Optimal coordination for an integrated multimodal and multi-operator transit system

Li, Shoujie

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OPTIMAL COORDINATION FOR AN INTEGRATED
MULTIMODAL AND MULTI-OPERATOR TRANSIT

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SYSTEM**

Li Shoujie

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**SCHOOL OF CIVIL AND ENVIRONMENTAL
ENGINEERING**

2008

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Optimal Coordination for an Integrated Multimodal and Multi-operator Transit System

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School of Civil and Environmental Engineering

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ABSTRACT

With rapid urbanisation in most parts of the world, modern transit systems have been made more and more convenient, reliable and integrated. In Singapore, more than 60% of the daily commuting trips in year 2005 are made in public transport and this share will likely continue to increase, due to the policy governing land transport. In the mean time the service level of public transport needs to continue to be enhanced to help achieve this goal. The aim is to make public transport, especially transit, seamless and sustainable. The public transport system in Singapore is currently operated by two multimodal transit operators on an integrated network with each of them providing services in different geographical areas. Currently this multimodal network is integrated mainly in infrastructure. The operation schedule is still in the domain of each individual operator which resulted in unnecessarily long delays for passengers. To provide better service especially during off-peak hours, it is necessary to coordinate the transit system. What the coordination results would be, when multi-operators co-exist in the same multimodal transit system with fully or partially overlapping services, originates this research topic.

This research focuses on optimal coordination of public transit operated by multiple operators. The optimal results would be achieved when the system reaches an equilibrium state that the operators and users would not modify their behaviour, i.e. operators would not adjust fare and schedule of transit services, while each groups of users keep on the same shortest paths. The objective function is the total cost which consists of operator cost and user cost. The formulation of the total cost in different coordination scenarios are analysed under three different operation policies, i.e. cooperation, competition and independence, respectively. By categorizing different groups of passengers in detail, the categorized passenger flows are traced on different paths that passengers have chosen. The optimal results in the equilibrium state can be obtained by the feedback optimisation algorithm developed in this study.

The objective functions in scenarios of non-coordination and common headway coordination, which are identified as nonlinear programming problems, are solved by considering the first order and second order conditions; whereas the objective function in integer-ratio headway coordination is identified as a mixed integer nonlinear

programming problem. All these problems are solved by coded AIMMS programmes developed, and the transit path choice model is applied to better estimate transit users' choices of their multimodal transit paths.

Using comparable transit data to Singapore transit system as input, the preliminary evaluation results show that integer-ratio headway coordination is a feasible and efficient coordination operation in a transit system. In addition, policy of cooperation between operators is an efficient strategy that can reduce the total cost of transit system and improve coordination. Whereas policy of competition between operators is the least efficient when making coordination operations within each operator's scope. The conclusion drawn from the sensitivity analysis and statistical analysis further confirms the preliminary evaluation results. In addition, through sensitive analysis, the conclusion can be drawn that the optimal coordination results are sensitive to transfer ratio, but are not sensitive to the variations of value of time, coefficient of in-vehicle value of time, standard deviation of bus arrival times and passenger OD rates of feeder bus services. Finally it can be concluded that integer-ratio headway coordination is an efficient and applicable method as it can greatly reduce the total cost and average transfer cost, when operators adopted cooperation policy between each other, whereas it is not recommended to adopt competition policy between each other, the coordination benefits are not obvious sometimes may even get worse. To be noted, the competition between operators is incomplete and unique to Singapore's context. The results would be beneficial to making regulatory policies as references in similiar circumstances to Singapore transit system.

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LIST OF NOTATIONS

The notations that will be used are listed as follows:

- P : the set of operators, {operator 1, operator 2, null} (when the mode is walking, there is no corresponding operator. Therefore the corresponding element in the set of operators is denoted by ‘null’).
- G : the directed graph representing a multi-modal transit network
- O : the origin nodes
- D : the destination nodes
- d : binary number ($d=0$ when in the upstream direction, $d=1$ when in the downstream direction)
- N : the set of all nodes
- A : the set of all arcs
- M : the set of modes, { bus, rail, walking }
- C : the total cost of the transit system (S\$)
- C^p : operator cost for operator p (S\$)
- C_{bf}^p : fixed cost of bus for operator p (S\$)
- C_{bv}^p : variable cost of bus for operator p (S\$)
- C_m^p : operator cost of mode m , $\forall m \in M$, $m \neq walk$ for operator p (S\$)
- C_{rf}^p : fixed cost of rail for operator p (S\$)
- C_{rv}^p : variable cost of rail for operator p (S\$)
- f_{ij}^m : the revenue from station i to j by mode m
- α_m : a constant for mode m
- β_m : the parameter based on the distance between station i and station j by mode m
- γ_i : the waiting time for passengers at bus stop i
- ψ_m^p : distance coefficient for mode m by operator p
- U^p : user cost for operator p (S\$)
- W_r^p : waiting cost for rail by operator p (S\$)

- W_b^p : waiting cost for bus by operator p (\$\$)
- F_m^p : transfer cost to mode m by operator p (\$\$)
- V_m^p : in-vehicle cost for mode m by operator p (\$\$)
- H_m^p : headway of mode m , $\forall m \in M$, $m \neq walk$ by operator p (min)
- K_d^p : offset for direction d , $d=0$ means upstream direction, $d=1$ means downstream direction for operator p (min) (upstream direction is which comes from a lower number station to a higher number station, downstream direction is the opposite direction)
- k_v : Coefficient of in-vehicle value of time
- $l_{i(i+1)}^{mp}$: the distance between transfer station i and station $i+1$ for mode m by operator p (km)
- N_b^p : the capacity of bus by operator p (person)
- N_r^p : the capacity of all the cars of rail in one fleet by operator p (person)
- B_{ik}^p : the vehicle cost of the one round trip time per fleet for bus route k at transfer station i by operator p (\$/bus · min)
- R^p : the vehicle cost of one round trip time per fleet for rail by operator p (\$/car · min)
- t_{ik}^p : the bus round trip time of k bus route that connects to rail transfer station i by operator p (min)
- ϕ^p : integer-ratio between bus and rail for operator p
- τ_b : Round trip time of bus (hr)
- τ_r : Round trip time of rail (hr)
- T_r : the total travel time of train in one round trip (min)
- γ_i : waiting time for bus at station i (min)
- V_m^p : average speed for mode m by operator p (km/hr)
- λ_{ij} : arrival rates of passengers who start from stations i to j (person/min)
- λ_{ij}^b : arrival rates of passengers who come by bus to stations i to j (person/min)

- λ_{ij}^r : arrival rates of passengers who come by rail from stations i to j (person/min)
- λ_{ij}^n : arrival rate of passengers who come from modes other than bus to station i to station j (person/min)
- λ_{ij}^{brp} : arrival rates of passengers who transfer from bus to rail at transfer station i to station j by operator p (person/min)
- λI_{ij}^{brp} : arrival rates of passengers who transfer from bus to rail at transfer stations i to j that transfer within operator p (person/min)
- λO_{ij}^{brp} : arrival rates of passengers who transfer from bus to rail at transfer stations i to j that transfer from the other operator to operator p (person/min)
- λ_{ij}^{bbp} : arrival rates of passengers who transfer from bus to bus at transfer stations i to j by operator p (person/min)
- λ_{ij}^{bn} : arrival rates of passengers who transfer from bus to modes other than bus and rail at transfer stations i to j (person/min)
- λ_{ij}^{rbp} : arrival rates of passengers who transfer from rail to bus at transfer stations i to j by operator p (person/min)
- λI_{ij}^{rbp} : arrival rates of passengers who transfer from rail to bus at transfer stations i to j that transfer within operator p (person/min)
- λO_{ij}^{rbp} : arrival rates of passengers who transfer from rail to bus at transfer stations i to j that transfer from the other operator to operator p (person/min)
- λ_{ij}^{rrp} : arrival rates of passengers who transfer from rail to rail at transfer stations i to j by operator p (person/min)
- λI_{ij}^{rrp} : arrival rates of passengers who transfer from rail to rail at transfer stations i to j that transfer within operator p (person/min)
- λO_{ij}^{rrp} : arrival rates of passengers who transfer from rail to rail at transfer stations i to j that transfer from the other operator to operator p (person/min)
- λ_{ij}^{rn} : arrival rates of passengers who transfer from rail to modes other than bus and rail at transfer stations i to j (person/min)

- λ_{ij}^{nbp} : arrival rates of passengers who come from modes other than bus and rail to bus at transfer stations i to j by operator p (person/min)
- λ_{ij}^{nrp} : arrival rates of passengers who come from modes other than bus and rail to rail at transfer stations i to j by operator p (person/min)
- λ_{ij}^{nn} : arrival rates of passengers who come from modes other than bus and rail to modes other bus and rail at transfer stations i to j (person/min)
- Y_{ij}^{mp} : the desired arrival time at destination node j from origin node i , when entering node i_1 on arc (k_1, i_1) on mode m by operator p . k_1 is adjacent node to i_1 (min)
- $V_{i_1 j_1}^{mp}$: the set of arc travel times on arc (i_1, j_1) on mode m by operator p
- $W_{ij i_1}^{mnpq}$: the delay at node i_1 , while origin node is i and destination node is j , when entering node i_1 from arc (k_1, i_1) on mode m by operator p and switching to arc (i_1, j_1) on mode n by operator q ; k_1 is adjacent node to i_1
- $PM_{ij i_1}^{mpk}$: the entering mode at node i_1 at k shortest path transferring from mode m by operator p , when origin node is i and destination node is j
- $PN_{ij i_1}^{mpk}$: the entering mode at node i_1 at k shortest path transferring from mode m by operator p , when origin node is i and destination node is j
- $PO_{ij i_1}^{mpk}$: the entering operator at node i_1 at k shortest path transferring from mode m by operator p , when origin node is i and destination node is j
- $PA_{ij i_1}^{mpk}$: the entering arc of which destination node is i_1 at k shortest path transferring from mode m by operator p , when origin node is i and destination node is j
- u : value of time (\$/ min· person)

CHAPTER 1 INTRODUCTION

This research topic was inspired from the special situation in Singapore transit system, which is a multimodal and multi-operator transit system. The public transport system in Singapore is currently operated by two multimodal transit operators on an integrated network with each of them providing services in different geographical areas, and fare integration has been facilitated in the whole transit system. Real time O-D transit data can be obtained from the counter machines on buses and at the entrance of transit stations. In this study, overlapping service areas for two multimodal operators are assumed and O-D flow rates from one transfer station to another by bus and by modes other than bus are assumed to be known. The background, problem statement, objectives and scope of this research topic are presented in the following sections.

1.1 The Background

With rapid urbanisation in the world, travel within an urban area has become a necessity for livelihood of urban dwellers. Developed countries predominately rely on private transport have seen their roads packed with congested traffic, while developing countries are gradually utilizing a higher percentage of motorised form of transport which leads to the same path of urban congestion. As a result, if such trend continues, the urban environment where we live and work may not be sustainable for long. Government officials, planners, operators and urban citizens have continued the implication of such trend and many have started to study alternative form to commute. Public transit, due to its higher energy efficiency per capita and lower impacts to the environment, has been considered as one of the most efficient tools to combat the degradation of our urban environment due to over-reliance on private transport. To attract more private car users, and to provide better services to basic transit users, modern transit systems have been made more and more convenient and reliable, normally integrating more than one mode of service. The aim is to make public transit seamlessly connected as a sustainable form of transport in urban areas.

Most transit systems consider integration at the infrastructure level by providing walkways and paths to make transfer between different modes more convenient. However, coordination of integrated transit services remains to be difficult as it requires the knowledge of real-time transit services and demand, limitation of supporting personnel and equipment, as well as the cooperation of multi-operators as normally these systems are run by different companies. Taking into consideration of coordination, the optimal coordination of transit system has the potential for the reduction of operator costs, enhancing congestion management especially during emergency, and improvement of mobility for passengers.

To allow for competition, transit services are normally provided by more than one operator. Deregulation policies had been put into practice firstly by British's bus industry of Transport Act of 1980, since then transit reform has been carried out in a lot of countries especially in Europe (Berechman,1993). In Singapore, in the 1950's, bus services were mainly operated at profitable corridors. Although there were multi-operators in bus transit system then, the vast majority of passengers were not benefited as the services were not coordinated and malign competition in service routes and fares led to low efficiency and high redundancy for the transit industry as a whole.

In 1970, at the first major re-organisation of bus system by Singapore government, ten Chinese bus companies were combined into three. Effort has been made to consolidate the numerous small bus companies to solve the plagued problems of poor management and labour unrest. With the re-organisation, the three bus companies each served a different geographical sector. In 1973, the three bus companies were further merged to form Singapore Bus Service (SBS), which later evolved to become what is now well known as SBS Transit Ltd. The new name, introduced in 2001, reflected SBS Transit's role as a multi-modal operator of bus, MRT (Mass Rapid Transit) and LRT (Light Rapid Transit). In 1983, Trans-Island Bus Services Limited (TIBS) became Singapore's 2nd public bus operator. TIBS later merged and is now known as SMRT Buses Ltd. The basic bus services run by SBS Transit together with SMRT Buses Ltd form the backbone of Singapore's bus network. The basic bus services are supplemented by express and rapid services, premier services, night services. These services help to meet commuting demands during peak hours.

Initial groundbreaking of MRT system was on Oct 22nd, 1983, further development has been undertaken through these years. Now Singapore MRT system consists of two lines: an east-west line from Pasir Ris to Boon Lay and a semi-loop from Jurong East to Marina Bay via Woodlands. It was designed to link residential areas with the central business district. The North-East LRT line, which interconnects with the previously existing East-West and North-South lines, runs from Harbour Front in the south to Punggol in the northeastern part of the city.

From the 1970's the Singapore government started to regulate closely the public transit market and later the rail operations. The effect was obvious: public transit corporations in Singapore have made profit since then, and it is now one of the few countries in the world whose public transit corporations make profit without any subsidy from the government. With the increased level of usage and demand for better services, there are two multimodal operators providing services from basic bus services to trunk rapid transit at present. The Singapore government encourages multi-operators coexisting in the public transit system at the same time regulates transit corporations closely.

1.2 Problem Statement

In a large metropolitan city, public transit plays a very important role in serving the transportation demand, as a result the planning and design of transit services is a very important process. From the demand perspective, public transit users are more concerned with the convenience and cost, thus making the coordination of public transit very important; whereas from the supply perspective public transit operators are more concerned with operating cost and profit. For the regulators, they need to safeguard the public interests by ensuring the public transit services are affordable, accessible and reliable, although this may need to be balanced with proper realization of certain business goals by private companies if they are to be attracted to operate the public transit. The balance between supply and demand of transit services therefore manifest how the three main stake holders are balanced and cooperated in a usually competitive market condition. Any slack in transportation services induce wastage in resources, while insufficient supply of transportation services leads to lower service levels, such as longer waiting times, more crowdedness in vehicles, which may even

lose some of the potential public transit users. The planning of public transit services therefore should consider the interests of these three stake holders. Under this circumstance, it is not only necessary to consider the efficiency and profit but also the convenience of the users for the public management process. Due to the operation policy during peak hours, the headways of bus and rail is quite short, the coordination between bus and rail is not so necessary. In this study, coordination is only considered during off-peak hours.

Planning of public transit services consists of four interrelated, and normally sequential process: (i) network route design, (ii) setting timetables, (iii) scheduling vehicles to trips, and (iv) assignment of crew (Ceder, 2003). More research has focused on network route design than those focused on optimisation of schedule operations. Although it is desirable to plan the four components simultaneously to maximise the whole system to the greatest efficiency, it is not practical to plan all the procedures quite often. Firstly the situation changes from time to time, what is optimal at the time of planning stage may not be optimal afterwards. Secondly it is difficult for the users to keep track of the latest transit routes with updated information during a very short time of period. Thirdly it is not realistic for the operator to adjust the transit facilities such as rail or bus routes and stops from time to time. Lastly, and more importantly, the methodology required to plan for the four processes requires consideration of complex interactions which are normally impossible to fully capture. Therefore it is more practical and realistic to coordinate the existing public transit systems focusing on the scheduling optimisation problems of setting timetables.

The problem of public transit system integration can be solved in three dimensions of analysis and evaluation. The first is integration in space, while the second integration in time and the third is fare standardization and charge integration (Gattuso et al., 2002). Unlike the first and the third method that need to take account of updating or construction of infrastructure in the planning process, the second method will take much of the consideration of operation strategies. The priority is obvious, and usually the solution is to shorten transfer time. There are basically two methods to reduce waiting times during transfers in a transportation network. The first is to decrease the headways on certain routes, at the same time increase the fleet

size and as a result it raises operating cost. The second is to improve transfer coordination through timed transfers so that to achieve better result of reducing waiting time for passengers. Apparently the second method uses resources more efficiently and may decrease the total user and operator's costs.

There are three kinds of dispatching policies that have been widely used. The first is C policy, which sends a vehicle as soon as it is filled to capacity. The second is T policy, which sends a vehicle every T periods while assuming an infinite capacity and the third is T - C policy, which sends a vehicle every T periods or whenever it is filled to capacity C, whichever comes first (Tapiero and Zuckerman, 1979). According to SBS and SMRT annual report, T policy is adopted by Singapore transit. In this research, after deriving the optimum headway and offsets, a timetable can be set. The dispatching policy adopted is T policy in such case.

Considering the operational cost, public transit economics should not be omitted in solving such kinds of problems. Most of the public transit systems in the world are operated by the state owned agencies or monopolistic private operators. Due to the monopoly and low efficiency of these operators, multiple operators are considered so that competition is aroused and efficiency can be raised from the economic point of view. In densely populated Asian countries, the demand for transit generally is large enough for multiple operators to coexist, and in certain countries, like Singapore, private operators (two of them), unlike their counterparts in other Asian countries, are providing multimodal services to the public, although in different service areas. The users of transit systems in Singapore normally travel in more than one modes operated by more than one operators to reach their destination. Therefore there exists a sophisticated relationship between operators, while they each strive for better profits and provide satisfactory services to the public at the same time. As such, the situation in Singapore presents a unique characteristic that deserves more detailed studies into how such relationships can be made beneficial for all stake holders of the system.

The optimal coordination of transit services consists of two main problems: the first is coordination of modal services in a multimodal public transit system, and the second is the problem of coexistence of multiple operators in the same transit market. Due

to the variant traffic conditions of roads, the scheduled bus services are difficult to be made punctual and delays caused by traffic jams or accidents are common, as a result it is not realistic and practical to coordinate between bus services. Rail services normally have their own right-of-way and are usually more reliable as there are no external interferences. Therefore it is more feasible to coordinate the rail with buses or rail instead. This is especially important in Singapore as rail is the trunk services that connects to feeder bus services at stations and carry a large number of commuters.

When multiple operators coexist in the same public transit system, competition is anticipated, but there will be other cases, such as when operators are in cooperation or independence. Taking all these factors into consideration, to derive optimum headway and proper offset between rail and bus for the public transit operators are critical problems in optimal coordination for a multi-operator and multimodal public transit system. Whereas in the case that multi-operator may adopt different strategies, the coordination optimisation may be different from each other, i.e. setting different headways of bus and rail besides different offsets between bus and rail.

Public transit systems can be operated more efficiently with available transit resources by adopting proper timetables and reasonable fares. In Singapore, due to the use of ez-link fare cards, the data of boarding and alighting of passengers can be obtained, thus a real-time inflow and outflow of passengers of terminals can be potentially obtained. It is reasonable to feed the network of the public transit with the inflow and outflow matrices that are generated by the computer simulation.

To find out what the situation may be when multiple operators with overlapping service areas coexist in the same public transit system and when all of them are required to provide coordination between transit modes, a multi-operator and multimodal public transit schedule optimisation model is developed in this research. Results on the optimal coordination for public transit in a market comprising multiple operators while considering different coordination operations schemes can be compared and analysed, and these outcomes would be beneficial to the making of regulatory policies as references.

1.3 Objectives

This research work focuses on setting timetables, in other words finding the optimum rail headways and offsets under the multi-operator and multimodal public transit system. The goal of this research work is to optimise the scheduling of the integrated public transit system so as to operate more efficiently and economically whereas provide more reliable and accessible service to the passengers under different strategies between operators.

Against this backdrop, this research aims to study the optimum coordination of transit services in a multi-operator and multimodal public transit system. More specifically, the objectives of this study include the followings:

- (i). To develop models for the optimisation of scheduling considering multi-operator operations and coordination constraints in a public transit network. The objective of scheduling optimisation is to minimize the cost for both operators and users. The models in three scenarios, non-coordination, common headway coordination and integer-ratio headway coordination are formulated under three different strategies between operators.
- (ii). To conduct sensitivity analysis to validate the results of the model and to study the impacts of coordination scenarios under different strategies between the multi-operators and compare different policy strategies by evaluating headways of bus and rail, the total cost, user cost, operator cost, waiting cost, transfer cost, average transfer cost and total transfer volume.

1.4 Scope

Scheduling problems include setting timetables, vehicle and crew scheduling. A number of studies have been carried out by researchers on transit network planning, vehicle and crew scheduling problems. In this study, the focus is on the establishment of operation headways and setting proper offsets between bus and rail

in an integrated multi-operator and multimodal public transit system during off-peak hours. Three operation scenarios are considered, i.e. non-coordination, common headway coordination and integer-ratio headway coordination. While these three different scenarios are compared under three different operation policies describing the market relationship of the operators, and the operator policies are cooperation, competition and independence. Taking account of multiple transit operators, the interactions between different operators in the whole transit system are further discussed.

Path choice is considered so that transit trip assignment can be assessed and the detailed OD matrices reflecting path preferences for different group of passengers can be obtained subsequently. Path choice model consists of two basic parts with different functions: (i) path search model; and (ii) path decision model (Xie, 2003). Firstly the K^{th} shortest path problem is employed to sort out the three shortest paths from origin station to the destination station, and then in discrete choice analysis, path size logit model is used to calibrate the probability for each of the three shortest paths reflecting path preferences of different groups of passengers. Therefore given OD matrices from origin trunk station to destination trunk station, transit trip assignment on specific paths can be estimated.

1.5 Organization

In this dissertation, the first Chapter introduces the background, problem statement, objectives and scope of this research topic. Chapter 2 reviews relevant literatures of previous methods and research. Chapter 3 presents the methodology which consists of assumptions, framework and optimisation algorithm of this research. Chapter 4 demonstrates the model formulations of different cost items and different scenarios under different strategies between transit operators. Chapter 5 evaluate the developed model under different operation policies employing the same typical data set that is comparable to Singapore transit. Sensitivity analysis is conducted from Chapter 6 to Chapter 8 to validate the evaluation results obtained in Chapter four. Sensitivity analysis of scenarios when operators are in cooperation is presented in Chapter 6. Sensitivity analysis of scenarios when operators are independent is presented in Chapter 7. Sensitivity analysis of scenarios when operators are in

competition is presented in Chapter 8. Analysis of scenarios in integer-ratio headway coordination under different operation policies is conducted in Chapter 9. Chapter 10, which is the last chapter, briefs summary and conclusions of this research, in which the contributions and limitations are highlighted, in addition, future research is recommended.

CHAPTER 2 LITERATURE REVIEW

Relevant research work are summarized in this chapter, with four major categories listed below: (i) public transit scheduling problems, (ii) transfer coordination of public transit system problems, (iii) multi-operator public transit system, (iv) multimodal transit trip assignment and finally a conclusion is drawn to propose further study of this research based on previous research work.

2.1 Public Transit Scheduling Problems

Public transportation planning process includes four basic components normally performed in sequence: (i) network design, (ii) setting timetables, (iii) vehicle scheduling, and (iv) crew assignment, as discussed by Ceder (2003).

As network design is to design transit routes and bus stops based on transit demands, it is started as the first step in public transportation planning process. Wirasinghe and Vandebona (1999) proposed of planning of operationally feasible transit networks by solving two sub-optimisation problems: (i) locations of stations to minimize the sum of station costs and access costs and (ii) choice of routes, based on given station locations. The advantage of this method is to consider both the operating cost and the capital cost of the system as well as the level of service provided to passengers. Wirasinghe and Seneviratne (1986) obtained optimum length of a rail line by considering minimum total transportation (user and operator) costs. Parajuli and Wirasinghe (2001) applied the similar approach by proposing a generic normative behavioural hybrid model which minimized cost associated with number and location of stations. Guan et al. (2003) improved transit network design by considering simultaneous optimisation of transit line configuration and passenger line assignment. It can be observed that all the previous research work mentioned above considered costs associated with either operators or users, or both. However, transfer cost within the same mode or between different modes was not included. In other words,

transfer coordination was not considered in transit network design stage in the research work.

A good summary of vehicle scheduling for public mass transit can be found in Daduna and Paixao (1993). The historical developments of scheduling problems were outlined and some structure and restrictions for vehicle scheduling problems were analysed. It was concluded that the vehicle scheduling process in vehicle scheduling cannot be done efficiently without human interference due to two reasons: one is that not all specific restrictions can be taken into account formulating the model, and the other is that there are independencies which result from actual situation of the combinatorial process. Thus practical experience of vehicle scheduling was summarized at the end of this paper.

Vehicle scheduling and crew scheduling have been well studied by many researchers. Their research differed mainly in the solution algorithms. Haghani et al. (2003) presented three vehicle scheduling models, which included one multiple depot and two single depot vehicle scheduling models. These models were compared using data from January 1998 MTA (Mass Transit Administration) operation of the city of the Baltimore. The comparison shows that, under certain conditions, a single depot vehicle scheduling model performs better. A sensitivity analysis with respect to two important parameters is also performed and the results are reported. The conclusion was drawn that deadheading speed was an important parameter in dealing with the scheduling problems. An optimal scheduled travel time on a non-stop transportation route, with a single time-point (check-point) at the destination was proposed by Wirasinghe (1993). The optimal scheduled travel time was obtained by minimizing the sum of costs of passengers and operators. It is noted that the cost of operators was classified into cost related to travel time budgeted for the trip, delayed travel time and a penalty associated with a delayed arrival. However, this cost based approach cannot be applied for multiple routes with many time-points in the real world.

Shen and Kwan (2000) stated that crew scheduling is an NP-hard problem, and they presented a Tabu Search heuristics to obtain the solution, and drew the conclusion that this algorithm was very fast, and was able to achieve results comparable to those based on mathematical programming approaches. Haase et al. (2001) proposed an

exact approach in solving the simultaneous vehicles and crew scheduling problem in urban mass transit. His approach consists of a column generation process integrated into a branch-and-bound scheme. The computation results showed that the size constraints on buses guarantee that an optimal vehicle assignment can be obtained in polynomial time. The limitation is that it studied the single depot case with a homogeneous fleet of vehicles. However, it gives the theoretical justification that for a particular timetable, simultaneous vehicles and crew scheduling problem can be achieved.

A detailed description of public transport timetabling and vehicle scheduling method is presented by Ceder (2003), in which a step function called deficit function was applied. The background and definition of deficit function were addressed in detail by Ceder and Stern (1981), in which the deficit function approach is described as:

Given a set of trips S (a fixed trip schedule) that must be carried out, it is possible to construct a step function for each terminal in the schedule. Such a function, say for terminal k , will take on a value at time t which represents the total number of trip departures from k less the total number of arrivals at k up to and including time t .

As this function represents the deficit number of vehicles required at the particular terminal in question in order to service the subset of trips using the terminal, it is called deficit function. The appealing advantage of deficit function approach is its visual simplicity and a practical tool in the development of heuristic procedures for handling more realistic scheduling problems. To construct a set of deficit functions, the only information needed is a timetable of required trips for which minimum number of vehicles is allocated. The value of exploiting such a technique was to achieve the greatest vehicle savings as well as complying with passenger demand, and it can be inferred that setting timetable is a very important process in the public transit planning procedure.

2.2 Transfer Coordination of Public Transit System Problems

In this section, the approach of transfer coordination of previous research is summarised in the first subsection. As passenger and vehicle arrival patterns are critical topics in analysing transfer coordination of public transit system, they are separated and described in the second subsection. Operator costs and user costs are two very important components of the total cost which is usually targeted as the objective function in optimal transfer coordination. Various formulations of operator costs and user costs by researchers are listed in the third and fourth subsections. Previous transfer coordination models are classified and presented in the last subsection.

2.2.1 Transfer Coordination Problems

Transfer coordination has been awarded by a lot of researchers. Wirasinghe and Liu (1995b) considered transfer coordination between buses by developing an analytical model for the determination of the number and locations of time points as well as the amount of slack times in transit design. Although the model is not applicable to bus routes with general passenger demand patterns, it is useful in the analysis of the contributing factors to the design of an economical, reliable and operational transit schedule. However, transfer coordination was only limited within one mode, i.e. bus in this research.

In 1970 the Urban Mass Transportation Assistance (UMTA) Act was enacted in the United States, in recognition of the widespread awareness that urban transportation has not been keeping pace with changing conditions. Subsequently, a conference was held at Mt. Pocono, Pennsylvania in 1971, with the focus of multimodal transportation planning (Weiner, 1987). This can be considered as a watershed event which started the era of multimodalism in public transport. At about the same time, integrated timed-transfer systems were beginning its use in Switzerland, Austria, and many regions of Germany. Clever (1997) reviewed in detail about the integrated timed-transfer systems, which differ from regular timed-transfer systems in which vehicles arrive at and depart from a station at approximately the same time to minimize waiting times for passengers. The conclusion was drawn that integrated

timed transfer was a scheduling concept being introduced in more and more regions in Europe. It was applicable to large, multicentered metropolitan areas and heavily congested corridors in North America.

Schumann (1997) stated that in multimodal transit systems, transfer coordination can be achieved, especially when rail systems become regional (or trunk) connections, linking via timed transfers at transit centres with feeders such as bus and paratransit. The pivotal function of rail systems to achieve sustainable and seamless public transit systems was noted by examining data from three cities: Zurich (Switzerland), Portland and Phoenix (USA). Crainic and Rousseau (1986) considered a multimode, multicommodity freight transportation problem in which their objective function was to determine and integrate decisions on the design of a service network, routes on this network, and terminal operation policies. A mixed integer programming was formulated with the objective function to minimize the total operation and delay costs associated with servicing all demands. The results included two parts; the first was the determination of the vehicle schedules, and the second was how to deliver goods given these schedules. Although the model was formulated for freight, it had brought out the problem of scheduling in a multimodal transportation system. Graham (1999) presented a model which integrated four modes of transportation, i.e. water, air, rail and road. The objective of this research was to establish a basic framework for mathematically modelling multimodal transportation systems, and to integrate the modelling framework with geographic information systems and global positioning systems so that the real-time model building and decision making become possible. A conclusion was drawn that the independent development and operation of the four major modes of transportation had limited the effectiveness and efficiency of America's transportation system, and this research contributed to the establishment of a national intermodal transportation system to solve this problem. The limitation of this research is that only a graphical framework of intermodal transportation system was developed. A systematic discussion on transfer coordination in public transit systems was given by Stern (1996), which covered nearly all the transfer issues, from transfer policy issues to transfer service designs. As this work was a summary of transfer issues and previous literature reviews, no further study was extended. It was concluded that there were no national level data on transfer usage rates, thus making the validation of transfer coordination models difficult. More research work has

focused on the optimisation of coordinated public transit systems. A mathematical model was developed by Salzborn (1972). This model determined the number of bus departures on a bus route by minimizing the fleet size and the total passenger waiting time for a given passenger arrival rate. Although this method can be extended to more than one route, the complexity of solving for the optimal results in such condition makes the task formidable to conquer. Another optimal model was presented by Palma and Lindsey (2001), analysing the optimal timetable for given public transport vehicles on a single transit line. Each transit rider was assumed to have an ideal time of a day to travel, and to incur a schedule delay cost from travelling earlier or later. Two location models are considered: the line model and the circle model. The optimal timetable was also derived for the circle model and compared with the timetable for the line model. There are two obvious limitations, the first is that buses were assumed to be able to adhere perfectly to the timetable while in reality this is hardly true; and the second is that each transit rider was assumed to have an ideal time of a day to travel. To control the randomness in transit operations, Barnett (1974) developed an algorithm for constructing a near-optimal dispatching strategy from the control station where optimality was defined both in terms of average passenger waiting time at the control stop and average delay for riders already boarded the vehicle. Three general ways to provide an effective regulation of the headway variation were proposed. Schedule synchronization in transit networks was the focus. The coordination strategy is to hold pickup vehicle until the feeder vehicle arrives. Finally the procedure for reducing headway variations due to operational randomness on a transit line was developed, and it was concluded that simple holding strategies in this paper might sometimes provide a feasible basis for improving transit operations.

Leblanc (1988) considered the overall network optimisation, and determined the frequencies of each transit line in a network to maximize the overall transit utilization. In his model, split assignment was studied with the use of transit frequencies to calculate transit access time and transfer time so that mode choices and link flows can be predicted more accurately. Further study considering scheduling synchronization was presented by Daduna and Vob (1993). The schedule synchronization problem was formulated as a quadratic semi-assignment problem (QSAP) to minimize the total transfer waiting time. With respect to service quality, the main objective of

minimizing the total waiting times of all passengers at transfer nodes in a public transit system was established. Although this research work only considered general optimisation of transfers, it was viewed as a good reference for coordination optimisation of public transit system.

An extensive analytical work of transfer optimisation in public transport was made by Adamski (1993). The static and dynamic transfer optimisation problems were formulated and solved for various objective functions. In the static condition, transfer waiting time was the main objective for optimisation. The analytical solutions with shifted truncated exponential randomness representation of the vehicle travel times were presented and compared using an example for two cases: no holding and punctuality holding (i.e. holding pickup vehicle according to timetables) control at the transfer point. In the dynamic condition a measure of off-schedule deviations was used as the objective function and optimal transfer synchronization problem was numerically illustrated by real cases. Ting (1997) dealt with transfer coordination in transit networks extensively. An optimisation model was developed for scheduling and coordinated timed transfers in a transit network. As vehicle arrivals at transfer terminals are stochastic, safety factors are built into the schedules, which are called 'slack' time. Thus slack time was introduced in order to reduce the probability of missed connections, even at the cost of additional delays for vehicles and some non-transfer passengers. Total system cost functions were formulated to assess the effectiveness of coordinated operations under various demand and arrival distributions. Three scenarios, including uncoordinated, fully coordinated and partially coordinated conditions were analysed. Uncoordinated scenario is that optimised schedule operation without considering coordination. Fully coordinated scenario is an ideal optimised schedule operation between bus and rail, in which headway of bus equals headway of rail. Partially coordinated scenario is more applicable and realistic schedule operation in which headway of rail divided by headway of bus is integer-ratio numbers. Considering the real time dispatching control to improve transfers in a coordinated transit network, an algorithm was developed to determine the holding times. The results showed that, within a range of arrival variances, coordinated operations were preferred even with smaller headways. The limitation of his research is that no distinction was made between different transportation modes, as they have different characteristics.

Chang et al. (2000) applied a multi-objective model for the planning of passenger train services on Taiwan's high-speed rail line. A multi-objective programming model for the optimal allocation of passenger train services on an intercity high-speed rail line without branches was developed, and the multi-objective function was to minimize the operating costs and passenger's total travel time loss. This study showed that an optimal set of schedule can always be obtained for a given travel demand. Thus a conclusion was reached that to achieve the best planning outcome, timetable scheduling should be flexibly planned according to the variation of demands. Chowdhury (2000) considered the dynamic dispatching of vehicles according to the fluctuation of the passenger demand, at the same time took the schedule synchronization into consideration. A model was constructed to optimise the coordination of an intermodal transit network under different demand situations. A numerical example was given to demonstrate the application of the dynamic dispatching model, in which optimum vehicle holding times were obtained and dispatching costs were analysed under different delay variations of coordinated vehicles arrival times. However, due to the widely accepted bus schedule policy which is demand based, this was not feasible in practice. The optimisation scope was too broad to calibrate and integer-ratio coordination that coordinated multimodal headways were integer-ratios of each other was not analysed either.

2.2.2 Passenger and Vehicle Arrival Patterns

Passenger and vehicle arrival patterns are very important factors in deriving the cost of users and operators in a coordinated public transit system. A large number of literatures focused on the passenger and vehicle arrival distributions. Analytical approaches have been extensively utilized in the analysis of service times by Adamski (1992), in which the main stochastic factor, which was stop time variability in bus operation, affecting the dwell times at bus stops was set as the objective. For this purpose, the parallel (i.e., simultaneous boarding and alighting) and series passenger service processes at bus stops were chosen and in those processes various random representations were used to determine if they were main determinants. Finally analytical expressions for density functions and moments of the stop time random variable were achieved which are validated by data from a real bus route. The results showed that the proposed models accurately reproduce the random

mechanisms governing the real service processes at bus stops. Jolliffe and Hutchinson (1975) made the observations at ten bus stops in suburban London on the arrival times of passengers and departure times of buses, and they made the observation at different bus stops at different times of the day and repeated at the same time on each bus stop of eight different days. It was found that passengers' waiting times were observed to be about 30 percent less than what they would have been had they arrived at random times. A behavioral explanation was given that considering three types of passengers: q of them whose arrival time is causally coincidental with the bus, $p(1-q)$ of them who arrive at the optimal time (the time at which the expected waiting time is the smallest), and the remaining proportion of $(1-p)(1-q)$ arrive at random. p was found to be larger at those bus stops observed in the peak hours than at those observed in the off-peak hours. The most commonly

Table 2-1 Summary of Previous Studies

Authors	Random Variable	Probability Distribution
Turnquist (1978)	Arrival Time	Lognormal
Bookbinder and Desilets (1992)	Arrival Time	Truncated Exponential
Strathman and Hopper (1993)	Arrival Time	Lognormal
Guenthner and Hamat (1983)	Arrival Time	Gamma
Abkowitz et al. (1987)	Arrival Time, Travel Time	Normal
Lee and Shonfeld (1991)	Arrival Time, Travel Time	Normal, Gumbel
Hall (1985)	Lateness, Delay	Exponential
Talley and Becker (1987)	Lateness, Delay	Exponential
Jenkins (1976)	Travel Time	Normal
Andersson et al. (1979)	Travel Time	Lognormal
Turnquist and Bowman (1980)	Travel Time	Gamma
Seneviratne (1990)	Travel Time	Normal, Gamma
Wirasinghe and Liu (1995a)	Travel Time	Gamma

adopted method of calculating passenger's waiting time is half of the vehicle headway, and this can be reasonable only when the headways are regular and passenger arrivals are uniformly distribution. This method cannot be applied especially when the transit headway is large and passenger arrivals are not so random (Black, 1995).

The probability models for vehicle arrival and travel time distributions of previous studies were illustrated by Hall et al. (1997) as shown in Table 2-1. Here the *lateness* is defined as the deviation from the schedule arrival time at a check point or stop, and *delay* as the deviation from the scheduled travel time over a segment, they are considered as variables classified as either *delay* (travel) or *lateness* (arrival). If only a single stop was considered, these two variables were the same. The probability distributions of these random variables are shown in the third column, it can be seen that arrival time is found to be Normal, Lognormal, Truncated Exponential distributed by different researchers respectively.

2.2.3. Operator Costs

An extensive analysis of the cost structure of an urban transit property was proposed by Berechman and Giuliano (1984). In their paper a general form of the cost model which placed very few presumptive restrictions on the bus operation and maintenance was presented, utilizing longitudinal data from one bus firm. This cost model (translog model), which met the neoclassic theoretical conditions for a cost function that takes prices as input, was estimated with one output factor (vehicle-miles or revenue passengers) and four input factors (capital, labour, fuel and maintenance) shown as follows:

$$\begin{aligned}
 \ln C = & \alpha_0 + \alpha_y \ln Y + \beta_k \ln P_k + \beta_L \ln P_L + \beta_F \ln P_F + \beta_M \ln P_M \\
 & + \frac{1}{2} \delta_{YY} (\ln Y)^2 + \frac{1}{2} \gamma_{KK} (\ln P_K)^2 + \frac{1}{2} \gamma_{LL} (\ln P_L)^2 + \frac{1}{2} \gamma_{FF} (\ln P_F)^2 + \frac{1}{2} \gamma_{MM} (\ln P_M)^2 \\
 & + \gamma_{KL} \ln P_K \ln P_L + \gamma_{KF} \ln P_K \ln P_F + \gamma_{KM} \ln P_K \ln P_M + \gamma_{LF} \ln P_L \ln P_F + \gamma_{LM} \ln P_L \ln P_M \\
 & + \rho_{FM} \ln P_F \ln P_M + \rho_{KY} \ln P_K \ln Y + \rho_{LY} \ln P_L \ln Y + \rho_{FY} \ln P_F \ln Y + \rho_{MY} \ln P_M \ln Y,
 \end{aligned}
 \tag{2.1}$$

where C is the total cost, P_K, P_L, P_F and P_M are prices of capital, labour, fuel and maintenance respectively. Y is vehicle-miles in the first estimation and revenue passengers in the second estimation. $\alpha_0, \alpha_y, \beta_k, \beta_L, \beta_F, \beta_M, \delta_{YY}, \gamma_{KK}, \gamma_{LL}, \gamma_{FF}, \gamma_{MM}, \gamma_{KF}, \gamma_{KM}, \gamma_{LF}, \gamma_{LM}, \rho_{FM}, \rho_{KY}, \rho_{LY}, \rho_{FY}$ and ρ_{MY} are all parameters.

The results of this study indicated that the nature of the data sample and the selected output measure have a significant effect on the results of an econometric analysis of bus transit. The use of two output measures (i.e., vehicle-miles and revenue passengers) in this study showed that results were affected by the type of output measure used. The cost structure of the cost function was found to be the same for both output measures. However, scale economies were found to be in striking contrast: diseconomies of scale with respect to vehicle miles, and economics of scale with respect to revenue passengers. To be noted, the authors added that this research was based on the behaviour of only one transit firm (Alameda-Contra Transit). Although it was typical of a great number of transit firms throughout the U.S., it may not be typical of the entire industry and those in other countries.

Most of the research in optimal coordination of the public transit constructed models minimizing the cost of operators or travellers, or of both. There are several types of operating cost models for bus systems. Barnett (1978) proposed a time related cost function for passengers. A rudimentary transport system was considered which the interaction between a transport company and its passengers was modelled as a cooperative game. The cost structure of the model takes the form of the following:

$$F(a, u, v) = Q(a) + R(u) + S(v), \quad (2.2)$$

where $a = w - t$; $u = t' - t$; $v = t'' - w$.

$F(a, u, v)$ = time related cost function.

w = the passenger's ideal alighting time at destination.

t = the time the passenger appears at boarding stop.

t' = the time the passenger actually leaves boarding stop.

t'' = the time the passenger actually reaches alighting stop.

$Q(a)$ = cost component based on the time the rider considers necessary to allow for his trip.

$R(u)$ = cost component relates solely to the inconvenience of waiting.

$S(v)$ = cost component that measures the rider's "disutility" of arriving at his destination v units late if v is negative, or $|v|$ units early if otherwise.

Gray and Hoel (1979) explained in detail the operation cost which consists of transportation expense (including station and fuel expense), maintenance and garage expenses, administrative and general expenses, etc. After defining the theoretical framework of the cost allocation model, an example was given applying such approach in Chicago metropolitan area. Cherwony et al. (1982) and Black (1995) both reported this model, as shown in Eq.(2.3).

$$OC = A(VM) + B(VH) + C(PV), \quad (2.3)$$

where OC is operating cost, VM is vehicle-miles of service, VH is vehicle-hours of service, PV is the number of peak-hour vehicles in service.

Usually values of the parameters (A , B and C) are estimated by the unit cost approach. This seems logical because major categories of the agency's accounts are assigned variables based on vehicle-miles, vehicle-hours or peak-hour number of vehicles in service, and then unit costs are calculated. For example, fuel, tires, parts and maintenance of vehicles are based on vehicle-miles. Wages of operators and supervisors are based on vehicle-hours, depending on the salary system of the operators. Administrative and overhead costs are considered a function of size of the system and are based on number of vehicles used in the peak hour, which is a standard measure of size used by the FTA (Federal Transit Administration) of USA. Similar models are used with more or less variables. An alternative to this unit cost approach is to use multiple regression analysis, which has been done by several researchers. The variables hypothesized were usually the same as in the unit cost approach, and the estimating equation calculated includes only the significant variables that emerge from the regression analysis, such as Roess et al. (1977) recommended a model as the following:

$$OC = A(CM) + B(CH) + C(PV) + D(RP), \quad (2.4)$$

Where OC is operating cost, CM is car-miles, CH is car-hours, PV is peak-period railcars in operation and RP is revenue passengers.

More exact models of calculating operating cost have been developed afterwards. Herzenberg (1983) developed models to estimate bus driver's wages considering the scheduling practice and labour contract provisions. Due to the widely adopted accounting format, reasonable models separate operation cost into fixed and variable costs. Such comprehensive models can be programmed and run on computers, e.g. Cervero (1982) and Stopher et al. (1987).

Wirasinghe and Seneviratne (1986) formulated operating costs in a different perspective. The formulation of operator costs is shown as follows:

$$C_o = \lambda_b \int_{x_R}^L (x - x_R) P(x) dx + \lambda_R x_R P^* \quad (2.5)$$

where λ_b is the cost of owning and operating a bus per passenger per unit distance,

λ_R is the operating cost of a train per passenger per unit distance,

x_R is the travel distance by rail,

$P(x)$ is the line density,

P^* is total corridor demand.

It can be seen from Eq.(2.5) that operator costs vary with operating cost of bus and rail per passenger per unit distance.

Wirasinghe and Vandebona (1999) proposed of formulation of operating costs which were proportional with floor area of transit stations. The construction and operating cost per day for a station at (x, y) was formulated by a linear function as follows:

$$Z_s(x, y) = \gamma_s(x, y) + c(x, y)a_s(x, y) \quad (2.6)$$

where, $\gamma_s(x, y)$ is fixed cost of a station per day at (x, y) ,
 $a_s(x, y)$ is total floor area of the station at (x, y) and
 $c(x, y)$ is marginal cost per unit-area of a station at (x, y) .

This formula is applicable to transit network design at planning stage. The optimal total operator costs can be obtained when minimum sum of passenger travel time, transfer costs and subway fleet are solved.

Lee and Shonfeld (1992) optimised coordination at multiple-route timed transfer terminals and the objective function is the total cost of transit system, in which operator cost is calibrated as sum of vehicle running cost for all routes in the format as follows:

$$C_o = \sum_{i=1}^n N_i B_i = \sum_{i=1}^n [2D_i B_i N_i H_i], \quad (2.7)$$

where n= number of routes,

B_i = vehicle operating cost on route i (S\$/vehicle minute),

H_i =vehicle headway on route i (vehicle/minute),

N_i =fleet size for route i (vehicles),

D_i =length of route i (miles),

V_i = average speed on route i (mile/min).

Ting (1997) examined the effectiveness of coordination under various demand and arrival distributions to formulate the total system cost function, in which the operator cost employed followed the vehicle operating cost function proposed by Jansson (1980). The formulation of operator cost and the constraint is shown as follows:

$$B_k = a_k + b_k S_k, \quad (2.8)$$

$$S_k = \frac{Q_k h_k}{l_k}, \quad (2.9)$$

where B_k = vehicle operating cost (S\$/min.),

a_k = fixed vehicle operating cost of route k (S\$/min.),

b_k = variable vehicle operating cost of k (S\$/seat-min.),

S_k = vehicle size of route k (seats),

Q_k = demand of route k (passengers/min.),

h_k = headway of route k (min.),

l_k = load factor of route k .

The research on intermodal coordination with dynamic dispatching control was proposed by Chowdhury (2000), whose dissertation calibrated operator cost by summarising operator cost for bus and for rail. The formulation is illustrated in Eq.(2.10) and (2.11) as follows:

$$C_{or} = \frac{T_r^R u_r}{H_r}, \quad (2.10)$$

where C_{or} = operator cost of rail (US\$/hr),

T_r^R = rail round trip time (US\$/hr),

u_r = average train operating cost (US\$180 /hr)

H_r = rail headway (hr)

$$C_{ob} = \sum_{i=1}^n \sum_{j=1}^m \frac{(T_{ij}^R + K_{ij}) u_b}{H_{ij}}, \quad (2.11)$$

s.t. $1 \leq i \leq n$,

$1 \leq j \leq m$

where C_{ob} = operator cost of bus (US\$/hr),

T_{ij}^R = round trip time of bus route j at station i (hr),

K_{ij} = slack time of bus route j at station i (hr),

u_b = average bus operating cost (US\$70 /hr),

H_{ij} = headway of bus route j at station i (hr).

As the input parameters were hypothetically created in this research, it is not meaningful to compare these assumed parameters (e.g. operator cost of bus or rail) with those in reality.

2.2.4. User Costs

Extensive review on user costs was summarized by Black (1995). User costs are associated with passengers. Although fares are costs to transit users, the fares are revenue to the transit operators, thus the sum of these two items should technically balance to zero in a transit system. Usually passengers' travel times are the major component of the user cost in most economic analysis. User costs are formulated as the product of passengers' travel times and monetary evaluation of travel times. Furthermore passengers' travel times consist of travel times to and fro transit stops, waiting times, transfer times and in-vehicle times. There are several approaches in estimating money valuation of travel time and they are mentioned below.

I. Travel Time

a. Travel Time to and fro Transit Stops

Travel time to and fro transit stops is estimated by passengers spent in arriving transit stops by automobile, bicycle or walking. Usually the average walking speed is used to estimate the walking time.

b. Waiting Time

The usual method of estimating the average waiting time for passengers is to divide the headway between buses or trains in half. This can be applied under two assumptions. The first is that vehicles arrive at equal intervals, and the other is that passengers arrive randomly at a transit stop according to a uniform distribution. Due to the arrival pattern of vehicles seldom uniformly distributed, evaluating waiting time using this method can be underestimated. An improved model for estimating the passenger waiting time was developed by Welding (1957) (cited by Chowdhury, 2000) taking the mean and variance of the headway into consideration as the equation shown in the following:

$$E(w) = \frac{1}{2} \left[E(H) + \frac{V(H)}{E(H)} \right], \quad (2.12)$$

where, $E(H)$ and $V(H)$ represent the mean and variance of vehicle headways respectively.

Later Osuna and Newell (1972) developed a model based on the Welding's. In his model, given any realization of random values of $H_1 \dots H_n$ (H_i : mean headway), between time t_o and t_n , if a customer arrives at some random time, uniformly distributed over t_o to t_n , the probability that he/she arrives in the K^{th} headway H_k is $H_k / (\sum_{i=1}^n H_i)$. Finally the expected waiting time can be derived,

$$E\{w\} = \frac{1}{2} E\{H\} [1 + C^2(H)], \quad (2.13)$$

$$\text{where } C\{H\} = \left[\frac{\text{var}\{H\}}{E^2\{H\}} \right]^{\frac{1}{2}}.$$

c. Transfer Time

Transfer time occurs due to passengers transferring from one route to another, and it consists two parts. The first component is the walking time from the platform passengers alighting to the platform that passengers boarding, and the second component is the time spent waiting for the pick up vehicle to arrive. Clearly the first component of transfer time cannot be reduced without changing infrastructure, thus minimizing the second component of transfer time is the only way to reduce transfer time. Transfer coordination is aimed to minimize the transfer cost for passengers. Several research works on transfer coordination have been conducted. General transfer coordination in formulating transfer time was carried out by Ting (1997), in which the transfer coordination is considered between two of the stochastic arrival of vehicles. Transfer cost in non-coordinated operation is straight forward which is calculated as the product of transfer waiting time and total number of transfer passengers. While transfer cost in coordinated operation is the sum of slack delay cost, missed connection cost and dispatching delay cost. Specific transfer time cost formulation in common headway coordination scenario was conducted extensively by Chowdhury (2000), in which the transfer time cost consists of bus to bus, bus to rail,

rail to bus and rail to rail costs. However the situation in his research is considered as ideal, as common headway coordination may be difficult to apply in practice, when headways of rail can be very short while headways of bus cannot be set so small.

d. In-Vehicle Time

In-vehicle time occurs when passengers stay in buses or trains during the trip. There are four factors affecting in-vehicle time: cruising speeds of vehicles; spacing of stations; acceleration and deceleration rates of vehicles and station dwell times (Black, 1995). Thus it can be seen that the first three factors are fixed, with the station dwell time flexible. In practice, when a bus route is fixed and the bus is operated steadily, the station dwell time is affected especially by irregular arrival of passengers to transit stops, and it in turn affects the total in-vehicle time.

II. Monetary Value of Travel Time Cost

To further derive transfer time cost, the estimation of money evaluation of travel time should be combined to obtain dollar outlays in an economic analysis. Since time is not a commodity traded in a market explicitly, shadow price (change in the objective value of the optimal solution of an optimization problem obtained by relaxing the constraint by one unit) or imputed price (the price assigned to an item for which the price is missing in a particular period.) is utilized in evaluating time cost in the cost-benefit analysis (Black, 1995).

There are several approaches to estimate the value of travel time. Usually two main methods are widely accepted. The first is called income, or productivity method, which bases on the value of one hour of travel time on the hourly wage rate, which is calibrated either by minimum hourly wage or average hourly wage. It is common to apply a uniform value of travel time in an economic analysis project. But there have been objections to this. Thomas and Thompson (1970) (cited by Black, 1995) argued that the value of time for commuters was a function of income and the amount of time saved. The authors supported their claims with data collected by Stanford Research Institute and estimated that those persons with an income of \$20,000 per year would pay four to five times as much as those with an income below \$4,000 per

year. Thus, if a high cost value of time cost placed by the rich was taken into account, it might lead to building transit infrastructures favoring the rich than the poor. Therefore using a constant value is more desirable and acceptable from the standpoint of equity.

Another approach is the willingness-to-pay method which is more preferred. It involves empirical studies to determine what people actually paid to save travel times. In such a survey study on revealed preference of travelers, some travelers must have a choice between an option that is faster and expensive and one alternative that is cheaper and slower. A sample of travelers need to be gotten on travelers' preference between times and money, and then a calculation is estimated on traveller's willingness to pay of travel time.

III. Other User Costs

Besides travel time cost, there are several types of user costs that are considered less important and are often ignored in cost-benefit analysis. These are intangible factors such as comfort, convenience, reliability and security. Although no one doubts that these factors do influence people's travel behavior to some extent, these factors are usually omitted as they are hard to measure in normal cost-benefit analysis projects.

2.2.5 Models for Multimodal Transfer Coordination

Previous studies have analysed multimodal transfer coordination which was coordinated between different transportation modes and transfer coordination within the same transportation mode. They can be broadly categorized into two kinds of models for transfer operations, i.e. deterministic and probabilistic models.

I. Deterministic Models

The variation of vehicle arrival times will reduce the benefit of coordination among transit routes, however there were still a lot of research work using deterministic models for transfer operations. Salzborn (1972) first developed a deterministic model to solve a bus-scheduling problem. Given a passenger arrival rate, the

optimal vehicle departure frequency was determined by minimizing the fleet size and the total passenger waiting time. Although this model was formulated for a single bus route, it was claimed that it could be extended to a pair of bus routes at a transfer terminal. Later Salzborn (1980) developed guidelines on scheduling a comprehensive system of connections between feeder lines and one inter-town bus route, and applied combinatorial group theory to construct timetables. However Salzborn did not consider the route demand and transfer cost, only presetting values for minimum and maximum passenger transfer times, stopping times at interchanges and minimum terminal times. Daduna and Vob (1993) formulated the schedule synchronization problem as a quadratic semi-assignment problem (QSAP) to minimize the total transfer waiting time. Their algorithm was designed to search the initial feasible solution with a regret heuristic and solved it by a Tabu search method.

Chakroborty et al. (1995) solved the scheduling problem at one transfer station by integer programming to minimize either the overall transfer time or waiting time or both, subjecting to constraints on fleet size, the minimum and maximum stopping times, and the maximum transfer time. The assumption was that the vehicle arrival times were deterministic and passenger arrival rates were different from bus to bus. The headway between two consecutive buses should be less than or equal to the pre-planned headway which was pre-set by the transit operator. They then used genetic algorithms (GA) to solve this integer programming problem.

II. Probabilistic Models

Some researches have developed probabilistic models to account for the stochastic nature of passenger demand and vehicle travel times, but typically at the expense of their analytic features, such as optimisation, realistic networks or objective functions. Hall (1985) developed an analytical model for timed transfer systems, but this model used an overly-simplified objective function by minimizing only the total waiting times for transfer passengers and using exponential distribution to describe vehicle arrivals.

Salzborn (1972) developed a mathematical model for a bus scheduling problem. Given the passenger arrival rate, the bus departure rate was determined by minimizing

the fleet size and total passenger waiting time, but this method can only deal with one bus route. As a result, the arrival times at stops varied and the average waiting time was a function of the headway distribution as described by Osuna and Newell (1972). Based on Newell's policy, Wirasinghe (1990) proposed uniform dispatching policy for a bus route with many to many time-varying demand, and obtained variable maximum load point with a capacity constraint based on the seats per unit time formulation of demand. However it is very difficult to get the data of seats per unit time on a bus route in real world which makes this approach inapplicable. Liu and Wirasinghe (2001) developed a simulation model of schedule design for a fixed transit route adopting the holding control strategy. By minimising the total cost associated with vehicle schedule, the model determines the locations of time points and the amount of slack time allocated to each time point. However it can be observed that the method in obtaining value of time for different cost components in data collection was not described. Adebisi (1986) developed a mathematical model for computing bus headway variances at various stopping stations on a fixed route with pre-planned schedule. He pointed out that the bus arrival distribution at a station was affected by route factor (upstream traffic condition) and bus loading factor (effects of variability in the loading situations encountered by successive buses), and then suggested that a careful decomposition of the route into segments in which each was truly homogeneous, could reduce the arrival variability. Talley and Becker (1987) used exponential distributions to compute the probability that buses would be more than some minutes earlier or later than the preset schedule. The probabilities may be interpreted as failure rates, and the probability equation allows flexibility in interpreting results and setting standards for on-time performance.

Guenthner and Sinha (1983) estimated the travel time based on delays due to passenger boarding and alighting, and the results showed that a Poisson distribution could be used only on routes with low level of ridership. Guenthner and Hamat (1983) studied the distribution of bus transit on-time performance. Their results showed that adjusted bus arrival times, which were the differences between observed arrival times and scheduled arrival times at a bus stop along a route, followed a gamma distribution.

Bookblinder and Desilets (1992) developed a model for optimizing vehicle departure times and showed how timed transfers could reduce the passenger waiting times, and how layover times could be added to vehicle schedules at transfer points to ensure successful connections. Chien (1995) developed a model for coordinating an intermodal transit system that consists of multiple bus routes at each transfer route at each transfer station and multiple transfer stations along a rail line. The planning stage of the whole public transit integration was presented. A probabilistic model was constructed assuming that bus arrival times follow the Normal distribution.

Knoppers and Muller (1995) investigated the possibility and limitation of coordinated transfers in public transit systems. The optimal transfer times were obtained while considering stochastic arrivals of feeder vehicles and deterministic arrivals of vehicles operating on the major transit route. They found that coordination was worthwhile when the standard deviation of arrival times on feeder lines at the transfer terminal is less than 40% of the headway on the major service network. When the frequency on the connecting line is higher, application of transfer planning becomes harder and the potential benefit becomes less significant. Although it can be extended to dual directions, only one directional transfer with common headway coordination was considered in their paper, and integer-ratio coordination was not considered. Ting (1997) constructed both deterministic and probabilistic models for non-coordinated and coordinated operations, but the models were general and theoretic that did not discriminate coordination within the same transit mode or between different transit modes. To be noted integer-ratio headway coordination was applied in transfer coordination. Simultaneous Perturbation Stochastic Approximation (SPSA) approach was applied to solve the stochastic scenario of coordination model, and this approach was first introduced and developed by Spall (1992). Chowdhury (2000) improved multimodal coordination research work and optimised for a public transit system, but the objective is too broad to achieve, as the demand based schedule of buses cannot be adjusted to dispatch just to meet the requirement of timed transfer. In this research work Powell's method was selected to solve the objective function in a coordinated operation scenario and a four-stage procedure was developed in searching all combinations of coordination.

2.3 Multi-operator Public Transit System

2.3.1 Deregulation of Public Transit

In a multi-operator public transit system, it cannot be omitted considering the situation of deregulation which was advanced by the European Union in 1999. The deregulation reform of public transit was dated from a 1980 Act which completely liberalized inter-urban bus transit markets. A historical date of transit industry was October 26, 1986, when the British government dismantled the regulatory structure of public transit (Berechman, 1993). Extensive research on deregulation has been conducted in the previous work. Daduna (2001) and Zubieta (1998) presented a new model for a deregulated transportation system with full representation of the city network, and assumed the case in which a few private bus companies provided the totality of the urban transportation services and each private company had exclusive rights to operate a particular transit line. Competition existed only in terms of the frequency of service as demand. The limitation is that transit fares were not considered interactively when competition in the companies existed; whereas it only considered transit fares when marginal revenue equalled marginal cost for each operation company themselves.

Another method used in the evaluation of competition among public transit operators can be found in Sanchez (2000). The results suggested that the separated supply of passenger and freight transportation by different firms would produce significant increases in the efficiency levels of rail industry. Besides, the results also indicate that at least for the monopoly company of a greater size, an efficiency increase, which was measured by lower costs that would be obtained from the breaking-up of the current monopoly company into two different companies of a smaller size.

2.3.2 Game Theory

Game theory is the mathematical study of conflict situations, born in 1944 with the publication of a single book *Theory of Games and Economic Behaviour* by von Neumann and Morgenstern (Jones, 2000).

A brief introduction of game theory on normal form games and Nash equilibrium was given by Creedy et al. (1992), in which the descriptions are illustrated as follows.

A normal form game consists of a set of n players, denoted $N = \{1, \dots, n\}$; a finite, pure strategy set for each player $i \in N$, denoted S_i , and a payoff function $\pi_i: S \rightarrow R$ for each $i \in N$. If each player j chooses a pure strategy $s_j \in S_j$, then player i 's payoff is given by $\pi_i(s_1, \dots, s_n) \equiv \pi_i(s) \equiv \pi_i(s_i, s_{-i})$. More generally, each player j may choose to randomize over the pure strategy from S_j to be played. Let ΔS_j denote the collection of probability mixtures over S_j . ΔS_j is then player j 's mixed strategy set. Let an element of ΔS_j be denoted m_j . If each player $j \in N$ chooses a mixed strategy m_j , then i 's payoff is given by

$$\sum_{s_i \in S_i} \sum_{s_{-i} \in S_{-i}} \pi_i(s_i, s_{-i}) m_i(s_i) m_{-i}(s_{-i}), \quad (2.14)$$

where $m_i(s_i)$ is the probability attached by mixed strategy m_i to be pure strategy s_i and $m_{-i}(s_{-i})$ is analogous. Eq. (2.14) can be written as $\pi_i(m_i, m_{-i})$.

An equilibrium concept is now required. Consider the following:

Definition 2.1 The strategy $m_i \in \Delta S_i$ is a (weekly) dominant strategy for player 1 if, for all $m_i' \in \Delta S_i$ with $m_i' \neq m_i$, we have

$$\pi_i(m_i, m_{-i}) \geq \pi_i(m_i', m_{-i}), \quad \forall m_{-i} \in \Delta S_{-i} \quad (2.15)$$

with strict inequality for at least one m_{-i} . The strategy $m_i \in \Delta S_i$ is a strictly dominant for player i if for all $m_i' \in \Delta S_i$ with $m_i' \neq m_i$:

$$\pi_i(m_i, m_{-i}) > \pi_i(m_i', m_{-i}), \quad \forall m_{-i} \in \Delta S_{-i}. \quad (2.16)$$

The attraction of a dominant strategy is that if player i has such a strategy, then playing this dominant strategy is payoff-maximizing for player i regardless of the strategy of i 's opponents. Player i can choose a strategy without engaging in any calculations or predictions of opponents' strategies.

Neither player has a dominant strategy in this game for which a less-demanding solution concept is required. The fundamental solution concept for non-cooperative games is that of a Nash equilibrium (Nash, 1951).

Definition 2.2 The strategies $m \in S$ are a Nash equilibrium if, for all $i \in N$,

$$\pi_i(m_i, m_{-i}) \geq \pi_i(m'_i, m_{-i}), \quad \forall m'_i \in \Delta S_i. \quad (2.17)$$

Nash (1951) showed the following:

Theorem 2.1 Every finite normal form game has a Nash equilibrium, though this equilibrium may require mixed strategies.

Game theory have been widely accepted and applied into many kinds of research areas these years, without exception in public transit research area. Game theory was firstly used in application for economy strategy, in areas concerning human behavior in adopting competition strategy to achieve the maximum utilities. Likewise game theory was adopted in research areas concerning cost associated with fares in public transit system in the previous work. A primary objective of game theory is the development of rational criteria for selecting a strategy. There are two assumptions: one is that both players are rational, and the other is that both players choose their strategies solely to promote their own welfare (no compassion for the opponent) (Hillier and Liberman, 2001). Bell (2000) considered a non-cooperative game between network user and an 'evil entity' that maximized the expected trip cost. When Nash mixed strategy equilibrium was reached, the user cannot reduce the expected trip cost and at the same time the evil entity cannot increase its expected trip cost either. It can be inferred that game players with conflicting interest can reach Nash equilibrium without cooperation. Later other researchers extended conflicting interests of multiple players in public transit systems. Lo et al. (2000) and Lo and

Yip (2002) analysed the situation under which the fare of transit services was deregulated using game theory. Their studies examined four scenarios of fare deregulation on transit services by considering three relevant participants of a public transit system: the service supplier, the service demander (the travelers and the society in general), with their respective objectives represented as revenue, travel utility and congestion. Analysing the resulted impacts on these three participants, the authors finally drew a conclusion that a different regulatory environment would favor or hurt a different set of participants; there was no clear win-win situation for all participants. The effect of value of time distributions on price and quality competition among differentiated bus services in a simple corridor was explored by Yang et al. (2001). Three travel modes were considered: private car, minibus and conventional bus in this paper. The behaviour of two types of buses was modeled as a Nash non-cooperative game. Competitive equilibrium is established in terms of each bus firm's optimal frequency and optimal fare, at the same time the profitability of two types of bus services was examined.

A bi-level transit fare equilibrium model for a deregulated transit system is proposed by Zhou et al (2005), in which bi-level equilibrium consists of an upper-level problem and a lower-level problem. Thus in the upper-level problem, the transit competition is portrayed as an n-player, non-cooperative game by changing the fare structure of each of a set of transit lines separately so as to maximize the profit of each transit operator within the oligopolistic market. It shows that there exists a generalized Nash game between transit operators, which can be formulated as a quasi-variational inequality problem. While in the lower-level problem, the passengers' response to the equilibrium fare structure of the transit operators is represented by the stochastic user equilibrium transit assignment model with elastic OD demand. Thus the bi-level transit fare equilibrium problem is presented in the Stackelberg form and solved by a heuristic solution algorithm based on a sensitivity analysis approach. A numerical example is given to illustrate the competition mechanism on the transit network and some useful findings are presented on competitive operations. Their paper considers fare adjustment purely from the market point of view without taking account of coordination operations.

As there are some noncooperative game theory problems that one of the players has the ability to enforce his strategy on the other player(s), one has to introduce a hierarchical equilibrium solution concept, called Stackelberg equilibrium which was elaborated by Başar and Olsder (1999). Following the original work of Stackelberg (1934), the players who holds the powerful position in such a decision problem is called leader, and the other players who react rationally to the leader's decision strategy are called followers. It is proved that the feedback Stackelberg solution features the "Stackelberg" property at every level of play, and this can be verified recursively (Başar and Olsder, 1999).

A game theoretical approach is applied to model the strategic interactions between the operators in a deregulated bus market by Wang and Yang (2005), taking into consideration of price competition as well as competition over service frequency. This paper investigated the competition of bus services in a deregulated bus market with vertical service differentiation without considering other transit modes. The decision variables are fare and service frequency of buses. The impact of deregulation of the bus market is then assessed in terms of profits and social cost to the society. A hypothetical case is constructed to test the model developed. Therefore this paper did not consider the coordination of multimodal transit system. Considering the competition among multi-operators of the public transit system, an analysis was derived by Lo et al. (2003), using game theory to analyse the competitive circumstances, in which the fare can be defined by the multi-operators. By means of a state augmentation technique, a multimodal network was transformed into so called State-Augmented Multimodal network (SAM). Through a case study their research work at last drew a conclusion that passengers generally paid higher fares for all of the public transit, and what is worse, the network got more congested especially on those already congested roadways, although their study did not consider the coordination of the multimodal transit system.

2.4 Multimodal Transit Trip Assignment

The path choice problem is to study how passengers choose paths in a multi-modal transit network that are optimal to them. For a transit user, there may be a large number of alternative transit routes available for him/her to use. The environment of

the decision maker determines what are called the universal set of alternatives. Any single decision maker considers a subset of this universal set, termed a choice set (Ben-Akiva and Lerman 1985). Ben-Akiva et al. (1989) discussed the generation of a finite choice set by labelling according to a single criterion such as the minimum time, distance, or fare. Besides travel time, distance and fare, there are a number of factors that may have a strong influence on a passenger's choice of transit paths; and these factors may include number of transfers, comfort and so on. The path decision model which was proposed by Xie (2003) is a multi-criteria path optimisation considering the above factors instead of the sole factor of travel time.

Nguyen and Pallottino (1988) developed a graph-theoretic framework for large scale transit networks and provided a method in the equilibrium transit assignment. A shortest hyperpaths problem was defined and computational techniques for shortest hyperpaths suggested, but the factors considered in the cost of hyperpaths are not clearly stated and discussed.

Integrating passengers' travel behaviour, Ben-Akiva and Lerman (1985) and Bekhor et al. (2001) divided path choice modelling into two parts. First, possible alternative paths are generated to form the choice set, and then the probability that a given path is chosen from a specified choice set is calculated. In Xie (2003), following the decision-making process as shown in Figure 2-1, the path choice model consisting of

Figure 2-1 the Path Choice Model Consisting of Path Search and Path Decision Models (Source: Xie, 2003)

two basic parts with different functions: (i) path search model; and (ii) path decision model, was illustrated. The first step, path search model, aims to obtain possible, finite alternative paths to form the choice set from a universal set. The finite choice

set may be labelled according to the single criteria such as minimum time, minimum distance, minimum fare, etc.

2.4.1 K^{th} Shortest Path Algorithm in a Multimodal Transit System

In a multimodal transit system, searching for the K^{th} shortest path problem closely resembles that in the field of graph theory. The shortest path algorithm has been developed for many years. Dijkstra Algorithm (Dijkstra, 1959) provided a solution to find the shortest path between origins and destinations. Several algorithms have been put forward since then. There are two types of problems in finding the shortest path, i.e. loopless paths and paths containing loops. It is obviously that there should be loopless in the K^{th} shortest path in a multimodal transit system. There are four types of algorithms in finding the K^{th} shortest paths, cited by Yen (1971).

The first type is to introduce an enumeration procedure for finding the K^{th} shortest path in a network, e.g. Rock et al. (1957).

The second type is to introduce a procedure for finding the K^{th} shortest path in a network, i.e. by Pollack (1961). To find the K^{th} shortest path this procedure first obtains $K-1$ shortest paths. Then the distance of each arc in each of the $1^{st}, 2^{nd}, \dots, (K-1)^{th}$ shortest paths is set, in turn, to infinity. The shortest path problem is solved for each case. The best of these resulting shortest paths is the desired K^{th} shortest path.

The third type is to introduce a branch and bound procedure for finding the K^{th} shortest paths in a network by Clarke et al. (1963). This procedure first finds the shortest path, and then finds the K^{th} shortest paths from all paths that branch out from the shortest path.

The fourth type is to find the first H ($H \geq K$) shortest paths that may contain loops, and then the H paths are scanned for the K shortest paths that contain no loops by Sakarovitch (1966).

Waiting, walking, transferring are important components in travelling in a multi-modal transit system. The search for the shortest path in this situation requires a more specific approach. In general, the major difficulties associated with computing paths in a multi-modal networks are: (i) the discontinuities of the fixed schedule lines, (ii) high number of transit lines/routes in some modes, with each line essentially operating on a different network, which means that each line has to be treated as a different mode, and (iii) the need to account for delays at switching points, i.e. transfer time (Ziliaskopoulos and Wardell, 2000).

The literature on multi-modal shortest path computation is mainly in static transit networks in planning applications. Spiess and Florian (1989) proposed an algorithm by adding arcs to represent waiting, walking and line switching (transfer). Probabilistic waiting and deterministic travel times are associated with every arc of the transformed network and the expected least time path is calculated on the expanded network with a static shortest path algorithm. Dial et al. (1979) introduced an extension to Moore's label correcting algorithm to be used as a ridership prediction tool by transit planners. The algorithm assumes that link travel times are static and that waiting time at transit stops is one-half of the inverses of the line frequency. These assumptions are appropriate for planning applications, where no actual schedules are available.

Ziliaskopoulos and Wardell (2000) presented an intermodal optimum path algorithm for multi-modal networks with dynamic arc travel times and switching delays. In the notation given as follows, Ziliaskopoulos and Wardell states the optimality equation that gives a necessary and sufficient condition to be optimum in finding shortest path in network $G = (V, A, T, M)$, as shown in Eq. (2.18) and Figure 2-2.

Based on the research work above, Xie (2003) extends the multimodal shortest paths problem to the K^{th} shortest paths problem by introducing the priority queue and sorting the SE (Scan Eligible) list to identify the K^{th} shortest path.

Notations:

V	the set of nodes
A	the set of arcs
T	the set of discrete departure times
M	the set of modes
$\tau_{ij}^x(t)$	the travel time on arc (i,j) for mode x for departing from i at time interval t
$\xi_{ikj}^{xy}(t)$	the delay at node i , when entering node i from arc (k,i) on mode x at time t and switching to arc (i,j) and mode y
$\lambda_{ik}^x(t)$	the path time from node i , when entering node i from arc (k,i) on mode x at time t , to destination node N and exit mode x_f
$\pi_{ik}^x(t)$	the path from node i , when entering node i from arc (k,i) on mode x at time t , to the destination node N and exit mode x_f
$\Gamma(i)$	the set of successor nodes to node i
$\Gamma^{-1}(i)$	the set of predecessor nodes to node i
N	the Destination node to which least time paths are found from every other node in the network
X_f	the Exit mode at destination node N

$$\lambda_{ik}^x(t) = \min\{\xi_{ikj}^{xy}(t) + \tau_{ij}^y(t + \xi_{ikj}^{xy}(t)) + \lambda_{ji}^y(t + \xi_{ikj}^{xy}(t) + \tau_{ij}^y(t + \xi_{ikj}^{xy}(t)))\}$$

$$\forall j \in \Gamma(i), \forall y \in M, \forall k \in \{\Gamma^{-1}(i), i'\}, \forall x \in M, \forall t \in T \text{ and } \forall i \in V$$

with conditions:

$$\lambda_{Nk}^x(t) = \xi_{NkN}^{xx_f}(t)$$

$$\forall k \in \{\Gamma^{-1}(i), N'\}, \forall x \in M, \forall t \in T$$
(2.18)

Figure 2-2 Illustration of the shortest path finding (Source: Ziliaskopoulos and Wardell, 2000)

2.4.2 Discrete Choice Analysis and Path Decisions

In general, the shortest path algorithm is based on a single criterion (e.g. the shortest travel time). In fact, there are a number of factors that may have strong influences on a passenger's decision. The factors include walking distance, number of transfers, fare, level of crowdedness, comfort, etc. Although in some applications, the various cost and time components are combined and converted into a generalized cost, the difficulties in estimating parameters in the generalised cost equation and the aggregate nature of the parameters make it not suitable for the type of personalised solutions envisaged in this research. Benshoof (1970) and Geehan and Deslauriers (1978) stated that some transit users did not actually choose the minimum time route. Besides time, many other path choice criteria, e.g. walking time, waiting time, etc, should be considered according to the result by Huber and Boutwell (1968). Comprehensive factors on transit path choice are included to model Level-of-Service (LOS) factors by Pursula and Weurlander (1999), which presented a quantitative analysis of the importance of the different service factors. These factors included in-vehicle time, transfer walking time, and other factors included walking time, waiting time, the first service headway, and transfer environment in public transportation in the Helsinki metropolitan area. Their research was in cooperation with one on transport price and level of service in path choice by Weurlander (1996) and Stated Preference (SP) study design for public transportation path choice by Merin (1996). Building upon these findings, a multi-criteria path choice model (or called path decision model in this study) is formulated to find out how passengers assess alternative paths based on travel behaviour and passengers' preference and make their decisions. Xie (2003) concluded that in a complex transit network, riders prefer to the first three shortest journey time paths. The models ranked three alternatives based on total journey times. Among the first three paths, the specification of the utility functions showed that riders make decision taking into consideration of different trip attributes and social economic factors. Therefore, the utility functions used in the PS (Path-Size) Logit model represent measures of trade-off among the attributes used in the functions.

The path choice behaviour of a decision maker can be described in terms of discrete choices and variables. If a decision maker is observed to choose from a discrete and

finite set of alternatives, one can specify a choice function for each one of the alternatives. Utility maximisation, as one of decision rules, is widely used in the development of predictive models of human behaviour (Ben-Akiva and Lerman, 1985). Discrete choice models, based on the random utility theory, postulate that the probability of an individual choosing a given option is a function of the socio-economic characteristics and the relative attractiveness of option (Domenich and McFadden, 1975).

The environment of the decision maker determines the universal set of alternatives (S). Any single decision maker considers a subset of this universal set, termed a **choice set** (C). A discrete choice model is written in parameterised form which consists of parameters (β) and attributes (X) of the choice alternatives and of the decision maker facing a choice set C . Each option $C \subseteq S$ is associated with a random utility U_{in} for individual n , which can be represented by two components:

$$U_{in} = V_{in} + \varepsilon_{in}, \quad (2.19)$$

- A measurable, systematic or representative part V_{in} which is a function of the measured attributes X and parameter β , and
- A random part ε_{in} following a particular statistical distribution.

If the decision rule to be used is utility maximisation, an individual n chooses the i^{th} alternative when $P_{in} = \Pr obabilty \{V_{jn} + \varepsilon_{jn} \geq V_{in} + \varepsilon_{in}, \forall i \neq j, C \subseteq S\}$. The models can take various forms, such as Multinomial Logit Model (MNL) or Multinomial Probit model (MNP) depending upon assumptions on the underlying statistical distributions.

As stated at the beginning of this section, the two-stage process used in the path choice model first considers possible alternative paths to form the choice set, then the probability a given path is chosen is calculated. Due to the numerous alternative paths a transit user may face, the choice set will be generated using a k-shortest path algorithm.

In transit path choice problems, different paths may have overlapping sections, thus Multinomial Logit Model (MNL) is not suitable to model. Alternatively Path-Size Logit Model (PSL) is a good choice, as a correction term is added to path utilities while the MNL model structure is maintained. The log of the path size is added to the path utility to form the Path-Size Logit model in Eq. (2.20) as follows.

$$P(i | C_n) = \frac{e^{\mu(V_{in} + \ln PS_{in})}}{\sum_{j \in C_n} e^{\mu(V_{jn} + \ln PS_{jn})}}, \quad (2.20)$$

where C_n is the choice set, V_{in} represents the systematic components of an PSL model, PS_{in} is termed as the size of path i for person n .

The definition by Ben-Akiva and Bierlaire (1999) for the Path-Size is shown in Eq. (2.21) as follows.

$$PS_{in} = \sum_{a \in \Gamma_i} \left(\frac{l_a}{L_i} \right) \frac{1}{\sum_{j \in C_n} \frac{L_{C_n}}{L_j} \delta_{aj}}, \quad (2.21)$$

where a is indexed links (arc), Γ_i is the set of links in path i , l_a is the length of link a , L_{C_n} is the length of the shortest path in C_n , L_i is the length of path i , δ_{ai} is binary, and equals to 1 if path j uses link a , and equals 0 otherwise.

V_{in} which represents the systematic components of an PSL model is calibrated based on the specific environment, and Xie (2003) formulated two models to estimate this variable based on data collected from survey in Singapore. The descriptions are illustrated as follows.

For n th person, let V_{in} be the systematic utility of the i th path ($i=1, 2, 3$). The models are given as follows.

Model 1:

The model specification is defined as follows:

$$V_i = \beta_{0,i} + \beta_1 walkdis_i + \beta_2 Ttrans_i + \beta_3 Tstnps_i + \beta_4 M_i, \quad (2.22)$$

where $Walkdis_i$ = total walking distance (km) by path i ,

$Ttrans_i$ = total number of transfer (km) by path i ,

$Tstnps_i$ = total number of MRT stations plus bus stop* taken by path i ,

M_i = a variable about proportion of trip by MRT during trip,

$\beta_1, \beta_2, \beta_3, \beta_4$ = unknown model parameters to be estimated,

$\beta_{0,i}$ = Path-specific constant (to path i , and $i=1,2$) and $\beta_{03}=0$.

The proportion of trip by MRT equals to number of MRT stations passed divided by total number of bus stops and MRT stations passed in the same journey. The variable M_i is the natural logarithm of the proportion of trip by MRT (M_i is set zero where there is no MRT stations passed during trip). Path sizes can be calculated based on the length of links within a path, and the relative lengths (time or distance) of paths that share a link according to Eq.(2.21). The length of link may be highly correlated with travel time, especially in un-congested networks. Bekhor et al. (2001) had tested the path size based on free-flow time instead of distance in private transport. As the expected (projected) travel time on each link of journey time by MRT, bus or walking obtained in data preparation, here the lengths of links are substituted by the expected travel time on each link by MRT, bus or walking. Then proceed to check every alternative combination for every individual route. The overlapping link by MRT, bus and walking is especially marked and their projected travel time was recorded, together with other links in alternative paths. If a path contains links that may be shared by other alternative paths, the size of a path with one or more shared links may be less than one according to Eq. 2.11. A path with no overlapping links needs no utility adjustment and has a size of one.

Model 2 :

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* The bus stops taken are counted based on TransitLink 1999. It also includes stops that bus may not dwell sometime because of no passenger boarding and alighting.

$\begin{aligned} V_1 &= \beta_{01} + \beta_1 \text{walkdis}_1 + \beta_2 \text{Ttrans}_1 + \beta_3 \text{Tstnps}_1 + \beta_4 M_1 + \beta_6 X_1 + \beta_7 Y \\ V_2 &= \beta_{02} + \beta_1 \text{walkdis}_2 + \beta_2 \text{Ttrans}_2 + \beta_3 \text{Tstnps}_2 + \beta_4 M_2 + \beta_6 X_2 + \beta_8 Y \\ V_3 &= \beta_1 \text{walkdis}_3 + \beta_2 \text{Ttrans}_3 + \beta_3 \text{Tstnps}_3 + \beta_4 M_3 + \beta_6 X_3 \end{aligned}$	(2.23)

where walkdis_i, Ttrans_i, Tstnps_i, M_i are defined as same as model 1.

$\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7$ are unknown model parameters to be estimated.

$\beta_{0,i}$ are *alternative-specific constant* (to path i).

X_i is fare indicator, $X_i = 1$ if total fare is more than (or equal to) \$2; $X_i = 0$ if otherwise.

$Y_i = \text{Ag} * \text{walkdis}_i$ Where Ag is age indicator, and Ag=1 when age is below 15 or above 55; Ag=0 if otherwise.

2.5 Conclusion

It can be concluded from the literature review of previous studies on the coordination of public transit system that deterministic models have been well developed whereas stochastic models were not fully considered. Few studies considered the multiple routes and multiple transfers in the public transit system. To be noted some of the previous studies concerning coordination defined a very broad analysis of coordination taking a lot of factors into consideration and the resulted model became too complicated to be practically applicable. Furthermore most of the studies only considered the common headway coordination scenario in which bus and rail share the same headway. To be noted, most of the previous studies on multimodal coordination did not consider transit trip assignment.

On the other hand, even if there were some studies considered coexisting of multi-operators, they only studied the case when competition existed among multiple operators without considering other cases that multi-operators may cooperate or operate independently. Those studies focusing on the multi-operators in a public transit system considered more of competition and fare deregulation problems without considering the coordination between transit modes. It can be found that no research work published has conducted on the optimal coordination for an integrated multi-operator and multimodal transit system considering traveller's path choice behaviour. Inspired from the situation in Singapore, this research topic can greatly

contribute to the policy making of public transit and be a good reference to public transit firms in other countries as well. Taking into account of the cooperation, independence, and competition between multi-operators and coordination of the multimodal public transit system, this study will formulate a model containing more of the important factors and use a suitable algorithm to solve it. Computation algorithms and optimisation procedure will be developed to allow for the solutions to be practically applicable; besides all the codes are implemented in AIMMS 3.6, the programmes are validated through sensitivity analysis of optimal decision variables and important cost components with respect to various values of input parameters and network configurations.

CHAPTER 3 METHODOLOGY

3.1 Introduction

This chapter discusses the methodology that will be adopted in this research. Firstly assumptions of this study are presented, followed by the framework and optimisation algorithm; secondly systematic analysis is carried out to prepare for model formulations. Lastly, a summary is addressed at the end of this chapter.

3.2 Methodology

In this section, some assumptions are proposed to make the model formulations reasonable. In the next, framework and optimisation algorithm are interpreted to make it clear of the processes of this research.

3.2.1 Assumptions

The assumptions of this research are listed as follows.

- i. The majority of the commuters travel along multimodal transit paths which may be operated by multimodal operators. It is assumed that the paths consist of two modes and there are two operators in the same transit system.
- ii. The demand distribution (inflow and outflow passenger matrices) during a unit time is known and assumed to be uniform within the period. In other words, two groups of passengers' rate are known, i.e. passengers that arrive by bus and those arrive by modes other than bus (e.g. walking, taxi, private car) at areas within walking distance to terminal stations.
- iii. Train services operating on the rail lines are assumed to be nearly deterministic, as trains are operated exclusively on the rail line and they are more easily controlled. While the buses' arrivals at transfer stations are considered

stochastic and follow Normal Distribution due to variations in traffic conditions, such as traffic jam and traffic accidents.

- iv. The passengers are assumed to be well-informed and knowledgeable about fare, travel time, and real time dispatching schedule.
- v. The operators when adopting a cooperative or independent strategies would adjust their strategies only in terms of headways of the rail and bus and offset between schedule of bus and train in the public transit system to attract more passengers, leaving other aspects, for example, accessibility of transit service unchanged; While competitive operators would adjust their strategies not only in terms of headways of rail and bus and offsets between schedule between bus and train but also in the fare which fluctuates within the boundary allowed by the transit administrator.
- vi. Coordination scope between operators varies with operation policies, i.e., when operators adopt cooperation, coordination is taken between operators. When operators adopt competition and independence, coordination is not considered between operators.

3.2.2 Framework and Optimisation Algorithm

The methodology adopted consists of systematic analysis of the research objective, model formulations in different scenarios, evaluation of models and sensitivity analysis. The framework of this research is shown in Figure 3-1, and the optimisation algorithm is shown in Figure 3-3.

Figure 3-1 Research Framework

According to the topic of this research, there are three keywords, i.e. coordination, multimodal and multi-operator. Multimodal means passengers have many options for their mode choices, thus classification of OD flows between or within modes should be clarified at first, e.g. OD flows from bus to rail, and OD flows within bus or rail.

The classification of OD matrices flow is explained in detail as follows. In a multimodal and multi-operator transit system, passengers entering station i and exiting station j can be classified into those taking bus (λ_{ij}^b), those taking rail (λ_{ij}^r) (if there exist) and passengers taking other modes (λ_{ij}^n). Considering path choice and trip assignment, passenger flows can be categorized in further detail. Thus three groups of passengers above can be further traced from their arrival modes, e.g. passengers taking bus (λ_{ij}^b) consisting of those passengers transferring from rail (λ_{ij}^{rbp}), those passengers transferring from bus (λ_{ij}^{bbp}) and those passengers transferring by other modes (λ_{ij}^{nb}). Further classification is carried out considering coordination scope, i.e. coordination within operator's scope or between operators, e.g. passengers entering station i and exiting station j arrive by rail and transferring to bus by operator p (λ_{ij}^{rbp}) consist of those arriving by rail and transferring to bus by the same operator p (λ_{ij}^{rbp}) and those arriving by rail and transferring to bus by another operator p (λ_{ij}^{rbp}). Passengers entering station i and exiting station j arriving by bus and transferring to train by operator p (λ_{ij}^{brp}) consist of those arriving by bus and transferring to train by the same operator p (λ_{ij}^{brp}) and those arriving by bus and transferring to train by another operator p (λ_{ij}^{brp}). The detailed groups of passengers are illustrated in Figure 3-2 as follows.

Multi-operator means multiple operators coexist in one transit system. Therefore they must adopt some policies between each other. In a well regulated transit system, these possible operation policies can be categorized as cooperation, independence, and competition. When operators are in cooperation, the coordination

is well operated just like one unique operator does. When operators are independent and competitive, the coordination is within each operator's scope without considering coordination between operators. Besides, fare adjustment during certain value of range is considered when operators are competitive. Obviously when operators adopt different operation policies, important decision variables, i.e. headways of bus and rail, and offset between schedule of bus and train, may vary accordingly, as a result coordination operation between or within operators and fare adjustment is allowed or forbidden are beneficial or not would be much needed to be investigated.

Figure 3-2 Categories of Passengers flows at Terminal Stations

Therefore it can be seen from Figure 3-1 that the framework of this study begins with systematic analysis of research objective which consists of different coordination operations, i.e. non-coordination, common headway coordination and integer-ratio headway coordination. Detailed classification of feeder OD flows which consists of feeder OD flows by bus and feeder OD flows by other modes than bus and rail is shown in Figure 3-2. Operation policies, which consist of cooperation, independence and competition, are to evaluate coordination effect reflecting operator relationship on the overall market condition of the transit system. To catch passengers' behaviour in path choice, path choice model is employed, and the

optimisation algorithm in Figure 3-3 is described to capture the dynamic equilibrium between operators and passengers. The next step is model formulations which consist of cost components formulation, transit path model and optimisation algorithm that is illustrated in Figure 3-3. In order to evaluate this model, transit data comparable to Singapore transit system are used, thereafter optimal results are derived to draw preliminary conclusion. To confirm this preliminary conclusion and validate the model formulation, sensitivity analysis of important decision variables and cost items with respect to important factors is carried out in different coordination operations when operators are in different operation policies. Finally a conclusion is drawn based on results from sensitivity analysis and comparison of integer-ratio headway coordination under different operation policies.

As it is assumed that passengers are well-informed and knowledgeable about fare, travel time, and real time dispatching schedule, passengers will make decisions in path choice accordingly, and it in turn makes transfer volume fluctuate. Thus transit operators are leader and passengers are followers, which can be represented in a Stackelberg form. This Stackelberg form of Nash equilibrium can be verified recursively according to Başar and Olsder (1999).

Therefore it can be seen in Figure 3-3 that this optimisation algorithm starts with initial input data that assume all the detailed OD matrices are evenly distributed, after solving the objective function, the optimum headways and offsets are derived. According to the optimum headways and offsets (if there exists), transit path choice model is employed to derive the adjusted OD matrices. Based on the adjusted OD matrices, optimum headways and offsets can be obtained by solving the objective function. If the total cost of this iteration equals that of the previous iteration, the current optimum results are the ultimate optimal solution of this problem. Whereas if the total cost of this iteration does not equal that of the previous iteration, transit path choice model is employed again according to the current optimum headways and offsets. Thus the adjusted OD matrices are obtained as well. Inputting this adjusted OD matrices, the objective function is solved with optimum results. By comparing the total cost of the current iteration with that of the previous iteration, the process is continued until the total cost converge.

Figure 3-3 Optimisation Algorithm

3.3 Modelling a Multimodal and Multi-operator Transit System

As it is stated in the previous section, in an integrated multimodal and multi-operator transit system, systematic analysis of this research consists of three parts, i.e. multimodal (classification of OD flows), multi-operator (different operation policies) and coordination (different coordination scenarios).

In an integrated public transit system, transfer coordination is the crux of the problem. In order to compare non-coordination with coordinated operations, optimal results are derived to make the comparisons. Thus coordination operations comprise two different scenarios: common headway coordination in which headways of bus and rail are the same, integer-ratio headway coordination in which the ratio of headways of bus with rail is integer number. The results obtained in non-coordinated scenario will be used as the base case for comparisons with these two coordinated scenarios.

The difficulty of transfer coordination may vary with different transit network configuration. In order to avoid peculiarity results that are inherent from a particular network configuration, two typical network configurations, i.e. linear and grid transit networks, are studied. The linear transit network named network I is shown in Figure 3-4, in which trunk rail lines with feeder buses are operated along linear corridor with parallel lines. While the grid transit network named network II is shown in Figure 3-5, in which circumferential rail lines with feeder buses are operated in grid manner.

Two multimodal operators are comparable and parallel in operation in network I, while in network II, operator 1 operates from stations 1, 2, 3, 4, and 5 as shown in black and red colour and operator 2 operates from stations 2, 3, 4, 6, 7 and 8 as shown in blue and green colour; both operators are multimodal operators and in partial overlapping from station 2 to 4. The combination of different operators and different modes can be extended as different arcs which can form different paths collectively to represent multimodal paths in a real network. The methodology developed can be extended to other configuration in a similar manner.

Figure 3-4 Linear transit network

Figure 3-5 Grid transit network

Another important part in this research is trip assignment in this multimodal and multi-operator transit system, as the operation of changing headways of bus or rail may result in alteration of the K^{th} shortest path, which will change the multimodal OD matrices. Thus the optimal coordination results were affected. The equilibrium can be reached by iteratively adjusting multimodal OD matrices by

employing transit path choice, thereafter obtaining optimal coordination results until the total cost of the current iteration equals that of the previous iteration.

It can be shown in Figure 3-1 that besides the keyword ‘multimodal’, other keywords are ‘multi-operator’ and ‘coordination’ in this research topic. Due to multiple operators coexisting in one transit system, they may adopt some policies between each other which may be viewed from market rule. Whereas transit agencies have some peculiarities, especially in a well regulated transit system, these possible operation policies can be categorized as cooperation, independence, and competition. When operators are in cooperation, the coordination operation will be taken just like one operator, e.g. harmonious coordination within bus, within rail, besides between bus and rail. When operators are independent, the coordination operations will be required to taken place only within each operator’s scope. Thus transfer within bus, within rail and between bus and rail by different operator may not benefit from coordination simply within each operator’s scope. This also happens when operators are in competition when coordination is required, in which operators adjust fare during the allowed range to achieve the most benefit.

3.4 Summary

In this chapter, the methodology of this research is analysed comprehensively. Firstly the methodology is proposed, with reasonable assumptions illustrated at the beginning, and then by the flow charts of research framework and optimisation algorithm, the overview of this research and methodology adopted are shown clearly. Secondly systematic analysis of this research objective is carried out in order to lay foundations for model formulations in the next chapter. Lastly modelling a multimodal and multi-operator transit system is presented.

CHAPTER 4 MODEL FORMULATIONS

4.1 Introduction

In this chapter the model formulation for a multimodal and multi-operator transit system are developed, with the objective function of minimizing the total cost of the transit system. The model formulation consists of three parts, i.e. when operators adopt three different strategies, cooperation, competition and independence, in which three operation scenarios, i.e. non-coordination, common headway coordination and integer-ratio headway are analysed respectively. Firstly general formulations of cost components are proposed; secondly transit path choice model is described, in which improved label algorithm is developed; and then model formulations in different strategies are shown respectively; lastly a summary is given at the end of this chapter.

4.2 Overview of the Model Formulations

There are two keywords in this research, one is multimodal and the other is multi-operator. Due to the pivotal function that rail takes in a public transit system, this research will focus on coexisting multi-operators in the urban transit system while bus routes are feeders for the rail transfer station. Thus the model formulations in this chapter will be categorized into three parts. The first part is the cost formulation when only a single operator of the rail in the multimodal transit system is considered, at the same time transit path choice is considered to take account of passengers' responses to the schedule of transit operation. The second part is the model formulation when multi-operators of the rail in the multimodal transit system are considered. Three different strategies, i.e. cooperation, competition and independence, between these two operators are analysed respectively.

4.3 Formulations of Cost Functions

In a multimodal public transit system, due to the pivotal functions of the rail, coordination is normally considered at the rail transfer stations. There are two

aspects of costs in a public transit system, the operator's and the users', in other words, the operator cost and user cost respectively. In this study, the scope of optimisation will consider both the operator cost and the user cost. The former contains that of rail and bus altogether, while user costs include waiting cost, transfer cost and in-vehicle cost. The formulation of both operator cost and user cost will be illustrated respectively in the following sections. Due to the stochastic characteristics in the operation of bus services, the arrival distribution of buses can be considered as a Normal Distribution according to Jenkins (1976), Abkowitz et al. (1987) and Lee and Shonfeld (1991). In contrast, due to the deterministic nature of operations of rail, the arrival of rail is considered to be accurate although there may be slight deviations in actual operations. Knoppers and Muller (1995) proposed concept of synchronization control margin. Other researchers, e.g. Chien (1995), proposed a holding strategy and called the offsets between different modes as slack time. In fact, Knoppers and Muller (1995) proposed a synchronization control margin to allow transfer passengers to meet the connecting vehicle, which, in real world, synchronization control margin is to allow passengers to have enough time to transfer from arrival platforms to leaving vehicle platforms.

Figure 4-1 Cost Structure

From the perspective of operators, their costs should be counted in, whereas from the perspective of users, user cost that is calibrated by the product of travel time and value of time should be considered. To emphasize the transfer between different modes, the transfer cost is taken out from waiting cost as a separate cost items in user cost.

Thus the user cost consists of waiting cost, transfer cost and in-vehicle cost. The cost structure of the transit system is shown in Figure 4-1.

In this section, the formulations of cost components are illustrated respectively.

4.3.1 Operator Cost

For a multimodal transit operator, the operator cost contains both rail costs and bus costs. The formulation of the rail costs and bus costs are illustrated as follows.

i. Operator Cost of Rail

The operator cost of rail consists of two parts, fixed cost (C_{rf}^p) and variable cost (C_{rv}^p). The fixed cost does not depend on the dispatching frequency and fleet size operated during one operation time period, which is formulated as a constant, i.e. C_{rf}^p for rail and C_{bf}^p for bus. The variable cost is the vehicle cost that correlates with the number of vehicles dispatched during one round trip, which is formulated as the product of number of trains per round trip and vehicle cost of rail during unit time (R^p) (to make the unit of time consistent, the unit time is defined as a minute), whereas the number of trains per trip can be obtained by dividing the rail round trip time (τ_r) by the headway of the rail (H_r^p). Thus the operator cost of rail during unit time can be formulated as:

$$C_r^p = C_{rf}^p + \frac{\tau_r}{H_r^p} R^p, \quad (4.1)$$

where C_{rf}^p , R^p and τ_r are all positive.

ii. Operator Cost of Buses

The operator cost of buses can be formulated in the same manner as that of the rail, which can be formulated as the sum of fixed cost (C_{bf}^p) and variable cost (C_{bv}^p). Likewise, the fixed cost (C_{bf}^p) is a constant. The variable cost (C_{bv}^p) is formulated as the product of the number of buses per round trip and vehicle cost of a bus per minute

(B_{ik}^p) of bus routes connecting to the rail transfer stations. Further the number of buses per round trip can be obtained from the round trip time (τ_b) of those buses connecting to rail transfer station i divided by the headway of the bus (H_b^p), hence the bus cost of one round trip time can be formulated as:

$$C_b^p = C_{bf}^p + \frac{\tau_b}{H_b^p} \sum_i \sum_\kappa B_{ik}^p, \quad (4.2)$$

Where B_{ik}^p and τ_b are all positive.

In summary, the cost of operator p (C^p) can be formulated as the sum of the operator cost of rail and bus as follows:

$$C^p = C_{rf}^p + \frac{\tau_r}{H_r^p} R^p + C_{bf}^p + \frac{\tau_b}{H_b^p} \sum_i \sum_\kappa B_{ik}^p, \quad (4.3)$$

4.3.2 User Cost

According to Black (1995), user costs are associated with passengers. Although fares are costs to transit users, the fares are revenue to the transit operators, thus the sum of these two items should technically balance to zero in a transit system. Usually passengers' travel times are the major component of the user cost in most economic analysis. User costs are formulated as the product of passengers' travel times and money valuation of travel times. The user cost includes three components: travel time costs; money valuation of travel time, and other user cost, which includes comfort, convenience, reliability and security, etc. These other user costs are hard to measure, as a result they are omitted in a number of studies. Although the research on money valuation of travel time has been in controversial, no one doubts that saving time is a primary justification of transportation improvements. In this study, the value of travel time (u) is considered as a constant for all users based on the principle that the utility of travel time of different people taking transit system is the same, although different users may belong to different income groups and may reveal their different preferences. To be consistent with the operator cost, the user cost incurred

during one minute is employed. The user cost per minute (C_u) can be defined in three categories based on the time users spend in different scenarios: user waiting cost (C_w), transfer cost (C_t), and in-vehicle cost (C_v).

As the user cost is defined as the total cost incurred by all users during one minute of operations of transit systems, it equals to the product of passenger flow during one minute and value of time. Different stages and the associated costs in a passenger's journey are identified to derive the user cost as follows.

i. Waiting Cost

There are two components of waiting cost; one is for the passengers' waiting cost for the bus (W_b^p), and the other is for the passengers' waiting cost for the rail (W_r^p).

(a) Waiting Cost for Rail

The waiting cost for rail (W_r^p) is for the passengers' waiting for arrival trains at train stations platform upon their arrival at the platform. Assuming that the rail services is nearly deterministic, thus average waiting time per passenger is equal to half of headway of rail (H_r^p). The waiting cost which is the total cost passengers spent waiting for arrival of trains can be formulated as the product of the average waiting time per passenger, passengers arrived during one minute (λ_{ij}^{nrp}) and value of time (u):

$$W_r^p = (t_{rw} + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2} H_r^p) \lambda_{ij}^{nrp} u, \quad (4.4)$$

Where $i \neq j$; H_r^p , λ_{ij}^{nrp} , u are all positive, n is positive integer number.

(b) Waiting cost for Bus

The waiting cost for bus (W_b^p) is obtained by assuming that passengers would randomly arrive at the bus stop. The waiting time for passengers at bus stop i (γ_i) can be formulated according to Welding (1957) (cited by Knoppers and Muller, 1995), assuming that the variance of the bus connecting to feeder transfer station i is σ_i^2 :

$$\gamma_i = \frac{1}{2} H_b [1 + (\frac{\sigma_i}{H_b})^2], \quad (4.5)$$

where H_b , σ_i are all nonnegative.

Thus the waiting cost for bus in a coordinated bus and rail scenario is the total waiting cost for buses in one headway operation. It can be derived as the product of waiting time during one headway session, number of passengers that wait for buses during one minute and value of time as follows:

$$W_b^p = (t_{bw} + \sum_{i=1}^n \gamma_i^p \sum_{j=1}^n \lambda_{ij}^{nbp})u = \sum_{i=1}^n \sum_{j=1}^n u[t_{bw} + (\frac{1}{2}H_b^p + \frac{\sigma_i^2}{2H_b^p})\lambda_{ij}^{nbp}], \quad (4.6)$$

where $H_b^p, u, \lambda_{ij}^{nbp}$ are all nonnegative, n is positive integer number.

Therefore the total waiting cost is the sum of those for train and buses.

$$W^p = (t_{rw} + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2}H_r^p)\lambda_{ij}^{nrp}u + \sum_{i=1}^n \sum_{j=1}^n u[t_{bw} + (\frac{1}{2}H_b^p + \frac{\sigma_i^2}{2H_b^p})\lambda_{ij}^{nbp}], \quad (4.7)$$

ii. Transfer Cost

Transfer cost (F^p) is generated due to passengers' waiting for transferring between routes or lines. In order to emphasize the effect of the coordination, the transfer cost is separated from the waiting cost. Thus transfer cost can be formulated as the sum of bus to rail transfer cost (F^{brp}), bus to bus transfer cost (F^{bbp}), rail to bus transfer cost (F^{rbp}) and rail to rail transfer cost (F^{rrp}).

If transit system is not coordinated, the transfer cost from bus to rail and rail to rail can be calibrated in the same manner as Eq.(4.4), transfer cost from bus to bus and rail to bus can be calibrated in the same manner as Eq.(4.6). Thus transfer cost in a non-coordinated scenario can be formulated as follows:

$$F^p = \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2}H_r^p (\lambda_{ij}^{brp} + \lambda_{ij}^{rrp})u + \sum_{i=1}^n \sum_{j=1}^n (\frac{1}{2}H_b^p + \frac{\sigma_i^2}{2H_b^p})(\lambda_{ij}^{bbp} + \lambda_{ij}^{rbp})u, \quad (4.8)$$

For a coordinated transit system, transfer time cannot simply be estimated as the average waiting time. There are two methods of coordination. One is common headway coordination, and the other is integer-ratio headway coordination. In a common headway coordination, bus and rail are synchronized by setting the same headways and offset between them, allowing sufficient time for passengers to transfer. Therefore transfer cost from bus to rail can be calculated as the missed

connection cost as illustrated in Figure 4-2. The right hand side red shadowed part, which is the product of passengers transferring from bus to rail during one minute, value of time and missed connection time.

$$F^{brp} = \sum_i^n \sum_j^n \lambda_{ij}^{brp} u H_r^p \int_{K_d}^{H_r^p} f(t) dt, \quad (4.9)$$

where $f(t)$ is the probability distribution function of the bus arrival time.

The transfer cost from rail to bus can be calibrated as missed connection cost from rail to bus as illustrated in Figure 4-2 which shows the left hand side black shadowed part. It is the total transfer cost of all passengers, which is equal to the product of passengers transferring from rail to bus during one minute, value of time and missed connection cost.

$$F^{rbp} = \sum_i^n \sum_j^n \lambda_{ij}^{rbp} u H_b^p \int_{-H_r^p}^{K_d} f(t) dt, \quad (4.10)$$

As coordination is not considered between bus routes, transfer cost from bus to bus can be evaluated as the waiting cost for bus, which is formulated in the same manner as Eq(4.6).

$$F^{bbp} = \sum_{i=1}^n \sum_{j=1}^n \lambda_{ij}^{bbp} u \left(\frac{1}{2} H_b^p + \frac{\sigma_i^2}{2 H_b^p} \right), \quad (4.11)$$

As transfer time from rail to rail can be neglected in the common headway coordination, the transfer cost from rail to rail can be viewed as zero in this situation.

Figure 4-2 Transfers from Bus to Rail at Station i in the Common Headway Coordination

Thus in the common headway coordination scenario, the transfer cost can be formulated as the summation above as:

$$F^p = \sum_{i=1}^n \sum_{j=1}^n u[\lambda_{ij}^{brp} H_r^p \int_{K_d}^{H_r^p} f(t) dt + \lambda_{ij}^{rbp} H_b^p \int_{-H_r^p}^{K_d} f(t) dt + \lambda_{ij}^{bbp} (\frac{1}{2} H_b^p + \frac{\sigma_i^2}{2 H_b^p})], \quad (4.12)$$

Then in integer-ratio headway coordination scenario, transfer cost from bus to rail can be calibrated as missed connection cost from bus to rail and waiting cost during other time sessions that are not coordinated as shown in Figure 4-3.

$$F^{brp} = \frac{1}{\phi^p} \sum_{i=1}^n \sum_{j=1}^n \lambda_{ij}^{brp} u[H_r^p \int_{K_d}^{H_r^p} f(t) dt + \frac{\phi^p - 1}{2} H_r^p], \quad (4.13)$$

Figure 4-3 Transfers from Bus to Rail at Station i in the Integer-ratio Headway Coordination

The transfer cost from rail to bus can be treated as missed connection cost from rail to bus and the waiting cost during other time periods that are not coordinated as shown in Figure 4-3.

$$F^{rbp} = \sum_i^n \sum_j^n \lambda_{ij}^{rbp} u \left[H_r^p \int_{-H_r^p}^{K_d} f(t) dt + \frac{\phi^p - 1}{2} H_r^p \right], \quad (4.14)$$

The transfer cost from bus to bus can be formulated as shown in Eq. (4.10). Thus the total transfer cost is the sum of Eq. (4.10), Eq. (4.13) and Eq. (4.14).

$$F^p = \sum_{i=1}^n \sum_{j=1}^n u \left[\frac{1}{\phi^p} \lambda_{ij}^{brp} H_r^p \left[\int_{K_d}^{H_r^p} f(t) dt + \frac{\phi^p - 1}{2} \right] + \lambda_{ij}^{rbp} H_r^p \left[\int_{-H_r^p}^{K_d} f(t) dt + \frac{\phi^p - 1}{2} \right] + \lambda_{ij}^{bbp} \left(\frac{1}{2} H_b^p + \frac{\sigma_i^2}{2 H_b^p} \right) \right], \quad (4.15)$$

iii. In-vehicle Cost

Those passengers who would transfer from bus to rail incur not only in-vehicle cost for rail, but in-vehicle cost for bus as well. Thus in-vehicle cost consists of two parts; one being the in-vehicle cost for rail and the other for bus. The in-vehicle cost, which is the total in-vehicle cost of all passengers, is formulated as the product of in-vehicle travel times, value of time and passenger volumes, and the formulations of each part are illustrated as follows. As value of time during in-vehicle is not the same as that of waiting cost and transfer cost, the value of riding is generally considered to be $k_v u$, where $0 < k_v < 1$.

In the integrated public transit system, the constraint of capacity of bus and rail should be considered to avoid the situation that too many passengers boarding vehicle during very long headway period. Therefore the capacity constraints are considered for each origin station i , when all the passengers boarding are less than the capacity of bus or rail. The in-vehicle cost consists of in-vehicle cost of bus and rail in dual directions. To make it clear, d which is a binary number representing the opposite directions is added in the formulation. Therefore in-vehicle cost of rail or bus equals all the passengers by rail or bus multiply by average riding time and value of in-vehicle time, which can be formulated as follows:

$$V^p = \sum_i^n \sum_j^n \sum_d k_v u [(\lambda_{ij}^{brp} + \lambda_{ij}^{rrp} + \lambda_{ij}^{nrp})[(1-d) \sum_{i=i}^{j-1} \frac{l_{i(i+1)}^r}{V_r^p} + d \sum_{i=i}^{j+1} \frac{l_{i(i-1)}^r}{V_r^p}] + (\lambda_{ij}^{bbp} + \lambda_{ij}^{rbp} + \lambda_{ij}^{nbp})[(1-d) \sum_{i=i}^{j-1} \frac{l_{i(i+1)}^b}{V_b^p} + d \sum_{i=i}^{j+1} \frac{l_{i(i-1)}^b}{V_b^p}]], \quad (4.16)$$

$$\text{s.t. } H_r^p \sum_{j=i+1}^{j>i} \lambda_{ij}^{rp} \leq N_r^p,$$

$$H_r^p \sum_{j=i-1}^{j<i} \lambda_{ij}^{rp} \leq N_r^p,$$

$$H_b^p \sum_{j=i+1}^{j>i} \lambda_{ij}^{bp} \leq N_b,$$

$$H_b^p \sum_{j=i+1}^{j>i} \lambda_{ij}^{bp} \leq N_b.$$

Where $i \neq j$, l_{ij}^m , H_r^p , k_v and u are all positive; λ_{ij}^{rp} , λ_{ij}^{brp} , λ_{ij}^{rrp} and λ_{ij}^{nrrp} are all nonnegative; i , j are all positive integer numbers; d is a binary number.

Hence the user cost can be formulated as:

$$U^p = W^p + F^p + V^p, \quad (4.17)$$

Thus the total cost can be derived as:

$$C = \sum_p (C^p + U^p), \quad (4.18)$$

In order to make user cost and operator cost comparable, the weight for user cost which is a positive integer number is added in the objective function. Therefore the objective function can be formulated as:

$$P: \text{minimize } f(C) = \text{minimize } \sum_p (C^p + w * U^p), \quad (4.19)$$

$$\text{s.t. } H_r^p \sum_{j=i+1}^{j>i} \lambda_{ij}^{rp} \leq N_r^p,$$

$$H_r^p \sum_{j=i-1}^{j<i} \lambda_{ij}^{rp} \leq N_r^p,$$

$$H_b^p \sum_{j=i+1}^{j>i} \lambda_{ij}^{bp} \leq N_b,$$

$$H_b^p \sum_{j=i+1}^{j>i} \lambda_{ij}^{bp} \leq N_b.$$

where $i \neq j$, R^p , B_{ik}^p , τ_b , τ_r , u , l_{ij}^m , K_d , ϕ^p , σ_i , N_b^p , N_r^p , H_r^p , H_b^p and V_m^p and all positive; λ_{ij}^{bbp} , λ_{ij}^{rbp} , λ_{ij}^{nbp} , λ_{ij}^{brp} , λ_{ij}^{rrp} and λ_{ij}^{nrrp} are all nonnegative; i , j and w are all positive integer numbers; d is a binary number.

It can be found that objective functions for non-coordination and common headway coordination scenarios are in the form of nonlinear programming problems, which can be solved by considering the first order and second order conditions. Evaluating the optimum value of the total cost will arrive at a local minimum value of the total cost, when the optimum values of the offset and headways of bus and rail services are derived. On the other hand, the objective function for integer-ratio headway coordination is identified as a mixed integer nonlinear programming problem. Making use of the built-in module AOA (Aimms Outer Approximation) in AIMMS 3.6, this MINLP (Mixed Integer Nonlinear Programming) problem can be solved by formulating this model into AIMMS project files.

4.4 Transit Path Choice Model

As only the input OD matrices of those who transfer from bus to terminal stations and those who come to transfer stations by modes other than bus are assumed to be known, the detailed transfer information, such as different combination of nodes, modes and operators, i.e. paths that users take should be sorted out, in order to assign the demand onto the multimodal and multi-operator transit network.

In this study, following the decision making process as shown in Figure 2-1 and adopting to transit system, the path choice model consists of two basic parts with different functions: (i) path search model; and (ii) path decision model (Xie, 2003). The detailed transfer information illustrated in Figure 3-2 can be identified.

4.4.1 Path Search Model

This algorithm solves for the first three shortest paths in a multimodal and multi-operator transit system. For example, in a transit network as shown in Figures 3-4 and 3-5, there are arcs between terminal stations if they exist, which are extended by different modes and operators. Each combination of mode and operator is denoted by an arc between adjacent terminal stations.

A simple network is proposed for illustration purposes in Figure 4-4. There are six stations, each of them is connected by arcs operated by different combinations of operator p and mode m .

Let $G = (O, D, M, P)$ be a directed graph representing a multi-operator and multimodal transit network. The travel time V_{ij}^{nq} is the arc travel times on arc (i, j) on mode n that operated by operator q , where mode $n = \{\text{bus, rail, walking}\}$, operator $q = \{\text{op1, op2, Null}\}$. $W_{i_1}^{mnpq}$ is the transfer time at node i_1 , when entering node i_1 from a path whose mode is m by operator p and switching to a path whose mode is n by operator q . The dwell time at the stop(s) for a vehicle can be captured implicitly by assuming any appropriate average speeds. Obviously, $W_{i_1}^{mnpq} = 0$ if $m = n$ and $p = q$, which means that there is no change of mode and operator at node i_1 . In order to discriminate those passengers transferring from bus with those passengers transferring from modes other than bus, three shortest paths must be searched under different assumptions from origin stations separately. Therefore it is necessary to set different initialization conditions according to different transfer categories of passenger flows. The definitions of transfer time ($W_{i_1}^{mnpq}$) for those who transfer from modes other than bus in different scenarios are shown from Tables A-1 to A-5; Table A-1 shows the transfer time in the non-coordinated scenario. Table A-2 shows the transfer time in the common headway coordination when operators are in cooperation. Table A-3 shows the transfer time in the integer-ratio headway coordination when operators are in cooperation. Table A-4 shows the transfer time in the common headway coordination when operators are independent and in competition. Table A-5 shows the transfer time in integer-ratio headway coordination when operators are in independent and in competition. The definitions of transfer time ($W_{i_1}^{mnpq}$) for those who transfer from bus in different scenarios can be formulated in the same manner, which are shown in Tables A-6 to A-10 respectively. All these definitions for transfer time are written in AIMMS codes.

Figure 4-4 Design of the K shortest path algorithm in a multimodal and multi-operator transit system

This algorithm starts at the origin station and in a label correcting fashion iteratively solves the optimality equation by scanning all the passing nodes from origin node to destination node. After finding the shortest path, each section arc on the shortest path is deleted at each iteration when searching the second shortest path by setting the travel time V_{ij}^{nq} to infinity. Likewise the third shortest path can be derived in the same manner. The design of the shortest path algorithm in a multimodal and multi-operator transit system is illustrated in Appendix B.

As this improved algorithm is based on integrating previously algorithms that have been proofed, the procedure of proof can be omitted. A simple example of this improved algorithm is demonstrated in the following to give a better view of the mechanics of this algorithm.

Example of Searching the shortest path in a Multimodal and Multi-operator Transit Network:

Based on the network of Figure 4-5, there are three modes: bus, rail and walking which are denoted by B, R, W. There are two operators who operate two modes of public transit, i.e. bus and rail. The two operators are denoted by 1 and 2 in the operator set. As walking is a human behaviour, the corresponding operator can be set as null. Therefore the operator of walking is denoted by N. To be more specific, the mode set $M = \{B, R, W\}$, operator set $P = \{1, 2, N\}$. The problem is to search the shortest path from station 1 to station 4 for those who arrive at station 1 by modes other than bus. Therefore the entering mode is 'walking', operator is

‘NULL’. The transit system is assumed to be non-coordinated, thus the transfer time can be derived as half of headway of bus or rail. Travel time between each station by each mode and each operator can be assumed and illustrated in Tables 4-1 to 4-5 (zero travel time means there is no eligible path between stations). The headways of bus and rail are assumed to be 9 minutes and 3 minutes respectively for operator 1; while the headways of bus and rail are assumed to be 10 minutes and 4 minutes respectively for operator 2.

Figure 4-5 A Simplified Multimodal and Multi-operator Network

Table 4-1 Travel Time between Each Station by bus for operator 1

	Station 1	Station 2	Station 3	Station 4
Station 1	0	9.92	7.36	33.6
Station 2	9.92	0	0	6.08
Station 3	7.36	0	0	7.36
Station 4	33.6	6.08	7.36	0

Table 4-2 Travel Time between Each Station by bus for operator 2

	Station 1	Station 2	Station 3	Station 4
Station 1	0	10.54	7.82	35.7
Station 2	10.54	0	0	6.46
Station 3	7.82	0	0	7.82
Station 4	35.7	6.46	7.82	0

Table 4-3 Travel Time between Each Station by Rail for operator 1

	Station 1	Station 2	Station 3	Station 4
Station 1	0	4.03	2.99	13.65
Station 2	4.03	0	0	2.47
Station 3	2.99	0	0	2.99
Station 4	13.65	2.47	2.99	0

Table 4-4 Travel Time between Each Station by Rail for operator 2

	Station 1	Station 2	Station 3	Station 4
Station 1	0	3.41	2.53	11.55
Station 2	3.41	0	0	2.09
Station 3	2.53	0	0	2.53
Station 4	11.55	2.09	2.53	0

Table 4-5 Travel Time between Each Station by Walking

	Station 1	Station 2	Station 3	Station 4
Station 1	0	15.5	11.5	52.5
Station 2	15.5	0	0	9.5
Station 3	11.5	0	0	11.5
Station 4	52.5	9.5	11.5	0

The steps in searching the shortest path are elaborated as follows:

Step 1: Initialize some variables, set $Y_{i_1 1}^{mp} = +\infty$, $Y_{i_1 i}^{mp} = 0$ for $\forall i_1 \in N, i_1 \neq 1, \forall m \in M$, $\forall p \in P$. The entering mode and operator are ‘walking’ and ‘NULL’ respectively.

Set Rednodes = {}, Bluenodes = {}; origin node at station 1. Insert station1 into Rednodes, and Bluenodes = set of all stations – Rednodes, thus Rednodes = {1}, Bluenodes = {2, 3, 4};

Step 2: the shortest path from station 1 to each station j_1 in Bluenodes, $Y_{1 j_1}$ is updated in each iteration and shown in Table 4-6. It can be seen that in each iteration, Rednodes are updated and displayed in red, while elements in Bluenodes are updated and displayed in blue; each time the minimum travel time from origin station

to the station Υ is obtained, the minimum travel time from station 1 to other stations along with mode, operator and predecessor station is updated as well.

Table 4-6 the Procedure in Searching Shortest Path

Iterations	Rednodes	Υ	Y_{111}	Y_{121}	Y_{131}	Y_{141}	Mode	Operator	Pre Node
1	1	1	0	5.41	4.53	13.55	R	2	1
2	1,3	3	0	5.41	4.53	7.06	R	2	1
3	1,3,2	2	0	5.41	4.53	7.06	R	2	3

Step 3: The shortest path from stations 1 to 4 can be traced back. It starts from stations 1 to 3 by rail operated by operator 2, and then from stations 3 to 4 still by rail operated by operator 2.

The calculations from steps 2 to 5 in the algorithm can be repeated in a similar manner and therefore they have been omitted in this example, as only the mechanics of this algorithm are demonstrated.

4.4.2 Path Decision Model

The path choice behaviour of a decision maker can best be described in terms of discrete choices and variables. If a decision maker is observed to choose from a discrete and finite set of alternatives, one can specify a choice function for each one of the alternatives. Utility maximization, as one of decision rules, is widely used in the development of predictive models of human behaviour (Ben-Akiva and Lerman, 1985). Discrete choice models, based on the random utility theory, postulate that the probability of individuals choosing a given option is a function of their socio-economic characteristics and the relative attractiveness of option (Domenrich and McFadden, 1975).

Path-Size Logit Model (PSL) is applicable in transit path choice problem; the formula is illustrated as follows according to Ben-Akiva and Bierlaire (1999).

$$P(i | C_n) = \frac{e^{\mu(V_{in} + \ln PS_{in})}}{\sum_{j \in C_n} e^{\mu(V_{jn} + \ln PS_{jn})}}, \quad (4.20)$$

Where C_n is the choice set, V_{in} represents the systematic components of a PSL model, PS_{in} is termed as the size of path i for person n .

The definition by Ben-Akiva and Bierlaire (1999) for the Path-Size is shown in Eq. (4.21) as follows.

$$PS_{in} = \sum_{a \in \Gamma_i} \left(\frac{l_a}{L_i} \right) \frac{1}{\sum_{j \in C_n} \frac{L_{C_n}}{L_j} \delta_{aj}}, \quad (4.21)$$

Where a is indexed links (arc), Γ_i is the set of links in path i , l_a is the length of link a , L_{C_n} is the length of the shortest path in C_n , L_i is the length of path i , δ_{ai} is binary, and equals to 1 if path j uses link a , and equals 0 otherwise.

As it was conclude by Xie (2003) that in a complex transit network, riders would prefer to the first three shortest (journeys) time paths. Let V_i be the systematic utility of the i^{th} path ($i=1, 2, 3$). The parameters in V_i are adopted from the models that presented by Xie (2003), and the models are shown in Eq.(4.22) and Eq. (4.23) respectively.

$$V_i = \beta_{0,i} + \beta_1 walkdis_i + \beta_2 Ttrans_i + \beta_3 Tstnps_i + \beta_4 M_i, \quad (4.22)$$

where $Walkdis_i$ is the total walking distance (km) by path i ,

$Ttrans_i$ is the total number of transfer (km) by path i ,

$Tstnps_i$ is the total number of MRT stations plus bus stop taken by path i ,

M_i is a variable about proportion of trip by MRT during trip,

$\beta_1, \beta_2, \beta_3, \beta_4$ are model parameters to be estimated.

$\beta_{0,i}$ is the path-specific constant (to path i , and $i=1,2$), and $\beta_{03}=0$

The system utility function in model 2 is shown as follows:

$$\begin{aligned} V_1 &= \beta_{01} + \beta_1 walkdis_1 + \beta_2 Ttrans_1 + \beta_3 Tstnps_1 + \beta_4 M_1 + \beta_6 X_1 + \beta_7 Y \\ V_2 &= \beta_{02} + \beta_1 walkdis_2 + \beta_2 Ttrans_2 + \beta_3 Tstnps_2 + \beta_4 M_2 + \beta_6 X_2 + \beta_8 Y \\ V_3 &= \beta_1 walkdis_3 + \beta_2 Ttrans_3 + \beta_3 Tstnps_1 + \beta_4 M_3 + \beta_6 X_3 \end{aligned} \quad (4.23)$$

where M_i is a variable about proportion of trip by MRT during trip,

X_i is fare indicator, $X_i = 1$ if total fare is more than (or equal to) \$2,

Y_i is $Ag * walkdis_i$, where Ag is age indicator, and $Ag = 1$ when age is below 15 or above 55; $Ag = 0$ if otherwise. As the systematic utility function is used to calculate for the average passengers, thus this variable can be omitted.

$\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7$ are unknown model parameters to be estimated,

$\beta_{0,i}$ is path-specific constant (to path i and $i = 1, 2$), and $\beta_{0,3} = 0$

The probabilities for each passenger selecting the first three shortest paths ($P(i)$) can be derived according to Eq.(4.20), the passenger volumes on each path can be obtained by multiplying $P(i)$ with the total passenger volumes from origin node to the destination node.

4.5 Model Formulations in Different Operator Strategies

In an integrated multi-operator and multimodal public transit system, coordination is, above all, the first step to be considered. To make the public transit system seamless for the passengers, proper strategies for transfer coordination should be adopted. The first is to find the proper offsets between bus and rail and the second is to set the optimum headways for each transit mode. The system optimisation is reached when the total cost of the users and the operators achieve the minimum value. Previous studies have not touched on the study of multi-operator and multimodal public transit system. Even if multiple operators were considered, the analyses were not thorough enough and resulted in different perspectives. In this study it is considered that there are two operators in the urban rail transit system, thus there will be three scenarios to be anticipated. The first is when the two operators cooperate with each other. The second scenario is when the two operators are in independent mode, they adopt strategies on their own without considering other operator's policy changes. The last is when the two operators compete with each other, in which case the whole public transit system will reach the Nash equilibrium only when it reaches an equilibrium state that the total costs of the two operators are the minimum while profits are the maximum and will remain for sometime before any of the operators adopt further

strategies. The model formulations in different operator strategies are illustrated as follows.

4.5.1 Cooperation

When the two operators cooperate with each other, transfers between different modes by different operators are the same as those in a monopoly operator in a transit system. In order to coordinate between different operators, rails are synchronized with the same headways so that to minimize the rail to rail transfer cost. The optimisation procedure is illustrated as follows.

- (i). OD matrices are assumed to be evenly assigned by all modes for all operators (i.e. OD matrices for bus by operator 1, for rail by operator 1, OD matrices for bus by operator 2, for rail by operator 2), together with other assumed parameters, the preliminary optimal headway and offset can be derived by Eq.(4.19) at the first step.
- (ii). Then the path choices for passengers in the transit system, the K^{th} ($k=1,2,3$) shortest path, is sorted out first. The system utility function for each path is then calculated by employing the parameters that was estimated by Xie (2003). The system utility function can be calculated, and path size can be derived accordingly. Thus trip assignment is carried out by multiplying the OD matrices with the probabilities of selecting the first three shortest paths. Passengers on the first three shortest paths by different modes and operators can be obtained by summing up all the passengers by the same mode and same operator on route. As waiting times for different groups of passenger flows differ, the three shortest paths are calibrated for those passengers arrived by transfer stations transferring from buses and those passengers by modes other than bus respectively.
- (iii). Inputting the new OD matrices by different mode and operator obtained in step (ii) in the objective function, the optimal headway and offset are derived again. The equilibrium is reached when the optimal total cost converges with that in the previous iteration.

Three different scenarios, i.e. non-coordination, common headway coordination and integer-ratio headway coordination are assessed following the same procedure above to make comparison between each scenario. To be noted, there are differences between different scenarios in path choice process besides formulating costs items, e.g. transfer time between different modes for non-coordinated and coordinated scenario, and it is assumed to be zero if coordinated or nonzero otherwise in this research.

4.5.2 Independence

When the two operators are independent, they do not coordinate between each other, but they coordinate within their own modes to attract more passenger flows. Thus the formulation for transfer cost is different from Eq.(4.12) for common headway coordination and Eq. (4.15) in integer-ratio headway coordination, while retaining the formulation of the other cost items in Eq.(4.3) to Eq.(4.19). In order to distinguish different transfer origin passenger flows, it is necessary to identify transfer passenger flows originating from the same operator or from other operator. Thus λ_{ij}^{brp} can be separated as λI_{ij}^{brp} and λO_{ij}^{brp} , λ_{ij}^{rrp} can be separated as λI_{ij}^{rrp} and λO_{ij}^{rrp} , λ_{ij}^{rbp} can be separated as λI_{ij}^{rbp} and λO_{ij}^{rbp} respectively. For those transfers occurring within the same operators, the formulation can be the same as Eq. (4.12) in common headway coordination and as Eq. (4.15) in integer-ratio headway coordination.

Therefore the transfer cost for operator p in the common headway coordination consists of two parts. The first is the transfer cost that is coordinated within the same operator according to Eq. (4.12). The second is the transfer cost that is not coordinated due to independence between different operators, thus the transfer cost can be formulated as the waiting cost for bus and rail. Therefore the formulation can be shown in Eq. (4.24) as follows.

$$F^p = \sum_{i=1}^n \sum_{j=1}^n u [\lambda I_{ij}^{brp} H_r^p \int_{K_d}^{H_r^p} f(t) dt + \lambda I_{ij}^{rbp} H_b^p \int_{-H_r^p}^{K_d} f(t) dt + \lambda_{ij}^{bbp} (\frac{1}{2} H_b^p + \frac{\sigma_i^2}{2 H_b^p})]$$

$$+ \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n u[(\lambda O_{ij}^{brp} + \lambda O_{ij}^{rrp})H_r^p + \lambda O_{ij}^{rbp}(H_b^p + \frac{\sigma_i^2}{H_b^p})], \quad (4.24)$$

Likewise transfer cost for operator p in integer-ratio headway coordination can be formulated in Eq. (4.25) in the same manner as follows.

$$F^p = \sum_{i=1}^n \sum_{j=1}^n [\lambda_{ij}^{brp} u H_r^p [\int_{K_d}^{H_r^p} f(t) dt + \frac{\phi^p - 1}{2}] + \lambda_{ij}^{rbp} u H_r^p [\int_{-H_r^p}^{K_d} f(t) dt + \frac{\phi^p - 1}{2}] + \lambda_{ij}^{bbp} u (\frac{1}{2} H_b^p + \frac{\sigma_i^2}{2 H_b^p})] \\ + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n u[(\lambda O_{ij}^{brp} + \lambda O_{ij}^{rrp})H_r^p + \lambda O_{ij}^{rbp}(H_b^p + \frac{\sigma_i^2}{H_b^p})], \quad (4.25)$$

Thus the objective function can be derived according to Eq. (4.19). Following the same optimisation procedure as that when operators are in cooperation, different scenarios, i.e. non-coordination, common headway coordination and integer-ratio headway coordination are analysed as well. In this section, as different operators are independent, there is no coordination between different operators; the coordination is within each operator's scope between different modes.

4.5.3 Competition

When the two operators are in competition, it may lead to certain degree of fare adjustment by operators. As the whole transit system is normally regulated by transit administration, fare adjustment can only fluctuate within a narrow range. Normally fare is distance based. For example, the fare adjustment with effect from July 1, 2005 in Singapore transit system released by Public Transport Council on May 30, 2005 (published on official website of Public Transport Council). The adjustment to bus and train fares under the new fare review framework revealed the year based new formula as follows:

$$\text{Maximum Fare Adjustment} = 0.5\text{CPI} + 0.5\text{WI} - 0.3\%, \quad (4.26)$$

where CPI refers to the change in Consumer Price Index over the preceding year, and WI refers to the change in Average Monthly Earnings (Annual National Average)

over the preceding year, adjusted to account for any change in the employer's CPF¹ contribution rate. The 0.3% is the productivity extraction to be used for the next 3 years. For this year, CPI and WI is 1.7% and 3.6% respectively.

The approved fare adjustment for adult train fares charged by EZ-link² system cited from public transport council is listed in Table 4-7 as follows.

Table 4-7 Adult Train Fares

EZ-Link Fares	1 cent increase for distances up to 3.2km; 2 cents increase for distances between 3.2km and 14.4km; and 3 cents increase for distances greater than 14.4km. The concession fare for senior citizens during off peak hours, which is currently pegged to the minimum adult fare, will increase by 1 cent correspondingly.

Thus a simple distance based fare structure is assumed in the same way as follows. Let $\beta_r = (\beta_{r1}, \dots, \beta_{rw}, \dots, \beta_{rs})$ denote the fares of s fare strategies by rail engaged in a fare competition. $\beta_b = (\beta_{b1}, \dots, \beta_{bw}, \dots, \beta_{bs})$ denoted the fares of s fare strategies by bus engaged in a fare competition. The fare from station i to station j (f_{ij}^m) can be formulated similar to fare structure by transit companies in Singapore.

$$f_{ij}^m = \alpha_m + \beta_m(l_{ij}), \quad (4.27)$$

where α_m is a constant for mode m , β_m is the variable based on the distance between station i and station j .

When two operators are in competition, both of them would seek the maximizing their individual revenues of which the sum is denoted by $\pi(\beta_m)$, a Nash Equilibrium is

¹ Central Provident Fund (CPF) is a comprehensive social security savings plan which provides many working Singaporeans with a sense of security and confidence in their old age

² Ez-link card is a thin, compact card that operates from electromagnetic energy received from the reader which forms the Ez-link system for Singapore public transport fare system.

achieved when the fare strategies for operator p_1 for rail and bus were denoted by β'_{rw} and β'_{bw} respectively, and the fare strategies for operator p_2 for rail and bus were denoted by β''_{rw} and β''_{bw} respectively, such that:

$$\pi(\beta'_{rw}, \beta'_{bw}, \beta''_{rw}, \beta''_{bw}) \geq \pi(\beta_{rw}, \beta_{bw}, \beta_{rw}, \beta_{bw}), \quad (4.28)$$

where $\beta_{rw}, \beta_{bw}, \beta_{rw}, \beta_{bw}$ cannot be the equal to $\beta'_{rw}, \beta'_{bw}, \beta''_{rw}, \beta''_{bw}$ respectively at the same time.

Furthermore the revenue for each operator is formulated as follows:

$$\pi_{p_1} = \sum_i \sum_j \sum_m (\alpha + \beta_m(l_{ij})) \lambda_{ij}^{mp_1} - \frac{\tau_r}{H_r^{p_1}} R^{p_1} - \frac{\tau_b}{H_b^{p_1}} \sum_i \sum_\kappa B_{ik}^{p_1}, \quad (4.29)$$

$$\pi_{p_2} = \sum_i \sum_j \sum_m (\alpha + \beta_m(l_{ij})) \lambda_{ij}^{mp_2} - \frac{\tau_r}{H_r^{p_2}} R^{p_2} - \frac{\tau_b}{H_b^{p_2}} \sum_i \sum_\kappa B_{ik}^{p_2}, \quad (4.30)$$

Thus the revenue for the two operators can be derived as:

$$\pi(\beta_{rw}, \beta_{bw}, \beta_{rw}, \beta_{bw}) = \pi_{p_1} + \pi_{p_2}, \quad (4.31)$$

When a Nash Equilibrium is achieved, the maximum revenue is derived as denoted by $\pi(\beta'_{rw}, \beta'_{bw}, \beta''_{rw}, \beta''_{bw})$. Maximizing the negative of the revenue gives it the minimum value. What is more, it can be treated as the cost for operators. Therefore the operator cost C^p when operators are in competition can be formulated as:

$$C^p = \frac{\tau_r}{H_r^p} R^p + \frac{\tau_b}{H_b^p} \sum_i \sum_\kappa B_{ik}^p - \sum_i \sum_j \sum_m (\alpha + \beta_m(l_{ij})) \lambda_{ij}^{mp}, \quad (4.32)$$

The objective function for this coordination problem can be formulated according to Eq. (4.19).

Likewise, the optimisation procedure follows that when the operators are in cooperation. The parameters in V_i as shown in Eq.(4.23) adopt those reported in Xie (2003).

Waiting times between different modes by different operators are assumed to be one half of headway of the pick-up vehicles in this case; while those between different modes within the same operators, e.g. waiting times from bus to rail and rail to bus are zero in a common headway coordination and in integer-ratio headway coordination. Waiting times from rail to bus are zero in common headway coordination and $\frac{\phi^p - 1}{2} H_r^p$ in integer-ratio headway coordination.

4.6 Summary

In this chapter, model formulations are carried out extensively according to the methodology that has been proposed. Firstly formulations of cost components for different scenarios are developed; and then transit path choice models consisting of path search model and path decision model are addressed to get rectified OD matrices as feedback to form the whole optimisation procedure. Lastly the formulations in different strategies are presented as well.

CHAPTER 5 Coordination under Different Strategies between Operators

5.1 Introduction

In this chapter, an evaluation optimisation case study comparable to Singapore Transit is carried out and the results are compared between different strategies. The objective is to find out which coordination method under which operation policy is the most advocable. The first implementation of AIMMS programmes is presented, and initial input parameters which are comparable to real data in Singapore are employed in this evaluation study. In order to compare the optimal results in different scenarios under different operation policies, important cost items and decision variables are illustrated to evaluate different coordination states under different operation policies. Finally a short summary on the evaluation study is described at the end of this chapter.

5.2 Implementation of Optimisation Model in AIMMS

Due to the complexity of this problem, an optimisation software is employed to solve the objective function. By coding in an improved label correcting algorithm, a feedback optimisation procedure is implemented to derive the optimal decision variables and important cost items. In this section, firstly the framework of AIMMS programmes is illustrated; secondly the design of optimisation model in AIMMS in different scenarios is described; lastly the structures of AIMMS model are shown.

5.2.1 Framework of AIMMS Programmes

In AIMMS 3.6, the GUI (Graphic User Interface) can be built by editing the page tree in the development environment. Adding scalars and tables attributed by decision variables (i.e. headways of bus and rail, offsets in different directions for different operators) and cost items, the optimal results of these variables and cost items can be shown in the scalars and tables. In order to coordinate between feeder buses and rail better, different transfer originated passenger flows are identified and calibrated. To

solve the objective function, five sections consisting of some procedures are developed in AIMMS 3.6. These five sections are preliminary optimal headway section, path selection section which consists of procedures of finding three shortest paths for those by feeder bus and for those by modes other than bus respectively, path decision section which consists of procedures of evaluating the probability of each three shortest paths for those by feeder bus and those by modes other than bus respectively, and the optimisation algorithm coded in AIMMS 3.6 can be shown in Figure 5-1.

5.2.2 Design of Optimisation Model in AIMMS in Different Scenarios

The policies between different operators in a multimodal and multi-operator transit system are categorized into three different strategies, i.e. cooperation, competition and independence. Each is analyzed in three scenarios, i.e. non-coordination, common headway coordination and integer-ratio headway coordination. Each scenario for each operator strategy is coded in separate AIMMS models. The objective functions in non-coordination and common headway coordination scenarios are nonlinear programming problems, whereas the objective function in integer-ratio headway coordination is a mixed integer nonlinear programming problem due to the integer property of the ratio between headway of bus and headway of rail.

The objective function in scenarios when operators are in competition is different from other scenarios when operators are in cooperation and independence. Besides operator cost and user cost, the revenue of each operator must be taken into consideration and added in the objective function under the policy of competition.

5.2.3 Structure of Optimisation Model in AIMMS

As all the structures of AIMMS models for all the scenarios are almost the same, AIMMS model in non-coordinated scenario is presented in this section for illustration purpose. This AIMMS model consists of seven sections, i.e. optimal headway, optimal cost results, path selection, path decision, path decision from bus transfer, feedback to get final results, optimal headway revision and one step to find optimal headway(this section integrate all the previous sections all together) shown in Figure

Figure 5-1 Flow Chart of Optimisation Procedure

5-2. In order to check the results step by step, the pages on which buttons for different “procedures” are developed which is shown in Figure 5-2. They are “Initial Optimal Headway”, “Path Selection”, “Path Decision”, “Path Decision from Bus Transfer”, “Transit Trip Assignment”, “Optimal Headway Revision” and “Final Optimal Headway” in sequence. The structure of optimization models in AIMMS is shown in Figure 5-3.

Figure 5-2 Main Interface of AIMMS model in Non-coordinated Scenario

It can be seen from Figure 5-3 that after the first page of the main interface of AIMMS model shown in Figure 5-2, the second page “Initial Optimal Headway” has two buttons inside, firstly the initial data are read from excel file, secondly the initial optimal results can be derived by AIMMS built in solver. The third page “Path Selection” has two buttons as well, which searches three shortest paths for those who transfer from modes other than bus and those who transfer from bus respectively. The fourth page “Path Decision” has three buttons, which calibrate path size and the probability of selecting each three shortest paths for those who transfer from modes other than bus. The fifth page “Path Decision from Bus Transfer” has three buttons, which calibrate path size and the probability of selecting each three shortest paths for

those who transfer from bus. By synthesizing all the trips on each shortest path, the rectified OD matrices can be derived in the sixth page “Transit Trip Assignment”. Thus the revised optimal results can be obtained in the seventh page “Optimal Headway Revision”. The last page “One Final Optimal Headway” is the integration of all the procedures of previous pages so that the equilibrium can be reached by setting the constraint that the difference between the total cost in the current iteration and that in the previous iteration is less than S\$0.5.

Due to the integer property of ratio between headway of bus and headway of rail, the objective function turns into a mixed integer nonlinear programming problem in integer-ratio headway coordination scenario. Therefore AOA (AIMMS Outer Approximation) solver is added in the AIMMS model in this case.

Figure 5-3 Structure of Optimisation Model in AIMMS in Non-Coordination

5.3 Initial Value of Input Parameters

To make the input parameters of this model comparable to real operation, parameters of transit system in Singapore are explored and presented in the section as follows.

5.3.1 Real Data of Singapore Transit System

- i *Value of time:* From 'Statistics Singapore', average monthly income is S\$3329 per person for the year 2004 in Singapore, and therefore average hourly income is: $3329 / (8 \times 22) = \text{S\$ } 18.9$, assuming average number of working days per month is 22 days.
- ii *OD rates:* The following data are obtained from official websites of SBS, SMRT (Singapore Mass Rail Transit) and LTA (Land Transport Authority) in Singapore. The transit network in Singapore shown in Figure 5-4 consists of East-West line, North-South line and Bukit Panjang LRT which are all operated by SMRT, and North-East line and Sengkang LRT which are operated by SBS. The round trip time of each rail line can be found from LTA (2006), e.g. the round trip time of North-South line is $2 \times 62 = 124$ minutes; the round trip time of East-West line is $2 \times 59 + 2 \times 7 = 132$ minutes, and that of North-East line is $2 \times 33 = 66$ minutes.

It can be found from SBS (2006) that at the end of 2004, SBS Transit operated a peak scheduled fleet of 2,418 buses on 192 service routes, commanding a 75% share of the scheduled bus market in Singapore. A majority of 75% of the buses were air-conditioned. It can be found from SMRT (2006) that SMRT Buses manage a bus fleet of more than 800 buses plying 72 routes including 46 trunk services, 19 feeder and intra-town services and seven night services.

The average daily ridership in 2004 of SBS Transit is shown in Table 5-1. As the operation hour is 18 hour/day, average ridership per minute of rail in 2004 can be derived as 209 person/min. There are 16 rail stations for SBS, thus average OD rates from one station to another can be obtained as $209 / (16 \times 16 - 16) = 0.87$ person/minute.

The average daily rideship of SMRT Transit is shown in Table 5-2. Similarly the average rail ridership per minute of operation from 2000 to 2004 can be derived as 1065 person/min. There are 26 rail stations for the East-West Line and 24 rail stations for the North-South Line, therefore the average OD rates from one station to another can be obtained as $1065/[(26*26-26)+(24*24-24)]=0.89$ person/minute.

Figure 5-4 Transit Network in Singapore (Source: SBS, 2006)

- iii *Operator Cost:* The fixed operator cost of rail (C_{rf}^p) can be derived from SBS (2007), which is S\$225.8 per minute. The fixed operator cost of bus (C_{bf}^p) can also be derived from SMRT (2007), which is S\$ 0.245 per minute. It can be seen from Table 5-1 that the railway operating costs per car km is 3.08 for the year 2004, and average speed of rail train is 40 km/h, thus the railway operating costs per car minute (R^p) is: $3.08*60/40=$ S\$4.62. The average fleet size is 6. For one fleet of cars, the railway operating cost is S\$27.72 per minute. It can be seen from Table 5-4 that the bus operating costs per bus km is S\$1.76 in 2004, and the average speed of rail train is 20 km/h, thus the bus operating cost per minute (B_{ik}^p)

is $1.76 \times 60 / 20 = S\$5.28$. Table 5-3 show the selected operating data of the MRT system and bus operations for the financial year ended 31 March for SMRT. To make it consistent, financial year 2004 is extracted as the reference of operation costs.

Table 5-1 Average Daily Ridership of SBS Transit (Source: SBS, 2006)

Average Daily Ridership						
Month	Bus			Rail		
	2003	2004	2005	2003	2004	2005
Jan	2,297,581	2,074,983	2,093,088	5,606	193,708	220,466
Feb	2,155,638	2,156,854	2,029,895	6,295	182,589	216,332
Mar	2,156,217	2,140,356	2,093,233	6,628	185,362	217,188
Apr	1,913,591	2,116,250	2,082,666	5,998	181,169	214,213
May	2,066,752	2,066,598	2,009,774	7,450	187,954	217,439
Jun	2,035,948	1,959,737	1,975,938	191,816	192,883	224,315
Jul	2,159,000	2,145,439	2,095,716	157,955	196,967	229,279
Aug	2,161,258	2,099,425	2,112,036	168,060	193,901	228,255
Sep	2,137,955	2,097,354	2,075,145	169,017	196,946	231,868
Oct	2,110,898	2,046,618	2,052,033	171,840	194,823	233,310
Nov	2,004,783	1,964,560	1,985,681	172,556	200,295	233,872
Dec	1,960,028	1,942,448	1,951,599	189,396	212,124	246,871

Table 5-2 Average Daily Ridership of SMRT (Source: SMRT, 2006)

Transit Modes Year	MRT/ LRT	Bus	Taxi
FY 95/96	760,000	3,032,000	940,000
FY 96/97	873,000	3,101,000	911,000
FY 97/98	923,000	3,121,000	923,000
FY 98/99	949,000	3,158,000	926,000
FY 99/00	1,017,000	3,197,000	917,000
FY 00/01	1,101,000	3,257,000	915,000
FY 01/02	1,108,000	3,214,000	871,000
FY 02/03	1,120,000	3,123,000	834,000
FY 03/04	1,272,000	2,972,000	827,000

iv *Age Structure in Taking Public Transit:* Age structures of the whole population in Singapore in 2000, 2004 and 2005 are illustrated in Table 5-5, which shows that the age structure varies very slightly. It can be seen that those age from 15 to 65 are 71.90% and 71.97% in 2004 and 2005 respectively, therefore it can be assumed that age structure in 2005 is almost the same as that in 2004. Due to the

restriction that only the detailed age groups in taking public transport in 2005 can be obtained, the results of age groups in taking public transport in 2005 would be used in 2004 accordingly. It can be seen from Table 5-6 that the population of working person (age from 15 to 65) in taking bus is 608643, and that in taking MRT is 413986. Those students who age over 15 in taking bus and MRT are 177621 and 90363 respectively. Therefore those who age from 15 to 65 of the whole population are the sum of these two groups. As the working person in taking bus and MRT are 22% and 25% respectively, those who age from 15 to 65 in taking bus and rail are 28.4% and 30.5% respectively.

Table 5-3 Operating Data of MRT System (Source: SMRT, 2006)

Rail YEAR ENDED 31 MARCH	2000	2001	2002	2003	2004	2005
Route length at period end (kilometres)	83.0	84.94	89.4	89.4	89.4	89.4
Total number of passengers (millions)	367.19	387.39	389.66	394.25	391.46	402.59
Growth in passenger numbers (%)	6.02	5.50	0.59	1.18	(0.71)	2.84
Car kilometers operated (millions)	74.1	75.3	81.38	89.62	77.39	75.11
Growth in car kilometers operated (%)	0.82	1.61	8.07	10.13	(13.64)	(2.96)
Average number of weekday passengers (thousands)	1,042.0	1,108.0	1,125.0	1,139.3	1,131.4	1,171.9
Growth in weekday passenger (%)	5.57	6.33	1.53	1.27	(0.69)	3.58
Aggregate passenger distance traveled per year (million kilometres)	4,687.0	4,935.0	4,963.0	4,988.9	4,861.3	4,928.3
Growth in passenger distance (%)	6.31	5.29	0.57	0.52	(2.56)	1.38
Average operating car occupancy (persons)	63.0	65.5	60.98	55.7	63.2	65.6
Growth in operating car occupancy (%)	5.00	3.97	(6.90)	(8.66)	13.46	3.79
Selected operating ratios						
Amounts per passenger carried	S\$	S\$	S\$	S\$	S\$	S\$
– Railway cash operating costs (a)	0.55	0.58	0.59	0.64	0.61	0.60
– Railway EBITDA (b)	0.44	0.46	0.45	0.39	0.38	0.36
Amounts per car km operated						
– Railway cash operating costs (a)	2.72	3.00	2.84	2.80	3.08	3.22
– Railway EBITDA (b)	2.19	2.37	2.17	1.73	1.90	1.95

5.3.2 Factors Considered in this Case Study

There are several important factors considered in this case study, i.e. off-peak vs. peak hours, network configurations, value of time, coefficient of in-vehicle value of time, transfer ratio, OD rates and social economic factors. Each of them is elaborated in the following sections.

I. Off-peak vs. peak hours

Table 5-4 Operating Data of Bus (Source: SMRT, 2006)

Bus YEAR ENDED 31 MARCH	2001	2002	2003	2004	2005
Route length at period end (kilometres)	2189.8	2393.6	2503.1	2463.5	2479.8
Total number of passengers (millions)	272.26	290.85	278.16	264.70	268.93
Growth in passenger numbers (%)	–	6.83	(4.37)	(4.84)	1.60
Bus kilometers operated (millions)	63.6	77.2	79.9	79.3	78.4
Growth in bus kilometers operated (%)	–	21.33	3.49	(0.68)	(1.16)
Aggregate passenger distance travelled per year (million kilometres)	1,706.4	1,796.7	1,689.4	1,591.0	1,613.0
Growth in passenger distance (%)	–	5.29	(5.97)	(5.83)	1.38
Load factor ^(a) (%)	29.0	25.2	22.1	22.6	22.8
Growth in load factor (%)	–	(12.82)	(12.30)	2.02	1.16
Selected operating ratios	S\$	S\$	S\$	S\$	S\$
Amounts per passenger carried					
– Bus cash operating costs ^(b)	0.43	0.45	0.51	0.53	0.53
– Bus EBITDA ^(c)	0.13	0.12	0.14	0.16	0.14
Amounts per bus km operated					
–Bus cash operating costs ^(b)	1.83	1.69	1.77	1.76	1.83
–Bus EBITDA ^(c)	0.54	0.45	0.48	0.53	0.47

Table 5-5 Age Structures of the Whole Population (Source: Statistics Singapore, 2006)

Age group Year	Those age 15 to 65 years old	Those age <15 and >65 years old
2000	71.20%	28.80%
2004	71.90%	28.10%
2005	71.97%	28.03%

Table 5-6 Public Transport Trips by Different Age Groups (Source: Statistics Singapore, 2006)

Public transport mode Age groups	Bus	MRT
Working Person	608643	413986
Students >15 years old	177621	90363
Those age 15 to 65 years old	786264	504349
Working person in taking public transport	22%	25%
Percentage of those age 15 to 65 years	28.4%	30.5%
Percentage of those age <15 and > 65 years	71.6%	69.5%

During peak hours, the headways of bus and rail are very short; the coordination between bus and rail is unnecessary, as waiting and transfer time are short. The operations of SBS transit in Singapore on LRT are illustrated in Table 5-7.

It can be seen from Table 5-7 that the average headway of rail is less than 5 minutes during peak hours and greater than 6.5 minutes during off-peak hours for LRT by SBS. While the headway of rail is 2 minutes to 4 minutes during peak hours, and 12 minutes during off-peak hours by SMRT. Besides the average headway of bus is less than 10 minutes during peak hours and greater than 12 minutes during off-peak hours for SBS (SBS, 2006). Whereas the headway of bus is 8 minutes to 14 minutes during peak hours, and 8 minutes to 15 minutes during off-peak hours for all bus routes for SMRT (SMRT, 2006). It can be seen that the average headway of rail and bus are rather short, therefore the coordination may not benefit too much for all the passengers. It is more necessary for the coordination during off-peak hours.

Table 5-7 Train Service Frequencies (in minutes) (Source: SBS, 2006 and SMRT, 2006)

HarbourFront/Punggol	Mon - Fri		Saturday	Sunday
	Average Peak	Average Off-Peak	No Specific Peak Period	No Specific Peak Period
	3.5 - 5	6.5 - 8		
East-West line, North-South line and Bukit Panjang LRT	2 - 4	>12	No Specific Peak Period	No Specific Peak Period

II. Network configuration

In this research, two typical network configurations are considered. One is linear transit network named network I shown in Figure 3-4, in which trunk rail lines with feeder buses are operated along linear corridor with parallel lines. The other is the grid transit network named network II shown in Figure 3-5, in which circumferential rail lines with feeder buses are operated in grid manner.

III. Value of Time

The evaluation of value of time is in controversial (Black, 1995). It is common to apply a uniform value of travel time, without exception, the average hourly income of S\$18.9 is applied in this study which was derived in the previous section. Therefore

it is necessary to conduct sensitivity analysis of important cost components with respect to value of time. As transit passengers are those with low or media income, the fluctuation range should not be large. The average hourly income in Singapore is S\$18.9, the fluctuation ranges from S\$13.9 to S\$22.9 which is within the scope of –S\$5.0 and +S\$4.0 of the average value in the sensitivity analysis study. As it is very special in Singapore that train passengers pay almost the same fare rate as to those who take buses, it is natural to adopt the same value of waiting time for bus and rail.

IV. Value of Time for Different Time Cost

As value of time of riding is lower than that of waiting and transferring, a coefficient of value of time (k_v) adjustment is made on in-vehicle value of time. A sensitivity analysis for a range from 0.1 to 0.9 on k_v value is conducted in this study.

V. Transfer Ratio

As transfer coordination is considered both between and within transit modes in an integrated transit system, transfer ratio from bus to rail is a very important factor to be included in the sensitivity analysis. Transfer ratio is defined to be those transferred from feeder bus divided by the whole feeder passenger flows. Therefore, transfer ratios are varied from 10% to 90% to cover nearly all the range of transfer ratios in the sensitivity analysis.

VI. OD rates

OD rates in this evaluation case study are made comparable to that of Singapore transit system. As it has been calibrated in the previous section, average OD rates from one station to another for SBS transit is 0.87 person/min and that for SMRT transit is 0.89 person/min. Therefore the OD rate from one station to another is rounded to be 0.9 person/min in this case study. As 0.9 person/minute is the average value, OD rates cannot go too far beyond from the average especially during off-peak hours. Therefore OD rates vary from 0.5 person/minute to 1.4 person/minute in the sensitivity analysis.

VII. Social Economic factor

Social economic factor, e.g. age, is considered in this research. As those below 15 and above 65 behave rather differently from those age from 15 to 65 as shown in the parameter in path choice, model 2 by Xie (2003) which is employed to capture the differences in path choices between different age groups of passengers in this research.

5.3.3 Initial Input Parameters in This Case Study

In this evaluation case study, the input parameters and values are assumed comparable to transit system in Singapore as follows:

- (i) Distance matrices are illustrated from Tables 5-8 to 5-12 for transit network I shown in Figure 3-4 and from Tables 5-13 to 5-17 for transit network II that is shown in Figure 3-5. Weight for user cost in the objective function is set to 20.

Table 5-8 Distances between Stations for Network I by Walking (km)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6
station_1	0	3.1	6.6	9.5	12.3	15.2
station_2	3.1	0	3.5	6.4	9.2	12.1
station_3	6.6	3.5	0	2.9	5.7	8.6
station_4	9.5	6.4	2.9	0	2.8	5.7
station_5	12.3	9.2	5.7	2.8	0	2.9
station_6	15.2	12.1	8.6	5.7	2.9	0

Table 5-9 Distances between Stations for Network I by Bus for Operator 1(km)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6
station_1	0	3.72	5.28	11.4	9.84	18.24
station_2	2.48	0	2.8	7.68	7.36	14.52
station_3	5.28	4.2	0	3.48	4.56	10.32
station_4	7.6	7.68	2.32	0	2.24	6.84
station_5	9.84	11.04	4.56	3.36	0	3.48
station_6	12.16	14.52	6.88	6.84	2.32	0

Table 5-10 Distances between Stations for Network I by Bus for Operator 2(km)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6
station_1	0	2.79	7.26	8.55	13.53	13.68
station_2	3.41	0	3.85	5.76	10.12	10.89
station_3	7.26	3.15	0	2.61	6.27	7.74
station_4	10.45	5.76	3.19	0	3.08	5.13
station_5	13.53	8.28	6.27	2.52	0	2.61
station_6	16.72	10.89	9.46	5.13	3.19	0

Table 5-11 Distances between Stations for Network I by Rail for Operator 1 (km)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6
station_1	0	3.72	6.6	11.4	12.3	18.24
station_2	3.1	0	3.5	7.68	9.2	14.52
station_3	6.6	4.2	0	3.48	5.7	10.32
station_4	9.5	7.68	2.9	0	2.8	6.84
station_5	12.3	11.04	5.7	3.36	0	3.48
station_6	15.2	14.52	8.6	6.84	2.9	0

Table 5-12 Distances between Stations for Network I by Rail for Operator 2(km)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6
station_1	0	3.1	7.26	9.5	13.53	15.2
station_2	3.41	0	3.85	6.4	10.12	12.1
station_3	7.26	3.5	0	2.9	6.27	8.6
station_4	10.45	6.4	3.19	0	3.08	5.7
station_5	13.53	9.2	6.27	2.8	0	2.9
station_6	16.72	12.1	9.46	5.7	3.19	0

Table 5-13 Distances between Stations for Network II by Walking (km)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6	station_7	station_8
station_1	0	3.1	6.6	9.5	12.3	5.7	8.4	11.6
station_2	3.1	0	3.5	6.4	9.2	2.6	5.7	8.5
station_3	6.6	3.5	0	2.9	5.7	5.8	2.7	5.7
station_4	9.5	6.4	2.9	0	2.8	9	5.6	2.8
station_5	12.3	9.2	5.7	2.8	0	11.8	8.4	5.6
station_6	5.7	2.6	5.8	9	11.8	0	3.1	6.3
station_7	8.4	5.7	2.7	5.6	8.4	3.1	0	3.2
station_8	11.6	8.5	5.7	2.8	5.6	6.3	3.2	0

Table 5-14 Distances between Stations for Network II by Bus for Operator 1 (km)

(NA: Not Applicable)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6	station_7	station_8
station_1	0	3.72	5.28	11.4	9.84	NA	NA	NA
station_2	2.48	0	2.8	7.68	7.36	NA	NA	NA
station_3	5.28	4.2	0	3.48	4.56	NA	NA	NA
station_4	7.6	7.68	2.32	0	2.24	NA	NA	NA
station_5	9.84	11.04	4.56	3.36	0	NA	NA	NA
station_6	NA	NA	NA	NA	NA	NA	NA	NA
station_7	NA	NA	NA	NA	NA	NA	NA	NA
station_8	NA	NA	NA	NA	NA	NA	NA	NA

Table 5-15 Distances between Stations for Network II by Bus for Operator 2(km)

(NA: Not Applicable)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6	station_7	station_8
station_1	NA	NA	NA	NA	NA	NA	NA	NA
station_2	NA	0	3.85	5.76	NA	2.34	6.27	7.65
station_3	NA	3.15	0	2.61	NA	5.22	2.97	5.13
station_4	NA	5.76	3.19	0	NA	8.1	6.16	2.52
station_5	NA	NA	NA	NA	NA	NA	NA	NA
station_6	NA	2.34	6.38	8.1	NA	0	3.41	5.67
station_7	NA	5.13	2.97	5.04	NA	2.79	0	2.88
station_8	NA	7.65	6.27	2.52	NA	5.67	3.52	0

Table 5-16 Distances between Stations for Network II by Rail for Operator 1 (km)

(NA: Not Applicable)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6	station_7	station_8
station_1	0	3.72	6.6	11.4	12.3	NA	NA	NA
station_2	3.1	0	3.5	7.68	9.2	NA	NA	NA
station_3	6.6	4.2	0	3.48	5.7	NA	NA	NA
station_4	9.5	7.68	2.9	0	2.8	NA	NA	NA
station_5	12.3	11.04	5.7	3.36	0	NA	NA	NA
station_6	NA	NA	NA	NA	NA	NA	NA	NA
station_7	NA	NA	NA	NA	NA	NA	NA	NA
station_8	NA	NA	NA	NA	NA	NA	NA	NA

Table 5-17 Distances between Stations for Network II by Rail for Operator 2 (km)

(NA: Not Applicable)

O \ D	station_1	station_2	station_3	station_4	station_5	station_6	station_7	station_8
station_1	NA	NA	NA	NA	NA	NA	NA	NA
station_2	NA	0	3.85	6.4	NA	2.6	6.27	8.5
station_3	NA	3.5	0	2.9	NA	5.8	2.97	5.7
station_4	NA	6.4	3.19	0	NA	9	6.16	2.8
station_5	NA	NA	NA	NA	NA	NA	NA	NA
station_6	NA	2.6	6.38	9	NA	0	3.41	6.3
station_7	NA	5.7	2.97	5.6	NA	3.1	0	3.2
station_8	NA	8.5	6.27	2.8	NA	6.3	3.52	0

- (ii) The parameters by different modes for different operators are assumed shown in Table 5-18. The capacity of high capacity bus (N_b) is 75 people, and speed (V_b) is 16~20 km/h; the capacity of rapid transit (N_r) is 2400 people, and operating speed at this capacity (V_r) is within 24~40 km/h; average walking speed can be

assumed to be 5 km/h (Vuchic, 2005). The operator costs for bus and rail have been derived in Section 5.3.1, i.e. the fixed operator cost of rail (C_{rf}^p) is S\$225.8 per minute, the fixed operator cost of bus (C_{bf}^p) is S\$ 0.245 per minute. Vehicle cost of rail per fleet (R^p) is S\$27.2 per minute, the vehicle cost of bus per fleet (B_{ik}^p) is S\$5.28 per minute.

Table 5-18 Parameters by different modes for different operators
(Source: Vuchic, 2005) (NA: Not Applicable)

Rail Lines	V_b (km/h)	V_r (km/h)	C_{rf}^p (S\$/min)	C_{bf}^p (S\$/min)	R^p (S\$/min)	B_{ik}^p (S\$/min)	N_b (person)	N_r (person)
Operator 1	20	40	225.8	0.245	27.2	5.28	75	2400
Operator 2	20	40	225.8	0.245	27.2	5.28	75	2400
Walking	5	5	5	5	NA	NA	NA	NA

(iii) The parameters of feeder buses at rail transfer stations are shown as follows.

Assuming that there are two bus routes at each rail transfer station. The distribution of arrival pattern for each bus connecting each rail transfer station is assumed to follow a Normal Distribution. The variances of this Normal Distribution (σ_i^2) are assumed to be 1 for all the stations. Sensitivity analysis with respect to the variance is carried out in the following chapters, in which the variances of this Normal Distribution (σ_i^2) vary from 1 to 100. In other words, the standard deviation of this Normal Distribution varies from 1 to 10 minute.

As it has been calibrated in the previous section, average OD rates from one station to another for SBS transit is 0.87 person/min and that for SMRT transit is 0.89 person/min. Therefore in this case study, the OD rates is rounded to be 0.9 person/min. Transfer ratio is assumed to be 50%. The value of time is assumed to be S\$18.9 per hour.

(iv) The parameters of system utility functions were calibrated by Xie (2003), i.e. the parameters in model 1, $\beta_{0,1} = 0.757$, $\beta_{0,2} = 0.085$, $\beta_{0,3} = 0$, $\beta_1 = -2.392$, $\beta_2 = -1.503$, $\beta_3 = -0.029$, $\beta_4 = 0.392$; the parameters in model 2,

$\beta_{0,1} = 0.817$, $\beta_{0,2} = 0.157$, $\beta_{0,3} = 0$, $\beta_1 = -2.271$, $\beta_2 = -1.417$,
 $\beta_3 = -0.029$, $\beta_4 = 0.302$, $\beta_5 = -1.501$, $\beta_6 = -0.578$, $\beta_7 = -1.499$, $\beta_8 = -1.842$. As
the coordination is preferred during off-peak hours, the percentage of those who
age from 15 to 65 must be less than the average value of percentage in taking
public transport during this period. Therefore the percentage of those from 15 to
65 can be assumed to 20% in this case study according to Table 5-6.

(v) Fare structures of transit modes are assumed to be the same as those in Singapore.

The constant for bus α_m is 0.5 and for rail α_m is 0.7 for all operators. Similar
to the fare adjustment posted on the website by Public Transport Council of
Singapore (PTC, 2006), the minor fare adjustment (*fareadjust*) is allowed as an
integer number fluctuating from 1 to 5 according to different distance between
stations i to j (l_{ij}), thus the fare structure is assumed to be as follows in the
format of AIMMS codes in Table 5-19.

5.4 Optimal Results under Different Strategies

In this section the optimal results in different scenarios (i.e. non-coordination,
common headway coordination and integer-ratio headway coordination) under
different strategies are derived based on the same multimodal and multi-operator
transit which are shown in Figure 3-4 (network I) and Figure 3-5 (network II)
respectively, using model formulations under different strategies with the same input
parameters as Section 5.3.3 describes. In each section, the optimal results in
different coordination scenarios comparing with non-coordinated scenarios on
different network configurations are shown to make comparisons. In order to find
the effect of the coordinate operations, the schedule operation in non-coordinated
scenario is still optimised. Although the percentages of transfer cost in the total cost
are quite small in all scenarios, at most 2.05% shown in Table 5-20, Table 5-22 and
Table 5-24 respectively, it makes sense that minimising transfer cost can greatly
enhance the level of service the users perceive.

I. Optimal Results of Scenarios When Operators are in Cooperation

Using the typical set of input parameters above, the optimal results can be derived by AIMMS models when operators are in cooperation. In order to compare optimal results in different scenarios, the headways of rail are set to equal for both operators in both non-coordination and coordination scenarios. Thus the optimal headways and different cost items in different scenarios based on different network configurations are summarized in Table 5-20. To be consistent, the percentages of cost items in common headway coordination and integer-ratio headway coordination compared with those in non-coordinated scenario are shown in Table 5-21.

Table 5-19 Fare Structure in AIMMS Codes

```

If  $l_{ij} \leq 3$  then
     $f_{ij}^m = \alpha_m + 0.1 * fareadjust ;$ 
elseif  $3 < l_{ij} \leq 6$  then
     $f_{ij}^m = \alpha_m + 0.2 * fareadjust ;$ 
elseif  $6 < l_{ij} \leq 9$  then
     $f_{ij}^m = \alpha_m + 0.3 * fareadjust ;$ 
elseif  $9 < l_{ij} \leq 12$  then
     $f_{ij}^m = \alpha_m + 0.4 * fareadjust ;$ 
elseif  $12 < l_{ij} \leq 15$  then
     $f_{ij}^m = \alpha_m + 0.5 * fareadjust ;$ 
elseif  $15 < l_{ij} \leq 18$  then
     $f_{ij}^m = \alpha_m + 0.6 * fareadjust ;$ 
elseif  $18 < l_{ij} \leq 21$  then
     $f_{ij}^m = \alpha_m + 0.7 * fareadjust ;$ 
elseif  $l_{ij} > 21$  then
     $f_{ij}^m = \alpha_m + 0.8 * fareadjust ;$ 
endif;

```

From Tables 5-20 and 5-21, it is shown that the total cost, operator cost, user cost, transfer cost and average transfer cost decrease for both networks in coordinated scenarios. It can be seen that waiting cost increases, while transfer cost decreases in coordinated scenarios for both networks. The improvement in reducing the total cost in common headway coordination is higher than in integer-ratio headway

coordination. It can be seen that transfer cost reduced while waiting cost increased, which means coordination can reduce

Table 5-20 Optimal Results in Different Scenarios
(NC: Non-coordination, CH: Common Headway Coordination, IR: Integer-ratio Headway Coordination)

Scenarios Optimal Results	Network I			Network II		
	NC	CH	IR	NC	CH	IR
H_r for both operators(min)	5.9	9.1	6.8	4.4	7.3	5.4
H_b^1 for Operator 1 (min)	13.6	9.1	13.6	7.6	7.3	10.8
H_b^2 for Operator 2 (min)	11.8	9.1	13.6	7.7	7.3	10.8
The total cost (S\$)	892.7	751.6	824.3	1363.2	1042.8	1160.9
Operator Cost (S\$)	773.2	641.6	710.9	1156.9	853.0	966.0
User Cost (S\$)	119.5	110.0	113.4	206.2	189.7	194.9
Waiting Cost (S\$)	56.2	61.9	58.0	96.9	106.7	101.7
Transfer Cost (S\$)	13.7	1.5	8.8	28.0	3.0	13.6
In-vehicle Cost (S\$)	49.6	46.6	46.6	81.3	80.1	79.6
Total Transfer Volume (person/min)	18.1	337.5	159.4	55.4	309.4	167.2
Transfer Share for Operator 1	51.03%	48.17%	48.63%	45.95%	47.29%	50.77%
Transfer Share for Operator 2	48.97%	51.83%	51.37%	54.05%	52.71%	49.23%
Average Transfer Cost (S\$/person)	0.7556	0.0043	0.0553	0.5050	0.0098	0.0813
Percentage of Transfer Cost in The total cost	1.53%	0.20%	1.07%	2.05%	0.29%	1.17%

transfer cost for some passengers at the cost of increase waiting cost for some other passengers, but the benefit is obvious, i.e. reducing the total cost for the whole transit system. It can be noticed that total Transfer Volume in coordinated scenarios for both networks increase greatly. This is due to good coordination operations that result in some passengers' more transfers than those in non-coordinated operation. It can be seen from Table 5-20 that the operator whose transfer share is higher in non-coordinated scenario gets less share of passengers than its opponent in coordinated operation on both networks.

It is indicated from Table 5-21 that common headway coordination is more efficient in reducing average transfer cost for the whole passengers in transit system. As common headway coordination is an ideal state that headway of bus equals that of rail, sometimes it cannot be feasible due to operation policies. Therefore it is more practical to realize integer-ratio headway coordination, what is more, the total cost, transfer cost and average transfer cost all reduced compared with those in non-coordinated operation, and it shows that integer-ratio headway coordination is feasible and efficient in reducing the total cost and average transfer cost in real coordination operation when operators are in cooperation.

Table 5-21 Optimal Results in Coordination Compared with Non-coordination

(CH: Common Headway Coordination, IR: Integer-ratio Headway Coordination)

Scenarios	Network I		Network II	
	CH	IR	CH	IR
The total cost (S\$)	-15.8%	-7.7%	-23.5%	-14.8%
Operator Cost (S\$)	-17.0%	-8.1%	-26.3%	-16.5%
User Cost (S\$)	-7.9%	-5.1%	-8.0%	-5.5%
Waiting Cost (S\$)	10.1%	3.2%	10.1%	5.0%
Transfer Cost (S\$)	-89.1%	-35.8%	-89.3%	-51.4%
Total Transfer Volume (person/min)	1764.6%	780.7%	458.5%	201.8%
Average Transfer Cost (S\$/person)	-99.4%	-92.7%	-98.1%	-83.9%

II. Optimal Results of Scenarios When Operators are Independent

Similarly based on the same data set above, the optimal results are derived using the AIMMS model in scenarios when operators are independent, thus the optimal headways and different cost items in different scenarios for both network configurations are summarized in Table 5-22. The percentages of cost items in common headway coordination and integer-ratio headway coordination compared with those in non-coordinated operation are shown in Table 5-23.

From Tables 5-22 & 5-23, it is shown that the total cost, operator cost, user cost transfer cost and average transfer cost decrease for both networks in coordinated scenarios. It can be seen that waiting cost increases in coordinated scenarios for

Table 5-22 Optimal Results in Different Scenarios
(NC: Non-coordination, CH: Common Headway Coordination, IR: Integer-ratio
Headway Coordination)

Scenarios Optimal Results	Network I			Network II		
	NC	CH	IR	NC	CH	IR
H_r for both operators(min)	5.9	9.1	6.8	4.4	6.3	4.7
H_b^1 for Operator 1 (min)	13.6	9.1	13.6	7.6	6.3	9.4
H_b^2 for Operator 2 (min)	11.8	9.1	13.6	7.7	6.3	9.4
The total cost (S\$)	892.7	754.7	827.4	1363.2	1123.8	1274.9
Operator Cost (S\$)	773.2	641.6	710.9	1156.9	957.8	1073.4
User Cost (S\$)	119.5	113.0	116.4	206.2	165.9	201.5
Waiting Cost (S\$)	56.2	61.9	58	96.9	104.5	98.9
Transfer Cost (S\$)	13.7	1.5	8.8	28.0	18.9	21.8
In-vehicle Cost (S\$)	49.6	49.6	49.6	81.3	42.5	80.8
Total Transfer Volume (person/min)	18.1	36.1	43.7	55.4	129.2	116.0
Transfer Share for Operator 1	51.03%	52.22%	50.92%	45.95%	50.00%	51.42%
Transfer Share for Operator 2	48.97%	47.78%	49.08%	54.05%	50.00%	48.58%
Average Transfer Cost (S\$/person)	0.7556	0.0403	0.2019	0.5050	0.1466	0.1880
Percentage of Transfer Cost in The total cost	1.53%	0.20%	1.06%	2.05%	1.68%	1.71%

both networks. The improvement in reducing the total cost in common headway coordination is higher than that in integer-ratio headway coordination. It can be found that transfer cost reduced while waiting cost increased, which means coordination can reduce transfer cost for some passengers at the cost of increase waiting cost for some other passengers, but the benefit is obvious, i.e. reducing the total cost for the whole transit system. Total transfer volume in coordinated operations in both networks increase a lot, as coordinated operations result in more passengers' transfers than those in non-coordinated scenarios. It can be seen from Table 5-22 that the operator whose transfer share is in priority in non-coordinated operation still remains more share of passengers in coordinated operation on network I, while the operator whose transfer share of passengers is not in priority in

non-coordinated operation get even share in common headway coordination, and gets less share of passengers than its opponent in integer-ratio headway coordination on network II. The total cost and average transfer cost in common headway decrease more than those in integer-ratio headway coordination. Therefore it indicates that common headway coordination is more efficient in reducing average transfer cost for the whole passengers in transit system.

Just as stated previously sometimes common headway coordination cannot be feasible due to operation policies, therefore it is more practical to realize integer-ratio headway coordination even when operators are independent. What is more, the total cost, transfer cost and average transfer cost all decreased compared with those in non-coordinated operation, therefore it can be concluded that integer-ratio headway coordination is feasible and efficient in reducing the total cost and average transfer cost in real coordination operation even when operators are independent.

Table 5-23 Optimal Results in Coordination Compared with Non-coordination (CH: Common Headway Coordination, IR: Integer-ratio Headway Coordination)

Scenarios	Network I		Network II	
	CH	IR	CH	IR
The total cost (\$\$)	-15.5%	-7.3%	-17.6%	-6.5%
Operator Cost (\$\$)	-17.0%	-8.1%	-17.2%	-7.2%
User Cost (\$\$)	-5.4%	-2.6%	-19.5%	-2.3%
Waiting Cost (\$\$)	10.1%	3.2%	7.8%	2.1%
Transfer Cost (\$\$)	-89.1%	-35.8%	-32.5%	-22.1%
Total Transfer Volume (person/min)	99.4%	141.4%	133.2%	109.4%
Average Transfer Cost (\$\$/person)	-94.7%	-73.3%	-71.0%	-62.8%

III. Optimal Results of Scenarios When Operators are in Competition

Employing the same data set above as the initial input parameter, the optimal results of scenarios when operators are in competition can be derived by developing AIMMS models. In order to compare the total cost with that when operators are in cooperation and independence, the total cost is calibrated as the summation of waiting cost, transfer cost, in-vehicle cost and operator cost that is not deducted revenue of operators. Therefore optimal headways and different cost items in different scenarios are sorted out and summarized in Table 5-24. The percentages of cost items in common headway coordination and integer-ratio headway coordination compared with those in non-coordinated scenario are shown in Table 5-25.

Table 5-24 Optimal Results in Different Scenarios
(NC: Non-coordination, CH: Common Headway Coordination, IR: Integer-ratio Headway Coordination)

Scenarios Optimal Results	Network I			Network II		
	NC	CH	IR	NC	CH	IR
H_r for both operators(min)	5.9	9.1	6.8	4.4	4.9	3.8
H_b^1 for Operator 1 (min)	13.6	9.1	13.6	7.6	4.9	7.6
H_b^2 for Operator 2 (min)	11.8	9.1	13.6	7.7	4.9	7.6
The total cost (\$\$)	892.7	754.6	827.3	1361.1	1347.5	1459.7
Operator Cost (\$\$)	773.2	641.6	710.9	1155.0	1156.9	1265.8
User Cost (\$\$)	119.5	113.0	116.4	165.5	190.6	193.9
Waiting Cost (\$\$)	56.2	61.9	58.0	96.9	97.4	95.1
Transfer Cost (\$\$)	13.7	1.5	8.8	28.0	12.1	17.9
In-vehicle Cost (\$\$)	49.6	49.6	49.6	81.3	81.1	80.9
Total Transfer Volume (person/min)	18.1	36.1	43.7	55.4	135.0	106.6
Transfer Share for Operator 1	51.03%	52.55%	51.06%	45.95%	48.50%	48.81%
Transfer Share for Operator 2	48.97%	47.45%	48.94%	54.05%	51.50%	51.19%
Average Transfer Cost (\$\$/person)	0.7556	0.0403	0.2019	0.5050	0.0899	0.1684
Percentage of Transfer Cost in The total cost	1.53%	0.20%	1.06%	2.05%	0.90%	1.23%

It can be seen from Table 5-24 and Table 5-25 that user cost, transfer cost and average transfer cost decrease for both networks in coordinated scenarios, while the total cost and operator cost decrease in coordinated operations on network I and common headway coordination on network II, but increase on network II in integer-ratio headway coordination. Waiting cost increases in coordinated operations on network I and common headway coordination on network II, but decrease in integer-ratio headway coordination on network II. Due to the coordination within each operator's scope, more passengers would prefer to transfer, thus total transfer volume in coordinated operations increase on both networks. It can be found from Table 5-24 that the operator who gets more transfer share of passengers in non-coordinated operation will keep the priority in coordinated operations on both networks when operators are in competition.

Table 5-25 Optimal Results in Coordination Compared with Non-coordination (CH: Common Headway Coordination, IR: Integer-ratio Headway Coordination)

Scenarios	Network I		Network II	
	CH	IR	CH	IR
The total cost (S\$)	-15.5%	-7.3%	-1.0%	7.2%
Operator Cost (S\$)	-17.0%	-8.1%	0.2%	9.6%
User Cost (S\$)	-5.4%	-2.6%	15.2%	17.2%
Waiting Cost (S\$)	10.1%	3.2%	0.5%	-1.9%
Transfer Cost (S\$)	-89.1%	-35.8%	-56.8%	-36.1%
Total Transfer Volume (person/min)	99.4%	141.4%	143.7%	92.4%
Average Transfer Cost (S\$/person)	-94.7%	-73.3%	-82.2%	-66.7%

As integer-ratio headway coordination cannot achieve consistent results on both networks when operators are in competition, it can be concluded that it is not effective in integer-ratio headway coordination for network II, while it is effective in coordinated operations for network I. The optimal results are the same as those when operators are independent on network I, which means small range of fluctuation of transit fare does not have much effect on path choice behaviour for transit users. This can be verified from the real operation in Singapore, when transit fares increase, there is no drastic decrease in the passenger volumes from the past statistics.

IV.Comparisons between Different Operation policies in Integer-ratio Headway Coordination

In order to make comparisons between different operation policies in integer-ratio headway coordination, optimal results in integer-ratio headway coordination under different operation policies are illustrated in Table 5-26. The variation percentages of cost items comparing with those when operators are in cooperation are shown in Table 5-27 for both network configurations.

Table 5-26 Optimal Results under Different Operation policies
(CP: Cooperation, ID: Independence, CM: Competition)

Scenarios Optimal Results	Network I			Network II		
	CP	ID	CM	CP	ID	CM
The total cost (S\$)	824.3	827.4	827.3	1160.9	1274.9	1459.7
Operator Cost (S\$)	710.9	710.9	710.9	966	1073.4	1265.8
User Cost (S\$)	113.4	116.4	116.4	194.9	201.5	193.9
Waiting Cost (S\$)	58	58	58	101.7	98.9	95.1
Transfer Cost (S\$)	8.8	8.8	8.8	13.6	21.8	17.9
In-vehicle Cost (S\$)	46.6	49.6	49.6	79.6	80.8	80.9
Total Transfer Volume (person/min)	159.4	43.7	43.7	167.2	116	106.6
Transfer Share for Operator 1	48.63%	50.92%	51.06%	50.77%	51.42%	48.81%
Transfer Share for Operator 2	51.37%	49.08%	48.94%	49.23%	48.58%	51.19%
Average Transfer Cost (S\$/person)	0.0553	0.2019	0.2019	0.0813	0.188	0.1684

In integer-ratio headway coordination, it can be seen from Table 5-27 that the total cost, user cost, in-vehicle cost and average transfer cost increase when operators are independent and in competition on both network configurations. Operator cost, waiting cost and transfer cost keep constant even if operation policy changes on network I. The variations of the total cost, operator cost, waiting cost, transfer cost and total transfer volume keep almost the same when operators are independent as when operators are in competition on network I, while the increases of the total cost and operator cost when operators are independent are less than those when operators are in competition on network II. Average transfer cost increases on both networks

when operators are both independent and in competition. Therefore it can be concluded that the integer-ratio headway coordination when operators are in competition is the least encouraged. It can be found that total transfer volume decrease for both network configurations, because there is no coordination between different operators when operators are independent and competitive, fewer passengers will choose to transfer between operators than those when operators are in cooperation. While it can be found from Table 5-26 that the operator which is superior in transfer share of passengers will become inferior when operators change policy from cooperation to independence or competition.

Table 5-27 Variation Percentages
(ID: Independence, CM: Competition)

Operation policies Optimal Results	Network I		Network II	
	ID	CM	ID	CM
The total cost (\$\$)	0.4%	0.4%	9.8%	25.7%
Operator Cost (\$\$)	0.0%	0.0%	11.1%	31.0%
User Cost (\$\$)	2.6%	2.6%	3.4%	-0.5%
Waiting Cost (\$\$)	0.0%	0.0%	-2.8%	-6.5%
Transfer Cost (\$\$)	0.0%	0.0%	60.3%	31.6%
In-vehicle Cost (\$\$/min)	6.4%	6.4%	1.5%	1.6%
Total Transfer Volume (person/min)	-72.6%	-72.6%	-30.6%	-36.2%
Average Transfer Cost (\$\$/person)	265.1%	265.1%	131.2%	107.1%

The preliminary conclusion can be drawn that based on this typical data set, integer-ratio headway coordination when operators are in cooperation is the most efficient and feasible in coordinating between bus and rail. The coordination results when operators are in competition are the worst compared with those when operators are in cooperation or independent. Therefore it can be concluded that in a multimodal and multi-operator transit system, integer-ratio headway coordination is an efficient and feasible method in coordination operations when operators are in cooperation.

5.5 Summary

Evaluation of coordination under different operation policies between operators is conducted step by step in this chapter. As optimal results are derived by AIMMS models, the implementations of AIMMS models are described initially. Firstly optimisation procedure implemented in AIMMS models is shown in a flow chart, and then design of AIMMS models in different scenarios is presented. The structures of these AIMMS models are briefly discussed in the subsequent section. In order to prepare a valid data set comparable to the real operation, real data of Singapore transit system are collected from annual reports of SBS, SMRT and LTA websites, and data analysis are accomplished to derive comparable data set. Using this data set, optimal results in different scenarios under different operation policies are obtained by running AIMMS models. The optimal results for both the important decision variables and cost items in different coordination operations under each operation policy based on different network configurations are compared. The preliminary conclusion can be drawn that integer-ratio headway coordination is a feasible and efficient coordination operation in a transit system when operators are in cooperation. By comparing cost items in integer-ratio headway coordination under different operation policies, it can be concluded that based on this typical data set, integer-ratio headway coordination when operators are in cooperation is most encouraged in coordination operations. The final conclusion in this section can be arrived that based on this typical data set, in an integrated multimodal and multi-operator transit system, integer-ratio headway coordination can be an efficient and feasible method in coordination operations, what is more, policy of cooperation between operators is an efficient strategy that can reduce the total cost of transit system and improve coordination; whereas policy of competition between operators is the least efficient when making coordination operations within each operator's scope.

All the results and preliminary conclusions above are based on the typical data set in Section 5.3.3. In order to verify the models and optimisation algorithm that proposed in Chapter 3, sensitivity analysis of important decision variables and cost items with respect to important factors is conducted in the next chapter.

CHAPTER 6 Sensitivity Analysis of Scenarios When Operators are in Cooperation

6.1 Introduction

In this chapter, extensive sensitivity analysis of essential decision variables and cost items with respect to important factors is carried out under different coordination operations when operators are in cooperation based on different transit network configurations. These important factors include value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, and passenger OD rates of feeder bus services. The optimal results are illustrated in Appendix C.

6.2 Sensitivity Analysis of Scenarios When Operators are in Cooperation

The sensitivity analysis of optimal decision variables and important cost items with respect to various input parameters is conducted in this chapter. The important cost items include the total cost, operator cost, user cost, waiting cost, transfer cost, total transfer volume and average transfer cost (transfer cost divided by total transfer volume). The input parameters include value of time, coefficient of in-vehicle value of time, transfer ratio of passengers from bus versus the whole feeder passenger flow, standard deviation of bus arrival times, passenger OD rates of feeder bus services. The variation range of each parameter and initial input data have been dealt with extensively in Sections 5.3.2 and 5.3.3 respectively. Sensitivity analysis in non-coordination, common headway coordination and integer-ratio headway coordination scenarios are shown in same figures so that their results can be compared directly.

6.2.1 Value of Time

Sensitivity analysis with respect to value of time was carried out from S\$13.9 to S\$22.9. With the increase of value of time, it can be seen in Figures 6-1 to 6-6 that headways of rail and bus for both operators on two networks decrease in non-coordination and coordinated operations. The headways of rail for non-coordinated cases are the lowest, while the headways of bus for common headway coordination are the lowest.

It can be seen in Figures 6-7 to 6-20 that the total cost, operator cost, user cost, waiting cost, transfer cost and average transfer cost in all three scenarios increase with the increase of value of time, while the costs are the highest for non-coordinated cases, followed by the integer-ratio headway coordination, and those in common headway coordination for the same value of time. It means that with the increase of value of time, coordination results are not affected. It can be found in Figures 6-17 and 6-18 that transfer volume in non-coordinated operation remained constant, while those in coordinated operation decreased slightly with the increase of value of time in both networks. Besides the order of Transfer Volume stayed constant, i.e. transfer volume in common headway coordination is the highest, followed by integer-ratio headway coordination, and non-coordinated operation. This is reasonable as more passengers would choose to transfer when the transit system is better coordinated, and transfer volumes would not be affected with the increase of value of time in non-coordinated operations. Therefore, it can be concluded that the fluctuation in value of time within normal range of value does not affect the coordination results when operators are in cooperation. Well coordinated transit systems would attract more passengers in transferring.

Statistical analysis is carried out based on the model that used the inference for the difference in means of two normal distributions, and the variances are unknown. Suppose there are two independent normal distribution with unknown means μ_1 and μ_2 , and unknown but equal variances, $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1: \mu_1 - \mu_2 \neq 0$$

$$t_0 = \frac{\overline{X}_1 - \overline{X}_2 - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

$$\text{where } S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Rejection area is $t_0 > t_{\alpha/2, n_1 + n_2 - 2}$ or $t_0 < -t_{\alpha/2, n_1 + n_2 - 2}$.

Therefore T test is carried out for the total cost in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 41.56 > t_{0.025, 18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, as $t_0 = 73.07 > t_{0.025, 18} = 2.101$, it can also be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. The alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in common headway coordination is greater than that in non-coordination in both networks at the 0.95 level of significance.

T test is also carried out for the total cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -21.99 < -t_{0.025, 18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -28.02 < -t_{0.025, 18} = -2.101$ it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the total cost in common headway coordination is less than that in non-coordination in both networks at the 0.95 level of significance.

T test is also carried out for the total cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 2.23 > t_{0.025, 18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 124.15 > t_{0.025, 18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level

of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in non-coordination is greater than that in integer-ratio headway coordination in both networks at the 0.95 level of significance.

T test is also carried out for the average transfer in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 9.34 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 7.38 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in common headway coordination in both networks at the 0.95 level of significance.

T test is also carried out for the average transfer cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -1.69 > -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. In network II, since $t_0 = -1.98 > -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. Therefore average transfer cost in common headway coordination equals that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the average transfer cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 8.24 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 5.77 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in integer-ratio headway coordination in both networks at the 0.95 level of significance.

Therefore T test on average transfer cost confirms with that on total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give best results when operators are in cooperation within normal range of value of time.

Figure 6-1 Headways of Rail vs. Value of Time (network I)

Figure 6-2 Headways of Rail vs. Value of Time (network II)

Figure 6-3 Headways of Bus for Operator 1 vs. Value of Time (network I)

Figure 6-4 Headways of Bus for Operator 1 vs. Value of Time (network II)

Figure 6-5 Headways of Bus for Operator 2 vs. Value of Time (network I)

Figure 6-6 Headways of Bus for Operator 2 vs. Value of Time (network II)

Figure 6-7 Total Cost vs. Value of Time (network I)

Figure 6-8 Total Cost vs. Value of Time (network II)

Figure 6-9 Operator Cost vs. Value of Time (network I)

Figure 6-10 Operator Cost vs. Value of Time (network II)

Figure 6-11 User Cost vs. Value of Time (network I)

Figure 6-12 User Cost vs. Value of Time (network II)

Figure 6-13 Waiting Cost vs. Value of Time (network I)

Figure 6-14 Waiting Cost vs. Value of Time (network II)

Figure 6-15 Transfer Cost vs. Value of Time (network I)

Figure 6-16 Transfer Cost vs. Value of Time (network II)

Figure 6-17 Transfer Volume vs. Value of Time (network I)

Figure 6-18 Transfer Volume vs. Value of Time (network II)

Figure 6-19 Average Transfer Cost vs. Value of Time (network I)

Figure 6-20 Average Transfer Cost vs. Value of Time (network II)

6.2.2 Transfer Ratio

Sensitivity analysis with respect to transfer ratio from 10% to 90% is carried out in this section. It is shown in Figures 6-21 to 6-40 that except waiting cost and transfer cost, the total cost, operator cost, user cost, total transfer volume and headways of bus and rail in non-coordinated operation remained constant, but the sum of waiting cost and transfer cost did not change at the same transfer ratio. This validates that this AIMMS programme is correct, and the model is reasonable that no matter how the transfer ratio of passengers from bus versus the whole feeder passenger flow changes, the optimal decision variables, i.e. headways of bus and headways of rail, some cost items, i.e. the total cost, operator cost, user cost, in-vehicle cost and summation of waiting cost and transfer cost do not change for the same OD rates, while waiting cost decreased and transfer cost increased at the same transfer ratio in non-coordinated operation. It can be seen in Figures 6-27 and 6-28 that the total costs in different scenarios are very close when the transfer ratio is low, e.g. 10%. The differences between the total costs in different scenarios become larger and larger when transfer ratio increases. It is reasonable that when transfer ratio is low, the coordination will not achieve obvious benefits, i.e. reducing the total cost. With the increase of transfer ratio, coordination operation would achieve better results. It can be seen in Figures 6-29 and 6-40 that average transfer costs increase with the increase of transfer ratio. It can be found in Figures 6-27, 6-28, 6-39 and 6-40 that the total cost and average transfer cost keep the same order in different scenarios at the same transfer ratio for both networks when the transfer ratio is higher than 10%, with the total cost and average transfer cost in non-coordinated operation being the highest, followed by integer-ratio headway coordination, and common headway coordination. This indicates that the integer-ratio headway coordination is moderate in effectiveness in reducing the total cost and average transfer cost when transfer ratio is greater than 10%.

T test is carried out for the total cost in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 36.88$

$> t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} > \mu_{t_2}$. In network II, since $t_0 = 28.42 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} > \mu_{t_2}$. Therefore the total cost in non-coordination is greater than that in common headway coordination at the 0.95 level of significance.

T test is also carried out for the total cost in common headway coordination (μ_{t_1}) and integer-ratio headway coordination (μ_{t_2}) in networks I and II respectively. In network I, since $t_0 = -18.03 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} < \mu_{t_2}$. In network II, since $t_0 = 14.77 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} < \mu_{t_2}$. Therefore the total cost in common headway coordination is less than that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the total cost in non-coordination (μ_{t_1}) and integer-ratio headway coordination (μ_{t_2}) in networks I and II respectively. In network I, since $t_0 = 22.68 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} > \mu_{t_2}$. In network II, since $t_0 = 65.14 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} > \mu_{t_2}$. Therefore the total cost in non-coordination is greater than that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the average transfer cost in non-coordination (μ_{t_1}) and in common headway coordination (μ_{t_2}) in networks I and II respectively. In network I, since $t_0 = 3.60 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t_1} = \mu_{t_2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t_1} > \mu_{t_2}$. In network II,

since $t_0 = 3.63 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in common headway coordination at the 0.95 level of significance.

T test is also carried out for the average transfer cost in common headway coordination (μ_{t1}) and in integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -0.87 > -t_{0.025,16} = -2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. In network II, since $t_0 = -1.06 > -t_{0.025,16} = -2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. Therefore average transfer cost in common headway coordination equals that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the average transfer cost in non-coordination (μ_{t1}) and in integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 3.25 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 2.85 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in integer-ratio headway coordination at the 0.95 level of significance.

Therefore T test on average transfer cost confirms with that of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give obvious benefit results when operators are in cooperation within normal range of transfer ratio on some network configuration.

Figure 6-21 Headways of Rail vs. Transfer Ratio (network I)

Figure 6-22 Headways of Rail vs. Transfer Ratio (network II)

Figure 6-23 Headways of Bus for Operator 1 vs. Transfer Ratio (network I)

Figure 6-24 Headways of Bus for Operator 1 vs. Transfer Ratio (network II)

Figure 6-25 Headways of Bus for Operator 2 vs. Transfer Ratio (network I)

Figure 6-26 Headways of Bus for Operator 2 vs. Transfer Ratio (network II)

Figure 6-27 Total Cost vs. Transfer Ratio (network I)

Figure 6-28 Total Cost vs. Transfer Ratio (network II)

Figure 6-29 Operator Cost vs. Transfer Ratio (network I)

Figure 6-30 Operator Cost vs. Transfer Ratio (network II)

Figure 6-31 User Cost vs. Transfer Ratio (network I)

Figure 6-32 User Cost vs. Transfer Ratio (network II)

Figure 6-33 Waiting Cost vs. Transfer Ratio (network I)

Figure 6-34 Waiting Cost vs. Transfer Ratio (network II)

Figure 6-35 Transfer Cost vs. Transfer Ratio (network I)

Figure 6-36 Transfer Cost vs. Transfer Ratio (network II)

Figure 6-37 Total Transfer Volume vs. Transfer Ratio (network I)

Figure 6-38 Total Transfer Volume vs. Transfer Ratio (network II)

Figure 6-39 Average Transfer Cost vs. Transfer Ratio (network I)

Figure 6-40 Average Transfer Cost vs. Transfer Ratio (network II)

6.2.3 Standard Deviation (SD) of Bus Arrival Times

Sensitivity analysis with respect to standard deviation of bus arrival times from 1 to 10 minutes is carried out in this section. It can be seen in Figures 6-41 to 6-47 that headways of bus and headways of rail increase, resulting in the operator cost decreased with the increase of standard deviation of bus arrival times shown in Figures 6-49 and 6-50. In contrast, waiting cost and transfer cost increase with the increase of standard deviation of bus arrival times as shown in Figures 6-53 to 6-56. Without any change in the order of value of the total cost and average transfer cost in different scenarios, the total cost decreases slightly with the increase of standard deviation of bus arrival times shown in Figures 6-47 and 6-48. This may be because the increase of user cost is insignificant compared with the decrease of operator cost. While it can be seen in Figures 6-47 to 6-60 that the total cost and average transfer cost increases with the increase of standard deviation of bus arrival times with the order of the total cost and average transfer cost remaining the same at the same standard deviation of bus arrival time. Thus it can be concluded that coordination operations can help to reduce the total cost of a transit system, and the coordination effects are not sensitive to the standard deviation of bus arrival times when the standard deviation of bus arrival times fluctuate within normal range, i.e. from 1 minute to 10 minutes.

T test is carried out for the total cost in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 123.57 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 157.80 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in non-coordination is greater than that in common headway coordination at the 0.95 level of significance.

T test is also carried out for the total cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In

network I, since $t_0 = -61.40 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -61.03 < -t_{0.025,18} = -2.101$ it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the total cost in common headway coordination is less than that in non-coordination in both networks at the 0.95 level of significance.

T test is also carried out for the total cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 65.65 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 107.91 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in non-coordination is greater than that in integer-ratio headway coordination in both networks at the 0.95 level of significance.

T test is also carried out for the average transfer in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 12.06 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 6.72 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance in network II, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in common headway coordination in both networks at the 0.95 level of significance.

T test is also carried out for the average transfer cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -2.58 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -2.23 < -t_{0.025,18} = -2.101$, it can be rejected

that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore average transfer cost in common headway coordination is less than that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the average transfer cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 11.15 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 5.84 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in integer-ratio headway coordination in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give best benefit results when operators are in cooperation within normal range of standard deviation of bus arrival times on some network configurations.

Figure 6-41 Headways of Rail vs. SD of Bus Arrival Time (network I)

Figure 6-42 Headways of Rail vs. SD of Bus Arrival Time (network II)

Figure 6-43 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network I)

Figure 6-44 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network II)

Figure 6-45 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network I)

Figure 6-46 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network II)

Figure 6-47 Total Cost vs. SD of Bus Arrival Time (network I)

Figure 6-48 Total Cost vs. SD of Bus Arrival Time (network II)

Figure 6-49 Operator Cost vs. SD of Bus Arrival Time (network I)

Figure 6-50 Operator Cost vs. SD of Bus Arrival Time (network II)

Figure 6-51 User Cost vs. SD of Bus Arrival Time (network I)

Figure 6-52 User Cost vs. SD of Bus Arrival Time (network II)

Figure 6-53 Waiting Cost vs. SD of Bus Arrival Time (network I)

Figure 6-54 Waiting Cost vs. SD of Bus Arrival Time (network II)

Figure 6-55 Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure 6-56 Transfer Cost vs. SD of Bus Arrival Time (network II)

Figure 6-57 Total Transfer Volume vs. SD of Bus Arrival Time (network I)

Figure 6-58 Total Transfer Volume vs. SD of Bus Arrival Time (network II)

Figure 6-59 Average Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure 6-60 Average Transfer Cost vs. SD of Bus Arrival Time (network II)

6.2.4 Passenger OD Rates of Feeder Bus Services

Sensitivity analysis with respect to passenger OD rates of feeder bus services from 0.1 to 0.9 person/min is carried out in this section. Figures 6-61 to 6-66 show that headways of rail and bus for both operators on both networks decrease with the increase of value in the OD rates in three scenarios. Figures 6-67 and 6-68 show that with value of OD rates increase, the total cost increases, and the differences of the total cost between different scenarios increases. It can be seen in Figures 6-69 to 6-78 that other cost items, e.g. operator cost, user cost, waiting cost, transfer cost and transfer volume increase with the increase of value in the OD rates. It can be seen in Figure 6-79 that the average transfer cost decreases with the increases in feeder OD rates of passengers in both networks. In fact, all the cost items in different coordination operations vary with the increase of OD rates without alternating the order of value of all the cost items for different coordination operations at the same value in feeder OD rates of passengers. Therefore it can be concluded that coordination results are not sensitive to feeder OD rates of passengers within normal range when operators are in cooperation. As such, the integer-ratio headway coordination can be considered feasible and efficient in reducing the total cost and average transfer cost, regardless of how the passenger OD rates of feeder bus services vary.

T test is carried out for the total cost in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 29.87 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, as $t_0 = 52.35 > t_{0.025,18} = 2.101$, it can also be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. The alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in common headway coordination is greater than that in non-coordination in both networks at the 0.95 level of significance.

T test is also carried out for the total cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In

network I, since $t_0 = -15.79 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -20.35 < -t_{0.025,18} = -2.101$ it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the total cost in common headway coordination is less than that in non-coordination in both networks at the 0.95 level of significance.

T test is also carried out for the total cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 14.13 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 31.77 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in non-coordination is greater than that in integer-ratio headway coordination in both networks at the 0.95 level of significance.

T test is also carried out for the average transfer in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 6.57 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 5.30 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in common headway coordination in both networks at the 0.95 level of significance.

T test is also carried out for the average transfer cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -3.68 < -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -2.98 < -t_{0.025,18} = -2.101$, it cannot be

rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore average transfer cost in common headway coordination is less than that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the average transfer cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 6.09 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 4.43 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore average transfer cost in non-coordination is greater than that in integer-ratio headway coordination in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give obvious benefit results when operators are in cooperation within normal range of OD rates of feeder bus services on some network configurations.

Figure 6-61 Headways of Rail vs. OD Rates (network I)

Figure 6-62 Headways of Rail vs. OD Rates (network II)

Figure 6-63 Headways of Bus for Operator 1 vs. OD Rates (network I)

Figure 6-64 Headways of Bus for Operator 1 vs. OD Rates (network II)

Figure 6-65 Headways of Bus for Operator 2 vs. OD Rates (network I)

Figure 6-66 Headways of Bus for Operator 2 vs. OD Rates (network II)

Figure 6-67 Total Cost vs. OD Rates (network I)

Figure 6-68 Total Cost vs. OD Rates (network II)

Figure 6-69 Operator Cost vs. OD Rates (network I)

Figure 6-70 Operator Cost vs. OD Rates (network II)

Figure 6-71 User Cost vs. OD Rates (network I)

Figure 6-72 User Cost vs. OD Rates (network II)

Figure 6-73 Waiting Cost vs. OD Rates (network I)

Figure 6-74 Waiting Cost vs. OD Rates (network II)

Figure 6-75 Transfer Cost vs. OD Rates (network I)

Figure 6-76 Transfer Cost vs. OD Rates (network II)

Figure 6-77 Total Transfer Volume vs. OD Rates (network I)

Figure 6-78 Total Transfer Volume vs. OD Rates (network II)

Figure 6-79 Average Transfer Cost vs. OD Rates (network I)

Figure 6-80 Average Transfer Cost vs. OD Rates (network II)

6.2.5 Coefficient of In-Vehicle Value of Time (VOT)

Sensitivity analysis of coefficient of in-vehicle value of time is carried out from 0.1 to 0.9 in this section. It can be observed in Figures 6-81 to 6-86 that all the decision variables remain constant with the increase of coefficient of in-vehicle value of time. Therefore it can be derived that waiting cost and transfer cost remain constant with the increase of coefficient of in-vehicle value of time which can be verified in Figures 6-93 to 6-96. It can also be seen in Figures 6-97 & 6-98 that transfer volumes remain constant when coefficient of in-vehicle value of time varies. Therefore it can be deducted that average transfer cost remain constant, which would also be confirmed in Figures 6-97 to 6-100. Therefore it can be concluded that variation of coefficient of in-vehicle value of time would not affect the optimal coordination decision variable results.

T test is carried out for the total cost in non-coordination (μ_{t1}) and common headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 58.32 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 102.25 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in non-coordination is greater than that in common headway coordination at the 0.95 level of significance.

T test is also carried out for the total cost in common headway coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -30.54 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -37.89 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the total cost in common headway coordination is less than that in integer-ratio headway coordination at the 0.95 level of significance.

T test is also carried out for the total cost in non-coordination (μ_{t1}) and integer-ratio headway coordination (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 28.28 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. In network II, since $t_0 = 64.65 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the total cost in non-coordination is greater than that in integer-ratio headway coordination at the 0.95 level of significance.

As average transfer cost for all the scenarios remains constant when the coefficient of in-vehicle value of time varies, there is no need to do statistical analysis. The results are obvious that average transfer cost in common headway coordination is the least, and that in non-coordination is the largest. Average transfer cost in integer-ratio headway coordination is in the medium value.

Therefore it can be concluded from T test on the total cost that would give moderate benefit results when operators are in cooperation within normal range of coefficient of in-vehicle value of time.

Figure 6-81 Headways of Rail vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-82 Headways of Rail vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-83 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT
(network I)

Figure 6-84 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT
(network II)

Figure 6-85 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT
(network I)

Figure 6-86 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT
(network II)

Figure 6-87 Total Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-88 Total Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-89 Operator Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-90 Operator Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-91 User Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-92 User Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-93 Waiting Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-94 Waiting Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-95 Transfer Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-96 Transfer Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-97 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-98 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network II)

Figure 6-99 Average Transfer Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure 6-100 Average Transfer Cost vs. Coefficient of In-Vehicle VOT (network II)

6.3 Summary

The sensitivity analysis of decision variables and cost items with respect to essential input parameters when operators are in cooperation have been presented in this chapter. These parameters include value of time, coefficient of in-vehicle value of time, transfer ratio of passengers from bus versus total feeder passenger flow, standard deviation of bus arrival times and passenger OD rates of feeder bus services. The optimal results of coordination are sensitive to transfer ratio, but are not sensitive to the variations of value of time, coefficient of in-vehicle value of time, standard deviation of bus arrival times and feeder OD rates, when operators are in cooperation. The conclusion drawn from the sensitivity analysis further confirms the results obtained in the evaluation study in Chapter 5 that integer-ratio headway coordination is feasible and efficient in reducing the total cost and average transfer cost in real coordination operation when operators are in cooperation.

CHAPTER 7 Sensitivity Analysis of Scenarios When Operators are Independent

7.1 Introduction

In this section, sensitivity analysis of essential decision variables and cost items with respect to important factors is carried out in different coordination operations when operators are independent based on different transit network configurations. These important factors include value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, and passenger OD rates of feeder bus services. The optimal results are illustrated in Appendix D.

7.2 Sensitivity Analysis of Scenarios When Operators are independent

When operators are independent, the coordination between bus and rail can be considered within each operator's scope individually. The sensitivity analysis may be different from that when operators are in cooperation as discussed in the previous chapter. The decision variables studied in this sensitivity analysis include headways of bus and headways of rail. The important cost items include the total cost, operator cost, user cost, waiting cost, transfer cost, total transfer volume and average transfer cost (the total transfer cost divided by the total transfer volume). The input parameters include value of time, coefficient of in-vehicle value of time, the transfer ratio of passengers from bus versus the whole feeder passenger flow, the standard deviation of bus arrival times, passenger OD rates of feeder bus services. The variation range of each parameter and initial input data have been dealt with extensively in Sections 5.3.2 and 5.3.3 respectively. The sensitivity analysis in non-coordination, common headway coordination and integer-ratio headway coordination scenarios are shown in same figures for easy comparisons. The sensitivity analysis of optimal decision variables and important cost items with respect to these five important input parameters are presented respectively in the following sections.

7.2.1 Value of Time

Sensitivity analysis with respect to value of time when operators are independent is carried out in this section. As each operator can set their own headways of bus and rail in coordinated scenarios when operators are independent, the optimal results may be different from those when operators are in cooperation. However it can be found in Figures F-1 to F-20 (Appendix F) that with the increase of value of time, the variations between different coordination scenarios are almost the same as those when operators are in cooperation. Therefore, it can be concluded that fluctuation of value of time within normal range of value does not affect the coordination results when operators are independent.

7.2.2 Transfer Ratio

Sensitivity analysis with respect to transfer ratio when operators are independent is carried out in this section. It can be seen in Figures 7-1 to 7-4 that the orders of value of headways of bus for different scenarios are different from those when operators are in cooperation shown in Figures 6-23 to 6-26. Still the headways of bus in non-coordination remain constant with the increase of transfer ratio. It is reasonable that with the same OD input but different transfer ratio, the optimal decision variables are the same in non-coordination since coordination is not considered in non-coordination scenario. It can be observed in Figures 7-5 and 7-6 that the variation of transfer cost with the increase of transfer ratio results in the same pattern in network I but different pattern in network II. Besides the transfer volumes are different from those when operators are in cooperation, which can be seen in Figures 7-7 and 7-8. Therefore it can be deducted that average transfer cost is different from that when operators are in cooperation which can be observed in Figures 7-9 and 7-10.

Other cost components, i.e. total cost, operator cost, user cost and waiting cost result in the same variation pattern as that when operators are in cooperation, which are illustrated in Figures F-21 to F-40 in Appendix F.

Figure 7-1 Headways of Bus for Operator 1 vs. Transfer Ratio (network I)

Figure 7-2 Headways of Bus for Operator 1 vs. Transfer Ratio (network II)

Figure 7-3 Headways of Bus for Operator 2 vs. Transfer Ratio (network I)

Figure 7-4 Headways of Bus for Operator 2 vs. Transfer Ratio (network II)

Figure 7-5 Transfer Cost vs. Transfer Ratio (network I)

Figure 7-6 Transfer Cost vs. Transfer Ratio (network II)

Figure 7-7 Total Transfer Volume vs. Transfer Ratio (network I)

Figure 7-8 Total Transfer Volume vs. Transfer Ratio (network II)

Figure 7-9 Average Transfer Cost vs. Transfer Ratio (network I)

Figure 7-10 Average Transfer Cost vs. Transfer Ratio (network II)

7.2.3 Standard Deviation (SD) of Bus Arrival Times

Sensitivity analysis with respect to standard deviation of bus arrival times when operators are independent is carried out in this section. It can be observed in Figures 7-11 and 7-12 that the variation of headways of bus with the increase of standard deviation of bus arrival times for different operators in network II results in different patterns from those when operators are in cooperation. It can be found in Figures 7-13 and 7-14 that user cost and transfer cost would be the highest value at the same value of standard deviation of bus arrival times (σ_i) when σ_i is higher than a certain value. Therefore it can be concluded that common headway coordination would not benefit when σ_i is higher than a certain value. Due to operators being independent, the coordination between operators is not well operated. Therefore transfer volumes are different from those when operators are in cooperation, which can be observed in Figures 7-15 and 7-16. In consequence, the variation of average transfer cost with the increase of σ_i is different from that when operators are in cooperation which is shown in Figures 7-17 and 7-18. It is noted that the value of transfer volume in integer-ratio headway coordination is the highest at the same value of σ_i . Other figures that show in the same patterns as those when operators are in cooperation can be found in Figures F-41 to F-60 in Appendix F.

Figure 7-11 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network II)

Figure 7-12 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network II)

Figure 7-13 User Cost vs. SD of Bus Arrival Time (network II)

Figure 7-14 Transfer Cost vs. SD of Bus Arrival Time (network II)

Figure 7-15 Total Transfer Volume vs. SD of Bus Arrival Time (network I)

Figure 7-16 Total Transfer Volume vs. SD of Bus Arrival Time (network II)

Figure 7-17 Average Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure 7-18 Average Transfer Cost vs. SD of Bus Arrival Time (network II)

7.2.4 Passenger OD Rates of Feeder Bus Services

Sensitivity analysis with respect to passenger OD rates of feeder bus services when operators are independent is carried out in this section. As operators are independent, transfer coordination between operators is not well operated. Therefore transfer volumes are different from those when operators are in cooperation, which can be seen in Figures 7-19 and 7-20. It is noted that the value of total transfer volumes in integer-ratio coordination scenario in network I is the highest at the same value of passenger OD rates of feeder bus services. Other cost components and decision variables result in the same variation pattern with the increase of passenger OD rates of feeder bus services, which can be found in Figures F-61 to F-80 in Appendix F. Therefore it can be concluded that coordination results are not sensitive to feeder OD rates of passengers within normal range when operators are in independence.

Figure 7-19 Total Transfer Volume vs. OD Rates (network I)

Figure 7-20 Total Transfer Volume vs. OD Rates (network II)

7.2.5 Coefficient of In-Vehicle Value of Time (VOT)

Sensitivity analysis of coefficient of in-vehicle value of time (k_v) when operators are independent is carried out in this section. It can be found in Figures 7-21 and 7-22 that transfer volumes in integer-ratio headway coordination is highest at the same value of coefficient of in-vehicle value of time (k_v). Other cost components and decision variables result in the same variation pattern with the increase of passenger OD rates of feeder bus services, which can be found in Figures F-61 to F-80 in Appendix F. Therefore it can be concluded that variation of coefficient of in-vehicle value of time would not affect the optimal coordination decision variable results.

Figure 7-21 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network I)

Figure 7-22 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network II)

7.3 Summary

The objective function of the total cost of transit system when operators are independent is different from that when operators are in cooperation, as the coordination between bus and rail is only considered within each operator's scope. The sensitivity analysis is carried out in the same way as discussed in Chapter 6 for comparison purposes. The variation parameters are the same, i.e. value of time, coefficient of in-vehicle value of time, transfer ratio, the standard deviation of bus arrival times, passenger OD rates of feeder bus services. By comparing decision variables and essential cost items at the same value of input parameters, it can be concluded that the optimal results of coordination operations will be affected more by transfer ratio, but are not sensitive to variations of value of time, coefficient of in-vehicle value of time, standard deviation of bus arrival times and feeder OD rates when operators are independent. The conclusion is consistent with conclusion drawn when operators are in cooperation in the previous chapter.

CHAPTER 8 Sensitivity Analysis of Scenarios When Operators are in Competition

8.1 Introduction

In this chapter, extensive sensitivity analysis of essential decision variables and cost items with respect to important factors is carried out in different coordination operations when operators are in competition based on different transit network configurations. These important factors include value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, and passenger OD rates of feeder bus services. The optimal results are illustrated in Appendix G.

8.2 Sensitivity Analysis of Scenarios When Operators are in Competition

When operators are in competition, fares may be adjusted in rather wide scope. In this study, it is assumed that fares are only allowed to adjust in a normal range so that transit users will not shift to other modes, e.g. private car or motorcycle. The sensitivity analysis may be different from that when operators are independent or in cooperation, as another cost item, i.e. revenue is counted in the objective function. To be consistent, the revenue is not counted in derivation of the final optimal total cost. The important cost items include the total cost, operator cost, user cost, waiting cost, transfer cost, total transfer volume and average transfer cost (transfer cost divided by total transfer volume). The input parameters include the value of time, coefficient of in-vehicle value of time, transfer ratio of passengers from bus versus the whole feeder passenger flow, standard deviation of bus arrival times, passenger OD rates of feeder bus services. The variation range of each parameter and initial input data have been dealt with extensively in Sections 5.3.2 and 5.3.3 respectively. The sensitivity analysis in non-coordination, common headway coordination and integer-ratio headway coordination scenarios are shown in same figures so that their results can be compared directly. The sensitivity analysis of decision variables and

cost items with respect to these five important input parameters are presented respectively in the following sections.

8.2.1 Value of Time

Sensitivity analysis with respect to value of time when operators are in competition is carried out in this section. As each operator can set their own headways of bus and rail in coordinated scenarios considering fare adjustment when operators are in competition, the optimal results may be different from those when operators are in cooperation and independence. With the increase of value of time, it can be seen in Figures G-1 to G-4 (Appendix G) that headways of bus and rail decrease with the increase of value of time for both operators in both networks. It can be observed from Figure 8-1 that the optimal headway of rail for integer-ratio headway coordination is the lowest in network II at the same value of value of time. While the optimal headway of bus for integer-ratio headway coordination is almost the same as that for non-coordination for operators 1&2 at the same value of time in network II which can be seen in Figures 8-2 and 8-3, the variation pattern is different from those when operators are in cooperation and independence. It can be observed in Figures 8-4 and 8-5 that the values of optimal total cost and operator cost for integer-ratio headway coordination are the highest at the same value of time in network II.

It can be seen in Figures 8-7 and 8-8 that total transfer volumes for coordinated scenarios decrease with the increase of value of time, while that for non-coordination keeps constant in both networks. This is due to the competition state between operators which discourage passengers from transferring between different operators with the increase of value of time. The order of value of transfer volumes in all scenarios remains constant in all cases. Other cost components and decision variables result in the same variation pattern with the increase of value of time, which can be found in Figures G-1 to G-20 in Appendix G. Therefore, it can be concluded that the fluctuation of value of time within normal range of value does not affect the coordination results when operators are in competition.

Figure 8-1 Headways of Rail vs. Value of Time (network II)

Figure 8-2 Headways of Bus for Operator 1 vs. Value of Time (network II)

Figure 8-3 Headways of Bus for Operator 2 vs. Value of Time (network II)

Figure 8-4 Total cost vs. Value of Time (network II)

Figure 8-5 Operator Cost vs. Value of Time (network II)

Figure 8-6 Waiting Cost vs. Value of Time (network II)

Figure 8-7 Total Transfer Volume vs. Value of Time (network I)

Figure 8-8 Total Transfer Volume vs. Value of Time (network II)

8.2.2 Transfer Ratio

Sensitivity analysis with respect to transfer ratio when operators are in competition is carried out in this section. It can be seen in Figures 8-11 to 8-13 that the orders of value of headways of bus for different scenarios are different from those when operators are in cooperation shown in Figures 6-23 to 6-26. Still the headways of bus in non-coordination remain constant with the increase of transfer ratio. It is reasonable that with the same OD input but different transfer ratio, the optimal decision variables are the same in non-coordination since coordination is not considered in non-coordination scenario.

It can be found in Figures 8-14 and 8-15 that the optimal total cost and operator cost for integer-ratio headway coordination is the highest at the same value of transfer ratio in network II. It can be observed in Figures 8-16 and 8-17 that the total transfer volume for integer-ratio headway coordination in network II is medium, while that in network I is medium when transfer ratio is below 30%, and it is the highest when transfer ratio is above 30%. Other cost components and decision variables result in the same variation pattern with the increase of transfer ratio, which can be found in Figures G-21 to G-40 in Appendix G.

In conclusion, when operators are in competition, integer-ratio headway coordination would not achieve obvious benefits in network II, i.e. reducing the total cost, whereas integer-ratio headway coordination can achieve obvious results for network I when the transfer ratio is higher than 10%.

Figure 8-9 Headways of Rail vs. Transfer Ratio (network II)

Figure 8-10 Headways of Bus for Operator 1 vs. Transfer Ratio (network II)

Figure 8-11 Headways of Bus for Operator 2 vs. Transfer Ratio (network II)

Figure 8-12 Total cost vs. Transfer Ratio (network II)

Figure 8-13 Operator Cost vs. Transfer Ratio (network II)

Figure 8-14 Total Transfer Volume vs. Transfer Ratio (network I)

Figure 8-15 Total Transfer Volume vs. Transfer Ratio (network II)

8.2.3 Standard Deviation (SD) of Bus Arrival Times

Sensitivity analysis with respect to standard deviation of bus arrival times (σ_i) when operators are in competition is carried out in this section. It can be seen in Figures 8-16 to 8-22 that the variation of optimal results of headways of rail, headways of bus, total cost, operator cost, user cost and waiting cost is different from that when operators are in cooperation or independence in network II. Total cost and operator cost for integer-ratio headway coordination in network II are at the highest value at the same value in standard deviation of bus arrival times as it can be observed in Figures 8-19 and 8-20. It can be seen in Figure 8-23 that the value in total transfer volumes in network I for integer-ratio headway coordination is the highest at the same value in standard deviation of bus arrival times. As it can be seen in Figure 8-19 that total cost for integer-ratio headway coordination would get closer to that for non-coordination with the increase of σ_i .

Other cost components and decision variables result in the same variation pattern with the the standard deviation of bus arrival times, which can be found in Figures G-41 to

G-60 in Appendix G. Therefore it can be concluded that coordination operations can help to reduce the total cost of a transit system on certain network configurations when operators are in competition within normal range of value in the standard deviation of bus arrival times.

Figure 8-16 Headways of Rail vs. SD of Bus Arrival Time (network II)

Figure 8-17 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network II)

Figure 8-18 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network II)

Figure 8-19 Total cost vs. SD of Bus Arrival Time (network II)

Figure 8-20 Operator Cost vs. SD of Bus Arrival Time (network II)

Figure 8-21 User Cost vs. SD of Bus Arrival Time (network II)

Figure 8-22 Waiting Cost vs. SD of Bus Arrival Time (network II)

Figure 8-23 Total Transfer Volume vs. SD of Bus Arrival Time (network I)

8.2.4 Passenger OD Rates of Feeder Bus Services

Sensitivity analysis with respect to passenger OD rates of feeder bus services when operators are in competition is carried out in this section. It can be seen in Figures 8-24 to 8-28 that variation of optimal value in headway of rail, headway of bus, total cost, operator cost and waiting cost with the increase of passenger OD rates of feeder bus services is different from that when operators are in cooperation and independence. It is noted in Figure 8-26 that the value in total cost in integer-ratio headway coordination is highest at the same value in passenger OD rates of feeder bus services.

Other cost components and decision variables result in the same variation pattern with the same passenger OD rates of feeder bus services, which can be found in Figures G-61 to G-80 in Appendix G. It can be observed in Figures G-61 to G-80 that all the cost items in different coordination operations vary with the increase of value in the OD rates without alternating their orders for different coordination operations in both networks. Therefore it can be concluded that fluctuation within normal range of value in the OD rates of passengers does not affect the results of coordination for some network configurations when operators are in competition.

Figure 8-24 Headways of Rail vs. OD Rates (network II)

Figure 8-25 Headways of Bus for Operator 2 vs. OD Rates (network II)

Figure 8-26 Total cost vs. OD Rates (network II)

Figure 8-27 Operator Cost vs. OD Rates (network II)

Figure 8-28 Waiting Cost vs. OD Rates (network II)

Figure 8-29 Total Transfer Volume vs. OD Rates (network I)

8.2.5 Coefficient of In-Vehicle Value of Time (VOT)

Sensitivity analysis of coefficient of in-vehicle value of time (k_v) when operators are in competition is carried out in this section. The sensitivity analysis results are shown in Figures G-81 to G100 in Appendix G. The variation patterns that are different from those when operators are in cooperation are shown in Figures 8-30 to 8-36 as follows. It can be observed that decision variables, i.e. headway of rail, headway of bus remain constant with the increase of k_v . It is noted in Figures 8-33 and 8-34 that the value in total cost and operator cost for integer-ratio headway coordination is highest at the same value of k_v in network II. However with the increase of k_v , the order of value of total cost and transfer cost remains constant. Therefore it can be concluded that integer-ratio headway coordination would not achieve obvious benefit for some network configuration, and the variation of coefficient of in-vehicle value of time would not affect the optimal coordination decision variable results.

Figure 8-30 Headways of Rail vs. Coefficient of In-Vehicle VOT (network II)

Figure 8-31 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT (network II)

Figure 8-32 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT (network II)

Figure 8-33 Total Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 8-34 Operator Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 8-35 Waiting Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure 8-36 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network I)

8.3 Summary

The model formulation of the total cost of the transit system when operators are in competition is different from that when operators are in cooperation or independent, as fare adjustment is taken into consideration in this circumstance. The sensitivity analysis is carried out in the same way as those in Chapters 6 and 7. The parameters studied are the same, i.e. value of time, coefficient of in-vehicle value of time, transfer ratio of passengers from bus versus the whole feeder passenger flow, standard deviation of bus arrival times, passenger OD rates of feeder bus services. By comparing decision variables and essential cost items at the same value of input parameters, it can be concluded that the optimal results of coordination operations would be affected more by transfer ratio and network configurations, but are not sensitive to the variations of value of time, standard deviation of bus arrival times and feeder OD rates, when operators are in competition.

CHAPTER 9 Comparison of Integer-ratio Headway Coordination

Improvement in Different Scenarios

9.1 Introduction

It can be drawn from evaluation study and the sensitivity analysis in Chapters 5 to 8 that integer-ratio headway coordination is feasible and efficient in reducing the total cost and average transfer cost when operators are in cooperation or independent and for some network configurations when operators are in competition. In this chapter, in order to determine which operation policy is the most efficient and feasible in integer-ratio headway coordination, comparisons of integer-ratio headway coordination improvement under different operation policies are illustrated when different input parameters varies. The changes in the total cost and average transfer cost based on the non-coordinated scenarios under each operation policy are shown in Appendix H.

9.2 Comparison of Integer-ratio Headway Coordination Improvement in Different Scenarios

It can be concluded from previous sections that integer-ratio headway coordination can be a feasible and efficient method in coordinating bus and rail in a multimodal and multi-operator transit system when operators are in cooperation. In order to compare the integer-ratio headway coordination operations under different operation policies, the changes in total cost and average transfer cost based on the non-coordinated scenarios are shown in the same figures for both networks respectively, when only one parameter varies. Similarly, these parameters are the value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, passenger OD rates of feeder bus services. The analysis of variation of total cost and average transfer cost with respect to these five important input parameters are presented respectively in the following sections.

9.2.1 Effects of Value of Time

The changes in total cost and average transfer cost under different operation policies for both networks when value of time varies are illustrated in Figures 9-1 to 9-4. The variations are based on the total cost in non-coordinated operation using the same input data under each operation policy. It can be seen that the decrease in total cost and average transfer cost are rather close when operators are either independent or in competition at each value of time in network I, while the results are opposite in network II. The average transfer cost under all operation policies decrease in both networks, while the total costs decrease under all operation policies in network I, but increase when operators are in competition in network II. It can also be seen in Figures 9-1 to 9-4 that the percentages of decrease in the total cost and the average transfer cost for integer-ratio headway coordination when operators are in cooperation are higher than those when operators are independent and in competition. Therefore it can be concluded that integer-ratio headway coordination may give best results when operators are in cooperation within normal range of value of time.

T test is carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -2.28 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -43.41 < -t_{0.025,18} = -2.101$, it can also be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in independence in both networks at the 0.05 level of significance.

T test is also carried out for the changes of total cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.05 < t_{0.025,18} = 2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. In network II, however since $t_0 = -77.60 < -t_{0.025,18} = -2.101$ it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost

when operators are in independence are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -2.23 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -124.15 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in competition in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -44.49 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -50.81 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in independence in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.00 < t_{0.025,18} = 2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = 8.90 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are greater than that when operators are in competition in network II at the 0.95 level of significance.

Figure 9-1 Changes in Total Cost vs. Value of Time (network I)

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_1) and competition (μ_2) in networks I and II respectively. In network I, since $t_0 = -44.49 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -37.32 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in competition in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give best results when operators are in cooperation within normal range of value of time. It is noted that integer-ratio headway coordination when operators are in independence and competition may give opposite results on different network configurations when value of time varies within normal range.

Figure 9-2 Changes in Total Cost vs. Value of Time (network II)

Figure 9-3 Changes in Average Transfer Cost vs. Value of Time (network I)

Figure 9-4 Changes in Average Transfer Cost vs. Value of Time (network II)

9.2.2 Effects of Transfer Ratio

The changes in total cost and average transfer cost under different scenarios for both networks when transfer ratio varies are shown in Figures 9-5 to 9-8. It is quite reasonable that when transfer ratio reduces, the decrease of total cost in integer-ratio headway coordination is less than that when operators are in cooperation or being independent. The increase in total cost in integer-ratio headway coordination is higher than that when operators are in competition. Thus it can be concluded that integer-ratio headway coordination can improve coordination results by greatly reducing the total cost at the same transfer ratio when operators are in cooperation. Besides, the higher percentage of transfer ratio, the more improvement it would achieve.

T test is carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -0.34 > -t_{0.025,16} = -2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. In network II, since $t_0 = -23.85 < -t_{0.025,16} = -2.120$, it can be rejected

that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in independence in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.01 < t_{0.025,16} = 2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However in network II, since $t_0 = -14.36 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in independence are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -0.34 > -t_{0.025,16} = -2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -36.96 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -22.46 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -21.83 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance in network II, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in

cooperation are less than that when operators are in independence in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.00 < t_{0.025,16} = 2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = 3.24 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are greater than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -22.46 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -18.61 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are less than that when operators are in competition in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give obvious benefit results when operators are in cooperation within normal range of transfer ratio on some network configuration.

Figure 9-5 Changes in Total Cost vs. Transfer Ratio (network I)

Figure 9-6 Changes in Total Cost vs. Transfer Ratio (network II)

Figure 9-7 Changes in Average Transfer Cost vs. Transfer Ratio (network I)

Figure 9-8 Changes in Average Transfer Cost vs. Transfer Ratio (network II)

9.2.3 Effects of Standard Deviation (SD) of Bus Arrival Times

The changes in total cost and average transfer cost under different scenarios when standard deviation of bus arrival times varies are shown in Figures 9-9 to 9-12. It can be seen that when operators are in cooperation, the changes in the reduction of total cost is the largest at the same value of standard deviation for the same transit network, whereas when operators are in competition, the changes in the increase of total cost is largest in Network II. It can be observed in Figures 9-11 and 9-12 that all the average transfer cost decreases, regardless of the operation policies, and the improvement in reducing average transfer cost when operators are in cooperation is the greatest. Therefore it can be concluded that integer-ratio headway coordination can achieve the best coordination benefit when operators are in cooperation within normal range of the standard deviation of bus arrival times.

T test is carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -1.69 > -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. In network II, since $t_0 = -24.56 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in independence in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.04 < t_{0.025,18} = 2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -22.94 < -t_{0.025,18} = -2.101$ it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in independence are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -1.65 > -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -40.50 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -111.74 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -42.83 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in independence in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.00 < t_{0.025,18} = 2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -5.61 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In

network I, since $t_0 = -111.74 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -48.24 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are less than that when operators are in competition in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give best benefit results when operators are in cooperation within normal range of standard deviation of bus arrival times on some network configurations.

Figure 9-9 Changes in Total Cost vs. SD of Bus Arrival Times (network I)

Figure 9-10 Changes in Total Cost vs. SD of Bus Arrival Times (network II)

Figure 9-11 Changes in Average Transfer Cost vs. SD of Bus Arrival Times (network I)

Figure 9-12 Changes in Average Transfer Cost vs. SD of Bus Arrival Times (network II)

9.2.4 Effects of Passenger OD Rates of Feeder Bus Services

The changes in total cost and average transfer cost in different scenarios when feeder OD rates varies are shown in Figures 9-13 to 9-16. It can be seen from Figures 9-13 and 9-14 that when operators are in cooperation, the improvement in reducing the total cost is the largest in network I, whereas the total cost when operators are in competition increases in Network II. While it can also be found from Figures 9-15 and 9-16 that with the average transfer cost decreases, regardless of the operation policies, and the improvement in reducing the average transfer cost when operators are in cooperation is the best. Therefore it can be concluded that integer-ratio headway coordination can achieve the best coordination benefit when operators are in cooperation within normal range of value in the passenger OD rates of feeder bus services.

T test is carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In networks I, since

$t_0 = -1.63 > -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -35.65 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in independence in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In networks I, since $t_0 = 0.04 < t_{0.025,18} = 2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -60.68 < -t_{0.025,18} = -2.101$ it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in independence are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -1.59 > -t_{0.025,18} = -2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -89.90 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -31.58 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -33.88 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$

at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in independence in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.00 < t_{0.025,18} = 2.101$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = 5.99 > t_{0.025,18} = 2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are greater than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In networks I, since $t_0 = -31.58 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -26.86 < -t_{0.025,18} = -2.101$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in competition in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give obvious benefit results when operators are in cooperation within normal range of OD rates of feeder bus services on some network configurations.

Figure 9-13 Changes in Total Cost vs. OD Rates (network I)

Figure 9-14 Changes in Total Cost vs. OD Rates (network II)

Figure 9-15 Changes in Average Transfer Cost vs. OD Rates (network I)

Figure 9-16 Changes in Average Transfer Cost vs. OD Rates (network II)

9.2.5 Effects of Coefficient of In-vehicle Value of Time (VOT)

The changes in total cost and average transfer cost in different scenarios when coefficient of in-vehicle value of time varies are shown in Figures 9-17 to 9-20. It can be seen from Figures 9-17 and 9-18 that when operators are in cooperation, the improvement in reducing the total cost is the largest in Network I, while the total cost when operators are in competition is increases in Network II. While it can also be found from Figures 9-19 and 9-20 that with the average transfer cost decreases, regardless of the operation policies, and the improvement in reducing the average transfer cost when operators are in cooperation is the best. Therefore it can be concluded that integer-ratio headway coordination can achieve the best coordination benefit when operators are in cooperation within normal range of value in coefficient of in-vehicle value of time.

T test is carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -1.86 < -t_{0.025,16} = -2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. In network II, since $t_0 = -31.42 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in independence in both networks at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.03 < t_{0.025,16} = 2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -63.57 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in independence are less than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of total cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -1.84 > -t_{0.025,16} = -2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = -80.02 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of total cost when operators are in cooperation are less than that when operators are in competition in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and independence (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = -46.25 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. In network II, since $t_0 = -44.10 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in independence in both networks at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in independence (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In network I, since $t_0 = 0.00 < t_{0.025,16} = 2.120$, it cannot be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance. However, in network II, since $t_0 = 7.02 > t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} > \mu_{t2}$. Therefore the changes of average transfer cost when operators are in independence are greater than that when operators are in competition in network II at the 0.95 level of significance.

T test is also carried out for the changes of average transfer cost when operators are in cooperation (μ_{t1}) and competition (μ_{t2}) in networks I and II respectively. In

network I, since $t_0 = -46.25 < -t_{0.025,16} = -2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. However, in network II, since $t_0 = -36.37 < -t_{0.025,16} = 2.120$, it can be rejected that $\mu_{t1} = \mu_{t2}$ at the 0.05 level of significance, the alternative hypothesis is accepted that $\mu_{t1} < \mu_{t2}$. Therefore the changes of average transfer cost when operators are in cooperation are less than that when operators are in competition in both networks at the 0.95 level of significance.

Therefore T test on changes of average transfer cost confirms with that on changes of total cost, on which a conclusion can be drawn that integer-ratio headway coordination may give best benefit results when operators are in cooperation within normal range of coefficient of in-vehicle value of time on some network configurations.

Figure 9-17 Changes in Total Cost vs. Coefficient of In-vehicle VOT (network I)

Figure 9-18 Changes in Total Cost vs. Coefficient of In-vehicle VOT (network II)

Figure 9-19 Changes in Average Transfer Cost vs. Coefficient of In-vehicle VOT (network I)

Figure 9-20 Changes in Average Transfer Cost vs. Coefficient of In-vehicle VOT (network II)

9.3 Summary

In order to determine which operation policy gives the best efficiency in integer-ratio headway coordination, comparisons of essential cost items, i.e. the total cost and average transfer cost are discussed in this chapter. It can be concluded that integer-ratio headway coordination is an efficient and feasible method when operators adopted a cooperation policy, whereas it may not be suitably to adopt a competition policy, as in this case, the coordination benefit is not obvious sometimes even worse. The results in this chapter also show that optimal results of coordination operations will be affected more by transfer ratio, but are not sensitive to the variations of value of time, coefficient of in-vehicle value of time, standard deviation of bus arrival times and passenger OD rates of feeder bus services.

Therefore from the numerical results and sensitivity analysis under different operation policies, not only can this model and optimisation procedures be verified, but also the conclusion can be confirmed that when operators are in cooperation, integer-ratio coordination in a multimodal and multi-operator transit system is the most efficient

and feasible method in coordination operation, while the competition between operators is not recommended.

CHAPTER 10 Summary and Conclusion

10.1 Summary and Conclusions

With rapid urbanisation of the world, public transit system has been more and more important in urban area especially these large metropolitan cities. As more and more people rely public transit system to work or in other travel purposes in urban areas, convenience, reliability, and easiness to transfer between modes are essential for the whole integrated transit system. The problem of public transit system integration can be solved in three dimensions of analysis and evaluation. The first is integration in space, while the second integration in time and the third is fare standardization and charge integration (Gattuso, et al., 2002). The public transit system in Singapore is one of the most successful transit systems in the world, which has considered the problem of transit integration in three dimensions. As multiple operators that do not overlay the service areas coexist in the public transit system in Singapore, what the situations will be like when multiple operators operate multimodal transit in the service areas under different operation policies originates this research topic.

A systematic analysis is conducted first which forms the basis of the methodology that focuses on formulating cost functions and identifying groups of passengers for analysis. The objective function is to minimize the total cost of the transit system, which consists of operator cost and users' waiting cost, transfer cost and in-vehicle cost. The groups of passenger flows entering terminal stations are categorised into three parts, one is those passengers transferring from bus, and then those passengers transferring from rail, and the last is those passengers transferring from modes other than bus and rail. Transit path choice is considered in the further detailed classification of groups of passenger flows. Two models, i.e. path search model and path decision model are employed to capture the travel behaviour of transit passengers in choosing their multimodal transit paths. The K^{th} shortest path algorithm is further developed by integrating the shortest path algorithm and the K^{th} shortest

path ordering algorithm, to search for the first three shortest paths. Improving the Label Algorithm, K^{th} shortest path algorithm in a multimodal and multi-operator transit system is developed. The path decision model is applied to get the volumes of passengers on each shortest path. Thus the detailed classification of groups of passenger flows can be obtained when the feedback system reaches Stackelberg equilibrium, at the same time the optimal results can be derived as well.

The evaluation of coordination in different operation strategies between operators was discussed extensively in Chapter 5. The Framework of AIMMS programmes was also presented. By employing a typical data set as initial input data, preliminary results and conclusions are drawn that in an integrated multimodal and multi-operator transit system, integer-ratio headway coordination can be an efficient and feasible method in coordination operations, what is more, policy of cooperation between operators is an efficient strategy that can reduce the total cost of transit system and improve coordination; whereas policy of competition between operators is the least efficient when making coordination operations within each operator's scope

To verify the model and optimisation procedure, the sensitivity analysis of decision variables (i.e. headways of bus and rail) and cost items with respect to the important input parameters under different operation policies was reported in Chapters 6 to 8. These important input parameters include the value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, passenger OD rates of feeder bus services. Extensive sensitivity analysis and statistical analysis were carried out in Chapter 6 presenting analysis results when operators are in cooperation. It was concluded that the coordination effect would not be affected when value of time, coefficient of in-vehicle value of time, standard deviation of bus arrival times, passenger OD rates of feeder bus services vary within normal value of range, but would be affected by transfer ratio. Integer-ratio headway coordination is a moderate coordinate method, which would give moderate benefit by reducing of total cost and average transfer cost compared with those in non-coordination.

Different sensitivity analysis results when operators are independent and in competition were presented in the Chapters 7 and 8 following Chapter 6. It was

concluded that coordination effect would be worse on some network configurations, e.g. network II when operators are in competition.

Comparison of integer-ratio headway coordination between different operation policies was conducted in Chapter 9. T test is carried out for changes of total cost and average transfer cost in this chapter. It was concluded that integer-ratio headway coordination would give the obvious coordination effect on some network configurations, when operators are in cooperation. The studies carried out from Chapters 6 to 9 further confirm the preliminary conclusions that have been drawn in Chapter 5 that integer-ratio headway coordination is an efficient and feasible method when operators adopted a cooperation policy, whereas it may not be suitably to adopt a competition policy, as in this case, the coordination benefit is not obvious sometimes even worse; besides, coordination results would be affected more by transfer ratio, but are not sensitive to the variations of value of time, coefficient of in-vehicle value of time, standard deviation of bus arrival times and passenger OD rates of feeder bus services.

In this research, the formulation of the objective function for coordinating a multimodal and multi-operator transit system is proposed. The detailed analysis of cost items and classifications of groups of OD matrices were presented during the formulation. Typical input data that is comparable to transit system in Singapore is employed to draw the preliminary conclusions based on different scenarios. The sensitivity analysis further confirm these conclusions especially that integer-ratio headway coordination can be considered as an efficient and feasible coordination method in a multimodal and multi-operator transit system, only when operators adopt cooperation policies between each other, while the competition between operators is not recommended. It is noted that the competition is not complete which is restricted in adjusting fare during certain range.

10.2 Contributions

The contributions in this research can be illustrated as follows.

I. Consideration of Different Operation Policies

Most of the previous research in this area focused on the operation policies of competition, usually Nash equilibrium is reached. As the state of equilibrium is not the optimal, other operation policies, i.e. competition and independence, are considered as well in this research. This can be verified from the sensitivity analysis that coordination results in integer-ratio headway coordination when operators are in coordination are much better than those when operators are in competition.

II. Improved K^{th} Shortest Path

Due to the special characteristics of transit trip assignment, the label correcting algorithm is improved so that the selected paths can be traced back. The K^{th} shortest path algorithm is proposed, and the first three shortest paths are obtained through programmes developed using AIMMS platform.

III. Schedule Coordination Considering Path Choice

No previous research work on coordination considered path choice by passengers which will affect the ultimate transfer volume of passengers. Social factors, e.g. age is considered in this path choice model to reflect that different age groups of passengers who may behave differently in their decision of which multimodal transit paths to take. These multimodal transit paths may consist of review from different operators and different modes.

IV. Algorithm for System Optimisation

As coordination and change of headways of bus and rail will affect the decision of those passengers en route, transfer volume will be affected consequently which will affect the optimal coordination results in return. The Stackelberg equilibrium state is described by an algorithm with feedback mechanism, when both transfer volume and schedule of important decision variables, i.e. headways of bus and rail, offset between bus and rail, reach the final stable results.

10.3 Limitations

The objective function of this research is to minimize the total cost of the integrated transit system, while the total cost being the sum of operator cost and user cost. The evaluation of the total cost and average transfer cost is the crux of this study. Using the demand data that are comparable to transit systems in Singapore, the optimisation study was carried out first, followed by extensive sensitivity analysis and statistical analysis of optimal decision variables and cost items with respect to important input parameters. The conclusions drawn from the sensitivity analysis and statistical analysis are consistent with the preliminary conclusion drawn from evaluation study. Although the model and programmes provide an ideal platform for evaluations, there are still some limitations of this research work that can be treated as follows:

I. Value of Time.

In order to simplify the problem, the value of time for passengers is assumed to be uniform in the whole trip. Value of time for passengers in waiting may not be the same as that for passengers transferring. Separate research work can be extended to evaluate the value of time for those passengers who take public transit system in different stages in the journey.

II. Distribution of Bus Arrival Times

The distribution of bus arrival times is assumed to be a Normal Distribution in this optimisation study and sensitivity analysis. Other relevant form of distribution of bus arrival time can be studied by analyzing collected data from real operations.

III. Vehicle Cost of Bus and Rail

The vehicle cost of bus and rail is calibrated comparable as the average variable cost for bus and rail of Singapore transit systems from annual reports by transit companies in Singapore. More precise calibration of vehicle cost of bus and rail shall be captured when detailed financial information are available.

IV. The Operator Policy of Competition between Operators is Incomplete

In this research, the competition between operators is incomplete, as the fare adjustment is not allowed out of a range, which mimics to the situation of Singapore transit system.

10.4 Further Research

Although this research has been conducted to cover an extensive area of topics, there are still some possible research areas that can be further extended. It should be noted that some of the extension may increase the complexity of model formulations, thus the extensive model may become formidable to be solved, in consequent, other solution techniques may need to be developed. These possible research areas are summarized as follows:

I. Estimation of Value of Time.

There may be differences among the values of time for passengers waiting for a vehicle, when making a transfer, and when travelling in vehicles. This issue should be investigated further.

II. Estimation of Distribution of Vehicle Arrivals

The normal distribution is assumed for bus arrival time, and trains of rail are assumed to be deterministic in this research. Future research may consider focusing on developing empirical distribution of vehicle arrivals based on data collected from real operations.

III. Developing Further Transit Path Choice Models

The transit path choice model that consists of path search model and path decision model is employed in this research. As it reflects the effect of the individual social-economic characteristics and how their decisions are changed, behaviour studies should be carried out more frequently and cover a wide spectrum of factors to enhance the robustness in the results of this model.

IV. Estimation of Vehicle Cost of Bus and Rail

More precise calibration of vehicle cost of bus and rail shall be derived when detailed financial information can be obtained.

V. Formulation of Objective Function

The objective function of this research work is formulated as the total cost which is summation of operator cost and user cost which is categorized as cost benefit analysis. Therefore other cost items, e.g. opportunity cost for operators and users, may be added to formulate the objective function. It should be noted that how to calibrate the opportunity cost of operators and users is still inconclusive.

VI. Studies of other Operation Scenarios

The model developed in this research looks into operation scenarios under a multimodal and multi-operator transit system. As some unique characteristics only relevant to Singapore's context were embedded in this model, future studies can be carried out to evaluate other scenarios, e.g. government subsidies, single operator multimodal, multi-operator and single mode market, as well as implication of opportunity costs.

VII. Studies of other Operator Policies

Considering the three possible operator policies in this study, the restriction of competition is set which makes the competition between operators an incomplete scenario. Further study may look into the completely deregulated transit market, at the same time coordination between different modes and operators are also considered.

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APPENDIX A Transfer Time in Different Scenarios

Table A-1 Transfer Time in Non-Coordinated Scenarios

```

if (  $i_1$  = Origin station ) then
  if(  $p \neq$  'NULL' or  $m \neq$  'walking') then
     $W_{i_1}^{mnpq} := +\infty$  ;
  else
    if(  $n =$  'walking') then
       $W_{i_1}^{mnpq} := 0$ ;
    elseif( $n =$  'bus') then
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    elseif (  $n =$  'rail') then
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif

elseif (  $i_1 \neq$  Origin station ) then
  if(  $p = q$  and  $m = n$  )then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif (  $n =$  'walking') then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif(  $n =$  'bus') then
     $W_{i_1}^{mnpq} := H_b^q / 2$  ;
  elseif (  $n =$  'rail') then
     $W_{i_1}^{mnpq} := H_r^q / 2$  ;
  endif
endif
endif

```

Table A-2 Transfer Time in Common Headway Coordination When Cooperation

```

if (  $i_1$  = Origin station ) then
  if(  $p \neq$  'NULL' or  $m \neq$  'walking') then
     $W_{i_1}^{mnpq} := +\infty$  ;
  else
    if(  $n$ ='walking') then
       $W_{i_1}^{mnpq} := 0$ ;
    elseif( $n$ ='bus') then
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    elseif (  $n$ ='rail') then
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
elseif (  $i_1 \neq$  Origin station ) then
  if(  $p=q$  and  $m=n$  )then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif (  $n$ ='walking') then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif(  $n$ ='bus') then
    if (  $m$ ='rail') then
       $W_{i_1}^{mnpq} := 0$ ;
    else
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    endif
  elseif (  $n$ ='rail') then
    if (  $m \neq$  'walking') then
       $W_{i_1}^{mnpq} := 0$ ;
    else
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif
endif

```

Table A-3 Transfer Time in Integer-ratio Headway Coordination When Cooperation

```
if (  $i_1$  = Origin station ) then
```

```
  if(  $p \neq$  'NULL' or  $m \neq$  'walking') then
```

$$W_{i_1}^{mpq} := +\infty ;$$

```
  else
```

```
    if(  $n =$  'walking') then
```

$$W_{i_1}^{mpq} := 0;$$

```
    elseif( $n =$  'bus') then
```

$$W_{i_1}^{mpq} := H_b^q / 2 ;$$

```
    elseif (  $n =$  'rail') then
```

$$W_{i_1}^{mpq} := H_r^q / 2 ;$$

```
    endif
```

```
  endif
```

```
elseif (  $i_1 \neq$  Origin station ) then
```

```
  if(  $p = q$  and  $m = n$  )then
```

$$W_{i_1}^{mpq} := 0;$$

```
  elseif (  $n =$  'walking') then
```

$$W_{i_1}^{mpq} := 0;$$

```
  elseif(  $n =$  'bus') then
```

```
    if (  $m =$  'rail') then
```

$$W_{i_1}^{mpq} := H_r^q * (\phi^q - 1) / 2 ;$$

```
    else
```

$$W_{i_1}^{mpq} := H_b^q / 2 ;$$

```
    endif
```

```
  elseif (  $n =$  'rail') then
```

```
    if (  $m \neq$  'walking') then
```

$$W_{i_1}^{mpq} := 0;$$

```
    else
```

$$W_{i_1}^{mpq} := H_r^q / 2 ;$$

```
    endif
```

```
  endif
```

```
endif
```

Table A-4 Transfer Time in Common Headway Coordination When Independence
and Competition

```

if (  $i_1$  = Origin station ) then
  if(  $p \neq \text{'NULL'}$  or  $m \neq \text{'walking'}$ ) then
     $W_{i_1}^{mnpq} := +\infty$  ;
  else
    if(  $n = \text{'walking'}$ ) then
       $W_{i_1}^{mnpq} := 0$ ;
    elseif( $n = \text{'bus'}$ ) then
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    elseif (  $n = \text{'rail'}$ ) then
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
elseif (  $i_1 \neq \text{Origin station}$  ) then
  if(  $p = q$  and  $m = n$  )then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif (  $n = \text{'walking'}$ ) then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif(  $n = \text{'bus'}$ ) then
    if (  $p = q$  and  $m = \text{'rail'}$ ) then
       $W_{i_1}^{mnpq} := 0$ ;
    else
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    endif
  elseif (  $n = \text{'rail'}$ ) then
    if (  $p = q$  and  $m \neq \text{'walking'}$ ) then
       $W_{i_1}^{mnpq} := 0$ ;
    else
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif
endif

```

Table A-5 Transfer Time in Integer-ratio Headway Coordination When Independence and Competition

```

if (  $i_1$  = Origin station ) then
  if(  $p \neq \text{'NULL'}$  or  $m \neq \text{'walking'}$ ) then
     $W_{i_1}^{mnpq} := +\infty$  ;
  else
    if(  $n = \text{'walking'}$ ) then
       $W_{i_1}^{mnpq} := 0$ ;
    elseif( $n = \text{'bus'}$ ) then
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    elseif (  $n = \text{'rail'}$ ) then
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
elseif (  $i_1 \neq$  Origin station ) then
  if(  $p = q$  and  $m = n$ ) then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif (  $n = \text{'walking'}$ ) then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif(  $n = \text{'bus'}$ ) then
    if (  $p = q$  and  $m = \text{'rail'}$ ) then
       $W_{i_1}^{mnpq} := H_r^q * (\phi^q - 1) / 2$  ;
    else
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    endif
  elseif (  $n = \text{'rail'}$ ) then
    if (  $p = q$  and  $m \neq \text{'walking'}$ ) then
       $W_{i_1}^{mnpq} := 0$ ;
    else
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif
endif

```

Table A-6 Transfer Time in Non-Coordinated Scenarios

```

if (  $i_1$  = Origin station ) then
  if(  $m \neq$  'bus') then
     $W_{i_1}^{mnpq} := +\infty$  ;
  else
    if(  $n$ ='walking') then
       $W_{i_1}^{mnpq} := 0$ ;
    elseif( $n$ ='bus') then
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    elseif (  $n$ ='rail') then
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif

elseif (  $i_1 \neq$  Origin station ) then
  if(  $p=q$  and  $m=n$  )then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif (  $n$ ='walking') then
     $W_{i_1}^{mnpq} := 0$ ;
  elseif(  $n$ ='bus') then
     $W_{i_1}^{mnpq} := H_b^q / 2$  ;
  elseif (  $n$ ='rail') then
     $W_{i_1}^{mnpq} := H_r^q / 2$  ;
  endif
endif
endif

```


Table A-7 Transfer Time in Common Headway Coordination When Cooperation

```

if (  $i_1$  = Origin station ) then
  if (  $m \neq$  'bus' ) then
     $W_{i_1}^{mnpq} := +\infty$  ;
  else
    if (  $n$  = 'walking' ) then
       $W_{i_1}^{mnpq} := 0$  ;
    elseif (  $n$  = 'bus' ) then
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    elseif (  $n$  = 'rail' ) then
       $W_{i_1}^{mnpq} := 0$  ;
    endif
  endif
elseif (  $i_1 \neq$  Origin station ) then
  if (  $p=q$  and  $m=n$  ) then
     $W_{i_1}^{mnpq} := 0$  ;
  elseif (  $n$  = 'walking' ) then
     $W_{i_1}^{mnpq} := 0$  ;
  elseif (  $n$  = 'bus' ) then
    if (  $m$  = 'rail' ) then
       $W_{i_1}^{mnpq} := 0$  ;
    else
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    endif
  elseif (  $n$  = 'rail' ) then
    if (  $m \neq$  'walking' ) then
       $W_{i_1}^{mnpq} := 0$  ;
    else
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif
endif

```

Table A-8 Transfer Time in Integer-ratio Headway Coordination When Cooperation

```

if (  $i_1$  = Origin station ) then
  if (  $m \neq$  'bus' ) then
     $W_{i_1}^{mnpq} := +\infty$  ;
  elseif (  $n$  = 'walking' ) then
     $W_{i_1}^{mnpq} := 0$  ;
  elseif (  $n$  = 'bus' ) then
     $W_{i_1}^{mnpq} := H_b^q / 2$  ;
  elseif (  $n$  = 'rail' ) then
     $W_{i_1}^{mnpq} := 0$  ;
  endif
endif
elseif (  $i_1 \neq$  Origin station ) then
  if (  $p=q$  and  $m=n$  ) then
     $W_{i_1}^{mnpq} := 0$  ;
  elseif (  $n$  = 'walking' ) then
     $W_{i_1}^{mnpq} := 0$  ;
  elseif (  $n$  = 'bus' ) then
    if (  $m$  = 'rail' ) then
       $W_{i_1}^{mnpq} := H_r^q * (\phi^q - 1) / 2$  ;
    else
       $W_{i_1}^{mnpq} := H_b^q / 2$  ;
    endif
  elseif (  $n$  = 'rail' ) then
    if (  $m \neq$  'walking' ) then
       $W_{i_1}^{mnpq} := 0$  ;
    else
       $W_{i_1}^{mnpq} := H_r^q / 2$  ;
    endif
  endif
endif
endif

```

Table A-9 Transfer Time in Common Headway Coordination When Independence and Competition

<pre> if (i_1 = Origin station) then   if($m \neq$ 'bus') then $W_{i_1}^{mnpq} := +\infty$;   else   if(n='walking') then $W_{i_1}^{mnpq} := 0$;   elseif(n='bus') then $W_{i_1}^{mnpq} := H_b^q / 2$;   elseif (n='rail') then   if ($p=q$) then $W_{i_1}^{mnpq} := 0$;   else $W_{i_1}^{mnpq} := H_r^q / 2$;   endif   endif   endif elseif ($i_1 \neq$ Origin station) then   if($p=q$ and $m=n$)then $W_{i_1}^{mnpq} := 0$;   elseif (n='walking') then $W_{i_1}^{mnpq} := 0$;   elseif(n='bus') then   if ($p=q$ and m='rail') then $W_{i_1}^{mnpq} := 0$;   else $W_{i_1}^{mnpq} := H_b^q / 2$;   endif   elseif (n='rail') then   if ($p=q$ and $m \neq$ 'walking') then $W_{i_1}^{mnpq} := 0$;   else $W_{i_1}^{mnpq} := H_r^q / 2$;   endif   endif endif </pre>

Table A-10 Transfer Time in Integer-ratio Headway Coordination When
Independence and Competition

<pre> if (i_1 = Origin station) then   if($m \neq$ 'bus') then $W_{i_1}^{mnpq} := +\infty$;   else   if(n='walking') then $W_{i_1}^{mnpq} := 0$;   elseif(n='bus') then $W_{i_1}^{mnpq} := H_b^q / 2$;   elseif (n='rail') then   if ($p=q$) then $W_{i_1}^{mnpq} := 0$;   else $W_{i_1}^{mnpq} := H_r^q / 2$;   endif   endif   endif elseif ($i_1 \neq$ Origin station) then   if($p=q$ and $m=n$)then $W_{i_1}^{mnpq} := 0$;   elseif (n='walking') then $W_{i_1}^{mnpq} := 0$;   elseif(n='bus') then   if ($p=q$ and m='rail') then $W_{i_1}^{mnpq} := H_r^q * (\phi^q - 1) / 2$;   else $W_{i_1}^{mnpq} := H_b^q / 2$;   endif   elseif (n='rail') then   if ($p=q$ and $m \neq$ 'walking') then $W_{i_1}^{mnpq} := 0$;   else $W_{i_1}^{mnpq} := H_r^q / 2$;   endif   endif endif </pre>

APPENDIX B The Shortest Path Algorithm in a Multimodal and Multi-operator Transit System

Step 1: Initialization

For every origin station i and every destination j

Set origin node = i , destination node = j ;

Set $Y_{i_1k}^{mp} = +\infty, Y_{i_1k}^{mp} = 0$ for $\forall i_1 \in N, i_1 \neq j, \forall m \in M, \forall p \in P$;

Set data set Rednodes and Bluenodes to empty.

Set $W_{i_1}^{mnpq} = +\infty$ for $i_1 = i, \forall m, n \in M, m \neq walk, \forall p, q \in P, p \neq Null$, if the shortest path is searched for those who transfer from modes other than bus.

Set $W_{i_1}^{mnpq} = +\infty$ for $i_1 = i, \forall m, n \in M, m \neq bus, \forall p, q \in P$, for those who transfer from bus.

Set $K=1$;

Insert element of origin node into set Rednodes; insert all elements of nodes other than set Rednodes into set Bluenodes.

Step 2:

For every node $j_1 \in \text{Bluenodes}$, if $V_{ij_1}^{nq} < 0$, do the following:

For every mode $m, n \in M$, do the following:

For every operator $p, q \in P$, do the following:

If $Y_{ij_1k} > W_i^{mnpq} + V_{ij_1}^{nq}$ Then

$$Y_{ij_1k} = W_i^{mnpq} + V_{ij_1}^{nq};$$

$$PM_{ij_1}^{mpk} = n;$$

$$PO_{ij_1}^{mpk} = q;$$

$$PN_{ij_1}^{mpk} = i;$$

Else

Do nothing

Step3:

If $j_1 < j$ and Rednodes is not empty, go to step 4, otherwise go to step 7;

Step 4:

Set minpath = $+\infty$, for every node $j_1 \in \text{Bluenodes}$,

If $Y_{ij_1k} < \text{minpath}$, then

$$\text{minpath} = Y_{ij_1k};$$

$$\Upsilon = j_1;$$

If minipath = $+\infty$, then there is no feasible path available, go to step 7; Otherwise insert element of Υ into set Rednodes, and delete element of Υ from set Bluenodes;

Step 5:

For every node $j_1 \in \text{Bluenodes}$, if $V_{ij_1}^{nq} < 0$, do the following:

For every mode $m, n \in M$, do the following:

For every operator $p, q \in P$, do the following:

If $Y_{ij,k} > Y_{i\gamma k} + W_{\gamma}^{mpq} + V_{\gamma j_l}^{nq}$ Then

$$Y_{ij,k} = Y_{i\gamma k} + W_{\gamma}^{mpq} + V_{\gamma j_l}^{nq};$$

$$PM_{ijj_l}^{mpk} = n;$$

$$PO_{ijj_l}^{mpk} = q;$$

$$PN_{ijj_l}^{mpk} = \gamma;$$

Else

Do nothing

Step 6.

The shortest path from i to j is Y_{ijk} , the path can be traced by $PN_{ijj_l}^{mpk}$.

Step 7. Terminate the algorithm.

APPENDIX C Sensitivity Analysis Results of Scenarios When Operators Are in Cooperation

1. Optimal Results When Value of Time Varies

Table C-1 Optimal Results in Non-coordinated (network I)

Items u (S\$/hr)	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
13.9	6.9	15.8	13.7	786.6	695.3	43.0	11.8	36.5	18.1	0.6476
14.9	6.7	15.3	13.2	808.8	711.8	45.7	12.2	39.1	18.1	0.6706
15.9	6.5	14.8	12.8	830.6	727.9	48.3	12.6	41.8	18.1	0.6928
16.9	6.3	14.3	12.4	851.8	743.4	51.0	13.0	44.4	18.1	0.7143
17.9	6.1	13.9	12.1	872.5	758.5	53.6	13.3	47.0	18.1	0.7352
18.9	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
19.9	5.8	13.2	11.5	912.6	787.4	58.8	14.1	52.3	18.1	0.7754
20.9	5.7	12.9	11.2	932.1	801.4	61.4	14.4	54.9	18.1	0.7948
21.9	5.5	12.6	10.9	951.3	815.0	64.0	14.8	57.5	18.1	0.8137
22.9	5.4	12.3	10.7	970.1	828.2	66.6	15.1	60.2	18.1	0.8321

Table C-2 Optimal Results in Common Headway Coordination (network I)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	10.6	10.6	9.8	10.6	10.6	10.6	10.6	665.9	582.5	47.9	1.2	34.3	384.0	0.0032
14.9	10.3	10.3	9.5	10.3	10.3	10.3	10.3	683.8	595.1	50.7	1.3	36.7	372.9	0.0035
15.9	9.9	9.9	9.2	9.9	9.9	9.9	9.9	701.4	607.3	53.6	1.3	39.2	362.8	0.0037
16.9	9.6	9.6	8.9	9.6	9.6	9.6	9.6	718.5	619.1	56.4	1.4	41.6	353.6	0.0039
17.9	9.4	9.4	8.6	9.4	9.4	9.4	9.4	735.2	630.5	59.2	1.4	44.1	345.2	0.0041
18.9	9.1	9.1	7.4	8.7	9.1	8.7	8.7	751.6	641.6	61.9	1.5	46.6	337.5	0.0043
19.9	8.9	8.9	8.2	8.9	8.9	8.9	8.9	767.7	652.5	64.7	1.5	49.0	330.4	0.0045
20.9	8.7	8.7	8.0	8.7	8.7	8.7	8.7	783.5	663.1	67.4	1.5	51.5	323.7	0.0047
21.9	8.5	8.5	7.8	8.5	8.5	8.5	8.5	799.1	673.4	70.2	1.6	54.0	317.6	0.0049
22.9	8.3	8.3	7.6	8.3	8.3	8.3	8.3	814.4	683.5	72.9	1.6	56.4	311.8	0.0052

Table C-3 Optimal Results in Integer-ratio Headway Coordination (network I)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	7.9	15.8	7.0	7.9	15.8	7.9	7.9	728.2	641.9	44.6	7.6	34.3	180.7	0.0418
14.9	7.6	15.3	6.8	7.6	15.3	7.6	7.6	748.4	656.6	47.3	7.8	36.7	175.6	0.0446
15.9	7.4	14.8	6.6	7.4	14.8	7.4	7.4	768.0	670.8	50.0	8.1	39.2	171.0	0.0473
16.9	7.2	14.3	6.6	7.2	14.3	7.2	7.2	787.2	684.5	52.7	8.3	41.6	166.8	0.0500
17.9	7.0	13.9	7.0	7.0	13.9	7.0	7.0	805.9	697.9	55.3	8.6	44.1	163.0	0.0527
18.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	824.3	710.9	58.0	8.8	46.6	159.4	0.0553
19.9	6.6	13.2	6.5	6.6	13.2	6.6	6.6	842.3	723.6	60.7	9.0	49.0	156.2	0.0579
20.9	6.4	12.9	6.4	6.4	12.9	6.4	6.4	860.0	735.9	63.3	9.3	51.5	153.2	0.0606
21.9	6.3	12.6	6.3	6.3	12.6	6.3	6.3	877.4	748.0	65.9	9.5	54.0	150.4	0.0631
22.9	6.2	12.3	6.2	6.2	12.3	6.2	6.2	894.4	759.7	68.6	9.7	56.4	147.7	0.0657

Table C-4 Optimal Results in Non-coordinated (network II)

Items u (S\$/hr)	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
13.9	5.1	8.9	9.0	1182.0	1024.6	73.6	24.0	59.8	55.5	0.4323
14.9	4.9	8.6	8.7	1220.1	1052.8	78.3	24.8	64.1	55.5	0.4477
15.9	4.8	8.3	8.4	1257.1	1080.0	83.0	25.7	68.4	55.5	0.4627
16.9	4.6	8.1	8.2	1293.3	1106.4	87.6	26.5	72.7	55.5	0.4772
17.9	4.5	7.9	8.0	1328.6	1132.0	92.3	27.2	77.0	55.4	0.4913
18.9	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
19.9	4.3	7.5	7.6	1397.1	1181.2	101.6	28.7	85.7	55.4	0.5183
20.9	4.2	7.3	7.4	1430.4	1204.8	106.2	29.4	90.0	55.4	0.5314
21.9	4.1	7.1	7.2	1463.1	1227.9	110.8	30.2	94.3	55.4	0.5442
22.9	4.0	7.0	7.0	1495.3	1250.5	115.4	30.8	98.6	55.4	0.5566

Table C-5 Optimal Results in Common Headway Coordination (network II)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	8.5	8.5	7.8	8.5	8.5	8.5	8.5	907.4	764.0	81.9	2.6	58.9	347.4	0.0074
14.9	8.2	8.2	7.6	8.2	8.2	8.2	8.2	935.7	783.0	86.9	2.7	63.1	338.3	0.0079
15.9	8.0	8.0	7.3	8.0	8.0	8.0	8.0	963.3	801.3	91.9	2.8	67.3	330.0	0.0084
16.9	7.7	7.7	7.1	7.7	7.7	7.7	7.7	990.4	819.1	96.9	2.9	71.6	322.5	0.0089
17.9	7.5	7.5	6.9	7.5	7.5	7.5	7.5	1016.8	836.3	101.8	2.9	75.8	315.7	0.0093
18.9	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1042.8	853.0	106.7	3.0	80.0	309.4	0.0098
19.9	7.1	7.1	7.0	7.1	7.1	7.1	7.1	1069.6	870.5	111.5	3.2	84.4	303.9	0.0106
20.9	6.9	6.9	6.9	6.9	6.9	6.9	6.9	1094.7	886.4	116.4	3.3	88.6	298.5	0.0111
21.9	6.8	6.8	6.8	6.8	6.8	6.8	6.8	1119.5	902.0	121.2	3.4	92.8	293.4	0.0116
22.9	6.6	6.6	6.6	6.6	6.6	6.6	6.6	1146.0	919.2	126.0	3.7	97.1	288.5	0.0127

Table C-6 Optimal Results in Integer-ratio Headway Coordination (network II)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	6.3	12.6	6.3	6.3	12.6	6.3	6.3	1005.8	858.1	77.7	11.4	58.5	181.4	0.0630
14.9	6.1	12.2	6.1	6.1	12.2	6.1	6.1	1037.5	880.4	82.6	11.8	62.8	176.8	0.0669
15.9	5.9	11.8	5.9	5.9	11.8	5.9	5.9	1070.3	903.7	87.3	12.4	66.9	175.8	0.0704
16.9	5.7	11.4	5.7	5.7	11.4	5.7	5.7	1100.7	924.6	92.2	12.8	71.1	171.9	0.0742
17.9	5.5	11.1	5.5	5.5	11.1	5.5	5.5	1131.8	946.2	96.9	13.2	75.4	170.5	0.0776
18.9	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1160.9	966.0	101.7	13.6	79.6	167.2	0.0813
19.9	5.3	10.5	5.3	5.3	10.5	5.3	5.3	1190.2	985.9	106.4	14.0	83.9	164.2	0.0851
20.9	5.1	10.2	5.1	5.1	10.2	5.1	5.1	1220.2	1006.5	111.1	14.5	88.2	164.0	0.0882
21.9	5.0	10.0	5.0	5.0	10.0	5.0	5.0	1249.4	1026.2	115.8	14.9	92.4	163.3	0.0914
22.9	4.9	9.7	4.9	4.9	9.7	4.9	4.9	1280.4	1047.5	120.6	15.4	96.9	163.2	0.0943

2. Optimal Results When Transfer Ratio Varies

Table C-7 Optimal Results in Non-coordinated (network I)

Items Transfer Ratio	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
10.0%	5.9	13.6	11.8	926.8	773.2	101.2	2.7	49.6	16.4	0.1668
20.0%	5.9	13.6	11.8	918.3	773.2	90.0	5.5	49.6	16.9	0.3252
30.0%	5.9	13.6	11.8	909.8	773.2	78.7	8.2	49.6	17.3	0.4758
40.0%	5.9	13.6	11.8	901.3	773.2	67.5	11.0	49.6	17.7	0.6190
50.0%	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
60.0%	5.9	13.6	11.8	884.2	773.2	45.0	16.5	49.6	18.6	0.8859
70.0%	5.9	13.6	11.8	875.7	773.2	33.7	19.2	49.6	19.0	1.0103
80.0%	5.9	13.6	11.8	867.2	773.2	22.5	21.9	49.6	19.4	1.1292
90.0%	5.9	13.6	11.8	858.7	773.2	11.2	24.7	49.6	19.9	1.2430

Table C-8 Optimal Results in Common Headway Coordination (network I)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	7.0	7.0	6.5	6.6	7.0	6.6	6.6	916.5	766.3	103.4	0.2	46.6	267.1	0.0008
20.0%	7.4	7.4	6.7	7.0	7.4	7.0	7.0	878.3	738.0	93.3	0.5	46.6	280.2	0.0017
30.0%	7.9	7.9	6.9	7.4	7.9	7.4	7.4	838.3	708.0	83.0	0.8	46.6	295.7	0.0026
40.0%	8.4	8.4	7.0	8.0	8.4	8.0	8.0	796.3	676.1	72.5	1.1	46.6	314.3	0.0034
50.0%	9.1	9.1	7.4	8.7	9.1	8.7	8.7	751.6	641.6	61.9	1.5	46.6	337.5	0.0043
60.0%	10.0	10.0	7.5	9.6	10.0	9.6	9.6	703.7	604.1	51.1	1.9	46.6	367.3	0.0052
70.0%	11.3	11.3	7.5	10.8	11.3	11.3	10.8	651.4	562.4	39.9	2.5	46.6	408.1	0.0061
80.0%	13.1	13.1	8.0	12.7	13.1	13.1	12.7	592.9	514.8	28.2	3.3	46.6	468.6	0.0071
90.0%	16.4	16.4	11.9	15.9	16.4	16.4	15.9	524.2	457.5	15.5	4.7	46.6	573.6	0.0081

Table C-9 Optimal Results in Integer-ratio Headway Coordination (network I)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	6.1	12.3	6.1	6.1	12.3	6.1	6.1	911.5	761.6	101.8	1.6	46.6	127.7	0.0125
20.0%	6.3	12.6	6.3	6.3	12.6	6.3	6.3	890.2	749.4	91.0	3.3	46.6	134.9	0.0242
30.0%	6.4	12.9	6.4	6.4	12.9	6.4	6.4	868.6	736.9	80.1	5.0	46.6	142.6	0.0352
40.0%	6.6	13.2	6.6	6.6	13.2	6.6	6.6	846.6	724.1	69.1	6.9	46.6	150.8	0.0456
50.0%	6.8	13.5	6.8	6.8	13.5	6.8	6.8	824.3	710.9	58.0	8.8	46.6	159.4	0.0553
60.0%	7.0	13.9	7.0	7.0	13.9	7.0	7.0	801.6	697.4	46.8	10.9	46.6	168.7	0.0645
70.0%	7.2	14.4	7.0	7.2	14.4	7.2	7.2	778.5	683.5	35.4	13.1	46.6	178.6	0.0732
80.0%	7.4	14.8	6.7	7.4	14.8	7.4	7.4	754.9	669.1	23.8	15.4	46.6	189.3	0.0815
90.0%	7.7	15.3	6.8	7.7	15.3	7.7	7.7	730.9	654.3	12.0	18.0	46.6	200.9	0.0894

Table C-10 Optimal Results in Non-coordinated (network II)

Items Transfer Ratio	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
10.0%	4.4	7.6	7.7	1425.7	1156.9	174.5	12.9	81.3	51.0	0.2538
20.0%	4.4	7.6	7.7	1410.1	1156.9	155.1	16.7	81.3	52.1	0.3206
30.0%	4.4	7.6	7.7	1394.5	1156.9	135.7	20.5	81.3	53.2	0.3846
40.0%	4.4	7.6	7.7	1378.9	1156.9	116.3	24.2	81.3	54.3	0.4460
50.0%	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
60.0%	4.4	7.6	7.7	1347.6	1156.9	77.6	31.8	81.3	56.6	0.5616
70.0%	4.4	7.6	7.7	1332.0	1156.9	58.2	35.5	81.3	57.7	0.6160
80.0%	4.4	7.6	7.7	1316.4	1156.9	38.8	39.3	81.3	58.8	0.6683
90.0%	4.4	7.6	7.7	1300.7	1156.9	19.4	43.1	81.3	59.9	0.7187

Table C-11 Optimal Results in Common Headway Coordination (network II)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	5.7	5.7	5.7	5.7	5.7	5.7	5.7	1293.1	1031.6	180.7	0.5	80.2	245.6	0.0021
20.0%	6.0	6.0	6.0	6.0	6.0	6.0	6.0	1235.3	991.5	162.5	1.1	80.2	257.9	0.0043
30.0%	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1175.2	949.2	144.1	1.7	80.2	272.2	0.0064
40.0%	6.7	6.7	6.7	6.7	6.7	6.7	6.7	1112.3	904.2	125.5	2.5	80.2	289.0	0.0086
50.0%	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1042.8	853.0	106.7	3.0	80.0	309.4	0.0098
60.0%	8.0	8.0	7.1	8.0	8.0	8.0	8.0	971.4	799.9	87.5	4.0	80.0	335.2	0.0118
70.0%	9.0	9.0	7.7	9.0	9.0	9.0	9.0	888.4	735.9	67.9	4.6	79.9	370.1	0.0125
80.0%	10.6	10.6	8.8	10.6	10.6	10.6	10.6	791.0	658.5	47.8	5.2	79.5	421.4	0.0124
90.0%	13.7	13.7	8.8	13.7	13.7	13.0	13.7	671.7	560.0	26.3	6.1	79.2	521.3	0.0117

Table C-12 Optimal Results in Integer-ratio Headway Coordination (network II)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	4.9	9.7	4.9	4.9	9.7	4.9	4.9	1085.7	823.5	179.0	3.0	80.2	127.3	0.0233
20.0%	5.0	10.0	5.0	5.0	10.0	5.0	5.0	1047.6	802.4	159.8	5.5	79.9	137.3	0.0398
30.0%	5.1	10.2	5.1	5.1	10.2	5.1	5.1	1011.2	782.6	140.6	8.1	79.9	147.0	0.0548
40.0%	5.2	10.5	5.2	5.2	10.5	5.2	5.2	973.2	761.5	121.3	10.7	79.7	156.0	0.0687
50.0%	5.4	10.8	5.4	5.4	10.8	5.4	5.4	935.1	740.3	101.7	13.6	79.6	167.2	0.0813
60.0%	5.6	11.1	5.6	5.6	11.1	5.6	5.6	897.1	719.0	81.9	16.7	79.5	179.3	0.0931
70.0%	5.7	11.5	5.7	5.7	11.5	5.7	5.7	856.4	695.3	61.9	19.8	79.4	189.1	0.1049
80.0%	5.9	11.9	5.9	5.9	11.9	5.9	5.9	816.6	672.4	41.6	23.4	79.3	202.6	0.1153
90.0%	6.2	12.4	6.2	6.2	12.4	6.2	6.2	772.0	644.7	21.0	26.9	79.3	211.7	0.1271

3. Optimal Results When the Standard Deviation (SD) of Bus Arrival Times Varies

Table C-13 Optimal Results in Non-coordinated (network I)

Items σ_i	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
1	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
2	5.9	13.7	11.9	892.1	772.4	56.3	13.8	49.6	18.1	0.7608
3	5.9	13.9	12.1	891.1	771.2	56.4	13.9	49.6	18.0	0.7691
4	5.9	14.1	12.4	889.8	769.7	56.5	14.0	49.6	17.9	0.7803
5	5.9	14.4	12.7	888.2	767.8	56.7	14.1	49.6	17.8	0.7937
6	5.9	14.8	13.2	886.5	765.7	56.8	14.3	49.6	17.7	0.8089
7	5.9	15.2	13.6	884.6	763.4	57.0	14.5	49.6	17.6	0.8253
8	5.9	15.7	14.2	882.7	761.0	57.3	14.7	49.6	17.5	0.8424
9	5.9	16.2	14.8	880.7	758.6	57.5	15.0	49.6	17.4	0.8597
10	5.9	16.8	15.4	878.8	756.2	57.7	15.2	49.6	17.4	0.8768

Table C-14 Optimal Results in Common Headway Coordination (network I)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	9.1	9.1	7.4	8.7	9.1	8.7	8.7	751.6	641.6	61.9	1.5	46.6	337.5	0.0043
2	9.1	9.1	9.1	9.1	9.1	9.1	9.1	750.7	640.6	62.0	1.5	46.6	338.2	0.0045
3	9.2	9.2	9.2	9.2	9.2	9.2	9.2	749.3	638.9	62.2	1.6	46.6	339.3	0.0047
4	9.2	9.2	9.2	9.2	9.2	9.2	9.2	747.3	636.6	62.4	1.7	46.6	341.0	0.0051
5	9.3	9.3	9.3	9.3	9.3	9.3	9.3	744.8	633.6	62.7	1.9	46.6	343.0	0.0055
6	9.4	9.4	9.4	9.4	9.4	9.4	9.4	741.8	630.0	63.1	2.1	46.6	345.5	0.0060
7	9.5	9.5	9.5	9.5	9.5	9.5	9.5	738.4	626.0	63.5	2.3	46.6	348.5	0.0066
8	9.6	9.6	9.6	9.6	9.6	9.6	9.6	734.5	621.4	64.0	2.6	46.6	351.8	0.0073
9	9.7	9.7	9.7	9.7	9.7	9.7	9.7	730.4	616.5	64.5	2.8	46.6	355.6	0.0080
10	9.8	9.8	9.8	9.8	9.8	9.8	9.8	725.9	611.1	65.1	3.2	46.6	359.7	0.0088

Table C-15 Optimal Results in Integer-ratio Headway Coordination (network I)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	6.8	13.5	6.8	6.8	13.5	6.8	6.8	824.3	710.9	58.0	8.8	46.6	159.4	0.0553
2	6.8	13.6	6.8	6.8	13.6	6.8	6.8	823.7	710.2	58.1	8.9	46.6	159.6	0.0555
3	6.8	13.6	6.8	6.8	13.6	6.8	6.8	822.7	709.1	58.2	8.9	46.6	159.9	0.0559
4	6.8	13.6	6.8	6.8	13.6	6.8	6.8	821.4	707.5	58.3	9.1	46.6	160.4	0.0565
5	6.9	13.7	6.9	6.9	13.7	6.9	6.9	819.7	705.4	58.5	9.2	46.6	160.9	0.0571
6	6.9	13.8	6.9	6.9	13.8	6.9	6.9	817.6	703.0	58.7	9.4	46.6	161.6	0.0580
7	6.9	13.9	6.9	6.9	13.9	6.9	6.9	815.2	700.1	58.9	9.6	46.6	162.3	0.0589
8	7.0	14.0	7.0	7.0	14.0	7.0	7.0	812.4	696.9	59.2	9.8	46.6	163.2	0.0600
9	7.0	14.1	7.0	7.0	14.1	7.0	7.0	809.4	693.3	59.5	10.0	46.6	164.3	0.0612
10	7.1	14.2	7.1	7.1	14.2	7.1	7.1	806.1	689.4	59.8	10.3	46.6	165.4	0.0625

Table C-16 Optimal Results in Non-coordinated (network II)

Items σ_i	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
1	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
2	4.4	7.8	7.9	1360.1	1153.3	97.2	28.3	81.3	54.9	0.5150
3	4.4	8.2	8.2	1355.4	1147.7	97.7	28.7	81.3	54.2	0.5305
4	4.4	8.6	8.7	1349.7	1140.8	98.3	29.3	81.3	53.3	0.5499
5	4.4	9.1	9.2	1343.6	1133.3	99.0	30.0	81.3	52.5	0.5717
6	4.4	9.7	9.7	1337.5	1125.6	99.7	30.8	81.3	51.8	0.5943
7	4.4	10.3	10.6	1331.4	1118.6	100.3	31.4	81.1	50.3	0.6234
8	4.4	11.0	11.3	1326.2	1111.7	101.2	32.2	81.1	50.0	0.6440
9	4.3	11.8	12.5	1321.3	1106.5	101.5	32.5	80.8	48.0	0.6765
10	4.3	12.5	13.6	1317.1	1101.5	102.0	32.9	80.6	46.9	0.7022

Table C-17 Optimal Results in Common Headway Coordination (network II)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1042.8	853.0	106.7	3.0	80.0	309.4	0.0098
2	7.3	7.3	7.3	7.3	7.3	7.3	7.3	1039.8	849.4	107.1	3.2	80.0	310.7	0.0103
3	7.4	7.4	7.4	7.4	7.4	7.4	7.4	1034.8	843.6	107.7	3.5	80.0	312.9	0.0112
4	7.5	7.5	7.5	7.5	7.5	7.5	7.5	1028.1	835.6	108.5	3.9	80.0	315.9	0.0124
5	7.6	7.6	7.6	7.6	7.6	7.6	7.6	1019.8	825.8	109.5	4.4	80.0	319.8	0.0139
6	7.8	7.8	7.8	7.8	7.8	7.8	7.8	1010.3	814.5	110.7	5.1	80.0	324.4	0.0156
7	7.9	7.9	7.9	7.9	7.9	7.9	7.9	999.8	801.9	112.1	5.7	80.0	329.8	0.0174
8	8.1	8.1	8.1	8.1	8.1	8.1	8.1	988.5	788.3	113.7	6.5	80.0	335.8	0.0194
9	8.4	8.4	8.4	8.4	8.4	8.4	8.4	976.8	774.0	115.4	7.4	80.0	342.5	0.0215
10	8.6	8.6	8.6	8.6	8.6	8.6	8.6	964.8	759.2	117.2	8.2	80.0	349.8	0.0236

Table C-18 Optimal Results in Integer-ratio Headway Coordination (network II)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1160.9	966.0	101.7	13.6	79.6	167.2	0.0813
2	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1159.1	964.0	101.9	13.7	79.6	167.6	0.0819
3	5.4	10.9	5.4	5.4	10.9	5.4	5.4	1156.2	960.5	102.2	13.9	79.6	168.1	0.0828
4	5.5	10.9	5.5	5.5	10.9	5.5	5.5	1152.2	955.8	102.6	14.2	79.6	168.9	0.0841
5	5.5	11.0	5.5	5.5	11.0	5.5	5.5	1147.2	949.9	103.2	14.6	79.6	169.9	0.0857
6	5.6	11.1	5.6	5.6	11.1	5.6	5.6	1141.2	942.8	103.8	15.0	79.6	171.1	0.0877
7	5.6	11.3	5.6	5.6	11.3	5.6	5.6	1133.6	934.1	104.6	15.3	79.6	170.2	0.0899
8	5.7	11.4	5.7	5.7	11.4	5.7	5.7	1126.2	925.3	105.5	15.9	79.6	171.8	0.0923
9	5.8	11.6	5.8	5.8	11.6	5.8	5.8	1118.1	915.6	106.5	16.5	79.6	173.5	0.0948
10	5.9	11.8	5.9	5.9	11.8	5.9	5.9	1109.5	905.3	107.5	17.1	79.6	175.5	0.0975

4. Optimal Results When Passenger OD Rates of Feeder Bus services Varies

Table C-19 Optimal Results in Non-coordinated (network I)

Items λ_{ij}	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.5	8.0	18.2	15.8	705.5	633.9	33.8	10.2	27.6	10.1	1.0127
0.6	7.3	16.6	14.4	756.6	672.8	39.5	11.2	33.1	12.1	0.9247
0.7	6.7	15.4	13.3	804.4	708.6	45.2	12.1	38.6	14.1	0.8563
0.8	6.3	14.4	12.5	849.7	741.9	50.7	12.9	44.1	16.1	0.8012
0.9	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
1.0	5.6	12.9	11.2	934.1	802.7	61.7	14.5	55.2	20.2	0.7170
1.1	5.4	12.3	10.6	973.8	830.9	67.1	15.2	60.7	22.2	0.6838
1.2	5.1	11.8	10.2	1012.3	857.7	72.5	15.8	66.2	24.2	0.6549
1.3	4.9	11.3	9.8	1049.6	883.5	77.9	16.5	71.7	26.2	0.6293
1.4	4.8	10.9	9.4	1085.9	908.3	83.3	17.1	77.2	28.2	0.6066

Table C-20 Optimal Results in Common Headway Coordination (network I)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	12.2	12.2	8.4	12.2	12.2	12.2	12.2	600.9	535.9	38.1	1.1	25.9	240.7	0.0045
0.6	11.2	11.2	9.2	11.2	11.2	11.2	11.2	641.8	565.4	44.2	1.2	31.1	267.0	0.0044
0.7	10.3	10.3	8.3	10.3	10.3	10.3	10.3	680.3	592.6	50.2	1.3	36.2	291.7	0.0044
0.8	9.7	9.7	7.4	9.7	9.7	9.7	9.7	716.8	617.9	56.1	1.4	41.4	315.1	0.0044
0.9	9.1	9.1	7.4	8.7	9.1	8.7	8.7	751.6	641.6	61.9	1.5	46.6	337.5	0.0043
1.0	8.7	8.7	7.5	8.7	8.7	8.7	8.7	785.1	664.1	67.7	1.5	51.8	359.0	0.0043
1.1	8.2	8.2	7.6	8.2	8.2	8.2	8.2	817.4	685.4	73.4	1.6	56.9	379.8	0.0043
1.2	7.9	7.9	7.2	7.9	7.9	7.9	7.9	848.8	705.8	79.1	1.7	62.1	399.9	0.0042
1.3	7.6	7.6	7.0	7.6	7.6	7.6	7.6	879.2	725.4	84.8	1.8	67.3	419.5	0.0042
1.4	7.3	7.3	6.7	7.3	7.3	7.3	7.3	908.9	744.2	90.4	1.8	72.5	438.6	0.0042

Table C-21 Optimal Results in Integer-ratio Headway Coordination (network I)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	9.1	18.2	8.6	9.1	18.2	9.1	9.1	655.1	587.5	35.2	6.6	25.9	112.9	0.0582
0.6	8.3	16.6	7.4	8.3	16.6	8.3	8.3	701.2	621.9	41.0	7.2	31.1	125.5	0.0574
0.7	7.7	15.4	6.8	7.7	15.4	7.7	7.7	744.4	653.7	46.7	7.8	36.2	137.3	0.0566
0.8	7.2	14.4	6.9	7.2	14.4	7.2	7.2	785.3	683.2	52.4	8.3	41.4	148.6	0.0559
0.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	824.3	710.9	58.0	8.8	46.6	159.4	0.0553
1.0	6.4	12.9	6.4	6.4	12.9	6.4	6.4	861.7	737.1	63.6	9.3	51.8	169.9	0.0547
1.1	6.1	12.3	6.1	6.1	12.3	6.1	6.1	897.8	762.1	69.1	9.8	56.9	179.9	0.0542
1.2	5.9	11.7	5.9	5.9	11.7	5.9	5.9	932.7	785.9	74.6	10.2	62.1	189.7	0.0537
1.3	5.6	11.3	5.6	5.6	11.3	5.6	5.6	966.6	808.7	80.0	10.6	67.3	199.3	0.0532
1.4	5.4	10.9	5.4	5.4	10.9	5.4	5.4	999.6	830.7	85.5	11.0	72.5	208.6	0.0528

Table C-22 Optimal Results in Non-coordinated (network II)

Items λ_{ij}	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.5	5.9	10.2	10.4	1042.9	919.5	57.4	20.8	45.2	30.9	0.6733
0.6	5.4	9.3	9.5	1130.1	985.6	67.4	22.8	54.2	37.0	0.6151
0.7	5.0	8.7	8.8	1212.5	1047.2	77.3	24.7	63.3	43.2	0.5717
0.8	4.6	8.1	8.2	1289.7	1103.8	87.2	26.4	72.3	49.3	0.5352
0.9	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
1.0	4.2	7.3	7.4	1433.7	1207.2	106.7	29.5	90.4	61.6	0.4794
1.1	4.0	6.9	7.0	1501.6	1254.9	116.3	31.0	99.4	67.7	0.4575
1.2	3.8	6.6	6.7	1567.3	1300.5	125.9	32.4	108.5	73.8	0.4383
1.3	3.6	6.4	6.4	1632.3	1345.4	135.5	33.8	117.6	80.5	0.4194
1.4	3.5	6.2	6.2	1694.3	1387.5	145.1	35.1	126.7	86.7	0.4044

Table C-23 Optimal Results in Common Headway Coordination (network II)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	9.8	9.8	7.6	9.8	9.8	9.8	9.8	802.5	691.3	64.8	2.0	44.4	215.2	0.0093
0.6	9.0	9.0	7.8	9.0	9.0	9.0	9.0	866.7	735.7	75.5	2.2	53.3	240.4	0.0092
0.7	8.3	8.3	7.6	8.3	8.3	8.3	8.3	930.1	779.2	85.9	2.7	62.3	264.5	0.0101
0.8	7.7	7.7	7.1	7.7	7.7	7.7	7.7	987.7	817.3	96.4	2.8	71.2	287.3	0.0099
0.9	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1042.8	853.0	106.7	3.0	80.0	309.4	0.0098
1.0	6.9	6.9	6.9	6.9	6.9	6.9	6.9	1097.2	888.0	116.9	3.3	89.0	331.1	0.0100
1.1	6.6	6.6	6.6	6.6	6.6	6.6	6.6	1150.8	922.2	127.0	3.7	98.0	351.5	0.0105
1.2	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1200.8	953.0	137.0	3.9	106.9	371.7	0.0104
1.3	6.1	6.1	6.1	6.1	6.1	6.1	6.1	1249.5	982.5	147.1	4.0	115.8	391.4	0.0103
1.4	5.8	5.8	5.8	5.8	5.8	5.8	5.8	1296.9	1010.9	157.1	4.2	124.7	410.8	0.0102

Table C-24 Optimal Results in Integer-ratio Headway Coordination (network II)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	7.3	14.6	6.5	7.3	14.6	7.3	7.3	886.3	771.5	61.0	9.8	44.1	108.5	0.0901
0.6	6.7	13.3	6.7	6.7	13.3	6.7	6.7	961.6	826.4	71.4	10.8	53.0	123.2	0.0874
0.7	6.1	12.3	6.1	6.1	12.3	6.1	6.1	1031.3	876.0	81.6	11.8	61.9	138.2	0.0851
0.8	5.7	11.5	5.7	5.7	11.5	5.7	5.7	1097.7	922.6	91.7	12.7	70.7	153.1	0.0831
0.9	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1160.9	966.0	101.7	13.6	79.6	167.2	0.0813
1.0	5.1	10.2	5.1	5.1	10.2	5.1	5.1	1223.0	1008.3	111.6	14.5	88.6	181.9	0.0797
1.1	4.8	9.7	4.8	4.8	9.7	4.8	4.8	1285.8	1051.1	121.5	15.5	97.8	198.9	0.0777
1.2	4.6	9.3	4.6	4.6	9.3	4.6	4.6	1341.9	1087.7	131.3	16.2	106.6	210.5	0.0767
1.3	4.5	8.9	4.5	4.5	8.9	4.5	4.5	1396.3	1122.8	141.1	16.8	115.5	221.8	0.0758
1.4	4.3	8.5	4.3	4.3	8.5	4.3	4.3	1454.0	1160.9	150.8	17.8	124.5	240.0	0.0743

5. Coefficient of In-vehicle Value of Time (VOT) Varies

Table C-25 Optimal Results in Non-coordinated (network I)

Items k_v	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.1	5.9	13.6	11.8	853.0	773.2	56.2	13.7	9.9	18.1	0.7556
0.2	5.9	13.6	11.8	863.0	773.2	56.2	13.7	19.9	18.1	0.7556
0.3	5.9	13.6	11.8	872.9	773.2	56.2	13.7	29.8	18.1	0.7556
0.4	5.9	13.6	11.8	882.8	773.2	56.2	13.7	39.7	18.1	0.7556
0.5	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
0.6	5.9	13.6	11.8	902.7	773.2	56.2	13.7	59.6	18.1	0.7556
0.7	5.9	13.6	11.8	912.6	773.2	56.2	13.7	69.5	18.1	0.7556
0.8	5.9	13.6	11.8	922.5	773.2	56.2	13.7	79.4	18.1	0.7556
0.9	5.9	13.6	11.8	932.5	773.2	56.2	13.7	89.4	18.1	0.7556

Table C-26 Optimal Results in Common Headway Coordination (network I)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	9.1	9.1	7.6	9.1	9.1	9.1	9.1	714.3	641.6	61.9	1.5	9.3	337.5	0.0043
0.2	9.1	9.1	7.9	9.1	9.1	9.1	9.1	723.7	641.6	61.9	1.5	18.6	337.5	0.0043
0.3	9.1	9.1	8.4	9.1	9.1	9.1	9.1	733.0	641.6	61.9	1.5	27.9	337.5	0.0043
0.4	9.1	9.1	8.4	9.1	9.1	9.1	9.1	742.3	641.6	61.9	1.5	37.3	337.5	0.0043
0.5	9.1	9.1	7.4	9.1	9.1	9.1	9.1	751.6	641.6	61.9	1.5	46.6	337.5	0.0043
0.6	9.1	9.1	8.4	9.1	9.1	9.1	9.1	760.9	641.6	61.9	1.5	55.9	337.5	0.0043
0.7	9.1	9.1	8.4	9.1	9.1	9.1	9.1	770.2	641.6	61.9	1.5	65.2	337.5	0.0043
0.8	9.1	9.1	8.4	9.1	9.1	9.1	9.1	779.6	641.6	61.9	1.5	74.5	337.5	0.0043
0.9	9.1	9.1	8.4	9.1	9.1	9.1	9.1	788.9	641.6	61.9	1.5	83.8	337.5	0.0043

Table C-27 Optimal Results in Integer-ratio Headway Coordination (network I)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	6.8	13.5	6.8	6.8	13.5	6.8	6.8	787.1	710.9	58.0	8.8	9.3	159.4	0.0553
0.2	6.8	13.5	6.8	6.8	13.5	6.8	6.8	796.4	710.9	58.0	8.8	18.6	159.4	0.0553
0.3	6.8	13.5	6.8	6.8	13.5	6.8	6.8	805.7	710.9	58.0	8.8	27.9	159.4	0.0553
0.4	6.8	13.5	6.8	6.8	13.5	6.8	6.8	815.0	710.9	58.0	8.8	37.3	159.4	0.0553
0.5	6.8	13.5	6.8	6.8	13.5	6.8	6.8	824.3	710.9	58.0	8.8	46.6	159.4	0.0553
0.6	6.8	13.5	6.8	6.8	13.5	6.8	6.8	833.6	710.9	58.0	8.8	55.9	159.4	0.0553
0.7	6.8	13.5	6.8	6.8	13.5	6.8	6.8	842.9	710.9	58.0	8.8	65.2	159.4	0.0553
0.8	6.8	13.5	6.8	6.8	13.5	6.8	6.8	852.3	710.9	58.0	8.8	74.5	159.4	0.0553
0.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	861.6	710.9	58.0	8.8	83.8	159.4	0.0553

Table C-28 Optimal Results in Non-coordinated (network II)

Items k_v	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.1	4.4	7.6	7.7	1298.2	1156.9	96.9	28.0	16.3	55.4	0.5050
0.2	4.4	7.6	7.7	1314.4	1156.9	96.9	28.0	32.5	55.4	0.5050
0.3	4.4	7.6	7.7	1330.7	1156.9	96.9	28.0	48.8	55.4	0.5050
0.4	4.4	7.6	7.7	1347.0	1156.9	96.9	28.0	65.1	55.4	0.5050
0.5	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
0.6	4.4	7.6	7.7	1379.5	1156.9	96.9	28.0	97.6	55.4	0.5050
0.7	4.4	7.6	7.7	1395.8	1156.9	96.9	28.0	113.9	55.4	0.5050
0.8	4.4	7.6	7.7	1412.0	1156.9	96.9	28.0	130.2	55.4	0.5050
0.9	4.4	7.6	7.7	1428.3	1156.9	96.9	28.0	146.4	55.4	0.5050

Table C-29 Optimal Results in Common Headway Coordination (network II)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	7.3	7.3	6.7	7.3	7.3	7.3	7.3	978.8	853.0	106.7	3.0	16.0	309.4	0.0098
0.2	7.3	7.3	6.7	7.3	7.3	7.3	7.3	994.8	853.0	106.7	3.0	32.0	309.4	0.0098
0.3	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1010.8	853.0	106.7	3.0	48.0	309.4	0.0098
0.4	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1026.8	853.0	106.7	3.0	64.0	309.4	0.0098
0.5	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1042.8	853.0	106.7	3.0	80.0	309.4	0.0098
0.6	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1058.8	853.0	106.7	3.0	96.1	309.4	0.0098
0.7	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1074.8	853.0	106.7	3.0	112.1	309.4	0.0098
0.8	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1090.8	853.0	106.7	3.0	128.1	309.4	0.0098
0.9	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1106.9	853.0	106.7	3.0	144.1	309.4	0.0098

Table C-30 Optimal Results in Integer-ratio Headway Coordination (network II)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1097.2	966.0	101.7	13.6	15.9	167.2	0.0813
0.2	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1113.2	966.0	101.7	13.6	31.8	167.2	0.0813
0.3	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1129.1	966.0	101.7	13.6	47.8	167.2	0.0813
0.4	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1145.0	966.0	101.7	13.6	63.7	167.2	0.0813
0.5	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1160.9	966.0	101.7	13.6	79.6	167.2	0.0813
0.6	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1176.8	966.0	101.7	13.6	95.5	167.2	0.0813
0.7	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1192.8	966.0	101.7	13.6	111.5	167.2	0.0813
0.8	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1208.7	966.0	101.7	13.6	127.4	167.2	0.0813
0.9	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1224.6	966.0	101.7	13.6	143.3	167.2	0.0813

APPENDIX D Sensitivity Analysis Results of Scenarios When Operators are Independent

1. Optimal Results When Value of Time Varies

Table D-1 Optimal Results in Non-coordinated (network I)

Items u (S\$/hr)	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
13.9	6.9	15.8	13.7	786.6	695.3	43.0	11.8	36.5	18.1	0.6476
14.9	6.7	15.3	13.2	808.8	711.8	45.7	12.2	39.1	18.1	0.6706
15.9	6.5	14.8	12.8	830.6	727.9	48.3	12.6	41.8	18.1	0.6928
16.9	6.3	14.3	12.4	851.8	743.4	51.0	13.0	44.4	18.1	0.7143
17.9	6.1	13.9	12.1	872.5	758.5	53.6	13.3	47.0	18.1	0.7352
18.9	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
19.9	5.8	13.2	11.5	912.6	787.4	58.8	14.1	52.3	18.1	0.7754
20.9	5.7	12.9	11.2	932.1	801.4	61.4	14.4	54.9	18.1	0.7948
21.9	5.5	12.6	10.9	951.3	815.0	64.0	14.8	57.5	18.1	0.8137
22.9	5.4	12.3	10.7	970.1	828.2	66.6	15.1	60.2	18.1	0.8321

Table D-2 Optimal Results in Common Headway Coordination (network I)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	10.6	10.6	9.8	10.6	10.6	10.6	10.6	668.2	582.5	47.9	1.2	36.5	37.7	0.0331
14.9	10.3	10.3	9.5	10.3	10.3	10.3	10.3	686.3	595.1	50.7	1.3	39.1	37.3	0.0346
15.9	9.9	9.9	9.2	9.9	9.9	9.9	9.9	703.9	607.3	53.6	1.3	41.8	37.0	0.0361
16.9	9.6	9.6	8.9	9.6	9.6	9.6	9.6	721.2	619.1	56.4	1.4	44.4	36.7	0.0375
17.9	9.4	9.4	8.6	9.4	9.4	9.4	9.4	738.1	630.5	59.2	1.4	47.0	36.4	0.0389
18.9	9.1	9.1	7.4	9.1	9.1	9.1	9.1	754.7	641.6	61.9	1.5	49.6	36.1	0.0403
19.9	8.9	8.9	8.2	8.9	8.9	8.9	8.9	770.9	652.5	64.7	1.5	52.3	35.9	0.0417
20.9	8.7	8.7	8.0	8.7	8.7	8.7	8.7	786.9	663.1	67.4	1.5	54.9	35.7	0.0430
21.9	8.5	8.5	7.8	8.5	8.5	8.5	8.5	802.6	673.4	70.2	1.6	57.5	35.5	0.0443
22.9	8.3	8.3	7.6	8.3	8.3	8.3	8.3	818.1	683.5	72.9	1.6	60.2	35.3	0.0455

Table D-3 Optimal Results in Integer-ratio Headway Coordination (network I)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	7.9	15.8	7.0	7.9	15.8	7.9	7.9	730.5	641.9	44.6	7.6	36.5	48.2	0.1569
14.9	7.6	15.3	6.7	7.6	15.3	7.6	7.6	750.8	656.6	47.3	7.8	39.1	47.1	0.1662
15.9	7.4	14.8	6.5	7.4	14.8	7.4	7.4	770.6	670.8	50.0	8.1	41.8	46.1	0.1753
16.9	7.2	14.3	6.8	7.2	14.3	7.2	7.2	789.9	684.5	52.7	8.3	44.4	45.2	0.1843
17.9	7.0	13.9	7.0	7.0	13.9	7.0	7.0	808.9	697.9	55.3	8.6	47.0	44.4	0.1932
18.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.4	710.9	58.0	8.8	49.6	43.7	0.2019
19.9	6.6	13.2	6.5	6.6	13.2	6.6	6.6	845.6	723.6	60.7	9.0	52.3	43.0	0.2106
20.9	6.4	12.9	6.4	6.4	12.9	6.4	6.4	863.4	735.9	63.3	9.3	54.9	42.3	0.2191
21.9	6.3	12.6	6.3	6.3	12.6	6.3	6.3	880.9	748.0	65.9	9.5	57.5	41.7	0.2275
22.9	6.2	12.3	6.2	6.2	12.3	6.2	6.2	898.2	759.7	68.6	9.7	60.2	41.2	0.2358

Table D-4 Optimal Results in Non-coordinated (network II)

Items u (S\$/hr)	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
13.9	5.1	8.9	9.0	1182.0	1024.6	73.6	24.0	59.8	55.5	0.4323
14.9	4.9	8.6	8.7	1220.1	1052.8	78.3	24.8	64.1	55.5	0.4477
15.9	4.8	8.3	8.4	1257.1	1080.0	83.0	25.7	68.4	55.5	0.4627
16.9	4.6	8.1	8.2	1293.3	1106.4	87.6	26.5	72.7	55.5	0.4772
17.9	4.5	7.9	8.0	1328.6	1132.0	92.3	27.2	77.0	55.4	0.4913
18.9	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
19.9	4.3	7.5	7.6	1397.1	1181.2	101.6	28.7	85.7	55.4	0.5183
20.9	4.2	7.3	7.4	1430.4	1204.8	106.2	29.4	90.0	55.4	0.5314
21.9	4.1	7.1	7.2	1463.1	1227.9	110.8	30.2	94.3	55.4	0.5442
22.9	4.0	7.0	7.0	1495.3	1250.5	115.4	30.8	98.6	55.4	0.5566

Table D-5 Optimal Results in Common Headway Coordination (network II)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	7.3	7.3	7.3	7.3	7.3	7.3	7.3	982.2	855.0	79.8	16.1	31.3	142.1	0.1135
14.9	7.0	7.0	7.0	7.0	7.0	7.0	7.0	1012.0	877.0	84.8	16.7	33.5	139.0	0.1203
15.9	6.8	6.8	6.8	6.8	6.8	6.8	6.8	1041.0	898.1	89.8	17.3	35.8	136.2	0.1270
16.9	6.6	6.6	6.6	6.6	6.6	6.6	6.6	1069.2	918.7	94.7	17.9	38.0	133.7	0.1336
17.9	6.4	6.4	6.4	6.4	6.4	6.4	6.4	1096.8	938.5	99.6	18.4	40.3	131.3	0.1401
18.9	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1123.8	957.8	104.5	18.9	42.5	129.2	0.1466
19.9	6.1	6.1	6.1	6.1	6.1	6.1	6.1	1150.3	976.6	109.4	19.5	44.8	127.2	0.1529
20.9	6.0	6.0	6.0	6.0	6.0	6.0	6.0	1176.2	994.9	114.3	20.0	47.0	125.4	0.1592
21.9	5.8	5.8	5.8	5.8	5.8	5.8	5.8	1201.6	1012.7	119.1	20.5	49.3	123.7	0.1654
22.9	5.7	5.7	5.7	5.7	5.7	5.7	5.7	1226.6	1030.1	124.0	21.0	51.5	122.1	0.1716

Table D-6 Optimal Results in Integer-ratio Headway Coordination (network II)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	5.5	11.1	5.5	5.5	11.1	5.5	5.5	1101.1	948.2	75.4	18.3	59.2	117.4	0.1561
14.9	5.3	10.7	5.3	5.3	10.7	5.3	5.3	1136.2	973.7	80.1	19.0	63.4	114.7	0.1654
15.9	5.2	10.3	5.2	5.2	10.3	5.2	5.2	1172.0	999.7	84.8	19.7	67.7	115.1	0.1712
16.9	5.0	10.0	5.0	5.0	10.0	5.0	5.0	1206.6	1024.7	89.5	20.4	72.0	114.9	0.1775
17.9	4.8	9.7	4.8	4.8	9.7	4.8	4.8	1242.6	1050.7	94.2	21.2	76.5	118.1	0.1797
18.9	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1274.9	1073.4	98.9	21.8	80.8	116.0	0.1880
19.9	4.6	9.2	4.6	4.6	9.2	4.6	4.6	1306.5	1095.5	103.6	22.4	85.0	114.1	0.1963
20.9	4.5	9.0	4.5	4.5	9.0	4.5	4.5	1337.5	1117.0	108.2	22.9	89.3	112.3	0.2044
21.9	4.4	8.8	4.4	4.4	8.8	4.4	4.4	1368.0	1138.1	112.9	23.5	93.6	110.6	0.2124
22.9	4.3	8.6	4.3	4.3	8.6	4.3	4.3	1399.8	1160.2	117.5	24.1	97.9	111.4	0.2168

2. Optimal Results When Transfer Ratio Varies

Table D-7 Optimal Results in Non-coordinated (network I)

Items Transfer Ratio	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
10.0%	5.9	13.6	11.8	926.8	773.2	101.2	2.7	49.6	16.4	0.1668
20.0%	5.9	13.6	11.8	918.3	773.2	90.0	5.5	49.6	16.9	0.3252
30.0%	5.9	13.6	11.8	909.8	773.2	78.7	8.2	49.6	17.3	0.4758
40.0%	5.9	13.6	11.8	901.3	773.2	67.5	11.0	49.6	17.7	0.6190
50.0%	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
60.0%	5.9	13.6	11.8	884.2	773.2	45.0	16.5	49.6	18.6	0.8859
70.0%	5.9	13.6	11.8	875.7	773.2	33.7	19.2	49.6	19.0	1.0103
80.0%	5.9	13.6	11.8	867.2	773.2	22.5	21.9	49.6	19.4	1.1292
90.0%	5.9	13.6	11.8	858.7	773.2	11.2	24.7	49.6	19.9	1.2430

Table D-8 Optimal Results in Common Headway Coordination (network I)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	7.0	7.0	6.9	7.0	7.0	7.0	7.0	919.6	766.3	103.4	0.2	49.6	28.4	0.0080
20.0%	7.4	7.4	7.1	7.4	7.4	7.4	7.4	881.4	738.0	93.3	0.5	49.6	30.0	0.0159
30.0%	7.9	7.9	7.3	7.8	7.9	7.8	7.8	841.4	708.0	83.0	0.8	49.6	31.7	0.0239
40.0%	8.4	8.4	7.4	8.4	8.4	8.4	8.4	799.3	676.1	72.5	1.1	49.6	33.8	0.0320
50.0%	9.1	9.1	7.4	9.1	9.1	9.1	9.1	754.7	641.6	61.9	1.5	49.6	36.1	0.0403
60.0%	10.0	10.0	7.5	10.0	10.0	10.0	10.0	706.8	604.1	51.1	1.9	49.6	39.1	0.0491
70.0%	11.3	11.3	8.1	11.2	11.3	11.3	11.2	654.5	562.4	39.9	2.5	49.6	42.8	0.0586
80.0%	13.1	13.1	9.9	13.1	13.1	13.0	13.1	595.9	514.8	28.2	3.3	49.6	48.0	0.0694
90.0%	16.4	16.4	12.9	16.3	16.4	16.4	16.3	527.3	457.5	15.5	4.7	49.6	56.5	0.0826

Table D-9 Optimal Results in Integer-ratio Headway Coordination (network I)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	6.1	12.3	6.1	6.1	12.3	6.1	6.1	914.6	761.6	101.8	1.6	49.6	21.4	0.0746
20.0%	6.3	12.6	6.3	6.3	12.6	6.3	6.3	893.3	749.4	91.0	3.3	49.6	26.6	0.1230
30.0%	6.4	12.9	6.4	6.4	12.9	6.4	6.4	871.6	736.9	80.1	5.0	49.6	32.0	0.1571
40.0%	6.6	13.2	6.6	6.6	13.2	6.6	6.6	849.7	724.1	69.1	6.9	49.6	37.7	0.1824
50.0%	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.4	710.9	58.0	8.8	49.6	43.7	0.2019
60.0%	7.0	13.9	7.0	7.0	13.9	7.0	7.0	804.7	697.4	46.8	10.9	49.6	50.0	0.2175
70.0%	7.2	14.4	7.2	7.2	14.4	7.2	7.2	781.6	683.5	35.4	13.1	49.6	56.8	0.2303
80.0%	7.4	14.8	7.0	7.4	14.8	7.4	7.4	758.0	669.1	23.8	15.4	49.6	64.1	0.2410
90.0%	7.7	15.3	6.9	7.7	15.3	7.7	7.7	733.9	654.3	12.0	18.0	49.6	71.8	0.2500

Table D-10 Optimal Results in Non-coordinated (network II)

Items Transfer Ratio	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
10.0%	4.4	7.6	7.7	1425.7	1156.9	174.5	12.9	81.3	51.0	0.2538
20.0%	4.4	7.6	7.7	1410.1	1156.9	155.1	16.7	81.3	52.1	0.3206
30.0%	4.4	7.6	7.7	1394.5	1156.9	135.7	20.5	81.3	53.2	0.3846
40.0%	4.4	7.6	7.7	1378.9	1156.9	116.3	24.2	81.3	54.3	0.4460
50.0%	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
60.0%	4.4	7.6	7.7	1347.6	1156.9	77.6	31.8	81.3	56.6	0.5616
70.0%	4.4	7.6	7.7	1332.0	1156.9	58.2	35.5	81.3	57.7	0.6160
80.0%	4.4	7.6	7.7	1316.4	1156.9	38.8	39.3	81.3	58.8	0.6683
90.0%	4.4	7.6	7.7	1300.7	1156.9	19.4	43.1	81.3	59.9	0.7187

Table D-11 Optimal Results in Common Headway Coordination (network II)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	5.6	5.6	5.6	5.6	5.6	5.6	5.6	1278.8	1049.7	183.2	3.4	42.5	64.4	0.0525
20.0%	5.7	5.7	5.7	5.7	5.7	5.7	5.7	1241.0	1027.7	163.8	6.9	42.5	79.2	0.0876
30.0%	5.9	5.9	5.9	5.9	5.9	5.9	5.9	1202.6	1005.1	144.3	10.7	42.5	94.8	0.1128
40.0%	6.1	6.1	6.1	6.1	6.1	6.1	6.1	1163.6	981.9	124.5	14.7	42.5	111.4	0.1318
50.0%	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1123.8	957.8	104.5	18.9	42.5	129.2	0.1466
60.0%	6.5	6.5	6.5	6.5	6.5	6.5	6.5	1083.3	933.0	84.3	23.5	42.5	148.3	0.1584
70.0%	6.7	6.7	6.6	6.7	6.7	6.7	6.7	1042.0	907.3	63.8	28.4	42.5	168.9	0.1682
80.0%	7.0	7.0	6.7	7.0	7.0	7.0	7.0	999.8	880.6	43.0	33.8	42.5	191.4	0.1764
90.0%	7.3	7.3	6.9	7.3	7.3	7.3	7.3	956.6	852.7	21.7	39.6	42.5	216.1	0.1834

Table D-12 Optimal Results in Integer-ratio Headway Coordination (network II)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	4.4	8.7	4.4	4.4	8.7	4.4	4.4	1407.6	1140.0	175.3	11.3	81.1	75.5	0.1493
20.0%	4.5	8.9	4.5	4.5	8.9	4.5	4.5	1374.9	1123.8	156.3	13.8	81.0	85.0	0.1618
30.0%	4.5	9.1	4.5	4.5	9.1	4.5	4.5	1341.9	1107.3	137.3	16.3	80.9	94.9	0.1720
40.0%	4.6	9.2	4.6	4.6	9.2	4.6	4.6	1308.6	1090.5	118.2	19.0	80.8	105.2	0.1807
50.0%	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1274.9	1073.4	98.9	21.8	80.8	116.0	0.1880
60.0%	4.8	9.6	4.8	4.8	9.6	4.8	4.8	1240.8	1055.9	79.4	24.7	80.7	127.2	0.1944
70.0%	4.9	9.9	4.9	4.9	9.9	4.9	4.9	1201.6	1033.8	59.9	27.5	80.3	131.6	0.2089
80.0%	5.1	10.1	5.1	5.1	10.1	5.1	5.1	1165.9	1014.8	40.2	30.7	80.2	142.9	0.2148
90.0%	5.2	10.4	5.2	5.2	10.4	5.2	5.2	1127.3	993.2	20.2	33.9	80.0	151.0	0.2246

3. Optimal Results When the Standard Deviation (SD) of Bus Arrival Times Varies

Table D-13 Optimal Results in Non-coordinated (network I)

Items σ_i	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
1	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
2	5.9	13.7	11.9	892.1	772.4	56.3	13.8	49.6	18.1	0.7608
3	5.9	13.9	12.1	891.1	771.2	56.4	13.9	49.6	18.0	0.7691
4	5.9	14.1	12.4	889.8	769.7	56.5	14.0	49.6	17.9	0.7803
5	5.9	14.4	12.7	888.2	767.8	56.7	14.1	49.6	17.8	0.7937
6	5.9	14.8	13.2	886.5	765.7	56.8	14.3	49.6	17.7	0.8089
7	5.9	15.2	13.6	884.6	763.4	57.0	14.5	49.6	17.6	0.8253
8	5.9	15.7	14.2	882.7	761.0	57.3	14.7	49.6	17.5	0.8424
9	5.9	16.2	14.8	880.7	758.6	57.5	15.0	49.6	17.4	0.8597
10	5.9	16.8	15.4	878.8	756.2	57.7	15.2	49.6	17.4	0.8768

Table D-14 Optimal Results in Common Headway Coordination (network I)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	9.1	9.1	7.4	9.1	9.1	9.1	9.1	754.7	641.6	61.9	1.5	49.6	36.1	0.0403
2	9.1	9.1	9.1	9.1	9.1	9.1	9.1	753.8	640.6	62.0	1.5	49.6	36.2	0.0418
3	9.2	9.2	9.2	9.2	9.2	9.2	9.2	752.4	638.9	62.2	1.6	49.6	36.2	0.0443
4	9.2	9.2	9.2	9.2	9.2	9.2	9.2	750.4	636.6	62.4	1.7	49.6	36.3	0.0477
5	9.3	9.3	9.3	9.3	9.3	9.3	9.3	747.9	633.6	62.7	1.9	49.6	36.3	0.0521
6	9.4	9.4	9.4	9.4	9.4	9.4	9.4	744.9	630.0	63.1	2.1	49.6	36.4	0.0573
7	9.5	9.5	9.5	9.5	9.5	9.5	9.5	741.4	626.0	63.5	2.3	49.6	36.5	0.0634
8	9.6	9.6	9.6	9.6	9.6	9.6	9.6	737.6	621.4	64.0	2.6	49.6	36.6	0.0701
9	9.7	9.7	9.7	9.7	9.7	9.7	9.7	733.5	616.5	64.5	2.8	49.6	36.7	0.0776
10	9.8	9.8	9.8	9.8	9.8	9.8	9.8	729.0	611.1	65.1	3.2	49.6	36.9	0.0857

Table D-15 Optimal Results in Integer-ratio Headway Coordination (network I)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.4	710.9	58.0	8.8	49.6	43.7	0.2019
2	6.8	13.6	6.8	6.8	13.6	6.8	6.8	826.8	710.2	58.1	8.9	49.6	43.7	0.2028
3	6.8	13.6	6.8	6.8	13.6	6.8	6.8	825.8	709.1	58.2	8.9	49.6	43.8	0.2043
4	6.8	13.6	6.8	6.8	13.6	6.8	6.8	824.5	707.5	58.3	9.1	49.6	43.9	0.2064
5	6.9	13.7	6.9	6.9	13.7	6.9	6.9	822.7	705.4	58.5	9.2	49.6	44.0	0.2091
6	6.9	13.8	6.9	6.9	13.8	6.9	6.9	820.6	703.0	58.7	9.4	49.6	44.1	0.2123
7	6.9	13.9	6.9	6.9	13.9	6.9	6.9	818.2	700.1	58.9	9.6	49.6	44.3	0.2160
8	7.0	14.0	7.0	7.0	14.0	7.0	7.0	815.5	696.9	59.2	9.8	49.6	44.5	0.2202
9	7.0	14.1	7.0	7.0	14.1	7.0	7.0	812.5	693.3	59.5	10.0	49.6	44.7	0.2248
10	7.1	14.2	7.1	7.1	14.2	7.1	7.1	809.2	689.4	59.8	10.3	49.6	44.9	0.2299

Table D-16 Optimal Results in Non-coordinated (network II)

Items σ_i	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
1	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
2	4.4	7.8	7.9	1360.1	1153.3	97.2	28.3	81.3	54.9	0.5150
3	4.4	8.2	8.2	1355.4	1147.7	97.7	28.7	81.3	54.2	0.5305
4	4.4	8.6	8.7	1349.7	1140.8	98.3	29.3	81.3	53.3	0.5499
5	4.4	9.1	9.2	1343.6	1133.3	99.0	30.0	81.3	52.5	0.5717
6	4.4	9.7	9.7	1337.5	1125.6	99.7	30.8	81.3	51.8	0.5943
7	4.4	10.3	10.6	1331.4	1118.6	100.3	31.4	81.1	50.3	0.6234
8	4.4	11.0	11.3	1326.2	1111.7	101.2	32.2	81.1	50.0	0.6440
9	4.3	11.8	12.5	1321.3	1106.5	101.5	32.5	80.8	48.0	0.6765
10	4.3	12.5	13.6	1317.1	1101.5	102.0	32.9	80.6	46.9	0.7022

Table D-17 Optimal Results in Common Headway Coordination (network II)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1123.8	957.8	104.5	18.9	42.5	129.2	0.1466
2	6.4	6.4	6.4	6.4	6.4	6.4	6.4	1108.7	939.8	106.1	20.3	42.5	131.2	0.1549
3	6.7	6.7	6.7	6.7	6.7	6.7	6.7	1086.1	912.5	108.5	22.5	42.5	134.4	0.1676
4	7.0	7.0	7.0	7.0	7.0	7.0	7.0	1058.7	879.1	111.7	25.4	42.5	138.7	0.1831
5	7.4	7.4	7.4	7.4	7.4	7.4	7.4	1029.3	842.5	115.5	28.8	42.5	143.9	0.2000
6	7.9	7.9	7.9	7.9	7.9	7.9	7.9	1000.0	805.2	119.8	32.6	42.5	150.0	0.2171
7	8.4	8.4	8.4	8.4	8.4	8.4	8.4	972.3	768.7	124.4	36.6	42.5	156.7	0.2338
8	9.0	9.0	9.0	9.0	9.0	9.0	9.0	946.8	734.2	129.2	40.9	42.5	163.9	0.2496
9	9.6	9.6	9.6	9.6	9.6	9.6	9.6	924.1	702.0	134.3	45.3	42.5	171.6	0.2643
10	10.3	10.3	10.3	10.3	10.3	10.3	10.3	904.2	672.4	139.4	49.9	42.5	179.6	0.2777

Table D-18 Optimal Results in Integer-ratio Headway Coordination (network II)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1274.9	1073.4	98.9	21.8	80.8	116.0	0.1880
2	4.7	9.5	4.7	4.7	9.5	4.7	4.7	1272.5	1070.6	99.1	22.0	80.8	116.2	0.1894
3	4.8	9.5	4.8	4.8	9.5	4.8	4.8	1268.6	1066.0	99.5	22.3	80.8	116.7	0.1915
4	4.8	9.6	4.8	4.8	9.6	4.8	4.8	1263.2	1059.7	100.0	22.8	80.8	117.2	0.1944
5	4.8	9.7	4.8	4.8	9.7	4.8	4.8	1256.5	1051.7	100.6	23.4	80.8	118.0	0.1981
6	4.9	9.8	4.9	4.9	9.8	4.9	4.9	1247.6	1041.6	101.4	23.9	80.7	116.2	0.2054
7	5.0	9.9	5.0	5.0	9.9	5.0	5.0	1238.0	1030.7	102.3	24.4	80.6	114.4	0.2132
8	5.0	10.1	5.0	5.0	10.1	5.0	5.0	1228.5	1019.4	103.3	25.2	80.6	115.4	0.2185
9	5.1	10.2	5.1	5.1	10.2	5.1	5.1	1218.0	1007.1	104.4	25.9	80.5	114.4	0.2263
10	5.2	10.4	5.2	5.2	10.4	5.2	5.2	1207.3	994.3	105.6	26.9	80.5	115.7	0.2322

4. Optimal Results When Passenger OD Rates of Feeder Bus services Varies

Table D-19 Optimal Results in Non-coordinated (network I)

Items λ_{ij}	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.5	8.0	18.2	15.8	705.5	633.9	33.8	10.2	27.6	10.1	1.0127
0.6	7.3	16.6	14.4	756.6	672.8	39.5	11.2	33.1	12.1	0.9247
0.7	6.7	15.4	13.3	804.4	708.6	45.2	12.1	38.6	14.1	0.8563
0.8	6.3	14.4	12.5	849.7	741.9	50.7	12.9	44.1	16.1	0.8012
0.9	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
1.0	5.6	12.9	11.2	934.1	802.7	61.7	14.5	55.2	20.2	0.7170
1.1	5.4	12.3	10.6	973.8	830.9	67.1	15.2	60.7	22.2	0.6838
1.2	5.1	11.8	10.2	1012.3	857.7	72.5	15.8	66.2	24.2	0.6549
1.3	4.9	11.3	9.8	1049.6	883.5	77.9	16.5	71.7	26.2	0.6293
1.4	4.8	10.9	9.4	1085.9	908.3	83.3	17.1	77.2	28.2	0.6066

Table D-20 Optimal Results in Common Headway Coordination (network I)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	12.2	12.2	8.4	12.2	12.2	12.2	12.2	602.6	535.9	38.1	1.1	27.6	21.8	0.050
0.6	11.2	11.2	9.2	11.2	11.2	11.2	11.2	643.9	565.4	44.2	1.2	33.1	25.5	0.047
0.7	10.3	10.3	8.3	10.3	10.3	10.3	10.3	682.7	592.6	50.2	1.3	38.6	29.1	0.044
0.8	9.7	9.7	7.4	9.7	9.7	9.7	9.7	719.5	617.9	56.1	1.4	44.1	32.6	0.042
0.9	9.1	9.1	7.4	9.1	9.1	9.1	9.1	754.7	641.6	61.9	1.5	49.6	36.1	0.040
1.0	8.7	8.7	7.5	8.7	8.7	8.7	8.7	788.5	664.1	67.7	1.5	55.2	39.6	0.039
1.1	8.2	8.2	7.6	8.2	8.2	8.2	8.2	821.2	685.4	73.4	1.6	60.7	43.1	0.037
1.2	7.9	7.9	7.2	7.9	7.9	7.9	7.9	852.9	705.8	79.1	1.7	66.2	46.6	0.036
1.3	7.6	7.6	7.0	7.6	7.6	7.6	7.6	883.6	725.4	84.8	1.8	71.7	50.0	0.035
1.4	7.3	7.3	6.7	7.3	7.3	7.3	7.3	913.7	744.2	90.4	1.8	77.2	53.4	0.034

Table D-21 Optimal Results in Integer-ratio Headway Coordination (network I)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	9.1	18.2	7.6	9.1	18.2	9.1	9.1	656.8	587.5	35.2	6.6	27.6	29.4	0.2233
0.6	8.3	16.6	7.3	8.3	16.6	8.3	8.3	703.2	621.9	41.0	7.2	33.1	33.2	0.2169
0.7	7.7	15.4	6.8	7.7	15.4	7.7	7.7	746.8	653.7	46.7	7.8	38.6	36.8	0.2113
0.8	7.2	14.4	6.8	7.2	14.4	7.2	7.2	788.0	683.2	52.4	8.3	44.1	40.3	0.2064
0.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.4	710.9	58.0	8.8	49.6	43.7	0.2019
1.0	6.4	12.9	6.4	6.4	12.9	6.4	6.4	865.2	737.1	63.6	9.3	55.2	47.0	0.1979
1.1	6.1	12.3	6.1	6.1	12.3	6.1	6.1	901.6	762.1	69.1	9.8	60.7	50.2	0.1943
1.2	5.9	11.7	5.9	5.9	11.7	5.9	5.9	936.8	785.9	74.6	10.2	66.2	53.4	0.1909
1.3	5.6	11.3	5.6	5.6	11.3	5.6	5.6	971.1	808.7	80.0	10.6	71.7	56.5	0.1877
1.4	5.4	10.9	5.4	5.4	10.9	5.4	5.4	1004.4	830.7	85.5	11.0	77.2	59.6	0.1848

Table D-22 Optimal Results in Non-coordinated (network II)

Items λ_{ij}	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.5	5.9	10.2	10.4	1042.9	919.5	57.4	20.8	45.2	30.9	0.6733
0.6	5.4	9.3	9.5	1130.1	985.6	67.4	22.8	54.2	37.0	0.6151
0.7	5.0	8.7	8.8	1212.5	1047.2	77.3	24.7	63.3	43.2	0.5717
0.8	4.6	8.1	8.2	1289.7	1103.8	87.2	26.4	72.3	49.3	0.5352
0.9	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
1.0	4.2	7.3	7.4	1433.7	1207.2	106.7	29.5	90.4	61.6	0.4794
1.1	4.0	6.9	7.0	1501.6	1254.9	116.3	31.0	99.4	67.7	0.4575
1.2	3.8	6.6	6.7	1567.3	1300.5	125.9	32.4	108.5	73.8	0.4383
1.3	3.6	6.4	6.4	1632.3	1345.4	135.5	33.8	117.6	80.5	0.4194
1.4	3.5	6.2	6.2	1694.3	1387.5	145.1	35.1	126.7	86.7	0.4044

Table D-23 Optimal Results in Common Headway Coordination (network II)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	8.4	8.4	6.9	8.4	8.4	8.4	8.4	873.8	773.5	62.7	14.0	23.6	86.5	0.1613
0.6	7.6	7.6	6.7	7.6	7.6	7.6	7.6	942.2	825.2	73.3	15.3	28.3	97.8	0.1568
0.7	7.1	7.1	6.4	7.1	7.1	7.1	7.1	1006.1	872.6	83.8	16.6	33.1	108.6	0.1530
0.8	6.6	6.6	6.6	6.6	6.6	6.6	6.6	1066.4	916.6	94.2	17.8	37.8	119.0	0.1496
0.9	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1123.8	957.8	104.5	18.9	42.5	129.2	0.1466
1.0	5.9	5.9	5.9	5.9	5.9	5.9	5.9	1178.7	996.7	114.8	20.0	47.2	139.2	0.1438
1.1	5.7	5.7	5.7	5.7	5.7	5.7	5.7	1231.5	1033.6	124.9	21.1	52.0	148.9	0.1414
1.2	5.4	5.4	5.4	5.4	5.4	5.4	5.4	1282.5	1068.7	135.1	22.0	56.7	158.5	0.1391
1.3	5.2	5.2	5.2	5.2	5.2	5.2	5.2	1331.9	1102.3	145.2	23.0	61.4	167.9	0.1371
1.4	5.0	5.0	5.0	5.0	5.0	5.0	5.0	1379.8	1134.5	155.2	23.9	66.1	177.2	0.1352

Table D-24 Optimal Results in Integer-ratio Headway Coordination (network II)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	6.4	12.8	6.4	6.4	12.8	6.4	6.4	971.6	852.2	58.9	15.9	44.6	70.3	0.2259
0.6	5.8	11.6	5.8	5.8	11.6	5.8	5.8	1051.9	911.9	69.1	17.4	53.5	79.2	0.2196
0.7	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1129.3	968.7	79.2	18.9	62.6	89.6	0.2103
0.8	5.0	10.0	5.0	5.0	10.0	5.0	5.0	1203.3	1022.3	89.1	20.3	71.6	102.4	0.1988
0.9	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1274.9	1073.4	98.9	21.8	80.8	116.0	0.1880
1.0	4.5	8.9	4.5	4.5	8.9	4.5	4.5	1340.6	1119.2	108.7	23.0	89.7	124.6	0.1847
1.1	4.3	8.5	4.3	4.3	8.5	4.3	4.3	1405.7	1164.2	118.4	24.3	98.8	135.8	0.1786
1.2	4.1	8.2	4.1	4.1	8.2	4.1	4.1	1467.1	1205.9	128.1	25.3	107.8	144.1	0.1759
1.3	3.9	7.8	3.9	3.9	7.8	3.9	3.9	1526.7	1245.7	137.8	26.4	116.8	152.3	0.1733
1.4	3.8	7.5	3.8	3.8	7.5	3.8	3.8	1586.7	1285.9	147.4	27.5	125.9	163.6	0.1683

5. Coefficient of In-vehicle Value of Time (VOT) Varies

Table D-25 Optimal Results in Non-coordinated (network I)

Items k_v	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.1	5.9	13.6	11.8	853.0	773.2	56.2	13.7	9.9	18.1	0.7556
0.2	5.9	13.6	11.8	863.0	773.2	56.2	13.7	19.9	18.1	0.7556
0.3	5.9	13.6	11.8	872.9	773.2	56.2	13.7	29.8	18.1	0.7556
0.4	5.9	13.6	11.8	882.8	773.2	56.2	13.7	39.7	18.1	0.7556
0.5	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
0.6	5.9	13.6	11.8	902.7	773.2	56.2	13.7	59.6	18.1	0.7556
0.7	5.9	13.6	11.8	912.6	773.2	56.2	13.7	69.5	18.1	0.7556
0.8	5.9	13.6	11.8	922.5	773.2	56.2	13.7	79.4	18.1	0.7556
0.9	5.9	13.6	11.8	932.5	773.2	56.2	13.7	89.4	18.1	0.7556

Table D-26 Optimal Results in Common Headway Coordination (network I)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	9.1	9.1	8.4	9.1	9.1	9.1	9.1	715.0	641.6	61.9	1.5	9.9	36.1	0.0403
0.2	9.1	9.1	8.4	9.1	9.1	9.1	9.1	724.9	641.6	61.9	1.5	19.9	36.1	0.0403
0.3	9.1	9.1	8.4	9.1	9.1	9.1	9.1	734.8	641.6	61.9	1.5	29.8	36.1	0.0403
0.4	9.1	9.1	8.4	9.1	9.1	9.1	9.1	744.8	641.6	61.9	1.5	39.7	36.1	0.0403
0.5	9.1	9.1	7.4	9.1	9.1	9.1	9.1	754.7	641.6	61.9	1.5	49.6	36.1	0.0403
0.6	9.1	9.1	8.4	9.1	9.1	9.1	9.1	764.6	641.6	61.9	1.5	59.6	36.1	0.0403
0.7	9.1	9.1	8.4	9.1	9.1	9.1	9.1	774.5	641.6	61.9	1.5	69.5	36.1	0.0403
0.8	9.1	9.1	8.4	9.1	9.1	9.1	9.1	784.5	641.6	61.9	1.5	79.4	36.1	0.0403
0.9	9.1	9.1	8.4	9.1	9.1	9.1	9.1	794.4	641.6	61.9	1.5	89.4	36.1	0.0403

Table D-27 Optimal Results in Integer-ratio Headway Coordination (network I)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	6.8	13.5	6.8	6.8	13.5	6.8	6.8	787.7	710.9	58.0	8.8	9.9	43.7	0.2019
0.2	6.8	13.5	6.8	6.8	13.5	6.8	6.8	797.6	710.9	58.0	8.8	19.9	43.7	0.2019
0.3	6.8	13.5	6.8	6.8	13.5	6.8	6.8	807.5	710.9	58.0	8.8	29.8	43.7	0.2019
0.4	6.8	13.5	6.8	6.8	13.5	6.8	6.8	817.5	710.9	58.0	8.8	39.7	43.7	0.2019
0.5	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.4	710.9	58.0	8.8	49.6	43.7	0.2019
0.6	6.8	13.5	6.8	6.8	13.5	6.8	6.8	837.3	710.9	58.0	8.8	59.6	43.7	0.2019
0.7	6.8	13.5	6.8	6.8	13.5	6.8	6.8	847.2	710.9	58.0	8.8	69.5	43.7	0.2019
0.8	6.8	13.5	6.8	6.8	13.5	6.8	6.8	857.2	710.9	58.0	8.8	79.4	43.7	0.2019
0.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	867.1	710.9	58.0	8.8	89.4	43.7	0.2019

Table D-28 Optimal Results in Non-coordinated (network II)

Items k_v	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.1	4.4	7.6	7.7	1298.2	1156.9	96.9	28.0	16.3	55.4	0.5050
0.2	4.4	7.6	7.7	1314.4	1156.9	96.9	28.0	32.5	55.4	0.5050
0.3	4.4	7.6	7.7	1330.7	1156.9	96.9	28.0	48.8	55.4	0.5050
0.4	4.4	7.6	7.7	1347.0	1156.9	96.9	28.0	65.1	55.4	0.5050
0.5	4.4	7.6	7.7	1363.2	1156.9	96.9	28.0	81.3	55.4	0.5050
0.6	4.4	7.6	7.7	1379.5	1156.9	96.9	28.0	97.6	55.4	0.5050
0.7	4.4	7.6	7.7	1395.8	1156.9	96.9	28.0	113.9	55.4	0.5050
0.8	4.4	7.6	7.7	1412.0	1156.9	96.9	28.0	130.2	55.4	0.5050
0.9	4.4	7.6	7.7	1428.3	1156.9	96.9	28.0	146.4	55.4	0.5050

Table D-29 Optimal Results in Common Headway Coordination (network II)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1089.8	957.8	104.5	18.9	8.5	129.2	0.1466
0.2	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1098.3	957.8	104.5	18.9	17.0	129.2	0.1466
0.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1106.8	957.8	104.5	18.9	25.5	129.2	0.1466
0.4	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1115.3	957.8	104.5	18.9	34.0	129.2	0.1466
0.5	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1123.8	957.8	104.5	18.9	42.5	129.2	0.1466
0.6	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1132.3	957.8	104.5	18.9	51.0	129.2	0.1466
0.7	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1140.8	957.8	104.5	18.9	59.5	129.2	0.1466
0.8	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1149.3	957.8	104.5	18.9	68.0	129.2	0.1466
0.9	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1157.8	957.8	104.5	18.9	76.5	129.2	0.1466

Table D-30 Optimal Results in Integer-ratio Headway Coordination (network II)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1210.3	1073.4	98.9	21.8	16.2	116.0	0.1880
0.2	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1226.4	1073.4	98.9	21.8	32.3	116.0	0.1880
0.3	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1242.6	1073.4	98.9	21.8	48.5	116.0	0.1880
0.4	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1258.7	1073.4	98.9	21.8	64.6	116.0	0.1880
0.5	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1274.9	1073.4	98.9	21.8	80.8	116.0	0.1880
0.6	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1291.0	1073.4	98.9	21.8	96.9	116.0	0.1880
0.7	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1307.2	1073.4	98.9	21.8	113.1	116.0	0.1880
0.8	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1323.3	1073.4	98.9	21.8	129.2	116.0	0.1880
0.9	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1339.5	1073.4	98.9	21.8	145.4	116.0	0.1880

APPENDIX E Sensitivity Analysis Results of Scenarios When Operators Are in Competition

1. Optimal Results When Value of Time Varies

Table E-1 Optimal Results in Non-coordinated (network I)

Items u (S\$/hr)	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
13.9	6.9	15.8	13.7	786.6	695.3	43.0	11.8	36.5	18.1	0.6476
14.9	6.7	15.3	13.2	808.8	711.8	45.7	12.2	39.1	18.1	0.6706
15.9	6.5	14.8	12.8	830.6	727.9	48.3	12.6	41.8	18.1	0.6928
16.9	6.3	14.3	12.4	851.8	743.4	51.0	13.0	44.4	18.1	0.7143
17.9	6.1	13.9	12.1	872.5	758.5	53.6	13.3	47.0	18.1	0.7352
18.9	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
19.9	5.8	13.2	11.5	912.6	787.4	58.8	14.1	52.3	18.1	0.7754
20.9	5.7	12.9	11.2	932.1	801.4	61.4	14.4	54.9	18.1	0.7948
21.9	5.5	12.6	10.9	951.3	815.0	64.0	14.8	57.5	18.1	0.8137
22.9	5.4	12.3	10.7	970.1	828.2	66.6	15.1	60.2	18.1	0.8321

Table E-2 Optimal Results in Common Headway Coordination (network I)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	10.6	10.6	0.0	1.0	10.6	9.5	10.6	668.1	582.5	47.9	1.2	36.5	37.7	0.0331
14.9	10.3	10.3	9.1	10.3	10.3	10.3	10.3	686.2	595.1	50.7	1.3	39.1	37.3	0.0346
15.9	9.9	9.9	8.8	9.9	9.9	9.9	9.9	703.9	607.3	53.6	1.3	41.7	37.0	0.0361
16.9	9.6	9.6	8.6	9.6	9.6	9.6	9.6	721.1	619.1	56.4	1.4	44.3	36.7	0.0375
17.9	9.4	9.4	8.4	9.4	9.4	9.4	9.4	738.0	630.5	59.2	1.4	47.0	36.4	0.0389
18.9	9.1	9.1		7.9	9.1		8.9	754.6	641.6	61.9	1.5	49.6	36.1	0.0403
19.9	8.9	8.9	7.9	8.9	8.9	8.9	8.9	770.9	652.5	64.7	1.5	52.2	35.9	0.0417
20.9	8.7	8.7	7.7	8.7	8.7	8.7	8.7	786.9	663.1	67.4	1.5	54.8	35.7	0.0430
21.9	8.5	8.5	7.5	8.5	8.5	8.5	8.5	802.6	673.4	70.2	1.6	57.5	35.5	0.0443
22.9	8.3	8.3	7.3	8.3	8.3	8.3	8.3	818.0	683.5	72.9	1.6	60.1	35.3	0.0455

Table E-3 Optimal Results in Integer-ratio Headway Coordination (network I)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	7.9	15.8	7.0	7.9	15.8	7.9	7.9	730.4	641.9	44.6	7.6	36.5	48.2	0.1569
14.9	7.6	15.3	6.7	7.6	15.3	7.6	7.6	750.8	656.6	47.3	7.8	39.1	47.1	0.1662
15.9	7.4	14.8	6.5	7.4	14.8	7.4	7.4	770.6	670.8	50.0	8.1	41.7	46.1	0.1753
16.9	7.2	14.3	6.8	7.2	14.3	7.2	7.2	789.9	684.5	52.7	8.3	44.3	45.2	0.1843
17.9	7.0	13.9	7.0	7.0	13.9	7.0	7.0	808.8	697.9	55.3	8.6	47.0	44.4	0.1932
18.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.3	710.9	58.0	8.8	49.6	43.7	0.2019
19.9	6.6	13.2	6.6	6.6	13.2	6.6	6.6	845.5	723.6	60.7	9.0	52.2	43.0	0.2106
20.9	6.4	12.9	6.4	6.4	12.9	6.4	6.4	863.3	735.9	63.3	9.3	54.8	42.3	0.2191
21.9	6.3	12.6	6.3	6.3	12.6	6.3	6.3	880.9	748.0	65.9	9.5	57.5	41.7	0.2275
22.9	6.2	12.3	6.2	6.2	12.3	6.2	6.2	898.1	759.7	68.6	9.7	60.1	41.2	0.2358

Table E-4 Optimal Results in Non-coordinated (network II)

Items u (S\$/hr)	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
13.9	5.1	9.0	9.0	1180.2	1023.0	73.6	23.9	59.7	55.2	0.4322
14.9	4.9	8.7	8.7	1218.1	1051.1	78.3	24.7	64.0	55.2	0.4476
15.9	4.8	8.4	8.4	1255.1	1078.3	83.0	25.5	68.3	55.2	0.4625
16.9	4.6	8.1	8.1	1291.2	1104.6	87.7	26.3	72.6	55.2	0.4770
17.9	4.5	7.9	7.9	1326.5	1130.2	92.3	27.1	76.9	55.2	0.4911
18.9	4.4	7.7	7.7	1361.1	1155.0	97.0	27.9	81.2	55.2	0.5048
19.9	4.3	7.5	7.5	1394.9	1179.2	101.6	28.6	85.5	55.2	0.5182
20.9	4.2	7.3	7.3	1428.1	1202.8	106.2	29.3	89.8	55.2	0.5313
21.9	4.1	7.2	7.2	1460.8	1225.8	110.8	30.0	94.1	55.2	0.5440
22.9	4.0	7.0	7.0	1492.9	1248.3	115.4	30.7	98.4	55.2	0.5565

Table E-5 Optimal Results in Common Headway Coordination (network II)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	5.7	5.7	5.7	5.7	5.7	5.7	5.7	1169.0	1024.9	74.0	10.4	59.7	146.9	0.0706
14.9	5.5	5.5	5.5	5.5	5.5	5.5	5.5	1206.4	1053.0	78.7	10.7	64.0	144.0	0.0746
15.9	5.4	5.4	5.4	5.4	5.4	5.4	5.4	1243.0	1080.2	83.4	11.1	68.2	141.5	0.0785
16.9	5.2	5.2	5.2	5.2	5.2	5.2	5.2	1278.6	1106.5	88.1	11.5	72.5	139.1	0.0824
17.9	5.1	5.1	5.1	5.1	5.1	5.1	5.1	1313.4	1132.0	92.8	11.8	76.8	137.0	0.0861
18.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	1347.5	1156.9	97.4	12.1	81.1	135.0	0.0899
19.9	4.8	4.8	4.8	4.8	4.8	4.8	4.8	1382.2	1182.3	102.0	12.5	85.4	133.6	0.0937
20.9	4.7	4.7	4.7	4.7	4.7	4.7	4.7	1415.1	1205.9	106.7	12.8	89.7	131.9	0.0974
21.9	4.6	4.6	4.6	4.6	4.6	4.6	4.6	1447.3	1228.9	111.3	13.2	94.0	130.3	0.1009
22.9	4.5	4.5	4.5	4.5	4.5	4.5	4.5	1479.0	1251.4	115.9	13.5	98.2	128.8	0.1045

Table E-6 Optimal Results in Integer-ratio Headway Coordination (network II)

Items u (S\$/hr)	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
13.9	4.5	9.0	4.5	4.5	9.0	4.5	4.5	1261.7	1115.1	72.0	15.2	59.3	112.7	0.1351
14.9	4.3	8.7	4.3	4.3	8.7	4.3	4.3	1304.2	1148.0	76.7	15.8	63.7	112.6	0.1407
15.9	4.2	8.4	4.2	4.2	8.4	4.2	4.2	1344.0	1178.4	81.3	16.4	67.9	110.2	0.1484
16.9	4.1	8.1	4.1	4.1	8.1	4.1	4.1	1382.8	1207.8	85.9	16.9	72.2	108.1	0.1561
17.9	4.0	7.9	4.0	4.0	7.9	4.0	4.0	1420.7	1236.3	90.5	17.4	76.5	106.2	0.1635
18.9	3.8	7.7	3.8	3.8	7.7	3.8	3.8	1459.7	1265.8	95.1	17.9	80.9	106.6	0.1684
19.9	3.7	7.5	3.7	3.7	7.5	3.7	3.7	1496.1	1292.9	99.7	18.4	85.2	104.9	0.1756
20.9	3.7	7.3	3.7	3.7	7.3	3.7	3.7	1531.8	1319.3	104.2	18.9	89.4	103.4	0.1827
21.9	3.6	7.1	3.6	3.6	7.1	3.6	3.6	1566.9	1345.0	108.8	19.3	93.7	101.9	0.1898
22.9	3.5	7.0	3.5	3.5	7.0	3.5	3.5	1602.9	1371.6	113.3	19.9	98.1	102.0	0.1946

2. Optimal Results When Transfer Ratio Varies

Table E-7 Optimal Results in Non-coordinated (network I)

Items Transfer Ratio	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
10.0%	5.9	13.6	11.8	926.8	773.2	101.2	2.7	49.6	16.4	0.1668
20.0%	5.9	13.6	11.8	918.3	773.2	90.0	5.5	49.6	16.9	0.3252
30.0%	5.9	13.6	11.8	909.8	773.2	78.7	8.2	49.6	17.3	0.4758
40.0%	5.9	13.6	11.8	901.3	773.2	67.5	11.0	49.6	17.7	0.6190
50.0%	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
60.0%	5.9	13.6	11.8	884.2	773.2	45.0	16.5	49.6	18.6	0.8859
70.0%	5.9	13.6	11.8	875.7	773.2	33.7	19.2	49.6	19.0	1.0103
80.0%	5.9	13.6	11.8	867.2	773.2	22.5	21.9	49.6	19.4	1.1292
90.0%	5.9	13.6	11.8	858.7	773.2	11.2	24.7	49.6	19.9	1.2430

Table E-8 Optimal Results in Common Headway Coordination (network I)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	7.0	7.0	6.5	6.4	7.0	6.4	6.4	919.6	766.3	103.4	0.2	49.6	28.4	0.0080
20.0%	7.4	7.4	6.6	6.8	7.4	6.8	6.8	881.3	738.0	93.3	0.5	49.6	30.0	0.0159
30.0%	7.9	7.9	7.3	7.6	7.9	7.6	7.6	841.3	708.0	83.0	0.8	49.6	31.7	0.0239
40.0%	8.4	8.4	7.2	8.2	8.4	8.2	8.2	799.3	676.1	72.5	1.1	49.6	33.8	0.0320
50.0%	9.1	9.1	7.9	8.9	9.1	8.9	8.9	754.6	641.6	61.9	1.5	49.6	36.1	0.0403
60.0%	10.0	10.0	8.6	9.8	10.0	9.8	9.8	706.7	604.1	51.1	1.9	49.6	39.1	0.0491
70.0%	11.3	11.3	9.4	11.0	11.3	11.0	11.0	654.4	562.4	39.9	2.5	49.6	42.8	0.0586
80.0%	13.1	13.1	9.8	12.9	13.1	12.9	12.9	595.9	514.8	28.2	3.3	49.6	48.0	0.0694
90.0%	16.4	16.4	11.5	16.1	16.4	16.4	16.1	527.3	457.5	15.5	4.7	49.6	56.5	0.0826

Table E-9 Optimal Results in Integer-ratio Headway Coordination (network I)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	6.1	12.3	6.1	6.1	12.3	6.1	6.1	914.5	761.6	101.8	1.6	49.6	21.4	0.0746
20.0%	6.3	12.6	6.3	6.3	12.6	6.3	6.3	893.2	749.4	91.0	3.3	49.6	26.6	0.1230
30.0%	6.4	12.9	6.4	6.4	12.9	6.4	6.4	871.6	736.9	80.1	5.0	49.6	32.0	0.1571
40.0%	6.6	13.2	6.6	6.6	13.2	6.6	6.6	849.6	724.1	69.1	6.9	49.6	37.7	0.1824
50.0%	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.3	710.9	58.0	8.8	49.6	43.7	0.2019
60.0%	7.0	13.9	7.0	7.0	13.9	7.0	7.0	804.6	697.4	46.8	10.9	49.6	50.0	0.2175
70.0%	7.2	14.4	7.2	7.2	14.4	7.2	7.2	781.5	683.5	35.4	13.1	49.6	56.8	0.2303
80.0%	7.4	14.8	7.0	7.4	14.8	7.4	7.4	758.0	669.1	23.8	15.4	49.6	64.1	0.2410
90.0%	7.7	15.3	6.9	7.7	15.3	7.7	7.7	733.9	654.3	12.0	18.0	49.6	71.8	0.2500

Table E-10 Optimal Results in Non-coordinated (network II)

Items Transfer Ratio	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
10.0%	4.4	7.7	7.7	1423.6	1155.0	174.6	12.8	81.2	50.7	0.2518
20.0%	4.4	7.7	7.7	1407.9	1155.0	155.2	16.5	81.2	51.9	0.3191
30.0%	4.4	7.7	7.7	1392.3	1155.0	135.8	20.3	81.2	53.0	0.3836
40.0%	4.4	7.7	7.7	1376.7	1155.0	116.4	24.1	81.2	54.1	0.4455
50.0%	4.4	7.7	7.7	1361.1	1155.0	97.0	27.9	81.2	55.2	0.5048
60.0%	4.4	7.7	7.7	1345.4	1155.0	77.6	31.6	81.2	56.3	0.5619
70.0%	4.4	7.7	7.7	1329.8	1155.0	58.2	35.4	81.2	57.4	0.6167
80.0%	4.4	7.7	7.7	1314.2	1155.0	38.8	39.2	81.2	58.5	0.6694
90.0%	4.4	7.7	7.7	1298.6	1155.0	19.4	43.0	81.2	59.6	0.7201

Table E-11 Optimal Results in Common Headway Coordination (network II)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	4.1	4.1	4.1	4.1	4.1	4.1	4.1	1599.5	1340.2	169.7	8.5	81.1	113.3	0.0752
20.0%	4.3	4.3	4.3	4.3	4.3	4.3	4.3	1539.9	1297.6	151.9	9.3	81.1	118.0	0.0789
30.0%	4.5	4.5	4.5	4.5	4.5	4.5	4.5	1478.3	1253.2	133.9	10.2	81.1	123.1	0.0825
40.0%	4.7	4.7	4.7	4.7	4.7	4.7	4.7	1414.8	1206.8	115.7	11.1	81.1	128.9	0.0863
50.0%	4.9	4.9	4.9	4.9	4.9	4.9	4.9	1347.5	1156.9	97.4	12.1	81.1	135.0	0.0899
60.0%	5.2	5.2	5.2	5.2	5.2	5.2	5.2	1279.0	1105.7	78.8	13.4	81.1	142.6	0.0938
70.0%	5.5	5.5	5.5	5.5	5.5	5.5	5.5	1207.2	1051.3	59.9	14.8	81.1	151.4	0.0979
80.0%	6.0	6.0	6.0	6.0	6.0	6.0	6.0	1128.2	990.2	40.6	16.4	80.9	161.1	0.1019
90.0%	6.5	6.5	6.5	6.5	6.5	6.5	6.5	1045.5	925.4	20.8	18.5	80.8	173.6	0.1064

Table E-12 Optimal Results in Integer-ratio Headway Coordination (network II)

Items Transfer Ratio	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
10.0%	3.6	7.1	3.6	3.6	7.1	3.6	3.6	1603.6	1344.4	169.0	9.2	81.0	69.7	0.1313
20.0%	3.6	7.3	3.6	3.6	7.3	3.6	3.6	1568.2	1325.3	150.7	11.2	81.0	78.4	0.1432
30.0%	3.7	7.4	3.7	3.7	7.4	3.7	3.7	1532.4	1305.8	132.2	13.4	81.0	87.4	0.1531
40.0%	3.8	7.5	3.8	3.8	7.5	3.8	3.8	1496.3	1286.0	113.7	15.6	80.9	96.8	0.1613
50.0%	3.8	7.7	3.8	3.8	7.7	3.8	3.8	1459.7	1265.8	95.1	17.9	80.9	106.6	0.1684
60.0%	3.9	7.9	3.9	3.9	7.9	3.9	3.9	1420.4	1243.1	76.4	20.3	80.7	114.2	0.1775
70.0%	4.0	8.0	4.0	4.0	8.0	4.0	4.0	1382.6	1221.7	57.5	22.8	80.6	124.4	0.1832
80.0%	4.1	8.2	4.1	4.1	8.2	4.1	4.1	1344.3	1199.8	38.5	25.4	80.6	135.1	0.1883
90.0%	4.2	8.4	4.2	4.2	8.4	4.2	4.2	1305.5	1177.4	19.3	28.2	80.5	146.3	0.1928

3. Optimal Results When the Standard Deviation (SD) of Bus Arrival Times Varies

Table E-13 Optimal Results in Non-coordinated (network I)

Items σ_i	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
1	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
2	5.9	13.7	11.9	892.1	772.4	56.3	13.8	49.6	18.1	0.7608
3	5.9	13.9	12.1	891.1	771.2	56.4	13.9	49.6	18.0	0.7691
4	5.9	14.1	12.4	889.8	769.7	56.5	14.0	49.6	17.9	0.7803
5	5.9	14.4	12.7	888.2	767.8	56.7	14.1	49.6	17.8	0.7937
6	5.9	14.8	13.2	886.5	765.7	56.8	14.3	49.6	17.7	0.8089
7	5.9	15.2	13.6	884.6	763.4	57.0	14.5	49.6	17.6	0.8253
8	5.9	15.7	14.2	882.7	761.0	57.3	14.7	49.6	17.5	0.8424
9	5.9	16.2	14.8	880.7	758.6	57.5	15.0	49.6	17.4	0.8597
10	5.9	16.8	15.4	878.8	756.2	57.7	15.2	49.6	17.4	0.8768

Table E-14 Optimal Results in Common Headway Coordination (network I)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	9.1	9.1	7.9	8.9	9.1	8.9	8.9	754.6	641.6	61.9	1.5	49.6	36.1	0.0403
2	9.1	9.1	9.1	9.1	9.1	9.1	9.1	753.7	640.6	62.0	1.5	49.6	36.2	0.0418
3	9.2	9.2	9.2	9.2	9.2	9.2	9.2	752.3	638.9	62.2	1.6	49.6	36.2	0.0443
4	9.2	9.2	9.2	9.2	9.2	9.2	9.2	750.3	636.6	62.4	1.7	49.6	36.3	0.0477
5	9.3	9.3	9.3	9.3	9.3	9.3	9.3	747.8	633.6	62.7	1.9	49.6	36.3	0.0521
6	9.4	9.4	9.4	9.4	9.4	9.4	9.4	744.8	630.0	63.1	2.1	49.6	36.4	0.0573
7	9.5	9.5	9.5	9.5	9.5	9.5	9.5	741.4	626.0	63.5	2.3	49.6	36.5	0.0634
8	9.6	9.6	9.6	9.6	9.6	9.6	9.6	737.6	621.4	64.0	2.6	49.6	36.6	0.0701
9	9.7	9.7	9.7	9.7	9.7	9.7	9.7	733.4	616.5	64.5	2.8	49.6	36.7	0.0776
10	9.8	9.8	9.8	9.8	9.8	9.8	9.8	728.9	611.1	65.1	3.2	49.6	36.9	0.0857

Table E-15 Optimal Results in Integer-ratio Headway Coordination (network I)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.3	710.9	58.0	8.8	49.6	43.7	0.2019
2	6.8	13.6	6.8	6.8	13.6	6.8	6.8	826.7	710.2	58.1	8.9	49.6	43.7	0.2028
3	6.8	13.6	6.8	6.8	13.6	6.8	6.8	825.7	709.1	58.2	8.9	49.6	43.8	0.2043
4	6.8	13.6	6.8	6.8	13.6	6.8	6.8	824.4	707.5	58.3	9.1	49.6	43.9	0.2064
5	6.9	13.7	6.9	6.9	13.7	6.9	6.9	822.7	705.4	58.5	9.2	49.6	44.0	0.2091
6	6.9	13.8	6.9	6.9	13.8	6.9	6.9	820.6	703.0	58.7	9.4	49.6	44.1	0.2123
7	6.9	13.9	6.9	6.9	13.9	6.9	6.9	818.2	700.1	58.9	9.6	49.6	44.3	0.2160
8	7.0	14.0	7.0	7.0	14.0	7.0	7.0	815.4	696.9	59.2	9.8	49.6	44.5	0.2202
9	7.0	14.1	7.0	7.0	14.1	7.0	7.0	812.4	693.3	59.5	10.0	49.6	44.7	0.2248
10	7.1	14.2	7.1	7.1	14.2	7.1	7.1	809.1	689.4	59.8	10.3	49.6	44.9	0.2299

Table E-16 Optimal Results in Non-coordinated (network II)

Items σ_i	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
1	4.4	7.7	7.7	1361.1	1155.0	97.0	27.9	81.2	70.7	0.3939
2	4.4	7.9	7.9	1357.9	1151.3	97.3	28.1	81.2	70.7	0.3980
3	4.4	8.2	8.2	1353.3	1145.8	97.7	28.6	81.2	70.7	0.4043
4	4.4	8.6	8.6	1347.6	1138.9	98.3	29.2	81.2	70.7	0.4126
5	4.4	9.1	9.1	1341.5	1131.4	99.0	29.9	81.2	70.7	0.4224
6	4.4	9.7	9.7	1335.3	1123.7	99.8	30.7	81.2	70.7	0.4334
7	4.4	10.4	10.6	1329.3	1116.7	100.4	31.2	81.0	70.9	0.4409
8	4.4	11.1	11.3	1324.1	1109.8	101.2	32.1	81.0	70.9	0.4527
9	4.4	11.8	12.5	1319.2	1104.7	101.5	32.4	80.6	71.1	0.4552
10	4.3	12.6	13.5	1315.0	1099.8	102.0	32.8	80.5	71.2	0.4605

Table E-17 Optimal Results in Common Headway Coordination (network II)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	4.9	4.9	4.9	4.9	4.9	4.9	4.9	1347.5	1156.9	97.4	12.1	81.1	130.7	0.0928
2	5.0	5.0	5.0	5.0	5.0	5.0	5.0	1339.5	1147.9	97.9	12.5	81.1	131.4	0.0955
3	5.0	5.0	5.0	5.0	5.0	5.0	5.0	1326.6	1133.5	98.7	13.2	81.1	132.5	0.0998
4	5.2	5.2	5.2	5.2	5.2	5.2	5.2	1309.6	1114.5	99.8	14.1	81.1	134.1	0.1054
5	5.3	5.3	5.3	5.3	5.3	5.3	5.3	1289.3	1091.7	101.2	15.3	81.1	136.1	0.1122
6	5.4	5.4	5.4	5.4	5.4	5.4	5.4	1266.6	1066.1	102.7	16.6	81.1	138.4	0.1198
7	5.6	5.6	5.6	5.6	5.6	5.6	5.6	1242.3	1038.6	104.5	18.1	81.1	141.1	0.1280
8	5.8	5.8	5.8	5.8	5.8	5.8	5.8	1217.9	1011.1	106.5	19.3	81.0	143.4	0.1347
9	6.1	6.1	6.1	6.1	6.1	6.1	6.1	1192.9	982.4	108.5	21.0	81.0	146.6	0.1432
10	6.3	6.3	6.3	6.3	6.3	6.3	6.3	1168.3	953.8	110.7	22.8	81.0	150.1	0.1516

Table E-18 Optimal Results in Integer-ratio Headway Coordination (network II)

Items σ_i	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
1	3.8	7.7	3.8	3.8	7.7	3.8	3.8	1459.7	1265.8	95.1	17.9	80.9	124.5	0.1441
2	3.9	7.7	3.9	3.9	7.7	3.9	3.9	1454.8	1260.4	95.4	18.2	80.9	124.9	0.1459
3	3.9	7.8	3.9	3.9	7.8	3.9	3.9	1446.8	1251.5	95.8	18.7	80.9	125.5	0.1487
4	3.9	7.9	3.9	3.9	7.9	3.9	3.9	1434.6	1238.3	96.5	19.1	80.8	125.4	0.1525
5	4.0	8.0	4.0	4.0	8.0	4.0	4.0	1421.5	1223.7	97.3	19.9	80.8	126.4	0.1571
6	4.1	8.1	4.1	4.1	8.1	4.1	4.1	1406.4	1206.6	98.2	20.7	80.8	127.6	0.1625
7	4.2	8.3	4.2	4.2	8.3	4.2	4.2	1389.4	1187.6	99.3	21.7	80.8	129.0	0.1686
8	4.2	8.5	4.2	4.2	8.5	4.2	4.2	1371.1	1167.0	100.5	22.9	80.8	130.5	0.1752
9	4.3	8.7	4.3	4.3	8.7	4.3	4.3	1351.8	1145.1	101.8	24.1	80.8	132.3	0.1822
10	4.5	8.9	4.5	4.5	8.9	4.5	4.5	1332.4	1123.5	103.2	25.0	80.7	133.0	0.1882

4. Optimal Results When Passenger OD Rates of Feeder Bus services Varies

Table E-19 Optimal Results in Non-coordinated (network I)

Items λ_{ij}	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.5	8.0	18.2	15.8	705.5	633.9	33.8	10.2	27.6	10.1	1.0127
0.6	7.3	16.6	14.4	756.6	672.8	39.5	11.2	33.1	12.1	0.9247
0.7	6.7	15.4	13.3	804.4	708.6	45.2	12.1	38.6	14.1	0.8563
0.8	6.3	14.4	12.5	849.7	741.9	50.7	12.9	44.1	16.1	0.8012
0.9	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
1.0	5.6	12.9	11.2	934.1	802.7	61.7	14.5	55.2	20.2	0.7170
1.1	5.4	12.3	10.6	973.8	830.9	67.1	15.2	60.7	22.2	0.6838
1.2	5.1	11.8	10.2	1012.3	857.7	72.5	15.8	66.2	24.2	0.6549
1.3	4.9	11.3	9.8	1049.6	883.5	77.9	16.5	71.7	26.2	0.6293
1.4	4.8	10.9	9.4	1085.9	908.3	83.3	17.1	77.2	28.2	0.6066

Table E-20 Optimal Results in Common Headway Coordination (network I)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	12.2	12.2	8.1	12.2	12.2	12.2	12.2	602.6	535.9	38.1	1.1	27.5	21.8	0.0495
0.6	11.2	11.2	9.6	11.2	11.2	11.2	11.2	643.9	565.4	44.2	1.2	33.1	25.5	0.0465
0.7	10.3	10.3	9.2	10.3	10.3	10.3	10.3	682.6	592.6	50.2	1.3	38.6	29.1	0.0441
0.8	9.7	9.7	8.6	9.7	9.7	9.7	9.7	719.4	617.9	56.1	1.4	44.1	32.6	0.0421
0.9	9.1	9.1	7.9	8.9	9.1	8.9	8.9	754.6	641.6	61.9	1.5	49.6	36.1	0.0403
1.0	8.7	8.7	7.6	8.7	8.7	8.7	8.7	788.4	664.1	67.7	1.5	55.1	39.6	0.0388
1.1	8.2	8.2	7.3	8.2	8.2	8.2	8.2	821.1	685.4	73.4	1.6	60.6	43.1	0.0375
1.2	7.9	7.9	7.0	7.9	7.9	7.9	7.9	852.8	705.8	79.1	1.7	66.1	46.6	0.0363
1.3	7.6	7.6	6.7	7.6	7.6	7.6	7.6	883.6	725.4	84.8	1.8	71.6	50.0	0.0352
1.4	7.3	7.3	7.1	7.3	7.3	7.3	7.3	913.6	744.2	90.4	1.8	77.1	53.4	0.0343

Table E-21 Optimal Results in Integer-ratio Headway Coordination (network I)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	9.1	18.2	7.6	9.1	18.2	9.1	9.1	656.7	587.5	35.2	6.6	27.5	29.4	0.2233
0.6	8.3	16.6	7.3	8.3	16.6	8.3	8.3	703.2	621.9	41.0	7.2	33.1	33.2	0.2169
0.7	7.7	15.4	6.8	7.7	15.4	7.7	7.7	746.7	653.7	46.7	7.8	38.6	36.8	0.2113
0.8	7.2	14.4	6.8	7.2	14.4	7.2	7.2	788.0	683.2	52.4	8.3	44.1	40.3	0.2064
0.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.3	710.9	58.0	8.8	49.6	43.7	0.2019
1.0	6.4	12.9	6.4	6.4	12.9	6.4	6.4	865.1	737.1	63.6	9.3	55.1	47.0	0.1979
1.1	6.1	12.3	6.1	6.1	12.3	6.1	6.1	901.5	762.1	69.1	9.8	60.6	50.2	0.1943
1.2	5.9	11.7	5.9	5.9	11.7	5.9	5.9	936.8	785.9	74.6	10.2	66.1	53.4	0.1909
1.3	5.6	11.3	5.6	5.6	11.3	5.6	5.6	971.0	808.7	80.0	10.6	71.6	56.5	0.1877
1.4	5.4	10.9	5.4	5.4	10.9	5.4	5.4	1004.3	830.7	85.5	11.0	77.1	59.6	0.1848

Table E-22 Optimal Results in Non-coordinated (network II)

Items λ_{ij}	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.5	5.9	10.2	10.3	1042.0	918.7	57.4	20.7	45.1	30.9	0.6711
0.6	5.4	9.3	9.4	1129.1	984.7	67.5	22.7	54.1	37.0	0.6131
0.7	5.0	8.7	8.7	1210.6	1045.6	77.4	24.6	63.2	43.0	0.5716
0.8	4.7	8.2	8.2	1287.7	1102.0	87.2	26.3	72.2	49.1	0.5351
0.9	4.4	7.7	7.7	1361.1	1155.0	97.0	27.9	81.2	55.2	0.5048
1.0	4.2	7.3	7.3	1431.4	1205.1	106.7	29.4	90.2	61.3	0.4793
1.1	4.0	7.0	7.0	1499.2	1252.8	116.3	30.8	99.2	67.4	0.4573
1.2	3.8	6.7	6.7	1564.7	1298.3	126.0	32.2	108.3	73.5	0.4382
1.3	3.7	6.4	6.4	1629.7	1343.1	135.6	33.6	117.4	80.2	0.4193
1.4	3.5	6.2	6.1	1691.5	1385.1	145.1	34.9	126.4	86.3	0.4043

Table E-23 Optimal Results in Common Headway Coordination (network II)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	6.6	6.6	6.6	6.6	6.6	6.6	6.6	1031.0	919.3	57.8	8.9	45.0	88.1	0.1012
0.6	6.0	6.0	6.0	6.0	6.0	6.0	6.0	1117.2	985.6	67.8	9.8	54.0	100.3	0.0977
0.7	5.6	5.6	5.6	5.6	5.6	5.6	5.6	1199.0	1047.5	77.8	10.7	63.1	112.5	0.0949
0.8	5.2	5.2	5.2	5.2	5.2	5.2	5.2	1275.1	1103.9	87.6	11.4	72.1	123.9	0.0922
0.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	1347.5	1156.9	97.4	12.1	81.1	135.0	0.0899
1.0	4.7	4.7	4.7	4.7	4.7	4.7	4.7	1418.3	1208.2	107.1	12.9	90.1	146.4	0.0879
1.1	4.4	4.4	4.4	4.4	4.4	4.4	4.4	1485.2	1255.8	116.8	13.5	99.1	157.1	0.0860
1.2	4.3	4.3	4.3	4.3	4.3	4.3	4.3	1549.9	1301.2	126.5	14.1	108.1	167.7	0.0843
1.3	4.1	4.1	4.1	4.1	4.1	4.1	4.1	1612.7	1344.7	136.1	14.7	117.1	178.2	0.0827
1.4	3.9	3.9	3.9	3.9	3.9	3.9	3.9	1673.7	1386.5	145.7	15.3	126.1	188.5	0.0812

Table E-24 Optimal Results in Integer-ratio Headway Coordination (network II)

Items λ_{ij}	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.5	5.2	10.4	5.2	5.2	10.4	5.2	5.2	1109.1	995.3	56.1	13.0	44.7	64.4	0.2024
0.6	4.7	9.4	4.7	4.7	9.4	4.7	4.7	1206.8	1072.6	66.0	14.5	53.8	77.5	0.1868
0.7	4.4	8.7	4.4	4.4	8.7	4.4	4.4	1296.1	1141.8	75.7	15.7	62.8	87.9	0.1789
0.8	4.1	8.2	4.1	4.1	8.2	4.1	4.1	1379.0	1204.9	85.4	16.8	71.8	96.3	0.1747
0.9	3.8	7.7	3.8	3.8	7.7	3.8	3.8	1459.7	1265.8	95.1	17.9	80.9	106.6	0.1684
1.0	3.6	7.3	3.6	3.6	7.3	3.6	3.6	1535.4	1321.9	104.7	18.9	89.9	114.7	0.1651
1.1	3.5	6.9	3.5	3.5	6.9	3.5	3.5	1609.7	1376.6	114.2	19.9	98.9	124.4	0.1604
1.2	3.3	6.7	3.3	3.3	6.7	3.3	3.3	1680.1	1427.6	123.8	20.8	107.9	132.2	0.1576
1.3	3.2	6.4	3.2	3.2	6.4	3.2	3.2	1749.2	1477.3	133.2	21.8	117.0	140.8	0.1545
1.4	3.1	6.2	3.1	3.1	6.2	3.1	3.1	1815.5	1524.3	142.7	22.6	126.0	148.4	0.1522

5. Coefficient of In-vehicle Value of Time (VOT) Varies

Table E-25 Optimal Results in Non-coordinated (network I)

Items k_v	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.1	5.9	13.6	11.8	853.0	773.2	56.2	13.7	9.9	18.1	0.7556
0.2	5.9	13.6	11.8	863.0	773.2	56.2	13.7	19.9	18.1	0.7556
0.3	5.9	13.6	11.8	872.9	773.2	56.2	13.7	29.8	18.1	0.7556
0.4	5.9	13.6	11.8	882.8	773.2	56.2	13.7	39.7	18.1	0.7556
0.5	5.9	13.6	11.8	892.7	773.2	56.2	13.7	49.6	18.1	0.7556
0.6	5.9	13.6	11.8	902.7	773.2	56.2	13.7	59.6	18.1	0.7556
0.7	5.9	13.6	11.8	912.6	773.2	56.2	13.7	69.5	18.1	0.7556
0.8	5.9	13.6	11.8	922.5	773.2	56.2	13.7	79.4	18.1	0.7556
0.9	5.9	13.6	11.8	932.5	773.2	56.2	13.7	89.4	18.1	0.7556

Table E-26 Optimal Results in Common Headway Coordination (network I)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	9.1	9.1	7.6	9.1	9.1	9.1	9.1	714.9	641.6	61.9	1.5	9.9	36.1	0.0403
0.2	9.1	9.1	7.9	9.1	9.1	9.1	9.1	724.9	641.6	61.9	1.5	19.8	36.1	0.0403
0.3	9.1	9.1	8.4	9.1	9.1	9.1	9.1	734.8	641.6	61.9	1.5	29.8	36.1	0.0403
0.4	9.1	9.1	8.4	9.1	9.1	9.1	9.1	744.7	641.6	61.9	1.5	39.7	36.1	0.0403
0.5	9.1	9.1	7.4	9.1	9.1	9.1	9.1	754.6	641.6	61.9	1.5	49.6	36.1	0.0403
0.6	9.1	9.1	8.4	9.1	9.1	9.1	9.1	764.5	641.6	61.9	1.5	59.5	36.1	0.0403
0.7	9.1	9.1	8.4	9.1	9.1	9.1	9.1	774.5	641.6	61.9	1.5	69.4	36.1	0.0403
0.8	9.1	9.1	8.4	9.1	9.1	9.1	9.1	784.4	641.6	61.9	1.5	79.3	36.1	0.0403
0.9	9.1	9.1	8.4	9.1	9.1	9.1	9.1	794.3	641.6	61.9	1.5	89.3	36.1	0.0403

Table E-27 Optimal Results in Integer-ratio Headway Coordination (network I)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	6.8	13.5	6.8	6.8	13.5	6.8	6.8	787.7	710.9	58.0	8.8	9.9	43.7	0.2019
0.2	6.8	13.5	6.8	6.8	13.5	6.8	6.8	797.6	710.9	58.0	8.8	19.8	43.7	0.2019
0.3	6.8	13.5	6.8	6.8	13.5	6.8	6.8	807.5	710.9	58.0	8.8	29.8	43.7	0.2019
0.4	6.8	13.5	6.8	6.8	13.5	6.8	6.8	817.4	710.9	58.0	8.8	39.7	43.7	0.2019
0.5	6.8	13.5	6.8	6.8	13.5	6.8	6.8	827.3	710.9	58.0	8.8	49.6	43.7	0.2019
0.6	6.8	13.5	6.8	6.8	13.5	6.8	6.8	837.2	710.9	58.0	8.8	59.5	43.7	0.2019
0.7	6.8	13.5	6.8	6.8	13.5	6.8	6.8	847.2	710.9	58.0	8.8	69.4	43.7	0.2019
0.8	6.8	13.5	6.8	6.8	13.5	6.8	6.8	857.1	710.9	58.0	8.8	79.3	43.7	0.2019
0.9	6.8	13.5	6.8	6.8	13.5	6.8	6.8	867.0	710.9	58.0	8.8	89.3	43.7	0.2019

Table E-28 Optimal Results in Non-coordinated (network II)

Items k_v	H_r (min)	Operator 1	Operator 2	Total cost (S\$)	Total operator cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	H_b (min)							
0.1	4.4	7.6	7.7	1296.1	1155.0	97.0	27.9	16.2	55.2	0.5048
0.2	4.4	7.6	7.7	1312.3	1155.0	97.0	27.9	32.5	55.2	0.5048
0.3	4.4	7.6	7.7	1328.6	1155.0	97.0	27.9	48.7	55.2	0.5048
0.4	4.4	7.6	7.7	1344.8	1155.0	97.0	27.9	65.0	55.2	0.5048
0.5	4.4	7.6	7.7	1361.1	1155.0	97.0	27.9	81.2	55.2	0.5048
0.6	4.4	7.6	7.7	1377.3	1155.0	97.0	27.9	97.4	55.2	0.5048
0.7	4.4	7.6	7.7	1393.5	1155.0	97.0	27.9	113.7	55.2	0.5048
0.8	4.4	7.6	7.7	1409.8	1155.0	97.0	27.9	129.9	55.2	0.5048
0.9	4.4	7.6	7.7	1426.0	1155.0	97.0	27.9	146.2	55.2	0.5048

Table E-29 Optimal Results in Common Headway Coordination (network II)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1282.6	1156.9	97.4	12.1	16.2	135.0	0.0899
0.2	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1298.9	1156.9	97.4	12.1	32.4	135.0	0.0899
0.3	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1315.1	1156.9	97.4	12.1	48.7	135.0	0.0899
0.4	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1331.3	1156.9	97.4	12.1	64.9	135.0	0.0899
0.5	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1347.5	1156.9	97.4	12.1	81.1	135.0	0.0899
0.6	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1363.8	1156.9	97.4	12.1	97.3	135.0	0.0899
0.7	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1380.0	1156.9	97.4	12.1	113.6	135.0	0.0899
0.8	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1396.2	1156.9	97.4	12.1	129.8	135.0	0.0899
0.9	7.3	7.3	6.7	7.3	7.3	7.3	7.3	1412.4	1156.9	97.4	12.1	146.0	135.0	0.0899

Table E-30 Optimal Results in Integer-ratio Headway Coordination (network II)

Items k_v	H_r (min)	Operator 1			Operator 2			Total Cost (S\$)	Total operator Cost (S\$)	Waiting Cost (S\$)	Transfer Cost (S\$)	In-vehicle Cost (S\$)	Transfer Volume (person)	Average Transfer Cost (S\$/person)
		H_b (min)	K_0 (min)	K_1 (min)	H_b (min)	K_0 (min)	K_1 (min)							
0.1	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1395.0	1265.8	95.1	17.9	16.2	106.6	0.1684
0.2	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1411.2	1265.8	95.1	17.9	32.3	106.6	0.1684
0.3	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1427.4	1265.8	95.1	17.9	48.5	106.6	0.1684
0.4	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1443.6	1265.8	95.1	17.9	64.7	106.6	0.1684
0.5	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1459.7	1265.8	95.1	17.9	80.9	106.6	0.1684
0.6	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1475.9	1265.8	95.1	17.9	97.0	106.6	0.1684
0.7	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1492.1	1265.8	95.1	17.9	113.2	106.6	0.1684
0.8	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1508.3	1265.8	95.1	17.9	129.4	106.6	0.1684
0.9	5.4	10.8	5.4	5.4	10.8	5.4	5.4	1524.4	1265.8	95.1	17.9	145.6	106.6	0.1684

APPENDIX F Sensitivity Analysis of Scenarios When Operators are Independent

The sensitivity analysis of optimal decision variables and important cost items with respect to value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, coefficient of in-vehicle value of time, and passenger OD rates of feeder bus services are presented respectively in the following sections.

F.1 Value of Time

Figure F-1 Headways of Rail vs. Value of Time (network I)

Figure F-2 Headways of Rail vs. Value of Time (network II)

Figure F-3 Headways of Bus for Operator 1 vs. Value of Time (network I)

Figure F-4 Headways of Bus for Operator 1 vs. Value of Time (network II)

Figure F-5 Headways of Bus for Operator 2 vs. Value of Time (network I)

Figure F-6 Headways of Bus for Operator 2 vs. Value of Time (network II)

Figure F-7 Total cost vs. Value of Time (network I)

Figure F-8 Total cost vs. Value of Time (network II)

Figure F-9 Operator Cost vs. Value of Time (network I)

Figure F-10 Operator Cost vs. Value of Time (network II)

Figure F-11 User Cost vs. Value of Time (network I)

Figure F-12 User Cost vs. Value of Time (network II)

Figure F-13 Waiting Cost vs. Value of Time (network I)

Figure F-14 Waiting Cost vs. Value of Time (network II)

Figure F-15 Transfer Cost vs. Value of Time (network I)

Figure F-16 Transfer Cost vs. Value of Time (network II)

Figure F-17 Total Transfer Volume vs. Value of Time (network I)

Figure F-18 Total Transfer Volume vs. Value of Time (network II)

Figure F-19 Average Transfer Cost vs. Value of Time (network I)

Figure F-20 Average Transfer Cost vs. Value of Time (network II)

F.2 Transfer Ratio

Figure F-21 Headways of Rail vs. Transfer Ratio (network I)

Figure F-22 Headways of Rail vs. Transfer Ratio (network II)

Figure F-23 Headways of Bus for Operator 1 vs. Transfer Ratio (network I)

Figure F-24 Headways of Bus for Operator 1 vs. Transfer Ratio (network II)

Figure F-25 Headways of Bus for Operator 2 vs. Transfer Ratio (network I)

Figure F-26 Headways of Bus for Operator 2 vs. Transfer Ratio (network II)

Figure F-27 Total cost vs. Transfer Ratio (network I)

Figure F-28 Total cost vs. Transfer Ratio (network II)

Figure F-29 Operator Cost vs. Transfer Ratio (network I)

Figure F-30 Operator Cost vs. Transfer Ratio (network II)

Figure F-31 User Cost vs. Transfer Ratio (network I)

Figure F-32 User Cost vs. Transfer Ratio (network II)

Figure F-33 Waiting Cost vs. Transfer Ratio (network I)

Figure F-34 Waiting Cost vs. Transfer Ratio (network II)

Figure F-35 Transfer Cost vs. Transfer Ratio (network I)

Figure F-36 Transfer Cost vs. Transfer Ratio (network II)

Figure F-37 Total Transfer Volume vs. Transfer Ratio (network I)

Figure F-38 Total Transfer Volume vs. Transfer Ratio (network II)

Figure F-39 Average Transfer Cost vs. Transfer Ratio (network I)

Figure F-40 Average Transfer Cost vs. Transfer Ratio (network II)

F.3 Standard Deviation (SD) of Bus Arrival Times

Figure F-41 Headways of Rail vs. SD of Bus Arrival Time (network I)

Figure F-42 Headways of Rail vs. SD of Bus Arrival Time (network II)

Figure F-43 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network I)

Figure F-44 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network II)

Figure F-45 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network I)

Figure F-46 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network II)

Figure F-47 Total cost vs. SD of Bus Arrival Time (network I)

Figure F-48 Total cost vs. SD of Bus Arrival Time (network II)

Figure F-49 Operator Cost vs. SD of Bus Arrival Time (network I)

Figure F-50 Operator Cost vs. SD of Bus Arrival Time (network II)

Figure F-51 User Cost vs. SD of Bus Arrival Time (network I)

Figure F-52 User Cost vs. SD of Bus Arrival Time (network II)

Figure F-53 Waiting Cost vs. SD of Bus Arrival Time (network I)

Figure F-54 Waiting Cost vs. SD of Bus Arrival Time (network II)

Figure F-55 Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure F-56 Transfer Cost vs. SD of Bus Arrival Time (network II)

Figure F-57 Total Transfer Volume vs. SD of Bus Arrival Time (network I)

Figure F-58 Total Transfer Volume vs. SD of Bus Arrival Time (network II)

Figure F-59 Average Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure F-60 Average Transfer Cost vs. SD of Bus Arrival Time (network II)

F.4 Passenger OD Rates of Feeder Bus Services

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Figure F-62 Headways of Rail vs. OD Rates (network II)

Figure F-63 Headways of Bus for Operator 1 vs. OD Rates (network I)

Figure F-64 Headways of Bus for Operator 1 vs. OD Rates (network II)

Figure F-65 Headways of Bus for Operator 2 vs. OD Rates (network I)

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Figure F-67 Total cost vs. OD Rates (network I)

Figure F-68 Total cost vs. OD Rates (network II)

Figure F-69 Operator Cost vs. OD Rates (network I)

Figure F-70 Operator Cost vs. OD Rates (network II)

Figure F-71 User Cost vs. OD Rates (network I)

Figure F-72 User Cost vs. OD Rates (network II)

Figure F-73 Waiting Cost vs. OD Rates (network I)

Figure F-74 Waiting Cost vs. OD Rates (network II)

Figure F-75 Transfer Cost vs. OD Rates (network I)

Figure F-76 Transfer Cost vs. OD Rates (network II)

Figure F-77 Total Transfer Volume vs. OD Rates (network I)

Figure F-78 Total Transfer Volume vs. OD Rates (network II)

Figure F-79 Average Transfer Cost vs. OD Rates (network I)

Figure F-80 Average Transfer Cost vs. OD Rates (network II)

F.5 Coefficient of In-Vehicle Value of Time (VOT)

Figure F-81 Headways of Rail vs. Coefficient of In-Vehicle VOT (network I)

Figure F-82 Headways of Rail vs. Coefficient of In-Vehicle VOT (network II)

Figure F-83 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT (network I)

Figure F-84 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT
(network II)

Figure F-85 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT
(network I)

Figure F-86 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT
(network II)

Figure F-87 Total Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure F-88 Total Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure F-89 Operator Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure F-90 Operator Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure F-91 User Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure F-92 User Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure F-93 Waiting Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure F-94 Waiting Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure F-95 Transfer Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure F-96 Transfer Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure F-97 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network I)

Figure F-98 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network II)

Figure F-99 Average Transfer Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure F-100 Average Transfer Cost vs. Coefficient of In-Vehicle VOT (network II)

APPENDIX G Sensitivity Analysis of Scenarios When Operators are in Competition

The sensitivity analysis of decision variables and cost items with respect to value of time, coefficient of in-vehicle value of time, coefficient of in-vehicle value of time, transfer ratio, standard deviation of bus arrival times, and passenger OD rates of feeder bus services are presented respectively in the following sections.

G.1 Value of Time

Figure G-1 Headways of Rail vs. Value of Time (network I)

Figure G-2 Headways of Rail vs. Value of Time (network II)

Figure G-3 Headways of Bus for Operator 1 vs. Value of Time (network I)

Figure G-4 Headways of Bus for Operator 1 vs. Value of Time (network II)

Figure G-5 Headways of Bus for Operator 2 vs. Value of Time (network I)

Figure G-6 Headways of Bus for Operator 2 vs. Value of Time (network II)

Figure G-7 Total cost vs. Value of Time (network I)

Figure G-8 Total cost vs. Value of Time (network II)

Figure G-9 Operator Cost vs. Value of Time (network I)

Figure G-10 Operator Cost vs. Value of Time (network II)

Figure G-11 User Cost vs. Value of Time (network I)

Figure G-12 User Cost vs. Value of Time (network II)

Figure G-13 Waiting Cost vs. Value of Time (network I)

Figure G-14 Waiting Cost vs. Value of Time (network II)

Figure G-15 Transfer Cost vs. Value of Time (network I)

Figure G-16 Transfer Cost vs. Value of Time (network II)

Figure G-17 Total Transfer Volume vs. Value of Time (network I)

Figure G-18 Total Transfer Volume vs. Value of Time (network II)

Figure G-19 Average Transfer Cost vs. Value of Time (network I)

Figure G-20 Average Transfer Cost vs. Value of Time (network II)

G.2 Transfer Ratio

Figure G-21 Headways of Rail vs. Transfer Ratio (network I)

Figure G-22 Headways of Rail vs. Transfer Ratio (network II)

Figure G-23 Headways of Bus for Operator 1 vs. Transfer Ratio (network I)

Figure G-24 Headways of Bus for Operator 1 vs. Transfer Ratio (network II)

Figure G-25 Headways of Bus for Operator 2 vs. Transfer Ratio (network I)

Figure G-26 Headways of Bus for Operator 2 vs. Transfer Ratio (network II)

Figure G-27 Total cost vs. Transfer Ratio (network I)

Figure G-28 Total cost vs. Transfer Ratio (network II)

Figure G-29 Operator Cost vs. Transfer Ratio (network I)

Figure G-30 Operator Cost vs. Transfer Ratio (network II)

Figure G-31 User Cost vs. Transfer Ratio (network I)

Figure G-32 User Cost vs. Transfer Ratio (network II)

Figure G-33 Waiting Cost vs. Transfer Ratio (network I)

Figure G-34 Waiting Cost vs. Transfer Ratio (network II)

Figure G-35 Transfer Cost vs. Transfer Ratio (network I)

Figure G-36 Transfer Cost vs. Transfer Ratio (network II)

Figure G-37 Total Transfer Volume vs. Transfer Ratio (network I)

Figure G-38 Total Transfer Volume vs. Transfer Ratio (network II)

Figure G-39 Average Transfer Cost vs. Transfer Ratio (network I)

Figure G-40 Average Transfer Cost vs. Transfer Ratio (network II)

G.3 Standard Deviation (SD) of Bus Arrival Times

Figure G-41 Headways of Rail vs. SD of Bus Arrival Time (network I)

Figure G-42 Headways of Rail vs. SD of Bus Arrival Time (network II)

Figure G-43 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network I)

Figure G-44 Headways of Bus for Operator 1 vs. SD of Bus Arrival Time (network II)

Figure G-45 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network I)

Figure G-46 Headways of Bus for Operator 2 vs. SD of Bus Arrival Time (network II)

Figure G-47 Total cost vs. SD of Bus Arrival Time (network I)

Figure G-48 Total cost vs. SD of Bus Arrival Time (network II)

Figure G-49 Operator Cost vs. SD of Bus Arrival Time (network I)

Figure G-50 Operator Cost vs. SD of Bus Arrival Time (network II)

Figure G-51 User Cost vs. SD of Bus Arrival Time (network I)

Figure G-52 User Cost vs. SD of Bus Arrival Time (network II)

Figure G-53 Waiting Cost vs. SD of Bus Arrival Time (network I)

Figure G-54 Waiting Cost vs. SD of Bus Arrival Time (network II)

Figure G-55 Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure G-56 Transfer Cost vs. SD of Bus Arrival Time (network II)

Figure G-57 Total Transfer Volume vs. SD of Bus Arrival Time (network I)

Figure G-58 Total Transfer Volume vs. SD of Bus Arrival Time (network II)

Figure G-59 Average Transfer Cost vs. SD of Bus Arrival Time (network I)

Figure G-60 Average Transfer Cost vs. SD of Bus Arrival Time (network II)

G.4 Passenger OD Rates of Feeder Bus Services

Figure G-61 Headways of Rail vs. OD Rates (network I)

Figure G-62 Headways of Rail vs. OD Rates (network II)

Figure G-63 Headways of Bus for Operator 1 vs. OD Rates (network I)

Figure G-64 Headways of Bus for Operator 1 vs. OD Rates (network II)

Figure G-65 Headways of Bus for Operator 2 vs. OD Rates (network I)

Figure G-66 Headways of Bus for Operator 2 vs. OD Rates (network II)

Figure G-67 Total cost vs. OD Rates (network I)

Figure G-68 Total cost vs. OD Rates (network II)

Figure G-69 Operator Cost vs. OD Rates (network I)

Figure G-70 Operator Cost vs. OD Rates (network II)

Figure G-71 User Cost vs. OD Rates (network I)

Figure G-72 User Cost vs. OD Rates (network II)

Figure G-73 Waiting Cost vs. OD Rates (network I)

Figure G-74 Waiting Cost vs. OD Rates (network II)

Figure G-75 Transfer Cost vs. OD Rates (network I)

Figure G-76 Transfer Cost vs. OD Rates (network II)

Figure G-77 Total Transfer Volume vs. OD Rates (network I)

Figure G-78 Total Transfer Volume vs. OD Rates (network II)

Figure G-79 Average Transfer Cost vs. OD Rates (network I)

Figure G-80 Average Transfer Cost vs. OD Rates (network II)

G.5 Coefficient of In-Vehicle Value of Time (VOT)

Figure G-81 Headways of Rail vs. Coefficient of In-Vehicle VOT (network I)

Figure G-82 Headways of Rail vs. Coefficient of In-Vehicle VOT (network II)

Figure G-83 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT
(network I)

Figure G-84 Headways of Bus for Operator 1 vs. Coefficient of In-Vehicle VOT
(network II)

Figure G-85 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT
(network I)

Figure G-86 Headways of Bus for Operator 2 vs. Coefficient of In-Vehicle VOT
(network II)

Figure G-87 Total Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure G-88 Total Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure G-89 Operator Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure G-90 Operator Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure G-91 User Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure G-92 User Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure G-93 Waiting Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure G-94 Waiting Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure G-95 Transfer Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure G-96 Transfer Cost vs. Coefficient of In-Vehicle VOT (network II)

Figure G-97 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network I)

Figure G-98 Total Transfer Volume vs. Coefficient of In-Vehicle VOT (network II)

Figure G-99 Average Transfer Cost vs. Coefficient of In-Vehicle VOT (network I)

Figure G-100 Average Transfer Cost vs. Coefficient of In-Vehicle VOT (network II)

APPENDIX H Comparisons between Different Coordination

Scenarios with Non-Coordinated Scenarios

1. Effects of Value of Time

Table H-1 Changes in the Total Cost for Different Scenarios and Networks

Value of time	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
13.9	-7.42%	-7.13%	-7.14%	-14.91%	-6.84%	6.91%
14.9	-7.47%	-7.17%	-7.17%	-14.97%	-6.88%	7.07%
15.9	-7.54%	-7.22%	-7.22%	-14.86%	-6.77%	7.08%
16.9	-7.58%	-7.27%	-7.27%	-14.89%	-6.70%	7.09%
17.9	-7.63%	-7.29%	-7.30%	-14.81%	-6.47%	7.10%
18.9	-7.66%	-7.31%	-7.33%	-14.84%	-6.48%	7.24%
19.9	-7.70%	-7.34%	-7.35%	-14.81%	-6.48%	7.26%
20.9	-7.74%	-7.37%	-7.38%	-14.70%	-6.49%	7.26%
21.9	-7.77%	-7.40%	-7.40%	-14.61%	-6.50%	7.26%
22.9	-7.80%	-7.41%	-7.42%	-14.37%	-6.39%	7.37%

Table H-2 Changes in the Average Transfer Cost for Different Scenarios and Networks

Value of time	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
13.9	-93.55%	-75.77%	-75.77%	-85.43%	-63.89%	-68.74%
14.9	-93.35%	-75.22%	-75.22%	-85.06%	-63.06%	-68.57%
15.9	-93.17%	-74.70%	-74.70%	-84.78%	-63.00%	-67.91%
16.9	-93.00%	-74.20%	-74.20%	-84.45%	-62.80%	-67.27%
17.9	-92.83%	-73.72%	-73.72%	-84.21%	-63.42%	-66.71%
18.9	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
19.9	-92.53%	-72.84%	-72.84%	-83.58%	-62.13%	-66.11%
20.9	-92.38%	-72.43%	-72.43%	-83.40%	-61.54%	-65.61%
21.9	-92.25%	-72.04%	-72.04%	-83.20%	-60.97%	-65.11%
22.9	-92.10%	-71.66%	-71.66%	-83.06%	-61.05%	-65.03%

2. Effects of Transfer Ratio

Table H-3 Changes in the Total Cost for Different Scenarios and Networks

Transfer Ratio	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
10.0%	-1.65%	-1.32%	-1.33%	-23.85%	-1.27%	12.64%
20.0%	-3.06%	-2.72%	-2.73%	-25.71%	-2.50%	11.39%
30.0%	-4.53%	-4.20%	-4.20%	-27.49%	-3.77%	10.06%
40.0%	-6.07%	-5.73%	-5.74%	-29.42%	-5.10%	8.69%
50.0%	-7.66%	-7.31%	-7.33%	-31.40%	-6.48%	7.24%
60.0%	-9.34%	-8.99%	-9.00%	-33.43%	-7.93%	5.57%
70.0%	-11.10%	-10.75%	-10.76%	-35.71%	-9.79%	3.97%
80.0%	-12.95%	-12.59%	-12.59%	-37.97%	-11.43%	2.29%
90.0%	-14.88%	-14.53%	-14.53%	-40.65%	-13.33%	0.53%

Table H-4 Changes in the Average Transfer Cost for Different Scenarios and Networks

Transfer Ratio	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
10.0%	-92.51%	-55.28%	-55.28%	-90.82%	-41.17%	-47.86%
20.0%	-92.56%	-62.18%	-62.18%	-87.59%	-49.53%	-55.12%
30.0%	-92.60%	-66.98%	-66.98%	-85.75%	-55.28%	-60.09%
40.0%	-92.63%	-70.53%	-70.53%	-84.60%	-59.48%	-63.79%
50.0%	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
60.0%	-92.72%	-75.45%	-75.45%	-83.42%	-65.38%	-68.41%
70.0%	-92.75%	-77.20%	-77.20%	-82.97%	-66.09%	-70.29%
80.0%	-92.78%	-78.66%	-78.66%	-82.75%	-67.86%	-71.87%
90.0%	-92.81%	-79.89%	-79.89%	-82.32%	-68.75%	-73.23%

3. Effects of Standard Deviation (SD) of Bus Arrival Times

Table H-5 Changes in the Total Cost for Different Scenarios and Networks

SD of Bus Arrival Time	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
1	-7.66%	-7.31%	-7.33%	-14.84%	-6.48%	7.24%
2	-7.67%	-7.32%	-7.33%	-14.78%	-6.44%	7.14%
3	-7.68%	-7.33%	-7.34%	-14.70%	-6.40%	6.91%
4	-7.69%	-7.34%	-7.35%	-14.63%	-6.41%	6.46%
5	-7.71%	-7.37%	-7.37%	-14.62%	-6.48%	5.96%
6	-7.77%	-7.43%	-7.43%	-14.68%	-6.72%	5.32%
7	-7.85%	-7.51%	-7.51%	-14.86%	-7.02%	4.52%
8	-7.96%	-7.61%	-7.62%	-15.08%	-7.37%	3.55%
9	-8.10%	-7.74%	-7.76%	-15.38%	-7.82%	2.47%
10	-8.27%	-7.92%	-7.93%	-15.76%	-8.34%	1.32%

Table H-6 Changes in Average Transfer Cost for Different Scenarios and Networks

SD of Bus Arrival Time	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
1	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-63.42%
2	-92.71%	-73.34%	-73.34%	-84.10%	-63.22%	-63.34%
3	-92.73%	-73.44%	-73.44%	-84.39%	-63.90%	-63.22%
4	-92.76%	-73.55%	-73.55%	-84.71%	-64.65%	-63.04%
5	-92.81%	-73.66%	-73.66%	-85.01%	-65.35%	-62.81%
6	-92.83%	-73.75%	-73.75%	-85.24%	-65.44%	-62.51%
7	-92.86%	-73.83%	-73.83%	-85.58%	-65.80%	-61.76%
8	-92.88%	-73.86%	-73.86%	-85.67%	-66.07%	-61.30%
9	-92.88%	-73.85%	-73.85%	-85.99%	-66.55%	-59.97%
10	-92.87%	-73.78%	-73.78%	-86.12%	-66.93%	-59.13%

4. Effects of Passenger OD rates of Feeder Bus Services

Table H-7 Changes in the Total Cost for Different Scenarios

OD rates	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
0.5	-7.14%	-6.90%	-6.92%	-15.02%	-6.84%	6.44%
0.6	-7.32%	-7.06%	-7.06%	-14.91%	-6.92%	6.88%
0.7	-7.46%	-7.16%	-7.17%	-14.94%	-6.86%	7.06%
0.8	-7.58%	-7.26%	-7.26%	-14.89%	-6.70%	7.09%
0.9	-7.66%	-7.31%	-7.33%	-14.84%	-6.48%	7.24%
1.0	-7.75%	-7.38%	-7.39%	-14.70%	-6.49%	7.27%
1.1	-7.80%	-7.41%	-7.42%	-14.37%	-6.39%	7.37%
1.2	-7.86%	-7.46%	-7.46%	-14.38%	-6.39%	7.38%
1.3	-7.91%	-7.48%	-7.49%	-14.46%	-6.47%	7.33%
1.4	-7.95%	-7.51%	-7.51%	-14.18%	-6.35%	7.33%

Table H-8 Changes in the Average Transfer Cost for Different Scenarios

OD rates	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
0.5	-94.25%	-77.95%	-77.95%	-86.62%	-66.45%	-69.84%
0.6	-93.79%	-76.54%	-76.54%	-85.79%	-64.30%	-69.53%
0.7	-93.39%	-75.32%	-75.32%	-85.11%	-63.21%	-68.70%
0.8	-93.02%	-74.24%	-74.24%	-84.47%	-62.86%	-67.35%
0.9	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
1.0	-92.37%	-72.40%	-72.40%	-83.38%	-61.47%	-65.55%
1.1	-92.07%	-71.59%	-71.59%	-83.02%	-60.96%	-64.92%
1.2	-91.80%	-70.85%	-70.85%	-82.50%	-59.87%	-64.03%
1.3	-91.55%	-70.17%	-70.17%	-81.93%	-58.68%	-63.15%
1.4	-91.30%	-69.54%	-69.54%	-81.63%	-58.38%	-62.35%

5. Effects of Coefficient of In-vehicle Value of Time,

Table H-9 Changes in the Total Cost for Different Scenarios and Networks

Transfer Ratio	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
0.1	-7.73%	-7.66%	-7.66%	-15.48%	-6.77%	7.63%
0.2	-7.72%	-7.58%	-7.58%	-15.31%	-6.70%	7.54%
0.3	-7.70%	-7.49%	-7.49%	-15.15%	-6.62%	7.44%
0.4	-7.68%	-7.40%	-7.41%	-15.00%	-6.56%	7.35%
0.5	-7.66%	-7.31%	-7.33%	-14.84%	-6.48%	7.24%
0.6	-7.65%	-7.24%	-7.26%	-14.69%	-6.42%	7.16%
0.7	-7.64%	-7.17%	-7.17%	-14.54%	-6.35%	7.08%
0.8	-7.61%	-7.08%	-7.09%	-14.40%	-6.28%	6.99%
0.9	-7.60%	-7.01%	-7.02%	-14.26%	-6.22%	6.90%

Table H-10 Changes in the Average Transfer Cost for Different Scenarios and Networks

Transfer Ratio	Network I			Network II		
	Cooperation	Independence	Competition	Cooperation	Independence	Competition
0.1	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.2	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.3	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.4	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.5	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.6	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.7	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.8	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%
0.9	-92.68%	-73.28%	-73.28%	-83.90%	-62.77%	-66.64%