Structural damage detection and diagnosis using time-domain data

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**STRUCTURAL DAMAGE DETECTION AND
DIAGNOSIS USING TIME-DOMAIN DATA**

GAO FENG

SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING

2007

STRUCTURAL DAMAGE DETECTION AND DIAGNOSIS USING TIME-DOMAIN DATA

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School of Civil & Environmental Engineering

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SUMMARY

Damage diagnosis and health monitoring based on structural dynamic data is a subject of wide attention in civil engineering applications, especially for important structures like high-rise buildings and long-span bridges. Methods that can directly apply on measurement data in the time domain provide potentially effective platform for quick damage diagnosis and monitoring. Recently such methods have been receiving increasing attention in the research community.

The most appealing advantage of time-domain methods perhaps lies on the fact that they do not require domain change, thus enable rapid processing of the measurement signals and also avoid some additional processing errors. Usually no mechanical model is required for the diagnosis analysis with a time domain method. On the other hand, however, the simplified approaches of typical time domain methods usually lead to relatively abstract and less-detailed results, rendering further securitization of damage to be difficult. Besides, most of the existing time domain methods require the availability of the external excitation signal, which however is often difficult to measure, especially under a monitoring setting. Susceptibility to measurement noises is another problem faced by typical time domain methods.

The present study aims at developing more robust time domain analysis approaches that are suitable for structural health monitoring and damage diagnosis in practice. Considering the various issues mentioned above, the objectives of this study are mainly three folds, 1) To tackle the problem with low sensitivity to damage and high dependence upon the availability of excitation signals. To this end, the formulation of the basic model will stem from system dynamic equations so that the features derived thereby become physically meaningful and potentially sensitive to parameter changes. From there, a novel scheme will be established for the utilization of easily measurable time-series signals (namely accelerations) and the disassociation from the actual excitation in the method. 2) To increase the

robustness of the method for application on noise-contaminated signals. To this end, a novel use of the well-known Kalman filter will be developed. 3) To develop a time domain method which can provide more precise identification of the location as well as the severity of damage. This method may be employed as a supplementary means to the simple above approaches for more advanced diagnosis.

Through this study, a new method for damage diagnosis using time series analysis of vibration signals has been developed. The method is based on the linear dynamic system formulation and is realized in a novel form of auto-regressive models (ARX) on acceleration response signals only. Through appropriate simplification, the model can be disassociated from the input excitation and this further enhances the potential robustness of the method in real-life applications.

For concerns with measurement noises, a procedure essentially based on Kalman filter is developed to extend the above method to more realistic environment with noise-contaminated vibration signals. The Kalman filter is incorporated to perform the stochastic processing of the input-response signals containing noise. Both the numerically simulated scenarios and an experimental study demonstrate that the effectiveness of the proposed approach under noisy measurement environment is satisfactory.

A residual generator method using time domain response signals is further developed for detecting and locating damage in a structure, and at the same time determining the severity of damage (in terms of stiffness reduction). Major advancement of this approach in the present study is the formulation to enable the use of acceleration signals, which are easier to measure in actual applications, instead of displacements, as well as the newly developed ability of determining the damage severity.

Numerical examples are used extensively in this study to demonstrate the implementation and effectiveness of the proposed approaches. An experimental

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study has also been conducted to provide further verification of the methods with real measurement data.

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LIST OF SYMBOLS

$\{ \}^T, []^T$	Transposed vector, matrix
$[]^{-1}, []^{-r}, []^{-l}$	Inverse, right-inverse, left-inverse matrix
$d()$	Dimension of matrix
$\oplus$	Internal direct sum of subspace
Im	Image of map
Ker	Kernel of map
X	Subspace of state-vector x
Y	Subspace of output-vector y
W	(C, A) -invariant subspace
S	(C, A) UnObservability Subspace
M_i	Subspace of disturbance m_i
$\mathbf{0}$	Null matrix
$\mathbf{I}$	Identity matrix
$\mathbf{y}(t), \mathbf{y}(k)$	Output time-series
$\hat{\mathbf{y}}(k)$	Predicted output
$\mathbf{y}_v(t), \mathbf{y}_v(k)$	Measured output with noise
$\mathbf{y}_{\text{vir}}(k)$	Virtual output
$\mathbf{y}^t(k)$	Output minus input
$\mathbf{x}(t), \mathbf{x}(k)$	State-vector time-series
$\hat{\mathbf{x}}(k)$	Predicted state-vector
$\mathbf{u}(t), \mathbf{u}(k)$	Input time-series, excitation signal of structure
$\mathbf{u}_v(t), \mathbf{u}_v(k)$	Measured input with noise
$\mathbf{e}(t), \mathbf{e}(k)$	Residual error time-series
$\mathbf{e}(t)$	State-vector of residual error model (for Chapter 5 only)
$m_i(t)$	The disturbance signal of i -th actuator

LIST OF SYMBOLS

$\mathbf{r}_i(t)$	The output of i -th residual generator
$\mathbf{w}(t)$	State-vector of residual generator
$\mathbf{p}_i$	Columns of decomposed i -th element stiffness matrix (for Chapter 5 only)
$\mathbf{q}_i$	Rows of decomposed i -th element stiffness matrix (for Chapter 5 only)
q	Shift operator
α_i	Stiffness reduction of i -th element
$\boldsymbol{\eta}(t), \boldsymbol{\eta}(k)$	Process noise
$\mathbf{v}(t), \mathbf{v}(k),$	Measurement noise
$\xi(t), \dot{\xi}(t), \ddot{\xi}(t)$	Structural responses as displacement, velocity and acceleration
Λ	Diagonal matrix
ω_i	Eigenvector value
Φ	Eigenvector matrix
Φ_i	Mode shape vector
σ	Standard deviation
$\mathbf{A}, \mathbf{A}_D$	State matrix in discrete form
$\mathbf{A}_c$	State matrix in continuous form
$\mathbf{A}_0$	Directly state coefficient of (C, A) u.o.s subspace (for Chapter 5 only)
$\mathbf{B}, \mathbf{B}_D$	Input matrix in discrete form
$\mathbf{B}_c$	Input matrix in continuous form
$\mathbf{C}$	Output influence matrix
$\mathbf{D}$	Direct transmission matrix
$\mathbf{D}, \mathbf{D}_0, \mathbf{D}_1$	Output injection map for (C, A) u.o.s. (for Chapter 5 only)
$\mathbf{E}$	Input coefficient for response of residual generator (for Chapter 5 only)
$\mathbf{E}(q)$	EXogenous input polynomial in ARMAX
$\mathbf{E}_0, \mathbf{E}_1, \dots, \mathbf{E}_i$	Input coefficient in ARMAX

LIST OF SYMBOLS

F	State coefficient of residual generator (for Chapter 5 only)
G	Input coefficient for process noise
G	Input coefficient for excitation of residual generator (for Chapter 5 only)
H	Direct transmission coefficient of residual generator (for Chapter 5 only)
H(q)	Error polynomial function in ARMAX
H₁, H₂,... H_i	Moving average coefficient of residual error in ARMAX
J	Normalized damping matrix
J₀	Global damping matrix
K	Normalized stiffness matrix
K₀	Global stiffness matrix
k	Element stiffness matrix
K_{0i}	Contribution of i-th element in global stiffness matrix
K(k)	Kalman Gain
$\bar{K}$	Converge Kalman Gain
L	Normalized input coefficient matrix
L₀	Input coefficient matrix
M(k)	Posteriori estimate error covariance
M	Output influence coefficient of residual generator (for Chapter 5 only)
M₀	Global mass matrix
O	Canonical projection (for Chapter 5 only)
P(k)	Priori estimate error covariance
P(q)	Auto-regression polynomial in ARMAX
P₁, P₂,... P_i	Auto-regression coefficient in ARMAX
P_i	Failure signature of i-th actuator failure (for Chapter 5 only)
Q(k)	Process noise covariance
R(k)	Measurement noise covariance
S	Arbitrary matrix (for Chapter 3 only)
T₁, T₂,... T_n	Natural periods

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1.1 Background

Most civil structures are constructed for long servicing life of 50 years to even hundreds of years. Over such a long time structural degradation takes place due to various reasons, for example corrosion of steel bars, creep of concrete, aging of rubber bearings, loss of prestress, incidents of external actions such as overloading, earthquakes, and collisions by vehicles or ships. Some of the damages are accumulated in the structure over time; others are accidental events but are still very possible to happen in the long servicing life. Some structures have even collapsed due to the lack of timely inspections and retrofits, causing injuries and deaths. So the interest in the ability to monitor a structure and detect damage is pervasive.

Traditional damage-detection methods are either visual or using localized experimental methods such as acoustic or ultrasonic methods, magnet field methods, radiographs, eddy-current methods or thermal field methods. But all of these experimental techniques require that the vicinity of the damage is known a priori and that the portion of the structure being inspected is readily accessible. It generates the needs for additional global damage detection methods that can be applied to complex structures. The topic of damage detection based on changes in the dynamic properties or response of structures under static or random loading has received considerable attention for the last 30 years. The basic idea is that modal parameters (normally frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness); when changes take place in

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the structures, they will cause changes in the modal properties. The recent rapid developments in the instrumentation, measurements and computer systems have made it possible to carry out detailed investigations of the dynamic behavior of structures at a reasonable cost. More and more applications are reported on important civil structures such as high rise buildings, towers, bridges, and offshore platforms.

For damage identification and health monitoring many different issues are critical to the success of using the mechanical vibration characteristics of a structure. Among the important issues are excitation and measurement considerations, including the selection of the type and location of sensors, the type and location of the excitations. Another important topic is signal processing, which includes such methods as Fourier analysis, time-frequency analysis and wavelet analysis. The area recently receiving much attention in the technical literature is called feature extraction. It is the process of identifying damage-sensitive properties derived from the measured vibration response that allows one to distinguish between undamaged and damaged structures.

According to the features used, the methods based on frequency, mode shapes and other dynamic parameters which correlate to frequency are categorized as methods in frequency domain. Frequency is the most common feature used for damage detection. The amount of literature using shifts in resonant frequencies is quite large. A rather comprehensive review of literature and reference list can be found in Doebling et al. (1996). However, it is also recognized that the frequency shifts have significant practical limitations for applications in various types of structures. The usually low sensitivity of frequency shifts to damage requires either very precise measurements or large levels of damage (Farrar et al., 1994). Mode shape is another common feature for damage detection. West (1984) presented what was possibly the first systematic use of mode shape information for the location of structural damage without the use of a prior FEM. Fox (1992) presented a method of scaling the relative changes in mode shapes to better identify the location of the damage. An alternative to using mode shapes to obtain spatial information about sources of vibration changes is to use

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mode shape derivatives, such as curvature. For beams, plates and shells, there is a direct relationship between curvature and bending strain. Pandey et al.(1991) demonstrated that absolute changes in mode shape curvature could be a good indicator of damage for the FEM beam structures. Stubbs et al. (1992) presented a method based on the decrease in modal strain energy between two structural DOFs, as defined by the curvature of the measured mode shapes. Pandey and Biswas (1994) used changes in flexibility to detect damage on beam model. Wahab and Roeck(1999) employed modal curvatures to detect damage on a real beam-like bridge. Cornwell et al. (1999) investigated the plate-like structures by using strain energy to detect and locate damage. However, the major limitation with the mode shape based methods lies on the difficulties in ensuring the accuracy of measured mode shapes.

The main advantage of frequency domain methods is that the features are only correlated to system characteristics and independent of the external excitation. The problems with this type of methods are mainly in two aspects. In data analysis part the features in frequency domain cannot reflect the structure condition directly. The information of damage factors needs to be interpreted from the change of features by some special procedure. The procedure would involve some dynamic mechanics model; and whether the model can accurately reflect the structural behavior or not will directly affect the interpretation. Another aspect of the problems is associated with the feature extraction. The features are either insensitive to damage or hard to be extracted in practice. Frequency domain methods also face the difficulty in identifying very closely spaced or pseudo-repeated frequencies.

In attempt to avoid the limitations of frequency domain analysis, some time domain methods have been developed. These methods do not require any domain change, thus enable rapid processing of the information and also avoid some additional errors. They also enable the use of short data records. But the simplified estimations could lead to relatively brief results, which may not contain enough information for detailed securitization of damage.

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In recent years, the time-domain methods have advanced towards better detection and identifying the location of damage. In a common time-domain procedure, a model is first established from the measured signals of undamaged system to predict time series data; the difference between the predicted results using the model and the measured one from unknown structural condition would be the clue to the detection of damage. One advantage of these methods is that a mechanical model is not required for data analysis. A comparison between the baseline model and the testing signals may suffice for the detection of damage. This type of quick procedures is sometimes called the signal approach which can be found in the works of Barai and Pandey (1995), Masri et al. (1996), Tsai and Hsu (2002), Fugate and Farrar (2001), Sohn and Farrar (2001). They used either the neural networks to train the model from reference signal for the prediction purpose, or the auto-regressive (AR) model family to train and predict signals. Ma et al. (2003) used a state space model and incorporated the geometric approach for the setting up of residual generators. This method could clearly indicate the damage occurrence on each individual component of a structure. All these works attempt to correlate the changes in the measured nodal time histories to various kinds of damage, and they require that data from undamaged structures to be available. Moreover, the tolerance to the variation of input excitations is very limited.

Some other works in the time domain are more oriented towards the actual identification of the physical parameters via time series models which are more explicitly related to the dynamic properties of the system. Typical works include those by Rhim and Lee (1994), Lee and Yun (1991). In these studies the models are based on Auto-Regressive and Moving Average model (ARMA), which links the excitation and response of the structure as input and output of the model. The ARMA model actually predicts the response at the current step by auto-regression of the response data at previous steps along with the moving average of the random input. On condition that the input to the structure can be measured, this method can work well

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and give satisfactory identification results. The auto-regressive (AR) model family is also used widely in the frequency domain in place of the FFT function.

In attempt to avoid the requirement on the availability of excitation input, as most of the time-domain methods do, Peeters and De Roeck (1999) and Bodeux and Golinval (2003) used subspace identification in which the excitation input is not explicitly required. However, a common assumption in these methods is that the “unknown” input of system is white noise with zero mean, which limits the applicability of this category of methods.

From the perspective of an ideal damage detection scheme, ultimately four levels of functions are expected (Doebeling et al., 1996): 1) to detect the occurrence of damage at a very early stage; 2) to locate the damage within the sensor resolution being used; 3) to provide some estimate of the severity of the damage; and 4) to predict the remaining useful life of the structure. Additional requirements include the automation of the method, which allows for the application in real-time Structural Health Monitoring (SHM). As typical time domain methods do not involve any mechanical models, they can usually only fulfill the first two levels of functions. As a matter of fact, even the ability in identifying the location of damage may be limited to some generic terms. This is because the damage information is extracted from the signals measured from selected degrees of freedom (DOF). When multiple members are connected to the same joint and share a common DOF, it is not possible to use these methods to separate the condition of each connecting member based on the information extracted from the common DOFs.

In summary, most of the existing time domain methods are limited to stationary excitations or white noise type excitations; otherwise require the availability of input to the structure, which however is difficult to measure in many cases. Although in the methods proposed by Fugate and Farrar (2001), Sohn and Farrar (2001) the input is not required, their models have no explicit physical meaning, and consequently, the

sensitivity in damage detection is limited. A more robust method that can identify general changes of a structure but not limited by the input excitation is desired. Such method may fulfill the aforementioned first and second levels of functions in the damage detection. It is also desired to fulfill the third level function for the identification of damage severity. As time series data is used for analysis, such method should also have certain tolerance of measurement noises.

1.2 Objectives and Scope of the Research

This study aims at developing a robust time domain analysis approach suitable for structural health monitoring and damage diagnosis. Specific objectives are enumerated as follows:

- 1) In view of the common limitation of the existing time domain methods, namely low sensitivity to damage and high dependence upon the availability of excitation signals, the formulation of the basic model will stem from system dynamic equations so that the features derived from the model become physically meaningful and sensitive to parameter changes. In this way, it will also be possible to make the model tolerant to certain variation in the excitation.
- 2) Extending from the basic model, a novel time-series analysis method will be established by means of auto-regressive techniques. The research will be focused on the utilization of easily measurable time-series signals (e.g., accelerations) and the disassociation from the actual excitation. Improvements in the ability in detecting, locating and correlating to the degree of damage will be investigated systematically.
- 3) Increasing the robustness of the methods for noise contaminated signals. Appropriate techniques, in particular the Kalman Filter technique, will be adapted in

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the time domain analysis scheme to minimize the influence of random measurement noises on the diagnosis of damage for practical applications.

4) Developing the method into fulfilling the second level and possibly the third level functions for locating the damage and assessing the damage severity. When necessary measurements are provided, the method should be capable of identifying the change of the time-series features due to different damage scenarios, as well as determining the severity of damage.

5) For the verification of the proposed methods, laboratory experiments will be conducted, in addition to numerical investigations, to test the validity of the proposed methodologies in real-life applications with comparison to some other existing methods. Various damage scenarios and excitation forms will be considered in the experiments, and an automated procedure incorporating the proposed time-series analysis model will be put forward to enable on-line monitoring and diagnosis, which may be extended to real life SHM applications.

It should be noted that the methodologies involved in this thesis are based on linear system assumption, as commonly adopted in modal testing and analysis for diagnosis purpose. However, this assumption does not restrict the applicability of the method only to structures without experiencing nonlinear process. The assumption of linearity of the system requires the system to respond linearly during the vibration tests from which signals representing the current state of the system are acquired for identifying the state parameters of the system.

1.3 Thesis Organization

Following the introduction given in this chapter, Chapter 2 presents the literature

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review, in which the system identification and health monitoring methods in time-domain are reviewed. The previous research works on the structural degradation due to fatigue and the monitoring of fatigue damage are also reviewed in this chapter.

In Chapter 3 a new time-series analysis method, which only requires the accelerations of the structure, is formulated based on linear system dynamics for damage diagnosis. Stemming from the linear dynamic system formulation, a prediction model of ARX (Auto-Regressive model with eXogenous input) is constructed from the reference signals. The input of the prediction model is the acceleration response at a specific location (“input” point), and the output of the model are the acceleration responses at other selected locations of the structure. The coefficients of the model are related to the structural dynamic parameters, thus the model could be sensitive to the change of the structure. When damage occurs to the structure, the prediction model set up from the signals of the undamaged structure can no longer predict the new time series measured from the damaged structure well, and the standard deviation (STD) of the residual errors can be used as a general indicator of the degree and location of the damage. As the model is based on the responses of the structure only, it can tolerate certain variation of the actual excitations. A numerical simulation study is included to illustrate the effectiveness of this method.

In Chapter 4, a procedure is presented for the time domain analysis of noise-contaminated vibration signals for global structural damage diagnosis. It extends from the response-only ARX model introduced in Chapter 3. The novel idea in the present procedure is to establish the intrinsic “input” and “measurement” pairs from noisy signals by using the Kalman filter, so that the underlying physical system is best presented to the subsequent diagnosis operation. The theoretical basis of representing the system by the raw measured input and the filtered response through the Kalman filter is discussed. When such raw input and filtered response signals are fed into the reference ARX model, the error feature becomes indicative of the change of the physical system. By analyzing the residual error, the damage status of the structure

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can be diagnosed. Applications to numerical and experimental examples demonstrate that the approach is effective in tackling the noises, and both the location and relative extent of damage can be assessed with appropriate damage features. The autoregressive and moving average with exogenous input (ARMAX) can be used instead of the Kalman filter in the actual filtering operation.

In Chapter 5 another time-domain method is introduced to detect and locate damage (in terms of stiffness changes) in a structure, and furthermore, to calculate the severity of damage. In this method, the change of structural stiffness from different components is looked on as Multi-failure events for state-space equation. For this disturbances decoupling problem (DDP), the geometric concept is used to design residual generators for all components of the structure. These generators process vibration signals of the structure and generate residual error signals. Each generator is only sensitive to the stiffness change of a particular component and is not affected by stiffness changes of other components. This makes it possible to detect and locate the occurrence of damage by monitoring the residual error signals in all the generators. By analyzing the residual error signal with a suitable error model, the damage severity of a component can also be obtained. Numerical results show that the method can effectively determine the occurrence and severity of damage in any members of a structural system.

In Chapter 6 a laboratory experimental study of a frame model is presented. Different forced vibration tests are carried out in the experiment, and various damage scenarios are simulated by physically modifying the (stiffness) properties of test model. With the experimental results and measured signals, the ARX-model based method introduced in Chapter 3 is applied for data analysis and damage diagnosis, and the noise-filtering approach presented in Chapter 4 is employed to improve the diagnosis results. The geometric method introduced in Chapter 5 is also applied to identify the damage, including the damage severity, in the test structure.

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Finally, the main conclusions of the study and the recommendations for future research are summarized in chapter 7.

CHAPTER 2 REVIEW OF THEORY AND PREVIOUS WORK

This chapter is organized into three parts. The first part provides an overview of research on structural damage identification and health monitoring. Several categories of methods are summarized and their merits and inherent limitations are discussed, with focus on the application of linear system theory and the family of auto-regressive models in damage identification and diagnosis in time domain. The remaining two parts review the research work on two special problems in damage diagnosis; one is signal processing for signals with noises, another concerns the identification of damage location in the time domain.

2.1 Overview of Structural Damage Identification and Health Monitoring

The general purposes of vibration-based damage identification are to detect, locate and characterize damage in structural and mechanical systems by examining changes in the measured vibration response. The basic idea behind this technology is that modal parameters (notably frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties will cause detectable changes in the modal properties. In line with the above general concept, the feature extraction has been receiving the most attention for purpose to quantify the changes of signals. Depending on the feature used, the methods can be categorized into frequency domain and time domain methods.

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Although the frequency domain analysis has been explored extensively and proved to be efficient in many cases, it is limited by the difficulty in identifying very closely spaced or pseudo-repeated frequencies, and it must use FFT to estimate modal parameters and apply some time windows. Examining the time histories directly for indications of damage is one way out. Time-history methods are usually non-modal based approaches that examine changes in the features derived directly from measured time histories or their corresponding spectra. In early times, methods like peak amplitude, RMS (root-mean-square) amplitude, crest factor analysis, kurtosis analysis and shock pulse counting have been used to detect the presence, location and type of fault, and the degree of damage. In general, the problem encountered in these methods is that if excitation sources change and/or environmental conditions change, this process becomes very difficult to produce useful diagnosis results.

On the other hand, the so called “time-domain” methods for system identification problems can generally be divided into two categories. The first category covers certain kinds of linear system methods to identify modal parameters of the dynamic systems, such as frequencies, damping ratios and mode shapes. These methods serve the purpose comparable to the FFT and are not strictly methods for the damage detection. Another category of time-domain methods are meant for detecting changes of the underlying systems by analyzing the measured time-history data. The study in this thesis is focused on this category of methods, and the relevant literatures are reviewed accordingly.

Yang et al. (1980, 1984) are among the pioneers who began analyzing time series for the damage identification more than two decades ago. In their proposed method, the random decrement technique was applied to identify damage in a scaled-model

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offshore platform. As the response of a linear system depends on the initial conditions and the applied forcing function, the response can be decomposed into three parts: that caused by the initial displacement, that caused by the initial velocity, and that caused by the forcing function. N number of time series are picked out from measured displacement data with the same time length T . The starting time for each series is taken such that the initial displacement is a constant, and the initial velocity is alternating from positive to negative value. The time series are added together for average. The response to initial velocity will diminish out because it alternates from positive to negative. If the input is random, the effect by the forcing function will also be averaged out. Therefore, only the effect by the initial displacements remains, so the average of time series is a decay curve. This vibration decay curve can be used to identify the resonant frequency of the structure and its damping. The technique requires 400 to 500 averages to produce a repeatable signature. The process does not locate the damage or give an estimate of its severity.

Martin (1989) utilized Kurtosis measurements of the acceleration amplitudes for detection of surface damage in roller bearings. If surface roughness attributes are used as an indicator of damage, then for a good surface, the profile is random corresponding to a Gaussian distribution with an infinite-sample theoretical value of 3.0. A Kurtosis value other than 3.0 denotes that the profile is no longer Gaussian, thus indicating the presence of damage.

Fritzen et al. (1995) described a vibrating system via FEM and reformulated it as a state-space model reduced by a modal transformation. Kalman filters were used to compare the time history response of the system to FEM predictions for the undamaged case and various damaged cases. The output was a distribution of probabilities indicating which of the test cases most likely corresponded to the

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actual measurements. Generally the filters were applied once to locate a damaged element or section and then a second time to further refine the location and quantify the damage. The work was all done in the time domain, and the technique applied directly to nonlinear damage. This method requires FEM models of possible damage states. As the feature indicating damage is a probabilistic factor, the method is not very sensitive to stiffness change and only nonlinear damage by FEM models is studied.

Koh et al. (1995) presented a condensation method for local damage detection of a multi-story frame building. In this paper the stiffness matrix of the multi-story frame building consisted of two parts: the static condensed stiffness and the remedial stiffness. First the condensed stiffness was calculated with the joint rotations accounted for. The remedial stiffness was treated as unknown parameters. Then an Extended Kalman Filtering process (EKF) was applied to identify the unknown coefficients that gave the closest predicted response to the measured response. So the result of pre-damage testing can be used as a benchmark for comparison of each testing. In this method the dynamic system was represented by a state-space model and EKF was applied to identify those non-time-varying parameters. A similar method using EKF was also applied to identify substructure property (Koh et al., 1991). The idea of using EKF for system identification was first presented by Hoshiya and Saito (1984).

Straser and Kiremidjian (1996) pointed out that it was not sufficient to monitor the elastic response before and after the event. In their study the analysis of measured responses was carried out by ordinary least squares, EKF and a substructure approach. Hysteretic models were used to represent the nonlinear behavior of the structure during an extreme event. The EKF was employed to estimate the parameters of the nonlinear hysteretic model from measured input and response

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data.

Barai and Pandey (1995) used back propagation neural networks to identify damage in a truss structure simulating a bridge. They used the neural network to identify the map from various nodal time histories to changes in stiffness. The time histories at small time intervals were used as inputs to the neural network for training. The signals were simulated by a finite element model of the bridge truss with a moving point force to simulate a vehicle at a constant velocity. The time history from a single, carefully selected node produced the best predictions. But the authors did not discuss whether this accuracy was on independent test data or the training set. Also, this method requires stationary excitation.

Also using neural networks, Masri et al. (1996) proposed an approach which could predict response signals of a structure for detecting damage. The vibration measurements from a 'healthy' system are required in this approach. The original signals were used to train a neural network for prediction purposes. Then the trained network was fed comparable vibration measurements from the same structure under different episodes of responses. By comparing the predictions of the neural network and the measured ones after potential structural changes (damage) occurred, quantifiable measures of the degree of fidelity of the predicted response measurements could be used to assess the degree of changes. The structure system considered in a numerical analysis example was a simulated mass-spring system with one degree of freedom (cantilever with concentrate mass at the free end). The inputs of neural network were the measured displacement and velocity, and the output (predicted response) was the restoring force, which could be obtained from the measurement of the exciting force and the response acceleration in this example. For identification, the RMS (root mean square) of error between the output of the network and the actual restoring force were obtained and compared with the RMS

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of the undamaged data. The RMS results had a magnitude that matches the corresponding change of system. But the authors did not explore the possibility of using the approach to detect damage location. Also the discussion was based on numerical study, and no physical experiment was provided.

Tsai and Hsu (2002) also used neural network for damage diagnosis purpose based on time history. The measured displacement time series were used as the characteristic measurements for the diagnosis. In the experiment the displacement-time histories of RC slabs were generated by same external loading that was controlled by a numerical control actuator. The loading condition required by the approach seems too strict to realize in field. This method is rather a laboratory testing method of individual component than a global detection method of structure.

Huang et al. (2003) used the back-propagation neural network approach to setup the model for damage diagnosis. The strategy was training an artificial neural network (ANN) to represent faithfully the linear responses of the undamaged system, using the acceleration responses and input base excitations observed in small energy earthquakes. The structure may experience varying degrees of damage in large earthquakes, exhibiting non-linear behaviour, thus yields notable prediction errors from ANN model. At the input layer of neural network model, there are acceleration responses $\ddot{x}(t-j)$ at time step $(t-j)$, $j=1, 2, \dots, m$, and base excitations $f(t-i)$, with $i = 0, 1, 2, \dots, n$. The step number m and n is dependent on the modes to be identified. The output of layer is acceleration $\ddot{x}(t)$ at the present time step. A five storey frame in shake table testing was examined by the behaviour under different weaken levels of the Kobe earthquake. The responses of each floor were recorded during the test. The ANN model is actually a prediction model using certain previous responses and excitation to predict current responses.

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Fugate and Farrar (2001) presented a vibration-based damage detection approach using statistical process control. The method was applied on laboratory testing of a reinforced-concrete column modeling the bridge column. The shaker rested on a steel plate attached to the top square block of the concrete column with horizontal loading. Forty accelerometers were mounted on the structure. First an auto-regressive (AR) model was fit to the measured acceleration time histories from an undamaged structure. An AR model with p autoregressive terms, $AR(p)$, can be written as:

$$y(t) = \sum_{j=1}^p \phi_j y(t-j) + e(t) \quad (2-1)$$

Details regarding the family of AR models will be introduced in Chapter 3. The above equation means the time-series $y(t)$ can be self predicted by the previous data $y(t-j)$. The coefficients are unknown parameters that can be estimated by measurements. The AR model is a self prediction model, so the acceleration measurements are analyzed separately. The residuals are obtained from fitting the AR model to the observed data. Here the feature extracted from the data was the statistical property of residuals. By analyzing the probability, damage could be detected. However, when the statistical feature changes, it did not necessarily indicate that the damage appeared but only that the system had varied to cause statistically significant changes in its vibration signature. This variability could be caused by a variety of environmental and operational conditions that the system of interest was subjected to. The statistical process control was used to identify the damage from the statistical feature changes of signals.

Sohn and Farrar (2001) extended the above method. Instead of using AR model, a two-step AR-ARX model was used for predication. The model was constructed by two step procedure with auto-regressive (AR) and auto-regressive with eXogenous input (ARX) model. In the first step, an $AR(p)$ model was set up by the

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measurements, and generated least error between the prediction and measurement; then, the data was predicted by a baseline $ARX(a, b)$ model with the eXogenous term. The input of the ARX was the error generated in the first step. Then another residual error was generated between the prediction of ARX and the measurement. The standard deviation (STD) of this new error is the damage feature. The time step of $AR(p)$ was set to be 30 based on a partial auto-correlation analysis described in Box et al. (1994). Ljung (1987) suggested keeping the sum of a and b smaller than p , $(a+b) < p$, so the $ARX(a, b)$ in this example was $ARX(5, 5)$. It was expected that when damage appeared, the largest increase of STD would appear near the damage location, so the method is presumably capable of detecting the damage location. The example lab experiment was an 8 DOF mass-spring system. The only vibration signals used in this method were response accelerations at the eight mass points. The method was applied to analyze the 8 signals independently, for detecting the nonlinear damage. The damage case was generated by putting a bumper between two adjacent masses. Both the simulation study and experimental data showed that the method can identify this kind of damage very well. But the simulation study also showed this method is only suitable when the stiffness change of system is from linear to non-linear. In Chapter 3 more details about the analysis procedure and the results of this method will be discussed when the method is used as a benchmark for comparison with the new method presented in Chapter 3.

Ma et al. (2003) applied the residual generator technique in the time-domain damage diagnosis. This approach can detect structural changes of specific components separately, without being affected by changes of other components. The formulation of the method starts from a state-space model of the structural dynamic system. When the baseline generators (state-space models) are set up for each component, the measured signals can be applied to generate residual errors. When a component is damaged, the error signal from the corresponding generator

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could be surging, while the errors from the other generators remain zero. The detail of this method will be reviewed in section 2.3 and Chapter 5. The results reported in Ma et al. (2003) showed that the method is applicable for both stationary and non-stationary excitations.

Almost all the methods described above require that data from undamaged structures is available as baseline. The general procedure of these methods may be summarized as follows: 1) a model is set up which can predict the time series data from the measured signals for the undamaged system; 2) a statistical variable between the predicted signals and the measured ones is employed as damage feature; 3) calculate the feature between the predicted one and the measured one of undamaged system as reference; 4) apply the “unseen” signals to the prediction model and compare the damage feature with the reference to detect damage. It can be seen that the prediction model is critical in these methods. Such prediction models can be realized by a neural network, an auto-regressive linear system, and so on. Among them, the linear model in the form of auto-regressive model or state-space model provides the simplest expression of the model. Also in some methods the linear system model is derived from the equation of motion, such that the model parameters can reflect more directly the relationship between input and output of a dynamic system. It provides clues for further analysis concerning the damage location and severity. As the estimation and prediction of auto-regressive linear system is widely used in digital control system and system identification, a standard procedure can be used to estimate the parameters directly.

2.2 Overview of Linear Models for Signal Analysis of Structural Dynamic Systems

As linear models, in particular the state-space model and AR models, can represent the dynamics of structural systems, they may be used as prediction models for damage diagnosis in the time domain. According to the requirements on the availability of excitation signals, the linear models may be classified into two categories: a) Excitation as input is available; c) Excitation is unavailable.

2.2.1 Excitation available

The equation of motion of a dynamic system may be written as:

$$\ddot{\xi}(t) + \mathbf{J}\dot{\xi}(t) + \mathbf{K}\xi(t) = \mathbf{L}u(t) \quad (2-2)$$

where $\mathbf{J} = \mathbf{M}_0^{-1}\mathbf{C}_0$, $\mathbf{K} = \mathbf{M}_0^{-1}\mathbf{K}_0$, and $\mathbf{L} = \mathbf{M}_0^{-1}\mathbf{L}_0$.

Based on Eq.(2-2), the continuous time state-space model representation of a structural dynamic system can be directly derived in a compact form as (refer to Chapter 3 for more detail):

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c u(t)$$

$$\mathbf{y}(t) = \mathbf{C} \mathbf{x}(t) + \mathbf{D} u(t) \quad (2-3)$$

$$\text{with the definition of } \mathbf{A}_c = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{K} & -\mathbf{J} \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} 0 \\ \mathbf{L} \end{bmatrix}, \quad \mathbf{x}(t) = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \end{bmatrix}. \quad (2-3a)$$

Consider the observed responses $\mathbf{y}(t)$ to be accelerations, $\mathbf{y} = \ddot{\xi}(t)$, the coefficients $\mathbf{C}$ and $\mathbf{D}$ in Eq.(2-3) can be obtained as $\mathbf{C} = [-\mathbf{K} \quad -\mathbf{J}]$, $\mathbf{D} = \mathbf{L}$ (Hoshiya and Saito, 1984; Quek, et al., 1999; Shi et al., 2000; Lardies and Larbi, 2001; Hung, et al., 2004). It should be noted that such state-space model as Eq.(2-3) is not unique in representing Eq.(2-2). There is an alternative state-space model with coefficients as

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(Lee and Yun, 1991):

$$\mathbf{A}_c = \begin{bmatrix} -\mathbf{J} & \mathbf{I} \\ -\mathbf{K} & 0 \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} -\mathbf{J}\mathbf{L} \\ -\mathbf{K}\mathbf{L} \end{bmatrix}, \quad \mathbf{C} = [\mathbf{I} \quad 0], \quad \mathbf{D} = \mathbf{L}. \quad (2-3b)$$

Consequently, the state-space model or its discretization can be used as an analysis model for the response signals of the dynamic system, as expressed in Eq.(2-3). One of the advantages using the state-space model is that some error function as Kalman filter can be applied. The application of Kalman filter improves the robustness of the model for noise-contaminated measurements. With adequate estimations, the model parameters can be used for modal analysis (Quek et al., 1999; Shi et al., 2000). Also the structural parameters such as stiffness matrix and damping matrix, which are time invariant, may be identified by EKF (Hoshiya and Saito, 1984; Koh et al., 1991; Koh et al., 1995). Various subspace methods can also be used to study the state-space model (Abdelghani et al., 1998; Ma et al., 2003).

The dynamic system of Eq.(2-2) can also be expressed by ARX (or called ARMA) model. A transfer function can be generated between responses and excitation based on Eq.(2-2). Then taking the Z-transformation of this transfer function, the ARX model can be obtained (refer to Chapter 3). The same ARX model can also be directly derived from the discrete state-space model. Unlike the state-space model, such ARX model can be uniquely expressed as:

$$\mathbf{y}(k) = \mathbf{P}_1\mathbf{y}(k-1) + \mathbf{P}_2\mathbf{y}(k-2) + \mathbf{D}\mathbf{u}(k) + \mathbf{E}_1\mathbf{u}(k-1) + \mathbf{E}_2\mathbf{u}(k-2) \quad (2-4)$$

Lee and Yun (1991) derived the relationship between the ARX coefficients and structural parameters (stiffness matrix and mass matrix). With the required data, the procedure first estimates the coefficients of ARX, and then identifies the structural parameters (stiffness matrix and mass matrix) from the coefficients by an appropriate algorithm. An application example was given with a 3-DOF shear frame tested on a shake table.

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Rhim and Lee (1994) used a back-propagation neural network to identify damage in a composite cantilevered beam. Before the neural network was applied, a preprocessing step was done on the data. An ARX model was performed on the transfer function of the beam from the force input to the displacement output. The advantage of doing the system identification first is that it reduces the body of data to a smaller number of physically meaningful parameters. The parameters of ARX model and the corresponding damage cases can form the training pairs.

2.2.2 Excitation unavailable

The linear models introduced above can be applied as analysis model for dynamic signals of structures. However, sometimes the excitation like ambient vibration is too complicated to be defined or measured. In such cases the model has to be expressed with the response signals only.

Brincker et al. (1995) applied an ARMA model from measured acceleration-time histories to estimate the modal parameters (resonant frequencies and modal damping) of a reinforced concrete offshore oil platform. The output of the ARMA model was defined as the measurement of response, while the moving average part was the residual error between measurement and prediction, which was assumed to be white noise. The AR parameters can be related to the modal frequencies and damping of the system. Following their work, Saito and Yokota (1996); He and Roeck (1997); Huang (2001); Lardies and Larbi (2001); Neild et al. (2003); Hung et al. (2004) applied and refined the method. In their work the AR models appeared to be good transfer functions of signals instead of the FFT analysis.

Saito and Yokota (1996) presented a method using earthquake response for the

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modal identification. A high-rise building was studied with its acceleration responses measured under low intensity earthquakes. The ARX model was set up with responses at the first level as input and the responses at middle and top levels of the building as output. The ARX model was used for signal processing and the modal identification.

He and De Roeck (1997) proposed a high order multivariate AR model for the modal analysis. By selecting a proper order of the model, the model could be established and model parameters estimated for the modal analysis. Similar model was used in Huang (2001), and a modification of the traditional estimation was presented as least-squares approach to determine the coefficients of the model. The AR model does not require the excitation information so the method was applied successfully to a three-span highway bridge from its ambient vibration due to traffic flow.

Peeters and De Roeck (1999) proposed a method using subspace identification with output-only measurements by ambient tests. In their method the ambient input was assumed to be white and uncorrelated random signals with zero mean. With this assumption the input was actually treated as noise. The Kalman filter is applied in this subspace model for the state estimation. Bodeux and Golinvall (2003) improved this method in model estimation by using ARMAV (Auto-Regressive Model with moving Average Vector).

Lardies and Larbi (2001) used a vector auto-regressive moving average VARMA model for the modal analysis. The output-only model using acceleration responses as the output and input of the system is looked on as an unknown white noise. First the method determines the model order using a combination of the multivariate minimum description length (MDL) criterion and an over-determined instrumental

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variable sequence. Then the AR parameters obtained by the instrumental variable sequence, frequencies and damping ratios can be estimated by an appropriate procedure.

Hung et al. (2004) proposed a modal identification system based on vector backward auto-regressive with exogeneous (VBARX) model. When the excitation signal is available, the ARX model can easily determine the true order of the system models from the spurious modes. When the input data are not available, the VBAR model is used instead with an increased order. Over-specifying the order of the model was found to be an effective method for increasing the accuracy of the eigenvalues estimated from noisy measurements when the signal to noise ratio was not particularly high. In order to identify the true modes from the spurious one, the eigenvalues estimated by a forward AR model should be studied. By a backward model the system eigenvalues are outside or on the unit circle, and spurious ones lie inside the unit circle. Numerical data were simulated with a 3-DOF dynamic system with various types of input and noise levels. The model used a combination of displacement, velocity as output, and then extracted the modal parameters, namely the resonant frequencies and damping ratios by the AR parameters.

The main purpose of the above mentioned models is limited to estimating the AR coefficients in order to determine the modal properties such as the natural frequencies and damping ratios. In case the excitation signal is not available, it is assumed as the Gaussian white noise, and subsequently the model is redefined as ARMA, with a Moving Average part of residual error instead of the eXogenous part of excitation (Lardies and Larbi, 2001; Hung et al., 2004). Such assumption may reduce the accuracy when the model is applied for signal prediction for damage diagnosis purpose. It would be more desirable if a model does not require explicitly the excitation signal.

2.3 Noise Filtering Process

In the process of data acquisition from physical testing, it is hardly possible to prevent the signals from being polluted by noise. Subsequent analysis based on the measured signals will inevitably be affected by the noise contents in the signals. For the general time-domain damage diagnosis methods introduced above, this problem could become very significant when performing the model estimation and signal prediction. In the model estimation stage, some statistic features could be altered by the noise, consequently the model parameters could be wrongly estimated. In the signal prediction stage, the noise is also an uncertain factor that affects the performance of the model. Therefore, a damage diagnosis method with a time-domain model should be examined under a realistic noisy condition.

As mentioned above, when noise is introduced in the signals, the reference model parameters could be wrongly estimated. For the purpose to minimize the noise effect, some methods have been proposed in the past for application in the analysis of AR models. One way is to increase the model order so as to increase the accuracy of the model in a noisy condition, as shown in the studies by He and Roeck (1997), Lardies and Larbi (2001), Huang (2001), and Hung et al. (2004). The model order increasing approach is commonly used in identifying the dynamic characteristics. But due to the use of a high order model, extra spurious modes will be brought in and affect the accuracy of prediction. Therefore this method is not suitable for signal predictions.

On the other hand, much attention has been given recently to subspace methods for identifying state-space models. Abdelghani et al. (1998) presented a detailed comparison study of these subspace identification methods, including the

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eigensystem realization algorithm with observer/Kalman filter Markov parameters computed from input/output data (ERA/OM), the robust version of the numerical algorithm for subspace state-space system identification (NJ4SID), and a refined version of the past outputs scheme of the multiple-output error state-space (MOESP) family of algorithms. The subspace models with discrete filter are applicable for linear structures and recordings with wide-band measurement noise, which is usually the case in real life situations.

In association with the state-space models, the Kalman Filter or Extended Kalman Filter (EKF) has been widely adopted in studying time-domain signals, as reported by Hoshiya and Saito (1984), Hoshiya et al. (1984), Koh et al. (1991), Koh et al. (1995), Quek et al. (1999), Shi et al. (2000). The Kalman filter invented by Kalman in 1960 is an effective tool for stochastic estimation of the state from noisy measurements. Because of its relative simplicity and robust nature, the Kalman filter has been widely used to obtain estimates of state variables in practice. The Extended Kalman Filter (EKF) is a generalization of the steady-state filter for time-varying systems or LTI (linear time-invariant) systems with nonstationary noise covariance.

The Kalman Filter can identify optimal parameter values of state space equations, because the Kalman Filter approach inherently has the flexibility of incorporating system dynamics equations into the algorithm as well as the provision for uncertainty in the system. Hoshiya and Saito (1984) presented a method using Extended Kalman Filter (EKF) to system identification problems of seismic structural systems. The basic model was a state-space model with displacement as output. The stiffness and mass parameters were looked on as time-invariant state vectors in the model. In order to obtain the stable and convergent solutions, a weighted global iteration procedure with an objective function was proposed for stable estimation, which was incorporated into the Extended Kalman Filter algorithm.

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Koh et al. (1991) used the above method to identify structural parameters. The substructure technique was employed to decompose the structural system into smaller subsystems for identification purposes. Then the EKF was applied to estimate structural parameters.

Koh et al. (1995) presented a condensation method for local damage detection of a multi-story frame building. In this method the stiffness matrix of the building K was combined by two parts, namely K_c - the stiffness static condensation, and K_r - the remedial stiffness matrix, which was identified by applying the EKF. By applying the EKF on a time window, the remedial one was identified and the stiffness matrices were updated. The method was verified experimentally by a laboratory model of 6-story steel-frame building.

Quek et al. (1999) introduced the eigenspace structural identification technique for tall buildings subjected to ambient excitations. The algorithm was for both displacement and velocity responses data. The model was a state-space model for the measured response of either displacement or velocity. The input was an unknown pure white noise. The measured responses were contaminated by white noise. In order to give good results for signals contaminated by noise, the algorithm was based on Kalman sequence to provide good estimation of model parameters. A series of numerical shear building models were used as examples to illustrate the application of the method.

Shi et al. (2000) also used Kalman filter algorithm to identify the model parameters in frequency domain. The Kalman filter was applied to identify power spectrum density of a single-DOF vibrational system. The Kalman filter based modeling approach showed good potential for simultaneously estimating both the system and

input characteristics. Both Shi et al. (2000) and Quek et al. (1999) adopted the following model assumption:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) = \mathbf{A}\mathbf{x}(k) + \mathbf{w}(k) \\ \mathbf{y}_v(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k)\end{aligned}\tag{2-5}$$

in which the excitation $\mathbf{u}(t)$ was regarded as immeasurable, and was looked on as a pure white noise input excitation $\mathbf{w}(t)$ passing through a filter.

Safak (1991) used some discrete filters as high order single-input single-output (SISO) AR model for system identification. The paper presented a method to extract frequencies and damping ratios from the AR parameters, which were discrete model parameters. The residual error was studied by auto-correlation and cross-correlation to test the validity of the model. Allen et al. (2003) used a similar procedure to optimize the order of time domain models. The underlying theory of this procedure was presented by Box et al. (1994).

2.4 Determination of Damage Location

The determination of damage location is one of the most challenging problems in system identification. It may be done in both frequency domain and time domain. In the frequency domain, the changes in mode shape curvature have been found as a good indicator of damage location. Pandey et al.(1991), Stubbs et al.(1992), Pandey and Biswas (1994), Wahab and Roeck (1999), Cornwell et al. (1999), Andrea et al.(2004) all used the mode shape or mode shape curvature in their methods for locating the damage.

Choi and Stubbs (2004) presented a method using time-domain response to locate and size the damage in a structure. The method used the mean strain energy for each

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element to represent the stiffness change. The basic idea of this method was to calculate strain energy of the structure, $U = \frac{1}{2} \mathbf{y}(k) \mathbf{K} \mathbf{y}(k)^T$, and the strain energy for the j th element, $U_j = \frac{1}{2} \mathbf{y}(k) \mathbf{K}_j \mathbf{y}(k)^T$. Based on the strain energy calculated, a series of steps can be performed to determine which element is damaged and the severity of damage. Due to the good sensitivity of strain energy to damage location, the method is presumably feasible. But further studies showed there could be difficulties in calculating the strain energy in the time-domain. In an example beam-like bridge model presented in the paper, the time-domain response $\mathbf{y}(k)$ was chosen as a column vector of translational displacements at the element nodes. Accordingly, the stiffness $\mathbf{K}$ must be a condensed stiffness matrix without the rotational DOFs, and $\mathbf{K}_{0j}$ is the contribution of j -th element stiffness to the global stiffness matrix. Even with a statically condensed matrix, it was found almost impossible to determine such contribution by one element to the system with only displacement measurements. There has not been any good method yet for locating damage in a continuous object as beams using time-domain information before rotational measurement becomes readily available.

Nevertheless, for a multi-degree-of-freedom (MDOF) system with lumped masses and shear (sway) type deformation, it is possible to locate damage using time-domain data (Sohn and Farrar, 2001; Lu and Gao, 2005). In such cases, the location of damage may be detected due to the fact that larger residual errors tend to occur near the actual damage locations. However, the ability of these methods in identifying the location of damage is limited by the sensor resolution. It is also limited to relatively simple systems, and in a “lump-sum” manner. This is because the damage information is extracted from the signals measured from selected degrees of freedom (DOF). When multiple members are connected to the same joint and share a common DOF, it is not possible to use these methods to separate the condition of each connecting member based on the information extracted from the

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common DOFs.

To overcome the above difficulty in using the time-domain methods, it is essential to find a more explicit way to link the damage of each individual component to a certain damage feature. Furthermore, it is desired that the damage severity of each component can be calculated by directly analyzing the time series of the vibration signals.

Ma, et al. (2003) presented a time-domain method for detecting structural changes of specific components separately. The study started from a system that was a representation of a 3-DOF linear lumped mass shear beam model as:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \Delta\mathbf{A}\mathbf{x}(t), \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t).\end{aligned}\tag{2-6}$$

where parameters $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ can all be determined from the mass, stiffness, and damping coefficients of the structure. The input of the system is the excitation from ground, while the output of the system are displacements, velocities or their linear combinations. $\Delta\mathbf{A}$ represents the damages in the structure; $\Delta\mathbf{A}$ is a zero matrix if there is no damage. $\Delta\mathbf{A}$ can be further written as $\sum_{i=1}^k \mathbf{L}_i \mathbf{C}_i$, where k is the number of stiffness coefficients in the structure; $\mathbf{L}_i$ and $\mathbf{C}_i$ are column and row vectors, respectively, which are decomposed from the contribution of the damage in the i -th component to the change of the global stiffness matrix. Defining $m_i(t) = \mathbf{C}_i \mathbf{x}(t)$, the model can be written as:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \sum_{i=1}^k \mathbf{L}_i m_i(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t).\end{aligned}\tag{2-7}$$

This becomes the standard form of the Extension of Fundamental Problem of

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Residual Generation (EFPRG) to multiple failure events. The EFPRG was initially defined by Massoumnia (1986) as a disturbances decoupling problem (DDP) in the control realm. A solution known as the residual generator technique was also established by using the geometric approach (Massoumnia, 1986; Massoumnia et al. 1989). Ma et al. (2003) used this technique to design three generators respectively for the three springs in the system concerned. By processing the measured signals each generator could detect the failure of individual components separately. The results showed that the method was applicable for both stationary (white noise) and non-stationary (El Centro earthquake) ground excitations. But the method was limited to using displacement or velocity vectors. Moreover, the paper did not provide any explicit algorithm for calculating the damage severity.

In this thesis further study has been done to extend the application of the residual generator technique to more practical situations. In particular, the acceleration responses of structure are used for analysis instead of displacement. A theoretical formulation and a calculation algorithm are also provided for explicit damage severity estimation in this thesis.

CHAPTER 3 A NEW AUTO-REGRESSIVE MODEL FOR STRUCTURAL HEALTH MONITORING AND DAMAGE DIAGNOSIS

In this Chapter a novel time series model is proposed for the diagnosis of structural damage. Two major issues need be addressed when considering time domain data for damage detection, one is a damage sensitive feature and the other concerns the fact that the input excitation usually is not measurable. The present approach stems from the linear dynamic system theory and it is formulated in the form of a prediction model of auto-regressive with eXogenous input (ARX). With some simplifications, the model is expressed such that only response (acceleration) signals are involved, with response at one location chosen as the “input” of the model. The model coefficients correlate with the dynamic properties of the structure and they can be established from reference-state response signals. The residual error of the established model when applied on actually measured signals reflects the structural change, and the standard deviation of the residual error is found to be a damage sensitive feature. Numerical examples demonstrate that the method can be applied for a rapid detection of structural changes and it can also indicate the damage locations. Furthermore, the model can tolerate certain variation of the actual excitation. The model provides a basis for developing more robust damage sensitive features for real applications.

3.1 Introduction of Linear Time-Invariant Model

As can be seen from the review of literature in Chapter 2, the linear model is widely used in time domain methods as it provides a simple expression of the system model. In fact, the Linear Time-Invariant system (LTI) constitutes probably the most important class of dynamical systems in practice. In the context of damage diagnosis, it also includes damaged systems in the sense that the occurrence of

certain degree of damage has resulted in a certain change of the physical properties, in particular the reduction of stiffness, of the structure, and hence, the damaged system becomes effectively a new linear system at the damaged state.

The linear system theory states that the output response of a linear system to a linear combination of inputs is the same as the linear combination of the output responses of the system to individual inputs. In this section the background of LTI models in the form of state-space models and auto-regression models is introduced. In particular, the formulation of the structural dynamic system with excitation input $\mathbf{u}(t)$ and responses output $\mathbf{y}(t)$ is given. This formulation provides the basis for the subsequent derivation of the ARX model in this Chapter and the geometric method presented in Chapter 5.

3.1.1 State-space model

The advantage of using state-space description is that differential equations can easily be written as state-space model in continuous form. Thus it becomes a good tool to describe a dynamic system with explicit physical meaning in its model parameters. It can then be used as a basis to derive ARX models. There are also methods incorporating discrete filters that are based on state-space model representation of the dynamic systems. These methods are applicable for linear structures with measurements containing wide-band noises, which is usually the case in real life situations. The method proposed in Chapter 4 is of this category. Some subspace models may also be based on state-space model, and they provide more detailed information about the status of the system. The model presented in Chapter 5 belongs to this category.

In a state-space model the relationship between the input, noise, and output signals is written as a system of first-order differential or difference equations using an auxiliary state vector $\mathbf{x}(t)$. The state-space model may be expressed in continuous time, such as

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad (3-1)$$

The model can also be expressed in discrete time. In discrete time state space, it is assumed that the input and response are constant over the time interval $kT \leq t < (k+1)T$, in which Δt is the sampling period. There is a discrete input signal:

$$\mathbf{u}(k \Delta t) = \mathbf{u}(t); \quad t = \Delta t, 2 \Delta t, \dots, N \Delta t$$

and a discrete output signal:

$$\mathbf{y}(k \Delta t) = \mathbf{y}(t); \quad t = \Delta t, 2 \Delta t, \dots, N \Delta t$$

Then the differential equation can be solved from $t = k \Delta t$ to $t = k \Delta t + \Delta t$, yielding (Phillips and Nagle, 1984):

$$\mathbf{x}(k \Delta t + \Delta t) = \mathbf{A}_D \mathbf{x}(k \Delta t) + \mathbf{B}_D \mathbf{u}(k \Delta t)$$

$$\text{where } \mathbf{A}_D = \sum_{j=0}^{\infty} \frac{(\mathbf{A}_c \Delta t)^j}{j!}, \text{ and } \mathbf{B}_D = \mathbf{A}_c^{-1} (\mathbf{A}_c - \mathbf{I}) \mathbf{B}_c.$$

The measurement equation can equivalently be given in the sampled-step form

$$\mathbf{y}(k \Delta t) = \mathbf{C}\mathbf{x}(k \Delta t) + \mathbf{D}\mathbf{u}(k \Delta t)$$

The discrete time state-space model thus becomes:

$$\mathbf{x}(k+1) = \mathbf{A}_D \mathbf{x}(k) + \mathbf{B}_D \mathbf{u}(k)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \quad (3-2)$$

The number of parameters in state-space model is usually too large to estimate directly. Besides, to represent a system the state-space model is not unique to express; with different definitions of vector space $\mathbf{x}(t)$ the model will be different. The common method is define $\mathbf{x}(t)$ first to restrict some of the parameters. Then a least square technique as described by Ljung (1987) can be applied to achieve the minimum error and estimate the rest of parameters.

3.1.2 Auto-Regressive model

Auto-Regressive (AR) model in essence is a transfer function. It assumes signals are related by a linear system. Unlike state-space model, an AR model can only be expressed in discrete time. But an AR model contains less model parameters and

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represents the same system with a simpler expression. Thus, the required computation for both model estimation and signal prediction is far simpler for an AR model than a state-space model. In practice it is widely used for time-series data analysis.

An example of AR models is as follows. Assuming the sampling interval is Δt , there is a discrete input signal:

$$\mathbf{u}(k \Delta t) = \mathbf{u}(t); \quad t = \Delta t, 2 \Delta t, \dots, N \Delta t$$

and a discrete output signal:

$$\mathbf{y}(k \Delta t) = \mathbf{y}(t); \quad t = \Delta t, 2 \Delta t, \dots, N \Delta t$$

Thus,

$$\mathbf{y}(k) - 1.5\mathbf{y}(k-1) + 0.7\mathbf{y}(k-2) = 0.9\mathbf{u}(k-2) + 0.5\mathbf{u}(k-3)$$

Such a relationship enables one to compute the output $\mathbf{y}(k)$. If the input is known and the disturbance can be ignored, the predicted output at current time is written as:

$$\hat{\mathbf{y}}(k) = 1.5\mathbf{y}(k-1) - 0.7\mathbf{y}(k-2) + 0.9\mathbf{u}(k-2) + 0.5\mathbf{u}(k-3)$$

The output at time $t=k \Delta t$ is thus computed as a linear combination of past outputs and past inputs. It follows, for example, that the output at time $t=k \Delta t$ depends on the input signal at many previous time instants.

Introducing a shift operator q^{-1} :

$$q^{-1}u(t) = u(t-1)$$

$$q^{-2}y(t) = y(t-2)$$

The general parametric model can be written as:

$$P(q)y(t) = E(q)u(t) + H(q)e(t) \quad (3-3)$$

where P, E and H are polynomials in the delay operator q^{-1} :

$$P(q) = 1 + P_1q^{-1} + \dots + P_{na}q^{-na}$$

$$E(q) = E_0 + E_1q^{-1} + \dots + E_{nb-1}q^{-(nb-1)}$$

$$H(q) = 1 + H_1q^{-1} + \dots + H_{nc}q^{-nc}$$

Here, the integer numbers na , nb and nc are the orders of the respective polynomials. The number nk is the number of delays from input to output. The model is usually written as:

$$P(q)y(t) = E(q)u(t - nk) + H(q)e(t) \quad (3-4)$$

The AR (auto-regressive) model is obtained for $nb=nc=0$. In this model only the previous outputs are used to calculate the new output. The ARMA (auto-regressive and moving average) model is obtained for $nb=0$. Both AR and ARMA describe a stationary process of a system using output data only. The MA (moving average) part provides a way of representing the disturbance, and it effects as a filter for the residual error. The ARX (auto-regressive with eXogenous input) model corresponds to $nc=0$. Such a model setting is particularly common in signal-processing applications. The ARMAX (auto-regressive and moving average with eXogenous input) structure takes the complete form of the Eq. (3-3), so it is the most common model and also represents the general form for types of linear state-space models. The ARMAX model has become a standard tool in control and econometrics for both system description and control design (Chatfield, 2001).

The least-square criterion, as the unique feature developed from the linear parametrization and the quadratic criterion, can be computed for parameters estimation (Golub and Van Loan, 1983; Ljung 1987).

3.2 Formulation of Linear Model for Structural Dynamic System

In this section the linear model is applied for the structural dynamic system. Based on the equation of motion, LTI model can be derived to represent idealizations of the dynamic processes with excitation input and responses output in structure.

For a structural system with n degrees of freedom (DOF), the equation of motion of the system can be described as:

$$\mathbf{M}_0 \ddot{\xi}(t) + \mathbf{J}_0 \dot{\xi}(t) + \mathbf{K}_0 \xi(t) = \mathbf{L}_0 \mathbf{u}(t), \quad (3-5)$$

where $\mathbf{u}(t)$ is an m -dimensional input vector; $\mathbf{M}_0$, $\mathbf{J}_0$, $\mathbf{K}_0$ are $n \times n$ mass, damping, and stiffness matrices; $\mathbf{L}_0$ is the $n \times m$ input coefficient matrix.

Eq. (3-5) can be rewritten as:

$$\ddot{\xi}(t) + \mathbf{J} \dot{\xi}(t) + \mathbf{K} \xi(t) = \mathbf{L} \mathbf{u}(t), \quad (3-6)$$

where $\mathbf{J} = \mathbf{M}_0^{-1}\mathbf{J}_0$, $\mathbf{K} = \mathbf{M}_0^{-1}\mathbf{K}_0$, and $\mathbf{L} = \mathbf{M}_0^{-1}\mathbf{L}_0$.

The dynamic system as Eq. (3-6) can be described as a state-space model, which has input defined as excitation $\mathbf{u}(t)$, and output as structural dynamic response signal like displacement $\xi(t)$, velocity $\dot{\xi}(t)$ or acceleration $\ddot{\xi}(t)$. As acceleration is the most common dynamic response for measurement in testing, the present study focuses only on models with acceleration responses. In order to set up the relationship between the excitation and the response, especially for multi-input and multi-output (MIMO) systems, the state space equation is considered first. With the definition of vector space as:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} \xi(t) \\ \dot{\xi}(t) \end{bmatrix},$$

and consider the observed responses $\mathbf{y}(t)$ to be accelerations, $\mathbf{y} = \ddot{\xi}(t)$, from Eq.(3-6) it follows:

$$\dot{\mathbf{x}}_1(t) = \mathbf{x}_2(t) \quad (3-7a)$$

$$\dot{\mathbf{x}}_2(t) = -\mathbf{K}\mathbf{x}_1(t) - \mathbf{J}\mathbf{x}_2(t) + \mathbf{L}\mathbf{u}(t) \quad (3-7b)$$

$$\mathbf{y}(t) = -\mathbf{K}\mathbf{x}_1(t) - \mathbf{J}\mathbf{x}_2(t) + \mathbf{L}\mathbf{u}(t) \quad (3-7c)$$

Defining $\mathbf{A}_c = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{K} & -\mathbf{J} \end{bmatrix}$, $\mathbf{B}_c = \begin{bmatrix} 0 \\ \mathbf{L} \end{bmatrix}$, $\mathbf{C} = [-\mathbf{K} \quad -\mathbf{J}]$, $\mathbf{D} = \mathbf{L}$, Eq. (3-7a, b, c) can be consolidated into the state-space form as:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned} \quad (3-8)$$

Eq. (3-8) is the state-space model representing the dynamic system described by Eq. (3-6).

The state-space model can have different expressions for a certain system with different definitions of the vector space. For the convenience of further analysis concerning the physical properties of the system, it is desired to formulate a state-space model which is the Observable Canonical Form (OCF) (Wonham, 1985) of the system stiffness and damping. Based on Eq.(3-6), a vector space is defined as $\mathbf{x}_1(t) = \ddot{\xi}(t) - \mathbf{L}\mathbf{u}(t)$, $\mathbf{x}_2(t) = -\mathbf{K}\dot{\xi}(t)$.

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The following can be obtained through the system filter:

$$\dot{\mathbf{x}}_1(t) + \mathbf{J}\mathbf{x}_1(t) - \mathbf{x}_2(t) = -\mathbf{J}\mathbf{L}\mathbf{u}(t) \quad (3-9)$$

Eq.(3-9) can be rewritten into a set of first-order differential equations as:

$$\begin{aligned} \dot{\mathbf{x}}_1(t) &= -\mathbf{J}\mathbf{x}_1(t) + \mathbf{x}_2(t) - \mathbf{J}\mathbf{L}\mathbf{u}(t) \\ \dot{\mathbf{x}}_2(t) &= -\mathbf{K}\mathbf{x}_1(t) - \mathbf{K}\mathbf{L}\mathbf{u}(t) \end{aligned} \quad (3-10)$$

With the state vector $\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix}$, the state-space equation of acceleration

becomes:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned} \quad (3-11)$$

where $\mathbf{A}_c = \begin{bmatrix} -\mathbf{J} & \mathbf{I} \\ -\mathbf{K} & \mathbf{0} \end{bmatrix}$, $\mathbf{B}_c = \begin{bmatrix} -\mathbf{J}\mathbf{L} \\ -\mathbf{K}\mathbf{L} \end{bmatrix}$, $\mathbf{C} = [\mathbf{I} \quad \mathbf{0}]$, $\mathbf{D} = \mathbf{L}$. This state-space form is called the OCF because the state of Eq.(3-11) is always observable regardless of $\mathbf{J}$ and $\mathbf{K}$.

For discrete-time response and input signals $\mathbf{y}(k\Delta t)$ and $\mathbf{u}(k\Delta t)$, according to Eq. (3-2) the state-space model in discrete time equations corresponding to Eq. (3-8) can be obtained as:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{aligned} \quad (3-12)$$

where $\mathbf{u}(k)$, $\mathbf{y}(k)$, and $\mathbf{x}(k)$ denote the sampled data at $t = k \Delta t$, and the coefficients can be expressed as:

$$\mathbf{A} = e^{\mathbf{A}_c \Delta t} \quad (3-12a)$$

$$\mathbf{B} = \int_0^{\Delta t} e^{\mathbf{A}_c \tau} d\tau \mathbf{B}_c = (\mathbf{A} - \mathbf{I})\mathbf{A}_c^{-1} \mathbf{B}_c \quad (3-12b)$$

For numerical applications, Eq.(3-12) can be used to calculate the parameter $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ with the stiffness and damping given. It could also be used in an inverse procedure for system identification (Lee and Yun, 1991) or modal identification (Hoshiya and Saito, 1984; Saito and Yokota, 1996; He and Roeck, 1997; Quek, et al., 1999; Shi, et al., 2000; Huang, 2001; Lardies and Larbi, 2001; Neild et al., 2003; Hung, et al., 2004). For these types of applications, some practical procedures may

be worked out to transform a state-space model to an ARX model or perform an inverse procedure. However, an explicit relationship between the ARX model parameters and the physical properties of a system has never been stated. In what follows an ARX model will be explored and developed with response-only signals in the input and output of the model.

3.3 Derivation of a Novel ARX Model for Damage Diagnosis

As discussed in Chapter 2, models using excitation as input and acceleration responses as output may be used for damage diagnosis in the time domain. But the application of such methods in structural monitoring is limited due to the requirement on the availability of excitation signals. To enhance the robustness of a time-series model for the monitoring of structural damage, it would be desirable if the model can be constructed from the measured response signals only. As such, the model may also become relatively independent from the types of excitation that the structure undergoes.

Before attempting to set up a model with the above desired feature, the ARX model with excitation input and structural response output is scrutinized to identify the intrinsic relationship between the model parameters and the physical parameters of the system. We begin with the discrete state-space model expressed by Eq. (3-12). The discrete-time matrices $\mathbf{A}$ and $\mathbf{B}$ in Eq. (3-12) may be computed by the following series expansions:

$$\mathbf{A} = \mathbf{I} + \mathbf{A}_c \Delta t + \frac{1}{2!}(\mathbf{A}_c \Delta t)^2 + \frac{1}{3!}(\mathbf{A}_c \Delta t)^3 + \dots \quad (3-13)$$

$$\mathbf{B} = \left[\mathbf{I} \Delta t + \frac{1}{2!} \mathbf{A}_c \Delta t^2 + \frac{1}{3!} \mathbf{A}_c^2 \Delta t^3 + \frac{1}{4!} \mathbf{A}_c^3 \Delta t^4 + \dots \right] \mathbf{B}_c \quad (3-14)$$

Let the state equation be written in a slightly different form. Add and subtract the term $\mathbf{S}\mathbf{y}(k)$ with an arbitrary matrix $\mathbf{S}$ to Eq. (3-12),

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{S}\mathbf{y}(k) - \mathbf{S}\mathbf{y}(k) \quad (3-15)$$

Substituting $\mathbf{y}(k)$ from Eq. (3-12) into the above,

$$\mathbf{x}(k+1) = (\mathbf{A} + \mathbf{S}\mathbf{C})\mathbf{x}(k) + (\mathbf{B} + \mathbf{S}\mathbf{D})\mathbf{u}(k) - \mathbf{S}\mathbf{y}(k) \quad (3-16)$$

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Define

$$\bar{\mathbf{A}} = \mathbf{A} + \mathbf{S}\mathbf{C}, \quad (3-17a)$$

$$\bar{\mathbf{B}} = [\mathbf{B} + \mathbf{S}\mathbf{D}, -\mathbf{S}], \quad (3-17b)$$

$$\text{and } \mathbf{v}(k) = \begin{bmatrix} \mathbf{u}(k) \\ \mathbf{y}(k) \end{bmatrix}$$

The original system now becomes

$$\begin{aligned} \mathbf{x}(k+1) &= \bar{\mathbf{A}}\mathbf{x}(k) + \bar{\mathbf{B}}\mathbf{v}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{aligned} \quad (3-18)$$

This is a state-space observer model of a dynamical system. Because matrix $\mathbf{S}$ can be arbitrarily chosen, $\bar{\mathbf{A}}$ may be made as asymptotically stable as desired with a proper $\mathbf{S}$.

Solving for the output $\mathbf{y}(k)$ from Eq. (3-18) with zero initial condition in terms of the previous input $\mathbf{u}(i)$ and outputs $\mathbf{y}(i)$ yields

$$\hat{\mathbf{y}}(k) = \sum_{i=1}^k \mathbf{C}(\mathbf{A} + \mathbf{S}\mathbf{C})^{i-1} (\mathbf{B} + \mathbf{S}\mathbf{D})\mathbf{u}(k-i) - \sum_{i=1}^k \mathbf{C}(\mathbf{A} + \mathbf{S}\mathbf{C})^{i-1} \mathbf{S}\mathbf{y}(k-i) + \mathbf{D}\mathbf{u}(k) \quad (3-19)$$

Define

$$\begin{aligned} \bar{\mathbf{Y}}_1 &= \mathbf{C}[\mathbf{B} + \mathbf{S}\mathbf{D} \quad -\mathbf{S}], \\ \bar{\mathbf{Y}}_2 &= \mathbf{C}[\mathbf{A} + \mathbf{S}\mathbf{C}][\mathbf{B} + \mathbf{S}\mathbf{D} \quad -\mathbf{S}], \\ \bar{\mathbf{Y}}_k &= \mathbf{C}[\mathbf{A} + \mathbf{S}\mathbf{C}]^{k-1} [\mathbf{B} + \mathbf{S}\mathbf{D} \quad -\mathbf{S}] \\ &= [\bar{\mathbf{Y}}_k^{(1)} \quad -\bar{\mathbf{Y}}_k^{(2)}] \end{aligned} \quad (3-20)$$

Then the estimated response $\mathbf{y}$ at the time step k becomes

$$\hat{\mathbf{y}}(k) + \sum_{i=1}^k \bar{\mathbf{Y}}_i^{(2)} \mathbf{y}(k-i) = \sum_{i=1}^k \bar{\mathbf{Y}}_i^{(1)} \mathbf{u}(k-i) + \mathbf{D}\mathbf{u}(k) \quad (3-21)$$

Eq. (3-21) is commonly called the linear difference model for multi-input / multi-output, linear, time-invariant systems. This is also often referred to as the ARX model where AR refers to the Auto-Regressive part (related to output data) and X refers to the eXogeneous part (related to input data). This form is commonly used in developing recursive system identification techniques (Ljung, 1987).

Define the series equation of matrix $\mathbf{X}$ as:

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$$\begin{aligned}\cos \mathbf{X} &= \frac{e^{i\mathbf{X}} + e^{-i\mathbf{X}}}{2} = \mathbf{I} - \frac{\mathbf{X}^2}{2!} + \frac{\mathbf{X}^4}{4!} - \dots \\ \sin \mathbf{X} &= \frac{e^{i\mathbf{X}} - e^{-i\mathbf{X}}}{2i} = \mathbf{X} - \frac{\mathbf{X}^3}{3!} + \frac{\mathbf{X}^5}{5!} - \dots\end{aligned}\quad (3-22)$$

They have the property (Golub and Van Loan, 1983): $\cos^2 \mathbf{X} + \sin^2 \mathbf{X} = \mathbf{I}$.

We also have:

$$\mathbf{K} = \mathbf{M}_0^{-1} \mathbf{K}_0 = \Phi \Lambda \Phi^T,$$

$$\text{where } \Lambda = \begin{bmatrix} \omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_n^2 \end{bmatrix}, \omega_i \text{ is eigenvalue, } \Phi \text{ is eigenvector matrix which}$$

satisfies $\Phi^T = \Phi^{-1}$. In dynamic problems ω_i and Φ_i are respectively the natural (circular) frequencies and the mass-normalized mode shape matrix. Then we have

$$\mathbf{K} = \Phi \Lambda^{1/2} \Phi^T \Phi \Lambda^{1/2} \Phi^T = (\Phi \Lambda^{1/2} \Phi^T)^2 \text{ and } \mathbf{K}^{1/2} = \Phi \Lambda^{1/2} \Phi^T.$$

To simplify the formulation, it is assumed at this point that the system is undamped, hence $\mathbf{A}_c$ in Eq. (3-8) can be expressed as:

$$\mathbf{A}_c = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K} & \mathbf{0} \end{bmatrix}$$

and

$$\begin{aligned}\mathbf{A}_c^2 &= \begin{bmatrix} -\mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{K} \end{bmatrix}, \quad \mathbf{A}_c^3 = \begin{bmatrix} \mathbf{0} & -\mathbf{K} \\ \mathbf{K}^2 & \mathbf{0} \end{bmatrix}, \\ \mathbf{A}_c^4 &= \begin{bmatrix} \mathbf{K}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}^2 \end{bmatrix}, \quad \mathbf{A}_c^5 = \begin{bmatrix} \mathbf{0} & \mathbf{K}^2 \\ -\mathbf{K}^3 & \mathbf{0} \end{bmatrix}, \dots\end{aligned}$$

Then, by Eq. (3-13) and (3-14) the matrices $\mathbf{A}$ and $\mathbf{B}$ can be calculated:

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} \mathbf{I} - \frac{1}{2!} \mathbf{K} \Delta t^2 + \frac{1}{4!} \mathbf{K}^2 \Delta t^4 - \dots & \mathbf{I} \Delta t - \frac{1}{3!} \mathbf{K} \Delta t^3 + \frac{1}{5!} \mathbf{K}^2 \Delta t^5 - \dots \\ -\mathbf{K} \Delta t + \frac{1}{3!} \mathbf{K}^2 \Delta t^3 - \frac{1}{5!} \mathbf{K}^3 \Delta t^5 + \dots & \mathbf{I} - \frac{1}{2!} \mathbf{K} \Delta t^2 + \frac{1}{4!} \mathbf{K}^2 \Delta t^4 - \dots \end{bmatrix} \\ &= \begin{bmatrix} \cos(\mathbf{K}^{1/2} \Delta t) & \mathbf{K}^{-1/2} \sin(\mathbf{K}^{1/2} \Delta t) \\ -\mathbf{K}^{1/2} \sin(\mathbf{K}^{1/2} \Delta t) & \cos(\mathbf{K}^{1/2} \Delta t) \end{bmatrix}\end{aligned}\quad (3-23)$$

$$\begin{aligned}
 \mathbf{B} &= \begin{bmatrix} \mathbf{I}\Delta t - \frac{1}{3!}\mathbf{K}\Delta t^3 + \frac{1}{5!}\mathbf{K}^2\Delta t^5 + \dots & \frac{1}{2!}\mathbf{I}\Delta t^2 - \frac{1}{4!}\mathbf{K}\Delta t^4 + \frac{1}{6!}\mathbf{K}^2\Delta t^6 + \dots \\ -\frac{1}{2!}\mathbf{K}\Delta t^2 + \frac{1}{4!}\mathbf{K}^2\Delta t^4 - \frac{1}{6!}\mathbf{K}^3\Delta t^6 + \dots & \mathbf{I}\Delta t - \frac{1}{3!}\mathbf{K}\Delta t^3 + \frac{1}{5!}\mathbf{K}^2\Delta t^5 + \dots \end{bmatrix} \mathbf{B}_c \\
 &= \begin{bmatrix} \mathbf{K}^{-1/2} \sin(\mathbf{K}^{1/2} \Delta t) & \mathbf{K}^{-1}(\mathbf{I} - \cos(\mathbf{K}^{1/2} \Delta t)) \\ -\mathbf{I} + \cos(\mathbf{K}^{1/2} \Delta t) & \mathbf{K}^{-1/2} \sin(\mathbf{K}^{1/2} \Delta t) \end{bmatrix} \mathbf{B}_c \\
 &= \begin{bmatrix} \mathbf{K}^{-1}(\mathbf{I} - \cos(\mathbf{K}^{1/2} \Delta t)) \\ \mathbf{K}^{-1/2} \sin(\mathbf{K}^{1/2} \Delta t) \end{bmatrix} \mathbf{L} \quad (3-24)
 \end{aligned}$$

Let the matrix $\mathbf{S}$ be chosen such that

$$\mathbf{S} = \begin{bmatrix} 2\mathbf{K}^{-1} \cos(\mathbf{K}^{1/2} \Delta t) \\ -\mathbf{K}^{-1/2}(\sin(\mathbf{K}^{1/2} \Delta t) - \sin^{-1}(\mathbf{K}^{1/2} \Delta t) \cos(\mathbf{K}^{1/2} \Delta t)) \end{bmatrix} \quad (3-25)$$

Note that matrices $\mathbf{K}$, $\mathbf{K}^{-1}$, $\mathbf{K}^{-1/2}$, $\cos(\mathbf{K}^{1/2} \Delta t)$, $\sin(\mathbf{K}^{1/2} \Delta t)$ are all symmetrical, so the sequence of matrices in the multiplication operation may be changed, e.g., $\mathbf{K}^{-1} \cos(\mathbf{K}^{1/2} \Delta t) = \cos(\mathbf{K}^{1/2} \Delta t) \mathbf{K}^{-1}$.

The matrices in Eq. (3-18) become

$$\begin{aligned}
 \overline{\mathbf{A}} &= \mathbf{A} + \mathbf{S}\mathbf{C} \\
 &= \begin{bmatrix} -\cos(\mathbf{K}^{1/2} \Delta t) & \mathbf{K}^{-1/2} \sin(\mathbf{K}^{1/2} \Delta t) \\ -\mathbf{K}^{-1/2} \cos^2(\mathbf{K}^{1/2} \Delta t) \sin^{-1}(\mathbf{K}^{1/2} \Delta t) & \cos(\mathbf{K}^{1/2} \Delta t) \end{bmatrix} \quad (3-26)
 \end{aligned}$$

$$\begin{aligned}
 \overline{\mathbf{B}} &= [\mathbf{B} + \mathbf{S}\mathbf{D}, -\mathbf{S}] \\
 &= \begin{bmatrix} \mathbf{K}^{-1}(\mathbf{I} + \cos(\mathbf{K}^{1/2} \Delta t))\mathbf{L} & -2\mathbf{K}^{-1} \cos(\mathbf{K}^{1/2} \Delta t) \\ \mathbf{K}^{-1/2} \cos^2(\mathbf{K}^{1/2} \Delta t) \sin^{-1}(\mathbf{K}^{1/2} \Delta t)\mathbf{L} & \mathbf{K}^{-1/2}(\sin(\mathbf{K}^{1/2} \Delta t) - \cos^2(\mathbf{K}^{1/2} \Delta t) \sin^{-1}(\mathbf{K}^{1/2} \Delta t)) \end{bmatrix} \quad (3-27)
 \end{aligned}$$

The matrix $\overline{\mathbf{A}}$ is asymptotically stable in the sense that it takes only a power of two for $\overline{\mathbf{A}}$ to decay completely, because $\overline{\mathbf{A}}^2 = \overline{\mathbf{A}}^3 = \dots = \overline{\mathbf{A}}^n = \mathbf{0}$.

Substituting the matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{G}$ into Eq. (3-19), and noting $\mathbf{J} = \mathbf{0}$ (and hence $\mathbf{C} = [-\mathbf{K}, \mathbf{0}]$), we have

$$\begin{aligned}
 \overline{\mathbf{Y}}_1 &= \mathbf{C}\overline{\mathbf{B}} = [-\mathbf{K}(\mathbf{I} + \cos(\mathbf{K}^{1/2} \Delta t))\mathbf{L}, \quad 2\cos(\mathbf{K}^{1/2} \Delta t)] \\
 \overline{\mathbf{Y}}_2 &= \mathbf{C}\overline{\mathbf{A}}\overline{\mathbf{B}} = [\cos(\mathbf{K}^{1/2} \Delta t)\mathbf{L}, \quad -\mathbf{I}]
 \end{aligned}$$

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$$\bar{\mathbf{Y}}_3 = \mathbf{C}\bar{\mathbf{A}}\bar{\mathbf{A}}\bar{\mathbf{B}} = [\mathbf{0} \quad \mathbf{0}]$$

$$\bar{\mathbf{Y}}_i = \mathbf{C}\bar{\mathbf{A}}^{i-1}\bar{\mathbf{B}} = [\mathbf{0} \quad \mathbf{0}], \text{ for } i > 3 \quad (3-28)$$

Then the ARX model in Eq. (3-21) can be written as:

$$\hat{\mathbf{y}}(k) - 2\cos(\mathbf{K}^{1/2}\Delta t)\mathbf{y}(k-1) + \mathbf{y}(k-2) = -(\mathbf{I} + \cos(\mathbf{K}^{1/2}\Delta t))\mathbf{L}\mathbf{u}(k-1) + \cos(\mathbf{K}^{1/2}\Delta t)\mathbf{L}\mathbf{u}(k-2) + \mathbf{L}\mathbf{u}(k) \quad (3-29)$$

Since $\mathbf{K}^{1/2} = \mathbf{\Phi}\mathbf{\Lambda}^{1/2}\mathbf{\Phi}^T$, $\cos(\mathbf{K}^{1/2}\Delta t)$ can be re-written as:

$$\begin{aligned} \cos(\mathbf{K}^{1/2}\Delta t) &= \mathbf{I} - \frac{1}{2!}(\mathbf{K}^{1/2}\Delta t)^2 + \frac{1}{4!}(\mathbf{K}^{1/2}\Delta t)^4 + \dots \\ &= \mathbf{I} - \mathbf{\Phi}\left(\frac{1}{2!}(\mathbf{\Lambda}^{1/2}\Delta t)^2\right)\mathbf{\Phi}^T + \mathbf{\Phi}\left(\frac{1}{4!}(\mathbf{\Lambda}^{1/2}\Delta t)^4\right)\mathbf{\Phi}^T + \dots \\ &= \mathbf{\Phi}\left(\mathbf{I} - \frac{1}{2!}(\mathbf{\Lambda}^{1/2}\Delta t)^2 + \frac{1}{4!}(\mathbf{\Lambda}^{1/2}\Delta t)^4 + \dots\right)\mathbf{\Phi}^T \\ &= \mathbf{\Phi}\cos(\mathbf{\Lambda}^{1/2}\Delta t)\mathbf{\Phi}^T \end{aligned} \quad (3-30)$$

$$\text{Here } \cos\mathbf{\Lambda}^{1/2}\Delta t = \begin{bmatrix} \cos\omega_1\Delta t & & 0 \\ & \ddots & \\ 0 & & \cos\omega_n\Delta t \end{bmatrix} \quad (3-30a)$$

Subsequently, Eq. (3-29) can be expressed as ARX model:

$$\mathbf{y}(k) = \mathbf{P}_1\mathbf{y}(k-1) + \mathbf{P}_2\mathbf{y}(k-2) + \mathbf{D}\mathbf{u}(k) + \mathbf{E}_1\mathbf{u}(k-1) + \mathbf{E}_2\mathbf{u}(k-2) \quad (3-31)$$

$$\text{where } \mathbf{P}_1 = 2\mathbf{\Phi}\cos(\mathbf{\Lambda}^{1/2}\Delta t)\mathbf{\Phi}^T, \quad (3-31a)$$

$$\mathbf{P}_2 = -\mathbf{I}, \quad (3-31b)$$

$$\mathbf{D} = \mathbf{L} \quad (3-31c)$$

$$\mathbf{E}_1 = -\mathbf{\Phi}[\mathbf{I} + \cos(\mathbf{\Lambda}^{1/2}\Delta t)]\mathbf{\Phi}^T\mathbf{L}, \quad (3-31d)$$

$$\mathbf{E}_2 = \mathbf{\Phi}\cos(\mathbf{\Lambda}^{1/2}\Delta t)\mathbf{\Phi}^T\mathbf{L}. \quad (3-31e)$$

It should be noted that the above explicit expression between the ARX model coefficients and the system dynamic properties have been derived under the assumption of zero damping. The coefficient matrices $[\mathbf{P}_1, \mathbf{P}_2, \mathbf{D}, \mathbf{E}_1, \mathbf{E}_2]$ of the ARX model can be estimated from a given set of response and input time series data. Subsequently, the system parameters $[\mathbf{J}, \mathbf{K}, \mathbf{L}]$ may be recovered through a numerical procedure. For damped systems, Lee and Yun (1991) also obtained a

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similar expression of the ARX model as Eq. (3-31). In that case, however, the model coefficient matrices are no longer related to the system dynamic properties in an explicit manner as shown in Eqs. (3-31a-e).

As each response can be regarded as propagation of excitation through the structure which acts as a filter, the response signals obviously contain both information of excitation and structure. If one of the response signals can be used to extract the excitation information, this response signal may be turned as the input of the model system, thus eliminating the real excitation term from the model.

It is possible to eliminate the external excitation input from the ARX model for cases whereby the excitation comes from one single source, be it force excitation with a single forcing function or base excitation. In such cases, $u(k)$ reduces to a single time series, and $\mathbf{L}$ becomes a column with n rows. Subsequently, Eq.(3-31) can be written as:

$$\begin{Bmatrix} y_1(k) \\ \vdots \\ y_n(k) \end{Bmatrix} = \mathbf{P}_1 \begin{Bmatrix} y_1(k-1) \\ \vdots \\ y_n(k-1) \end{Bmatrix} + \mathbf{P}_2 \begin{Bmatrix} y_1(k-2) \\ \vdots \\ y_n(k-2) \end{Bmatrix} + \mathbf{L}u(k) + \mathbf{E}_1 u(k-1) + \mathbf{E}_2 u(k-2) \quad (3-32)$$

Suppose the excitation acts on j -th DOF (or simply take the j -th DOF as a reference point in case of base excitation), then the collocated response at the same DOF, j , can be written as:

$$y_j(k) = l_j u(k) + \sum_{i=1}^n p_{1,ji} y_i(k-1) + \sum_{i=1}^n p_{2,ji} y_i(k-2) + e_{1,j} u(k-1) + e_{2,j} u(k-2) \quad (3-33)$$

As l_j is not zero, $u(k)$ can be expressed as:

$$u(k) = \frac{1}{l_j} \left[y_j(k) - \sum_{i=1}^n p_{1,ji} y_i(k-1) - \sum_{i=1}^n p_{2,ji} y_i(k-2) - e_{1,j} u(k-1) - e_{2,j} u(k-2) \right] \quad (3-34)$$

Define $\bar{\mathbf{y}}$ as vector $\bar{\mathbf{y}} = \{y_1, \dots, y_{j-1}, y_{j+1}, \dots, y_i\}^T$

$$\begin{aligned} \bar{\mathbf{y}}(k) &= \bar{\mathbf{P}}_1 \mathbf{y}(k-1) + \bar{\mathbf{P}}_2 \mathbf{y}(k-2) + \bar{\mathbf{L}}u(k) + \bar{\mathbf{E}}_1 u(k-1) + \bar{\mathbf{E}}_2 u(k-2) \\ &= \bar{\mathbf{P}}_1 \mathbf{y}(k-1) + \bar{\mathbf{P}}_2 \mathbf{y}(k-2) + \bar{\mathbf{E}}_1 u(k-1) + \bar{\mathbf{E}}_2 u(k-2) \\ &\quad + \frac{\bar{\mathbf{L}}}{l_j} \left(y_j(k) - \sum_{i=1}^n p_{1,ji} y_i(k-1) - \sum_{i=1}^n p_{2,ji} y_i(k-2) - e_{1,j} u(k-1) - e_{2,j} u(k-2) \right) \end{aligned}$$

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$$\begin{aligned}
 &= \bar{\mathbf{P}}_1 \mathbf{y}(k-1) + \bar{\mathbf{P}}_2 \mathbf{y}(k-2) + \frac{\bar{\mathbf{L}}}{l_j} y_j(k) - \frac{\bar{\mathbf{L}}}{l_j} \sum_{i=1}^n p_{1,ji} y_i(k-1) - \frac{\bar{\mathbf{L}}}{l_j} \sum_{i=1}^n p_{2,ji} y_i(k-2) \\
 &+ \left(\bar{\mathbf{E}}_1 - \frac{\bar{\mathbf{L}}}{l_j} \mathbf{e}_{1,j} \right) u(k-1) + \left(\bar{\mathbf{E}}_2 - \frac{\bar{\mathbf{L}}}{l_j} \mathbf{e}_{2,j} \right) u(k-2)
 \end{aligned} \tag{3-35}$$

where matrices $\bar{\mathbf{P}}_1$, $\bar{\mathbf{P}}_2$ refer to the matrices $\mathbf{P}_1$ and $\mathbf{P}_2$ without j -th row; and vectors $\bar{\mathbf{L}}$, $\bar{\mathbf{E}}_1$ and $\bar{\mathbf{E}}_2$ are vectors $\mathbf{L}$, $\mathbf{E}_1$ and $\mathbf{E}_2$ without j -th element.

Under the condition that $\omega_n \Delta t$ is small enough (for relatively stiff systems, this could be achieved by using a sufficiently small Δt), the matrix $\cos(\Lambda^{1/2} \Delta t)$ as expressed in Eq. (3-30a) will approach an identity matrix. For typical structures, there should be no particular difficulty to achieve this requirement. As a general guidance, it is suggested that the time step be set below $T_n/30$ (T_n is the natural period of the highest mode of interest). According to Eq.(3-35), the induced error is no more than $(1 - \cos(\omega_n \Delta t))u(k)$, which is about $0.02u(k)$ when $\Delta t = 0.03T_n$. An error of $0.02u(k)$ would lead to less than 2% increase in the Residual Error STD ratios, $\sigma(e_m)/\sigma(e_0)$, in the final result.

According to Eq. (3-31d~e), the reduction of $\cos(\Lambda^{1/2} \Delta t)$ to an identity matrix will lead to $\mathbf{E}_1 \approx -2\mathbf{L}$, $\mathbf{E}_2 \approx \mathbf{L}$, $\bar{\mathbf{E}}_1 \approx -2\bar{\mathbf{L}}$, $\bar{\mathbf{E}}_2 \approx \bar{\mathbf{L}}$ and $e_{1,j} \approx -2l_{1,j}$, $e_{2,j} \approx l_{2,j}$. Then in Eq. (3-35) the coefficients of $u(k-1)$ and $u(k-2)$ approaches zero, and subsequently the equation reduces to include only the time series $\mathbf{y}(k-1)$, $\mathbf{y}(k-2)$, $y_j(k)$, $y_j(k-1)$ and $y_j(k-2)$, without the excitation terms. Writing the equation in a standard ARX form yields:

$$\bar{\mathbf{y}}(k) = \mathbf{P}_1^* \bar{\mathbf{y}}(k-1) + \mathbf{P}_2^* \bar{\mathbf{y}}(k-2) + \mathbf{E}_0^* y_j(k) + \mathbf{E}_1^* y_j(k-1) + \mathbf{E}_2^* y_j(k-2) + \mathbf{e}(k) \tag{3-36}$$

where $\mathbf{e}(k)$ denotes the residual error of the model, and

$$\begin{aligned}
\mathbf{P}_1^* &= \begin{bmatrix} p1_{1,1} & \cdots & p1_{1,j-1} & p1_{1,j+1} & \cdots & p1_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p1_{j-1,1} & \cdots & p1_{j-1,j-1} & p1_{j-1,j+1} & \cdots & p1_{j-1,n} \\ p1_{j+1,1} & \cdots & p1_{j+1,j-1} & p1_{j+1,j+1} & \cdots & p1_{j+1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p1_{n,1} & \cdots & p1_{n,j-1} & p1_{n,j+1} & \cdots & p1_{n,n} \end{bmatrix} - \frac{\bar{\mathbf{L}}}{l_j} [p1_{j,1} \quad \cdots \quad p1_{j,j-1} \quad p1_{j,j+1} \quad \cdots \quad p1_{j,n}] \\
\mathbf{P}_2^* &= \begin{bmatrix} p2_{1,1} & \cdots & p2_{1,j-1} & p2_{1,j+1} & \cdots & p2_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p2_{j-1,1} & \cdots & p2_{j-1,j-1} & p2_{j-1,j+1} & \cdots & p2_{j-1,n} \\ p2_{j+1,1} & \cdots & p2_{j+1,j-1} & p2_{j+1,j+1} & \cdots & p2_{j+1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ p2_{n,1} & \cdots & p2_{n,j-1} & p2_{n,j+1} & \cdots & p2_{n,n} \end{bmatrix} + \frac{\bar{\mathbf{L}}}{l_j} [p2_{j,1} \quad \cdots \quad p2_{j,j-1} \quad p2_{j,j+1} \quad \cdots \quad p2_{j,n}] \\
\mathbf{E}_0^* &= \frac{\bar{\mathbf{L}}}{l_j}, \quad \mathbf{E}_1^* = -\frac{p1_{j,j}}{l_j} \bar{\mathbf{L}} + [p1_{1,j} \quad \cdots \quad p1_{j-1,j} \quad p1_{j+1,j} \quad \cdots \quad p1_{n,j}]^T \\
\mathbf{E}_2^* &= -\frac{p2_{j,j}}{l_j} \bar{\mathbf{L}} + [p2_{1,j} \quad \cdots \quad p2_{j-1,j} \quad p2_{j+1,j} \quad \cdots \quad p2_{n,j}]^T
\end{aligned}$$

For distinguishing purposes, the parameters of the new ARX model are expressed as $\mathbf{P}_i^*$, and $\mathbf{E}_i^*$, in this Chapter.

With the new model form of Eq. (3-36), the response measurements taken from the undamaged state of a structure can be used as reference signals to estimate the coefficient matrices of the ARX model through a standard procedure of the least-squares approach. As demonstrated earlier, the coefficients $\mathbf{P}_1^*$, $\mathbf{P}_2^*$, $\mathbf{E}_0^*$, $\mathbf{E}_1^*$ and $\mathbf{E}_2^*$ of the ARX model are functions of the dynamic parameters, they may be utilized for the system identification purpose. The present study, however, focuses on the capability of this ARX model as a time series analysis tool for the damage diagnosis of structural systems.

Based on this ARX model, an analysis scheme can be established with the acceleration at a particular degree of freedom (DOF) as the “input” while the accelerations at other measured DOFs as the “measurements” (or responses). The diagnosis procedure can be depicted by a block diagram as shown in Fig. 3.1. In this procedure, first a reference model is estimated using measurements from a reference

state (preferably the original state) of the structure. Then relative changes of the structural conditions with respect to the reference state can be diagnosed by feeding the current measurements to the reference model and analyzing the residual error features.

Figure 3.1. ARX model based procedure for damage diagnosis

Due to the close relationship with the underlying dynamic system, it can be expected that the present model is sensitive to the changes of the physical parameters (stiffness in particular) to serve the purpose of damage diagnosis, especially in terms of their relative changes. Taking one particular measurement point from the ARX model as an example, the model function is actually a linear combination of response of the same DOF at past times and responses at other DOFs. As a result, the model function becomes intrinsically related to relative changes in adjacent parts of the system; and the closer a damaged element is to the particular model point, the more sensitive its (stiffness) change will affect the prediction error, thus becomes more detectable from the model. By checking the distribution of the residual errors at different measurement points, the damage location can be identified. Furthermore, as there is neither assumption of excitation type nor excitation term involved, the model is expected to be robust to tolerate certain variation in the excitation so that major prediction errors are mainly related to the change of the structural properties.

In the subsequent numerical investigation, it will be shown that the standard deviation (STD) of the residual errors in the predicted signals for an actually

measured state, $\sigma(e_x)$, as compared to the STD of the residual error for the reference (undamaged) state, $\sigma(e_y)$, is a suitable feature for the diagnosis of damage. For MDOF systems, larger residual errors occur at the measurement points closer to the damage, and this enables the detection of the damage location. It will also be shown that the present model has much improved sensitivity to the structural stiffness change as compared to a previous two-step time series model proposed by Sohn and Farrar (2001).

3.4 Application of the New Model and Numerical Examples

In order to investigate the effectiveness of the new model, a numerical simulation study is conducted. Although the explicit expression of ARX as in Eq.(3-31) is only for undamped systems (otherwise the relationship between the state-space model or ARX model parameters and the physical properties of a system cannot be expressed explicitly), the framework of the model is still valid for both damped and undamped cases. In this numerical study, two structural models are considered, one is a 2-DOF mass-spring system and another is an 8-DOF system. The effect of the presence of damping on the diagnosis results will also be demonstrated.

Random excitations are generated from either Gaussian white noise or random noise. The white noise time series have a power of 1 dB. The first two random time-series have the power of 0 dB and standard deviation set to be 1.0, and another time-series has a power of 6 dB and standard deviation of 2.0. For each type of excitation, the response time series are simulated by calculating the response of the system using standard numerical procedure. Table 3.1 summarizes the properties of five excitation signals. The sampling time step Δt is taken equal to 0.01sec and the number of data points is chosen to be 4000 for each sample piece. The same time step and duration are applied in sampling the response signals. In the presentation that follows, Wgn1, Wgn2, Randn1, Randn2 and Randn3 will be used to label the simulation study corresponding to the respective excitations. The unit of input time series is kN for force excitation and m/sec^2 for base excitation.

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Table 3.1 List of input time series

Case	mode	Standard deviation
Wgn1	White Noise	1.1287
Wgn2	White Noise	1.1273
Randn1	Random	1.0040
Randn2	Random	1.0039
Randn3	Random	2.0080

3.4.1 Two-DOF system

Figure 3.2 A two-DOF dynamic system

The 2-DOF mass spring system considered is shown in Fig. 3.2, without or with damping (assuming 5% damping ratio). Each point mass is 419.4 kg and the initial spring stiffness k_1 and k_2 are both 56.7 KNm^{-1} . The natural periods of the system are calculated to be $T_1 = 0.874 \text{ sec}$ and $T_2 = 0.334 \text{ sec}$. The excitation on the structure is imposed at the base by random acceleration as described above.

The acceleration responses at m_1 and m_2 , denoted as y_1 and y_2 , are sampled from the numerical calculations. In the prediction model, y_1 is regarded as input and y_2 as output. According to Eq. (3-36), the ARX model can be written as:

$$y_1(k) = P_1^* y_1(k-1) + P_2^* y_1(k-2) + E_0^* y_2(k) + E_1^* y_2(k-1) + E_2^* y_2(k-2) + e(k)$$

Using the responses measured at m_1 and m_2 when the system is subjected to a particular excitation, a least-squares approach is applied to determine the

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coefficients. The procedure is performed using Matlab system identification toolbox. Taking the excitation case Wgn1 for example, the ARX model coefficients are found to be:

$$P^*_1=1.973; P^*_2=1.000; E^*_0=1.002; E^*_1=-1.964; E^*_2=1.002.$$

The reference feature $\sigma(e_y)$ is found to be 9.6301e-06. The coefficients for the system under the other three base excitations are almost the same as the above, and the corresponding reference features are shown in Table 3.2 along the right diagonal line. Each of the 5 ARX models so established is then used to predict the system response under the other four excitations. The predicted results are compared with the actually measured signals at m1, and the residual errors and their standard deviations are calculated. Table 3.2 summarizes the STDs of the residual errors for all the combinations. Similarly, for the 5%-damped system, the ARX models are constructed based on the respective response data, and the residual errors are also shown in Table 3.2.

Table 3.2 STD of residual error for the undamaged state of the 2-DOF system under different input excitations

Reference	Undamped system				
Model	Wgn1	Wgn2	Randn1	Randn2	Randn3
Wgn1	9.6301e-6	1.0038e-5	9.7156e-6	9.8430e-6	1.0098e-5
Wgn2	9.6374e-6	1.0030e-05	9.7230e-6	9.8428e-6	1.0061e-5
Randn1	9.6347e-6	1.0045e-5	9.7120e-6	9.8390e-6	1.0090e-5
Randn2	9.6396e-6	1.0039e-5	9.7152e-6	9.8356e-6	1.0052e-5
Randn3	9.6286e-6	1.0014e-5	9.7030e-6	9.8172e-6	1.0037e-5
Reference	5%-Damped system				
Model	Wgn1	Randn1	Randn3		
Wgn1	9.8948e-6	9.7179e-6	9.6089e-6		
Randn1	9.0647e-6	9.9095e-6	9.6347e-6		
Randn3	9.6054e-6	9.8959e-6	9.7136e-6		

As can be seen from Table 3.2, all the residual errors are very small; and they are

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very close to each other regardless whether the excitation is the same as that used for establishing the reference model. In other words, each ARX model is able to predict the response signals under different excitations with practically the same accuracy. This behavior is significant in the sense that once the ARX model is established using signals from a particular random excitation, it can be applied for diagnosis purpose without requiring exactly the same type of excitation. Besides, for the damped system, the ARX model can be established to a comparable degree of satisfaction as that for the undamped system.

Now three damage scenarios are introduced: a) the stiffness of k_1 is reduced to 51.03 KNm^{-1} (90% of the original); b) the stiffness of k_1 is reduced to 45.36 KNm^{-1} (80% of the original value); and c) a bumper is inserted between m_1 and m_2 to cause nonlinear response. The bumper effect is simulated by a nonlinear element of a gap nature with an opening of 0.01m and a close-up stiffness of 500 KNm^{-1} . For each damage scenario, the system is subjected respectively to the aforementioned base excitations (Table 3.1) and the response at m_1 and m_2 are recorded. The response signals are then fed to predict the response at m_2 and the corresponding residual errors are obtained. Subsequently, the STD of the residual errors, $\sigma(e_m)$, are calculated and compared with reference STD error, $\sigma(e_0)$. For comparison, Table 3.3 shows the ratios between the STD of the residual errors in several damage scenarios to that of the reference errors ($\sigma(e_m)/\sigma(e_0)$). Included in the table is also a damage scenario for the damped system for illustrative purpose.

From Table 3.3, it can be clearly observed that the residual error when using the reference ARX model on the damaged scenarios are 2 to 5 orders of magnitude higher than the reference residual error. This indicates that the model is sensitive to the presence of damage in the dynamic system. In general, higher residual errors occur for larger degree of damage, while the presence of non-linearity in the measured signals results in a further increased residual error. For the same state of the structure, the residual errors from different excitations are not exactly the same, but maintain a good consistency. For a damped system with damage represented by a stiffness change (e.g., the 10% stiffness reduction case), the presence of damage

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can also be detected by the considerable increase of the prediction residual error using the respective reference model, although the magnitude of the residual error STD ratio is somewhat lower as compared to the undamped situation.

Table 3.3 Residual error STD ratios, $\sigma(e_m)/\sigma(e_0)$, for various damage scenarios of the 2-DOFsystem

Reference Model	Undamped system		5%-damped system	
	Scenario (a) : 10% stiffness reduction Wtn-1	Randn-1	Scenario (a) : 10% stiffness reduction Wtn-1	Randn-1
Wtn-1	6.023e+2	7.269e+2	3.177e+2	3.226e+2
Randn-1	6.020e+2	7.265e+2	3.238e+2	3.289e+2
	Scenario (b): 20% stiffness reduction			
Wtn-1	1.121e+3	1.879e+3		
Randn-1	1.121e+3	1.878e+3		
	Scenario (c): bumper			
Wight-1	2.637e+5	3.852e+5		
Random-1	2.623e+5	3.832e+5		

Fig. 3.3 shows a typical comparison of the ARX predicted and the actual response time histories for the reference state and a damaged state. To highlight the residual errors, the error histories are also plotted in the figure. As can be seen, the reference residual error is very small and it shows an apparent random characteristic. On the contrary, the residual error when the model is applied on the damaged state is considerably larger; and moreover, its waveform clearly contains the system response as opposed to the random error at the reference state. This feature tends to imply that it may be possible to deduce more detailed damage information from the system response content in the residual error for the damaged state, in addition to what can be understood from the increased magnitude of the error. Further study is to be conducted to analyze the feature of the residual error for a damaged state in a more comprehensive manner.

The subsequent example will demonstrate the ability of the model in detecting the location, as well as the presence, of damage in an MDOF system.

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Figure 3.3. Typical time histories of simulated and predicted acceleration responses and residual errors. (a) Reference state; (b) damaged state (Scenario (b): 20% stiffness reduction).

3.4.2 Eight-DOF system

a) General description

An 8-DOF mass spring system is set up, as shown in Fig. 3.4, to investigate the applicability of the proposed model in multiple DOF systems and its ability of locating the damage. The system has uniform mass of 419.4g at each node and a uniform spring stiffness of 56.7KNm^{-1} throughout, with 5% damping ratio. The first three natural periods of the system are calculated to be $T_1=2.931$ sec, $T_2=0.9882$ sec and $T_3=0.6067$ sec. Two excitation schemes are considered, one is by acceleration from the base, and another is by random force acting at m_1 which is closest to the base. For each excitation scheme, both Wgn1 and Randn1 are used as the input time series. The force excitation scheme is adopted for the consideration that a controlled

random force excitation at a DOF would be easier to realize in practice than by base excitation. Since the vibration input to the structure is not required in the current method, it is expected that both excitation schemes would lead to similar diagnosis results. It should be noted that, as the input coefficient matrix $\mathbf{L}_0$ in Eq.(3-5) are different under different excitation schemes, the respective model coefficients will relate to the system's dynamic properties in different ways; this to a certain extent could affect the model sensitivity.

Figure 3.4 The eight-DOF dynamic system under consideration

After all the acceleration response signals are simulated by means of a dynamic analysis procedure, the ARX model of Eq. (3-36) is established. When the response at m_1 is used as “input”, and the response at other locations are regard as “output”, the ARX model is expressed as:

$$\bar{\mathbf{y}}(k) = \mathbf{P}_1^* \bar{\mathbf{y}}(k-1) + \mathbf{P}_2^* \bar{\mathbf{y}}(k-2) + \mathbf{E}_0^* y_1(k) + \mathbf{E}_1^* y_1(k-1) + \mathbf{E}_2^* y_1(k-2) + \mathbf{e}(k)$$

where $\bar{\mathbf{y}}(k) = \{y_2(k), \dots, y_8(k)\}^T$, $y_1 \sim y_8$ are acceleration responses of joints m_1 to m_8 , and $\mathbf{e}(k)$ is a 7-row vector residual error.

Table 3.4 STD of residual errors of undamaged structure by two excitation methods

Reference model	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
<i>By base excitation</i>							
$\text{Wgn1} \sigma(e_y)$	5.1259e-5	5.1311e-5	5.1328e-5	5.1333e-5	5.1322e-5	5.1318e-5	5.1292e-5
Randn1	5.5282e-5	5.5391e-5	5.5366e-5	5.5376e-5	5.5339e-5	5.5384e-5	5.5375e-5
<i>By random force acting on m_1</i>							
$\text{Wgn1} \sigma(e_y)$	1.2018e-4	6.9199e-6	6.9898e-6	6.7692e-6	7.0903e-6	7.1103e-6	6.9457e-6
Randn1	1.3008e-4	7.1687e-6	6.943e-6	7.1435e-6	7.0635e-6	6.7957e-6	6.9499e-6

Table 3.4 shows the STD of the reference residual errors (undamaged state) at the seven mass points with two excitation schemes. It is noted that with the second excitation scheme (force excitation at m_1) the STD at m_2 is larger than those at other locations. This reflects that the response at this point is somehow masked by the excitation input that takes place at the adjacent point m_1 .

b) Damage at locations away from the “input” response point

When one spring stiffness has been altered, the largest increase in the residual error (in terms of STD) is expected to occur at the nearest measurement points. Here two damage scenarios are simulated, respectively, namely: (1) case-a: the stiffness of spring k_7 between m_6 and m_7 is reduced, and (2) case-b: the stiffness of spring k_3 , between m_2 and m_3 is reduced by 20%. Four sets of responses are simulated for each damage scenario; two under base excitations of Wtn1 and Randn1, and two under the exciting forces of Wtn1 and Randn1 at m_1 . Figs. 3.5 and 3.6 show the STD ratios of the residual errors between the damaged and the original reference states under base excitation and force excitation, respectively.

Figure 3.5 STD ratio $(\sigma(e_m)/\sigma(e_0))$ for two damage scenarios by base excitation.

Figure 3.6 STD ratio $(\sigma(e_m)/\sigma(e_0))$ for of two damage scenarios by force excitation at m_1 .

From Figs. 3.5 and 3.6 the locations of damage can be clearly identified since the STD ratio of the residual errors increase abruptly at the points near damage. Both excitation methods can provide good diagnosis, except that in the case of forced excitation imposed at m_1 (see Fig. 3.6), the response at the adjacent point m_2 is not as sensitive to damage at the nearby k_3 , due to the “masking” effect mentioned earlier. Overall, the proposed ARX model is able to detect the locations of the damage in the MDOF system.

To highlight the improvement of the present method in detecting this kind of damage, a direct comparison is made between the performance of the present model as shown above and the model proposed by Sohn and Farrar (2001) when applied on the same 8-DOF system. The method proposed by Sohn and Farrar, as summarized in Chapter 2, can be described as a two-step AR-ARX model. The procedure analyzes the acceleration response of each point independently. In the first step, the data is predicted by AR model; then, the data is predicted by ARX model using the residual error of the AR model in the first step as the eXogenous input. The STD of errors between the prediction data in the second step and the measured one is used as the damage feature. The STD ratios are then used to detect damage location. The method has been shown to work out quite successfully for cases where nonlinearity (such as a bumper) is involved in the measured response

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signals. However, when the structural changes are limited to a stiffness reduction so that the measured signals from the damaged structure remains to be linear, such as the two damage scenarios mentioned above in the present study, their method does not appear to produce satisfactory results.

Table 3.5 shows the results (the STD ratios) from the above two-step model when it is applied on the 8-DOF system under consideration, for two stiffness degradation scenarios (Case-a and Case-b). Note that only the results from the base excitation scheme are presented. As can be seen, the damage features (STD ratios) are almost identical at all measurement points and they are close to unity, which means no damage is detected while stiffness degradation actually took place, respectively, at two individual springs. In comparison, the results from the present ARX model, as described earlier, clearly capture the presence and the location of the damage regardless whether it is stiffness degradation or a nonlinear effect.

Table 3.5 STD ratios from the two step AR-ARX model

Damage scenario	Y_8	Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1
Scenario (a)	0.993	1.003	0.998	1.001	1.002	1.010	0.998	0.999
Scenario (b)	1.005	1.018	1.010	1.007	1.002	1.001	1.009	1.005

c) Damage at location near the “input” response point

In this section, it will be shown that when the damage location is close to the point from which the response signal is taken as “input” to the ARX model, the diagnosis could be complicated. For this purpose, two more damage scenarios are simulated, one (Case-c) is a degradation of stiffness by 20% at spring k_2 between m_2 and m_3 ; the other (Case-d) is a degradation of stiffness by 20% at spring k_1 between m_1 and the base support. The response at m_1 is again used as the “input” in the ARX model.

Table 3.6 shows the STD ratios of the residual errors for the damaged structure when the excitation is imposed from the base. For damage at k_2 (Case-c), the STD ratios at all points are very large. Although the damage at k_2 still appears to be

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detectable as the residual error at m_2 (y_2) is higher than the remaining errors, the high STD ratios at all locations tend to indicate an abnormal behavior of the model. This is further evidenced in the case of damage at k_1 (Case-d), for which all points show large residual errors and no clear pattern is observed.

Table 3.7 shows the STD ratios when the system is subjected to a force excitation at m_2 . It can be seen that when damage occurs at k_2 , it is detectable because only the STD ratio at m_2 is significantly large. When damage is at k_1 , however, no feature appears to reflect the occurrence and location of the damage.

Table 3.6 STD ratio of damaged structure by base excitation

Reference of Model	Residual error ratio for damage case -c: 20% stiffness reduction at k_7						
	Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1
Wgn1	2.18E+02	1.09E+02	1.09E+02	1.09E+02	1.09E+02	1.09E+02	1.09E+02
	Residual error ratio for damage case-d: 20% stiffness reduction at K_8						
Wgn1	6.83E+01	7.02E+01	7.01E+01	7.01E+01	7.01E+01	7.02E+01	7.02E+01

Table 3.7 STD ratio of damaged structure by force excitation

Reference of Model	Residual error ratio for damage case-c: 20% stiffness reduction at K_7						
	Y_7	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1
Wgn1	2.48E+02	5.80E+00	1.31E+00	1.35E+00	1.28E+00	1.28E+00	1.10E+00
	2.44E+02	5.96E+00	1.35E+00	1.32E+00	1.35E+00	1.38E+00	1.11E+00
	Residual error ratio for damage case-d: 20% stiffness reduction at K_8						
Wgn1	1.01E+00	1.07E+00	1.06E+00	1.11E+00	1.06E+00	1.06E+00	1.03E+00
	1.01E+00	1.01E+00	1.05E+00	1.03E+00	1.04E+00	1.08E+00	1.03E+00

Apparently, the abnormal behavior of the model for the above cases is associated

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with the fact that the location of the damage is close to the response point that is used as “input” of the ARX model. As such, a natural way to get rid of the problem would be carrying out two separate runs of the procedure using two different “input” response locations. A correct detection of any arbitrary damage location should be achieved by inspecting the results from both runs.

As an example, when the response at m_8 is used as the “input”, the model becomes:

$$\bar{\mathbf{Y}}(k) = \mathbf{A}_1 \bar{\mathbf{Y}}(k-1) + \mathbf{A}_2 \bar{\mathbf{Y}}(k-2) + \mathbf{B}_0 y_8(k) + \mathbf{B}_1 y_8(k-1) + \mathbf{B}_2 y_8(k-2) + \mathbf{e}(k)$$

where $\bar{\mathbf{Y}}(k) = \{y_1(k), \dots, y_7(k)\}^T$. Accordingly, for the force excitation scheme, the random force is to be applied at point m_8 .

Figure 3.7 STD ratio $(\sigma(e_m)/\sigma(e_0))$ for damage Case-d (20% reduction of k_1), using m_8 as ARX model “input”: (a) by base excitation; (b) by force excitation at m_8

Fig. 3.7 shows the STD ratios for damage Case-d with the above new ARX model under a random base excitation, or a force excitation at m_8 . The location of the damage at k_1 , which is near the base but far away from the new “input” response location (m_8), can now be clearly identified.

The above observations further confirm that using the force excitation scheme, the ability of the AR model in locating the damage is rather independent of the relative locations of the damage and the “input” point. Using the base excitation scheme, however, care should be taken to avoid selecting an input point which may be near the potential damage. To get rid of this uncertainty, it is recommended that at least two AR models be set up using two different “input” points located distinctively

away from each other to carry out separate diagnosis. The combination of results from the two models should lead to a definite identification of the damage location regardless of what excitation scheme is adopted.

3.5 Conclusions and Discussion

A new method for damage diagnosis using time series analysis of vibration signals is presented in this chapter. The method is based on the linear dynamic system formulation and is realized in a novel form of auto-regressive models (ARX) on acceleration response signals. The model coefficients are directly associated with the system physical parameters and hence are expected to provide sensitive and practically consistent features for the diagnosis of damage. Through appropriate simplification, the model can be disassociated from the input excitation and this further enhances the potential robustness of the method in real-life applications. At the present stage, the independence of the method from excitation has been achieved successfully within the category of random excitations.

The standard deviation of the residual error, which is the difference between the measured signals from an unknown state of the system and the predicated signals from the ARX model whose coefficients are determined from undamaged reference measurements, is found to be a suitable damage-sensitive feature. Numerical simulation study demonstrated that using the standard deviation of the residual errors as a feature, the occurrence of damage can be easily detected; and moreover, in a MDOF system the location of damage can also be identified as larger STD of the residual errors tend to occur near the actual damage locations.

As the establishment of the model requires response signals from the structure, one response signal as the model input and the others as the output, the selection of the response location to be used as “input” signal requires special attention. It has been found that when damage appears near the point where the response is specified as input to the model, the results become complicated. A simple way to get rid of the problem is to set up two AR models using two distinctive sets of “input” and

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“output” points which are far apart from each other in physical locations to carry out separate diagnosis. The combination of results from the two models is found to lead to the correct identification of the damage location.

It should be pointed out the formulation of the ARX model in this chapter does not take into account the noises in the measured signals. To extend the framework of the model to noisy measurement environment, appropriate ways to tackle the noise contamination and restore the intrinsic relationships between the “input” and “response” signals are necessary. Chapter 4 will present a novel scheme to deal with the noise problem in applying the ARX model for damage diagnosis.

CHAPTER 4 A KALMAN FILTER BASED APPROACH FOR TIME DOMAIN DAMAGE DIAGNOSIS WITH NOISY SIGNALS

In this chapter, an approach is presented for the time domain analysis of noise-contaminated vibration signals for global structural damage diagnosis. It extends from the response-only ARX model introduced in Chapter 3. The novel idea in the present procedure is to establish the intrinsic “input” and “measurement” pairs from noisy signals by using the Kalman filter, so that the underlying physical system is best represented in the signals pairs and passed on to the subsequent diagnosis operation. The theoretical basis of representing the system by the raw measured input and the filtered response through the Kalman filter is discussed. When such raw input and filtered response signals are fed into the reference ARX model, the error feature becomes indicative of the change of the physical system. By analyzing the residual error, the damage status of the structure can be diagnosed. Applications to numerical and experimental examples demonstrate that the approach is effective in tackling the noises, and both the location and relative extent of damage can be assessed with appropriate damage features.

4.1 Introduction

In the process of acquiring vibration signals from physical testing, a certain level of noise contamination is inevitable. Subsequent analysis based on the measured signals will more or less be affected by the noise contents in the signals. This problem could become more significant when performing model estimation and

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signal prediction in the time domain. In the time-domain model estimation stage, some statistic features can be altered by the noise; consequently the model parameters could be wrongly estimated. In the signal prediction stage, the noise in the measurement certainly affects the performance of the model.

A time domain analysis approach based on Auto-Regressive with eXogenous input (ARX) was proposed in Chapter 3 for global structural damage diagnosis. The diagnosis procedure is depicted by a block diagram in Fig. 3.1. It was shown that under certain conditions, a process can be established with the acceleration at a particular degree of freedom (DOF) as the “input” while the accelerations at other measured DOFs as the “measurements” (or responses). The model coefficients are related to the dynamic properties of the structural system under consideration. A reference model can be established using measurements from a reference state (preferably the original state) of the structure. Relative changes of the structural conditions with respect to the reference state can be diagnosed by feeding the current measurements to the reference model and analyzing the residual error features.

The approach was shown to work well under noise-free measurement conditions in detecting and locating the damage. However, the performance of the model would deteriorate under noisy measurement conditions, apparently due to the errors introduced by the noise in both the reference model estimation stage and the current signal prediction stage. In order to improve the situation, both aspects of the problem need to be addressed. At this juncture, it is important to note that the current damage diagnosis is based on the relative changes of the system; as such, the essential objective in processing the signals from the noisy measurements is to preserve the intrinsic input-output relationship that represents the underlying physical system. Therefore, processing the signals as a system rather than individual

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series out of the noisy measurements becomes the most important consideration in the current effort to minimize the effect of noise.

For the general purpose of minimizing the noise effect, much attention has been given to subspace methods for identifying state-space models. The methods based on subspace models with discrete filter are applicable for linear structures and recordings with wide-band measurement noise, which is usually the case in real life situations. Among the subspace models, the Kalman filter and Extended Kalman filtering (EFK) are widely adopted in studying time-domain signals (Hoshiya and Saito, 1984; Koh, et al, 1991; Safak, 1991; Koh, et al, 1995; Quek, et al, 1999; Shi, et al, 2000).

In this chapter, the ARX model proposed in the previous study is applied in conjunction with the Kalman filter technique to perform the structural damage diagnosis with noisy measurement signals. The ARX model is first expressed in a state-space form, so that the noise terms can be introduced in to evaluate their effects on the model performance. The expression of the model in a state-space form facilitates the application of the Kalman filter. It is demonstrated that using the raw input signal as the Kalman filter input to process the measured response signals, the raw input and the processed response signals gives rise to a desired input-output system that well represent the underlying state of the structure. This input-output signal system can then be fed to the reference model for damage diagnosis using appropriate residual error features.

4.2 Overview of the Kalman Filter

The Kalman Filter is an effective tool for stochastic estimation of the state from

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noisy measurements. Because of its relative simplicity and robust nature, the Kalman filter has been widely used to obtain estimates of state variables in practice.

The Kalman filter approach inherently has the flexibility of incorporating system dynamics equations into the algorithm as well as the provision for uncertainty in the system. Shi, et al. (2000) used Kalman filter algorithm to identify model parameters in frequency domain. Hoshiya and Saito (1984) demonstrated the application of EKF to the problem of identifying system parameters in frequency domain. Koh et al. (1995) presented a condensation method for local damage detection of a multi-story frame building, in which the remedial stiffness matrix was identified by applying the EKF. Quek et al. (1999) used the Eigenspace Algorithm produces good prediction of structural responses. In order to give good results for signals contaminated by noise, the algorithm is based on Kalman filter sequence.

The Kalman filter is essentially a set of mathematical equations (Harvey, 1989) that aims at minimizing the estimated error covariance in the state estimator. The Kalman Filter proceeds with a given process and measurement equations as:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{G}\boldsymbol{\eta}(k)$$

$$\mathbf{y}_v(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \quad (4-1)$$

where $\mathbf{u}(k)$ is the process input of the system, $\mathbf{y}_v(k)$ represents the measurement (output), $\boldsymbol{\eta}(k)$ and $\mathbf{v}(k)$ are input noise and output noise respectively (they are also called “process noise” and “measurement noise” or “sensor noise”).

The algorithm of the Kalman filter effectively resembles that of a prediction-correction algorithm. The filter estimates the process state and obtains the feedback from the measurement which is noisy. The Kalman filter thus consists of two groups of equations, one performs the prediction (called “time update”), by which the *a priori* estimates for the current step is obtained based on the previous

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state and error covariance, and the other does the correction (called “measurement update”) to obtain an improved *a posteriori* estimate. These equations are recursive nature and can be written (Harvey, 1989; Chatfield, 1989) as:

Time update equations:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}\hat{\mathbf{x}}(k-1|k-1) + \mathbf{B}\mathbf{u}(k-1) \quad (4-2)$$

$$\mathbf{P}(k|k-1) = \mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^T + \mathbf{G}\mathbf{Q}(k-1)\mathbf{G}^T \quad (4-3)$$

$$\mathbf{Q}(k) = E(\boldsymbol{\eta}(k-1)\boldsymbol{\eta}(k-1)^T) \quad (4-4)$$

$$\mathbf{R}(k) = E(\mathbf{v}(k-1)\mathbf{v}(k-1)^T) \quad (4-5)$$

Measurement update equations:

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{M}(k)(\mathbf{y}_v(k) - \mathbf{C}\hat{\mathbf{x}}(k|k-1)) \quad (4-6)$$

$$\mathbf{M}(k) = \mathbf{P}(k|k-1)\mathbf{C}^T(\mathbf{R}(k) + \mathbf{C}\mathbf{P}(k|k-1)\mathbf{C}^T)^{-1} \quad (4-7)$$

$$\mathbf{P}(k|k) = (\mathbf{I} - \mathbf{M}(k)\mathbf{C})\mathbf{P}(k|k-1) \quad (4-8)$$

With Kalman Gain given by:

$$\mathbf{K}(k) = \mathbf{A}\mathbf{M}(k) \quad (4-9)$$

Combining Eq.(4-2) and (4-6) yields:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}\hat{\mathbf{x}}(k-1|k-2) + \mathbf{K}(k-1)(\mathbf{y}_v(k-1) - \mathbf{C}\hat{\mathbf{x}}(k-1|k-2)) + \mathbf{B}\mathbf{u}(k-1) \quad (4-10)$$

In these equations, $\hat{\mathbf{x}}$ denotes the estimated process state, $\mathbf{P}$ represents the *a priori* estimate error covariance, $\mathbf{M}$ represents the *a posteriori* estimate error covariance. Given initial conditions $\hat{\mathbf{x}}(1|0)$ and $\mathbf{P}(1|0)$, one can iterate these equations to perform the filtering. The filter can produce an optimal estimate of the true response measurements, $\mathbf{y}_e = \hat{\mathbf{y}}(k|k-1)$, by the following equation:

$$\hat{\mathbf{y}}(k|k-1) = \mathbf{C}\hat{\mathbf{x}}(k|k-1) \quad (4-11)$$

The block diagram shown in Fig. 4.1 depicts the flowchart of a standard procedure of stochastic signal processing with Kalman filter.

Figure 4.1 Typical procedure of stochastic processing of signals with Kalman filter

In the next section the Kalman filter will be applied in conjunction with the proposed ARX model for filtering the measured noisy signals for subsequent damage diagnosis purpose.

4.3. Analysis of the Effect of Using Kalman Filter on the ARX Model

In this section the ARX model concerned is examined for its susceptibility to the noise content in the measured signals. The appropriate way of using the Kalman filter to process the noisy signals, the ultimate objective as to preserve the state information, and the effectiveness of such an approach are demonstrated mathematically.

In Chapter 3, an ARX model was established to model the process of a linear system with input as an acceleration at a specific DOF and response (measurement) as accelerations at other DOFs. The damage diagnosis is achieved by analyzing the residual errors in the predicted response using the reference model. The ARX model as Eq.(3-36) is written as:

$$\mathbf{y}(k) = \mathbf{P}_1 \mathbf{y}(k-1) + \mathbf{P}_2 \mathbf{y}(k-2) + \mathbf{D} \mathbf{u}(k) + \mathbf{E}_1 \mathbf{u}(k-1) + \mathbf{E}_2 \mathbf{u}(k-2) \quad (4-12)$$

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where both the model output $\mathbf{y}(k)$ and input $\mathbf{u}(k)$ are acceleration responses of the structure. $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{D}$, $\mathbf{E}_1$, $\mathbf{E}_2$ are the model parameters.

In order to analyze the noise effect on the system represented by the ARX model, it is advantageous to express the system in state-space form. This can be done through a standard procedure (see for example Chatfield, 1989). By introducing a variable vector $\mathbf{x}(k)$, the ARX model of Eq.(4-12) can be expressed in a state-space form, as: Introduce a variant vector $\mathbf{x}(k)$

$$\mathbf{x}(k) = \begin{Bmatrix} \mathbf{x}_1(k) \\ \mathbf{x}_2(k) \\ \mathbf{x}_3(k) \end{Bmatrix}$$

such that $\mathbf{x}(k)$ satisfies the following four equations:

$$\mathbf{y}(k-1) = \mathbf{x}_1(k) \quad (4-13)$$

$$\mathbf{x}_1(k) = \mathbf{x}_2(k-1) + \mathbf{D}\mathbf{u}(k-1) \quad (4-14)$$

$$\mathbf{x}_2(k) = \mathbf{P}_1\mathbf{x}_2(k-1) + \mathbf{x}_3(k-1) + (\mathbf{E}_1 + \mathbf{P}_1\mathbf{D})\mathbf{u}(k) \quad (4-15)$$

$$\mathbf{x}_3(k) = \mathbf{P}_2\mathbf{x}_2(k-1) + (\mathbf{E}_2 + \mathbf{P}_2\mathbf{D})\mathbf{u}(k-1) \quad (4-16)$$

Here we notice the $\mathbf{x}_1(k)$ is one step ahead of $\mathbf{y}(k)$, but in order to give a standard form of the state space model, we write

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \quad \text{and} \quad \mathbf{C} = [\mathbf{I} \quad \mathbf{0} \quad \mathbf{0}]$$

This implies that when the model is applied, the $\mathbf{y}(k)$ should be shifted one step ahead, and add $\mathbf{y}(1) = \{\mathbf{0}\}$.

Then we get the state-space model

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k),$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k). \quad (4-17)$$

where the state parameters $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are related to the ARX model parameters as follows

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_1 & \mathbf{I} \\ \mathbf{0} & \mathbf{P}_2 & \mathbf{0} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{D} \\ \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix}, \quad \mathbf{C} = [\mathbf{I} \quad \mathbf{0} \quad \mathbf{0}] \quad (4-17a)$$

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$$\mathbf{Q}_1 = \mathbf{E}_1 + \mathbf{P}_1 \mathbf{D}, \quad \mathbf{Q}_2 = \mathbf{E}_2 + \mathbf{P}_2 \mathbf{D}, \quad (4-17b)$$

The process and measurement noises can be easily incorporated into the above state-space model. Consider firstly a noise term in the measurement, we have:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}_v(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \end{aligned} \quad (4-18)$$

where $\mathbf{y}_v$ refers to the measured output, $\mathbf{u}$ is supposed to be the real input signal. But as $\mathbf{u}$ in the present model is actually an acceleration response at a particular DOF of the structure, only a measured signal with noise, $\mathbf{u}_v$, is available,

$$\mathbf{u}_v(k) = \mathbf{u}(k) + \boldsymbol{\eta}(k), \text{ where } \boldsymbol{\eta}(k) \text{ is the noise content in the measured "input" signal.}$$

Consequently, the system should be re-written as:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}_v(k) - \mathbf{B}\boldsymbol{\eta}(k) \\ \mathbf{y}_v(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{v}(k) \end{aligned} \quad (4-19)$$

Herein $\boldsymbol{\eta}(k)$ and $\mathbf{v}(k)$ are assumed to be white noise processes with zero means and finite covariance of $\Delta\boldsymbol{\eta}=\mathbf{Q}$ and $\Delta\mathbf{v}=\mathbf{R}$, respectively.

The Kalman filter can now be applied on the model described by Eq.(4-19). But it is important to note that, because the raw input $\mathbf{u}_v$ has been used in place of the real input $\mathbf{u}$ which is not known, the filtered response signals $\hat{\mathbf{y}}$ is not really an optimal estimate of the real responses $\mathbf{y}$ in Eq.(4-17). The achieved estimate of the response signals, $\hat{\mathbf{y}} = \hat{\mathbf{y}}(k|k-1)$, is in fact an optimal estimate of the "virtual" response of the system, $\mathbf{y}_{vir}$, as if the the noisy input signal were the "real" input, i.e.,

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}_v(k) \\ \mathbf{y}_{vir}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned} \quad (4-20)$$

where the state parameters $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are those of Eq. (4-19).

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As a matter of fact, here lies the novelty of the present approach. Indeed, in the context of the current damage diagnosis scheme which looks upon the deviation from a reference state model, it is not the quality of each individual signal but a system relationship formed by the input-output signals that ultimately determines the outcome of the diagnosis. With an inevitable noisy $\mathbf{u}_v$ instead of the real $\mathbf{u}$, what we are looking for is actually a good representation of the process described by Eq.(4-20) rather than that described by Eq.(4-19). In what follows, we will demonstrate the $\hat{\mathbf{y}}$ produced from the Kalman filter using $\mathbf{u}_v$ as input is indeed the closest estimate of $\mathbf{y}_{vir}$ in Eq.(4-20).

Assuming the “noise” in the virtual measured counterpart for $\mathbf{y}_{vir}$ of Eq.(4-20), denoted as $\mathbf{y}_{v,vir}$, is $\boldsymbol{\gamma}$. It has

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}_v(k), \quad (4-21a)$$

$$\mathbf{y}_{v,vir}(k) = \mathbf{C}\mathbf{x}(k) + \boldsymbol{\gamma}(k) \quad (4-21b)$$

$$\mathbf{H}(k) = E(\boldsymbol{\gamma}(k-1)\boldsymbol{\gamma}(k-1)^T) \quad (4-21d)$$

Application of the Kalman filter on this model delivers the optimal estimator by the following equations:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}\hat{\mathbf{x}}(k-1|k-1) + \mathbf{B}\mathbf{u}_v(k-1) \quad (4-22a)$$

$$\mathbf{P}(k|k-1) = \mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^T \quad (4-22b)$$

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{M}(k)(\mathbf{y}_{v,vir}(k) - \mathbf{C}\hat{\mathbf{x}}(k|k-1)) \quad (4-22c)$$

$$\mathbf{M}(k) = \mathbf{P}(k|k-1)\mathbf{C}^T(\mathbf{H}(k) + \mathbf{C}\mathbf{P}(k|k-1)\mathbf{C}^T)^{-1} \quad (4-22d)$$

$$\mathbf{P}(k|k) = (\mathbf{I} - \mathbf{M}(k)\mathbf{C})\mathbf{P}(k|k-1) \quad (4-22e)$$

With the same definition of $\mathbf{K}(k)$ in Eq.(4-9), the recursion of the error covariance matrix is:

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$$\mathbf{P}(k+1|k) = \mathbf{A}\mathbf{P}(k|k-1)\mathbf{A}^T - \mathbf{K}(k)\mathbf{C}\mathbf{P}(k|k-1)\mathbf{A}^T \quad (4-23)$$

The $\mathbf{P}$ in the Kalman filter has the definition as:

$$\mathbf{P}(k+1|k) = E\left[\left(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\right)\left(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\right)^T\right] \text{ and}$$

$$\mathbf{P}(k|k) = E\left[\left(\mathbf{x}(k) - \hat{\mathbf{x}}(k|k)\right)\left(\mathbf{x}(k) - \hat{\mathbf{x}}(k|k)\right)^T\right]$$

The recursion of the optimal estimator becomes:

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{A}\hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\mathbf{e}(k) + \mathbf{B}\mathbf{u}_v(k) \quad (4-24a)$$

$$\hat{\mathbf{y}}(k|k-1) = \mathbf{C}\hat{\mathbf{x}}(k|k-1) \quad (4-24b)$$

$$\mathbf{e}(k) = \mathbf{y}_{v, \text{vir}}(k) - \mathbf{C}\hat{\mathbf{x}}(k|k-1) \quad (4-24c)$$

The residual error between $\hat{\mathbf{y}}(k+1|k)$ and $\mathbf{y}_{v, \text{vir}}(k+1)$ can be evaluated by combining Eqs.(4-20) and (4-24b), as:

$$\mathbf{y}_{v, \text{vir}}(k+1|k) - \hat{\mathbf{y}}(k+1|k) = \mathbf{C}(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)) \quad (4-25a)$$

By Eqs.(4-21a) and (4-24a) the error for the state vector is:

$$\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k) = \mathbf{A}(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k|k-1)) - \mathbf{K}(k)\mathbf{e}(k) \quad (4-25b)$$

Following the definition of $\mathbf{P}$ in the Kalman filter, the covariance error matrix of Eqs.(4-25a) and (4-25b) is:

$$E\left[\left(\mathbf{y}_{v, \text{vir}}(k+1) - \hat{\mathbf{y}}(k+1|k)\right)\left(\mathbf{y}_{v, \text{vir}}(k+1) - \hat{\mathbf{y}}(k+1|k)\right)^T\right] = \mathbf{C}\mathbf{P}(k+1|k)\mathbf{C}^T \quad (4-26a)$$

$$E\left[\left(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\right)\left(\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\right)^T\right] = \mathbf{P}(k+1|k) \quad (4-26b)$$

It has been demonstrated by Harvey (1989) that $\hat{\mathbf{x}}(k|k-1)$ is the minimum mean square linear estimator of $\mathbf{x}(t)$ based on observations $\mathbf{u}_v(k)$ and $\mathbf{y}_v(k)$ up to time $k-1$. This estimator is unconditionally unbiased and the unconditional covariance matrix of the estimation error is the $\mathbf{P}(k|k-1)$ given by the Kalman filter. Then in this

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special case, $\hat{\mathbf{y}}(k|k-1)$ is the closest estimator to $\mathbf{y}_{\text{vir}}(k)$, as the error of $(\hat{\mathbf{y}}(k|k-1) - \mathbf{y}_{\text{vir}}(k))$ is directly related to the error of $(\hat{\mathbf{x}} - \mathbf{x})$.

Unfortunately it is usually difficult to obtain an explicit solution of $\mathbf{P}(k+1|k)$; hence it is difficult to evaluate how close the $\hat{\mathbf{y}}(k|k-1)$ is to $\mathbf{y}_{\text{vir}}(k)$ by an explicit equation. To give an idea, let's take a look of the random walk with noise example given by Harvey (1989). In this example, when the real input is known, the solution of $\mathbf{P}(k+1|k)$ yields $\mathbf{P}(k+1|k) = \mathbf{P}(k|k-1) / (\alpha^2 + \mathbf{P}(k|k-1))$, where α^2 is the variance of sensor noise. In general $\mathbf{P}(k+1|k)$ converges with increase of time, t , following α^2/t , regardless of the initial condition. For the case we study in this chapter, as the system is stable, $\mathbf{P}(k+1|k)$ will converge even faster than the random walk system which is not stable.

4.4 Implementation of the Kalman Filter in the ARX based Damage Diagnosis

4.4.1 ARX model estimation and signal processing using Kalman filter

When the signals $\mathbf{y}(k)$ and $\mathbf{u}(k)$ are contaminated with noise, the least-squares algorithm of the ARX model may lead to inaccurate estimation of the model coefficients. A common method in overcoming the noise influence on the model estimation is using the approximate maximum likelihood (ML) estimators to estimate the model parameters, based on the Kalman filter (Harvey, 1989). For a stable system, the error covariance $\mathbf{P}$ will converge such that the Kalman filter eventually has a time invariant solution. This solution can be transformed into an ARMAX (auto-regressive and moving average with eXogenous input) model, which can be directly used for model estimation and prediction to the same effect as

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directly using the Kalman filter (Lee and Yun, 1991). Further elaboration follows.

For a stable system, the error covariance $\mathbf{P}$ and the Kalman gain $\mathbf{K}$ converge to the steady-state values $\bar{\mathbf{P}}$ and $\bar{\mathbf{K}}$ as $k \rightarrow \infty$ (Harvey, 1989), i.e., $\lim_{t \rightarrow \infty} \mathbf{P}(t+1|t) = \bar{\mathbf{P}}$ and $\lim_{t \rightarrow \infty} \mathbf{K}(k) = \bar{\mathbf{K}}$. Specializing the Kalman filter on the model of Eq.(4-24), it can be written as:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}\hat{\mathbf{x}}(k-1|k-2) + \bar{\mathbf{K}}\mathbf{e}(k-1) + \mathbf{B}\mathbf{u}_v(k-1) \quad (4-27a)$$

$$\hat{\mathbf{y}}(k|k-1) = \mathbf{C}\hat{\mathbf{x}}(k|k-1) \quad (4-27b)$$

$$\mathbf{e}(k-1) = \mathbf{y}_v(k-1) - \mathbf{C}\hat{\mathbf{x}}(k-1|k-2) \quad (4-27c)$$

By taking the Z-transform of Eqs.(4-27a, b), the following equation can be obtained (Chatfield, 1989):

$$\begin{aligned} \hat{\mathbf{y}}(k+1|k) &= \mathbf{P}_1\hat{\mathbf{y}}(k|k-1) + \mathbf{P}_2\hat{\mathbf{y}}(k-1|k-2) + \mathbf{D}\mathbf{u}_v(k) + \mathbf{E}_1\mathbf{u}_v(k-1) + \mathbf{E}_2\mathbf{u}_v(k-2) \\ &\quad + \mathbf{K}_1\mathbf{e}(k) + (\mathbf{K}_2 - \mathbf{P}_1\mathbf{K}_1)\mathbf{e}(k-1) + (\mathbf{K}_3 - \mathbf{P}_2\mathbf{K}_1)\mathbf{e}(k-2) \end{aligned} \quad (4-28)$$

It can be further written as an ARMAX model:

$$\begin{aligned} \mathbf{y}_v(k+1) &= \mathbf{P}_1\mathbf{y}_v(k) + \mathbf{P}_2\mathbf{y}_v(k-1) + \mathbf{D}\mathbf{u}_v(k) + \mathbf{E}_1\mathbf{u}_v(k-1) + \mathbf{E}_2\mathbf{u}_v(k-2) \\ &\quad + \mathbf{e}(k+1) + \mathbf{H}_1\mathbf{e}(k) + \mathbf{H}_2\mathbf{e}(k-1) + \mathbf{H}_3\mathbf{e}(k-2) \end{aligned} \quad (4-29)$$

Where

$$\mathbf{E}_1 = \mathbf{Q}_1 - \mathbf{P}_1\mathbf{D}, \quad \mathbf{E}_2 = \mathbf{Q}_2 - \mathbf{P}_2\mathbf{D}, \quad (4-29a)$$

$$\mathbf{H}_1 = \mathbf{K}_1 - \mathbf{P}_1, \quad \mathbf{H}_2 = \mathbf{K}_2 - \mathbf{P}_1\mathbf{K}_1 - \mathbf{P}_2, \quad \mathbf{H}_3 = \mathbf{K}_3 - \mathbf{P}_2\mathbf{K}_1 \quad (4-29b)$$

$$\bar{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_1 \\ \mathbf{K}_2 \\ \mathbf{K}_3 \end{bmatrix}, \quad (4-29c)$$

The process parameter matrices $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{D}$, $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{Q}_1$, and $\mathbf{Q}_2$ are identical to those in Eq.(4-12) and Eq.(4-17). The prediction $\hat{\mathbf{y}}(k+1|k)$ here is the same as expressed by the steady-state Kalman filter.

For stable systems under consideration, the ARMAX model of Eq.(4-29) can be used to fulfill the function of the Kalman filter in a more straightforward way. It has been found that a method utilizing ARMAX model usually yields good estimations for the model parameters even under the condition of a relatively large noise level (Lardies and Larbi 2001; Hung, et al., 2004; Lee and Yun, 1991). A least-square approach can be applied to determine the model coefficients $[\mathbf{P}_1, \mathbf{P}_2, \mathbf{D}, \mathbf{E}_1, \mathbf{E}_2, \mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3]$ of the ARMAX model (and hence the state parameters $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ according to Eqs.(4-17a) and (4-17b)). The block diagram of Fig. 4.2 shows the flowchart of the standard procedure of model estimation with ARMAX model. Some trial numerical simulations have been performed, and the results confirmed the satisfactory accuracy of using the ARMAX model in the estimation of the state parameters.

Figure 4.2 Standard procedure of model estimation with ARMAX model

4.4.2 Implementation for damage diagnosis

The block diagram in Fig.4.3 shows the implementation procedure of the Kalman

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filter (ARMAX model) on the measured input-output signals and the subsequent damage diagnosis operations.

a) Generating new Input/Output (filtering through Kalman filter)

b) Damage diagnosis with new I/O

Figure 4.3 Procedure of damage diagnosis

The procedure can be summarized into the following essential steps:

- i) Select the “input” acceleration signal (a response at a particular DOF) and the output “response” acceleration signals as outlined in Chapter 3.
- ii) For a selected reference state of the structure, estimate the model parameters using the ARMAX scheme on the corresponding set of signals. Dropping the moving average (MA) part gives rise to the reference ARX model.
- iii) For any other state of the structure, acquire the input-response signals. Establish the current ARMAX model from the set of signals acquired. Then perform the stochastic processing of the signals through the current ARMAX model (Kalman filter) with the raw $u_v(k)$ as input, get the virtual input-response set.
- iv) Apply the reference ARX model established in step i) on the above virtual input-response set, and analyze the residual error of the ARX results and assess

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the damage using an appropriate residual error feature

- v) Repeat step ii) to iii) for any other state of the structure requiring diagnosis

For the diagnosis by residual error feature, here the new statistical feature CRE (Covariance of the Residual Error) is used instead of STD ratio in Chapter 3. The CRE is a better feature than the STD ratios, $\sigma(e_m)/\sigma(e_0)$ introduced in Chapter 3. Because the model is a good representation of dynamic system, $\sigma(e_0)$ could be near zero value. It is not proper to use such value as denominator. More importantly, the error $\sigma(e_m)$ would vary with magnitude of response $\sigma(y)$ for the same structure status. So the CRE gives a relatively stable value as a damage indicator. The CRE is defined as the percentage of the error variation as:

$$\text{CRE} = \sigma(e_m)/\sigma(y) * 100\% \quad (4-30)$$

which means the percentage of unfitness of the signal to the model.

In the next section some numerically simulated examples and an experimental study will be given to illustrate the implementation procedure and the effectiveness of the approach in reducing the effect of noise on the residual error for the damage diagnosis.

4.5 Numerical Examples

In this section the effectiveness of the model under noisy condition is studied using numerical simulation.

The same two-DOF mass spring model, as shown in Fig.3.2, is considered here for the numerical simulation study. Each point mass is 419.4 kg and the initial spring stiffness k_1 and k_2 are both 56.7 KNm^{-1} . The damping ratio is assumed to be 5%. The natural periods of the system are calculated to be $T_1 = 0.874 \text{ sec}$ and $T_2 = 0.334 \text{ sec}$. The excitation on the structure is imposed at the base by random acceleration.

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The noise components $\eta(k)$ and $v(k)$ are added into the responses after the generation of the acceleration responses at m_1 and m_2 by the numerical simulations. In accordance with the assumption stated earlier, the artificial noises $\eta(k)$ and $v(k)$ are white noise, with zero mean and the covariance $E(\eta\eta^T) = Q$, $E(vv^T) = R$, $E(\eta v^T) = 0$.

4.5.1 Model estimation

The ARX model of the two-DOF mass spring system can be expressed as:

$$P(q)y(k) = E(q)u(k)$$

Here $y(k)$ is defined as the response at m_2 and $u(k)$ is the response of m_1 . q^{-1} is the delay operator.

A least-squares approach is applied to determine the model parameters. The procedure is performed using Matlab system identification toolbox. When the noise-free signal are used, the model parameters are estimated as:

$$P(q) = 1 - 1.939 q^{-1} + 0.9794 q^{-2}$$

$$E(q) = 0.9952 - 1.948 q^{-1} + 0.9798 q^{-2}$$

This model can be looked on as the exact model of the system. It gives a covariance of residual error against the pure signal as small as $1.0e^{-05}$.

When 10% noise ($E(vv^T) = 0.01E(yy^T)$) is added into the output signal $y(k)$, the ARX model estimated from the noisy reference signal becomes:

$$P(q)y(k) = E(q)u(k)$$

$$P(q) = 1 - 1.387 q^{-1} + 0.4391 q^{-2};$$

$$E(q) = 0.9591 - 1.389 q^{-1} + 0.4549 q^{-2};$$

With this model the covariance of the residual error is around 0.5. Comparing with

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the “exact” model, the coefficients estimated by the noisy reference signals deviate considerably from the expected values. Consequently, the model will not predict the signal to a satisfactory accuracy. This example demonstrates that a direct model estimation is unable to produce an accurate ARX model from noisy reference signals.

The ARMAX model, as mentioned in Section 4.4.1, is now considered. The ARMAX model estimated from the signal with 50% noise ($E(vv^T) = 0.25E(yy^T)$) in the output signal $y(k)$ are estimated as:

$$P(q)y(t) = E(q)u(t) + H(q)e(t)$$

$$P(q) = 1 - 1.94 q^{-1} + 0.9803 q^{-2}$$

$$E(q) = 0.9945 - 1.948 q^{-1} + 0.9802 q^{-2}$$

$$H(q) = 1 - 1.903 q^{-1} + 0.9061 q^{-2} + 0.0379 q^{-3}$$

It can be observed that the coefficients $P(q)$ and $E(q)$ are very close to the “exact” ARX model. This shows that the coefficients of the ARX model can be estimated quite accurately even when the noise content in the signal is high.

To further illustrate the influence of the noise content on the model estimation and the performance of the estimated model, several scenarios of reference signals are explored, namely (refer to Table 4.1, first column): 1) white noise excitation, noise-free signals; 2) white noise excitation, 10% noise in the model output (acceleration at m2) only; 3) random excitation, 10% noise in the model output only; 4) white noise excitation, 10% noise in both the model input (acceleration at m1) and output signals; and 5) white noise excitation, 20% noise in both the model input and output signals. From each set of the reference signals, the ARX model parameters are estimated using the ARMAX scheme as mentioned above. Subsequently, three sets of test signals with no noise, 10% noise and 20% noise, respectively, as indicated in Table 4.1, are applied on each of the above five reference ARX models. The residual errors are analyzed in terms of the standard

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deviation (STD) and the Covariance of the Residual Error (CRE), respectively, and CRE value are summarized in Table 4.1.

From Table 4.1 it can be seen that the errors are closely correlated to the level of noise in the test signals (viewing the table row-wise), but are relatively independent of the scenarios from which the reference model has been established (viewing the table column-wise). The interpretations can be two folds, 1) the reference models are estimated satisfactorily using the ARMAX scheme; and 2) the performance of the reference model then depends primarily on the quality of the current signals supplied for diagnosis. When the current signals contain high noise level, the residual errors can be high even when the structural state remains unchanged. The results here also indicates that such a procedure using self-generated ARMAX model for prediction can be applied to study the noise level contained in a signal.

Table 4.1 CRE(%) by reference models for different noise level

Reference models		Wgn1		Randn1
		noise-free signal	10% noise output +10% input	20% noise output +20% input
Wgn1	Noise free signal	8.63e-004	34.24	70.56
	10%noise in output	0.41	34.18	70.50
	10%noise output +10%input	3.21	33.93	70.12
Randn1	20%noise output+20% input	2.36	34.26	69.7
	50%noise in output	1.22	34.09	70.40

4.5.2 Noise filtering (stochastic processing) of signals

Following the procedure described in Section 4.4.2, the three sets of test signals are processed through the Kalman filter (the ARMAX model derived from each set itself) to overcome the noise effect. The processed sets of input (raw u_v) and

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response ($\hat{y}$) signals are then applied on the reference models, and the predicted $\hat{y}_e$ are compared with the above $\hat{y}$ as the residual error (refer to Fig.4.3). Table 4.2 summarizes the resulting residual errors in terms of CRE for three reference models. Comparing with the corresponding results in Table 4.1, it can be seen that the error feature (CRE) are drastically reduced, from as much as 70% without the “filtering” process to generally below 3% using the “filtered” signals. This level of accuracy for the same state of the structure indicates that the approach should be workable to diagnose sensible damages.

Table 4.2 CRE(%) from signals with different noise levels after implementation of the proposed procedure

Reference models	Wgn1		Randn1
	noise-free signal	10% noise output +10% input	20% noise output +20% input
Wgn1	Noise free signal	8.63e-004	2.65
	10%noise output +10%input	3.21	2.98
Randn1	20%noise output+20% input	2.36	0.53

Now two damage scenarios are introduced for testing the diagnosis ability of the approach, including a) a stiffness reduction of 20% on K_2 , (b) a stiffness reduction of 10% on K_2 . The same three reference models as mentioned in the previous paragraph are examined. Three different sets of test signals from each of the two damage scenarios are considered. The test signals are processed first through the Kalman filter and then applied on the reference models. The results of the residual errors in terms of CRE are summarized in Table 4.3 and Table 4.4, respectively, and representative error results are also plotted in Fig. 4.4

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Table 4.3 CRE(%) from signals of damage scenario (a)

Reference models	Wgn1		Randn1
	noise-free signal	10% noise output +10% input	20% noise output +20% input
Wgn1	Noise free signal	23.69	24.72
	10%noise output +10%input	23.31	24.08
Randn1	20%noise output+20% input	22.92	23.18

Table 4.4 CRE(%) from signals of damage scenario (b)

Reference models	Wgn1		Randn1
	noise-free signal	10% noise output +10% input	20% noise output +20% input
Wgn1	Noise free signal	13.75	15.35
	10%noise output +10%input	13.32	14.54
Randn1	20%noise output+20% input	13.13	13.53

It can be observed that the residual error feature (CRE) for the same damage scenario are very consistent among those assuming different levels of noise. The “filtering” process renders the results in terms of CRE from noisy measurements to be as good as that from noise-free signals. Furthermore, the CRE value shows a good correlation with the degree of damage; the CRE is about 23% for 20% stiffness reduction, and about 14% for 10% stiffness reduction.

Figure 4.4 Damage feature (CRE) for three states of structure based on stochastically processed virtual input-response signals

4.5.3 Application on a MDOF model

Following the procedure described in Section 4.4.2, an eight-DOF mass spring system, same as the example shown in Fig.3.4, is studied. The system has uniform mass of 419.4 kg at each node and a uniform spring stiffness of 56.7KNm-1 throughout. The first three natural periods of the system are calculated to be $T_1=2.931$ sec, $T_2=0.9882$ sec and $T_3=0.6067$ sec. The excitation schemes are base excitation by acceleration series Wgn-1 and Randn-1 (Refer to Table 3.1).

Two physical states of the structural system are examined to test the diagnosis ability of the approach, including the undamaged state, and a stiffness reduction of 50% on K_5 (refer to Fig.3.4). After all the acceleration response signals for the undamaged and damaged systems are simulated by means of a standard dynamic analysis procedure, 10% noise level is introduced in each response signal.

The response at m1 is used as input of the model. The reference model as base-line is estimated from the response of Wgn-1 excitation on undamaged structure. For each scenario (undamaged and damage on K_5) two set of signals with excitation

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Wgn-1 and Randn-1 are processed through the procedure shown in Fig.4.3. The representative error results are illustrated in Fig.4.5 (b). For a comparison, the results produced by applying directly the ARX model following the diagram in Fig. 3.1 without performing the noise treatment are shown in Fig. 4.5 (a).

(a) Damage diagnosis without noise filtering

(b) Damage diagnosis with noise filtering

Figure 4.5 Damage feature (CRE) for two states of the 8-DOF system

It can be clearly seen that with the approach proposed in this chapter, the results as represented by the distribution of CRE throughout the system achieved considerable

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improvement as compared with the results from the direct use of ARX model on the raw response signals. The results from the direct use of ARX model (Fig. 4.5 (a)) basically provide no clue on the damage and its location, because of the mixing up of damage information with the noise. With the implementation of the present procedure (Fig. 4.5 (b)), the CRE of the undamaged state of the system exhibits satisfactorily low values (less than 5%), whereas the CRE for the damaged state increases drastically. The CRE values at the mass locations close to the damaged element shows exceptionally high value, about an order of magnitude higher than the CRE before damage, making the location of damage easily identifiable from the distribution of CRE.

The above numerically simulated scenarios show clearly the effectiveness of the proposed approach depicted in Fig.4.3 in using noisy time domain acceleration signals for the diagnosis of damage. In the next section the approach is further tested on a physical experiment case.

4.6 Experimental Example

A two-storey concrete model frame was fabricated and tested on a shake table for its dynamic response. The test frame was made of micro-concrete and has similar mechanical properties as the normal concrete frame except the reduced dimensions. Fig. 4.6(a) shows the dimensions and the reinforcement arrangement of the model frame. Beam flanges were provided to represent the effective slab effects and also accommodate the artificial masses required by the model similitude laws. Fig. 4.6 (b) shows the test set-up. Complementary random vibration tests were conducted before and after each dynamic load test to get the random vibration signals for damage diagnosis purpose. In what follows, the acceleration signals acquired from the random vibration tests for the initial state and an intermediate state with a degree of equivalent to about 10% stiffness reduction (Lu, et al, 2002) are analyzed using the proposed approach. The sampling rate for the data acquisition was

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1000Hz.

(a) Configuration of the test model frame

(b) Test set-up of the model frame

Figure 4.6 Shake-table test of two-storey concrete frame model

Table 4.5 lists the four sets of random vibration response signals taken from the test data for the present application. S1-1 and S1-2 are two sets of signals from the original state of the structure, while S2-1 and S2-2 are two sets of signals from a same damage state of the structure. For a cross comparison, two reference ARX models are established, using signals from the original state (S1) and the damaged

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state of the structure (S2), respectively.

Table 4.5 Summary of measured signals from the test frame

Name of time series	Description	Power of base excitation (STD)	Power of floor-1 acceleration (STD)	Power of floor-2 acceleration (STD)
S1-1	Initial state, time series 1	0.0636	0.0384	0.0444
S1-2	Initial state, time series 2	0.0651	0.0349	0.0366
S2-1	Damage state, time series 1	0.0475	0.0311	0.0342
S2-2	Damage state time series 2	0.0497	0.0317	0.0408

The residual error CREs after implementing the procedure of Fig. 4.3(b) only (without “filtering”) and of Fig. 4.3 (a) (with “filtering”), respectively, are plotted in Fig. 4.7. For the cases without “filtering” the signals (Fig. 4.7a), the residual error CREs do not show clear changes of the state of the structure from the reference state. However, from the error CRE results shown in Fig. 4.7b) after implementing the filtering operation, it can be clearly observed the change of the state of the structure as compared to the reference state. For example, when the reference state is established from S1 signals, around 8% CRE is obtained for another set of signals from the same state of the structure, whereas as much as 20% CRE is observed for a set of signals from the damage state of the structure. Vice versa, when the reference state is established using the damage state signals, the CRE is about 10% for another set of signals from the same state of the structure and increases to about 25% for signals from the original state of the structure.

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Figure 4.7 Residual error feature (CRE) before and after implementing the proposed procedure

It should be mentioned that the experimental case used here was originally conducted for investigation of high frequency ground shock effects on structures, and for that purpose a rather complicated arrangement of auxiliary masses was adopted (Lu et al., 2002). The results for this particular experimental case are not as good as in the previous numerical simulated scenarios. The reason is believed not primarily because of the quality of the real signals; but rather because of some complication of the test model configuration. The attachment of the additional masses to the model was not ideal (although served its original purpose) concerning a modal vibration probe. However, this real experimental case represented an interesting but also a challenging scenario for damage assessment, and the diagnosis result achieved by the current model can be considered as quite successful albeit many uncertainties in the experimental setup.

Further experimental verification of the approach will be carried out as more experimental data are available as shown in Chapter 6.

4.7 Conclusions

A procedure essentially based on Kalman filter is presented for the time domain analysis of noise-contaminated vibration signals for structural damage diagnosis. The approach stems from the previously proposed ARX model, in which the acceleration signals measured at various degrees of freedoms of a structure forms the basic input-response system. The Kalman filter is incorporated to perform the stochastic processing of the input-response signals containing noise. It is demonstrated that using the raw input signal and the processed response signals to form the virtual input-response system, the underlying physical state of the structure is well preserved. By presenting the above virtual input-response signals to the reference model, the residual error becomes reasonably indicative to the degree of damage to the structure.

For a better correlation with the degree of damage, the new feature called Covariance of the Residual Error (CRE) is used as the damage feature instead of the STD ratio $\sigma(e_m)/\sigma(e_0)$. The CRE defined as $\sigma(e_m)/\sigma(y) * 100\%$ is a better feature than the STD ratios and it gives a relatively stable value as a damage indicator.

The effectiveness of the proposed approach is demonstrated using numerically simulated scenarios and an experimental study. It is shown that using the proposed new procedure, the noise content of even a quite high level has little effect on the outcome of the damage diagnosis, and the residual error feature in terms of CRE using the processed set of virtual input - response signals correlate favorably with the degree of damage.

CHAPTER 5 TIME DOMAIN DAMAGE DIAGNOSIS USING RESIDUAL GENERATOR TECHNIQUE WITH ACCELERATION SIGNALS

The methods proposed in Chapter 3 and 4 have been shown to work out satisfactorily for global damage diagnosis to detect the occurrence and location of damage, as well as in a general sense the level of damage. To more precisely identify the location and the severity of damage, an alternative approach would have to be considered. This Chapter is aimed at such a method in the time domain which can be applied for more precise determination of the location and particularly the severity of damage in a structure.

The proposed method extends from a previous work by Ma et al (2003) based on the residual generator technique for time domain damage diagnosis. In this method, the change of structural stiffness from different components is looked on as Multi-failure events for the state-space equation. For this disturbances decoupling problem (DDP), the geometric concept is used to design residual generators for all components of the structure. These generators process vibration signals of the structure and generate residual error signals. Each generator is only sensitive to the stiffness change of a particular component and is not affected by stiffness changes of other components. This makes it possible to detect and locate the occurrence of damage by monitoring the residual error signals in all the generators. By analyzing the residual error signal with a suitable error model, the damage severity of a component can also be obtained.

The essential development in the present work on the residual generator technique lies with the formulation of the model such that the response acceleration signals, rather than the displacement, can be used as the basic measurement for the analysis. This largely enhances the robustness of the method for practical applications. Furthermore, an explicit approach is developed for the determination of the damage severity.

5.1 Introduction

Typical time domain damage diagnosis methods follow a similar general procedure. A mathematical model representing the system is set up first from the signals measured from the undamaged state of the structure as the baseline. The baseline model is then applied on the signals measured from an unknown state of the same structure. If there is no structural change to the system, the model should fit the measured signals within a minimum error. However, if the dynamic properties of the system have changed due to damage, sensible errors (also called residual error) will occur when fitting the model to the actual measured signals. By analyzing the residual errors, a suitable damage feature can be extracted for the diagnosis of the damage (e.g., Masri et al., 1996; Sohn and Farrar, 2001). The methods presented in Chapter 3 and 4 of this thesis also adopted this general approach, with however enhanced capabilities due to the fact the only response (acceleration) signals are required and the problem with noise is tackled through a novel processing scheme based on incorporating Kalman filter.

However, generally speaking the ability of such methods in identifying the location of damage is limited to relatively simple systems, and in a “lump-sum” manner. The cause of this problem to a large extent may be attributed to the fact that the damage information is extracted from the signals measured from selected degrees of freedom (DOF). When multiple members are connected to the same joint and share a common DOF, it is not possible to use these methods to separate the condition of each connecting member based on the information extracted from the common DOFs.

To overcome the above difficulty in using the time-domain methods, it is essential to find a more explicit way to link the damage of each individual component to a certain damage feature. Furthermore, it is desired that the damage severity of each component can be calculated by directly analyzing the time series of the vibration signals.

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Ma, et al (2003) presented a time-domain method for detecting structural changes in individual components. The study specifically analyzed a 3-DOF linear lumped mass system representing a shear beam model using the following state space equations:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) + \Delta \mathbf{A} \mathbf{x}(t), \\ \mathbf{y}(t) &= \mathbf{C} \mathbf{x}(t).\end{aligned}\tag{5-1}$$

where parameters $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}$ can all be determined from the mass, stiffness, and damping coefficients of the structure. The input of the system is the excitation from ground, while the output of the system are displacements, velocities or their linear combinations. $\Delta \mathbf{A}$ represents the damages in the structure; $\Delta \mathbf{A}$ is a zero matrix if there is no damage. $\Delta \mathbf{A}$ can be further written as $\sum_{i=1}^k \mathbf{P}_i \mathbf{C}_i$, where k is the number of stiffness coefficients in the structure; $\mathbf{P}_i$ and $\mathbf{C}_i$ are column and row vectors, respectively, which are decomposed from the contribution of the damage in the i -th component to the change of the global stiffness matrix. Defining $m_i(t) = \mathbf{C}_i \mathbf{x}(t)$, the model can be written as:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) + \sum_{i=1}^k \mathbf{P}_i m_i(t) \\ \mathbf{y}(t) &= \mathbf{C} \mathbf{x}(t).\end{aligned}\tag{5-2}$$

This becomes the standard form of the Extension of Fundamental Problem of Residual Generation (EFPRG) to multiple failure events. The EFPRG was initially defined by Massoumnia (1986) as a disturbances decoupling problem (DDP) in the control theory. A solution known as the residual generator technique was also established by using the geometric approach (Massoumnia, 1986; Massoumnia et al., 1989). Ma et al. (2003) in the aforementioned study used this technique to design three generators respectively for the three springs in the system concerned. By processing the measured signals each generator could detect the failure of individual components separately. The results showed that the method was applicable for both stationary (white noise) and non-stationary (El Centro earthquake) ground excitations. But the method is limited to using displacement or velocity vectors. Moreover, the paper did not provide any explicit algorithm for calculating the damage severity.

The present study attempts to broaden the application of the residual generator technique in the damage diagnosis. The acceleration responses of structure, which can be more easily measured in practice, are considered in the analysis instead of the displacement. Furthermore, a theoretical derivation and an implementation algorithm are provided for the explicit calculation of the damage severity.

The following study includes four parts. In the first part, a brief introduction of EFPRG and its solution is presented. In the second part the dynamic system of a structure is studied in the time domain. The study is based on a state space model that can reflect the relationship between the excitation and the structural acceleration responses. After introducing the damage effect to different components and by appropriate transformation, this model can be written as the standard form of EFPRG. As such, the technique to solve EFPRG can be employed for the structural damage diagnosis. The third part deals with the identification of the damage severity in individual structural components. In the last part two numerical examples are given to demonstrate the implementation and the validity of the approach in locating the damage and identifying the damage severity.

5.2 Introduction of the Standard Form of EFPRG to Multiple Failure Events

Generally speaking, the Extension of Fundamental Problem of Residual Generation (EFPRG) is a disturbances decoupling problem (DDP) originated in the control theory (Massoumnia 1986; Massoumnia et al., 1989). In this section an introduction of EFPRG is given briefly and the solution known as the residual generator technique is also described.

The standard form of EFPRG with multiple failure events is written as:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) + \sum_{i=1}^k \mathbf{P}_i m_i(t) \\ \mathbf{y}(t) &= \mathbf{C} \mathbf{x}(t)\end{aligned}\tag{5-2}$$

Eq. (5-2) describes a *system* (C, A, B) . The nominal input $\mathbf{u}(t)$ and the output $\mathbf{y}(t)$ are assumed to be known. The signals $m_i(t)$ are arbitrary and unknown functions of time, as the i -th actuator failure mode. The *map* $\mathbf{P}_i: m_i \rightarrow \mathbf{X}$ is referred to as the i -th actuator failure signature. The range of $\mathbf{P}_i$, is looked on as disturbance from the i -th actuator. If there is no disturbance from the i -th actuator, the signal $m_i(t)$ is a zero signal.

Figure 5.1 Block diagram of a residual generator

The EFPRG may be solved by designing k residual generators, and a standard procedure for such purpose was developed by Massoumnia et al.(1989). It is desired that a nonzero $m_i(t)$ should affect the residual signal of the i -th residual generator, $r_i(t)$, and only this generator. At the same time, other non-zero $m_j(t)$ for $j \neq i$ should not affect $r_i(t)$ of this residual. The process of one generator is depicted in Fig. 5.1.

According to Massoumnia et al.(1989), such residual generator has a general form for a realizable linear time-invariant (LTI) processor that takes the observables $\mathbf{y}(t)$ and $\mathbf{u}(t)$ as inputs and generates a residual $\mathbf{r}_i(t)$ is:

$$\dot{\mathbf{w}}(t) = \mathbf{F}\mathbf{w}(t) - \mathbf{E}\mathbf{y}(t) + \mathbf{G}\mathbf{u}(t)$$

$$\mathbf{r}_i(t) = \mathbf{M}\mathbf{w}(t) - \mathbf{H}\mathbf{y}(t) . \quad (5-3)$$

The residual generator represented by Eq. (5-3) may be designed for a specific actuator, so that when the actuator failure mode is present, a nonzero signal $m_i(t)$ can lead to a nonzero $\mathbf{r}_i(t)$. Thus, $\mathbf{r}_i(t)$ calculated from Eq. (5-3) becomes the monitor of the i -th actuator. A geometric approach may be used to develop these monitors (Massoumnia et al., 1989), i.e., to determine the coefficients in Eq. (5-3).

When the system with its parameters $[A_c, B_c, C, D, P_i \text{ (} i=1,k \text{)}]$ is provided and satisfies the solution condition (Appendix 3), a procedure can calculate the parameters $[E, F, G, M, H]$ of the monitor as Eq. (5-3). Further information on the procedure is given in Appendix 5.3.

As the geometric approach is based on the concept of *unobservability* subspace (see Appendix 5.2), here we can define $e(t) = w(t) - O\mathbf{x}(t)$ when the generator's parameters $[E, F, G, H, M]$ are solved through the procedure described in Appendix 5.3. From Eq. (5-2) and (5-3) the following can be obtained (Massoumnia et al, 1989; see also Appendix 5.4):

$$\begin{aligned}\dot{e}(t) &= \mathbf{F}e(t) - \mathbf{O}P_i m_i(t) \\ r_i(t) &= \mathbf{M}e(t) = \mathbf{M}w(t) - \mathbf{H}y(t)\end{aligned}\tag{5-4}$$

In Eq. (5-4), $\dot{e}(t)$ is expressed with $m_i(t)$ as the only input. It follows that the system relating $m_i(t)$ to $e(t)$ is *input observable*; obviously other disturbances $m_{j \neq i}(t)$ are *unobservable*. Hence, $r_i(t)$ is not affected by signals $m_{j \neq i}(t)$. Only when $m_i(t)$ exhibits failure of the i -th actuator, $r_i(t)$ will become distinctively non-zero. Thus, $r_i(t)$ satisfies the requirement of the solution as an indicator of the i -th actuator.

5.3 Problem Formulation for Structural Damage Diagnosis

A state space model can be used to describe a structural dynamic system, using the excitation on the structure as input and the structural dynamic responses (e.g. displacement, velocity, or acceleration) as output. In such a model the parameters are easily related to the physical properties of the structure, thus allowing for the physical damage of the structure to be introduced by an explicit way. If the damage effects of different components in the model can be treated as failure of different actuators following different signatures, then the solution of EFPRG can be applied to identify the damage to individual components of the structure. As introduced by

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Ma et al. (2003), a state-space model based on displacement or velocity responses can be formulated for EFPRG to be applied. Considering the fact that acceleration signals are the most common measurements in an actual testing of a structure, the present study focuses on the state space model with acceleration as the measurement. Unlike the models based on displacement or velocity, in the acceleration based model, the model parameters $\mathbf{A}_c$ and $\mathbf{B}_c$ are both affected when damage is introduced in terms of a stiffness change. Hence, the failure modes and disturbances cannot be figured out in a straightforward manner and a pertinent formulation is required, as described in what follows.

Refer to Chapter 3, based on the equation of motion as expressed in Eq. (3-5) or (3-6) the state space representation of a structural dynamic system can be written as follows:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C} \mathbf{x}(t) + \mathbf{L} \mathbf{u}(t)\end{aligned}\tag{3-11}$$

where $\mathbf{u}(t)$ is the excitation and $\mathbf{y}(t)$ is the structural acceleration responses. Confining $\mathbf{y}(t)$ as accelerations, the model coefficients can be determined as:

$$\mathbf{A}_c = \begin{bmatrix} -\mathbf{J} & \mathbf{I} \\ -\mathbf{K} & \mathbf{0} \end{bmatrix}, \mathbf{B}_c = \begin{bmatrix} -\mathbf{J}\mathbf{L} \\ -\mathbf{K}\mathbf{L} \end{bmatrix}, \mathbf{C} = [\mathbf{I} \quad \mathbf{0}].$$

where $\mathbf{K}$, $\mathbf{J}$, $\mathbf{L}$ are defined as $\mathbf{K} = \mathbf{M}_0^{-1}\mathbf{K}_0$, $\mathbf{J} = \mathbf{M}_0^{-1}\mathbf{C}_0$, and $\mathbf{L} = \mathbf{M}_0^{-1}\mathbf{L}_0$, from the dynamic equation:

$$\mathbf{M}_0 \ddot{\boldsymbol{\xi}}(t) + \mathbf{C}_0 \dot{\boldsymbol{\xi}}(t) + \mathbf{K}_0 \boldsymbol{\xi}(t) = \mathbf{L}_0 \mathbf{u}(t). \tag{3-5}$$

When the structural changes are limited only to the stiffness change, $\mathbf{J}$ and $\mathbf{L}$ will remain unchanged. Hence, $\mathbf{L}\mathbf{u}(t)$ can be moved to the left hand side in the second equation of Eq. (3-11), so that $\mathbf{y}(t) - \mathbf{L}\mathbf{u}(t)$ can be used as the output of the model instead of $\mathbf{y}(t)$. In the following study, we define $\mathbf{y}^t(t) = \mathbf{y}(t) - \mathbf{L}\mathbf{u}(t)$.

When damage occurs in the structure with some stiffness changes, the model parameters $\mathbf{A}_c$ and $\mathbf{B}_c$ will change because they are related to the stiffness matrix $\mathbf{K}$. Thus, the system with damage can be written as:

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$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) + \Delta \mathbf{A} \mathbf{x}(t) + \Delta \mathbf{B} \mathbf{u}(t)$$

$$\mathbf{y}^t(t) = \mathbf{C} \mathbf{x}(t). \quad (5-5)$$

Define $\Delta \mathbf{K}$ as the change of the stiffness matrix $\mathbf{K}$ due to damage in some components; it can be decomposed (see Appendix 6.1) as:

$$\Delta \mathbf{K} = \sum_{i=1}^k \mathbf{p}_i \mathbf{q}_i \alpha_i \quad (5-5a)$$

where k is the number of components that may be subject to damage; α_i denotes the damage severity of the i -th component; $\mathbf{p}_i$ and $\mathbf{q}_i$ are the decomposed matrices from the contribution of the i -th component stiffness in the global stiffness matrix.

$\Delta \mathbf{A}$ and $\Delta \mathbf{B}$ can be written as:

$$\Delta \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -\Delta \mathbf{K} & \mathbf{0} \end{bmatrix} = -\sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} [\mathbf{q}_i, \mathbf{0}] \alpha_i \quad (5-5b)$$

$$\Delta \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\Delta \mathbf{K} \mathbf{L} \end{bmatrix} = -\sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} \mathbf{q}_i \mathbf{L} \alpha_i, \quad (5-5c)$$

and $\Delta \mathbf{A} \mathbf{x}(t) + \Delta \mathbf{B} \mathbf{u}(t)$ can be written as:

$$\begin{aligned} \Delta \mathbf{A} \mathbf{x}(t) + \Delta \mathbf{B} \mathbf{u}(t) &= -\sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} [\mathbf{q}_i, \mathbf{0}] \alpha_i \mathbf{x}(t) - \sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} \mathbf{q}_i \mathbf{L} \alpha_i \mathbf{u}(t) \\ &= -\sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} \alpha_i ([\mathbf{q}_i, \mathbf{0}] \mathbf{x}(t) + \mathbf{q}_i \mathbf{L} \mathbf{u}(t)) \\ &= \sum_{i=1}^k \mathbf{P}_i m_i(t). \end{aligned} \quad (5-6)$$

Here the failure signatures are: $\mathbf{P}_i = [\mathbf{0} \quad \mathbf{p}_i]^T$ for each component, and the disturbance $m_i(t)$ has the form:

$$\begin{aligned} m_i(t) &= -\alpha_i (\mathbf{q}_i [\mathbf{I}, \mathbf{0}] \mathbf{x}(t) + \mathbf{q}_i \mathbf{L} \mathbf{u}(t)) \\ &= -\alpha_i \mathbf{q}_i (\mathbf{C} \mathbf{x}(t) + \mathbf{L} \mathbf{u}(t)) \\ &= -\alpha_i \mathbf{q}_i \mathbf{y}(t) \end{aligned} \quad (5-6a)$$

Although this definition of $m_i(t)$ is not necessary for EFPRG, it will be used in the next section for assessing the damage severity.

Substitution of Eq. (5-6) in Eq. (5-5) yields the model with multi-failures events as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t) + \sum_{i=1}^k \mathbf{P}_i m_i(t)$$

$$\mathbf{y}^t(t) = \mathbf{C} \mathbf{x}(t).$$
(5-7)

a) Block diagram for the design of residual generator

b) Block diagram for residual generation for i th component

Figure 5.2 Standard procedure of design of residual generators and residual generation

Through the above transformation of equations, the damage effect on different components can be described as the actuator failure of *system* $(\mathbf{C}, \mathbf{A}, \mathbf{B})$. Now the geometric approach can be used to design the residual generators of Eq. (5-7), which has the standard form of EFPRG as Eq. (5-2). For the EFPRG problem, when *system* $(\mathbf{C}, \mathbf{A}, \mathbf{B})$ is given, $\mathbf{P}_i$ (refer to Appendix 5.1) for the damage pattern on the i -th component is fixed by a simple calculation; therefore, the residual generator for each component can be designed following the algorithm presented in Appendix 5.3 at the end of this Chapter. The diagrams for designing the residual generators and the procedure of residual generation are shown in Fig. 5.2. Then simply examining

the status of $\mathbf{r}_i(t)$ from the i -th generator (whether or not it is a zero signal), the damage status of the i -th component can be identified.

5.4 Evaluation of Damage Severity

When the residual generators have been designed for possible damage in individual components, there will be a series of monitors and every monitor has the form of Eq.(5-3) with its own coefficients. As the residual error $\mathbf{r}_i(t)$ only represents a specific component, it is possible to calculate the damage severity by further examining the residual error and the model coefficients.

Using the definition of $\mathbf{m}_i(t)$ in Eq. (5-6a), from Eq. (5-4) we get:

$$\dot{\mathbf{e}}(t) = \mathbf{F}\mathbf{e}(t) + \alpha_i \mathbf{O}_i \mathbf{P}_i \mathbf{q}_i \mathbf{y}(t)$$

$$\mathbf{r}_i(t) = \mathbf{M}\mathbf{e}(t) \quad (5-8)$$

With the generator parameters obtained as mentioned in Section 5.3, and $\mathbf{y}(t)$, $\mathbf{u}(t)$ available from the measurement, $\mathbf{r}_i(t)$ can be obtained from the i -th generator by Eq. (5-3). When parameter $\mathbf{u}_i$ is also provided (refer to Appendix 5.1), the damage severity factor α_i as the only unknown parameter in Eq. (5-8) can be easily estimated. The block diagram of evaluating the damage severity factor α_i is shown in Fig. 5.3.

Figure 5.3 Block diagram for evaluation of damage severity

5.5 Numerical Examples

In this section the proposed approach is applied on a three-storey lumped-mass system and a truss for damage diagnosis. As joint rotations are difficult to measure in real practice, the selection of the numerical examples takes into consideration of this constraint, so that no joint rotations are included. But in principle, the method is also applicable for structures with joint rotations so long as the rotational response signals can be measured.

The three-storey lumped mass system herein is identical to the example used by Ma et al. (2003); however, herein acceleration signals are used instead of displacements and the damage severity is also calculated. The simply supported truss system is employed to further demonstrate the applicability of the method on a more complex structure.

In these examples, the mass, stiffness, and damping of the undamaged system are pre-selected. The time series of the acceleration measurements under various damage scenarios are simulated by a numerical analysis procedure. These signals are used as the available measurement for the damage diagnosis.

In implementing the models for the generators and damage severity, the continuous formulation described in the previous sections are transformed into the discrete models by zero-order-hold discretization. The discretization of state-space model can be expressed (Phillips and Nagle 1984) as:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k).\end{aligned}\tag{3-2}$$

where $\mathbf{A} = \sum_{j=0}^{\infty} \frac{(\mathbf{A}_c \Delta t)^j}{j!}$, and $\mathbf{B} = \mathbf{A}_c^{-1}(\mathbf{A}_c - \mathbf{I})\mathbf{B}_c$

5.5.1 Three-story lumped mass shear frame

The three-story lumped mass shear frame, shown in Fig. 5.4, is assumed to have uniform properties in all stories, with mass $M_i = 1000$ kg, stiffness $K_i = 980$ kN/m,

and damping coefficient $C_i = 1.407$ kN-s/m. The natural frequencies of the frame are obtained to be 2.22, 6.21 and 8.97 Hz, respectively. The damage is represented by stiffness changes. The simulated damage scenarios include single-damage and multiple-damage cases with different damage severity. A broad-band white noise acceleration with a standard deviation of 1 m/s^2 is considered as the ground excitation. The measurements are assumed to be the accelerations at the three storey levels.

Figure 5.4 A three-story lumped mass shear frame model

Applying the procedure shown in Appendix 5.1, the failure signatures for this example are found to be:

$$\mathbf{P}_1 = [0, 0, 0, 0, 0, 1]^T$$

$$\mathbf{P}_2 = [0, 0, 0, 0, -1, 1]^T$$

$$\mathbf{P}_3 = [0, 0, 0, -1, 1, 0]^T$$

Also, referring to Appendix 5.3, as $k_i = 1$ for actuators 1, 2 and 3, and $l = 3$, $v = 3$, $n = 6$, the generic solvability conditions $v \leq n$ and $v - \min\{k_i, i \in k\} < l$ are satisfied. Then three monitors designated as No. 1, 2, and 3 are designed to monitor the

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stiffness coefficients K_1 , K_2 , and K_3 , respectively. The design of the residual generator $\mathbf{r}_2(t)$ is described as an example in what follows.

First, the subspace W_2^* of (C, A) -invariant is given by the following **CAISA**[(C, A)-Invariant Subspace Algorithm] (refer to Appendix 5.2):

$$W_2^* = P_1 \oplus P_3 \oplus A_c P_1 \oplus A_c P_3;$$

Thus the insertion map $W_2: W_2^* \rightarrow X$ is $W_2 = [P_1, P_3, A_c P_1, A_c P_3]$.

S_2^* as (C, A) -u.o.s. (**UnObservability Subspace**) is obtained by the following **UOSA** (**UnObservability Subspace Algorithm**) (refer to Appendix 5.2):

$$S_2^* + \text{Ker } C = W_2^* + \text{Ker } C.$$

Step 1: Find a D that satisfies $D_0 \in \underline{D}(S_2^*)$. According to **UOSA**, such D can be found by $(A_c + DC)W_2^* \subseteq W_2^*$. One such D is given by Massoumnia (1986) as:

$$D_0 = -A_c^{-2} [P_1, P_3] (CA_c [P_1, P_3])^{-1}$$

$$D_0 = \begin{bmatrix} 1.4 & 0 & 0 \\ -1.4 & 0 & 0 \\ 0 & 0 & 1.4 \\ 1960 & 0 & 0 \\ -2940 & 0 & -980 \\ 980 & 0 & 1960 \end{bmatrix}$$

Step 2: Find O as: $X \rightarrow X/S_2^*$. Referring to the definition of the *canonical projection* in Appendix 6.2, $\text{Ker } O = S_2^*$. One such O can be extracted from $[P_1, P_2, P_3, A_c P_1, A_c P_2, A_c P_3]^{-1}$ by its second row and fifth row as:

$$O = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

And let $A_0 = A_c + D_0 C$: X/S_2^* be the map induced on the factor space X/S_2^* ,

$$A_0 = \begin{bmatrix} -1.407 & 1 \\ -980 & 0 \end{bmatrix}$$

Step 3: H is a solution of: $\text{Ker } HC = S_2^* + \text{Ker } C$.

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Hence, $\mathbf{H}$ can be calculated by:

$$\mathbf{H} = \mathbf{I} - \mathbf{CA}_c[\mathbf{P}_1, \mathbf{P}_3](\mathbf{CA}_c[\mathbf{P}_1, \mathbf{P}_3])^{-1}$$

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 4: $\mathbf{M}$ is the unique solution of $\mathbf{MO} = \mathbf{HC}$

$$\mathbf{M} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Step5: Let $\text{eig}(\mathbf{F}) = [-1, 0; 0, -1]$ that satisfies the condition of self-conjugate with negative eigenvalue; then $\mathbf{F}$ satisfying $\mathbf{F} = \mathbf{A}_0 + \mathbf{D}_1\mathbf{M}$ can be found, as

$$\mathbf{F} = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$$

Hence $\mathbf{D}_1$ is found as:

$$\mathbf{D}_1 = \begin{bmatrix} 0 & -0.593 & 0 \\ 0 & 979 & 0 \end{bmatrix}$$

Step 6: With $\mathbf{O}^r$ denoting a right inverse of $\mathbf{O}$, calculate $\mathbf{D} = \mathbf{D}_0 + \mathbf{O}^r\mathbf{D}_1\mathbf{H}$, $\mathbf{E} = \mathbf{OD} = \mathbf{OD}_0 + \mathbf{D}_1\mathbf{H}$, $\mathbf{G} = \mathbf{OB}_c$

$$\mathbf{E} = \begin{bmatrix} -0.593 & -0.593 & 0 \\ -1 & 979 & -980 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} -2.8140 \\ 0 \end{bmatrix}$$

Then the generator of $\mathbf{r}_2(t)$ that takes the observables $\mathbf{y}(t)$ and $\mathbf{u}(t)$ as inputs is completely established following Eq. (5-3) as:

$$\dot{\mathbf{w}}(t) = \mathbf{F}\mathbf{w}(t) - \mathbf{E}\mathbf{y}(t) + \mathbf{G}\mathbf{u}(t)$$

$$\mathbf{r}_2(t) = \mathbf{M}\mathbf{w}(t) - \mathbf{H}\mathbf{y}(t) .$$

The generator $\mathbf{r}_1(t)$ and $\mathbf{r}_3(t)$ are designed following the same procedure. In order to keep the same magnitude of three residual signals, the $\text{eig}(\mathbf{F}) = [-1, 0; 0, -1]$ is adopted for each generator.

Fig. 5.5 and Fig. 5.6 show the output of the residual generators for different damage scenarios. Each figure includes the signals from three residual generators for a particular damage scenario. When a residual signal from a generator shows an appreciable magnitude, it indicates the occurrence of damage on the structure at the component represented by the corresponding generator. On the other hand, if a residual signal is of negligible magnitude, no change takes place at the corresponding component. From the results shown in Fig. 5.5 and Fig. 5.6, the various damage scenarios can be clearly determined.

a) Reduction of K_1 by 10%

b) Reduction of K_2 by 10%

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c) Reduction of K_3 by 10%

Figure 5.5 Residual error of acceleration signals for single-damage scenarios

a) Reduction of K_1 and K_2 by 10% each

b) Reduction of K_1 and K_3 by 10% each

c) Reduction of K_1 , K_2 and K_3 by 10% each

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d) Reduction of K_2 and K_3 by 20% each

Figure 5.6 Residual error of acceleration signals for multiple-damage scenarios

With the residual signal generated, the damage severity can be calculated following the block diagram shown in Fig. 5.3. The values of α_i for the three springs are listed in Table 5.1. It can be seen that the α_i values reflect quite accurately the actual damage severity in terms of the percentage reduction of the spring stiffness.

Table 5.1 Results of damage severity (α values) for the shear frame example

Damage Scenarios	α_1 for K_1	α_2 for K_2	α_3 for K_3
No damage	0.15%	-0.04%	-0.02%
K_1 10% damage	10.12%	-0.06%	-0.02%
K_2 10% damage	0.15%	9.97%	-0.02%
K_3 10% damage	0.14%	-0.03%	9.98%
K_1 and K_2 10% damage	10.12%	9.97%	-0.02%
K_1 and K_3 10% damage	10.12%	-0.04%	9.98%
K_2 and K_3 10% damage	0.14%	9.97%	9.98%
K_2 and K_3 20% damage	0.16%	19.97%	19.98%
K_1, K_2, K_3 10% damage	10.12%	9.97%	9.98%

5.5.2 Truss System

Figure 5.7 Truss with component code and DOF code

The truss is shown in Fig. 5.7. This is a case whereby the traditional time-domain method could have difficulty in locating the damage if signals measured at the joints are considered. This is because multiple members are joined together at certain joints; therefore it is difficult to pinpoint changes on individual members based on the residual errors from the signals measured at these joints. However, using the residual generator technique it becomes possible to determine the damage status of each member.

In this truss example, each member is 0.5 m long, and all the members are made by aluminium pipe with an outer radius of 20 mm and a wall thickness of 2 mm. The modulus of elasticity of aluminium is 6.96×10^7 KN/m². Joints J1, J2 and J3 are allocated with equal concentrated mass of 50 kg. The damping ratio is assumed to be 0.5% for all modes. The truss is excited at J1 with a vertical dynamic force of the broad band white noise type with a standard deviation of 50 N. The acceleration at the 6 DOFs shown in Fig. 5.7 are taken as the measurement.

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Six residual generators are designed, one for each member of the truss. With the acceleration at the joints provided, the damage at each component can be indicated by the corresponding residual generator.

Two damage scenarios are chosen to show the results, one with a stiffness reduction of 20% in member 2, another with a stiffness reduction of 20% in both members 2 and 4. For such damage scenarios, the traditional method will meet difficulty in judging which member may have damage. For example, signals from joint J1 and J2 are affected by both members; therefore non-zero residual errors at these joints can cause confusion as it is hard to distinguish specific contributions from individual members. The use of the residual generators for individual members overcomes this problem.

a) Damage 20% on both K_2 and K_4

b) Damage 20% on K_2

Figure 5.8 Residual error of acceleration signals from generators of K_2 and K_4

The output from generators for K_2 and K_4 are plotted in Fig. 5.8. The signals from other generators are not shown because they are almost zero in these cases. From that figure, the damaged members can be easily identified from the large amplitude output signals. The values of α_i for the three truss members are listed in Table 5.2. It can be observed that the percentage stiffness changes are very well identified.

Table 5.2 Results for damage severity (α values) for the truss example

Damage Scenarios	α_2 for K_2	α_4 for K_4
K_2 and K_4 20% damage	21.73%	20.54%
K_2 20% damage	20.46%	0.50%

5.6 Conclusions

An extended residual generator technique is presented for time-domain damage diagnosis using acceleration signals. In addition to detecting the occurrence and precisely locating the damage in individual elements, the method also makes it possible to explicitly calculate the severity of damage in terms of stiffness reduction. The basic idea is that, through appropriate transformation, the structural dynamic system with damage can be expressed as multi-failure events in state space model. This disturbance decoupling problem can be written in the standard form of EFPRG, so that the solution of EFPRG can be used to design the residual generators for all individual components of the structure. In this way, each monitor is associated only with the stiffness change of a specific component and is not affected by the stiffness changes in other components. Thus, when damage occurs to certain components, the corresponding monitors will produce non-zero residual error signals and so the damaged members can be identified easily.

Necessary formulation is worked out to enable the use of acceleration signals, which are easier to measure in actual applications as compared to displacements, in the analysis. By further analyzing the residual signals generated by EFPRG, a procedure is derived to calculate the damage severity on each component in terms

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of stiffness change. Numerical results show that the method can effectively determine the damage location and severity in a frame with rigid floors and a truss system.

The limitation of the method in its present form is that it requires measured signals at all degrees of freedom of the structure at the discretized nodes. For example, it cannot be applied to diagnose the damage in a beam with only translational acceleration signals but without rotational measurement. In such cases, the required $(\mathbf{C}, \mathbf{A})$ – *invariant* subspace may not exist, and then the EFPRG does not have a solution. Although the application of the method to beam like systems may be made possible with the technological developments in the rotational measurements, it will be desired if the method can be extended to using other auxiliary information in lieu of rotational measurements for the damage diagnosis. To this end, further development of the method is required.

Appendix of Chapter 5

Appendix 5.1. Decomposition of stiffness matrix

In order to satisfy the generic solvability conditions for EFPRG (Refer to Appendix 5.3), failure signatures $\mathbf{P}_i$ should contain as less number of columns as possible. So the decomposition of the damage effect of the model parameters is necessary.

A component stiffness matrix can be decomposed into the product of matrices with fewer columns and fewer rows, respectively. For example,

The stiffness matrix of spring element:

$$\mathbf{k} = k_0 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k_0 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \end{bmatrix}$$

The stiffness matrix of truss element:

$$\mathbf{k} = \frac{EA}{l} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} = \frac{EA}{l} \begin{bmatrix} c \\ s \\ -c \\ -s \end{bmatrix} \begin{bmatrix} c & s & -c & -s \end{bmatrix}$$

The stiffness matrix of beam element:

$$\mathbf{k} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} = \frac{2EI}{l^3} \begin{bmatrix} 1 & 1 \\ l & 0 \\ -1 & -1 \\ 0 & l \end{bmatrix} \begin{bmatrix} 3 & 2l & -3 & l \\ 3 & l & -3 & 2l \end{bmatrix}$$

In the global stiffness matrix, the distribution of each component matrix can be looked on as the component matrix putting in an empty matrix with the dimension of the global stiffness matrix. So the distribution of the i -th component as $\mathbf{K}_{0i}$ also can be decomposed in the same way. Take the structure in Section 5.5.1 as an example, the decomposition of the truss $\mathbf{K}_2$ in the global stiffness matrix is:

$$\mathbf{K}_{02} = k_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} k_2$$

Subsequently, $\Delta \mathbf{K}$ can be written as below for all possible damage on these components,

$$\Delta \mathbf{K} = \mathbf{M}_0^{-1} \sum_{i=1}^k \alpha_i \mathbf{K}_{0i} = \sum_{i=1}^k \mathbf{p}_i \mathbf{q}_i \alpha_i$$

where k is the number of components that may be subject to damage, α_i denotes the damage severity of the i -th component.

Considering Eq. (3-27), finally $\Delta \mathbf{A}$ and $\Delta \mathbf{B}$ can be written as:

$$\Delta \mathbf{A} = \begin{bmatrix} 0 & 0 \\ -\Delta \mathbf{K} & 0 \end{bmatrix} = -\sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} [\mathbf{q}_i, \mathbf{0}] \alpha_i$$

$$\Delta \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\Delta \mathbf{K} \mathbf{L} \end{bmatrix} = -\sum_{i=1}^k \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix} [\mathbf{q}_i] \mathbf{L} \alpha_i$$

The failure signature for the i -th component is:

$$\mathbf{P}_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_i \end{bmatrix}.$$

Appendix 5.2. Notation and background information

A quick summary of the relevant notions, linear algebra and the rudiments of linear dynamic systems is provided in this section. It mainly serves the purpose for the presentation of mathematical problem that this paper is concerned. Interested readers may refer to (Wonham, 1985; Massoumnia, 1986; Massoumnia et al., 1989) for more details.

Space and subspace

Denote the real vector spaces by $X, Y, Z, \dots$, and typical elements by $x, y, z, \dots$; $d(X)$ is the dimension of X . A subspace S of the linear space X is presented as: $S \subset X$. It is a subset of X which is a linear space under the operations of vector addition and scalar multiplication inherited from X .

If $R, S \subset X$, the internal direct sum of subspaces is written as: $S \oplus R$. It means the subspaces being added are known, or claimed, to be independent. It has $d(S \oplus R) = d(S) + d(R) - d(S \cap R)$.

Matrix and Map

If $\mathbf{B}$ is *monic*, then $\mathbf{B}^{-1}$ denotes a left inverse of $\mathbf{B}$ (i.e., $\mathbf{B}^{-1}\mathbf{B} = \mathbf{I}$). If $\mathbf{C}$ is *epic*, then $\mathbf{C}^r$ denotes a left inverse of $\mathbf{C}$ (i.e., $\mathbf{C}\mathbf{C}^r = \mathbf{I}$).

Let $\{x_i, i \in n\}$ be a basis for $\mathbf{X}$ and $\{y_j, j \in p\}$ a basis for $\mathbf{Y}$. If $\mathbf{C}: \mathbf{X} \rightarrow \mathbf{Y}$ is a *map*,

$$\mathbf{C}x_i := c_{1i}y_1 + c_{2i}y_2 + \cdots + c_{pi}y_p, \quad i \in n$$

The symbol “:=” means equality by definition. Observe that if $x \in \mathbf{X}$ then $\mathbf{C}x$ is completely determined by the $\mathbf{C}x_i$ due to linearity. Matrices and linear maps are denoted by $\mathbf{A}, \mathbf{B}, \mathbf{C}$; the same symbol is used both for a matrix and its map.

As $\mathbf{C}: \mathbf{X} \rightarrow \mathbf{Y}$, $\text{Im } \mathbf{C}$ denotes the image of $\mathbf{C}$:

$$\text{Im } \mathbf{C} := \{\mathbf{C}x : x \in \mathbf{X}\} \subset \mathbf{Y}$$

and $\text{Ker } \mathbf{C}$ denotes the kernel of $\mathbf{C}$:

$$\text{Ker } \mathbf{C} := \{x : x \in \mathbf{X} \ \& \ \mathbf{C}x = 0\} \subset \mathbf{X}.$$

The maps $\mathbf{A}: \mathbf{X} \rightarrow \mathbf{X}$, $\mathbf{B}: \mathbf{U} \rightarrow \mathbf{X}$, and $\mathbf{C}: \mathbf{X} \rightarrow \mathbf{Y}$ ($d(\mathbf{X}) = n$, $d(\mathbf{Y}) = l$, $d(\mathbf{U}) = m$) are fixed throughout and are associated with the “*system* ($\mathbf{C}, \mathbf{A}, \mathbf{B}$)”:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t),$$

$$y(t) = \mathbf{C}x(t).$$

Factor space

Let $\mathbf{S} \subset \mathbf{X}$. Call vectors $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ equivalent mod $\mathbf{S}$ if $x - y \in \mathbf{S}$. The *factor space* $\mathbf{X}/\mathbf{S}$ is defined as the set of all equivalence classes

$$\bar{x} := \{y; y \in \mathbf{X}, y - x \in \mathbf{S}\}, \quad x \in \mathbf{X}.$$

The function $x \mapsto \bar{x}$ is a map $\mathbf{O}: \mathbf{X} \rightarrow \mathbf{X}/\mathbf{S}$ called the *canonical projection* of $\mathbf{X}$ on $\mathbf{X}/\mathbf{S}$. Clearly $\mathbf{O}$ is epic, and $\text{Ker } \mathbf{O} = \mathbf{S}$.

Invariant

A subspace $W \subseteq X$ is termed *A-invariant* if $AW \subseteq W$. Let $W \subseteq X$ be *A-invariant*; write $A|_W$ for the restriction of A to W , and $A: X/W$ for the map induced by A on the factor space X/W .

If a subspace $W \subseteq X$ is (C, A) -invariant there exists a map $D: Y \rightarrow X$ such that $(A + DC)W \subseteq W$. Let W be (C, A) -invariant; we denote by $\underline{D}(W)$ the class of all maps D such that $(A + DC)W \subseteq W$. The W is (C, A) -invariant if and only if $A(W \cap \text{Ker} C) \subseteq W$. Let $L \subseteq X$; here define the family of (C, A) -invariant subspaces containing L by $\underline{W}(L)$. Referring to Massoumnia (1986), the family $\underline{W}(L)$ is closed under intersection; hence $\underline{W}(L)$ contains an infimal element $W^* := \inf \underline{W}(L)$. Also $W^* = \lim W^k$ where W^k is given by the following recursive algorithm [(C, A)-Invariant Subspace Algorithm] (Massoumnia 1986):

$$CAISA: W^{k+1} = L + A(W^k \cap \text{Ker} C), \quad W^0 = 0$$

Unobservable and observable

If the subspace $B = \text{Im} B$ and $\langle A|B \rangle = B + AB + \dots + A^{n-1}B$ for the infimal *A-invariant* subspace containing B , i.e., the reachable subspace of (A, B) . Write $K = \text{Ker} C$ and $\langle K|A \rangle = K \cap A^{-1}K \cap \dots \cap A^{n-1}K$ for the supremal *A-invariant* subspace contained in K , i.e., the *unobservable subspace* of (C, A) .

A subspace $S \subseteq X$ is a (C, A) *UnObservability Subspace* (*u.o.s.*) if $S = \langle \text{Ker} HC|A+DC \rangle$ for some output injection map $D: Y \rightarrow X$ and measurement mixing map $H: Y \rightarrow Y$. Note that S is the unobservable subspace of the pair $(HC, A+DC)$, and the spectrum of $A+DC: X/S$ can be assigned to an arbitrary symmetric set by appropriate choice of D . Here use the notation $\underline{S}(L)$ for the class of *u.o.s.* containing L . The class of *u.o.s.* $\underline{S}(L)$ is closed under intersection; therefore, it contains an infimal element $S^* := \inf \underline{S}(L)$. Also $S^* = \lim S^k$ where S^k is given by

the following recursive algorithm (*UnObservability Subspace Algorithm*) (Massoumnia 1986):

$$UOSA: \mathbf{S}^{k+1} = \mathbf{W}^* + (\mathbf{A}^{-1} \mathbf{S}^k) \cap \text{Ker} \mathbf{C}, \quad \mathbf{S}^0 = \mathbf{X}$$

Moreover, $\mathbf{S}^* = \langle \text{Ker} \mathbf{C} + \mathbf{W}^* | \mathbf{A} + \mathbf{D} \mathbf{C} \rangle$ for $\mathbf{W}^* := \inf \underline{\mathbf{W}}(\mathbf{L})$ and $\mathbf{D} \in \underline{\mathbf{D}}(\mathbf{W}^*)$. It follows that $\underline{\mathbf{D}}(\mathbf{W}^*) \subseteq \underline{\mathbf{D}}(\mathbf{S}^*)$, and $\text{Ker} \mathbf{C} + \mathbf{W}^* = \text{Ker} \mathbf{C} + \mathbf{S}^*$.

A system $(\mathbf{C}, \mathbf{A}, \mathbf{B})$ is *input observable* if $\mathbf{B}$ is monic and $\mathbf{S} \cap \mathbf{B} = 0$, $\mathbf{S}$ is the *u.o.s.* of $(\mathbf{C}, \mathbf{A})$.

Appendix 5.3 The Extension of Fundamental Problem of Residual Generation (EFPRG) to Multiple Failure Events and Monitor Design Procedure

In this section the formulation of the Extension of Fundamental Problem of Residual Generation (EFPRG) to multiple failure events is described. A geometric approach is used to solve the EFPRG to multiple failure events, and an algorithm is presented to design residual generators for multiple failure events as monitors which are only sensitive to their specific failure but insensitive to the others (Wonham, 1985; Massoumnia, 1986; Massoumnia et al., 1989).

Consider the EFPRG form with multiple failure events (Eq. (5-2) in the main text):

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \sum_{i=1}^k \mathbf{P}_i m_i(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{5-2}$$

Eq. (5-2) describes a system $(\mathbf{C}, \mathbf{A}, \mathbf{B})$. The observables are the known actuation signal $\mathbf{u}(t)$ and the output $\mathbf{y}(t)$. The maps $\mathbf{P}_i: \mathbf{M}_i \rightarrow \mathbf{X}$ is referred as the i -th actuator failure signatures. The range of $\mathbf{P}_i$ is $\mathbf{P}_i$. The signal $m_i(t)$ is the disturbance from the i -th actuator, which is unknown. If there is no disturbance from the i -th actuator, the signal $m_i(t)$ is zero signal.

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Here the generic solvability conditions for EFPRG should be stated (Massoumnia 1986, Massoumnia et al. 1989): With $\mathbf{A}$, $\mathbf{C}$ and the $\mathbf{P}_i$ being arbitrary matrices of dimensions $n \times n$, $l \times n$, $n \times k_i$, respectively, and let $v = \sum_{i=1}^k k_i$, the EFPRG generically has a solution if and only if $v \leq n$ and $v - \min\{k_i, i \in k\} < l$. So the $\mathbf{P}_i$ should contain as less number of columns as possible.

The most general form for a realizable LTI (linear time-invariant) processor that takes the observables $\mathbf{y}(t)$ and $\mathbf{u}(t)$ as inputs and generates a residual $\mathbf{r}(t)$ is

$$\begin{aligned}\dot{\mathbf{w}}(t) &= \mathbf{F}\mathbf{w}(t) - \mathbf{E}\mathbf{y}(t) + \mathbf{G}\mathbf{u}(t) \\ \mathbf{r}_i(t) &= \mathbf{M}\mathbf{w}(t) - \mathbf{H}\mathbf{y}(t).\end{aligned}\tag{5-3}$$

Eq. (5-3) is a residual generator only designed for a specify actuator. The following procedure is to decide the coefficients of Eq. (5-3) so that when the i -th actuator failure mode is present, the nonzero signal $\mathbf{m}_i(t)$ can lead to the nonzero $\mathbf{r}_i(t)$. So $\mathbf{r}_i(t)$ calculated from Eq. (5-3) is the monitor of the i -th actuator.

General speaking, the algorithm of this solution is that, by choosing $\mathbf{D}_0$ and $\mathbf{H}$ appropriately, one can change the observability properties of $(\mathbf{H}\mathbf{C}, \mathbf{A} + \mathbf{D}_0\mathbf{C})$ in such a way that failure of all actuators are *invariant*, and the failure of all actuators are *unobservable* except i -th actuator. The first step is to keep the failure in their own subspace; the second step is to make the failure of the i -th actuator be observed only.

Let $\mathbf{S}_i^*$ be a $(\mathbf{C}, \mathbf{A})$ UnObservability Subspace, EFPRG has a solution if and only if $\mathbf{S}_i^* \cap \mathbf{P}_i = 0$, where $\mathbf{S}_i^* := \inf \underline{\mathbf{S}}(\sum_{j \neq i} \mathbf{P}_j)$.

The design of a monitor for a specific component is as follows:

Step 1: Find $\mathbf{D}_0 \in \underline{\mathbf{D}}(\mathbf{S}_i^*)$ using a pole assignment procedure described in (Massoumnia 1986);

Step 2: Let $\mathbf{O}: \mathbf{X} \rightarrow \mathbf{X}/\mathbf{S}_i^*$ be the canonical projection, and let $\mathbf{A}_0 = \mathbf{A} + \mathbf{D}_0\mathbf{C}: \mathbf{X}/\mathbf{S}_i^*$;

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Step 3: Let $\mathbf{H}$ denote a solution of $\text{Ker } \mathbf{HC} = \mathbf{S}_i^* + \text{Ker } \mathbf{C}$;

Step 4: Let $\mathbf{M}$ be the unique solution of $\mathbf{MO} = \mathbf{HC}$;

Step 5: Find $\mathbf{D}_1$ such that $\text{eig}(\mathbf{F}) = \Lambda$; where $\mathbf{F} = \mathbf{A}_0 + \mathbf{D}_1\mathbf{M}$, and Λ is an arbitrary self-conjugate set;

Step 6: With $\mathbf{O}^r$ denoting a right inverse of $\mathbf{P}$, let $\mathbf{D} = \mathbf{D}_0 + \mathbf{O}^r\mathbf{D}_1\mathbf{H}$, $\mathbf{E} = \mathbf{OD}$, $\mathbf{G} = \mathbf{OB}$ and $\mathbf{K} = \mathbf{0}$.

Appendix 5.4 Derivation of residual error equation (5-4)

Using Eq. (5-2) and (5-3), and arranging, it follows

$$\begin{aligned}\dot{\mathbf{w}}(t) - \mathbf{O}\dot{\mathbf{x}}(t) &= \mathbf{F}\mathbf{w}(t) - (\mathbf{OD}_0 + \mathbf{D}_1\mathbf{H})\mathbf{y}(t) + \mathbf{G}\mathbf{u}(t) - \mathbf{OA}\mathbf{x}(t) - \mathbf{OB}\mathbf{u}(t) - \mathbf{O}\sum_{i=1}^k \mathbf{P}_i m_i(t) \\ &= \mathbf{F}\mathbf{w}(t) - (\mathbf{OD}_0 + \mathbf{D}_1\mathbf{H})\mathbf{C}\mathbf{x}(t) + \mathbf{OB}\mathbf{u}(t) - \mathbf{OA}\mathbf{x}(t) - \mathbf{OB}\mathbf{u}(t) - \mathbf{O}\sum_{i=1}^k \mathbf{P}_i m_i(t) \\ &= \mathbf{F}\mathbf{w}(t) - \mathbf{O}(\mathbf{A} + \mathbf{D}_0\mathbf{C})\mathbf{x}(t) - \mathbf{D}_1\mathbf{HC}\mathbf{x}(t) - \mathbf{O}\sum_{i=1}^k \mathbf{P}_i m_i(t)\end{aligned}$$

Referring to the definition of the *canonical projection* $\mathbf{O}$ (Wonham 1985), here it has

$$\mathbf{O}(\mathbf{A} + \mathbf{D}_0\mathbf{C})\mathbf{x}(t) = \mathbf{A}_0\mathbf{O}\mathbf{x}(t); \mathbf{MO} = \mathbf{HC}; \text{ and } \mathbf{OP}_j = \mathbf{0} \text{ for } j \neq i.$$

So the above equation can be rewritten as:

$$\dot{\mathbf{w}}(t) - \mathbf{O}\dot{\mathbf{x}}(t) = \mathbf{F}\mathbf{w}(t) - (\mathbf{A}_0 + \mathbf{D}_1\mathbf{M})\mathbf{O}\mathbf{x}(t) - \mathbf{OP}_i m_i(t)$$

Define $\mathbf{e}(t) = \mathbf{w}(t) - \mathbf{O}\mathbf{x}(t)$. As $\mathbf{F} = \mathbf{A}_0 + \mathbf{D}_1\mathbf{M}$, the above equation can be written as:

$$\dot{\mathbf{e}}(t) = \mathbf{F}\mathbf{e}(t) - \mathbf{OP}_i m_i(t)$$

along with

$$\mathbf{r}_i(t) = \mathbf{M}\mathbf{w}(t) - \mathbf{H}\mathbf{y}(t) = \mathbf{M}\mathbf{w}(t) - \mathbf{MO}\mathbf{x}(t) = \mathbf{M}\mathbf{e}(t)$$

Thus, the residual signal from the i -th generator has the expression with the state vector $\mathbf{e}(t)$ as:

$$\dot{\mathbf{e}}(t) = \mathbf{F}\mathbf{e}(t) - \mathbf{O}\mathbf{P}_i m_i(t)$$

$$\mathbf{r}_i(t) = \mathbf{M}\mathbf{e}(t) \quad (5-4)$$

In Eq. (5-4), $\dot{\mathbf{e}}(t)$ is expressed with $m_i(t)$ as the only input. It follows that the system relating $m_i(t)$ to $\mathbf{e}(t)$ is *input observable*; and other disturbances $m_{j \neq i}(t)$ are *unobservable*. Thus, the non-zero $m_i(t)$ could lead to non-zeros $\mathbf{r}_i(t)$; $m_{i \neq i}(t)$ didn't effect $\mathbf{r}_i(t)$. So the generator of $\mathbf{r}_i(t)$ satisfies the requirement of the solution as an indicator of the i -th actuator.

CHAPTER 6 EXPERIMENTAL STUDIES

6.1 Introduction

The time series methods presented in the preceding chapters proved to work satisfactorily in numerically simulated case studies, given the required response signals for the respective approaches. The ARX model incorporating Kalman filter was also verified with an experimental case study. In this chapter, the laboratory experiments on a physical model frame for different physical conditions and under different excitation schemes will be presented. The methods concerned will be applied on the experimental cases to verify their workability and effectiveness under real-life structural and measurement conditions. More attention is placed on the real measurements rather than the complexity of the structural system, and for this reason a relatively simple model structure form is chosen.

The basic information about the experiment is introduced first, including the equipments used, the test set-up and data acquisition system, and the test procedure and recording of signals. This is followed by standard modal analysis to extract the basic dynamic properties such as the natural frequencies and mode shapes, using respectively standard frequency-domain and time-domain methods. The frequency-domain and time-domain methods give similar results for the modal parameters.

Subsequently, the damage diagnosis method introduced in Chapter 3 is examined. Considering the fact that the measured signals inevitably contain noises, the ARMAX model is applied instead of ARX model in the method to guarantee the accuracy of model estimation, as discussed in Chapter 4. The CRE feature instead of STD ratio is used for the damage diagnosis.

In order to verify the effectiveness of the procedure proposed in Chapter 4 for restoring the virtual input-output pairs from noisy signals, a testing with a hard tip

for impact was performed to purposely boost the noise level in the measured signals. Comparison is made between the results with and without performing such “filtering” procedure on the measured signals.

Finally, the residual generator approach described in Chapter 5 is examined with the experimental cases, and the results are discussed.

6.2 Test Structure

As the time-series methods under consideration mainly concern mass-spring kind of systems rather than continuous systems like beams, the test model structure is designed as a multi-storey frame with rigid beams and flexible columns. Under horizontal excitation, the frame will undergo lateral vibration and the system can be approximately regarded to have only translational DOF in the horizontal direction of each storey. The damage scenarios are created by changing certain columns with smaller cross-section dimensions. As the methods concerned are presumably not dependent upon the type of excitation for vibration, two different types of excitations, namely the ground excitation and impact excitation with a hammer, are adopted in the experiment.

The test structure is a steel frame, as shown in Fig. 6.1. The beams are selected as hollow box section with dimensions of 50mm×50mm×3mm. The columns are made of steel strips having cross section dimensions of 50mm×6mm, with the weaker side aligned in the frame plane so that the test structure is sufficiently flexible at the reduced scale to preserve the frequency characteristics of an actual steel frame. The beams and columns are connected via welded joints. The frame is affixed on a strong steel base plate at the bottom of the columns by bolts through angle bars that are pre-welded onto the base plate. The steel base plate is then bolted on the platform of a shake table.

Figure 6.1 Three-storey steel frame model

The out-of-plane rigidity of the columns is purposely made large so that the vibration of the system is practically limited only within the frame plane. Comparing with columns, the box-section beams can be considered as rigid bodeis. As a result, the rotation at joints can be neglected such that the test frame has only translational DOF in the horizontal direction. Based on the mechanical properties and the measured dimensions of columns, the stiffness matrix of this 3 DOF system is calculated as (N/m):

$$K_0 = 10^4 \times \begin{bmatrix} 5.070 & -5.070 & 0 \\ -5.070 & 11.070 & -6.000 \\ 0 & -6.000 & 10.040 \end{bmatrix}$$

In addition to the tests on the bare frame, another series of tests were performed on the test frame with additional masses attached on the floor levels. The test model with added masses represents building frames in which large concentrated masses are found at the floor levels. The additional masses, weighing 3.8kg, 4.08kg, 4.08kg for the 1st, 2nd, and 3rd storey levels, respectively, were made of thick iron pieces and they were tied firmly on the beams. The actual amount of mass of the test frame

was weighted at the tests. Considering columns as beam elements with distributed mass, the consistent-mass matrix can be obtained by the principle of virtual displacement (Chopra, 2001). Then the global mass matrix of the system without additional masses is found to be (kg):

$$M_0 = \begin{bmatrix} 1.8114 & 0.2186 & 0 \\ 0.2186 & 2.4800 & 0.2314 \\ 0 & 0.2314 & 3.0114 \end{bmatrix}$$

For the system with additional mass, the mass matrix is (kg):

$$M_0 = \begin{bmatrix} 5.8914 & 0.2186 & 0 \\ 0.2186 & 6.5600 & 0.2314 \\ 0 & 0.2314 & 6.8114 \end{bmatrix}$$

After the initial tests on the original test frame were completed, the test structure was modified to represent a damaged state by replacing a third storey column with a new column element of a reduced cross section equal to 32mm×6mm. The resulting reduction of the stiffness of the third storey was approximately 18%. In the description that follows, this damaged scenario will be simply referred to as the damaged state as opposed to the undamaged state of the structure

6.3 Experimental Setup and Data Acquisition

In the experiments two excitation methods are respectively applied, as Forced Vibration Testing (FVT). The hammer impact is used as impulse excitation, and the shake table is used for long duration base excitation with wide band random motion.

6.3.1 Introduction of Equipments

Impact Hammer is embedded with a force transducer behind the tip. Since the force is an impulse, the amount of the energy imparted to the structure is a function of the mass and the velocity of the impact. It is difficult to control the velocity, so the force level is usually controlled by varying the mass. The frequency content of

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the energy applied to the structure is a function of the stiffness of the hammer tip and, to a lesser extent, the mass of the hammer. The harder the tip, the shorter the pulse duration, and thus the higher the frequency content. Therefore, a soft tip is necessary in order to avoid generating high frequency noise. Additional masses can also be attached to the back of the hammer head to increase the excitation force. More detailed considerations on impact hammer can be seen in Corelli and Brown (1984).

In the present study, an impact hammer, Model 086C03 shown in Fig. 6.2, is used for testing. It comes with three plastic tips of different hardness such that the frequencies within the bandwidth of interest for the frames can be excited. The hammer has the properties: Sensitivity ($\pm 15\%$) 2.53 mV/N (with Extender Mass); Measurement Range ± 2200 N (peak); Frequency Range (-10 dB) (Super soft Tip) 600Hz; Resonant Frequency ≥ 22 kHz; Hammer Mass 0.16 kg; Head Diameter 1.57 cm; Tip Diameter 0.63 cm; Hammer Length 21.6 cm; Electrical Connection Position Bottom of Handle; and Extender Mass Weight 75 gm.

Figure 6.2 Impact Hammer, Model 086C03

Shake Table used in the tests is a hydraulic driven shaker for simulating earthquake ground motions. The motion of the table is controlled by a control computer, allowing a wide range of motions to be simulated for testing a structure. Fig. 6.3 shows the shake table. The shaker can generate uniaxial ground motions horizontally. Other properties of the shake table include: Size 3m×3m Steel table

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top; Frequency Range 0~50 with extension to over 100 Hz; Acceleration 4.0 g with no payload, and 1.3g with the maximum allowable 10 ton payload. The shaker can be used to simulate Sine, Random, Impulse, Arbitrary, and some other special vibration waveforms.

Figure 6.3 Shake Table ANCOR R-149

Accelerometers were used to measure the acceleration response. The measured acceleration signals and the impact force signal were then processed to obtain the modal parameters. There are many types of accelerometers in terms of such properties as working mechanism, sensitivity and mass. Accelerometers must be chosen carefully to suit an intended application. For the current applications, the accelerometers chosen had test weight of 5 grams and a sensitivity of 100mV/g.

Figure 6.4 Accelerometers: Kistler Sensor Type 8636C50

Fig. 6.4 shows the Kistler Sensor Type 8636C50 accelerometers used in the tests. These accelerometers have the following properties: Measuring Range $\pm 50g$; Sensitivity 100mV/g; Transverse sensitivity 1.0%; Resonant Frequency 22.0 KHz; Operating Temperature 0-65°C.

Figure 6.5 Data Logger NI PXI-1000B

Data Logger used in this experiment is National Instruments PXI-1000B as shown in Fig.6.5. It has 32 channels. The sampling rate for the measurements was chosen to be 500Hz.

6.3.2 Test Set-up, Instrumentation and Data Acquisition

Fig. 6.6 shows the settings for the impact hammer testing and the ground excitation testing, respectively. During the testing, the accelerometers were fixed at floor levels, and they are designated as Channel 1 to 3 in Fig. 6.6, for measurement of accelerations at every storey level of the frame. For each impact hammer testing, horizontal impacts were applied at a joint, and the impact position was varied at different storey levels. The impact force was recorded by Channel 4. For ground excitation testing, an extra accelerometer was fixed on the shake table to record the achieved acceleration of the table, also through Channel 4.

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a) Ground excitation testing

b) Impact hammer testing

c) Data Transducer

Figure 6.6 Test set-up, instrumentation and data acquisition

6.3.3 Testing and Signals

The base excitation testing was performed by a random signal of uniform spectrum with a frequency band of 0 to 60 Hz for a duration of 20 seconds. The RMS is set equal to 0.030. The same excitation was repeated for three times for each test. The actually achieved base acceleration time history and its Fourier amplitude spectrum

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are shown in Fig.6.7. The corresponding measured acceleration responses on the structure are shown in Fig. 6.8.

a) Time history of base acceleration b) Fourier Spectra of base acceleration

Figure 6.7 Signal of base excitation

a) Responses at top storey, Channel-1 b) Responses at second storey, Channel-2

c) Responses at first storey, Channel-3

Figure 6.8 Acceleration response signals of test structure under random base excitation

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For the impact hammer testing, a super soft tip was chosen to add on the hammer in most of the tests (unless stated otherwise). The bandwidth of the hammer impact with such a super soft tip is 0~200 Hz. As the natural frequencies of the test frame are all below 50Hz, this tip was considered appropriate for the testing. Fig. 6.9 shows the measured impact force (N), its auto-spectrum, and the measured acceleration responses (m/s^2) of the structure, when the impact position of hammer is at the top level. The tests for impact position at second storey and first storey levels were also carried out for each damage scenario.

e) Responses at Channel 3

Figure 6.9 Time history of impact force and responses

6.4 Modal Analyses

Classical modal analyses are performed to obtain the basic dynamic properties of the test frame at each test stage. Although the actual diagnosis of the test structure will be carried out using the various time domain methods concerned, the extracted modal data can provide general information about the dynamic properties of the systems being tested.

6.4.1 Modal Analyses in Frequency Domain

Simple modal analysis is performed to identify the modal parameters. By peak picking of the Fourier spectra and the decay curve-fitting, a set of natural frequencies and damping ratios are obtained (Juang and Pappa, 1985; Friswell and Mottershead, 1995). Fig. 6.10 shows a typical FFT spectrum (Michael and Greenberg, 1998) of acceleration response recorded at the first storey by hammer testing for the undamaged system with additional weights. Tables 6.1 and 6.2 summarize the frequencies and damping ratios of the two systems (without/with added weights) for undamaged and damaged states.

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Figure 6.10 FFT Spectra of acceleration response

Table 6.1 Frequencies and damping ratios of test structure without additional masses

Mode	Undamaged		Damaged	
	Freq(Hz)	Damping (%)	Freq(Hz)	Damping (%)
1	9.77	0.03	9.66	0.05
2	26.68	0.04	25.60	0.04
3	41.70	0.04	40.11	0.06

Table 6.2 Frequencies and damping ratios of test structure with additional masses

Mode	Undamaged		Damaged	
	Freq(Hz)	Damping (%)	Freq(Hz)	Damping (%)
1	5.99	0.02	5.92	0.01
2	16.31	0.01	15.61	0.02
3	24.56	0.05	24.02	0.01

The frequency response function (FRF) curves for the impact tests are also calculated from the recorded impact force and acceleration response signals. Based on the statistics of the measured FRFs, the mode shapes can be extracted. The Rational Fraction Polynomial Method (Richardson and Formenti, 1985) is used here for this purpose. The idea of the rational fraction polynomial method is to express an FRF in terms of rational fraction polynomials, and through numerical manipulations, the coefficients of these polynomials can be identified. The links between these coefficients and the modal parameters of the FRF can be established. The mode shapes for each mode in each scenario are given in Table 6.3.

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Table 6.3 Mode shapes of the two systems before and after damage

System without added masses			
Mode	Mode shape	Undamaged	Damaged
1	Top Storey	1.00	1.00
	Second Storey	1.49	0.80
	First Storey	0.85	0.46
2	Top Storey	1.00	1.00
	Second Storey	-0.41	-0.31
	First Storey	-1.67	-0.86
3	Top Storey	1.00	1.00
	Second Storey	-2.67	-1.71
	First Storey	1.61	1.21
System with added masses			
Mode		Undamaged	Damaged
1	Top Storey	1.00	1.00
	Second Storey	0.78	0.76
	First Storey	0.45	0.43
2	Top Storey	1.00	1.00
	Second Storey	-0.50	-0.59
	First Storey	-1.04	-0.98
3	Top Storey	1.00	1.00
	Second Storey	-2.20	-2.59
	First Storey	1.83	2.39

6.4.2 Modal Analysis in Time Domain

Appropriate Auto-Regression model in time domain can also be used to identify the modal parameters. Many researchers have applied and refined the algorithm using AR models for modal parameters identification (Brincker et al., 1995; Saito and Yokota, 1996; He and Roeck, 1997; Huang, 2001; Lardies and Larbi, 2001; Neild et al., 2003; Hung et al., 2004)

In the present experiment, as the excitation of the structure was measured, the ARMAX model can be applied. The ARMAX model used for this example can be written as:

$$\begin{aligned}
 \begin{Bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{Bmatrix} &= \mathbf{P}_1 \begin{Bmatrix} y_1(k-1) \\ y_2(k-1) \\ y_3(k-1) \end{Bmatrix} + \mathbf{P}_2 \begin{Bmatrix} y_1(k-2) \\ y_2(k-2) \\ y_3(k-2) \end{Bmatrix} + \mathbf{D}u(k) + \mathbf{E}_1 u(k-1) + \mathbf{E}_2 u(k-2) \\
 &+ \mathbf{e}(k) + \mathbf{H}_1 \mathbf{e}(k-1) + \mathbf{H}_2 \mathbf{e}(k-2).
 \end{aligned}$$

Here y_1 , y_2 and y_3 are responses of acceleration at the three storey levels, recorded

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through Channel 1~3. Channel 4 provided the excitation signal, as $u(k)$. For the tests with ground excitation testing, $u(k)$ is the acceleration of the shake table; and for impact hammer tests, $u(k)$ is the impact force of the hammer.

The modal parameters, namely the frequencies, mode shapes and damping matrix, can be identified from the AR part parameters of the model (Juang, 1994). The modal parameters identified by this method are found to be almost the same as those identified from the frequency domain analysis described in Section 6.4.1. Taking the original system with additional mass as example, the three natural frequencies are found from the AR parameters to be equal to 5.97 Hz, 16.32Hz, and 24.57 Hz respectively. Compared with results shown in the first column of Table 6.1, the frequencies are almost the same. The mode shapes are also very close to the results obtained from the frequency domain analysis. Fig. 6.11 compares the mode shapes identified by the two methods.

Figure 6.11 Mode Shapes of undamaged system with additional masses

6.5 Damage Diagnosis with ARMAX model in time domain

In this section, the experimental measurements are employed to investigate the effectiveness of the damage diagnosis method introduced in Chapter 3. In this method, the ARX model is used as a prediction model and the prediction errors when the reference model is applied on the actual measured response from an unknown state reflects the change of the structural system. The reference model is set up from the measured response signals at the undamaged state of the structure. The model uses response (acceleration) signals only.

As shown in Chapter 4, under actual measurement conditions where noise is inevitable, the use of ARMAX model provides a better estimation of the model parameters. So the ARMAX model is applied here instead of ARX model for the sake of more precise model estimation. The damage diagnosis follows the general procedure shown in Fig. 3.1, with ARMAX model as the operation model instead of ARX model. Herein the CRE is considered as a damage indicator.

6.5.1 Application on Base Excitation Test without Additional Masses

For the test structure excited by base motion, the response at the first storey (Channel 3) is used as input of the model, while the responses at second (Channel 2) and third (Channel 1) storey levels are the output of the model.

To get an idea about the noise level contained in the measured signals, one set of signals out of the total three sets from the repeated tests on each scenario of the test system is chosen as reference signal to setup an ARMAX model. Then all three sets of the measured signals for the same scenario are predicted by this reference model. The CRE values from this prediction exercise for the same state of the test structure (“self-prediction”) serves as a good indicator of the accuracy of the ARMAX model estimation as well as the noise level in the measured signals. The “self-prediction” CRE results are summarized in Table 6.4. It can be seen that the CRE values are

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very low, indicating that the ARMAX models represent well the structural system at the respective states and the noise level in the measured signals is low.

Table 6.4 CRE of “self-prediction” signals in different damage scenarios

Scenarios	Test set	Channel	Channel 2
Undamaged system	Y1	1.2%	2.8%
	Y2	1.3%	2.4%
	Y3	1.0%	2.2%
Damaged system	YA1	1.5%	4.5%
	YA2	1.0%	3.0%
	YA3	1.2%	2.7%

The set of signals of undamaged system designated as Y3 in Table 6.4 is used (any other set of signals can also be used) as baseline for the damage diagnosis. The model obtained by using this set of signals is the baseline (reference) model for the subsequent damage diagnosis. Table 6.5 shows the CRE values when using the reference model on the signals measured from the undamaged and damaged scenarios. It can be observed that the CRE results for the same state of the structure with three repeated tests are consistent, indicating that the diagnosis model performs in a stable manner.

Table 6.5 CRE of testing signals predict by baseline reference modal

Scenarios	Test set	Channel	Channel 2
Undamaged system	Y1	1.2%	2.8%
	Y2	1.3%	2.4%
	Y3	1.0%	2.2%
Scenario a) 18% at K1	YA1	12.6%	48.6%
	YA2	13.5%	34.2%
	YA3	14.4%	30.1%

From Table 6.5, it can be seen that the CRE results for the damaged scenario are considerably high as compared to the undamaged state, indicating the occurrence of damage around second to third storey levels. This is indicative to the actual damage on the third storey. The comparable CRE results between the two storey levels do not seem to make much sense here with regard to the relative influence of the

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reduction of the third storey column on the responses at the third (channel 1) and second (channel 2) storey levels. This is not surprising as the present diagnosis model using ARX is more a qualitative rather than quantitative indicator of damage.

6.5.2 Base Excitation Test of Structure with Additional Masses

For the test frame with additional masses attached to the beams, tests were conducted on the undamaged and damaged cases. Table 6.6 shows the results of the damage diagnosis.

Table 6.6 CRE of testing signals predicted by shake table testing base-line modal

Scenarios	Testing Signal	Channel	Channel 2
Undamaged system	Y1	1.6%	3.4%
	Y2	1.7%	2.1%
	Y3	1.8%	3.8%
Damaged system	Ya1	22.4%	6.5%
	Ya2	22.4%	6.5%
	Ya3	22.4%	6.5%

Results in Table 6.6 clearly indicate the occurrence of damage and its location on the third storey. This shows that the method generally works well in this case. But again, the relative CRE values between Channel 1 and Channel 2 do not necessarily correlate well the actual effect of the damaged member on the response at these two levels.

6.5.3 Impact Hammer Testing of System with Additional Masses

The test frame with added masses was also tested with hammer impact excitation. The tests were performed with the hammer impact applied at different storey levels. Table 6.7 shows the CRE results when the diagnosis model is applied on the measured response signals from the hammer impact tests.

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Table 6.7 CRE of testing signals predict by impact hammer testing base-line modal

Impact position	Scenario		Channe2	Channel 3
Top storey	Undamaged		0.79%	0.43%
	Damaged		3.0%	1.4%
Impact position	Scenario	Channel		Channel 3
Second storey	Undamaged	0.45%		0.34%
	Damaged	7.3%		1.0%
Impact position	Scenario	Channel	Channel 2	
First storey	Undamaged	0.43%	0.25%	
	Damaged	7.4%	2.0%	

From Table 6.7, it can be seen that the CRE values for the undamaged structure with different impact position are very small and they are basically indicative of the noise levels. Thus, the noise contained in signal was below 1%, and this is attributable to the use of a soft tip of the hammer that generated relatively low frequency excitation.

The CRE for the damaged structure exhibits noticeably higher values, especially for Channel 1 (top storey) signals. This is consistent with the actual location of damage on the third storey.

As mentioned in Chapter 3, when the impact position and the response signal used as input in the ARX model is close to the location of damage, the diagnosis results could be confused. This is further confirmed from the current experiment as seen from Table 6.7 for the impact position at the top level. In this case, the CRE values for the damaged structure, although somewhat higher than the reference error, are less indicative of the damage as compared to the other two impact situations.

6.6 Verification of Effect of ARX Incorporating Kalman Filter

As described in Chapter 4, this scheme is proposed for the time domain diagnosis with noise-contaminated vibration signals. With the pairing of raw input signals and the “filtered” response signals through the Kalman filter, the underlying physical

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system can be well represented in the subsequent diagnosis operation with minimized effect of noise. In this section some real measured signals with noise will be used to demonstrate that the above noise “filtering” procedure is effective in tackling the noises.

The results presented in the preceding sections have shown that the noise levels in the measured signals for base excitation and impact with a soft hammer tip tests were quite low as evidenced by small CRE values for undamaged structure (Table 6.6~6.7). In order to examine the method with some significant level of noise, an additional test was conducted with purposely raised noise level in the measurements by using impact hammer with a hard tip. Using a hard tip generates higher frequency excitation, for which the measurements are more prone to noises. Fig. 6.12 shows the auto-power spectrum of impact force, indicating a broad frequency band up to 250Hz. This particular test was done on the undamaged system with additional masses.

When the raw signals recorded from the above test are presented to the respective baseline reference models, the resulting CRE values are noticeably high, as can be seen from Table 6.8. The reference models here are the same as those used in Table 6.7, and they are estimated from the same undamaged system by low frequency impact tests. Comparing with the CRE values in Table 6.7, the residual errors shown in Table 6.8 for the undamaged system are considerably higher and cannot be ignored. This confirms that the measurements from hard-tip hammer impact test contain higher level of noise.

To get rid of the noise effect on the diagnosis result, the “noise” filtering procedure shown in Fig. 4.3 is applied to generate new pairs of signals with the raw measured input and the filtered response output through the Kalman filter. The processed pairs of signals are then presented to the reference model. The resulting CRE values are compared with those without the “filtering” process in Table 6.8. It can be clearly observed that after the noise “filtering” process the prediction errors are reduced significantly. This indicates that the proposed “filtering” scheme is

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effective in reducing the noise effect on the diagnosis for the test model under consideration.

a) Impacting force time history b) Auto-Power spectrum of impact force

Figure 6.12 Impact force by hard tip hammer

Table 6.8 Comparison of CRE for hammer testing without/with Kalman filtering procedure

Impact position	Scenario	Channe2	Channel 3
Top storey	Raw data	12.1%	10.2%
	Aft Filtering	2.6%	2.8%
Impact position	Scenario	Channel	Channel 3
Second storey	Raw data	6.7%	8.8%
	Aft Filtering	1.1%	1.1%
Impact position	Scenario	Channel	Channel 2
First storey	Raw data	3.1%	6.5%
	Aft Filtering	1.6%	3.4%

6.7 Application of Time Domain Geometric Method

In Chapter 5, numerical examples of a lumped mass-spring system and a truss system were used to test the geometric method. In this section, the geometric

method is examined with the laboratory experiments on the three-storey swaying frame.

The three-storey frame is effectively a three-DOF system. The damage is represented by the stiffness change at one third-storey column. Both undamaged and damaged states of the test frame are considered for the application of the methods. For simplicity, only the results from the base excitation for the test frame without added mass, and the results from hammer impact excitation for the test frame with added mass are presented and discussed herein.

6.7.1 Application on Base Excitation Test Data

In some common cases the undamaged structure can only be described by an FE model. With the dimension survey of the structure and the estimation of mass on the structure, such an FE model can be set up to resemble the initial state of the structure. This section describes such a case where the stiffness matrix and mass matrix of undamaged system can be estimated from the measured properties (see section 6.2). The time domain geometric method is applied to design the residual generators from the FE model. Then the measured signals from the test structure under base excitation are applied for damage diagnosis. Both undamaged and damaged states (damage on one third-storey column) are considered.

As described in Section 6.2, the stiffness matrix of this 3 DOF system is calculated as (N/m):

$$K_0 = 10^4 \times \begin{bmatrix} 5.070 & -5.070 & 0 \\ -5.070 & 11.070 & -6.000 \\ 0 & -6.000 & 10.040 \end{bmatrix}$$

Here the stiffness of each Column from bottom to top is K_3 : 40400 (N/m), K_2 : 60000 (N/m), K_1 : 50700 (N/m).

The mass matrix of the system without additional masses is (kg):

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$$\mathbf{M}_0 = \begin{bmatrix} 1.8114 & 0.2186 & 0 \\ 0.2186 & 2.4800 & 0.2314 \\ 0 & 0.2314 & 3.0114 \end{bmatrix}$$

The damping ratio taken is 2.0% for all modes. According to Eq.(3-11) the reference FE model (baseline) can be written in state-space model as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C} \mathbf{x}(t) + \mathbf{D} \mathbf{u}(t)$$

Thus $\mathbf{A} =$

$$\begin{bmatrix} -6.1607 & -6.8008 & -0.6709 & 1 & 0 & 0 \\ 4.6651 & -9.9702 & -5.5599 & 0 & 1 & 0 \\ -0.3585 & -4.7510 & -7.0952 & 0 & 0 & 1 \\ -30803 & 34004 & -3354 & 0 & 0 & 0 \\ 23326 & -49851 & 27800 & 0 & 0 & 0 \\ -1792 & 23755 & -35476 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = [0.0307, -0.25492, 2.7027, 153.8, -1274.6, 13514]$$

The failure signatures are:

$$\mathbf{P}_1 = [0, 0, 0, \mathbf{M}_0^{-1}[1, -1, 0]^T]^T$$

$$\mathbf{P}_2 = [0, 0, 0, \mathbf{M}_0^{-1}[0, 1, -1]^T]^T$$

$$\mathbf{P}_3 = [0, 0, 0, \mathbf{M}_0^{-1}[0, 0, 1]^T]^T$$

Referring to Chapter 5, as $\kappa_i = 1, 2, 3$, and $l = 3$, $v = 3$, $n = 6$, the generic solvability conditions $v \leq n$ and $v - \min\{k_i, i \in k\} < l$ are satisfied. Then three monitors dedicated for K_1 , K_2 , and K_3 , respectively, are designed to monitor the stiffness of the columns in the three storeys, k_1 , k_2 , and k_3 , respectively. The detailed design of the residual generators follows the steps described in Appendix 5.3

The measurements from two scenarios (undamaged and damaged at third storey) of the structure are tested by the generators and the residual error signals are examined. Fig.6.13 lists the error signals generated by each monitors for the undamaged and damaged structure. The damage severities of individual storeys are calculated and shown in Table 6.9.

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a) The error signals of K_1 generator for two scenarios

b) The error signals of K_2 generator for two scenarios

c) The error signals of K_3 generator for two scenarios

Figure 6.13 Error Signals from base excitation test

The results shown in Fig. 6.13 clearly indicate that K_1 is changed between the two scenarios. At the same time, K_2 and K_3 show insensitive differences between the two scenarios. These results agree well with the actual changed between the two scenarios.

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It should be pointed out that the FE model, despite using the measured physical properties, cannot be an exact representation of the initial state of the system (unless with a stringent model updating procedure), particularly in terms of the stiffness due to imperfect connections and other reasons. With the geometric method, the deviation of the FE model from the actual initial structure can be estimated by looking at the “damage severity” for the undamaged state of the model with respect to the baseline FE model. Table 6.9 shows the calculated severities of stiffness changes for both undamaged and damaged scenarios using the FE model as a baseline. It can be seen that the baseline FE model deviates from the undamaged state quite markedly. Nevertheless, the relative difference between the two scenarios gives a favourable indication of the level of damage on the third storey (K_1).

Table 6.9 Damage severity identified for two scenarios of shake table testing

	FE model (N/M)	no damage		K_1 - 18%	
		severity	stiffness(N/M)	severity	stiffness(N/M)
K1	50700	16.37%	42398	27.41%	36804
K2	60000	10.77%	53540	10.20%	53882
K3	40400	-14.79%	46374	-12.49%	45444

6.7.2 Application on Impact Hammer Test Data

The geometric method can be designed directly based on measured data from a reference (undamaged) state of the structure. In this case study with impact hammer testing on the test structure, the baseline model is constructed from signals measured on the undamaged structure, and the measured data from undamaged and damaged (K_1 -18%) are then presented to the monitors for diagnosis. The measurements are taken from the test with the hammer impact position at first storey level.

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Based on the measured signals from the undamaged structure, the state-space model as *system* (**C**, **A**, **B**) can be identified via an inverse procedure (Friswell and Mottershead, 1995; Lee and Yun, 1991):

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{u}(t)$$

$$\mathbf{y}'(t) = \mathbf{C} \mathbf{x}(t),$$

where $\mathbf{y}'(t) = \mathbf{y}(t) - \mathbf{L} \mathbf{u}(t)$.

$$\text{Thus, } \mathbf{A}_c = \begin{bmatrix} -1.485 & -1.545 & -0.722 & 1 & 0 & 0 \\ 0.6387 & -0.197 & -0.289 & 0 & 1 & 0 \\ -0.322 & -0.08 & -0.108 & 0 & 0 & 1 \\ -7079 & 7463 & -263 & 0 & 0 & 0 \\ 6401 & -14270 & 7955 & 0 & 0 & 0 \\ -179 & 7744 & -14371 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_c = [-0.2378 \quad 0.1061 \quad -0.0527 \quad -1195.6 \quad 1107.5 \quad -53]$$

$$\mathbf{L} = [0.1646 \quad -0.0046 \quad -0.0002]$$

The mass, stiffness, and damping of the undamaged system are required in order to set up the generators. Based on the above established *system* (**C**, **A**, **B**), the baseline quantities of the above parameters are identified as:

$$\mathbf{M}_0 = \begin{bmatrix} 6.0803 & 0.2086 & 0.0206 \\ 0.1809 & 6.4216 & 0.3216 \\ 0.0274 & 0.7075 & 7.6402 \end{bmatrix}$$

$$\mathbf{K}_0 = 10^4 \times$$

$$\begin{bmatrix} 4.0304 & -4.0847 & -0.0188 \\ -4.0404 & 9.0190 & -4.8801 \\ -0.0263 & -4.9446 & 9.6046 \end{bmatrix}$$

$$\mathbf{J}_0 = \begin{bmatrix} 8.9009 & 9.4381 & 4.4504 \\ -3.7294 & 1.5712 & 2.0181 \\ 2.0500 & 0.7948 & 1.0499 \end{bmatrix}$$

The failure signatures for this example are:

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$$\begin{aligned}
 \mathbf{P}_1 &= [0, 0, 0, \mathbf{M}_0^{-1}[1 \ -1 \ 0]^T]^T \\
 &= [0, 0, 0, 0.1699, -0.1612, 0.0143]^T \\
 \mathbf{P}_2 &= [0, 0, 0, \mathbf{M}_0^{-1}[0 \ 1, -1]^T]^T \\
 &= [0, 0, 0, -0.0051, 0.1632, -0.1460]^T \\
 \mathbf{P}_3 &= [0, 0, 0, \mathbf{M}_0^{-1}[0 \ 1, -1]^T]^T \\
 &= [0, 0, 0, -0.0002, -0.0066, 0.1315]^T
 \end{aligned}$$

The following procedure is the same as described in the previous subsection for base excitation test. The solvability conditions are satisfied. Subsequently three generators are designed, and the generators analyze the measured data of two scenarios and produce residual signals. Fig. 6.14 shows $\mathbf{r}_i(t)$ of the residual generators for both undamaged and damaged states of the structure. Each figure consists of the signals from three residual generators for the same state of the structure.

From the results shown in Fig. 6.14 a), the three residual errors exhibit similarly a small error for the undamaged structure, which is indicative of the fact that there was no change on any part of the structure. In Fig. 6.14b), the magnitude of K_1 is observed to be apparently higher than the other two generators, indicating the occurrence of changes (stiffness reduction) on K_1 .

a) Residual error of undamaged system

b) Residual error of damaged system

Figure 6.14 Error Signals from Impact Hammer test

With the residual signal generated, the damage severity can be calculated. The values of α_i for the stiffness of the three storeys (columns) are listed in Table 6.10. It can be seen that the α_i values reflect quite reasonably the actual damage severity in terms of the percentage reduction of the storey stiffness.

Table 6.10 Damage severity identified by geometric method

	no damage	K1 - 18%
K1	0.74%	15.52%
K2	2.53%	-0.48%
K3	-1.19%	0.16%

6.8 Conclusions

Laboratory experiments were conducted on a three-storey steel frame model, for the undamaged state and a damage state represented by the reduction of stiffness on one third storey column. Two excitation schemes were applied; one with base excitation and another with hammer impact excitation. The measurements included the acceleration signals at all floor levels as well as the base excitation or impact force signals.

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The proposed time domain diagnosis methods are applied on the measured signals to examine their effectiveness in a real testing and measurement environment. The following conclusions may be drawn:

- 1) The basic method using ARMAX model is effective for damage diagnosis under relatively low noise conditions (base excitation and hammer impact with a softer tip herein). With consideration of only response signals, the method proved to be able to detect the occurrence of damage, as well as generally the location of damage.
- 2) The procedure incorporating the Kalman filter for processing high noise contaminated response signals is observed to produce satisfactory results. After applying the procedure, the residual error due to noise effect is considerably reduced, thus facilitating the subsequent damage diagnosis operation.
- 3) The application of the geometric method on the experiments also proved to be quite successful. Although this method requires the excitation signal, it can locate the damage in individual members (in the present case the storeys as only one DOF was considered for each storey at the floor level) in a more precise manner; and moreover, the damage severity can be estimated quite accurately.

It should be pointed out that the present experimental study was mainly for the purpose to demonstrate the workability and effectiveness of the proposed methods in a realistic measurement environment. As is well known, actual civil engineering structures are complex and involve a lot of structural and non-structural complexities and uncertainties. For a more effective application of the proposed methods and identification of other issues that may arise from application of these methods on real complex structures, further experimental and application investigations will be required.

CHAPTER 7 SUMMARY AND CONCLUSIONS

Damage diagnosis methods in time domain, despite many inherent advantages, have found limited applications in practice in the past due primarily to the usual requirement on the availability of input excitation information, and the susceptibility to noises in measured signals. The detection of damage location with time domain analysis methods has also been difficult, while the assessment of damage severity could hardly be achieved.

This study aims to develop general methodologies and implementation schemes to enhance the robustness of the time domain methods for damage diagnosis and extend the ability to the assessment of damage severity. Major contributions of this study include 1) the development of a novel time-series analysis method, which requires only acceleration response signals from the structure, for detection of the occurrence and location of damage; 2) extension of the above method to noise-contaminated vibration signals, with a novel scheme incorporating Kalman filter to establish the virtual Input-Output signal pairs that essentially represent the underlying physical system; and 3) formulation of a residual generator technique, based on the geometric concept for disturbances decoupling problem (DDP), for detecting and locating the damage using acceleration measurements instead of displacements. The method also enables quantitative estimation of the severity of damage in individual elements of in systems with multiple members jointed at the same joints.

Detailed conclusions on each aspect of the investigations have been provided in the individual chapters of this thesis. The following summarizes the major conclusions

and observations.

A. ARX model based damage diagnosis method in time-domain

A new method for damage diagnosis using time series analysis of vibration signals has been developed. The method is based on the linear dynamic system formulation and is realized in a novel form of auto-regressive models (ARX) on acceleration response signals only. Through appropriate simplification, the model can be disassociated from the input excitation and this further enhances the potential robustness of the method in real-life applications. At the present stage, the independence of the method from excitation has been achieved successfully with response signals from random excitations.

The standard deviation (STD) of the residual error is chosen as a basic damage-sensitive feature. From there, a feature called CRE, defined as a function of the ratio of STD of the residual error to that of the output signal is introduced to indicate damage in the later study. The reason for using the CRE is that it is a relative quantity and so is less affected by the amplitude of vibration. With these features, numerical simulation studies demonstrated that the occurrence of damage can be easily detected; and moreover, in a MDOF system the location of damage can also be identified.

B. A novel scheme for restoring intrinsic relationships between signals based on Kalman filter

A procedure essentially based on Kalman filter is developed to extend the above method to more realistic environments with noise-contaminated vibration signals. The Kalman filter is incorporated to perform the stochastic processing of the input-response signals containing noise.

The novelty in this procedure is to establish the virtual “input” and “measurement” pairs from noisy signals, so that the underlying physical system is best represented and passed on to the subsequent diagnosis operation. The theoretical basis of the method is established through appropriate formulation, and it is shown that when the raw input and filtered response signals are fed into the reference ARX model, the error feature is essentially indicative of the change of the physical system. By analyzing the residual error, the damage status of the structure can be diagnosed.

Both the numerically simulated scenarios and experimental studies demonstrate that the effectiveness of the proposed approach under noisy measurement environment is satisfactory. Through this new procedure, noises of even a quite high level has little effect on the outcome of the damage diagnosis, and the residual error feature in terms of CRE using the processed set of virtual input-output signals correlate well with the level of damage.

C. Residual generator approach with geometric concept for locating and determining the severity of damage from acceleration response signals

A residual generator method using time domain response signals is further developed for detecting and locating damage in a structure, and at the same time determining the severity of damage (in terms of stiffness reduction). Through appropriate transformation, the structural dynamic system with excitation as input and acceleration responses as output can be expressed as multi-failure events in state space model. Thus, the solution of EFPRG can provide residual generators for all individual components of the structure, and each monitor is associated only with the stiffness change of a specific component and is not affected by the stiffness changes in other components. Major advancement of the method in the present study is the formulation to enable the use of acceleration signals, which are easier to measure in actual applications, instead of displacements, as well as the newly developed ability of determining the damage severity.

CHAPTER 7 SUMMARY AND CONCLUSIONS

Numerical results show that the method can successfully determine the damage location and severity in frames with rigid floors and truss systems. The application of the method on the experimental cases also demonstrates that the damage severity calculation can be workable in real measurement environments.

In summary, the three major developments on the time-domain analysis for damage diagnosis complement each other and can serve for different demands of monitoring and diagnosis of damage. The basic ARX-model based method provides a simple and easy way to detect structure damage, and it is advantageous also because it requires only acceleration response signals. With the incorporation of the Kalman-filter based scheme for establishing the virtual input-output signal pairs, the method can tackle noisy measurement environments in a satisfactory manner. This method can be applied for continuous monitoring purpose and enables rapid detection of occurrence and to a certain extent the location of damage. The main shortcoming is that it can provide only some qualitative indication of the level of damage.

The geometric approach complements the above ARX based method mainly in two ways, the first is that with a unique residual generator (monitor) for each individual member, the location of damage can be precisely determined; the second is the quantitative determination of severity of damage (in terms of reduction of stiffness).

It has to be pointed out that, however, the geometric method requires the excitation signal. Therefore, a possible integrated use of the ARX method and the geometric method in practice could be for the ARX method to be applied for constant monitoring; and once a damage occurrence is detected, a more detailed diagnosis exercise is conducted with a dedicated vibration testing on the structure, allowing measurement of excitation signals, so that the geometric method can be applied for further identification of the damage location and severity.

CHAPTER 7 SUMMARY AND CONCLUSIONS

Future developments of the proposed time-domain methods/schemes shall focus on enhancing the robustness of the methods on different structural settings and excitation conditions. As has been shown from the experimental study, the geometric method, despite the inherent ability in identifying the location and severity of damage, can still be susceptible to measurement noises. At the present stage, the methods have been tested with relatively simple systems due to the assumption that rotational DOF are not measured. In the future, when the measurement of rotation DOF becomes practicable, these methods can be extended for more complex systems.

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