Reliable robust aircraft control against faults and saturations

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Reliable Robust Aircraft Control Against Faults and Saturations

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School of Electrical & Electronic Engineering

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in fulfillment of the requirements for the degree of

Doctor of Philosophy

2006

Statement of Originality

I hereby certify that the content of this thesis is the result of work done by me and has not been submitted for a higher degree to any other University or Institution.

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Date

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FENG LE

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Summary

This thesis studies the reliable robust aircraft flight control problem for linear time-invariant (LTI) systems and presents seven novel approaches. Several types of faults are considered, including actuator outage/stuck faults and control/wing/tail surface impairment faults. Input saturation nonlinearity and output/state hard constraints are also studied by different means.

For traditional H_2 and/or H_∞ approaches, parameter dependent Lyapunov functions and additional slack variables are adopted to reduce the design conservativeness. Anti-windup technique is also used to compensate for the input nonlinearity. Both state feedback and output feedback cases are covered.

For ease of considering hard constraints together with input saturations, model predictive control (MPC) technique is introduced from Chapter 6. Quadratic programming (QP) based optimization problem is formulated and solved on-line to handle faults and constraints.

To address polytopic uncertainties, disturbances and to reduce the heavy on-line computational burden, Chapter 7 presents two novel linear matrix inequality (LMI) based robust multi-objective MPC approaches that can move a large part of on-line computation off-line. In Chapter 9, a more general uncertainty description is adopted and a novel robust MPC with structured uncertainty is proposed.

The reliable controllers obtained by the proposed approaches in this thesis guarantee the closed-loop stability, robustness and an optimal tracking performance in the nomi-

nal condition while maintaining an acceptable downgraded tracking performance in the presence of faults. The effectiveness of the proposed approaches is demonstrated by the examples provided in this thesis, using computer simulation of high fidelity nonlinear 6 degree of freedom (DOF) model of a high performance fighter aircraft.

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Acronyms and symbols

Acronyms

BMI	Bilinear Matrix Inequality
DOF	Degree of Freedom
FCS	Flight Control System
FDI	Fault Detection and Isolation
ILMI	Iterative Linear Matrix Inequality
LFT	Linear Fractional Transformation
LMI	Linear Matrix Inequality
LQ	Linear Quadratic
LQG	Linear Quadratic Gaussian
LTI	Linear Time Invariant
MIMO	Multi-input and Multi-output
MPC	Model Predictive Control
RHC	Receding Horizon Control
SRFCS	Self-repairing Flight Control System

Mathematical symbols

$\in$	belong to.
$\triangleq$	defined as.
$\text{diag}\{a_1, \dots, a_n\}$	an $n \times n$ diagonal matrix with a_i as its i_{th} diagonal element.
$\ G\ _2$	H_2 norm of system G .
$\ G\ _\infty$	H_∞ norm of system G .
I	identity matrix.
$I_{n \times n}$	$n \times n$ identity matrix.
A^T	transpose of matrix A .
A^{-1}	inverse of matrix $A \in \mathbb{R}^{n \times n}$.
$\mathbb{R}^n$	n -dimensional real Euclidean space.
$\mathbb{R}^{n \times m}$	$n \times m$ real matrix.
$\text{trace}(A)$	trace of A .
$P \geq 0$	symmetric positive semi-definite matrix $P \in \mathbb{R}^{n \times n}$.
$P > 0$	symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$.
$P \geq Q$	$P - Q \geq 0$ for symmetric positive semi-definite matrices $P, Q \in \mathbb{R}^{n \times n}$.
$P > Q$	$P - Q > 0$ for symmetric positive semi-definite matrices $P, Q \in \mathbb{R}^{n \times n}$.
$*$	symmetric entries of a symmetric matrix.

Aircraft symbols

m	Aircraft mass, kg .
I_x	Moment of inertia about X body axis, $kg \cdot m^2$.
I_y	Moment of inertia about Y body axis, $kg \cdot m^2$.
I_z	Moment of inertia about Z body axis, $kg \cdot m^2$.
I_{xz}	Product of inertia with respect to X and Z body axes, $kg \cdot m^2$.
p	Aircraft roll rate about X body axis, rad/sec .
q	Aircraft pitch rate about Y body axis, rad/sec .
r	Aircraft yaw rate about Z body axis, rad/sec .
u	Components of aircraft velocity along X body axis, m/sec .
v	Components of aircraft velocity along Y body axis, m/sec .
w	Components of aircraft velocity along Z body axis, m/sec .
V_t	Total aircraft velocity, m/sec .
δ_{al}	Left aileron deflection, positive for downward, <i>degree</i> .
δ_{ar}	Right aileron deflection, positive for downward, <i>degree</i> .
δ_{el}	Left elevator deflection, positive for downward, <i>degree</i> .
δ_{er}	Right elevator deflection, positive for downward, <i>degree</i> .
δ_r	Rudder deflection, positive for downward, <i>degree</i> .
δ_T	Engine throttle, %.
α	Angle of attack (AOA), <i>degree</i> .
β	Angle of sideslip, <i>degree</i> .

$\bar{c}$	Wing mean aerodynamic chord, m .
X_{cg}	Center of gravity, fraction of $\bar{c}$.
θ	Pitch angle, <i>degree</i> .
ϕ	Roll angle, <i>degree</i> .
ψ	Yaw angle, <i>degree</i> .
T	Engine thrust, N .
P_s	Static pressure, N/m^2 .
$\bar{q}$	Dynamic pressure, N/m^2 .
S	Wing area, m^2 .
ρ	Atmosphere density, kg/m^3 .
g	Acceleration due to gravity, $g = 9.81m/sec^2$.
H	Altitude, m .
b	Wing span, m .
$C_{x,t}, C_{y,t}, C_{z,t}$	Total X-axis, Y-axis, and Z-axis force coefficients.
$C_{m,t}, C_{n,t}, C_{l,t}$	Total pitching, yawing and rolling moment coefficients.
W_{bx}, W_{by}, W_{bz}	Components of wind disturbance along X, Y and Z body axes, m/sec .
H_e	Engine angular momentum, $kg \cdot m^2/sec$.

Chapter 1

Introduction

This thesis is concerned with designing reliable robust tracking controllers to maneuver an F-16 aircraft safely and efficiently, in the presence of nominal condition as well as fault conditions and input saturations. This chapter provides an introduction of motivation and related literature.

1.1 Motivation

Today, most aircrafts adopt flight control systems (FCS) to achieve good performance. Plenty of approaches have been proposed and applied in the literature. In the design of either classical or modern flight control systems, there are two most important factors to be considered: faults and saturations. Ignoring faults in design procedure can result in a controller that works well in nominal condition but experiences a significant performance degradation in the presence of faults. Therefore we are motivated to investigate how to design a controller that is not sensitive to predefined faults.

Input saturation is another important factor that affects the performance of designed controller. During our research on unconstrained controller design, windup phenomena

occurs in some particular occasions (e.g., severe faults, high angle maneuvers etc.). Failing to handle actuator saturation generally leads to tracking performance degradation, eventually losing control of the aircraft. This motivates our research on how to handle actuator saturations.

This thesis will treat theoretical and practical results within reliable robust tracking controller design problems against faults and saturations.

1.2 Reliable robust control

In today's literature, it is often presumed that all the system components are working in a good condition. Therefore, the resulting controller usually works pretty well in nominal condition but may not maintain an acceptable performance in the presence of faults. In aircraft flight control area, for example, actuator outage/stuck fault and control/wing/tail impairment may be quite popular in some circumstances such as in a battle field. Based on these, fault handling ability is of more and more importance in flight control area.

A control system designed to tolerate faults of actuators, sensors or other system components, while maintaining an acceptable level of the closed-loop system stability/performance, is called a reliable control system [69]. Reliable control strategy is particularly suitable for safety-critical systems, such as aircraft, nuclear power plants where safety and reliability are sometimes of more importance than good performance solely. Numerous approaches and applications have been reported, such as [35, 49, 51, 71, 79, 82, 86], etc.

For an aircraft, normally there are some inherent redundancies in multi-input and multi-output systems, which often make it possible to design controllers such that faults in some system components do not cause immediate threats to the safety of the overall system. For example, conventional aircrafts have mainly three sets of control surfaces: the elevator (horizontal stabilizer) for pitch control; the differential aileron for roll control; and the rudder for yaw (directional) control. For some advanced tactical fighters,

there are usually additional canard and flaps for pitch and yaw control. In F. Liao *et al.* [47], the movement of the left and right elevators and ailerons are made independent from each other. Hence, the control surface and actuator redundancy of aircraft control surfaces can be made use of to tolerate the loss of control surfaces or malfunction of actuators.

To exploit the inherent redundancy built in aircraft and achieve the reliability for aircraft flight control, two branches of methods are mainly studied. One is called *active* approach or self-repairing flight control system (SRFCS) [76]. Such an approach includes real-time fault detection procedure and isolation procedure (FDI), and control system reconfiguration procedure in the presence of faults. Due to the flexibility offered by the control system reconfiguration procedure, this type of reliable control is *active* in the event of component faults. In the cases that severe damage such as a heavily damaged control surface happens, these *active* approaches can offer promising stabilizing ability, if provided sufficient time for FDI and reconfiguration procedure. Many researchers are attracted to relevant areas, for example, Tao Gang *et al.* [67], Eslinger *et al.* [23], Wu *et al.* [75, 76], M. Gopinathan *et al.* [30, 31], Wang *et al.* [72] and Liu *et al.* [53, 54]. It should also be noticed that identifying the fault and reconfiguring the controller may result in time delay, which may not be tolerable in many safety-critical applications.

An alternate reliable flight control system design methodology is to design a fixed controller which can tolerate a set of faults by solely exploiting the physical redundancy of aircraft control surfaces. As the resulting controller remains fixed in the presence of faults as well as in nominal condition, this type of reliable control is named *passive* approach. In this type of control strategy, the component faults have been taken into account at the controller design stage. Based on the the inherent redundancies built in aircraft, normally in hardware form (i.e. duplicate actuators, duplicate sensors, multiple control computers, etc), the *passive* approaches are quite promising for especially mild faults.

Compared to *active* designs, one of the advantages of these fixed reliable controllers is

that they guarantee satisfactory system performance not only during normal operations but also under various faults without spending time on fault detection, isolation, and controller reconfiguration. This is very important in cases that the available reaction time for the system is very limited after the occurrence of a fault.

To-date, considerable attention has been attracted and plenty of approaches have been developed, such as [51, 81, 82] etc. In this thesis, one of the primary focuses is on *passive* reliable control problems.

1.3 Anti-windup compensation

Another main issue studied in this thesis is input saturations. In today's literature, there are fairly rich theories for designing controllers - both linear (for example, PI/PID, LQG, LTR, H_2/H_∞ , pole placement, etc.) and nonlinear (for example, nonlinear H_∞ , feedback linearization, sliding mode control, gain scheduling, etc.). There are also plenty of theories dealing with the effect of model uncertainty on controller performance in the context of linear systems, and attempts in developing similar theories for nonlinear systems. However, all real world control systems must deal with constraints but none of these popular controller design techniques account for the presence of input and output constraints (saturations). Although occasionally these constraints can be neglected, in general, they lead to design and operating problems in the final controller implementation.

It is well recognized that actuator saturation degrades the performance of the control system and may even lead to instability [3, 33, 57, 74]. A classic example of the detrimental effect of neglecting saturations comes from the nuclear industry, namely, the Chernobyl unit 4 nuclear power plant disaster in 1986. One of the major causes of this mishap was attributed to the fact that the speed at which the control rods could be moved in and out of the nuclear reactor core was limited. However, the controller did not respect this limit. As the reaction started accelerating, the controller tried to push the rods

into the core faster and faster to slow down the reaction. Shortly the speed of the control rod movement reached limit, and eventually an uncontrolled runaway reaction happened [65]. In flight control area, the destabilizing effects of actuator saturation has also been proved that it will not only cause windup in controller with slow or unstable dynamics [3, 74], but also have been cited as contributing factors in several mishaps involving high performance aircraft [19].

Due to these cases, the constrained control problem receives more and more consideration in recent years. Not surprisingly, quite some activity in this area has been reported. For example, Professor E. Gilbert very clearly outlined the various issues involved in controlling linear systems with “point-wise-in-time constraints” [29] in the 1992 American Control Conference. For the nonlinear systems, Professor David Mayne elucidated numerous open issues with constraints in [15, 56].

In the flight control area, actuator saturation is of particular concern in reliable flight control system design as the remaining actuators have to increase control activity, likely resulting in saturation to compensate for the failed actuator. To address the constrained control problems for linear systems, this thesis will discuss the anti-windup compensation technique.

Windup problems were originally encountered in an industrial context when using PI/PID controllers for controlling systems with input nonlinearities. Later on, it was recognized that integrator windup is only a special case of a far more general problem. Actually, any controller with slow or unstable modes will experience windup problems when implemented on a constrained system [20].

The basic philosophy of anti-windup control technique employs a two-step design paradigm [57, 74]: *Design first the linear controller ignoring control input nonlinearities and then add anti-windup compensation to minimize the adverse effects of any control input saturation on closed-loop performance.* Such anti-windup compensation schemes provide a computationally efficient technique for existing unconstrained controllers to mitigate against input saturation thereby eliminating “controller windup” problem.

Some significant works have been developed, for example. Professor Mayuresh V. Kothare [57], Professor S. R. Arief and other researchers in anti-windup area [23, 33, 74].

1.4 Robust model predictive control

It is also important to notice that although the two stage anti-windup paradigm provides a simple, computational cheap but powerful tool to address input saturation problem, it is not perfect. One drawback is that the added anti-windup compensator to minimize the adverse effects caused by saturation will change the overall closed-loop system. Therefore the desired performance requirements in the first stage may not be guaranteed automatically in the presence of actuator saturation. Another drawback is that anti-windup technique can not handle state or output constraints which are also usually met in flight control area. These two issues are main motivation for our research on Model Predictive Control (MPC) technique.

1.4.1 Standard on-line MPC

Model Predictive Control, also known as Receding Horizon Control (RHC), can be traced back to 1960's and 1970's, when several excellent textbooks emerged on the optimal control theory [4, 12, 46]. In particular, Lee and Markus [46] had an interesting paragraph that described a hypothetical method for obtaining a closed-loop controller from open-loop trajectories. Their novel idea leads to the modern version of MPC technology.

Basically, MPC is a control strategy that solves at each sampling time, a trajectory optimization problem (typically, a linear or quadratic program) to compute optimal control inputs over a fixed/infinite future time “horizon”, using a model to predict future plant outputs. Although more than one control moves are generated, only the first element

is implemented. At the next sampling time, the whole procedure is repeated with new measurement obtained from the system. In this way this becomes a closed loop approach.

The essential features of predictive control include: an explicit internal model, the receding horizon idea, and computation of the control signal by optimizing predicted plant behavior. These fundamental attributes differentiate MPC from other control methods.

There is a plethora of names denoting particular variants of predictive control, usually with corresponding acronyms. Examples of these are:

- Dynamic Matrix Control (DMC),
- Quadratic Dynamic Matrix Control (QDMC),
- Shell Multi-variable Optimizing Control (SMOC),
- Identification-Command (IDCOM),
- Identification-Command (multi-input/output) (IDCOM-M),
- Set-point Multi-variable Control Architecture (SMCA),
- Predictive Control Technology (PCT),
- Robust Multi-variable Predictive Control Technology (RMPCT)

and so on.

The ability to handle constraints is probably the most important reason for the success of MPC in the industry. Constraints appear in nearly every application, e.g., actuators are limited, safety limits apply etc. Generally maximal efficiency is obtained when operating near these constraints. Nevertheless, methods for handling constraints have not been abundant, and MPC is perhaps the most successful technique in which constraints are explicitly handled in a systematic way.

MPC is usually formulated in the state space. The system to be controlled can be described by the difference equation,

$$x(k+1) = f(x(k), u(k)), \quad x(0) = x_0, \quad (1.1)$$

where $x \in \mathbb{R}^{n_x}$ and $u \in \mathbb{R}^{n_u}$ denote the state and control input at time k , respectively. The control objective is usually to steer the state to the origin under state and input constraints:

$$x(k) \in \mathbb{X}, \quad u(k) \in \mathbb{U}, \quad (1.2)$$

where $\mathbb{X}$ is a convex closed subset of $\mathbb{R}^{n_x}$ and $\mathbb{U}$ is a convex compact subset of $\mathbb{R}^{n_u}$, both containing the origin in the interior. Let N denote the prediction horizon. A terminal constraint is sometimes imposed for stability reasons,

$$x(k+N) \in X_f \subset \mathbb{X}. \quad (1.3)$$

A typical formulation of the open loop optimization problem is given as follows.

$$J(x_k) = \min_{\mathbf{u}(k)} \left\{ E(x(k+N)) + \sum_{i=1}^{N-1} l(x(k+i|k), u(k+i|k)) \right\} \quad (1.4)$$

subject to

$$x(k+i+1|k) = f(x(k+i|k), u(k+i|k)), \quad i = 1, 2, \dots, N-1, \quad (1.5)$$

$$x(k+i|k) \in \mathbb{X}, \quad i = 1, 2, \dots, N-1, \quad (1.6)$$

$$x(k+N|k) \in X_f, \quad (1.7)$$

$$x(k|k) = x_k, \quad (1.8)$$

$$u(k+i|k) \in \mathbb{U}, \quad i = 1, 2, \dots, N-1, \quad (1.9)$$

where $l(x(i), u(i))$ is the stage cost, and $E(x(k+N))$ is the terminal cost.

For linear MPC a linear model $f(x(k), u(k)) = Ax(k) + Bu(k)$ is used to predict the system states on the horizon. The cost function is quadratic in the states and inputs, and the constraints are linear. The resulting optimization problem becomes a convex quadratic program (QP), for which reliable commercial software exists. In nonlinear MPC the idea is the same as in linear MPC, but with a nonlinear model describing the process dynamics. The resulting optimization problem becomes a nonlinear program (NLP).

1.4.2 Robust MPC

Explicitly incorporating constraints in the implicit definition of the MPC control law (through QP) makes the analysis of stability and robustness very difficult. This motivated much activity in analyzing MPC strategy's stability, such as S. S. Keerthi and E. G. Gilbert [37] proposing an approach to guarantee stability for MPC using concept of terminal cost, J. B. Rawlings *et al.* [62] using an infinite prediction horizon idea, F. Allgöwer [13, 14] using a quasi-infinite horizon to guarantee closed-loop stability, etc.

Compared with stability research, issues of robustness, and in particular, MPC design in the presence of plant-model uncertainty have been addressed to a much less extent. In 1996, Kothare *et al.* [44] proposed an outstanding approach to address polytopic and structured uncertainty using Linear Matrix Inequality (LMI). A broad class of model uncertainty descriptions were addressed with an upper bound to the “worst-case” infinite horizon objective function derived in [44]. And the problem of minimizing this upper bound is formulated in an LMI framework, subject to input and output constraints. Solving such a convex optimization problem at each time step yields a state feedback controller which guarantees closed-loop stability and robustness. Later on, Wu [73], and Cuzzola, *et al.* [17] developed some improved approaches based on [44]. Meanwhile, two drawbacks of these approaches are also brought into attention. One of which is the

on-line computational complexity. Due to the on-line optimization which involves many parameters to calculate, the application of MPC is restricted for a long time to “slow” response processes where the on-line computation can be accomplished between two sampling instances. In order to address such problems, Wan and Kothare [70] proposed an off-line MPC formulation in 2003 and in the following year another improved off-line approach was developed in [24]. The other drawback of the current LMI based MPC approaches is the lack of analysis on systems with disturbances. As we know, disturbances are widely met and often adversely affect performance and stability. Therefore, guaranteeing an H_∞ norm bound for systems with disturbances is also a very important issue in MPC area. In this thesis, a novel LMI based robust multi-objective MPC technique is proposed for guaranteeing an H_∞ norm bound with an optimized H_2 performance.

1.5 Objectives

In this thesis, the main objectives are as follows:

- (1) To develop the reliable robust tracking controller designs based on H_2 and/or H_∞ robust control theories;
- (2) To develop the reliable robust tracking controller designs based on model predictive control theories;
- (3) To develop the reliable robust tracking controller designs to handle actuator faults and aircraft control/wing/tail surface impairment faults;
- (4) To develop the reliable robust tracking controller designs that take the input saturation nonlinearities into account;
- (5) To guarantee robust stability of the resulting closed-loop systems in the nominal condition as well as when fault occurs;

- (6) To guarantee robust stability of the resulting closed-loop systems in the presence of input saturations;
- (7) To achieve optimized tracking performance in the nominal condition and simultaneously maintain an acceptable tracking performance degradation in the presence of faults and/or input saturations.
- (8) To eliminate the steady-state tracking errors for step reference signals;
- (9) To reduce the design conservativeness of the performance/stability analysis and synthesis;
- (10) To reduce the on-line computational complexity for MPC designs;
- (11) To demonstrate the proposed methods on a high fidelity six degree-of-freedom nonlinear aircraft flight simulation system.

1.6 Major contributions

Major contributions of this thesis are summarized as follows:

- (1) Developed a new reliable H_∞ flight tracking controller based on vertex-dependent Lyapunov functions. The design covers state-feedback and output-feedback cases;
- (2) Developed a new reliable H_2 dynamic output-feedback controller design problem for discrete-time linear systems based on the iterative linear matrix inequality (ILMI) approach. Additional slack variables are adopted in this approach therefore the design conservativeness is further reduced;
- (3) Developed a mixed H_2/H_∞ reliable controller against faults and actuator saturation. The design covers mixed H_2/H_∞ controller design based on vertex-dependent Lyapunov functions and an improved anti-windup compensator design;

- (4) Developed a new on-line model predictive flight controller to safely land an aircraft against faults and input saturations;
- (5) Developed a multi-objective MPC approach to handle faults and constraints. The proposed approach addresses polytopic uncertainty and disturbances simultaneously. With an optimized H_2 performance and a guaranteed H_∞ norm bound, the proposed robust multi-objective MPC approach can cover a broad class of systems. Parameter dependent Lyapunov functions, one corresponding to each and every vertex of the uncertainty polytope, are employed in order to reduce the design conservativeness.
- (6) Developed two novel off-line model predictive controller in order to reduce the on-line computational complexity. Faults and saturations are also considered;
- (7) Developed an improved robust model predictive control with structural uncertainty. The design conservativeness is also reduced by employing vertex-dependent Lyapunov functions.

1.7 Organization

This thesis is organized as follows:

Chapter 1 introduces the background and latest research work on reliable flight control, fault tolerate control, anti-windup compensation and MPC problems. Major contributions are also summarized.

Chapter 2 constructs the six degree-of-freedom nonlinear model for an F-16 fighter. The aircraft model is used in the subsequent chapters as a platform to illustrate the proposed approaches. The fault models and the reliable tracking problem is also formulated in this chapter.

Chapter 3 investigates a reliable H_∞ controller design methods via vertex-dependent

Lyapunov functions. Both state feedback and output feedback cases are covered with reduced design conservativeness.

Chapter 4 investigates a reliable H_2 controller design method. Dynamic output feedback controller is designed against faults. The proposed approach employs additional slack variables and vertex-dependent Lyapunov functions to further reduced the design conservativeness.

Chapter 5 studies mixed H_2/H_∞ reliable aircraft flight controller against faults and actuator saturation. Other than the mixed H_2/H_∞ controller design based on vertex-dependent Lyapunov functions, a new anti-windup compensator design method is discussed to address the input saturations. The aircraft maneuverability is extraordinarily improved through this approach.

Chapter 6 introduces model predictive control to flight control area. An on-line model predictive flight controller is designed to safely land an aircraft in the presence of faults and input saturations.

Chapter 7 investigates the multi-objective MPC problem. The proposed approach can handle polytopic uncertainty and disturbances simultaneously, with an optimized H_2 performance and a guaranteed H_∞ norm bound. Two off-line MPC formulations are proposed in this chapter to significantly reduce the on-line computational complexity.

Chapter 8 presents a novel robust MPC approach with structured uncertainty. The proposed approach can address a broad class of uncertain systems.

Chapter 9 concludes all the approaches proposed in this thesis and gives a brief comparison. Some future research directions are also included.

Chapter 2

Aircraft model and problem formulation

2.1 Aircraft model

2.1.1 Physical aircraft model

The candidate plant selected for study is a high fidelity nonlinear F-16 model that simulates the dynamics of the real aircraft.

Body axis

The body axes of F-16 model are conventional and can be described as in Fig. 2.1. The X-axis is positive out the nose of the aircraft. The Y-axis is positive out the right wing of the aircraft. Z-axis is positive vertically downward. Fig. 2.1 also indicates the positivity of the body rates (p for roll rate, q for pitch rate and r for yaw rate), which obeys the right hand rule on each axis.

Figure 2.1: Body-fixed axis system of F-16 aircraft

Aero-dynamic equations

The equations used to describe the motions of the aircraft are nonlinear six-degree-of-freedom, rigid-body equations and are given as follows.

Forces equations:

$$\dot{u} = rv - qw - g \sin \theta + \frac{\bar{q}S}{m} C_{X,t} + \frac{T}{m}, \quad (2.1)$$

$$\dot{v} = pw - ru + g \cos \theta \sin \phi + \frac{\bar{q}S}{m} C_{Y,t}, \quad (2.2)$$

$$\dot{w} = qu - pv + g \cos \theta \cos \phi + \frac{\bar{q}S}{m} C_{Z,t}. \quad (2.3)$$

Moment equations:

$$\dot{p} = \frac{I_Y - I_Z}{I_X} qr + \frac{I_{XZ}}{I_X} (\dot{r} + pq) + \frac{\bar{q} S b}{I_X} C_{l,t}, \quad (2.4)$$

$$\dot{q} = \frac{I_Z - I_X}{I_Y} pr + \frac{I_{XZ}}{I_Y} (r^2 - p^2) + \frac{\bar{q} S \bar{c}}{I_Y} C_{m,t} - \frac{H_e}{I_Y} r, \quad (2.5)$$

$$\dot{r} = \frac{I_X - I_Y}{I_Z} pq + \frac{I_{XZ}}{I_Z} (\dot{p} - qr) + \frac{\bar{q} S b}{I_Z} C_{n,t} + \frac{H_e}{I_Z} q. \quad (2.6)$$

Kinematic equations:

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi), \quad (2.7)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi, \quad (2.8)$$

$$\dot{\psi} = (q \sin \phi + r \cos \phi) / \cos \theta. \quad (2.9)$$

Auxiliary equations:

$$\alpha = \tan^{-1} \left(\frac{w}{u} \right), \quad (2.10)$$

$$\beta = \sin^{-1} \left(\frac{v}{V_t} \right), \quad (2.11)$$

$$V_t = \sqrt{u^2 + v^2 + w^2}, \quad (2.12)$$

$$a_n = \frac{qu - pv + g \cos \theta \cos \phi - \dot{w}}{g}, \quad (2.13)$$

$$a_y = \frac{-pw + ru - g \cos \theta \sin \phi + \dot{v}}{g}, \quad (2.14)$$

$$\bar{q} = \frac{1}{2} \rho V^2. \quad (2.15)$$

Here u , v and w are the components of aircraft velocity along X , Y and Z body axes

respectively; p , q and r are the aircraft roll rate, pitch rate and yaw rate along X , Y and Z body axes respectively; ϕ , θ and ψ are the aircraft roll angle, pitch angle and yaw angle along X , Y and Z body axes respectively. In the equations above, $C_{X,t}$, $C_{Y,t}$ and $C_{Z,t}$ represent the total force coefficients and $C_{l,t}$, $C_{m,t}$ and $C_{n,t}$ represent the total moment coefficients, along X , Y and Z body axes respectively. They are functions of control surfaces such as elevators, ailerons and rudder, as well as α , β , V_t etc. The aerodynamic data used in these equations were acquired from low-speed static and dynamic wind-tunnel tests conducted at the NASA Ames and Langley Research Center [59]. The static aerodynamic data is given in tabular form for angle of attack and sideslip over the ranges $-20^\circ \leq \alpha \leq 90^\circ$ and $-30^\circ \leq \beta \leq 30^\circ$, respectively. For the purpose of brevity only the X -axis force coefficient $C_{X,t}$ is illustrated as follows. We refer the readers to [59] for more details.

$$C_{X,t} = C_X(\alpha, \beta, \delta_e) + \Delta C_{X,lef} \left(1 - \frac{\delta_{lef}}{25}\right) + \Delta C_{X,sb}(\alpha) \left(\frac{\delta_{sb}}{60}\right) + \frac{\bar{c}q}{2V_t} \left[C_{Xq}(\alpha) + \Delta C_{Xq,lef}(\alpha) \left(1 - \frac{\delta_{lef}}{25}\right) \right] \quad (2.16)$$

where

$$\Delta C_{X,lef} = C_{X,lef}(\alpha, \beta) - C_X(\alpha, \beta, \delta_e = 0^\circ) \quad (2.17)$$

with $C_X(\alpha, \beta, \delta_e)$, $C_{X,lef}(\alpha, \beta)$, $\Delta C_{X,sb}(\alpha)$, $C_{Xq}(\alpha)$ and $\Delta C_{Xq,lef}(\alpha)$ all given in tabular form.

Control inputs

The F-16 aircraft employs engine thrust (δ_T), elevator (δ_e), aileron (δ_a) and rudder (δ_r) as control inputs. Except positive engine thrust increases acceleration in X -axis, other positive control inputs decrease the body rates in corresponding axes. For example, deflecting downward of the right aileron and deflecting upward of the left aileron decrease the roll rate p , which means positive aileron movement. Consequently, a positive elevator movement (downward) results in a decrease in pitch rate q . A positive rudder

movement results in a decrease in yaw rate r , which can be described as a deflection to right. Fig. 2.2 gives a clear illumination on the positive deflections of control inputs.

Figure 2.2: The positive deflection of control surfaces

One additional control input is the leading edge flap, which is designed for a high angle of attack flight. The leading edge flap can not be manipulated directly by the pilot or flight control computer but is scheduled with angle of attack and the static and dynamic pressures as follows,

$$\delta_{lef} = 1.38 \frac{2S + 7.25}{S + 7.25} \alpha - 9.05 \frac{\bar{q}}{p_s} + 1.45. \quad (2.18)$$

Apparently, these control surfaces have saturation limits that should be considered, listed in Table 2.1,

Control inputs	Min	Max
Engine Thrust	1000 lbs	19000 lbs
Elevators	-25 deg	25 deg
Ailerons	-21.5 deg	21.5 deg
Rudder	-30 deg	30 deg
Leading edge flap	0 deg	25 deg

Table 2.1: Saturation limits of control inputs

2.1.2 Wind models

As wind disturbance is a main factor that can cause degradation in the tracking performance during maneuver and landing progress, in this section we will construct wind models for further study in following chapters.

Deterministic wind

Consider the ICAO (International Civil Aeronautics Organization) standard atmosphere with a typical idealized wind profile represented by

$$\begin{cases} V_w = V_{w9.15} \frac{H^{0.2545} - 0.4097}{1.3470}, & 0 < H < 300\text{m} \\ V_w = 2.86585V_{w9.15}, & H \geq 300\text{m} \end{cases} \quad (2.19)$$

where $V_{w9.15}$ stands for the wind speed at altitude of 9.15m. Fig. 2.3 shows the wind profile when $V_{w9.15} = 1\text{m/s}$.

Figure 2.3: The deterministic wind profile

Wind turbulence

Dryden spectrum is used to model wind turbulence in many works. As longitudinal wind turbulence is of much more importance than latitudinal wind turbulence in maneuver and landing process, only horizontal direction turbulence is considered in our study. Mathematically, the longitudinal wind turbulence is represented by

$$\begin{aligned}\dot{u}_{g1} &= 0.2|u_{gc}|\sqrt{2a_u}N_1 - a_u u_{g1} \\ u_g &= u_{g1} + u_{gc}\end{aligned}\tag{2.20}$$

where u_{g1} is the turbulence component and u_{gc} is the mean wind with

$$u_{gc} = \begin{cases} -u_0 \left(1 + \frac{\ln \frac{H}{510}}{\ln 51} \right) & H \geq 10m \\ 0 & H < 10m \end{cases}\tag{2.21}$$

with u_0 is 6m/s and N_1 is the Gaussian random noises with zero mean and variance of 100, and

$$a_u = \begin{cases} \frac{U_0}{100\sqrt[3]{H}} & H > 230m \\ \frac{U_0}{600} & H \leq 230m \end{cases}\tag{2.22}$$

and U_0 refers to the trimmed aircraft speed (eg, 152.4m/s for level flight).

2.1.3 Linearized aircraft model

For ease of analyzing nonlinear system dynamics, trimming and linearization are both efficient tools. Trimming is used to determine a steady state condition followed by linearization to find a linear model at the trimmed state condition. Using small perturbation assumptions from steady-state condition, a set of linear constant-coefficient state equations can be derived at the corresponding equilibrium conditions. For example, the trim condition of straight and level flight in this work is as follows,

δ_{el}	-0.5831°	δ_{er}	-0.5831°	δ_{al}	0.0°	δ_{ar}	0.0°
δ_r	0.1464	δ_r	0.0°	ϕ	0.0°	p	0.0°
θ	2.3703°	q	0.0°	ψ	0.0°	r	0.0°
α	2.3703°	β	0.0°	H	500 m	V_t	152.4 m/s

Table 2.2: Trim condition for straight level flight

The resulting continuous time aircraft model is in the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + B^u u(t) + B^w w(t) \\ y(t) = Cx(t) \end{cases} \quad (2.23)$$

where $x = [u, w, q, v, p, r]^T \in \mathbb{R}^{n_x}$ is the state, control input $u = [\delta_{el}, \delta_{er}, \delta_{al}, \delta_{ar}, \delta_r]^T \in \mathbb{R}^{n_u}$ means left and right elevators, left and right ailerons and rudder, $w \in \mathbb{R}^{n_w}$ is the bounded disturbance and $y = [q, \dot{\mu}_{rat}, r, \alpha, \beta]^T \in \mathbb{R}^{n_y}$ is the output with $\dot{\mu}_{rat}$ representing stability axis roll rate. Although the nonlinear F-16 aircraft model utilizes 10 states ($[H, \theta, \phi, \psi, u, v, w, p, q, r]$) and 5 control inputs, it should also be noted in the subsequent chapters that the whole or partial set of these states and control inputs can be used as x and u in (2.23).

2.2 Problem formulation

As a fighter aircraft, F-16 experiences complicated and dangerous missions in various circumstances. Therefore it's not wise to assume that all the components onboard can always work properly. In this section, we will discuss how to modify the F-16 model to account for some typical fault conditions and how to formulate a zero steady state error tracking problem.

2.2.1 Fault conditions

The most frequently met faults in flight control area are actuator faults which include actuator outage faults, stuck faults and efficiency loss, etc. Another type of faults is control surface impairment.

Actuator outage faults

An actuator is a device that manipulates the control variable, and delivers the control action to the physical system. Failures in actuators can have severe consequences on the performance of the system. In this section, we will discuss the outage fault of actuators.

Compared to surface impairment faults that we will discuss in the next sub-section, actuator outage faults are mild faults that normally will not change the aircraft dynamics (non-dynamics related). Without loss of generality, such fault scenarios can be modeled by relating the actuator output signal and the actuator input signal. Let $u^F(t)$ represent the control surface input vector after actuator outage faults have occurred. Then, the following actuator fault model is adopted

$$u^F(t) = \omega_L u(t), \quad L = 0, 1, \dots, l_p, \quad l_p \leq 2^{n_u} - 1 \quad (2.24)$$

where $\omega_L \in \mathbb{R}^{n_u \times n_u}$ is the scaling factor satisfying

$$\omega_L \in \Omega \triangleq \{\text{diag}\{\omega_{L1}, \omega_{L2}, \dots, \omega_{Ln_u}\}, \quad \omega_{Lj} = 0 \text{ or } 1, \quad j = 1, 2, \dots, n_u\}. \quad (2.25)$$

From equations (2.24) and (2.25), it is clear that $\omega_{Lj} = 0$ ($1 \leq j \leq n_u$) represents the outage fault case of the j^{th} actuator and $\omega_{Lj} = 1$ corresponds to nominal condition of the j^{th} control channel. Definitions of the above parameters can be found in nomenclature. We also define $L = 0$ to correspond to the nominal condition, i.e., $\omega_0 = I_{n_u \times n_u}$. The following possible control surface actuator faults are considered in this study:

1. $\omega_0 = \text{diag}\{1, 1, 1, 1, 1\} \rightarrow$ Nominal condition;
2. $\omega_1 = \text{diag}\{0, 1, 1, 1, 1\} \rightarrow$ Left elevator actuator outage fault;
3. $\omega_2 = \text{diag}\{1, 0, 1, 1, 1\} \rightarrow$ Right elevator actuator outage fault;
4. $\omega_3 = \text{diag}\{1, 1, 0, 1, 1\} \rightarrow$ Left aileron actuator outage fault;
5. $\omega_4 = \text{diag}\{1, 1, 1, 0, 1\} \rightarrow$ Right aileron actuator outage fault;
6. $\omega_5 = \text{diag}\{0, 1, 0, 1, 1\} \rightarrow$ Left elevator and left aileron outage faults;
7. $\omega_6 = \text{diag}\{0, 1, 1, 0, 1\} \rightarrow$ Left elevator and right aileron outage faults;
8. $\omega_7 = \text{diag}\{1, 0, 0, 1, 1\} \rightarrow$ Right elevator and left aileron outage faults;
9. $\omega_8 = \text{diag}\{1, 0, 1, 0, 1\} \rightarrow$ Right elevator and right aileron outage faults.

Control/wing/tail surface impairment faults

Another type of fault is the control/wing/tail surface impairment which is frequently encountered in a battlefield (e.g., elevators damaged in an explosion). In most situations, such fault is regarded as dynamics-related fault since the fault will change physical parameters of the system, e.g., weight of an aircraft, drag and lift, etc. These changes will manifest themselves as parameter changes in the mathematical model describing the aircraft system (i.e., for a linear aircraft model, A , B^u , B^w , C will change). Control/wing/tail surface impairment can be characterized by the percentage loss of the total control/wing/tail surface area. For a number of percentage losses of certain control/wing/tail surfaces, linear models of aircraft can be obtained by trimming and linearizing the nonlinear model of the aircraft with such impairments. For intermediate percentage of surface impairment faults, linear interpolations among all trimmed and linearized models can be used for approximation. Namely, we adopt the following poly-

topic uncertainties in the matrices A, B^u, B^w, C

$$\begin{aligned} A(\theta) &= \sum_{i=0}^l A_i \theta_i, & B^u(\theta) &= \sum_{i=0}^l B_i^u \theta_i, \\ C(\theta) &= \sum_{i=0}^l C_i \theta_i, & B^w(\theta) &= \sum_{i=0}^l B_i^w \theta_i \end{aligned} \quad (2.26)$$

where the parameter $\theta = [\theta_0, \theta_1, \dots, \theta_l]^T$ satisfying

$$\theta \in \Theta \triangleq \left\{ \theta \in \mathbb{R}^{l+1} : \theta_i \geq 0, \sum_{i=0}^l \theta_i = 1 \right\}. \quad (2.27)$$

The vertex matrices A_i, B_i^u, B_i^w, C_i ($i = 0, \dots, l$) are known matrices of appropriate dimensions, which represent the vertices of possible control/wing/tail surface impairment (including the nominal case of no impairment faults). Without loss of generality, suppose that $i = 0$ correspond to the nominal case and $i = 1, \dots, l$, the fault cases.

Due to the restriction imposed by the available aerodynamic data, only symmetric impairment of the control surfaces are considered here, i.e., $u = [\delta_e, \delta_a, \delta_r]^T$ with $\delta_e = \delta_{er} = \delta_{el}$, $\delta_a = \delta_{ar} = \delta_{al}$. In this study, the control/wing/tail surface impairment faults are modeled as efficiency loss to the control surface, for example, 75% efficiency loss to the elevator, symmetrically. Since the aerodynamic characteristics of the aircraft will change nonlinearly from nominal condition to 75% loss of elevator, in order to achieve better modeling accuracy in interpolating polytope vertices, two additional vertices are considered (25% and 50% efficiency loss to the elevator, symmetrically) besides the existing two. The trim conditions for 25%, 50%, 75% loss are, respectively,

- 25% efficiency loss to the elevator:

δ_e	1.0618°	δ_a	0.0°	δ_r	0.0°	δ_T	0.1025
ϕ	0.0°	p	0.0°	θ	2.3877°	q	0.0°
ψ	0.0°	r	0.0°	α	2.3877°	β	0.0°
H	500 m	V_t	152.4 m/s				

Table 2.3: Trim condition for 25% efficiency loss to the elevator

- 50% efficiency loss to the elevator:

δ_e	4.2830°	δ_a	0.0°	δ_r	0.0°	δ_T	0.0682
ϕ	0.0°	p	0.0°	θ	2.4728°	q	0.0°
ψ	0.0°	r	0.0°	α	2.4728°	β	0.0°
H	500 m	V_t	152.4 m/s				

Table 2.4: Trim condition for 50% efficiency loss to the elevator

- 75% efficiency loss to the elevator:

δ_e	15.8683°	δ_a	0.0°	δ_r	0.0°	δ_T	0.0817
ϕ	0.0°	p	0.0°	θ	3.3419°	q	0.0°
ψ	0.0°	r	0.0°	α	3.3419°	β	0.0°
H	500 m	V_t	152.4 m/s				

Table 2.5: Trim condition for 75% efficiency loss to the elevator

Hence, integrating the actuator outage faults (2.24) and control/wing/tail surface impairment faults (2.26), the aircraft dynamics can be characterized as

$$\begin{cases} \dot{x}(t) = A(\theta)x(t) + B^u(\theta)u^F(t) + B^w(\theta)w(t) \\ y(t) = C(\theta)x(t) \end{cases} \quad (2.28)$$

with the polytopic coefficient matrices as given in (2.26). Note that there is a certain degree of over-design in adopting the polytopic uncertainty model (2.28). However, this ensures that the designed performance can be achieved for intermediate percentage of surface losses (as will be demonstrated by the nonlinear aircraft simulation results in following chapters).

2.2.2 Reliable tracking problem formulation

Consider the aircraft model (2.28) with both actuator outage fault (2.24) and control/wing/tail surface impairment faults (2.26), the reliable controller design problem aims to find a controller such that:

- (1) The closed-loop system is robustly stable for all $\omega_L \in \Omega$ and $\theta \in \Theta$.

- (2) In the event of no actuator outage faults or control/wing/tail surface impairment faults, the output signal $Sy(k)$ tracks the reference signal $r(k)$ without steady-state error, that is:

$$\lim_{k \rightarrow \infty} e(k) = 0, \quad e(k) = r(k) - Sy(k) \quad (2.29)$$

and with optimized closed-loop performance. Here S is a known matrix of appropriate dimension to determine which outputs are required to track the reference signal.

- (3) In the case of actuator outage faults and/or control/wing/tail surface impairment faults, the output signal $Sy(k)$ tracks the reference signal $r(k)$ without steady-state error and with an acceptable degradation in tracking performance.
- (4) In the presence of disturbances (wind gust, etc.), the closed-loop system is stable without much tracking performance degradation.

In order to obtain a reliable flight controller with zero steady state tracking error for step input, an integral action is introduced on the tracking error and the following continuous time augmented state-space description is constructed.

$$\begin{aligned} \begin{bmatrix} e(t) \\ \dot{x}(t) \end{bmatrix} &= \begin{bmatrix} 0 & -SC(\theta) \\ 0 & A(\theta) \end{bmatrix} \begin{bmatrix} \int_0^t e(t) dt \\ x(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B^u(\theta) \end{bmatrix} \omega_L u(t) \\ &\quad + \begin{bmatrix} I & 0 \\ 0 & B^w(\theta) \end{bmatrix} \begin{bmatrix} r(t) \\ w(t) \end{bmatrix}, \quad L = 0, 1, \dots, l_p \quad (2.30) \\ \begin{bmatrix} \int_0^t e(t) dt \\ y(t) \end{bmatrix} &= \begin{bmatrix} I & 0 \\ 0 & C(\theta) \end{bmatrix} \begin{bmatrix} \int_0^t e(t) dt \\ x(t) \end{bmatrix}. \end{aligned}$$

Assuming a sampling period of T second, the augmented system can be discretized and the resulting discrete-time system can be written as

$$\begin{cases} x_a(k+1) = A_a(\theta)x_a(k) + B_a^u(\theta)\omega_L u(k) + B_a^w(\theta)w_a(k) \\ y(k) = C_a(\theta)x_a(k) \end{cases} \quad (2.31)$$

where the augmented state vector $x_a(k)$ and the augmented disturbance vector $w_a(k)$ are given by

$$x_a(k) = \begin{bmatrix} x_c(k) \\ x(k) \end{bmatrix}, \quad w_a(k) = \begin{bmatrix} r(k) \\ w(k) \end{bmatrix},$$

with $x_c(k) = \int_0^{kT} e(t)dt$ and the matrices $A_a(\theta)$, $B_a^u(\theta)$, $B_a^w(\theta)$ and $C_a(\theta)$ are the discretizations of the corresponding matrices in (2.28). Note that the equations defining $A_a(\theta)$, $B_a^u(\theta)$ and $B_a^w(\theta)$ are inherently nonlinear. However, by selecting the sampling period T to be fast enough (say, much faster than the short period dynamics) then polytopic structure of the continuous-time model (2.28) can be preserved with high accuracy, namely,

$$\begin{aligned} A_a(\theta) &\approx \sum_{i=0}^l A_{ai}\theta_i, & B_a^u(\theta) &\approx \sum_{i=0}^l B_{ai}^u\theta_i, \\ B_a^w(\theta) &\approx \sum_{i=0}^l B_{ai}^w\theta_i, & C_a(\theta) &\approx \sum_{i=0}^l C_{ai}\theta_i, \end{aligned} \quad (2.32)$$

where

$$\begin{aligned} A_{ai} &= \exp \left\{ \begin{bmatrix} 0, & -SC_i \\ 0, & A_i \end{bmatrix} T \right\} \triangleq \begin{bmatrix} A_{ci}, & B_{ci} \\ 0, & A_{pi} \end{bmatrix}, \\ B_{ai}^u &= \int_0^T \exp \left\{ \begin{bmatrix} 0, & -SC_i \\ 0, & A_i \end{bmatrix} t \right\} dt \cdot \begin{bmatrix} 0 \\ B_i^u \end{bmatrix} \triangleq \begin{bmatrix} B_{ci}^u \\ B_{pi}^u \end{bmatrix}, \\ B_{ai}^w &= \int_0^T \exp \left\{ \begin{bmatrix} 0, & -SC_i \\ 0, & A_i \end{bmatrix} t \right\} dt \cdot \begin{bmatrix} I & 0 \\ 0 & B_i^w \end{bmatrix} \triangleq \begin{bmatrix} B_{ci}^r & B_{ci}^w \\ 0, & B_{pi}^w \end{bmatrix}, \\ C_{ai} &= [0, \quad C_i], \end{aligned} \quad (2.33)$$

with A_i , B_i^u , B_i^w and C_i as given in (2.26) and the matrices A_{ci} , B_{ci} , B_{ci}^u , B_{ci}^r , B_{ci}^w , A_{pi} , B_{pi}^u , B_{pi}^w are partitioned from A_{ai} , B_{ai}^u , B_{ai}^w for anti-windup compensator design in Chapter 5. If we obtain a controller to stabilize the augmented system (2.31), then it also stabilizes the original system (2.23) and guarantees a zero steady state error.

Chapter 3

Reliable robust H_∞ aircraft control against faults

In this chapter, the reliable H_∞ flight tracking controller design problem is studied, in the presence of actuator outage faults and control/wing/tail surface impairment faults. Each fault is defined as a vertex to a polytopic uncertain system, and the design goal is to find a controller to stabilize this uncertain system. Comparing with approaches that use only one common Lyapunov function on all vertices in controller design, parameter dependent Lyapunov functions, one for each and every vertex of the uncertainty polytope, are employed in order to reduce the controller's conservativeness. Multi-objective optimization methodology is applied to optimize the tracking performance during nominal condition and to maintain acceptable degradations when faults occur.

Before proceeding to the controller design, the uncertain system is reproduced here for ease of reading.

$$\begin{cases} x_a(k+1) = A_a(\theta)x_a(k) + B_a^u(\theta)\omega_L u(k) + B_a^w(\theta)w_a(k) \\ y_a(k) = C_a(\theta)x_a(k) \\ z_\infty(k) = C_\infty x_a(k) + D_\infty u(k) \end{cases} \quad (3.1)$$

and

$$\begin{aligned} A_a(\theta) &= \sum_{i=0}^l A_{ai} \theta_i, & B_a^u(\theta) &= \sum_{i=0}^l B_{ai}^u \theta_i, \\ C_a(\theta) &= \sum_{i=0}^l C_{ai} \theta_i, & B_a^w(\theta) &= \sum_{i=0}^l B_{ai}^w \theta_i, \end{aligned} \quad (3.2)$$

where z_∞ is the regulated output signal and C_∞ and D_∞ are weight matrices of appropriate dimension.

3.1 H_∞ flight controller design using parameter-dependent Lyapunov functions

For convenience, we first give the customary H_∞ performance indices over an infinite time horizon: consider system (3.1) and a real number $\gamma_0 > 0$, the exogenous signal w_a is attenuated by γ_0 if, assumed $x(0) = 0$,

$$\sum_{k=0}^{\infty} \|z_\infty(k)\|^2 < \gamma_0 \sum_{k=0}^{\infty} \|w_a(k)\|^2. \quad (3.3)$$

Hereafter, we will design state/output feedback reliable flight controllers in an H_∞ framework by using parameter-dependent Lyapunov functions.

3.1.1 State feedback controller design

For the augmented polytopic system (3.1), consider the static state feedback controller

$$u(k) = K_s x_a(k). \quad (3.4)$$

The closed-loop system of (3.1) becomes

$$\begin{cases} x_a(k+1) = [A_a(\theta) + B_a^u(\theta) \omega_L K_s] x_a(k) + B_a^w(\theta) w_a(k) \\ z(k) = [C_\infty + D_\infty K_s] x_a(k) \end{cases}. \quad (3.5)$$

Theorem 3.1.1 Consider the discrete time linear system (3.1) with given fault cases H_∞ bounds γ_L , $L > 0$. There exists a static state feedback control law (3.4) that robustly stabilizes system (3.5) and satisfies the H_∞ constraint (3.3) in nominal case if there exist matrices $Q_i = Q_i^T > 0$, nonsingular matrix G and matrix V such that for $i = 0, \dots, l$, and $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$,

$$\begin{bmatrix} Q_i - G - G^T & * & * & * \\ A_{ai}G + B_{ai}^u \omega_L V & -Q_i & * & * \\ C_\infty G + D_\infty V & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_L I \end{bmatrix} < 0 \quad (3.6)$$

The state feedback gain K_s is given by

$$K_s = VG^{-1}. \quad (3.7)$$

Proof: Given $Q_i = Q_i^T > 0$ and G the additional matrix of suitable dimension, then $(G^T - Q_i)Q_i^{-1}(G^T - Q_i) \geq 0$, consequently:

$$G + G^T - Q_i \leq G^T Q_i^{-1} G \quad (3.8)$$

holds true. Hence, (3.6) is implied by the following LMI,

$$\begin{bmatrix} -G^T Q_i^{-1} G & * & * & * \\ A_{ai}G + B_{ai}^u \omega_L V & -Q_i & * & * \\ C_\infty G + D_\infty V & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_L I \end{bmatrix} < 0 \quad (3.9)$$

where $V = K_s G$. Then, as G is nonsingular and G^{-1} always exists, multiply (3.9) from the left by $\text{diag}\{Q_i G^{-T}, I, I, I\}$ and from the right by $\text{diag}\{G^{-1} Q_i, I, I, I\}$, the following LMI is obtained

$$\begin{bmatrix} -Q_i & * & * & * \\ A_{ai}Q_i + B_{ai}^u \omega_L W_i & -Q_i & * & * \\ C_\infty Q_i + D_\infty W_i & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_L I \end{bmatrix} < 0 \quad (3.10)$$

where $W_i = K_s Q_i$. This is nothing but the formulation of H_∞ constraint based on one common Lyapunov function in [28]. ■

3.1.2 Output feedback controller design

Since state is not always measurable in some cases. In this section we consider the static output feedback controller

$$u(k) = Ky_a(k) = KC_a x(k). \quad (3.11)$$

Assume that C_a is of full row rank.

The closed-loop system of system (3.1) becomes

$$\begin{cases} x_a(k+1) = [A_a(\theta) + B_a^u(\theta) \omega_L KC_a] x_a(k) + B_a^w(\theta) w_a(k) \\ z(k) = [C_\infty + D_\infty KC_a] x_a(k) \end{cases} \quad (3.12)$$

Then the following robust approach for output feedback case is presented.

Theorem 3.1.2 *Consider the discrete time linear system (3.1) with given fault cases H_∞ bounds γ_L , $L > 0$. There exists a static output feedback control law (3.11) robustly stabilizing system (3.12) and fulfilling the H_∞ constraint (3.3) if there exist matrices $Q_i = Q_i^T > 0$, nonsingular matrix G and matrices K , G_0 and K_0 such that for $i = 0, \dots, l$, and $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$,*

$$\begin{bmatrix} Z_{11} & * & * & * & * & * \\ A_{ai}G & Z_{22} & * & * & * & * \\ C_\infty G & 0 & Z_{33} & * & * & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_L I & * & * \\ G & (B_{ai}^u \omega_L K C_a)^T & 0 & 0 & -I & * \\ G & 0 & (D_\infty K C_a)^T & 0 & 0 & -I \end{bmatrix} < 0 \quad (3.13)$$

where

$$Z_{11} = Q_i - G - G^T - 2G^T G_0 - 2G_0^T G + 2G_0^T G_0,$$

$$Z_{22} = -Q_i - B_{ai}^u \omega_L K C_a (B_{ai}^u \omega_L K_0 C_a)^T - B_{ai}^u \omega_L K_0 C_a (B_{ai}^u \omega_L K C_a)^T + B_{ai}^u \omega_L K_0 C_a (B_{ai}^u \omega_L K_0 C_a)^T,$$

$$Z_{33} = -I - D_\infty K C_a (D_\infty K_0 C_a)^T - D_\infty K_0 C_a (D_\infty K C_a)^T + D_\infty K_0 C_a (D_\infty K_0 C_a)^T.$$

Proof: By using the Schur lemma, it is possible to rewrite (3.13) as follows:

$$\begin{bmatrix} Z_{11} & * & * & * \\ A_{ai}G & Z_{22} & * & * \\ C_\infty G & 0 & Z_{33} & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_L I \end{bmatrix} + \begin{bmatrix} G^T G & * & * & * \\ B_{ai}^u \omega_L K C_a G & B_{ai}^u \omega_L K C_a \times (B_{ai}^u \omega_L K C_a)^T & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} G^T G & * & * & * \\ 0 & 0 & * & * \\ D_\infty K C_a G & 0 & D_\infty K C_a (D_\infty K C_a)^T & * \\ 0 & 0 & 0 & 0 \end{bmatrix} < 0. \quad (3.14)$$

Then

$$\begin{aligned}
 & \begin{bmatrix} Q_i - G - G^T & * & * & * \\ (A_{ai} + B_{ai}^u \omega_L K C_a) G & -Q_i & * & * \\ (C_\infty + D_\infty K C_a) G & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_L I \end{bmatrix} + \\
 & \begin{bmatrix} (G - G_0)^T & & & & & & & \\ \times (G - G_0) & * & & * & * & * & & \\ 0 & (B_{ai}^u \omega_L K C_a - B_{ai}^u \omega_L K_0 C_a) \times & & & * & * & & \\ & (B_{ai}^u \omega_L K C_a - B_{ai}^u \omega_L K_0 C_a)^T & & & * & * & & \\ 0 & & 0 & & 0 & * & & \\ 0 & & 0 & & 0 & 0 & & \end{bmatrix} + \\
 & \begin{bmatrix} (G - G_0)^T & & & & & & & \\ \times (G - G_0) & * & & * & * & * & & \\ 0 & 0 & & * & * & * & & \\ & & (D_\infty K C_a - D_\infty K_0 C_a) \times & & * & & & \\ & & (D_\infty K C_a - D_\infty K_0 C_a)^T & & * & & & \\ 0 & 0 & & 0 & 0 & & & \end{bmatrix} < 0. \tag{3.15}
 \end{aligned}$$

Because the second and third part of (3.15) is nonnegative due to the assumption, then the first part of (3.15) is negative definite. And as what has been shown by Theorem 3.1.1, it implies the closed-loop stability of the output feedback system (3.12). ■

Remark 3.1.1 Theorem 3.1.2 gives a sufficient condition to achieve closed-loop stability and H_∞ performance via a static output feedback control law. Actually, (3.13) is not an LMI. However, if G_0 and K_0 are given, then (3.13) is an LMI in G , K and Q_i , $i = 1, \dots, l$. And the conservativeness lies in the differences of $G - G_0$ and $K - K_0$. By using the following iterative algorithm, this conservativeness can be minimized.

Algorithm 3.1.1 (1) Calculate a stabilizing static output feedback controller K_0^0 . If there is no stabilizing controller that can be found, then stop. The details of finding

- K_0^0 can be found in (Algorithm 5.2 of [52]).
- (2) With K_0^0 from step (1), let $K = K_0^0$ and $G = G_0^0$, solving a feasibility problem (3.15) can yield G_0^0 .
 - (3) Choose independent performance upper bounds γ_L for $L = 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$. Notice that γ_0 is the performance upper bound in nominal condition which is to be minimized.
 - (4) At j^{th} iteration, set $G_0 := G_0^{j-1}$, $K_0 := K_0^{j-1}$ and minimize γ_0^j subject to linear matrix inequality (3.13). This gives G^j , K^j and γ_0^j .
 - (5) If $|\gamma_0^j - \gamma_0^{j-1}| < \varepsilon$ where $\varepsilon > 0$ is a given error tolerance, set the obtained K^j as the static output feedback controller K . Otherwise, set $G_0^j := G^j$, $K_0^j := K^j$ and return to Step 3.

Remark 3.1.2 As the LMI (3.13) is solvable, the given algorithm is convergent. Compared with approaches where only one common Lyapunov function is adopted, the proposed reliable aircraft flight controller design method is based on vertex dependent Lyapunov functions each one corresponding to a different vertex of the uncertainty polytope. By this method, the conservativeness can be reduced. The design and simulation in Section 3.2 also illuminate the advantage.

3.2 Design and simulation

In modern aircraft flight controller design, more and more challenging maneuvers such as high angle of attack flight or high roll rate flight under a low air speed condition are considered. Under these conditions, significant kinematic and inertial couplings are prevalent and must be handled properly. Therefore, three variables must be stabilized and tracked closely, namely, stability axis roll rate $\dot{\mu}_{rat}$, angle of attack α and sideslip angle β .

In this section, two examples of reliable tracking control for a nonlinear F-16 aircraft model are given to demonstrate the proposed method. The nonlinear dynamic model for this F-16 aircraft can be found in Chapter 2. The following parameters are used for the two examples,

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, C_\infty = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 6} \\ 0_{3 \times 3} & 0_{3 \times 6} \end{bmatrix}, D_\infty = \begin{bmatrix} 0_{3 \times 3} \\ 0.1I_{3 \times 3} \end{bmatrix},$$

$$\gamma_{Li} = 1, \quad i = 0, \dots, l, \quad L = 0, \dots, l_p, \quad Li \neq 00, \quad l_p \leq 2^{n_u} - 1,$$

where S determines the outputs required to track, i.e., $\dot{\mu}_{rat}$, α and β .

3.2.1 Example 1 (Actuator outage faults)

First, only actuator outage faults are considered. The trim condition is in Table 2.2. Then, the F-16 aircraft with independent control/wing/tail surfaces can be approximately modelled as follows,

$$\begin{cases} x(k+1) = Ax(k) + B^u \omega_L u(k) + B^w w(k) \\ y(k) = Cx(k) \quad L = 0, \dots, 8 \end{cases} \quad (3.16)$$

where $x = [u, w, q, v, p, r]^T$, $y = [q, \dot{\mu}_{rat}, r, \alpha, \beta]^T$, w represents the vertical gust disturbance and A , B^u , B^w , C are system matrices trimmed from the nonlinear aircraft model and discretized with a sampling period of 0.01s, given as below.

$$\begin{aligned}
A &= \begin{bmatrix} -0.0136 & 6.3495 & -0.4616 & 0 & -0.7888 & 0 \\ -0.0002 & -0.6599 & 0.0012 & 0 & 0.9391 & 0 \\ -0.0000 & -0.0096 & -0.1914 & 0.0794 & 0 & -0.9918 \\ 0.0002 & -0.6375 & -22.1194 & -2.2486 & 0 & 0.4935 \\ 0.0008 & -2.0330 & -0.0437 & 0 & -0.8827 & 0 \\ -0.0000 & 0.0541 & 4.8092 & -0.0454 & 0 & -0.3190 \end{bmatrix}; \\
B^u &= \begin{bmatrix} 0.0387 & 0.0387 & 0 & 0 & 0 \\ -0.0014 & -0.0014 & 0 & 0 & 0 \\ 0 & 0 & -0.0002 & 0.0002 & 0.0005 \\ -0.0012 & -0.0012 & 0.4279 & -0.4279 & 0.0812 \\ -0.1188 & -0.1188 & 0 & 0 & 0 \\ 0.0015 & 0.0015 & 0.0188 & -0.0188 & -0.0404 \end{bmatrix}; B^w = \begin{bmatrix} 0.00048 \\ -0.00960 \\ 0.00005 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \\
C &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2468 & 2.3696 \\ 0 & 0 & 0 & 0 & -2.3696 & 57.2468 \\ -0.0155 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

In this example, the actuator outage faults described in Chapter 2 are considered. By using the proposed approach in Section 3.1.1, a reliable state feedback tracking controller is designed to tolerate the above outage faults. For comparison purpose, we also designed a standard (without considering the actuator outage faults) state feedback controller by using the same LMI formulation. The numerical values of these two controller are given as below.

$$\begin{aligned}
K_{reliable} &= \begin{bmatrix} 0.061995 & -70.693 & 1.7135 & -0.040303 & 1156.9 & -33.369 \\ 0.061995 & -70.693 & 1.7135 & -0.040303 & 1156.9 & -33.369 \\ 27.64 & 2.3034 & -59.425 & 0.0009500 & -36.25 & 1101.7 \\ -27.64 & -2.3034 & 59.425 & -0.0009500 & 36.25 & -1101.7 \\ -2.133 & -9.7226 & 667.63 & -0.0064891 & 208.25 & -12023 \end{bmatrix} \\
&\quad \begin{bmatrix} 1.2932 & 169.05 & 6.2959 \\ 1.2932 & 169.05 & 6.2959 \\ -151.64 & -7.2475 & -209.75 \\ 151.64 & 7.2475 & 209.75 \\ -112.12 & 28.294 & 2174.2 \end{bmatrix}, \\
K_{standard} &= \begin{bmatrix} -0.13408 & -57.923 & 2.2733 & -0.023733 & 812.14 & -45.696 \\ -0.13408 & -57.923 & 2.2733 & -0.023733 & 812.14 & -45.696 \\ 19.313 & 0.99684 & -57.405 & 0.0004559 & -19.304 & 1061.5 \\ -19.313 & -0.99684 & 57.405 & -0.0004559 & 19.304 & -1061.5 \\ -4.3243 & -9.9125 & 668.01 & -0.0066897 & 212.2 & -12027 \end{bmatrix} \\
&\quad \begin{bmatrix} -0.10218 & 98.182 & 8.3681 \\ -0.10218 & 98.182 & 8.3681 \\ -49.378 & -2.7589 & -193.96 \\ 49.378 & 2.7589 & 193.96 \\ -91.491 & 29.67 & 2176.6 \end{bmatrix}.
\end{aligned}$$

Following figures, Fig. 3.1 to Fig. 3.8, describe the performance of two controllers in nominal case and eight different outage faults $\omega_1 - \omega_8$. The tracking commands are unit steps and the simulations are carried out on the nonlinear F-16 model.

Figure 3.1: Nonlinear simulation results comparison: nominal case and fault case 1

Figure 3.2: Nonlinear simulation results comparison: nominal case and fault case 2

Figure 3.3: Nonlinear simulation results comparison: nominal case and fault case 3

Figure 3.4: Nonlinear simulation results comparison: nominal case and fault case 4

Figure 3.5: Nonlinear simulation results comparison: nominal case and fault case 5

Figure 3.6: Nonlinear simulation results comparison: nominal case and fault case 6

Figure 3.7: Nonlinear simulation results comparison: nominal case and fault case 7

Figure 3.8: Nonlinear simulation results comparison: nominal case and fault case 8

From these figures above, it can be seen that both the standard and reliable controllers perform well in nominal condition and standard controller is to some extent better. However, the reliable controller gives much better tracking performance in the presence of any of the eight actuator outage faults. Especially, when elevators are in fault condition the standard controller leads to an evident degradation of the tracking performance.

On the other hand, benefited from the properly chosen output signals and the ILMI method proposed in Section 3.1.2, the output feedback controller performs the same as the state feedback controller. Therefore the simulation results are omitted here for brevity.

3.2.2 Example 2 (Control/wing/tail surface impairment)

In this example, control/wing/tail surface impairment faults (up to 75% efficiency loss to the elevator, symmetrically) are considered. Since the aerodynamic characteristics of the aircraft will change nonlinearly from nominal condition to 75% loss of elevator, in order to achieve better modeling accuracy in interpolating polytope vertices, we consider two additional vertices (25% and 50% efficiency loss to the elevator, symmetrically) besides the existing two (trim conditions are shown in Chapter 2). Then, the aircraft model with control/wing/tail surface impairment can be written as

$$\begin{cases} x(k+1) = A(\theta)x(k) + B^u(\theta)u(k) + B^w(\theta)w(k) \\ y(k) = C(\theta)x(k) \end{cases} \quad (3.17)$$

where

$$\begin{aligned} A(\theta) &= \sum_{i=0}^3 A_i \theta_i, & B^u(\theta) &= \sum_{i=0}^3 B_i^u \theta_i, \\ C(\theta) &= \sum_{i=0}^3 C_i \theta_i, & B^w(\theta) &= \sum_{i=0}^3 B_i^w \theta_i. \end{aligned} \quad (3.18)$$

The system matrices are given as below, corresponding to nominal condition and 3 fault conditions as discussed in Chapter 2.

$$A_0 = \begin{bmatrix} -0.0136 & 6.3495 & -0.4616 & 0 & -0.7888 & 0 \\ -0.0002 & -0.6599 & 0.0012 & 0 & 0.9391 & 0 \\ -0.0000 & -0.0096 & -0.1914 & 0.0794 & 0 & -0.9918 \\ 0.0002 & -0.6375 & -22.1194 & -2.2486 & 0 & 0.4935 \\ 0.0008 & -2.0330 & -0.0437 & 0 & -0.8827 & 0 \\ -0.0000 & 0.0541 & 4.8092 & -0.0454 & 0 & -0.3190 \end{bmatrix};$$

$$B_0^u = \begin{bmatrix} 0.0387 & 0 & 0 \\ -0.0014 & 0 & 0 \\ 0 & 0.0002 & 0.0005 \\ -0.0012 & -0.4279 & 0.0812 \\ -0.1188 & 0 & 0 \\ 0.0015 & -0.0188 & -0.0404 \end{bmatrix}; \quad B_0^w = \begin{bmatrix} 0.00048 \\ -0.00960 \\ 0.00005 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$C_0 = \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2468 & 2.3696 \\ 0 & 0 & 0 & 0 & -2.3696 & 57.2468 \\ -0.0155 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}.$$

$$A_1 = \begin{bmatrix} -0.0131 & 8.3675 & -0.6233 & 0 & -0.96240 \\ -0.0002 & -0.5693 & -0.0001 & 0 & 0.95090 \\ -0.0000 & -0.0003 & -0.1542 & 0.1041 & 0 - 0.9906 \\ -0.0000 & 0.8462 & -18.8048 & -1.7934 & 00.4876 \\ 0.0007 & -2.0227 & 0.0020 & 0 & -0.72560 \\ -0.0000 & -0.0285 & 3.8209 & -0.0454 & 0 - 0.2525 \end{bmatrix};$$

$$B_1^u = \begin{bmatrix} 0.0112 & 0 & 0 \\ -0.0012 & 0 & 0 \\ 0 & 0.0002 & 0.0004 \\ -0.0008 & -0.3378 & 0.0642 \\ -0.0948 & 0 & 0 \\ 0.0012 & -0.0149 & -0.0322 \end{bmatrix}; \quad B_1^w = \begin{bmatrix} 0.00025 \\ -0.00709 \\ 0.00001 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2460 & 2.3871 \\ 0 & 0 & 0 & 0 & -2.3871 & 57.2460 \\ -0.0157 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}.$$

$$\begin{aligned}
A_2 &= \begin{bmatrix} -0.0128 & 5.4569 & -0.8683 & 0 & -1.1730 & 0 \\ -0.0002 & -0.3857 & -0.0022 & 0 & 0.9687 & 0 \\ -0.0000 & 0.0029 & -0.1028 & 0.1742 & 0 & -0.9822 \\ -0.0004 & -0.1808 & -16.0589 & -1.1081 & 0 & 0.5274 \\ 0.0005 & -1.5655 & 0.0367 & 0 & -0.4877 & 0 \\ -0.0000 & -0.0918 & 2.2985 & -0.0512 & 0 & -0.1492 \end{bmatrix}; \\
B_2^u &= \begin{bmatrix} -0.0368 & 0 & 0 \\ -0.0009 & 0 & 0 \\ 0 & 0.0001 & 0.0003 \\ 0.0001 & -0.2094 & 0.0385 \\ -0.0593 & 0 & 0 \\ 0.0006 & -0.0094 & -0.0198 \end{bmatrix}; \quad B_2^w = \begin{bmatrix} 0.00001 \\ -0.00453 \\ 0.00004 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \\
C_2 &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2424 & 2.4720 \\ 0 & 0 & 0 & 0 & -2.4720 & 57.2424 \\ -0.0162 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}. \\
A_3 &= \begin{bmatrix} -0.0221 & -16.8235 & 4.2538 & 0 & -1.2501 & 0 \\ -0.0002 & -0.1404 & 0.0217 & 0 & 0.9861 & 0 \\ -0.0000 & 0.0073 & -0.0574 & 0.3481 & 0 & -0.9370 \\ -0.0013 & 2.8718 & -16.7660 & -0.4682 & 0 & 0.1478 \\ 0.0005 & -0.3377 & 0.2112 & 0 & -0.1816 & 0 \\ 0.0001 & -0.7063 & 0.4240 & -0.0165 & 0 & -0.0735 \end{bmatrix}; \\
B_3^u &= \begin{bmatrix} -0.0200 & 0 & 0 \\ -0.0004 & 0 & 0 \\ 0 & 0.0000 & 0.0001 \\ -0.0000 & -0.1137 & 0.0193 \\ -0.0294 & 0 & 0 \\ -0.0002 & -0.0007 & -0.0107 \end{bmatrix}; \quad B_3^w = \begin{bmatrix} 0.00021 \\ -0.00192 \\ 0.00009 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \\
C_3 &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.1983 & 3.3400 \\ 0 & 0 & 0 & 0 & -3.3400 & 57.1983 \\ -0.0219 & 0.3753 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

In this example, the control/wing/tail surface impairment faults described in Chapter 2 are considered. By using the proposed approach in Section 3.1.1, a reliable state feed-

back tracking controller is designed to tolerate these faults. For comparison purpose, we also designed a standard (without considering the faults) state feedback controller by using the same LMI formulation. The numerical values of the controllers are given as below.

$$K_{reliable} = \begin{bmatrix} -0.0018 & -36.6792 & 0.1328 & -0.6142 & 7.1768 & 293.4076 \\ -11.7712 & 0.0041 & 2.8889 & -0.0000 & 0.0004 & 0.0602 \\ 2.0357 & 0.0204 & 20.1053 & -0.0006 & 0.0053 & 0.6501 \\ -0.0252 & -0.0047 & 0.1799 & & & \\ -0.6375 & 43.9843 & 23.4762 & & & \\ -2.9578 & -11.5442 & 133.1401 & & & \end{bmatrix},$$

$$K_{standard} = \begin{bmatrix} -0.0000 & -16.9992 & 0.0245 & -0.1455 & 2.1498 & 57.7607 \\ -11.7114 & 0.0033 & 3.1470 & 0.0000 & -0.0004 & 0.0001 \\ 2.0433 & 0.0186 & 17.8935 & 0.0001 & -0.0022 & -0.0001 \\ 0.0000 & 0.0003 & -0.0127 & & & \\ -0.6659 & 43.7695 & 22.8552 & & & \\ -2.1625 & -10.4762 & 107.0869 & & & \end{bmatrix}.$$

Following figures, Fig. 3.9 to Fig. 3.11, describe the performance of two controllers in nominal case and three different fault cases. The tracking commands are unit steps and the simulations are carried out on the nonlinear F-16 model.

From these figures above, we can see that in nominal condition where no efficiency loss has occurred on the elevator, both the standard controller and the reliable controller are working well and the standard controller is to some extent better (faster response when tracking α). However, when fault occurs, the standard controller experiences a major performance degradation, especially when the efficiency loss percentage is beyond 50%, resulting in a large overshoot (sub-figure 5 and 6 in Fig. 3.10) and an acute oscillation (Fig. 3.11). Meanwhile, the reliable controller can be found not sensitive to these faults. For nominal case and all three fault cases the reliable controller provides good tracking ability and maintains the closed-loop stability.

Figure 3.9: Nonlinear simulation results comparison: nominal case and 25% efficiency loss

Figure 3.10: Nonlinear simulation results comparison: nominal case and 50% efficiency loss

Figure 3.11: Nonlinear simulation results comparison: nominal case and 75% efficiency loss

Furthermore, following the ILMI method proposed in Section 3.1.2, a reliable output feedback controller is designed and applied to the nonlinear F-16 aircraft model. A Herbst-like maneuver is performed, which consists of a large angle-of-attack pull-up, followed by a stability axis roll. The scenario also includes a 30% efficiency loss on elevator from $t = 2$ s, a 40% efficiency loss from $t = 5$ s, a 50% efficiency loss from $t = 8$ s followed by a 65% efficiency loss from $t = 30$ s. For comparison purpose, the standard output feedback controller is also applied to the nonlinear aircraft model. The simulation results are given in Fig. 3.12 and Fig. 3.13.

Figure 3.12: Nonlinear simulation of Herbst-like maneuver by reliable output feedback controller

Figure 3.13: Nonlinear simulation of Herbst-like maneuver by standard output feedback controller

Clearly, from Fig. 3.12 and Fig. 3.13, it can be seen that the standard controller can not stabilize the aircraft in the presence of control/wing/tail surface impairment although it performs well in nominal condition, while the reliable controller can still work properly with acceptable degradation in performance.

Summarizing the two examples presented in this section, it is noted that the reliable controller design method proposed can significantly improve the system performance in the presence of faults, without much sacrifice of performance in nominal condition.

3.3 Conclusion

This chapter discusses the reliable flight controller design problems subject to H_∞ constraint. The actuator outage faults and control surface impairment have been considered by using polytopic fault models. To reduce the controller's conservativeness, the proposed approaches are based on several parameter dependent Lyapunov functions, each one corresponding to a different vertex of the uncertainty polytope. Both state feedback and output feedback cases are covered in this chapter. By using an ILMI based method, the output feedback case can be solved. Finally, the proposed methods are applied to a nonlinear F-16 aircraft model and the effectiveness is illuminated by the simulation results.

Chapter 4

Improved reliable H_2 output feedback controller design for F-16 aircraft against faults

In Chapter 3 we have discussed how to design a state feedback controller and proposed an ILMI based approach for *static* output feedback case. In this chapter, we will further study how to design a *dynamic* output feedback controller. The achievements include not only finding an internally stabilizing output feedback controller that optimizes the nominal performance and respects the reliability constraint (in terms of stability and performance) under fault conditions, but also deriving a less-conservative matrix inequality formulation than most of the existing sufficient conditions. At the end of this chapter an application based on a nonlinear F-16 aircraft model with faults is presented to illustrate the effectiveness of the proposed approach.

First of all the linearized model is re-produced here for ease of reading. By discretizing the continuous time uncertain system in (2.23) with sampling time 0.01s, the following

discrete time model is obtained,

$$\begin{cases} x(k+1) = A(\theta)x(k) + B^u(\theta)u(k) + B^w(\theta)w(k) \\ y(k) = C(\theta)x(k) \\ z(k) = C_2x(k) + D_2u(k) \end{cases} \quad (4.1)$$

where C_2 and D_2 are weight matrices that define H_2 performance variable z and $\theta = [\theta_0, \theta_1, \dots, \theta_l]^T$ satisfies

$$\theta \in \Theta \triangleq \left\{ \theta \in \mathbb{R}^{l+1} : \theta_i \geq 0, \sum_{i=0}^l \theta_i = 1 \right\}. \quad (4.2)$$

and

$$\begin{aligned} A(\theta) &= \sum_{i=0}^l A_i \theta_i, & B^u(\theta) &= \sum_{i=0}^l B_i^u \theta_i, \\ C(\theta) &= \sum_{i=0}^l C_i \theta_i, & B^w(\theta) &= \sum_{i=0}^l B_i^w \theta_i. \end{aligned} \quad (4.3)$$

Assume that, a dynamic output feedback controller for the given plant is of the following form

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c y(k) \\ u(k) = C_c x_c(k) + D_c y(k) \end{cases}. \quad (4.4)$$

Denote $x_\zeta = [x^T \ x_c^T]^T$. The resulting closed-loop system can be expressed as follows

$$\begin{cases} x_\zeta(k+1) = A_\zeta(\theta)x_\zeta(k) + B_\zeta(\theta)w(k) \\ z(k) = C_\zeta(\theta)x_\zeta(k) + D_\zeta(\theta)w(k) \end{cases}, \quad (4.5)$$

where $\theta \in \Theta$ and

$$\begin{aligned} A_\zeta(\theta) &= \sum_{i=0}^l A_{\zeta i} \theta_i, & B_\zeta(\theta) &= \sum_{i=0}^l B_{\zeta i} \theta_i, \\ C_\zeta(\theta) &= \sum_{i=0}^l C_{\zeta i} \theta_i, & D_\zeta(\theta) &= \sum_{i=0}^l D_{\zeta i} \theta_i, \end{aligned} \quad (4.6)$$

with

$$\begin{aligned} A_{\zeta i} &= \begin{bmatrix} A_i + B_i^u D_c C_i & B_i^u C_c \\ B_c C_i & A_c \end{bmatrix}, & B_{\zeta i} &= \begin{bmatrix} B_i^w \\ 0 \end{bmatrix}, \\ C_{\zeta i} &= [C_2 + D_2 D_c C_i \quad D_2 C_c], & D_{\zeta i} &= 0. \end{aligned} \quad (4.7)$$

The closed-loop transfer matrix from w to z is given by

$$T_{zw} = T_{zw}(z) \triangleq C_{\zeta}(zI - A_{\zeta})^{-1}B_{\zeta} + D_{\zeta}. \quad (4.8)$$

4.1 Review of robust H_2 control

Given a discrete time linear system (4.1) with scalar $\gamma > 0$, the control goal is to find an internally stabilizing output feedback controller (4.4) such that $\|T_{zw}\|_2 < \gamma$ holds for all defined uncertainties (i.e., $\forall \theta \in \Theta$).

The H_2 problem has been studied extensively for many years by many researchers. The following formulation proposed by Shaked *et al.* [63] and Geromel *et al.* [27] is adopted in this study. And is reproduced here for convenience.

Suppose that the matrices $(A_{\zeta}, B_{\zeta}, C_{\zeta}, D_{\zeta})$ in (4.5) are given. The H_2 norm of system (4.5) can be computed by solving the following optimization problem with $P = P^T > 0$:

$$\min_P \text{trace}(C_{\zeta} P C_{\zeta}^T + D_{\zeta} D_{\zeta}^T) \quad (4.9)$$

subject to

$$A_{\zeta} P A_{\zeta}^T - P + B_{\zeta} B_{\zeta}^T < 0. \quad (4.10)$$

Note that (4.10) can be rewritten as

$$\begin{bmatrix} A_\zeta & B_\zeta \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_\zeta^T \\ B_\zeta^T \end{bmatrix} - P < 0, \quad (4.11)$$

which, by Schur complement, is equivalent to

$$\begin{bmatrix} A_\zeta^T \\ B_\zeta^T \end{bmatrix} Q \begin{bmatrix} A_\zeta & B_\zeta \end{bmatrix} - \begin{bmatrix} Q & 0 \\ 0 & I \end{bmatrix} < 0, \quad (4.12)$$

where $Q = P^{-1} > 0$.

In this chapter, motivated by the work of Peaucelle *et al.* [60] together with Xie *et al.* [77], an improved design method for the H_2 output feedback control problem is proposed.

4.2 Reliable H_2 controller design

This section first presents a less-conservative analysis result for evaluating the upper bound of the H_2 norm of uncertain discrete time systems. Then the result is applied to derive a less-conservative design for the reliable dynamic output feedback controller.

Lemma 4.2.1 (Peaucelle *et al.* [60]) *There exists a matrix $Q = Q^T > 0$ to (4.12) if and only if there exists a solution (F, G, Q) where G is nonsingular and F is of appropriate dimension, such that*

$$\begin{bmatrix} - \begin{bmatrix} Q & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} A_\zeta^T \\ B_\zeta^T \end{bmatrix} F + F^T \begin{bmatrix} A_\zeta & B_\zeta \end{bmatrix} & * \\ -F + G^T \begin{bmatrix} A_\zeta & B_\zeta \end{bmatrix} & Q - (G + G^T) \end{bmatrix} < 0. \quad (4.13)$$

Proof: Omitted for brevity. Please refer to Peaucelle *et al.* [60] for details. ■

Lemma 4.2.2 (Xie et al. [77]) *Given a dynamic controller in (4.4), the upper bound of the H_2 norm of the augmented system (4.5) can be evaluated by the following optimization:*

$$\min_{(F, G, Q^{(i)}, i=0,1,2,\dots,l)} \text{trace}(S) \quad (4.14)$$

subject to

$$\begin{bmatrix} -\begin{bmatrix} Q^{(i)} & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} A_{\zeta i}^T \\ B_{\zeta i}^T \end{bmatrix} F + F^T \begin{bmatrix} A_{\zeta i} & B_{\zeta i} \end{bmatrix} & * \\ -F + G^T \begin{bmatrix} A_{\zeta i} & B_{\zeta i} \end{bmatrix} & Q^{(i)} - (G + G^T) \end{bmatrix} < 0 \quad (4.15)$$

and

$$\begin{bmatrix} S & * & * \\ C_{\zeta i}^T & Q^{(i)} & 0 \\ D_{\zeta i}^T & 0 & I \end{bmatrix} > 0 \quad (4.16)$$

for $i = 0, 1, 2, \dots, l$, where $(A_{\zeta i}, B_{\zeta i}, C_{\zeta i}, D_{\zeta i})$ are the matrices in (4.7), and symmetric matrices $Q^{(i)} > 0$, nonsingular matrix G and matrix F are decision variables of appropriate dimension.

Proof: Omitted for brevity. Please refer to Xie et al. [77] for details. ■

Remark 4.2.1 *The result from Lemma 4.2.2 implies the equivalence between (4.9)-(4.10) and (4.14)-(4.16). Furthermore, suppose $(A_{\zeta i}, B_{\zeta i}, C_{\zeta i}, D_{\zeta i})$ are known, (4.14)-(4.16) would render a less-conservative evaluation of the upper-bound of the H_2 norm of system (4.5), since the extra freedom given by slack parameters F , G and the vertex-dependent $Q^{(i)}$. Note that when F is setting to 0, Lemma 4.2.2 reduces to the result in Shaked et al. [63] and Geromel et al. [27]. This additional matrix parameter enables us to derive a less-conservative design than that of Shaked et al. [63] and Geromel et al. [27].*

Theorem 4.2.1 Consider the discrete time system (4.1). A dynamic output feedback controller in (4.4) that gives a guaranteed H_2 performance exists if there exists a solution $(R, W, S_A, S_B, S_C, S_D, T, S, \hat{Q}_{11}^{(i)}, \hat{Q}_{21}^{(i)}, \hat{Q}_{22}^{(i)}, \lambda_1, \lambda_2, R_0, W_0, S_{C,0}, S_{D,0})$ to the following optimization problem

$$\min_{(R, W, S_A, S_B, S_C, S_D, T, S, \hat{Q}_{11}^{(i)}, \hat{Q}_{21}^{(i)}, \hat{Q}_{22}^{(i)}, i=0,1,2,\dots,l, \lambda_1, \lambda_2, R_0, W_0, S_{C,0}, S_{D,0})} \text{trace}(S)$$

subject to

$$\begin{bmatrix} Z_{11} & * & * & * & * & * & * \\ Z_{21} & Z_{22} & * & * & * & * & * \\ \lambda_1 B_i^{wT} R & \lambda_1 B_i^{wT} W & -I & * & * & * & * \\ R^T A_i - \lambda_1 R & -\lambda_1 W - \lambda_2 T^T & R^T B_i^w & Z_{44} & * & * & * \\ W^T A_i + S_B C_i + S_A & -S_A + \lambda_2 T^T & W^T B_i^w & Z_{54} & Z_{55} & * & * \\ \lambda_1 R + B_i^u (S_D C_i + S_C) & \lambda_1 W - B_i^u S_C & 0 & 0 & 0 & -I & * \\ B_i^u (S_D C_i + S_C) & -B_i^u S_C & 0 & R & W & 0 & -I \end{bmatrix} < 0, \quad (4.17)$$

and

$$\begin{bmatrix} S & * & * & * \\ C_2^T + C_i^T S_D^T D_2^T + S_C^T D_2^T & \hat{Q}_{11}^{(i)} & * & * \\ -S_C^T D_2^T & \hat{Q}_{21}^{(i)} & \hat{Q}_{22}^{(i)} & * \\ 0 & 0 & 0 & I \end{bmatrix} > 0, \quad (4.18)$$

where

$$\begin{aligned} Z_{11} = & \lambda_1 (A_i^T R + R^T A_i) - \hat{Q}_{11}^{(i)} - \lambda_1^2 R^T R_0 - \lambda_1^2 R_0^T R + \lambda_1^2 R_0^T R_0 \\ & - 2 (B_i^u S_D C_i + B_i^u S_C)^T (B_i^u S_{D,0} C_i + B_i^u S_{C,0}) \\ & - 2 (B_i^u S_{D,0} C_i + B_i^u S_{C,0})^T (B_i^u S_D C_i + B_i^u S_C) \\ & + 2 (B_i^u S_{D,0} C_i + B_i^u S_{C,0})^T (B_i^u S_{D,0} C_i + B_i^u S_{C,0}), \end{aligned} \quad (4.19)$$

$$\begin{aligned}
Z_{21} = & \lambda_1 W^T A_i + \lambda_2 (S_B C_i + S_A) - \hat{Q}_{21}^{(i)} - \lambda_1^2 W_0^T R - \lambda_1^2 W^T R_0 + \lambda_1^2 W_0^T R_0 \\
& + 2 (B_i^u S_{C,0})^T (B_i^u S_D C_i + B_i^u S_C) + 2 (B_i^u S_C)^T (B_i^u S_{D,0} C_i + B_i^u S_{C,0}) \\
& - 2 (B_i^u S_{C,0})^T (B_i^u S_{D,0} C_i + B_i^u S_{C,0}), \quad (4.20)
\end{aligned}$$

$$\begin{aligned}
Z_{22} = & -\lambda_2 (S_A + S_A^T) - \hat{Q}_{22}^{(i)} - \lambda_1^2 W_0^T W - \lambda_1^2 W^T W_0 + \lambda_1^2 W_0^T W_0 \\
& - 2 (B_i^u S_{C,0})^T (B_i^u S_C) - 2 (B_i^u S_C)^T (B_i^u S_{C,0}) + 2 (B_i^u S_{C,0})^T (B_i^u S_{C,0}), \quad (4.21)
\end{aligned}$$

$$Z_{44} = \hat{Q}_{11}^{(i)} - (R + R^T) - R_0^T R - R^T R_0 + R_0^T R_0, \quad (4.22)$$

$$Z_{54} = \hat{Q}_{21}^{(i)} - (T + W^T) - W_0^T R - W^T R_0 + W_0^T R_0, \quad (4.23)$$

$$Z_{55} = \hat{Q}_{22}^{(i)} + (T + T^T) - W_0^T W - W^T W_0 + W_0^T W_0, \quad (4.24)$$

for $i = 0, 1, 2, \dots, l$. The dynamic controller is given by

$$A_c = T^{-1} S_A, \quad B_c = T^{-1} S_B, \quad C_c = S_C, \quad D_c = S_D. \quad (4.25)$$

Proof: Specify the matrix F as

$$F = [\Lambda G \quad 0_{2n_x \times n_u}], \quad (4.26)$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 I_{n_x} & 0 \\ 0 & \lambda_2 I_{n_x} \end{bmatrix}, \quad (4.27)$$

and λ_1, λ_2 are real scalars and $\lambda_1 > 0$.

Hence (4.15) can be rewritten as

$$\begin{bmatrix} -Q^{(i)} + A_{\zeta i}^T \Lambda G + G^T \Lambda A_{\zeta i} & * & * \\ B_{\zeta i}^T \Lambda G & -I & * \\ -\Lambda G + G^T A_{\zeta i} & G^T B_{\zeta i} & Q^{(i)} - (G + G^T) \end{bmatrix} < 0. \quad (4.28)$$

Observe from (4.28) that G is invertible since $G + G^T > Q^{(i)} > 0$. Denote

$$G = \begin{bmatrix} X & M \\ M_1 & U \end{bmatrix}, \quad G^{-1} = \begin{bmatrix} Y & N \\ N_1 & V \end{bmatrix}, \quad (4.29)$$

and

$$R = Y^{-1}, \quad J = \begin{bmatrix} I & 0 \\ N_1 R & -N_1 R \end{bmatrix}. \quad (4.30)$$

Multiply from the left and the right of (4.28) by $\text{diag}\{J^T, I, J^T\}$ and $\text{diag}\{J, I, J\}$, respectively, and apply (4.7). We obtain

$$\Delta_1 + \Delta_2 < 0, \quad (4.31)$$

where

$$\Delta_1 = \begin{bmatrix} \lambda_1 (A_i^T R + R^T A_i) - \hat{Q}_{11}^{(i)} & * & * \\ \lambda_1 W^T A_i + \lambda_2 (S_B C_i + S_A) - \hat{Q}_{21}^{(i)} & -\lambda_2 (S_A + S_A^T) - \hat{Q}_{22}^{(i)} & * \\ \lambda_1 B_i^{wT} R & \lambda_1 B_i^{wT} W & -I \\ R^T A_i - \lambda_1 R & -\lambda_1 W - \lambda_2 T^T & R^T B_i^w \\ W^T A_i + S_B C_i + S_A & -S_A + \lambda_2 T^T & W^T B_i^w \\ * & * & * \\ * & * & * \\ * & * & * \\ \hat{Q}_{11}^{(i)} - (R + R^T) & * & * \\ \hat{Q}_{21}^{(i)} - (T + W^T) & \hat{Q}_{22}^{(i)} + (T + T^T) & * \end{bmatrix}, \quad (4.32)$$

$$\Delta_2 = \begin{bmatrix} \lambda_1 R^T B_i^u (S_D C_i + S_C) + \lambda_1 (S_D C_i + S_C)^T B_i^{uT} R & * & * & * & * \\ \lambda_1 W^T B_i^u (S_D C_i + S_C) - \lambda_1 S_C^T B_i^{uT} R & -\lambda_1 W^T B_i^u S_C - \lambda_1 S_C^T B_i^{uT} W & * & * & * \\ 0 & 0 & * & * & * \\ R^T B_i^u (S_D C_i + S_C) & -R^T B_i^u S_C & * & * & * \\ W^T B_i^u (S_D C_i + S_C) & -W^T B_i^u S_C & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ 0 & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix}, \quad (4.33)$$

and

$$\begin{aligned} \hat{Q}^{(i)} &= J^T Q^{(i)} J, \\ S_A &= M_1^T A_c N_1 R, \quad S_B = M_1^T B_c, \quad S_C = C_c N_1 R, \\ S_D &= D_c, \quad T = M_1^T N_1 R, \quad W = X - R. \end{aligned} \quad (4.34)$$

Let

$$\Upsilon = [R \ W], \quad \Psi = [B_i^u S_D C_i + B_i^u S_C \quad -B_i^u S_C], \quad (4.35)$$

then

$$\Delta_2 = \begin{bmatrix} \lambda_1 \Upsilon^T \Psi + \lambda_1 \Psi^T \Upsilon & * & * \\ 0 & 0 & * \\ \Upsilon^T \Psi & 0 & 0 \end{bmatrix}. \quad (4.36)$$

Define

$$\Delta_3 = \begin{bmatrix} (\lambda_1 \Upsilon)^T + \Psi^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (\lambda_1 \Upsilon)^T + \Psi^T \\ 0 \\ 0 \end{bmatrix}^T = \begin{bmatrix} \lambda_1^2 \Upsilon^T \Upsilon + \Psi^T \Psi + \lambda_1 \Upsilon^T \Psi + \lambda_1 \Psi^T \Upsilon & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix}, \quad (4.37)$$

$$\Delta_4 = \begin{bmatrix} \Psi^T \\ 0 \\ \Upsilon^T \end{bmatrix} \begin{bmatrix} \Psi^T \\ 0 \\ \Upsilon^T \end{bmatrix}^T = \begin{bmatrix} \Psi^T \Psi & * & * \\ 0 & 0 & * \\ \Upsilon^T \Psi & 0 & \Upsilon^T \Upsilon \end{bmatrix}, \quad (4.38)$$

$$\Delta_5 = \begin{bmatrix} \lambda_1^2 (-\Upsilon^T \Upsilon_0 - \Upsilon_0^T \Upsilon + \Upsilon_0^T \Upsilon_0) + 2(-\Psi^T \Psi_0 - \Psi_0^T \Psi + \Psi_0^T \Psi_0) & * & * \\ 0 & * & * \\ 0 & 0 & -\Upsilon^T \Upsilon_0 - \Upsilon_0^T \Upsilon + \Upsilon_0^T \Upsilon_0 \end{bmatrix}, \quad (4.39)$$

where

$$\Upsilon_0 = [R_0 \ W_0], \quad \Psi_0 = [B_i^H S_{D,0} C_i + B_i^H S_{C,0} \ -B_i^H S_{C,0}], \quad (4.40)$$

then

$$\Delta_3 + \Delta_4 + \Delta_5 = \Delta_2 + \begin{bmatrix} \lambda_1^2 (\Upsilon - \Upsilon_0)^T (\Upsilon - \Upsilon_0) + 2(\Psi - \Psi_0)^T (\Psi - \Psi_0) & * & * \\ 0 & 0 & * \\ 0 & 0 & (\Upsilon - \Upsilon_0)^T (\Upsilon - \Upsilon_0) \end{bmatrix} \geq \Delta_2. \quad (4.41)$$

Consequently, (4.31) holds true if the following inequality is satisfied

$$\Delta_1 + \Delta_3 + \Delta_4 + \Delta_5 < 0. \quad (4.42)$$

By Schur complement, (4.17) can be obtained.

Secondly, (4.18) can be derived by multiply inequality (4.16) from left and right by $\text{diag}\{I, J^T, I\}$ and $\text{diag}\{I, J, I\}$, respectively.

Notice that T , M_1 , N_1 and R are invertible, in view of (4.29) and (4.30). Hence, it follows that the controller is of the transfer function

$$\begin{aligned}
 \text{tf}(z) &= C_c(zI - A_c)^{-1}B_c + D_c \\
 &= S_C R^{-1} N_1^{-1} (zI - M_1^{-T} S_A R^{-1} N_1^{-1})^{-1} M_1^{-T} S_B + S_D \\
 &= S_C (zT - S_A)^{-1} S_B + S_D \\
 &= S_C (zI - T^{-1} S_A)^{-1} T^{-1} S_B + S_D
 \end{aligned} \tag{4.43}$$

which gives the controller parameters of (4.25). This finishes the proof. $\blacksquare$

Remark 4.2.2 *Theorem 4.2.1 gives a sufficient condition to achieve closed-loop stability and H_2 performance via dynamic output feedback. Observe that the existence of solutions R , W , S_C , S_D , R_0 , W_0 , S_{C0} , S_{D0} in (4.17) and (4.18) is equivalent to the existence of solutions Q^i , F , G , S in (4.15) and (4.16). Therefore, Theorem 4.2.1 does not introduce any additional conservativeness other than those in Lemma 4.2.2. It should also be noted that for given R_0 , W_0 , S_{C0} , S_{D0} and λ_1 , λ_2 , (4.17) is LMI in R , W , S_A , S_B , S_C , S_D , T , $\hat{Q}_{11}^{(i)}$, $\hat{Q}_{21}^{(i)}$, $\hat{Q}_{22}^{(i)}$, $i = 0, 1, 2, \dots, l$. Then it can be solved efficiently by numerical LMI toolboxes, i.e., LMILAB [26], SEDUMI [66] and PENSDP [45]. Hence, in this sense, Theorem 4.2.1 converts a non-convex optimization problem into a convex one and its conservativeness can be minimized when R_0 tends to R , W_0 tends to W , S_{C0} tends to S_C and S_{D0} tends to S_D .*

To reduce the conservativeness, Theorem 4.2.1 leads to the following iterative algorithm for designing reliable output feedback controllers.

Algorithm 4.2.1 (1) Calculate a stabilizing dynamic output feedback controller R_0^0 , W_0^0 , S_{C0}^0 , S_{D0}^0 . Some detail can be found from Algorithm 5.2 of [52] and Algorithm

3.5 of [84] and is omitted here for brevity. If there is no stabilizing controller can be found, then stop and the algorithm fails to get a solution.

(2) At the j^{th} ($j = 1, 2, \dots$) iteration, set $R_0 = R_0^{j-1}$, $W_0 = W_0^{j-1}$, $S_{C0} = S_{C0}^{j-1}$, $S_{D0} = S_{D0}^{j-1}$ and minimize $\text{trace}(S)$ subject to linear matrix inequality (4.17) and (4.18). This gives R^j , W^j , S_C^j , S_D^j and S^j .

(3) If $|\text{trace}(S^{j-1} - S^j)| < \Delta$ where $\Delta > 0$ is a given error tolerance, the obtained controller is the optimal dynamic output feedback controller. Otherwise, set $R_0^j = R^j$, $W_0^j = W^j$, $S_{C0}^j = S_C^j$, $S_{D0}^j = S_D^j$, $j = j + 1$ and return to Step 2.

Remark 4.2.3 Note that, in Step 1 of Algorithm 4.2.1, finding a stabilizing dynamic output feedback controller is usually not a tractable problem and will influence the further development. However, we can translate the dynamic controller design problem into a static one, by making some augmentation to the original system (Section 2 in [84]), and then follow the instructions in Section 5 of [52]. Another approach is to formulate a vertex-independent problem, which is simpler but more conservative, and solve it in BMI framework by using PENSDP [45].

Theorem 4.2.2 Given an initial stabilizing output feedback controller as stated in Step 1 of Algorithm 4.2.1, for $j = 1, 2, \dots$, LMI (4.17)-(4.18) is feasible and the sequence $\{\text{trace}(S^j)\}_{j=1}^{\infty}$ is convergent.

Proof: As the given output feedback controller is stabilizing, LMI (4.17)-(4.18) is feasible at $j = 1$ with the resulting optimizers $(R^1, W^1, S_A^1, S_B^1, S_C^1, S_D^1, S^1)$. For $j \geq 2$, $(R^{j-1}, W^{j-1}, S_A^{j-1}, S_B^{j-1}, S_C^{j-1}, S_D^{j-1}, S^{j-1})$ is always a feasible solution to Step 2 of Algorithm 4.2.1, while $(R^j, W^j, S_A^j, S_B^j, S_C^j, S_D^j, S^j)$ is the optimal solution. Therefore, $0 < \text{trace}(S^j) \leq \text{trace}(S^{j-1})$, $j = 2, \dots$. This ends the proof. ■

Remark 4.2.4 Theorem 4.2.2 guarantees that Algorithm 4.2.1 is convergent. It should also be noted that Algorithm 4.2.1 is guaranteed to achieve local minimum only. There is

no guarantee for achieving global minimum. The result depends on the initial stabilizing controller which is chosen to start the computation. The impact of the initial stabilizing controller is still under investigation.

Remark 4.2.5 *It should be mentioned that when $\lambda_1 = \lambda_2 = 0$, Theorem 4.2.1 recovers the existing vertex-dependent approach, i.e., Shaked et al. [63] and Geromel et al. [27]. It is then clear that the proposed approach is guaranteed to be less-conservative in general due to the extra degrees of freedom in optimization provided by the free parameters λ_1 and λ_2 . A set of numerical optimization algorithm, such as the program `fminsearch` in MATLAB optimization toolbox [16] and Genetic Algorithm can be used to solve this problem. The optimization procedure can be efficient because only two parameters are optimized. In next section, this approach is demonstrated by an F-16 aircraft with control surface impairment faults.*

4.3 Design and simulation

In this section, an example of reliable tracking control is given to demonstrate the proposed method. The nonlinear dynamic model for this F-16 aircraft can be found in Chapter 2. In this example, control surface impairment (up to 75% efficiency loss to the elevator, symmetrically) is considered. Since the aerodynamic characteristics of the aircraft will change nonlinearly from nominal condition to 75% loss of elevator, in order to achieve better modelling accuracy in the polytopic system model, we consider two additional vertices (25% and 50% efficiency loss to the elevator, symmetrically) besides the existing two. Each fault is defined as a vertex to a polytopic uncertain system, and the design goal is to find a dynamic output feedback controller to stabilize this system and meet the performance requirement.

The aircraft equation with elevator impairment is modelled as (4.1) with state $x = [u, w, q]^T$ (i.e., forward airspeed, vertical airspeed and pitch rate), the output $y = [q, \alpha]^T$

(pitch rate and angle of attack), and the control input $u = \delta_e$ (i.e. elevator). The disturbance w represents the vertical gust.

The uncertainty polytope includes the following 4 ($i=0,1,2,3$) vertices, linearized from the nonlinear dynamic model and discretized with a sampling time of 0.01s.

- (1) The trim condition of the F-16 aircraft equations in the nominal operation is as in Table 2.2 with

$$A_0 = \begin{bmatrix} 0.9998 & 0.0005 & -0.0588 \\ -0.0009 & 0.9905 & 1.3700 \\ 0.0000 & 0.0000 & 0.9899 \\ 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}, \quad B_0^u = \begin{bmatrix} 0.0005 \\ -0.0046 \\ -0.0017 \end{bmatrix}, \quad B_0^w = \begin{bmatrix} 0.0005 \\ -0.0095 \\ 0.0000 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}.$$

- (2) The trim condition of the F-16 aircraft equations with 25% loss of elevator is as in Table 2.3 with

$$A_1 = \begin{bmatrix} 0.9999 & 0.0002 & -0.0594 \\ -0.0009 & 0.9930 & 1.3717 \\ 0.0000 & 0.0000 & 0.9898 \\ 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}, \quad B_1^u = \begin{bmatrix} -0.0003 \\ -0.0041 \\ -0.0014 \end{bmatrix}, \quad B_1^w = \begin{bmatrix} 0.0002 \\ -0.0070 \\ 0.0000 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}.$$

- (3) The trim condition of the F-16 aircraft equations with 50% loss of elevator is as in Table 2.4 with

$$A_2 = \begin{bmatrix} 0.9999 & 0.0000 & -0.0617 \\ -0.0009 & 0.9955 & 1.3731 \\ 0.0000 & -0.0000 & 0.9898 \\ 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}, \quad B_2^u = \begin{bmatrix} -0.0002 \\ -0.0028 \\ -0.0009 \end{bmatrix}, \quad B_2^w = \begin{bmatrix} 0.0000 \\ -0.0045 \\ -0.0000 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}.$$

- (4) The trim condition of the F-16 aircraft equations with 75% loss of elevator is as in Table 2.5 with

$$A_3 = \begin{bmatrix} 0.9999 & -0.0002 & -0.0840 \\ -0.0010 & 0.9980 & 1.3718 \\ 0.0000 & -0.0001 & 0.9898 \\ 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}, \quad B_3^u = \begin{bmatrix} -0.0004 \\ -0.0008 \\ -0.0003 \end{bmatrix}, \quad B_3^w = \begin{bmatrix} -0.0002 \\ -0.0020 \\ -0.0001 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}.$$

Furthermore, an additional integral action is introduced to eliminate the steady state tracking error in the output. It will be incorporated into the optimization cost function to penalize the tracking error. Therefore, the other two system matrices are as follows,

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}. \quad (4.44)$$

Using the same initial values as that in [47], Algorithm 4.2.1 is convergent. By using the MATLAB toolbox *fminsearch* [16], the optimal scaling parameters are obtained as $\lambda_1 = -0.500004$ and $\lambda_2 = -0.506237$. The optimal guaranteed H_2 cost is 0.2530. Note that it is much smaller than 0.8293 which is obtained from [47]. The corresponding dynamic output feedback controller is given as follows,

$$A_c = \begin{bmatrix} 0.1594 & 0.0122 & -0.3030 & -0.3954 \\ -0.0030 & 0.9983 & 0.0412 & 0.7374 \\ 0.0326 & 0.0385 & 0.0318 & -0.3664 \\ -0.0165 & -0.0039 & 0.0950 & -0.0169 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0.8525 & 0.0061 & 0.7945 \\ 0.0089 & -0.0153 & -0.1085 \\ -0.0368 & 0.0319 & 2.5566 \\ 0.0150 & 0.0181 & -0.2584 \end{bmatrix},$$

$$C_c = \begin{bmatrix} -0.0102 & -0.0791 & 0.0240 & -0.1725 \end{bmatrix},$$

$$D_c = \begin{bmatrix} -7.2086 & 1.2733 & 5.4265 \end{bmatrix}.$$

Fig. 4.1 shows the linear simulation results of the reliable dynamic output feedback controller. The first row in this figure depicts the output signal α where the tracking commands are unit steps. The second row is the control input signal δ_e . Four columns stand for the nominal case and 3 fault cases correspondingly.

Figure 4.1: Linear simulation results of the nominal condition and control surface impairment faults with reliable dynamic output feedback controller

From the result we can see the dynamic output feedback controller is not sensitive to these faults. It provides a good tracking performance in the nominal condition as well as fault conditions.

The following simulation scenario is based on the nonlinear F-16 aircraft model (as discussed in Chapter 2). A Herbst-like maneuver, which consists of a large angle-of-attack pull-up and then returning to the level flight, is performed with the reliable dynamic output feedback controller. A 30% efficiency loss of the elevator is introduced at 5s, followed by a 60% efficiency loss at 15s and a 70% efficiency loss at 25s. The simulation

result is given as below.

Figure 4.2: Nonlinear simulation of Herbst-like maneuver by reliable dynamic output feedback controller

Through this example, the following comments can be made about the proposed reliable design scheme:

- (1) The design conservativeness is reduced as the optimal guaranteed H_2 cost is much smaller than the common approach. This is due to the extra optimization freedom provided by the vertex-dependent Lyapunov functions and the other slack parameters we introduced to the LMI formulation.
- (2) The proposed scheme optimizes the nominal performance.
- (3) The proposed design scheme is able to satisfy the desired reliability requirements when fault occurs (as long as the optimization problem is feasible).

4.4 Conclusion

In this chapter we address the reliable dynamic output feedback controller design problem of linear uncertain systems with polytopic uncertainty. A less-conservative design is presented by adopting vertex-dependent Lyapunov function and some additional slack parameters. The solution, if exists, provides optimized nominal performance while retaining the desired reliability goal in fault conditions. Finally, the proposed method is applied to a nonlinear F-16 aircraft model and the effectiveness is illuminated by the simulation results.

Chapter 5

Mixed H_2/H_∞ reliable aircraft flight controller design against faults and actuator saturations

In Chapter 3 and Chapter 4 we have discussed unconstrained reliable H_∞ and H_2 approaches. In the following chapters we will further extend this topic to a more complicated and realistic circumstance, i.e., from now on we will consider an aircraft system with actuator saturations.

The motivation to do this is that actuator limits physically exist on an aircraft and these limits can not be exceeded in any case. In many current research [50, 51, 52, 75, 80] as well as Chapter 3 and Chapter 4 in this thesis, these actuator limits are neglected for the ease of analysis and synthesis. Such unconstrained approaches are generally applicable for most of cases, including level flight, low angle pull-up, coordinate turn, etc. But for some challenging maneuvers (high angle Herbst, Cobra) or during fault situations (the remaining surfaces have to increase control activity to compensate for the failed ones), the actuator saturations are more likely to occur (e.g., elevator actuator saturated in Fig. 3.12). The consequence of actuator saturations includes performance degradation and

even instability [3, 33, 57, 74]. The destabilizing effects of actuator saturations not only cause a windup to any controller with slow or unstable dynamics [3, 74], but also have been cited as contributing factors in several mishaps involving high performance aircraft [19]. Therefore, extending our study to considering actuator saturations as well as faults is meaningful to the literature.

In this chapter, we aim at designing a reliable robust controller that can handle actuator saturation problems as well as various of faults defined in Chapter 2. Based on the work in Chapter 3 and Chapter 4, the most straight forward way to address actuator saturations is the anti-windup technique. This method employs a two-step design paradigm [57, 74]: *Design first the linear controller ignoring control input nonlinearities and then add anti-windup compensation to minimize the adverse effects of any control input saturations on closed-loop performance.* Such anti-windup compensation schemes provide a computationally efficient technique for existing unconstrained controllers to mitigate input saturations thereby eliminating “controller windup” problem.

Hereafter, we revisit the reliable robust tracking controller design problem for aircraft against actuator outage faults and control/wing/tail surface impairment faults. A less-conservative mixed H_2/H_∞ formulation is presented based on vertex-dependent Lyapunov functions. A novel anti-windup compensation technique is also proposed to handle input saturations. Finally, numerical simulations are performed on a nonlinear F-16 model considering control/wing/tail surface impairment faults and input saturations. The results illustrate the enhanced maneuverability and the effectiveness of the proposed approach.

5.1 Mixed H_2/H_∞ flight controller design using vertex-dependent Lyapunov functions

Most of the practical control problems are actually multi-objectives in nature. In this section, we seek a set of robust control strategies to stabilize the high performance aircraft system and to achieve good tracking performance in nominal and fault cases. For ease of reading, we reproduce the linear aircraft system as follows,

$$\begin{cases} x_a(k+1) = A_a(\theta)x_a(k) + B_a^u(\theta)\omega_L u(k) + B_a^w(\theta)w_a(k) \\ y_a(k) = C_a(\theta)x_a(k) \\ z_\infty(k) = C_\infty x_a(k) + D_\infty u(k) \\ z_2(k) = C_2 x_a(k) + D_2 u(k) \end{cases}, \quad (5.1)$$

where C_2 , D_2 and C_∞ , D_∞ define the H_2 and H_∞ performance variables z_2 and z_∞ , respectively. And

$$\begin{aligned} A_a(\theta) &= \sum_{i=0}^l A_{ai}\theta_i, & B_a^u(\theta) &= \sum_{i=0}^l B_{ai}^u\theta_i, \\ C_a(\theta) &= \sum_{i=0}^l C_{ai}\theta_i, & B_a^w(\theta) &= \sum_{i=0}^l B_{ai}^w\theta_i. \end{aligned} \quad (5.2)$$

with the polytopic coefficient matrices as given in (2.26).

5.1.1 H_∞ optimal controller design

The H_∞ optimal synthesis problem is discussed in Chapter 3. For ease of reading, the result is reproduced here with new system description (5.1).

For nominal case, consider system (5.1) and a real number $\gamma_{00} > 0$, the exogenous signal w_a is attenuated by γ_{00} if, assuming $x_a(0) = 0$,

$$\sum_{k=0}^{\infty} \|z_\infty(k)\|^2 < \gamma_{00} \sum_{k=0}^{\infty} \|w_a(k)\|^2. \quad (5.3)$$

For fault cases, the exogenous signal w_a is attenuated by γ_{Li} as follows,

$$\sum_{k=0}^{\infty} \|z_\infty(k)\|^2 < \gamma_{Li} \sum_{k=0}^{\infty} \|w_a(k)\|^2, \quad i = 0, \dots, l, \quad L = 0, 1, \dots, l_p, \quad l_p \leq 2^{n_u} - 1, \quad Li \neq 00. \quad (5.4)$$

Consider the static state feedback controller

$$u(k) = Kx_a(k). \quad (5.5)$$

The closed-loop system is given by

$$\begin{cases} x_a(k+1) = [A_a(\theta) + B_a^u \omega_L K]x_a(k) + B_a^w(\theta)w_a(k) \\ z_\infty(k) = [C_\infty + D_\infty K]x_a(k) \\ z_2(k) = [C_2 + D_2 K]x_a(k) \end{cases}. \quad (5.6)$$

Theorem 5.1.1 (H_∞ optimal synthesis) Consider the discrete time linear system (5.1). There exists a static state feedback control law (5.5), such that the closed-loop system (5.6) is stable and the H_∞ constraint (5.3), (5.4) are fulfilled for the nominal case as well as for all faults described in (2.24)-(2.26), if there exist matrices $Q_{\infty, Li} = Q_{\infty, Li}^T > 0$, nonsingular matrix G and V such that for $i = 0, \dots, l$, and $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$,

$$\begin{bmatrix} Q_{\infty, Li} - G - G^T & * & * & * \\ A_{ai}G + B_{ai}^u \omega_L V & -Q_{\infty, Li} & * & * \\ C_\infty G + D_\infty \omega_L V & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I \end{bmatrix} < 0. \quad (5.7)$$

The state feedback gain K is given by

$$K = VG^{-1}. \quad (5.8)$$

Remark 5.1.1 Please note that γ_{Li} , $i = 0, \dots, l$, $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$, $Li \neq 00$ are the H_∞ performance upper bounds for the fault cases. In H_∞ optimal controller

design, we can optimize the nominal case performance γ_{00} under the fault case performance constraints γ_{Li} .

5.1.2 H_2 optimal controller design

Chapter 5 proposed a dynamic output feedback H_2 controller design approach. In this section, static state feedback H_2 optimal controller design problem is reviewed. Suppose $T_{z_2 w_a}$ is the closed-loop transfer matrices from w_a to z_2 , the design method based on H_2 optimization is to find a feedback gain matrix (5.5) that can minimize $\|T_{z_2 w_a}\|_2$ in nominal case (upper bounded by parameter λ_{00}) without violation of $\|T_{z_2 w_a}\|_2 < \lambda_{Li}$ in fault cases, where λ_{Li} are given upper bounds for $i = 0, \dots, l$, $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$, $Li \neq 00$.

Theorem 5.1.2 (H_2 optimal synthesis [28]) *Consider the discrete time linear system (5.1). There exists a static state feedback control law (5.5), such that the closed-loop system (5.6) is stable and the closed-loop norm $\|T_{z_2 w_a}\|_2$ is upper bounded by λ_{Li} for nominal case and all fault cases described in (2.24)-(2.26), if there exist matrices $Q_{2, Li} = Q_{2, Li}^T > 0$, nonsingular matrix G and V such that for $i = 0, \dots, l$, and $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1$,*

$$\begin{bmatrix} Q_{2, Li} - G - G^T & * & * \\ A_{ai}G + B_{ai}^u \omega_L V & -Q_{2, Li} & * \\ 0 & (B_{ai}^w)^T & -I \end{bmatrix} < 0, \quad (5.9)$$

$$\begin{bmatrix} G + G^T - Q_{2, Li} & * \\ C_2 G + D_2 \omega_L V & \lambda_{Li} I \end{bmatrix} > 0. \quad (5.10)$$

The state feedback gain K is given by

$$K = VG^{-1}.$$

Remark 5.1.2 *Similar to Remark 5.1.1 and as was done in [51] for the continuous time case, here we can optimize the nominal performance λ_{00} under the constraints*

$$\|T_{z_2 w_a}(z)\|_2 < \lambda_{Li}, \quad i = 0, \dots, l, \quad L = 0, 1, \dots, l_p, \quad l_p \leq 2^{n_u} - 1, \quad Li \neq 00. \quad (5.11)$$

5.1.3 Simultaneous H_2/H_∞ control synthesis using vertex-dependent Lyapunov functions

In this subsection we combine Theorem 5.1.1 and 5.1.2 to formulate the mixed H_2/H_∞ synthesis for the reliable robust tracking controller design problem. In some approaches [78, 85], where no slack matrix (G in (5.7), (5.9) and (5.10)) is utilized, a set of matrices ($Q_{\infty, Li}, Q_{2, Li}, K$) simultaneously satisfying the LMIs (5.7), (5.9) and (5.10) is not convex in general. Commonly it requires that the Lyapunov matrices in all specifications should be imposed by the constraints $Q_{\infty, Li} = Q_{2, Li}$ [11, 42]. These constraints clearly lead to design conservativeness. By using slack matrix G and vertex-dependent Lyapunov functions, the constraints $Q_{\infty, Li} = Q_{2, Li}$ are no longer required, hence the design conservativeness can be reduced through the following theorem.

Theorem 5.1.3 (*Mixed H_2/H_∞ control synthesis*) *Consider the discrete time linear system (5.1). There exists a static state feedback control law (5.5), such that the closed-loop norm $\|T_{z_2 w_a}\|_2$ in nominal case is minimized while the H_2 constraints (5.11) are satisfied for all faults described in (2.24)-(2.26) and H_∞ constraints are satisfied for the nominal case, i.e. (5.3) as well as for all fault cases, i.e. (5.4), if there exist symmetric matrices $Q_{2, Li} > 0$, $Q_{\infty, Li} > 0$ and nonsingular matrices G and V such that for given γ_{Li} and λ_{Li} $i = 0, \dots, l$, and $L = 0, 1, \dots, l_p$, $l_p \leq 2^{n_u} - 1, Li \neq 00$,*

$$\min_{G, V, Q_{2, Li}, Q_{\infty, Li}} \lambda_{00} \quad \text{subject to LMIs (5.7), (5.9), (5.10).} \quad (5.12)$$

The mixed H_2/H_∞ approach can provide optimal H_2 tracking performance with guaranteed disturbance attenuation ability, in nominal case and in the presence of faults.

5.2 Anti-windup compensation

As well known, actuator saturations are probably the most widely encountered and most dangerous nonlinearity in control systems. In Section 5.1, we have discussed the unconstrained closed-loop aircraft system where the control surface deflection limits are not included in the controller design procedure. However, in realistic circumstances, this assumption is almost impossible to satisfy. As in the aircraft model [48, 59], for example, the elevator actuators (δ_{el} , δ_{er}) are modeled as 0.0495s first-order lags with rate limits of $60^\circ/\text{sec}$ and the surface deflection limits of $\pm 25^\circ$, the aileron actuators (δ_{al} , δ_{ar}) are modeled as 0.0495s first-order lags with rate limits of $80^\circ/\text{s}$ and the surface deflection limits of $\pm 20^\circ$, the rudder actuator (δ_r) are modeled as 0.0495s first-order lags with rate limits of $120^\circ/\text{s}$ and the surface deflection limits of $\pm 30^\circ$. Considering such input saturations, we include an anti-windup compensation procedure into the control system design scheme in this section, for both nominal condition and fault conditions.

As described in [57, 74], anti-windup compensation scheme employs a two-step design paradigm. Section 5.1 studies the first step (controller design without considering saturations). In this paradigm, the second step (compensator design) is discussed in the following framework as shown in Fig. 5.1.

Figure 5.1: Anti-windup compensation framework

Within this framework, the block $N \neq I$ represents input saturations, r represents reference signal and e for tracking error, u and $\hat{u}$ represent the input signal before and after saturation nonlinearity and v is the difference between u and $\hat{u}$. The block ‘Plant’

represents the nonlinear F-16 system while the block ‘*Controller*’ takes the following realization, for $i = 0, \dots, l$

$$\begin{aligned} x_c(k+1) &= A_{ci}x_c(k) + B_{ci}x(k) + B_{ci}^u u(k) + B_{ci}^r r(k) + B_{ci}^w w(k) + \xi_1(k) \\ u(k) &= C_c x_c(k) + D_c x(k) + \xi_2(k) \\ \xi(k) &= [\xi_1(k)^T \quad \xi_2(k)^T]^T = -\Lambda v(k) \end{aligned} \quad (5.13)$$

where $v = u - N(u)$ and $A_{ci}, B_{ci}, B_{ci}^u, B_{ci}^r, B_{ci}^w, i = 0, \dots, l$ are constant matrices in (2.33). Matrices C_c and D_c satisfy $[C_c \ D_c] = K$ with K the state-feedback controller acquired from Section 5.1.3, and Λ is the anti-windup compensator to be designed.

Define the nonlinearity $\Delta = I - N$, the system in Fig. 5.1 can be transformed to the equivalent system as shown in Fig. 5.2,

Figure 5.2: Anti-windup compensation framework with shifted nonlinearity

By using the linear fractional transformation (LFT), interconnection of linear elements in Fig. 5.1 can be represented as shown in Fig. 5.3.

Figure 5.3: LFT form of anti-windup compensation framework

In Fig. 5.3, W represents the overall system given as below, for $i = 0, \dots, l$

$$\begin{aligned} x_a(k+1) &= A_{Wi}x_a(k) + \left(B_{Wi}^v - B_{Wi}^\xi \Lambda\right)v(k) + B_{Wi}^w w_a(k), \\ u(k) &= C_{Wi}^u x_a(k) + \left(D_{Wi}^{uv} - D_{Wi}^{\xi u} \Lambda\right)v(k) + D_{Wi}^{uw} w_a(k), \\ z_{aw}(k) &= C_{Wi}^z x_a(k) + \left(D_{Wi}^{zv} - D_{Wi}^{\xi z} \Lambda\right)v(k) + D_{Wi}^{zw} w_a(k). \end{aligned} \quad (5.14)$$

where $x_a(k) = [x_c(k)^T, x(k)^T]^T$, $w_a(k) = [r(k)^T, w(k)^T]^T$ and

$$\begin{aligned} A_{Wi} &= \begin{bmatrix} A_{ci} + B_{ci}^u C_c & B_{ci} + B_{ci}^u D_c \\ B_{ci}^u C_c & A_i + B_i^u D_c \end{bmatrix}, \quad B_{Wi}^v = \begin{bmatrix} 0 \\ -B_i^u \end{bmatrix}, \quad B_{Wi}^\xi = \begin{bmatrix} I & B_{ci}^u \\ 0 & B_i^u \end{bmatrix}, \\ B_{Wi}^w &= \begin{bmatrix} B_{ci}^r & B_{ci}^w \\ 0 & B_i^w \end{bmatrix}, \quad C_{Wi}^u = [C_c \quad D_c], \quad D_{Wi}^{uv} = 0, \\ D_{Wi}^{\xi u} &= [0 \quad I], \quad D_{Wi}^{uw} = 0, \end{aligned}$$

with A_i , B_i^u , B_i^w , $i = 0, \dots, l$ obtained from (2.33). The matrices C_{Wi}^z , D_{Wi}^{zv} , $D_{Wi}^{\xi z}$ and D_{Wi}^{zw} are chosen to define the performance variable z_{aw} to be regulated in the presence of saturations, which should be selected in consistence with the H_2 and H_∞ performance variables z_2 and z_∞ in (5.1) in general (e.g., $C_{Wi}^z = C_\infty$, $D_{Wi}^{zv} = 0$, $D_{Wi}^{\xi z} = 0$ and $D_{Wi}^{zw} = D_\infty^w$).

Remark 5.2.1 When the input saturations are not activated, (i.e., $v = 0$), the resulting anti-windup compensator has no effect on the system in Fig. 5.1. The mixed H_2/H_∞ performance index will be guaranteed through the reliable controller K designed in Section 5.1.2. In the presence of input saturations, where $v \neq 0$, the following theorem guarantees robust stability of the overall system and a weighted induced $\mathcal{L}_2$ gain less than $\sqrt{\Gamma}$ (Γ is a given scalar) from w_a to z_{aw} .

As stated in [2, 57], choosing the form in Fig. 5.3 allows us to recast the synthesis problem into the following LMIs.

Theorem 5.2.1 (Robust anti-windup compensator design) The system in Fig. 5.3 is $\mathcal{L}_2$

stabilizable for all Δ and has a weighted induced $\mathcal{L}_2$ gain less than $\sqrt{\Gamma}$ in nominal case as well as in fault cases, if there exists nonsingular matrix E and matrices $F_i = F_i^T > 0$, $M = \text{diag}\{M_1, M_2, \dots, M_{n_u}\} \in \mathbb{R}^{n_u \times n_u}$ with $M > 0$, $X := \Lambda M$, and scalar $\delta > 0$, $\Gamma > 0$, such that for $i = 0, \dots, l$,

$$\begin{bmatrix} F_i - E - E^T & * & * & * & * & * \\ 0 & -\Gamma I & * & * & * & * \\ C_{Wi}^u E & D_{Wi}^{uw} & -2M - D_{Wi}^{u\xi} X - X^T D_{Wi}^{u\xi T} & * & * & * \\ C_{Wi}^z E & D_{Wi}^{zw} & 0 & -\Gamma I & * & * \\ A_{Wi} E & B_{Wi}^w & B_{Wi}^v M - B_{Wi}^\xi X & 0 & -F_i & * \\ 0 & 0 & M & 0 & 0 & -\delta I \end{bmatrix} < 0. \quad (5.15)$$

The anti-windup compensator is given by $\Lambda = XM^{-1}$.

Proof: Given $F_i = F_i^T > 0$ and the additional nonsingular matrix E of suitable dimension, $(E^T - F_i)F_i^{-1}(E - F_i)$ is nonnegative definite, and consequently $0 < E + E^T - F_i \leq E^T F_i^{-1} E$ holds true. Hence, the following LMI holds,

$$\begin{bmatrix} -E^T F_i^{-1} E & * & * & * & * & * \\ 0 & -\Gamma I & * & * & * & * \\ C_{Wi}^u E & D_{Wi}^{uw} & -2M - D_{Wi}^{u\xi} X - X^T D_{Wi}^{u\xi T} & * & * & * \\ C_{Wi}^z E & D_{Wi}^{zw} & 0 & -\Gamma I & * & * \\ A_{Wi} E & B_{Wi}^w & B_{Wi}^v M - B_{Wi}^\xi X & 0 & -F_i & * \\ 0 & 0 & M & 0 & 0 & -\delta I \end{bmatrix} < 0. \quad (5.16)$$

Then, as E is nonsingular and E^{-1} always exists, we can multiply (5.16) from the left by $\text{diag}\{F_i E^{-T}, I, I, I, I, I\}$ and from the right by $\text{diag}\{E^{-1} F_i, I, I, I, I, I\}$, the following

LMI is obtained

$$\begin{bmatrix} -F_i & * & * & * & * & * \\ 0 & -\Gamma I & * & * & * & * \\ C_{Wi}^u F & D_{Wi}^{uw} & -2M - D_{Wi}^{u\xi} X - X^T D_{Wi}^{u\xi T} & * & * & * \\ C_{Wi}^z F & D_{Wi}^{zw} & 0 & -\Gamma I & * & * \\ A_{Wi} F & B_{Wi}^w & B_{Wi}^v M - B_{Wi}^{\xi} X & 0 & -F_i & * \\ 0 & 0 & M & 0 & 0 & -\delta I \end{bmatrix} < 0. \quad (5.17)$$

LMI (5.17) is the existing result proposed in [2] and the proof for it is omitted here for brevity. ■

5.3 Design and simulation

In this section we will test the designed mixed H_2/H_∞ controller and the anti-windup compensator upon a nonlinear F-16 model through three different scenarios. Similar to the former chapters, three variables, namely, stability axis roll rate $\dot{\mu}_{rat}$, angle of attack α and sideslip angle β are to be stabilized and tracked closely. The following parameters are used during simulation,

$$\begin{aligned} S &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 6} \\ 0_{3 \times 3} & 0_{3 \times 6} \end{bmatrix}, \quad D_2^u = \begin{bmatrix} 0_{3 \times 3}, \\ 0.1 I_{3 \times 3} \end{bmatrix}, \quad D_2^w = 0_{6 \times 4}, \\ C_\infty &= \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 6} \\ 0_{3 \times 3} & 0_{3 \times 6} \end{bmatrix}, \quad D_\infty^u = \begin{bmatrix} 0_{3 \times 3}, \\ 0.1 I_{3 \times 3} \end{bmatrix}, \quad D_\infty^w = 0_{6 \times 4}, \\ \gamma_{Li} &= 1, \quad i = 0, \dots, l, \quad L = 0, 1, \dots, l_p, \quad l_p \leq 2^{n_u} - 1, \\ \lambda_{Li} &= 100, \quad i = 0, \dots, l, \quad L = 0, 2, \dots, l_p, \quad Li \neq 00, \quad l_p \leq 2^{n_u} - 1, \end{aligned}$$

where S determines the outputs required to track, $\dot{\mu}_{rat}$, α and β . Besides, a 5m/s vertical gust disturbance is considered in the simulation to verify the H_∞ attenuation ability.

By using the proposed approach in Section 5.1.3, a reliable state feedback tracking controller can be designed to tolerate the above control/wing/tail surface impairment faults. Values of controller gain are given as below,

$$K_{reliable} = \begin{bmatrix} 21.518 & -160.22 & -6.2023 & -0.14388 & 4782.7 & 438.41 \\ -11.206 & -2.8265 & -45.722 & -0.035923 & 237.24 & 1596.7 \\ -35.54 & 12.35 & 304.82 & 0.057444 & -534.17 & -11320 \\ -32.992 & 1291.1 & -75.923 & & & \\ 207.45 & 35.481 & -400.74 & & & \\ 1.4006 & -26.347 & 3493.3 & & & \end{bmatrix}.$$

Furthermore, in order to account for the input saturation nonlinearity, an anti-windup compensator is designed by using the method proposed in Section 5.2 with following design parameters,

$$C_{Wi} = [I_{3 \times 3} \quad 0_{3 \times 6}], \quad D_{Wi}^{zv} = 0_{3 \times 3}, \quad D_{Wi}^{z\xi} = 0_{3 \times 3}, \quad D_{Wi}^{zw} = 0_{3 \times 4}, \quad i = 0, 1, 2, 3$$

i.e., the performance variable $z_{aw}(k) = x_c(k) = \int_0^{kT} e(t)dt$. A performance value of $\sqrt{\Gamma} = 8.9612$ is achieved and the corresponding compensator Λ is given as below,

$$\Lambda = \begin{bmatrix} -2.0427\text{E}-08 & -1.3082\text{E}-06 & +1.3207\text{E}-07 \\ -8.6015\text{E}-07 & +2.7466\text{E}-07 & -1.2243\text{E}-08 \\ +3.7006\text{E}-08 & -2.0191\text{E}-06 & -3.0419\text{E}-07 \\ -0.99949 & +8.3292\text{E}-05 & +0.0001071 \\ +2.0845\text{E}-05 & -0.99963 & +4.4275\text{E}-05 \\ +0.00012656 & +0.00046431 & -0.99952 \end{bmatrix}.$$

5.3.1 Scenario 1: low-angle Herbst maneuver

First of all, a low-angle Herbst maneuver is performed by using the designed controller and anti-windup compensator. The maneuver consists of a pull-up of angle of attack from $t = 5\text{s}$ to $t = 25\text{s}$ (from 0° to 8°), and a stability axis roll from $t = 5\text{s}$ to $t = 10\text{s}$ (from 0° to 5°). The scenario also includes a 30% efficiency loss of the elevator is introduced at 12s, followed by a 60% efficiency loss at 15s and a 70% efficiency loss at

18s. The simulation result is given in Fig. 5.4.

Figure 5.4: Nonlinear simulation results comparison: low-angle Herbst maneuver

From the simulation result we can see that for low-angle Herbst maneuver the control actions have not reached their limits, meaning that the anti-windup compensator is not invoked during this maneuver (i.e., $v(k) = 0$ in Fig. 5.1). Therefore two trajectories (with/without anti-windup compensator) coincide.

5.3.2 Scenario 2: mid-angle Herbst maneuver

In this subsection, a mid-angle Herbst maneuver is performed. Compared to Scenario 1, here we need alpha of attack track 11° from $t = 5\text{s}$ to $t = 25\text{s}$. The simulation result is given in Fig. 5.5.

Clearly, the standard controller failed to stabilize the nonlinear aircraft model from $t = 18\text{s}$. This is mainly because that when the 70% efficiency loss fault is introduced at 18s, actuator pushes the elevator to act harder and harder to compensate for the effi-

Figure 5.5: Nonlinear simulation results comparison: mid-angle Herbst maneuver

ciency loss, eventually induces actuator saturation. Since this situation is not considered in reliable controller design procedure, the system starts to oscillate and finally loses stability. Benefited from the designed anti-windup compensator, we can see from Fig. 5.5 that the overall closed-loop stability can still be guaranteed in the presence of actuator saturation.

5.3.3 Scenario 3: high-angle Herbst maneuver

In this subsection, a high-angle Herbst maneuver is performed. Compared to the first two scenarios, alpha of attack will track 30° from $t = 5\text{s}$ to $t = 25\text{s}$ with the stability axis roll rate $\dot{\mu}_{rat}$ tracking $20^\circ/\text{s}$ simultaneously. The simulation result is given in Fig. 5.6.

We can see from this figure that the reliable controller together with anti-windup compensator work properly even during this high-angle maneuver. This means that the manoeuvrability has been greatly improved.

Figure 5.6: Nonlinear simulation results comparison: high-angle Herbst maneuver

Summarizing up the three scenarios presented in this section, the proposed mixed H_2/H_∞ reliable controller and the anti-windup compensator design method not only provides good disturbance attenuation ability but also significantly improves the system performance in the presence of faults as well as actuator saturations, with minimal compromise in the nominal performance.

5.4 Conclusion

This chapter presents a reliable flight controller design problem subject to mixed H_2/H_∞ constraints. The actuator outage faults and control/wing/tail surface impairment faults have been considered by using polytopic fault models. To reduce the controller's conservativeness, the proposed approach is based on several vertex-dependent Lyapunov functions, each one corresponding to a different vertex of the uncertainty polytope. Furthermore, improved anti-windup technology (vertex-dependent Lyapunov functions are

used) is applied to handle input saturations. Finally, the proposed method is applied to a nonlinear high fidelity F-16 aircraft model and the effectiveness is validated by simulation results.

Chapter 6

Reliable aircraft control against faults and saturations: an on-line MPC approach

In Chapter 3 and 4 we have discussed unconstrained H_2 and/or H_∞ control methodologies and extended them to constrained system in Chapter 5. The two-step design paradigm has been proven theoretically to be efficient and robust to handle input saturations and validated throughout nonlinear simulations. With no doubt, such paradigm reinforces the existing reliable approaches with an easy and powerful way to avoid potential instability caused by saturations. The advantage also includes practicality, that is, all the designing effort can be done off-line (which is technically easy and cheap), resulting in a controller and a compensator to be applied for on-line simulations.

Nevertheless, there are two disadvantages regarding this methodology that should not be neglected. One is on the two-step design paradigm. As stated in [57, 74], a controller is first designed ignoring input saturations and then add anti-windup compensation to minimize the adverse effects caused by saturations. However, the additional compensator may change the overall closed-loop system which in turn can not guarantee the desired

performance requirements in the first stage in the presence of actuator saturations.

The other disadvantage is that anti-windup compensation can only handle input saturations. It is not applicable for some other types of constraints, like state and output constraints or more specifically, flight envelope constraints in flight control area.

In view of this we are trying to find a control methodology that will not jeopardize the original performance requirements in the presence of actuator saturations, and can handle various types of constraints. As reviewed in Chapter 1, Model Predictive Control (MPC) is such a candidate. In this chapter we will introduce MPC into flight control area. We consider another simulation scenario, an auto-landing process in the presence of actuator stuck faults. Compared to the Herbst maneuver scenario, the auto-landing process requires to consider more types of constraints, i.e., state/input/output constraints.

6.1 Performance requirements

First of all, the performance requirements of the auto-landing control system is revisited here (please refer to [48] and [59] for more details):

- Fast response to command inputs is required. The maximum rise time and settling time are given in Table 6.1:

Type of Response	Rise Time (s)	Settling Time (s)	Over-shoot	Steady State Error
Lateral Deviation	10	30	5% to 30%	0
Altitude	4	15	5% to 30%	
Heading Angle	3	10	5% to 30%	
Flight Path Angle	2	7	5% to 30%	
Airspeed	4	15	5% to 30%	0

Table 6.1: The maximum rise time and settling time

- Deviation error requirements:

- (1) During the final phase of landing, the lateral deviation (Δy) from the desired flight path should not exceed (unit in meter):

$$\Delta y = \begin{cases} 5 & h \leq 30 \\ 5 + 15(h - 30)/120 & 30 \leq h \leq 150 \\ 20 & 150 \leq h \end{cases} \quad (6.1)$$

- (2) During the final phase of landing, the vertical deviation (Δh) from the desired flight path should not exceed (unit in meter):

$$\Delta h = \begin{cases} 1.5 & h \leq 30 \\ 1.5 + 4.5(h - 30)/120 & 30 \leq h \leq 150 \\ 6 & 150 \leq h \end{cases} \quad (6.2)$$

- Robustness requirements:

Stability and performance should be maintained in the presence of the following variations, shown in Table 6.2:

Center of Gravity Variation	Horizontal: 15% to 31% of c Vertical: 0 to 21% of c
Mass Variation	$\pm 20\%$
Time Delay	up to 100ms
Speed Variation	At least 140kt to 170kt

Table 6.2: Stability and performance requirements

- Safety criteria is shown in Table 6.3:

Airspeed	$> 1.05V_{stall}$
Angle of Attack	$< 12deg$
Roll Angle	$< 30deg$
Sideslip Angle	minimized

Table 6.3: Safety Criteria

6.2 MPC problem formulation

6.2.1 System model

The system to be controlled is usually described, or approximated, by a difference linear/nonlinear equation in the MPC literature, as follows:

$$\begin{aligned} x(k+1) &= f(x(k), u(k)), \\ y(k) &= h(x(k)), \end{aligned} \tag{6.3}$$

where k is a discrete time index, and $f(\cdot)$ is implicitly defined by the original differential equation (linear/nonlinear) that has an equilibrium point at the origin $f(0,0) = 0$. The control and state sequences must satisfy

$$\begin{aligned} u(k) &\in \mathbb{U}, \\ x(k) &\in \mathbb{X}, \end{aligned} \tag{6.4}$$

where, usually, $\mathbb{U}$ is a convex, compact subset of $\mathbb{R}^{n_u}$ and $\mathbb{X}$ is a convex, closed subset of $\mathbb{R}^{n_x}$, each set containing the origin in its interior. The control objective is usually to steer the state to the origin or to an equilibrium state x_r for which the output $y_r = h(x_r) = r$ where r is the constant reference. A suitable change of coordinates reduces the second problem to the first which therefore we consider in the sequel.

6.2.2 Constraints

In practical environments, there exist limits on control inputs and outputs, sometimes even on the rate of change of inputs. One of MPC's outstanding advantages is that it can handle these constraints easily and effectively. In this study we write these constraints in the following form which is easy to describe with MPC toolbox of Matlab.

$$E_s \begin{bmatrix} \Delta \hat{u}(k|k) \\ \vdots \\ \Delta \hat{u}(k + H_u - 1|k) \\ 1 \end{bmatrix} \leq \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad (6.5)$$

$$F_s \begin{bmatrix} \hat{u}(k|k) \\ \vdots \\ \hat{u}(k + H_u - 1|k) \\ 1 \end{bmatrix} \leq \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \quad (6.6)$$

$$G_s \begin{bmatrix} \hat{x}(k + H_w|k) \\ \vdots \\ \hat{x}(k + H_p|k) \\ 1 \end{bmatrix} \leq \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \quad (6.7)$$

Here E_s , F_s and G_s denotes constraints matrices of appropriate dimension, $\hat{u}(k + i|k)$ denotes a future value (at time $k + i$) of the input u , which based on the measurement at time k and $\Delta \hat{u}(k|k) = \hat{u}(k|k) - u(k - 1)$. $\hat{x}(k + i|k)$ denotes the prediction, made at time k , of the variables x , at time $k + i$, on the assumption that some sequence of inputs $\hat{u}(k + j|k)$ ($j = 0, 1, \dots, i - 1$) would have occurred. H_p is length of the *prediction horizon*, H_u is length of the *control horizon*. H_w is the so-called ‘*window*’ parameter from where the derivations of the predicted controlled outputs $\hat{y}(k + i|k)$ from a (vector) reference trajectory $r(k + i|k)$ is penalized.

6.2.3 Cost function

The cost function J penalizes deviations of the predicted controlled outputs $\hat{y}(k + i|k)$ from a (vector) reference trajectory $r(k + i|k)$. Again the notation indicates that this reference trajectory may depend on measurements made up to time k ; in particular, its initial point may be the output measurement $y(k)$, as mentioned in Section 6.2.1. But it may also be a fixed set-point, or some other predetermined trajectory. We define the

cost function to be:

$$J(k) = \sum_{i=H_w}^{H_p-1} \|\hat{y}(k+i|k) - r(k+i|k)\|_{Q(i)}^2 + \sum_{i=0}^{H_u-1} \|\Delta\hat{u}(k+i|k)\|_{R(i)}^2 \quad (6.8)$$

where $Q(i) > 0$ and $R(i) > 0$ are time variant weight matrices.

There are two points to be noted here:

- The prediction horizon has length H_p , but we do not necessarily start penalizing deviations of y from r immediately (if $H_w > 1$). This is because that there may be some delay between applying an input and seeing any effect on output.
- It is always assumed that $H_u \leq H_p$, and that $\Delta\hat{u}(k+i|k) = 0$ for $i \geq H_u$, so that $\hat{u}(k+i|k) = \hat{u}(k+H_u-1|k)$ for all $i \geq H_u$.

Equation (6.8) is a general form of cost function. But in most cases, it is common to set $H_w = 1$ and matrices $Q(i)$ and $R(i)$ time-invariant, say, Q and R .

Minimizing the cost function J at time k subject to the input, output, and state constraints is called the open-loop optimal control problem. Solving such problem yields a control sequence with H_u elements. Only the first element is applied and the whole procedure is repeated in next time instant.

6.3 Model predictive controller design for FCS

The discrete time linear nominal aircraft model is used, by the form of:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k). \end{aligned} \quad (6.9)$$

According to [48], the landing trajectory consists of four segments—“*Cruise*”, “*Coordinated turn*”, “*Glide Slope*” and “*Flare*”. Then, three linear models can be acquired

based on these different conditions, as the following $(x = [u \ \psi \ \theta \ \phi \ v \ w \ H \ p \ q \ r]^T$, $u = [\delta_{el} \ \delta_{er} \ \delta_{al} \ \delta_{ar} \ \delta_r \ \delta_T]^T$ and $C = I$):

- Aircraft Model under Wing-Level Flight:

δ_{el}	-1.8829°	δ_{er}	-1.8829°	δ_{al}	0.0°	δ_{ar}	0.0°
δ_T	0.1881	δ_r	0.0°	ψ	-90°	p	0.0°
θ	11.0°	q	0.0°	ϕ	0.0°	r	0.0°
α	11.0°	β	0.0°	H	600 m	V_t	82.88 m/s

Table 6.4: Wing-Level Flight Conditions

$$\begin{aligned}
 A &= \begin{bmatrix} u & \psi & \theta & \phi & v & w & H & p & q & r \\
 -0.0210 & 0 & -9.6306 & 0 & -0.0012 & 0.1302 & -0.0002 & 0 & -15.0366 & 0 \\
 0 & 0 & -0.0000 & -0.0000 & 0 & 0 & 0 & 0 & -0.0000 & 1.0186 \\
 0 & 0 & 0 & 0.0000 & 0 & 0 & 0 & 0 & 1.0000 & 0.0000 \\
 0 & 0 & -0.0000 & -0.0000 & 0 & 0 & 0 & 1.0000 & -0.0000 & 0.1939 \\
 0.0000 & 0 & 0.0000 & 9.6306 & -0.1474 & -0.0000 & -0.0000 & 15.9723 & 0 & -80.7102 \\
 -0.1178 & 0 & -1.8676 & 0.0000 & -0.0054 & -0.6136 & 0.0009 & 0 & 73.6667 & 0 \\
 0.1904 & 0 & 82.8800 & -0.0000 & 0.0000 & -0.9817 & 0 & 0 & 0 & 0 \\
 -0.0000 & 0 & 0 & 0 & -0.1687 & 0.0000 & -0.0000 & -1.7570 & 0.0003 & 0.8766 \\
 0.0001 & 0 & 0 & 0 & -0.0001 & 0.0001 & 0.0000 & 0 & -0.6460 & -0.0029 \\
 -0.0000 & 0 & 0 & 0 & 0.0224 & 0.0000 & -0.0000 & -0.0482 & 0.0025 & -0.2402
 \end{bmatrix}, \\
 B &= \begin{bmatrix} \delta_{el} & \delta_{er} & \delta_{al} & \delta_{ar} & \delta_r & \delta_T \\
 0.0075 & 0.0075 & -0.0067 & -0.0067 & 0 & 7.1522 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 -0.0154 & 0.0154 & 0.0007 & -0.0007 & 0.0336 & 0 \\
 -0.0674 & -0.0674 & -0.0597 & -0.0597 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0.0562 & -0.0562 & 0.0749 & -0.0749 & 0.0340 & 0 \\
 -0.0301 & -0.0301 & 0.0018 & 0.0018 & 0 & 0 \\
 0.0084 & -0.0084 & -0.0006 & 0.0006 & -0.0169 & 0
 \end{bmatrix}. \tag{6.10}
 \end{aligned}$$

- Aircraft Model under Glide slope Flight:

δ_{el}	-1.8068°	δ_{er}	-1.8068°	δ_{al}	0.0°	δ_{ar}	0.0°
δ_T	0.0465	δ_r	0.0°	ψ	0.0°	p	0.0°
θ	5.15°	q	0.0°	ϕ	0.0°	r	0.0°
α	11.15°	β	0.0°	H	600 m	V_t	82.88 m/s

Table 6.5: Glide slope Flight Conditions

$$\begin{aligned}
 A &= \begin{bmatrix}
 u & \psi & \theta & \phi & v & w & H & p & q & r \\
 -0.0217 & 0 & -9.7708 & 0 & -0.0012 & 0.1312 & -0.0001 & 0 & -15.2498 & 0 \\
 0 & 0 & 0.0000 & -0.0000 & 0 & 0 & 0 & 0 & -0.0000 & 1.0040 \\
 0 & 0 & 0 & -0.0000 & 0 & 0 & 0 & 0 & 1.0000 & 0.0000 \\
 0 & 0 & 0.0000 & -0.0000 & 0 & 0 & 0 & 1.0000 & -0.0000 & 0.0897 \\
 0.0000 & 0 & 0.0000 & 9.7708 & -0.1472 & -0.0000 & -0.0000 & 16.1869 & 0 & -80.6686 \\
 -0.1195 & 0 & -0.8767 & 0.0000 & -0.0052 & -0.6146 & 0.0009 & 0.0000 & 73.6335 & 0 \\
 0.0894 & 0 & 82.4260 & -0.0000 & 0.0000 & -0.9960 & 0 & 0 & 0 & 0 \\
 -0.0000 & 0 & 0 & 0 & -0.1699 & 0.0000 & -0.0000 & -1.7543 & 0.0003 & 0.8784 \\
 0.0001 & 0 & 0 & 0 & -0.0001 & -0.0003 & 0.0000 & 0 & -0.6482 & -0.0029 \\
 -0.0000 & 0 & 0 & 0 & 0.0223 & 0.0000 & -0.0000 & -0.0482 & 0.0025 & -0.2418
 \end{bmatrix}, \\
 B &= \begin{bmatrix}
 \delta_{el} & \delta_{er} & \delta_{al} & \delta_{ar} & \delta_r & \delta_T \\
 0.0073 & 0.0073 & -0.0067 & -0.0067 & 0 & 7.1522 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 -0.0154 & 0.0154 & 0.0008 & -0.0008 & 0.0336 & 0 \\
 -0.0675 & -0.0675 & -0.0598 & -0.0598 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0.0563 & -0.0563 & 0.0749 & -0.0749 & 0.0340 & 0 \\
 -0.0302 & -0.0302 & 0.0018 & 0.0018 & 0 & 0 \\
 0.0084 & -0.0084 & -0.0006 & 0.0006 & -0.0169 & 0
 \end{bmatrix}. \tag{6.11}
 \end{aligned}$$

• Aircraft Model under Flare Flight:

δ_{el}	-4.1910°	δ_{er}	-4.1910°	δ_{al}	0.0°	δ_{ar}	0.0°
δ_T	0.0994	δ_r	0.0°	ψ	0.0°	p	0.0°
θ	8.46°	q	0.0°	ϕ	0.0°	r	0.0°
α	11.73°	β	0.0°	H	12 m	V_t	76.19 m/s

Table 6.6: Flare Flight Conditions

$$\begin{aligned}
 A &= \begin{bmatrix}
 u & \psi & \theta & \phi & v & w & H & p & q & r \\
 -0.0215 & 0 & -9.7075 & 0 & -0.0026 & 0.1504 & -0.0196 & 0 & -14.5273 & -0.0001 \\
 0 & 0 & -0.0000 & -0.0000 & 0 & 0 & 0 & 0 & -0.0000 & 1.0106 \\
 0 & 0 & 0 & 0.0000 & 0 & 0 & 0 & 0 & 1.0000 & 0.0000 \\
 0 & 0 & -0.0000 & -0.0000 & 0 & 0 & 0 & 1.0000 & -0.0000 & 0.1457 \\
 0.0000 & 0 & 0.0000 & 9.7075 & -0.1314 & -0.0000 & -0.0000 & 15.4426 & 0 & -74.0115 \\
 -0.1320 & 0 & -1.4144 & 0.0000 & -0.0092 & -0.6269 & 0.0965 & 0.0001 & 67.1915 & 0 \\
 0.1442 & 0 & 76.0663 & 0.0001 & 0.0000 & -0.9896 & 0 & 0 & 0 & 0 \\
 -0.0000 & 0 & 0 & 0 & -0.1604 & 0.0000 & -0.0000 & -1.6994 & 0.0003 & 0.8595 \\
 0.0015 & 0 & 0 & 0 & 0.0016 & -0.0068 & 0.0142 & 0 & -0.6365 & -0.0029 \\
 0.0000 & 0 & 0 & 0 & 0.0206 & 0.0000 & -0.0000 & -0.0467 & 0.0025 & -0.2395
 \end{bmatrix}, \\
 B &= \begin{bmatrix}
 \delta_{el} & \delta_{er} & \delta_{al} & \delta_{ar} & \delta_r & \delta_T \\
 0.0059 & 0.0059 & -0.0061 & -0.0061 & 0 & 7.5156 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 -0.0138 & 0.0138 & 0.0007 & -0.0007 & 0.0300 & 0 \\
 -0.0601 & -0.0601 & -0.0536 & -0.0536 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0.0506 & -0.0506 & 0.0669 & -0.0669 & 0.0304 & 0 \\
 -0.0277 & -0.0277 & 0.0017 & 0.0017 & 0 & 0 \\
 0.0075 & -0.0075 & -0.0006 & 0.0006 & -0.0151 & 0
 \end{bmatrix}. \tag{6.12}
 \end{aligned}$$

Assume that the whole state vector is measured and there are no disturbances or measurement noise. Then the predicted state based on (6.9) is given as follows:

$$\begin{aligned}\hat{x}(k+1|k) &= Ax(k) + B\hat{u}(k|k), \\ &\vdots \\ \hat{x}(k+H_p|k) &= A\hat{x}(k+H_p-1|k) + B\hat{u}(k+H_p-1|k) \\ &= A^{H_p}x(k) + A^{H_p-1}B\hat{u}(k|k) + \dots + B\hat{u}(k+H_p-1|k).\end{aligned}$$

Because the input will only change at times $k, k+1, \dots, k+H_u-1$, and will remain constant after that. So we have $\Delta\hat{u}(k+i|k) = 0$ for $H_u \leq i \leq H_p-1$. Then the input sequence can be rewritten as:

$$\begin{aligned}\hat{u}(k|k) &= \Delta\hat{u}(k|k) + u(k-1), \\ &\vdots, \\ \hat{u}(k+H_u-1|k) &= \Delta\hat{u}(k+H_u-1|k) + \dots + \Delta\hat{u}(k|k) + u(k-1),\end{aligned}$$

and hence it's easy to get the predicted stats in following matrix-vector form,

$$\begin{aligned}\hat{\mathcal{X}}(k) &= \Psi x(k) + \Upsilon u(k-1) + \Theta \Delta \hat{\mathcal{U}}(k), \\ \begin{bmatrix} \hat{x}(k+1|k) \\ \vdots \\ \hat{x}(k+H_u|k) \\ \hat{x}(k+H_u+1|k) \\ \vdots \\ \hat{x}(k+H_p|k) \end{bmatrix} &= \begin{bmatrix} A \\ \vdots \\ A^{H_u} \\ A^{H_u+1} \\ \vdots \\ A^{H_p} \end{bmatrix} x(k) + \begin{bmatrix} B \\ \vdots \\ \sum_{i=0}^{H_u-1} A^i B \\ \sum_{i=0}^{H_u} A^i B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i B \end{bmatrix} u(k-1) + \\ &\quad \begin{bmatrix} B & \dots & 0 \\ AB+B & \dots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_u-1} A^i B & \dots & B \\ \sum_{i=0}^{H_u} A^i B & \dots & AB+B \\ \vdots & \vdots & \vdots \\ \sum_{i=0}^{H_p-1} A^i B & \dots & \sum_{i=0}^{H_p-H_u} A^i B \end{bmatrix} \begin{bmatrix} \Delta\hat{u}(k|k) \\ \vdots \\ \Delta\hat{u}(k+H_u-1|k) \end{bmatrix}.\end{aligned}\tag{6.13}$$

where $\hat{\mathcal{X}}(k) = [\hat{x}(k+1|k)^T, \dots, \hat{x}(k+H_p|k)^T]^T$ and $\Delta\hat{\mathcal{U}}(k) = [\Delta\hat{u}(k|k)^T, \dots, \Delta\hat{u}(k+H_u-1|k)^T]^T$.

Suppose the constraints are in the form:

$$\begin{aligned} E_s \begin{bmatrix} \Delta\mathcal{U}(k) \\ 1 \end{bmatrix} &\leq 0, \\ F_s \begin{bmatrix} \mathcal{U}(k) \\ 1 \end{bmatrix} &\leq 0, \\ G_s \begin{bmatrix} \mathcal{X}(k) \\ 1 \end{bmatrix} &\leq 0, \end{aligned} \quad (6.14)$$

and F_s has the form:

$$F_s = [F_{s1}, F_{s2}, \dots, F_{sH_u}, f_s] \quad (6.15)$$

where each F_{si} is of size $q \times n_u$, and f_s has size $q \times 1$, so that (6.14) can be rewritten as:

$$\sum_{i=1}^{H_u} F_{si} \hat{u}(k+i-1|k) + f_s \leq 0. \quad (6.16)$$

Since

$$\hat{u}(k+i-1|k) = u(k-1) + \sum_{j=0}^{i-1} \Delta\hat{u}(k+j|k), \quad (6.17)$$

then from (6.14) we can get:

$$\begin{aligned} \sum_{j=1}^{H_u} F_{sj} \Delta\hat{u}(k|k) &+ \sum_{j=2}^{H_u} F_{sj} \Delta\hat{u}(k+1|k) + \dots + F_{sH_u} \Delta\hat{u}(k+H_u-1|k) \\ &+ \sum_{j=1}^{H_u} F_{sj} u(k-1) + f_s \leq 0 \end{aligned}$$

Define $\mathcal{F}_i = \sum_{j=i}^{H_u} F_{sj}$ and $\mathcal{F} = [\mathcal{F}_1, \dots, \mathcal{F}_{H_u}]$, (6.14) can be rewritten as:

$$\mathcal{F} \Delta\mathcal{U}(k) \leq -\mathcal{F}_1 u(k-1) - f_s, \quad (6.18)$$

where the right hand side of this inequality is a vector known at time k . So we have converted (6.14) into a linear inequality constraint on $\Delta\mathcal{U}(k)$. Recall (6.13), we can rewrite (6.14) as:

$$G_s \begin{bmatrix} \Psi x(k) + \Upsilon u(k-1) + \Theta \Delta\mathcal{U}(k) \\ 1 \end{bmatrix} \leq 0. \quad (6.19)$$

Now let $G_s = [\mathcal{G}, g_s]$, where g_s is the last column of G_s , we have:

$$\mathcal{G}[\Psi x(k) + \Upsilon u(k-1) + \Theta \Delta\mathcal{U}(k)] + g_s \leq 0 \quad (6.20)$$

and hence:

$$\mathcal{G}\Theta\Delta\mathcal{U}(k) \leq -\mathcal{G}[\Psi x(k) + \Upsilon u(k-1)] - g_s. \quad (6.21)$$

After put inequality (6.14) into the form:

$$\mathcal{E}\Delta\mathcal{U}(k) \leq e_s, \quad (6.22)$$

we can assemble inequalities (6.18), (6.21) and (6.22) into the single inequality:

$$\begin{bmatrix} \mathcal{F} \\ \mathcal{G}\Theta \\ \mathcal{E} \end{bmatrix} \Delta\mathcal{U}(k) \leq \begin{bmatrix} -\mathcal{F}_1 u(k-1) - f_s \\ -\mathcal{G}[\Psi x(k) + \Upsilon u(k-1)] - g_s \\ e_s \end{bmatrix} \quad (6.23)$$

Recall the cost function $J(k)$ we defined in formula (6.8). We have to solve the following constrained optimization problem:

$$\min_{\Delta u} \Delta\mathcal{U}(k)^T H \Delta\mathcal{U}(k) - \mathcal{G}_f^T \Delta\mathcal{U}(k) \quad (6.24)$$

subject to the inequality constraints (6.23), where

$$H = \Theta^T Q \Theta + R, \quad (6.25)$$

and

$$\begin{aligned}
 \mathcal{G}_f &= 2\Theta^T Q \varepsilon(k) \\
 \varepsilon(k) &= \mathcal{T}(k) - \Psi x(k) - \Upsilon u(k-1) \\
 \mathcal{T}(k) &= \begin{bmatrix} \hat{r}(k+H_1|k) \\ \vdots \\ \hat{r}(k+H_p|k) \end{bmatrix}.
 \end{aligned} \tag{6.26}$$

Obviously, this is a standard optimization problem know as QP problem, and standard algorithms are available for its solution [26].

6.4 Design and simulation

In this section, a high fidelity nonlinear F-16 aircraft model is controlled by the designed MPC and an existing H_∞ controller to perform this auto-landing process. Input constraints include $-25^\circ \leq \delta_e \leq 25^\circ$, $-20^\circ \leq \delta_a \leq 20^\circ$, $-30^\circ \leq \delta_r \leq 30^\circ$ and $0^\circ \leq \delta_T \leq 100\%$. Input changing rate constraints include $-60^\circ/\text{s} \leq \dot{\delta}_e \leq 60^\circ/\text{s}$, $-80^\circ/\text{s} \leq \dot{\delta}_a \leq 80^\circ/\text{s}$ and $-120^\circ/\text{s} \leq \dot{\delta}_r \leq 120^\circ/\text{s}$. Meanwhile, we require the state constraint $-40^\circ \leq \phi \leq 40^\circ$ to be satisfied.

Simulation results are given as below.

MPC: nominal case

First of all, we consider the nominal case where every actuator is well functional.

Figure 6.1: Auto-landing process with MPC controller, states and outputs in nominal case

Figure 6.2: Auto-landing process with MPC controller, control inputs in nominal case

The first stage in this auto-landing process is level flight stage (from $t = 0$ s to $t = 50$ s) where the controller tries to maintain velocity, height and path angle. The second stage is from $t = 50$ s to $t = 120$ s, which is named coordinate turn. In this stage, our controller need to manipulate the aircraft to turn from $\psi = -90^\circ$ to $\psi = 0^\circ$ without changing height and velocity, as depicted in the (1,3) entry of Fig. 6.1. The third stage is glide slope stage, from $t = 120$ s to $t = 225$ s. In this stage we aim at tracking the pre-defined trajectory as close as possible, shown in the (4,3) entry of Fig. 6.1. The last stage is named flare stage, starting from $t = 225$ s and ending at $t = 250$ s. In this stage, forward velocity needs to be reduced to setting point, as shown in the (3,1) entry and (4,1) entry of Fig. 6.1.

Fig. 6.2 gives the input signals. We can see from these two figures that all the constraints (including input saturations and state constraint) are satisfied during the simulation.

MPC: fault case

In fault case, the left aileron is stuck at 20° from $t = 15$ s (during level flight stage).

Figure 6.3: Auto-landing process with MPC controller, states and outputs in fault case

Figure 6.4: Auto-landing process with MPC controller, control inputs in fault case

Because of the stuck fault introduced from $t = 15$ s, the system performance is degraded. We can see from the (4,3) entry of Fig. 6.3 that the tracking performance is worse nominal case but still satisfy the requirements defined in Section 6.1. In Fig. 6.4 the (2,1) entry depicts the left aileron experiencing a stuck fault at 20° from 15s. Thereafter, the remaining 4 control surfaces (δ_{el} , δ_{er} , δ_{ar} and δ_r) have to push harder to counteract the effect led by the fault. Similar to the nominal case, all the constraints are not violated during the simulation.

6.5 Comparison with a standard H_∞ controller

In [51], H_∞ approach has been fully developed and results in a good tracking performance. In this section we compare the proposed MPC approach with H_∞ approach in [51], considering same scenarios in above section.

6.5.1 H_∞ Controller: nominal case

Figure 6.5: Auto-landing process with H_∞ controller, states and outputs in nominal case

Figure 6.6: Auto-landing process with H_∞ controller, control inputs in nominal case

Because H_∞ approach can not take into account of state constraint, the coordinate turn stage is neglected. The simulation consists of only two stages, level flight (from $t = 0$ s to $t = 25$ s) and glide slope stage (from $t = 0$ s to $t = 25$ s to end).

6.5.2 H_∞ Controller: fault case

In fault case, the left aileron is stuck at 20° from $t = 10$ s.

Figure 6.7: Auto-landing process with H_∞ controller, states and outputs in fault case

Figure 6.8: Auto-landing process with H_∞ controller, control inputs in fault case

6.5.3 Discussion and conclusion

From Fig. 6.1 and Fig. 6.5, both in nominal case, we can see that the maximum vertical tracking error of MPC approach is 2.4m while that of H_∞ approach is 16m. The maximum resultant velocity variation is 2m/s and 7m/s, for MPC and H_∞ approach, respectively.

In fault case, that is, when the left aileron is stuck at 20° , according to Fig. 6.3 and Fig. 6.7, we can find that the maximum vertical error of MPC approach is 3.8m while that of H_∞ approach is more than 16m. The maximum resultant velocity variation is 2m/s and 7m/s, MPC and H_∞ approach, respectively. Further more, notice that in the flare stage, MPC approach only leads to a tracking error no more than 1m while that of H_∞ approach is 5m. Recall the deviation error requirements, that “during the final phase of landing, the vertical deviation from the desired flight path should not exceed 1.5m”, we can find that the MPC approach can satisfy the performance requirements.

Also, from Fig. 6.3, the fault tolerance capability of constrained MPC has been illustrated. Without an *FDI* or controller reconfiguration, the system can still be stabilized and the performance degradation is not significant. This is because that when the left aileron is jammed at the limit, after a few ‘seconds’, the controller ‘thinks’ that it has driven the aileron to the limit, and starts to move the other control surfaces to meet the setting trajectory, although the controller has no knowledge of the fault at all.

On the other hand, MPC approach can guarantee various types of constraints. Compared to the anti-windup compensation approach, MPC seems more capable. Nevertheless, the MPC approach introduced here is an on-line optimization algorithm. The computational burden is much heavier than the H_2/H_∞ or anti-windup approaches which can be determined off-line. The heavy on-line computational burden limits MPC for a long time to the ‘slow’ dynamic systems. Benefited from availability of faster computer in these decades, on-line MPC approaches become more and more popular in other ‘rapid’ response system like aircraft. In the following chapters, we will introduce some efficient off-line MPC approaches to theoretically address this disadvantage.

Chapter 7

Reliable aircraft control against faults and saturations: on-line and off-line robust MPC approaches

From Chapter 3 to Chapter 5 we discussed H_2/H_∞ control strategy and anti-windup technique to address input saturation problem. In order to consider more types of constraints in one-step design methodology we introduced a typical on-line MPC approach to aircraft reliable control area in Chapter 6. Such on-line optimization method provide good constraints and fault handling ability in the auto-landing process simulation. Nevertheless, three potential disadvantages have also been raised.

First of all, the fault handling ability in Chapter 6 is achieved by exploring the inherent robustness of the MPC, which can be further improved by considering system behavior in fault conditions as well as in nominal condition, i.e., robust MPC. Robust MPC is to study MPC problem over linear systems with polytopic uncertainty. In recent years, many researchers have made great contribution in developing robust MPC approaches for these systems, such as Kothare *et al.* [44], Wu, [73], Cuzzola, *et al.* [17], etc. Via such approaches, a broad class of model uncertainty descriptions are addressed with

guaranteed robust stability and satisfied performance.

Second of all, disturbances have not been considered in Chapter 6. As we know, disturbances are widely encountered and often adversely affect performance and stability. Therefore, guaranteeing an H_∞ norm bound for systems with disturbances is also a very important issue in MPC area.

Last but not least, the on-line procedure typically leads to a huge computational effort that has to be finished in each and every sampling period. This is the foremost factor that keeps preventing MPC from being populated on fast response systems. Although this problem is being relieved with the rapid development of computer power, it is also necessary to investigate how to reduce such on-line computational demand theoretically.

According to these disadvantages mentioned above, in this chapter we aim at constructing a robust multi-objective MPC formulation to address both polytopic uncertainty and disturbances. Further more, the concept of *the asymptotically stable invariant ellipsoids* is used to reduce the computational effort dramatically, leading to two efficient off-line MPC approaches. These two approaches are applied on a high fidelity nonlinear F-16 aircraft model with control surface impairment faults and various types of constraints.

For ease of reading, we represent the augmented system with disturbances as follows,

$$\begin{aligned} x(k+1) &= A(k)x(k) + B^u(k)u(k) + B^w(k)w(k) \\ y(k) &= C(k)x(k) + D(k)u(k) \\ z(k) &= C_\infty x(k) + D_\infty u(k) \end{aligned} \tag{7.1}$$

where w is assumed energy bounded. The weight matrices C_∞ , D_∞ define the performance variable $z(k)$. The system matrices are consistent with (2.32) and (2.33). The uncertainty polytope is defined as

$$[A(k) \ B^u(k) \ B^w(k) \ C(k) \ D(k)] \in \Omega \tag{7.2}$$

where Ω is a convex hull with vertices $[A_j \ B_j^u \ B_j^w \ C_j \ D_j]$, $j = 0, 1, \dots, l$:

$$\Omega := Co\{[A_1 \ B_1^u \ B_1^w \ C_1 \ D_1], \dots, [A_l \ B_l^u \ B_l^w \ C_l \ D_l]\} \quad (7.3)$$

and for $\forall k$ there exist l nonnegative coefficients $\theta_j(k)$ (with $j = 0, 1, \dots, l$) such that $\sum_{j=0}^l \theta_j(k) = 1$ and

$$\begin{aligned} A(k) &= \sum_{j=0}^l A_j \theta_j(k), & B^u(k) &= \sum_{j=0}^l B_j^u \theta_j(k), & B^w(k) &= \sum_{j=0}^l B_j^w \theta_j(k), \\ C(k) &= \sum_{j=0}^l C_j \theta_j(k), & D(k) &= \sum_{j=0}^l D_j \theta_j(k). \end{aligned} \quad (7.4)$$

7.1 On-line multi-objective robust MPC

Consider the following discrete time linear system (7.1), with $C_\infty^T D_\infty = 0$. In this section, we aim at achieving the following multi-objective robust performance,

$$\min_{u(k+i|k), i=0,1,\dots,\infty} \max_{[A(k) \ B^u(k) \ B^w(k) \ C(k) \ D(k)] \in \Omega} J_\infty(k) \quad (7.5)$$

subject to

$$\|T_{zw}\|_\infty^2 < \delta, \quad (7.6)$$

with

$$J_\infty(k) = \sum_{i=0}^{\infty} z(k+i|k)^T z(k+i|k), \quad (7.7)$$

where $\delta > 0$ is the given H_∞ performance bound.

7.1.1 Constraints

Input constraints

In this section the constraints on the present and the entire horizon of future input signals are considered, as follows, at each sampling time k ,

$$\|H_m u(k+i|k)\|^2 \leq \hat{u}_m^2, \quad i \geq 0, \quad m = 1, 2, \dots, n_u, \quad (7.8)$$

where $H_m, m = 1, 2, \dots, n_u$ are given matrices that select the m^{th} element of u , and $\hat{u}_m$ is given element-wise input constraints.

State constraints

Constraints on the one-step ahead state signals are given as below,

$$\|E_m x(k+1)\|^2 \leq \hat{x}_m^2, \quad m = 1, 2, \dots, n_x, \quad (7.9)$$

where $E_m, m = 1, 2, \dots, n_x$ are given matrices that select the m^{th} element of x , and $\hat{x}_m$ is given element-wise state constraints.

7.1.2 Multi-objective robust MPC problem formulation

Consider a quadratic function $V(x) = x^T P_j x, P_j > 0, j = 0, 1, \dots, l$ and state feedback control law $u(k+i|k) = F(k)x(k+i|k), i \geq 0$. We have

$$\begin{aligned} V(x(k+i+1|k)) - V(x(k+i|k)) = \\ \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix}^T \begin{bmatrix} A_{cl}(k)^T P_j A_{cl}(k) - P_j & * \\ B^w(k)^T P_j A_{cl}(k) & B^w(k)^T P_j B^w(k) \end{bmatrix} \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix} \end{aligned} \quad (7.10)$$

where $A_{cl}(k) = A(k) + B^u(k)F(k)$.

Since the system is robustly stabilizable, we can assume $x(\infty|k) = 0$ and hence, $V(x(\infty|k)) = 0$. In view of this, summing (7.10) from $i = 0$ to $i = \infty$, we can obtain

$$-V(x(k|k)) = \sum_{i=0}^{\infty} \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix}^T \begin{bmatrix} A_{cl}(k)^T P_j A_{cl}(k) - P_j & * \\ B^w(k)^T P_j A_{cl}(k) & B^w(k)^T P_j B^w(k) \end{bmatrix} \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix}. \quad (7.11)$$

As $C_{\infty}^T D_{\infty} = 0$, (7.7) is equivalent to

$$J_{\infty}(k) = \delta \sum_{i=0}^{\infty} w(k+i)^T w(k+i) + \sum_{i=0}^{\infty} \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix}^T \begin{bmatrix} C_{\infty}^T C_{\infty} + F(k)^T D_{\infty}^T D_{\infty} F(k) & * \\ 0 & -\delta \end{bmatrix} \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix}. \quad (7.12)$$

Adding (7.11) to (7.12), following equation can be obtained,

$$J_{\infty}(k) = V(x(k|k)) + \delta \sum_{i=0}^{\infty} w(k+i)^T w(k+i) + \sum_{i=0}^{\infty} \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix}^T \Lambda \begin{bmatrix} x(k+i|k) \\ w(k+i) \end{bmatrix} \quad (7.13)$$

where

$$\Lambda = \begin{bmatrix} A_{cl}(k)^T P_j A_{cl}(k) - P_j + C_{\infty}^T C_{\infty} + F(k)^T D_{\infty}^T D_{\infty} F(k) & * \\ B^w(k)^T P_j A_{cl}(k) & B^w(k)^T P_j B^w(k) - \delta I \end{bmatrix}.$$

Using bounded real lemma [21, 34], it can be shown that (7.6) is satisfied if and only if $\Lambda \leq 0$, which in turn results in

$$J_{\infty}(k) \leq V(x(k|k)) + \delta \sum_{i=0}^{\infty} w(k+i)^T w(k+i). \quad (7.14)$$

Since $w(k)$ is assumed to be energy bounded, i.e., $\sum_{i=0}^{\infty} w(k+i)^T w(k+i) \leq \hat{w}^2 < \infty$, let $V(x(k|k)) \leq \gamma$, therefore

$$J_{\infty}(k) \leq \gamma + \delta \hat{w}^2. \quad (7.15)$$

Summing up the above development, the multi-objective MPC problem described in (7.5), (7.6) becomes

$$\min_{F(k), P_j} \gamma \quad (7.16)$$

subject to

$$\Lambda \leq 0. \quad (7.17)$$

7.1.3 Main result

In this section we are ready to present the proposed multi-objective robust MPC approach as the following theorem.

Theorem 7.1.1 (*Multi-objective MPC*) Consider the discrete time linear system (7.1) with polytopic uncertainty (7.4) at time k and let $x(k|k)$ be the measured state $x(k)$.

- (a) *Existence:* There exists a state feedback control law $u(k+i|k) = F(k)x(k+i|k)$, $i \geq 0$ that guarantees the H_{∞} constraint (7.6) if there exist $l+1$ matrices $Q_j = Q_j^T > 0$, and a pair of matrices $\{Y, G\}$ with G is nonsingular, satisfying the following LMIs, $\forall j = 0, 1, \dots, l$

$$\begin{bmatrix} Q_j - G - G^T & * & * & * & * \\ 0 & -\delta\gamma I & * & * & * \\ A_j Q_j + B_j^u Y & \gamma B_j^w & -Q_j & * & * \\ C_\infty Q_j & 0 & 0 & -\gamma I & * \\ D_\infty Y & 0 & 0 & 0 & -\gamma I \end{bmatrix} \leq 0, \quad (7.18)$$

$$\begin{bmatrix} 1 & * \\ x(k|k) & Q_j \end{bmatrix} \geq 0. \quad (7.19)$$

(b) *Optimization:* Suppose matrices Q_j , Y and G are defined as in (a), the robust H_2 tracking performance is optimized by solving the following LMI problem:

$$\min_{Q_j, Y, G} \gamma \quad (7.20)$$

subject to LMIs (7.18) and (7.19) for $j = 0, 1, \dots, l$.

(c) *Controller:* The state feedback matrix $F(k)$ of the form $u(k+i|k) = F(k)x(k+i|k)$, $i \geq 0$ is given by:

$$F(k) = YG^{-1}. \quad (7.21)$$

(d) *Invariant ellipsoids:* Suppose the LMI optimization problem (7.20) has a solution and the state feedback matrix $F(k)$ of (7.21) is adopted, then

$$\mathcal{E} = \{x^T Q_e^{-1} x \leq 1\} \quad (7.22)$$

is an invariant ellipsoid for the predicted states of the uncertain system where the matrix Q_e can be obtained as the solution of the following LMI optimization

problem:

$$\max_{Q_e} \text{trace}(Q_e) \quad (7.23)$$

subject to the following constraints:

$$Q_e \leq Q_j, \quad \forall j = 0, 1, \dots, l. \quad (7.24)$$

(e) *Input constraints: input constraints (7.8) are satisfied if the following inequalities can be fulfilled, for $m = 1, 2, \dots, n_u$,*

$$\begin{bmatrix} \hat{u}_m^2 & * \\ Y^T H_m^T & G + G^T - Q_j \end{bmatrix} \geq 0. \quad (7.25)$$

(f) *State constraints: state constraints (7.9) are satisfied if the following inequalities can be fulfilled, for $m = 1, 2, \dots, n_x$,*

$$\begin{bmatrix} \hat{x}_m^2 & * & * \\ B_j^{wT} E_m^T & \frac{1}{1+\bar{w}^2} & * \\ (A_j G + B_j^u Y)^T E_m^T & 0 & \frac{1}{1+\bar{w}^2} (G + G^T - Q_j) \end{bmatrix} \geq 0. \quad (7.26)$$

Proof:

(a) By Shur complement, $\Lambda \leq 0$ is equivalent to

$$\begin{bmatrix} -P_j & * & * & * & * \\ 0 & -\delta I & * & * & * \\ A_j + B_j F(k) & B_j^w & -P_j^{-1} & * & * \\ C_\infty & 0 & 0 & -I & * \\ D_\infty F(k) & 0 & 0 & 0 & -I \end{bmatrix} \leq 0, \quad (7.27)$$

for $j = 0, 1, \dots, l$.

Pre and post multiplying $\text{diag}\{P_j^{-1}, I, I, I, I\}$ to (7.27), we get for $j = 0, 1, \dots, l$

$$\begin{bmatrix} -P_j^{-1} & * & * & * & * \\ 0 & -\delta I & * & * & * \\ (A_j + B_j F(k))P_j^{-1} & B_j^w & -P_j^{-1} & * & * \\ C_\infty P_j^{-1} & 0 & 0 & -I & * \\ D_\infty F(k)P_j^{-1} & 0 & 0 & 0 & -I \end{bmatrix} \leq 0. \quad (7.28)$$

Defining $Q_j = \gamma P_j^{-1}$, pre and post multiplying (7.28) with $\text{diag}\{G^T Q_j^{-1}, I, I, I, I\}$ and $\text{diag}\{Q_j^{-1} G, I, I, I, I\}$ results in

$$\begin{bmatrix} -G^T Q^{-1} G & * & * & * & * \\ 0 & -\delta \gamma I & * & * & * \\ (A_j + B_j Y & \gamma B_j^w & -Q & * & * \\ C_\infty G & 0 & 0 & -\gamma I & * \\ D_\infty Y & 0 & 0 & 0 & -\gamma I \end{bmatrix} \leq 0. \quad (7.29)$$

where $Y := F(k)G$. Since the additional matrix G is nonsingular, $(G^T - Q_j)Q_j^{-1}(G - Q_j)$ is nonnegative definite, and consequently $G + G^T - Q_j \leq G^T Q_j^{-1} G$ holds true.

Thus (7.29) holds true if (7.18) is satisfied.

By Shur complement, (7.19) can be rewritten as $x(k|k)^T Q_j^{-1} x(k|k) \leq 1$. Since $Q_j = \gamma P_j^{-1}$, then $x(k|k)^T P_j x(k|k) \leq \gamma$, which implies the upper bound assumption (7.15). This finish the proof to part (a) of Theorem 7.1.1.

- (b) From (7.5) and (7.15), it's clear that with given scalars δ and $\hat{w}$, $J_\infty(k)$ is upper bounded by $\gamma + \delta \hat{w}^2$. Thus, minimizing $\gamma + \delta \hat{w}^2$ is equivalent to minimizing γ .
- (c) As we define $Y = F(k)G$ and G is nonsingular, (7.21) can be obtained.
- (d) Proof of claim (d) follows Cuzzola *et al.* [17] and is omitted here for brevity.
- (e) Applying similar technique to the LMI conditions of input/state constraints in Kothare *et al.* [44], (7.25) and (7.26) can be obtained.
- (f) Proof to claim (f) consists of two parts. Firstly, we prove the case that only one common Lyapunov function is adopted, which means, $G = Q_j = Q$, $j = 0, 1, \dots, l$.

Then, (7.26) is straightforward to be proved.

Since we have defined that $w(k)$ is energy bounded by $\hat{w}^2$, then

$$\begin{aligned}
\|E_m x(k+1)\|^2 &= \|E_m[(A(k) + B^u(k)YQ^{-1})Q^{\frac{1}{2}} \quad B^w(k)][x(k)^T Q^{-\frac{1}{2}} \quad w(k)^T]^T\|^2 \\
&\leq \|E_m[(A(k) + B^u(k)YQ^{-1})Q^{\frac{1}{2}} \quad B^w(k)]\|^2 \| [x(k)^T Q^{-\frac{1}{2}} \quad w(k)^T]^T \|^2 \\
&\leq (1 + \hat{w}^2) \|E_m[(A(k) + B^u(k)YQ^{-1})Q^{\frac{1}{2}} \quad B^w(k)]\|^2 \\
&= (1 + \hat{w}^2) \|E_m[(A(k)Q + B^u(k)Y)Q^{-\frac{1}{2}} \quad B^w(k)]\|^2 \\
&= (1 + \hat{w}^2) E_m(A(k)Q + B^u(k)Y)Q^{-1}(A(k)Q + B^u(k)Y)^T E_m^T \\
&\quad + (1 + \hat{w}^2) E_m B^w(k) B^w(k)^T E_m^T
\end{aligned}$$

Following state constraints (7.9), by Shur complement, following LMI can be obtained, for $m = 1, 2, \dots, n_x$

$$\begin{bmatrix} \hat{x}_m^2 & * & * \\ B^w(k)^T E_m^T & \frac{1}{1+\hat{w}^2} & * \\ (A(k)Q + B^u(k)Y)^T E_m^T & 0 & \frac{1}{1+\hat{w}^2} Q \end{bmatrix} \geq 0 \quad (7.30)$$

Using similar technique of in proving claim (a), (7.26) can be derived.

This complete the proof. ■

Remark 7.1.1 *Instead of restricting ourselves to the trajectory tracking problem, the proposed multi-objective robust MPC approach can be popularly adopted in many fields. The exogenous input w can be replaced by all signals which enter the system from its environment, such as commands, disturbances and sensor noises as well as reference signals.*

Theorem 7.1.2 (Robust Stability) *Let $x(k|k) = x(k)$ be the state of uncertain system (7.1) measured at time k and assume that the LMI optimization problem (7.20) admits a*

solution at the same sampling time. Then, the feasible MPC state feedback control law obtained from Theorem 7.1.1 can robustly asymptotically stabilize system (7.1).

Proof: Kothare *et al.* proposed a robust MPC approach in [44] and proved that it can also robustly asymptotically stabilize the system in the presence of energy bounded disturbances. Hereafter we will show that all the conditions in [44] can be guaranteed through Theorem 7.1.1.

Firstly, $\Lambda \leq 0$ implies that

$$A_{cl}(k)^T P_j A_{cl}(k) - P_j + C_\infty^T C_\infty + F(k)^T D_\infty^T D_\infty F(k) \leq 0,$$

which can be recasted as follows,

$$\begin{bmatrix} Q_j & * & * & * \\ A_j Q_j + B_j^u Y & Q_j & * & * \\ C_\infty Q_j & 0 & \gamma I & * \\ D_\infty Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0.$$

As stated in [44], satisfying this LMI and (7.19) guarantees robustly asymptotically stability in the presence of energy bounded disturbances. Therefore, our result can also robustly asymptotically stabilize the uncertain system (7.1). ■

Theorem 7.1.3 (Feasibility) Let $x(k|k) = x(k)$ be the state of system (7.1) measured at time k and assume that the LMI optimization problem (7.20) admits a solution at the same sampling time. Then the solution at time k is a feasible (but not necessarily optimal) solution for problem (7.20) at time $k + 1$.

Proof: The proof directly follows Lemma 2 in [44] and is omitted here for brevity. ■

7.2 Off-line robust MPC based on parameter dependent Lyapunov functions

Although the robust MPC approach in [17] reduces the conservativeness by adopting parameter dependent Lyapunov functions, it also aggravates the on-line computational burden. In this section, inspired by Wan and Kothare [70], two off-line approaches for this robust constrained MPC are presented, by using the concept of *the asymptotically stable invariant ellipsoids*. The subsequent off-line formulation is based on polytopic uncertain systems described by (7.1) and applied to an F-16 model in the next section.

7.2.1 Asymptotically stable invariant ellipsoid

First, for convenience, we recall the definition of *the asymptotically stable invariant ellipsoids*:

Definition 7.2.1 (Kothare *et. al.* [44]) *Given a discrete time dynamical system $x(k+1) = f(x(k))$, a subset $\mathcal{E} = \{x \in \mathbb{R}^{n_x} | x^T Q^{-1} x \leq 1\}$ with $Q > 0$ of the state space $\mathbb{R}^{n_x}$, or in short, Q , is said to be an asymptotically stable invariant ellipsoid if it has the property that whenever $x(k_1) \in \mathcal{E}$, then $x(k) \in \mathcal{E}$ for all times $k \geq k_1$ and $x(k) \rightarrow 0$ as $k \rightarrow \infty$.*

From Theorem 7.1.1, we know that $\mathcal{E} = \{x \in \mathbb{R}^{n_x} | x^T Q_e^{-1} x \leq 1\}$ is an asymptotically stable invariant ellipsoid.

7.2.2 Off-line robust MPC formulation: interpolation method

For an input constrained system, if the initial state is far from the origin, to satisfy (7.8), the resulting Q_e^{-1} should have more constraints, and hence lead to a more constrained state feedback matrix. From Kothare *et. al.* [44], and Wan and Kothare [70], we know

that it is not necessary to keep the feedback matrix F constant while the state is converging to the origin. The closer it goes to the origin, the less constraints it will be imposed. By pre-defining *asymptotically stable invariant ellipsoids* one inside another, we can schedule the corresponding feedback matrices based on the distance ($\|x\|_{Q_e^{-1}}$) between the state and the origin. The modified algorithm from that of Wan and Kothare [70] is shown as follows.

Algorithm 7.2.1 (*Off-line robust constrained MPC based on parameter dependent Lyapunov functions*). Consider the uncertain system (7.1) subject to input and state constraints (7.8), (7.9).

Off-line (only need to do once): In this procedure, a set of canonical states x^i ($i = 1, \dots, N$) will be used to calculate the corresponding optimizers.

- (1) Given an initial feasible state x^1 , let $i := 1$;
- (2) With x^i , solve the optimization problem (7.20) subject to LMIs (7.18), (7.19), input constraint (7.8), state constraint (7.9), and l additional constraints $Q_j^{i-1} > Q_j^i$ (ignored at $i = 1$), for all $j = 1, 2, \dots, l$. This will yield the minimizers γ^i , G^i , X^i , Y^i , Z^i and Q_j^i ;
- (3) Solve the optimization problem (7.23) subject to constraint (7.24) and record the resulting Q_e^i in a look-up table with G^i , $F^i (= Y^i(G^i)^{-1})$;
- (4) Choose a state x^{i+1} satisfying $\|x^{i+1}\|_{(Q_e^i)^{-1}}^2 < 1$ while $i < N$ and go to step (2).

On-line: Given the optimizers from the off-line part, in real-time implementation,

- (1) Given an initial state $x(0)$ satisfying $\|x(0)\|_{(Q_e^1)^{-1}}^2 \leq 1$, set time step $k := 0$;
- (2) Perform a bisection search over Q_e^i in the look-up table to find the largest index i , which is equivalent to the smallest asymptotically stable ellipsoid $\mathcal{E}^i = \{x \in \mathbb{R}^{n_x} | x^T (Q_e^i)^{-1} x \leq 1\}$ that $x(k)$ belongs to;

- (3) Apply the control law $u(k) = F^i x(k)$ to the system, set $k = k + 1$ and move to step (2).

Corollary 7.2.1 *Assuming the optimization problem described in the off-line part of Algorithm 7.2.1 is feasible for x^i , it is always feasible for all $x^{i+1}, \dots, x^N$.*

Proof: Assuming the optimization problem in the step (2) (off-line part of Algorithm 7.2.1) for x^i is feasible and a set of optimizers $\gamma^i, G^i, X^i, Y^i, Z^i$ and Q_j^i are available.

To prove this corollary, we need to find a set of optimizers which satisfy the LMIs (7.18), (7.19), (7.8), (7.9) and the additional constraints

$$Q_j^i > Q_j^{i+1}, \quad \forall j = 1, 2, \dots, l. \quad (7.31)$$

From step (4) (off-line part of Algorithm 7.2.1) we know that

$$\|x^{i+1}\|_{(Q_e^i)^{-1}}^2 < 1. \quad (7.32)$$

Now, $\exists \delta > 1$ such that

$$\|\delta x^{i+1}\|_{(Q_e^i)^{-1}}^2 = 1, \quad (7.33)$$

and $\gamma^{i+1} = \frac{\gamma^i}{\delta^2}$, $G^{i+1} = \frac{G^i}{\delta^2}$, $X^{i+1} = \frac{X^i}{\delta^2}$, $Y^{i+1} = \frac{Y^i}{\delta^2}$, $Z^{i+1} = \frac{Z^i}{\delta^2}$, $Q_j^{i+1} = \frac{Q_j^i}{\delta^2}$ is a feasible solution which satisfies the LMIs (7.18), (7.19), (7.8), (7.9) and (7.31). This argument can be continued for time $i + 2, i + 3, \dots$ to complete the proof. ■

Remark 7.2.1 (Optimality) *From Kothare et. al. [44], it is known that the optimal robust MPC control laws and the corresponding asymptotically stable invariant ellipsoids depend on the states. Notice that in steps (2) and (3) (on-line part of Algorithm 7.2.1), we kind of “find” the smallest asymptotically stable ellipsoid $\mathcal{E}^i$ the current state $x(k)$ belongs to and “apply” the corresponding control law F^i to the system. Although all states in the $\mathcal{E}^i$ can be asymptotically stabilized, the control law is not necessarily*

optimal. This means we sacrifice the optimality for significantly reducing the on-line computational demand. By this way, we can extend MPC to some novel fields where on-line optimization approaches are hardly implementable.

Remark 7.2.2 Notice in step (4) (off-line part of Algorithm 7.2.1), every randomly chosen state x^{i+1} may yield an unique Q_e^{i+1} and state feedback gain F^{i+1} , and hence affect the result of on-line implementation. Then, how to choose a “better” initial state x^{i+1} is still a problem for further study. In order to avoid one single state that affects too much on the MPC control law, we can use a set of randomly chosen states x^{i+1} instead to balance their effects.

Hereafter, we give the closed-loop analysis of this off-line MPC approach.

Theorem 7.2.1 For a discrete time polytopic uncertain system (7.1) with an initial state $x(0)$ which satisfies $\|x(0)\|_{(Q_e^1)^{-1}}^2 \leq 1$, the control law from Algorithm 7.2.1 for the robust constrained MPC based on parameter dependent Lyapunov functions can robustly asymptotically stabilize the closed-loop system.

Proof: For the off-line minimization at x^i , $i = 2, \dots, N$, the additional constraints $Q_j^{i-1} > Q_j^i$, $j = 1, 2, \dots, l$ implies the constructed asymptotically stable invariant ellipsoid $\mathcal{E}_j^i = \{x \in \mathbb{R}^{n_x} | x^T (Q_j^i)^{-1} x \leq 1\}$, $j = 1, 2, \dots, l$ is inside $\mathcal{E}_j^{i-1}$. That means, for each vertex j ($j = 1, 2, \dots, l$) of the uncertainty polytope, we have $\mathcal{E}_j^i \subset \mathcal{E}_j^{i-1}$. And because of constraint (7.24), it is sure that the asymptotically stable invariant ellipsoid $\mathcal{E}^i = \{x \in \mathbb{R}^{n_x} | x^T (Q_e^i)^{-1} x \leq 1\}$ is inside $\mathcal{E}^{i-1}$.

In the on-line procedure, given system (7.1) with an initial state $x(0)$ which satisfies $\|x(0)\|_{(Q_e^1)^{-1}}^2 \leq 1$, the closed-loop system can be written as follows:

$$x(k+1) = \begin{cases} (A(k) + B(k)F^i)x(k) & \text{if } \|x(k)\|_{(Q_e^i)^{-1}} \leq 1 \text{ and} \\ & \|x(k)\|_{(Q_e^{i+1})^{-1}} > 1, \quad i \neq N, \\ (A(k) + B(k)F^N)x(k) & \text{if } \|x(k)\|_{(Q_e^N)^{-1}} \leq 1. \end{cases} \quad (7.34)$$

At time interval k , if $x(k)$ satisfies $\|x(k)\|_{(Q_e^i)^{-1}}^2 \leq 1$, and $\|x(k)\|_{(Q_e^{i+1})^{-1}} > 1$, $i \neq N$, because $\mathcal{E}^i$ is the asymptotically stable invariant ellipsoid, then the corresponding state feedback control law $u(k) = F^i x(k)$ is guaranteed to keep this state $x(k)$ within $\mathcal{E}^i$ and to drive it into ellipsoid $\mathcal{E}^{i+1}$, hence to the smallest ellipsoid $\mathcal{E}^N$ and to the origin. This completes the proof. ■

Remark 7.2.3 The state feedback matrices generated by Algorithm 7.2.1 are constant between two adjacent asymptotically stable invariant ellipsoids. It is not continuous on the boundary of each asymptotically stable invariant ellipsoid. To establish the continuity of state feedback matrix over the state space, some modification on Algorithm 7.2.1 is made and Algorithm 7.2.2 is represented as follows.

Algorithm 7.2.2 (Modification of Algorithm 7.2.1)

Off-line:

The same as that of Algorithm 7.2.1 with following additional inequalities satisfied for each x^i ($i = 1, \dots, N-1$),

$$\begin{bmatrix} G + G^T - Q_j^i & * \\ (A_j + B_j F^{i+1}) G & Q_j^i \end{bmatrix} \geq 0, \quad \forall j = 1, 2, \dots, l. \quad (7.35)$$

On-line:

- (1) Given an initial state $x(0)$ satisfying $\|x(0)\|_{(Q_e^1)^{-1}}^2 \leq 1$;
- (2) At time k , perform a bisection search over Q_e^i in the look-up table to find the largest index i , which is equivalent to the smallest asymptotically stable ellipsoid $\mathcal{E}^i = \{x(k) \in \mathbb{R}^{n_x} | x(k)^T (Q_e^i)^{-1} x(k) \leq 1\}$;
- (3) If $i < N$, calculate α^i ($0 \leq \alpha^i \leq 1$) by solving

$$x(k)^T \left(\alpha^i (Q_e^i)^{-1} + (1 - \alpha^i) (Q_e^{i+1})^{-1} \right) x(k) = 1, \quad (7.36)$$

If $i = N$, set $\alpha^i = 1$;

(4) Apply the control law

$$u(k) = (\alpha^i F^i + (1 - \alpha^i) F^{i+1}) x(k)$$

to the system, set $k = k + 1$ and move to step (2).

Theorem 7.2.2 For the discrete time polytopic uncertain system (7.1) with an initial state $x(0)$ which satisfies $\|x(0)\|_{(Q_e^1)^{-1}}^2 \leq 1$, the control law from Algorithm 7.2.2 for the robust constrained MPC based on parameter dependent Lyapunov functions can robustly asymptotically stabilize the closed-loop system.

Proof: In the on-line procedure of Algorithm 7.2.2, given the system (7.1) with an initial state $x(0)$ which satisfies $\|x(0)\|_{(Q_e^1)^{-1}}^2 \leq 1$, the closed-loop system can be written as follows:

$$x(k+1) = \begin{cases} (A(k) + B(k)F(\alpha^i))x(k) & \text{if } \|x(k)\|_{(Q_e^i)^{-1}} \leq 1 \text{ and} \\ & \|x(k)\|_{(Q_e^{i+1})^{-1}} > 1, \quad i \neq N, \\ (A(k) + B(k)F^N)x(k) & \text{if } \|x(k)\|_{(Q_e^N)^{-1}} \leq 1 \end{cases} \quad (7.37)$$

where $F(\alpha^i) = \alpha^i F^i + (1 - \alpha^i) F^{i+1}$ with α^i satisfying (7.36).

The satisfaction of (7.18) and (7.35) for x^i implies the following,

$$\begin{bmatrix} G + G^T - Q_j^i & * \\ (A_j + B_j F(\alpha^i)) G & Q_j^i \end{bmatrix} \geq 0, \quad \forall j = 1, 2, \dots, l, \quad (7.38)$$

hence implies

$$\begin{bmatrix} G^T (Q_j^i)^{-1} G & * \\ (A_j + B_j F(\alpha^i)) G & Q_j^i \end{bmatrix} \geq 0, \quad \forall j = 1, 2, \dots, l, \quad (7.39)$$

because $(G^T - Q_j^i) (Q_j^i)^{-1} (G - Q_j^i)$ is positive semi-definite.

By multiplying (7.39) from the left with $\begin{bmatrix} G^{-T} & 0 \\ 0 & I \end{bmatrix}$ and from the right with $\begin{bmatrix} G^{-1} & 0 \\ 0 & I \end{bmatrix}$ gives

$$\begin{bmatrix} (Q_j^i)^{-1} & * \\ A_j + B_j F(\alpha^i) & Q_j^i \end{bmatrix} \geq 0, \quad \forall j = 1, 2, \dots, l. \quad (7.40)$$

Actually, (7.40) is equivalent to

$$x(k)^T \left[(Q_j^i)^{-1} - (A_j + B_j F(\alpha^i))^T (Q_j^i)^{-1} (A_j + B_j F(\alpha^i)) \right] x(k) \geq 0 \quad (7.41)$$

hence

$$x(k+1)^T (Q_j^i)^{-1} x(k+1) \geq x(k)^T (Q_j^i)^{-1} x(k). \quad (7.42)$$

This means the control law $u(k) = F(\alpha^i)x(k)$ is guaranteed to keep the state within $\mathcal{E}^i$ and drive it into $\mathcal{E}^{i+1}$. Lastly, the control law $u(k) = F^N x(k)$ will keep the state within $\mathcal{E}^N$ and drive it to the origin. ■

Corollary 7.2.2 *The feedback matrix F implemented in Algorithm 7.2.2 is a continuous function of the state x .*

Proof: The feedback matrix F implemented in Algorithm 7.2.2 is

$$F = \begin{cases} \alpha^i F^i + (1 - \alpha^i) F^{i+1} & \text{if } \|x(k)\|_{(Q_e^i)^{-1}} \leq 1 \text{ and} \\ & \|x(k)\|_{(Q_e^{i+1})^{-1}} > 1, \quad i \neq N, \\ F^N & \text{if } \|x(k)\|_{(Q_e^N)^{-1}} \leq 1 \end{cases} \quad (7.43)$$

where α^i satisfies (7.36). Consider two ring regions

$$\begin{aligned}\mathcal{B}^{i-1} &= \left\{ x \in \mathbb{R}^{n_x} \left| \|x(k)\|_{(Q_e^{i-1})^{-1}} \leq 1, \|x(k)\|_{(Q_e^i)^{-1}} > 1 \right. \right\}, \\ \mathcal{B}^i &= \left\{ x \in \mathbb{R}^{n_x} \left| \|x(k)\|_{(Q_e^i)^{-1}} \leq 1, \|x(k)\|_{(Q_e^{i+1})^{-1}} > 1 \right. \right\}.\end{aligned}$$

Within the region $\mathcal{B}^i$, the solution to (7.36) is

$$\alpha^i = \frac{1 - x(k)^T (Q_e^{i+1})^{-1} x(k)}{x(k)^T (Q_e^i)^{-1} x(k) - x(k)^T (Q_e^{i+1})^{-1} x(k)}.$$

Therefore, α^i is a continuous function of $x(k)$ in $\mathcal{B}^i$, hence implies that $F(\alpha)$ is a continuous function of $x(k)$ in $\mathcal{B}^i$.

With the same reason

$$\alpha^{i-1} = \frac{1 - x(k)^T (Q_e^i)^{-1} x(k)}{x(k)^T (Q_e^{i-1})^{-1} x(k) - x(k)^T (Q_e^i)^{-1} x(k)}$$

is a continuous function of $x(k)$ in $\mathcal{B}^{i-1}$, hence implies that $F(\alpha)$ is a continuous function of $x(k)$ in $\mathcal{B}^{i-1}$. Then on the boundary between $\mathcal{B}^i$ and $\mathcal{B}^{i-1}$, the following holds true

$$\lim_{\alpha^{i-1} \rightarrow 0} F(\alpha^{i-1}) = \lim_{\alpha^i \rightarrow 1} F(\alpha^i) = F^i,$$

which establishes the continuity of x between $\mathcal{B}^i$ and $\mathcal{B}^{i-1}$. And it can be concluded that the feedback matrix F implemented in Algorithm 7.2.2 is a continuous function of the state x . ■

7.2.3 Off-line robust MPC formulation: convex combination method

Following Section 7.2.2, we still aim to reduce the on-line computational complexity of robust MPC. In this section, we propose a novel convex combination method to move a

significant part of the computation off-line, leading to a reduced on-line computational complexity. This goal is achieved by calculating *the asymptotically stable invariant ellipsoids*, controllers, costs and constraints off-line. A time-varying controller is obtained by on-line making convex combinations based on these ingredients. To some extent, the approach proposed here is an extension of the result in Section 7.2.2 and more general. Therefore, the problem formulation is retained. The new algorithm is given as follows.

Algorithm 7.2.3 (*Convex combination method*). Consider the uncertain system (7.1) subject to input and state constraints (7.8), (7.9).

Off-line:

The same as that of Algorithm 7.2.1.

On-line: Given the optimizers from the off-line part, in real-time implementation,

(1) Given an initial feasible state $x(0)$, set time step $k := 0$;

(2) Solve the following optimization problem

$$\min_{\alpha^i, i=1,2,\dots,N} \sum_{i=1}^N \alpha^i \gamma^i \quad (7.44)$$

subject to, $\forall i = 1, 2, \dots, N$,

$$\alpha^i \geq 0, \quad (7.45)$$

$$\sum_{i=1}^N \alpha^i \leq 1, \quad (7.46)$$

$$\begin{bmatrix} 1 & * \\ x(k) & \sum_{i=1}^N \alpha^i Q_e^i \end{bmatrix} > 0. \quad (7.47)$$

(3) Calculate the control law $F(k) = \sum_{i=1}^N \alpha^i F^i$ and apply $u(k) = F(k)x(k)$ to the system, set $k = k + 1$ and move to step (2).

Remark 7.2.4 (Convex combinations) Assume canonical states $x^i, i = 1, 2, \dots, N$ are given, and optimizers $\gamma^i, G^i, X^i, Y^i, Z^i$ and Q_j^i , for all $j = 1, 2, \dots, l$ and $i = 1, 2, \dots, N$

have been calculated. It can be easily proven that any convex combination,

$$\begin{aligned}\gamma &\equiv \sum_{i=1}^N \alpha^i \gamma^i, \quad G \equiv \sum_{i=1}^N \alpha^i G^i, \quad Q_j \equiv \sum_{i=1}^N \alpha^i Q_j^i, \\ X &\equiv \sum_{i=1}^N \alpha^i X^i, \quad Y \equiv \sum_{i=1}^N \alpha^i Y^i, \quad Z \equiv \sum_{i=1}^N \alpha^i Z^i,\end{aligned}\tag{7.48}$$

of these optimizers, with $\alpha^i \geq 0$, $\sum_{i=1}^N \alpha^i \leq 1$ is a feasible solution to the multi-objective MPC problem described in (7.20). Note that in the strictest sense, convex combinations only allow $\sum_{i=1}^N \alpha^i = 1$, but since we can always find an arbitrarily small solution γ^0 for $x^0 = 0$, $\sum_{i=1}^N \alpha^i \leq 1$ is still valid.

Hereafter, we give the closed-loop analysis of this method.

Theorem 7.2.3 *For a discrete time polytopic uncertain system (7.1) with an initial feasible state $x(0)$ which satisfies $x(0) \in \{x | \exists i \text{ s.t. } x^T (Q_e^i)^{-1} x \leq 1, i = 1, 2, \dots, N\}$, state feedback gain F generated Algorithm 7.2.3 can robustly asymptotically stabilize the closed-loop system.*

Proof: $x(0) \in \{x | \exists i \text{ s.t. } x^T (Q_e^i)^{-1} x \leq 1, i = 1, \dots, N\}$ guarantees that the optimization problem (7.44) is initially feasible. From geometrical property of *asymptotically stable invariant ellipsoids*, it can be proven that the optimization problem (7.44) is feasible for $k > 0$ if it is feasible for $k = 0$. Then, satisfaction to (7.45)-(7.47) guarantees a feasible solution (7.48) of problem (7.20). Consequently, $\mathcal{E}(k) = \{x | x(k)^T (\sum_{i=1}^N \alpha^i Q_e^i)^{-1} x(k) \leq 1\}$ is an *asymptotically stable invariant ellipsoid*, which guarantees the asymptotical stability of the closed-loop system. ■

7.2.4 Computational complexity analysis

Table 7.1 lists the number of variables to be optimized at every time instant, both off-line and on-line, for different existing approaches and the two methods proposed in this chapter.

Method	Off-line optimization variables	On-line optimization variables
Single Lyapunov function MPC [44]	0	$\frac{n_x(n_x+1)}{2} + n_u n_x + \frac{n_y(n_y+1)}{2} + \frac{n_u(n_u+1)}{2}$
Multi Lyapunov function MPC [17]	0	$\frac{n_x(n_x+1)}{2} l + n_x^2 + \frac{n_y(n_y+1)}{2} + n_u n_x + \frac{n_u(n_u+1)}{2}$
Interpolation method (Section 7.2.2)	$\frac{n_x(n_x+1)}{2} (l+1) + \frac{n_y(n_y+1)}{2} + n_u n_x + \frac{n_u(n_u+1)}{2} + n_x^2$	2
Convex combination method (Section 7.2.3)	$\frac{n_x(n_x+1)}{2} (l+1) + \frac{n_y(n_y+1)}{2} + n_u n_x + \frac{n_u(n_u+1)}{2} + n_x^2$	l

Table 7.1: Off-line and on-line computational complexity analysis

From the above table, it is easy to see that both two methods proposed in this chapter moves a significant part of computation to off-line. On-line, only 2 (for interpolation method) and l (number of vertices of uncertain systems for convex combination method) variables are required to be determined. Moreover, the on-line computational complexity is independent of the order of state n_x as well as n_y and n_u .

7.3 Design and simulation

In this section, we present an example to illustrate the effectiveness of the proposed off-line approaches. This example is based on a high fidelity nonlinear F-16 model considering control surface impairment faults.

The aircraft model with control surface impairment can be written as the form of (7.1) with $l = 3$ and $x = [u, w, q, v, p, r]^T$ is the state, $y = [q, \dot{\mu}_{rat}, r, \alpha, \beta]^T$ is the output, and $u = [\delta_e, \delta_a, \delta_r]^T$ is the control input and w is the disturbance. The system matrices are given as follows. All these system matrices are trimmed from a high fidelity nonlinear F-16 aircraft model under different conditions mentioned in Chapter 2.

$$\begin{aligned}
A_0 &= \begin{bmatrix} 0.9984 & 0.0045 & -0.5338 & 0.0002 & 0.0001 & -0.0015 \\ -0.0086 & 0.9116 & 12.5484 & 0.0012 & 0.0004 & -0.0118 \\ 0 & 0.0004 & 0.9057 & 0 & 0 & 0.0001 \\ 0 & 0 & -0.0018 & 0.9343 & 0.5355 & -14.3601 \\ 0 & 0 & 0 & -0.0149 & 0.7058 & 0.1715 \\ 0 & 0 & 0.0002 & 0.0043 & -0.0012 & 0.9227 \end{bmatrix}, \\
B_0^u &= \begin{bmatrix} 0.0095 & 0 & 0 \\ -0.1461 & 0 & 0 \\ -0.0166 & 0 & 0 \\ 0 & 0.0064 & 0.0582 \\ 0 & -0.0588 & 0.0100 \\ 0 & -0.0028 & -0.0057 \end{bmatrix}, \quad B_0^w = \begin{bmatrix} 0.00048 \\ -0.00960 \\ 0.00005 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
C_0 &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2468 & 2.3696 \\ 0 & 0 & 0 & 0 & -2.3696 & 57.2468 \\ -0.0155 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}, \\
A_1 &= \begin{bmatrix} 0.9989 & 0.0023 & -0.5529 & 0.0002 & 0.0001 & -0.0015 \\ -0.0088 & 0.9327 & 12.6983 & 0.0016 & 0.0005 & -0.0146 \\ 0 & 0.0001 & 0.9032 & -0.0001 & 0 & 0.0002 \\ 0 & 0 & -0.0018 & 0.9434 & 0.5368 & -14.4054 \\ 0 & 0 & 0 & -0.0117 & 0.7068 & 0.1465 \\ 0 & 0 & 0.0002 & 0.0033 & -0.0010 & 0.9307 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
B_1^u &= \begin{bmatrix} 0.0077 & 0 & 0 \\ -0.1203 & 0 & 0 \\ -0.0130 & 0 & 0 \\ 0 & 0.0064 & 0.0583 \\ 0 & -0.0588 & 0.0100 \\ 0 & -0.0028 & -0.0057 \end{bmatrix}, \quad B_1^w = \begin{bmatrix} 0.00025 \\ -0.00709 \\ 0.00001 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2460 & 2.3871 \\ 0 & 0 & 0 & 0 & -2.3871 & 57.2460 \\ -0.0157 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.9993 & 0.0002 & -0.5883 & 0.0002 & 0.0001 & -0.0015 \\ -0.0089 & 0.9529 & 12.8406 & 0.0020 & 0.0006 & -0.0181 \\ 0 & -0.0004 & 0.9000 & -0.0001 & 0 & 0.0002 \\ 0 & 0 & -0.0018 & 0.9525 & 0.5544 & -14.4496 \\ 0 & 0 & 0 & -0.0087 & 0.7078 & 0.1241 \\ 0 & 0 & 0.0002 & 0.0022 & -0.0008 & 0.9389 \end{bmatrix}, \\
B_2^u &= \begin{bmatrix} 0.0002 & 0 & 0 \\ -0.0810 & 0 & 0.0001 \\ -0.0087 & 0 & 0 \\ 0 & 0.0058 & 0.0585 \\ 0 & -0.0588 & 0.0101 \\ 0 & -0.0028 & -0.0057 \end{bmatrix}, \quad B_2^w = \begin{bmatrix} 0.00001 \\ -0.00453 \\ 0.00004 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.2424 & 2.4720 \\ 0 & 0 & 0 & 0 & -2.4720 & 57.2424 \\ -0.0162 & 0.3756 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
A_3 &= \begin{bmatrix} 0.9994 & -0.0017 & -0.8133 & 0.0003 & 0.0001 & -0.0017 \\ -0.0090 & 0.9751 & 12.9686 & 0.0027 & 0.0012 & -0.0239 \\ 0.0001 & -0.0008 & 0.8970 & -0.0001 & 0 & 0.0005 \\ 0 & 0 & -0.0018 & 0.9616 & 0.7593 & -14.4726 \\ 0 & 0 & 0 & -0.0069 & 0.7094 & 0.1163 \\ 0 & 0 & 0.0002 & 0.0010 & -0.0017 & 0.9480 \end{bmatrix}, \\
B_3^u &= \begin{bmatrix} -0.0026 & 0 & 0 \\ -0.0244 & 0 & 0.0001 \\ -0.0026 & 0 & 0 \\ 0 & -0.0020 & 0.0601 \\ 0 & -0.0591 & 0.0101 \\ 0 & -0.0027 & -0.0057 \end{bmatrix}, \quad B_3^w = \begin{bmatrix} 0.00021 \\ -0.00192 \\ 0.00009 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
C_3 &= \begin{bmatrix} 0 & 0 & 57.2958 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 57.1983 & 3.3400 \\ 0 & 0 & 0 & 0 & -3.3400 & 57.1983 \\ -0.0219 & 0.3753 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3760 & 0 & 0 \end{bmatrix}
\end{aligned}$$

The first step of designing off-line robust MPC controllers for this uncertain system is to find a set of discrete states as described in Algorithms 7.2.1, 7.2.2 and 7.2.3. Here, we use the proposed multi-objective MPC approach to produce a trajectory and discretize it into several discrete states, like shown as follows, with $N = 28$,

$$x^{set} = \left\{ \begin{array}{l} \{0.8063 \quad -40.4470 \quad -0.2751 \quad -26.5723 \quad -0.1765 \quad 0.0253\} \\ \{0.5513 \quad -37.0758 \quad 0.1093 \quad -24.3582 \quad 0.0825 \quad -0.1945\} \\ \{0.1657 \quad -30.0682 \quad 0.3567 \quad -19.5276 \quad 0.0690 \quad -0.3374\} \\ \{-0.2459 \quad -21.6820 \quad 0.4564 \quad -13.5818 \quad 0.0830 \quad -0.3807\} \\ \{-0.6001 \quad -13.8692 \quad 0.4254 \quad -7.8491 \quad 0.0703 \quad -0.3455\} \\ \{-0.8591 \quad -7.7702 \quad 0.3270 \quad -3.2057 \quad 0.0537 \quad -0.2648\} \\ \{-1.0340 \quad -3.4855 \quad 0.2348 \quad -0.1267 \quad 0.0324 \quad -0.1591\} \\ \{-1.1521 \quad 0.0631 \quad 0.0424 \quad 1.7344 \quad 0.0005 \quad -0.0029\} \\ \{-1.1524 \quad 0.2966 \quad 0.0036 \quad 1.4622 \quad -0.0054 \quad 0.0263\} \\ \{-1.1446 \quad 0.2298 \quad -0.0078 \quad 0.9656 \quad -0.0068 \quad 0.0332\} \\ \{-1.1371 \quad 0.1166 \quad -0.0077 \quad 0.4905 \quad -0.0057 \quad 0.0280\} \\ \{-1.1319 \quad 0.0398 \quad -0.0047 \quad 0.1456 \quad -0.0038 \quad 0.0185\} \\ \{-1.1262 \quad -0.0059 \quad -0.0010 \quad -0.1304 \quad -0.0006 \quad 0.0028\} \\ \{-1.1241 \quad -0.0060 \quad -0.0006 \quad -0.1343 \quad 0.0002 \quad -0.0010\} \\ \{-1.1221 \quad -0.0035 \quad -0.0005 \quad -0.1024 \quad 0.0005 \quad -0.0025\} \\ \{-1.1201 \quad -0.0014 \quad -0.0006 \quad -0.0617 \quad 0.0005 \quad -0.0026\} \\ \{-1.1180 \quad -0.0002 \quad -0.0007 \quad -0.0273 \quad 0.0004 \quad -0.0019\} \\ \{-1.1158 \quad 0.0003 \quad -0.0007 \quad -0.0048 \quad 0.0002 \quad -0.0012\} \\ \{-1.1137 \quad 0.0004 \quad -0.0007 \quad 0.0061 \quad 0.0001 \quad -0.0005\} \\ \{-1.1115 \quad 0.0004 \quad -0.0007 \quad 0.0088 \quad 0 \quad 0\} \\ \{-1.1094 \quad 0.0004 \quad -0.0007 \quad 0.0074 \quad 0 \quad 0.0001\} \\ \{-1.1072 \quad 0.0003 \quad -0.0007 \quad 0.0048 \quad 0 \quad 0.0002\} \\ \{-1.1051 \quad 0.0004 \quad -0.0007 \quad 0.0024 \quad 0 \quad 0.0001\} \\ \{-1.1030 \quad 0.0004 \quad -0.0007 \quad 0.0009 \quad 0 \quad 0.0001\} \\ \{-1.0987 \quad 0.0005 \quad -0.0007 \quad -0.0002 \quad 0 \quad 0\} \\ \{-1.0945 \quad 0.0004 \quad -0.0007 \quad -0.0001 \quad 0 \quad 0\} \\ \{-1.0924 \quad 0.0003 \quad -0.0007 \quad 0 \quad 0 \quad 0\} \\ \{-0.0744 \quad -0.0005 \quad -0.0001 \quad 0 \quad 0 \quad 0\} \end{array} \right\}.$$

7.3.1 Interpolation method

Using these discrete states, following the instruction described in the off-line part of Algorithm 7.2.1 and Algorithm 7.2.2, the hyper-ellipsoids defined by $\|x\|_{(Q_j^i)^{-1}}$, $j = 0, \dots, 3$ and $i = 1, \dots, 28$, and the asymptotically stable invariant hyper-ellipsoids defined by $\|x\|_{(Q_e^i)^{-1}}$, $i = 1, \dots, 28$ can be found.

A Herbst maneuver, which consists of a large angle-of-attack pull-up, followed by a stability axis roll is performed (Fig. 7.1). In the simulation, a 30% loss of the elevator is introduced at 12s, followed by a 60% loss at 15s and a 70% loss at 18s.

Figure 7.1: Response curves of the Herbst maneuver with the interpolation MPC method

From this figure, we can see that the off-line MPC controller performs well in both normal condition and the fault conditions. The angle of attack (α) can successfully track 33° ((1,1) entry of Fig. 7.1) while $\dot{\mu}_{rot}$ tracks 20° ((1,2) entry of Fig. 7.1). From (2,1) entry of Fig. 7.1 we can find that the elevator saturated from $t = 25$ s to around 27s. This is mainly because that the aircraft was experiencing a 70% efficiency loss on the elevator. Therefore, the wounded elevator has to make more action in order to track

a step command from $\alpha = 33^\circ$ to the trimmed value. From above we can conclude that the off-line MPC can stabilize the overall system and provide a good tracking ability in the presence of faults and saturations.

Compared with the on-line robust MPC approaches, the proposed off-line algorithm is more than 1000 times faster with slight performance degradation which provides a promising methodology to apply MPC to systems with fast dynamics.

7.3.2 Convex combination method

Considering the same example and parameter settings in Section 7.2.2, in this subsection we apply the convex combination method to the F-16 model. The simulation result of a Herbst maneuver is shown as below. Also, a 30% loss of the elevator is introduced at 12s, followed by a 60% loss at 15s and a 70% loss at 18s.

Figure 7.2: Response curves of the Herbst maneuver with the convex combination method

From this figure, it can be seen that there is only slight changes in between the convex combination method and the interpolation method. Similar conclusion as above subsection can be made.

7.4 Conclusion

This chapter presents a novel multi-objective MPC formulation with optimized H_2 performance and guaranteed H_∞ norm bounds. The proposed MPC technique can be widely used in trajectory tracking, disturbance attenuation etc.

To introduce MPC to the fast response systems like FCS, two off-line MPC approaches are formulated in LMI frameworks, with guaranteed closed-loop stability. The main advantages of these off-line methods are the dramatically relaxed computational burden compared with on-line MPC approaches and the reduced conservativeness compared with an existing off-line MPC method. The effectiveness of these approaches are validated by using a nonlinear F-16 model with control surface impairment faults and input saturations.

Chapter 8

Improved robust MPC with structured uncertainty

Other than polytopic uncertainty studied in past chapters, we will discuss another type of uncertainty: structured uncertainty in this chapter. Motivation to do this comes from that structured uncertainty is a more common paradigm for robust control which consists of an LTI system with uncertainties or perturbations appearing in the feedback loop. In flight control area, whether to model an aircraft system as a polytopic system or a system with structured uncertainty depends on a number of factors, such as the underlying physical model of the system, available model identification and validation techniques, etc. In the literature, research work has been done both on polytopic uncertain systems [25, 51, 50, 53] and structured uncertain systems [36, 61, 83]. In view of this, it's also needed to discuss robust MPC with structured uncertainty.

In 2001, Wu [73] extended the LMI-based robust MPC technique to a general class of uncertain linear systems with time-varying, linear fractional transformation (LFT) perturbations. With this uncertainty description, some promising properties (more accurate modeling of nonlinear system *etc.*) can be expected (see [73] for details). However, one drawback of [73] is that only one single Lyapunov function is enforced in controller

design, which leads to some conservativeness. Inspired by some recent dilated LMI techniques, such as [1, 17, 18, 27, 22], we consider a new approach where parameter dependent Lyapunov functions are adopted for each and every structured uncertainty.

8.1 Problem formulation

The problem formulation follows that of [73] and is reproduced here for ease of reading. The system under consideration consists of linear time-invariant part G and structured uncertain part Δ , shown in Figure 8.1,

Figure 8.1: Linear system with structured uncertainty

and is described by

$$\begin{aligned}
 x(k+1) &= Ax(k) + B_0w(k) + B_2u(k), \\
 z(k) &= C_0x(k) + D_{00}w(k) + D_{02}u(k), \\
 y(k) &= C_2x(k) + D_{20}w(k) + D_{22}u(k), \\
 w(k) &= \Delta(k)z(k)
 \end{aligned} \tag{8.1}$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ is the control signal, $y \in \mathbb{R}^{n_y}$ is the output, $z, w \in \mathbb{R}^{n_w}$ are the regulated output and perturbation, respectively. The time-varying, structured uncertainty is given by

$$\begin{aligned}
 \Delta \in \Pi := \Big\{ \text{diag}\{\delta_1 I_{r_1}, \dots, \delta_S I_{r_S}, \Delta_{S+1}, \dots, \Delta_{S+F}\} \mid \delta_i : l_2 \rightarrow l_2, \|\delta_i\| \leq 1, \\
 \Delta_{S+j} : l_2^{m_j} \rightarrow l_2^{m_j}, \|\Delta_{S+j}\| \leq 1, i = 1, \dots, S, j = 1, \dots, F \Big\}.
 \end{aligned} \tag{8.2}$$

where the operator norm on δ_i and Δ_i is the induced l_2 norm and $\sum_{i=1}^S r_i + \sum_{j=1}^F m_j = n_w$. Consequently, another scaling set $\mathcal{D}$ is reproduced from [73] as follows,

$$D \in \mathcal{D} := \{\text{diag}\{D_1, \dots, D_S, d_{S+1}I_{m_1}, \dots, d_{S+F}I_{m_F}\}, \mid D_i = D_i^T > 0, d_{S+j} > 0 \\ i = 1, \dots, S, j = 1, \dots, F\}. \quad (8.3)$$

Therefore, it is clear that for any $\Delta \in \Pi$ and $D \in \mathcal{D}$, $\Delta D = D\Delta$ holds.

Following [73], it is assumed that

(A1) State x is measurable in real-time.

(A2) The uncertain system (8.1) is robustly stabilizable, i.e. there exist matrices $X = X^T > 0$, $F \in \mathbb{R}^{n_u \times n_x}$ and $S \in \mathcal{D}$, such that

$$\begin{bmatrix} X^{-1} & 0 \\ 0 & S^{-1} \end{bmatrix} - \begin{bmatrix} (A + B_2 F)^T & (C_0 + D_{02} F)^T \\ B_0^T & D_{00}^T \end{bmatrix} \begin{bmatrix} X^{-1} & 0 \\ 0 & S^{-1} \end{bmatrix} \begin{bmatrix} A + B_2 F & B_0 \\ C_0 + D_{02} F & D_{00} \end{bmatrix} > 0.$$

In addition, following constraints are considered in this chapter

$$\max_{i \geq 0} \|u_\xi(k+i|k)\| \leq u_{\xi, \max}, \quad \xi = 1, \dots, p \quad (8.4)$$

with p defining a p -tuple of integers $\{\ell_1, \dots, \ell_p\}$ which satisfies $\sum_{\xi=1}^p \ell_\xi = n_u$, and

$$\max_{i \geq 0} \|y_\eta(k+i|k)\| \leq y_{\eta, \max}, \quad \eta = 1, \dots, q \quad (8.5)$$

with q defining a q -tuple of integers $\{\lambda_1, \dots, \lambda_q\}$ which satisfies $\sum_{\eta=1}^q \lambda_\eta = n_y$.

By implementing the input/output constraints defined above, both component-wise peak-bounded and Euclidean norm-bounded constraints can be studied.

8.2 Robust MPC with structured uncertainty based on parameter dependent Lyapunov functions

In this section an LMI approach based on parameter dependent Lyapunov functions is proposed for the robust MPC problem defined as follows,

$$\min_{u(k+i|k), i=0,1,\dots,\infty} \max_{\Delta \in \Pi} J_{\infty}(k) \quad (8.6)$$

with

$$J_{\infty}(k) := \sum_{i=0}^{\infty} \begin{bmatrix} x(k+i|k) \\ u(k+i|k) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q} & 0 \\ 0 & \mathcal{R} \end{bmatrix} \begin{bmatrix} x(k+i|k) \\ u(k+i|k) \end{bmatrix} \quad (8.7)$$

where $\mathcal{Q} > 0$ and $\mathcal{R} > 0$ are weight matrices of appropriate dimensions.

The goal is to find a state feedback control law of the type

$$u(k+i|k) = F(k)x(k+i|k), \quad i = 0, \dots, \infty \quad (8.8)$$

to minimize the cost function $J_{\infty}(k)$ subject to input and output constraints as described in (8.4) and (8.5). The LMI conditions can be formulated by defining a parameter dependent Lyapunov function

$$V(i, k) = x(k+i|k)^T P(i, k) x(k+i|k), \quad i = 0, \dots, \infty \quad (8.9)$$

with $P(i, k) > 0 \forall k, i \geq 0$ and requiring that

$$[x(k+i|k)^T \mathcal{Q} x(k+i|k) + u(k+i|k)^T \mathcal{R} u(k+i|k)] \leq V(i, k) - V(i+1, k). \quad (8.10)$$

Here index i implies that there is a different Lyapunov function corresponding to each and every step of the prediction time. Comparing with single common Lyapunov function in [73], more design freedom is introduced in the following optimization problem

which we expect to produce a less conservative controller.

By summing up the above inequality (8.10) from $i = 0$ to ∞ , an upper bound to the cost function $J_\infty(k)$ follows

$$\max_{\Delta \in \Pi} J_\infty(k) \leq V(0, k) \triangleq \gamma \quad (8.11)$$

where γ is a nonnegative parameter that is to be minimized in the optimization problem.

Now we are ready to present the proposed LMI formulation based on parameter-dependent Lyapunov functions.

Theorem 8.2.1 *Consider the linear system (8.1) with structured uncertainty Π . Suppose state x is measurable in current time and $x(k|k)$ is the measured state $x(k)$. There exists a state feedback control law of (8.8) satisfying the performance constraints (A2) and the input/output constraints (8.4) (8.5) at step k if there exist matrices $X_j \in \mathcal{D}$, $S_j \in \mathcal{D}$, $T_\eta \in \mathcal{D}$ and matrices H , Y and nonsingular matrix G of appropriate dimensions, such that for $j = 1, 2, \dots, S + F$, $\xi = 1, \dots, p$ and $\eta = 1, \dots, q$*

$$\begin{bmatrix} G + G^T - X_j & * & * & * & * & * \\ 0 & H + H^T - S_j & * & * & * & * \\ \mathcal{R}^{\frac{1}{2}}Y & 0 & \gamma & * & * & * \\ \mathcal{Q}^{\frac{1}{2}}G & 0 & 0 & \gamma & * & * \\ AG + B_2Y & B_0H & 0 & 0 & X_j & 0 \\ C_0G + D_{02}Y & D_{00}H & 0 & 0 & 0 & S_j \end{bmatrix} > 0, \quad (8.12)$$

$$\begin{bmatrix} 1 & * \\ x(k|k) & X_j \end{bmatrix} \geq 0, \quad (8.13)$$

$$\begin{bmatrix} u_{\xi, \max}^2 I & * \\ Y_{\xi}^T & X_j \end{bmatrix} \geq 0, \quad \xi = 1, \dots, p, \quad (8.14)$$

$$\begin{bmatrix} y_{\eta, \max}^2 (G + G^T - X_j) & * & * & * \\ 0 & T_{\eta} & * & * \\ C_{2, \eta} G + D_{20, \eta} Y & D_{20, \eta} Y & I & * \\ C_0 G + D_{02} Y & D_{00} T_{\eta} & 0 & T_{\eta} \end{bmatrix} \geq 0, \quad \eta = 1, \dots, q. \quad (8.15)$$

The resulting robust state feedback controller is given by $F = YG^{-1}$.

Proof: Given $X_j = X_j^T > 0$, $S_j = S_j^T > 0$ and the additional nonsingular matrices $\{G, H\}$ of suitable dimensions, the following inequalities hold,

$$\begin{aligned} (G^T - X_j)X_j^{-1}(G - X_j) &\geq 0 \\ (H^T - S_j)S_j^{-1}(H - S_j) &\geq 0. \end{aligned} \quad (8.16)$$

consequently,

$$\begin{aligned} G + G^T - X_j &\leq G^T X_j^{-1} G \\ H + H^T - S_j &\leq H^T S_j^{-1} H. \end{aligned} \quad (8.17)$$

Then the satisfaction of (8.12) implies the satisfaction of

$$\begin{bmatrix} G^T X_j^{-1} G & * & * & * & * & * \\ 0 & H^T S_j^{-1} H & * & * & * & * \\ \mathcal{R}^{\frac{1}{2}} Y & 0 & \gamma & * & * & * \\ \mathcal{Q}^{\frac{1}{2}} G & 0 & 0 & \gamma & * & * \\ AG + B_2 Y & B_0 H & 0 & 0 & X_j & 0 \\ C_0 G + D_{02} Y & D_{00} H & 0 & 0 & 0 & S_j \end{bmatrix} > 0. \quad (8.18)$$

As $\{G, H\}$ are nonsingular, we can multiply (8.18) from the left by $\text{diag}\{X_j G^{-T}, S_j H^{-T}, I, I, I, I\}$ and from the right by $\text{diag}\{G^{-1} X_j, H^{-1} S_j, I, I, I, I\}$, and the following LMI is obtained.

$$\begin{bmatrix} X_j & * & * & * & * & * \\ 0 & S_j & * & * & * & * \\ \mathcal{R}^{\frac{1}{2}} F X_j & 0 & \gamma & * & * & * \\ \mathcal{Q}^{\frac{1}{2}} X_j & 0 & 0 & \gamma & * & * \\ AX_j + B_2 F X_j & B_0 S_j & 0 & 0 & X_j & 0 \\ C_0 X_j + D_{02} F X_j & D_{00} S_j & 0 & 0 & 0 & S_j \end{bmatrix} > 0. \quad (8.19)$$

This is nothing but the formulation of robust MPC approach based on one common Lyapunov function that is proposed in [73]. This concludes the proof for (8.12).

Consider the system and the pair of constraints (8.10) and (8.11). By resorting to the Schur lemma it is simple to prove that $V(i, k) < \gamma$ can be rewritten as

$$\begin{bmatrix} 1 & * \\ x(k|k) & X_j \end{bmatrix} \geq 0$$

where $X_j = \gamma P_j^{-1}$. This proves LMI condition (8.13). Following similar technique as above, LMIs (8.14)-(8.15) can also be proved.

Finally, the (1,1)-block of (8.18) implies that $0 < G^T X_j^{-1} G$, i.e., G is nonsingular. Hence, $F = YG^{-1}$ is valid. This complete the proof. ■

Remark 8.2.1 *As stated in the remark of [73], enforcing a single X to all the structured uncertainties is potentially conservative. Following the work of de Oliveira, Bernussou, and Geromel [18], we introduce additional matrices G and H to reduce the design conservatism, which in practice means that the value γ is expected to be always smaller than the one achieved by Wu in [73]. As slack matrices G , H and parameter dependent Lyapunov variables X_j , S_j are employed in the LMI formulation, more freedom are allowed in solving such optimization problem, therefore, reduces the potential design conservativeness. In particular, when $G = G^T = X_1 = X_2 = \dots = X_j$ and $H = H^T = S_1 = S_2 = \dots = S_j$, we recover Wu's solution [73]. On the other hand, employing additional matrices G and H may lead to more decision variables to be calculated in the on-line optimization procedure. As the variables in G , H and X_j , S_j grow rapidly with increasing system dimension, the on-line computational burden can be quite heavy when applied to a high dimensional system. To reduce this on-line computational burden, we can explore moving part of the on-line computation work off-line to enforce the practicability [24, 70].*

Remark 8.2.2 *The feasibility property of step by step LMI optimization problem (8.12-8.15) is another important result. The new LMI approach also admits the feasibility lemma in [44]. Any feasible solution of the optimization problem described in Theorem 8.2.1 at time k is also feasible for time $k+1, k+2, \dots$, which means if the optimization problem is feasible at the first step, it will always be feasible in the sequel.*

8.3 Numerical example

To illustrate the effectiveness of the proposed robust MPC approach, we will revisit the longitudinal third order F-16 example used in Chapter 4.

8.3.1 Aircraft model with structured uncertainty

Considering nominal condition and fault condition (75% efficiency loss of elevator), the system matrices are given as below,

$$\begin{aligned}
 A &= \begin{bmatrix} 0.9999 & 0.0002 & -0.0714 \\ -0.0010 & 0.9943 & 1.3709 \\ 0 & -0.0001 & 0.9899 \end{bmatrix}, \quad B_0 = \begin{bmatrix} -0.0009 & -0.0450 \\ 0.0100 & 0.1900 \\ -0.0001 & 0.0700 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0000 \\ -0.0027 \\ -0.0010 \end{bmatrix}, \\
 C_0 &= \begin{bmatrix} 0 & 0 & 57.2958 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{00} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{02} = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, \\
 C_2 &= \begin{bmatrix} 0 & 0 & 57.2958 \\ -0.0155 & 0.3756 & 0 \end{bmatrix}, \quad D_{20} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
 \end{aligned}$$

Two structured uncertainties $\delta_1(k)$ and $\delta_2(k)$ are modeled as

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \leq \Delta(k) = \begin{bmatrix} \delta_1(k) & 0 \\ 0 & \delta_2(k) \end{bmatrix} \leq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

8.3.2 Design and simulation

Case 1: First of all, no structured uncertainty is introduced to the aircraft system. The simulation result is given in Figure 8.2. The control objective is to track a step signal $\alpha = 10^\circ$. The first three sub-figures depict output signals q (degree/sec), α (degree) and u (m/sec) respectively while the in the fourth γ (the upper bound) from two approaches are compared. The unit for x axis is second. As stated in the above sections, the proposed approach can reduce the conservativeness in controller design procedure. This has been proved from Figure 8.2. The upper bound from the proposed approach (solid line) is always less than that of [25] (dashed line).

Figure 8.2: Simulation result with no uncertainty: solid line for proposed approach, dashed line for approach in [25].

Case 2: In this case, structured uncertainties are set to $\delta_1(k) = 1$, $\delta_2(k) = 1$ and $\delta_1(k) = -1$, $\delta_2(k) = -1$, as shown in Figure 8.3 and Figure 8.4 in the following. In both of these figures, we can see that the upper bound from the proposed approach (solid line) is always less than that of [25] (dashed line), which proves that the design conservativeness is reduced by the proposed approach.

Figure 8.3: Simulation result with uncertainty $\delta_1(k) = 1$, $\delta_2(k) = 1$: solid line for proposed approach, dashed line for approach in [25].

Figure 8.4: Simulation result with uncertainty $\delta_1(k) = -1$, $\delta_2(k) = -1$: solid line for proposed approach, dashed line for approach in [25].

Case 3: Considering random structured uncertainty $-1 \leq \delta_1(k), \delta_2(k) \leq 1$, the simulation result is illustrated in Figure 8.5. Both the proposed approach and the approach in [25] are using a same uncertainty profile. From the fourth sub-figure, the same conclusion as in Case 1 and Case 2 can be obtained.

Figure 8.5: Simulation result with random uncertainty $-1 \leq \delta_1(k), \delta_2(k) \leq 1$: solid line for proposed approach, dashed line for approach in [25].

8.4 Conclusion

This chapter discusses the robust MPC for a class of linear systems with structured time-varying uncertainties. To reduce the controller design conservativeness, a method

based on several parameter dependent Lyapunov functions is proposed. The proposed approach is applied to a longitudinal F-16 model. Through computer simulation results it can be seen that the proposed approach can stabilize the system with an improved performance. However, because of introducing additional slack variables into the on-line optimization, the computational load may be increased.

Chapter 9

Conclusion and future work

9.1 Conclusion

This thesis studies the reliable robust aircraft flight control problem for LTI systems. Seven novel approaches have been proposed to address several types of faults (actuator outage/stuck faults and control/wing/tail surface impairment faults) as well as input saturation nonlinearity and output/state hard constraints.

The thesis is organized as problem oriented. First of all, in Chapter 3 and Chapter 4 an improved H_∞ approach and a dynamic output-feedback H_2 approach are presented. Both of these two approaches aim at reducing the design conservativeness by introducing parameter-dependant Lyapunov functions and extra slack variables into the tracking problem formulation.

However, input saturation is observed in the simulation (especially for high-angle maneuvers and severe fault conditions), causing closed-loop performance degradation, which cannot be addressed by using the proposed methods of Chapters 3 and 4. In order to further improve the tracking performance of F-16 aircraft, we introduce a novel anti-windup compensation technique to a mixed H_2/H_∞ approach in Chapter 5. Input satu-

ration problem has been fully addressed and the aircraft maneuverability has been enhanced significantly.

The approach proposed in Chapter 5 can satisfy most of our requirements. Since the controller and the compensator can be optimized off-line. It works pretty well on a fast-response system, nevertheless, such a scheme lacks in ability to handle output/state constraints which inspires the work in Chapter 6 to Chapter 9. On-line MPC is investigated in Chapter 6, leading to many promising properties, such as inherent robustness, hard input/output constraints handling ability, disturbance rejection ability, etc.

In Chapter 7 multi-objective robust MPC is studied and polytopic uncertainty and disturbances are both considered in an LMI framework. The proposed approach guarantees an H_∞ norm bound with an optimized LQ performance. In order to reduce the heavy on-line optimization effort brought with MPC approaches, an off-line procedure has been proposed using the concept of *the asymptotically stable invariant ellipsoids*. Two off-line MPC formulations are presented and validated through nonlinear simulations.

In Chapter 8, structured uncertainty has been studied and an improved robust MPC approach has been proposed. Since structured uncertainty is also frequently met in FCS, the proposed method is quite promising in our research.

Concluding all the approaches proposed in this thesis, we can see the reliable robust aircraft control problem has been studied mainly in two directions: H_2 and/or H_∞ with anti-windup compensation, and robust MPC technique. Both of them work well in case of fault and input saturation but MPC has the added capability to also handle state/output constraints at the expense of higher computational complexity. We will highlight the applicability of these two design approaches using the two simulation scenarios studied in this thesis.

The first one is known as Herbst maneuver, which is done mostly in high altitude. In this case, what we focus on is the movement of control surfaces. In other words, only input saturation problem has to be addressed carefully, especially in fault conditions,

where the remaining actuators will experience increased control activity (likely leading to control actuator saturation) to compensate for the malfunctioned actuator(s). Towards this goal, the author prefers H_2 and/or H_∞ with anti-windup compensation because it is not computationally intensive and has sufficient capability to satisfy this requirement.

The other simulation scenario is the auto-landing process, where more than input saturation has to be considered. In this case we normally have state constraints, output constraints and performance constraints. Because of the outstanding constraints handling ability, robust MPC is authors' recommendation in this kind of scenarios. The only disadvantage is that it may be hard to obtain an optimal solution within the short sampling period of a system like aircraft. Although we have proposed two off-line robust MPC approaches to address this inadequacy, the solution is not guaranteed to be globally optimal. Such shortcoming can be avoided by using explicit robust MPC, which is a future direction as discussed later.

9.2 Future work

9.2.1 Outer-loop guidance and navigation

Classical controller design techniques aim to find a feedback controller that could stabilize the aircraft and achieve certain tracking criteria. Much successful work has been reported in this area. However, most of these works still have theoretical limits, such as respect to various constraints, resulting controller complexity etc. Other than these "inner-loop" designs, in recent years, out-loop MPC, which can also be taken as guidance or navigation, has received more and more consideration. Plenty of significant works have emerged, such as Richard M. Murray *et. al.* [58], Kim [43], Keviczky, Balas, *et. al.* [10, 38, 39, 40, 41].

An outer-loop MPC structure is shown as below, First, the inner-loop controller K is designed to ensure the stability of the aircraft in case of faults. Then based on this inner

Figure 9.1: Outer-loop MPC structure

closed-loop, our goal is to design an outer-loop MPC tracking controller which computes appropriate guidance inputs to feed the inner dynamic inversion flight controller. The flight controller will utilize information from the navigation equipments on the vehicles position and speed relative to a reference trajectory, as well as information from other sensors onboard the flight vehicle. Meanwhile, the outer-loop MPC controller must respect to various constraints, such as actuator limits and state bounds, flight envelop, and mission/tactical requirements. Some of these are hard constraints (actuator limits) which can not be exceed at all, some are soft constraints (mission/tactical requirements) which can be softened in particular environments. Fault is another important issue to be considered. In addition to designing a passive fault tolerant controller K , active approaches are also promising candidate in outer-loop MPC controller design. Compared with traditional inner-loop design approaches, outer-loop MPC controller will enable target aircrafts capable of

- Improved tracking capability of legacy controllers;
 - Increased aggressiveness of maneuvers compared with current techniques; (In Chapters 5, the maneuver ability has been enhanced compared with legacy controllers considering control surface limits. With this outer-loop MPC technique, more improvement are expected.)
 - Component and flight envelope constraints respected; (Constraints are quite hard for classical techniques to consider. For example, in Chapter 5, input

saturation problems are discussed. Normally, an anti-windup compensator could be designed against saturations after controller K is designed. Thereafter, the overall system could no longer guarantee performance criteria that K is designed for in case of saturations. Another way to address constraints is to use MPC as inner-loop controller (Chapter 5-7). This has been broadly adopted in slow-response plants, usually in chemical engineering. But for flight control, to the best of our knowledge, MPC is not yet capable for an nonlinear 6 degree freedom aircraft model which also requires 100Hz or faster response. In our previous work in Chapter 7, some methods are used to reduce the online computational burden for inner-loop MPC controller. This outer-loop idea could be a very interesting issue to address all the problems we have met.)

- Increased mission effectiveness;
 - Accommodation to changes in mission/tactical goals;
 - Accommodation to changes in environmental conditions failures and constraints.

(By means of communication, the changes in mission/tactical goals, environmental conditions, failures and constraints can easily be handled by changing the cost functions in MPC controller. There is an interesting chapter in professor Maciejowskis book [55] discussing this issue.)

In the future, the main promising aspects to be studied in outer-loop MPC design could be,

- Constraints (input saturations/state limits/flight envelope/mission requirements);
- Faults (actuator or sensor failures/control surface impairments);
- Disturbances rejection (winds/sensor noises);
- Efficient LMI based MPC algorithms.

9.2.2 Explicit MPC in FCS control area

To address the on-line implementation problem of MPC, [7, 8] proposes a new approach, where the optimization effort is moved off-line. This approach is based on multi-parametric programming, through which the optimal solution is given in an explicitly defined piecewise linear function with dependence on the current state vector x_k . Thus, the on-line control computation reduces to the simple evaluation of the piecewise linear function.

In our point of view, explicit MPC can significantly reduce the on-line computational effort and is very promising in FCS area. In the following we will briefly introduce how it works and plan to apply it on an F-16 aircraft in the future.

Multi-parametric programming

Consider the following optimization problem

$$\min_{\mathbf{u}} \left\{ J(\mathbf{u}; x) \triangleq \ell(\mathbf{u}, x) \right\} \quad (9.1a)$$

subject to

$$G\mathbf{u} \leq W + Ex, \quad (9.1b)$$

where $\mathbf{u} \in \mathbb{R}^{n_u}$ is the optimization variable, $x \in \mathbb{R}^{n_x}$ is the parameter, and G, W, E are constant matrices with appropriate dimensions.

In multi-parametric programming, the objective is to obtain the optimizer $\mathbf{u}^*$ as an explicit function of the parameter x . The term ‘multi’ is used to indicate that the parameter x is a vector instead of a scalar. Depending on whether the cost function $\ell(\mathbf{u}, x)$ is linear or quadratic in the decision variable $\mathbf{u}$, the terminology ‘multi-parametric linear programming’ (mp-LP) or ‘multi-parametric quadratic programming’ (mp-QP) is adopted.

Properties of multi-parametric programming

Suppose $\ell(x, \mathbf{u}) = \mathbf{u}^T H \mathbf{u}$ with $H \succ 0$. For solving problem (9.1) in the j^{th} state space partition, the first-order Karush-Kuhn-Tucker (KKT) conditions are given by

$$H\mathbf{u} + G_j^T \lambda_j = 0, \quad j \in \mathcal{J}, \quad (9.2a)$$

$$\lambda_j^r (G_j^r \mathbf{u} - W_j^r - E_j^r x) = 0, \quad j \in \mathcal{J}, \quad r = 1, \dots, m, \quad (9.2b)$$

$$\lambda_j \geq 0, \quad j \in \mathcal{J}, \quad (9.2c)$$

$$G_j \mathbf{u} \leq W_j + E_j x, \quad j \in \mathcal{J}, \quad (9.2d)$$

where the superscript index r denotes a subset of the rows of a matrix or vector, the subscript index j denotes the index of the final state space partitions, $\mathcal{J}$ is the union of j and $\lambda_j = [\lambda_j^1, \dots, \lambda_j^m]^T \in \mathbb{R}^m$ are the Lagrange multipliers.

Definition 9.2.1 *Active constraints* are defined as the constraints that satisfy $G_j^r \mathbf{u} = E_j^r x + W_j^r$, while the *inactive constraints* are defined as the constraints that satisfy $G_j^r \mathbf{u} < E_j^r x + W_j^r$.

From (9.2a), $\mathbf{u} = -H^{-1} G_j^T \lambda_j$ is derived. Substituting it into (9.2b) gives $\lambda_j^r (-G_j^r H^{-1} G_j^T \lambda_j - W_j^r - E_j^r x) = 0, \forall r = 1, \dots, m$. Consider a combination of active constraints $\hat{G}_j, \hat{W}_j$ and $\hat{E}_j$ and assume that the rows of $\hat{G}_j$ are linearly independent. Let $\hat{\lambda}_j$ be the Lagrange multipliers corresponding to the active constraints. For active constraints, $\hat{\lambda}_j > 0$, it follows $-\hat{G}_j H^{-1} \hat{G}_j^T \hat{\lambda}_j - \hat{W}_j - \hat{E}_j x = 0$. Then,

$$\hat{\lambda}_j = -(\hat{G}_j H^{-1} \hat{G}_j^T)^{-1} (\hat{W}_j + \hat{E}_j x). \quad (9.3)$$

Substitute (9.3) into $\mathbf{u} = -H^{-1} G_j^T \lambda_j$, we have

$$\mathbf{u} = H^{-1} G_j^T (\hat{G}_j H^{-1} \hat{G}_j^T)^{-1} (\hat{E}_j x + \hat{W}_j). \quad (9.4)$$

From (9.3) and (9.4) we can see that $\hat{\lambda}_j$ and $\mathbf{u}$ are both affine functions of x . If $\mathbf{u}$ of (9.4) is optimal, it must satisfy constraint (9.2d):

$$G_j H^{-1} G_j^T (\hat{G}_j H^{-1} \hat{G}_j^T)^{-1} (\hat{E}_j x + \hat{W}_j) \leq W_j + E_j x \quad (9.5)$$

and the Lagrange multipliers must remain non-negative, which is expressed as

$$-(\hat{G}_j H^{-1} \hat{G}_j^T)^{-1} (\hat{W}_j + \hat{E}_j x) \geq 0. \quad (9.6)$$

Let $\mathcal{R}_j$ be the set of states x such that conditions (9.5)-(9.6) hold, i.e.,

$$\mathcal{R}_j = \{x \in \mathbb{R}^{n_x} : (9.5) - (9.6) \text{ are satisfied} \}. \quad (9.7)$$

From (9.4) and (9.7), the optimal solution of (9.1) at x is

$$\mathbf{u}^*(x) = \mathbf{L}_j x + \mathbf{g}_j, \text{ if } x \in \mathcal{R}_j, \quad (9.8)$$

where $\mathbf{L}_j = H^{-1} G_j^T (\hat{G}_j H^{-1} \hat{G}_j^T)^{-1} \hat{E}_j$, and $\mathbf{g}_j = H^{-1} G_j^T (\hat{G}_j H^{-1} \hat{G}_j^T)^{-1} \hat{W}_j$. A full discussion of the computation of the matrices $\mathbf{L}_j$, $\mathbf{g}_j$ and the corresponding region $\mathcal{R}_j$ is given in [8, 68]. Let Z be the set of states x for which the optimization problem (9.1) is feasible, i.e.,

$$Z := \{x \in \mathbb{R}^{n_x} : \exists \mathbf{u} \text{ such that } \mathbf{G}\mathbf{u} \leq \mathbf{W} + \mathbf{E}x\}. \quad (9.9)$$

Theorem 9.2.1 (Properties of mp-QP [8])

Consider the multi-parametric quadratic program (9.1). Then, the feasible set Z is convex, the optimizer $\mathbf{u}^(x)$ is continuous and piecewise affine, i.e.,*

$$\mathbf{u}^*(x) = \mathbf{L}_j x + \mathbf{g}_j, \text{ if } x \in \mathcal{R}_j, \quad \forall j \in \mathcal{J}, \quad (9.10)$$

and the optimal value function is continuous, convex and piecewise quadratic, where

Figure 9.2: Partition for an mp-QP and mp-LP problem.

each $\mathbf{L}_j \in \mathbb{R}^{n_u \times n_x}$ and $\mathbf{g}_j \in \mathbb{R}^{n_u}$ are associated with a polytope $\mathcal{R}_j$. The polytopes $\mathcal{R}_j$ have mutually disjoint interiors and $Z = \bigcup_{j \in \mathcal{J}} \mathcal{R}_j$.

Note that the evaluation of the optimal piecewise affine solution (9.10) of the optimization problem provides the same result as using quadratic programming (QP). For a detailed description of mp-LP, we refer the readers to [6]. The properties of mp-LP solutions are stated below.

Theorem 9.2.2 (Properties of mp-LP [6])

Consider the multi-parametric linear program (9.1) with a linear $\ell(x, \mathbf{u})$. Then, the feasible set Z is convex, the optimizer $\mathbf{u}^*(x)$ is continuous and piecewise affine, i.e.,

$$\mathbf{u}^*(x) = \mathbf{L}_j x + \mathbf{g}_j, \text{ if } x \in \mathcal{R}_j, \quad \forall j \in \mathcal{J}, \quad (9.11)$$

and the optimal value function is continuous, convex and piecewise affine, where each $\mathbf{L}_j \in \mathbb{R}^{n_u \times n_x}$ and $\mathbf{g}_j \in \mathbb{R}^{n_u}$ are associated with a polytope $\mathcal{R}_j$. The polytopes $\mathcal{R}_j$ have mutually disjoint interiors and $Z = \bigcup_{j \in \mathcal{J}} \mathcal{R}_j$.

For a general mp-QP (mp-LP) problem, the region partition $\mathcal{R}_j, \forall j \in \mathcal{J}$ and value function can be depicted as in Figure 9.2 and 9.3. Normally mp-LP solution comprises more regions than mp-QP solution does subject to the same constraints (e.g., Figure 9.2), see

Figure 9.3: Partition and value function for an mp-QP and mp-LP problem.

[32]. This is because mp-LP problem is subject to more constraints and has optimizers of higher dimensions than their mp-QP counterpart. In addition, mp-QP is less susceptible to numerical problems since dual degeneracies cannot occur ([9]).

Outline of the mp-QP algorithm

A brief outline of a generic mp-QP algorithm is given below. For detailed discussion on mp-QP algorithms we refer the reader to the literature [5, 8, 64, 68]. An mp-QP computation scheme ([8]) consists of the following three main steps:

- (1) **Determine initial state vector** The first step of the algorithm is to find one initial state vector x_0 , such that the QP problem (9.1) is initially feasible. This will yield the optimizer $\mathbf{u}^*(x_0)$ and the initial active constraints set $\hat{G}_0$, $\hat{W}_0$ and $\hat{E}_0$.
- (2) **Region computation** The rows indexed by the active constraints $\hat{G}_0$, $\hat{W}_0$, $\hat{E}_0$ are extracted from the constraint matrices G , W and E . If $\hat{G}_0$ is linearly independent, the optimizer $\mathbf{u}^*$ is computed by (9.8) and $\mathcal{R}_0$ is computed by (9.7).
- (3) **State space exploration** In the sequel, the rest of state space, $\mathcal{R}_{\text{rest}} = Z - \mathcal{R}_0$ (normally non-convex), needs to be partitioned into several convex regions (say, Ω_j) and then to be explored iteratively until the entire feasible state space Z is covered with controller regions, i.e., $Z = \cup_{j \in \mathcal{J}} \mathcal{R}_j$.

Some efficient schemes have been reported in the literature to address step 3. In [8], $\mathcal{R}_{\text{rest}}$ is subdivided by reversing each and every row of active constraints one by one. However, the main drawback in this algorithm is that some $\mathcal{R}_j$ s may be split into several parts. This in general requires a merging procedure afterward to unify the fragmented parts and obtain a compact representation. In view of this, [68, Theorem 2] proposes the facet-to-facet method, by which the optimal active constraints set of a region can be determined with its adjacent region and their common facet. And in [64], an improved facet-to-facet method is also presented. These algorithms are currently available in MATLAB (Multi-Parametric Toolbox).

It should also be noticed that we assumed in (9.5) and (9.6) that the rows of $\hat{G}$ are linearly independent. When this assumption is violated, then the mp-QP is so called **degenerate** and the KKT conditions do not lead directly to the description of the regions. Several approaches in solving the degenerate mp-QP have emerged in these years, such as projection method of the high dimensional polyhedra, and method of arbitrarily selection of a subset from active constraints. For detailed information on handling the degeneration please refer to [8] and [68].

Publication list

Journal Papers:

1. **Le Feng**, Jianliang Wang, Engkee Poh, “Multi-objective Robust Model Predictive Control: An LMI based approach”, Submitted to International Journal of Control, November 2006.
2. **Le Feng**, Jianliang Wang, Engkee Poh, Fang Liao, “Digital Reliable Aircraft Flight Controller Design Against Faults With Enhanced Maneuverability: A Mixed H_2/H_∞ Approach”, Submitted to Control Engineering Practice, July 2006.
3. **Le Feng**, Jianliang Wang, Engkee Poh, “Improved Robust Model Predictive Control With Structural Uncertainty”, Journal of Process Control, Volume 17, Issue 8, September 2007, Pages 683-688.

Conference Papers:

1. **Le Feng**, Jianliang Wang, Engkee Poh, Fang Liao, Dan Sui, “Improved Reliable H_2 Output Feedback Controller Design for F-16 Aircraft Against Faults”, Accepted for publication by IEEE Conference on Control Applications 2007.
2. **Le Feng**, Jianliang Wang, Engkee Poh, Fang Liao, Dan Sui, “Aircraft Landing Controller Design Against Actuator Stuck Fault: An MPC Approach”, Accepted for publication by IEEE Conference on Control Applications 2007.

3. **Le Feng**, Jianliang Wang, Engkee Poh, “Multi-objective Robust Model Predictive Control: Trajectory Tracking Problem Through LMI Formulation”, American Control Conference 2007, pp. 5589-5594, New York, NY, USA, 9-13 July 2007.
4. **Le Feng**, Jianliang Wang, Engkee Poh, “Robust Model Predictive Control with Structured Uncertainty Using Vertex-dependent Lyapunov Functions”, the Ninth International Conference on Control, Automation, Robotics and Vision (ICARCV2006), pp. 77-81, Singapore, December 6-9, 2006;
5. **Le Feng**, Jianliang Wang, Engkee Poh, “Computational Complexity Reduction for Robust Model Predictive Control”, The 5th International Conference on Control & Automation, pp. 522-527, Hungarian Academy of Science, Budapest, Hungary, June 27-29, 2005.
6. **Le Feng**, Jianliang Wang, Engkee Poh, Fang Liao, “Reliable H_∞ Aircraft Flight Controller Design Against Faults with State/output Feedback”, American Control Conference, pp. 2664-2669, Oregon Portland, USA, June 8-10, 2005.
7. **Le Feng**, Jianliang Wang, Engkee Poh, “Off-line Formulation Of Robust Model Predictive Control Based On Several Lyapunov Functions”, the Eighth International Conference on Control, Automation, Robotics and Vision (ICARCV2004), pp. 1705-1710, Kunming, China, December 6-9, 2004.

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