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SEISMIC DAMAGE ASSESSMENT BASED ON STRUCTURAL RESPONSE & ENERGY APPROACH

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
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2005

**Seismic Damage Assessment
Based on Structural Response & Energy Approach**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

A Thesis Submitted to the Nanyang Technological University
in Fulfilment of the Requirement for the Degree of
Doctor of Philosophy

2005

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SUMMARY

A study on seismic damage assessment using response and energy approach is presented in this thesis. A new structural damage assessment model is proposed based on modified force analogy method, and some special features, characteristics and applications of this model are investigated.

The proposed damage assessment model considers structural damage as a combination of monotonic effects and cycling effects in nature, and therefore it evaluates structural damage from both response and energy point of view. This model includes a total damage index and several local damage indices. The total damage index gives the overall damage conditions of a structure during an earthquake, while the local damage indices give insights into the locations within the structure where most damage has been suffered during an earthquake, and they also help to reveal the damage process of a structure progresses from partial damage to total collapse. Thus, the damage conditions of structures can be inspected wholly and locally through this damage assessment model.

The concepts of damage index spectra are discussed. Comparisons are made between the damage assessment results obtained from inelastic time history analysis and from inelastic spectra analysis for various frames. It is concluded that the damage conditions of MDOF structures can be predicted using SDOF damage index spectra embedded with normal mode method at a certain accurate level.

The limit state of the proposed total damage index at different performance levels are then drawn from probability theories. It gives recommended limit values of the proposed damage index from minor damage to severe collapse. The correlation between the proposed damage index and various ground motion intensity indices are also illustrated.

Finally, applications of the proposed damage assessment model in structural modification and active control are demonstrated.

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LIST OF SYMBOLS

A	Continuous time state space matrix
ABS	Maximum absolute response method
ADOOF	Alternative degree of freedom
$\mathbf{a}(t)$	Continuous time earthquake ground acceleration vector in 6 directions
$\mathbf{a}_k$	Discrete time earthquake ground acceleration vector in 6 directions
a_{rms}	Root mean square acceleration
a_{rs}	Root of the integral of the squared acceleration
B	$2n \times p$ control force transition matrix
C	Damping matrix
$\mathbf{C}_{mn}$	Damping sub-matrix corresponds to ODOF
CDF	Cumulative density function
CE	Control energy
CQC	Complete quadratic combination response method
C1-C6	Control actuators
$\mathbf{C}_{1(OLC)}, \mathbf{C}_{2(OLC)}$	$p \times n$ constant feedback control matrices
$\hat{\mathbf{c}}_i$	Uncoupled damping matrix in the i th mode
D	Control force distribution matrix
$\mathbf{D}_a$	Control force distribution matrix corresponds to ADOF
$\mathbf{D}_n$	Control force distribution matrix corresponds to ODOF
$\overline{\mathbf{D}}_n$	Reduced $\mathbf{D}_n$ matrix
D_y	Yield displacement of structure
D_{yi}	Yield displacement in the i th mode
DDOF	Displacement degree of freedom
DE	Damping energy
DI	Damage index

$DI(i)$	Damage index in the i th mode
DI_{BB1}, DI_{BB2}	Bozorgnia and Bertero improved damage indices
DI_i	Local damage index at i th PHL
DI_K	Stiffness damage index
DI_K^i	Stiffness damage index of the i th storey
$DI_{\max}$	Maximum values of DI
DI_{PA}	Park and Ang' local damage index
DI_{Total}	Total damage index
DI_{μ}	Powell and Allahabadi damage index
DR	Drift ratio
$DR_{\max}$	Maximum values of DR
$DR_{roof}(t)$	Roof drift ratio time history
DOF	Degree of freedom
d_1, d_2	Damage Parameters in Banon and Veneziano damage index
d_{rms}	Root mean square displacement
d_{rs}	Root of the integral of the squared displacement
dE	Incremental absorbed hysteretic energy
E_a	Integral of the squared acceleration
$E_c(u)$	Hysteretic energy per loading cycle at deformation u
E_d	Integral of the squared displacement
E_h	Plastic strain energy or hysteretic energy
E_{hmon}	Hysteretic energy capacity under monotonically increasing lateral deformation
E_v	Integral of the squared velocity
ELC	1940 El-Centro earthquake
$e^{\Delta t}$	$2n \times 2n$ state transition matrix
$\mathbf{F}_a(t)$	Force vector applied to cancel restoring forces
$\mathbf{F}_p$	$2n \times n$ discrete time inelastic transition matrix

$\mathbf{F}_p^c$	$2n \times n$ inelastic transition matrix
$\mathbf{F}_{RF}(t)$	Restoring force vector
$\mathbf{F}_s$	$2n \times 2n$ discrete time state transition matrix
$F_s(t)$	Force in spring at any time
F_y	Yield force
FMS	Fundamental mode shape response method
$\mathbf{f}_c(t)$	Control force vector
$\mathbf{f}_{c,k}$	Discrete time control force vector
$\mathbf{G}$	$2n \times n$ discrete time control force transition matrix
g	Acceleration of gravity
$\mathbf{H}$	$2n \times 6$ continuous time ground acceleration transition matrix
H	Height of the structure
$\mathbf{H}_d$	$2n \times 6$ discrete time ground acceleration transition matrix
H_k	Hamiltonian function
HE	Hysteretic energy
HE_n	Energy dissipated under the negative force portion of the hysteretic loop
HE_p	Energy dissipated under the positive force portion of the hysteretic loop
HE_t	Total hysteretic energy
$\mathbf{h}$	Earthquake distribution matrix
$\mathbf{I}$	Identity matrix
$I_A(\zeta)$	Arias' earthquake intensity
I_c	Characteristic intensity
I_F	Fajfar's intensity index
ID	Interstorey drift
IE	Input energy
IE'	Relative input energy
$IE_{\max}$	Maximum value of input energy

$IE(i)_{\max}$	Maximum value of input energy in the i <u>th</u> mode
ISA	Inelastic spectral analysis
ITHA	Inelastic time history analysis
J	Optimal linear control cost function
$\mathbf{K}$	Structure stiffness matrix
$\mathbf{K}_{nn}, \mathbf{K}_{na}, \mathbf{K}_{an}, \mathbf{K}_{aa}$	Global stiffness sub-matrices
$\bar{\mathbf{K}}_{nn}$	Reduced global stiffness matrix
$\mathbf{K}_p$	Matrix that relates plastic rotation with applied force
$\bar{\mathbf{K}}_p$	Reduced $\mathbf{K}_p$ matrix
$\mathbf{K}_R$	Matrix that relates plastic rotation with moments
$\bar{\mathbf{K}}_R$	Reduced $\mathbf{K}_R$ matrix
$\bar{\mathbf{K}}_D$	Modified $\mathbf{D}_a$ matrix
K_{final}	Final stiffness of structures after cycles of loading
K_{final}^i	Final stiffness of the i <u>th</u> storey after cycles of loading
$K_{initial}$	Initial stiffness of structures
$K_{initial}^i$	Initial stiffness of the i <u>th</u> storey
KE	Kinetic energy
$k, k+1$	Time step
k_e	Initial stiffness
$\hat{k}_i$	Uncoupled stiffness matrix in the i <u>th</u> mode
k_t	Post-yield stiffness
L	Lagrange function
l	The number of earthquake acceleration components
$\mathbf{M}$	Mass matrix
$\mathbf{M}_{nn}$	Mass sub-matrix corresponds to ODOF
$\mathbf{M}(t)$	Total moment vector
$\mathbf{M}'(t)$	Elastic moment vector
$\mathbf{M}''(t)$	Residual moment vector

$\mathbf{M}_{EQ}^+(t)$	Positive moment capacities reserved for earthquake
$\mathbf{M}_{EQ}^-(t)$	Negative moment capacities reserved for earthquake
$\mathbf{M}_p(t)$	Moment vector at potential PRDOF due to $\mathbf{F}_a(t)$
$\mathbf{M}_{RF}(t)$	Restoring moment vector
$\mathbf{M}_k$	Discrete time total moment vector
M_i	Total moment at <u>i</u> th PHL
M'_i	Elastic moment at <u>i</u> th PHL
M''_i	Inelastic moment at <u>i</u> th PHL
m	The number of PHLs
$\hat{m}_i$	Uncoupled mass matrix in the <u>i</u> th mode
N	The number of total DOFs
NOR	1994 Northridge earthquake
n	The number of ODOFs
n_a	The number of ADOFs
ODOF	Original degree of freedom
OLC	Optimal linear control
$\mathbf{P}$	Steady-state Riccati matrix
P	Earthquake power index
P_a	Mean-square acceleration
P_D	Earthquake potential destructiveness
P_d	Mean-square displacement
P_v	Mean-square velocity
PDF	Probability density function
PE	Plastic energy
$PE_{\max}$	Maximum value of plastic energy
$PE(i)_{\max}$	Maximum value of plastic energy in the <u>i</u> th mode
PE_i	Plastic energy at <u>i</u> th PHL
$(PE_i)_{\max}$	Maximum value of plastic energy at <u>i</u> th PHL

PGA	Peak ground acceleration time history
PGV	Peak ground velocity time history
PGD	Peak ground displacement time history
PHL	Plastic hinge location
POF	Probability of failure
PRDOF	Plastic rotation degree of freedom
p	Number of control actuators
$\mathbf{Q}$	$2n \times 2n$ positive semi-definite weighting matrix in the cost function
$\mathbf{Q}(t)$	Modal displacement vector
$\dot{\mathbf{Q}}(t)$	Modal velocity vector
$\ddot{\mathbf{Q}}(t)$	Modal acceleration vector
$\mathbf{Q}''(t)$	Modal inelastic displacement vector
Q_y	Yield strength
$q_i(t)$	i th modal displacement
$\dot{q}_i(t)$	i th modal velocity
$\ddot{q}_i(t)$	i th modal acceleration
$q_i'(t)$	i th modal elastic displacement
$\mathbf{R}$	$p \times p$ positive definite weighting matrix in the cost function
R_s	Over strength factor
R_μ	Ductility-related force reduction factor
r	Optimal linear control parameter
S_a	Spectral acceleration
S_{DI}	Spectral damage index
S_{DR}	Spectral drift ratio
S_d	Spectral displacement
$S_I(\zeta)$	Housner's spectral intensity
S_{IE}	Spectral input energy
S_{PE}	Spectral plastic energy

S_v	Spectral velocity
S_{vp}	Pseudo spectral velocity
SE	Strain energy
SMRF	Steel moment resisting frame
SR	Slope ratio
SRSS	Square root of the sum of the square response method
T	Time according to $X_{roof}^{''+}$ or $X_{roof}^{''-}$ which one is used in the proposed local damage index
$T(i)$	Natural period of vibration in the i th mode
T_n	Undamped natural period of vibration
t_0	Starting time
t_5	The time at which 5% of $I_A(\zeta)$ occurs
t_{95}	The time at which 95% of $I_A(\zeta)$ occurs
t_d	Strong motion duration
t_f	Total duration of earthquake time histories
t_k	Time at step k
t_{k+1}	Time at step $k+1$
UNC	Uncontrolled condition
$u_{elastic}$	Maximum elastic portion of deformation
u_{max}	Maximum deformation
u_{mon}	Ultimate deformation capacity under monotonic loading
u_y	Yield deformation
V_y	Lateral yield force
v_0	The number of zero-crossings per unit of time of the accelerogram
v_e	Equivalent velocity
v_{rms}	Root mean square velocity
v_{rs}	Root of the integral of the squared velocity

$\mathbf{X}(t)$	Total displacement vector
$\dot{\mathbf{X}}(t)$	Relative velocity vector
$\ddot{\mathbf{X}}(t)$	Relative acceleration vector
$\mathbf{X}'(t)$	Elastic displacement vector
$\mathbf{X}''(t)$	Inelastic displacement vector
$\mathbf{X}_a(t)$	Displacement vector corresponds to ADOF
$\dot{\mathbf{X}}_a(t)$	Velocity vector corresponds to ADOF
$\ddot{\mathbf{X}}_a(t)$	Structural relative acceleration vector corresponds to ADOF
$\mathbf{X}'_a(t)$	Elastic displacement vector corresponds to ADOF
$\mathbf{X}''_a(t)$	Inelastic displacement vector corresponds to ADOF
$\mathbf{X}_n(t)$	Displacement vector corresponds to ODOF
$\dot{\mathbf{X}}_n(t)$	Velocity vector corresponds to ODOF
$\ddot{\mathbf{X}}_n(t)$	Structural relative acceleration vector corresponds to ODOF
$\mathbf{X}'_n(t)$	Elastic displacement vector corresponds to ODOF
$\mathbf{X}''_n(t)$	Inelastic displacement vector corresponds to ODOF
$\mathbf{X}_g(t)$	Earthquake ground displacement vector
$\dot{\mathbf{X}}_g(t)$	Earthquake ground velocity vector
$\ddot{\mathbf{X}}_g(t)$	Earthquake ground acceleration at each ODOF
$X_{roof}(t)$	Total roof displacement
$X''_{roof}(t)$	Inelastic roof displacement
$(X_{roof}^+)_{\max}$	Maximum value of positive roof total displacement
$(X_{roof}^-)_{\max}$	Maximum value of negative roof total displacement
$(X''_{roof})^+_{\max}$	Maximum value of positive roof inelastic displacement
$(X''_{roof})^-_{\max}$	Maximum value of negative roof inelastic displacement
$X''_{roof,i,T}$	Roof inelastic displacement caused by i th PHL at time T
$X^+(i)_{\max}$	Maximum value of positive total displacement in the i th mode

$X^-(i)_{\max}$	Maximum value of negative total displacement in the i <u>th</u> mode
$X''^+(i)_{\max}$	Maximum value of positive inelastic displacement in the i <u>th</u> mode
$X''^-(i)_{\max}$	Maximum value of negative inelastic displacement in the i <u>th</u> mode
$\mathbf{X}_k$	Discrete time total displacement vector
$\dot{\mathbf{X}}_k$	Discrete time relative velocity vector
$\ddot{\mathbf{X}}_k$	Discrete time relative acceleration vector
$\mathbf{X}'_k$	Discrete time elastic displacement vector
$\mathbf{X}''_k$	Discrete time inelastic displacement vector
$\mathbf{X}_{a,k}$	Discrete time total displacement vector at ADOF
$\mathbf{X}'_{a,k}$	Discrete time elastic displacement vector at ADOF
$\mathbf{X}''_{a,k}$	Discrete time inelastic displacement vector at ADOF
$\mathbf{X}_{n,k}$	Discrete time total displacement vector at ODOF
$\mathbf{X}'_{n,k}$	Discrete time elastic displacement vector at ODOF
$\mathbf{X}''_{n,k}$	Discrete time inelastic displacement vector at ODOF
$(\mathbf{X}''_{n,k})_i$	Discrete time inelastic displacement vector at ODOF due to the plastic rotation at i <u>th</u> PHL
$x(t)$	Total displacement
$x'(t)$	Elastic displacement
$x''(t)$	Inelastic displacement
$\dot{x}(t)$	Relative velocity time history
$\ddot{x}_g(t)$	Earthquake ground acceleration time history
$(\ddot{x}_g)_{\max}$	Peak ground acceleration time history
$x_{roof}(t)$	Roof total displacement time history
$\mathbf{Y}(t)$	Absolute displacement vector
$\dot{\mathbf{Y}}(t)$	Absolute velocity vector

$\ddot{\mathbf{Y}}(t)$	Absolute acceleration vector
$\dot{\mathbf{Y}}_n(t)$	Absolute velocity vector corresponds to ODOF
$\ddot{\mathbf{Y}}_n(t)$	Absolute acceleration vector corresponds to ODOF
$\mathbf{z}(t)$	State variable vector
$\dot{\mathbf{z}}(t)$	Differential of state variable vector
$\mathbf{z}_k$	Discrete time state variable vector
$\Theta''(t)$	Plastic rotation vector
Θ''_k	Discrete time plastic rotation vector
$\Delta\Theta''$	Incremental form of plastic rotation vector
$\theta''_i(t)$	Continuous time plastic rotation at <u>i</u> th PHL
$\theta''_{i,k}$	Discrete time plastic rotation at <u>i</u> th PHL
$\Delta\theta''_i$	Incremental form of plastic rotation at <u>i</u> th PHL
$\mathbf{0}$	Zero matrix
Δt	Time step size
α_1, α_2	Parameters in Bozorgnia and Bertero damage indices
α, β	Weighting parameters in proposed total damage index and local damage index
α^*	Plastic strain energy to total input energy ratio
β^*	Strength deterioration parameter
$\bar{\alpha}, \bar{\beta}$	Non-negative parameters in Pak and Ang local damage index
ω_n	Structural undamped natural frequency
ζ	Damping ratio
ζ_i	Damping ratio in the <u>i</u> th mode
σ	Parameter in PDF
σ_y	Yield stress
ε_y	Yield strain
γ	Reduction factor of the deformation capacity of structures due to low-cycle fatigue

λ_k	Lagrange multiplier
μ	Displacement ductility ratio
μ_c	Cyclic displacement ductility ratio
μ_e	Elastic ductility ratio
μ_{mon}	Monotonic displacement ductility ratio
μ_X	Mean of PDF
Φ	Modal matrix with the collection of mode shapes
Γ_i	<u>i</u> th modal participation factor
Ω_a	Elastic response acceleration amplification factor

CHAPTER ONE

INTRODUCTION

1.1 Background of This Research

There is renewed interest in damage analysis for applications in the seismic assessment, rehabilitation of existing buildings and performance-based engineering design. The idea of describing the state of damage in the structure by one number on a defined scale in the form of a damage assessment model is attractive because of its simplicity. However, its development is complex since such kind of study requires great background knowledge and comprehensive understanding of analytic tools, material characteristics, experimental methods, index quantification, practical implementation, and etc. To date, although several structural damage assessment models based on different conceptual assumptions have been proposed, many of them can only be met with some parts of satisfaction. As a result, there is still a very big space left to explore.

Along with evaluating seismic damage only from characterization of ground motions, most of the damage assessment approaches need a good inspection of the structures themselves. Among them, most of the approaches are either based on the comparison

between the vibrational characteristics of a structure before and after a damaging event or based on the comparison between capacities and demand of components imposed by ground motions. Since there exist great uncertainties in both the demand and capacity predictions, limit state method, which only involves uncertainties in demand prediction, is more preferable in this research. Besides, with the rapid popularization of high-speed computers, inelastic dynamic analysis should be one of the most reasonable solutions to earthquake engineering problems due to its advantages of more directly, more accurate and less confusion over elastic static analysis and pushover analysis, therefore it is adopted here.

In structural design, most commonly used seismic damage assessment models are mainly from response point of view, such as ductility ratio and drift ratio. However, researchers who have developed seismic damage assessment models recently believe that energy is a better parameter, since earthquake effects on structural vibration can be considered as energy input. Therefore, assessing seismically-excited damage in structures using energy approach should be more reasonable than those only considering response aspects due to its efficient evaluation of cumulative effects of repeated cycles of inelastic responses. Considering the real seismically-excited structural damage, most of the damage is a combination of moderate deformation and moderate number of cyclic reversals instead of a single large deformation or a single large cumulative energy alone. As a result, developing a damage assessment model, which can represent the damaging effects in both response and energy points of view, as well as avoiding the complicated procedures in element capacity prediction, should be the most attractive. This is exactly the motivation of this study. A clear description of the position of this research on structural seismic damage assessment is shown in Figure 1-1.

In this research, a seismic damage assessment model as afore-mentioned, which is on the basis of both inelastic structural dynamic response and plastic energy dissipation, is proposed to meet the practical requirements for structural damage assessment of steel frames in earthquake engineering.

Figure 1-1 Position of the current research in seismic damage Assessment

1.2 Main Contribution

It is difficult if not impossible to define a universal damage parameter for structures of different types, members and materials under different loading condition because the damage in such structures may follow different damage mechanisms. The work in this thesis is limited to the damage assessment of steel frame structures subjected to earthquakes. The proposed seismic damage assessment model is referred as *damage potential*, which is mainly for design purpose. The main contribution of this research includes:

- A computation method is derived to characterize quantitatively the energy and transfer among various energy forms in multi-degree-of-freedom (MDOF) structures during earthquakes. By modifying the classic force analogy method in which the mass matrix is invertible with static condensation, the method can now be used to analyze real structures where rotational mass moment of inertia is often ignored.
- A damage assessment model denoted as damage potential is proposed to evaluate the structural damage in steel frame works. The main characteristics of this model are as follows:
 1. The damage potential model consists of inelastic information using both displacement and energy, therefore it accounts for the damage effects of both excessive and repeated inelastic deformations.
 2. The damage potential model is calculated from inelastic time history analysis, which is only based on seismic demand prediction. By setting the limit state values, damage can be assessed without capacity prediction, which avoids the great uncertainties in the member capacity estimation.
 3. The damage potential model is constituted of local damage indices and total damage index. The former gives a clear indication of the weakest part in a structure, and the latter gives an immediate impression to what extent the damage in overall structure will suffer during an earthquake. This concept is very useful in both the structural design process and after-shock hazard mitigation.
- Energy density spectra and damage potential spectra are developed corresponding to different damping ratios, yielding levels and earthquake scalars. Embedded with normal mode method, the proposed spectra can be applied to MDOF structures and the damage conditions can be estimated in a relatively accurate extent.
- The correlation between proposed damage potential and various ground motion characteristics is studied. Structural modification and optimal linear control are also introduced into this research to give a glimpse of the prospective application of the proposed damage assessment model.

The steel special-moment-resisting frame (SMRF) is selected in this research because

it is used extensively in seismic regions for its versatility and economy. Force analogy method is adopted to perform the inelastic structural analysis, since it is based on a changing in displacement instead of stiffness. Because the state transition matrix in this method needs to be calculated only once, using initial stiffness only can save a lot of time and costs of numerical calculation.

Examples are given to illustrate the practical application of the proposed damage potential model. Results show that this damage potential model works well as a reasonable index, either in designing structures such that energies are distributed among components of the structure according to designer's objective or in applying active control to reduce or avoid structural damage.

1.3 Organization

After a general overview of this research in this chapter, the basic theory and other researcher's works are reviewed in Chapter 2. The literature review emphasizes on those that are in the same area as this research.

The basic methodology of force analogy method, state space method and static condensation procedures are introduced in Chapter 3 as a theoretical basis of inelastic structural analysis. After introducing static condensation concepts into classic force analogy method and state space method, the inelastic dynamic analysis for real moment-resisting steel frames whose rotational mass moment of inertia are often ignored is greatly simplified.

Energy balance equations for inelastic dynamic solution and the damage assessment model based on response and energy approach are proposed in Chapter 4. Through a typical MDOF frame example, the concepts of inelastic responses, various energy forms, total and local damage indices are illustrated.

Inelastic spectra analysis, parallel to the inelastic time history analysis introduced in the first four chapters, is presented in Chapter 5 as another option in dynamic analysis

of structures. The damage potential spectra is deduced and proved applicable to MDOF frames using normal mode method. Numerical cases with different series of combination of structural parameters are studied.

Probability study is firstly performed in Chapter 6 to show the advantages of using the proposed damage potential over drift ratio in structural damage evaluation. Recommended limit values of the proposed damage potential at different performance levels are then derived. The correlation between the proposed damage potential and various ground motion intensity indices are investigated too.

In addition to damage assessment, applications of the proposed damage potential in design aspects are presented in Chapter 7. Both structural modification and optimal linear control procedures are introduced based on the proposed damage potential. Comprehensive results from numerical examples are given to show the prospective application of the proposed damage assessment model.

Finally, general conclusions of this research are presented in Chapter 8. Recommendations are also given for a possible future research.

CHAPTER TWO

LITERATURE REVIEW

2.1 Background of Structural Damage Assessment

It has been said that structural dynamics is the topic of 21st century in structural engineering. Due to the increasing cost of modern buildings, these buildings will be designed and constructed taller and slender in order to maximize space and minimize costs. At the same time, structural damage due to different types of loading will become more tremendous and hazardous, which can be observed from the collapse of the twin tower of the World Trade Center in the “9.11” event. Therefore, how to assess the structural damage accurately and apply modern hazard control technology in a most effective way becomes an important topic among structural engineers worldwide today.

Among various types of loading, strong ground motions are one of the most devastating environmental loading to structures. When subjected to strong earthquake ground motions, structures will generally exhibit inelastic behavior and result in damage to structural members or even collapse if such damage is excessive (Park and Ang, 1985). But certain degree of damage is generally permitted in earthquake

resistant design; otherwise an elastic structure solution will be too expensive. Therefore, suitable definition and categorization of seismic structural damage are not easy. In order to use mathematical terminology to represent seismic “damage” in structures, introducing “damage parameters” or “damage functions” relevant to the structural capacities under cyclic loads is necessary.

Most of the existing seismic damage measures to date are based on established theories in terms of displacement and forces. Ductility factor, which is the ratio of maximum displacement to the yield displacement, is traditionally used to depict the grade of nonlinearity and damage (McCabe and Hall, 1992). It is not only a basis for constructing response spectra based on prescribed levels of nonlinear maximum response but also an important parameter to construct inelastic design spectra from elastic design spectra (Newmark and Hall, 1982). Ductility ratio works well for damage evaluation under monotonic loading because it represents the maximum response, or envelope, of the demand on the structure. However, it has been widely recognized that the level of structural damage due to earthquakes does not depend only on the maximum displacement, and the results from numbers of inelastic cycles should also be taken into consideration. Some researchers took into account the time-history process to interpret the progressive damage (Chai et al., 1998), while others constructed damage indices by incorporating energy in plastic elements (Pinelli et al., 1996).

Recent approaches have suggested the use of earthquake input energy as an alternative to the ductility-based seismic design. Since the earthquake input energy is determined through the integration of nonlinear structural response over the duration of the ground motion, the approach is capable of accounting for all the inelastic cycles that can be sustained by the structures during strong ground motions. By considering its capacity to deal explicitly with the cumulative effect, energy can logically be an alternative parameter along with the ground acceleration and structural capacities such as strength and ductility. But due to the difficulty in evaluating the different energy forms during the energy transferring process, most of the research works were done only on a qualitative level and not on a quantitative level. Difficulty also lies in

developing a comprehensive damage description representative of actual structural behavior, and at the same time developing measures that are practical and simple to use in engineering applications. Though improvement in the estimation of input and hysteretic energy demands on structures has been achieved in recent research, none has ever included these updating results in structural damage assessment. Therefore, in order to apply the concept of energy in practical damage assessment and hazard mitigation, representing and evaluating the energy dissipation in a quantitative level is necessary. Besides, a new seismic structural damage model based on a more comprehensive understanding of energy and damage is worthy of developing.

2.2 Summary of Various Damage Assessment Methods

As previously illustrated, much work has already been done in damage assessment area, but the research results still cannot meet the requirements of structural engineers because there is no general agreement on what parameters should be used in assessing structural damage. To develop more effective damage assessment criteria, a good understanding in both earthquake engineering and structural dynamics is necessary in order to achieve good results. The following sub-sections provide a good overview of the previous works in this field that is related to the present study.

2.2.1 Damage Assessment with Spectra

The design spectrum approach is the most commonly used method of analysis in earthquake engineering. Elastic design spectra are often given in aseismic codes or in the case of important structures are determined by a seismic hazard analysis at a specific site. However, how to constitute inelastic spectra has been one of the research preoccupations of many researchers for the past 20 years. Two main different approaches have been investigated: (a) reduction of the elastic spectra through the use of reduction factors, and (b) direct derivation of inelastic spectra through statistical studies of the response spectra obtained by the nonlinear dynamic analysis of structures subjected to available ground motions. Whichever approach it is, since the

time when response spectrum method of analysis had been extended to include studying the structural response in the inelastic range, many explorations has been made such that these spectra not only provide information about the design forces and maximum inelastic displacements but also about the associated damage. According to the opinion of Vidic et al. (1994), strength, displacement, input and hysteretic energy spectra represents the basic spectra which define seismic demand in earthquake-resistant design.

Mahin and Bertero (1981) carried out time-history analysis to obtain inelastic spectra and studied the effects of various governing factors on these spectra. Fajfar (1992) proposed equivalent ductility factors based on damage due to low-cycle fatigue and used these factors for constructing the inelastic spectra. Vidic et al. (1994) proposed strength and displacement compatible spectra by carrying out parametric studies for several ground motions. Fajfar and Vidic (1994) incorporated the parameters of input and hysteretic energy in estimating the inelastic spectra for strength and displacement. Chai et al. (1998) developed a duration-dependent inelastic spectrum which assumes that the lateral strength of the structure can be increased to compensate for the increased number of inelastic cycles that may be imposed on the structure under a long duration ground motion. Their study has shown that for a set of nonlinear spectra, the ductility demand and the number of yield events (equal to the number of inelastic excursions) are quite sensitive to the parametric variations as well as to the type of nonlinear oscillator under consideration.

Mostaghel and Hernried (1986) proposed bounds on the inelastic spectral values but their results were conservative for the ranges of natural period of an elasto-plastic oscillator. Miranda (1993) studied the effects of different sites and soil conditions on the inelastic spectra. Murakami and Penzien (1977) presented constant strength probability inelastic spectra based on artificially generated accelerograms. Iwan (1980) proposed relations to obtain the inelastic spectra from the elastic spectra by using the equivalent linearization theory. Cosenza et al. (1993) used different damage models to obtain collapse spectra (i.e. the spectra corresponding to the complete collapse) and carried out a comparison between the corresponding damage functions. Several other

researchers have also contributed significantly to the concept of damage-based inelastic spectra. However all of these studies have seldom explicitly considered the cumulative damage occurring from the repeated inelastic excursions.

Basu and Gupta (1995) proposed a probabilistic model to estimate the damage in a structure with a given amount of ductility by using the order statistics of the higher-order peaks. Damage was assumed to accumulate in cyclic application of large strains in structures, and a failure law was developed accordingly. In 1996, they proposed another stochastic damage-dependent inelastic spectrum (Basu and Gupta, 1996). The uniform inelastic spectra in their study were considered to consist of a set of curves representing the variation of linear design forces with equivalent period of a nonlinear oscillator. Each curve was obtained for a uniform value of maximum inelastic displacement, cumulative damage, damping ratio, or the duration of the ground motion. The cumulative damage was normalized with respect to the total cumulative damage that occurs when all the displacement peaks exceed the yield displacement level. These approaches are useful in the design of structural systems with better balance and control, keeping in view several practical factors such as the frequency of earthquakes, importance of the structure and the economical losses associated with the structural failure.

Recently, the concept of damage spectra has been introduced to quantify the damage potential of recorded earthquake ground motion. Damage spectra represents variation of a damage index versus structural period for a series of inelastic single-degree-of-freedom systems subjected to a recorded ground motion. The damage spectra proposed by Bozorgnia and Bertero (2001, 2003) are based on a combination of normalized hysteretic energy and deformation ductility. This method is promising not only for assessment of the performance-based seismic vulnerability of existing structures but also for carrying out performance-based design of new structures. With respect to damage control, much effort has been put to improve the response spectra by incorporating elements of cyclic loading to limit deformation or displacement. In this respect, these research works are deformation-based damage assessment.

The followings are some recommendation on damage evaluation proposed by various researchers on the concept of damage indices.

2.2.2 Damage Assessment with Damage Indices

Damage indices, established analytical relationships between the maximum and/or cumulative response of structural and nonstructural components and the level of damage they exhibit, have been developed and quantified in recent years. The idea of describing the state of structural damage by a single number on a defined scale in the form of damage index is very attractive because of its simplicity. However, developing such a scale is generally very complex since the index should be applicable to structural systems at different stages of inelastic deformation up to collapse. The damage state of a structure can be defined in many ways, and two of which are summarized: (1) a binary damage state (failure/no failure) and (2) a discrete valued damage state using qualitative indicators such as none, minor, repairable, severe, and failure. Empirical and theoretical approaches have been applied to yield various estimates of structural damage (Ghobarah et al., 1999).

The empirical damage models are based on statistics of observed structural damage due to earthquakes. Though they provide useful qualitative information on the overall structural performance during earthquakes, they do not lend themselves well to rationally predicting the reserved strength and response characteristics of a structure with a specified degree of damage. The analytical damage models can be addressed from two alternative yet equivalent standpoints: the strength-based and response/energy-based, each of which has its own advantages and disadvantages. Shiga et al. (1968) first proposed the strength-based assessment of structures. These indices depend on the geometry of structural elements and their general material properties. In the prevalent design practice, it is common to reduce the linear design force during a severe earthquake shaking. However, although structures can be designed to resist severe earthquakes, it is not feasible economically to design buildings to elastically withstand earthquakes of the greatest foreseeable intensity. In

modern design procedures, damage analysis is usually based on response/energy-based damage models.

The response/energy-based damage indices can be divided into three groups according to what the index accounts for: (1) maximum deformation; (2) cumulative damage; and (3) both maximum deformation and cumulative damage.

2.2.2.1 Damage indices based on maximum deformation

Structural performance is usually gauged in terms of deformation demands being limited within certain defined levels. Here, “deformation” generally refers to both global and local responses of the structure (Bazzurro et al., 1998).

(1) Ductility Ratio

The level of inelastic deformation expected in a structure under the design earthquake is often characterized by a displacement ductility ratio μ , which is defined as $\mu = u_{\max} / u_y$, where $u_{\max}$ and u_y are the maximum and yield deformation, respectively. It has been used extensively in seismic analysis to evaluate the capacity of structures undergoing inelastic deformation and develop inelastic response spectra, but it does not account for the effect of the duration and frequency content of the ground motion.

(2) Powell and Allahabadi Index

Based on plastic deformation, Powell and Allahabadi (1988) composed their damage index from the definition of ductility ratio. The damage index was defined as:

$$DI_{\mu} = (u_{\max} - u_y) / (u_{\text{mon}} - u_y) = (\mu - 1) / (\mu_{\text{mon}} - 1) \quad (2-1)$$

where u_{mon} is the maximum deformation capacity of the system under a monotonically increasing lateral deformation, and $\mu_{\text{mon}} = u_{\text{mon}} / u_y$ is the monotonic ductility capacity. Since it is also based on ductility ratio, it cannot reveal information on the repeated cycles of inelastic deformation and energy dissipation demand.

(3) Interstorey Drift

The interstorey drift (*ID*) is the maximum relative displacement between two stories normalized to the storey height. According to Sozen (1981), the percentage of damage is given by:

$$\% \text{ of damage} = 50 \times (\text{maximum interstorey drift in percentage}) - 25 \quad (2-2)$$

From the analysis of test data on components and small-scale structures, it was found that an *ID* value smaller than 1% corresponds to damage of non-structural components while values of *ID* larger than 4% might result in irreparable structural damage or collapse. Collapse is considered to occur when *ID* exceeds 6%. Similarly, the interstorey drift does not account for effects of cumulative damage due to repeated inelastic deformation either.

(4) Slope Ratio

The slope ratio (*SR*) is a measure of damage due to stiffness degradation during seismic loading. It is defined as the ratio of the slope of loading branch to the slope of unloading branch in the force-displacement diagram. From tests on small-scale structural systems, *SR* with values of 1.0 and 0.2 has been determined correspond to safe structural behavior and critically damaged structures, respectively (Toussi and Yao, 1982).

(5) Flexural Damage Ratio

Roufaiel and Meyer (1981) suggested that the ratio of initial stiffness to the reduced secant stiffness at the maximum displacement could be used as a measure of damage. It does not account for effects of cumulative damage caused by repeated load reversals. Besides, their use in the prediction of seismic damage for structures with characteristics significantly different from those used in the critical values calibration process requires caution.

(6) Maximum Permanent Drift

Both Toussi and Yao (1982) and Stephens and Yao (1987) introduced a qualitative classification of damage, which defined four levels of structural damage as: (1) *Safe* with storey drift not exceeding 0.1% of the storey height and no permanent drift;

(2) *Lightly damaged* with permanent displacement of approximately 0.5% of the storey height; (3) *Damaged* with permanent displacement of 1% of the storey height; and (4) *Critically damaged* with top storey displacement showing some aperiodicity at the end of the record with poor correlation between base shear and top level displacement. However, a permanent drift of 0.5% does not necessarily indicate light damage. For example, a damaged non-ductile frame structure may exhibit no permanent drift, even after severe inelastic deformation.

2.2.2.2 Damage indices based on cumulative damage

2.2.2.2.1 Cumulative damage evaluated by response modification factor

In general, response modification factor is used to include the cumulative effects. But as demonstrated by experimental studies, the damage indices that include the factor still do not account for the effect of cumulative damage due to repeated inelastic deformation (Chai et al., 1998). The following damage indices consider the effect of cumulative damage due to repeated inelastic deformation in structural members.

(1) Normalized Cumulative Rotation

A simple measure of structural deterioration is the sum of all inelastic excursions experienced by the structure. The value of this measure depends on the duration and intensity of the earthquake. The normalized cumulative rotation is defined as the ratio of the sum of plastic rotations during half cycles to yield rotation. Statistical analysis shows that damage indices based only on cumulative inelastic deformation or dissipated energy may be inadequate to characterize the complex process of damage propagation and subsequent failure in concrete members.

(2) Low Cycle Fatigue

The theory of low-cycle fatigue has been applied to the analysis of structures due to strong earthquake ground motion to estimate the state of damage (Stephens, 1985). The determination of the damage index is somewhat complex and involves the entire

response history. However, the index does not account for the effect of maximum inelastic deformation.

(3) Pushover Damage Analysis

When a reinforced concrete member is subjected to cyclic loading at constant displacement level, the member stiffness will decrease with the increase in loading cycles. To overcome the difficulties in the change in period as measure of damage, Ghobarah et al. (1999) proposed the pushover analysis approach for determining changing stiffness of the structure. The damage analysis was conducted in three steps: (1) a static nonlinear pushover analysis; (2) the frame response to an applied earthquake; and (3) a second pushover analysis. The stiffness damage index DI_K of the total frame is defined as $DI_K = 1 - (K_{final} / K_{initial})$ where K_i represents the stiffness of the structure at the i th stage, and the stiffness damage index of the i th storey is defined as $DI_K^i = 1 - (K_{final}^i / K_{initial}^i)$. A value of zero represents no damage while a damage index value of one corresponds to collapse. The advantages of this index include:

- The calculation is based on rational response analysis with unnecessary calibration.
- The damage can be calculated at any stage of loading without guessing of any maximum displacement.
- The information regarding the element and storey damage and the sequence of element damage and failure can be obtained.
- The storey and global damage indices can be obtained without an averaging or weighting procedure.
- The modeling damage due to mechanisms other than flexural yielding can be attained.

The disadvantage of this method is that its application and reliability are affected by the limitation of pushover analysis technique.

2.2.2.2.2 Cumulative damage evaluated by energy

Housner (1956, 1959) had proposed a limit design type of analysis to ensure that there was sufficient energy-absorbing capacity to give an adequate factor of safety against collapse in the event of extremely strong ground motion. However, for about three decades the energy concept was ignored in earthquake-resistant design because of the apparent complexities in the quantification of energy demands and capacities and their implementation in the design process. It is only recently that energy concept has attracted extensive attention in the research community (Fajfar and Vidic, 1994).

Input energy is found to be related to the cumulative damage potential of earthquake ground motions (Bertero and Uang, 1992). In 1985, Akiyama (1985) proposed a structural design procedure based on energy concept. Over the following few years, several relevant publications have appeared, which discussed different problems related to energy concept. In 1992, two papers were discussed at the International Workshop in Bled, Slovenia, one of which was titled “Energy concepts and damage models” (Krawinkler and Fajfar, 1992). As the results of research performed in recent years, important improvements in the estimation of input and hysteretic energy demands on structures have been achieved.

(1) Qualitative Energy Investigation

Nakashima et al. (1996) performed and presented a qualitative investigation of the energy behavior of structures in which hysteretic dampers are incorporated in the structural model. However, the energy exerted on the structure and the hysteretic energy dissipated during the ground motion were taken as a form of kinetic energy, which means that they assumed the velocity rather than the deformation was critical to damage.

(2) Equivalent Ductility Factor Procedure

Fajfar and Vidic (1994) presented a procedure based on equivalent ductility factors where the reduction due to cumulative damage is controlled by two non-dimensional parameters γ and HE/IE' , where IE' is the relative input energy, HE is the hysteretic energy, and HE/IE' represents a normalization of dissipated hysteretic energy, which is influenced by damping, ductility factor and the hysteretic behavior.

The parameter γ , which controls the reduction of the deformation capacity of structures due to low-cycle fatigue, depends on the natural periods of the system, prescribed ductility factor, hysteretic behavior and ground motion characteristics. The hysteretic energy is derived from the elastic design (pseudo-acceleration) spectrum using γ and from the spectrum for the reduction factors. The relative input energy is determined directly from the elastic input energy spectrum. However, the derivation of these equations assumed completely cyclic reversal in response with equal amplitude and frequency. Furthermore, since the response process is random in nature, the ductility is also a random variable; therefore there is much uncertainty in this procedure.

(3) Hysteretic Ductility Procedure

McCabe and Hall (1989) observed that plastic ductility is responsible for damage and developed a damage index that depends on the energy dissipated under both positive and negative hysteretic loops. They first presented the equivalent hysteretic cycles, and then applied the low-cycle fatigue concept to determine the hysteretic ductility. A damage index, DI , is:

$$DI = \left(\frac{HE_p + HE_n}{HE_t} \right)^2 + \left(\frac{HE_p - HE_n}{HE_t} \right)^2 \quad (2-3)$$

where HE_p , HE_n are the energy dissipated under the positive and negative force portion of the hysteretic loop, respectively, and HE_t is the total hysteretic energy.

(4) Spectral Input Energy Procedure

Chai et al. (1998) presented a procedure for formulating the duration-dependent seismic design spectrum. They used the spectral input energy in addition to the acceleration spectra of ground motion as the seismic energy demand. In this approach, the energy damage parameter assumed that collapse occurs when the dissipated energy of structures equals to a limit value. The input energy was given by $IE = -\int_0^{t_f} m \ddot{x}_g \dot{x} dt$, where t_f is the total duration of the earthquake, m is the mass of the oscillator, $\ddot{x}_g$ is the ground acceleration time history, and $\dot{x}$ is the time history

of relative velocity response. The input energy at the end of ground motion instead of the maximum input energy was used as the parameter for cumulative damage assessment. For design purpose, the spectra input energy of a single degree of freedom (SDOF) system can be written as: $IE/m \equiv v_e^2/2$, and the equivalent velocity v_e spectrum in terms of the duration of the ground motion was adopted for design. The model of Park and Ang (1985) and later modified by Chai et al. (1995) was used to estimate the plastic strain energy capacity. Finally, the formula for calculating the plastic strain energy density was proposed as:

$$\frac{E_h}{m} = \left[\frac{\Omega_a (\ddot{x}_g)_{\max} T(1)}{2\pi R_\mu} \right]^2 \left[\mu_{mon} \left(\frac{1}{\beta^*} + 1 \right) - \frac{1}{\beta^*} \mu_c - 1 \right] \quad (2-4)$$

The duration-dependent inelastic seismic design spectra correspond to the strength-level spectra was then developed and should be divided by the over strength factor R_s . The lateral yield force is:

$$V_y = \frac{\Omega_a (\ddot{x}_g)_{\max} m}{R_\mu} \quad (2-5)$$

where Ω_a is the elastic response acceleration amplification factor, $T(1)$ is the fundamental period of structure, R_μ is the ductility-related force reduction factor, $(\ddot{x}_g)_{\max}$ is the peak ground acceleration, μ_{mon} and μ_c are monotonic and cyclic displacement ductility ratios respectively, and β^* is the strength deterioration parameter for energy-based damage model. For design, the energy capacity must be greater than energy demand, that is: $E_h/m \geq \alpha^* \times IE/m$, where α^* is the plastic strain energy to total input energy ratio. Thus, the cyclic plastic energy was related to the ground motion parameters as well as the cumulative damage characteristics of the structure. For ductility parameter, this method considers the cycles with the maximum plastic deformation while neglecting the others to evaluate structural safety, which is provided by a comparison between the maximum plastic excursion and the allowable value. At the same time, this method considers the contribution to energy dissipation of all cycles independently from the cyclic amplitude.

2.2.2.3 Damage indices accounting for both maximum deformation and cumulative damage

Although structural damage is related to the magnitude of inelastic deformation and inelastic cycles, using either factor alone as a parameter to characterize the damage potential of the ground motion is generally inadequate. This is because either factor implicitly assumes that structural damage occurs only due to the maximum deformation or the number of inelastic cycles and is independent of the other that would occur during a major earthquake. Thus damage indices considering the combination of the two factors were developed.

(1) Damage Modifier

Chung et al. (1987) proposed a damage model, which considers the difference in response of members to positive and negative moments. The effect of loading history is taken into consideration by a damage modifier, which includes the change in stiffness and bending moment sustained up to the calculation cycle. However, the damage index definition does not explicitly account for the damage caused by the maximum deformation experienced by the element.

(2) Maximum Softening

Dipasquale and Cakmak (1988) developed a damage model based on the evolution of the natural period of a time-varying linear system equivalent to the actual nonlinear system for a series of non-overlapping time windows. This global damage index depends on a combined effect of stiffness degradation and plastic deformation. However, the input ground acceleration and the acceleration at another location such as at the top of the structure is necessary in order to compute the maximum softening. In addition, this maximum softening index does not explicitly account for the dissipated hysteretic energy and strength deterioration and therefore it does not provide information concerning the extent of local damage sustained by the members.

(3) Final Softening

Dipasquale and Cakmak (1988) used the change in the fundamental period of the structure as a measure of the change in the stiffness caused by the earthquake. The

advantage of the final softening is that it can be evaluated from the initial natural period and the final period determined from vibration field-testing after the earthquake. In effect, it is not necessary to know the actual structural response. However, the final period is affected by the changes in the fundamental mode of the structure due to inelastic response, which will lead to final softening index that is no longer representative of the global stiffness deterioration. A recognized difficulty in the calculation of the final period is due to the idealization used in the analytical procedure. The period calculation at the final time step of the earthquake loading may be affected by the randomness of the instantaneous tangent stiffness at the end of the dynamic load. In addition, the stiffness at the zero load position may differ from the stiffness at the loaded positions (Ghobarah et al., 1999).

(4) Banon and Veneziano Damage Index

Since the concept of energy has been introduced to evaluate the cumulative damage, damage indices accounting for maximum deformation and cumulative damage could be defined as some combinations of ductility and energy. The analysis performed by Banon and Veneziano (1982) is set in a probabilistic context where the plastic energy is normalized with respect to the absorbed energy at the elastic limit. Damage parameters d_1 and d_2 are expressed as:

$$d_1 = \frac{u_{\max}}{u_y} = \mu, \quad d_2 = \frac{E_h}{(1/2)Q_y u_y} \quad (2-6)$$

where Q_y is the calculated yield strength. Thus, the damage function is defined as:

$$DI = \sqrt{(d_1 - 1)^2 + (ad_2^b)^2} \quad (2-7)$$

where a and b characterize the structural problem and are to be defined by experiments. For reinforced concrete elements, $a = 1.1$ and $b = 0.38$.

(5) Park and Ang Local Damage Index

Park and Ang (1985) presented a damage index based on elastic displacement and energy dissipation capacity and used it for the design of reinforced concrete structures. Until now, it has been considered the most widely used damage index in the technical literature. In this model, seismic structural damage is expressed as a linear

combination of the damage caused by excessive deformation and that contributed by repeated cyclic loading effect. This is represented in terms of a damage index:

$$DI_{PA} = \frac{u_{\max}}{u_{\text{mon}}} + \frac{\bar{\beta}}{Q_y u_{\text{mon}}} \int dE \quad \text{or} \quad DI_{PA} = \frac{u_{\max}}{u_{\text{mon}}} + \bar{\beta} \int \left(\frac{u}{u_{\text{mon}}} \right)^{\bar{\alpha}} \frac{dE}{E_c(u)} \quad (2-8)$$

where $E_c(u)$ is the hysteretic energy per loading cycle at deformation u , dE is the incremental absorbed hysteretic energy, and $\bar{\alpha}$ and $\bar{\beta}$ are non-negative parameters. The parameter $\bar{\beta}$ can be considered as the free deterioration parameter that characterizes the structural elements.

DI_{PA} has been calibrated against numerous experimental results and field observations in earthquakes (Park et al., 1985). For example, a damage index value of $DI_{PA} < 0.4$ to 0.5 has been reported as the limit of repairable damage. However, the damage index has drawbacks: the parameter $\bar{\beta}$ is not specified and has to be obtained by calibration using laboratory or field data, so this index is strongly dependent on the hysteretic model of the elements. Moreover, the damage function is not normalized; that is to say, the given expression does not provide the value 0 when $u_{\max} = u_y$ and does not provide the value 1 in the case of failure under monotonic loading (Cosenza et al., 1993).

(6) Bozorgnia and Bertero Improved Damage Indices

Due to the drawbacks of the Park and Ang's damage index, Bozorgnia and Bertero (2001) introduced two improved damage indices for a generic inelastic SDOF system. These damage indices are as follows:

$$DI_{BB1} = [(1 - \alpha_1)(\mu - \mu_e)/(\mu_{\text{mon}} - 1)] + \alpha_1(E_h / E_{h\text{mon}}) \quad (2-9)$$

$$DI_{BB2} = [(1 - \alpha_2)(\mu - \mu_e)/(\mu_{\text{mon}} - 1)] + \alpha_2(E_h / E_{h\text{mon}})^{1/2} \quad (2-10)$$

where

$$\begin{aligned} \mu_e &= u_{\text{elastic}} / u_y = \text{Maximum elastic portion of deformation}/u_y \\ &= 1 \text{ for inelastic behavior; and} \\ &= \mu \text{ if the response remains elastic} \end{aligned} \quad (2-11)$$

and E_h is hysteretic energy demanded by the earthquake ground motion, E_{hmon} is hysteretic energy capacity under monotonically increasing lateral deformation, and $0 \leq \alpha_1 \leq 1$ and $0 \leq \alpha_2 \leq 1$ are constants.

2.2.3 Global Damage Indices

Most of the above-mentioned damage indices such as ductility ratio, normalized cumulative rotation, local cycle fatigue, maximum softening as well as the popular Park and Ang's local damage index represent the local damage sustained by the individual structural elements. In order to give a clear idea of a structural system's response to a given input ground motion from a long list of element damage indices, global indices are introduced for post-earthquake evaluation of structures, reliability studies, and applications in the performance-based engineering approach. The global damage index is obtained from special combinations of local damage measures. The simplest technique for combining local damage indices is to use a weighting scheme. The weighting factor can reflect the replacement cost and/or the relative importance of the member or substructure in maintaining the integrity of the structure. Due to the integration of detailed damage information of an entire structure into a single global estimator, much information is lost thus allowing only a crude estimate of structural performance during seismic events. Both Park and Ang (1985) and Chung et al. (1987) once presented their own approach to calculate the global damage index. However, the use of weighted average procedure to calculate global damage index does not properly account for the local concentration of damage, does not distinguish between a column and a beam, and may lead to misleading results.

2.3 Further Discussion on Seismic Damage Assessment

Generally speaking, the ground shaking and response parameters which have been commonly used to quantify the damage potential of recorded earthquake ground motion can be classified into the following three categories:

(1) Parameters that are purely measures of free-field ground motions:

These include peak ground acceleration and velocity, which are amongst the most widely used parameters to measure the severity of ground shaking. However, they are totally independent of any data about the behavior and response of structural systems therefore these parameters alone have limitations in reliably identifying the structural damage due to earthquakes.

(2) Parameters that are related to elastic response of structures:

These include elastic response spectra, the spectrum intensity and so on. Although these are important measures of damage potential of ground motions, which applications have been extensive, they do not include effects of amplitude and number of cycles of inelastic structural deformations.

(3) Parameters that are related to inelastic response of structures:

These include almost all of the afore-mentioned damage indices, such as ductility ratio, drift ratio, Park and Ang's damage index as well as inelastic response/energy/damage spectra. Parameters in this category are much better reliable in evaluating seismic damage than those in the former two. But as stated above, various disadvantages do exist. Considering the popular Park and Ang's damage index for example, the determinations of parameters in this damage model are very complicated, and the ultimate deformation capacity under monotonic loading should be calculated after carrying a push over analysis case by case, which induces a lot of uncertainties in this model. Therefore, there is still much space to explore in this area.

After the 1994 Northridge earthquake, more than 100 moment resisting steel frame building in Los Angeles reported damage in the welds connecting the beams to the columns. The fundamental design concept of a moment resisting steel frame is to dissipate the energy from the earthquake through plastic rotations in the beams. However, it seems clear that for moment resisting steel frames of the type built in Southern California, this design goal was not achieved. Therefore, there is a demand for engineers to quantitatively evaluate the energy dissipation in the plastic hinge locations and develop a suitable damage index to predict the damage performance of

the welded connection in this type of structures. Since the existing damage indices seldom give information in this respect, proposing such a new damage assessment method is worthy of doing, which is sure of great significance and challenges.

CHAPTER THREE

METHODOLOGY

3.1 Introduction to Inelastic Structural Analysis

Beams, columns, walls, and braces are most commonly found in buildings, and these members will experience inelastic deformation when the building is subjected to excessive loading such as earthquakes. The current method of analyzing inelastic structural behavior is to extend the well-established knowledge gained from elastic analysis and to proceed on to inelastic analysis by varying the structural members' stiffness. Although extensive research has been done in this area for cases where the materials exhibit strain-hardening properties (see Figure 3-1(a)), it appears inefficient in analyzing structures with either strain-softening (see Figure 3-1(b)) or elastic-plastic (see Figure 3-1(c)) materials properties. This is mainly due to the problems of structures being statically unstable once there are sufficient numbers of locations in the structure that experience yielding such that no additional force can be resisted without structural collapse.

Figure 3-1 Comparison of three types of material properties

For studying the dynamic stability in structural time history analysis, either numerical differentiation using a finite-difference approach or numerical integration using a state space approach can be used. Since numerical integration in general takes advantages of higher accuracy over numerical differentiation such as Newmark β -Method or Wilson θ -Method, the state space method of numerical analysis has been gaining acceptance in engineering applications. However, a drawback of this method occurs when inverting the stiffness matrix especially if the system is large. It could be very time consuming and economic costly in calculating the state transition matrix and inverting the mass matrix, therefore using the state space method by changing the stiffness in inelastic analysis is generally inappropriate.

To overcome some of the difficulties in using the state space method of dynamic analysis, the force analogy method is incorporated to study the inelastic structural behavior using only the initial stiffness. In brief, the concept of the force analogy method is based on a change in displacement instead of a change in stiffness to give the total force. This gives a governing inelastic equation that can be written in a single

equation. There come several advantages in doing so: (1) it handles the aforementioned dynamic stability of structures in a much better way, which means it has a better capability in dealing with both the elastic-plastic properties or strain-softening properties; (2) it makes the state space numerical integration methods fit naturally into the inelastic analysis when structural dynamics is considered in order to achieve a high degree of accuracy; (3) it greatly reduces computational efforts because the state transition matrix in the state space method is based on initial stiffness and remains unchanged, and therefore it only needs to be calculated once.

Even though the force analogy method of changing displacement has its advantages, the original method (Lin, 1968) only found limited acceptance because it was developed at approximately the same time that researchers were focusing their attention on studying the deformation of solids using a finite-element method with the inelastic behavior defined by changing stiffness. The finite-element method uses force and displacement, which are first-rank tensors, while the force analogy method uses stress and strain, which are second-rank tensors. But in the algorithm stated below, the stress-strain relationship originally used in the force analogy method is transformed into force and displacement on the structure. So the comparison is brought to the same tensor level. As a result, the force analogy method may occasionally have advantages over using the finite-element method with changing stiffness. Considering its other advantages such as fulfilling the requirements for characterization of energy distribution of structures, it is adopted as a basic methodology in this research.

3.2 Force Analogy Method

The force analogy method illustrated has been demonstrated in detail by Wong and Yang (1999). Consider a SDOF system with the force-displacement relationship shown in Figure 3-2. The concept of the force analogy method is to extend the initial stiffness line OA until it reaches the force $F_s(t)$ at point B. Defining the displacement according to point B as the elastic displacement $x'(t)$, then the difference between the

total displacement $x(t)$ and the elastic displacement is the inelastic displacement $x''(t)$. Representing this concept in mathematical form gives

$$x(t) = x'(t) + x''(t) \quad (3-1)$$

$$\text{if } \begin{cases} x(t) \leq x_y \\ x(t) > x_y \end{cases} \text{ then } \begin{cases} x'(t) = F_s(t)/k_e, & x''(t) = 0 \\ x'(t) = F_s(t)/k_e, & x''(t) > 0 \end{cases} \quad (3-2)$$

Therefore, the force in the spring $F_s(t)$ at any displacement $x(t)$ is always equal to the displacement multiplied by the initial stiffness, i.e.

$$F_s(t) = k_e x'(t) = k_e (x(t) - x''(t)) \quad (3-3)$$

Note that in force analogy method, the inelastic structural response focuses on the change in displacement, not in stiffness, to give the same level of force. This is the main idea of the force analogy method.

Figure 3-2 System with bilinear force-displacement relationship

Assume that the inelastic displacement is due to inelastic deformation of structural members. For an n -DOF system, Eq. (3-1) can be written as:

$$\mathbf{X}(t) = \mathbf{X}'(t) + \mathbf{X}''(t) = \begin{Bmatrix} x'_1(t) \\ x'_2(t) \\ \vdots \\ x'_n(t) \end{Bmatrix} + \begin{Bmatrix} x''_1(t) \\ x''_2(t) \\ \vdots \\ x''_n(t) \end{Bmatrix} \quad (3-4)$$

where $\mathbf{X}(t)$ is total displacement vector; $\mathbf{X}'(t)$ is elastic displacement vector; $\mathbf{X}''(t)$ is inelastic displacement vector. Each term in the total displacement vector represents a displacement degree of freedom (DDOF) of the structure. After the inelastic displacement is defined, the elastic displacement can also be defined according to Figure 3-2.

Generally speaking, inelastic displacement is caused by any inelastic deformation of structural members at certain locations in the structure. This inelastic deformation includes nonlinear extension of the braces in a braced frame, plastic rotation of the beams in a moment resisting frame, or yielding of the base isolators in a base isolation system. Plastic rotations of the structural members in a moment-resisting structural system at the locations of plastic hinges are used in the following discussion because this type of structural system is the most commonly used around the world. A plastic rotation degree of freedom (PRDOF) is assigned to each of these plastic hinge locations (PHLs). The total moment $\mathbf{M}(t)$ at the locations where plastic hinges may form are described by the equation

$$\mathbf{M}(t) = \mathbf{M}'(t) + \mathbf{M}''(t) = \begin{Bmatrix} M'_1(t) \\ M'_2(t) \\ \vdots \\ M'_m(t) \end{Bmatrix} + \begin{Bmatrix} M''_1(t) \\ M''_2(t) \\ \vdots \\ M''_m(t) \end{Bmatrix} \quad (3-5)$$

where $\mathbf{M}'(t)$ is elastic moment vector due to elastic displacement; $\mathbf{M}''(t)$ is residual moment vector due to inelastic displacement. The value m represents the total number of potential plastic hinge locations (PHLs).

Consider first the residual moment vector $\mathbf{M}''(t)$. When plastic rotations occur at certain PHLs in the structure, these plastic rotations are replaced with a set of fictitious forces. Define the plastic rotation vector $\boldsymbol{\Theta}''(t)$ as

$$\boldsymbol{\Theta}''(t) = \begin{Bmatrix} \theta''_1(t) \\ \theta''_2(t) \\ \vdots \\ \theta''_m(t) \end{Bmatrix} \quad (3-6)$$

An example of plastic rotations in two PHLs is shown in Figure 3-3(a). As shown in Figure 3-3(a), this state of the structure can never exist because it violates the compatibility condition. To make the structure deform in a compatible way, the member with plastic rotations is first isolated from the structure and restoring forces are applied to bring this member back to the original undeformed shape, as shown in Figure 3-3(b). This induces internal restoring forces within the member. At the DDOFs, the restoring forces are:

$$\mathbf{F}_{RF}(t) = \begin{Bmatrix} F_{RF1}(t) \\ F_{RF2}(t) \\ \vdots \\ F_{RFn}(t) \end{Bmatrix} = -\mathbf{K}_p \boldsymbol{\Theta}''(t) \quad (3-7)$$

where $\mathbf{K}_p$ is the $n \times m$ matrix that relates the PRDOFs with the applied force on the DDOFs. Similarly, at all the potential PHLs, the restoring moments due to the plastic rotation are

$$\mathbf{M}_{RF}(t) = \begin{Bmatrix} M_{RF1}(t) \\ M_{RF2}(t) \\ \vdots \\ M_{RFm}(t) \end{Bmatrix} = -\mathbf{K}_R \boldsymbol{\Theta}''(t) \quad (3-8)$$

where $\mathbf{K}_R$ is the $m \times m$ matrix that relates the PRDOFs with the moments at these DOFs. The minus signs appear in Eq. (3-7) and Eq. (3-8) are because negative restoring forces must be applied to rotate a positive plastic rotation back to zero.

Figure 3-3 Force analogy method

Now the member is assembled back into the structure and the deformation of this member is compatible with that of the joint. However the restoring forces as shown in Figure 3-3(b) are actually not present globally, and as a result equal and opposite forces must be applied to the structural DDOFs to cancel these restoring forces (Figure 3-3(c)). These applied forces are described by the equation

$$\mathbf{F}_a(t) = -\mathbf{F}_{RF}(t) = \mathbf{K}_p \boldsymbol{\Theta}''(t) = \mathbf{K} \mathbf{X}''(t) \quad (3-9)$$

where $\mathbf{K}$ is the $n \times n$ global structural stiffness matrix; and $\mathbf{F}_a(t)$ is the applied force on the structural DDOFs due to plastic rotations. Through this process, the structure remains in equilibrium. Solving for inelastic displacement vector in Eq. (3-9) gives permanent deformation of the structure due to the plastic rotation, and the permanent deformation is represented by the inelastic displacement vector, i.e.,

$$\mathbf{X}''(t) = \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}''(t) \quad (3-10)$$

The moments at the potential PHLs, $\mathbf{M}_p(t)$, due to $\mathbf{F}_a(t)$ are related to the inelastic displacement field by the equation

$$\mathbf{M}_p(t) = \mathbf{K}_p^T \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}''(t) \quad (3-11)$$

Recall that $\mathbf{K}_p$ represents the relationship between plastic rotation at the PRDOFs and the applied force at the DDOFs. Therefore, from Maxwell's reciprocal relationship, it follows that $\mathbf{K}_p^T$ represents the relationship between the inelastic displacement at the DDOFs and the moments at the PRDOFs.

Finally, the residual moment vector is

$$\mathbf{M}''(t) = \mathbf{M}_{RF}(t) + \mathbf{M}_p(t) = -(\mathbf{K}_R - \mathbf{K}_p^T \mathbf{K}^{-1} \mathbf{K}_p) \boldsymbol{\Theta}''(t) \quad (3-12)$$

Equation (3-12) represents the residual moment vector due to the plastic rotations within the structure with no external applied force. For example, if an earthquake causes plastic rotations within the structure, then the residual moments represent the forces remaining in the members after the earthquake.

Elastic moments in the PRDOFs relate directly to the elastic displacement in the DDOFs through the $\mathbf{K}_p$ matrix. This relationship is:

$$\mathbf{M}'(t) = \mathbf{K}_p^T [\mathbf{X}(t) - \mathbf{X}''(t)] = \mathbf{K}_p^T [\mathbf{X}(t) - \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}''(t)] \quad (3-13)$$

Finally, substituting Eqs. (3-12) and (3-13) into Eq. (3-5), the total moment vector becomes

$$\mathbf{M}(t) = \mathbf{M}'(t) + \mathbf{M}''(t) = \mathbf{K}_p^T \mathbf{X}(t) - \mathbf{K}_R \boldsymbol{\Theta}''(t) \quad (3-14)$$

Rearranging terms gives

$$\mathbf{M}(t) + \mathbf{K}_R \boldsymbol{\Theta}''(t) = \mathbf{K}_p^T \mathbf{X}(t) \quad (3-15)$$

which is the governing equation of the force analogy method.

3.3 Inelastic Dynamic State Space Response

Based on the force analogy method developed for calculating the inelastic behavior of the structure, state space method can now be used to study the inelastic dynamic behavior of structures subjected to strong earthquakes by recognizing that the stiffness matrix multiplying the elastic displacement gives the elastic force. Consider the dynamic equilibrium equation

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}'(t) = \mathbf{M}\mathbf{h}\mathbf{a}(t) \quad (3-16)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are $n \times n$ mass, damping, and stiffness matrices respectively; $\ddot{\mathbf{X}}(t)$, $\dot{\mathbf{X}}(t)$ and $\mathbf{X}'(t)$ are $n \times 1$ relative acceleration, relative velocity and elastic displacement vectors respectively; $\mathbf{a}(t)$ is $l \times 1$ ($l \leq 6$) ground acceleration vector, where l equals to the number of earthquake acceleration components considered in the analysis. If all the six components of the ground acceleration are considered, namely $a_x(t)$, $a_y(t)$, $a_z(t)$, $a_r(t)$, $a_s(t)$ and $a_t(t)$ (where the subscripts x , y and z denote translations in the Cartesian coordinate system and r , s and t denote rotations in the same coordinate system), then

$$\mathbf{a}(t) = \begin{Bmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \\ a_r(t) \\ a_s(t) \\ a_t(t) \end{Bmatrix} \text{ and } \mathbf{h} \text{ is } n \times l \text{ earthquake distribution matrix where } \mathbf{h} = \begin{Bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_j \\ \vdots \\ \mathbf{h}_n \end{Bmatrix};$$

For $l = 6$,

$$\mathbf{h}_j = \begin{cases} [-1 & 0 & 0 & 0 & 0 & 0] & \text{if } a_j = a_x \\ [0 & -1 & 0 & 0 & 0 & 0] & \text{if } a_j = a_y \\ [0 & 0 & -1 & 0 & 0 & 0] & \text{if } a_j = a_z \\ [0 & 0 & 0 & -1 & 0 & 0] & \text{if } a_j = a_r \\ [0 & 0 & 0 & 0 & -1 & 0] & \text{if } a_j = a_s \\ [0 & 0 & 0 & 0 & 0 & -1] & \text{if } a_j = a_t \end{cases} \quad j = 1, \dots, n$$

The elastic displacement $\mathbf{X}'(t)$ in Eq. (3-16) is defined as the total force divided by the initial stiffness. Therefore, multiplying the elastic displacement by the stiffness matrix gives the total force, and this total force contributes to the balancing of forces in the equilibrium equation given by Eq. (3-16). Solving for the elastic displacement vector in Eq. (3-4) and substituting it to Eq. (3-16) gives:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{M}\mathbf{h}\mathbf{a}(t) + \mathbf{K}\mathbf{X}''(t) \quad (3-17)$$

The state space method analyzes the response of the system using both the displacement and the velocity as independent variables, and these variables are called *states*. Define the state vector $\mathbf{z}(t)$ to be:

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{X}(t) \\ \dot{\mathbf{X}}(t) \end{Bmatrix} \quad (3-18)$$

Then it follows from Eq. (3-17) that

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{H}\mathbf{a}(t) + \mathbf{F}_p \mathbf{X}''(t) \quad (3-19)$$

where

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{X}(t) \\ \dot{\mathbf{X}}(t) \end{Bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ \mathbf{h} \end{bmatrix}, \quad \mathbf{F}_p = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{K} \end{bmatrix}$$

and $\mathbf{0}$ is the zero matrix, $\mathbf{I}$ is the identity matrix,.

Equation (3-19) is a first-order linear matrix differential equation of motion, and it is called the continuous state space equation of motion, in which $\mathbf{A}$ is state space matrix and $\mathbf{H}$ is earthquake distribution matrix. In contrast, Eq. (3-17) is a second-order linear differential equation. The solution to Eq. (3-19) is

$$\mathbf{z}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{z}(t_0) + e^{\mathbf{A}t} \int_{t_0}^t e^{-\mathbf{A}s} [\mathbf{H}\mathbf{a}(s) + \mathbf{F}_p \mathbf{X}''(s)] ds \quad (3-20)$$

where e^{At} is called the state transition matrix, and it can be computed using the Taylor series expansion as follows:

$$e^{At} = \mathbf{I} + \mathbf{A}t + \frac{1}{2!}(\mathbf{A}t)^2 + \frac{1}{3!}(\mathbf{A}t)^3 + \dots \quad (3-21)$$

The earthquake ground acceleration, $\mathbf{a}(s)$, embedded in the integral of Eq. (3-20) is very irregular in time and has a wide band of frequency content. Hence, direct integration of Eq. (3-20) is practically impossible. The best solution for performing the integration in Eq. (3-20) is to use numerical discretization. Let $t_{k+1} = t$, $t_k = t_0$ and $\Delta t = t - t_0$. Then it follows from Eq. (3-20) that

$$\mathbf{z}_{k+1} = e^{A\Delta t} \mathbf{z}_k + e^{A t_{k+1}} \int_{t_k}^{t_{k+1}} e^{-As} [\mathbf{H}\mathbf{a}(s) + \mathbf{F}_p \mathbf{X}''(s)] ds \quad (3-22)$$

where $\mathbf{z}_k$ is the digitized function of $\mathbf{z}(t)$.

The objective of the numerical analysis using the integration method is to integrate the forcing function. Two methods commonly used to represent the forcing function are the Delta forcing function method and the constant forcing function method.

In the Delta forcing function method, both the earthquake ground acceleration $\mathbf{a}(s)$ and the inelastic displacement $\mathbf{X}''(s)$ are digitized using a series of Delta functions:

$$\mathbf{a}(s) = \mathbf{a}_k \delta(s - t_k) \Delta t, \quad \mathbf{X}''(s) = \mathbf{X}_k'' \delta(s - t_k) \Delta t, \quad t_k \leq s < t_{k+1} \quad (3-23)$$

where $\mathbf{a}_k$ and $\mathbf{X}_k''$ are the digitized function of $\mathbf{a}(s)$ and $\mathbf{X}''(s)$ respectively. Now substituting Eq. (3-23) into Eq. (3-22) gives

$$\mathbf{z}_{k+1} = e^{A\Delta t} \mathbf{z}_k + e^{A t_{k+1}} \mathbf{H} \Delta t \mathbf{a}_k + e^{A t_{k+1}} \mathbf{F}_p \Delta t \mathbf{X}_k'' \quad (3-24)$$

In the constant forcing function method, both the earthquake ground acceleration $\mathbf{a}(s)$ and the inelastic displacement $\mathbf{X}''(s)$ are digitized using a constant forcing function within the time interval from t_k to t_{k+1} , that is,

$$\mathbf{a}(s) = \mathbf{a}_k, \quad \mathbf{X}''(s) = \mathbf{X}_k'', \quad t_k \leq s < t_{k+1} \quad (3-25)$$

Therefore, substituting Eq. (3-25) into Eq. (3-22) gives

$$\mathbf{z}_{k+1} = e^{\mathbf{A}\Delta t} \mathbf{z}_k + \mathbf{A}^{-1} (e^{\mathbf{A}\Delta t} - \mathbf{I}) \mathbf{H} \mathbf{a}_k + \mathbf{A}^{-1} (e^{\mathbf{A}\Delta t} - \mathbf{I}) \mathbf{F}_p \mathbf{X}_k'' \quad (3-26)$$

As shown in Eq. (3-24) or Eq. (3-26), given all the information at time step k , both the displacement and velocity vectors at time step $k+1$ can be calculated. The displacement vector, $\mathbf{X}_{k+1}$, defined the total displacement at all degrees of freedom and is embedded in $\mathbf{z}_{k+1}$. The inelastic displacements at time step $k+1$ (i.e., $\mathbf{X}_{k+1}''$) can be determined using $\mathbf{X}_{k+1}$ and the force analogy method discussed in Section 3.2 above. From the substitution of Eq. (3-15),

$$\mathbf{M}_{k+1} + \mathbf{K}_R \mathbf{\Theta}_{k+1}'' = \mathbf{K}_P^T \mathbf{X}_{k+1} \quad (3-27)$$

The term $\mathbf{\Theta}_{k+1}''$, which represents the plastic rotation at each potential plastic hinge location, depend on the past history of $\mathbf{\Theta}_{k+1}''$. Let

$$\mathbf{\Theta}_{k+1}'' = \mathbf{\Theta}_k'' + \Delta \mathbf{\Theta}'' \quad (3-28)$$

Then Eq. (3-27) becomes

$$\mathbf{M}_{k+1} + \mathbf{K}_R \Delta \mathbf{\Theta}'' = \mathbf{K}_P^T \mathbf{X}_{k+1} - \mathbf{K}_R \mathbf{\Theta}_k'' \quad (3-29)$$

Using Eq. (3-29) the moments $\mathbf{M}_{k+1}$ and the change in plastic rotation $\Delta \mathbf{\Theta}''$ are solved simultaneously based on pre-defined moment versus plastic rotation relationships for each PHL. Finally, $\mathbf{X}_{k+1}''$ can be calculated using Eq. (3-10):

$$\mathbf{X}_{k+1}'' = \mathbf{K}^{-1} \mathbf{K}_P (\mathbf{\Theta}_k'' + \Delta \mathbf{\Theta}'') \quad (3-30)$$

Once these states are calculated, the relative acceleration at each DOF at time step $k+1$ can be calculated by the formula:

$$\ddot{\mathbf{X}}_{k+1} = \mathbf{h} \mathbf{a}_{k+1} - \mathbf{M}^{-1} \mathbf{C} \dot{\mathbf{X}}_{k+1} - \mathbf{M}^{-1} \mathbf{K} \mathbf{X}_{k+1} + \mathbf{M}^{-1} \mathbf{K} \mathbf{X}_{k+1}'' \quad (3-31)$$

Extensive research has been done on how to efficiently calculate the state transition matrix $e^{\mathbf{A}\Delta t}$. One method is to expand Eq. (3-21) directly with a specified time step, that is,

$$e^{\mathbf{A}\Delta t} = \mathbf{I} + \mathbf{A}\Delta t + \frac{1}{2!} (\mathbf{A}\Delta t)^2 + \frac{1}{3!} (\mathbf{A}\Delta t)^3 + \dots \quad (3-32)$$

However, the disadvantage of using this method is that the number of terms required in the solution is large if the entries in $e^{A\Delta t}$ matrix are large. Moler and Van Loan (1994) evaluated different methods of calculating the state transition matrix $e^{A\Delta t}$, and the most efficient method they recommended is the scaling and squaring method. The key to this method is the mathematical property that

$$e^{A\Delta t} = (e^{A\Delta t/m})^m \quad (3-33)$$

where m is an integer. When the matrix $A\Delta t$ is divided by m , the result is a matrix whose entries contain a maximum value of ε_m , which is usually less than the predefined limit ε_{lim} , and this ε_{lim} is usually taken to be 1.0. Consider the Taylor series expansion,

$$e^{A\Delta t/m} = \mathbf{I} + \frac{A\Delta t}{m} + \frac{1}{2!} \left(\frac{A\Delta t}{m} \right)^2 + \frac{1}{3!} \left(\frac{A\Delta t}{m} \right)^3 + \dots \quad (3-34)$$

The matrix $e^{A\Delta t/m}$ usually converges very quickly using Eq. (3-34), then the matrix $e^{A\Delta t/m}$ is multiplied by itself m times to obtain the desired result.

Recall from Eq. (3-22) that when calculating the structural response at time step t_{k+1} (i.e. $\mathbf{z}_{k+1}$) using the information at time step t_k (i.e. $\mathbf{z}_k$), the operation is exact. Only the integration of the earthquake acceleration function (i.e. $\mathbf{a}(s)$) and inelastic displacement (i.e. $\mathbf{X}''(s)$) introduces a numerical error, and this error will be small when a small time step size is used. Therefore, accurate results can be obtained when the time step is as large as $\Delta t \leq (T_n/5)$, where T_n is the undamped natural period of vibration. However, in general it is recommended that $\Delta t \leq (T_n/10)$ (Bathe, 1996; Clough and Penzien, 1982; Hart and Wong, 1999).

Because only the initial stiffness is used in the force analogy method, the state transition matrix $e^{A\Delta t}$ remains constant, and therefore it requires only one calculation, which can be done before the time history analysis begins. That is the main reason why the efficiency of the force analogy method and the accuracy of the state space method fit very naturally into one another, producing an algorithm that is both accurate and time efficient in inelastic structural dynamic analysis.

3.4 Inelastic Dynamic Response with Static Condensation

In real moment-resisting frames, joint rotations must be considered in order to capture the actual deformation behavior of the structure. However, in dynamic time history analysis of the moment-resisting frames using the force analogy method presented above, the mass matrix is considered invertible; therefore it is not quite appropriate for use in analyzing real structures where rotation mass moment of inertia is often ignored. Hence the force analogy method discussed in Section 3.2 is here known as classical force analogy method. Recognizing the complexities of the actual structural problems, one major contribution of this research is presented in this section on the modified force analogy method with static condensation to reduce the degrees of freedom without masses back to the original number of DOFs, which is n -DOFs. At the same time, modified governing dynamic equations are also rewritten in this section.

Consider a two-dimensional moment resisting frame using conventional modeling of counting all translations and rotations as degrees of freedom to give a total of N DOFs. Let n represents the number of original DOF (ODOFs) that remain after static condensation (i.e. n DOFs contain masses) and n_a represents the number of alternative DOF (ADOFs) that are to be condensed out by static condensation (i.e. n_a DOFs contain zero masses). Then the total number of DOFs becomes $N = n + n_a$. For the case where damping is ignored at the ADOFs, the governing equation of motion is given by the formula

$$\begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_n(t) \\ \ddot{\mathbf{X}}_a(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{X}}_n(t) \\ \dot{\mathbf{X}}_a(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}'_n(t) \\ \mathbf{X}'_a(t) \end{Bmatrix} = - \begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_g(t) \\ \mathbf{0} \end{Bmatrix} \quad (3-35)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{X}'$ is the elastic displacement vector, $\dot{\mathbf{X}}$ is the velocity vector, $\ddot{\mathbf{X}}$ is the acceleration vector, and $\ddot{\mathbf{X}}_g$ is the ground acceleration vector. The subscript n denotes the matrices or vectors corresponding to the ODOFs (which contains n DOFs) and the subscript a

denotes the matrices or vectors corresponding to the ADOFs (which contains n_a DOFs).

Consider the second equation of the matrix equation given in Eq. (3-35), which is

$$\mathbf{K}_{an}\mathbf{X}'_n(t) + \mathbf{K}_{aa}\mathbf{X}'_a(t) = \mathbf{0} \quad (3-36)$$

Solving for the elastic displacement vector at ADOFs, $\mathbf{X}'_a(t)$, gives

$$\mathbf{X}'_a(t) = -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an}\mathbf{X}'_n(t) \quad (3-37)$$

which represents the dependency of the elastic displacement at ADOFs on the elastic displacement at ODOFs. Now substituting Eq. (3-37) back into the first equation of Eq. (3-35) gives

$$\mathbf{M}_{nn}\ddot{\mathbf{X}}_n(t) + \mathbf{C}_{nn}\dot{\mathbf{X}}_n(t) + \bar{\mathbf{K}}_{nn}\mathbf{X}'_n(t) = -\mathbf{M}_{nn}\ddot{\mathbf{X}}_g(t) \quad (3-38)$$

where

$$\bar{\mathbf{K}}_{nn} = \mathbf{K}_{nn} - \mathbf{K}_{na}\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \quad (3-39)$$

Equation (3-38) is similar to Eq. (3-16), and the solution procedure is the same as that presented in Section 3.3. The Delta forcing function method is adopted here, where from Eq. (3-24):

$$\mathbf{z}_{k+1} = \mathbf{F}_s\mathbf{z}_k + \mathbf{H}_d\mathbf{a}_k + \mathbf{F}_p\mathbf{X}_{n,k}'' \quad (3-40)$$

where

$$\mathbf{F}_s = e^{\mathbf{A}\Delta t}, \quad \mathbf{H}_d = e^{\mathbf{A}\Delta t}\mathbf{H}\Delta t, \quad \mathbf{F}_p = e^{\mathbf{A}\Delta t}\mathbf{F}_p^c\Delta t$$

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{X}_n(t) \\ \dot{\mathbf{X}}_n(t) \end{Bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{nn}^{-1}\bar{\mathbf{K}}_{nn} & -\mathbf{M}_{nn}^{-1}\mathbf{C}_{nn} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{h} \end{bmatrix}, \quad \mathbf{F}_p^c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{nn}^{-1}\bar{\mathbf{K}}_{nn} \end{bmatrix}$$

Again, the matrix $\mathbf{H}$ contains 6 columns, where each column corresponds to the ground acceleration in 6 different directions, and $\mathbf{a}_k$ is the discretized form of a 6×1 earthquake ground acceleration vector.

Using Eq. (3-37) and that total displacement is the sum of elastic displacement and inelastic displacement given in Eq. (3-4), the displacement at all degrees of freedom $\mathbf{X}_{k+1}$ (including both ODOFs and ADOFs) can be written as:

$$\begin{aligned}
\mathbf{X}_{k+1} &= \begin{bmatrix} \mathbf{X}_{n,k+1} \\ \mathbf{X}_{a,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{n,k+1} \\ \mathbf{X}'_{a,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{n,k+1} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_{n,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{X}_{n,k+1} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} (\mathbf{X}_{n,k+1} - \mathbf{X}''_{n,k+1}) + \mathbf{X}''_{a,k+1} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{X}''_{n,k+1} \\ \mathbf{X}''_{a,k+1} \end{Bmatrix} \\
&= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{X}''_{k+1}
\end{aligned} \tag{3-41}$$

Now substituting Eq. (3-30) into Eq. (3-41) gives

$$\mathbf{X}_{k+1} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{K}^{-1} \mathbf{K}_P \boldsymbol{\Theta}''_{k+1} \tag{3-42}$$

Consider now the multiplication of the two matrices in the second term on the right hand side of Eq. (3-42). Note that:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \tag{3-43}$$

Post-multiplying Eq. (3-43) by $\mathbf{K}^{-1}$ gives

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{K}^{-1} \tag{3-44}$$

Therefore, substituting the result in Eq. (3-44) into Eq. (3-42), it follows that

$$\mathbf{X}_{k+1} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}''_{k+1} \tag{3-45}$$

Equation (3-45) gives the displacement at all the DOFs in terms of the displacement at ODOFs and the plastic rotations at all the plastic hinges. Substituting Eq. (3-45) into the governing equation of the force analogy method presented in Eq. (3-29) and simplifying gives

$$\mathbf{M}_{k+1} + \bar{\mathbf{K}}_R \Delta \boldsymbol{\Theta}'' = \bar{\mathbf{K}}_P^T \mathbf{X}_{n,k+1} - \bar{\mathbf{K}}_R \boldsymbol{\Theta}''_k \tag{3-46}$$

where

$$\bar{\mathbf{K}}_R = \mathbf{K}_R - \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \tag{3-47}$$

$$\bar{\mathbf{K}}_P^T = \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \quad (3-48)$$

Equation (3-46) is the modification to Eq. (3-29) after static condensation is performed.

Similar procedure can be performed on Eq. (3-30) to obtain a new condensed equation. Pre-multiplying both sides of Eq. (3-30) by $\mathbf{K}$ and pre-multiplying the equation again by the transpose of the bracket term in Eq. (3-48) gives

$$\begin{bmatrix} \mathbf{I} & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_{n,k+1}'' \\ \mathbf{X}_{a,k+1}'' \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_{k+1}'' \quad (3-49)$$

The multiplication of the first two stiffness matrix terms on the right hand side of Eq. (3-49) gives exactly $\bar{\mathbf{K}}_P$. Simplifying the matrix on the left hand side of Eq. (3-49) gives

$$\begin{aligned} \bar{\mathbf{K}}_P \boldsymbol{\Theta}_{k+1}'' &= \begin{bmatrix} \mathbf{I} & \vdots & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_{n,k+1}'' \\ \mathbf{X}_{a,k+1}'' \end{Bmatrix} \\ &= \begin{bmatrix} \mathbf{K}_{nn} - \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \vdots & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_{n,k+1}'' \\ \mathbf{X}_{a,k+1}'' \end{Bmatrix} \\ &= \begin{bmatrix} \mathbf{K}_{nn} - \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}'' \\ &= \bar{\mathbf{K}}_{nn} \mathbf{X}_{n,k+1}'' \end{aligned} \quad (3-50)$$

Finally multiplying both sides of Eq. (3-50) by $\bar{\mathbf{K}}_{nn}^{-1}$ gives

$$\mathbf{X}_{n,k+1}'' = \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P (\boldsymbol{\Theta}_k'' + \Delta \boldsymbol{\Theta}'') \quad (3-51)$$

Equation (3-51) gives the effect of inelastic displacement at each ODOF resulted from the deformation of all the PHLs. Thus, the contribution of the plastic rotation at the i th PHL to the overall inelastic displacement at each ODOF can be written as

$$\left(\mathbf{X}_{n,k+1}'' \right)_i = \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \begin{pmatrix} 0 \\ \vdots \\ \theta_{i,k}'' + \Delta \theta_i'' \\ \vdots \\ 0 \end{pmatrix} \quad i = 1, \dots, m \quad (3-52)$$

and that the overall inelastic displacement at each ODOF can then be computed as

$$\mathbf{X}_{n,k+1}'' = \sum_{i=1}^m (\mathbf{X}_{n,k+1}'')_i \quad (3-53)$$

This concept will be used in establishing seismic damage model in the following chapter.

Both Eqs. (3-12) and (3-13) can also be modified in a similar manner using the currently proposed modified force analogy method with static condensation. Consider first the inelastic displacement $\mathbf{X}''(t)$ by rearranging the terms in Eq. (3-4). Then substituting Eqs. (3-37) and (3-45) in the inelastic displacement equation gives:

$$\begin{aligned} \mathbf{X}_k'' &= \mathbf{X}_k - \mathbf{X}_k' = \begin{bmatrix} \mathbf{X}_{n,k} \\ \mathbf{X}_{a,k} \end{bmatrix} - \begin{bmatrix} \mathbf{X}_{n,k}' \\ \mathbf{X}_{a,k}' \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_k'' - \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k}' \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k}'' + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_k'' \end{aligned} \quad (3-54)$$

Now substituting the result of Eqs. (3-51) and (3-54) into Eq. (3-12) gives

$$\begin{aligned} \mathbf{M}_{k+1}'' &= -\mathbf{K}_R \boldsymbol{\Theta}_{k+1}'' + \mathbf{K}_P^T \mathbf{X}_{k+1}'' \\ &= -\mathbf{K}_R \boldsymbol{\Theta}_{k+1}'' + \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}'' + \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_{k+1}'' \\ &= \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}'' + \left(\mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P - \mathbf{K}_R \right) \boldsymbol{\Theta}_{k+1}'' \\ &= \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \boldsymbol{\Theta}_{k+1}'' + \left(\mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P - \mathbf{K}_R \right) \boldsymbol{\Theta}_{k+1}'' \\ &= - \left[\left(\mathbf{K}_R - \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \right) - \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \right] \boldsymbol{\Theta}_{k+1}'' \\ &= - \left[\bar{\mathbf{K}}_R - \bar{\mathbf{K}}_P^T \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \right] \boldsymbol{\Theta}_{k+1}'' \end{aligned} \quad (3-55)$$

Finally, substituting the result of Eqs. (3-37) and (3-51) into Eq. (3-13) gives

$$\begin{aligned} \mathbf{M}_{k+1}' &= \mathbf{K}_P^T \mathbf{X}_{k+1}' = \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}' \\ &= \bar{\mathbf{K}}_P^T \mathbf{X}_{n,k+1}' = \bar{\mathbf{K}}_P^T [\mathbf{X}_{n,k+1} - \mathbf{X}_{n,k+1}''] \\ &= \bar{\mathbf{K}}_P^T [\mathbf{X}_{n,k+1} - \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \boldsymbol{\Theta}_{k+1}''] \end{aligned} \quad (3-56)$$

In summary, Eqs. (3-29) and (3-30) in the classical force analogy method have been replaced by Eqs. (3-46) and (3-51) in the modified force analogy method after static condensation is performed. Together with the equations of motion given in Eq. (3-40), the inelastic time history analysis can be performed the same way as the classical force analogy method. This demonstrates the usefulness of force analogy method in analyzing real moment-resisting frames. In the above derivation, the main advantage of using the condensed forms is that the size of the problem can be significantly reduced, while the stiffness matrices $\mathbf{K}$, $\mathbf{K}_P$ and $\mathbf{K}_R$ in the classical force analogy method are replaced by $\bar{\mathbf{K}}_{nn}$, $\bar{\mathbf{K}}_P$ and $\bar{\mathbf{K}}_R$ in the modified force analogy method with static condensation. In other words, instead of storing and operating with the full $\mathbf{K}$, $\mathbf{K}_P$ and $\mathbf{K}_R$ matrices in the computer, only storing and operating with the condensed $\bar{\mathbf{K}}_{nn}$, $\bar{\mathbf{K}}_P$ and $\bar{\mathbf{K}}_R$ matrices is necessary. Therefore, the memory required for storing the stiffness matrices and computing time can be significantly reduced.

CHAPTER FOUR

DAMAGE ASSESSMENT USING RESPONSE & ENERGY APPROACH

4.1 Energy Concepts

It has been widely accepted that the procedure of structural vibrations due to earthquakes is actually an energy transferring process. Energy produced by the collision of underground plates is partly transferred to structures to induce structural vibration by the movement of the ground. This input energy is then transferred to various forms such as strain energy and kinetic energy. Therefore, it is reasonable to study the energy characteristics and energy transfer in structures during earthquakes.

In practical applications, evaluating energy quantitatively still remains as a rather difficult job, and one main concern in developing a method of characterizing energy is that whether absolute energy or relative energy should be used in the analysis. The study of Uang and Bertero (1990) showed that the use of absolute energy in the energy balance equation is physically more meaningful. Therefore, the energy

equation derived here will be based on absolute energy for a multi-degree of freedom (MDOF) system using the modified force analogy method discussed in Section 3.4.

Define the absolute acceleration $\ddot{\mathbf{Y}}(t)$ at both ODOFs and ADOFs to be

$$\ddot{\mathbf{Y}}(t) = \ddot{\mathbf{X}}(t) + \ddot{\mathbf{X}}_g(t) \quad (4-1)$$

Then it follows from Eq. (3-35) that

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}'(t) = \mathbf{0} \quad (4-2)$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \quad (4-3)$$

Integrating both sides of Eq. (4-1) over the path of structural response from time zero to time t_k gives

$$\int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X} = 0 \quad (4-4)$$

Since $d\mathbf{X}(t) = d\mathbf{Y}(t) - d\mathbf{X}_g(t)$ where $\mathbf{Y}(t)$ is the absolute displacement and $\mathbf{X}_g(t)$ is the ground displacement, substituting this result into the first term of Eq. (4-4) gives

$$\int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{Y} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X} = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X}_g \quad (4-5)$$

In addition, since $d\mathbf{X}(t) = d\mathbf{X}'(t) + d\mathbf{X}''(t)$, substituting this result into the third term of Eq. (4-5) gives

$$\begin{aligned} \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{Y} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} \\ + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}' + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}'' = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X}_g \end{aligned} \quad (4-6)$$

Note that the first three terms in the left-hand side are

$$\int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{Y} = \frac{1}{2} \dot{\mathbf{Y}}(t)^T \mathbf{M} \dot{\mathbf{Y}}(t) - \frac{1}{2} \dot{\mathbf{Y}}(0)^T \mathbf{M} \dot{\mathbf{Y}}(0) \quad (4-7)$$

$$\int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} = \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} \dot{\mathbf{X}} dt \quad (4-8)$$

$$\int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}' = \frac{1}{2} \mathbf{X}'(t)^T \mathbf{K} \mathbf{X}'(t) - \frac{1}{2} \mathbf{X}'(0)^T \mathbf{K} \mathbf{X}'(0) \quad (4-9)$$

where $\dot{\mathbf{Y}}(t) = \dot{\mathbf{X}}(t) + \dot{\mathbf{X}}_g(t)$ is the absolute velocity and $\dot{\mathbf{X}}_g(t)$ is the earthquake ground velocity. The structure is generally assumed to be at rest at the instant when the earthquake ground motion begins, and therefore both $\dot{\mathbf{Y}}(0)$ and $\mathbf{X}'(0)$ in Eqs. (4-7) and (4-9) are zero. Now consider the fourth term in the left-hand side of Eq. (4-6), substituting Eqs. (3-4), (3-10) and (3-13) into this term gives

$$\begin{aligned} \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}'' &= \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} \cdot \mathbf{K}^{-1} \mathbf{K}_p d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K}_p d\boldsymbol{\Theta}'' \\ &= \int_{t=0}^{t=t_k} (\mathbf{K}_p^T \mathbf{X}')^T d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' \end{aligned} \quad (4-10)$$

The right hand side of this equation suggests that the integrand involves the multiplication of elastic moment with the change in plastic rotation. Further simplification to this term can be performed using Eqs. (3-5) and (3-12), i.e.

$$\begin{aligned} \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}'' &= \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} (\mathbf{M} - \mathbf{M}'')^T d\boldsymbol{\Theta}'' \\ &= \int_{t=0}^{t=t_k} \mathbf{M}^T d\boldsymbol{\Theta}'' + \int_{t=0}^{t=t_k} \boldsymbol{\Theta}''^T (\mathbf{K}_R - \mathbf{K}_p^T \mathbf{K}^{-1} \mathbf{K}_p)^T d\boldsymbol{\Theta}'' \\ &= \int_{t=0}^{t=t_k} \mathbf{M}^T d\boldsymbol{\Theta}'' + \frac{1}{2} \boldsymbol{\Theta}''^T (\mathbf{K}_R - \mathbf{K}_p^T \mathbf{K}^{-1} \mathbf{K}_p)^T \boldsymbol{\Theta}'' \\ &= \int_{t=0}^{t=t_k} \mathbf{M}^T d\boldsymbol{\Theta}'' - \frac{1}{2} \mathbf{M}''^T \boldsymbol{\Theta}'' \end{aligned} \quad (4-11)$$

where $\boldsymbol{\Theta}''(t=0)$ is assumed to be zero in performing the integration in Eq. (4-11).

Finally, substituting Eq. (4-7) to Eq. (4-11) into Eq. (4-6) gives

$$\begin{aligned} \frac{1}{2} \dot{\mathbf{Y}}^T \mathbf{M} \dot{\mathbf{Y}} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} \dot{\mathbf{X}} dt + \frac{1}{2} \mathbf{X}'^T \mathbf{K} \mathbf{X}' \\ + \int_{t=0}^{t=t_k} \mathbf{M}^T d\boldsymbol{\Theta}'' - \frac{1}{2} \mathbf{M}''^T \boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X}_g \end{aligned} \quad (4-12)$$

Note that based on the definition of mass and damping matrices in Eq. (4-3), Eq. (4-12) can be simplified as:

$$\begin{aligned} \frac{1}{2} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} \dot{\mathbf{Y}}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} \dot{\mathbf{X}}_n dt + \frac{1}{2} \mathbf{X}'^T \mathbf{K} \mathbf{X}' \\ + \int_{t=0}^{t=t_k} \mathbf{M}^T d\boldsymbol{\Theta}'' - \frac{1}{2} \mathbf{M}''^T \boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g \end{aligned} \quad (4-13)$$

where $\dot{\mathbf{Y}}_n(t) = \dot{\mathbf{X}}_n(t) + \dot{\mathbf{X}}_g(t)$, $\ddot{\mathbf{Y}}_n(t) = \ddot{\mathbf{X}}_n(t) + \ddot{\mathbf{X}}_g(t)$. Similarly, based on the definition of stiffness matrix and Eq. (3-37), the third term of Eq. (4-13) can be simplified as:

$$\begin{aligned} \frac{1}{2} \mathbf{X}'^T \mathbf{K} \mathbf{X}' &= \frac{1}{2} \begin{bmatrix} \mathbf{X}'_n \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n \end{bmatrix}^T \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{bmatrix} \mathbf{X}'_n \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} \mathbf{X}'_n \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n \end{bmatrix}^T \begin{bmatrix} (\mathbf{K}_{nn} - \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{K}_{an}) \mathbf{X}'_n \\ \mathbf{0} \end{bmatrix} \\ &= \frac{1}{2} \mathbf{X}'_n{}^T \bar{\mathbf{K}}_{nn} \mathbf{X}'_n \end{aligned} \quad (4-14)$$

Substituting Eq. (4-14) back into Eq. (4-13) gives

$$\begin{aligned} \frac{1}{2} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} \dot{\mathbf{Y}}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} \dot{\mathbf{X}}_n dt + \frac{1}{2} \mathbf{X}'_n{}^T \bar{\mathbf{K}}_{nn} \mathbf{X}'_n \\ + \int_{t=0}^{t=t_k} \mathbf{M}^T d\Theta'' - \frac{1}{2} \mathbf{M}''^T \Theta'' = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g \end{aligned} \quad (4-15)$$

Equation (4-15) shows the different forms of energy in the structure. These forms include:

- Kinetic Energy (*KE*): The first term on the left-hand side of Eq. (4-15) represents the kinetic energy. Since the mass matrix is a positive definite matrix, the kinetic energy will always be a positive value. i.e.:

$$KE = \frac{1}{2} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} \dot{\mathbf{Y}}_n \quad (4-16)$$

- Damping Energy (*DE*): The second term on the left-hand side of Eq. (4-15) represents the damping energy, which is the energy dissipated during the course of vibration. Since the damping matrix is a positive definite matrix, the integrand is always a positive value and thus more energy will be dissipated as time progressed. i.e.:

$$DE = \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} \dot{\mathbf{X}}_n dt \quad (4-17)$$

- Strain Energy (*SE*): The third term on the left-hand side of Eq. (4-15) represents the strain energy. This strain energy contains the elastic displacement vector, which is the portion of the total displacement that can be recovered upon unloading. Since the stiffness matrix is a positive definite matrix, the strain energy will always be a positive value. i.e.:

$$SE = \frac{1}{2} \mathbf{X}_n'^T \overline{\mathbf{K}}_{nn} \mathbf{X}_n' \quad (4-18)$$

- Plastic Energy (PE): The fourth and fifth terms on the left-hand side of Eq. (4-15) represents the plastic energy, which is the energy dissipated due to the formation of plastic hinges. i.e.:

$$PE = \int_{t=0}^{t=t_k} \mathbf{M}'^T d\Theta'' = \int_{t=0}^{t=t_k} \mathbf{M}^T d\Theta'' - \frac{1}{2} \mathbf{M}''^T \Theta'' \quad (4-19)$$

The fourth term represents the accumulation of plastic energy due to each plastic hinge formation, where the plastic energy at the i th PHL can be computed by using $\int_{t=0}^{t=t_k} M_i d\theta_i''$. In this integral, there will be no incremental plastic rotation when the response is elastic and therefore the accumulated amount of plastic rotation remains the same as the previous time step. When the response is inelastic and plastic rotation occurs, the amount of energy dissipated is exactly equal to the incremental plastic rotation multiplied by the total moments in these PHLs. The fifth term represents the energy dissipated due to the cross-relationship between any two PHLs. This term considers the distribution of plastic energy accumulation among the individual plastic hinges, and it is generally small because this term is not accumulative and the residual moment is usually small. However, it is not small enough to be ignored, as a comparison will be demonstrated in the subsequent example. Therefore, the total plastic energy that the i th plastic hinge can dissipate is given by the equation

$$PE_i = \int_{t=0}^{t=t_k} M_i' d\theta_i'' = \int_{t=0}^{t=t_k} M_i d\theta_i'' - \frac{1}{2} M_i'' \theta_i'' \quad (4-20)$$

Since excessive positive moment causes positive plastic rotation and excessive negative moment causes negative plastic rotation, plastic energy will therefore always be dissipated and never be absorbed.

- Input Energy (IE): The term on the right-hand side of Eq. (4-15) represents the input energy, and it is always positive because it is equal to the sum of kinetic, damping, strain, and plastic energies. i.e.:

$$IE = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g \quad (4-21)$$

However, input energy does not always increase with time because, as shown in Eq. (4-15), the change in ground displacement may be in the opposite direction to the absolute acceleration. When this happens, earthquake acts as a reactive force to help the structure balancing itself.

In summary, Eq. (4-15) can be written as

$$KE + DE + SE + PE = IE \quad (4-22)$$

4.2 Definition of Proposed Damage Potential Index

From the view of structural damage mechanism, the damage process of a structure progresses from partial damage to total collapse. In most cases, a structure will experience reversals in vibration several times, and even if its displacement does not reach the maximum displacement it will still collapse or suffer significant damage due to excessive energy dissipation resulted from irrecoverable deformation. Hence the ways that damage can be excessive at the two extremes are:

- When the earthquake causes structural inelastic deformation in only one direction without any reversals of plastic rotations, damage may become excessive with large inelastic deformation but without much plastic energy dissipation.
- When the earthquake causes small structural inelastic deformation but with frequent reversals of plastic rotations, damage may become excessive with small inelastic deformation but with a tremendous amount of plastic energy that needs to be dissipated.

In general, moderate inelastic deformation is likely to occur with moderate reversals of plastic rotations during major earthquakes, and therefore damage can be excessive anywhere between the two above-mentioned extremes.

Since the inelastic roof displacement $X''_{roof}(t)$ and the total roof displacement $X_{roof}(t)$ can be calculated using Eq. (3-51) and Eq. (3-40), respectively, together with the total plastic energy PE and total input energy IE calculated using Eq. (4-19) and Eq. (4-21), a new damage potential index (DI) in evaluating the total damage condition of

structures from both displacement and energy point of view, which is based on the maximum value of these four terms is proposed as:

$$DI_{Total} = \alpha \times \frac{PE_{\max}}{IE_{\max}} + \beta \times \max \left\{ \frac{(X_{roof}^{''+})_{\max}}{(X_{roof}^{+})_{\max}}, \frac{(X_{roof}^{''-})_{\max}}{(X_{roof}^{-})_{\max}} \right\} \quad (4-23)$$

where DI_{Total} is the total damage potential index. The terms $PE_{\max}$ and $IE_{\max}$ are the maximum values of plastic energy and input energy, respectively, and $(X_{roof}^{''+})_{\max}$, $(X_{roof}^{+})_{\max}$, $(X_{roof}^{''-})_{\max}$, and $(X_{roof}^{-})_{\max}$ are the maximum values of positive inelastic roof displacement, positive total roof displacement, negative inelastic roof displacement, and negative total roof displacement, respectively. α and β are weighting parameters satisfying $\alpha + \beta = 1$, which are assumed to be $\alpha = \beta = 0.5$ in this research. Note that DI_{Total} in Eq. (4-23) consists of two terms:

- The first term on the right hand side of Eq. (4-23) represents the portion of input energy that causes inelastic effects in the structures during an earthquake, and this can be viewed as evaluating structural damage from the energy point of view.
- The second term on the right hand side of Eq. (4-23) represents the portion of total displacement that causes inelastic effect in the structures during an earthquake, and this can be viewed as evaluating structural damage from the response point of view.

The robustness of the proposed model is guaranteed since either the approach from plastic energy to input energy (first item) or from inelastic displacement to total displacement (second item) to a great level indicates the occurrence of damage, whatever the structures are. It should be pointed out here that the maximum value of roof inelastic displacement occurs approximately simultaneously with the maximum value of roof total displacement with only a small hysteresis due to vibrations of earthquakes, thus the use of $(X_{roof}^{''+})_{\max} / (X_{roof}^{+})_{\max}$ or $(X_{roof}^{''-})_{\max} / (X_{roof}^{-})_{\max}$ in Eq. (4-23) is reasonable to represent the inelastic effect from the response point of view. The easy application can also be shown in the following numerical simulation examples.

Note that the contribution of the two terms in Eq. (4-23) to the total damage is assumed to be equal here with the choice of $\alpha = \beta = 0.5$. This is acceptable in this preliminary work, further research in defining the real values of the weighting parameters of response part and energy part is still necessary. Therefore, the total damage potential index DI_{Total} given by Eq. (4-23) considering both aspects of response and energy is reasonable. In addition, the proposed DI_{Total} satisfies $0 \leq DI_{Total} < 1$, where $DI_{Total} = 0$ means no damage and with an increase of DI_{Total} suggests that the damage of structures becomes more significant until total collapse.

The damage process of a structure progresses from partial damage to total collapse mainly due to the fact that structural damage becoming more intensive and the number of plastic hinge formation increases. Thus evaluating the local damage at each plastic hinge location has a good significance and gives insights into the locations of the structure where most damage has been suffered in the structure. Based on Eqs. (3-51), (3-52), (4-19) and (4-20), the local damage potential index in i th PHL is defined as:

$$DI_i = \alpha \times \frac{(PE_i)_{\max}}{PE_{\max}} + \beta \times \left\{ \left| \frac{X''_{roof,i,T}}{(X''_{roof})_{\max}^+} \right| \text{ or } \left| \frac{X''_{roof,i,T}}{(X''_{roof})_{\max}^-} \right| \right\} \quad (4-24)$$

where α and β are the same weighting parameters with Eq. (4-23), which are assumed to be 0.5 also. $(PE_i)_{\max}$ is the maximum value of the plastic energy at the i th PHL, $X''_{roof,i,T}$ is the inelastic roof displacement caused by i th PHL at time T , where

- For the ratio $X''_{roof,i,T} / (X''_{roof})_{\max}^+$, T is defined as the first time at which X''_{roof} achieves its maximum value when $(X''_{roof})_{\max}^+ / (X''_{roof})_{\max}^+$ is chosen for calculating DI_{Total} in Eq. (4-23). Thus $X''_{roof,i,T}$ is the inelastic roof displacement caused by i th PHL at that time T .
- For the ratio $X''_{roof,i,T} / (X''_{roof})_{\max}^-$, T is defined as the first time at which X''_{roof} achieves its maximum value when $(X''_{roof})_{\max}^- / (X''_{roof})_{\max}^-$ is chosen for calculating DI_{Total} in Eq.(4-23). Thus $X''_{roof,i,T}$ is also the inelastic roof displacement caused by i th PHL at that time T .

This definition of local damage potential index shows clearly the contribution of each PHL to the overall damage of structures from both energy and response points of view. It should also be addressed here that the local damage index does not give quantitative damage of the structural element, but it gives the relative contribution of the element damage to the global damage. In addition, the reason why inelastic displacement is adopted in the local damage potential index instead of the conventional approach which uses the plastic rotation as the local damage parameter lies in the consideration of the consistency of the global damage potential index and local damage potential indices. The relationship between the local damage potential index and overall damage of the structure is:

$$\begin{cases} \sum_{i=1}^m DI_i \geq 1 & (DI_{Total} > 0) \\ \sum_{i=1}^m DI_i = 0 & (DI_{Total} = 0) \end{cases} \quad (4-25)$$

4.3 Example: Numerical Simulation of a MDOF Steel Model

Based on the theories derived in Chapter 3 and Sections 4.1 and 4.2, the seismic inelastic response time histories and energy distributions of MDOF structures can be obtained. In addition, the damage potentials of these structures can be predicted readily using the proposed damage model. In this section, a six-story hospital building located in Woodland Hills, California, is used as an example to demonstrate the theories illustrated above. A moment resisting frame in this building, as shown in Figure 4-1(a), is selected in this research. The model of this frame contains 30 degrees of freedom system (includes 6 ODOFs and 24 ADOFs), and using static condensation the structural stiffness matrix $\bar{\mathbf{K}}_{nn}$ is reduced to a 6×6 matrix. The floor masses of 630,400 kg per floor are selected to give a long first period of vibration of 1.77 seconds. The damping matrix is assumed to be a uniform diagonal matrix, i.e., $\mathbf{C} = c \times \mathbf{I}$, where $c = 210.1$ kN-s/m.

Figure 4-1 Six-story moment resisting frame model

Forty plastic hinge locations are identified, with six PHLs per floor and four PHLs at the bottom of the ground columns. These plastic hinges are assumed to deform in an elastic-plastic behavior as in Figure 4-1(b) with yield moments computed based on the plastic modulus of the members and a yield stress of steel of 248.2 MPa. Assume that a uniform gravity load of 21.89 kN/m is acting on every floor; the moment capacities of the beams and ground columns reserved for earthquake are summarized in Table 4-1, where M_{EQ}^+ is the positive moment capacity and M_{EQ}^- is the negative moment capacity reserved for earthquake actions. The 1994 Northridge earthquake time history at Sylmar as shown in Figure 4-2 is used in the simulation process.

Table 4-1 Beam and ground column moment capacity calculations

Type	Z (cm ³)	M_p (kN-m)	M_{RF} (kN-m)	M_{EQ}^+ (kN-m)	M_{EQ}^- (kN-m)
W36×210	13,650	3,388	106	3,282	3,494
W36×150	9,521	2,363	106	2,257	2,469
W36×135	8,341	2,070	106	1,964	2,176
W27×94	4,556	1,131	106	1,025	1,237
W14×283	8,880	2,204	—	2,204	2,204
W14×500	17,200	4,269	—	4,269	4,269

Figure 4-2 1994 Northridge earthquake ground acceleration time history

4.3.1 Inelastic Response

Figure 4-3 shows the relative displacement, relative velocity and absolute acceleration time histories of the roof level. Besides the total displacement time history, the inelastic displacement time history is also given in Figure 4-3(a). It can be found that the maximum value of inelastic displacement occurs approximately simultaneously with that of the total displacement. This is because most of the earthquake ground motions have low frequency contents, which excites the structure mainly in the first mode shape. Thus a large total displacement also indicates a large moment and, more importantly, a large inelastic displacement. Figure 4-4 gives the plastic rotation time histories at three selected PHLs. Results show that all levels of the frame have yielded,

which lead to plastic energy dissipated at all levels as illustrated in the following section.

(a)

(b)

(c)

Figure 4-3 Roof response time history of a 6-story frame due to Northridge EQ

(a)

(b)

(c)

Figure 4-4 Plastic rotation time history at selected plastic hinge locations (PHLs)

4.3.2 Energy

Figure 4-5 shows the energy distribution time history of different energy forms for the frame shown in Figure 4-1 subjected to the Northridge earthquake shown in Figure 4-2. It is found that most of the input energy is dissipated by plastic and damping energies, while the kinetic and strain energies of this building are not so large. In addition, from the figure the characteristics of each energy form are demonstrated. At the beginning of an earthquake, all the strain energy, kinetic energy, damping energy, plastic energy and their summation, which is equal to input energy, are approximately zero, which means that at this time the earthquake is so small that it can be neglected. With the coming of the earthquake's peak acceleration, all the energies increase greatly. The strain energy and kinetic energy change with the change of displacement and velocity, while the damping energy and plastic energy keep increasing for which they are lost and irrecoverable during the energy transferring process. The input energy, which is the outer envelop line, remains positive but sometimes drops by a small amount because it equals to the sum of the four energies. At the end of the earthquake duration, the strain energy and kinetic energy become approximately zero because the earthquake excitation diminishes, while the damping energy and plastic energy converge to certain constants. The input energy also converges to a constant, which is approximately equal to the sum of the final damping energy and plastic energy.

Figure 4-5 Energy distribution among various forms

Figure 4-6 shows the plastic energy time histories at the three selected PHLs. According to Eq. (4-19), the accumulation of plastic energy is calculated by the elastic moment multiplying the change in plastic rotations (i.e. fourth and fifth terms in Eq. (4-15)). However, the classical thinking is that calculating dissipated plastic energy is simply measuring the enclosed area within the moment versus plastic rotation response, which is computed by the total moment multiplying the change in plastic rotations (i.e. only the fourth term in Eq. (4-15)).

As shown in Figure 4-6, comparisons between plastic energy calculations using elastic moment and total moment give the difference, particularly at the PHLs in the lower stories. The difference is exactly equal to the fifth term of Eq. (4-15), where the residual moment is usually larger at the lower stories because larger plastic energy dissipation usually occurs in the lower stories. Based on the numerical results obtained in Figure 4-6, it can be concluded that elastic moment should be used in calculating plastic energy, as it is derived analytically above.

(a)

(b)

(c)

Figure 4-6 Comparison of plastic energy time history calculated using elastic moment and total moment

4.3.3 Damage

Using Eqs. (4-23) and (4-24), the damage situation of this structure due to Northridge earthquake can be predicted both in total aspect and in local contributions. In Eq. (4-23), the total structural damage potential index is defined as a combination of inelastic contributions from both the response portion and the energy portion. Some researchers may wonder if the ratio between the maximum of inelastic displacement and maximum of total displacement can represent structural damage from response point of view, what the situation will be if the total displacement does not achieve its maximum value while the inelastic displacement does. In fact, there should be no problem in doing so since evaluating structural damage in response aspects means just considering the condition at which the structure achieves its maximum total displacement, while the damage contribution due to cyclic reversals have been taken into account in the energy portion in the proposed damage assessment model. Besides the phenomenon that the maximum of inelastic displacement always occurs at a slight delay of that of the total displacement in the case of earthquake vibration, as demonstrated in Section 4.3.1, it can be concluded that the damage potential index defined in the form of Eq. (4-23) is quite reasonable. Based on the equations aforementioned, the total damage potential index is calculated to be $DI_{Total} = 0.6875$ for the structure shown in Figure 4-1 due to Northridge earthquake ground motion shown in Figure 4-2.

After the total damage potential index is defined, Eq. (4-24) gives the local damage potential prediction of the frame. The contribution of the i th PHL to the inelastic roof displacement can be obtained using Eq. (3-52), results of the three selected PHLs are shown in Figure 4-7. In this case, the time T in Eq. (4-24) is defined as the first time at which X''^+_{roof} achieves its maximum value since $(X''^+_{roof})_{\max} / (X^+_{roof})_{\max}$ has been chosen for calculating DI_{Total} in Eq. (4-23). It can be found from Figure 4-3(a) that $T = 4.04$ s.

Figure 4-7 Roof inelastic displacement caused by i th plastic rotation

The local damage potential index distribution of the 6-story frame subjected to Northridge earthquake is shown in Figure 4-8. It can be observed that the PHLs at the same floor contribute to the total damage approximately in the same level except the four PHLs at the bottom columns. The local damage contributions of the floors also decrease with the increase of the floor height, which proves that the lower stories generally require stronger beams and columns than the upper stories. One thing should be emphasized is that the contributions from PHLs #2 and #3 are the largest among all the forty PHLs to the total structural damage and they are even much larger than that from PHLs #1 and #4 in the same floor. This means that the two inner base columns may be too weak to distribute the damage, and a better damage distribution design of this frame may be achieved by substituting the two inner base columns with stronger ones.

Figure 4-8 Local DI (%) of the 6-story frame subjected to Northridge EQ

In this case, the total damage potential index of the 6-story frame is calculated to be $DI_{Total} = 0.6875$. But what this number means? Does it mean that the structure suffers severe unreparable damage, repairable damage or no damage at all? Therefore, only if these numbers are categorized according to their performance levels, the total damage potential index will have its meaning and application significance. The next chapter mainly focuses on this point.

CHAPTER FIVE

SPECTRA ANALYSIS

5.1 Introduction

Modern building codes, such as the Uniform Building Code (UBC), present three options for the dynamic analysis of structures: Linear Time History Analysis, Nonlinear (Inelastic) Time History Analysis and Response Spectra Analysis. Among them, the response spectra analysis is regarded as the most popular one because it provides an approximate, but relatively accurate method of analysis in the form of a static analysis. The nonlinear time history analysis has been demonstrated in details in Chapters 3 and 4, while in this section, the response spectra analysis is introduced to deal with structural dynamic problems.

As it is known, earthquake ground motions have spectral properties similar to waves and sound. Due to this nature, engineers must consider a dynamic system, which is dependent upon the frequencies, associated with the motions of the ground as well as those of the structure. A practical solution was found to be the response spectrum, i.e., a system is considered to be a collection of ideal, damped single-degree-of-freedom

oscillators, and the response spectrum presents the maximum response of that system at various frequencies. For any linear SDOF structural systems, all responses can be computed for a given ground acceleration time history $\mathbf{a}(t)$, the structure's natural frequency ω_n and damping ratio ζ . Though the maximum values of the response do not contain the time information, they can reflect the combined effects of the properties of both the ground motion and structures. With ω_n moving along the coordinate axis, the maximum responses for continuous frequencies can be obtained. Representing the idea with formula are:

$$S_d(\omega_n, \zeta) = |x(t)|_{\max} \quad (5-1)$$

$$S_v(\omega_n, \zeta) = |\dot{x}(t)|_{\max} \quad (5-2)$$

$$S_a(\omega_n, \zeta) = |\ddot{y}(t)|_{\max} \quad (5-3)$$

where $S_d(\omega_n, \zeta)$ is relative spectral displacement, $S_v(\omega_n, \zeta)$ is relative spectral velocity, and $S_a(\omega_n, \zeta)$ is absolute spectral acceleration.

Every response spectrum curve is the result of the maximum dynamics response of many structures with different properties to a ground motion time history. Response spectra reflect the properties of the amplitude and frequency of ground motions, i.e. the shape of response spectra reflects the relative relationship between different frequency components of ground motions. On the other hand, because response spectra describe ground motion from the special perspective of the maximum response of SDOF systems by “moving” the structural property along the frequency domain, this description thus establishes a bridge between the ground motions and structural responses. Recognition of ground motion can lead to structural responses through every point on the response spectra.

Ever since the concept of response spectrum was developed in the 1930's, it has been widely used to estimate demands on structures imposed by earthquake ground motions. In modern earthquake design, response spectra have also become the most commonly used method of structural analysis. The primary reason for this popularity is that it provides the designer with a rational and simple basis for specifying the earthquake

loading. Another reason often cited is that the method is computationally economical since eliminating the time variable greatly reduces the quantity of response data that must be stored in the computer and/or post-processor. However, the major defect of this method arises from the fact that the temporal information is lost in the process of evaluating the spectrum only.

There have been many related studies that, by considering a large number of recorded ground motions, have investigated the characteristics of linear elastic response spectra (Newmark and Hall, 1982). However, considering buildings designed using elastic response spectra are likely to experience significant inelastic deformations during major earthquakes, which may lead to great economic losses. Thus, study on inelastic response spectra becomes necessary.

5.2 Inelastic Spectra Analysis

Generating inelastic response spectra due to various earthquake excitations requires not only the two parameters of period of vibration (or frequency) and damping ratio as those generally used in generating elastic response spectra, but also requires one additional parameter which can capture the inelastic behavior of the structure. It is observed in Chapter 3 that yield displacement relates to the yield moment at the plastic hinge of the moment-resisting framed structure quite well, and it can be calculated prior to any dynamic analysis is performed. In other words, this yield displacement parameter gains an advantage over other parameters such as ductility in uncoupling the effect of earthquake characteristics. Therefore, in this research, yield displacement is chosen as the third parameter. By using all these three parameters, Eqs. (5-1) to (5-3) can be modified to be:

$$S_d(\omega_n, \zeta, D_y) = |x(t)|_{\max} \quad (5-4)$$

$$S_v(\omega_n, \zeta, D_y) = |\dot{x}(t)|_{\max} \quad (5-5)$$

$$S_a(\omega_n, \zeta, D_y) = |\ddot{y}(t)|_{\max} \quad (5-6)$$

and these three equations represent the governing equations of inelastic spectral analysis (ISA). It is possible to very rapidly calculate by computer the spectral

displacement, spectral velocity and spectra acceleration for a given earthquake ground motion time history for different values of ω_n , ζ and D_y . If this is done, the response can be summarized in a tabular or graphic form. In either form, most structural engineers prefer to express S_d , S_v and S_a in terms of the undamped natural period of vibration, T_n , instead of the undamped natural frequency of vibration, ω_n .

Besides the three response spectra considering displacement, velocity and acceleration, drift ratio spectra can also be developed. As explained in Chapter 2, drift ratio is a very important and commonly used index in structural damage evaluation and especially in design aspect because of its easy application. There are various types of drift ratios, such as single storey drift ratios and inter-storey drift ratios, but the one used here is one kind of single storey drift ratio so called roof drift ratio, which has the form:

$$DR_{roof} = \frac{|x_{roof}(t)|_{\max}}{H} \quad (5-7)$$

where H is the height of the structure. Thus, the roof drift ratio spectra can be defined as:

$$S_{DR}(\omega_n, \zeta, D_y) = DR_{roof} \quad (5-8)$$

Illuminated by the beauty of response spectra, energy spectra are here constructed in a similar fashion. As in the case for response spectra, energy spectra can provide information on energy dissipation of different structures in a specified earthquake. Based on the quantitative energy forms in Eqs. (4-16) to (4-21), energy density spectra are defined to be the maximum values of kinetic energy, strain energy, damping energy, plastic energy or input energy divided by the mass of structures. The reason for dividing the individual energy terms by mass is to uncouple the effect of structural mass in the spectra. Since the proposed damage index emphasizes plastic energy and input energy, the following equations representing the plastic energy density spectra and input energy density spectra are given:

$$S_{PE}(\omega_n, \zeta, D_y) = \left(\frac{PE(t)}{M} \right)_{\max} \quad (5-9)$$

$$S_{IE}(\omega_n, \zeta, D_y) = \left(\frac{IE(t)}{M} \right)_{\max} \quad (5-10)$$

Moreover, with the powerful tool of response spectra analysis, damage evaluation can also take a clear and objective form. Based on the definition of DI_{Total} in Eq. (4-23), considering all the three parameters, i.e., (1) period of vibration, (2) damping ratio, (3) yield displacement, spectra of total damage potential index can be generated resultantly. The equation for damage index spectra is:

$$S_{DI}(\omega_n, \zeta, D_y) = DI_{Total} \quad (5-11)$$

Thus, after the tabular or graphic forms are plotted, the structural damage conditions due to a specific ground motion can be predicted using the damage index spectra given a specific combination of ω_n , ζ and D_y . It should be pointed out that the DI_{Total} is defined to indicate the final status of damage. Thus a combination of two ratios of maximum values until final status is used in Eq.(4-23). As long as the ground motion with certain duration is defined, the value of DI_{Total} will be calculated to be a number instead of a variable changing with time, therefore, there is no subscript of $_{max}$ in Eq. (5-11) as in Eqs.(5-9) and (5-10).

Consider the SDOF structure as shown in Figure 5-1, where the plastic hinge is located at a distance of 0.25 m from the joint of beam and column. Assume that the column exhibits ideally elastic-plastic behavior with a predefined yield displacement D_y . Subjected to the recorded 1994 Northridge earthquake ground motion time histories as shown in Figure 4-2 and the magnified 1940 El-Centro earthquake ground motion time histories with scalar of 1.8 as shown in Figure 5-2, the drift ratio spectra, plastic energy density spectra and damage index spectra for various cases are generated as shown in Figures 5-3 to 5-8. These figures show the effects of various yield displacement levels, damping ratios, and earthquake scaling factors, respectively, and the results are briefly discussed in the following sub-sections.

Figure 5-1 Single degree of freedom system

Figure 5-2 Magnified 1940 El-Centro earthquake ground acceleration time history (Scalar=1.8)

5.2.1 Variation of Yield Displacement Level

Subjected to the Northridge earthquake as shown in Figure 4-2, the 5% damped spectra of drift ratio (DR), spectra of plastic energy density (*PE* Density) and spectra of damage index (DI) with different yield displacement levels are shown in Figure 5-3. Since longer period structures always experience larger displacement response, the DR spectra experience a generally consistent increasing trend. As shown in the Figure 5-3(a), there exist no consistent trend for the DR values with the increase of yield displacement. For example, it is observed for structures with long period of vibration over 2.5 s that DR decreases as yield displacement increases. However, for structure

with period of vibration ranging from 0.5 s to 2.5 s, larger DR is observed for structure with yield displacement of 18 cm than that of 9 cm. This is because structures with very large yield levels tend to be too soft and therefore have large displacement responses as well as DR values.

Figure 5-3(b) shows the *PE* Density spectra, and observation suggests that plastic energy dissipated in structures with smaller yield displacement is larger than those with larger yield displacement for the periods up to approximately 1.3 s. It is reasonable because structures with smaller yield displacement tend to yield more often when subjected to the same earthquake ground motion, and thus more plastic energy must be dissipated. Beyond periods of 1.3 s the observation is completely opposite, where the plastic energy dissipated is larger for structures with larger yield displacement. This is also because that longer period structures always experience larger displacement response and thus giving larger plastic energy dissipation.

Similar results are obtained when the El-Centro earthquake shown in Figure 5-2 is used, where the DR spectra and *PE* Density spectra in Figure 5-4 exhibit the same phenomena as those shown in Figure 5-3. The only difference lies in the period at which the *PE* Density changes trend is now about 2.3 s. From Figures 5-3 and 5-4, it is reasonable to conclude that both DR spectra and *PE* Density spectra are very dependent on the characteristic of ground motions and neither of them have a simple indication which can be used as a satisfactory index of structural damage.

However, the dependency on characteristic of ground motions is not observed for the spectra of DI. Considering the DI spectra in Figures 5-3(c) and 5-4(c) for both earthquakes, the DI values for short period structures are generally smaller than those for long period structures, and DI values decrease with the increase of yield displacement for both of two earthquake time histories. This means that the proposed DI spectra are much more independent on the characteristic of ground motions. Comparing these two figures at period of 1.5 s and yield displacement of 9 cm, DI is about 70% for Northridge earthquake and 51% for El-Centro earthquake. In addition, DI remains at 0 until period at 0.36 s for Northridge earthquake and 0.5 for El-Centro

earthquake, which means structures with period less than 0.36 s will suffer no damage during Northridge earthquake and less than 0.5 s will suffer no damage during El-Centro earthquake. More importantly, as shown in these figures, the DI spectra approach stable values over an extended range of periods. For example, DI values remain at approximately 74% for structures with periods larger than 2.0 s and a yield displacement of 9 cm subjected to Northridge earthquake. This is because the chosen ground motions are so large that structures at these periods and over these periods all experience large inelastic deformation and may even collapse as a whole, thus the phenomenon of DI approaching stable values reflects such results.

5.2.2 Variation of Damping Ratio

The DR spectra, *PE* density spectra, and DI spectra at yield displacement of 9 cm with different damping ratios subjected to Northridge earthquake and El-Centro earthquake are shown in Figures 5-5 and 5-6, respectively. It has been well known that structures with the same yield displacement and damping ratio but of different natural period will dissipate different amount of plastic energy, and this can be verified from Figures 5-5(b) and 5-6(b). Moreover, the period at which the peak values of plastic energy occur tend to be consistent corresponding to different damping ratios. For example, the structures' period at which the plastic energy reaches its peak value is approximately 1.3 s for Northridge earthquake in Figure 5-5(b). But this consistency is highly dependent on the characteristics of ground motions since the value is found to be 0.8 s for El-Centro earthquake in Figure 5-6(b).

From these figures, it can also be observed that structures with larger damping ratio tend to be more effective in resisting vibration, and therefore all values of DR, *PE* Density, and DI decrease with the increase of damping ratio. For example, DI decreases from 0.63 to 0.31 when damping ratio increases from 2% to 15%, which means that structures will suffer less damage when damping ratio increases. This observation is consistent with the basic principles that increasing damping in the structure will help in resisting earthquakes.

5.2.3 Variation of Recorded Earthquake Scalar

Subjected to Northridge earthquake ground motion with duration of 20 s in Figure 4-2, the 5% damped DR spectra, *PE* density spectra and DI spectra with different scaling factors of the earthquake ground motion at yield displacement of 9 cm are shown in Figure 5-7. In addition, Figure 5-8 shows the spectra for the same structure but subjected to magnified El-Centro earthquake ground motion as shown in Figure 5-2.

These two figures show that larger earthquake accelerations cause more severe structural damage in general. The scaling factors affect the values of each line for the same ground motion, but do not affect much on the contours. For example, the DI spectra approach stable values over an extended range of periods especially for the cases with large scalars. The reason is just like afore-mentioned that the original ground motions as shown in Figures 4-2 and 5-2 are major earthquakes to begin with, and therefore when it is magnified by a large scalar such as 1.2 and 1.5 the ground acceleration are so large that structures at these periods experience too large inelastic deformation and may even collapse as a whole. Thus, the phenomenon of DI approaching stable values is quite reasonable and reflects such results well.

Therefore, from the discussions in Sections 5.2.1, 5.2.2 and 5.2.3, it can be concluded, simply based on these two earthquake ground motions, that the proposed damage index is more effective in evaluating structural damage than drift ratio or plastic energy alone since it can reflect more clearly the effects of increasing damping ratio and yield displacement as well as decreasing magnitude of ground motions on the hazard resistant abilities of structures.

(a)

(b)

(c)

Figure 5-3 5% damped spectra at different yield displacement levels due to Northridge earthquake

(a)

(b)

(c)

Figure 5-4 5% damped spectra at different yield displacement levels due to El-Centro earthquake

(a)

(b)

(c)

Figure 5-5 Spectra at yield displacement of 9cm with different damping ratios due to Northridge earthquake

(a)

(b)

(c)

Figure 5-6 Spectra at yield displacement of 9cm with different damping ratios due to El-Centro earthquake

(a)

(b)

(c)

Figure 5-7 5% damped spectra at yield displacement of 9cm with different earthquake scalars due to Northridge earthquake

(a)

(b)

(c)

Figure 5-8 5% damped spectra at yield displacement of 9cm with different earthquake scalars due to El-Centro earthquake

5.3 Practical Application to MDOF Systems

As mentioned in above sections, in modern earthquake engineering design the response spectra method has emerged as the most commonly used method of structural analysis because of its simplicity while giving accurate results. These spectra should not only be applicable to single degree of freedom (SDOF) structures, but they can also be used to predict the inelastic deformation of multi-degree of freedom (MDOF) structures with acceptable accuracy. According to the characteristics of energy density spectra and damage index spectra discussed in Section 5.2, study of their application to real structures is a possible new approach to perform more accurate and fast prediction of structural damage of structures suffered from specific pre-defined ground motion vibrations. Hence in the following four sub-sections, a method of estimating different forms of energy using energy density spectra and evaluating seismic damage using damage index spectra in MDOF structures is firstly presented using normal mode method, which is demonstrated in Section 5.3.1. Then, based on the numerical studies performed using several different models illustrated in Section 5.3.2 and several cases of ground motions described in Section 5.3.3, the accuracy of the predicted damage conditions will be summarized by comparing the results with that obtained using inelastic time history analysis in Section 5.3.4.

5.3.1 Normal Mode Method

To study the modal energy dissipation of a multi-degree of freedom structures, various parameters must be considered. Starting with the dynamic equilibrium equation:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\ddot{\mathbf{X}}_g(t) \quad (5-12)$$

Replacing the displacement vector by $\mathbf{X}(t) = \Phi\mathbf{Q}(t)$ and pre-multiplying the resulting equation by Φ^T gives

$$\Phi^T \mathbf{M} \Phi \ddot{\mathbf{Q}}(t) + \Phi^T \mathbf{C} \Phi \dot{\mathbf{Q}}(t) + \Phi^T \mathbf{K} \Phi \mathbf{Q}(t) = -\Phi^T \mathbf{M} \ddot{\mathbf{X}}_g(t) + \Phi^T \mathbf{K} \mathbf{X}''(t) \quad (5-13)$$

where Φ is the modal matrix with the collection of mode shapes and $\mathbf{Q}$ is the modal response vector. After performing matrix multiplication, Eq. (5-12) can be decomposed into n nearly uncoupled single degree of freedom equations, i.e.,

$$\hat{m}_i \ddot{q}_i(t) + \hat{c}_i \dot{q}_i(t) + \hat{k}_i q_i(t) = -\Gamma_i \ddot{X}_g(t) + \Phi^T \mathbf{K} \mathbf{X}''(t) \quad (5-14)$$

where $\hat{m}_i = \Phi_i^T \mathbf{M} \Phi_i$, $\hat{c}_i = \Phi_i^T \mathbf{C} \Phi_i$, $\hat{k}_i = \Phi_i^T \mathbf{K} \Phi_i$, $\Gamma_i = \Phi_i^T \mathbf{M} \{\mathbf{I}\}$, $q_i(t)$ is the i th modal response, $\ddot{X}_g(t)$ is the earthquake acceleration corresponding to the direction of the i th mode, and $\{\mathbf{I}\}$ is the column vector with 1's in all entries for two-dimensional frame analysis. The last term on the right hand side of Eq. (5-14) represents the coupling between the modal responses when the structure becomes inelastic. To simplify the equation, assume the inelastic displacement follows the same relationship as total displacement, i.e., $\mathbf{X}''(t) = \Phi \mathbf{Q}''(t)$, where $\mathbf{Q}''(t)$ is the modal inelastic displacement vector. Doing so Eq. (5-14) becomes

$$\ddot{q}_i(t) + \frac{\hat{c}_i}{\hat{m}_i} \dot{q}_i(t) + \frac{\hat{k}_i}{\hat{m}_i} q_i(t) = -\frac{\Gamma_i}{\hat{m}_i} \ddot{X}_g(t) \quad (5-15)$$

where $q_i'(t)$ represents the elastic displacement of the i th mode. Numerical error will be introduced while making such an uncoupling assumption of inelastic displacement and the magnitude of error will be investigated when the results are compared with those obtained from the accurate inelastic time history analysis.

Equation (5-15) is the single degree of freedom version of Eq. (5-12), and therefore the energy in each mode can be calculated using the balance of energy summarized in Chapter 4. Note that $\Gamma_i / \hat{m}_i$ in Eq. (5-15) represents the earthquake scalar that has been studied and presented in Section 5.2, and this shows the importance of studying the effects of earthquake scalars on structural responses shown in Figures 5-7 and 5-8.

After the responses and energy forms in each mode are calculated, the damage potential index in each mode can be defined as:

$$DI(i) = \alpha \times \frac{PE(i)_{\max}}{IE(i)_{\max}} + \beta \times \max \left\{ \frac{X^{''+}(i)_{\max}}{X^{+}(i)_{\max}}, \frac{X^{''-}(i)_{\max}}{X^{-}(i)_{\max}} \right\} \quad (5-16)$$

where the bracket (i) represents the values of the symbols in the i th mode. And here $\alpha = \beta = 0.5$. Thus, the damage potential index in the i th mode can be easily obtained from the combination of response in the i th mode and energy in the i th mode, which is the major advantage in reducing the MDOF problem to a series of SDOF problems.

There are many methods that can be used to approximate the total damage potential index of a MDOF system from the damage potential index obtained from the individual modes. These methods include fundamental mode shape response method (FMS), maximum absolute response method (ABS), square root of the sum of the square response method (SRSS), and complete quadratic combination response method (CQC). Each method has its own advantages and disadvantages. Since the fundamental mode shape of structures play the most important role in seismic studies, as well as the fact that the values of all natural frequencies of vibration are generally very far from each other, only FMS and SRSS will be considered and made comparisons in this study.

The equation of calculating total damage potential index of a MDOF system from FMS method is:

$$DI_{Total} = DI(1) \quad (5-17)$$

which means only the fundamental damage potential index is considered in predicting the system's damage condition.

The equation of calculating total damage potential index of a MDOF system from SRSS method considering the first three mode shapes is:

$$DI_{Total} = \sqrt{DI(1)^2 + DI(2)^2 + DI(3)^2} \quad (5-18)$$

which means the damage potential indices in the first three modes, including the fundamental damage potential index are considered in predicting the damage condition of the system. The reason why only the first three modes are considered is that the higher modes often contribute insignificantly to the structural vibrations.

5.3.2 Design of Frames

The steel moment resisting frame (SMRF) is selected for this study. The SMRF system is used extensively in seismic regions for its versatility and economy. The seismic performance of this type of frames can best demonstrate both success and failure of the philosophy of relying on the inelastic deformation capacity to dissipate the seismic energy. Three buildings with 6, 3, and 10 stories in height are used as shown in Figures 5-9, 5-10, and 5-11, respectively. The main purpose of choosing various stories in different models is to study the influence of height on the damage potential index.

The 6-story 3-span frame shown in Figure 5-9(a) is the same with the one shown in Figure 4-1. Two mass properties are studied: (1) floor masses of 630,400 kg per floor, giving a long fundamental period of vibration of 1.77 s, and (2) floor masses of 210,133 kg per floor, giving a medium fundamental period of vibration of 1.02 s.

The 3-story frame shown in Figure 5-10 has floor masses of 52,500 kg per floor, which gives a short fundamental period of vibration of 0.51 s. The 10-story frame shown in Figure 5-11 has floor masses of 210,133 kg per floor, which gives a long fundamental period of vibration of 1.47 s.

Two different damping properties, ζ_i , are used to study the influence of damping ratio on the damage potential index. Both 2% and 5% damping ratios are assumed in all modes for all three structural models.

In addition, a fourth model is created by extending the 6-story 3-span frame shown in Figure 5-9(a) to 6-span as shown in Figure 5-9(b). The member sizes of the newly created 6-story-6-span frame in Figure 5-9(b) are the same as those in the 6-story-3-span frame in Figure 5-9(a). In addition, the floor masses of 630,400 kg per floor are selected to make a comparison between the responses of the frames in Figures 5-9(a) and (b) due to the influence of span.

The periods of vibration T_i for each of the structural model are calculated and summarized in Tables 5-1 to 5-6. In summary, six parameters are considered in this study: (1) building height; (2) building span; (3) floor mass; (4) damping ratio; (5) ground motion; and (6) earthquake magnifying factor.

The yield displacement in the i th mode D_{yi} , are calculated based on the mass normalized mode shapes correspondingly. Pre-multiplying $\mathbf{K}_p^T$ matrix (see Eq. 3-13) to each mode shape vector gives the moment demand at each plastic hinge corresponding to this mode shape. For example, for the 6-story 3-span frame shown in Figure 5-9(a) with a floor mass of 630,400 kg, the moment demand per unit of the first mode shape at PHL #1 is 3,934 kN-m/m, and that at PHL #2 is 8,299 kN-m/m. Dividing this moment demand by the moment capacity of the corresponding plastic hinge gives the scaling factor of the mode shape required to cause plastic rotation at each plastic hinge, which represents the yield displacement level at which the plastic hinge forms. For example, dividing the moment demand of 3,934 kN-m/m by the moment capacity at PHL #1 gives a yield displacement level of 0.5602 m, while dividing the moment demand of 8,299 kN-m/m by the moment capacity at PHL #2 gives a yield displacement level of 0.5144 m. Similar calculation for all modes are performed for each and every plastic hinge in the frame. Finally, the calculated yield displacement levels corresponding to each plastic hinge are scanned to obtain a minimum yield displacement level, and this yield displacement levels for all vibrating modes in different cases are summarized in Tables 5-1 to 5-6.

(a) Three spans

(b) Six spans

Figure 5-9 Six-story frame models

Figure 5-10 Three-story frame model

Figure 5-11 Ten-story frame model

5.3.3 Selection of Earthquake Ground Motions

In order to accurately predict the damage conditions using inelastic spectra analysis (ISA) demonstrated above by comparing the results with that obtained using inelastic time history analysis (ITHA), all the frame models described in Section 5.3.2 are

vibrated by a set of pre-defined ground motions. As the characteristic of ground motions should be considered in the research, the two earthquakes popular known and with different vibration characteristics are used again, which are the 1994 Northridge earthquake as shown in Figure 4-2 and the magnified 1940 El-Centro earthquake with scalar of 1.8 as shown in Figure 5-2. In addition, forty-nine synthetic time histories as shown in Figure 5-12 are designed based on the earthquake simulation technology (Nigam and Narayanan, 1994) to simulate a repeat of the 1994 Northridge earthquake, which will also be used in section 5.3.4.

Figure 5-12 Forty-nine synthetic acceleration time histories of 20 seconds (in 'g')

5.3.4 Comparison between ITHA & ISA

Based on the theory of normal mode method for the frame models subjected to ground motions described above, the practical application of inelastic spectra analysis to MDOF systems are shown in Tables 5-1 to 5-6. For example, the contents in Table 5-1 represent the simulation results of the 6-story 3-span frame in Figure 5-9(a) with floor mass of 630,400 subjected to ground motions. There are six modes of vibration for this model and the fundamental period is 1.77 s. The damping ratio at each mode is assumed to be equal to either 2% or 5%. The earthquake scalar $\Gamma_i / \hat{m}_i$ and yield displacement D_{yi} for each mode has been calculated accordingly. Two earthquakes are included to consider the effects of the characteristics of ground motions. The “ELC” presents the El-Centro earthquake and the “NOR” presents the Northridge earthquake.

As shown in Table 5-1, when damping ratio equals to 5% the damage potential indices in the first three modes are 0.41, 0.2655, and 0 for ELC and 0.6525, 0.2889, and 0.0363 for NOR. When SRSS method is used, the total DIs for the system are 0.4885 and 0.7145 respectively, which are obtained from using Eq. (5-18); when FMS method is used, the total DIs for the system are 0.41 and 0.6525 respectively, which equal to that in the fundamental mode. It is noted that the DIs in the third mode are small, and therefore the DIs in all modes beyond 3 are ignored. Similarly calculations have been performed for other structural models, and the results are shown in Tables 5-2 to 5-6.

The DI values from inelastic time history analysis (ITHA) are also given so that the accuracy of ISA can be approximated and compared. The absolute difference and relative difference of DI between SRSS and ITHA are listed in the first two columns of error in Tables 5-1 to 5-6, and the one between FMS and ITHA are given in the last two columns. It is observed from Table 5-1 that when damping ratios equal to 5%, the DIs from ITHA are 0.4638 and 0.6527, respectively, for the two earthquakes. Thus, the relative error between SRSS and ITHA is **5.33%** for ELC and **9.47%** for NOR; and the relative error between FMS and ITHA are **-11.6%** and **-0.03%**, respectively.

From the seismic damage evaluation point of view, it seems that these errors are quite acceptable. However, further study is required in order to find out whether SRSS or FMS is better in approximating the responses obtained from ITHA.

From Tables 5-2 to 5-6, the same definition of symbols and the same methods are used to carry out simulations for several different combination cases. These combinations include not only different damping ratios, ground motions as shown in Table 5-1, but also different earthquake magnifying factors as in Table 5-3. In Table 5-3, it is observed that the increase of DI is not proportional to the increase of earthquake magnifying factor. This is reasonable since if the earthquake ground motion is large to begin with, it may have caused structural to collapse already. At this time, magnifying the earthquake ground motion will bring no further destruction to the structure, which means the value of DI cannot be increased anymore. It is also observed from Table 5-3 that the largest relative error of DI is -37.57%, which is quite large but occurs in the case where DI calculated from ITHA is only 0.0535. While the DI from SRSS is 0.0334, this shows that the results are actually very close in the absolute sense. Furthermore, both of them are quite close to zero, which corresponds to the condition of no structural damage at all. Therefore, the large error in this range is ignorable.

The effects of variation in floor mass, building height and building span are now considered by comparing the results from different tables. For example, when the floor mass of the 6-story 3-span frame shown in Figure 5-9(a) decreases from 630,400 kg to 210,133 kg, the simulation results change greatly from Table 5-1 to Table 5-2. As the fundamental period decreases from long period of 1.77 s to middle period of 1.02 s, the increases of the anti-vibration ability of structures and the decrease of DI values in all four rows of results prove it well. When damping ratio equals to 5%, it can be seen that only the fundamental mode contribute to the overall structural damage subjected to Northridge earthquake, thus the error between FMS and ITHA is the same with that between SRSS and ITHA. However, the increase of accuracy in approximating ITHA cannot count on the decrease of DI values.

When the building span of the 6-story frame increase from three as shown in Figure 5-9(a) to six as shown in Figure 5-9(b), the simulation results change greatly from Table 5-1 to Table 5-4. The increase of span number enhances the structural ability in defending against the earthquake, and therefore all the DI values decrease in Table 5-4 as compared to Table 5-1. The contribution from higher modes to overall system damage also shrinks. However, there is no improvement on the accuracy in approximating ITHA responses, as the errors remain within 25%.

Buildings with different heights are also studied to make a comparison, Tables 5-2 and 5-6 summarize the simulation results for the 6-story 3-span model shown in Figure 5-9(a) and the 10-story 3-span model shown in Figure 5-11 respectively, while both models have the same floor mass of 210,133 kg. It can be observed that the 10-story 3-span model performs much better during the El-Centro earthquake, where the small DIs reflect that there is nearly no damage under this condition. The large error of -70.82% between FMS and ITHA for ELC and 2% damping ratio occurs since in this instance the second mode contributes the most to the structural damage, thus the SRSS method has the advantages over FMS method in some sense. The simulation results of the 3-storey 4-span model shown in Figure 5-10 is listed in Table 5-5 for reference. For this short period model, the fundamental mode plays the most important role in damage potential prediction, and therefore there's no difference between SRSS and FMS at all.

In order to study the effects of the characteristics of ground motions in detail, the 49 synthesis ground acceleration time histories as shown in Figure 5-12 are used to vibrate the typical 6-storey 3-span building as shown in Figure 5-9(a). Figure 5-13 gives the total DI values obtained from ITHA, SRSS and FMS subjected to the 49 ground motions, while the absolute error and relative error of DI between SRSS and ITHA or between FMS and ITHA are shown in Figures 5-14 and 5-15, respectively. From these three figures it is observed that most of the relative errors between ISA and ITHA, no matter which method of SRSS or FMS is used, are within 25%. On this condition the FMS performs better than SRSS, which coincides with the results shown in Table 5-1. Several factors may have contributed to the errors between ISA and

ITHA, and one main factor is that ideal elastic-plastic behavior is assumed for ISA for each mode of vibration. The original structures contain tens of plastic hinges, and these hinges deform inelastically at different levels of structural displacements. Therefore, representing all the plastic hinges using only the SDOF oscillator with ideal elastic-plastic behavior in ISA may only give approximate results. Finally, as shown in Figures 5-13 to 5-15, there seems to be more chances that the ISA overestimates the overall structural damage than ITHA. The main reason for this overestimation of damage potential is because of the assumption where the coupling of inelastic displacement among each mode is forced to be uncoupled in Eq. (5-15), and as discussed the error can be as much as 25%. However, since the amount of structural damage during an earthquake are overestimated, the ISA method based on the proposed damage potential index will therefore ensure a conservative result when used in structural design.

From the above discussion, it seems that evaluating damage of structures using ISA method based on the proposed damage potential index is quite economical in computational cost while achieving sufficient accuracy. For various floor masses, damping ratios, building heights and spans, characteristic and magnitude of the ground motions, all relative errors between ISA and ITHA are within 25%. This shows that using ISA method in assessing structural damage conditions also has a general application and can achieve a consistent accuracy. Once damage index spectral ordinates for a set of recorded ground motions are computed, it is possible to estimate the variation of damage index spectra with source-to-site distance. In addition, the proposed damage index spectra can be used to quantify the damage potential of any given recorded ground motion and relate it to seismic structural performance categories. This makes the proposed damage index spectra together with ISA promising for performance-based seismic vulnerability assessment of existing structures. For example, following an earthquake, generating contour maps of damage index spectral ordinates can be done rapidly at selected structural periods of vibration to provide useful information on the spatial distribution of damage potential in various structures for the specific recorded earthquake ground motion. Utilization of an up-to-

date inventory of existing structures enhances the reliability of the spatial distribution of damage spectra to identify the damaged areas in the structure.

Finally, it should be point out that damage assessment of steel structures due to earthquakes is fuzzy instead of rigorous work in some sense. There exists great uncertainties, such as the soil conditions, the type and design of the frames, the unseen characteristics of the earthquakes, and so on, no one can tell exactly which damage the structure will suffer, and the proposed damage potential index only serves as an indication to approximate the damage levels of the building. Even if there is a 25% relative error of DI between ISA and ITHA, it may still be accurate in terms of reflecting the appropriate structural damage level. As the advantages of ISA outweigh the accuracy of this method, ISA seems satisfactory and acceptable.

In a summary, assessing seismic damage of steel structures using the ISA method based on the proposed damage potential index can be widely applied and have a great significance. As for the practical application of the damage indices, i.e., the relationship between the damage indices and the structural performance categories, it will be discussed in the next chapter.

Table 5-1 Comparison between ITHA & ISA for 6Story-3Span frame (M=630,400 kg)

			MODE						SRSS	ITHA	ERROR			
			1	2	3	4	5	6			SRSS		FMS	
												%		%
T _i			1.77	0.64	0.37	0.26	0.20	0.17	\	\	\	\	\	\
EQ SCALAR			4.23	1.48	0.90	0.64	0.48	0.25						
D _{yi} (cm)			51.44	13.94	8.18	6.76	6.91	9.38						
DI	$\zeta=2\%$	ELC	0.5279	0.4039	0	\	\	\	0.6647	0.6258	0.0389	6.22	-0.0979	-15.64
		NOR	0.7442	0.3497	0.1740	\	\	\	0.8405	0.7479	0.0926	12.38	-0.0037	-0.49
	$\zeta=5\%$	ELC	0.4100	0.2655	0	\	\	\	0.4885	0.4638	0.0247	5.33	-0.0538	-11.60
		NOR	0.6525	0.2889	0.0363	\	\	\	0.7145	0.6527	0.0618	9.47	-0.0002	-0.03

* Considering variation in (1) damping ratio; (2) ground motion.

Table 5-2 Comparison between ITHA & ISA for 6Story-3Span frame (M=210,133 kg)

			MODE						SRSS	ITHA	ERROR			
			1	2	3	4	5	6			SRSS		FMS	
												%		%
T_i			1.02	0.37	0.21	0.15	0.12	0.10	\	\	\	\	\	\
EQ SCALAR			2.44	0.85	0.52	0.37	0.28	0.14						
D_{yi} (cm)			29.70	8.05	4.72	3.90	3.99	5.41						
DI	$\zeta=2\%$	ELC	0.4571	0	0	\	\	\	0.4571	0.4307	0.0264	6.13	\	\
		NOR	0.4511	0.1322	0	\	\	\	0.4701	0.5873	-0.1172	-19.96	-0.1362	-19.96
	$\zeta=5\%$	ELC	0.3545	0	0	\	\	\	0.3545	0.2963	0.0582	19.64	\	\
		NOR	0.3963	0	0	\	\	\	0.3963	0.5019	-0.1056	-21.04	\	\

* Considering variation in (1) damping ratio; (2) ground motion; (3) floor mass (compare with Table 5-1).

Table 5-3 Comparison between ITHA & ISA for 6Story-3Span frame with $\zeta=5\%$ (M=630,400 kg)

			MODE						SRSS	ITHA	ERROR			
			1	2	3	4	5	6			SRSS		FMS	
												%		%
T _i			1.77	0.64	0.37	0.26	0.20	0.17	\	\	\	\	\	\
EQ SCALAR			4.23	1.48	0.90	0.64	0.48	0.25						
D _{yi} (cm)			51.44	13.94	8.18	6.76	6.91	9.38						
DI	ELC	×0.5	0.0334	0	0	\	\	\	0.0334	0.0535	-0.0201	-37.57	\	\
		×0.8	0.2886	0.1328	0	\	\	\	0.3177	0.3301	-0.0124	- 3.76	-0.0415	-12.57
		×1.0	0.4100	0.2655	0	\	\	\	0.4885	0.4638	0.0247	5.33	-0.0538	-11.60
		×1.2	0.4945	0.3348	0	\	\	\	0.5972	0.5497	0.0475	8.64	-0.0552	-10.04
		×1.5	0.5936	0.3693	0	\	\	\	0.6991	0.6217	0.0774	12.45	-0.0281	-4.52
	NOR	×0.5	0.5511	0	0	\	\	\	0.5511	0.5025	0.0486	9.67	\	\
		×0.8	0.6144	0.1324	0	\	\	\	0.6285	0.5808	0.0477	8.21	0.0336	5.79
		×1.0	0.6525	0.2889	0.0363	\	\	\	0.7145	0.6527	0.0618	9.47	-0.0002	-0.03
		×1.2	0.6853	0.3854	0.2084	\	\	\	0.8134	0.7135	0.0999	14.00	-0.0282	-3.95
		×1.5	0.7317	0.4745	0.3703	\	\	\	0.9474	0.7513	0.1961	26.10	-0.0196	-2.61

* Considering variation in (1) ground motion; (2) earthquake magnifying factor.

Table 5-4 Comparison between ITHA & ISA for 6Story-6Span frame (M=630,400 kg)

			MODE						SRSS	ITHA	ERROR			
			1	2	3	4	5	6			SRSS		FMS	
												%		%
T_i			1.25	0.45	0.26	0.18	0.14	0.12	\	\	\	\	\	\
EQ SCALAR			4.23	1.48	0.90	0.65	0.48	0.25						
D_{yi} (cm)			51.57	19.86	11.14	8.29	6.91	9.33						
DI	$\zeta=2\%$	ELC	0.4909	0	0	\	\	\	0.4909	0.4562	0.0347	7.61	\	\
		NOR	0.6708	0.0249	0	\	\	\	0.6713	0.7011	-0.0298	- 4.25	-0.0303	- 4.32
	$\zeta=5\%$	ELC	0.3309	0	0	\	\	\	0.3309	0.2711	0.0598	22.06	\	\
		NOR	0.5604	0	0	\	\	\	0.5604	0.5811	-0.0207	- 3.56	\	\

* Considering variation in (1) damping ratio; (2) ground motion; (3) building span (compare with Table 5-1).

Table 5-5 Comparison between ITHA & ISA for 3Story-4Span frame (M=52,525 kg)

			MODE			SRSS	ITHA	ERROR			
			1	2	3			SRSS		FMS	
									%		%
T _i			0.51	0.17	0.08	\	\	\	\	\	\
EQ SCALAR			0.88	0.32	0.17						
D _{yi} (cm)			6.70	2.15	1.33						
DI	ζ=2%	ELC	0.3843	0	0	0.3843	0.3433	0.041	11.94	\	\
		NOR	0.4924	0	0	0.4924	0.4320	0.0604	13.98	\	\
	ζ=5%	ELC	0.2438	0	0	0.2438	0.1910	0.0528	27.64	\	\
		NOR	0.3538	0	0	0.3538	0.2741	0.0797	29.08	\	\

* Considering variation in (1) damping ratio; (2) ground motion; (3) building height (compare with Table 5-1 and 5-6).

Table 5-6 Comparison between ITHA & ISA for 10Story-3Span frame (M=210,133 kg)

			MODE										SRSS	ITHA	ERROR			
			1	2	3	4	5	6	7	8	9	10			SRSS		FMS	
																%		%
T _i			1.47	0.54	0.32	0.23	0.17	0.14	0.12	0.10	0.09	0.08	\	\	\	\	\	\
EQ SCALAR			3.07	1.15	0.73	0.52	0.41	0.32	0.25	0.22	0.20	0.20						
D _{yi} (cm)			58.52	14.95	8.76	6.19	4.85	4.78	5.84	5.04	4.76	3.56						
DI	$\zeta=2\%$	ELC	0.0724	0.1770	0	\	\	\	\	\	\	\	0.1912	0.2481	-0.0569	-22.93	-0.1757	-70.82
		NOR	0.6533	0.0464	0	\	\	\	\	\	\	\	0.6549	0.6398	0.0151	2.36	0.0135	2.11
	$\zeta=5\%$	ELC	0	0	0	\	\	\	\	\	\	\	0	0.0539	-0.0539	\	\	\
		NOR	0.5527	0	0	\	\	\	\	\	\	\	0.5527	0.5170	0.0357	10.39	\	\

* Considering variation in (1) damping ratio; (2) ground motion; (3) building height (compare with Table 5-1 and 5-5).

Figure 5-13 Comparison of DI between ITHA & ISA subjected to 49 synthetic acceleration time histories

Figure 5-14 Error of DI between ITHA & ISA subjected to 49 synthetic acceleration time histories

Figure 5-15 Error of DI (in %) between ITHA & ISA subjected to 49 synthetic acceleration time histories

CHAPTER SIX

CHARACTERISTICS OF PROPOSED DAMAGE POTENTIAL INDEX

6.1 Damage Potential Index Limits at Different Performance Levels

In recent years, large numbers of research works focus on performance-based earthquake engineering (PBEE), and the Pacific Earthquake Engineering Research (PEER) Center has PBEE as its principal focus for research and development. Performance objectives are statements of acceptable performance of the structure. The performance target can be specified limits on any response parameter such as stresses, strains, displacements, accelerations, drift ratio, and etc. It is appealing to express the performance objective in terms of a specific damage state or the probability of failure against a prescribed probability demand level. Performance-based earthquake engineering is a more encompassing concept that includes performance-based design, evaluation and construction engineering. Some progress has been made over the past several years in the development of a global framework and in various research aspects. In this research, the problem of interest is mainly from the design point of

view. The objective is to give an indication based on the newly proposed damage potential index that can limit the structural damage to certain desirable levels during the next major earthquake, which is unknown at the time of design. Since drift ratio has been leveled to different limits corresponding to different performance levels in some engineering manuals such as “SEAOC Blue Book” as a performance-based structural design basis (Ghobarah, 2001), recommended values for proposed damage potential index for different performance levels should be provided here. Therefore, probability analysis is performed to correlate drift ratio with the presently proposed damage potential index defined in Eq. (4-23).

6.1.1 Probabilistic Analysis

While the structural damage assessment is still in the early development stage, probability theory has been used extensively in different areas of research as well as in practical applications. Earthquakes are often modeled as a stochastic process where the input ground motion to the structure is considered as a white noise process or some other random processes. However, the complexity of this approach is generally not appropriate to extend the analysis to inelastic structural response. Therefore, in order to use probability theory on structures responding in the inelastic domain, an alternative approach must be used.

This research considers the position that in the damage assessment and design of civil structures, structural engineers will be required to take an ensemble of earthquake ground motion records and perform nonlinear time history Monte Carlo simulations. The selection of an ensemble of earthquake ground motions is a decision made by the structural engineer or by the building code. For example, the UBC defines a design basis earthquake that has a 10% probability of being exceeded in a 50 year time period. Once this design level and earthquake ground motion is selected, the geotechnical engineer can then provide the ensemble of earthquake time histories where the average of this ensemble agrees favorably with the design basis earthquake response spectra that can occur at a specific site with soil condition that can be obtained through field measurements.

The design level of earthquake ground motion used in this study is the earthquake ground motion at the site for a repeat of the Northridge earthquake. Forty-nine synthetic time histories were developed and shown in Figure 5-12. These 49 earthquake time histories represent the ensemble of records for this study and they are labeled from 1 to 49. Performing 49 dynamic time history analyses gives 49 sets of results, which include 49 *PE* density spectra, *IE* density spectra, DR spectra, and DI spectra. Since the objective here is to find a relationship between drift ratio and proposed damage potential index, let the random variables be DR (drift ratio) and DI (damage index). Assume that the two variables take a Rayleigh probability density function (PDF) of the form

$$f(X) = \begin{cases} \frac{X}{\sigma^2} e^{-X^2/2\sigma^2} & , \quad X \geq 0 \\ 0 & , \quad X < 0 \end{cases} \quad (6-1)$$

where X represents either one of the two random variables DR or DI, and σ is the only parameter of the PDF to be determined by the sample data obtained through numerical simulations. This PDF is appropriate since both DR and DI are always positive given that an earthquake has occurred. The mean of the PDF in Eq. (6-1) is

$$\mu_X = \sigma \times \sqrt{\pi/2} \quad (6-2)$$

If the mean of the PDF is estimated by mean of the responses calculated based on an ensemble of earthquake ground motions, then solving for σ in Eq. (6-2) gives

$$\sigma = \mu_X \times \sqrt{2/\pi} \quad (6-3)$$

The cumulative distribution function (CDF) corresponding to the PDF in Eq. (6-1) is

$$F(X) = 1 - e^{-\frac{1}{2\sigma^2}X^2} \quad (6-4)$$

Based on the cumulative distribution function as given in Eq. (6-4), the probability of failure (POF) for each random variable DR and DI are

$$POF(DR > DR_{\max}) = e^{-\frac{1}{2\sigma_{DR}^2}DR_{\max}^2} \quad (6-5)$$

$$POF(DI > DI_{\max}) = e^{-\frac{1}{2\sigma_{DI}^2}DI_{\max}^2} \quad (6-6)$$

Therefore, the probability of failure (e.g., that repair work is necessary) can be computed by selecting the maximum probability calculated using Eqs. (6-5) and (6-6).

6.1.2 Performance-based Design Criteria

Based on the DR spectra and DI spectra calculated using the ensemble of 49 earthquake time histories, the PDFs of both DR and DI are generated corresponding to structures with periods of 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 seconds. Here the three different vibration parameters of yield displacement, damping ratio, and earthquake scalar are considered, and the results are shown in Figures 6-1 to 6-4 corresponding to different parameters listed in the figures. As shown in these four figures, the abscissa value of peak point represents the value of σ defined in Eq. (6-3). It can be observed that larger values of σ correspond to longer structural periods of vibration. Thus, it is known from Eq. (6-2) that the mean values of the PDF of both DR and DI are larger for the cases with longer period, and this also means longer period structures tend to suffer more damage as long as yield displacement remains unchanged. In addition, the mean values of the PDF of DR and DI both decrease with an increase either of damping ratio or yield displacement level. This is reasonable since both increasing structural damping ratio and increasing the elastic response domain benefit the seismic resistant ability of structures.

The third column of Table 6-1 gives the suggested performance levels and damage state expressed by drift ratio (Ghobarah, 2001). Based on the mean values computed from the 49 samples, σ_{DR} and σ_{DI} in Eqs. (6-5) and (6-6) can be calculated using Eq. (6-3). Once this is done, the probability of failure of DR and DI can be calculated. Since $0 \leq DI < 1$, maximum DIs (i.e., $DI_{\max}$) equal to 0.1, 0.2, ..., 0.9 are considered, while maximum DRs (i.e., $DR_{\max}$) equal to five values corresponding to the different performance levels are considered:

$$DI_{\max} = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} \quad (6-7)$$

$$DR_{\max} = \{0.005, 0.01, 0.015, 0.02, 0.025\} \quad (6-8)$$

Setting the probability of failure of DI to be consistent with the failure probability based on the DR corresponding to the five performance levels, the recommended values for the proposed damage potential index corresponding to different performance levels can be developed, which is a primary objective of this research.

Subjected to the ensemble of 49 earthquake time histories, the probability of failure spectra for DR exceeding five levels of $DR_{\max}$ and DI exceeding nine levels of $DI_{\max}$ are shown in Figures 6-5 and 6-6, respectively. The effects of different damping ratios and yield displacements are also indicated in the figures. For structures with periods of 0.5, 1.0, and 1.5 seconds, the probability of failure (POF) for DR and DI at different levels are compared in Tables 6-2 and 6-3. In addition, Figures 6-7 and 6-8 show the probability of failure spectra for DR and DI respectively subjected to the same earthquake ensembles but with a scalar of 0.5, while Tables 6-4 and 6-5 compare the values at periods of 1.0, 1.5 and 2.0. Similarly, Figures 6-9 and 6-10 show the probability of failure spectra for DR and DI, respectively, subjected to the same earthquake ensembles but with a scalar of 0.25, while the values for structures with periods equal to 1.5, 2.0 and 2.5 are summarized in Tables 6-6 and 6-7.

In Tables 6-2 to 6-7, Columns (a), (b), and (c) denotes the three cases of (a) damping ratio of 5% and yield displacement of 9cm; (b) damping ratio of 8% and yield displacement of 9cm; and (c) damping ratio of 5% and yield displacement of 12cm, as shown in Figures 6-5 to 6-10. The choice of different periods in Tables 6-2 to 6-7 is because, with the decrease of ground motions (earthquake scalar reducing from 1.0 to 0.5 and then to 0.25), the period at which structure can survive without damage becomes longer and longer. For example, the probability of failure of DI is zero only at period of 0.5 with an earthquake scalar of 1.0 as shown in Figure 6-6, but is zero at both period 0.5 and 1.0 with an earthquake scalar of 0.25 as shown in Figure 6-10. Because of the minimal damage that structure is suffering for the three periods of 0.5, 1.0, and 1.5 for the case with an earthquake scalar of 0.25, the results from comparisons between the POF of DR and DI will probably make no sense.

From the six tables, it is observed that seismically-excited structural damage evaluated by the proposed damage potential index is generally smaller than that evaluated by drift ratio. In other words, using drift ratio often overestimates the real damage. Now considering Tables 6-2, 6-4, and 6-6, the effects of different damping ratio, yield displacement and earthquake scalar on POF values of DR are not clear. On the

contrary, the advantages of increased damping ratio and yield displacement as well as the effects of earthquake magnitude are clearly shown by the proposed damage potential index in Tables 6-3, 6-5, and 6-7. This explains well the proposed damage evaluation method is more rational in predicting damage than drift ratio on certain occasions. However, it is observed from Tables 6-3, 6-5, and 6-7 that the difference in POF values between one period and another is very large, which means the damage condition of structures within two periods can change very greatly and rapidly. For example, in Table 6-3, there is no damage at all for structures with period of 0.5 second, while for structures with period of 1.0 second; there is a probability of 94% for DI exceeding 10% and a probability of 5% for DI exceeding 70% for the same vibration. This phenomenon still requires further investigations in the future.

Learning from the results in Tables 6-2 to 6-7, the correlation between drift ratios with the presently proposed damage potential index are achieved based on approximate comparative levels of probability of failure. Thus, the tentative values for the proposed damage potential index corresponding to different performance levels are also listed in the fourth column of Table 6-1. It can be observed that structures are at life-safe damage state with damage index less than 50%. When the value of damage index exceeds 70%, structures will suffer total collapses. Therefore, Table 6-1 could not only be used to evaluate the structural damage during an earthquake, but also be regarded as a design method using performance-based design corresponding to the proposed damage potential index. However, these results are still in their preliminary study and further work is needed.

Recall for the conclusions obtained at the end of Chapter 5, where almost all the errors in approximating ITHA using ISA are within 25%. As it was concluded that even if there is a 25% error of DI between ISA and ITHA, it may still be accurate in terms of reflecting the appropriate structural damage level. It can be easily proven here. For example, the largest error is as much as -21.04% in Table 5-2, though the DI values obtained from ITHA and ISA are 39.63% and 50.19%, respectively, they still satisfy the third performance level and within life-safe damage range. Therefore, the accuracy using ISA method in assessing structural damage conditions can be guaranteed.

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

Figure 6-1 Rayleigh PDF of drift ratio (EQ scalar = 1.0)

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

Figure 6-2 Rayleigh PDF of damage index (EQ scalar = 1.0)

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

Figure 6-3 Rayleigh PDF of drift ratio (EQ scalar = 1.5)

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

Figure 6-4 Rayleigh PDF of damage index (EQ scalar = 1.5)

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

**Figure 6-5 Probability of failure for different drift ratio levels
(EQ scalar=1.0)**

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

Figure 6-6 Probability of failure for different damage index levels (EQ scalar=1.0)

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

**Figure 6-7 Probability of failure for different drift ratio levels
(EQ scalar=0.5)**

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

Figure 6-8 Probability of failure for different damage index levels (EQ scalar=0.5)

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

**Figure 6-9 Probability of failure for different drift ratio levels
(EQ scalar=0.25)**

(a) Damping Ratio=5%, Yield Displacement=9cm

(b) Damping Ratio=8%, Yield Displacement=9cm

(c) Damping Ratio=5%, Yield Displacement=12cm

**Figure 6-10 Probability of failure for different damage index levels
(EQ scalar = 0.25)**

Table 6-1 Performance levels, drift ratio and damage index limits

Performance level	Damage state	Drift ratio	Damage index
Fully operational, Immediate occupancy	No damage	<0.2%	<10%
Operational, Damage control, Moderate	Repairable	<0.5%	<30%
Life safe-Damage state	Irreparable	<1.5%	<50%
Near collapse, Limited safety, Hazard reduced	Severe	<2.5%	<70%
Collapse		>2.5%	>70%

Table 6-2 Probability of failure for drift ratio (EQ scalar=1.0)

DR	Probability of Failure (%)								
	0.5 second			1.0 second			1.5 second		
	a	b	c	a	b	c	a	b	c
0.005	96	95	97	99	99	97	100	100	100
0.01	86	81	87	98	97	87	99	99	99
0.015	72	62	73	95	93	73	99	98	99
0.02	56	43	57	91	88	57	97	97	97
0.025	40	27	41	86	83	41	96	95	96

Table 6-3 Probability of failure for damage index (EQ scalar=1.0)

DI	Probability of Failure (%)								
	0.5 second			1.0 second			1.5 second		
	a	b	c	a	b	c	a	b	c
0.1	0	0	0	94	90	83	98	98	97
0.2	0	0	0	78	66	48	93	91	90
0.3	0	0	0	57	40	19	84	80	79
0.4	0	0	0	37	19	5	73	67	66
0.5	0	0	0	21	8	1	62	54	53
0.6	0	0	0	11	2	0.1	50	41	40
0.7	0	0	0	5	0.6	0.01	39	30	28
0.8	0	0	0	2	0.1	0.0008	29	21	19
0.9	0	0	0	0.7	0.06	0	21	14	13

Table 6-4 Probability of failure for drift ratio (EQ scalar=0.5)

DR	Probability of Failure (%)								
	1.0 second			1.5 second			2.0 second		
	a	b	c	a	b	c	a	b	c
0.005	97	97	97	99	99	99	100	99	100
0.01	90	87	90	97	97	97	98	98	98
0.015	79	73	78	94	93	94	96	95	97
0.02	66	57	65	90	87	90	93	92	94
0.025	52	41	51	85	81	85	89	87	91

Table 6-5 Probability of failure for damage index (EQ scalar=0.5)

DI	Probability of Failure (%)								
	1.0 second			1.5 second			2.0 second		
	a	b	c	a	b	c	a	b	c
0.1	10	0	0	95	90	86	97	95	95
0.2	0.009	0	0	80	66	55	88	83	81
0.3	0	0	0	61	39	26	75	66	63
0.4	0	0	0	42	18	9	61	47	44
0.5	0	0	0	26	7	2	46	31	28
0.6	0	0	0	14	2	0.5	32	18	16
0.7	0	0	0	7	0.6	0.07	22	10	8
0.8	0	0	0	3	0.1	0.007	14	5	4
0.9	0	0	0	1	0.02	0.0006	8	2	2

Table 6-6 Probability of failure for drift ratio (EQ scalar=0.25)

DR	Probability of Failure (%)								
	1.5 second			2.0 second			2.5 second		
	a	b	c	a	b	c	a	b	c
0.005	98	97	98	99	98	99	98	98	99
0.01	91	87	91	95	93	95	94	93	94
0.015	80	73	81	88	85	89	87	85	87
0.02	68	57	68	80	75	82	78	75	79
0.025	54	42	55	71	63	73	68	63	69

Table 6-7 Probability of failure for damage index (EQ scalar=0.25)

DI	Probability of Failure (%)								
	1.5 second			2.0 second			2.5 second		
	a	b	c	a	b	c	a	b	c
0.1	6	0	0	82	50	26	74	55	11
0.2	0.002	0	0	46	6	0.4	31	9	0.02
0.3	0	0	0	17	0.2	0.0005	7	0.5	0
0.4	0	0	0	4	0.001	0	0.9	0.007	0
0.5	0	0	0	0.8	0	0	0.06	0	0
0.6	0	0	0	0.09	0	0	0.003	0	0
0.7	0	0	0	0.008	0	0	0.0001	0	0
0.8	0	0	0	0.0004	0	0	0	0	0
0.9	0	0	0	0	0	0	0	0	0

6.2 Correlation between the Proposed Damage Potential Index and the Characteristics of Ground Motion

It is well known that structural damage observed during earthquakes has a close relationship with the characteristics of ground motions. For a rather long time, researchers were trying to evaluate seismic damage only from characterization of ground motions. Thus, it is necessary to develop such relationship in order to investigate the characteristics of the proposed damage potential index in detail. The intensity of ground motion cannot be satisfactorily characterized by a single parameter. It is expected that different intensity measures are more suitable in the three characteristic spectra regions: short period (acceleration sensitive systems), intermediate period (velocity controlled responses) and long period (displacement sensitive systems). Consequently, a number of indices need to be considered.

The most generally used ground motion intensity indices are peak ground acceleration (*PGA*), peak ground velocity (*PGV*) and peak ground displacement (*PGD*). These indices are defined as the peak values of ground motion acceleration, ground motion velocity and ground motion displacement time histories respectively, i.e.:

$$PGA = (\ddot{x}_g)_{\max} \quad (6-9)$$

$$PGV = (\dot{x}_g)_{\max} \quad (6-10)$$

$$PGD = (x_g)_{\max} \quad (6-11)$$

Arias (1970) proposed a measure of earthquake intensity that relates to the sum of the energies dissipated per unit of mass by a population of damped oscillators of all natural frequencies:

$$I_A(\zeta) = \frac{\cos^{-1} \zeta}{g\sqrt{1-\zeta^2}} \int_0^{t_f} \ddot{x}_g^2(t) dt \quad (6-12)$$

where t_f is the total duration of the ground motion and g is the acceleration of gravity.

Housner (1975) argued that a measure of seismic destructiveness could be given by the average rate of buildup of the total energy per unit mass input to structures. By considering that the integral of the squared ground acceleration is proportional to the total input energy, he proposed the earthquake power index

$$P = \frac{1}{t_1 - t_2} \int_{t_1}^{t_2} \ddot{x}_g^2(t) dt \quad (6-13)$$

where t_1 and t_2 are the limits of the strong portion of motion. The popular definition of significant duration of motion, i.e. the interval between instants t_5 and t_{95} at which 5% and 95% of the integral in Eq. (6-12), was proposed by Trifunac and Brady (1975) in their study. Hereafter, the significant duration is designated as $t_d = t_{95} - t_5$. Then the indices mean-square acceleration P_a , mean-square velocity P_v , and mean-square displacement P_d can be defined as:

$$P_a = \frac{1}{t_{95} - t_5} \int_{t_5}^{t_{95}} \ddot{x}_g^2(t) dt \quad (6-14)$$

$$P_v = \frac{1}{t_{95} - t_5} \int_{t_5}^{t_{95}} \dot{x}_g^2(t) dt \quad (6-15)$$

$$P_d = \frac{1}{t_{95} - t_5} \int_{t_5}^{t_{95}} x_g^2(t) dt \quad (6-16)$$

For simplicity, without regard to duration or to the constants involved in Eq. (6-12), the integrals of the squared ground motions have been also used as indices in the form:

$$E_a = \int_0^{t_f} \ddot{x}_g^2(t) dt \quad (6-17)$$

$$E_v = \int_0^{t_f} \dot{x}_g^2(t) dt \quad (6-18)$$

$$E_d = \int_0^{t_f} x_g^2(t) dt \quad (6-19)$$

The root-mean-square values of the ground motions have also been considered as potential measures of earthquake intensities, i.e.:

$$a_{rms} = \sqrt{P_a} \quad (6-20)$$

$$v_{rms} = \sqrt{P_v} \quad (6-21)$$

$$d_{rms} = \sqrt{P_d} \quad (6-22)$$

as well as the square root of the integral of the squared ground motions, i.e.:

$$a_{rs} = \sqrt{E_a} \quad (6-23)$$

$$v_{rs} = \sqrt{E_v} \quad (6-24)$$

$$d_{rs} = \sqrt{E_d} \quad (6-25)$$

Other indices, which are based on the previously discussed quantities and include new parameters, have been proposed as descriptors of earthquake intensity. Araya and Saragoni (1984) defined the potential destructiveness of an earthquake as:

$$P_D = \frac{I_A}{v_0^2} \quad (6-26)$$

where I_A is given by Eq. (6-12) and v_0 is the number of zero-crossings per unit of time of the accelerogram. The significance of this parameter is the incorporation of the frequency content of the ground motion through v_0 .

Park et al. (1985) developed the “characteristic intensity”:

$$I_C = a_{rms}^{1.5} t_d^{0.5} \quad (6-27)$$

which is a reasonable representation of the destructiveness of ground motions because it correlates well with structural damage expressed in terms of their damage index. Fajfar et al. (1990) proposed the expression

$$I_F = PGV t_d^{0.25} \quad (6-28)$$

as a measure of the ground motion capacity to damage structures with fundamental periods in the intermediate period range.

All the above indices depend only on the ground motions and they are used to test their correlation with the newly proposed damage potential index. Only one response-related parameter, Housner's spectral intensity, is used as well. Housner (1952) argued that the pseudo-velocity (*PSV*) response spectrum itself was a measure of the severity of the earthquake, therefore defined the spectral intensity:

$$S_I(\zeta) = \int_{0.1}^{2.5} S_{vp}(\zeta, T) dT \quad (6-29)$$

where the pseudo spectral velocity S_{vp} is defined as:

$$S_{vp}(\omega_n, \zeta) = \omega_n S_d(\omega_n, \zeta) \quad (6-30)$$

and S_d is the spectral displacement defined in Eq. (5-1).

In the present study, inelastic time history analysis is carried out on a 6-story building frame subjected to the 49 synthetic ground motions as shown in Figure 5-12. The 6-story frame used here is the one used in Chapter 4 (see Figure 4-1), which has a fundamental period of 1.77 seconds. To visualize the correlation among newly proposed damage potential index and the intensity indices, plots are constructed for all the indices and some of them are shown in Figures 6-11 to 6-15. In these figures, each asterisk corresponds to each of the 49 earthquake records, and therefore the relationship between DI and PGA , PGV , PGD , a_{rms} , v_{rms} , d_{rms} , a_{rs} , v_{rs} , d_{rs} , I_A , I_C , I_F , P_D , and S_I are shown graphically. It can be seen that DI correlates well with all the target intensity indices for this building frame, which means the proposed DI has the ability to describe the influence of the characteristic variations of ground motions to the seismic damage of structures.

To make further studies of such influences, the magnified 1940 El-Centro earthquake as shown in Figure 5-2, the 1994 Northridge earthquake as shown in Figure 4-2, and the 1995 Kobe earthquake as shown in Figure 6-16 are applied to the 6-storey 3-span frame to carry out the inelastic time history analysis. The frame is subjected to the increasing intensity of these earthquake ground motions with the peak ground

accelerations (PGA) ranging from 0.1g to 1.0g with a 0.1g increments. Figure 6-17 gives the normalized spectral acceleration of these three earthquakes, and the damage index versus PGA is illustrated in Figure 6-20. It can be observed from Figure 6-20 that the seismic damage of a given frame consistently becomes larger with the increase of PGA for all the three earthquakes, which coincides with the fact that DI correlates well with PGA. However, the seismic damage is dramatically different from one ground motion to another, which prove that ground motion intensity itself cannot be used alone to predict structural damage. In detail, the DI values subjected to Northridge earthquake are always the largest among the three cases of vibration, while the two lines of DI correspond to El-Centro earthquake and Kobe earthquake cross each others after PGA increases to 0.5g, which means the damage caused by Kobe earthquake is larger than that caused by El-Centro earthquake when the PGA of these two earthquakes is smaller than 0.5g, but the damage caused by El-Centro earthquake is larger than that caused by Kobe earthquake with the PGA beyond 0.5g. In addition, Figure 6-20 shows that almost all the DIs are smaller than 0.7 even if with PGA as large as 1.0g. Therefore, it has been shown that this building is well designed in preventing collapse during all these three earthquakes according to the performance limits of DIs proposed in the previous section.

Figure 5-12 gives the 49 synthetic time histories as a repeat of Northridge earthquake, while the mean and envelopes of spectral acceleration of these 49 time histories are plotted in Figure 6-18. By randomly selecting 7 time histories from this ensemble of 49 earthquakes, the characteristic influence of the same earthquake on structural damage can be studied. Figure 6-19 gives the normalized spectral acceleration of the 7 selected time histories, whose labels are #1, #2, #3, #15, #25, #33, and #48 according to Figure 5-12. By applying this ensemble of 7 time histories to different frames of 6-storey 3-span (see Figure 5-9(a)), 6-storey 6-span (see Figure 5-9(b)), and 10-story 3-span (see Figure 5-11), the influence on structural damage from the variations of structural height and span can also be studied. These frames are also subjected to the increasing intensity of these earthquake ground motions with the peak ground accelerations (PGA) ranging from 0.1g to 1.0g with a 0.1g increment.

Figures 6-21, 6-22, and 6-23 show the results for 6-storey 3-span, 6-storey 6-span, and 10-story 3-span frames, respectively. From Figures 6-21 to 6-23, it is observed that the damage indices change considerably in the different frames with each individual ground motions. Figure 6-24 shows the average curves of the lines in Figures 6-21 to 6-23, which illustrates that there are significant differences in damage indices among the frames having different heights and spans. Taller structures seem to suffer less damage during a series of earthquakes with the same magnitude, expanding in spans also enhances the anti-earthquake ability of structures. Besides, all the three lines in Figure 6-24 tend to become stabilized as ground motion intensity increases, which means that the proposed DI is more sensitive to small and medium magnitude earthquakes. This is reasonable since structures that are already suffering great damage during severe earthquakes will not suffer much more damage if magnitude of the severe earthquakes is increased further.

Some scatters of results are observed in Figs. 6.20 to 6.24. The scatter of the results is more due to the varying characteristics of the ground motions than due to the consistency of the damage assessment model. The situation may be improved by a more comprehensive damage model and more appropriate scaling of the ground motions.

It is usually not advisable or recommended to draw many conclusions based on preliminary work. However, one thing seems certain is that it is unreliable to predict the structural damage in the frames using the characteristics of the ground motions only. Therefore, the proposed damage potential index can be used to show the structural damage more clearly and accurately in different frames subjected to different ground motions.

(a)

(b)

(c)

Figure 6-11 DI vs. PGA, PGV, and PGD

(a)

(b)

(c)

Figure 6-12 DI vs. a_{rms} , v_{rms} , and d_{rms}

(a)

(b)

(c)

Figure 6-13 DI vs. a_{rs} , v_{rs} , and d_{rs}

(a)

(b)

(c)

Figure 6-14 DI vs. I_A , I_C , and I_F

(a)

(b)

Figure 6-15 DI vs. P_D, S_I

Figure 6-16 1995 Kobe earthquake ground acceleration time history

Figure 6-17 Normalized response spectra of El-Centro EQ, Northridge EQ and Kobe EQ

Figure 6-18 Mean and envelopes of the acceleration spectra of the 49 synthetic ground motions

Figure 6-19 Normalized response spectra of the selected 7 ground motions

Figure 6-20 Variation of DI for 6-storey 3-span frame subjected to El-Centro EQ, Northridge EQ and Kobe EQ

Figure 6-21 Variation of DI for 6-storey 3-span frame subjected to selected 7 ground motions

Figure 6-22 Variation of DI for 6-storey 6-span frame subjected to selected 7 ground motions

Figure 6-23 Variation of DI for 10-storey 3-span frame subjected to selected 7 ground motions

Figure 6-24 Comparison of the average variations of the frames

6.3 Conclusion

Probability study is firstly performed in this chapter to show the advantage of using proposed damage potential index over drift ratio. Rayleigh PDF is chosen here to simulate the distribution of drift ratio and proposed damage potential index, and this PDF is appropriate since both drift ratio and proposed damage potential index are always positive given that an earthquake has occurred. However, owing to the lack of data from experiments, the assumption of Rayleigh PDF need to be verified in future research.

After the definitions of PDFs of drift ratio and proposed damage potential index, probability of failure (POF) of drift ratio and proposed damage potential index for various possible cases is calculated. The POF spectra of drift ratio and proposed damage potential index are generated and compared with each other. The effects of different vibration parameters such as damping ratio, yield displacement, and earthquake scaling factor are also studied. Results show that proposed damage potential index is more effective in evaluating structural damage than drift ratio since it can reflect more clearly the effects of increasing damping ratio and yield displacement as well as decreasing the magnitude of ground motions on the hazard resistant abilities of structures.

Meanwhile, in the end of the first part of this chapter, the tentative limits of the proposed damage potential index are given corresponding to each performance level, DI of 10%, 30%, 50% and 70% are defined to be the limits correspond to performance levels from minor damage to collapse.

In the second half of this section, the advantages of the proposed damage potential index over various forms of ground motion intensity are illustrated through numbers of numerical simulations. Not only the proposed damage potential index coincides well with the ground motion intensity indices, it is also found to be more meaningful and helpful in evaluating structural damage. Therefore, these features enable the proposed damage potential index, which evaluates structural damage in both response

and energy points of view, to be prospective in lieu of practical engineering applications.

However, further research is necessary to study the characteristics of this proposed damage potential index in other aspects. For example, the effects of different earthquake records with different soil conditions and different distances from the fault on the proposed damage potential index spectra as well as the POF spectra need to be further investigated for taking into considerations more properties of this damage index. Moreover, the limit value of propose damage index at each performance level need to be developed more precisely based on larger number of calculations and calibrations of various real structures subjected to real recorded ground motions when all these resources become available.

CHAPTER SEVEN

STRUCTURAL MODIFICATION AND ACTIVE CONTROL BASED ON THE PROPOSED DAMAGE POTENTIAL INDEX

7.1 Introduction

After introducing the required methodologies to calculate the responses and energies of structures as well as proposing damage potential indices to evaluate structural damage subjected to earthquake excitation in Chapters 3 to 6, better understanding of the structural dynamic behaviors is now obtained for a definite structure. However, the aim for engineers to study the response and energy time histories or spectra is to search for a better method to control the responses and energies, meanwhile making a good design in preventing structures from severe damages or even collapse. There are several ways to achieve such effects:

- From structural modification point of view, which mainly focuses on redesigning structures or replacing the weaker elements, such that loads and energies can be allocated more evenly among different structural members;

- From structural control point of view, which itself could be divided into passive structural control and active structural control.

Passive structural control is a method placing structural elements, or members manufactured in plants away from the building site in the structures to achieve certain defined objectives. Engineers control the response of the structures with the number and design of these passive control devices such as base isolators or damping elements. In general for passive control, the passive control devices are installed in the structure and then wait for extreme events (e.g., earthquake) to occur. The structure then responds to the induced forces without any modification in performance characteristics of the manufactured elements during the time history of the event. Active structural control, on the other hand, is when the performance characteristics of the manufactured structural element are modified to adjust for the particular forcing function as it occurs. In other words, active control of structures senses the lateral loading of the structure and actively applies external forces to overcome a negative earthquake response. Therefore, in this chapter, the structural modification method and active structural control method together with the newly proposed damage potential index will be combined to study the structural dynamic responses subjected to earthquake excitations, while leaving passive control to researchers in the future work.

Some people tend to think that in addition to the input energy imposed on the structure due to an earthquake, applying active control force injects more energy into the structure. In this case, the structure may be jeopardized when using active control because the structure must now dissipate control energy in addition to the input energy from the earthquake. As a matter of fact, Tzan and Pantelides (1998) have shown that the presence of optimal control reduces the input energy. In this chapter, one of the most popular algorithms, i.e., the Optimal Linear Control (OLC) algorithm will be used in conjunction with the damage index proposed to further confirm this observation. By giving numerical examples of applying active control, good supportive results on damage reduction in the system will be investigated to demonstrate the effectiveness of active control.

7.2 Structural Modification

The newly proposed damage evaluation model includes both the total damage index and local damage indices. The proposal of local damage indices supplies a probability for engineers to have a damage distribution figure of a structure in mind even before a ground vibration occurs. At this rate, engineers can determine the weakest parts of the structure during an earthquake and strengthen or adjust them in advance to avoid severe structural damage or even collapse. In this sense, the structure has to be modified to have better performances during earthquakes according to the objectives of engineers. The once used six-storey three-span moment-resisting frame as shown in Figure 4-1 is again used here as an example to demonstrate the effectiveness of the proposed damage index in structural modification aspects.

The parameters of this frame model are all the same with that used in Chapter 4, which have 30 degrees of freedom and 40 plastic hinge locations with floor masses of 630,400 kg per floor. Let this structural configuration be denoted as the “benchmark model”. The 1994 Northridge earthquake time history at Sylmar shown in Figure 4-2 and the magnified El-Centro earthquake time history with a scale factor of 1.8 shown in Figure 5-2 are used in the numerical simulation process. After the time history analysis is performed using Eqs. (3-40), (3-46), and (3-51), the energy responses and the damage indices in Eqs. (4-23) and (4-24) can also be calculated. Based on Eq. (4-23), the global damage index is found to be $DI_{Total} = 0.6875$ when Northridge earthquake is selected as the input ground motion, and $DI_{Total} = 0.5811$ when El-Centro is used. This shows that the damage experienced in the structure is larger for Northridge earthquake than that for El-Centro earthquake, which is reasonable since the peak ground acceleration of the Northridge earthquake is larger than that of the El-Centro earthquake. Based on Eq. (4-24), the local damage index DI_i at the i th PHL in the structure subjected to both earthquakes are computed and shown in Figures 7-1(a) and 7-1(b). Both figures show that the two locations where largest local damage occurs are at PHLs #2 and #3, since these are the locations where the local DI_i are at

maximum. This signifies that these two PHLs are the most vulnerable locations in the structure and will be the locations of major damage during any sizable earthquakes.

Based on this result, it is reasonable to conclude that the structure is most vulnerable at the base of the interior columns (i.e., W14×500), and using larger sections at these locations may be justified to improve the overall performance of this structure. Therefore, the two interior columns running from the ground to the third story are here replaced by two larger wide flange sections (i.e., W14×605), and let this new structural model be denoted as the “New Model”. The earthquake time histories shown in Figures 4-2 and 5-2 are again used in the numerical simulations to calculate the responses and damage indices, and the DI_{Total} subjected to these two earthquakes are 0.6952 and 0.5789, respectively. This again shows that damage on this New Model will be also greater for Northridge earthquake than that for El-Centro earthquake. The local DI_i at each PHL is shown in Figures 7-2(a) and 7-2(b). It can be observed that the local DI_i at both PHLs #2 and #3 decrease greatly for the New Model, from 7.88% in Figure 7-1(a) to 6.30% in Figure 7-2(a) for Northridge earthquake, and from 9.23% in Figure 7-1(b) to 7.26% in Figure 7-2(b) for El-Centro earthquake. Comparison between the Benchmark Model and the New Model among all 40 PHLs shows that 26 local DI_i increase, 12 local DI_i decrease, 2 local DI_i remain constant corresponding to Northridge earthquake. For El-Centro earthquake, 22 local DI_i increase and 18 local DI_i decrease. Although the DI_{Total} almost remains relatively unchanged when the structure is strengthened from the Benchmark Model to the New Model (i.e., from 0.6875 to 0.6952 for Northridge earthquake and from 0.5811 to 0.5789 for El-Centro earthquake), the local DI_i have *redistributed* more evenly among all the PHLs in the New Model, and this gives the chance for the structure to absorb more seismic damage locally, especially at the two interior bottom columns.

Comparing Figures 7-1(a) and 7-1(b) shows that larger portions of local damage contribution to total structural damage are different corresponding to different earthquake vibration characteristics. For example, larger DI_i in Figure 7-1(a) are

found at the ground, 2nd, and 3rd floors, but larger DI_i in Figure 7-1(b) are found at the ground, 2nd, and 6th floors. Similar results are also obtained when comparing Figures 7-2(a) and 7-2(b). This means that even if the overall damage extent is almost the same, the distribution of local damage may be quite different for different earthquakes. Thus it shows that the proposed DI_{Total} and local DI_i not only predict whether a structure will suffer major damage during an earthquake, but it also gives good indications to which part of the structure is most vulnerable and needs urgent attentions. More importantly from the practical point of view based on the proposed DI_{Total} and local DI_i , good structural design should reflect an even distribution of local DI_i among all PHLs. This will highly enhance the structure's overall capacity and reducing seismic damage.

Figure 7-1 Local DI (%) of Benchmark Model subjected to different earthquakes

(a) Northridge Earthquake

(b) El-Centro Earthquake

Figure 7-2 Local DI (%) of New Model subjected to different earthquakes

(↑ = increase, ↓ = decrease, — = constant comparing with Benchmark Model)

7.3 Active Control

Instead of changing some components of the structure in structural modification, another method of improving the structural performance and reducing the seismic damage is the use of active control. In recent years, the concept of active structural control has been advancing rapidly in preventing structural damage from earthquakes. Many different control algorithms have been proposed by various researchers (Housner et al., 1997). As an example of the most attractive control algorithms accepted by researchers, the basic theory and application of optimal control algorithm has been demonstrated in Yang (1975), Yang, Akbarpur and Ghaemmaghami (1987) and Soong (1990). Since few studies on the energy mechanism of active control have been done, Wong and Yang (2001) studied the two most popular linear control algorithms: optimal linear control and instantaneous optimal control, from energy point of view and proved that using active control in structures to resist earthquakes not only helps to stabilize the structure but also helps to dissipate large amount of energy that otherwise relying only on structural damping may not be sufficient. Therefore, the optimal linear control is employed in this section to verify the effectiveness of the newly proposed damage evaluation model.

7.3.1 Modified Equation of Motion with Active Control

Applying control forces to the structure changes the dynamic equilibrium equation, and thus some related equations introduced in the previous chapters must be modified to the new forms. Consider the dynamic equilibrium equation with static condensation in Eq. (3-35), applying control forces to this equation becomes:

$$\begin{aligned} \begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_n(t) \\ \ddot{\mathbf{X}}_a(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{X}}_n(t) \\ \dot{\mathbf{X}}_a(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}'_n(t) \\ \mathbf{X}'_a(t) \end{Bmatrix} \\ = - \begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_g(t) \\ \mathbf{0} \end{Bmatrix} + \begin{bmatrix} \mathbf{D}_n \\ \mathbf{D}_a \end{bmatrix} \mathbf{f}_c(t) \end{aligned} \quad (7-1)$$

where $\mathbf{D}_n$ and $\mathbf{D}_a$ are the $n \times p$ and $n_a \times p$ control force distribution matrices, respectively, $\mathbf{f}_c(t)$ is the $p \times 1$ control force vector and p is the number of control actuators in the active control system.

Consider the second equation of the matrix equation given in Eq. (7-1), which is

$$\mathbf{K}_{an} \mathbf{X}'_n(t) + \mathbf{K}_{aa} \mathbf{X}'_a(t) = \mathbf{D}_a \mathbf{f}_c(t) \quad (7-2)$$

Solving for the elastic displacement vector at ADOFs $\mathbf{X}'_a(t)$ gives

$$\mathbf{X}'_a(t) = -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n(t) + \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c(t) \quad (7-3)$$

Now substituting Eq. (7-3) into the first equation of Eq. (7-1) gives

$$\mathbf{M}_{nn} \ddot{\mathbf{X}}_n(t) + \mathbf{C}_{nn} \dot{\mathbf{X}}_n(t) + \bar{\mathbf{K}}_{nn} \mathbf{X}'_n(t) = -\mathbf{M}_{nn} \ddot{\mathbf{X}}_g(t) + \bar{\mathbf{D}}_n \mathbf{f}_c(t) \quad (7-4)$$

where

$$\bar{\mathbf{D}}_n = \mathbf{D}_n - \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{D}_a \quad (7-5)$$

It can be observed that Eq. (7-4) is similar to Eq. (3-38) with an additional control force term on the right hand side. Using state space numerical integration method of analysis, Eq. (7-4) becomes

$$\dot{\mathbf{z}}(t) = \mathbf{A} \mathbf{z}(t) + \mathbf{H} \mathbf{a}_g(t) + \mathbf{B} \mathbf{f}_c(t) + \mathbf{F}_p^c \mathbf{X}_n''(t) \quad (7-6)$$

where

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{nn}^{-1} \bar{\mathbf{D}}_n \end{bmatrix} \quad (7-7)$$

and other notations used in Eq. (7-6) is similar to those used in Eqs. (3-19) and (3-40).

The solution to this equation is

$$\mathbf{z}(t) = e^{\mathbf{A}(t-t_0)} \mathbf{z}(t_0) + e^{\mathbf{A}t} \int_{t_0}^t e^{-\mathbf{A}s} [\mathbf{H} \mathbf{a}_g(s) + \mathbf{B} \mathbf{f}_c(s) + \mathbf{F}_p^c \mathbf{X}_n''(s)] ds \quad (7-8)$$

Let $t_{k+1} = t$, $t_k = t_0$, $\Delta t = t - t_0$ and represent the ground acceleration vector and the inelastic displacement vector using pulses within the small time step Δt . Performing the integration in Eq. (7-8) gives

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' + \mathbf{G} \mathbf{f}_{c,k} \quad (7-9)$$

where

$$\mathbf{G} = \mathbf{A}^{-1} (e^{\Delta t} - \mathbf{I}) \mathbf{B} \quad (7-10)$$

and other notations used in Eq. (7-9) are similar to those used in Eq. (3-40). Comparing Eq. (7-9) with Eq. (3-40) shows that the only difference is the additional control force term appearing on the right hand side of Eq. (7-9).

As presented in Section 3.4, the total displacement must be decomposed into the displacements at both the ODOFs and ADOFs, where the displacements at the ADOFs must be rewritten in the form of plastic rotations in order to apply the force analogy method. Accordingly, similar derivation must be performed here in order to include the effect of control force. Using Eq. (7-3) and the total displacement is the sum of elastic displacement and inelastic displacement presented in Eq. (3-4), the displacements at all degrees of freedom can be rewritten as:

$$\begin{aligned} \mathbf{X}_{k+1} &= \begin{bmatrix} \mathbf{X}_{n,k+1} \\ \mathbf{X}_{a,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{n,k+1} \\ \mathbf{X}'_{a,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{n,k+1} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_{n,k+1} + \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_{c,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{X}_{n,k+1} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} (\mathbf{X}_{n,k+1} - \mathbf{X}''_{n,k+1}) + \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_{c,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{X}''_{n,k+1} \\ \mathbf{X}''_{a,k+1} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{D}_a \end{bmatrix} \mathbf{f}_{c,k+1} \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{X}''_{k+1} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{D}_a \end{bmatrix} \mathbf{f}_{c,k+1} \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{K}_P^{-1} \mathbf{K}_P \boldsymbol{\Theta}''_{k+1} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{D}_a \end{bmatrix} \mathbf{f}_{c,k+1} \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}''_{k+1} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{D}_a \end{bmatrix} \mathbf{f}_{c,k+1} \end{aligned} \quad (7-11)$$

Substituting Eq. (7-11) into the force analogy method equation given in Eq. (3-29), the modified governing equation for the force analogy method becomes

$$\mathbf{M}_{k+1} + \overline{\mathbf{K}}_R \Delta \boldsymbol{\Theta}'' = \overline{\mathbf{K}}_P^T \mathbf{X}_{n,k+1} - \overline{\mathbf{K}}_R \boldsymbol{\Theta}''_k + \overline{\mathbf{K}}_D^T \mathbf{f}_{c,k+1} \quad (7-12)$$

where

$$\overline{\mathbf{K}}_D^T = \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{D}_a \end{bmatrix} \quad (7-13)$$

Note that Eq. (7-12) is similar to Eq. (3-46) but with an additional control force term on the right hand side of Eq. (7-12). This shows the effects of applying active control on the inelastic response of the structure.

In addition, Eqs. (3-54), (3-55) and (3-56) can also be modified to include control effects. First consider the inelastic displacement in Eq. (3-54), substituting Eqs. (7-3) and (7-11) into the inelastic displacement equation gives:

$$\begin{aligned}\mathbf{X}_k'' &= \mathbf{X}_k - \mathbf{X}_k' = \mathbf{X}_k - \begin{Bmatrix} \mathbf{X}_{n,k}' \\ \mathbf{X}_{a,k}' \end{Bmatrix} = \mathbf{X}_k - \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k}' - \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1}\mathbf{D}_a \end{bmatrix} \mathbf{f}_{c,k} \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_k'' - \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k}' \\ &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k}'' + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_k''\end{aligned}\quad (7-14)$$

Then using Eqs. (3-51) and (7-14), the inelastic moment vector becomes

$$\begin{aligned}\mathbf{M}_{k+1}'' &= -\mathbf{K}_R \boldsymbol{\Theta}_{k+1}'' + \mathbf{K}_P^T \mathbf{X}_{k+1}'' \\ &= -\mathbf{K}_R \boldsymbol{\Theta}_{k+1}'' + \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}'' + \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_{k+1}'' \\ &= \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}'' + \left(\mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P - \mathbf{K}_R \right) \boldsymbol{\Theta}_{k+1}'' \\ &= \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \boldsymbol{\Theta}_{k+1}'' + \left(\mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P - \mathbf{K}_R \right) \boldsymbol{\Theta}_{k+1}'' \\ &= - \left[\left(\mathbf{K}_R - \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \right) - \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \right] \boldsymbol{\Theta}_{k+1}'' \\ &= - \left[\bar{\mathbf{K}}_R - \bar{\mathbf{K}}_P^T \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \right] \boldsymbol{\Theta}_{k+1}''\end{aligned}\quad (7-15)$$

Similarly, based on Eq. (7-3), the elastic moment vector becomes:

$$\begin{aligned}\mathbf{M}_{k+1}' &= \mathbf{K}_P^T \mathbf{X}_{k+1}' = \mathbf{K}_P^T \begin{Bmatrix} \mathbf{X}_{n,k+1}' \\ \mathbf{X}_{a,k+1}' \end{Bmatrix} \\ &= \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1}\mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}' + \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} \\ \mathbf{K}_{aa}^{-1}\mathbf{D}_a \end{bmatrix} \mathbf{f}_{c,k+1} \\ &= \bar{\mathbf{K}}_P^T \mathbf{X}_{n,k+1}' + \bar{\mathbf{K}}_D^T \mathbf{f}_{c,k+1}\end{aligned}\quad (7-16)$$

Recall from Eq. (3-5) that $\mathbf{M}_{k+1} = \mathbf{M}'_{k+1} + \mathbf{M}''_{k+1}$, and based on the results obtained in Eqs. (7-15) and (7-16), the same equation as shown in Eq. (7-12) can be obtained.

Now consider the energy balance equation when active control is applied to the structure. Since the effect of applying active control on the dynamic equilibrium equation as presented in Eq. (7-4) is an additional control force term, the result is simply adding an additional energy term in the energy equation. Similarly to Eq. (4-2), Eq. (7-1) can be written as:

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}'(t) - \mathbf{D}\mathbf{f}_c(t) = \mathbf{0} \quad (7-17)$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_n \\ \mathbf{D}_a \end{bmatrix} \quad (7-18)$$

$\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ matrices are defined in Eq. (4-3), and $\ddot{\mathbf{Y}}(t)$ is defined in Eq. (4-1).

Integrating both sides of Eq. (7-17) over the path of structural response from time zero to time t_k and following the similar derivation from Eq. (4-4) to Eq. (4-13), the resulting energy equation becomes:

$$\begin{aligned} \frac{1}{2} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} \dot{\mathbf{Y}}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} \dot{\mathbf{X}}_n dt + \frac{1}{2} \mathbf{X}'^T \mathbf{K} \mathbf{X}' \\ + \int_{t=0}^{t=t_k} \mathbf{M}'^T d\Theta'' - \int_{t=0}^{t=t_k} \mathbf{f}_c^T \mathbf{D}^T d\mathbf{X} = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g \end{aligned} \quad (7-19)$$

Similarly, based on the definition of stiffness matrix and Eq. (7-3), the third term of Eq. (7-19) can be simplified as:

$$\begin{aligned}
 \frac{1}{2} \mathbf{X}'^T \mathbf{K} \mathbf{X}' &= \frac{1}{2} \begin{bmatrix} \mathbf{X}'_n \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n + \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c \end{bmatrix}^T \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \\
 &\quad \begin{bmatrix} \mathbf{X}'_n \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n + \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c \end{bmatrix} \\
 &= \frac{1}{2} \begin{bmatrix} \mathbf{X}'_n \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n + \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c \end{bmatrix}^T \\
 &\quad \begin{bmatrix} (\mathbf{K}_{nn} - \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{K}_{an}) \mathbf{X}'_n + \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c \\ \mathbf{K}_{an} \mathbf{X}'_n - \mathbf{K}_{an} \mathbf{X}'_n + \mathbf{D}_a \mathbf{f}_c \end{bmatrix} \\
 &= \frac{1}{2} (\mathbf{X}'_n{}^T \bar{\mathbf{K}}_{nn} \mathbf{X}'_n + \mathbf{f}_c^T \mathbf{D}_a^T \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c)
 \end{aligned} \tag{7-20}$$

Substituting Eq. (7-20) back into Eq. (7-19) gives

$$\begin{aligned}
 \frac{1}{2} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} \dot{\mathbf{Y}}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} \dot{\mathbf{X}}_n dt + \frac{1}{2} \mathbf{X}'_n{}^T \bar{\mathbf{K}}_{nn} \mathbf{X}'_n + \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' \\
 - \int_{t=0}^{t=t_k} \mathbf{f}_c^T \mathbf{D}^T d\mathbf{X} + \frac{1}{2} \mathbf{f}_c^T \mathbf{D}_a^T \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g
 \end{aligned} \tag{7-21}$$

Therefore, comparing with Eq. (4-15), the additional terms due to applying active control appears in the fifth and the sixth terms on the left hand side of Eq. (7-21), which are defined as the control energy (*CE*). i.e.:

$$CE = - \int_{t=0}^{t=t_k} \mathbf{f}_c^T \mathbf{D}^T d\mathbf{X}_n - \int_{t=0}^{t=t_k} \mathbf{f}_c^T \mathbf{D}_a^T d\mathbf{X}_a + \frac{1}{2} \mathbf{f}_c^T \mathbf{D}_a^T \mathbf{K}_{aa}^{-1} \mathbf{D}_a \mathbf{f}_c \tag{7-22}$$

Finally, Eq. (4-22) can be modified to:

$$KE + DE + SE + PE + CE = IE \tag{7-23}$$

7.3.2 Optimal Linear Control Algorithm

The control force at time step k (i.e. $\mathbf{f}_{c,k}$) depends on the response of the structure. It can be seen that a large control force is generally required when large displacement or velocity response occurs at any time step (Hart and Wong, 1999). To determine the control force that gives an optimal linear control (OLC), define the cost function J to be

$$J = \frac{1}{2} \sum_{k=0}^N \left(\mathbf{z}_k^T \mathbf{Q} \mathbf{z}_k + \mathbf{f}_{c,k}^T \mathbf{R} \mathbf{f}_{c,k} \right) \quad (7-24)$$

where $\mathbf{Q}$ is a $2n \times 2n$ positive semi-definite weighting matrix and $\mathbf{R}$ is a $p \times p$ positive definite weighting matrix, and N represents the final time step over the control period. Note that the objective of optimal structural control is to select a set of control forces that will minimize the cost function J over the entire duration of the earthquake.

The procedure used to minimize J in Eq. (7-24) subject to the constraint in Eq. (7-9) involves the use of a Lagrangian. Define the Lagrangian L as:

$$L = \sum_{k=0}^{N-1} \left[\frac{1}{2} \left(\mathbf{z}_k^T \mathbf{Q} \mathbf{z}_k + \mathbf{f}_{c,k}^T \mathbf{R} \mathbf{f}_{c,k} \right) + \boldsymbol{\lambda}_{k+1}^T \left(\mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' + \mathbf{G} \mathbf{f}_{c,k} - \mathbf{z}_{k+1} \right) \right] \quad (7-25)$$

where $\boldsymbol{\lambda}_{k+1}$ is Lagrange multiplier. Note that the Lagrangian is a function of $\boldsymbol{\lambda}_{k+1}$, $\mathbf{z}_k$ and $\mathbf{f}_{c,k}$, and therefore minimization must be performed over these three variables.

Now define the Hamiltonian function to be

$$H_k = \frac{1}{2} \left(\mathbf{z}_k^T \mathbf{Q} \mathbf{z}_k + \mathbf{f}_{c,k}^T \mathbf{R} \mathbf{f}_{c,k} \right) + \boldsymbol{\lambda}_{k+1}^T \left(\mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' + \mathbf{G} \mathbf{f}_{c,k} \right) \quad (7-26)$$

Then substituting Eq. (7-26) into Eq. (7-25) gives:

$$L = \sum_{k=0}^{N-1} \left[H_k - \boldsymbol{\lambda}_{k+1}^T \mathbf{z}_{k+1} \right] = H_0 - \boldsymbol{\lambda}_N^T \mathbf{z}_N + \sum_{k=1}^{N-1} \left[H_k - \boldsymbol{\lambda}_k^T \mathbf{z}_k \right] \quad (7-27)$$

Taking the variation of the Lagrangian in Eq. (7-27) and setting the result equal to zero, it follows that

$$\begin{aligned} \delta L = & \frac{\partial H_0}{\partial \mathbf{z}_0} \delta \mathbf{z}_0 + \frac{\partial H_0}{\partial \mathbf{f}_{c,0}} \delta \mathbf{f}_{c,0} + \frac{\partial H_0}{\partial \boldsymbol{\lambda}_1} \delta \boldsymbol{\lambda}_1 - \delta \boldsymbol{\lambda}_N^T \mathbf{z}_N - \boldsymbol{\lambda}_N^T \delta \mathbf{z}_N \\ & + \sum_{k=1}^{N-1} \left[\left(\frac{\partial H_k}{\partial \mathbf{z}_k} - \boldsymbol{\lambda}_k^T \right) \delta \mathbf{z}_k + \frac{\partial H_k}{\partial \mathbf{f}_{c,k}} \delta \mathbf{f}_{c,k} + \frac{\partial H_k}{\partial \boldsymbol{\lambda}_{k+1}} \delta \boldsymbol{\lambda}_{k+1} - \mathbf{z}_k^T \delta \boldsymbol{\lambda}_k \right] = 0 \end{aligned} \quad (7-28)$$

Note that the control force at time zero is zero (i.e., $\mathbf{f}_{c,0} = 0$). Therefore, Eq. (7-28) becomes

$$\begin{aligned}
 \delta L &= \frac{\partial H_0}{\partial \mathbf{z}_0} \delta \mathbf{z}_0 + \frac{\partial H_0}{\partial \boldsymbol{\lambda}_1} \delta \boldsymbol{\lambda}_1 - \delta \boldsymbol{\lambda}_N^T \mathbf{z}_N - \boldsymbol{\lambda}_N^T \delta \mathbf{z}_N \\
 &+ \sum_{k=1}^{N-1} \left[\left(\frac{\partial H_k}{\partial \mathbf{z}_k} - \boldsymbol{\lambda}_k^T \right) \delta \mathbf{z}_k + \frac{\partial H_k}{\partial \mathbf{f}_{c,k}} \delta \mathbf{f}_{c,k} + \frac{\partial H_k}{\partial \boldsymbol{\lambda}_{k+1}} \delta \boldsymbol{\lambda}_{k+1} - \mathbf{z}_k^T \delta \boldsymbol{\lambda}_k \right] \\
 &= \frac{\partial H_0}{\partial \mathbf{z}_0} \delta \mathbf{z}_0 - \boldsymbol{\lambda}_N^T \delta \mathbf{z}_N + \sum_{k=1}^{N-1} \left[\left(\frac{\partial H_k}{\partial \mathbf{z}_k} - \boldsymbol{\lambda}_k^T \right) \delta \mathbf{z}_k + \frac{\partial H_k}{\partial \mathbf{f}_{c,k}} \delta \mathbf{f}_{c,k} \right] + \sum_{k=0}^{N-1} \left[\left(\frac{\partial H_k}{\partial \boldsymbol{\lambda}_{k+1}} - \mathbf{z}_{k+1}^T \right) \delta \boldsymbol{\lambda}_{k+1} \right] = 0
 \end{aligned} \tag{7-29}$$

In order to satisfy Eq. (7-29), each term in Eq. (7-29) must be equal to zero, i.e.

$$\left(\frac{\partial H_k}{\partial \boldsymbol{\lambda}_{k+1}} \right)^T - \mathbf{z}_{k+1} = \mathbf{0}, \quad \boldsymbol{\lambda}_N = \mathbf{0} \tag{7-30}$$

$$\left(\frac{\partial H_k}{\partial \mathbf{z}_k} \right)^T - \boldsymbol{\lambda}_k = \mathbf{0}, \quad \mathbf{z}_0 = \mathbf{z}(0) \tag{7-31}$$

$$\left(\frac{\partial H_k}{\partial \mathbf{f}_{c,k}} \right)^T = \mathbf{0} \tag{7-32}$$

where H_k is given in Eq. (7-26). Equations (7-30) to (7-32) represent a set of three simultaneous difference equations with three unknown vectors, $\boldsymbol{\lambda}_{k+1}$, $\mathbf{z}_k$ and $\mathbf{f}_{c,k}$. Note that the boundary condition in Eq. (7-31) is $\mathbf{z}(0) = \mathbf{z}_0$ (i.e., the initial condition of the structure), and therefore the variation will be zero, i.e., $\delta \mathbf{z}(0) = \mathbf{0}$. In addition, Eqs. (7-30) and (7-31) together represent a two-point boundary value problem, in which the boundary condition of one variable is given in the initial state and the boundary condition of another variable is given in the final state.

Note also that in Eq. (7-30), N is generally unknown in real time control. Therefore, this value is generally taken to be very large. Since damping always exists in the structure, the structural response always diminishes to zero for this large value of N . Thus the boundary condition $\boldsymbol{\lambda}_N = \mathbf{0}$ can be dropped in Eq. (7-30). Substituting Eq. (7-26) into Eqs. (7-30) to (7-32) and performing differentiation, it follows that

$$\left(\frac{\partial H_k}{\partial \boldsymbol{\lambda}_{k+1}} \right)^T - \mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' + \mathbf{G} \mathbf{f}_{c,k} - \mathbf{z}_{k+1} = \mathbf{0} \tag{7-33}$$

$$\left(\frac{\partial H_k}{\partial \mathbf{z}_k} \right)^T - \boldsymbol{\lambda}_k = \mathbf{Q}\mathbf{z}_k + \mathbf{F}_s^T \boldsymbol{\lambda}_{k+1} - \boldsymbol{\lambda}_k = \mathbf{0} \quad (7-34)$$

$$\left(\frac{\partial H_k}{\partial \mathbf{f}_{c,k}} \right)^T = \mathbf{R}\mathbf{f}_{c,k} + \mathbf{G}^T \boldsymbol{\lambda}_{k+1} = \mathbf{0} \quad (7-35)$$

Solving for $\mathbf{f}_{c,k}$ in Eq. (7-35) gives the control law, which is

$$\mathbf{f}_{c,k} = -\mathbf{R}^{-1} \mathbf{G}^T \boldsymbol{\lambda}_{k+1} \quad (7-36)$$

Substituting Eq. (7-36) into Eq. (7-33), it follows from Eqs. (7-33) and (7-34) that

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' - \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \boldsymbol{\lambda}_{k+1}, \quad \mathbf{z}(0) = \mathbf{z}_0 \quad (7-37)$$

$$\boldsymbol{\lambda}_k = \mathbf{Q}\mathbf{z}_k + \mathbf{F}_s^T \boldsymbol{\lambda}_{k+1} \quad (7-38)$$

Now define $\mathbf{P}_k$ to be a $2n \times 2n$ matrix with the property that

$$\boldsymbol{\lambda}_k = \mathbf{P}_k \mathbf{z}_k \quad (7-39)$$

Substituting Eq. (7-39) into Eqs. (7-37) and (7-38), it follows that

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' - \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \mathbf{z}_{k+1} \quad (7-40)$$

$$\mathbf{P}_k \mathbf{z}_k = \mathbf{Q}\mathbf{z}_k + \mathbf{F}_s^T \mathbf{P}_{k+1} \mathbf{z}_{k+1} \quad (7-41)$$

Solving for $\mathbf{z}_{k+1}$, Eq. (7-40) becomes

$$\begin{aligned} \mathbf{z}_{k+1} = & \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{F}_s \mathbf{z}_k + \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{H}_d \mathbf{a}_k \\ & + \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{F}_p \mathbf{X}_{n,k}'' \end{aligned} \quad (7-42)$$

In this equation, the matrix $\mathbf{P}_{k+1}$ is unknown and must be determined using Eq. (7-41).

Substituting Eq. (7-42) into Eq. (7-41), it follows that

$$\begin{aligned} \mathbf{P}_k \mathbf{z}_k = & \mathbf{Q}\mathbf{z}_k + \mathbf{F}_s^T \mathbf{P}_{k+1} \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{F}_s \mathbf{z}_k \\ & + \mathbf{F}_s^T \mathbf{P}_{k+1} \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_s^T \mathbf{P}_{k+1} \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{F}_p \mathbf{X}_{n,k}'' \end{aligned} \quad (7-43)$$

where $\mathbf{P}_k$ is known as the discrete time Riccati matrix. Note that both $\mathbf{a}_k$ and $\mathbf{X}_{n,k}''$ appear on the right-hand side of Eq. (7-43). Assuming that the Riccati difference equation is independent of both the earthquake and the inelasticity of the structure, then Eq. (7-43) becomes

$$\mathbf{P}_k \mathbf{z}_k = \mathbf{Q}\mathbf{z}_k + \mathbf{F}_s^T \mathbf{P}_{k+1} \left(\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1} \right)^{-1} \mathbf{F}_s \mathbf{z}_k \quad (7-44)$$

Note that $\mathbf{z}_k$ occurs in all terms on both sides of Eq. (7-44). Therefore, it follows that

$$\mathbf{P}_k = \mathbf{Q} + \mathbf{F}_s^T \mathbf{P}_{k+1} (\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P}_{k+1})^{-1} \mathbf{F}_s \quad (7-45)$$

Equation (7-45) is known as the discrete time Riccati equation. Assuming the Riccati matrix to be constant (i.e., $\mathbf{P}_k = \mathbf{P}_{k+1} = \mathbf{P}$), it follows from Eq. (7-45) that

$$\mathbf{P} = \mathbf{Q} + \mathbf{F}_s^T \mathbf{P} (\mathbf{I} + \mathbf{G} \mathbf{R}^{-1} \mathbf{G}^T \mathbf{P})^{-1} \mathbf{F}_s \quad (7-46)$$

Equation (7-46) is known as the steady-state Riccati equation, and $\mathbf{P}$ is known as the steady-state Riccati matrix. This equation is a quadratic function of $\mathbf{P}$, and therefore there are two solutions of $\mathbf{P}$. One of the two solutions gives a $\mathbf{P}$ matrix that is a positive definite matrix, and this is the desired solution.

Once $\mathbf{P}$ is solved, the control law can be obtained by substituting Eq. (7-39) into Eq. (7-36):

$$\mathbf{f}_{c,k} = -\mathbf{R}^{-1} \mathbf{G}^T \mathbf{P} \mathbf{z}_{k+1} \quad (7-47)$$

Substituting Eq. (7-9) into Eq. (7-47), it follows that

$$\mathbf{f}_{c,k} = -\mathbf{R}^{-1} \mathbf{G}^T \mathbf{P} (\mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' + \mathbf{G} \mathbf{f}_{c,k}) \quad (7-48)$$

Solving for $\mathbf{f}_{c,k}$ in Eq. (7-48), therefore, gives

$$\begin{aligned} \mathbf{f}_{c,k} = & -(\mathbf{G}^T \mathbf{P} \mathbf{G} + \mathbf{R})^{-1} \mathbf{G}^T \mathbf{P} \mathbf{F}_s \mathbf{z}_k - (\mathbf{G}^T \mathbf{P} \mathbf{G} + \mathbf{R})^{-1} \mathbf{G}^T \mathbf{P} \mathbf{H}_d \mathbf{a}_k \\ & - (\mathbf{G}^T \mathbf{P} \mathbf{G} + \mathbf{R})^{-1} \mathbf{G}^T \mathbf{P} \mathbf{F}_p \mathbf{X}_{n,k}'' \end{aligned} \quad (7-49)$$

According to Eq. (7-49), the control force $\mathbf{f}_{c,k}$ depends not only on $\mathbf{z}_k$ but also on $\mathbf{a}_k$ and $\mathbf{X}_{n,k}''$. A time delay always exists in practical control application because measuring the earthquake ground acceleration and at the same time applying a control force that depends on the measured ground acceleration at the same time step is physically impossible. Therefore, by taking a one-step time delay, the ground acceleration is equal to zero in determining the control force. In addition, the purpose of active control is to minimize the inelastic displacement, and therefore the $\mathbf{X}_{n,k}''$ term will be small and can be neglected in Eq. (7-49). Then it follows from Eq. (7-49) that

$$\mathbf{f}_{c,k} = -(\mathbf{G}^T \mathbf{P} \mathbf{G} + \mathbf{R})^{-1} \mathbf{G}^T \mathbf{P} \mathbf{F}_s \mathbf{z}_k \quad (7-50)$$

Note that the control force depends linearly on the state vector $\mathbf{z}_k$, which consists of the displacement and velocity responses. Therefore, Eq. (7-50) can be simplified as

$$\mathbf{f}_{c,k} = -\mathbf{C}_{1(OLC)}\mathbf{X}_k - \mathbf{C}_{2(OLC)}\dot{\mathbf{X}}_k \quad (7-51)$$

where $\mathbf{C}_{1(OLC)}$ and $\mathbf{C}_{2(OLC)}$ are $p \times n$ constant feedback control matrices computed by performing matrix multiplication as given in Eq. (7-50). From a practical point of view, these constant matrices will contain mostly positive entries, since the control force is generally applied in the direction opposite to the displacement and velocity in order for the control system to be effective in controlling structural responses.

7.3.3 Numerical Example

The six-story moment-resisting frame shown in Figure 4-1 is again selected with the same parameters but with an addition of six control actuators installed on every floor and labeled from C1 to C6 as shown in Figure 7-3. Doing so, the control force distribution matrix $\mathbf{D}_n$ is a 6×6 matrix, and it is of the form:

$$\mathbf{D}_n = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7-52)$$

Optimal linear control algorithm is used, and let the weighting matrices be of the forms:

$$\mathbf{Q} = \begin{bmatrix} \bar{\mathbf{K}}_{nn} & 0 \\ 0 & \mathbf{M}_{nn} \end{bmatrix}, \quad \mathbf{R} = r \times \bar{\mathbf{K}}_{nn}^{-1} \quad (7-53)$$

where r is the control parameter. Here $r = 100.0$, 1.0 , and 0.1 are chosen to make a comparison of structural damage potential index with the uncontrolled condition (UNC). Note that a smaller r in Eq. (7-53) means that less restriction is placed on the control force (see Eq. (7-24)) and therefore larger response reduction is expected.

Assume again that the structure is subjected to both the 1994 Northridge earthquake (Figure 4-2) and the magnified 1940 El-Centro earthquake (Figure 5-2). Figure 7-4 shows a comparison of DI_{Total} for both Benchmark Model and New Model with different control parameter and earthquake ground motions. It can be seen that after applying optimal linear control to the structure, the DI_{Total} decreases greatly, which means that the structural damage becomes less severe. In particular, when there is no control, the DI_{Total} for both Benchmark Model and New Model subjected to Northridge earthquake and El-Centro earthquake are all between 50% and 70%, which corresponds to damage conditions between irreparable and severe. However, when $r = 1$, the DI_{Total} for both models subjected to Northridge earthquake reduce to between 30% and 50%, which corresponds to high levels of safety with damage conditions between repairable and irreparable; and the ones subjected to El-Centro earthquake greatly reduce to below 10%, which corresponds to no damage conditions. In addition, when $r = 0.1$, the DI_{Total} for both two models subjected to the El-Centro earthquake even reduce to zero, which means there is no damage at all. Therefore, this shows that the proposed total damage index has good meaning in evaluating the extent of structural damage.

Figures 7-5 to 7-8 give the local damage indices values at all 40 PHLs for the four cases of different values of control parameter r (where $r = \infty$ means UNC) and different earthquakes. Generally, it is observed that using active control will increase the damage of the PHLs located in middle floors (i.e., 3rd, 4th, and 5th floors) and at the same time decrease the damage of the PHLs locates in bottom and top floors (i.e., ground, 2nd floor, and roof). This observation is consistent with the result obtained by Park and Ang (1985), in which structural damages based on the analysis of damaged buildings due to past earthquakes occur in the bottom floors. Thus decreasing the local DI_i of PHLs in bottom floors greatly decrease the DI_{Total} , and this point is shown in Figure 7-4. When $r = 0.1$, the DI_{Total} is equal to zero for both models subjected to El-Centro earthquake (see Figure 7-4), and this also shows up in Figures 7-6 and 7-8 where the local DI_i at all PHLs for these two cases are equal to zero. It is also observed that Figures 7-7 and 7-8 are similar to Figures 7-5 and 7-6, and the only

difference lies in the local DI_i values where the values at PHLs #2 and #3 are smaller for the New Model than that of the Benchmark Model. This shows that strengthening the interior columns mainly help in redistributing the local damage in the structure, and it does not affect the overall DI_{Total} of the structure.

Looking in more details into Figures 7-5 to 7-8, it is observed that the local DI_i at certain PHLs have no trend of increasing or decreasing after applying active control. For example, the local DI_i at PHLs in the upper two floors (i.e., from PHLs #29 to #40) decrease with a small control force based on the control parameter $r = 100$, but these local DI_i increase with larger control forces based on the control parameter $r = 1$ or 0.1 . Moreover, the magnitude of increase or decrease of the local DI_i for different PHLs is different. This result importantly shows that active control should be used in a more strategic manner in order to prevent the negative effects of increasing responses and damages which otherwise may not happen.

Figure 7-3 Six-story moment resisting frame model with control devices

Figure 7-4 Comparison of DI_{Total} for two models and two earthquakes

Figure 7-5 Local DI of Benchmark Model subjected to 1994 Northridge earthquake

Figure 7-6 Local DI of Benchmark Model subjected to 1940 El-Centro earthquake

Figure 7-7 Local DI of New Model subjected to 1994 Northridge earthquake.

Figure 7-8 Local DI of New Model subjected to 1940 El-Centro earthquake.

7.4 Conclusion

The objective of this chapter is to verify the effectiveness of newly proposed total and local damage potential indices as damage indications in structural damage evaluation, structural modification and active structural control. After performing dynamic time history analysis of structures subjected to a specific earthquake, both DI_{Total} of the entire structure and local DI_i at each PHL can be calculated to give a figure of the damage conditions of the structure. The DI_{Total} gives general information of whether the structure will suffer major damage. At the same time, the local DI_i at each PHL shows the damage distribution within the entire structure, and points out which member or which floor will suffer the most damage and becomes the likely starting point of structural collapse. According to the information obtain by the proposed damage assessment model, different methods of enhancing the structural performance can be used, such as increasing the size of most vulnerable members or introducing suitable control devices. Numerical examples are supplied to demonstrate the application of the proposed damage assessment model in these aspects.

A good damage index should have the ability to estimate the level of structural damage suffered during an earthquake. The currently proposed damage indices have this ability, where the structure has “no damage” if $DI_{Total} = 0$ and the structure will “collapse” if $DI_{Total} = 1$. Besides, according to the performance level limits defined in Chapter 6, it also serves well as an indication of the enhancement of safety levels of structures with large amount of control forces applied. However, further research is still necessary in passive structural control and other control areas to prove the wide application of this damage evaluation model with more numerical simulations using different earthquake records and experimental verifications.

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The course of earthquake-induced vibration is in fact an energy transfer from earthquake input energy to other energy forms. In this research, a computation method is firstly derived to characterize quantitatively the energy and transfer among energy forms of kinetic energy, strain energy, damping energy, plastic energy and control energy in structures during earthquakes. Energy dissipated by irrecoverable inelastic deformation is defined, and the plastic energy is derived based on the plastic rotations of steel moment-resisting frames computed using the force analogy method. Based on the modified force analogy method with static condensation, together with the concept of inelastic displacement, the calculation of responses and energies of a MDOF system becomes very simple.

Using this computation method of characterizing energy, a structural damage assessment model is proposed based on structural response and energy approach to meet the requirements of evaluating seismic damage of steel framed structures considering both monotonic and cycling effects of structural damage in earthquake

engineering. This damage assessment model is proposed to reflect structural damage globally and locally, in which the total damage index is defined as a combination of inelastic effects in displacement and energy, while the local damage indices give the local damage contribution at each plastic hinge location to the total damage of structures.

Afterwards, the concepts of energy density spectra and proposed total damage index spectra are discussed and illustrated through numerical examples. Major research works have been performed to understand the effects of different natural periods of vibration, damping, and yielding levels on these spectra. Embedded with normal mode method, applications of the proposed total damage index spectra in predicting structural damage in MDOF systems is presented. Meanwhile, probability studies are carried out to define the limit states of proposed damage potential index at different performance levels, which exposes the real meanings of these damage index numbers between 0 and 1. Some characteristics of the proposed damage potential index are also investigated via the study of correlation between proposed damage potential index and various ground motion intensities. Finally, the significance of the proposed damage assessment model is demonstrated through numerical examples of structural modification and active control.

However, the proposed damage assessment criteria are only developed based on purely conceptual qualification without any justification. Therefore, further research on the quantification of damage assessment criteria from both experimental and analytical perspectives is necessary. Based on the comparisons of the results, some conclusions are drawn and listed below:

- Input energy equals to the sum of kinetic energy, strain energy, damping energy, plastic energy and control energy. Kinetic energy and strain energy vanish as earthquake diminishes over time, while damping energy, plastic energy and control energy are dissipated during the earthquake period and cannot be recovered. The final input energy is approximately equal to the sum of damping energy, plastic energy and control energy.

- The proposed damage assessment model evaluates structural damage from both response and energy point of view. The total damage index DI_{Total} gives an overview of the total damage conditions of structures, which also satisfies the requirement of $0 \leq DI_{Total} < 1$, where $DI_{Total} = 0$ means no damage and with an increase of DI_{Total} suggests that the damage of structures becomes more significant until total collapse. The local damage index DI_i at the i th PHL gives insights into the locations of the structure where most damage has been suffered in the structure, which also helps to reveal the damage process of a structure as it progresses from partial damage to total collapse.
- The effects of natural period, damping ratio, yield displacement level and ground motion scalar on drift ratio spectra, plastic energy density spectra and proposed damage index spectra are quite different. However, it can be concluded that the proposed damage index is more effective in evaluating structural damage than drift ratio or plastic energy alone since it can reflect more clearly the effects of increasing damping ratio and yield displacement as well as decreasing magnitude of ground motions on the hazard resistant abilities of structures.
- Evaluating damage of MDOF structural models using inelastic spectra analysis based on the proposed damage index through normal mode method is quite economical both in time and in cost while achieving sufficient accuracy. For various floor masses, damping ratios, building heights and spans, characteristic and magnitude of the ground motions, all relative errors between the inelastic spectra analysis and inelastic time history analysis are within 25%.
- From design point of view, recommended limit state values of proposed damage index for different performance levels are provided based on approximate comparative levels of probability of failure with drift ratio using probability theory. It is observed that damage index values of 10%, 30%, 50%, and 70% are the limits correspond to performance levels of minor damage, repairable damage, irreparable

damage, and collapse, respectively. Since these results are based on a series of synthetic ground motions which simulate a repeat of Northridge earthquake but considering uncertainties in vibration characteristics, structural parameters and soil conditions, the recommended limit values should be applicable to other earthquakes. However, design codes adopted in different countries may implement different limit values corresponding to the same performance level.

- Not only the proposed damage index coincides well with various ground motion intensity indices, it is also found to be more meaningful and helpful in evaluating structural damage. Therefore, these features enable the proposed damage index, which shows the structural damage more clearly and accurately in different frames subjected to different ground motions, to be prospective in lieu of practical engineering applications.
- The newly proposed total and local damage indices serve effectively as structural damage evaluation, structural modification and active control indications. According to the information obtained by the proposed damage assessment model, different methods of enhancing the structural performance can be used, such as increasing the size of most vulnerable members from structural modification point of view or introducing suitable control devices from active/passive control point of view. Therefore, it is straightforward to see the significance of this newly proposed damage assessment model not only in damage evaluation aspects but also in hazard mitigation aspects.

8.2 Recommendations

This preliminary study shows that the proposed damage assessment model evaluating structural damage from response and energy approach contains special features that may be useful in engineering applications. However, as mentioned in the course of developing respective chapters, further research involving more practical details and implementations should be investigated.

One consideration is in the derivation of the proposed damage assessment model. It is quite necessary to define the exact form of the total damage index and local damage indices, i.e., the weighting parameters of response part and energy part in the proposed damage indices need to be defined more carefully from experiments. It is acceptable to assume the damage effects from the two parts are equal in present work because the focus of the present study is the development of a framework for the damage assessment of steel frames. The current choice of $\alpha = \beta = 0.5$ is although preliminary but does not affect the validity of the proposed approach. However, more strict values must be achieved from tremendous experiment results in the future.

Another extension is that the effects of different earthquake records with different soil conditions and different distances from the fault on the proposed damage index spectra as well as the probability of failure spectra need to be investigated for taking into considerations more properties of this damage index. Moreover, the limit value of propose damage index at each performance level need to be developed more precisely based on larger number of calculations and calibrations of various real structures subjected to real recorded ground motions when all these resources become available. Study on different design codes adopted by different countries should also be involved in the future work.

Finally, it is also meaningful to widen the dynamic load to other types of ground motions. Only earthquake ground motions are adopted as the dynamic load inputs in this research. There are other types of ground excitations such as environmental dynamic loads, explosive forces, impetus forces, synthetic waves, and etc. Thus, further exploration can be performed by taking these excitations into consideration in attempt to make the proposed damage assessment model applicable in a wider scope.

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