Analysis of metallic cellular materials for blast mitigation

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2007

Ye, Z. Q. (2007). Analysis of metallic cellular materials for blast mitigation. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/12152>

<https://doi.org/10.32657/10356/12152>

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ANALYSIS OF METALLIC CELLULAR MATERIALS FOR BLAST MITIGATION

YE ZIQING

**SCHOOL OF
CIVIL AND ENVIRONMENTAL ENGINEERING**

NANYANG TECHNOLOGICAL UNIVERSITY

SINGAPORE 2007

Analysis of Metallic Cellular Materials for Blast Mitigation

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B. Eng., M. Eng.

**A thesis submitted to Nanyang Technological University
in fulfillment of the requirements for
the degree of Doctor of Philosophy**

School of Civil and Environmental Engineering

2007

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Dr. Ma Guowei, for his profound vision, research philosophy and methodology, invaluable guidance, and encouragement in conducting this research work. His unwavering enthusiasm and interest in scientific research is much appreciated and unforgettable. It has been a great privilege and honor to work with him.

The research scholarship offered by Nanyang Technological University is gratefully acknowledged.

Finally, I must acknowledge the constant understanding, patience, encouragement and support provided by my wife, my family and my friends.

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Abstract

A mesoscale numerical model is firstly developed to investigate the deformation mode and the dynamic compressive strength of cellular materials. The loading rate effect on the nominal stresses at the impact side and the stationary side of a cellular material sample are discussed. Previous experimental studies on the dynamic behavior of cellular materials are reviewed and analyzed with the mesoscale numerical model. The resistance of the cellular material against blast loading is also studied at the mesoscale. The blast resisting capacity of the cellular material is analytically predicted based on a shock wave theory. A double-layer foam cladding is then proposed to fulfill different structural protection purposes. The energy absorption capacity of the double-layer foam cladding under blast load is analytically derived based on the shock wave theory. Good agreement is found between the analytical and numerical results.

To consider the stiffness of the protected structure, an analytical Load-Cladding-Structure (LCS) model is further developed. Two non-dimensional parameters are introduced to describe the relations between the foam cladding and the protected structure. Based on the LCS model, the maximum allowable blast load for the structure with the protection of a particular foam cladding can be predicted. It is seen that the maximum deflection of the protected structure subjected to a certain explosive load varies with the two non-dimensional parameters of the foam cladding. The foam cladding can be appropriately designed to achieve structural retrofit against blast loads. Two structural models with sacrificial foam claddings are numerically simulated using LS-DYNA. The numerical results conform to the conclusion based on the LCS model that the foam cladding should be designed appropriately with reference to the potential blast load and the protected structure. The present LCS model can be conveniently used for the design and evaluation of foam claddings to retrofit military and civil structures.

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List of Symbols

Symbols used in Chapter 3

A_0	Area of the 2D Voronoi diagram
C, P	Parameters of Cowper-Symonds model
H	Thickness of the foam sample
L_0	Length of cell edge
V	Impact velocity
V_s	Propagation speed of shock front
d_0	Distance of two adjacent nuclei of a regular 2D Voronoi diagram
h	Thickness of cell wall
n	Number of cells
α	Irregularity degree of Voronoi diagram
β	Shape coefficient of cell wall
Δ	Lateral offset distance of cell wall
δ	Minimum distance of two adjacent nuclei of a irregular 2D Voronoi diagram
δ_1	Relative shortening of the first half of the foam sample
δ_2	Relative shortening of the second half the foam sample
$\dot{\epsilon}$	Strain rate of the base material
ρ_0	Density of the cellular material
ρ_s	Density of the base material
σ_y^d	Dynamic yield stress of the base material
σ_y^s	Quasi-static yield stress of the base material
σ_0	Plateau stress of the cellular material
σ_D	Stress behind the shock front

Symbols used in Chapter 4

A	Cross-sectional area of the cover plate
I_0	Total input impulse
$P(t)$	Blast pressure
P_0	Initial peak pressure
U	Initial kinetic energy of the cover plate
W	Absorbed energy absorbed per unit volume up to densification under quasi-static condition
l	Thickness of the cellular material sample
m_1	Mass of the cover plate
m_f	Mass of the cellular material
Δm	Mass of the densified part of cellular material
t_0	Blast loading duration
$u, \dot{u}, \ddot{u}$	Displacement, velocity and acceleration of the cover plate
σ_0	Plateau stress of the cellular material
σ_D	Stress behind the shock front
ε_D	Densification strain
ρ_0	Density of the cellular material

Symbols used in Chapter 5

A	Cross-sectional area of the cover plate
E	Total energy imparted to the foam cladding by blast loading
I_0	Impulse of the blast loading
I_1, I_2, I_3, I_4	Maximum impulse that can be resisted by claddings
K	Kinetic energy of foam cladding system under blast loading
P_0	Peak pressure of the blast loading
Q_s	Energy absorption capacity of foam cladding under quasi-static state

ΔQ	Energy absorbed by per unit mass of the foam at the shock front
Q_1, Q_2, Q_3	Absorbed energy by foam cladding under blast loading
U_1', U_2'	Dimensionless deformation of front foam and rear foam, respectively
l_1, l_2	Thickness of the foam panels
m_1, m_2	Mass of the cover plates
m_f, m_{f1}, m_{f2}	Mass of the foams
$\Delta m, \Delta m_1, \Delta m_2$	Mass of the densified parts of foams
t_0	Duration of the blast loading
$u, \dot{u}, \ddot{u}$	Displacement, velocity and acceleration of the cover plate
$\varepsilon_D, \varepsilon_{D1}, \varepsilon_{D2}$	Densification strain of the foam material
ρ_s	Density of metallic composition of foam
ρ_0, ρ_1, ρ_2	Initial density of the foam materials
σ_{ys}	Yield stress of metallic composition of foam
$\sigma, \sigma_1, \sigma_2$	Stress immediately ahead of the shock front
$\sigma_0, \sigma_{01}, \sigma_{02}$	Plateau stress of the foam material
$\sigma_D, \sigma_{D1}, \sigma_{D2}$	Stress immediately behind the shock front

Symbols used in Chapter 6

A	Cross-sectional area of the foam cladding
F_m, F_s, F_l, F_{lm}	Mass, stiffness, load factor and load-mass factor, respectively
I_0	Impulse of the blast load
$P(t)$	Blast pressure
P_0	Initial peak pressure of the blast load
T	Natural period of the equivalent structure
i	Non-dimensional impulse
k	Stiffness of the main structure

l	Thickness of the foam layer
m	Mass of the main structure
m_{se}	Equivalent mass of the equivalent SDOF structure
m_l	Mass of the cover plate
m_f	Mass of the foam layer
Δm	Mass of the densified part of foam
p	Non-dimensional initial peak pressure
t_0	Duration of the blast loading
t_m	Duration of transmitted pressure by the foam cladding
$u, \dot{u}, \ddot{u}$	Displacement, velocity and acceleration of the cover plate
$y, \dot{y}, \ddot{y}$	Deflection, velocity and acceleration of the SDOF structure
y_c	Maximum allowable deflection of the SDOF structure
ρ_0	Initial density of the foam material
σ_0	Plateau stress of the foam material
σ_D	Stress immediately behind the shock front
σ_t	Transmitted pressure by the foam layer
ε_D	Densification strain of the foam material
κ, τ	Two non-dimensional parameters of the foam cladding

Symbols used in Chapter 7

D	Distance from the centroid of the foam attached pendulum to the rotation axis
E	Young's modulus
F_{lm}	Load-mass factor of the SDOF model
I	The second moment of inertia of the beam
I_1	Impulse imparted to the pendulum with the foam cladding
I_2	Impulse imparted to the pendulum without the foam cladding
L	Length of the beam.
M	Mass of the bare pendulum
M_f	Mass of the foam cladding

M_N	Negative moment capacity of the beam
M_P	Positive moment capacity of the beam
P_{E1}	The potential energy acquired by the pendulum with the foam cladding
P_{E2}	The potential energy acquired by the pendulum without the foam cladding
R	Elastic resistance of the beam
d	Distance from the centroid of the bare pendulum to the rotation axis
k	Equivalent stiffness of the beam
y_c	Critical deflection of the SDOF structure
κ, τ	Two non-dimensional parameters of the foam cladding

Chapter 1 Introduction

1.1 Background

Structural retrofit using metallic cellular materials is of special interest to engineering communities and government agencies against possible impact and blast threats. The superior microstructure of metallic cellular materials endows them with the ability to undergo large deformation at nearly constant nominal stress. In compression they can absorb remarkable energy by plastic deformation, in addition to the advantage of their ultra-light and nonflammable characteristics. It is expected that they can be used as sacrificial claddings in the retrofit of structures against impacts and blast loads.

The mechanical properties of cellular materials vary in a wide range, depending on their base material composition, relative bulk density and production method. Although some basic data for cellular materials are provided by the suppliers, the deformation and failure mechanism of these materials under high-strain-rate loading, such as impact and blast loading, are still not fully understood.

The implementation of cellular materials in blast mitigation still struggles in the premature stage. Limited theoretical and experimental research work has been done on a cellular material under blast loading. Guidelines are currently not available for the design of blast mitigating structures based on cellular materials.

Some recent reports stated that sacrificial foam layer intended to mitigate blast loading may work unexpectedly to aggravate the damage of the protected structure, if the sacrificial cladding was not properly designed. Thus, it is of importance to study the dynamic behavior of cellular materials under high velocity impact and blast loading and to optimize the design of sacrificial claddings for structure protection.

1.2 Objectives and scope

The objectives of the study are,

- to investigate the dynamic behavior of cellular materials under impact and blast loads;
- to study the shock wave propagation mechanism in cellular materials;
- to investigate the efficiency of cellular materials for structure protection;
- to optimize the design of sacrificial cladding by using cellular materials for blast mitigation.

The compressive strength of cellular material under quasi-static and dynamic conditions will be first investigated by mesoscale numerical simulations. The loading rate effect on the dynamic strength will be clarified. Further, shock wave propagation in cellular material will be investigated. Blast resistant capacity of the cellular material will be analytically and numerically predicted. The protective effect of cellular material for structure protection will be studied analytically by considering the characteristics of blast loading, cladding made of cellular material and the protected structure. An optimized design of sacrificial cladding or blast barrier by using cellular material to attenuate blast loading will be obtained and it will then be numerically validated.

1.3 Thesis organization

The thesis consists of eight chapters. The first chapter gives a brief introduction of the background, objectives and scope of the present study.

Chapter 2 reviews experimental and theoretical studies on the static and dynamic behavior of cellular materials. Focus is placed on the dynamic compressive strength of cellular materials and wave propagation in cellular materials. The available material models for cellular materials are summarized. The current status of impact and blast barrier designs using cellular materials is also presented.

In Chapter 3, a mesoscale numerical model is developed to investigate the dynamic behavior of cellular materials. The deformation mode of the cellular material is characterized. The nominal crushing stresses at both ends of the material sample are investigated. Several factors including the loading rate and inertia that may influence the nominal crushing stress of the cellular material are discussed. Various previous experimental studies on the dynamic behavior of cellular materials are reviewed and analyzed with the mesoscale numerical model.

In Chapter 4, the behavior of cellular material under blast loading is studied at the meso-scopic scale. The crushing of the cellular material is approximated with a shock wave theory. The blast resisting capacity of a single-layer cellular material is predicted numerically and analytically.

Chapter 5 proposes a double-layer foam cladding to enhance the energy absorption capacity. The energy absorption capacity of the double-layer foam cladding under blast loading is analytically derived. Numerical simulation with finite element method is also carried out to verify the analytical solutions.

In Chapter 6, an analytical Load-Cladding-Structure model is developed to evaluate the effectiveness of foam claddings for protecting the main structure. The maximum allowable blast load for the protected structure with and without a foam cladding is predicted. It is suggested that the foam cladding should be properly designed to match the load and structure properties.

In Chapter 7, numerical simulations are carried out on the sacrificial claddings for structure protection. The numerical simulations are based on some experimental studies. The numerical results verify the analytical prediction based on the Load-Cladding-Structure model.

Finally, Chapter 8 gives the conclusions from the PhD work and recommends the areas for the future work.

Chapter 2 Literature Review

2.1 Cellular materials

Cellular materials are made up of interconnected networks of solid struts or plates which form the edges and faces of cells (Gibson and Ashby 1997). Two-dimensional cellular materials are called honeycombs, as shown in Fig. 2.1(a). Three-dimensional cellular materials are referred as foams. If a foam has both edges and wall, it is called closed-cell foam, as seen in Fig. 2.1(b); otherwise, it is called open-cell foam, as shown in Fig. 2.1(c) (Gibson and Ashby 1997).

(a)Honeycomb

(b)Closed-cell foam

(c) Open-cell foam

Fig. 2.1 Categories of cellular materials

Artificial cellular materials can be made from various compositions, including polymers, metals and metallic alloys. Polymeric foams include polyethylene foam, polyurethane foam, and so on. With the advantages of light weight, impacting buffering and thermal insulation, they have been widely used in industrial packaging and everyday life.

Metallic cellular materials, made of aluminum or magnesium or alloys, have been commercially available for decades. Metallic foams are recyclable, non-flammable, and, most attractively, capable of absorbing high energy during plastic deformation. Some aluminum foams such as Alporas, Alulight and ERG (Duocel) have been gaining increasing attention of researchers.

The mechanical properties of cellular materials are related to their base material composition, relative bulk density and production method. Generally a cellular material in compression shows a linear-elastic regime followed by a plateau of roughly constant stress, leading into a final regime of steeply rising stress, as shown in Fig. 2.2.

Fig. 2.2 Compressive stress–strain curves for several aluminum foams (Andrews et al. 1999)

Due to their low densities and novel physical, mechanical, thermal, electrical and acoustic properties, cellular materials have been used in industrial, military and civil applications such as thermal insulation, packaging, structural use, impact buffering and buoyancy (Gibson and Ashby 1997; Ashby et al. 2000). However, only the applications arising from the physical and mechanical superiorities of cellular materials are in the scope of the present study.

2.2 Static deformation of cellular materials

2.2.1 Deformation mechanism of cellular materials

When a honeycomb is subjected to in-plane compression, the cell walls first bend elastically. But when a critical stress is reached the cells begin to collapse. At last, at high strains, the cells collapse sufficiently that opposing cell walls touch and further deformation give the densification portion of stress- strain curve. The honeycomb is much stiffer and stronger in out-of-plane compression. When subjected to out-of-plane loading, the honeycomb has a short linear elastic regime, followed by buckling regime and finally crushing. In tensile deformation, the honeycomb deforms elastically until it tears, yields plastically or fractures.

Similar to the deformation of honeycombs, foams show a linear elastic property at low stress followed by a long collapse plateau regime, truncated by a regime of densification. In compression, the cell wall bends and cell face, if the cells are closed, stretches at the linear elastic stage. Then the plateau regime is associated with collapse of the cells – by the formation of plastic hinges in an elastic-plastic foam or by brittle fracture in a brittle foam. At high strain the foam reaches densification regime; the opposing cell walls touch and further straining compresses the solid itself (Gibson and Ashby 1997). In tensile deformation, foams also behave like honeycombs. Some foam, however, is elastic-plastic in compression while brittle in tension (Hanssen et al. 2002b).

2.2.2 Static compressive strength of cellular materials

At the macroscopic scale, metallic honeycombs and foams have been regarded as continuum materials. The compressive strengths of these materials are very important and can be measured experimentally.

Papka and Kyriakides (1994) carried out experiments to study the in-plane crushing behavior of honeycombs under quasi-static condition. Their study gave the stress-strain curves of the honeycomb over elastic, plastic as well as densification regimes.

Baumeister et al. (1997) measured the Young's modulus and compressive strength of aluminium foams with densities varying from 400 kg/m³ to 1000 kg/m³. A non-linear dependency of these properties on the density was found and discussed.

It has been found that the mechanical properties of cellular materials are relevant to their compositions, manufacture methods and densities. Based on laboratory tests of different foams, Gibson and Ashby (1997) and Ashby et al. (2000) have given the phenomenological models of metallic foams, some of which are cited below.

- Young's modulus $\frac{E}{E_s} = \alpha_2 \left(\frac{\rho}{\rho_s} \right)^n$, where n has a value between 1.8 and 2.2 and α_2 between 0.1 and 4.
- The plateau stress $\frac{\sigma_{pl}}{\sigma_{ys}} = C_1 \left(\frac{\rho}{\rho_s} \right)^m$, C_1 is a constant with a value often between 0.25 and 0.35. And the parameter m lies between 1.5 and 2.0.
- Densification strain $\varepsilon_D = 1 - \alpha_1 \frac{\rho}{\rho_s}$, α_1 lies between 1.4 and 3.0.

where ρ is density of the foam and ρ_s is the density of the base material or matrix material.

Andrews et al. (1999) investigated the Young's modulus and strength of five foams, i.e., Alcan, Alporas, Fraunhofer, Alulight and Duocel. The uniaxial compressive and tensile modulus and strength of several aluminum foams were compared with models for cellular solids (Gibson and Ashby 1997). The open cell foam was well described by the model. However, the closed cell foams had modulus and strengths that fell well below the expected values.

Lu and Ong (2001) studied the deformation behavior of two types of aluminum alloy foam processed at the Southeast University in China. The measured stiffness and strength as functions of relative foam density were compared to those predicted by the phenomenological model. Overall, the prediction by Gibson and Ashby (1997) agreed well with the experimental measurements. Yet the morphological

imperfections such as fractured cell walls and holes should be considered. And the phenomenological model should be improved to account for cracking and softening mechanisms.

It is also of interest to study how the mechanical properties of cellular materials are affected by the meso-scopical structural parameters such as relative density, cell size and cell morphology. Gibson and Ashby (1997) developed skeleton cubic cells to analytically predict the mechanical properties of the foam material. Shu and Goh (2001) also used the cubic cell model to study the Young's modulus of foam materials.

In recent years, mesoscale finite element (FE) simulations have been carried out to investigate the influence of the micro-structural parameters of metallic honeycombs and foams on their compressive behavior. Note that the process in which the cellular materials are manufactured introduces geometric imperfections. Thus, mesoscale simulations considering the spatial distribution of such imperfections are necessary for quantitative accuracy.

Silva et al. (1995) generated FE models of 2D Voronoi foam. They investigated the influence of the variability of the arrangement of cell walls on the elastic moduli of the foam. Silva and Gibson (1997) continued the investigation on the influence of random cellular microstructures and missing cell walls on the Young's modulus and uniaxial yield strength of 2D Voronoi honeycombs.

Papka and Kyriakides (1998) simulated the honeycomb in full scale using ABAQUS. The cell sides were discretized with three-noded, quadratic beam elements. By incorporating key geometric, material and processing parameters in the numerical model, the numerical simulations of the expansion and crushing of honeycomb could reproduce the experimental results both qualitatively and quantitatively.

Further, Santosa and Wierzbicki (1998) developed a model of truncated cube which approximated better the morphology of foam. The geometrical model of the foam was established using mesh generator program HYPER-MESH, and the calculation of the numerical simulation was performed within PAM-CRASH. The effects of the thickness of cell wall and the cell size on the crushing behavior of foam were investigated and good predictions were given by the numerical analysis.

Chen et al. (1999) studied the influence of each of the six different types of morphological imperfection, namely, waviness, non-uniform cell wall thickness, cell-size variations, fractured cell walls, cell-wall misalignments, and missing cells, on the yielding of 2D cellular solids. It was found that fractured cell edges produce the largest knock-down effect on the yield strength of 2D foams, followed in order by missing cells, wavy cell edges, cell edge misalignments, Voronoi cells, and non-uniform wall thickness.

Meguid et al. (2002) employed a modified and representative unit cell model to study the crushing behaviour of closed-cell metallic foams with varying spatial density distribution. The non-uniform deformation patterns were found to be in good agreement only if the appropriate density distribution was taken into account.

2.3 Dynamic deformation of cellular materials

2.3.1 Experimental techniques for high rate compression test

Techniques used to obtain the mechanical properties of materials at high strain rate include dropweight machines, split Hopkinson pressure bars (SHPB), Taylor impact and so on.

SHPB technique, as shown in Fig. 2.3, has been widely used to determine the dynamic properties of a variety of engineering materials. SHPB experiments can provide complete dynamic stress-strain curve instead of a single point data.

However, if the sample is a soft material with low impedance such as a cellular material, the conventional SHPB technique needs to be modified before producing reliable dynamic data. Because of the drastic impedance mismatch, the transmitted signal can be too weak to be accurately measured. Viscoelastic bars, magnesium bars or hollow aluminum bars have been used to obtain a transmitted signal with sufficiently high signal-to-noise ratio (Zhao and Gary 1997). Deshpande and Fleck (2000a) adopted unsymmetrical SHPB setup, i.e. the output/transmitter bar was PMMA, while the striker and input/incident bars were steel.

Fig. 2.3 Configuration of Split Hopkinson Pressure Bars

On the other hand, the sample must have at least three to four cells in length to ensure it is statistically representative. The size of some available aluminum foams can be so large that the sample for test should be thicker than 40 mm which is much larger than conventional size. And it takes a longer time for the specimen to achieve dynamic equilibrium. Zhao (1998) suggested that the specimen thickness effect was resulted from the radial inertia and friction effect. However, the previous studies on specimen thickness effect only considered a specimen with thickness less than 15 mm (Chen et al. 2002; Zhao 1998)

2.3.2 Dynamic strength of cellular materials

When a cellular material is subjected to dynamic compression, the mechanical properties may be influenced by strain rate, inertia and entrapped air.

Mukai and co-workers (Kanahashi et al. 2000; Mukai et al. 1999a; Mukai et al. 1999b; Mukai et al. 2006) experimentally investigated the energy absorption in closed-cell aluminium foam, ALPORAS. The study concluded that the plateau stress of foam material exhibited remarkable strain rate sensitivity. This implied that foam material was able to absorb higher energy with higher strain rate. The absorption of energy at a strain rate of $2.5 \times 10^3 \text{ s}^{-1}$ was about 50% higher than that at a strain rate of $2.5 \times 10^{-3} \text{ s}^{-1}$.

The strain-rate sensitivity of closed-cell aluminium foam (ALPORAS) was also discussed by Paul and Ramamurty (2000). They concluded that the properties of metallic foam were highly sensitive to the strain rate. These results were explained with the aid of suitable micromechanical models such as micro-inertial effects against the bucking of cell walls at high strain rates.

Dannemann and Lankford (2000) also observed the strain rate sensitivity of the closed-cell aluminium foam (ALPORAS). They suggested that the strain rate effect could be related to fluid (air) flow through the ruptured cell walls. The open-cell Duocel foam was found to be rate-insensitive in their study.

Yi et al. (2001) tested an open-cell aluminum alloy foam fabricated by a powder metallurgical method. Within the strain rate range from 10^{-3} to 2600 s^{-1} , the experimental results showed that the yield strength and the energy absorbed increase with an increase of strain rate. It is noted that the sample thickness used in quasi-static test and dynamic test was 25 mm and 10 mm, respectively. This sample thickness effect was not discussed.

Zhang et al. (2002) further tested the dynamic compression properties of porous aluminum. The open-cell porous specimens were prepared by an air pressure infiltration process. It was found that the plateau stress at dynamic compression was much higher than that at quasi-static compression. They argued that at high rates the cells near the impact surface were more tightly compressed during densification and the highly localized nature of crushing results in very large strain-rates in the

crushing band. The two processes cause the enhancement of plateau stress. The specimens in the dynamic test had a diameter of 35 mm and a height of 8 mm. The uniaxial quasi-static compression tests with a strain rate 10^{-3} s^{-1} were carried out on an 810 Material Test System (MTS) for comparison. The specimens were 25 mm in diameter and 30 mm in height.

Park and Nutt (2002) tested a steel foam fabricated by a powder metallurgical process. It was shown that the yield strength of foam samples had strain rate dependence at higher strain rates, while the energy absorption increased linearly with strain rate.

Yu et al. (2006) experimentally investigated the in-plane quasi-static and dynamic behavior of circular-cell aluminum alloy honeycombs. The influence of strain rate (10^2 – 10^3 s^{-1}) on the localized deformation mode and the plateau stress has been found.

However, opposite statement has also been given that there was no or little strain-rate sensitivity in foam materials. Deshpande and Fleck (2000a) investigated the high strain rate compressive behavior of two cellular aluminum alloys (Alulight and Duocel) using the split Hopkinson pressure bar and direct impact tests. It was found that the plateau stress was almost insensitive to strain rate, for strain rates up to 5000 s^{-1} . The dynamic strength is 10% higher when the strain rate is 10^3 s^{-1} , but it was less than the scatter band.

Hall et al. (2000) also found that the strain rate had little or no influence to the plateau stress of Aluminum close-celled foams (Fraunhofer). Ashby et al. (2000) also indicated that the dependence of plateau stress on strain rate was not strong and it was important to separate the effect of strain rate and material inertia effect.

Ruan et al. (2002) also tested the compressive behaviour of CYMAT aluminium foams. An MTS machine was employed to apply a compressive load at strain rates of 10^{-3} – 10 s^{-1} to these closed-cell aluminium foams. It was observed that

deformation was not uniform over the whole sample: it first occurred in the weakest band, followed by the next weakest bands after the first one had been completely crushed. It was found that the plateau stress was insensitive to the strain rate.

Montanini (2005) tested the plateau stress of three types of aluminium foams (M-PORE, CYMAT, SCHUNK) using a bi-pendulum impact machine. It was shown that the dependence of the plateau stress on strain rate (up to 100 s^{-1}) could be considered negligible for M-PORE and CYMAT foams while it was quite remarkable for SCHUNK foams. Moreover, it was found that the peak stress of CYMAT foams is highly sensitive to loading rate.

Different from others, Reid and co-workers (Harrigan et al. 1999; Harrigan et al. 2005; Reid and Peng 1997; Reid et al. 1993; Tan et al. 2002; Tan et al. 2005a; Tan et al. 2005b) used a direct impact technique to study the dynamic uniaxial deformation of cellular materials. Wood specimens or foam specimens were fired directly to a transmitter bar, and then the crushing stress history was obtained from the measurement on the transmitter bar. A shock-like deformation mode in the specimen was observed and a significant enhancement of the crushing stress under dynamic loading conditions was found. They explained this stress enhancement by using a shock wave theory based on a rate-insensitive rigid-perfectly plastic-locking (RPPL) material model. It was argued that the micro-inertia of the matrix material played an important role in the shock wave theory.

Moreover, Gibson and Ashby (1997) discussed the entrapped fluid had influence on the material property. There was insufficient time for the air to escape through the small cracks and voids in the cell walls in high strain rate compression tests. Deshpande and Fleck (2000a) argued that the elevation of plateau stress due to the squeezing of gas was less than 1.5% of the static strength, and was thus within the scatter associated with these materials. They concluded that the compression of entrapped air gave a negligible contribution to the strength of Alulight closed-cell foam.

However, Dannemann et al. (2002) reported that the gas entrapped in the closed cells of Alporas foam could influence the failure pattern of the cell walls and consequently led to the strain rate strengthening effect.

2.3.3 Wave propagation in cellular materials

Reid et al. (1993) did some experiments on clusters of metal ring system. Crushing wave propagation effects had been identified when the ring system was subjected to high velocity impact.

Fatima Vaz et al. (1995) analytically and experimentally investigated the initiation and propagation of cell collapse in dynamic compression of cellular materials. The analytical work adopted one-dimensional mass-spring model. The element at which collapse initiated depended on the imposed velocity. At a large imposed velocity, the elements collapsed sequentially. At a low imposed velocity, the elastic wave effect made the collapse band distributed randomly, and usually originated at the fixed end due to the reflection of elastic wave.

Reid and Peng (1997) investigated the dynamic response of wood subjected to impact at high velocities (up to 300 m/s). It was observed that the highly localized crushing band propagated through the wood specimen. The propagation of the crushing wave was analogous to plastic shock wave in solid metals. A simple shock wave theory has been developed when the stress-strain curve of wood was idealized as rigid, perfectly-plastic, locking (RPPL) model.

Ashby et al. (2000) indicated that when the impact velocity was very high up to 50 m/s, a shock wave rose in the cellular material and its effect was expected to become significant and the response became dependent upon impact velocity. However, all the previous analytical and numerical studies relied on various simplifications.

Li and Meng (2002a) numerically studied the shock wave transmission in a cellular material employing one-dimensional mass-spring model. It was found that both elastic and shock wave exist in the wave transmission in cellular solid.

In the work done by Lopatnikov et al. (2003), the direct impact of closed-cell aluminum foam cylinders onto the output bar in a single pressure bar set-up was considered. It was observed that shock wave formed in the foam bar when the impact velocity was sufficiently high, as shown in Fig. 2.4.

Fig. 2.4 Deformed foam after high velocity impact (Lopatnikov et al. 2003)

2.4 Available material models

2.4.1 Continuum model

For numerical simulation, most researchers treated the cellular material as continuum (Deshpande and Fleck 2000b; Miller 2000; Schreyer et al. 1994). Hanssen et al. (2002b) reviewed several finite element constitutive models of the

commercially available software, LS-DYNA and ABAQUS, and the models proposed by other researchers, i.e. Miller (2000) and Deshpande and Fleck (2000b). These models had different formulas for yield surface, hardening rule and plastic flow rule, while none of them took fracture into account.

There are several cellular material models within LS-DYNA (Hallquist 2001), e.g., #26 for honeycomb, #63 for crushable foam, #75 for bilkhu/dubois foam, and #126 for the modified honeycomb. Hanssen et al. (2002b) calibrated and evaluated the four models and concluded that each model appeared to have its fault depending on the loading conditions and should be used with care. Recently, #163 material model for modified crushable foam has been developed and it can incorporate the strain rate effect.

The crushable foam model of ABAQUS appears to be equal with the model of LS-DYNA #75 apart from the description of the hardening of the yield surface. Both uniaxial and triaxial test data have to be used in the two models. While the model of Schreyer et al. (1994) only requires a knowledge of the uniaxial compressive yield stress.

Most models for cellular materials assume that the materials are isotropic. In between are the models by LS-DYNA #75, ABAQUS, Deshpande and Fleck (2000b) and finally Miller (2000). Limited evidence has suggested that this approximation was appropriate.

2.4.2 Lump mass-spring model

Mass-spring model is often used to simplify uniaxial analysis (Li and Meng 2002a; Shim and Yap 1997; Tu et al. 2001). The essential idea of this model is to idealize the material as a series of masses interconnected by plastic springs. The model is not only suitable for a system consisting of periodic structures such as a ring or a tube system, it is also capable of representing a macroscopically continuous cellular material. Its limitation is that it is greatly restricted to one-dimensional phenomena.

And it is worth noting that the discrete lump mass number may have influence on the performance when a finite number of masses are used to represent a cellular material.

Tu et al. (2001) even extended the spring-mass model to 2D studies. They discretized a block of polymer foam into contiguous element volumes, each represented by a mass point. Interactions between a node and its neighbors were accounted for by defining ‘connections’ that represent their interfaces which transmit stresses. The model even took the failure of cellular model into account. But further use of 2D mass-spring model has not been seen.

Depending on the mechanical properties of cellular material, the stress-strain curve of springs has three regions: linear-elastic region, plateau region and densification region. However, there is no publication to show the reloading behavior after unloading. Thus, the reloading stress is usually assumed to be zero before the current strain reaches the maximum strain it ever undergoes.

The mass-spring model could face computational instability when it is involved with shock wave propagation. The local strain of an individual spring could overflow 1.0. In order to deal with this problem, some viscosity of the element may be incorporated in the model, as in most commercial finite element packages.

2.5 Impact and blast barrier design using cellular materials

2.5.1 Cellular materials for impact buffering

Since 1970s, considerable attention has been paid to the large plastic deformation of metal structures. Such structures are suitable in energy absorption applications including the crashworthiness of vehicle, blast container, and so on (Jones and Abramowicz 1985; Reid 1985). It is the plastic crushing of metal tubes that dissipate the impact energy, including the lateral and axial compression of circular tubes, deformation of tubular beams subjected to transverse load. Filled tubes, such as polyurethane foam filled tube and sand filled tubular beam under axial

compression and transverse bending, have larger energy absorption capacity for the interaction effect between tube and filler.

Polymer foams and wood could not be used at high temperature environment. Metallic honeycombs and foams, however, have great advantages at this point of view. The long plateau region of a honeycomb or foam allows considerable energy absorption at nearly constant stress level. The work done in the plateau region is completely dissipated as plastic work. It is possible, as indicated by Baumeister et al. (1997), to use integral foams or composite materials made of aluminum foams to further improve the energy absorption capacity of the composite structure.

Aluminum foam-filled tubes (Hanssen et al. 1999) showed excellent energy absorbing properties. It has been found that the interaction of foam-wall enhanced the crushing strength of the foam-filled structures. Therefore, the crash box of car may be improved with the introduction of aluminum foam and composites, gaining less weight and higher compactness with even small modifications of existing geometry.

In the study of Evans et al. (1999), metal tubes have been used for energy absorption applications including the crashworthiness of vehicle, barrier design, and so on. However, tubes absorbed efficiently only upon axial loading. They were much less effective when impacted obliquely. By contrast, cellular media were nearly isotropic and omni-directional. Accordingly, when impacts from a range of directions were expected, foams were attractive. Upon impact, kinetic energy from the object must be dissipated by plastic work. The impact could be fully absorbed, without exceeding the plateau stress, provided the foam had an ample thickness.

Hanssen et al. (2001) experimentally assessed the optimum design of foam-filled column. It was found that the optimum foam filled columns compared to the traditionally designed non-filled columns showed smaller cross section dimensions in addition to less weight.

Armour integrated with an inserted foam layer (Gama et al. 2001) was a good attempt. In the composite plate, high hardness steel, aluminum foam, alumina ceramic were bonded together with epoxy adhesive. The composite structural armor plate could delay and attenuate the stress wave induced by a ballistic impact. However, the reduction of the amplitude of stress pulse transferred to the backing plate was not significant.

2.5.2 Cellular materials for blast mitigation

When an explosion happens, a spherical front of blast wave develops. The spherical pressure front can be approximated as a plane pressure wave if the distance of explosion is large as compared with the dimensions of the target surface which is subjected to the blast loading. The blast pressure decays exponentially, but it can be approximated as an equivalent triangular pressure pulse (Smith and Hetherington 1994). The nature of blast loading is always impulsive, a high intensity loading with a very short duration.

Some researchers (Ashby et al. 2000; Evans et al. 1999) have introduced the concept of blast amelioration by using cellular materials. A cover plate was attached at the front of the energy absorber made of cellular material. They assumed that the cover plate acquired the momentum first, and then compressed the cellular material absorber which dissipated all the energy from the blast shock. The stress applied on the protected objects would not exceed the plateau stress of the cellular material. The impulse from the explosion could be calculated according to the mass and standoff distance of the charge and then the critical thickness of the energy absorber could be derived.

There is no doubt that cellular material is in advantage to reduce structural damages from low velocity impact, which has been supported by experimental, numerical and theoretical research works as well as practical applications in various engineering fields. However, when a cellular material is used in protective structures to resist intensive blast load, the protective efficiency of cellular materials

has not been fully understood. Theoretical and experimental studies are necessary to validate the blast mitigating efficiency of cellular materials.

Guruprasad and Mukherjee (2000a) referred to the energy absorber made of metallic foam against blast load as a sacrificial cladding. In the event of a nearby explosion, the sacrificial layer undergoes a significant amount of deformation and in doing so absorbs energy. The main structure behind the sacrificial cladding is therefore protected. They also carried out free-field experiments on sacrificial layers subjected to blast loading (Guruprasad and Mukherjee 2000b), as shown in Fig. 2.5. The sacrificial layers consisted of unit cells of mild steel, which could be regarded as honeycomb showing near-perfect plastic collapse behavior. This sacrificial layer appeared to efficiently prevent the main structure from damage.

Fig. 2.5 A sacrificial cladding under blast loading
(Guruprasad and Mukherjee 2000b)

Li and Meng (2002a) numerically confirmed that the cellular material could attenuate the blast pressure to a lower level with a longer duration when the

intensity of the blast load was below a critical value. However, when the blast load was beyond the resistance capacity of the cellular material, the transmitted pressure could be larger than the plateau stress of the foam or even larger than the input peak pressure.

Furthermore, Ben-Dor et al. (2003) did another free-field experiment on two simply-supported concrete plates subjected to a moderate explosive load. One was covered with a foam panel; the other was not. The damage of the two plates was compared. It was observed after the explosion that the fractures at the backside of the foam protected plate was much less than that of bare plate.

However, Hanssen et al. (2002a) observed that the global response of the protected structure was increased by using a sacrificial foam layer. In their field tests on a pendulum subjected to a close-range blast loading, as shown in Fig. 2.6, the pendulum with a foam cladding swung to a higher position compared with that when the pendulum was not covered with a foam cladding.

Fig. 2.6 A pendulum with foam cladding under close-range blast loading
(Hanssen et al. 2002a)

According to Hanssen et al. (2002a), the transmitted pressure is reduced to a low level by the foam layer. However, the impulse is not reduced due to conservation of momentum. The natural period of the pendulum is much longer than the duration of the blast loading. In this case, the global response of the pendulum is directly proportional to the value of the impulse, i.e., the shape of the impulse is irrelevant. Since the impulse transferred to the main structure is equal with or without a sacrificial layer, this implies that the global response can not be reduced by the use of a sacrificial-layer cladding. It was then argued that the increase in the energy and impulse transfer was due to that the shape of the foam panel surface had changed into a dish shape during the blast event.

The concept of effective mass proposed by Hulton (2003) seemed to be able to help the understanding of the increased energy transfer of the cellular material layer. According to Hulton (2003), the energy transferred varied inversely with effective mass. The addition of the foam layer decreased the effective mass of target; hence the target absorbed more energy. The effective mass, however, is confusing and difficult to quantify.

Chapter 3 Loading Rate Effect on the Dynamic Behavior of Metallic Cellular Materials

3.1 Introduction

The deformation mode and the crushing strength of metallic cellular materials under dynamic compression are of great interest to researchers and engineers. Many impact tests have been conducted to investigate the dynamic behaviour of cellular materials. However, sometimes different or even controversial conclusions about the loading rate or strain rate effect on the crushing stress have been drawn (Deshpande and Fleck 2000a; Mukai et al. 1999a).

Ruan et al. (2003) carried out mesoscale finite element simulations to investigate the effect of impact velocity ranging from 3.5 m/s to 140 m/s on the deformation mode and the plateau crushing stress of a regular honeycomb. Zheng et al. (2005) extended the work of Ruan et al. (2003) by considering the irregularity of the cell geometries. Both studies showed that the crushing stress was relevant to the impact velocity. It left an impression that the crushing stress of the cellular material was sensitive to the loading rate.

In this chapter, an extensive literature review of the experimental results on the dynamic crushing of cellular materials is conducted. To understand the mechanism of the dynamic crushing of cellular materials, numerical simulations by employing a mesoscale Voronoi tessellation model are carried out. The foam specimen used in the numerical simulation is sandwiched between a moving rigid wall and a stationary rigid wall. This approximates the situation in an impact test for cellular materials. The crushing stress is measured at the impact side as well as the distal side. Some important factors that may influence the nominal crushing stress are discussed. Loading rate effect on the stress-strain relation of the cellular material is

clarified. Rate sensitivity of cellular materials is also discussed by comparing the results based on the mesoscale simulation with the results of a continuum model and a shock wave theory.

3.2 Experimental studies on the dynamic crushing of cellular materials

Equipment used to obtain the dynamic mechanical properties of cellular materials include split Hopkinson pressure bars (SHPB) (Fig. 3.1(a)), Taylor impact (Fig. 3.1(b) and (c)), Bi-pendulum (Fig. 3.1(d)), MTS (material testing system) and so on.

The SHPB technique provides an effective test method to obtain the stress-strain relations of tested materials at relatively high strain rates. The advantage of this technique is that the stress and strain of the specimen can be derived from the wave signals measured on the input and output pressure bars. However, due to the low impedance of the cellular materials, the forces at the specimen-steel input bar interface can not be resolved accurately because the incident and the reflected waves on the input bar have comparable magnitudes and they can even cancel each other. Meanwhile the strain signal on the transmitter bar is too weak to be measured accurately. Viscoelastic bars have been used to obtain a transmitted signal with sufficiently high signal-to-noise ratio (Zhao et al. 1997). However, dispersion and attenuation of stress waves in the viscoelastic pressure bars may bring uncertainties into the data.

There are some other requirements for the cellular material specimens in the SHPB test. A relatively thinner specimen is often required to achieve rapid stress equilibrium within the specimen and to achieve high strain rate and large final strain. However, the size of the specimen should contain at least 5 to 6 cells through the thickness in order to be statistically representative (Andrews et al. 2001).

Reid and co-workers (Reid and Peng 1997; Harrigan et al. 1999; Tan et al. 2002) used a direct impact technique, as shown in Fig. 3.1(b) to study the dynamic uniaxial

deformation of cellular materials. In their studies, wood or foam specimens were fired with very high velocities on an output bar, and then the stress history at the impact side was obtained. Further, Lee et al. (2006) studied the stresses at both the impact and distal sides by using direct and reverse impact tests, as shown in Fig. 3.1(b) and (c). Table 3.1 summarizes the test results done by different researchers.

(a) Split Hopkinson Pressure Bars (SHPB)

(b) Direct impact

(c) Reverse impact

(d) Bi-pendulum system

Fig. 3.1 Techniques for testing dynamic mechanical properties of materials

Table 3.1 Experimental studies on the dynamic behavior of cellular materials

No	Researcher	Sample	Density (kg/m ³)	Sample size (mm)	Apparatus & strain rate	Observation	Remark
1	(Zhao and Gary 1998)	Aluminum honeycomb	100~130	36×36×36 (L×W×H)	SHPB 50~770	Rate insensitive for in- plane crushing	
2	(Mukai et al. 1999a)	ALPORAS (Closed-cell)	270	9×9×6	SHPB ~2500	Plateau stress was 50% higher than Q-S data	Sample size 30×30×20 used in Q-S test
3	(Dannemann and Lankford 2000)	ALPORAS (Closed-cell)	189 or 405	Ø23.6×25.4*	SHPB 400~2500	Rate sensitive	
4	(Dannemann and Lankford 2000)	DUOCEL (Open-cell)	189	Ø9.5×19	SHPB 400~2500	Rate insensitive	
5	(Deshpande and Fleck 2000a)	DUOCEL (Open-cell)	189	Ø10×10	SHPB 1000~5000	Rate insensitive	A criterion of 20% was adopted
6	(Deshpande and Fleck 2000a)	ALULIGHT (Closed-cell)	459~1080	Ø10×10	SHPB 1000~5000	Rate insensitive	

Note: *Ø23.6×25.4 stands for 23.6 in diameter and 25.4 in thickness.

Table 3.1 Experimental studies on the dynamic behavior of cellular materials (continued)

No	Researcher	Sample	Density (kg/m ³)	Sample size (mm)	Apparatus & strain rate	Observation	Remark
7	(Hall et al. 2000)	Fraunhofer (Closed-cell)	340~821	Ø18×12	SHPB 300~2000	Rate insensitive	
8	(Paul and Ramamurty 2000)	ALPORAS (Closed-cell)	216~270	25×25×50	MTS $3.3 \times 10^{-5} \sim 1.6 \times 10^{-1}$	Rate sensitive	
9	(Yi et al. 2001)	Open-cell foam	405~783	Ø35×10	SHPB 700~1600	Rate sensitive	Sample size Ø35×25 in Q-S test
10	(Zhang et al. 2002)	Porous aluminum	1107~1161	Ø35×8	SHPB 600~2000	Rate sensitive	Sample size Ø25×30 in Q-S test
11	(Park and Nutt 2002)	Steel foam	3770	20×18×17	Instron, ~16	Rate sensitive	Base material is steel
12	(Ruan et al. 2002)	CYMAT (Closed-cell)	135~540	50×50×25	MTS ~10	Dynamic plateau stress increased by 5%	

Table 3.1 Experimental studies on the dynamic behavior of cellular materials (continued)

No	Researcher	Sample	Density (kg/m ³)	Sample size (mm)	Apparatus & strain rate	Observation	Remark
13	(Montanini 2005)	M-PORE (Open-cell)	143~221	50×50×25	Bi-pendulum ~100	Rate insensitive	
14	(Montanini 2005)	CYMAT (Closed-cell)	259~640	50×50×25	Bi-pendulum ~100	Initial peak stress greatly increased	
15	(Montanini 2005)	SCHUNK (Closed-cell)	194~750	50×50×25	Bi-pendulum ~100	Rate sensitive	
16	(Reid and Peng 1997)	Wood	260-1200	Ø75×75	30-300 m/s [#]	Crushing stress was significantly increased	Direct impact
17	(Harrigan et al. 1999)	Aluminum honeycomb	42	Ø45×30	20-300 m/s [#]	Crushing stress was significantly increased	Direct impact
18	(Tan et al. 2002)	Hydro foam (Closed-cell)	107-511	Ø45×45	10-210 m/s [#]	Crushing stress was significantly increased	Direct impact
19	(Lee et al. 2006)	DUOCEL (Open-cell)	189	17.8×17.8×25.4	63.5-83.8 m/s [#]	Stress was increased in direct impact but not in reverse impact	Direct and reverse impact

Note: [#] The impact velocity of the striker bar

The direct and reverse impact techniques can achieve high strain rate and large strain of sample. However, the velocity history of the striker is difficult to record. From a personal point of view, the bi-pendulum system is simple and easy to use. However, the measurement of the impact acceleration and velocity is not very accurate.

From Table 3.1, it is seen that the experimental studies on cellular materials vary in a very wide range in terms of material type, sample size, experimental setup and so on. Practically the stress-strain relation of a cellular material is from the measurement of the nominal stress at the impact side or the distal side of a certain specimen. Different experimental setups may affect the result of stress-strain relation significantly.

Mukai et al. (1999a) used a very small specimen $9 \times 9 \times 6$ mm (L $\times$ W $\times$ H) in their test while the mean cell size of the foam was 2.6 mm. The specimen did not contain enough cells through the thickness. Another two studies (Yi et al. 2001; Zhang et al. 2002) used thicker samples in the quasi-static tests while used thinner samples in the SHPB test. In their tests, the dynamic strengths of the foam were obviously higher than the quasi-static strengths. The sample size effect on the experimental studies was not addressed.

It is known that the strength of cellular materials depends on its relative density. The density variation of the samples undoubtedly leads to the strength variation. Therefore, Deshpande and Fleck (2000a) adopted a criterion of 20% to determine whether the dynamic crushing strength had an enhancement compared to a quasi-static test.

It is noted most of the tested metallic cellular materials are aluminum foams, except the steel foam in the study by Park and Nutt (2002). The steel material itself shows obvious rate sensitivity, which could attribute to the overall rate-sensitivity of the bulk foam.

It is seen from Table 3.1 that Ruan et al. (2002) and Montanini (2005) both tested the closed-cell foam CYMAT by using MTS and bi-pendulum, respectively. While Ruan et al. (2002) claimed that CYMAT had little rate-dependence, Montanini (2005) observed that the initial peak stress of CYMAT had a high sensitivity on the loading rate.

The rate-dependence of the closed-cell foam ALPORAS appears to be confirmed when several researchers (Dannemann and Lankford 2000; Mukai et al. 1999a; Paul and Ramamurty 2000) repeatedly tested it and obtained consistent results. But the mechanism of the rate sensitivity of ALPORAS is still unknown. Dannemann et al. (2002) argued that the entrapped air in the closed cells plays an important role under dynamic compression. However, the entrapped air in the cells is difficult to be modeled numerically.

In order to investigate the deformation behavior of cellular materials under different loading rates, a 2-D Voronoi structure is used to model the crushing process of cellular materials. Several factors which may influence the nominal stress-strain relation of the cellular material are discussed, namely, (i) sample size; (ii) inertia (iii) cell-wall shape; and (iv) rate-sensitivity of base material.

3.3 Mesoscale modeling of cellular material

Some researchers (Chen et al. 1999; Papka and Kyriakides 1998; Silva and Gibson 1997; Silva et al. 1995) have conducted mesoscale finite element simulation to study the quasi-static behavior of cellular materials. Although these studies were limited in quasi-static conditions or restricted to the behavior over small strain regime, they provided an effective method to qualitatively and quantitatively simulate a cellular material at the mesoscale.

A two-dimensional Voronoi diagram will be constructed to represent the cellular material. The Voronoi diagram is similar to the physical structure of foam materials. In many foaming processes, gas bubbles nucleate throughout the liquid metal and

grow, initially as spheres but later as polyhedral cells when the bubbles start to interact.

3.3.1 Construction of Voronoi diagram

The first step of construction of a Voronoi diagram is to generate n nuclei in a square area A_0 . Nucleation points are created in the square by generating x and y coordinates independently from pseudo-random numbers which are distributed evenly between 0 and 1. After the first point is specified, each subsequent random point is accepted only if it is greater than a minimum allowable distance from any existing point, until n nuclei are seeded in the square. A Voronoi diagram is then generated by constructing the perpendicular bisectors related to each pair of adjacent points, and trimming the lines (cell walls) where they intersected (Fig. 3.2).

Fig. 3.2 Voronoi diagram

In the current study, the construction of a Voronoi diagram is implemented by using Matlab program. When the Voronoi diagram is constructed, cell edges that are too short are stretched to a certain length. This can avoid the time step in the finite element computation being too small. The Voronoi diagram is then extruded along the normal direction of the plane so that a 3-dimensional cellular skeleton is constructed. Shell elements are used for the cell walls in the numerical simulation. The out-of-plane deformation of the cells is restricted.

3.3.2 Definition of cell regularity

According to Zhu et al. (2001), a regular 2-D foam material, composed of identical cells having six sides and vertex angles of 120, is a fully ordered 2-D Voronoi structure. For a regular 2-D Voronoi structure with n cells in the area A_0 , the distance d_0 between any two adjacent nuclei is constant and given by

$$d_0 = \sqrt{\left(\frac{2A_0}{n\sqrt{3}}\right)} \quad (3.1)$$

The regularity of a 2D random Voronoi foam can be measured by (Zhu et al. 2001)

$$\alpha = \frac{\delta}{d_0} \quad (3.2)$$

where δ is the minimum distance between any two nuclei. Fig.3.3 presents samples with different degrees of regularity: $\alpha=0.1$; 0.4 and 0.7.

Fig. 3.3 Voronoi foams with various regularities

The effect of the irregularity α has been studied by other researchers, for example, Zheng et al. (2005). Obviously the irregularity has an important influence on the static and dynamic strength of cellular materials. In the thesis, the focus is on the dynamic effect on the cellular material with the same irregularity and α is fixed as 0.4.

3.3.3 Imperfection of cell wall

Note that the process in which the cellular materials are manufactured introduces some geometric imperfections. Some researchers (Chen et al. 1999; Simone and Gibson 1998) have considered the influence of morphological imperfection of the individual cell wall of a regular honeycomb. In the current study, we continue to consider the imperfection of the individual cell wall of a random Voronoi structure. Two different types of imperfection are added to the cell walls. One imperfection stems from the fracture of cell wall. In this case, a certain number of random chosen shell elements are deleted from the finite element model. The other imperfection stems from the curved cell wall or corrugated cell wall. In this case, the originally straight cell edge is divided into several segments, each of the length about 0.5 mm. Then the endpoints of these segments are laterally shifted according to the following equation,

$$\Delta = \beta L_0 \sin\left(\frac{m\pi L}{L_0}\right), \quad m = 1, 2 \quad (3.3)$$

where Δ is the lateral offset distance; β is a coefficient and $\beta=0.05$ is used in the present study. L_0 is the length of the original cell edge. L is the local coordinate of the short segment which varies from 0 to L_0 . When $m=1$, the cell wall is called “curved wall”, as shown in Fig. 3.4(b). When $m=2$, the cell wall is called “corrugated wall”, as shown in Fig. 3.4(c).

Fig. 3.4 Voronoi structure with various shapes of cell walls

3.3.4 Finite element model

The finite element mesh is generated by using the commercial finite element software ANSYS. Each cell wall is discretized with several shell elements

according to its length, with the mean element length of about 0.5 mm, as shown in Fig. 3.5. There is only one segment along the z axis.

Fig. 3.5 Finite element model of the Voronoi foam

The node information and element information are written to a LS-DYNA keyword file. The keyword file can be edited and more control input can be added, such as material properties and contact definitions. Then the keyword file is run by the explicit LS-DYNA code. There are many options of shell element in LS-DYNA. In the current study, the fully integrated shell element (ELFORM=16 in LS-DYNA) is selected after some trial simulations. In the trial simulations, a regular honeycomb is simulated; with the shell element option ELFORM being 2, 9 or 16. The last choice gives the best results, with the crushing stress in good agreement with the prediction by (Gibson and Ashby 1997).

A Voronoi foam is constructed in an area of 100mm×100mm with 600 nuclei. A specimen of the size 50mm×50mm is tailored for the numerical crushing simulation. Each model consists of about 2800 shell elements. Due to the random construction of the Voronoi foam, the simulation is repeated for three times and the final crushing stress is taken to be the average value. The specimens with cell regularity degree $\alpha=0.4$, and with some imperfection in the cell walls (curved wall and 1% fractured) are discussed throughout the study unless otherwise specified.

The cell wall thickness is initially set as $h=0.12$ mm. The properties of the cell wall material are selected to be those of aluminum. An elastic-plastic material model (*Mat_plastic_kinematic in LS-DYNA) is employed, with isotropic hardening and von Mises yield criterion. Its density, Young's modulus, Poisson's ratio, yield stress, tangent modulus and failure strain are assigned as 2.7×10^3 kg/m³, 69 GPa, 0.3, 76 MPa, 0.69 GPa and 0.15, respectively.

In the simulation, the Voronoi foam is sandwiched between two rigid walls, as shown in Fig. 3.6. When crushed in the vertical direction, the top rigid wall moves downward with a constant velocity while the bottom rigid wall is held stationary. The lateral direction is not confined, namely, the left and right edges are free. Automatic single surface contact is applied to all the cell surfaces. And its contact edges can slip on both rigid walls with slight friction. The friction coefficient is taken to be 0.2.

Fig. 3.6 Schematic of Voronoi foam under impact (x-y plane)

Usually for a dynamic compression simulation (strain rate= 1.0×10^2 to 1.0×10^4 s⁻¹), the computational time is normally between 5 minutes and 5 hours. Therefore, there is no need to scale the impact velocity and material density. The time step size can

be determined automatically by LS-DYNA and be expressed as the following equation.

$$\Delta t = L_e / C = L_e \sqrt{\rho / E} \quad (3.12)$$

where L_e is the smallest element size, C is the speed of elastic wave in the base material, and ρ_s and E are respectively the density and Young's modulus of the base material (Aluminium or Steel).

For a quasi-static compression (strain rate= $1.0 \times 10^{-3} \text{ s}^{-1}$), the compression velocity is about $1.0 \times 10^{-5} \text{ m/s}$. It is necessary to reduce the excessively long computational time commonly encountered when dealing with the quasi-static problems. In the study, the rigid wall velocity is first scaled up by a factor of 1000 (the impact velocity is 0.05 m/s for a quasi-static simulation). Second, the density of the model is scaled up by a factor of 100 and consequently the time step is increased by a factor of 10. This so-called 'mass scaling approach' significantly reduces the total computational time. It should be mentioned that the velocity of rigidwall for the quasi-static compression (after scaling up by 1000) is at the order of 10^{-2} m/s , hence, much less than the speed of stress wave.

3.4 Deformation mode

It has been pointed out by Ruan et al. (2003) that the deformation mode of the foam is primarily influenced by the impact velocity. In the current study, we catalogued three deformation modes, namely, random mode, transitional mode and progressive mode, corresponding to increasing impact velocities, as shown in Fig. 3.7 (a)~(c).

Fig. 3.7 Deformation modes of foam corresponding to different impact velocities

Under low velocity impact, the deformation is firstly localized in one or two bands of collapsing cells, which are the weakest regions of the sample material. Then the

crushing band begins to diffuse and then all the cells collapse until they are fully densified. Basically, the deformation is randomly distributed.

Under moderate impact velocities, the inertia of the cells becomes important and the cells tend to collapse progressively starting from the impact side. Finally, when a foam material is impacted at a sufficiently high velocity, the deformation band is highly localized at the impact side (the deformation band appears within a cell). A crushing band initiated at the impact end and propagates along the impact direction. It is also called the shock wave phenomenon in the cellular material.

In order to quantitatively plot the deformation process, the cellular material is equally divided into ten parts in the impact direction (see Fig. 3.6). The displacements of five selected nodes at every boundary of the ten parts are traced. Then the strains of each part are approximately derived. Figs. 3.8 (a)~(c) show the strain profile of the ten parts under low, moderate and high velocity impact. The transformation of deformation pattern from the random mode to progressive mode with increasing impact velocity is clearly plotted.

Fig. 3.8 (a) Strain distribution under low velocity impact
($V=5$ m/s, $H=50$ mm)

Fig. 3.8 (b) Strain distribution under moderate velocity impact
($V=25$ m/s, $H=50$ mm)

Fig. 3.8 (c) Strain distribution under high velocity impact
($V=50$ m/s, $H=50$ mm)

The differentiation of deformation mode is quite subjective in previous studies, e.g., (Ruan et al. 2003; Zheng et al. 2005). In the current study, a criterion by comparing the shortenings of the first half and the second half is introduced to determine the specific deformation mode. The relative shortening of the first half (at the impact side) is expressed as δ_1 and the relative shortening of the second half (at the stationary side) is written as δ_2 . The criterion of the deformation mode, as below, is defined as the ratio of the relative shortening of the two halves at the moment when the overall shortening is 50%, i.e., $(\delta_1 + \delta_2) / 2 = 0.5$.

$$\frac{\delta_1}{\delta_2} < 1.5, \quad \text{Random mode} \quad (3.4a)$$

$$1.5 \leq \frac{\delta_1}{\delta_2} < 3.0, \quad \text{Transitional mode} \quad (3.4b)$$

$$\frac{\delta_1}{\delta_2} \geq 3.0, \quad \text{Progressive mode} \quad (3.4c)$$

The criteria for random deformation ($\delta_1/\delta_2=1.5$) has taken account of the non-uniform nature of the cellular material. The criteria for the progressive deformation is simply double the value for the random deformation. The values of the criteria should be assessed in the future.

Fig. 3.9 Deformation mode map for the Voronoi foam

In the study, we vary the thickness of the cell wall to see how the relative density influences the deformation mode. When the thickness of the cell wall is 0.09 mm, 0.12 mm, 0.2 mm, the corresponding relative density of the foam specimen is 4.4%, 5.9% and 9.8%, respectively. It is obvious that an individual cell wall becomes stronger in bending when its thickness increases. It is seen from Fig. 3.9 that the critical velocity that differentiates the deformation modes increases with an increase of thickness of cell wall. This trend is consistent with Tan et al. (2005b).

3.5 Crushing strength of cellular materials

3.5.1 Quasi-static compressive strength

It has been proved that the quasi-static compressive strength of cellular material is a function of its relative density. In the current study, the thickness of the cell walls (h) is varied to change the relative density of the bulk foam. Each shell element has the same thickness in the current numerical model, which is a simplification of a true heterogeneous cellular material. The strengths of cellular materials with different relative densities are plotted in Fig. 3.10. It is seen that the strength of the cellular material increases with an increase of its density.

Fig. 3.10 Quasi-static strength of cellular material ($H=50$ mm)

3.5.2 Dynamic nominal stress and sample size effect

In the present study, the nominal stress and the nominal strain up to densification are measured from the numerical model. The nominal stress is derived from the reaction force at the rigid wall divided by the bonding area between the rigid wall and the sample. The nominal stress may be different at the impact end and the distal end. The nominal strain is calculated as the total compression of the sample divided by the sample height (H).

Fig. 3.11 gives the nominal stress at the impact end vs. nominal strain at different velocities. It is seen that initial peak stress and plateau stress (at the plastic region) increases with an increase of impact velocity. This is consistent with the studies by (Ruan et al. 2003; Zheng et al. 2005). It is also interesting to see that while the nominal stress has a strain-hardening trend in quasi-static condition, the nominal stress under high velocity impact appears to fluctuate over a constant value.

Fig. 3.11 Nominal stress vs. nominal strain at the impact side
(H=50 mm, h=0.12 mm)

The spikes of the curves in Fig. 3.11 are also seen in the test results in the literature, for example, Radford et al. (2005). The reason of the spikes could be that the local buckling of a cell or a cell wall under a high velocity impact have a higher mode (with unstable higher stiffness) compared to that under a low velocity impact.

The nominal stress at the stationary side under high velocity impact is so different from that at the impact side, as shown in Fig. 3.12. It is seen that the plateau stress at the stationary side over small strain range is nearly the same as the quasi-static value no matter what the impact velocity is. However, the stress over large strain range deviates from the quasi-static value. It should be mentioned that the stress at stationary side has not been discussed by Ruan et al. (2003) or Zheng et al. (2005).

Fig. 3.12 Nominal stress vs. nominal strain at the stationary side
($H=50$ mm, $h=0.12$ mm)

The nominal stress-strain curve may be different when the sample size is changed. In the following discussion, we will tailor the original 50×50 mm specimen into two specimens of equal thickness ($H=25$ mm), and then further sub-divide into four equal thin specimen ($H=12.5$ mm). The width of the samples remains 50 mm. The thin sample contains insufficient cells to statistically represent the cellular material.

Hence four thin samples are tested and the average nominal stress-strain curve is adopted.

Fig. 3.13 gives the nominal stress-strain curves at the impact side of the thin sample. For a specific impact velocity, the nominal stress at the impact surface is almost at the same level for the thin and thick samples by comparing Fig. 3.11 and Fig. 3.13. This reveals that the nominal stress at the impact side is primarily related to the impact velocity (V) rather than the apparent strain rate (V/H).

The nominal stress at the stationary side of the thin sample is shown in Fig. 3.14. It is seen that the nominal stress-strain relation at the stationary side has no appreciable dependency on the impact velocity (V) or apparent strain rate (V/H). If only the nominal stress at the stationary side is considered, rate-insensitive of the dynamic strength may be concluded. The apparent strain rate in Fig. 3.14 is up to 4000 /s for the 50 m/s impact.

Fig. 3.13 Nominal stress vs. nominal strain at the impact side
($H=12.5$ mm, $h=0.12$ mm)

Fig. 3.14 Nominal stress vs. nominal strain at the stationary side
($H=12.5$ mm, $h=0.12$ mm)

3.5.3 Inertia effect

The loading rate effect on the nominal stress at the impact and stationary sides may be caused by the micro-inertia of cell walls and the strain rate sensitivity of the cellular material. It is difficult to differentiate the inertia effect and the strain rate effect in a physical impact tests. The loading rate effect may thus be misunderstood as the strain rate sensitivity of the material. This section is to investigate the variation of the nominal stress merely due to the inertia effect. A solid continuum model (50×50 mm) is developed with LS-DYNA, as shown in Fig. 3.15. The continuum material model #63 is used to represent the cellular material. The quasi-static stress-strain curve in Fig.3.11 is adopted for the solid material. The stress-strain relationship used in the simulation of the continuum material is strain rate insensitive. The continuum model is also sandwiched in two rigid walls, with the same loading condition as that of the Voronoi model.

Fig. 3.15 Continuum finite element model

Fig. 3.16 gives the nominal stress-strain curves at the impact side. It is seen that the initial peak stress and plateau stress increase with the increase of the impact velocity. The increase of the nominal stress of the continuum model is merely due to the inertia effect since the constitutive material of the continuum model is rate insensitive. The magnitude of the plateau stress of the continuum model agrees with that of the Voronoi model in Fig. 3.11. One can thus claim that the loading rate effect of dynamic stress-strain relationship of the Voronoi foam is mainly from the inertia effect.

Fig. 3.16 Nominal stress vs. nominal strain at the impact side of the solid continuum model

Fig. 3.17 gives the nominal stress-strain curves at the stationary side. It is seen again that the magnitude of the plateau stress of the continuum model agrees with that of the Voronoi model in Fig. 3.12. However, there is a delay of the stress at the initial stage probably due to that an effective Young's modulus is used in the continuum model and the propagation speed of the elastic stress wave is much smaller than that in the Voronoi model.

Fig. 3.17 Nominal stress vs. nominal strain at the stationary side of the solid continuum model

It should be mentioned that in a typical SHPB test on solid metals, the average value of the stresses at both ends of the sample is used to calculate the nominal stress of the sample. The inertia force may be negligible for a solid metal in the SHPB test. However, it is conjectured that the inertia force may significantly affect the calculated stress-strain curve for a cellular material. The density of cellular material is in the order 1/10 of the solid metal while the quasi-static compressive strength is in the order of 1/100 of the solid metal.

Undoubtedly, if the average nominal stress of the sample is used, the cellular material will show considerable dependency on the impact velocity (V) as well as

the apparent strain rate (V/H). This may be a possible reason that some experimental studies by using SHPB claimed that the dynamic strength of a cellular material increases with the increase of the apparent strain rate.

This inertia effect can also explain why Ruan et al. (2002) and Montanini (2005) gave different conclusions when both tested the closed-cell foam CYMAT. The reason may be that Montanini (2005) measured the stress at the impact side (see Fig. 3.1(d)) while Ruan et al. (2002) measured the stress at the stationary side.

Zhao and Gary (1998), Deshpande and Fleck (2000a) and Lee et al. (2006) took the nominal stress at the sample-output bar interface to calculate the strength of a foam material. It seems reasonable considering that the nominal stress at the sample-output bar interface (almost stationary) is less affected by the inertia force.

3.5.4 Effect of cell wall shape

When the cellular material is subjected to an impact, the cell walls buckle and then progressively collapse. Different shapes of cell walls could lead to different deformation modes and show different overall crushing strengths. In the current study, we consider three different shapes of cell walls, namely, perfectly straight, curved and corrugated. It should be noted that the cell walls of the real foam is much more irregular.

The difference among the nominal stress-strain curves of the cellular materials with different cell wall shapes is negligible, as shown in Fig. 3.18. This reveals that the overall morphology (structure) of a cell and the overall organization of the cells are more important for the compressive strength of the cellular material.

Fig. 3.18 Influence of cell wall shape on the nominal stress at the stationary side
($V=15$ m/s, $H=25$ mm, $h=0.12$ mm)

3.5.5 Rate sensitivity of base material

All the cellular materials given in Table 3.1 except the steel foam used by Park and Nutt (2002) are made of aluminum alloy. For example, ALPORAS is made of Al-Ca-Ti alloy and Alulight is made of Al-Si alloy. In order to incorporate the strain-rate sensitivity of the base material, the well-known Cowper-Symonds relation is adopted for the plastic deformation of the material, i.e.,

$$\sigma_y^d = \sigma_y^s \left(1 + \frac{\dot{\epsilon}}{C}\right)^{\frac{1}{P}} \quad (3.5)$$

where σ_y^d denotes the dynamic yield stress of the material; σ_y^s denotes the quasi-static yield stress and $\dot{\epsilon}$ is the strain rate. Usually aluminum alloy shows weak rate independence. The two parameters for aluminum alloy can be taken as $C=6500$ s⁻¹ and $P=4$, which is consistent with Su et al. (1995). For comparison purpose, it is assumed that the base material could be as rate sensitive as for a mild steel with $C=40$ s⁻¹ and $P=5$.

The nominal stresses at the stationary end of the Voronoi structure in Fig. 3.7 are compared for the base materials having different rate sensitivities, as shown in

Fig. 3.19. In the numerical model for the three cases, only the coefficients of Cowper-Symonds relation (C and P) are different, while the other material parameters, such as the Young's modulus and the yield stress of the matrix material (base material), are exactly the same. It is seen in Fig. 3.19 the overall bulk foam becomes appreciably rate sensitive when the base material is as rate sensitive as for mild steel. It should be mentioned that the stress-strain curves at the impact side have the same tendency for different C and P values.

Fig. 3.19 Influence of rate sensitivity of base material on the nominal stress at the stationary side ($V=15$ m/s, $H=25$ mm, $h=0.12$ mm)

In order to quantify the rate sensitivity, Fig. 3.20 gives the internal energy of the three foams versus the plastic deformation. When the base material has weak rate dependency ($C=6500$ and $P=4$), the energy absorbed by the foam is 109% of that in the baseline case for which the base material is rate insensitive. When the base material has strong rate dependency ($C=40$ and $P=5$), the energy absorbed by the foam is 133% of that in the baseline case. It should be noted that the relative increase of the absorbed energy may vary with the cell size, cell wall thickness and so on.

Fig. 3.20 Influence of rate sensitivity of base material on the absorbed energy
($V=15$ m/s, $H=25$ mm, $h=0.12$ mm)

3.6 Comparison with shock wave theory

Reid and Peng (1997) have developed a shock wave theory to explain the dynamic enhancement of the crushing stress at the impact end. The shock wave theory relies on an idealized rigid-perfectly plastic-locking (RPPL) stress-strain model, as shown in Fig.3.21.

The RPPL stress-strain curve for the material has a constant plastic stress σ_0 (plateau stress) without elastic deformation. The material is then locked at the densification strain ε_D and no further deformation is allowed. The two parameters of the RPPL model can be determined with reference to Li et al. (2005),

$$\varepsilon_D = 1 - \lambda \rho_0 / \rho_s \quad (3.6)$$

$$\sigma_0 = \frac{1}{\varepsilon_D} \int_0^{\varepsilon_D} \sigma d\varepsilon \quad (3.7)$$

where ρ_0 is the density of the cellular material, and ρ_s is the density of the solid aluminum alloy. λ is a coefficient and $\lambda=3.0$ is adopted in the present study. Then the two important parameters for the RPPL idealization are derived as, $\varepsilon_D=0.82$,

$\sigma_0 = 0.167$ MPa. It should be noted that the strain rate effect is not considered in the shock wave theory.

Fig. 3.21 Idealization of the quasi-static stress-strain curve (RPPL material model)

A schematic view of the shock wave propagation in a cellular material sample is shown in Fig. 3.22.

Fig. 3.22 Schematic view of shock wave model

Upon impact, a shock wave forms and propagates through the material. Behind the shock wave, the material has fully collapsed. The strain is ε_D and the density is $\rho_0/(1-\varepsilon_D)$ in the collapsed region. The particle velocity in this region can be approximated to be equal to V and the stress σ_D , which is assumed constant, is unknown. Ahead of the shock wave, the material is assumed to be at rest, with deformation $\varepsilon=0$ and particle velocity $V_p=0$. This region is under a constant stress σ_0 .

Conservation of mass across the shock front yields the relationship as follows,

$$\rho_0 V_s = \frac{\rho_0}{1-\varepsilon_D} (V_s - V) \quad (3.8)$$

or,

$$V_s = \frac{V}{\varepsilon_D} \quad (3.9)$$

Eq. (3.9) provides the shock wave speed as a function of impact velocity and material densification strain.

As the shock wave progresses, the material immediately ahead of the shock front will be involved into the shock front and be accelerated instantaneously. The shock wave accelerates a mass equal to $\rho_0 V_s$ per unit time to the velocity of V , which results in a momentum transfer of $\rho_0 V_s V$. The conservation of momentum provides the following equation,

$$\sigma_D - \sigma_0 = \rho_0 V_s V \quad (3.10)$$

From the above equation, we can see that the mechanism of the increase of the stress across the shock front is associated with the inertia of the material at the shock front. In other words, the shock wave theory is essentially considering the inertia effect.

Eliminating V_s from Eqs. (3.9) and (3.10) yields the stress in the densified region, namely,

$$\sigma_D = \sigma_0 + \frac{\rho_0 V^2}{\varepsilon_D} \quad (3.11)$$

The average value of the nominal stresses (over the strain range 0~0.8) at the impact end based on the mesoscale model and the continuum model are given in Fig. 3.23 and compared with the prediction by the shock wave theory. It is seen that the shock theory slightly overpredicts the stress at the impact end. The discrepancy may result from the idealized RPPL model, which neglects the elastic deformation. The nominal stress at the stationary side can be derived as a constant based on the shock wave theory.

Fig. 3.23 Influence of impact velocity on the nominal stress at the impact side

The mesoscale model, the continuum model and the shock wave model are all useful for modeling of the cellular materials. The mesoscale model for a regular honeycomb has been validated by Papka, and Kyriakides (1998). However, it is difficult to accurately simulate an irregular foam by using mesoscale model. The continuum model is usually based on some static test results on samples. A continuum model is normally uniform (each element has the overall stress-strain characteristics of the sample) and the random characteristics of a foam can not be

captured. The shock wave model uses a rigid-perfectly plastic-locking stress-strain model, which is less suitable for a cellular material with a slope plastic behaviour.

3.7 Summary

A Voronoi structure has been numerically simulated to investigate the loading rate effect on the deformation mode and compressive strength of cellular materials. Three deformation modes are categorized. It is seen the impact velocity play a dominant role in the overall deformation mode. Under high velocity impact, the crushing band initiates at the impact side and propagates along the impact direction.

The nominal stresses at the impact end and the stationary end are both discussed in the current study. Several factors that can lead to an increased nominal stress under dynamic conditions have been discussed. The enhancement of the nominal stress at the impact side under dynamic conditions is primarily due to the micro-inertia of cell walls. It is important to exclude the inertia force from the measured nominal stress at the impact side. The nominal stress at the stationary side is found to be insensitive to loading rate unless the base material has strong rate-dependency.

The appreciable rate-sensitivity of some cellular materials, eg., ALPORAS, can not be explained by the current study. One possible factor causing the rate sensitivity is that the entrapped air in the closed cells has an influence on the deformation of the cell wall. The entrapped air, however, is not modeled in the present study.

In summary, the experimental observations in Table 3.1 can be clarified into the following categories based on the present numerical simulations using the mesoscale Voronoi model.

- Rate insensitive when the stress at the stationary end or sample-output bar interface is used to calculate the material strength, such as in Test No. 1, 4, 5, 6, 7, and 12;
- Rate sensitive when the base material is rate sensitive, such as in Test No. 11.

- Pseudo rate sensitive when the stress at the impact side is used to calculate the material strength, such as in Test No. 14 and 15;
- Significant increase of nominal stress due to inertia effect in a direct impact, which can be approximated by the shock wave theory, such as in Test No. 16~19.
- Rate sensitive of foam ALPORAS due to uncertain factors, for Test No. 2, 3, and 8.
- Other uncertainties, for Test No. 9, 10 and 13.

Chapter 4 Resistance of Cellular Material against Blast Loading

4.1 Introduction

It has been found that when a cellular material is subjected to a high velocity impact, a shock wave initiates and the cellular material may undergo progressive collapse. Blast loading is one of the possible causes of the shock waves. However, limited research work has been done on the deformation analysis for cellular materials under blast loading.

In this chapter, the deformation behavior of cellular material subjected to blast loading is studied numerically by the mesoscale model and the continuum model and analytically by the shock wave theory. The blast resistant capacity of the cellular material is predicted.

4.2 Mesoscale numerical modeling of cellular material under blast loading

Usually a blast loading can be simplified as a triangular pulse that has the form of,

$$P(t) = \begin{cases} P_0(1 - t/t_0) & \text{for } t \leq t_0 \\ 0 & \text{for } t > t_0 \end{cases} \quad (4.1)$$

where P_0 is the initial peak pressure of the blast load, t_0 is the loading duration.

The input total impulse per unit area is $I_0 = \frac{1}{2} P_0 t_0$.

In the numerical analysis, a 2D Voronoi foam sample (50×50 mm) is modeled at the mesoscale as shown in Fig. 4.1. The density of the cellular material is 158 kg/m³, the mean cell size is about 4.4 mm. The horizontal expansion of the cellular

material is constrained and the cellular material undergoes uniaxial strain deformation along the loading direction.

Fig. 4.1 Mesoscale model of a cellular material under blast loading

A cover plate is added at the front surface of the cellular material. The steel cover plate is 0.4 mm thick. It is assumed that the blast load acts on the cover plate uniformly. The blast loading discussed in Section 4.4 is $P_0=4.0$ MPa, $t_0=0.1$ ms. The input impulse per unit area is 0.2 MPa-ms.

The cover plate, the cellular material sample and the rigid wall are connected in two possible ways, which will be respectively discussed in Sections 4.2.1 and 4.2.2. In a trial study in Section 4.2.1, the cover plate and the cellular material are simply placed together without bonding (simply connected).

4.2.1 Simply connected sample

Figs. 4.2 (a)~(d) show the deformation sequence of the cellular material under the blast loading. The deformation pattern of the cellular material seems to be a combination of the progressive mode and the random mode described in Chapter 3. When the cover plate moves fast, the cellular material deforms progressively. When

the cover plate slows down with the decay of the blast pressure, the cellular material deforms in the random mode.

The cellular material reaches the maximum compressive state (Fig. 4.2(c)) when the velocity of the cover plate reduces to zero. It is interesting to see that the cellular material elastically recovers (springback) from its maximum strain. Then the cover plate detaches from the cellular material and the cellular material detaches from the bottom rigid wall (rebound), as shown in Fig. 4.2(d).

(a)

(b)

(c)

Fig. 4.2 Compression, springback and rebound of the cellular material (simply connected) under blast loading

Fig. 4.3 gives the transmitted pressure history by the cellular material. The average pressure is approximately equal to the plateau stress of the cellular material and much lower than the input peak pressure. Integration of the pressure in Fig. 4.3 over the response time gives the impulse imparted to per unit area of the rigid wall, as given in Fig. 4.4. The transferred impulse is about 10% larger than the input impulse. This is due to that some extra impulse is transferred to the rigid wall when the cellular material and the cover plate rebound upwards (the cover plate and the cellular material have an upward momentum).

Fig. 4.3 Transmitted pressure by the cellular material on the rigid wall

Fig. 4.4 Impulse applied on per unit area of the rigid wall

4.2.2 Tightly connected sample

The cellular material is glued to the cover plate and the rigid wall or the protected object (tightly connected). In the numerical model, the top nodes of the cellular material are tied to the cover plate and the bottom nodes are constrained to the rigid wall. By doing so, the cellular material and the cover plate shall not rebound from the rigid wall. Instead, they will be held back when they tend to move backwards, as seen in Fig. 4.5(d).

Fig. 4.5 Compression, springback and tension of the cellular material (tightly connected) under blast loading

Fig. 4.6 gives the transmitted pressure history by the cellular material. It is seen the pressure is positive prior to the time of 2 ms. Then there is a negative pressure regime when the cellular material springbacks from the rigid wall. The cellular material is in tension then.

The impulse imparted to the rigid wall during the positive pressure period is also 10% higher than the input impulse, as seen in Fig. 4.7. The impulse imparted to the rigid wall over time from 0 to 5 ms is equal to the input impulse (The cellular

material and the cover plate cease motion at the time of 5 ms). This is consistent with the law of momentum conservation.

Fig. 4.6 Transmitted pressure by the cellular material on the rigid wall

Fig. 4.7 Impulse applied on per unit area of the rigid wall

4.3 Analytical modeling of cellular material under blast loading

4.3.1 Analytical model

In the analytical model, the cover plate is considered as a rigid body, with a mass of m_1 . It is assumed that the cover plate is bonded with the cellular material sample and the cellular material sample is bonded with the rigid wall. The cellular material can be idealized as a RPPL material model with a plateau stress σ_0 , as presented in Section 3.6.

It is assumed that the cellular material layer always deforms progressively from the loaded end, as shown in Fig. 4.8, and a shock wave forms. Undoubtedly this assumption is reasonable when the cover plate has a very high velocity. It should be noted that the deformation pattern of the cellular material becomes random mode when the velocity of the cover plate becomes low. However, the energy absorption of the assumed progressive collapse is nearly the same as the random deformation under quasi-static and low velocity impact conditions, as proved in Lopatnikov et al. (2006).

Based on the assumption of progressive deformation, at the upstream of the shock front (Fig. 4.8), the cellular material is compacted up to the locking strain ε_D and its stress increases to σ_D . At the downstream of the shock front, the strain is zero and the stress is σ_0 .

Fig. 4.8 Schematic of a cellular material under blast loading

The densified part of the cellular material moves together with the cover plate, which is considered as a rigid body. The relationship between the mass of the densified part of the cellular material and the displacement of the cover plate is given by,

$$\Delta m = \frac{\rho_0 A}{\varepsilon_D} u \quad (4.2)$$

According to the mass and momentum conservation laws for the transition part at the shock front, which is the increment of the densified part as shown in Fig. 4.8, σ_D can be derived as (refer to Eq. (3.11)),

$$\sigma_D = \sigma_0 + \frac{\rho_0}{\varepsilon_D} \dot{u}^2 \quad (4.3)$$

In light of the Newton's second law the equation of motion for the densified cellular material can be expressed as,

$$[m_1 + \Delta m] \ddot{u} + (\sigma_D - P(t))A = 0 \quad (4.4)$$

where $P(t)$ is the blast loading, which is expressed in Eq. (4.1). Considering Eqs. (4.2) and (4.3), the above equation can be rewritten as,

$$\left[m_1 + \frac{\rho_0 A}{\varepsilon_D} u \right] \ddot{u} + \frac{\rho_0 A}{\varepsilon_D} \dot{u}^2 + (\sigma_0 - P(t))A = 0 \quad (4.5)$$

The above equation can be solved with the aid of a mathematical tool Maple 5.1 (Maple 1998),

(i) Phase 1: $0 < t \leq t_0$, where t_0 is the duration of the blast loading.

The solution of Eq. (4.5) is derived as,

$$u(t) = \frac{-m_1 + \sqrt{m_1^2 + \rho_0 A^2 t^2 (P_0 - \sigma_0 - P_0 t / 3t_0) / \varepsilon_D}}{\rho_0 A / \varepsilon_D} \quad (4.6)$$

$$\dot{u}(t) = \frac{At(P_0 - \sigma_0 - P_0 t / 2t_0)}{\sqrt{m_1^2 + \rho_0 A^2 t^2 (P_0 - \sigma_0 - P_0 t / 3t_0) / \varepsilon_D}} \quad (4.7)$$

If $P_0 \leq 2\sigma_0$, the cover plate will cease motion at $t = \frac{2(P_0 - \sigma_0)}{P_0} t_0$.

Otherwise, the cover plate and the densified part has a velocity of V_1 and a displacement of U_1 when $t = t_0$,

$$\dot{u}(t_0) = V_1 = \frac{At_0(P_0 - 2\sigma_0)}{2\sqrt{m_1^2 + \rho_0 A^2 t_0^2 (2P_0/3 - \sigma_0) / \varepsilon_D}} \quad (4.8)$$

$$u(t_0) = U_1 = \frac{-m_1 + \sqrt{m_1^2 + \rho_0 A^2 t_0^2 (2P_0/3 - \sigma_0) / \varepsilon_D}}{\rho_0 A / \varepsilon_D} \quad (4.9)$$

The total mass including the cover plate and the densified part of the cellular material is,

$$M_1 = m_1 + \frac{U_1}{\varepsilon_D} \rho_0 A = \sqrt{m_1^2 + \rho_0 A^2 t_0^2 (2P_0/3 - \sigma_0) / \varepsilon_D} \quad (4.10)$$

(ii) Phase 2: $t_0 < t \leq \frac{P_0 t_0}{2\sigma_0}$.

At this phase, the overpressure $P(t)$ has dropped to zero. Similar to Eq. (4.5), the motion equation of the cover plate is then expressed as,

$$\left[M_1 + \frac{\rho_0 A}{\varepsilon_D} u' \right] \ddot{u}' + \frac{\rho_0 A}{\varepsilon_D} \dot{u}'^2 + \sigma_0 A = 0 \quad (4.11)$$

where, $u' = u - U_1$, and $U_1 = \frac{M_1 - m_1}{\rho_0 A / \varepsilon_D}$ (See Eq. (4.10)).

The solution of Eq.(4.11) can be derived as,

$$u(t) = U_1 + \frac{-M_1 + \sqrt{M_1^2 + 2\rho_0 A J_1 t' / \varepsilon_D - \sigma_0 \rho_0 A^2 t'^2 / \varepsilon_D}}{\rho_0 A / \varepsilon_D} \quad (4.12)$$

where $t' = t - t_0$, and $J_1 = M_1 V_1$.

Substituting Eqs. (4.8–10) into Eq. (4.12) yields,

$$u(t) = \frac{-m_1 + \sqrt{m_1^2 + \rho_0 A^2 P_0 t_0^2 \left[-\frac{1}{3} + \frac{t}{t_0} - \frac{\sigma_0}{P_0} \left(\frac{t}{t_0} \right)^2 \right] / \varepsilon_D}}{\rho_0 A / \varepsilon_D} \quad (4.13)$$

$$\dot{u}(t) = \frac{A(P_0 t_0 / 2 - \sigma_0 t)}{\sqrt{m_1^2 + \rho_0 A^2 P_0 t_0^2 \left[-\frac{1}{3} + \frac{t}{t_0} - \frac{\sigma_0}{P_0} \left(\frac{t}{t_0} \right)^2 \right] / \varepsilon_D}} \quad (4.14)$$

From Eq. (4.14), we can see that the cover plate will cease motion at $t = \frac{P_0}{2\sigma_0} t_0$.

4.3.2 Comparisons with numerical results

To verify the analytical solution, the deformation of the cellular material under blast loading derived by the shock wave theory is compared to the results predicted by the mesoscale model and an equivalent continuum model. The 50×50×0.5 mm cellular material sample shown in Fig. 4.1 has the same cell size as that in Section 3.5.1. Hence the stress-strain curve can be referred to Fig. 3.10 (see the stress-strain curve when relative density=5.9%). The equivalent continuum model is similar to the continuum model in Fig. 3.15. However, a 0.4 mm thick plate is covered on the

top of the cellular material. The material property of the continuum model can also be referred to Fig. 3.10. In the analytical model, the idealized plateau stress of the cellular material is 0.167 MPa and the densification strain is 0.82 (refer to Fig. 3.21).

The velocity history of the cover plate under different blast loads predicted by the shock theory, the mesoscale model and the continuum model, are given in Figs. 4.9(a)~(c). It is seen that the results derived by the three methods are in good agreement. However, the cover plate in the mesoscale model and the continuum model has a negative velocity regime. This is because that the cellular material elastically recovers (springback) when it reaches the maximum compression state. There is no springback in the analytical model as rigid unloading is assumed.

Fig. 4.9 (a) Velocity history of the cover plate ($P_0=2.0$ MPa, $t_0=0.1$ ms)

Fig. 4.9 (b) Velocity history of the cover plate ($P_0=4.0$ MPa, $t_0=0.1$ ms)

Fig. 4.9 (c) Velocity history of the cover plate ($P_0=6.0$ MPa, $t_0=0.1$ ms)

The displacement history of the cover plate under different blast loads are predicted by the shock theory, the mesoscale model and the continuum model, as shown in Figs. 4.10(a)~(c). The predicted final maximum displacement of the cover plate by the three methods is in good agreement (the difference is less than 10%). This indicates the overall shortening of the cellular material is almost the same.

However, it should be noted that the deformation of the cellular material in the mesoscale model is mixed with progressive deformation (when the cover plate moves fast) and random deformation (when the cover plate slows down). However, the deformation in the analytical model is assumed to be always localized at the loaded end.

Fig. 4.10(a) Displacement history of the cover plate ($P_0=2.0$ MPa, $t_0=0.1$ ms)

Fig. 4.10(b) Displacement history of the cover plate ($P_0=4.0$ MPa, $t_0=0.1$ ms)

Fig. 4.10(c) Displacement history of the cover plate ($P_0=6.0$ MPa, $t_0=0.1$ ms)

4.4 Blast resistant capacity of cellular material

4.4.1 Prediction by Ashby et al. (2000)

The maximum blast impulse has been preliminarily analyzed in the book by Ashby et al. (2000). The blast impulse is assumed to suddenly accelerate the cover plate to a velocity V .

$$V = \frac{I_0}{m_1} \quad (4.15)$$

Then the cover plate has an initial kinetic energy,

$$U = \frac{1}{2} m_1 V^2 = \frac{I_0^2}{2m_1} \quad (4.16)$$

It is conservatively assumed by Ashby et al. (2000) that the energy absorption capacity of the cellular material under dynamic condition is the same as that under

quasi-static condition and the energy absorbed per unit volume up to densification by the cellular material is expressed as,

$$W = \sigma_0 \varepsilon_D \quad (4.17)$$

The initial kinetic energy is assumed to be absorbed in the plastic deformation of the cellular material

$$U = lAW \quad (4.18)$$

where l is the thickness, and A is the cross-sectional area of the cellular material. Consider Eqs. (4.16)~ (4.18), the maximum impulse that can be absorbed by the cellular material is

$$I_0 = \sqrt{2m_1 \sigma_0 \varepsilon_D l A} \quad (4.19)$$

4.4.2 Prediction by the shock theory

According to the analytical model in the previous section, we can derive the maximum impulse that can be resisted by the cellular material prior to full densification ($u_{\max} = l\varepsilon_D$).

If $1 < \frac{P_0}{\sigma_0} \leq 2$, the cover plate will cease motion at $t = \frac{2(P_0 - \sigma_0)}{P_0} t_0$, as seen from

Eq. (4.7). The maximum displacement of the cover plate is calculated by Eq. (4.6),

$$u_{\max} = l\varepsilon_D = \frac{-m_1 + \sqrt{m_1^2 + 4\rho_0 A^2 t_0^2 (P_0 - \sigma_0)^3 / 3P_0^2 \varepsilon_D}}{\rho_0 A / \varepsilon_D} \quad (4.20)$$

Consider the total mass of the cellular material $m_f = \rho_0 l A$ and the maximum

allowable impulse $I_0 = \frac{1}{2} P_0 t_0 A$, the above equation can be rewritten as,

$$(m_f + 2m_1)lA = \frac{16I_0^2 (P_0 - \sigma_0)^3}{3P_0^4 \varepsilon_D} \quad (4.21)$$

The maximum impulse that can be absorbed by the cellular material is therefore derived as

$$I_0 = \sqrt{\frac{(m_f + 2m_1)\varepsilon_D P_0 l A}{\frac{16}{3}(1 - \frac{\sigma_0}{P_0})^3}}, \quad \text{for } 1 < \frac{P_0}{\sigma_0} \leq 2 \quad (4.22)$$

If $\frac{P_0}{\sigma_0} > 2$, the cover plate will cease motion at $t = \frac{P_0}{2\sigma_0} t_0$ as seen from Eq. (4.14).

The maximum displacement of the cover plate is calculated by Eq. (4.13),

$$u_{\max} = l\varepsilon_D = \frac{-m_1 + \sqrt{m_1^2 + \rho_0 A^2 P_0 t_0^2 (-\frac{1}{3} + \frac{P_0}{4\sigma_0})/\varepsilon_D}}{\rho_0 A / \varepsilon_D} \quad (4.23)$$

Consider the total mass of the cellular material $m_f = \rho_0 l A$ and the maximum allowable impulse $I_0 = \frac{1}{2} P_0 t_0 A$, the above equation can be rewritten as,

$$(m_f + 2m_1)lA = \frac{4I_0^2}{P_0 \varepsilon_D} (-\frac{1}{3} + \frac{P_0}{4\sigma_0}) \quad (4.24)$$

From the above equation, the maximum impulse that can be absorbed by the cellular material is

$$I_0 = \sqrt{\frac{(m_f + 2m_1)\varepsilon_D \sigma_0 l A}{1 - 4\sigma_0 / 3P_0}}, \quad \text{for } \frac{P_0}{\sigma_0} > 2 \quad (4.25)$$

When $P_0 \gg \sigma_0$, the above equation can be expressed as,

$$I_0 = \sqrt{(m_f + 2m_1)\varepsilon_D \sigma_0 l A}, \quad \text{for } \frac{P_0}{\sigma_0} \gg 1 \quad (4.26)$$

Compared to (4.19), Eq. (4.26) has one additional term in the square root.

4.5 Summary

The behavior of the cellular material under blast loading has been modeled at the mesoscale. The transmitted pressure and impulse by the cellular material have been investigated. The transmitted pressure can be reduced to a low level. However, the rebound of the cellular material can lead to an increased transmitted impulse. The cellular material is suggested to be bonded to the protected structure, and the transmitted impulse is then equal to the input impulse. The deformation of the cellular material under blast loading has been approximated by the shock wave theory. The overall shortening of the cellular material predicted by the shock wave theory is found to be in good agreement with the mesoscale model and the continuum model. Therefore the shock wave theory can be effectively used to predict the blast resistant capacity of the cellular material.

Chapter 5 Energy Absorption of Double-Layer Foam Claddings for Blast Alleviation

5.1 Introduction

Loading rate effect and shock wave propagation in cellular materials have been respectively analyzed by using a mesoscale model in Chapters 3 and 4. It is clear that the deformation mode of cellular material will be different under different impact velocities. A shock wave will be formed corresponding to the progressive collapse mode of the material. The shock wave theory with an RPPL model can approximately estimate the cellular material deformation and energy absorption in the progressive collapse mode. In this chapter, the shock wave theory will be used for analyzing energy absorption of double-layer foam cladding for blast alleviation. The analysis will be verified by numerical simulations.

5.2 Deformation of a double-layer foam cladding

The blast load can be simplified as a triangular pulse with P_0 of the initial peak pressure and t_0 of the load duration. For a blast load with a relatively large stand-off distance, it is reasonable to assume that the load acts on the structure uniformly. Therefore, a unit strip along the thickness of the foam cladding is sufficient to analyze the energy absorption of the cladding based on one-dimensional wave propagation theory.

For convenience of analysis, the foam (3D cellular material) is idealized as a rigid-perfectly plastic-locking (RPPL) material, as shown in Fig. 5.1.

Fig. 5.1 Rigid-perfectly plastic-locking model of foam material

For a cladding made of two foam layers as shown in Fig. 5.2, it is assumed that the front layer (FL) and the rear layer (RL) are bonded perfectly. The FL receives the blast load first, while the RL is attached to the protected structure. Each foam layer has a cover plate. Considering a unit strip along the thickness direction of the cladding, the initial foam density ρ , total mass m_f , plateau stress σ_0 , and densification strain ε_D of the foam and the mass m of the cover plate are denoted respectively with subscript 1 for the FL and 2 for the RL. The two cover plates are assumed rigid. A single-layer foam cladding can be treated as a special case of the double-layer cladding.

Based on the RPPL foam model, which is more accurate for low density foams (Reid and Peng 1997), the deformation of the foams can be calculated from the displacements of the two cover plates, i.e., u_1 and u_2 . The initial conditions of the two cover plates are

$$u_1(0) = u_2(0) = 0 \quad (5.1)$$

$$\dot{u}_1(0) = \dot{u}_2(0) = 0 \quad (5.2)$$

Once a blast wave is applied to the layered cladding, the stresses in the two foam layers will increase instantaneously. The two foams deform simultaneously under the blast load and two shock fronts then appear as shown in Fig. 5.2. There are a velocity jump and a stress jump across each shock wave front. As the shock front

proceeds, the foam behind the shock front is compressed up to the locking strain and moves together with the cover plate.

Fig.5.2 Schematic of a double-layer foam cladding under blast loading

Conservation of mass for the foam leads to the following relationship between the mass of the densified part and the displacement of the cover plate,

$$\Delta m_1 = \frac{\rho_1 A}{\varepsilon_{D1}} (u_1 - u_2), \quad u_1 - u_2 \geq 0 \quad (5.3a)$$

$$\Delta m_2 = \frac{\rho_2 A}{\varepsilon_{D2}} u_2, \quad u_2 \geq 0 \quad (5.3b)$$

According to the momentum conservation law for the transition part at the shock front, the stress at the upstream of the shock wave front can be derived as

$$\sigma_{D1} = \sigma_1 + \frac{\rho_1}{\varepsilon_{D1}} (\dot{u}_1 - \dot{u}_2)^2, \quad \dot{u}_1 - \dot{u}_2 \geq 0 \quad (5.4a)$$

$$\sigma_{D2} = \sigma_2 + \frac{\rho_2}{\varepsilon_{D2}} \dot{u}_2^2, \quad \dot{u}_2 \geq 0 \quad (5.4b)$$

where σ_1 and σ_2 are respectively the stresses at the downstream of the shock fronts in the FL and RL. The foams at the downstream of the shock fronts remain intact. Therefore, $\sigma_1 \leq \sigma_{01}$ and $\sigma_2 \leq \sigma_{02}$.

In light of Newton's second law the motion equation of the densified part of the FL foam can be expressed as follows,

$$[m_1 + \Delta m_1] \ddot{u}_1 + (\sigma_{D1} - P(t))A = 0 \quad (5.5)$$

With reference to Eq. (5.3a) and (5.4a), Eq. (5.5) expands to

$$\left[m_1 + \frac{\rho_1 A}{\varepsilon_{D1}} (u_1 - u_2) \right] \ddot{u}_1 + \frac{\rho_1 A}{\varepsilon_{D1}} (\dot{u}_1 - \dot{u}_2)^2 + (\sigma_1 - P(t))A = 0 \quad (5.6)$$

The intact part of the FL foam ($m_{f1} - \Delta m_1$) moves together with the RL cover plate and the densified part of the RL foam. Hence the motion equation of the RL can be expressed as

$$[m_{f1} - \Delta m_1 + m_2 + \Delta m_2] \ddot{u}_2 + (\sigma_{D2} - \sigma_1)A = 0 \quad (5.7)$$

With reference to Eq. (5.3b) and (5.4b), Eq. (5.7) is then rewritten as

$$\left[m_{f1} - \frac{\rho_1 A}{\varepsilon_{D1}} (u_1 - u_2) + m_2 + \frac{\rho_2 A}{\varepsilon_{D2}} u_2 \right] \ddot{u}_2 + \frac{\rho_2 A}{\varepsilon_{D2}} \dot{u}_2^2 + (\sigma_2 - \sigma_1)A = 0 \quad (5.8)$$

When the shock wave in the FL propagates completely through the FL foam and reaches the cover plate of the RL, the FL foam becomes fully compacted, i.e.,

$\Delta m_1 = m_{f1}$ or $\frac{u_1 - u_2}{l_1 \varepsilon_{D1}} = 1.0$. It is assumed that the fully compacted FL will move

together with the cover plate of the RL at the same velocity. Thus there will be a momentum transition at the instant of the collision. Since the velocity of the FL is initially larger than that of the RL, the momentum of the FL will be partially transferred to the RL. Based on momentum conservation of the system, it has

$$MV = (m_1 + m_{f1})\dot{u}_1 + (m_2 + \Delta m_2)\dot{u}_2 \quad (5.9)$$

where M and V are respectively the mass and velocity of the combined part of the two layers that is moving with a consistent velocity, and $M = m_1 + m_{f1} + m_2 + \Delta m_2$.

In the case that the FL foam has a higher plateau than the RL foam, the FL foam does not deform but moves together with the RL cover plate at the same velocity. Eqs. (5.6) and (5.8) are not effective any more and the motion equation of the RL and the FL are, respectively

$$\dot{u}_1 = \dot{u}_2 \quad (5.10a)$$

$$\left[m_1 + m_{f1} + m_2 + \frac{\rho_2 A}{\varepsilon_{D2}} u_2 \right] \ddot{u}_2 + \frac{\rho_2 A}{\varepsilon_{D2}} \dot{u}_2^2 + (\sigma_2 - P(t))A = 0 \quad (5.10b)$$

where σ_2 is the stress at the downstream of the shock front in the RL foam, and

i) if $P(t) > \sigma_{02}$, which indicates that the foam material at the shock front is yielding or is about to yield, then σ_2 in Eq. (5.10b) is equal to σ_{02} ;

ii) if $P(t) \leq \sigma_{02}$, the foam material at the shock front ceases crushing with the decaying of the external pressure, thus σ_2 in Eq. (5.10b) is equal to $P(t)$ based on the stress equilibrium.

The stress applied on the protected structure will exceed the plateau stress of the RL foam once it becomes fully compacted. The behavior of the cladding after the RL foam becomes fully compacted, however, is not covered in the current study. For the convenience of discussion, the configurations of the double-layer foam claddings are classified into two categories in terms of the plateau stresses in the two foams, i.e., $\sigma_{01} \leq \sigma_{02}$ and $\sigma_{01} > \sigma_{02}$.

5.2.1 Category 1, $\sigma_{01} \leq \sigma_{02}$.

i) If $P_0 \leq \sigma_{01}$, the stress in the FL and RL foams rises up to P_0 at the instant $t=0$ based on the RPPL model, i.e., $\sigma_1 = P_0$ and $\sigma_2 = P_0$. Since the stress is lower than the yield stress of the foams, neither of the two foams crushes.

ii) If $P_0 > \sigma_{01}$, the FL foam begins to crush and a shock wave forms in the FL with $\sigma_1 = \sigma_{01}$. Meanwhile, the RL foam is compressed by a stress the same as that in the intact part of the FL foam, i.e., σ_2 is equal to σ_{01} which is smaller than σ_{02} . The RL foam will not deform until the FL foam becomes fully compacted. The governing equation of the deformation of FL foam is Eq. (5.6), where $u_2 = 0$ and $\dot{u}_2 = 0$.

In the case that the blast impulse is low, $\dot{u}_1$ could drop to zero with the decaying of the external pressure and the FL foam ceases crushing before the shock front reaches the RL. σ_1 will decrease to $P(t)$ where $P(t) < \sigma_{01}$.

In the other case that the blast impulse is sufficiently high, the FL foam may become fully compacted, and the shock front is then transferred to the RL foam. The momentum transition equation (Eq. (5.9)) where $\Delta m_2 = 0$ can then be used. When the RL foam deforms, the governing equation becomes Eq. (5.10a, b).

From the above analysis, we can see that the two foams with $\sigma_{01} \leq \sigma_{02}$ deform progressively when the foam cladding is subjected to a high blast impulse.

5.2.2 Category 2, $\sigma_{01} > \sigma_{02}$.

i) If $P_0 \leq \sigma_{02}$, the stress in each foam rises up to P_0 at the instant $t=0$, i.e., $\sigma_1 = \sigma_2 = P_0$ and neither of the two foams deforms under the blast pressure $P(t)$.

ii) If $\sigma_{02} < P_0 \leq \sigma_{01}$, the stress in the FL foam is lower than its yield stress, i.e., $\sigma_1 < \sigma_{01}$ and thus the FL foam does not deform. Only the RL foam begins to deform and $\sigma_2 = \sigma_{02}$. The governing motion equation of the system follows Eq. (5.10a, b).

iii) If $P_0 > \sigma_{01}$, both the FL and RL foams start to deform and $\sigma_1 = \sigma_{01}$, $\sigma_2 = \sigma_{02}$.

The governing equations of the deformations are Eqs. (5.6) and (5.8). The FL foam could become fully compacted earlier than the RL foam and subsequently collides with the RL. The momentum transition between the FL and RL follows Eq. (5.9) which is based on the momentum conservation law during the collision. Another scenario is that the velocity of the FL cover plate decreases to be the same as that of the RL cover plate. For both cases, the FL cover plate will move together with the RL cover plate at the same velocity, and the governing motion equation follows Eq. (5.10a, b).

For a given blast impulse, the deformation of the two foams can be calculated from the governing equations (5.6), (5.8), (5.10a) and (5.10b), which can be solved by using the Wilson- θ algorithm. The transmitted pressure by the foam cladding to the protected structure is less than or equal to the plateau stress of the RL foam unless the RL foam is fully compacted. When the RL foam is fully compacted, the pressure transferred to the protected structure could be much larger than the plateau stress of the RL foam or even larger than the input peak pressure P_0 , which induces the stress amplification (Li and Meng 2002a). Therefore, the maximum blast impulse that the cladding can resist is considered to be the critical value when the RL foam initially becomes fully compacted.

5.3 Blast resistant capacity of a double-layer foam cladding

Based on the current analytical model, the blast resistant capacity of a double-layer foam cladding, i.e., the maximum impulse that the foam cladding may withstand without stress amplification can be predicted.

In the case of a blast load with high initial peak pressure and very short duration (t_0 is shorter than the shock front traveling time in the FL foam), the blast resistant capacity of a double-layer foam cladding with $\sigma_{01} \leq \sigma_{02}$ (Category 1) can be explicitly derived with the aid of the mathematical tool Maple (1998). As indicated

in Section 5.2, the two foams subjected to blast loads will deform progressively and the deformation of the foams can be divided into three phases,

(i) Phase 1: $0 < t \leq t_0$, where t_0 is the duration of the blast load.

At this stage, only the FL foam deforms and the motion equation of the FL cover plate follows Eq. (5.6), which can be simplified as

$$\left[m_1 + \frac{\rho_1 A}{\varepsilon_{D1}} u_1 \right] \ddot{u}_1 + \frac{\rho_1 A}{\varepsilon_{D1}} \dot{u}_1^2 + (\sigma_{01} - P(t))A = 0 \quad (5.11)$$

The solution to Eq. (5.11) is

$$u_1(t) = \frac{-m_1 + \sqrt{m_1^2 + \rho_1 A^2 t^2 (P_0 - \sigma_{01} - P_0 t / 3t_0) / \varepsilon_{D1}}}{\rho_1 A / \varepsilon_{D1}} \quad (5.12)$$

$$\dot{u}_1(t) = \frac{At(P_0 - \sigma_{01} - P_0 t / 2t_0)}{\sqrt{m_1^2 + \rho_1 A^2 t^2 (P_0 - \sigma_{01} - P_0 t / 3t_0) / \varepsilon_{D1}}} \quad (5.13)$$

If $\sigma_{01} < P_0 \leq 2\sigma_{01}$, the cover plate ceases motion, and the FL foam stops crushing within the loading duration t_0 . The protected structure is thus not disturbed.

Considering the case that $P_0 > 2\sigma_{01}$, the FL plate has a velocity of V_1 and a displacement of U_1 at $t = t_0$,

$$u_1(t_0) = U_1 = \frac{-m_1 + \sqrt{m_1^2 + \rho_1 A^2 t_0^2 (2P_0 / 3 - \sigma_{01}) / \varepsilon_{D1}}}{\rho_1 A / \varepsilon_{D1}} \quad (5.14)$$

$$\dot{u}_1(t_0) = V_1 = \frac{At_0(P_0 / 2 - \sigma_{01})}{\sqrt{m_1^2 + \rho_1 A^2 t_0^2 (2P_0 / 3 - \sigma_{01}) / \varepsilon_{D1}}} \quad (5.15)$$

The total mass including the cover plate and the densified part of foam is

$$M_1 = m_1 + \frac{U_1}{\varepsilon_{D1}} \rho_1 A = \sqrt{m_1^2 + \rho_1 A^2 t_0^2 (2P_0 / 3 - \sigma_{01}) / \varepsilon_{D1}} \quad (5.16)$$

(ii) Phase 2: $t_0 < t \leq t_1$, where t_1 is the time when the FL foam is fully compacted.

Provided that the external blast pressure has dropped to zero before the shock wave reaches the RL cover plate, the motion equation of the FL plate is then expressed as

$$\left[M_1 + \frac{\rho_1 A}{\varepsilon_{D1}} u'_1 \right] \ddot{u}'_1 + \frac{\rho_1 A}{\varepsilon_{D1}} \dot{u}'_1{}^2 + \sigma_{01} A = 0 \quad (5.17)$$

where, $u'_1 = u_1 - U_1$, and $U_1 = \frac{M_1 - m_1}{\rho_1 A / \varepsilon_{D1}}$. The solution to Eq.(5.17) is obtained as

$$u_1(t) = U_1 + \frac{-M_1 + \sqrt{M_1^2 + 2\rho_1 A M_1 V_1 t' / \varepsilon_{D1} - \sigma_{01} \rho_1 A^2 t'^2 / \varepsilon_{D1}}}{\rho_1 A / \varepsilon_{D1}} \quad (5.18)$$

where $t' = t - t_0$. Substituting Eqs. (5.14–5.16) into Eq. (5.18) yields

$$u_1(t) = \frac{-m_1 + \sqrt{m_1^2 + \frac{\rho_1 A^2 P_0 t_0^2}{\varepsilon_{D1}} \left[-\frac{1}{3} + \frac{t}{t_0} - \frac{\sigma_{01}}{P_0} \left(\frac{t}{t_0} \right)^2 \right]}}{\rho_1 A / \varepsilon_{D1}} \quad (5.19)$$

When $t = t_1$, the FL foam is considered fully compacted, and there have

$u_1(t_1) = l_1 \varepsilon_{D1}$ and $\Delta m_1 = \frac{\rho_1 A u_1(t_1)}{\varepsilon_{D1}} = m_{f1}$. Therefore, we have

$$m_{f1} = -m_1 + \sqrt{m_1^2 + \frac{\rho_1 A^2 P_0 t_0^2}{\varepsilon_{D1}} \left[-\frac{1}{3} + \frac{t_1}{t_0} - \frac{\sigma_{01}}{P_0} \left(\frac{t_1}{t_0} \right)^2 \right]} \quad (5.20)$$

From Eq. (5.20), the time t_1 is obtained as

$$\frac{t_1}{t_0} = \frac{P_0}{2\sigma_{01}} - \sqrt{\frac{1}{4} \left(\frac{P_0}{\sigma_{01}} \right)^2 - \frac{P_0}{3\sigma_{01}} - \frac{\varepsilon_{D1}(m_{f1}^2 + 2m_{f1}m_1)}{\rho_1 \sigma_{01} A^2 t_0^2}} \quad (5.21)$$

When $\frac{P_0}{\sigma_{01}} \geq \frac{2}{3} \left(1 + \sqrt{1 + \frac{9\varepsilon_{D1}(m_{f1}^2 + 2m_{f1}m_1)}{\rho_1 \sigma_{01} A^2 t_0^2}} \right)$, the square root of the above

equation is valid. From Eq. (5.21), the assumption of $\frac{t_1}{t_0} > 1$ also requires

$$t_0 < \sqrt{\frac{3\varepsilon_{D1}(m_{f1}^2 + 2m_{f1}m_1)}{\rho_1 A^2 (2P_0 - 3\sigma_{01})}}.$$

The momentum of the moving part of the cladding is equal to the impulse imparted by the blast load I_0 subtracting that exerted by the protected structure. The stress in the RL foam and the stress in the protected structure behind the RL foam is σ_{01} when the FL foam deforms. Therefore, at the instant that the shock wave reaches the RL cover plate, i.e., $t = t_1$, the momentum of the moving part is

$$J_2 = I_0 - \sigma_{01}At_1 \quad (5.22)$$

Substituting Eq. (5.21) into Eq. (5.22) and considering $\frac{P_0At_0}{2} = I_0$, J_2 can be rewritten as

$$J_2^2 = I_0^2 \left(1 - \frac{4\sigma_{01}}{3P_0}\right) - \frac{\sigma_{01}\varepsilon_{D1}}{\rho_1} (m_{f1}^2 + 2m_{f1}m_1) \quad (5.23)$$

(iii) Phase 3: $t_1 < t \leq t_m$, where t_m is the time when the whole system ceases the motion.

Once the FL foam is fully compacted, FL as a whole will collide and then move together with the RL cover plate (m_2). The momentum J_2 is conserved during the bonding process. Immediately after the bonding, and the total mass of the moving part becomes M_2 and the velocity is V_2 . Then the RL foam starts to deform and the stress in the protected structure behind the RL foam is σ_{02} .

The motion equation of the RL plate is Eq. (5.10b), which is simplified as

$$\left[M_2 + \frac{\rho_2 A}{\varepsilon_{D2}} u_2 \right] \ddot{u}_2 + \frac{\rho_2 A}{\varepsilon_{D2}} \dot{u}_2^2 + \sigma_{02} A = 0 \quad (5.24)$$

Again by using the mathematical tool, the solution of the above equation is derived as follows,

$$u_2(t) = \frac{-M_2 + \sqrt{M_2^2 + 2\rho_2 AM_2 V_2 t'' / \varepsilon_{D2} - \sigma_{02} \rho_2 A^2 t''^2 / \varepsilon_{D2}}}{\rho_2 A / \varepsilon_{D2}} \quad (5.25)$$

where $t'' = t - t_1$, $M_2 = m_1 + m_{f1} + m_2$ and $V_2 = J_2 / M_2$.

The whole system, due to momentum conservation, will cease motion at $t'' = J_2 / \sigma_{02} A$, i.e., $t_m = t_1 + J_2 / \sigma_{02} A$. When the moving part ceases motion, the mass of the densified part of the RL foam is

$$\Delta m_2 = \frac{\rho_2 A u_2(t_m)}{\varepsilon_{D2}} = -M_2 + \sqrt{M_2^2 + \rho_2 J_2^2 / \sigma_{02} \varepsilon_{D2}} \quad (5.26)$$

For the RL foam with a total mass of m_{f2} , let the right hand side of Eq. (5.26) equal to m_{f2} , then the maximum momentum J_2 that can be resisted is derived from Eq. (5.26) as follows,

$$J_2^2 = (m_{f2}^2 + 2m_{f2}M_2)\sigma_{02}\varepsilon_{D2} / \rho_2 \quad (5.27)$$

Therefore, combining Eqs. (5.23) and (5.27), the maximum impulse I_0 of the blast load which can be resisted by the whole cladding is derived as

$$I_0 = \sqrt{\frac{(m_{f1}^2 + 2m_{f1}m_1)\frac{\sigma_{01}\varepsilon_{D1}}{\rho_1} + [m_{f2}^2 + 2m_{f2}(m_1 + m_{f1} + m_2)]\frac{\sigma_{02}\varepsilon_{D2}}{\rho_2}}{1 - 4\sigma_{01} / 3P_0}} \quad (5.28)$$

It should be noted that the blast resistant capacity of the foam cladding in Category 2 has to be obtained by trial and error numerically.

5.5 Energy absorption of a double-layer foam cladding

The energy absorption capacities under quasi-static state of a double-layer cladding can be calculated as (Gibson and Ashby 1997)

$$Q_s = l_1 A \varepsilon_{D1} \sigma_{01} + l_2 A \varepsilon_{D2} \sigma_{02} \quad (5.29)$$

Under blast loading, the transition part at the shock front consumes the kinetic energy while the densified part and the intact part have no contribution. With reference to the Rankine-Hugoniot relations of shock wave (Asay and Shahinpoor

1993; Kolsky 1963), the absorbed energy per unit mass of the transition part at the shock front is given as

$$\Delta Q = \frac{1}{2}(\sigma_D + \sigma_0)\left(\frac{1}{\rho_0} - \frac{1}{\rho_D}\right) \quad (5.30)$$

where ρ_D is the density of the densified foam and $\rho_D = \rho_0/(1 - \varepsilon_D)$. Considering Eqs. (5.4a) and (5.4b), the energy absorbed per unit mass of the transition part of each foam is

$$\Delta Q_1 = \frac{\sigma_{01}\varepsilon_{D1}}{\rho_1} + \frac{(\dot{u}_1 - \dot{u}_2)^2}{2}, \quad \dot{u}_1 - \dot{u}_2 > 0 \quad (5.31a)$$

$$\Delta Q_2 = \frac{\sigma_{02}\varepsilon_{D2}}{\rho_2} + \frac{\dot{u}_2^2}{2}, \quad \dot{u}_2 > 0 \quad (5.31b)$$

The first term of the right-hand side of Eqs. (5.31a) and (5.31b) is the absorbed energy per unit mass of the foam in the quasi-static state (Gibson and Ashby 1997). It is shown in Eqs. (5.31a) and (5.31b), due to the shock wave effect, the foam can absorb greater energy under blast load.

The transition part at the shock front is the increment of the densified part of the foam layer. Therefore, the energy absorbed by the foam can be integrated as

$$Q_i(t) = \int_0^t (\Delta Q_i) \frac{d(\Delta m_i)}{dt'} dt', \quad i = 1, 2 \quad (5.32)$$

With reference to Eq. (5.3a) and (5.31a), the energy absorbed by the FL foam is derived as

$$Q_1(t) = \int_0^t \left(\frac{\sigma_{01}\varepsilon_{D1}}{\rho_1} + \frac{(\dot{u}_1(t') - \dot{u}_2(t'))^2}{2} \right) \frac{(\dot{u}_1(t') - \dot{u}_2(t'))}{\varepsilon_{D1}} \rho_1 A dt' \quad (5.33)$$

Considering Eq. (5.3b) and (5.31b), the energy absorbed by the RL foam is

$$Q_2(t) = \int_0^t \left(\frac{\sigma_{02}\varepsilon_{D2}}{\rho_2} + \frac{\dot{u}_2(t')^2}{2} \right) \frac{\dot{u}_2(t')}{\varepsilon_{D2}} \rho_2 A dt' \quad (5.34)$$

If the FL foam is fully compacted, the energy lost during the collision process between the two layers is

$$Q_3 = \frac{1}{2}(m_1 + m_{f1})\dot{u}_1^2 + \frac{1}{2}(m_2 + \Delta m_2)\dot{u}_2^2 - \frac{1}{2}MV^2 \quad (5.35)$$

in which $M = m_1 + m_{f1} + m_2 + \Delta m_2$ and V is calculated by Eq. (5.9).

The kinetic energy of the whole foam cladding can be expressed as

$$K(t) = \frac{1}{2}(m_1 + \Delta m_1)\dot{u}_1(t)^2 + \frac{1}{2}(m_{f1} - \Delta m_1 + m_2 + \Delta m_2)\dot{u}_2(t)^2 \quad (5.36)$$

The total energy imparted by the blast impulse equals the work done on the cladding by the external impulse, which has the form of

$$E(t) = \int_0^t P(t') A \dot{u}_1(t') dt' \quad (5.37)$$

5.6 Case studies

In the current case study, aluminum foam is used as the foam material, while steel is used for the cover plate. The thickness of the foam layers is 10 cm, both of the cover plates have a thickness of 0.5 cm.

It is known that the properties of foam materials are functions of the foam density (Andrews et al. 1999; Ashby et al. 2000; Baumeister et al. 1997). The plateau stress σ_0 and the densification strain of foams can be estimated using the phenomenological model for the metallic foams (Ashby et al. 2000) as follows,

$$\frac{\sigma_0}{\sigma_{ys}} = \alpha_1 \left(\frac{\rho}{\rho_s} \right)^m \quad (5.38)$$

where α_1 is a constant usually ranging from 0.25 to 0.35, the parameter m goes between 1.5 and 2.0, and the densification strain has the form of

$$\varepsilon_D = 1 - \alpha_2 \frac{\rho}{\rho_s} \quad (5.39)$$

where α_2 ranges from 1.4 to 3.0.

According to Andrews et al. (1999), the yield stress and density of the metallic composition and other parameters in the Eqs. (5.38) and (5.39) are given as $\sigma_{ys} = 358$ MPa, $\rho_s = 2700$ kg/m³, $\alpha_1 = 0.35$, $m = 1.7$, and $\alpha_2 = 2.0$, respectively. Two foams are selected for the present study. Foam I has a density of 200 kg/m³, plateau stress of 1.5 MPa and densification strain 0.85. Foam II has a density of 300 kg/m³, plateau stress of 3.0 MPa and densification strain 0.78.

The blast alleviation behaviors of four different configurations of the cladding are investigated, as presented in Table 5.1. Cladding-1 has a single-layer of foam I. Cladding-2 has two identical layers and each layer is the same as Cladding-1. In Cladding-3, the RL foam of Cladding-2 changes to foam II, which means that the RL foam has higher plateau than the FL foam. In Cladding-4, the foams of the two layers in Cladding-3 exchange. It is calculated that under quasi-static conditions, the energy absorption capacity of Cladding-2 is exactly twice as that of Cladding-1. Cladding-3 and Cladding-4 have an equal capacity which is 42% higher than that of Cladding-2.

Table 5.1 Blast resistant capacities of foam claddings

No.	Configuration	Energy absorption capacity under quasi-static load(J)	Transmitted pressure ^c (MPa)	Analytical		LS-DYNA	
				Max. impulse ^a (Pa-s)	Absorbed energy (J)	Max. impulse ^{a,b} (Pa-s)	Absorbed energy (J)
1	Single layer (foam I)	127,500	1.5	3,690	154,550	3,525	142,000
2	Two layers (foam I+I)	255,000	1.5	6,480	483,500	6,000	419,200
3	Two layers (foam I+II)	361,500	3.0	8,250	781,070	7,950	734,000
4	Two layers (foam II+I)	361,500	1.5	6,690	480,000	6,195	416,000

^a The duration of the blast load is fixed as 0.3 milli-sec.

^b The maximum impulses that can be resisted by the four claddings (simulated by LS-DYNA) are denoted as $I_1 \sim I_4$.

The transmitted pressure is calculated from the analytical model.

Based on the analytical model, the blast resistant capacity (maximum allowable impulse) of Cladding-2 is 76% larger than that of Cladding-1. Compared with Cladding-2, the blast resistant capacity of Cladding-3 is further increased by 27%. On the other hand, the pressure transferred to the protected structure is twice as much of Cladding-2 and the weight of Cladding-3 is 8% higher. Cladding-4 is expected to withstand much higher blast impulse. However, the results indicate that the blast resistant capacity of Cladding-4 has no significant enhancement compared to Cladding-2. Energy absorption of the four claddings has also been simulated by using LS-DYNA. The results are shown in Table 5.1 as well.

Table 5.1 also shows the analytical predictions of the absorbed energy for the four claddings under the maximum blast impulses the claddings can take. It is seen that the claddings absorb much more energy under blast loads than under the quasi-static condition. Compared with the energies absorbed under the quasi-static loads, there are respectively 21%, 90%, 116% and 33% increment of the absorbed energy for the four claddings. It is worth noting that the absorbed energy is related to the loading conditions as well as the properties of the cladding.

Figs. 5.3 and 5.4 demonstrate the effect of the foam density on the blast resistant capacity of the cladding. In Fig. 5.3, the density of the FL foam is fixed, while that of the RL foam increases (the plateau stress and densification strain of the foam also vary with its density). The blast resistant capacity of the cladding increases almost linearly with an increase of the density of the RL foam. However, with the increase of the plateau stress of the RL foam, the transferred pressure to the protected structure increases correspondingly.

In Fig. 5.4, the density of RL foam is fixed while the density of the FL foam varies. The blast resistant capacity of the cladding initially increases but then decreases. The reason for the fluctuating blast resistant capacity of the cladding may be that the mass, the plateau stress and the densification strain of the foams have complex

influence. If the plateau stress of the FL foam is larger than that of the RL foam, the RL foam tends to be fully compacted earlier than the FL foam. As a result, the FL foam is not fully utilized and the blast resistance of the whole cladding could decrease.

Fig.5.3 Blast resistant capacity vs. the density of the rear foam (Density of FL foam is fixed as 200 kg/m³)

Fig.5.4 Blast resistant capacity vs. the density of the front foam (Density of RL foam is fixed as 200 kg/m³)

It is also found that the blast resistant capacity of the cladding varies with the loading duration and initial peak pressure. As seen from Figs. 5.3 and 5.4, the longer the loading duration t_0 (associated with a lower initial peak pressure P_0), the higher the blast resistant capacity of foam cladding.

5.7 Comparison between analytical and numerical results

In order to verify the current analytical solution, numerical simulation of the cladding subjected to blast load is performed by using the explicit finite element code LS-DYNA. Both the cover plate and the foams are modeled as brick elements, and the cover plates and foams are bonded perfectly, as shown in Fig. 5.5. The lateral degrees of freedom of the nodes on the four side faces are constrained. Only axial deformation is allowed to simulate the uniaxial compression. A continuum foam model (#63) from LS-DYNA material library is used to represent the foam material behavior. A stress-strain curve similar to the RPPL model shown in Fig.5.1 is applied for the analysis. However, the infinite stiffness in the initial regime as well as in the densification regime is avoided. In the LS-DYNA model, the Young's modulus in both elastic and densification regimes are 1.5GPa for Foam I and 3.0 GPa for Foam II. The plateau regime is perfectly plastic, with a constant stress of 1.5 MPa for Foam I and 3.0 MPa for Foam II. The deformation of a single-layer foam will increase by 8% if the Young's modulus decrease from 1.5 GPa to 0.075 GPa (20 times), which indicates that the results are not very sensitive to different Young's modulus values.

Fig.5.5 Finite element model of the double-layer foam cladding

The numerically simulated maximum blast impulses ($I_1 \sim I_4$) that the claddings can withstand have been obtained by trial and error. As given in Table 5.1, all the numerical results of the cladding capacity are slightly lower than the counterparts from the analytical predictions. This is probably due to the elastic stress wave reflections in the numerical simulations where the stress-strain curve contains an initial elastic regime and a slope densification regime. According to Kolsky (1963), when an elastic stress wave with a magnitude of σ_0 traveling in a perfectly plastic solid (with a yield stress $= \sigma_0$) reflects from a rigid boundary, the stress magnitude at the boundary will not become $2\sigma_0$ as in the elastic solid. Instead, the stress magnitude is still σ_0 and the reflection results in some plastic strain, which increases the overall deformation of the foam.

Figs. 5.6–5.7 compare the non-dimensional deformations versus the non-dimensional time of the three double-layer claddings subjected to the blast impulses $I_2 \sim I_4$ (see Table 5.1). The non-dimensional time is defined as the real time over the loading duration t_0 . The non-dimensional deformations of the FL and RL foam are defined as

$$U'_1 = (u_1 - u_2) / l_1 \varepsilon_{D1} \quad (5.40a)$$

$$U'_2 = u_2 / l_2 \varepsilon_{D2} \quad (5.40b)$$

Consistent with the analytical analysis, the double-layer cladding deforms progressively when the FL foam is weaker as shown in Figs. 5.6 and 5.7. However, if the FL foam has a higher plateau stress than the RL foam, the two foams deform simultaneously once the cladding is loaded and the RL foam is prone to be fully compacted earlier than the front layer, as shown in Fig. 5.8.

Fig.5.6 Non-dimensional deformation of Cladding-2 vs. non-dimensional time under the blast impulse of I_2

Fig.5.7 Non-dimensional deformation of Cladding-3 vs. non-dimensional time under the blast impulse of I_3

Fig.5.8 Non-dimensional deformation of Cladding-4 vs. non-dimensional time under the blast impulse of I_4

Figs. 5.9–5.11 compare the non-dimensional total energies, absorbed energies and kinetic energies versus the non-dimensional time of the three double-layer claddings subjected to blast loads $I_2 \sim I_4$. The non-dimensional energy is defined as the energy value over the maximum energy absorbed by the cladding under quasi-static conditions (see Table 5.1).

It is observed that the analytical results are again in very good agreement with the numerical results. The total energy is equal to the work done on the cladding by the blast load. As seen from Figs. 5.9–5.11, there are about 64%, 103% and 15% enhancement of the energy absorption respectively for the three double-layer claddings comparing to the quasi-static counterparts. The kinetic energy of the cladding increases initially, then drops eventually to zero.

Fig.5.9 Non-dimensional energies of Cladding-2 vs. non-dimensional time under the blast impulse of I_2

Fig.5.10 Non-dimensional energies of Cladding-3 vs. non-dimensional time under the blast impulse of I_3

Fig.5.11 Non-dimensional energies of Cladding-4 vs. non-dimensional time under the blast impulse of I_4

It is shown in Figs. 5.9 and 5.10 that there is a sudden drop of the kinetic energy and a sharp increase of the absorbed energy when the front layer impacts and bonds with the RL plate immediately after the FL foam becomes fully compacted. This impact and bonding process could be approximated as a perfectly inelastic collision. In the collision process, the momentum is conserved; however, the kinetic energy is partially converted to irreversible internal energy (absorbed energy). Lim and Stronge (1999) discussed the energy loss mechanism during the impact between elastic-plastic bodies. As indicated by Lim and Stronge (1999), the kinetic energy of the colliding bodies can be transformed to the energy of elastic stress waves, the work for plastic deformation and friction dissipation. The approximation as a perfectly inelastic collision could lead to overestimation of the kinetic energy loss. Thus, the value calculated by the present analytical model is slightly higher than that simulated by LS-DYNA.

5.8 Summary

Analytical solutions for the blast mitigation and energy absorption of double-layer foam claddings have been derived. The total energy, kinetic energy and absorbed

energy of four representative foam claddings under critical blast loads are calculated. The energy absorbed by the cladding is much larger than that under quasi-static conditions due to shock wave effect. The current analytical solution is based on the idealized rigid-perfectly plastic-locking foam model which neglects the elastic behavior of the foam. Despite the limitation, the present analytical model is efficient to estimate the blast alleviation and energy absorption capacity of a double-layer foam cladding. Comparison between the numerical and analytical results shows that they are in good agreement. Therefore, the current analytical solution is considered to be efficient and could be very useful for the design of double-layer foam sacrificial cladding.

Chapter 6 Effectiveness of Foam Cladding for Structural Retrofit: Analytical Derivation

6.1 Introduction

In the previous chapters, the behavior of foam materials under impact and blast loads has been discussed. It has been shown that the foam material may be used to protect structures from blast loads. However all the existing theoretical studies, e.g., Karagiozova and Jones (2000) and Li and Meng (2002a), focused on the sacrificial foam claddings only. The effect of the deformation behavior of the protected structure is not taken into consideration.

In this chapter, an analytical Load-Cladding-Structure (LCS) model is developed by considering the properties of the blast load, foam cladding as well as protected structure, as shown in Fig. 6.1. Two non-dimensional parameters for the foam cladding are introduced to describe the relations between the foam cladding and the protected structure. Based on the LCS model and the pressure-impulse diagram analysis, the maximum allowable blast load can be determined for a given cladding protected structure. In turn, the maximum deflection of the protected structure subjected to a certain explosive load can be predicted. Equally, the appropriate foam cladding can be obtained to achieve structural retrofit against blast loads.

6.2 Analytical Load-Cladding-Structure model

The blast loading can be approximated as a plane pressure wave if the distance of the explosion is much larger comparing with the dimensions of the protected structure. The time history of the loading can be simplified as an equivalent triangular pressure pulse (Evans et al. 1999). The nature of the blast loading is always impulsive (a high initial peak pressure P_0 with a very short duration t_0).

The compressive deformation of the metallic foams can be approximately characterized into three regions, i.e., a linear-elastic region, a plateau region and a densification region. According to Reid and Peng (1997) and previous chapters, the foam material can be idealized as a rigid-perfectly plastic-locking (RPPL) material with a plateau stress of σ_0 . At the strain of ε_D , the foam locks into a rigid solid and the stress rises to σ_D . The RPPL foam model has been shown very effective in the analytical prediction of the deformation of the foam material.

Fig. 6.1 Analytical Load-Cladding-Structure model

In an elastic analysis, a flexible structure, e.g., beam, column, etc., can be converted to an equivalent single-degree-of-freedom (SDOF) structural model (Mays and Smith 1995; Smith and Hetherington 1994). Using the SDOF model is a relatively simple approach in obtaining the maximum response of the structure subjected to blast loading. However, the accuracy of this method relies heavily on the representation of the real system with the equivalent system. It is difficult to incorporate all the bending, shear and membrane effects into a SDOF structural model. Sometimes the SDOF model is non-conservative, in that it underestimates the response. The motion equation of the equivalent SDOF structural model (without foam cladding) can be expressed as

$$F_m m \ddot{y} + F_s k y = F_l P(t) A \quad (6.1)$$

where m and k are respectively the total mass and stiffness of the real structure; $P(t)$ is the exerted uniform pressure on the structure; A is the bearing area against the blast load; and F_m , F_s and F_l are respectively the mass, stiffness and load transformation factors. According to Smith and Hetherington (1994), the mass factor, stiffness factor and load factor for a fully clamped beam under uniformly distributed pressure is respectively 0.5, 0.64 and 0.64.

According to Smith and Hetherington (1994), the stiffness and load transformation factors are usually equal, i.e., $F_s = F_l$. The above equation of motion can be further simplified by introducing a load-mass factor F_{lm} which is defined as $F_{lm} = F_m / F_l$. Dividing Eq. (6.1) by the load factor gives a simpler form

$$m_{se}\ddot{y} + ky = P(t)A \quad (6.2)$$

where the equivalent lump mass m_{se} is equal to $F_{lm}m$.

The foam material is assumed to start deforming at the loaded end and a shock wave forms in the foam layer. When the shock front progresses, the foam material behind the shock front is compacted up to the densification strain ε_D (densified) while that ahead of the shock front is not disturbed (undeformed). The densified part moves with the same velocity as the rigid cover plate. The mass of the densified part of the foam layer is conserved, which is written as

$$\Delta m = \frac{\rho_0 A}{\varepsilon_D}(u - y) \quad (6.3)$$

in which u is the displacement of the cover plate of the cladding, y is the deflection of the structure and ρ_0 is the density of the foam material.

There is a stress jump across the shock wave front. Based on the momentum conservation of the shock front (Fig. 6.1), the stress at the upstream of the shock front is

$$\sigma_D = \sigma_0 + \frac{\rho_0}{\varepsilon_D}(\dot{u} - \dot{y})^2 \quad (6.4)$$

Eq. (6.4) is very similar to the result derived in Chapter 4, in which the foam layer is fixed to a rigid wall and the velocity $\dot{y}$ is zero.

In light of Newton's second law the motion equation of the densified foam can be expressed as

$$[m_1 + \Delta m]\ddot{u} + (\sigma_D - P(t))A = 0 \quad (6.5)$$

Considering Eqs. (6.3) and (6.4), the above equation can be rewritten as

$$\left[m_1 + \frac{\rho_0 A}{\varepsilon_D}(u - y) \right] \ddot{u} + \frac{\rho_0 A}{\varepsilon_D}(\dot{u} - \dot{y})^2 + (\sigma_0 - P(t))A = 0 \quad (6.6)$$

The undeformed part of the foam layer ($m_f - \Delta m$) moves together with the lump mass (m_{se}) of the equivalent SDOF structure. The stress at the downstream of the shock front (σ_0) is now the load for the undeformed foam and the lump mass while the resistance of the spring in Fig. 6.1 is ky . In light of Newton's second law the motion equation for the undeformed foam and the lump mass is written as

$$\left[m_f - \frac{\rho_0 A}{\varepsilon_D}(u - y) + m_{se} \right] \ddot{y} + ky - \sigma_0 A = 0 \quad (6.7)$$

When the shock wave in the foam propagates completely through the foam, the foam becomes fully compacted, i.e., $\Delta m = m_f$ or $\frac{u - y}{l\varepsilon_D} = 1.0$. It is assumed that the

fully compacted foam and the cover plate will move together with the equivalent mass (m_{se}) at the same velocity. Since the velocity of the cover plate and the densified foam is initially larger than that of the equivalent mass, the momentum of the foam cladding will be partially transferred to the equivalent mass. Based on momentum conservation of the system, it has

$$MV = (m_1 + m_f)\dot{u} + m_{se}\dot{y} \quad (6.8)$$

Where M and V are respectively the mass and velocity of the combined body of the foam cladding and the equivalent mass, moving with a consistent velocity V , and the total mass $M = m_1 + m_f + m_{se}$.

At the instant that the foam layer is fully compacted (completely densified through the thickness direction), or the velocities of the cover plate and the equivalent mass are equal, the foam cladding will move together with the equivalent mass of the structure. Another case is that the foam will never deform when $P_0 \leq \sigma_0$. For all these cases, the motion equation can then be simplified as

$$\left[m_1 + m_f + m_{se} \right] \ddot{y} + ky - P(t)A = 0 \quad (6.9)$$

In Eqs. (6.6–9), u , y and $P(t)$ are time dependent. Hence Eqs. (6.6–9) are non-linear differential equations and they can be solved by using the Wilson- θ algorithm.

6.3 Non-dimensional parameters

6.3.1 Non-dimensional parameters p and i

In assessing the behavior of the blast loaded structure, the maximum deflection of the structure is the major concern. The maximum deflection $y_{\max}$ is normally used to define the structural damage, namely,

$$y_{\max} = y_c \quad (6.10)$$

where y_c is the critical deflection. In the present study, if a structure deforms within the critical deflection it indicates a safe scenario.

It is known that the initial peak pressure P_0 and impulse I_0 of the blast load determine the maximum deflection of a SDOF structure. According to Mays and Smith (1995), the initial peak pressure and impulse of the blast load can be normalized as

$$p = \frac{P_0 A}{k y_c / 2} \quad \text{and} \quad i = \frac{I_0}{y_c \sqrt{k m_{se}}} \quad (6.11a,b)$$

6.3.2 Non-dimensional parameter κ

To achieve an effective structural protection, the plateau stress of the foam material should be appropriately selected to match with the resistance of the structure. The resistance of the structure system can be denoted as $\frac{1}{2} k y_c$ and the plateau stress of the foam can be selected by comparing with the resistance of the structure.

Considering that a distributed pressure with a magnitude of P_0 is applied to the cladding-protected structure, if the plateau stress of the foam σ_0 is very low, e.g., $\sigma_0 A \ll \frac{1}{2} k y_c$, the foam will deform even under a lower load $P_0 A$ than the structural resistance, i.e., $\sigma_0 A < P_0 A < \frac{1}{2} k y_c$. However, under such a load, the structure will not produce any damage even without the foam cladding. Therefore, the foam cladding is not economically utilized in such a circumstance.

On the contrary, if the plateau stress of the foam panel is very high, e.g., $\sigma_0 A > \frac{1}{2} k y_c$, the pressure transferred to the structure, which is approximately equal to the plateau stress of the foam, is higher than the allowable load based on the maximum allowable deflection of the structure system. In this case, the structure may experience a maximum deflection beyond y_c even with the attached foam cladding.

Therefore, based on the above preliminary analysis, a proper selection of the foam material for the cladding should satisfy

$$\sigma_0 A \approx \frac{1}{2} k y_c \quad (6.12)$$

To describe the relation between the plateau stress of the foam and the resistance of the structure, a non-dimensional parameter is defined as follows,

$$\kappa = \frac{\sigma_0 A}{ky_c / 2} \quad (6.13)$$

6.3.3 Non-dimensional parameter τ

To achieve a safe design, the foam layer should have a sufficient thickness. According to the analysis in Chapter 4, see Eq. (4.25), the thickness of the foam layer fixed at a rigid reaction wall to completely absorb the blast impulse without being fully compacted is derived as

$$l \geq \frac{I_0^2}{(m_f + 2m_1)\varepsilon_D \sigma_0 A} \left(1 - \frac{4\sigma_0}{3P_0}\right), \quad \text{for } \frac{P_0}{\sigma_0} > 2 \quad (6.14)$$

where m_f is the mass of the foam layer ($m_f = \rho A l$), m_1 is the mass of the cover plate and I_0 is the incident blast impulse.

When $4\sigma_0/3P_0$ is approximately zero which means that the peak pressure P_0 is much larger than the plateau stress σ_0 , the minimum thickness of the foam layer is approximated as

$$l = \frac{I_0^2}{(m_f + 2m_1)\varepsilon_D \sigma_0 A} \quad (6.15)$$

During the time when the foam layer, with a thickness of l , deforms until it becomes fully compacted, the fixed distal end of the foam layer experiences a constant pressure of $\sigma_0 A$; and the foam layer transfers an maximum impulse of I_0 which is equal to the incident blast impulse, where

$$I_0 = \sigma_0 A t_m \quad (6.16)$$

in which t_m is the maximum duration of the transmitted rectangular pressure. Combining Eqs. (6.15) and (6.16) yields

$$t_m = \sqrt{\left(m_f + 2m_1\right) \frac{l \varepsilon_D}{\sigma_0 A}} \quad (6.17)$$

As seen from Eqs. (6.15) and (6.17), the maximum impulse I_0 increases and the corresponding maximum duration of the transmitted pressure t_m also increases when the thickness of the foam layer increases.

On the other hand, a proper design of the thickness of the foam layer depends on the deformation property of the protected main structure as well. An optimal design of the foam layer thickness is that the foam layer becomes fully compacted at the instant when the structure experiences the maximum allowable deflection.

Consider an elastic SDOF structural model subjected to a rectangular pulse load with a constant force of $\sigma_0 A$, the deflection time history of the SDOF structure system can be expressed as,

$$y(t) = \frac{\sigma_0 A}{k} \left(1 - \cos \frac{2\pi}{T} t\right) \quad (6.18)$$

where T is the natural period of the equivalent structure system, and $T = 2\pi \sqrt{m_{se} / k}$.

At the time $t=T/2$, the structure reaches the maximum deflection. It is then reasonable to suggest that the foam layer has such a thickness that the foam layer becomes fully compacted at the instant of $t=T/2$. This indicates that $t_m=T/2$, i.e.,

$$\sqrt{(m_f + 2m_1) \frac{l\varepsilon_D}{\sigma_0 A}} = \frac{T}{2} \quad (6.19)$$

The maximum duration of the transmitted pressure t_m varies with the foam material property, the thickness of the foam layer and the mass of cover plate. The second non-dimensional parameter is therefore defined as $\frac{t_m}{T/2}$, namely,

$$\tau = \frac{\sqrt{(m_f + 2m_1) \frac{l\varepsilon_D}{\sigma_0 A}}}{T/2} \quad (6.20)$$

6.4 A case study using LCS model

Assume that a steel wide flange I-beam (W14×426) is to be protected. The beam is 6 m long, with both ends fully clamped. When the beam is converted to an SDOF structural model, the equivalent mass and stiffness are respectively 2720 kg and 1.12×10^8 N/m. The critical deflection is assumed to be 9.1 mm. The following example first considers the case with $\kappa=1$ and $\tau=1$. For $\kappa=1$ and $\tau=1$, the steel cover plate is 1.5 cm thick and the foam layer is 23 cm thick. The density, plateau stress, and densification strain of the foam are respectively 225 kg/m^3 , 1.25 MPa and 0.8.

Fig. 6.2 Typical non-dimensional input pressure on the foam cladding and non-dimensional transmitted pressure on the structure

Fig. 6.2 gives the typical non-dimensional input pressure on the foam cladding and the non-dimensional transmitted pressure on the structure. The input pressure is a simplified blast loading with a high initial peak pressure and a short duration. The transmitted pressure σ_t (contact pressure at the interface between the foam layer and the structure) is reduced to a roughly constant magnitude of σ_0 with a longer duration. Provided that the foam layer has not been fully compacted, the transmitted

pressure is approximately equal to the plateau stress of the foam material. It might be slightly lower than the plateau stress due to the movement of the structure.

Figs. 6.3 and 6.4 plot the typical non-dimensional deformation of the foam cladding $[(u-y)/l \varepsilon_D]$ and the non-dimensional deflection of the structure (y/y_c) . It is seen that the foam layer undergoes large plastic deformation when it is subjected to the blast load. Simultaneously, the structure deforms due to the transmitted pressure by the foam cladding. The deflection and velocity of the equivalent lump mass in the analysis are relatively small compared to those of the cover plate of the foam cladding. The deflection (y) of the equivalent SDOF structure is in the order of 1 cm and the velocity ($\dot{y}$) is within 5 m/s while the displacement (u) of the cover plate of the foam cladding is between 5 cm and 25 cm, and the velocity ($\dot{u}$) is up to 50m/s. It is worth noting that the foam deforms slightly less when the additional y is included than when treating the protected structure as rigid.

Fig. 6.3 Typical non-dimensional deformation of the foam cladding

Fig. 6.4 Typical non-dimensional deflection of the structure

It is found that the foam layer is partially compacted while the rear end of the foam remains in the plateau region and the deflection of the structure is below the critical value when the blast load is at a low level. When the peak blast load increases to a certain value, the foam layer becomes fully compacted and the deflection of the structure may exceed the allowable value.

It is also found that the structure can withstand higher intensity of the blast load with the attached foam cladding, compared with a bare structure without a cladding. The maximum allowable blast load is plotted by using the widely used pressure-impulse ($p-i$) diagram of the SDOF structural model (Mays and Smith 1995). The $p-i$ curve represents various combinations of the initial peak pressure p and the impulse i of the external load that will cause the structure experience a maximum deflection of y_c .

As shown in Fig. 6.5, the double-dotted-dash curve represents various combinations of the initial peak pressure p and the impulse i of the external load that will cause the same maximum deflection y_c of the structure without a cladding. The impulse asymptote of the curve is $i=1.0$ and the quasi-static asymptote is $p=1.0$ (Li and Meng 2002b; Mays and Smith 1995; Smith and Hetherington 1994).

When the foam cladding is attached to the structure, the solid curve in Fig. 6.5 is obtained based on the LCS model, which is called the non-dimensional $p-i$ diagram

of the cladding-protected structure in the present study. Different combinations of the pressure and impulse of the external load on the curve will cause the same maximum deflection y_c to the cladding-protected structure. Combinations of the pressure and impulse of the external load that fall below the curve indicate no failure of the structure, while those above the curve will produce damage for the structure.

In addition, the uniformly-dotted curve in Fig. 6.5 is derived which represents the critical pressure-impulse combinations that will cause the foam layer to be fully compacted. Combinations of the peak pressure and the impulse of the external load that fall below the curve indicate that the foam layer is partially compacted, while those on and above the curve will cause the foam layer to be fully compacted.

Both the solid curve and the uniformly-dotted curve are obtained by trial and error based on the LCS model and the error percentage of the deflection of the structure is 0.5%.

Fig. 6.5 Non-dimensional p - i diagrams of the structure ($\kappa = 1.0$, $\tau = 1.0$)

It is seen that the p - i diagram of the cladding-protected structure lies on the right and above of that without a cladding and the impulse asymptote of the curve is

$i=1.61$. This indicates that the structure can withstand 61% higher impulse with the foam cladding under a very high peak pressure load. The protective effect of the cladding is even more notable under moderate and low peak pressure loads.

The uniformly-dotted $p-i$ curve indicating that the foam layer becomes fully compacted locates closely below the $p-i$ diagram of the cladding-protected structure. It means that the structure reaches the critical deflection just after the foam cladding becomes fully compacted.

6.5 Maximum allowable blast load

The maximum allowable blast load can be determined for various cladding-structure configurations. For different κ and τ , the thickness of the cover plate varies from 0.5 cm to 3.5 cm and the thickness of the foam layer varies from 7 cm to 35 cm. The investigation can be classified into the following three categories.

6.5.1 Foam cladding with $\kappa=1$, while $\tau \neq 1$

Fig. 6.6 depicts the $p-i$ diagrams in the case of $\tau=0.67$ for the structure that has a longer natural period or the foam layer is relatively thinner than the case when $\tau=1$ in Section 6.4. It is seen that the foam cladding will become fully compacted under comparatively lower intensity of blast loads. Consequently, the maximum allowable intensity for the cladding protected structure decreases compared with the case that $\tau=1$. The impulse asymptote of the $p-i$ diagram of the cladding-protected structure is $i=1.15$.

Fig.6.6 Non-dimensional $p-i$ diagrams of the structure ($\kappa = 1.0$, $\tau = 0.67$)

Fig. 6.7 gives the $p-i$ diagrams in the case of $\tau=1.33$ for the structure that has a shorter natural period, or the foam layer is thicker compared with the case that $\tau=1$. As seen, the foam cladding will become fully compacted under comparatively higher intensity of blast loads. As a result, the protective effectiveness of the cladding is improved. The impulse asymptote of the $p-i$ diagram of the cladding-protected structure is $i=2.20$.

Fig. 6.7 Non-dimensional $p-i$ diagrams of the structure ($\kappa = 1.0$, $\tau = 1.33$)

6.5.2 Foam cladding with $\tau=1$, while $\kappa \neq 1$

Fig. 6.8 shows the p - i diagrams in the case of $\kappa=0.67$ for the plateau stress of the foam material is lower or, equivalently, the resistance capacity of the structure is larger compared with the case when $\kappa=1$. It is seen that the foam cladding is relatively easier to be fully compacted. As a result, the structure may experience the critical deflection after the foam cladding becomes fully compacted under a comparatively lower blast load. As seen, the impulse asymptote of the p - i diagram with $\kappa=0.67$ is $i=1.26$.

Fig. 6.8 Non-dimensional p - i diagrams of the structure ($\kappa = 0.67$, $\tau = 1.0$)

On the other hand, a foam cladding with $\kappa > 1$ means that the strength of the foam layer is higher or, equivalently, the resistance capacity of the structure is smaller compared with the case when $\tau=1$ and $\kappa=1$. Comparatively, the foam cladding is harder to be fully compacted while the transmitted pressure on the structure is higher.

Fig. 6.9 shows the $p-i$ diagrams in the case of $\kappa=1.33$. It is interesting to note that the $p-i$ diagram of the cladding-protected structure is a straight line when $p>2.0$ and the abscissa of the line is initially fixed at $i=1.18$. This is because all the external $p-i$ combinations locating on the vertical straight line have been converted to the same rectangular loading. It is seen that the foam cladding has no influence on the $p-i$ diagram when $p<2.0$ because the foam undergoes little or no deformation under such a low peak pressure and thus does not have any protective effect.

It is seen in Fig. 6.9 that the uniformly-dotted curve representing the critical $p-i$ combinations which cause the foam layer to be fully compacted locates above the $p-i$ diagram of the cladding-protected structure. This indicates that the foam layer is only partially compacted when the structure reaches the critical deflection. In this case, the use of the present LCS model is especially important because the protected structure is more vulnerable than the foam cladding. It is not appropriate to treat the protected structure as rigid, as done by previous studies.

Fig. 6.9 Non-dimensional $p-i$ diagrams of the structure ($\kappa = 1.33$, $\tau = 1.0$)

6.5.3 Foam cladding with $\kappa \neq 1$ and $\tau \neq 1$

For other configurations of cladding-structure corresponding to varying κ and τ , a protection effective factor of the foam cladding is defined as

$$n = \frac{i_{with} - i_{without}}{i_{without}} \quad (6.21)$$

where $i_{without}$ and i_{with} are the non-dimensional critical impulses associated with a very large non-dimensional peak pressure p ($p=20$ in the following case study) for the structure without and with the foam cladding, respectively.

For a fixed τ , the protective effect of the foam cladding varies with κ , as illustrated in Fig. 6.10. If τ is larger than or equal to 1.0 ($\tau=2.0, 1.5$ and 1.0 in Fig. 6.10), n increases with an increase of κ until it reaches a peak value at $\kappa=1.0$. When κ becomes larger than 1.0, n drops abruptly and then keeps decaying with a further increase of κ . If τ is smaller than 1.0 ($\tau=0.75$ and 0.5 in Fig. 6.10), the protective effect of the foam cladding is comparatively less satisfactory and it varies with κ in a similar way, i.e., n increases at first, and then decreases with the increase of κ . In this case, the peak value of n occurs at a value of κ larger than 1.0.

Fig. 6.10 Influence of κ and τ on the protective effect of the foam cladding

It is also seen in Fig. 6.10 that the protection effective factor n of the foam cladding increases with the increase of τ . The influence of τ on the protective effect of the foam cladding is significant if κ is smaller than or equal to 1.0 and the increasing tendency of n with the increase of τ is obvious. However, in the case of κ is larger than 1.0, the increase of τ induces little or no increase of n .

When τ is smaller than 0.5 (the duration of the transmitted pressure is less than $T/4$), the protection effective factor n of the foam cladding is nearly zero. This indicates that the main structure will have almost the same deflection with or without the foam cladding. This is consistent with Clough and Penzien (1975) which indicates that the deflection of the structure is proportional to the total incident pulse when the loading duration is less than one quarter of the natural period of the structure.

6.6 Maximum deflection of the protected structure

Based on the LCS model, the deflection of the foam-protected structure can be predicted. Figs. 6.11(a)~(d) plot the normalized maximum deflection of the equivalent structure under different loads. The non-dimensional maximum deflection of the structure is defined as the maximum deflection divided by the critical deflection y_c . For comparison purpose, the maximum deflection of a bare SDOF structure without a foam cladding is obtained based on the pressure-impulse diagram method (Mays and Smith 1995). The non-dimensional maximum deflections of a bare structure under different p and i corresponding to the Figs. 6.11 (a)~(d) are 1.15, 1.79, 1.19 and 1.97, respectively.

Fig. 6.11 Normalized maximum deflection of the foam protected structure under different blast loads (a) $p=4$, $i=1.2$ (b) $p=4$, $i=2.0$ (c) $p=10$, $i=1.2$ and (d) $p=10$, $i=2.0$

It is seen from Figs. 6.11(a)~(d) that the normalized maximum deflection of the foam-protected structure varies with the two non-dimensional parameters (κ and τ) of the foam cladding. The maximum deflections corresponding to different κ and τ form a three-dimensional surface, and it looks like the terrain of a valley. The lowest maximum deflection of the structure corresponds to a small κ and a large τ . As seen, the maximum deflection of the structure is nearly constant for a fixed κ as long as τ is sufficiently large and the foam cladding is ensured to withstand the blast load without being fully compacted. The maximum deflection of the structure begins to increase when τ is lower than a certain value, as shown in the left slope of the “valley” in Figs. 6.11(a)~(d). This is because the foam cladding with a small τ has low blast resistant capacity (see Eq. (6.20)) and it may become fully compacted

and subsequently impact to the structure with a considerable velocity. In this case, the foam-protected structure may experience a larger maximum deflection than a bare SDOF structure.

For a fixed and sufficiently large τ , the maximum deflection of the structure increases with the increase of κ (the increase of κ means the increasing of the plateau stress of the foam and the pressure transmitted to the protected structure), as seen in the right slope of the “valley” in Figs. 6.11(a)~(d). When κ is very large, the maximum deflection of the foam protected structure is nearly equal to the maximum deflection of the bare structure. This is because the foam does not deform and acts as a rigid body which moves together with the protected structure when κ is larger than p .

6.7 Design map of the foam cladding

Appropriate foam claddings can be designed to protect the structure against blast loads based on the LCS model. Figs. 6.12(a)~(d) give four design maps of the foam claddings under different blast loads. The design map can be divided into four regions. In Region I, distinct from the other three regions, κ is larger than a certain value, which indicates that the plateau stress of the foam, hence the transmitted pressure to the structure, is comparatively large. The foam cladding in this region is unable to prevent the structure from deflecting beyond the critical deflection no matter how large τ is. In Region II, τ is smaller than required and the foam cladding in this region is easier to be fully compacted and not able to prevent the structure from damage either.

The foam cladding in Region III and IV (boundaries inclusive) can reduce the maximum deflection of the structure lower or equal to the critical deflection. These two regions are separated by the dashed curve as shown in Figs. 6.12(a)~(d). The difference between Region III and Region IV is that the foam in Region III is fully compacted and the foam in Region IV is partly compacted when the equivalent SDOF structure experiences the maximum deflection. It should be realised that the

structure could undergo an enhanced pressure load as soon as the foam is fully compacted.

Fig. 6.12 Design of foam claddings under different blast loads (a) $p=4$, $i=1.2$ (b) $p=4$, $i=2.0$ (c) $p=10$, $i=1.2$ and (d) $p=10$, $i=2.0$

The boundaries among the four regions depend on p and i . In Figs. 6.12(a)~(d), the boundary between Region I and the other three regions is distinguished by κ_0 which equals 1.27, 1.0, 1.25 and 1.0, respectively. The two boundaries distinguishing Region II, III and IV meet at a joint point (τ_0, κ_0) . The value of τ_0 in Figs. 6.12(a)~(d) is equal to 0.44, 1.02, 0.54, and 1.18, respectively.

To be conservative, we can design the foam cladding inside Region IV. The scope of Region IV can be roughly curve fitted as below,

$$\kappa \leq 1 \text{ and } \tau \geq \frac{i\sqrt{1-\kappa/p}}{1.6\kappa} \quad (6.22a, b)$$

When we look into the experimental study on the pendulum system by Hanssen et al. (2002a), the critical deflection y_c is referred to the maximum rotation angle (in radian) when the foam panel is not attached to the pendulum (Test Series I). It is known that the resistance force of the pendulum is the gravity and the stiffness of the pendulum can be written as $k = mg$, where g is the acceleration of gravity. The non-dimensional parameter of the foam cladding used in the test is derived as $\kappa=680$, which is much larger than 1.0. The other non-dimensional parameter τ is very small, i.e., $\tau \approx 0.001$. Therefore, the foam cladding used in the pendulum test in Hanssen et al. (2002a) may locate in Region I of the design map, which indicates an inappropriate design. The observation that the foam layer was unable to reduce the global response of the pendulum is thus understandable.

6.8 Summary

An analytical Load-Cladding-Structure (LCS) model, incorporating the properties of the blast load, the foam cladding and the protected structure, has been developed to evaluate the blast alleviation efficiency of foam claddings for structural retrofit. The present model differs from other models in that it considers the response of the protected structure. Two non-dimensional parameters, i.e., κ and τ have been introduced to describe the relations between the foam cladding and the protected structure. Knowing any two of the three components of the LCS model, the third one can be determined or predicted. Despite the limitations such as the idealized RPPL foam model and the equivalent SDOF structural model, the LCS model gives a good guidance to the design and evaluation of foam claddings for structural retrofit against blast loads.

For a given cladding-protected structure, the maximum allowable blast loads have been predicted and plotted by the non-dimensional $p-i$ diagrams of the equivalent SDOF structural model. It is seen from the analytical results that when the non-

dimensional κ is close to and not larger than 1.0 and τ is as large as possible, the foam protected structure has a much larger blast resistant capacity than the bare structure. It is also seen that the maximum deflection of foam protected structure under blast loading varies with two non-dimensional parameters, i.e., κ and τ of the foam cladding. To protect the structure from damage, the foam cladding should locate in two of the four regions of the design map, where the boundaries of the four regions depend on the blast load (non-dimensional p and i).

Chapter 7 Effectiveness of Foam Cladding for Structural Retrofit: Numerical Simulation

7.1 Introduction

To test the effectiveness of sacrificial foam claddings, some free-field blast experiments have been carried out. Guruprasad and Mukherjee (2000b) reported that the sacrificial cladding achieved good pressure attenuation. However, Hanssen et al. (2002a) reported that a foam panel amplified the response of the protected structure. It was argued by Hanssen et al. (2002a) that the increased energy and impulse transfer was due to that the shape of the foam cladding surface had changed into a double-curved shape (dishing shape) during the blast event. Undoubtedly, more numerical and experimental studies are needed to better understand the role that the foam cladding plays in blast mitigation.

As known, full scale blast field tests are expensive, very complicated to perform and sometimes exclusive. The explicit finite element code LS-DYNA is widely used to predict the dynamical response of structures under blast loads. The overpressure due to a blast event can be calculated by the empirical function CONWEP (1986) or by an Arbitrary Lagrangian-Eulerian coupling model. The first approach is empirical and relatively simple. The second approach can model the propagation of the blast wave in the air and the interaction between the blast wave and the structures.

In this chapter, two numerical structural models have been developed with LS-DYNA to verify the analytical results on the foam claddings based on the LCS model. Some aspects neglected in the LCS model such as the stand-off distance of the explosive and the dishing shape of the cover plate will be reflected in the numerical analysis. One numerical model is based on the experimental study by

Hanssen et al. (2002a), which is a ballistic pendulum with and without a foam cladding subjected to close-range blast loading. Explanations are given on the unexpected observation that the global response of the pendulum was increased by the attached foam panel. The other model is a steel beam with and without a foam cladding under blast loading. Foam claddings are designed based on the method introduced in Chapter 6, and the protective efficiency varies with the two non-dimensional parameters of the foam cladding.

7.2 Full-scale simulation of the pendulum test

7.2.1 Lagrangian model

One simple way to model the blast test by Hanssen et al. (2002a) is to model the pendulum structure (with and without foam cladding) merely by using Lagrangian elements. The blast overpressure can be calculated by the empirical CONWEP air blast function available within LS-DYNA. The fluid-structure interaction is not considered by the CONWEP function.

The Lagrangian model of the pendulum is established based on the data given by Hanssen et al. (2002a). The finite element model was generated with the pre-processor of ANSYS/LS-DYNA. As shown in Fig. 7.1, the 2.35 m long steel pendulum can rotate freely around a static axle. The pendulum system consists of a solid bob and four cylindrical bars. The bob is measured as $180 \times 700 \times 684 \text{ mm}^3$ and its mass is 676.5 kg. The cross-sectional area of each bar is 0.005 m^2 . The total mass of the pendulum is 935 kg and the distance from the centroid to the rotational axis is 1.645 m, both values are exactly the same as in the test. The gravity is applied in the y-direction.

The foam cladding consists of a foam panel and a cover plate made of aluminium alloy AA6061. The cross sectional area of the cladding is tailored to be the same as strike face of the pendulum bob ($700 \times 684 \text{ mm}^2$). The thickness of the cover plate and the foam panel are respectively 1 cm and 6 cm. All data are consistent with Test Series B by Hanssen et al. (2002a).

There are 3456 solid hexahedron elements and 4 beam elements (truss) in the Lagrangian model. Typical edge length of the element for the bob is 3 cm. The element size is $1 \times 3 \times 3$ m for the cover plate and the foam. The computational time is about two hours with a Pentium-IV personal computer.

Fig. 7.1 Lagrangian model of the pendulum with the foam cladding

The foam material has a density of 150 kg/m^3 , the plateau stress is 1.0 MPa and the densification strain is 0.7. The steel material in the pendulum is assumed to be elastic, with the density 7850 kg/m^3 , Young's modulus 200 GPa and Poisson's ratio 0.3. The aluminum alloy in the cover plate is also assumed to be elastic, with the density 2700 kg/m^3 , Young's modulus 70 GPa and Poisson's ratio 0.3.

The charge used in the Test B and Test I by Hanssen et al. (2002a) is 1.0 kg PE4, which is equivalent to 1.19 kg TNT. The air blast loading can be automatically

calculated by the *LOAD_BLAST card, which has been implemented in LS-DYNA based on CONWEP (1986).

The CONWEP function is an empirical formula for calculating free field blast loading. For a given mass of TNT, the overpressure time history at a certain range can be approximately derived. The incident and reflected overpressure history in the positive phase of the air blast are expressed as,

$$P_{in}(t) = P_{s0} \left(1 - \frac{t-t_a}{t_0}\right) e^{-\frac{t-t_a}{a}}, \quad \text{for } t_a \leq t \leq (t_a + t_0) \quad (7.1a)$$

$$P_{ref}(t) = P_{r0} \left(1 - \frac{t-t_a}{t_0}\right) e^{-\frac{t-t_a}{b}}, \quad \text{for } t_a \leq t \leq (t_a + t_0), \quad (7.1b)$$

where P_{r0} is the peak reflected pressure and P_{s0} is the peak incident pressure, t_0 is the duration of the positive phase, t_a is the arrival time. a and b are decay coefficients.

The *LOAD_BLAST takes into consideration of the incident angle of blast (θ), the reflected pressure ($P_{ref}(t)$) and incident pressure ($P_{in}(t)$), as below,

$$P(t) = P_{ref}(t) \cos^2 \theta + P_{in}(t)(1 + \cos^2 \theta - 2 \cos \theta) \quad (7.2)$$

In the Test Series B by Hanssen et al. (2002a), a foam cladding (a foam panel with a cover plate) is attached to the pendulum. Hanssen et al. (2002a) also did a reference test of the bare pendulum—Test Series I. Table 7.1 cites some of the experimental data from their paper.

Table 7.1 Experimental observation of the pendulum test (Hanssen et al 2002a)

Case	Stand-off distance (cm)	Foam cladding	Maximum rotation angle α (deg)	Maximum potential energy (J)	Imparted impulse (Ns)
Test B	50	With	9.13±0.31	195.2±13.4	609.3±20.9
Test I	---	Without	8.28±0.25	157.5±9.5	542.6±16.4

Corresponding to Test Series B, we assumed that the foam is perfectly bonded to the pendulum. The boundary nodes of the foam and the pendulum are merged in the numerical model. We are uncertain about the stand-off distance in the Test Series I. We simulate two possible situations corresponding to Test Series I, namely, Case L-I1 and Case L-I2.

- Case L-B, the foam cladding is attached to the pendulum. The stand-off distance from the charge to the cover plate is 50 cm.
- Case L-I1, the foam cladding is removed from the pendulum. The charge is shifted by 7 cm to the pendulum, and thus the stand-off distance from the charge to the striker face of the pendulum remains at 50 cm.
- Case L-I2, the foam cladding is removed from the pendulum. The charge and the pendulum remain in the same positions as in Case L-B. The stand-off distance from the explosive to the striker face of the pendulum is 57 cm.

Fig. 7.2 gives the displacement history in the x axis direction of the bottom of the pendulum. The maximum amplitude of the displacement reads as 36.84 cm, 36.69 cm and 32.94 cm, respectively for Case L-B, L-I1 and L-I2. As seen, the maximum response of the pendulum in Case L-B and L-I1 are almost the same, but both are obviously larger than that in Case L-I2.

Fig. 7.2 Displacement time history of the bottom of the pendulum

The maximum rotation angle of the pendulum can be calculated with the x-displacement and the length of the pendulum, as given in Table 7.2.

Table 7.2 Numerical results of the Lagrangian model

Case	Stand-off distance (cm)	Foam cladding	Maximum rotation angle α (deg)	Maximum potential energy (J)	Imparted impulse (Ns)
L-B	50	With	9.02	190.5	602.7
L-I1	50	Without	8.98	184.8	592.3
L-I2	57	Without	8.06	148.8	531.4

The potential energy acquired by the pendulum with and without the foam cladding, as given in Table 7.2, is calculated from the maximum rotation angle, following Eqs. (7.3a) and (7.3b),

$$P_{E1} = (M + M_f)gD(1 - \cos \alpha_{\max}) \quad (7.3a)$$

$$P_{E2} = Mgd(1 - \cos \alpha_{\max}) \quad (7.3b)$$

where M is the mass of the pendulum (935 kg), M_f is the mass of the foam cladding (17.2 kg). D is the distance from the centroid of the foam-attached pendulum to the rotation axis (1.651 m) and d is the distance from the centroid of the bare pendulum to the rotation axis (1.645 m).

The impulse imparted by the blast loading to pendulum with and without the foam cladding can be calculated by Eqs. (7.4a) and (7.4b),

$$I_1 = \sqrt{2(M + M_f)^2 gD(1 - \cos \alpha_{\max})} \quad (7.4a)$$

$$I_2 = \sqrt{2M^2 gd(1 - \cos \alpha_{\max})} \quad (7.4b)$$

It is seen from Tables 7.1 and 7.2 that the simulation result of Case L-B is very close to the experimental result of Test B. And the simulation result of Case L-I2 is very close to the experimental results of Test I.

Comparing of Case L-B and L-I1, the global response of the pendulum is almost same. The attached foam cladding is not able to reduce the global response of the pendulum. This is consistent with the analytical prediction in Chapter 6.

In Chapter 6, we have introduced two non-dimensional parameters of the foam cladding, namely, κ and τ . The non-dimensional parameter of the foam cladding used in Case L-B is derived as $\kappa=680$, which is much larger than 1.0. The other non-dimensional parameter τ is very small, i.e., $\tau\approx0.001$. Based on the analytical LCS model in Chapter 6, the protection effective factor of the foam cladding $n=0$, namely, the pendulum's blast resistant capacity has almost no improvement or reduction when the foam panel is added.

Comparing of Case L-I1 and L-I2, we can see that the global response of the bare pendulum is very sensitive to the stand-off distance in such a close-range blast event. The impulse imparted to the pendulum is increased by 11.5% when the stand-off distance is decreased by 7 cm.

Comparing of Case L-B and L-I2, it is seen that the attached foam cladding does not improve the blast resistant capacity of the pendulum. On the other hand, the attachment of foam cladding makes the stand-off distance shorter, and hence makes the incident blast loading more intensive.

The energy absorbed by the foam panel in Case L-B can be obtained from the numerical results, which read as 10861 J. The energy absorbed by the foam panel is greatly larger than the energy acquired by the pendulum (less than 200 J). We can see that the blast loading imparts much more energy to the whole structural system when the pendulum is covered with a foam cladding, compared with the bare pendulum. Although the foam can dissipate much energy in plastic deformation, the energy transferred to the pendulum is not necessarily lower than that when the pendulum is directly subjected to blast loading.

7.2.2 ALE model

Another way to model the full scale blast test is to use arbitrary Lagrangian-Eulerian (ALE) coupling method. In the arbitrary Lagrangian-Euler model, as shown in Fig. 7.3, the explosive and the air are modeled using Eulerian elements and the pendulum is modeled using Lagrangian elements. By using ALE method, the explosive detonation and the process of shock propagation in the air can be modeled. The fluid-structure interaction can be captured by the ALE model. Therefore, the dishing shape effect on the blast wave can be evaluated. With the ALE model, the three cases mentioned in the previous section are simulated again. The three cases here are called Case A-B, Case A-I1 and Case A-I2.

In the ALE model, 1.0 kg PE4 detonates in the air, 0.7 m above the ground and at a distance of 0.5 m from the cover plate of the foam. The explosive is a sphere with a diameter of 10.64 cm. The air, modeled as a rectangular prism, is sufficiently large to cover the bob of the pendulum. The non-reflecting boundary conditions on the top and side surfaces of the Euler's domain were assumed. The boundary condition for the ground is modeled as a rigid surface.

The multi-material ALE formulation (ELFORM=11) is adopted for the explosive and the air. The Eulerian elements allow materials mixing and mass transfer. The Eulerian mesh consists of 132,480 hex elements (including 16,000 for the explosive), with a minimum edge length of 0.2 cm for the PE4 to a maximum 2.5 cm for the air and a suitable intermediate zone between them, as shown in Fig. 7.4. The Lagrangian mesh for the pendulum is identical to that in the previous section.

Coupling between the Eulerian and Lagrangian elements was accomplished by an appropriate LS-DYNA feature called *CONSTAINED_LAGRANGIAN_IN_SOLID. The interaction between the pendulum and the air blast was modeled by penalty algorithm.

Fig. 7.3 Arbitrary Lagrangian Eulerian coupling model

Fig. 7.4 Finite element model for Euler's domain (side view)

The Jones-Wilkins-Lee (JWL) equation of state is used for detonation products of PE4, as following

$$p = A(1 - \frac{\omega}{R_1 V})e^{-R_1 V} + B(1 - \frac{\omega}{R_2 V})e^{-R_2 V} + \frac{\omega E}{V} \quad (7.5)$$

where the constants are given in Table 7.3 with reference to Lu and Kennedy (2003).

Table 7.3 Constants of JWL equation of station for PE4

Parameter	Value	Description
ρ_0 (kg/m ³)	1590	Mass density
D_{cj} (m/s)	7900	Detonation velocity
P_{cj} (Pa)	2.4E10	Chapman-Jouget pressure
A (Pa)	7.74E11	
B (Pa)	8.677E9	
R_1	4.837	
R_2	1.074	
ω	0.284	
E_0 (J/m ³)	9.381E9	Initial internal energy
V_0	1.0	Initial relative volume

The linear polynomial equation of state is used for the air.

$$p = C_0 + C_1 \mu + C_2 \mu^2 + C_3 \mu^3 + (C_4 + C_5 \mu + C_6 \mu^2)E \quad (7.6)$$

where $\mu = \frac{\rho}{\rho_0} - 1$, and $\frac{\rho}{\rho_0}$ is the ratio of current density to initial density. The

constants are given in Table 7.4.

Table 7.4 Constants of linear polynomial equation of station for the air

Parameter	Value	Description
ρ_0 (kg/m ³)	1.29	Mass density
C_0 (Pa)	-1.0E5	
C_1 (Pa)	0	

C_2 (Pa)	0	
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Table 7.4 Constants of linear polynomial equation of station for the air
(Continued)

Parameter	Value	Description
C_3 (Pa)	0	
C_4	0.4	
C_5	0.4	
C_6	0	
E_0 (J/m ³)	2.5E5	Initial internal energy
V_0	1.0	Initial relative volume

When the explosive is detonated, a spherical blast wave develops. It takes about 100 μ s for the wave front to reach the structural surface. The structure will be engulfed and pushed by the blast wave. Fig. 7.5 shows the iso-surfaces of the overpressure in the air after 200 μ s from the initiation of detonation.

Fig. 7.5 Blast wave activated by the high explosive detonation

The blast wave vanishes and the overpressure drops to zero at the time of about 750 μs . We can delete the parts of the explosive and the air from the numerical model at this moment. The pendulum has acquired some energy and continues to swing until it reaches the highest position at the time of 0.72 s.

The cover plate in Case A-B acquires a considerable kinetic energy in a very short time. Fig. 7.6 shows the velocity contour on the cover plate at the time of 200 μs . The center part has a larger velocity than the side part. At the center (Point O), the velocity is about 30 m/s. At Point A, which is 0.25 m above Point O, the velocity is about 10 m/s. The average velocity of the cover plate is much larger than that of the bare bob in Case A-I1 at the same time after the detonation, where the latter is less than 1 m/s.

Fig. 7.6 Velocity distribution on the cover plate in Case A-B ($t=200 \mu\text{s}$)

Dishing shape of the foam cladding is not obvious during the interaction between the blast wave and the foam cladding, as shown in Fig. 7.7(a)~(c). At the time of $300 \mu\text{s}$ when the overpressure is at its peak value, the angle between the side and center points (A, O and B in Fig. 7.7(a)) is 179.0° . This indicates that the cover plate is almost flat. At the time of $600 \mu\text{s}$ when the blast wave almost disappears, the angle between the same three points is 178.0° , as shown in Fig. 7.7(b). It is seen that the dishing shape of cover plate is not significant when the blast wave acts on the cover plate.

It should be noted that the velocity distribution on the cover plate is not uniform at the moment and the dishing shape continues to grow after the blast overpressure drops to zero. What can be normally seen in an experiment is the final shape when the foam stops deformation. The final angle between the three points in Fig. 7.7(c) at the time of 0.8 s is 176.2° .

Fig. 7.7 The dishing shape of the foam cladding

The overpressure history at the center point of the cover plate (Case A-B) or the striker face of the bare bob (Case A-I1 and Case A-I2) is plotted in Fig. 7.8. The peak overpressure at the center of the cover plate in Case A-B is slightly lower than that at the center of the bare bob in Case A-I1. This is due to the cover plate moving backwards at a much larger velocity. As indicated in Smith and Hetherington (1994), the reflecting pressure at a backward moving surface is lower compared to that at a rigid fixed surface. It is also noted that the overpressure at the center of the bare bob in Case A-I2 is much lower due to a larger stand-off distance.

Fig. 7.8 Overpressure at the center of the cover plate or the striker face of the bare bob

The overpressure history at the side point of the cover plate (Case A-B) or the striker face of the bare bob (Case A-I1 and Case A-I2) is plotted in Fig. 7.9. The side point is 0.25 m above the center point. The overpressure at the side point in Case A-B is almost the same as that in Case A-I1. Again the overpressure in Case A-I2 is much lower due to a larger stand-off distance.

Fig. 7.9 Overpressure at the side point of the cover plate or the striker face of the bare bob

We can see from the overpressure history that the dishing shape of the cover plate and the movement of the cover plate during the loading process have trivial effect on the blast wave. As a result, the impulse imparted to the structural system by the blast wave is almost the same in Case A-B and Case A-I1.

Table 7.5 Numerical results of the ALE model

Case	Stand-off distance (cm)	Foam cladding	Maximum rotation angle α (deg)	Maximum potential energy (J)	Imparted impulse (Ns)
A-B	50	With	7.62	136.2	509.4
A-I1	50	Without	7.79	139.4	514.1
A-I2	57	Without	6.78	105.3	447.0

Table 7.5 gives the maximum rotation angle and maximum potential energy of the pendulum measured in the ALE model. The global responses of the pendulum in Case A-B is very slightly less than that in A-I1. The global response of the pendulum in Case A-I2 is much smaller than the other two cases, mainly due to a larger stand-off distance.

It should be noted that the global response predicted by the ALE model is smaller than the experimental observation (Table 7.1). The maximum potential energy acquired by the pendulum system (with the foam cladding) is 69.8% of the experimental value. And the imparted impulse to the pendulum system is 83.6% of the experimental value. This discrepancy may partially result from the algorithm of LS-DYNA.

7.3 Analysis of a beam subjected to blast loading

In the second analysis, we assume that a steel wide flange I-beam (W14×426) is to be protected. The 6 m long steel beam is W14×426 model (the depth, width, flange thickness and web thickness are respectively 0.46 m, 0.41 m, 74 mm and 46 mm)

and both ends of the beam are fully clamped. As shown in Fig. 7.10, half of the beam is numerically modeled with LS-DYNA.

The density of the steel is 7850 kg/m^3 . The steel is considered as an elastic, linearly hardening plastic material. The Young's modulus is 200 GPa, the Poisson's ratio is 0.3, the tangent modulus in the plastic regime is 2.0 GPa and the yield stress is 220 MPa. The positive moment capacity of the beam M_p can hence be obtained as $2.3 \times 10^6 \text{ Nm}$.

Fig. 7.10 Finite element model of wide flange I-beam with foam cladding

With reference to TM5-1300 (1990), the elastic resistance of the beam can be evaluated as

$$R = \frac{8(M_N + M_p)}{L} \quad (7.7)$$

where M_N is the negative moment capacity ($M_N = M_p$) and L is the length of the beam. With the value of moment capacity, the elastic resistance of the beam is obtained as $6.14 \times 10^6 \text{ N}$.

The equivalent stiffness of the beam is evaluated as

$$k = \frac{307EI}{L^3} \quad (7.8)$$

where E is the Young's modulus and I is the second moment of cross section area. Substituting all the values, the stiffness is obtained as $6.74 \times 10^8 \text{ N/m}$.

The allowable deflection of the beam without plastic strain is derived as,

$$y_c = \frac{R}{k} \quad (7.9)$$

which yields 9.1 mm.

According to TM5–1300 (1990), the load-mass factor F_{lm} for the fully clamped beam is 0.77 and the equivalent mass of the beam (converted to an equivalent SDOF model) is 2720 kg.

As known, the mechanical properties of foam materials are functions of the foam density. The stress-strain curves of different foams are determined according to the phenomenological model for the metallic foams in Ashby et al. (2000). Suitable foam claddings are designed to satisfy different values of κ and τ .

For $\kappa=1$ and $\tau=1$, the steel cover plate is 1.5 cm thick and the foam layer is 23 cm thick. The density, plateau stress, and densification strain of the foam are respectively 225 kg/m³, 1.25 MPa and 0.8. For different values of κ and τ , the thickness of the steel cover plate varies from 0.5 cm to 3.5 cm and the thickness of the foam layer varies from 7 cm to 35 cm. With the increase of κ , the density and the plateau stress of foam increase while the densification strain decreases. With the increase of τ , the thickness of the foam or/and the cover plate increases.

The steel beam, the foam and the steel cover plate are all modeled as brick elements. *MAT_PLASTIC_KINEMATIC and *MAT_CRUSHABLE_FOAM are used to model the steel and the aluminum foam, respectively.

It is assumed that the load is from the detonation of a 1000 kg TNT at a standoff distance of 8 m relative to the center of the beam's top surface. The blast load is calculated by the CONWEP equations implemented in LS-DYNA, which is usually accurate for predicting overpressure on simple structural surfaces. The pressure applied on the foam cladding is roughly uniform. The average peak pressure applied on the top surface of the beam is 8.89 MPa, and the impulse per unit area is

6194 Pa-s. According to Eqs. (6.11a, b), the non-dimensional peak pressure and impulse can be derived as $p=6.87$ and $i=1.23$, respectively.

It is found that the pressure and total impulse imparted to the structural system is slightly sensitive to the stand-off distance. When the stand-off distance from the TNT to the beam is 8 m, the impulse can be increased by about 5% because the distance is reduced by the attached foam cladding with a thickness of 20~30 cm.

Figs. 7.11-13 compare the pressure time history at the center point of the top surface of beam with and without a cladding. The foam cladding corresponding to Fig. 7.11 locates in Region I of the design map introduced in Chapter 6 (see Figs. 6.12(a)~(d)). The magnitude of the transmitted pressure by the foam is 1.5 MPa, which is still too high for the beam to deflect elastically.

Fig. 7.11 Pressure time history at the center point of the top surface of beam
($\kappa=1.2$ and $\tau=1.0$)

The foam cladding corresponding to Fig. 7.12 locates in Region II of the design map introduced in Chapter 6. The blast resistant capacity of the foam cladding itself is not adequate. The magnitude of the transmitted pressure is at first 0.75 MPa, then it becomes much higher when the foam is densified. Under such a high peak pressure, deflection of the beam is mostly due to shear deformation (Jones 1989).

Fig. 7.12 Pressure time history at the center point of the top surface of beam
($\kappa=0.6$ and $\tau=1.0$)

The foam cladding corresponding to Fig. 7.13 locates in Region IV of the design map introduced in Chapter 6. The magnitude of the transmitted pressure is nearly constant as 1.0 MPa. Under such a low pressure, the beam deflects elastically.

Fig. 7.13 Pressure time history at the center point of the top surface of beam
($\kappa=0.8$ and $\tau=1.25$)

Table 7.6 gives the maximum deflection and maximum von Mises stress of the beam protected with different foam claddings. Compared with the analytical prediction, the numerically derived maximum deflection is slightly larger than the corresponding analytical prediction. This discrepancy could result from the shear deformation of the beam in addition to the flexural deformation. The shear deformation of the beam, however, is not considered in the analytical model in Chapter 6.

Consistent with the analytical prediction in Chapter 6, the maximum deflection of the beam varies with the two parameters of the foam cladding. From the numerical results, it is seen that the parameter κ should be less than a certain value and, meanwhile, the parameter τ should be sufficiently large to prevent the beam from plastic damage.

According to the analytical model in Chapter 6, when $\kappa=0.8$, the parameter τ should be greater than 0.91 (see Eq. (6.22b)). When $\kappa=0.6$, τ should be greater than 1.23. However, it is needed to be more conservative according to the numerical simulation. When $\kappa=0.8$, the parameter τ should be greater than 1.05 according the numerical model. When $\kappa=0.6$, τ should be greater than 1.35 according the numerical model to achieve proper protection effect for the steel beam. When κ is greater than 1.0, the beam seems to always deform beyond the critical deflection.

Table 7.6 Maximum deflection of the beam subjected to blast loading

κ	τ	Analytical	Numerical				
		Max deflection (mm)	Max deflection (mm)	Max stress (MPa)	Local yield [#]	Average strain of foam	Foam densified
0.6	0.5	11.9	14.8	230.0	Y	0.97	Y
	0.75	10.3	13.6	228.6	Y	0.95	Y
	1.0	8.6	12.1	228.1	Y	0.91	Y
	1.25	6.8	9.5	227.1	Y	0.88	Y
	1.5	5.6	6.2	175.9	N	0.78	N
0.8	0.5	10.6	13.6	227.9	Y	0.96	Y
	0.75	9.2	12.9	228.2	Y	0.91	Y
	1.0	7.5	9.3	225.4	Y	0.86	Y
	1.25	7.5	8.3	218.3	N	0.61	N
	1.5	7.5	8.3	218.4	N	0.45	N
1.0	0.5	10.3	13.4	228.4	Y	0.86	Y
	0.75	8.7	11.4	228.0	Y	0.81	Y
	1.0	8.7	10.2	223.9	Y	0.55	N
	1.25	8.7	10.1	223.9	Y	0.37	N
	1.5	8.7	10.1	223.9	Y	0.25	N
1.2	0.5	9.9	13.2	228.0	Y	0.78	Y
	0.75	9.4	11.1	224.5	Y	0.55	N
	1.0	9.3	11.1	224.7	Y	0.32	N
	1.25	9.2	11.2	225.0	Y	0.24	N
	1.5	9.2	11.2	225.3	Y	0.13	N
---	---*	11.1	13.1	226.9	Y	---	---

Note: * For the beam without a foam cladding.

[#]N: No; Y: Yes

7.4 Summary

A pendulum model and a beam model have been developed in LS-DYNA to investigate the protective effect of the foam cladding. Some aspects neglected in the analytical analysis such as the stand-off distance of the explosive and the dishing shape of the cover plate have been discussed in the numerical analysis. It is found that the pendulum with the foam cladding always swings to a larger rotation angle compared to a bare pendulum. This is because inappropriate foam cladding can not improve the pendulum's blast resistant capacity while the reduced stand-off distance makes the incident load more intensive.

It is shown in the second numerical model that both the local stress and the global deflection of the beam are successfully mitigated by the foam cladding when it is appropriately designed. It is noted that the numerically simulated maximum deflection of the beam is higher than the analytically predicted value. This is due to the shear deformation which is not included in the analytical LCS model. Nevertheless, the numerical results basically conform to the conclusion based on the LCS model that the foam cladding should be designed appropriately by considering the blast load as well as the protected structure.

Chapter 8 Conclusions and Recommendations

8.1 Conclusions

A mesoscale finite element model has been developed to investigate the loading effect on the deformation mode and compressive strength of cellular materials. Several factors including the apparent strain rate and inertia that can lead to an increased nominal stress under dynamic conditions have been discussed. By discussing the nominal stress at the impact side and the stationary side, the controversy of some experimental studies on the dynamic strength of the cellular material can be explained. The enhancement of the nominal stress at the impact side is primarily due to the micro-inertia of cell walls. A continuum model with rate insensitive constitutive material model has been developed and compared with the mesoscale model, by which the inertia effect has been demonstrated. The nominal stress at the stationary side is found to be insensitive to loading rate unless the base material has strong rate-dependency. It is concluded that most cellular materials can be considered as rate insensitive.

The behavior of the cellular material under blast loading has also been modeled at the mesoscale. The transmitted pressure and impulse by the cellular material have been investigated. The transmitted pressure can be reduced to a low level. However, the rebound of the cellular material can lead to an increased transmitted impulse. The cellular material is suggested to be bonded to the protected structure, and the transmitted impulse is then equal to the input impulse. The deformation of the cellular material under blast loading has been approximated by the shock wave theory. The overall shortening of the cellular material predicted by the shock wave theory is found to be in good agreement with the mesoscale model and the continuum model.

Analytical solutions for the blast mitigation and energy absorption of double-layer foam claddings have also been derived. The total energy, kinetic energy and absorbed energy of four representative foam claddings under critical blast loads are calculated. The current analytical solution is based on the idealized rigid-perfectly plastic-locking foam model which neglects the elastic behavior of the foam. Despite the limitation, the present analytical model is efficient to estimate the blast alleviation and energy absorption capacity of a double-layer foam cladding. Comparison between the numerical and analytical results shows that they are in good agreement. Therefore, the analytical solution is considered to be efficient and could be very useful for the design of double-layer foam sacrificial cladding.

An analytical Load-Cladding-Structure (LCS) model, incorporating the properties of the blast load, the foam cladding and the protected structure, has been developed to evaluate the blast alleviation efficiency of foam claddings for structural retrofit. The present model differs from other models in that it considers the response of the protected structure. Two non-dimensional parameters have been introduced to describe the relations between the foam cladding and the protected structure. Knowing any two of the three components of the LCS model, the third one can be determined or predicted. Despite the limitations such as the idealized RPPL foam model and the equivalent SDOF structural model, the LCS model gives a good guidance to the design and evaluation of foam claddings for structural retrofit against blast loads.

Two numerical structural models with sacrificial claddings have been developed with LS-DYNA. Some aspects neglected in the analytical analysis such as the stand-off distance of the explosive and the dishing shape of the cover plate have been discussed in the numerical analysis. From one of the models, the unexpected observation of the pendulum test by other researchers is explained. The other model demonstrates the positive protective effect of the foam claddings. If the foam cladding is appropriately designed, both the local stress and the deflection of the protected structure are mitigated. The numerical simulation results conform to the

conclusion based on the LCS model that the foam cladding should be designed appropriately with reference to the blast load and the protected structure.

8.2 Recommendations and future work

Future work may focus on the following five parts:

1. A three-dimensional (3D) mesoscale finite element model of cellular material can be developed to further investigate the material's static and dynamic behavior. The 3D model should be representative of a true cellular material. The irregular shape of an individual finite element in a 3D numerical model should be avoided. Furthermore, the entrapped air in a closed-cell foam can be considered.
2. The shock wave theory in the cellular material may be improved by considering the strain hardening of the plastic regime. The explicit solution of the wave propagation and reflection in a cellular material layer with finite thickness may be obtained with numerical methods.
3. In the analytical analysis of the protective effect of foam cladding on the main structure, shear deformation of the protected structure can be considered beside the flexural deformation. More parameters of the foam cladding as well as the protected structure need to be taken into account.
4. Simple structures, e.g., a beam, with and without a foam cladding has been considered in the current study. More complex structures can be considered in the future work. The foam may not necessarily be placed in front of the protected structure. It may be integrated flexibly with the main structure.
5. Field blast tests on the effectiveness of sacrificial foam claddings should be carried out, when experiment conditions are permitted.

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Appendix A: Publications

Papers in journals

1. Li Q.M., Ma G.W. and **Ye Z.Q.** “An elastic-plastic model on the dynamic response of composite sandwich beam subjected to mass impact.” *Composite Structures*, 2006; 72(1): 1-9.
2. Ma G.W. and **Ye Z.Q.** “Energy Absorption of Double-Layer Foam Cladding for Blast Alleviation.” *International Journal of Impact Engineering*. 2007; 34(2): 329-347.
3. **Ye Z.Q.** and Ma G.W. “Effect of Foam Claddings for Structure Protection against Blast Loads.” *ASCE Journal of Engineering Mechanics*. 2007; 133(1): 41-47.
4. Li Q.M., **Ye Z.Q.**, Ma G.W. and Reid S.R.. “Influence of Overall Structural Response on Perforation of Concrete Targets.” *International Journal of Impact Engineering*. 2007; 34(5): 926-941.
5. Ma G.W. and **Ye Z.Q.** “Analysis of Foam Claddings for Blast Alleviation.” *International Journal of Impact Engineering*. 2007; 34(1): 60-70.
6. Li Q. M., **Ye Z.Q.**, Ma G.W. and Jones N. “The influence of elastic shear deformation on the transverse shear failure of a fully clamped beam subjected to blast load.” *International Journal of Solids and Structures*. (Submitted)

Papers in conference proceedings

1. Ma GW, Ye ZQ. “Energy absorption capacity of layered foam cladding.” The International Conference of Computational Methods, December 2004, Singapore.
2. Ma GW, Ye ZQ. “Analysis of Foam Claddings for Blast Alleviation.” International Conference on Impact Loading of Lightweight Structures, May 2005, Brazil.
3. Ma GW, Ye ZQ. “Analytical prediction and numerical simulation of foam cladding for blast mitigation.” 6th Asia-Pacific Conference on Shock & Impact Loads on Structures, December 2005, Australia.
4. Ma GW, Ye ZQ. “Effect of Foam Cladding for Blast Mitigation: Numerical Simulation.” 1st International Conference on Analysis and Design of Structures against Explosive and Impact Loads, September 2006, Tianjin, China.

Appendix B: Pressure-impulse diagram

In assessing the behavior of the blast loaded structure, as shown in Fig. B.1, the calculation of the final states of destruction is of major concern. The maximum deflection rather than the detailed deflection-time history of the structure determines the failure criterion of the structure.

Fig. B.1 Single degree of freedom (SDOF) equivalent structural model subjected to idealized blast pulse

A common way to evaluate the structural damage is to develop a pressure-impulse ($p-i$) diagram of the equivalent SDOF structural model (Smith and Hetherington 1994; Mays and Smith 1995). Once the critical deflection y_c of the structure is specified, a curve can be obtained, as shown in Fig. B.2, which indicates various combinations of the non-dimensional initial peak load p and the impulse i of the external load that will cause the same deflection of the structure. The non-dimensional pressure and impulse are defined as $p = \frac{P_0 A}{k y_c / 2}$ and $i = \frac{I_0}{y_c \sqrt{k m_{se}}}$.

The impulsive asymptote of the curve is $i = 1.0$ and the quasi-static asymptote is $p = 1.0$. The $p-i$ combinations of the incident load below the curve will not cause any damage of the structure while those above the curve will induce failure of the structure.

Fig. B.2 Non-dimensional $p-i$ diagram of an equivalent SDOF structural model

Appendix C: Procedure of design and evaluation of foam claddings for structure protection

In Chapter 6, an analytical Load-Cladding-Structure (LCS) model has been developed considering the properties of the blast load, the foam cladding as well as the protected structure. The initial peak pressure and the impulse of the blast load can be normalized as Eqs. (6.11a, b), which are written again for convenience as follows,

$$p = \frac{P_0 A}{ky_c / 2} \quad (C.1)$$

$$i = \frac{I_0}{y_c \sqrt{km_{se}}} \quad (C.2)$$

Two non-dimensional parameters for the foam cladding are expressed in Eqs. (6.13) and (6.20), which are written again as follows,

$$\kappa = \frac{\sigma_0 A}{ky_c / 2} \quad (C.3)$$

$$\tau = \frac{\sqrt{(m_f + 2m_1) \frac{l\varepsilon_D}{\sigma_0 A}}}{T / 2} \quad (C.4)$$

Knowing any two of the three components of the LCS model, the third one can be determined or predicted, as detailed below.

C. 1 Design of foam cladding

When the blast load and the main structure are specified, an appropriate foam cladding can be designed according to the following five steps.

1. Convert the main structure to an equivalent SDOF structural model. The equivalent stiffness, equivalent mass, the critical deflection shall be known.
2. Calculate the non-dimensional p and i according to Eqs. (C.1) and (C.2). If $p \geq 1$ and $i \geq 1$, a foam cladding is needed to prevent the main structure from damage.
3. Decide the scope of the two non-dimensional parameters, namely, κ and τ of the foam claddings, according to Eqs. (6. 22a, b), which are cited here.

$$\kappa \leq 1 \quad (C.5)$$

$$\tau \geq \frac{i\sqrt{1-\kappa/p}}{1.6\kappa} \quad (C.6)$$

4. Base on the scope of κ , select suitable foam material in terms of the plateau stress (σ_0) by using of Eq. (C.3). The plateau stress of the foam is related to the type and the density of the foam. The densification strain (ε_D) is also a related parameter to the density.
5. Base on the scope of τ , design the thickness and mass (l and m_f) of the foam and the thickness/mass (m_1) of the cover plate by using of Eq. (C.4).

C. 2 Determination of allowable blast load

When the foam and the main structure are specified, the maximum allowable intensity of the blast load can be determined according to the following four steps.

1. Convert the main structure to an equivalent SDOF structural model. The equivalent stiffness, equivalent mass, the critical deflection shall be known.
2. Calculate the non-dimensional κ and τ according to Eqs. (C.3) and (C.4).
3. Numerically determine the maximum allowable combination of the two non-dimensional parameters, namely, p and i of the blast load by using of the p - i diagram analysis. Usually p is larger than 1.0 considering the nature of the blast load. If $\kappa \leq 1$, the scope of i can be approximated by Eq. (C.7).

$$i \leq \frac{1.6\kappa\tau}{\sqrt{1-\kappa/p}}, \quad \text{for } \kappa \leq 1 \text{ and } p > 1 \quad (\text{C.7})$$

4. The maximum allowable initial peak pressure and impulse are then derived by Eqs. (C.8) and (C.9).

$$P_0 = \frac{ky_c p}{2A} \quad (\text{C.8})$$

$$I_0 = i y_c \sqrt{km_{se}} \quad (\text{C.9})$$

C. 3 Structural damage evaluation

When the foam and the blast load are known, the maximum deflection of the foam protected structure can be predicted numerically base on the LCS model. If the maximum deflection is smaller than the critical deflection, the structure remains elastic. If the maximum deflection is larger than the critical deflection, the structure has some plastic damage.