Numerical simulation on rock dynamic failure by considering heterogeneity effect

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**NUMERICAL SIMULATION ON ROCK
DYNAMIC FAILURE BY CONSIDERING
HETEROGENEITY EFFECT**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2006

Numerical Simulation on Rock Dynamic Failure By Considering Heterogeneity Effect

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A thesis submitted to Nanyang Technological University
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SUMMARY

Understanding the dynamic fracture behavior of brittle materials is of great importance in many engineering practices including protective technology, mining industry and earthquake research, etc. With the rapid development of computing methods, numerical simulations are becoming more and more important in gaining the insight of the rock fracture process instead of the expensive, time-consuming and dangerous experiments especially under dynamic loads.

As a meshless Lagrangian method, the smoothed particle hydrodynamics (SPH) method has distinct advantages over the conventional methods such as the finite element method in simulation of the fracture and fragmentation of rock-like materials under dynamic loads. The heterogeneity from the microstructure point of view greatly influences the failure behavior of the rock-like materials. A statistical approach using Weibull distribution of the material properties is adopted in this study to model the heterogeneity effect. The statistical model is implemented into the SPH method embedded in the commercial code Autodyn-2D.

The SPH method by considering material heterogeneity is then employed to perform two series of numerical simulations of rock-like materials under dynamic compressive and tensile loads respectively. Various pulse loads with respected to different strain rates and heterogeneity indices are applied to the specimens. The fracture processes of the specimens are shown clearly based on the numerical simulation results. It is found that the heterogeneity index has significant effect to the failure pattern and the failure strength of the brittle specimens. The dynamic increase factors (DIF) of the specimens under both compressive and tensile loads increases with the increase of the strain rate. It is clear that the strain rate effect is because of the microstructure distributions or the heterogeneity of the material. The numerical results will help to understand the dynamic failure behavior of brittle materials with different heterogeneity indices.

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LIST OF SYMBOLS

SYMBOL	MEANING
σ	Stress
τ	Shear stress
c	Cohesive strength
σ_n	Normal stress
ϕ	Angle of friction
σ_c, σ_t	Uniaxial compressive and tensile strength
σ_1, σ_3	Maximum and minimum principal stresses
$\sigma_x, \sigma_y, \sigma_z$	Normal stresses
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	Shear stresses
E, ν, G	Young's modulus, Poisson's ratio and Shear modulus
μ	Bulk strain
K	Bulk modulus
I_1	First invariant of stress tensor
P	Equivalent hydrostatic stress
D	Damage variable
x, y, z	Coordinates of coordinate system
C_d	Crack density
$\sigma^*, \sigma_i^*, \sigma_f^*$	Normalized equivalent stress, intact equivalent stress, fracture stress
P^*, T^*	Normalized pressure, maximum tensile hydrostatic pressure
$\varepsilon, \varepsilon_p^f$	Strain, plastic strain to fracture
$\dot{\varepsilon}$	Strain rate

HEL	Hugoniot elastic limit
P_{HEL}, σ_{HEL}	Pressure and equivalent stress at the Hugoniot elastic limit
C	Strength parameter (for strain rate dependence)
T	Uniaxial tensile strength
t	Time
f_{III}, f_t, f_c	Cut-off hydro-tensile strength, uniaxial tensile strength and uniaxial compressive strength
m	Heterogeneity index
ρ	Density
$\ddot{x}, \ddot{y}, \ddot{z}$	Acceleration
e	Energy
h	Smoothing length of SPH particle
c_0	Bar velocity
v_i, v_r, v_t	Particle velocities of incident wave, reflected wave and transmitted wave
r, R	Current radius of polar coordinate and radius of the Brazilian disc

Chapter 1 Introduction

1.1 Background

Understanding the dynamic behavior of brittle materials is of great importance in many engineering practices. Rock and concrete structures have been extensively constructed for various infrastructures and facilities concerning dynamic loads. In protective engineering, materials need to be designed against weapon attacks and explosive loads. In mining industry, the material dynamic behavior should be studied in order to optimize the efficient breaking of rocks and ores. Underground excavation and earthquake research also involve dynamic properties of rocks and rock-like materials. The failure process and the fracture mechanism of brittle materials under dynamic loads are very different from that under static loads.

Nowadays although many experimental works have been done to study the behavior of rocks under dynamic loads, the fracture process and failure mechanism of brittle and heterogeneous materials is still poorly understood. Split Hopkinson pressure bar (SHPB) test is an important facility used to study the dynamic behavior of materials. However, it is difficult to capture the fracture process occurred in the specimen because the specimen is fragmented or pulverized instantly. High speed camera is still not extensively employed in the experiments because of the limitations of graphic resolution. The mechanism of rock fracture under dynamic loads is still not fully understood and the progressive fracture process of the specimens cannot be visually shown.

With the rapid development of computing techniques and interactive computer graphics, numerical simulations have become a popular tool to study the fracture process in order to explore the fracture mechanism under dynamic loads. Numerical modeling is versatile for either different specimen geometry or different loading conditions. The specimen

geometries cover from regular to irregular. And the dynamics loads vary in a wide range from low to high strain rates. Moreover, numerical simulations can present the failure process conveniently. The fracture pattern and stress-strain curves can also be obtained to reveal the dynamic fracture mechanism of brittle materials. Numerical simulation provides an alternative tool of scientific investigation, instead of carrying out expensive, time-consuming and sometimes even dangerous experiments in laboratories or on site. The numerical tool are often more useful than the traditional experimental methods in terms of providing insightful and complete information that cannot be directly measured or observed, or difficult to acquire via other means.

As one of the most popular numerical methods, the finite element method (**FEM**) has been the dominant method and has achieved great successes in various areas since its invention at 1950s. Although FEM is powerful, its dependence on a mesh makes it not well suited to the treatment of severe deformation and discontinuous problems with free surfaces, deformable boundaries and moving interfaces which do not coincide with the original mesh lines. In such cases, in order to prevent the severe distortion of meshes and allow mesh lines to remain coincident with the discontinuities, the most viable strategy is to rezone these meshes throughout the evolution of the problem. Although great progress has been made in such conditions, meshing and the subsequent adjusting process is still a difficult task. Moreover, additional inaccuracies could be brought into the numerical simulation during such processes due to the mesh distortions and the state variables mapping transformations.

Recently, a new class of numerical methods called meshless methods has been developed to ameliorate the difficulties in FEM. Smooth particle hydrodynamics (**SPH**) is one of the most popular numerical meshless methods which is adopted in this study. It is a meshless Lagrangian method which has been employed in various numerical simulations nowadays. The main advantage of SPH, as a meshless technique, is that it does not require a numerical grid. This avoids the problems associated with mesh tangling and distortion, which usually occur in finite element method analyses for large deformation

impact and explosive loading events. This method no longer needs to realize fracture simulation by means of erosion as finite element method.

SPH is initially developed to solve astrophysical problems (Gingold and Monaghan, 1977; Lucy, 1977). Then the earliest applications of SPH were mainly focused on fluid dynamics related areas which includes elastic flow, magneto-hydrodynamics and heat transfer, etc. Benz and Asphaug (1994; 1995) extended SPH to the simulation of the fracture of brittle solids. Bonet and Kulasegaram (2000) applied SPH to the simulation of metal forming. The SPH method has been very attractive in simulating large deformation and impulsive loading events. A variety of shock problems have been studied and the SPH algorithm gives very satisfactory results for shocks (Monaghan and Gingold, 1983; Monaghan, 1997). Another important application of SPH method is the explosion phenomena arising from the detonation of high explosive (Wang, et al., 2005).

From the point of solid mechanics, the most appealing attribute of SPH lies in its ability to fracture in a realistic manner while without unduly compromising its ability in subsequent computation. Benz and Asphaug (1994; 1995) showed how SPH could be applied to the fracture of brittle solids, where it gives much better results than the finite element or the finite difference methods. SPH also provides a simple and robust technique for simulating complex fracture in geological rock formations and in brittle materials (Gray, et al., 2001). Great efforts have been made to incorporate damage and fracture algorithms into SPH which take the advantage of its mesh free attributes while instilling physical realism in the mechanics of progressive damage and ultimate fracture.

In the past, it was common in numerical modeling to assume that the distribution of material properties is uniform over the model. However, it is not true for brittle materials, including rocks and concrete, which are composed by composites or microstructures. These materials may also have preexisting stress induced defects in the forms of voids, micro cracks and weak interfaces. Fractures are inclined to develop via the formation, growth and coalescence of microcracks in rocks. Failure behavior of rocks is greatly

influenced by their heterogeneity. Analytical model before are usually simplified and sometimes the simplification ignores important factors influencing the material behavior (Liu, et al., 2004). Heterogeneity is one of such factors for rock-like materials. It is of significance that the heterogeneity can be implemented into the numerical modeling in order to understand the dynamic fracture mechanism of brittle materials.

The virtues of SPH make it very promising in simulations on the fracture process of brittle materials. The advantage of SPH that requires no grid makes it popular in numerical simulations of material fracture and fragmentation under dynamic loads including impact, shocks and explosives, etc. SPH method involving heterogeneity is not only capable to simulate fracture and fragmentations in a realistic manner but also effective to reflect the heterogeneous characteristics of rock-like materials from the microstructure point of view.

Another important issue is about the strain rate effect for brittle materials. Based on the experimental results (Goldsmith, et al., 1966; Ross, et al., 1989; Zhao, 1998; Ross et al., 1995; Tedesco and Ross, 1998; Grote, et al., 2001), it has been commonly understood that the strain rate has significant effect on the material strength after it exceeds a certain level. However, it was doubted recently by Li and Meng (2003) that the strain rate effect for brittle materials was actually introduced due to the lateral confinement. Application of the dynamic increase factor of the material strength in the constitutive models may double count the confinement effect because most of the strength models for brittle materials are pressure-sensitive. The confinement effect has usually already been reflected in the strength model.

1.2 Objectives and scopes

The present study is to investigate the dynamic compressive and tensile failure behaviors of rock-like materials with heterogeneity effect by using the SPH method. The objectives

are outlined as follows,

- 1) to study the failure behavior of rock-like materials under dynamic compressive and tensile loads with different heterogeneity indices. Failure pattern and stress-strain curves of the specimens will be discussed.
- 2) to investigate the strain rate effect of the heterogeneous rock-like materials.
- 3) to examine the validity of the dynamic Brazilian split test.

The scopes of the present study include,

- 1) implementation of the material heterogeneity into the SPH model of the commercial software -Autodyn;
- 2) carrying out numerical simulations on dynamic compressive fracture process of rock-like materials by considering heterogeneity effect;
- 3) performing numerical simulations on dynamic tensile fracture process on rock-like materials by considering heterogeneity effect.

1.3 Thesis organization

This thesis consists of seven chapters. Chapter 1 introduces the background, objectives and scopes of the present study. Chapter 2 reviews constitutive models of rock-like materials and determine the material parameters for the adopted constitutive model. In Chapter 3, literature review on the existing heterogeneous models is firstly carried out. A Monte-Carlo method is introduced for the implementation of the material heterogeneity effect. Chapter 4 derives the SPH method by considering the heterogeneity model. It is then incorporated into the dynamic software-Autodyn. Numerical simulations are conducted for the dynamic compressive failure of rock-like materials in Chapter 5. Dynamic Brazilian split tests are simulated in Chapter 6. Chapter 7 draws some conclusions and points out the future works of the present study.

Chapter 2 Constitutive models of rock-like materials

2.1 Introduction

Constitutive models are the fundamentals in rock mechanics, which represent mathematical models that describe the macroscopic material behavior resulting from the internal constitution of a material. The study of the constitutive models has been enhanced significantly with the development and application of modern computer-based techniques. In engineering analysis, these constitutive models play a crucial role providing reliability to the results obtained from the numerical procedures.

The commonly used elasto-plastic and perfectly brittle models are simplifications of the actual behavior of a material. Fig. 2.1 (Ma, et al., 1998) shows schematically the elastic brittle and elasto-plastic properties of rock. The unloading path (a), (b) and (c) are for perfectly brittle, elastic-plastic and plastic damage models, respectively.

Elasto-plastic theory has been well established in the past. Some pressure sensitive strength criteria such as the Mohr-Coulomb criterion and the Drucker-Prager criterion were suggested and used to model the plastic flow of brittle materials such as rock, concrete and soil. To model the non-linear relationship between the effective stress and the hydrostatic pressure, some modified Mohr-Coulomb criteria and modified Drucker-Prager criteria have been suggested. It should be noted, nevertheless, that these models do not consider the elastic degradation and the rate-dependent properties of rock mass.

It is well known that mechanical properties of most engineering materials are affected by strain rates. Brittle materials such as rock and concrete also show the trend that the dynamic strength increases with the increase of strain rate. Study of rock dynamic constitutive model is necessary in engineering construction, especially in protective

engineering. Many constitutive models considering the strain rate effects have also been proposed with the development of rock rheological theory and damage mechanics.

Fig. 2.1 Elastic brittle and elasto-plastic properties of rock (Ma, et al., 1998)

2.2 Constitutive models without strain rate effect

2.2.1 Mohr-Coulomb strength criteria

One of the most popular strength theories was advanced by Coulomb (1773), which was proposed to explain shear fractures (Fig. 2.2). The criterion in shear and normal stress (τ, σ_n) space is

$$\tau = c + \sigma_n \tan \phi \quad (2.1)$$

where c is the cohesive strength of the cemented surface and ϕ is the angle of friction.

The uniaxial compressive strength and tensile strength can be written as functions of the cohesion strength and the angle of friction as:

$$\sigma_c = 2c \cos \phi / (1 - \sin \phi) \quad (2.2a)$$

$$\sigma_t = 2c \cos \phi / (1 + \sin \phi) \quad (2.2b)$$

For practical use purposes, in rock mechanics, the uniaxial tensile strength (σ_t) often has

a cut-off value or is taken as 0.

Fig. 2.2 Mohr-Coulomb strength criterion: (a) shear failure on plane ab, (b) strength envelope of shear and normal stresses, and (c) strength envelope of principal stresses.

2.2.2 Griffith fracture theory

The Griffith theory of fracture (1921) was originally developed to explain why the mechanical tensile strength measured in test for a brittle material such as glass is much less than the theoretical molecular bond strength of the material. Griffith postulated the existence of micro cracks (or flaws) randomly distributed within a material that might appear to be intact. Using the Inglis solution for stress distribution around an elliptical crack, Griffith showed that, under uniaxial tension, the criterion of crack propagation is

$$\sigma_1 = \sqrt{\frac{\Omega E}{\pi c}} \quad (2.3)$$

where Ω is the energy released per unit crack surface area; E is the modulus of elasticity; and c is half of the crack length.

Under compressive stresses, the Griffith criterion in principal stress (σ_1, σ_3) space is

$$(\sigma_1 - \sigma_3)^2 - 8\sigma_t(\sigma_1 + \sigma_3) = 0 \quad \text{if } \sigma_1 + 3\sigma_3 \geq 0 \quad (2.4a)$$

$$\sigma_t = -\sigma_3 \quad \text{if } \sigma_1 + 3\sigma_3 < 0 \quad (2.4b)$$

where σ_t is the tensile strength.

The criterion in shear and normal stress (τ, σ_n) space is

$$\tau^2 - 4\sigma_t\sigma_n = 4\sigma_t^2 \quad (2.5)$$

It should be noted that this theory is applicable to field problem in which the stress range is tensile or involves only low compressive stresses.

2.2.3 Hoek-Brown failure criterion

The Hoek-Brown criterion (1980) is an empirical criterion originally developed for applications in underground excavation design.

$$\sigma_1 = \sigma_3 + \sigma_c \left(m\sigma_3 / \sigma_c + s \right)^a \quad (2.6)$$

where m , s and a are parameters that depend upon the characteristics of the rock mass; σ_c is the uniaxial compressive strength of the intact rock material; and σ_1 and σ_3 are the “axial” and “confining” principal stresses respectively.

The peak triaxial compressive strengths of a wide range of isotropic intact rock materials can be described by the following equation ($s = 1$, $a = 1/2$):

$$\sigma_1 = \sigma_3 + \sigma_c \left(m\sigma_3 / \sigma_c + 1 \right)^{1/2} \quad (2.7)$$

The parameter m has been found increasing with the rock categories in the following general way:

- (1) $m \approx 7-10$ for marble (carbonate rocks), $m \approx 10$ for mudstone (lithified sedimentary rocks),
- (2) $m \approx 15$ for quartzite (strong crystals and poorly developed crystal cleavage rocks),
- (3) $m \approx 17$ for dolerite (fine-grained crystalline rocks) and $m \approx 25$ for granite (coarse-grained igneous and metamorphic rocks).

2.2.4 Drucker-Prager criterion

The Drucker-Prager model (Drucker and Prager, 1952) has been extended to model the behavior of brittle materials. The model is capable of providing phenomenological account for the pressure dependent flow due to the internal friction, a typical feature of brittle materials. And it allows the evolution of the deformation to be tracked through the strain hardening and the strain softening within the framework of finite deformation kinematics.

Fig. 2.3 Piecewise linear Drucker-Prager model

In triaxial stress state, the Drucker-Prager strength criterion has an advantage to represent the strength property of a material since it includes all the six stress components in its expression. It has the following form

$$f = \alpha I_1 + \sqrt{J_2} - K = 0 \quad (2.8)$$

where $\sqrt{J_2} = \sqrt{\frac{1}{2}s_{ij}s_{ij}}$, is the second invariant of the stress deviator tensor s_{ij} ;

$I_1 = \sigma_1 + \sigma_2 + \sigma_3$, is the first invariant of stress tensor σ_{ij} ; $P = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ is the equivalent hydrostatic stress; α and K are material constants. Fig. 2.3 shows the meridian of a modified Drucker-Prager criterion which has piecewise linear relation between the effective stress and the hydrostatic pressure.

2.3 Constitutive models with strain rate effects

It is well known that mechanical properties of most engineering materials are affected by strain rates, which may further influence the structural behaviors when the structures is subjected to rapidly changed loads, such as high velocity impact and blast. Thus, it is necessary to take into account the strain rate effect on the material constitutive models. Extensive studies have been carried out on the mechanical response of concrete materials in compression under high strain rate loading. Fig. 2.4 shows the dynamic strength increase ratio. And the trend that the concrete strength increased with the increase of strain rate is noticeable. Fig. 2.5 shows the dynamic tensile strength also increases with the increase of the strain rate.

Fig. 2.4 Relative concrete compression strength versus strain rate (Bischoff, 1988)

Fig. 2.5 Relative concrete tension strength versus strain rate (Bachmann, 1993)

In recent years, strain rate dependency of rocks has attracted more and more attentions associated with the widely employment of underground tunnels and chambers. Many researchers (Gary and Bailly, 1998; Zhao, et al., 1999 & 2000; Shan, et al., 2000; Frew, et al., 2001; Cho, et al., 2003 & 2004 and Lok and Zhao, 2004) carried out various rate-dependence studies for different rock-like materials. The general conclusion has been drawn that the dynamic compressive and tensile strength of rocks increase with the increase of the strain rate. However, the origin of the strain rate effect needs be explored.

Zhao, et al. (1999 & 2000) studied the dynamic compressive and tensile tests on granite (Figs. 2.6 and 2.7). The strain rate effect on the uniaxial compressive and tensile strength can be seen clearly.

Fig. 2.6 Variation of the uniaxial compressive strength with the loading rate

Fig. 2.7 Brazilian and 3-point flexural tensile strength at different loading rates

There are different kinds of methods to study the dynamic behavior of rock-like materials, which are mainly divided into three categories. The first category is an empirical method fitting experimental results, in which researchers summarize the dynamic strength increase ratio under different strain rates. Based on rheological theory, the second category regards rock as a coupled body of elastic, plastic and viscous parts. The last category is based on continuum damage mechanics, which is in popularity and a perspective method for developing dynamic constitutive models.

2.3.1 Dynamic strength increase ratio from high strain rate tests

Different experiment techniques have been applied in determining dynamic mechanical properties, as well as failure modes of materials at high strain rates. Split Hopkinson pressure bar (SHPB), which was developed originally to test the dynamic stress-strain relation of metallic specimens, has been widely applied to study the dynamic compressive strength of concrete and rocks at high strain rate from 10^1 to 10^3 s^{-1} (Goldsmith, et al., 1966; Ross, et al., 1989; Zhao, 1998; Ross et al., 1995; Tedesco and Ross, 1998; Grote, et al., 2001). SHPB-based experimental results suggested that the strain rate influence on the DIF becomes significant when the strain-rate is greater than a critical value that is in the range of 10^1 to 10^2 s^{-1} (Tedesco and Ross, 1998; Grote, et al., 2001).

Olsson (1991) gave the relation of the compressive strength of tuff as a function of the strain rate from 10^{-6} to 10^3 s^{-1} ,

$$\sigma_d \propto \begin{cases} \dot{\epsilon}^{0.007} & \dot{\epsilon} \leq 76 \text{ s}^{-1} \\ \dot{\epsilon}^{0.35} & \dot{\epsilon} \geq 76 \text{ s}^{-1} \end{cases} \quad (2.9)$$

The dependence of DIF on strain rate has been recommended by CEB for concrete, i.e.,

$$DIF = \frac{f_{cd}}{f_{cs}} = \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_s} \right]^{1.026\alpha_s} \quad \text{for } \dot{\epsilon} \leq 30 \text{ s}^{-1} \quad (2.10a)$$

$$DIF = \gamma_s \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_s} \right]^{\frac{1}{3}} \quad \text{for } \dot{\epsilon} > 30 \text{ s}^{-1} \quad (2.10b)$$

where f_{cs} and f_{cd} are the unconfined uniaxial compressive strength in quasi-static and dynamic loading, respectively, γ_s and α_s are material constants.

A DIF regression equation was suggested as follows by Tedesco and Ross (1998)

$$DIF = 0.00965 \log \dot{\epsilon} + 1.058 \geq 1.0 \quad \text{for } \dot{\epsilon} \leq 63.1 \text{ s}^{-1} \quad (2.11a)$$

$$DIF = 0.758 \log \dot{\epsilon} - 0.289 \leq 2.5 \quad \text{for } \dot{\epsilon} > 63.1 \text{ s}^{-1} \quad (2.11b)$$

The dynamic strength increase factor can be used to modify the existing static strength

models. For example, to take the strain rate effect into account, the yield function of Drucker-Prager model in Eq. (2.8) is modified as

$$f = \sqrt{J_2} - (\alpha I_1 - k)(1 + C \ln \dot{\varepsilon}) = 0 \quad (2.12)$$

where C is a material constant.

2.3.2 Constitutive models based on rock rheological theory

In rock rheological theory, constitutive models are formed by elastic, plastic and viscous parts. The viscous part involves the rate-sensitivity into the constitutive model and makes the coupled model more reasonable. Some basic models with respect to the rock rheological theory are introduced as follows:

1) St. Venant model

St. Venant model consists of elastic and plastic bodies, which is an ideal elastic-plastic model as shown in Fig. 2.8.

Fig. 2.8 St.Venant mechanical model

The constitutive relation is

$$\begin{cases} \sigma < \sigma_s & \varepsilon = \frac{\sigma}{k} \\ \sigma \geq \sigma_s & \varepsilon \rightarrow \infty \end{cases} \quad (2.13)$$

2) Maxwell constitutive relation

The Maxwell model consists of a series-connection of elastic and plastic bodies as shown in Fig. 2.9.

Fig. 2.9 Maxwell mechanical model

The constitutive equation is

$$\dot{\varepsilon} = \frac{1}{k} \dot{\sigma} + \frac{1}{\eta} \sigma \quad (2.14)$$

3) Kelvin constitutive model

The Kelvin model is a viscous-elastic body which is a parallel connection of elastic spring and a damper (Fig. 2.10).

Fig. 2.10 Kelvin mechanical model

The constitutive relation is:

$$\sigma = k\varepsilon + \eta\dot{\varepsilon} \quad (2.15)$$

A few more complex constitutive models are proposed coupled from the elastic, plastic and viscous parts in order to study the strain rate dependence of materials, some of which are simply introduced as follows:

4) Modified Kelvin constitutive model

A Kelvin model part and a spring are series connected to form a modified Kelvin model and its constitutive equation is:

$$\frac{\eta}{k_1} \dot{\sigma} + \left(1 + \frac{k_2}{k_1}\right) \sigma = \eta \dot{\varepsilon} + k_2 \varepsilon \quad (2.16)$$

5) Ideal viscous-plastic model

Ideal viscous-plastic model is one parallel connected body from one viscous part and one damper. Its constitutive relation is:

$$\begin{cases} \varepsilon = 0, & \sigma < \sigma_s \\ \dot{\varepsilon} = \frac{\sigma - \sigma_s}{\eta}, & \sigma \geq \sigma_s \end{cases} \quad (2.17)$$

6) Poyting-Thomson model

One Maxwell body is parallel connected with one elastic spring, which forms Poyting-Thomson model. Its constitutive relation is:

$$\dot{\sigma} + \frac{k_1}{\eta} \sigma = (k_1 + k_2) \dot{\varepsilon} + \frac{k_1 k_2}{\eta} \varepsilon \quad (2.18)$$

Besides, a number of researchers (Shan, 1997; Eibl and Schmidt-Hurtienne, 1999; Zhao, 2003 and Georgin and Reynouard, 2003) have contributed to develop coupled constitutive models in the view of the rheological theory.

Shan (1997) proposed a rock dynamic constitutive model from statistic damage theory. The constitutive model (Fig. 2.11) for granite is suggested as:

$$\sigma = E \varepsilon \exp\left(-\frac{\varepsilon^m}{\alpha}\right) + \eta \frac{d\varepsilon}{dt} \quad (2.19)$$

where m , α , η are material constants.

Fig. 2.11 Mechanic damage model

Georgin and Reynouard (2003) proposed a viscoplastic model in a natural way to take into account the rate effect of concrete under impact as shown in Fig. 2.12. The total strain rate vector can be expressed as:

$$\{\dot{\epsilon}\} = \{\dot{\epsilon}^e\} + \{\dot{\epsilon}^{vp}\} \quad (2.20)$$

where $\{\dot{\epsilon}^e\}$ is an elastic strain rate vector, $\{\dot{\epsilon}^{vp}\}$ is an viscoplastic strain rate vector.

Fig. 2.12 Uniaxial model description

2.3.3 Constitutive models based on damage mechanics

Damage mechanics can be applied to study the intrinsic properties from the views of statistics and micromechanics. It can propose continuum models to study the properties of materials including static and dynamic aspects. And it is one of the most important theories in developing rock constitutive model for both static and dynamic loads.

Damage evolution equations and constitutive equations of damaged materials can be

expressed as:

$$D = F(\sigma, \varepsilon, \dot{\varepsilon}, D_0, T, \dots) \quad (2.21)$$

The damage variable was defined in scalar form by Hult (1974),

$$D = \frac{A_0 - A_n}{A_0} \quad (2.22)$$

where A_0 is the original cross-sectional area without microcracks and A_n is the net area of the cross section which is still capable of supporting and transmitting load. The scalar value $D = 0$ corresponds to damage “free” material, while $D = 1$ corresponds to total disintegration.

The compressibility of an elastic solid containing cracks is higher than that of an identical solid without cracks and is strongly influenced by the presence of cracks (Walsh and Brace, 1966). Therefore, Young’s modulus of an elastic solid containing cracks tends to be lower than that of an identical solid without cracks. Hence, the change in the Young’s modulus or compressibility of rock can reflect the internal damage development. Lemaitre (1984) suggests that material damage can be experimentally evaluated by measuring the modulus degradation ratio as follows:

$$D = 1 - \frac{E_n}{E_0} \quad (2.23)$$

where E_0 and E_n can be the Young’s moduli of the pre-and postshock specimen, respectively. Eq. (2.23) implies that the change in the elastic modulus allows one to describe material damage.

Damage models are generally used to show a degradation of the material due to microcracking characterized by loss of rigidity. In recent years, a number of researchers (Dube, et al., 1996; Liu and Katsabanis, 1997; Kim and McCarter, 1998; Ma, et al., 1998; Fichant, et al., 1999; Chen, 1999; Le Nard and Bailly, 2000; Hao, et al., 2002; de Borst, 2002; Hari Manoj Simha, et al., 2002; Tang, et al., 2002; Li, et al., 2003; Li and Meng, 2003; Liu, et al., 2003; Soh, et al., 2003; Zhang and Hao, 2003; and Cheng and Dusseault,

2004) proposed some new methods using different damage variables to describe the complex damage evolution of rock-like materials. The majority of the models describe the damage evolution based on the damage variables, in which the stiffness is degraded with the damage process.

In dynamic damage of rocks, damage dynamic accumulation will also be reflected by the microcracks density. Rate of the damage evolution determination represents an even more complex problem which in an essential manner depends on the statistical distribution of the microstructural heterogeneities. Microcrack distribution is widely used to represent damage distribution.

Under dynamic loading, microcracks contained in a brittle material may grow and new cracks may be generated. The crack density then increases, and stiffness and strength of the material decrease. Ma, et al. (1998) proposed a damage model using crack density as the damage variable. Characterization of dynamic fracture requires the definition of some measure of the level of fracture damage incurred. Damage will be defined as a scalar parameter D . If the occurrence in the volume V of a single microcrack of critical severity is assumed to produce fracture of this volume, then the probability of fracture is given as

$$P_f = 1 - e^{-C} \quad (2.24)$$

where the parameter $C = C_d V$ with C_d being the crack density.

The damage scalar can be defined as the probability of fracture

$$D = P_f = 1 - e^{-C} \quad (2.25)$$

The increase of the crack number depends on deformation of the solid, which is assumed in the form

$$dC / dt = \alpha \langle \varepsilon - \varepsilon_{cr} \rangle^\beta \quad (2.26)$$

where t is time, α and β are parameters, ε_{cr} is the critical strain.

2.4 Johnson-Holmquist (J-H) strength model

2.4.1 Introduction

With the rapid development of the computer technologies, finite element method and material constitutive theory, numerical analysis becomes more and more important in engineering structural design and analysis. In this section, a rate and pressure dependent strength model, Johnson-Holmquist (J-H) model (Johnson and Holmquist, 1994 and Holmquist, et al., 2001) is introduced to simulate rock dynamic failure under different loading conditions. The J-H model was first introduced for ceramic material under shock loads (Johnson and Holmquist, 1994). It has the advantage to properly account for the strength degradation from intact to damaged material by employing a damage scalar. A pressure-cutoff failure criterion is used to modify the J-H model in simulation of volumetric tensile failure. The implementation of the modified J-H model exhibits the capacity to simulate both tension induced cracks and compression-shear dominated crushing zone of rock failure.

The J-H model is schematically presented in Fig. 2.13. The model includes a representation of the intact and fractured strength, a pressure-volume relationship that can include bulking, and a damage model that transitions the material from an intact state to a fractured state. The normalize equivalent stress for the strength is

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*) \quad (2.27)$$

where σ_i^* is the normalized intact equivalent stress, σ_f^* is the normalized fracture stress, and D is the damage ($0 \leq D \leq 1.0$).

The normalized equivalent stresses ($\sigma^*, \sigma_i^*, \sigma_f^*$) have the general form

$$\sigma^* = \sigma / \sigma_{HEL} \quad (2.28)$$

where σ is the actual equivalent stress and σ_{HEL} is the equivalent stress at the

Hugoniot elastic limit (HEL). The general form of the equivalent stress is

$$\sigma = \sqrt{1/2 \left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right]} \quad (2.29)$$

where σ_x , σ_y , and σ_z are the three normal stresses, and τ_{xy} , τ_{xz} , and τ_{yz} are the three shear stresses.

The normalized intact strength is given by

$$\sigma_i^* = A(P^* + T^*)^N (1 + C \ln \dot{\epsilon}^*) \quad (2.30)$$

and the normalized fractured strength is given by

$$\sigma_f^* = B(P^*)^M (1 + C \ln \dot{\epsilon}^*) \quad (2.31)$$

The material constants are A , N , B , M , C , T , and $\sigma_{f \max}^*$. The normalized pressure is

$P^* = P / P_{HEL}$, where P is the actual pressure and P_{HEL} is the pressure at the HEL.

The normalized maximum tensile hydrostatic pressure is $T^* = T / P_{HEL}$, where T is the maximum tensile hydrostatic pressure the material can withstand.

The dimensionless strain rate is defined as $\dot{\epsilon}^* = \dot{\epsilon} / \dot{\epsilon}_0$, where $\dot{\epsilon}$ is the actual equivalent strain rate and $\dot{\epsilon}_0 = 1.0 s^{-1}$ is the reference strain rate. The equivalent strain rate is analogous to the equivalent stress and is expressed as

$$\dot{\epsilon} = \sqrt{\frac{2}{9} \left[(\dot{\epsilon}_x - \dot{\epsilon}_y)^2 + (\dot{\epsilon}_x - \dot{\epsilon}_z)^2 + (\dot{\epsilon}_y - \dot{\epsilon}_z)^2 + \left(\frac{3}{2}\right) (\dot{\gamma}_{xy}^2 + \dot{\gamma}_{xz}^2 + \dot{\gamma}_{yz}^2) \right]} \quad (2.32)$$

Fig. 2.13 Description of the J-H strength model

The damage variable is expressed as

$$D = \sum \Delta \varepsilon_p / \varepsilon_p^f \quad (2.33)$$

where $\Delta \varepsilon_p$ is the effective plastic strain during a cycle of integration and $\varepsilon_p^f = f(P)$ is the plastic strain to fracture under a constant pressure P . The specific expression is

$$\varepsilon_p^f = D_1 (P^* + T^*)^{D_2} \quad (2.34)$$

where D_1 and D_2 are two damage constants. The material cannot undertake any plastic strain at $P^* = -T^*$, but ε_p^f increases as P^* increases.

A polynomial equation of state is employed in J-H model. The pressure variable is

calculated as,

$$p = k_1\mu + k_2\mu^2 + k_3\mu^3 + \Delta p \quad (2.35)$$

where k_1 , k_2 and k_3 are constants (k_1 is the bulk modulus), μ is a compression variable, and $\mu = \rho / \rho_0 - 1$ for current density ρ and initial density ρ_0 . For tension with $\mu < 0$, the above equation is replaced by $p = k_1\mu$. Δp in Eq. (2.35) is an additional incremental pressure determined from energy considerations to include the dilatation effect after compression failure.

It should be highlighted that rate dependence of J-H model is implemented by the dynamic strength increase factor (DIF) as follows:

$$DIF = 1 + C \ln \dot{\varepsilon}^* \quad (2.36)$$

And the strength model can be expressed if strain rate effect is not involved:

$$\sigma_i^* = A(P^* + T^*)^N \quad (2.37)$$

The J-H strength model is a Drucker-Prager-like strength model. The advantage of the J-H model is that the strength degradation with material damage is considered in a proper way. The damage variable depends on the accumulated plastic strain over the fracture plastic strain. While, the fracture plastic strain is pressure-dependent. The higher the hydrostatic pressure, the larger the fracture plastic strain. In an equivalent tensile stress state (the hydrostatic pressure is negative), the material is completely brittle, and the fracture plastic strain is exactly zero. The Hugoniot elastic limit (HEL) in the J-H strength model was introduced for the cases of high velocity impact or shock loads. In the present study, it is replaced by the uniaxial compressive strength to normalize the intact and damage strength functions (Eqs. (2.30) and (2.31)). The strain rate effect on the failure strength can be considered through magnifying the effective stress by a dynamic increase factor which is associated to a normalized strain rate.

2.4.2 Determination of J-H model constants for granite

The determination of material constants is not a straightforward process because some of the constants cannot be determined explicitly. Indirect method to determine the constants will be introduced in the later sections.

The intact J-H strength (Drucker-Prager-like) model can be expressed as:

$$\sigma = A \frac{(P + T)^N}{P_{HEL}^N} \sigma_{HEL} \quad (2.38)$$

It should be noted that the original J-H model is doubted to overestimate the volumetric tensile strength. The ultimate tensile pressure given by the J-H model is

$$P = -T \quad (2.39)$$

where T is the uniaxial tensile strength. However, in uniaxial tensile condition, the pressure at failure is $P_t = -\frac{1}{3}T$ with a deviatoric stress $\sigma_{dev} = T$. Thus, completely brittle failure may happen at a lower volumetric tensile pressure. In this study, the ultimate tensile pressure is given as:

$$P_t = -\frac{1}{3}T \quad (2.40)$$

Four sets of static experimental data, i.e.,

- (1) cut-off hydro-tensile strength f_{III} ($f_{III} = \sigma_1 = \sigma_2 = \sigma_3$, $f_{III} = \frac{1}{3}f_t$);
- (2) uniaxial tensile strength f_t ($f_t = \sigma_1, \sigma_2 = \sigma_3 = 0$);
- (3) uniaxial compressive strength f_c ($\sigma_1 = \sigma_2 = 0, \sigma_3 = -f_c$);
- (4) confinement compressive strength (f_{ccc}, f'_{cc}) ($0 > \sigma_1 = \sigma_2 = -f'_{cc}, 0 > \sigma_3 = -f_{ccc}$) are used to determined the parameters.

For the granite used in the present study, the uniaxial compressive strength f_c and the uniaxial tensile f_t are 147.5 and 16.1 MPa, respectively. A confinement compressive

strength of $(2.7f_c, 0.5f_c)$ is used as one set of the strength parameters.

The mean pressure is given as:

$$P = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (2.41)$$

The equivalent yield stress is given as:

$$Y = \sqrt{3J_2} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (2.42)$$

Thus, the points in the J-H curve can be calculated respectively as (-5.37, 0), (-5.37, 16.1), (49.17, 147.5), (181.92, 324.5) without involving strain rate effect.

The J-H strength model is an exponential curve, which can be regressed from the existing scattered experiment data points. $A=2.55$ and $N=0.64$ are obtained for J-H strength curve from the data regression. It is assumed that the damaged rock has about 30 percentage of intact strength. So we choose $B=0.8$, $M=0.6$. Strain rate constant and the damage parameters are referring to that of concrete material ($D_1=0.04$, $D_2=1.0$). The regressed J-H model is plotted in Fig. 2.14.

Impact tests to determine the equation of state of the rock material were conducted by using a light gas gun (Zhao, et al., 1999). The polynomial equation of state is:

$$P = 95.0\mu + 613.3\mu^2 + 1249.4\mu^3 \quad (\text{GPa}) \quad (2.43)$$

Properties of granite used in this study are summarized in Table 2.1.

(a) Strength model

(b) Damage model

Fig. 2.14 J-H model for granite

In all, the J-H model consists of a pressure-dependent elasto-plastic strength model, a pressure-dependent cumulative damage model and a polynomial equation of state.

In this study, the J-H model without involving the strain rate effect will be used which means that microstructures or the particles in the SPH method are insensitive to the strain rate. Based on this model, the rock dynamic failure behavior in the macro level will be

investigated. It is interesting that similar phenomena as given by Li and Meng (2003) were observed. Based on the strain-insensitive J-H model in the particle level, the strain rate effect in the macro level for the heterogeneous materials is identified. Detail numerical simulations will be discussed in Chapter 4 and Chapter 5 respectively for uniaxial tension and compression for rock-like specimens.

Table 2.1 Properties of granite in J-H strength model

Properties	Symbol	Unit	Granite
Density	ρ	kg/m ³	2670
Uniaxial compressive strength of intact rock	σ_{ci}	MPa	147.5
Tensile strength of intact rock	σ_{ti}	MPa	16.1
Young's modulus	E	GPa	75.2
Shear modulus	G_i	GPa	32.4
Possion's ratio	ν	-	0.16
Intact normalized strength parameter	A	-	2.55
Fractured normalized strength parameter	B	-	0.8
Strength parameter	C	-	-
Fractured strength parameter(pressure exponent)	M	-	0.6
Intact strength parameter(pressure exponent)	N	-	0.64
Maximum tensile strength	T	MPa	16.1
Hydro tensile limit	t	MPa	5.37
Parameter for plastic strain to fracture	D_1	-	0.04
Parameter for plastic strain to fracture(exponent)	D_2	-	1.0
First pressure coefficient(equivalent to the bulk modulus)	K_1	-	95.0
Second pressure coefficient	K_2	-	613.3
Third pressure coefficient	K_3	-	1249.4

Chapter 3 Rock heterogeneity and its implementation in AUTODYN

3.1 Characterization of rock heterogeneity

In the past, it was common in numerical modeling to assume that the distribution of material properties is uniform over the model. However, it is not true for brittle materials, including rock and concrete. The methods based on fracture mechanics or damage mechanics assume that the rock-like materials are homogeneous at the macro level, which ignore the strength randomness at the microstructure level. Although the existing methods based on homogenized material models can simulate the stress-strain curves of brittle materials in a certain range of accuracy, they can not reflect the real microstructures of rocks and the fracture mechanism of the heterogeneous material, especially when subjected to dynamic loads.

Rock-like materials have a heterogeneous microstructure that fractures via the formation, growth and coalescence of microcracks. The characteristics of heterogeneity of rocks can be seen with the naked eye or with a microscope at various magnifications as shown in Fig. 3.1 (Liu, et al., 2004). The rock-like materials also have pre-existing or stress-induced defects in the form of voids, microcracks and weak interfaces. All these features show that microstructures are more or less heterogeneous and random. Moreover, the fracture mechanism and fractures development are significantly influenced by the heterogeneous microstructure. Consequently, it is necessary to involve the heterogeneity into the study of fracture behavior of rock-like materials under static or dynamic loads.

Fig. 3.1 Microstructures of a granite specimen. M, Q, F and O represent Mica, Quartz, Feldspar and the undistinguished mineral, respectively. (Liu, et al., 2004)

A great deal of work has been done experimentally to describe the process of deformation and fracture of rock in compression (Batzle, et al., 1980; Brace, et al., 1966; Halbauer, et al., 1973; Kranz, 1979; Locker, et al., 1991; Wawersik and Fairhurst, 1970; Wong, 1982). These experimental results indicate that the overall strength and the static and dynamic properties of the rock are controlled both by the spatial arrangement of the grains and their microgeometry, and by the mechanical properties of the individual grains and cementing materials (Blair and Cook (1998)). Macro-nonlinear behavior of brittle rock is due to the heterogeneity. Rock contains structural weakness at all scales. During fracturing, heterogeneity plays a marked influence in determining the fracture paths and the resulting fracture patterns. However, the exact nature of how grain-scale heterogeneity in the shape, loading and strength of grains affects the macroscopic behavior and the fracture process is poorly understood. In the experiments, measurements of displacement, pore volume change, acoustic emission and seismic velocities and attenuation are destructive to specimen and expensive to conduct a large number of experiments.

Numerical simulations have become the most popular methods for rock fracture process

with the development of computing power, interactive computer graphics and topological data structures during the past few years. Numerical methods coupled with the fracture criterion and a heterogeneous material model have become an effective tool to investigate the complicated fracture behavior of rock-like materials such as the studies of Blair and Cook (1998), Tang et al. (2000), Kou, et al. (2001), Liu, et al. (2004) and Zhu and Tang (2002, 2004 and 2006). The capability of providing the visual fracture process has attached great interests of researchers and makes the numerical methods a potential future.

3.2 Existing numerical simulations involving heterogeneity

It is of great significance to study the fracture mechanism of brittle materials including rocks and concrete. Though experimental observations have provided a great deal of insight into the fracture process, the fracture mechanism is not completely understood and the progressive failure process can not be visually shown for the specimen during the loading process. While for the numerical analysis, it can consider the real microstructure of rock material not just simplified as an analytical homogeneous model. Heterogeneity plays a key role in the failure process of rock, especially under dynamic loads. Thus in order to simulate fractures in a real material heterogeneity has to be implemented into the material model.

Heterogeneity is involved from the point of statistical modeling to reflect the real random microstructures, which is particularly introduced in the section 3.3. In this part, the work concerned about the rock fracture with considering heterogeneity effect will be extensively reviewed.

3.2.1 Uniaxial compression and triaxial compression

In rock-like materials, the presence of the heterogeneity results in the formation and

growth of microcracks under compressive forces. This kind of materials is observed to display three distinct modes of failure, depending on the magnitude of the lateral compression, which may accompany the axial load. These failure modes are (1) axial splitting when no confining pressure exists, (2) faulting or (macroscopic) shear failure when some moderate (often less than 20% of maximum axial stress) confining pressure is applied and (3) plastic flow when the confining pressure is suitably large. The strength of these materials is also strongly affected by the confining pressure.

Splitting mode is the basic failure pattern of rock specimen under static and dynamic loadings. The fracture mechanism cannot be well understood until now. A typical result of rock material under compression is presented in Fig. 3.2(a). The initiation of crack and the development of crack growth reflect the fracture mechanism under compressive loadings. For uniaxial compression, the splitting mode is dramatically shown in the progressive fracture process of the specimen. The crack initiates from around the middle part of the specimen and propagates parallel to the applied loading. This failure phenomenon needs further discussion and it is of great importance to study the fracture mechanism of brittle material under uniaxial compression.

The fracture process of rock under triaxial compression is shown in Fig. 3.2(b). The nucleation of initial cracks forms the shear fracture in macroscale finally. It investigates the influence of confining pressure to the failure pattern of rocks. Confining rocks are often encountered in the site field. Consequently, the study of rock behavior under lateral confinement is also necessary.

Expect for the above result from Liu, et al. (2005), a great deal of work is found in many literatures such as Tang and Kou (1998), Zhu and Tang (2004), Tang, et al. (2000), Fang and Harrison (2002) to simulate the basic rock tests with heterogeneity effect. Tang et al. (2000) conducted numerical simulations to study the influence of microstructure on rock failure in uniaxial compression including heterogeneity effect and constraint, slenderness and size effect of rock specimen. Fang and Harrison (2002) developed a local

degradation approach to the modeling of brittle fracture in heterogeneous rocks. The results show the failure process of rock material visually proving the numerical methods a powerful approach in the research of rock properties.

Fig. 3.2 Simulated fracture process of a rock specimen under (a) uniaxial compression and (b) triaxial compression (the confining pressure is 20MPa), respectively (Liu, et al., 2005)

3.2.2 Brazilian tensile test

Brazilian tensile test has been widely used for the determination of static tensile strength of rock-like materials because of its simplicity and steadiness of the results. Rock is split along the diameter because of the tensile stress in the perpendicular direction to the diameter.

Tang et al. (2001) studied the fracture process of static Brazilian test and presented the splitting process visually shown in Fig. 3.3. The numerical results indicate that the dominant mode of failure is catastrophic splitting and progressive crushing. The paper also showed the horizontal and vertical stresses along the diameter, which convinces the tensile fracture mechanism.

Fig. 3.3 Failure process in the Brazilian disk under static diametric loading (Tang, et al., 2001)

To meet the development of protective technology, dynamic tensile Brazilian test is used to determine the tensile strength of rock-like materials. Tedesco, et al. (1989) conducted numerical split Hopkinson pressure bar test to investigate the effects of strain rate on the tensile strength of concrete. The paper showed the dynamic states of stress occurring in the split cylinder prior to failure as well as the mode of failure. Hughes, et al. (1993) provided the fracture process of concrete disc under high strain rate loading. However, it didn't give the stress distribution along the diameter under stress wave. Gomez, et al.

(2001) studied the static and dynamic behavior of concrete and granite in tension with damage. Photoelastic dynamic tensile splitting experiments were performed to establish the validity of using static relations for the determination of dynamic tensile strength.

To obtain the real fracture process by considering the microstructure of rocks, Zhu and Tang (2006) studied the deformation and failure process of a Brazilian disc of heterogeneous rock when subjected to static and dynamic loading conditions. The dynamic failure process of rock specimen is shown in Fig. 3.4. The numerical simulation also identifies the failure mechanism of rock during dynamic failure processes that are closely related to the propagation of stress wave.

Fig. 3.4 Failure process of rock specimen subjected to one stress wave (Zhu and Tang, 2006)

3.2.3 Numerical simulations of cracked rock and brittle porous solids

Fractures in forms of joints and microcracks are commonly found in natural rocks, and their failure mechanism strongly depends on the crack coalescence pattern between

pre-existing flaws. Tang and Kou (1998) conducted some numerical simulations to study the crack propagation and coalescence in brittle materials under compression. It confirms that under axial compression, wing-cracks nucleate at the tips of the pre-existing flaws, grow with increasing compression, and become parallel to the direction of the maximum far-field compression. Coalescence of the wing-cracks may be in either tensile mode or shear mode, or a combination of both modes. Wong et al. (2001) and Tang et al. (2001) carried out some analysis of crack coalescence in rock-like materials containing preexisting flaws using experimental and numerical approach, respectively. The patterns of crack coalescence and propagation are presented and seem that the later development boosts the splitting mode under uniaxial compression. Wong, et al. (2002) also obtained similar results on splitting failure in brittle rocks containing pre-existing flaws under uniaxial compression.

In heterogeneous porous brittle solids, splitting failure is observed in the numerical simulations done by Tang, et al. (2005). When porous specimens are loaded in compression, cracks may nucleate from these flaws, which propagate along an orientation parallel to the direction of the axial compression. Their propagation and interaction may lead to the collapse of the solid in a splitting mode. The cases can be used to study the mechanisms of compression-induced axial splitting.

3.2.5 Rock cutting simulations

Kou, et al. (1999) conducted numerical simulations of disc cutting of inhomogeneous rocks. Fracture pattern, chipping process, loading displacement response and influence of heterogeneity are investigated in detail. Based on this study, Liu, et al. (2002) carried out numerical simulations to study the fracture process in cutting heterogeneous brittle materials. The study can be utilized to improve the understanding of tool-rock interaction and rock failure mechanisms under the action of mechanical tools. In turn, the work should help in the design of cutting equipment and fragmentation operation.

In another paper, Liu, et al. (2002) conducted numerical analysis of heterogeneous rock fragmentation process induced by indenters. Rock fragmentation processes induced by single and double indenters were examined by numerical modeling. The chipping process, formation of crack and force-penetration curve are presented to explore the fracture mechanism under indentation. It is thought that the numerical simulation method will contribute to an improved knowledge of rock fragmentation in indentation, which will in turn help to enhance mining and drilling efficiency through the improved design of mining tools and equipment.

3.2.5 Fracture processes in heterogeneous rock under dynamic impact and explosive loading

Cho, et al. (2003) investigated the strain rate dependency of the dynamic tensile strength of rock. The fracture processes under various loading conditions were analyzed using a proposed finite element method to verify the differences between the dynamic and static tensile strengths and the strain-rate dependency. It is believed the strain-rate dependency of the dynamic tensile strength was caused by the heterogeneity of the rock. Then, Cho, et al. (2005) conducted numerical simulations to estimate the strength properties of granitic rock subjected to dynamic loading, in which strain rate effect is further studied.

Cho and Kaneko (2004) conducted numerical simulations to examine the dynamic fracture process in order to verify the dynamic fracture mechanism related to blast-induced borehole breakdown. It is proved that rock heterogeneity plays an influence to the fracture pattern as well as the applied pressure waveform.

3.2.6 Summary

Besides the literatures reviewed above, the numerical method involved heterogeneity effect draw much interests in many fields such as underground excavation (Zhu, et al., 2005) and comminution industry (Kou, et al., 2001). As a real material, the heterogeneity

of rock from the point of microstructure can not be ignored, which greatly influences the failure process of rocks under both static and dynamic loadings. Heterogeneity has to be involved in the numerical method in order to simulate the real rock failure.

3.3 Heterogeneity implementation in commercial software

3.3.1 Statistical model description of rock heterogeneity

Heterogeneity has to be implemented to simulate the fracture process correctly. Different techniques have been used in the past to implement heterogeneity. These techniques all have their own application scale to obtain good results. The different options include:

- 1) Randomly assigning different properties, possibly following some kind of distribution, to the elements in the numerical model (Batrouni and Hansen, 1988; Tang, 1997);
- 2) Using a mesh with a random geometry, but equal properties for the elements;
- 3) Generating a microstructure and projecting the microstructure on regular elements, assigning different properties to the elements depending on their position;
- 4) Using a combination of a random geometry and a generated grain structure (Bazant, et al, 1990). However, none of these techniques correlate their parameters describing heterogeneity with the practical physical-mechanical parameters of the specific rock.

In 1939, Weibull proposed statistical distribution of brittle material local strength, which is called Weibull distribution later. Weibull (1951) discussed the wide applicability of the statistical distribution function. Hudson and Fairhurst (1969) described the Weibull theory and its general statistical approach to rock failure. The above studies indicate that the Weibull distribution describes very well the experimental data for the distribution of micro-defects within rock. Therefore, the Weibull statistical distribution is introduced below in detail to characterize the rock heterogeneity.

For heterogeneity, the material properties for elements are randomly distributed throughout the specimen by following a Weibull distribution function:

$$\varphi = \frac{m}{\sigma_0} \left(\frac{\sigma}{\sigma_0} \right)^m \exp \left[- \left(\frac{\sigma}{\sigma_0} \right)^m \right] \quad (3.1)$$

where σ is the element strength and σ_0 is the mean strength of the elements for the specimen. For an elastic modulus E , the same distribution is used.

The presence of the shape parameter allows Weibull function to take a wide variety of shapes. As shown in Fig. 3.5, for $m=1$, this distribution is exponential; at about $m=1.5$, the distribution is nearly log-normal; and at about $m=4$, it closely approximates a normal distribution. The shape parameter m can be defined as the heterogeneity index of the rock. A larger m implies a more homogeneous material and vice versa.

Fig. 3.5 Probability density function for different homogeneous indices (Liu, et al. 2002)

In the present study, the concept of the Weibull distribution of material properties is adopted. The J-H model is selected as the constitutive model for the rock-like materials. Smoothed particle hydrodynamics (SPH) method is used to model the specimen. The particles carrying different material properties are randomly distributed over the specimen. The parameters following Weibull distribution include the tensile strength, the

compressive strength and the Young's modulus, etc.

3.3.2 Implementation of heterogeneity in Commercial software-Autodyn through its user subroutines

Monte Carlo method is suitable to achieve the random distributions of the elemental parameters in the physical space (Liu, et al., 2002). A sample space is formed after we specify the parameters following the Weibull distribution. However, even if the parameters have the same distribution function, the spatial distribution may be different.

The disorder of the probability distribution in physical space can be achieved by the Monte Carlo method. The simplest method is to generate a series of random data which have a uniform distribution between 0 and 1. Because the Weibull distribution is non-monotone, the integral distribution function is derived. That is:

$$q(\sigma) = \int_0^{\sigma} \varphi(x) dx = 1 - e^{-1(\sigma/\sigma_0)^m} \quad (3.2)$$

The Weibull random numbers are generated by Monte Carlo method as follows: σ is the random variable that is Weibull distributed with mean σ_0 , then

$$F(\sigma) = 1 - e^{-\sigma^m/\sigma_0} \quad (3.3)$$

Weibull distributed σ is

$$\sigma = \sigma_0 \{-\ln[U(0,1)]\}^m \quad (3.4)$$

where σ_0 is the scale parameter, m is the shape parameter. $U(0,1)$ represents the random data uniformly distributed between 0 and 1, which is actually rand (0,1) in the computer language. σ can be substituted by other material property parameters, in order to implement heterogeneity into the material properties, including compressive strength σ_c , tensile strength σ_t , Young's modulus E and Poisson's ration ν , etc.

The heterogeneity is implementation into the commercial software-Autodyn by translating Eq. (3.4) into Fortran Language. Eq. (3.5) gives the basic grammar of the heterogeneity implementation.

$$\text{Weibullrandomnumbers} = \text{scale} * ((-\log(\text{rand}()))^{1/\text{shape}}) \quad (3.5)$$

3.3.3 Heterogeneity implementation into J-H strength model

The J-H strength model which has been introduced in Chapter 2 is used in the present study. The envelope is a smoothed curve with a strength function with respect to pressure. The J-H model without involving strain rate effect is used in this study which is composed by an elasto-plastic strength model, a damage model and an equation of state. This rate-insensitive constitutive model assumes that the microstructures of the distributed particles is rate-insensitive to dynamic loadings.

Fig. 3.6 Strength envelope of J-H model (Holmquist, et al. 2001)

Without involving the strain rate effect, the intact strength and fracture strength of the material in the J-H model are expressed as follows:

$$\sigma_i = a(p + t)^n \quad (3.6)$$

$$\sigma_f = b(p)^n \quad (3.7)$$

Both the intact strength and fracture strength of the material is defined to conform to the Weibull distribution function to realize the heterogeneity. After involving the

heterogeneity using Weibull random numbers, the intact strength and fracture strength can respectively be expressed as:

$$\sigma_i = a(p + t)^n \{-\ln[U(0,1)]\}^m \quad (3.8)$$

$$\sigma_f = b(p)^n \{-\ln[U(0,1)]\}^m \quad (3.9)$$

Thus, the strengths of the particles in the specimen conform to the Weibull distribution function. It can be seen that both tension-cutoff and the effective compressive strength follow the Weibull distribution based on Eq. (3.8) and Eq. (3.9). The tensile strength and the corresponding effective tensile strain of a material are often considered to be the key factor influencing initiation of fractures. So it is reasonable and necessary to implement the heterogeneity into the tensile strength. Moreover, the Young's modulus E can also be realized into heterogeneous by this method:

$$E = E_0 \{-\ln[U(0,1)]\}^m \quad (3.10)$$

The Poisson's ratio, however, is considered a constant for all the particles.

Chapter 4 Smooth Particle Hydrodynamics (SPH) method by considering material heterogeneity

4.1 Introduction

4.1.1 Role of numerical simulation

Numerical simulation using computers or computational simulation has increasingly become a very important approach for solving complex practical problems in engineering and science. Numerical simulation translates important aspects of a physical problem into a discrete form of mathematical description, recreates and solves the problem on a computer, and reveals phenomena virtually according to the requirements of the analysts. Rather than adopting the traditional theoretical practice of constructing layers of assumptions and approximations, this modern numerical approach attacks the original problems in all its detail without making too many assumptions, with the help of the increasing computer power.

Numerical simulation provides an alternative tool of scientific investigation, instead of carrying out expensive, time-consuming or even dangerous experiments in laboratories or on site. The numerical tool are often more useful than the traditional experimental methods in terms of providing insightful and complete information that cannot be directly measured or observed, or difficult to acquire via other means.

4.1.2 Lagrangian grid-based method

There are two fundamental frames for describing the physical governing equations: the Eulerian description and the Lagrangian description. The Eulerian description is a spatial description, and the Lagrangian description is a material description. The Lagrangian grid-based methods have several advantages.

1. Since no convective term exists in the related partial differential equations, the code is conceptually simpler and should be faster as no computational effort is

- necessary for dealing with the convective terms.
2. Since the grid is fixed on the moving material, the entire time history of all the field variables at a material point can be easily tracked and obtained.
 3. In the Lagrangian computation, some grid nodes can be placed along boundaries and material interfaces. The boundary conditions at free surfaces, moving boundaries, and material interfaces are automatically imposed, tracked and determined simply by the movement of these grid nodes.
 4. Irregular or complicated geometries can be conveniently treated by using an irregular mesh.
 5. Since the grid is required only within the problem domain, no additional grids beyond the problem domain is required, and hence the Lagrangian grid-based methods are computationally efficient.

Due to these advantages, Lagrangian methods are very popular and successful in solving computational solid mechanics problems, where the deformation is not as large as that in the fluid flows.

However, Lagrangian grid-based methods are practically very difficult to apply for cases with extremely distorted mesh, because their formulation is always based on mesh. When the mesh is heavily distorted, the accuracy of the formulation and hence the solution will be severely affected. In addition, the time step, which is controlled by the smallest element size, can become too small to be efficient for the time marching, and may even lead to the breakdown of the computation. A Lagrangian meshfree method, whose solution does not depend on a mesh and hence is not affected by the heavy movement of the nodes, is indeed desirable.

4.1.3 Lagrangian meshfree method-Smoothed particle hydrodynamic (SPH)

In order to overcome the calculation difficulties in Lagrangian-based method, meshfree methods were developed, which are expected to be superior to the conventional grid-based finite element method for many applications. Because meshfree methods do not have a rigid connectivity, they are the ideal ones for modeling the problems which are not well suited to conventional mesh-based methods, such as extremely large deformation of the mesh and propagation of cracks with arbitrary and complex paths.

Smoothed particle hydrodynamics (**SPH**) (Lucy, 1977; Gingold and Monaghan, 1977), as a meshfree and particle-based method, was originally invented for modeling astrophysical phenomena, and later widely extended for applications to problems of continuum solid and fluid mechanics. Because of the distinct advantages of the particle method, soon after its debut, the SPH method was widely adopted as one of the efficient computational techniques to solve applied mechanics problems. Nowadays, the SPH method has been incorporated into many commercial codes. Fig. 4.1 shows the image of SPH for example, which is generated by the software-Autodyn.

Fig. 4.1 Smoothed particle hydrodynamics method

The principles and advantages of the SPH method will be introduced in detail in the following sections.

4.2 Advantages of SPH over traditional methods

4.2.1 Advantages in computation

The SPH method has been adopted in various numerical simulations. SPH is a meshfree technique as shown in Fig. 4.1, for solving computational continuum dynamics problems and has several potential advantages over the grid based Lagrange and Euler processors:

- (1) SPH does not require a numerical grid
 - i) No grid tangling problems as in the Lagrange processor.
 - ii) No need for erosion to obtain efficient solutions.
- (2) SPH is a Lagrangian technique
 - i) Allows efficient tracking of material deformation and history dependent behavior.
 - ii) Efficient compared with Eulerian techniques since only need to model regions where material exists, not where material will flow.

- iii) Specific complex constitutive models can be included with relative ease compared with Eulerian technique.

SPH is a method for obtaining approximate numerical solutions of the equations of fluid dynamics by replacing the fluid with a set of particles. For the mathematician, the particles are just interpolation points from which properties of the fluid can be calculated. For the physicist, the SPH particles are material particles which can be treated like any other particle system. Either way, the method has a number of attractive features.

The first of these is that pure advection is treated exactly. For example, if the particles are given a color, and the velocity is specified, the transport of color by the particle system is exact. Modern finite difference methods give reasonable results for advection but the algorithms are not Galilean invariant so that, when a large constant velocity is superposed, the results can be badly corrupted. The second advantage is that with more than one material, each described by its own set of particles, interface problems are often trivial for SPH but different for finite difference schemes. The third advantage is that particle methods bridge the gap between the continuum and fragmentation in a natural way. As a consequence, the best current method for the study of brittle fracture and subsequent fragmentation in damaged solids is SPH (Benz and Asphaug (1994, 1995)). A fourth advantage is that the resolution can be made to depend on position and time, which makes the method very attractive for most astrophysical and many geophysical problem. Fifth, SPH has the computational advantage, particularly in problems involving fragments, drops or stars, that the computation is only where the matter is, with a consequent reduction in storage and calculation. Finally, because of the close similarity between SPH and molecular dynamics, it is often possible to include complex physics easily.

4.2.2 Wide Applications

Firstly, SPH is ideal for astrophysical problems, which is initially developed to solve these problems (Gingold and Monaghan, 1977; Lucy, 1977). Then the earliest applications of SPH were mainly focused on fluid dynamics related areas which includes elastic flow, magneto-hydrodynamics and heat transfer, etc.

Benz and Asphaug (1994; 1995) extended SPH to the simulation of the fracture of brittle solids. Bonet and Kulasegaram (2000) applied SPH to the simulation of metal forming. The SPH method has been very attractive in simulating large deformation and impulsive loading events. When combined with a simple but effective viscosity, and a form of the thermal energy equation that guarantees that the viscous dissipation increases both the thermal energy and the entropy, a variety of shock problems have been studied (Monaghan and Gingold, 1983; Monaghan, 1997). The SPH algorithm gives very satisfactory results for shocks. One significant application area is high velocity impact problems concerning the effects of projectiles impacting upon space assets (Johnson et al., 1993; Johnson and Bessiel, 1996a, b). Another important application of SPH method is the explosion phenomena arising from the detonation of high explosive (Wang, et al., 2005).

Moreover, another class of problems suitable for the SPH algorithm arises in elasticity and fracture. It is of great importance in our investigation of dynamic fracture process and failure behavior of rock-like materials in this study.

4.2.3 Capability in Fracture and Fragmentation simulation

From the point view of solid mechanics, the most appealing attribute of SPH lies in its ability to fracture in a realistic manner while without unduly compromising its ability in subsequent computation. Great efforts have been made to incorporate damage and fracture algorithms into SPH which take the advantage of its meshfree attributes while instilling physical realism in the mechanics of progressive damage and ultimate fracture.

Benz and Asphaug (1994, 1995) showed how SPH could be applied to the fracture of brittle solids, where it gives much better results than the finite element or the finite difference methods. These methods have been applied to the breakup of planetesimals and the formation of asteroid families (Michel et al., 2004). In these simulations, the ease with which the SPH particles can describe the transition from a continuum to a set of fragments gives it a computational edge over other numerical methods. Commercial software packages such as Dyna3D and Autodyn for simulation impact now incorporate SPH. Elastic SPH also provides a simple and robust technique for simulating complex fracture in geological rock formations and in brittle materials (Gray, et al., 2001).

4.3 SPH methodology

4.3.1 Governing equations

The conservation equations of continuum mechanics are:

Mass:
$$\rho = \frac{\rho_0 V_0}{V} = \frac{m}{V} \quad (4.1)$$

Momentum:

$$\begin{aligned} \rho \ddot{x} &= \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \\ \rho \ddot{y} &= \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} \\ \rho \ddot{z} &= \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \end{aligned} \quad (4.2)$$

Energy:
$$\dot{e} = \frac{1}{\rho} (\sigma_{xx} \dot{\epsilon}_{xx} + \sigma_{yy} \dot{\epsilon}_{yy} + \sigma_{zz} \dot{\epsilon}_{zz} + 2\sigma_{xy} \dot{\epsilon}_{xy} + 2\sigma_{yz} \dot{\epsilon}_{yz} + 2\sigma_{zx} \dot{\epsilon}_{zx}) \quad (4.3)$$

The moving equation for particles:
$$\dot{x} = v \quad (4.4)$$

4.3.2 Method of approximation-Kernel function

SPH is relatively new numerical technique for the approximate integration of partial differential equations. In the SPH methodology, the material is represented by fixed mass particles to follow its motion. Unlike the grid based methods, which assume a connectivity between nodes to construct spatial derivatives, SPH uses a kernel approximation that is based on randomly distributed interpolation points. The particles carry material quantities such as mass m , velocity vector v , position vector x , etc. and form the computational frame for the conservation equations.

A rationale for SPH method was provided by invoking the notion of a kernel approximation for $u(x)$ on a domain Ω is generated by

$$u^h(x) = \int_{\Omega} w(x-y, h) u(y) d\Omega_y \quad (4.5)$$

where $u^h(x)$ is the approximation, $w(x-y, h)$ is a kernel or weight function, and h

is a measure of the size of the support; in SPH words, it is often called smoothing function in SPH. According to Monaghan (1982), the kernel is required to satisfy the following conditions:

- (1) $w(x - y, h) > 0$ on a subdomain of Ω , Ω_1 ,
- (2) $w(x - y, h) = 0$ outside the subdomain Ω_1 ,
- (3) a normality property: $\int_{\Omega} w(x - y, h) d\Omega = 1$,
- (4) $w(s, h)$ is a monotonically decreasing function, where $s = \|x - y\|$.
- (5) $w(s, h) \rightarrow \delta(s)$ as $h \rightarrow 0$, where $\delta(s)$ is the Dirac delta function.

For SPH, the popular used weight function is B-spline function. The argument of $w(x)$ is $s = \|x - y\|$; let $\bar{s} = s / s_{\max}$, where $s_{\max}$ is the radius of the support.

$$w(s, h) = \alpha_D \begin{cases} \frac{2}{3} - q^2 + \frac{1}{2}q^3, & q \leq 1 \\ \frac{1}{6}(2 - q)^3, & 1 < q \leq 2 \\ 0, & q > 2 \end{cases} \quad (4.6)$$

where $q = s / h$. α is $\frac{1}{h}$, $\frac{15}{7}\pi h^2$, $\frac{3}{2}\pi h^3$ in one, two and three-dimension, respectively.

4.3.3 Implementation of kernel function into governing equations

For the mass conservation equations, there are two popular kinds of approximation forms for the density approximation. One is summation density,

$$\rho_i = \sum_{j=1}^N m_j W_{ij} \quad (4.7)$$

The inherent benefit of the SPH formulation is the transformation of complex partial differential equations into their corresponding ordinary differential equations via construction of integral equations with a smoothing (kernel) function. A kernel density example is shown in Fig. 4.2 (Autodyn, 1997). The density at particle I can be calculated using an expression such as

$$\rho^I = \sum_{J=1}^N m^J W^{IJ} (x^I - x^J, h) \quad (4.8)$$

where m^J is the mass of particle J, W^{IJ} is a weighting function, x is the position of the center of a particle, h is known as the smoothing length or particle size.

Fig. 4.2 Kernel density function (Autodyn manual, 1997)

Therefore, to calculate the value of a function at particle I, in this case density, we sum the value of the function at all neighboring particles (interpolation points J1, J2, I, J3, J4) multiplied by a weighting function (the Kernel function).

Another one is the time dependent form as

$$\frac{d\rho_i}{dt} = \sum_{j=1}^N m_j v_{ij}^\alpha \frac{\partial W_{ij}}{\partial x_i^\alpha} \quad (4.9)$$

When two or more kinds of materials are involved in the calculation, Gary, et al. (2001) proposed a more reasonable density approximation as follows:

$$\frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^N \frac{m_j}{\rho_j} v_{ij}^\alpha \frac{\partial W_{ij}}{\partial x_i^\alpha} \quad (4.10)$$

Two popular approximation forms of the different equation of momentum are

$$\frac{dv^\alpha}{dt} = - \sum_{j=1}^N m_j \left(\frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} \right) \frac{\partial W_{ij}}{\partial x_i^\beta} \quad (4.11)$$

and,

$$\frac{dv^\alpha}{dt} = -\sum_{j=1}^N m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} \right) \frac{\partial W_{ij}}{\partial x_i^\beta} \quad (4.12)$$

The energy equation also has two approximation forms as follows:

$$\frac{de_i^\alpha}{dt} = \frac{1}{2} \sum_{j=1}^N m_j v_{ij}^\beta \left(\frac{\sigma_i^{\alpha\beta} + \sigma_j^{\alpha\beta}}{\rho_i \rho_j} \right) \frac{\partial W_{ij}}{\partial x_i^\beta} \quad (4.13)$$

$$\frac{de_i^\alpha}{dt} = \frac{1}{2} \sum_{j=1}^N m_j v_{ij}^\beta \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} \right) \frac{\partial W_{ij}}{\partial x_i^\beta} \quad (4.14)$$

4.4 Heterogeneity implementation in commercial software Autodyn

The J-H strength model is popular in numerical simulations of brittle materials under dynamic loadings (Johnson and Holmquist, 1994 and Holmquist, et al., 2001) which consists of an elasto-plastic strength model, a cumulative damage model and a polynomial equation of state. It will be adopted in our study to study the failure behavior of rock-like materials under dynamic compressive and tensile loadings. It should be mentioned that the strain rate effect is not involved in the J-H model to assume the material of each computation particle is insensitive to the strain rate.

The actual goal of numerical simulation is to simulate fracture process and strain rate effect in a real material. Real materials such as rock and concrete are non-homogeneous. So the fracture process of these materials should be influenced by the heterogeneous microstructures. Heterogeneity has to be implemented to simulate the rock dynamic fragmentation process.

The J-H constitutive model and the method of heterogeneity implementation in the computer program are introduced in Chapter 2 and Chapter 3, respectively. In general, it has been assumed that the strengths of the particles including compressive and tensile strengths and Young's modulus in the specimen conform to the Weibull distribution function. The realization of heterogeneity through the Weibull distribution function in the commercial software Autodyn by linking its user subroutines will be introduced in detail in the following section.

4.4.1 Autodyn's subroutines involved

The dynamic analysis software provides users with a number of standard alternatives for option such as equation of state, strength models, failure models, boundary conditions, etc. However, users need to write the user subroutines to realize these functions in FORTRAN language. The user subroutines all start with the characters "EX". The user subroutines involved in this study include:

- (1) EXBULK-custom bulk modulus;
- (1) EXSHR-custom shear modulus;
- (2) EXYLD-custom yield and/or shear model;
- (3) EXFAIL-custom failure criteria;
- (4) EXDAM-custom damage parameter;

Shear modulus can be modified in the subroutine EXSHR. In the subroutine EXYLD, new strength model can be added. Failure criterion developed by users can be involved in EXFAIL. Damage output is realized through the invoking of EXDAM. Heterogeneity is realized for the Young's modulus through the subroutine EXSHR and compressive and tensile strength through the user subroutines EXYLD and EXFAIL, respectively.

4.4.2 Revised user subroutines for SPH method

The user subroutines are used to involve heterogeneity in the J-H constitutive model. The user subroutines are presented here with the bold part being added to realize the material heterogeneity:

- (1) This is a user supplied subroutine which can be used to compute the bulk modulus for a material

```
SUBROUTINE EXBULK (BULK)
USE kindf
USE ijknow
USE locell
USE matdef
USE wrapup
use locsub
```

```
use mdgrid, only: ijset, var01, pn

IMPLICIT NONE
REAL (REAL8):: BULK

integer(int4)::n, defined, times
real(real8):: weibull(10000), rand
common/weibull_daa/weibull, defined, times
integer(int4):: number_cells
number_cells=jmax

if(defined.eq.0)then
    do n=1, number_cells
        weibull(n)=((-log(rand()))**(1.000/2.000))
    end do
defined=1
endif

bulk=bulk*weibull(ijk)
bulk=bulk*(1-var01(ijk))

RETURN
END SUBROUTINE EXBULK
```

(2) This is a user supplied subroutine which can be used to compute the shear modulus for a material

```
SUBROUTINE EXSHR (PRES,XMU,EPS,EPSP,TEMP,SHRMOD)
  USE kindf
  USE ijknow
  USE locell
  USE matdef
  USE wrapup
  use locsub
  use mdgrid, only: ijset, var01
  use cycvar

  IMPLICIT NONE
```

```

REAL (REAL8):: EPSD, EPS, PRES, SHRMOD, TEMP, XMU
integer(int4):: n,defined
integer::times
real(real8):: weibull(10000),rand
common/weibull_daa/weibull, defined

shrmod=shrmod*weibull(ijk)
shrmod=shrmod*(1-var01(ijk))

RETURN
END SUBROUTINE EXSHR

```

(3) Subroutine EXYLD is a user supplied subroutine which can be used to compute the yield stress for a material.

```

SUBROUTINE EXYLD (PRES,TT1,TT2,TT3,XMU,EPS,EPSD,TEMP,YIELD,IFAIL)
  USE kindf
  USE ijknow
  USE matdef
  USE wrapup
  use locell
  use locsub
  use mdgrid, only: ijset, var01, pn
  use cycvar

  IMPLICIT NONE

  INTEGER (INT4) ::  IFAIL
  REAL (REAL8) ::  EPSD,    EPS,    PRES,    TEMP,    TT1,    TT2
  REAL (REAL8) ::  TT3,    XMU,    YIELD

  real(real8):: yield_i, yield_f, p_cutoff
  integer(int4)::n, defined
  real(real8):: weibull(10000), rand
  common/weibull_daa/weibull, defined

  p_cutoff=sc(5)*weibull(ijk)

```

```

    a_jh=sc(6)
    e_jh=sc(7)
    if(pres>=p_cutoff) then
        yield_i=a_jh*(((pres/1.0e6)-(p_cutoff/1.0e6))**sc(7))*1.0e6
        yield_i=yield_i*weibull(ijk)
    else
        yield_i=0.0
    endif

    if(pres>=0.0) then
        yield_f=1.20*((pres/1.0e6)**0.6)*1000000.0
        yield_f=yield_f*weibull(ijk)
    else
        yield_f=0.0
    endif

    yield=yield_i-var01(ijk)*(yield_i-yield_f)

```

```

900  RETURN
    END SUBROUTINE EXYLD

```

(4) Subroutine EXFAIL is a user supplied subroutine which can be used to compute failure of a material.

```

SUBROUTINE EXFAIL
(NFCRIT,TT1,TT2,TT3,TT12,ET1,ET2,ET3,ET12,PANG,IFAIL)
    USE cycvar
    USE locell
    USE matdef
    USE mdgrid
    USE wrapup
    use ijknow

    IMPLICIT NONE
    INTEGER (INT4) :: IFAIL, NFCRIT
    REAL (REAL8)   :: ET1, ET12, ET2, ET3, PANG, TT1, TT12
    REAL (REAL8)   :: TT2, TT3, FT11U, FT12U
    real(real8) :: eps_f, D1, D2, phel, t_cut

```

```
real(real8) :: pres, var1
real(real8) :: p_cutoff
integer(int4)::n, defined
integer(int4):: times
real(real8):: weibull(10000), rand
common/weibull_daa/weibull, defined,times

      t_cut=fc(3)*weibull(ijk)
      D1=fc(4)
      D2=fc(5)
      if(pn(ijk).gt.p_cutoff.and.var01(ijk).ne.1.0) then
          var02(ijk)=D1*((pn(ijk)/phel+t_cut/phel)**D2)
          var1=abs(eps(ijk)/var02(ijk))
          if(var1.gt.var01(ijk)) var01(ijk)=var1
          if(var01(ijk).gt.0.999) var01(ijk)=1.0
      endif

      if(pn(ijk).le.0.0) then
          if(var01(ijk).eq.1.0) then
              txx(ijk)=0.0
              tyy(ijk)=0.0
              txy(ijk)=0.0
              ttt(ijk)=0.0
              pn(ijk)=0.0
          endif

          if(pn(ijk).le.p_cutoff)then
              pn(ijk)=0.0
              txx(ijk)=0.0
              tyy(ijk)=0.0
              txy(ijk)=0.0
              ttt(ijk)=0.0
              var01(ijk)=1.0
          endif
      endif

900  RETURN
      END SUBROUTINE EXFAIL
```


(5) Subroutine EXDAM is a user supplied subroutine which can be used to define the damage parameter for a cell or particle.

```
SUBROUTINE EXDAM (DAMAGE)
```

```
USE kindf
```

```
USE ijknow
```

```
USE matdef
```

```
USE wrapup
```

```
use mdgrid
```

```
use locell
```

```
use locsub
```

```
IMPLICIT NONE
```

```
REAL (REAL8) :: DAMAGE
```

```
dam(ijk)=var01(ijk)
```

```
RETURN
```

```
END SUBROUTINE EXDAM
```

Chapter 5 Rock dynamic compressive behavior with heterogeneity effect

5.1 Introduction

Rocks and rock-like materials, which are brittle and heterogeneous, have been extensively utilized in civil engineering constructions. For protective structures, impact and blast loads are one of the major concerns. They have relatively high amplitude and short duration. The fracturing behavior of brittle materials under impact and blast loads is very different from that by a static load. It is of great importance to investigate the dynamic strength, especially the dynamic fracturing and fragmentation process for rock and rock-like materials. The present chapter focuses on numerical simulations of the fracturing and fragmentation process of rock-like materials under high strain rate loadings. The modified version of AUTODYN which has been described in Chapter 4 will be adopted. SPH approximations by considering material heterogeneity effect are used in the simulations.

5.2 Experiments on strain rate effect

Split Hopkinson pressure bar (SHPB) experiment is the primary tool to study the dynamic properties of materials. It can accurately reflect the confinement, magnitude of stress change, and the time scale of loading that material would undergo in the real field. With the development of computer technology, numerical technique, as an effective method, is being vastly used to investigate the progressive failure of rocks under static and dynamic loadings. This chapter mainly uses numerical simulations to study rock properties under high strain rate loadings.

The technique of SHPB was introduced by Kolsky (1949). It is now an important facility used to study dynamic behaviour of materials. The conventional configuration of SHPB is shown in Fig. 5.1.

Fig. 5.1 Diagnostic schematic of the SHPB Set-Up

The strain histories measured on the surface of the incident and transmitter bars can be used to deduce the strain, stress and strain rate histories experienced by the specimen. The classical method is based on elementary wave theory of an infinitesimally slender rod. The strain history induced by a wave propagating in a single direction can be measured at one location and moved to another location without changing its shape.

Thus the specimen's strain, strain rate and stress time histories can be derived from the strain pulses obtained at the location of strain gages using one-dimensional wave theory. Particle velocities of three fundamental waves are related to the strain records as:

$$v_i = -c_0 \varepsilon_i \quad (5.1)$$

$$v_r = -c_0 \varepsilon_r \quad (5.2)$$

$$v_t = -c_0 \varepsilon_t \quad (5.3)$$

where c_0 is the bar velocity of the incident and transmitter bars, subscripts “ i ”, “ r ” and “ t ” represent “incident wave”, “reflected wave” and “transmitted wave”, respectively. So, the particle velocities of the steel bar at two interfaces, which are also the velocities of the specimen at the same location because of the continuity in motion, are expressed as:

$$v_1 = -c_0 \varepsilon_i + c_0 \varepsilon_r \quad (5.4)$$

$$v_2 = -c_0 \varepsilon_t \quad (5.5)$$

where v_1 refers to the velocity at the incident bar/specimen interface, and v_2 to the specimen-transmitter bar interface.

Strain rate of the specimen is defined as strain per unit time, which is the deformation per unit time per unit length of the specimen. It is calculated from:

$$\dot{\varepsilon}(t) = \frac{v_2 - v_1}{L_s} = \frac{c_0}{L_s} [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)] \quad (5.6)$$

where L_s is the length of the specimen. The average strain in the specimen is then calculated by integrating the strain rate history as:

$$\varepsilon(t) = \frac{c_0}{L_s} \int_0^t [\varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_t(t)] dt \quad (5.7)$$

The stress in the specimen is determined by calculating the total force of the cross section of steel bars at interfaces. Total forces on the specimen sides of two interfaces are equal to those on the steel bar sides, and calculated as:

$$P_1 = AE(\varepsilon_i + \varepsilon_r) \quad (5.8)$$

$$P_2 = AE\varepsilon_t \quad (5.9)$$

where A and E are the cross sectional area and Young's modulus of steel bars.

Finally, the average stress of the specimen is expressed as:

$$\sigma(t) = \frac{P_1 + P_2}{2A_s} = \frac{AE}{2A_s} [\varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_t(t)] \quad (5.10)$$

where A_s is specimen's cross sectional area. Eqs. (5.6), (5.7) and (5.10) are the basic formulae for conventional processing of SHPB test data.

The incident wave of conventional SHPB test is a rectangular wave. Typical experimental results for granite subjected to a conventional rectangular waveform are shown in Fig. 5.2. The measured waveform contains some oscillation which is caused by three-dimensional effect of the steel bars with finite diameter. It is apparent that the higher the loading rate, the more serious is the oscillation. In addition, impact in tests can not ensure a perfect contact. This delays the rise-time of the incident pulse. On the assumption of one-dimensional wave propagation, the measured waveform will be rectangular without effect from wave dispersion and with a perfect contact which is important to the accuracy of experimental results.

Fig. 5.3 shows the failure patterns of specimen undergoing different strain rates. It can be seen that the splitting mode is the major failure pattern of the specimens under dynamic loadings. At low strain rate, specimens are split into two pieces. With the increase of strain rate, the number of pieces increases after the specimen is impacted. At a relatively

high strain rate, specimens are broke into many pieces with splitting failure mode.

Fig. 5.2 Incident and reflected waves for granite specimens (Li, X.B., et al., 2000)

(a) Specimen undergone relatively low strain rate impact loading

(b) Specimen undergone relatively high strain rate impact loading

Fig. 5.3 Specimen undergone dynamic loading

5.3 Numerical model description

Numerical simulation is currently the most popular method used for modeling deformation behavior of rock before failure. Even though progress has been made in numerical simulation of failure occurring in rocks, there is a lack of tools which can simulate the failure process in a more visual way to get a better understanding to the failure mechanism of rock-like materials under dynamic loadings.

Numerical simulations will be conducted to study the fracture behavior of rock-like materials under dynamic compressive loadings. In all cases, the numerical simulations are conducted in plane strain conditions. The geometries of the specimens are 5 cm high and 5 cm wide if the geometry is not specially given. The geometry of the specimen is shown in Fig. 5.4. Based on the theory of SHPB, a rectangular velocity boundary is applied at the top end of the specimen with the bottom end fixed. The equivalent strain rate is

$$\dot{\varepsilon} = \frac{v}{L} \quad (5.11)$$

where v is the impact velocity, L is the length or height of the rock sample.

Fig. 5.4 Loading description and applied velocity waveform

It should be mentioned that the applied impact load in the present study is simplified from the SHPB impact test. A rectangular velocity pulse is applied at the top of the specimen and the strain rate effect can be reflected conveniently based on the magnitude of the velocity.

The numerical model of the rock sample is composed by 10000 SPH particles. Each of these particles has the same diameter of 0.5 mm. They are arranged in the rectangular way. Fig. 5.5 shows the arrangement of the particles with the zoomed rectangle allowing for better recognition of the arrangement.

Fig. 5.5 Schematic of the SPH particles arrangement

In the present study, numerical simulations by considering material heterogeneity are carried out to study the progressive failure process visually. The mean value of the material properties of Bukit Timah granite is presented in Table 5.1. Different heterogeneity indices are introduced in the simulations to investigate the heterogeneity effect. The material properties such as the material strength σ and elastic modulus E are randomly distributed throughout the specimen by following a Weibull distribution function (see Eq. (3.1)). The heterogeneity index m is of importance to the assignment of the material properties. A larger value of m implies a more homogeneous material. The heterogeneity index m is used to examine the heterogeneity effect on the failure behavior of rocks under dynamic compressive loadings.

Table 5.1 Properties of Bukit Timah granite

Mean compressive strength (MPa)	147.5
Mean tensile strength (MPa)	16.1
Density(Kg/m ³)	2670
Poisson ratio, ν	0.2
Mean elastic modulus, E (GPa)	67.2
Heterogeneity index m	∞ , 50, 20, 10, 5, 3, 2

Fig. 5.6 presents some 2D numerical specimens, which are all composed of 10000 particles, produced randomly according to the Weibull distributions with different heterogeneity indices. In Fig. 5.6, the different degree of gray color corresponds to a different magnitude of Young's modulus of the elements with the zoomed rectangle allowing for better recognition of the arrangement. The mean value of the Young's modulus is 67.2 GPa.

(a) $m=1.1$ (The values range from 0.01 to 506)

(b) $m=2$ (The values range from 0.5 to 194)

(c) $m=3$ (The values range from 2.6 to 144)

(d) $m=5$ (The values range from 11.3 to 105)

(e) $m=50$ (The values range from 56.6 to 70.4)

Fig. 5.6 Numerical specimens with different heterogeneity index m . (Unit: GPa)

A series of numerical simulations have been carried out to investigate the failure behaviors of the rock specimens under dynamic compressive loadings. The failure behaviors of rocks include many aspects, such as fracture process, failure pattern, volumetric deformation and strain rate effect which helps for us to understand the fracture mechanism of rocks under dynamic compressive loadings. Five groups of specimens were numerically simulated, they are

- 1) One homogeneous specimen used to study the dynamic compressive fracture process, failure pattern, volumetric deformation and strain rate effect.
- 2) One specimen with the heterogeneity index, $m=10$, to study the dynamic compressive fracture process, failure pattern, volumetric deformation and strain rate effect.
- 3) Three specimens with the same heterogeneity index ($m=10$ and 50, respectively) to study the effect of the spatial distribution of material properties.
- 4) Five specimens with different heterogeneity indices, i.e., $m=\infty$, 50, 20, 10, 5, 2, representing materials from homogeneous to very heterogeneous to study the heterogeneity effect to the failure pattern and stress strain curves.

It should be mentioned that the damaged particles are denoted with different degrees of dark colors. The damage parameter D of the J-H constitutive model (see Chapter 2) varies between 0 and 1 with the color varies from light grey to black, which is of importance to obtain the progressive fracture process in the visible manner.

5.4 Dynamic compressive failure behavior of homogeneous granite

Rock-like materials can be divided into different categories based on the heterogeneity index m . In this section, a homogeneous rock specimen is used to simulate its dynamic failure behavior which serves a baseline of the dynamic compressive failure of the heterogeneous specimens. Numerical simulations are conducted on one homogeneous rock specimen to study the failure behaviors of rock including the fracture process, stress strain curve and volumetric deformation under the impact velocity 2.5 m/s with the strain rate at 50 s^{-1} . The validation of the numerical tests is verified by the discussion of stress uniformity.

5.4.1 Fracture process of the rock specimen

Fig. 5.7 shows the fractures initiation and propagation of homogeneous rock specimen under the impact loading velocity 2.5 m/s. After the velocity boundary acts on the top end, the stress wave propagates toward the bottom end and then reflects in the specimen. It can be seen that the failure is first initiated by two side cracks from the impacted end. The fractures propagate downward which satisfies the Griffith fracture theory. It is interesting that one crack initiates at the middle of the specimen when $t = 12\mu\text{s}$. The median crack propagates in both directions along the vertical line. The median crack reaches the bottom of the specimen when $t = 18\mu\text{s}$. With the increase of the vertical displacement, the three major cracks reach the bottom of the specimen and the specimen is thoroughly split when $t = 40\mu\text{s}$.

The splitting mode agrees with the SHPB experiments results. It seems that the middle part of the specimen is influenced by the external loading because of stress concentration. The initiation of cracks induces the development of tensile failure in a splitting mode.

Fig. 5.7 Failure process of rock specimen under $v=2.5\text{m/s}$ ($m=\infty$)

Fig. 5.8 shows the stress-strain curve of the homogeneous rock correspondingly. The stress is the average stress of all particles. And the strain is calculated by integrating Eq. (5.11). The overall stress and strain definition in the stress-strain curves is same for all the simulations. It reveals that the stress-strain relation of the rock specimen can be modeled well by the modified version of Autodyn. As the axial strain increases, the stress-strain curve is nearly linear at the initial stage until the peak stress is reached at $t = 40\mu\text{s}$. The specimen is split at around $t = 40\mu\text{s}$ when the axial strain reaches 2 ‰. The Young's modulus obtained from the linear portion of the stress-strain curve is about 80 GPa, which is a bit larger than the mean value of the Young's modulus of 67.2 GPa assigned to the particles. The peak stress of 160 MPa is also a little larger than the average compressive strength of the particles which is 147.5 MPa. After $t = 40\mu\text{s}$, the stress drops gradually to nearly zero.

It is seen that the post-failure of the specimen is clearly shown in the numerically simulated stress-strain curve. It is sometimes difficult to be obtained in a physical compressive test for rock. After the peak stress is reached, the stress drops very fast which reflects that the material is very brittle.

Fig. 5.8 Stress-strain curve of homogeneous granite specimen

5.4.2 Stress uniformity-Validation of the numerical test

One important factor about the validity of the numerical results is the uniformity of stress and strain inside the specimen along the loading direction. Propagation of the stress wave inside the specimen takes a certain amount of time. The stresses within the specimen are initially not uniform, with one end being loaded and the other end being free of stress. Propagation of the stress wave from one end to the other and the subsequent multiple reflections make the stress distribution becoming uniform gradually.

Fig. 5.9 compares with the stress histories of the specimen, at the top end and the bottom end under the loading velocity of 2.5 m/s. It can be seen that the stress at the top of the specimen occur instantly as soon as the specimen is loaded, while the stress at the bottom of the specimen is generated at about $t = 6\mu s$. The overall average stress of the specimen, however, increases linearly from the starting time. The stresses at both the top and the bottom vibrate around the overall average stress. The difference between the two average stresses at the bottom and the top ends is not considerable so that the stresses in the specimen can be regarded as uniform. The overall average stress-strain curve, on the other hand, can give a good representation of the dynamic behavior of tested specimen.

Fig. 5.9 Stress histories at different cross sections of the specimen under $v=2.5\text{m/s}$

5.4.3 Volumetric strain

Volumetric expansion or dilation is an important factor for rocks after failure. The volumetric strain is defined as

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad (5.7)$$

In plane strain condition, $\varepsilon_z = 0$, consequently,

$$\varepsilon_v = \varepsilon_x + \varepsilon_y \quad (5.8)$$

Fig. 5.10 shows the strain components ε_{xx} , ε_{yy} and the volumetric strain ε_v of the homogeneous granite specimen, where ε_{yy} is in the loading direction, ε_{xx} the lateral direction. The volumetric strain (positive in compression) increases almost linearly with the increase of the stress in the elastic state. With the increase of vertical deformation of the specimen, the volumetric strain starts to decrease. After the stress reaches the peak stress, the volumetric strain further decreases to negative, which implies the occurrence of the volumetric expansion. From the numerical results, it is clear that the volumetric expansion or dilation of the material can be simulated by the SPH method which is identical with the acknowledged test result (Cai, et al., 2002).

Fig. 5.10 Volumetric deformation of homogeneous granite

5.5 Dynamic failure behavior of heterogeneous rock (m=10)

One rock specimen with the heterogeneity index $m=10$, for example is used to study the failure behavior of heterogeneous rock specimen under the loading velocity of 2.5 m/s with the strain rate of 50 s^{-1} . The fracture process, stress-strain curve and the volumetric deformation are well studied. Moreover, the numerical test is validated by studying the stress uniformity in the specimen.

5.5.1 Fracture process of the rock specimen

Fig. 5.11 presents the failure process of rock specimen with the heterogeneity index $m=10$ under impact velocity of 2.5 m/s. When the velocity boundary is applied onto the specimen, scattered damaged particles distributed at the impact end of the specimen when $t = 6.5\mu\text{s}$. With the increase of time, damage propagates to the fixed end of the specimen roughly along the vertical direction. It means that the coalescence of the major cracks is parallel to the loading applied. At $t = 11\mu\text{s}$, cracks begin to be developed at the bottom of the specimen. At $t = 18\mu\text{s}$, the peak stress of the specimen is reached and the specimen is damaged in a splitting mode.

Fig. 5.12 presents the stress-strain curve of the heterogeneous granite specimen. The

Young's modulus obtained from the linear part of the stress-strain curve is about 50 GPa, which is 25% smaller than the mean value of 67.2 GPa of the particles. In addition, the peak stress is only 50 MPa, which is also much smaller than the mean value of 147.5 MPa for the particles. The overall stress-strain curve of the specimen showed a certain degree of strain-softening. The unstable failure associated with large stress drop is well represented in the simulated curve. The residual strength of the specimen exists but relatively very small.

Fig. 5.11 Failure process of the rock specimen under $v= 2.5\text{m/s}$ with $m=10$

Fig. 5.12 Stress-strain curve of heterogeneous rock with $m=10$

5.5.2 Stress uniformity-validation of the numerical test

Fig. 5.13 compares the stress histories at the top and bottom ends of the heterogeneous specimen ($m=10$) under loading velocity of 2.5m/s.

Fig. 5.13 Stress histories of the heterogeneous specimen under loading $v=2.5$ m/s ($m=10$)

It can be seen that, similar to the homogeneous rock, the stress at the top of the specimen occur instantly after the loading is applied. The stress at the bottom of the specimen is generated at about $t = 6\mu s$. The average stress of the specimen increases linearly at the initial time. After $t = 7\mu s$, both the stresses at the top and bottom vibrate around the average stress.

The numerical results for the heterogeneous rock specimen show that the heterogeneity index has significant effect on the peak stress, the Young's modulus, failure pattern of the specimen as well as the strain-softening behavior; it has a little influence on the stress wave propagation and reflection in the specimen and the non-uniformity of the stress.

5.5.3 Volumetric strain

The stress-volumetric strain curve of the specimen is plotted in Fig. 5.14 together against ε_{xx} and ε_{yy} . It is found that the volumetric expansion for the specimen with $m=10$ becomes more significant. It is reasonable because much more cracks occurred in the

heterogeneous specimen comparing with the homogeneous one.

Fig. 5.14 Volumetric deformation of heterogeneous granite under $v=2.5\text{m/s}$ ($m=10$)

5.6 Effect of spatial distribution of material properties

With the same Weibull distribution law of the material properties, the spatial distribution of particles can be stochastically different. In order to investigate the randomness of the particles, three specimens whose material properties are specified randomly according to the same Weibull distribution are studied. The loading velocity is 2.5 m/s with the strain rate at 50 s^{-1} .

Fig. 5.15 presents the stress-strain curves for three different specimens with same heterogeneity index ($m=10$ and 50 , respectively in Fig. 5.15 (a) and Fig. 5.15 (b)). The numerical results show that the overall Young's modulus and peak stress are very close for the three samples. The stress-strain curves are also very close to each other. It indicates that although the spatial distribution of the particles may be different, it has minor effect to the overall stress-strain curve of the specimens providing that the heterogeneity index of the specimens is the same. In other words, a specimen with certain heterogeneity index can give a very good representation of the overall stress-strain curves.

(a) $m=10$

(b) $m=50$

Fig. 5.15 Stress-strain curves of three specimens with same homogeneity index

Fig. 5.16 compares the failure patterns of the specimens with same heterogeneity index. Owing to the randomness of the spatial distribution of material properties, the crack distribution of different specimens is different. However, the failure patterns all appear very similar. The specimens all undergo splitting failure along the loading direction. The crack density of the specimen with a heterogeneity index of $m=10$ is much higher than that with $m=50$. The results are reasonable because more inherent weak particles exist in the more heterogeneous specimen, which results in lower modulus and peak stress, and

denser cracks of the specimen. The difference actually reflects the influence of spatially stochastic distribution of particles. The spatial arrangement of particles with different mechanical properties greatly influences which element attains its strength at first, leading to different crack distributions. The specimens all started with tensile failure of discrete particles, and the damaged particles coalesce with others, until, eventually, the macroscopic failure band is formed. For specimens with high homogeneity index, the macroscopic failure bands are more concentrated compared to the specimens with low homogeneity index.

Fig. 5.16 Failure patterns of specimens with same heterogeneity index

5.7 Heterogeneity effect on failure behavior of rocks

A parameter study is conducted to evaluate the heterogeneity effect on the fracture processes and strength characteristics of rock-like materials under dynamic uniaxial compressive load. Based on experimental observations, material constitutive models can be constructed. The constitutive models can be either analytical or numerical. The analytical models lead to correct answers without necessarily considering the real reality including the microstructure of the material. Analytical models have to be simplified and sometimes this simplification ignores important factors influencing the material behavior. Heterogeneity is such an example for rocks. From this point of view, numerical tools

incorporating a heterogeneity capability can provide a better simulation of the mechanical behavior.

In laboratory experiments for rock, two failure modes are typically observed. In dynamic uniaxial compression conditions, one or more individual microfractures expand into one or more large cleavage fractures (extension or splitting mode). On the other hand, the confinement conditions in triaxial tests will inhibit the expansion of individual microfractures and lead to the coalescence of a large number of microfractures into a shear fracture zone at higher loading.

5.7.1 Heterogeneity effect on the failure process

Different heterogeneity indices are used in the simulations to study the heterogeneity effect to the failure patterns and stress-strain properties of the rock specimens. Figs. 5.17-5.22 show the failure processes and stress-strain relations for rock specimens with the heterogeneity index m of 50, 5 and 2, respectively. The results of $m=10$ have been shown in Figs. 5.11 and 5.12. A larger m implies a more homogeneous material. It is shown that splitting modes occurred in the failure process of all the specimens. Theoretically, brittle fracture must be accomplished by subaxial (parallel to loading direction) fracturing at the micro-, as well at the macro-level. It is known as axial splitting and cleavage at the microscopic and the macroscopic, respectively. The development of the fractures from axial cleavage to splitting is numerically vividly shown in the failure processes of specimen.

In a homogeneous material, failure will always initiate from the high stress site. However, in a heterogeneous material, failure may start at a point where the stresses are not highest, but the local strength is much lower due to pore microfractures, grain boundaries, etc., this is the reason why Fairhurst (1998) introduced the notion ‘stress severity’. The severity represents the ration of the theoretical stress at the failure moment to the stress that would theoretically be necessary for a failure in a given point. Heterogeneity is the main reason for failure occurring in the location where the stresses are not necessarily the highest, this problem can only be handled by a numerical method in which the heterogeneity of material can be considered.

Figs. 5.17 and 5.18 shows respectively the fracture process and the stress-strain curve of the rock specimen when $m=50$. The failure mode is similar to that of homogeneous material even if the heterogeneity effect is considered. The specimen shows that its brittleness is still relatively high. From Fig. 5.18, it can be seen that the peak stress is obtained at $t = 27 \mu s$ when the specimen is damaged in the splitting mode.

Fig. 5.17 Failure process of the rock specimen under $v=2.5\text{m/s}$ ($m=50$)

Fig. 5.18 Stress-strain curve of the rock specimen under $v= 2.5 \text{ m/s}$ ($m=50$)

Figs. 5.19 and 5.20 presents respectively the failure process and the stress-strain curve of the rock specimen with $m=5$. Denser cracks are experienced in the specimen. A fault

mode in splitting failure is observed in the specimen. At $t = 10.4 \mu s$, the specimen reaches the peak stress while the specimen can be considered as complete failure of the specimen at $t = 15 \mu s$.

Fig. 5.19 Failure process of the rock specimen under $v=2.5$ m/s ($m=5$)

Fig. 5.20 Stress-strain curve of the rock specimen under $v=2.5$ m/s ($m=5$)

The failure process and stress-strain curve of the rock specimen with a heterogeneity index of $m=2$ are given in Figs. 5.21 and 5.22 respectively. More microcracks occurred after the boundary velocity is applied. The cracks distribute over the specimen. At $t = 9 \mu s$, the peak strength is obtained. At $t = 9.5 \mu s$, the cracks reach the bottom end of

the specimen. At $t = 13\mu s$, the specimen is split into many pieces.

Fig. 5.21 Failure process of the rock specimen under $v=2.5$ m/s ($m=2$)

Fig. 5.22 Stress-strain curve of rock specimen under $v=2.5$ m/s ($m=2$)

5.7.2 Heterogeneity effect on the failure patterns

Fig. 5.23 compares the failure patterns of specimens with different heterogeneity indices, i.e., $m=\infty$, 50, 20, 10, 5 and 2, respectively. The failure patterns vary significantly with the increase of the heterogeneity index. Axial splitting and faulting are the major failure patterns in dynamic uniaxial compression. The more heterogeneous specimen shows a

more random distribution of the microfractures and a rougher fracture surface than a relatively homogeneous specimen. It is suggested that the variation in microfracture distribution or fracture surface roughness may serve as an indicator of the degree of the rock brittleness and the heterogeneity of the specimen.

Fig. 5.23 Failure patterns of specimens with different heterogeneity index

5.7.3 Complete stress-strain curves of rocks with different heterogeneity indices

Fig. 5.24 Stress-strain curves of specimens with different heterogeneity index

In order to compare the heterogeneity effect to the dynamic behavior of rocks, Fig. 5.24 plots the stress-strain curves of the specimens with different heterogeneity indices. It is clear that the stress-strain relation and the strength characteristics depend strongly on the heterogeneity of the specimen. It can be seen that, under dynamic uniaxial compression, the shape of heterogeneous rock has a gentler post-peak behavior. The peak stress of the specimens is also related to the homogeneity index. The higher the heterogeneity index, the higher the peak stress of the specimen. The post-peak behaviors of the specimens are also different. The homogeneous rock can be regarded as completely brittle and almost no strain-softening occurs. While the heterogeneous rock ($m=2$) shows dramatic strain-softening phenomenon and the residual strength is relatively high.

5.8 Strain rate effect on homogeneous specimens ($m=\infty$)

The strain rate effect on the strength of various rock-like material, e.g. rocks, concrete and mortar, has become an important factor in both the material model and the design of structures that may experience high strain-rate load in a range of applications when impact or blast loading is involved. It is generally accepted that there is an apparent increase of the dynamic strength when the rock-like material is subjected to high strain rate load. The dynamic increase factor (**DIF**), defined by the ratio of the dynamic strength to the quasi-static strength in a uniaxial tensile or compressive condition, has been widely accepted as an important parameter to measure the strain rate effect on the strength of rock-like materials. It is interesting to know whether the test results at high strain rates purely reflect the strain rate dependence of the material itself, or the dynamic effect due to the test set-up and the method of the measurement have great influence on the strain rate sensitivity of the rock-like materials.

In this section, the J-H model which has been described in Chapter 2 is used in the numerical simulations in order to examine the strain rate effect phenomenon. However, the rate effect in the model is not involved for the particles. It means that it is assumed, at the particle level, the strength of the material is strain rate insensitive. The selected loading velocities include 1.25, 2.5, 5 and 10 m/s with the corresponding strain rate at 25, 50, 100 and 200 s^{-1} , respectively.

5.8.1 Failure behavior of rock specimen under $v=1.25$ m/s

Figs. 5.25 and 5.26 show respectively the progressive failure process and the corresponding stress-strain curve of a homogeneous rock specimen under the loading velocity of 1.25 m/s with the strain rate at 25 s^{-1} .

Fig. 5.25 Failure process of homogeneous rock specimen under $v=1.25$ m/s ($m=\infty$)

Fig. 5.26 Stress-strain curve of homogeneous rock specimen under $v=1.25$ m/s
($m=\infty$)

It is interesting that the major crack initiates at the median and close to the top end of the

specimen at $t = 15\mu s$ with the same phenomenon found in the paper of Repetto, et al. (2000). The crack subsequently propagates in both directions along the vertical line. At $t = 37\mu s$, the crack reaches the top end of the specimen. With the further increase of time, the specimen is split into two halves at $t = 60\mu s$. Another interesting phenomenon appears when the average overall stress in the specimen is 147.5 MPa, which is exactly the static strength of the granite specimen. In the physical experiments, the specimen cannot bear load any more as soon as it is split. However, the strength of the specimen can still increase with the increase of time in the numerical simulation. It can be seen from Fig. 5.26 that the stress increases linearly with the increase of time until $t = 113\mu s$. At $t = 113\mu s$, the strength in the specimen increases to the peak value, i.e., 269.6 MPa. The similar failure pattern can be found in the literatures of Liu, et al. (2005).

5.8.2 Failure behavior of the rock specimen under $v=5$ m/s ($m=\infty$)

Fig. 5.27 and Fig. 5.28 show respectively the failure process and the corresponding stress-strain curve of rock specimen under a load velocity of 5 m/s with the strain rate of 100 s^{-1} . The propagation velocity of the major cracks increases with the increases of the loading velocity. The failure pattern of the specimen can still be regarded as splitting mode although a small angle is oriented for major cracks with respect to the vertical direction when the loading rate is relatively high. The specimen reaches its peak stress at $t = 20\mu s$.

Fig. 5.27 Failure process of homogeneous rock specimen under $v=5$ m/s ($m=\infty$)

Fig. 5.28 Stress-strain curve of homogeneous rock specimen under $v=5$ m/s ($m=\infty$)

5.8.3 Failure behavior of homogeneous rock specimen under $v=10$ m/s

The corresponding failure process and the stress-strain curve are shown in Figs. 5.29 and 5.30, respectively. A stress wave is generated and propagates in the specimen after the loading is applied. At $t = 7\mu s$, a pair of symmetric cracks initiates from the top of the specimen and propagate to the bottom. At $t = 9.9\mu s$, the peak stress of the specimen is achieved whereas the specimen is not completely damaged. At $t = 14.5\mu s$, the specimen fails because of splitting.

Fig. 5.29 Failure process of homogeneous rock specimen under $v=10$ m/s ($m=\infty$)

Fig. 5.30 Stress-strain curve of homogeneous rock specimen under $v = 10 \text{ m/s}$ ($m = \infty$)

5.8.4 Strain rate effect on failure behavior of homogeneous specimen

Fig. 5.31 presents the stress-strain curves of homogeneous specimens under different strain rates. Failure of the specimens is mainly in a splitting mode. The specimen is split into two pieces by the crack at the axis at a low strain rate, e.g. 25 s^{-1} . The specimen is split into more pieces when the strain rate becomes higher. For example, four pieces are generated at the strain rate 50 s^{-1} . The central cracks mainly propagate downward which generates more cracks at the bottom of the specimen due to wave reflection. At a very high strain rate 200 s^{-1} , two major side cracks are produced with shear cracks inside.

Fig. 5.31 Failure patterns of four homogeneous specimens under different strain rates

Fig. 5.32 compares the stress-strain curves for the specimens under different strain rates. The phenomenal strength at 25 s^{-1} is the highest in the numerical simulation which may not occur in the real experiments. The peak strength decreases with the increase of the

strain rate at both 50 s^{-1} and 100 s^{-1} . At the strain rate, 200 s^{-1} , the strength of rock becomes higher than that at 100 s^{-1} .

Fig. 5.32 Stress-strain curves of homogeneous rock specimen under different strain rates

It seems that the numerical results for the homogeneous rock do not agree with the general understandings of the strain rate effect although they can reflect the failure patterns of brittle specimens. However, it is worth noting that the numerical simulations are able to simulate the entire stress-strain curve for different failure patterns. The physical experiments usually measure the material response until the first major failure occurs. Furthermore, the above numerical simulations considered homogeneous material only, the heterogeneity effect has not been included in the numerical models yet.

5.9 Strain rate effect on heterogeneous specimens with $m=10$

Rock-like materials are always heterogeneous from the microstructure point of view. For example, granite is regarded as relatively homogeneous material while concrete is relatively more heterogeneous. This section is to investigate the strain rate effect on the compressive failure behavior of rock-like specimens with the heterogeneity index of $m=10$.

Many rock and concrete structures may be submitted to high rate dynamic loadings. So it is necessary to know the behavior of this material in order to predict the response of the

structure. Under dynamic loading, an increase of apparent compressive strength is observed. This may be due to a rheological effect or to a structural effect. In the numerical simulation, J-H model is used but not involves the strain rate effect which means the material of the particle scale is insensitive to the strain rate. The J-H model consists of an elasto-plastic strength model, a damage model and a polynomial equation of state, which is pressure-dependent but not strain-rate sensitive. So the numerical model only takes into account the main features of the behavior, the damage and the softening. If DIF still exist, the structural effect as an inertial confinement will be responsible.

5.9.1 Failure process of heterogeneous rock specimen under different loading rates

Failure processes for rock specimens under some selected loading rates are presented in the following. They are divided into five categories based on the specimen failure,

- (I) 0.5 m/s
- (II) 1 m/s
- (III) 2.5 m/s and 5 m/s
- (IV) 10 m/s and 15 m/s
- (V) 25m/s and 50m/s

Fig. 5.33 Failure process of rock specimen under $v=0.5\text{m/s}$ ($m=10$)

Fig. 5.33 shows the failure process of the specimen under a loading velocity of 0.5 m/s. At $t=67\ \mu\text{s}$, a few particles are damaged. With the increase of time, more scattered

cracks are developed. It is worth noting that most of the cracks are caused by tensile failure and propagate along the vertical direction. The tensile failures can be divided into two types: (I) splitting mode locating around the vertical symmetric axis and (II) spalling failure occurring at the sides of the specimen. With the nucleation of the cracks, a shearing fault is formed by the linkage of the scattered tensile cracks at $t=106\ \mu\text{s}$.

Fig. 5.34 presents the corresponding stress-strain curve of the heterogeneous rock specimen under $v=0.5\text{m/s}$. The stress increases linearly with the increase of the displacement until it reaches the peak value. Then it drops relatively fast with the continuous increases of the displacement. It should be mentioned that the peak stress occurs $t=82\ \mu\text{s}$, the corresponding microcracks can be found in Fig. 5.33. After that, the stress decreases with the increase of the displacement because of the linkage of the damaged particles. When $t=106\ \mu\text{s}$, the rock specimen is completely damaged with the formation of the shearing band.

Fig. 5.34 Stress-strain curve of rock specimen under $v=0.5\text{ m/s}$ ($m=10$)

Fig. 5.35 present respectively the fracture process of a heterogeneous rock specimen with the heterogeneity index of $m=10$. Cracks initiate near the top end of the specimen at $t=9\ \mu\text{s}$ and propagate downwards to the bottom end of the rock specimen. At $t=10\ \mu\text{s}$, two major splitting cracks starting from the top end of the specimen are formed. The splitting cracks propagate subsequently downwards parallel to the vertical direction.

Finally at $t=44\ \mu\text{s}$ one tensile splitting fault controls the damage of the specimen while the other splitting is not connected. The stress strain curve of the specimen under $v=1\ \text{m/s}$ with $m=10$ is shown in Fig. 5.36. The stress increases linearly with the increase of the strain and then drops gradually to the residual strength. It should also be mentioned that the peak stress occurs at $t=44\ \mu\text{s}$.

Fig. 5.35 Failure process of rock specimen under $v=1\ \text{m/s}$ ($m=10$)

Fig. 5.36 Stress-strain curve of rock specimen under $v=1\ \text{m/s}$ ($m=10$)

With the increase of the strain rate, another category of the fracture process occurs. The fracture processes under a loading velocity of $2.5\ \text{m/s}$ and $5\ \text{m/s}$ are quite similar. Figs.

5.37 and 5.39 show the fracture process with the loading velocity of 2.5 and 5 m/s, respectively. Cracks initiate from the top end of the specimen as soon as the velocity boundary is applied. Two symmetric fracture bands are formed and then propagate to the bottom end of the sample. The two fracture bands finally connect the two ends of the specimen and form a splitting tensile failure mode.

Although the failure patterns under 2.5 m/s and 5 m/s are similar, they are also influenced by the loading rate. At the loading velocity of 2.5 m/s, the rock specimen is more brittle than that of 5 m/s. The fracture band is wider under higher strain rate load which causes the denser cracks inside the specimen.

Fig. 5.37 Failure process of rock specimen under $v=2.5$ m/s ($m=10$)

Fig. 5.38 Stress-strain curve of rock specimen under $v=2.5$ m/s ($m=10$)

Figs. 5.38 and 5.40 show the stress-strain curves of the heterogeneous rock specimen under the impact velocity of 2.5 m/s and 5 m/s, respectively. The stress increases with the increase of the displacement although the curve is not exactly linear. The stress reaches its peak when the specimen is at $t=18\ \mu\text{s}$ under 2.5m/s and $t=15\ \mu\text{s}$ under 5 m/s, respectively.

Fig. 5.39 Failure process of rock specimen under $v=5\ \text{m/s}$ ($m=10$)

Fig. 5.40 Stress-strain curve of rock specimen under $v=5\ \text{m/s}$ ($m=10$)

The failure process and failure patterns under 10 m/s and 15 m/s with the strain rates at $200\ \text{s}^{-1}$ and $300\ \text{s}^{-1}$ respectively belong to another category. Figs. 5.41 and 5.42 present the fracture process and the stress-strain curve for the specimen under the loading velocity

10 m/s, while Figs. 5.43 and 5.44 for the specimen under 15 m/s. Cracks initiate from the top end and propagate downwards to the bottom end. However, the specimen seems to be completely crushed with two major fracture zones along the loading direction. The peak stress occurs at the instant $t=6\ \mu\text{s}$ under the loading velocity of 10 m/s, and at $t=6\ \mu\text{s}$ for the loading velocity of 15 m/s when the specimen has not been splitted yet. The phenomenon is caused by strain rate effect which is worth being discussed to explore the insight of the strain rate effect on the material strength. Moreover, higher strain rate ($300\ \text{s}^{-1}$) results in more severe damage or much denser cracks in the specimen.

Fig. 5.41 Failure process of rock specimen under $v=10\ \text{m/s}$ ($m=10$)

Fig. 5.42 Stress-strain curve of rock specimen under $v=10\ \text{m/s}$ ($m=10$)

Fig. 5.43 Failure process of rock specimen under $v=15$ m/s ($m=10$)

Fig. 5.44 Stress-strain curve of rock specimen under $v=15$ m/s ($m=10$)

When the loading rate reaches a very high value, the damage of the rock-like material is completely different with that under low strain rate. Figs. 5.45 and 5.46 shows the failure process and the stress strain curve under the loading velocity 25 m/s with a strain rate of 500 s^{-1} , and Figs. 5.47 and 5.48 for the loading velocity 50m/s with a strain rate of 1000 s^{-1} , respectively.

Cracks are generated once the dynamic loading is applied. Shear bands are formed in

about 45° directions with respect to the loading direction. In the middle of the specimen no damage is produced at the initial stage. The damage propagates downwards to the bottom end of the specimen. Finally the rock specimen is completely damaged by crushing. Under high strain rate loadings, the rock specimen does not seem classically brittle. Moreover, the peak stress occurs before rock failure under these very high strain rate loads. The phenomenon is probably caused by the dynamic stress propagation in the rock specimen which is of importance to explore the strain rate effect of rock-like materials.

Fig. 5.45 Failure process of rock specimen under $v= 25$ m/s ($m=10$)

Fig. 5.46 Stress-strain curve of rock specimen under $v=25$ m/s ($m=10$)

Fig. 5.47 Failure process of rock specimen under $v=50$ m/s ($m=10$)

Fig. 5.48 Stress-strain curve of rock specimen under $v=50$ m/s ($m=10$)

5.9.2 Strain rate effect on the dynamic compressive behavior of heterogeneous rock specimen

From Figs. 5.34, 5.36, 5.38 and 5.40, it can be seen that the strain rate slightly influences the overall compressive strength of the heterogeneous rock specimen under 0.5, 1, 2.5 and 5 m/s. The conclusion can be drawn that the failure of the specimens is in a quasi-static pattern when it is subjected to impact loads with a strain rate below 100 s^{-1} . The effect of the strain rate on the overall strength of the specimen is minor. However, the failure strain increases with the increase of the strain rate in this range which makes

the rock specimen a bit ductile under higher strain rates.

Fig. 5.49 shows the strain rate effect on the failure behavior of the rock specimen with both the height and width being 5 cm under different strain rates 100, 200, 300, 400, 500 and 1000 s^{-1} . It can be seen that both the strength and failure strain of the heterogeneous rock specimen increase with increase of the strain rate. The rock specimen behaves a little bit ductile under the strain rates in this range.

Fig. 5.49 Strain rate effect to the heterogeneous rock specimen (5cmx5cm)

Fig. 5.50 presents the DIF of the heterogeneous rock specimen under dynamic compressive loading. The peak stress of the specimen under the strain rate of 1 s^{-1} is taken as the quasi-static compressive strength. It can be seen that the strength of the rock specimen almost keeps constant around the static strength value under the strain rate below 100 s^{-1} . Then DIF increases fast with the increase of the strain rate. DIF increases to about 6 when the strain rate is 1000 s^{-1} . Le Nard and Bailly (2000) carried out dynamic compressive tests to investigate the dynamic increase factor of concrete using SHPB as well as numerical simulations. The experimental results and simulations show that the structural effect as an inertial confinement responsible for the apparent increase in strength. Our simulation results are identical with that of Le Nard and Bailly (2000) which also presents an apparent increase of dynamic compressive strength. Because the constitutive model for the numerical simulations-J-H model which is an elasto-plastic

model with a damage law does not consider the strain rate effect into the material which is to say that the model is independent of the strain rate. J-H model only takes into account the main features of the behavior. The DIF is also caused by the structural effect as the inertia confinement.

Fig. 5.50 DIF of rock specimen with $m=10$

From the above discussion, the structural effect as inertial confinement is responsible for the strain rate effect of the rock specimen. Le Nard and Bailly (2000) pointed out that it is not necessary to introduce rate sensitivity in a constitutive model in order to take into account the experiment compressive strength increase. However, further study is still in need to compare our numerical results to the experiments results to prove that.

Chapter 6 Rock dynamic tensile behavior with heterogeneity effect

6.1 Introduction

For rock and rock-like brittle materials, the tensile strength is much lower than the compressive strength. It will greatly affect the failure of materials so that tensile stresses are usually avoided in the service life of the constructed structures.

A good understanding of the dynamic tensile failure behaviors is of great importance in many areas from rock dynamics to protective structure design and mining engineering. It is essential to understand how fractures initiate and propagate under high strain rate loadings. Moreover, the tensile strength of rock and rock-like materials is regarded as strain-rate sensitive. Cho, et al. (2003) performed dynamic tension tests on rocks to investigate the strain-rate dependency of the dynamic tensile strength of rock. Brara and Klepaczko (2005) carried out tensile tests on concrete specimens at different strain rates which clearly show the strain rate effect to the dynamic tensile strength.

Tensile tests for brittle materials are more difficult to perform comparing to compressive tests. Tensile tests are generally differentiated into two types, i.e., direct tension and indirect tension. The direct tensile test is difficult to be implemented because of the complexity of the connection between the specimen and the fixtures. The indirect tensile test, usually referred to as the Brazilian split test has become the most widely used test method to determine the tensile strength for rock and rock-like materials. It is relatively simple and stable for the test results.

Numerical simulations on the dynamic Brazilian test have been carried out by Zhu and Tang (2006) to investigate the fracturing mechanism and process of the rock split failure. Material heterogeneity has been considered in their studies. However, the simulations adopt continuum meshes and the material failure is characterized by erosion of the elements which is rather arbitrary in determination of the parameters. In this chapter, the smoothed particle hydrodynamics (SPH) is again applied to simulate the splitting failure

of rock samples. Material heterogeneity is also considered by employing Weibull distributions of the mechanical properties for the distributed particles. Strain rate and heterogeneity effects for the tensile splitting cases are discussed.

6.2 Brazilian tensile test

6.2.1 The conventional Brazilian tensile test

Brazilian test is the most widely used method to determine the tensile strength for rock and rock-like materials. A schematic plot of the Brazilian test is shown in Fig. 6.1 where the load is illustrated with bold arrows. The compressive load produces the tensile stress at the centre of the specimen. When the stress reaches the tensile limit of Brazilian specimen, the disc will be split into two halves. To prevent the failure due to stress concentrations in the specimen at the two loading points, curved anvils are usually used in the experiments. Since the test is mainly used to determine the static tensile strength, the recommended loading rate for the Brazilian tensile is limited within 0.1~1 MPa/s.

Fig. 6.1 Schematic plot of the Brazilian test, load illustrated with bold arrows

The Hertzian solution (Hertz, 1883) serves as a theoretical basis of Brazilian tensile test which supposes a force P is applied on the strip with a finite width $m = 2a$ (Fig. 6.2). Consequently, Hondros (1959) reached a complete analytical solution for the width of the loading strip $m = 2a$ again for an elastic state. Along the vertical diameter, the distributions of the two normal stresses are

$$\sigma_x = \sigma_\theta = -\frac{2p}{\pi} \left[\frac{(1 - r^2/R^2) \sin 2\alpha}{1 - (2r^2/R^2) \cos 2\alpha + r^4/R^4} - \tan^{-1} \frac{1 + r^2/R^2}{1 - r^2/R^2} \tan \alpha \right] \quad (6.1)$$

$$\sigma_y = \sigma_r = -\frac{2p}{\pi} \left[\frac{(1 - r^2/R^2) \sin 2\alpha}{1 - (2r^2/R^2) \cos 2\alpha + r^4/R^4} + \tan^{-1} \frac{1 + r^2/R^2}{1 - r^2/R^2} \tan \alpha \right] \quad (6.2)$$

where $p = P/2aL$ (L is the thickness of the disc), r is the current radius of the polar coordinate, R is the radius of the Brazilian disc.

At the disc center

$$\sigma_x = \sigma_\theta = -\frac{2p}{\pi} (\sin 2\alpha - \alpha) \approx -\frac{2p\alpha}{\pi} = -\frac{2P}{\pi DL} \quad (6.3)$$

$$\sigma_y = \sigma_r = \frac{6P}{\pi DL} \quad (6.4)$$

since $p\alpha = P/DL$. Here, D is the diameter of the disc.

Fig. 6.2 Quantities participating in solutions of Hondros

When L is commensurable with D , the specimen stress state approaches the plane strain. The stress component σ_z exists,

$$\sigma_z = \nu(\sigma_x + \sigma_y) \quad (6.5)$$

and, consequently,

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \quad (6.6)$$

Thus, from Eqs. (6.3), (6.4), (6.5) and (6.6) we can obtain

$$\varepsilon_x = -\frac{P}{\pi RL} \frac{1+\nu}{E} \left[(1-2\nu) + \nu \frac{4R^2}{R^2 - y^2} \right] \quad (6.7)$$

It is believed that the specific strain energy $\omega_e = (1/E)(\sigma_1 + \nu\sigma_3)\sigma_1$ is the minimum at

the center of the disc and splitting starts precisely here, at $y = 0$. Then

$$\varepsilon_x = \frac{2P}{\pi DEL}(1 + \nu)(1 + 2\nu) \quad (6.8)$$

Hence, at the disc center,

$$\sigma_x = E\varepsilon_x = \frac{2P}{\pi DL}(1 + \nu)(1 + 2\nu) \quad (6.9)$$

The Brazilian tensile test is applicable when the primary fracture initiates from the center of the disc. The tensile strength can be evaluated by calculating the horizontal stress σ_x that corresponds to the applied peak load. Eqs. (6.3) and (6.9) can be used to estimate the tensile strength indirectly from the applied peak load, under plane stress and plane strain states, respectively.

The above analytical solution determines the theoretical point of failure in an ideal, homogeneous, isotropic material. The tensile strength is calculated on the assumptions that failure occurs at the center. However, rock and rock-like materials are actually heterogeneous in the microstructure point of view. So in practice, failure may actually first occur at some other points where, although the calculated stress condition is not the most severe, the material is locally weaker due, for example, to a flaw, inclusion, grain boundary etc.

Failure patterns of rock and rock-like materials under static loadings in experiments are shown in Fig. 6.3 (Andreev, 1991). It can be seen that the failure pattern is in a splitting mode with respect to the static cases. However, the fracture process can not be presented. Tang, et al. (2001) carried numerical simulations to study the fracture process of heterogeneous Brazilian test under static loadings and presented the splitting process visually. The numerical results indicate that the dominant mode of failure is catastrophic splitting and progressive crushing. However, in dynamic tests, the failure patterns may be very different depending on the loading rate and heterogeneities. The validity of the Hertzian solution should be reviewed.

Fig. 6.3 Failure patterns of rock-like material under static loadings (Andreev, 1991)

6.2.2 Dynamic Brazilian tensile test

Dynamic tensile failure behavior of materials has attracted wide attentions in the last few decades. The Brazilian splitting test can be conducted by using the split Hopkinson pressure bar (SHPB) apparatus. Although there are still some difficulties in data recording and analysis, the SHPB test method has become the most helpful tool to study the dynamic tensile failure of rock and rock-like materials.

The principle of the direct tension is to generate a tensile stress in the specimen when a compressive stress wave reflects back from the free surface, which is shown in Fig. 6.4 (a). Tedesco, et al. (1991) conducted direct tension tests of plain concrete on a SHPB to investigate the effects of increasing strain rate on the concrete tensile strength. The results of the numerical analyses disclose the dynamic states of stress in the specimen prior to failure as well as the mode of failure. Brara and Klepaczho (2006) used SHPB to perform tensile test on cylindrical concrete specimens. The experiments demonstrated a significant increase of the tensile strength with the increase of the strain rate for the specimens.

The indirect tension test, i.e., dynamic Brazilian tensile test, by using SHPB is shown in Fig. 6.4 (b). Failure patterns of some granite samples under dynamic loadings are shown

in Fig. 6.5. It can be seen that the failure patterns are still splitting, however, the Hertzian solution may be no longer valid with respect to a high strain rate loading.

(a) Direct tension test

(b) Dynamic Brazilian tensile test

Fig. 6.4 Schematic of dynamic tensile test using SHPB: (a) Direct tension; (b) Indirect tension

Fig. 6.5 Failure patterns of Brazilian tests under dynamic loadings in experiments

A lot of works have been done to study the dynamic tensile failure using numerical simulations. Hughes, et al. (1993) carried out a series of numerical simulations of high strain rate Brazilian tensile tests to investigate the crack growth characteristics, and failure mode in concrete specimens. The typical fracture process is shown in Fig. 6.6. Tedesco, et al. (1989) carried out numerical analysis of dynamic split cylinder tests to investigate strain rate effect on the tensile strength of concrete on SHPB which also

revealed the failure process of the concrete specimen. Tang, et al. (2001) and Zhu and Tang (2006) studied the static and dynamic splitting process of rock disk with considering heterogeneity effect.

Fig. 6.6 Cracking sequence for splitting tensile test in numerical simulations
(Hughes, et al., 1993)

6.3 Numerical model description

The SPH method is again used to simulate the dynamic Brazilian splitting test because of its advantages in simulating rock fracture and fragmentation. It will help to understand the fracture mechanism of rocks under dynamic tensile loadings. Numerical simulations by considering material heterogeneity are conducted to study the progressive failure process of rock-like materials. In all cases, the numerical simulations are conducted for plane strain conditions. The J-H material model which has been introduced in detail in Chapter 2 is used as the constitutive law for the rock specimens which consists of an elasto-plastic strength model, a damage model and a polynomial equation of state. J-H model does not consider the strain rate which means each particle is insensitive.

The schematic view of the Brazilian disc and the applied velocity waveform are shown in Fig. 6.7. The diameter of the Brazilian disc is 5 cm. The loading width in all the

numerical simulations of this study is 7 mm. Then the angle α can be calculated as:

$$\alpha = \tan^{-1}\left(\frac{a}{R}\right) = \tan^{-1}\left(\frac{3.5}{25}\right) = 8^\circ \quad (6.10)$$

A velocity boundary is applied at the top end of the specimen with the other end fixed. The equivalent strain rate is

$$\dot{\varepsilon} = \frac{v}{D} \quad (6.11)$$

where v is the impact velocity, D is the diameter of the rock disc.

Fig. 6.7 Loading description and applied velocity waveform

When the velocity boundary is applied, stresses are produced and propagates on the rock disc. During the stress propagation, two response stresses p_1 and p_2 occur at the two boundaries shown in Fig. 6.8. The two stresses may not be in equilibrium due to the inertia effect and wave propagation in the specimen. The average stress of the two boundaries of the rock disc is

$$p = (p_1 + p_2) / 2 \quad (6.12)$$

Consequently, the stress history and the stress-strain curve in the rock disc along the vertical diameter can be determined based on the theory of the SHPB test and Eqs. (6.11) and (6.12).

Fig. 6.8 Response stresses at the two ends of the Brazilian disc

The numerical model of the disc is composed by 7852 particles. Each of these particles has the same diameter of 0.5 mm. They are arranged in the rectangular way. Fig. 6.9 shows the arrangement of the particles with the zoomed part allowing for better recognition of the arrangement.

Fig. 6.9 Illustration of the SPH particles arrangement

The first concern of the present numerical simulations is about the effect of the material heterogeneity. The material properties of the rock specimens in this chapter are the same as those given in Chapter 5 (see Table 5.1). The material properties such as tensile strength σ and elastic modulus E for the particles are randomly distributed throughout the specimen following a Weibull distribution law (see Eq. (3.1)). In the Weibull distribution function, the parameter m defines the shape of the distribution function. From the properties of the Weibull distribution, a larger value of m implies a

more homogeneous material. Therefore, the parameter m is called heterogeneity index in the present study. Parametric studies with respect to the index m are carried out to examine the heterogeneity effect to the failure behavior of rocks under dynamic tensile loadings.

Fig. 6.10 presents some 2D numerical rock discs, which are all composed of 7852 particles. The particles carry different material properties according to the Weibull distribution law with different heterogeneity indices. In Fig. 6.9, the different degree of gray color corresponds to a different magnitude of Young's modulus of the particles. The zoomed rectangle gives a better recognition of the arrangement. The mean value of the Young's modulus is 67.2 GPa.

(a) $m=1.1$ (The values range from 0.02 to 602)

(b) $m=2$ (The values range from 0.6 to 213)

(c) $m=3$ (The values range from 6.5 to 135)

(d) $m=5$ (The values range from 10.3 to 106)

(e) $m=50$ (The values range from 56.9 to 70.2)

Fig. 6.10 Numerical specimens with different heterogeneity index m . (Unit: GPa)

A series of numerical simulations are carried out to investigate the failure behavior of rock specimen under dynamic splitting loadings. Four groups of specimens were numerically simulated, they are

- 1) One homogeneous rock disc used to study the dynamic tensile fracture process, failure pattern and strain rate effect.
- 2) One specimen with the heterogeneity index, $m=10$, to study the dynamic tensile fracture process, failure pattern and strain rate effect.

- 3) Three specimens with the same heterogeneity index ($m=10, 5$ and 3 , respectively) to study the effect of the spatial distribution of material properties.
- 4) Four specimens with different heterogeneity indices, i.e., $m=\infty, 50, 10, 5, 3$ and 2 , representing materials from homogeneous to very heterogeneous to study the heterogeneity effect to the failure pattern and average compressive stress on the loading boundaries.

Similar to Chapter 5, damaged particles are displayed with different degrees of dark colors. The damage parameter D of the J-H constitutive model (see Chapter 2) varies between 0 and 1 which is represented in different colors from light grey to black.

6.4 Dynamic failure process of homogeneous rock disc ($m=\infty$)

Although rock-like material can be regarded as heterogeneous material, they can be divided into different categories based on the heterogeneity index m . A larger m implies a more homogeneous material. In this section, a homogeneous rock is used to simulate the dynamic failure behavior of rock disc which serves a baseline of the dynamic tensile failure of the heterogeneous specimens.

Numerical simulations in this section are conducted for a homogeneous rock disc to study the failure behavior of rock under different strain rates. The failure processes of the rock specimens under 0.1, 1 and 5 m/s are investigated in detail. The validity of dynamic Brazilian tensile test is discussed. Strain rate effect on the failure pattern and the tensile failure strength are also studied at some selected loading velocities. The selected loading velocities include 0.1, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7.5 and 10 m/s with the corresponding strain rate $\dot{\epsilon}=2, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 150$ and 200 s^{-1} .

6.4.1 Failure process of rock specimen when $v=0.1\text{ m/s}$ ($\dot{\epsilon}=2\text{ s}^{-1}$)

Fig. 6.11 shows the fracture process of splitting tensile test when the applied velocity is 0.1 m/s with the equivalent strain rate of 2 s^{-1} .

Fig. 6.11 Cracking sequence for splitting tensile test under $v = 0.1 \text{ m/s}$

At this strain rate, quasi-static failure mode occurs. Before $t = 625.6 \mu\text{s}$, the stress firstly propagates from the top to the bottom of the rock disc and then reflects between the two ends. At $t = 625.6 \mu\text{s}$, the stress in the rock sample reaches equilibrium. With the time going on, at $t = 814.0 \mu\text{s}$, the stress responses at the two ends of the Brazilian disc is equal to each other just before the initiation of rock fracture. At $t = 815.0 \mu\text{s}$, one major crack initiates at the center of the disc. The crack propagates quickly in both directions along the vertical diameter. At $t = 826.4 \mu\text{s}$, the rock sample is split into two symmetric halves along the vertical diameter. The crack formation can be regarded to be occurred instantly during the short period from $t = 815.0 \mu\text{s}$ to $t = 826.4 \mu\text{s}$. Moreover, some flexural tensile cracks have developed more extensively at the sides of the disc. This phenomenon has not been described in quasi-static splitting tests. However, it agrees the numerical results of Hughes, et al.(1993).

The corresponding stress distributions along the loading diameter are shown in Fig. 6.12. Fig. 6.12 not only presents the stress distribution before rock failure but also gives the stress development during the fracture process. After the initiation of the major crack, the tensile stress σ_x at the fractured particles vanishes. The particles can not bear tensile and compressive stresses anymore if they are completely damaged. As a result, a major crack is developed and the stress field redistributes. Then the external forces will be carried by other particles on the disc. It is interesting to see that both σ_x and σ_y changes with the development of the diametric crack.

Fig. 6.12 Profiles for the normal stresses along the vertical diameter

Fig. 6.13 compares the distributions of the analytical and simulated stresses σ_x and σ_y for the homogeneous rock specimen at the time of crack initiation under the loading velocity 0.1 m/s. It can be seen that the numerically simulated tensile and compressive stress distributions show a very good agreement with the theoretical results of Hondros (1959) which is based on the assumptions of the linear elastic and quasi-static loading. At the center and the edges of the rock disc, discrepancy exists between the numerical and analytical values for σ_y , which is probably caused by the applied boundary conditions and the analytical solution may oversimplify the stress condition.

Fig. 6.13 Analytical and simulated stress distribution along the vertical diameter at $t=814.0 \mu s$

The average compressive stress on the top and bottom boundaries is calculated from Eq. (6.12). It derives the splitting force by multiplying the loading width. The equivalent tensile stress (a splitting stress) at the disc center can then be calculated from Eqs. (6.9). The splitting stress at the disc center can also be obtained from the stress time-history at the disc center. Fig. 6.14 shows the average compressive stress on the boundary particles and the corresponding calculated splitting stress time-histories at the disc center. All the curves have the same shape. The stress is basically linear until the peak load. The specimen then loses its load carrying capacity substantially with the fracture process as shown in Fig. 6.11. The splitting stress calculated from the average compressive stress at the two boundaries is compared with the stress-strain time history at the disc center in Fig. 6.14(b). It can be seen that they are almost identical. The peak splitting stresses when $v=0.1 \text{ m/s}$ (strain rate $\dot{\epsilon} = 2 \text{ s}^{-1}$) is respectively 17.3 and 16.9 MPa from average stress and the output stress-time history at the disc center. They are a little larger than the tensile strength assigned for the particles, 16.1 MPa.

(a)

(b)

Fig. 6.14 Stress histories during the loading process: (a) Average compressive stress at the two boundaries; (b) Splitting stresses at the disc center

6.4.2 Failure process of rock sample when $v=1$ m/s ($\dot{\epsilon}=20$ s⁻¹)

The cracking sequences predicted by the numerical analyses, from crack initiation until complete failure are illustrated in Fig. 6.15 when the velocity at the boundary is 1 m/s with the strain rate at 20 s⁻¹. The corresponding normal stresses σ_x and σ_y along the

vertical diameter are shown in Fig. 6.16.

The stress in the Brazilian disc propagates from the top to the bottom and then reflects back. After a few of reflections the stress in the Brazilian disc reaches equilibrium and the forces on the two ends of the specimen almost get to be equal. From about $20\ \mu s$ to $84\ \mu s$, the forces at the two ends of the specimen remain in equilibrium. The normal stresses σ_x and σ_y increase with the increase of time.

Fig. 6.15 Cracking sequence for splitting tensile test under $v=1\text{ m/s}$

It can be seen that the damage occurs only after both the top and the bottom ends of the specimen are loaded. Therefore, the failure pattern is similar to that under static loading. At $t = 85.1\ \mu s$, the tensile failure initiates from the disc center. The crack propagates

along the vertical diameter based on the Griffith theory which conforms to the existing literatures (Hughes, et al., 1993). Then the crack propagates fast in both directions and reaches the two ends of the disc instantly. Additionally, flexural tensile cracks have developed on both sides of the Brazilian disc. Finally, at the time $t = 93.8\mu s$, the crack has propagated to both ends along the vertical diameter, and flexural tensile cracks have developed more extensively at the sides of the disc.

The profiles of the normal stresses σ_x and σ_y are presented in Fig. 6.16. It convinces the fracture process in the disc. At the initial stage, the pressure propagates from the top to the bottom of the disc. With the increase of time, the forces on the two ends become consistent and both increase with the increase of time. At the time of crack initiation ($t = 85.1\mu s$), the damaged particles can not bear the horizontal and vertical stresses any more. The normal stresses σ_x and σ_y at the damaged particles dropped to zero. At $t = 93.8\mu s$, the normal stresses almost vanish along the loading diameter and the specimen was split completely.

Fig. 6.16 Profiles for the normal stresses along the vertical diameter

The numerical result with a loading velocity of 1 m/s ($\dot{\epsilon}=20 s^{-1}$) is compared with the analytical solution in Fig. 6.17 at peak load when $t=84.6 \mu s$. Again, the numerical and analytical results show a good agreement except at the center and the two edges of rock disc.

Fig. 6.17 Analytical and simulated stress distribution along the vertical diameter at $t = 84.6 \mu s$

Because of the good agreement of numerical and analytical results, the equivalent tensile strength of the specimen can be calculated from the average stress from the two ends. Fig. 6.18 (a) shows the average compressive stress on the boundaries. Fig. 6.18 (b) compares the splitting stress calculated from the average compressive stress and the splitting stress time-history at the disc center. The average compressive stress increases almost linearly before the peak stress. The peak average stress is 394 MPa which gives a corresponding splitting stress of 18.1 MPa at $t = 89.2 \mu s$. The splitting stress time-history at the disc center vibrates because of the propagation of the dynamic stress between the two ends of the rock disc, however, the trend of the splitting stress increases almost linearly with the increase of the time. The splitting stress time history gives a peak stress of 16.6 MPa. It is a little different from that calculated from the average compressive stress on the boundaries, which is 18.5 MPa. It is found that the Hertz and Hondros solution of static Brazilian tensile test may become invalid for the dynamic splitting test.

(a)

(b)

Fig. 6.18 Stress history during the loading process: (a) Average compressive stress of the two boundaries; (b) Splitting stress from calculation and splitting stress at the disc center

6.4.3 Failure process of rock sample under $v=5\text{m/s}$ ($\dot{\epsilon}=100\text{s}^{-1}$)

At the lower strain rates (2 s^{-1} and 20 s^{-1}), the numerical results showed a relatively good agreement with the theoretical results which is based on the elastic analysis of homogeneous material under static loading conditions. In this section, the loading

velocity is taken as 5 m/s with the effective strain rate $\dot{\epsilon} = 100 \text{ s}^{-1}$.

Figs. 6.19 and 6.20 present the fracture process and the stress distribution along the vertical diameter, respectively at the selected instants during the whole loading process. Once the velocity boundary is applied, the stress wave begins to propagate from the top to the bottom of the Brazilian disc. At the time $t = 4.0 \mu\text{s}$, the stress wave does not reach the bottom of the disc and the bottom end is unstressed. With the increase of time, the bottom of the disc is loaded with stress and it reflects between the two ends. However, a considerable difference between the stresses at the two ends of the rock disc exists. At $t = 12.2 \mu\text{s}$, a crack initiates with an offset from the disc center and nearer to the bottom end of the disc. The crack is prolonged in both directions along the vertical diameter. At $t = 26.3 \mu\text{s}$, the splitting mode is formed. A cone zone is also found at the bottom of the disc. Similar to the rock failure under 0.1 m/s or 1 m/s velocities, the primary crack will result in the stress redistributions over the particles.

Fig. 6.20 shows the stress distribution over the vertical diameter during the fracture process. The stress distributions of σ_x and σ_y show clearly that the stresses at the two ends are obviously unbalanced. It is due to the inertia effect. It is found that the Hertzian solution is no longer valid since the stress state of the disc is very different from that of the quasi-static case. Two major problems can be observed for the invalidity of the Brazilian tensile test, (1) the stress responses at the two ends can not satisfy the quasi-static equilibrium condition and the discrepancy between them can not be simply ignored; (2) the rock failure does not occur at calculated peak splitting force which can be seen later in the stress-strain curve.

Fig. 6.19 Cracking sequence for splitting tensile test under $v=5$ m/s

Fig. 6.20 Profiles for the normal stresses along the vertical diameter

Fig. 6.21 shows the theoretical and simulated stress distributions along the vertical diameter at $t = 12.2 \mu s$ and $t = 18.6 \mu s$, respectively. It can be seen that large

discrepancies exist between the theoretical and simulated stress distribution results under the current loading velocity. The Hertz and Hondros solutions derived from the static Brazilian tensile test are no longer valid when the loading velocity of 5 m/s is applied.

(b) $t=18.6 \mu s$

Fig. 6.21 Analytical and simulated stress distribution along the vertical diameter at
(a) $t=12.2 \mu s$ and (b) $t=18.6 \mu s$, respectively

Fig. 6.22 (a) shows the average compressive stress on the boundaries. The splitting stresses calculated from the boundary stress and the stress time-history at the disc center are shown in Fig. 6.22 (b).

(a)

(b)

Fig. 6.22 Stress time-history during the loading process: (a) Average compressive stress of the two boundaries; (b) Splitting stress from calculation and splitting stress at the disc center

It can be seen that the average compressive stress increases to a certain value instantly after the velocity boundary is applied. The splitting stress calculated from the average compressive stress and the splitting stress at the disc center are compared. The splitting stress obtained from the stress time-history reaches its peak value, 16 MPa, at the time $t = 12 \mu s$ while the calculated peak splitting stress from boundary stresses is 19 MPa,

occurred at $t = 19 \mu s$. The discrepancy is 19% for the splitting stress at the disc center. It should be mentioned that dynamic experiments usually measure the overall deformation along the specimen and the equivalent force applied to the specimen. The splitting stress calculated from the compressive stresses on the boundaries reflect the measurement result in a dynamic test. However, the stress time-history gives the real stress response of the material. Based on the comparison in Fig. 6.22 and the stress distribution shown in Fig. 6.20, the dynamic splitting tests for rock specimens using the traditional SHPB method are invalid when the impact velocity is 5 m/s or, in another word, the strain rate is $100 s^{-1}$.

Based on the above results when $v=5$ m/s or a strain rate of $100 s^{-1}$, it can be summarized,

- (1) From the fracture process of the rock disc, the major crack does not initiate from the center but nearer to the bottom of the disc which implies that the Hertz and Hondros solutions are no longer applicable, although the rock disc fails in a splitting mode. Consequently, the average stress on the top and bottom boundaries can not be used to derive the tensile stress at the disc center.
- (2) From the stress distribution along the vertical diameter, it can be seen that the specimen is not in a quasi-static equilibrium state. The inertia plays an important role in determination of the average compressive stress on the boundaries.
- (3) By the comparison of the splitting stresses respectively calculated from the compressive stresses on the boundaries and the stress time history at the disc center, it can be seen that under low strain rate such as $2 s^{-1}$, the Brazilian tensile test is applicable in determination of the dynamic tensile strength. However, under a strain rate as high as $100 s^{-1}$, a relatively large discrepancy exists. Of course, the difference may also be introduced from other assumptions, for example, Eq. (6.3) takes α as a very small angle but α used in the present study is 8° . Eq. (6.9) transforms plane stress to plane strain where the Poisson's ratio may also influence the value of the tensile strength.

6.4.4 Strain rate effect on the failure process of homogeneous rock disc

Strain rate effect has been a long-time discussion for the material properties under

dynamic loading. In this section, strain rate effect is to be investigated with respect to the fracture process and tensile strength of the homogeneous rock disc.

Fig. 6.23 presents the fracture process of rock disc under different strain rates. For the cases of 0.1 m/s, 0.5 m/s and 1 m/s, the fracture process is similar to that of static Brazilian tensile splitting. A major crack initiates at the center of the rock disc and propagates in both directions along the vertical diameter.

With the increase of the strain rate, the failure pattern of the rock disc changes although it still shows a splitting failure mode. When $v=1.5$ m/s, the major crack is developed offset from the center and closer to the bottom of rock disc. Then bifurcations occur at both ends of the crack. The bifurcation initiating from the top tip of the crack propagates faster than that initiating from the bottom tip of the crack. Finally the specimen is split. It is interesting that a cone is formed at the top part of the rock disc because of the development of the bifurcated fractures.

When the loading velocity is 2.0 m/s, the primary crack initiates offset from the disc center but nearer to the top end of the rock disc. The crack also propagates in both directions along the vertical diameter. The cone cannot be formed completely and the tensile crack propagates and almost reaches both ends of rock disc.

When the loading velocity is further increased to 2.5 m/s, the failure pattern is something similar to that of 1.5 m/s. With the increase of loading velocity from 3 m/s to 6 m/s, another fracture pattern is found. The primary crack initiates first near the bottom end of the rock disc and mainly propagates upwards to form the splitting mode. The crack bifurcated later to form a conical zone near the bottom end of the disc.

At higher loading velocities of 7.5 m/s and 10 m/s, the initiation of the primary crack always occurs near the bottom of the disc. The crack tends to bifurcate at the top tip and form a conical zone at the top end of the rock specimen. Moreover, the vertical crack becomes wider because of the development of the crack bifurcation.

It is worth noting that, although the splitting failure is the major mode of the specimen, the initiation of the primary crack is different depending on the loading velocity.

Bifurcation of the primary crack and flexural cracks at the two sides of the disc exist. It is clear that at a higher loading velocity, the assumptions of the quasi-static and elastic solution becomes invalid. For example, the initiation of the primary crack offsets from the disc center. The primary crack occurs before the disc reaches a quasi-static equilibrium state.

(d) $v=1.5$ m/s

(e) $v=2.0$ m/s

(f) $v=2.5$ m/s

(g) $v=3.0$ m/s

(h) $v=3.5$ m/s

(i) $v=4.0$ m/s

(j) $v=4.5$ m/s

(k) $v=5.0$ m/s

Fig. 6.23 Fracture process of homogenous rock sample under different strain rates

Fig. 6.24 shows the splitting stress at the disc center versus the vertical displacement of the specimen. It can be seen that the shapes of the curves are greatly influenced by the dynamic loadings. Under the velocities of 0.1 and 1.0 m/s, the splitting stress σ_x at the disc center increases almost linearly until the peak stress and then falls to zero. With the increase of the loading rate, the occurrence of fracture at the disc center as well as the shape of the stress-displacement curve is influenced by the dynamic loadings. Under the loading velocities of 2.5, 5.0, 7.5 and 10 m/s, σ_x is a compressive stress first and then changes to a tensile stress and fails by the tensile stress finally. However, the tensile

strength at the disc center is almost the same which means that the strain rate has no influence to the tensile strength.

Fig. 6.24 Splitting stress with displacement curve at the disc center

Fig. 6.25 gives the splitting stress σ_x calculated from the averages stress at the two boundaries versus the displacement along the loading diameter during the whole splitting process with different loading velocities. It can be seen that the displacement at the peak stress increases with the increase of the loading velocity or the equivalent strain rate. The lower the loading velocity, the faster the stress drops. The peak average stresses are different from each other but the difference is not significant. The peak stress increase with the increase of loading velocity from 0.1 m/s to 2.5 m/s. However the peak stress decreases when the loading velocity further increased from 5 m/s to 10 m/s. It should be mentioned that the splitting stress is calculated based on Eq. (6.9) which is derived from a quasi-static and elastic solution. It may be invalid when the loading velocity exceeds a certain range. In the present case, when the loading velocity is smaller than 1 m/s or the equivalent strain rate is lower than 20 s^{-1} , the failure mode is similar to the quasi-static case, for which the primary crack initiates exactly from the disc center. When the loading velocity is larger than 1 m/s, the initiation of the primary crack becomes uncertain.

Fig. 6.25 Average compressive stress of two loadings boundaries with displacement curve during the loading process under different loading velocities

Fig. 6.26 DIF of tensile strength of the homogeneous rock disc

Fig. 6.26 shows the DIF of calculated tensile strength based on the average compressive stress on the boundaries. The DIF of the tensile strength varies between the range of 1.0 and 1.2. Based on this curve, the strain rate effect is not apparent for the homogeneous specimen. The DIF of the tensile strength based on the stress time-history at the disc center is a constant of 1, which means that the strain rate has no effect to the dynamic tensile strength.

From the above discussion, the strain rate effect to the DIF of the homogeneous rock disc is not apparent and can be ignored.

6.5 Heterogeneity effect on rock tensile failure

Rock-like materials are heterogeneous from the view of microstructure which may influence the fracture behavior of these materials. Heterogeneity is of importance to the tensile behavior as well as the compressive behavior including the failure pattern and the stress responses. In order to investigate the influence of the microstructure on dynamic tensile failure, rock specimens with different heterogeneity index $m=50, 10, 5, 3$ and 2 are used in the following numerical simulations. The selected loading velocity is 1 m/s with the strain rate of 20 s^{-1} . The progressive failure processes of the specimens are shown in Figs. 6.27, 6.28, 6.29, 6.30 and 6.31, respectively.

Fig. 6.27 Failure process of the specimen with the heterogeneity index $m=50$

Fig. 6.28 Failure process of the specimen with the heterogeneity index $m=10$

Figs. 6.27 and 6.28 presents the cracking growth process when $m=50$ and $m=10$, respectively, which is similar to that of the homogeneous rock disc. The crack initiates from the disc center and propagates in both directions to the two ends of the disc. It should be mentioned that the primary crack does not occur exactly at the disc center because of the material heterogeneity. Flexural cracks also appear at the sides of the disc when the vertical displacement is big enough.

Fig. 6.29 shows the failure process for the specimen with the heterogeneity index $m=5$. At the loading velocity of 1 m/s, scattered cracks initiate at the center of the disc. After that, one primary crack propagates to the two ends of the disc along the vertical diameter. The formation of the major crack along the vertical diameter proves that the failure mode keeps almost unchanged as splitting even if the specimen is relatively heterogeneous.

Fig. 6.29 Failure process of the specimen with the heterogeneity index $m=5$

Fig. 6.29 presents the failure process of the specimen with the heterogeneity index $m=5$. The pressure propagates from the top to the bottom during which damaged particles appeared near the vertical diameter. More cracks initiate at the disc center than other places of the disc and then propagate to the disc ends along the vertical diameter. Finally a cracked band is formed in the vertical direction. A few long flexural cracks appear at the sides of the disc. It is interesting that the zone beneath the loading is undamaged because of the high pressure.

Figs. 6.30 and 6.31 show the failure process of rock specimen with the heterogeneity index $m=3$ and 2, respectively. The failure process is similar with the relatively homogeneous specimen. Scattered cracks initiate around the center of the disc and propagate along the vertical direction. It presents that the more homogeneous rock

specimen shows a more brittle property. The specimens all begin from the tensile damage of scattered particles, then the damaged particles coalesce with others, until, eventually, the macroscopic fracture band is formed. For specimens with higher homogeneity, the macroscopic failure bands are more concentrated compared to specimens with lower homogeneity.

In summary, the failure processes of all specimens with different heterogeneity index are similar under the same velocity $v=1\text{m/s}$. The initiation of cracks is around the center of the disc and propagates to the two ends of the disc along the vertical direction. However, the failure process is also significantly influenced by the heterogeneity index. The homogeneous specimen seems more brittle than the heterogeneous specimen.

Fig. 6.30 Failure process of the specimen with the heterogeneity index $m=3$

Fig. 6.31 Failure process of the specimen with the heterogeneity index $m=2$

Fig. 6.32 Average compressive stress of the two boundaries versus displacement curves under different heterogeneity index

Fig. 6.32 presents the average compressive stress of the two boundaries with displacement curves with different heterogeneity index. The mean strength of the particles of the rock discs is identical for different heterogeneity indices. It can be seen that the peak stress response increases with the increase of the homogeneity of the rock specimens. It means that the overall tensile strength of the rock specimens with the same mean strength in the Weibull distributions for the particles increases with the increase of the homogeneity index.

6.6 Effect of spatial distribution of material properties

This part reviews the spatial distribution effect of the material properties, similar to that done in Chapter 5. Even with the same heterogeneity index, for each specimen, the spatial distribution of material properties of the particles may be stochastically different. In order to investigate the effect of the randomness due to the spatial distributions, three numerical specimens whose material properties are specified randomly according to the same Weibull distribution parameters are studied under the loading velocity of 1.0 m/s.

Fig. 6.33 shows the fracture processes of the three different specimens with $m=10$ during the Brazilian tensile test. Fig. 6.34 gives the average stress of the stresses at the boundaries in the three specimens correspondingly. It can be seen that the failure process of the three specimens are similar to each other. Scattered cracks initiate and then nucleate around the center of the rock disc at $t=26.6 \mu s$. At $t=35.5 \mu s$, the peak stress is obtained and the splitting mode can be clearly observed. With the increase of displacement, the fracture band is formed and causes the splitting failure of the rock disc. Flexural cracks occur on the sides of rock disc and the failure patterns of three specimens all undergo splitting failure. However, owing to the randomness of spatial distribution of the material properties, the cracking patterns of different numerical specimens are a bit different. From Fig. 6.34, the average compressive stresses at the two boundaries are also a bit different. As a result, the peak stress and the overall tensile strength of the specimens are different, which can reflect the effect of the randomness of the particles. The discrepancy of the peak stress, however, is within an acceptable range. Any of them can give a good representation of the failure pattern and the overall tensile strength.

Fig. 6.33 Failure patterns of three different samples with $m=10$

Fig. 6.34 Average compressive stress histories of the two boundaries in the three samples with the same heterogeneity index $m=10$

Similar results have been obtained for the rock specimens with heterogeneity indices of $m=5$ and $m=3$ respectively. Figs. 6.35 and 6.36 presents the fracture process and average compressive stress at the boundaries histories for rock specimens with $m=5$ while Figs. 6.37 and 6.38 are for the rock specimens with $m=3$. The initiation and nucleation of the primary cracks are from the central area of the disc. They propagate in both directions along the vertical diameter and form the tensile splitting failure mode. For the rock samples with $m=3$, the crack band width becomes much bigger than that of discs with $m=10$. Under the Brazilian tensile test, the rock specimens with the heterogeneity index $m=10$ exhibit very brittle failure behavior by examining the stress-displacement curves. Flexural tensile cracks are also observed for all the heterogeneous rock specimens.

It can be seen from Figs. 6.36 and 6.38 that the average compressive stress at the two boundaries curves are different from each other with the same heterogeneity index concerned about the distributions of the material properties. There are some differences of the peak stress which reflects the effect of the randomness of the distributed particles.

Here it should be highlighted that a relatively big difference exists for the peak stresses when $m=3$ (see Fig. 6.38). The peak stress of sample 2 is bigger than the other two samples which can be reflected from the failure pattern of sample 2. For sample 2, the cracks do not exactly initiate at the center of the disc. At $t = 16.2\mu s$, the undamaged disc center probably cause the relatively big peak stress.

Both the fracture processes and time-history of the average compressive stress at the boundaries for different rock discs with the same heterogeneity index of $m=10$, 5 and 3 respectively presents that the more homogeneous rock specimen shows a more brittle property. The difference actually reflects the influence of the spatial stochastic distribution of the material properties. The spatial distribution of the particles with different mechanical properties greatly influences which particle first damaged, and the cracking patterns. The specimens all begin from the tensile damage of scattered particles, then the damaged particles coalesce with others, until, eventually, the macroscopic fracture band is formed. For specimens with higher homogeneity, the macroscopic failure bands are more concentrated compared to specimens with lower homogeneity.

Fig. 6.35 Failure patterns of three different samples with $m=5$

Fig. 6.36 Average compressive stress histories of the two boundaries in the three samples with the same heterogeneity index $m=5$

Fig. 6.37 Failure patterns of three different samples with $m=3$

Fig. 6.38 Average compressive stress histories of the two boundaries of the three samples with the same heterogeneity index $m=3$

6.7 Strain rate effect on heterogeneous rock specimen

From the section of 6.4.4, it can be seen that although the failure pattern of the homogeneous rock specimen can be visually presented, the strain rate effect observed from the experiments can not be well studied by the numerical simulation. This may be caused by the real heterogeneous microstructure of the rock-like materials. Heterogeneity is of importance to the failure behavior of brittle materials including failure pattern and failure strength. The objective of this section is to investigate the strain rate effect to the failure process of heterogeneous rock specimens. The heterogeneity index m is taken as 5 or 10. And the loading velocities are 0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0 and 5.0 m/s, respectively. The corresponding effective strain rates are respectively 2, 10, 20, 30, 40, 50, 60, 80 and 100 s^{-1} .

The failure processes for the specimens with $m=5$ are shown in Fig. 6.39. The failure patterns of the rock discs under different loading rates changes with the increase of the loading velocity. The locations of the primary cracks are influenced by the loading velocity or, in another word, the strain rate. At $v=0.1\text{ m/s}$, the fracture patterns of the rock disc is similar to that under static loadings with cracks initiating at the center of the disc. With the increase of strain rate, the location of crack initiation moves upwards closer to the top end of the rock sample which dominates the formation process of the fractures. The fracture processes can be divided into three categories:

- (I) (a), (b) and (c) ;
- (II) (d) ;
- (III) (e), (f), (g), (h) and (i)

For category I, the fracture processes can be regarded the same with that of the disc under quasi-static loadings. Cracks initiate around the center of the rock disc and coalesce to form a fracture band along the vertical diameter. A few of relatively long flexural tensile cracks occur at the sides of the rock disc.

For category II at $v=1.5\text{ m/s}$, the scattered cracks initiate near the top end of the rock disc because of the relatively high loading rate. Then a few fractures is formed which is not the same as the fracture under quasi-static loadings. Finally a wide fracture band which

consists of three major fractures is formed along the vertical diameter. Although the failure pattern is different from that of type I, the theory of Brazilian tensile test seems to be still suitable under this condition because the failure pattern can be regarded as a quasi-static failure mode.

For category III, the failure pattern is very different with that under static loadings. The cracks occur immediately near the top end of the rock disc after the loading is applied. The stresses at the two ends of the disc can not satisfy equilibrium which results in the non-symmetry of fractures with respect to x axis. Consequently, the theory of the Brazilian tensile failure such as Hertz and Hondros solutions are no longer suitable for the analysis of the numerical results.

(d) $v=1.5$ m/s

(e) $v=2.0$ m/s

(f) $v=2.5$ m/s

(g) $v=3.0$ m/s

Fig. 6.39 Failure processes of heterogeneous specimen ($m=5$) at different loading rates

Fig. 6.40 shows the average compressive stress at the boundaries versus vertical displacement of rock specimens under different loading velocities with $m=5$. The peak compressive stress increases with the increase of the strain rate. The failure strain of the rock specimens also increases with the increase of the strain rate.

Fig. 6.40 Average compressive stress of the two boundaries with displacement curves of rock samples under different loading velocities with $m=5$

Fig. 6.41 DIF of tensile strength with strain rate curve of rock with $m=5$

Fig. 6.41 shows DIF at different strain rates of one heterogeneous rock specimen with $m=5$. It should be mentioned that the peak tensile stress at the strain rate of $2 s^{-1}$ happens identical with the mean tensile strength of the particles. It is assumed that the tensile strength can be calculated from the theory of the Brazilian tensile test under static loadings. It can be seen that DIF increases with the increase of the strain rate. DIF increases to about 5 at the strain rate of $200 s^{-1}$.

Similar analysis is carried out to investigate the strain rate effect to the failure behavior of heterogeneous rock disc with the heterogeneity $m=10$. Fig. 6.40 shows the failure processes under different strain rate of the rock disc. The failure patterns under the different strain rates also can be divided into three categories:

- (I) (a), (b), (c) and (d)
- (II) (e), (f) and (g)
- (III) (h) and (i)

For category I, the failure pattern is of quasi-like splitting failure mode. One crack initiates around the center of the rock disc and propagates in both directions along the vertical diameter. Finally the rock disc is split into two halves. Flexural tensile cracks also occur at the sides of the rock disc.

For category II, two or three major cracks instead of only one primary crack occur at the

disc center and propagate in both directions along the vertical diameter. Although the failure pattern is not the same as that under static loadings, the stresses at the two ends seem in equilibrium which results in a symmetric failure pattern.

For category III at relatively high strain rate loadings, dramatic difference exist between the two ends of the disc which can be reflected from the failure patterns. In these conditions of high strain rate loadings, the Hertz and Hondros solutions will not be applicable anymore.

(a) $v=0.1$ m/s

(b) $v=0.5$ m/s

(c) $v=1.0$ m/s

(d) $v=1.5$ m/s

(e) $v=2.0$ m/s

(f) $v=2.5$ m/s

(g) $v=3.0$ m/s

Fig. 6.42 Failure processes of heterogeneous specimen at different strain rates with $m=10$

Fig. 6.43 shows the average compressive stress at the boundaries versus the vertical displacement under different loading velocities when $m=10$. The peak stress increases with the increase of the strain rate. The failure strain of the rock specimens also increases with the increase of the strain rate.

Fig. 6.43 Average compressive stress of the two boundaries with displacement curves of rock specimens under different loading velocities with $m=10$

Fig. 6.44 shows DIF at different strain rates of one heterogeneous rock sample with $m=10$. The tensile strength at the strain rate of 1 s^{-1} is considered identical with the static tensile strength. And it is also assumed that the tensile strength can be calculated from the theory of Brazilian tensile test under static loadings. It can be seen that DIF increases with the strain rate effect. DIF increases by 4.3 at the strain rate of 200 s^{-1} .

Fig. 6.44 DIF of tensile strength of heterogeneous rock specimen with $m=10$

Chapter 7 Conclusions and future work

7.1 Conclusions

The SPH method of the commercial code AUTODYN-2D is used to investigate the fracture process of rock-like materials with heterogeneity effect under dynamic compressive and tensile loads. The J-H strength model, which consists of an elasto-plastic strength model, a cumulative damage model and polynomial equation of state, is used as the material model of the brittle specimens. The J-H model does not involve the strain rate effect which means the microscale element is rate insensitive. The Weibull distribution is utilized to model the material heterogeneity, which is implemented into the SPH method through the subroutines of AUTODYN to reflect the microstructure characteristics of rock-like materials. Two series of numerical simulations were carried out with respect to dynamic compressive and tensile tests, respectively.

The following conclusions can be drawn based on the numerical simulations of the compressive tests.

- 1) Splitting mode is the basic failure pattern of the rock-like specimens used in the present study under dynamic compressive loads.
- 2) The volumetric strain of the specimens increases first and then decreases with the increase of the displacement in the compressive test which conforms to the physical experimental results.
- 3) Spatial distribution of the material properties with the same heterogeneity index slightly influences the stress-strain curves and the failure patterns of the rock specimens.
- 4) The heterogeneity index influences significantly the failure pattern and the failure strength of the specimens. Relatively, the more homogeneous specimens are more brittle with higher compressive strength and larger failure strain.

- 5) Different strain rate loads affect the failure pattern of the heterogeneous rocks. Although the splitting mode is the basic failure pattern for the compressive pulse load, the tensile cracks result in a shearing band at a relatively low strain rate, laterally splitting failure at a medium strain rate and splitting failure with severe crushing at relatively high strain rate. The dynamic increase factor (DIF) is merely a constant with the strain rates below 100 s^{-1} and it increases with the increase of strain rate if the strain rate is above 100 s^{-1} . The structural effect as inertial confinement is thought to be responsible for the strain rate effect.

The conclusions drawn from the numerical simulations on the dynamic Brazilian tensile tests are as follows.

- 1) The fracture process and stress distribution along the diameter of the homogeneous disc at relatively low strain rate conforms to the theoretical results.
- 2) Different strain rates cause different failure patterns and tensile strengths of the homogeneous discs. The major cracks are initiated from different locations of the rock disc under different strain rate loads and splitting failure mode occurs in all the tests. However, strain rate effect to the tensile strength of the homogeneous Brazilian disc is not showed.
- 3) The spatial distribution of material properties with the same heterogeneity index slightly influences the failure pattern of the specimens.
- 4) Heterogeneity index has significant effect to the failure pattern and the tensile strength of rock-like materials. The more homogeneous the specimen is, the higher the tensile strength is.
- 5) Strain rate influences the failure pattern and calculated tensile strength of the heterogeneous specimens. DIF increases with the increase of the strain rate in the range studied. The strain rate effect is because of the heterogeneous microstructure of the specimens.
- 6) The Brazilian tensile test may be invalided under relatively high strain rate based on the failure pattern and the stress distribution in the homogeneous or heterogeneous rock disc.

7.2 Future work

The future work of the present study may include

- 1) To compare the numerical results to the experimental results further.

The specimens used in the present numerical simulations do not represent a real material but adopt an assumed heterogeneity index of the material properties. The numerical results, including the failure pattern, stress-strain curves and strain rate effect, etc., need be verified by real physical experiments

- 2) To improve the material heterogeneity model

As a statistical approach, the Weibull distribution is a very promising method to reflect the heterogeneous microstructure of rock-like materials. The numerical modeling involving the heterogeneity effect can simulate well the rock fracture under dynamic compressive and tensile loads. However, the real distribution of the material properties may not exactly agree the Weibull distribution. In the future, a numerical model may consider the real material heterogeneity by analyzing the microstructure digital image.

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