Fire resistance of high strength concrete columns under axial restraint

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**NANYANG
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FIRE RESISTANCE OF HIGH STRENGTH
CONCRETE COLUMNS UNDER AXIAL RESTRAINT

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UNDER AXIAL RESTRAINT**

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Fire Resistance of High Strength Concrete Columns Under Axial Restraint

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Abstract

This study aims to study the structural behavior of axially-restrained high strength concrete (HSC) columns under fire. To achieve this, a material model for HSC under high temperature is developed. The model includes transient strain in an explicit way, and is an extension of Kodur's work. An existing FEM program is extended by incorporating the proposed material model. Structural behavior of HSC columns under fire is predicted through this program. When an axially-restrained column is heated, great axial force develops. Since transient strain is significantly related to external force, in this case, the method of calculating transient strain has a considerable effect on structural behavior. In other words, implicit or explicit inclusion of transient strain yields different results for the development of internal compression force. Therefore, axial restraint and transient strain are of principal interest in this research. By comparison of different material models, the nature of transient strain is investigated. The author proposed an equation to calculate transient strain. Based on this equation, a HSC material model which explicitly includes transient strain is established. The validity of the material model is verified by some material tests. The interaction between axial restraint and transient strain is analyzed using the FEM program. The FEM program is further verified by some case studies. Through the comparison of numerical and test results, it is demonstrated that when there is no axial restraint, implicit and explicit models give similar results. However, if axial restraint is present, explicit model gives more reasonable results than implicit model.

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List of Symbols

A	=	Cross section area (mm^2)
A_s	=	Area of steel reinforcement (mm^2)
Bi	=	Biot number
c	=	Specific heat ($J/kg/K$)
c_{peak}	=	Constant specific heat of concrete situated between 100°C and 115°C ($J/kg/K$)
E	=	Elastic Modulus of concrete (MPa)
E_0	=	Elastic Modulus of concrete at room temperature (MPa)
E_s	=	Elastic Modulus of steel at room temperature (MPa)
E^-	=	Tangent modulus in descending branch of stress-strain curve (MPa)
e	=	Load eccentricity (mm)
f_{cu}	=	Characteristic strength of concrete (MPa)
f'_c	=	Compressive strength of concrete (MPa)
f_{sp}	=	Proportional limit of steel (MPa)
f_{sy}	=	Maximum stress level of steel (MPa)
g	=	Increase of elasticity due to preload before heating in Schneide's model
$g(x,t)$	=	Impulse Green's functions
$G(x,t)$	=	Step Green's functions
h	=	Convective heat transfer coefficient (W/m^2k)

$J(\sigma, T)$	=	Compliance function in Schneider's model
k	=	Thermal conductivity ($W/m/K$)
$k_t(T)$	=	Coefficient allowing for decrease of tensile strength of concrete at elevated temperatures
k_{tr}	=	Material constant for transient strain in Anderberg's model
$LFTS$	=	Load-free thermal strain in Khoury's model
$LITS$	=	Load-induced thermal strain in Khoury's model
m	=	Bending moment ($kN \cdot m$)
p	=	Axial force (kN)
q	=	Constant value for thermal boundary condition
R_a	=	Axial restraint parameter of the restraint system (kN/m)
T_a	=	Ambient temperature ($^{\circ}C$)
T_c	=	Fire Temperature when column collapse ($^{\circ}C$)
T_g	=	Gas Temperature ($^{\circ}C$)
T_i	=	Initial temperature ($^{\circ}C$)
ΔT	=	Change of temperature ($^{\circ}C$)
t	=	Fire exposure time (s)
t_c	=	Collapse time (s)
$\bar{t}$	=	Dimensionless time in step system
$\bar{x}$	=	Dimensionless spatial coordinate in step system
w	=	Moisture content (%)
α	=	Thermal diffusivity ($W \cdot m^2/J$)

α_s	=	Coefficient of thermal expansion ($^{\circ}\text{C}^{-1}$) of steel
β	=	Plastic magnifications of stress-strain diagram in Schneider's model
Φ	=	Empirical creep functions in Schneider's model
$\theta^A(x, t)$	=	Temperature difference between solid and ambient air of step system with normalized initial condition ($^{\circ}\text{C}$)
$\theta^B(x, t)$	=	Temperature difference between solid and its initial temperature of step system with normalized boundary condition ($^{\circ}\text{C}$)
$\bar{\theta}^A, \bar{\theta}^B$	=	Dimensionless temperature in step system
ε	=	Strain
ε_0	=	Permanent plastic strain
ε_1	=	Strain at the transition point of Anderberg's model
ε_{cr}	=	Creep strain
ε_{crush}	=	Crushing strain of concrete
ε_e	=	Elastic strains
ε_e^{20}	=	Elastic strains at 20°C
ε_{ep}	=	Elastic and plastic strains, the same as mechanical strain
ε_{full}	=	Full strains, the summation of mechanical strain and transient strain
ε_m	=	Longitudinal strain in the mid-plane of column
ε_t	=	Cracking strain of concrete
ε_{th}	=	Free thermal strain
ε_{tot}	=	Total strain
ε_{tr}	=	Transient strain

ϵ_{ult}	=	Ultimate compressive strain at temperature T
ϵ_{uT}	=	Peak compressive strain at temperature T
ϵ_{σ}	=	Mechanical strain
ρ	=	Density (kg/m^3)
σ	=	Stress (MPa)
σ_1	=	Stress at the transition point of Anderberg's model (MPa)
σ_F	=	External load before heating (MPa)
σ_t	=	Tensile strength of concrete at temperature T (MPa)
σ_t^0	=	Tensile strength of concrete at room temperature (MPa)
σ_{u0}	=	Peak compressive stress at room temperature (MPa)
σ_{uT}	=	Peak compressive stress at temperature T (MPa)
χ	=	Curvature (m^{-1})

Chapter 1

Introduction

1.1 Background

Reinforced concrete (RC) structures have been widely used in the world due to high strength, durability and economy. In the interest of life safety and property protection, building structures are designed to withstand adverse environmental loads. Fire is one of these extreme loads. Load bearing components, such as beams, columns or structural walls, are required to remain structurally sound during a fire and retain adequate load bearing capacity after the fire. Among these components, columns form the main load bearing components in a building. Failure of columns usually leads to damage or even collapse of the whole structure. Hence, there is a need to understand structural behavior of RC columns under fire conditions. Research on advanced analysis and design of RC columns under fire has attracted more and more attention due to terrorist threats.

In recent years, the construction industry has shown significant interest in the use of high strength concrete (HSC). This is due to its improvement in structural performance compared to traditional normal strength concrete (NSC). Generally, concrete up to a compressive strength of 55MPa is referred to as NSC, while concrete with compressive strength in excess of 55MPa is classified as HSC (Kodur 2003a). High Strength Concrete is widely used all over the world as it offers significant economic and architectural advantages over ordinary concrete.

Research on RC columns in fire condition has started a few decades ago. Many tests have been conducted on RC columns under high temperatures. Since it is too expensive to conduct fire tests on a whole building, normal practice is to relate the fire test performance on a single construction element to the behavior of whole building. However, when an actual column in a structure is heated up, surrounding structure exerts restraint on the heated member. This may have an adverse or beneficial effect on the fire resistance of the column. Therefore, it is necessary to take into account the effects of thermal and physical boundary conditions. Physical boundary conditions normally refer to both axial and rotational restraint. Rotational restraint is beneficial to a heated member because it reduces flexural deformation of the member. On the other hand, axial restraint not only prevents axial deformation but more importantly, induces additional axial force in the heated member itself. This effect is adverse on isolated columns, particularly, during the heating phase, which is the focus of current study. The scope does not include beyond the failure state when axial restraint can have a positive, stabilizing effect on weakened columns. Hence, for convenience, physical boundary condition in this thesis refers exclusively to axial restraint only. Up till now, design approach on fire resistance of RC columns is essentially based on tabulated data without considering physical boundary conditions. Besides, most concrete design standards do not offer any guidelines for fire resistant design of RC columns made of HSC. Results of some fire tests in a number of laboratories have shown that there are significant differences between the properties of HSC and NSC at elevated temperatures (Kodur and McGrath 2001). Clearly, a better understanding and a more accurate prediction of structural behavior of axially-restrained HSC columns at high

temperatures are urgent and useful to structural engineers.

To simulate structural response under fire conditions, numerical techniques are required because these problems are far too complex and highly nonlinear. Besides, laboratory testing is very costly. The numerical technique, which has achieved the greatest degree of popularity and success, is Finite Element Method (FEM). This approach will be adopted to analyze HSC columns under fire conditions.

1.2 FEM for Analyzing Structures in Fire

Fire resistant design has achieved great progress in the last few years. Comprehensive experimental programs have been carried out. Among the early studies were those of Abrams (1971), Malhotra (1956), and Schneider (1985; 1988) on normal strength concrete, and those of Phan and Carino (1998), Khoury and Algar (1999), Kodur and McGrath (2003c) on high strength concrete. Both analytical and empirical approaches have been proposed by different researchers (Anderberg 1983; Bresler et al. 1985; Franssen et al. 1995; Dotreppe et al. 1999; Tan and Yao 2003; Tan and Tang 2004). Sophisticated finite element methods (FEM) have been successfully developed and applied to analyze thermal and structural responses of building components. Available FEM programs include:

1. FIRE-RC (Becker and Bresler 1974) and FIRES-T3 (Bresler et al. 1977) at UC Berkeley;
2. CONFIRE (Forsen 1982) at Lund Institute of Technology, Sweden;
3. SAFIR (Franssen 1999) at University of Liege;
4. The Canadian model (Kodur et al. 2004) at Institute for Research in

Construction, National Research Council of Canada;

5. HITECO (Khoury et al. 2002) at Imperial College, University of London.

Currently, these programs are mainly used for research purpose. Practical codes have been developed. Design guides and regulations are available. ECCS-TC3 (1983), BS (1990) and Eurocode (1995b) are some widely-used codes for steel structures, while ACI (1994), ACI (2000), CEB-FIP (1991) and Eurocode (1995a) are applicable for reinforced concrete structures. ECCS-TC3 (1988) is used for composite structures. These codes allow both analytical and empirical methods to be used to predict the fire resistance of structures.

With the aid of modern high-speed computer, there is rapid development of numerical programs that can accurately predict the fire resistance of structural members. The following stages are required (Lie and Chabot 1993a) in a fire resistance analysis using FEM:

1. Calculation of fire-gas temperature. Fire temperature is to be expressed as a function of time. It can be determined from a heat balance calculation, or a standard fire curve such as ISO 834 or ASTM-E119 (Bresler et al. 1985).
2. Prediction of temperature distribution in structural members. Heat transfer models are applied to determine temperature distribution within columns.
3. Determination of structural response. Some mechanical and thermal properties of material are highly temperature-dependent. Additional axial forces are generated by boundary conditions.

The basic idea is to find collapse temperature T_c - the fire temperature at which a column collapses. The corresponding fire exposure duration is defined as collapse

time t_c . Although this research focuses on the fire resistance of columns, the concepts are also applicable to other structural members, such as beams and slabs.

1.2.1 Standard Fire Curve

There are a number of factors determining the temperature development of a fire, either hinder or favor fire propagation. It is difficult to predict its mean temperatures. Several standard fire curves have been established as a common, standard basis for comparison. The ISO 834 (1975) fire curve is commonly accepted by European countries, while the ASTM-E119 (1995) fire curve is widely used in the USA and Canada (Fig. 1.1):

$$T_g = 20 + 345 \log_{10}(480t + 1) \quad (\text{ISO 834}) \quad (1.1)$$

$$T_g = 20 + 750[1 - e^{-3.79553\sqrt{t}}] + 170.41\sqrt{t} \quad (\text{ASTM-E119}) \quad (1.2)$$

where T_g is the gas temperature in °C; t is the exposure time in hours.

As shown in Fig. 1.1, both the ISO 834 and ASTM-E119 standard fire curves increase monotonically with time. The gas temperature of the ISO 834 fire is slightly greater than that of the ASTM-E119 fire. Fig. 1.1 also shows a severe fire in which the gas temperature rises much faster than both the ISO 834 and the ASTM-E119 standard fires. Nevertheless, the standard fire curves are still widely used for the following reasons. First, they provide a common basis for comparison. Secondly, the standard fire curves can be related to real fire curves by the concept of equivalent time. Thirdly, the formula can be used in analytical heat flow calculations because it is integrable.

Figure 1.1 Standard fire curves

1.2.2 Temperature of Columns in Fire

Temperature in a RC column is generally different from fire-gas temperature as it takes time for the heat transfer process to take place. It can be determined by a heat balance analysis. Radiation, convection and conduction are the three heat transfer modes involved. It is generally assumed that the calculation of temperature response of an element may be entirely decoupled from structural analysis. Major factors affecting temperature distribution of a column include:

1. Column location: the location of a column will affect the amount of heat transferred by radiation and convection to the column.
2. Construction type: steel has a relatively high thermal conductivity, and temperature generally does not vary significantly across the section; for concrete, the temperature varies significantly from outer surface to inner core.
3. Fire exposure: the temperature distribution also depends on whether the column is fully or partially exposed to fire.
4. Section size and shape: for concrete columns, the larger the section size, the

lower is the heating rate.

5. High temperature material properties of constituent materials.
6. Occurrence of fire induced spalling in concrete.

1.2.3 Structural Response in Fire

Structural response of an RC column to fire condition depends on the following:

1. Reduction of material strength and stiffness. Hence, strength and stability of columns will be adversely affected.
2. Thermal expansion of heated elements. Thermal stresses are important only in columns fully or partially restrained at the ends.
3. Thermal gradient within a section: this effect is to cause some fix-end forces and moments, which will affect structural behavior.

Under fire conditions, different types of strains are induced in concrete members, including mechanical strain, thermal strain, creep strain, transient strain, etc. These strains have different effects on the structural behavior of RC columns. The complicated structural behavior is commonly analyzed by FEM programs. Besides FEM programs, some researchers also developed simple methods for predictions of fire resistance of RC columns. Dotreppe et al. (1999) extended the Perry-Robertson formula for RC columns under fire conditions. They resorted to curve fitting to determine the empirical coefficients. Tan and Yao (2003) also extended the Rankine approach to include the fire resistance predictions of both reinforced concrete filled steel columns and plain concrete filled steel columns. It

shows that the agreement with test results is reasonably good. Tan and Tang (2004) presented an approach for RC columns in fire conditions based on the ACI method for column design at ambient temperature. This approach is suitable for both axially- and eccentrically-loaded columns. Experimental results show that the extended ACI approach yields relatively accurate and conservative predictions.

1.3 Objectives and Scope

The purpose of this research is to extend a FEM program to investigate the influence of axial restraint on structural behavior of HSC column under high temperature. Hitherto, little attention has been paid to the effect of physical boundary conditions. Only axial restraint effect is considered since flexural restraint is beneficial to heated members. Most of previous work is conducted on single columns without considering axial restraint. Since axial restraint is always present, in order to obtain a more accurate result, axial restraint must be taken into account in the analysis. This motivates the author's interest for this research. The main objectives of the research include:

- To explore the theoretical foundation of current material models and establish a new HSC material model that can be used for modeling columns with axial restraint.
- To incorporate the new material model into an established FEM program. To use the program to investigate the structural behavior of axially-restrained HSC columns under high temperature.

Material model is a prerequisite for structural analysis. Currently, there are

some material models for normal strength concrete. These models are not suitable for structural analysis of axially-restrained HSC column under fire. Therefore, a new material model is needed for HSC. In order to establish a robust HSC material model, the first job is to compare current NSC models and to find out the distinction among them. Through the comparison, an understanding of their advantages and disadvantages can be obtained so that the merits can be retained and extended, while the shortcomings can be avoided.

In the material models, there are different strain components. Among them, transient strain is the trickiest one, which only develops under compression force as temperature increases. This strain component occurs almost independent of time. Essentially, It is defined as the strain difference between two heating and loading sequences applied to a column, namely, one column is loaded before heating, and the other one loaded after heating. Detailed discussion will be presented in presented in Chapter 3. Although its nature and significance are not well understood, it accounts for a large amount of total deformation. This motivates the author to investigate transient strain and its influence on structural behavior of axially-restrained column under high temperature. Up till now, to the best of the author's knowledge, there is only one HSC material model (Kodur et al. 2004), but this model does not include transient strain explicitly. Therefore, the author intends to find out the transient strain effect on high strength concrete and complete Kodur's HSC model.

When an RC column is heated up, material properties of concrete and rebar are all affected by temperature. This complicates the structural behavior. The situation

is exacerbated by uneven temperature distribution across the section. Analytic solution for this type of problem is impossible. To obtain an accurate result, a FEM program is necessary. The second job of this research is to extend a FEM program (Huang 2002) for structural analysis. The established material model will be incorporated into this program.

In previous work, computer programs make use of a simplified model of restraint, where either pin or free ends are considered at the ends of a column, although neither of which is likely to occur. Using this program, effects of axial restraint on fire resistance of HSC column can be evaluated. In addition, specific examples help to illustrate the complicated problems.

It should be noted that this research program mainly focuses on structural analysis. Material model is only to provide the necessary parameters to support structural analysis. Thus, it will only involve the same type of HSC used in structural testing. The deterioration of material properties of NSC and reinforcement is assumed to follow Eurocode (1995a; 1995b), and for HSC, is assumed to follow the equations proposed by Kodur et al. (2004).

1.4 Overview

There are five chapters in this thesis. The first Chapter is an introduction about the research background, research objectives and scope. Chapter 2 presents a literature review of previous work on FEM analysis concrete columns under fire. Some material and structural tests are also introduced because they will be used for

verification. In Chapter 3, some available material models are presented and compared. Basic theories on material models are presented. The nature of transient strain is investigated. A new material model for HSC is established based on the comparison study of different material models. Available material tests are used to verify the new material model. In Chapter 4, structural model that is used in FEM program for analysis is introduced. Relevant theories and assumptions are presented. An existing FEM program is extended by incorporating the proposed new material model into it. Results from FEM program and available NSC and HSC experiments are compared to verify the numerical approach. Significance of transient strain and axial restraint on structural response of concrete columns under fire is studied using this program. Chapter 5 presents the conclusions and recommendations for future work.

Chapter 2

Literature review

2.1 Introduction

Calculations of fire resistance of building elements usually consist of three steps. The first step is developing relevant time-temperature scenario resulting from a given fire exposure. In the second step, heat transfer is conducted to determine temperature distribution in elements. The third step is to determine structural response to heating, as manifest by the decline of load-bearing capacity, up to the point of structural failure. In order to obtain reasonable structural response, accurate temperature prediction is necessary. Therefore, some available techniques for temperature prediction are presented first. To understand the structural response of RC columns under fire conditions, knowledge of the influence of elevated temperatures on concrete and reinforcement material properties is also an important prerequisite. The second part of this chapter introduces the effect of elevated temperatures on the material and thermal properties of concrete (NSC and HSC) and reinforcement. Because material and structural tests will be used as important tools for verification in this research, previous work in this field is introduced as well. The objective of this research is to extend an established FEM program for structural analysis. Therefore, a review of previous numerical programs for structural members at elevated temperatures is presented in the fourth part. Information about a FEM program-SAFIR, which will be used for numerical verification in Chapter 4, is also included in this chapter.

2.2 Thermal Analysis

Several methods exist for temperature prediction of structural members that are exposed to fire, normally experimental, analytical, numerical and graphical methods. Of all these methods, numerical method is most often used. The most common two numerical tools are finite difference method (Lie 1977) and finite element method (FEM) (Zienkiewicz and Cheung 1967). An efficient analytical approach based on Green's function (GF) solution is also developed by Wang et al. (2005a).

2.2.1 Analytical Method

A number of research works has been carried out for analytical formulation of heat conduction in multi-dimensional solids, among which the Green's function approach (Beck et al. 1992) is most commonly adopted. Different series expansions based on different mathematical techniques are used for the small time and the large time representations of the analytical solution, respectively, together with time partitioning strategy to improve the convergence properties of analytical solutions.

2.2.2 Finite Difference Method

In this method, a member cross section is divided into several elementary regions. A heat balance equation is established for each region. With the aid of these equations, temperature of each element can be successively evaluated. Finite difference method is suitable for the calculation of temperatures in monolithic

building components such as concrete columns, beams, and walls (Lie 1992).

2.2.3 Finite Element Method

Finite element method is a more recent numerical technique developed after the 1950s. The rising popularity of the FEM is attributed to its great flexibility in handling problems involving complex geometrical shapes, several different materials, and mixed boundary conditions. A number of publications are available on this subject (Zienkiewicz and Cheung 1967; Becker et al. 1974).

2.3 Properties of Concrete at Elevated temperatures

Different aggregate types have different performance under elevated temperature. This has already been reflected in Eurocode (1995a) as the variation of compressive strength, thermal expansion and other thermal properties with temperature (as discussed later in Section 2.3). The relationship between material properties (e.g. compressive strength) and temperature for concrete made of siliceous aggregates are different from those made of calcareous aggregates. These different effects for different aggregate have been included in the FEM program.

Concrete material properties comprise thermal and mechanical parts. Thermal properties include conductivity and specific heat, whereas mechanical properties include yield strength, elastic modulus and stress-strain relationship. In this research, thermal and mechanical properties under elevated temperatures is are in accordance with Eurocode (1995a; 1995b).

2.3.1 Compressive Strength

Degradation of compressive strength of concrete due to short-term exposure to elevated temperature has been studied as early as the 1950s. Researchers have recommended some strength-temperature relationships. The relative compressive strength curves at elevated temperatures are shown in Fig. 2.1. It is noted from this figure that the material properties of high strength concrete are significantly different from those of normal strength concrete. These differences become less significant at temperatures above 400°C. Therefore it is inappropriate to use the strength-temperature relationship recommended by Eurocode for HSC.

Figure 2.1 Relative compressive strength of concrete at elevated temperature

2.3.2 Modulus of Elasticity

A limited number of publications consider the elastic properties of concrete at high temperatures because there are some quantitative discrepancies among the

results obtained from different tests. This is due partly to ambiguity in the definition of elastic modulus, and partly to the effect of simultaneous loss of moisture. Although elastic moduli are different for different concrete strengths (for example, HSC is stiffer than NSC and has a higher modulus of elasticity), there are no significant differences for concretes with different strength grades (Harmathy 1993; Bazant 1996). Purkiss (1996) proposed the following equation for elastic modulus:

$$E(T) = \begin{cases} E_0 & T \leq 60^\circ \text{C} \\ \frac{800-T}{740} E_0 & 60^\circ \text{C} < T \leq 800^\circ \text{C} \end{cases} \quad (2.1)$$

where E is modulus of elasticity in MPa ; T is temperature in $^\circ\text{C}$.

Figure 2.2 Relative modulus of elasticity of concrete at elevated temperature

2.3.3 Tensile Strength

The reduction of characteristic tensile strength of concrete can be calculated by a coefficient $k_t(T)$ using the simplified method given in ENV 1992-1-2/2004/:

$$\sigma_t(T) = k_t(T) \sigma_t^0 \quad (2.2a)$$

$$k_t = \begin{cases} 1.0 & 20^\circ C \leq T \leq 100^\circ C \\ 1.0 - 1.0(T - 100)/500 & 100^\circ C < T \leq 600^\circ C \end{cases} \quad (2.2b)$$

2.3.4 Constitutive Model

A constitutive model is a stress-strain model used to describe the mechanical properties of concrete. Original research in this field was carried out by Anderberg (1976a), but his analysis has been questioned by Schneider (1986) because Anderberg neglected preload effect. An alternative approach based on Anderberg's method for calculating transient strain was proposed by Khennane and Baker (1993). Another model using the concept of load-induced thermal strain (LITS) was developed by Khoury et al. (1985a; 1985b; 1995). Kodur et al. (1996; 2004) and Cheng et al. (2004b) proposed a simple stress-strain relationship expressed as a function of temperature. Detailed discussion on the difference of these models will be presented in Chapter 3.

2.3.5 Thermal properties

Traditionally, free thermal strain ε_{th} is expressed as a function of temperature. The difference among different concrete strength grades on ε_{th} is relatively small and the thermal strain model given in ENV 1992-1-2/2004/ can be used depending on the type of concrete:

Siliceous Aggregates Concrete:

$$\varepsilon_{th}(T) = \begin{cases} -1.8 \times 10^{-4} + 9 \times 10^{-6}T + 2.3 \times 10^{-11}T^3 & 20^\circ C \leq T \leq 700^\circ C \\ 14.0 \times 10^{-3} & 700^\circ C < T \leq 1200^\circ C \end{cases} \quad (2.3a)$$

Calcareous Aggregate Concrete:

$$\varepsilon_{th}(T) = \begin{cases} -1.2 \times 10^{-4} + 6 \times 10^{-6}T + 1.4 \times 10^{-11}T^3 & \dots\dots\dots 20^\circ C \leq T \leq 805^\circ C \\ 14.0 \times 10^{-3} & \dots\dots\dots 805^\circ C < T \leq 1200^\circ C \end{cases} \quad (2.3b)$$

where T is temperature in $^\circ C$.

Other thermal properties of NSC in this research are evaluated according to ENV 1992-1-2/2004/. Thermal conductivity of NSC for $20^\circ C \leq T \leq 1200^\circ C$ can be interpolated between the lower and the upper limit values given below:

$$\text{Upper limit:} \quad k_c = 2 - 0.2451(T/100) + 0.0107(T/100)^2 \quad (2.3c)$$

$$\text{Lower limit:} \quad k_c = 1.36 - 0.136(T/100) + 0.0057(T/100)^2 \quad (2.3d)$$

Thermal conductivity of HSC recommended by Kodur et al. (2004) are given:

$$\text{Siliceous concrete:} \quad k_c = (2 - 0.0011T) \times 0.85 \quad (2.3e)$$

$$\text{Carbonate concrete:} \quad k_c = \begin{cases} (2 - 0.0013T) \times 0.85 & \dots\dots\dots T \leq 300^\circ C \\ (2.21 - 0.002T) \times 0.85 & \dots\dots\dots T > 300^\circ C \end{cases} \quad (2.3f)$$

where k_c is in $W/(mK)$; T is temperature in $^\circ C$.

Specific heat of NSC may be modeled by a constant value, c_{peak} between $100^\circ C$ and $115^\circ C$, with a linear variation to $1000 J/kg/K$ at $200^\circ C$. It then linearly increases to $1100 J/kg/K$ at $400^\circ C$ and becomes constant beyond that temperature.

$$c_{peak} = 900 J/kg/K \text{ for moisture content of } 0\% \text{ of concrete weight;}$$

$$c_{peak} = 1470 J/kg/K \text{ for moisture content of } 1.5\% \text{ of concrete weight;}$$

$$c_{peak} = 2020 J/kg/K \text{ for moisture content of } 3.0\% \text{ of concrete weight;}$$

For other moisture contents a linear interpolation is acceptable.

The product of specific heat and density is called thermal capacity. Thermal capacity of HSC proposed by Kodur et al. (2004) is given in Table 2.1. It should be noted that HSC is denser than NSC. As a result, thermal conductivity, specific heat

and density of HSC are higher than that of NSC per cubic meter. It is important to notice that specific heat per kilogram is the same for HSC and NSC, as these materials consists of the same gradients. A higher thermal conductivity means that heat conducts faster in HSC than in NSC, resulting in a higher temperature for a given time interval.

Table 2.1 Thermal capacity ($J/m^3 \text{ } ^\circ C$) of HSC

Siliceous aggregate concrete		Carbonate aggregate concrete	
$0 \leq T \leq 200^\circ C$	$\rho c = (0.005T + 1.7) \times 10^6$	$0 \leq T \leq 400^\circ C$	$\rho c = 2.45 \times 10^6$
$200 < T \leq 400^\circ C$	$\rho c = 2.7 \times 10^6$	$400 < T \leq 475^\circ C$	$\rho c = (0.026T - 12.85) \times 10^6$
$400 < T \leq 500^\circ C$	$\rho c = (0.013T - 2.5) \times 10^6$	$475 < T \leq 650^\circ C$	$\rho c = (0.0143T - 6.295) \times 10^6$
$500 < T \leq 600^\circ C$	$\rho c = (-0.013T + 10.5) \times 10^6$	$650 < T \leq 735^\circ C$	$\rho c = (0.1894T - 120.11) \times 10^6$
$T > 600^\circ C$	$\rho c = 2.7 \times 10^6$	$735 < T \leq 800^\circ C$	$\rho c = (-0.263T + 212.4) \times 10^6$
		$T > 800^\circ C$	$\rho c = 2.0 \times 10^6$

2.4 Properties of Steel at Elevated Temperatures

Mechanical properties of reinforcement are of great importance when structural elements are subjected to fire, because reinforcement accounts for moment capacity, but is vulnerable to fire. Therefore, it is important to know the material properties of steel under elevated temperatures.

2.4.1 Relative Strength

The strength reduction with respect to elevated temperature depends on the magnitude of residual strain considered. The reduction is shown in Fig. 2.3, in which f_{sy} is the maximum stress level and f_{sp} is the proportional limit.

Figure 2.3 Relative strength of steel according to Eurocode

2.4.2 Modulus of Elasticity

Modulus of elasticity of rebar is an important parameter controlling the displacement and stability of structure. The modulus of elasticity will reduce at elevated temperatures. Reduction factor with respect to temperature is determined from the initial slopes of stress-strain curves tested at different temperatures.

Figure 2.4 Relative modulus of steel according to Eurocode

2.4.3 Constitutive Model

Total strain can be described as follows:

$$\varepsilon_{tot}(T) = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) \quad (2.4)$$

where ε_{tot} is total strain; ε_{th} is thermal strain; ε_{σ} is mechanical strain; ε_{cr} is creep strain. Thermal strain is normally the largest one. The stress-strain relationship of reinforcement is defined by two parameters as depicted in Fig. 2.5: the proportional limit f_{sp} and the maximum stress level f_{sy} . The formulation of stress-strain relationships may also be applied for reinforcing steel in compression. In particular, at temperature over 450°C, steel displays a significant creep phenomenon. Therefore, the effect of creep has to be modeled carefully. In this research, Eurocode model for reinforcement is adopted because it is easy to use. In this model, creep is implicitly included.

Figure 2.5 Mathematical model for stress-strain relationships of reinforcing steel

2.4.4 Thermal Properties

Thermal strain of steel is an important factor in the calculation of total deformation of a structural member exposed to fire. Thermal strain varies with T according to Eq. 2.10 (ENV 1992-1-2/2004/).

Reinforcing steel:

$$\varepsilon_{th} = \begin{cases} -2.416 \times 10^{-4} + 1.2 \times 10^{-5} T + 0.4 \times 10^{-8} T^2 & \dots\dots\dots 20^\circ C \leq T \leq 750^\circ C \\ 11.0 \times 10^{-3} & \dots\dots\dots 750^\circ C \leq T \leq 860^\circ C \\ -6.2 \times 10^{-3} + 2.0 \times 10^{-5} T & \dots\dots\dots T > 860^\circ C \end{cases} \quad (2.5)$$

where T is temperature of rebar in $^\circ C$.

Other thermal properties of steel are shown in Fig. 2.6. Values are given in ENV 1993-1-2/2005/.

Figure 2.6 Variation of thermal properties of steel with temperature

2.5 Review of Previous Research Work

2.5.1 Material Tests

Many investigations on the effect of fire on material properties of concrete have been reported during the last four decades. Not only uniaxial material tests have been established (Anderberg 1976a; Schneider 1986; Bazant and Chern 1987), biaxial and triaxial tests on concrete under elevated temperature have also been carried out (Thelandersson 1983; Bazant 1996; Thienel and Rostasy 1996). 3-D material models were proposed by Thelandersson (1987) and Bazant and Chern (1987; 1996). Some experiments to measure transient strain on concrete cylinders have been conducted in different laboratories (Colina and Sercombe 2004; Mindeguia et al. 2006; Persson 2006).

2.5.2 Structural Tests

Tests on RC columns exposed to fire conditions started several decades ago for developing the design codes. Various works carried out during 1920~1956 in Germany, the UK, and the USA were reviewed by Weeks (1985). During the last twenty years, a large amount of experimental work has been carried out in many countries on structural elements, mostly following the standard fire curve.

During 1984 to 1988, Lie, Lin, Allen and Abrams performed a series of experiments including 41 RC columns at the National Research Council (NRC) and the Portland Cement Association (Lie and Lin 1985). The following parameters were studied in their tests: area of cross-section, shape of cross section, thickness of concrete cover, percentage of longitudinal reinforcing steel, lateral reinforcement, type of aggregate, concrete strength, moisture content, axial and rotational restraint,

load eccentricity and fire exposure.

To determine the major parameters affecting the behavior of RC columns under fire conditions, Dotreppe et al. (1997) performed experiments on 21 simply supported columns. A special method was adopted instead of the usual procedure for fire tests. For each column a load corresponding to R_f 1h or R_f 2h was defined, where R_f 1h and R_f 2h of ENV 1992-1-2 refer to the maximum loads that a column can withstand while providing the required fire resistance (1 hour or 2 hours).

In 1993 at the Lund Institute in Sweden, 5 tests were carried out for slender columns under fire attack (Holmberg and Anderberg 1993). Fire related factors included the standard test curve ISO 834 (1975), applied axial load and initial eccentricity (or moment). Test data indicated that using simplified 400-isotherm curve for HSC gives conservative results.

In 2002, a collaborative research project was undertaken between the National Research Council of Canada (NRCC) and the National Chiao Tung University (NCTU), Taiwan to develop guidelines for fire resistant design and construction of HSC columns and fiber-reinforced HSC columns (Kodur et al. 2004). Effects of tie spacing, tie configuration and extent of spalling were investigated. Experimental results showed that ties with 135° hook and cross ties have beneficial effects on fire resistance of HSC columns. Presence of steel or polypropylene fibers in HSC columns can reduce spalling and enhance their fire resistance.

2.5.3 Finite Element Method in Fire Analysis

In the past few decades, with the development of computer technology, Finite Element Method (FEM) has been widely used to analyze RC structures.

Scordelis (1967) first analyzed shear resistance of RC beams under fire by FEM. They divided both concrete and steel parts of the beam into triangular elements and analyzed them as a linear elastic plane stress problem.

In 1972, Dougill (1972a; 1972b) used a FEM program to study the role of loading and restraint on the failure modes of concrete panels at high temperatures. He concluded that different degrees of rotational restraint and axial restraint have significant effects on structural response.

Allen and Lie (1974) analyzed the behavior of reinforced concrete columns based on load deflection analysis. Lateral deflection is calculated from the assumption that the column is fixed at both ends, and the curvature diagram varies linearly from mid-height to both supports. They assumed that the curvatures at each support are equal and opposite.

The first widely-known FEM programs for analyzing the structural response of RC frames in a fire was FIRES-RC by Becker and Bresler (1974). A revised version of the program was presented by Iding in 1977 (Bresler et al. 1977) called FIRES-T3. At the same time, Anderberg and Thelandersson (1976a) developed a special version of FIRES-RC which included new material models developed at Lund Institute of Technology in Sweden.

Forsen (1982) considered geometrical and material non-linearity developed at the Lund Institute of Technology in his Finite Element program-CONFIRE. This program employed a beam element with three degrees of freedom at each node and one internal axial degree of freedom, where total strain is taken as linear over the cross section. Newton-Raphson iteration method was used to obtain time-dependent stresses, strains and displacements.

In 1991, Franssen (1999) developed a program called SAFIR. This program can perform both thermal and structural analysis for 2D or 3D model exposed to fire. 2D solid elements, 3D solid elements, beam elements, shell elements and truss elements can be used. The stress-strain material laws are generally linear-elliptical for steel and non-linear for concrete. Eurocode model for steel and concrete is included in this program. The post processor of this program is user-friendly and illustrative. Besides, this software is readily available to the author. All these features make the author decide to use it as a numerical comparison to verify the FEM program that will be developed in his research. Using this software, the first step involves predicting the temperature distribution within structural members. Then the thermal analysis results are input for transient mechanical analysis. Large displacement and nonlinear material properties can be considered in the mechanical analysis.

All the computer programs mentioned above assumed that heat transfer is completely decoupled from structural analysis. Khoury et al. (2002) developed a three dimensional non-linear program named HITECO at Imperial College,

University of London, where thermal, hydro and mechanical analyses were performed in a fully interactive and coupled fashion, i.e., deformation of the solid due to thermal expansion or stresses will influence the heat transfer process. The spalling effect of concrete can be predicted using this program.

2.6 Summary

In this chapter, processes of FEM analysis of concrete columns under fire are introduced, as well as the basic theories and previous work in this field. Some material models have been developed from the material tests described in section 2.5.1. These material models formed the foundation for structural analysis of RC members under fire. However, there are obvious differences among these material models, i.e., the presence of transient strain. Before carrying out further study, the author intends to find out the nature and significance of transient strain by a comparison of existing material models for NSC. Nature of transient strain and the necessity to include transient strain explicitly in a material model will be investigated in the next chapter. Besides, most existing material models for NSC are not suitable for HSC. As the popularity of HSC is increasing, there is an urgent need to develop a HSC material model, which includes transient strain explicitly.

Structural tests described in section 2.5.2 have all been carried out according to the standard fire condition. The results can serve as a quantitative comparison. One important point to note is that the boundary conditions of columns in a structure are vastly different from those in the idealized furnace tests. A simplified arrangement is employed in the furnace where there is either no axial restraint or nearly full axial

restraint. However, in reality neither condition is likely to exist. The real structural behavior of a column exposed to fire is different from that in a furnace test. It is generally agreed that unrestrained tests give a lower bound solution for a column under normal restraint (Lie and Lin 1985). But how axial restraint will affect the structural behavior of RC columns at elevated temperature remains unknown. There are some tests conducted on the effect of axial restraint arising on floors and beams over the last few years (Gordon 2001; Linus 2004). However, there are relatively fewer publications on the effect of axially restrained columns. This then becomes the focus of current study. The effect of axial restraint on structural behavior of concrete columns under fire will be studied in Chapter 4.

Chapter 3

Material Models of Concrete

3.1 Introduction

This chapter begins with an introduction of available material models under elevated temperature effect. All strain components are introduced and a new material model will be developed in Section 3.4. Since there is only limited work has been done on HSC material model, NSC material models are used as reference. Therefore, existing NSC models are compared first to find out the differences among them. Because transient strain is a dominant strain, it requires special

attention. The nature of transient strain will be investigated through the comparison study. After the new material model is established, material tests of HSC cylinders at elevated temperature will be collected from literature to verify the proposed material model. To incorporate the proposed material model into a FEM program, the former must take account of compression, tension and failure criteria. However, transient strain is only included under compression strain. Therefore, the proposed material model must be extended to a complete material model before it can be used for FEM programming.

3.2 Comparison of Existing Material Model

In a thermal stress analysis using FEM, the unknowns are nodal displacements. The corresponding strain is the total strain, which includes four components, namely, free thermal strain caused by a change of temperature, instantaneous stress-related strain caused by externally applied stress, creep strain caused by dislocation of microstructures of the material and transient strain caused by a change of chemical composition. Mathematically, total strain can be expressed as:

$$\varepsilon_{tot} = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) + \varepsilon_{tr}(\sigma, \Delta T) \quad (3.1)$$

where σ is the corresponding stress of total strain, T is temperature, ΔT is change of temperature, and t is time. ε_{th} is free thermal strain which is temperature-dependent, ε_{σ} is instantaneous stress-related strain related to stress and temperature, ε_{cr} is creep strain that is a function of stress, temperature and time and ε_{tr} is transient strain that occurs only under compression as temperature increases. Some researches doubted that transient strain is present in tension as well, which is ignored in this thesis, since transient strain in tension is relatively small

and insignificant compared to that in compression. Please note ε_{ir} is essentially permanent, irrecoverable and only occurs under first heating. Transient strain is a function of instantaneous stress and temperature increment. This strain only occurs when temperature is increased under compressive load (ΔT and σ), and its development is almost instantaneous. Note that the stress-related strain ε_{σ} is also dependent on stress history, which means that the stress-strain relationship is elasto-plastic. When a material becomes plastic the strain states corresponding to two identical stress states will be different if the two stress states have different loading histories.

As mentioned in Section 2.3.5, free thermal strain ε_{th} is expressed as a function of temperature:

Siliceous Aggregates Concrete:

$$\varepsilon_{th}(T) = \begin{cases} -1.8 \times 10^{-4} + 9 \times 10^{-6} T + 2.3 \times 10^{-11} T^3 & \dots\dots\dots 20^{\circ} C \leq T \leq 700^{\circ} C \\ 14.0 \times 10^{-3} & \dots\dots\dots 700^{\circ} C < T \leq 1200^{\circ} C \end{cases} \quad (3.2a)$$

Calcareous Aggregate Concrete:

$$\varepsilon_{th}(T) = \begin{cases} -1.2 \times 10^{-4} + 6 \times 10^{-6} T + 1.4 \times 10^{-11} T^3 & \dots\dots\dots 20^{\circ} C \leq T \leq 805^{\circ} C \\ 14.0 \times 10^{-3} & \dots\dots\dots 805^{\circ} C < T \leq 1200^{\circ} C \end{cases} \quad (3.2b)$$

where T is temperature in $^{\circ}C$.

Creep, transient, and instantaneous stress-related strains are functions of stress, and temperature. However, only creep strain is assumed to be time-dependent. This makes it quite difficult to separate them during an experiment. Therefore, some of existing models use only two strains, namely, stress-induced strain and transient creep strain, the latter including both transient and creep strains. In some other models, apart from ε_{th} , all of the three strains are taken together as a single strain

on the right hand side of Eq. 3.1.

3.2.1 Anderberg and Thelandersson Model

Anderberg (1976b) and Thelandersson (1987) proposed a concrete model in which free thermal strain, stress-related strains, creep strain, transient strain and are separately accounted. The constitutive equation of this model is the same as Eq. 3.1. Based on experimental data, the instantaneous stress-related strain is assumed to be governed by a parabolic curve in the ascending branch and a linear curve in the descending branch:

For $0 \leq \varepsilon_\sigma \leq \varepsilon_1$

$$\sigma = E(\varepsilon_\sigma - \frac{\varepsilon_\sigma^2}{2\varepsilon_{uT}}) \quad (3.3a)$$

for $\varepsilon_1 \leq \varepsilon_\sigma \leq \varepsilon_{ult}$

$$\sigma = \sigma_1 + E^-(\varepsilon_\sigma - \varepsilon_1) \quad (3.3b)$$

in which

$$\sigma_1 = E(\varepsilon_1 - \frac{\varepsilon_1^2}{2\varepsilon_{uT}}), \quad \varepsilon_1 = \varepsilon_{uT}(1 - \frac{E^-}{E}), \quad \varepsilon_{uT} = \frac{2\sigma_{uT}}{E} \quad (3.3c)$$

where E is the initial tangent modulus of stress-strain curve at temperature T ; $E^- = -880MPa$ is the tangent modulus of the stress-strain curve in the descending branch and is assumed temperature-independent; ε_{ult} is the ultimate compressive strain at temperature T which increases linearly from 0.005 at 20°C to 0.015 at 1000°C; σ_{uT} is the peak compressive stress at temperature T , ε_{uT} is the corresponding strain; σ_1 and ε_1 is the stress and strain at the transition point between the parabolic branch and the linear branch. Fig. 3.1 shows the normalized stress-strain curve, while the peak stress, initial tangent modulus and ultimate compressive strain are temperature-dependent.

Figure 3.1 Normalized stress-strain curve in Anderberg-Thelandersson's model

Explicit classical creep strain for concrete is assumed as:

$$\epsilon_{cr} = -0.00053 \left(\frac{\sigma}{\sigma_{uT}} \right) \left(\frac{t}{180} \right)^{0.5} \exp(0.00304(T - 20)) \quad (3.4)$$

where the unit for time t is minutes. The classical creep strain is often very small compared to the other three strains due to the short period of fire. It had been indicated that, practically, it might be neglected. A comparison of creep strain and transient strain of Anderberg's model for two loading levels is shown in Table 3.1.

Transient strain develops under compressive stress when temperature increases. The strain is essentially permanent, irrecoverable and occurs only under the first heating (RILEM 1998). In transient tests this strain component is often dominant; it is approximately independent of time. Furthermore, it is temperature-dependent, very similar to that of free thermal strain and it has a linear relation to compressive stress level. Transient strain ϵ_{tr} as defined by Anderberg can be expressed by the following equations:

$$\text{For } T \leq 550^\circ C$$

$$\varepsilon_{tr} = -k_{tr} \left(\frac{\sigma}{\sigma_{u0}} \right) \varepsilon_{th} \quad (3.5a)$$

For $T \geq 550^\circ C$

$$\Delta \varepsilon_{tr} = -0.0001 \left(\frac{\sigma}{\sigma_{u0}} \right) \Delta T \quad (3.5b)$$

where k_{tr} is a constant between 1.8 and 2.35 and σ_{u0} is the peak compressive stress at room temperature. It is likely that the kind of aggregate and concrete mix used will influence the value of k_{tr} .

From Table 3.1, it can be seen, when the time is less than 2 hours, creep strain accounts for less than 10% of transient strain except when the temperature is lower than $200^\circ C$. For the ISO 834 fire, the temperature increases so fast that it is already $600^\circ C$ in the first 10 minutes. Therefore, creep strain can be assumed negligible in a fire. However, it should be noted that when the duration is longer than 5 hours, creep strain plays a more important role and cannot be ignored.

Table 3.1 Comparison of creep strain and transient strain

Time (mins)	Loading Level	Temperature ($^\circ C$)	Creep Strain, ε_{cr} (10^{-3})	Transient Strain, ε_{tr} (10^{-3})	Percentage $\varepsilon_{cr} / \varepsilon_{tr}$ (%)
120	$0.1 \sigma_{u0}$	200	0.058	0.424	13.7
		400	0.106	1.150	9.3
		600	0.196	2.690	7.3
		800	0.359	4.700	7.6
	$0.3 \sigma_{u0}$	200	0.183	1.272	14.4
		400	0.336	3.450	9.7
		600	0.618	8.072	7.7
		800	1.130	14.072	8.0

Khennane and Baker (1993) proposed a similar model in which creep and transient strains are taken as the same form as those in Anderberg and Thelandersson's model. The only difference is the instantaneous stress-related strain, which is split into two parts referred to as elastic and plastic strains. However, this model does not allow for a descending branch of the stress-strain curve. Diederichs (1987) adopted a similar model as Anderberg and Thelandersson's one, except that classical creep strain is ignored and instantaneous elastic strain is calculated using the ambient modulus of elasticity. In Diederichs's model transient strain is calculated by a simple expression.

3.2.2 Schneider Model

The model used by Schneider (1986) is based on a unit stress compliance function, i.e., creep transient strain is considered to be linear with respect to stress. An important simplification to the general compliance function approach that account for creep in a fire is that, the duration of between one-half and four hours is short compared with the age of concrete, and thus any time-dependence in the model can be ignored. Therefore, in this model, apart from thermal strain, there are only two other strain components; one is the transient creep strain, which combines the transient and creep strains together, and the other is the instantaneous stress-related strain. The stress-strain constitutive equation is taken as follows:

$$\varepsilon_{tot} - \varepsilon_{th} = \varepsilon_{\sigma} + \varepsilon_{tr} + \varepsilon_{cr} = J(\sigma, T) \frac{\sigma}{E} \quad (3.6)$$

where $J(\sigma, T)$ is the compliance function which represents the stress-dependent strain per unit stress, i.e. the strain response to a sustained constant unit imposed stress applied at temperature T , written as:

$$J(\sigma, T) = \frac{(1 + \beta)}{g} + \frac{\Phi}{g} \quad (3.7)$$

where β function accounts for plastic magnifications in a stress-strain diagram, and may be neglected within the elastic range, i.e. load level less than 0.5 of the strength limit; the g function allows for an increase of elasticity due to external load before heating; Φ are empirical creep functions dependent upon both the stress and temperature.

$$\beta(\sigma, T) = \frac{1}{n-1} \cdot \left(\frac{\varepsilon}{\varepsilon_{uT}} \right)^n \quad (3.8a)$$

Within the elastic range, Eq. 3.8a can be approximated by:

$$\beta(\sigma, T) = \frac{1}{n-1} \cdot \left(\frac{\sigma}{\sigma_{uT}} \right)^5 \quad (3.8b)$$

where $n=2.5$ for lightweight concrete and $n=3.0$ for normal concrete.

$$g(\sigma_F, T) = \begin{cases} 1 + \frac{\sigma_F}{\sigma_{u0}} \cdot \frac{T-20}{100} \dots\dots\dots \frac{\sigma_F}{\sigma_{u0}} \leq 0.3 \\ 1 + \frac{0.3(T-20)}{100} \dots\dots\dots \frac{\sigma_F}{\sigma_{u0}} > 0.3 \end{cases} \quad (3.9)$$

where σ_F is the compressive stress generated by external force before concrete is heated up.

$$\Phi(\sigma, T) = \begin{cases} g\varphi + \frac{\sigma}{\sigma_{uT}} \cdot \frac{T-20}{100} \dots\dots\dots \frac{\sigma}{\sigma_{uT}} \leq 0.3 \\ g\varphi + \frac{0.3(T-20)}{100} \dots\dots\dots \frac{\sigma}{\sigma_{uT}} > 0.3 \end{cases} \quad (3.10a)$$

$$\varphi = C_1 \tanh[\gamma_w(T-20)] + C_2 \tanh[\gamma_0(T-T_g)] + C_3 \quad (3.10b)$$

$$\gamma_w = (0.3w + 2.2) \times 10^{-3} \quad (3.10c)$$

where w is the moisture content, γ_0 , T_g , C_1 , C_2 and C_3 are constants defined in Table 3.2.

Table 3.2 Parameters for Schneider's model

Parameter	Dimension	Quartzite	Limestone	Lightweight
-----------	-----------	-----------	-----------	-------------

		Concrete	Concrete	Concrete
C_1	1	2.60	2.60	2.60
C_2	1	1.40	2.40	3.00
C_3	1	1.40	2.40	3.00
γ_0	$^{\circ}\text{C}^{-1}$	7.5×10^{-3}	7.5×10^{-3}	7.5×10^{-3}
T_g	$^{\circ}\text{C}$	700	650	600

Figure 3.2 Normalized stress-strain curve in Schneider's model

3.2.3 Khoury and Terro Model

Khoury et al. (1985a; 1985b; 1995) and Terro (1998) used a three term formulation for total strain. Their formulation is different from Schneider's model. In Khoury and Terro's model, thermal strain that develops during first heating under load consists of 'load-free', 'load-induced' and initial elastic components, as expressed in Eq. (3.11):

$$\epsilon_{tot} = LFTS + LITS + \epsilon_e^{20} \quad (3.11)$$

in which, ϵ_e^{20} is the elastic strain at 20°C . Within the elastic range, say $0.5\sigma_{u0}$,

ϵ_e^{20} approximates ϵ_σ^{20} , the mechanical strain. Each of these components has its distinct properties.

Load-free thermal strain (LFTS) is measured directly as the ‘total’ strain of an unloaded specimen. Basically, The LFTS can be considered as equal to free thermal expansion without any applied load. i.e.:

$$LFTS = \epsilon_{tot}^0 = \epsilon_{th} \quad (3.12a)$$

where superscript ‘0’ indicates the 0% stress level, subscript ‘tot’ indicates total measured strain.

Load-induced thermal strain (LITS) is evaluated indirectly as the difference between the ‘total’ strain of loaded and unloaded specimens. The LITS comprises all the load-induced strain components that developed during the first heating of concrete under load (e.g. transitional thermal creep (TTC), drying creep, ‘basic’ creep and changes in the ‘elastic’ components):

$$LITS = \epsilon_{tot}^0 - \epsilon_{tot}^\sigma - \frac{\sigma}{E_0} \quad (3.12b)$$

where superscript ‘ σ ’ indicates the stress level as a percentage of initial cold strength.

Khoury reported only the experimental results for different concretes. Later, Terro developed an empirical formula by curve fitting and by assuming the LITS to be a linear function of applied stress. The proposed equation is:

$$LITS(0.3\sigma_{u0}, T) = (43.87 - 2.73T - 6.35 \times 10^{-2}T^2 + 2.19 \times 10^{-4}T^3 - 2.77 \times 10^{-7}T^4) \times 10^{-6} \quad (3.12c)$$

This equation is related to a stress level equal to $0.3\sigma_{u0}$. For other stress levels,

the strain is determined using the following approximate equation:

$$LITS(\sigma, T) = LITS(0.3\sigma_{u0}, T) \times (0.032 + 3.226 \frac{\sigma}{\sigma_{u0}}) \quad (3.12d)$$

For gravel (siliceous) aggregate concrete Eq. (3.12c) is modified to:

$$LITS(0.3\sigma_{u0}, T) = 1.48 \times 10^{-6} (1098.5 - 39.21T - 0.43T^2) - 1.48 \times 10^{-9} (2.44T^3 - 6.27 \times 10^{-3}T^4 + 5.95 \times 10^{-6}T^5) \quad (3.12e)$$

This equation is for concrete containing 65% of aggregate by volume. For concrete with a volume fraction of aggregate V_a , the load-induced thermal-strain is given by:

$$LITS(\sigma, T)|_{V_a} = LITS(\sigma, T)|_{65\%} \frac{V_a}{0.65} \quad (3.12f)$$

where V_a is the volume fraction of aggregate in the concrete. It should be noted that all of the equations in the model for calculating LITS only apply to temperatures up to 590°C.

Due to the assumptions used in this model, the stress-strain curves are straight lines. Besides, although the slopes of these straight lines decrease with an increase of temperature, there are no descending branches when the stress reaches the maximum compressive stress.

3.2.4 Kodur's Model

In this model, a part of creep is implicitly taken into account in the stress-strain relationship. While the effects of load and thermal expansion are significant in the early stage, the effect of creep becomes pronounced in the later stage. This model does not mention about transient strain. The constitutive equation is expressed as:

$$\mathcal{E}_{tot} = \mathcal{E}_{th} + \mathcal{E}_{\sigma} \quad (3.13)$$

It is ambiguous whether transient strain is included or not. But according to his test

method (Cheng et al. 2004b), it is the author's view that transient strain is already included in ε_σ . Detailed explanation will be presented in Section 3.3 and 3.4.

For normal strength concrete:

$$\sigma = \begin{cases} \sigma_{uT} [1 - (\frac{\varepsilon - \varepsilon_{uT}}{\varepsilon_{uT}})^2] \dots \dots \dots \varepsilon \leq \varepsilon_{uT} \\ \sigma_{uT} \{1 - (\frac{\varepsilon_{uT} - \varepsilon}{3\varepsilon_{uT}})^2\} \dots \dots \dots \varepsilon > \varepsilon_{uT} \end{cases} \quad (3.14a)$$

$$\begin{cases} \sigma_{uT} = \sigma_{u0} \dots \dots \dots 0^\circ \text{C} < T \leq 450^\circ \text{C} \\ \sigma_{uT} = \sigma_{u0} [2.011 - 2.353(\frac{T - 20}{1000})] \dots \dots \dots 450^\circ \text{C} < T \leq 874^\circ \text{C} \\ \sigma_{uT} = 0 \dots \dots \dots T > 874^\circ \text{C} \end{cases} \quad (3.14b)$$

$$\varepsilon_{uT} = 0.0025 + (6.0T + 0.04T^2) \times 10^{-6} \quad (3.14c)$$

For high strength concrete:

$$\sigma = \begin{cases} \sigma_{uT} [1 - (\frac{\varepsilon_{uT} - \varepsilon}{\varepsilon_{uT}})^H] \dots \dots \dots \varepsilon \leq \varepsilon_{uT} \\ \sigma_{uT} \{1 - [\frac{30(\varepsilon_{uT} - \varepsilon)}{(130 - \sigma_{uT})\varepsilon_{uT}}]^2\} \dots \dots \dots \varepsilon > \varepsilon_{uT} \end{cases} \quad (3.15a)$$

$$\begin{cases} \sigma_{uT} = \sigma_{u0} [1.0 - 0.003125(T - 20)] \dots \dots \dots 0^\circ \text{C} < T \leq 100^\circ \text{C} \\ \sigma_{uT} = 0.75\sigma_{u0} \dots \dots \dots 100^\circ \text{C} < T \leq 400^\circ \text{C} \\ \sigma_{uT} = \sigma_{u0} [1.33 - 0.00145T] \dots \dots \dots T > 400^\circ \text{C} \end{cases} \quad (3.15b)$$

$$\varepsilon_{uT} = 0.0018 + (6.7\sigma_{u0} + 6T + 0.03T^2) \times 10^{-6} \quad (3.15c)$$

$$H = 2.28 - 0.012\sigma_{u0} \quad (3.15d)$$

Figs. 3.3 and 3.4 show the normalized stress-strain curve for normal strength concrete and high strength concrete.

Figure 3.3 Normalized NSC stress-strain curve in Kodur's model

Figure 3.4 Normalized HSC stress-strain curve in Kodur's model

3.2.5 Design Code Provisions for Stress-Strain Behavior

Full stress-strain-temperature relationships are only needed when a full elasto-plastic analysis is required to determine the fire performance of a concrete structure. When an 'end point' calculation is sufficient, a stress-strain curve that allows for some creep may be used. ENV 1992-1-2 and ENV 1994-1-2 give such a set of curves providing data for a stress-strain relationship that may be used for the

analysis of concrete sections. The analytical form of the curve is the same as that in Schneider's model with $n = 3$. The stress-strain relationships are presented in Fig. 3.5. The constitutive equation of this model takes the same form as Kodur's model (i.e., $\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{\sigma}$). The stress-strain relationships of concrete at different temperatures are shown in Fig. 3.6. The parameters used to define the stress-strain relationships are given in Table 3.3.

$\varepsilon \leq \varepsilon_{uT}$	$\sigma = 3\sigma_{uT} \cdot (\varepsilon / \varepsilon_{uT}) / (2 + (\varepsilon / \varepsilon_{uT})^3)$
$\varepsilon_{uT} < \varepsilon \leq \varepsilon_{ult}$	For numerical purposes a descending branch should be adopted. Linear or nonlinear models are permitted.

Figure 3.5 Stress-strain curves in Eurocode

Figure 3.6 Stress-strain curves at different temperatures

Table 3.3 Values for the parameters of the stress-strain relationships of concrete in compression at elevated temperatures

Temperature °C	Siliceous Aggregates			Calcareous Aggregates		
	$\sigma_{uT} / \sigma_{u0}$	ϵ_{uT}	ϵ_{ult}	$\sigma_{uT} / \sigma_{u0}$	ϵ_{uT}	ϵ_{ult}
20	1.00	0.0025	0.0200	1.00	0.0025	0.0200
100	1.00	0.0040	0.0225	1.00	0.0040	0.0225
200	0.95	0.0055	0.0250	0.97	0.0055	0.0250
300	0.85	0.0070	0.0275	0.91	0.0070	0.0275
400	0.75	0.0100	0.0300	0.85	0.0100	0.0300
500	0.60	0.0150	0.0325	0.74	0.0150	0.0325
600	0.45	0.0250	0.0350	0.60	0.0250	0.0350
700	0.30	0.0250	0.0375	0.43	0.0250	0.0375
800	0.15	0.0250	0.0400	0.27	0.0250	0.0400
900	0.08	0.0250	0.0425	0.15	0.0250	0.0425
1000	0.04	0.0250	0.0450	0.06	0.0250	0.0450
1100	0.01	0.0250	0.0475	0.02	0.0250	0.0475
1200	0.00			0.00		

3.3 Material Models of NSC

3.3.1 Classification of Material Models

Up till now, a brief review of the most commonly used material models for concrete at high temperature is presented. A summary comparison of these material models is given in Table 3.4. It should be noted that creep strain component is ignored in all the material models. The most obvious difference among these models lies in transient strain. Some models include it, while the others do not.

Table 3.4 Summary comparison of material models

Models	Transient strain	Constitutive equation	Preload Effect	Stress-strain curve	
				Ascending branch	Descending branch
Anderberg	Explicitly included (Explicit Model)	$\epsilon_{tot} = \epsilon_{th} + \epsilon_{\sigma} + \epsilon_{tr}$	N.A.	Nonlinear curve	Straight line, independent of temperature
Schneider			Considered	Nonlinear curve	Nonlinear curve, almost independent of temperature
Khoury	ϵ_{lts} includes transient strain and increment of mechanical strain	$\epsilon_{tot} = \epsilon_{th} + \epsilon_{\sigma}^{20} + \epsilon_{lts}$	N.A.	Straight line	N.A.

Kodur	Implicitly included (Implicit Model)	$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{\sigma}$ ($\varepsilon_{\sigma} = \varepsilon_{full}$)	N.A.	Nonlinear curve	Nonlinear curve, dependent on temperature
Eurocode			N.A.	Nonlinear curve	Nonlinear curve, dependent on temperature

Since all these models have been verified by various tests, it is reasonable to believe that all the models should be correct and therefore they should yield similar results in spite of their different forms. Total strains ε_{tot} in all the models have the same physical meaning, which is the final strain of a concrete specimen in a material test. By comparing the constitutive equations in Table 3.4, clearly, three strain components are present in Anderberg or Schneider's model (ε_{th} , ε_{σ} and ε_{tr} but ε_{cr} is neglected), and only two strain components in Kodur's and Eurocode model (ε_{th} and ε_{σ}). Since free thermal strain ε_{th} is only determined by temperature increment, ε_{th} is the same for all models at the same temperature increment. Based on the discussion above, the author assumes that the 'mechanical strain' ε_{σ} in Kodur's or Eurocode model is the summation of mechanical strain ε_{σ} and transient strain ε_{tr} in Anderberg or Schneider's model. For convenience, from now on, the 'mechanical strain' ε_{σ} in Kodur's or Eurocode model will be called 'full strain' ε_{full} to differentiate from the mechanical strain ε_{σ} in Anderberg or Schneider's model. Stress-strain curves in Kodur's or Eurocode model will be called full stress-strain curves. Anderberg or Schneider's model that explicitly includes transient strain will be called explicit material model. On the other hand, material model such as Kodur's and Eurocode model, which implicitly

includes transient strain, will be called implicit material model. It can also be seen from Table 3.4 that load-induced thermal strain ϵ_{lis} in Khoury's model actually include transient strain and the increment part of mechanical strain as temperature increases from 20°C to T . The relationship can be expressed as: $\epsilon_{\sigma} + \epsilon_{tr}$ (Anderberg and Scheider's model) = ϵ_{full} (Kodru's and Eurocode model) = $\epsilon_{lis} + \epsilon_{\sigma}^{20}$ (Khoury's model) = $\epsilon_{tot} - \epsilon_{th}$ (All models).

However, as mentioned in section 3.2, transient strain not only depends on the final stress and final temperature, but also on the stress and temperature history. From Eqs. 3.5 and 3.10, transient strain is expressed as a function of σ and ΔT in Anderberg or Schneider's model. Transient strain remains constant with increasing stress and constant T ($\Delta\sigma$ and T), and only changes when stress is present while T is increasing (σ and ΔT). However, in Kodur's or Eurocode model, full strain is expressed as a function of T and σ . No matter which loading and heating scenario it is (loading with constant temperature, or, heating with constant stress), full strain always changes. Since full strain is comprised of transient strain and mechanical strain, it is difficult to tell whether the change comes from transient strain, mechanical strain or even both. But actually, there is no change of transient strain in loading with constant temperature ($\Delta\sigma$ and T) by definition. Fig. 3.7 shows the relationship between transient strain and other strain components. Preload effect, i.e., increased elastic modulus due to a preload prior to heating up, is shown as well. On the right of the ordinate axis, strain is in compression or contraction. On the left, strain is in tension or expansion. Creep strain is neglected in this figure.

Figure 3.7 Strain components of concrete at high temperatures

There are two heating-loading scenarios, one is along 1-5-6, and the other one along 1-2-3-4. These two heating-loading processes are shown in Fig. 3.8 and 3.9, respectively. Path 1-5-6 is heating up first. Temperature is increased from point 1 to point 5 to get free thermal strain. Loading is applied from point 5 to point 6. Temperature is kept constant during this period. This test cannot consider the effect of increased elasticity due to preload. The horizontal displacement from point 5 to point 6 is just the mechanical strain at isothermal temperature T . The curve 5-6 is shifted from point 6 to point 3 as a dash line to facilitate a comparison between these two paths. Path 1-2-3-4 is applying loading first at 20°C, then heating up from point 2 to 3 and finally unloading at that constant temperature. Compressive force is applied first from point 1 to point 2 at 20°C. From 1-2, mechanical strain at room temperature is obtained. Then loading is kept constant with increased temperature from point 2 to point 3. Compression strain is increased in this stage. Finally, from

3-4, specimen is unloaded at constant temperature T . Mechanical strain at this temperature can also be obtained from the horizontal displacement between point 4 and point 3.

Figure 3.8 Test method of Path 1-2-3-4

Figure 3.9 Test method of 1-5-6

To obtain transient strain, two specimens are needed. The first specimen is tested along path 1-5-6 to obtain mechanical strain ϵ_{σ}^{5-6} (superscript '5-6' means ϵ_{σ} is obtained from abscissa on path 5-6) at elevated temperature and free thermal strain ϵ_{th} from path 1-5, and then another specimen is tested along path 1-2-3 to obtain total strain ϵ_{tot}^3 (superscript 3 denotes the final state at point 3) from the horizontal displacement between point 1 and 3. Transient strain can be obtained from $\epsilon_{tot} - \epsilon_{\sigma}^{5-6} - \epsilon_{th}$. If the second specimen is tested along path 1-2-3-4, at point 4, a different mechanical strain ϵ_{σ}^{3-4} (superscript '3-4' means ϵ_{σ} is obtained from

abscissa on path 3-4) can also be obtained. Transient strain can be obtained by the same method: $\varepsilon_{tot} - \varepsilon_{\sigma}^{3-4} - \varepsilon_{th}$. However, in the second test, since loading is applied before heating up, elastic modulus of concrete is higher than that along path 1-5-6, and mechanical strain $\varepsilon_{\sigma}^{3-4}$ is smaller than $\varepsilon_{\sigma}^{5-6}$. The two alternate methods yield different transient strain due to different mechanical strain $\varepsilon_{\sigma}^{5-6}$ and $\varepsilon_{\sigma}^{3-4}$, which is caused by pre-load effect.

It is also shown in Fig. 3.7, that LITS (load-induced thermal strain) from Khoury's model is actually the summation of transient strain and the increment of mechanical strain from 20°C to T . The increment is due to the reduction of elastic modulus. Stress is held constant. As elastic modulus reduces, elastic strain increases. It should be noted that plastic strain is ignored in Khoury's model.

The two paths are compared to illustrate the obtaining of transient strain by different test methods. In Fig. 3.7, all strain components except free thermal strain are in compression. Transient strain, or load-induced thermal strain in Khoury's model, or full strain in Kodur's and Eurocode model has to counteract the free thermal strain (in expansion) to make the final total strain in compression. It can also be seen from the figure that although concrete becomes ductile when heated up, the stress-strain curve at high temperature has a greater slope if preload exists. Even different pre-load levels may influence the stress-strain relationships markedly. Many researchers have conducted tests to investigate this feature but this phenomenon is not fully understood yet. Some researchers think pre-load effect may be non-conservative. The most commonly used stress-strain relationship in many FEM programs is the heating without preload case because it usually results

in a greater reduction in strength with temperature than those with preload. Furthermore, as temperature increases, the reduction of elastic modulus becomes greater and greater; the preload effect becomes less and less significant. Considering these reasons, the pre-load effect is ignored in this research. When the pre-load effect is ignored, the two mechanical strains $\varepsilon_{\sigma}^{5-6}$ from path 1-5-6 and $\varepsilon_{\sigma}^{3-4}$ from 1-2-3-4 become equal, or their difference is negligible. Thus, the author concludes that transient strain is actually the strain difference of the two test specimens at the same final state (same σ and same T). When the same final state is reached along two different paths: 1-5-6 and 1-2-3, there are two different final strains: one at point 6 and the other one at point 3. Both strains are total strains. Strain ε at point 6 is ε_{total}^6 , which is the horizontal displacement from point 1 to point 6. Since along path 1-5-6, there is no transient strain, $\varepsilon_{total}^6 = \varepsilon_{\sigma} + \varepsilon_{th}$. ε at point 3 ε_{total}^3 is total strain as well. But along path 1-2-3, there is transient strain, $\varepsilon_{total}^3 = \varepsilon_{\sigma} + \varepsilon_{th} + \varepsilon_{tr}$. Transient strain accounts for the difference between the two total strains ε_{total} corresponding to two different heating-loading processes.

In Eurocode model, all the temperature-related stress-strain curves were obtained from non-steady-state creep tests, which were conducted with sufficiently large numbers of different temperature and load levels (Hosser et al. 1994). These so-called non-steady-state creep tests were carried out such that the tension or compression specimens were mechanically loaded before being heated to the same temperature. The initial stress was kept constant in each test. Strains were recorded as total strain. Free thermal strain was subtracted from total strain to obtain thermal stress-induced strain. By changing the preload level, different strains corresponding to different stress levels were obtained. The stress-strain curve at this temperature

was constructed by connecting these 'stress-producing strains'. The process is detailed in Fig. 3.10. The process is repeated at different temperature levels, and then the whole family of stress-temperature-strain curves is constructed. These strains implicitly include transient strain and some creep strain. Since the model is formulated without reference to time, the load-bearing behavior of a structure can also be determined directly, without analyzing the preceding times. This is a helpful consideration when using these stress-strain curves in simplified calculations. The constitutive equation of Eurocode model has the same format as Kodur's model. Moreover, values of the 'stress-producing strains' in Eurocode model are close to values of the full strain in Kodur's model (this will be shown in section 3.3.2). Therefore, the 'stress-producing strains' are called full strain.

Figure 3.10 Process of non-steady-state creep tests

It is the author's view that through this test method, there is a pre-load effect

which will cause a higher elastic modulus. However, at each point (e.g., point 1, 2 and 3 in Fig. 3.10) the strain is the sum of transient strain and mechanical strain (with pre-load effect) and is greater than mechanical strain (without pre-load effect) alone. By connecting all these points together, the new curve has a lower elastic modulus because at each point strain is greater. This makes it more conservative.

Up till now, the definitions of transient strain and full strain are introduced. It is the author's opinion that when mechanical strain is subtracted from full strain, the remainder is transient strain (preload effect is ignored so that mechanical strain is not related to test method). Their relationship is shown in Fig. 3.11. At 20°C, a specimen is loaded to point A. Then the specimen is heated up to temperature T under a constant stress. In this stage, strain increases from A to C just as in Fig. 3.7 from point 2 to 3. Now use another specimen and directly heat up to temperature T without any loading. After the target temperature is reached, the second specimen is loaded to the same stress level as the first specimen. The compression strain due to the loading is the horizontal distance between the origin and point B as shown in Fig. 3.11. The counterpart of point B in Fig. 3.7 is point 6. The stress-strain curve at T (w/o preload) in Fig. 3.11 corresponds to curve 5-6 in Fig. 3.7. When the mechanical strain at T is removed, the remaining BC is transient strain.

Figure 3.11 Transient strain

Before deducing a formula to calculate transient strain, some assumptions are needed. Plastic strain is assumed to be less affected by temperature or the influence of T is almost negligible. This is reasonable. For example, in the stress-strain equation $\sigma = 3\sigma_{uT} \cdot (\varepsilon / \varepsilon_{uT}) / (2 + (\varepsilon / \varepsilon_{uT})^3)$, σ_{uT} is the peak strength at temperature T , and ε_{uT} is the corresponding strain. When stress $\sigma = 0.3\sigma_{uT}$, elastic strain $\varepsilon_e = 0.2\varepsilon_{uT}$, plastic strain $\varepsilon_p = 0.0008\varepsilon_{uT}$, i.e., ε_p is only 0.4% of ε_e ; when stress $\sigma = 0.5\sigma_{uT}$, elastic strain $\varepsilon_e = 0.33\varepsilon_{uT}$, plastic strain $\varepsilon_p = 0.02\varepsilon_{uT}$, ε_p is only 6% of ε_e . Since plastic strain only accounts for a very small proportion of elastic strain, even if plastic strain is affected by temperature, relative to elastic strain, the increase in ε_p is insignificant. Changes of total strain come from free thermal strain, increment of elastic strain due to reduction of elastic modulus and transient strain, as shown in Fig. 3.12. In this figure, stress-strain curve at T (w/o preload) is obtained by path 1-5-6, i.e., heating up first, then applying loading. Full stress-strain curve at T is from Kodur's or Eurocode model. It should be noted that the point A, B and C in Fig. 3.12 is the same as their counterparts in Fig. 3.11. Consider Point A on stress-strain curve at 20°C with corresponding stress σ . Temperature is increased from 20°C to T while stress is kept constant. Then point A moves to point C as in Fig 3.7 from point 2 to 3. The portion AB corresponds to an increase of elastic strain due to reduced elastic modulus at T , and BC is transient strain. Their relation can be expressed as:

$$\varepsilon_{tr} = \varepsilon_{tot}^T - \varepsilon_{tot}^{20} - \varepsilon_{th} - \left(\frac{\sigma}{E_T} - \frac{\sigma}{E_{20}} \right) \quad (3.16)$$

In this formula, E_T is the elastic modulus at temperature T obtained from the mechanical stress-strain curve. Fig. 3.11 and Fig. 3.12 are very similar, the former

shows the composition of full strain by transient strain and mechanical strain. Based on the composition, Fig. 3.12 is used to deduce a formula to calculate transient strain.

Figure 3.12 Derivation of transient strain

Using this method, it is very convenient to retrieve transient strain from total strain, the latter can be easily obtained from test. This method can be applied to both NSC and HSC. Later, this method will be applied to Kodur's implicit HSC material model to derive transient strain for HSC at high temperature (Eq. 3.15a~d).

3.3.2 Transient Strain & Full Strain

In section 3.3.1, the material models are compared and discussed on a theoretical level. In this section, a specific concrete specimen will be used to compare these models in order to support the assumptions and statements the author made in section 3.3.1. The purpose of the comparison is to illustrate that full strain in Kodur and Eurocode's model is the summation of mechanical strain and transient strain in Anderberg and Schneider's model (see Table 3.4).

In order to compare the models introduced in section 3.2, reduction factors for initial tangent modulus, E , as recommended by CEB-FIP (1991) are used. For the peak compressive stress, σ_{uT} , values given in Eurocode for siliceous concrete are used.

$$E(T) = \begin{cases} E_0 & \dots\dots\dots T \leq 60^\circ C \\ 1 - \frac{0.9(T-60)}{640} & \dots\dots\dots 60 \leq T \leq 700^\circ C \end{cases} \quad (3.17)$$

$\sigma_{u0} = 30MPa$ and $\varepsilon_{u0} = 0.0025$ are initial compressive strength and peak strain at room temperature, respectively. Free thermal strain ε_{th} is calculated from the equation in Eurocode for siliceous concrete. This term is not involved in the comparison study. The following constants are assumed: $k_{tr} = 2.35$ in Anderberg's model and $C_1 = 2.6$, $C_2 = 1.4$, $C_3 = 1.4$, $\gamma_0 = 0.0075$, $T_g = 700$, $w = 0.3$ in Schneider's model. For simplicity, creep strain in Anderberg's model and increase of elasticity due to preload in Schneider's model are ignored (i.e., $\varepsilon_{cr} = 0$ and $g(\sigma_F, T) = 0$).

Firstly, the mechanical stress-strain curves of Anderberg and Schneider's model, the elastic stress-strain curves in Khoury's model, and the full stress-strain curves in Kodur's and Eurocode model are shown in Fig. 3.13. Since in Khoury's model there is only elastic strain, the curves are straight lines. From the comparison, it is noted that at room temperature, all the five models yield similar curves due to the same material properties of concrete (i.e., compressive strength, peak strain and elastic modulus). However, as temperature is increased from $100^\circ C$ to $600^\circ C$, the curves begin to separate into two groups. The first group consists of curves from Anderberg, Schneider and Khoury's model, and the second group consists of

Kodur's and Eurocode model. In each group, the curves are very close to one another in the ascending branch, while the strain from the second group is larger than the strain from the first group. The gap between the two groups grows as temperature increases. Although in the descending branch, there is considerable divergence among these curves, this still confirms what the author has discussed in Section 3.3.1, that is, the mechanical strain component from Anderberg, Schneider and Khoury's model are 'real' mechanical strain. It should be noted that stress in Khoury's model is within the elastic range, say, $0.5 \sigma_{u0}$. Thus, plastic strain can be neglected. On the other hand, in Kodur's and Eurocode model, the full strain component includes mechanical strain and transient strain.

Figure 3.13 Comparisons of mechanical stress-strain curves of five models at various isothermal conditions

Secondly, curves from Kodur's and Eurocode model remain unchanged (still full stress-strain curves). Curves from Anderberg and Schneider's model are transformed to full stress-strain curves with the addition of explicit transient strain. Curves from Khoury's model are also changed to full stress-strain curves by adding LITS to elastic strain at 20°C (LITS is the summation of transient strain and the increment of elastic strain. See section 3.3.1 for more information). All the new curves are compared again in Fig. 3.14. Because in the second comparison, full stress-strain curves are used for all the models, these curves should be more close to one another, equal to the subtraction of free thermal strain (in expansion) from total strain (see Fig. 3.7). This can be expressed as: $\epsilon_{tot} - \epsilon_{th} = \epsilon_m + \epsilon_{tr}$ (in Anderberg or Schneider's model) $= \epsilon_e^{20} + \epsilon_{lits}$ (elastic strain at 20°C plus load-induced thermal strain in Khoury's model) $= \epsilon_{full}$ (full strain in Kodur's and Eurocode model).

Figure 3.14 Comparisons of full stress-strain curves of five models at various isothermal conditions

Due to the assumptions used in Khoury's models, stress-strain curves provided by his model are straight lines representing elastic strains at room temperature plus load-induced thermal strain at high temperatures. Besides, although the slopes of Khoury's straight lines decrease with an increase of temperature, there is no descending branch when the stress reaches the maximum compression. In contrast, the models proposed by Schneider and Anderberg not only provide nonlinear stress-strain relationships but also account for the descending branches after the stress has reached the maximum compressive stress. In Fig. 3.13 and Fig. 3.14, all the loading curves are very close in ascending branch at room temperature, but major differences in descending branches are found. Since there are no experimental data published in the literature for describing the descending branches it is difficult to judge which model is more accurate.

It is noticed from Fig. 3.14 that, there are growing differences in the ascending branch among different curves as temperature increases. However, compared to

Fig.3.13, the difference is relatively less. This confirms what the author has discussed, that is the ‘mechanical strain’ component in Kodur’s and Eurocode model is the full strain, which is the summation of the mechanical strain and transient strain in Anderberg and Schneider’s model. Consider there are two different concrete specimens with the same final state, i.e., the same final stress and temperature. But the path to reach this final state is different, namely, one is stressed first and then heating up; the other is stressed after being heated up. Although the two concrete specimens reach the same final state, their deformations are totally different as their strains are different. Therefore, transient strain is defined as the strain difference between two heating and loading sequences applied to a column, namely, one column is loaded before heating, and the other one loaded after heating. Following this definition, Eq. 3.16 can be used to obtain transient strain from full strain provided in Kodur’s and Eurocode model.

The loading curves provided by Anderberg and Khoury are very close when the temperature is below 400°C and give higher values of strain than the other models. Kodur’s model and Eurocode produce the smallest strains within this temperature range. But when the temperature is raised beyond 400°C, full strain provided by Schneider’s model becomes far greater compared to the other models, while Anderberg’s model gives lower values of strain at the same stress level. The loading curves in Kodur’s model and Eurocode are reasonably close for the temperature range considered. The peak strain in Khoury’s model fits very well to Eurocode especially above 400°C. In the descending branch the stress reduction in Anderberg’s model is generally slower than the other models.

It is also seen from Fig. 3.14 that, with an increase of temperature, the nonlinearity of the loading curves decreases and the slope of the descending branch increases. This is attributed to the contribution of transient strain (Li and Purkiss 2005). The transient strain is a linear function of stress. At elevated temperatures the dominant strain is the transient strain and this is why the stress-strain curve in the ascending part becomes almost linear. On the other hand, once stress exceeds the peak strength, it will decrease. However, even if compressive stress has reduced, transient strain will not reduce because it is irrecoverable. In Fig. 3.14, transient strain is calculated from the corresponding stress level no matter whether the stress is before or after the peak strength. The actual transient strain after the peak strength must be greater than the one before peak strength, which is equal to the calculated transient strain. This is why the curves in Fig. 3.14 drop sharply when stress is beyond the peak strength. In actual case, the gradient should be gentler as strain increases, just like the curves in Eurocode model. Unfortunately, there are no available experimental data to validate this strain behavior as it will be difficult, if not impossible, to carry out experimental verification.

Fig. 3.15 shows transient strain calculation in Anderberg and Schneider's model and load-induced thermal strains in Khoury's model for different temperatures. Transient strain is a linear function of stress and it increases with temperature according to their assumptions. Transient strain in Schneider's model increases more rapidly with temperature than the other two models. It is obvious according to Fig. 3.7, that LITS from Khoury's model is the largest one, because it is the summation of transient strain and increment of elastic strain. This can be seen from Fig. 3.15, although at 700°C it is exceeded by Schneider's model. The reason

is that the preload effect in Schneider's model is not included. The preload effect increases the elastic modulus and compressive strength, which lead to a significant reduction of mechanical strain and transient strain. This will be discussed next.

Figure 3.15 Comparisons of transient strains (LITS in Khoury's model) of three models

So far, the effect of preload has not been included in the above discussions. In reality, structural members are always stressed first before a fire occurs. The increase in elasticity due to external load before heating leads to a smaller mechanical and transient strain. In Fig. 3.14 and 3.15, strain prediction by Schneider's model will become lower and closer to the other models when the preload effect is considered. For better understanding, three preload cases are considered, namely, $0.2f'_c$, $0.4f'_c$ and $0.6f'_c$, respectively. These preloads are

kept constant during the heating stage. The increase of strain with temperature is plotted in Fig. 3.16.

Figure 3.16 Comparisons of full strains of five models for various preload

As shown in Fig. 3.16, full strain in Schneider's model reduces significantly as the preload level increases. Khoury's model always gives the highest value of strain among the five models. Within the elastic range, say at a load level less than 0.5 of the strength limit at room temperature as in most cases, all these models produce similar results than the one $\sigma_F = 0.6f'_c$ (Fig. 3.16c) which is beyond the elastic range. Once again, this confirms the author's view that the summation of mechanical strain and transient strain (or Load-induced thermal strain in Khoury's model) in Anderberg and Schneider's model is equal to the full strain in Kodur's and Eurocode model. Transient strain is the strain component, which allows for different deformations between two cases with the same final state but different loading-heating paths.

3.3.3 Conclusion

Based on the comparison presented in Section 3.3.1, the nature of transient strain has been investigated. Differences among the material models of NSC have been clarified. A useful equation (Eq. 3.16) is deduced for calculating transient strain. Therefore, transient strain of HSC can be calculated through this way. Based on Kodur's implicit HSC model, full strain can be divided into transient strain component and mechanical strain component. As a result, an explicit HSC model can be established.

Creep effect is assumed negligible in this research. In fact, of the different strains, creep strain can be very significant just prior to failure of a member (last few minutes of fire exposure). It is true that creep strain may become very large

even for an hour long duration. The testing process to obtain stress-strain curves in Eurocode or Kodur's model normally takes hours (refer to Fig. 3.7 and Fig. 3.10 in Section 3.3.1). Hence, creep strain inevitably develops during the process. The full strain in these models includes not only transient strain and mechanical strain, but also some creep strain which develops during the testing process. When the author uses his method to calculate induced transient strain from Kodur's model, creep strain taking place in the early hours is already included. That means transient strain and mechanical strain obtained from the author's method are greater than the actual transient strain and mechanical strain because of the inclusion of creep strain. Since transient strain and mechanical strain are both affected by elastic modulus (refer to Eq. 3.16 in Section 3.3.1), and elastic modulus are affected by creep strain as well, it is hard to extract actual creep strain from transient strain defined in this way. Although these two strain effects cannot be clearly distinguished in the proposed model, the total amount of strain is the same.

3.4 Implicit and Explicit Models of HSC

Since there are limited number of material models for HSC, Kodur's implicit HSC material model is chosen as the starting point of the author's work.

3.4.1 Material Models

Stress-strain curves of HSC at elevated temperature were developed by Kodur, etc. (Kodur et al. 2004). The strain component in these stress-strain curves is the so-called 'full strain', which implicitly includes transient strain and mechanical

strain. Kodur's material model cannot be used where axial force in a column is not constant (Please refer to Section 4.2.4 for detailed explanation). To analyze structural behavior of HSC members with axial restraint at elevated temperature, transient strain must be obtained from 'full strain'. Eq. 3.16 and Eq. 3.15 are reproduced here for ease of reading. Eq. 3.16 is applied to separate transient strain from full strain:

$$\varepsilon_{tr} = \varepsilon_{tot}^T - \varepsilon_{tot}^{20} - \varepsilon_{th} - \left(\frac{\sigma}{E_T} - \frac{\sigma}{E_{20}} \right) \quad (3.16)$$

ε_{tot}^T , ε_{tot}^{20} is the full strain at temperature T and 20°C , respectively, and can be calculated from Kodur's HSC stress-strain curves as Eq. 3.15:

$$\sigma = \begin{cases} \sigma_{uT} \left[1 - \left(\frac{\varepsilon_{uT} - \varepsilon}{\varepsilon_{uT}} \right)^H \right] & \dots \varepsilon \leq \varepsilon_{uT} \\ \sigma_{uT} \left\{ 1 - \left[\frac{30(\varepsilon_{uT} - \varepsilon)}{(130 - \sigma_{uT})\varepsilon_{uT}} \right]^2 \right\} & \dots \varepsilon > \varepsilon_{uT} \end{cases} \quad (3.15a)$$

$$\begin{cases} \sigma_{uT} = \sigma_{u0} [1.0 - 0.003125(T - 20)] & \dots 0^\circ\text{C} < T \leq 100^\circ\text{C} \\ \sigma_{uT} = 0.75\sigma_{u0} & \dots 100^\circ\text{C} < T \leq 400^\circ\text{C} \\ \sigma_{uT} = \sigma_{u0} [1.33 - 0.00145T] & \dots T > 400^\circ\text{C} \end{cases} \quad (3.15b)$$

$$\varepsilon_{uT} = 0.0018 + (6.7\sigma_{u0} + 6T + 0.03T^2) \times 10^{-6} \quad (3.15c)$$

$$H = 2.28 - 0.012\sigma_{u0} \quad (3.15d)$$

ε_{th} is free thermal strain. In Kodur's model coefficient of thermal expansion is calculated as:

$$\alpha = (0.008T + 6) \times 10^{-6} \quad (3.18)$$

For elastic modulus of HSC at elevated temperature, Eq. 3.17 from CEB-FIP (1991) can be used:

$$E(T) = \begin{cases} E_0 & \dots T \leq 60^\circ\text{C} \\ 1 - \frac{0.9(T - 60)}{640} & \dots 60 \leq T \leq 700^\circ\text{C} \end{cases} \quad (3.17)$$

Since deformation of NSC and HSC at elevated temperature is different, i.e.,

total strain is different, transient strain may be different as well. Therefore different equations should be used to calculate transient strain for NSC and HSC. However, for elastic modulus there are no significant differences for concrete with different strength grades. In fact, from Fig. 2.2, the difference of elastic modulus of NSC and HSC at elevated temperature is not so significant.

Three categories HSC (i.e., compressive strength at room temperature is 60MPa , 70MPa and 80MPa) with five pre-load levels (i.e., $0.1\sigma_{u0}$, $0.2\sigma_{u0}$, ..., $0.5\sigma_{u0}$) are used to obtain explicit transient strains under different temperatures from 20°C to 900°C . The loads are applied at room temperature, and then concrete is heated up to target temperature under the constant load. The results are shown in Fig. 3.17.

(a) whole temperature range

(b) 0~450°C temperature range

(c) 450~700°C temperature range

Figure 3.17 Transient strain of HSC from 20°C to 900°C

From Fig. 3.17, it can be seen that transient strain of HSC is proportional to external load level (ratio of external loading to compressive strength at room temperature), and transient strain increases with temperature. Here it is assumed that transient strain is independent of concrete compressive strength at room temperature but solely dependent on load level. As the load level increases, concrete fails at a lower temperature; only when the load level is $0.1 \sigma_{u0}$, can transient strain curve exceed above 800°C. All these features are similar to those in Anderberg or Schneider's model for NSC. However, transient strain of HSC is slightly different from that of NSC. In Anderberg's model, transient strain development is divided

into two stages, viz. before and after 550°C. However, Fig. 3.17 shows that the development of transient strain for HSC with temperature is a three-stage process. First, from 20°C to 400°C (Fig. 3.17(b)), the development of transient strain with temperature is relatively small and almost linear. From 400°C to 700°C (Fig. 3.17(c)), the development becomes more rapid. At the end of this stage, the increment of transient strain becomes highly nonlinear. This phenomenon is more obvious when the load level is high. In the third stage, the growth of transient strain reaches its maximum value.

Using Eq. 3.16, transient strain is calculated from total strain. However, in FEM analysis, the unknown is total strain. Transient strain must be calculated from other parameters. Since transient strain is found to be proportional to external load level and temperature increment, curve fitting is used to obtain transient strain from these parameters. The mathematical format of the formulae for transient strain in Anderberg's NSC model is adopted in this research program. The first reason is that, development of transient strain of HSC with temperature and load level in all three stages is basically linear. This complies with the transient strain formulae in Anderberg's NSC model. The second reason is that the mathematical format of transient strain in Anderberg's model is simple enough to be programmed. A similar mathematical expression will be used, i.e., a linear relationship with external load level and free thermal strain increment (or temperature increment). For simplicity, the nonlinear part of transient strain development at high load level is ignored, and will be approximated by a linear equation. The variations of transient strain with temperature and loading level ratio in the three stages are shown in Fig. 3.18(a) to (c). In (a) and (b), $K_{tr} = \varepsilon_{tr} / ((\sigma / \sigma_{u0}) / \Delta \varepsilon_{th})$; in (c), $K_{tr} = \varepsilon_{tr} / ((\sigma / \sigma_{u0}) / \Delta T)$.

Figure 3.18 Transient strain developments with temperature and load level

As in Anderberg's model with temperature less than 550°C , the coefficient K_{tr}

in stage 1 and stage 2 is obtained by dividing transient strain by load level σ/σ_{u0} and increment of free thermal strain $(\varepsilon_{th}^{T1} - \varepsilon_{th}^{T2})$. In stage 3, due to an accelerated temperature effect on transient strain change of temperature ΔT is used instead of increment of free thermal strain. This is similar to Anderberg's NSC model when temperature is above 550°C.

When temperature is less than 400°C, the mean value of K_{tr} is 1.76 with a maximum value of 2.01 and a minimum value of 1.56. The coefficient of variation is 7%. In stage 2, K_{tr} varies between 2.60 to 3.08 with a mean value of 2.81 and a coefficient of variation 9%. When temperature is higher than 700°C, there are limited points for K_{tr} due to concrete failure under a high load level. The average K_{tr} in this temperature region is 1.171e-4 with a coefficient of variation 12%. Thus the author proposed the following equations to describe transient strain of HSC:

For $T \leq 400^\circ C$

$$\varepsilon_{tr} = -1.76\left(\frac{\sigma}{\sigma_{u0}}\right)\varepsilon_{th} \quad (3.19a)$$

For $400^\circ C < T \leq 700^\circ C$

$$\varepsilon_{tr} = -2.81\left(\frac{\sigma}{\sigma_{u0}}\right)\varepsilon_{th} \quad (3.19b)$$

For $T \geq 700^\circ C$

$$\Delta\varepsilon_{tr} = -0.0001171\left(\frac{\sigma}{\sigma_{u0}}\right)\Delta T \quad (3.19c)$$

HSC transient strain under a combination of any load and any temperature can now be calculated from Eq. 3.19. When transient strain is extracted from full strain, the remaining component is mechanical strain. Using this method, Kodur's implicit material model for high strength concrete can be extended to an explicit model for transient strain. In the explicit model, the constitutive equation for mechanical

strain takes the same mathematical form as in Eurocode, because it accounts for mechanical stress-strain curves and is well developed. Normally, a mechanical stress-strain curve of concrete at room temperature is a second-order parabola (Eurocode 1995a). It is assumed that under high temperature, the curve is still a second-order parabola, with some parameters to be adjusted. The slope of descending branch of mechanical strain becomes flatter as strain increases (refer to Fig. 3.6). This ties in with the common observation of descending branch of stress-strain curve of concrete. The variation of peak compressive strength at high temperature follows the same rule as that in Kodur's model. The peak compressive strain at temperature T is obtained by subtracting transient strain at T with an external load level $\sigma_{uT} / \sigma_{u0}$ from full strain, which corresponds to 'peak compressive strain' at temperature T in Kodur's model. The mechanical stress-strain curve of concrete at different temperatures is expressed in Eq. 3.20, and the parameters at different temperatures are listed in Table 3.5.

$$\sigma = 3\sigma_{uT} \cdot (\varepsilon / \varepsilon_{uT}) / (2 + (\varepsilon / \varepsilon_{uT})^3) \quad (3.20)$$

Table 3.5 Parameters of stress-strain relationships of HSC
in compression at elevated temperatures

Temperature °C	$\sigma_{uT} / \sigma_{u0}$	ε_{uT}	
		Implicit model	Explicit model
20	1.00	0.0023	0.0023
100	0.75	0.0031	0.0027
200	0.75	0.0046	0.0032
300	0.75	0.0067	0.0041
400	0.75	0.0094	0.0051
500	0.61	0.0127	0.0058
600	0.46	0.0166	0.0097

700	0.32	0.0211	0.0140
800	0.17	0.0262	0.0182
900	0.03	0.0319	0.0220

Based on Kodur's material model, a new material model is established for HSC, which includes transient strain explicitly. The constitutive equation for this model is expressed as:

$$\varepsilon_{tot} = \varepsilon_{th} + \varepsilon_{\sigma} + \varepsilon_{tr} \quad (3.21)$$

Creep strain is ignored in this model. Free thermal strain can be calculated according to relevant standards codes, as Eq. 3.2. Mechanical strain and transient strain can be calculated from Eq. 3.20. and Eq. 3.19, respectively.

For better understanding, the implicit material model (Kodur's model) and the proposed explicit material model of HSC will be compared. The compressive strength of concrete at room temperature is 60 MPa . Full stress-strain curves of these two types of material models are shown in Fig. 3.19. Mechanical stress-strain curves from the explicit material model are also included in this figure. It should be noted that the differences between mechanical stress-strain curves of explicit model and full stress-strain curves of explicit model are termed as transient strain at the corresponding stress level.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 3.19 Comparisons of full stress-strain curves of implicit and explicit HSC material models

In Figure 3.19(a), there are only two curves at 20°C since there is no transient strain at room temperature. The two curves coincide with each other in ascending branch. In descending branch, a concave shape is adopted for the explicit model because this is closer to observations in experiments. For different isothermal conditions temperatures, mechanical stress-strain curves and full stress-strain curves are shown in Fig. 3.19. Transient strain, which is the horizontal difference between mechanical stress-strain curve and full stress-strain curve of explicit model, grows rapidly as temperature increases. The full stress-strain curves of implicit and

explicit model are very close to each other. This is self-evident since the explicit model is directly extended from the implicit model (Kodur's model). Although there is an obvious divergence in the descending branch, it is believed that the explicit model reflects a more realistic material behavior of concrete at high temperature.

3.4.2 Material Test

Some material tests of HSC at high temperature are used to verify the proposed explicit material model. Three series of material tests are collected from literature. These were conducted by Cheng et al. (2004b), Fu et al. (2005), and Castillo and Durrani (1990). The new material model is based on Kodur's HSC material model, which was developed from the test conducted by Cheng et al. (2004b), therefore this test series is chosen as reference. The other two tests are chosen because the ambient compressive strength of these test specimens is within the range of 60MPa ~ 80MPa , the same strength range from which the new material model is developed. All the specimens were made from ordinary Portland cement.

3.4.2.1 Series 1 (Cheng et al. 2004b)

Two types of HSC specimens were used in the test, namely, specimens made from siliceous and carbonate aggregate concrete. Two batches: TWN1 and TWN2 were prepared for each type. TWN1 was made of siliceous aggregates and TWN2 of carbonate aggregates. For each batch of specimens, there were 40 small cylinders of 100mm in diameter and 200mm in length, 10 large cylinders of 150mm in

diameter and 300mm in length. The specimen strength at 28 days was 75.5MPa and 74.5MPa. Granite was used for siliceous aggregate and limestone for carbonate aggregates. In all batches, general-purpose Portland cement for construction of concrete structures was used. The fine aggregates for all four batches consisted of silica-based sand.

The compression tests at room temperature were conducted on large cylinders (150×300mm) at 7, 28, and 91 days, respectively, for each type of concrete. The compressive strengths of concrete are given in Table 3.6. Afterwards, 100mm diameter cylinders of each concrete type were tested under high temperature. For each type of concrete and target test temperature, three specimens were tested and the average of these results was used as the final result. The target test temperatures were set to 100, 200, 400, 600 and 800°C of uniform temperature distribution, respectively. All tests were conducted under ‘hot conditions’. The cylindrical specimens were heated without any load and then they were loaded until failure. Therefore the stress-strain curves obtained from this test were mechanical strain only, excluding transient strain.

Table 3.6 Compressive strength of high strength concrete specimens

	TWN 1	TWN 2
7 Days	67.8	60.5
28 Days	75.5	74.5
91 Days	79	78.3

The stress-strain curves obtained from the test are normalized and compared with mechanical stress-strain curves from the proposed explicit model in Fig. 3.20.

Figure 3.20 Stress-strain curves (Cheng et al. 2004b) of HSC

It can be seen from this figure that the explicit material models are more

accurate at higher temperature ($\geq 400^{\circ}\text{C}$). When temperature is lower than 400°C , mechanical strain from the explicit material model is smaller than measure strains.

3.4.2.2 Series 2 (Fu et al. 2005)

Three batches of concrete with different mix proportions were prepared in this program. The mix proportions are shown in Table 3.7. The compressive strength at ambient temperature was 76MPa , 72MPa and 75MPa , respectively. Ordinary Portland cement complying with BS12 was used as the principal binder. Commercially available fly ash and one type of metakaolin were used as mineral additives to replace part of cement. The specimens were cylinders with 75mm in diameter and 150mm in length. They were cast according to ASTM C192C/192M-95. For each set of tests two specimens from the same batch were used to determine the mechanical properties at different constant temperatures (room temperature, 100°C , 200°C , 400°C and 600°C). Both unstressed and stressed tests were performed, but as mentioned in Section 3.3.1, the effects of increased elastic modulus and strength due to pre-load are ignored. Hence, only the unstressed tests are considered here. In the unstressed tests, the specimens were heated up to the desired temperature and then maintained at that temperature for 60 minutes to attain a steady state condition. The specimens were then loaded until failure. Mechanical stress-strain curves from the proposed explicit model and test results are shown in Fig. 3.21. Please note the stress is normalized in this figure.

Table 3.7 Concrete mix proportions

Mix Proportions (kg/m^3)	HSC-PC	HSC-FA30	HSC-MK5
	w/c = 0.3	w/c = 0.3	w/c = 0.3

Cement (OPC)	500	350	475
Water (kg/m ³)	150	150	150
Crushed Granite (20mm)	712	682	710
Coarse Aggregate (10mm)	356	341	355
Fine Aggregate (sand)	750	750	750
Mineral (PFA)	-	150 (30%)	-
Addition (MK)	-	-	25 (5%)

Figure 3.21 Stress-strain curves (Fu et al. 2005) of HSC

When temperature is in a lower range (20°C to 200°C), the divergence between material model and test results is noticeable. At this temperature range, the divergence of test results for different concrete batches is also noticeable. However, the material model agrees better with test results when it is above 400°C.

3.4.2.3 Series 3 (Castillo and Durrani 1990)

In this series of tests, high strength concrete of 62.1MPa nominal strength was subjected to uniform temperature up to 800°C and then loaded to failure under axial compression. The specimens were made from ordinary Portland cement, natural river sand, and crushed limestone. A commercially available sulfonated naphthalene formaldehyde type superplasticizer was used to obtain high strength. Properties of the aggregate and the dry weight mix proportions are given in Table 3.8.

Table 3.8 Properties of aggregate and mix proportions

Coarse Aggregate	Unit Weight (lb/ft^3)	99.0
	Absorption (%)	0.1
	Fineness Modulus	2.86

Fine Aggregate	Unit Weight (lb/ft^3)	110.3
	Absorption (%)	0.85
	Fineness Modulus	2.51
Mix proportions	Water/Cement Ratio	0.327
	Proportions by Weight	1:1.54:2.31
	Cement (lb/yd^3)	786
	Superplasticizer ($fl\ oz/100\ lb\ of\ cement$)	14.7

The test specimens consisted of 2-inch diameter and 4-inch height ($51 \times 102\text{mm}$) cylinders. For each set of tests at a given temperature, three specimens from the same batch were tested at room temperature. The target temperatures varied from 100 to 800°C at 100°C increment. The tests were conducted in an ‘unstressed’ state.

(c)

(d)

(e)

(f)

(g)

(h)

Figure 3.22 Stress-strain curves (Castillo and Durrani 1990) of HSC (i)

From the three series of material tests, it can be seen that mechanical stress-strain curves from the proposed explicit model basically agree with test results. However, the divergence is more noticeable at lower temperature ($\leq 400^\circ\text{C}$). The explicit material model is insensitive to the mix proportion of concrete, it can be used to any type of HSC with acceptable tolerance.

3.5 Complete Stress-Strain Relationship

Since the new model only accounts for compression, to incorporate it into a FEM program, tension behavior, unloading path and failure criteria also need to be specified. Ideally, the material model should allow for occurrence of cracking and possibility of crack closure. Since these are beyond the scope of this research, rules recommended by Eurocode (1995a) is adopted:

1. Behavior during unloading from (or reloading to) a previously obtained value of compressive stress is linear with $d\sigma/d\varepsilon_\sigma|_{\varepsilon_\sigma} = 0$, that is, at the initial Young's modulus at temperature T .
2. Before the occurrence of cracking, behavior in tension is linear, and the slope of

the stress-strain curve is equal to $E(T)$.

3. The unloading branch on tension side always leads to origin or to ϵ_0 , which is the plastic strain depending on the stress history.
4. Cracking occurs if tensile stress reaches a critical value equal to the tensile strength of concrete.
5. Immediately after occurrence of cracking in any region, the stiffness of concrete in tension in that region is reduced to zero.
6. The occurrence of cracking in any region has no effect upon subsequent performance in compression.
7. Failure criteria recommend by Eurocode (1995a) is adopted (Table 3.3).

The parameters governing the complete stress-strain relationships are presented in Fig. 3.23. They lead to an idealized behavior of HSC material.

Determination of all parameters for a given temperature is described in section 3.2 and 3.4. The most serious deviation from test results arises from the unloading and reloading assumptions in compression zone. From experimental results, the assumption adopted will lead to an over-estimate of stiffness during unloading. To better represent the HSC stress-strain curves, the effects of temperature and time should be included in unloading. Furthermore, knowledge about the stress-strain relationship during cooling is essential to establish a complete theory. However, existing experimental data are insufficient. As an approximation it is reasonable to assume that the stress-strain curve used at the maximum temperature level remains invariant during a subsequent cooling. This implies that numerical results are likely to be relevant only during the initial stages of heating and cooling, before creep strain becomes significant or before material properties begin to recover. It is very

convenient that fires do in fact occur only in a very short period of time and are not repetitive. Besides, unlike steel material, concrete properties after subjected to temperature above 200°C (Phan 1996) do not recover upon cooling.

Figure 3.23 Complete stress-strain relationship for concrete

3.6 Summary

In this chapter, the nature of transient strain is investigated. By comparing some existing NSC material models, the author defines transient strain to be the strain difference between two cases with the same final stage (σ and T) but different test methods (heating first and loading first). Then an equation is developed to calculate transient strain from Kodur's implicit HSC model. A new HSC material model, which explicitly includes transient strain, is established. The proposed material model is verified by three series of HSC material tests at elevated temperature. A complete material model including tension and compression behavior, loading and unloading path, and failure criteria, is constructed based on the new model.

Next, this HSC model will be incorporated into an existing FEM program to study the structural behavior of axially-restrained HSC columns under fire. Existing implicit material models will also be incorporated into the program so that column fire resistance prediction based on implicit and explicit models will be compared and analyzed. Thus, significance of transient strain based on explicit and implicit material models will be revealed.

Chapter 4

Structural Model & FEM Program

4.1 Introduction

Up till now, we only consider how to calculate transient strain and how to develop an explicit material model. But the reason why transient strain should be

calculated explicitly has not been investigated. This issue will be addressed hereafter. Structural behavior of axially restrained columns under fire will be studied. For this purpose, an existing FEM program for steel structures (Huang (2002)) is extended to cater for concrete structures. Using this program, results from both implicit and explicit model will be compared to investigate the significance of transient strain. In FEM programs, a column is represented by a simplified model. The structural model includes boundary conditions (thermal and mechanical) and axial restraint. The structural model will be introduced first, followed by a theoretical analysis to show different results from implicit and explicit material models for an axially restrained column. For better understanding, different results from implicit and explicit models will be illustrated by some examples using the same FEM program. These examples include some structural tests and numerical problems. They are summarized in Table 4.1.

Table 4.1 Summary of Illustration Examples

Section	Case Study	Objective
4.4.1	NSC structural tests	Verify the FEM program for NSC columns
4.4.2	NSC numerical problems	Investigate structural response of axially restrained NSC column; Compare the difference between implicit and explicit NSC models

4.5.1	HSC structural tests	Verify the FEM program for HSC columns; Verify the proposed explicit HSC material model
4.5.2	HSC numerical problems	Investigate structural response of axially restrained HSC column; Compare the difference between implicit and explicit HSC models

4.2 Structural Model

4.2.1 Heating & Loading Process

In general, structural members subjected to fire conditions experience combined actions of varying stress and elevated temperature. Thus, loading-heating process can be simplified into two different processes, as shown in Fig. 4.1. One process is heating with constant loading, and the other one is loading with constant temperature. If increments in both processes are kept sufficiently small, it will better approximate the actual situation. Therefore, it is necessary to study the properties of concrete for these processes.

Figure 4.1 Loading-heating processes

In the heating process, a change of total strain of concrete includes increased mechanical strain due to a reduction of elasticity, free thermal strain and transient strain. In the loading process, free thermal strain and transient strain are constant since temperature is constant. Thus, variation of total strain is due to mechanical strain only, which is due to a change of stress. This is different from that in heating process. If the material model adopted implicitly includes transient strain, such as Kodur's or Eurocode model, these two processes make no difference. Full strain always changes because of the experimental process mentioned in section 3.3.1. In this type of model, full strain is only a function of temperature and stress, not related to actual heating and loading history. Therefore, full strain is only decided by the final stress and temperature. In this type of model, transient strain is not dependent on history. However, loading-heating process becomes very important when adopting material model which explicitly includes transient strain, such as Anderberg or Schneider's model. Different scenarios yield different transient strains. Transient strain comes about due to an increase in temperature with constant loading. Thus, transient strain is path-dependent.

Consider a column heated uniformly on four sides. Shortly after the start of heating, the column interior will be in tension and the material near the external heated surfaces will be in compression. Thermal gradient across section is exacerbated as the respective size of tension zone and compression zone changes with duration of heating. The occurrence of transient strain and cracking make this situation more complicated. All these factors are governed by external loading and a rise in temperature. The column will fail if the applied stress exceeds its strength at

a given temperature. Therefore, it is necessary to include all these features into this research program to yield satisfactory comparison with test results.

4.2.2 Equilibrium Equations & Assumptions

Consider a square column with thickness h , as shown in Fig. 4.2. It is subjected to a point load P and a couple M . These loadings are the combined effects of external force and restraint system specified by Eq. 4.1.

$$P = p + R_a h \epsilon_m \quad (4.1a)$$

$$M = m + P e \quad (4.1b)$$

where R_a is an axial restraint parameter (refer to Section 4.2.3 for detailed explanation); ϵ_m is longitudinal strain in the centroid of the mid-span cross section, e is the load eccentricity, subscript m indicates the location at the mid-plane of column; p and m are the respective imposed axial force and bending moment.

Figure 4.2 Details of a heated column

The finite element analysis is based on the following assumptions:

1. plane sections always remain plane during deformation;
2. column lateral deflections are small in comparison with its length;
3. longitudinal stress at any point in the column is dependent only upon longitudinal mechanical strain at that point;
4. property of the restraint system is known and it is temperature-independent;
5. deformations due to shear forces are negligible;
6. during the heating process, external force p and m remain unchanged.
7. there is no fire induced spalling in RC member.

According to the plane section assumption, longitudinal strain at any point of a cross section can be expressed as:

$$\varepsilon = \varepsilon_m + (y - h/2)\chi \quad (4.2)$$

The equilibrium condition can be expressed as:

$$\int_0^h \alpha dy - P = 0 \quad (4.3a)$$

$$\int_0^h \sigma(y - h/2) dy - M = 0 \quad (4.3b)$$

The relationship between stress σ and strain ε depends upon the constitutive model adopted. Mechanical strain can be separated from the total strain ε_{tot} as follows:

$$\varepsilon_\sigma = \varepsilon_{tot} - \varepsilon_{th} - \varepsilon_{tr} \quad (4.4a)$$

$$\sigma = f(\varepsilon_\sigma, T) \quad (4.4b)$$

In Eq. 4.4, f denotes a function determined by the state of material and temperature. The state of material indicates whether it is in compression or tension, loading or unloading, cracked or intact, etc.

4.2.3 Restraint Systems

In Eq. 4.1a, R_a denotes the stiffness of axial restraint. The axial restraint is provided by surrounding structures, which tends to resist the thermal expansion of a column when it is heated up. The axial restraint can be simplified as a linear elastic spring, whose stiffness is related to the stiffness of beams and floors above the heated column. Hence, the stiffness of axial restraint of a heated column increases with the number of floors above it. The force induced by axial restraint is proportional to its stiffness and the vertical end displacement of the heated column.

In order to calculate the force generated by axial restraint, its force-displacement relationship (stiffness) must be known. A typical force-displacement relation for a reinforced concrete beam-column joint is shown in Fig. 4.3 where it shows increasing displacement U with applied force F . The actual force-displacement diagram indicates only small deformations occurring prior to the concrete joint becoming nonlinear (Weeks 1985). As the maximum force is approached much larger deformations occur but the curve is still nearly linear between F_L and F_{max} . After the peak is reached the load cannot keep up with the displacement; this results in a typical falling branch. The falling branch behavior may be due to unloading material behavior, or inertia of the testing system, or a combination of both. To simplify the analysis, an ideal linear elastic spring model is used in this research, without modeling the falling branch.

Figure 4.3 Typical force-displacement relationship (Weeks 1985)

4.2.4 Theoretical Analysis of Structural System

Figure 4.4 shows a simplified model to represent the interaction between a simply supported column and its boundary condition, i.e., a linear spring of constant stiffness K_s . This structural model will be used in the extended FEM program. The stiffness of the spring is assumed to be independent of temperature rise since the beams and floors above the fire-exposed column are shielded from the fire below. The column is assumed to be uniformly heated on all four sides.

The column out-of-straightness is represented by an initial value α_i . Spring K_s follows the linear model according to Fig. 4.3 and is independent of temperature. The structure behaves in an elasto-plastic manner.

Figure 4.4 Structural model

First consider there is no axial restraint, i.e., R_a in Eq. 4.1 is set to zero. During heating process, P and M do not change with temperature. In this situation, σ at any point remains constant. The whole process only comprises heating with constant loading process. Transient strain keeps increasing during the whole process. In this situation, no matter either implicit or explicit material model is adopted, the results will be identical. But when axial restraint is not zero, as in actual situations, results from these two material models will be significantly different. Consider four processes. In the first process temperature is increased from T_1 to T_2 with a constant stress level σ_1 , then in the second process temperature is constant, stress is increased to σ_2 . The third process is a heating process from T_2 to T_3 with constant stress σ_2 . The fourth one is a loading process again and stress is up to σ_3 with temperature remaining at T_3 . The four processes are shown in Fig. 4.5.

Figure 4.5 The four processes

In implicit model, as discussed in Section 3.3.1, full strain is the summation of transient strain and mechanical strain ($\epsilon_{full} = \epsilon_{\sigma} + \epsilon_{tr}$). The full strain is directly calculated from the final temperature and stress, not related to any loading and heating history. Since at the end of the fourth process, stress is σ_3 and temperature is T_3 , the whole process is equivalent to a heating process from T_1 to T_3 with a constant stress level σ_3 (refer to Fig. 3.10 for the method to construct full stress-strain curves). Transient strain cannot be separated from full strain. But here it is assumed that transient strain among full strain can be calculated through Anderberg's formulae (not Anderberg's model). Thus, the transient strain in the four processes is given by:

$$\epsilon_{tr} = k \frac{\sigma_3}{\sigma_{u0}} (\epsilon_{th}(T_3) - \epsilon_{th}(T_1)) \quad (4.5a)$$

If explicit model is used instead, transient strain at the end of the fourth process is the summation of transient strain that comes about in each step. Anderberg's formulae are also used to calculate transient strain:

$$\epsilon_{tr} = k \frac{\sigma_1}{\sigma_{u0}} (\epsilon_{th}(T_2) - \epsilon_{th}(T_1)) + k \frac{\sigma_2}{\sigma_{u0}} (\epsilon_{th}(T_3) - \epsilon_{th}(T_2)) \quad (4.5b)$$

Since stress and temperature in each step is different, the two results from Eqs. 4.5(a) and 4.5(b) are very different. By definition, there is no transient strain in the

second and fourth process. However, if implicit model is used, transient strain is present in all the four processes. On the other hand, in explicit model, transient strain is only present in the first and third process. This complies with actual situation.

Form the above discussions, it can be concluded that implicit material model can only yield comparable result with explicit material model when there is no axial restraint. If axial restraint exists, implicit material model should not be used. Although explicit material model is difficult to use (or to be programmed) than the implicit one, it should yield reasonable results for both situations.

The argument presented so far is only a qualitative argument. It is abstract. To illustrate the effect of restraint system and transient strain on structural behavior of columns under fire, a FEM program is needed.

4.3 FEM Program

A FEM program will be extended based on an existing FEM program in this section to reveal the differences in fire resistant predictions between implicit and explicit models. Basic theories and assumptions about the FEM program are introduced first. The material model and structural model respectively developed in Chapter 3 and Section 4.2 will be incorporated into the program. This program will be applied to both NSC and HSC columns.

4.3.1 Introduction of the Computer Program

A FEM program based on FEMFAN (Version 2) is extended to predict the structural response of reinforced concrete columns at elevated temperature. FEMFAN (Version 2) is a nonlinear finite element program for analyzing 2-D steel frames at ambient and elevated temperatures. For ambient temperature analysis, a steel frame is subjected to monotonically increasing external loads until the structure fails. At elevated temperature, a frame is subjected to varying temperatures but constant external loads.

FEMFAN (Version 2) was written by Dr. Huang (2002) as part of his doctoral work. This is an elasto-plastic creep program. The solution is based on tangent stiffness approach. The program is written in FORTRAN 90 language and compiled on Compaq Visual FORTRAN 6.0 platform. FEMFAN is of small-strain large-displacement capability. The derivation of element stiffness matrix and the flow chart of FEMFAN are presented in Huang's Ph.D. thesis. Currently, for NSC, Eurocode model (implicit) and Anderberg's model (explicit) are incorporated. For HSC, Kodur's model (implicit) and proposed model in Chapter 3 (explicit) are incorporated. Uneven temperature distribution of a reinforced concrete cross section is implemented in this program. The extended program is called FEMFAN-CONC.

4.3.2 Finite Element Model

The proposed finite element model is applicable to 2D RC frames subjected to nodal loads. Besides those presented in Section 4.2.2, additional assumptions adopted in the FEM are:

1. External loads are placed at nodal positions only.
2. Only rectangular section is considered. The element is straight and prismatic.
3. The beam or column cross section is discretized into 225 sub-areas (See Fig. 4.6) to allow for uneven temperature distribution within a concrete section. Within each sub-area, temperature is assumed constant and represented by the temperature at its centroid. The black sub-areas can represent rebar or concrete. Circular cross section of a rebar is converted to an equivalent square with the same area.
4. No thermal gradient along the longitudinal direction.
5. Neutral axis is parallel to either Y or Z axis (Fig. 4.6).
6. Concrete models are listed in Table 4.2.
7. Complete stress-strain curve (including loading and unloading process) proposed in Section 3.5 is adopted for concrete. Failure criteria recommend by Eurocode (1995a) is adopted (Table 3.3).

Figure 4.6 Discretization of cross section

8. Material behavior of steel in tension is the same as that in compression.
Reinforcement material model recommended by Eurocode (1995b) is adopted.
9. Creep effect of concrete is ignored.
10. Spalling effect of concrete is not considered.

Table 4.2 Material models in FEMFAN-CONC

	Implicit	Explicit
NSC	Eurocode	Anderberg
HSC	Kodur	Author

In general, neutral axis may not be parallel to one of the axes when subjected to partial exposure to fire. This will cause conflict with the above assumptions. Thus, the computer program at present can only deal with 1-Face, 3-Face and 4-Face symmetric heating.

Both the material and geometric nonlinearities are taken into account in the modeling. The geometric nonlinearities are formulated at two levels: the local element level and the global structural level. A simple co-rotational beam element is adopted to include geometric nonlinearity effect (Huang 2002).

Creep of concrete is ignored in this program because the fire exposure time is relatively short, not more than 3 hours. Compared to transient strain, creep strain

does not yield a noticeable effect on structural response. Temperature-time relationship of each sub-area is required as an input file in a series of discrete time steps. Temperature between each time step is assumed to vary linearly.

4.4 Program Verification for NSC

Before the program can be used for illustration, it should be verified first. For this reason, some structural tests are collected from literature. Since there are more NSC columns subjected fire, this program will be applied to NSC first. Subsequently, its predictions are compared to a well-established software-SAFIR for verification.

4.4.1 NSC Structural tests

Eurocode model (implicit) and Anderberg's model (explicit) for NSC are adopted in FEMFAN-CONC. In SAFIR, there is no explicit material model, and only Eurocode model is adopted. Eurocode model is adopted for rebar in both programs. For all the tests, heat transfer analysis is conducted by SAFIR, and the temperature distributions are used as input file for structural analysis. The temperature profiles are compared with test data. The same number of beam/column elements is used in both programs. It is found that mesh density has little effect on the accuracy of final results. Normally, 10 or 12 elements for one column are sufficient to obtain satisfactory results without too much computer time. As a result, all structural members used for verification are divided into 10 elements, unless otherwise indicated. Due to inherent imperfection in actual tests, a 5mm

eccentricity in both FEM programs is assumed to represent imperfections for columns that were loaded concentrically, which is $1/60$ of length of side for a $300 \times 300 \text{ mm}$ cross section.

4.4.1.1 Test 1 (Anderberg 1982)

Since Anderberg's model is used in this program, his tests are chosen as a reference. In this test, two RC columns were exposed on three sides to ISO 834 standard fire. Load was applied eccentrically. Material data of the column were as follows:

moisture content: $w = 6\%$ by weight;

reinforcement: $K_s \ 40\phi 16$, $f_{0.2} = 450 \text{ MPa}$, $f_u = 716 \text{ MPa}$;

siliceous concrete: $f'_c = 34 \text{ MPa}$;

thermal conductivity and enthalpy: as depicted in Eurocode.

Cross section of the column was $200 \times 200 \text{ mm}^2$, whereas the column length was 2 m . Boundary condition at the bottom was a two-way pin, and at the top was a horizontal pin. The details of the cross section and the arrangement of the test are shown in Fig. 4.7. Both implicit and explicit material models in FEMFAN-CONC are used. Implicit material model is used in SAFIR. The self-weight of both columns is modeled as a downward axial load. Density of concrete is assumed to be 24 kN/m^3 .

Figure 4.7 Test arrangement

The predicted and measured temperatures of four points at the mid-height section of the column are shown in Fig. 4.8. There is excellent agreement with SAFIR prediction. The temperature profile is input into both programs.

Figure 4.8 Measured and predicted temperatures

The first column was loaded to $0.6MN$ with an eccentricity of $6cm$. The load corresponded to 63% of the ultimate load at normal condition. In the test the measurements were stopped after $0.5h$ due to a support failure (which was a mishap), but a comparison is still of interest. Fig. 4.9 illustrates the mid-span deflection and axial elongation as a function of time in the test. In the legend, ‘FEMFAN’ denotes results from FEMFAN-CONC, and ‘SAFIR’ is self-evident. ‘Ex’ in the parentheses means results from Anderberg’s model (explicit). Similarly, ‘Im’ means results from Eurocode model (implicit). These rules apply to all the experimental and numerical verifications hereafter.

Figure 4.9 Measured and predicted displacement of first column

The second column was loaded to $0.3MN$ with the same eccentricity but in a different direction. This load corresponded to 31% of the original ultimate strength at ambient temperature. The comparison of test and program results is shown in Fig. 4.10. It is noted that the column expanded axially in FEM programs and the test during the first 1.7h, following which, it went into contraction. The sudden change

is caused by a rapid inward movement of the compression zone into the cross section, which means that the reinforcement is yielded at elevated temperature and causes the loading to transfer to the core intact concrete.

Figure 4.10 Measured and predicted displacement of second column

From the comparison, we can see that the predicted displacements in the two FEM programs are in very good agreement with test results. Since there is no axial restraint for column elongation, as discussed in section 4.2.2, both implicit and explicit models give identical results. In this case, Eurocode and Anderberg's models give the same predictions.

4.4.1.2 Test 2 (Lie and Irwin 1993b)

This test was performed by Lie and Irwin (1993b) on an axially-loaded reinforced concrete column. The material properties of the specimen and the boundary conditions are similar to those in Test 1. This makes it convenient for comparison, and therefore, it is chosen. The geometric and loading data are given in

Fig. 4.11. Self-weight is input as an external vertical load at each node.

Figure 4.11 Test arrangement

The column was exposed to hot gas with temperature (generated by the furnace) following the ASTM fire curve. The yield strength of reinforcing bars was $f_y = 420 \text{ MPa}$, while the strength of siliceous aggregate concrete at room temperature was $f'_c = 36 \text{ MPa}$. Thermal parameters, such as conductivity k_c , convection heat transfer coefficient h_c , and emissivity ε_r , were not given in the document and were selected in such a manner that the calculated temperatures in concrete agreed as much as possible with temperature measurements. The selected values were $\varepsilon_r = 0.3$, $h_c = 20 \text{ W/mK}$, and k_c is given as in Eurocode. The other material and thermal parameters required in the numerical analysis were estimated on the basis of Eurocode. Fig. 4.12 shows the measured and calculated temperature distributions in concrete cross section along its centerline at various times. Clearly, the calculated and measured temperatures agree well.

Figure 4.12 Measured and predicted temperatures

The column was axially-loaded in the test. Fig. 4.13 shows a comparison of measured and calculated displacement of axial expansion at the end point and deflection at mid-span. The axial displacement was increasing during the first 120 min. This corresponded to column elongation caused by growing thermal strain, which dominated behavior at the beginning. Then the axial displacement started decreasing due to a rapid increase of transient strain and compressive mechanical strain due to reduction of elastic modulus and strength. The same phenomenon occurs in Test 1. This kind of behavior is typical for reinforced concrete columns in fire.

Figure 4.13 Measured and predicted displacement of the column

Fig. 4.13 shows that the agreement of FEM program and test result is not as good as in Anderberg's test. Since the column was axially loaded in this test, the mid-span deflection was very small at the beginning, but increased very fast when it approached failure. Due to imperfection, a small eccentricity must exist in actual column. That distinguishes it from a concentrically-loaded straight column. Since an initial eccentricity (5mm) has to be assumed in the two programs, this explains why there are some differences between the results from FEMFAN-CONC and test. There is no axial restraint in this test, once again, Eurocode and Anderberg's model give similar results.

4.4.1.3 Test 3 (Lie and Lin 1985)

This test is chosen because the columns were axially restrained during testing. From this test, the differences between implicit and explicit model become apparent. The second reason is that the geometry and material properties of the columns are almost the same as Test 2 column. The end condition was fully restrained and was obtained by initially applying the maximum allowable load on the column and preventing the column from expansion during the tests by adjusting the load so that the axial displacement at the end was always zero. The lengths of the columns were kept constant. The applied loading increased initially, then reduced later and returned to its original value. Then the load was kept constant until the column failed. The maximum allowable load was determined according to ACI 318-83, using a live-to-dead load ratio of 0.4 and actual cylinder strength of concrete on the

test date.

The specimens were square, tied, reinforced concrete columns. All were 3810mm long and had a cross section of 305×305mm, which consisted of 25mm diameter longitudinal rebars and 10mm diameter ties. The cross section was almost the same as in Test 2 except that the concrete cover is 48mm. The yield strength of the main bars was 444MPa and that of the ties was 427MPa. The ultimate strength was 730MPa for the main bars and 671MPa for the ties. The designed compressive strength of concrete was 35MPa, and the average cylinder strength was 42.6MPa for column A and 36.7MPa for column B. The relative humidity at the center of each column was approximately 75%, which is used for temperature prediction in SAFIR.

The columns were fixed at both ends, which were idealized as pins in two directions. The effective length was the same as in Test 2. The loading was applied concentrically. During the test the heat input was controlled so that the average gas temperature follows as closely as possible with the standard temperature-time relation specified in ASTM-E119. The load was applied in a displacement-controlled mode, so that the length was kept constant. Changes of the applied loading were recorded and compared with predictions from FEMFAN-CONC and SAFIR.

Because axial restraint is present, axial force is not constant in this test. From Fig. 4.14, both numerical and test results reflect the same trend for axial force. It increases first and then decreases. Adopting explicit model yields results closer to

the test. Although implicit model can also produce similar trend for axial force (increased first then decreased), the value is different, and the deflection curve may be different as well. Unfortunately, deflection results are not available in this test. Later in numerical verification (refer to Section 4.4.2.1 and 4.4.2.2), it can be seen that in an axially restrained heated column, even though axial force value from implicit and explicit models is similar, deflection curve is quite different.

Figure 4.14 Measured and predicted axial forces of the columns

4.4.2 NSC Numerical Verification

In Section 4.4.1, some NSC structural tests have been used to verify FEMFAN-CONC. Next, some NSC numerical problems are conducted to investigate the structural response of an axially restrained column due to limited published test results. Once again, predictions from implicit and explicit material models will be compared to show their differences. A simply-supported column and a beam are chosen because they are common in real structures, and they are also easy to analyze.

4.4.2.1 Problem 1: A Simply Supported Column

A simply supported RC column is exposed to ISO 834 fire at 4 faces, while subjected to an eccentric compression force, as illustrated in Fig. 4.15. The external compression force is $100kN$, and load eccentricity is $100mm$. The compressive strength of concrete at room temperature f'_c is $30MPa$; the diameter ϕ of the rebars is $20mm$; the Young's modulus and the yield strength of the rebar are $210GPa$ and $410MPa$, respectively.

Figure 4.15 Characteristic data for problem 1

The predicted temperature distribution from SAFIR at 30mins, 60mins, 90mins and 120mins are shown in Fig. 4.16. In the first case, there is no axial restraint. The column can expand freely in the axial direction. The axial displacement at the upper end and the lateral displacements at the mid-span are shown in Fig 4.17. This figure illustrates that predictions from implicit model (Eurocode) of SAFIR, implicit (Eurocode) and explicit (Anderberg's) model of FEMFAN-CONC are very close.

Anderberg's model yields very slightly smaller displacements than Eurocode's model. In order to show the significance of transient strain, it is excluded from Anderberg's model in FEMFAN-CONC, and the results obtained are also shown in Fig. 4.17, which is indicated as 'No TR'. These results are considerably different from those of other models because transient strain is present during the whole process and contributes a large amount to the total displacement.

Figure 4.16 Predicted temperature profiles at the cross section

In the second case, the column is the same as in the first case except that it is axially restrained. The lateral displacements at the mid-span and the development of

the restraint axial forces are shown in Fig. 4.17.

Figure 4.17 Results comparison of problem 1

When the column is axially restrained, significant axial force will develop. In this case, explicit model yields different results from those of implicit model from FEMFAN-CONC and SAFIR. This confirmed what the author has discussed in Section 4.2.2. In the second case, although the axial force development from implicit and explicit model is similar, there is a noticeable divergence between the deflection curves as stated in Test 3.

When transient strain is ignored, it leads to an earlier failure of that column. Since transient strain is in compression, the presence of transient strain reduces the thermal expansion so that it somewhat relieves the axial force. This means the $P-\delta$ effect in the heated column is smaller too, resulting in a smaller lateral deflection. Thus, the effect of explicit model is more significant in more slender columns than in stocky columns. On the other hand, if transient strain is excluded, axial force becomes greater, therefore lateral deflection grows more rapidly.

4.4.2.1 Problem 2: A Simply Supported Beam

This problem is about a simply supported beam with bottom and side faces exposed to ISO 834 fire. It is showed the interaction between axial restraint and transient strain can be applied not only to columns but also to beams. The beam supports a lateral distributed loading (see Fig. 4.18). The material properties are the same as those in Problem 1. The predicted temperature distribution from SAFIR at 30mins, 60mins, 90mins and 120mins are shown in Fig. 4.19.

Figure 4.18 Characteristic data for problem 2

Figure 4.19 Predicted temperature profiles at the cross section

In the first case, horizontal expansion at the right end of the beam is allowed. Results from both programs are close irregardless of material model adopted. The mid-span lateral displacements are shown in Fig. 4.20. The downward displacement increases monotonically. In the second case, the right end of the beam is restrained in both directions. The mid-span lateral displacements and the axial forces are also shown in Fig. 4.20. In the second case, the mid-span lateral displacements from

implicit model are different from those from explicit model. Because transient strain helps to relieve axial force, when it is ignored, axial force increases more rapidly.

Case 1: No axial restraint

Case 2: Axially restrained

Figure 4.20 Results comparison of problem 2

4.4.3 Discussion

From the comparison in Fig. 4.17 and Fig. 4.20, FEMFAN-CONC yields almost the same results as SAFIR for NSC when implicit (Eurocode) model is

adopted, FEMFAN-CONC also agrees well with experimental results as shown in Section 4.4.1. When there is no axial restraint, axial force remains constant; both implicit (Eurocode) and explicit (Anderberg's) material models give very similar results. Results from implicit model are more conservative than that from explicit model. In problems where columns or beams are axially restrained, significant axial force develops and varies during the fire. The generated axial restraint force may be relieved by the presence of transient strain, ignoring it leads to an earlier failure of structural members. Results from implicit model and explicit model are significantly different. It is therefore concluded that axial restraint and transient strain interact with each other and both of them affect the structural behavior significantly.

4.5 Program Verification for HSC

So far, the validity of FEMFAN-CONC has been verified for NSC columns. Next, the program will be applied to HSC columns based on proposed HSC explicit material model.

4.5.1 HSC Structural tests

Four series of HSC column tests under fire condition (Kodur et al. 2004; 2003c; 2000; Benmarce and Guenfoud 2005) are collected from literature. Since Kodur's HSC model and the author's model (developed based on Kodur's HSC model) are used in FEMFAN-CONC, some tests conducted by Kodur are chosen as a case study for validation. The concrete used in these tests are similar to those in the

material test 1 (Cheng et al. 2004b), from which Kodur's HSC model is developed. Besides, Benmarce's test is chosen because it was an axially-restrained column test. From this test, difference between implicit and explicit model for axially restrained columns can be revealed.

Temperature distribution across the column section is predicted by SAFIR, and the results are used as input file for FEMFAN-CONC. All structural members used for verification are divided into 10 elements. A 5mm eccentricity at mid-height represents the imperfection for concentrically-loaded columns.

4.5.1.1 Test 4 (Kodur et al. 2004)

The two columns were 3810mm long and had a square cross section of 305mm. The dimensions of the column cross section and relevant details are given in Table 4.3. Both columns had four 25mm longitudinal bars that were tied with 10mm links at a spacing of 75mm at both ends and 145mm in the middle. The main reinforcing bars and ties had characteristic yield strength of 420 and 280MPa, respectively. Fig. 4.21 shows the elevation and cross-sectional details of the columns. Two batches of concrete were used in fabricating the columns. Columns THC4 and THC8 were fabricated from Batch 1 and Batch 2, respectively.

The moisture conditions of columns THC4 and THC8 on the day of the test were around 78% and 67% relative humidity, respectively, at room temperature. The columns were installed in the furnace. The end conditions of the columns were fixed-fixed. For each column, the length exposed to fire was approximately

3000mm. At high temperature, the stiffness of unheated column ends, which was much greater in comparison to that of the heated portion of column, contributed to a reduction in the effective length. In previous studies, it was found by Kodur et al. (2004) that, for columns tested with fixed ends, an effective length of 2000mm represented experimental behavior. Thus, in the FEM modeling, only the effective length is considered.

Table 4.3 Test parameters and results of Test 4

Column	Column Dimensions (mm)	Concrete Strength		Factored Resistance (kN)	Test Load (kN)	Load Intensity	Fire Resistance (h:min)
		28 days (MPa)	Test days (MPa)				
THC4	305×305	60.6	99.6	3697	2000	0.54	3:22
THC8	305×305	60.4	72.7	2805	2000	0.71	5:05

The two columns were tested under concentric loads. The load intensity, defined as a ratio of applied load to column resistance at ambient temperature, is given in Table 4.3. The load was applied approximately 45 minutes before the start of the fire test and was maintained until there was no further increase of axial deformation. This was selected as the initial condition for the column axial deformation. During the test, the column was exposed to heat, controlled in such a way that the average temperature in the furnace followed, as closely as possible, the ASTM E119-88 standard temperature-time curve. The load was maintained constant throughout the test. The columns were considered to have failed and the tests terminated when the hydraulic jack, which had a maximum speed of 76mm/min, could no longer maintain the load.

Figure 4.21 Test arrangement

In the FEM program, the support condition is modeled as pinned-pinned. The temperature distribution is calculated by SAFIR. A rebar is transformed into equivalent square area with the centroid at the same location. The ties are ignored in the FEM modeling due to the limitation of the program.

The predicted and measured temperatures are illustrated in Fig. 4.22, which gives the temperature of four locations from the column centre. There is excellent agreement with SAFIR prediction. The predicted temperature distribution is taken as the input file for FEMFAN-CONC structural analysis.

Figure 4.22 Measured and predicted temperatures

The FEM calculation results and the test results are shown in Fig. 4.23. In this figure, ‘Kodur (Im)’ represents the results from Kodur’s HSC material model. ‘Author (Ex)’ represents the results from the explicit HSC material model proposed in Chapter 3. The numerical and test results are very close. Implicit (Kodur’s) and explicit (the author’s) models yield almost the same results since there is no axial restraint. This is reasonable because the explicit model is extended based on the implicit model. The explicit model can only exhibit its effect when axial restraint is present or if axial force is not constant. The column expands in the beginning due to heating, and then contracts as mechanical strain and transient strain (compression) increase. When external load is small compared to its load capacity, this trend is more obvious. If applied load ratio is large, the expansion is small and the duration is short. The FEM program predicts this trend very well.

It is interesting to note that in THC 8 column test, although the load intensity is larger, the fire resistance is 5 hours and 5 minutes, much longer than that in THC 4 column test (3 hours and 22 minutes). This unusual phenomenon contradicts findings from researchers that fire resistance of a concrete column is inversely

proportional to the load intensity applied on it. The greater the load intensity, the lower the column fire resistance (Wu et al. 2005, Harmathy 1993, Lie 1992). One of the partial reason for higher fire resistance of THC 8 is due to carbonate aggregate which gives higher fire resistance than for a siliceous aggregate concrete column (THC 4).

Figure 4.23 Measured and predicted displacement of the columns

4.5.1.2 Test 5 (Kodur and McGrath 2003c)

This experimental program consisted of two HSC columns of 3810mm long exposed to fire. One column was of 406mm square cross section, and the other was 305mm. The dimensions of the columns and other details are given in Table 4.4. Longitudinal reinforcement consisted of 25mm (designated as No.8 or 25M) or 16mm (designated as No.5 or 15M) bars. The ties were of $\phi 8mm$ (No.3) bars for one column and $\phi 6mm$ bars for the other. Deformed bars meeting the requirements of ASTM A615-80 were used as reinforcement and had characteristic yield strength of 414MPa. Figure 4.24 shows the elevation and cross-sectional

details of the columns.

Table 4.4 Test parameters and results of Test 5

Column	Column Dimensions (mm)	Rebar	Concrete Strength (Test days) (MPa)	Factored Resistance (kN)	Test Load (kN)	Load Intensity	Fire Resistance (min)
HSC 2	406×406	8-25M	86	5900	2406	0.54	224
HSC 6	305×305	8-15M	120	3145	2954	0.94	266

Figure 4.24 Test arrangement

The 28-day compressive strength determined from tests on 152×305mm cylinders, was 81MPa and 107MPa, respectively. The columns were installed inside the furnace by bolting the endplates to a loading head at the top and to a hydraulic jack at the bottom. Therefore the end condition was fixed-fixed, i.e., restrained against rotation and horizontal translation. The length exposed to fire was approximately 3000mm. As in Test 1, the effective length is 2000mm and only this

part is modeled in the FEM program.

The columns were tested under a concentric load, and the applied load on the columns was 54% and 94% of full service load, respectively (factored column compressive resistance determined according to the CSA Standard CSA-A23.3-M94), respectively. The factored compressive resistance was calculated based on effective length factor, $K=0.65$ for fixed ends. Material strength was based on compressive strength of cylinders on the day of the test.

Moisture content, corresponding to approximately 86% and 64% relative humidity for HSC 2 and HSC 6, respectively, was measured. Relative humidity is input into SAFIR for temperature prediction. The load was applied approximately 45 min before the start of the fire test, and was maintained until there was no further increase of axial deformation. This was selected as the initial condition for the column axial deformation.

During the test, the columns were exposed to heating which is controlled in such a way that the average temperature in the furnace followed, as closely as possible, the ASTM E119-88 standard temperature-time curve. The load was maintained constant throughout the test. The columns were considered to have failed, and the tests terminated, when the hydraulic jack, which had a maximum speed of 76 mm/min, could no longer maintain the load.

Fig. 4.25 compares the test and FEM results of temperature development at different locations within the cross section of HSC 2. The two temperature profiles

agree with each other very well. Then the FEM results of temperature profile are input into FEMFAN-CONC for structural analysis.

Figure 4.25 Measured and predicted temperatures of HSC 2

The FEM and the test results are shown in Fig. 4.26. It can be seen from the figure that HSC 2 collapsed during expansion. The implicit and explicit models yield similar results due to the absence of axial restraint. In HSC 6, since external load ratio is larger than that in HSC 2, the column barely experienced expansion then developed into contraction. Although the load intensity of HSC 6 is higher than HSC 2, its fire resistance is longer than the latter. This contradicts the usual conclusions from researchers (Wu et al. 2005, Harmathy 1993, Lie 1992).

Figure 4.26 Measured and predicted displacement of the columns

4.5.1.3 Test 6 (Kodur et al. 2000)

Ten HSC columns were fabricated and tested under fire conditions. The length and cross section were the same as in Test 1 and Test 2 specimens. Concrete cover for HS1 to HS6 was 38mm, while for the other columns was 41mm. Other relevant parameters are given in Table 4.5. Seven columns were tested with both ends fixed, i.e., restrained against rotation and horizontal translation. This was achieved by bolting the column end plates to the loading head and the hydraulic jack. Column HS 2, HS 4 and HS 10 were tested under hinged end conditions, i.e., with restraint against horizontal translation only. The hinged condition was obtained by bolting the end plates to receiving plates with vertical roller bearings. All columns were tested under a concentric load, except HS 10, where load eccentricity was 25mm. In the FEM modeling, this is converted to an equivalent bending moment. The applied load on the columns ranged from 54% to 183% of full service load (factored column compressive resistance). The temperature profile and the comparison of test results and numerical results are shown in Fig. 4.27 and Fig. 4.28.

Table 4.5 Test parameters and results of Test 6

	Concrete Strength(MPa)		RH ⁺ (%)	Rebar	Cross Section (mm)	Test Load (kN)	Load Intensity	Fire Resistance (mins)
	28-day	Test						
HS1	91.7	-	75	8M25	406×406	45*	-	248
HS2	105.1	-	67	8M25	406×406	2913	0.60	204

HS3	83.7	99.7	69	8M25	406×406	3080	0.63	239
HS4	74.9	89.6	61	8M25	406×406	2934	0.73	145
HS5	80.9	86.0	86	8M25	406×406	2406	0.54	224
HS6	84.0	96.0	57	8M25	406×406	4919	0.90	104
HS7	107.0	119.7	50	8M15	305×305	1979	0.82	266
HS8	107.0	119.7	68	8M15	305×305	2363	0.98	290
HS9	107.0	119.7	64	8M15	305×305	2954	1.23	266
HS10	107.0	119.7	64	12M15	305×305	2954	1.83	49

+ Relative humidity

* At 240 minutes the load on the column was increased at a rate of 445kN/min. Due to the limitation of FEMFAN-CONC, the calculation is only proceeded to 240 minutes.

Figure 4.27 Temperature Profiles of HS 1 and HS 8

Figure 4.28 Measured and predicted displacement of the columns

From Fig. 4.28, it is shown that when axial restraint is not present, implicit and explicit models make no difference. Both models can give satisfactory results compared with test results. A typical behavior for concentrically-loaded column under fire is observed, that is, it expanded first and then come contracted. The same phenomenon occurs in NSC column tests. The reason for this behavior is that at the beginning, expansion is caused by free thermal strain. Later, when temperature is higher, elastic modulus is lower. This increases mechanical strain (compression) and transient strain significantly and they dominate the total deformation. As a result, columns get into contraction phase. When external load ratio is larger,

compressive mechanical strain and transient strain increase as well. Therefore, this type of column barely experiences expansion. From test results of HSC columns, FEMFAN-CONC for HSC columns has been verified. For columns without axial restraint, both implicit and explicit models can be used.

4.5.1.3 Test 7 (Benmarce and Guenfoud 2005)

Six HSC columns were tested in fire under two heating rates at different load ratios and restraint levels. Axial restraint was imposed against thermal expansion at the column ends. Axial restraint caused additional axial force to be generated, and therefore axial force was not constant. In this case, implicit and explicit material models yield different results. Three different load ratios, 20%, 40% and 60% of design load from BS8110 code, at two restraint ratios (ratio of column axial stiffness to restraint stiffness from surrounding structure) of 0.1 and 0.2, and two heating rates (BS476 and a lower rate) were investigated. Restraining ratio governs the rate and magnitude of restraint force generated, and the column collapse temperature. The closer the column stiffness to that of the structure, the greater the restraint ratio, and thus the greater the axial force imposed onto the column when it starts to expand. The details about the test are shown in Table 4.6.

All of the specimens were identical: $125 \times 125 \text{ mm}^2$ square cross-sections and a height of 1.80 m . Each column was reinforced with four 12 mm diameter steel bars and connected with ties (6 mm diameter) at 100 mm intervals in the middle and 200 mm at both ends. The concrete cover was 20 mm . Concrete strength at 28 days was 108 MPa . The axial restraint was applied by a frame system as shown in Fig.

4.29. The expansion of column was restrained by a lateral beam which is fixed on threaded bars. To provide the variable stiffness required for the test program, rubber springs were used between the lateral beam and its fixity on the threaded bars. By choosing springs with different stiffness, different restraint values could be achieved.

Figure 4.29 Test Arrangement

Table 4.6 Test parameters and results of Test 7

	Restraint Ratio	Load Ratio	Heating Rate	Fire Resistance
HSC 1	0.1	0.2	High	67
HSC 3	0.1	0.4	High	46
HSC 5	0.1	0.6	High	28
HSC 8	0.2	0.2	Low	124
HSC 10	0.2	0.4	Low	79
HSC 12	0.2	0.6	Low	51

The columns were simply supported at both ends and then loading was applied concentrically. Each column was tested under two heating regimes. The first curve

representing the BS476 fire curve was defined as the high heating rate (indicated as ‘HH’ in the legend); the second curve with a lower intensity was defined as the low heating rate (indicated as ‘LH’ in the legend). The temperature profile from SAFIR heat transfer analysis is input for structural analysis. It is shown in Fig. 4.30.

In FEM modeling, one beam element was placed on top of the column representing axial restraint. The elastic modulus of this element is ten times or twenty times that of column elements and its section properties are the same as those of columns. Therefore, its ‘ EA ’ (E is elastic modulus and A is area) is ten times or twenty times that of column elements. The restraint ratio is 0.1 or 0.2 accordingly. This element is unaffected by fire (temperature remains at 20°C during the whole process). The end condition is pinned-pinned. The measured and predicted axial displacement of the test columns is shown in Fig. 4.31

Figure 4.30 Temperature Profile

Figure 4.31 Measured and predicted displacement of the columns

From Fig. 4.31, it is obvious that when the column is axially restrained, explicit model can yield more accurate results than implicit model, Different material models yield different results, although both models show the same trend for axial displacement (expansion followed by contraction). A simply supported column is a

very simple structure. In a statically indeterminate RC frame, structural behavior is much more complex. Columns and beams are restrained by nearby members when they are exposed to local fire conditions. Relative displacement at beam-column joints cause stress redistribution. Different joints displacement may lead to a totally different structural behavior. In this case, implicit model may be non-conservative and cannot obtain a reasonable result. Therefore, explicit material model should be used instead of implicit model.

4.5.2 HSC Numerical Verification

Numerical verification is used to demonstrate the difference between implicit and explicit model. Like NSC members, these two types of models give the same results only when axial force is constant. However, in actual cases, axial force in structural members varies with temperature. To obtain more accurate results, explicit model should be used. Two numerical case studies are conducted due to limited number of axially-restrained column tests. All aspects are the same as in normal strength concrete analysis (Section 4.4.2) including rebar material model, and temperature distribution except that high strength concrete material models are applied.

4.5.2.1 Problem 3: A Cantilever Column

An HSC column with one end fixed end and the other end free is exposed to ISO 834 fire at 4 faces, while subjected to an eccentric compression force as shown in Fig. 4.32. In the first instance, there is no axial restraint at the top of the column.

The applied compression force is $80kN$ with an eccentricity of $50mm$. The compressive strength of concrete at room temperature f'_c is $60MPa$; the diameter of rebars is $20mm$; the Young's modulus and the yield strength of rebars are $210GPa$ and $410MPa$, respectively.

The temperature distribution is the same as NSC column in Section 4.4.2.1. The axial and lateral displacements at the free end are shown in Fig 4.32. In the absence of axial restraint, implicit (Kodur's) model and explicit (the author's) model yield similar results.

Figure 4.32 Characteristic data for problem 3

In the second case, the axial elongation at the free end is restrained. A very slight bending moment $2kN\cdot m$ is applied on the column top to induce lateral displacement. The lateral displacement at the free end and the development of restraint force are shown in Fig. 4.33.

Case 1: No axial restraint

Case 2: Axially restrained

Figure 4.33 Results comparison of problem 3

When the column is axially restrained, significant axial force will develop. At the beginning of 20 minutes, the development of axial force in both columns is almost the same. Then after 30 minutes, the axial force begins to reduce due to softening of concrete. As a result, lateral displacement increases dramatically. In this case, the axial force is not constant. Implicit and explicit models yield different results.

4.5.2.2 Problem 4: A Simply Supported Beam

Using implicit and explicit models, FEMFAN-CONC is used to analyze beam problems to show the effects of different types of material models on axially-restrained beams. This problem is identical with the beam problem in Section 4.4.2.2, except HSC is used. The temperature distribution in Problem 2 (refer to section 4.4.2.2) is used here. In the first case, there is no axial restraint. The mid-span lateral displacement is shown in Fig. 4.34. It is observed that the results from implicit and explicit model are very close. In the second case, axial restraint is present. The mid-span lateral displacement and the axial force are shown in Fig. 4.35. For this whole process, the development of axial force from both models is similar. However, there is divergence in the mid-span lateral displacements from the two models.

Figure 4.34 Characteristic data for problem 4

Figure 4.35 Results comparison of problem 4

4.5.2.3 Discussion

From experimental and numerical verification, it is demonstrated that numerical results from FEMFAN-CONC agree well with test results. The validity of FEMFAN-CONC can be verified. If a column is not axially restrained, implicit (Kodur's) and explicit (the author's) material models yield almost the same results. However, when a column is axially restrained, more reasonable result can only be obtained by adopting the explicit model.

4.6 Conclusion

In this chapter, significance of transient strain is investigated. It is demonstrated that transient strain significantly affects the final results when axial force is not constant. The simultaneous heating and loading processes can be separated into two independent steps: heating only (constant loading) and loading only (constant temperature). Using implicit material model, transient strain is always present in the two steps. By definition, transient strain only occurs in heating only (constant loading) step. Using explicit model can avoid this mistake. A theoretical analysis is carried out to show the difference between implicit and explicit models.

To reinforce the argument made from theoretical analysis, a FEM program named FEMFAN-CONC is extended for numerical case studies. Implicit (Eurocode and Kodur's) and explicit (Anderberg's and the author's) material models for NSC and HSC are incorporated into this program. Structural tests for NSC columns are collected from literature and used to verify the validity of FEMFAN-CONC. Numerical problems are used to demonstrate that when axial restraint is present, implicit and explicit material model yield different results. Numerical verification also shows the interaction between axial restraint and transient strain in axially-restrained beam problems. After that, HSC structural tests are used to verify the explicit model proposed by the author. Due to limited number of axially restrained HSC columns tests, two numerical problems are used to demonstrate the difference between implicit and explicit model. Through the comparison and illustration, it is concluded that when a concrete column is heated with axial restraint, transient strain helps to relieve the increase of axial force. Consequently, a

higher fire resistance is achieved compared to analyses without transient strain. In this case, only explicit model can yield reasonable results. When there is no axial restraint, implicit model can be used.

Chapter 5

Conclusion and Recommendation

5.1 Conclusion

When concrete is loaded under high temperature, normally four types of strains occur. They are free thermal strain, mechanical strain, creep strain and transient strain. The characteristics of these strain components are introduced in Chapter 3. A few material models for concrete under high temperature are also presented in this Chapter. According to how transient strain is included, these material models can be classified as implicit or explicit. In implicit model, transient strain is implicitly included in ‘mechanical strain’. However, for explicit model, transient strain is calculated separately. For material models such as Kodur’s and Eurocode, ‘mechanical strain’ already includes ‘transient strain’. This ‘mechanical strain’ is called ‘full strain’ in this thesis. As a result, Kodur’s and Eurocode model is called implicit model. In Anderberg’s and Schneider’s material model, ‘mechanical strain’ is the actual strain from mechanical action, and transient strain is calculated independently. Therefore, these two models are called explicit models.

Through comparison, it is also found that transient strain accounts for the strain difference of concrete with the same final state (same temperature, same stress level) but different loading-heating paths. One is loaded first, then heated up to target temperature, while the other is heated up first, and then stressed. A useful equation to calculate transient strain is developed. Using this equation, Kodur’s implicit

material model for HSC is converted to an explicit material model in which transient strain is calculated independently. Some material tests of HSC specimens are used to verify this new HSC model.

For better illustration, an existing FEM program is extended in Chapter 4. The structural model is introduced. A theoretical analysis based on this model is carried out to show that transient strain influences the final results considerably when axial force is not constant. This situation is quite common, since in a structural system, members are always restrained by surrounding members. Thus, axial force in heated members varies with temperature. Implicit and explicit models yield different results for this case.

The significance of transient strain is demonstrated by applying specific examples to the FEM program. Current material models and the new proposed HSC model are adopted in the program (for NSC, Eurocode model as implicit and Anderberg's model as explicit; for HSC, Kodur's HSC model as implicit and the author's model as explicit). Through comparison of NSC column tests, the validity of this program is verified first. Then numerical verification is conducted to show the significance of explicit model, which yields different results from those of implicit model when there is an axial restraint. HSC column tests are used to verify that FEMFAN-CONC can be applied to HSC columns as well. As a result, the validity of the author's HSC explicit model is verified. When there is no axial restraint, both implicit and explicit models give similar results. However, when axial restraint is present, explicit model gives more reasonable results than implicit model. At last, some HSC numerical verification is used to strengthen the author's

statements about the interaction between axial restraint and transient strain. The statements can also be applied to axially-restrained RC beams under fire.

5.2 Recommendation for Future Work

The explicit material model for high strength concrete has been developed. Material tests need to be conducted to verify this model. The material tests should include not only ordinary HSC, but also HSC with steel fibers or HSC with different mix proportions. Based on the test results, the explicit material model can be made more robust to cater not only for ordinary HSC, but also other types of HSC, e.g., HSC with steel fibers, which are more and more popular in industry.

Creep model and increased elastic modulus and strength due to pre-load effect are not considered in this research. They should be included in this model to make it more comprehensive. Therefore, the explicit model can be applied to structural members exposed to elevated temperature for a longer period of time.

Structural tests of NSC and HSC members with axial restraints are required to verify the explicit material model. The effect of axial restraint should be investigated to differentiate between the implicit and explicit material models. Different axial restraint ratios and their effect on structural behavior of RC members should also be studied.

The FEM program has to be extended. Firstly, the lateral confining effect from steel ties is ignored in current version. Since this effect plays an important role on

structural behavior at ambient temperature, it could be important at elevated temperature as well. To achieve this, a 3D FEM program is needed. Secondly, spalling of HSC under high temperature is very common, and has significant effect on structural behavior. This effect cannot be considered at current stage. Thirdly, the discretization of cross section should be improved to include circular section or even composite sections as well, such as concrete filled steel column.

Appendix A

Analytical Solution for Thermal Analysis

A.1 Introduction

Heat transfer problem is basically a partial differential equation with some specific boundary condition within finite spatial domains. In most cases, it is impossible to obtain analytical solution. However, by approximation to some extent, analytical solution can be found by different mathematical techniques. Different analytical solutions for heat conduction problem are available in textbooks. Some of these methods are introduced by Ozisik (1993). An efficient numerical approach based on analytical Green's function (GF) solution is developed by Wang et al. (2005a). Because it can be implemented easily, this numerical algorithm is adopted in this research and incorporated into the FEM program-FEMFAN-CONC for thermal analysis.

A.2 Basic Correlations

In rectangular coordinates, the law of conservation of energy is expressed by the following equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (\text{A.1})$$

where α is thermal diffusivity of the solid,

$$\alpha = \frac{k}{\rho \cdot c} \quad (\text{A.2})$$

where k is thermal conductivity (W/m/K); ρ is density of solid (kg/m^3); c is

specific heat of solid ($J/kg/K$); T is temperature (K) and t is time (s).

The initial condition for temperature distribution in the body is assumed uniform and equal to the temperature of the environment:

$$T(x,0) = T_i \quad (A.3)$$

Thermal boundary condition is given by:

$$k \frac{\partial T}{\partial x} \Big|_{x=\pm L} + hT(\pm L, t) = q \quad (A.4)$$

where h is convective heat transfer coefficient and q is a prescribed constant.

The heat transfer problem in Eq. (A.1) coupled with the initial/boundary conditions in Eq. (A.3), (A.4) can be solved by Green's function. For homogeneous problems, i.e., thermal properties of the solids are assumed to be isotropic and temperature-independent, and the initial temperature of the problem domain is normalized to zero and the prescribed boundary temperature (with respect to the Dirichlet model) or heat flux (with respect to the Neumann model) is unity, solutions are referred to as "impulse" Green's functions, denoted by $g(x,t)$; and the temporal integrals of the impulse Green's function are referred to as the "step" Green's functions, denoted by $G(x,t)$. The complete solutions for the temperature response in the problem domain are therefore obtained using Duhamel's integral by incorporating the time-varying boundary conditions, i.e. the standard fire (Wang and Tan 2005b).

A.3 Step Functions

There are two different kinds of "step" systems commonly used:

1. Step system with normalized initial condition

Let $\theta^A(x, t) = T(x, t) - T_a$ denote the temperature difference between the solid and the ambient air, where superscript 'A' denotes the properties of the step system with normalized initial condition, T_a is ambient temperature. A dimensionless temperature in this system is given by

$$\bar{\theta}^A = \frac{\theta^A}{\theta_i^A} = \frac{T - T_a}{T_i - T_a} \quad (\text{A.5})$$

where T_i is initial temperature of the solid. In addition, two other dimensionless parameters are introduced for normalization, namely, the dimensionless spatial coordinate

$$\bar{x} = \frac{x}{L} \quad (\text{A.6})$$

and the dimensionless time

$$\bar{t} = \frac{\alpha t}{L^2} = Fo \quad (\text{A.7})$$

where the dimensionless time $\bar{t}$ is equivalent to the Fourier number. The initial condition becomes:

$$\bar{\theta}^A(\bar{x}, 0) = \bar{\theta}_i^A = 1 \quad (\text{A.8})$$

and the ambient temperature is

$$\bar{\theta}_a^A = \frac{T_a - T_a}{T_i - T_a} = 0 \quad (\text{A.9})$$

2. Step system with normalized boundary condition

In this system, let $\theta^B(x, t) = T(x, t) - T_i$ denote the temperature difference between the plate and the initial temperature inside the solid, where superscript 'B' denotes the properties of the step system with normalized boundary condition. The dimensionless temperature in this system is given by

$$\bar{\theta}^B = \frac{\theta^B}{\theta_a^B} = \frac{T - T_i}{T_a - T_i} \quad (\text{A.10})$$

Using the same dimensionless spatial coordinate and dimensionless time as defined

in Eq. (2.6) and (2.7), the initial condition is

$$\bar{\theta}^B(\bar{x}, 0) = \bar{\theta}_i^B = 0 \quad (\text{A.11})$$

and the ambient temperature is

$$\bar{\theta}_a^B = \frac{T_a - T_i}{T_a - T_i} = 1 \quad (\text{A.12})$$

In a similar way, the step functions in an infinite cylinder can be derived. The two systems are depicted in Fig. A.1.

Figure A.1 Two step systems

The above two step systems can be physically interpreted as the phenomenon where a plate with thickness $2L$ or an infinite cylinder with radius r_0 and normalized initial conditions, $\bar{\theta}_i^A = 1$ (or $\bar{\theta}_i^B = 0$) is immersed in a fluid at normalized boundary conditions $\bar{\theta}_a^A = 0$ (or $\bar{\theta}_i^B = 1$).

The canonical form of series solution of the step system A by separation of variables is given in Table A.1(Wang and Tan 2006). The two step systems are correlated by: $\bar{\theta}^A = 1 - \bar{\theta}^B$.

Table A.1 Canonical form of series solution of the step system A

$$\bar{\theta}^A = \sum_{n=1}^{\infty} C_n \exp(-\xi_n^2 Fo) \psi(\xi_n \bar{x})$$

Solids	kind of B.C.	Coefficients	Eigen condition	
		C_n	ξ_n	$\psi(\cdot)$
Plate, $\bar{\theta}_p^A$	first	$C_n = \frac{4}{\pi} \frac{(-1)^{n+1}}{(2n-1)}$	$\xi_n = \frac{(2n-1)\pi}{2}$	$\cos(\cdot)$
	third	$C_n = \frac{4 \sin(\xi_n)}{2\xi_n + \sin(2\xi_n)}$	$\xi_n \tan(\xi_n) = Bi$	$\cos(\cdot)$
Infinite cylinder, $\bar{\theta}_c^A$	first	$C_n = \frac{2}{\xi_n J_1(\xi_n)}$	$J_0(\xi_n) = 0$	$J_0(\cdot)$
	third	$C_n = \frac{2 J_1(\xi_n)}{\xi_n [J_0^2(\xi_n) + J_1^2(\xi_n)]}$	$\frac{\xi_n J_1(\xi_n)}{J_0(\xi_n)} = Bi$	$J_0(\cdot)$

in which, $J_n(\cdot)$ is the Bessel function of the first kind of order n , $Bi = hL/k$ is the Biot number in rectangular plate, while $Bi = hr_0/k$ in cylinder.

Using the technique of separation of variables, it can be shown that the dimensionless temperature distribution in a 3D solid body may be expressed as a product of the respective 1D solution in different directions without consideration of coupling effect, as shown in Eq. (A.13).

$$\bar{\theta}^A(x, y, z, t) = \bar{\theta}^A(x, t) \times \bar{\theta}^A(y, t) \times \bar{\theta}^A(z, t) \quad (\text{A.13})$$

The validity of the use of product rule to cater for multi-dimensional problems was discussed by Beck et al. (1992), provided the heat conduction problem is linear, the system is homogenous and the geometry is “orthogonal”. Although the presence of rebars and concrete per se introduces non-homogeneity, this solution can still be applied with acceptable error (Wang 2006).

A.4 Incorporation of Fire Condition

Since uniform boundary conditions are assumed in the step functions, i.e. all surfaces of structural members are equally exposed to fire, the incorporation of the time-varying fire boundary can be readily obtained using Duhamel's integral (Ozisik 1993), as

$$T = T_i + \int_0^t \theta_g(t - \tau) \frac{\partial \bar{\theta}}{\partial \tau} d\tau \quad (\text{A.14})$$

where $\theta_g = T_g - T_i$, T_g is the imposed fire temperature curves as proposed by ISO 834 or ASTM-E119. Here, the expression proposed by SBN (1975) is adopted, which is:

$$\theta_g = \sum_{i=0}^3 B_i \exp(-\beta_i t) \quad (\text{A.15})$$

Therefore, the analytical solution for a 2D rectangular domain, $-L \leq x \leq L$ and $-W \leq y \leq W$ can be obtained as (Wang and Tan 2006):

$$T_R(x, y, t) = T_i + \sum_{i=0}^3 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{B_i C_{mx} \cos(\xi_{mx} \bar{x}) C_{ny} \cos(\xi_{ny} \bar{y}) \Phi_{mn}}{\beta_i - \Phi_{mn}} \times [\exp(-\Phi_{mn} t) - \exp(-\beta_i t)] \quad (\text{A.16})$$

where

$$\Phi_{mn} = \alpha \xi_{mx}^2 / L^2 + \alpha \xi_{ny}^2 / W^2 \quad (\text{A.17})$$

The final solution in the 2D cylinder subjected to standard fire can be obtained as:

$$T_{CC}(r, z, t) = T_i + \sum_{i=0}^3 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{B_i C_{mr} J_0(\xi_{mx} \bar{r}) C_{nz} \cos(\xi_{ny} \bar{z}) \Phi_{mn}}{\beta_i - \Phi_{mn}} \times [\exp(-\Phi_{mn} t) - \exp(-\beta_i t)] \quad (\text{A.18})$$

where

$$\Phi_{mn} = \alpha \xi_{mr}^2 / r_0^2 + \alpha \xi_{nz}^2 / H^2 \quad (\text{A.19})$$

And for the 3D rectangular parallelepiped, the analytical solution is given by

$$T_{CC}(x, y, z, t) = T_i + \sum_{i=0}^3 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \frac{B_i C_{mx} \cos(\xi_{mx} \bar{x}) C_{ny} \cos(\xi_{ny} \bar{y}) C_{lz} \cos(\xi_{lz} \bar{z}) \Phi_{mnl}}{\beta_i - \Phi_{mnl}} \times [\exp(-\Phi_{mnl} t) - \exp(-\beta_i t)]$$

(A.20)

where

$$\Phi_{mn} = \alpha \xi_{mx}^2 / L^2 + \alpha \xi_{ny}^2 / W^2 + \alpha \xi_{lz}^2 / H^2 \quad (\text{A.21})$$

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