Deterministic and stochastic dynamic study on inelastic structures with force analogy method

Wang, Zhe

2005

Wang, Z. (2005). Deterministic and stochastic dynamic study on inelastic structures with force analogy method. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/12100>

<https://doi.org/10.32657/10356/12100>

Nanyang Technological University

Downloaded on 26 Apr 2023 10:54:04 SGT

**DETERMINISTIC AND STOCHASTIC DYNAMIC
STUDY ON INELASTIC STRUCTURES WITH FORCE
ANALOGY METHOD**

WANG ZHE

**SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY**

2005

Deterministic and Stochastic Dynamic Study on Inelastic Structures with Force Analogy Method

Wang Zhe

School of Civil & Environmental Engineering

**A thesis submitted to the Nanyang Technological University in fulfillment of
the requirements for the degree of Doctor of Philosophy**

2005

ACKNOWLEDGEMENTS

The work presented in this thesis is done under the supervision of Associate Professor Kevin Wong, Kai Fai. During the course of it, Dr. Wong has given the author utmost guidance, assistance and encouragement. For this, the author wishes to express sincere gratitude to Dr. Wong.

The acknowledgements of the author are also given to the School of Civil and Environmental Engineering of Nanyang Technological University for the good research environment it provides.

In addition, the help and suggestion of all friends are highly appreciated.

Finally, a special gratitude of the author goes to her family members, her parents, Wang Shijie and Zang Quantong, especially to her husband, Dr. Liu Xianbin and her pretty daughter, Liu Siming. Only with the great assistance and constant support of them, the fulfillment of this work becomes possible.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS.....	I
TABLE OF CONTENTS.....	II
SUMMARY.....	VI
LIST OF FIGURES.....	VIII
LIST OF TABLES.....	XIII
LIST OF SYMBOLS.....	XV
CHAPTER 1 INTRODUCTION.....	1
1.1 Background of This Research.....	1
1.2 Main Contributions.....	3
1.3 Arrangement of Contents.....	4
CHAPTER 2 LITERATURE REVIEW.....	7
2.1 General Statement.....	7
2.2 Energy-Based Seismic Analysis Method.....	7
<i>2.2.1 Evaluation of Energy Dissipation Capacity of Structures.....</i>	<i>8</i>
<i>2.2.2 Energy-Based Damage Index.....</i>	<i>13</i>
2.3 Seismic Beam-Column Joint models of Moment Resisting Steel	
Frames.....	15
<i>2.3.1 Connections.....</i>	<i>16</i>
<i>2.3.2 Panel Zones.....</i>	<i>19</i>
2.4 Nonlinear Stochastic Dynamic Study.....	24
<i>2.4.1 Analytical Methods.....</i>	<i>25</i>
<i>2.4.2 Numerical Method.....</i>	<i>31</i>
2.5 Discussion on This Research.....	32

CHAPTER 3	SEISMIC ANALYSIS OF INELASTIC STRUCTURES WITH FORCE ANALOGY METHOD.....	35
3.1	General Statement.....	35
3.2	Force Analogy Method.....	36
3.2.1	Basic Concepts.....	36
3.2.2	Force Analogy Method.....	38
3.3	Inelastic Dynamic State Space Analysis Using The Force Analogy Method.....	41
3.4	Modified Force Analogy Method with Static Condensation.....	43
3.5	Energy Evaluation Method of Inelastic Structures under Earthquake Loading.....	48
3.5.1	General Energy Formulation.....	48
3.5.2	Energy Formulation with Static Condensation.....	50
3.6	Summary.....	52
CHAPTER 4	DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH RIGID END OFFSETS.....	54
4.1	General Statement.....	54
4.2	Structural Modeling with Rigid End Offsets.....	55
4.3	Member Stiffness Matrix with Rigid End Offsets.....	56
4.4	Numerical Examples.....	63
4.4.1	Example 1: One-Story One-Bay Moment Resisting Frame.....	64
4.4.2	Example 2: Six-Story Moment Resisting Frame.....	71
4.5	Summary.....	80
CHAPTER 5	DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH PANEL ZONE DEFORMATION.....	82
5.1	General Statement.....	82
5.2	Panel Zone Deformation Model.....	83
5.3	Numerical Examples.....	85
5.3.1	Example 1: One-Story One-Bay Moment Resisting Frame.....	85
5.3.2	Example 2: Six-Story Moment Resisting Frame.....	91

5.4	Summary.....	121
CHAPTER 6	STOCHASTIC DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH THE FORCE ANALOGY METHOD.....	124
6.1	General Statement.....	124
6.2	Stochastic Force Analogy Method (SFAM).....	125
6.2.1	<i>Solution to the Equation of Motion Based on the Force Analogy Method and State Space Method.....</i>	<i>125</i>
6.2.2	<i>Recursive Equation Used in the Stochastic Analysis.....</i>	<i>127</i>
6.2.3	<i>Expressions of Random Variables and Boundary of Domains Based on the Force Analogy Method.....</i>	<i>131</i>
6.2.4	<i>Calculation of Joint Probability Density Function.....</i>	<i>138</i>
6.2.5	<i>Calculation of the Variance and the Covariance.....</i>	<i>140</i>
6.3	Numerical Example.....	146
6.4	Summary.....	151
CHAPTER 7	SIMULATION AND RELIABILITY STUDY ON INELASTIC STRUCTURES WITH RIGID END OFFSETS AND DEFORMABLE PANEL ZONES.....	154
7.1	General Statement.....	154
7.2	Simulation of the Earthquake Ground Acceleration.....	155
7.2.1	<i>The Random Model of Ground Acceleration.....</i>	<i>155</i>
7.2.2	<i>Generation of Artificial Ground Acceleration.....</i>	<i>157</i>
7.3	Monte Carlo Simulation Study on Inelastic Structures with Rigid End Offsets Using the Force Analogy Method.....	164
7.4	Monte Carlo Simulation Study on Inelastic Structures with Deformable Panel Zones Using the Force Analogy Method.....	175
7.5	Reliability Analysis of Inelastic Structures with Rigid End Offsets and Deformable Panel Zones.....	191
7.5.1	<i>Reliability Analysis of Inelastic Structures with Rigid End Offsets</i>	<i>193</i>
7.5.2	<i>Reliability Analysis of Inelastic Structures with Deformable Panel Zones.....</i>	<i>196</i>

7.6	Summary.....	204
CHAPTER 8	CONCLUSIONS AND RECOMMENDATIONS.....	206
8.1	Conclusions.....	206
8.2	Recommendations.....	208
REFERENCES.....		211

SUMMARY

Severe earthquakes in most cases will cause significant inelastic deformation in conventional building structures. Furthermore, due to the inherent uncertainty of the earthquake excitation, random process should be a more appropriate model to represent this type of ground motion. Therefore, from the 1950's, seismic analysis of structures has been divided into two different directions, deterministic and stochastic. Although in the past fifty years stochastic dynamics and reliability analysis based on random vibration have made great progress in various aspects, the traditional deterministic design procedures is still in the dominance. More effort in the field of nonlinear stochastic dynamic study is necessary with the objective of general application of stochastic dynamic methodology.

In this research, the force analogy method is used as a main tool to analyze the inelastic dynamic response and energy dissipation of structures in both deterministic and stochastic fields. A simple analytical method of modeling the panel zone deformation is proposed. The model of rigid end offsets is also included to give a more accurate structural model. New stiffness matrices for members with two ends being rigid are obtained using the unit displacement method. With the aid of static condensation, each term in the stiffness matrices using in force analogy method can be derived simply through multiplying by a scale factor, which only depends on the sizes of rigid end and offset lengths.

Based on the proposed model, numerical simulations are performed on a one-story one-bay frame and a six-story moment resisting frame. Comparison of results, such as global responses, global energy dissipation, local responses, and local energy dissipation, between different models shows that rigid end offsets as well as elastic and inelastic deformation of panel zones have significant effects on dynamic behavior of inelastic structures subjected to earthquake excitation. In addition, the Monte Carlo simulation method using 500 sample ground acceleration time histories performed on a six-story moment resisting frame based on the proposed model also indicates the significant influence of rigid end offsets and panel zone deformation on structural inelastic dynamic behaviors in a statistic sense.

This type of significant effect is further verified by investigating the probability of failure of individual plastic hinge location based on the concept of the plastic energy dissipated at individual plastic hinge location.

Finally, a new stochastic dynamic analysis method for inelastic structures based on the force analogy method is proposed for the first time. The force analogy method combined with the state space method has been proven to be efficient in solving the equation of motion in inelastic dynamic analysis, since in the recursive process only initial stiffness is involved, while the inelastic property of structures is reflected by the inelastic displacement. This advantage of the force analogy method is also evident in stochastic inelastic dynamic analysis. The proposed stochastic force analogy method can produce the covariance functions of displacement, velocity, inelastic displacement, and plastic rotation at individual plastic hinge location. The reasonability of the proposed method is demonstrated by means of the agreement of results of the proposed method with those of the Monte Carlo simulation method.

LIST OF FIGURES

Figure 3.1	Bilinear Stiffness Relationship.....	37
Figure 3.2	Force Analogy Method.....	41
Figure 4.1	Additional DOFs at Plastic Hinges.....	56
Figure 4.2	Model of Member with Rigid End Offsets.....	56
Figure 4.3	Derivation of Element Stiffness Matrix for Member 1.....	58
Figure 4.4	Derivation of Element Stiffness Matrix with Two Ends Being Rigid.....	60
Figure 4.5	Calculation of Moments at Plastic Hinges Due to Different Unit Displacement Patterns.....	62
Figure 4.6	Ground Acceleration Time History of 1940 El-Centro Earthquake.	63
Figure 4.7	Acceleration, Velocity, and Displacement Spectra for 1940 El-Centro Earthquake.....	64
Figure 4.8	One-Story One-Bay Moment Resisting Frame.....	65
Figure 4.9	Structural Global Responses of One-Story One-Bay Frame for Cases NEO and REO.....	68
Figure 4.10	Some Local Responses of One-Story One-Bay Frame for Cases NEO and REO.....	69
Figure 4.11	Structural Energy Dissipation of One-Story One-Bay Frame for Cases NEO and REO.....	70
Figure 4.12	Six-Story Moment Resisting Frame.....	72
Figure 4.13	Model of Six-Story Moment Resisting Frame.....	73
Figure 4.14	Structural Global Responses of MDOF Structure for Cases NEO and REO.....	75
Figure 4.15	Energy Dissipation of MDOF Structure for Cases NEO and REO..	76
Figure 4.16	Plastic Rotation of MDOF Structure at Selected PHLs for Cases NEO and REO.....	78

Figure 4.17	Plastic Energy Dissipation of MDOF Structure at Selected PHLs for Cases NEO and REO.....	79
Figure 5.1	Structural Model with Rotational Springs.....	83
Figure 5.2	Effect of Rotational Spring on Joint Deformation.....	84
Figure 5.3	One-Story One-Bay Moment Resisting Frame.....	86
Figure 5.4	Structural Global Responses of One-Story One-Bay Frame for Different Rigidities in Panel Zones.....	88
Figure 5.5	Some Local Responses of One-Story One-Bay Frame for Different Rigidities in Panel Zones.....	89
Figure 5.6	Structural Energy Dissipation of One-Story One-Bay Frame for Different Rigidities in Panel Zones.....	90
Figure 5.7	Model of Six-Story Moment Resisting Frame.....	92
Figure 5.8	Structural Global Responses of MDOF Structure with Different Rotational Spring Stiffness.....	94
Figure 5.9	Energy Dissipation of MDOF Structure with Different Rotational Spring Stiffness.....	95
Figure 5.10	Plastic Rotation of MDOF Structure at Selected PHLs with Different Rotational Spring Stiffness.....	99
Figure 5.11	Plastic Energy Dissipation of MDOF Structure at Selected PHLs with Different Rotational Spring Stiffness.....	100
Figure 5.12	Maximum Plastic Rotation and Plastic Energy at Each PHL with Softer Rotational Spring $k_{s1} = 4.80 \times 10^5$ kN-m/rad.....	103
Figure 5.13	Maximum Plastic Rotation and Plastic Energy at Each PHL with Soft Rotational Spring $k_{s2} = 5.55 \times 10^5$ kN-m/rad.....	105
Figure 5.14	Maximum Plastic Rotation and Plastic Energy at Each PHL with Stiff Rotational Spring $k_{s3} = 1.55 \times 10^6$ kN-m/rad.....	107
Figure 5.15	Maximum Plastic Rotation and Plastic Energy at Each PHL with Stiffer Rotational Spring $k_{s4} = 3.40 \times 10^6$ kN-m/rad.....	109
Figure 5.16	Maximum Plastic Rotation and Plastic Energy at Each PHL with Fully Rigid Spring $k_{s5} = \infty$ kN-m/rad.....	111

Figure 5.17	Comparison of Maximum Global Responses between Models with Different Stiffness and Yield Moment of Rotational Springs.....	113
Figure 5.18	Comparison of Maximum Energy Dissipation between Models with Different Stiffness and Yield Moment of Rotational Springs.....	114
Figure 5.19	Comparison of Maximum Local Responses in Columns between Models with Different Stiffness and Yield Moment of Rotational Springs.....	115
Figure 5.20	Comparison of Maximum Local Responses in Selected Beams between Models with Different Stiffness and Yield Moment of Rotational Springs.....	116
Figure 5.21	Comparison of Maximum Local Responses in Selected Beam-Column Joints between Models with Different Stiffness and Yield Moment of Rotational Springs.....	117
Figure 6.1	Single degree of freedom of System with PHL at the Base of the Column.....	147
Figure 6.2	Time Histories of Response Statistics of SDOF System under Non-Stationary Excitation.....	149
Figure 7.1	Deterministic Temporal Modulation Function.....	156
Figure 7.2	Kanai-Tajimi Power Spectrum.....	156
Figure 7.3	Simulated Earthquake Ground Motion Time History-Sample 1...	160
Figure 7.4	Simulated Earthquake Ground Motion Time History-Sample 2...	161
Figure 7.5	Simulated Earthquake Ground Motion Time History-Sample 3...	162
Figure 7.6	Simulated Earthquake Ground Motion Time History-Sample 4...	163
Figure 7.7	Standard Deviation of Displacement per Floor of MDOF Structure for Cases NEO and REO.....	168
Figure 7.8	Standard Deviation of Velocity per Floor of MDOF Structure for Cases NEO and REO.....	169
Figure 7.9	Standard Deviation of Acceleration per Floor of MDOF Structure for Cases NEO and REO.....	170
Figure 7.10	Mean and Standard Deviation of Energy Dissipation of MDOF Structure for Cases NEO and REO.....	171

Figure 7.11	Standard Deviation of Plastic Rotation of MDOF Structure at Selected PHLs for Cases NEO and REO.....	173
Figure 7.12	Mean and Standard Deviation of Plastic Energy Dissipation of MDOF Structure at Selected PHLs for Cases NEO and REO.....	174
Figure 7.13	Maximum Mean and Standard Deviation of Global Response and Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	179
Figure 7.14	Standard Deviation of Global Response of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	180
Figure 7.15	Mean of Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	181
Figure 7.16	Standard Deviation of Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	182
Figure 7.17	Maximum STD of Plastic Rotation and Maximum Mean and STD of Plastic Energy at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	183
Figure 7.18	Maximum STD of Plastic Rotation at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	184
Figure 7.19	Maximum Mean of Plastic Energy at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	185
Figure 7.20	Maximum STD of Plastic Energy at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	186
Figure 7.21	Standard Deviation of Plastic Rotation of MDOF Structure at Selected PHLs with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	187
Figure 7.22	Mean of Plastic Energy Dissipation of MDOF Structure at Selected PHLs with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	188

Figure 7.23	Standard Deviation of Plastic Energy Dissipation of MDOF Structure at Selected PHLs with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	189
Figure 7.24	Typical $M - \theta''$ Curve Samples of Some Plastic Hinges.....	190
Figure 7.25	Histograms of Plastic Energy at Some Selected PHLs for Case NEO.....	194
Figure 7.26	Histograms of Plastic Energy at Some Selected PHLs for Case REO.....	195
Figure 7.27	Probability of Failure at Individual PHL for Cases NEO and REO.....	196
Figure 7.28	Histograms of Plastic Energy at Selected PHLs for $M_s = 1695$ kN-m with Stiffness = 1.55×10^6 kN-m/rad.....	199
Figure 7.29	Histograms of Plastic Energy at Selected PHLs for $M_s = 3050$ kN-m with Stiffness = 1.55×10^6 kN-m/rad.....	200
Figure 7.30	Histograms of Plastic Energy at Selected PHLs for $M_s = 5875$ kN-m with Stiffness = 1.55×10^6 kN-m/rad.....	201
Figure 7.31	Histograms of Plastic Energy at Selected PHLs for $M_s = 11000$ kN-m with Stiffness = 1.55×10^6 kN-m/rad.....	202
Figure 7.32	Probability of Failure at each PHL of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad.....	203

LIST OF TABLES

Table 4.1	Rigid End and Offset Properties of Each Member	66
Table 4.2	Maximum Structural Global and Local Responses of One-Story One-Bay Frame for Cases NEO and REO	67
Table 4.3	Maximum Global Responses and Energy Dissipation of MDOF Structure for Cases NEO and REO	74
Table 4.4	Maximum Local Responses and Energy Dissipation of MDOF Structure for Cases NEO and REO	77
Table 5.1	Maximum Structural Global and Local Responses of One-Story One-Bay Frame with Different Rotational Spring Stiffness.....	87
Table 5.2	Maximum Global Responses and Energy Dissipation of MDOF Structure with Different Rotational Spring Stiffness.....	93
Table 5.3	Maximum Local Responses and Energy Dissipation of MDOF Structure with Different Rotational Spring Stiffness.....	96
Table 5.4	Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness $k_{s1} = 4.80 \times 10^5$ kN-m/rad	102
Table 5.5	Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness $k_{s2} = 5.55 \times 10^5$ kN-m/rad	104
Table 5.6	Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness $k_{s3} = 1.55 \times 10^6$ kN-m/rad	106
Table 5.7	Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness $k_{s4} = 3.40 \times 10^6$ kN-m/rad	108
Table 5.8	Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness $= \infty$ kN-m/rad	110
Table 7.1	Maximum Mean and Standard Deviation of Global Response and Energy Dissipation of MDOF Structure for Cases NEO and REO	167

Table 7.2	Maximum Mean and Standard Deviation of Local Response and Energy Dissipation of MDOF Structure for Cases NEO and REO 172
Table 7.3	Maximum Mean and Standard Deviation of Global Response and Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad178

LIST OF SYMBOLS

$a_g(t)$	Earthquake ground acceleration
a_k	Discrete form of earthquake ground acceleration
$\mathbf{a}_g(t)$	Earthquake ground acceleration vector
A	Cross sectional area of member
$\mathbf{A}$	State space matrix
c	Damping
C	Damping
$\mathbf{C}$	Damping matrix
d	Depth of beam
DE	Damping energy
E	Young's modulus
$E[x_k \dot{x}_k]$	Covariance of relative displacement and relative velocity
$E[x_k x_k'']$	Covariance of relative displacement and inelastic displacement
$E[x_k \theta_k'']$	Covariance of relative displacement and plastic rotation
$E[\dot{x}_k x_k'']$	Covariance of relative velocity and inelastic displacement
$E[\dot{x}_k \theta_k'']$	Covariance of relative velocity and plastic rotation
$E[x_k'' x_k'']$	Covariance of inelastic displacement
$E[\mathbf{z}_k x_k'']$	Covariance of state vector and inelastic displacement
$E[\mathbf{z}_k \mathbf{z}_k^T]$	Covariance of Structural state vector
$E[\mathbf{z}_k \theta_k'']$	Covariance of state vector and plastic rotation
$\bar{E}[x(t)]$	Mean of samples
$F_s(t)$	Force at any time
F_y	Yield strength
$\mathbf{F}_p^c$	Inelastic displacement transition matrix
$\mathbf{F}_{RF}(t)$	Restoring force vector

g	Gravity acceleration
$G_{xx}(\omega)$	One-sided power spectral density function
$\mathbf{H}$	Earthquake transition matrix
I_b	Moment of inertia of the beam
I_c	Moment of inertia of the column
IE	Input energy
$\mathbf{I}$	Identity matrix
k	Time step
k_e	Initial stiffness
k_s	Spring rotational stiffness
k_t	Post-yield stiffness
K	Stiffness
KE	Kinetic energy
$\mathbf{K}$	Stiffness matrix
$\overline{\mathbf{K}}_{nn}$	Stiffness matrix after static condensation
K_p	Structural member recovery force
$\mathbf{K}_p$	Structural member recovery force matrix
$\overline{\mathbf{K}}_p$	Structural member recovery force matrix after static condensation
K_R	Structural member restoring moment
$\mathbf{K}_R$	Structural member restoring moment matrix
$\overline{\mathbf{K}}_R$	Structural member restoring moment matrix after static condensation
L	Length of member
L_b	Length of beam
L_c	Length of column
m	(1) Total number of potential plastic hinge locations (2) Mass
M	Mass
M_b	Yield moment of the beam

M_c	Yield moment of the column
M_k	Discrete form of total moment
M_s	Yield moment of the rotational spring
$\mathbf{M}$	Mass matrix
$\mathbf{M}(t)$	Total moment vector
$\mathbf{M}'(t)$	Elastic moment vector
$\mathbf{M}''(t)$	Inelastic moment vector
$\mathbf{M}_k$	Discrete form of total moment vector
$\mathbf{M}_{RF}(t)$	Restoring moment vector
n	Structural degree of freedom
N	(1) Original degree of freedom of structure (2) Number of samples
P_f	Probability of failure of individual plastic hinge location (PHL)
PE	Plastic energy
PE_{capacity}	Capacity of plastic energy dissipation at individual PHL
PE_i	Plastic energy at the i th plastic hinge
PE_{max}	Maximum plastic energy dissipated at individual PHL
$S(t)$	Stationary process
SE	Strain energy
$\bar{S}[x(t)]$	Standard deviation of samples
t	Time
t_0	Initial time
t_k	Time at step k
t_{k+1}	Time at step $k+1$
T	(1) Natural period of structure (2) Time duration
T_n	Natural period of structure
$\bar{V}[x(t)]$	Variance of samples

$x(t)$	(1) Relative displacement (2) Displacement
x_k	Discrete form of relative displacement
x_y	Yield displacement
$\mathbf{X}(t)$	Relative displacement vector
$\mathbf{X}_k$	Discrete form of relative displacement vector
$\dot{x}(t)$	Relative velocity
$\dot{x}_k$	Discrete form of relative velocity
$\dot{\mathbf{X}}(t)$	Relative velocity vector
$\ddot{x}(t)$	Relative acceleration
$\ddot{\mathbf{X}}(t)$	Relative acceleration vector
$x'(t)$	Elastic displacement
$\mathbf{X}'(t)$	Elastic displacement vector
$x''(t)$	Inelastic displacement
x''_k	Discrete form of inelastic displacement
$\mathbf{X}''(t)$	Inelastic displacement vector
$\mathbf{X}''_k$	Discrete form of inelastic displacement vector
$\mathbf{X}_g(t)$	Ground displacement vector
$\ddot{\mathbf{X}}_g(t)$	Earthquake ground acceleration vector
$\mathbf{Y}(t)$	Absolute displacement vector
$\dot{\mathbf{Y}}(t)$	Absolute velocity vector
$\ddot{\mathbf{Y}}(t)$	Absolute acceleration vector
$\mathbf{z}(t)$	Structural state vector
$\mathbf{z}_k$	Discrete form of structural state vector
$\dot{\mathbf{z}}(t)$	Differential of structural state vector
Δt	Time interval
$\Delta \omega$	Frequency increment
α_g, β_g	Constants in deterministic modulation function

σ_y	Yield stress
$\sigma_{a_k}^2$	Variance of ground acceleration
$\sigma_{x_k}^2$	Variance of relative displacement
$\sigma_{\dot{x}_k}^2$	Variance of relative velocity
$\sigma_{\theta_k''}^2$	Variance of plastic rotation
μ_{a_k}	Mean of ground acceleration
μ_{x_k}	Mean of relative displacement
$\mu_{\dot{x}_k}$	Mean of relative velocity
$\mu_{\theta_k''}$	Mean of plastic rotation
ε_μ	Ultimate strain
ζ	Damping ratio
ζ_g	Damping ratio of ground
ω_g	Natural frequency of ground
ω_k	Discrete form of frequency
ω_l	Left boundary of frequency interval
ω_n	Natural frequency of structure
ω_r	Right boundary of frequency interval
$\phi(t)$	Deterministic modulation function
Φ_0	Spectrum level
$\Phi_{xx}(\omega)$	Two-sided power spectral density function
φ_k	Random phase angles
$\theta''(t)$	Plastic rotation
θ_k''	Discrete form of plastic rotation
$\Theta''(t)$	Plastic rotation vector
$\Delta\Theta''(t)$	Incremental form of plastic rotation vector
Θ_k''	Discrete form of plastic rotation vector

CHAPTER 1 INTRODUCTION

1.1 Background of This Research

For the past century, many research works in structural engineering have focused on how to protect structures from natural or man-made destructive forces. Among these forces, earthquake is considered to be one of the most destructive dynamic forces. As a result, structural performance during earthquake excitation becomes one of the most concerned topics in the area of structural engineering.

When structures are subjected to strong earthquake ground motions, structural members will first exhibit inelastic behavior and then go into damage phase or even finally the entire structure collapses if such damage is excessive. Due to economic reasons, certain degree of inelastic deformation or damage is permitted in earthquake resistant design (Park and Ang, 1985).

In traditional inelastic structural analysis, the displacement ductility factor, which is the ratio of maximum displacement to the yield displacement of the structure, has been used as a tool to evaluate the degree of nonlinearity and damage. It has been widely recognized, however, that the displacement ductility factor is insufficient to reflect the actual level of structural damage due to earthquake, and the inherent effect of cumulative damage due to substantial non-peak inelastic cycles should also be taken into account in evaluating the inelastic structural behaviors. In practice, however, it is very difficult to directly estimate cumulative damage by means of the whole inelastic deformation process, since the details of the whole process are not easy to be understood clearly (Fajfar and Vidic, 1994; Chai et al., 1998). Recent trends in seismic analysis methods are directed to energy-based method as an alternative to the ductility-based method in order to include the effect of cumulative damage. In this field, the energy concepts (including input energy and hysteretic energy), which are considered to be related to the cumulative damage potential of ground motion, are used to further ensuring satisfactory performance of the structure (Bertero and Uang, 1992; Krawinkler and Nassar, 1992; Bertero, 1997). However, due to the difficulty in evaluating energy during the energy transferring process, there still exist amounts of problems that prevent the concept of energy from being applied into practical design and damage assessment.

Chapter 1 Introduction

Therefore, more efforts on the study of how to evaluate the energy dissipation more accurately and how to describe the relationship between the energy dissipation and structural damage more clearly are necessary.

In addition, the question of shear strength and stiffness of joint panel zones in seismic design of steel frame structures has been discussed frequently over the last three decades. The seismic design code requirement of shear strength and stiffness of joint panel zones has changed from very loose to very severe and then relaxed some controls again in recent years before the 1994 Northridge earthquake (Krawinkler and Mohasseb, 1987). This change is based on increasing evidences (Krawinkler and Popov, 1982; Popov, 1987; Lee and Lu, 1989) that panel zones possess reliable shear ductility and energy dissipation capacity and thereby they can participate in the seismic resistance of structures to some extent. As a result, during severe earthquake ground motion, joint panel zones are allowed to have some degree of shear deformations and take part in energy dissipation, and this will induce a significant effect on the strength and stiffness of the structure and on the distribution of inelastic deformations in the frame structure. Although after the 1994 Northridge Earthquake large panel zone deformations are believed to be the critical responsibility for brittle fracture of beam-to-column connections and as a consequence some new seismic codes (NEHRP, 1997a and 1997b; AISC, 1997) again require a shear-strength-stronger panel zone to restrict the shear yielding in panel zones, they are still allowed to have some degree of shear yielding to dissipate portion of the energy in the whole structure on the premise of a good balance between the strength of beams and panel zones. In practice, however, the member-centerline dimensional model or rigid end member model are the most general ones for steel frame structures, apparently both of them do not consider beam-column panel zone deformation explicitly (Tsai and Popov, 1990). Therefore, in order to analyze responses and energy dissipation of a frame structure subjected to earthquake excitations more accurately, the effect of panel zones deformation should be studied carefully.

Finally, due to the inherent uncertainty of the earthquake excitation, random process should be a more appropriate model for the earthquake ground acceleration rather than only one deterministic acceleration time history. Therefore, from the

1950's, seismic analysis of structures has been divided into two different directions, deterministic and stochastic. Although in the past fifty years stochastic dynamics and reliability analysis based on the former have made great progress in various aspects, such as theory derivation, numerical computation, experiment technique, and practical application, the traditional deterministic design procedures is still in the dominant status in practice (Schueller, 1998), and moreover there are still many problems worth to be discussed in the field of nonlinear stochastic dynamic study. More effort in this area is needed in order to eventually achieve the objective of general application of stochastic dynamic methodology.

1.2 Main Contributions

In this study, the dynamic analysis of inelastic structures is based on the force analogy method with the aid of the state space method. Entire research refers two main fields: deterministic and stochastic. The main contribution of this research includes:

- Extending the dynamic analysis of inelastic structures based on force analogy method to analyze real moment resisting frames with rigid end offsets. New stiffness matrices for members with two ends being rigid are derived using the unit displacement method. Static condensation is then used to simplify the analysis by condensing out the unnecessary degrees of freedom. Through condensation, each term in the stiffness matrices of the force analogy method is modified simply through multiplying by a scale factor, which only depends on the sizes of rigid end and offset lengths.
- Performing numerical simulation on the proposed analysis procedure for both one-story one-bay frame and six-story moment resisting frame. The effect of rigid end offsets on dynamic behaviors of inelastic structures are studied by comparing the responses and energy dissipation obtained by the model with rigid end offsets with those obtained by the model with no end offset in both deterministic and statistic senses. Conclusions are drawn.
- Proposing a simple analytical method of modeling the panel zone deformation using rotational springs, which can model both shear deformation and inelastic

behavior of the panel zones at the same time. Modified force analogy method with static condensation is employed to study the effect of elastic and inelastic deformation of panel zones, while rigid end offsets are also considered.

- Performing numerical simulation on the proposed analytical method on panel zone deformation for both one-story one-bay frame and six-story moment resisting frame. The effect of elastic and inelastic deformation of the panel zones on dynamic behaviors of inelastic structures is studied by comparing the responses and energy dissipation between modeling structures with various panel zone rigidities and yield moments in the rotational springs in both deterministic and statistic senses. Conclusions are drawn.
- Proposing a new stochastic dynamic analysis method for inelastic structures based on the force analogy method, which is named here as the stochastic force analogy method (SFAM). The merit of the force analogy method is time-saving, due to the reason that only initial stiffness is involved in the recursive process, while the inelastic property of structures is reflected by the inelastic displacement. This advantage is remained in the stochastic procedure. The proposed stochastic force analogy method can produce the covariance functions of displacement, velocity, inelastic displacement, and plastic rotation at individual plastic hinge location. The reasonable agreement between the results obtained by the proposed method and those obtained by the Monte Carlo simulation method shows that the proposed stochastic force analogy method is a reasonable one.
- First use the plastic energy dissipation at individual plastic hinge location as an index to evaluate the probability of failure of these locations. Numerical simulation on a six-story moment resisting frame with rigid end offsets and deformable panel zones is performed. Conclusions are drawn.

1.3 Arrangement of Contents

After a general overview of this research in this chapter, the basic theory and the works done by other researchers, which is relevant to this research, are reviewed in Chapter 2.

Chapter 1 Introduction

The force analogy method, which is a main tool to analyze dynamic behaviors of inelastic structures in this research, is discussed in Chapter 3. The solution to the equation of motion is based on the state space method. The evaluation method of energy dissipation of inelastic structures under earthquake excitations based on the force analogy method is briefly stated. In addition, modifications to the formulations of all these methods after considering static condensation are discussed.

In Chapter 4, dynamic analysis of inelastic structures based on the force analogy method is extended for the first time to analyze real moment resisting frames with rigid end offsets. New stiffness matrices for members with two ends being rigid are derived using the unit displacement method. Through static condensation, each term of the stiffness matrices in the force analogy method is modified simply through multiplying by a scale factor, which depends only on the sizes of rigid end and offset lengths. Numerical simulations are performed on a one-story one-bay frame and a six-story moment resisting frame. Comparisons of the responses and energy dissipation are done between the model of structure with no end offset and that with rigid end offsets to show the effect of rigid end offsets on dynamic behaviors of inelastic structures.

A simple analytical method of modeling the panel zone deformation using rotational springs is proposed in Chapter 5. Both shear deformation and inelastic behavior of the panel zone are modeled. Modified force analogy method with static condensation is employed to provide the responses and energy dissipation of inelastic structures with deformable panel zones as well as rigid end offsets. Numerical simulations are performed on a one-story one-bay frame and a six-story moment resisting frame with various panel zone rigidities and yield moments in the rotational springs to show the effects of elastic and inelastic deformation of panel zones on dynamic behaviors of inelastic structures.

A new stochastic dynamic analysis method for inelastic structures based on the force analogy method is proposed for the first time in Chapter 6. The entire derivation process is presented. Numerical simulation is performed on a single degree of freedom system to show the reasonability of the proposed method by

Chapter 1 Introduction

comparing results obtained by the proposed method with those obtained by the Monte Carlo simulation method.

In Chapter 7, the investigation of the effects of rigid end offsets and panel zone deformation on dynamic behaviors of inelastic structures is performed based on the Monte Carlo simulation method using 500 sample ground acceleration time histories. The structural reliability is analyzed in terms of the probability of failure of individual plastic hinge location based on plastic energy dissipation at each plastic hinge location.

Finally, general conclusions of this research are drawn in Chapter 8. Recommendations are also given for a possibly future research.

CHAPTER 2 LITERATURE REVIEW

2.1 General Statement

The principle objective of this Chapter is to do some literature review on the works for seismic analysis field and provide a clear background for the study of this thesis. In fact, seismic analysis methods have experienced a long period of evolutionary process, and the methods can be classified from different viewpoints:

- Linear analysis or nonlinear analysis;
- Elastic analysis or inelastic analysis;
- Response spectra method or time history analysis;
- Ductility-based method or energy-based method;
- Passive control or active control;
- Deterministic analysis or probabilistic analysis (including stochastic analysis and reliability study);
- etc.

Among these methods there are significant achievements that are worth to be recognized. However, for the purpose of this research, only limited research works from several points of view, such as energy-based method, models of seismic beam-column joints of steel moment resisting frames, and nonlinear stochastic dynamic study are discussed.

2.2 Energy-Based Seismic Analysis Method

Energy-based method in seismic design has been recently emphasized as a tool for better estimation of seismic demand for structures. This method considers the loading effect of earthquakes on structures as an energy input and the structural resistance to be the energy absorption capacity. It is performed through the use of an energy balance among input energy, kinetic energy, recoverable elastic strain energy, viscous damped energy, and irrecoverable hysteretic energy. The kinetic energy and the strain energy at the end of the ground motion are generally small in comparison with other dissipative energies. Consequently, the basic equation of

interest is the energy balance between total input energy and the energies dissipated by viscous damping and inelastic deformations.

Energy-based design method was first proposed by Housner (1956, 1959). In his research works, he calculated two kinds of input energy, one is responsible for the whole building response and the other is responsible for the damage in the elastic-plastic system. He then proposed an adequate factor of safety to ensure sufficient energy absorbing capacity against collapse under extremely strong ground motion (Gioncu and Mazzolani, 2001).

However, this proposal has been ignored until the 80's of last century. The study of Zahrah and Hall (1984) demonstrated that the emphasis on displacement ductility, traditionally used to characterize inelastic spectra, ignores the cumulative damage that results from numerous cycles of inelastic deformations. Akiyama (1985) introduced the energy concept into structural design and developed a design method for both single-degree of freedom (SDOF) and multi-degree of freedom (MDOF) systems on the basis of the energy concept. Since then, this approach has gained extensive attention. Several relevant publications appeared, which discussed different problems related to the energy concept (Darwin and Nmai, 1986; Tembulkar and Nau, 1987; Uang and Bertero, 1988; Fajfar et al., 1989; Kuwamura and Galambos, 1989; Uang and Bertero, 1990).

In 1992, a workshop on nonlinear seismic analysis of reinforced concrete buildings was held in Beld, Slovenia (Fajfar and Krawinkler, 1992), and in this workshop several papers were presented to discuss about how to combine energy terms into cumulative damage models for the purpose of damage assessment and seismic resistant design. This workshop prompted further study on energy method in seismic analysis. As a result of research performed in recent years (Kuwamura et al., 1994; Sucuoglu and Nuttug, 1995; Nakashima et al., 1996; Decanini and Mollaioli, 1998; Riddell and Garcia, 2001), a great improvement in the estimation of input and hysteretic energy demands on structures has been achieved.

2.2.1 Evaluation of Energy Dissipation Demand of Structures

Proper seismic design implies that the energy dissipation capacity of the building structure should exceed the energy demand imposed by the ground motion.

The seismic energy entering the building structure during strong earthquake ground motion (Input Energy) is dissipated in part by damping (Damping Energy) and in part by yielding of the structural components (Hysteretic Energy). It is well known that hysteretic energy is the structural response parameter, which is often correlated to cumulative damage. Consequently, exact evaluation of input energy and hysteretic energy is an important event in inelastic seismic analysis.

(1) *Energy equations proposed by Uang and Bertero*

Uang and Bertero (1990) pointed out that two types of energy equation are usually used in structural seismic study.

The first method uses an absolute formulation:

$$\frac{m(\dot{y})^2}{2} + \int c\dot{x}dx + \int f_s dx = \int m\ddot{y}dx_g$$

where y , $\dot{y}$ and $\ddot{y}$ are absolute displacement, absolute velocity and absolute acceleration, respectively, and x_g is the ground displacement. The first left-hand-side term of the above equation is the ‘absolute’ kinetic energy E_k . The right-hand-side term is the ‘absolute’ input energy E_i .

The second method uses a relative formulation:

$$\frac{m(\dot{x})^2}{2} + \int c\dot{x}dx + \int f_s dx = -\int m\ddot{x}_g dx$$

where x and $\dot{x}$ are relative displacement and relative velocity, respectively, and $\ddot{x}_g$ is the ground acceleration. In the above equation, the first left-hand-side term is the ‘relative’ kinetic energy E'_k and the right-hand-side term is the ‘relative’ input energy E'_i .

Although the relative formulation has been appeared in the majority of previous investigations, the study of Uang and Bertero illustrated that the absolute energy equation is physically more meaningful.

(2) *Inelastic design spectra-hysteretic and input energy by Fajfar and Vidic*

The study of Fajfar and Vidic (1994) focused on the inelastic spectra of hysteretic energy and input energy. They proposed two dimensionless parameters γ

and E_h/E'_i , where E_h is the hysteretic energy and E'_i is the relative input energy. While the E_h -spectrum and E'_i -spectrum are derived from the γ -spectrum, E_h -spectrum is based on E_h/E'_i . The whole procedure was validated by comparing the approximate and 'exact' spectra determined by nonlinear dynamic time-history analysis.

The parameter γ , which controls the reduction of the deformation capacity of structures due to low-cycle fatigue, depends on the natural period of the system, prescribed ductility factor, the hysteretic behavior and the ground motion characteristics. It is defined by the formula:

$$\gamma = \frac{\sqrt{(E_h/m)}}{\omega D}$$

where m is the mass of the system, ω is the natural frequency and D is the maximum relative displacement of the system.

Based on the results of the parametric studies, they proposed a simple approximate formula for E_h/E'_i :

$$\frac{E_h}{E'_i} = c_E \frac{(\mu - 1)^{c_H}}{\mu}$$

where c_E and c_H are constants which depend on the hysteretic model and on the damping, and μ is the ductility factor (maximum displacement divided by the yield displacement).

However, the proposed procedure cannot be applied to the case of ground motions with extremely narrow-band spectra, like those recorded on very soft soil deposits.

(3) *Duration-dependent inelastic seismic design spectrum by Chai et al.*

Chai et al. (1998) proposed a procedure for developing the duration-dependent inelastic seismic design spectrum. The approach is established by requiring that the cyclic plastic strain energy capacity of the structure is larger than or equal to the seismic energy demand, which is the portion of the seismic input energy contributing to cumulative damage.

Here, the seismic energy demand E'_i is calculated as the relative input energy at the end of the ground motion:

$$E'_i = -\int_0^{t_0} m \ddot{x}_g \dot{x} dt$$

where t_0 is the time at end of ground motion; m is the mass of the oscillator; $\ddot{x}_g$ is the base acceleration; $\dot{x}$ is the relative velocity. For design purposes, the spectral input energy for a SDOF system (per unit mass) can be written as:

$$\frac{E'_i}{m} = \frac{1}{2} v_e^2$$

where v_e is the equivalent velocity for the input energy. It was related to the peak ground velocity $(\dot{x}_g)_{\max}$ via an amplification factor Ω_v .

The plastic strain energy capacity of the structure E_h is estimated using the model proposed by Park and Ang (1985) and subsequently modified by Chai et al. (1995). It depends on the ground motion parameters, the lateral strength parameters and the cumulative damage characteristics of the structure. The expression of the cyclic plastic strain energy capacity was given by:

$$\frac{E_h}{m} = \left[\frac{\Omega_a (\ddot{x}_g)_{\max} T}{2\pi R_\mu} \right]^2 \left[\mu_m \left(\frac{1}{\beta^*} + 1 \right) - \frac{1}{\beta^*} \mu_c - 1 \right]$$

where the formula $\frac{\Omega_a (\ddot{x}_g)_{\max} m}{R_\mu}$ represents the lateral yield force of the structure and Ω_a is the elastic response acceleration amplification factor, R_μ is the ductility-related force reduction factor, and $(\ddot{x}_g)_{\max}$ is the peak-ground-acceleration.

Finally, for safe seismic design of structures, the cyclic plastic strain energy capacity of the structure must be equal to or greater than the seismic energy demand contributing to damage, i.e.

$$\frac{E_h}{m} \geq \alpha \frac{E'_i}{m}$$

(4) *Simplified representation of hysteretic and input energy spectra by Manfredi*

Manfredi (2001) proposed a simplified representation of hysteretic and input energy spectra, which is based on an equivalent number of cycles. The peak values of the hysteretic and input energy can be obtained using the proposed procedure. The hysteretic energy expression is:

$$\frac{E_h}{m} = (\mu_c - 1)n_{eq} \left(\frac{S_a(T)}{\omega} \right)^2 \left(\frac{1}{R} \right)^2$$

where μ_c is cyclic ductility, ω is the angular frequency of the oscillator, n_{eq} is the equivalent number of cycles representing the number of cycles at the maximum plastic displacement that the structure can develop, which is correlated to the earthquake characteristics by a seismic index I_D , $S_a(T)$ is the elastic spectral acceleration and R is the reduction factor. The seismic index I_D can be expressed as

$$I_D = \frac{I_E}{\text{PGA PGV}}$$

where PGA and PGV are the peak ground acceleration and the peak ground velocity, respectively, while I_E is equal to:

$$I_E = \int_0^{t_E} a(t)^2 dt$$

where $a(t)$ is the ground acceleration and t_E is the earthquake duration. At the same time, the input energy expression is:

$$\frac{E_i}{m} = 1.4\mu_c n_{eq} \left(\frac{S_a(T)}{\omega} \right)^2 \left(\frac{1}{R} \right)^2$$

From these approximate formulations, it is possible to obtain the simplified expressions of the maximum value of the input energy and hysteretic energy:

$$\begin{aligned} \left(\frac{E_h}{m} \right)_{\max} &= 0.32 \left(\frac{\sqrt{\mu_c - 1}}{\mu_c} + 0.23 I_D \frac{\mu_c - 1}{\mu_c} \right) \left(\frac{S_a(T)}{\omega} \right)_{\max}^2 \\ \left(\frac{E_i}{m} \right)_{\max} &= 0.45 \left(\frac{1}{\sqrt{\mu_c - 1}} + 0.23 I_D \right) \left(\frac{S_a(T)}{\omega} \right)_{\max}^2 \end{aligned}$$

2.2.2 Energy-Based Damage Index

Damage index is a useful and convenient tool for damage assessment in seismic analysis. Recently, several models of damage index were developed based on energy concept to include the effect of cumulative damage. This kind of energy-based model combines the energy together with the displacement to represent a more reasonable model, which considers the damage influence of the maximum deformation and non-peak inelastic cycles simultaneously. A comparison between different damage index models has been done by Cosenza et al. (1993). The followings summarize some energy-based damage indices.

(1) *Energy damage index proposed by McCabe and Hall*

McCabe and Hall (1989) pointed out that hysteretic energy would be a more complete indicator of the deformation process. They developed a damage index using hysteretic energy by combining two completely new damage measures, one based on the concept of equivalent hysteretic cycles of deformation and the other on the concept derived from low-cycle fatigue theory. According to the proposed damage index, one can evaluate damage based not only on ductility principles, but also on the repetitive cyclic nature of the deformation.

The damage index is described as the ratio of the actual hysteretic energy dissipated to the hysteretic energy corresponding to complete damage. It was found that a second-degree hysteretic energy ratio appeared to provide more reasonable results. Finally, the damage index definition includes two terms, one corresponding to symmetrical response and the other corresponding to nonsymmetrical response which can lead to a residual stress and further damage. Accordingly, the damage index DI was chosen to be:

$$DI = \left[\frac{H_p + H_n}{H_t} \right]^2 + \left[\frac{H_p - H_n}{H_t} \right]^2$$

where H_p is the positive resistance hysteretic energy, H_n is the negative resistance hysteretic energy and H_t is the total hysteretic energy. When the damage index reaches or exceeds a value of 1.0, the structure is considered to be completely damaged.

(2) *Banon and Veneziano damage index*

The damage index introduced by Banon and Veneziano (1982) consists of two damage parameters d_1 and d_2 , which are defined as the ratio of stiffness at yielding point to secant stiffness at failure point and the ratio of the total plastic energy to the absorbed energy at the yielding point, respectively. If the model is elastic-plastic, d_1 is equal to the ratio of the maximum displacement to the yielding displacement. Therefore, damage parameters d_1 and d_2 are expressed as:

$$d_1 = \frac{x_{max}}{x_y}, \quad d_2 = \frac{E_h}{(1/2)F_y x_y}$$

Finally, combining the above two parameters, the damage function D_B is defined as

$$D_B = \sqrt{(d_1 - 1)^2 + (a d_2^b)^2}$$

where a and b are two structural parameters obtained by experiments. For reinforced concrete members, $a = 1.1$ and $b = 0.38$. When D_B reaches a value of 1.0 means that the model collapse occurs.

(3) *Park and Ang's damage model*

The damage model of Park and Ang (1985) is the most popular one in the technical literature. It can be utilized to evaluate potential damage of reinforced concrete structures under earthquake ground motions. In this model, seismic structural damage is expressed as a linear function of the maximum deformation and hysteretic energy:

$$D = \frac{\delta_M}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \quad \text{or} \quad D = \frac{\delta_M}{\delta_u} + \beta \int \left(\frac{\delta}{\delta_u} \right)^\alpha \frac{dE}{E_c(\delta)}$$

in which δ_M is the maximum deformation under earthquake, δ_u is the ultimate deformation under monotonic loading, Q_y is the calculated yield strength, $E_c(\delta)$ is the hysteretic energy per loading cycle at deformation δ , dE is the incremental absorbed hysteretic energy, and α and β are non-negative parameters. A value $D \geq 1.0$ signifies complete collapse or total damage.

The parameter β represents the effect of cyclic loading on structural damage. Because β is not specified and has to be obtained by many laboratory or

field data, it is strongly dependent on the hysteretic model of the elements. Furthermore, this damage model is not normalized, which means, it does not equal to 0 in case of $x_{\max} = x_y$ and does not equal to 1 in the case of failure under monotonic loading (Cosenza et al., 1993).

2.3 Seismic Beam-Column Joint models of Moment Resisting Steel Frames

In moderate to severe seismic regions, moment resisting steel frames are common seismic resisting systems for low- to mid-rise buildings. Under strong ground motion, the structures will inevitably enter inelastic deformation phase. These inelastic behaviors most likely occur around the beam-to-column joints, including the beam section, the column section, and the beam-to-column joint which consists of the beam-to-column connection and the joint panel zone. The role of beam-to-column joints is guaranteeing reliable transfer of forces and moments between the beam and the column. According to the criterion of design code, i.e. “strong column-weak beam” and “strong joint-weak member”, most of the inelastic deformations are expected to occur in the beams. But in practice this criterion will produce a heavy burden on the joints which must own sufficient strength and ductility to permit the development of plastic hinges in the beams. As a result, joints may also be experiencing inelastic deformation or damage, and this possibility will further affect the amount and distribution of inelastic behavior and damage of the whole structure. The joint model is one of the most difficult problems in seismic analysis. Conventional analytical models of joints such as nominally pinned without any strength and fully rigid with full strength without any deformation do not explicitly consider the inelastic deformations of joints. Therefore, some recent studies emphasize explicit modeling of joints. Since connection and panel zone are two different parts composing of beam-to-column joint in moment resisting steel frames, many studies focus on each of them respectively, and there have appeared several models for both of them.

2.3.1 Connections

Connection is the location at which two members are connected and the means of this interconnection (e.g. welds, bolts, plates, stiffeners, and etc). In high seismic regions, WWWF and BWWF are two general types for beam-to-column connection in steel moment resisting frames, where WWWF means welded-web-welded-flange, and BWWF means bolted-web-welded-flange. From the point of view of safety, both of them are experimental satisfactory in terms of strength and ductility to permit the development of plastic hinges in the beams, although the low cycle fatigue life of the BWWF connection is somewhat erratic. From the point of view of economy, BWWF is less expensive than WWWF. Due to the above reasons, BWWF is a strongly recommended type of connection in the past (Krawinkler and Popov, 1982; Popov et al., 1986; Tsai et al., 1995).

However, the qualification of these traditional types of connections was doubted during the 1994 Northridge Earthquake. Because during this earthquake, the ductility performance of beam-to-column connections was found to be very poor and numbers of them are observed to experience brittle fractures at or near the flange weld of the beam-to-column connection (Roeder and Foutch, 1996). Since then, extensive investigations have been attempted to provide a rational explanation, achieve a possible solution to the failure of connections, and explore its effect on the seismic behavior of whole steel frames (SAC, 1994; Roeder and Foutch, 1996; El-Tawil et al., 2000; Luco and Cornell, 2000; Roeder, 2002a and 2002b; Lee and Foutch, 2002). This task is rather difficult since there is plenty of effect factors among which even exists interaction. Nevertheless, researchers have made a great progress in terms of new and more reliable seismic design codes.

The behavior of connections is somewhat between fully rigid and perfectly flexible. Thus, the analytic models of connections can be divided into three major categories of fully-rigid, semi-rigid, and fully-flexible.

(1) *Fully-rigid model*

This type of model assumes that the beam-to-column connections have sufficient rigidity to hold on the original angles between the connecting members without any change, i.e. the connections have no deformation. This model will lead

to an underestimate to frame drift and overestimate to frame strength. Nevertheless, its application is relatively popular because of its simplicity (Lui and Chen, 1986).

(2) *Semi-rigid model*

This type of model assumes that the beam-to-column connections possess certain moment capacity and intermediate rigidity between fully-rigid and fully-flexible to connect the intersecting members. The connection itself can experience elastic as well as inelastic deformation. Till now, various models of semi-rigid connections representing moment-rotation curves ($M-\theta$) have been proposed in the literature.

(a) Linear Model

The linear models include single-linear, bilinear, piecewise-linear (more than two segments) models. The common characteristics of them are that they all have unchanged stiffness within one straight-line segment. The difference is that some of them only have one stiffness term within the entire range of loading, while some uses two or more stiffness terms to describe the behavior of connection. The merit of using of these types of models is offset by the jumps of stiffness at the transition points (Council, 1993).

(b) Polynomial Model

A polynomial function developed by Frye and Morris (1975) can simulate the experimental results of moment-rotation behavior well:

$$\theta = C_1(KM)^1 + C_2(KM)^3 + C_3(KM)^5$$

where θ is the rotation, M is the moment, K is the scaling factor which is a function of connection size, and C_i 's are curve-fitting constants which are determined by the method of least squares. Since it may produces unreasonable negative stiffness, the utility of the model should be careful (Council, 1993).

(c) Power Model

Krishnamurthy et al. (1979) proposed a simple two-parameter power model for end-plate connections:

$$\theta = aM^b$$

where a and b are two curve-fitting parameters with the conditions $a > 0$ and $b > 1$.

Ang and Morris (1984) extended a standardized Ramberg-Osgood function to a power form,

$$\frac{\theta}{(\theta)_0} = \left| \frac{KM}{(KM)_0} \right| \left[1 + \left(\left| \frac{KM}{(KM)_0} \right| \right)^{n-1} \right]$$

where $(\theta)_0$, $(KM)_0$, and n are system parameters and K is the standardization parameter which relates to connection type and geometry.

(d) Exponential Model

Lui and Chen (1986) proposed a multi-parameter exponential model

$$M = M_0 + \sum_{j=1}^n C_j \left[1 - \exp \left(-\frac{|\theta|}{2j\alpha} \right) \right] + K_s |\theta|$$

where M_0 is initial moment, K_s is strain-hardening stiffness, α is scaling factor, and C_j is curve-fitting parameters.

The exponential model of Yee and Melchers (1986) has four-parameter,

$$M = M_p \left| 1 - \exp \left[-\frac{(K_i - K_s + C\theta)\theta}{M_p} \right] \right| + K_s \theta$$

where M_p is plastic moment capacity, K_i is initial elastic stiffness, K_s is strain-hardening stiffness, and C is constant correlating to the slope of curve. This model is proposed for bolted connections.

A more refined three-parameter exponential model is proposed for top and seat-angle connections with and without double web angle (Council, 1993). The relationship of moment and rotation are given as:

$$\frac{M}{M_u} = n \ln \left(1 + \frac{\theta}{n\theta_0} \right)$$

or

$$\frac{\theta}{\theta_0} = n \left[\exp \left(\frac{M}{nM_u} \right) - 1 \right]$$

where M_u is idealized elastic-plastic moment capacity, K_i is initial stiffness, θ_0 is reference rotation and $\theta_0 = M_u / K_i$, and n is shape parameter. The shape parameter n is a curve-fitting parameter which can be determined from test data. This model is applicable to those connections whose $M - \theta$ relationship can be approximated as elastic-plastic.

(3) *Fully-Flexible (Simple) model*

This simple connection is assumed to be ideally-pinned and thus it is able to rotate freely. This model is applicable to simulate the behavior of relatively flexible connections within the steel frames under low earthquake loading, particularly in terms of the lateral drifts and the corresponding $P-\Delta$ effects (Astaneh-Asl et al., 1989; Nader et al., 1996). With this type of model the beams will be overestimated while the columns will be underestimated (Lui and Chen, 1986a).

2.3.2 *Panel Zones*

Panel zone is the portion of column web corresponding to the connection dimensions. During the past three decades, the shear behavior of joint panel zone and its effect on the whole structure have been investigated experimentally by some researchers (Krawinkler et al., 1975; Krawinkler and Popov, 1982; Popov et al., 1986; Popov, 1987; Lee and Lu, 1989; Schneider et al., 1993; Tsai et al., 1995). According to these studies, some conclusions can be made as follows:

- Well-detailed panel zones exhibit significant reserve strength beyond yielding of the panel zone.
- Well-detailed panel zones possess considerably stable ductility and hysteretic behaviors under cyclic loadings.
- Well-detailed panel zones own excellent energy dissipation capacity.
- Panel zone deformation has a significant effect on the behaviors of the whole structure.
- Extremely large panel zone deformations may lead to local kinking in beam and column flanges and may cause crack propagation and fracture at the beam flange welds.

Chapter 2 Literature Review

- The excessive reduction of shear strength of panel zone may lead to an undesirable strength degradation in the whole structure.

Based on these investigations it can be concluded that panel zone participation in energy dissipation is desirable when the steel frame structure is subjected to severe earthquake. As a result, several modern building codes (*Recommended*, 1988; SANZ: “Code”, 1988; *Uniform*, 1991; *Specifications*, 1992) have relaxed the requirement of shear strength of panel zone and allow yielding development in panel zone to some extent.

However, due to the disadvantage influence of inelastic deformation of panel zone on the whole structure and the connection welds which was first observed in experiments and then was proven during the 1994 Northridge Earthquake, there exists a problem on how to balance the inelastic deformation taken by the beams and by the panel zones in order to utilize the energy dissipation capacity of panel zones as well as control the detrimental effect of panel zone deformation to the utmost extent. This problem has been noticed or explored (Krawinkler and Mohasseb, 1987; Popov et al., 1989; Schneider et al., 1993; Tsai et al., 1995; Schneider and Amidi, 1998; El-Tawil et al., 1999) and balance conditions for some kinds of connections have been proposed (Roeder, 2002b). Nevertheless, the investigation on this problem is only at the primary stage, and more efforts are needed for a further and thorough understanding.

Since weak panel zones are widely believed to have contributions to the connection damage which are generally observed in moment resisting steel frames after the Northridge Earthquake, new seismic design codes (AISC, 1997; NEHRP, 1997a and 1997b) require stronger panel zones in shear than the last generations and expect that flexural yielding first occur in beams, while limited shear yielding is still permitted in panel zones after beam yields. Under this circumstance, the effects of panel zone elastic and inelastic deformations on dynamic response should be considered explicitly in seismic analysis for more accurate estimation. Some research has been done with explicitly modeling the panel zone deformations in seismic analysis model of moment resisting steel frames (Lui and Chen, 1986b; Krawinkler and Mohasseb, 1987; Kato et al., 1988; Popov et al., 1989; Tsai and

Chapter 2 Literature Review

Popov, 1990; Osman et al., 1995; Khudada and Geschwindner, 1997; Schneider and Amidi, 1998; El-Tawil et al., 1999; Gupta and Krawinkler, 2000; Foutch and Yun, 2002; Wong and Wang, 2004a; Wong and Wang, 2004b).

The study of Krawinkler and Mohasseb (1987) showed that joint panel zone shear deformations do not have an apparent influence on the global dynamic response of frame structures, whereas they have a considerable effect on local inelastic deformation response of the individual members. Therefore, in order to predict the behavior of steel frames more accurately, the effects of joint shear strength and stiffness should be included in the mathematical model, especially for frames with weak joints.

According to Tsai and Popov (1990), a weaker panel zone can induce larger story drift in moment resisting steel frame, and as a result the frame will be much more flexible than the conventional one with rigid end zones. Therefore, the effect of panel zone deformations must be considered in the evaluation of elastic story drift. However, introducing flexibility of the panel zones in traditional models will produce additional degrees of freedom. An approximate formulation was proposed to conveniently calculate the story drift of a given floor due to the panel zone shear deformation without introducing additional degrees of freedom. The story drift of a given floor due to the panel zone shear deformation is approximately equal to the average panel zone deformation for the same floor:

$$\theta_{\gamma} = \frac{\sum_{j=1}^{nc} \gamma^j}{nc}$$

where θ_{γ} is the story drift of a given floor due to the panel zone shear deformation, γ^j is panel zone shear deformation at the j th column line at a given floor, and nc is number of the column in the frame at the floor being considered. Consequently, the total story drift can be obtained by directly adding θ_{γ} into the results of traditional rigid end model. This amplifying influence of weaker panel zone on the frame's lateral drift is also observed by Osman et al. (1995), and they suggested a design criterion on panel zone, i.e. stable hysteretic behavior, high post-elastic stiffness, reasonable over-strength, global ductility and limited lateral deflections.

Schneider and Amidi (1998) developed a tri-linear analytical model to simulate the strength and stiffness of the panel zone by combining two bilinear rotational springs, where one corresponding to the strength of the column web, and the other for the strength of the column flanges. Through comparing the results among four types of model, it was found that simple analytic models that do not include panel zone deformation would underestimate the elastic drift and the rotational ductility of beam-column joints, and whereas overestimate the base shear capacity for moment-resisting steel frames.

Detailed finite element analyses (El-Tawil et al., 1999) clearly demonstrated that a weak panel zone can lead to a greater potential for brittle and/or ductile fracture at larger plastic rotations, and large inelastic panel zone deformations are needed to develop the ultimate plastic shear capacity of panel zone.

Till now all of the models of panel zone proposed in the literature describe panel zone shear characteristic, i.e. the relationship between shear force and shear deformation. Some of them are summarized in followings.

(1) *Krawinkler Model*

Krawinkler (1978) modeled the panel zone as an elastic-plastic shear panel surrounded by four rigid elements. The effective width and height of the panel are taken as 95% of the column and beam depths, respectively, and the rigid elements are connected by rotational springs at the corners. The elastic shear stiffness K_e and the yield shear strength V_y of panel zone are

$$K_e = 0.95d_c t_{cw} G$$

$$V_y = 0.55F_y d_c t_{cw}$$

where F_y is the steel yield stress, G is shear modulus, and d_c and t_{cw} are the column depth and column web thickness, respectively. Based on some experimental results, it is shown that due to the contributions of the column flanges, panel zones own certain post-yield shear strength V_p , which is greater than the yield shear strength V_y . Therefore, Krawinkler proposed a formula of the post-yield shear strength V_p which corresponds to the ultimate shear distortion equal to four times

the yield shear distortion, γ_y . At the same time the post-yield shear stiffness K_p can also be obtained:

$$V_p = 0.55F_y d_c t_{cw} \left(1 + 3.45 \frac{b_{cf} t_{cf}^2}{d_c d_b t_{cw}} \right)$$

$$K_p = \frac{1.095 b_{cf} t_{cf}^2 G}{d_b}$$

where d_b is beam depth, b_{cf} and t_{cf} are the column flange width and thickness, respectively. The term inside parenthesis in the formula for V_p represents the increase in strength beyond V_y due to the contributions of the column flanges.

(2) Kato Model

In this Kato model (Kato et al., 1988), the contributions of the column flange are also included in the shear strength of the joint panel zone V_p , and the formula are given as:

$$V_p = \frac{1}{\sqrt{3}} \phi d_c t_{cw} F_y$$

$$\phi = \frac{\alpha^2 + 2(1+\nu)(1+2\beta)}{\alpha^2 + 2(1+\nu)}$$

$$\alpha = \frac{d_b}{b_{cf}}, \quad \beta = \frac{b_{cf} t_{cf}}{d_c t_{cw}}$$

where F_y is the steel yield stress, d_c is the column depth, t_{cw} is the column web thickness, ϕ is the parameter corresponding to the column flange, ν is Poisson's ratio, d_b is the beam depth, and b_{cf} and t_{cf} are the column flange width and thickness, respectively.

(3) A model proposed by Schneider et al.

The work of Schneider et al. (1998) is based on theoretical derivation and not on experimental data. The shear strength of panel zone is considered to be equal to the sum of the strength of the column web part and the column flange part. First

they are calculated respectively, and then by added together. The total shear strength of the panel zone V_p is:

$$V_p = 0.577F_y t_{cw} (d_c - t_{cf}) \left[1 + 1.73 \frac{b_{cf} t_{cf}^2}{t_{cw} (d_c - t_{cf}) (d_b - t_{bf})} \right]$$

where F_y is the steel yield stress, d_c is the column depth, t_{cw} is the column web thickness, b_{cf} and t_{cf} are the column flange width and thickness, respectively, d_b is the beam depth, and t_{bf} is the beam flange thickness.

If the effective column depth $(d_c - t_{cf})$ and the effective beam depth $(d_b - t_{bf})$ are approximated to be $0.95d_c$ and $0.95d_b$, respectively, the shear strength of panel zone can be simplified to

$$V_p = 0.55F_y d_c t_{cw} \left(1 + 1.92 \frac{b_{cf} t_{cf}^2}{d_c d_b t_{cw}} \right)$$

2.4 Nonlinear Stochastic Dynamic Study

Up to date in dominant traditional fields, deterministic design procedures are still considered to be conventional. In nature, however, physical phenomena, which own uncertainty features, are common, especially for those under dynamic loadings. Many dynamic loadings, such as earthquakes, winds, sea waves, jet or rocket noises, and roadway unevenness, have strong randomness and more accurate models should be random functions of time as well as the responses of the structures subjected to such excitations (Wen, 1989). This fact has been recognized and taken attention in scientific groups dated back to Crandall (1958, 1963) and Caughey (1959). From then on, research on random vibration theory and applications has made significant progress over the past four decades, accompanied by more and more special conferences, journals and books. The books in this area, for example, Crandall and Mark (1963); Lin (1967); Clough and Penzien (1975); Elishakoff (1983); Nigam (1983); Yang (1986); Roberts and Spanos (1990); Soong and Grigoriu (1993); Nigam and Narayanan (1994); Lin and Cai (1995) are

excellent literatures for reference. The random vibration theory for linear systems has been rather mature (Schueller, 1998) and will not be discussed here.

When structures are excited by severe dynamic loadings, it is most likely that they will entry nonlinear or inelastic deformation phase, and finally these inelastic deformations may lead to serious damage and collapse in structures. Under this circumstance, the models including structural nonlinear properties are necessary and they will highly depend on the time history of the response and may deteriorate in strength or stiffness, or both (Wen, 1989). It is well known that it is not an easy task to describe the model of the inelastic structure and estimate their response under random excitation. However, there already have been plenty of achievements in this area. The methods can be divided into two directions, analytical and numerical. Analytical methods include Fokker-Planck-Kolmogorov (FPK) diffusion equation method, the perturbation method, stochastic averaging techniques, equivalent linearization method, equivalent nonlinear systems method, moment equations and closure techniques, etc. Because of the limitation of analytical methods in accuracy, efficient, and size of the system, the need for developing numerical methods has increased. In this part, the Monte Carlo simulation technique is available.

2.4.1 Analytical Methods

(1) FPK Diffusion Equation Method

When subjected to Gaussian white noise, the response of nonlinear systems owns Markov properties, called as diffusion Markov process or diffusion process. The transition probability density of the response may satisfy the Fokker-Planck-Kolmogorov (FPK) equation (Zhu, 1992; Schueller, 1998):

$$\frac{\partial p}{\partial t} + \sum_{i=1}^N \frac{\partial}{\partial x_i} [a_i(\mathbf{x}, t) p] - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{\partial^2}{\partial x_i \partial x_j} [b_{ij}(\mathbf{x}, t) p] = 0$$

where $\mathbf{x}$ represents the state vector of the response, p represents the transition probability density of the response and a_i and b_{ij} represent the drift and diffusion terms of the response, respectively.

The exact solutions of the FPK equation can be obtained for only few problems (Caughey and Ma, 1982; Zhu et al., 1990), and its application in practice

is restricted. On the other hand, the approximate solutions of the FPK equation are available and applicable for engineering problems. The approaches to obtain the approximate solutions are different. With the aid of eigenfunction-expansion of the transition probability density, Johnson and Scott (1980) obtained the approximate transient transition probability density of Duffing's oscillator. Galerkin's method can be applied to solve Duffing's oscillator, hysteretic system and higher dimension system, and the results were rather consistent with those of numerical simulation (Wen, 1975 and 1976; Soize, 1988). However, convergent rate of this method for strongly-nonlinear systems is unacceptable (Zhu, 1992). Based on Galerkin's technique, Langley (1985) proposed a finite element method for nonlinear random vibration. It was shown that the solution for Duffing's oscillator agreed perfectly with the exact one. The computing-time of FEM is less than Galerkin's method for strongly-nonlinear systems, but is still extremely long for systems of the order higher than two (Zhu, 1992; Schueller, 1998). The path integral method is a kind of numerical one which can be used to solve higher dimensional FPK equation (Wehner and Wolfer, 1983). This method is efficient and easily extended to multi-dimension system.

(2) *Perturbation Method*

Perturbation method is a classic approach in deterministic mechanics. It is a series solution method and can also be called small parameter method. It was first introduced into the field of nonlinear random vibration theory by Crandall in 1963, after which some researchers (Tung, 1967; Soni and Surrendran, 1975) applied it in studying the dynamic responses of some kinds of nonlinear systems. The thought of this method is to take advantage of the results of linear theory to solve nonlinear equations. As a result of finding a small parameter ε , the solution to the nonlinear differential equations can be replaced by an approximate one in terms of a power series in ε . Substituting the approximate solution expressing in ε into the initial nonlinear equations, an infinite set of linear equations can be obtained. These linear equations can be solved step by step, and finally approximate solutions to the initial nonlinear differential equations can be obtained (Zhu, 1992; Soong and Grigoriu, 1993).

Although it is an applicable method for general type of random excitations and multi-degree of freedom systems, perturbation method is only limited in weakly nonlinear behavior.

(3) *Stochastic Averaging Techniques*

The method of stochastic averaging was first formally introduced by Stratonovich (1963), and in his book he derived the formula of the drift and diffusion coefficients of average FPK equation. A few years later the principle of stochastic averaging was strictly proven in mathematics (Khasminskii, 1966; Papanicolaou and Kohler, 1974).

Based on the principle of stochastic averaging, the stochastic response of nonlinear systems excited by non-white noise process can be approximated as a diffusion process under certain condition. Through appropriate stochastic averaging of kinetic equation of given dynamic system, the drift and diffusion coefficients of FPK equation of this approximate diffusion process can be determined. Finally, the approximate response statistics of the original system can be derived by solving the averaging FPK equation. After averaging, FPK equation can be greatly simplified in terms of fewer dimensions and furthermore less solving difficulty than the initial one. Because of the above benefit, this method is widely used in nonlinear random vibration (Zhu, 1992). However, it is still limited to systems with very few degrees of freedom.

Roberts obtained the stationary response and energy envelope of nonlinear oscillators subjected to random excitation based on stochastic averaging method in 1977 and 1978, respectively. There already have been several papers to review stochastic averaging method and its application in random vibration (Roberts and Spanos, 1986; Zhu, 1988).

(4) *Equivalent Linearization Method*

Equivalent linearization is one of the most popular approximate analytical methods in studying stochastic behavior of nonlinear dynamic systems. In this method the nonlinear system is replaced by a linear one and the linearization coefficients are determined by minimizing some statistical measure error between

the two systems. An excellent summary of the method was given by Spanos (1981) and Roberts and Spanos (1990).

The method was first proposed to solve nonlinear random vibration problems by Caughey in 1959, who used the mean square error minimization criterion to find the parameters of the equivalent linear system for bilinear systems under white noise excitation. Since then the method has been developed greatly and generalized to multi-degree of freedom nonlinear system and non-stationary response case (Iwan and Yang, 1972; Iwan, 1973). The results have been further extended by Atalik and Utku (1976). The contributions of them were the closed form expressions of the parameters of the optimal linear system, which replaces the nonlinear one with nonlinear inertia under Gaussian excitation.

It was a significant and successful practice for equivalent linearization method to apply to hysteretic systems. Bilinear model is a simpler hysteretic one and some work are based on this kind of model (Asano and Iwan, 1984; Iwan and Paparizos, 1988). They proposed an analytical representation of the bilinear hysteretic characteristic that enables the equivalent linearization technique to be applied.

On the other hand, the study of Wen and his co-workers (Wen, 1980; Baber and Wen, 1981 and 1982) based on another hysteretic model, namely the smooth hysteretic restoring force model, which was initially proposed by Bouc (1967) and generalized by Wen (1976). For a single degree-of-freedom system, the equation of motion is

$$\ddot{x} + Q(x, \dot{x}, t) = f(t) \quad (2.1)$$

where $Q(x, \dot{x}, t)$ is the restoring force and consists of two parts

$$Q(x, \dot{x}, t) = g(x, \dot{x}) + h(x) \quad (2.2)$$

The former term in the right hand side of Eq. (2.2) is a non-hysteretic component, which is an algebraic function of the instantaneous displacement x and velocity $\dot{x}$:

$$g(x, \dot{x}) = c\dot{x} + \alpha kx \quad (2.3)$$

where c is the linear viscous damping, k is the initial stiffness, and α is the ratio of post-yielding to pre-yielding stiffness. Meanwhile, the latter term in the right hand side of Eq. (2.2) is the hysteretic component, a function of the time history of x and can be expressed as

$$h(x) = (1 - \alpha)kz \quad (2.4)$$

where z is an auxiliary variable and satisfies the following nonlinear differential equation

$$z = \frac{1}{\eta} \left[A\dot{x} - \nu \left(\beta |\dot{x}| |z|^{n-1} z - \gamma \dot{x} |z|^n \right) \right] \quad (2.5)$$

Parameters A , β , ν , η , γ and n govern the features of the hysteretic loop, and the adjustment of the values of them provides the model different shapes.

Performing the linearization on the smooth hysteretic model given in Eq. (2.5) can be done in closed form, which is a difficult task for other hysteretic models and consequently it facilitated the applicability of the linearization method to: multi-degree-of-freedom systems (Baber and Wen, 1982); non-zero mean case (Baber, 1984); strength and stiffness degrading systems (Baber and Wen, 1981; Baber and Noori, 1985); two-dimensional structure under biaxial load case (Park et al., 1986); three-dimensional frames (Casciati and Faravelli, 1988); nonlinear finite element analysis of the continuum (Simulescu et al., 1989), and biaxial and torsional structural response (Wen and Yeh, 1989).

The accuracy of equivalent linearization method for predicting the mean square of displacement and velocity is generally good when comparing with the analytical or simulation results as well as with system that has no exact relationship with the strength of nonlinearity. However, the method still has several deficiencies. One of them is the Gaussian distribution assumption of the response of nonlinear system, which cannot correctly estimate the tail behavior of the distribution. As a result, the mean square of displacement and velocity are often underestimated and the results will tend to be unsafe. Meanwhile, the following reliability analysis based on the distribution would be erroneous. In addition, equivalent linearization method is not applicable to those nonlinear systems with inherent nonlinear phenomena, such as jumps, limit cycles and parametrically excited vibration, because of their inherent non-Gaussian distribution (Socha and Soong, 1991; Zhu, 1992; Soong and Grigoriu, 1993; Schueller, 1998).

(5) *Equivalent Nonlinear Systems Method*

Due to the drawbacks of equivalent linearization method mentioned above, some researchers considered to develop more accurate and applicable method, but at the same time do not discard the basis idea of equivalent linearization method. On this background equivalent nonlinear systems method was proposed. This method approximates the original nonlinear system by another nonlinear one, which contains known and exact steady-state solution in some statistical sense.

The first utility of the method dated back to Lutes (1970), who directly analyzed the response of an equivalent nonlinear non-hysteretic system instead of a hysteretic system under the condition of minimizing the mean square error of the two systems. From then on, the method was applied to different nonlinear systems (Caughey, 1986; Cai and Lin, 1988; Zhu and Yu, 1989).

Equivalent nonlinear systems method has overcome some severe limitations of equivalent linearization method in terms of being applicable to those systems that contain inherent nonlinear phenomena. It is illustrated that equivalent nonlinear systems method can provide more satisfying results than equivalent linearization method, especially when the response is severely amplitude limited. Nowadays, however, the method still limits to single-degree-of-freedom nonlinear systems excited by white noise, the extension of its applicability is necessary in future (Nigam, 1983; Zhu, 1992).

(6) *Moment Closure Techniques*

For some kinds of nonlinear random vibration problem, response moments could be obtained by directly solving the moment equations. However, due to the nonlinearity, these moment equations form an infinite hierarchy and thus must be truncated before solutions can be obtained. These techniques are referred to as moment closure methods. Till now, in the field of random vibration, there are already three sorts of closure scheme: Gaussian closure, cumulant closure, and non-Gaussian closure (Zhu, 1992; Soong and Grigoriu, 1993).

Gaussian closure scheme is the simplest and most general scheme among the three. It is based on the hypothesis that response $X(t)$ is a Gaussian process. As a result, the moment equations can be closed on second order level, and then the

probability distribution and all higher order moments of random process $X(t)$ can be defined uniquely by the first two moments. This method has been applied to second order nonlinear elastic systems (Iyengar, 1975), hysteretic systems (Iyengar and Dash, 1978), stochastic time-varying systems (Iyengar and Dash, 1976), and parametric random vibration (Ibrahim, 1985).

Cumulant closure scheme is an extension of Gaussian closure scheme. Due to the assumption that all of the cumulants of response process $X(t)$ with orders higher than k are zero, the moment equations are closed on k th order level and thus all higher order moments can be expressed in terms of lower order moments. This means if $k = 2$, the scheme goes back to Gaussian closure scheme. It has been employed to investigate the parametrically and externally excited systems (Wu and Lin, 1984) and it has been shown that the results are better than those produced by Gaussian closure. The study of Bergman et al. (1995) has demonstrated that it is possible to select any closure level for this method and it will perform well for the case that the distribution is not too far from Gaussian, otherwise the method may lead to an erroneous solution, irrespective of the selected closure level.

Non-Gaussian closure scheme was first introduced by Crandall (1980; 1985). The procedure is to first approximating a non-Gaussian probability density with some unknown parameters, which can be derived from the moment equations, for the response of nonlinear system. Then the response statistics can be obtained by using a hypothetical probability density (Zhu, 1992; Soong and Grigoriu, 1993). The application of the method has been developed in the analysis of the non-stationary response of a Duffing oscillator (Liu and Davis, 1988). It was shown (Crandall, 1985) that unfit construction of the replaced non-Gaussian distribution may lead to incorrect solution.

2.4.2 Numerical Method

Monte Carlo simulation technique is the major as well as significant numerical method for the analysis of random vibration, especially for the study of nonlinear random vibration. The method is based on the principle of probability theory, i.e. the statistics of the system response can be approximately derived from

large amounts of response samples. Meanwhile, larger sample size will lead to more accurate results. According to this principle, it has three main procedures:

- Generating samples of excitation based on the statistic characteristics of random excitation or random field;
- Calculating the sample of system response under each sample of excitation one by one based on the method of deterministic analysis;
- Obtaining the needed statistic information of response, such as moment, probability density, spectral density, and so on, from substantial response samples.

In principle, Monte Carlo simulation method is applicable to any kinds of system and any kinds of excitation. Till now, it has been considered as the unique means to many complex problems, such as strong nonlinear, severe excitations and serious unevenness of materials. In addition, it also has been a powerful calibration to other various approximate methods. The computing effort of this method, however, is so considerable that the error decreases at the rate of $1/\sqrt{N}$, where N is the sample size. This means improving the accuracy slightly must be accompanied by increasing the computing-time a hundred times (Zhu, 1992; Schueller et al., 1997; Schueller, 1998). In order to solve the time-consuming problem, “importance sampling procedures” (Schueller et al., 1991; Bayer and Bucher, 1996), which are applicable for the random excitation with the model of few variables, and “controlled MCS procedures” (Pradlwarter and Kliemann, 1996; Pradlwarter and Schueller, 1999), which are applicable for Markov processes (e.g. white noise), have been proposed. With the aid of these procedures, the sample size, which is required to achieve an expectant accuracy, can be reduced. However, both of them are still in the preliminary stage and more research efforts in this area are needed.

2.5 Discussion on This Research

The objective of this research is to investigate the effect of elastic and inelastic deformation of panel zones on dynamic behaviors of inelastic structures in both deterministic and statistic senses. To achieve this objective, a simple analytical

Chapter 2 Literature Review

method of modeling the panel zone deformation using rotational springs is proposed. This model can simulate shear deformation as well as inelastic behavior of the panel zone at the same time. The inelastic analysis is based on the force analogy method, while the equation of motion is solved by means of the state space method. The study involving statistic field is based on the Monte Carlo simulation method. In the analysis, the model of rigid end offsets is considered to give a more accurate structural model. The effect of rigid end offsets on behaviors of inelastic structures is also investigated in both deterministic and statistic senses by comparing the results obtained using the model with rigid end offsets with those obtained using the model with no end offset. The results presented in this research include global responses, such as displacement, velocity, acceleration and drift ratio; global energy dissipation, such as kinetic energy, strain energy, damping energy, plastic energy, and input energy; local response (i.e., plastic rotation at individual plastic hinge location), and local energy dissipation (i.e., plastic energy dissipated at individual plastic hinge location). In this research, it is attempted to analyze the problems from the point of view of energy dissipation in structures. In addition, at the end of this study, the plastic energy dissipated at individual plastic hinge location is used as a calibration to estimate the probability of failure of individual plastic hinge location based on the results obtained by the Monte Carlo simulation method to further investigate the effects of rigid end offsets and panel zone deformation.

In addition, the present study also tries to make a progress in the field of stochastic dynamic analysis of inelastic structures. Since the force analogy method has been proven to be efficient in inelastic dynamic analysis in the deterministic field, it seems significant to introduce it into the field of stochastic inelastic dynamic analysis. In this research, the force analogy method is successfully combined for the first time with the procedure of stochastic analysis, and the newly proposed method is named as the stochastic force analogy method. It is a preliminary effort since the theoretical deduction presented in this research is only involving single degree of freedom system with one plastic hinge location. The production of the proposed stochastic force analogy method includes the covariance function of displacement, velocity, inelastic displacement, and plastic rotation at

Chapter 2 Literature Review

individual plastic hinge location. The reasonability of the proposed method is demonstrated based on the rather agreement between results of the proposed method and those of the Monte Carlo simulation method.

CHAPTER 3 SEISMIC ANALYSIS OF INELASTIC STRUCTURES WITH FORCE ANALOGY METHOD

3.1 General Statement

Under severe earthquake excitation, civil engineering structures will undergo inelastic deformation and enter into the inelastic phase. During this phase of structural behavior, the relationship between force and deflection will no longer be linear, but instead it will follow a nonlinear relationship. The general structural dynamic equation of motion subjected to earthquake is:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\ddot{\mathbf{X}}_g(t) \quad (3.1)$$

where $\mathbf{M}$ is mass matrix, $\mathbf{C}$ is damping matrix, $\mathbf{K}$ is stiffness matrix, $\mathbf{X}(t)$ is relative displacement vector, $\dot{\mathbf{X}}(t)$ is relative velocity vector, $\ddot{\mathbf{X}}(t)$ is relative acceleration vector, and $\ddot{\mathbf{X}}_g(t)$ is earthquake ground acceleration vector. Based on this equation, the objective is to solve for the responses of structures (i.e. displacement, velocity, and acceleration).

In Eq. (3.1), the nonlinear relationship between force and deflection during inelastic deformation is mainly reflected by stiffness matrix $\mathbf{K}$ which will no longer be a constant, but a variable that varies with deflection. For many years, major research works have concentrated on establishing more accurate models of the changing $\mathbf{K}$ matrix in order to obtain more accurate results on structural responses. However, due to the complexity of structures, it is often very difficult to gain a relative precise model of the $\mathbf{K}$ matrix. In addition, the $\mathbf{K}$ matrix will have different value at each time step because of its changing property as a function of time in the process of solving the equation. This results in costly computing effort.

It is worth to note that the goal of changing the $\mathbf{K}$ matrix is to obtain an accurate force representation in the product $\mathbf{K}\mathbf{X}(t)$ as shown in Eq. (3.1). About more than three decades ago, Lin (1968) first proposed the force analogy method to calculate the force. Recently, Hart and Wong (2000) extended this force analogy method into dynamic analysis of nonlinear structures, while Wong and Yang (1999) proposed a new algorithm based on the force analogy method. The basic concept of

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

this method is to achieve the same objective of calculating the force based on $\mathbf{K}\mathbf{X}(t)$ through varying displacement field instead of varying stiffness. Because only initial stiffness is used while changing the displacement, the algorithm is very time efficient, and it achieves high numerical accuracy when incorporating the use of state space numerical integration method in dynamic analysis.

In this chapter, some basic concepts and the derivation of the force analogy method are first introduced. Then the procedure of inelastic dynamic analysis combining the state space approach and the force analogy method is presented. Finally, the energy evaluation in inelastic structures and the transfer of energy among various forms over the duration of an earthquake, which were first proposed by Wong and Yang (2002), are presented.

In structural dynamic analysis, many degrees of freedom are usually modeled without any mass in order to simplify the analysis (Hart and Wong, 2000). Therefore, static condensation, which is very popular and well-established method in elastic analysis, is employed in the following derivations to reduce those DOFs without mass. In doing so, it will be shown here that modification to the force analogy method is needed when incorporating the static condensation to calculate the dynamic response and energy dissipation of inelastic structures.

3.2 Force Analogy Method

3.2.1 Basic Concepts

Before presenting the force analogy method in detail, it is necessary to introduce several basic concepts. Consider a single-degree-of-freedom (SDOF) system with a curve of force versus displacement as shown in Figure 3.1. In this figure, k_e is the initial stiffness, k_t is the post-yield stiffness, F_y and x_y represent the yield force and yield displacement, respectively. Let $F_s(t)$ and $x(t)$ represent the force and displacement of the system at any instant in time, respectively.

In Figure 3.1, initial stiffness line OA is extended to reaches the force $F_s(t)$ at point B. The displacement according to point B is defined as *elastic displacement*, denoted by $x'(t)$. The difference between the total displacement $x(t)$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

Figure 3.1 Bilinear Stiffness

and the elastic displacement $x'(t)$ is defined as *inelastic displacement*, denoted by $x''(t)$. Elastic displacement $x'(t)$ represents the displacement that will be recovered after unloading, whereas inelastic displacement represents the permanent deformation that will not be recovered after unloading. This concept can be expressed in mathematical form as

$$x(t) = x'(t) + x''(t) \quad (3.2)$$

$$\text{if } \begin{cases} x(t) \leq x_y \\ x(t) > x_y \end{cases} \text{ then } \begin{cases} x'(t) = F_s(t) / k_e, & x''(t) = 0 \\ x'(t) = F_s(t) / k_e, & x''(t) > 0 \end{cases} \quad (3.3)$$

Therefore, it can be seen that the force $F_s(t)$ at any displacement $x(t)$ is always equal to the elastic displacement multiplied by the initial stiffness, i.e.,

$$F_s(t) = k_e x'(t) \quad (3.4)$$

Substituting Eq. (3.2) into Eq. (3.4) gives

$$F_s(t) = k_e (x(t) - x''(t)) \quad (3.5)$$

Here it should be noticed that this elastic displacement is not equal to the yield displacement, i.e., $x'(t) \neq x_y$. Wong and Yang (1999) has presented that the force analogy method is applicable to both strain-hardening property (i.e., $k_t > 0$) and strain-softening property (i.e., $k_t < 0$).

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

In the force analogy method, the inelastic structural response is computed through a change in displacement, not in stiffness, to give the same level of force. This is the main idea of the force analogy method.

3.2.2 Force Analogy Method

The overall procedure of the force analogy method is discussed in Wong and Yang (1999), and it is briefly summarized here.

For an n -DOF system, the displacements can be expressed as:

$$\mathbf{X}(t) = \mathbf{X}'(t) + \mathbf{X}''(t) = \begin{Bmatrix} x'_1(t) \\ x'_2(t) \\ \vdots \\ x'_n(t) \end{Bmatrix} + \begin{Bmatrix} x''_1(t) \\ x''_2(t) \\ \vdots \\ x''_n(t) \end{Bmatrix} \quad (3.6)$$

where $\mathbf{X}(t)$ is the total displacement vector, $\mathbf{X}'(t)$ is the elastic displacement vector, and $\mathbf{X}''(t)$ is the inelastic displacement vector. Here the size of $\mathbf{X}(t)$, i.e., n , is refer to a displacement of degree of freedom (DDOF) of the structures.

In general, inelastic displacement of the structure is caused by any inelastic deformation of the structural members at certain locations in the structure. In the following presentation of the force analogy method, plastic rotation of the beams in a moment frame is used as the inelastic deformation of the structural members since this type of structural system is most commonly used around the world.

In a moment resisting structural system excited by any severe lateral dynamic loadings, the moments at the two ends of the members often exceed the moment capacity. At the locations where the moments exceed their corresponding yield moment, yielding occurs and these locations are called plastic hinge locations (PHLs). A plastic rotation degree of freedom (PRDOF) is assigned to each of these PHLs.

The total moments $\mathbf{M}(t)$ at the locations where plastic hinges may form are described as

$$\mathbf{M}(t) = \mathbf{M}'(t) + \mathbf{M}''(t) = \begin{Bmatrix} M'_1(t) \\ M'_2(t) \\ \vdots \\ M'_m(t) \end{Bmatrix} + \begin{Bmatrix} M''_1(t) \\ M''_2(t) \\ \vdots \\ M''_m(t) \end{Bmatrix} \quad (3.7)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

where $\mathbf{M}'(t)$ is the elastic moment vector due to elastic displacement, $\mathbf{M}''(t)$ is the residual moment vector due to inelastic displacement. The value m represents the total number of potential PHLs, i.e., the total number of PRDOFs.

First consider the residual moment vector $\mathbf{M}''(t)$. It is assumed that plastic rotations occur at certain PHLs in the structure. Here the plastic rotation vector, $\mathbf{\Theta}''(t)$, is defined as

$$\mathbf{\Theta}''(t) = \begin{Bmatrix} \theta_1''(t) \\ \theta_2''(t) \\ \vdots \\ \theta_m''(t) \end{Bmatrix} \quad (3.8)$$

Figure 3.2(a) illustrates a moment resisting framed structure, which has plastic rotations in certain PHLs. These plastic rotations will induce internal restoring forces within the member, as shown in Figure 3.2(b). At the DDOFs, the restoring forces are

$$\mathbf{F}_{RF}(t) = \begin{Bmatrix} F_{RF1}(t) \\ F_{RF2}(t) \\ \vdots \\ F_{RFn}(t) \end{Bmatrix} = -\mathbf{K}_p \mathbf{\Theta}''(t) \quad (3.9)$$

where $\mathbf{K}_p$ is the $n \times m$ matrix that relates the PRDOFs with the applied force on the DDOFs. Similarly, at all of the potential PHLs, the restoring moments due to the plastic rotation are

$$\mathbf{M}_{RF}(t) = \begin{Bmatrix} M_{RF1}(t) \\ M_{RF2}(t) \\ \vdots \\ M_{RFm}(t) \end{Bmatrix} = -\mathbf{K}_R \mathbf{\Theta}''(t) \quad (3.10)$$

where $\mathbf{K}_R$ is the $m \times m$ matrix that relates the plastic rotation vector with the moments at the PHLs. In order to balance the restoring forces as shown in Figure 3.2(b), opposite forces must be applied to the structural DDOFs (see Figure 3.2(c)). These applied forces are described as

$$\mathbf{F}_a(t) = -\mathbf{F}_{RF}(t) = \mathbf{K}_p \mathbf{\Theta}''(t) = \mathbf{KX}''(t) \quad (3.11)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

where $\mathbf{K}$ is the $n \times n$ global stiffness matrix, and $\mathbf{F}_a(t)$ is applied force on the structural DDOFs due to inelastic displacement. Solving for the inelastic displacement vector in Eq. (3.11) gives

$$\mathbf{X}''(t) = \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}''(t) \quad (3.12)$$

The moments at the potential PHLs, $\mathbf{M}_p(t)$, due to $\mathbf{F}_a(t)$, are related to the inelastic displacement field by the equation

$$\mathbf{M}_p(t) = \mathbf{K}_p^T \mathbf{X}''(t) = \mathbf{K}_p^T \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}''(t) \quad (3.13)$$

Finally, the residual moment can be obtained as

$$\begin{aligned} \mathbf{M}''(t) &= \mathbf{M}_{RF}(t) + \mathbf{M}_p(t) \\ &= -(\mathbf{K}_R - \mathbf{K}_p^T \mathbf{K}^{-1} \mathbf{K}_p) \boldsymbol{\Theta}''(t) \end{aligned} \quad (3.14)$$

Equation (3.14) represents the residual moment vector due to plastic rotations within the structure with no external applied force. For example, if an earthquake causes plastic rotations within the structure, then the residual moments represent the forces remaining in the members after the earthquake.

The elastic moment vector $\mathbf{M}'(t)$ can be related directly to the elastic displacement vector by $\mathbf{K}_p$ matrix

$$\mathbf{M}'(t) = \mathbf{K}_p^T \mathbf{X}'(t) \quad (3.15)$$

Substituted Eq. (3.6) into Eq. (3.15) gives

$$\mathbf{M}'(t) = \mathbf{K}_p^T [\mathbf{X}(t) - \mathbf{X}''(t)] \quad (3.16)$$

Then substituting Eq. (3.12) into Eq. (3.16) gives

$$\mathbf{M}'(t) = \mathbf{K}_p^T [\mathbf{X}(t) - \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}''(t)] \quad (3.17)$$

Thus, the total moment vector becomes

$$\mathbf{M}(t) = \mathbf{M}'(t) + \mathbf{M}''(t) = \mathbf{K}_p^T \mathbf{X}(t) - \mathbf{K}_R \boldsymbol{\Theta}''(t) \quad (3.18)$$

Rearranging terms gives

$$\mathbf{M}(t) + \mathbf{K}_R \boldsymbol{\Theta}''(t) = \mathbf{K}_p^T \mathbf{X}(t) \quad (3.19)$$

Equations (3.12) and (3.19) represent the essence of the force analogy method. Expressing these equations in discrete form gives

$$\mathbf{M}_{k+1} + \mathbf{K}_R \boldsymbol{\Theta}_{k+1}'' = \mathbf{K}_p^T \mathbf{X}_{k+1} \quad (3.20)$$

$$\mathbf{X}_{k+1}'' = \mathbf{K}^{-1} \mathbf{K}_p \boldsymbol{\Theta}_{k+1}'' \quad (3.21)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

Define $\Delta\Theta_k''$ as the change in plastic rotation between time steps k and $k+1$, then

$\Theta_{k+1}'' = \Theta_k'' + \Delta\Theta_k''$. This gives

$$\mathbf{M}_{k+1} + \mathbf{K}_R \Delta\Theta_k'' = \mathbf{K}_P^T \mathbf{X}_{k+1} - \mathbf{K}_R \Theta_k'' \quad (3.22)$$

$$\mathbf{X}_{k+1}'' = \mathbf{K}^{-1} \mathbf{K}_P (\Theta_k'' + \Delta\Theta_k'') \quad (3.23)$$

Figure 3.2 Force Analogy Method

3.3 Inelastic Dynamic State Space Analysis Using The Force Analogy Method

Eq. (3.1) can be changed to the following form by recognizing that the initial stiffness matrix multiplying the elastic displacement gives the total force:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}'(t) = -\mathbf{M}\ddot{\mathbf{X}}_g(t) \quad (3.24)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

where $\mathbf{K}$ is the initial stiffness matrix and $\mathbf{X}'(t)$ is the elastic displacement vector. Solving for the elastic displacement vector $\mathbf{X}'(t)$ in Eq. (3.6) and substituting the result into Eq. (3.24) gives

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\ddot{\mathbf{X}}_g(t) + \mathbf{K}\mathbf{X}''(t) \quad (3.25)$$

With the aid of Eq. (3.25), the force analogy method can be used in dynamic analysis of inelastic structures subjected to earthquake loadings. Here the state space numerical integration method is used. Define

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{X}(t) \\ \dot{\mathbf{X}}(t) \end{Bmatrix}$$

Then Eq. (3.25) becomes

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{H}\mathbf{a}_g(t) + \mathbf{F}_p^c \mathbf{X}''(t) \quad (3.26)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{1} \end{bmatrix}, \quad \mathbf{F}_p^c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{K} \end{bmatrix}$$

and $\mathbf{0}$ is the zero matrix, $\mathbf{I}$ is the identity matrix, and $-\mathbf{1}$ is the matrix with either 0's or -1 's in all entries. The $\mathbf{H}$ matrix contains 6 columns, where each column corresponds to the ground acceleration in 6 different directions, and $\mathbf{a}_g(t)$ is the 6×1 earthquake ground acceleration vector. The solution to Eq. (3.26), which is a first order linear differential equation, is

$$\mathbf{z}(t) = \mathbf{e}^{\mathbf{A}(t-t_0)} \mathbf{z}(t_0) + \mathbf{e}^{\mathbf{A}t} \int_{t_0}^t \mathbf{e}^{-\mathbf{A}s} [\mathbf{H}\mathbf{a}_g(s) + \mathbf{F}_p^c \mathbf{X}''(s)] ds \quad (3.27)$$

Let $t_{k+1} = t$, $t_k = t_0$, and $\Delta t = t - t_0$, then the discrete form of Eq. (3.27) can be expressed as

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_k'' \quad (3.28)$$

where

$$\mathbf{F}_s = \mathbf{e}^{\mathbf{A}\Delta t}, \quad \mathbf{H}_d = \mathbf{e}^{\mathbf{A}\Delta t} \mathbf{H} \Delta t, \quad \mathbf{F}_p = \mathbf{e}^{\mathbf{A}\Delta t} \mathbf{F}_p^c \Delta t \quad (3.29)$$

and $\mathbf{a}_k$ and $\mathbf{X}_k''$ are the discretized forms of $\mathbf{a}_g(t)$ and $\mathbf{X}''(t)$, respectively.

Note that Eq. (3.28) is a recursive equation. For a given earthquake time history (i.e., $\mathbf{a}_k$) and assuming that all the structural information is known at time step k (i.e., $\mathbf{z}_k$, $\mathbf{M}_k$, $\mathbf{\Theta}_k''$, and $\mathbf{X}_k''$), the state vector at time step $k+1$ (i.e., $\mathbf{z}_{k+1}$) is

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

first calculated using Eq. (3.28). The displacement vector, $\mathbf{X}_{k+1}$ defines the total displacements at all the DOFs and is embedded in $\mathbf{z}_{k+1}$. Based on this computed $\mathbf{X}_{k+1}$, both the moment vector and the plastic rotation vector at time step $k+1$ (i.e., $\mathbf{M}_{k+1}$ and $\mathbf{\Theta}_{k+1}''$) are solved simultaneously by using the force analogy method described by Eq. (3.20) together with the moment versus plastic rotation relationship defined at each PHL. In this study, the relationship of force and displacement is assumed to be elastic-plastic. Therefore, the process of calculating $\mathbf{M}_{k+1}$ and $\mathbf{\Theta}_{k+1}''$ can be described as following: First assume $\mathbf{\Theta}_{k+1}'' = \mathbf{\Theta}_k''$, i.e., assume $\Delta\mathbf{\Theta}_k'' = 0$, where $\mathbf{\Theta}_{k+1}'' = \mathbf{\Theta}_k'' + \Delta\mathbf{\Theta}_k''$. Compute $\mathbf{M}_{k+1}$ by Eq. (3.20) with known $\mathbf{X}_{k+1}$ and $\mathbf{\Theta}_{k+1}''$, then compare $\mathbf{M}_{k+1}$ to $\mathbf{M}_s$, where $\mathbf{M}_s$ is yield moment. If $\mathbf{M}_{k+1} < \mathbf{M}_s$, it can be known that the assumption of $\Delta\mathbf{\Theta}_k'' = 0$ is correct. On the other hand, if $\mathbf{M}_{k+1} \geq \mathbf{M}_s$, according to the elastic-plastic relationship let $\mathbf{M}_{k+1} = \mathbf{M}_s$, then $\mathbf{\Theta}_{k+1}''$ can be calculated by Eq. (3.20) with known $\mathbf{X}_{k+1}$ and $\mathbf{M}_{k+1}$. The inelastic displacement vector at time step $k+1$ (i.e., $\mathbf{X}_{k+1}''$) is then computed by using the computed $\mathbf{\Theta}_{k+1}''$ in Eq. (3.21). This completes the calculation of all the structural information at time step $k+1$ (i.e., $\mathbf{z}_{k+1}$, $\mathbf{M}_{k+1}$, $\mathbf{\Theta}_{k+1}''$, and $\mathbf{X}_{k+1}''$) and the recursive calculation can be advanced to the next time step.

3.4 Modified Force Analogy Method with Static Condensation

Although static condensation is a simple and well-established technique in elastic structural analysis, it is not guaranteed to work efficiently and appropriately when the structure behaves inelastically. The reason is because inelastic dynamic analysis using updating stiffness method changes the stiffness matrix during inelastic deformation and therefore static condensation must be inefficiently performed at every time step. However, force analogy method uses the method of updating displacement instead of stiffness to give the same level of force, and therefore performing static condensation only once at the beginning of the analysis is necessary.

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

Consider an N -DOF system, let n represents those degrees of freedom with mass that remain after static condensation (n -DOFs), and a represents the other degrees of freedom without mass that are to be condensed out by static condensation (a -DOFs). This gives the equation $N = n + a$. For simplicity, consider the case where damping at the a -DOFs is ignored. The governing equation of motion is given by the formula:

$$\begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_n(t) \\ \ddot{\mathbf{X}}_a(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{X}}_n(t) \\ \dot{\mathbf{X}}_a(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}'_n(t) \\ \mathbf{X}'_a(t) \end{Bmatrix} = - \begin{bmatrix} \mathbf{M}_{nn} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_g(t) \\ \mathbf{0} \end{Bmatrix} \quad (3.30)$$

The subscript n denotes the matrices or vectors corresponding to the n -DOFs and the subscript a denotes the matrices or vectors corresponding to the a -DOFs.

Consider the second equation of the matrix equation given in Eq. (3.30), which is

$$\mathbf{K}_{an} \mathbf{X}'_n(t) + \mathbf{K}_{aa} \mathbf{X}'_a(t) = \mathbf{0} \quad (3.31)$$

Solving for the elastic displacement vector at a -DOFs (i.e., $\mathbf{X}'_a(t)$) gives

$$\mathbf{X}'_a(t) = -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n(t) \quad (3.32)$$

which represents the dependency of the elastic displacement at a -DOFs on the elastic displacements at n -DOFs. Now substituting Eq. (3.32) back into the first equation of Eq. (3.30) gives

$$\mathbf{M}_{nn} \ddot{\mathbf{X}}_n(t) + \mathbf{C}_{nn} \dot{\mathbf{X}}_n(t) + \bar{\mathbf{K}}_{nn} \mathbf{X}'_n(t) = -\mathbf{M}_{nn} \ddot{\mathbf{X}}_g(t) \quad (3.33)$$

where

$$\bar{\mathbf{K}}_{nn} = \mathbf{K}_{nn} - \mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \quad (3.34)$$

Since the elastic displacement $\mathbf{X}'(t)$ is equal to the total displacement $\mathbf{X}(t)$ minus inelastic displacement $\mathbf{X}''(t)$ based on the force analogy method given in Eq. (3.6), it follows similarly for the elastic displacement at n -DOFs, i.e., $\mathbf{X}'_n(t) = \mathbf{X}_n(t) - \mathbf{X}''_n(t)$. Substituting this result into Eq. (3.33) gives

$$\mathbf{M}_{nn} \ddot{\mathbf{X}}_n(t) + \mathbf{C}_{nn} \dot{\mathbf{X}}_n(t) + \bar{\mathbf{K}}_{nn} \mathbf{X}_n(t) = -\mathbf{M}_{nn} \ddot{\mathbf{X}}_g(t) + \bar{\mathbf{K}}_{nn} \mathbf{X}''_n(t) \quad (3.35)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

Equation (3.35) represents the equation of motion after performing static condensation, where only the displacements at the n -DOFs are considered. Using state space numerical integration method of analysis, Eq. (3.35) becomes

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{H}\mathbf{a}_g(t) + \mathbf{F}_p^c \mathbf{X}_n''(t) \quad (3.36)$$

where

$$\mathbf{z}(t) = \begin{Bmatrix} \mathbf{X}_n(t) \\ \dot{\mathbf{X}}_n(t) \end{Bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{nn}^{-1}\overline{\mathbf{K}}_{nn} & -\mathbf{M}_{nn}^{-1}\mathbf{C}_{nn} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{1} \end{bmatrix}, \quad \mathbf{F}_p^c = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_{nn}^{-1}\overline{\mathbf{K}}_{nn} \end{bmatrix}$$

The solution to Eq (3.36) is

$$\mathbf{z}(t) = \mathbf{e}^{\mathbf{A}(t-t_0)}\mathbf{z}(t_0) + \mathbf{e}^{\mathbf{A}t} \int_{t_0}^t \mathbf{e}^{-\mathbf{A}s} [\mathbf{H}\mathbf{a}_g(s) + \mathbf{F}_p^c \mathbf{X}_n''(s)] ds \quad (3.37)$$

Let $t_{k+1} = t$, $t_k = t_0$, and $\Delta t = t - t_0$, and then represent the ground acceleration vector and the inelastic displacement vector using pulses within the small time step Δt . Performing the integration in Eq. (3.37) gives

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d \mathbf{a}_k + \mathbf{F}_p \mathbf{X}_{n,k}'' \quad (3.38)$$

where

$$\mathbf{F}_s = \mathbf{e}^{\mathbf{A}\Delta t}, \quad \mathbf{H}_d = \mathbf{e}^{\mathbf{A}\Delta t} \mathbf{H} \Delta t, \quad \mathbf{F}_p = \mathbf{e}^{\mathbf{A}\Delta t} \mathbf{F}_p^c \Delta t \quad (3.39)$$

and $\mathbf{a}_k$ and $\mathbf{X}_{n,k}''$ are the discretizing forms of $\mathbf{a}_g(t)$ and $\mathbf{X}_n''(t)$, respectively.

Again, the time history analysis can be performed using the recursive equation given in Eq. (3.38) and the force analogy method. Assume that all the information at time step k are known, the states for all n -DOFs at time step $k+1$ can be computed. However, note that

$$\mathbf{z}_{k+1} = \begin{Bmatrix} \mathbf{X}_{n,k+1} \\ \dot{\mathbf{X}}_{n,k+1} \end{Bmatrix}$$

Since only the displacements at n -DOFs (i.e., $\mathbf{X}_{n,k+1}$ which is embedded in $\mathbf{z}_{k+1}$) are obtained based on Eq. (3.38), the information is incomplete in solving for the moments and plastic rotations increment in Eq. (3.20) or Eq. (3.22) because the displacements at all degrees of freedom (i.e., $\mathbf{X}_{k+1}$, which contains both $\mathbf{X}_{n,k+1}$ and $\mathbf{X}_{a,k+1}$) are required. Therefore, modification to both Eqs. (3.20) and (3.22) is necessary.

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

Using Eqs. (3.6) and (3.32), the displacements at all the degrees of freedom (including both n -DOF and a -DOF) can be written as:

$$\begin{aligned}
 \mathbf{X}_{k+1} &= \begin{bmatrix} \mathbf{X}_{n,k+1} \\ \mathbf{X}_{a,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{n,k+1} \\ \mathbf{X}'_{a,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{n,k+1} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_{n,k+1} + \mathbf{X}''_{a,k+1} \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{X}_{n,k+1} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} (\mathbf{X}_{n,k+1} - \mathbf{X}''_{n,k+1}) + \mathbf{X}''_{a,k+1} \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{X}''_{n,k+1} \\ \mathbf{X}''_{a,k+1} \end{Bmatrix} \\
 &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{X}''_{k+1}
 \end{aligned} \tag{3.40}$$

Now substituting Eq. (3.21) into Eq. (3.40) gives

$$\mathbf{X}_{k+1} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{K}^{-1} \mathbf{K}_P \boldsymbol{\Theta}''_{k+1} \tag{3.41}$$

Consider the multiplication of the two matrices in the second term on the right hand side of Eq. (3.41). Note that:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \tag{3.42}$$

Now post-multiplying Eq. (3.42) by $\mathbf{K}^{-1}$ gives

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} & \mathbf{I} \end{bmatrix} \mathbf{K}^{-1} \tag{3.43}$$

Therefore, substituting the result in Eq. (3.43) into Eq. (3.41), it follows that

$$\mathbf{X}_{k+1} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}''_{k+1} \tag{3.44}$$

Equation (3.44) gives the displacements at all DOFs in terms of the displacements at n -DOF and the plastic rotations at the plastic hinges. Substituting Eq. (3.44) into Eq. (3.20) and simplify gives

$$\mathbf{M}_{k+1} + \bar{\mathbf{K}}_R \boldsymbol{\Theta}''_{k+1} = \bar{\mathbf{K}}_P^T \mathbf{X}_{n,k+1} \tag{3.45}$$

where

$$\bar{\mathbf{K}}_R = \mathbf{K}_R - \mathbf{K}_P^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \tag{3.46}$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

$$\bar{\mathbf{K}}_P^T = \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \quad (3.47)$$

Let $\boldsymbol{\Theta}_{k+1}'' = \boldsymbol{\Theta}_k'' + \Delta \boldsymbol{\Theta}''$, this gives

$$\mathbf{M}_{k+1} + \bar{\mathbf{K}}_R \Delta \boldsymbol{\Theta}'' = \bar{\mathbf{K}}_P^T \mathbf{X}_{n,k+1} - \bar{\mathbf{K}}_R \boldsymbol{\Theta}_k'' \quad (3.48)$$

Equations (3.45) and (3.48) are the modification to Eqs. (3.20) and (3.22) after static condensation is performed, respectively.

Similar procedure can be performed on Eq. (3.21) to obtain a modified equation that directly calculates $\mathbf{X}_{n,k+1}''$ based on $\boldsymbol{\Theta}_{k+1}''$. Taking advantage of the identify presented in Eq. (3.47), pre-multiplying both sides of Eq. (3.21) by $\mathbf{K}$ and pre-multiplying the resulting equation again by the transpose of the bracket term in Eq. (3.47) gives

$$\begin{bmatrix} \mathbf{I} & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_{n,k+1}'' \\ \mathbf{X}_{a,k+1}'' \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \end{bmatrix} \mathbf{K}_P \boldsymbol{\Theta}_{k+1}'' \quad (3.49)$$

The stiffness matrix on the right hand side of Eq. (3.49) gives exactly $\bar{\mathbf{K}}_P$. Simplifying the matrix on the left hand side of Eq. (3.49) gives

$$\begin{aligned} \bar{\mathbf{K}}_P \boldsymbol{\Theta}_{k+1}'' &= \begin{bmatrix} \mathbf{I} & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_{n,k+1}'' \\ \mathbf{X}_{a,k+1}'' \end{Bmatrix} \\ &= \begin{bmatrix} \mathbf{K}_{nn} & -\mathbf{K}_{na} \mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}_{n,k+1}'' \\ &= \bar{\mathbf{K}}_{nn} \mathbf{X}_{n,k+1}'' \end{aligned} \quad (3.50)$$

Finally multiplying both sides of Eq. (3.50) by $\bar{\mathbf{K}}_{nn}^{-1}$ gives

$$\mathbf{X}_{n,k+1}'' = \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P \boldsymbol{\Theta}_{k+1}'' \quad (3.51)$$

Since $\boldsymbol{\Theta}_{k+1}'' = \boldsymbol{\Theta}_k'' + \Delta \boldsymbol{\Theta}''$, this gives

$$\mathbf{X}_{n,k+1}'' = \bar{\mathbf{K}}_{nn}^{-1} \bar{\mathbf{K}}_P (\boldsymbol{\Theta}_k'' + \Delta \boldsymbol{\Theta}'') \quad (3.52)$$

Equations (3.51) and (3.52) are the modification to Eqs. (3.21) and (3.23) after static condensation is performed, respectively.

In summary, Eqs. (3.22) and (3.23) in the original force analogy method are replaced by Eqs. (3.48) and (3.52) after performing static condensation, which represent the essence of modified force analogy method with static condensation. The main difference is that the stiffness matrices $\mathbf{K}$, $\mathbf{K}_P$, and $\mathbf{K}_R$ in the original

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

force analogy method are replaced by $\overline{\mathbf{K}}_{nn}$, $\overline{\mathbf{K}}_p$, and $\overline{\mathbf{K}}_R$ in the modified force analogy method, while the solution procedure remains the same.

3.5 Energy Evaluation Method of Inelastic Structures under Earthquake

Loading

From the energy point of view, the structural response under earthquake excitation is actually an energy transferring process. The excitation energy due to collision of underground plates is transferred through the soil to the structural systems so that the whole structure starts vibration. The excitation energy inputting into the whole structural system is then transferred to other types of energy forms such as kinetic energy, strain energy, damping energy, plastic energy, and etc.

In the process of characterizing energy in structures, one debate is that which concept is proper in the analysis, absolute energy or relative energy. The study of Uang and Bertero (1990) demonstrated that absolute concept is physically more meaningful than relative concept. Therefore, the following derivation uses the absolute energy representation. The overall procedure for calculating the energy with the concept of force analogy method has been presented in Wong and Yang (2002), and it is briefly summarized here.

3.5.1 General Energy Formulation

The derivation of the energy equation starts from Eq. (3.24). Let $\ddot{\mathbf{Y}}(t) = \ddot{\mathbf{X}}(t) + \ddot{\mathbf{X}}_g(t)$, where $\ddot{\mathbf{Y}}(t)$ is absolute acceleration. Substituting $\ddot{\mathbf{Y}}(t)$ into Eq. (3.24), we have

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}'(t) = \mathbf{0} \quad (3.53)$$

Integrating both sides of Eq. (3.53) over the path of structural response from time 0 to time t_k gives

$$\int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X} = 0 \quad (3.54)$$

From $d\mathbf{X}(t) = d\mathbf{Y}(t) - d\mathbf{X}_g(t)$ where $\mathbf{Y}(t)$ is the absolute displacement vector and $\mathbf{X}_g(t)$ is the ground displacement vector, we have

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

$$\int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{Y} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X} = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X}_g \quad (3.55)$$

From $d\mathbf{X}(t) = d\mathbf{X}'(t) + d\mathbf{X}''(t)$, it can be obtained that

$$\int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{Y} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}' + \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}'' = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X}_g \quad (3.56)$$

The first three terms in the left hand side of Eq. (3.56) are be rewritten as

$$\int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{Y} = \frac{1}{2} \dot{\mathbf{Y}}(t_k)^T \mathbf{M} \dot{\mathbf{Y}}(t_k) - \frac{1}{2} \dot{\mathbf{Y}}(0)^T \mathbf{M} \dot{\mathbf{Y}}(0) \quad (3.57)$$

$$\int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} d\mathbf{X} = \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} \dot{\mathbf{X}} dt \quad (3.58)$$

$$\int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}' = \frac{1}{2} \mathbf{X}'(t_k)^T \mathbf{K} \mathbf{X}'(t_k) - \frac{1}{2} \mathbf{X}'(0)^T \mathbf{K} \mathbf{X}'(0) \quad (3.59)$$

where $\dot{\mathbf{Y}}(t)$ in Eq. (3.57) is the absolute velocity vector. According to the assumption that the structure is at rest at the initial instant, it is known that both $\dot{\mathbf{Y}}(0)$ and $\mathbf{X}'(0)$ in Eqs (3.57) and (3.59) are zero. Substituting Eqs. (3.12) and (3.15) into the fourth term in the left hand side of Eq. (3.56) gives

$$\int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K} d\mathbf{X}'' = \int_{t=0}^{t=t_k} \mathbf{X}'^T \mathbf{K}_p d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' \quad (3.60)$$

Finally substituting Eqs. (3.57) to (3.60) into Eq. (3.56), it follows that

$$\frac{1}{2} \dot{\mathbf{Y}}^T \mathbf{M} \dot{\mathbf{Y}} + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}^T \mathbf{C} \dot{\mathbf{X}} dt + \frac{1}{2} \mathbf{X}'^T \mathbf{K} \mathbf{X}' + \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}^T \mathbf{M} d\mathbf{X}_g \quad (3.61)$$

Equation (3.61) is the general energy formulation and each term of it represents a kind of energy form:

- *Kinetic Energy (KE)*: The first term on the left hand side of Eq. (3.61) represents the kinetic energy.
- *Damping Energy (DE)*: The second term on the left hand side of Eq. (3.61) represents the damping energy.
- *Strain Energy (SE)*: The third term on the left hand side of Eq. (3.61) represents the strain energy. This strain energy contains the elastic displacement vector, which can be recovered after unloading.
- *Plastic Energy (PE)*: The fourth term on the left hand side of Eq. (3.61) represents the plastic energy, which is the energy dissipated due to the

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

formation of plastic hinges. If it is viewed in terms of each plastic hinge, we have:

$$PE = \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \sum_{i=1}^m M'_i d\theta''_i = \sum_{i=1}^m \int_{t=0}^{t=t_k} M'_i d\theta''_i = \sum_{i=1}^m PE_i \quad (3.62)$$

where PE_i is the plastic energy dissipation at the i th plastic hinge. It is implied that structural plastic energy PE is equal to the sum of individual plastic energies at each plastic hinge.

- *Input Energy (IE)*: The term on the right hand side of Eq. (3.61) represents the input energy.

In summary, Eq. (3.61) can be written as

$$KE + DE + SE + PE = IE \quad (3.63)$$

3.5.2 Energy Formulation with Static Condensation

When static condensation as discussed in Section 3.4 is involved in the analysis of the response, the general energy formulation in Eq. (3.61) should be changed accordingly. Let $\mathbf{Y}_n(t)$ be the absolute displacement vector of the n -DOFs of a structure, where n again represents the degrees of freedom that remain after static condensation. Thus, $\mathbf{Y}_n(t)$ can be written as the sum of relative displacement $\mathbf{X}_n(t)$ and ground displacement $\mathbf{X}_g(t)$. The equations for displacement, velocity, and acceleration of a condensed n -DOFs system can be written as:

$$\mathbf{Y}_n(t) = \mathbf{X}_n(t) + \mathbf{X}_g(t) \quad (3.64)$$

$$\dot{\mathbf{Y}}_n(t) = \dot{\mathbf{X}}_n(t) + \dot{\mathbf{X}}_g(t) \quad (3.65)$$

$$\ddot{\mathbf{Y}}_n(t) = \ddot{\mathbf{X}}_n(t) + \ddot{\mathbf{X}}_g(t) \quad (3.66)$$

From Eq. (3.33), the dynamic equilibrium equation subjected to earthquake excitations is given by the formula:

$$\mathbf{M}_{nn} \ddot{\mathbf{Y}}_n(t) + \mathbf{C}_{nn} \dot{\mathbf{X}}_n(t) + \bar{\mathbf{K}}_{nn} \mathbf{X}'_n(t) = \mathbf{0} \quad (3.67)$$

Integrating both sides of Eq. (3.67) over the path of structural response from time 0 to time t_k give

$$\int_{t=0}^{t=t_k} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} d\mathbf{X}_n + \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \bar{\mathbf{K}}_{nn} d\mathbf{X}_n = 0 \quad (3.68)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

Since $d\mathbf{X}_n(t) = d\mathbf{Y}_n(t) - d\mathbf{X}_g(t)$ based on Eq. (3.64), substituting this equation into the first term on the left hand side of Eq. (3.68) gives

$$\int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{Y}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} d\mathbf{X}_n + \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \overline{\mathbf{K}}_{nn} d\mathbf{X}_n = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g \quad (3.69)$$

In addition, since $d\mathbf{X}_n(t) = d\mathbf{X}'_n(t) + d\mathbf{X}''_n(t)$ based on Eq. (3.6), substituting this equation into the third term on the left hand side of Eq. (3.69) gives

$$\begin{aligned} \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{Y}_n + \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} d\mathbf{X}_n + \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \overline{\mathbf{K}}_{nn} d\mathbf{X}'_n \\ + \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \overline{\mathbf{K}}_{nn} d\mathbf{X}''_n = \int_{t=0}^{t=t_k} \ddot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{X}_g \end{aligned} \quad (3.70)$$

The first three terms on the left hand side of Eq. (3.70) are similar to those given in Eqs. (3.57) to (3.59), and therefore these three terms represent the kinetic energy, damping energy, and strain energy, respectively. Now consider the fourth term on the left hand side of Eq. (3.70). Based on Eq. (3.51), the change in inelastic displacement can be written as

$$d\mathbf{X}''_n(t) = \overline{\mathbf{K}}_{nn}^{-1} \overline{\mathbf{K}}_p d\boldsymbol{\Theta}''(t) \quad (3.71)$$

In addition, note that

$$\mathbf{X}'(t) = \begin{Bmatrix} \mathbf{X}'_n(t) \\ \mathbf{X}'_a(t) \end{Bmatrix} \quad (3.72)$$

Substituting Eq. (3.72) into Eq. (3.15) gives

$$\mathbf{M}'(t) = \mathbf{K}_P^T \begin{Bmatrix} \mathbf{X}'_n(t) \\ \mathbf{X}'_a(t) \end{Bmatrix} \quad (3.73)$$

Then, substituting Eq. (3.32) into Eq. (3.73) gives

$$\mathbf{M}'(t) = \mathbf{K}_P^T \begin{bmatrix} \mathbf{X}'_n(t) \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \mathbf{X}'_n(t) \end{bmatrix} = \mathbf{K}_P^T \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{aa}^{-1} \mathbf{K}_{an} \end{bmatrix} \mathbf{X}'_n(t) \quad (3.74)$$

Now substituting Eq. (3.47) into Eq. (3.74) gives

$$\mathbf{M}'(t) = \overline{\mathbf{K}}_P^T \mathbf{X}'_n(t) \quad (3.75)$$

Therefore, the fourth term on the left-hand side of Eq. (3.70) can be simplified using Eqs. (3.71) and (3.75), and it gives

$$\int_{t=0}^{t=t_k} \mathbf{X}_n'^T \overline{\mathbf{K}}_{nn} d\mathbf{X}''_n = \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \overline{\mathbf{K}}_P d\boldsymbol{\Theta}'' = \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' \quad (3.76)$$

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

In summary, the four terms on the left-hand side of Eq. (3.70) represent kinetic energy (KE), damping energy (DE), strain energy (SE), and plastic energy (PE), respectively, and the term on the right-hand side of Eq. (3.70) represents the input energy (IE). Assuming that the structure is at rest at time equals to zero, then these energy forms become:

$$KE = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}_n^T \mathbf{M}_{nn} d\mathbf{Y}_n = \frac{1}{2} \dot{\mathbf{Y}}_n^T(t_k) \mathbf{M}_{nn} \dot{\mathbf{Y}}_n(t_k) \quad (3.77)$$

$$DE = \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T \mathbf{C}_{nn} d\mathbf{X}_n = \int_{t=0}^{t=t_k} \dot{\mathbf{X}}_n^T(t) \mathbf{C}_{nn} \dot{\mathbf{X}}_n(t) dt \quad (3.78)$$

$$SE = \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \bar{\mathbf{K}}_{nn} d\mathbf{X}_n' = \frac{1}{2} \mathbf{X}_n'^T(t_k) \bar{\mathbf{K}}_{nn} \mathbf{X}_n'(t_k) \quad (3.79)$$

$$PE = \int_{t=0}^{t=t_k} \mathbf{X}_n'^T \bar{\mathbf{K}}_{nn} d\mathbf{X}_n'' = \int_{t=0}^{t=t_k} \mathbf{M}'^T d\boldsymbol{\Theta}'' = \sum_{i=1}^m \int_{t=0}^{t=t_k} M_i'(t) d\theta_i''(t) = \sum_{i=1}^m PE_i \quad (3.80)$$

$$IE = \int_{t=0}^{t=t_k} \dot{\mathbf{Y}}_n^T(t) \mathbf{M}_{nn} d\mathbf{X}_g(t) \quad (3.81)$$

3.6 Summary

In this Chapter the basic analytical procedures, which will be used as a basis for doing further study on inelastic dynamic behaviors of several kinds of structural models under deterministic and stochastic earthquake excitations in the following Chapters, are presented as a preparation. The core of this analytical procedure is the force analogy method. The force analogy method, which was first combined with the state space method and introduced into inelastic dynamic analysis by Wong and Yang (1999), is proven to be a power tool in the field of inelastic dynamic analysis due to its computing efficient and good accuracy. The method focuses on changing displacement instead of changing stiffness, which is conventional in the traditional inelastic analysis, to achieve the same force level and avoid the heavy labor of changing stiffness at each time step in the inelastic phase.

At the beginning of this Chapter, some basic concepts and the derivation procedure of the force analogy method are introduced. Then the whole inelastic dynamic analytical procedure, which combines the state space approach and the force analogy method, is presented. In order to delete those degrees of freedom

Chapter 3 Seismic Analysis of Inelastic Structures with Force Analogy Method

(DOFs) without mass to reduce the size of problems, static condensation is introduced into the inelastic dynamic state space procedure. As a result, the forms of stiffness matrices, which will appear in the force analogy method, are modified, while the solution procedure remains the same. Finally, the analytical procedure is extended to evaluate energy dissipation of inelastic structures under earthquake excitation. Similarly, the formula of energy dissipation will have some kind of difference between the models with and without static condensation.

CHAPTER 4 DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH RIGID END OFFSETS

4.1 General Statement

Two major problems exist in the conventional models of moment resisting frames in inelastic dynamic analysis. One problem is that plastic hinges, which occur or are expected to occur at the beams and columns, are often modeled at the two ends of the structural member. In this kind of modeling, the distance away from the joints where plastic hinges are designed to form is ignored, and in this study this distance is defined as *end offsets*. As a result of the ignorance of the dimensions of panel, the evaluation error is inevitable. In addition, with this kind of model where the flexibility of panel zones is ignored, the deformation of panel zones, irrespective of elastic or inelastic, cannot be studied in detail. Therefore, in order to explore the effects of panel zone deformation on the dynamic response of inelastic structures, first including end offsets of plastic hinges in the members in the structural models is necessary.

The second problem is that the rigidity of panel zones in beam-column joints is often ignored. In practice, the stiffness of panel zones is often larger than the connected members, but in the study models this discrepancy is usually ignored, i.e., the stiffness of panel zones is considered to be same as those of members. In order to model the structure and predict the responses more accurately, the rigidity of panel zone should be considered by modifying the stiffness matrix to include the rigidity of panel zones.

In this study, both problems discussed above are addressed simultaneously. To account for end offsets, the simplest method is to place additional joints at locations where plastic hinges are expected to occur. In doing so, plastic hinges will then occur at the ends of the shortened member. However, introducing additional joints means increasing the degrees of freedom and therefore increasing the size of the problem. The problem becomes even more complicated when rigidity of panel zones is considered because additional considerations on the compatibility of plastic hinges at the slaved joint must be addressed.

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

In this chapter, the modified force analogy method with static condensation discussed in Chapter 3 is used to study the effects of plastic hinges end offsets on inelastic dynamic behavior of structures. The uniqueness of this research is that additional joints placed at the plastic hinges with additional degrees of freedom will be condensed out by the method of static condensation. Through this process, *each term in the stiffness matrices used in the force analogy method is modified simply through multiplying by a scale factor*. The result is that the dynamic analysis contains the same number of degrees of freedom and the same number of plastic hinges. In addition, to account for the rigidity of beam-column joints, the new stiffness matrix is derived for members with two ends being rigid using the unit displacement method. This is an improvement to the original force analogy method or other conventional methods, where the condensed form is derived analytically and at the same time reduce the size of structural problem by eliminating the unnecessary additional degrees of freedom that must be placed at the plastic hinge.

4.2 Structural Modeling with Rigid End Offsets

In well-designed real moment resisting frames using the strong-column weak-beam concept, plastic hinges are expected to form in the beam with certain distance offset from the beam-column joints. In order to model the plastic hinges with rigid end offsets, additional joints with extra degrees of freedom (DOFs) are introduced at the plastic hinges. For example as shown in Figure 4.1, two plastic hinges in the beam member are expected to occur under loading, one at a distance dL from the left end and the other at a distance fL from the right end. This beam member originally contains 4 degrees of freedom at the two ends and is modeled using rigid end with infinite rigidity of lengths aL from the left end and cL from the right end. Adding 2 extra joints at the location of plastic hinges gives 4 extra DOFs to the beam, and these extra DOFs are indicated as $n+1$, $n+2$, $n+3$, and $n+4$. Introducing additional degrees of freedom at the plastic hinges increases the size of the problem. After adding the additional degrees of freedom (ADOFs) a to the original degrees of freedom (ODOFs) n , the degrees of freedom of the problem become $N = n + a$. Then, the analysis procedure discussed in Chapter 3 about

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

modified force analogy method with static condensation can be used to obtain the responses of the structures with rigid end offsets.

Figure 4.1 Additional DOFs at Plastic Hinges

4.3 Member Stiffness Matrix with Rigid End Offsets

Now consider that the beam member shown in Figure 4.1 is located somewhere inside a moment resisting frame. Isolating this beam from the frame as shown in Figure 4.2, the connectivity of this member to other part of the frame can be modeled as springs. Only the translation and rotational degrees of freedom are considered. Hence there are 8 DOFs, with 4 DOFs originally present in the model (Degrees of Freedom #1, 2, 3, and 4) and 4 DOFs added at the plastic hinges (Degrees of Freedom #5, 6, 7, and 8). The two plastic hinge locations (PHLs) are located at the two ends of length eL .

Figure 4.2 Model of Member with Rigid End Offsets

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

To formulate the stiffness matrix for this beam, the entire length of the member is first subdivided into three members of lengths dL (Member 1), eL (Member 2), and fL (Member 3). The stiffness matrices for these three members are:

$$\mathbf{K}_1 = \begin{bmatrix} \frac{12EI}{(a'L)^3} & \frac{12EI}{(a'L)^3}(aL) + \frac{6EI}{(a'L)^2} & -\frac{12EI}{(a'L)^3} & \frac{6EI}{(a'L)^2} \\ \frac{12EI}{(a'L)^3}(aL) + \frac{6EI}{(a'L)^2} & \frac{12EI}{(a'L)^3}(aL)^2 + \frac{12EI}{(a'L)^2}(aL) + \frac{4EI}{a'L} & -\frac{12EI}{(a'L)^3}(aL) - \frac{6EI}{(a'L)^2} & \frac{6EI}{(a'L)^2}(aL) + \frac{2EI}{a'L} \\ -\frac{12EI}{(a'L)^3} & -\frac{12EI}{(a'L)^3}(aL) - \frac{6EI}{(a'L)^2} & \frac{12EI}{(a'L)^3} & -\frac{6EI}{(a'L)^2} \\ \frac{6EI}{(a'L)^2} & \frac{6EI}{(a'L)^2}(aL) + \frac{2EI}{a'L} & -\frac{6EI}{(a'L)^2} & \frac{4EI}{a'L} \end{bmatrix}$$

$$\mathbf{K}_2 = \begin{bmatrix} 12EI/(eL)^3 & 6EI/(eL)^2 & -12EI/(eL)^3 & 6EI/(eL)^2 \\ 6EI/(eL)^2 & 4EI/(eL) & -6EI/(eL)^2 & 2EI/(eL) \\ -12EI/(eL)^3 & -6EI/(eL)^2 & 12EI/(eL)^3 & -6EI/(eL)^2 \\ 6EI/(eL)^2 & 2EI/(eL) & -6EI/(eL)^2 & 4EI/(eL) \end{bmatrix}$$

$$\mathbf{K}_3 = \begin{bmatrix} \frac{12EI}{(c'L)^3} & \frac{6EI}{(c'L)^2} & -\frac{12EI}{(c'L)^3} & \frac{12EI}{(c'L)^3}(cL) + \frac{6EI}{(c'L)^2} \\ \frac{6EI}{(c'L)^2} & \frac{4EI}{c'L} & -\frac{6EI}{(c'L)^2} & \frac{6EI}{(c'L)^2}(cL) + \frac{2EI}{c'L} \\ -\frac{12EI}{(c'L)^3} & -\frac{6EI}{(c'L)^2} & \frac{12EI}{(c'L)^3} & -\frac{12EI}{(c'L)^3}(cL) - \frac{6EI}{(c'L)^2} \\ \frac{12EI}{(c'L)^3}(cL) + \frac{6EI}{(c'L)^2} & \frac{6EI}{(c'L)^2}(cL) + \frac{2EI}{c'L} & -\frac{12EI}{(c'L)^3}(cL) - \frac{6EI}{(c'L)^2} & \frac{12EI}{(c'L)^3}(cL)^2 + \frac{12EI}{(c'L)^2}(cL) + \frac{4EI}{c'L} \end{bmatrix}$$

where the derivation of the stiffness matrix for Member 1 with length dL using the unit displacement method is shown in Figure 4.3. Figure 4.3(a) shows the forces required for a unit lateral displacement on the left end, while Figure 4.3(b) shows the forces required for a unit rotation at the left end. At the right end, Figure 4.3(c) shows the forces required for a unit lateral displacement, and Figure 4.3(d) shows the forces required for a unit rotation. The stiffness matrix for Member 3 can also be derived following a similar procedure as that for Member 1.

Assembling the member stiffness matrices into the global stiffness matrix $\mathbf{K}$ gives

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{na} \\ \mathbf{K}_{an} & \mathbf{K}_{aa} \end{bmatrix} \quad (4.1)$$

where

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

$$\mathbf{K}_{nn} = \begin{bmatrix} \frac{12EI}{(a'L)^3} + k_1 & \frac{6EI}{(a'L)^2} \left(1 + 2 \frac{a}{a'}\right) & 0 & 0 \\ \frac{6EI}{(a'L)^2} \left(1 + 2 \frac{a}{a'}\right) & \frac{4EI}{a'L} \left(1 + 3 \frac{a}{a'} + 3 \frac{a^2}{a'^2}\right) + k_2 & 0 & 0 \\ 0 & 0 & \frac{12EI}{(c'L)^3} + k_3 & -\frac{6EI}{(c'L)^2} \left(1 + 2 \frac{c}{c'}\right) \\ 0 & 0 & -\frac{6EI}{(c'L)^2} \left(1 + 2 \frac{c}{c'}\right) & \frac{4EI}{c'L} \left(1 + 3 \frac{c}{c'} + 3 \frac{c^2}{c'^2}\right) + k_4 \end{bmatrix}$$

$$\mathbf{K}_{na} = \mathbf{K}_{an}^T = \begin{bmatrix} -\frac{12EI}{(a'L)^3} & \frac{6EI}{(a'L)^2} & 0 & 0 \\ -\frac{6EI}{(a'L)^2} \left(1 + 2 \frac{a}{a'}\right) & \frac{2EI}{a'L} \left(1 + 3 \frac{a}{a'}\right) & 0 & 0 \\ 0 & 0 & -\frac{12EI}{(c'L)^3} & -\frac{6EI}{(c'L)^2} \\ 0 & 0 & \frac{6EI}{(c'L)^2} \left(1 + 2 \frac{c}{c'}\right) & \frac{2EI}{c'L} \left(1 + 3 \frac{c}{c'}\right) \end{bmatrix}$$

$$\mathbf{K}_{aa} = \begin{bmatrix} \frac{12EI}{(a'L)^3} + \frac{12EI}{(eL)^3} & -\frac{6EI}{(a'L)^2} + \frac{6EI}{(eL)^2} & -\frac{12EI}{(eL)^3} & \frac{6EI}{(eL)^2} \\ -\frac{6EI}{(a'L)^2} + \frac{6EI}{(eL)^2} & \frac{4EI}{a'L} + \frac{4EI}{eL} & -\frac{6EI}{(eL)^2} & \frac{2EI}{eL} \\ -\frac{12EI}{(eL)^3} & -\frac{6EI}{(eL)^2} & \frac{12EI}{(eL)^3} + \frac{12EI}{(c'L)^3} & -\frac{6EI}{(eL)^2} + \frac{6EI}{(c'L)^2} \\ \frac{6EI}{(eL)^2} & \frac{2EI}{eL} & -\frac{6EI}{(eL)^2} + \frac{6EI}{(c'L)^2} & \frac{4EI}{eL} + \frac{4EI}{c'L} \end{bmatrix}$$

The subscript 'n' corresponds to the original degrees of freedom (ODOFs) to be retained after static condensation and the subscript 'a' corresponds to the additional degrees of freedom (ADOFs) to be condensed out.

Figure 4.3 Derivation of Element Stiffness Matrix for Member 1

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Based on the configuration of the beam shown in Figure 4.2, the stiffness matrices based on the original force analogy method are:

$$\mathbf{K}_P^T = \begin{bmatrix} 0 & 0 & 0 & 0 & \vdots & \frac{6EI}{(eL)^2} & \frac{4EI}{eL} & -\frac{6EI}{(eL)^2} & \frac{2EI}{eL} \\ 0 & 0 & 0 & 0 & \vdots & \frac{6EI}{(eL)^2} & \frac{2EI}{eL} & -\frac{6EI}{(eL)^2} & \frac{4EI}{eL} \end{bmatrix} \quad (4.2)$$

$$\mathbf{K}_R = \begin{bmatrix} \frac{4EI}{eL} & \frac{2EI}{eL} \\ \frac{2EI}{eL} & \frac{4EI}{eL} \end{bmatrix} \quad (4.3)$$

where $\mathbf{K}_P$ is the $N \times m$ matrix that relates the plastic rotation vector with the restoring forces at the structural degrees of freedom (DOFs) and $\mathbf{K}_R$ is the $m \times m$ matrix that relates the plastic rotation vector with the restoring moments at the PHLs.

Recognizing the problem of increasing problem size by introducing additional degrees of freedom at the plastic hinges, modified force analogy method with static condensation discussed in Chapter 3 is now employed to reduce the additional degrees of freedom back to the original number of DOFs, which is n -DOFs. Performing static condensation using the equations given in Eqs. (3.26), (3.38) and (3.39), the new stiffness matrices are obtained using software MAPLE (2002) as:

$$\overline{\mathbf{K}}_{nn} = \begin{bmatrix} \frac{12EI}{(bL)^3} + k_1 & \frac{6EI}{(bL)^2} (1 + 2\frac{a}{b}) & -\frac{12EI}{(bL)^3} & \frac{6EI}{(bL)^2} (1 + 2\frac{c}{b}) \\ \frac{6EI}{(bL)^2} (1 + 2\frac{a}{b}) & \frac{4EI}{bL} (1 + 3\frac{a}{b} + 3\frac{a^2}{b^2}) + k_2 & -\frac{6EI}{(bL)^2} (1 + 2\frac{a}{b}) & \frac{2EI}{bL} (1 + 3\frac{a}{b} + 3\frac{c}{b} + 6\frac{ac}{b^2}) \\ -\frac{12EI}{(bL)^3} & -\frac{6EI}{(bL)^2} (1 + 2\frac{a}{b}) & \frac{12EI}{(bL)^3} + k_3 & -\frac{6EI}{(bL)^2} (1 + 2\frac{c}{b}) \\ \frac{6EI}{(bL)^2} (1 + 2\frac{c}{b}) & \frac{2EI}{bL} (1 + 3\frac{a}{b} + 3\frac{c}{b} + 6\frac{ac}{b^2}) & -\frac{6EI}{(bL)^2} (1 + 2\frac{c}{b}) & \frac{4EI}{bL} (1 + 3\frac{c}{b} + 3\frac{c^2}{b^2}) + k_4 \end{bmatrix} \quad (4.4)$$

$$\overline{\mathbf{K}}_P^T = \begin{bmatrix} \frac{6EI}{(bL)^2} (1 - 2\frac{a'}{b}) & \frac{4EI}{bL} (1 - 1.5\frac{a'-a}{b} - 3\frac{a'a}{b^2}) & -\frac{6EI}{(bL)^2} (1 - 2\frac{a'}{b}) & \frac{2EI}{bL} (1 - 3\frac{a'-c}{b} - 6\frac{a'c}{b^2}) \\ \frac{6EI}{(bL)^2} (1 - 2\frac{c'}{b}) & \frac{2EI}{bL} (1 - 3\frac{c'-a}{b} - 6\frac{c'a}{b^2}) & -\frac{6EI}{(bL)^2} (1 - 2\frac{c'}{b}) & \frac{4EI}{bL} (1 - 1.5\frac{c'-c}{b} - 3\frac{c'c}{b^2}) \end{bmatrix} \quad (4.5)$$

$$\overline{\mathbf{K}}_R = \begin{bmatrix} \frac{4EI}{bL} (1 - 3\frac{a'}{b} + 3\frac{a'^2}{b^2}) & \frac{2EI}{bL} (1 - 3\frac{a'}{b} - 3\frac{c'}{b} + 6\frac{a'c'}{b^2}) \\ \frac{2EI}{bL} (1 - 3\frac{a'}{b} - 3\frac{c'}{b} + 6\frac{a'c'}{b^2}) & \frac{4EI}{bL} (1 - 3\frac{c'}{b} + 3\frac{c'^2}{b^2}) \end{bmatrix} \quad (4.6)$$

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.4 Derivation of Element Stiffness Matrix with Two Ends Being Rigid

First consider the stiffness matrix $\bar{\mathbf{K}}_m$ in Eq. (4.4). This matrix is exactly the stiffness matrix of the structural member in Figure 4.2 using only Degrees of Freedom #1, 2, 3, and 4. The derivation of this stiffness matrix using the unit displacement method is summarized in Figure 4.4. In Figure 4.4(a), a unit lateral

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

displacement at the left end causes Member 2 to deform with lateral displacement of $v_{21} = 1$. Based on equilibrium and compatibility conditions of the beam member, calculating the member end forces give the first column of the stiffness matrix in Eq. (4.4). In Figure 4.4(b), a unit rotation at the left end causes Member 2 to deform with lateral displacement $v_{21} = aL$ in addition to the unit rotation $\theta_{21} = 1$. Based on equilibrium and compatibility conditions of the beam member, calculating the member end forces give the second column of the stiffness matrix in Eq. (4.4). Similarly, using the unit displacement patterns given in Figures 4.4(c) and 4.4(d) gives the third and fourth columns of the stiffness matrix in Eq. (4.4), respectively.

Now consider the $\overline{\mathbf{K}}_p^T$ matrix in Eq. (4.5). Each column of this matrix represents the moment at the two plastic hinges shown in Figure 4.2 due to a unit displacement or unit rotation at the corresponding degree of freedom. For example, due to a unit displacement at DOF #1 as shown in Figure 4.5(a), both ends of the member with length bL will experience moments of $6EI/(bL)^2$. By similar triangle, the moments at the plastic hinges are:

$$M_a = \frac{6EI}{(bL)^2} \times \frac{bL/2 - a'L}{bL/2} = \frac{6EI}{(bL)^2} \left(1 - 2\frac{a'}{b} \right) \quad (4.7)$$

$$M_c = \frac{6EI}{(bL)^2} \times \frac{bL/2 - c'L}{bL/2} = \frac{6EI}{(bL)^2} \left(1 - 2\frac{c'}{b} \right) \quad (4.8)$$

which is the same as the first column of the $\overline{\mathbf{K}}_p^T$ matrix in Eq. (4.5). Similarly, due to a unit rotation at DOF #2, the left end of member with length bL will experience a lateral displacement of aL in addition to the unit rotation. Hence the moments at both ends of the member will experience a combination of moments as shown in Figure 4.5(a) and 4.5(b). Then by similar triangle, the moments at the plastic hinges are computed as:

$$M_a = \frac{4EI}{bL} \times \frac{2bL/3 - a'L}{2bL/3} + \frac{6EI}{(bL)^2} (aL) \times \frac{bL/2 - a'L}{bL/2} = \frac{4EI}{bL} \left(1 - 1.5\frac{a' - a}{b} - 3\frac{a'a}{b^2} \right) \quad (4.9)$$

$$M_c = \frac{2EI}{bL} \times \frac{bL/3 - c'L}{bL/3} + \frac{6EI}{(bL)^2} (aL) \times \frac{bL/2 - c'L}{bL/2} = \frac{2EI}{bL} \left(1 - 3\frac{c' - a}{b} - 6\frac{c'a}{b^2} \right) \quad (4.10)$$

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

which is the same as the second column of the $\bar{\mathbf{K}}_p^T$ matrix in Eq. (4.5). Similar procedure can be used to obtain the results for the remaining columns of the $\bar{\mathbf{K}}_p^T$ matrix in Eq. (4.5) (see Figures 4.5 (c) and 4.5(d)).

Figure 4.5 Calculation of Moments at Plastic Hinges Due to Different Unit Displacement Patterns

Finally, let's consider the $\bar{\mathbf{K}}_R$ matrix in Eq. (4.6). This matrix relates the additional moments imposed on the plastic hinges when plastic rotation occurs at certain PHL. The coefficient in each term of this $\bar{\mathbf{K}}_R$ matrix represents a reduction of moments imposed on the plastic hinges, which is difficult to obtain when only classical beam theory is used. While comparing the $\bar{\mathbf{K}}_{mn}$ matrix in Eq. (4.4) with the $\bar{\mathbf{K}}_R$ matrix in Eq. (4.6), it is interesting to note that similar pattern exists for the

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

coefficients in the $4EI/(bL)$ and $2EI/(bL)$ terms. This pattern suggests that while rigid ends increase the structural stiffness by magnifying the $\bar{\mathbf{K}}_{mn}$ matrix, end offsets reduce the interdependency of plastic hinges in the same member by reducing the coefficients in the $\bar{\mathbf{K}}_R$ matrix.

In summary, each term of the stiffness matrices (i.e., $\mathbf{K}$, $\mathbf{K}_p$, and $\mathbf{K}_R$) in the force analogy method with static condensation is modified simply through multiplying by a scale factor. When considering the case of plastic hinges with rigid end offsets, this scale factor depends only on the sizes of rigid end and offset lengths.

4.4 Numerical Examples

Numerical studies are carried out to analyze the effects of rigid end offsets on inelastic structural dynamic behaviors. The 1940 El-Centro earthquake excitation is used. Figure 4.6 shows this acceleration versus time ground motion and Figure 4.7 shows plots of the corresponding spectral displacement, spectral velocity, and spectral acceleration versus period with two different damping ratios, i.e., $\zeta = 0.02$, and $\zeta = 0.05$. Meanwhile, two structural models, i.e. one-story one-bay and six-story three-bay moment resisting frames are selected to study the differences in response between models with no end offset (NEO) and with rigid end offsets (REO).

Figure 4.6 Ground Acceleration Time History of 1940 El-Centro Earthquake

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.7 Acceleration, Velocity, and Displacement Spectra for 1940 El-Centro Earthquake

4.4.1 Example 1: One-Story One-Bay Moment Resisting Frame

Consider the one-story one-bay moment resisting steel frame as shown in Figure 4.8. Assume that the length of columns is $L_c = 3$ m, the length of beam is $L_b = 5$ m, and $E = 200\text{kN/mm}^2$, and the plastic hinges in all three members exhibit elastic-plastic behavior with yield moments computed based on the plastic modulus

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Z of the members and a yield stress of 248.2 MPa. The interaction between axial force and moment in the columns is not considered here for simplicity. In addition, assume that both the beam and columns are rigid along the axial direction, then the frame becomes a 3 DOFs system (with 1 ODOF and 2 ADOFs) with 6 plastic hinges.

Figure 4.8 One-Story One-Bay Moment Resisting Frame

First consider the structure with no end offset (NEO), the stiffness matrices (i.e., $\mathbf{K}$, $\mathbf{K}_P$, and $\mathbf{K}_R$) without any modification are as followings:

$$\mathbf{K} = \begin{bmatrix} 24EI_c/L_c^3 & 6EI_c/L_c^2 & 6EI_c/L_c^2 \\ 6EI_c/L_c^2 & 4EI_c/L_c + 4EI_b/L_b & 2EI_b/L_b \\ 6EI_c/L_c^2 & 2EI_b/L_b & 4EI_c/L_c + 4EI_b/L_b \end{bmatrix} \quad (4.11)$$

$$\mathbf{K}_P = \begin{bmatrix} 6EI_c/L_c^2 & 6EI_c/L_c^2 & 0 & 0 & 6EI_c/L_c^2 & 6EI_c/L_c^2 \\ 2EI_c/L_c & 4EI_c/L_c & 4EI_b/L_b & 2EI_b/L_b & 0 & 0 \\ 0 & 0 & 2EI_b/L_b & 4EI_b/L_b & 4EI_c/L_c & 2EI_c/L_c \end{bmatrix} \quad (4.12)$$

$$\mathbf{K}_R = \begin{bmatrix} 4EI_c/L_c & 2EI_c/L_c & 0 & 0 & 0 & 0 \\ 2EI_c/L_c & 4EI_c/L_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 4EI_b/L_b & 2EI_b/L_b & 0 & 0 \\ 0 & 0 & 2EI_b/L_b & 4EI_b/L_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 4EI_c/L_c & 2EI_c/L_c \\ 0 & 0 & 0 & 0 & 2EI_c/L_c & 4EI_c/L_c \end{bmatrix} \quad (4.13)$$

where E , I , and L are the elastic modulus, moment of inertia, and length of the structural member, respectively, and subscript b denotes the beam and subscript c denotes the columns. Let the structural mass and damping be $m = 7.0 \times 10^6$ kg and

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

$c = 2,100$ kN-s/m, respectively, this gives a period of vibration $T = 1.068$ s and a damping ratio $\zeta = 2.54\%$. Assume the system is subjected to the 1940 El-Centro earthquake time history (see Figure 4.6) with a scale factor of 0.5, and that $\Delta t = 0.02$ s. The structural responses are plotted in Figures 4.9 to 4.11. These responses include global responses such as relative displacement, relative velocity, absolute acceleration of the mass, kinetic energy (KE), strain energy (SE), damping energy (DE), plastic energy (PE), and input energy (IE). In addition, Figures 4.9 and 4.10 also show selected local responses such as the plastic rotation and plastic energy dissipated at PHL #1, and the moment versus plastic rotation relationships for PHLs #2 and #3. Furthermore, the maximum responses of the structures due to El-Centro earthquake time history with a scale factor of 0.5 are summarized in Table 4.2.

Now consider the structure with rigid end offset (REO) where the length of rigidity is equal to half the panel zone dimension, the offset lengths are assumed to be half the column depth plus half the beam depth, and the stiffness modification factors used in Eqs. (4.4) to (4.6) for each of the members to compute stiffness matrices are summarized in Table 4.1. Assume that the same mass and damping values as in the case of NEO are used, the parameters of the structural model are:

$$m = 7.0 \times 10^6 \text{ kg} \quad , \quad c = 2,100 \text{ kN-s/m} \quad , \quad T = 0.892 \text{ s} \quad , \quad \zeta = 2.13\%$$

Subjected to the El-Centro earthquake time history (shown in Figure 4.6) with a scale factor of 0.5, the structural responses with rigid end offsets (REO) are also shown in Figures 4.9 to 4.11, and the maximum responses of the structures are also summarized in Table 4.2.

Table 4.1 Rigid End and Offset Properties of Each Member

	Member 1	Member 2	Member 3
a	0.0	0.0435	0.1532
b	0.8468	0.9130	0.8468
c	0.1532	0.0435	0.0
d	0.0725	0.1354	0.2257
e	0.7018	0.7292	0.7018
f	0.2257	0.1354	0.0725
a'	0.0725	0.0919	0.0725
c'	0.0725	0.0919	0.0725

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Table 4.2 Maximum Structural Global and Local Responses of One-Story One-Bay Frame for Cases NEO and REO

Maximum Response	NEO	REO	Percent Difference
Structural Parameters:			
Period (s)	1.068	0.892	-19.7%
Damping Ratio	2.54%	2.13%	-19.2%
Global Response:			
Displacement (cm)	6.19	4.47	-38.5%
Velocity (cm/s)	20.66	23.88	13.5%
Acceleration (g)	0.053	0.070	24.3%
Kinetic Energy (kJ)	83.3	129.5	35.7%
Strain Energy (kJ)	21.8	28.6	23.8%
Damping Energy (kJ)	150.0	174.7	14.1%
Plastic Energy (kJ)	525.6	776.5	32.3%
Input Energy (kJ)	718.7	986.4	27.1%
Local Response:			
Plastic Rotation at PHL #1 (rad)	0.0172	0.0126	-36.5%
Plastic Rotation at PHL #2 (rad)	0.0149	–	–
Plastic Rotation at PHL #3 (rad)	–	0.0145	–
Plastic Energy at PHL #1 (kJ)	192.4	231.0	16.7%
Plastic Energy at PHL #2 (kJ)	70.4	–	–
Plastic Energy at PHL #3 (kJ)	–	157.2	–

Comparison of results in Figures 4.9 to 4.11 and in Table 4.2 between NEO and REO shows that the responses between the two cases are quite different. In general, responses such as velocity, acceleration, and energy dissipation with rigid end offsets are consistently larger than those with no end offsets. The percent of increase when rigid end offsets are considered ranges from 13.5% to 35.7%. In addition, the local response, such as the plastic rotation and plastic energy dissipation at PHL #1, are quite different. This is particularly true for the maximum responses in Table 2 for PHLs #2 and #3. For NEO, PHL #2 has yielded while PHL #3 remains elastic, but PHL #3 has yielded for REO while PHL #2 remains elastic. This phenomenon occurs because the moment in column reaches its moment capacity before that in the beam when no end offset is considered, but the moment in beam reaches its moment capacity before that in the column when rigid end

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

offset is considered. This shows that rigid end offsets not only affect the stiffness of the structure, but also the inelastic response of the structure. Therefore, modeling rigid end offsets is very important in capturing the true responses of the structure.

Figure 4.9 Structural Global Responses of One-Story One-Bay Frame for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.10 Some Local Responses of One-Story One-Bay Frame for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.11 Structural Energy Dissipation of One-Story One-Bay Frame for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

4.4.2 Example 2: Six-Story Moment Resisting Frame

A six-story moment resisting frame shown in Figure 4.12 is selected to study the differences in response between models with no end offset and rigid end offsets in a multi-degree of freedom (MDOF) system. The floor masses of 630,400 kg per floor are selected to give a long first period of vibration of 1.94 seconds. In this study, the structural damping is assumed to be non-proportional, and remain unchanged in the whole process of analysis. The damping matrix is a uniform diagonal matrix, i.e., $\mathbf{C} = c\mathbf{I}$ where $c = 210.1$ kN-s/m. The structural model is shown in Figure 4.13, where there are 40 plastic hinge locations, with 6 PHLs per floor and 4 PHLs at the base on the columns. These plastic hinges are assumed to deform in an elastic-plastic behavior with yield moments computed based on the plastic modulus of the members and a yield stress of steel of 248.2 MPa. Assume that the beams are subjected to a uniform gravity load of 21.89 kN/m on every floor, and the interaction between axial force and moment in the columns is not considered.

Figure 4.14 and 4.15 shows the global response time histories of the structure when subjected to the El-Centro earthquake excitation with a scale factor of 1.8. Comparison of results between structures with NEO and REO shows that the responses are quite different due to the increase in stiffness for the structure with REO, particularly for the energy responses. In general, energy dissipations are larger for structure with REO than that with NEO, except the maximum kinetic energy. Table 4.3 compares the maximum values of the global responses due to the magnified El-Centro earthquake time history. Results show that percentage difference ranges from 0.4% to 50.0%, with the largest difference in maximum drift ratio between the 3rd and 4th stories. In addition, due to an increase in stiffness for structure with REO, the maximum displacement response is consistently smaller for structure with REO than that with NEO.

Figure 4.16 and 4.17 shows the local response time histories at some plastic hinges of the structure subjected to the magnified El-Centro earthquake ground motion with a scale factor of 1.8. At the same time, Table 4.4 summarized the maximum values of the local responses for the frames with NEO and REO, respectively. The results demonstrate that for plastic hinges at the bases of the

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

columns on the first floor, plastic rotations and plastic energy dissipation in structure with NEO are larger than those in structure with REO; for plastic hinges at the beams on all the floors, the situation is reversed, with an exception for all PHLs on beams at the roof level and plastic energy dissipation for PHL #11, #13, #15 on beams at the 3rd floor. It can be implied that for relative flexible structures more force will be allocated on the roots, while for relative rigid ones more force will be allocated on the up-floors. The maximum percentage difference in plastic rotation is

Figure 4.12 Six-Story Moment Resisting Frame

55.6% at PHL #23, while the maximum percentage difference in plastic energy dissipation is 139.3% at PHL #1. This demonstrates that although the difference of

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

the global plastic energy dissipation between the structures with NEO and REO is not very apparent to be noticed, the local plastic energy dissipation and damage can be different by over 100%. This means that considering only the global response without carefully studying the local response in assessing structural damage is absolutely insufficient and may lead to wrong conclusions.

Figure 4.13 Model of Six-Story Moment Resisting Frame

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Table 4.3 Maximum Global Responses and Energy Dissipation of MDOF Structure for Cases NEO and REO

	NEO	REO	Percent Difference
Displacement (cm)			
2nd	8.40	7.28	-15.4%
3rd	15.45	12.89	-19.9%
4th	20.56	16.31	-26.1%
5th	24.89	22.91	-8.6%
6th	29.43	29.15	-1.0%
Roof	33.94	33.09	-2.6%
Velocity (cm/s)			
2nd	38.3	31.0	-23.5%
3rd	59.8	56.0	-6.8%
4th	79.3	67.3	-17.8%
5th	88.8	75.8	-17.2%
6th	93.0	103.6	10.2%
Roof	101.9	130.4	21.9%
Drift Ratio (cm/cm)			
Gr - 2nd	0.0197	0.0170	-15.9%
2 - 3rd	0.0165	0.0131	-26.0%
3 - 4th	0.0120	0.0080	-50.0%
4 - 5th	0.0101	0.0155	34.8%
5 - 6th	0.0106	0.0146	27.4%
6 - Roof	0.0106	0.0092	-15.2%
Acceleration (g)			
2nd	0.688	0.685	-0.4%
3rd	0.749	0.650	-15.2%
4th	0.801	0.639	-25.4%
5th	0.733	0.762	3.8%
6th	0.712	0.736	3.3%
Roof	0.694	0.671	-3.4%
Energy (kJ)			
<i>KE</i>	801	767	-4.4%
<i>SE</i>	332	361	8.0%
<i>DE</i>	1633	1763	7.4%
<i>PE</i>	2568	2869	10.5%
<i>IE</i>	4519	4872	7.2%

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.14 Structural Global Responses of MDOF Structure for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.15 Energy Dissipation of MDOF Structure for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

**Table 4.4 Maximum Local Responses and Energy Dissipation of MDOF
Structure for Cases NEO and REO**

PHL #	Plastic Rotation (rad)			Plastic Energy (kJ)		
	NEO	REO	Percent Difference	NEO	REO	Percent Difference
1	0.0156	0.0128	-21.9%	115.1	48.1	-139.3%
2	0.0162	0.0135	-20.0%	291.3	130.5	-123.2%
3	0.0162	0.0135	-20.0%	291.3	130.5	-123.2%
4	0.0155	0.0127	-22.0%	115.5	48.5	-138.1%
5	0.0111	0.0126	11.9%	78.4	84.9	7.7%
6	0.0121	0.0135	10.4%	81.4	86.5	5.9%
7	0.0114	0.0129	11.6%	85.2	93.5	8.9%
8	0.0121	0.0136	11.0%	82.7	91.2	9.3%
9	0.0114	0.0128	10.9%	83.8	88.7	5.5%
10	0.0122	0.0135	9.6%	74.9	81.5	8.1%
11	0.0061	0.0067	9.0%	25.0	23.6	-5.9%
12	0.0073	0.0076	3.9%	25.6	27.9	8.2%
13	0.0066	0.0071	7.0%	28.4	27.9	-1.8%
14	0.0073	0.0077	5.2%	26.0	30.1	13.6%
15	0.0066	0.0069	4.3%	28.0	25.7	-8.9%
16	0.0070	0.0075	6.7%	21.5	26.1	17.6%
17	0.0053	0.0095	44.2%	16.9	38.5	56.1%
18	0.0064	0.0104	38.5%	22.6	42.7	47.1%
19	0.0057	0.0098	41.8%	20.6	41.9	50.8%
20	0.0064	0.0105	39.0%	22.8	44.2	48.4%
21	0.0057	0.0097	41.2%	20.3	40.5	49.9%
22	0.0063	0.0103	38.8%	19.9	41.2	51.7%
23	0.0064	0.0144	55.6%	53.3	149.0	64.2%
24	0.0079	0.0155	49.0%	63.4	155.7	59.3%
25	0.0069	0.0147	53.1%	61.9	158.3	60.9%
26	0.0078	0.0157	50.3%	64.3	160.6	60.0%
27	0.0068	0.0145	53.1%	61.0	153.5	60.3%
28	0.0079	0.0156	49.4%	56.7	151.8	62.6%
29	0.0111	0.0146	24.0%	64.9	88.3	26.5%
30	0.0126	0.0159	20.8%	70.8	91.5	22.6%
31	0.0113	0.0148	23.6%	68.8	91.6	24.9%
32	0.0126	0.0160	21.3%	71.2	93.2	23.6%
33	0.0113	0.0147	23.1%	68.4	89.2	23.3%
34	0.0126	0.0158	20.3%	67.7	90.8	25.4%
35	0.0077	0.0077	0.0%	33.7	15.0	-124.7%
36	0.0113	0.0109	-3.7%	37.9	18.2	-108.2%
37	0.0083	0.0082	-1.2%	35.2	16.0	-120.0%
38	0.0113	0.0110	-2.7%	38.1	18.6	-104.8%
39	0.0083	0.0081	-2.5%	35.1	15.6	-125.0%
40	0.0117	0.0112	-4.5%	37.5	18.3	-104.9%

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.16 Plastic Rotation of MDOF Structure at Selected PHLs for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

Figure 4.17 Plastic Energy Dissipation of MDOF Structure at Selected PHLs for Cases NEO and REO

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

4.5 Summary

The deviation of force analogy method has been extended to analyze real moment resisting frames with rigid end offsets. Instead of using the method of slaving the movements of panel zone to the master joint, a new member stiffness matrix is derived to replace the mastering-slaving procedure. Static condensation is performed to calculate the modified stiffness matrices used in the force analogy method. It is found that the new member stiffness matrix can be obtained by modifying the original member stiffness matrix with a scale factor in each term, which can be seen in Eqs. (4.4) to (4.6). This is an improvement to the original force analogy method or other conventional methods of inelastic structural dynamic analysis with rigid end offsets, because (1) the new stiffness matrix can be readily used in structural analysis and dynamic computations, (2) the condensed forms reduce the size of the structural problem by eliminating the unnecessary additional degrees of freedom that must be placed at the plastic hinges, and (3) the memory required for storing the stiffness matrices can be significantly reduced, and hence the computational speed can be increased. But more importantly, the stiffness matrices used in the inelastic analysis are theoretically derived.

The force analogy method presented in Chapter 3 combined with the procedure presented in this Chapter is then used to investigate the inelastic structural response differences between modeling structures with no end offset (NEO) and with rigid end offsets (REO). Comparison of results shows that differences in global responses can go up to as much as 38% for a one-story one-bay moment resisting frame. The modeling of structures with rigid end offsets consistently gives larger responses than that with no end offset.

Finally, a six-story moment resisting frame is considered as a multi-degree of freedom structure to study the differences in response between structures with NEO and REO. Subjected to the magnified El-Centro earthquake time history, results show that the percentage differences in global responses can go up to as much as 50%. In addition, it is implied that for relative flexible structures more force will be allocated on the roots, while for relative rigid ones more force will be allocated on the up-floors. The maximum percentage difference in local responses between structures with no end offset and rigid end offsets can be over 100%. This

Chapter 4 Dynamic Analysis of Inelastic Structures with Rigid End Offsets

observation shows that considering only the global responses such as maximum displacement and drift in assessing structural damage is insufficient. Damage occurs locally in the structure, thus local responses such as plastic rotation and plastic energy dissipation should also be used to measure structural damage.

The objective of this chapter is to show how to include plastic hinges end offsets and rigidity of panel zones in the structural models with the force analogy method. The study of this chapter is not the ultimate goal of this research, and it is only the earlier stage work for next chapter, in which the influence of panel zone deformation on dynamic behaviors of inelastic structures is investigated.

CHAPTER 5 DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH PANEL ZONE DEFORMATION

5.1 General Statement

Modern structural seismic design of building structures emphasizes on one principle, which is weak beam, strong column, and strong joint. Structural damage is expected to begin at the beams, and at the same time it is hoped that there is no deformation or damage in the joints. For this reason, beam-column joints are frequently assumed to be completely rigid in structural analysis. However, there exists a potential possibility that structural damage due to severe earthquake excitation may be shifted to the joints because of various factors such as building problems arising from the construction process. To date, although some researches (Popov et al., 1989; Khudada and Geschwindner, 1997; Schneider and Amidi, 1998; El-Tawil et al., 1999; Gupte and Krawinkler, 2000; Foutch and Yun, 2002) have been done in addressing problems of rigidity and inelastic deformation of the panel zones in moment resisting steel frame structures simultaneously, there still exist many problems, such as the effect of panel zone deformation to the structural response, how to balance the strength of panel zones and other members, and so on, that need to be investigated more comprehensively.

In order to study the inelastic behavior of panel zones, these panel zones should be modeled as deformable. In this Chapter, a simple analytical method of modeling the panel zone deformation is proposed to study the influence of elastic and inelastic deformation of panel zones on dynamic behavior of inelastic structures. This proposed method is based on the force analogy method discussed in Chapter 3, where the stiffness contribution of the panel zones is included in the original stiffness matrix before static condensation is performed. The effect of rigid end offsets discussed in Chapter 4 is also included to accurately model the shear deformation in the panel zones.

Numerical simulations are performed on two types of structure models, i.e., a one-story one-bay frame and a six-story frame, to demonstrate the application of the proposed approach. Different rigidity and yield moment in panel zones are

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

considered. It is found that rigidity and inelastic deformation of panel zones have significant influence on dynamic responses of inelastic structures.

5.2 Panel Zone Deformation Model

In order to study damage in panel zones, panel zone deformation must be modeled accurately. Due to deformation in panel zones, the rotations between the member ends connecting to a beam-column joint are usually not compatible, while the displacement of the member ends connecting to the joint is compatible. Therefore, to model the panel zone deformation accurately, modification to the global stiffness matrix is needed in order to capture the differences in rotations between member ends connecting to the beam-column joint.

Figure 5.1 shows the structural model of a one-story one-bay moment resisting frame used in this chapter. Assuming that all the members are rigid in the axial direction, there is only one translational degree of freedom, labeled x_1 , which means that both beam-column joints (i.e., Joints #3 and #4) will move horizontally by the same amount. However, because the columns rotate differently from the beam at these two joints when panel zone deformation is considered, two rotational degrees of freedom are used to represent the movements of left joint (labeled x_2 and x_3) and two more rotational degrees of freedom are used to represent the movements of the right joint (labeled x_4 and x_5).

Figure 5.1 Structural Model with Rotational Springs

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

In order to induce a relative rotation between the beam end and the column end while satisfying equilibrium at the joint, the connection between the beam and the column at the beam-column joint is modeled using a rotational spring with stiffness k_s . Using this rotational spring, the equilibrium condition will be satisfied because any moment occurring in the beam will be transferred to the spring and then transferred to the column. Hence the beam, column, and the spring at the beam-column joint will experience the same magnitude of moment. Once the rotational spring is subjected to a moment, relative rotation will be induced between the beam and the column.

The advantage of using a rotational spring to model panel zone deformation is that shear deformation can be captured. Consider the structural displacement shown in Figure 5.2, any horizontal movement of the structure will cause the beam-column joint to deform. When the beam and the column are modeled using member with rigid ends as discussed in Chapter 4, the relative rotation between the beam and the column will cause the panel zone to deform in a parallelogram pattern as shown in the joint displacement profile. For any rotation where $x_4 \neq x_5$, the panel zone becomes a parallelogram, which can be used to simulate shear deformation in the panel zone.

Figure 5.2 Effect of Rotational Spring on Joint Deformation

Once shear deformation in the panel zone is captured, damage in the joint can then be modeled. By setting the rotational spring to behave inelastically with certain moment capacity M_s , the effect of structural damage in the panel zone can

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

therefore be estimated. For example, a plastic rotation θ_s'' in the rotational spring causes permanent deformation between the beam and the column. Based on this θ_s'' , the inelastic shear strain, ε_{12}'' , can be computed by the formula $\varepsilon_{12}'' = \theta_s''/2$.

5.3 Numerical Examples

To demonstrate the analysis procedure in studying the dynamic behavior of inelastic structures with panel zone deformation, two structural models with rigid end offsets are used.

5.3.1 Example 1: One-Story One-Bay Moment Resisting Frame

The structural model is shown in Figure 5.3. A total of 5 degrees of freedom (see Figure 5.1) and 8 plastic hinges (which include 2 plastic hinges for each of the 3 member ends and 2 beam-column joints) is used. Hence, the sizes of stiffness matrices based on the force analogy method are 5×5 for the original $\mathbf{K}$ matrix, 5×8 for $\mathbf{K}_p$ matrix, and 8×8 for $\mathbf{K}_R$ matrix. Assume that the mass of the structure is $m = 7.0 \times 10^6$ kg, damping is $c = 2,100$ kN-s/m, and time step is $\Delta t = 0.02$ s. In addition, assume that the inelastic behavior of columns, beam, and joints are elastic-plastic, and the interaction between axial force and moment in the columns is ignored.

First consider the case where the rotational spring stiffness of the structure is $k_s = 4.80 \times 10^5$ kN-m/rad. A horizontal displacement imposed on the structure gives a ratio of relative rotations between the beam and column connected to the beam-column joint of 0.3433 (i.e., $x_3/x_2 = x_4/x_5 = 0.3433$, or 34.33% rigidity). Based on this rotational spring stiffness, the period of the structure is calculated to be $T = 1.164$ s and a damping ratio of $\zeta = 2.78\%$. The yield moment of rotational springs is selected to be $M_s = 6,780$ kN-m, and therefore plastic hinges are formed in the beam with yield moment $M_b = 2,730$ kN-m instead of forming at the joint or at the column. Subjected to the El-Centro earthquake time history (see Figure 4.6) with a scale factor of 0.5, the responses are shown in Figures 5.4 to 5.6.

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Now consider the case where the rotational spring stiffness is $k_s = 1.55 \times 10^6$ kN-m/rad. A horizontal displacement imposed on the structure gives a ratio of relative rotations between the beam and column connected to the beam-column joint of 0.8322 (i.e., $x_3/x_2 = x_4/x_5 = 0.8322$, or 83.22% rigidity). Based on this rotational stiffness, the period of the structure is calculated to be $T = 0.999$ s and a damping ratio of $\zeta = 2.39\%$. The yield moment of rotational springs is again selected to be $M_s = 6,780$ kN-m. Subjected to the El-Centro earthquake time history (see Figure 4.6) with a scale factor of 0.5, the responses are also shown in Figures 5.4 to 5.6.

Figure 5.3 One-Story One-Bay Moment Resisting Frame

Comparison of results shown in Figure 5.4 to Figure 5.6 shows that structure with stiffer rotational springs (i.e., 83.22% rigidity) gives slightly larger acceleration response than that with softer rotational springs (i.e., 34.33% rigidity), while the displacement and the velocity response remains about the same for both types of spring. In terms of energy, more input energy enters the structure with stiffer rotational spring and therefore more plastic energy dissipation is needed. In addition, more kinetic energy is stored in the structure with stiffer rotational springs, while damping energy remains about the same as compared to the structure with softer rotational springs.

Table 5.1 summarizes the maximum response of different rotational spring stiffness shown in Figure 5.4 to Figure 5.6. The maximum responses for completely

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

rigid joint (i.e., 100% rigidity) and completely flexible joint (i.e., 0% rigidity) are also presented in Table 5.1. Comparison of maximum global responses shows that maximum acceleration increases as the rigidity in panel zones increases, and more input energy enters a structure with stiffer rotational springs than that with softer rotational springs, and therefore more plastic energy must be dissipated in the stiffer structure. Similarly, more kinetic energy is stored in a structure with stiffer rotational springs than that with softer rotational springs. For strain energy, the maximum case take place for structure with 34.33% rigidity in panel zones, while the damping energy dissipation is approximately the same for all cases. Due to the flexibility of the structure with 0% rigidity, the maximum displacement response is significantly larger when compared with the other three cases of rigidities. Comparison of local responses shows that more plastic energy is dissipated at PHL #1 for structure with 83.22% rigidity in panel zones, while maximum plastic rotation at PHL #1 remains about the same for all cases.

Table 5.1 Maximum Structural Global and Local Responses of One-Story One-Bay Frame with Different Rotational Spring Stiffness

Rotational Spring Stiffness k_s (kN-m/rad)	0	4.80×10^5	1.55×10^6	∞
Structural Rigidity	0%	34.33%	83.22%	100%
Structural Parameters:				
Period (s)	1.855	1.164	0.999	0.892
Damping Ratio	4.43%	2.78%	2.39%	2.13%
Global Maximum Response:				
Displacement (cm)	6.35	4.07	4.77	4.47
Velocity (cm/s)	18.72	22.67	24.43	23.88
Acceleration (g)	0.0313	0.0670	0.0690	0.0696
Kinetic Energy (kJ)	84	109	121	130
Strain Energy (kJ)	19	49	36	29
Damping Energy (kJ)	154	181	183	175
Plastic Energy (kJ)	315	467	629	776
Input Energy (kJ)	502	696	853	986
Local Maximum Response:				
Plastic Rotation at PHL #1 (rad)	0.0149	0.0111	0.0136	0.0126
Plastic Rotation at PHL #3 (rad)	0	0	0	0
Plastic Rotation at PHL #4 (rad)	0	0.0019	0.0127	0.0145
Plastic Energy at PHL #1 (rad)	158	230	238	231
Plastic Energy at PHL #3 (rad)	0	0	0	0
Plastic Energy at PHL #4 (rad)	0	4	76	157

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.4 Structural Global Responses of One-Story One-Bay Frame for Different Rigidities in Panel Zones

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.5 Some Local Responses of One-Story One-Bay Frame for Different Rigidities in Panel Zones

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.6 Structural Energy Dissipation of One-Story One-Bay Frame for Different Rigidities in Panel Zones

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

5.3.2 Example 2: Six-Story Moment Resisting Frame

The six-story moment resisting steel frame used in Chapter 4 is again selected to study the differences in response between various amounts of rotational spring stiffness and yield moments in a multi-degree of freedom (MDOF) system. Figure 5.7 shows the structural model with a total of 64 potential plastic hinges, which include 6 PHLs in the beams per floor, 4 PHLs at the base on the columns, and 24 PHLs at each beam-column joint.

Rigid end offsets are included for all the members, and each offset length is equal to half the column depth plus half the beam depth. Rotational springs are used to model the panel zone deformation. At each beam-column joint, columns are modeled as continuous where moments are allowed to transfer between the top and bottom columns. In addition, beams are modeled as continuous where moments are allowed to transfer between the left and right beams. Rotational springs are used to connect the two beams with the two columns.

Two cases are considered to investigate the influence of elastic deformation and inelastic deformation of rotational springs (model of panel zones) on the dynamic behaviors of inelastic structures. One is corresponding to the case of elastic rotational springs, which is reflected by choosing a large yield moment in rotational springs and consequently plastic hinges are forced to form in the beams while the beam-column joints remain elastic. The other corresponds to the case of inelastic rotational springs, which is reflected by using smaller yield moments in rotational springs, and as a result it is possible for plastic hinges to form in either beams or beam-column joints.

Case 1:

For this case, yield moment of rotational springs in all the beam-column joints is selected to be $M_s = 11,000 \text{ kN-m}$. Because of using this large yield moment in the rotational springs, plastic hinges are formed in the beams while the beam-column joints remain elastic. Two values of rotational spring stiffness are used in this study, and they are: (1) a larger stiffness of $k_s = 4.80 \times 10^5 \text{ kN-m/rad}$ (stiffer rotational spring) and (2) a smaller stiffness of $k_s = 1.55 \times 10^6 \text{ kN-m/rad}$ (softer rotational spring).

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.7 Model of Six-Story Moment Resisting Frame

Figures 5.8 and 5.9 show the global response time histories of the two cases of using different rotational spring stiffness when subjected to the El-Centro earthquake ground motion with a scale factor of 1.8. Comparison of results shows that the responses are quite different due to the difference in rotational spring stiffness. Larger roof displacement, roof velocity, strain energy, kinetic energy, and damping energy take place for structure with softer rotational springs, while larger input energy and plastic energy dissipation take place for structure with stiffer rotational springs. The roof acceleration remains about the same for both structures.

Table 5.2 compares the maximum values of the global responses due to the magnified El-Centro earthquake time history for structures with stiffer and softer rotational springs. In addition, maximum responses for structure with completely

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

rigid rotational spring stiffness (i.e., $k_s \rightarrow \infty$) are also shown in the table for comparisons.

Table 5.2 Maximum Global Responses and Energy Dissipation of MDOF Structure with Different Rotational Spring Stiffness

	Softer Springs $k_s = 4.80 \times 10^5$ kN-m/rad	Stiffer Springs $k_s = 1.55 \times 10^6$ kN-m/rad	Rigid Springs $k_s \rightarrow \infty$
Displacement (cm)			
2nd	8.60	7.13	7.28
3rd	16.83	12.62	12.89
4th	27.05	17.68	16.31
5th	37.48	23.36	22.91
6th	46.68	29.33	29.15
Roof	53.36	34.48	33.09
Velocity (cm/s)			
2nd	44.7	39.8	31.0
3rd	60.5	56.3	56.0
4th	96.3	79.6	67.3
5th	116.1	86.0	75.8
6th	124.5	98.4	103.6
Roof	149.2	119.2	130.4
Drift Ratio (cm/cm)			
Gr - 2nd	0.0201	0.0167	0.0170
2 - 3rd	0.0193	0.0129	0.0131
3 - 4th	0.0239	0.0119	0.0080
4 - 5th	0.0244	0.0133	0.0155
5 - 6th	0.0215	0.0140	0.0146
6 - Roof	0.0156	0.0121	0.0092
Acceleration (g)			
2nd	0.745	0.791	0.685
3rd	0.720	0.780	0.650
4th	0.858	0.889	0.639
5th	0.672	0.615	0.762
6th	0.698	0.839	0.736
Roof	0.642	0.735	0.671
Energy (kJ)			
<i>KE</i>	1337	841	767
<i>SE</i>	908	501	361
<i>DE</i>	2154	2083	1763
<i>PE</i>	1807	2222	2869
<i>IE</i>	4067	4560	4872

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.8 Structural Global Responses of MDOF Structure with Different Rotational Spring Stiffness

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.9 Energy Dissipation of MDOF Structure with Different Rotational Spring Stiffness

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

**Table 5.3 Maximum Local Responses and Energy Dissipation of MDOF
Structure with Different Rotational Spring Stiffness**

PHL #	Plastic Rotation (rad)			Plastic Energy (kJ)		
	Softer Springs $k_s = 4.80 \times 10^5$ kN-m/rad	Stiffer Springs $k_s = 1.55 \times 10^6$ kN-m/rad	Rigid Springs $k_s \rightarrow \infty$	Softer Springs $k_s = 4.80 \times 10^5$ kN-m/rad	Stiffer Springs $k_s = 1.55 \times 10^6$ kN-m/rad	Rigid Springs $k_s \rightarrow \infty$
1	0.0153	0.0126	0.0128	149.28	124.88	48.09
2	0.0159	0.0133	0.0135	297.50	279.81	130.48
3	0.0159	0.0133	0.0135	297.61	280.00	130.45
4	0.0152	0.0125	0.0127	149.68	125.57	48.55
5	0	0	0	0	0	0
6	0.0060	0.0077	0.0126	29.77	93.33	84.92
7	0.0002	0.0059	0.0135	0.53	31.05	86.54
8	0	0	0	0	0	0
9	0	0.0051	0.0129	0	22.75	93.48
10	0	0.0058	0.0136	0	24.65	91.20
11	0	0	0	0	0	0
12	0	0.0053	0.0128	0	29.11	88.74
13	0.0073	0.0087	0.0135	25.88	90.63	81.49
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0.0104	0.0057	0.0067	94.09	53.05	23.58
17	0.0027	0.0040	0.0076	8.40	12.48	27.93
18	0	0	0	0	0	0
19	0.0011	0.0031	0.0071	2.68	8.68	27.86
20	0.0018	0.0038	0.0077	4.33	10.52	30.13
21	0	0	0	0	0	0
22	0.0020	0.0033	0.0069	6.57	10.51	25.71
23	0.0117	0.0067	0.0075	90.84	50.84	26.14
24	0	0	0	0	0	0
25	0	0	0	0	0	0
26	0.0120	0.0049	0.0095	57.47	21.43	38.52
27	0.0045	0.0032	0.0104	14.34	9.86	42.68
28	0	0	0	0	0	0
29	0.0030	0.0023	0.0098	7.21	6.43	41.89
30	0.0037	0.0030	0.0105	8.84	8.26	44.15
31	0	0	0	0	0	0
32	0.0039	0.0025	0.0097	12.46	7.90	40.49
33	0.0133	0.0058	0.0103	54.01	24.43	41.16
34	0	0	0	0	0	0
35	0	0	0	0	0	0
36	0.0165	0.0093	0.0144	104.89	131.46	149.03
37	0.0117	0.0082	0.0155	29.35	45.10	155.69
38	0	0	0	0	0	0
39	0.0102	0.0071	0.0147	20.20	35.17	158.33
40	0.0112	0.0081	0.0157	22.14	37.34	160.58

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Continue Table 5.3						
41	0	0	0	0	0	0
42	0.0108	0.0073	0.0145	27.20	42.84	153.46
43	0.0182	0.0108	0.0156	109.32	134.71	151.78
44	0	0	0	0	0	0
45	0	0	0	0	0	0
46	0.0086	0.0119	0.0146	26.82	78.61	88.33
47	0.0050	0.0116	0.0159	10.29	42.81	91.54
48	0	0	0	0	0	0
49	0.0034	0.0104	0.0148	5.76	35.84	91.25
50	0.0045	0.0115	0.0160	7.69	37.99	93.24
51	0	0	0	0	0	0
52	0.0038	0.0104	0.0147	8.09	40.56	89.24
53	0.0102	0.0132	0.0158	29.35	79.25	90.75
54	0	0	0	0	0	0
55	0	0	0	0	0	0
56	0.0098	0.0087	0.0077	28.54	28.59	14.99
57	0.0103	0.0112	0.0109	11.55	25.20	18.17
58	0	0	0	0	0	0
59	0.0074	0.0084	0.0082	7.53	21.69	15.95
60	0.0102	0.0112	0.0110	10.17	24.24	18.57
61	0	0	0	0	0	0
62	0.0076	0.0084	0.0081	10.21	22.59	15.62
63	0.0139	0.0124	0.0112	26.30	32.25	18.27
64	0	0	0	0	0	0

Comparison of results in Table 5.2 shows that the maximum responses of the structure using stiffer springs (i.e., $k_s = 1.55 \times 10^6$ kN-m/rad) are very similar to those using rigid springs (i.e., $k_s \rightarrow \infty$). In addition, the results in Table 5.2 also show that using softer springs generally gives larger maximum displacement, velocity, drift responses, but for acceleration the maximum values for most of the floors take place for the case with $k_s = 1.55 \times 10^6$. In terms of energy, due to larger maximum displacement and velocity responses for structure with softer springs, the maximum kinetic energy, strain energy, and damping energy are also larger for structure with softer springs. However, for plastic energy and input energy, stiffer structure tends to dissipate more than softer structure.

Figures 5.10 and 5.11 show the local response (include plastic rotation and plastic energy) time histories at some selected PHLs of the two cases of using different rotational spring stiffness when subjected to the El-Centro earthquake

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

ground motion with a scale factor of 1.8. Table 5.3 compares the maximum values of the local responses (include plastic rotation and plastic energy) for each PHL for the above three cases of rotational spring stiffness, i.e., softer spring, stiffer spring and fully rigid spring.

Comparison of local response results in Figure 5.10, 5.11 and Table 5.3 demonstrates that more plastic rotation and more plastic energy take place in the columns for structure with softer springs, while for beams the situation is not consistent. In general, most of the local responses (include plastic rotation and plastic energy) are observed to be larger for beams in structure with stiffer springs than those for beams in structure with softer springs. For example, for plastic rotation and plastic energy in all beam PHLs on the second, sixth and roof floors, and in the interior beam PHLs on the third, and moreover for plastic energy in all beam PHLs on the fifth floor. However, there exists a few exceptions, such as plastic rotation and plastic energy in all beam PHLs on the forth floor, and in the exterior beam PHLs on the third floor, plastic rotation in all beam PHLs on the fifth floor. Consequently, it is implied that as the rigidity of structures increases, damage will shift from columns to beams. In addition, it is noticed that although for global responses the discrepancy between the two cases of $k_s = 1.55 \times 10^6$ and $k_s \rightarrow \infty$ is small, the situation is not applicable for local responses, where the difference between the two cases of $k_s = 1.55 \times 10^6$ and $k_s \rightarrow \infty$ can not be ignored. It also can be noted that for some PHLs the change tendency of plastic rotation and plastic energy is not consistent. The explanation for this phenomenon can be derived from Eq. (3.62), in which the plastic energy depends on both elastic moment and plastic rotation. Therefore, the increase in elastic moment in the rotational spring can offset the decrease in plastic rotation to give larger plastic energy dissipation.

In general, it is demonstrated that carefully investigating the local response owns extremely important status in the whole analysis process. Otherwise the conclusion will become misleading.

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.10 Plastic Rotation of MDOF Structure at Selected PHLs with Different Rotational Spring Stiffness

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.11 Plastic Energy Dissipation of MDOF Structure at Selected PHLs with Different Rotational Spring Stiffness

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Case 2:

For this case, panel zones are allowed to deform inelastically. Five values of rotational spring stiffness are selected to compare the different effects of spring rigidity on the structural dynamic responses, and they are: (1) $k_{s1} = 4.80 \times 10^5 \text{ kN-m/rad}$ (softer panel zones); (2) $k_{s2} = 5.55 \times 10^5 \text{ kN-m/rad}$ (soft panel zones); (3) $k_{s3} = 1.55 \times 10^6 \text{ kN-m/rad}$ (stiff panel zones); (4) $k_{s4} = 3.40 \times 10^6 \text{ kN-m/rad}$ (stiffer panel zones); (5) $k_{s5} = \infty \text{ kN-m/rad}$ (fully rigid panel zones). Meanwhile for each case of rotational spring stiffness, four values of yield moments in the rotational springs are considered: (1) $M_s = 1,695 \text{ kN-m}$; (2) $M_s = 3,050 \text{ kN-m}$; (3) $M_s = 5,875 \text{ kN-m}$; and (4) $M_s = 11,000 \text{ kN-m}$, and consequently plastic hinges can take place either in beams or in beam-column joints.

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

$$(1) \quad k_{s1} = 4.80 \times 10^5 \text{ kN-m/rad}$$

The maximum global responses are summarized in Table 5.4, while the maximum local responses are summarized in Figure 5.12.

Table 5.4 Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 4.80×10^5 kN-m/rad

	Case 1	Case 2	Case 3	Case 4
M_s (kN-m)	1695	3050	5875	11000
Displacement (cm)				
2nd	4.16	5.03	7.89	8.60
3rd	10.25	11.47	15.48	16.83
4th	16.31	19.30	25.63	27.05
5th	22.06	26.45	36.19	37.48
6th	24.26	31.39	44.56	46.68
Roof	25.14	35.30	50.99	53.36
Velocity (cm/s)				
2nd	29.9	38.3	44.7	44.7
3rd	51.6	55.3	60.5	60.5
4th	86.6	83.8	97.8	96.3
5th	78.1	99.0	115.5	116.1
6th	89.5	104.9	122.9	124.5
Roof	104.5	131.3	148.8	149.2
Drift Ratio (cm/cm)				
Gr - 2nd	0.0097	0.0118	0.0185	0.0201
2 - 3rd	0.0143	0.0151	0.0178	0.0193
3 - 4th	0.0142	0.0183	0.0238	0.0240
4 - 5th	0.0135	0.0168	0.0247	0.0244
5 - 6th	0.0051	0.0116	0.0196	0.0215
6 - Roof	0.0021	0.0092	0.0151	0.0156
Acceleration (g)				
2nd	0.775	0.817	0.790	0.745
3rd	0.717	0.683	0.720	0.720
4th	0.672	0.799	0.858	0.858
5th	0.694	0.723	0.707	0.672
6th	0.709	0.693	0.698	0.698
Roof	0.529	0.633	0.642	0.642
Energy (kJ)				
KE	456	736	1281	1337
SE	180	358	725	908
DE	1137	1597	2073	2154
PE	2548	2300	1888	1807
IE	3788	4144	4172	4067

Figure 5.12 Maximum Plastic Rotation and Plastic Energy at Each PHL with Softer Rotational Spring $k_{s1} = 4.80 \times 10^5 \text{ kN-m/rad}$

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

$$(2) \quad k_{s2} = 5.55 \times 10^5 \text{ kN-m/rad}$$

The maximum global responses are summarized in Table 5.5, while the maximum local responses are summarized in Figure 5.13.

Table 5.5 Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 5.55×10^5 kN-m/rad

	Case 1	Case 2	Case 3	Case 4
M_s (kN-m)	1695	3050	5875	11000
Displacement (cm)				
2nd	4.03	5.17	8.16	9.28
3rd	10.03	12.64	15.73	17.69
4th	16.09	20.23	24.95	26.98
5th	21.99	26.27	33.89	35.92
6th	24.07	31.36	40.84	43.57
Roof	24.43	34.79	46.33	48.91
Velocity (cm/s)				
2nd	28.3	34.8	43.0	43.0
3rd	52.5	57.9	66.8	66.1
4th	86.1	83.3	90.5	83.8
5th	75.6	97.0	105.7	108.4
6th	87.0	103.6	104.0	105.5
Roof	101.6	127.3	125.9	125.3
Drift Ratio (cm/cm)				
Gr - 2nd	0.0094	0.0121	0.0191	0.0217
2 - 3rd	0.0141	0.0175	0.0177	0.0197
3 - 4th	0.0142	0.0178	0.0216	0.0217
4 - 5th	0.0138	0.0141	0.0209	0.0209
5 - 6th	0.0049	0.0119	0.0163	0.0179
6 - Roof	0.0009	0.0080	0.0129	0.0125
Acceleration (g)				
2nd	0.763	0.798	0.729	0.742
3rd	0.674	0.665	0.708	0.708
4th	0.654	0.816	0.887	0.887
5th	0.668	0.697	0.646	0.648
6th	0.675	0.635	0.697	0.697
Roof	0.506	0.608	0.632	0.632
Energy (kJ)				
KE	455	752	1108	1182
SE	162	321	651	799
DE	1096	1497	1946	2009
PE	2605	2380	2080	2019
IE	3800	4113	4198	4115

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.13 Maximum Plastic Rotation and Plastic Energy at Each PHL with Soft Rotational Spring $k_{s2} = 5.55 \times 10^5 \text{ kN-m/rad}$

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

$$(3) \quad k_{s3} = 1.55 \times 10^6 \text{ kN-m/rad}$$

The maximum global responses are summarized in Table 5.6, while the maximum local responses are summarized in Figure 5.14.

Table 5.6 Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

	Case 1	Case 2	Case 3	Case 4
M_s (kN-m)	1695	3050	5875	11000
Displacement (cm)				
2nd	4.16	3.98	6.71	7.13
3rd	10.40	9.44	13.57	12.62
4th	16.63	15.34	20.23	17.68
5th	22.11	20.27	25.40	23.36
6th	24.70	24.71	30.16	29.33
Roof	26.12	27.02	34.08	34.48
Velocity (cm/s)				
2nd	20.0	31.3	36.4	39.8
3rd	56.1	55.8	53.6	56.3
4th	77.7	71.0	76.1	79.6
5th	73.0	74.6	81.4	86.0
6th	84.8	86.0	95.9	98.4
Roof	90.3	91.6	110.6	119.2
Drift Ratio (cm/cm)				
Gr - 2nd	0.0097	0.0093	0.0157	0.0167
2 - 3rd	0.0146	0.0128	0.0161	0.0128
3 - 4th	0.0146	0.0138	0.0156	0.0118
4 - 5th	0.0128	0.0115	0.0121	0.0133
5 - 6th	0.0061	0.0104	0.0112	0.0140
6 - Roof	0.0033	0.0054	0.0092	0.0121
Acceleration (g)				
2nd	0.628	0.679	0.799	0.791
3rd	0.575	0.546	0.760	0.780
4th	0.538	0.788	0.852	0.889
5th	0.540	0.493	0.625	0.615
6th	0.532	0.571	0.814	0.839
Roof	0.441	0.575	0.728	0.735
Energy (kJ)				
KE	415	609	846	841
SE	96	197	377	501
DE	893	1115	1897	2083
PE	2823	2817	2262	2222
IE	3841	4102	4516	4560

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.14 Maximum Plastic Rotation and Plastic Energy at Each PHL with Stiff Rotational Spring $k_{s3} = 1.55 \times 10^6 \text{ kN-m/rad}$

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

$$(4) \quad k_{s4} = 3.40 \times 10^6 \text{ kN-m/rad}$$

The maximum global responses are summarized in Table 5.7, while the maximum local responses are summarized in Figure 5.15.

Table 5.7 Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 3.40×10^6 kN-m/rad

	Case 1	Case 2	Case 3	Case 4
M_s (kN-m)	1695	3050	5875	11000
Displacement (cm)				
2nd	4.27	3.90	6.90	7.01
3rd	10.33	9.63	13.12	12.48
4th	16.77	15.44	17.53	15.88
5th	21.15	20.49	22.64	20.05
6th	23.60	25.50	26.86	25.93
Roof	25.35	26.94	30.08	31.19
Velocity (cm/s)				
2nd	23.2	23.7	36.4	36.4
3rd	54.5	51.2	61.0	63.0
4th	72.8	63.7	69.0	73.3
5th	76.6	71.1	77.8	82.4
6th	82.6	79.0	96.2	102.4
Roof	84.3	89.0	115.1	126.6
Drift Ratio (cm/cm)				
Gr - 2nd	0.0100	0.0091	0.0162	0.0164
2 - 3rd	0.0142	0.0134	0.0146	0.0128
3 - 4th	0.0151	0.0136	0.0103	0.0080
4 - 5th	0.0103	0.0118	0.0120	0.0098
5 - 6th	0.0057	0.0117	0.0099	0.0138
6 - Roof	0.0041	0.0034	0.0075	0.0123
Acceleration (g)				
2nd	0.636	0.609	0.786	0.724
3rd	0.529	0.634	0.771	0.685
4th	0.522	0.636	0.715	0.781
5th	0.492	0.485	0.675	0.655
6th	0.499	0.558	0.739	0.724
Roof	0.424	0.556	0.720	0.722
Energy (kJ)				
KE	403	525	634	801
SE	78	164	331	401
DE	814	963	1385	1561
PE	2804	2848	2486	2312
IE	3729	3902	4131	4029

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.15 Maximum Plastic Rotation and Plastic Energy at Each PHL with Stiffer Rotational Spring $k_{s4} = 3.40 \times 10^6$ kN-m/rad

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

(5) $k_{s5} = \infty$ kN-m/rad

Finally consider the case where the rotational spring is fully rigid. The maximum global responses are summarized in Table 5.8, while the maximum local responses are summarized in Figure 5.16.

Table 5.8 Maximum Responses of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness $= \infty$ kN-m/rad

	Case 1	Case 2	Case 3	Case 4
M_s (kN-m)	1695	3050	5875	11000
Displacement (cm)				
2nd	3.98	3.67	6.85	7.28
3rd	9.75	8.11	12.78	12.89
4th	15.71	12.72	19.04	16.31
5th	19.49	18.55	26.22	22.91
6th	21.38	23.60	32.24	29.15
Roof	22.95	25.01	35.01	33.09
Velocity (cm/s)				
2nd	26.8	25.4	33.0	31.0
3rd	53.2	57.4	60.9	56.0
4th	67.4	64.0	66.3	67.3
5th	78.0	66.4	72.5	75.8
6th	86.3	80.3	95.4	103.6
Roof	81.3	94.3	120.4	130.4
Drift Ratio (cm/cm)				
Gr - 2nd	0.0093	0.0086	0.0160	0.0170
2 - 3rd	0.0135	0.0104	0.0139	0.0131
3 - 4th	0.0140	0.0108	0.0146	0.0080
4 - 5th	0.0089	0.0137	0.0168	0.0155
5 - 6th	0.0044	0.0118	0.0141	0.0146
6 - Roof	0.0037	0.0033	0.0065	0.0092
Acceleration (g)				
2nd	0.638	0.697	0.573	0.685
3rd	0.474	0.644	0.761	0.650
4th	0.448	0.685	0.830	0.639
5th	0.458	0.585	0.621	0.762
6th	0.468	0.623	0.662	0.736
Roof	0.363	0.525	0.702	0.671
Energy (kJ)				
KE	381	500	610	767
SE	66	128	291	361
DE	764	917	1451	1763
PE	2823	3032	3031	2869
IE	3723	4037	4775	4872

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.16 Maximum Plastic Rotation and Plastic Energy at Each PHL with Fully Rigid Spring $k_{s5} = \infty \text{ kN-m/rad}$

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

In addition, maximum global responses, such as roof displacement, roof velocity, fourth floor acceleration, drift ratio between fourth and fifth stories, kinetic energy, strain energy, damping energy, plastic energy, and input energy for five cases of stiffness and four cases of yield moment in rotational springs are plotted together in Figure 5.17 and 5.18 for making comparison more conveniently. Similarly, maximum local responses (i.e., plastic rotation and plastic energy) at selected PHLs in columns, beams, and beam-column joints for five cases of stiffness and four cases of yield moment in rotational springs are also plotted in Figure 5.19 to 5.21 for the purpose of comparison.

The results shown in Table 5.4 to Table 5.8, and Figure 5.12 to Figure 5.21 demonstrate some trends, and they can be summarized in the following paragraphs. First consider the global responses:

- For displacement and velocity (see Tables 5.4 to 5.8 and Figure 5.17), the values generally increase as the floors increase for all cases, and the maximum values take place at the roof level. This is consistent with others' observations where the displacement and velocity follow closely to the vibration of the first mode shape of the structure. Through comparing roof displacement between different cases, it can be seen that for each of the five cases of rotational spring stiffness the maximum values generally increase as the yield moment in rotational springs increases, except for k_{s5} (i.e., fully rigid spring) where the roof displacement is largest for $M_s = 5875$ kN-m. The maximum displacement value takes place in k_{s1} (i.e., softer spring), while the values remain about the same for k_{s3} , k_{s4} , and k_{s5} . As for roof velocity, it is also shown that the values increase as the yield moment in rotational springs increases, and k_{s1} also gives the largest velocity response.
- For absolute acceleration (see Tables 5.4 to 5.8 and Figure 5.17), the maximum values take place at the 4th floor for almost all five cases of different spring stiffness, and the values of acceleration increase as the yield moment in rotational springs increases for most cases. However, there are some exceptions, for example, the maximum acceleration value at the 4th floor for k_{s5} is observed in the case of $M_s = 5875$ kN-m.

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.17 Comparison of Maximum Global Responses between Models with Different Stiffness and Yield Moment of Rotational Springs

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.18 Comparison of Maximum Energy Dissipation between Models with Different Stiffness and Yield Moment of Rotational Springs

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.19 Comparison of Maximum Local Responses in Columns between Models with Different Stiffness and Yield Moment of Rotational Springs

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.20 Comparison of Maximum Local Responses in Selected Beams between Models with Different Stiffness and Yield Moment of Rotational Springs

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

Figure 5.21 Comparison of Maximum Local Responses in Selected Beam-Column Joints between Models with Different Stiffness and Yield Moment of Rotational Springs

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

- For drift ratio (see Tables 5.4 to 5.8 and Figure 5.17), the general trend is not apparent. For certain rotational spring stiffness, such as k_{s1} and k_{s2} , the drift ratio increases as the yield moment in rotational springs increases for almost all cases. On the other hand, for the other rotational spring stiffness, i.e., k_{s3} , k_{s4} , and k_{s5} , as the yield moment in rotational springs increases the drift ratio will increase at certain floors, but decrease at other floors. In general, the drift ratio between 3rd to 4th floors and 4th to 5th floors are largest, while the maximum drift ratio is found between 4th to 5th floors for the case $M_s = 5875$ kN-m of k_{s1} .
- For kinetic energy (KE), strain energy (SE), and damping energy (DE) (see Figure 5.18), the trend is consistent and clear. For all five cases of rotational spring stiffness, the energy dissipation for KE , SE , and DE increases as the yield moment in rotational springs increases, and the maximum value takes place for the case of k_{s1} (i.e., softer spring).
- For plastic energy (PE) (see Figure 5.18), in the cases of k_{s1} , k_{s2} , and k_{s3} , the energy dissipation decreases as the yield moment in rotational springs increases. While in the cases of k_{s4} and k_{s5} , the energy dissipation value for $M_s = 3050$ kN-m is the largest compared to the other three cases. The maximum value takes place for the case of k_{s5} (fully rigid spring).
- For input energy (IE) (see Figure 5.18), the energy dissipation for all five cases of rotational spring stiffness is generally the same, while the maximum value takes place in the case of k_{s5} (i.e., fully rigid spring). The input energy increases as the yield moment in rotational springs increases for most cases, except for k_{s1} , k_{s2} , and k_{s4} , where the maximum input energy value takes place at $M_s = 5875$ kN-m.

In general, it can be seen that in terms of global responses, rotational springs with smaller yield moments tend to produce smaller responses. The reason maybe exists in that under the same earthquake ground motion structures with weak panel zones will yield prematurely comparing with those with strong panel zones. As a result, for structures with weak panel zones input energy will be dissipated in terms

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

of plastic energy at an earlier stage which otherwise will be stored as either strain energy or kinetic energy. By lowering the amount of strain energy or kinetic energy, weaker structures thus tend to have smaller global response.

Now let's investigate the local responses. According to Figures 5.12 to 5.16, it can be summarized that as the yield moment in rotational springs increases, major damage shifts from the beam-column joints to the beams and the columns. The structural models with five cases of rotational spring stiffness shown in Figures 5.12 to 5.16 have the same location pattern of plastic hinge formation as long as the yield moment in rotational springs remains constant. This implies that the rotational spring stiffness in panel zones have little effect on the plastic hinge distribution within the structure, whereas the yield moment of rotational springs plays a critical role on the formation of plastic hinges. In addition, it is noticed that plastic hinges will not occur at the beams and the beam-column joints simultaneously, i.e., plastic hinges occur either at the beams' PHL or at the adjacent joints' PHL. This phenomenon is consistent with the fact that when plastic hinges occur at the beams or the joints first, the transfer of moments to the adjacent PHL will cease and therefore the moment in these adjacent PHL are kept below its yield level.

In the following paragraphs, the observation of plastic hinge formation in columns, beams, and joints will be discussed respectively.

- For the base of columns (see Figures 5.12 to 5.16 and Figure 5.19), in the case of $M_s = 1695$ kN-m of all five cases of rotational spring stiffness where yield moment in the rotational springs is small, all inelastic deformation occurs at the beam-column joints and the base of the columns. Larger plastic rotations occur at the beam-column joints on the third and fourth stories, while only minimum plastic rotations with small plastic energy dissipation occur at the base of the columns. This is mainly because once the beam-column joints deform inelastically, the transfer of moments between the beams to the columns ceases and therefore reduces the moment demands at the base of the columns. As the yield moment in the rotational spring increases for all five cases of rotational spring stiffness, the plastic rotation and plastic energy dissipation at the base of columns increase due to the shift of plastic hinge formation from the joints to

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

the beams. The maximum values of both plastic rotation and plastic energy take place in the case of k_{s2} (i.e., soft spring).

- For beams (see Figures 5.12 to 5.16 and Figure 5.20), the plastic rotation and plastic energy dissipation generally increase as the yield moment in the rotational spring increases for almost all five cases of rotational spring stiffness. This is again mainly due to the shift of plastic hinge formation from the joints to the beams as the yield moment in the rotational spring increases. It is also observed that the plastic rotation and plastic energy dissipation also generally increase as the stiffness in the rotational springs increase for most of the PHLs in the beams.
- For beam-column joints (see Figures 5.12 to 5.16 and Figure 5.21), the situation is complicated. For all exterior joints, both plastic rotation and plastic energy dissipation decrease as the yield moment in the rotational spring increases, and the maximum values of both plastic rotation and plastic energy generally take place in the case of k_{s4} . For the interior joints, however, the plastic rotation of the joints at 2nd, 3rd, 6th, and roof floors for all of the five cases of rotational spring stiffness decrease as the yield moment in the rotational spring increases. In terms of plastic energy dissipation, results show that plastic energy dissipation at 2nd, 3rd, 6th, and roof floors for the cases of k_{s1} and k_{s2} also decrease as the yield moment in the rotational spring increases. But for the cases of k_{s3} , k_{s4} , and k_{s5} , larger plastic energy dissipations in the joints at 2nd, 3rd, 6th, and roof floors occur in the case of $M_s = 3050$ kN-m. For the joints at 4th and 5th floors, larger plastic rotation and plastic energy dissipation are also found in the case of $M_s = 3050$ kN-m for all of the five cases of rotational spring stiffness. It is also shown that larger panel zone stiffness tends to dissipate larger plastic energy in the beam-column joints. The reason for the phenomenon of the plastic energy dissipation at some beam-column joints increases while the plastic rotation at these joints decreases is that the plastic energy depends on both elastic moment and plastic rotation as shown in Eq. (3.62), where the increase in elastic moment in the rotational spring offsets the decrease in plastic rotation to give a larger plastic energy dissipation.

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

In general, it is demonstrated that one way to minimize damage at the base of the columns and at the beams is to reduce the capacity of the beam-column joints. However, whether it is desirable to form plastic hinges at the beam-column joints and further more if the joints are permitted to yield, how to balance the capacity of beams and joints to achieve an economical and safe structure remain an important issue, and further research in this area is necessary.

5.4 Summary

A simple analytical method of modeling the panel zone deformation with rigid end offsets is proposed to study the dynamic behavior of inelastic structures. Both shear deformation and inelastic deformation in the panel zone are modeled in order to accurately capture the actual behavior of the overall structural responses. Based on this proposed analytical method, numerical simulation is first performed on a one-story one-bay moment resisting frame. Comparison of results shows that rigidity of panel zones plays a major role in the overall structural responses during earthquake excitations.

Numerical simulations are then performed to demonstrate the use of proposed method on multi-degree of freedom structures. It is shown that there exist significant differences in both global and local response and also in energy dissipation between models with different panel zone rigidities and yield moments in the beam-column joints. Several conclusions can be drawn based on the results obtained from the multi-degree of freedom structures:

- Structure with softer rotational springs (i.e., smaller panel zone stiffness) generally gives larger maximum displacement, velocity, and drift responses;
- Structure with softer rotational springs produces more plastic rotation and dissipates more plastic energy at the base of column;
- Structure with softer rotational springs generally gives larger maximum strain energy, kinetic energy, and damping energy;
- Structure with stiffer rotational springs (i.e., larger panel zone stiffness) tends to dissipate more plastic energy and input energy;
- Structure with larger panel zone stiffness tends to produce larger plastic rotation and plastic energy in most of the beams and beam-column joints;

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

- When the panel zones remain elastic, structure with larger panel zone stiffness tends to dissipate larger plastic energy in the beams while structure with smaller panel zone stiffness tends to dissipate larger plastic energy in the columns;
- Structure with smaller yield moment in rotational springs tends to produce smaller global responses, which include displacement, velocity, drift, and acceleration;
- Structure with smaller yield moment in rotational springs generally has smaller maximum strain energy, kinetic energy, damping energy, and input energy;
- Structure with larger yield moment in rotational springs tends to dissipate less plastic energy;
- Structure with smaller yield moment in rotational springs mainly suffers larger local damages in the beam-column joints, while only minimum plastic rotations with small plastic energy dissipation occur at the base of the columns;
- As the yield moment in the rotational spring increases, the plastic rotation and the plastic energy decreases at most of the beam-column joint;
- As the yield moment in the rotational spring increases, the plastic rotation and plastic energy dissipation at the base of the columns also increase;
- As the yield moment in the rotational spring increases, the plastic rotation and plastic energy dissipation in most of the beams also increase.

It has been previously thought that damage will be more severe if yielding happens at the beam-column joint since the capacity of plastic energy dissipation in the beam-column joint is in general much less than that in the beam. Therefore, it is desirable to dissipate plastic energy in the beam, which is one reason why modern structural design focuses on weak beam with strong column and panel zone. However, current research implies that permit yielding in panel zone to certain extent may result in smaller structural response, including certain global responses, such as displacement, velocity, acceleration, drift ratio, kinetic energy, strain energy, damping energy, input energy, and certain local responses, such as plastic rotation and plastic energy at the base of the columns and at the beams. Hence, it may be desirable and rewarding to further investigate the effect of the yielding in panel zones on the dynamic behaviors of inelastic structures and find a good balance of

Chapter 5 Dynamic Analysis of Inelastic Structures with Panel Zone Deformation

yield moment capacity between the beams and beam-column joints to provide a more economical and safer design.

CHAPTER 6 STOCHASTIC DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH THE FORCE ANALOGY METHOD

6.1 General Statement

Starting from this chapter, the current research work will be streamed into the random field. Due to inherent uncertainty in earthquake excitation, random process is generally a more reasonable assumption for this type of ground motion. As a result, the responses of the structures under this type of loads should also be treated as random processes. In this chapter, the force analogy method presented in Chapter 3, which has been proven to be an efficient method to analyze the dynamic responses of inelastic structures in the deterministic field (Wong and Yang, 1999), is introduced to the procedure of stochastic analysis for the first time. In doing so, the stochastic force analogy method (SFAM) is developed, which is one of the major contributions of this research work. The characteristic of the force analogy method, which is described by varying the displacement field instead of varying the stiffness matrices to describe the inelastic behaviors of structures, remains in the stochastic procedure. Therefore, the efficiency in terms of numerical computation of the SFAM can be expected. As was done for the deterministic cases, in this section the state space method is also used to solve the equation of motion, and the whole process of calculating the covariance functions of the structural response involves a recursive process until the entire time history of the dynamic analysis is performed.

The proposed SFAM can produce the covariance function and hence a variance function of the inelastic structural response subjected to stochastic excitation. These responses include displacement, velocity, inelastic displacement of the whole structures, and plastic rotation at each plastic hinge location (PHL). Due to the complicity in the analysis, the entire theoretical development of this chapter will only involve a single degree of freedom (SDOF) system with one PHL. Multi-degree of freedom (MDOF) system with multiple PHLs will be considered in Chapter 7.

After the theoretical derivation, a numerical example is presented to demonstrate the accuracy of the stochastic force analogy method (SFAM) through

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

comparing the results obtained by the stochastic force analogy method (SFAM) with those obtained by the Monte Carlo simulation method (MCSM).

6.2 Stochastic Force Analogy Method (SFAM)

6.2.1 Solution to the Equation of Motion Based on the Force Analogy Method and State Space Method

Due to the complicity in the theoretical development of this study, a SDOF system with one PHL is first considered. For a SDOF system, the equation of motion used in the force analogy method based on the Eq. (3.25) is

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -Ma_g(t) + Kx''(t) \quad (6.1)$$

where M is mass, C is damping, K is stiffness, $x(t)$ is relative displacement, $\dot{x}(t)$ is relative velocity, $\ddot{x}(t)$ is relative acceleration, $x''(t)$ is inelastic displacement which has been explained in Chapter 3, and $a_g(t)$ is earthquake ground acceleration. Note that since SDOF system is used, all the parameters appeared in Eq. (6.1) are scalars and not vectors. By using the state space method to solve for Eq. (6.1), define

$$\mathbf{z}(t) = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} \quad (6.2)$$

Then Eq. (6.1) becomes

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{H}a_g(t) + \mathbf{F}_p^c x''(t) \quad (6.3)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -K/M & -C/M \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad \mathbf{F}_p^c = \begin{bmatrix} 0 \\ K/M \end{bmatrix} \quad (6.4)$$

Equation (6.3) is the governing first order linear differential equation, and the solution to this equation is

$$\mathbf{z}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{z}(t_0) + e^{\mathbf{A}t} \int_{t_0}^t e^{-\mathbf{A}s} [\mathbf{H}a_g(s) + \mathbf{F}_p^c x''(s)] ds \quad (6.5)$$

Let $t_{k+1} = t$, $t_k = t_0$, and $\Delta t = t - t_0$, then Eq. (6.5) becomes

$$\mathbf{z}(t_{k+1}) = e^{\mathbf{A}\Delta t}\mathbf{z}(t_k) + e^{\mathbf{A}t_{k+1}} \int_{t_k}^{t_{k+1}} e^{-\mathbf{A}s} [\mathbf{H}a_g(s) + \mathbf{F}_p^c x''(s)] ds \quad (6.6)$$

A transformation of $s = t_k + \tau$ in Eq. (6.6) leads to

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$\mathbf{z}(t_{k+1}) = e^{\mathbf{A}\Delta t}\mathbf{z}(t_k) + e^{\mathbf{A}\Delta t} \int_0^{\Delta t} e^{-\mathbf{A}\tau} \left[\mathbf{H}\mathbf{a}_g(t_k + \tau) + \mathbf{F}_p^c x''(t_k + \tau) \right] d\tau \quad (6.7)$$

which is equivalent to

$$\mathbf{z}(t_{k+1}) = e^{\mathbf{A}\Delta t}\mathbf{z}(t_k) + e^{\mathbf{A}\Delta t} \int_0^{\Delta t} e^{-\mathbf{A}\tau} \mathbf{H}\mathbf{a}_g(t_k + \tau) d\tau + e^{\mathbf{A}\Delta t} \int_0^{\Delta t} \mathbf{F}_p^c x''(t_k + \tau) d\tau \quad (6.8)$$

Representing the inelastic displacement using pulses within the small time step Δt while keeping the integral in the second term of the right hand side of Eq. (6.8) unchanged, then performing the integration in the third term of the right hand side of Eq. (6.8) gives

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{F}_s \int_0^{\Delta t} \mathbf{H}_g(\tau) a_g(t_k + \tau) d\tau + \mathbf{F}_p x_k'' \quad (6.9)$$

where

$$\mathbf{z}_k = \begin{Bmatrix} x_k \\ \dot{x}_k \end{Bmatrix} \quad (6.10)$$

and

$$\mathbf{F}_s = e^{\mathbf{A}\Delta t}, \quad \mathbf{H}_g(\tau) = e^{-\mathbf{A}\tau} \mathbf{H}, \quad \mathbf{F}_p = e^{\mathbf{A}\Delta t} \mathbf{F}_p^c \Delta t \quad (6.11)$$

where $\mathbf{z}_k$, x_k , $\dot{x}_k$ and x_k'' are the discretized forms of $\mathbf{z}(t)$, $x(t)$, $\dot{x}(t)$ and $x''(t)$ at time step k , respectively. Note that Eq. (6.9) is a recursive equation. The transpose to the vector $\mathbf{z}_{k+1}$ leads to

$$\begin{aligned} \mathbf{z}_{k+1}^T &= \left(\mathbf{F}_s \mathbf{z}_k + \mathbf{F}_s \int_0^{\Delta t} \mathbf{H}_g(\tau) a_g(t_k + \tau) d\tau + \mathbf{F}_p x_k'' \right)^T \\ &= (\mathbf{F}_s \mathbf{z}_k)^T + \left(\mathbf{F}_s \int_0^{\Delta t} \mathbf{H}_g(\tau) a_g(t_k + \tau) d\tau \right)^T + (\mathbf{F}_p x_k'')^T \\ &= \mathbf{z}_k^T \mathbf{F}_s^T + \left(\int_0^{\Delta t} \mathbf{H}_g^T(\tau) a_g(t_k + \tau) d\tau \right) \mathbf{F}_s^T + x_k'' \mathbf{F}_p^T \end{aligned} \quad (6.12)$$

Multiplying $\mathbf{z}_{k+1}$ by $\mathbf{z}_{k+1}^T$, one obtains

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$\begin{aligned}
 \mathbf{z}_{k+1} \mathbf{z}_{k+1}^T = & \mathbf{F}_s \left[\mathbf{z}_k \mathbf{z}_k^T \right] \mathbf{F}_s^T + \mathbf{F}_p \left[x_k'' x_k'' \right] \mathbf{F}_p^T + \mathbf{F}_s \left[\mathbf{z}_k x_k'' \right] \mathbf{F}_p^T + \mathbf{F}_p \left[x_k'' \mathbf{z}_k^T \right] \mathbf{F}_s^T \\
 & + \mathbf{F}_s \left\{ \int_0^{\Delta t} \int_0^{\Delta t} \left\{ \mathbf{H}_g(\tau_1) \left[\mathbf{H}_g(\tau_2) \right]^T \left[a_g(t_k + \tau_1) a_g(t_k + \tau_2) \right] \right\} d\tau_1 d\tau_2 \right\} \mathbf{F}_s^T \\
 & + \mathbf{F}_s \left\{ \int_0^{\Delta t} \mathbf{H}_g(\tau) \left[a_g(t_k + \tau) \right] d\tau \right\} \mathbf{z}_k^T \mathbf{F}_s^T \\
 & + \mathbf{F}_s \left\{ \int_0^{\Delta t} \mathbf{H}_g(\tau) \left[a_g(t_k + \tau) \right] d\tau \right\} x_k'' \mathbf{F}_p^T \\
 & + \mathbf{F}_p x_k'' \left\{ \int_0^{\Delta t} \left[a_g(t_k + \tau) \right] \left[\mathbf{H}_g(\tau) \right]^T d\tau \right\} \mathbf{F}_s^T \\
 & + \mathbf{F}_s \mathbf{z}_k \left\{ \int_0^{\Delta t} \left[a_g(t_k + \tau) \right] \left[\mathbf{H}_g(\tau) \right]^T d\tau \right\} \mathbf{F}_s^T
 \end{aligned} \tag{6.13}$$

There are totally nine terms in the right hand side of Eq. (6.13).

6.2.2 Recursive Equation Used in the Stochastic Analysis

In the present study, the ground acceleration $a_g(t)$ is idealized as a stochastic process with zero mean. Thus, the mean of all the structural responses can be assumed to be zero. As a result, the variance functions equal to the autocorrelation functions, while the covariance functions between different variables equal to the cross-correlation functions. Based on Eq. (6.13), the covariance matrix of the response state space vector $\mathbf{z}(t)$ can be computed in the following form:

$$\begin{aligned}
 \mathbf{E} \left[\mathbf{z}_{k+1} \mathbf{z}_{k+1}^T \right] = & \mathbf{F}_s \mathbf{E} \left[\mathbf{z}_k \mathbf{z}_k^T \right] \mathbf{F}_s^T + \mathbf{F}_p \mathbf{E} \left[x_k'' x_k'' \right] \mathbf{F}_p^T \\
 & + \mathbf{F}_s \mathbf{E} \left[\mathbf{z}_k x_k'' \right] \mathbf{F}_p^T + \mathbf{F}_p \mathbf{E} \left[x_k'' \mathbf{z}_k^T \right] \mathbf{F}_s^T \\
 & + \mathbf{F}_s \left\{ \int_0^{\Delta t} \int_0^{\Delta t} \mathbf{H}_g(\tau_1) \left[\mathbf{H}_g(\tau_2) \right]^T \mathbf{E} \left[a_g(t_k + \tau_1) a_g(t_k + \tau_2) \right] d\tau_1 d\tau_2 \right\} \mathbf{F}_s^T
 \end{aligned} \tag{6.14}$$

Equation (6.14) is an approximation because it disregards the covariance function of all the cross products involving the earthquake ground acceleration $a_g(t)$, i.e., the covariance function of the last four terms in the right hand side of Eq. (6.13). It is implied that the ground acceleration at the next time step is uncorrelated with all the structural responses at the current time step. Note that Eq. (6.14) is a recursive equation, i.e., if all the covariance information at time step k is known, the covariance matrix of the state space vector at time step $k+1$ (i.e., $\mathbf{E} \left[\mathbf{z}_{k+1} \mathbf{z}_{k+1}^T \right]$) can be calculated by using Eq. (6.14).

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Now consider the fifth term in the right hand side of Eq. (6.14). This term represents the variance of the ground acceleration. Two cases are discussed in the following.

(1) Stationary Process

If the ground acceleration $a_g(t)$ is a stationary process with zero mean, then its covariance function, which initially is a function of two different time points t_1 and t_2 , will be simplified to be the function of the difference between the two different time points t_1 and t_2 , i.e., $t_1 - t_2$, and it can be written as

$$E[a_g(t_k + \tau_1)a_g(t_k + \tau_2)] = R_{a_g a_g}(\tau_1 - \tau_2) \quad (6.15)$$

Then substituting Eq. (6.15) into Eq. (6.14) gives

$$\begin{aligned} E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T] &= \mathbf{F}_s E[\mathbf{z}_k\mathbf{z}_k^T] \mathbf{F}_s^T + \mathbf{F}_p E[x_k''x_k''] \mathbf{F}_p^T \\ &\quad + \mathbf{F}_s E[\mathbf{z}_k x_k''] \mathbf{F}_p^T + \mathbf{F}_p E[x_k''\mathbf{z}_k^T] \mathbf{F}_s^T \\ &\quad + \mathbf{F}_s \left\{ \int_0^{\Delta t} \int_0^{\Delta t} \mathbf{H}_g(\tau_1) [\mathbf{H}_g(\tau_2)]^T R_{a_g a_g}(\tau_1 - \tau_2) d\tau_1 d\tau_2 \right\} \mathbf{F}_s^T \end{aligned} \quad (6.16)$$

Here, note that the fifth term in the right hand side of Eq. (6.16) only need to be calculated once at the beginning, since it is not the function of time step, t_k . In the subsequent recursive calculation process, within each time step, to obtain the covariance matrix of the state space vector at time step $k + 1$ shown on the left term in the Eq. (6.16), only the first four terms in the right hand side of Eq. (6.16) need to be calculated

(2) Non-Stationary Process

If the ground acceleration $a_g(t)$ is a non-stationary process with zero mean, then it can be modeled as a function constructed by multiplying a stationary process by a deterministic modulation function (Simulescu et al., 1989)

$$a_g(t) = \phi(t) \cdot S(t) \quad (6.17)$$

where $S(t)$ represents the stationary process and $\phi(t)$ represents the deterministic modulation function. With the aid of Eq. (6.17), the covariance function of the ground acceleration $a_g(t)$ is obtained as

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$E[a_g(t_k + \tau_1)a_g(t_k + \tau_2)] = \phi(t_k + \tau_1)\phi(t_k + \tau_2)R_{SS}(\tau_1 - \tau_2) \quad (6.18)$$

Substituting Eq. (6.18) into Eq. (6.14) gives

$$\begin{aligned} E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T] &= \mathbf{F}_s E[\mathbf{z}_k\mathbf{z}_k^T] \mathbf{F}_s^T + \mathbf{F}_p E[x_k''x_k''] \mathbf{F}_p^T \\ &\quad + \mathbf{F}_s E[\mathbf{z}_k x_k''] \mathbf{F}_p^T + \mathbf{F}_p E[x_k'' \mathbf{z}_k^T] \mathbf{F}_s^T \\ &\quad + \mathbf{F}_s \left\{ \int_0^{\Delta t} \int_0^{\Delta t} \mathbf{H}_g(\tau_1) [\mathbf{H}_g(\tau_2)]^T \phi(t_k + \tau_1)\phi(t_k + \tau_2) R_{SS}(\tau_1 - \tau_2) d\tau_1 d\tau_2 \right\} \mathbf{F}_s^T \end{aligned} \quad (6.19)$$

Here, assuming that Δt is small, then the following approximation is introduced:

$$\phi(t_k + \tau) \approx \phi(t_k + \frac{\Delta t}{2}) \quad (6.20)$$

where $0 \leq \tau \leq \Delta t$. Substituting Eq. (6.20) into Eq. (6.19) gives

$$\begin{aligned} E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T] &= \mathbf{F}_s E[\mathbf{z}_k\mathbf{z}_k^T] \mathbf{F}_s^T + \mathbf{F}_p E[x_k''x_k''] \mathbf{F}_p^T \\ &\quad + \mathbf{F}_s E[\mathbf{z}_k x_k''] \mathbf{F}_p^T + \mathbf{F}_p E[x_k'' \mathbf{z}_k^T] \mathbf{F}_s^T \\ &\quad + \mathbf{F}_s \left\{ \left[\phi\left(t_k + \frac{\Delta t}{2}\right) \right]^2 \int_0^{\Delta t} \int_0^{\Delta t} \mathbf{H}_g(\tau_1) [\mathbf{H}_g(\tau_2)]^T R_{SS}(\tau_1 - \tau_2) d\tau_1 d\tau_2 \right\} \mathbf{F}_s^T \end{aligned} \quad (6.21)$$

Here, note that the fifth term in the right hand side of Eq. (6.21) is the function of time step, t_k , and therefore in the recursive calculation all five terms in the right hand side of Eq. (6.21) are needed to obtain the information at the next step. However, it also should be noted that the integral portion within the fifth term in the right hand side of Eq. (6.21) is not the function of time step, t_k , which is the same as the case of stationary process. Therefore, the integral part only needs to be calculated once at the beginning of the calculation process, and it remains unchanged in the subsequent calculation.

In the present research study, the ground acceleration is assumed to be a non-stationary process, which means that Eq. (6.21) is the key equation in the following theoretical development. Note that Eq. (6.21) is a recursive equation. For a given ground motion stochastic process (i.e., $\phi(t_k)$ and $R_{SS}(\tau_1 - \tau_2)$), if all the structural covariance information at time step k (i.e., $E[\mathbf{z}_k\mathbf{z}_k^T]$, $E[x_k''x_k'']$, $E[\mathbf{z}_k x_k'']$) is known, the covariance matrix of the response state space vector at time step $k+1$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

(i.e., $E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T]$) can be calculated using Eq. (6.21). The calculations of the variance of the inelastic displacement at time step $k+1$ (i.e., $E[x_{k+1}''x_{k+1}'']$) and the covariance of the state space vector and the inelastic displacement at time step $k+1$ (i.e., $E[\mathbf{z}_{k+1}x_{k+1}'']$) require the aid of the force analogy method, and this will be discussed in detail as follows. When the calculations of all the structural covariance information at time step $k+1$ (i.e., $E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T]$, $E[x_{k+1}''x_{k+1}'']$, $E[\mathbf{z}_{k+1}x_{k+1}'']$) are completed, the recursive calculation can be advanced to the next time step.

It is here noted that the key point in this study is to calculate $E[x_{k+1}''x_{k+1}'']$ and $E[\mathbf{z}_{k+1}x_{k+1}'']$ under the premise that $E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T]$ has been known. Here, substituting Eq. (6.10) into $E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T]$ gives

$$\begin{aligned} E[\mathbf{z}_{k+1}\mathbf{z}_{k+1}^T] &= E\left[\begin{pmatrix} x_{k+1} \\ \dot{x}_{k+1} \end{pmatrix} \begin{pmatrix} x_{k+1} & \dot{x}_{k+1} \end{pmatrix}\right] \\ &= E\begin{bmatrix} (x_{k+1})^2 & (x_{k+1}\dot{x}_{k+1}) \\ (x_{k+1}\dot{x}_{k+1}) & (\dot{x}_{k+1})^2 \end{bmatrix} \\ &= \begin{bmatrix} E[(x_{k+1})^2] & E[x_{k+1}\dot{x}_{k+1}] \\ E[x_{k+1}\dot{x}_{k+1}] & E[(\dot{x}_{k+1})^2] \end{bmatrix} \\ &= \begin{bmatrix} \sigma_{x_{k+1}}^2 & E[x_{k+1}\dot{x}_{k+1}] \\ E[x_{k+1}\dot{x}_{k+1}] & \sigma_{\dot{x}_{k+1}}^2 \end{bmatrix} \end{aligned} \quad (6.22)$$

With the aid of the force analogy method, the random variables at time step $k+1$ (i.e., $\mathbf{z}_{k+1}$ (i.e., x_{k+1} and $\dot{x}_{k+1}$), x_{k+1}'' , and θ_{k+1}'') and their probability distribution can be expressed as the functions of the random variables at the former time step k , i.e., $\mathbf{z}_k$ (i.e., x_k and $\dot{x}_k$), θ_k'' , and a_k . Here θ_k'' and a_k are the discretized values of plastic rotation $\theta''(t)$ and ground acceleration $a_g(t)$, respectively, at time step k . Consequently, the variance and the covariance at time step $k+1$ (i.e., $E[x_{k+1}''x_{k+1}'']$ and $E[\mathbf{z}_{k+1}x_{k+1}'']$) can be calculated based on the known information about the variance and the covariance at time step k .

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Therefore, the next research is to learn first how to express the random variables (i.e., $\mathbf{z}_{k+1}$, x''_{k+1} , and θ''_{k+1}) as the functions of the random variables at the former time step k .

6.2.3 Expressions of Random Variables and Boundary of Domains Based on the Force Analogy Method

Before the research, the definitions of $E[x''_{k+1}x''_{k+1}]$ and $E[\mathbf{z}_{k+1}x''_{k+1}]$ should be introduced in advance.

The variance of the inelastic displacement at time step $k+1$ (i.e., $E[x''_{k+1}x''_{k+1}]$) is defined as

$$E[x''_{k+1}x''_{k+1}] = E[(x''_{k+1})^2] = \int_{\Omega_1} (x''_{k+1})^2 f(x''_{k+1}) dx''_{k+1} \quad (6.23)$$

where $f(x''_{k+1})$ is the probability density function of x''_{k+1} , and Ω_1 is the corresponding sample space. On the other hand, the covariance of the state space vector and the inelastic displacement at time step $k+1$ (i.e., $E[\mathbf{z}_{k+1}x''_{k+1}]$) is a vector. It consists of two components, the first one is the covariance of the relative displacement and the inelastic displacement, and the second one is the covariance of the relative velocity and the inelastic displacement, i.e.

$$E[\mathbf{z}_{k+1}x''_{k+1}] = \begin{bmatrix} E[x_{k+1}x''_{k+1}] \\ E[\dot{x}_{k+1}x''_{k+1}] \end{bmatrix} \quad (6.24)$$

These two components can be computed separately and the covariance of the state space vector and the inelastic displacement at time step $k+1$ (i.e., $E[\mathbf{z}_{k+1}x''_{k+1}]$) can then be obtained. Define the first component $E[x_{k+1}x''_{k+1}]$ as

$$E[x_{k+1}x''_{k+1}] = \int_{\Omega_2} (x_{k+1})(x''_{k+1}) f(x_{k+1}, x''_{k+1}) dx_{k+1} dx''_{k+1} \quad (6.25)$$

where $f(x_{k+1}, x''_{k+1})$ is the joint probability density function of x_{k+1} and x''_{k+1} , and Ω_2 is the corresponding sample space. Similarly, define the second component $E[\dot{x}_{k+1}x''_{k+1}]$ as

$$E[\dot{x}_{k+1}x''_{k+1}] = \int_{\Omega_3} (\dot{x}_{k+1})(x''_{k+1}) f(\dot{x}_{k+1}, x''_{k+1}) d\dot{x}_{k+1} dx''_{k+1} \quad (6.26)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

where $f(\dot{x}_{k+1}, x_{k+1}'')$ is the joint probability density function of $\dot{x}_{k+1}$ and x_{k+1}'' , and Ω_3 is the corresponding sample space.

According to the results in Chapter 3, theoretically all the random variables, which includes x_{k+1} , $\dot{x}_{k+1}$, x_{k+1}'' and θ_{k+1}'' , can be expressed as the functions of the relevant random variables at the former step, then respectively the variance and the covariances given in Eq. (6.23), Eq. (6.25) and Eq. (6.26) can be rewritten as

$$\begin{aligned} E[x_{k+1}'' x_{k+1}''] &= E[(x_{k+1}'')^2] \\ &= \int_{\Omega} g_1(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{\Omega} g_1(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.27)$$

$$\begin{aligned} E[x_{k+1} x_{k+1}''] &= \int_{\Omega} g_2(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{\Omega} g_2(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.28)$$

$$\begin{aligned} E[\dot{x}_{k+1} x_{k+1}''] &= \int_{\Omega} g_3(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{\Omega} g_3(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.29)$$

where $g_1(x_k, \dot{x}_k, \theta_k'', a_k)$, $g_2(x_k, \dot{x}_k, \theta_k'', a_k)$, and $g_3(x_k, \dot{x}_k, \theta_k'', a_k)$ are the expressions of $(x_{k+1}'')^2$, $x_{k+1} x_{k+1}''$, and $\dot{x}_{k+1} x_{k+1}''$, respectively, $f(x_k, \dot{x}_k, \theta_k'', a_k)$ is the joint probability density function of x_k , $\dot{x}_k$, θ_k'' , and a_k , and Ω is the sample space for the relevant random vector $(x_k, \dot{x}_k, \theta_k'', a_k)$.

In order to give the explicit expressions of $g_1(x_k, \dot{x}_k, \theta_k'', a_k)$, $g_2(x_k, \dot{x}_k, \theta_k'', a_k)$, and $g_3(x_k, \dot{x}_k, \theta_k'', a_k)$, the results obtained in Chapter 3 should be reviewed *a priori*. Based on Eq. (3.28), the relationship between the states at time step $k+1$ and all the information at time step k is

$$\mathbf{z}_{k+1} = \mathbf{F}_s \mathbf{z}_k + \mathbf{H}_d a_k + \mathbf{F}_p x_k'' \quad (6.30)$$

where

$$\mathbf{H}_d = e^{\mathbf{A}\Delta t} \mathbf{H} \Delta t \quad (6.31)$$

Since

$$\mathbf{z}_{k+1} = \begin{Bmatrix} x_{k+1} \\ \dot{x}_{k+1} \end{Bmatrix} \quad (6.32)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

then x_{k+1} and $\dot{x}_{k+1}$ can be expressed as

$$x_{k+1} = \mathbf{F}'_s \mathbf{z}_k + H'_d a_k + F'_p x''_k \quad (6.33)$$

$$\dot{x}_{k+1} = \mathbf{F}''_s \mathbf{z}_k + H''_d a_k + F''_p x''_k \quad (6.34)$$

where

$$\mathbf{F}_s = \begin{bmatrix} \mathbf{F}'_s \\ \mathbf{F}''_s \end{bmatrix}, \quad \mathbf{H}_d = \begin{bmatrix} H'_d \\ H''_d \end{bmatrix}, \quad \mathbf{F}_p = \begin{bmatrix} F'_p \\ F''_p \end{bmatrix} \quad (6.35)$$

Equation (3.21), which is one of the two essential equations in the force analogy method, can be written in SDOF system form as

$$x''_{k+1} = K^{-1} K_p \theta''_k \quad (6.36)$$

where K is the stiffness and K_p is the parameter that relates the plastic rotation with the global degree of freedom. From Eq. (6.36), for the time step k it can be deduced that

$$x''_k = K^{-1} K_p \theta''_k \quad (6.37)$$

Substituting Eq. (6.37) into Eq. (6.33) and Eq. (6.34), then the explicit expressions of x_{k+1} and $\dot{x}_{k+1}$ as the functions of the random variables x_k , $\dot{x}_k$, θ''_k , and a_k are obtained as

$$x_{k+1} = \mathbf{F}'_s \mathbf{z}_k + H'_d a_k + F'_p K^{-1} K_p \theta''_k \quad (6.38)$$

$$\dot{x}_{k+1} = \mathbf{F}''_s \mathbf{z}_k + H''_d a_k + F''_p K^{-1} K_p \theta''_k \quad (6.39)$$

In addition, introducing two notations for simplicity as

$$\xi_k = \mathbf{F}'_s \mathbf{z}_k + H'_d a_k + F'_p K^{-1} K_p \theta''_k \quad (6.40)$$

$$\zeta_k = \mathbf{F}''_s \mathbf{z}_k + H''_d a_k + F''_p K^{-1} K_p \theta''_k \quad (6.41)$$

In this research the inelastic behavior of the plastic hinge is assumed to deform in elastic-plastic behavior as shown in Figure 6.1(b). Based on this assumption, there are three possible deformation states for θ''_k , and then the sample space Ω is divided into three subsets, i.e.,

$$\Omega = R_1 + R_2 + R_3 \quad (6.42)$$

wherein each subset is defined respectively as

$$R_1 = \{(x_k, \dot{x}_k, \theta''_k, a_k) : |M_{k+1}| \leq M_s, -\infty < x_k, \dot{x}_k, a_k < +\infty\} \quad (6.43)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$R_2 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : M_{k+1} > M_s, -\infty < x_k, \dot{x}_k, a_k < +\infty\} \quad (6.44)$$

$$R_3 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : M_{k+1} < -M_s, -\infty < x_k, \dot{x}_k, a_k < +\infty\} \quad (6.45)$$

where M_s is the positive yield moment at plastic hinge location, and M_{k+1} is the calculated moment in plastic hinge location at time step $k+1$. Thus, the variance in Eq. (6.27) can be rewritten as

$$\begin{aligned} E[x_{k+1}'' x_{k+1}''] &= E[(x_{k+1}'')^2] \\ &= E_{R_1}[(x_{k+1}'')^2] + E_{R_2}[(x_{k+1}'')^2] + E_{R_3}[(x_{k+1}'')^2] \end{aligned} \quad (6.46)$$

where

$$\begin{aligned} E_{R_1}[(x_{k+1}'')^2] &= \int_{R_1} g_1(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{R_1} g_1(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.47)$$

$$\begin{aligned} E_{R_2}[(x_{k+1}'')^2] &= \int_{R_2} g_1(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{R_2} g_1(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.48)$$

$$\begin{aligned} E_{R_3}[(x_{k+1}'')^2] &= \int_{R_3} g_1(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{R_3} g_1(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.49)$$

and similarly the covariance in Eq. (6.28) can be rewritten as

$$E[x_{k+1} x_{k+1}''] = E_{R_1}[x_{k+1} x_{k+1}''] + E_{R_2}[x_{k+1} x_{k+1}''] + E_{R_3}[x_{k+1} x_{k+1}''] \quad (6.50)$$

where

$$\begin{aligned} E_{R_1}[x_{k+1} x_{k+1}''] &= \int_{R_1} g_2(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{R_1} g_2(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.51)$$

$$\begin{aligned} E_{R_2}[x_{k+1} x_{k+1}''] &= \int_{R_2} g_2(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{R_2} g_2(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.52)$$

$$\begin{aligned} E_{R_3}[x_{k+1} x_{k+1}''] &= \int_{R_3} g_2(\mathbf{z}_k, \theta_k'', a_k) f(\mathbf{z}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{R_3} g_2(x_k, \dot{x}_k, \theta_k'', a_k) f(x_k, \dot{x}_k, \theta_k'', a_k) dx_k d\dot{x}_k d\theta_k'' da_k \end{aligned} \quad (6.53)$$

For the covariance in Eq. (6.29), the result is also the same, i.e.

$$E[\dot{x}_{k+1} x_{k+1}''] = E_{R_1}[\dot{x}_{k+1} x_{k+1}''] + E_{R_2}[\dot{x}_{k+1} x_{k+1}''] + E_{R_3}[\dot{x}_{k+1} x_{k+1}''] \quad (6.54)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

where

$$\begin{aligned} E_{R_1} [\dot{x}_{k+1} x''_{k+1}] &= \int_{R_1} g_3(\mathbf{z}_k, \theta''_k, a_k) f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{R_1} g_3(x_k, \dot{x}_k, \theta''_k, a_k) f(x_k, \dot{x}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \end{aligned} \quad (6.55)$$

$$\begin{aligned} E_{R_2} [\dot{x}_{k+1} x''_{k+1}] &= \int_{R_2} g_3(\mathbf{z}_k, \theta''_k, a_k) f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{R_2} g_3(x_k, \dot{x}_k, \theta''_k, a_k) f(x_k, \dot{x}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \end{aligned} \quad (6.56)$$

$$\begin{aligned} E_{R_3} [\dot{x}_{k+1} x''_{k+1}] &= \int_{R_3} g_3(\mathbf{z}_k, \theta''_k, a_k) f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{R_3} g_3(x_k, \dot{x}_k, \theta''_k, a_k) f(x_k, \dot{x}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \end{aligned} \quad (6.57)$$

Next, the objective is to determine the domains of R_1 , R_2 and R_3 within the sample space Ω and meanwhile express the random variable x''_{k+1} and θ''_{k+1} as the explicit functions of the random variables x_k , $\dot{x}_k$, θ''_k , and a_k corresponding to each subset R_1 , R_2 , and R_3 .

$$(1) \quad R_1 = \{(x_k, \dot{x}_k, \theta''_k, a_k) : |M_{k+1}| \leq M_s, -\infty < x_k, \dot{x}_k, a_k < +\infty\}$$

Equation (3.20), which is the second of the two essential equations of the force analogy method, can be written in SDOF system form as

$$M_{k+1} + K_R \theta''_{k+1} = K_P^T x_{k+1} \quad (6.58)$$

where K_R is the parameter that relates the plastic rotation with the moment at the plastic hinge location (PHL). Based on the assumption of elastic-plastic behavior of plastic hinge, if $|M_{k+1}| \leq M_s$, then $\Delta \theta''$ at time step k should be equal to zero, and thus the expression of θ''_{k+1} in the domain R_1 is determined as

$$\theta''_{k+1} = \theta''_k \quad (6.59)$$

Substituting Eq. (6.59) into Eq. (6.36), gives the form of x''_{k+1} in the domain R_1 as

$$x''_{k+1} = K^{-1} K_P \theta''_k \quad (6.60)$$

Substituting Eqs. (6.38) and (6.59) into Eq. (6.58), and rearranging the items gives

$$\begin{aligned} M_{k+1} &= K_P^T x_{k+1} - K_R \theta''_{k+1} \\ &= K_P^T (\mathbf{F}'_s \mathbf{z}_k + H'_d a_k + F'_p K^{-1} K_P \theta''_k) - K_R \theta''_k \\ &= \mathbf{K}_1 \mathbf{z}_k + K_2 a_k + K_3 \theta''_k \end{aligned} \quad (6.61)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

where

$$\mathbf{K}_1 = K_p^T \mathbf{F}'_s, \quad K_2 = K_p^T H'_d, \quad K_3 = K_p^T F'_p K^{-1} K_p - K_R \quad (6.62)$$

With the results given in Eq.(6.61), the boundary of the domain R_1 , which is determined completely by the following inequality

$$|M_{k+1}| \leq M_s \quad (6.63)$$

can then be expressed as

$$|\mathbf{K}_1 \mathbf{z}_k + K_2 a_k + K_3 \theta_k''| \leq M_s \quad (6.64)$$

Under the condition of $K_3 < 0$, it gives

$$K_3^{-1} (M_s - \mathbf{K}_1 \mathbf{z}_k - K_2 a_k) \leq \theta_k'' \leq -K_3^{-1} (M_s + \mathbf{K}_1 \mathbf{z}_k + K_2 a_k) \quad (6.65)$$

Here, denote

$$L_1 = K_3^{-1} (M_s - \mathbf{K}_1 \mathbf{z}_k - K_2 a_k) \quad (6.66)$$

$$L_2 = -K_3^{-1} (M_s + \mathbf{K}_1 \mathbf{z}_k + K_2 a_k) \quad (6.67)$$

Then the domain R_1 can be expressed as

$$R_1 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : L_1 \leq \theta_k'' \leq L_2, -\infty < x_k, \dot{x}_k, a_k < +\infty\} \quad (6.68)$$

$$(2) \quad R_2 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : M_{k+1} > M_s, -\infty < x_k, \dot{x}_k, a_k < +\infty\}$$

Next consider the second domain of R_2 , which is determined by the following inequality

$$M_{k+1} > M_s \quad (6.69)$$

Substituting Eq. (6.61) into Eq. (6.69), results in

$$\mathbf{K}_1 \mathbf{z}_k + K_2 a_k + K_3 \theta_k'' > M_s \quad (6.70)$$

Then, under the condition of $K_3 < 0$, it gives

$$\theta_k'' < K_3^{-1} (M_s - \mathbf{K}_1 \mathbf{z}_k - K_2 a_k) \quad (6.71)$$

Substituting Eq. (6.66) in Eq. (6.71), then the domain of R_2 is finally determined as

$$R_2 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : -\infty < \theta_k'' < L_1, -\infty < x_k, \dot{x}_k, a_k < +\infty\} \quad (6.72)$$

Within this domain, based on the assumption of elastic-plastic behavior of plastic hinge, if $M_{k+1} > M_s$, then it has

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$M_{k+1} = M_s \quad (6.73)$$

Substituting Eq. (6.73) into Eq. (6.58), gives

$$M_s + K_R \theta_{k+1}'' = K_P^T x_{k+1} \quad (6.74)$$

to which the solution is

$$\theta_{k+1}'' = K_R^{-1} (K_P^T x_{k+1} - M_s) \quad (6.75)$$

Then substituting Eq. (6.38) into Eq. (6.75) gives the expression of θ_{k+1}'' in the domain R_2 as:

$$\theta_{k+1}'' = K_R^{-1} \left[K_P^T (\mathbf{F}'_s \mathbf{z}_k + H'_d a_k + F'_p K^{-1} K_P \theta_k'') - M_s \right] \quad (6.76)$$

which can be simplified by inserting Eq. (6.40) into Eq. (6.76), as

$$\theta_{k+1}'' = K_R^{-1} (K_P^T \xi_k - M_s) \quad (6.77)$$

Now substituting Eq. (6.77) into Eq. (6.36) gives the expression of x_{k+1}'' in the domain R_2 as:

$$x_{k+1}'' = K^{-1} K_P K_R^{-1} (K_P^T \xi_k - M_s) \quad (6.78)$$

$$(3) \quad R_3 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : M_{k+1} < -M_s, -\infty < x_k, \dot{x}_k, a_k < +\infty\}$$

The third domain R_3 is determined by the following inequality

$$M_{k+1} < -M_s \quad (6.79)$$

i.e.,

$$K_1 \mathbf{z}_k + K_2 a_k + K_3 \theta_k'' < -M_s \quad (6.80)$$

which, under the condition of $K_3 < 0$, is equivalent to

$$\theta_k'' > -K_3^{-1} (M_s + K_1 \mathbf{z}_k + K_2 a_k) \quad (6.81)$$

Substituting Eq. (6.67) into Eq. (6.81), then the domain R_3 can be expressed as

$$R_3 = \{(x_k, \dot{x}_k, \theta_k'', a_k) : L_2 < \theta_k'' < +\infty, -\infty < x_k, \dot{x}_k, a_k < +\infty\} \quad (6.82)$$

Within this domain, under the assumption of elastic-plastic behavior of plastic hinge, if $M_{k+1} < -M_s$, then

$$M_{k+1} = -M_s \quad (6.83)$$

Inserting Eq. (6.83) into Eq. (6.58), one obtains

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$-M_s + K_R \theta_{k+1}'' = K_P^T x_{k+1} \quad (6.84)$$

Solving for θ_{k+1}'' leads to

$$\theta_{k+1}'' = K_R^{-1} (K_P^T x_{k+1} + M_s) \quad (6.85)$$

Then substituting Eq. (6.38) into Eq. (6.85) gives the expression of θ_{k+1}'' in the domain R_3 as:

$$\theta_{k+1}'' = K_R^{-1} \left[K_P^T (F_s' z_k + H_d' a_k + F_p' K^{-1} K_P \theta_k'') + M_s \right] \quad (6.86)$$

which can be simplified by substituting Eq. (6.40) into Eq. (6.86), i.e.

$$\theta_{k+1}'' = K_R^{-1} (K_P^T \xi_k + M_s) \quad (6.87)$$

Finally in domain R_3 , the expression of x_{k+1}'' can be obtained by substituting Eq. (6.87) into Eq. (6.36)

$$x_{k+1}'' = K^{-1} K_P K_R^{-1} (K_P^T \xi_k + M_s) \quad (6.88)$$

6.2.4 Calculation of Joint Probability Density Function

In this research, it is assumed that the stochastic vector of $(x_k, \dot{x}_k, \theta_k'')$ is independent of the random variable a_k , which means that the joint probability density function for the random vector (z_k, θ_k'', a_k) can be written as

$$\begin{aligned} f(z_k, \theta_k'', a_k) &= f(x_k, \dot{x}_k, \theta_k'', a_k) \\ &= f(x_k, \dot{x}_k, \theta_k'') f(a_k) \end{aligned} \quad (6.89)$$

In addition, the random variable a_k is assumed to be in the form of Gaussian distribution, in which the probability density function is defined as

$$f(a_k) = \frac{1}{\sqrt{2\pi}\sigma_{a_k}} \exp \left[-\frac{(a_k - \mu_{a_k})^2}{2\sigma_{a_k}^2} \right], \quad -\infty < a_k < +\infty \quad (6.90)$$

where the mean of a_k is

$$\mu_{a_k} = 0 \quad (6.91)$$

This is under the assumption that the ground acceleration $a_g(t)$ is a non-stationary Gaussian process with zero mean. Based on Eq. (6.18), the variance of a_k is given as

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$\begin{aligned}\sigma_{a_k}^2 &= E\left[(a_k)^2\right] \\ &= \phi^2(t_k)R_{SS}(t_k - t_k) \\ &= \phi^2(t_k)R_{SS}(0)\end{aligned}\quad (6.92)$$

In addition, the three random variables x_k , $\dot{x}_k$, and θ_k'' are assumed to be jointly Gaussian distributed. As stated in the book of Lin (1967), for those random variables $(x_1, x_2, \dots, x_n)$, which are said to be jointly Gaussian distributed, their joint probability density function is defined as

$$f(x_1, x_2, \dots, x_n) = \frac{1}{(2\pi)^{\frac{n}{2}} |\mathbf{S}|^{\frac{1}{2}}} \exp \left[-\frac{1}{2|\mathbf{S}|} \sum_{j=1}^n \sum_{k=1}^n |\mathbf{S}|_{jk} (x_j - \mu_{x_j})(x_k - \mu_{x_k}) \right] \quad (6.93)$$

where $|\mathbf{S}|$ is the determinant of the matrix of covariances, which is

$$\mathbf{S} = \begin{bmatrix} \sigma_{x_1}^2 & k_{x_1 x_2} & \cdots & k_{x_1 x_n} \\ k_{x_2 x_1} & \sigma_{x_2}^2 & \cdots & k_{x_2 x_n} \\ \cdots & \cdots & \cdots & \cdots \\ k_{x_n x_1} & k_{x_n x_2} & \cdots & \sigma_{x_n}^2 \end{bmatrix} \quad (6.94)$$

and

$$\mu_{x_j} = E(x_j), \quad \sigma_{x_j}^2 = E\left[(x_j - \mu_{x_j})^2\right], \quad k_{x_j x_k} = E\left[(x_j - \mu_{x_j})(x_k - \mu_{x_k})\right] \quad (6.95)$$

which denote the mean, variance and covariance, respectively. Therefore, the joint probability density function of the random variables $(x_k, \dot{x}_k, \theta_k'')$ is given by

$$f(x_k, \dot{x}_k, \theta_k'') = \frac{1}{(2\pi)^{\frac{3}{2}} |\mathbf{S}_k|^{\frac{1}{2}}} \exp \left[-\frac{1}{2} (\mathbf{x}_k - \boldsymbol{\mu}_{\mathbf{x}_k})^T \mathbf{S}_k^{-1} (\mathbf{x}_k - \boldsymbol{\mu}_{\mathbf{x}_k}) \right] \quad (6.96)$$

where

$$\left\{ \mathbf{x}_k - \boldsymbol{\mu}_{\mathbf{x}_k} \right\} = \begin{bmatrix} x_k - \mu_{x_k} \\ \dot{x}_k - \mu_{\dot{x}_k} \\ \theta_k'' - \mu_{\theta_k''} \end{bmatrix} \quad (6.97)$$

in which μ_{x_k} , $\mu_{\dot{x}_k}$, and $\mu_{\theta_k''}$ are the mean of x_k , $\dot{x}_k$, and θ_k'' , respectively, and

$$\mathbf{S}_k = \begin{bmatrix} \sigma_{x_k}^2 & E[x_k \dot{x}_k] & E[x_k \theta_k''] \\ E[x_k \dot{x}_k] & \sigma_{\dot{x}_k}^2 & E[\dot{x}_k \theta_k''] \\ E[x_k \theta_k''] & E[\dot{x}_k \theta_k''] & \sigma_{\theta_k''}^2 \end{bmatrix} \quad (6.98)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

where σ_{x_k} , $\sigma_{\dot{x}_k}$, and $\sigma_{\theta_k''}$ are the standard deviation of x_k , $\dot{x}_k$, and θ_k'' , respectively, and $E[\bullet]$'s are the covariance of two different random variables. Since the mean of the responses are assumed to be zero, it gives

$$f(x_k, \dot{x}_k, \theta_k'') = \frac{1}{(2\pi)^{\frac{3}{2}} |\mathbf{S}_k|^{\frac{1}{2}}} \exp \left[-\frac{1}{2} \mathbf{x}_k^T \mathbf{S}_k^{-1} \mathbf{x}_k \right] \quad (6.99)$$

where

$$\mathbf{x}_k = (x_k, \dot{x}_k, \theta_k'')^T \quad (6.100)$$

Here it is noted that in Eq. (6.98) there are totally six terms which are needed to be calculated at time step k , and they are $\sigma_{x_k}^2$, $\sigma_{\dot{x}_k}^2$, $\sigma_{\theta_k''}^2$, $E[x_k \dot{x}_k]$, $E[x_k \theta_k'']$, and $E[\dot{x}_k \theta_k'']$. Among these six terms, $\sigma_{x_k}^2$, $\sigma_{\dot{x}_k}^2$, and $E[x_k \dot{x}_k]$ are embedded in $E[\mathbf{z}_k \mathbf{z}_k^T]$, which have been calculated using Eq. (6.21) based on all the terms at time step $k-1$, whereas $\sigma_{\theta_k''}^2$, $E[x_k \theta_k'']$, and $E[\dot{x}_k \theta_k'']$ must be calculated extra at time step k .

6.2.5 Calculation of the Variance and the Covariance

In order to calculate $E[x_{k+1}'' x_{k+1}'']$ (i.e., $\sigma_{x_{k+1}}^2$) and $E[\mathbf{z}_{k+1} x_{k+1}'']$ (i.e., $E[x_{k+1} x_{k+1}'']$ and $E[\dot{x}_{k+1} x_{k+1}'']$) at time step $k+1$, six terms at time step k must be known, and they are $\sigma_{x_k}^2$, $\sigma_{\dot{x}_k}^2$, $\sigma_{\theta_k''}^2$, $E[x_k \dot{x}_k]$, $E[x_k \theta_k'']$, and $E[\dot{x}_k \theta_k'']$. In other words, it can be said that during the recursive calculation process, at each time step there are totally nine terms which need to be determined, and they are $\sigma_{x_k}^2$, $\sigma_{\dot{x}_k}^2$, $\sigma_{\theta_k''}^2$, $\sigma_{x_k''}^2$, $E[x_k \dot{x}_k]$, $E[x_k \theta_k'']$, $E[\dot{x}_k \theta_k'']$, $E[x_k x_k'']$, and $E[\dot{x}_k x_k'']$. Then the calculation can be advanced to the next time step, which includes the calculation of $E[\mathbf{z}_{k+1} \mathbf{z}_{k+1}^T]$ (i.e., $\sigma_{x_{k+1}}^2$, $\sigma_{\dot{x}_{k+1}}^2$, $E[x_{k+1} \dot{x}_{k+1}]$), $E[x_{k+1}'' x_{k+1}'']$ (i.e., $\sigma_{x_{k+1}}^2$), $E[\mathbf{z}_{k+1} x_{k+1}'']$ (i.e., $E[x_{k+1} x_{k+1}'']$ and $E[\dot{x}_{k+1} x_{k+1}'']$), $E[\theta_{k+1}'' \theta_{k+1}'']$ (i.e., $\sigma_{\theta_{k+1}}^2$), and $E[\mathbf{z}_{k+1} \theta_{k+1}'']$ (i.e., $E[x_{k+1} \theta_{k+1}'']$ and $E[\dot{x}_{k+1} \theta_{k+1}'']$).

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Here, the calculation analyses for six terms, which include $E[x_{k+1}''x_{k+1}'']$ (i.e., $\sigma_{x_{k+1}}^2$), $E[\mathbf{z}_{k+1}''x_{k+1}'']$ (i.e., $E[x_{k+1}''x_{k+1}'']$ and $E[\dot{x}_{k+1}''x_{k+1}'']$), $E[\theta_{k+1}''\theta_{k+1}'']$ (i.e., $\sigma_{\theta_{k+1}}^2$), and $E[\mathbf{z}_{k+1}''\theta_{k+1}'']$ (i.e., $E[x_{k+1}''\theta_{k+1}'']$ and $E[\dot{x}_{k+1}''\theta_{k+1}'']$), are needed to be explained in detail.

(1) Calculation of $E[x_{k+1}''x_{k+1}'']$ (i.e., $\sigma_{x_{k+1}}^2$ or $E[(x_{k+1}'')^2]$)

From Eq. (6.46), one knows that

$$E[(x_{k+1}'')^2] = E_{R_1}[(x_{k+1}'')^2] + E_{R_2}[(x_{k+1}'')^2] + E_{R_3}[(x_{k+1}'')^2] \quad (6.101)$$

The expression for the first term of the right hand side of Eq. (6.101) is obtained by substituting Eq. (6.60), Eq. (6.68) and Eq. (6.89) into Eq. (6.47), i.e.

$$\begin{aligned} E_{R_1}[(x_{k+1}'')^2] &= \int_{R_1} (x_{k+1}'')^2 f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_1}^{L_2} \left\{ (K^{-1}K_P\theta_k'')^2 f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.102)$$

Then substituting Eqs. (6.72), (6.78), and (6.89) into Eq. (6.48) gives the expression of the second term of the right hand side of Eq. (6.101), as

$$\begin{aligned} E_{R_2}[(x_{k+1}'')^2] &= \int_{R_2} (x_{k+1}'')^2 f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{-\infty}^{L_1} \left\{ \left[K^{-1}K_PK_R^{-1} (K_P^T\xi_k - M_s) \right]^2 \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.103)$$

For the third term in the right hand side of Eq. (6.101), the substitution of Eq. (6.82), Eq. (6.88) and Eq. (6.89) into Eq. (6.49) leads to

$$\begin{aligned} E_{R_3}[(x_{k+1}'')^2] &= \int_{R_3} (x_{k+1}'')^2 f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_2}^{+\infty} \left\{ \left[K^{-1}K_PK_R^{-1} (K_P^T\xi_k + M_s) \right]^2 \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.104)$$

(2) Calculation of $E[\mathbf{z}_{k+1}''x_{k+1}'']$ (i.e., $E[x_{k+1}''x_{k+1}'']$ and $E[\dot{x}_{k+1}''x_{k+1}'']$)

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

First consider $E[x_{k+1}x_{k+1}'']$. From Eq. (6.50), it gives

$$E[x_{k+1}x_{k+1}''] = E_{R_1}[x_{k+1}x_{k+1}''] + E_{R_2}[x_{k+1}x_{k+1}''] + E_{R_3}[x_{k+1}x_{k+1}''] \quad (6.105)$$

Substituting Eqs. (6.38), (6.40), (6.60), (6.68), and (6.89) into Eq. (6.51), one obtains the expression of the first term in the right hand side of Eq. (6.105) as

$$\begin{aligned} E_{R_1}[x_{k+1}x_{k+1}''] &= \int_{R_1} x_{k+1}x_{k+1}'' f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_1}^{L_2} \left\{ \xi_k \left(K^{-1} K_p \theta_k'' \right) f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.106)$$

The expression of the second term in the right hand side of Eq. (6.105) can be evaluated by substituting Eqs. (6.38), (6.40), (6.72), (6.78), and (6.89) into Eq. (6.52), i.e.,

$$\begin{aligned} E_{R_2}[x_{k+1}x_{k+1}''] &= \int_{R_2} x_{k+1}x_{k+1}'' f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{-\infty}^{L_1} \left\{ \xi_k \left[K^{-1} K_p K_R^{-1} \left(K_p^T \xi_k - M_s \right) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.107)$$

For the third term in the right hand side of Eq. (6.105), substituting Eqs. (6.38), (6.40), (6.82), (6.88), and (6.89) into Eq. (6.53) gives

$$\begin{aligned} E_{R_3}[x_{k+1}x_{k+1}''] &= \int_{R_3} x_{k+1}x_{k+1}'' f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_2}^{+\infty} \left\{ \xi_k \left[K^{-1} K_p K_R^{-1} \left(K_p^T \xi_k + M_s \right) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.108)$$

Now consider $E[\dot{x}_{k+1}x_{k+1}'']$. From Eq. (6.54), it gives

$$E[\dot{x}_{k+1}x_{k+1}''] = E_{R_1}[\dot{x}_{k+1}x_{k+1}''] + E_{R_2}[\dot{x}_{k+1}x_{k+1}''] + E_{R_3}[\dot{x}_{k+1}x_{k+1}''] \quad (6.109)$$

For the three terms in the right hand side of Eq. (6.109), first substituting Eqs. (6.39), (6.41), (6.60), (6.68), and (6.89) into Eq. (6.55) gives

$$\begin{aligned} E_{R_1}[\dot{x}_{k+1}x_{k+1}''] &= \int_{R_1} \dot{x}_{k+1}x_{k+1}'' f(\mathbf{z}_k, \theta_k'') f(a_k) dx_k d\dot{x}_k d\theta_k'' da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_1}^{L_2} \left\{ \zeta_k \left(K^{-1} K_p \theta_k'' \right) f(x_k, \dot{x}_k, \theta_k'') f(a_k) \right\} d\theta_k'' \end{aligned} \quad (6.110)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Then substituting Eqs. (6.39), (6.41), (6.72), (6.78), and (6.89) into Eq. (6.56) gives

$$\begin{aligned} E_{R_2} [\dot{x}_{k+1} x''_{k+1}] &= \int_{R_2} \dot{x}_{k+1} x''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{-\infty}^{L_1} \left\{ \zeta_k \left[K^{-1} K_P K_R^{-1} (K_P^T \xi_k - M_s) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.111)$$

Finally, substituting Eqs. (6.39), (6.41), (6.82), (6.88), and (6.89) into Eq. (6.57) gives

$$\begin{aligned} E_{R_3} [\dot{x}_{k+1} x''_{k+1}] &= \int_{R_3} \dot{x}_{k+1} x''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_2}^{+\infty} \left\{ \zeta_k \left[K^{-1} K_P K_R^{-1} (K_P^T \xi_k + M_s) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.112)$$

(3) Calculation of $E[\theta''_{k+1} \theta''_{k+1}]$ (i.e., $\sigma_{\theta''_{k+1}}^2$ or $E[(\theta''_{k+1})^2]$)

Based on the same reason as for $E[x''_{k+1} x''_{k+1}]$, $E[\theta''_{k+1} \theta''_{k+1}]$ can be written as

$$\begin{aligned} E[\theta''_{k+1} \theta''_{k+1}] &= E[(\theta''_{k+1})^2] \\ &= E_{R_1} [(\theta''_{k+1})^2] + E_{R_2} [(\theta''_{k+1})^2] + E_{R_3} [(\theta''_{k+1})^2] \end{aligned} \quad (6.113)$$

where

$$E_{R_1} [(\theta''_{k+1})^2] = \int_{R_1} (\theta''_{k+1})^2 f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.114)$$

$$E_{R_2} [(\theta''_{k+1})^2] = \int_{R_2} (\theta''_{k+1})^2 f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.115)$$

$$E_{R_3} [(\theta''_{k+1})^2] = \int_{R_3} (\theta''_{k+1})^2 f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.116)$$

For the three terms in the right hand side of Eq. (6.113), first substituting Eqs. (6.59), (6.68), and (6.89) into Eq. (6.114) gives

$$\begin{aligned} E_{R_1} [(\theta''_{k+1})^2] &= \int_{R_1} (\theta''_{k+1})^2 f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_1}^{L_2} \left\{ (\theta''_k)^2 f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.117)$$

Then substituting Eqs. (6.72), (6.77), and (6.89) into Eq. (6.115) gives

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$\begin{aligned} E_{R_2} \left[(\theta''_{k+1})^2 \right] &= \int_{R_2} (\theta''_{k+1})^2 f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{-\infty}^{L_1} \left[K_R^{-1} (K_p^T \xi_k - M_s) \right]^2 \\ &\quad \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \} d\theta''_k \end{aligned} \quad (6.118)$$

Finally, substituting Eqs. (6.82), (6.87), and (6.89) into Eq. (6.116) gives

$$\begin{aligned} E_{R_3} \left[(\theta''_{k+1})^2 \right] &= \int_{R_3} (\theta''_{k+1})^2 f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_2}^{+\infty} \left[K_R^{-1} (K_p^T \xi_k + M_s) \right]^2 \\ &\quad \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \} d\theta''_k \end{aligned} \quad (6.119)$$

(4) Calculation of $E[\mathbf{z}_{k+1} \theta''_{k+1}]$ (i.e., $E[x_{k+1} \theta''_{k+1}]$ and $E[\dot{x}_{k+1} \theta''_{k+1}]$)

First consider $E[x_{k+1} \theta''_{k+1}]$. Based on the same reason as for $E[x_{k+1} x''_{k+1}]$,

$E[x_{k+1} \theta''_{k+1}]$ can be written as

$$E[x_{k+1} \theta''_{k+1}] = E_{R_1} [x_{k+1} \theta''_{k+1}] + E_{R_2} [x_{k+1} \theta''_{k+1}] + E_{R_3} [x_{k+1} \theta''_{k+1}] \quad (6.120)$$

where

$$E_{R_1} [x_{k+1} \theta''_{k+1}] = \int_{R_1} x_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.121)$$

$$E_{R_2} [x_{k+1} \theta''_{k+1}] = \int_{R_2} x_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.122)$$

$$E_{R_3} [x_{k+1} \theta''_{k+1}] = \int_{R_3} x_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.123)$$

For the three terms in the right hand side of Eq. (6.120), first substituting Eqs. (6.38), (6.40), (6.59), (6.68), and (6.89) into Eq. (6.121) gives

$$\begin{aligned} E_{R_1} [x_{k+1} \theta''_{k+1}] &= \int_{R_1} x_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_1}^{L_2} \{ \xi_k \theta''_k f(x_k, \dot{x}_k, \theta''_k) f(a_k) \} d\theta''_k \end{aligned} \quad (6.124)$$

Then substituting Eqs. (6.38), (6.40), (6.72), (6.77), and (6.89) into Eq. (6.122) gives

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

$$\begin{aligned} E_{R_2} [x_{k+1} \theta''_{k+1}] &= \int_{R_2} x_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{-\infty}^{L_1} \left\{ \xi_k \left[K_R^{-1} (K_p^T \xi_k - M_s) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.125)$$

Finally, substituting Eqs. (6.38), (6.40), (6.82), (6.87), and (6.89) into Eq. (6.123) gives

$$\begin{aligned} E_{R_3} [x_{k+1} \theta''_{k+1}] &= \int_{R_3} x_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_2}^{+\infty} \left\{ \xi_k \left[K_R^{-1} (K_p^T \xi_k + M_s) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.126)$$

Now consider $E[\dot{x}_{k+1} \theta''_{k+1}]$. Based on the same reason with $E[\dot{x}_{k+1} x''_{k+1}]$,

$E[\dot{x}_{k+1} \theta''_{k+1}]$ can be written as

$$E[\dot{x}_{k+1} \theta''_{k+1}] = E_{R_1} [\dot{x}_{k+1} \theta''_{k+1}] + E_{R_2} [\dot{x}_{k+1} \theta''_{k+1}] + E_{R_3} [\dot{x}_{k+1} \theta''_{k+1}] \quad (6.127)$$

where

$$E_{R_1} [\dot{x}_{k+1} \theta''_{k+1}] = \int_{R_1} \dot{x}_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.128)$$

$$E_{R_2} [\dot{x}_{k+1} \theta''_{k+1}] = \int_{R_2} \dot{x}_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.129)$$

$$E_{R_3} [\dot{x}_{k+1} \theta''_{k+1}] = \int_{R_3} \dot{x}_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k, a_k) dx_k d\dot{x}_k d\theta''_k da_k \quad (6.130)$$

For the three terms in the right hand side of Eq. (6.127), first substituting Eqs. (6.39), (6.41), (6.59), (6.68), and (6.89) into Eq. (6.128), gives

$$\begin{aligned} E_{R_1} [\dot{x}_{k+1} \theta''_{k+1}] &= \int_{R_1} \dot{x}_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_1}^{L_2} \left\{ \zeta_k \theta''_k f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.131)$$

Then substituting Eqs. (6.39), (6.41), (6.72), (6.77), and (6.89) into Eq. (6.129) gives

$$\begin{aligned} E_{R_2} [\dot{x}_{k+1} \theta''_{k+1}] &= \int_{R_2} \dot{x}_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{-\infty}^{L_1} \left\{ \zeta_k \left[K_R^{-1} (K_p^T \xi_k - M_s) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.132)$$

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Finally, substituting Eqs. (6.39), (6.41), (6.82), (6.87), and (6.89) into Eq. (6.130) gives

$$\begin{aligned} E_{R_3} [\dot{x}_{k+1} \theta''_{k+1}] &= \int_{R_3} \dot{x}_{k+1} \theta''_{k+1} f(\mathbf{z}_k, \theta''_k) f(a_k) dx_k d\dot{x}_k d\theta''_k da_k \\ &= \int_{-\infty}^{+\infty} da_k \int_{-\infty}^{+\infty} dx_k \int_{-\infty}^{+\infty} d\dot{x}_k \int_{L_2}^{+\infty} \left\{ \zeta_k \left[K_R^{-1} (K_p^T \xi_k + M_s) \right] \right. \\ &\quad \left. \times f(x_k, \dot{x}_k, \theta''_k) f(a_k) \right\} d\theta''_k \end{aligned} \quad (6.133)$$

In this study, the multiple integrals presented above are calculated by the Matlab programs. This part can be referred to “Advanced Mathematics and Mechanics Applications Using Matlab” (p.146), written by H. B. Wilson and L. H. Turcotte. With regard to the infinite integral up and low bounds of the integral variables, $\pm 4\sigma_x$ is used instead of $\pm\infty$, where σ_x is the standard deviation of the corresponding integral variables. This assumption is based on the fact that the probability of variables appearing beyond $\pm 4\sigma_x$ will be rather small and can be ignored. As a result, the original multiple integrals are transformed from infinite limits to finite limits.

6.3 Numerical Example

For the purpose of demonstrating the stochastic force analogy method (SFAM) proposed above, an example problem involving a simple structure is considered. The structure is shown in Figure 6.1(a), which has single degree of freedom (SDOF) in the horizontal displacement direction on the top of the column and one plastic hinge location (1 PHL) at the base of the column. Assume the structure properties contain a mass of $M = 1.75 \times 10^5$ kg, a damping ratio of $\zeta = 5\%$, and period of $T_n = 0.5$ s. Then

$$\omega_n = 2\pi/T_n = 4\pi \text{ rad/s} \quad (6.134)$$

$$C = 2\zeta\omega_n M = 220.1 \text{ kN-s/m} \quad (6.135)$$

$$K = M\omega_n^2 = 2.77 \times 10^4 \text{ kN/m} \quad (6.136)$$

The plastic hinge is assumed to deform in an elastic-plastic behavior as shown in Figure 6.1(b). Because only SDOF system is used, the three matrices

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

involving in the force analogy method, i.e., $\mathbf{K}$, $\mathbf{K}_p$, and $\mathbf{K}_R$ are now scalar, and based on the properties of the structure, they are

$$K = \frac{3EI}{L^3}, \quad K_p = \frac{3EI}{L^2}, \quad K_R = \frac{3EI}{L} \quad (6.137)$$

Figure 6.1 SDOF System with PHL at the Base of the Column

As a numerical example, let

$$L = 0.25 \text{ m} \quad (6.138)$$

Then substituting Eqs. (6.136) and (6.138) into Eq. (6.137) gives

$$K_p = LK = 7.02 \times 10^3 \quad (6.139)$$

$$K_R = L^2 K = 1.78 \times 10^3 \quad (6.140)$$

In addition, assume that the column can take a maximum moment corresponding to a force on the mass of 1.75×10^5 kg, i.e.

$$M_s = MgL = (1.75 \times 10^5)(9.81)(0.25) = 429.5 \text{ kN-m} \quad (6.141)$$

Let the time interval be $\Delta t = 0.03$ s for a time duration of $T = 12$ s. The variances and covariances of responses of SDOF system under non-stationary excitation calculated by the proposed stochastic force analogy method (SFAM) are shown in Figure 6.2.

In order to validate the results computed by the proposed method, a Monte Carlo simulation is performed using 5000 sample ground acceleration time histories. After computing the variances and covariances on the basis of these 5000 values, the sample variances and covariances are also plotted in Figure 6.2 and

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

compared with those obtained from the proposed analytical method. Regarding to the artificial ground acceleration used in the Monte Carlo simulation method (MCSM), its generation procedure will be discussed in detail in the next chapter. Here, only a brief introduction to the ground acceleration is given. As mentioned in the former section, the ground acceleration is idealized as a non-stationary Gaussian process

$$a_g(t) = \phi(t) \cdot S(t) \quad (6.142)$$

where $\phi(t)$ represents a deterministic modulation function, and the model proposed by Shinozuka and Sato (1967) is used here:

$$\phi(t) = \frac{e^{-\alpha_g t} - e^{-\beta_g t}}{\phi(t_0)} \quad (6.143)$$

where α_g and β_g are constants and $\phi(t_0)$ is the normalizing factor which is the maximum value of $\phi(t)$ that occurs at $t = t_0$. For this example, $\alpha_g = 0.25/\text{s}$ and $\beta_g = 0.50/\text{s}$, which results in t_0 equal to 2.78s. In Eq. (6.142), $S(t)$ represents the stationary process whose spectral density is assumed to be Kanai-Tajimi spectrum (Kanai, 1957; Tajimi, 1960)

$$\Phi_{ss}(\omega) = \frac{\omega_g^4 + (2\zeta_g \omega_g \omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\zeta_g \omega_g \omega)^2} \Phi_0 \quad (6.144)$$

The auto-correlation function corresponding to this power spectral density function is (Simulescu et al., 1989):

$$\begin{aligned} R_{ss}(t_1, t_2) &= R_{ss}(t_1 - t_2) \\ &= \Phi_0 \pi \frac{\omega_g}{2\zeta_g} e^{-\zeta_g \omega_g |\tau|} \left[(1 + 4\zeta_g^2) \cos(\omega_g \tau \sqrt{1 - \zeta_g^2}) + (1 - 4\zeta_g^2) \frac{\zeta_g}{\sqrt{1 - \zeta_g^2}} \sin(\omega_g |\tau| \sqrt{1 - \zeta_g^2}) \right] \end{aligned} \quad (6.145)$$

where $\tau = t_1 - t_2$. For this example, $\Phi_0 = 0.1 \text{ m}^2/\text{s}^3$, $\omega_g = 4\pi$, and $\zeta_g = 0.64$.

In Figure 6.2, ‘MCSM’ means the Monte Carlo simulation method and ‘SFAM’ means the stochastic force analogy method, x represents relative displacement, v represents relative velocity, x'' represents inelastic displacement in the force analogy method, and q'' represents plastic rotation at the PHL. It is shown that the agreement appears to be satisfactory. The reason for discrepancy, which

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

still exists in the covariance time histories of displacement and velocity between MCSM and SFAM to some extent, maybe resulted from the assumption of Gaussian distribution for intrinsically non-Gaussian response quantities. In this respect, it is noted that the Monte Carlo simulation method does not require the Gaussian approximation.

Figure 6.2 Time Histories of Response Statistics of SDOF System under Non-Stationary Excitation

(Continues on the next page)

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

Figure 6.2 Time Histories of Response Statistics of SDOF System under Non-Stationary Excitation

(Continues on the next page)

Figure 6.2 Time Histories of Response Statistics of SDOF System under Non-Stationary Excitation

6.4 Summary

In this chapter, the force analogy method has been introduced for the first time into the stochastic dynamic analysis of inelastic structures, and the proposed method is named as the stochastic force analogy method (SFAM). As stated by Wong and Yang (1999), inelastic analysis with the force analogy method is performed based on varying the structural displacement field, which differs from the conventional method of changing stiffness. Consequently, the algorithm is very time efficient due to only initial stiffness being used. This merit is still remained in the stochastic dynamic analysis of inelastic structures. In addition, the force analogy method can achieve high numerical accuracy when incorporating the use of a state space numerical integration method in dynamic analysis.

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

The entire calculation is dependent on a recursive equation given in Eq. (6.21), which is based on the state space method. The ground acceleration is assumed to be a non-stationary Gaussian process. It is found that the key point within each recursive step is how to calculate the covariance terms, which are related to the inelastic displacement x'' and the plastic rotation θ'' at plastic hinge location. These two terms are especially emphasized in the force analogy method and at the same time completely describe the inelastic property of inelastic structures. The calculation of the covariances relating to these two terms is based on the force analogy method. With aid of the force analogy method, all the random variables at time step $k+1$ can be expressed as the functions of the random variables at time step k , and as a result the covariances of the random variables at time step $k+1$ can be calculated with the aid of the known covariances of the random variables at time step k .

Under the assumption of the inelastic behavior based on an elastic-plastic model, the sample space of the random vector $(x_k, \dot{x}_k, \theta_k'', a_k)$ is divided into three subdomains. Within different subdomains, the expressions of the random variable θ_{k+1}'' are expressed respectively as different functions of $x_k, \dot{x}_k, \theta_k''$ and a_k . Meanwhile, the boundaries for the three subdomains are also expressed as the functions of $x_k, \dot{x}_k, \theta_k''$ and a_k . Because of the fact that a_k , which represents the ground acceleration at the time step k , has no influence on the other three random variables of $x_k, \dot{x}_k$ and θ_k'' , which are the structural responses at the time step k caused by the excitation of a_{k-1} , a_k is assumed to be independent with $x_k, \dot{x}_k$ and θ_k'' . For the problem to be tractable, a_k is assumed as a Gaussian random variable, and meanwhile $x_k, \dot{x}_k$ and θ_k'' are assumed to be jointly Gaussian distributed. Based on these preparations, the covariances of all the random variables at time step $k+1$ can be evaluated, and the recursive calculation can be advanced to the next time step until the entire time history of the dynamic analysis is performed.

The proposed stochastic force analogy method (SFAM) produces a covariance function, and hence a variance function, of the relative displacement, relative velocity, inelastic displacement, and plastic rotation of inelastic structures under stochastic excitation. These covariance and variance functions play an

Chapter 6 Stochastic Dynamic Analysis of Inelastic Structures with the Force Analogy Method

important role in estimating structural safety and reliability, which is actually the ultimate purpose of performing such a stochastic analysis.

The covariance functions developed on the basis of the present analysis show a reasonable agreement with those constructed by means of Monte Carlo simulation. Therefore, it can be concluded that the proposed stochastic force analogy method, which is an analytical method, gives reasonable results. It is noted, however, that some discrepancy between the two methods still exist. The reason may lie in the assumption of Gaussian distribution of structural responses, for which the premise is that the structure is under small excitations. In this study, however, the structure is assumed to be subject to a severe ground excitation, under which the structural responses may not obey Gaussian distribution. Therefore, the assumption of Gaussian distribution for the structural responses under severe ground excitations will inevitably lead to error to some extent. Nevertheless, it is shown that the error is still within the acceptable range according to the results plotted in Figure 6.2.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End
Offsets and Deformable Panel Zones

**CHAPTER 7 SIMULATION AND RELIABILITY STUDY ON
INELASTIC STRUCTURES WITH RIGID END OFFSETS AND
DEFORMABLE PANEL ZONES**

7.1 General Statement

In the previous chapter a new analytical method for stochastic dynamic analysis of inelastic structures based on the force analogy method is proposed. However, due to the complexity of the entire analysis process, so far the proposed method is only applicable to single-degree of freedom system, and its further extension to multi-degree of freedom system should be an important task in the future.

On the other hand, it is well known that the Monte Carlo simulation method in principle is a universal method for stochastic analysis of various types of nonlinear problems, irrespective of the size of degree of freedom (Schueller, 1998). Therefore, in this chapter the Monte Carlo simulation method is used to further investigate the effects of rigid end offsets and deformable panel zones on dynamic behaviors of inelastic structures with multi-degree of freedom under random excitations.

First, the method of how to generate the artificial ground acceleration is presented. In this study, the ground acceleration is idealized as a non-stationary Gaussian process, and several samples of the ground acceleration are shown. Then the Monte Carlo simulation is performed on inelastic structures subjected to random excitations, and the force analogy method is again used to analyze the inelastic dynamic behaviors of the structures. The time histories and the maximum values of standard deviation of responses as well as the time histories and the maximum values of mean and standard deviation of energy dissipation are presented.

The Monte Carlo simulation is first performed on inelastic structures with the model of rigid end offsets. Comparison of results is made between the structures with no end offset (NEO) and with rigid end offsets (REO) in order to show the possible influences of rigid end offsets on dynamic behaviors of inelastic structures in a statistic sense.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Next, the Monte Carlo simulation is performed on inelastic structures with the model of deformable panel zones. Comparison of results is made among the structures with different yield moments in the rotational springs in order to demonstrate the effects of elastic and inelastic deformation of panel zones on dynamic behaviors of inelastic structures in a statistic sense.

Finally, as mentioned in the previous chapter, the goal of such stochastic study is to estimate structural safety and reliability. Therefore, at the end of this chapter, with the aid of the data obtained by the Monte Carlo simulation method, a new preliminary effort to the reliability analysis on inelastic structures with the model of rigid end offsets and deformable panel zones is performed based on the concept of plastic energy at each plastic hinge to evaluate the probability of failure of individual plastic hinge location (PHL).

7.2 Simulation of the Earthquake Ground Acceleration

7.2.1 The Random Model of Ground Acceleration

In this part of research, the ground acceleration $a_g(t)$ is considered as a random process, which is modeled as a non-stationary Gaussian process with zero mean. It is constructed by multiplying a stationary random process $S(t)$ by a deterministic temporal modulation function $\phi(t)$ (Simulescu et al., 1989)

$$a_g(t) = \phi(t) \cdot S(t) \quad (7.1)$$

Here, the deterministic temporal modulation function $\phi(t)$ is to account for the non-stationary characteristics of earthquake excitation, and the formulation proposed by Shinozuka and Sato (1967) is used here as

$$\phi(t) = \frac{e^{-\alpha_g t} - e^{-\beta_g t}}{\phi(t_0)} \quad (7.2)$$

where α_g and β_g are constants, $\phi(t_0)$ is the normalizing factor which is the maximum value of $\phi(t)$ that occurs at $t = t_0$.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

The stationary process $S(t)$ is assumed to have zero mean and possesses the two-sided power spectral density function known as the Kanai-Tajimi spectrum (Kanai, 1957; Tajimi, 1960)

$$\Phi_{ss}(\omega) = \frac{\omega_g^4 + (2\zeta_g \omega_g \omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\zeta_g \omega_g \omega)^2} \Phi_0 \quad (7.3)$$

where Φ_0 represents the spectrum level (normalized to unit mass) of the broad-band excitation at the base, ω_g represents the natural frequency of the ground, ζ_g represents the ratio of damping to the critical damping of the ground, respectively.

Figure 7.1 Deterministic Temporal Modulation Function

Figure 7.2 Kanai-Tajimi Power Spectrum

In this chapter, the parameters in the random model of ground acceleration are determined in the following values. The deterministic temporal modulation function $\phi(t)$ defined in Eq. (7.2) has the parametric values $\alpha_g = 0.25$ /s and

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

$\beta_g = 0.50$ /s, which results in t_0 equal to 2.78s. The deterministic temporal modulation function is then plotted in Figure 7.1. The parameters of the Kanai-Tajimi spectrum are chosen so that $\Phi_0 = 0.1 \text{ m}^2/\text{s}^3$, $\omega_g = 4\pi$, and $\zeta_g = 0.64$. The spectrum using these parameter values is plotted in Figure 7.2. The reason for choosing $\Phi_0 = 0.1 \text{ m}^2/\text{s}^3$ is that the artificial ground accelerations generated based on this value are comparable with the El-Centro earthquake with a scale factor of 1.8. This represents a severe earthquake for the structural model, i.e., a six-story moment resisting steel frame to be used in this chapter as a numerical example. There are certainly other methods of selecting a more reasonable value for Φ_0 , which represents the spectrum level, but this is beyond the scope of this research.

7.2.2 Generation of Artificial Ground Acceleration

Once the parameters in the random model of ground acceleration are determined, the next step is to generate sample functions of the process. For this purpose, the method of simulating the random process as a sum of cosine functions proposed by Shinozuka (1972) is employed.

(1) Stationary Process

First consider the stationary case. The sample functions of the process can be expressed as

$$x(t) = \sum_{k=1}^N A_k \cos(\omega_k t + \varphi_k) \quad (7.4)$$

where N is a positive integer; φ_k ($k = 1, 2, \dots, N$) are independent and identically distributed random phase angles with a uniform density function $1/2\pi$ over the interval $[0, 2\pi]$, i.e., φ_k are uniform random numbers distributed in the region $[0, 2\pi]$; and ω_k ($k = 1, 2, \dots, N$) are the frequencies within each term of the series. Furthermore,

$$\omega_k = \omega_l + \left(k - \frac{1}{2}\right) \Delta\omega \quad (7.5)$$

$$A_k = \sqrt{2G_{XX}(\omega_k) \Delta\omega} \quad (7.6)$$

$$\Delta\omega = (\omega_r - \omega_l) / N \quad (7.7)$$

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

where $G_{xx}(\omega)$ is the one-sided power spectral density function of the random process to be simulated (i.e., target spectral density function), $\Delta\omega$ is the frequency increment, and ω_l and ω_r are frequency bounds such that $G_{xx}(\omega)$ have significant values in the region $\omega_l < \omega < \omega_r$. Let $\Phi_{xx}(\omega)$ represents the two-sided power spectral density function so that $G_{xx}(\omega)$ and $\Phi_{xx}(\omega)$ have the following relationship

$$G_{xx}(\omega) = 2\Phi_{xx}(\omega) \quad (7.8)$$

It can be proven that based on the central limit theorem, the process simulated by Eq. (7.4) tends to a Gaussian and ergodic random process as $N \rightarrow \infty$ (Nigam and Narayanan, 1994). Furthermore, the autocorrelation function and the power spectral density will converge to the target autocorrelation and target spectral density of the random process when $N \rightarrow \infty$ at the rate of $1/\sqrt{N}$, respectively.

(2) Non-Stationary Process

In this research, the non-stationary portion of the process is represented by the deterministic temporal modulation function $\phi(t)$ and it can be simply added in the above generating procedure of sample functions of the stationary process.

It can be noticed that if the deterministic modulation function is only the function of time, the sample functions $x(t)$ of the non-stationary process can be obtained by simply multiplying the sample functions of the stationary process, which are simulated by spectral density $\Phi_{xx}(\omega)$, by $\phi(t)$ (Nigam and Narayanan, 1994). The procedure is shown in the following:

$$\begin{aligned} x(t) &= \sum_{k=1}^N \sqrt{\phi^2(t, \omega)} A_k \cos(\omega_k t + \varphi_k) \\ &= \sum_{k=1}^N \sqrt{\phi^2(t)} A_k \cos(\omega_k t + \varphi_k) \\ &= \sum_{k=1}^N \phi(t) A_k \cos(\omega_k t + \varphi_k) \\ &= \phi(t) \sum_{k=1}^N A_k \cos(\omega_k t + \varphi_k) \end{aligned} \quad (7.9)$$

Thus, the sample functions of a non-stationary process are obtained.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

(3) Simulation of Random Process Using Fast Fourier Transform (FFT)

It is appropriate for the method of sum of cosine functions to simulate the stationary processes having various spectral density functions. However, this method will involve large amounts of cosine function operations, and this type of operation is rather time consuming on the computer (Zhu, 1992). Therefore, in order to improve the speed of calculation, Shinozuka (1974) and other researchers introduced Fast Fourier Transform (FFT) into the method of sum of cosine functions. The procedure can be shown in the following.

Let the sample points be $x(n\Delta t)$, where Δt is time interval, then

$$\begin{aligned} x(n\Delta t) &= \sum_{k=1}^N A_k \cos(\omega_k n\Delta t + \varphi_k) \\ &= \text{Re} \left\{ \sum_{k=1}^N A_k e^{i\varphi_k} e^{i\omega_k n\Delta t} \right\} \end{aligned} \quad (7.10)$$

Let $N = 2^m$, and $\omega_l = 0$, $\omega_r = 2\pi/\Delta t$, $\Delta\omega = \omega_r/N$, $\omega_k = \omega_l + (k-1)\Delta\omega$ ($k = 1, 2, \dots, N$), then

$$\begin{aligned} x(n\Delta t) &= \text{Re} \left\{ \sum_{k=1}^N A_k e^{i\varphi_k} e^{i2\pi(k-1)n/N} \right\} \\ &= \text{Re} \left\{ \sum_{k=0}^{N-1} A_{k+1} e^{i\varphi_{k+1}} e^{i2\pi kn/N} \right\} \\ &= \text{Re} \left\{ \sum_{k=0}^{N-1} C_k e^{i2\pi kn/N} \right\} \\ &= \text{Re} \{ \text{DFT}(C_k) \} \end{aligned} \quad (7.11)$$

where

$$\begin{aligned} C_k &= A_{k+1} e^{i\varphi_{k+1}} = A_{k+1} (\cos \varphi_{k+1} + i \sin \varphi_{k+1}) \\ &\quad (k = 0, 1, 2, \dots, N-1) \end{aligned} \quad (7.12)$$

Here, ‘DFT’ represents discrete Fourier transform. In the process of simulation, first calculating C_k , and then obtaining $x(n\Delta t)$ through FFT.

(4) Numerical Examples for the Earthquake Ground Acceleration

In this study, the Monte Carlo simulation is performed using 500 sample ground acceleration histories. For this research, 500 samples represent a reasonable number to achieve the expected accuracy, and at the same time satisfy reasonable

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

computing time and size. Selected samples of the earthquake ground acceleration using the parameters mentioned above, and the corresponding ground velocity and ground displacement are shown in the following figures, i.e., Figures 7.3 to 7.6.

Figure 7.3 Simulated Earthquake Ground Motion Time History-Sample 1

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.4 Simulated Earthquake Ground Motion Time History-Sample 2

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.5 Simulated Earthquake Ground Motion Time History-Sample 3

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.6 Simulated Earthquake Ground Motion Time History-Sample 4

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

7.3 Monte Carlo Simulation Study on Inelastic Structures with Rigid End Offsets Using the Force Analogy Method

To demonstrate the Monte Carlo simulation study on inelastic structures with rigid end offsets with the aid of the force analogy method, the model of a six-story moment resisting steel frame with rigid end offsets used in Chapter 4 (see Figure 4.11 and Figure 4.12) is again selected as an example. The objective is to investigate the effect of rigid end offsets on the global and local response of inelastic structure in a statistic sense through analyzing the differences in the statistics (such as mean and standard deviation) of response between multi-degree of freedom structures with no end offset and with rigid end offsets under non-stationary random earthquake excitation. The structural properties remain the same as those used in Chapter 4.

The Monte Carlo simulation process is performed based on 500 samples. The procedure of generating the ground acceleration has been presented in the above section. After generating 500 random earthquake excitation samples, with the aid of the analysis procedure based on the force analogy method presented in Chapter 3 and the structural model presented in Chapter 4, the 500 samples of structural response can be obtained. Based on these response samples, the statistics of response, such as the mean and the variance can be calculated as

$$\bar{E}[x(t)] = \frac{1}{N} \sum_{i=1}^N x_i(t) \quad (7.13)$$

and

$$\bar{V}[x(t)] = \frac{1}{N-1} \sum_{i=1}^N \left[(x_i(t) - \bar{E}[x(t)])^2 \right] \quad (7.14)$$

where N is the number of samples, $x_i(t)$ is the i th response sample, $\bar{E}[x(t)]$ is the mean of the response samples, and $\bar{V}[x(t)]$ is the variance of the response samples. According to $\bar{V}[x(t)]$, the standard deviation (STD) of the response samples can be given as

$$\bar{S}[x(t)] = \sqrt{\bar{V}[x(t)]} \quad (7.15)$$

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Since the ground acceleration in this chapter is assumed as a non-stationary Gaussian process with zero mean, the mean of the responses, such as displacement, velocity, acceleration, and plastic rotation at individual plastic hinge location (PHL), are approximated to be zero. Therefore, in this study only the standard deviations (STD) of these responses are shown in the corresponding tables and figures. However, for the energy dissipation, such as kinetic energy, strain energy, damping energy, plastic energy, input energy, and plastic energy at individual PHL, their mean values will not be zero, and therefore both the mean and standard deviation of energy dissipation are shown in the corresponding tables and figures.

Figures 7.7 to 7.9 compare the time histories of the standard deviation of the global response, i.e., displacement, velocity, and acceleration at each floor, between the structures with no end offset (NEO) and rigid end offsets (REO). Figure 7.10 compares the time histories of the mean and the standard deviation of the energy dissipation, i.e., kinetic energy, strain energy, damping energy, plastic energy, and input energy, between the structures with NEO and REO. The maximum values of the standard deviation of global responses and the maximum values of the mean and the standard deviation of the energy dissipation for both cases are summarized in Table 7.1 for convenience in making comparisons.

The analysis of the results presented in Figures 7.7 to 7.10 and Table 7.1 shows that the standard deviation of the global response is generally smaller for the structure with REO than that with NEO, whereas the mean and the standard deviation of the energy dissipation are generally larger for the structure with REO than that with NEO. The result is consistent with the deterministic one obtained in Chapter 4. Nevertheless it is also shown that the differences in standard deviation of the global response and the differences in mean and the standard deviation of the energy dissipation between the two cases are generally small, except for the standard deviation of velocity at the 2nd floor, the standard deviation of acceleration at the roof floor, and the mean of plastic energy, for which case the difference is beyond 10%. Therefore, it seems reasonable to conclude that as long as the evaluation of the global responses and the global energy dissipation is in a statistic sense, the effect of rigid end offsets on these global responses can be ignored.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

In terms of local responses, Figure 7.11 compares the time histories of the standard deviation of plastic rotation at selected plastic hinge locations (PHLs) between the structures with no end offset (NEO) and rigid end offsets (REO). Figure 7.12 compares the time histories of the mean and the standard deviation of the plastic energy dissipation at selected PHLs between the structures with NEO and REO. Meanwhile, Table 7.2 summarizes the maximum values of the standard deviation of plastic rotation at individual PHL and the maximum values of the mean and the standard deviation of plastic energy at individual PHL for both cases of NEO and REO.

As shown in Table 2, it is implied that for relative flexible structures more force will be allocated on the roots of the structures, while for relative rigid ones more force will be allocated on the up-floors. Larger standard deviation of plastic rotation occurs in the roots of columns on the first floor in structures with NEO accompanied by larger mean and standard deviation of the plastic energy dissipation than in structures with REO. On the other hand, larger standard deviation of plastic rotation occur in the beams in structures with REO accompanied by larger mean and standard deviation of the plastic energy dissipation than in structures with NEO, with an apparent exception for beams at the roof level. The result is also generally consistent with the deterministic one obtained in Chapter 4. At the same time, it is found that many statistics of local response at individual PHL exhibits differences between the structures with NEO and REO beyond 10%.

Furthermore, it is noted that the larger discrepancy of the standard deviation of plastic rotation and the mean and the standard deviation of the plastic energy dissipation at individual PHL between the two cases NEO and REO mainly concentrates at the beams at the 2nd, 3rd, 4th and roof levels. The maximum percentage difference in the standard deviation of plastic rotation is 18.9% at PHLs #7 and #8, and the maximum percentage difference in the mean of plastic energy is 55.7% at PHL #36, while the maximum percentage difference in the standard deviation of plastic energy is 46.4% at PHL #36. This implies that although the statistics of global response is approximately equal for both the structure with NEO and that with REO, the statistics of the local response can be different by over 50%. Therefore, it can be concluded that the influence of rigid end offsets on the local

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

response, such as plastic rotation and plastic energy at individual PHL, cannot be disregarded in a statistic sense, especially for plastic energy at individual PHL which will be used as a calibration to investigate the probability of failure at each plastic hinge location in the following section. In addition, on the basis of the deterministic study presented in Chapter 4, the research based on the Monte Carlo simulation method presented in this chapter further demonstrates that only considering the global response without carefully studying the local response in assessing structural damage is absolutely insufficient and may lead to wrong conclusions.

Table 7.1 Maximum Mean and Standard Deviation of Global Response and Energy Dissipation of MDOF Structure for Cases NEO and REO

	NEO		REO		Percent Difference	
Displacement	Standard Deviation (STD)					
2nd	5.53		5.17		-7.0%	
3rd	10.53		9.98		-5.5%	
4th	14.46		13.59		-6.4%	
5th	17.52		16.27		-7.7%	
6th	20.21		18.68		-8.2%	
Roof	22.37		20.58		-8.7%	
Velocity (cm/s)	Standard Deviation (STD)					
2nd	20.8		18.1		-14.9%	
3rd	32.0		31.7		-0.9%	
4th	42.1		40.0		-5.3%	
5th	50.1		46.6		-7.5%	
6th	56.4		54.0		-4.4%	
Roof	64.1		62.8		-2.1%	
Acceleration (g)	Standard Deviation (STD)					
2nd	0.339		0.345		1.7%	
3rd	0.322		0.320		-0.6%	
4th	0.324		0.328		1.2%	
5th	0.343		0.333		-3.0%	
6th	0.337		0.336		-0.3%	
Roof	0.317		0.358		11.5%	
Energy (kJ)	Mean			Standard Deviation (STD)		
	NEO	REO	Percent Difference	NEO	REO	Percent Difference
<i>KE</i>	379	412	8.0%	396	429	7.7%
<i>SE</i>	199	193	-3.1%	108	111	2.7%
<i>DE</i>	1913	1815	-5.4%	318	311	-2.3%
<i>PE</i>	4368	4912	11.1%	1386	1537	9.8%
<i>IE</i>	6279	6705	6.4%	1667	1783	6.5%

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.7 Standard Deviation of Displacement per Floor of MDOF Structure for Cases NEO and REO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.8 Standard Deviation of Velocity per Floor of MDOF Structure for Cases NEO and REO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.9 Standard Deviation of Acceleration per Floor of MDOF Structure for Cases NEO and REO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.10 Mean and Standard Deviation of Energy Dissipation of MDOF Structure for Cases NEO and REO

**Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End
Offsets and Deformable Panel Zones**

**Table 7.2 Maximum Mean and Standard Deviation of Local Response and
Energy Dissipation of MDOF Structure for Cases NEO and REO**

PHL #	Plastic Rotation (rad)			Plastic Energy (kJ)					
	Standard Deviation (STD)			Mean			Standard Deviation (STD)		
	NEO	REO	Percent Difference	NEO	REO	Percent Difference	NEO	REO	Percent Difference
1	0.0127	0.0123	-3.3%	139.4	126.6	-10.1%	57.3	54.0	-6.1%
2	0.0128	0.0126	-1.6%	332.8	299.5	-11.1%	122.9	115.6	-6.3%
3	0.0128	0.0126	-1.6%	332.8	299.5	-11.1%	122.9	115.6	-6.3%
4	0.0127	0.0123	-3.3%	139.4	126.6	-10.1%	57.3	54.0	-6.1%
5	0.0116	0.0142	18.3%	137.0	211.8	35.3%	73.2	94.9	22.9%
6	0.0116	0.0142	18.3%	143.7	218.3	34.2%	75.0	96.4	22.2%
7	0.0116	0.0143	18.9%	146.1	226.3	35.4%	76.1	99.1	23.2%
8	0.0116	0.0143	18.9%	146.1	226.4	35.5%	76.3	99.1	23.0%
9	0.0116	0.0142	18.3%	143.7	218.2	34.1%	74.9	96.4	22.3%
10	0.0116	0.0142	18.3%	136.9	212.0	35.4%	73.4	94.9	22.7%
11	0.0098	0.0116	15.5%	105.8	130.4	18.9%	57.4	67.2	14.6%
12	0.0099	0.0116	14.7%	115.8	136.0	14.9%	59.6	68.3	12.7%
13	0.0099	0.0117	15.4%	117.6	141.0	16.6%	60.3	70.0	13.9%
14	0.0099	0.0117	15.4%	117.6	141.1	16.7%	60.3	70.1	14.0%
15	0.0099	0.0116	14.7%	115.8	135.8	14.7%	59.5	68.3	12.9%
16	0.0098	0.0116	15.5%	105.7	130.6	19.1%	57.4	67.2	14.6%
17	0.0076	0.0089	14.6%	61.0	62.9	3.0%	36.7	36.1	-1.7%
18	0.0077	0.0089	13.5%	68.1	66.6	-2.3%	38.7	37.1	-4.3%
19	0.0077	0.0089	13.5%	69.0	69.4	0.6%	39.2	38.1	-2.9%
20	0.0077	0.0089	13.5%	69.2	69.4	0.3%	39.2	38.1	-2.9%
21	0.0077	0.0089	13.5%	67.9	66.6	-2.0%	38.7	37.1	-4.3%
22	0.0076	0.0089	14.6%	61.3	63.0	2.7%	36.8	36.1	-1.9%
23	0.0084	0.0092	8.7%	122.4	137.7	11.1%	40.9	43.7	6.4%
24	0.0085	0.0092	7.6%	128.5	140.7	8.7%	41.5	43.8	5.3%
25	0.0085	0.0093	8.6%	129.7	145.1	10.6%	41.9	45.0	6.9%
26	0.0085	0.0093	8.6%	129.7	145.1	10.6%	41.9	44.8	6.5%
27	0.0085	0.0092	7.6%	128.5	140.7	8.7%	41.6	44.0	5.5%
28	0.0084	0.0092	8.7%	122.3	137.7	11.2%	40.7	43.4	6.2%
29	0.0085	0.0090	5.6%	90.0	93.1	3.3%	33.9	35.5	4.5%
30	0.0085	0.0090	5.6%	93.3	94.5	1.3%	34.4	35.8	3.9%
31	0.0085	0.0090	5.6%	93.9	97.1	3.3%	34.7	36.5	4.9%
32	0.0085	0.0090	5.6%	93.9	96.9	3.1%	34.6	36.5	5.2%
33	0.0085	0.0090	5.6%	93.3	94.6	1.4%	34.5	35.8	3.6%
34	0.0085	0.0090	5.6%	90.0	92.9	3.1%	33.8	35.5	4.8%
35	0.0086	0.0081	-6.2%	29.3	19.2	-52.6%	16.1	11.1	-45.0%
36	0.0086	0.0081	-6.2%	30.2	19.4	-55.7%	16.4	11.2	-46.4%
37	0.0086	0.0081	-6.2%	30.6	20.2	-51.5%	16.6	11.5	-44.3%
38	0.0086	0.0081	-6.2%	30.4	20.0	-52.0%	16.4	11.5	-42.6%
39	0.0086	0.0081	-6.2%	30.3	19.6	-54.6%	16.5	11.3	-46.0%
40	0.0086	0.0081	-6.2%	29.0	19.0	-52.6%	15.9	11.1	-43.2%

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.11 Standard Deviation of Plastic Rotation of MDOF Structure at Selected PHLs for Cases NEO and REO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.12 Mean and Standard Deviation of Plastic Energy Dissipation of MDOF Structure at Selected PHLs for Cases NEO and REO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

7.4 Monte Carlo Simulation Study on Inelastic Structures with Deformable Panel Zones Using the Force Analogy Method

To demonstrate the Monte Carlo simulation on inelastic structures with deformable panel zones with the aid of the force analogy method, the model of a six-story moment resisting steel frame with deformable panel zones used in Chapter 5 (see Figure 4.11 and Figure 5.7) is again selected as an example. The objective is to investigate the effect of elastic and inelastic deformation of panel zones on the global and local responses of inelastic structure in a statistic sense through demonstrating the differences in the statistics (such as mean and standard deviation) of response of multi-degree of freedom structure under non-stationary random earthquake excitation with different yield moments in the rotational springs.

The main structural properties remain the same as those in Chapter 5. In this part of study, panel zones are allowed to deform inelastically. The rotational spring stiffness is chosen to be $k_s = 1.55 \times 10^6$ kN-m/rad, which represents a stiff panel zone. Meanwhile, four values of yield moments in the rotational springs are considered: (1) $M_s = 1,695$ kN-m; (2) $M_s = 3,050$ kN-m; (3) $M_s = 5,875$ kN-m; and (4) $M_s = 11,000$ kN-m, and consequently plastic hinges can take place in either beams or beam-column joints.

The Monte Carlo simulation process is performed based on 500 samples. The procedure of generating the ground acceleration has been presented in Section 7.2. After generating 500 random earthquake excitation samples, with the aid of the analysis procedure based on the force analogy method presented in Chapter 3 and the structural model presented in Chapter 5, the 500 samples of structural response are obtained. Based on these response samples, the statistics of the structural response, such as the mean, variance, and standard deviation can be calculated with the aid of Eqs. (7.13), (7.14), and (7.15), respectively.

Since the ground acceleration in this chapter is assumed to be a non-stationary Gaussian process with zero mean, the mean of the responses, such as roof displacement, roof velocity, roof acceleration, and plastic rotation at individual plastic hinge location (PHL), are approximated to be zero. Therefore, in this study only the standard deviations of these responses are presented in the corresponding

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

tables and figures. However, for the energy dissipation, such as kinetic energy, strain energy, damping energy, plastic energy, input energy, and plastic energy at individual PHL, their mean values will not be zero, and therefore both the mean and standard deviation of energy dissipation are shown in the corresponding tables and figures.

Figure 7.14 compares the time histories of the standard deviation of global response, i.e., roof displacement, roof velocity, and roof acceleration, among the structures with four cases of yield moment in the rotational springs. Figure 7.15 compares the time histories of the mean of energy dissipation, i.e., kinetic energy, strain energy, damping energy, plastic energy, and input energy, among the structures with four cases of yield moment in the rotational springs. Finally, Figure 7.16 compares the time histories of the standard deviation of energy dissipation among the structures with four cases of yield moment in the rotational springs. In addition, the maximum values of the standard deviation of global response, i.e., displacement, velocity, and acceleration at each floor and the maximum values of the mean and standard deviation of energy dissipation for the four cases of yield moment in the rotational springs are summarized in Table 7.3. The maximum values are also plotted in Figure 7.13 in order to present graphically the increasing and decreasing trends in the variation of yield moment in the rotational springs.

The analysis of the results presented in Figures 7.13 to 7.16 and Table 7.3 shows that as yield moments in the rotational springs increase from Case 1 to Case 4, the standard deviation of the global response, i.e., displacement, velocity, and acceleration, the mean and the standard deviation of the energy dissipation also increase for most cases. The results are generally consistent with the deterministic ones obtained in Chapter 5. It is shown that in terms of global responses, rotational springs with smaller yield moments tend to give smaller responses. In terms of energy, it can be noted that larger mean is accompanied by larger standard deviation. This means that larger values tend to be accompanied by high uncertainty. Therefore, it is important to evaluate seismic responses of inelastic structures with different yield moments in the rotational springs in a statistic sense. In addition, it is indicated that excessive conservatism on designing the yield capacity of panel zones

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

will lead to higher responses. How to balance the yield capacity of panel zones with the global responses is worth to be investigated carefully in the future.

Figure 7.17 summarizes the maximum values of the standard deviation of plastic rotation at individual PHL and the maximum values of the mean and the standard deviation of plastic energy at individual PHL for the four cases of yield moment in the rotational springs. The maximum values are also presented in Figure 7.18, Figure 7.19, and Figure 7.20 in order to present graphically the increasing and decreasing trends in the variation of yield moment in the rotational springs. At the same time, Figure 7.21 compares the time histories of the standard deviation of plastic rotation at selected PHLs among the structures with four cases of yield moment in the rotational springs. Figure 7.22 compares the time histories of the mean of the plastic energy dissipation at selected PHLs among the structures with four cases of yield moment in the rotational springs. Figure 7.23 compares the time histories of the standard deviation of the plastic energy dissipation at selected PHLs among the structures with four cases of yield moment in the rotational springs. Finally, Figure 7.24 shows typical $M - \theta''$ curve samples of some plastic hinges.

In general, it is indicated that as the yield moments in the rotational springs increase from Case1 to Case 4, the local damages have been shifted from beam-column joints to beams and columns. The result is also generally consistent with the deterministic one obtained in Chapter 5. The following is a concrete analysis. For joints, as the yield moment in the rotational springs increases from Case 1 to Case 4, the standard deviation of plastic rotation as well as the mean of plastic energy decrease for most cases, whereas the standard deviation of plastic energy increase for most cases. This implies that for plastic energy at PHLs in joints, large values do not necessarily mean high uncertainty, i.e., the divergence of larger plastic energy is small. Meanwhile, for beams and columns, as the yield moment in the rotational springs increases from Case 1 to Case 4, the standard deviation of plastic rotation as well as the mean and the standard deviation of plastic energy increase for most cases. It implies that for plastic energy at PHLs in beams and column, large values are usually companied by high uncertainty. Therefore the divergence of larger plastic energy for beams and columns should be noticed.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

The statistic analysis on the local responses further demonstrates that one way to minimize damage at the base of the columns is to reduce the yield capacity of the beam-column joints. However, excessive reduction to the yield capacity of the joints will lead to other problems, such as severe damage in panel zones, brittle fracture in connections, and so on. Therefore, it is significant to develop an optimal seismic design method for inelastic structures in order to achieve a better balance.

Table 7.3 Maximum Mean and Standard Deviation of Global Response and Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

	Case 1		Case 2		Case 3		Case 4	
M_s (kN-m)	1695		3050		5875		11000	
Displacement (cm)	Standard Deviation (STD)							
2nd	3.12		4.04		4.98		5.20	
3rd	7.74		8.74		9.98		10.03	
4th	12.74		13.38		14.04		13.85	
5th	17.15		17.03		17.14		17.03	
6th	20.32		19.40		19.53		19.98	
Roof	22.18		20.80		21.38		22.42	
Velocity (cm/s)	Standard Deviation (STD)							
2nd	13.5		17.0		20.9		23.4	
3rd	25.4		28.7		32.8		33.9	
4th	38.0		41.3		45.1		45.7	
5th	46.3		49.0		52.1		54.0	
6th	49.0		52.7		57.6		61.5	
Roof	52.1		56.6		65.1		70.0	
Acceleration (g)	Standard Deviation (STD)							
2nd	0.324		0.349		0.360		0.356	
3rd	0.275		0.307		0.336		0.336	
4th	0.269		0.309		0.335		0.338	
5th	0.263		0.302		0.332		0.343	
6th	0.258		0.306		0.341		0.344	
Roof	0.215		0.290		0.350		0.361	
Energy (kJ)	Mean	STD	Mean	STD	Mean	STD	Mean	STD
KE	234	242	305	313	393	403	451	445
SE	53	23	112	53	218	124	269	168
DE	1139	267	1441	273	2019	338	2248	404
PE	3501	736	4060	1043	4163	1418	4227	1589
IE	4759	1034	5601	1331	6191	1716	6423	1918

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.13 Maximum Mean and Standard Deviation of Global Response and Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.14 Standard Deviation of Global Response of MDOF Structure with Different Yield Moments of Rotational Springs
with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.15 Mean of Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.16 Standard Deviation of Energy Dissipation of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Figure 7.17 Maximum STD of Plastic Rotation and Maximum Mean and STD of Plastic Energy at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.18 Maximum STD of Plastic Rotation at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.19 Maximum Mean of Plastic Energy at each PHL with Different Yield Moments of Rotational Springs with Stiffness $= 1.55 \times 10^6$ kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.20 Maximum STD of Plastic Energy at each PHL with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.21 Standard Deviation of Plastic Rotation of MDOF Structure at Selected PHLs with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.22 Mean of Plastic Energy Dissipation of MDOF Structure at Selected PHLs with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.23 Standard Deviation of Plastic Energy Dissipation of MDOF Structure at Selected PHLs with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.24 Typical $M - \theta''$ Curve Samples of Some Plastic Hinges

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

7.5 Reliability Analysis of Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

In this section, a preliminary study on the reliability analysis on inelastic structures with rigid end offsets and deformable panel zones is performed in order to further investigate the effects of rigid end offsets and deformable panel zones on the dynamic behaviors of inelastic structures under non-stationary earthquake excitation. The objective of reliability analysis is to find out the probability of failure of individual plastic hinge location (PHL), which means the excessiveness of local damage occurring at any of the plastic hinge locations under earthquake excitations such that repair work is necessary. The probability of failure of individual PHL is calculated based on the concept of plastic energy dissipation at the corresponding locations, which is a new effort in the field of structural damage assessment.

To evaluate the structural failure two failure criteria are available, they are ductility-based one and energy-based one. Ductility-based failure criterion is conventional. In this criterion, the displacement ductility factor, which is the ratio of maximum displacement to the yield displacement of the structure, is often used as a tool to evaluate the degree of nonlinearity and damage. It gives

$$\mu = \frac{u_{\max}}{u_y}$$

where μ = the displacement ductility factor; $u_{\max}$ = maximum displacement experienced by the structure; u_y = characteristic yield displacement of the structure.

The ductility-based failure criterion can be expressed as

$$\mu \leq \mu_u$$

However, some studies (Zahrah and Hall, 1984; Akiyama, 1985) indicated that using the displacement ductility factor alone is inadequate in structural assessment due to its inability to reflect the effect of non-peak inelastic cycles, i.e., the cumulative damage; whereas, the energy (including input energy and plastic energy) is considered to be able to account for all of the inelastic cycles that are sustained by the structures in a strong earthquake ground motion. As a result, the energy concept has been introduced in this field as an auxiliary or an independent

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

means to evaluate the structural damage (Park and Ang, 1985; Park et al., 1985; Uang and Bertero, 1988; Fajfar and Fischinger, 1990), and the criterion based on the energy concept is known as energy-based failure criterion. This criterion can be expressed as

$$E_h \leq E_{h,u}$$

where E_h = hysteretic energy dissipated by structures; $E_{h,u}$ = the allowed hysteretic energy of structures. Therefore, in this study the plastic energy dissipated at individual PHL is chosen as a calibration to estimate the structural damage by means of calculating the probability of failure of individual PHL.

The reliability study is based on the data obtained from the Monte Carlo simulation study with 500 samples presented in the former sections. It is noted that the structural responses in this part of study are random processes due to the random excitation. However, it can also be noticed that the plastic energy dissipated at individual PHL is a random variable. In this study, it is used as a calibration to evaluate the probability of failure of individual PHL.

In the former two sections, which discuss about the Monte Carlo simulation study, the first and second order statistics (i.e., mean and standard deviation) of the structural response have been obtained. This type of information can be utilized in the reliability study. However, due to the lack of experimental data as well as the knowledge of the probability distribution of structural response, this study considers the calculation of the probability of failure of individual PHL directly based on the histograms of 500 response samples, which is more accurate than that based on the mean and the standard deviation of the response.

On the other hand, it is assumed that the capacity of a structural system to dissipate hysteretic energy under cyclic loading is equal to its energy dissipation capacity under monotonic loading. In addition, the capacity of dissipating plastic energy at individual PHL is assumed to be equal to the ultimate energy supplied by a block of the volume corresponding to the plastic hinge. Here, the length of the plastic hinge is assumed to be related to the depth of the cross section of the corresponding structural member, while the area of the plastic hinge is assumed to be equal to the area of the cross section of the corresponding member. Therefore,

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

the capacity of the plastic energy dissipation at individual PHL, PE_{capacity} , is defined as

$$PE_{\text{capacity}} = \sigma_y \cdot \varepsilon_\mu \cdot A \cdot d \quad (7.16)$$

where σ_y represents the yield stress, ε_μ represents the ultimate strain corresponding to damage under monotonic loading, A is the cross sectional area of the member, and d equals to 0.7 times of the depth of the cross section of the member. In this study, the material is steel, thus the yield stress and the ultimate strain are assumed to be $\sigma_y = 248.2$ MPa and $\varepsilon_\mu = 0.035$, respectively.

Therefore, the limit state function reads

$$g(PE_{\text{capacity}}, PE_{\text{max}}) = PE_{\text{capacity}} - PE_{\text{max}} \quad (7.17)$$

where PE_{max} represents the maximum plastic energy dissipated at each PHL. Consequently, the probability of failure of individual PHL, P_f , is defined as

$$P_f = p\{PE_{\text{max}} > PE_{\text{capacity}}\} \quad (7.18)$$

which means the probability that damage occurs at certain plastic hinge location.

7.5.1 Reliability Analysis of Inelastic Structures with Rigid End Offsets

The model of a six-story moment resisting steel frame with rigid end offsets used in Chapter 4 (see Figure 4.11 and Figure 4.12) is again selected as an example. The result of 500 samples for the maximum plastic energy dissipated at individual PHL for the structure with no end offset (NEO) and with rigid end offsets (REO) is taken from Section 7.3. The probability of failure at individual PHL is calculated based on these 500 samples.

Figure 7.25 presents the histograms of plastic energy dissipated at some selected PHLs for the structure with NEO. Figure 7.26 shows the histograms of plastic energy dissipated at selected PHLs for structure with REO. Figure 7.27 compares the probability of failure at individual PHL between the structures with no end offset (NEO) and with rigid end offsets (REO). It is further verified the results obtained from the former calculation that more force will be allocated on the roots for relative flexible structures, while more force will be allocated on the up-floors for relative rigid ones. This means that larger plastic energy is dissipated at the roots

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.25 Histograms of Plastic Energy at Some Selected PHLs for Case NEO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.26 Histograms of Plastic Energy at Some Selected PHLs for Case REO

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.27 Probability of Failure at Individual PHL for Cases NEO and REO

of columns on the first floor with higher probability of failure for the structure with NEO, whereas larger plastic energy is dissipated at beams on the up-floors with higher probability of failure for the structure with REO. Therefore, it can be concluded that the model of rigid end offsets has certain effect on the local response and should be included in the modeling of structures.

7.5.2 Reliability Analysis of Inelastic Structures with Deformable Panel Zones

The model of a six-story moment resisting steel frame with deformable panel zones used in Chapter 5 (see Figure 4.11 and Figure 5.7) is again selected as an example. The main structural properties remain the same as those in Chapter 5. In this part of study, panel zones are allowed to deform inelastically in some cases. The rotational spring stiffness is chosen to be $k_s = 1.55 \times 10^6$ kN-m/rad, which represents a stiff panel zone. Meanwhile, four values of yield moments in the rotational springs are considered: (1) $M_s = 1,695$ kN-m; (2) $M_s = 3,050$ kN-m; (3) $M_s = 5,875$ kN-m; and (4) $M_s = 11,000$ kN-m, and consequently plastic hinges can take place either in beams or in beam-column joints.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

For beam-column joints, the capacity of the plastic energy dissipation at individual PHL, PE_{capacity} , is still based on Eq. (7.16). Since the beam-column joints are allowed to deform inelastically, it is necessary to explain the meaning of A and d appearing in Eq. (7.16) for beam-column joints in detail. As yield moments in rotational springs increase from Case 1 to Case 4, the capacity of the plastic energy dissipation at joints, PE_{capacity} , should also increase. In this study, this difference is embodied in the parameter A for joints, i.e., for different yield moments in rotational springs the parameter A for joints is bestowed on different values. For Case 1, i.e., $M_s = 1,695$ kN-m, A is chosen to be 1.3 times of the area of the cross section of the corresponding beams; for Case 2, i.e., $M_s = 3,050$ kN-m, A is chosen to be 1.5 times of the area of the cross section of the corresponding beams; for Case 3, i.e., $M_s = 5,875$ kN-m, A is chosen to be 1.8 times of the area of the cross section of the corresponding beams; and finally for Case 4, i.e., $M_s = 11,000$ kN-m, A is chosen to be 2 times of the area of the cross section of the corresponding beams. The reason of approximating the area of joints by multiplying the cross sectional area of the corresponding beams by a factor is due to the general existence of thick stiffening plates in joints. Here, for different cases of yield moment in rotational springs, the depth of the plates in joints is assumed to be different. Meanwhile for all cases of yield moments in rotational springs, the parameter d for joints is always chosen to be the depth of the cross section of the corresponding columns.

The result of 500 samples for the maximum plastic energy dissipated at individual PHL for the structures with four cases of yield moments in the rotational springs is taken from Section 7.4. The probability of failure at individual PHL is calculated based on these 500 samples.

Figures 7.28 to 7.31 show the histograms of plastic energy dissipated at selected PHLs for the structures with four cases of yield moment in rotational springs. It is found that as the yield moments in the rotational springs increase, the larger local damage has shifted from the joints to the beams and columns. As the yield moments in rotational springs increase, the probability of failure of beam-

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

column joints decreases, whereas the probability of failure of beams and columns increases. The result is consistent with the one obtained in the former chapter.

Figure 7.32 compares the probability of failure at individual PHL for the structures with four cases of yield moment in rotational springs. It is demonstrated that for the models with the two smaller yield moments in rotational springs, i.e., $M_s = 1,695$ kN-m, and $M_s = 3,050$ kN-m, the probability of failure at individual PHL are very high. In other words, it means that the models with extremely small yield moments in the panel zones will dissipate excessively large plastic energy at beam-column joints, and the structures with this type of model tend to be unsafe, as a result are not applicable in practice. Comparison of results with the two larger yield moments in the rotational springs, i.e., $M_s = 5,875$ kN-m, and $M_s = 11,000$ kN-m, in Figure 7.32 shows that excessive conservatism in the resistant capacity of beam-column joints will lead to high probability of failure in beams and columns, which agree with the conclusion presented in the former chapter. According to Figure 7.32, it appears that the model with $M_s = 5,875$ kN-m in the rotational springs is more reasonable with the relatively smaller probability of failure in all PHLs than those with the other three cases of yield moment in rotational springs. Therefore, it again implies that allowing moderate damage in beam-column joints appears to mitigate the pressure of resistance in beams and columns. Nevertheless, how to balance the damage in beam, column and joint in order to achieve an optimal design will be an important task in the future.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.28 Histograms of Plastic Energy at Selected PHLs for $M_s = 1695$ kN-m with Stiffness $= 1.55 \times 10^6$ kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.29 Histograms of Plastic Energy at Selected PHLs for $M_s = 3050$ kN-m with Stiffness $= 1.55 \times 10^6$ kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.30 Histograms of Plastic Energy at Selected PHLs for $M_s = 5875$ kN-m with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.31 Histograms of Plastic Energy at Selected PHLs for $M_s = 11000$ kN-m with Stiffness $= 1.55 \times 10^6$ kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

Figure 7.32 Probability of Failure at each PHL of MDOF Structure with Different Yield Moments of Rotational Springs with Stiffness = 1.55×10^6 kN-m/rad

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

7.6 Summary

The study of this chapter is based on the Monte Carlo simulation method using 500 sample ground acceleration time histories. The ground acceleration is assumed to be a non-stationary Gaussian random process. The objective of the study is to further investigate the effects of rigid end offsets and deformable panel zones on the dynamic behaviors of inelastic structures in a statistic sense on the basis of the deterministic study performed in the former chapters. In addition, a preliminary effort of the reliability analysis is made on the structures with rigid end offsets and deformable panel zones in terms of the probability of failure of individual PHL based on the plastic energy dissipated at each PHL. At the end of the study, several conclusions are drawn as follows:

- The standard deviation of the global response is generally smaller for the structure with REO than that with NEO, whereas the mean and standard deviation of energy dissipation are generally larger for structure with REO than that with NEO.
- For relative flexible structures more force will be allocated on the roots, while for relative rigid ones more force will be allocated on the up-floors. On the roots of columns at the first floor, larger standard deviation of plastic rotation accompanied by larger mean and standard deviation of plastic energy dissipation occur in the structure with NEO than that in the structure with REO. On the contrary, on most beams at the up-floors, larger standard deviation of plastic rotation accompanied by larger mean and standard deviation of plastic energy dissipation occur in the structure with REO than that in the structure with NEO.
- Higher probability of failure occurs at columns in the structure with NEO than that with REO, whereas higher probability of failure occurs at beams in the structure with REO than that with NEO.
- As the yield moments in the rotational springs increase, the standard deviation of global responses, and the mean and standard deviation of energy dissipation also increase for most cases.
- As the yield moments in rotational springs increase, local damages have been shifted from beam-column joints to beams and columns.

Chapter 7 Simulation and Reliability Study on Inelastic Structures with Rigid End Offsets and Deformable Panel Zones

- For joints, as the yield moments in rotational springs increase, the standard deviation of plastic rotation as well as the mean of plastic energy decrease for most cases, whereas the standard deviation of plastic energy increase for most cases.
- For both beams and columns, as the yield moments in the rotational springs increase, the standard deviation of plastic rotation as well as the mean and standard deviation of plastic energy increase for most cases.
- As the yield moments in rotational springs increase, the probability of failure of beam-column joints decreases, whereas the probability of failure of beams and columns increases.

In a statistic sense, although the differences of the global response and global energy dissipation between the structures with REO and NEO are generally small, the differences of local responses and plastic energy dissipated at individual PHL between the structures with REO and NEO are generally large. Therefore, the effect of rigid end offsets on those local responses should be taken into account carefully.

Excessive conservatism on the yield capacity of panel zones will lead to higher responses and maybe result in potential hazard in seismic resistance. However, excessive reduction in the yield capacity of joints will lead to other problems, such as severe damage in panel zones and brittle fracture in connections. Meanwhile, the study implies that allowing moderate inelastic deformation in panel zones appears to mitigate the pressure of resistance in beams and columns. Therefore, how to balance the damage in beams, columns and joints in order to achieve an optimal design will be an important task in the future.

CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

In this study, the dynamic analysis of inelastic structures is based on the force analogy method with the aid of the state space method. Entire research refers two main fields: deterministic and stochastic.

In deterministic part, first, inelastic dynamic analysis of structures based on force analogy method is extended to analyze real moment resisting frames with rigid end offsets. To account for rigid end offsets, each term in the modified stiffness matrices of the force analogy method is only multiplied by a scale factor, which depends only on the sizes of rigid end and offset lengths. Second, a simple analytical method of modeling the panel zone deformation using rotational springs is proposed. This model can simulate both shear deformation and inelastic behavior of the panel zones at the same time. The effects of rigid end offsets and both elastic and inelastic deformation of the panel zones on dynamic responses and energy dissipation of inelastic structures are investigated based on numerical studies.

In stochastic part, the force analogy method is first introduced into the stochastic dynamic analysis of inelastic structures. The proposed method is named as the stochastic force analogy method (SFAM). This method can produce a covariance function of displacement, velocity, inelastic displacement, and plastic rotation at plastic hinge locations (PHLs) of inelastic structures subjected to a non-stationary random process. Comparison between the results obtained by the proposed method and Monte Carlo simulation method shows a reasonable agreement. Then, the Monte Carlo simulation is performed on structures with models of rigid end offsets and deformable panel zones under non-stationary Gaussian random earthquake excitation. The influences of rigid end offsets and deformable panel zones on the dynamic behaviors of inelastic structures are further explored in a statistic sense. Finally, a reliability analysis about the probability of failure of the individual PHL is proposed on the structures with the models of rigid end offsets and deformable panel zones based on the concept of plastic energy dissipated at each PHL.

Chapter 8 Conclusions and Recommendations

Based on numerical study presented in this thesis, several conclusions can be drawn as follows:

- The structure with rigid end offsets (REO) generally produces larger global responses and dissipates larger energy than that with no end offset (NEO) in both deterministic and statistic senses.
- More force will be allocated on the roots for relative flexible structures, while more force will be allocated on the up-floors for relative rigid ones. In both deterministic and statistic senses, larger damage occurs on the roots of columns at the first floor with larger plastic rotation and plastic energy dissipation for structure with no end offset, while larger damage occurs in beams at the up-floors with larger plastic rotation and plastic energy dissipation for structure with rigid end offsets.
- Higher probability of failure occurs on the roots of columns at the first floor in the structure with NEO than that with REO, whereas higher probability of failure occurs at beams on the up-floors in the structure with REO than that with NEO.
- Rigid end offsets should be embodied in the structural model, and its effect on the local dynamic responses of inelastic structures should be investigated carefully regardless of studying in deterministic sense or in statistic sense.
- Structure with softer rotational springs (i.e., smaller panel zone stiffness) generally gives larger maximum displacement, velocity, and drift responses. This type of structure also produces more plastic rotation and dissipates more plastic energy at the base of column, and gives larger maximum strain energy, kinetic energy, and damping energy.
- Structure with stiffer rotational springs (i.e., larger panel zone stiffness) tends to dissipate more plastic energy and input energy, and produces more plastic rotation as well as plastic energy in most of the beams and beam-column joints.
- When the panel zones remain elastic, structure with larger panel zone stiffness tends to dissipate larger plastic energy in the beams while structure with smaller panel zone stiffness tends to dissipate larger plastic energy in the columns.

Chapter 8 Conclusions and Recommendations

- As the yield moments in rotational springs increase, both global responses and energy dissipation also increase for most cases in both deterministic and statistic senses.
- As the yield moments in rotational springs increase, local damages shift from beam-column joints to beams and columns. Both plastic rotation and plastic energy decrease for beam-column joints, whereas they increase for beams and columns in both deterministic and statistic senses.
- As the yield moments in rotational springs increase, the probability of failure of beam-column joints decreases, whereas the probability of failure of beams and columns increases.
- Excessive conservatism on the yield capacity of panel zones will lead to higher responses in structures. At the same time, excessive reduction to the yield capacity of the joints will lead to severe damage in panel zones. Allowing moderate inelastic deformation in panel zones appears to mitigate the pressure of resistance in beams and columns.
- The proposed stochastic force analogy method is shown to be a reasonable one, and can be applied to perform stochastic dynamic analysis on inelastic structures.

8.2 Recommendations

Although a detailed study about the effects of rigid end offsets and elastic and inelastic deformation of panel zones on the dynamic behavior of inelastic structures has been done in both deterministic and statistic senses, and meanwhile an inelastic stochastic dynamic analytical method based on the force analogy method is successfully proposed, there still exist some limitations in this study. Therefore in order to overcome these limitations further studies are needed

The model of structure needs to be consummated. In this study the axial force-moment interaction in the columns is ignored. The inelastic behavior of panel zone is idealized as elastic-plastic, ignoring the effect of strain-hardening and the deterioration of stiffness. In addition, geometric nonlinear effect, which is an important factor in predicting response levels and collapse vulnerability, is not

Chapter 8 Conclusions and Recommendations

considered in this study. The applicability of the force analogy method in the case of geometric nonlinear of the structures being included in the models remains to be studied in the future.

The results obtained based on the structural model with rigid end offsets are only compared with those obtained based on the structural model with no end offset. More comparisons as well as more experiments should be done to investigate the effect of rigid end offsets more comprehensively.

The model of deformable panel zones as rotational springs proposed in this study should be consummated further. In this study, the stiffness and yield moment of panel zone are assigned with constant values to the entire frame for simplicity. In order to investigate the effect of panel zone deformation in more detail, the different sizes of the members should be included in the model meanwhile the effect of elastic and inelastic deformation of panel zones on behaviors of beam-column connection should be noticed. In addition, it is significant to develop an optimal design method to balance the capacity of beam-column joints to the capacity of beams and columns in order to achieve the objective of safety as well as economy.

In this study, the evaluation to the capacity of individual plastic hinge location in terms of the concept of plastic energy dissipated at these locations is only based on the theoretical hypothesis, in the future some experiments should be done to provide a more reliable estimation. In addition, regarding to the probability distribution of plastic energy dissipated at individual plastic hinge location, experiments are also required to propose a more accurate model.

The conclusions of the deterministic part are drawn only based on the study of one SDOF system and one six-story frame. In addition, in the deterministic part only one real earthquake ground motion (El Centro) was used. In the future, more types of structures and more earthquake ground motions should be involved in the study to make the conclusion more general.

In this study, the generation of artificial ground accelerations used in the Monte Carlo simulation method is based on the random model of non-stationary Gaussian process. In the future, non-stationary non-Gaussian process, which is a more accurate description to the inherent property of earthquake ground motions, can be used as the random model of earthquake excitation.

Chapter 8 Conclusions and Recommendations

Due to the complexity of stochastic dynamic analysis of inelastic structures, the proposed stochastic force analogy method in this study is derived only based on single degree of freedom system with one plastic hinge location. Therefore, its extension to multi-degree of freedom system with multiple plastic hinge locations should be studied in the future. Furthermore, so far the production of the proposed stochastic force analogy method is just confined in the covariance function of responses, i.e., displacement, velocity, inelastic displacement, and plastic rotation at individual PHL, but how to produce the covariance function of energy dissipation is also an important task. In addition, in the deducing process of the stochastic force analogy method in this study, the jointly probability distribution of displacement, velocity, and plastic rotation at individual PHL is assumed to be jointly Gaussian distribution. However, under severe earthquake excitation the jointly probability distribution of these responses should be intrinsic non-Gaussian. This assumption is bound to result in error to some extent. Therefore, in the future the jointly probability distribution of inelastic structural responses subjected to severe earthquake excitation should be investigated carefully in order to provide more accurate results.

REFERENCES

AISC. (1997), *Seismic Provisions for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, USA.

Akiyama, H. (1985), *Earthquake Resistant Limit-State Design for Buildings*, University of Tokyo Press, Tokyo.

Ang, K. M. and Morris, G. A. (1984), "Analysis of Three-Dimensional Frames with Flexible Beam-Column Connections", *Canadian Journal of Civil Engineers*, Vol. 11, pp. 245-254.

Asano, K. and Iwan, W. D. (1984), "An Alternative Approach to the Random Response of Bilinear Hysteretic Systems", *Earthquake Engineering and Structural Dynamics*, Vol. 12, No. 2, pp. 229-236.

Astaneh, A., Nader, M. N. and Malik, L. (1989), "Cyclic Behavior of Double Angle Connections", *Journal of Structural Engineering*, ASCE, Vol. 115, No. 5, pp. 1101-1118.

Atalik, T. S. and Utku, S. (1976), "Stochastic Linearization of Multi-Degree-of-Freedom Nonlinear Systems", *Earthquake Engineering and Structural Dynamics*, Vol. 4, pp. 411-420.

Baber, T. T. (1984), "Nonzero Mean Random Vibration of Hysteretic Systems", *Journal of Engineering Mechanics*, ASCE, Vol. 110, No. 7, pp. 1036-1049.

Baber, T. T. and Noori, M. N. (1985), "Random Vibration of Degrading, Pinching Systems", *Journal of Engineering Mechanics*, ASCE, Vol. 111, No. 8, pp. 1010-1026.

References

Baber, T. T. and Wen, Y. K. (1981), "Random Vibration of Hysteretic, Degrading Systems", *Journal of Engineering Mechanics Division*, ASCE, Vol. 107, No. 6, pp. 1069-1087.

Baber, T. T. and Wen, Y. K. (1982), "Stochastic Response of Multistory Yielding Frames", *Earthquake Engineering and Structural Dynamics*, Vol. 10, pp. 403-416.

Banon, H. and Veneziano, D. (1982), "Seismic Safety of Reinforced Concrete Members and Structures", *Earthquake Engineering and Structural Dynamics*, Vol. 10, pp. 179-193.

Bayer, V. and Bucher, C. G. (1996), "A Simulation Procedure for First Passage Problems of Nonlinear Structures", *Proceedings of ASCE Speciality Conference on Probabilistic Mechanics and Structural Reliability*, D. M. Frangopol and M. D. Grigoriu (Eds.), Worcester, Massachusetts, August 7-9, pp. 816-819.

Bergman, L. A., Wojtkiewicz, S. F., Johnson, E. A. and Spencer, Jr. B. F. (1995), "Some Reflections on the Efficiency of Moment Closure Methods", *Computational Stochastic Dynamics* (Spanos, P. D. ed.), pp. 87-96.

Bertero, V. V. (1997), "Performance-Based Seismic Engineering: a Critical Review of Proposed Guidelines", *Seismic Design Methodologies for the Next Generation of Codes*, P. Fajfar and H. Krawinkler (Eds.), Balkema, Rotterdam, pp. 1-32.

Bertero, V. V. and Uang, C. M. (1992), "Issues and Future Directions in the Use of an Energy Approach for Seismic-Resistant Design of Structure", *Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings*, P. Fajfar and H. Krawinkler (Eds.), Elsevier, London, pp. 3-22.

Bouc, R. (1967), "Forced Vibration of Mechanical Systems with Hysteresis", *Proceedings of the 4th Conference on Nonlinear Oscillation*, Prague, Czechoslovakia.

References

Cai, G. Q. and Lin, Y. K. (1988), "A New Approximate Solution Technique for Randomly Excited Nonlinear Oscillators", *International Journal of Non-Linear Mechanics*, Vol. 23, No. 2/3, pp. 409-420.

Casciati, F. and Faravelli, L. (1988), "Stochastic Equivalent Linearization for 3-D Frames", *Journal of Engineering Mechanics*, ASCE, Vol. 114, No. 10, pp. 1760-1771.

Caughey, T. K. (1959), "Response of a Linear String to Random Loading", *Journal of Applied Mechanics*, ASME, Vol. 26, pp. 341-344.

Caughey, T. K. (1986), "On the Response of Nonlinear Oscillators to Stochastic Excitation", *Probabilistic Engineering Mechanics*, Vol. 1, No. 1, pp. 2-4.

Caughey, T. K. and Ma, F. (1982), "The Exact Steady-State Solution of a Class of Nonlinear Stochastic Systems", *International Journal of Non-Linear Mechanics*, Vol. 17, pp. 137-142.

Chai, Y. H., Fajfar, P. and Romstad, K. M. (1998), "Formulation of Duration-Dependent Inelastic Seismic Design Spectrum", *Journal of Structural Engineering*, ASCE, Vol. 124, No. 8, pp. 913-921.

Chai, Y. H., Romstad, K. M. and Bird, S. M. (1995), "Energy-Based Linear Damage Model for High-Intensity Seismic Loading", *Journal of Structural Engineering*, ASCE, Vol. 121, No. 5, pp. 857-864.

Clough, R. W. and Penzien, J. (1975), *Dynamics of Structures*, McGraw-Hill, Inc., New York, N. Y.

"Code for Design of Steel Structures", (1988), *NZS 3404*, Standards Association of New Zealand, Wellington, New Zealand.

References

Cosenza, E., Manfredi, G. and Ramasco, R. (1993), "The Use of Damage Functionals in Earthquake Engineering: A Comparison between Different Methods", *Earthquake Engineering and Structural Dynamics*, Vol. 22, pp. 855-868.

Council on Tall Buildings and Urban Habitat. (1993), *Semi-Rigid Connections in Steel Frames*, W. F. Chen, ed., McGraw-Hill Book Co., Inc., New York, N. Y.

Crandall, S. H. ed. (1958), *Random Vibration*, Vol. 1. The MIT Press, Cambridge, M. A.

Crandall, S. H. ed. (1963), *Random Vibration*, Vol. 2. The MIT Press, Cambridge, M. A.

Crandall, S. H. (1963), "Perturbation Techniques for Random Vibration of Nonlinear Systems", *Journal of the Acoustical Society of America*, Vol. 35, pp. 1700-1705.

Crandall, S. H. (1980), "Non-Gaussian Closure for Random Vibration of Nonlinear Oscillators", *International Journal of Non-Linear Mechanics*, Vol. 15, pp. 303-313.

Crandall, S. H. (1985), "Non-Gaussian Closure Techniques for Stationary Random Excitation", *International Journal of Non-Linear Mechanics*, Vol. 20, pp. 1-8.

Crandall, S. H. and Mark, W. D. (1963), *Random Vibration in Mechanical Systems*, Academic Press, New York, N. Y.

Darwin, D. and Nmai, C. K. (1986), "Energy Dissipation in RC Beams Under cyclic Load", *Journal of Structural Engineering*, ASCE, Vol. 112, No. 8, pp. 1829-1846.

Decanini, L. D. and Mollaioli, F. (1998), "Formulation of Elastic Earthquake Input Energy Spectra", *Earthquake Engineering and Structural Dynamics*, Vol. 27, pp. 1503-1522.

References

Elishakoff, I. (1983), *Probabilistic Methods in The Theory of Structures*, John Wiley, Inc., New York, N. Y.

El-Tawil, S., Vidarsson, E., Mikesell, T. and Kunnath, S. K. (1999), “Inelastic Behavior and Design of Steel Panel Zones”, *Journal of Structural Engineering*, ASCE, Vol. 125, No. 2, pp. 183-193.

El-Tawil, S., Vidarsson, E., Mikesell, T. and Kunnath, S. K. (2000), “Effect of Local Details and Yield Ratio on Behavior of FR Steel Connections”, *Journal of Structural Engineering*, ASCE, Vol. 126, No. 1, pp. 79-87.

Fajfar, P. and Fischinger, M. (1990), “Earthquake Design Spectra Considering Duration of Ground Motion”, *Proceedings of the 4th U. S. National Conference on Earthquake Engineering*, Vol. 2, pp. 15-24.

Fajfar, P. and Krawinkler, H. (eds) (1992), *Nonlinear Seismic Analysis and Design of Reinforced Concrete Building*, Elsevier Applied Science, London.

Fajfar, P. and Vidic, T. (1994), “Consistent Inelastic Design Spectra: Hysteretic and Input Energy”, *Earthquake Engineering and Structural Dynamics*, Vol. 23, pp. 523-537.

Fajfar, P., Vidic, T. and Fishinger, M. (1989), “Seismic Demand in Medium and Long-Period Structures”, *Earthquake Engineering and Structural Dynamics*, Vol. 18, pp. 1133-1144.

Foutch, D. A. and Yun, S. Y. (2002), “Modeling of Steel Moment Frames for Seismic Loads”, *Journal of Constructional Steel Research*, Vol. 58, pp. 529-564.

Frye, M. J. and Morris, G. A. (1975), “Analysis of Flexibly Connected Steel Frames”, *Canadian Journal of Civil Engineering*, Vol. 2, No. 3, pp. 280-291.

References

Gioncu, V. and Mazzolani, F. M. (2001), *Ductility of Seismic Resistant Steel Structures*, Spon Press, Taylor & Francis Group.

Gupta, A. and Krawinkler, H. (2000), "Behavior of Ductile SMRFs at Various Seismic Hazard Levels", *Journal of Structural Engineering*, ASCE, Vol. 126, No. 1, pp. 98-107.

Hart, G. C. and Wong, K. K. F. (2000), *Structural Dynamics for Structural Engineers*, John Wiley & Sons, Inc., New York.

Housner, G. W. (1956), "Limit Design of Structures to Resist Earthquake", *Proceedings of the 1st World Conference of Earthquake Engineering*, Berkeley, California, 5.1-5.13.

Housner, G. W. (1959), "Behavior of Structures during Earthquake", *Journal of Engineering Mechanical Division*, ASCE, Vol. 85, No. 4, pp. 109-129.

Ibrahim, R. A. (1985), *Parametric Random Vibration*, John Wiley and Sons, Inc., New York, N. Y.

Iwan, W. D. (1973), "A Generalization of the Concept of Equivalent Linearization", *International Journal of Non-Linear Mechanics*, Vol. 8, pp. 279-287.

Iwan, W. D. and Paparizos, L. G. (1988), "The Stochastic Response of Strongly Yielding Systems", *Probabilistic Engineering Mechanics*, Vol. 3, No. 2, pp. 75-82.

Iwan, W. D. and Yang, I. M. (1972), "Application of Statistical Linearization Techniques to Nonlinear Multi-Degree-of-Freedom Systems", *Journal of Applied Mechanics*, ASME, Vol. 39, pp. 545-550.

Iyengar, R. N. (1975), "Random Vibration of a Second Order Nonlinear Elastic System", *Journal of Sound and Vibration*, Vol. 40, pp. 155-165.

References

Iyengar, R. N. and Dash, P. K. (1976), "Random Vibration Analysis of Stochastic Time-Varying Systems", *Journal of Sound and Vibration*, Vol. 45, pp. 69-89.

Iyengar, R. N. and Dash, P. K. (1978), "Study of the Random Vibration of Nonlinear Systems by the Gaussian Closure Technique", *Journal of Applied Mechanics*, ASME, Vol. 45, pp. 393-399.

Johnson, J. P. and Scott, R. A. (1980), "Extension of Eigenfunction-Expansion Solutions of a Fokker-Planck Equation-II. Second Order System", *International Journal of Non-Linear Mechanics*, Vol. 15, pp. 41-56.

Kanai, K. (1957), "Seismic-Empirical Formula for the Seismic Characteristics of the Ground", *Bull. Earthquake Research Institute*, Japan, Vol. 35, pp. 309-325.

Kato, B., Chen, W. F. and Nakao, M. (1988), "Effects of Joint-Panel Shear Deformation on Frames", *Journal of Constructional Steel Research*, Vol. 10, pp. 269-320.

Khasminskii, R. Z. (1966), "A Limit Theorem for the Solutions of Differential Equations with Random Right-Hand Sides", *Theo. Prob. Appl.*, Vol. 11, pp. 390-405.

Khudada, A. E. and Geschwindner, L. F. (1995), "Nonlinear Dynamic Analysis of Steel Frames by Modal Superposition", *Journal of Structural Engineering*, ASCE, Vol. 123, No. 11, pp. 1519-1527.

Krawinkler, H. (1978), "Shear in Beam-Column Joints in Seismic Design of Steel Frames", *Engineering Journal*, AISC, Vol. 15, No. 3, pp. 82-91.

Krawinkler, H., Bertero, V. V. and Popov, E. P. (1975), "Shear Behavior of Steel Frame Joints", *Journal of Structural Division*, ASCE, Vol. 101, No. 11, pp. 2317-2336.

References

- Krawinkler, H. and Mohasseb, S. (1987), "Effects of Panel Zone Deformations on Seismic Response", *Journal of Constructional Steel Research*, Vol. 8, pp. 233-250.
- Krawinkler, H. and Nassar, A. A. (1992), "Seismic Design Based on Ductility and Cumulative Damage Demands and Capacities", *Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings*, P. Fajfar and H. Krawinkler (Eds.), Elsevier, London, pp. 23-40.
- Krawinkler, H. and Popov, E. P. (1982), "Seismic Behavior of Moment Connections and Joints", *Journal of the Structural Division*, ASCE, Vol. 108, No. 2, pp. 373-391.
- Krishnamurthy, N., Huang, H. T., Jeffrey, P. K. and Avery, L. K. (1979), "Analytical $M-\theta$ Curves for End-Plate Connections", *Journal of Structural Division*, ASCE, Vol. 105, No. 1, pp. 133-145.
- Kuwamura, H. and Galambos, T. V. (1989), "Earthquake Load for Structural Reliability", *Journal of Structural Engineering*, ASCE, Vol. 115, No. 6, pp. 1446-1462.
- Kuwamura, H., Kirino, Y. and Akiyama, H. (1994), "Prediction of Earthquake Energy Input from Smoothed Fourier Amplitude Spectrum", *Earthquake Engineering and Structural Dynamics*, Vol. 23, pp. 1125-1137.
- Langley, R. S. (1985), "A Finite Element Method for the Statistics of Non-linear Random Vibration", *Journal of Sound and Vibration*, Vol. 101, pp. 41-54.
- Lee, K. and Foutch, D. A. (2002), "Seismic Performance Evaluation of Pre-Northridge Steel Frame Buildings with Brittle Connections", *Journal of Structural Engineering*, ASCE, Vol. 128, No. 4, pp. 546-555.

References

Lee, S. J. and Lu, L. W. (1989), "Cyclic Tests of Full-Scale Composite Joint Sub-Assemblages", *Journal of Structural Engineering*, ASCE, Vol. 115, No. 8, pp. 1977-1998.

Lin, T. H. (1968), *Theory of Inelastic Structures*, Wiley, New York.

Lin, Y. K. (1967), *Probabilistic Theory of Structural Dynamics*, McGraw-Hill, Inc., New York, N. Y. (Second Edition, 1976).

Lin, Y. K. and Cai, G. Q. (1995), *Probabilistic Structural Dynamics: Advanced Theory and Applications*, McGraw-Hill, Inc., New York, Tokyo, Singapore.

Liu, Q. and Davis, H. G. (1988), "Application of Non-Gaussian Closure to the Non-Stationary Response of a Duffing Oscillator", *International Journal of Non-Linear Mechanics*, Vol. 23, pp. 241-250.

Luco, N. and Cornell, C. A. (2000), "Effects of Connection Fractures on SMRF Seismic Drift Demands", *Journal of Structural Engineering*, ASCE, Vol. 126, No. 1, pp. 127-136.

Lui, E. M. and Chen, W. F. (1986a), "Analysis and Behavior of Flexibly Jointed Frames", *Engineering Structures*, Vol. 8, No. 2, pp. 107-118.

Lui, E. M. and Chen, W. F. (1986b), "Frame Analysis with Panel Deformation", *International Journal of Solids and Structural*, Vol. 22, No. 12, pp. 1599-1986.

Lutes, L. D. (1970), "Approximate Technique for Treating Random Vibration of Hysteretic Systems", *Journal of the Acoustical Society of America*, Vol. 48, No. 1, pp. 299-306.

Manfredi, G. (2001), "Evaluation of Seismic Energy Demand", *Earthquake Engineering and Structural Dynamics*, Vol. 30, pp. 485-499.

References

MAPLE 7 (2002), *Essential Maple 7: An Introduction for Scientific Programmers* by R. M. Corless, Springer-Verleg, NY, USA.

McCabe, S. L. and Hall, W. J. (1989), “Assessment of Seismic Structural Damage”, *Journal of Structural Engineering*, ASCE, Vol. 115, No. 9, pp. 2166-2183.

Nader, M. N. and Astaneh-Asl, A. (1996), “Shaking Table Tests of Rigid, Semirigid, and Flexible Steel Frames”, *Journal of Structural Engineering*, ASCE, Vol. 122, No. 6, pp. 589-596.

Nakashima, N., Saburi, K. and Tsuji, B. (1996), “Energy Input and Dissipation Behavior of Structures with Hysteretic Dampers”, *Earthquake Engineering and Structural Dynamics*, Vol. 25, pp. 483-496.

NEHRP. (1997a), *Recommended Provisions for Seismic Regulations for New Buildings and Other Structures, Part 1-Provisions*, FEMA 302, Federal Emergency Management Agency, Washington, DC, USA.

NEHRP. (1997b), *Recommended Provisions for Seismic Regulations for New Buildings and Other Structures, Part 2-Commentary*, FEMA 303, Federal Emergency Management Agency, Washington, DC, USA.

Nigam, N. C. (1983), *Introduction to Random Vibrations*, The MIT Press, Cambridge, M. A.

Nigam, N. C. and Narayanan, S. (1994), *Applications of Random Vibrations*, Springer-Verlag, Berlin.

Osman, A., Ghobarah, A. and Korol, R. M. (1995), “Implications of Design Philosophies for Seismic Response of Steel Moment Frames”, *Earthquake Engineering and Structural Dynamics*, Vol. 24, No. 1, pp. 127-143.

References

Papanicolaou, G. C. and Kohler, W. (1974), "Asymptotic Theory of Mixing Stochastic Ordinary Differential Equations", *Commun. Pure Appl. Math.*, Vol. 27, pp. 641-668.

Park, Y. J. and Ang, A. H. S. (1985), "Mechanistic Seismic Damage Model for Reinforced Concrete", *Journal of Structural Engineering*, ASCE, Vol. 111, No. 4, pp. 722-738.

Park, Y. J., Ang, A. H. S. and Wen, Y. K. (1985), "Seismic Damage Analysis of Reinforced Concrete Buildings", *Journal of Structural Engineering*, ASCE, Vol. 111, No. 4, pp. 740-757.

Park, Y. J., Wen, Y. K. and Ang, A. H. S. (1986), "Random Vibration under Bidirectional Ground Motion", *Earthquake Engineering and Structural Dynamics*, Vol. 14, pp. 543-557.

Popov, E. P. (1987), "Panel Zone Flexibility in Seismic Moment Joints", *Journal of Constructional Steel Research*, Vol. 8, pp. 91-118.

Popov, E. P., Amin, N. R., Louis, J. J. C. and Stephen, R. M. (1986), "Cyclic Behavior of Large Beam-Column Assemblies", *Engineering Journal*, AISC, Vol. 23, No. 1, pp. 9-23.

Popov, E. P., Tsai, K. C. and Engelhardt, M. D. (1989), "On Seismic Steel Joints and Connections", *Engineering Structures*, Vol. 11, No. 3, pp. 148-162.

Pradlwarter, H. J. and Kliemann, W. (1996), "First Exit Times in Nonlinear Dynamical Systems", *Proceedings of the 11th ASCE Engineering Mechanics Conference*, Fort Lauderdale, FL, May 19-22, Y. K. Lin and T. C. Su (Eds.), ASCE, New York, USA, pp. 531-534.

References

Pradlwarter, H. J. and Schueller, G. I. (1999), "Assessment of Low Probability Events of Dynamical Systems by Controlled Monte Carlo Simulation", *Probabilistic Engineering Mechanics*, Vol. 14, No. 3, pp. 213-227.

Recommended Lateral Force Requirements and Tentative Commentary, (1988), Seismology Committee, Structural Engineers Association of California, San Francisco, California, USA.

Ridell, R. and Garcia, J. E. (2001), "Hysteretic Energy Spectrum and Damage Control", *Earthquake Engineering and Structural Dynamics*, Vol. 30, pp. 1791-1816.

Roberts, J. B. (1977), "Stationary Response of Oscillators with Nonlinear Damping to Random Excitation", *Journal of Sound and Vibration*, Vol. 30, pp. 145-156.

Roberts, J. B. (1978), "The Energy Envelope of a Randomly Excited Nonlinear Oscillator", *Journal of Sound and Vibration*, Vol. 60, pp. 177-185.

Roberts, J. B. and Spanos, P. D. (1986), "Stochastic Averaging: An Approximate Method of Solving Random Vibration Problems", *International Journal of Non-Linear Mechanics*, Vol. 21, pp. 111-134.

Roberts, J. B. and Spanos, P. D. (1990), *Random Vibration and Statistical Linearization*, John Wiley, Inc., New York, N. Y.

Roeder, C. W. (2002a), "General Issues Influencing Connection Performance", *Journal of Structural Engineering*, ASCE, Vol. 128, No. 4, pp. 420-428.

Roeder, C. W. (2002b), "Connection Performance for Seismic Design of Steel Moment Frames", *Journal of Structural Engineering*, ASCE, Vol. 128, No. 4, pp. 517-525.

References

Roeder, C. W. and Foutch, D. A. (1996), "Experimental Results for Seismic Resistant Steel Moment Frame Construction", *Journal of Structural Engineering*, ASCE, Vol. 122, No. 6, pp. 581-588.

SAC Joint Venture, Sacramento, Calif. (1994), "Invitational Workshop on Steel Seismic Issues", *Proc.*, Los Angeles, Calif.

Schneider, S. P. and Amidi, A. (1998), "Seismic Behavior of Steel Frames with Deformable Panel Zones", *Journal of Structural Engineering*, ASCE, Vol. 124, No. 1, pp. 35-42.

Schneider, S. P., Roeder, C. W. and Carpenter, J. E. (1993), "Seismic Behavior of Moment-Resisting Steel Frames: Experimental Study", *Journal of Structural Engineering*, ASCE, Vol. 119, No. 6, pp. 1885-1902.

Schueller, G. I., et al. (1997), "A State-of-the-Art Report on Computational Stochastic Mechanics", *Probabilistic Engineering Mechanics*, Special Issue, Vol. 12, No. 4, pp. 197-321.

Schueller, G. I. (1998), "Structural Reliability-Recent Advances", *Proceedings of the 7th International Conference on Structural Safety and Reliability*, Kyoto, Japan, pp. 3-35.

Schueller, G. I., Pradlwarter, H. J. and Bucher, C. G. (1991), "Efficient Computational Procedures for Reliability Estimate of MDOF-Systems", *International Journal of Non-Linear Mechanics*, Vol. 26, No. 6, pp. 961-974.

Shinozuka, M. (1972), "Monte Carlo Solution of Structural Dynamics", *Tech. Rep. 19*, Department of Civil Engineering and Engineering Mechanics, Columbia University.

References

Shinozuka, M. (1974), "Digital Simulation of Random Processes in Engineering Mechanics with the Aid of FFT Techniques", *Stochastic Problems in Mechanics*, Ariaratnam, S. T. and Leipholtz, H. H. E., eds., Study 10. Solid Mech. Div., University of Waterloo Press, pp. 277-286.

Shinozuka, M. and Sato, Y. (1967), "Simulation of Nonstationary Random Process", *Journal of the Engineering Mechanics Division*, ASCE, Vol. 93, No. EM1, pp. 11-40.

Simulescu, I., Mochio, T. and Shinozuka, M. (1989), "Equivalent Linearization Method in Nonlinear FEM", *Journal of Engineering Mechanics*, ASCE, Vol. 115, No. 3, pp. 475-492.

Socha, L. and Soong, T. T. (1991), "Linearization in Analysis of Nonlinear Stochastic Systems", *Applied Mechanics Reviews*, ASME, Vol. 44, No. 10, pp. 399-422.

Soize, C. (1988), "Steady State Solution of Fokker-Planck Equation in Higher Dimension", *Probabilistic Engineering Mechanics*, Vol. 3, No. 4, pp. 196-206.

Soni, S. R. and Surrendran, K. (1975), "Transient Response of Nonlinear Systems to Stationary Random Excitation", *Journal of Applied Mechanics*, Vol. 42, pp. 891-893.

Soong, T. T. and Grigoriu, M. (1993), *Random Vibration of Mechanical and Structural Systems*, Prentice Hall, Inc., Englewood Cliffs, N. J.

Spanos, P. D. (1981), "Stochastic Linearization in Structural Dynamics", *Applied Mechanics Reviews*, ASME, Vol. 34, No. 1.

Specifications of Seismic Provisions for Structural Steel Buildings. (1992), American Institute of Steel Construction, Chicago, USA, No. I11.

References

Stratonovich, R. L. (1963), *Topics in the Theory of Random Noise*, Vol. I, Gordon and Breach, N. Y.

Sucuoglu, H. and Nurtug, A. (1995), "Earthquake Ground Motion Characteristics and Seismic Energy Dissipation", *Earthquake Engineering and Structural Dynamics*, Vol. 24, pp. 1195-1213.

Tajimi, H. (1960), "A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake", *Proceedings of the 2nd World Conference on Earthquake Engineering*, Japan, pp. 781-798.

Tembulkar, J. M. and Nau, J. M. (1987), "Inelastic Modeling and Seismic Energy Dissipation", *Journal of Structural Engineering*, ASCE, Vol. 113, No. 6, pp. 1373-1377.

Tsai, K. C. and Popov, E. P. (1990), "Seismic Panel Zone Design Effect on Elastic Story Drift in Steel Frames", *Journal of Structural Engineering*, ASCE, Vol. 116, No. 12, pp. 3285-3301.

Tsai, K. C., Wu, S. and Popov, E. P. (1995), "Experimental Performance of Seismic Steel Beam-Column Moment Joints", *Journal of Structural Engineering*, ASCE, Vol. 121, No. 6, pp. 925-931.

Tung, C. C. (1967), "The Effects of Runway Roughness on the Dynamic Response of airplanes", *Journal of Sound and Vibration*, Vol. 5, pp. 164-172.

Uang, C. M. and Bertero, V. V. (1988), "Use of Energy as a Design Criterion in Earthquake Resistant Design", *Rep. No. UCB/EERC-88/18*, Earthquake Engrg. Res. Ctr., Univ. of California, Berkeley, Calif.

Uang, C. M. and Bertero, V. V. (1990), "Evaluation of Seismic Energy in Structures", *Earthquake Engineering and Structural Dynamics*, Vol. 19, pp. 77-90.

References

Uniform Building Code. (1991), International Conference of Building Officials, Whittier, California, USA.

Wehner, M. F. and Wolfer, W. G. (1983), "Numerical Evaluation of Path-Integral Solutions to the Fokker-Planck Equations", *Physics Review*, Vol. A27, pp. 2663-2670.

Wen, Y. K. (1975), "Approximate Methods for Nonlinear Random Vibration", *Journal of Engineering Mechanics Division*, ASCE, Vol. 101, pp. 389-401.

Wen, Y. K. (1976), "Method for Random Vibration of Hysteretic Systems", *Journal of Engineering Mechanics Division*, ASCE, Vol. 102, No. 2, pp. 249-264.

Wen, Y. K. (1980), "Equivalent Linearization of Hysteretic Systems under Random Excitation", *Journal of Applied Mechanics*, ASME, Vol. 47, No. 1, pp. 150-154.

Wen, Y. K. (1989), "Methods of Random Vibration for Inelastic Structures", *Applied Mechanics Reviews*, ASME, Vol. 42, No. 2, pp. 39-52.

Wen, Y. K. and Yeh, C-H (1989), "Biaxial and Torsional Response of Inelastic Structures under Random excitation", *Structural Safety*, Vol. 6, pp. 137-152.

Wilson, H. B. and Turcotte, L. H. (1997), *Advanced Mathematics and Mechanics Applications Using Matlab*, CRC Press, LLC.

Wong, K. K. F. and Yong, R. (1999), "Inelastic Dynamic Response of Structures Using Force Analogy Method", *Journal of Engineering Mechanics*, ASCE, Vol. 125, No. 10, pp. 1190-1199.

Wong, K. K. F. and Yong, R. (2002), "Earthquake Response and Energy Evaluation of Inelastic Structures", *Journal of Engineering Mechanics*, ASCE, Vol. 128, No. 3, pp. 308-317.

References

Wong, K. K. F. and Wang, Z. (2004a), "Seismic Analysis of Inelastic Moment Resisting Frames, Part I: End Offsets", *Journal of Engineering Mechanics*, ASCE, in review.

Wong, K. K. F. and Wang, Z. (2004b), "Seismic Analysis of Inelastic Moment Resisting Frames, Part II: Deformable Panel Zones", *Journal of Engineering Mechanics*, ASCE, in review.

Wu, W. F. and Lin, Y. K. (1984), "Cumulant-Neglect Closure for Nonlinear Oscillators under Random Parametric and External Excitations", *International Journal of Non-Linear Mechanics*, Vol. 19, pp. 349-362.

Yang, C. Y. (1986), *Random Vibration of Structures*, John Wiley, Inc., New York, N. Y.

Yee, Y. L. and Melchers, R. E. (1986), "Moment-Rotation Curves for Bolted Connections", *Journal Structural Engineering*, ASCE, Vol. 112, No. 3, pp. 615-635.

Zahrah, T. F. and Hall, W. J. (1984), "Earthquake Energy Absorption in SDOF Structures", *Journal of Structural Engineering*, ASCE, Vol. 110, No. 8, pp. 1757-1772.

Zhu, W. Q. (1988), "Stochastic Averaging Methods in Random Vibration", *Applied Mechanics Reviews*, ASME, Vol. 41, No. 5, pp. 189-199.

Zhu, W. Q. (1992), *Random Vibration*, Science Press, Inc., Beijing, China, in Chinese.

Zhu, W. Q., Cai, G. Q. and Lin, Y. K. (1990), "On Exact Stationary Solutions of Stochastically Perturbed Hamiltonian Systems", *Probabilistic Engineering Mechanics*, Vol. 5, pp. 84-87.

References

Zhu, W. Q. and Yu, J. S. (1989), “The Equivalent Nonlinear System Method”, *Journal of Sound and Vibration*, Vol. 129, pp. 385-395.