag{5.126a}$$

$$\eta_2 = (\theta'' - \theta') + \tan \theta' - \tan \beta' - \tan \theta'' + \tan \left(\frac{\pi}{2} - \beta'' \right) \tag{5.126b}$$

Figure 5.28 Ineffective area of failure surface due to opening location (d) by using method AII for case (c)

(e) Opening location perpendicular and adjacent to the shorter side of column

The η_1 and η_2 terms for Equations 5.111 and 5.112 related to this ineffective area of failure surface due to opening by using method AII are:

$$\eta_1 = C_1 \tag{5.127a}$$

$$\eta_2 = 2 \beta' \tag{5.127b}$$

Figure 5.29 Ineffective area of failure surface due to opening location (e) by using method AII

5.6.3 Slab-column connection with shear reinforcement

The punching shear capacity for slab-column connection without shear reinforcement has been discussed in section 5.6.1. When the slab-column connections without opening have shear reinforcement, the same equations for punching shear capacity of the slab-column connection without shear reinforcement can be used by superimposing the contribution of shear reinforcement on the tensile strength of concrete in Equations 5.49 and 5.50. In this case, the dowel action of flexural reinforcement is neglected because it is assumed to contribute only the stresses in its axial direction. The contribution of shear reinforcement should be superimposed on the tensile strength of concrete.

The failure surface in punching shear failure of slab observed in practice involves separation failure as well as sliding failure. The sliding is resisted by the compressive and shear stress along the failure surface. The tensile strength of concrete is against the separation. The shear reinforcement can strengthen against separation. When the shear reinforcement is used, the tensile strength of slab by the contribution of concrete and shear reinforcement is:

$$f_t = f_{ct} + f_{st} \quad (5.128)$$

where f_t = tensile strength of slab

f_{ct} = tensile strength of concrete

f_{st} = tensile strength of shear reinforcement

However, the concrete is weak in tension. The ultimate punching shear failure in the slab with shear reinforcement occurs after cracking of concrete. After cracking, the concrete can resist no tension (Collins, 1978). When the slab has shear reinforcement, no contribution of concrete in tension is considered. Only shear reinforcement is assumed to resist tension. Thus,

$$f_{ct} = 0 \quad (5.129)$$

Therefore, Equation 5.128 becomes:

$$f_t = f_{st} \quad (5.130)$$

where f_{st} is tensile strength contributed by shear reinforcement.

Thus, the tensile strength in Equations 5.49, 5.50, and 5.130 for the punching shear capacity of the slab-column connection without shear reinforcement is:

$$f_t = f_{st} = \rho_s f_{sy} \sin \alpha_{avg} \quad (5.131)$$

$$A_f = \frac{2}{A^2 \pi} \left(-\sinh^{-1}(r'(0)) - r'(0) \sqrt{1 + (r'(0))^2} + \sinh^{-1}(r'(h)) + r'(0) \sqrt{1 + (r'(h))^2} \right) \quad (5.132)$$

$$\rho_s = \frac{A_s}{A_f} \quad (5.133)$$

$$\alpha_{avg} = \tan^{-1} \left(\frac{1}{2} A \pi e^{A h \pi} \left(1 + e^{A h \pi} \right) \left(-C + B e^{A h \pi} \right) \right) \quad (5.134)$$

where ρ_s = ratio of shear stud reinforcement passing through the failure surface

A_s = total area of shear studs passing through the failure surface

A_f = total area of failure surface of slab without opening after punching
 α_{avg} = average slope of failure surface

Detailed derivation of area of failure surface A_f and average slope of failure α_{avg} are presented in appendix B.

After substituting Equation 5.131 into L and M from Equations 5.49 and 5.50 and using it, the punching shear capacity for the slab-column connection with shear reinforcement is as follows:

(a) If the failure surface is a curve i.e. the initial slope of failure surface at the compression side of the slab is equal or greater than 37 degree, the punching shear capacity for the slab-column connection with shear reinforcement is:

$$P_s = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1 + (r_h')^2} \right) - \left(\sinh^{-1}(r_0') + r_0' \sqrt{1 + (r_0')^2} \right) \right) - M \left((r_h')^2 - (r_0')^2 \right) \right) \quad (5.135a)$$

where r_0' and r_h' are $r'(0)$ and $r'(h)$ from Equations 5.76 and 5.78, respectively. Or,

$$P_s = \frac{f_c'}{2A^2\pi} \left(L \left(-2A\Delta\sqrt{-1 + 4A^2\Delta^2} - \ln \left(2A\Delta + \sqrt{-1 + 4A^2\Delta^2} \right) + \ln \left(\sqrt{A^2(-2\Delta - \pi r_h)^2} + \sqrt{-1 + A^2(-2\Delta - \pi r_h)^2} \right) \right) \right. \\ \left. \ln \left(-\sqrt{A^2(-2\Delta - \pi r_h)^2} \sqrt{-1 + A^2(-2\Delta - \pi r_h)^2} \right) + M \left(-4A^2\pi\Delta r_h - A^2\pi^2 r_h^2 \right) \right) \quad (5.135b)$$

where r_h is obtained from Equation 5.77.

(b) If the failure surface is a combination of a straight line and a curve i.e. the initial slope of failure surface at the compression side of the slab is less than 37 degree, the punching shear capacity for the slab-column connection with shear reinforcement is:

$$P_s = P_{LS} + P_{CS} \quad (5.136)$$

$$P_{LS} = \frac{1}{32} (5L - 3M) f_c' h_0 (16\Delta + 3\pi h_0) \quad (5.136a)$$

$$P_{CS} = \frac{f'_c}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_{h0}) + r'_{h0} \sqrt{1 + (r'_{h0})^2} \right) \right) - M \left((r'_h)^2 - (r'_{h0})^2 \right) \right) \quad (5.136b)$$

where P_s = punching shear capacity for the slab-column connection with shear reinforcement

P_{LS} = punching shear capacity for the slab-column connection with shear reinforcement for straight portion of failure surface

P_{CS} = punching shear capacity for the slab-column with shear reinforcement for curve portion of failure surface

Then, r'_{h0} and $r'(h_0)$ can be obtained from Equations 5.101 and 5.102.

5.6.4 Punching shear capacity for slab-column connection with opening and shear reinforcement

When the slab-column connection has opening and shear reinforcement, the punching shear capacity of slab-column connection is

$$P_{op+s} = \bar{P}_s - \bar{P}_{op} \quad (5.137)$$

where P_{op+s} = punching shear capacity of slab-column connection with opening and shear reinforcement

$\bar{P}_s$ = punching shear capacity of slab-column connection without opening and with shear studs having steel ratio $\bar{\rho}_s$ (About $\bar{\rho}_s$ is explained latter.)

$\bar{P}_{op}$ = punching shear capacity of slab-column connection lost due to opening (previously described in above section 5.5.2)

Using Equations 5.135a or 5.135b and 5.136, $\bar{P}_s$ for the punching shear capacity of slab-column connections without opening and with shear reinforcement can be obtained as follows:

(a) If the failure surface is a curve or the initial slope of failure surface α_0 is equal or greater than 37° ,

$$\bar{P}_s = \frac{f'_c}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_0) + r'_0 \sqrt{1 + (r'_0)^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \quad (5.137a)$$

(b) If the failure surface is a combination of a straight line and a curve, or the initial slope of failure surface α_0 is less than 37° ,

$$\bar{P}_{LS} = \frac{1}{32} (5L - 3M) f'_c h_0 (16\Delta + 3\pi h_0) \quad (5.137b)$$

$$\bar{P}_{CS} = \frac{f'_c}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_{h0}) + r'_{h0} \sqrt{1 + (r'_{h0})^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \quad (5.137c)$$

$$\bar{P}_s = \bar{P}_{LS} + \bar{P}_{CS} \quad (5.137d)$$

However, in determining $\bar{P}_s$ regarding to the contribution of shear reinforcement, the ratio of shear stud is considered based on the net area of failure surface of slab without including the ineffective area of failure surface lost due to opening. Thus, the tensile strength f_t from Equations 5.49 and 5.50 is:

$$f_t = \bar{\rho}_s f_{sy} \sin \alpha_{avg} \quad (5.138)$$

$$\bar{A}_f = \frac{1}{2A^2\pi^2} \left[2A \eta'_1 \{ -r'(0) + r'(h) \} + \eta'_2 \{ -\sinh^{-1}(r'(0)) + \sinh^{-1}(r'(h)) + 4A\Delta r'(0) - r'(0) \sqrt{1 + (r'(0))^2} - 4A\Delta r'(h) + r'(h) \sqrt{1 + (r'(h))^2} \} \right] \quad (5.138a)$$

$$\bar{\rho}_s = \frac{A_s}{\bar{A}_f} \quad (5.138b)$$

where $\bar{\rho}_s$ = ratio of shear stud reinforcement passing through the failure surface of slab based on the net area of failure surface $\bar{A}_f$ without including ineffective area of failure surface

α_{avg} = average slope of failure surface from Equation 5.134

$\bar{A}_f$ = net area of failure surface of slab without including ineffective area of failure surface lost due to opening

A_s = total area of shear studs passing through the failure surface

The difference between P_s from Equations 5.135a or 5.135b and 5.136 and $\bar{P}_s$ from Equations 5.137, 5.137a and 5.137d is that Equations 5.131 to 5.134 where the ratio of shear stud is based the total area of failure surface of slab without opening are used to obtained P_s from Equation 5.135a or 5.135b and 5.136, while Equations 5.138 to 5.138b where the ratio of shear stud is based the net area of failure surface without including the ineffective area of failure surface due to opening are used to obtained $\bar{P}_s$ from Equations 5.137, 5.137a and 5.137d.

5.6.5 Critical radius of failure surface

In determining the punching shear capacity by plasticity model, the radius of failure surface at the bottom of slab needs to be determined to achieve the minimum punching shear capacity of slab. Therefore, the critical radius of failure surface is defined as the radius of failure surface at the tension side of slab that can be able to give the minimum punching shear capacity of the slab-column connections. When the radius of failure surface becomes larger, the slope of failure surface is flatter. The vertical components of normal and shear stresses become smaller as the radius of failure surface becomes larger. Thus, the punching shear capacity of slab decreases as the radius of failure surface increases.

In this case, the radius of failure surface beyond the support of the slab is not reasonable. Thus, there is a need to know the critical radius of failure surface not greater than the support radius, which gives the least punching shear capacity of slab-

column connections. If the critical radius of failure surface is beyond the support radius, the radius of failure surface which gives the minimum punching shear capacity is equal to support radius of slab.

The critical radius of failure surface is derived as follows:

The punching shear capacity from Equation 5.82 and 5.81 is differentiated by r_h , and $r'(0)$ and $r'(h)$, respectively, and equated to zero. The r_h is radius of failure surface at tension side of slab. The $r'(0)$ and $r'(h)$ are tangent values of failure surface at slab thickness zero and h , respectively.

From Equation 5.83,

$$\frac{\partial P}{\partial r_h} = 0 \quad (5.139)$$

By solving Equation 5.139, the critical radius of failure surface is:

$$r_{cr} = \frac{1}{\pi} \left(\frac{M}{A_{cr} \sqrt{M^2 - L^2}} - 2\Delta \right) \quad (5.140)$$

where

r_{cr} = critical radius of failure surface at tension face of slab

A_{cr} = constant that give critical radius of failure surface at tension face of slab

L , M and Δ can be obtained by using Equation 5.49, 5.50 and 5.56.

From Equation 5.81,

$$\frac{\partial P}{\partial r'(0)} = 0 \quad (5.141)$$

By solving Equation 5.141,

$$r'(0) = \frac{L}{\sqrt{M^2 - L^2}} \quad (5.142)$$

And then,
$$\frac{\partial P}{\partial r'(h)} = 0 \quad (5.143)$$

Similarly,
$$r'(h) = \frac{L}{\sqrt{M^2 - L^2}} \quad (5.144)$$

Therefore,
$$r'(0) = r'(h) \quad (5.145)$$

By substituting $r'(0)$ and $r'(h)$ from Equation 5.76 and 5.78, respectively,

$$A_{cr} = -\frac{1}{2 \Delta \csc h\left(\frac{A_{cr} h \pi}{2}\right)} \quad \text{or} \quad A_{cr} = \frac{1 - \cosh(A_{cr} h \pi)}{8 \Delta^2} \quad (5-146)$$

where A_{cr} , B_{cr} and C_{cr} are the constant terms related to critical radius of failure surface.

5.7 Procedure for calculation of punching shear capacity

The procedure in calculating the punching shear capacity of slab-column connections is as follows:

(I) Punching shear capacity of slab-column connection without opening and without shear reinforcement, P.

- (1) Input C_1 , C_2 , h , f'_c , f_t , Sdia-1 and SDia-2

C_1 & C_2 = shorter and longer column width, respectively

h = thickness of slab

f'_c = cylinder compressive strength of concrete

f_t = tensile strength of concrete

Sdia-1 = Support diameter of slab in C_1 direction

SDia-2 = Support diameter of slab in C_2 direction

- (2) Calculate minimum r_h , A and SRad.

$$\text{minimum } r_h = \frac{3}{4} h$$

$$\Delta = \frac{1}{2} (C_1 + C_2)$$

$$\text{SRad} = \frac{1}{2} \left(\frac{1}{2} (\text{SDia-1} - C_1) + \frac{1}{2} (\text{SDia-2} - C_2) \right)$$

- (3) Assume r_h ($\geq$ Minimum r_h & $\leq$ SRad).

- (4) Assume $B \left(< \frac{2\Delta}{\pi} \right)$.

- (5) Calculate C, A, and r_h .

$$C = B - \frac{2\Delta}{\pi} \quad (5.75)$$

Chapter 5 Punching shear capacity of slab-column connection by theory of plasticity

$$A = \frac{1}{2\pi\sqrt{BC}} \quad (5.73)$$

$$r_h = -\frac{2\Delta}{\pi} + B e^{A\pi h} + C e^{-A\pi h} \quad (5.77)$$

(6) Check if assumed r_h = calculated r_h .

(a) If assumed $r_h \neq$ calculated r_h , repeat step 4 with another trial value of B.

(b) If assumed r_h = calculated r_h , r_h = assumed r_h .

Take A, B, and C.

(c) Calculate A,

$$A_{cr} = \frac{1 - \cosh(Ah\pi)}{8\Delta^2} \quad (5.146)$$

(d) Check if $A_{cr} = A$.

(e) If $A_{cr} \neq A$, the radius of failure surface r_h does not reach to critical radius of failure surface r_c .

(i) Repeat step (3) with another trial r_h until to reach $r_h = r_c$.

(ii) If $r_h = r_c$, $r_{cr} = r_h$.

(f) If $A_{cr} = A$, the radius of failure surface r_h have reached to critical radius of failure surface r_c .

$$r_{cr} = r_h.$$

(7) Calculate the punching shear capacity P.

$$\varphi = 37^\circ$$

$$k = \frac{1 + \sin \varphi}{1 - \sin \varphi} \quad (5.8)$$

$$L = 1 - 2 \frac{f_t}{f_c'} \frac{\sin \varphi}{1 - \sin \varphi} = 1 - \frac{f_t}{f_c'} (k - 1) \quad (5.49)$$

$$M = 1 - 2 \frac{f_t}{f_c'} \frac{1}{1 - \sin \varphi} = 1 - \frac{f_t}{f_c'} (k + 1) \quad (5.50)$$

Calculate α_0 and limited r_h

$$\alpha_0 = \tan^{-1}(A\pi(B - C)) \quad (5.76)$$

$$\text{limited } r_h = \frac{8A\Delta(1 + \cosh(A\pi h)) + 3\sinh(A\pi h)}{4A\pi} \quad (5.88)$$

Check if $\alpha_0 \geq 37^\circ$ and $r_h \leq$ limited r_h

- (a) If $\alpha_0 \geq 37^\circ$, $r_h \leq$ limited r_h , the shape of failure surface is a catenary curve.

$$r_0' = A\pi(B - C) \quad (5.76)$$

$$r_h' = A\pi(Be^{A\pi h} - Ce^{-A\pi h}) \quad (5.78)$$

$$P = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1 + (r_h')^2} \right) - \left(\sinh^{-1}(r_0') + r_0' \sqrt{1 + (r_0')^2} \right) \right) - M \left((r_h')^2 - (r_0')^2 \right) \right) \quad (5.80)$$

- (b) If $\alpha_0 < 37^\circ$, $r_h >$ limited r_h , the shape of failure surface is a combination of a straight line and a curve.

(i) Assume h_0 ($0 < h_0 \leq h$)

(ii) Calculate h_0 .

$$h_0 = \frac{4}{3} \left(-\frac{2\Delta}{\pi} + \left(\frac{2\Delta}{\pi} - C \right) e^{A\pi h_0} + C e^{-A\pi h_0} \right) \quad (5.96)$$

Check if $r_{ho} = h_0 \tan 37^\circ$.

(iii) If not O.K., repeat step (i).

(iv) If O.K.,

Calculate new values of A, B and C

$$C = \frac{-8\Delta(-1 + e^{A\pi h_0}) + 3\pi h_0}{4\pi(e^{-A\pi h_0} - e^{A\pi h_0})} \quad (5.97)$$

$$B = \frac{2\Delta}{\pi} - C \quad (5.98)$$

$$A = \frac{1}{2\pi\sqrt{BC}} \quad (5.99)$$

$$P_L = \frac{1}{32} (5L - 3M) f_c' h_0 (16\Delta + 3\pi h_0) \quad (5.92)$$

$$P_C = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1 + (r_h')^2} \right) - \left(\sinh^{-1}(r_{ho}') + r_{ho}' \sqrt{1 + (r_{ho}')^2} \right) \right) - M \left((r_h')^2 - (r_0')^2 \right) \right) \quad (5.103)$$

$$P = P_L + P_C \quad (5.104)$$

When the slab-column connection without opening has shear reinforcement, the program continues as follows.

(II) Punching shear capacity of slab-column connection without opening and with shear reinforcement, P_s .

From the above step (1) to (7), the minimum punching shear capacity of slab-column connection without opening and without shear reinforcement, P , and its radius of failure surface are known. Constants A , B and C for radius of failure surface, L , M , α_0 , r'_0 and r'_h are also known. Here, the type of shear reinforcement used is shear stud (SSR).

- (a) Input total number of SSR, A , per one stud, and yield strength of SSR f_{sy} .
- (b) Calculate A_s , A_f , f_t , ρ_s and α_{avg}

A_s = number of SSR $\times A_s$ per one stud

$$A_f = \frac{2}{A^2 \pi} \left(-\sinh^{-1}(r'(0)) - r'(0) \sqrt{1 + (r'(0))^2} + \sinh^{-1}(r'(h)) + r'(0) \sqrt{1 + (r'(h))^2} \right) \quad (5.132)$$

$$\rho_s = \frac{A_s}{A_f} \quad (5.133)$$

$$\alpha_{avg} = \tan^{-1} \left(\frac{1}{2} A \pi e^{A h \pi} \left(1 + e^{A h \pi} \right) \left(-C + B e^{A h \pi} \right) \right) \quad (5.134)$$

$$f_t = \rho_s f_{sy} \sin \alpha_{avg} \quad (5.131)$$

- (c) Calculate P_s .

- (i) If the failure surface is a curve or $\alpha_0 \geq 37^\circ$,

$$P_s = \frac{f'_c}{2 A^2 \pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_0) + r'_0 \sqrt{1 + (r'_0)^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \quad (5.135a)$$

- (ii) If the failure surface is a combination of a straight line and a curve, or $\alpha_0 < 37^\circ$,

$$P_{LS} = \frac{1}{32} (5L - 3M) f'_c h_0 (16\Delta + 3\pi h_0) \quad (5.136a)$$

$$P_{CS} = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1 + (r_h')^2} \right) - \left(\sinh^{-1}(r_{h0}') + r_{h0}' \sqrt{1 + (r_{h0}')^2} \right) \right) - M \left((r_h')^2 - (r_0')^2 \right) \right) \quad (5.136b)$$

$$P_S = P_{LS} + P_{CS} \quad (5.136)$$

For the slab-column connection with opening and without shear reinforcement, the program continues as follows:

(III) Punching shear capacity of slab-column connection with opening and without shear reinforcement, P_{op} .

From the above step (1) to (7), the minimum punching shear capacity of slab-column connection without opening and without shear reinforcement, P , and its radius of failure surface are known. Constants A , B and C for radius of failure surface, L , M , r_0' and r_h' are also known.

- (a) Input description defined based on opening location and column shape: square or rectangular, dimensions of opening parallel to shorter column width O_{c1} , longer column width O_{c2} .

Descriptions of opening location are (i) "1" for slab-column specimen with square column, without opening and without SSR, (ii) "2" for slab-column specimen with square column, without opening and with SSR, (iii) "3" for slab-column specimen with rectangular column, without opening and without SSR, (iv) "4" for slab-column specimen with rectangular column, without opening and with SSR, (v) "5" for slab-column specimen with rectangular column, with opening and without SSR, and (vi) "6" for slab-column specimen with rectangular column, with opening and with SSR.

- (b) Calculate η_1 and η_2 related to location of opening. The η_1 and η_2 terms related to for five kinds of opening locations considered in this study are presented in section 5.5.2.1.

- (c) Calculate P_{op} .

$$\bar{P}_{op} = \frac{f_c'}{2} \left(\eta_1 P_{\eta_1} + \eta_2 P_{\eta_2} \right) \quad (5.112)$$

$$P_{\eta_1} = \frac{1}{A\pi} \left(L(r_h' - r_0') + M \left(-\sqrt{1 + (r_h')^2} + \sqrt{1 + (r_0')^2} \right) \right) \quad (5.113)$$

$$P_{\eta_2} = \frac{1}{2A^2\pi^2} \left(L \left(\sin^{-1}(r_h') - \sin^{-1}(r_0') + r_h' \sqrt{1 + (r_h')^2} - r_0' \sqrt{1 + (r_0')^2} \right) + 4A\Delta(r_0' - r_h') \right) \quad (5.114)$$

$$P_{op} = P - \bar{P}_{op} \quad (5.107)$$

Note that the formulae in this step (III) for punching shear capacity of slab-column connection lost due to opening $\bar{P}_{op}$, are only for that when the shape of failure surface is a catenary curve. When the loaded area or column section area is not a point, the shape of failure surface is usually in catenary curve.

For the slab-column connection with opening and shear reinforcement, the program continues as the following.

(IV) Punching shear capacity of slab-column connection with opening and shear reinforcement, P_{op+s}

From the above step (1) to (7), the minimum punching shear capacity of slab-column connection without opening and without shear reinforcement, P , and its radius of failure surface are known. Constants A , B and C for radius of failure surface, L , M , α_{avg} , r_0' and r_h' are also known. And from step (II(b)), A_s has been known. And then, from the step (III(c)), the punching shear capacity of slab-column connection lost due to opening $\bar{P}_{op}$ has been known. When the slab-column connection has opening and shear reinforcement, the procedure is:

(a) Calculate $\bar{A}_f$, f_i and $\bar{\rho}_s$.

$$\begin{aligned} \bar{A}_f = \frac{1}{2A^2\pi^2} & \left(2A\eta_1'(-r'(0) + r'(h)) + \eta_2' \left(-\sinh^{-1}(r'(0)) + \sinh^{-1}(r'(h)) \right) \right. \\ & \left. + 4A\Delta r'(0) - r'(0)\sqrt{1 + (r'(0))^2} - 4A\Delta r'(h) + r'(h)\sqrt{1 + (r'(h))^2} \right) \end{aligned} \quad (5.138a)$$

$$\bar{\rho}_s = \frac{A_s}{A_f} \quad (5.138b)$$

$$f_t = \rho_s f_{sy} \sin \alpha_{avg} \quad (5.138)$$

(c) calculate $\bar{P}_s$.

(i) If the failure surface is a curve or the initial slope of failure surface α_0 is equal or greater than 37° ,

$$\bar{P}_s = \frac{f'_c}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_0) + r'_0 \sqrt{1 + (r'_0)^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \quad (5.137a)$$

(ii) If the failure surface is a combination of a straight line and a curve, or the initial slope of failure surface α_0 is less than 37° ,

$$\bar{P}_{LS} = \frac{1}{32} (5L - 3M) f'_c h_0 (16\Delta + 3\pi h_0) \quad (5.137b)$$

$$\begin{aligned} \bar{P}_{CS} = \frac{f'_c}{2A^2\pi} & \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) \right. \right. \\ & \left. \left. - \left(\sinh^{-1}(r'_{h0}) + r'_{h0} \sqrt{1 + (r'_{h0})^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \end{aligned} \quad (5.137c)$$

$$\bar{P}_s = \bar{P}_{LS} + \bar{P}_{CS} \quad (5.137d)$$

$$P_{op+s} = \bar{P}_s - \bar{P}_{op} \quad (5.137)$$

Using the procedure described above, the punching shear capacities for slab-column connections with square or rectangular column shape, with or without opening and with or without shear reinforcement, which are selected from the literatures and experimental program presented in chapter three, are calculated and, the results and discussions are presented in the next chapter.

To make easier to follow the above procedure, the following flow charts are prepared.

Figure 5.30 Flow chart for calculation of punching shear capacity of slab-column connections without opening and without shear reinforcement

Known values: Minimum P (without opening and without shear reinforcement),

Figure 5.31 Flow chart for calculation of punching shear capacity of slab-column connections with opening and without shear reinforcement

Known values: Minimum P (without opening and without shear reinforcement),

Figure 5.32 Flow chart for calculation of punching shear capacity of slab-column connections without opening and with shear reinforcement

Known values: Minimum P (without opening and without shear reinforcement),

Figure 5.33 Flow chart for calculation of punching shear capacity of slab-column connections with opening and shear reinforcement

CHAPTER 6

RESULTS AND DISCUSSION ON PUNCHING SHEAR CAPACITY OF SLAB-COLUMN CONNECTION BY THEORY OF PLASTICITY

Chapter 6

Results and discussions on punching shear capacity of slab-column connections by theory of plasticity

6.1 Introduction

The formula of upper bound solution for punching shear capacity of slab-column connections with square and rectangular columns, with or without shear reinforcement, and with or without opening positioned adjacent to a column have been derived based on theory of plasticity in the previous chapter.

In the plasticity solutions, the softening effect of concrete on the compressive and tensile strength of concrete will significantly influence the accuracy of the predicted punching shear capacity. In considering the softening effect of concrete on the compressive strength of concrete, a so-called softening effect of concrete – the effectiveness factor on the cylinder or cube compressive strength of concrete can be used. Various effectiveness factors on the compressive strength of concrete related to the plasticity models have been proposed in the literature.

However, the effectiveness factors previously proposed for plasticity models in the literature are only for predicting the punching shear capacity of circular slab-column connections. Thus, for the plasticity model described in chapter five, there is a need to propose an effectiveness factor on the compressive strength of concrete. The punching shear capacity of slab by the proposed plasticity model depends on compressive and tensile strength of concrete, slab thickness and column width. Depending on these factors, a new effectiveness factor on the compressive strength of concrete for the proposed plasticity model will be recommended in the next section.

Concrete is weak in tension and no tensile strength will be assumed when concrete cracks. Thus, no tensile strength of concrete is assumed for the prediction of punching shear capacity of slab-column connections by the proposed plasticity model.

The tensile strength of the slab can be contributed by the use of shear reinforcement. Although, there are many types of shear reinforcement, due to the advantages of shear stud reinforcement described in chapter two, only shear stud reinforcement (SSR) is considered in verifying the upper-bound punching shear capacity derived in previous chapter. In addition, the plastic solution derived in previous chapter will be used to predict the increase in punching shear capacity due to the presence of SSR. The effectiveness factor on the tensile strength contributed by SSR will also be recommended for the plastic upper bound solution derived in previous chapter.

In this chapter, five locations of openings and two methods to calculate the ineffective area of failure surface due to opening considered in verifying the proposed plasticity model are explained first. Secondly, the effectiveness factor on the compressive strength of concrete is proposed. Thirdly, the effectiveness factor on the tensile strength of slab contributed by SSR to be used for the proposed plasticity model is recommended. Finally, the upper bound punching shear capacity of slab-column connection previously derived will be verified by calculating the punching shear capacity of slab-column connections:

- (1) without shear reinforcement
 - (a) with square column
 - (b) with rectangular column
 - (c) with rectangular column and opening
- (2) with shear reinforcement
 - (a) with square column
 - (b) with rectangular column and with or without opening

The calculated punching shear capacity will be compared with the test results.

After discussions are made on the comparison between the punching shear capacity of slab-column connections predicted by plasticity model and the test results,

conclusions are drawn. The calculation program prepared by Matlab software is presented in appendix C.

6.2 Consideration on Openings

The five locations of opening adjacent to a column are considered. They are (a) opening location at adjacent corner of column (type I), (b) opening location parallel and adjacent to the shorter side of column (type II), (c) opening location perpendicular and adjacent to the longer side of column end (type III), (d) opening location parallel and adjacent to the middle of longer side of column (type IV), and (e) opening location perpendicular and adjacent to the shorter side of column (type V). All opening locations are shown in Figure 6.1.

Figure 6.1 Five locations of opening

The ineffective areas of punching shear failure surface due to opening are considered by two methods; AI and AII. Method AI is that the projection lines to confine the ineffective area of failure surface propagate from the center of column section through the two corners of opening and intersect the circumference of failure surface at the tension side of the slab. While method AII is that the projection lines to confine the ineffective area of failure surface propagate from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening, which is equally divided along the longer column width. The methods to determine the ineffective area of failure surface due to opening were presented in detail in chapter 5 and appendix B. The ineffective area of failure surface due to opening for the opening located adjacent to the shorter column width presented by these two methods are shown in Figure 6.2. For each opening location, the upper bound formula derived previously used to predict the reduction of punching shear capacity due to opening is discussed.

Depending on the locations of opening, the reduction of punching shear capacity varies.

Figure 6.2 Ineffective area of failure surface due to opening

6.3 Effectiveness factor on the compressive strength of concrete

The punching shear capacity of slab-column connections predicted by Equations 5.80, 5.82 and 5.104 depends on the parameters; column width, slab thickness and effective compressive strength of concrete. As the area of column section, aspect ratio of column widths, slab thickness, and effective compressive strength of concrete increase, the punching shear capacity of slab increases. In the calculation of punching shear capacity of slab-column connections by theory of plasticity, the effectiveness factor on the compressive strength of concrete is proposed as:

$$v_c = \frac{0.8}{\sqrt{f'_c}} \left(10 + \ln \left(\frac{1}{h} \right) \right) \left(\frac{C_2}{C_1} \right)^{-\frac{1}{8}} \quad (6.1)$$

where v_c = proposed effectiveness factor on compressive strength of concrete

h = slab thickness in mm

C_1 & C_2 = shorter and longer column width, respectively, in mm

f'_c = cylinder compressive strength of concrete in MPa

6.4 Effectiveness factor on the yield strength of SSR

The tensile strength of slab contributed by concrete is assumed as zero, while the tensile strength of slab contributed by shear stud reinforcements is considered. In the plasticity model derived in chapter 5, all shear stud reinforcements (SSR) are considered to reach its yield strength when punching failure occurs. In practice, all shear reinforcements do not yield at the failure of slab. Seible et al. (1980) recommended that because of the stress level in the shear reinforcement at failure load did not reach the yield strength in all shear reinforcements, the reduced strength of shear reinforcement should be accounted for in calculating the ultimate strength. Therefore, the effective tensile strength contributed by shear reinforcement should be considered. The effectiveness factor on the tensile strength contributed by SSR for the plastic upper bound solution derived in previous chapter is proposed as

$$v_{st} = \frac{1}{12} \quad (6.2)$$

where v_{st} is the effectiveness factor on the tensile strength contributed by SSR.

6.5 Punching shear capacity of slab-column connections without shear reinforcement

6.5.1 With square column

The punching shear capacity of twelve slab-column specimens with square columns are calculated. All slabs have no opening and no shear reinforcement. They failed in pure punching failure. Details of square slab-column specimens are shown in Table 6.1. The punching shear capacity of square slab-column specimens predicted by theory of plasticity are shown in Table 6.2. Equations 6.1 proposed for effectiveness factor on the compressive strength of concrete is used and the tensile strength of concrete is assumed as zero.

Generally, the upper bound formula derived by theory of plasticity can predict the punching shear capacity quite well compared with test results for slab-column

connections with square column as shown in Table 6.2. The predicted punching shear capacity for slabs with square column are well within $\pm 15 \%$ of the test results. The proposed effectiveness factor v_c on the compressive strength concrete of Equation 6.1 and zero tensile strength of concrete can give acceptable prediction on punching shear capacity of slabs with square column.

Table 6.1 Detail of slab-column specimens with square column

Author	Spec.	C (mm)	h (mm)	d (mm)	SDia (mm)	f_c' (MPa)	f_{fy} (MPa)	ρ_f (%)	P (test) (kN)
Marzouk & Hussein (1991)	HS7	150	120	95	1500	74	490	1.193	356
	HS3	150	120	95	1500	69	490	1.473	356
	NS2	150	150	120	1500	30	490	0.944	396
	HS6	150	150	120	1500	70	490	0.944	489
	HS9	150	150	120	1500	74	490	1.611	543
	HS10	150	150	120	1500	80	490	2.333	645
	HS14	150	120	95	1500	72	490	1.473	498
	HS15	220	120	95	1500	71	490	1.473	560
Mokhtar et al. (1985)	AB1	250	150	132	1800	36	490	1.013	408
Moe (1961)	H1	254	150	114	1778	26	328	1.15	371
	S2-60	254	150	114	1778	22	399	1.53	356
	S3-60	254	150	114	1778	23	399	2.3	364

C = column width

h & d = thickness and effective depth of slab, respectively

SDia = distance between supports of slab

f_c' & f_{fy} = cylinder compressive strength of concrete and flexural steel yield strength, respectively

ρ_f = flexural steel ratio

P (test) = test result of punching shear capacity of slab-column specimen

The punching shear capacity decreases with increasing radius of failure surface. The smaller the support radius of slab, the more the punching shear capacity of slab increases. This is due to the facts that (1) when the area of failure surface is larger, the shear stress and normal stresses on this failure surface become smaller, and (2) when the radius of failure surface is larger, the vertical component of stress on the failure surface becomes smaller. The radius of failure surface at which the punching shear capacity of slab converges to minimum is usually beyond the support radius. This means that the failure surface always extend all the way to the support. Therefore, the critical radius of failure surface at which the punching shear capacity converges to minimum is finally set less than or equal to support radius which is a distance from column face to support.

Table 6.2 Punching shear capacity predicted by theory of plasticity for slab column specimens with square column

No	Spec.	f_c' (MPa)	C (mm)	h (mm)	$\frac{R_{CR}}{h}$	$\frac{SRad}{h}$	$P(Pred)$ (kN)	$\frac{P(Pred)}{P(Test)}$
1	HS7	74	150	120	5.60	5.63	355.80	1.00
2	HS3	69	150	120	5.60	5.63	343.57	0.97
3	NS2	30	150	150	4.50	4.50	334.48	0.84
4	HS6	70	150	150	4.50	4.50	510.93	1.04
5	HS9	74	150	150	4.50	4.50	525.33	0.97
6	HS10	80	150	150	4.50	4.50	546.21	0.85
7	HS14	72	150	120	5.33	5.33	422.66	0.85
8	HS15	71	220	120	5.00	5.00	503.45	0.90
9	AB1	36	250	150	5.15	5.17	434.54	1.07
10	H1	26	254	150	5.08	5.08	374.20	1.01
11	S2-60	22	254	150	5.08	5.08	344.21	0.97
12	S3-60	23	254	150	5.08	5.08	351.95	0.97

Mean 0.95

Standard deviation 0.08

COV 0.08

- SRad = support radius of slab
- R_{CR} = critical radius of failure surface at which the punching shear capacity of slab is minimum
- P (Pred) = predicted punching shear capacity of slab-column specimen by theory of plasticity

6.5.2 With rectangular column

The punching shear capacity of twelve slab-column specimens with rectangular column are calculated. All slabs have no opening and no shear reinforcement. All slabs failed in pure punching. Detail of selected rectangular slab-column specimens are shown in Table 6.3.

The punching shear capacity for the selected slab-column specimens predicted by plasticity model are shown in Table 6.4. The effectiveness factor for compressive strength of concrete of Equation 6.1 is used and tensile strength of concrete is taken as zero.

From Table 6.4, it can be seen that the punching shear capacity of slab increases as the aspect ratio of longer to shorter column width and slab thickness increase and as in the case of slab-column specimens with square column, the critical radii of failure surface that give the minimum punching shear capacity of slab-column specimens are equal to the support radii of slab. Table 6.4 shows that the ratio of the predicted punching shear capacity and test results for slabs with rectangular column falls well within $\pm 13\%$. It indicates that the proposed plasticity model with proposed effectiveness factor on the compressive strength of concrete of Equation 6.1 and consideration of zero tensile strength of concrete can give the acceptable results on the punching shear capacity of rectangular slab-column connections.

Chapter 6 Results and discussions on punching shear capacity of slab-column connection by plasticity model

Table 6.3 Detail of slab-column specimens with rectangular column

Author	Spec.	C ₁ (mm)	C ₂ (mm)	h (mm)	d (mm)	SDia-C ₁ (mm)	SDia-C ₂ (mm)	f' _c (MPa)	f _{sp} (MPa)	ρ _f (%)	P (test) (kN)
Kuang (2001)	OC13	200	600	150	107	1800	2000	36	452	1.71	568
	OC15	200	1000	150	103	1800	2500	40	452	1.76	649
	OC13_1.6	200	600	150	110	1800	2000	33	470	1.67	508
IHPT project (1999)	C13F22	250	750	200	155	2000	2400	36	460	1.66	792
	C15F22	250	1250	200	160	2000	3000	35	460	1.64	1056
Hawkins (1971)	2	203	406	152	117	2083	2083	26	460	1.12	351
	3	152	457	152	117	2083	2083	32	460	1.12	333
	4	114	494	152	117	2083	2083	31	460	1.12	331
	8	114	494	152	120.6	2083	2083	26	460	0.81	314
	9	152	305	152	120.6	2083	2083	30	460	0.77	317
Moe (1961)	R1	152	457	152	114	1780	1780	28	328	1.14	394
Hognestad (1956)	R1	152	457	152	114	1780	1780	28	329	1.64	397

C₁ & C₂ = shorter and longer column width, respectively
SDia-C₁ = distance between supports of slab parallel to shorter column width
SDia-C₂ = distance between supports of slab parallel to longer column width
P (Test) = test result of punching shear capacity for slab-column connections

Table 6.4 Upper bound solution of punching shear capacity by plasticity model for slab-column specimens with rectangular column

Spec.	$\frac{C_2}{C_1}$	h (mm)	f_c' (MPa)	$\frac{R_{CR}}{h}$	$\frac{SRad}{h}$	P (Pred) (kN)	P (test) (kN)	$\frac{P (Pred)}{P (Test)}$
OC13	3.00	150	36	5.00	5.00	482.86	568	0.85
OC15	5.00	150	40	5.17	5.17	587.37	649	0.91
OC13_1.6	3.00	150	33	5.00	5.00	462.16	508	0.91
C13F22	3.00	200	36	4.25	4.25	843.87	792	1.07
C15F22	5.00	200	35	4.38	4.38	978.02	1056	0.93
2	2.00	152	26	5.85	5.85	359.01	351	1.02
3	3.01	152	32	5.85	5.85	376.51	333	1.13
4	4.33	152	31	5.82	5.84	355.4	331	1.07
8	4.33	152	26	5.82	5.84	326.63	314	1.04
9	2.01	152	30	6.10	6.10	329.58	317	1.04
R1	3.00	152	28	4.84	4.84	382.32	394	0.97
R1	3.00	152	28	4.84	4.84	382.32	397	0.96

Mean 0.99

Standard deviation 0.08

COV 0.08

SRad = support radius of slab

SRad =support radius of slab averaging in two directions parallel to shorter and longer column width

$$= \frac{1}{2} \left(\left(\frac{SDia - C_1 - C_1}{2} \right) + \left(\frac{SDia - C_2 - C_2}{2} \right) \right)$$

P (Pred) = predicted punching shear capacity of slab-column connections by plasticity model

P (Test) = test result of punching shear capacity of slab-column connections

6.5.3 With rectangular column and opening

Twelve slab-column specimens with rectangular column and with various locations of openings near the column stubs are considered. All slab specimens have no shear reinforcement. All slab-column specimens failed in punching. Details of the specimens used are shown in Table 6.5. For the specimens ended with 1.6, the ratio of bending moments in two perpendicular directions of the slab is 1.6 while for other specimens, the bending moments in two perpendicular directions of slab is the same.

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There are five kinds of opening location near the column stubs as shown by figures in Table 6.6. The opening dimensions in all slab specimens are 200 mm x 400 mm. For specimens started with OC13, the slab dimensions between supports are 1800 mm and 2000 mm parallel to shorter and longer column widths, respectively. For specimens started with OC15, the slab dimensions between supports are 1800 mm and 2500 mm parallel to shorter and longer column widths, respectively. For A1 and B1 specimens, the slab dimensions between supports are 2000 mm and 3000 mm parallel to shorter column and longer column widths, respectively.

Table 6.5 Details of slab-column connections with rectangular column and opening

Author	Spec.	C ₁ (mm)	C ₂ (mm)	h (mm)	d (mm)	f_c' (Map)	f_{fy} (Map)	ρ_f (%)	P (test) (kN)
Kuang (2001)	OC13H50	200	600	150	109.5	36.29	452.48	1.67	443
	OC13V43	200	600	150	113.5	36.62	452.48	1.76	467
	OC13V23	200	600	150	108	36.94	452.48	1.70	484
	OC13V40	200	600	150	108.5	43.03	452.48	1.69	340
	OC13H50_1.6	200	600	150	114.5	33.09	470.23	1.60	428
	OC13V43_1.6	200	600	150	111	33.21	470.23	1.65	383
	OC13H02_1.6	200	600	150	113.5	37.53	470.23	1.61	420
	OC13H02	200	600	150	111.75	43.09	452.48	1.12	512
	OC15V43	200	1000	150	109	35.97	470.23	1.66	612
	OC15H70	200	1000	150	108.25	37.89	470.23	1.67	529
NTU- BCA Project (2001)	A1	250	1000	200	160	38.2	540	1.65	740
	B1	250	1000	200	160	36.3	540	1.65	580

C₁ = longer column width

C₂ = shorter column width

When the slab-column connections has opening, the punching shear capacity of slab-column connection is considered as the punching shear capacity sustained in the net area of failure surface without the inclusion of the ineffective area of failure surface

due to opening. The ineffective punching shear failure surface area due to opening is considered by two methods; AI and AII, which were described previously.

Using the proposed effectiveness factor on the compressive strength of concrete from Equation 6.1 and taking the tensile strength of concrete as zero, the punching shear capacities of rectangular slab-column specimens with opening are calculated. The predicted punching shear capacity of slab-column specimens with rectangular column and opening by plasticity model together with its parameters; the compressive strength of concrete, slab thickness, aspect ratio of longer to shorter column width and critical radius of failure surface are shown in Table 6.6 where the terms used are defined as follows:

$P(\text{Test})$ = test result of punching shear capacity of slab-column connection

$P(\text{Pred})$ = predicted punching shear capacity of slab-column connection

P_{op} = predicted punching shear capacity of slab-column connections with opening

AI = method AI to calculate the ineffective area of failure surface due to opening

AII = method AII to calculate the ineffective area of failure surface due to opening

R_{AI} = ratio of punching shear capacity of slab-column specimen from using method AI predicted by plasticity model and the test results

$$= \frac{P_{op}}{P(\text{test})}$$

R_{AII} = ratio of punching shear capacity of slab-column specimen from using method AII predicted by plasticity and the test results

$$= \frac{P_{op}}{P(\text{test})}$$

Chapter 6 Results and discussions on punching shear capacity of slab-column connection by plasticity model

Table 6.6 Punching shear capacity for slab-column connections with rectangular column and opening (Methods AI & AII)

Type	Slab-column connection with opening	Spec	f'_c (MPa)	$\frac{C_2}{C_1}$	h (mm)	$\frac{R_{CR}}{h}$	$\frac{SR_{ad}}{h}$	P (Test) (kN)	P (Pred) (kN)	AI		AII	
										P_{op}	R_{AI}	P_{op}	R_{AII}
I		OC13V43	36.62	3	150	5.00	5.00	467	487.00	413.68	0.89	425.10	0.91
		OC13V43_1.6	33.21	3	150	5.00	5.00	383	463.77	393.94	1.03	404.82	1.06
		AI	38.2	4	200	4.69	4.69	740	904.77	790.42	1.07	806.67	1.09
II		OC13V40	43.03	3	150	5.00	5.00	340	527.91	410.54	1.21	388.38	1.14
		BI	36.3	4	200	4.69	4.69	580	881.98	759.37	1.31	703.25	1.21
III		OC13V23	36.94	3	150	5.00	5.00	484	489.12	405.4	0.84	390.81	0.81
		OC15V43	35.97	5	150	5.17	5.17	612	555.96	466.96	0.76	479.24	0.78
IV		OC13H02	43.09	3	150	5.00	5.00	512	528.27	353.74	0.69	442.86	0.86
		OC13H02_1.6	37.53	3	150	5.00	5.00	420	493.01	330.13	0.79	413.30	0.98
V		OC13H50	36.29	3	150	5.00	5.00	443	484.80	429.43	0.97	395.27	0.89
		OC13H50_1.6	33.09	3	150	5.00	5.00	428	462.93	410.06	0.96	377.44	0.88
		OC15H70	37.89	5	150	4.33	4.33	529	720.54	583.98	1.10	556.97	1.05

Mean 0.97
Standard deviation 0.19
COV 0.19

0.97
0.14
0.14

From Table 6.6, it can be seen that generally the punching shear capacities of slab-column specimens with rectangular column and opening predicted by theory of plasticity are compatible with the test results. It indicates that the proposed plasticity model with the proposed effectiveness factor on the compressive strength of concrete of Equation 6.1 and taking the tensile strength of concrete as zero is acceptable and reasonable to predict the punching shear capacity of rectangular slab-column connections with opening. In addition, the proposed plasticity model can also predict the punching shear capacity for the rectangular slab-column connections with opening location types I, IV and V when the ratio of applied bending moments in two perpendicular directions of slab is 1.6.

Regarding the ineffective area of failure surface due to opening by the two methods; AI and AII,

- (1) In opening location type I, the predicted punching shear capacity of slab-column specimens falls between + 11 % of the test results. For the specimen OC13V43_1.6, the ratio of bending moments in two perpendicular directions of slab is 1.6. Generally both methods to calculate the ineffective area of failure surface are suitable for this type of opening location as well as for that when the ratio of bending moments in two perpendicular directions of slab is 1.6.
- (2) In opening location type II, generally the punching shear capacity of slab-column specimens overestimates the test results. This is due to the fact that the estimated ineffective areas of failure surface due to opening by two methods for these specimens are small. The position of opening adjacent and perpendicular to the shorter side of column in specimens OC13V40 and B1 affects the punching shear capacity of the slab most significantly among other opening locations.
- (3) In opening location type III, for specimen OC13V23, both methods that determine the ineffective area of failure surface due to opening are just on the acceptable margin in predicting the reduced punching shear capacity of the

slab due to opening. However, for specimen OC15V43, the punching shear capacity of slab-column specimens underestimates the test results. It indicates that the ineffective area of failure surface due to this type of opening location determined by both methods AI and AII are inconsistently larger as the aspect ratio of longer to shorter column width increases.

- (4) In opening location type IV, generally methods AII is reasonably good in predicting the reduced punching shear capacity of slab due to opening while method AI underestimates the reduced punching shear capacity of the slab due to opening. This is due to the fact that the location of opening adjacent to the middle of longer column side does not affect the punching shear capacity of slab very much.
- (5) When the opening is located in adjacent and perpendicular to the shorter side of column (opening location type V), the punching shear capacity of slab-column specimens with opening are in fairly good agreement with the test results.
- (6) Generally, method AII is more reasonable in predicting the ineffective area of failure surface due to opening for all opening location type considered in this study.

The ineffective punching shear capacity $\bar{P}_{op}$ due to opening by using method AI and AII, and the percentage reduction in predicted punching shear capacity by theory of plasticity are shown in Table 6.7. The percentage reduction in punching shear capacity of slab-column connections due to opening predicted by theory of plasticity and the test results are also compared and shown in Tables 6.8 and 6.9. Because of no control specimens for specimens A1 and B1, no percentage reduction in punching shear capacity of slab-column connections due to opening for these specimens is included in these tables.

Table 6.7 shows that the percentage reduction of predicted punching shear capacity lost due to opening by theory of plasticity using the two methods, for opening location

type I, II, III and V, the similar observation is that the ineffective punching shear capacity of slab due to opening predicted by theory of plasticity by using method AII is larger than that by using method AI, while for opening location type IV, the ineffective punching shear capacity of slab due to opening predicted by theory of plasticity by using of method AII is less than that by using of method AI.

Table 6.7 Percentage reduction of predicted punching shear capacity due to opening by theory of plasticity using the two methods

Type	Spec.	P (Pred) (kN)	AI				AII			
			$\overline{P}_{op}$	P_{op}	% r_{AI}	R_{AI}	$\overline{P}_{op}$	P_{op}	% r_{AII}	R_{AII}
I	OC13V43	487.00	73.33	413.68	15.1	0.89	61.90	425.10	17.1	0.91
	OC13V43_1.6	463.77	69.83	393.94	15.1	1.03	58.95	404.82	17.1	1.06
	A1	904.77	114.35	790.42	12.6	1.07	98.10	806.67	15.8	1.09
II	OC13V40	527.91	117.36	410.54	22.2	1.21	139.52	388.38	26.4	1.14
	B1	881.98	122.61	759.37	13.9	1.31	178.73	703.25	20.3	1.21
III	OC13V23	489.12	83.72	405.4	17.1	0.84	98.31	390.81	20.1	0.81
	OC15V43	555.96	89.00	466.96	16.0	0.76	76.72	479.24	13.8	0.78
IV	OC13H02	528.27	174.53	353.74	33.0	0.69	85.41	442.86	16.1	0.86
	OC13H02_1.6	493.01	162.88	330.13	33.0	0.79	79.71	413.30	16.1	0.98
V	OC13H50	484.80	55.37	429.43	11.4	0.97	89.54	395.27	23.3	0.89
	OC13H50_1.6	462.93	52.87	410.06	11.4	0.96	85.50	377.44	23.3	0.88
	OC15H70	720.54	47.56	583.98	7.5	1.10	74.56	556.97	16.0	1.05

Mean 0.97 0.97

Standard deviation 0.19 0.14

COV 0.19 0.14

$\overline{P}_{op}$ = predicted punching shear capacity of slab-column connection lost due to opening

% r_{AI} & % r_{AII} = percentage reduction in punching shear capacity of slab due to opening by method AI and AII, respectively

R_{AI} & R_{AII} = ratios of predicted punching shear capacity by using method AI and AII, to the test results, respectively

As shown in Table 6.8, the punching shear capacity of all specimens predicted by method AI falls within $\pm 24\%$ of the test results. While Table 6.9 shows that the

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punching shear capacity of all specimens predicted by method AII falls within $\pm 20\%$ of the test results. In Figure 6.3, the percentage reductions in punching shear capacity of slab-column connections due to opening for five opening location types considered are shown. Figure 6.3 shows that the trend of percentage reductions in punching shear capacity of slab-column connections due to opening for five opening location types is compatible with the trend of percentage reductions observed in the tests. Therefore, it can be concluded that method AII is more reasonable than method AI.

Table 6.8 Comparison in percentage reduction in punching shear capacity for slab-column connections due to opening (method AI)

Type	Spec:	P (test) (kN)	% r (test)	AI				Difference in % r = % r _{AI} - % r(test)
				P	P_{op}	% r _{AI}	R _{AI}	
-	OC13	568	-	-	-	-	-	-
-	OC15	649	-	-	-	-	-	-
I	OC13V43	467	17.8	73.33	413.68	15.1	0.89	-2.7
	OC13V43_1.6	383	24.6	69.83	393.94	15.1	1.03	-9.5
	A1	740	-	114.35	790.42	12.6	1.07	-
II	OC13V40	340	40.2	117.36	410.54	22.2	1.21	-18
	B1	580	-	122.61	759.37	13.9	1.31	-
III	OC13V23	484	14.8	83.72	405.4	17.1	0.84	2.3
	OC15V43	612	5.7	89.00	466.96	16.0	0.76	10.3
IV	OC13H02	512	9.9	174.53	353.74	33.0	0.69	23.1
	OC13H02_1.6	420	17.3	162.88	330.13	33.0	0.79	15.7
V	OC13H50	443	22.0	55.37	429.43	11.4	0.97	-10.6
	OC13H50_1.6	428	15.8	52.87	410.06	11.4	0.96	-4.4
	OC15H70	529	18.5	47.56	583.98	7.5	1.10	-11

Mean 0.97

Standard deviation 0.19

COV 0.19

OC13 and OC15 are control slab-column specimens without opening.

Table 6.9 Comparison in percentage reduction of punching shear capacity for slab-column connections due to opening (method AII)

Type	Spec:	P (test) (kN)	% r (test)	AII				Difference in % r = % r_{AII} - % r (test)
				P (Pred)	P_{op} (Pred)	% r_{AII}	R_{AII}	
-	OC13	568	-	-	-	-		-
-	OC15	649	-	-	-	-	-	-
I	OC13V43	467	17.8	61.90	425.10	17.1	0.91	-0.7
	OC13V43_1.6	383	24.6	58.95	404.82	17.1	1.06	-7.5
	A1	740	-	98.10	806.67	15.8	1.09	-
II	OC13V40	340	40.2	139.52	388.38	26.4	1.14	-13.8
	B1	580	-	178.73	703.25	20.3	1.21	-
III	OC13V23	484	14.8	98.31	390.81	20.1	0.81	5.3
	OC15V43	612	5.7	76.72	479.24	13.8	0.78	8.1
IV	OC13H02	512	9.9	85.41	442.86	16.1	0.86	6.2
	OC13H02_1.6	420	17.3	79.71	413.30	16.1	0.98	-1.2
V	OC13H50	443	22.0	89.54	395.27	23.3	0.89	1.3
	OC13H50_1.6	428	15.8	85.50	377.44	23.3	0.88	7.5
	OC15H70	529	18.5	74.56	556.97	16.0	1.05	-2.5
Mean							0.97	
Standard deviation							0.14	
COV							0.14

Figure 6.3 Percentage reduction in punching shear capacity due to opening

6.6 Punching shear capacity of slab-column connection with shear reinforcement

6.6.1 With square column

There are twelve slab-column specimens with shear reinforcement selected to corroborate the predicted punching shear capacity by theory of plasticity derived in previous chapter. Details of specimens are shown in Table 6.10. All specimens have square column and no opening, and they are reinforced with shear studs. All specimens failed in pure punching.

The arrangements of shear studs in all specimens are shown in Figure 6.4. There are three types of shear stud arrangement used in the specimens. They are (1) arrangement of shear stud rails in perpendicular directions of column widths (2) arrangement of shear stud rails in radial directions of slab, and (3) arrangement of shear stud rails in parallel directions of column widths. The shear studs in the slab specimens tested by Mokhtar et al.(1985) were arranged in perpendicular directions of column widths. The shear studs in the slab specimens tested by Ronaldo et al. (2000) and from the NTU-BCA project (2003) were arranged in radial directions of the slab. For slab specimens tested by Frieder et al. (1980), the shear studs in one slab was arranged in parallel direction of column widths while the shear studs in two slabs were arranged in perpendicular directions of the column widths.

The predicted punching shear capacities of selected specimens are shown in Table 6.11. Through the calculation of punching shear capacity of slab-column specimens by theory of plasticity, the proposed effectiveness factors on the compressive strength of concrete and tensile strength of slab contributed by shear studs of Equations 6.1 and 6.2, respectively, are used.

Figure 6.4 Arrangements of shear studs used in square slab-column specimens

Table 6.10 Details of slab-column specimens with square column and shear reinforcement

Author	Spec	C (mm)	h (mm)	d (mm)	SDia (mm)	f'_c (MPa)	f_{fy} (MPa)	Steel Ratio (%)	f_{ssry} (MPa)	No of SSR	Φ	P (test) (kN)
Mokhtar et al. (1985)	AB3	250	150	116	1800	23	516	2.0	278	12 × 8	9.5	545
	AB5	250	150	116	1800	40	516	2.0	278	12 × 8	9.5	583
	AB6	250	150	116	1800	29	516	2.0	489	12 × 6	9.5	541
	AB7	250	150	116	1800	35	448	2.0	278	12 × 6	9.5	572
	AB8	250	150	116	1800	30	448	2.0	489	12 × 5	9.5	508
Ronaldo et al. (2000)	L301	200	200	160	2700	37.8	678	1.32	602	8 × 6	10	830
	L302	200	200	160	2700	34.2	678	1.32	602	8 × 12	10	790
Frieder et al. (1980)	SC11	305	152	121	1749	33.6	450	1.17	500	8 × 8	6.4	596
	SC12	305	152	121	1749	33.6	450	1.17	500	16 × 4	6.4	595
	SC13	305	152	121	1749	33.6	450	1.17	500	16 × 4	6.4	580
NTU-BCA (2003)	A3	250	200	160	2121	39.8	515	1.725	513	7 × 8	10	914
	A4	250	200	160	2121	40.3	515	1.725	513	7 × 10	10	1058

Number of SSR is presented as row x studs per rail.

Φ = diameter of shear stud reinforcement in mm

f_{fy} = yield strength of flexural steel

f_{ssry} = yield strength of shear stud reinforcement

Table 6.11 shows that the predicted punching shear capacity for slabs with square column and shear studs falls within $\pm 13 \%$ of the test results. Thus, the upper-bound formula derived by theory of plasticity can predict the punching shear capacities well for slab-column connections with square column and shear studs. In addition, using the recommended effectiveness factor v_{st} on the tensile strength contributed shear studs of Equation 6.2, the plasticity model can give the punching shear capacity of slab-column specimens with shear studs in good agreement with test results, Furthermore, the plasticity model in predicting the punching shear capacity contributed by the shear studs is good enough for all amount and arrangements of shear studs in square slab-column specimens as shown in Figure 6.4.

Table 6.11 Punching shear capacity of slab-column specimens with square column and shear reinforcement

Spec:	h (mm)	f'_c (MPa)	$\frac{R_{CR}}{h}$	$\frac{SRad}{h}$	P (Pred) (kN)	P (test) (kN)	$\frac{Pu(Pred)}{Pu(Test)}$
AB3	150	23	5.15	5.17	487.91	545	0.90
AB5	150	40	5.15	5.17	604.97	583	1.03
AB6	150	29	5.15	5.17	571.57	541	1.07
AB7	150	35	5.15	5.17	543.51	572	0.94
AB8	150	30	5.15	5.17	552.08	508	1.09
L301	200	37.8	6.25	6.25	777.85	830	0.89
L302	200	34.2	6.25	6.25	896.59	790	1.12
SC11	152	33.6	4.73	4.74	583.04	596	0.95
SC12	152	33.6	4.73	4.74	583.04	595	0.95
SC13	152	33.6	4.73	4.74	583.04	580	0.98
A3	200	39.8	4.68	4.68	932.61	914	0.95
A4	200	40.3	4.68	4.68	973.18	1058	0.87

Mean 0.98
Standard deviation 0.08
COV 0.08

6.6.2 With rectangular column and with or without opening

Eight slab-column specimens with rectangular column and with shear stud reinforcements are selected to verify whether the plasticity model derived in chapter 5 be able to predict the contribution of shear reinforcement in strengthening the punching shear capacity of slab-column connections. Four slab-column specimens have no opening but they are reinforced by shear studs with different arrangement to increase the punching shear capacity of slab. The remaining four slab-column specimens have opening and shear stud reinforcements with different arrangement to recover the punching shear capacity of slab lost due to opening. Details of slab-column specimens are shown in Table 6.12. All slab specimens failed in punching. In specimens B3 (C 13F22VS1) and B4 (C 13F22VS2a), the distance between supports are 2000 mm and 2400 mm in shorter and longer column width direction, respectively. In other specimens, the distances between supports are 2000 mm and 3000 mm in shorter and longer column width direction, respectively. The arrangements of shear studs in all specimens are shown in Figure 6.5. The diameter of shear stud used in all slab-column specimens is 10 mm.

The predicted punching shear capacities of specimens are shown in Table 6.13. In addition, the predicted punching shear capacities of slab-column specimens lost due to opening and contributed by shear studs are presented. The effectiveness factor for the compressive strength of concrete presented in Equation 6.1 is used, while the effectiveness factor on the tensile strength contributed of shear studs of Equation 6.2 is applied.

Table 6.12 Details of slab-column connection with rectangular column, with or without opening, and with shear stud reinforcement

Author	Spec:	C ₁ (mm)	C ₂ (mm)	h (mm)	d (mm)	f_c' (MPa)	f_{sxy} (MPa)	Steel ratio (%)	P (test) (kN)	Opening
										No of SSR
BCA- NTU Project (2003)	B3 (C13F22VS1)	250	750	200	160	39.5	515	1.742	1168	Without 8 × 7
	B4 (C13F22VS2a)	250	750	200	160	40.6	515	1.742	1266	Without 12 × 7
	C3 (C15F22VS1)	250	1250	200	160	38.3	515	1.759	1245	Without 12 × 7
	C4 (C15F22VS2a)	250	1250	200	160	39.7	515	1.759	1477	Without 16 × 7
BCA- NTU Project (2003)	A2	250	1000	200	160	38.2	540	1.65	908	With 5 × 8
	A3	250	1000	200	160	38.3	540	1.65	948	With 6 × 5
	B2	250	1000	200	160	36.3	540	1.65	848	With 6 × 8
	B3	250	1000	200	160	36.3	540	1.65	692	With 6 × 5

Number of SSR is presented as row × studs per rail.

Figure 6.5 Location of opening and arrangement of SSR in the rectangular slab-column specimens

From Table 6.13, it can be seen that the predicted punching shear capacity for slabs with rectangular column, with or without opening and with shear studs falls around $\pm 20\%$ of the test results. Thus, generally the upper-bound formula derived by theory of plasticity can predict the punching shear capacity well for slabs with rectangular column, and shear studs, with or without opening.

Table 6.13 Punching shear capacity of slab-column connections with rectangular column, with or without opening and with shear stud reinforcement

Spec:	P (Test) (kN)	P (Pred) (kN)	$\bar{P}_{op}$ (kN)	P_{op} (kN)	P_{op+S} (Pred) (kN)	$\bar{P}_{SSR}$ (kN)	P_{SSR} (kN)	$\frac{P_{op+S} \text{ (Pred)}}{P \text{ (Test)}}$
B3 (C13F22VS1)	1168	888.89	–	–	–	138.67	1053.97	0.90
B4 (C13F22VS2a)	1266	901.18	–	–	–	208.00	1148.8	0.91
C3 (C15F22VS1)	1245	1017.29	–	–	–	207.54	1264.36	1.02
C4 (C15F22VS2a)	1477	1035.72	–	–	–	276.72	1365.15	0.92
A2	908	904.77	114.35	790.42	908.93	118.51	–	1.00
			98.1	806.67	925.18	118.51	–	1.02
A3	948	904.77	114.35	790.42	879.3	88.88	–	0.93
			98.1	806.67	895.55	88.88	–	0.94
B2	848	881.98	122.61	759.37	890.01	130.64	–	1.05
			178.73	703.25	833.89	130.64	–	0.98
B3	692	881.98	122.61	759.37	841.02	81.65	–	1.22
			178.73	703.25	784.9	81.65	–	1.13

Mean 1.00
Standard deviation 0.10
COV 0.10

- $P \text{ (Test)}$ = test result of punching shear capacity of slab-column connection with shear reinforcement and with or without opening
- $P \text{ (Pred)}$ = predicted punching shear capacity of slab-column connection without opening and without shear reinforcement
- P_{op+S} = predicted punching shear capacity for the slab-column connection with opening and shear stud reinforcement

- P_{op} = predicted punching shear capacity of slab-column connection with opening and without shear stud reinforcement
- $\bar{P}_{op}$ = predicted punching shear capacity of slab-column connection lost due to opening
- P_{SSR} = predicted punching shear capacity for the slab-column connection without opening and with shear stud reinforcement

It can be observed from Table 6.13 that the predicted punching shear capacities of all slab specimens without opening are compatible with the test results. The plasticity model is acceptable in predicting the punching shear capacity of the rectangular slab-column connections with shear studs arranged as shown in Figure 6.5.

It can be also observed from Table 6.13 that for the slab specimens B3, the predicted shear capacities by theory of plasticity with both methods to calculate the ineffective area of failure surface due to opening are overestimated while for the slab specimen B2, the predicted shear capacities by theory of plasticity with both methods are in good agreement with test results. This is due to the fact that the amount and arrangement of shear studs in specimens B3 is not as effective as the arrangement of shear studs in specimens B2 in recovering the punching shear capacity lost due to opening. It can be concluded that the proposed plasticity model gives a lesser punching shear capacity of slab-column connections with opening and shear stud reinforcement as the shear studs reinforced in the slabs is more effective.

6.7 Conclusions

Regarding the plasticity model proposed in chapter 5 to predict the punching shear capacity of slab-column connections, the following conclusions are drawn:

1. Generally the plasticity model can predict the punching shear capacity quite well for slab-column connections with square and rectangular column, with or without opening and with or without shear stud reinforcement.
2. The proposed effectiveness factors on the compressive strength of concrete of Equation 6.1 is applicable to use in calculating the punching shear capacity of

square and rectangular slab-column connections with the thickness of the slab ranged less than 250 mm and aspect ratio of longer to shorter column widths less than 5.

3. The proposed effectiveness factors on the yield strength of shear studs of Equation 6.2 is applicable to use in calculating the punching shear capacities of square and rectangular slab-column connections with shear reinforcement.
4. The plasticity model can also predict the punching shear capacities of slab-column connections reduced due to opening and contributed by SSR. Therefore, from the predicted punching shear capacities of slab-column connections reduced due to opening by theory of plasticity, the quantity of shear stud reinforcement to recover the reduction of punching shear capacity due to opening can be recalculated for design purpose.
5. In predicting the ineffective punching shear capacity of slab due to opening by the proposed plasticity model, the method AII to calculate the ineffective area of failure surface due to opening is more reasonable.
6. The punching shear failure surface obtained by plasticity model is a curve and has continuous circumference around the column.
7. The proposed plasticity model is valid for slab-column connections failed in punching failure mode.
8. The proposed plasticity model is only for concentric punching load.

CHAPTER 7

SUMMARY, CONCLUSION AND

RECOMMENDED FUTURE

WORKS

Chapter 7

Summary, conclusion and recommended future works

7.1 Summary

In this study, the experimental results of 6 slab specimens which were tested to investigate the behavior of rectangular slab-column connections with opening and shear stud reinforcement are presented. There are two types of opening locations in the slab specimens; at the corner of column and at the shorter side of column width. Except the two control specimens, all specimens have different amount and arrangement of shear stud reinforcement. The specimens were loaded to fail under monotonic uniformly distributed loading.

Based on the theory of plasticity, a model to predict the upper bound punching shear capacity of square and rectangular slab-column connections with or without opening and with or without shear stud reinforcement is proposed. The punching shear failure criterion of concrete is established. The collapse mechanism and generatrix of punching shear failure surface for different slab-column connections are considered. The plastic strain along the punching crack is derived based on the established failure criterion. The dissipation energy in the punching shear failure surface is derived and the upper bound solution for punching shear capacity of slab-column connections is achieved by using virtual work principle and variational calculus.

To take into account the softening effect of concrete, an effectiveness factor on the compressive strength of concrete, which depends on slab thickness, aspect ratio of longer and shorter column width and compressive strength of concrete, is proposed. Because all shear reinforcements in the slab may not yield at the punching shear failure, an effectiveness factor on the tensile strength contributed by shear stud reinforcement is also recommended.

7.2 Conclusion

Based on the results obtained from the experimental programme described in chapter four, and the plasticity model proposed in chapter five and its verification by predicting the punching shear capacity of square and rectangular slab-column connections with and without opening, and with and without shear stud reinforcements, the following conclusions can be drawn.

7.2.1 Conclusion on experimental program

From the experimental results and discussion from the experimental programme, the conclusions are:

- (1) The concentrated stresses near the region around the shorter column width are higher than the concentrated stresses near the region around the longer column width.
- (2) If the slab has opening near the column, the region near the opening is the weakest portion in the slab regardless the slab is reinforced with shear reinforcements.
- (3) The location of opening at the shorter column width affect the punching shear capacity of slab more than the location of opening at the corner of column.
- (4) The shear stud reinforcements can significantly recover the reduced punching shear capacity of the slabs due to openings.
- (5) For the rectangular slab-column connections with opening and shear stud reinforcements, the circumference of failure surface is within the distance of $2d$ and $4d$ from the column face, where d is the effective depth of slabs.
- (6) The angle of inclined cracks in the slabs with opening at the shorter side of column is steeper than angle of inclined cracks in the slabs with opening at

the corner of column. The more the opening affect the punching shear capacity of slab, the steeper the angle of inclined crack in the slab.

- (7) When the opening is located at the corner of column in the interior slab-column connection with rectangular column, more number of shear stud reinforcements with shorter length of rails should be evenly placed around the column face and near the opening.
- (8) When the opening is located at the shorter column width in the interior slab-column connection with rectangular column, the shear stud reinforcements with longer length of rails tilting near the opening should be used.

7.2.2 Conclusion on proposed plasticity model

Regarding the proposed plasticity model to predict the punching shear capacity of slab-column connections, the conclusions are:

- (1) The proposed plasticity model is able to predict the punching shear capacity for slab-column connections with square and rectangular column, with or without opening and with or without shear stud reinforcement well compared to the test results.
- (2) The proposed plasticity model is valid for slab-column connections under concentric load and failed in punching shear failure mode.
- (3) The proposed effectiveness factors on the compressive strength of concrete of Equation 6.1 is applicable in calculating the punching shear capacities for square and rectangular slab-column connections with thickness of slab less than 250 mm and aspect ratio of longer and shorter column widths less than 5.
- (4) The proposed effectiveness factors on the tensile strength of slab contributed by shear stud reinforcement of Equation 6.2 with the proposed plasticity

model is applicable in calculating the punching shear capacities of square and rectangular slab-column connections with shear stud reinforcement.

- (5) The punching shear capacity of slab-column connections reduced due to opening and contributed by SSR can be reasonably predict by the proposed plasticity model.
- (6) The quantity of shear stud reinforcement to recover the reduced punching shear capacity due to opening can be recalculated by the proposed plasticity model for design purpose.
- (7) The proposed plasticity model gives a lesser punching shear capacity of slab-column connections with opening and shear stud reinforcement as the shear stud reinforcement is more effective.
- (8) In predicting the ineffective punching shear capacity of slabs due to opening by the proposed plasticity model, the method AII explained in chapter six to calculate the ineffective area of failure surface due to opening is more reasonable than method AI.
- (9) For five kinds of opening location described in chapter six, the proposed plasticity model with method AII to calculate the ineffective area of failure surface due to opening, can reasonably predict the ineffective punching shear capacity of slab-column connections due to opening.

7.3 Recommended future works

Regarding to the proposed plasticity model, the future works are recommended as follows.

- (1) Although the proposed plasticity model to predict the punching shear capacity of slab-column connections with square or rectangular column can give relatively good results compared to the test results, the procedure is quite long.

For design purpose, it may be simplified by considering the straight failure surface from compression to tension face of slab.

- (2) The proposed plasticity model is for pure punching of slabs only. It should be extended for flexural punching failure.
- (3) The proposed plasticity model needs to be extended for the exterior and corner slab-column connections.
- (4) The proposed plasticity model to predict the punching shear capacity of slab-column connections with square or rectangular column is valid for the slope of failure surface at the compressive side of the slab greater than or equal to the internal friction angle of concrete of 37 degree. But in practice, the slope of failure surface at the compressive side of the slab may be less than 37 degree. To overcome this, the parabolic Mohr failure criterion described in chapter 2 instead of modified Mohr-Coulomb failure criterion for concrete should be used. Thus, the proposed plasticity model can be extended for all slope of failure surface at the compressive side of the slab by employing parabolic Mohr-Coulomb failure criterion for concrete.
- (5) Although the flexural reinforcement does not significantly affect on the shear strength of heavily reinforced laterally restrained slabs, the punching shear capacity can be enhanced by the effect of flexural reinforcement in lightly reinforced laterally restrained slabs. This is due to the effect of compressive membrane action of laterally restrained slabs. The proposed plasticity model to predict the punching shear capacity of slab-column connections with square or rectangular column should be extended to predict the punching shear capacity of laterally restrained slab-column connections.

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APPENDIX A

LVDT AND STRAIN GAUGE MEASUREMENTS FROM THE EXPERIMENTAL PROGRAMME

(b) Arrangement of shear studs or strain gauge locations on shear studs

(c) Strain gauge locations on tension bars

Figure A1 LVDT and strain gauge locations in the slab-column specimens

Table A1 Recorded test data for slab-column specimen A1

Load Cell 1	Load Cell 2	Load Cell 3	Load Cell 4	Nullspace	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D16	D18	Control	R01	R02	R03	R04	R05	R06
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
6	5	5.4	5	21.4	0	0	0	0	0	0	0	0	0.1	0.1	0	0	0.1	0.2	0	0	0	0	0	0	0	0	
11	10	10.6	10	41.6	0	0	0	0	0	0	0	0.1	0.1	0.2	0	0.1	0.2	0.3	0.2	0.1	0	0	0	0	0	0.1	
16	15.4	14.6	14.6	57.6	0	0	0	0	0	0	0	0.1	0.1	0.2	0	0.1	0.2	0.3	0.2	0.1	0	0	0	0	0	0.1	
21	20.2	19.3	20.3	73.6	0.1	0.1	0.1	0.1	0.1	0.3	0.1	0.2	0.2	0.3	0.2	0.3	0.3	0.5	0.5	0.5	0.1	0	0	0	0	0.2	
26	24.8	25	24.8	100.8	0.1	0.2	0.1	0.1	0.1	0.4	0.2	0.3	0.3	0.3	0.3	0.6	0.6	0.7	0.7	0.7	0.1	0	0	0	0	0.3	
31	30.3	29.4	30.3	129.7	0.1	0.2	0.1	0.2	0.3	0.4	0.2	0.4	0.5	0.7	0.7	0.7	0.9	0.9	0.8	0.8	0.1	0	0.1	0	0.1	0.2	
36	35.8	34.8	35.8	158.6	0.2	0.2	0.2	0.2	0.4	0.5	0.2	0.5	0.7	0.9	0.9	0.9	1.2	1.2	1.2	1.2	0.1	0.1	0.1	0.1	0.2	0.3	
41	40.3	40	40.3	167.3	0.2	0.3	0.2	0.2	0.4	0.6	0.2	0.6	0.7	1.1	1.1	1.1	1.4	1.4	1.4	1.4	0.2	0.1	0.1	0.1	0.2	0.3	
46	45.1	44	45.1	176.3	0.2	0.3	0.2	0.2	0.4	0.6	0.2	0.6	0.7	1.1	1.1	1.1	1.4	1.4	1.4	1.4	0.2	0.1	0.1	0.1	0.2	0.3	
51	50	50	50	202	0.3	0.4	0.2	0.2	0.6	0.8	0.3	0.8	1.1	1.1	1.4	1.5	1.5	1.5	1.5	1.5	0.2	0.1	0.2	0	0	0.4	
56	55	61.3	60.4	243.7	0.3	0.5	0.3	0.3	0.7	1.1	0.6	1.1	1.1	1.8	1.1	1.9	2.3	2.3	2.2	2.4	0.3	0	0	0	0.4	0.8	
61	60.4	60.4	60.4	285.4	0.4	0.5	0.3	0.4	0.8	1.2	0.6	1.2	1.2	2.3	1.2	2.4	3.1	3.1	3.1	3.1	0.3	0.1	0.2	0	0.1	0.5	
66	65.7	65.7	65.7	327.1	0.4	0.5	0.3	0.4	0.8	1.2	0.6	1.2	1.2	2.3	1.2	2.4	3.1	3.1	3.1	3.1	0.3	0.1	0.2	0	0.1	0.5	
71	70.7	70.7	70.7	369.2	0.4	0.5	0.3	0.4	0.8	1.2	0.6	1.2	1.2	2.3	1.2	2.4	3.1	3.1	3.1	3.1	0.3	0.1	0.2	0	0.1	0.5	
76	75.7	75.7	75.7	410.9	0.4	0.5	0.3	0.4	0.8	1.2	0.6	1.2	1.2	2.3	1.2	2.4	3.1	3.1	3.1	3.1	0.3	0.1	0.2	0	0.1	0.5	
81	80.7	80.7	80.7	452.6	0.4	0.6	0.4	0.6	1.1	2	1.2	2.1	2.1	3.5	2.2	4	4.8	4.8	4.8	4.8	0.5	0.2	0.4	0	0.2	0.7	
86	85.7	85.7	85.7	494.3	0.5	0.6	0.4	0.6	1.1	2	1.2	2.1	2.1	3.5	2.2	4	4.8	4.8	4.8	4.8	0.5	0.2	0.4	0	0.2	0.7	
91	90.7	90.7	90.7	536.0	0.5	0.6	0.4	0.6	1.1	2	1.2	2.1	2.1	3.5	2.2	4	4.8	4.8	4.8	4.8	0.5	0.2	0.4	0	0.2	0.7	
96	95.7	95.7	95.7	577.7	0.5	0.6	0.5	0.5	0.9	2	1.2	2.1	2.1	3.5	2.2	4	4.8	4.8	4.8	4.8	0.5	0.2	0.4	0	0.2	0.7	
101	100.7	100.7	98.9	598.6	0.5	0.8	0.4	0.8	1.4	2.4	1.3	2.5	2.7	4.2	2.5	4.8	6.1	5.7	5.3	6.1	0.4	0.1	0.4	0	0.1	2	
106	105.7	105.7	103.9	639.8	0.6	1	0.5	0.7	1.7	2.9	1.4	3	3.3	5	2.8	5.6	7.4	6.9	6.2	7.1	0.4	0.2	0.6	0.1	0.3	1.3	
111	110.7	110.7	108.9	681.1	0.6	1.1	0.6	0.8	1.9	3.3	1.5	3.5	3.8	5.5	3.2	6.4	7.9	7.4	7.3	7.4	0.4	0.2	0.6	0.1	0.3	2.4	
116	115.7	115.7	113.9	722.3	0.6	1.2	0.7	0.9	2.1	3.7	1.5	3.7	4	5.8	3.4	7.5	8.7	8.3	8.3	8.3	0.5	0.1	0.6	0	0.4	3.2	
121	120.7	120.7	118.9	763.6	0.7	1.3	0.8	0.9	2.3	4.1	1.5	4.5	4.6	7.4	4.7	8.6	11.1	9.9	9.6	11.1	0.5	0.2	0.8	0.1	0.4	3.6	
126	125.7	125.7	123.9	804.9	0.7	1.4	0.9	1	2.6	4.6	2.5	5.3	6.3	8.2	5.2	9.8	12.6	11.1	10.6	12.5	0.6	0.1	0.8	0	0.4	2	
131	130.7	130.7	128.9	846.2	0.8	1.5	0.9	1.1	2.8	4.9	2.5	5.5	6.5	8.4	5.4	10.3	13.1	12.3	12.3	13.1	0.6	0.1	0.8	0	0.4	2.4	
136	135.7	135.7	133.9	887.5	0.8	1.6	1	1.1	3.1	5.1	2.5	5.8	6.8	8.6	5.6	11.2	14.1	13.3	13.3	14.1	0.6	0.2	0.9	0.1	0.5	2.6	
141	140.7	140.7	138.9	928.8	0.8	1.7	0.8	1.2	3.3	5.9	3.6	7.4	7	10.3	7.4	13.7	17.4	14.1	17.4	17.4	0.6	0.2	1.1	0.2	0.6	2.7	
146	145.7	145.7	143.9	970.1	0.8	1.7	0.8	1.2	3.3	5.9	3.6	7.4	7	10.3	7.4	13.7	17.4	14.1	17.4	17.4	0.6	0.2	1.1	0.2	0.6	2.7	
151	150.7	150.7	148.9	1011.4	0.8	1.7	0.8	1.2	3.6	6.1	3.9	6.2	7.6	11.3	8	15.1	19.1	14.7	14.8	19.1	0.7	0.1	0.1	0.5	2.9	5.4	
156	155.7	155.7	153.9	1052.7	0.9	1.8	0.9	1.3	3.8	6.4	4.1	8.2	8.3	11.6	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
161	160.7	160.7	158.9	1094	0.9	1.8	0.9	1.3	4.1	6.7	4.1	8.5	8.4	11.8	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
166	165.7	165.7	163.9	1135.3	0.9	1.9	0.9	1.3	4.4	6.9	4.1	8.8	8.4	12.1	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
171	170.7	170.7	168.9	1176.6	0.9	1.9	0.9	1.3	4.7	7.1	4.1	9.1	8.4	12.4	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
176	175.7	175.7	173.9	1217.9	0.9	1.9	0.9	1.3	4.9	7.3	4.1	9.4	8.4	12.6	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
181	180.7	180.7	178.9	1259.2	0.9	1.9	0.9	1.3	5.2	7.5	4.1	9.7	8.4	12.8	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
186	185.7	185.7	183.9	1300.5	0.9	1.9	0.9	1.3	5.5	7.7	4.1	10	8.4	13	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
191	190.7	190.7	188.9	1341.8	0.9	1.9	0.9	1.3	5.8	7.9	4.1	10.3	8.4	13.2	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
196	195.7	195.7	193.9	1383.1	0.9	1.9	0.9	1.3	6.1	8.1	4.1	10.6	8.4	13.4	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
201	200.7	200.7	198.9	1424.4	0.9	1.9	0.9	1.3	6.4	8.3	4.1	10.9	8.4	13.6	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
206	205.7	205.7	203.9	1465.7	0.9	1.9	0.9	1.3	6.7	8.5	4.1	11.2	8.4	13.8	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
211	210.7	210.7	208.9	1507	0.9	1.9	0.9	1.3	7	8.7	4.1	11.5	8.4	14	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
216	215.7	215.7	213.9	1548.3	0.9	1.9	0.9	1.3	7.3	8.9	4.1	11.8	8.4	14.2	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
221	220.7	220.7	218.9	1589.6	0.9	1.9	0.9	1.3	7.6	9.1	4.1	12.1	8.4	14.4	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
226	225.7	225.7	223.9	1630.9	0.9	1.9	0.9	1.3	7.9	9.3	4.1	12.4	8.4	14.6	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
231	230.7	230.7	228.9	1672.2	0.9	1.9	0.9	1.3	8.2	9.5	4.1	12.7	8.4	14.8	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
236	235.7	235.7	233.9	1713.5	0.9	1.9	0.9	1.3	8.5	9.7	4.1	13	8.4	15	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
241	240.7	240.7	238.9	1754.8	0.9	1.9	0.9	1.3	8.8	9.9	4.1	13.3	8.4	15.2	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
246	245.7	245.7	243.9	1796.1	0.9	1.9	0.9	1.3	9.1	10.1	4.1	13.6	8.4	15.4	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
251	250.7	250.7	248.9	1837.4	0.9	1.9	0.9	1.3	9.4	10.3	4.1	13.9	8.4	15.6	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
256	255.7	255.7	253.9	1878.7	0.9	1.9	0.9	1.3	9.7	10.5	4.1	14.2	8.4	15.8	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
261	260.7	260.7	258.9	1920	0.9	1.9	0.9	1.3	10	10.7	4.1	14.5	8.4	16	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
266	265.7	265.7	263.9	1961.3	0.9	1.9	0.9	1.3	10.3	10.9	4.1	14.8	8.4	16.2	8.4	16.3	20.3	16.3	20.3	20.3	0.6	0.3	1.2	0.3	0.4	3.5	
271	270.7	270																									

Table A1 Recorded test data for slab-column specimen A1 (Continued)

[illegible]

RD1, RD2, ... = relative deflection of slab at the locations shown in Figure A1
V1, V2, ... = strain occurred in the tension bars in the direction of shorter column width shown in Figure A1
H1, H2, ... = strain occurred in the tension bars in the direction of longer column width shown in Figure A1
P1, P2, ... = strain occurred in the tension bars around the opening shown in Figure A1

Relative deflection is the difference between respective LVDT reading and Center that is central deflection of slab

Table A2 Recorded data for slab-column specimen A2

Load Cell 1	Load Cell 2	Load Cell 3	Load Cell 4	Ultimate	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	Centre
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	5	6.1	4.3	20.4	0	0	0	0	0	0	0	0	0	0	0.1	0.1	0	0	0	0.1	0
10	10	10	9.7	39.7	0	0	0	0	0	0	0.1	0.1	0	0.2	0.2	0.2	0.1	0.1	0.3	0.4	0
15	15	15.1	14.3	59.4	0	0	0	0	0	0.2	0.1	0.2	0.1	0.3	0.3	0.4	0.3	0.3	0.4	0.6	0
20	20	19.7	19.4	79.1	0	0.1	0.1	0.1	0	0.2	0.2	0.3	0.1	0.4	0.4	0.6	0.5	0.4	0.7	0.9	0
26	24	25.1	25	100.1	0.1	0.1	0.1	0.1	0.1	0.4	0.4	0.4	0.1	0.6	0.6	0.8	0.5	0.5	1	1.2	0
30	30	29.7	30.6	120.3	0.1	0.2	0.2	0.2	0.1	0.4	0.4	0.6	0.2	0.7	0.7	1	0.9	0.8	1.2	1.5	0
41	39	40	39.4	159.4	0.1	0.2	0.2	0.3	0.2	0.6	0.6	0.8	0.3	1	1.1	1.6	1.4	0.9	1.6	2.2	0.1
52	49	51	50.7	202.7	0.1	0.3	0.3	0.4	0.2	0.8	0.9	1.2	0.4	1.5	1.6	2.2	1.9	1.3	2.4	3.3	0.1
60	59	61.6	60	240.6	0.1	0.4	0.4	0.5	0.2	1.1	1.2	1.5	0.4	1.9	2.1	2.8	2.5	1.7	3.2	4.3	0.1
70	69	71.7	69.4	280.1	0.2	0.4	0.4	0.6	0.4	1.4	1.4	1.9	0.7	2.4	2.5	3.6	3.3	2.3	4	5.4	0.1
81	78	83.6	80.7	323.3	0.2	0.5	0.5	0.7	0.5	1.8	1.8	2.5	1	3.3	3.4	4.8	4.5	3.2	5.4	7.2	0.2
90	88	94.6	90.4	363	0.2	0.6	0.6	0.8	0.6	2.2	2.2	3.1	1.2	3.9	4.2	5.9	5.5	3.6	6.5	8.9	0.2
99	98	104.6	100.1	401.7	0.2	0.7	0.6	0.9	0.7	2.5	2.4	3.6	1.5	4.6	4.7	6.9	6.6	4.5	7.5	10.3	0.2
113	110	110.1	109.8	442.9	0.4	0.9	0.6	0.9	1.4	3.4	2.5	3.8	3	6.3	5.2	7.6	8.3	7.2	8.9	10.9	0.3
122	120	119.8	119.2	481	0.4	0.9	0.6	0.9	1.4	3.6	2.7	4	3.1	6.6	5.5	8	8.7	7.6	9.5	11.5	0.3
131	131	129.8	129.2	521	0.4	1	0.7	0.9	1.4	3.9	2.9	4.3	3.3	7.2	5.9	8.5	9.2	8.2	10.3	12.3	0.3
141	140	139.8	139.2	560	0.4	1.1	0.7	1	1.6	4.3	3.1	4.6	3.8	8.1	6.3	9.1	10.1	9.3	11.4	13.1	0.3
149	150	149.8	150.5	599.3	0.5	1.2	0.8	1	1.9	4.8	3.3	5	4.3	8.9	6.8	10	11.1	10.4	12.5	14.1	0.3
163	163	160.5	158.3	640.8	0.5	1.3	0.8	1.1	2.2	5.3	3.6	5.6	5.1	9.9	7.4	11.1	12.6	11.8	13.7	15.5	0.4
170	169	170.2	169.9	679.1	0.5	1.3	0.9	1.2	2.3	5.8	4	6.1	5.4	10.8	8.1	12.1	13.5	12.6	14.9	16.9	0.4
180	179	181.2	180.9	721.1	0.5	1.4	0.9	1.2	2.5	6.2	4.4	6.6	5.8	11.8	8.9	13.1	14.6	13.5	16.2	18.4	0.4
191	190	192.5	190.3	763.8	0.5	1.5	1	1.3	2.7	6.7	4.7	7.3	6.3	12.6	9.7	14.5	16	14.5	17.5	20.2	0.4
195	194	196.7	195.9	781.6	0.6	1.5	1	1.4	2.8	6.9	5	7.7	6.6	13.2	10.2	15.1	16.7	15	18.2	21.1	0.4
199	199	202.1	200.3	800.4	0.6	1.5	1.1	1.4	2.9	7.1	5.2	8.2	6.7	13.6	10.8	15.9	17.4	15.4	18.9	22.1	0.4
205	204	205.4	203.7	818.1	0.6	1.5	1.1	1.4	3.1	7.3	5.4	8.7	7.2	13.9	11.1	16.8	18.5	15.9	19.3	23.1	0.4
209	209	209.6	209.3	836.9	0.6	1.5	1.1	1.4	3.3	7.5	5.5	9.1	7.6	14.4	11.4	17.6	19.4	16.5	19.7	24	0.4
214	214	214.7	213.7	856.4	0.6	1.5	1.1	1.5	3.5	7.7	5.7	9.6	8	14.8	11.7	18.4	20.3	17	20.2	24.9	0.4
220	218	221.9	219.7	879.6	0.6	1.6	1.2	1.5	3.6	8	6	10.2	8.1	15.3	12.4	19.4	21.2	17.4	21.1	26.3	0.4
226	219	229	224.1	898.1	0.6	1.6	1.2	1.5	3.7	8.2	6.4	11.2	8.4	15.7	13.2	20.8	22.4	17.7	21.9	28.1	0.4
226	222	225.7	225.3	899	0.7	1.6	1.2	1.5	4	8.3	6.7	12.2	8.9	15.8	13.6	22.3	23.9	17.9	22	29.8	0.4
228	222	227.3	230.9	908.2	0.7	1.6	1.3	1.5	4.4	8.3	7.4	14.8	9.7	15.8	15	26.3	27.3	17.9	22.3	34.4	0.4
228	226	221	160.8	136.4	744.2	0.9	1.1	0.9	1.5	6.9	12	10.1	19.7	17.8	21.4	33.3	32.4	18.2	24.9	42.8	0.5
132	150	123	130.5	535.5	0.8	1	0.9	1.1	7.8	22.6	10.8	15.1	16	30.7	24.5	25	23.4	30.1	36.6	34	0.5
131	146	121.7	129.8	530.5	0.8	1	0.9	1	8.3	25	10.9	13.9	18.1	37.3	26	21.9	20.6	36.6	42.8	30.6	0.5
136	154	126.6	134.5	551.1	0.8	1	0.9	1.1	9	26.7	11.2	13.5	20.2	43	27.9	20.5	19.3	42.3	48.4	29.1	0.5
140	158	130.1	138.6	566.7	0.8	1	0.9	1.1	9.6	29	11.3	13.3	22	48.2	30	19.1	17.9	47.3	53.6	27.8	0.5
140	159	131.14	138.9	569.04	0.8	1	1	1.1	9.9	28.6	11.2	13.3	23.1	50.6	30.8	18.6	17.5	49.7	55.8	27.2	0.5

Ultimate total ultimate load of four load cells in one application
D1, D2,..... Deflection of slab measured by linear variable transducer (LVDT) at the locations shown in Figure A1
Ccenter central deflection of slab measured by LVDT D17 located at the bottom of the slab shown in Figure A1

Table A2 Recorded test data for slab-column specimen A2 (Continued)

RD1	RD2	RD3	RD4	RD5	RD6	RD7	RD8	RD9	RD10	RD11	RD12	RD13	RD14	RD15	RD16	RD17	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16	V17	V18	V19	V20	V21	V22	V23	V24	V25	V26	V27	V28	V29	V30	V31	V32	V33	V34	V35	V36	V37	V38	V39	V40	V41	V42	V43	V44	V45	V46	V47	V48	V49	V50	V51	V52	V53	V54	V55	V56	V57	V58	V59	V60	V61	V62	V63	V64	V65	V66	V67	V68	V69	V70	V71	V72	V73	V74	V75	V76	V77	V78	V79	V80	V81	V82	V83	V84	V85	V86	V87	V88	V89	V90	V91	V92	V93	V94	V95	V96	V97	V98	V99	V100	V101	V102	V103	V104	V105	V106	V107	V108	V109	V110	V111	V112	V113	V114	V115	V116	V117	V118	V119	V120	V121	V122	V123	V124	V125	V126	V127	V128	V129	V130	V131	V132	V133	V134	V135	V136	V137	V138	V139	V140	V141	V142	V143	V144	V145	V146	V147	V148	V149	V150	V151	V152	V153	V154	V155	V156	V157	V158	V159	V160	V161	V162	V163	V164	V165	V166	V167	V168	V169	V170	V171	V172	V173	V174	V175	V176	V177	V178	V179	V180	V181	V182	V183	V184	V185	V186	V187	V188	V189	V190	V191	V192	V193	V194	V195	V196	V197	V198	V199	V200	V201	V202	V203	V204	V205	V206	V207	V208	V209	V210	V211	V212	V213	V214	V215	V216	V217	V218	V219	V220	V221	V222	V223	V224	V225	V226	V227	V228	V229	V230	V231	V232	V233	V234	V235	V236	V237	V238	V239	V240	V241	V242	V243	V244	V245	V246	V247	V248	V249	V250	V251	V252	V253	V254	V255	V256	V257	V258	V259	V260	V261	V262	V263	V264	V265	V266	V267	V268	V269	V270	V271	V272	V273	V274	V275	V276	V277	V278	V279	V280	V281	V282	V283	V284	V285	V286	V287	V288	V289	V290	V291	V292	V293	V294	V295	V296	V297	V298	V299	V300	V301	V302	V303	V304	V305	V306	V307	V308	V309	V310	V311	V312	V313	V314	V315	V316	V317	V318	V319	V320	V321	V322	V323	V324	V325	V326	V327	V328	V329	V330	V331	V332	V333	V334	V335	V336	V337	V338	V339	V340	V341	V342	V343	V344	V345	V346	V347	V348	V349	V350	V351	V352	V353	V354	V355	V356	V357	V358	V359	V360	V361	V362	V363	V364	V365	V366	V367	V368	V369	V370	V371	V372	V373	V374	V375	V376	V377	V378	V379	V380	V381	V382	V383	V384	V385	V386	V387	V388	V389	V390	V391	V392	V393	V394	V395	V396	V397	V398	V399	V400	V401	V402	V403	V404	V405	V406	V407	V408	V409	V410	V411	V412	V413	V414	V415	V416	V417	V418	V419	V420	V421	V422	V423	V424	V425	V426	V427	V428	V429	V430	V431	V432	V433	V434	V435	V436	V437	V438	V439	V440	V441	V442	V443	V444	V445	V446	V447	V448	V449	V450	V451	V452	V453	V454	V455	V456	V457	V458	V459	V460	V461	V462	V463	V464	V465	V466	V467	V468	V469	V470	V471	V472	V473	V474	V475	V476	V477	V478	V479	V480	V481	V482	V483	V484	V485	V486	V487	V488	V489	V490	V491	V492	V493	V494	V495	V496	V497	V498	V499	V500	V501	V502	V503	V504	V505	V506	V507	V508	V509	V510	V511	V512	V513	V514	V515	V516	V517	V518	V519	V520	V521	V522	V523	V524	V525	V526	V527	V528	V529	V530	V531	V532	V533	V534	V535	V536	V537	V538	V539	V540	V541	V542	V543	V544	V545	V546	V547	V548	V549	V550	V551	V552	V553	V554	V555	V556	V557	V558	V559	V560	V561	V562	V563	V564	V565	V566	V567	V568	V569	V570	V571	V572	V573	V574	V575	V576	V577	V578	V579	V580	V581	V582	V583	V584	V585	V586	V587	V588	V589	V590	V591	V592	V593	V594	V595	V596	V597	V598	V599	V600	V601	V602	V603	V604	V605	V606	V607	V608	V609	V610	V611	V612	V613	V614	V615	V616	V617	V618	V619	V620	V621	V622	V623	V624	V625	V626	V627	V628	V629	V630	V631	V632	V633	V634	V635	V636	V637	V638	V639	V640	V641	V642	V643	V644	V645	V646	V647	V648	V649	V650	V651	V652	V653	V654	V655	V656	V657	V658	V659	V660	V661	V662	V663	V664	V665	V666	V667	V668	V669	V670	V671	V672	V673	V674	V675	V676	V677	V678	V679	V680	V681	V682	V683	V684	V685	V686	V687	V688	V689	V690	V691	V692	V693	V694	V695	V696	V697	V698	V699	V700	V701	V702	V703	V704	V705	V706	V707	V708	V709	V710	V711	V712	V713	V714	V715	V716	V717	V718	V719	V720	V721	V722	V723	V724	V725	V726	V727	V728	V729	V730	V731	V732	V733	V734	V735	V736	V737	V738	V739	V740	V741	V742	V743	V744	V745	V746	V747	V748	V749	V750	V751	V752	V753	V754	V755	V756	V757	V758	V759	V760	V761	V762	V763	V764	V765	V766	V767	V768	V769	V770	V771	V772	V773	V774	V775	V776	V777	V778	V779	V780	V781	V782	V783	V784	V785	V786	V787	V788	V789	V790	V791	V792	V793	V794	V795	V796	V797	V798	V799	V800	V801	V802	V803	V804	V805	V806	V807	V808	V809	V810	V811	V812	V813	V814	V815	V816	V817	V818	V819	V820	V821	V822	V823	V824	V825	V826	V827	V828	V829	V830	V831	V832	V833	V834	V835	V836	V837	V838	V839	V840	V841	V842	V843	V844	V845	V846	V847	V848	V849	V850	V851	V852	V853	V854	V855	V856	V857	V858	V859	V860	V861	V862	V863	V864	V865	V866	V867	V868	V869	V870	V871	V872	V873	V874	V875	V876	V877	V878	V879	V880	V881	V882	V883	V884	V885	V886	V887	V888	V889	V890	V891	V892	V893	V894	V895	V896	V897	V898	V899	V900	V901	V902	V903	V904	V905	V906	V907	V908	V909	V910	V911	V912	V913	V914	V915	V916	V917	V918	V919	V920	V921	V922	V923	V924	V925	V926	V927	V928	V929	V930	V931	V932	V933	V934	V935	V936	V937	V938	V939	V940	V941	V942	V943	V944	V945	V946	V947	V948	V949	V950	V951	V952	V953	V954	V955	V956	V957	V958	V959	V960	V961	V962	V963	V964	V965	V966	V967	V968	V969	V970	V971	V972	V973	V974	V975	V976	V977	V978	V979	V980	V981	V982	V983	V984	V985	V986	V987	V988	V989	V990	V991	V992	V993	V994	V995	V996	V997	V998	V999	V1000	V1001	V1002	V1003	V1004	V1005	V1006	V1007	V1008	V1009	V1010	V1011	V1012	V1013	V1014	V1015	V1016	V1017	V1018	V1019	V1020	V1021	V1022	V1023	V1024	V1025	V1026	V1027	V1028	V1029	V1030	V1031	V1032	V1033	V1034	V1035	V1036	V1037	V1038	V1039	V1040	V1041	V1042	V1043	V1044	V1045	V1046	V1047	V1048	V1049	V1050	V1051	V1052	V1053	V1054	V1055	V1056	V1057	V1058	V1059	V1060	V1061	V1062	V1063	V1064	V1065	V1066	V1067	V1068	V1069	V1070	V1071	V1072	V1073	V1074	V1075	V1076	V1077	V1078	V1079	V1080	V1081	V1082	V1083	V1084	V1085	V1086	V1087	V1088	V1089	V1090	V1091	V1092	V1093	V1094	V1095	V1096	V1097	V1098	V1099	V1100	V1101	V1102	V1103	V1104	V1105	V1106	V1107	V1108	V1109	V1110	V1111	V1112	V1113	V1114	V1115	V1116	V1117	V1118	V1119	V1120	V1121	V1122	V1123	V1124	V1125	V1126	V1127	V1128	V1129	V1130	V1131	V1132	V1133	V1134	V1135	V1136	V1137	V1138	V1139	V1140	V1141	V1142	V1143	V1144	V1145	V1146	V1147	V1148	V1149	V1150	V1151	V1152	V1153	V1154	V1155	V1156	V1157	V1158	V1159	V1160	V1161	V1162	V1163	V1164	V1165	V1166	V1167	V1168	V1169	V1170	V1171	V1172	V1173	V1174	V1175	V1176	V1177	V1178	V1179	V1180	V1181	V1182	V1183	V1184	V1185	V1186	V1187	V1188	V1189	V1190	V1191	V1192	V1193	V1194	V1195	V1196	V1197	V1198	V1199	V1200	V1201	V1202	V1203	V1204	V1205	V1206	V1207	V1208	V1209	V1210	V1211	V1212	V1213	V1214	V1215	V1216	V1217	V1218	V1219	V1220	V1221	V1222	V1223	V1224	V1225	V1226	V1227	V1228	V1229	V1230	V1231	V1232	V1233	V1234	V1235	V1236	V1237	V1238	V1239	V1240	V1241	V1242	V1243	V1244	V1245	V1246	V1247	V1248	V1249	V1250	V1251	V1252	V1253	V1254	V1255	V1256	V1257	V1258	V1259	V1260	V1261	V1262	V1263	V1264	V1265	V1266	V1267	V1268	V1269	V1270	V1271	V1272	V1273	V1274	V1275	V1276	V1277	V1278	V1279	V1280	V1281	V1282	V1283	V1284	V1285	V1286	V1287	V1288	V1289	V1290	V1291	V1292	V1293	V1294	V1295	V1296	V1297	V1298	V1299	V1300	V1301	V1302	V1303	V1304	V1305	V1306	V1307	V1308	V1309	V1310	V1311	V1312	V1313	V1314	V1315	V1316	V1317	V1318	V1319	V1320	V1321	V1322	V1323	V1324	V1325	V1326	V1327	V1328	V1329	V1330	V1331	V1332	V1333	V1334	V1335	V1336	V1337	V1338	V1339	V1340	V1341	V1342	V1343	V1344	V1345	V1346	V1347	V1348	V1349	V1350	V1351	V1352	V1353	V1354	V1355	V1356	V1357	V1358	V1359	V1360	V1361	V1362	V1363	V1364	V1365	V1366	V1367	V1368	V1369	V1370	V1371	V1372	V1373	V1374	V1375	V1376	V1377	V1378	V1379	V1380	V1381	V1382	V1383	V1384	V1385	V1386	V1387	V1388	V1389	V1390	V1391	V1392	V1393	V1394	V1395	V1396	V1397	V1398	V1399	V1400	V1401	V1402	V1403	V1404	V1405	V1406	V1407	V1408	V1409	V1410	V1411	V1412	V1413	V1414	V1415	V1416	V1417	V1418	V1419	V1420	V1421	V1422	V1423	V1424	V1425	V1426	V1427	V1428	V1429	V1430	V1431	V1432	V1433	V1434	V1435	V1436	V1437	V1438	V1439	V1440	V1441	V1442	V1443	V1444	V1445	V1446	V1447	V1448	V1449	V1450	V1451	V1452	V1453	V1454	V1455	V1456	V1457	V1458	V1459	V1460	V1461	V1462	V1463	V1464	V1465	V1466	V1467	V1468	V1469	V1470
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Table A2 Recorded data for slab-column specimen A2 (Continued)

[illegible]

A21a	70	345	1865	1865	1756	3031	2701	1296	223
A21b.....	= strain occurred in the shear studs located shown in Figure A1								

Table A3 Recorded data for slab-column specimen A3

Load Cell 1	Load Cell 2	Load Cell 3	Load Cell 4	Vulnitate	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	Ccentre
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	4	6.7	5.3	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	9	10.9	9.3	40.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	15	14.8	15.3	60.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	20	20.9	19.7	81.6	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.3	0.2	0.1	0.1	0.1	0.3	0.3	0.1	0.2	0
26	24	25.1	23.7	98.8	0.1	0.1	0.1	0.2	0.1	0.2	0.2	0.4	0.2	0.3	0.4	0.7	0.7	0.4	0.5	0.9	0.1
31	30	29.4	28.4	120.4	0.1	0.1	0.1	0.2	0.2	0.3	0.3	0.5	0.4	0.4	0.4	0.9	1	0.6	0.6	1.1	0.1
36	34	35.2	33.8	139	0.1	0.1	0.1	0.3	0.4	0.4	0.3	0.8	0.7	0.6	0.6	1.1	1.4	0.9	0.7	1.2	0.1
41	39	39.4	38.8	158.2	0.2	0.2	0.1	0.3	0.5	0.5	0.3	0.9	0.8	0.7	0.6	1.4	1.7	1.1	0.9	1.6	0.1
52	48	48.7	47.5	197.2	0.2	0.3	0.2	0.4	0.7	0.7	0.4	1.2	1.3	0.9	0.6	1.6	1.9	1.3	1.1	1.8	0.1
61	59	59.4	58.2	237.6	0.2	0.3	0.2	0.4	0.7	0.9	0.6	1.5	1.4	1.7	1.1	2.1	2.8	1.9	1.5	2.3	0.2
72	69	70.7	70.7	282.4	0.3	0.5	0.2	0.5	0.9	1.4	0.7	1.9	1.8	2.5	1.5	3.5	4.4	3.4	2.2	3.2	0.1
81	78	80.4	80.1	319.5	0.3	0.5	0.2	0.6	1	1.7	1	2.2	2.1	3.1	2	4.2	5.2	4.1	3.2	4.2	0.2
91	89	89.7	90.1	359.8	0.3	0.6	0.3	0.7	1.2	2.1	1.1	2.7	2.6	3.9	2.4	5.1	6.4	5	5.2	6.4	0.2
102	99	100.7	99.5	401.2	0.3	0.7	0.3	0.8	1.5	2.6	1.4	3.2	3.1	4.7	2.9	6.1	7.6	6.1	6.2	7.6	0.2
113	110	111.4	110.4	444.8	0.4	0.8	0.3	1	1.8	3.1	1.6	3.8	3.7	5.7	3.5	7.2	9.2	7.2	7.5	9	0.3
119	120	118.8	119.8	477.6	0.4	0.9	0.4	1.1	1.9	3.5	1.9	4.3	4.1	6.4	4.1	8.3	10.4	8.1	8.6	10.3	0.3
129	129	129.5	128.3	516.8	0.5	0.9	0.4	1.2	2.2	3.9	2.1	4.8	4.7	7.2	4.6	9.3	11.8	9	9.6	11.6	0.3
139	140	139.2	139.2	557.4	0.5	1	0.4	1.3	2.5	4.3	2.3	5.4	5.3	8	5	10.5	13.3	10.1	10.6	12.9	0.3
150	149	150.1	149.3	598.4	0.5	1	0.5	1.4	2.7	4.7	2.6	6	5.9	8.8	5.6	11.5	14.7	11	11.7	14.2	0.3
161	159	159.2	158.6	637.8	0.6	1.1	0.5	1.6	3.1	5.2	2.8	6.8	6.8	9.8	6.2	13	16.6	12.3	12.9	15.7	0.4
173	170	170.5	169.6	683.1	0.6	1.1	0.5	1.7	3.4	5.6	3	7.4	7.5	10.6	6.7	13.9	18.2	13.4	13.8	17	0.4
181	179	179.5	178.4	717.9	0.7	1.2	0.5	1.8	3.7	6	3.2	8	8	11.4	7.2	15.1	19.6	14.3	14.8	18.3	0.4
190	191	188.6	188.3	758.9	0.7	1.2	0.6	1.9	4	6.5	3.6	8.8	8.7	12.3	8	16.5	21.3	15.3	16.1	20.1	0.4
199	197	199.6	198.7	794.3	0.7	1.3	0.6	1.9	4.2	6.9	3.9	9.3	9.2	13.1	8.6	17.4	22.3	16.2	17.3	21.2	0.4
205	204	206	203.4	818.4	0.7	1.3	0.6	2	4.4	7.3	4.2	9.9	9.7	13.8	9.1	18.4	23.5	16.9	18.1	21	0.4
210	210	211.2	209.3	840.5	0.7	1.4	0.6	2.1	4.6	7.6	4.4	10.4	10.2	14.4	9.6	19.3	24.6	17.6	18.9	23.5	0.5
214	213	215.1	215	857.1	0.7	1.4	0.7	2.1	4.8	7.8	4.5	10.6	10.4	14.9	9.9	19.8	25.2	18.1	19.5	24.1	0.5
219	219	220.2	219.7	877.9	0.8	1.4	0.7	2.2	4.9	8	4.6	11	10.8	15.2	10.1	20.4	26.1	18.6	19.9	24.8	0.5
223	225	224.4	223.7	896.1	0.8	1.5	0.7	2.2	5.1	8.2	4.8	11.6	11.2	15.6	10.6	21.4	27.3	19.1	20.5	25.9	0.5
228	227	227.8	227.3	909.8	0.8	1.5	0.7	2.3	5.4	8.5	4.9	11.9	11.7	16	10.7	22	28.1	19.7	20.9	26.4	0.5
235	234	235.7	234.4	939.1	0.8	1.5	0.7	2.3	5.6	8.8	5.1	12.6	12.2	16.6	11.2	23.2	29.6	20.4	21.6	27.4	0.5
233	232	227	239.7	931.7	0.8	1.5	0.7	2.3	5.9	8.9	5.4	13.9	12.8	16.8	11.6	25.2	31.5	20.7	21.7	28.7	0.5
232	235	231.2	240	938.2	0.8	1.5	0.8	2.3	6.3	8.9	5.7	15.3	13.4	16.7	12.1	27.2	33.6	20.6	21.7	31.8	0.5
223	214	221.5	234.7	893.2	0.8	1.5	0.8	2.1	6.6	8.7	6.3	16.6	14	16.2	12.9	29.1	35.1	20.1	21.4	33.7	0.5
201	194	188.6	215.9	798.5	0.9	1.4	0.6	2	7.1	7.8	7.5	18.7	15.3	14.1	15.6	35.1	38.5	18.1	19.5	40.5	0.4
194	189	183.7	212.8	779.5	0.9	1.4	0.6	2	7.3	7.4	7.7	19.8	16.3	13.2	16.3	38.1	42.7	17.2	18.6	43.1	0.4
180	176	173.1	198.7	727.8	0.8	1.3	0.6	1.9	7.4	6.6	7.8	20.2	17.6	11.6	16.5	40.9	46	15.9	16.6	44.7	0.4
168	165	162.4	189	684.4	0.8	1.3	0.6	1.8	7.5	6	7.8	21.3	19.4	9.8	16.9	44.2	49.8	14.6	14.3	47.1	0.4

Vulnitate = total ultimate load of four load cells in one application
D1, D2,..... = deflection of slab measured by linear variabletransducer (LVDT) at the locations shown in Figure A1
Ccentre = central deflection of slab measured by LVDT D17 located at the bottom of the slab shown in Figure A1

Table A3 Recoreded data for slab-column specimen A3 (Continued)

MDT1	MDT2	MDT3	MDT4	MDT5	MDT6	MDT7	MDT8	MDT9	MDT10	MDT11	MDT12	MDT13	MDT14	MDT15	MDT16	MDT17	MDT18	MDT19	MDT20	MDT21	MDT22	MDT23	MDT24	MDT25	MDT26	MDT27	MDT28	MDT29	MDT30	MDT31	MDT32	MDT33	MDT34	MDT35	MDT36	MDT37	MDT38	MDT39	MDT40	MDT41	MDT42	MDT43	MDT44	MDT45	MDT46	MDT47	MDT48	MDT49	MDT50	MDT51	MDT52	MDT53	MDT54	MDT55	MDT56	MDT57	MDT58	MDT59	MDT60	MDT61	MDT62	MDT63	MDT64	MDT65	MDT66	MDT67	MDT68	MDT69	MDT70	MDT71	MDT72	MDT73	MDT74	MDT75	MDT76	MDT77	MDT78	MDT79	MDT80	MDT81	MDT82	MDT83	MDT84	MDT85	MDT86	MDT87	MDT88	MDT89	MDT90	MDT91	MDT92	MDT93	MDT94	MDT95	MDT96	MDT97	MDT98	MDT99	MDT100	MDT101	MDT102	MDT103	MDT104	MDT105	MDT106	MDT107	MDT108	MDT109	MDT110	MDT111	MDT112	MDT113	MDT114	MDT115	MDT116	MDT117	MDT118	MDT119	MDT120	MDT121	MDT122	MDT123	MDT124	MDT125	MDT126	MDT127	MDT128	MDT129	MDT130	MDT131	MDT132	MDT133	MDT134	MDT135	MDT136	MDT137	MDT138	MDT139	MDT140	MDT141	MDT142	MDT143	MDT144	MDT145	MDT146	MDT147	MDT148	MDT149	MDT150	MDT151	MDT152	MDT153	MDT154	MDT155	MDT156	MDT157	MDT158	MDT159	MDT160	MDT161	MDT162	MDT163	MDT164	MDT165	MDT166	MDT167	MDT168	MDT169	MDT170	MDT171	MDT172	MDT173	MDT174	MDT175	MDT176	MDT177	MDT178	MDT179	MDT180	MDT181	MDT182	MDT183	MDT184	MDT185	MDT186	MDT187	MDT188	MDT189	MDT190	MDT191	MDT192	MDT193	MDT194	MDT195	MDT196	MDT197	MDT198	MDT199	MDT200	MDT201	MDT202	MDT203	MDT204	MDT205	MDT206	MDT207	MDT208	MDT209	MDT210	MDT211	MDT212	MDT213	MDT214	MDT215	MDT216	MDT217	MDT218	MDT219	MDT220	MDT221	MDT222	MDT223	MDT224	MDT225	MDT226	MDT227	MDT228	MDT229	MDT230	MDT231	MDT232	MDT233	MDT234	MDT235	MDT236	MDT237	MDT238	MDT239	MDT240	MDT241	MDT242	MDT243	MDT244	MDT245	MDT246	MDT247	MDT248	MDT249	MDT250	MDT251	MDT252	MDT253	MDT254	MDT255	MDT256	MDT257	MDT258	MDT259	MDT260	MDT261	MDT262	MDT263	MDT264	MDT265	MDT266	MDT267	MDT268	MDT269	MDT270	MDT271	MDT272	MDT273	MDT274	MDT275	MDT276	MDT277	MDT278	MDT279	MDT280	MDT281	MDT282	MDT283	MDT284	MDT285	MDT286	MDT287	MDT288	MDT289	MDT290	MDT291	MDT292	MDT293	MDT294	MDT295	MDT296	MDT297	MDT298	MDT299	MDT300	MDT301	MDT302	MDT303	MDT304	MDT305	MDT306	MDT307	MDT308	MDT309	MDT310	MDT311	MDT312	MDT313	MDT314	MDT315	MDT316	MDT317	MDT318	MDT319	MDT320	MDT321	MDT322	MDT323	MDT324	MDT325	MDT326	MDT327	MDT328	MDT329	MDT330	MDT331	MDT332	MDT333	MDT334	MDT335	MDT336	MDT337	MDT338	MDT339	MDT340	MDT341	MDT342	MDT343	MDT344	MDT345	MDT346	MDT347	MDT348	MDT349	MDT350	MDT351	MDT352	MDT353	MDT354	MDT355	MDT356	MDT357	MDT358	MDT359	MDT360	MDT361	MDT362	MDT363	MDT364	MDT365	MDT366	MDT367	MDT368	MDT369	MDT370	MDT371	MDT372	MDT373	MDT374	MDT375	MDT376	MDT377	MDT378	MDT379	MDT380	MDT381	MDT382	MDT383	MDT384	MDT385	MDT386	MDT387	MDT388	MDT389	MDT390	MDT391	MDT392	MDT393	MDT394	MDT395	MDT396	MDT397	MDT398	MDT399	MDT400	MDT401	MDT402	MDT403	MDT404	MDT405	MDT406	MDT407	MDT408	MDT409	MDT410	MDT411	MDT412	MDT413	MDT414	MDT415	MDT416	MDT417	MDT418	MDT419	MDT420	MDT421	MDT422	MDT423	MDT424	MDT425	MDT426	MDT427	MDT428	MDT429	MDT430	MDT431	MDT432	MDT433	MDT434	MDT435	MDT436	MDT437	MDT438	MDT439	MDT440	MDT441	MDT442	MDT443	MDT444	MDT445	MDT446	MDT447	MDT448	MDT449	MDT450	MDT451	MDT452	MDT453	MDT454	MDT455	MDT456	MDT457	MDT458	MDT459	MDT460	MDT461	MDT462	MDT463	MDT464	MDT465	MDT466	MDT467	MDT468	MDT469	MDT470	MDT471	MDT472	MDT473	MDT474	MDT475	MDT476	MDT477	MDT478	MDT479	MDT480	MDT481	MDT482	MDT483	MDT484	MDT485	MDT486	MDT487	MDT488	MDT489	MDT490	MDT491	MDT492	MDT493	MDT494	MDT495	MDT496	MDT497	MDT498	MDT499	MDT500	MDT501	MDT502	MDT503	MDT504	MDT505	MDT506	MDT507	MDT508	MDT509	MDT510	MDT511	MDT512	MDT513	MDT514	MDT515	MDT516	MDT517	MDT518	MDT519	MDT520	MDT521	MDT522	MDT523	MDT524	MDT525	MDT526	MDT527	MDT528	MDT529	MDT530	MDT531	MDT532	MDT533	MDT534	MDT535	MDT536	MDT537	MDT538	MDT539	MDT540	MDT541	MDT542	MDT543	MDT544	MDT545	MDT546	MDT547	MDT548	MDT549	MDT550	MDT551	MDT552	MDT553	MDT554	MDT555	MDT556	MDT557	MDT558	MDT559	MDT560	MDT561	MDT562	MDT563	MDT564	MDT565	MDT566	MDT567	MDT568	MDT569	MDT570	MDT571	MDT572	MDT573	MDT574	MDT575	MDT576	MDT577	MDT578	MDT579	MDT580	MDT581	MDT582	MDT583	MDT584	MDT585	MDT586	MDT587	MDT588	MDT589	MDT590	MDT591	MDT592	MDT593	MDT594	MDT595	MDT596	MDT597	MDT598	MDT599	MDT600	MDT601	MDT602	MDT603	MDT604	MDT605	MDT606	MDT607	MDT608	MDT609	MDT610	MDT611	MDT612	MDT613	MDT614	MDT615	MDT616	MDT617	MDT618	MDT619	MDT620	MDT621	MDT622	MDT623	MDT624	MDT625	MDT626	MDT627	MDT628	MDT629	MDT630	MDT631	MDT632	MDT633	MDT634	MDT635	MDT636	MDT637	MDT638	MDT639	MDT640	MDT641	MDT642	MDT643	MDT644	MDT645	MDT646	MDT647	MDT648	MDT649	MDT650	MDT651	MDT652	MDT653	MDT654	MDT655	MDT656	MDT657	MDT658	MDT659	MDT660	MDT661	MDT662	MDT663	MDT664	MDT665	MDT666	MDT667	MDT668	MDT669	MDT670	MDT671	MDT672	MDT673	MDT674	MDT675	MDT676	MDT677	MDT678	MDT679	MDT680	MDT681	MDT682	MDT683	MDT684	MDT685	MDT686	MDT687	MDT688	MDT689	MDT690	MDT691	MDT692	MDT693	MDT694	MDT695	MDT696	MDT697	MDT698	MDT699	MDT700	MDT701	MDT702	MDT703	MDT704	MDT705	MDT706	MDT707	MDT708	MDT709	MDT710	MDT711	MDT712	MDT713	MDT714	MDT715	MDT716	MDT717	MDT718	MDT719	MDT720	MDT721	MDT722	MDT723	MDT724	MDT725	MDT726	MDT727	MDT728	MDT729	MDT730	MDT731	MDT732	MDT733	MDT734	MDT735	MDT736	MDT737	MDT738	MDT739	MDT740	MDT741	MDT742	MDT743	MDT744	MDT745	MDT746	MDT747	MDT748	MDT749	MDT750	MDT751	MDT752	MDT753	MDT754	MDT755	MDT756	MDT757	MDT758	MDT759	MDT760	MDT761	MDT762	MDT763	MDT764	MDT765	MDT766	MDT767	MDT768	MDT769	MDT770	MDT771	MDT772	MDT773	MDT774	MDT775	MDT776	MDT777	MDT778	MDT779	MDT780	MDT781	MDT782	MDT783	MDT784	MDT785	MDT786	MDT787	MDT788	MDT789	MDT790	MDT791	MDT792	MDT793	MDT794	MDT795	MDT796	MDT797	MDT798	MDT799	MDT800	MDT801	MDT802	MDT803	MDT804	MDT805	MDT806	MDT807	MDT808	MDT809	MDT810	MDT811	MDT812	MDT813	MDT814	MDT815	MDT816	MDT817	MDT818	MDT819	MDT820	MDT821	MDT822	MDT823	MDT824	MDT825	MDT826	MDT827	MDT828	MDT829	MDT830	MDT831	MDT832	MDT833	MDT834	MDT835	MDT836	MDT837	MDT838	MDT839	MDT840	MDT841	MDT842	MDT843	MDT844	MDT845	MDT846	MDT847	MDT848	MDT849	MDT850	MDT851	MDT852	MDT853	MDT854	MDT855	MDT856	MDT857	MDT858	MDT859	MDT860	MDT861	MDT862	MDT863	MDT864	MDT865	MDT866	MDT867	MDT868	MDT869	MDT870	MDT871	MDT872	MDT873	MDT874	MDT875	MDT876	MDT877	MDT878	MDT879	MDT880	MDT881	MDT882	MDT883	MDT884	MDT885	MDT886	MDT887	MDT888	MDT889	MDT890	MDT891	MDT892	MDT893	MDT894	MDT895	MDT896	MDT897	MDT898	MDT899	MDT900	MDT901	MDT902	MDT903	MDT904	MDT905	MDT906	MDT907	MDT908	MDT909	MDT910	MDT911	MDT912	MDT913	MDT914	MDT915	MDT916	MDT917	MDT918	MDT919	MDT920	MDT921	MDT922	MDT923	MDT924	MDT925	MDT926	MDT927	MDT928	MDT929	MDT930	MDT931	MDT932	MDT933	MDT934	MDT935	MDT936	MDT937	MDT938	MDT939	MDT940	MDT941	MDT942	MDT943	MDT944	MDT945	MDT946	MDT947	MDT948	MDT949	MDT950	MDT951	MDT952	MDT953	MDT954	MDT955	MDT956	MDT957	MDT958	MDT959	MDT960	MDT961	MDT962	MDT963	MDT964	MDT965	MDT966	MDT967	MDT968	MDT969	MDT970	MDT971	MDT972	MDT973	MDT974	MDT975	MDT976	MDT977	MDT978	MDT979	MDT980	MDT981	MDT982	MDT983	MDT984	MDT985	MDT986	MDT987	MDT988	MDT989	MDT990	MDT991	MDT992	MDT993	MDT994	MDT995	MDT996	MDT997	MDT998	MDT999	MDT1000
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										

Table B1 Recorded test data for slab-column specimen B1

Load Cell 1	Load Cell 2	Load Cell 3	Load Cell 4	Vullimate	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	Ccentre	RD1	RD2	RD3
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
5	5	5	5	20.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
10	10	10	10	40.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
15	15	15	15	60.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
20	20	20	20	79.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
25	25	25	25	101.5	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
30	30	30	30	123.7	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
35	35	35	35	145.9	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
40	40	40	40	168.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
45	45	45	45	190.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	
50	50	50	50	212.5	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	
55	55	55	55	234.7	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	
60	60	60	60	256.9	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	
65	65	65	65	279.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
70	70	70	70	301.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
75	75	75	75	323.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
80	80	80	80	345.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
85	85	85	85	367.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
90	90	90	90	390.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
95	95	95	95	412.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
100	100	100	100	434.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
105	105	105	105	456.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
110	110	110	110	478.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
115	115	115	115	501.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
120	120	120	120	523.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
125	125	125	125	545.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
130	130	130	130	567.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
135	135	135	135	589.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
140	140	140	140	612.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
145	145	145	145	634.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
150	150	150	150	656.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
155	155	155	155	678.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
160	160	160	160	700.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
165	165	165	165	723.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
170	170	170	170	745.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
175	175	175	175	767.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
180	180	180	180	789.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
185	185	185	185	811.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
190	190	190	190	834.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
195	195	195	195	856.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
200	200	200	200	878.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
205	205	205	205	900.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
210	210	210	210	922.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
215	215	215	215	945.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
220	220	220	220	967.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
225	225	225	225	989.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
230	230	230	230	1011.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
235	235	235	235	1033.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
240	240	240	240	1056.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
245	245	245	245	1078.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
250	250	250	250	1100.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
255	255	255	255	1122.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
260	260	260	260	1144.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
265	265	265	265	1167.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
270	270	270	270	1189.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
275	275	275	275	1211.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
280	280	280	280	1233.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
285	285	285	285	1255.9	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
290	290	290	290	1278.1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
295	295	295	295	1300.3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
300	300	300	300	1322.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
305	305	305	305	1344.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	
310	310	310	310	1																				

Table B2 Recorded test data for slab-column specimen B2 (Continued)

RD1	RD2	RD3	RD4	RD5	RD6	RD7	RD8	RD9	RD10	RD11	RD12	RD13	RD14	RD15	RD16	RD17	RD18	RD19	RD20	RD21	RD22	RD23	RD24	RD25	RD26	RD27	RD28	RD29	RD30	RD31	RD32	RD33	RD34	RD35	RD36	RD37	RD38	RD39	RD40	RD41	RD42	RD43	RD44	RD45	RD46	RD47	RD48	RD49	RD50	RD51	RD52	RD53	RD54	RD55	RD56	RD57	RD58	RD59	RD60	RD61	RD62	RD63	RD64	RD65	RD66	RD67	RD68	RD69	RD70	RD71	RD72	RD73	RD74	RD75	RD76	RD77	RD78	RD79	RD80	RD81	RD82	RD83	RD84	RD85	RD86	RD87	RD88	RD89	RD90	RD91	RD92	RD93	RD94	RD95	RD96	RD97	RD98	RD99	RD100	RD101	RD102	RD103	RD104	RD105	RD106	RD107	RD108	RD109	RD110	RD111	RD112	RD113	RD114	RD115	RD116	RD117	RD118	RD119	RD120	RD121	RD122	RD123	RD124	RD125	RD126	RD127	RD128	RD129	RD130	RD131	RD132	RD133	RD134	RD135	RD136	RD137	RD138	RD139	RD140	RD141	RD142	RD143	RD144	RD145	RD146	RD147	RD148	RD149	RD150	RD151	RD152	RD153	RD154	RD155	RD156	RD157	RD158	RD159	RD160	RD161	RD162	RD163	RD164	RD165	RD166	RD167	RD168	RD169	RD170	RD171	RD172	RD173	RD174	RD175	RD176	RD177	RD178	RD179	RD180	RD181	RD182	RD183	RD184	RD185	RD186	RD187	RD188	RD189	RD190	RD191	RD192	RD193	RD194	RD195	RD196	RD197	RD198	RD199	RD200	RD201	RD202	RD203	RD204	RD205	RD206	RD207	RD208	RD209	RD210	RD211	RD212	RD213	RD214	RD215	RD216	RD217	RD218	RD219	RD220	RD221	RD222	RD223	RD224	RD225	RD226	RD227	RD228	RD229	RD230	RD231	RD232	RD233	RD234	RD235	RD236	RD237	RD238	RD239	RD240	RD241	RD242	RD243	RD244	RD245	RD246	RD247	RD248	RD249	RD250	RD251	RD252	RD253	RD254	RD255	RD256	RD257	RD258	RD259	RD260	RD261	RD262	RD263	RD264	RD265	RD266	RD267	RD268	RD269	RD270	RD271	RD272	RD273	RD274	RD275	RD276	RD277	RD278	RD279	RD280	RD281	RD282	RD283	RD284	RD285	RD286	RD287	RD288	RD289	RD290	RD291	RD292	RD293	RD294	RD295	RD296	RD297	RD298	RD299	RD300	RD301	RD302	RD303	RD304	RD305	RD306	RD307	RD308	RD309	RD310	RD311	RD312	RD313	RD314	RD315	RD316	RD317	RD318	RD319	RD320	RD321	RD322	RD323	RD324	RD325	RD326	RD327	RD328	RD329	RD330	RD331	RD332	RD333	RD334	RD335	RD336	RD337	RD338	RD339	RD340	RD341	RD342	RD343	RD344	RD345	RD346	RD347	RD348	RD349	RD350	RD351	RD352	RD353	RD354	RD355	RD356	RD357	RD358	RD359	RD360	RD361	RD362	RD363	RD364	RD365	RD366	RD367	RD368	RD369	RD370	RD371	RD372	RD373	RD374	RD375	RD376	RD377	RD378	RD379	RD380	RD381	RD382	RD383	RD384	RD385	RD386	RD387	RD388	RD389	RD390	RD391	RD392	RD393	RD394	RD395	RD396	RD397	RD398	RD399	RD400	RD401	RD402	RD403	RD404	RD405	RD406	RD407	RD408	RD409	RD410	RD411	RD412	RD413	RD414	RD415	RD416	RD417	RD418	RD419	RD420	RD421	RD422	RD423	RD424	RD425	RD426	RD427	RD428	RD429	RD430	RD431	RD432	RD433	RD434	RD435	RD436	RD437	RD438	RD439	RD440	RD441	RD442	RD443	RD444	RD445	RD446	RD447	RD448	RD449	RD450	RD451	RD452	RD453	RD454	RD455	RD456	RD457	RD458	RD459	RD460	RD461	RD462	RD463	RD464	RD465	RD466	RD467	RD468	RD469	RD470	RD471	RD472	RD473	RD474	RD475	RD476	RD477	RD478	RD479	RD480	RD481	RD482	RD483	RD484	RD485	RD486	RD487	RD488	RD489	RD490	RD491	RD492	RD493	RD494	RD495	RD496	RD497	RD498	RD499	RD500	RD501	RD502	RD503	RD504	RD505	RD506	RD507	RD508	RD509	RD510	RD511	RD512	RD513	RD514	RD515	RD516	RD517	RD518	RD519	RD520	RD521	RD522	RD523	RD524	RD525	RD526	RD527	RD528	RD529	RD530	RD531	RD532	RD533	RD534	RD535	RD536	RD537	RD538	RD539	RD540	RD541	RD542	RD543	RD544	RD545	RD546	RD547	RD548	RD549	RD550	RD551	RD552	RD553	RD554	RD555	RD556	RD557	RD558	RD559	RD560	RD561	RD562	RD563	RD564	RD565	RD566	RD567	RD568	RD569	RD570	RD571	RD572	RD573	RD574	RD575	RD576	RD577	RD578	RD579	RD580	RD581	RD582	RD583	RD584	RD585	RD586	RD587	RD588	RD589	RD590	RD591	RD592	RD593	RD594	RD595	RD596	RD597	RD598	RD599	RD600	RD601	RD602	RD603	RD604	RD605	RD606	RD607	RD608	RD609	RD610	RD611	RD612	RD613	RD614	RD615	RD616	RD617	RD618	RD619	RD620	RD621	RD622	RD623	RD624	RD625	RD626	RD627	RD628	RD629	RD630	RD631	RD632	RD633	RD634	RD635	RD636	RD637	RD638	RD639	RD640	RD641	RD642	RD643	RD644	RD645	RD646	RD647	RD648	RD649	RD650	RD651	RD652	RD653	RD654	RD655	RD656	RD657	RD658	RD659	RD660	RD661	RD662	RD663	RD664	RD665	RD666	RD667	RD668	RD669	RD670	RD671	RD672	RD673	RD674	RD675	RD676	RD677	RD678	RD679	RD680	RD681	RD682	RD683	RD684	RD685	RD686	RD687	RD688	RD689	RD690	RD691	RD692	RD693	RD694	RD695	RD696	RD697	RD698	RD699	RD700	RD701	RD702	RD703	RD704	RD705	RD706	RD707	RD708	RD709	RD710	RD711	RD712	RD713	RD714	RD715	RD716	RD717	RD718	RD719	RD720	RD721	RD722	RD723	RD724	RD725	RD726	RD727	RD728	RD729	RD730	RD731	RD732	RD733	RD734	RD735	RD736	RD737	RD738	RD739	RD740	RD741	RD742	RD743	RD744	RD745	RD746	RD747	RD748	RD749	RD750	RD751	RD752	RD753	RD754	RD755	RD756	RD757	RD758	RD759	RD760	RD761	RD762	RD763	RD764	RD765	RD766	RD767	RD768	RD769	RD770	RD771	RD772	RD773	RD774	RD775	RD776	RD777	RD778	RD779	RD780	RD781	RD782	RD783	RD784	RD785	RD786	RD787	RD788	RD789	RD790	RD791	RD792	RD793	RD794	RD795	RD796	RD797	RD798	RD799	RD800	RD801	RD802	RD803	RD804	RD805	RD806	RD807	RD808	RD809	RD810	RD811	RD812	RD813	RD814	RD815	RD816	RD817	RD818	RD819	RD820	RD821	RD822	RD823	RD824	RD825	RD826	RD827	RD828	RD829	RD830	RD831	RD832	RD833	RD834	RD835	RD836	RD837	RD838	RD839	RD840	RD841	RD842	RD843	RD844	RD845	RD846	RD847	RD848	RD849	RD850	RD851	RD852	RD853	RD854	RD855	RD856	RD857	RD858	RD859	RD860	RD861	RD862	RD863	RD864	RD865	RD866	RD867	RD868	RD869	RD870	RD871	RD872	RD873	RD874	RD875	RD876	RD877	RD878	RD879	RD880	RD881	RD882	RD883	RD884	RD885	RD886	RD887	RD888	RD889	RD890	RD891	RD892	RD893	RD894	RD895	RD896	RD897	RD898	RD899	RD900	RD901	RD902	RD903	RD904	RD905	RD906	RD907	RD908	RD909	RD910	RD911	RD912	RD913	RD914	RD915	RD916	RD917	RD918	RD919	RD920	RD921	RD922	RD923	RD924	RD925	RD926	RD927	RD928	RD929	RD930	RD931	RD932	RD933	RD934	RD935	RD936	RD937	RD938	RD939	RD940	RD941	RD942	RD943	RD944	RD945	RD946	RD947	RD948	RD949	RD950	RD951	RD952	RD953	RD954	RD955	RD956	RD957	RD958	RD959	RD960	RD961	RD962	RD963	RD964	RD965	RD966	RD967	RD968	RD969	RD970	RD971	RD972	RD973	RD974	RD975	RD976	RD977	RD978	RD979	RD980	RD981	RD982	RD983	RD984	RD985	RD986	RD987	RD988	RD989	RD990	RD991	RD992	RD993	RD994	RD995	RD996	RD997	RD998	RD999	RD1000	RD1001	RD1002	RD1003	RD1004	RD1005	RD1006	RD1007	RD1008	RD1009	RD1010	RD1011	RD1012	RD1013	RD1014	RD1015	RD1016	RD1017	RD1018	RD1019	RD1020	RD1021	RD1022	RD1023	RD1024	RD1025	RD1026	RD1027	RD1028	RD1029	RD1030	RD1031	RD1032	RD1033	RD1034	RD1035	RD1036	RD1037	RD1038	RD1039	RD1040	RD1041	RD1042	RD1043	RD1044	RD1045	RD1046	RD1047	RD1048	RD1049	RD1050	RD1051	RD1052	RD1053	RD1054	RD1055	RD1056	RD1057	RD1058	RD1059	RD1060	RD1061	RD1062	RD1063	RD1064	RD1065	RD1066	RD1067	RD1068	RD1069	RD1070	RD1071	RD1072	RD1073	RD1074	RD1075	RD1076	RD1077	RD1078	RD1079	RD1080	RD1081	RD1082	RD1083	RD1084	RD1085	RD1086	RD1087	RD1088	RD1089	RD1090	RD1091	RD1092	RD1093	RD1094	RD1095	RD1096	RD1097	RD1098	RD1099	RD1100	RD1101	RD1102	RD1103	RD1104	RD1105	RD1106	RD1107	RD1108	RD1109	RD1110	RD1111	RD1112	RD1113	RD1114	RD1115	RD1116	RD1117	RD1118	RD1119	RD1120	RD1121	RD1122	RD1123	RD1124	RD1125	RD1126	RD1127	RD1128	RD1129	RD1130	RD1131	RD1132	RD1133	RD1134	RD1135	RD1136	RD1137	RD1138	RD1139	RD1140	RD1141	RD1142	RD1143	RD1144	RD1145	RD1146	RD1147	RD1148	RD1149	RD1150	RD1151	RD1152	RD1153	RD1154	RD1155	RD1156	RD1157	RD1158	RD1159	RD1160	RD1161	RD1162	RD1163	RD1164	RD1165	RD1166	RD1167	RD1168	RD1169	RD1170	RD1171	RD1172	RD1173	RD1174	RD1175	RD1176	RD1177	RD1178	RD1179	RD1180	RD1181	RD1182	RD1183	RD1184	RD1185	RD1186	RD1187	RD1188	RD1189	RD1190	RD1191	RD1192	RD1193	RD1194	RD1195	RD1196	RD1197	RD1198	RD1199	RD1200	RD1201	RD1202	RD1203	RD1204	RD1205	RD1206	RD1207	RD1208	RD1209	RD1210	RD1211	RD1212	RD1213	RD1214	RD1215	RD1216	RD1217	RD1218	RD1219	RD1220	RD1221	RD1222	RD1223	RD1224	RD1225	RD1226	RD1227	RD1228	RD1229	RD1230	RD1231	RD1232	RD1233	RD1234	RD1235	RD1236	RD1237	RD1238	RD1239	RD1240	RD1241	RD1242	RD1243	RD1244	RD1245	RD1246	RD1247	RD1248	RD1249	RD1250	RD1251	RD1252	RD1253	RD1254	RD1255	RD1256	RD1257	RD1258	RD1259	RD1260	RD1261	RD1262	RD1263	RD1264	RD1265	RD1266	RD1267	RD1268	RD1269	RD1270	RD1271	RD1272	RD1273	RD1274	RD1275	RD1276	RD1277	RD1278	RD1279	RD1280	RD1281	RD1282	RD1283	RD1284	RD1285	RD1286	RD1287	RD1288	RD1289	RD1290	RD1291	RD1292	RD1293	RD1294	RD1295	RD1296	RD1297	RD1298	RD1299	RD1300	RD1301	RD1302	RD1303	RD1304	RD1305	RD1306	RD1307	RD1308	RD1309	RD1310	RD1311	RD1312	RD1313	RD1314	RD1315	RD1316	RD1317	RD1318	RD1319	RD1320	RD1321	RD1322	RD1323	RD1324	RD1325	RD1326	RD1327	RD1328	RD1329	RD1330	RD1331	RD1332	
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Table B3 Recorded test data for slab-column specimen B3

Load Cell 1	Load Cell 2	Load Cell 3	Load Cell 4	Volume	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	Centre	R01	R02	R03	R04	R05	R06	R07	R08	R09	R10	R11	R12	R13
3	5	5.1	5.8	18.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
8	10	9.3	9.7	37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
15	14	15.3	14.7	55.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
25	26	25.1	25.6	102.7	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	
28	28	28.1	28.6	102.7	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	
35	30	29.7	29.7	116.4	0.2	0.2	0.1	0.4	0.4	0.5	0.2	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.1	0.1	0	0	0	0	0	0	0	0	0	0	0	0
38	40	38	41	159	0.2	0.3	0.2	0.7	0.5	0.7	0.3	1.3	1	1.2	0.6	1.7	2.3	1.6	1.4	1.8	0.2	0	0	0	0	0	0	0	0	0	0	0	0	0
49	48	48.4	48.3	195.2	0.3	0.4	0.2	0.9	0.7	0.9	0.5	1.7	1.2	1.6	1	2.3	3.1	2.1	2	2.7	0.2	0.1	0.2	0	0	0	0	0	0	0	0	0	0	0
69	68	67.9	68.1	261.1	0.4	0.5	0.4	1.2	0.9	1.1	0.6	2.1	1.5	2.1	1.2	4.2	5.8	2.8	2.7	3.7	0.2	0.1	0.2	0	0	0	0	0	0	0	0	0	0	0
70	69	70	70.3	260.1	0.4	0.5	0.4	1.6	1	1.6	1.1	3.1	1.9	2.9	2.2	4.2	6.3	4.2	4.9	3.7	0.3	0.1	0.3	0	0	0	0	0	0	0	0	0	0	0
80	79	81.3	78.1	318.8	0.4	0.6	0.4	2	1.2	1.9	1.4	5	2.9	4.4	3.6	5.8	6.6	6.2	6.3	8.5	0.3	0.1	0.4	0.2	0	0	0	0	0	0	0	0	0	0
90	89	91.7	89.1	360.2	0.4	0.7	0.5	2.4	1.5	2.4	1.8	5	2.9	4.4	3.6	5.8	6.6	6.2	6.3	8.5	0.3	0.1	0.4	0.2	0	0	0	0	0	0	0	0	0	0
100	100	100.1	100.4	395.5	0.5	0.7	0.6	2.9	1.7	2.7	2	6	3.5	5.2	4.2	6.2	10.4	6.1	7.4	9.9	0.3	0.2	0.4	0.3	0.2	0	0	0	0	0	0	0	0	0
110	110	110.1	110.4	430.8	0.5	0.7	0.6	3.2	2	2.7	2	6	3.5	5.2	4.2	6.2	10.4	6.1	7.4	9.9	0.3	0.2	0.4	0.3	0.2	0	0	0	0	0	0	0	0	0
119	120	119.8	119.8	478.6	0.5	0.8	0.7	4.1	2.2	3.6	2.8	8.2	4.7	6.6	5.8	11.2	14.3	8	9.7	13.8	0.4	0.1	0.4	0.3	0.3	0.1	0	0	0	0	0	0	0	0
128	130	129.5	130.2	517.7	0.6	1	0.8	4.8	2.5	4	3.2	9.5	5.3	7.5	6.7	13	16.1	8.8	10.7	15.6	0.4	0.2	0.5	0.4	0.4	0.1	0	0	0	0	0	0	0	0
138	139	139.2	140.5	557.7	0.6	1	0.8	5.5	2.8	4.4	3.6	10.7	6.1	8.3	7.5	14.7	18.2	9.8	11.8	17.5	0.4	0.2	0.5	0.4	0.4	0.1	0	0	0	0	0	0	0	0
148	149	149.1	150.4	597.8	0.6	1	0.8	6.3	3.2	4.8	4	12.1	6.9	9.1	8.4	16.6	20.5	10.8	12.9	19.7	0.4	0.2	0.6	0.5	0.5	0.2	0	0	0	0	0	0	0	0
154	154	154.1	154.6	617.3	0.6	1	0.8	6.3	3.2	4.8	4	12.1	6.9	9.1	8.4	16.6	20.5	10.8	12.9	19.7	0.4	0.2	0.6	0.5	0.5	0.2	0	0	0	0	0	0	0	0
159	159	159.5	159.9	637.4	0.7	1.1	0.9	7.4	3.8	5.1	4.6	13.9	7.6	9.7	9.4	19.1	23.4	11.9	13.8	21.3	0.5	0.2	0.6	0.4	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
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164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
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164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5	0.3	0	0	0	0	0	0	0	0
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164	164	164.1	164.3	660.3	0.7	1.1	1	8.1	4.1	5.3	4.8	15.1	8.9	10.1	10	20.8	25.2	12	14.3	23.9	0.5	0.2	0.6	0.5	0.5									

APPENDIX B

DERIVATION OF THE PUNCHING SHEAR CAPACITY OF SLAB-COLUMN CONNECTION BY THEORY OF PLASTICITY

Appendix B

Derivation for punching shear capacity of slab-column connection by plasticity model

B.1 Punching shear capacity of slab-column connection without opening and without shear reinforcement

In deriving the punching shear capacity of slab-column connection based on the theory of plasticity, MathematicaSoftware is used.

Consider a slab-column connection with square or rectangular column under concentric punching load, P. The slab is simply supported along its edges. The slab is reinforced in such a way that flexural failure is prevented. A failure mechanism is considered such that the slab portion bounded by the failure surface is punched out vertically to a distance u and the remaining slab portion remains as rigid. The generatrix of failure surface is axisymrnetricand described by a function $r = r(y)$. The strain in the circumferential direction is considered zero and the dowel action of flexural reinforcement is neglected. The punching failure surface for simply supported slab-column connections under concentric punching load is shown in Figure B- 1.

The dissipation energy per one unit area of failure surface W_A is

$$W_A = \frac{1}{2} f'_c u (L - M \sin \alpha) \quad (\text{B-1})$$

$$L = 1 - 2 \frac{f_t}{f'_c} \frac{\sin \varphi}{1 - \sin \varphi} = 1 - \frac{f_t}{f'_c} (k - 1) \quad (\text{B-1a})$$

$$M = 1 - 2 \frac{f_t}{f'_c} \frac{1}{1 - \sin \varphi} = 1 - \frac{f_t}{f'_c} (k + 1) \quad (\text{B-1b})$$

$$k = \frac{1 + \sin \varphi}{1 - \sin \varphi} \quad (\text{B-1c})$$

By substituting W_A from Equation B-1 into Equation B-2, the total dissipation energy of the failure surface is

$$W_T = \frac{1}{2} f'_c u \int_A (L - M \sin \alpha) dA \quad (B-3)$$

External virtual work EW due to concentric load P that punches the slab portion confined by the failure surface to a vertical displacement u is

$$EW = P \times u \quad (B-4)$$

According to virtual work principle, the external virtual work is equal to the total dissipation energy in the failure surface. Therefore,

$$P \times u = \frac{1}{2} f'_c u \int_A (L - M \sin \alpha) dA \quad (B-5)$$

$$P = \frac{1}{2} f'_c \int_A (L - M \sin \alpha) dA \quad (B-6)$$

From Figure B-1, the infinitesimal area dA of punching shear failure surface is

$$dA = 4(dA_I + dA_{II} + dA_{III}) \quad (B-6a)$$

$$dA_I = \Delta_1 \frac{dy}{\cos \alpha} \quad (B-6b)$$

$$dA_{II} = \Delta_2 \frac{dy}{\cos \alpha} \quad (B-6c)$$

$$dA_{III} = \frac{1}{2} \pi r(y) \frac{dy}{\cos \alpha} \quad (B-6d)$$

where Δ_1 and Δ_2 are the components of circumferential length of failure surface parallel to shorter and longer column width, respectively, as shown in Figure B-1.

By substituting Equations B-6b, B-6c and B-6d into Equation B-6a, dA is

$$dA = 4 \left(\frac{1}{2} \pi r(y) + \Delta_1 + \Delta_2 \right) \frac{dy}{\cos \alpha} \quad (B-7)$$

Then, $\Delta = \Delta_1 + \Delta_2 \quad (B-8)$

From Figure B-1a,

$$\Delta_1 = \frac{C_1}{2} - \left(\frac{C_2}{2} + r_h \right) \tan \frac{\beta_1}{2} \quad (B-8a)$$

$$\Delta_2 = \frac{C_2}{2} - \left(\frac{C_1}{2} + r_h \right) \tan \frac{\beta_2}{2} \quad (\text{B-8b})$$

$$\beta_1 = 2 \tan^{-1} \left(\frac{c_{i1}}{C_2 + 2r_h} \right) \quad (\text{B-8c})$$

$$\beta_2 = 2 \tan^{-1} \left(\frac{c_{i2}}{C_1 + 2r_h} \right) \quad (\text{B-8d})$$

where C_1 & C_2 = shorter and longer column width, respectively

r_h = radius of failure surface at the tension face of slab

c_{i1} & c_{i2} = incremental length of failure surface shown in Figure B-1

$\beta_1, \beta_2, \Delta_1$ and A , can be seen in Figure B-1.

By substituting Equation B-8 into Equation B-7,

$$dA = 4 \left(\frac{1}{2} \pi r(y) + \Delta \right) \frac{dy}{\cos \alpha} \quad (\text{B-9})$$

And, Equation B-9 can be changed into

$$dA = 4L_f \frac{dy}{\cos \alpha} \quad (\text{B-9a})$$

and

$$L_f = \frac{1}{2} \pi r(y) + \Delta \quad (\text{B-9b})$$

where L_f is $\frac{1}{4}$ of circumferential length of failure surface as shown in Figure B-1.

By substituting dA from Equation B-9 into Equation B-6, the punching shear capacity of slab-column connection presented in integral equation form is obtained:

$$P = 2f_c' \int_0^h \frac{(L - M \sin \alpha)}{\cos \alpha} \left(\frac{1}{2} \pi r(y) + \Delta \right) dy \quad (\text{B-10})$$

Equation B-10 is also a functional equation with variables $r(y)$ for the punching shear capacity of slab-column connection. Equation B-10 can be changed into as

$$P = 2f_c' \int_0^h F(r(y)) dy \quad (\text{B-11})$$

and

$$F(r(y)) = \frac{(L - M \sin \alpha)}{\cos \alpha} \left(\frac{1}{2} \pi r(y) + \Delta \right) \quad (\text{B-12})$$

We have the relations related to radius of failure surface $r(y)$ as:

$$r'(y) = \tan \alpha \quad (\text{B-13})$$

$$\sin \alpha = \frac{r'(y)}{\sqrt{1 + (r'(y))^2}} \quad (\text{B-14})$$

$$\cos \alpha = \frac{1}{\sqrt{1 + (r'(y))^2}} \quad (\text{B-15})$$

By substituting Equations B-13, B-14 and B-15 into Equation B-12, the function $F(r(y))$ becomes functions of $r(y)$ and $r'(y)$.

$$F(r, r') = \frac{1}{2} (\pi r(y) + 2\Delta) \left(L \sqrt{1 + (r'(y))^2} - M r'(y) \right) \quad (\text{B-16})$$

By substituting Equation B-16 into Equation B-11,

$$P = f_c' \int_0^h (\pi r(y) + 2\Delta) \left(L \sqrt{1 + (r'(y))^2} - M r'(y) \right) dy \quad (\text{B-17})$$

To obtain the shape of failure surface that gives the least upper bound value of punching shear capacity, the functional Equation B-17 is minimized by $r(y)$ according to calculus of variation. In this case, Euler's equation is used. Euler's equation is

$$F(r, r') - r' F_{r'}(r, r') = C1 \quad (\text{B-18})$$

where C1 is constant.

By differentiating Equation B-16 by $r'(y)$,

$$F_{r'}(r, r') = \frac{1}{2} (\pi r(y) + 2\Delta) \left(\frac{L r'(y)}{\sqrt{1 + (r'(y))^2}} - M \right) \quad (\text{B-19})$$

By using Equations B-16 and B-19,

$$F(r, r') - r' F_{r'}(r, r') = \frac{L (\pi r(y) + 2\Delta)}{2\sqrt{1 + (r'(y))^2}} \quad (\text{B-20})$$

By substituting Equation B-20 into Euler's equation,

$$\frac{L (\pi r(y) + 2\Delta)}{2\sqrt{1 + (r'(y))^2}} = C1 \quad (\text{B-21})$$

$$\frac{L}{2C1} \frac{(\pi r(y) + 2\Delta)}{\sqrt{1 + (r'(y))^2}} = 1 \quad (\text{B-22})$$

By replacing $A = \frac{L}{2 C1}$ in Equation B-22,

$$\frac{A (\pi r(y) + 2\Delta)}{\sqrt{1 + (r'(y))^2}} = 1 \quad (\text{B-23})$$

$$A (\pi r(y) + 2\Delta) = \sqrt{1 + (r'(y))^2} \quad (\text{B-24})$$

$$(A (\pi r(y) + 2\Delta))^2 = 1 + (r'(y))^2 \quad (\text{B-25})$$

$$1 + (r'(y))^2 = A^2 (\pi r + 2\Delta)^2 \quad (\text{B-26})$$

From Equation B-26, the relational equation of $r(y)$ and $r'(y)$ can be obtained as

$$r(y) = \frac{1}{\pi} \left(\frac{\sqrt{1 + (r'(y))^2}}{A} - 2\Delta \right) \quad (\text{B-27})$$

$$r'(y) = \sqrt{A^2 (2\Delta + \pi r(y))^2 - 1} \quad (\text{B-28})$$

But,
$$r'(y) = \frac{dr(y)}{dy} = \tan \alpha \quad (\text{B-29})$$

Therefore,
$$dy = \frac{dr(y)}{r'(y)} = \frac{1}{\sqrt{A^2 (2\Delta + \pi r(y))^2 - 1}} dr(y) \quad (\text{B-30})$$

$$dy = \frac{1}{\sqrt{A^2 (2\Delta + \pi r(y))^2 - 1}} dr(y) \quad (\text{B-31})$$

By differentiating Equation B-26 by y ,

$$2 r'(y) r''(y) = 2 A^2 \pi (\pi r(y) + 2\Delta) r'(y) \quad (\text{B-32})$$

From Equation B-32, the second-order differential equation of radius of failure surface can be obtained as

$$r''(y) = A^2 \pi (\pi r(y) + 2\Delta) \quad (\text{B-33})$$

However,
$$r''(y) = \frac{dr'(y)}{dy} \quad (\text{B-34a})$$

Therefore,
$$\frac{dr'(y)}{dy} = A^2 \pi (\pi r(y) + 2\Delta) \quad (\text{B-34b})$$

$$dr'(y) = A^2 \pi (\pi r(y) + 2\Delta) dy \quad (\text{B-35})$$

$$dy = \frac{dr'(y)}{A^2 \pi (\pi r(y) + 2\Delta)} \quad (B-36)$$

By substituting equation B-27 into Equation B-36, the relationship between dy and $r'(y)$ is

$$dy = \frac{dr'(y)}{A^2 \pi \left(\left(\frac{\sqrt{1 + (r'(y))^2}}{A} - 2\Delta \right) + 2\Delta \right)} \quad (B-37)$$

$$dy = \frac{dr'(y)}{A^2 \pi \left(\sqrt{1 + (r'(y))^2} \right)} \quad (B-38)$$

By solving the second-order differential equation B-33, the curvature of failure surface is obtained as

$$r(y) = -\frac{2\Delta}{\pi} + B e^{A\pi y} + C e^{-A\pi y} \quad (B-39)$$

$$r'(y) = A\pi (B e^{A\pi y} - C e^{-A\pi y}) \quad (B-40)$$

where A , B & C are constants.

Determination of constants A, B and C

By substituting $r(y)$ and $r'(y)$ from Equations B-39 and B-40 into Equation B-26 and solving it, the relationship between A , B and C is obtained as follows.

$$4A^2\pi^2 BC = l \quad (B-41)$$

Therefore,
$$A = \frac{1}{2\pi\sqrt{BC}} \quad (B-42)$$

From Figure B-1, two boundary conditions for the failure surface $r(y)$ are:

$$(1) \ y = 0, \ r(0) = 0 \quad (B-43)$$

$$(2) \ y = h, \ r(h) = r_h = -\frac{2\Delta}{\pi} + B e^{A\pi h} + C e^{-A\pi h} \quad (B-44)$$

where r_h is radius of failure surface at the tension face of slab.

By substituting Equation B-43 from boundary condition (1) into Equations B-39 and B-40,

$$B = \frac{2\Delta}{\pi} - C \quad (\text{B-45})$$

$$r'(0) = A\pi(B - C) \quad (\text{B-46})$$

By substituting Equation B-44 from the boundary condition (2) into Equations B-39 and B-40,

$$r_h = -\frac{2\Delta}{\pi} + B e^{A\pi h} + C e^{-A\pi h} \quad (\text{B-47})$$

$$r'(h) = A\pi(B e^{A\pi h} - C e^{-A\pi h}) \quad (\text{B-48})$$

By using trial and error method, the constants A , B and C and r_h can be determined by satisfying Equations B-42, B-45 and B-47. After determining the constant terms related to the radius of failure surface $r(y)$ from Equation B-39, by substituting Equations B-39 and B-40 into Equation B-17 and solving it, the punching shear capacity of slab-column can be obtained.

However, the slope of failure surface at the compression side of slab should not be less than 37 degree because the internal friction angle of concrete is 37° . Therefore, when the initial slope of failure surface at $y = 0$ is equal or greater than 37 degree, the failure surface of slab is considered as a **curve** from top to bottom of slab while the failure surface of slab is considered as a combined shape of a straight line and a curve when the initial slope of failure surface at $y = 0$ is less than 37 degree. Thus, there are two cases in determining the punching shear capacity of slab; (1) when the initial slope of failure surface at $y = 0$ is equal or greater than 37 degree, and (2) when the initial slope of failure surface at $y = 0$ is less than 37 degree.

Accordingly, the punching capacity of slab for each case is derived as follows.

B.1.1 Initial slope of failure surface at the compression face of slab is equal or greater than 37°

When the initial slope of failure surface at $y = 0$ is equal or greater than 37° degree, the failure surface of slab is considered as a curve from the top to the bottom of slab. The failure surface with slope at the compression face of slab equal or greater than 37° is shown in Figure B-2.

Figure B-2 Punching shear failure surface in curve shape for simply supported slab-column connection under concentric load

By substituting Equations B-39 and B-40 into B-17,

$$P = \frac{f'_c}{A^2 \pi} \int_{r'(0)}^{r'(h)} \left(L \sqrt{1 + (r')^2} - M r' \right) dr' (y) \quad (\text{B-49})$$

By solving the integration,

$$P = \frac{f'_c}{2 A^2 \pi} \left[L \left(\sinh^{-1} r' + r' \sqrt{1 + (r')^2} \right) - M (r')^2 \right]_{r'(0)}^{r'(h)} \quad (\text{B-50})$$

And by inserting the limits, the punching shear capacity of slab-column connection is obtained as the functions of $r'(0)$ and $r'(h)$.

$$P = \frac{f'_c}{2 A^2 \pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_0) + r'_0 \sqrt{1 + (r'_0)^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \quad (\text{B-51})$$

where r'_0 and r'_h are $r'(0)$ and $r'(h)$ from Equations B-46 and B-48, respectively.

In addition, by substituting Equation B-31 and B-39 into Equation B-17,

$$P = f_c' \int_0^{r(h)=r_h} (\pi r(y) + 2\Delta) \left(L \sqrt{A^2 + (\pi r(y) + 2\Delta)^2} - M \sqrt{-1 + A^2 + (\pi r(y) + 2\Delta)^2} \right) dr(y) \quad (B-52)$$

By solving the integration and inserting the limits, the punching shear capacity of slab-column connection is obtained as a function of r_h ,

$$P = \frac{f_c'}{2A^2\pi} \left(L \left(-2A\Delta\sqrt{-1+4A^2\Delta^2} - \ln(2A\Delta + \sqrt{-1+4A^2\Delta^2}) \right) + \ln \left(\sqrt{A^2(-2\Delta - \pi r_h)^2} + \sqrt{-1+A^2(-2\Delta - \pi r_h)^2} \right) - \sqrt{A^2(-2\Delta - \pi r_h)^2} \sqrt{-1+A^2(-2\Delta - \pi r_h)^2} \right) + M \left(-4A^2\pi\Delta r_h - A^2\pi^2 r_h^2 \right) \quad (B-53)$$

where r_h is obtained from Equation B-47.

Equations B-51 or B-53 gives the punching shear capacity of slab if the failure surface of slab is considered as catenary curve shape from the top to the bottom of slab when the initial slope of failure surface at $y = 0$ is equal or greater than 37 degree. The limitation to check if the shape of failure surface is a catenary curve is derived as follows.

Limitation to check if the shape of failure surface is a catenary curve

When the initial slope of failure surface at $y = 0$ is equal or greater than 37 degree, the slope of failure surface at the compression face of slab can be expressed as

$$r'(0) \geq \frac{3}{4} \quad (B-54)$$

By substituting Equation B-46 into B-54,

$$A\pi(B-C) \geq \frac{3}{4} \quad (B-55)$$

By substituting $B = \frac{2\Delta}{\pi} - C$ from Equation B-45 into Equation B-55, the constant C is

$$C \leq \frac{8A\Delta - 3}{8A\pi} \quad (\text{B-56})$$

By substituting $B = \frac{2\Delta}{\pi} - C$ from Equation B-45 into Equation B-47,

$$r_h = r(h) = \frac{2}{\pi} \left(\Delta(1 - e^{A\pi h}) + C\pi \sinh(A\pi h) \right) \quad (\text{B-57})$$

By solving Equation B-57, the constant C is

$$C = \frac{2\Delta(1 - e^{A\pi h}) + \pi r_h}{2\pi \sinh(A\pi h)} \quad (\text{B-58})$$

By substituting Equation B-56 into Equation B-58,

$$\frac{2\Delta(1 - e^{A\pi h}) + \pi r_h}{2\pi \sinh(A\pi h)} \leq \frac{8A\Delta - 3}{8A\pi} \quad (\text{B-59})$$

By solving Equation B-59, the radius of failure surface r_h at the tension face of slab is

$$r_h \leq \frac{8A\Delta(1 + \cosh(A\pi h)) + 3\sinh(A\pi h)}{4A\pi} \quad (\text{B-60})$$

However, the radius of failure surface at the tension face of the slab is equal or greater than $\frac{3}{4}h$, where h is thickness of slab, when the initial slope of failure surface at $y = 0$ is equal or greater than 37 degree. Therefore,

$$r_h \geq \frac{3}{4}h \quad (\text{B-61})$$

By using Equation B-60 and B-61, the radius of failure surface at the tension face of slab is

$$\frac{3}{4}h \leq r_h \leq \frac{8A\Delta(1 + \cosh(A\pi h)) + 3\sinh(A\pi h)}{4A\pi} \quad (\text{B-62})$$

Thus, when the radius of failure surface at the bottom of slab r_h satisfies Equation B-62, the shape of failure surface for $0 \leq y \leq h$ is a catenary curve. Therefore, when the radius of failure surface at the tension face of slab is a catenary curve, the punching shear capacity of slab can be obtained by using Equation B-51 or B-53.

B.1.2 Initial slope of failure surface at the compression face of slab is less than 37°

When the initial slope of failure surface at $y = 0$ is less than 37° , the failure surface of slab is considered as a combination of two portions; (a) straight portion with slope of 37° and (b) curved portion. The straight portion is from compression face of slab to h_0 at which straight portion intersects with curved portion, where h_0 is the vertical distance from the top of slab to the intersection point of straight portion and curved portion. The curved portion is from h_0 to the tension face of slab. The failure surface having the combination of a straight line and a curve is shown in Figure B-3.

Figure B-3 Punching shear failure surface composed with a straight line and a curve for simply supported slab-column connection under concentric load

(a) For straight portion of failure surface from $y = 0$ to h_0

As mentioned above, the slope of straight portion of failure surface is 37° . Therefore, by using Equations B-39 and B-40, the radius of failure surface and its derivative can be expressed as

$$r(y) = \frac{3}{4}y \quad \text{for } 0 \leq y \leq h_0 \quad (\text{B-63a})$$

$$r'(y) = \frac{3}{4} \quad \text{for } 0 \leq y \leq h_0 \quad (\text{B-63b})$$

For straight portion of failure surface, there are two boundary conditions. They are as follows.

$$(1) \quad y = 0, \quad r(0) = 0 \quad (\text{B-64a})$$

$$y = h_0, \quad r(h_0) = \frac{3}{4} h_0 \quad (\text{B-64b})$$

$$(2) \quad y = 0, \quad r'(0) = \frac{5}{4} \quad (\text{B-65a})$$

$$y = h_0, \quad r'(h_0) = \frac{3}{4} \quad (\text{B-65b})$$

By substituting Equations B-63a and B-63b into Equation B-17,

$$P_L = f_c' \int_0^{h_0} \left(2\Delta + \frac{3}{4}\pi y \right) \left(L \sqrt{1 + \left(\frac{3}{4} \right)^2} - \frac{3}{4}M \right) dy \quad (\text{B-66})$$

By solving Equation B-66 and inserting the integration limits into it, the punching shear capacity for straight portion of failure surface is

$$P_L = \frac{1}{32} (5L - 3M) f_c' h_0 (16\Delta + 3\pi h_0) \quad (\text{B-67})$$

where h_0 is the vertical distance from the top of slab to intersection point of straight and curved portion of failure surface shown in Figure B-3.

(b) For curved portion of failure surface from $y = h_0$ to h

As mentioned above, the straight portion and the curved portion of failure intersect at the point in a vertical distance h_0 from the top of slab. Therefore, the curve portion of failure surface is from the intersection point of straight portion and curved portion to the tension face of the slab. By using Equations B-39 and B-40, the radius of failure and its derivative for curved portion of failure surface are

$$r(y) = -\frac{2\Delta}{\pi} + B e^{A\pi y} + C e^{-A\pi y} \quad \text{for } h_0 \leq y \leq h \quad (\text{B-68})$$

$$r'(y) = A\pi (B e^{A\pi y} - C e^{-A\pi y}) \quad \text{for } h_0 \leq y \leq h \quad (\text{B-69})$$

For curved portion of failure surface, there are two boundary conditions. They are as follows.

$$(1) \quad y = h_0, \quad r(h_0) = r_{h_0} = -\frac{2\Delta}{\pi} + B e^{A\pi h_0} + C e^{-A\pi h_0} \quad (\text{B-70a})$$

$$y = h, \quad r(h) = r_h = -\frac{2\Delta}{\pi} + B e^{A\pi h} + C e^{-A\pi h} \quad (\text{B-70b})$$

$$(2) \quad y = h_0, \quad r'(h_0) = r'_{h_0} = A\pi(B e^{A\pi h_0} - C e^{-A\pi h_0}) \quad (\text{B-71a})$$

$$y = h, \quad r'(h) = r'_h = A\pi(B e^{A\pi h} - C e^{-A\pi h}) \quad (\text{B-71b})$$

By equating the radii of failure surface from Equations B-64b and B-70a at $y = h_0$

and substituting $B = \frac{2\Delta}{\pi} - C$, we get

$$\frac{3}{4}h_0 = -\frac{2\Delta}{\pi} + \left(\frac{2\Delta}{\pi} - C\right)e^{A\pi h_0} + C e^{-A\pi h_0} \quad (\text{B-72})$$

From Equation B-72, the following new constant C for curved portion of failure surface is obtained.

$$C = \frac{-8\Delta(-1 + e^{A\pi h_0}) + 3\pi h_0}{4\pi(e^{-A\pi h_0} - e^{A\pi h_0})} \quad (\text{B-73})$$

The other new constants A and B related to new constant C from Equation B-68 can be obtained by using Equations B-42 and B-45 for curved portion of failure surface from h_0 to the tension face of slab.

The r'_{h_0} and r'_h from Equations B-71a and B-71b are substituted into Equation B-51 instead of r'_0 and r'_h , respectively, and the punching shear capacity for curved portion of failure surface is

$$P_C = \frac{f'_c}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_{h_0}) + r'_{h_0} \sqrt{1 + (r'_{h_0})^2} \right) \right) \right. \\ \left. - M \left((r'_h)^2 - (r'_0)^2 \right) \right) \quad (\text{B-74})$$

Finally, the punching shear capacity P for the slab-column connection when the initial slope of failure surface at $y = 0$ is less than 37 degree can be obtained by summing up P_L from Equation B-67 and P_C from Equation B-74.

$$P = P_L + P_C \quad (B-75)$$

Equation B-75 gives the punching shear capacity for slab-column connection if the failure surface of slab is considered as a combination of a straight line and a curve from the top to the bottom of slab when the initial slope of failure surface at $y = 0$ is less than 37 degree. The limitation to check if the shape of failure surface is a combination of a straight line and a curve from the top to the bottom of slab is derived as follows.

Limitation to check if the shape of failure surface is a combination of a straight line and a curve

When the initial slope of failure surface at $y = 0$ is less than 37 degree, the slope of failure surface at the compression face of slab can be expressed as

$$r'(0) < \frac{3}{4} \quad (B-76)$$

By substituting Equation B-46 into B-76,

$$A\pi(B - C) < \frac{3}{4} \quad (B-77)$$

By substituting $B = \frac{2A}{\pi} - C$ from Equation B-45 into Equation B-77, the constant C is

$$C > \frac{8A\Delta - 3}{8A\pi} \quad (B-78)$$

By substituting Equation B-58 into Equation B-78,

$$\frac{2\Delta(1 - e^{A\pi h}) + \pi r_h}{2\pi \sinh(A\pi h)} > \frac{8A\Delta - 3}{8A\pi} \quad (B-79)$$

By solving Equation B-79, the radius of failure surface r_h at the tension face of slab is

$$r_h > \frac{8A\Delta(1 + \cosh(A\pi h)) + 3\sinh(A\pi h)}{4A\pi} \quad (B-80)$$

Thus, when the radius of failure surface at the bottom of slab satisfies Equation B-80, the equation of failure surface for $0 \leq y \leq h$ is a combination of a straight line and a curve from the top to the bottom of slab. Therefore, when the radius of failure surface at the tension face of slab is a combination of a straight line and a curve, the punching shear capacity of slab can be obtained by using Equation B-75.

B.2 Punching shear capacity of slab-column connection with opening

When the slab-column connection has opening adjacent to the column, the punching shear capacity of slab-column connection decreases because the concrete area against punching is reduced due to opening. In the same way, the area of failure surface is reduced due to opening. Therefore, some parts of failure surface around the opening are considered ineffective to resist punching due to opening. The punching shear capacity sustained by the ineffective area of failure surface is defined as the ineffective punching shear capacity of slab due to opening. Thus, the punching shear capacity of slab-column connection with opening can be obtained as follows.

$$P_{op} = P - \bar{P}_{op} \quad (\text{B-81})$$

where P_{op} = punching shear capacity of slab-column connection with opening

P = punching shear capacity of slab-column connection

$\bar{P}_{op}$ = ineffective punching shear capacity of slab-column connection due to opening

The ineffective punching shear capacity of slab-column connection due to opening can be determined as follows.

$$P_{op} \times u = P \times u - \bar{P}_{op} \times u \quad (\text{B-82})$$

$$\bar{P}_{op} \times u = \int_{A_{op}} W_A dA_{op} \quad (\text{B-82a})$$

where A_{op} = ineffective area of failure surface due to opening

W_A = dissipation energy per unit area of failure surface from Equation B-1

u = vertical displacement of slab portion confined by the area of failure surface

By substituting W_A from Equation B-1 into Equation B-82a,

$$\bar{P}_{op} \times u = \frac{1}{2} f_c' u \int_{A_{op}} (L - M \sin \alpha) dA_{op} \quad (\text{B-83})$$

The integral form of ineffective punching shear capacity of slab due to opening is

$$\bar{P}_{op} = \frac{1}{2} f_c' \int_{A_{op}} (L - M \sin \alpha) dA_{op} \quad (\text{B-84})$$

$$dA_{op} = (\eta_1 + \eta_2 r(y)) \frac{dy}{\cos \alpha} \quad (\text{B-85})$$

$$dA_{op} = L_{op} \frac{dy}{\cos \alpha} \quad (\text{B-85a})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-85b})$$

where L_{op} is the circumferential length of ineffective area of failure surface due to opening and η_1 and η_2 are terms related to the circumferential length of ineffective area of failure surface due to opening.

By substituting Equation B-85 into Equation B-84,

$$\bar{P}_{op} = \frac{1}{2} f_c' \int_0^h (L - M \sin \alpha) (\eta_1 + \eta_2 r(y)) \frac{dy}{\cos \alpha} \quad (\text{B-86})$$

Equation B-86 can be written as

$$\bar{P}_{op} = \frac{1}{2} f_c' (\bar{P}\eta_1 + \bar{P}\eta_2) \quad (\text{B-87})$$

$\bar{P}_{op}\eta_1$ and $\bar{P}_{op}\eta_2$ can be expressed as

$$\bar{P}\eta_1 = \int_0^h \eta_1 (L - M \sin \alpha) \frac{dy}{\cos \alpha} \quad (\text{B-87a})$$

$$\bar{P}\eta_2 = \int_0^h \eta_2 r(y) (L - M \sin \alpha) \frac{dy}{\cos \alpha} \quad (\text{B-87b})$$

By substituting Equations B-13, B-14, B-15 and B-27 into B-87a,

$$\bar{P}\eta_1 = \frac{\eta_1}{A\pi} \int_{r'(0)}^{r'(h)} \left(L - \frac{M r'(y)}{\sqrt{1 + (r'(y))^2}} \right) dr'(y) \quad (\text{B-88a})$$

By integrating and substituting the limits,

$$P_{\eta_1} = \frac{1}{A\pi} \left(L(r_h' - r_0') + M \left(-\sqrt{1 + (r_h')^2} + \sqrt{1 + (r_0')^2} \right) \right) \quad (\text{B-88b})$$

where r_0' and r_h' are $r'(0)$ and $r'(h)$ from Equations B-46 and B-48, respectively.

By substituting Equations B-13, B-14, B-15 and B-27 into B-87b,

$$\bar{P}_{\eta_2} = \frac{\eta_1}{A^2 \pi^2} \int_{r'(0)}^{r'(h)} \left(\frac{\left(-2A\Delta + \sqrt{1 + (r'(y))^2} \right) \left(-Mr'(y) + \sqrt{1 + (r'(y))^2} \right)}{\sqrt{1 + (r'(y))^2}} \right) dr'(y) \quad (\text{B-89a})$$

By integrating and substituting the limits,

$$P_{\eta_2} = \frac{1}{2A^2 \pi^2} \left(L \left(\sin^{-1}(r_h') - \sin^{-1}(r_0') + r_h' \sqrt{1 + (r_h')^2} - r_0' \sqrt{1 + (r_0')^2} + 4A\Delta(r_0' - r_h') \right) \right. \\ \left. + M \left((r_0')^2 - (r_h')^2 + 4A\Delta \left(\sqrt{1 + (r_h')^2} - \sqrt{1 + (r_0')^2} \right) \right) \right) \quad (\text{B-90b})$$

where r_0' and r_h' are $r'(0)$ and $r'(h)$ from Equations B-46 and B-48, respectively.

The punching shear capacity of slab-column connection reduced due to opening can be obtained by using Equation B-87. However, L_{op} varies depending on the location of opening and method used to determine the ineffective area of failure surface due to opening. In addition, even for same opening location, L_{op} may vary depending on the position of intersection points of two projection lines with the circumference of failure surface. Thus, L_{op} needs to be derived for various location of opening and for the method used to determine the ineffective area of failure surface.

The ineffective area of failure surface due to opening is defined as the area under the plan area of failure surface bounded by two projection lines which propagate from the inside of column section through two corners of opening and intersect at two points on the circumference of failure surface. Regarding to these two projection lines that confine the ineffective area of failure surface due to opening, there are two methods of propagation of two projection lines considered in this study. They are classified as method AI and method AII depending on their starting points inside the column section. In method AI, the projection lines start from the center of column

while in method AII, the projection lines start from the center of $\frac{1}{3}$ area of column section, which is divided along the longer column width, close to the shorter column width nearest to the opening. For example, for the slab with opening located parallel and adjacent to the shorter column width, the ineffective areas of failure surface due to opening confined by two projection lines using by methods AI and AII are presented by the shaded areas shown in Figure B-4.

Figure B-4 Ineffective area of failure surface in the slab due to opening confined by two projection lines

There are five kinds of opening locations considered in this study. They are (a) opening location at adjacent corner of column, (b) opening location parallel and adjacent to the shorter side of column, (c) opening location perpendicular and adjacent to the longer side of column, (d) opening location parallel and adjacent to the middle of longer side of column, and (e) opening location perpendicular and adjacent to the shorter side of column. Five kinds of opening location considered in this study are shown in Figure B-5. For these five kinds of opening location shown in Figure B-5, L_{op} , η_1 and η_2 terms for Equation B-85, B-85a and B-85b are derived as follows.

Figure B-5 Different kinds of opening locations

B.2.1 Ineffective area of failure surface due to opening by using method AI

The ineffective area of failure surface due to opening by using method AI is the area under the plan area of failure surface bounded by two projection lines which propagate from the center of column through two comers of opening and extend up to intersect the circumference of failure surface. For this kind of ineffective area of failure surface due to opening by using method AI for five kinds of opening location described above, L_{op} , η_1 and η_2 terms are derived as follows.

(a) Opening location at adjacent corner of column

Opening in the slab is located at adjacent corner of column. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure B-6. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two comers of opening at

B and G and they intersect the circumference of failure surface at the tension face of the slab at D and E.

**Figure B-6 Ineffective area of failure surface due to opening location (a)
by using method AI**

From Figure B-6, the circumferential length L_{op} of ineffective area of failure surface due to opening is

$$L_{op} = \left(O_{c2} \tan \beta' + r(y) \tan \theta' + r(y) \left(\frac{\pi}{2} - (\theta' + \theta'') \right) + r(y) \tan \theta'' + O_{c1} \tan \beta'' \right) - (r(y) \tan \beta' + r(y) \tan \beta'') \quad (\text{B-91})$$

In Equation B-91, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = (O_{c2} \tan \beta' + O_{c1} \tan \beta'') + r(y) \left(\frac{\pi}{2} - \theta' - \theta'' - \tan \beta' - \tan \beta'' + \tan \theta' + \tan \theta'' \right) \quad (\text{B-92})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-93})$$

$$\eta_1 = O_{c2} \tan \beta' + O_{c1} \tan \beta'' \quad (\text{B-94})$$

$$\eta_2 = \frac{\pi}{2} - \theta' - \theta'' - \tan \beta' - \tan \beta'' + \tan \theta' + \tan \theta'' \quad (\text{B-95})$$

$$\beta' = \tan^{-1} \left(\frac{C_1}{C_2 + 2O_{c2}} \right) \quad (\text{B-96})$$

$$\beta'' = \tan^{-1} \left(\frac{C_2}{C_1 + 2O_{c1}} \right) \quad (\text{B-97})$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (\text{B-98})$$

$$\theta'' = \sin^{-1} \frac{r_{21}}{r_h} \quad (\text{B-99})$$

where r_{11} and r_{21} are the coordinates of D and E, respectively. The r_{11} and r_{21} can be determined as follows.

From Figure B-6, the equation of the projection line OD can be obtained as

$$r_{11} = m_{11} r_{12} + m_{12} \quad (\text{B-100})$$

where the first subscript present the coordinate axis passed by the projection line and the second subscript present the coordinate axis parallel to the components of respective projection line. The m_{11} and m_{12} are determined by known boundary conditions of projection line OD. The boundary conditions of projection line OD are

$$(1) \text{ when } r_{11} = O_{c2}, r_{12} = 0$$

$$(2) \text{ when } r_{11} = 0, r_{12} = O_{c2} \tan \beta'$$

By using these boundary condition in Equation B-100,

$$m_{11} = \tan \beta' \quad (\text{B-101})$$

$$m_{12} = O_{c2} \quad (\text{B-102})$$

By using circle equation,

$$r_1^2 + r_2^2 = r_h^2 \quad (\text{B-103})$$

By equating equation B-100 and B-103 at $r_1 = r_{11}$,

$$m_{11} r_{12} + m_{12} = \sqrt{r_h^2 - r_{12}^2} \quad (\text{B-104})$$

Equation B-104 can be solved as,

$$r_{12}^2 (m_{11}^2 + 1) + 2m_{11} m_{12} r_{12} + (m_{12}^2 - r_h^2) = 0 \quad (\text{B-105})$$

$$r_{12}^2 (a_1) + b_1 r_{12} + (c_1) = 0 \quad (\text{B-106})$$

$$a_1 = m_{11}^2 + 1 \quad (\text{B-107})$$

$$b_1 = 2m_{11} m_{12} \quad (\text{B-108})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-109})$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (\text{B-110})$$

Similarly, r_{21} is determined as follows.

From Figure B-6, the equation of the projection line OE can be obtained as

$$r_{21} = m_{21} r_{22} + m_{22} \quad (\text{B-111})$$

$$r_{22} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2 c_2}}{2a_2} \quad (\text{B-112})$$

$$a_2 = m_{21}^2 + 1 \quad (\text{B-113})$$

$$b_2 = 2m_{21}m_{22} \quad (\text{B-114})$$

$$c_2 = m_{22}^2 - r_h^2 \quad (\text{B-115})$$

By using these boundary conditions of the projection line OE,

$$m_{21} = \frac{\left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \right)}{O_{c1}} \quad (\text{B-116})$$

$$m_{22} = -\left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \right) \quad (\text{B-117})$$

(b) Opening location parallel and adjacent to the shorter side of column

Opening in the slab is located parallel and adjacent to the shorter side of column. The ineffective area of failure surface due to this location of opening presented by using method AI is shown in Figure B-7. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two comers of opening at B and G and they intersect the circumference of failure surface at the tension face of the slab at C and F.

From Figure B-7, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = C_1 + 2 \left(r(y) \left(\frac{\pi}{2} - \theta' \right) + \left(\frac{O_{c1}}{2} - \frac{C_1}{2} \right) \tan \beta' + r(y) \tan \theta' \right) \quad (\text{B-118})$$

Figure B-7 Ineffective area of failure surface due to opening location (b) by using method AII

In Equation B-118, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = (C_1 + (O_{c1} - C_1) \tan \beta') + 2r(y) \left(\tan \theta' + \left(\frac{\pi}{2} - \theta' \right) - \tan \beta' \right) \quad (B-119)$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (B-120)$$

$$\eta_1 = (C_1 + (O_{c1} - C_1) \tan \beta') \quad (B-121)$$

$$\eta_2 = 2 \left(\tan \theta' + \left(\frac{\pi}{2} - \theta' \right) - \tan \beta' \right) \quad (B-122)$$

$$\beta' = \beta'' = \tan^{-1} \left(\frac{C_2}{O_{c1}} \right) \quad (B-123)$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (B-124)$$

The equation of projection OC or OF is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (B-125)$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (B-126)$$

$$a_1 = m_{11}^2 + 1 \quad (B-127)$$

$$b_1 = 2m_{11}m_{12} \quad (\text{B-128})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-129})$$

By using the boundary conditions of projection line OC or OE,

$$m_{11} = \frac{\left(\frac{C_1}{2} - \frac{C_2}{2} \tan \beta' \right)}{\left(\frac{O_{c2}}{2} - \frac{C_1}{2} \right)} \quad (\text{B-130})$$

$$m_{12} = -\left(\frac{C_1}{2} - \frac{C_2}{2} \tan \beta' \right) \quad (\text{B-131})$$

(c) Opening location perpendicular and adjacent to the longer side of column

Opening in the slab is located perpendicular and adjacent to the longer side of column. For this type of opening location (c), according to where two projection lines propagating from the center of column through two comers of opening intersect the circumference of failure surface, there may have two possible cases regarding to the ineffective area of failure surface due to opening location type (c).

Case (a)

For this case, the ineffective area of failure surface due to this location of opening by using method AI is shown in Figure B-8. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two corners of opening at B and F and they intersect the circumference of failure surface at the tension face of the slab at C and E. The intersection point C of one projection line is at the circumference of failure surface between the extended lines from the comers of column along the shorter column widths while the intersection point E of the remaining projection line is at the circumference of failure surface between the extended lines from the corner of column along the longer and shorter column width as shown in Figure B-8.

**Figure B-8 Ineffective area of failure surface due to opening location (c)
by using method AI for case (a)**

From Figure B-8, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = \left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' + r(y) \left(\frac{\pi}{2} - \beta' \right) \right) - r(y) \tan \beta'' \quad (\text{B-132})$$

In Equation B-132, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = \left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \right) + r(y) \left(\frac{\pi}{2} - \beta' - \tan \beta'' \right) \quad (\text{B-133})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-134})$$

$$\eta_1 = \frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \quad (\text{B-135})$$

$$\eta_2 = \frac{\pi}{2} - \beta' - \tan \beta'' \quad (\text{B-136})$$

$$\beta' = \tan^{-1} \left(\frac{C_1}{C_2} \right) \quad (\text{B-137})$$

$$\beta'' = \tan^{-1} \left(\frac{\frac{C_2}{2} - O_{c2}}{\frac{C_1}{2} + O_{c1}} \right) \quad (\text{B-138})$$

Case (b)

The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure B-9. To confine the ineffective area of failure surface due to opening, two projection lines propagate from the center of the column through two corners of opening at B and F and they intersect the circumference of failure surface at the tension face of the slab at D and E. In contrast to case (a) for this type of opening location (c), both intersection points of two projection lines are at the circumference of failure surface between the extended lines from the corner of column along the longer and shorter column width as shown in Figure B-9.

**Figure B-9 Ineffective area of failure surface due to opening location (c)
by using method AI for case (b)**

From Figure B-9, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = \left(\left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \right) + r(y) \tan \theta' + r(y) \left(\frac{\pi}{2} - \beta' - \theta' \right) \right) - r(y) \tan \beta'' \quad (\text{B-139})$$

In Equation B-139, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = \left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \right) + r(y) \left(\tan \theta' + \left(\frac{\pi}{2} - \beta' - \theta' \right) - \tan \beta'' \right) \quad (\text{B-140})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-141})$$

$$\eta_1 = \frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \quad (\text{B-142})$$

$$\eta_2 = \frac{\pi}{2} - \beta' - \theta' + \tan \theta' \quad (\text{B-143})$$

$$\beta' = \tan^{-1} \left(\frac{C_1}{C_2} \right) \quad (\text{B-144})$$

$$\beta'' = \tan^{-1} \left(\frac{\frac{C_2}{2} - O_{c2}}{\frac{C_1}{2} + O_{c1}} \right) \quad (\text{B-145})$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (\text{B-146})$$

The equation of projection line OD is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (\text{B-147})$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (\text{B-148})$$

$$a_1 = m_{11}^2 + 1 \quad (\text{B-149})$$

$$b_1 = 2m_{11} m_{12} \quad (\text{B-150})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-151})$$

By using the boundary conditions of projection line OD,

$$m_{11} = \tan \left(\frac{\pi}{2} - \beta'' \right) \quad (\text{B-152})$$

$$m_{12} = - \left(\frac{C_2}{2} - \frac{C_1}{2} \tan \beta'' \right) \quad (\text{B-153})$$

(d) Opening location parallel and adjacent to the middle of longer side of column

Opening in the slab is located parallel and adjacent to the middle of longer side of column. Similar to opening location (c), as for this type of opening location (d), according to where two projection lines propagating from the center of column through two comers of opening and intersect the circumference of failure surface, there may have two possible cases regarding to the ineffective area of failure surface due to opening.

Case (a)

The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure B-10. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two comers of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E which are between the extended lines from the comers of column along the longer and shorter column width as shown in Figure B-10.

**Figure B-10 Ineffective area of failure surface due to opening location (d)
by using method AI for case (a)**

From Figure B-10, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = O_{c2} + r(y) \left(2 \left(\theta' - \sin \theta' + \tan \left(\frac{\pi}{2} - \beta' \right) \right) \right) \quad (B-154)$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (B-155)$$

$$\eta_1 = O_{c2} \quad (B-156)$$

$$\eta_2 = 2 \left(\theta' - \sin \theta' + \tan \left(\frac{\pi}{2} - \beta' \right) \right) \quad (B-157)$$

$$\beta' = \beta'' = \tan^{-1} \left(\frac{C_1}{O_{c2}} \right) \quad (\text{B-158})$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (\text{B-159})$$

The equation of projection OB or OE is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (\text{B-160})$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (\text{B-161})$$

$$a_1 = m_1^2 + 1 \quad (\text{B-162})$$

$$b_1 = 2m_{11} m_{12} \quad (\text{B-163})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-164})$$

By using the boundary conditions of projection line OB or OE,

$$m_{11} = \frac{1}{\tan \beta'} \quad (\text{B-165})$$

$$m_{12} = \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \quad (\text{B-166})$$

Case (b)

The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure B-11. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two corners of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E. In contrast to case (a) of this type of opening location, both intersection points of two projection lines are at the circumference of failure surface between the extended lines from the corners of column along the shorter column widths as shown in Figure B-11.

From Figure B-11, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as

$$L_{op} = O_{c2} + r(y) \left(2 \left(\frac{\pi}{2} - \beta' \right) \right) \quad (\text{B-167})$$

**Figure B-11 Ineffective area of failure surface due to opening location (d)
by using method AI for case (b)**

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (B-168)$$

$$\eta_1 = O_{c2} \quad (B-169)$$

$$\eta_2 = 2 \left(\frac{\pi}{2} - \beta' \right) \quad (B-170)$$

$$\beta' = \beta'' = \tan^{-1} \left(\frac{C_1}{O_{c2}} \right) \quad (B-171)$$

(e) Opening location perpendicular and adjacent to the shorter side of column

Opening in the slab is located perpendicular and adjacent to the shorter side of column. The ineffective area of failure surface due to this location of opening by using method AI is shown in Figure B-12. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of the column through two comers of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E.

**Figure B-12 Ineffective area of failure surface due to opening location (e)
by using method AI**

From Figure B-12, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = C_1 + r(y)(2\beta^i) \quad (\text{B-172})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-173})$$

$$\eta_1 = C_1 \quad (\text{B-174})$$

$$\eta_2 = 2 \beta' \quad (\text{B-175})$$

$$\beta' = \tan^{-1} \left(\frac{C_1}{C_2} \right) \quad (\text{B-176})$$

B.2.2 Ineffective area of failure surface due to opening by using method AII

The ineffective area of failure surface due to opening by using method AII is the area under the plan area of failure surface bounded by two projection lines which start from the center of $\frac{1}{3}$ area of column section, which is close to the shorter column width nearest to the opening, through two corners of opening, through two corners of opening and extend up to intersect the circumference of failure surface.

(a) Opening position at adjacent corner of column

Opening in the slab is located at adjacent corner of column. The ineffective area of failure surface due to this location of opening by using method AII is shown in

Figure B-13. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening through two corners of opening at B and G and intersect with the circumference of failure surface at the tension face of the slab at D and E.

**Figure B-13 Ineffective area of failure surface due to opening location (a)
by using method AII**

From Figure B-13, the circumferential length L_{op} of ineffective area failure surface due to opening can be obtained as,

$$L_{op} = \left(O_{c_2} \tan \beta' + r(y) \tan \theta' + r(y) \left(\frac{\pi}{2} - (\theta' + \theta'') \right) + r(y) \tan \theta'' + O_{c_1} \tan \beta'' \right) - (r(y) \tan \beta' + r(y) \tan \beta'') \quad (\text{B-177})$$

In Equation B-177, the terms related and not related to $\mathbf{r}(\mathbf{y})$ are separated as follows:

$$L_{op} = (O_{c2} \tan \beta' + O_{c1} \tan \beta'') + r(y) \left(\frac{\pi}{2} - \theta - \theta' - \tan \beta' - \tan \beta'' + \tan \theta + \tan \theta' \right) \quad (\text{B-178})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-179})$$

$$\eta_1 = O_{c2} \tan \beta' + O_{c1} \tan \beta'' \quad (\text{B-180})$$

$$\eta_2 = \frac{\pi}{2} - \theta' - \theta'' - \tan \beta' - \tan \beta'' + \tan \theta' + \tan \theta'' \quad (\text{B-181})$$

$$\beta' = \tan^{-1} \left(\frac{3C_1}{C_2 + 6O_{c2}} \right) \quad (\text{B-182})$$

$$\beta'' = \tan^{-1} \left(\frac{C_2}{3C_1 + 6O_{c1}} \right) \quad (\text{B-183})$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (\text{B-184})$$

$$\theta'' = \sin^{-1} \frac{r_{21}}{r_h} \quad (\text{B-185})$$

The equation of projection line OD is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (\text{B-186})$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (\text{B-187})$$

$$a_1 = m_{11}^2 + 1 \quad (\text{B-188})$$

$$b_1 = 2m_{11} m_{12} \quad (\text{B-189})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-190})$$

By using the boundary conditions of the projection line OD,

$$m_{11} = \frac{O_{c2}}{\left(\frac{C_1}{2} - \frac{C_2}{6} \tan \beta' \right)} \quad (\text{B-191})$$

$$m_{12} = O_{c2} \quad (\text{B-192})$$

The equation of projection line OE is

$$r_{21} = m_{21} r_{22} + m_{22} \quad (\text{B-193})$$

$$r_{22} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2 c_2}}{2a_2} \quad (\text{B-194})$$

$$a_2 = m_{21}^2 + 1 \quad (\text{B-195})$$

$$b_2 = 2m_{21} m_{22} \quad (\text{B-196})$$

$$c_2 = m_{22}^2 - r_h^2 \quad (B-197)$$

By using the boundary conditions of the projection line OE,

$$m_{21} = \frac{\left(\frac{C_2}{6} - \frac{C_1}{2} \tan \beta'' \right)}{O_{c1}} \quad (B-198)$$

$$m_{22} = -\left(\frac{C_2}{6} - \frac{C_1}{2} \tan \beta'' \right) \quad (B-199)$$

(b) Opening position parallel and adjacent to the shorter side of column

Opening in the slab is located parallel and adjacent to the shorter side of column. The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-14. To confined the ineffective area of failure surface due to opening, two projection lines are propagated from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening through two comers of opening at B and G and intersect with the circumference of failure surface at the tension face of the slab at C and F.

Figure B-14 Ineffective area of failure surface due to opening location (b) by using method AII

From Figure B-14, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = C_1 + 2r(y)\left(\frac{\pi}{2} - \theta'\right) + 2r(y) \tan \theta' + 2\left(\frac{O_{c1}}{2} - \frac{C_1}{2}\right) \tan \beta' \quad (\text{B-200})$$

In Equation B-200, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = (C_1 + (O_{c1} - C_1) \tan \beta') + r(y) \left(2\left(\frac{\pi}{2} - \theta' + \tan \theta'\right) \right) \quad (\text{B-201})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-202})$$

$$\eta_1 = C_1 + (O_{c1} - C_1) \tan \beta' \quad (\text{B-203})$$

$$\eta_2 = 2\left(\frac{\pi}{2} - \theta' + \tan \theta'\right) \quad (\text{B-204})$$

$$\beta' = \tan^{-1}\left(\frac{C_2}{3O_{c1}}\right) \quad (\text{B-205})$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (\text{B-206})$$

The equation of projection line OC or OF is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (\text{B-207})$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (\text{B-208})$$

$$a_1 = m_{11}^2 + 1 \quad (\text{B-209})$$

$$b_1 = 2m_{11} m_{12} \quad (\text{B-210})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-211})$$

By using the boundary conditions of the projection line OD or OF,

$$m_{11} = \frac{\left(\frac{C_1}{2} - \frac{C_2}{6} \tan \beta'\right)}{\left(\frac{O_{c2}}{2} - \frac{C_1}{2}\right)} \quad (\text{B-212})$$

$$m_{12} = -\left(\frac{C_1}{2} - \frac{C_2}{6} \tan \beta'\right) \quad (\text{B-213})$$

(c) Opening location perpendicular and adjacent to longer side of column

Opening in the slab is located perpendicular and adjacent to the longer side of column. For this type of opening location (c), according to where two projection lines propagating from the center of column through two comers of opening and intersect with the circumference of failure surface, there may have two possible different cases regarding to the ineffective area of failure surface due to opening.

Case (a)

The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-15. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening through two comers of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E. The intersection point B of one projection line is at the circumference of failure surface between the extended lines from the comers of column along the shorter column widths while the intersection point E of the remaining projection line is at the circumference of failure surface between the extended lines from the comer of column along the longer and shorter column width as shown in Figure B-15.

From Figure B-15, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = O_{c2} + r(y) \left(\left(\frac{\pi}{2} - \beta' \right) + \tan \left(\frac{\pi}{2} - \beta'' \right) \right) \quad (B-214)$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (B-215)$$

$$\eta_1 = O_{c2} \quad (B-216)$$

$$\eta_2 = \left(\frac{\pi}{2} - \beta' \right) + \tan \left(\frac{\pi}{2} - \beta'' \right) \quad (B-217)$$

$$\beta' = \tan^{-1} \left(\frac{3 C_1}{C_2} \right) \quad (B-218)$$

$$\beta'' = \tan^{-1} \left(\frac{\frac{C_1}{2}}{\left| O_{c2} - \frac{C_2}{6} \right|} \right) \tag{B-219}$$

Figure B-15 Ineffective area of failure surface due to opening location (c) by using method AII for case (a)

Case (b)

The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-16. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center of $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening through two comers of opening at A and F and intersect with the circumference of failure surface at the tension face of the slab at C and E. In contrast to case (a) for this type of opening location (c), each intersection point of two projection lines is at each circumference of failure surface between the extended lines from the comers of column along the longer and shorter column width as shown in Figure B-16.

From Figure B-16, the circumferential length L_{op} of ineffective area of failure surface due to opening can be obtained as,

$$L_{op} = \left(C_2 + r(y) \tan\left(\frac{\pi}{2} - \beta''\right) + r(y) \theta' \right) - \left((C_2 - O_{c2}) + r(y) \tan \theta' - r(y) \left(\frac{\pi}{2} - \beta' \right) \right) \quad (B-220)$$

**Figure B-16 Ineffective area of failure surface due to opening location (c)
by using method AII for case (b)**

In Equation B-220, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = O_{c2} + r(y) \left(\left(\frac{\pi}{2} - \beta' \right) + \theta' - \tan \theta' + \tan\left(\frac{\pi}{2} - \beta''\right) \right) \quad (B-221)$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (B-222)$$

$$\eta_1 = O_{c2} \quad (B-223)$$

$$\eta_2 = \left(\frac{\pi}{2} - \beta' \right) + \theta' - \tan \theta' + \tan\left(\frac{\pi}{2} - \beta''\right) \quad (B-224)$$

$$\beta' = \tan^{-1} \left(\frac{3 C_1}{C_2} \right) \quad (\text{B-225})$$

$$\beta'' = \tan^{-1} \left(\frac{\frac{C_1}{2}}{\left| O_{c2} - \frac{C_2}{6} \right|} \right) \quad (\text{B-226})$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (\text{B-227})$$

The equation of projection line OC is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (\text{B-228})$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4 a_1 c_1}}{2 a_1} \quad (\text{B-229})$$

$$a_1 = m_{11}^2 + 1 \quad (\text{B-230})$$

$$b_1 = 2 m_{11} m_{12} \quad (\text{B-231})$$

$$c_1 = m_{12}^2 - r_h^2 \quad (\text{B-232})$$

By using the boundary conditions of the projection line OC,

$$m_{11} = \frac{1}{\tan \beta''} \quad (\text{B-233})$$

$$m_{12} = -(C_2 - O_{c2}) \quad (\text{B-234})$$

(d) Opening location parallel and adjacent to the middle of longer side of column

Opening in the slab is located parallel and adjacent to the middle of longer side of column. Similar to opening location (c), as for this type of opening location (d), according to where two projection lines propagating **from** the center of column through two comers of opening and intersect with the circumference of failure surface, there may have three possible cases regarding to the ineffective area of failure surface due to opening.

Case (a)

The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-17. To confine the ineffective area of failure

surface due to opening, two projection lines are propagated from the $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening through two comers of opening at A and F and they intersect with the circumference of failure surface at the tension face of the slab at B and E, each of which is at each circumference of failure surface between the extended lines from the comers of column along the longer and shorter column widths as shown in Figure B-17.

From Figure B-17, the circumferential length L_{op} of ineffective failure surface due to opening is

$$L_{op} = O_{c2} + r(y) \left(\theta' + \theta'' - \tan \theta' - \tan \theta'' + \tan \left(\frac{\pi}{2} - \beta' \right) + \tan \left(\frac{\pi}{2} - \beta'' \right) \right) \quad (\text{B-235})$$

**Figure B-17 Ineffective area of failure surface due to opening location (d)
by using method AII for case (a)**

In Equation B-235, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-236})$$

$$\eta_1 = O_{c2} \quad (5.237)$$

$$\eta_2 = \theta' + \theta'' - \tan \theta' - \tan \theta'' + \tan\left(\frac{\pi}{2} - \beta'\right) + \tan\left(\frac{\pi}{2} - \beta''\right) \quad (5.238)$$

$$\beta' = \tan^{-1} \left(\frac{\frac{C_1}{2}}{\frac{C_2}{6} - \frac{C_2}{2} + \frac{O_{c2}}{2}} \right) \quad (B-239)$$

$$\beta'' = \tan^{-1} \left(\frac{\frac{C_1}{2}}{\frac{5C_2}{6} - \frac{C_2}{2} + \frac{O_{c2}}{2}} \right) \quad (B-240)$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (B-241)$$

$$\theta'' = \sin^{-1} \frac{r_{21}}{r_h} \quad (B-242)$$

The equation of projection OE is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (B-243)$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (B-244)$$

$$a_1 = m_{11}^2 + 1 \quad (B-245)$$

$$b_1 = 2m_{11} m_{12} \quad (B-246)$$

$$c_1 = m_{12}^2 - r_h^2 \quad (B-247)$$

By using the boundary conditions of projection line OE,

$$m_{11} = \frac{1}{\tan \beta'} \quad (B-248)$$

$$m_{12} = -\left(\frac{C_2}{3} - \frac{O_{c2}}{2} \right) \quad (B-249)$$

The equation of projection OB is

$$r_{21} = m_{21} r_{22} + m_{22} \quad (B-250)$$

$$r_{22} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2 c_2}}{2a_2} \quad (B-251)$$

$$a_2 = m_{21}^2 + 1 \quad (B-252)$$

$$b_2 = 2 m_{21} m_{22} \quad (\text{B-253})$$

$$c_2 = m_{22}^2 - r_h^2 \quad (\text{B-254})$$

By using the boundary conditions of projection line OE,

$$m_{21} = \frac{1}{\tan \beta''} \quad (\text{B-255})$$

$$m_{12} = -\left(\frac{C_2}{2} - \frac{O_{c2}}{2}\right) \quad (\text{B-256})$$

Case (b)

The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-18. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the $1/3$ column section area close to the shorter column width nearest to the opening through two corners of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and D. In contrast to case (a) for this type of opening location (d), the intersection point B of one projection line is at the circumference of failure surface between the extended lines from the corners of column along the longer and shorter column width while the intersection point D of other projection line is at the circumference of failure surface between the extended lines from the corner of column along the shorter column widths as shown in Figure B-18.

From Figure B-18, the circumferential length L_{op} of ineffective failure surface due to opening can be obtained as,

$$L_{op} = \left(\frac{5}{6} C_2 - \frac{C_1}{2} \tan \beta' + r(y) \theta'' \right) - r(y) \tan \beta' - \left(r(y) \tan \theta'' + \left(\frac{O_{c2}}{2} - \frac{C_2}{2} \right) - r(y) \tan \left(\frac{\pi}{2} - \beta'' \right) \right) \quad (\text{B-257})$$

In Equation B-257, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = \left(\frac{5}{6} C_2 - \frac{C_1}{2} \tan \beta' - \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \right) + r(y) \left(\theta'' - \tan \theta'' - \tan \beta' + \tan \left(\frac{\pi}{2} - \beta'' \right) \right) \quad (\text{B-258})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-259})$$

$$\eta_1 = \frac{5}{6}C_2 - \frac{C_1}{2} \tan \beta' - \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \quad (5.260)$$

$$\eta_2 = \theta'' - \tan \theta'' - \tan \beta' + \tan \left(\frac{\pi}{2} - \beta'' \right) \quad (5.261)$$

$$\theta'' = \sin^{-1} \frac{r_{21}}{r_h} \quad (\text{B-262})$$

**Figure B-18 Ineffective area of failure surface due to opening location (d)
by using method AII for case (b)**

The equation of projection OB is

$$r_{21} = m_{21} r_{22} + m_{22} \quad (\text{B-263})$$

$$r_{22} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2} \quad (\text{B-264})$$

$$a_2 = m_{21}^2 + 1 \quad (\text{B-265})$$

$$b_2 = 2m_{21}m_{22} \quad (B-266)$$

$$c_2 = m_{22}^2 - r_h^2 \quad (B-267)$$

By using the boundary conditions of projection line OB,

$$m_{21} = \frac{1}{\tan \beta''} \quad (B-268)$$

$$m_{12} = -\left(\frac{C_2}{2} - \frac{O_{c2}}{2}\right) \quad (B-269)$$

Case (c)

The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-19. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the $\frac{1}{3}$ column section area close to the shorter column width nearest to the opening through two corners of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at C and D. In contrast to case (a) and (b), for this type of opening location (d), both intersection points of two projection lines are at the circumference of failure surface between the extended lines from the corner of column along the longer and shorter column width as shown in Figure B-19.

Figure B-19 Ineffective area of failure surface due to opening location (d) by using method AII for case (c)

From Figure B-19, the circumferential length L_{op} of ineffective failure surface due to opening can be obtained as,

$$L_{op} = \left(\frac{5}{6} C_2 - \frac{C_1}{2} \tan \beta' + r(y) \tan \theta' + r(y) (\theta'' - \theta') \right) - \left(r(y) \tan \beta' + \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) + r(y) \tan \theta'' - r(y) \tan \left(\frac{\pi}{2} - \beta'' \right) \right) \quad (B-270)$$

In Equation B-270, the terms related and not related to $r(y)$ are separated as follows:

$$L_{op} = \left(\frac{5}{6} C_2 - \frac{C_1}{2} \tan \beta' - \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \right) + r(y) \left((\theta'' - \theta') + \tan \theta' - \tan \beta' - \tan \theta'' + \tan \left(\frac{\pi}{2} - \beta'' \right) \right) \quad (B-271)$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (B-272)$$

$$\eta_1 = \frac{5}{6} C_2 - \frac{C_1}{2} \tan \beta' - \left(\frac{C_2}{2} - \frac{O_{c2}}{2} \right) \quad (B-273)$$

$$\eta_2 = (\theta'' - \theta') + \tan \theta' - \tan \beta' - \tan \theta'' + \tan \left(\frac{\pi}{2} - \beta'' \right) \quad (B-274)$$

$$\theta' = \sin^{-1} \frac{r_{11}}{r_h} \quad (B-275)$$

$$\theta'' = \sin^{-1} \frac{r_{21}}{r_h} \quad (B-276)$$

The equation of projection line OD is

$$r_{11} = m_{11} r_{12} + m_{12} \quad (B-277)$$

$$r_{12} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (B-278)$$

$$a_1 = m_1^2 + 1 \quad (B-279)$$

$$b_1 = 2m_{11}m_{12} \quad (B-280)$$

$$c_1 = m_{12}^2 - r_h^2 \quad (B-281)$$

By using the boundary conditions of projection line OC,

$$m_{11} = \frac{1}{\tan\left(\frac{\pi}{2} - \beta'\right)} \quad (\text{B-282})$$

$$m_{12} = -\left(\frac{5C_2}{6} - \frac{C_1}{2} \tan \beta'\right) \quad (\text{B-283})$$

The equation of projection OC is

$$r_{21} = m_{21} r_{22} + m_{22} \quad (\text{B-284})$$

$$r_{22} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2 c_2}}{2a_2} \quad (\text{B-285})$$

$$a_2 = m_{21}^2 + 1 \quad (\text{B-286})$$

$$b_2 = 2m_{21} m_{22} \quad (\text{B-287})$$

$$c_2 = m_{22}^2 - r_h^2 \quad (\text{B-288})$$

By using the boundary conditions of projection line OC,

$$m_{21} = \frac{1}{\tan \beta''} \quad (\text{B-289})$$

$$m_{12} = -\left(\frac{C_2}{2} - \frac{O_{c2}}{2}\right) \quad (\text{B-290})$$

(e) Opening location perpendicular and adjacent to the shorter side of column

Opening in the slab is located perpendicular and adjacent to the shorter side of column. The ineffective area of failure surface due to this location of opening by using method AII is shown in Figure B-20. To confine the ineffective area of failure surface due to opening, two projection lines are propagated from the center $\frac{1}{2}$ column section area close to the shorter column width nearest to the opening through two comers of opening at A and F and they intersect the circumference of failure surface at the tension face of the slab at B and E.

**Figure B-20 Ineffective area of failure surface due to opening location (e)
by using method AII**

From Figure B-20, the circumferential length L_{op} of ineffective failure surface due to opening can be obtained as:

$$L_{op} = C_1 + r(y)(2\beta^i) \quad (\text{B-291})$$

$$L_{op} = \eta_1 + \eta_2 r(y) \quad (\text{B-292})$$

$$\eta_1 = C_1 \quad (\text{B-293})$$

$$\eta_2 = 2 \beta' \quad (\text{B-294})$$

$$\beta' = \tan^{-1} \left(\frac{6C_1}{C_2} \right) \quad (\text{B-295})$$

B.3 Punching shear capacity of slab-column connection without opening and with shear reinforcement

The punching shear capacity for slab-column connection has been derived in section B.1. When the slab-column connection has shear reinforcement, Equations B-51 or B-53 and B-75 for punching shear capacity of the slab-column connection from section B.1 can be used by adding the contribution of shear reinforcement to the tensile strength of concrete in Equations B- 1a and B- 1b.

The failure surface in punching shear failure of slab observed in practice involves separation failure as well as sliding failure. The sliding is resisted by the compressive and shear stress along the failure surface. The tensile strength of concrete is against the separation. The shear reinforcement can strengthen against separation. In this case, the dowel action of flexural reinforcement is neglected because it is assumed to contribute only the stresses in its axial direction. The contribution of shear reinforcement is derived as follows:

When the shear reinforcement is used, the tensile strength of slab contributed by concrete and shear reinforcement is:

$$f_t = f_{ct} + f_{st} \quad (\text{B-296})$$

where f_t = tensile strength of slab

f_{ct} = tensile strength contributed by concrete

f_{st} = tensile strength contributed by shear reinforcement

However, the concrete is weak in tension. When the slab has shear reinforcement, no contribution of concrete in tension is considered. Only shear reinforcement is assumed to resist tension. Thus,

$$f_{ct} = 0 \quad (\text{B-296a})$$

Therefore, Equation B-296 becomes:

$$f_t = f_{st} \quad (\text{B-296b})$$

where f_{st} is tensile strength contributed by shear reinforcement.

Thus, the tensile strength f_t in Equations B-1a and B-1b for the punching shear capacity of the slab-column connection with shear reinforcement is equal to the contribution of tensile strength by shear reinforcement. The tensile strength of slab contributed by shear reinforcement can be obtained as,

$$f_t = \rho_s f_{sy} \sin \alpha_{avg} \quad (\text{B-297})$$

$$\rho_s = \frac{A_s}{A_f} \quad (\text{B-298})$$

where

- ρ_s = ratio of shear stud reinforcement passing through the failure surface
 A_s = total area of shear stud reinforcement passing through the failure surface
 A_f = total area of failure surface of slab after punching
 α_{avg} = average slope of failure surface

The total area of failure surface of slab after punching A_f can be derived as follows:

$$A_f = \int_{A_f} dA_f \quad (\text{B-299})$$

From Equation B-9,

$$dA = 4 \left(\frac{\pi r(y)}{2} + \Delta \right) \frac{dy}{\cos \alpha} = dA_f \quad (\text{B-300})$$

By substituting Equations B-15, B-27 and B-38 into Equation B-300,

$$dA_f = \frac{2}{A^2 \pi} \sqrt{1 + (r'(y))^2} dr'(y) \quad (\text{B-301})$$

and substituting it into Equation B-299,

$$A_f = \int_{r'(0)}^{r'(h)} \frac{2}{A^2 \pi} \sqrt{1 + (r'(y))^2} dr'(y) \quad (\text{B-302})$$

By solving Equation B-302, the total area of failure surface is

$$A_f = \frac{2}{A^2 \pi} \left(-\sinh^{-1}(r'(0)) - r'(0) \sqrt{1 + (r'(0))^2} + \sinh^{-1}(r'(h)) + r'(h) \sqrt{1 + (r'(h))^2} \right) \quad (\text{B-303})$$

Equation B-303 gives the total area of failure surface around the column due to concentric punching load.

Average tangent value α of failure surface from compression to tension face of slab is derived as follow.

$$\tan \alpha_{avg} = \frac{\int_0^s r'(y) ds}{\int_0^s ds} \quad (\text{B-304a})$$

By substituting $ds = \frac{dy}{\cos \alpha}$, $dy = \frac{dr'(y)}{A\pi\sqrt{1+r'(y)^2}}$ and $\cos \alpha = \frac{1}{\sqrt{1+(r')^2}}$, and after solving,

$$\tan \alpha_{avg} = \frac{1}{2} A \pi e^{Ah\pi} \left(1 + e^{Ah\pi}\right) \left(-C + B e^{Ah\pi}\right) \quad (\text{B-304b})$$

$$\alpha_{avg} = \tan^{-1} \left(\frac{1}{2} A \pi e^{Ah\pi} \left(1 + e^{Ah\pi}\right) \left(-C + B e^{Ah\pi}\right) \right) \quad (\text{B-305})$$

The Equation B-310 gives the average value of slope of failure surface.

After substituting Equation B-297 into L and M from Equations B-1a and B-1b and using them, the punching shear capacity for the slab-column connection with shear reinforcement is

(a) If the failure surface is a curve i.e. the initial slope of failure surface at the compression side of the slab is equal or greater than 37 degree, the punching shear capacity for the slab-column connection with shear reinforcement is:

$$P_s = \frac{f_c'}{2A^2\pi} \left(L \left(\left(\sinh^{-1}(r_h') + r_h' \sqrt{1+(r_h')^2} \right) - \left(\sinh^{-1}(r_0') + r_0' \sqrt{1+(r_0')^2} \right) \right) - M \left((r_h')^2 - (r_0')^2 \right) \right) \quad (\text{B-306})$$

where r_0' and r_h' are $r'(0)$ and $r'(h)$ from Equations B-46 and B-48, respectively.

Or,

$$P_s = \frac{f_c'}{2A^2\pi} \left(L \left(-2A\Delta\sqrt{-1+4A^2\Delta^2} - \ln \left(2A\Delta + \sqrt{-1+4A^2\Delta^2} \right) + \ln \left(\sqrt{A^2(-2\Delta-\pi r_h)^2} + \sqrt{-1+A^2(-2\Delta-\pi r_h)^2} \right) - \sqrt{A^2(-2\Delta-\pi r_h)^2} \sqrt{-1+A^2(-2\Delta-\pi r_h)^2} \right) + M \left(-4A^2\pi\Delta r_h - A^2\pi^2 r_h^2 \right) \right) \quad (\text{B-307})$$

where r_h is obtained from Equation B-47.

(b) If the failure surface is a combination of a straight line and a curve i.e. the initial slope of failure surface at the compression side of the slab is less than 37 degree, the punching shear capacity for the slab-column connection with shear reinforcement is:

$$P_s = P_{LS} + P_{CS} \quad (\text{B-308})$$

$$P_{LS} = \frac{1}{32} (5L - 3M) f'_c h_0 (16\Delta + 3\pi h_0) \quad (\text{B-309})$$

$$P_{CS} = \frac{f'_c}{2A^2\pi} \left(L \left(\sinh^{-1}(r'_h) + r'_h \sqrt{1 + (r'_h)^2} \right) - \left(\sinh^{-1}(r'_{h0}) + r'_{h0} \sqrt{1 + (r'_{h0})^2} \right) \right) - M \left((r'_h)^2 - (r'_0)^2 \right) \quad (\text{B-310})$$

where P_s = punching shear capacity for the slab-column connection with shear reinforcement

P_{LS} = punching shear capacity for the slab-column connection with shear reinforcement for straight portion of failure surface

P_{CS} = punching shear capacity for the slab-column connection with shear reinforcement for curve portion of failure surface

Then, r'_{h0} and r'_h can be obtained from Equations B-71a and B-71b.

B.4 Punching shear capacity of slab-column connection with opening and with shear reinforcement

The punching shear capacity for the slab-column connection with opening has been derived in section B.2. The punching shear capacity for the slab-column connection with shear reinforcement has been derived in section B-3. When the slab-column connection has opening and shear reinforcement, the punching shear capacity of slab-column connection is

$$P_{op+s} = \bar{P}_s - \bar{P}_{op} \quad (\text{B-311})$$

where P_{op+s} = punching shear capacity of slab-column connection with opening and shear reinforcement

$\bar{P}_s$ = punching shear capacity of slab-column connection without opening and with shear reinforcement based on the ratio of shear

stud reinforcements passing through the area of failure surface ,
which is based on the area of failure surface A_f of slab without
ineffective area of failure surface A_{op} due to opening

$\bar{P}_{op}$ = ineffective punching shear capacity of slab-column connection due
to opening

The ratio of shear stud reinforcements $\bar{\rho}_s$ used in determining $\bar{P}_s$ by using Equations
B-307 and B-308 is considered based on the area of failure surface of slab without
opening.

$$\bar{\rho}_s = \frac{A_s}{\bar{A}_f} \quad (\text{B-312})$$

where $\bar{\rho}_s$ = ratio of shear stud reinforcement passing through the area of failure
surface based on the area of failure surface $\bar{A}_f$ of slab without
ineffective area of failure surface A_{op} due to opening

$\bar{A}_f$ = area of failure surface of slab without ineffective area of failure
surface A_{op} due to opening

A_s = total area of shear studs passing through the failure surface

Therefore, the contribution of shear reinforcements is

$$f_t = \bar{\rho}_s f_{sy} \sin \alpha_{avg} \quad (\text{B-313})$$

The area of failure surface $\bar{A}_f$ of slab without ineffective area of failure surface
 A_{op} due to opening is

$$d\bar{A}_f = dA_f - dA_{op} \quad (\text{B-314})$$

From Equation 8-85, $dA_{op} = (\eta_1 + \eta_2 r(y)) \frac{dy}{\cos \alpha}$

From Equation B-300, $dA_f = 4 \left(\frac{\pi r(y)}{2} + \Delta \right) \frac{dy}{\cos \alpha}$

By substituting dA_f and dA_{op} into Equation B-314,

$$d\bar{A}_f = 4 \left(\frac{\pi r(y)}{2} + \Delta \right) \frac{dy}{\cos \alpha} - (\eta_1 + \eta_2 r(y)) \frac{dy}{\cos \alpha} \quad (\text{B-315})$$

$$d\bar{A}_f = ((4\Delta + \eta_1) + (2\pi - \eta_2) r(y)) \frac{dy}{\cos \alpha} \quad (\text{B-316})$$

$$d\bar{A}_f = (\eta_1' + \eta_2' r(y)) \frac{dy}{\cos \alpha} \quad (\text{B-317})$$

$$\eta_1' = 4\Delta + \eta_1 \quad (\text{B-318})$$

$$\eta_2' = 2\pi - \eta_2 \quad (\text{B-319})$$

By substituting Equations B-15, B-27 and B-38 into Equation B-317,

$$d\bar{A}_f = \frac{1}{A^2 \pi^2} \left(A \pi \eta_1' + \eta_2' \left(-2A\Delta + \sqrt{1 + (r'(y))^2} \right) \right) dr'(y) \quad (\text{B-320})$$

$$\bar{A}_f = \int_{r'(0)}^{r'(h)} \frac{1}{A^2 \pi^2} \left(A \pi \eta_1' + \eta_2' \left(-2A\Delta + \sqrt{1 + (r'(y))^2} \right) \right) dr'(y) \quad (\text{B-321})$$

By solving Equation B-321,

$$\begin{aligned} \bar{A}_f = \frac{1}{2A^2 \pi^2} & \left(2A \eta_1' (-r'(0) + r'(h)) + \eta_2' \left(-\sinh^{-1}(r'(0)) + \sinh^{-1}(r'(h)) + 4A\Delta r'(0) \right. \right. \\ & \left. \left. - r'(0) \sqrt{1 + (r'(0))^2} - 4A\Delta r'(h) + r'(h) \sqrt{1 + (r'(h))^2} \right) \right) \end{aligned} \quad (\text{B-322})$$

where

$\bar{A}_f$ = area of failure surface of slab without ineffective area of failure surface A_f due to opening

A_{op} = ineffective area of failure surface due to opening

The η_1 and η_2 are the terms related to circumferential length L_{op} of ineffective area of failure surface due to opening and same as in the case for slab-column connections with opening and without shear reinforcement derived in section B.2.

B.5 Critical radius of failure surface

The critical radius of failure surface, which gives the least punching shear capacity of slab-column connection predicted by plasticity model, is a distance of failure surface at tension face of slab measured radially from column face. The punching shear capacity from Equations B-53 and B-51 is differentiated by r_h , and $r'(0)$ and $r'(h)$,

respectively, and equated to zero. The r_h is radius of failure surface at tension side of slab. The $r'(0)$ and $r'(h)$ are tangent vales of failure surface at slab thickness zero and h , respectively.

From differentiating Equation B-53 by r_h and equating to zero,

$$\frac{\partial P}{\partial r_h} = 0 \quad (\text{B-323})$$

$$\begin{aligned} \frac{\partial P}{\partial r_h} = \frac{\partial}{\partial r_h} & \left(\frac{f'_c}{2A^2\pi} \left(L \left(-2A\Delta\sqrt{-1+4A^2\Delta^2} - \ln(2A\Delta + \sqrt{-1+4A^2\Delta^2}) \right. \right. \right. \\ & + \ln\left(\sqrt{A^2(-2\Delta - \pi r_h)^2} + \sqrt{-1+A^2(-2\Delta - \pi r_h)^2}\right) - \sqrt{A^2(-2\Delta - \pi r_h)^2} \\ & \left. \left. \left. \sqrt{-1+A^2(-2\Delta - \pi r_h)^2} \right) + M(-4A^2\pi\Delta r_h - A^2\pi^2 r_h^2) \right) \right) \end{aligned} \quad (\text{B-324})$$

$$\frac{\partial P}{\partial r_h} = f'_c(2\Delta + \pi r_h) \left(-M + \frac{L}{\sqrt{1 - \frac{1}{A^2(2\Delta + \pi r_h)^2}}} \right) = 0 \quad (\text{B-325})$$

$$f'_c(2\Delta + \pi r_h) \left(-M + \frac{L}{\sqrt{1 - \frac{1}{A^2(2\Delta + \pi r_h)^2}}} \right) = 0$$

$$-M + \frac{L}{\sqrt{1 - \frac{1}{A^2(2\Delta + \pi r_h)^2}}} = 0 \quad (\text{B-326})$$

$$\begin{aligned} \frac{L}{\sqrt{1 - \frac{1}{A^2(2\Delta + \pi r_h)^2}}} &= M \\ \sqrt{1 - \frac{1}{A^2(2\Delta + \pi r_h)^2}} &= \frac{L}{M} \end{aligned} \quad (\text{B-327})$$

$$A^2(2\Delta + \pi r_h)^2 = 1 - \frac{L^2}{M^2} \quad (\text{B-328})$$

$$r_h = \frac{1}{\pi} \left(\sqrt{\frac{1}{A^2} \left(1 - \frac{L^2}{M^2} \right)} - 2\Delta \right) \quad (\text{B-329})$$

$$r_h = \frac{1}{\pi} \left(\frac{M}{A\sqrt{M^2 - L^2}} - 2\Delta \right) \quad (\text{B-330})$$

$$r_{cr} = \frac{1}{\pi} \left(\frac{M}{A_{cr}\sqrt{M^2 - L^2}} - 2\Delta \right) \quad (\text{B-331})$$

where

r_{cr} = critical radius of failure surface at tension face of slab

A_{cr} = constant that give critical radius of failure surface at tension face of slab

L, M and A can be obtained by using Equations B-1a, b-1b and B-8, respectively.

By differentiating Equation B-51 by $r'(0)$ and equating to zero,

$$\frac{\partial P}{\partial r'(0)} = 0 \quad (\text{B-332})$$

$$\begin{aligned} \frac{\partial P}{\partial r'(0)} = \frac{\partial}{\partial r'(0)} & \left(\frac{f'_c}{2A^2\pi} \left(L \left(-\sinh^{-1}(r'(0)) - r'(0)\sqrt{1+(r'(0))^2} + \sinh^{-1}(r'(h)) \right. \right. \right. \\ & \left. \left. \left. + r'(h)\sqrt{1+(r'(h))^2} \right) \right) + M \left((r'(0))^2 - (r'(h))^2 \right) \right) \end{aligned} \quad (\text{B-333})$$

$$\frac{\partial P}{\partial r'(0)} = \frac{f'_c}{A^2\pi} \left(M r'(0) - L \sqrt{1+(r'(0))^2} \right) = 0 \quad (\text{B-334})$$

$$\frac{f'_c}{A^2\pi} \left(M r'(0) - L \sqrt{1+(r'(0))^2} \right) = 0 \quad (\text{B-335})$$

$$M r'(0) - L \sqrt{1+(r'(0))^2} = 0 \quad (\text{B-336})$$

$$M r'(0) = L \sqrt{1+(r'(0))^2} \quad (\text{B-337})$$

$$\begin{aligned} (M r'(0))^2 &= \left(L \sqrt{1+(r'(0))^2} \right)^2 \\ M^2 (r'(0))^2 &= L^2 (1+(r'(0))^2) \end{aligned} \quad (\text{B-338})$$

$$(r'(0))^2 \left(1 - \frac{L^2}{M^2} \right) = \frac{L^2}{M^2} \quad (\text{B-339})$$

$$\begin{aligned}(r'(0))^2 &= \frac{L^2}{M^2 - L^2} \\ r'(0) &= \frac{L}{\sqrt{M^2 - L^2}}\end{aligned}\tag{B-340}$$

By differentiating Equation B-51 by $r'(h)$ and equating to zero,

$$\frac{\partial P}{\partial r'(h)} = 0\tag{B-341}$$

$$\begin{aligned}\frac{\partial P}{\partial r'(h)} &= \frac{\partial}{\partial r'(h)} \left(\frac{f'_c}{2A^2\pi} \left(L \left(-\sinh^{-1}(r'(0)) - r'(0)\sqrt{1+(r'(0))^2} + \sinh^{-1}(r'(h)) \right. \right. \right. \\ &\quad \left. \left. \left. + r'(h)\sqrt{1+(r'(h))^2} \right) \right) + M((r'(0))^2 - (r'(h))^2) \right) = 0\end{aligned}\tag{B-342}$$

Similarly,

$$r'(h) = \frac{L}{\sqrt{M^2 - L^2}}\tag{B-343}$$

$$r'(0) = r'(h)\tag{B-344}$$

But, we have $r'(0) = A\pi(B - C)$ and $r'(h) = A\pi(Be^{A\pi h} - Ce^{-A\pi h})$,

$$A\pi(B - C) = A\pi(Be^{A\pi h} - Ce^{-A\pi h})\tag{B-345}$$

$$B - C = Be^{A\pi h} - Ce^{-A\pi h}\tag{B-346}$$

$$\left(\frac{2\Delta}{\pi} - C \right) (1 - e^{A\pi h}) = C(1 - e^{-A\pi h})\tag{B-347}$$

$$\frac{2\Delta}{\pi} (1 - e^{A\pi h}) = C(1 - e^{-A\pi h}) + C(1 - e^{A\pi h})$$

$$\frac{2\Delta}{\pi} (1 - e^{A\pi h}) = 2C - C(e^{-A\pi h} - e^{A\pi h})$$

$$\frac{2\Delta}{\pi} (1 - e^{A\pi h}) = 2C - C \cosh(A\pi h)$$

$$\frac{2\Delta}{\pi} (1 - e^{A\pi h}) = C(2 - \cosh(A\pi h))\tag{B-348}$$

$$C = \frac{2\Delta}{\pi} \frac{(1 - e^{A\pi h})}{(2 - \cosh(A\pi h))}\tag{B-349}$$

$$C_{cr} = \frac{2\Delta}{\pi} \frac{(1 - e^{A_{cr} \pi h})}{(2 - \cosh(A_{cr} \pi h))} \quad (\text{B-350})$$

$$B_{cr} = \frac{2\Delta}{\pi} \left(1 - \frac{(1 - e^{A_{cr} \pi h})}{(2 - \cosh(A_{cr} \pi h))} \right) \quad (\text{B-351})$$

$$4 A_{cr}^2 B_{cr} C_{cr} \pi^2 = 1 \quad (\text{B-352})$$

$$4 A_{cr}^2 B_{cr} C_{cr} \pi^2 = 4 A_{cr}^2 \left(\frac{2\Delta}{\pi} \left(1 - \frac{(1 - e^{A_{cr} \pi h})}{(2 - \cosh(A_{cr} \pi h))} \right) \right)^2 \left(\frac{2\Delta}{\pi} \left(\frac{(1 - e^{A_{cr} \pi h})}{(2 - \cosh(A_{cr} \pi h))} \right) \right)^2 \pi^2 \quad (\text{B-353})$$

$$4 A_{cr}^2 B_{cr} C_{cr} \pi^2 = -4 A_{cr}^2 \Delta^2 \left(\operatorname{csch} \left(\frac{A_{cr} h \pi}{2} \right) \right)^2 \quad (\text{B-354})$$

$$1 = -4 A_{cr}^2 \Delta^2 \left(\operatorname{csch} \left(\frac{A_{cr} h \pi}{2} \right) \right)^2 \quad (\text{B-355})$$

$$A_{cr} = - \frac{1}{2 \Delta \operatorname{csch} \left(\frac{A_{cr} h \pi}{2} \right)} \quad (\text{B-356})$$

where

A_{cr} , B_{cr} & C_{cr} = constant terms related to critical radius of failure surface

APPENDIX C

MATLAB PROGRAM TO CALCULATE THE PUNCHING SHEAR CAPACITY OF SLAB-COLUMN CONNECTION BY THEORY OF PLASTICITY

Appendix C

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Main program %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Punching shear capacity of slab-column connection %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      This program is to determine the punching shear capacity of slab-column
%      connection (1) with square column, without opening and with or without
%      SSR and (2) with rectangular column, with or without opening and with or
%      without SSR by means of plasticity model derived in chapter 5.
%      SSR is shear stud reinforcement.
%      Data files in which the details of slab-column specimens are
%      included have already been prepared.
%      Names of data files are (1) Square_NoOp_NoSSR, for slab-column
%      specimen with square column, without opening and without SSR, (2)
%      Square_NoOp_SSR, for slab-column specimen with square column, without
%      opening and with SSR, (3) Rectangular_NoOp_NoSSR, for slab-column
%      specimen with rectangular column, without opening and without SSR,
%      (4) Rectangular_NoOp_SSR, for slab-column specimen with rectangular
%      column, without opening and with SSR, and (5) Rectangular_Op_SSR, for
%      slab-column specimen with rectangular column, with opening and with SSR.
%      c1          = shorter column width in mm
%      c2          = longer column width in mm
%      h           = thickness of slab in mm
%      sdia_c1     = distance between supports parallel to c1 in mm
%      sdia_c2     = distance between supports parallel to c2 in mm
%      fc_prime    = cylinder compressive strength of concrete in MPa
%      fssry       = yield strength of SSR in MPa
%      no_ssr      = total numbers of SSR
%      ssr_dia     = diameter of SSR in mm
%      P_Test      = punching shear capacity of slab-column connection
%                  obtained by test in kN
%      Oc1         = opening dimension parallel to c1 in mm
%      Oc2         = opening dimension parallel to c2 in mm
%      vc          = effectiveness factor on the compressive strength of
%                  concrete, i.e., a so-called softening of concrete
%      vt          = effectiveness factor on the tensile strength
%                  contributed by SSR
%      ho          = vertical distance from the compression side of slab to
%                  intersection point of straight line failure surface
%                  with 37 degree slope and curved failure surface in mm
%      SRad        = distance between column face and support in mm
%      rh          = radius of failure surface at which the punching shear
%                  capacity predicted by plasticity is minimum
%      P_line_curve = punching shear capacity of slab-column connection
%                  sustained by combination of straight and curved
%                  shape failure surface in kN
%      P_curve     = punching shear capacity of slab-column connection
%                  sustained by curved shape failure surface in kN
%      P_NoOp_NoSSR = punching shear capacity of slab-column connection
%                  without opening and without SSR in kN
%      P_NoOp_SSR  = punching shear capacity of slab-column connection
%                  without opening and with SSR in kN
%      P_Op_NoSSR  = punching shear capacity of slab-column connection with
%                  opening and without SSR in kN
%

```

```
%      P_Op_SSR      = punching shear capacity of slab-column connection
%
%      with opening and with SSR in kN
%
%      P_Op_SSR_Area_1 = punching shear capacity of slab-column connection
%      with opening and with SSR from method AI in kN
%
%      P_Op_SSR_Area_2 = punching shear capacity of slab-column connection with
%      opening and with SSR from method AII in kN
%
%      P_Op_Area_1      = punching shear capacity of slab-column connection lost
%      due to opening from method AI in kN
%
%      P_Op_Area_2      = punching shear capacity of slab-column connection lost
%      due to opening from method AII in kN
%
%      method AI      = method to calculate ineffective area of failure
%      surface between two projection lines extended from
%      center of column section through the two corners of
%      opening
%
%      method AII      = method to calculate ineffective area of
%      failure surface between two projection lines extended
%      from 1/3 center of column section through the
%      two corners of opening
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fprintf('Enter related description of file  with respect to type of slab-column
connection\n')
fprintf('When data in file is\n')
fprintf('(1) for slab-column specimen with square column, without opening and
without SSR, enter number 1.\n')
fprintf('(2) for slab-column specimen with square column, without opening and
with SSR, enter number 2.\n')
fprintf('(3) for slab-column specimen with rectangular column, without opening
and without SSR, enter number 3.\n')
fprintf('(4) for slab-column specimen with rectangular column, without opening
and with SSR, enter number 4.\n')
fprintf('(5) for slab-column specimen with rectangular column, with opening and
without SSR, enter number 5.\n')
fprintf('(6) for slab-column specimen with rectangular column, with opening and
with SSR, enter number 6.\n')
fprintf('\n')
file_description=input('Number : ');
fprintf('Running of program for\n')
if (file_description==1)
    fprintf('punching shear capacity for slab-column specimen with square
column, without opening and without SSR\n')
    Square_NoOp_NoSSR
elseif (file_description==2)
    fprintf('punching shear capacity for slab-column specimen with square
column, without opening and with SSR\n')
    Square_NoOp_SSR
elseif (file_description==3)
    fprintf('punching shear capacity for slab-column specimen with rectangular
column, without opening and without SSR\n')
    Rectangular_NoOp_NoSSR
elseif (file_description==4)
    fprintf('punching shear capacity for slab-column specimen with rectangular
column, without opening and with SSR\n')
    Rectangular_NoOp_SSR
elseif (file_description==5)
    fprintf('punching shear capacity for slab-column specimen with rectangular
column, with opening and without SSR\n')
```



```

    Rectangular_Op_NoSSR
elseif (file_description==6)
    fprintf('punching shear capacity for slab-column specimen with rectangular
    column, with opening and with SSR\n')
    Rectangular_Op_SSR
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Starting the execution %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i =1:1:no
    c1=s(i,1);
    c2=s(i,2);
    h=s(i,3);
    sdia_c1=s(i,4);
    sdia_c2=s(i,5);
    fc_prime=s(i,6);
    P_Test=s(i,7);
    if (file_description==2)|(file_description==4)
        fssry=s(i,8);
        no_ssr=s(i,9);
        ssr_dia=s(i,10);
        ssr_area=ssr_dia^2*pi/4;
    elseif (file_description==5)
        Oc1=s(i,8);
        Oc2=s(i,9);
        Op_type=s(i,10);
    elseif (file_description==6)
        fssry=s(i,8);
        no_ssr=s(i,9);
        ssr_dia=s(i,10);
        Oc1=s(i,11);
        Oc2=s(i,12);
        Op_type=s(i,13);
        ssr_area=ssr_dia^2*pi/4;
    end

    %%%%Initialization values.
    vc=(0.8/sqrt(fc_prime))*(10+log(1/h))*(c2/c1)^(-0.125);
    vt=1/12;
    fc=vc*fc_prime;
    ft=0;
    M=1.-2.*(ft/fc)/(1.-sin(37.*pi/180.));
    L=1.-2.*(ft/fc)*sin(37.*pi/180.)/(1.-sin(37.*pi/180.));
    rMin=0.75*h;
    if (c1==c2)
        rMax=((sdia_c1-c1)/2);
    elseif (c1~=c2)
        rMax=0.5*((sdia_c1-c1)/2+(sdia_c2-c2)/2);
    end
    delta_1=c1/2;
    delta_2=c2/2;
    delta=delta_1+delta_2;
    n1=0;
    n2=0;
    n3=0;
    rh_trial1=rMin;
    increment_r=2;
    remainder=mod((rMax-rMin),increment_r);

```

C4

```

    if (n==1)
        n1=n1+1;
        rh_i(n1)=rhs;
        A_i(n1)=As;
        B_i(n1)=Bs;
        C_i(n1)=Cs;
        rMax_limit_i(n1)=rMax_limit_trials;
        alpha_zero_i(n1)=alpha_zeros;
    end
    rh_trial1=rh_trial1+increment_r;
    if (abs(rMax-rh_trial1)==abs(remainder))
        rh_trial1=rMax;
    end
end
for i4=1:n1
    if (i4~=n1)
        if (rh_i(i4)~=rh_i(i4+1))
            n2=n2+1;
            rhi_temp(n2)=rh_i(i4);
            Ai_temp(n2)=A_i(i4);
            Bi_temp(n2)=B_i(i4);
            Ci_temp(n2)=C_i(i4);
            rMax_limiti_temp(n2)=rMax_limit_i(i4);
            alpha_zeroi_temp(n2)=alpha_zero_i(i4);
        end
    elseif (i4==n1)
        if (rh_i(i4-1)~=rh_i(i4))
            n2=n2+1;
            rhi_temp(n2)=rh_i(i4);
            Ai_temp(n2)=A_i(i4);
            Bi_temp(n2)=B_i(i4);
            Ci_temp(n2)=C_i(i4);
            rMax_limiti_temp(n2)=rMax_limit_i(i4);
            alpha-zeroi-temp(n2)=alpha_zero_i(i4);
        end
    end
end
end

%%%%Aleminate the r value with unreasonable alpha value.
n3=n3+1;
rhi(n3)=rhi_temp(1);
Ai(n3)=Ai_temp(1);
Bi(n3)=Bi_temp(1);
Ci(n3)=Ci_temp(1);
rMax_limiti(n3)=rMax_limiti_temp(1);
alpha-zeroi(n3)=alpha_zeroi_temp(1);
for i5=2:n2
    if (alpha-zeroi-temp(i5)>=alpha_zeroi_temp(1:(i5-1)))
        n3=n3+1;
        rhi(n3)=rhi_temp(i5);
        Ai(n3)=Ai_temp(i5);
        Bi(n3)=Bi_temp(i5);
        Ci(n3)=Ci_temp(i5);
        rMax_limiti(n3)=rMax_limiti_temp(i5);
        alpha_zeroi(n3)=alpha_zeroi_temp(i5);
    end
end
end

```

```

%% Determination whether critical radius rh_cr is within the support radius
or not.

for i5=1:n3
    Acr_RHS(i5)=(1-cosh(A_i(i5)*h*pi))/(8*delta^2);
    Acr_LHS(i5)=(A_i(i5))^2;
    RHS_per_LHS=abs(Acr_RHS(i5)/Acr_LHS(i5)-1);
    if (RHS_per_LHS>=0)&(RHS_per_LHS<=0.2)
        rh_cr_exist=1;
        no_cr=i5;
    else
        rh_cr_exist=0;
        no_cr=n3;
    end
end
end
%% Radius of failure surface has been successfully determined.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Determination of punching shear capacity %%%%%%%%%%
***** If the slab - column specimen without opening has no SSR,
rh=rhi(no - cr) ;
A=Ai(no - cr) ;
B=Bi(no - cr) ;
C=Ci(no - cr) ;
rMax_limit=rMax_limiti(no_cr);
alpha_zero=alpha_zeroi(no_cr);
rTan_zero=A*pi*(B-C);
rTan_h=A*pi*(B*exp(A*h*pi)-C*exp(-A*h*pi));
alpha_half_h=atan(A*pi*(B*exp(A*pi*h/2)-C*exp(-A*pi*h/2)))*180/pi;
alpha_h=atan(A*pi*(B*exp(A*pi*h)-C*exp(-A*pi*h)))*180/pi;
if (0<alpha_zero)&(alpha_zero<37)&(rMax_limit<rh)
    % The failure surface is in shape having combination of line and
    curve.
    % Determination of ho
    for i7=0.5:0.5:h
        y=i7;
        r_ho_trial1=0.75*y;
        r_ho_trial2=-(2*delta/pi)+B*exp(A*pi*y)+C*exp(-A*pi*y);

        if (0<=abs(r_ho_trial1-r_ho_trial2)&abs(r_ho_trial1...
            -r_ho_trial2)<=0.2)&(r_ho_trial2>0)
            ho_line=y;
        end
    end
    B=(-1+coth(A*pi*h_line))*...
        (-8*delta+exp(A*pi*h_line)*(8*delta+3*pi*h_line))/(8*pi);
    C=exp(A*pi*h_line)*(-1+coth(A*pi*h_line))*...
        (8*(-1+exp(A*pi*h_line)*delta-3*pi*h_line))/(8*pi);
    A=1/(2*pi*sqrt(B*C));
    rTan_ho=A*pi*(B*exp(A*h_line*pi)-C*exp(-A*h_line*pi));
    ho=ho_line;
    P_line=(1/32)*fc*(5*L-3*M)*h_line*(16*delta+3*pi*h_line);
    P_curve=(1/1000)*(fc/(2*A^2*pi))*(L*(asinh(rTan_h)...
        +rTan_h*sqrt(1+rTan_h^2)-asinh(rTan_ho)...
        -rTan_ho*sqrt(1+rTan_ho^2))+M*(-rTan_h^2+rTan_ho^2));
    P_line_curve=P_line+P_curve;
    P=P_line_curve;
elseif (37<=alpha_zero)&(rh<=rMax_limit)
    % The failure surface is in curved shape from top to bottom.

```

```

P_curve=(1/1000)*(fc/(2*A^2*pi))*((-L*asinh(rTan_zero)+M*rTan_zero^2...
-L*rTan_zero*sqrt(1+rTan_zero^2)+L*asinh(rTan_h)-M*rTan_h^2...
+L*rTan_h*sqrt(1+rTan_h^2)));
P=P_curve;
end
SRad(i)=rMax;
r_h(i)=rh;
r_h_limit(i)=rMax_limit;
P_NoOp_NoSSR(i)=P;
AvgAlpha=atan(0.5*A*pi*exp(-A*pi*h)*(1+exp(A*pi*h))*(-C+B*exp(A*pi*h)));
if (file_description==1)|(file_description==3)
    Ratio(i)= P_NoOp_NoSSR(i)/P_Test;
end

%%%% If the slab-column specimen without opening has SSR,
if (file_description==2)|(file_description==4)
    Af=(1/(A^2*pi))*(-asinh(rTan_zero)+asinh(rTan_h)-rTan_zero+rTan_h...
-rTan_zero*sqrt(1+rTan_zero^2)+rTan_h*sqrt(1+rTan_h^2));
    steel_ratio=no_ssr*ssr_area/Af;
    ft1=vt*steel_ratio*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR(i)=(1/1000)*(fc/(2*A^2*pi))*((-L1*asinh(rTan_zero)...
+M1*rTan_zero^2-L1*rTan_zero*sqrt(1+rTan_zero^2)...
+L1*asinh(rTan_h)-M1*rTan_h^2+L1*rTan_h*sqrt(1+rTan_h^2)));
    Ratio(i)=P_NoOp_SSR(i)/P_Test;
end

***** If the slab-column specimen with opening has no SSR,
if (file_description==5)|(file_description==6)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Opening location type I %%%%%%%%%%%%%%%%%%%%%%%%%
    if (Op_type==1)
        %%%% For OC13V43,OC13V43_1.6,and A1
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AI %%%%%%%%%%%%%%%%%%%%%%%%%
        beta_1_1=atan(c1/(c2+2*Oc2));
        m1_11=1/tan(beta_1_1);
        m1_12=Oc2;
        a1_1=1+m1_11^2;
        b1_1=2*m1_11*m1_12;
        c1_1=m1_12^2-rh^2;
        r1_11=(-b1_1+sqrt(b1_1^2-4*a1_1*c1_1))/(2*a1_1);
        r1_12=(-b1_1-sqrt(b1_1^2-4*a1_1*c1_1))/(2*a1_1);
        if (r1_11>0)
            r_1_12=r1_11;
        elseif (r1_12>0)
            r_1_12=r1_12;
        end
        r_1_11=m1_11*r_1_12+m1_12;
        theta_1_1=asin(abs(r_1_12)/rh);
        beta_1_2=atan(c2/(c1+2*Oc1));
        m1_21=tan(beta_1_2);
        m1_22=-Oc1*tan(beta_1_2);
        a1_2=1+m1_21^2;
        b1_2=2*m1_21*m1_22;
        c1_2=m1_22^2-rh^2;
        r1_21=(-b1_2+sqrt(b1_2^2-4*a1_2*c1_2))/(2*a1_2);
        r1_22=(-b1_2-sqrt(b1_2^2-4*a1_2*c1_2))/(2*a1_2);
        if (r1_21>0)

```

```

        r_1_22=r1_21;
elseif (r1_22>0)
    r_1_22=r1_22;
end
r_1_21=m1_21*r_1_22+m1_22;
theta_1_2=asin(abs(r_1_21)/rh);
theta_1=theta_1_1+theta_1_2;
zeta_1_1=Oc2*tan(beta_1_1)+Oc1*tan(beta_1_2);
zeta_1_2=0.5*pi-theta_1-tan(beta_1_1)+tan(theta_1_1)...
        -tan(beta_1_2)+tan(theta_1_2);
P_Op_Area_1_1(i)=(1/1000)*(zeta_1_1/(A*pi))*(L...
        *(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)...
        -sqrt(1+rTan_h^2)));
P_Op_Area_1_2(i)=(1/1000)*(0.5*zeta_1_2/(A^2*pi^2))
        *(L*(-asinh(rTan_zero)+asinh(rTan_h)+4*A*delta...
        *(rTan_zero-rTan_h)-Tan_zero*sqrt(1+rTan_zero^2)...
        +rTan_h*sqrt(1+rTan_h^2))+M*(rTan_zero^2...
        -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
        +4*A*delta*sqrt(1+rTan_h^2)));
P_Op_Area_1(i)=0.5*fc*(P_Op_Area_1_1(i)+P_Op_Area_1_2(i));
P_Op_NoSSR_Area_1(i)=P_NoOp_NoSSR(i)-P_Op_Area_1(i);
Ratio-Area-1(i)= P_Op_NoSSR_Area_1(i)/P_Test;

%%%% If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta1_1=zeta_1_1+delta;
    zeta2_1=2*pi-zeta_1_2;
    Af_1=(1/(A^2*pi^2))*(zeta1_1*2*A*pi*(-asinh(rTan_zero)...
        +asinh(rTan_h))+zeta2_1*(-asinh(rTan_zero)+asinh(rTan_h)...
        +4*A*delta*(rTan_zero-rTan_h)...
        -rTan_zero*sqrt(1+rTan_zero^2)+rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_1=no_ssr*ssr_area/Af_1;
    ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_1(i)=(1/1000)*(fc/(2*A^2*pi))*...
        *((-L1*asinh(rTan_zero)+M1*rTan_zero^2 ...
        -L1*rTan_zero*sqrt(1+rTan_zero^2)...
        +L1*asinh(rTan_h)-M1*rTan_h^2...
        +L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_1(i)=P_NoOp_SSR_1(i)-P_Op_Area_1(i);
    Ratio-Area-1(i)= P_Op_SSR_Area_1(i)/P_Test;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AII %%%%%%%%%
beta_2_1=atan((c1/2)/(c2/6+Oc2));
m2_11=1/tan(beta_2_1);
m2_12=Oc2;
a2_1=1+m2_11^2;
b2_1=2*m2_11*m2_12;
c2_1=m2_12^2-rh^2;
r2_11=(-b2_1+sqrt(b2_1^2-4*a2_1*c2_1))/(2*a2_1);
r2_12=(-b2_1-sqrt(b2_1^2-4*a2_1*c2_1))/(2*a2_1);
if (r2_11>0)
    r_2_12=r2_11;
elseif(r2_12>0)
    r_2_12=r2_12;
end

```

```

r_2_11=m2_11*r_2_12+m2_12;
theta_2_1=asin(abs(r_2_12)/rh);
beta_2_2=atan((c2/6)/(c1/2+Oc1));
m2_21=tan(beta_2_2);
m2_22=-Oc1*tan(beta_2_2);
a2_2=1+m2_21^2;
b2_2=2*m2_21*m2_22;
c2_2=m2_22^2-rh^2;
r2_21=(-b2_2+sqrt(b2_2^2-4*a2_2*c2_2))/(2*a2_2);
r2_22=(-b2_2-sqrt(b2_2^2-4*a2_2*c2_2))/(2*a2_2);
if (r2_21>0)
    r_2_22=r2_21;
elseif (r2_22>0)
    r_2_22=r2_22;
end
r_2_21=m2_21*r_2_22+m2_22;
theta_2_2=asin(abs(r_2_21)/rh);
theta_2=theta_2_1+theta_2_2;
zeta_2_1=Oc2*tan(beta_1_1)+Oc1*tan(beta_1_2);
zeta_2_2=0.5*pi-theta_2-tan(beta_2_1)+tan(theta_2_1)-tan(beta_2_2)...
    +tan(theta_2_2);
P_Op_Area_2_1(i)=(1/1000)*(zeta_2_1/(A*pi))*(L*(-rTan_zero+rTan_h)...
    +M*(sqrt(1+rTan_zero^2)-sqrt(1+rTan_h^2)));
P_Op_Area_2_2(i)=(1/1000)*(0.5*zeta_2_2/(A^2*pi^2))*...
    *(L*(-asinh(rTan_zero)+asinh(rTan_h))...
    +4*A*delta*(rTan_zero-rTan_h)...
    -rTan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1+rTan_h^2))+M*(rTan_zero^2...
    -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
    +4*A*delta*sqrt(1+rTan_h^2)));
P_Op_Area_2(i)=0.5*fc*(P_Op_Area_2_1(i)+P_Op_Area_2_2(i));
P_NoOp_NoSSR_Area_2(i)=P_NoOp_NoSSR(i)-P_Op_Area_2(i);
Ratio-Area-2(i)= P_NoOp_NoSSR_Area_2(i)/P_Test;

##### If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta1_2=zeta_1_1+delta;
    zeta2_2=2*pi-zeta_2_2;
    Af_2=(1/(A^2*pi^2))*(zeta1_2^2*A*pi*(-asinh(rTan_zero)...
        +asinh(rTan_h))+zeta2_2*(-asinh(rTan_zero)...
        +asinh(rTan_h)+4*A*delta*(rTan_zero-rTan_h)...
        -rTan_zero*sqrt(1+rTan_zero^2)+rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_2=no_ssr*ssr_area/Af_2;
    ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_2(i)=(1/1000)*(fc/(2*A^2*pi))*...
        *((-L1*asinh(rTan_zero)...
        +M1*rTan_zero^2-L1*rTan_zero*sqrt(1+rTan_zero^2)...
        +L1*asinh(rTan_h)-M1*rTan_h^2...
        +L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_2(i)=P_NoOp_SSR_2(i)-P_Op_Area_2(i);
    Ratio-Area-2(i)= P_Op_SSR_Area_2(i)/P_Test;
end
#####

##### Opening location type II #####
elseif (Op_type==2)

```

```

Specimens OC13V40 & B1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AI %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta_1_1=atan(c2/Oc1);
m1_11=tan(beta-1-1);
m1_12=-(Oc1/2-c1/2)*tan(beta-1-1);
a1_1=1+m1_11^2;
b1_1=2*m1_11*m1_12;
c1_1=m1_12^2-rh^2;
r1_11=(-b1_1+sqrt(b1_1^2-4*a1_1*c1_1))/(2*a1_1);
r1_12=(-b1_1-sqrt(b1_1^2-4*a1_1*c1_1))/(2*a1_1);
if (r1_11>0)
    r_1_12=r1_11;
elseif (r1_12>0)
    r_1_12=r1_12;
end
r_1_11=m1_11*r_1_12+m1_12;
theta_1_1=asin(abs(r_1_12)/rh);
zeta_1_1=c1+(Oc1-c1)*tan(beta-1-1);
zeta_1_2=2*(theta_1_1-tan(beta_1_1)+tan(pi/2-theta_1_1));
P_Op_Area_1_1(i)=(1/1000)*(zeta_1_1/(A*pi))*(L...
    *(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)...
    -sqrt(1+rTan_h^2)));
P_Op_Area_1_2(i)=(1/1000)*(0.5*zeta_1_2/(A^2*pi^2))
    *(L*(-asinh(rTan_zero)+asinh(rTan_h)+4*A*delta...
    *(rTan_zero-rTan_h)-Tan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1+rTan_h^2))+M*(rTan_zero^2...
    -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
    +4*A*delta*sqrt(1+rTan_h^2)));
P_Op_Area_1(i)=0.5*fc*(P_Op_Area_1_1(i)+P_Op_Area_1_2(i));
P_Op_NoSSR_Area_1(i)=P_NoOp_NoSSR(i)-P_Op_Area_1(i);
Ratio_Area_1(i)= P_Op_NoSSR_Area_1 (i)/P_Test;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta11_1=zeta_1_1+delta;
    zeta22_1=2*pi-zeta_1_2;
    Af_1=(1/(A^2*pi^2))*(zeta11_1^2*A*pi*(-asinh(rTan_zero)...
        +asinh(rTan_h))+zeta22_1*(-asinh(rTan_zero)+asinh(rTan_h)...
        +4*A*delta*(rTan_zero-rTan_h)-rTan_zero*sqrt(1+rTan_zero^2)...
        +rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_1=no_ssr*ssr_area/Af_1;
    ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_1(i)=(1/1000)*(fc/(2*A^2*pi))...
        *((-L1*asinh(rTan_zero)+M1*rTan_zero^2...
        -L1*rTan_zero*sqrt(1+rTan_zero^2)+L1*asinh(rTan_h)...
        -M1*rTan_h^2+L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_1(i)=P_NoOp_SSR_1(i)-P_Op_Area_1(i);
    Ratio-Area-1(i)= P_Op_SSR_Area_1(i)/P_Test;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AII %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta_2_1=atan(c2/(3*Oc1));
m2_11=tan(beta_2_1);
m2_12=-(Oc1/2-c1/2)*tan(beta_2_1);
a2_1=1+m2_11^2;
b2_1=2*m2_11*m2_12;

```



```

c2_1=m2_12^2-rh^2;
r2_11=(-b2_1+sqrt(b2_1^2-4*a2_1*c2_1))/(2*a2_1);
r2_12=(-b2_1-sqrt(b2_1^2-4*a2_1*c2_1))/(2*a2_1);
if (r2_11>0)
    r_2_12=r2_11;
elseif (r2_12>0)
    r_2_12=r2_12;
end
r_2_11=m2_11*r_2_12+m2_12;
theta_2_1=asin(abs(r_2_12)/rh);
zeta_2_1=c1+(Oc1-c1)*tan(beta_2-1);
zeta_2_2=2*(theta_2_1-tan(beta_2_1)+tan(pi/2-theta_2_1));
P_Op_Area_2_1(i)=(1/1000)*(zeta_2_1/(A*pi))*...
    *(L*(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)..
    -sqrt(1+rTan_h^2)));
P_Op_Area_2_2(i)=(1/1000)*(0.5*zeta_2_2/(A^2*pi^2))*...
    *(L*(-asinh(rTan_zero)+asinh(rTan_h)...
    +4*A*delta*(rTan_zero-rTan_h)
    -rTan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1.0+rTan_h^2))+M*(rTan_zero^2...
    -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
    +4*A*delta*sqrt(1+rTan_h^2)));
P_Op_Area_2(i)=0.5*fc*(P_Op_Area_2_1(i)+P_Op_Area_2_2(i));
P_Op_NoSSR_Area_2(i)=P_NoOp_NoSSR(i)-P_Op_Area_2(i);
Ratio_Area_2(i)= P_Op_NoSSR_Area_2(i)/P_Test;

~~~~~ If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta11_2=zeta_1_1+delta;
    zeta22_2=2*pi-zeta_2_2;
    Af_2=(1/(A^2*pi^2))*(zeta11_2*2*A*pi*(-asinh(rTan_zero)...
        +asinh(rTan_h))+zeta22_2*(-asinh(rTan_zero)...
        +asinh(rTan_h)+4*A*delta*(rTan_zero-rTan_h)...
        -rTan_zero*sqrt(1+rTan_zero^2)+rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_2=no_ssr*ssr_area/Af_2;
    ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_2(i)=(1/1000)*(fc/(2*A^2*pi))*...
        *((-L1*asinh(rTan_zero)...
        +M1*rTan_zero^2-L1*rTan_zero*sqrt(1+rTan_zero^2)...
        +L1*asinh(rTan_h)-M1*rTan_h^2...
        +L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_2(i)=P_NoOp_SSR_2(i)-P_Op_Area_2(i);
    Ratio-Area-2(i)= P_Op_SSR_Area_2(i)/P_Test;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Opening location type IIII %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
elseif (Op_type==3)
    ~~~~~ For specimen OC13V23 & OC15V43
    ~~~~~ For method AI ~~~~~
    beta_1_1=atan((c1/2)/(c2/2));
    beta_1_2=atan((c2/2-Oc2)/(c1/2+Oc1));
    if (c2/2*tan(pi/2-beta_1_2)-c1/2 < rh)
        m1_21=1/tan(pi/2-beta_1_2);
        m1_22=-(c2/2-c1/2*tan(beta_1-1-21));
        a1_2=1+m1_21^2;

```

```

b1_2=2*m1_21*m1_22;
c1_2=m1_22^2-rh^2;
r1_21=(-b1_2+sqrt(b1_2^2-4*a1_2*c1_2))/(2*a1_2);
r1_22=(-b1_2-sqrt(b1_2^2-4*a1_2*c1_2))/(2*a1_2);
if r1_21>0
    r_1_22=r1_21;
elseif (r1_22>0)
    r_1_22=r1_22;
end
r_1_21=m1_21*r_1_22+m1_22;
theta_1_2=asin(abs(r_1_21)/rh);
zeta_1_1=c2/2-c1/2*tan(beta_1_2);
zeta_1_2=(0.5*pi-beta_1_1)+theta_1_2-tan(beta_1_2);
else
    zeta_1_1=c2/2-c1/2*tan(beta_1_2);
    zeta_1_2=pi/2-beta_1_1-tan(beta_1_2);
end
P_Op_Area_1_1(i)=(1/1000)*(zeta_1_1/(A*pi))*...
    *(L*(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)...
    -sqrt(1+rTan_h^2)));
P_Op_Area_1_2(i)=(1/1000)*(0.5*zeta_1_2/(A^2*pi^2))*...
    *(L*(-asinh(rTan_zero)+asinh(rTan_h)...
    +4*A*delta*(rTan_zero-rTan_h)...
    -rTan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1+rTan_h^2))+M*(rTan_zero^2...
    -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
    +4*A*delta*sqrt(1+rTan_h^2)));
P_Op_Area-1(i)=0.5*fc*(P_Op_Area_1_1(i)+P_Op_Area_1_2(i));
P_Op_NoSSR_Area_1(i)=P_NoOp_NoSSR(i)-P_Op_Area_1(i);
Ratio-Area-1(i)= P_Op_NoSSR_Area_1(i)/P_Test

%%%% If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta11_1=zeta_1_1+delta;
    zeta22_1=2*pi-zeta_1_2;
    Af_1=(1/(A^2*pi^2))*(zeta11_1*2*A*pi*(-asinh(rTan_zero)...
    +asinh(rTan_h))+zeta22_1*(-asinh(rTan_zero)+asinh(rTan_h)...
    +4*A*delta*(rTan_zero-rTan_h)-rTan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_1=no_ssr*ssr_area/Af_1;
    ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_1(i)=(1/1000)*(fc/(2*A^2*pi))*...
    *((-L1*asinh(rTan_zero)+M1*rTan_zero^2...
    -L1*rTan_zero*sqrt(1+rTan_zero^2)+L1*asinh(rTan_h)...
    -M1*rTan_h^2+L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_1(i)=P_NoOp_SSR_1(i)-P_Op_Area_1(i);
    Ratio-Area-1(i)= P_Op_SSR_Area_1(i)/P_Test;
end

%%%%%%%%%%%% For method AII %%%%%%%%%%%%%%
beta_2_1=atan((c1/2)/(c2/6));
beta_2_2=atan((c1/2)/abs(oc2-c2/6));
if (5/6*c2*tan(beta_2_2-c1/2)>=rh)
    zeta_2_1=oc2;
    zeta_2_2=(pi/2-beta_2_1)+tan(beta_2-21);
else

```



```

beta_1_1=atan((c1/2)/(Oc2/2));
m1_11=1/tan(beta_1_1);
m1_12=-(c2/2-Oc2/2);
a1_1=1+m1_11^2;
b1_1=2*m1_11*m1_12;
c1_1=m1_12^2-rh^2;
r1_11=(-b1_1+sqrt(b1_1^2-4*a1_1*c1_1))/(2*a1_1);
r1_12=(-b1_1-sqrt(b1_1^2-4*a1_1*c1_1))/(2*a1_1);
if (r1_11>0)
    r_1_12=r1_11;
elseif(r1_12>0)
    r_1_12=r1_12;
end
if ((c2/2-Oc2/2)*tan(beta-1-1)<rh)
    r_1_11=m1_11*r_1_12+m1_12;
    theta_1_1=asin(abs(r_1_11)/rh);
    zeta_1_1=Oc2;
    zeta_1_2=2*(theta_1_1+tan(pi/2-beta_1_1)-tan(theta_1_1));
else
    zeta_1_1=Oc2;
    zeta_1_2=2*(pi/2-beta_1_1);
end
P_Op_Area_1_1(i)=(1/1000)*(zeta_1_1/(A*pi))...
    *(L*(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)
    -sqrt(1+rTan_h^2)));
P_Op_Area_1_2(i)=(1/1000)*(0.5*zeta_1_2/(A^2*pi^2))...
    *(L*(-asinh(rTan_zero)+asinh(rTan_h)...
    +4*A*delta*(rTan_zero-rTan_h)...
    -rTan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1+rTan_h^2))+M*(rTan_zero^2...
    -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
    +4*A*delta*sqrt(1+rTan_h^2)));
P_Op_Area-1(i)=0.5*fc*(P_Op_Area_1_1(i)+P_Op_Area_1_2(i));
P_Op_NoSSR_Area_1(i)=P_NoOp_NoSSR(i)-P_Op_Area_1(i);
Ratio-Area-1(i)= P_Op_NoSSR_Area_1(i)/P_Test

%%%% If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta1_1=zeta_1_1+delta;
    zeta2_1=2*pi-zeta_1_2;
    Af_1=(1/(A^2*pi^2))*(zeta1_1*2*A*pi*(-asinh(rTan_zero)...
    +asinh(rTan_h))+zeta2_1*(-asinh(rTan_zero)+asinh(rTan_h)...
    +4*A*delta*(rTan_zero-rTan_h)-rTan_zero*sqrt(1+rTan_zero^2)...
    +rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_1=no_ssr*ssr_area/Af_1;
    ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
    L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_1(i)=(1/1000)*(fc/(2*A^2*pi))...
    *((-L1*asinh(rTan_zero)+M1*rTan_zero^2...
    -L1*rTan_zero*sqrt(1+rTan_zero^2)+L1*asinh(rTan_h)...
    -M1*rTan_h^2+L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_1(i)=P_NoOp_SSR_1(i)-P_Op_Area_1(i);
    Ratio-Area-1(i)= P_Op_SSR_Area_1(i)/P_Test;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AII %%%%%%%%%%%%%%%
beta_2_1=atan((c2/2-c1/6-Oc2/2)/(c1/2+Oc1));

```

[illegible]

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Opening location type V %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
elseif (Op_type==5)
    %%%% For specimen OC13H50, OC13H50-1.6, and OC15H70
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AI %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    beta_1_1=atan(c1/c2);
    zeta_1_1=c1;
    zeta_1_2=2*beta_1_1;
    P_Op_Area-1-1(i)=(1/1000)*(zeta_1_1/(A*pi))...
        *(L*(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)...
            -sqrt(1+rTan_h^2)));
    P_Op_Area-1-2(i)=(1/1000)*(0.5*zeta_1_2/(A^2*pi^2))...
        *(L*(-asinh(rTan_zero)+asinh(rTan_h)...
            +4*A*delta*(rTan_zero-rTan_h)...
            -rTan_zero*sqrt(1+rTan_zero^2)...
            +rTan_h*sqrt(1+rTan_h^2))+M*(rTan_zero^2...
            -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
            +4*A*delta*sqrt(1+rTan_h^2)));
    P_Op_Area_1(i)=0.5*fc*(P_Op_Area_1_1(i)+P_Op_Area_1_2(i));
    P_Op_NoSSR_Area_1(i)=P_NoOp_NoSSR(i)-P_Op_Area_1(i);
    Ratio-Area-1(i)= P_Op_NoSSR_Area_1(i)/P_Test

    %%%% If the slab-column specimen with opening has SSR,
    if (file_description==6) % Due to the inclusion of SSR
        zeta1_1=zeta_1_1+delta;
        zeta2_1=2*pi-zeta_1_2;
        Af_1=(1/(A^2*pi^2))*(zeta1_1*2*A*pi*(-asinh(rTan_zero)...
            +asinh(rTan_h))+zeta2_1*(-asinh(rTan_zero)+asinh(rTan_h)...
            +4*A*delta*(rTan_zero-rTan_h)-rTan_zero*sqrt(1+rTan_zero^2)...
            +rTan_h*sqrt(1+rTan_h^2)));
        steel_ratio_1=no_ssr*ssr_area/Af_1;
        ft1=vt*steel_ratio_1*fssry*sin(AvgAlpha);
        M1=1-2*(ft1/fc)/(1-sin(37*pi/180));
        L1=1-2*(ft1/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
        P_NoOp_SSR_1(i)=(1/1000)*(fc/(2*A^2*pi))...
            *((-L1*asinh(rTan_zero)+M1*rTan_zero^2...
            -L1*rTan_zero*sqrt(1+rTan_zero^2)+L1*asinh(rTan_h)...
            -M1*rTan_h^2+L1*rTan_h*sqrt(1+rTan_h^2)));
        P_Op_SSR_Area_1(i)=P_NoOp_SSR_1(i)-P_Op_Area_1(i);
        Ratio-Area-1(i)= P_Op_SSR_Area_1(i)/P_Test;
    end

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% For method AII %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    beta_2_1=atan(c1/(c2/6));
    zeta_2_1=c1;
    zeta_2_2=2*beta_2_1;
    P_Op_Area_2_1(i)=(1/1000)*(zeta_2_1/(A*pi))...
        *(L*(-rTan_zero+rTan_h)+M*(sqrt(1+rTan_zero^2)...
            -sqrt(1+rTan_h^2)));
    P_Op_Area_2_2(i)=(1/1000)*(0.5*zeta_2_2/(A^2*pi^2))...
        *(L*(-asinh(rTan_zero)+asinh(rTan_h)...
            +4*A*delta*(rTan_zero-rTan_h)...
            -rTan_zero*sqrt(1+rTan_zero^2)...
            +rTan_h*sqrt(1.0+rTan_h^2))+M*(rTan_zero^2...
            -rTan_h^2-4*A*delta*sqrt(1+rTan_zero^2)...
            +4*A*delta*sqrt(1+rTan_h^2)));
    P_Op_Area_2(i)=0.5*fc*(P_Op_Area_2_1(i)+P_Op_Area_2_2(i));
    P_Op_NoSSR_Area_2(i)=P_NoOp_NoSSR(i)-P_Op_Area_2(i);

```

```

Ratio_Area_2(i)= P_Op_NoSSR_Area_2(i)/P_Test;

##### If the slab-column specimen with opening has SSR,
if (file_description==6)
    zeta11_2=zeta_1_1+delta;
    zeta22_2=2*pi-zeta_2_2;
    Af_2=(1/(A^2*pi^2))* (zeta11_2*2*A*pi*(-asinh(rTan_zero)...
        +asinh(rTan_h))+zeta22_2*(-asinh(rTan_zero)...
        +asinh(rTan_h)+4*A*delta*(rTan_zero-rTan_h)...
        -rTan_zero*sqrt(1+rTan_zero^2)+rTan_h*sqrt(1+rTan_h^2)));
    steel_ratio_2=no_ssr*ssr_area/Af_2;
    ftl=vt*steel_ratio_1*fssry*sin(AvgAlpha);
    M1=1-2*(ftl/fc)/(1-sin(37*pi/180));
    L1=1-2*(ftl/fc)*sin(37*pi/180)/(1-sin(37*pi/180));
    P_NoOp_SSR_2(i)=(1/1000)*(fc/(2*A^2*pi))...
        *((-L1*asinh(rTan_zero)...
            +M1*rTan_zero^2-L1*rTan_zero*sqrt(1+rTan_zero^2)...
            +L1*asinh(rTan_h)-M1*rTan_h^2...
            +L1*rTan_h*sqrt(1+rTan_h^2)));
    P_Op_SSR_Area_2(i)=P_NoOp_SSR_2(i)-P_Op_Area_2(i);
    Ratio_Area-2(i)= P_Op_SSR_Area_2(i)/P_Test;
end
#####
end

##### Saving the results #####
if (file_description==1)
    save Results_Sq_NoOp_NoSSR P_NoOp_NoSSR Rator_h_limit r_h SRad
elseif (file_description==2)
    save Results_Sq_NoOp_SSR P_NoOp_NoSSR Pu_NoOp_SSR Ratio r_h_limit...
        r_h SRad
elseif (file_description==3)
    save Results_Rect_NoOp_NoSSR P_NoOp_NoSSR Ratio r_h_limit r_h SRad
elseif (file_description==4)
    save Results_Rect_NoOp_SSR P_NoOp_NoSSR P_NoOp_SSR Ratio r_h_limit...
        r_h SRad
elseif (file_description==5)
    save Results_Rect_Op_NoSSR P_NoOp_NoSSR P_Op_Area_1...
        P_Op_NoSSR_Area_1 Ratio-Area-1 P_Op_Area_2 r_h_limit...
        r_h P_Op_NoSSR_Area_2 Ratio-Area-2 SRad
elseif (file_description==6)
    save Results_Rect_Op_SSR P_NoOp_NoSSR P_NoOp_SSR_1...
        P_Op_Area_1 P_Op_SSR_Area_1 Ratio-Area-1 P_NoOp_SSR_2 ...
        P_Op_Area_2 P_Op_SSR_Area_2 Ratio-Area-2 r_h_limit r_h SRad
end
#####

##### End of execution #####

fprintf ('Running of program has successfully finished.\n');

#####

```

```
% Data file for slab-column connection with square column, without opening %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      = column width c1 in horizontal direction on plan area (mm)    %
%      = column width c2 in vertical direction on plan area (mm)     %
%      = thickness 03 slab (mm)                                     %
%      = distance between supports parallel to c1 ( mm )            %
%      = distance between supports parallel to c2 ( mm )             %
%      = cylinder compressive strength of concrete (MPa)           %
%      = punching shear capacity 03 slab-column connection obtained %
%      by test (kN)                                                %
%      no                    = total numbers of specimens          %
%      filename              = Square_NoOp_NosSR                  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
spec_name=['HS7','HS3 ','NS2 ','HS6 ',...
'HS9 ','HS10 ','SH14 ','HS15 ',...
'H1 ','S2-60','S3-60','AB1 ',];
no=12;
s=[150 TSO TZO OTO 0 Lf 356   % LSH
150 TSO TZO 1500 0 6f 356   % ESH
150 TSO OTO 0 OE 96        % ZSN
150 TSO 150 OL 8P         % SH6
150 STO 150 PL EPS       % SH6
150 STO 150 0 ZO 645     % OTSH
ZZO ZZO TZO 1500 0 LZ 998 % PTSH
OOE OOZ EOZO 1500 0 fL 560 % STSH
TH                                % TH
SZF SZF TSOT LTTL 0 ZF 356 % SZ-60
ZF SF 364                     % SF-60
250 ZSO 150 1800 0 36 408], % AB1(Mokhtar,A.B.(1985))
save Square_NoOp_NosSR s spec_name
```



```
#####
%% Data file for slab-column connection with square column, without opening %%
##### and with SSR #####
%
column 1      = column width c1 in horizontal direction on plan area (mm)
column 2      = column width c2 in vertical direction on plan area (mm)
column 3      = thickness of slab (mm)
column 4      = distance between supports parallel to c1 (mm)
column 5      = distance between supports parallel to c2 (mm)
column 6      = cylinder compressive strength of concrete (Mpa)
column 7      = punching shear capacity of slab-column connection obtained
                by test (kN)
column 8      = yield strength of shear stud reinforcement (Mpa)
column 9      = total numbers of shear stud reinforcement
column 10     = diameter of shear stud reinforcement (mm)
no            = total numbers of specimens
file name     = Square_NoOp_SSR
#####
spec_name=['AB3 ','AB5 ','AB6 ','AB7 ','...
           'AB8 ','L301','L302','SC11','...
           'SC12','SC13','A3 ','A4 '];

no=12;
s=[250  250  150  1800 0 23   545  278 96 9.5      % A83
   250  250  150  1800 0 40   583  278 96 9.5 % AB5
   250  250  150  1800 0 29   541  489 72 9.5      % AB6
   250  250  150  1800 0 35   572  278 72 9.5      % AB7
   250  250  150  1800 0 30   508  489 60 9.5      % A88
   200  200  200  2700 0 37.8 830  602 48 10      % L301
   200  200  200  2700 0 34.2 790  602 96 10      % L302
   304.8 304.8 152.4 1749 0 33.6 596  500 64 6.4      % SC11
   304.8 304.8 152.4 1749 0 33.6 595  500 64 6.4      % SC12
   304.8 304.8 152.4 1749 0 33.6 580  500 64 6.4 % SC13
   250  250  200  2121 0 39.8 914  513 56 10      % A3
   250  250  200  2121 0 40.3 1058 513 70 101;      % A4

save  Square_NoOp_SSR s spec-name
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Data file for slab-column connection with rectangular column, %%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% without opening and without SSR %%%%%%%%%%
%      column 1      = column width c1 in horizontal direction on plan area (mm)
%      column 2      = column width c2 in vertical direction on plan area (mm)
%      column 3      = thickness of slab (mm)
%      column 4      = distance between supports parallel to c1 (mm)
%      column 5      = distance between supports parallel to c2 (mm)
%      column 6      = cylinder compressive strength of concrete (Mpa)
%      column 7      = punching shear capacity of slab-column connection obtained
%                    by test (kN)
%      no            = total numbers of specimens
%      file name     = Rectangular_NoOp_NoSSR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
spec_name=['OC13      ','OC15      ','OC13_1.6    ','C13F22      ',...
          'C15F22      ','2        ','3         ',...
          '8          ','9         ','R1(Moe)    ','R1(Hognestad)'];

no=12;
s=[200  600  150  107   1800 2000 36   568      % OC13
   200  1000 150  103   1800 2500 40.15 649      % OC15
   200  600  150  109.75 1800 2000 32.98 508      % OC13_1_6
   250  750  200  155   2000 2400 35.60 792      % C13F22
   250  1250 200  160   2000 3000 35.4  1056 % C15F22
   203  406  152  117   2083 2083 26.3  351      % slab-2
   152  457  152  117   2083 2083 32.03 333      % slab-3
   114  494  152.4 117   2083 2083 30.90 331      % slab-4
   114  494  152.4 120.65 2083 2083 26.10 314      % slab-8
   152  305  152  120.65 2083 2083 29.50 317      % slab-9
  152.4 457.2 152.4 114   1780 1780 27.58 394      % R1(Moe,1961)
  152.4 457.2 152.4 114   1780 1780 27.58 3971; % R1(Hognestad)

save  Rectangular_NoOp_NoSSR s spec_name
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%% Data file for slab-column connection with rectangular column, %%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% without opening and with SSR %%%%%%%%%
%      column 1      = column width c1 in horizontal direction on plan area (mm)
%      column 2      = column width c2 in vertical direction on plan area (mm)
%      column 3      = thickness of slab (mm)
%      column 4      = distance between supports parallel to c1 (mm)
%      column 5      = distance between supports parallel to c2 (mm)
%      column 6      = cylinder compressive strength of concrete (Mpa)
%      column 7      = punching shear capacity of slab-column connection obtained
%                      by test (kN)
%      column 8      = yield strength of shear stud reinforcement (Mpa)
%      column 9      = total numbers of shear stud reinforcement
%      column 10     = diameter of shear stud reinforcement (mm)
%      no            = total numbers of specimens
%      file name     = Rectangular_NoOp_SSR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
spec_name=['B3_C13f22Vs1 ','B4_C13f22Vs1 ','C3_C15f22Vs2a','C4_C15f22Vs2a'];
no =4;
s=[250 750 200 2000 2400 39.5 1168 515 56 10          % B3_C13f22Vs1
   250 750 200 2000 2400 40.6 1266 515 84 10          % B4_C13f22Vs1
   250 1250 200 2000 3000 38.3 1245 515 84 10         % C3_C15f22Vs2a
   250 1250 200 2000 3000 39.70 1477 515 112 101;    % C4_C15f22Vs2a

save  Rectangular_NoOp_SSR s spec_name

```

```
#####
##### Data file for slab-column connection with rectangular column, #####
##### with opening and without SSR #####
%      column 1      = column width c1 in horizontal direction on plan area (mm)
%      column 2      = column width c2 in vertical direction on plan area (mm)
%      column 3      = thickness of slab (mm)
%      column 4      = distance between supports parallel to c1 (mm)
%      column 5      = distance between supports parallel to c2 (mm)
%      column 6      = cylinder compressive strength of concrete (Mpa)
%      column 7      = punching shear capacity of slab-column connection obtained
%                      by test (kN)
%      column 8      = opening dimension parallel to c1 (mm)
%      column 9      = opening dimension parallel to c2 (mm)
%      column 10     = type of opening location
%                      1 is meant opening location type I that the opening is
%                      positioned adjacent to the corner of column.
%                      2 is meant opening location type II that the opening is
%                      positioned adjacent to the shorter side of column with
%                      longer side of opening being parallel to shorter side of
%                      opening.
%                      3 is meant opening location type III that the opening is
%                      positioned adjacent to the end of longer column side with
%                      shorter side of opening being parallel to the longer
%                      column side)
%                      4 is meant opening location type IV that the opening is
%                      positioned adjacent to the middle of longer column side
%                      with longer side of opening being parallel to the longer
%                      column side)
%                      5 is meant opening location type V that the opening is
%                      positioned adjacent to the shorter column side with
%                      shorter side of opening being parallel to the shorter
%                      column side)
%      no            = total numbers of specimens
%      file name     = Rectangular_Op_NoSSR
#####
spec_name=['OC13V43      ','OC13V43_1.6','A1          ','OC13V40      '];
          'B1          ','OC13V23      ','OC15V43      ','OC13H02      '];
          'OC13H02-1.6','OC13          ','OC13H50      ','OC15H70      '];

no =12;
s=[200 600 150 1800 2000 36.62 467 400 200 1      % OC13V43
   200 600 150 1800 2000 33.21 383 400 200 1      % OC13V43_1.6
   250 1000 200 2000 3000 38.20 740 400 200 1      % A1
   200 600 150 1800 2000 43.03 340 400 200 2      % OC13V40
   250 1000 200 2000 3000 36.30 580 400 200 2      % B1
   200 600 150 1800 2000 36.94 484 400 200 3      % OC13V23
   200 1000 150 1800 2500 35.97 612 400 200 3      % OC15V43
   200 600 150 1800 2000 43.09 512 200 400 4      % OC13H02
   200 600 150 1800 2000 37.53 420 200 400 4      % OC13H02-1.6
   200 600 150 1800 2000 36.29 443 200 400 5      % OC13H50
   200 600 150 1800 2000 33.09 428 200 400 5      % OC13H50-1.6
   200 1000 150 1800 2000 37.89 529 200 400 5]; % OC15H70
save Rectangular_Op_NoSSR s spec-name
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%% Data file for slab-column connection with rectangular column, %%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% with opening and with SSR %%%%%%%%%%
%      column 1      = column width c1 in horizontal direction on plan area (mm)
%      column 2      = column width c2 in vertical direction on plan area (mm)
%      column 3      = thickness of slab (mm)
%      column 4      = distance between supports parallel to c1 (mm)
%      column 5      = distance between supports parallel to c2 (mm)
%      column 6      = cylinder compressive strength of concrete (Mpa)
%      column 7      = punching shear capacity of slab-column connection obtained
%                      by test (kN)
%      column 8      = yield strength of shear stud reinforcement (Mpa)
%      column 9      = total numbers of shear stud reinforcement
%      column 10     = diameter of shear stud reinforcement (mm)
%      column 11     = opening dimension parallel to c1 (mm)
%      column 12     = opening dimension parallel to c2 (mm)
%      column 13     = type of opening location
%
%      1 is meant opening location type I that the opening is
%      positioned adjacent to the corner of column.
%
%      2 is meant opening location type II that the opening is
%      positioned adjacent to the shorter side of column with
%      longer side of opening being parallel to shorter side of
%      opening.
%
%      no            = total numbers of specimens
%      file name     = Rectangular Op SSR
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
spec_name=['A2';'A3';'B2';'B3'];
no=4;
s=[250 1000 200 2000 3000 38.20 908 540 40 10 400 200 1 % A2
  250 1000 200 2000 3000 38.20 948 540 30 10 400 200 1 % A3
  250 1000 200 2000 3000 36.30 848 540 48 10 400 200 2 % B2
  250 1000 200 2000 3000 36.30 692 540 30 10 400 200 2]; % B2

save Rectangular_Op_NoSSR s spec_name
```

Punching shear capacities of slab-column specimens with square column, without opening and without SSR

Spec:	C1 (mm)	C2 (mm)	h (mm)	f _c (MPa)	P (Test) (kN)	P _{NoOp} (Pred) (kN)	R _{cr} /h	S _{Rad} /h	Ratio
HS7	150	150	120	74	356	360	5.60	5.63	1.01
HS3	150	150	120	69	356	348	5.60	5.63	0.98
NS2	150	150	150	30	396	354	4.50	4.50	0.89
HS6	150	150	150	70	489	540	4.50	4.50	1.10
HS9	150	150	150	74	543	555	4.50	4.50	1.02
HS10	150	150	150	80	645	577	4.50	4.50	0.90
HS14	220	220	120	72	498	428	5.33	5.33	0.86
HS15	300	300	120	71	560	509	5.00	5.00	0.91
H1	254	254	150	26	371	396	5.08	5.08	1.07
S2-60	254	254	150	22	356	364	5.08	5.08	1.02
S3-60	254	254	150	23	364	372	5.08	5.08	1.02
AB1	250	250	150	36	408	459	5.15	5.17	1.13

Punching shear capacities of slab-column connections with square column, without opening and with SSR

Spec:	C1 (mm)	C2 (mm)	h (mm)	f _c (MPa)	f _{ssry} (MPa)	no of SSR	P (Test) (kN)	P _{NoOp} (Pred) (kN)	R _{cr} /h	S _{Rad} /h	Ratio
AB3	250	250	150	23	278	96	545	367.21	5.15	5.17	0.90
AB5	250	250	150	40	278	96	583	484.26	5.15	5.17	1.04
AB6	250	250	150	29	489	72	541	412.34	5.15	5.17	1.06
AB7	250	250	150	35	278	72	572	452.99	5.15	5.17	0.95
AB8	250	250	150	30	489	60	508	419.39	5.15	5.17	1.09
L301	200	200	200	38	602	48	830	628.44	6.25	6.25	0.94
L302	200	200	200	34	602	96	790	597.77	6.25	6.25	1.13
SC11	305	305	152	34	500	64	596	518.44	4.73	4.74	0.98
SC12	305	305	152	34	500	64	595	518.44	4.73	4.74	0.98
SC13	305	305	152	34	500	64	580	518.44	4.73	4.74	1.01
A3	250	250	200	40	513	56	914	784.89	4.68	4.68	1.02
A4	250	250	200	40	513	70	1058	789.81	4.68	4.68	0.92

Punching shear capacities of slab-column specimens with rectangular column, without opening and without SSR

Spec:	C1	C2	h	fc (Test) (MPa)	P (kN)	P NoOp (Pred) (kN)	Rcr/h	SRad/h	Ratio
	(mm)	(mm)	(mm)			(kN)			
OC13	200	600	150	36	568	543.33	5.00	5.00	0.96
OC15	200	1000	150	40	649	670.16	5.17	5.17	1.03
OC13_1.6	200	600	150	33	508	520.05	5.00	5.00	1.02
C13F22	250	750	200	36	792	843.41	4.25	4.25	1.06
C15F22	250	1250	200	35	1056	991.12	4.38	4.38	0.94
2	203	406	152	26	351	339.24	5.85	5.85	0.97
3	152	457	152	32	333	365.07	5.85	5.85	1.10
4	114	494	152	31	331	349.06	5.82	5.84	1.05
8	114	494	152	26	314	320.80	5.82	5.84	1.02
9	152	305	152	30	317	311.50	6.10	6.10	0.98
R1 (Moe)	152	457	152	28	394	370.78	4.84	4.84	0.94
R1 (Hognestad)	152	457	152	28	397	370.78	4.84	4.84	0.93

Punching shear capacities of slab-column specimens with rectangular column, without opening and with SSR

Spec:	C1	C2	h	fc (MPa)	fssry (MPa)	no of SSR	P (Test) (kN)	P NoOp (kN)	NoSSR (Pred) (kN)	P_NoOp SRad/h (kN)	Rcr/h	Ratio
	(mm)	(mm)	(mm)									
B3 C13f22Vs1	250	750	200	40	515	56	1168	888.40	1027.07	4.25	4.25	0.88
B4_C13f22Vs1	250	750	200	41	515	84	1266	900.69	1108.69	4.25	4.25	0.88
C3_C15f22Vs2a	250	1250	200	38	515	84	1245	1030.92	1238.46	4.38	4.38	0.99
C4_C15f22Vs2a	250	1250	200	40	515	112	1477	1049.59	1326.31	4.38	4.38	0.90